THREE ESSAYS ON MONETARY ECONOMICS

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Abstract

The thesis consists of three essays on monetary economics. In particular, I focus on using modern monetary theory with explicit microfoundations to address issues in macroeconomics concerning the effects of inflation and the coexistence of multiple assets.

The first essay is motivated by the observation that economies undergoing high inflation often experience a reduction of variety in the marketplace. Existing models study how inflation affects quantity, but few have studied how inflation affects variety. In a monetary model with explicit microfoundations, I analyze how inflation affects variety as well as quantity. I consider two pricing mechanisms – bargaining and price posting with directed search. I show that inflation reduces both quantity and variety under both pricing mechanisms. Quantitatively, the model implies that the total welfare cost of 10% inflation ranges from 4.77% to 8.4% under bargaining and is 1.52% under price posting.

In the second essay, I study an economy in which money and credit coexist as means of payment and the settlement of credit requires money. The model extends recent developments in microfounded monetary theory to address the choice of payment methods and the effects of inflation. Whether a buyer uses money or credit depends on the fixed cost of credit and the inflation rate. Based on quantitative analysis, the model suggests that the relationship between inflation and credit exhibits an inverse U-shape which is broadly consistent with the evidence. Compared to an economy without credit, allowing credit as a means of payment affects the economy's money demand, welfare and the welfare cost of inflation.

In modern monetary theory, money is viewed as a substitute for the record-keeping technology. In the third essay, my coauthor and I investigate whether one money constitutes a perfect substitute for the record-keeping technology in a quasi-linear environment, where private information and limited commitment are present. We adopt the mechanism design

approach and solve a planner's problem subject to various constraints. The result is that when money is divisible, concealable and in variable supply, one money may not be sufficient to replace the record-keeping technology. We further show that two monies are a perfect substitute for the record-keeping technology.

Keywords: Inflation; Variety; Welfare; Money; Credit; Mechanism Design

To my parents, Baokun and Xinmin, and my husband, Yong: for their love and support.

I love you.

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Chapter 1

Inflation and Variety[1]

1.1 Introduction

It is a stylized fact that extended periods of high inflation are associated with low levels of economic activity. A classic example is the German hyperinflation of 1919-1923 where shops remained empty and suppliers, unable to sell their wares, reduced production; see Guttman and Meehan (1975). More recent examples include many Latin American economies, where chronically high rates of inflation are associated with prolonged periods of depressed economic activity; see McKenzie and Schargrodsky (2003), and Midrigan (2007). Conventional economic theory can easily account for the inverse relationship observed between inflation and the level of economic activity. That is, inflation essentially acts as a distortionary tax in markets that rely heavily on money to facilitate exchange. The distortion caused by inflation tends to induce agents to substitute non-market activities for market activities. However, in quantitative terms, the estimated welfare cost of inflation is generally low. In Cash-in-Advance models, the estimated welfare cost of inflation is less than 1%; see Lucas (2000). Recently, search theoretic based monetary models generate the welfare cost of inflation up to 3%; see Lagos and Wright (2005).

One dimension that is typically ignored by conventional models is that of product variety. As it turns out, there is ample evidence to suggest that high inflation is associated with a contraction not only in the quantity of goods produced, but also in the variety of goods offered for sale. For example, Heymann and Leijonhufvud (1995) report that during periods of high inflation, fewer transactions were realized, the markets became thinner, and some markets were thinned out of existence altogether. Shevchenko (2004) discusses how, during the Russian inflation in the 1990s, a significant reduction was noticed in both the quantity and the variety of goods offered for sale; see also Guardiano (1993). McKenzie and Schargrodsky (2003) find that product variety – as measured by the number of Stock Keeping Units offered in supermarkets – fell by almost 15% during the high inflation period in Argentina during the early 2000s. For the same country, Midrigan (2007) documents that the rate of net product creation fell from 19% in June 2001 to -8% in June 2002. During Zimbabwe's hyperinflation in 2007, many consumer items have disappeared altogether, forcing supermarkets to fill their shelves with empty packaging behind the few goods on display.

There are good reasons to believe that product variety enhances welfare. Indeed, in much of the international trade literature, enhanced product variety is highlighted as a major source of the welfare improvements that stemmed from freer trade. For example, Broda and Weinstein (2006) estimate that the gains from the greater variety provided from imported goods is on the order of 3% of GDP per annum for the U.S. from 1972-2001.[2] Since variety is evidently important for welfare, and since inflation appears to be related to product variety, it seems sensible to explore how current estimates of the welfare cost of inflation might be affected by explicitly modeling the variety dimension. This is the primary goal of this paper.

There is some discussion of variety in economics. Lancaster (1990) provides a review of the existing literature. However, few papers in monetary theory discuss variety. Shevchenko (2004) analyzes the choice of variety in a model with middlemen, but there is no money in his model. Burdett and Shevchenko (2007) study money and variety in a model with indivisible money and indivisible goods. Because of the indivisibility, they cannot discuss

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[2]The theoretical model in Broda and Weinstein (2006) is based on the love-of-variety model. However, their model is different from the basic love-of-variety model in that the gains from variety occur not only because individuals simultaneously consume multiple types of goods, but also because individuals enjoy higher utilities from certain types of goods than other types.

issues related to inflation. Corbae and Narajabad (2007) combine a monetary search model with a Hotelling model in order to address the choice of variety using a mechanism design approach. However, they do not analyze the effect of inflation on variety. In a multiple matching model, Laing, Li and Wang (2007) show that inflation may increase or decrease the product variety offered by a seller depending on parameter values. In equilibrium, the aggregate measure of variety is fixed at 1, independent of the inflation rate.

Other papers studying money and specialization include Kiyotaki and Wright (1993), Shi (1995), Camera et al. (2003) and Ghossoub and Reed (2005). At the individual level, if an agent is more specialized, it implies that he can produce less varieties of goods. In this sense, papers that study specialization can also be viewed as studying the choice of variety at the individual level. Due to the structure of their models, the measure of variety at the aggregate level is fixed. In contrast, I construct my model to address variety at the aggregate level. In addition, specialization is often assumed to reduce the marginal cost of production, e.g. Camera et al. (2003), whereas in my model less varieties only decreases the average cost of production, but not the marginal cost. It seems more natural to use my model to study variety instead of specialization. In terms of the welfare cost of inflation, Ghossoub and Reed (2005) find that the welfare cost of inflation can be higher when specialization is more important. In their model, inflation leads to less specialization.

To estimate the welfare cost of inflation with endogenous product variety, I consider a variant of the monetary framework developed by Lagos and Wright (2005) and Rocheteau and Wright (2005). In particular, I consider a world where households and firms are matched randomly in a decentralized market and where standard frictions make money essential. Prior to matching, firms must invest in a potential set of varieties. Each household experiences an idiosyncratic preference shock that determines which variety the household values. Conditional on a match taking place, if the household's taste shock corresponds to the firm's ability to supply the desired variety, the household trades money for goods. Firms then take their accumulated money balances and distribute their profits to households in a future centralized market, while households reaccumulate money balances. As in the standard model, inflation will affect the return to accumulating money and hence will affect the quantity produced. Moreover, inflation will now affect the variety of products offered for sale in the decentralized market.

Several different pricing mechanisms can be considered in the decentralized market. I begin with bargaining, which is fairly common in monetary theory. Then I consider price

posting with directed search, which is also called competitive search and has been previously used in labor economics; see Moen (1997) and Acemoglu and Shimer (2001) for references. Price posting seems to be a very realistic assumption for the application in this paper. It also delivers sharp analytical results, especially for comparative statics since the equilibrium is unique. Additionally, the Friedman rule achieves the constrained efficient allocation in price posting equilibrium.

Not surprisingly, I find that inflation reduces both equilibrium quantity and variety, which is consistent with the observations stated above. The main intuition is that inflation reduces the surplus from each trade, which in turn lowers the marginal benefit of investing in variety. As variety is costly, inflation reduces variety. Since my concern is also to measure the additional welfare cost of inflation when product variety is endogenous, the model calibrated to the U.S. money demand data suggests that these additional costs can be substantial, although it depends on the pricing mechanism in the decentralized market. In bargaining equilibrium, the welfare cost of inflation due to the reduction of variety can account for more than half of the total welfare cost of inflation, depending on households' bargaining power. In price posting equilibrium, the welfare cost of inflation due to the reduction of variety is very small. As a theoretical extension, I allow households to consume the alternate varieties if they do not find the desired variety.

The rest of the paper is organized as follows. Section 2 describes the environment of the theoretical model. Section 3 analyzes bargaining equilibrium. Section 4 analyzes price posting equilibrium. Section 5 studies the model quantitatively and assesses the welfare cost of inflation. I consider an extension of the model in section 6. Finally, section 7 concludes.

1.2 Environment

The economy is populated by two types of agents: households and firms. There is a $[0, 1]$ continuum of each type. The set of households and the set of firms are $\mathcal{H}$ and $\mathcal{F}$, respectively. Time is discrete and the horizon is infinite. Households and firms are infinitely lived. During each period, a centralized market (hereafter CM) and a decentralized market (hereafter DM) open sequentially. The CM and the DM are distinguished as follows. Households and firms in the CM are centrally located; whereas in the DM, they must meet one-on-one according to a random search process.

Households work in the CM, consume a general good in the CM and a special good in

the DM; while firms maximize profits by producing the general good in the CM and the special goods in the DM. Firms are equally owned by all households. Households discount across periods at the rate β where $0 < \beta < 1$ and firms discount across periods at the rate $\frac{1}{1+r}$ where r is the net real interest rate. All goods are non-storable.

In what follows, I restrict attention to stationary allocations. Let x denote the quantity of the general good consumed by a household and let y denote the hours worked by a household in any CM. The momentary utility payoff associated with a household is $v(x) - y$, where $v'' < 0 < v'$, $\lim_{x \to 0} v'(x) = \infty$, and $\lim_{x \to \infty} v'(x) = 0$.

In the CM, firms distribute their last period's realized profits, hire labor and produce the general good. At the same time, firms also make investment decisions for production in the following DM. I assume that the production technology in the CM is such that one unit of labor input produces one unit of the general good. In the DM, firms may potentially produce a wide variety of goods, but each firm f, $f \in \mathcal{F}$ can produce a unique set of special goods Φ_f. That is, if a special good is in Φ_f, it is not in $\Phi_{f'}$, for all $f' \neq f$. Each special good in the set represents a distinct variety.[3] For simplicity, the measure of Φ_f is N, $N \in \mathbb{R}_+$ for all $f \in \mathcal{F}$.

A firm's ability to produce goods in a given set of varieties depends on its investment in the CM. In particular, by investing a measure $n \in [0, N]$ of variety at cost $k(n)$ in terms of the general good, the firm leaves itself with enough flexibility to produce any special good $j \in [0, n]$. Assume that $k', k'' > 0$, $k(0) = 0$, and $k'(0) = 0$. If the firm turns out to produce q units of a special good j, $j \in [0, n]$ in the DM, the cost of production is $c(q)$.[4] As usual, $c' > 0$, $c'' \geq 0$ and $c(0) = 0$.

In the DM, households will have a desire to consume some special goods. I assume that prior to their meetings, households have the same distribution of preference over each firm's set of special goods. Exactly which variety of good is desired is determined by an idiosyncratic preference shock, which is realized after households are matched with firms. Preference shocks are *i.i.d.* across households and across time. Given that a household meets

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[3]There is no consensus on the terminology of variety. Normally variety is measured by a certain classification criteria. Broadly speaking, as pointed out in White (1977), a particular variety of good might involve a difference in quality, or can pertain to preferences such as desiring a red shirt but not a blue shirt.

[4]A previous version of the paper assumes that firms invest in capital in the CM for the production in the DM. In that version, if a firm invests in b units capital in the CM, the firm's production capacity is constrained, i.e., $c(q) \leq b$. Assuming that firms have the transformation technology to convert 1 unit capital in the DM into $1 + r$ units of the general good in the following CM, firms would invest in enough capital for the DM production. All the results hold with this slightly more complicated environment.

a firm, let $\xi(j)$ be the probability that the household likes the firm's good j. Meeting with a firm who has invested in n varieties, the probability for a household to find the good that he likes is $\sigma(n)$, where $\sigma'' \leq 0 < \sigma'$. One simple way to model this is to assume that each household's preference shock is distributed uniformly over the interval $[0, N]$ in each Φ_f for all $f \in \mathcal{F}$. Hence, $\xi(j) = \frac{1}{N}$ and $\sigma(n) = \frac{n}{N}$.[5] A household with a desire to consume good j of q units has utility $u(q(j))$, with $u'' < 0 < u'$, $u'(0) = \infty$ and $u(0) = 0$.

The matching technology in the DM is constant return to scale, $\mathcal{M} : \mathbb{R}_+^2 \rightarrow [0, 1]$, where $\mathcal{M}$ denotes the aggregate measure of matches that occur between households and firms. Let $\alpha = \mathcal{M}$ denote the fraction of households/firms who make contact with a firm/household. At the individual level, α is the probability that any given household makes contact with a firm, or is the probability that any given firm makes contact with a household. Since all firms specialize in their own production set and the measure of the firms is 1, the aggregate measure of the actual product variety is $\alpha\sigma(n)$ in this model.

I now consider as a benchmark, the allocation that would be chosen by a planner who weights all households equally. In each period, the planner must assign the general good consumption x and labor y to the household. The planner also instructs all firms to invest in variety $n \in [0, N]$ for the production of the special goods. Since the investment in variety is costly, it will generally be desirable to choose some $n < N$.

Subsequent to the investment n, households and firms are matched together in a random manner. I assume that the planner must respect the matching technology in the sense that he cannot insure households against the risk of not finding a match. Given the ex post realization of each household's preference shock, only the fraction $\alpha\sigma(n)$ of households consume the special goods and only the fraction $\alpha\sigma(n)$ of firms produce the special goods. In the cases where: [1] a household and a firm are matched; and [2] the household desires a good in the set of varieties $[0, n]$, the household will be assigned consumption q in the DM, and the firm will produce q units output. In all other cases, the household receives zero and

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[5] Another example of preference distribution models households' preferences as a symmetrically truncated normal distribution over the goods $[0, N]$ in each Φ_f for $f \in \mathcal{F}$;

$$\xi(j) = \frac{\frac{1}{\delta}\lambda(\frac{j-\mu}{\delta})}{\Lambda(\frac{N-\mu}{\delta}) - \Lambda(\frac{0-\mu}{\delta})}$$

where $\lambda(\cdot)$ is the probability density function of a standard normal distribution and $\Lambda(\cdot)$ is the cumulative density function. The mean corresponds to the good that is most likely to be chosen by households and the variance represents the dispersion of the ex ante preference. In this situation, $\sigma(n) = \int_a^{a+n} \xi(j)dj$ such that $\xi(a) = \xi(a+n)$. One can show that $\sigma'(n) > 0$ and $\sigma''(n) < 0$.

the firm produces nothing.

The planner's objective can be stated as follows:

$$\max_{x,n,q} \left\{ v(x) - x - k(n) + \alpha\sigma(n)[u(q) - c(q)] \right\}. \tag{1.1}$$

At an interior solution (assuming N is sufficiently large), the optimal allocation is characterized by

$$v'(x^*) \quad = \quad 1, \tag{1.2}$$

$$u'(q^*) \quad = \quad c'(q^*), \tag{1.3}$$

$$k'(n^*) \quad = \quad \alpha\sigma'(n^*)[u(q^*) - c(q^*)]. \tag{1.4}$$

Note that the planner's solution has the flavor of a credit arrangement. In particular, households who find the desired special goods want to make a purchase. By construction, they have nothing to offer the firm, except the implicit promise of producing a quantity of the general good the next period (an object that the firm does value).

1.3 Monetary Equilibrium with Bilateral Bargaining

As is standard, I introduce an essential role for money by assuming that agents are anonymous and lack commitment. Let M denote the aggregate money supply at any given date. The money supply grows at a gross rate γ. Money is injected (or withdrawn) via a lump-sum transfer (or tax) only to households at the beginning of each period. The transfer is $\tau = (\gamma - 1)M_-$, where M_- denotes money supply in the previous period.

As all agents are centrally located in the CM, I assume that it is a competitive spot market (where money is exchanged for the general good). In the CM, households will be induced to accumulate money balances as money will be the only way in which they can purchase their desired special goods later. When households and firms meet individually in the DM, I assume that the exchange of money for good is dictated by a generalized Nash bargaining solution concept.

1.3.1 Households

Let ϕ denote the competitive-determined value of money in the CM (i.e., the inverse of the price level). Let m denote the nominal money balance for a household at the beginning

of the CM. Likewise, let $\hat{m}$ denote the money balance carried forward into the DM by a household. Let W and V denote the value functions associated with a household in the CM and the DM, respectively. Finally, let π denote a firm's current period expected profit measured in terms of the general good. Firms' profits are realized at the beginning of the next period, so let Π denote the realized aggregate profit measured in terms of the next period's general good.[6] Since firms are owned by households, each household receives Π_- at the beginning of the CM, where Π_- denotes the realized aggregate profit from last period.

At the beginning of each period, a household's choice problem is

$$W(m) = \max_{x,y,\hat{m}} \{v(x) - y + V(\hat{m})\}$$

$$\text{s.t. } \phi(\hat{m} - m - \tau) + x = y + \Pi_-,$$

or

$$W(m) = \phi(m + \tau) + \Pi_- + \max_{x,\hat{m}} \{v(x) - x - \phi\hat{m} + V(\hat{m})\}. \tag{1.5}$$

The first order conditions are

$$v'(x) = 1, \tag{1.6}$$
$$V'(\hat{m}) = \phi. \tag{1.7}$$

The optimal x is determined in (1.6), which corresponds to the planner's choice. The optimal $\hat{m}$ in (1.7) does not depend on m. From the envelope condition, $W'(m) = \phi$. Note that $W(m)$ is linear in m. In particular, $W(m) = W(0) + \phi m$.

As the household enters into the DM, the value function is

$$V(\hat{m}) = \alpha\sigma(n)[u(q) + \beta W_+(\hat{m} - d)] + [1 - \alpha\sigma(n)]\beta W_+(\hat{m}), \tag{1.8}$$

where W_+ denotes the value function in the next period. Each household has probability α of being matched with a firm. Given that a household and a firm meet, the probability that a household obtains the desired special good is $\sigma(n)$. In such an event, the household spends d units of money in exchange for q units of the special good.

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[6]In aggregate, $\Pi = (1 + r) \sum_{f \in \mathcal{F}} \pi_f$, where r is the real interest rate.

1.3.2 Firms

Firms in this environment face a sequence of static problems. At the beginning of each period, a firm may or may not bring money into the CM depending on whether the firm made a sale in the previous DM. In any case, firms distribute all of their profits to the households. The objective of a firm is to maximize the current period's expected profit.

If a firm is matched with a household and is able to provide the household's desired variety, it produces q units of output and receives d units of money from the household. The value of a sale is $-c(q) + \frac{1}{1+r}\phi_+ d$ for the firm. Note that in this economy, the gross real interest rate $1 + r$ is implicitly given by $\frac{1}{\beta}$. A firm's choice problem is

$$\pi = \max_n \left\{ -k(n) + \alpha\sigma(n)[-c(q) + \frac{1}{1+r}\phi_+ d] \right\}. \tag{1.9}$$

In general, the terms of trade (q, d) may depend on n, because forward looking households and firms should internalize the impact of n on the terms of trade. Therefore, I proceed to solve the generalized Nash problem to get (q, d).

1.3.3 Equilibrium

Let $\theta \in (0, 1]$ be the household's bargaining power. The generalized Nash problem is

$$\max_{q,d}[u(q) - \beta\phi_+ d]^\theta [-c(q) + \beta\phi_+ d]^{1-\theta} \tag{1.10}$$

$$\text{s.t. } d \leq \hat{m}.$$

If $d < \hat{m}$, the solution for q is given by $u'(q) = c'(q)$ or $q = q^*$. If $d = \hat{m}$, the first order condition with respect to q can be reduced to

$$\beta\phi_+ \hat{m} = \frac{(1-\theta)u(q)c'(q) + \theta u'(q)c(q)}{\theta u'(q) + (1-\theta)c'(q)} \equiv g(q). \tag{1.11}$$

Following Lagos and Wright (2005), one can prove that the constraint $d \leq \hat{m}$ always binds in equilibrium. So the terms of trade are given by $d = \hat{m}$ and $\beta\phi_+ \hat{m} = g(q)$. It follows that $\frac{dq}{d\hat{m}} = \frac{\beta\phi_+}{g'(q)} > 0$, where $g'(q) > 0$.

It is interesting to note that the choice of (q, d) does not depend on n. Households and firms act as if they do not take the choice of n into consideration during bargaining. This is not surprising since at the time that a household and a firm decide to trade, the firm has already made the investment in n. The household does not internalize the cost of variety

during the bargaining process. This is the classical holdup problem associated with the bargaining solution.

Rewrite (1.8) and (1.9) as follows:

$$V(\hat{m}) = \beta W_+(0) + \alpha\sigma(n)u(q) + [1 - \alpha\sigma(n)]\beta\phi_+\hat{m}, \tag{1.12}$$

$$\pi = \max_n \left\{ -k(n) + \alpha\sigma(n)[\frac{1}{1+r}\phi_+\hat{m} - c(q)] \right\}. \tag{1.13}$$

Since n does not depend on individual $\hat{m}$, the envelope condition gives

$$\frac{\partial V(\hat{m})}{\partial \hat{m}} = \alpha\sigma(n)u'(q)\frac{dq}{d\hat{m}} + [1 - \alpha\sigma(n)]\beta\phi_+. \tag{1.14}$$

Given that (q, d) do not depend on n, the first order condition for an interior n is

$$\alpha\sigma'(n)[g(q) - c(q)] = k'(n). \tag{1.15}$$

To derive the equilibrium, substitute (1.7) into (1.14),

$$\phi = \beta\phi_+[1 - \alpha\sigma(n)] + \alpha\sigma(n)u'(q)\frac{\beta\phi_+}{g'(q)}. \tag{1.16}$$

As new money is injected only to households, each household carries $\hat{m} = M$ into the DM. In the steady state, the gross inflation rate is $\frac{\phi}{\phi_-} = \frac{M}{M_-} = \gamma$. Substituting $\phi_+ = \frac{g(q)}{\beta\hat{m}}$ into (1.16) and using $\frac{\phi}{\phi_-} = \gamma$, (1.16) is reduced to

$$\frac{u'(q)}{g'(q)} = 1 + \frac{i}{\alpha\sigma(n)}, \tag{1.17}$$

where i is the nominal interest rate defined by the Fisher equation $1 + i = (1 + r)\gamma$.

Definition 1.1 *Bargaining equilibrium is a list (q, d, n) such that (q, n) solves (1.15) and (1.17), and $d = \hat{m}$, where $\hat{m}$ is given by (1.11).*

To prove the main results in bargaining equilibrium, I adopt the following assumption.

Assumption 1: (i) $\lim_{q\to 0}\frac{u'(q)}{g'(q)} = +\infty$; (ii) $\frac{u'(q)}{g'(q)}$ is strictly decreasing in q.

Proposition 1.2 *A bargaining monetary equilibrium exists iif*

$$\max_q \left\{ -ig(q) + \alpha\sigma(\bar{n})[u(q) - g(q)] \right\} > 0,$$

where $\bar{n} = \arg\max_{n\in[0,N]} \left\{ -k(n) + \alpha\sigma(n)[g(q) - c(q)] \right\}.$

In short, the condition needed in order for a monetary equilibrium to exist rules out $q = n = 0$ as a solution. This seems to be obvious, but this is what can be concluded without imposing any functional forms of $u(q)$, $c(q)$, $\sigma(n)$ and $k(n)$. If one is willing to consider some special form of these functions, the condition for the existence of a monetary equilibrium is more concrete. For example, if $u(q) = \frac{1}{\rho}q^{\rho}$, $c(q) = q$, $\sigma(n) = n$ and $k(n) = \frac{1}{2}n^2$ for $0 < \rho < 1$ and $N = 1$, I can show that a monetary equilibrium exists when i is not too big.[7]

Quantity q can be viewed as the intensive margin. Variety n affects the frequency of trade and hence can be viewed as the extensive margin. In the literature, endogenizing entry decisions or search intensity can also affect the extensive margin. Rocheteau and Wright (2005) endogenize the entry decisions by sellers. They show that in bargaining equilibrium, if monetary equilibrium exists, there must be multiple equilibria. In contrast to their result, I can only prove that a bargaining equilibrium exists under certain conditions.[8] Bargaining equilibrium can be unique, although I have no formal proof for uniqueness. Lagos and Rocheteau (2005) endogenize search intensity by buyers. There may be multiple equilibria in their bargaining equilibrium and they argue that there are several ways to ensure a unique equilibrium. It seems that different types of extensive margins lead to very different properties of the equilibrium although they all affect the frequency of trade.

In Rocheteau and Wright (2005), entry decisions by sellers have a thick market effect on buyers and a congestion effect on sellers. More sellers entering the market directly makes buyers better off and sellers worse off. Similarly in Lagos and Rocheteau (2005), buyers' search intensity has a thick market effect on sellers and a congestion effect on buyers. If a buyer increases his search intensity, it benefits sellers and hurt other buyers. In this paper, more varieties benefits both households (buyers) and firms (sellers). If a firm offers more varieties, it increases the probability of trading for both parties in this meeting, but it does not directly affect other firms' trading probabilities. In this sense, variety does not have the "direct" congestion effect.

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[7]A sketch of the proof is given here. Define $f(q) = \frac{u'(q)}{g'(q)} - 1 - \frac{i}{\alpha\sigma(n)}$ where n is given by (1.15). A monetary equilibrium solves $f(q) = 0$ and $q > 0$. It is straightforward that monetary equilibrium exists when $i = 0$. As i shifts $f(q)$ down, one can see that for any small ε, there is always an i that makes $f(q) < 0$ for all $q > \varepsilon$. As ε approaches 0, $\lim_{q\to 0} f(q) = -\infty$ with these specific functional forms. So when i is big enough, $f(q) < 0$ for all $q > 0$. There is no monetary equilibrium.

[8]If assuming that $k'(0) > 0$ and $\sigma'(0) < \infty$, then there exists a $q_0 \in (0, q^*]$ such that $n(q_0) = 0$. In this particular case, one can show that if monetary equilibrium exists, there must be multiple equilibria.

1.3.4 Inflation and Variety

Since bargaining equilibrium may not be unique and since as always when there exist multiple equilibria, the comparative statics results are different for different equilibria, I assume that either equilibrium is unique, or if multiple equilibria exist, I focus on the equilibrium with the highest q.

Proposition 1.3 $\frac{\partial q}{\partial \gamma} < 0$ *and* $\frac{\partial n}{\partial \gamma} < 0$.

Inflation has negative effects on both the intensive margin and the extensive margin. When inflation goes up, it distorts households' incentives to hold money, which in turn has a negative impact on activities that require money. In general, quantity per trade q decreases. For firms, producing a smaller quantity lowers the marginal benefit of investing in variety. Since the marginal cost of variety is increasing in n, the choice of n decreases as q decreases. In this model, there is no double coincidence of wants meeting in the DM. All transactions of the special goods have to be done with money. In other words, there are only single coincidence meetings. Once money is less valuable, it is natural that the market of the special goods shrinks. Previous models that study money and specialization usually have the property that less specialization (more varieties at the individual level) leads to a higher probability of double coincidence meetings. Given that inflation lowers the trade surplus from single coincidence meetings, inflation may lead to less specialization because double coincidence meetings become more desirable. While this is interesting to study, it seems appropriate to assume the absence of double coincidence meetings in the current application.

Proposition 1.4 $q < q^*$ *and* $n < n^*$. *The Friedman rule is the optimal monetary policy, but the efficient allocation cannot be achieved.*

In bargaining equilibrium, distortions come from bargaining and inflation. Before bargaining, households invest in money and firms invest in variety. For $\gamma > \beta$, the double sided investments create a double holdup problem, which cannot be solved by varying bargaining power. Acemoglu and Shimer (1999) show that bargaining equilibrium is inefficient in a labor search model when a double holdup problem exists. Hosios' condition does not give rise to the efficient outcome in the presence of a double holdup problem. At the Friedman rule, money is not subject to the holdup problem as noted by Aruoba et al. (2007). However,

investment in variety is still subject to the holdup problem unless $\theta = 0$. Since $\theta = 0$ in general precludes the existence of a monetary equilibrium, I do not consider $\theta = 0$ in this paper. As a result, the holdup problem still exists and the efficient allocation cannot be achieved at the Friedman rule.

1.4 Monetary Equilibrium with Price Posting

As an alternative to bargaining, one can consider price posting in the DM, where prices are posted before meetings, and agents are able to direct their search towards favorable terms of trade. There are a variety of ways to formalize this. In particular, one can assume that prices are posted by firms, or households, or market makers.[9] I adopt the market maker version, where market makers design and open a set of submarkets Ω. At the beginning of each period, each market maker announces the terms of trade (q, d) and the variety n for his particular submarket $\omega \in \Omega$. Note that market makers only announce variety n without specifying the exact subset of special goods. After exiting the CM, households and firms get to choose which submarket to visit. Those who direct their search to the same (q, d, n) form an active submarket. Trade is bilateral and there is random matching in each submarket. Let H_ω and F_ω be the measure of households and the measure of firms in submarket ω for $\omega \in \Omega$. The market tightness of submarket ω is $Q_\omega = \frac{H_\omega}{F_\omega}$. Since Q_ω affects agents' matching probabilities, agents anticipate Q_ω when they choose among the submarkets. In equilibrium, the actual Q_ω should be consistent with agents' rational expectations.

In the CM, a household has the same value function as (1.5) and a firm has the same profit function as (1.9). For a household in the DM,

$$V(\hat{m}) = \max_{\omega \in \Omega} \left\{ \alpha_h(Q)\sigma(n)[u(q) + \beta W_+(\hat{m} - d)] + [1 - \alpha_h(Q)\sigma(n)]\beta W_+(\hat{m}) \right\}. \qquad (1.18)$$

Here $\alpha_h(Q_\omega) = \frac{\mathcal{M}(H_\omega, F_\omega)}{H_\omega} = \frac{\mathcal{M}(Q_\omega, 1)}{Q_\omega}$ is the probability that a household meets a firm in submarket ω. I omit the subscript ω of (q, d, n, Q) to reduce notations. For a firm,

$$\pi = \max_{\omega \in \Omega} \left\{ -k(n) + [-\alpha_f(Q)\sigma(n)c(q) + \frac{1}{1+r}\alpha_f(Q)\sigma(n)\phi_+ d] \right\}, \qquad (1.19)$$

where $\alpha_f(Q_\omega) = \frac{\mathcal{M}(H_\omega, F_\omega)}{F_\omega} = \mathcal{M}(Q_\omega, 1)$ is the probability that a firm meets a household.

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[9]Here market makers represent a third party that is not involved in actual trading. Competition among market makers makes them earn zero profit.

Using the results from the previous section,

$$W(m) = \phi(m+\tau) + \Pi_- + \max_{\hat{m}} \left\{ -\phi\hat{m} + \beta\phi_+\hat{m} + \alpha_h(Q)\sigma(n)[u(q) - \beta\phi_+ d] + \beta W_+(0) \right\}. \tag{1.20}$$

1.4.1 Equilibrium

Equilibrium requires that there is no possible submarket that makes some firms better off without making households worse off. Market makers choose (q, d, n, Q) to maximize π such that households get the equilibrium expected utility $\bar{U}$ from the submarkets. The market maker's problem is

$$\max_{q,d,Q,n} \pi \quad \text{s.t. } W(m) = \bar{U}.$$

It can be shown that households choose to bring just enough money d to the DM. Ignoring the constants in (1.19) and (1.20), the market maker's problem is reformulated as

$$\max_{q,d,Q,n} \left\{ \alpha_f(Q)\sigma(n)[-c(q) + \beta\phi_+ d] - k(n) \right\} \tag{1.21}$$

$$\text{s.t. } -[i + \alpha_h(Q)\sigma(n)]\beta\phi_+ d + \alpha_h(Q)\sigma(n)u(q) = \hat{U}. \tag{1.22}$$

where $\hat{U} = \bar{U} - \beta W_+(0) - \phi(m+\tau) - \Pi_-$.

Since market makers take $\bar{U}$ and hence $\hat{U}$ as given, I can denote the set of solutions as $\Upsilon(\hat{U}) = \{q(\hat{U}), d(\hat{U}), Q(\hat{U}), n(\hat{U})\}$. Lemma 1 establishes the existence of a solution.

Lemma 1.5 $\Upsilon(\hat{U})$ *is nonempty and upper-hemicontinuous.*

Substituting the expression of $\beta\phi_+ d$ from (1.22) into (1.21), the unconstrained market maker's problem is

$$\max_{q,Q,n} \left\{ \alpha_f(Q)\sigma(n)[-c(q) + \frac{\alpha_h(Q)\sigma(n)u(q) - \hat{U}}{i + \alpha_h(Q)\sigma(n)}] - k(n) \right\}. \tag{1.23}$$

Define the elasticity $\eta(Q) = \mathcal{M}_H(H,F)\frac{H}{\mathcal{M}(H,F)} = \mathcal{M}_Q(Q,1)\frac{Q}{\mathcal{M}(Q,1)}$ as households' contribution to firms' probability of matching. Firms' contribution to households' probability of matching is $1 - \eta(Q) = \mathcal{M}_F(H,F)\frac{F}{\mathcal{M}(H,F)} = [\mathcal{M}(Q,1) - Q\mathcal{M}_Q(Q,1)]\frac{1}{\mathcal{M}(Q,1)}$. The first order

conditions for an interior solution of the unconstrained market maker's problem are

$$q \quad : \quad \frac{u'(q)}{c'(q)} - 1 - \frac{i}{\alpha_h(Q)\sigma(n)} = 0, \tag{1.24}$$

$$Q \quad : \quad u(q) - \frac{\hat{U}}{\alpha_h(Q)\sigma(n)} - \frac{u'(q)}{c'(q)} \frac{\eta(Q)u'(q)c(q) + [1-\eta(Q)]c'(q)u(q)}{\eta(Q)u'(q) + [1-\eta(Q)]c'(q)} = 0, \tag{1.25}$$

$$n \quad : \quad \alpha_f(Q)\sigma'(n)\frac{c'(q)[u(q)-c(q)]}{\eta(Q)u'(q) + [1-\eta(Q)]c'(q)} - k'(n) = 0. \tag{1.26}$$

The next assumption is used to ensure the uniqueness of price posting equilibrium.

Assumption 2: (i) $\frac{qu'(q)}{u(q)}$ is weakly decreasing in q; (ii) $\frac{qc'(q)}{c(q)}$ is weakly increasing in q; (iii) $\eta(Q)$ is weakly decreasing in Q.

Commonly used utility functions such as CRRA and CARA utilities satisfy (i). Log concavity of $u'(q)$ is also sufficient to guarantee (i). Cost functions such as $c(q) = q^a$, $a \geq 1$ satisfy (ii). With regard to (iii), $\eta(Q)$ is decreasing in Q for the most frequently used matching functions, such as the Cobb-Douglas matching function, the matching function in Kiyotaki and Wright (1993) and the urn-ball matching function as in Burdett, Shi and Wright (2001).

Lemma 1.6 $Q(\hat{U})$ *is nonempty, upper-hemicontinuous and strictly decreasing in* $\hat{U}$.

In price posting equilibrium, households and firms choose the submarket that yields the highest expected payoff. The formal definition of a price posting equilibrium is given below.

Definition 1.7 *A price posting equilibrium is a list of* $(q_\omega, Q_\omega, d_\omega, n_\omega, F_\omega)$ *and a* $\bar{U} \geq 0$ *such that given* $\bar{U}$, $(q_\omega, Q_\omega, d_\omega, n_\omega, F_\omega)$ *maximize the firm's expected profit subject to the constraint that households get* $\bar{U}$, *where* $\bar{U}$ *satisfies* $\sum_\omega F_\omega = 1$, *and* $\sum_\omega F_\omega Q_\omega = 1$.

Proposition 1.8 *A price posting monetary equilibrium exists if*

$$\max_{q,n} \left\{ \alpha_f(Q)\sigma(n)[-c(q) + \frac{\alpha_h(Q)\sigma(n)u(q)}{i + \alpha_h(Q)\sigma(n)}] - k(n) \right\} > 0,$$

where $Q = Q(0)$. *Moreover, it is unique.*

Similar to bargaining equilibrium, I need an extra condition to ensure the existence of a monetary equilibrium, i.e., $q > 0$ and $n > 0$. In contrast to bargaining equilibrium, the monetary equilibrium is unique. If $Q(0) \geq 1$, the market tightness Q should be consistent with the exogenous supply of households and firms and $Q = 1$ in equilibrium. Once Q is

determined, (1.24) and (1.26) jointly determine (q, n). In equilibrium, $\hat{U}$ satisfies (1.25) and then one can get the market value of a household $\bar{U}$. If $Q(0) < 1$, the only equilibrium requires $\hat{U} = 0$ and $Q = Q(0)$. Here households are indifferent whether to participate the submarket or not. Again, (1.24) and (1.26) jointly determine (q, n). In either case, the solution of $\beta\phi_+d$ is

$$\beta\phi_+d = \frac{\eta(Q)u'(q)c(q) + [1 - \eta(Q)]c'(q)u(q)}{\eta(Q)u'(q) + [1 - \eta(Q)]c'(q)} \equiv h(q), \tag{1.27}$$

where I use (1.22) and (1.25).

1.4.2   Inflation and Variety

A nice property of the price posting equilibrium is that it is unique. Hence, I can study the effects of inflation on quantity and variety without additional assumptions.

Proposition 1.9 $\frac{\partial q}{\partial \gamma} < 0$ and $\frac{\partial n}{\partial \gamma} < 0$.
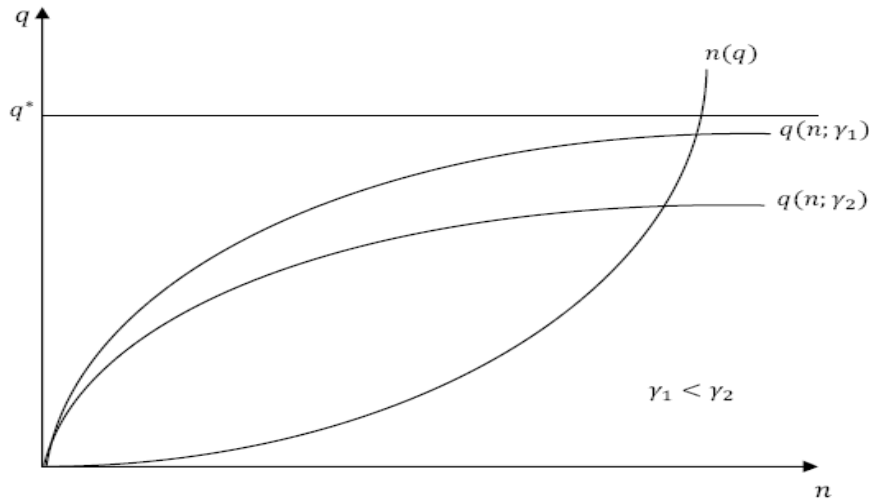
Figure 1.1: The Effect of an Increase in Inflation

Graphically, (1.24) determines $q(n)$ and (1.26) determines $n(q)$. Increasing γ shifts $q(n)$ down and leaves $n(q)$ unchanged. Figure 1 illustrates how inflation affects q and n in price posting equilibrium. Consistent with the comparative statics results in bargaining equilibrium, inflation reduces both quantity and variety. This is in contrast to Lagos and Rocheteau

(2005), where the price posting equilibrium generates different comparative statics results from the bargaining equilibrium. In their paper, inflation increases buyers' search intensity at low inflation rates because buyers' surplus from trade increase at low inflation rates. With endogenous variety, inflation always lowers firms' surplus from trade. Therefore, investment in variety also decreases.

Proposition 1.10 $n \leq n^*$ *and* $q \leq q^*$. *The Friedman rule achieves* (q^*, n^*).

In the absence of inflation distortion, the price posting equilibrium achieves the efficient allocation. This result is in line with the results of Acemoglu and Shimer (1999). The key factors that make price posting an efficient pricing mechanism include both posting prices and directed search. Terms of trade (q, d) and variety n are announced before a household and a firm meet. Households and firms can direct their search towards the submarkets that are most desirable to them. The timing makes households and firms internalize the impact of the choices of money holding and variety on the terms of trade, and thereby avoids the double holdup problem. Suppose that the monetary authority can run the Friedman rule in order to avoid inflation distortion. Price posting with directed search can solve all other inefficiencies arising in this environment.

1.5 Quantitative Analysis

The current model predicts that inflation affects both quantity and variety. The main purpose of this section is to quantify the welfare cost of inflation due to the reduction of both quantity and variety. Moreover, I decompose the total welfare cost of inflation into the cost due to quantity and the cost due to variety. It helps to understand how endogenizing variety contributes to the total welfare cost of inflation.

Suppose that a household's DM consumption and variety are q_τ and n_τ at $\tau\%$ inflation. In the steady state, let the aggregate welfare at $\tau\%$ inflation be $\mathcal{W}(\tau)$. The welfare cost of $\tau\%$ inflation is measured by Δ, which is the fraction of consumption that a household would like to give up to have 0% inflation rather than $\tau\%$ inflation.[10] Formally, Δ is implicitly

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[10]Here I measure the welfare cost of $\tau\%$ inflation as the fraction of consumption a household would like to give up for the economy to have 0% inflation rather than $\tau\%$. This is a little different from existing welfare studies using monetary search models, where there is either one type of agent or an endogenous buyer-seller ratio. With only one type of agent, the welfare cost of inflation is simply the fraction of consumption an agent would be willing to give up. With endogenous choice of being a buyer or a seller, e.g. Rocheteau and

given by

$$\mathcal{W}(\tau) = \frac{1}{1-\beta} \left\{ v[x^*(1-\Delta)] - x^* + \alpha\sigma(n_0)\{u[q_0(1-\Delta)] - c(q_0)\} - k(n_0) \right\}.$$

I first fix variety at n_τ and determine the fraction of consumption a household is willing to give up in order to have q_0 instead of q_τ. This can be viewed as the welfare cost due to quantity and it is measured by Δ_q, where Δ_q is from

$$\mathcal{W}(\tau) = \frac{1}{1-\beta} \left\{ v[x^*(1-\Delta_q)] - x^* + \alpha\sigma(n_\tau)\{u[q_0(1-\Delta_q)] - c(q_0)\} - k(n_\tau) \right\}.$$

Next I fix quantity at q_τ and determine the fraction of consumption a household is willing to give up to have n_0 instead of n_τ. This is the welfare cost due to variety, which is measured by Δ_n from the following equation.

$$\mathcal{W}(\tau) = \frac{1}{1-\beta} \left\{ v[x^*(1-\Delta_n)] - x^* + \alpha\sigma(n_0)\{u[q_\tau(1-\Delta_n)] - c(q_\tau)\} - k(n_0) \right\}.$$

Money demand and nominal interest rate data are from Craig and Rocheteau (2007). The standard money demand data is computed from $L(i) = M/PY$, where M is measured by $M1$ and PY is the nominal GDP. The nominal interest rate i is annual short term commercial paper rate. Let $v(x) = a \log x$, $u(q) = \frac{q^{1-\rho}}{1-\rho}$ where $0 < \rho < 1$, $c(q) = Aq$, $\sigma(n) = \frac{n}{n+1}$ and $k(n) = \kappa n^2$. I use the urn-ball matching function so that $\alpha = \mathcal{M} = 1 - e^{-1}$. I set the real interest rate $r = 0.04$, which implies that $\beta = 0.9615$. The model's "money demand" is:

$$L(i) = M/PY = \frac{1}{\frac{a}{(1+i)g(q)} + \alpha\sigma(n)}.$$

The strategy is to choose (a, ρ, A, κ) to fit the standard money demand observations. Ideally, parameters related to variety should be calibrated from data related to variety. As it is hard to obtain these data, I choose all parameters through fitting the money demand. Due to the identification problem, I fix $A = 1$, $\kappa = 0.01$ and fit (a, ρ) to the data.[11] In the following, I

Wright (2004), buyers and sellers get the same expected utility in equilibrium. So the welfare cost of inflation is also measured by the fraction of consumption a buyer or a seller would like to give up. In my model, firms are owned by households. So it is reasonable to study the welfare cost of inflation as the fraction of consumption that households would give up.

[11]I begin by finding values of (a, ρ, κ) together. For price posting equilibrium, the results are quite robust. For bargaining equilibrium, the results are sensitive to the initial guess of (a, ρ, κ) and the value of κ is sometimes found to be 0, which makes the equilibrium problematic and hence the welfare calculation impossible. I choose to fix κ at 0.01, which is in the range of its values in the above exercise. Because fixing $\kappa = 0.01$ causes very small loss of fit, and because the welfare results from choosing (a, ρ, κ) if available, are very similar to what are reported below, I use the results from fixing κ and only choosing (a, ρ).

study the welfare cost of 10% inflation. As a benchmark, I consider a quantitative version of the model where there is no endogenous variety choice. That is, $\sigma(n) = 1$ and $k(n) = 0$.

In bargaining equilibrium, I choose (a, ρ) with different θs. Table 1 reports the parameter values from the model without variety choice and the model with variety choice. Table 2 contains the estimated welfare costs, where the 1st column is the welfare cost of inflation when there is no variety choice. The 2nd column through the 4th column contain the total welfare cost of inflation, the welfare cost of inflation due to quantity (variety is fixed), and the welfare cost of inflation due to variety (quantity is fixed).

Interestingly, the total welfare cost of inflation is decreasing in θ in the model without variety choice. However, the total welfare cost is non-monotonic in θ when the choice of variety is endogenous. This is because of the double holdup problem in bargaining equilibrium. When θ is very low, quantity is more distorted because households have too little bargaining power. The welfare cost of inflation is relatively more due to quantity. If θ is very high, variety is more distorted because firms have too little bargaining power. This is consistent with the result that most of the welfare cost is due to variety. In the example, $\theta = 0.5$ causes the least welfare cost of 10% inflation. Rocheteau and Wright (2004) also find that the welfare cost of inflation is non-monotonic in θ in their model when inflation affects both the intensive margin and the extensive margin.

Table 1.1: Parameter Values in Bargaining Equilibrium

	a (without n)	ρ (without n)	a	ρ
θ=0.1	1.0048	0.4754	0.8988	0.5515
θ=0.5	1.6733	0.1938	1.9033	0.3694
θ=0.9	1.7052	0.1143	2.1766	0.3758

Table 1.2: Welfare Cost in Bargaining Equilibrium

	Δ (without n)	Δ	Δ_q	Δ_n
θ=0.1	0.0927	0.084	0.0579	0.0063
θ=0.5	0.0321	0.0477	0.0312	0.0052
θ=0.9	0.0175	0.0589	0.0165	0.0212

Table 3 and table 4 report the results in price posting equilibrium. The total welfare cost of 10% inflation is only 1.52% of total consumption.[12] This is because price posting with

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[12] As suggested by a referee, the welfare cost of inflation can also be measured as the area underneath the

directed search is an efficient pricing mechanism in comparison to bargaining. In the model without variety choice, the welfare cost is only 1.47%. Surprisingly, if I fix quantity at the 10% inflation level and only consider the welfare cost of variety, Δ_n is negative. It seems that variety alone does not cause a large welfare cost in price posting equilibrium. Total welfare is increasing in variety in equilibrium, but not in general. The marginal benefit of variety depends on quantity. With a relatively low quantity level, simply increasing variety may not improve the economy's welfare.

Table 1.3: Parameter Values in Price Posting Equilibrium

a (without n)	ρ (without n)	a	ρ
1.8163	0.0978	2.2796	0.1875

Table 1.4: Welfare Cost in Price Posting Equilibrium Equilibrium

Δ (without n)	Δ	Δ_q	Δ_n
0.0147	0.0152	0.0131	-0.0023

1.6 Extension

So far I assume that if a household does not find the desired variety, the household does not consume at all. In this section, I relax this restriction. More specifically, if the household finds the ideal variety, the household gets utility $\delta u(q)$ from consuming q units of the ideal good where $\delta > 1$. If the household does not find the ideal variety, the household gets utility $u(q)$ from consuming q units of any non-ideal good. This specification is similar to Burdett and Shevchenko (2007). Each household receives the preference shock determining which variety is the ideal variety.

The household's problem in the CM remains the same as before. The value function in the DM is modified as

$$V(\hat{m}) = \alpha\sigma(n)[\delta u(q_1) + \beta W_+(\hat{m} - d_1)] \tag{1.28}$$
$$+\alpha[1 - \sigma(n)][u(q_0) + \beta W_+(\hat{m} - d_0)] + (1 - \alpha)\beta W_+(\hat{m}).$$

money demand curve. Following this method, the welfare cost of 10% inflation is 1.69% in price posting equilibrium, which is fairly close to the consumption equivalence measure.

Here (q_1, d_1) are the terms of trade if the household finds the ideal variety and (q_0, d_0) are the terms of trade if the household consumes a non-ideal variety. The probability for a household to find the ideal variety is $\alpha\sigma(n)$ and the probability for a household to consume the non-ideal variety is $\alpha[1 - \sigma(n)]$. With probability $1 - \alpha$, households are not matched and consume nothing. The firm's expected profit becomes

$$\pi = \max_n \left\{ -k(n) + \alpha\sigma(n)[-c(q_1) + \frac{1}{1+r}\phi_+ d_1] + \alpha[1 - \sigma(n)][-c(q_0) + \frac{1}{1+r}\phi_+ d_0] \right\}. \tag{1.29}$$

In the bargaining stage, there are two types of meetings. I use type 1 meeting to refer to a meeting where a household finds the ideal variety and use type 0 meeting to refer to a meeting where a household does not find the ideal variety. For a type j, $j \in \{0, 1\}$ meeting, the generalized Nash problem is

$$\max_{q_j, d_j} [\delta_j u(q_j) - \beta\phi_+ d_j]^\theta [-c(q_j) + \beta\phi_+ d_j]^{1-\theta} \tag{1.30}$$

$$\text{s.t. } d_j \leq \hat{m},$$

where $\delta_0 = 1$ and $\delta_1 = \delta$.

One can show that depending on (δ, θ), there are two types of solutions.[13] When δ and θ are not too low and when inflation is not too high, households that find the ideal variety spend all of the money in the DM. Households that do not find the ideal variety only spend part of the money and consume the efficient amount of the non-ideal goods. I can prove that inflation does not affect the consumption of the non-ideal varieties, but it still lowers the consumption of the ideal varieties. Therefore, inflation lowers expected surplus from trade and reduces variety. When δ and θ are not too low and when inflation is high, all households spend all of their money balances. Inflation tends to lower the consumption of both the ideal varieties and the non-ideal varieties in the DM. When firms receive the same real balance no matter which goods they sell, the marginal benefit of providing variety comes from less production associated with producing the ideal variety. Whether inflation reduces variety depends on whether inflation hurts the consumption of the non-ideal varieties more. As θ is not too small, it is more likely that inflation hurts the consumption of the non-ideal varieties more, which implies that the marginal benefit of variety decreases. So inflation can still reduce variety.

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[13] I only provide a summary of the findings in the extension. Detailed arguments are available upon request.

The other type of solution occurs when δ or θ is too low. All households spend all of their money balances. Inflation tends to reduce the consumption in the DM. However, when θ is low enough, inflation might increase variety. The intuition for this result is that inflation hurts the consumption of the ideal varieties more when θ is very low. As mentioned above, the marginal benefit of providing the ideal variety is from less production cost. If inflation hurts the consumption of the ideal varieties more, it implies that the marginal benefit of variety increases. Inflation may induce firms to increase variety to take advantage of the lower production. This result is different from the result in the baseline model.

1.7 Conclusion

This paper is motivated by the observation that inflation reduces variety in the marketplace. In a microfounded monetary model, I analyzed the effects of inflation on quantity and variety. With the two market structures that I considered – bargaining and price posting with directed search, inflation reduces both quantity and variety.[14] While the qualitative predictions from the two market structures are similar, the quantitative results are quite different. The welfare cost of inflation due to variety is negligible in price posting equilibrium, but it is much bigger in bargaining equilibrium. The paper also studied a theoretical extension where households are allowed to consume non-ideal varieties. This setup tends to lower the marginal benefit of variety for firms. As a result, depending on parameter values, inflation may or may not reduce variety.

There are several extensions of this research that are worth pursuing. First of all, it would be more desirable to explicitly model product variety following the literature on variety in international trade. Another extension is to allow free entry by firms. It would be useful to study how the two types of extensive margins interact when the inflation rate increases. In this paper, the model predicted that inflation monotonically reduced variety. When considering Japan in the 1990s and the U.S. in Great Depression, it seems that product variety also decreased during those deflationary episodes. Inflation and variety might have an inverse U-shape relationship. This conjecture requires more careful empirical support. Theoretically, the current model should be modified to study deflation and variety. All these

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[14]For completeness, I also considered Walrasian price taking in the DM. All of the main results remain valid with price taking. The only extra restriction is that $c(q)$ should not be linear. Otherwise, firms always offer the minimum variety since there is no profit from selling the product. Because I used a linear specification of $c(q)$ in the quantitative analysis, I did not discuss Walrasian price taking in detail in my paper.

extensions are left for future work.

1.8 Appendix

1.8.1 Proof of Proposition 1.2

Proof. In what follows, I restrict the attention to $q \in [0, \tilde{q}]$, where $\tilde{q}$ solves $\frac{u'(q)}{g'(q)} = 1$. For a household, the choice of $\hat{m}$ can also be formulated as a choice problem of q, where the household takes n as given and

$$\max_q \left\{ -ig(q) + \alpha\sigma(n)[u(q) - g(q)] \right\}. \tag{1.31}$$

Recall that the firm takes $\hat{m}$ as given and chooses n to maximize its expected profit in each CM. From the Nash bargaining solution, the firm's problem is equivalent to

$$\max_n \left\{ -k(n) + \alpha\sigma(n)[g(q) - c(q)] \right\}. \tag{1.32}$$

The solution of (q, n) from (1.31) and (1.32) is a Nash equilibrium. Let $\chi(q; n)$ be the first derivative of the objective function in (1.31),

$$\chi(q; n) = -ig'(q) + \alpha\sigma(n)[u'(q) - g'(q)].$$

Dividing both sides of the above expression by $u'(q)$, I have

$$\frac{\chi(q; n)}{u'(q)} = \alpha\sigma(n) - [i + \alpha\sigma(n)]\frac{g'(q)}{u'(q)}. \tag{1.33}$$

From assumption 1, the RHS of (1.33) is strictly decreasing in q. As $u(q)$ is strictly concave, $\frac{1}{u'(q)}$ is increasing in q. It follows that $\chi(q; n)$ must be decreasing in q, which further implies that $\chi'(q; n) < 0$. Since $\chi'(q; n)$ is the second derivative of (1.31), the household's objective function in (1.31) is concave. On the other hand, the firm's objective function in (1.32) is also concave because it is immediate to verify that $-k''(n) + \alpha\sigma''(n)[g(q) - c(q)] < 0$.

Notice that [1] both the household's objective function in (1.31) and the firm's objective function in (1.32) are continuous functions and concave; and [2] I only focus on $q \in [0, \tilde{q}]$ and $n \in [0, N]$. By the Nash's Existence Theorem (Ok, p348), the solution of (q, n) to (1.31) and (1.32) as a Nash equilibrium must exist.

The next step is to establish the existence of a monetary equilibrium under certain conditions. For $q = 0$, the household's objective function in (1.31) is 0 for any n. Similarly, for $n = 0$, the firm's objective function in (1.32) is 0 for any q. These two observations imply that (1.31) and (1.32) should not be negative in any equilibrium. Moreover,

it can be shown that for any $q > 0$, $-ig(q) + \alpha\sigma(n)[u(q) - g(q)] > 0$. As a result, if $\max_{q}\{-ig(q) + \alpha\sigma(n)[u(q) - g(q)]\} > 0$, it implies that the solution of q must not be 0 and hence $q > 0$. The only if part of the proposition is obvious. Once $q > 0$, $n > 0$ because $g(q) > c(q)$ ($\theta = 1$ implies that $q = n = 0$ in equilibrium.) in (1.15). ∎

1.8.2 Proof of Proposition 1.3

Proof. Define $f(q;i) = \frac{u'(q)}{g'(q)} - 1 - \frac{i}{\alpha\sigma[n(q)]}$, where $n(q)$ is implicitly defined by (1.15). Notice that $f(q;i) = 0$ gives the solution of bargaining equilibrium. As I can show that $g'(q) > c'(q)$, I know that from (1.15), $n(0) = 0$ and $\frac{dn}{dq} > 0$. By assumption 1, $\frac{g'(q)u''(q)-u'(q)g''(q)}{[g'(q)]^2} < 0$. From (1.17), $\frac{dq}{dn} \geq 0$. Also knowing that $\frac{dn}{dq} > 0$, the equilibrium with the highest q is also the equilibrium with the highest n. Differentiate (1.15) and (1.17) with respect to i,

$$\alpha[g'(q) - c'(q)]\frac{dq}{di} - \frac{\sigma'(n)k''(n) - k'(n)\sigma''(n)}{[\sigma'(n)]^2}\frac{dn}{di} = 0,$$

$$\frac{g'(q)u''(q) - u'(q)g''(q)}{[g'(q)]^2}\frac{dq}{di} + \frac{i\sigma'(n)}{\alpha[\sigma(n)]^2}\frac{dn}{di} = \frac{1}{\alpha\sigma(n)}.$$

The sign of $\frac{dn}{di}$ is the same as the sign of $\frac{dq}{di}$. After rearranging, $\frac{dq}{di}$ takes the sign of

$$\frac{g'(q)u''(q) - u'(q)g''(q)}{[g'(q)]^2} + \frac{i\sigma'(n)}{[\sigma(n)]^2}\frac{[g'(q) - c'(q)][\sigma'(n)]^2}{\sigma'(n)k''(n) - k'(n)\sigma''(n)}.$$

Differentiate $f(q;i)$ with respect to q

$$f'(q;i) = \frac{g'(q)u''(q) - u'(q)g''(q)}{[g'(q)]^2} + \frac{i\sigma'[n(q)]\frac{dn}{dq}}{\alpha\{\sigma[n(q)]\}^2}.$$

When $i \to 0$, $\lim_{i\to 0} f'(q;i) < 0$. Recall that $\tilde{q}$ is the solution of $\frac{u'(q)}{g'(q)} = 1$. The solution q to $f(q;i) = 0$ must lie between 0 and $\tilde{q}$. It can be shown that $0 < \tilde{q} \leq q^*$ and $n(\tilde{q}) > 0$. At $q = \tilde{q}$, $f(\tilde{q};i) = -\frac{i}{\alpha_b\sigma[n(\tilde{q})]} < 0$ for $i > 0$. Since $q \in (0, \tilde{q})$ for $i > 0$, the equilibrium with the highest q must satisfy $f'(q;i) < 0$. Therefore, $\frac{dq}{di} < 0$ and $\frac{dn}{di} < 0$. ∎

1.8.3 Proof of Proposition 1.4

Proof. From (1.17), $u'(q) = g'(q)$ at the Friedman rule. Hence, q is efficient if and only if $g(q) = c(q)$, which is true when $\theta = 1$. However, the choice of n goes to the corner solution since from (1.15), $-k'(n) < 0$. The optimal n is the minimum n. To get the efficient n from (1.15), one needs $g(q) - c(q) = u(q^*) - c(q^*)$. This is true only when $\theta = 0$ and

$q = q^*$. However, when $\theta = 0$, monetary equilibrium does not exist. To summarize, it is not possible to have both efficient q and efficient n in bargaining equilibrium. In (1.15), $g(q) - c(q) < u(q) - c(q) \leq u(q^*) - c(q^*)$. Since $\frac{k'(n)}{\sigma'(n)}$ is increasing in n, $n < n^*$.

In equilibrium, total welfare is increasing in both q and n. As $\frac{\partial q}{\partial \gamma} < 0$ and $\frac{\partial n}{\partial \gamma} < 0$ in equilibrium with the highest (q, n), inflation reduces total welfare. Therefore the Friedman rule is the optimal monetary policy. ∎

1.8.4 Proof of Lemma 1.5

Proof. I rewrite (1.21) as

$$\max_{q,d,Q,n} \left\{ \alpha_f[\alpha_h^{-1}(Q)]\sigma(n)[-c(q) + \beta\phi_+ d] - k[\sigma^{-1}(n)] \right\}$$

$$\text{s.t.} - [i + \alpha_h(Q)\sigma(n)]\beta\phi_+ d + \alpha_h(Q)\sigma(n)u(q) = \hat{U},$$

and restrict the constraint to the following compact set: $\Gamma(\hat{U}) = \{(q, d, n, \alpha_h(Q)) \in \mathbb{R}^4$, such that $q \in [0, q^*]$, $\beta\phi_+ d \in [c(q), u(q)]$, $\sigma(n) \in [0, 1]$, $\alpha_h(Q) \in [0, 1]$ and $-[i + \alpha_h(Q)\sigma(n)]\beta\phi_+ d + \alpha_h(Q)\sigma(n)u(q) \geq \hat{U}\}$. $\Gamma(\hat{U})$ is continuous and compact valued correspondence. The objective function is continuous. By the theorem of the maximum, the set of solutions $\Upsilon(\hat{U})$ is nonempty and upper-hemicontinuous. ∎

1.8.5 Proof of Lemma 1.6

Proof. It is obvious that $Q(\hat{U})$ is nonempty and upper-hemicontinuous from lemma 1. Define

$$\Psi(q, Q, n; \hat{U}) = \alpha_f(Q)\sigma(n)[-c(q) + \frac{\alpha_h(Q)\sigma(n)u(q) - \hat{U}}{i + \alpha_h(Q)\sigma(n)}] - k(n).$$

Consider $\hat{U}_1 > \hat{U}_0 > 0$. Hence, $\Upsilon(\hat{U}_1) = (q_1, d_1, Q_1, n_1)$ and $\Upsilon(\hat{U}_0) = (q_0, d_0, Q_0, n_0)$. First note that when $Q_0 = 0$, it must be true that $Q_1 = 0$. Now considering $Q_0 > 0$, it must be true that $\Psi(q_1, Q_1, n_1; \hat{U}_1) \geq \Psi(q_0, Q_0, n_0; \hat{U}_1)$ and $\Psi(q_0, Q_0, n_0; \hat{U}_0) \geq \Psi(q_1, Q_1, n_1; \hat{U}_0)$ It implies that $\Psi(q_1, Q_1, n_1; \hat{U}_1) - \Psi(q_1, Q_1, n_1; \hat{U}_0) \geq \Psi(q_0, Q_0, n_0; \hat{U}_1) - \Psi(q_0, Q_0, n_0; \hat{U}_0)$. From the definition of $\Psi(q, Q, n; \hat{U})$, I have

$$\frac{\alpha_f(Q_0)\sigma(n_0)}{i + \alpha_h(Q_0)\sigma(n_0)} \geq \frac{\alpha_f(Q_1)\sigma(n_1)}{i + \alpha_h(Q_1)\sigma(n_1)}.$$

Notice that $\alpha_f(Q)$ is increasing in Q, $\alpha_h(Q)$ is decreasing in Q and $\sigma(n)$ is increasing in n. For (Q_0, n_0) and (Q_1, n_1), the only possible cases are: 1) $Q_0 > Q_1$ and $n_0 > n_1$; 2)

$Q_0 > Q_1$ and $n_0 < n_1$; 3) $Q_0 < Q_1$ and $n_0 > n_1$; 4) $Q_0 > Q_1$ and $n_0 = n_1$; 5) $Q_0 = Q_1$ and $n_0 > n_1$; 6) $Q_0 = Q_1$ and $n_0 = n_1$.

Since $\hat{U}_1 > \hat{U}_0 > 0$, it must be true that $q_1 > 0$ and $q_2 > 0$ from (1.22). I will show in the next step that case 3), 5) and 6) can be ruled out. In cases 3), 5) and 6), $Q_1 \geq Q_0 > 0$ and thus it follows that (q_0, Q_0) and (q_1, Q_1) should satisfy (1.24) and (1.25). From (1.24), since $\frac{u'(q)}{c'(q)}$ is decreasing in q, $\alpha_h(Q)$ is decreasing in Q and $\sigma(n)$ is increasing in n, $q_0 > q_1$ in case 3), 5) and $q_0 = q_1$ in case 6). Rearranging (1.25),

$$u(q) - \frac{u'(q)}{c'(q)} \frac{\eta(Q)u'(q)c(q) + [1 - \eta(Q)]c'(q)u(q)}{\eta(Q)u'(q) + [1 - \eta(Q)]c'(q)} = \frac{\hat{U}}{\alpha_h(Q)\sigma(n)}. \tag{1.34}$$

Define the LHS of (1.34) as $G(q, Q)$.

$$G(q, Q) = u(q)\left\{1 - \frac{\eta(Q)\frac{u'(q)c(q)}{c'(q)u(q)} + [1 - \eta(Q)]}{\eta(Q) + [1 - \eta(Q)]\frac{c'(q)}{u'(q)}}\right\}.$$

By assumption 2 (i) and (ii), I can show that $\frac{u'(q)c(q)}{c'(q)u(q)}$ is decreasing in q, which further implies that $\frac{\partial G(q,Q)}{\partial q} > 0$. By assumption 2, $\frac{\partial G(q,Q)}{\partial Q} \leq 0$. In case 3) and 5), $G(q_0, Q_0) > G(q_1, Q_1)$. In case 6), $G(q_0, Q_0) = G(q_1, Q_1)$. Now consider the RHS of (1.34). In case 3) and 5), $\frac{\hat{U}_0}{\alpha_h(Q_0)\sigma(n_0)} < \frac{\hat{U}_1}{\alpha_h(Q_1)\sigma(n_1)}$, which is a contradiction with $G(q_0, Q_0) > G(q_1, Q_1)$. In case 6), $\frac{\hat{U}_0}{\alpha_h(Q_0)\sigma(n_0)} < \frac{\hat{U}_1}{\alpha_h(Q_1)\sigma(n_1)}$, which is also a contradiction with $G(q_0, Q_0) = G(q_1, Q_1)$. To summarize, given that $\hat{U}_0 < \hat{U}_1$, it is only possible that $Q_0 > Q_1$. So $Q(\hat{U})$ is strictly decreasing in Q. ∎

1.8.6 Proof of Proposition 1.8

Proof. In equilibrium, $\sum_\omega F_\omega = 1$, $\sum_\omega F_\omega Q_\omega = 1$. It implies that 1 belongs to $\tilde{Q}(\hat{U})$, where $\tilde{Q}(\hat{U})$ is the convex hull of $Q(\hat{U})$. $\tilde{Q}(\hat{U})$ is convex-valued and upper hemicontinuous. By lemma 2, $\tilde{Q}(\hat{U})$ is strictly decreasing in $\hat{U}$. If $Q(0) < 1$, the only equilibrium is to have $\hat{U} = 0$. Let $Q(0)$ be the demand for households when $\hat{U} = 0$ and it can be solved from (1.24)-(1.26). When $\hat{U} > u(q^*) - c(q^*)$, $Q(\hat{U}) = 0$ and $\tilde{Q}(\hat{U}) = \{0\}$. Since households are indifferent whether participating or not, $Q = Q(0)$ and (q, n) solves (1.24) and (1.26). If $Q(0) \geq 1$, $\tilde{Q}(\hat{U}) = 1$ determines a unique $\hat{U}$ because $\tilde{Q}(\hat{U})$ is strictly decreasing in $\hat{U}$. Note that the unique $\hat{U}$ may admit multiple Q. From now on, I focus on symmetric equilibrium. In a symmetric equilibrium, $\hat{U}$ determines a unique Q. Notice that $\hat{U}$ also determines a

unique Q if $Q(\hat{U})$ is convex-valued or if $Q(\hat{U})$ is a function. Once Q is unique and $Q(0) > 1$, $Q = 1$. To summarize, $Q = \min\{Q(0), 1\}$ in equilibrium.

Knowing Q, (1.24) and (1.26) can be used to solve for interior (q, n). From (1.24), $\frac{dq}{dn} > 0$. Let $\varphi(q) = \frac{c'(q)[u(q)-c(q)]}{\eta(Q)u'(q)+[1-\eta(Q)]}$. It is immediate that $\varphi'(q) > 0$ for $q \in (0, q^*]$. From (37), $\frac{dn}{dq} > 0$. If there exist multiple solutions of (q, n), these solutions also need to satisfy (1.25). Since $\frac{dq}{dn} > 0$ and $\frac{dn}{dq} > 0$, it is not possible to have multiple solutions that satisfy (1.25). Therefore, (q, n) is also unique. The equilibrium with interior solutions is also unique.

To focus on monetary equilibrium, one needs to rule out $q = n = 0$ as a solution. If $Q(0) > 1$, $Q = 1$ and equilibrium $\hat{U} > 0$, which implies that $q > 0$ from (1.22). That is, monetary equilibrium must exist if $Q(0) > 1$. If $Q(0) \leq 1$, then $Q = Q(0)$ and $\hat{U} = 0$ in equilibrium. In this case, if $\max_{q,n} \left\{ \alpha_f(Q)\sigma(n)[-c(q) + \frac{\alpha_h(Q)\sigma(n)u(q)}{i+\alpha_h(Q)\sigma(n)}] - k(n) \right\} > 0$ where $Q = Q(0)$, q must not be 0 and the solution entails $q > 0$. ■

1.8.7 Proof of Proposition 1.9

Proof. Differentiate (1.24) and (1.26) with respect to i,

$$\frac{c'(q)u''(q) - u'(q)c''(q)}{[c'(q)]^2}\frac{dq}{di} + \frac{i\sigma'(n)}{\alpha_h(Q)[\sigma(n)]^2}\frac{dn}{di} = \frac{1}{\alpha_h(Q)\sigma(n)},$$

$$\alpha_f(Q)\varphi'(q)\frac{dq}{di} - \frac{\sigma'(n)k''(n) - k'(n)\sigma''(n)}{[\sigma'(n)]^2}\frac{dn}{di} = 0.$$

It follows that $\frac{dq}{di}$ and $\frac{dn}{di}$ take the same sign. After rearranging, the sign of $\frac{dq}{di}$ depends on the sign of

$$\frac{c'(q)u''(q) - u'(q)c''(q)}{[c'(q)]^2} + \frac{i\sigma'(n)}{\alpha_h(Q)[\sigma(n)]^2}\frac{\alpha_f(Q)\varphi'(q)[\sigma'(n)]^2}{\sigma'(n)k''(n) - k'(n)\sigma''(n)}.$$

Let $f(q; i) = \frac{u'(q)}{c'(q)} - 1 - \frac{i}{\alpha_h(Q)\sigma[n(q)]}$, where $n(q)$ is implicitly defined by (1.26). The solution of q is given by $f(q; i) = 0$. Differentiate $f(q; i)$ with respect to q:

$$f'(q; i) = \frac{c'(q)u''(q) - u'(q)c''(q)}{[c'(q)]^2} + \frac{i\sigma'(n)}{\alpha_h(Q)[\sigma(n)]^2}\frac{dn}{dq}.$$

At $q = q^*$, $f(q^*; i) = -\frac{i}{\alpha_h(Q)\sigma[n(q^*)]} \leq 0$. Since I restrict the solution to $q \in (0, q^*]$, it implies that $f'(q; i) < 0$ when $f(q; i) = 0$ for $q \in (0, q^*)$. As the solution of q converges to q^* only at $i \to 0$, $\lim_{i \to 0} f'(q^*; 0) = \frac{c'(q^*)u''(q^*) - u'(q^*)c''(q^*)}{[c'(q^*)]^2} < 0$. For solution $q \in (0, q^*]$, $f'(q; i) < 0$. Therefore, $\frac{dq}{di} < 0$ and furthermore, $\frac{dn}{di} < 0$. It is equivalent to conclude that $\frac{dq}{d\gamma} < 0$ and $\frac{dn}{d\gamma} < 0$. ■

1.8.8 Proof of Proposition 1.10

Proof. From (1.24), q is efficient when $i \to 0$. The Friedman rule achieves the efficient q. For n, as long as q is efficient, n is also efficient. When $i > 0$, (1.24) implies that $q < q^*$. Since $u'(q) > c'(q)$ and $u(q) - c(q) < u(q^*) - c(q^*)$ for $q \in (0, q^*)$, $\varphi(q)[u(q) - c(q)] < u(q^*) - c(q^*)$. Because $\frac{k'(n)}{\sigma'(n)}$ is increasing in n, in general $n < n^*$. $\blacksquare$

1.9 References

Acemoglu, Daron and Robert Shimer, "Holdups and Efficiency with Search Frictions," *International Economic Review*, Vol. 40 (1999), pp. 827-849.

Aruoba, Boragan, Guillaume Rocheteau and Christopher J. Waller, "Bargaining and the Value of Money," *Journal of Monetary Economics*, Vol. 54 (2007), pp. 2636-2655.

Broda, Christian and David Weinstein, "Globalization and the Gains from Variety", *Quarterly Journal of Economics*, Vol. 121 (2006), pp. 541-585.

Burdett, Ken and Andrei Shevchenko, "Money and the Variety of Goods," manuscript, Michigan State University, 2007.

Burdett, Ken, Shouyong Shi and Randall Wright, "Pricing and Matching with Frictions," *Journal of Political Economy*, Vol. 109 (2001), pp. 1060-1085.

Camera, Gabriele, Robert R. Reed and Christopher J. Waller, "Jack of All Traders or a Master of One? Specialization, Trade, and Money," *International Economic Review*, Vol. 44 (2003), pp. 1275-1294.

CCR, "Evolución de la Cantidad de SKUs Vendidos Antes y Después de la Devaluación en las Cadenas de Hipermercados, Supermercados y Hard Discounts en la Argentina", 2003.

Corbae, Dean and Borghan Narajabad, "A Hotelling Model with Money," manuscript, The University of Texas at Austin, 2007.

Craig, Ben and Guillaume Rocheteau, "Inflation and Welfare: A Search Approach," *Journal of Money, Credit, and Banking*, Vol. 40 (2008), pp. 89-119.

Ghossoub, Edgar and Robert R. Reed, "Money and Specialization in a Neoclassical Growth Model," *Journal of Money, Credit, and Banking*, Vol. 37 (2005), pp. 969-975.

Guardiano, John, "Exploding the Myth about Economic Reform in Russia," Backgrounder #930, The Heritage Foundation, 1993.

Guttmann, William and Patricia Meehan, The Great Inflation, Germany 1919-23 (Farnborough, Eng.: Saxon House, 1975).

Heymann, Daniel and Axel Leijonhufvud, High Inflation (Oxford University Press, 1995).

International Herald Tribune, "Inflation in Zimbabwe Surges to Record 3714 Percent, the Highest in the World", the Associate Press, May 17th, 2007.

Kiyotaki, Nobuhiro and Randall Wright, "A Search-Theoretic Approach to Monetary Economics," *American Economic Review*, Vol. 83 (1993), pp. 63-77.

Lagos, Ricardo and Guillaume Rocheteau, "Inflation, Output, and Welfare," *International Economic Review*, Vol. 46 (2005), pp. 495-522.

Lagos, Ricardo and Randall Wright, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, Vol. 113 (2005), pp. 463-484.

Laing, Derek, Victor Li and Ping Wang, "Inflation and Productive Activity in a Multiple-Matching Model of Money," *Journal of Monetary Economics*, 54 (2007), pp. 1949-1961.

Lancaster, Kelvin, "The Economics of Product Variety: A Survey," *Marketing Science*, 9 (1990), 189-206.

Lucas, Robert E., "Inflation and Welfare," *Econometrica*, Vol. 68 (2000), pp. 247-274.

McKenzie, David and Ernesto Schargrodsky, "Buying Less, but Shopping More: the Use of Non-Market Labor During a Crisis," BREAD Working Paper, No. 92, 2006.

Midrigan, Virgiliu, "Comments on Monetary Policy and Business Cycle with Endogenous Entry and Product Variety by Bilbiie, Ghironi and Metlitz," manuscript, New York University, 2007.

Moen, Espen, ”Competitive Search Equilibrium,” *Journal of Political Economy*, Vol. 105 (1997), pp. 385-411.

Ok. Efe A., Real Analysis with Economic Applications (Princeton and Oxford: Princeton University Press, 2007).

Rocheteau, Guillaume and Randall Wright, ”Inflation and Welfare in Models with Trading Frictions,” in D. Altig and E. Nosal, eds., Monetary Policy in Low Inflation Economies (Cambridge University Press, forthcoming).

Rocheteau, Guillaume and Randall Wright, ”Money in Search Equilibrium, in Competitive Equilibrium and in Competitive Search Equilibrium,” *Econometrica*, Vol. 73 (2005), pp. 175-202.

Shevchenko, Andrei, ”Middleman,” *International Economic Review*, Vol. 45 (2004), pp. 1-24.

Shi, Shouyong, ”Money and Specialization,” *Economic Theory*, Vol. 10 (1997), pp. 99-113.

White, Lawrence J., ”Market Structure and Product Varieties,” *American Economic Review*, Vol. 67 (1977), pp. 179-182.

Chapter 2

Money and Costly Credit[1]

2.1 Introduction

This paper develops a model of money and credit in order to study issues in monetary economics concerning the choice of payment methods and the effects of inflation. Not too long ago, consumers typically paid for things using either cash or checks. A check is essentially an IOU for a future cash payment; to be drawn from the consumer's bank account and credited to the merchant's bank account. In recent decades, the payment instruments available to consumers have expanded to include debit and credit cards. A debit transaction is essentially the electronic equivalent of a cash transaction. A credit transaction in some ways resembles a check transaction; except that the credit is now offered by some third party (rather than the merchant), with this third party willing to postpone debt settlement to the indefinite future (typically at very high rates of interest). The key difference between credit and the other forms of means of payment is that the acquisition of money is not necessary to make a purchase using credit, whereas money must be acquired prior to purchasing with the other forms of means of payment.[2] In some sense, credit transactions are settled "later", while transactions using the other means of payment are settled "now".

[2]Checks can be viewed as a short-term form credit. Since the time until settlement is typically very short for checks, checks are more like cash or debit cards although postal checks resemble credit cards from a historical perspective.

Based on a report from Bank for International Settlements, in 2005 the percentage of total number of transactions using cards with a credit function was 23.4% in the U.S., 13% in the UK and 24.8% in Canada. As Figure 1 shows, both the ratio of consumer revolving credit to GDP and the ratio of consumer revolving credit to M1 have been increasing in recent decades in the U.S..[3] It appears obvious that credit has become increasingly important as a means of payment.

As money is also an important means of payment, one may wonder how the introduction of credit affects money.[4] In particular, does credit decrease money demand? How does credit affect the transmission of monetary policy? What is the effect of monetary policy on credit? These questions are interesting and important to the conduct of monetary policy in an economy in which credit is a popular means of payment.
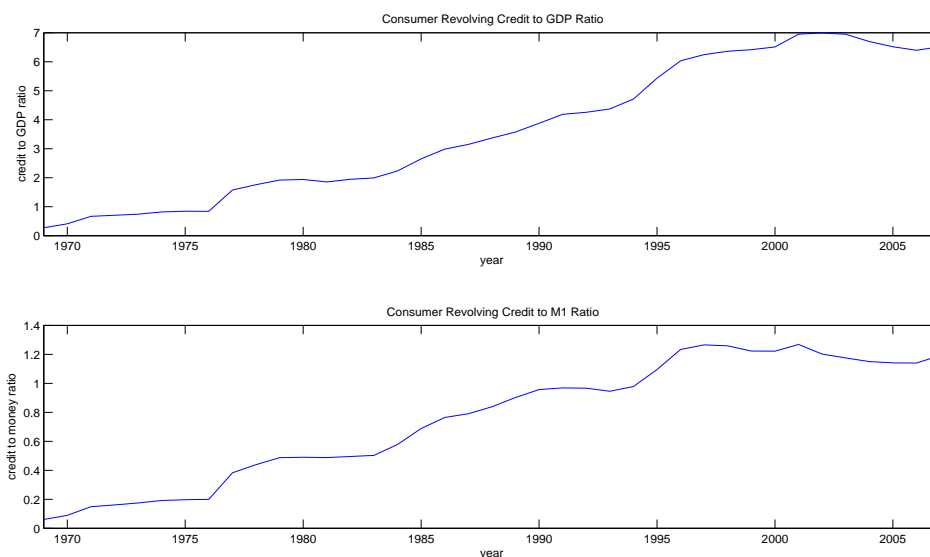
Figure 2.1: Credit to GDP Ratio and Credit to Money Ratio

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[3]The data are from the Federal Reserve Bank of St. Louis. Consumer revolving credit is a stock variable that measures outstanding credit balances. It would have been ideal to use the volume of credit card transactions to illustrate the trend increase in the usage of consumer credit. Due to the data availability, I use the stock variable. Table 3 in the appendix shows that outstanding consumer revolving credit accounts for around 35% of the total credit card transaction volume in 2000 and 2005. If this ratio is stable, it is reasonable to believe that this stock variable can approximately measure the increasing popularity of credit card transactions.

[4]Money in this paper includes cash, checks and debit cards, which are settled "now" as opposed to credit, which is settled "later".

It seems clear enough that technological advances in electronic record-keeping have facilitated the use of consumer credit. Specialized intermediaries are now able to offer consumer credit balances with limits that can vary with each person's recorded credit history. These technological advances allow credit to substitute for money as a means of payment. Indeed, this appears to be supported by the evidence. For example, Duca and Whitesell (1995) estimate based on U.S. household-level data that for every 10% increase in the probability of owning a credit card, checking balances are reduced by 9%.

The evidence on how inflation affects the money-credit margin is less clear. Empirical investigation here is hampered to some extent by data limitations and the recent secular improvements in credit card technology. Nevertheless, the evidence from various high-inflation episodes suggests that high inflation hampers the use of credit as a means of payment. Credit cards gained widespread popularity in Brazil following the successful reduction of inflation to sustainable levels, with the number of cards in force growing by 88% between 2000 and 2004. In Colombia, because of lower inflation and lending rates, the proportion of households with formal access to credit was expected to increase by 25% from 2004 to 2008. High inflation episodes also delayed the adoption and widespread use of credit cards in Turkey. Even in Australia, households' debts increased dramatically due to the lower inflation rates and thus lower cost of borrowing during the 1990s.[5] Reducing the inflation rate is perceived to promote the use of credit.

Other empirical evidence regarding the relationship between inflation and credit is based on a broader measure of credit – namely the ratio of total private credit to GDP.[6] Using a sample of 97 countries, Boyd et al. (2001) conclude that inflation has a negative impact on credit. See also Boyd and Champ (2003). Later, Khan et al. (2006) also use a large cross-country sample, but they find that there is a threshold effect of inflation on credit. Inflation has a negative impact on credit when it exceeds a threshold. The evidence based on total private credit suggests that inflation tends to have a negative impact on credit at high rates, but not at low rates.

In this paper, I propose a model that is able to replicate the evidence on inflation,

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[5]The inflation rate was on average around 8% in Australia in the 1980s and was reduced to around 3% in the 1990s.

[6]Total private credit may be too broad in comparison to consumer credit. Given that [1] it is difficult to obtain data on consumer credit for a large sample of countries over an adequately long time span; and [2] different measures of credit tend to highly comove, as can be seen from the U.S. data, it seems useful and reasonable to review this evidence.

money demand and credit. The model is built on Lagos and Wright (2005). In monetary theory, frictions that render money essential make credit arrangements impossible. In order for credit to exist, I assume that there exist competitive financial intermediaries that can identify agents and have access to a record-keeping technology. There are two frictions associated with credit arrangements. First, arranging credit is costly. In a bilateral trade, if a buyer wants to use credit, he must incur a fixed utility cost in order to make the seller and himself identified to a financial intermediary. When buyers have heterogeneous preferences, the fixed cost of credit will endogenously determine the fraction of buyers using credit. Second, the settlement of credit is available only at a particular time in each period, during which the financial intermediaries accept repayment of credit and settle debts. Due to the timing structure of the model, settlement is "delayed" and money becomes the only means of settlement.

These two features of the model allow some interesting interactions between money and credit. Inflation tends to increase the fraction of buyers using credit at low inflation rates, but decrease the fraction of buyers using credit at high inflation rates. Compared to an economy without credit, the model has three implications: [1] the real demand for money is lower at low to moderate inflation rates; [2] social welfare is higher when the inflation rate exceeds a specific threshold; and [3] for a given inflation rate, the welfare cost of inflation can be higher for some reasonable values of the credit cost parameter.

Several recent papers have attempted to allow the coexistence of money and credit.[7] In general, some imperfections associated with credit should be incorporated to sustain the essentiality of money and permit the existence of credit. Sanches and Williamson (2008) adopt the notion of limited participation in the sense that only an exogenous subset of agents can use credit. While the banks in Berentsen et al. (2007) can record financial history, they cannot record goods transaction history so that credit takes the form of bank loans and bank loans must be taken in the form of fiat money. Telyukova and Wright (2008) build a model where agents can use money and credit to explain the credit card debt puzzle. Their market structure determines that agents do not use money and credit simultaneously. A more related paper is Chiu and Meh (2008). They study how banks as in Berentsen et al.

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[7]The literature on money and credit is vast and thus I cannot hope to review it in its totality. Rather, I will only mention a few papers that are more recent or are more or less directly related to this paper. For example, for recent attempts to rationalize the coexistence of inside and outside money, see Cavalcanti and Wallace (1998), Kocherlakota and Wallace (1998), Mills (2007), Sun (2007). A more recent paper by Lester et al. (2008) studies the coexistence of multiple assets which differ in their return and liquidity.

(2007) affect allocations and welfare in an economy where ideas (or projects) are traded among investors and heterogeneous entrepreneurs. The role of banking is similar to credit, but money is the only means of payment in their paper. In this paper, both money and credit can serve as means of payment, and the choice of payment methods is endogenous.

In terms of the model's prediction on inflation and credit, the paper's result is similar to the result of Azariadis and Smith (1996). The key friction for their result is asymmetric information associated with using credit. This paper instead considers different frictions that affect the use of credit. Several papers have used the notion of costly credit in the Cash-in-Advance model or in the OLG model.[8] With the fixed cost, it is not surprising that inflation always decreases money demand and increases credit demand. I label the effect of inflation on credit through the fixed cost channel as the fixed cost effect. The "delayed" settlement has been used in Ferraris (2006), where money and credit are complements. In fact, this idea can be traced back to Stockman (1981), where he shows that inflation reduces the capital stock if money and capital are complements. The delayed settlement effect of inflation on credit is that inflation should reduce credit. As credit is subject to both frictions in this paper, it turns out that the fixed cost effect dominates at low inflation rates and the delayed settlement effect dominates at high inflation rates. This prediction is consistent with the empirical evidence cited above.

The rest of the paper is organized as follows. Section 2 lays out the physical environment. Section 3 solves for the equilibrium and analyzes the equilibrium when the repayment of credit can be enforced. I numerically study the model in Section 4. In Section 5, I consider monetary equilibrium when the repayment of credit cannot be enforced. Finally, Section 6 concludes. All proofs are provided in the Appendix.

2.2 Environment

Time is discrete and runs forever. In each period, there are three submarkets that open sequentially. The first submarket is characterized by bilateral trades and is labelled as market 1. The second submarket is characterized by a centrally located competitive spot market and is labelled as market 2. No trade occurs in the third submarket and it is labelled as market 3. The activity in the third submarket will be described in detail later. There

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[8]For Cash-in-Advance models, see Lacker and Shreft (1996), Aiyagari et al. (1998), and English (1999) for examples. For the OLG framework, see Freeman and Huffman (1991) for an example.

are two permanent types of agents – buyers and sellers, each with measure 1. Buyers are those who want to consume in market 1 and sellers are those who produce in market 1. All agents are anonymous and lack commitment. Each buyer receives a preference shock ε at the beginning of each period, which determines the buyer's preference in market 1.[9] The preference shock ε is drawn from a *c.d.f.* $G(\varepsilon)$. The preference shocks are *iid* across buyers and across time. The realization of these preference shocks is public information. There are two types of goods. Goods that are produced and consumed in market 1(2) are called good 1(2). All goods are nonstorable.

In market 1, buyers and sellers are matched randomly according to a matching technology. The probability that a buyer (seller) meets a seller (buyer) is σ with $0 < \sigma \le 1$. Given that a buyer and a seller meet, the terms of trade are determined by the buyer's take-it-or-leave-it offer. After exiting market 1, all agents enter market 2. Buyers supply labor for production and consume good 2. Sellers only consume good 2. For simplicity, the production technology in market 2 is assumed to be linear and 1 unit labor can be converted into 1 unit of good 2.

The preference of a buyer with a preference shock ε is $\varepsilon u(q) + v(x) - h$, where $\varepsilon u(q)$ is the buyer's utility from consuming q units of good 1. As usual, $u(0) = 0$, $u'(0) = \infty$ and $u''(q) < 0 < u'(q)$. In market 2, the buyer's utility from consuming x units of good 2 is $v(x)$, where $\lim_{x \to 0} v'(x) = \infty$ and $v''(x) < 0 < v'(x)$. The buyer's disutility from working is h. The preference of a seller is $-c(q) + y$, where $c(q)$ is the seller's disutility from producing q units of good 1 with $c(0) = 0$, $c'(0) = 0$, $c'(q) > 0$ and $c''(q) \ge 0$. The seller has a linear utility in market 2, where y is the amount of consumption of good 2. All agents discount between market 3 and the next market 1. The discount rate is β.

Now I consider a planner's problem as the benchmark allocation. Suppose that the planner weights all agents equally and is subject to the random matching technology. I restrict the attention to stationary allocations in what follows. In market 1 of each period, given a buyer's preference shock ε and the buyer meeting a seller, the planner instructs the seller to produce $q(\varepsilon)$ for the buyer. Those agents who do not find a match consume and produce nothing. In market 2 of each period, the planner assigns the consumption of good 2 x, y and the labor supply h subject to the resource constraint. Formally, the planner's

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[9]There are a variety of ways to model the heterogeneity in this model. For example, one can model heterogeneous sellers that have different cost functions or model heterogeneity as match specific shocks.

problem is

$$\max_{q(\varepsilon),x,y,h} \left\{ \sigma \int [\varepsilon u(q(\varepsilon)) - c(q(\varepsilon))]dG(\varepsilon) + v(x) - h + y \right\} \tag{2.1}$$

$$\text{s.t. } x + y = h.$$

The optimal $(x^*, q^*(\varepsilon))$ are characterized by $v'(x) = 1$ and $\varepsilon u'(q(\varepsilon)) = c'(q(\varepsilon))$ for all ε. The optimal $q(\varepsilon)$ is increasing in ε. In fact, the optimal allocation features a slight indeterminacy. That is, given the quasi-linear preference structure, h and y are indeterminate as long as $h - y = x^*$.

The planner's allocation cannot be implemented in the economy since agents are anonymous and lack commitment. As a result, bilateral trade credit is not possible and thus money is essential. I assume that there exists a monetary authority that controls the supply of money. Let M denote the aggregate money supply at any given date. It grows at a gross rate $\gamma > 0$, i.e., $\hat{M} = \gamma M$. Here the hat denotes the variable in the next period. I will consider $\gamma > \beta$ and $\gamma \to \beta$ from above. New money is injected (or withdrawn) via a lump-sum transfer (or tax) to each buyer at the beginning of each period and the transfer is $\hat{\tau} = (\gamma - 1)M$.

Besides the monetary authority, there exist competitive financial intermediaries. These financial intermediaries possess a record-keeping technology, which allows them to identify agents and keep track of goods market transaction history. Clearly the availability of the record-keeping technology makes credit arrangements through the financial intermediaries possible in this economy. To sustain the essentiality of money, I assume two frictions associated with the record-keeping technology. The first friction is that the record-keeping technology or the financial intermediaries are not available in market 2. This restriction implies that agents may arrange credit transactions in market 1, but cannot settle their debts in market 2. As the financial intermediaries are available in market 3, buyers who have used credit in market 1 repay their debts and sellers who have extended credit get repayment in market 3. One can think of market 3 as an overnight market for settlement. Without such a restriction, agents would want to settle their debts in market 2. In some sense, the settlement of debts is delayed. Since goods are nonstorable, money becomes the only means of settlement. The second friction associated with the record-keeping technology is that it is costly. As all agents are anonymous, the buyer in a match in market 1 can incur a fixed utility cost k to make the pair identifiable to a financial intermediary so that the seller can

extend credit to the buyer.[10] Without incurring the fixed cost, the buyer and the seller remain anonymous and cannot make credit arrangements. I provide a timeline of events in Figure 2.
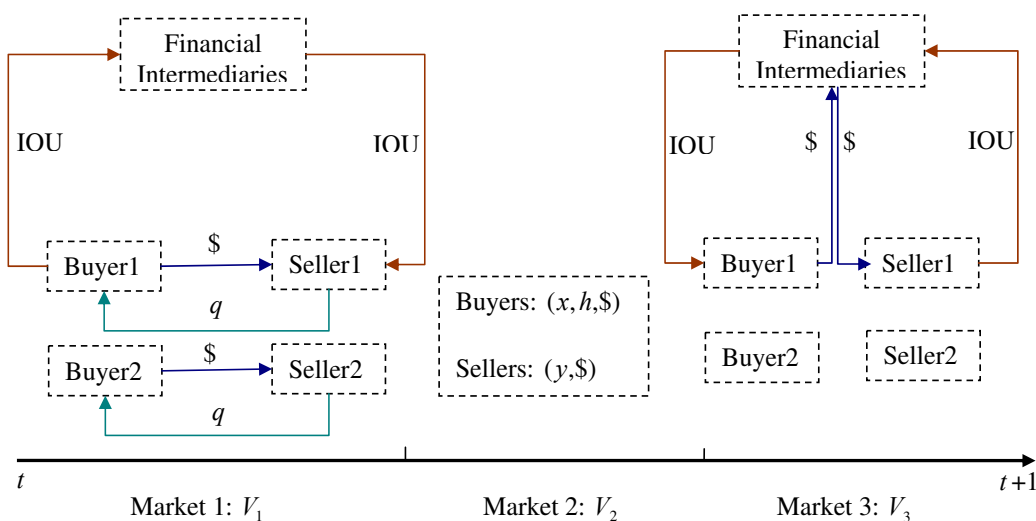
Figure 2.2: Timeline of Events

2.3   Monetary Equilibrium with Enforcement

In this section, I assume that there is perfect enforcement in the economy. It implies that the financial intermediaries can enforce the repayment of credit, so there is no credit limit for buyers. It also implies that the monetary authority can impose lump-sum taxes, i.e., $\gamma < 1$ is feasible.

2.3.1   Buyers

To facilitate the analysis, I begin with buyers in market 2. Suppose that in nominal terms, a buyer carries money balance m and debt ℓ at the beginning of market 2. Let $V_2^b(m, \ell)$ and $V_3^b(z, \ell)$ be the value functions for a buyer in market 2 and 3, respectively.

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[10]One may argue that in reality, sellers actually pay the cost of using credit. The model can be modified to have the seller pay the fixed cost in a match. All the main results hold.

Notice that the buyer cannot pay off his debt in market 2 because the financial intermediaries are not available. However, the buyer can accumulate money balance. Let z denote the money balance that the buyer carries to market 3. The buyer's choice problem is

$$V_2^b(m, \ell) = \max_{x,h,z} \left\{ v(x) - h + V_3^b(z, \ell) \right\} \tag{2.2}$$

$$\text{s.t. } x + \phi z = \phi m + h,$$

where ϕ is the inverse of the price level (or the value of money). Substituting h from the buyer's budget constraint into (2.2), the unconstrained problem is

$$V_2^b(m, \ell) = \phi m + \max_{x,z} \{ v(x) - x - \phi z + V_3^b(z, \ell) \}.$$

The first order conditions for interior solutions are $v'(x) = 1$ and

$$\frac{\partial V_3^b(z, \ell)}{\partial z} = \phi. \tag{2.3}$$

As is standard, the choice of z does not depend on m; however, it depends on ℓ. Intuitively, if the buyer incurs more debt in market 1, he must accumulate more money in market 2 for repayment in market 3. The envelope conditions imply

$$\frac{\partial V_2^b(m, \ell)}{\partial m} = \phi, \tag{2.4}$$

$$\frac{\partial V_2^b(m, \ell)}{\partial \ell} = \frac{\partial V_3^b(z, \ell)}{\partial \ell}. \tag{2.5}$$

Note that $V_2^b(m, \ell)$ is linear in m.

For the buyer entering market 3, the value function is

$$V_3^b(z, \ell) = \beta \int \hat{V}_1^b(z - \ell + \hat{\tau}, 0; \varepsilon) dG(\varepsilon).$$

The only activity for the buyer in market 3 is to repay his debt. Due to the quasilinear structure of the buyer's preference, the buyer should be indifferent between repaying the debt in the current market 3 or in any future market 3. I assume that if the buyer has any debt, he repays in the current market 3. To simplify notations, let $\hat{m} = z - \ell + \hat{\tau}$ be the buyer's money holding at the beginning of the next period. For a buyer receiving a preference shock ε at the beginning of the next period, let $\hat{V}_1^b(\hat{m}, 0; \varepsilon)$ be the buyer's value function. Since $\hat{V}_1^b(\hat{m}, 0; \varepsilon)$ depends on ε, I take the expected value for the buyer in market

3 and discount it by β. The envelope conditions yield

$$\frac{\partial V_3^b(z,\ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^b(\hat{m},0;\varepsilon)}{\partial \hat{m}} dG(\varepsilon), \tag{2.6}$$

$$\frac{\partial V_3^b(z,\ell)}{\partial \ell} = -\beta \int \frac{\partial \hat{V}_1^b(\hat{m},0;\varepsilon)}{\partial \hat{m}} dG(\varepsilon). \tag{2.7}$$

Combining (2.3) and (2.5) with (2.6) and (2.7),

$$\frac{\partial V_3^b(z,\ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^b(\hat{m},0;\varepsilon)}{\partial \hat{m}} dG(\varepsilon) = -\frac{\partial V_3^b(z,\ell)}{\partial \ell} = -\frac{\partial V_2^b(m,\ell)}{\partial \ell} = \phi. \tag{2.8}$$

From (2.8), $V_2^b(m,\ell)$ is linear in ℓ and $V_2^b(m,\ell) = \phi m - \phi \ell + V_2^b(0,0)$.

After exiting market 3, each buyer realizes a preference shock ε. For a buyer with ε, the value function in market 1 is

$$V_1^b(m,0;\varepsilon) = \sigma[\varepsilon u(q) - k \cdot I(a) + V_2^b(m - d, a \cdot I(a))] + (1 - \sigma)V_2^b(m,0),$$

where (q,d,a) are the terms of trade. With probability σ, the buyer spends d units of money and uses a units of credit in nominal terms in exchange for q units of good 1 from the seller. An indicator function $I(a)$ is such that $I(a) = 1$ if $a > 0$ and $I(a) = 0$ if $a = 0$. With probability $1 - \sigma$, the buyer is not matched and carries his money to market 2.

2.3.2 Sellers

Let $V_2^s(m,\ell)$ and $V_3^s(z,\ell)$ be a seller's value functions in market 2 and 3, respectively. Since the seller is the creditor, ℓ should be either 0 or negative. The seller's value function in market 2 is

$$V_2^s(m,\ell) = \max_{y,z} \{y + V_3^s(z,\ell)\} \tag{2.9}$$

$$\text{s.t. } y + \phi z = \phi m.$$

By substituting y from the constraint into (2.9), the first order condition of the unconstrained problem is

$$\frac{\partial V_3^s(z,\ell)}{\partial z} \leq \phi, \text{ and } z = 0 \text{ if } \frac{\partial V_3^s(z,\ell)}{\partial z} < \phi. \tag{2.10}$$

The envelope conditions yield

$$\frac{\partial V_2^s(m,\ell)}{\partial m} = \phi, \tag{2.11}$$

$$\frac{\partial V_2^s(m,\ell)}{\partial \ell} = \frac{\partial V_3^s(z,\ell)}{\partial \ell}. \tag{2.12}$$

Again, $V_2^s(m, \ell)$ is linear in m.

For the seller in market 3, the value function is

$$V_3^s(z, \ell) = \beta \int \hat{V}_1^s(z - \ell, 0; \varepsilon) dG(\varepsilon).$$

If the seller has extended any credit in the previous market 1, the seller will receive repayment from the financial intermediary in market 3. I take the expected value function of the seller because the seller anticipates that a potential buyer he will meet in the next market 1 may have a preference shock ε drawn from $G(\varepsilon)$. Let $\hat{m} = z - \ell$ denote the seller's money holding at the beginning of the next market 1. The envelope conditions are

$$\frac{\partial V_3^s(z, \ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon), \tag{2.13}$$

$$\frac{\partial V_3^s(z, \ell)}{\partial \ell} = -\beta \int \frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon). \tag{2.14}$$

Now combining (2.12) with (2.13) and (2.14), I obtain

$$\frac{\partial V_3^s(z, \ell)}{\partial z} = \beta \int \frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon) = -\frac{\partial V_3^s(z, \ell)}{\partial \ell} = -\frac{\partial V_2^s(m, \ell)}{\partial \ell}. \tag{2.15}$$

For the seller who potentially meets a buyer with ε, the value function in market 1 is

$$V_1^s(m, 0; \varepsilon) = \sigma[-c(q) + V_2^s(m + d, -a)] + (1 - \sigma)V_2^s(m, 0). \tag{2.16}$$

If the seller meets a buyer, the seller sells q units of good 1, receives d units of money and extends credit with the nominal value a if the buyer chooses to use credit.

2.3.3 Equilibrium

Take-it-or-Leave-it Offer

Before deriving the equilibrium conditions, I solve for the terms of trade in market 1. The terms of trade in a match are determined by the buyer's take-it-or-leave-it offer.[11] There are two types of trades in market 1, depending on whether the buyer in a match uses credit or not.

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[11]It will be interesting to generalize the buyer's bargaining power from 1 to less than 1. It will also be interesting to study other pricing mechanisms that have been used in the literature such as competitive pricing and price posting. In this paper, I only focus on the buyer's take-it-or-leave-it offer to get the main intuition from the model and leave those extensions for future work.

Suppose that a buyer with ε only uses money. Recall that V_1^b and V_1^s are linear in m. The buyer's problem is

$$\max_{q,d}[\varepsilon u(q) - \phi d]$$

$$\text{s.t. } c(q) = \phi d \text{ and } d \leq m,$$

where m is the buyer's money holding. Let λ_1 and λ_2 be the Lagrangian multipliers associated with the two constraints.

$$\mathcal{L} = \max_{q,d,\lambda_1,\lambda_2} [\varepsilon u(q) - \phi d] + \lambda_1[\phi d - c(q)] + \lambda_2(m - d).$$

It is straightforward that the solution is the following.

$$\begin{cases} \lambda_2 = 0: & (q,d) \text{ are given by } \varepsilon u'(q) = c'(q) \text{ and } \phi d = c(q), \\ \lambda_2 > 0: & (q,d) \text{ are given by } d = m \text{ and } c(q) = \phi d. \end{cases}$$

Suppose that the buyer with ε uses credit. From (2.8), V_2^b is also linear in ℓ. However, it is not clear that V_2^s must be linear in ℓ at this stage. So I define the buyer's problem as

$$\max_{q,d,a}[\varepsilon u(q) - k - \phi d - \phi a]$$

$$\text{s.t. } c(q) = \phi d + V_2^s(0, -a) - V_2^s(0,0) \text{ and } d \leq m.$$

It is obvious that the seller's money holding does not appear in the above problem. Therefore, the terms of trade with credit do not depend on the seller's money holding. In addition, recall that the terms of trade without credit do not depend on the seller's money holding. It follows from (2.16) that

$$\frac{\partial \hat{V}_1^s(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} = \sigma\hat{\phi} + (1 - \sigma)\hat{\phi} = \hat{\phi}. \tag{2.17}$$

From (2.15) and (2.17),

$$\frac{\partial V_3^s(z, \ell)}{\partial z} = -\frac{\partial V_3^s(z, \ell)}{\partial \ell} = -\frac{\partial V_2^s(m, \ell)}{\partial \ell} = \beta\hat{\phi}. \tag{2.18}$$

Two results follow from (2.18). First, V_2^s is linear in ℓ and $V_2^s(m, \ell) = \phi m - \beta\hat{\phi}\ell + V_2^s(0,0)$. Second, sellers choose $z = 0$. One can show that the gross inflation rate is $\frac{\phi}{\hat{\phi}} = \gamma$ in the steady state. As I only consider $\gamma > \beta$ and $\gamma \to \beta$ from above, the second result is derived from (2.10) and (2.18).

Using (2.18), the Lagrangian is

$$\mathcal{L} = \max_{q,d,a,\lambda_1,\lambda_2} [\varepsilon u(q) - k - \phi d - \phi a] + \lambda_1[\phi d + \beta\hat{\phi}a - c(q)] + \lambda_2(m - d).$$

It turns out that the inequality constraint is always binding, and thus $d = m$. The solutions for (q, a) are

$$\varepsilon u'(q) = \frac{\gamma}{\beta}c'(q), \tag{2.19}$$

$$\beta\hat{\phi}a = c(q) - \phi d. \tag{2.20}$$

It is interesting to note that q depends on γ. In this economy, if a buyer uses credit in market 1, he will accumulate money for debt repayment in market 2. However, for the seller who extends the credit in the match, he will not be paid in the same market 2. Instead, the seller must wait to get settled in market 3. After receiving the money, the seller carries the money to the next market 1, but he cannot spend it because he does not want to consume. Hence, the seller actually spends the money one period after the buyer accumulates the money. There is an asymmetry between the time at which the buyer accumulates the money for repayment and the time at which the seller can spend the money from repayment. The buyer must compensate the seller for the loss in the value of money. From (2.20), the buyer essentially borrows $\frac{\beta\hat{\phi}}{\phi}a$ and repays a in nominal terms. The nominal interest rate of credit is $1 + i = \frac{\phi}{\beta\hat{\phi}} = \frac{\gamma}{\beta}$. As γ is higher, credit is more costly in nominal terms. For any given ε, q is decreasing in γ. Credit transactions are subject to inflation distortion.

As the structure of the model implies that money is the only means to settle credit, one may think that it is natural inflation affects a credit transaction's terms of trade. However, this is not necessarily true. The key feature that makes credit subject to inflation distortion is the inability of sellers to spend the money from repayment right away. Imagine an environment in which financial intermediaries exist in market 2 and the settlement of credit requires money. Sellers receive the repayment in the form of money and can spend it in market 2. It is clear that a credit transaction's terms of trade do not depend on the inflation rate in this scenario although money is imposed as the only means of settlement.

Money versus Credit

Having solved the terms of trade, I proceed to find the condition that determines whether a buyer uses credit or not. For a buyer with ε in market 1, if he only uses money,

$$V_1^b(m, 0; \varepsilon) = \sigma[\varepsilon u(q) - c(q)] + \phi m + V_2^b(0, 0).$$

If the buyer uses credit,

$$V_1^b(m,0;\varepsilon) = \sigma[\varepsilon u(q) - \frac{\gamma}{\beta}c(q) + (\frac{\gamma}{\beta} - 1)\phi m - k] + \phi m + V_2^b(0,0).$$

Let $T(\varepsilon)$ be the net benefit of using credit for the buyer, where

$$T(\varepsilon) = \sigma[\varepsilon u(q^c) - \frac{\gamma}{\beta}c(q^c) + (\frac{\gamma}{\beta} - 1)\phi m - k] - \sigma[\varepsilon u(q^m) - c(q^m)]. \qquad (2.21)$$

I use q^c to denote the quantity traded with credit and q^m to denote the quantity traded without credit. For the rest of the paper, I assume that ε is uniformly distributed, $\varepsilon \sim U(0,1]$.

Lemma 2.1 *For any given inflation rate γ, there exist two threshold values of ε, ε_0 and ε_1 such that*

$$\begin{cases} 0 \leq \varepsilon \leq \varepsilon_0, \text{ the buyer spends } d < m, \ a = 0 \text{ and consumes } q^* \text{ where } \varepsilon u'(q^*) = c'(q^*), \\ \varepsilon_0 \leq \varepsilon \leq \varepsilon_1, \text{ the buyer spends } d = m, \ a = 0 \text{ and consumes } q \text{ where } c(q) = \phi m, \\ \varepsilon_1 \leq \varepsilon \leq 1, \text{ the buyer spends } d = m, \ a > 0 \text{ and consumes } q^c \text{ where } \varepsilon u'(q^c) = \frac{\gamma}{\beta}c'(q^c). \end{cases}$$

Lemma 1 is very intuitive. If a buyer receives a very low ε, he has enough money at hand to afford q^*, which is the optimal consumption for him. Here ε_0 is the threshold that determines whether a buyer is liquidity constrained. For a buyer who receives an intermediate ε, the money may not be enough to afford his q^*. The buyer is liquidity constrained. Using credit can relax the buyer's liquidity constraint, but this is costly. Therefore, buyers with intermediate εs find it optimal not to use credit, because the benefit from using credit is not enough to cover the fixed cost. For those buyers who have large εs, paying the fixed cost to relax their liquidity constraints becomes optimal. The threshold ε_1 determines whether a buyer uses credit.

The decision to use credit is endogenous in this environment. Buyers use credit for large purchases. This result is in accordance with the evidence on consumers' choices of payment methods. Empirically, the mean value of cash purchases is smaller than the mean value of credit purchases. In English (1999), the mean values of credit card purchases and cash purchases are $54 and $11, respectively. Klee (2008) documents that these respective mean values are $30.85 and $14.2.

Monetary Equilibrium

With different groups of buyers in terms of their choices of payment methods, I can now characterize the equilibrium. I define (q_0, q_1) such that

$$\varepsilon_0 u'(q_0) = c'(q_0), \qquad (2.22)$$

$$\varepsilon_1 u'(q_1) = \frac{\gamma}{\beta} c'(q_1). \qquad (2.23)$$

Notice that $c(q_0) = \phi m$ represents the transaction demand for money. In market 3, the expected marginal benefit of 1 unit money $\beta \int \frac{\partial \hat{V}_1^b(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} dG(\varepsilon)$ is

$$\beta \hat{\phi} \left\{ \int_0^{\varepsilon_0} dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} [\sigma \varepsilon \frac{u'(q_0)}{c'(q_0)} + (1 - \sigma)] dG(\varepsilon) + \int_{\varepsilon_1}^{1} [\sigma \frac{\gamma}{\beta} + (1 - \sigma)] dG(\varepsilon) \right\}.$$

From (2.8), the marginal cost of 1 unit money is ϕ. Using $\frac{d\hat{m}}{dm} = 1$, the optimal q_0 is determined by

$$\varepsilon_0 + \frac{1}{2} \frac{u'(q_0)}{c'(q_0)} (\varepsilon_1^2 - \varepsilon_0^2) + \frac{\gamma}{\beta} (1 - \varepsilon_1) = 1 + \frac{\gamma - \beta}{\beta \sigma}. \qquad (2.24)$$

The last condition that completes the characterization of the equilibrium is derived from $T(\varepsilon) = 0$,

$$\varepsilon_1 u(q_1) - \frac{\gamma}{\beta} c(q_1) - k = \varepsilon_1 u(q_0) - \frac{\gamma}{\beta} c(q_0). \qquad (2.25)$$

Lemma 2.2 *When γ is close to β or approaches ∞, $\varepsilon_1 = 1$.*

Following Lemma 2, it is possible that no buyer would want to use credit. When γ is close to β, the rate of return of money is high enough so that there is no need to use credit. As γ is higher, the terms of trade with credit become worse. When γ approaches ∞, the gain from using credit cannot cover the fixed utility cost k. In (2.21), $T(\varepsilon)$ is negative. Depending on the parameter values of (γ, k, σ), there are two types of monetary equilibrium.

Definition 2.3 *When repayment of credit can be enforced, a monetary equilibrium with credit is characterized by $(\varepsilon_0, \varepsilon_1, q_0, q_1)$ satisfying (2.22), (2.23), (2.24) and (2.25). A monetary equilibrium without credit is characterized by $\varepsilon_1 = 1$ and (ε_0, q_0) satisfying (2.22) and (2.24).*

Proposition 2.4 *For any inflation rate above the Friedman rule $(\gamma > \beta)$, monetary equilibrium exists and the equilibrium is unique for generic values of σ. The optimal monetary policy is the Friedman rule $(\gamma \to \beta)$.*

In Proposition 1, I establish the existence and uniqueness of a monetary equilibrium. It is not surprising that the Friedman rule is the optimal monetary policy. If the monetary authority can run the Friedman rule, there is no cost to hold money.[12] Buyers would hold enough money to afford the optimal q. Credit is driven out as a means of payment.

In the model, the fixed cost k of using credit affects a buyer's choice of payment methods. A lower k can be viewed as an improvement in credit transaction technology, which is likely to promote the use of credit and contract the transaction demand for money. Proposition 2 establishes the related results.

Proposition 2.5 *(The Effect of the Fixed Cost) In a monetary equilibrium with credit, the thresholds are increasing in k, i.e., $\frac{d\varepsilon_0}{dk} > 0$ and $\frac{d\varepsilon_1}{dk} > 0$. Moreover, $\frac{dq_0}{dk} > 0$ and $\frac{dq_1}{dk} > 0$.*

If k is too big, no buyer will use credit because it is too costly. The economy would function as the one where money is the only means of payment. The other extreme case is where credit is not costly.

Corollary 2.6 *When $k = 0$, $\varepsilon_0 = \varepsilon_1 = q_0 = 0$. In equilibrium, credit becomes the only means of payment and money only functions as the means of settlement.*

If credit is available without any cost, money is driven out by credit as a means of payment. The transaction demand for money is 0. However, the total demand for money is not 0 as money is needed for settlement. Monetary equilibrium still exists, but money is only a means of settlement. Given that both money and credit are means of payment, it seems that they substitute each other. Since the settlement of credit requires money, money and credit are also complements. What is the effect of credit on money demand? The total money demand in this economy is

$$\phi M = c(q_0) + \frac{\sigma}{\beta} \int_{\varepsilon_1}^{1} [c(q^c(\varepsilon) - c(q_0)]dG(\varepsilon), \tag{2.26}$$

where $c(q_0)$ reflects the transaction demand for money. From Proposition 2, the introduction of credit lowers q_0, which in turn lowers the transaction demand for money. It does not follow that the total money demand must be lower as k decreases. Since money is the only means

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[12]If money is subject to theft, inflation lowers the rate of return of money and hence reduces theft. As a result, the optimal monetary policy may deviate from the Friedman rule. See Sanches and Willamson (2008) for an example. Similarly, introducing counterfeiting is another way that may generate the optimal monetary policy above the Friedman rule.

of settlement, the second term in (2.26) represents the repayment demand for money. It may increase as k decreases because a lower k makes ε_1 smaller and induces more buyers to use credit. Therefore, the overall effect of k on the total money demand is ambiguous.

Another parameter of interest is the trading probability σ. A higher σ implies less trading frictions in goods market. If it is easier to find a trade, will more buyers use credit as a means of payment? Proposition 3 addresses this question.

Proposition 2.7 *(The Effect of the Trading Probability) In a monetary equilibrium with credit, the thresholds are increasing in σ, i.e., $\frac{d\varepsilon_0}{d\sigma} > 0$ and $\frac{d\varepsilon_1}{d\sigma} > 0$. Moreover, $\frac{dq_0}{d\sigma} > 0$ and $\frac{dq_1}{d\sigma} > 0$.*

It turns out that money becomes more popular as a means of payment when the trading probability increases. The search friction in goods market promotes the use of credit. Recall that a key difference between money and credit is that money has to be acquired before making a purchase. In the case of not finding a trade, the value of money depreciates when the money growth rate is above the Friedman rule. Credit allows buyers to avoid such a distortion because the money required for repayment is accumulated after making a purchase. If it is easier to find a trade, holding money is less costly so that money is more desirable. However, even if $\sigma = 1$, credit may still be useful as a means of payment depending on the inflation rate.

As the paper is motivated by the observations on inflation and credit, I analyze the effects of monetary policy on this economy in the next proposition.

Proposition 2.8 *(The Effect of the Inflation Rate) In a monetary equilibrium with credit, when $\sigma = 1$ or $\gamma < 2\beta$, $\frac{d\varepsilon_0}{d\gamma} < 0$ and $\frac{dq_0}{d\gamma} < 0$.*

In Proposition 4, ε_0 and q_0 are decreasing in γ under certain conditions. It is easy to show that $\frac{d\varepsilon_0}{d\gamma}$ and $\frac{dq_0}{d\gamma}$ always have the same sign. However, it is not clear how $(\varepsilon_0, \varepsilon_1, q_0, q_1)$ depend on γ in general. Intuitively, inflation should have negative impacts on ε_0 and q_0 because inflation is a tax on money. The effects of inflation on ε_1 and q_1 are less clear. The two frictions associated with using credit generate two channels through which γ affects ε_1. A higher γ lowers the rate of return of money and makes more buyers liquidity constrained. As a result, more buyers may find that the gain from relaxing the liquidity constraint by using credit can cover the fixed cost. Through the fixed cost channel, γ decreases ε_1.

Indeed, this type of effect has been predicted by many other models using the Cash-in-Advance framework or the OLG framework. The other friction associated with credit is the delayed settlement. From (2.19), γ affects the marginal benefit of using credit. When γ is higher, terms of trade using credit become worse. Therefore, buyers have less incentive to use credit. Through the delayed settlement channel, γ increases ε_1.

Having analyzed these two channels, it would be interesting to know the effect from which channel dominates. From Lemma 2, ε_1 hits the boundary 1 when either $\gamma \to \beta$ or $\gamma \to \infty$. Thus, the total effect of γ on ε_1 should be non-monotonic. In fact, it is likely that the effect displays a U-shape. I will rely on numerical results in the next section to verify these conjectures.

2.3.4 Welfare

In order to analyze the effect of monetary policy on aggregate welfare, I define aggregate welfare in this economy as $\mathcal{W}$ and

$$(1-\beta)\mathcal{W} = \sigma\Psi(\varepsilon_0, \varepsilon_1, q_0) + [v(x^*) - x^*] - \sigma(\frac{1-\beta}{\beta})\int_{\varepsilon_1}^{1}[c(q^c(\varepsilon)) - c(q_0)]dG(\varepsilon), \quad (2.27)$$

where $\Psi(\varepsilon_0, \varepsilon_1, q_0)$ is

$$\int_0^{\varepsilon_0}[\varepsilon u(q^*(\varepsilon)) - c(q^*(\varepsilon))]dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1}[\varepsilon u(q_0) - c(q_0)]dG(\varepsilon) + \int_{\varepsilon_1}^{1}[\varepsilon u(q^c(\varepsilon)) - c(q^c(\varepsilon)) - k]dG(\varepsilon).$$

Note that aggregate welfare is also buyers' aggregate welfare since sellers in this economy earn 0 surplus from trades and their aggregate welfare is 0. The first and second terms in the aggregate welfare function are standard. What's new in (2.27) is the third term, which is the production distortion from using credit in the following sense. After a seller extends credit in market 1, he receives payment from the financial intermediary in market 3 and must wait until the next market 2 to spend the money. As discussed earlier, the buyer who uses credit should pay the nominal interests to compensate the seller. Since buyers receive monetary transfers from the monetary authority in each period, the actual extra payment the buyer has to accumulate by working is the real interest rate. This part is reflected in the third term, which can be viewed as the production distortion from using credit. Without knowing how γ affects $(\varepsilon_0, \varepsilon_1)$, it is not obvious how aggregate welfare responds to γ. Analytically, I can show that $\frac{d\mathcal{W}}{d\gamma} < 0$ when $\frac{d\varepsilon_1}{d\gamma} > 0$. Again, I leave a more general analysis in the next section.

2.4 Quantitative Analysis

In this section, I numerically study the model to obtain more implications. For the numerical exercise, I adopt some specific functional forms for $u(q)$, $c(q)$ and $v(x)$ that have been used in the literature. Let $u(q) = \frac{1}{\rho}q^\rho$, $v(x) = B \log x$, and $c(q) = q$, where $0 < \rho < 1$. In market 1, the matching technology that I specify is the urn-ball matching function, where $\sigma = 1 - e^{-1}$. There are four parameters (β, B, ρ, k) to be determined. The period length in this model is set to 1 year mainly to facilitate comparisons with past work on the welfare cost of inflation.

The time preference parameter β is set $\beta^{-1} = 1.04$, so the implied annual real interest rate is 0.04. For the other parameters, I follow Lucas (2000) and Lagos and Wright (2005) and fit the model's money demand to the U.S. money demand data by nonlinear least square. The data covers annual nominal interest rate and the real demand for money (or the inverse of the velocity of money) for the period $1900 - 2000$.[13] The real money demand predicted by the model is

$$L(i) = \frac{M}{PY} = \frac{c(q_0) + \frac{\sigma}{\beta} \int_{\varepsilon_1}^{1} [c(q^c(\varepsilon) - c(q_0)]dG(\varepsilon)}{Y_c + \sigma[\int_{0}^{\varepsilon_0} c(q^*(\varepsilon))dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} c(q_0)dG(\varepsilon) + \int_{\varepsilon_1}^{1} c(q^c(\varepsilon))dG(\varepsilon)]}.$$

where

$$Y_c = x + \sigma\left[\int_{0}^{\varepsilon_0} c(q^*(\varepsilon))dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} c(q_0)dG(\varepsilon) + \int_{\varepsilon_1}^{1} c(q^c(\varepsilon))dG(\varepsilon)\right]$$
$$+ \frac{\sigma(\beta - 1)}{\beta} \int_{\varepsilon_1}^{1} [c(q^c(\varepsilon)) - c(q_0)]dG(\varepsilon).$$

The parameters from the best fit are in Table 1.

Table 2.1: Parameter Values

Parameters	ρ	B	k
Values	0.4732	1.4436	0.0739

The values of (ρ, B) are in the ballpark of existing studies. To evaluate the plausibility of the value of k, I use a consumption equivalence measure. The utility cost $k = 0.0739$ is worth of 1% of consumption for buyers. Based on these parameter values, I numerically solve the model and show the results in Figure 3. The upper-left and upper-right panels are

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[13]The data are originally from Craig and Rocheteau (2007).

the effects of inflation on the threshold ε_1 and the credit to GDP ratio, respectively.[14] The lower panels are the comparisons with a no-credit economy. The lower-left panel presents the total demand for money in the credit economy and the no-credit economy. The lower-right panel shows the welfare improvement of having credit based on a consumption equivalence measure. That is, the number on the vertical axis is the fraction of consumption that a buyer is willing to give up to live in a credit economy instead of a no-credit economy.
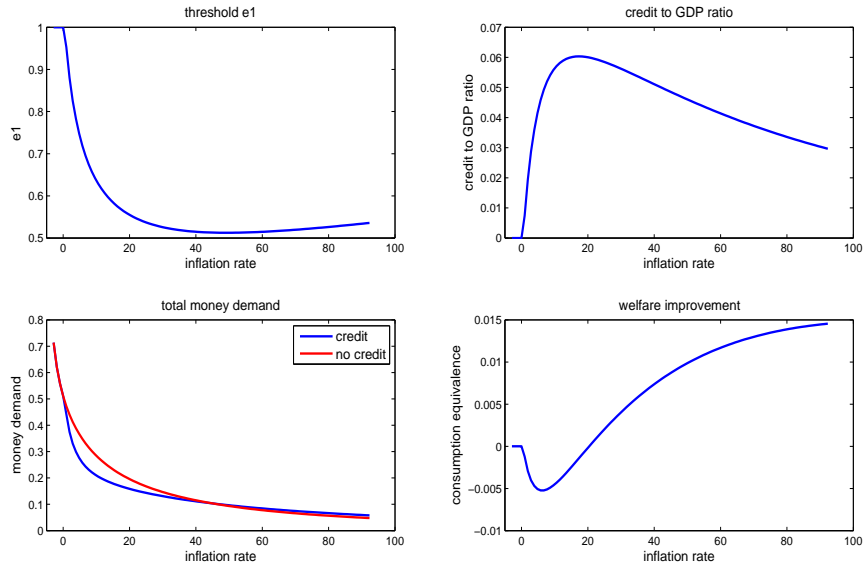
Figure 2.3: The Effect of Inflation - Benchmark

There are several interesting findings from Figure 3. It is clear that inflation induces more buyers to use credit at low inflation rates and less buyers to use credit at high inflation rates. Moreover, the credit to GDP ratio predicted by the model has an inverse U-shape against inflation. As discussed in the previous section, inflation has two effects on ε_1. The fixed cost effect implies that inflation makes more buyers use credit. This is because high inflation causes more buyers liquidity constrained so that more buyers may find using credit

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[14]The predicted credit to GDP ratio from the model is

$$\frac{\sigma \int_{\varepsilon_1}^1 [c(q^c(\varepsilon)) - c(q_0)]dG(\varepsilon)}{Y_c + \sigma[\int_0^{\varepsilon_0} c(q^*(\varepsilon))dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} c(q_0)dG(\varepsilon) + \int_{\varepsilon_1}^1 c(q^c(\varepsilon))dG(\varepsilon)]}.$$

beneficial enough to cover the fixed cost. The delayed settlement effect on the other hand lowers incentives for buyers to use credit because of a deterioration in the terms of trade. According to numerical results, the fixed cost effect dominates the delayed settlement effect at low inflation rates, but the delayed settlement effect dominates the fixed cost effect at high inflation rates. Intuitively, in the presence of a very high γ, using credit involves high repayment and hence unfavorable terms. This exactly describes the consumer credit market in Brazil during the late 80s.[15]

Compared to a no-credit economy, credit lowers money demand at low to moderate inflation rates, but slightly increases money demand at high inflation rates. One can show that the transaction demand in a credit economy is always lower than in a no-credit economy. As the repayment of credit also requires money, money demand from the repayment channel may increase as the inflation rate increases. It seems that credit and money are substitutes at low to moderate inflation rates, but are complements at high inflation rates. The first half of the result can be supported by the empirical work using U.S. data, since the inflation rates in the U.S. have been low to moderate in recent decades. See Duca and Whitesell (1995) for an example. The latter half of the result, however, has not been verified empirically.

The lower-right panel reveals that having credit does not always benefit the society in terms of aggregate welfare. Since individuals optimally choose to use money versus credit, it seems a little puzzling that credit can hurt the economy. From (2.27), credit improves welfare by relaxing the liquidity constraint for some buyers, but may hurt welfare because of the production distortion. Besides these direct effects, credit affects welfare through the general equilibrium effect as well. As analyzed above, credit may lower the demand for money and thus the value of money, which will generate a negative externality on agents who use money. On the other hand, credit may increase money demand and the value of money, which will generate a positive externality on agents who use money. Since credit lowers money demand at low to moderate inflation rates, the general equilibrium effect implies that credit may hurt welfare at low to moderate inflation rates, but improve welfare at high inflation rates. Similar results appear in Chiu and Meh (2008). Overall, credit improves aggregate welfare when the inflation rate exceeds a threshold.

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[15]Due to the long time delay in credit card charges clearing through the banking system, vendors have been documented to normally add on a 20 to 30 percent surcharge to the price of the purchased item. In this way, vendors can protect themselves from the depreciation of money during the time the vendors are waiting to be paid by the credit card companies.

In terms of the effect of monetary policy, the model predicts that aggregate welfare and aggregate output are decreasing in the inflation rate. This is not surprising although the model does introduce a channel through which inflation may potentially increase output levels at low inflation rates by encouraging more buyers to use credit. However, the effect from this channel does not appear to be strong.

In Figure 3, the threshold value for credit to improve welfare is around 20% inflation rate, which is fairly high. The potential problem is that fitting (ρ, B, k) together implicitly assumes that these parameters do not change over the hundred years. However, it is hard to believe that the cost of credit transactions stays constant over time. Nevertheless, since there is no direct data that measures how k evolves over time and the focus of the paper is not to match any moment in the data, I take a simple approach to evaluate the model's predictions by varying k and fixing (ρ, B). To highlight the effect of changing k, I show in Figure 4 the credit to GDP ratio and the welfare improvement when $k = 0.01$, 0.05 and 0.1.
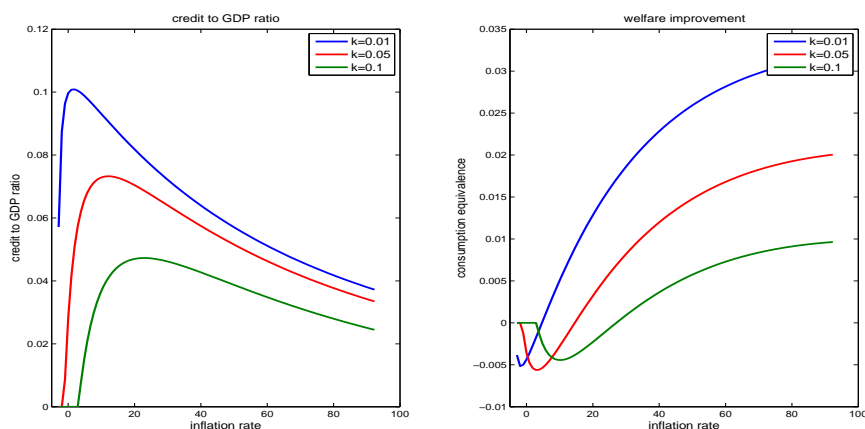
Figure 2.4: Comparative Statics - Varying k

According to Figure 4, a lower cost of credit promotes the use of credit. Using the average inflation rate 7.387 from $1969 - 2000$, the predicted credit to GDP ratio is 0.26% and the predicted real money demand is 0.395 when $k = 0.1$. For $k = 0.01$, the predicted credit to GDP ratio is 10.02% and the predicted real money demand is 0.201. The low cost credit regime is featured by more credit and less real money demand. It has been noted that there is a trend decline in real money demand in the recent decade, which has been viewed

as a shift of the money demand curve.[16] Clearly, improvements in the credit transaction technology contribute to the trend decline in real money demand.

In terms of welfare, more costly credit makes credit less beneficial to the society. One can see that the threshold for credit to be welfare-improving is higher when k is higher. In the real world, if sellers receive repayment in the form of money, they may put the money into their saving accounts to avoid any inflation distortion, which makes credit more beneficial. This type of argument can be built into the model by allowing a fraction of agents to settle in market 2 and the rest to settle in market 3. While this is a nice extension, the current model still serves as a benchmark for analyzing the effect of inflation on credit in a world in which credit is not entirely free of inflation distortion.

As a robustness check, I choose values of (ρ, B) by varying the sample period and evaluate the model's predictions. In these experiments, the value of ρ varies from 0.387 to 0.589, but the value of B does not change much, which is around 1.4. In terms of the model's predictions, the patterns emerging from Figure 3 are very robust.

To study how introducing credit affects the welfare cost of inflation, I compute the welfare cost of 10% inflation based on the parameters given in Table 1. The measure of the welfare cost follows the recent literature by using the consumption equivalence measure. The numbers reported in Table 2 is the fraction of consumption a buyer is willing to give up to have 0% inflation rather than 10% inflation. As a benchmark, I compute the welfare cost for a no-credit economy, i.e., $k = \infty$ and hence $\varepsilon_1 = 1$. The welfare cost of 10% inflation is 1.12% in the benchmark economy, which is relatively small because I use take-it-or-leave-it offer by buyers during bargaining.[17]

Table 2.2: Welfare Cost of 10% inflation

k	benchmark	0.01	0.05	0.0739	0.1
Welfare Cost	1.12%	0.22%	0.85%	1.32%	1.40%

I then compute the welfare cost of inflation for different values of k. The introduction

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[16]Considering an economy without credit, I can find the values of (ρ, B) by fitting the money demand curve. Starting in the 1980s, the predicted money demand diverges from the data. If one is willing to assume a specific functional form of the trend of financial innovation and assume that each year is in a steady state, then fitting the money demand data can generate the values of (ρ, B) and the trend of financial innovation. By doing such an exercise, I found that the predicted money demand in recent years is much closer to the money demand data. A similar exercise is in Faig and Jerez (2006).

[17]In Lagos and Wright (2005), the welfare cost of 10% inflation is 1.4% when buyers have all the bargaining power. The current result does not deviate from their estimate.

of credit can raise the welfare cost when credit is costly enough. For low values of k, the cost for buyers to substitute credit for money is relatively low. Therefore, inflation does not generate a large welfare loss. On the contrary, if the cost for buyers to switch from money to credit is high, inflation can result in a higher welfare loss compared to the benchmark economy. Note that if k is too big, no buyer uses credit and the economy is essentially the benchmark economy. Dotsey and Ireland (1996) and Lacker and Shreft (1996) both emphasize that credit costs are quantitatively important as a component of the welfare cost of inflation. The results in Table 2 further confirm their results.

2.5 Monetary Equilibrium without Enforcement

So far I have assumed that financial intermediaries can enforce the repayment of credit, which implies that buyers are not credit constrained. In this section, I relax the assumption of perfect enforcement. Financial intermediaries can identify agents and keep records of goods market transactions, but they cannot enforce the repayment of credit. As in Berentsen et al. (2007) and Sanches and Williamson (2008), the punishment for default is permanent exclusion from the financial system. That is, if a buyer defaults, the buyer will never be able to use credit in any future period. Given the punishment, the amount of credit extended to a buyer is consistent with the buyer's incentive to repay. In an environment without enforcement, the government (or the monetary authority) cannot enforce buyers to pay taxes either. It implies that $\gamma \geq 1$.

With this modification, buyers and sellers face the same choice problems as before in market 2 and 3. Only in market 1, the buyer's take-it-or-leave-it offer should be reformulated.

$$\max_{q,d,a}[\varepsilon u(q) - \phi d - \phi a]$$

$$\text{s.t. } c(q) = \phi d + \beta \hat{\phi} a, \ d \leq m \text{ and } a \leq \bar{a}.$$

Here $\bar{a}$ is the credit limit faced by the buyer. An individual buyer takes $\bar{a}$ as given. In equilibrium, $\bar{a}$ will be endogenously determined. The Lagrangian is

$$\mathcal{L} = \max_{q,d,a,\lambda_1,\lambda_2,\lambda_3}[\varepsilon u(q) - \phi d - \phi a] + \lambda_1[\phi d + \beta \hat{\phi} a - c(q)] + \lambda_2(m - d) + \lambda_3(\bar{a} - a).$$

If the credit constraint is not binding, i.e., $\lambda_3 = 0$, then $d = m$ and (q, a) are given by

$$\varepsilon u'(q) = \frac{\gamma}{\beta} c'(q),$$
$$\beta \hat{\phi} a = c(q) - \phi d.$$

If the credit constraint is binding, i.e., $\lambda_3 > 0$, I have $d = m$, $a = \bar{a}$ and q solving

$$c(q) = \phi d + \beta \hat{\phi} \bar{a}.$$

Introducing the credit limit may cause some buyers to be credit constrained. If such buyers exist, there are potentially four groups of buyers.

Lemma 2.9 *For any given inflation rate γ, there exist three thresholds of ε, ε_0, ε_1 and ε_2 such that*

$$\begin{cases} 0 \leq \varepsilon \leq \varepsilon_0, \text{ the buyer spends } d < m, \ a = 0, \text{ and consumes } q^* \text{ where } \varepsilon u'(q^*) = c'(q^*), \\ \varepsilon_0 \leq \varepsilon \leq \varepsilon_1, \text{ the buyer spends } d = m, \ a = 0 \text{ and consumes } q \text{ where } c(q) = \phi m, \\ \varepsilon_1 \leq \varepsilon \leq \varepsilon_2, \text{ the buyer spends } d = m, \ a < \bar{a} \text{ and consumes } q^c \text{ where } \varepsilon u'(q^c) = \frac{\gamma}{\beta} c'(q^c), \\ \varepsilon_2 \leq \varepsilon \leq 1, \text{ the buyer spends } d = m, \ a = \bar{a} \text{ and consumes } q \text{ where } c(q) = \phi m + \beta \hat{\phi} \bar{a}. \end{cases}$$

The expected marginal value of money in market 3 is

$$\beta \int \frac{\partial \hat{V}_1(\hat{m}, 0; \varepsilon)}{\partial \hat{m}} G(\varepsilon) = \beta \hat{\phi} \left\{ \int_0^{\varepsilon_0} dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} [\sigma \frac{\varepsilon u'(q_0)}{c'(q_0)} + (1 - \sigma)] dG(\varepsilon) \right.$$
$$\left. + \int_{\varepsilon_1}^{\varepsilon_2} [\sigma \frac{\gamma}{\beta} + (1 - \sigma)] dG(\varepsilon) + \int_{\varepsilon_2}^1 [\sigma \frac{\varepsilon \gamma}{\varepsilon_2 \beta} + (1 - \sigma)] dG(\varepsilon) \right\},$$

where q_0 is defined in (2.22). Combining with (2.8), the optimal q_0 is implicitly given by

$$\varepsilon_0 + \frac{1}{2} \frac{u'(q_0)}{c'(q_0)} (\varepsilon_1^2 - \varepsilon_0^2) + \frac{\gamma}{\beta} (\varepsilon_2 - \varepsilon_1) + \frac{1}{2} \frac{\gamma}{\beta} (\frac{1}{\varepsilon_2} - \varepsilon_2) = 1 + \frac{\gamma - \beta}{\beta \sigma}. \tag{2.28}$$

Define q_2 such that

$$\frac{\varepsilon_2 u'(q_2)}{c'(q_2)} = \frac{\gamma}{\beta}, \tag{2.29}$$

and q_2 satisfies

$$c(q_2) = c(q_0) + \beta \hat{\phi} \bar{a}. \tag{2.30}$$

Now suppose that $\varepsilon_2 < 1$. Consider a buyer with ε carrying debt a into market 2. The buyer must have spent all of his money. If he repays the debt, the buyer should work to

accumulate the money for repayment. His value function is $V_2^b(0, a)$. If the buyer defaults, he does not need to work as much in market 2. Denote the payoff from default by $V_2^{bD}(0, a)$ where

$$V_2^{bD}(0, a) = v(x^*) - x^* + \max_{z^D} \left\{ -\phi z^D + V_3^{bD}(z^D, 0) \right\}.$$

The superscript D represents variables associated with default. The real credit limit is the value of $\beta\hat{\phi}\bar{a}$ that solves $V_2^b(0, a) = V_2^{bD}(0, a)$. After some algebra, the real credit limit is

$$
\begin{aligned}
\beta\hat{\phi}\bar{a} &= \frac{\beta^2\sigma}{\gamma(1-\beta)} \left\{ \int_0^{\varepsilon_0} [\varepsilon u(q^*(\varepsilon)) - c(q^*(\varepsilon))]dG(\varepsilon) + \int_{\varepsilon_0}^{\varepsilon_1} [\varepsilon u(q_0) - c(q_0)]dG(\varepsilon) \quad (2.31) \right. \\
&\quad + \int_{\varepsilon_1}^{\varepsilon_2} [\varepsilon u(q^c(\varepsilon)) - k - \frac{\gamma}{\beta}c(q^c(\varepsilon)) + (\frac{\gamma}{\beta} - 1)c(q_0)]dG(\varepsilon) \\
&\quad + \int_{\varepsilon_2}^1 [\varepsilon u(q_2) - k - \frac{\gamma}{\beta}c(q_2) + (\frac{\gamma}{\beta} - 1)c(q_0)]dG(\varepsilon) \\
&\quad \left. - \int_0^{\varepsilon_0^D} [\varepsilon u(q^*(\varepsilon) - c(q^*(\varepsilon))]dG(\varepsilon) - \int_{\varepsilon_0^D}^1 [\varepsilon u(q_0^D) - c(q_0^D)]dG(\varepsilon) \right\} \\
&\quad + \frac{(\gamma - \beta)\beta}{\gamma(1-\beta)} [c(q_0^D) - c(q_0)].
\end{aligned}
$$

When the repayment of credit cannot be enforced, a monetary equilibrium with constrained credit is defined by a list of $(\varepsilon_0, \varepsilon_1, \varepsilon_2, q_0, q_1, q_2, \beta\hat{\phi}\bar{a})$ characterized by (2.22), (2.23), (2.25), (2.28), (2.29), (2.30) and (2.31). It is possible that $(\varepsilon_1, \varepsilon_2)$ hit the boundary 1 in equilibrium. In particular, there are two special cases: [1] $\varepsilon_2 = 1$ and [2] $\varepsilon_1 = \varepsilon_2 = 1$. In case [1], the equilibrium corresponds to the monetary equilibrium with credit in Section 3. In case [2], the equilibrium corresponds to the monetary equilibrium without credit. Notice that in these two cases, the endogenous credit limit still exists, but it is not binding.

It becomes very complicated to derive any analytical results. Hence, I use a numerical example to show how endogenizing credit limit affects equilibrium and welfare. Based on the numerical exercise in Section 4, I again set $\rho = 0.4732$ and $B = 1.4436$. As for k, I choose $k = 0.03$. The left panel in Figure 5 shows the endogenous credit limit and the maximum amount of borrowing for various inflation rates. The main finding is that buyers are credit constrained only at very low inflation rates, which is consistent with Berentsen et al. (2007). High inflation rates help to relax the credit constraint because punishment becomes more severe. Since inflation also lowers the surplus from credit trades in this model, the overall effect of inflation on credit limit is that inflation first relaxes and then tightens the credit limit. At high inflation rates, because buyers borrow less, the credit constraint does not
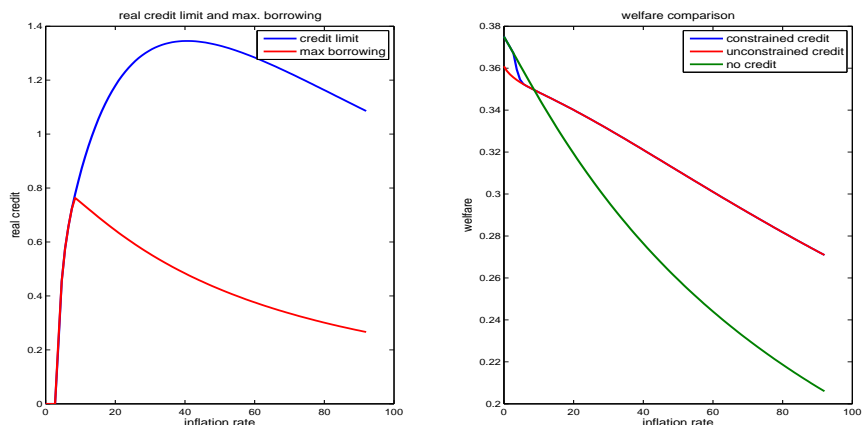
bind although the credit limit is lower.

Figure 2.5: The Effect of Inflation - Endogenous Credit Limit

In the right panel of Figure 5, I compare welfare in an economy with endogenous credit limit with welfare in an economy with either unconstrained credit or no credit. The implication is that having the endogenous credit limit can improve social welfare at very low inflation rates. It sounds counterintuitive that imposing a constraint would make the allocation better. However, money functions well as a means of payment at low inflation rates. It is more likely that buyers would choose to default because the punishment may make them better off. If the credit limit reaches zero, the economy converts into a pure monetary economy. At very low inflation rates, I demonstrated in Section 4 that a monetary equilibrium without credit is better in terms of aggregate welfare than a monetary equilibrium with credit. Having a zero credit limit is essentially welfare-enhancing.

2.6   Conclusion

Both money and credit are widely used as means of payment. It is important to understand how credit affects money demand and hence the transmission of monetary policy. I constructed a model in which money and credit coexist as means of payment and money is the means of settlement. There are two frictions associated with using credit – a fixed utility cost and the delayed settlement. In this environment, a buyer's choice of payment methods is endogenous. Credit lowers money demand at low to moderate inflation, but it slightly increases money demand at high inflation rates. The relationship between inflation

and the credit to GDP ratio exhibits an inverse U-shape which is broadly consistent exist-
ing evidence. Costly credit does not always improve social welfare. Depending on the fixed
utility cost of credit, allowing credit as a means of payment may raise the welfare cost of
inflation.

In a modified environment where enforcement is imperfect, the endogenous credit limit
tends to increase at low inflation rates, but decrease at high inflation rates. However, credit
constraints only bind at very low inflation rates. Interestingly, imperfect enforcement may
improve social welfare because it avoids socially inefficient borrowing.

One testable implication from the model is that the relationship between inflation and
the credit to GDP ratio has an inverse U-shape. There exist many studies testing the long
run effect of inflation on credit. Most of them use the total private credit to GDP ratio as
the measure of credit. To complement this paper, it would be ideal to use the consumer
credit to GDP ratio and test its long run relationship with inflation. As I only obtain the
data on consumer credit in Canada and the U.S., I will pursue this empirical study with
more cross-country data in my future work.[18]

2.7 Appendix

2.7.1 Use of Credit Card Data

Table 3 describes the U.S. households' credit card spending volume and credit card debt
outstanding in 2000 and 2005. The numbers reported for 2010 are projections. Data are
from U.S. Statistical Abstract.

2.7.2 Proof of Lemma 2.1

Proof. As discussed in the paper, if a buyer uses credit, he must spend all his money. Define
$\bar{q}$ as the solution to $\bar{\varepsilon}u'(q) = c'(q)$. One can show that $\phi m < c(\bar{q})$ as long as $\gamma > \beta$. It implies
that there exists a threshold ε_0 such that $\varepsilon_0 u'(q_0) = c'(q_0)$ where q_0 is from $c(q_0) = \phi z$. For

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[18]I tested the long run relationship between inflation and the private credit to GDP ratio following Bullard
and Keating (1995). The results moderately support the inverse U-shape prediction. In addition, I used
Canada and the U.S. consumer credit data to perform the same test. For these two low inflation countries, a
permanent increase in inflation increases the consumer credit to GDP ratio, which is consistent with the left
half of the inverse U-shape. However, due to data availability, I do not have any consumer credit data for
high inflation countries. Therefore, it is hard to verify the right half of the U-shape at this stage. A detailed
description of the estimation is available upon request.

Table 2.3: Credit Card - Spending Volume and Debt Outstanding

	Credit Card Spending Volume (billions of dollars)	Credit Card Debt Outstanding (billions of dollars)
2000	1458	680
2005	2052	832
2010	3378	1091

buyers with $\varepsilon > \varepsilon_0$,

$$T'(\varepsilon) = \sigma[\varepsilon u'(q^c) - \frac{\gamma}{\beta}c'(q^c)]\frac{dq^c}{d\varepsilon} - \sigma[\varepsilon u'(q^m) - c'(q^m)]\frac{dq^m}{d\varepsilon} + \sigma[u(q^c) - u(q^m)].$$

Notice that $\varepsilon u'(q^c) - \frac{\gamma}{\beta}c'(q^c) = 0$ from (2.19) and $\frac{dq^m}{d\varepsilon} = 0$. Since $q^c > q^m$, it follows that $T'(\varepsilon) > 0$ for $\varepsilon > \varepsilon_0$. Define ε_1 such that $T(\varepsilon_1) = 0$. If an interior ε_1 exists, then [1] buyers with $\varepsilon_0 < \varepsilon < \varepsilon_1$ are liquidity constrained and do not use credit; and [2] buyers with $\varepsilon > \varepsilon_1$ are liquidity constrained and use credit. ∎

2.7.3 Proof of Lemma 2.2

Proof. When γ is close to β, $\varepsilon_0 \to 1$, $T(\varepsilon) \to \sigma[\varepsilon u(q^c) - c(q^c) - k] - \sigma[\varepsilon u(q^m) - c(q^m)]$ and $q^c \to q^m$. It implies that $T(\varepsilon) < 0$ for all $\varepsilon > \varepsilon_0$, which means $\varepsilon_1 = 1$.

Recall that $T'(\varepsilon) > 0$ for $\varepsilon > \varepsilon_0$. For $\varepsilon > \varepsilon_0$, it is true that $\phi m = c(q^m)$. The maximum of $T(\varepsilon)$ is achieved at $\varepsilon = 1$ and $T(1) = \sigma[u(q^c) - \frac{\gamma}{\beta}c(q^c) - k] - \sigma[u(q^m) - \frac{\gamma}{\beta}c(q^m)]$. When γ approaches ∞, $T(1) \to -\sigma k$. It is clear that $\varepsilon_1 = 1$ when $\gamma \to \infty$. ∎

2.7.4 Proof of Proposition 2.4

Proof. Notice that (2.24) determines q_0, where ε_0 and ε_1 are functions of q_0. From (2.22), one can solve for $\varepsilon_0(q_0)$ as a function and $\frac{d\varepsilon_0}{dq_0} > 0$. To solve for ε_1, both (2.23) and (2.25) are used. In (2.23), $\frac{d\varepsilon_1}{dq_1} > 0$. In (2.25), $\frac{d\varepsilon_1}{dq_1} = 0$. It follows that $\varepsilon_1(q_0)$ is also a function and $\frac{d\varepsilon_1}{dq_0} > 0$. The equilibrium problem can be viewed as a choice problem of q_0, where the first order condition is rewritten from (2.24) as the following

$$\frac{1}{2}\frac{c'(q_0)}{u'(q_0)} + \frac{1}{2}\frac{u'(q_0)}{c'(q_0)}\varepsilon_1^2(q_0) - \frac{\gamma}{\beta}\varepsilon_1(q_0) = 1 + \frac{\gamma(1-\sigma)-\beta}{\beta\sigma}. \tag{2.32}$$

Since I focus on $q_0 \in [0, q^*]$ with q^* given by $u'(q^*) = c'(q^*)$, a solution of q_0 must exist by the theorem of maximum. For any $\gamma > \beta$, $q_0 > 0$ so that monetary equilibrium must exist.

Similar to the proof in Wright (2008), one can show that equilibrium q_0 is unique for generic values of σ. However, it is not obvious that equilibrium q_0 is unique for generic values of γ in this model. ∎

2.7.5 Proof of Proposition 2.5

Proof. Total differentiate (2.24), (2.22), (2.23) and (2.25) with respect to k:

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2)\frac{dq_0}{dk} + [\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}]\frac{d\varepsilon_1}{dk} = 0,$$

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{[c'(q_0)]^2}\frac{dq_0}{dk} + \frac{1}{\varepsilon_0^2}\frac{d\varepsilon_0}{dk} = 0,$$

$$\frac{c'(q_1)u''(q_1) - u'(q_1)c''(q_1)}{[c'(q_1)]^2}\frac{dq_1}{dk} + \frac{\gamma}{\beta\varepsilon_1^2}\frac{d\varepsilon_1}{dk} = 0,$$

$$[u(q_1) - u(q_0)]\frac{d\varepsilon_1}{dk} - 1 - [\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]\frac{dq_0}{dk} = 0.$$

From (2.25), one can show that $q_1 > q_0$. Together with (2.22) and (2.23), $\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta} > 0$. It follows that $\frac{d\varepsilon_0}{dk}$, $\frac{dq_0}{dk}$, $\frac{d\varepsilon_0}{dk}$ and $\frac{d\varepsilon_1}{dk}$ are equal in sign. Moreover,

$$\{\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2) + \frac{[\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}][\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]}{[u(q_1) - u(q_0)]}\}\frac{dq_0}{dk} = \frac{1}{u(q_0) - u(q_1)}. \tag{2.33}$$

Define

$$f(q_0; \gamma) = \varepsilon_0(q_0) + \frac{1}{2}\frac{u'(q_0)}{c'(q_0)}[\varepsilon_1^2(q_0) - \varepsilon_0^2(q_0)] + \frac{\gamma}{\beta}[1 - \varepsilon_1(q_0)] - 1 - \frac{\gamma - \beta}{\beta\sigma},$$

where $\varepsilon_0(q_0)$ is from (2.22) and $\varepsilon_1(q_0)$ is from (2.23) and (2.25). As the equilibrium q_0 must be local maximum, it must be true that $f'(q_0; \gamma) < 0$, so

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2) + \frac{[\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}][\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]}{[u(q_1) - u(q_0)]} < 0. \tag{2.34}$$

From (2.33), I have $\frac{dq_0}{dk} > 0$ and hence, $\frac{d\varepsilon_0}{dk} > 0$, $\frac{d\varepsilon_1}{dk} > 0$ and $\frac{dq_1}{dk} > 0$. ∎

2.7.6 Proof of Proposition 2.7

Proof. The proof is similar to the proof of Proposition 2. Total differentiate (2.24), (2.22), (2.23) and (2.25) with respect to σ:

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2)\frac{dq_0}{d\sigma} + [\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}]\frac{d\varepsilon_1}{d\sigma} = -\frac{\gamma - \beta}{\beta\sigma^2},$$

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{[c'(q_0)]^2}\frac{dq_0}{d\sigma} + \frac{1}{\varepsilon_0^2}\frac{d\varepsilon_0}{d\sigma} = 0,$$

$$\frac{c'(q_1)u''(q_1) - u'(q_1)c''(q_1)}{[c'(q_1)]^2}\frac{dq_1}{d\sigma} + \frac{\gamma}{\beta\varepsilon_1^2}\frac{d\varepsilon_1}{d\sigma} = 0,$$

$$[u(q_1) - u(q_0)]\frac{d\varepsilon_1}{d\sigma} - [\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]\frac{dq_0}{d\sigma} = 0.$$

It is clear that $\frac{d\varepsilon_0}{d\sigma}$, $\frac{dq_0}{d\sigma}$, $\frac{d\varepsilon_1}{d\sigma}$ and $\frac{d\varepsilon_1}{d\sigma}$ have the same sign. Using (2.34), $\frac{dq_0}{d\sigma} > 0$. Moreover, $\frac{d\varepsilon_0}{dk} > 0$, $\frac{d\varepsilon_1}{dk} > 0$ and $\frac{dq_1}{dk} > 0$. ∎

2.7.7 Proof of Proposition 2.8

Proof. The proof is similar to the proofs of Proposition 2 and Proposition 3. Total differentiate (2.24), (2.22), (2.23) and (2.25) with respect to γ:

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{2[c'(q_0)]^2}(\varepsilon_1^2 - \varepsilon_0^2)\frac{dq_0}{d\gamma} + [\varepsilon_1\frac{u'(q_0)}{c'(q_0)} - \frac{\gamma}{\beta}]\frac{d\varepsilon_1}{d\gamma} = \frac{1}{\beta\sigma} - \frac{1 - \varepsilon_1}{\beta},$$

$$\frac{c'(q_0)u''(q_0) - u'(q_0)c''(q_0)}{[c'(q_0)]^2}\frac{dq_0}{d\gamma} + \frac{1}{\varepsilon_0^2}\frac{d\varepsilon_0}{d\gamma} = 0,$$

$$\frac{c'(q_1)u''(q_1) - u'(q_1)c''(q_1)}{[c'(q_1)]^2}\frac{dq_1}{d\gamma} + \frac{\gamma}{\beta\varepsilon_1^2}\frac{d\varepsilon_1}{d\gamma} = \frac{1}{\beta\varepsilon_1},$$

$$[u(q_1) - u(q_0)]\frac{d\varepsilon_1}{d\gamma} - [\varepsilon_1 u'(q_0) - \frac{\gamma}{\beta}c'(q_0)]\frac{dq_0}{d\gamma} = \frac{1}{\beta}[c(q_1) - c(q_0)].$$

Using (2.34), the sign of $\frac{dq_0}{d\gamma}$ is the same as the sign of $h(\varepsilon_0, \varepsilon_1, q_0, q_1)$, where

$$h(\varepsilon_0, \varepsilon_1, q_0, q_1) = \frac{1 - \varepsilon_1}{\beta} + (\frac{\varepsilon_1}{\varepsilon_0} - \frac{\gamma}{\beta})\frac{c(q_1) - c(q_0)}{\beta[u(q_1) - u(q_0)]} - \frac{1}{\beta\sigma}. \tag{2.35}$$

From (2.25),

$$\frac{c(q_1) - c(q_0)}{\beta[u(q_1) - u(q_0)]} < \frac{\varepsilon_1}{\gamma}.$$

It follows that $h(\varepsilon_0, \varepsilon_1, q_0, q_1) < \frac{1}{\beta}(1 - \frac{1}{\sigma} - 2\varepsilon_1 + \varepsilon_1\frac{\varepsilon_1\beta}{\varepsilon_0\gamma})$. Deriving $\frac{\varepsilon_1^2}{\varepsilon_0}$ from (2.24),

$$\frac{1}{\beta}(1 - \frac{1}{\sigma} - 2\varepsilon_1 + \varepsilon_1\frac{\varepsilon_1\beta}{\varepsilon_0\gamma}) = \frac{1}{\gamma}[(2 - \frac{\gamma}{\beta})(1 - \frac{1}{\sigma}) - \varepsilon_0].$$

One can show that $h(\varepsilon_0, \varepsilon_1, q_0, q_1) < 0$ when either $\sigma = 1$ or $\frac{\gamma}{\beta} \leq 2$. As a result, $\frac{dq_0}{d\gamma} < 0$ and $\frac{d\varepsilon_0}{d\gamma} < 0$ when $\sigma = 1$ or $\frac{\gamma}{\beta} \leq 2$. ∎

2.7.8   Proof of Lemma 2.9

Proof. The proof is similar to the proof of Lemma 1. From (2.19), q is increasing in ε for any given γ. An individual buyer takes the credit limit $\bar{a}$ as given. If ε is large enough, the buyer may face a binding credit limit and the maximum amount q that the buyer can consume is given by $c(q) = \phi m + \beta \hat{\phi} \bar{a}$. ∎

2.8   References

Aiyagari, Rao, Anton Braun and Zvi Eckstein (1998), "Transaction Services, Inflation, and Welfare", *Journal of Political Economy*, Vol. 106 (6), pp. 1274-1301.

Azariadis, Costas and Bruce Smith (1996), "Private Information, Money, and Growth: Indeterminacy, Fluctuations, and the Mundell-Tobin Effect", *Journal of Economics Growth*, 1, pp. 309-332.

Berentsen, Aleksander, Gabriele Camera and Christopher Waller (2007), "Money, Credit and Banking", *Journal of Economic Theory,* Vol. 135 (1), pp. 171-195.

Boyd, John and Bruce Champ (2003), "Inflation and Financial Market Performance: What Have We Learned in the Last Ten Years?", Federal Reserve Bank of Cleveland, working paper, 03-17.

Boyd, John, Ross Levine and Bruce Smith (2001), "The Impact of Inflation on Financial Sector Performance", *Journal of Monetary Economics*, 47, pp. 221-248.

Bullard, James and Keating (1995), "The Long-run Relationship between Inflation and Output in Postwar Economies", *Journal of Monetary Economics*, 36, pp. 477-496.

Cavalcanti, Ricardo and Neil Wallace (1999), "Inside and Outside Money as Alternative Media of Exchange", *Journal of Money, Credit and Banking*, Vol 31 (3), pp. 443-457.

Chiu, Jonathan and Cesaire Meh (2008), "Banking, Liquidity and Inflation", Bank of Canada, working paper.

Craig, Ben and Guillaume Rocheteau (2007), "Inflation and Welfare: A Search Approach", *Journal of Money, Credit and Banking*, forthcoming.

Dotsey, Michael and Peter Ireland (1996), "The Welfare Cost of Inflation in General Equilibrium", *Journal of Monetary Economics*, Vol. 37 (1), pp. 29-47.

Duca, John V. and William C. Whitesell (1995), "Credit Cards and Money Demand: A Cross-sectional Study", *Journal of Money, Credit and Banking*, Vol. 27, No. 2, pp. 604-623.

English, William, (1999), "Inflation and Financial Sector Size", *Journal of Monetary Economics*, 44 (3), pp. 379-400.

Faig, Miquel and Belen Jerez (2006), "Precautionary Balance and the Velocity of Circulation of Money", *Journal of Money, Credit and Banking*, 39 (4), pp. 843-873.

Ferraris, Leo (2006), "On the Complementarity of Money and Credit", Universidad Carlos III de Madrid, working paper.

Freeman, Scott and Gregory Huffman (1991), "Inside Money, Output, and Causality", *International Economic Review*, 32, pp. 645-667.

Kahn, Mohsin, Abdelhak Senhadji and Bruce Smith (2006), "Inflation and Financial Depth", *Macroeconomic Dynamics*, 10, pp. 165-182.

Klee, Elizabeth (2008), "How People Pay: Evidence from Grocery Store Data", *Journal of Monetary Economics*, 55, pp. 526-541.

Kocherlakota, Narayana and Neil Wallace (1999), "Incomplete Record-Keeping and Optimal Payment Arrangements", *Journal of Economic Theory,* Vol. 81, Issue 2, pp. 272-289.

Lacker, J. and Stacey Shreft (1996), "Money and Credit as Means of Payment", *Journal of Monetary Economics*, 38, pp. 3-23.

Lagos, Ricardo and Randall Wright (2005), "A Unified Framework for Monetary Theory and Policy Analysis", *Journal of Political Economy*, Vol. 113 (3), pp. 463-484.

Lester, Ben, Andrew Postlewaite and Randall Wright (2008), "Information, Liquidity and Asset Prices", PIER working paper, 08-039.

Lucas, Robert E. (2000), "Inflation and Welfare", *Econometrica*, Vol. 68, No. 2, pp. 247-274.

Mills, David (2007), "A Model in which Outside and Inside Money are Essential", *Macroeconomics Dynamics*, 11, pp. 347-366.

Sanches, Daniel and Stephen Williamson (2008), "Money and Credit with Limited Commitment and Theft", Washington University in St. Louis, working paper.

Stockman, Alan (1981), "Anticipated Inflation and the Capital Stock in a Cash-in-Advance economy", *Journal of Monetary Economics*, 8, pp. 387-393.

Sun, Hongfei (2007), "Banking, Inside Money and Outside Money", QED working paper, No. 1146.

Telyukova, Irina and Randall Wright (2008), "A Model of Money and Credit, with Application to the Credit Card Debt Puzzle", *Review of Economic Studies*, 75 (2), pp. 629-647.

Wright, Randall (2008), "A Uniqueness Proof for Monetary Steady State", University of Pennsylvania, working paper.

Chapter 3

One or Two Monies?[1]

3.1 Introduction

Micro-founded monetary theory explains how an intrinsically useless object can be valued in exchange. Recent advances in the literature seem to have reached a consensus that the role of money is to make up for the missing memory or the record-keeping technology, i.e., see Kocherlakota (1998a, b). A natural question to follow is whether money constitutes a perfect substitute for the record-keeping technology. Most micro-founded monetary models feature one single money and there is no welfare-enhancing role for a second money. In this paper, we show that when money is divisible, concealable and in variable supply, a single money might or might not be sufficient to replace the record-keeping technology. We then show that in the latter case, introducing a second money improves welfare, and that two monies act as a perfect substitute for the record-keeping technology.

We construct a heterogeneous agent model in a quasilinear environment as introduced by Lagos and Wright (2005). There are two types of agents and two locations indexed by a and b. In every period, a location/preference shock randomly assigns agents to one of the two locations and determines their marginal utilities from consumption. Type a (b) agents have high marginal utilities at location a (b) and low marginal utilities at location b (a).

Since agents at the same location are endowed with the same amount of goods, the first-best allocation requires that agents with low marginal utilities transfer some of their endowment to agents with high marginal utilities.

There are two frictions in the economy: limited commitment and private information about types. In the presence of the these frictions, an implementable allocation must be incentive compatible to ensure participation and truthful revelation of types. Throughout the paper, we adopt a mechanism design approach and solve the conditions under which the first-best allocation satisfies the relevant incentive constraints.

We first analyze mechanisms with a perfect record-keeping technology. The mechanism can directly record nonparticipation and impose perpetual autarky as the punishment for nonparticipation.[2] With regard to private information, the mechanism asks agents to report their types, records the information and uses it later on to infer agents' marginal utilities. Due to the symmetry structure of the preferences, agents have the incentive to truthfully report their types *ex ante* (we call this 'early-sorting' because the incentive is aligned before the realization of the location shock) at the first-best allocation. The first-best allocation can be achieved as long as the participation constraint is satisfied, or agents are patient enough.

Next, we assume that the record-keeping technology is not available, but the society has access to one fiat money. By rewarding participants with more money and requiring an ever increasing amount of money for future participation, one-money mechanisms can deal with limited commitment as effectively as mechanisms with the record-keeping technology. However, one-money mechanisms are not as powerful in dealing with private information. With a single concealable money, encoding and passing information on *ex ante* type reporting becomes problematic. Different reports can only be encoded into different money balances, which however, can be hidden to prevent credible information communication. In this case, the only effective way to deal with private information is to induce agents to reveal their types/marginal utilities after the period location shock (we call this 'late-sorting').

Due to quasilinearity, the late-sorting mechanism involves no *ex ante* welfare cost; it, however, imposes an extra constraint on the patience parameter. As a result, the restriction on the patience parameter to implement the first-best allocation is more stringent than with the record-keeping technology. There exists a positive measure of the patience parameter

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[2]We assume that the most severe societal penalty is ostracism.

such that the first-best allocation can be achieved with the record-keeping technology, but not with one money. Hence, one money is not a perfect substitute for the record-keeping technology in the quasilinear environment that we consider.

We then investigate mechanisms when a second money is introduced. We find that having two monies allows the mechanism to use monetary portfolios and total money balances to record *ex ante* reporting about types. Moreover, the information can be credibly passed into the future. It follows that two monies act as a perfect substitute for the record-keeping technology. We also extend the above results to an environment with more than two types of agents. We argue in general that two monies are a perfect substitute for the record-keeping technology so that there is no need for a third money.

Our work is most closely related to Kocherlakota and Krueger (1999), and Kocherlakota (2002) which also study the essentiality of multiple monies.[3]

Kocherlakota and Krueger (1999) share with us a common feature that a second money improves welfare in that a second money serves as a signalling device to deal with private information. Their model, however, builds on Trejos and Wright (1995) with indivisible money. The result that there is no need for a third money cannot be extended to multiple-type-agent models. Moreover, the quasilinear preferences in our model introduce an additional way (the late-sorting mechanism) to align incentives. It follows that a second money is inessential if agents are patient enough (because late-sorting is powerful enough to deal with both private information and limited commitment).

Kocherlakota (2002) correctly points out that when money is concealable, it is necessary to establish a monotonic relationship between 'proper' behavior and money balances. In Kocherlakota (2002), limited commitment makes it impossible to establish such a relationship and renders the need for a second money. This conclusion, however, hinges critically on the assumption of a fixed money supply. When money supply is fixed, the only way to record whether an agent behaves properly is to transfer some money to him from somebody else (who might also behave properly). Agents' money holdings will differ in general. The first-best allocation, however, requires that future allocation should not discriminate those with less money balances. The mechanism that we propose circumvents the problem by

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[3]There is another strand of literature investigating whether multiple currencies can coexist or circulate at the same time. Examples are Trejos and Wright (2001), Camera and Winkler (2003), Camera *et al.* (2004), and Craig and Waller (2004). Our paper's goal is to study the welfare enhancing role of multiple currencies, or whether multiple currencies are essential.

increasing money supply to reward all properly behaved participants. Limited commitment thus does not justify a role for a second money if money supply is allowed to change.

The rest of the paper proceeds as follows. Section 2 lays out the physical environment and characterizes the first-best allocation. Section 3 introduces private information and limited commitment. We solve for the condition to achieve the first-best allocation when the society has access to a record-keeping technology. Section 4 studies the optimal monetary mechanisms when the record-keeping technology is absent, and establishes the condition under which a second money is essential. Section 5 extends the results to a multi-type-agent model. Section 6 argues in general that two monies constitute a perfect substitute for the record-keeping technology. We conclude and suggest directions for future research in section 7.

3.2   The Physical Environment

The framework that we adopt is the quasi-linear environment suggested by Lagos and Wright (2005) without the search friction. Time is discrete and runs from 0 to ∞. Each period consists of two stages: day and night. There are two locations a and b. Inter-location interaction is allowed during the day but prohibited at night. There are three non-storable goods, one day good and two location-specific night goods indexed by 1 and 2. Good 1 is local to location a and good 2 is local to location b. There are two types of agents – each is of measure 1.

During the day, all agents can produce or consume the day good. They have the same linear preference over the good. Let z be the amount of production (consumption if z is negative). The disutility of production (utility of consumption if z is negative) is $-z$.

At night, agents consume one of the two night goods. The two types of agents are distinguished by their preferences over the two night goods. Type a value good 1 more than good 2, and type b value good 2 more than good 1. It might be helpful to think of type a as local consumers of good 1 and foreign consumers of good 2; similarly, think of type b as local consumers of good 2 and foreign consumers of good 1. The utility of a local consumer is $\delta u(c)$ and the utility of a foreigner is $u(c)$, where $\delta > 1$, $u(0) = 0$, $u'' < 0 < u'$ and $u'(0) = +\infty$. Which night good an agent consumes is determined by a preference shock realized upon entering the night stage. With probability $1/2$, an agent becomes a local consumer and has a high valuation of the night good. With probability $1/2$, the agent

becomes a foreign consumer and has a low valuation of the night good. We assume that each agent is endowed with y units of the location-specific good after the realization of the preference shock. Note that at the night stage, each location is inhabited by two types of agents who value the night good differently. Refer to Figure 1 for a graphical illustration of the environment.

Figure 3.1: Environment

The life-time expected utility of a type a agent $i \in (0,1)$ is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^a(i) + \frac{1}{2} \left[\delta u(c_{1,t}^a(i)) + u(c_{2,t}^a(i)) \right] \right\}.$$

where $0 < \beta < 1$ is the discount factor, $z_t^a(i)$ is the production (consumption if negative) of the day good, and $c_{1,t}^a(i)$ and $c_{2,t}^a(i)$ are the consumption of (night) good 1 and 2 respectively. Similarly, the life-time expected utility of a type b agent $j \in (0,1)$ is
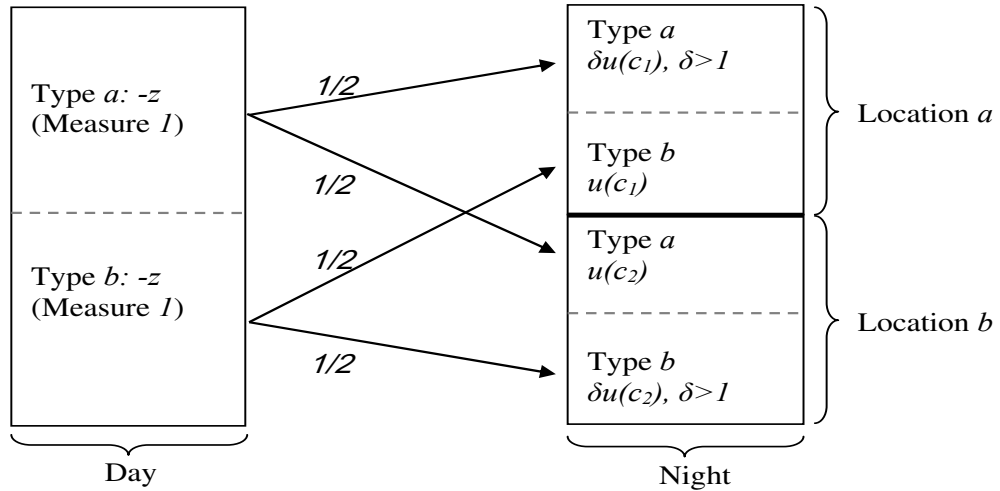
$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^b(j) + \frac{1}{2} \left[\delta u(c_{2,t}^b(j)) + u(c_{1,t}^b(j)) \right] \right\}.$$

The resource constraints are given by

$$\int_0^1 z_t^a(i) di + \int_0^1 z_t^b(j) dj = 0$$

at the day stage, and

$$\int_0^1 c_{1,t}^a(i)I_t^a(i)di + \int_0^1 c_{1,t}^b(j)I_t^a(j)dj \quad = \quad y \text{ at location } a,$$

$$\int_0^1 c_{2,t}^a(i)I_t^b(i)di + \int_0^1 c_{2,t}^b(j)I_t^b(j)dj \quad = \quad y \text{ at location } b,$$

at the night stage for all $t \geq 0$. $I_t^k(\bullet)$ is an indicator function and is equal to 1 if the agent is at location $k \in \{a,b\}$ at date t.

We will focus on symmetric stationary allocations where for all i and $j \in (0,1)$ and $k \in \{a,b\}$,

- $c_{1,t}^a(i) = c_{2,t}^b(j) = c_h$, $c_{2,t}^a(j) = c_{1,t}^b(i) = c_\ell$ with $c_h + c_\ell = 2y$ for $t \geq 0$;

- $z_0^a(i) = z_0^b(j) = 0$;

- $z_t^k(\cdot) = \begin{cases} z_h, & \text{if the agent consumed } c_h \text{ at the night stage of time } t-1; \\ z_\ell, & \text{if the agent consumed } c_\ell \text{ at the night stage of time } t-1; \end{cases}$

  with $z_h + z_\ell = 0$ for $t \geq 1$.

The social planner's problem is to choose $(c_h, c_\ell, z_h, z_\ell)$ to maximize the *ex ante* utility

$$W(c_h, c_\ell, y) = \frac{1}{2}\frac{1}{1-\beta}\left[\delta u(c_h) + u(c_\ell)\right] \tag{3.1}$$

$$\text{s.t. } c_h + c_\ell = 2y.$$

The solution is characterized by

$$\delta u'(c_h^*) \quad = \quad u'(c_\ell^*),$$

$$c_h^* + c_\ell^* \quad = \quad 2y,$$

$$W^* \quad = \quad \frac{1}{2}\frac{1}{1-\beta}\left[\delta u(c_h^*) + u(c_\ell^*)\right].$$

Note that since $\delta > 1$, $c_h^* > y > c_\ell^*$. The planner can instruct the night stage low-valuation agents to 'lend' $\tau^* \equiv y - c_\ell^*$ units of his endowment to high-valuation agents. For the day stage allocation, note that since z_t enters linearly in preferences, any $z_t^a(i)$ and $z_t^b(j)$ that satisfy $E_0 z_t^a(i) = E_0 z_t^b(j) = 0$ would satisfy the day stage resource constraint and entail no *ex ante* welfare loss. In the current context, one such allocation is $z_t^a(i) = z_t^b(j) = 0$ for all i and j and $t \geq 0$.

The first-best allocation can be achieved if agents' types are public information, and agents are able to commit to sticking with the allocation.

3.3 Limited Commitment and Private Information

Assume that agents cannot commit, and agents' types and thus their valuations of the night goods are private information. In this case, a record-keeping technology becomes essential to overcome the frictions caused by limited commitment and private information (see Kocherlakota, 1998a, b).

In the presence of limited commitment and private information, an implementable allocation must satisfy individual rationality or participation constraints (so that individuals have the incentive to stick with the mechanism), and the incentive constraints (so that individuals have the incentive to truthfully reveal their private information).[4]

When agents cannot commit, the allocation $(c_h, c_\ell, z_h, z_\ell)$ must respect *ex post* rationality. Assuming that the punishment for nonparticipation is autarky, the welfare is

$$W_0 = \frac{1}{2}\frac{1}{1-\beta}[\delta u(y) + u(y)]. \tag{3.2}$$

It is straightforward that $W^* > W_0$. At the night stage, there are two individual rationality conditions: one for high-valuation agents and one for low-valuation agents,

$$\delta u(c_h) + \beta(-z_h + W) \geq \delta u(y) + \beta W_0, \tag{3.3}$$

$$u(c_\ell) + \beta(-z_\ell + W) \geq u(y) + \beta W_0, \tag{3.4}$$

where W is as defined in (1). At the day stage, there are also two individual rationality conditions

$$-z_h + W \geq W_0, \tag{3.5}$$

$$-z_\ell + W \geq W_0. \tag{3.6}$$

Note that if $c_h > c_\ell$, for night stage high-valuation agents, the day stage individual rationality condition (3.5) implies the night stage individual rationality condition (3.3). For night stage low-valuation agents, the night stage individual rationality condition (3.4) implies the day stage individual rationality condition (3.6). An implementable allocation with $c_h > c_\ell$ must satisfy (3.5) and (3.4), which we rewrite and label as (IRH) and (IRL)

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[4]We assume that group deviation and side trades can be prevented to avoid extra constraints incurred by the market structure.

respectively. To simplify notation, let $z = z_h = -z_\ell$.

$$z \leq W - W_0, \tag{IRH}$$

$$z \geq W_0 - W + \frac{u(y) - u(c_\ell)}{\beta}. \tag{IRL}$$

If a mechanism prescribes higher night consumption for high-valuation agents (which is the case at the first-best allocation), low-valuation agents will have the incentive to claim to be high-valuation agents. Private information about types implies that agents can potentially lie about their valuations of the night goods.

Due to the structure of the preference shocks, there are two ways to deal with the incentive problem caused by private information. Since the two types of agents always value the same night good differently, the planner can induce agents to truthfully reveal their types at the day stage and use the information to infer an agent's valuation of the night good. For example, if an agent reports to be a type a agent and shows up at location b, the planner can infer that the agent is a low-valuation agent. Note that since recorded information can be passed into the infinite future, the mechanism only needs to ask agents to report their types once at the day stage of periods 0. The information will then be used in all the following periods.[5] We call this mechanism the early-sorting mechanism because information used to identify agents' valuations is revealed before the realization of the preference shocks. To use the early-sorting mechanism, the following constraint needs to be satisfied

$$\frac{1}{1-\beta}\left\{\frac{\delta u(c_h) - \beta z_h}{2} + \frac{u(c_\ell) - \beta z_\ell}{2}\right\} \geq \frac{1}{1-\beta}\left\{\frac{\delta u(c_\ell) - \beta z_\ell}{2} + \frac{u(c_h) - \beta z_h}{2}\right\}, \tag{ICT}$$

which holds if $c_h > c_\ell$.[6]

Alternatively, the planner can skip type reporting and try to induce the agents to truthfully report their valuations of the night goods by resorting to variations in production/consumption at the following day stage. We call this the late-sorting mechanism because information used to identify agents' valuations is revealed after the realization of the preference shocks. Notice that the late-sorting mechanism is effective if and only if the

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[5]This explains why we use life-time utilities in the *ICT* below.

[6]If $c_h > c_\ell$, using *ICT* does not impose extra constraints on the day stage allocation z other than the resource constraint.

following two conditions are satisfied

$$\delta u(c_h) + \beta(-z_h + W) \geq \delta u(c_\ell) + \beta(-z_\ell + W),$$
$$u(c_\ell) + \beta(-z_\ell + W) \geq u(c_h) + \beta(-z_h + W).$$

We can rearrange the two incentive constraints as (again, let $z = z_h = -z_\ell$ to simplify notation)

$$z \leq \frac{\delta[u(c_h) - u(c_\ell)]}{2\beta}, \tag{ICH}$$

$$z \geq \frac{u(c_h) - u(c_\ell)}{2\beta}. \tag{ICL}$$

The first constraint ensures that high-valuation agents do not want to imitate low-valuation agents (note that this means that type a agents do not want to imitate type b agents at location a, and that type b agents do not want to imitate type a agents at location b). The second constraint ensures that low-valuation agents do not want to imitate high-valuation agents.

Proposition 1 states the condition under which the first-best allocation can be achieved when the planner has access to a record-keeping technology,

Proposition 3.1 *When agents lack commitment and hold private information about their types, a record-keeping technology can achieve the first-best allocation if and only if $\beta \geq \beta_0$ where β_0 is defined as*

$$\beta_0 = \frac{u(y) - u(c_\ell^*)}{\delta[u(c_h^*) - u(y)]}.$$

Proof. With a record-keeping technology, the planner can use ICT to deal with private information. Since $c_h^* > c_\ell^*$, the first-best allocation meets ICT automatically. The first-best can be achieved if and only if there exists a z such that IRH and IRL are satisfied, or

$$W_0 - W^* + \frac{u(y) - u(c_\ell)}{\beta} \leq W^* - W_0,$$

which, with some manipulation, can be rewritten as

$$\beta \geq \frac{1}{\delta} \frac{u(y) - u(c_\ell^*)}{u(c_h^*) - u(y)} \equiv \beta_0. \blacksquare$$

Proposition 1 states that the first-best allocation is implementable when agents are patient enough. Andolfatto (2008) has a similar result. The key friction that generates this result is limited commitment. Private information can be overcome since the first-best allocation entails $c_h^* > c_\ell^*$. As long as a record-keeping technology is available, ICT is automatically satisfied. There is no need to use ICH and ICL.

3.4   Monetary Mechanisms

Now suppose that the society has no access to the record-keeping technology. Then it is impossible to directly pass information across time. In this case, the planner uses tokens – which we call money – as a substitute for the missing record-keeping technology to communicate information across stages.[7] We assume that money is perfectly divisible, concealable and in variable supply.

3.4.1   One-Money Mechanisms

We first assume that there is a single money available and study if one money is a perfect substitute for the record-keeping technology. One-money mechanisms can deal with limited commitment as follows. By rewarding participants with money and increasing the amount of money required for future participation, a one-money mechanism can effectively catch nonparticipants and cast them into perpetual autarky. The individual rationality constraints remain the same as in the case with a record-keeping technology. Note that the concealability of money balances does not pose a problem here since the proposed mechanism establishes a monotonically increasing relationship between participation and money balances so that people do not have the incentive to hide money.

Now we show how one-money mechanisms deal with private information. The question we ask is whether the planner can encode type reports into money holdings and use them later on to identify agents' valuations of the night goods. The answer is no. It means that early-sorting cannot be used in one-money mechanisms.

With one money, the only way to encode type reports is to associate different types with different money balances. For example, the planner can give those who report to be type a more money. To use early-sorting, the planner is supposed to give high consumption to those with more money (or those reported to be type a) at location a, and those with less money (or those reported to be type b) at location b. The problem is that at location b, those with more money have the incentive and ability to mimic those with less money to demand for higher consumption. A quick examination of figure 2 shows that holding more money is strictly preferred to holding less money, so all agents will report to be type a agents at the day stage of period 0. All agents will hold the same amount of money. The

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[7]The society, though, has access to a contemporaneous memory technology which can remember agents' actions *within* a stage.

planner will not be able to infer agents' valuations of the night goods based on their money holdings.
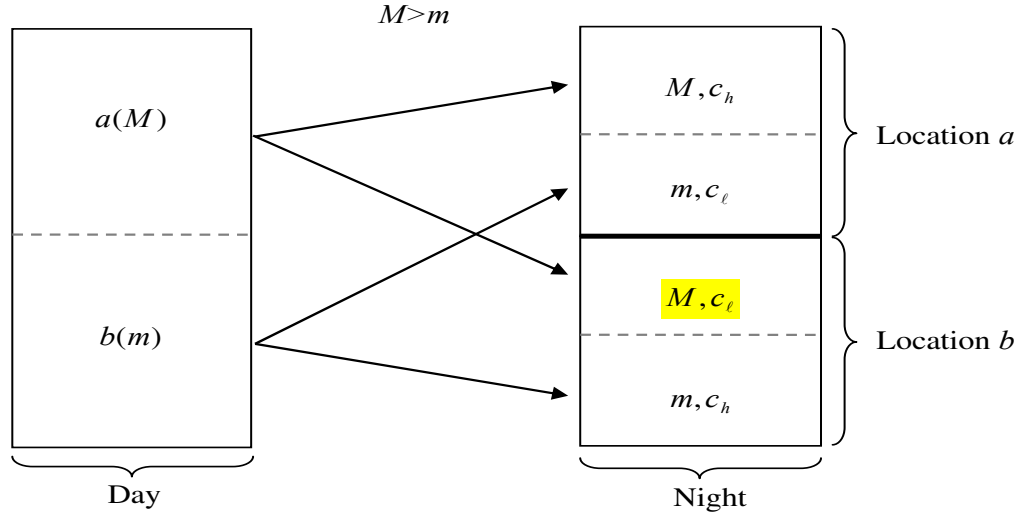
Figure 3.2: Early Sorting Ineffective with a Single Money

To induce agents to truthfully reveal their valuations of the night goods, the planner must rely on the late-sorting mechanism. The planner can give all agents the same amount of money at the day stage of period 0. High-valuation agents can choose to consume more at the night stage, but they need to work more in the future. They will leave the night stage with less money and work more in the following day stage to accumulate more money. In this case, we need to replace ICT by ICH and ICL.

Proposition 2 states the condition under which one-money mechanisms can achieve the first-best allocation.

Proposition 3.2 *When agents lack commitment and hold private information about their types, one-money mechanisms can achieve the first-best allocation if and only if $\beta \geq \beta_1$, with β_1 given by*

$$\beta_1 = \frac{u(c_h^*) - u(c_\ell^*)}{(\delta + 1)[u(c_h^*) - u(y)]} > \beta_0.$$

Proof. Consider the following mechanism.

Let $0 < \rho_h < \rho_\ell < 1$.

At date 0 **day stage,** the mechanism endows each agent with one unit of money (\$).

At date 0 **night stage,** after preference shocks are realized, the mechanism offers agents the following choices.

Show 1 \$, $\begin{cases} \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \text{ \$; or} \\ \text{receive } \tau^* \text{ good (1 or 2) and } \rho_h \text{ \$.} \end{cases}$

With this mechanism, non-participants leave with 1 \$ and participants leave with more than 1 \$. Participants receiving transfers consume more and leave with lower money balances; those giving up endowment consume less and leave with higher balances.[8]

At date 1 day stage, the mechanism offers agents the following options.

Show $(1 + \rho_h)$ \$, use z day good to exchange for $(1 - \rho_h)$ \$;

Show $(1 + \rho_\ell)$ \$, receive z day good and $(1 - \rho_\ell)$ \$.

With this mechanism, all participants leave the stage with 2 units of money and non-participating agents leave with less than 2 units of money. Participants entering with lower balances work to earn extra money.

At date 1 night stage, the choices are

Show 2 \$, $\begin{cases} \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_\ell \text{ \$; or} \\ \text{receive } \tau^* \text{ good (1 or 2) and } \rho_h \text{ \$.} \end{cases}$

At date $t \geq 2$ **day stage,** the choices are

Show $(t + \rho_h)$ \$, use z day good to exchange for $(1 - \rho_h)$ \$;

Show $(t + \rho_\ell)$ \$, receive z day good and $(1 - \rho_\ell)$ \$.

At date $t \geq 2$ **night stage,** the choices are

Show $(t + 1)$ \$, $\begin{cases} \text{use } \tau^* \text{ good (1 or 2) to exchange for } \rho_h \text{ \$; or} \\ \text{receive } \tau^* \text{ good (1 or 2) and } \rho_\ell \text{ \$.} \end{cases}$

Note that under this mechanism, if an agent skips a stage, his money balance will fall short of the required balances to participate in all of the following stages. The mechanism effectively catches non-participants and casts them into perpetual autarky. The individual rationality conditions thus remain the same as in the case with the record-keeping technology.

The first-best allocation can be achieved if and only if there exists a z such that at (c_h^*, c_ℓ^*, z),

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[8]Since there is a contemporaneous memory technology, the mechanism can prevent agents from participating more than once.

ICH, ICL, IRH and IRL are satisfied, or

$$\frac{u(c_h^*) - u(c_\ell^*)}{2\beta} \leq z \leq \frac{\delta[u(c_h^*) - u(c_\ell^*)]}{2\beta}, \tag{IC}$$

$$\frac{u(y) - u(c_\ell^*)}{\beta} + W_0 - W^* \leq z \leq W^* - W_0. \tag{IR}$$

z is non-empty if and only if

$$\frac{u(c_h^*) - u(c_\ell^*)}{2\beta} \leq W^* - W_0,$$

which can be rearranged as

$$\beta \geq \frac{u(c_h^*) - u(c_\ell^*)}{(\delta + 1)[u(c_h^*) - u(y)]} \equiv \beta_1.$$

If follows from $\delta u(c_h^*) + u(c_\ell^*) > (1 + \delta)u(y)$ that $\beta_1 > \beta_0$. ∎

The one-money mechanism outlined above deals with frictions caused by limited commitment and private information as follows. By rewarding participants with newly issued money and increasing the money balances required for future participation, the mechanism effectively catches non-participants and casts them into perpetual autarky. By requiring previous high-valuation agents to work for previous low-valuation agents at the day stage, the mechanism induces agents to truthfully reveal private information and signal preferences by choosing different money balances at the night stage. The first-best allocation can be implemented if and only if $\beta \geq \beta_1$. When $\beta_0 < \beta < \beta_1$, the one-money mechanism cannot implement the first-best allocation, while the mechanism with a record-keeping technology can. In this sense, the one-money mechanism is less powerful in dealing with private information about types.

3.4.2 Two-Money Mechanisms

Given that one-money mechanisms cannot fully replicate the allocations that are implementable with a record-keeping technology, we introduce a second money in this subsection and show that two monies constitute a perfect substitute for the record-keeping technology.

Label the two monies as "red" and "green". Similar to the one-money mechanism, the two-money mechanism can reward participants with more money balances and effectively exclude nonparticipants from the mechanism forever. The individual rationality conditions thus stay the same as in the case with the record-keeping technology.

What is different from the one-money mechanism is that two-money mechanisms make *ICT* feasible again. The planner can encode type reports into monetary portfolios with the same total balances but different compositions of the two monies, and request agents to show the same total balances at the night stage. For example, suppose that the planner gives those reporting as type a more red money and those reporting as type b more green money. At the following night stage, the planner requires more red money for high consumption at location a and more green money for high consumption at location b. By requesting all agents to show the same total money balances, agents will not be able to juggle their portfolios to renege on their earlier reports. The early-sorting mechanism is thus reinstated (see Figure 3). As in the case with record-keeping technology, the first-best allocation can be achieved if and only if $\beta \geq \beta_0$.

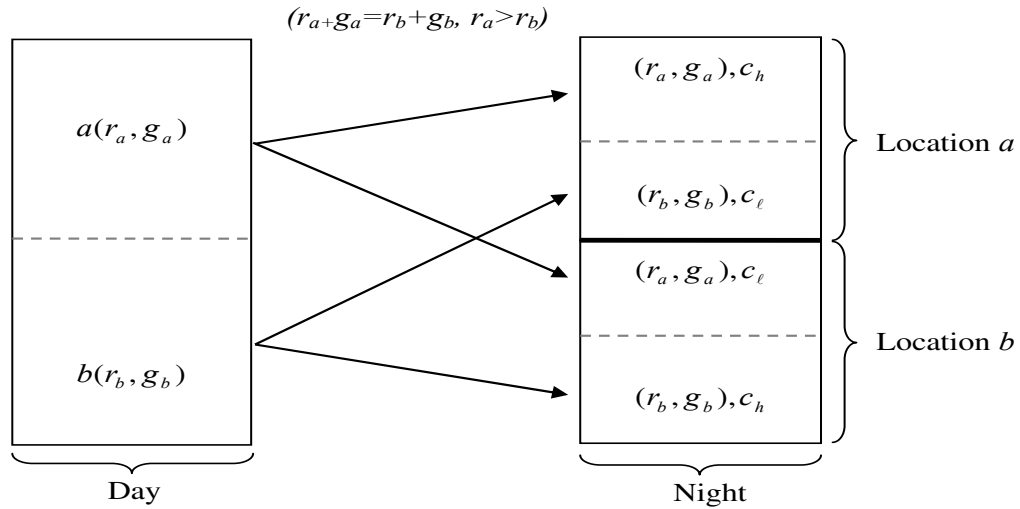

Figure 3.3: Early Sorting Effective with Two Monies

Proposition 3.3 *When agents lack commitment and hold private information about their types, two monies act as a prefect substitute for the record-keeping technology and can achieve the first-best allocation if and only if $\beta \geq \beta_0$.*

Proof. Call the two monies red (R) and green (G). Consider the following mechanism.

At date 0 **day stage,** the mechanism asks agents to choose from two monetary portfolios: 1 R or 1 G.

At date 0 night stage, after the shocks are realized, the mechanism offers agents the following choices.

 At location a,

  Show R, receive τ^* good 1 and ρ R, where $0 < \rho < 1$;

  Show G, use τ^* good 1 to exchange for ρ R;

 At location b,

  Show G, receive τ^* good 2 and ρ G;

  Show R, use τ^* good 2 to exchange for ρ G.

 With this mechanism, non-participants leave the night stage with 1 unit of money and participants leave with more than 1 unit of money. Participants with higher consumption leave the night stage with a single type of money (i.e., $1 + \rho$ units of R at location a); participants with lower consumption leave with two types of money (for example, 1 unit of G and ρ units of R at location a). All participants exit the night stage with the same total money balances $1 + \rho$.

At date 1 **day stage,**

  Show $(1 + \rho)$ R, use z day output to exchange for $(1 - \rho)$ R;

  Show $(1 + \rho)$ G, use z day output to exchange for $(1 - \rho)$ G;

  Show $R + \rho$ G, use ρ G to exchange for z day output and 1 R;

  Show $G + \rho$ R, use ρ R to exchange for z day output and 1 G.

At date 1 **night stage**,

 At location a,

  Show 2 R, receive τ^* good 1 and ρ R;

  Show 2 G, use τ^* good 1 to exchange for ρ R;

 At location b,

  Show 2 G, receive τ^* good 2 and ρ G;

  Show 2 R, use τ^* good 2 to exchange for ρ G.

At date t $\geq$ 2 day **stage**,

  Show $(t + \rho)$ R, use z day output to exchange for $(1 - \rho)$ R;

  Show $(t + \rho)$ G, use z day output to exchange for $(1 - \rho)$ G;

  Show t $R + \rho$ G, use ρ G to exchange for z day output and 1 R;

  Show t $G + \rho$ R, use ρ R to exchange for z day output and 1 G.

At date t $\geq$ 2 night stage,

At location a,

Show $(t+1)$ R, receive τ^* good 1 and ρ R;

Show $(t+1)$ G, use τ^* good 1 to exchange for ρ R;

At location b,

Show $(t+1)$ G, receive τ^* good 2 and ρ G;

Show $(t+1)$ R, use τ^* good 2 to exchange for ρ G.

The two-money mechanism described here rewards participants with more money balances, effectively catches non-participants and bars them from participating in the mechanism forever. The individual rationality conditions thus stay the same as in the case with the record-keeping technology.

The two-money mechanism can induce the two types of agents to hold different monetary portfolios with the same total balances. At the night stages, low-valuation agents will not be able to falsely claim to be high-valuation agents. For example, suppose that type a choose to hold red money and type b choose to hold green money at date 0. At the following night stage, a type a agent at location b is a low-valuation agent and cannot claim to be a high-valuation agent since he does not have the green money required for higher consumption. We verify in the following that the two types of agents indeed have the incentive to differentiate themselves from each other by choosing different monetary portfolios at date 0. Take type a agents as an example. The expected life-time utility from holding the red money is[9]

$$W_r^a = \frac{1}{2}\frac{1}{1-\beta}[\delta u(c_h^*) + u(c_\ell^*)],$$

and the expected utility from holding the green money is

$$W_g^a = \frac{1}{2}\frac{1}{1-\beta}[\delta u(c_\ell^*) + u(c_h^*)].$$

It is straightforward that $W_r^a > W_g^a$ so that type a agents prefer holding the red money. The mechanism outlined above can achieve the first-best allocation if the allocation (c_h, c_ℓ, z) satisfies ICT, IRH and IRL at (c_h^*, c_ℓ^*). As in the case with a record-keeping technology, the first-best allocation can be achieved if and only if $\beta \geq \beta_0$.∎

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[9]Note that under the proposed mechanism, agents hold the same color of money while entering all night stages; they basically make only one type reporting choice when they decide what portfolio to hold at the day stage of period 0. This is why we compare the *life-time* utilities from holding different monetary portfolios.

The two-money mechanism induces the two types of agents to hold different monetary portfolios. Since the two portfolios feature the same total balances, it is impossible to juggle one's portfolio to renege on earlier type reports. The early-sorting mechanism (ICT) is thus reinstated and two monies provide a perfect substitute for the record-keeping technology. The introduction of a second money improves welfare when $\beta < \beta_1$.[10]

3.5 Extension to Multi-type-agent Models

In this section, we show that as in Townsend (1987), two monies consist of a perfect substitute for the record-keeping technology even when there are more than two types of agents.[11] The optimal two-money mechanism is to let different types hold different combinations of the red and green monies, with all combinations giving the same total money balances.

There are $N < +\infty$ symmetric locations and N location specific night goods. There are N types of agents distinguished by their preferences over the night goods. A type $m \in \{1, 2, ..., N\}$ agent derives utility $\delta_{mn} u(c_{mn})$ from consuming c_{mn} units of night good $n \in \{1, 2, ..., N\}$, where $\delta_{mn} = \delta_{m-n+I(m<n)N}$, $\delta_0 > \delta_1 > \delta_2 > ... > \delta_{N-1} > 0$, and $I(m < n) = 1$ if $m > n$ and 0 otherwise. For example, type 1 agents derive utility $\delta_0(c)$ from goods at location 1, $\delta_{N-1}u(c)$ from goods at location 2,..., and $\delta_1 u(c)$ from goods at location N; type N agents derive utility $\delta_0 u(c)$ from goods at location N, $\delta_{N-1}u(c)$ from goods at location 1, ..., and $\delta_1 u(c)$ from goods at location $N-1$. See table 1 for an illustration of the structure of the preferences. The columns represent the types of goods and the rows represent the types of agents.

During the day, all agents can produce and consume the day good. At night, each agent is subject to a preference shock and goes to each of the N locations with the same probability $1/N$. After agents are relocated, at location $n \in \{1, 2, ..., N\}$, all agents are endowed with y units of good n, but they differ in their valuations of the good. Agents cannot commit and agents' types and thus their valuations of the night goods are private information.

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[10]Note that when $\beta < \beta_0$, the first-best allocation cannot be achieved even with a record-keeping technology. It can be shown that two monies are still a perfect substitute for the record-keeping technology and two-money mechanisms strictly improve welfare over one-money mechanisms.

[11]Kocherlakota and Krueger (1999) also mention that two monies are sufficient in their model which has only two types of agents. With indivisible monies, however, more monies will be needed if there are more than two types of agents.

Table 3.1: Preference Shocks

	1	2	...	n	...	$N-1$	N
1	δ_0	δ_{N-1}	...	δ_{1-n+N}	...	δ_2	δ_1
2	δ_1	δ_0	...	δ_{2-n+N}	...	δ_3	δ_2
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
m	δ_{m-1}	δ_{m-2}	...	$\delta_{m-n+I(m<n)N}$	...	δ_{m+1}	δ_m
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
$N-1$	δ_{N-2}	δ_{N-3}	...	δ_{N-1-n}	...	δ_0	δ_{N-1}
N	δ_{N-1}	δ_{N-2}	...	δ_{N-n}	...	δ_1	δ_0

We focus on symmetric stationary allocations where all agents with the same valuations of night goods consume the same amount, or for any $m, n \in \{1, 2, ..., N\}$,

- $c_{mnt} = c_{mn} = c_{m-n+I(m<n)N}$ for $t \geq 0$ where c_{mnt} is the consumption of type m agents at location n in period t night stage;

- $z_{m0} = 0$;

- $z_{mnt} = z_{m-n+I(m<n)N}$ if the agent consumed c_{mn} at the night stage of time $t-1$ for $t \geq 1$;

The first-best night stage consumption is characterized by

$$\delta_q u'(c_q^*) = \delta_{q'} u'(c_{q'}^*) \text{ for all } q \neq q' \in \{0, 1, ..., N-1\},$$
$$\sum_{q=0}^{N-1} c_q^* = Ny.$$

Any day stage allocation $(z_0, z_1, .., z_{N-1})$ satisfying $\sum_{q=0}^{N-1} z_q = 0$ satisfies the resource constraint and is consistent with the first-best allocation. The first-best life-time welfare of a representative agent is

$$W^{N*} = \frac{1}{N} \frac{1}{1-\beta} \left[\sum_{q=0}^{N-1} \delta_q u(c_q^*) \right].$$

We first characterize the condition under which the first-best can be achieved when the planner has access to a record-keeping technology. Suppose that $c_0^* > c_1^* > ... > c_{B-1}^* > y >$

$c_B^* > ... > c_{N-1}^*$ so that at the first-best allocation, B types of agents are borrowers who consume more than their endowment, and $N - B$ types of agents are lenders who consume less than their endowment.

To deal with limited commitment, the following $2N$ individual rationality conditions must be satisfied: for all q, $q' \in \{0, 1, ..., N - 1\}$ and $q \neq q'$,

$$\delta_q u(c_q) + \beta(-z_q + W^N) \geq \delta_q u(y) + \beta W_0^N,$$
$$-z_q + W^N \geq W_0^N,$$

where

$$W^N = \frac{1}{N} \frac{1}{1 - \beta} \left[\sum_{q=0}^{N-1} \delta_q u(c_q) \right] \text{ and}$$

$$W_0^N = \frac{1}{N} \frac{1}{1 - \beta} u(y) \sum_{q=0}^{N-1} \delta_q.$$

There are two ways to deal with the friction caused by private information. If early-sorting is used, the following $(N^2 - N)$ constraints must be satisfied

$$\frac{1}{N} \frac{1}{1 - \beta} \left[\sum_{n=1}^{N} \delta_{mn} u(c_{mn}) \right] \geq \frac{1}{N} \frac{1}{1 - \beta} \left[\sum_{n=1}^{N} \delta_{mn} u(c_{m'n}) \right],$$

for all $m, m' \in \{1, 2, ..., N\}$ and $m' \neq m$. If late-sorting is used, the following $(N^2 - N)$ constraints must be satisfied

$$\delta_q u(c_q) + \beta(-z_q + W^N) \geq \delta_q u(c_{q'}) + \beta(-z_{q'} + W^N),$$

for all $q \neq q' \in \{0, 1, ..., N - 1\}$.

Following the steps in section 3, it can be shown that the *ICT*s hold at the first-best allocation. When there is a record-keeping technology, the first-best allocation can be achieved if and only if

$$\beta \geq \beta_0^N = \frac{\sum_{q=B}^{N-1} \delta_q \left[u(y) - u(c_q^*) \right]}{\sum_{q=0}^{B-1} \delta_q \left[u(c_q^*) - u(y) \right]}.$$

In the absence of a record-keeping technology, one-money mechanisms must resort to late-sorting to align incentives, and the first-best allocation can be achieved if and only if

$$\beta \geq \beta_1^N = \frac{\sum_{q=1}^{N-1}(N - q)\delta_q \left[u(c_{q-1}^*) - u(c_q^*) \right]}{\sum_{q=1}^{N-1}(N - q)\delta_q \left[u(c_{q-1}^*) - u(c_q^*) \right] + \sum_{q=0}^{N-1} \delta_q \left[u(c_q^*) - u(y) \right]} \geq \beta_0^N.$$

The following proposed mechanism with two monies (R and G) shows that two monies act as a perfect substitute for the missing record-keeping technology. Two-money mechanisms improve welfare over one-money mechanisms when $\beta < \beta_1^N$.

At the day stage of date 0**,** the mechanism asks agents to choose from N monetary portfolios

$$r_m\ R + (1 - r_m)\ G,$$

with $0 < r_m < 1$ for all $m \in \{1, 2, ..., N\}$ and $r_m \neq r_{m'}$ for all $m \neq m'$.

At the night stage of date 0**,** after the shocks are realized, the planner offers each agent the following choices.

At location $n \in \{1, 2, ..., N\}$,

Show $r_m\ R + (1 - r_m)\ G$, get $c_{mn} = c_{m-n+I(m<n)N}$ good n and $\varepsilon[r_n\ R + (1 - r_n)\ G]$ where $0 < \varepsilon < \min\limits_{m \neq m' \in \{1,2,...,N\}} \{|r_m - r_{m'}|\}$.

The mechanism requires agents to show 1 unit of money to participate in the stage, and proposes consumption contingent on the composition of monetary portfolios held by agents. The mechanism rewards participating agents with ε units of money, the composition of which differs across locations. We restrict ε to ensure that agents of different types and consuming at different locations exit the night stage with different monetary portfolios.

At the day stage of the t $\geq$ 1 period,

Show $t[r_m\ R + (1 - r_m)\ G] + \varepsilon[r_n\ R + (1 - r_n)\ G]$, use $z_{mn} = z_{m-n+I(m<n)N}$ day output and $\varepsilon[r_n\ R + (1 - r_n)\ G]$ to exchange for $[r_m\ R + (1 - r_m)\ G]$.

At the night stage of the $t \geq 1$ **period,**

At location n,

Show $(t+1)[r_m\ R + (1 - r_m)\ G]$, get $c_{mn} = c_{m-n+I(m<n)N}$ good n and $\varepsilon[r_n\ R + (1 - r_n)\ G]$.

The two-money mechanism outlined here deals with limited commitment and private information exactly the same way as the two-money mechanism with two types of agents. As long as N is finite, we can see that two monies are always a perfect substitute for the record-keeping technology. If money is indivisible as in Kocherlakota and Krueger (1999), we will need at least N monies to replace the record-keeping technology when there are N types of agents.

3.6 Two Monies as A Perfect Substitute for the Record-Keeping Technology

In our model's environment, two monies are sufficient to replace the record-keeping technology. Townsend (1987) and Kocherlakota (2002) have similar results in different environments. Here we develop an intuitive argument to show that two monies are always sufficient as a substitute for the record-keeping technology so there is no need for a third money.

If money balances are not concealable, there is a one-to-one match between records and money balances, money balances will thus carry the relevant information into future periods. When money balances are concealable, however, the one-to-one match will be destroyed since individuals can change balances by hiding money. Or, the concealability of money balances makes it possible for individuals to change records to their own benefits. The introduction of a second money solves the problem by encoding records into different monetary portfolios with the same total balances and different compositions of the two monies. Agents will not be able to juggle their monetary portfolios to mimic other portfolios by concealing money. Note that when money is divisible, it is possible to encode any finite number of records into different monetary portfolios so a third money will not be needed.

3.7 Conclusion

In this paper, we show in a quasi-linear environment that in the presence of private information and limited commitment, a second money can potentially improve welfare by providing an efficient way to pass information across time.

In the absence of a record-keeping technology, monetary mechanisms (with either a single money or two monies) can effectively deal with limited commitment by rewarding participants with more money balances and requiring ever increasing money for future participation. The individual rationality conditions stay the same as in the case with the record-keeping technology where defectors are directly caught and forced into perpetual autarky.

There are two options to deal with private information about preferences. The first is to induce agents to truthfully report their types before the realization of the preference shocks and use the information later on to infer agents' valuations of the night goods. We call

this early-sorting. The second option is to induce agents to report their valuations after the realization of the preference shocks, and use the day stage consumption/production to align the incentives. We call this late-sorting.

Mechanisms with a single concealable money rule out early-sorting. When agents are patient enough, the late-sorting mechanism effectively aligns the incentive by inducing agents to leave the night stage with different money balances and produce/consume different amounts in the following day stage. When agents are not patient enough, the late-sorting mechanism is not powerful enough to align incentives. The introduction of a second money permits the early-sorting mechanism and allows agents to signal their preferences by holding different monetary portfolios (with the same total money balances).

We intend to extend the paper in the following way. In the paper, we take the mechanism design approach and there is no market in the mechanisms proposed in the paper. In particular, monies do not circulate as a medium of exchange. We would like to follow Waller (2007) to see if the allocations can be decentralized with market mechanisms (and with the help of monetary and fiscal policies). In cases that a second money can enhance welfare, we can then further investigate whether the second money merely serves as a receipt or we also require the second money to circulate.

3.8   References

Andolfatto, David (2008), "The Simple Analytics of Money and Credit in a Quasi-linear Environment," manuscript, Simon Fraser University.

Camera, Gabriele, Ben Craig and Christopher J. Waller, (2004), "Currency Competition in a Fundamental Model of Money," *Journal of International Economics*, 64, pp. 521-544.

Camera, Gabriele and Johannes Winkler, (2003), "International Monetary Trade and the Law of One Price," *Journal of Monetary Economics*, 50, pp. 1531-1553.

Craig, Ben and Christopher J. Waller, (2004), "Dollarization and Currency Exchange," *Journal of Monetary Economics*, 51, pp. 671-689.

Kocherlakota, Narayana (1998a), "The Technological Role of Fiat Money," *Federal Reserve Bank of Minneapolis Quarterly Review*, 22 (3), pp. 2–10.

Kocherlakota, Narayana (1998b), "Money is Memory," *Journal of Economic Theory*, 81(2), pp. 232–251.

Kocherlakota, Narayana and Thomas Krueger (1999), "A Signaling Model of Multiple Currencies," *Review of Economic Dynamics*, 2, pp. 231-244.

Kocherlakota, Narayana (2002). "The Two-money Theorem," *International Economic Review*, 43(2), pp. 333-346.

Lagos, Ricardo and Randall Wright (2005), "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), pp. 463-484.

Townsend, Robert M. (1987), "Economic Organization with Limited Communication," *The American Economic Review*, 77(5), pp. 954–971.

Trejos, Alberto and Randall Wright (1995), "Search, Money, Bargaining, and Prices," *Journal of Political Economy*, 103, 118-141.

Trejos, Alberto and Randall Wright (2001), "International Currency," *Advances in Macroeconomics*, Vol 1, Issue 1, Article 3.

Waller, Christopher J. (2007), "Dynamic Taxation, Private Information and Money," manuscript, University of Notre Dame.