EXPLORING MATHEMATICS-RELATED BELIEF SYSTEMS

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Abstract

The purpose of this thesis is to explore student's mathematics-related beliefs systems and the use of a "mathematics-related belief questionnaire" (MRBQ) to measure such systems. 73 students from three Mathematics 11 classes are used as subjects for the study. The results of part one of this thesis support the existence of three subcategories of mathematics-related beliefs that include beliefs about self, mathematics the object, and the context in which they learn mathematics. An exploratory factor analysis of the MRBQ determined five factors that fit into this framework, and a sixth factor (*I like mathematics*) that suggested, in part, an emotional component.

An analysis of the correlations among the factors suggest a "core" set of beliefs with strong interconnections that might be used to categorize student's mathematical dispositions and which may be connected to achievement in mathematics. Gender differences are considered and limitations of the experiment are discussed in detail.

Keywords: Mathematics-related Beliefs, MRBQ, Belief Systems, Mathematics Education, Affective Domain.

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1 Introduction

The key to effective mathematics instruction is more than the clear ‘delivery’ of a well thought out lesson plan. It is more than a simple exchange between teacher and student where knowledge is ‘transferred’ from one individual to another. This traditional “teacher-centered” approach to mathematics instruction is what most mathematics educators today experienced as students themselves going through grade school. Some of these educators chose to use the same traditional approaches in their own teaching practice. However, with the movement to reform mathematics instruction, many are broadening their strategies to included classroom activities that help students develop *understanding* in mathematics.

In the first five years of my teaching career I developed a traditional style of teaching mathematics that worked for some and left others with almost no understanding of the mathematics they were doing. With the best intentions, I spent endless hours ‘fine-tuning’ the sequences of notes and examples that I would deliver to my students. In many cases, I would even create alternate explanations to deliver if the first appeared to fail. If students were struggling I would ask myself “how can I teach this to them in a way that they will understand?” In the process of creating these “teacher-centered” lessons I was merely giving my students a set of ‘recipes’ to follow. Each recipe was a set of step-by-step routines designed to accomplish a mathematical task. As I reflect on my own teaching practice in those first years of my career, I know that my belief was that I could teach any student if I could find the right recipe to give them for the problem at hand - students would understand the concepts if the lesson was clear and delivered effectively. After five years of traditional instruction I realized that it was impossible to create a recipe for a mathematical concept that would work for every students in the class. Even after writing and re-writing several sets of instructions that could be followed by many, when faced with non-routine problems, many students found themselves lost because they lacked the necessary mathematical understanding - when the recipe failed there was nothing else. I was only teaching procedural knowledge. After realizing the limitations of a strictly traditional approach, I began to research reformed approaches to mathematics instruction as a graduate student. I found myself learning about progressive approaches to teaching mathematics. As I experienced these approaches first hand, I quickly discovered the benefits they could have for my students and began to experiment with strategies that enabled students to build ideas for themselves.

The defining characteristic that sets my new approach apart from a more traditional style is that instead of students being *given* the mathematical processes, they are guided in the construction of mathematical processes. The construction of a new concept involves students making connections between new ideas and prior knowledge. It is the prior knowledge that gives meaning to the new concepts that a student constructs for him or herself. Orchestrating the building of new ideas in a diverse classroom involves teaching in a way that forces students to access their individual sets of prior knowledge while engaged in learning activities.

Implementing progressive approaches to teaching mathematics comes with many challenges. As students are encouraged to construct new concepts for themselves, they are faced with unfamiliar problems to solve. Hiebert et al. (1996, as cited by: Van De Walle & Folk, 2005) explain that the most important principle for reform in mathematics lies in allowing students to make mathematics problematic. By problematic, these authors mean “allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities” (p. 39). This suggests that both the mathematics curriculum and mathematics instruction should begin with problems, dilemmas, and questions for students. Even with the guidance of an experienced teacher, students facing these types of problems, dilemmas, and non-routine questions will often experience a wide range of affective responses having both positive and negative effects on persistence. Persistence may be even more difficult for those students who have only experienced a traditional mathematics classroom and are exposed to a reformed constructivist approach for the first time. These students, when faced with the uncertainties of a problematic classroom, will struggle with the absence of teacher-directed procedures that provide, in some cases, a false sense of understanding. When students find a way to persist through the “unfamiliar,” progressive approaches to instruction can successfully enable students to understand the mathematical concepts needed to become effective problem solvers.

Teaching students to become problem solvers involves guiding them through the unknown, and persistence through the unfamiliar requires risk taking. The integrated resource package for mathematics educators in British Columbia (Ministry of Education, 2007), states in its’ rational that “problem solving is the cornerstone of mathematical instruction.” It continues: “becoming a mathematical problem solver requires a willingness to take risks and persevere when faced with problems that do not have an immediate apparent solution” (p. 1). So why are so many students hesitant to take risks when solving non-routine mathematical problems? This is an important question. In order for students to construct an understanding of new ideas in mathematics they must be active participants. Active participation means that students must "wrestle" with new ideas to fit them into existing networks (Van De Walle & Folk, 2005).

McLeod (1992) states that "if students are going to be active learners of mathematics, who willingly attack non-routine problems, their affective responses to mathematics are going to be much more intense than if they are merely expected to achieve satisfactory levels of performance in low-level computational skills" (p. 575). This suggests that teachers who are interested in encouraging their students to take risks on non-routine problems should have an understanding of affect in general and the affective experiences that their students will likely encounter while attacking these types of problems. Op't Eynde & De Corte (2004) explain that in order for students to become competent problem solvers they must develop a "mathematical disposition" where affect plays a major role. More specifically, it is a student's mathematics-related belief system that forms a central component of a mathematical disposition and has a strong impact on learning and problem solving. This belief system is made up of the many interconnections between mathematics-related beliefs from different categories and influences both thought and behavior. Op't Eynde & De Corte (2003) explain that "several studies have demonstrate how beliefs about the nature of mathematics and mathematical learning and problem solving determine how one chooses to approach a problem and which techniques and cognitive strategies will be used" (p. 3). It is with this knowledge that teachers can begin to effectively implement progressive teaching approaches in the classroom.

The aim of this study is to explore the mathematics-related beliefs that play a critical role in forming students' mathematical dispositions. In particular, this study looks at the connections between beliefs that make up students' mathematics-related beliefs systems. After exploring these belief systems using data from a larger sample, this study includes a case study involving three students from the sample. Here, students are asked about their beliefs in the context of a real problem solving experience. After observing the each student, their belief systems and mathematical dispositions are compared and contrasted.

The second chapter of this thesis serves as an overview of the affective domain. It is here that I review literature on the components of affect such as emotions, attitudes and beliefs. I look at the role of cognition in affect and cognitive frameworks such as Mandler's (1989) framework. Finally, I review the role of beliefs in affect and look more specifically at goal-orientation and self-perceptions.

In chapter 3, I review mathematics-related beliefs and belief systems. After discussing the role of beliefs in the affective domain in the previous chapter I take a closer look at research and literature pertaining to beliefs related to mathematics and mathematics education. Here, I consider frameworks for analyzing the connections between mathematics-related beliefs found in mathematics-related belief systems. One such framework is Op't Eynde and Decorte's (2003) triangular model that breaks mathematics-related beliefs into three sub-categories: beliefs about self (as a mathematics learner), beliefs about mathematics education, and beliefs about classroom context. After looking at frameworks for mathematics-related belief systems, I look at factors that can influence these belief systems such as parental influences and gender differences. Finally, I discuss methods for measuring mathematics-related beliefs and the connections between beliefs in a mathematics-related belief system.

The study is split into two parts. Part 1 involves the distribution of a mathematics-related belief questionnaire to three grade 11 mathematics classes. In chapter four I describe the sample population and then discuss how the questionnaire is designed and administered. Chapter 5 is the first results chapter. Here, I provide a detailed analysis of the questionnaire results. I look at the factor analysis and discuss how each of the factors is determined. I look at correlations between the factors and identify significant correlations that suggest a "core" set of beliefs. I also look at how factors such as achievement and gender may be connected to student's mathematics-related beliefs. The data from this results chapter is then used to select three students for part 2 of the study.

In part 2 of the study, three students with differing "core" beliefs are chosen and given a non-routine mathematics problem to solve. In chapter 6 I describe the methodology used in the collection of data describing each student's problem solving experience. The chapter starts with a detailed analysis of each student's questionnaire results showing why each student was chosen for the second part of the study. Next, the method for gathering and analyzing the video and video-based recall data is described. After describing the methodology, the results of part 2 of the study are discussed in chapter 7. Here, each student's solution process is analyzed. The video data, think-aloud data, interview data, and video-based recall data are aligned and summarized for each student. Each student's experience is discussed as it relates to his or her mathematics-related belief system (as determined by the questionnaire in part 1).

The last two chapters of this thesis will include conclusions, and possible teaching implications. It is important to note here that the nature of this thesis is exploratory. The accurate measurement of student's mathematics-related belief systems is a difficult task that has been attempted by few researchers. The primary goal of this study is to gain insight into the mathematics-related belief systems that form student's mathematical dispositions. It is my hope that through the exploration of student's mathematics-related belief systems, I might learn to refine "affective pedagogy" in a way that enables students to benefit from a progressive teaching approach.

2 The Affective Domain

Leder & Forgasz (2006) explain that the interconnective nature of concepts such as affect, feelings, emotions, and attitudes and their link to behavior can be seen in definitions spanning some 50 years. It is this overlapping and connective nature of the components of affect that have given rise to a multitude of theoretical frameworks. McLeod (1992) explains that "the affective domain refers to a wide range of beliefs, feelings and moods that are generally regarded as going beyond the domain of cognition" (p. 576). It is because affect goes beyond cognition that McLeod says it is more difficult to describe and measure. He describes the affective domain using three components: emotions, attitudes and beliefs. He makes distinctions among these components by describing emotions as the most intense and least stable, beliefs as the most stable and least intense, and attitudes as somewhere in between. He explains that "we can think of beliefs, attitudes and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (p. 579).

DeBellis and Goldin (1997) add "values" as a fourth component to affect and suggest a tetrahedral model where all components interact with one another. They explain that a student's values, morals, and ethical judgments are powerful motivators that interact with problem decision-making. Student's personal value systems develop from a young age, giving them a sense of what is 'good' and 'bad.' For example, a student could value 'correctness' in their day-to-day work, seeing error as something that is 'bad.' DeBellis and Goldin (1997) explain that this type of value is much more than a belief about what mathematics is, or how one should go about solving a mathematics problem. DeBellis and Goldin also divide affect into two components: “global” and “local.” Local affect involves changing states of feeling during problem solving; whereas, global affect describes more general feelings and attitudes reinforced by belief structures (DeBellis & Goldin, 1997; Goldin, 2000). Goldin (2000) explains that the attitudes and belief structures characterized by global affect are like “personality traits.” He argues that it is these personality traits that are typically addressed abroad in mathematics education research. The rapidly changing emotional states that are a part of local affect have been addressed less frequently in mathematics education research. It is these local affective states that interact with cognition and impact more stable affective structures. Goldin (2000) says that stable affective structures are complex, and part of their complexity lies in the pathways of local affective representation that they incorporate.

Malmivuuori (2004) explains that newly discovered theoretical constructs, such as consciousness, metacognition and self-regulation, suggest one must consider cognition as more closely linked to affect and behavior in learning and education. Malmivuuori’s (2004) “dynamic viewpoint” on affect emphasizes the role of personal constructive and self-regulatory aspects of affective responses in social, contextual and situational environments. She explains that “the related unique evaluations and judgments of the self in a mathematical situation are accompanied by affective arousals and constructive or directive processes with affect and behaviors, implying important affective ‘self-states’ for doing and learning mathematics” (p. 116). Students’ affective self-states are regulated by students’ self-systems and self-system processes during the problem solving process. Students' self-systems include "self-beliefs and self-knowledge systems, mathematical beliefs and belief systems, related affective responses, and the related behavioral patterns in mathematics situations" (Malmivuuori, 2004, p.116). Self-system processes refer to how students' self-systems function in unique social mathematics situations with active self-regulation. These self-system processes involve varying levels of self-awareness and point to the "unconstrained path" that lies between cognition and affect.

2.1 Cognition and Affect

With its multitude of theoretical frameworks, research on affect has been faced with the problem of understanding the complex interaction between affect and cognition (Hannula, 2004). Many in the field agree that there is a strong link between cognition and affect (DeBellis & Goldin, 1997, Hannula, 2002, McLeod, 1992, Mandler, 1989, Schlogmann, 2005) but have different views with regards to how the two interact with one another during the problem solving process. McLeod (1992) describes the "beliefs" component of the affective domain as largely cognitive in nature and stable over time, whereas emotions may involve little cognitive appraisal and may appear and disappear rather quickly. In contrast, DeBellis & Goldin (1997) suggest that there is a high level of cognitive activity associated with emotion during problem solving, though the thoughts that interact with "fleeting" emotions may be difficult to identify. They suggest that the "affective system" is one of five representational systems at work during mathematical problem solving and that it "encodes" essential information for problem solving. The five representational systems that interact with each other are: (a) a verbal/syntactic system, (b) imagistic systems, (c) formal notational systems, (d) a system of planning and executive control, and (e) an affective system. They regard the affective system as its own system of representation that, while interacting with other modes of representation, encodes essential information for problem solving.

Mandler (1989) describes a cognitive framework for affect where the problem solver starts with a plan that arises from the activation of one, or many, problem solving schemas. Each schema initiates an action sequence that carries its own set of anticipated steps. When there is a discrepancy or interruption in the anticipated action sequence, the problem solver experiences some visceral arousal that can be either joyful or unhappy depending on whether the action sequence was changed in a positive or negative manner. Mandler (1989) explains that arousal, combined with an ongoing evaluative cognition, produces the subjective experience of emotion. In other words, emotions experienced by a student while problem solving are a result of how they interpret the arousal caused by discrepancies or interruptions in their anticipated problem solving schema. McLeod (1992) explains that the student's knowledge and beliefs play a significant role in the interpretation of such discrepancies and interruptions. Also, when students experience the same action sequence repeatedly, their responses become more automatic and stable resulting in both positive and negative attitudes towards mathematics. Hannula (2004) adds that "it is well known that emotions are not only consequences of cognitive processing; they also effect cognition in several ways: emotions bias attention and memory and activate action tendencies" (p. 108).

Goldin (2000) suggests that there are two 'well-travelled' pathways of affect when students problem solve in mathematics. According to his model, each of the two affective pathways begin with the same three stages:

> In this (idealized) model, the initial feelings are *curiosity*. If the problem has significant depth for the solver, a sense of *puzzlement* will follow, as it proves impossible to satisfy the curiosity quickly. Puzzlement does not in itself have unpleasant odor – but *bewilderment*, the next state in the sequence, may. The later can include disorientation, a sense of having "lost the thread of the argument," of being "at sea" in the problem (p. 212).

Once a student has reached bewilderment it is possible for the student to travel down one of two affective pathways. The first is a sequence of predominantly positive affect including feelings of: encouragement, pleasure, elation, and satisfaction. The second path is one of negative affect that includes: frustration, anxiety, and fear/despair (Goldin, 2000). Goldin also explains that as students move through affective pathways they use affect to make heuristic choices.

DeBellis and Goldin (1997) describe meta-affect as: "(a) emotions about emotional states, and emotions about or within cognitive states, and (b) the monitoring and regulation of emotion" (p. 214). Schloglmann (2005) explains that from the neuroscientific point of view, there exist two different systems: cognition and emotion. Although the two systems can interact with one another, the existence of two separate systems implies that there is a difference between feeling and knowing that we are feeling (Damasio, 1999; as cited by Schlolmann, 2005). Schloglmann (2005) also explains that when students assimilate and accommodate new information about affect, affective-cognitive schemata are developed. These affective-cognitive schemata can then help students assess affective states during problem solving and develop strategies for handling the emotions they experience. Goldin (2004) explains that:

> Consideration of meta-affect suggests that the most important goals in mathematics are not to eliminate frustrations or to make mathematical activity easy and fun. Rather they are to develop meta-affect where feelings about emotions associated with impasse or difficulty are productive (p. 113).

Lazarus (1991) believes that emotion "is *always* a response to cognitive activity, which generates meaning regardless of how meaning is achieved" (p. 353). He defines the cognitive process by which emotion is generated as "appraisal" and states that cognition is both a necessary and sufficient condition of emotion. He also adds that as an emotion is generated through appraisal, that new emotion becomes "food" for the next appraisal of emotion. Lazarus (1991) describes his causal view of the role of cognition in emotion as the strongest position possible, and the most controversial since there is a deep-rooted tradition in western culture for separating emotion and cognition dating back to the ancient Greeks. Lazarus (1991) writes:

> In the Apollonian Greek ideal, which the medieval church also adopted, rationality was enthroned as godlike. Passion was regarded as animal-like, and people were enjoined to control their animal natures by reason.

He suggests "social scientists get uncomfortable about the treatment of appraisal as both a cause and a part of the (emotional) effect" (p. 353).

Each of these frameworks suggests that the feelings students experience and their interpretations of those feelings have a major impact on their ability to solve non-routine problems. Goldin (2000) suggests that as teachers we have the ability to transform the quality of students' affect with minimal heuristic suggestions at times of discouragement. If we as teachers can help students become more aware of the feelings they experience during problem solving they may be able to reconstruct cognition and affect so that negative emotions can be used in a positive way. McLeod (1989) says that "once students understand that problem solving involves interruptions and blockages, they should be able to view their frustrations as a normal part of problem solving, not as a sign they should quit" (p. 578).

2.2 Beliefs

Beliefs play a key role in the construction of emotion and the development of attitudes according to Mandler's (1989) cognitive framework. D'Andrade (1981; as cited by McLeod, 1992) suggests that beliefs develop over time much like "guided discovery" where students develop beliefs that are in line with their personal experiences. These personal experiences happen in the cultural setting of the classroom, where many researchers believe student beliefs are developed. There is no single, exact definition for the term "belief," but many researchers have attempted to define the term. McLeod (1992) categorizes student's beliefs into four categories related to the object of belief: beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the context in which mathematics education occurs.

Beliefs about mathematics might include the belief that mathematics is mainly memorizing or that most math problems can be solved in two minutes or less. In Schoenfeld's (1989) study, "Explorations of Students' Mathematical Beliefs and

Behavior", he discovered that students believe that learning mathematics is mostly about memorizing and requires lots of practice in following rules. He also discovered that students believed that a typical homework problem should take about two minutes to solve and if they found themselves working longer than about ten minutes, they believed that the problem was impossible. Schoenfeld (1989) mentions that the troubling aspect to his study was that many of the beliefs students professed were not in line with their actual behaviors. Schoenfeld (1989) suggests that the students in his study have "come to separate school mathematics – the mathematics they know and experience in their classrooms – from abstract mathematics, the discipline of creativity, problem solving, and discovery, about which they are told but which they have not experienced" (p. 349). This suggests that measuring student's beliefs about mathematics is problematic. Regardless, researchers agree that the beliefs that students have about mathematics that guide behavior in the classroom can have a major impact in a students ability to solve non-routine math problems (McLeod, 1992).

Research on self-concept, confidence, and causal attributions related to mathematics tends to focus on beliefs about the self (McLeod, 1992). Malmivuori's (2004) dynamic viewpoint on affect emphasizes the role of personal constructive and self-regulatory aspects of affective responses in social, contextual and situational environments. She explains that student's self-perceptions are especially important with respect to their affective arousals and experiences. She calls these highly influential responses self-affects and says that they are connected with students' experiences of self-esteem, self-worth and/or personal control with respect to mathematics. The self-system and self-system processes are described by Malmivouri as the basic concepts that characterize the affective responses in mathematics learning. The self-system includes the following stable internal structures (Malmivuori, 2004):

- content-based mathematical knowledge
- learned socio-cultural beliefs about mathematics, its learning and problem solving
- beliefs about the self in mathematics
- affective schemata
- habitual behavioral patterns in mathematical situations

These situation-specific self-systems impact students' metacognitive, cognitive and affective capacities during mathematical thinking and therefore the quality of their experiences learning mathematics.

The role of self-beliefs in mathematics education is made clear by Schoenfeld (1989) when he explains that research has consistently shown correlations between confidence and achievement. This correlation could exist because the strength of people's convictions in their own effectiveness is likely to affect whether they will even try to cope with given situations (Bandura,1977). Self-efficacy is one's belief in one's ability to complete a task. Efficacy expectations dictate how much effort a student is willing to expend when faced with a difficult problem. Bandura (1977) explains that "expectations of personal efficacy are based on four major sources of information: performance accomplishments, vicarious experience, verbal persuasion, and physiological states" (p. 195).

Self-confidence is another aspect of self-perception that has been found to have a stronger correlation with achievement in mathematics than with other variables (Fennema, 1984; Meyer & Fennema, 1986; as cited by Kloosterman, 1988).

Kloosterman's (1988) research on self-confidence and motivation in mathematics suggests that students who are self-confident have an incremental view of intelligence and are mastery oriented. Mastery oriented students feel that success is controllable and that failure is due to unstable causes such as lack of effort or bad luck. On the other end of the spectrum are students described as learned helpless. Learned helpless students view success as out of their control causing them to view effort as useless. As such, failure is attributed to lack of ability (Diener & Dweck, 1978; as cited by Kloosterman, 1988).

The beliefs that students hold are impacted by the role models in their lives – parents in particular (Parson et al; 1982). Parson (1982) found that parents' beliefs about their childrens' ability in mathematics were of a more critical mediator for the child's self-concept than the child's own math performance. She also explains that parents' beliefs about their own mathematical abilities can have an impact on their childrens' beliefs. As teachers model behaviors for their students every day, their beliefs about themselves and their students have an impact on student beliefs in their classroom. Teacher beliefs also have an impact on their own instructional practices. Thompson (1984) states that "teachers' beliefs, views, and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behavior" (p. 124).

2.3 Affect and Personal Goals

Students' beliefs about the tasks they confront in classrooms and their general goal orientation to the classroom have a major influence on students' engagement in learning, their motivation, and their performance (Ames, 1992; as cited by Garcia & Pintrich, 1994). This influence could be due to the strong connection between emotion and personal goals that many researchers agree on (e.g. Buck, 1999; Lazarus, 1991; Power & Dalgleish, 1997; Mandler, 1989; as cited by Hannula, 2002). While a student is engaged in mathematical problem solving, he or she is continuously evaluating the situation unconsciously in terms of personal goals. Hannula (2002) states that evaluation is represented as an emotion: when one proceeds towards a goal, positive emotions are induced, while obstacles that block the progress may induce fear, anger, sadness or other negative emotions. This analysis of emotion seems to be in line with Mandler's framework; however, Mandler (1989) also explains that if students work in a routine manner, after time the action sequences used will become automatic and there will be little or no arousal experienced by the student. When this occurs, proceeding towards the goal of problem completion could have no effect on emotion or possibly create negative emotions linked to boredom. The idea that success at routine tasks can have a negative impact on students is also supported by research in motivation. Research indicates that continued success on personally easy tasks is ineffective in producing stable confidence, challenge seeking, and persistence (Dweck, 1975; Relich, 1983; as cited by Dweck 1986).

Another aspect of personal goals that has an effect on problem solving and is connected to the affective domain is goal orientation. Dweck (1986) explains that in the area of achievement, motivation goals including competence appear to fall into two classes: (a) learning goals, where individuals attempt to increase their competence, to understand or master something new, and (b) performance goals, where individuals attempt to gain favorable judgments of their competence or avoid negative judgments of their competence. A third motivational orientation is also present in more recent literature, namely ego defensive goals, where individuals seek to avoid public failure (Linnenberg & Pintrich, 2000; Lemos, 1999; as cited by Hannula, 2002). A student's goal orientation is based on their beliefs about intelligence. Those students who believe that intelligence is fixed tend to be performance goal oriented and those who believe intelligence is a malleable quality tend to be oriented towards learning goals. The research has shown that when students are performance goal oriented their perceptions of their own ability must be high before they will engage in a challenging task. In contrast, when students are learning goal orientated, even when they perceive their own ability as low, they will engage in challenging tasks that foster learning (M. Bandura & Dweck, 1985; Elliot & Dweck, 1985; as cited by Dweck, 1986).

Hannula (2002) explains that goals are always self-chosen and internal for students, and that their behavior is always goal oriented. In this approach, student behavior is viewed as both goal-directed and self-regulated. He describes self-regulation as a psychological process that is primarily automatic and part of every goal-driven action. In other words, students' personal goals drive their self-regulatory behavior in the classroom. Hannula (2002) adds that even though all student behavior is goal orientated, the goals of students may differ from the learning goals set by the teacher. Also, when it comes to their goal directed behavior, some students may be more flexible than others. Based on this theory, Hannula (2002) concludes that in order for change in behavior to take place, students need goals that motivate change and beliefs that support change. Examples of beliefs that could support change in behavior are: (1) the belief that the new goals set are accessible and (2) positive self-efficacy beliefs. Hannula (2002) explains that automated emotional reactions play a critical role in students' goal choice. Because of this, students may need support in regulating emotional reactions when new goals are set to motivate change. One common emotional reaction is often described as mathematics anxiety.

May (1977; as cited by Hembre, 1990) says that the feelings associated with anxiety are helplessness and uncertainty in the face of danger. In the mathematics classroom the perceived danger could take the form of a non-routine problem on a test or daily assignment. Hembre (1990) discusses anxiety in terms of test anxiety and general mathematics anxiety and says that both forms have an emotional and cognitive element. Hebree (1990) found that high achievement is linked to a reduction is mathematics anxiety. Clute (1984) discovered that students with high mathematics anxiety tend to have lower scores on mathematics achievement tests. Walen and Williams (2002) argue that visceral arousal caused by feelings of anxiety produce negative emotions. These negative emotions can cause students to focus attentional conscious capacity on the aspects of the situation that he or she feels is important. Walen and William's (2002) study shows that when the constraint of time is lifted from the mathematical task at hand, students appeared to be able to focus more attention on the mathematics rather than the time constraint. Clute (1984) also explains that mathematics anxiety has a strong negative correlation to confidence. She noticed that students with low mathematics anxiety were more confident in solving problems in a discovery environment. Pekrun et al (2002) found that students' emotions are strongly related to motivation, cognitive resources, learning strategies, self-regulation, and achievement, as well as to personality and classroom antecedents. These findings imply that if a progressive approach is to be implemented in the classroom, the learning environment must be such that it reduces anxiety to improve student's confidence.

In the next chapter I take a closer look at mathematics-related beliefs. In particular, I consider frameworks for mathematics-related belief systems. I also discuss how researchers have attempted to measure mathematics-related beliefs and belief systems.

3 Mathematics-Related Belief Systems

As mentioned in chapter 2, McLeod (1992) categorizes student's beliefs into four categories related to the object of belief: beliefs about mathematics, beliefs about self, beliefs about mathematics teaching, and beliefs about the context in which mathematics education occurs. This framework for mathematics-related beliefs has been suggested, adopted and adapted by a number of researchers in the field (Hannula et al., 2005; Op't Eynde & Decort, 2003; Lampert, 1990; Seegers & Boekaerts, 1993). For example, Schoenfeld (1983) explains that student's cognitive actions are impacted by both consciously and unconsciously held beliefs about the task they are working on (mathematics), the social environment where the task is taking place (context), and the student's self-perceptions in relation to the task (self).

Belief systems can be characterized by how beliefs in multiple categories work together to influence thought and guide behavior. Schoenfeld (1985, as cited by Op't Eynde 2004) describes a mathematical belief system as: "one's mathematical world view, the perspective which one approaches mathematics and mathematical tasks" (p. 2). He continues to say that "one's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. Beliefs establish the context within which resources, heuristics, and control operate" (p. 2).

Epistemological beliefs are students' general beliefs about knowledge, knowing, and in some cases learning (Op't Eynde & De Corte 2006). Op't Eynde & De Corte (2006) argue that epistemological beliefs are domain specific and can therefore describe a specific set of mathematics-related beliefs. They also say that "students' beliefs about knowledge and knowing in classroom mathematics seem to be very much the exponents of interactions between the individual and the context, rather than the logical consequence of the epistemological beliefs students hold at a more general level" (p. 9). So, in general, student's beliefs are fundamentally social as they are grounded in their social life (Op't Eynde & Decort, 2003). Mathematics-related beliefs are determined by the vast social-historical context in which students discuss and experience mathematics. This context, in which students experience mathematics, takes place both inside and outside of the classroom. Inside the classroom it is the students and teachers of the class that provide the context in which a student experiences mathematics. Outside the classroom parents can also provide experiences that influence beliefs. An example of this can be found in a study by Kurtz et al. (1988) where 354 elementary school students' attributional beliefs were measured. In general, attributional beliefs refer to beliefs about the causes of events. Here, Kurtz et al. look more specifically at student and parent's beliefs about the causes of both academic success and failure. Parents' were given a questionnaire with regards to attributional beliefs they ascribe to their children. They found that parental attributional beliefs paralleled children's attributional beliefs. Also, the attributional beliefs of the parents were connected to children's metacognitive knowledge. Finally, children's self-perceptions, combined with parental instruction of learning strategies and metoacognitive knowledge accounted for differences in learning performance.

Op't Eynde & Decort (2003) say: "the analysis of the nature and the structure of beliefs and belief systems points to the social context, the self and the object in the world that the beliefs relate to, as constitutive for the development and functioning of these systems" (p. 4). They suggest a triangular representation of students mathematics-related belief system (see figure 1). In other words, students' mathematics-related belief systems are characterized by their beliefs about mathematics education, beliefs about self, and beliefs about the class context (Op't Eynde & De Corte, 2003). This framework is similar to that described by McLeod (1992). One difference between the two frameworks is McLeod's fourth category: beliefs about mathematics teaching. This category could be part of the object mathematics education as it relates to teaching practices or it could be part of classroom context if the beliefs are related to one's personal experiences being taught mathematics.

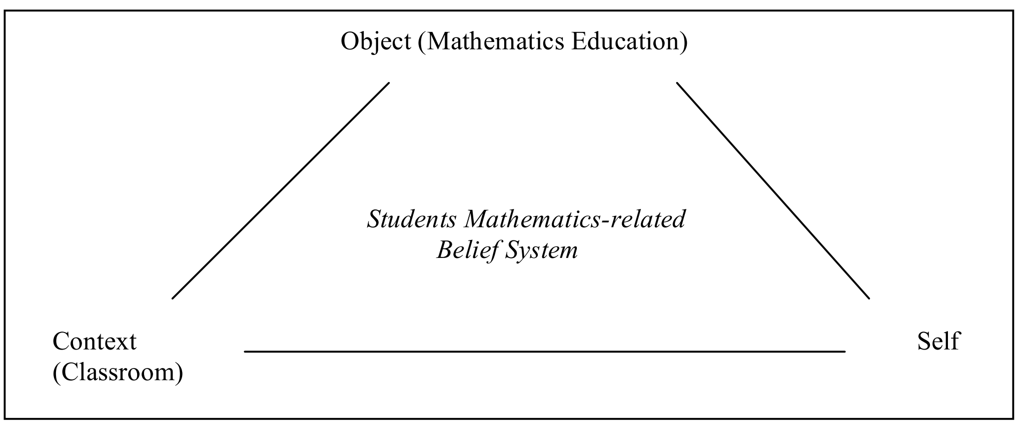


Figure 3.1: Triangular model of students mathematics-related belief systems

Op't Eynde & De Corte (2003) describes mathematics education beliefs as containing: "(1) students' beliefs about mathematics, (2) about mathematical learning and problem solving, and (3) their beliefs about mathematical teaching" (p. 4). Students' beliefs about self refer to "(1) their intrinsic goal orientation beliefs related to mathematics, (2) extrinsic goal orientation beliefs, (3) task value beliefs, (4) control beliefs, and (5) self-efficacy beliefs" (p. 4). Finally, classroom context beliefs include: "(1) beliefs about the role and functioning of their teacher, (2) beliefs about the role and functioning of the students in their own class, and (3) beliefs about the socio-mathematical norms and practices in their class" (p. 4). With this framework, Op't Eynde & De Corte (2003) define mathematics-related beliefs systems as:

> The implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematics learners, and about the mathematics class context. These beliefs determine in close interaction with each other and with student's prior knowledge their mathematical learning and problem-solving activities in class (p. 4).

This framework for mathematics-related belief systems, which has been adopted by other researchers, has been validated through the analysis of mathematics-related beliefs questionnaires (Op't Eynde & De Corte 2003, Hannula et al., 2005). It is this framework that will too be adopted here, and described in more detail in chapter 4.

3.1 Measuring Mathematics-Related Beliefs

Little work has been done in the area of *measuring* the mathematics-related belief systems of students. The focus of research on measuring beliefs has focused primarily on measuring subsets of beliefs in isolation of other belief clusters such as attributional beliefs or self-efficacy beliefs. The development of a comprehensive mathematics-related belief questionnaire (or MRBQ) has been the subject of only a few research studies (Hannula, 2005; Op't Enyde & De Corte, 2004).

After what they describe as a profound literature review Op't Eynde & De Corte (2004) suggest their triangular theoretical framework that splits mathematics-related beliefs into the object, the self, and the context. After developing the theoretical framework, they gather questions that fit into each of their three categories spanning the full spectrum of beliefs. They start with existing questionnaires that were designed to measure only one kind of belief (like beliefs about math, or beliefs about self), and then develop a more "integrated instrument that asked students about their beliefs on mathematics education, on self in relation to mathematics, and on the social context in their specific math classroom." They note in their study that beliefs about social context were limited to beliefs about the role and functioning of the teacher. After a factor analysis of the survey results they found four categories of beliefs existed in their questionnaire: (1) role and functioning of teacher (belief of social context) (2) significance of and their own competence in mathematics (these items related to task/value and self-efficacy beliefs) (3) mathematics as a social activity (math is grounded in human practices and is dynamic). Factor 3 also included a socio-constructivist perspective on learning and problem solving (anyone can learn/several ways to find solution) (4) math as a domain of excellence (importance to excel and extrinsic goal orientation).

In Hannula et al.'s (2005) study, a mathematics-related belief questionnaire was developed that consisted primarily of items that were generated in a qualitative study on student teacher's mathematical beliefs (Pietila, 2002; as cited by Hannula et al., 2005) it also included a self-confidence scale containing 10 items from the Fennema-Sherman mathematics attitude scale (Fennema & Sherman, 1976; as cited by Hannula et al., 2005), and four items from a success orientation scale found in a study with pupils of comprehensive school (Nurmi, Hannula, Maijala & Pehkonen, 2003; as cited by Hannula, 2005). Hannula et al. (2005) explain that the items chosen for their questionnaire were structured around five topics: (1) experience as a mathematics learner, (2) image of oneself as a mathematics learner, (3) view of mathematics, learning of mathematics, and teaching of mathematics, (4) view of oneself as mathematics teacher, and (5) experiences as teacher of mathematics. After a factor analysis they found a ten-factor solution that they described as follows:

> Two of the principle components relate primarily to the student teacher's past experiences (*My family encouraged me, I had a poor teacher*), three to the personal beliefs (*I am not talented in mathematics, I am hard working and conscientious, and mathematics is difficult*), one to emotions (*I like mathematics*), and two to the person's expectations about future success (*I am insecure as a mathematics teacher, I can do well in mathematics*) (p. 92).

Hannula et al. (2005) claim that the subcategories (beliefs about mathematics education, beliefs about self, and beliefs about social context) suggested by Op't Eynde & DeCorte (2003) are present in their analysis of the questionnaire. They also suggest that there is evidence in their analysis of the ten factors for the distinction between cognition, motivation, and emotion. This evidence is in the separation of one emotional component (*I like mathematics*) and one motivational (*I am motivated*) from the cognitive components.

In the case study of Frank, Op't Eynde and Hannula (2006) use the *Mathematics Related Belief Questionnaire* (MRBQ) developed by Op't Eynde and DeCore (2003) to measure the beliefs of a student named Frank. The data gathered from the MRBQ, in this case, is used to investigate how Frank's beliefs may be connected to his problem solving behavior and related affect. They explain that although the MRBQ needs further development it can provide a useful indication of Frank's mathematical beliefs.

3.2 Clusters and Correlations

Clearly, the spectrum of student beliefs that could fit into Op't Eynde and DeCorte's (2003) framework is vast. Hannula et al. (2005) explains that this vast spectrum of mathematics-related beliefs usually group into clusters that can influence each other within a student's belief system.

The analysis of the relations between students' beliefs has been a topic of research for many years (Fennema, 1989; Schoenfeld, 1989; Hembree, 1990; Op't Eynde & De Corte, 2003; Hannula et al. 2005). Schoenfeld (1989) found in his study that students' overall academic performance, their expected mathematical performance, and their sense of their own mathematical ability are all correlated. Schoenfeld primarily looks at the correlations between students' self-beliefs. The strongest correlation was between expected mathematical performance and perceived mathematical ability. This finding suggests that regardless of a student's goal orientation or personal beliefs about the nature of intelligence, as described by Dweck (1986), belief in one's *own* mathematical ability is strongly tied to the expectation that one will do well. Without a strong belief in one's ability, the performance goal orientated student who believes intelligence is fixed will have difficulty engaging in challenging tasks and will likely expect poor performance.

The learning goal oriented student is likely to engage in challenging tasks whether he or she believes in his or her mathematical ability. This type of student would likely expect to perform well if he or she believed in his or her mathematical ability. McLeod (1992) explains that "It seems likely that success in problem solving will engender a belief in one's capacity for doing mathematical problems, leading to an increase in confidence, which correlates positively with achievement in mathematics" (p. 584).

Shoenfeld (1989) also found that, "students who think less of their mathematical ability tend more to attribute their mathematical success to luck and their failures to lack of ability whereas those who think themselves good at mathematics attribute their success to their abilities" (p. 347). Finally, he discovered a correlation between a self-belief and a belief about the object: "those who see themselves as being good at mathematics also tend to find the subject interesting" (p. 348).

Op't Eynde and De Corte (2003, 2004) also found significant correlations between mathematics-related beliefs. One such correlation was that students who had a more social-dynamic view of mathematics See mathematics as valuable and have more confidence in their mathematical ability. Moreover, they tend to have more positive beliefs about the functioning of the teacher. In this analysis we see correlations between each of the three components of their hypothetical framework for a mathematics-related belief system. Within that system of beliefs they isolated two factors related to beliefs about mathematics (*the object*). One factor was student's belief that mathematics is a social activity, or not, and the other factor pertained to student's view of mathematics as a domain of excellence. Op't Eynde and De Corte found that these two factors, although correlated to other factors, were only mildly related to each other. Surprisingly, the socio-constructivist view of mathematics was present in one of the factors and the absolutist view on mathematical learning and problem solving was present in the other factor. One would expect these two theoretically opposite views to have a negative correlation with each other – not a mild positive correlation. Op't Eynde and De Corte (2003) suggest that "the orientation toward achievement and grading, that up to a certain point always characterizes a mathematical school context, might account for the presence and acceptance of certain absolutist characteristics" (p. 9). Object beliefs *were* strongly connected to beliefs about self. For example, students who are confident in their ability are the ones who are convinced about the relevance of mathematics.

In their (2005) study, Hannula et al. looked closely at correlations between each of his eight factors. After analysis, they determined that within the many correlations between the ten factors, three of the factors were very closely related and formed what they labeled a "core" of the person's view of mathematics. The three core components forming the core set of beliefs were: (F1) "I am not talented in mathematics" – a self-belief, (F7) "I like mathematics" – showing an emotional relation to the object, and (F8) "Mathematics is difficult" – an object belief. Factors 1 and 8 were each negatively correlated to factor 7. The other five factors situated around the core beliefs were related primarily to the core with only some secondary relations to one another. Beliefs that showed a significant connection to the core beliefs included: (F6) "I can do well in mathematics" (positive correlation) and (F4) "I had a poor teacher in mathematics" (negative correlation). The other factors that had only a minor effect on the core factors were factors 2, 3 and 5. These included beliefs about their personal diligence, family encouragement, and abilities as a mathematics educator.

After consideration of the strong correlations between the core beliefs of the students in their study, Hannula et al. (2005) explain that: "a person with a positive view believes oneself to be talented in mathematics, believes mathematics to be easy, and likes mathematics. The person with a positive view is usually also confident on being able to do well in mathematics, hard-working, and satisfied with the teaching he or she had in mathematics" (p. 97). Using student scores on the core beliefs, Hannula et al. (2005) separate students into three groups: (1) positive core beliefs, (2) negative core beliefs, and (3) neutral core beliefs. In this cluster analysis, each of the three groups is also given two sub-groups. They divided students with positive core beliefs into either *autonomous* or *encouraged* – based on the level of family encouragement. They divided students with negative core beliefs into either *lazy* or *hopeless* – based on how hard they have tried. Finally, they divided students with neutral core beliefs into either *pushed* or *diligent*, based on the level of family encouragement. By performing a cluster analysis, Hannula et al. (2005) were able to determine that 43% of their students had positive views towards mathematics, 36% had a neutral view and 22% had a negative view. They discovered that each of the three clusters showed differences in both mathematics achievement and motivation. They add that "some of the students with a negative view were seriously impaired as they felt that they have tried hard and failed. Consequently, they have adopted a belief that they can not learn mathematics" (p. 97).

The concept of the existence of a core set of mathematics-related beliefs deserves some attention in mathematics education research. Pietila (as cited by Hannula, 2005) says that it's only the experiences that penetrate the core that can change a student's view of mathematics in an essential way. This suggests that if mathematics educators want to help their students develop positive views of mathematics that enable them to succeed, they must focus on student's core beliefs before any others. To begin this process, teachers must have a way to accurately measure the belief systems of their students.

3.3 Gender Differences

In General, research has shown that the mathematics-related beliefs of males are more positive than females (Fennema, 1989; Pehkonen, 1997). Fennema, (1989) in a summary of her research, explains that males perceive mathematics as a more useful subject than females. In terms of beliefs related to self, there are substantial differences between males and females. Reyes (1984; as cited by McLeod, 1992) and Meyer and Fennema (1988; as cited by McLeod, 1992) explain that males show more confidence than females, even when females should feel more confident based on their performance. Also, females tend to experience mathematics anxiety more often than males (Frost, Hyde & Fennema, 1976).

There are also gender differences in terms of beliefs related to the attribution of success and failure. McLeod (1992) states that males are more likely to attribute mathematical success to ability, whereas females are more likely to attribute their failures to lack of ability. Also, more than males, females tend to attribute their success to extra effort, whereas males tend to attribute their failures to lack of effort more than females do.

In a more recent study, Nurmi et al. (2007) look at gender differences in self-confidence in mathematics. Using a questionnaire they gather information with regards to both male and female beliefs about self-confidence in mathematics. After a factor analysis of the data they split the items into three components: (1) self-confidence, (2) success orientation (or willingness to succeed), and (3) defense orientation (fear of embarrassing and avoiding behavior). They found that the boys in the sample scored significantly higher in self-confidence, however, there was no statistically significant difference in male and female scores on success and defense orientation. Finally, contrary to earlier research results (e.g. Minkkinen, 2001; as cited by Nurmi et al., 2007), Nurmi et al discovered that the differences in self-confidence were significant even when the most skillful girls were compared with the most skillful boys.

Finally, in Andrews et al's (2007) evaluation of Op't Eynde and DeCorte's (2003) MRBQ with regards to sensitivity to nationality, gender, and age, Andrews et al. discover that girls, regardless of age or nationality, are less positive in their beliefs about their own competence than boys. In terms of mathematics being inaccessible and elitist, they found that both males and females shared a negative view, however females had a significantly more negative viewpoint. Finally, they found that both the boys and the girls in their study were equally positive in terms of their teachers as facilitators of their learning and of the relevance of mathematics to their lives.

After reviewing the literature I have narrowed my research questions to the following: (1) What can the factor analysis of a mathematics-related belief questionnaire (MRBQ) tell us about the structure of mathematics-related belief systems? (2) What can the analysis of an MRBQ tell us about gender differences in mathematics related belief systems? (3) Are student responses to the MRBQ consistent with beliefs expressed in the context of problem solving? In the next chapter I discuss in more detail how questions from both Op’t Eynde and DeCorte’s (2003) and Hannula’s (2005) MRBQ combined to form the questionnaire used in this study.

4 Methodology (1)

This study is designed to explore students' mathematics-related beliefs systems and the use of a "mathematics-related belief questionnaire" (MRBQ) to measure such systems. The data was collected in two parts. Part 1 involved the administration of a mathematics related beliefs questionnaire. Part 2 involved both video and interview data related to a problem solving activity. In this chapter I look at how the questionnaire data was gathered in the first part of this study.

4.1 Sample

Participants in this study were students from three Principles of Mathematics 11 classes at an independent Catholic school located the Lower Mainland of British Columbia. With subsidized tuition and no entrance exam, this school hosts an eclectic mix of students from varying socio-economic backgrounds.

Grade 11 students, rather than juniors, were chosen for this study for their ability to articulate their thoughts and feelings. A second reason these students were chosen is that Mathematics 11 is the last required mathematics course for graduation so students enrolled range in both ability and interest in the subject. At the time of the study, students were approximately one third of the way through the content of the course. Student achievement in the course was measured using a wide range of assessment strategies including: standard tests, journal writing, group projects, observation, and peer evaluation. In the three classes used for this study, student achievement ranged from scores of 30% to 100%, with the average of the sample being 67%.

Participation in the study was optional for students in the three Mathematics classes. Students were told that participation in the study would have no impact on their grade in the course even though the questionnaire would be written during class time. Of the 87 students in the three classes, 73 chose to participate. It should be noted that students participating in this study were students of the researcher. Conducting research on one's own students poses a number of problems in terms of the validity of the data collected. Precautions taken to limit these problems will be discussed in more detail in this chapter and proceeding chapters.

4.2 Mathematics-related belief questionnaire

In part 1 of the study, students responded to a 43 question, mathematics-related beliefs questionnaire. The purpose of the questionnaire in this study was two-fold. The first purpose of it was to gather information about the belief systems of students and how they might be "clustered" into "belief subsets." The second purpose of the questionnaire was in the selection of a few students exhibiting different belief systems to participate in the second part of the study.

It was impossible to keep the survey data completely anonymous, as it would be used to select students for part 2 of the study. This posed a problem. To encourage truthfulness in survey response, each survey was numbered and students were instructed to not record their names on the paper. A separate record was kept that matched names to numbers. Students were told that the survey data would be analyzed anonymously and that only a few students selected for part two of the experiment would have their names revealed to the researcher. Students were also told that once the three students were chosen for part two of the study, the key that matched names to numbers would be destroyed. The process that was used for selecting three students for part two of the study is described in the second methodology chapter.

The questionnaire was designed to measure students' beliefs about: (1) self, (2) mathematics education, and (3) the social context in which they learn mathematics. As discussed in chapter 3, it is these three components that form students' mathematics-related belief systems (Op't Eynde & De Corte, 2004, Hannula, 2005). These overall categories have a wide range of subcategories. For example, beliefs about self include such components as self-efficacy, goal orientation and task-value. Beliefs about mathematics education include subcategories such as mathematics as a subject, mathematical learning and mathematics teaching in general. Finally, beliefs about the social context in which students learn mathematics includes subcategories such as the role and functioning of their mathematics teacher and the influences of their parents.

The questionnaire consisted mainly of items pulled from two sources. The first is the "Mathematics-related belief questionnaire" used in Op't Eynde and De Corte's (2004) study on junior high students (see appendix A for a complete list of the items). All of the questions on the questionnaire were designed to gather information with regards to the three overall belief categories described above. In their analysis of the questionnaire, they found a four-factor solution. Factor 1 included items related to students' beliefs about the role and functioning of their own teacher – an example of classroom context beliefs.

Factor 2 included items related to beliefs about self, more specifically, task-value and self-efficacy beliefs. Factors 3 and 4 (*mathematics as a social activity and mathematics as a domain of excellence*) were clearly in the domain of beliefs about mathematics education. Op't Eynde and De Corte (2003) explain that even though the empirical factors determined by the analysis are not entirely constituted as they theoretically expected, the three main categories differentiated in the hypothetical framework are at least identified in the four-factor model. Perhaps the most significant finding in their study was the discovery of an important subcategory of classroom context beliefs – the role and functioning of the teacher. Op't Eynde and De Corte explain: "the way students feel accepted by the teacher and find him sensitive to their needs, seems to be related to how motivating they perceive their teacher to be and how he organizes instruction, since items referring to these subcategories are significantly loaded on the same factor" (p. 8).

Asking students to truthfully respond to questions regarding the role and functioning of their teacher when the teacher is also the researcher could cause problems with regards to the validity of the results. So, although Op't Eynde and DeCorte (2004) found that the role and functioning of one's teacher is an important component of context-related beliefs, questions from factor 1 were the only ones not included in this study.

All of the questions from factor 2 (*Beliefs about the significance of and their own competence in mathematics*) were included in the questionnaire for this study. Op't Eynde and De Corte (2004) explain that it was this factor, as determined by the analysis of their questionnaire, that included items related to student's self-beliefs. Questions measuring student's beliefs about their competence in mathematics include: "If I try hard enough then I will understand the course material of the mathematics class."[1] In addition to items related to self-beliefs, factor 2 also contains a number of items pertaining to beliefs about the significance of mathematics such as: "To me, mathematics is an important subject." Op't Eynde and De Corte add that "the clustering of these two subcategories in one factor indicates that students who are confident about their mathematical ability are most of the time the ones who are convinced about the relevance of mathematics" (p. 4).

Factors 3 and 4 from Op't Eynde and De Corte's questionnaire primarily relate to beliefs about mathematics. Like factor 2, none of the items in these factors are related to beliefs about the role and functioning of the teacher, so all of the questions from these two factors were included in the questionnaire for this study. Factor 3 (*Mathematics as a social activity*) includes items such as "Mathematics enables men to better understand the world he lives in" and "Solving a mathematics problem is demanding and requires thinking, also from smart students." Op't Eynde and De Corte insist that the socio-constructivist view of mathematics is present in this factor. Factor 4 (*Mathematics as a domain of excellence*) includes questions such as "My major concern when learning mathematics is to get a good grade" and "I want to do well in mathematics to show the teacher and my fellow students how good I am in it." After analysis of these two factors related to student's beliefs about mathematics, Op't Eynde and De Corte found that these two dimensions of mathematics-related beliefs were only mildly related to one another.

Excluding questions about the role and functioning of the teacher meant that almost one third of the questions from Op't Eynde and De Corte's questionnaire could not be used. So, to supplement Op't Eynde and De Corte's mathematics-related belief

[1] The wording of each question used is consistent with Op't Eynde and De Corte's (2004) study.

questionnaire, questions were drawn from a second source (see appendix B). In Hannula et al's (2005) study, a questionnaire was used to gather information with regards to elementary school teacher's mathematics-related beliefs. Although subjects in this study were pre-service teachers, many of the items included in the survey were applicable to high school mathematics students as well. Hannula's questions originated from a number of sources (Pietila, 2002, Fennema & Sherman 1976, Nurmi, Hannula, Maijala & Pehkonen, 2003). As in Op't Eynde and De Corte's (2003) study, each of the factors fit into one of the three components of the hypothetical framework discussed in chapter 3 of this thesis.

The component analysis of Hannula's questionnaire produced ten factors: (F1) I am not talented in mathematics, (F2) I am hard-working and conscientious, (F3) My family encouraged me, (F4) I had a poor teacher in mathematics, (F5) I am insecure as a mathematics teacher, (F6) I can do well in mathematics, (F7) I like mathematics, (F8) mathematics is difficult, (F9) Mathematics is calculations, and (F10) I am motivated.

Using all of the questions from Hannula's questionnaire in addition to Op't Eynde & De Corte's questionnaire would produce a survey with too many questions so a limited number of questions are selected here to supplement Op't Eynde and De Corte's items. Since factors 9 and 10 on Hannula's questionnaire had only two items each, they are not taken to further analysis in Hannula's study and are not used as part of the questionnaire in this study. Questions from factor 5 were also not used in this study because they pertained to experiences as a teacher rather than a student. Finally, questions from factors 4 and 8 were not used in this study because they involved the role and functioning of the teacher. Questions from the remaining five factors were chosen to supplement

Op't Eynde and De Corte's questionnaire: (F1) I am not talented in mathematics, (F2) I am hard working and conscientious, (F3) My family encouraged me, (F6) I can do well in mathematics, and (F7) I like mathematics

Factors 1, 2 and 6 were related to beliefs about self. Items from factor 1 (*I am not talented in mathematics*) included "I'm no good in math" and "math has been my worst subject." Like factor 2 of Op't Eynde and De Corte's (2004) questionnaire, these items were designed to measure student's beliefs about their self-competence in mathematics. Factor 2 (*I am hard working and conscientious*) included items such as "I am hard working by nature" and "I always prepare myself carefully for exams." These items related to self beliefs in the domain of control. Factor 6 (*I can do well in mathematics*) included items such as "I am sure I can learn math" and "I can get good grades in math." These items, like factor 1, relate to competence.

Hannula et al (2005) explain that factor 3 (*My family encouraged me*) is in the domain of "beliefs about the social context" in which mathematics is learned. In particular, he uses the item "My family encouraged me" as an example of beliefs of social norms. All three items from factor 3 were used on the questionnaire for this study. In addition to these three questions related to family encouragement, two more were added by the researcher in this domain: "31. My parents enjoy helping me with mathematics problems" and "33. My parents expect that I will get a good grade in mathematics." These questions were added because of the limited number of questions available in the domain of social context.

Finally, factor 7 (*I like mathematics*) provided additional items in the domain of "beliefs about mathematics." Items from this factor included "Mathematics is a mechanical and boring subject" and "Mathematics is my favorite subject." Hannula (2004) explains that the items from this factor are focused on the student's emotional relationship with mathematics. In total, 14 items from Hannula's (2004) questionnaire were used to supplement the 27 items taken from Op't Eynde and De Corte's (2005) MRBQ. With 2 additional items written added by the researcher, the total number of items on the questionnaire was 43.

Students responded to each question using a five point Likert scale ranging from "strongly disagree" to "strongly agree." The neutral choice on the scale was "neither agree or disagree." See figure 4.1 below for the complete questionnaire. The questionnaires were administered at the beginning of a regular class and students were given as much time as they needed to complete it.

Since the first goal of the questionnaire was to gather information about the belief systems of students and how they might be "clustered" into "belief subsets," an exploratory principal component analysis of the inter-correlations among the items was carried out. After the principle component analysis, factors were determined and possible correlations between them were analyzed. In addition to looking at the results of the entire group, male and female subgroups were also analyzed to determine any possible gender differences. These results will be discussed in the next chapter.

Mathematics-Related Beliefs Questionnaire ID: ________________

	Please respond to each statement	Strongly agree	Some-what agree	Neither agree or disagree	Some-what disagree	Strongly disagree
1.	Making mistakes is part of learning mathematics.	O	O	O	O	O
2.	Group work helps me learn mathematics.	O	O	O	O	O
3.	Mathematics learning is mainly memorizing.	O	O	O	O	O
4.	The importance of competence in mathematics has been emphasized at my home.	O	O	O	O	O
5.	Anyone can learn mathematics.	O	O	O	O	O
6.	There are several ways to find the correct solution of a mathematical problem.	O	O	O	O	O
7.	I am hard working by nature.	O	O	O	O	O
8.	Solving a mathematics problem is demanding and requires thinking, even from smart students.	O	O	O	O	O
9.	Mathematics is continuously evolving. New things are still discovered.	O	O	O	O	O
10.	There is only one way to find the correct solution of a mathematics problem.	O	O	O	O	O
11.	Mathematics is used by a lot of people in their daily life.	O	O	O	O	O
12.	My family has encouraged me to study mathematics.	O	O	O	O	O
13.	I'm only satisfied when I get a good grade in mathematics..	O	O	O	O	O
14.	The example of my parent(s) has had a positive influence on my motivation.	O	O	O	O	O
15.	I believe that I will receive this year an excellent grade for mathematics.	O	O	O	O	O
16.	By doing the best I can in mathematics I want to show the teacher that I'm better than most of the other students.	O	O	O	O	O
17.	I like doing mathematics.	O	O	O	O	O

Figure 4.1 Mathematics-Related Belief Questionnaire (MRBQ)

	Please respond to each statement	Strongly agree	Some-what agree	Neither agree or disagree	Some-what disagree	Strongly disagree
18.	I want to do well in mathematics to show the teacher and my fellow students how good I am in it.					
19.	I can understand the course material in mathematics.					
20.	To me mathematics is an important subject.					
21.	I prefer mathematics tasks for which I have to exert myself in order to find the solution.					
22.	I know I can do well in math					
23.	If I try hard enough, then I will understand the course material of the mathematics class.					
24.	When I have the opportunity, I choose mathematical assignments that I can learn from even if I'm not at all sure of getting a good grade.					
25.	I'm very interested in mathematics.					
26.	Taking in to account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics.					
27.	I think I will be able to use what I learn in mathematics also in other courses.					
28.	I am no good in math.					
29.	Those who are good in mathematics can solve problems in a few minutes.					
30.	I am not the type to do well in math.					
31.	My parents enjoy helping me with mathematics problems.					
32.	It is a waste of time when the teacher makes us think on our own about how to solve a new mathematical problem.					
33.	My parents expect that I will get a good grade in mathematics.					

Figure 4.1 Mathematics-Related Belief Questionnaire (MRBQ)

Please respond to each statement	Strongly agree	Some-what agree	Neither agree or disagree	Some-what disagree	Strongly disagree
34. I expect to get good grades on assignments and tests of mathematics.	O	O	O	O	O
35. Math has been my worst subject.	O	O	O	O	O
36. Mathematics enables people to better understand the world they live in.	O	O	O	O	O
37. I have not worked very hard in math.	O	O	O	O	O
38. Mathematics is a mechanical and boring subject.	O	O	O	O	O
39. I can get good grades in math.	O	O	O	O	O
40. I always prepare myself carefully for exams.	O	O	O	O	O
41. Mathematics has been my favorite subject.	O	O	O	O	O
42. I am sure that I can learn math.	O	O	O	O	O
43. My major concern when learning mathematics is to get a good grade.	O	O	O	O	O

Figure 4.1 Mathematics-Related Belief Questionnaire (MRBQ)

5 Results and Discussion (1)

5.1 Factor analysis

An exploratory principal component analysis of the inter-correlations among the items was carried out using Varimax with Kaiser normalization (N = 73). The rotated solution converged in 13 iterations. The 43-question Mathematics-related belief questionnaire was analyzed and found to have a reliability of 0.869. The rotated component matrix produced 13 factors for 76.0% of the total variance (see Appendix C).

Of the 13 factors produced, 7 were made up of only one or two items. Questions in these 7 factors that showed significant pattern coefficients in multiple factors were kept and those that did not were removed before running a second exploratory component analysis. In total, 11 questions were removed, reducing the number of factors produced to 9 and raising the reliability of the questionnaire to 0.908. See table 5.1 for a list of the 11 items removed.

After the 11 items were removed, the rotated component matrix produced 9 factors accounting for 74.1% of the total variance (see Appendix D). Factor I received salient loadings (pattern coefficients > .40) on each of the 7 items.[2] Items 15, 26 and 34 from this factor came from factor 2 (*Beliefs about the significance of and competence in mathematics*) of Op't Eynde and De Corte's (2004) questionnaire. These three items are not related to the significance of mathematics, rather the student's perceived competence.

[2] Roman numerals will be used when listing the factors determined in this study.

Also, two of the three items taken from factor 6 (*I can do well in mathematics*) on Hannula et al's (2005) questionnaire received significant loadings on this factor.

Table 5.1: 11 items removed before second exploratory factor analysis.

2. Group work helps me learn mathematics.
3. Mathematics learning is mainly memorizing.
8. Solving a mathematical problem is demanding and requires thinking, even from smart students.
13. I am only satisfied when I get a good grade in mathematics.
16. By doing the best I can in mathematics I want to show the teacher that I 'm better than most of the other students.
18. I want to do well in mathematics to show the teacher and my fellow students how good I am in it.
23. If I try hard enough, then I will understand the course material of the mathematics class.
24. When I have the opportunity, I choose mathematical assignments that I can learn from even if I'm not sure of getting a good grade.
29. Those who are good in mathematics can solve problems in a few minutes.
32. It is a waste of time when the teacher makes us think on our own about how to solve a new mathematical problem.
43. My major concern when learning mathematics is to get a good grade.

Items 35 and 33 received the least significant loadings but also point to students beliefs about their competence in mathematics. Item 33 "*My parents expect that I will get a good grade in mathematics*," was the only item in factor one connected to parental influences, and suggests that student's perceptions of their own competence is tied to their perceptions of their parents beliefs. With each of the of the seven items in this factor being connected to students beliefs about their competence in mathematics, factor I was defined as "*I can do well in mathematics*," like factor 6 on Hannula et al's (2005) questionnaire.

Factor II received salient loadings on 6 items. Four of the six items were from factor 2 of Op't Eynde and De Corte's (2004) questionnaire. These four items are related more to beliefs about the "significance of mathematics" rather than beliefs about self-competence and include items such as: 20. *To me mathematics is an important subject*. Items such as 17. "*I like doing mathematics*" from Op't Eynde and De Corte's factor 2 that eluded the "enjoyment" of mathematics had the highest loadings on this factor. Also included with these four items were two items from factor 7 (*I like mathematics*) of Hannula et al's (2005) questionnaire. Like factor 7 from Hannula's questionnaire, factor II of the analysis of this questionnaire was defined as "*I enjoy mathematics*."

Factor III received salient loadings on 5 items and was defined as "*I am talented in mathematics*." Items 28 and 30 were from factor 1 of Hannula et al's questionnaire (*I am not talented in mathematics*). Each of these items received significant negative loadings. Items 19 and 42 were also related to student's beliefs about their competence in mathematics. Item 42, "*I am sure that I can learn math*," received loadings on both factor I and III of this study, suggesting that students who believe they have mathematical talent believe they can do well in the subject. Item 5, "*Anyone can learn mathematics*", received the highest loading in this factor and suggests that students who believe they have mathematical talent believe that it is possible for anyone to learn mathematics.

Factor IV received high loadings on three items: (37) *I have not worked very hard in mathematics*, (7) *I am hard working by nature*, and (40) *I always prepare myself careful for exams*. All three of these items were taken from factor 2 (*I am hard-working and conscientious*) of Hannula et al's (2005) questionnaire. Factor IV of this analysis was thus defined the same way.

Factor V received salient loadings on 4 items and was defined as "*My family encourages me*." Three of the four items in this factor were from factor 3 (*My family* *encourages me*) of Hannula et al's questionnaire. Item 31, *my parents enjoy helping me with math problems,* also received a high loading on this factor.

Factor VI received salient loadings on 4 items. Each of the four items were from factors 3 and 4 of Op't Eynde and De Corte's (2004) questionnaire. Op't Eynde and De Corte explain that it is these two factors of his questionnaire that include student's beliefs about mathematics. They label factors 3 and 4 "*Mathematics as a social activity*" and "*Mathematics as a domain of excellence*" respectively. The items in this factor are related to a dynamic, verses an inert, view of mathematics.[3] Factor VI was defined as "*Mathematics is dynamic*."

Factors VII - IX each contained only one item each accounting for 14.8% of the total variance. It should be noted that the 3 items in factors VII - IX were from the seven factors in the original factor analysis that contained two or less items. Table 5.2 shows a summary of the Varimax factor rotation with factors 7-9 removed.

Table 5.2: Summary of Results of Varimax Factor Rotation

Item	Coefficient
Factor I: *I can do well in mathematics*	
39. I can get good grades in math	.813
34. I expect to get good grades on assignments and tests of mathematics	.766
15. I believe that I will receive this year an excellent grade for mathematics	.739
26. Taking into account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics	.709
22. I know I can do well in mathematics	.636
33. My parents expect that I will get a good grade in mathematics	.627
35. Math has been my worst subject	-.526

[3] Here, the term "dynamic" is meant to describe mathematics as a subject that is growing, changing and has life outside of the context of the classroom.

Factor II: ***I like mathematics***

25. I'm very interested in mathematics	.853
17. I like doing mathematics	.835
41. Mathematics has been my favorite subject	.810
38. Mathematics is a mechanical and boring subject	-.702
21. I prefer mathematics tasks for which I have to exert myself in order to find the solution	.515
20. To me mathematics is an important subject	.453

Factor III: ***I am talented in mathematics***

5. Anyone can learn mathematics	.807
42. I am sure that I can learn math	.735
28. I am no good in math	-.587
19. I can understand the course material in mathematics	.554
30. I am not the type to do well in math	-.439

Factor IV: ***I am hard working and conscientious***

37. I have not worked very hard in mathematics	-.879
7. I am hard working by nature	.720
40. I always prepare myself carefully for exams	.703

Factor V: ***My family encourages me***

12. My family has encouraged me to study mathematics	.799
31. My parents enjoy helping me with mathematics problems	.686
4. The importance of competence in mathematics has been emphasized at my home	.539
14. The example of my parent(s) has had a positive influence on my motivation	.499

Factor VI: ***Mathematics is dynamic***

10. There is only one way to find the correct solution of a mathematics problem	-.725
6. There are several ways to find the correct solution of a mathematics problem	.703
9. Mathematics is continuously evolving. New things are still discovered	.591
27. I think I will be able to use what I learn in mathematics also in other courses	.463

Factor VII: Will not be used in further analysis

11. Mathematics is used by a lot of people in their daily life	.508

Factor VIII: Will not be used in further analysis

36. Mathematics enables people to better understand the world they live in	.892
Factor IX: Will not be used in further analysis	
1. Making mistakes is part of learning mathematics	.788

In comparing the factor analysis of this questionnaire with the analyses of the two questionnaires that were combined to form it, there are some interesting results. First, it should be noted that the items receiving significant loadings on the factors determined here were items from the original two questionnaires that received the highest loadings. Looking specifically at the factors from Op't Eynde and De Corte's (2004) analysis, it appears that items from their factor 2 (*Beliefs about the significance of and competence in mathematics*) were split into three factors all related to beliefs about self. Looking at the items from factors 3 and 4 from Op't Eynde and De Corte's that relate to beliefs about mathematics, it is clear that only 8 out of the 15 items used received salient loadings. Three of these 8 items were not considered for further analysis as they were the three items forming the last three factors. Of the five items left relating to beliefs about mathematics (the object) from their questionnaire, four of them did form factor VI "Mathematics is dynamic."

Looking at the items selected from Hannula et al's (2005) questionnaire, it should be noted that items chosen from each of the factors were the ones with high loadings. Each of these items too received significant loading on the factors that they were placed into in the current analysis. Not only were all of the items included in the six factors of this thesis, but the items from Hannula et al's questionnaire grouped into essentially the same factors. It is for this reason that many of the factor titles from Hannula et al's study were also used here.

In general, there appears to be a number of similarities between factors determined in the three analyses (including the current analysis) in the domain of student's self-beliefs such as self-confidence, perceived talent and effort level. When it comes to beliefs about mathematics education (the object), the variance here in factors formed by the same items suggests that isolating beliefs about the object mathematics in an MRBQ may be more problematic than isolating beliefs about self or social context.

After determining 6 significant factors from the survey, each student was assigned a score on each of the 6 factors. The score was determined using the 5-point Likert scale from the questionnaire. "Strongly agree" to "strongly disagree" were assigned numerical values of 1 through 5 respectively for each question. For items with negative loadings the numerical values were reversed. Next, the average of all items in a factor was calculated for each student, resulting in a mean score for each factor ranging from 1 to 5.

For factor I (*I can do well in mathematics*), mean scores ranged from 1 to 4.4. A score of 1 on factor I indicated that the student believes he or she has been successful in the past and will continue to do well in mathematics. Conversely, a score of 5 on factor I indicated that the student believes math has been, and will continue to be, his or her worst subject. The average score on factor I for the entire population was 2.0 with a standard deviation of 0.834.

On Factor II (*I like mathematics*), student's mean scores ranged from 1 to 5 with a population average score of 2.6 and a standard deviation of 0.976. A score of 1 on factor II indicated that the student believes mathematics is both interesting and enjoyable. A score of 5 on factor II indicated the student believes mathematics is a mechanical and boring subject.

Scores ranged from 1 to 4.6 for factor III (*I am talented in mathematics*). The population average score for this factor was 1.8 and the standard deviation was 0.777. A score of 1 on factor III indicated that the student believed they have high mathematical ability. A score of 5 indicated that the student believed they were not able to do well in mathematics.

For factor IV (*I am hard working and conscientious*), a score of 1 indicated that the student believed he or she works hard in mathematics including careful preparation for exams. A score of 5 indicated that the student believed he or she did not put a lot of effort into the course. Mean scores on factor IV ranged from 1 to 4.3 with a population average of 2.2 and a standard deviation of 0.867.

On Factor V (*My family encourages me*), mean scores also ranged from 1 to 4.3. The population average was 2.5, with a standard deviation of 0.820. A score of 1 on factor V indicated they believed that they were encouraged to do well in mathematics and supported by their parents. A score of 5 indicated that the student believed there was little support and encouragement from home.

Finally, on factor VI (*Mathematics is dynamic*), student mean scores ranged from 1 to 3.3 with an population average of 1.8 and a standard deviation of 0.528. A score of 1 on factor VI indicated that students believed that mathematics is dynamic subject that is evolving and useful. A score of 5 would indicate that students believed that mathematics is a static subject where there is only one way to find the solution to a problem. Table 2 below shows the range, average, and standard deviation for the mean scores on each of the six factors.

In general, the average scores for each factor were all less than 3 indicating that students from the three classes appear to have somewhat positive views towards mathematics.

Table 5.3: Range, average, and standard deviation of mean factor scores.

Factor	Minimum	Maximum	Average	Standard Deviation
I. *I am successful at mathematics*	1	4.4	2	0.834
II. *I enjoy mathematics*	1	5	2.6	0.976
III. *I have mathematical ability*	1	4.6	1.8	0.777
IV. *I am hard working and conscientious*	1	4.3	2.2	0.867
V. *My family encourages me*	1	4.3	2.5	0.820
VI. *Mathematics is evolving and useful*	1	3.3	1.8	0.528

For each of the factors (except factor VI: *Mathematics is evolving and useful*) student scores ranged from strong positive to strong negative beliefs suggesting a wide range of beliefs in the student population. For factor VI, student beliefs were either positive or neutral.

5.2 Factor correlations

After determining individual student's average scores on each of the six factors, the data was used to investigate possible correlations between the factors. The Pearson product-moment correlation coefficient was calculated for each possible pair of factors and the results are shown in table 3 below. To determine the significance of each correlation, *t*-

values were determined for each using a two-tailed test and $p = 0.01$ Correlations of $r > 0.304$ were deemed significant and are shown in bold in the table below.

Table 5.4: Correlations between the factors. *Statistically significant correlations in bold*

Correlation between factors	(r)
I/II. *I can do well in mathematics / I like mathematics*	**0.613**
I/III. *I can do well in mathematics / I am talented in mathematics*	**0.613**
I/IV. *I can do well in mathematics / I am hard working and conscientious*	0.123
I/V. *I can do well in mathematics / My family encourages me*	0.265
I/VI. *I can do well in mathematics / Mathematics is dynamic*	0.231
II/III. *I like mathematics / I am talented in mathematics*	**0.652**
II/IV. *I like mathematics / I am hard working and conscientious*	0.027
II/V. *I like mathematics / My family encourages me*	0.242
II/VI. *I like mathematics / Mathematics is dynamic*	**0.469**
III/IV. *I am talented in mathematics / I am hard working and conscientious*	0.051
III/V. *I am talented in mathematics / My family encourages me*	0.213
III/VI. *I am talented in mathematics / Mathematics is dynamic*	**0.351**
IV/V. *I am hard working and conscientious / My family encourages me*	**0.355**
IV/VI. *I am hard working and conscientious / Mathematics is dynamic*	0.129
V/VI. *My family encourages me / Mathematics is dynamic*	0.199

When we look at the correlations between the factors we see that three of the factors are closely related. These three factors appear to form a core of the person's view of mathematics. Factors I (*I can do well in mathematics*) and III (*I am talented in mathematics*) both focus on student's self-beliefs, whereas factor II (*I like mathematics*) focuses more on the student's emotional connection to mathematics.

In Hannula et al.'s (2005) study a similar set of core beliefs was found consisting of three components: (1) *I am not talented in mathematics*, (2) *Mathematics is difficult*, and (3) *I like mathematics*. Also, Hannula et al add that there was a forth factor which also had a strong connection to what they identified as the core. That factor was *I can do well in mathematics*. They explain that it is this factor relates to students "personal expectations to do well" and differs from the core factor *I am not talented in mathematics* in that "the element of effort has a greater role in it.

In comparing the four factors that had the strongest correlations in Hannula et al's study with the three core factors here we see that every item used from Hannula et al's questionnaire appearing in factors with strong correlations are present in the core beliefs here. In fact, all three of the core factors identified here (*I can do well in mathematics*, *I like mathematics* and *I am talented in mathematics*) are the same as three of the factors with the strongest correlations in Hannula et al.'s study (with an opposite loading on the factor related to talent). One difference between the "core" beliefs in these two studies is that Hannula et al include *mathematics is difficult* as part of the "core." Items from this factor were not included in the present questionnaire because they asked students to comment on the functioning of their teacher and former high school experience (see appendix B). Regardless, the results from both studies suggest that at the core of student's mathematics-related beliefs are strong correlations between beliefs about mathematical ability, expectancy beliefs (possibly due to perceived difficulty), and beliefs about their emotional relation to the object.

The strong correlation between "expected mathematical performance and perceived mathematical ability" found by Schoenfeld (1989), and discussed in chapter 3, is supported by the core beliefs discovered here and in Hannula et al's (2005) study. There is also evidence in Schoenfeld's study that supports the connection seen here between self-beliefs (*I can do well in mathematics*) and a more emotional connection to the subject (*I like mathematics*). This evidence lies in the connection he found between perceived mathematical ability and interest in the subject. As, discussed in chapter 2, many researchers believe in the strong connection between emotion and cognition so it should not be a surprise to see it manifest itself in both Hannula et al's and the present study. Hannula et al (2005) argue that the presence of an "emotional" component in the core beliefs as a separate factor from the more cognitive core factors, while producing a strong correlation between the two components of affect, is evidence for the distinction between cognition and emotion.

In contrast, Lazarus' (1991) stance on the nature of emotion (as described in chapter 2) suggests that since emotion is always a response to cognitive activity, the core factor here that seems to be emotional in nature is merely the result of some belief that is highly cognitive in nature. Regardless of your viewpoint on the nature of emotion, one must agree that there is an emotional element to one of the factors of the "core."

The strong correlation seen here between expected performance and an emotional connection to the subject may also be evidence of the connection between emotion and personal goals agreed upon by many researchers (e.g. Buck, 1999; Lazarus, 1991; Power & Dalgleish, 1997; Mandler, 1989; as cited by Hannula, 2002) that is described in chapter 2. If the first step in moving towards a personal goal is the belief that you can do so, then expecting mathematical performance might be the critical first step in proceeding towards goals and inducing the positive emotions the accompany that process.

The 3 remaining factors surround the core, each relating primarily to the core factors with the exception of factor IV (*I am hard working and conscientious*). Although no significant connection was detected between the core and factor IV, there was a connection determined between factor IV and factor V (*My family encourages me*). These correlations, between parental support and other mathematics related beliefs such as self-efficacy beliefs and perceived effort support Person et al's (1982) findings discussed in chapter 2. Clearly, the role of the parent is an important consideration when determining teaching practices that promote positive mathematics related belief systems in students. The role of the teacher is equally important, as seen in Op't Eynde and De Corte's (2003) study. Unfortunately, questions regarding the role of the teacher were not used here (for reasons described in chapter 5), and the connection between the role and functioning of the teacher and the core beliefs can not be discussed.

The strongest connection to the core was factor VI (*Math is dynamic*). This finding echoes Op't Eynde and De Corte's (2003, 2005) findings that students who hold a more social-dynamic view of mathematics attach more value to mathematics and have more confidence in their mathematical ability. Figure 5.1 shows the structure of student's views of mathematics based on the correlations between factors.

5.3 Correlations to achievement

Next, each student's course percentage at the time the survey was tabulated and an analysis of how each factor might be correlated to course achievement was conducted. Again, the Pearson product-moment correlation coefficient was determined and the results are shown in table 5.5 below. Again, to determine the significance of each correlation, t-values were determined for each using a two-tailed test and p=0.01. Correlations of r>0.304 were deemed significant. Looking at the correlations between

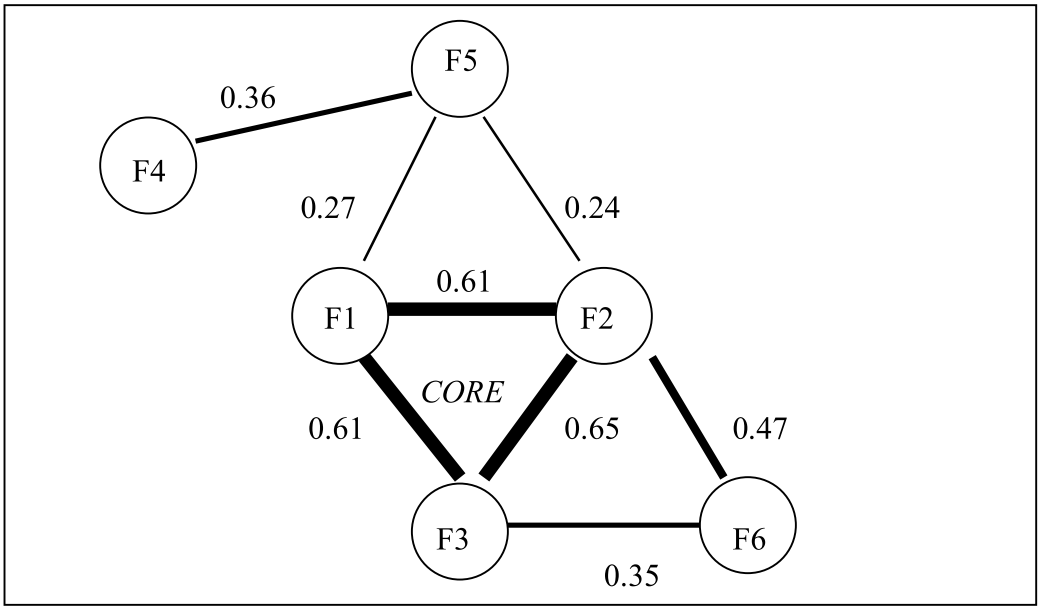


Figure 5.1 Structure of student's view of mathematics.

The connection weights are Pearson correlations. The factors are: (F1) *I can do well in mathematics*, (F2) *I like mathematics*, (F3) *I am talented in mathematics*, (F4) *I am hard working and conscientious*, (F5) *My family encourages me*, and (F6) *Mathematics is dynamic*.

Table 5.5: Correlations between achievement and factors. *Statistically significant correlations in bold.*

Factor	Correlation with achievement
I. *I can do well in mathematics*	**0.509**
II. *I like mathematics*	0.219
III. *I am talented in mathematics*	**0.358**
IV. *I am hard working and conscientious*	0.224
V. *My family encourages me*	0.083
VI. *Mathematics is dynamic*	0.051

achievement and the six factors, we see that two of the factors showed a significant correlation to achievement. The two factors exhibiting the most significant correlation were factor I (*I can do well in mathematics*) and III (*I am talented in mathematics*). Each of these self-beliefs is included in the "core" beliefs mentioned above. Here, there is evidence of the strong connection between perceived ability and achievement as discussed in chapter 2. It is the students with a "mastery orientation" that feel success is within their control – these students believe that they posses talent in mathematics and that they have the ability to do well. It is these self-confident, mastery-oriented students that experience higher achievement in mathematics (Diener & Dweck, 1978; as cited by Kloosterman, 1988). Also, whether a student is performance goal orientated (where belief in ability is a necessary condition for task engagement) or learning goal orientated (where perceived ability has less of an impact on whether or not a student will engage in a task), it is clear that self confidence could have a positive effect on achievement.

Factors II (*I like mathematics*) and IV (*I am hard working and conscientious*) both showed only a weak positive correlation to achievement. Factors V (*My family encourages me*) and VI (*Mathematics is dynamic*) both showed no correlation to achievement. These results support research that says self confidence, when compared with other variables, is the belief that has the strongest correlation with achievement in mathematics (Fennema, 1984; Meyer & Fennema, 1986; as cited by Kloosterman, 1988).

5.4 Further Analysis of Student Views

Using Hannula et al.'s (2005) study as a model, a further analysis was performed after the factor analysis. The factor analysis in this study showed a strong correlation between a set of three "core" factors: (I) *I can do well in mathematics*, (II) *I like mathematics*, and (III) *I am talented in mathematics*. As in Hannula et al.'s (2005) study, students in this study were each placed into one of three groups based on the core of their view of mathematics: (1) positive view, (2) neutral view, and (3) negative view. To determine which group each student belonged in, the three core factor scores (ranging from 1-5 where 1 is extreme positive and 5 is extreme negative) were averaged so that each student would have a core belief average score. Scores ranging from 1-2.6 were placed in the "positive" group, scores ranging from 2.6-3.4 were placed in the "neutral" groups and scores ranging from 3.4-5 were placed in the "negative" group.

After students were placed in each group they were put in one of two subcategories for the group based on their scores for factors IV (I am hard working and conscientious) and V (my family encourages me). The subgroups were modeled after Hannula et al.'s (2005) study and are described in chapter 3. The first part of the cluster analysis produced some interesting results. First, 81% of the students in the sample appear to have a positive view of mathematics, based on the core beliefs. Again, this means that 81% of the students in the sample, when their average score on each of the three core factors was averaged, fell within the range of 1 (most positive) to 2.6 (slightly positive). Of the 81%, 58% of these students are in the encouraged subcategory and 23% fell into the autonomous subcategory. This result speaks to the high parent involvement at the high school that these students attend. Only 13% of the students seem to have a neutral belief of mathematics. Hannula et al (2005) describe these students as having "modest confidence in their own talent and they neither liked nor hated mathematics." Of the 13% in the neutral category, 5% were "pushed" and 8% were "diligent," as described in Hannula et al's (2005) study. Finally, in the negative category, there were 5%. Of the 5% none fell into what Hannula et al. (2005) describe as "lazy." The four students in this category all reported that they believed they had put some effort into their coursework.

5.5 Correlations and gender differences

The sample of 73 students was made up of 31 males and 42 females. The average achievement of the class at the time of the survey was 73%. No significant difference was found when the average achievement of the males was calculated separate from the females. The average score on each of the six factors was calculated for the males and then the females. The results are shown in table 5.6.

Table 5.6: Gender differences in factor scores. Male and female average scores for each factor were compared and t-values were calculated for each with $p = 0.01$. Factors with significantly different scores are in **bold**.

Factor	Male mean	Male σ	Female Mean	Female σ
I. *I can do well in mathematics*	**1.7**	**.576**	**2.2**	**.927**
II. *I like mathematics*	2.3	.689	2.8	1.098
III. *I am talented in mathematics*	**1.5**	**.603**	**2**	**.839**
IV. *I am hard working and conscientious*	2.4	.909	2.1	.826
V. *My family encourages me*	2.3	.886	2.6	.760
VI. *Mathematics is dynamic*	1.6	.508	1.9	.518

Again, the general trend was somewhat positive beliefs for both males and females. T-scores were used to determine whether or not the differences in mean factor scores were statistically significant. Analysis of the t-scores suggests that differences in factors I and III in the core beliefs are of particular statistical significance. It should be noted also that t-scores for factors II and IV were within two tenths of being deemed significant at $p = 0.01$ as well.

After looking at the mean scores separately, the first thing we see is that for the "core" beliefs (factors I-III) male students in the sample have more positive views. This difference is not a surprise based on the research discussed in chapter 3. As in the research cited by Nurmi et al. (2007), we see here that females are generally less confident than males even when the average achievement for both groups is similar. Based on performance, the females in the population have no reason to feel less confident than the males, however, average scores on the three core factors suggest that females are slightly less positive in their beliefs about their mathematical talent, their beliefs about their ability to do well and their like of the subject.

Another difference we see here is that males in the sample had a slightly more positive view of mathematics as a dynamic subject. With two of the items in factor VI (mathematics is dynamic) referring to mathematics as a useful and transferable subject, these results are in agreement with Fennema's (1989) findings where he describes males as perceiving mathematics as a more useful subject than females.

Next, correlations between course achievement and each factor were calculated for both male and female groups individually. The results are shown below in table 5.7. Here we see little difference in statistically significant correlations. We do see that males and females alike have the strongest connection to achievement within the core factors, particularly the two factors related to self-confidence. Although their individual correlations to achievement were deemed insignificant is it worth noting that for females the correlation with perceived effort (factor 4) and achievement is higher than that for males. This difference could support the significance of the role of perceived effort in female student achievement.

Table 5.7: Gender differences in achievement-factor correlations. *Significant correlations in bold.*

Factor	Male achievement corr.	Female achievement corr.
I. *I can do well in mathematics*	**0.459**	**0.574**
II. *I like mathematics*	0.214	0.229
III. *I am talented in mathematics*	0.337	0.386
IV. *I am hard working and conscientious*	0.167	0.284
V. *My family encourages me*	0	0.147
VI. *Mathematics is dynamic*	0.128	-0.013

Also, correlations between the six factors were determined for the males and females separately. The results are shown in table 5.8. To determine the significance of each correlation, *t*-values were determined for each using a two-tailed test and p=0.01. Correlations of r>0.453 and r>0.401 were deemed significant for the male and female groups respectively.

When looking at the gender differences in factor correlations the following results were produced. The core factors (I-III) again show significant positive correlations to one another for both the male and female students when analyzed separately. Within the core factors, male students showed the strongest correlation between factors I (*I can do well in mathematics*) and III (*I am talented in mathematics*). This strong correlation is further evidence of the role of self-confidence in male student's mathematics-related belief systems. Male students who believe they have the ability to do well are confident in the fact that that ability will produce results.

Table 5.8: Gender differences in factor correlations. *Statistically significant correlations in bold.*

Correlation between factors	Male (r)	Female (r)
I/II. *I can do well in mathematics / I like mathematics*	**0.463**	**0.611**
I/III. *I can do well in mathematics / I am talented in mathematics*	**0.871**	**0.572**
I/IV. *I can do well in mathematics / I am hard working and conscientious*	0.297	0.139
I/V. *I can do well in mathematics / My family encourages me*	0.227	0.247
I/VI. *I can do well in mathematics / Mathematics is dynamic*	0.228	0.142
II/III. *I like mathematics / I am talented in mathematics*	0.360	**0.711**
II/IV. *I like mathematics / I am hard working and conscientious*	0.201	0.013
II/V. *I like mathematics / My family encourages me*	0.221	0.218
II/VI. *I like mathematics / Mathematics is dynamic*	0.410	**0.453**
III/IV. *I am talented in mathematics / I am hard working and conscientious*	0.272	0.010
III/V. *I am talented in mathematics / My family encourages me*	0.191	0.175
III/VI. *I am talented in mathematics / Mathematics is dynamic*	0.231	0.348
IV/V. *I am hard working and conscientious / My family encourages me*	**0.557**	0.234
IV/VI. *I am hard working and conscientious / Mathematics is dynamic*	0.289	0.091
V/VI. *My family encourages me / Mathematics is dynamic*	0.247	0.100

Although correlations for female students also show a connection between perceived ability and expectations for success, female students showed the strongest correlation between factors II (*I like mathematics*) and III (*I am talented in mathematics*). Here, it appears that within the core, the emotional component play a more significant role for females. It could be that, for females, perceived ability results in a like for the subject. For male students, the correlation between liking mathematics and perceived mathematical ability was insignificant, suggesting that the emotional component is plays less of a role in the core.

Outside the core beliefs, female students showed a significant positive correlation between factors II (*I like Mathematics*) and VI (*Math is dynamic*). This indicates that for female students, perceived usefulness may foster a like for the subject or a like for the subject may cause the student to believe that it is useful.

Differences between male and female students also occurred outside the core beliefs. Only male students showed a significant correlation between factors IV and V indicating a connection between effort and family encouragement. Here the difference between males and females is notable and suggests that male students who perceive their parents as influential, when it come to mathematics, too believe that they put effort into their studies. This might suggest that males in this study are more influenced by the support of their parents than females. Figures 5.2 and 5.3 below show how the structures of student's views of mathematics vary slightly between male and female students in the study.

In the next chapter of this thesis I describe the methodology for part 2. The purpose of part two is to explore further the utility of the MRBQ by interviewing and observing students in the context of mathematical problem solving. Here I will look at three students with different core beliefs (as determined by the questionnaire) and examine their beliefs in more detail. After looking at each of the three students

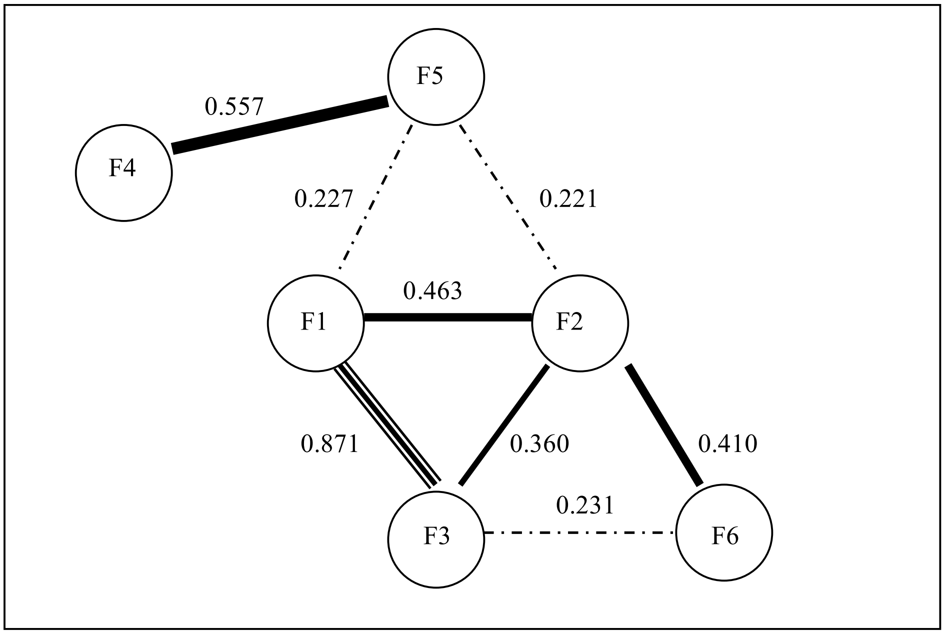


Figure 5.2 Structure of male student's view of mathematics.

The connection weights are Pearson correlations. The factors are: (F1) *I can do well in mathematics*, (F2) *I like mathematics*, (F3) *I am talented in mathematics*, (F4*) I am hard working and conscientious*, (F5) *My family encourages me*, and (F6) *Mathematics is dynamic*.

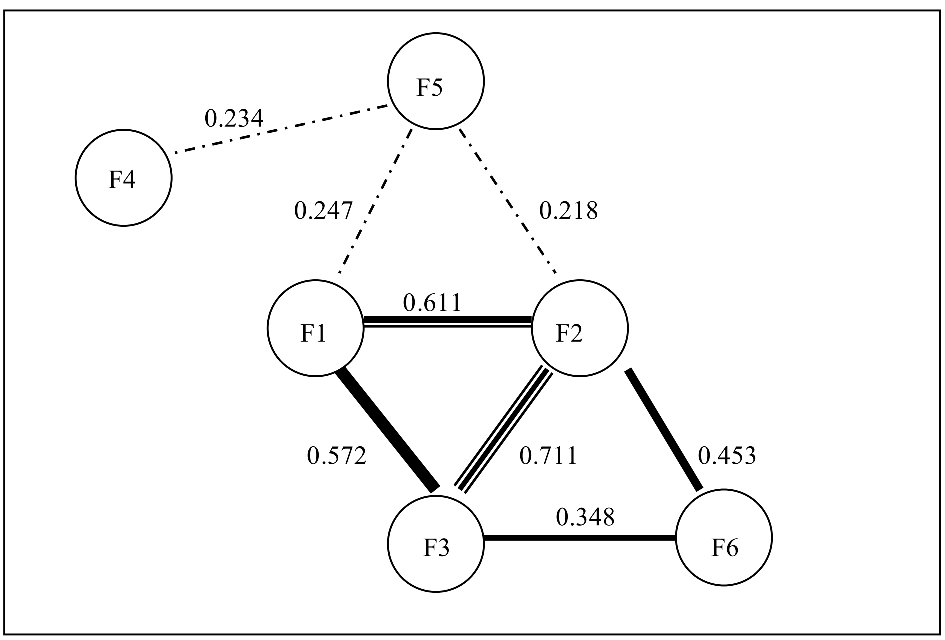


Figure 5.3 Structure of female student's view of mathematics.

The connection weights are Pearson correlations. The factors are: (F1) *I can do well in mathematics*, (F2) *I like mathematics*, (F3) *I am talented in mathematics*, (F4*) I am hard working and conscientious*, (F5) *My family encourages me*, and (F6) *Mathematics is dynamic*.

individual survey responses, I will compare those results to both student's responses in an interview and observations made by myself and the student during a video-based recall interview.

6 Methodology (2)

6.1 Student selection

After analyzing the questionnaire data, three students from the sample were chosen as case studies for part 2 of the experiment. The purpose here was to look in more detail at the beliefs of three students with different "core" beliefs (as indicated by the MRBQ). The students were chosen based on their core belief factor scores. Amanda was chosen for her positive core beliefs, Susan for her negative core beliefs, and Rory for his neutral core beliefs. Table 6.1 shows a summary of how each student selected for the second part of the study scored on each of the six factors.

Table 6.1: Student's scores on the questionnaire compared to mean group score. *Scores range 1-5.*

Factors	Amanda	Susan	Rory	Mean
I. *I can do well in mathematics*	1.4	4.1	3.1	2
II. *I like mathematics*	1.0	4.8	2.8	2.6
III. *I am talented in mathematics*	1.8	3.8	3.4	1.8
IV. *I am hard working and conscientious*	2.0	2.0	2.7	2.2
V. *My family encourages me*	3.7	2.7	3.7	2.5
VI. *Math is dynamic*	1.5	3.3	1.8	1.8

After the three students with three different sets of core beliefs were chosen, their individual results on the MRBQ were analyzed in more detail. Here, I look at how responses on individual items support (or not) the assignment of that student to a particular cluster group (such as positive core beliefs). These results will be discussed in the next chapter.

6.2 Pre-problem interview

After selecting the three students, each was individually asked to voluntarily participate in a short interview, which would include work on a mathematics problem. When students arrived for the problem solving session they were first asked a series of questions in a taped interview. Questions were designed to gather information about student's task-specific perceptions just before starting a task (Op't Eynde & Hannula, 2006). The questions related to perceptions of confidence, strategy use, and emotional state. The purpose in these questions was to gather information about the students' beliefs just before attempting a mathematical task for a comparison with the MRBQ results. Table 6.2 is a list of the questions used in the "pre-problem" interview.

Table 6.2: Questions used in the pre-problem interview.

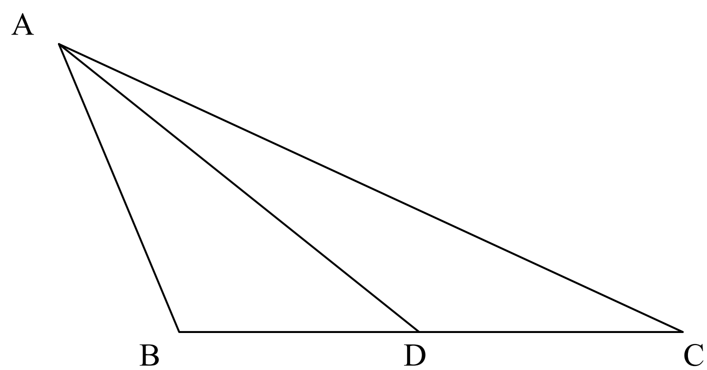
Q1.	Think back to math problems you've tried in the past. Have you been a successful problem solver?
Q2.	In a minute, I'm going to give you a math problem to do. Do you think you will be able to solve it?
Q3.	Without seeing the question, what kinds of strategies do you think you are going to use to solve it?
Q4.	Are you looking forward to doing this problem?
Q5.	In general, do you enjoy solving problems in mathematics?

Question 1 was designed to first have student's reflect on their past experiences in mathematics and then comment about their ability to solve mathematics problems in the past. Question 2 was designed to measure student's expectations for success on the problem at hand. Question 3 asks about the problem solving strategies the student might use and was included here to gather more information about self-confidence. Finally, questions 4 and 5 are related more to the emotional connection to mathematics. Together, these questions were designed to gather information with regards to each student's core beliefs when faced with a mathematics problem.

6.3 Problem solving experience

After the short interview, each student was instructed to solve a problem given on a white board. The whiteboard was located on one of the walls in the classroom and the student was to stand while solving the problem. One video camera was placed approximately six feet from the student on an angle so that both the whiteboard and the student could be seen. Students were asked to think aloud while the problem solving process was video taped. Students were encouraged to vocalize not only their solution processes but also any thoughts or feelings they might be experiencing at the time whether they were working on the problem or standing still in front of the board. Students were also told that they had as much time as they needed to work on the problem and that they should indicate when they had finished the problem or had gotten as far as they could. Also, it was made clear that I, the researcher, could not be used as a resource in solving the problem – the only questions that would be answered would be those of clarification.

The problem chosen for students was a challenging geometry problem. The question was chosen for a few reasons. First, the students were studying geometry at the time and at a first glance it would feel familiar. The purpose in this was to allow students the chance to al least get started on the problem. Second, geometry problems typically contain multiple solution paths so students would not have a step-by-step algorithm in place. The students in this study would have to look at the problem from different angles in order to determine a course of action. Finally, the question is such that students can work through a few steps to begin with, however, they quickly realize that the problem is challenging. Figure 6.1 shows the geometry problem chosen.



Problem: In triangle ABC, drawAD, where D is the midpoint of BC. If angle ACB is 30 degrees and angle ADB is 45 degrees find angle ABC.

Figure 6.1: Triangle problem

Immediately after finishing the problem, each student sat with the researcher for a "Video Based Stimulated Recall Interview" as described in Op't Eynde and Hannula's (2006) study. Here, the student was first asked some general questions with regards to how they thought they did on the problem, what their initial thoughts on the problem were, and how they thought their effort was in solving the problem. After answering these questions, the video data was played back for each student to watch. While watching the video, students were asked to recall their thoughts and feelings throughout the solution process. Each time the video was paused, the student was asked "What were you thinking at this point?" or "How were you feeling here?" Finally, after watching and commenting on the video data, the student was asked one last question about family influences. The entire video based stimulated recall interview was recorded on audio tape.

The pre-problem interview, thinking aloud video and post problem interview were all transcribed. A time coded chart was then used to compile the data so that the think aloud video data (what the student was saying while solving the problem), observations during the solution process (made by the researcher), and video based stimulated recall interview data (student comments on their thought and feelings during the solution process) could placed in chronological order and investigated all at once. Table 13 shows an example of the time coded data chart.

The first column of the data table indicates the time in minutes and seconds. The clock on the timer started in each case when the researcher gave the initial description of the problem to the student and he or she began working on the problem. In the second column, "Think Aloud Video Data," every word spoken by the student during the solution process was recorded. Each student was encouraged to "think aloud" during the entire problem so the data consists of not only a description of what they are doing on the board, but also the students thoughts about how their solution process is progressing. In some cases, students also comment on how they are feeling at a certain point during the solution process. These data were recorded in **bold** font.

Table 6.3: Time coded data chart *(sample)*

Time min:sec	**Think Aloud Video Data**	*Observed physical behaviors*	"Video stimulated recall"
2:00	**So all together it has to equal 180, so …30 and 15 and this 180 are cancelled out … um …**		
2:30	**I'm not seeing the significance of these being the same length (laughs)** **(laughs)**	*Smile* *Playing with pen, left hand clenched, deep breath. Looking over problem, right arm raised, laughs, plays with hair.*	"Well I'm trying to figure out if the sides and lengths are relevant in some way or if the side and lengths go with anything like … I was kind of confused about the whole thing."
3:00	**I'm officially completely lost (laugh)**	*Staring at problem, playing with pen.*	"I find that when I get confused or frustrated I like do this … I'll play with the hair and bite my lips and stuff like that."
3:30	**Well I know ADC, ACD, DAC, (laughs) it's definitely not a right angle.** So this one would be the same in both triangles.		

While watching the video data, I inserted observed behaviors into the time coded data chart in column 3. Physical behaviors such as hand movements (other than writing on the board), fidgeting, pacing, and deep breathing were observed and recorded on the chart in line with the time that they occurred. It was difficult to record detailed facial expressions because the video camera was positioned on an angle and at a distance that would enable recording of the student's whole body and, at the same time, what was being written on the board. Where possible, simple facial expressions were recorded such as a smile or a frown. All of the observed behavior data was recorded in *italicized* font.

The fourth column shows the video stimulated recall data. Directly after doing the problem, the student was shown the video data and was asked to comment on his or her thoughts and feelings at various points during the solution process. As the video was played back, it was paused at several points during the process. Before asking the student to comment, the time was stated so that the recall data could be properly aligned with the rest of the video data. The first question asked each time the video was paused was “How were you feeling at this point?” After the student had time to think and respond to the question, further questions were asked if clarification was needed. This portion of the interview was recorded on audio tape. Afterwards, the data was transcribed and put in the data chart in “quotation” marks.

The next chapter includes the results from part 2 of this thesis. Here, I examine each student’s individual MRBQ responses and the data gathered from the problem solving experience. I will then explore connections between the survey results and the beliefs reported by students during the interview process in each of the three cases. Starting with Amanda, I will explore connections between the survey data and the interview data as they relate to each student’s mathematics related beliefs.

7 Results (2)

7.1 Amanda

Amanda was chosen because her scores on factors I-III were the lowest in the population suggesting an extremely positive set of "core beliefs." Amanda enjoys mathematics and believes that she is a student who is capable of doing well. Table 7.1 below shows her scores on each of the factors compared to the average scores of the population.

Table 7.1: Amanda's factor scores compared to mean group score. *Range: 1-5.*

Factors	Amanda	Mean
I. *I am successful at mathematics*	1.4	2
II. *I enjoy mathematics*	1	2.6
III. *I have mathematical ability*	1.8	1.8
IV. *I am hard working in mathematics*	2	2.2
V. *My family encourages me*	3.7	2.5
VI. *Math is evolving and useful*	1.5	1.8

Table 7.2 below shows Amanda's choice for each of the items in the six factors. A score of 1.4 on factor I suggests that she believes she has the ability to do well in mathematics. Amanda expects to get good grades on her assignments and tests and believes that she will receive an excellent grade for mathematics this year. Amanda's score on this factor also suggests that she either believes that her current grade of C+ is a good grade buy her own standards, or if it is not, she has the ability to improve on it buy the end of the year. Amanda likes mathematics. Her score of 1.0 on factor II is the highest possible score for this factor and suggests that she likes doing mathematics

Table 7.2: Amanda's questionnaire results for factors I-VI.
Questions with a negative loading (shown in parenthesis) were given a reversed score when determining the overall factor score.

Item		Score 1 (strongly agree) – 5 (strongly disagree)
Factor I: *I can do well in mathematics*	**factor score**	**1.4**
39. I can get good grades in math		1
34. I expect to get good grades on assignments and tests of mathematics		2
15. I believe that I will receive this year an excellent grade for mathematics		2
26. Taking into account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics		1
22. I know I can do well in mathematics		1
33. My parents expect that I will get a good grade in mathematics		2
35. Math has been my worst subject (*negative loading*)		1 (5)
Factor II: *I like mathematics*	**factor score**	**1.0**
25. I'm very interested in mathematics		1
17. I like doing mathematics		1
41. Mathematics has been my favorite subject		1
38. Mathematics is a mechanical and boring subject (*negative loading*)		1 (5)
21. I prefer mathematics tasks for which I have to exert myself in order to find the solution		1
20. To me mathematics is an important subject		1
Factor III: *I am talented in mathematics*	**factor score**	**1.8**
5. Anyone can learn mathematics		1
42. I am sure that I can learn math		1
28. I am no good in math (*negative loading*)		1 (5)
19. I can understand the course material in mathematics		1
30. I am not the type to do well in math (*negative loading*)		5 (1)
Factor IV: *I am hard working and conscientious*	**factor score**	**2.0**
37. I have not worked very hard in mathematics (*negative loading*)		2 (4)
7. I am hard working by nature		2
40. I always prepare myself carefully for exams		2
Factor V: *My family encourages me*	**factor score**	**3.7**
12. My family has encouraged me to study mathematics		3

31. My parents enjoy helping me with mathematics problems	5
4. The importance of competence in mathematics has been emphasized at my home	2
14. The example of my parent(s) has had a positive influence on my motivation	5
Factor VI: *Mathematics is dynamic* **factor score**	**1.5**
10. There is only one way to find the correct solution of a mathematics problem	2 (4)
6. There are several ways to find the correct solution of a mathematics problem	2
9. Mathematics is continuously evolving. New things are still discovered	1
27. I think I will be able to use what I learn in mathematics also in other courses	1

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because she is interested in the subject. Amanda describes mathematics as a subject that is one of her favorites and arrives in class most days with a positive attitude towards her studies. Amanda's high score on factor II is an example of the significant role of the emotional component reported for females in chapter 5. When asked in the pre-problem interview if she has been a successful problem solver her response suggests that she believes her success is connected to the fact that she enjoys the class: "I think I'm pretty good at solving mathematical problems because I absorb the laws, and stuff like that, pretty well because I actually like this class (*laughs*), I pay attention."

A score of 1.8 on factor III suggests that Amanda believes that she has mathematical talent. She believes that she is the type of person that can learn mathematics. Amanda's choice of "strongly agree" for item 30 "I am not the type to do well in mathematics" is the only item that opposes a positive set of core beliefs. This choice could be due to an error in reading the question or it could suggest that even though Amanda seems to believe that she is capable of doing well, she believes she is not the "type" of student that typically does well in mathematics. Either way, the fact that item 30 (I am not the type to do well in math) received the lowest loading for factor three in the original factor analysis suggests that the wording of the item may need to be revised.

Amanda believes that she is hard working and conscientious. Her score of 2.0 on factor IV suggests that she believes she is hard working by nature. Amanda's work ethic presents itself consistently in the classroom. She works hard on assignments whether they are individual or group exercises and always completes the suggested review work for tests and quizzes. She describes her effort in the course as having little to do her parents support, rather her desire to be accepted into a program where mathematics 11 and 12 are prerequisites: "math is a bigger part too because the course I want to get into you have to have Math 11 and Math 12 and stuff, so I've really been trying on it."

Amanda's score of 3.7 on factor V indicates that she has slightly negative beliefs in terms of the encouragement she gets from her family with mathematics. Although Amanda somewhat agrees that the importance of mathematics has been emphasized at home, she does not believe that her parents are able to help her or motivate her by providing a positive example. When asked about the support from her parents, Amanda's response is: "Well I think that it's (math) emphasized, but more on my brother because he's really excelling at it, so that's awesome but I'm kind of just doing well in it." Amanda's response here suggests that her parents have had more of a positive influence on her brother than her. We see evidence of this in factor IV of her questionnaire responses. Three of the four items in factor IV use the word "me" or "my" emphasizing the role of the parents directed towards Amanda as an individual. On each of these items Amanda had a negative or neutral response. On item 4, "The importance of competence in mathematics has been emphasized at my home," the influence of the parents is described as more general, and in Amanda's case her positive response may reflect the emphasis on her brother rather than her.

Finally, Amanda's score of 1.5 on factor VI suggests that she views mathematics as a dynamic subject that applies in other subject areas and is continuously evolving. The low score on factor VI correlating to the low scores on the core beliefs is consistent with the overall findings in the population. Amanda's dynamic view of mathematics presents itself in her response to the question: "do you look forward to trying something that could potentially be challenging?" Amanda responds: "Yah, because usually if it's something you don't get you learn something from it." In general, we see here that Amanda's answers to the interview questions support her responses on the questionnaire. Next, we take a brief look at Amanda's problem solving process and how her beliefs may or may not manifest themselves during the process.

When faced with the question: "Without looking at the problem, do you think you will be able to solve it?" Amanda appears hopeful but a little nervous: "Well I think it would be easier if I knew what kind of problem, Like if it were a certain unit or something because I probably would have looked something over, but, um, (*laughs*) I think um yes … hopefully?" She explains also that her strategy for solving the problem will include "reading it over a couple of times" and thinking back to the "laws" that she has learned. Amanda appears to believe that her success in problem solving will depend heavily on her ability to recall the appropriate law or rule that applies. That said, she feels confident in her ability to apply the "different laws and stuff" in a non-routine problem solving situation. As she looks at the problem for the first time she says that she is looking forward to attempting to solve it.

Amanda spends a total of 4 minutes and 20 seconds on the geometry problem. During that time she describes a variety of emotions and engages in a number of problem solving strategies. She begins the problem feeling hopeful that through the effective recall of specific geometric laws she will be able to find the solution to the problem. She starts by filling in missing angles using simple geometric properties:[4]

1. **O.K., so this must be 180 degrees here, so that's 135. Um, I'm not sure …**
2. **would this angle be the same? …**
3. **no, because they're not parallel.**
4. "I'm trying to figure out if I can put the 135 on the other side and then I realize that they're not parallel. Darn! (*laughs*). So I put the 15 at the top because I know that to find the other two angles you subtract from 180, then I finished off and I was like … hmm."
5. *Amanda takes a step back, looks at the board and smiles.*
6. **O.K.?** (*laughs*)
7. *Amanda is looking over the problem.*
8. "At this point here I'm trying to figure out if I've missed something. If I have, if there's an angle then I can use some kind of rule or law like I said so I can somehow get that other angle or like I was trying to figure out … I was looking at the two triangles individually and then I was thinking maybe I should look at it as a whole … it didn't help (l*augh*s). And for I while I thought it was isosceles."

After a minute and thirty seconds, Amanda is having difficulties finding geometric laws that can be used to take her further through the problem:

[4] In the transcript the following conventions have been used (as used in Op't Eynde & Hannula, 2006): **Thinking aloud data**, from the videotaped problem solving written in **bold**. Student's comments in the "video based stimulated recall interview" are placed in "quotations." *Observation data* of the student's expressive behavior are written in *italics.*

9. **I'm not seeing the significance of these being the same length.** (*laughs*)
10. *Amanda is playing with the pen, her left hand is clenched, and she takes a deep breath.*
11. *Laughs.*
12. "I'm trying to figure out if the sides and lengths are relevant in some way of if the sides and lengths go with anything like … I was kind of confused about the whole thing."
13. **I'm officially completely lost.** (*laughs)*
14. *Amanda laughs again and plays with her hair.*

At two minutes Amanda attempts a new strategy while continuing to exhibit outward signs that she is confused:

15. **Well I know ADC, ACD, DAC** *(laughs)* ... **it's definitely not a right triangle**.
16. *Amanda is staring at the problem and playing with her hair.*
17. **So this one would be the same in both triangles.**
18. "I'm just like … I have to have missed something …you have to be able to solve it right?"
19. **Ok, I'm going to give them fake numbers. This one here can be a 1 and this can be a three a guess.** *(laughs)* **that would just find the sides in the end though.**
20. "I was feeling like I might still be able to do something because I haven't even said that the center is the same yet because I even tried to put in numbers for sides at one point then I was like – wait, what the hell, how is that going to help?"

After three minutes, Amanda's appears frustrated with the problem but attempts one last strategy:

21. *Amanda steps back, takes a deep breath, and plays with the pen. She takes another step back, tilts her head and touches the pen to her mouth.*

22. **It looks like an isosceles … I'm going to assume.** *(laughs)*

23. *Amanda adds an angle of 120 degrees to the diagram.*

24. *Teacher explains that she cannot make that assumption.*

25. **I know, assuming makes an ass of you and me.** *(laughs).* **I'm completely stumped right now.**

26. *Amanda is laughing, she touches the back of her neck and her hands fall to her sides.*

27. "Well I was just kind of like "Oh Man," (*laughs*) I wish I could have like gotten it, but I ran out of ideas. At least when I was standing there I was still having things run through my head like I was thinking the sine law and all that stuff and like thinking about the geometry supplementary and complementary things and stuff like that and then I was like "OK, I'm done."

When faced with a challenging non-routine mathematics problem, Amanda is capable of applying more than one strategy in an attempt to solve the problem. After one minute and thirty seconds, Amanda says that she is "completely lost" but continues to apply at least two more strategies to the problem. Amanda describes her own feelings of confusion leading to frustration throughout the process until the time reaches four minutes and twenty seconds and she decides that she has done all that she can do. In the end Amanda describes the frustration as too much to handle: "I got frustrated, I kind of just was like – ok, I don't know what to do and I just had to like even stop thinking about different ways to solve it – I just kept thinking about how frustrated I was."

7.2 Susan

In contrast to Amanda, Susan's high scores on factors I-III suggest that she has a negative set of "core beliefs." In Table 7.3 we see how Susan's factor scores compare to those of the population. Here we see a significant difference between Susan's scores on each of the three core factors and factor VI.

Table 7.3: Susan's factor scores compared to mean group score. *Range: 1-5.*

Factors	Susan	Mean
I. *I am successful at mathematics*	4.1	2
II. *I enjoy mathematics*	4.8	2.6
III. *I have mathematical ability*	3.8	1.8
IV. *I am hard working in mathematics*	2	2.2
V. *My family encourages me*	2.7	2.5
VI. *Math is evolving and useful*	3.3	1.8

Table 7.4 below shows Susan's choice for each of the items in the six factors. An average score of 4.1 on factor I suggests that Susan believes that she is not able to do well in mathematics. Even though Susan's current grade level is the same as Amanda's, Susan expects to get a poor grade in mathematics and believes that mathematics has been her worst subject. She believes that in general she has been unsuccessful in mathematics, particularly when it comes to problem solving. In the pre-problem interview Susan explains: "If it's a problem solving question I find it really really difficult for me, but if it's a formula where you just plug in the numbers I find it really really easy. I understand more of the 'formula' math way more than the 'thinking' stuff." Susan's desire for

Table 7.4: Susan's questionnaire results for factors I-VI.
Questions with a negative loading (shown in parenthesis) were given a reversed score when determining the overall factor score.

Item		Score 1 (strongly agree) – 5 (strongly disagree)
Factor I: *I can do well in mathematics*	**factor score**	**4.1**
39. I can get good grades in math		5
34. I expect to get good grades on assignments and tests of mathematics		5
15. I believe that I will receive this year an excellent grade for mathematics		5
26. Taking into account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics		5
22. I know I can do well in mathematics		2
33. My parents expect that I will get a good grade in mathematics		2
35. Math has been my worst subject (*negative loading*)		5 (1)
Factor II: *I like mathematics*	**factor score**	**4.8**
25. I'm very interested in mathematics		5
17. I like doing mathematics		5
41. Mathematics has been my favorite subject		5
38. Mathematics is a mechanical and boring subject (*negative loading*)		5 (1)
21. I prefer mathematics tasks for which I have to exert myself in order to find the solution		5
20. To me mathematics is an important subject		4
Factor III: *I am talented in mathematics*	**factor score**	**3.8**
5. Anyone can learn mathematics		1
42. I am sure that I can learn math		4
28. I am no good in math		5 (1)
19. I can understand the course material in mathematics		4
30. I am not the type to do well in math		5 (1)
Factor IV: *I am hard working and conscientious*	**factor score**	**2.0**
37. I have not worked very hard in mathematics (*negative loading*)		2 (4)
7. I am hard working by nature		2
40. I always prepare myself carefully for exams		2
Factor V: *My family encourages me*	**factor score**	**2.7**

12. My family has encouraged me to study mathematics	2
31. My parents enjoy helping me with mathematics problems	3
4. The importance of competence in mathematics has been emphasized at my home	2
14. The example of my parent(s) has had a positive influence on my motivation	4
Factor VI: *Mathematics is dynamic* **factor score**	**3.3**
10. There is only one way to find the correct solution of a mathematics problem	4 (2)
6. There are several ways to find the correct solution of a mathematics problem	4
9. Mathematics is continuously evolving. New things are still discovered	3
27. I think I will be able to use what I learn in mathematics also in other courses	2

mathematical problem solving process that involve ‘formulas’ rather than open ended ‘thinking stuff’ may contribute to (or be a result of) her non-dynamic view of mathemaics. Susan’s answer “somewhat agree” to item 22 “I know I can do well in mathematics” appears to contradict her responses to the rest of the items in factor one. This response could be due to error or it might suggest that even though she believes mathematics has been her worst subject, there is always a possibility for someone to do well and improve.

Susan does not like mathematics. Her score of 4.8 on factor II indicates that she finds mathematics a mechanical and boring subject. During class, she is usually on task but frequently expresses her dislike for what she is doing. We see here that Susan’s negative feelings towards mathematics are strong as indicated by a 5 on every item except one.

Susan’s score on factor III of the core beliefs was 3.8. This score, although not as high as the first two scores, also suggests a negative belief. Here, we see that Susan believes she lacks mathematical talent. Susan indicates strong negative beliefs for each of the items in this factor with the exception of item 5 “anyone can learn mathematics” where she strongly agreed. This supports the idea that although Susan has strong negative beliefs about herself with regards to mathematics, she believes that mathematics is a subject that anyone can learn.

On factors IV and V, Susan's scores are similar to those of Amanda's. Susan's score of 2.0 on factor IV suggests that she too believes that she is a hard worker. In class, Susan puts effort into most of the assigned tasks even though she dislikes the subject. She participates in all of the group activities and takes extra time at home to prepare for exams. In terms of family support, Susan believes that some emphasis is put on mathematics at home but her parents have not provided an example that is motivating. Susan believes that she works hard for herself in mathematics and although her paretns see math as important, they are unable to provide support: "My dad, he really pushes the whole math situation … he gets really frustrated and he starts to yell at me when I can't get an equation so I don't really ask my dad for help. My mom … she's fine … she's like the sane one – she doesn't help cause she doesn't remember from highschool. So no parents – I work for myself."

Finally, Susan's score of 3.3 on factor VI suggests that she has slightly negative beliefs with regards to mathematics as a dynamic subject. Susan's beliefs in this factor are not strong either way. She somewhat agrees that she will be able to use mathematics in other subject areas and somewhat disagrees that there several ways to solve mathematics problems. Although her score on this factor is only slightly negative, it should be noted that her score is the highest for the population that ranged from 1 to 3.3. The fact that the population appears on a whole to believe that mathematics is dynamic may be a reflection of the teacher's strong belief that mathematics is a dynamic subject.

The idea of multiple solution processes (for example) has been emphasized by the teacher throughout the course. It might be Susan's set of negative core beliefs that pushes her average score for this factor the closest to the negative end of the spectrum in the population.

In the pre-problem interview, Susan appears nervous and unsure as to whether or not she will be able to solve the problem: "I think I can do it? (*laughs*) I'll try. I'll try. But if I don't succeed that's fine." Here, her uncertainty and nervousness is evidence of the lack of self confidence she describes. Also, it appears that Susan's expectations for success may already be negative as she adds she will be 'fine' if she does not succeed. When asked about the strategies she might use to solve the problem, Susan has trouble thinking of general stragegies for problem solving other than attempting to "see if there is a formula." Without a formula or algorithm she explains that she "doesn't really have any strategies." Again, Susan's rigid view of mathematics present itself. Also, it seems as though her view serves to impede her expectations for success on the problem at hand. Before even looking at the problem, she belives that without a step-by-step formula, she will have no strategy.

Susan spends a total of 12 minutes 44 seconds on the problem. She begins the problem feeing hopeful but after less than a minute she describes herself as feeling nervous:

1. **O.K. So I'm trying to find angle X. Is there a time limit?**
2. *Sarah is staring at the board. She takes a deep breath, sighs, and then flicks her pen.*
3. **Um … I'm trying to see if there's any kind of triangle properties that I learned in class** *(laughs)* **try to figure out this triangle … um …**

4. “Here I was nervous … I was like Susan, you can do this! I was like … oh my gosh , I don’t get this question … why can’t I figure out what an angle is … any angle! Just the nerves – it took me a really long time to figure out the 180 one.”

After two minuts of work Susan discovers that she can use supplementary angles to determine one of the other angles in the diagram:

5. **O.K., 180 degrees. 180-45 is 135. That’s correct right? So this angle is 135.**
6. *Susan points to the angle on the board. Susan sighs.*
7. “Once I got that angle I was like YES, I can figure out the top angle … I was really happy!

Soon after feeling happy about discovering some of the missing angles in the diagram, Susan realizes that she can get no further and she describes her emotions as changing from happiness to confusion and then frustration:

8. **this angle is 15, and then … O.K. …**
9. *Susan is mumbling, and dancing as she takes a step back to look at the board.*
10. **O.K., I was thinking that 135 could be the same as this but I’m thinking not because this is 30 and this is 45 so …**
11. *Susan sways back and forth*
12. **But that could be debatable**
13. *Susan sighs, scratches her neck, and points to the problem.*
14. *Susan swaysback and forth, sighs again, and closes her eyes.*
15. **I want to say that’s 15 but probably not because it’s different angles … um …**
16. “I was actually feeling really frustrated that I couldn’t get an angle or find an equation to figure it out … and I couldn’t remember what to do … It’s like an exam when you just blank

out. I was looking at the board just staring … just hoping that a number would just pop up in the corner and I'd be like – that's X!"

After five minutes of work, Susan makes an uncertain hypothesis:

17. **If this is … lets say** *(sigh)* **… lets say this side is equal to this whole side then this whole angle would be 30 and that would be 15, and then IF this was 15 then 180 minus 15 minus 45 would give us angle X … but since there is no line here … then I don't know … um.**
18. *Susan is dancing*
19. **Now that I'm into the problem I want to solve it now**
20. "I'm looking at the problem and I feel like it's so close that I just want to keep on trying."
21. **It seems so simple** *(laughs)* **It's probably the pressure.**
22. *Susan flicks her pen, turns her head up-side-down, laughs and lets outa big sigh.*
23. **I don't know Mr. Physick**
24. "I was getting frustrated … um … It's like … I can't … I'm tired …I can't figure it out."

Finally, after several minutes of frustration, Susan attempts one last strategy. In the the post problem interview she explains that even though she is frustrated she is determined to solve the problem:

25. **180 minus 2X minus 25? Solve for X? Oh wait, but it's equal, or can be 2X plus 45 equals 180. I don't know … we're going to do both.**
26. *Sarah is working on the board*
27. "I feel that when I'm working on a problem my determination builds up as I work on it"
28. **O.K., so if 2X plus 45 equals 180. Then to solve for X … so 2X equals 180 minus 45 … so X equals 135 divided by 2 … Oh Crap!**

Susan explains that in a regular classroom situation she would be determined but would not spend the same amount of time working on a problem in a frustrated state. Although Susan explains that she had finished her "last ditch effort" 10 minutes in to the problem, she goes on to say that she "probably would have stood there all day" until she "figured it out." In the post-problem interview Susan explains that the research setting impacted her motivation and effort. When asked if she would exert the same amount of effort in a classroom situation her response was: "Well, maybe not as much effort because this is like a study, so like it determined me to finish the question, but in class I would probably try and figure out the question until like 'ok class now I will give you the answer' kind of thing."

7.3 Rory

Rory's score on factors I-III suggest a "neutral" set of core beliefs – neither positive nor negative. In table 7.5, Rory's factor scores (compared to those of the population) are shown. Rory's scores on each factor lie close to three which is a neutral position.

Table 7.5: Rory's factor scores compared to mean group score. *Range: 1-5.*

Factors	Rory	Mean
I. *I am successful at mathematics*	3.1	2
II. *I enjoy mathematics*	2.8	2.6
III. *I have mathematical ability*	3.4	1.8
IV. *I am hard working in mathematics*	2.7	2.2
V. *My family encourages me*	3.7	2.5
VI. *Math is evolving and useful*	1.8	1.8

Also, the population averages are not neutral, rather slightly positive in nature. Next, table 7.6 shows Rory's choice for each of the items in the six factors. For factor I, Rory chose "neither agree or disagree' for every item except 15 "I believe that I will receive this year an excellent grade for mathematics," where he chose "somewhat disagree." It might be the use of the word "excellent" when referring to grades (as opposed to good) that elicited this non-neutral response.

Table 7.6: Rory's questionnaire results for factors I-VI.
Questions with a negative loading (shown in parenthesis) were given a reversed score when determining the overall factor score.

Item		Score 1 (strongly agree) – 5 (strongly disagree)
Factor I: *I can do well in mathematics*	**factor score**	**3.1**
39. I can get good grades in math		3
34. I expect to get good grades on assignments and tests of mathematics		3
15. I believe that I will receive this year an excellent grade for mathematics		4
26. Taking into account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics		3
22. I know I can do well in mathematics		3
33. My parents expect that I will get a good grade in mathematics		3
35. Math has been my worst subject (*negative loading*)		3 (3)
Factor II: *I like mathematics*	**factor score**	**2.8**
25. I'm very interested in mathematics		2
17. I like doing mathematics		3
41. Mathematics has been my favorite subject		3
38. Mathematics is a mechanical and boring subject (*negative loading*)		3 (3)
21. I prefer mathematics tasks for which I have to exert myself in order to find the solution		5
20. To me mathematics is an important subject		1
Factor III: *I am talented in mathematics*	**factor score**	**3.4**
5. Anyone can learn mathematics		4

42. I am sure that I can learn math			4
28. I am no good in math			3 (3)
19. I can understand the course material in mathematics			3
30. I am not the type to do well in math			3 (3)
Factor IV: *I am hard working and conscientious*		**factor score**	**2.7**
37. I have not worked very hard in mathematics (*negative loading*)			1 (5)
7. I am hard working by nature			4
40. I always prepare myself carefully for exams			3
Factor V: *My family encourages me*		**factor score**	**3.7**
12. My family has encouraged me to study mathematics			3
31. My parents enjoy helping me with mathematics problems			4
4. The importance of competence in mathematics has been emphasized at my home			5
14. The example of my parent(s) has had a positive influence on my motivation			3
Factor VI: *Mathematics is dynamic*		**factor score**	**1.8**
10. There is only one way to find the correct solution of a mathematics problem			1 (5)
6. There are several ways to find the correct solution of a mathematics problem			2
9. Mathematics is continuously evolving. New things are still discovered			1
27. I think I will be able to use what I learn in mathematics also in other courses			3

For factor II, Rory had neutral responses for three out of the six items. He neither likes nor dislikes mathematics but appears to strongly believe that mathematics is an important subject, as indicated by his response to item 20. For Rory, the belief in the importance of mathematics does not appear to have a positive impact on his core beliefs. It should be noted here that item 20 had the lowest loading on factor II, suggesting that the idea of 'importance of the subject' may not correlate with the core in other students besides Rory. Rory also indicates that he does not enjoy mathematical tasks where he has to exert himself. Here, although the average for the factor is neutral, Rory's belief is strong when it come to his dislike of challenging math problems.

For factor III, Rory's average score is 3.4 again indicating a neutral set of beliefs. On two of the items in this factor Rory "somewhat disagreed." Here, his responses suggest that he is leaning towards the belief that not everyone can learn mathematics and that he might be one of those people.

On factor IV Rory's somewhat positive score of 2.7 suggests that he is believes that he might be a hard worker. Although the average score is positive, a further look into the individual item responses reveals mixed beliefs in terms of Rory's beliefs about his work ethic. It is on items 37 "I have not worked very hard in mathematics" and 7 "I am hardworking by nature" that Rory reveals the belief that even though he might not consider himself hard working by nature, he knows he has worked hard in mathematics this year. It might be Rory's strong belief in the importance in mathematics as a subject that drives him to put more effort into this course that his others.

Rory's average score of 3.7 on factor V is somewhat negative. Rory's responses on two of the items are neutral and the other two are negative in nature. The strongest negative response for Rory in this factor was to Item 4. Rory believes that the importance of competence in mathematics has not been emphasized at home. In the interview, Rory explains that "My parents aren't huge on school and stuff like that … they're not that crazy about mathematics and the role of it." This result is interesting because Rory himself believes that mathematics is an important subject. Here we see a disconnect between parental beliefs and student beliefs that is contrary to literature that emphasizes the role of the parent. It could be that Rory's beliefs have been influenced by another positive role model or that he has developed this belief on his own accord.

Finally, Rory's score of 1.8 on factor VI suggests that, like Amanda and much of the population, he has a dynamic view of mathematics. Rory has a strong belief that there are multiple solution paths when solving mathematics problems. He also believes that mathematics is a subject that is evolving and useful. Here, Rory's dynamic view of mathematics does not correlate to a positive set of core beliefs. This example may be an indication of why the correlation between these two factors for the entire population, although present, is weak.

When faced with the problem, Rory's responses to the interview questions suggest that his core beliefs may not be neutral as indicated in the analysis of his MRBQ. Rory believes that he has been somewhat unsuccessful at mathematics in the past. When asked if he believes he has been a successful problem solver his response was: "I have the knowledge to do it, but it's more the process and chronology of how to do things that I get confused on – and that's the biggest problem." When asked in the pre-problem interview if he thinks he will be able to solve the upcoming question there is evidence that Rory is lacking confidence in mathematics (at least at this moment). His answer to the question is: "probably not." He does not believe he is going to be successful on the problem and without seeing the question has trouble commenting on general strategies he might use. When asked if he enjoys problem solving in general he response is: "I'd say generally I'm not that fond of it but it depends on the case."

Rory spends a total of 11 minutes on the problem. At first glance of the problem he explains that he is not a fan of geometry because there are "so many different perspectives on how you could see the problem" but he feels hopeful because it was at least something familiar. In beginning the problem, he appears hopeful that he will find a way to solve it and sets out a plan of action:

1. **Well looking at this I think it's safe to say that this line here would be 180 degrees so I'm going to do 180 minus 45 which would be 135.**
2. **Um … yah … 135. So my idea here is to find as much information as I can just to solve whatever angles I have and hopefully, eventually that will lead me to solving the whole question. Um … O.K.**
3. *Rory touches his face and takes a deep breath.*
4. **"**at this point I thought what I was doing would eventually lead me to solving angle ABC"

After making a false assumption one minute into the problem, Rory is feeling "less confident" but quickly discovers that he can use a geometric property to find some missing angles in the diagram:

5. **I'm just trying to think what I can do with three angles.**
6. *Rory is making a clicking noise from his mouth, then he takes a deep breath and crosses hi arms.*
7. **Well I can find the angle for this** *(smile)* **because I have these two angles here. So that would be 165 up here so 75 and 50. That should be 15 degrees.**
8. "I felt I just missed something pretty obvious … I was a little more gratified that I found that."

After success, Rory becomes confused and then frustrated that he can't get further in the problem:

9. **I'm thinking that if I draw something else in this triangle there's a way I could get the other angles … but I'm not really sure what to do for that because …**
10. *Rory folds his arms and tilts his head*
11. **O.K.?**
12. *Rory takes a deep breath*

Immediately after taking a deep breath, Rory begins a new strategy – introducing variables into the diagram. He works with this strategy for two minutes and then realizes that he is confused and becomes frustrated when he can't determine what to do next:

13. **This um … hmm … I don't even know if I've done the right approach with writing this out … um …**
14. *Rory is playing with his pen and is touching his face.*
15. **Lets see … um**
16. *Rory takes a deep breath*
17. **I've run into a little bit of a stall here** *(raises eyebrows, little smile)* **because I don't think that this is really the way to go about it, but looking at the problem …** *(sigh)* **… Um … to find X and Y …**
18. *Big sigh and arms crossed. Hand touches chin.*
19. "here I'm feeling frustrated more than anything because NOTHING seems to be working at that point."

At eight minutes into the problem, Rory continues in trying two more strategies while in a state of frustration until he decides that he has done all he can do. Like Susan, Rory explains that his effort on this problem is not typical of the effort he put into the same sort of problem in a regular classroom situation. He explains that the research setting added pressure that induced effort but caused the problem solving process to be more difficult because of the stress associated with the pressure.

7.4 Beliefs in the context of problem solving

After looking at student responses during the problem solving experience it appears that many of the beliefs expressed on the MRBQ are present. Looking first at the core beliefs, the responses of Amanda (positive core beliefs) and Susan (negative core beliefs) can be compared. Here, although both students describe feelings of nervousness due to the research setting, Amanda approaches the problem with the confidence that might be expected from a student with positive core beliefs. She believes that even though she is feeling nervous, she can do it because she is "pretty good at solving math problems." In addition to expressing her belief in her own ability, Amanda adds that she "likes mathematics" which corresponds to her responses on factor II of the core factors.

In contrast, Susan approaches the problem with little confidence and appears to believe that she does not expect to succeed on the problem. Her negative core beliefs present themselves when she explains that she will probably not be able to succeed because she finds problem solving "really difficult." Here, Susan adds that she prefers (and has had some success with) mathematics questions where a formula can be followed. This response suggests that when looking at mathematics-related beliefs, core beliefs could change depending on mathematical content or problem type.

Although Amanda and Susan's responses during the problem solving experience appear the support their core beliefs determined by the MRBQ, there is evidence in

Rory's problem solving experience that his core beliefs may be more negative, rather than neutral (as indicated on the MRBQ). Before even looking at the problem Rory explains that he will "probably not" be able to solve it. This suggests that (at least for this problem) Rory has negative expectations for success that would characterize "negative core" beliefs. It could be that Rory is anticipating a "challenging" problem, which he has expressed a dislike for both on the MRBQ and in the problem solving interview. This negative response may also suggest that because the 5-point Likert scale allowed for neutral responses, Rory's neutral MRBQ result may not be true to his actual core beliefs. Had the scale been 6 point and not allowed for a neutral response, Rory's MRBQ responses might have indicated the negative beliefs we see in the problem solving situation.

When looking at beliefs outside of the "core" we see again that many of the beliefs indicated on the student's MRBQ parallel their responses during the problem solving experience. When looking at Factor 4 (*I am hard working and conscientious*) we see that all three students indicated on the MRBQ that they believed they were hard working individuals. This response was echoed during the problem solving experience. In addition to explaining that they generally work hard in mathematics, each student also added that there effort during the problem solving interview exceeded the effort they would normally put into their every day class work. Again, it should be noted that when looking at the beliefs of the entire population we saw that generally, most of the students reported that they were working hard in mathematics (regardless of their positive, negative or neutral outlook on mathematics). This general result combined with results described above suggest that although the MRBQ results appear to be supported by responses during the problem solving process for this factor, the fact that researcher is also the teacher may have skewed the results. It could be that the pressure of the research setting and the desire of students to make a positive impression on their teacher influenced their responses on this factor.

Student responses on the MRBQ with regards to family influences (factor V) appear to be supported by their responses in the context of the problem solving interview. Amanda's MRBQ results suggest that believes her parents (although seeing the importance in the subject) are unable to provide her with support. During the interview process more insight is gained into why this might be the case when she explains that the importance of mathematics is emphasized more for her brother than herself. In Susan's case, her negative score on factor V of the MRBQ is a true representation of the fact that she believes both her Mom and her Dad are incapable of helping her and that she does mathematics "for herself." Like Amanda and Susan, Rory's negative beliefs expressed on the MRBQ with regards to family support are also confirmed in the interview when he says that his parents are not crazy about mathematics and the role of it.

Finally, there is evidence in the problem solving interview for all three students that confirm the beliefs expressed on factor VI of the MRBQ. Factor VI in the MRBQ relates to students beliefs about multiple solution processes and the utility of the subject. In the case of Amanda, her positive score on factor VI is supported in the interview when she explains that when a problem is difficult she knows that she can learn something from trying. This suggests that Amanda realizes that even if she sets out to attempt a problem one way and fails, she will learn something by considering a second solution process. Susan's MRBQ results on factor VI are also present in the problem solving interview. In contrast to Amanda, Susan has a negative score on factor VI and explains in the problem solving interview that without a formula to follow she has no strategy for solving the given problem. Finally, Rory's strong belief that there are multiple solution processes in mathematics is too present in the problem solving interview when he explains that although he does not enjoy geometry problems, he recognizes that the reason he does not enjoy them is because there are so many different ways to look at those types of problems.

To summarize, it appears that although the interview data points to some limitations of the MRBQ, generally, the beliefs expressed on the questionnaire hold true in the context of a problem solving situation. In the next chapter I will discuss the conclusions that I have drawn from both parts of this thesis. Limitations of both parts will be addressed and suggestions for future research in the area of measuring student's mathematics–related belief systems will be discussed.

8 Conclusion

The primary purpose of this thesis was to explore student's mathematics-related beliefs systems and the use of a "mathematics-related belief questionnaire" (MRBQ) to measure such systems. It is my belief that if the beliefs of students could be measured with such a device, then mathematics teachers could use this knowledge to create a classroom environment that fosters learning and mathematical problem solving.

The thesis here was split into two separate sections. The first section included the factor analysis on of the MRBQ where a number of dimensions to student's mathematics-related beliefs systems were analyzed and discussed. Part 2 served as both an in-depth look at the intricacies of the items within the factors for three students with different core beliefs and an investigation as to whether or not student beliefs expressed on the MRBQ would hold true in the context of a problem solving experience.

What can the factor analysis of a mathematics-related belief questionnaire (MRBQ) tell us about the structure of mathematics-related belief systems? The results of part one of this thesis clearly support the existence of the subcategories of mathematics-related beliefs suggested in Op't Eynde & De Corte's (2004) framework. In the exploratory factor analysis described above, we see six factors, five of which fit into one of Op't Eynde & De Corte's three subcategories:

(1) Beliefs about mathematics.

Factor VI: Mathematics is dynamic.

(2) Beliefs about self.

Factor I: I can do well in mathematics.

Factor III: I am talented in mathematics.

Factor IV: I am hard working and conscientious.

(3) Beliefs about context.

Factor VI: My family encourages me.

In addition to these five factors that fit into the framework, a sixth factor was determined (*I like mathematics*) that suggested, in part, an emotional component. The factors determined here are similar to the factors determined and Hannula et al.'s (2005) study and similar factor titles were used. Hannula et al. also identify the factor "I like mathematics" as an emotional component and add that this factor is evidence for the distinction between cognition and emotion. It should be noted again that the interaction between cognition and emotion is complex. For the purposes of this, emotion is considered to be present within factor II (I like mathematics) but no hypothesis is made as to the relationship between emotion and cognition for students in the population.

In comparing the factors determined in this study to the factors determined in the two studies from which the items were borrowed, it appears that items related to self-beliefs group in a similar way. Items related to beliefs about mathematics (the object) grouped differently in each of the three studies suggesting that measuring and analyzing student's beliefs about mathematics (the object) could be more challenging than measuring mathematics-related self-beliefs such as self-confidence. The challenge in measuring beliefs about the object mathematics may lie in the fact that there is such a wide variety of ways to interpret it. Mathematics the object may be seen as the subject we learn in school, some abstract concept, or something that mathematicians do.

Analysis of the average scores on each of the six factors for the population revealed that, even though student's individual scores ranged from extreme positive to extreme negative, in general, the students in these three mathematics classes have somewhat positive views towards mathematics. This finding suggests that in order to get a more complete picture student's mathematics-related belief systems (particularly those with negative beliefs) future research should involve a much larger and diverse population.

After analyzing the correlations among the factors, it appears that strong correlations exist between three of the factors: (I) *I can do well in mathematics,* (II) *I like mathematics,* and (III) *I am talented in mathematics.* As in Hannula et al's (2005) study, these closely related elements are labeled the "core." Student's with a positive set of core beliefs believe that he or she is talented in mathematics, expects to do well, and likes the subject. Those students with negative core beliefs believe that they are not able to do well, expect that this will be the case and express a dislike for the subject. These strong connections at the "core" of a student's mathematics-related belief system are supported by a variety of research (Schoenfeld, 1989; Hannula, 2005; Lazarus, 1991, Mandler, 1989).

The three factors determined in this thesis that were not included as part of the core were found to have a close connection to the core (with the exception of factor IV: *I am hard working and conscientious*). The strongest of these was the correlation between the core and factor VI (*Math is dynamic*) supporting Op't Eynde and De Corte's (2003, 2005) findings that suggest student's with a dynamic view of mathematics consider the subject more valuable and have confidence in their ability in the subject.

After looking at correlations between the factors, each factor's relation to achievement was considered and strong connections between perceived ability, expectancies for success and achievement were noted. Essentially, it is the "core" factors that have the strongest correlation to achievement. This connection between self-confidence and achievement in mathematics has been reported on numerous occasions in mathematics education research. Here, this widely accepted theory presents itself in the exploratory factor analysis of a mathematics-related belief questionnaire. The results here suggest that positive core beliefs relating to self-confidence are essential for all students regardless of their goal orientation. It should be noted that measuring correlations between beliefs and achievement may be problematic because student's perceptions of "success" may differ from the teacher. This may be apparent here as two students in part 2 of the study at opposite ends of the spectrum in terms of beliefs had the same achievement score (letter grade) in the course. In order to gather valuable information with regards to correlations to student achievement it may be necessary to measure student achievement using tools other than letter grade and including the student's perception of their current success in the course at the time of the survey.

In the further analysis of student views it appears clear that this particular sample of students has positive mathematics-related beliefs. In fact, 81% of the population has positive core beliefs. 58% of the population falling in the "encouraged" subcategory, points to the high parent involvement in this particular school. Of the few students with negative core beliefs, all of them believe that they put effort into their studies, so none of them fell into the "lazy" subcategory described by Hannula et al (2005). The absence of students falling into the "lazy" category could be a result of student knowing the researcher is also their teacher. In a school with high parent involvement, it could also be evidence of the correlation between parent support and perceived effort found in the factor analysis. Regardless, the analysis produces valuable information about the trends in the sample.

What can the analysis of an MRBQ tell us about gender differences in mathematics-related beliefs? When looking at the gender differences in the sample there is evidence of the higher self-confidence in male students than female students reported by many mathematics education researchers. This difference is apparent when comparing the average scores on the "core" factors. There is also evidence here of male students finding the subject of mathematics more dynamic than females. This finding too echoes previous mathematics education research and supports the evidence shown here of the strong correlation between self-confidence and a dynamic view of mathematics. Within the "core" beliefs determined here, female students show a much stronger connection between factors II (*I like mathematics*) and III (*I am talented in mathematics*) than male students indicating that the emotional component plays a more significant role for females. On the other hand, male students were the only ones to show a correlation between perceived effort and family support, suggesting that parents may have more of an influence on male students than female.

After considering the role of parents in student's mathematics related belief systems we are reminded that the impact of another important role model was not included in this survey – namely the teacher. Clearly, based on the research described in chapter 3, student's beliefs with regards to the role and functioning of their teacher plays an important part of their beliefs about the context in which they learn mathematics. These types of questions were not used in this study because the researcher was also the teacher. However, in future research, questions regarding student's beliefs about their teacher should be included. In fact, after analysis of the questionnaire data and further consideration of mathematics-related belief categories, I am left with the realization that the MRBQ could evolve by adding items in a number of areas such as beliefs about goal orientation, and the sociological context of the classroom.

The purpose of part 2 of this thesis was to address the third research question: "Are student responses to the MRBQ consistent with beliefs expressed in the context of problem solving." First, the individual item responses of three students from the sample were analyzed for their support of average factor scores. Second, the beliefs of students in the context of a problem solving experience and how these beliefs may or may not be in line with the beliefs expressed on the MRBQ were considered. Three students with different core beliefs were chosen (as indicated by the MRBQ results) to participate in a short case study. The results from part two provide some insight into both the utility and limitations of the MRBQ. As far as making comparisons between three students with different core beliefs in terms of such constructs as ability to solve non-routine problems or affective responses during the problems solving process, it appears as though this methodology is not sufficient for connecting behavior to beliefs. I can only make hypotheses that would require a more rigorous approach to arrive at definitive conclusions.

In general, student responses during the pre-problem interview, the problem solving process and the post-problem interview support the results from the MRBQ for all three students. This suggests that the MRBQ may be an accurate measure of student's beliefs in different environments. In particular, it can be seen that the core beliefs identified in the MRBQ are clearly present in the interview answers and the think aloud data during the problem solving process. With both Amanda and Susan, the emotional connection to the subject is seen in their like and dislike expressed for the subject respectively supporting the general findings for the female population in the sample. The importance of the emotional connection is quite clear in Amanda's response when she explains that her attention to the subject is due to her like for the class.

In Amanda and Susan's responses there is evidence of the correlation between positive core beliefs and a dynamic view of mathematics that was also discovered in the analysis of the questionnaire results. When describing the strategies she is going to use to solve the problem, Amanda explains that she is going to think back through the properties she has learned and then lists a number of different possibilities. In contrast, Susan explains that she is more comfortable with problems where she can follow a "formula" and is uncomfortable with problems that involve "the thinking stuff." Susan's only slightly negative score on factor VI (math is dynamic) suggests that although Susan prefers problems with step-by-step procedures, she may recognize that these same problems can be solved by other means. With multiple solution processes as an emphasis in these student's classes by the teacher, these results could point to the impact of teacher beliefs on student's beliefs.

Even though Rory's questionnaire results indicate that he has a "neutral" set of core beliefs, it appears that in the context of a problem solving situation, Rory lacks self confidence and may be better described as having slightly negative core beliefs. This result suggests that the use of a 6-point Likert scale may better serve the purposes of such a study. Had Rory not been able to give a neutral response, his factor scores on the core factors may have produced a slightly negative average that would be more in line with the beliefs expressed in the face of an actual problem solving experience.

After analyzing the individual item responses of each student, it is clear that a few item responses for each student that are not in line with the overall factor average score that was determined for each student. For example, Rory, who had a neutral score on factor 2 (I like mathematics), believes strongly in the importance of mathematics. This results points to one of the limitations of determining average factor scores from the MRBQ. Although analysis of the correlations between factors depends on assigning student scores on each factor, valuable information with regards to the intricacies of individual student's beliefs may be lost in such a process.

In looking at these individual survey responses it appears that when there is a discrepancy within a factor it is likely with the item that had the lowest loading on that factor. This result raises questions about the structure of such items. Should these items be revised to better reflect the theme of the factor, or do the discrepancies point to the need for further factor analysis? Either way, the results point to a clear picture of the complexity of measuring beliefs and belief systems.

Very little can be said here about how performance on the problem or affective states and responses to those states may or may not be connected to student belief systems. Each student reported in some way that the problem-solving scenario lacked the true characteristics of a real classroom situation. Each student felt extra pressure to perform on the problem and because of that described the experience as stressful. In the face of this stress there is evidence of each student progressing through the affective pathways described by Goldin (2000) but it is impossible to know whether the sustained effort exerted by each student is true to a real classroom experience. Susan explains that if it were a real classroom situation she would not have put the same amount of effort into the problem, rather she would have turned to another member of her group for help or waited for the teacher to explain.

After the initial factor analysis of the questionnaire and from the data gather in part two of this thesis it is my opinion that the MRBQ, although it is in need of some additions and refinements, is a useful tool for gathering information with regards to students mathematics-related belief systems. In the final chapter of this thesis I discuss how the MRBQ, could be used by teachers to gather information that could be used to inform their affective teaching pedagogy.

9 Teaching Implications

The results of this thesis and similar research in the realm of measuring mathematics-related beliefs, suggest that with continued research and refinement the MRBQ could be a useful tool for mathematics educators. Here, and in similar studies, the factor analysis of MRBQ results confirm trends in mathematics-related beliefs that have been the subject of numerous research studies. These trends include the positive correlation between self-confidence and achievement, and the impact that a dynamic view of mathematics can have on one's like of the subject. Also, the MRBQ confirms some findings in research on gender differences in mathematics-related beliefs and also uncovers results that could become the topic of future research in gender differences such as the stronger impact of the emotional component for female students.

If there exists a core to student's mathematics related beliefs that could impact beliefs outside the core and other constructs such as motivation, then teachers interested in how the beliefs of their students impact understanding and achievement should put those core beliefs at the forefront of their affective pedagogy. If perceived ability, expectations for success and a "like" for the subject play a critical role in students developing a positive mathematical disposition, then teachers should work deliberately, to structure experiences in the classroom that foster positive beliefs in these areas.

When student's perceptions of their ability in math are low, changing beliefs in this area is difficult. In my own teaching experience, the single most challenging part of motivating students to try something new lies in convincing them that they *can* do it. How do you create experiences in the classroom where students who believe they have no ability are suddenly able? This is the craft of teaching. Knowing the prior knowledge of your students is critical so that activities can be designed to fit within their zone of proximal development.

How do we help students have positive expectations for success when they have had little success in the past? In my experience, students will only begin to expect success when they start to experience it. To experience success, the first step is often in setting an attainable goal. If teachers can first know what attainable might mean for individuals in the class they might be able to assist students in goal setting. When goals are set and met, students begin to feel like success is possible and motivation begins to drive the next goal to completion.

When students begin to feel like they have ability and can experience success, they may begin to "like" doing mathematics. Conversely, if students feel like mathematics is something they "like," they may be more willing to engage in the activities (such as goal setting) that will have positive influences on self-confidence. How do we help students "like" doing mathematics? To start, I believe that we as mathematics teachers must "like" doing math ourselves and convey that to our students. Our love for the subject is portrayed when we take the time to plan and orchestrate activities that reveal mathematics as a dynamic subject.

The MRBQ is a device that could give teachers insight into the beliefs of their students so that they can work to analyze, reflect on, and then use the information to inform teaching practice. Here, I obtained valuable information with regards to three classes of grade 11 students, however, more work must be done in the improvement and addition of questions that measure the wide range of beliefs that fall into the framework discussed in this thesis. In particular, questions with regards to the role and functioning of the teacher could provide useful information about the context in which students learn mathematics.

Future research in this area should include the analysis of a new questionnaire that includes a wider range of questions on a much larger sample of students. The next step might include the refinement of current items, the addition of new items and then the analysis of such a MRBQ using a much larger population. After analysis and then refinement and then another study using a different sample one might select an entire school district to participate in a similar study. If an MRBQ can be developed that accurately determines the mathematical dispositions of students, this information could be invaluable to a district that is interested in improving their mathematics program. The MRBQ could inform classroom teachers and entire school districts alike by revealing trends in mathematics-related beliefs. It is my hope to be a part of such a project.

10 References

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Appendices

Appendix A: Factor analysis of Op't Eynde & De Corte's (2004) Mathematics-related belief questionnaire (MRBQ) and corresponding item numbers and factor location for current thesis

Factor 1: Beliefs about the role and the functioning of their own teacher		**item # / factor for current thesis**
A.1.1	Our teacher is friendly to us (.884)	Not used
A.1.2	Our teacher listens carefully when we ask for something (.849)	Not used
A.1.3	Our teacher understands the problems and difficulties we experience (.826)	Not used
A.1.4	Our teacher does not really care how we feel in class. She/he is totally absorbed with the content on this mathematics course (-.811)	Not used
A.1.5	Out teacher cares how we feel in the mathematics lessons (.806)	Not used
A.1.6	Our teacher appreciates it when we have tried hard, even if our results are not so good (.742)	Not used
A.1.7	Our teacher really wants us to enjoy learning new things (.730)	Not used
A.1.8	Our teacher wants us to understand the content of this mathematics course, not just memorize it (.674)	Not used
A.1.9	Our teacher tries to make mathematics lessons interesting (.654)	Not used
A.1.10	Our teacher gives us the time to really explore new problems and try out possible solution strategies (.652)	Not used
A.1.11	Our teacher thinks mistakes are okay as long as we are learning (.643)	Not used
A.1.12	Our teacher thinks she/he knows everything best (-.616)	Not used
A.1.13	Our teacher first shows step by step how we have to solve a specific mathematical problem, before he gives us similar exercises (.585)	Not used
A.1.14	Our teacher explains why mathematics is important (.519)	Not used
A.1.15	We are not allowed to ask fellow students for help during class work (-.506)	Not used
A.1.16	We do a lot of group work in this mathematics class (.405)	Not used
Factor 2: Beliefs about the significance of and competence in mathematics		
A.2.1	I like doing mathematics (.850)	17 – F2
A.2.2	I believe that I will receive this year an excellent grade for mathematics (.844)	15 – F1
A.2.3	I'm very interested in mathematics (.830)	25 – F2
A.2.4	Taking into account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics (.798)	26 – F1
A.2.5	I can understand the course material in mathematics (.682)	19 – F3
A.2.6	I expect to get good grades on assignments and tests of mathematics (.673)	34 – F1
A.2.7	If I try hard enough, then I will understand the course material of the mathematics class (.540)	23 – n/a
A.2.8	To me mathematics is an important subject (.538)	20 – F2
A.2.9	I prefer mathematics asks for which I have to exert myself in order to find a solution (.527)	21 – F2
A.2.10	Mathematics learning is mainly memorizing (-.516)	3 – n/a
A.2.11	It is a waste of time when the teacher makes us think on our own about how to solve a new mathematical problem (-.438)	32 – n/a
A.2.12	Group work facilitates the learning of mathematics (-.414)	2 – n/a
Factor 3: Mathematics as a social activity		
A.3.1	I think I will be able to use what I learn in mathematics also in other courses (.568)	27 – F6
A.3.2	Mathematics enables men to better understand the world he lives in (.545)	36 – n/a
A.3.3	Solving a mathematics problem is demanding and requires thinking, also from smart students (.492)	8 – n/a
A.3.4	Mathematics is used by a lot of people in their daily life (.478)	11 – n/a
A.3.5	Mathematics is continuously evolving. New things are still discovered (.463)	9 – F6

A.3.6	There are several ways to find the correct solution of a mathematics problem (.448)	6 – F6
A.3.7	Anyone can learn mathematics (.431)	5 – F3
A.3.8	When I have the opportunity, I choose mathematical assignments that I can learn from even if I'm not at all sure of getting a good grade (.409)	24 – n/a
A.3.9	Making mistakes is part of learning mathematics (.402)	1 – n/a
Factor 4: Mathematics as a domain of excellence		
A.4.1	By doing the best I can in mathematics I want to show the teacher that I'm better than most of the other students (.664)	16 – n/a
A.4.2	I want to do well in mathematics to show the teacher and my fellow students how good I am in it (.633)	18 – n/a
A.4.3	My major concern when learning mathematics is to get a good grade (.603)	43 – n/a
A.4.4	There is only one way to find the correct solution of a mathematics problem (.544)	10 – F6
A.4.5	Those who are good in mathematics can solve any problem in a few minutes (.540)	29 – n/a
A.4.6	I'm only satisfied when I get a good grade in mathematics (.521)	13 – n/a

Appendix B: Factor analysis of Hannula's (2005) Mathematics-related belief questionnaire (MRBQ) and corresponding item numbers and factor location for current thesis

Factor 1: I am not talented in mathematics		**item # / factor for current thesis**
B.1.1	I am sure I could do advanced work in math (-.573)	Not used
B.1.2	I'm no good in math (.554)	28 – F3
B.1.3	I'm not the type to do well in math (.522)	30 – F3
B.1.4	I am sure of myself when I do math (-.460)	Not used
B.1.5	Math is hard for me (.438)	Not used
B.1.6	Math has been my worst subject (.431)	35 – F1
B.1.7	I made it well in mathematics (-.418)	Not used
B.1.8	Being compared with others made me anxious (.326)	Not used
Factor 2: I am hard working and conscientious		
B.2.1	I am hard working by nature (.812)	7 – F4
B.2.2	I have not worked hard enough (-.761)	37 – F4
B.2.3	I always prepare myself carefully for exams (.690)	40 – F4
B.2.4	I worked hard to learn mathematics (.588)	Not used
B.2.5	My attitude is wrong (-.477)	Not used
Factor 3: My family encouraged me		
B.3.1	My family encouraged me to study mathematics (.870)	12 – F5
B.3.2	The importance of competence in mathematics was emphasized at my home (.839)	4 – F5
B.3.3	The example of my parent(s) had a positive influence on my motivation (.678)	14 – F5
Factor 4: I had a poor teacher in mathematics		
B.4.1	My teacher did not inspire to study mathematics (.754)	Not used
B.4.2	My teacher was a positive example (-.727)	Not used
B.4.3	I would have needed a better teacher (.697)	Not used
B.4.4	The teacher could not explain the things we were studying (.636)	Not used
B.4.5	The teacher created appropriately challenging learning situations (-.598)	Not used
B.4.6	The teacher did not explain what for we needed the things we were learning (.530)	Not used
B.4.7	During the mathematics lessons we did only tasks from our mathematics book (.490)	Not used
B.4.8	During the lessons I usually did not want to ask for advice, in order not to be labeled stupid by classmates (.264)	Not used
Factor 5: I am insecure as a mathematics teacher		
B.5.1	I am insecure (as a mathematics teacher) (.767)	Not used
B.5.2	I am inexperienced in teaching mathematics (.690)	Not used
B.5.3	I am not able to give pupils clear enough explanations (.657)	Not used
B.5.4	My level of competence in mathematics causes me problems (.419)	Not used
Factor 6: I can do well in mathematics		
B.6.1	I am sure that I can learn math (.916)	42 – F3
B.6.2	I can get good grades in math (.711)	39 – F1
B.6.3	I know I can do well in math (.516)	22 – F1
B.6.4	I think I could handle more difficult math (.354)	Not used
Factor 7: I like mathematics		
B.7.1	Doing exercises was pleasant (.728)	Not used
B.7.2	It was boring to study mathematics (-.653)	Not used
B.7.3	Mathematics is a mechanical and boring subject (-.590)	38 – F2
B.7.4	Mathematics was my favorite subject (.524)	41 – F2
B.7.5	To study mathematics was something of a chore (-.483)	Not used

B.7.6	Mathematics was the most unpleasant part of studying (-.468)	Not used
B.7.7	I enjoyed pondering mathematical exercises (.420)	Not used
B.7.8	My attitude towards mathematics helps me as a teacher (.301)	Not used
Factor 8: Mathematics is difficult		
B.8.1	I did not understand teacher's explanations (.556)	Not used
B.8.2	Mathematics was difficult in high school (.412)	Not used
B.8.3	The teacher hurried ahead (.380)	Not used
B.8.4	Mathematics is difficult (.369)	Not used
B.8.5	Learning mathematics requires a lot of effort (.364)	Not used
B.8.6	Mathematics was a clear a precise subject to study (-.207)	Not used
Factor 9: Mathematics is calculations		
B.9.1	Mathematics is numbers and calculations (.634)	Not used
B.9.2	One learns mathematics through doing exercises (.526)	Not used
Factor 10: I am motivated		
B.10.1	It is important to me to get a good grade in mathematics (.574)	Not used
B.10.2	For me the most important thing in learning mathematics is to understand (.308)	Not used

Appendix C: Original rotated component matrix. 13 factors for 76.0% of the total variance. No items removed.

Question	Component												
	1	2	3	4	5	6	7	8	9	10	11	12	13
Q 28	.854	-.042	.009	-.041	.016	.167	.096	.201	-.141	-.038	-.017	-.170	.070
Q 35	.793	-.073	.141	.159	.126	-.060	-.095	-.081	.012	-.097	.071	-.317	-.104
Q 34	.738	.157	.157	.457	.019	.052	.050	-.055	.109	-.153	-.043	.155	.046
Q 26	.732	.362	.067	.227	-.025	-.112	.109	-.075	.099	-.032	.075	.065	.020
Q 15	.710	.261	-.030	.259	-.099	-.130	.037	-.130	.128	-.050	.181	-.052	-.105
Q 25	.702	.048	.375	-.105	-.003	.096	-.122	-.024	.223	.114	.037	.329	.015
Q 30	.686	-.010	.086	.181	.038	.266	.162	.212	-.246	-.116	.146	-.145	.169
Q 19	.684	.184	.032	-.091	-.127	.075	.312	.148	.217	.094	-.192	.006	-.063
Q 17	.673	-.121	.240	-.162	.074	.133	.103	.040	.225	.349	-.076	.250	.102
Q 42	.656	-.074	-.061	-.127	.031	.230	.504	.125	.004	-.122	-.077	.134	-.062
Q 38	.643	-.069	.184	-.084	.142	.209	.032	.214	.317	.189	.018	.140	.015
Q 41	.639	-.075	.487	-.036	.176	.126	-.013	-.008	.124	.152	-.040	.205	.076
Q 39	.630	-.018	.033	.509	-.024	.156	.329	-.203	.039	-.026	-.019	.063	-.025
Q 21	.526	-.092	.165	-.022	.193	.039	.158	.307	.192	.424	.247	.044	.066
Q 20	.482	-.026	.386	.054	.279	.393	.111	.254	.236	-.053	-.117	-.137	.024
Q 37	.039	.813	-.065	-.098	-.081	.103	-.153	-.110	-.058	.254	-.067	-.057	-.033
Q 7	-.022	.799	-.064	.032	.070	-.067	.145	.001	.134	-.119	.007	-.006	.017
Q 40	.119	.680	.077	.090	.370	.039	-.083	-.045	-.172	.076	-.079	.284	.159
Q 14	.204	.524	.245	.163	.086	.269	.097	.070	-.104	-.386	.162	.125	.053
Q 18	.146	-.026	.827	.076	.026	-.007	.067	-.055	.012	-.021	-.075	-.057	-.104
Q 16	.206	.003	.804	.022	.026	-.165	-.135	-.117	-.045	-.105	.023	-.175	.108
Q 13	-.043	.080	-.076	.752	-.145	-.143	.042	.152	.129	-.027	-.031	-.105	-.072
Q 43	.312	-.023	.157	.674	-.099	.048	-.137	-.240	-.193	-.038	-.094	-.078	-.127
Q 33	.302	-.055	.239	.513	.249	-.097	.183	.147	.041	.130	.320	-.071	.212
Q 4	-.028	.062	-.017	-.002	.874	.054	.190	-.003	.059	.187	.031	.193	-.017
Q 12	.063	.099	.062	-.144	.833	.026	-.083	-.005	.046	-.207	.019	-.054	-.061
Q 10	.042	-.102	-.048	-.022	-.003	.707	-.066	.095	-.081	.023	.006	.372	-.068
Q 6	.255	.176	.001	-.002	.019	.639	.159	-.314	.168	.023	.180	.058	-.110
Q 9	.064	.164	-.141	-.277	.094	.575	-.114	.015	.128	.198	-.060	-.166	.103
Q 27	.343	-.009	.063	.221	.109	.456	.116	.215	.345	.147	-.082	-.231	.181
Q 23	.155	.012	-.024	.010	.136	-.019	.785	-.008	.070	-.074	-.092	-.029	.006
Q 22	.475	.029	.017	.259	-.093	-.068	.593	.041	-.112	.216	.155	.089	-.123
Q 32	.045	-.162	-.169	.059	.046	.022	-.056	.789	.049	-.022	-.126	.101	-.033
Q 5	.407	.106	-.001	-.193	-.060	-.096	.286	.543	.020	-.119	.002	.110	-.292
Q 24	-.236	-.289	-.457	-.072	.119	-.106	-.086	-.500	-.185	-.011	-.327	.002	-.091
Q 36	.200	.014	.017	.091	.011	.060	-.034	.021	.873	-.103	.010	-.047	-.064
Q 11	.158	-.066	.026	-.176	.366	.181	.244	.169	.520	.120	-.143	.174	.162
Q 29	.008	.080	-.074	-.005	-.037	.148	-.035	-.069	-.078	.844	-.004	-.094	-.006
Q 8	-.005	-.202	-.207	-.061	-.113	-.003	-.224	-.179	-.080	.100	.703	.077	.006
Q 31	.069	.273	.180	-.007	.397	.093	.142	.082	.023	-.261	.620	.009	-.124
Q 1	.049	.143	-.235	-.132	.181	.117	.055	.152	.005	-.159	.088	.737	-.006
Q 3	.040	.075	-.054	-.228	-.066	-.129	-.188	.052	.043	-.083	-.173	-.056	.805
Q 2	.016	-.046	-.101	-.154	.021	-.178	-.233	.285	.082	-.140	-.275	-.103	-.598

Appendix D: Rotated component matrix after 11 questions removed. 9 factors produced for 74.1% of the total variance.

Rotated Component Matrix

	COMPONENT								
	1	2	3	4	5	6	7	8	9
Q39	0.8130	0.1963	0.1247	-0.0672	-0.0238	0.2000	0.0655	0.0710	-0.0067
Q34	0.7664	0.3877	0.1330	0.0915	0.1320	0.0683	-0.0776	0.1384	0.0105
Q15	0.7391	0.2286	0.1567	0.2000	0.0130	-0.0713	-0.1380	0.1619	-0.0634
Q26	0.7093	0.3186	0.2018	0.3284	0.0529	-0.0319	-0.0574	0.1172	0.0132
Q22	0.6361	0.1242	0.3262	-0.0095	-0.1544	-0.0152	0.2881	-0.1838	0.0839
Q33	0.6268	0.0659	-0.0537	-0.1213	0.2022	-0.0505	0.5011	-0.0133	-0.1871
Q30	0.5354	0.2350	0.4390	-0.0729	0.2089	0.2635	-0.0247	-0.2151	-0.2457
Q35	0.5263	0.4279	0.2401	-0.0726	0.1953	-0.0705	-0.1182	0.0277	-0.4262
Q25	0.2892	0.8527	0.1099	0.0916	0.0528	0.0708	-0.1020	0.1066	0.1106
Q17	0.1885	0.8349	0.2248	-0.0157	-0.0952	0.1453	0.1979	0.0638	0.0367
Q41	0.3077	0.8100	0.0433	-0.0348	0.1277	0.1188	0.1139	-0.0038	-0.0221
Q38	0.1848	0.7022	0.3074	-0.0291	0.0959	0.1750	0.1327	0.2272	-0.0419
Q21	0.1942	0.5148	0.3633	-0.0388	0.0486	0.0295	0.4722	0.0562	-0.1090
Q20	0.2003	0.4534	0.3119	-0.0977	0.3835	0.3090	0.0685	0.2608	-0.2638
Q5.	0.1063	0.1028	0.8066	0.0292	0.0714	-0.1441	-0.0172	0.0823	0.1591
Q42	0.2997	0.2876	0.7352	-0.0626	0.0244	0.2228	0.1043	0.0140	0.1145
Q28	0.4196	0.4100	0.5874	-0.0434	0.0836	0.1728	-0.0618	-0.1023	-0.2639
Q19	0.3605	0.3755	0.5544	0.2321	-0.1988	0.1262	0.1085	0.2103	-0.0483
Q37	-0.0307	0.0246	-0.0395	0.8785	-0.0764	0.1230	-0.0891	-0.0696	-0.0781
Q7.	0.1615	-0.2436	0.0945	0.7196	0.1861	-0.0555	0.0305	0.2224	0.0958
Q40	0.0836	0.1819	-0.0774	0.7025	0.3398	0.0475	0.1436	-0.1966	0.1857
Q12	-0.1813	0.1737	-0.0024	0.0599	0.7986	-0.0393	0.1396	0.0661	-0.0177
Q31	0.2353	-0.0715	0.0533	0.1009	0.6860	0.0520	0.0944	0.0131	0.1135
Q14	0.3550	-0.0036	0.1422	0.3634	0.4988	0.2013	-0.2451	-0.0117	0.1067
Q10	-0.0162	0.1085	0.0333	-0.1659	0.0530	0.7253	-0.0646	-0.0948	0.3538
Q6.	0.2878	0.0962	-0.0124	0.1793	0.0378	0.7032	0.0492	0.1427	0.0843
Q9.	-0.3039	0.1900	0.0709	0.2586	0.0077	0.5908	0.0541	0.0290	-0.1992
Q27	0.2155	0.2445	0.1682	-0.0168	0.0396	0.4626	0.2645	0.3064	-0.3481
Q4.	-0.0799	0.1416	-0.0782	0.0925	0.5385	0.0590	0.6640	-0.0080	0.1861
Q11	-0.1662	0.2706	0.3338	0.0040	0.1156	0.1870	0.5083	0.4624	0.1247
Q36	0.1400	0.1838	0.0185	-0.0286	0.0395	0.0559	0.0080	0.8920	-0.0330
Q1.	-0.0359	0.0698	0.1816	0.0931	0.1991	0.1379	0.0523	0.0021	0.7875