A COMMENTARY ON ASSUMPTIONS OF DISCOUNTING PARAMETERS IN THE STERN REVIEW ON THE ECONOMICS OF CLIMATE CHANGE

By

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Bachelor of Economics, Zhejiang University of China, 2006

PROJECT SUBMITTED IN PARTIAL FULFULLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

In the
Department
of
Economics

SIMON FRASER UNIVERSITY
Summer 2008

APPROVAL

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Degree: **Master of Arts**

Title of Project: **A Commentary on Assumptions of Discounting Parameters in The Stern Review on the Economics of Climate Change**

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Date Defended/Approved: June 13, 2008

Abstract

The Stern Review concludes that human beings can avoid a loss of 20% of the present value of consumption due to climate change if nations immediately reduce GHG emissions with a loss of only 1% of consumption per year. Since this conclusion varies from earlier studies, many reviews of the Stern Review focus on criticizing this conclusion focusing on its extreme assumption of discounting rates. However, only a few have justified their arguments through empirical work. Thus, this paper aims to provide empirical evidence to support their theories. The Review's original IAM model and methodology are improved and its results are replicated. Its conclusions can barely survive with alternative conventional values such as higher discount rates. The paper conducts comparative tests demonstrating the effect these other parameter values can have on the loss of the consumption. These tests reveal large changes in the Review's estimate of a 20% loss.

Acknowledgements

First of all, I want to thank my examining committee: Prof. Terry Heaps, Prof. Richard Harris, for the tremendous support and valuable advice they have given me in this paper. Prof. Heaps directed me to a wide range of topics in nature resources. He answered all of my questions as well as asked me questions that inspired me to solve every theoretical problem. I also want to thank Dave Cox for his patience in revising my paper and gave me many helpful suggestions.

Secondly, I want to thanks all my friends, who support me always. Many thanks to Lihua, Ben, Chris and Jiamin.

Dedication

To my parents, for your love

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1. Introduction

The Stern review on the Economics of Climate Change, which is a precious treasure for human generations, vividly introduces us to the basic climate change issues and the way global warming will affect world GDP and people's everyday life from now on. The chief author, Sir Nicholas Stern, is an academic economist as well as a distinguished economic advisor for the United Kingdom government. After working as the second permanent secretary at H.M. Treasury[1], he has dedicated himself to the review of the economic effects of climate change. On 30th October 2006, H.M. Treasury presented a scientific-and-economic comprehensive analysis of climate change to the public. In a short time, it has attracted worldwide attention from economists, environmental scientists and politicians.

1.1 Overall Review of the Study in Climate Change and the Methodology in the Stern Review

Climate change research is one of the most interesting studies but is also a formidable challenge in the environmental science area. It is fascinating because it combines many analytical facets in environmental and economic science. It is challenging because it encompasses so many further uncertain issues in climatic facts as well as dynamic economic analysis across time, regions and countries. First, for the uncertainty part, each

[1] H.M. Treasury, with the full name of Her Majesty's Treasury, is the United Kingdom government department responsible for developing and executing the British government's public finance policy and economic policy.

uncertain climatic parameter is presented by a probability distribution. Most results are then attained by Monte Carlo methods, which require at least 1000 simulation runs[2]. Generally speaking, the number of uncertain parameters is around 80. Those parameters crucially determine the simulation results predicting climate change analysis. For example, the uncertain parameters of greenhouse gas (hereafter GHG) emissions and the proportion of GHG that resides in the atmosphere will affect the atmospheric temperature, rainfall, sea level, and ocean temperature. Secondly, for the dynamic economic analysis part, the remaining work after simulation programming is to undertake a dynamic benefit-cost analysis[3]. This process should also take account of many issues of uncertainties, like technology advancement, which would probably decrease the emission of GHG, and increase the human adaptation to the climate change.

Therefore, climate change research requires a model that can encompass the considerable large numbers of uncertain parameters and accurately simulate the economic impacts of global climate change by combining geophysical climate dynamics and economic dynamics.

An integrated assessment model, known short as IAM, is designed to meet this analytical purpose. As stated in IPCC Technical papers, "IAMs are based on the integration of models that simulate the most critical processes of the climate system (human emissions, biosphere, oceans and atmosphere), and are used to explore the

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[2] Results of the simulation programming are provided in the IPCC reports (2001).

[3] This analysis is to compare the "business as usual" (BAU) economy scenario with the scenario with the climate change damage, so as to predict how the climate change would influence the life of human beings.

impacts of diverse emissions scenarios generated by alternative energy sources, different land-use changes, pollution control, and population policies[4]".

Based on multiple equation programming, an IAM can "constructively force multiple dimensions of the climate change problem into the same framework, and quantify the relative importance of climate change in the context of other environmental and non-environmental problems facing mankind[5]". In fact, many academics have already developed IAMs to simulate the economic impacts of climate change, for instant, the AIM model by Morita et al in 1994, the DICE model by Nordhaus in 2007, the FUND model by Tol et al in 2001, the PAGE model by CEC in 1992 and by Chris Hope in 2007, the RICE model by the Nordhaus and Yang in 1996, and so on.

After assessing the 'Mendelsohn' model, the 'Tol' model and the 'Nordhaus' model, the Stern Review does not believe they can accurately capture the economic and social costs of climate change, since they "omit other potential important factors---such as social and political instability and cross-sectoral impacts, and they have not yet incorporated the newest evidence on damaging warming effects[6]". Such shortages in other existing IAMs inspired the Stern Review to adopt PAGE2002 as its foundation IAM, since "the PAGE2002 IAM can take account of the range of risks by allowing outcomes

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4 IPCC Second Assessment Report, 1997, pp30.

5 Weyant, et. al., 1996.

6 The Stern Review, chapter 6, pp151.

to vary probabilistically across many model runs, with the probabilities calibrated to the latest scientific quantitative evidence on particular risks"[7].

However, the Stern Review does not include detailed information about the programming process and technical explanations of its "attractive" quantitative results.

First, the method used to calibrate uncertain parameters is not clearly documented. As we know, IAM modeling involves dozens of uncertain parameters, thus, the way to calibrate these parameters is the essential step for the whole modeling procedure. In the Stern Review, there are 2×3 scenarios[8] indicating that there should be probability distributions for 80 uncertain parameters[9] in each of the six scenarios in the Stern Review. It only explains that the 'Monte Carlo' simulation method was used to generate probability distributions for scenario outcomes. However, it skips over answering many essential questions about what equations it has derived to simulate the uncertainties. How does it define its six scenarios during the very first step of simulation and what are the mean values for the crucial uncertain parameters? Thus, besides the fascinating results and policy recommendations, what a reader wants to know is the radical reasoning and

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7 The Stern Review, chapter 6, pp153 .

8 The Stern Review classified the levels of temperature change into two categories: baseline climate and High climate. In chapter 6, it states that Baseline climate scenario "produces a mean warming of 3.9°C relative to pre-industrial in 2100 and a 90% confidence interval of 2.4-5.8°C " and High Climate scenario "is designed to explore the level of temperature change is pushed to higher levels through the action of amplifying feedbacks in the climate system and the 90% confidence interval increases to 2.6-6.5°C ".

The categories of climate change impacts comprehensively include economic impact and welfare impact. In particular, these three categories are market impacts, risk of catastrophic events and non-market impacts considering the affects on human health and the environment.

9 PAGE2002 model, adpoted by the Stern Review, contains 80 uncertain parameters.

and policy recommendations, what a reader wants to know is the radical reasoning and quantitative analysis supporting those conclusion. There are no details about these assumptions in any or the chapters or in the technical appendix. That is also the reason why there is no Review of the Stern Review that has successfully replicated its results under those 6 scenarios.

Secondly, the IAM programming in the Stern Review is loosely tied to PAGE2002. Although at the very beginning of IAM programming introduction in the Review, it clearly states that, "we use PAGE2002 IAM…it is flexible enough to include market impacts and non-market impacts as well as the possibility of catastrophic climate impacts…we present results base results based on different assumptions along 2×3 dimensions[10]". However, the connection between the Review's IAM and PAGE2002 is quite ambiguous. PAGE2002 includes both market impacts and non-market impacts. More specifically, Hope [11] incorporates market and non-market impacts in the 'Computing the Value of Global Warming Impacts' section of PAGE2002 model, by combining these two sectors' impacts as one joint impact on the global GDP. Thus, he reports on only one scenario in PAGE2002 model. The Stern Review treats the market and non-market impacts as separate economic impact scenarios. This is a big difference between the results of two models: one only has one final result under one scenario while the other includes six scenarios and claimed those results are directly derived from the

[10] The Stern Review, Chapter 6, pp153

[11] Chris Hope updates PAGE97 to PAGE2002 to capture the new findings in the third assessment report of IPCC.

former model. Unless it clarifies how it expands the one-scenario model into a six-scenario IAM, the Stern Review's results and conclusions are questionable.

Thirdly, the `Expected-utility' analysis of the global cost of climate change in the Stern Review loses consistency. By using `Monte Carlo' simulation method together with PAGE2002 IAM, the Stern Review generates two consumption paths, one with climate change and one without climate change. Subsequently, with those two consumption paths, the Review calculates the present value of global welfare using a special unity utility function. The expected utility is given by the following equation, where the utility function is $U = \ln C(t)$.

$$W = \sum_{1}^{2200} N(t) \ln C(t) e^{-\delta t} + \left(\frac{N_T \ln C_T}{\delta} + \frac{N_T g}{\delta^2}\right) e^{-\delta T} \quad (1.1)$$ [12]

Then, the Stern Review adopts the idea of "Balanced Growth Equivalent"[13] (hereafter BGE) since the dollar value calculated by the BGE method is a more easy-understood unit than the `utils' in representing global social welfare. Developing this idea, the Stern Review gets the value of BGE consumption per capita using equation (1.2), where, obviously, the utility function changes to $U = \frac{C(t)^{1-\eta}}{1-\eta}$.

$$W = \sum_{1}^{2200} N(t) \left(\frac{C_{BGE}^{1-\eta}}{1-\eta} + gt\right) e^{-\delta t} + \left(\frac{N(t)\frac{(C_{BGE}+200g)^{1-\eta}}{1-\eta}}{\delta - g(1-\eta)}\right) e^{-\delta t} \quad (1.2)$$

[12] The Stern Review, Chapter 6, pp162.

[13] The BGE method first calibrates the welfare from a consumption part [C(t)]. Then by setting it equivalent in welfare terms, the balanced growth path yields BGE consumption that grews at a constant rate from now to future.

setting W(1.2) = W(1.1), equation (1.2) determines the value of BGE consumption C_{BGE}. Since we have two series of consumption path c(t), one with climate change and one without, hence, we can represent the influence of the climate change on our global social welfare by comparing the two BGE consumption C_{BGE}, which represents equivalent social welfare. This BGE method sounds very reasonable, however, to be consistent, equation (1.2) should use the same utility function used in equation (1.1) while, in fact, it adopts another general CRRA utility function form. It should also be noted that it again fails to provide the crucial explanation about the derivation of the equation (1.1) and (1.2).

Finally, the procedure used for cost-benefit analysis in Review is also problematic. Chapter 6 of the Stern Review explains that it uses an IAM to estimate "the cost of BAU[14]" and "Benefit from Regulation". In this analytical procedure, the first round estimation is to assess the aggregate business-as-usual cost without considering regulation of GHG emissions. As a result, the Review finds that this cost is much higher than that in other previous analyses. The Review argues that the reason might be that its model has taken account of new-found scientific "facts", including "increased direct impacts on the environment and human health", "amplifying feedbacks in the climate system" and "a disproportionate burden of climate change impacts fall on poor regions of the world"[15]. All these new facts apparently help to increase the aggregate cost of BAU,

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[14] Definition of "BAU" refers to footnote 3.

[15] The Stern Review, chapter 6, pp143.

which plays such a critical role in arriving at the Review's extreme conclusion. However, as pointed out by its critics, all those new facts are unconvincing because there barely are any references to support them in the Review. Together with the Review's questionable assessment of BAU cost, we can find his analysis of "benefit from Regulation" is also problematic. Overall, "Benefit from Regulation" measures the aggregate value of reduced damage when government take political actions to control climate change which result in the sacrifice of economic growth. The empirically proven fact of "climate-policy-ramp"[16] assumes that the outcome of climate change control only produces benefit far in the future so these benefits are heavily discounted. However, if more weight is put on future generations, then we will expect a larger climate control should be taken in current period; and this is what the Stern Review has proposed.

Trying to clarify thus unclear analytical reasoning, what I propose to do in the following sections is to recast the essential analytical reasoning and detailed procedure used in the Stern Review's methodology. The whole procedure will strictly be tied to PAGE2002 IAM.

1.2 Literature Review of Critiques on the Stern Review

Since the release of the Stern Review, this report has been criticized by many other climate change researchers, especially economists. Their core arguments are about (1) the

[16] This strategy is viewed as milestone in climate change research. David L. Kelly and Charles D. Kolstad initially found this policy through their IAM work in 1999. Then, William D. Nordlaus has empirically proved it with DICE model.

Review's strong call for early and urgent action to reduce GHG emission, which contradicts many accepted analyses of climate change, and (2) about the values of several economic parameters used in the Review, especially the value of the rate of pure time preference.

Martin L. Weitzman has made his review of the Stern Review focusing on theoretical and financial issue (Weitzman, 2007). He argues that it's so hard to make readers or economists accept the Review's conclusion because it relies upon such an extremely low discount rate. Weitzman bases his critique on the famous Frank Ramsey equation. From this equation, he calculates that the annual interest rate in the Review is 1.4% which is much lower than the market value of 6%. Such a numerical disparity directly leads to a difference of 'two orders of magnitude'[17] in the estimated damage cost of climate change over two hundred years. Although Weitzman has made his point by using generally accepted values for the parameters in Ramsey equation, he also feels worried about the omission of uncertainty in this equation. In the last part of his critique, he has extended the Ramsey equation by treating the consumption growth rate g as an i.i.d. (independent and identically distributed) random variable. Then using financial theory, Weitzman has theoretically enriched the oversimplified treatment of uncertainty in Stern's Review.

Also questioning the Stern Review's recommendation of immediate action for the sharp reduction of GHG emission, is Nordhaus(2007). He focuses on finding the reasons that lead to the Review's strikingly different conclusions about the best strategy for

[17] Weitzman, Martin L., 2007.

effectively controlling climate change. Nordhaus finds the Review's assumptions about the social welfare function and other economic parameters to be quite unconvincing. First, as its social welfare function stems from the British utilitarian tradition, it is hardly consistent with the real intertemporal decision making mechanism. It's hard to argue that future generations will behave in exactly the same maner as current generations, while the Stern Review assumes that all of them will behave the same according to the same utility function. Secondly, it is known that the real discount rate is very important to balance present and future. However, this crucial parameter is casually discussed, with neither apparent reference nor justification. In order to prove that the Stern Review's decisive conclusion stems from its choice of social utility function and other extreme parameters' value, Nordhaus employs the DICE-2007 model to see whether using different values of the consumption elasticity and the time discount rate will result in different conclusions about the economic impacts of GHG emission. The empirical work with the DICE model consists of three runs with different parameter values. Run one's combination is the baseline bundle with a time discount rate of 1.5% and a consumption elasticity of 2. Run two is based on the Review's extreme value of 0.1% and elasticity of 1. The last run recalibrates the elasticity of marginal utility while using a zero discount rate and a desired real interest rate. The DICE model experiments prove that the Stern Review's extreme values for the economic parameters are the central causes in producing a conclusion substantially distinct from mainstream conclusions. Also, it again proves that Nordhaus own climate change policy, known as Policy Ramp[18], in the climate change issue, is one of the most effective policies found to control climate change so far.

Besides Weitzman and Nordhaus, many other economists also have criticized the Stern Review. These include Robert Mendelsohn, Partha Dasgupta, Richard Tol, Hal Varian and Kenneth Arrow. Mendelsohn believes that the Stern Review's questionable conclusion arises from several assumptions, such as the inconsistent discount rate, extreme weather scenarios, inequity uncertainty and so on. Also Mendelsohn questions the Stern Review's arguments about low abatement costs saying that he ignored the fact that regulation of emissions should have a different cost at different level of GHG concentration. Dasgupta criticizes the Stern Review in a theoretical way. He argues that the new scientific and economic facts have not driven the Review's political suggestions on climate change issue. Moreover, Dasgupta also believes that the value of elasticity of the marginal utility to consumption should depend on the level of consumption while the Stern Review keeps this elasticity constant. As he states in his review, "what we should have expected from the Review is a study of the extent to which its recommendations are sensitive to the choice of *eta*"[19].

Looking at all the reviews on the Stern Reviews, most make their statements without providing any empirical evidence. Only Dr. Nordhaus engage in empirical work.

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[18] The controls on climate change should be put into effect in an increasing but gradual manner to reduce GHG emissions in near term while a sharp reduction is required in the medium and long term. This climate-policy suggestion is opposite to the Review's political recommendation.

[19] Partha Dausgupta, 2007, pp6

However, he uses his own IAM known as DICE rather than the PAGE2002 used in the Review, because there is insufficient evidence to allow the replication of the original programming. This paper will follow Nordhaus's idea to replicate the Stern Review's aggregate global welfare analysis by adjusting the numerical figures for the Ramsey equation parameters. The main differences with Dr. Nordhaus' work are the following. First, Nordhaus uses the DICE model rather than the PAGE2002 which is used in Stern's Review. However, it's better to adopt the original model to replicate the original results. Secondly, most existing critiques focus only on arguing that the value of pure time discounting rate is too small in the Review, including Nordhaus' paper. However, in this paper, I attempt to find out the quantitative influence of all the essential parameters, including the consumption growth rate, the time discounting rate and the elasticity of marginal utility to consumption.

2. Discounting Rates in the Long-term Climate Change Issue

Since IAMs incorporate an economic growth model, in economic growth theories, the real interest rate lies at the heart of these economic frameworks for it balances benefits of present generations with those of future generations through discounting. Most crucial results, conclusions, and suggestions of dynamic analysis depend on the values of those discounting rates; even an insignificant change might lead to a reversal of final implications and conclusions. Tjalling Charles Koopmans has already warned us of this in his famous work on growth theory, "The problem of optimal growth is too complicated, or at least too unfamiliar, for one to feel comfortable in making an entirely a priori choice of a time discount rate before one knows the implications of alternative choices"[20]. As Koopmans reminds us, no matter whether developing our own or examining others' intertemporal growth models, we should be very circumspect about the assumptions made about the the discount rates.

Noticing the importance of discounting rate to economic growth models, it is also an essential determinant in the climate change research since such research is also a long run problem involving intertemporal analysis. Consequently, discounting rate might be the key that drives the Stern Review's questionable conclusion. Before we examine the rationality of Stern's conclusion, we should initially focus on examining the assumptions

[20] Koopmans, Tjalling C, 1965.

about the discount rate and find out if it is the reason that leads to the Review's distinct conclusion.

2.1 The Theoretical Background of the Economic Model in the Stern Review

The Stern Review operates intertemporal analysis and Business-As-Usual (BAU) cost-benefit methodology in its Economic model of climate-change impacts. Obviously, it adopts the dynamic analysis of the Ramsey-Koopmans-Cass model to determine the real interest rate in the the following aggregate work. Then, what is the mechanism in Ramsey-Koopmans-Cass model that determine the real interest rate?

Ramsey-Koopmans-Cass (RCK) model, which is the joint work by Ramsey (1928), Koopmans (1965) and Cass (1965), has been considered as the central organization model in neo-classical optimal growth theory. This model assumes that an economy only has two production inputs, labour and capital, owned by households who would produce a homogenous good. Also it assumes that the representative household maximizes the aggregate present value of future utility by choosing his or her optimal consumption, saving and labour supply. In continuous time, the optimization problem in the RKC model can be interpreted in the "per capita" version,

$$\max_{c \in C[0,\infty)} \int_0^{\infty} U[c(t)]e^{-\delta t}\, dt$$

$$\text{Subject to: } \dot{k} = f(k) - (n + d)k - c,$$

$$k(0) = k_0,$$

$$\liminf_{t\to\infty} k(t) \geq 0,$$

$$f(t) \geq c(t) \geq 0,$$

(2.1)

where individual utility U[.] depends on a single variable, household's consumption c(t). The core constraint for this intertemporal optimization problem is $\dot{k} = f(k) - (n + d)k - c$; the growth of the capital stock per capita is social investment (f(k)-c) after depreciation ((n+d)k, where n denotes the growth rate of labor and d denotes the depreciation rate, both of which are exogenous variables).

Here, we have our first essential parameter, δ, which is called the pure time preference or time discounting rate. The important distinction between time discounting rate δ and real interest rate r is that the former one is used for discounting utility while the later one is more primitive rate that discounts "dollar-valued" parameters, like consumption in this model. Thus, in equation (2.1), we designate δ to discount social welfare, U[c(t)].

Further, for mathematical convenience, RCK model features a specific form for utility function, which has a constant elasticity of the marginal utility to consumption, which has been conventionally called a CRRA utility function.

$$U[c(t)] = \frac{c(t)^{1-\eta}}{1-\eta}, \text{ for } \eta>0$$

(2.2)

In this utility function, comes our second important parameter, elasticity of marginal utility to consumption η. Such elasticity can be represented as the relative curvature of the utility function. Besides the elasticity, η can be equivalently called the coefficient of relative risk aversion. Like δ captures households' time preference, η captures consumers' risk preference and attitudes towards interpersonal inequality. Generally, the larger value of η, the more aversion to risk and inequality consumers will behave.

Lastly, the per capita production function has the famous form as Cobb-Douglas production function.

$$f(k) = Ae^{\mu t}k^{\alpha}, \text{here k is per capita capital} \tag{2.3}$$

To solve the problem, we use the Homitalian:

$$H = \frac{c(t)^{1-\eta}}{1-\eta}e^{-\delta t} + \lambda(f(k) - (n + d)k - c) \tag{2.4}$$

The conduction for the time paths are the:

$$\frac{dH}{dc(t)} = 0 \Rightarrow c^{-\eta}e^{-\delta t} - \lambda = 0 \tag{2.5}$$

$$\frac{dH}{dk} = \dot{\lambda} \Rightarrow -\lambda(f'(k) - (n + d)) = \dot{\lambda} \tag{2.6}$$

Rearrange equation (2.5) and (2.6), we can derive:

$$\text{Equation}(2.5) \Leftrightarrow \lambda = c^{-\eta}e^{-\delta t}$$

$$\Leftrightarrow \frac{d\lambda}{dt} = \dot{\lambda} = -\eta\dot{c}c^{-\eta-1}e^{-\delta t} - \delta c^{-\eta}e^{-\delta t}$$

$$\Leftrightarrow \frac{\dot{\lambda}}{\lambda} = -\eta\frac{\dot{c}}{c} - \delta$$

$$\text{Equation}(2.6) \Leftrightarrow \frac{\dot{\lambda}}{\lambda} = -f'(k) + (n + d)$$

$$\text{Equation}(2.5) = \text{Equation}(2.6)$$

$$\Leftrightarrow -\eta\frac{\dot{c}}{c} - \delta = -f'(k) + (n + d)$$

$$\Leftrightarrow \eta\frac{\dot{c}}{c} + \delta = f'(k) + (n + d)$$

(2.7)

Here, η and δ have been mentioned above; $\frac{\dot{c}}{c}$ is the growth rate of the per capita consumption, which will is denoted as g in the following. The right hand of the equation is the price of per capita capital $(f'(k) + (n + d))$, which is well known as the real interest rate, r.

Equation (2.7) leads us to the famous Frank Ramsey Equation:

$$r = \delta + \eta g \tag{2.8}$$

As stated in equation (2.8), the dynamic real interest rate, or equivalently speaking, the rate of return on capital r is determined by three parameters, the rate of pure time preference δ, the product of the elasticity of marginal utility η and the growth rate of consumption g. If, as stated before, δ and η feature behavior aspects of personal tastes, then, the growth rate of consumption g represents the growth trajectory of technical advancement facets in the economic development. To be more precisely, if we use Cobb-Douglas per capita production function $y = Ae^{\mu t}k^{\alpha}$, then this will show us that the growth rate of economy $\hat{y} = g$ is determined by $(\mu + \alpha\hat{k})$[21]. Since in our scenario of Balanced Equivalent Growth model, the growth rate of per capita output y, per capita consumption c, and per capita capital k are all the same, then, the growth rate of economy g equals to $\frac{\mu}{1-\alpha}$ along a balanced growth path. Thus, under the assumption of Cobb-Douglas production function in this project, we will treat g as a constant parameter rather than a random variable as argued in Weitzman's critique. Consequently, given a

[21] $y = Ae^{\mu t}k^{\alpha} \Longrightarrow \dot{y} = Ak^{\alpha}e^{\mu t}\mu + Ae^{\mu t}\alpha k^{\alpha-1}\dot{k} \Longrightarrow \hat{y} = \mu + \alpha\hat{k}$

constant g, δ and η, the real interest rate r would also be constant parameter in Ramsey Equation.

In the following section, I will discuss the Stern Review's argument about its critical choices for η, δ and g.

2.2 Commentary on Assumptions of Discounting Parameters in the Stern Review

Chapter 6 of the Stern Review explains that it use an Integrated Assessment Model to estimate "the cost of BAU" and the "Benefit from Regulation" using a cost-benefit analysis. In this analytical procedure, the first round estimation is to assess the aggregate business-as-usual cost without considering climate change damage. The Review finds that such cost is much higher than that in other previous analysis. It argues that the reason might be that its model has taken account of new-found scientific "facts", including "increased direct impacts on the environment and human health", "amplifying feedbacks in the climate system" and "a disproportionate burden of climate change impacts fall on poor regions of the world"[22]. All these new facts apparently help to increase the aggregate cost of BAU, which plays a critical role that in driving the Review's extreme conclusion. However, as pointed out by its critiques, all those new facts are unconvincing because there barely are any references to support them in Review. Together with the Review's questionable assessment of BAU cost, we can find his analysis of the "benefit from

[22] The Stern Review, chapter 6, pp143.

Regulation" is also problematic. Overall, "Benefit from Regulation" measures the aggregate value of reduced damage if governments take political actions to control climate change at a scary price of economic growth in current periods. Since such political adaption follows the empirically proved fact of "climate-policy ramp"[23], which indicates the outcome of climate change control can only appear after a considerable long period, hence, the benefit of political adaption has to be discounted far in the future. Therefore, if we put more weight on the future generations, we will expect a larger climate control should be taken in current period; and this is what the Stern Review has proposed.

Chapter 6 of the Stern Review conducts a economic cost-benefit analysis to assess the benefit from reduction of GHG emissions in current period. In this intertemporal analysis, the discount rate critically determines the outcome of the "cost-benefit analysis". Even an insignificant change could result in a different inverse conclusion because after compounding over decades and centuries, such "an insignificant change" can be aggregate into a considerably large factor. Thus, examining Review's assumption about its discounting rate determines whether or not we accept its outcome of IAM analysis and political recommendation of rapid reduction on GHG emissions in current period.

Following the present-discussion of the Ramsey Equation in section 2.1, consequently I will separately discuss the Review's assumptions on the critical parameters, δ, η and g, which together give the value of the real interest rate r.

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[23] See footnote 13.

2.2.1 Per Capital Growth Rate of Consumption, g

Parameter, g, measures the global growth rate of consumption, and also it is the growth rate of per capita income in Stern's IAM model. The Stern Review assumes that after two centuries, our economy will achieve the steady state growing at constant rate of 1.3% per annual into the far off future. It further explains that, "We use 1.3% per annum, which is the annual average projection from 2001 to 2200 in the PAGE2002's baseline world without climate change"[24].

An alternative would be to use the current growth rate of 3%. However, as g here denotes an average measure of the global economy growth, we will expect such an average term should be lower than the current growth rate of 3% that measures the recent rapidly expanding global economy. Secondly, this growth rate is also consistent with the "A2 scenario" in the International Panel on Climate Change's 2000 special report, in which the scenario assumes the per capital consumption will grow at the constant rate of 1.3% [25]per year.

Also in part III, I will empirically prove that the value of g is not as critical as the other two parameters since it only loosely affects the final outcome of PAGE2002 programming.

2.2.2 Pure Time Discounting Rate, δ

[24] The Stern Review, Chapter 6, box6.3, pp161.

[25] This consumption growth rate is consistent with the average growth rate calculated in PAGE2002 IAM.

The pure time discounting rate is one of the parameters designated to measure individual preference. δ specifically reflects the attitude towards future generations by current generations; the lower the value, the more weight we will put on the future well-being. As the Stern Review argues, "Attaching little weight to the future, simply because it is in the future, would produce low estimates of cost-but if you care little for the future you will not wish to take action on climate change"[26]. It chooses a near-zero pure time discounting value(δ=0.1%) to demonstrate fairness to every generation. Does such an extreme low pure time preference really play an egalitarian role in this long run issue? In fact, this low pure time discounting rate indicates unfairness for current generation in this cost-benefit analysis. We can consider this problem through intuition and mathematics way. By intuition, if our global economy grows at the rate of 1.3% as assumed in the Stern Review, after 200 years, global GDP will become 7 billion while current GDP is 45 million only. However, with this low pure time discounting rate, the present value of global GDP in 2200 almost stays constant. If one of Stern Review's aims is to take account of "distributional and ethical equity judgments systematically and explicitly", then why should we use a poor generation's income to benefit the rich generation of the future? With this low pure time discounting rate, it has already put unfairness on current generations. Mathematically, the Stern Review suggests that with 1% reduction of current GDP to control GHG emissions, such action will lead to 5% increase in GDP after two centuries. Obviously, present value of this cost is 1% while the present

[26] The Stern Review, Chapter 6, pp143.

value of future benefit is $5\% \times e^{-200\delta}$. When δ is close to 0, $e^{-200\delta}$ approximates to 1 which gives us the cost-benefit ratio $\frac{B}{C}$ is 5 as stated in the Stern Review. Without doubt, this benefit should attract us to take emergent action to reduce GHG emissions, but if we increase δ to some value that makes this benefit ration equals 1%, the $\frac{B}{C}$ ratio will become 1. Then, why should we sacrifice current well beings' 1 dollar to generate also 1 dollar valued benefit for the well beings after 200 decades? Thus, the Stern Review's extreme conclusion is driving its extreme assumption; once changing the value of this critical parameter, its model and methodology will generate a different result.

2.2.3 Elasticity of Marginal Utility of Consumption, η

Following the discussion in section 2.2.2, the other parameter that measures individual preference is η, the elasticity of marginal utility of consumption, or equivalently, the coefficient of relative risk aversion. As its definition stated, η quantitatively represents the level of personal sensitivity to interpersonal inequality and risk aversion to consumption. Thus, η can also tell authors' attitude towards social equality; how individuals wish to transfer their consumption across time and how the societies judge about the consumption transformation across time and persons. However, authors of the Stern Review ignore this important issue and assume the value of η=1 without giving any reference and supporting arguments. Then, does this unit value of η really help to deliver Stern Review's equalitarian attitudes? Based on previous studies of CRRA[27] utility function, the Stern

[27] Constant relative risk aversion (CRRA) utility function, $U(c) = \frac{c^{1-\eta}}{1-\eta}$.

Review has chosen the baseline guess of the value of η which is at the lowest bound of the conventional guess range for this risk aversion parameter. Such unity value of η tells us that it is worthwhile to spend current wealth for the future generation because the low value indicates that the wealth distribution does not matter that much even though future generations are much richer than us. Also, this unity value demostrates a risk-neutral attitude among the consumers. Thus, although they may expect a risk of future damage caused by the climate change, they still would like to invest in the future periods regardless of the expected risk. Under such an interpretation, it's more clear now why the Stern Review's model conclusion suggests us to take rapid action to offset the climate change. This conclusion may be based on the unconventional guess of η which put unfairness in favors of future generations over current generation.

2.3 Long Term Discounting Rate, r

In section 2.2, I focused on discussing the assumptions of the three determinant parameters separately. In this section, I will examine the rationality of the Review's assumption about the real interest rate.

According to the Ramsey Equation derived in section 2.1, the long term discount rate r equals the sum of δ and ηg, which gives us 1.4% under the Review's assumptions. This figure is another contradiction to the conventional economist's best-guess; most of the economists suggest the real interest rate should around 4% in the "business as usual" environment.

Weitzman also claimed that this numerical figure of 1.4% that will lead to us an approximate 100% saving rate[28]. However, with the assumption of the Cobb-Douglas production function with technology advancement, I have derived the saving rate depends α, g and r, that is $s = \frac{\alpha g}{r}$[29]. If given $g \approx r$ consistent to the Review, s will approximate to α, which is around 0.3 to 0.4 in the economic literature. Thus, the Review's assumptions actually give us a 30% saving rate, which is quite normal and conventional. The reason why Weitzman argued the Review would generate a 100% saving rate with its assumption is because he ignored the "technology factor α"[30].

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[28] Weitzman, 2007.

[29] See Appendix.

[30] In Weitzman's Review, he suggests the saving rate is $s = \frac{g}{r}$.

3. Empirical Adjustments of Model Parameters in the Stern Review

Most Reviews of the Stern Review focus on criticizing its extreme conclusion and its assumptions about the model parameters but a few of them have also questioned the Review's methodology. As Nordhaus states in his review, "understanding the analysis of the Stern Review is made even more difficult because the detailed calculations behind the Review have not been made available"[31]. The absence of the details of the Stern Review's methodological process means most of the reviewers had to argue in a theoretical way while the only empirical work written by Nordhaus(2007) adopts an alternative model, DICE2007, to examine the Stern Review's discounting strategies.

After reviewing the economic and geophysical rationale in the Review together with the PAGE2002 Model programming developed by Dr. Chris Hope, I will aim to replicate the Stern Review's results and then adjust the model parameters' values. There are two incentives inspiring me to do so. First, economists of those critiques all agree that the most crucial problem in this hastily finished political document lies in its assumptions about model parameters, which are inconsistent with the conventional real interest rate and the market saving rate. Thus, I will follow the mainstream's best-guess and choose those rates sensibly in replicating the Review's analytical process. Second, as most of the critiques stop at the theoretical argument, I will move forward into IAM modeling to find

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[31] Nordhaus, William D., 2007。

if there is empirical evidence to support those critiques' arguments. Although Nordhaus(2007) also did such empirical modeling, the comparison of his results with Review's conclusion is to some extent questionable since their results are obtained using totally different IAM models. To be consistent, it's better to follow the original analytical process in the Stern Review and generate comparable results.

In the following sections, I will first review Stern's theoretical analysis and his welfare calculations since this process is very unclearly stated in the Review[32]. Second, I will explain the process of replicating the PAGE2002 model used in the Review's methodology.

3.1 Welfare analysis

3.1.1 'Expected Utility' analysis in the Stern Review

The Stern Review adopts the PAGE2002 model to calculate the global cost of climate change. By introducing the "Balanced Growth Equivalent" (BGE) method, the current BGE consumption loss is calculated to represent the loss in global social welfare due to climate change. As explained in the Stern Review, the idea of "BGE" "measures the utility generated by a consumption path in terms of the consumption now that, if it grew at a constant rate, would generate the same present value utility"[33]. The Review calculate the present value using a CRRA utility function $U(t) = \frac{C^{1-\eta}}{1-\eta}$ in the special case of η=1. The formula used is:

[32] Calculation process is stated in box 6.3, chapter 6 of the Stern Review.

[33] The Stern Review, chapter 6, pp.160.

$$W = \sum_{t=1}^{2200} N(t) \ln C(t) e^{-\delta t} + (\frac{N_T \ln C_T}{\delta} + \frac{N_T g}{\delta^2}) e^{-\delta T} \quad ^{34} \tag{3.1}$$

The first term is the present value of total global utility from 2000 to 2200 while the second term is the present value of aggregate global welfare from 2200 to infinity which is discounted to 2000. In other words, with two paths of consumption with and without climate change calculated by PAGE2002 model, we can calculate the total social welfare under two scenarios, a world affected or not by climate change.

Afterwards, using the same amount for the present value of utility, the Review "immediately" find the BGE equation to calculate the constant consumption path in the BGE scenario, which is:

$$W = \sum_{t=1}^{2200} N(t) \left(\frac{C_{BGE}^{1-\eta}}{1-\eta} + gt \right) e^{-\delta t} + \left(\frac{N(t) \frac{(C_{BGE} + 200g)^{1-\eta}}{1-\eta}}{\delta - g(1-\eta)} \right) e^{-\delta T} \quad ^{35} \tag{3.2}$$

The idea is very straightforward, that is, by calculating the two scenarios' C_{BGE} we can find the difference between two C_{BGE} (equivalent current consumption loss) to represent the cost of climate change. However, the mathematical explanation is quite confused. First, there is no detail about how these two essential equations are derived. It's hard to convince readers about its final conclusion which decisively depends on these two welfare equations. Secondly, the Stern Review chooses the assumption of unit elasticity of marginal utility to consumption which gives a logarithm utility function instead of the

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[34] There is a misleading parameter t here. "t" should starts from 2001 not 1 to present the first 200 hundred years starting from 2001.

[35] Another misleading parameter here is N(t) in the second term, which should be N(T).

general CRRA form. Why does it change to use general CRRA utility form to derive the C_{BGE} in equation (3.2)? By setting different levels of η, it would change the basic attitude towards personal risk and equality preference. Since I will totally follow Review's method in deriving the results by adjusting values of the model parameters and in order to avoid any other typos I did not see at the first glance, it's better to derive these two essential equations starting from the original idea and assumptions.

3.1.2 Deriving the BGE social welfare

From the PAGE2002 model programming, we will expect to generate three time series. One is the time path of world population $N(t)$ and the other two are the time paths of consumption $C(t)$ from 2000 to 2200, with and without climate change. Also, we can designate values to the critical parameters, η, δ and g. Consistent to the Stern Review's assumption, we take accounts of growing population while ignoring regional effects by assuming there is only one region (the world) in the model. Then automatically weighted by world population, the original form of aggregate global utility should be:

$$W = \int_{t=1}^{\infty} N(t)\, U(t)\, e^{-\delta t} dt \tag{3.3}$$

Here, the utility function is exclusively determined by C(t), and is the well known CRRA utility function.

$$U(t) = \frac{C^{1-\eta}}{1-\eta} \tag{3.4}$$

We need to derive the BGE social welfare equations with two utility functions; one is the logarithm function $U(t) = \ln C(t)$ used in the Review's model, and the other uses the general CRRA form when we assume the value of η is not one.

BGE Equation within Logarithm Utility Function

When η=1, the general CRRA utility function will approximate to the logarithm function $U(t) = \ln C(t)$. Thus equation (3.3) can be written as:

$$W = \int_{t=1}^{\infty} N(t) \ln C(t) e^{-\delta t} dt \quad (3.5)$$

The Stern Review separate the whole time period into two parts; the first part is 2001 to 2200, for which we have the consumption per capita and world population data time path from PAGE2002 model, and the last part is 2200 to infinity which assumes that the global economy has developed into a balanced growth path with constant consumption growth rate, g, and constant world population, N(T). Then, we can transform equation (3.5) into the sum of these separate aggregate utilities:

$$W = \sum_{t=1}^{200} N(t) \ln C(t) e^{-\delta t} + \sum_{t=1}^{\infty} N(T) \frac{\ln(C_T(1+g)^t)}{(1+\delta)^t} e^{-\delta T} \quad (3.6)$$ [36]

The mathematical meaning of the first term is quite straightforward as discussed earlier. The second term measures the present value of aggregate global utility from 2200 to infinity. I first discount each year's per capita utility $\ln(C_T(1+g)^t)$ to 2200 through the discounting factor $(1+\delta)^t$. Then I sum the utilities weighted by the constant world population, giving $\sum_{t=1}^{\infty} N(t) \frac{\ln(C_T(1+g)^t)}{(1+\delta)^t} e^{-\delta T}$. For the last step, I discount this term to 2000 through one discounting factor $e^{-\delta T}$.

Since g is considerably small, we can approximate $\ln(1+g)$ to g. Then, Equation (3.6) can be future deducted to:

[36] N(t) is the world population at time t. while T refers to 200

$$W = \sum_{t=1}^{200} N(t) \ln C(t) e^{-\delta t} + N(T) e^{-\delta T} \sum_{t=1}^{\infty} \frac{\ln C_T + \ln(1+g)^t}{(1+\delta)^t}$$

$$= \sum_{t=1}^{200} N(t) \ln C(t) e^{-\delta t} + \left(\frac{N(T) \ln C_T}{\delta} + \frac{N(T) g (1+\delta)}{\delta^2} \right) e^{-\delta T} \quad [37] \quad (3.7)$$

For this aggregate global welfare, the difference between Stern's equation and mine lies in the last term; I have (1+δ) in the numerator while Review does not. The best guess is that Review ignores the value of δ since it is so small under its assumption. However, in my empirical model, I would increase the value of δ to more conventional values. Thus, it's better to keep this term in this global welfare equation.

Next, I will derive the BGE equation when utility function is in logarithm form. The basic idea is quite similar to the previous work, except changing the randomly varied consumption path over time with a constant growing path of $[C_{BGE}, C_{BGE}(1+g), C_{BGE}(1+g)^2, C_{BGE}(1+g)^3 \ldots]$. Then the equation (3.7) would be transformed to:

$$W = \sum_{t=1}^{200} N(t) \ln[C_{BGE}(1+g)^t]\, e^{-\delta t} + \sum_{t=1}^{\infty} N(T) \frac{\ln(C_{BGE}(1+g)^{200+t})}{(1+\delta)^t} e^{-\delta T} \quad [38]$$

$$= \sum_{t=1}^{200} N(t) \left[(\ln C_{BGE} + gt) e^{-\delta t} \right] + \left[\frac{(\ln C_{BGE} + 200g)}{\delta} + \frac{(1+\delta)g}{\delta^2} \right] N(T) e^{-\delta T} \quad (3.8)$$

However, it's impossible to compare this equation with that in the Stern Review since the Review only derive the BGE utility equation within the general CRRA utitlity function not the logarithm one.

BGE Utility Equation within CRRA utility function

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[37] The specific derivations for all such equations refer to the appendix.

[38] Here $C_T = C_{BGE}(1+g)^{200+t}$

In this part, we will change the utility function back to the original CRRA form. Then, equation (3.7) will change to:

For this deriving process, I did not use any approximations and assumptions. Again, it's better to keep the original derivation form.

$$W = \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + \sum_{t=1}^{\infty} N(T)\frac{[C_T(1+g)^t]^{1-\eta}}{(1-\eta)(1+\delta)^t}e^{-\delta T}$$ [39]

$$= \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + N(T)e^{-\delta T}\frac{C_T^{1-\eta}}{1-\eta} \times \frac{(1+g)^{1-\eta}}{1+\delta-(1+g)^{1-\eta}} \qquad (3.9)$$

Also, adopting CRRA utility function and BGE scenario, I transform the equation (3.7) into:

$$W = \sum_{t=1}^{200} N(t)\frac{[C_{BGE}(1+g)^t]^{1-\eta}}{1-\eta}e^{-\delta t} + \sum_{t=1}^{\infty} N(T)\frac{[C_{BGE}(1+g)^{200+t}]^{1-\eta}}{(1-\eta)(1+\delta)^t}e^{-\delta T}$$

$$= \sum_{t=1}^{200} N(t)\frac{[C_{BGE}(1+g)^t]^{1-\eta}}{1-\eta}e^{-\delta t} + N(T)e^{-\delta T}\frac{[C_{BGE}(1+g)^{200}]^{1-\eta}}{(1-\eta)}$$

$$\times \frac{1}{(1+\delta)(1+g)^{\eta-1}-1} \qquad (3.10)$$ [40]

This derives a different result than equation (3.2) which is the Review's result. Thus, I will use equation (3.10) instead of equation (3.2) in the programming since I did not use any approximation.

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[39] Here $U(t) = \frac{C(t)^{1-\eta}}{1-\eta}$.

[40] By using the approximation $(1+g)^x \cong 1 + \ln(1+g)\,x \approx 1 + gx$, this will give $(1+\delta)(1+g)^{\eta-1} - 1 \cong (1+\delta)(1+(\eta-1)g) - 1 \cong \delta - g(1-\eta)(\delta+1)$. However, in the Review's equation (3.2), it omit $(\delta+1)$ for the last product since it assumes $\delta \approx 0$.

3.2 Testing Different Parameter Assumption Using PAGE2002 Modeling

The Stern Review's BGE results are derived using the PAGE2002 model which was originally developed by Dr Chris Hope. In order to see how the Review's results change with change in parameter assumptions, I need to first generate two paths of GDP for 200 years. There are the paths with and without climate change and are the outcomes calculated directly from PAGE2002 model. Secondly, I calculate the current consumption loss corresponding to different sets of parameter values. In test one, I will adopt Stern's choice of $[\delta,\eta,g] = [0.001,1,1.3]$; in test two, I will choose Weitzman's assumption of "trio of twos", $[\delta,\eta,g] = [0.2,2,2]$. This provides an empirical test of his theoretical analysis. In the following part, I aim to use comparative tests to find out which parameters have a significant influence on the final results.

3.2.1 PAGE2002 Model Programming

In order to capture IPCC's new findings published in its third assessment report, Dr. Chris Hope, who was one of the authors who developed PAGE95 IAM, updated PAGE95 into a new version PAGE2002 IAM. This model has three sections including 53 equations 80 variables, most of which are three-dimension variables. My replication includes only the two sections, "Computing the Temperature Rise" and "Computing the Value of Global Warming Impacts". The third section, which is about simulation of carbon price and implementing adaptive cost, is irrelevant here.

Computing the Temperature Rise

The effects of GHG emissions on global temperature occur through two processes. One is the excess concentration and the other one is radioactive mainly forcing, which together will determine the crucial value of the parameter "Global Released Temperature". Figure 1 shows the time path of concentration of CO_2 and according to PAGE2002.

Figure 1(a)[41]

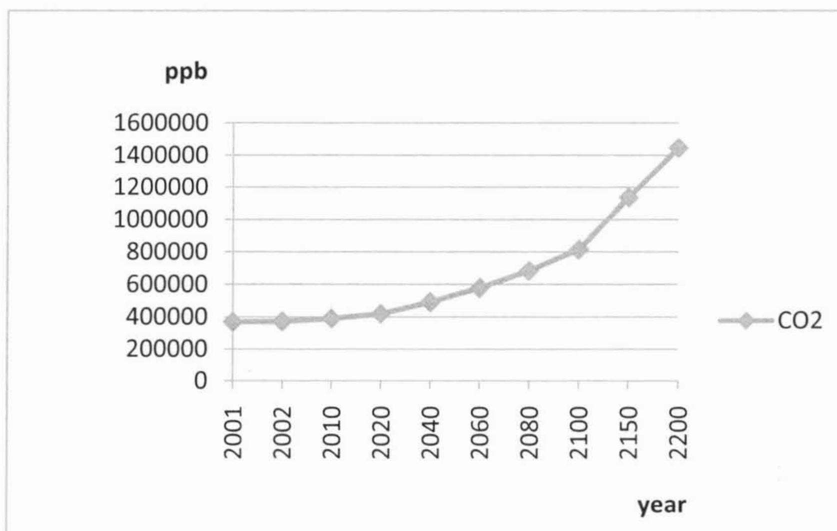
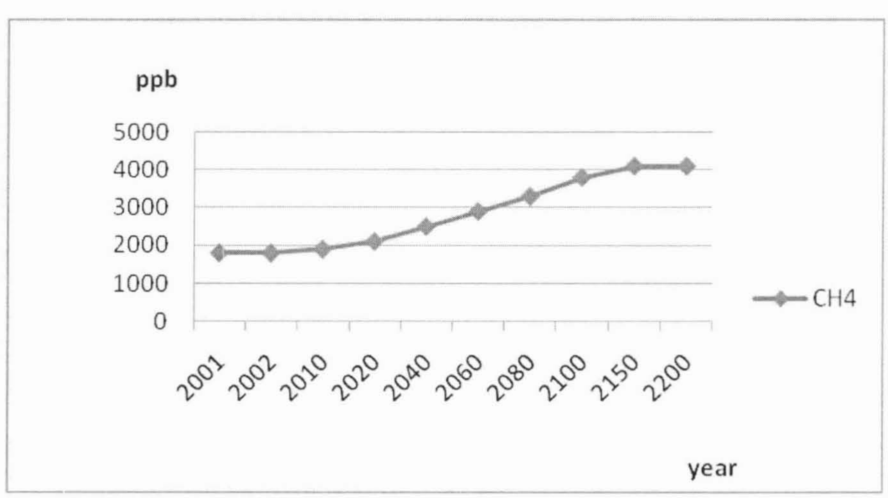


Figure1 (b)

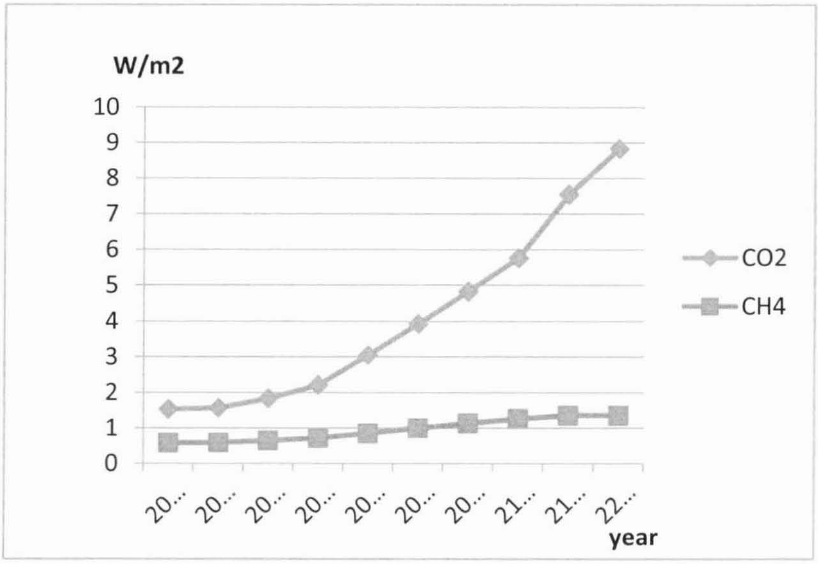


41 Unit ppb stands for parts per billion.

An excess concentration of GHG emissions will raise the global temperature and then this higher level of temperature will influence global economy. In fact, only a portion of the GHG emissions stay in the air leading to new environment equilibrium between atmosphere and ocean with a higher level of temperature. As a consequence, the increasing global temperature will make the ocean and land less able to absorb the GHG emissions, and will lead to an even higher concentration of GHG in the air. Thus, we will expect a convex-concave shape of increasing GHG concentration curve with respect to time.

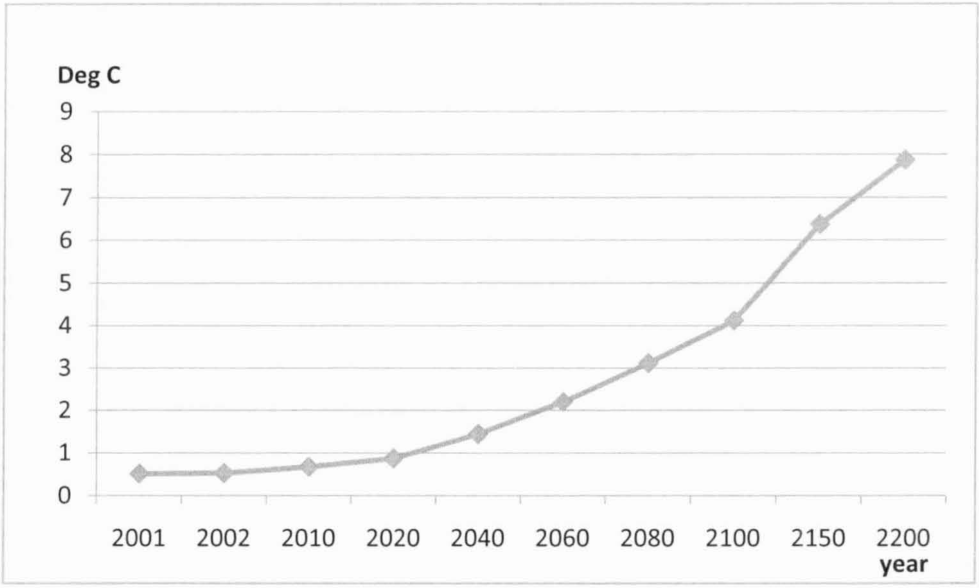
Another process by which GHG influences global temperature is radioactive forcing. In contrast to the concentration process, the radioactive forcing from CO_2 and CH_4 increase at a constant growth rate, as shown in Figure 2.

Figure 2



Putting together these two effects, Figure 3 shows the change in the global temperature by year, which is also a convex curve. The model predicts that, we can expect the temperature to rise faster in the future than in the current period.

Figure 3



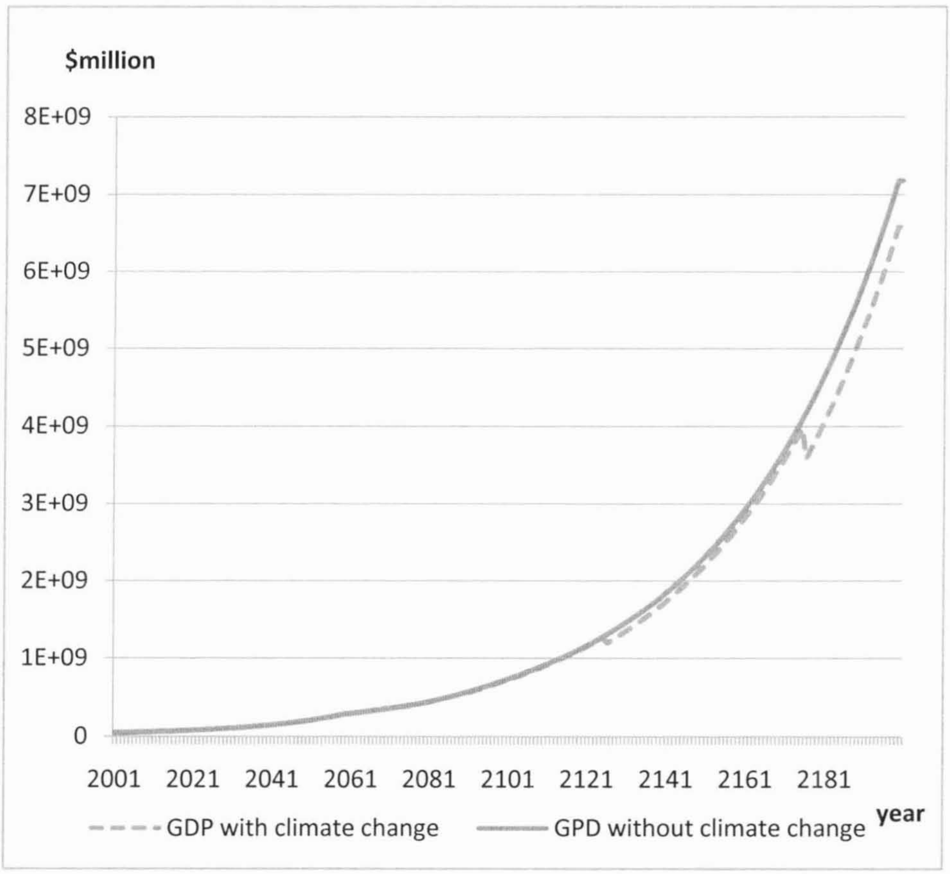
Computing the Economic Impacts of Climate Change

PAGE2002 model keeps accounts of two types of damages, economic and noneconomic. Depending on the damage types and regions, PAGE2002 model can generate a time path of aggregate damage.

PAGE2002 also assumes that damage impacts only happen when the increasing temperature has exceeded the tolerable values. Within the documented value of the damage loss for a 2.5 deg warming in terms of percentage of GDP, then, we can calculate the economic impacts as a damage function of excess temperature. Also, we take account of the original policy impacts. Different regions, time periods and economic sectors have a different impact factor. The damage function is defined as a power function of increased temperature:

$$\text{Economy impact} = \left(\frac{\text{increased temperature}}{2.5°}\right)^{1.76} \times (\text{GDP lost for } 2.5° \text{ warming}) \times (1 - \text{regulation impact})\text{GDP}^{42} \quad (3.11)$$

Figure 4



[42] Impact function exponent is consistent with IPCC TAR which set it as 1.76.

To be consistent with IPCC TAR, for the economic sector, GDP lost for 2.5° warming is 0.5% while for the non-economic sector it is 0.73%.

The regulation impact is a $2 \times 8 \times 10$ matrix according to different impact sectors(2), regions(8) and time periods(10).

Also a relative research about economic valuation from the political control on ecosystems and biodiversity has been released recently

(available on internet: http://ec.europa.eu/environment/nature/biodiversity/economics /index_en.htm).

Here, I extend PAGE2002 to a version which is more suitable for the Stern Review's analysis. Since in the Review, all the welfare and BGE equations require 200 years GDP data but PAGE2002 integrates these impacts into 10 periods, there might be a loss of accuracy in the calculation of welfare using the aggregated impacts. Thus, I strictly conform to PAGE2002 equations but calculate every year's damage. Thus, instead of 10 numerical numbers for 10 periods, I generate a time path of GDP data for 200 years. First I do not consider climate change effects. Then, I use the GDP to calculate climate change damage and use this difference to calculate the GDP with climate change. Figure 4 shows the GDP growth time path in each scenario.

3.2.2 Comparative Tests of Alternative Parameter Assumptions

Using PAGE2002 model, I have generated two paths of global GDP for 200 years, with and without considering climate change affects. Then since the Stern Review has assumed that the exogenous rate of saving is 20%, I can transform each GDP value into global consumption per capita. With these two time paths of consumption per capita, we can then calculate BGE consumption for different scenarios so as to calculate the current consumption loss of climate change.

Run one

In the first run, I adopt Review's parameters values, $[\delta,\eta,g]=[0.001,1,0.013]$, to replicate the Review's calculation of, "Losses in current per-capita consumption from six scenarios of climate change and economic impacts"[43].

Since no details are given about the Review's six scenarios as noted before, I follow PAGE2002 model's scenario which combines the economic and non-economic impacts together. The result of the standard PAGE2002 model under Review's assumption is quite similar to that in the Review.

Table 1

Model	Scenario	BGE:% loss in current consumption due to climate change
Stern Review	Climate + Economic	2.1
	Market impacts+risk of catastrophe	5
	Market impacts+risk of catastrophe+non-market impacts	10.9
PAGE2002	Economic and Non-economic	7.86

Table 1 shows that the replicating number is 7.86% which is between the lowest value 2.1% in the simplest scenario and the highest value 10.9% in the most complicated scenario in the Stern Review. Since all the following runs are basing on the same methodology, scenario and modeling, figure 7.86% to some extent can represent Review's result as the baseline value in the following comparisons.

43 Table 6.1 in Chapter 6, the Stern Review, pp163.

Run Two

Based on the analysis in earlier Reviews, most of the economic critics of the Review agree that a very low time discounting rate leads to Review's conclusion about the need for an immediate reduction in GHG emissions. Therefore, in run two, I will calculate the BGE consumption loss by using Nordhaus's[44] and Weitzman's suggestions about the discounting parameters.

Weitzman prefers the "trio of twos" assumption, $[\delta,\eta,g]=[0.02,2,0.02]$, which leads to a discount rate of 6% while Nordhaus adopts $[\delta,\eta,g]=[0.015,2,0.02]$ yielding an equilibrium discount rate of 5.5%. Since both of them designate the elasticity of marginal utility to consumption to 2 not 1, I have to change the utility function back to the normal CRRA utility form. Thus, in run two, I use equations (3.9) and (3.10) to calculate BGE consumption. The results show a considerable difference from the baseline model.

Table 2

Model	Parameter $[\delta\ \eta\ g]$	Yielded real interest rate	BGE:% loss in current consumption due to climate change
Baseline Model	[0.001 1 0.013]	1.4%	7.86
Dr.Nordaus	[0.015 2 0.02]	5.5%	0.43
Dr. Weitzmen	[0.02 2 0.02]	6%	0.30

[44] Although Dr. Nordhaus has also tested his assumption by using his own model, however, he has followed Dr Chris Hope analysis about Carbon cost not the expected utility . Thus, I will test his assumption again by adopting the Review's expected utility analysis approach.

From Table 2, the results in run two confirm the theoretical intuition of earlier publications that the considerable high welfare cost of an approach BAU climate change are caused by a low real interest rate. In Baseline model, the low discount interest rate of 1.4% puts a high weight on future damages leading to a considerable high loss in social welfare by an equivalent amount of cutting consumption per capita by 7.86%. Once we adjust the parameters' value to more sensible numbers, it dramatically changes the loss from 7.86% to 0.43% under [δ η g]=[0.015 2 0.02] and 0.30% under [δ η g]=[0.02 2 0.02]. Thus, the influence of climate change on our global economy is not as terrible as Review concludes.

Run Three

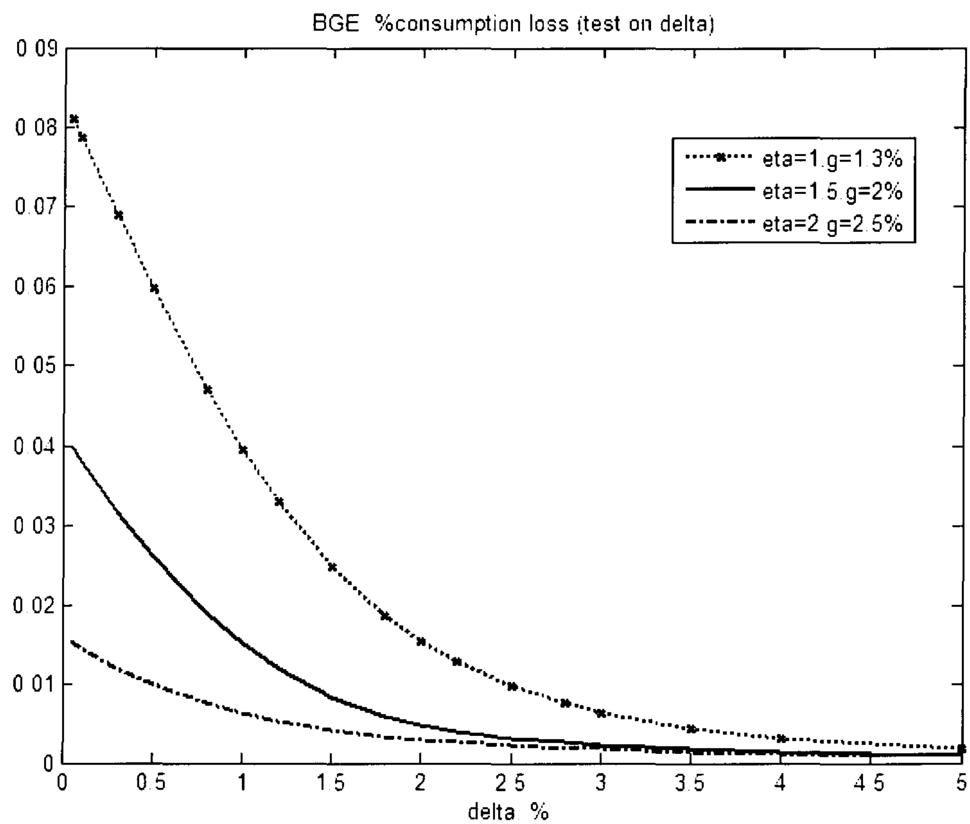
In Run Two, the results show that by adjusting parameter values, the loss caused by climate change can be considerably different. But, which parameter plays the essential role that might directly control the final results? Is it the time discount rate δ which has been criticized in most of the reviews? In the following runs, I will find the answers through comparative tests.

<u>Adjusting one parameter</u>

First, I take look at the most controversial parameter, δ. By controlling η and g with in three cases as [1, 1.3%] [1.5, 2%] and [2, 2.5%], I adjust δ from 0.5% to 5% to see how the consumption loss is influenced.

From Figure 5, the three curves according to different sets for η and g all demonstrate a similar decreasing trend in BGE consumption loss. The highest point appears at the smallest value of δ. In other words, if consumption in near periods or far future make no difference to individual utility and social welfare, then, the climate change impact will be large enough to draw our attention to take effective action immediately like what have been suggested in the Stern Review. The more one is concerned about our future generations, the more we should sacrifice in current periods to benefit them.

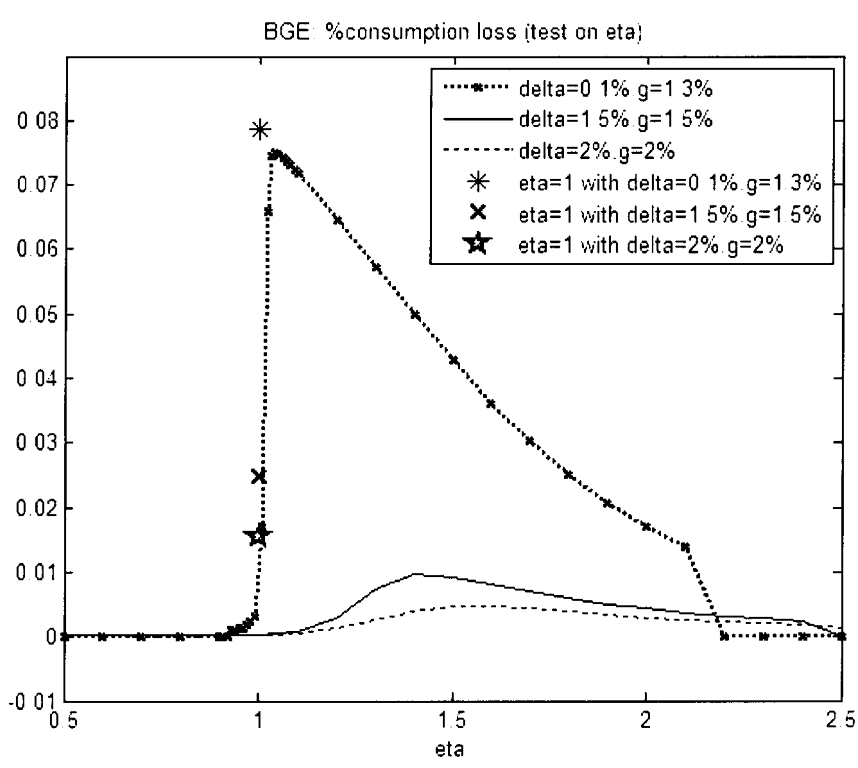
Figure 5



Also, those curves show that change in δ can have a high impact on current consumption. Especially when δ is assigned with values smaller than one, the loss dramatically decreases even with an insignificant increase in δ. Consequently, if we want to generate a significant loss from climate change, choosing a small number for δ is very effective because a little larger number will probably reverse the whole conclusion. Now, it's clearer why the Stern Review sticks to such a controversial value even after other critics have pointed out this problem.

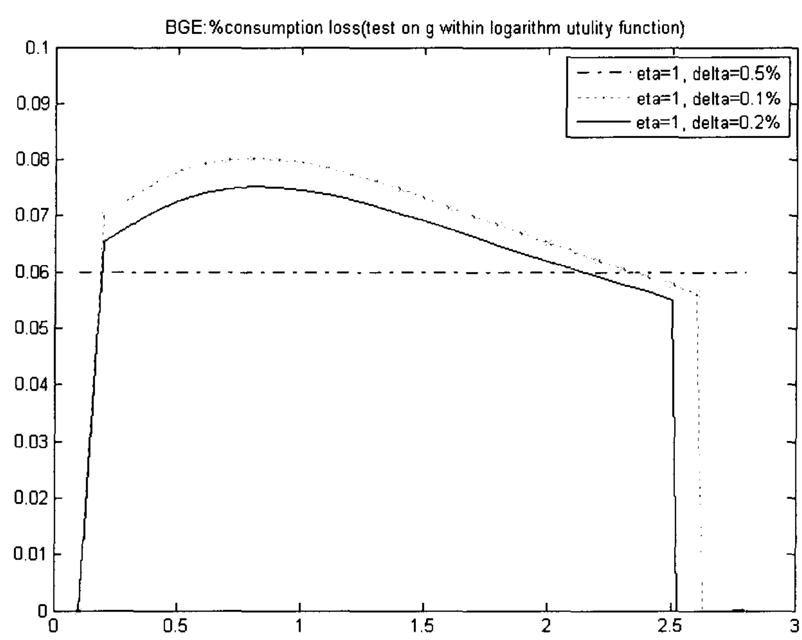
Secondly, I control δ and g instead with value of [0.1%, 1.3%], [1.5%, 1.5%] and [2%, 2%], and adjust η from 0.5 to 2.5 giving figure 6.

Figure 6



The shapes of these three curves are different from those in the test of adjusting δ. They share a similar shape which all have one common significant maximum peak point around η=1. For the first curve which takes Review's choice about δ and g, it's very obvious that it only has one peak when η=1 while the other two curves have a local maximum point in the range of [1.4% 1.6%]. So, the Review choice of this controversial value of unity for η favors its results. Once we increases η a little bit from one, it will lead to a huge difference; take curve one for example, when η increases to 1.5, the loss of current consumption will decrease from 8% to 4%, that is from a alarming level about climate change damage to a level that can be considered as a normal reaction of industrial development. Thus, like δ, a little change in η will significantly influence our final results.

Figure 7



In the last test, I find out that the effect of changing g is different from the former two; the difference depends on the forms of utility function. Thus, I will separate the examination into two groups, one with a logarithm function and the other considering the general CRRA form. For the first group, I set η and δ to [1, 0.1%] [1, 0.2%] and [1, 0.5%] while for the second group, η and δ are set to [1.5, 0.1%] [1.5, 1.5%] and [2, 2%]. Then, I adjust g from 0.1% to 3% meanwhile the two GDP growth paths generated in 3.2 will be automatically adjusted according to difference values of GDP growth rate, g. Thus, the first set gives Figure 7 and the second set gives Figure 8.

Figure 7 demonstrates that parameter g has very little influence on the results when $\eta=1$. In fact, I have taken some more runs with δ set at a bigger value than 0.5% all of which leads to a constant loss given to any level of g. However, if we assume delta is a small value that is smaller than 0.5%, changing in g will significantly influence the consumption loss as what has been shown on the graph. However, it is still delta not g leads to this significant change. Given a near-zero value to delta, the future wealth will be equally valued as current wealth. When the economy grows at very small growth rate, the damage from the climate change will overweight the growth in the economy. We will expect the consumption loss due to climate change should be significantly high. Thus, with a logarithm utility function, the final results are still determined by the other two model parameters.

Figure 8

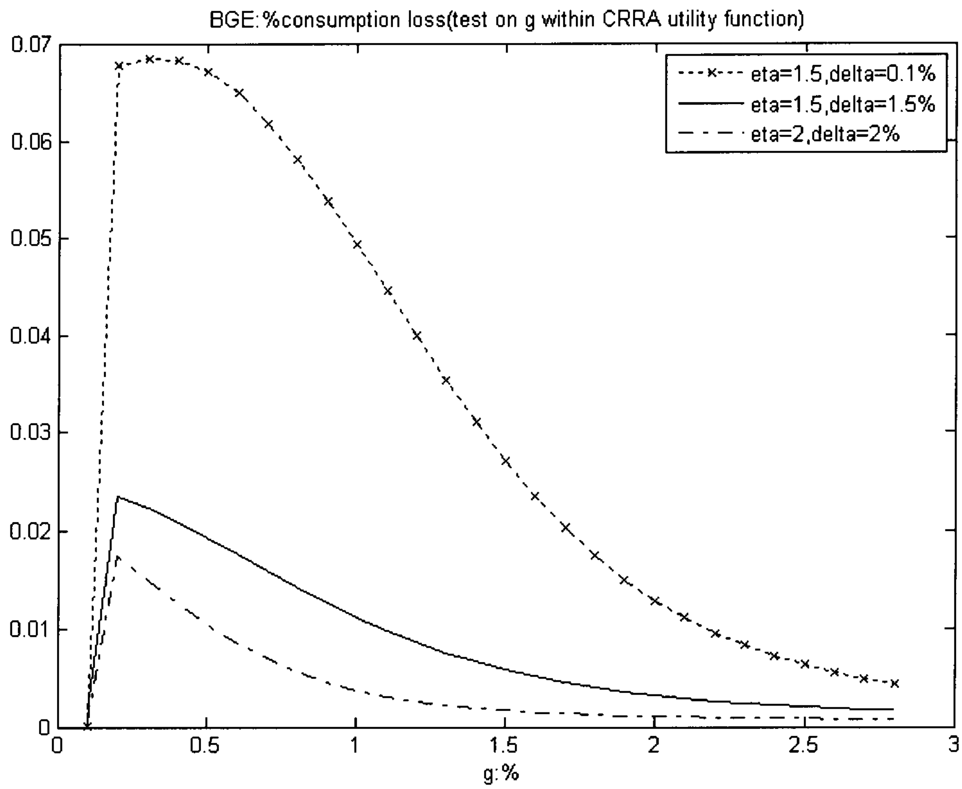


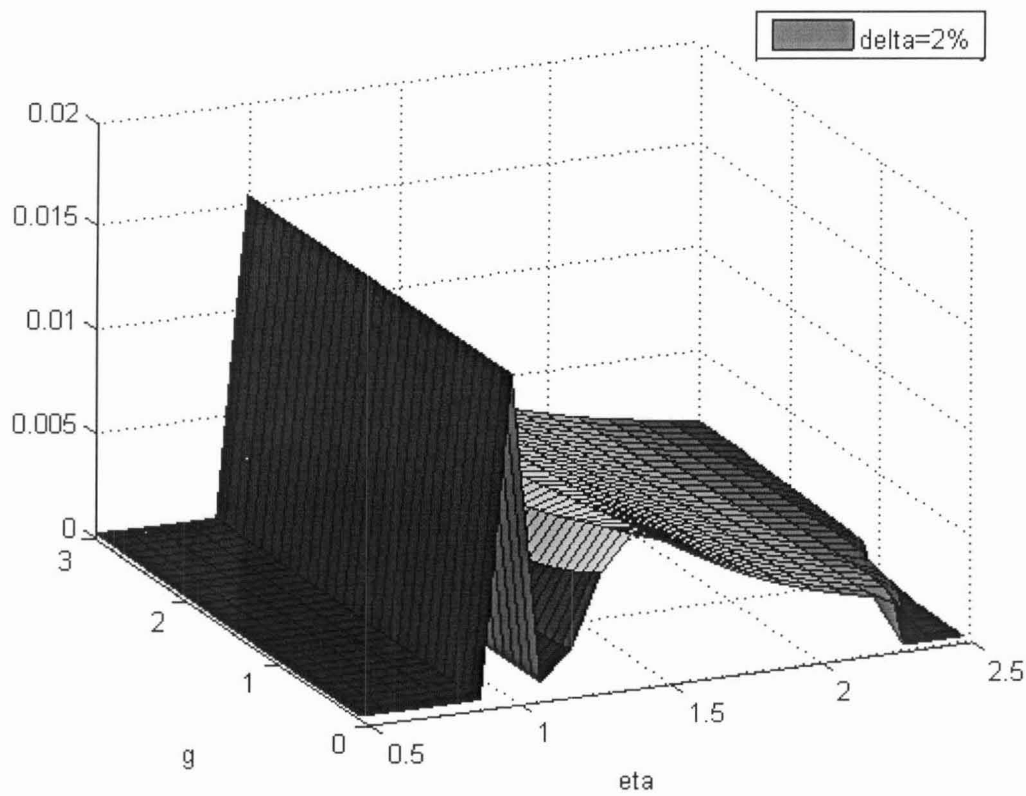
Figure 8 also shows consistently decreasing curves. In normal CRRA utility form, g seems to have significantly influences the consumption loss. While the second and third curve behave smoothly changing in the range of 0 to 2.3%, however, if we control δ to the near-zero value, the consumption loss ration could achieve the gradient of 6.8%, which is already the alarming level of the baseline. As discussed before, a smaller value of δ, especially one less than one, will significantly influence the final result. Thus, the first curve shows that a near-zero δ still leads us to a significant loss in current consumption with a low economic growth rate.

Adjusting two parameters

The first part of the comparative tests has confirmed that the crucial determinants are η and δ while g mostly has an insignificant effect. The smaller δ, the larger the loss while a unit value of η leads to the highest impact on the final results.

After testing individual influence, in the following runs, I will aim to find out what combination of these three parameters would cause the highest consumption loss.

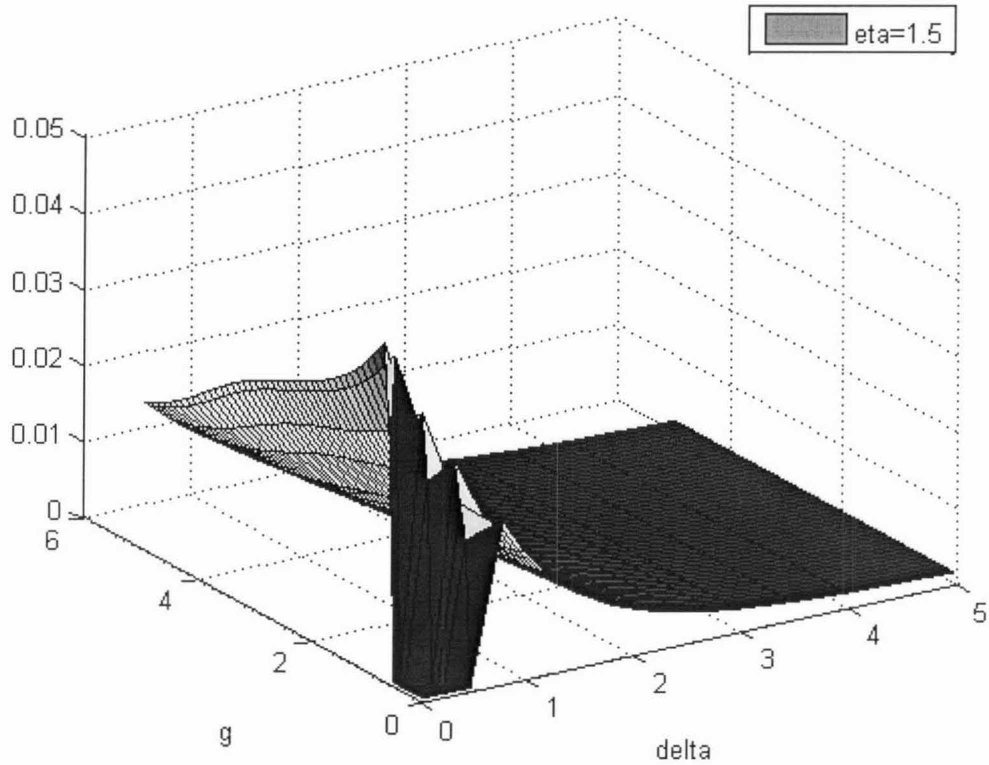
Figure 9



First, I designate δ with the conventional value 2% per year. Then, which combination of g and η would result in a large enough consumption loss, like the 7.8% per year in

Review, and push us to sharply and immediately decrease GHG emissions? Figure 9 shows that unity η still leads to the highest loss no matter what the growth rate the economy is. However, even at the highest peak level, the loss is around 0.9% rather than 7.8%. Thus, setting δ at the conventional value of 2%, it cannot generate the Review's high loss as 7.8% per year cannot be generated no matter what the combination of η and g is. This test again has confirmed that the conventional choice of δ barely meets Review's conclusion.

Figure 10



Next, by controlling η with a constant value of 1.5, Figure 10 shows the best combination of δ and g to generate the largest climate change damage is δ equaling 0.05%

and g with value of 0.7% per year. This result confirms Figure 8 which has shown that within near-zero δ and a CRRA utility function; there is a sharp increase of current consumption loss when g is close to 0.7%. It also demonstrates that the shape along the δ axis show dramatic changes around the value of one while the curves along the g axis is very flat except for the sharp jump when g is around 0.7% and δ is smaller than 0.5%. Furthermore, when δ exceeds the value around 2.56%, the impact on consumption loss almost equals zero no matter what the economic growth rate is. Although the combination of δ and g with [0.05%, 0.7%] increases the largest impact to a higher level compared with the previous test, however, it is still below Review's level of 7.8%. Thus, with such conventional values of η, it's also hard to generate such a significant loss as the Stern Review does.

4. Conclusion

Acknowledging the fact that industrial development will worsen the global climate condition, how to balance the benefit from the development and the cost of climate change for the future generation is one of the most essential questions for most countries. Since President George W. Bush announced that United States opposed the Kyoto Protocol which helps to restrict GHG emissions, politicians and scientists in climate change research began a debate about whether it is worthwhile to sacrifice our welfare to benefit future generations. In these benefit-cost debates, the Stern Review, released as a political document, has strongly and unambiguously advocated such a policy of sharp reduction on GHG emission right away or it will result "a 20% cut in per-capita consumption, now and forever"[45]. On the ethical side, Review put a high weight on future generations and considers this long-term issue in such a dynamic work frame. On the economic side, however, the Stern Review adopts a controversial assumption of a near-zero time discount rate and unity utility function, all of which will favor its extreme results. As the empirical tests in this paper show, the Review's conclusion barely survives once we adjust the discounting parameters' values to some more conventional ones.

Thus, debates about how to control climate change issue will be continued since there are still no absolute answers to what the values for the economic parameters should be. However, this paper has shown that the crucial discounting parameters controlling the

[45] The Stern Review, Chapter 6, pp163.

economic analysis results are the time discounting rate and the elasticity of marginal utility to consumption. We probably should pay more attention to these two parameters in the following research in this field.

Without doubt, controlling the GHG emissions is an important issue for the whole world. But, how fast and how intensively should we take action on the reduction of the GHG emission? To answer this question, we still need more scientific findings, economic analysis and improved models.

Appendix

Footnote 26, saving rate equation:

When the economy is at steady state, by using Cobb-Douglas Production Function we get:

$$y = Ae^{\mu t}k^{\alpha} \Longrightarrow \dot{y} = Ak^{\alpha}e^{\mu t}\mu + Ae^{\mu t}\alpha k^{\alpha-1}\dot{k}$$

$$\Longrightarrow \frac{\dot{y}}{y} = \mu + \alpha\frac{\dot{k}}{k}$$

At steady state, $\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{c}}{c}$, then

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\mu}{1-\alpha}$$

From equation(2.7), we have

$$\eta\frac{\dot{c}}{c} = \alpha\frac{y}{k} - \delta - d$$

$$\Leftrightarrow \eta\frac{\mu}{1-\alpha} = \alpha\frac{y}{k} - \delta - d$$

$$\Leftrightarrow \frac{y}{k} = \frac{1}{\alpha}\left[\frac{\eta\mu}{1-\alpha} + \delta + d\right]$$

From the constraint in equation(2.1), we can derive:

$$\dot{k} = y - nk - c \Longrightarrow \frac{\dot{k}}{k} = \frac{y}{k} - n - \frac{c}{k}$$

$$\Longrightarrow \frac{c}{k} = \frac{y}{k} - n - \frac{\mu}{1-\alpha}$$

$$\Longrightarrow \frac{c}{k} = \frac{1}{\alpha}\left[\frac{\eta\mu}{1-\alpha} + \delta + d\right] - n - \frac{\mu}{1-\alpha}$$

$$= \frac{(\eta-\alpha)\frac{\mu}{1-\alpha} + \delta + (1+\alpha)d}{\alpha}$$

$$\Longrightarrow \frac{c}{k} \times \frac{k}{y} = \frac{(\eta-\alpha)\frac{\mu}{1-\alpha} + \delta + (1-\alpha)d}{\frac{\eta\mu}{1-\alpha} + \delta + d}$$

$$\Rightarrow \frac{c}{y} = 1 - \frac{s}{y} = 1 - \frac{\alpha\left[\frac{\mu}{1-\alpha} + d\right]}{\frac{\eta\mu}{1-\alpha} + \delta + d}$$

$$\Rightarrow \frac{s}{y} = \frac{\alpha\left[\frac{\mu}{1-\alpha} + d\right]}{\frac{\eta\mu}{1-\alpha} + \delta + d}$$

At the steady state, n=0, then

$$\frac{s}{y} = \frac{\alpha\mu}{\eta\mu + (1-\alpha)\delta} = \frac{\alpha g}{\eta g + \delta} = \frac{\alpha g}{r}$$

Equation (3.7):

$$W = \sum_{t=1}^{200} N(t)\ln C(t)e^{-\delta t} + N(T)e^{-\delta T}\sum_{t=1}^{\infty}\frac{\ln C_T + \ln(1+g)^t}{(1+\delta)^t}$$

$$= \sum_{t=1}^{200} N(t)\ln C(t)e^{-\delta t} + N(T)e^{-\delta T}\sum_{t=1}^{\infty}\frac{\ln C_T + gt}{(1+\delta)^t}$$

$$= \sum_{t=1}^{200} N(t)\ln C(t)e^{-\delta t} + N(T)e^{-\delta T}(\ln C_T\sum_{t=1}^{\infty}\frac{1}{(1+\delta)^t} + g\sum_{t=1}^{\infty}\frac{t}{(1+\delta)^t})$$

$$= \sum_{t=1}^{200} N(t)\ln C(t)e^{-\delta t} + N(T)e^{-\delta T}(\frac{\ln C_T}{\delta} + \frac{g(1+\delta)}{\delta^2})$$ [46]

$$= \sum_{t=1}^{200} N(t)\ln C(t)e^{-\delta t} + \left(\frac{N(T)\ln C_T}{\delta} + \frac{N(T)g(1+\delta)}{\delta^2}\right)e^{-\delta T}$$

Equation (3.8):

$$W = \sum_{t=1}^{200} N(t)\ln[C_{BGE}(1+g)^t]\,e^{-\delta t} + \sum_{t=1}^{\infty} N(T)\frac{\ln(C_{BGE}(1+g)^{200+t})}{(1+\delta)^t}e^{-\delta T}$$ [47]

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46 $\sum_{t=1}^{\infty} tx^t = \frac{x}{(1-x)^2}$, when $x \in (0,1)$

47 Here $C_T = C_{BGE}(1+g)^{200+t}$

$$= \sum_{t=1}^{200} N(t)\,[(\ln C_{BGE} + gt)e^{-\delta t}] + N(T)e^{-\delta T} \sum_{t=1}^{\infty} \frac{\ln C_{BGE} + (200 + t)g}{(1 + \delta)^t}$$

$$= \sum_{t=1}^{200} N(t)\,[(\ln C_{BGE} + gt)e^{-\delta t}]$$

$$+ N(T)e^{-\delta T}\left[(\ln C_{BGE} + 200g)\sum_{t=1}^{\infty}\frac{1}{(1+\delta)^t} + g\sum_{t=1}^{\infty}\frac{t}{(1+\delta)^t}\right]$$

$$= \sum_{t=1}^{200} N(t)\,[(\ln C_{BGE} + gt)e^{-\delta t}] + N(T)e^{-\delta T}\left[(\ln C_{BGE} + 200g)\frac{1}{\delta} + g\frac{1+\delta}{\delta^2}\right]$$

$$= \sum_{t=1}^{200} N(t)\,[(\ln C_{BGE} + gt)e^{-\delta t}] + \left[\frac{N(T)(\ln C_{BGE} + 200g)}{\delta} + \frac{N(T)(1+\delta)g}{\delta^2}\right] e^{-\delta T}$$

Equation (3.9):

$$W = \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + \sum_{t=1}^{\infty} N(T)\frac{[C_T(1+g)^t]^{1-\eta}}{(1-\eta)(1+\delta)^t}e^{-\delta T}$$ [48]

$$= \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + N(T)e^{-\delta T}\frac{C_T^{1-\eta}}{1-\eta}\sum_{t=1}^{\infty}\left[\frac{(1+g)^{1-\eta}}{(1+\delta)}\right]^t$$ [49]

$$= \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + N(T)e^{-\delta T}\frac{C_T^{1-\eta}}{1-\eta} \times \frac{(1+g)^{1-\eta}}{1+\delta-(1+g)^{1-\eta}}$$

Equation (3.10):

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48 Here $U(t) = \frac{C(t)^{1-\eta}}{1-\eta}$.

49 $\frac{(1+g)^{1-\eta}}{(1+\delta)} < 1 \Leftrightarrow \eta > 1 - \ln(\delta - g)$, this inequation determines the particular discount parameters values for the empircial test in Section 4.

$$W = \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + \sum_{t=1}^{\infty} N(T)\frac{[C_T(1+g)^t]^{1-\eta}}{(1-\eta)(1+\delta)^t}e^{-\delta T}$$ [50]

$$= \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + N(T)e^{-\delta T}\frac{C_T^{1-\eta}}{1-\eta}\sum_{t=1}^{\infty}\left[\frac{(1+g)^{1-\eta}}{(1+\delta)}\right]^t$$ [51]

$$= \sum_{t=1}^{200} N(t)\frac{C(t)^{1-\eta}}{1-\eta}e^{-\delta t} + N(T)e^{-\delta T}\frac{C_T^{1-\eta}}{1-\eta} \times \frac{(1+g)^{1-\eta}}{1+\delta-(1+g)^{1-\eta}} \qquad (3.9)$$

[50] Here $U(t) = \frac{C(t)^{1-\eta}}{1-\eta}$.

[51] $\frac{(1+g)^{1-\eta}}{(1+\delta)} < 1 \Leftrightarrow \eta > 1 - \ln(\delta - g)$, this inequation determines the particular discount parameters values for the empircial test in Section 4.

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