THREE ESSAYS ON MONEY AND BANKING

by

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Abstract

The dissertation consists of three studies on money and banking in the presence of uncertainty.

In the first paper, agents face uncertain future liquidity needs and the bank is formed to provide liquidity insurance to depositors. The bank holds cash reserves to meet depositors' liquidity needs and as an insurance against uncertain return on the bank's assets. The paper analyzes the effect of inflation on banking crises. The main result is that when the bank has access to a stable foreign currency, inflation has a threshold effect on the incidence of banking crises: higher inflation reduces the likelihood of crises when inflation is below the threshold; the reverse occurs when inflation exceeds the threshold. This result appears to be broadly consistent with available evidence.

The second paper is an experimental study about depositors' behavior under a demand deposit contract when they face uncertainty over other depositors' actions; and investigates whether bank runs can occur as the result of pure coordination failures. It is found that bank runs can occur as a result of pure coordination failures, but only when coordination is difficult. I compare the experimental results and the simulation results from a learning algorithm modified from Temzelides (1997), and find that learning offers a good approximation to observed lab behavior.

In the third paper, agents face uncertainty over future preferences. The paper takes the mechanism design approach and studies the essentiality of multiple currencies - which act as substitute for the missing record-keeping technology - in the presence of limited commitment and private information about the realization of preferences. When money balances are concealable, a single money is sufficient to solve the problem of limited commitment, and can deal with the private information problem if agents are patient enough; in which case, money balances serve as a preference signalling device. When agents are sufficiently impatient, a second money is essential and allows agents to signal their preferences by holding different monetary portfolios all giving rise to the same total money balances.

Dedication

To my parents, Quanzhong and Qilian, and husband, Xiang.

Acknowledgements

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Chapter 1

Banking Crises in Monetary Economies[1]

This paper analyzes the effect of inflation on banking crises in a model in which money and banks play essential roles. The model's equilibrium replicates some key features of actual banking crises; namely, the partial suspension of payments, and the desire to hold cash even in the absence of pressing liquidity needs. When banks have access to a stable foreign currency, inflation has a threshold effect on banking crises: higher inflation reduces the likelihood of crises when inflation is below the threshold; the reverse happens when inflation exceeds the threshold. This result appears to be broadly consistent with available evidence.

1.1 Introduction

This paper is concerned with explaining what appears to be a U-shaped relationship between banking crises and inflation; that is, the fact that banking crises appear much more likely to occur in either very 'low' inflation environments or in very 'high' inflation environments. Low inflation (and deflation) environments like the depression-era U.S., or Japan throughout its 'lost decade' of the 1990s, for example, were infamous for their widespread banking-sector troubles.[2] Similar banking-sector problems appear to be present in several

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[2]Jonker and van Zanden (1995) study banking crises in the inter-war period and point out that '... As a rule it would seem as if crises occurred in countries which, following the collapse of the post-war boom, implemented delfationary policies in the run-up towards restoration of the gold standards. ... In Denmark,

Table 1.1: Banking Crises in High Inflation Environments

Banking crises	Net annual inflation (%) at start of crises
Argentina 1980-1982	100.76
Argentina 1989-1990	3079.81
Bolivia 1986-1988	276.34
Brazil 1990	2947.73
Brazil 1994-1999	2075.89
Israel 1983-1984	145.64
Lebanon 1988-1990	127.84
Peru 1983-1990	111.15
Sierra Leone 1990-1993	110.95
Turkey 1994	106.26

SOURCE: IMF/IFS.

NOTES: The crisis episodes are identified in Demirguc-Kunt and Detragiache (2005).

economies that feature very high rates of inflation; see Table 1, and the formal econometric investigation in Demirguc-Kunt and Detragiache (1998, 2005). Why this should be the case is not known. The purpose of this paper is to develop a possible rationale for this phenomenon.

To address the issue at hand, it seems clear that any passable theory will have to include, at the very least, the following three elements: [1] a role for banks; [2] a role for money; and [3] a set of shocks that can potentially trigger 'crisis' events. There are several potential modelling choices that one may take here concerning the role/nature of each one of these elements. There is also a question concerning the actual definition of what constitutes a banking 'crisis.' Let me first briefly review some of the relevant literature and then justify the general approach that I take.

First, it is almost conventional wisdom to suppose that banks are somehow 'different' from other private agencies (including other intermediaries, like insurance companies or pension funds). This difference stems from the peculiar liability structure of those agencies we label 'banks;' i.e., the demandable nature of their debt instruments. While this liability structure obviously has an economic purpose (with bank liabilities serving as an important payment instrument, and their demandable nature serving as a low-cost form of insurance against idiosyncratic liquidity needs), it allegedly opens the door to a form of inherent 'fragility' or 'instability.' That is, if everyone (for some unexplained reason) chooses to exercise their redemption option simultaneously (i.e., make a 'run' on the bank), then the bank

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Sweden, and the Netherlands delfationary policies were introduced which combined with the depression to produce banking crises. Between 1920 and 1923 the price level in these countries fell by 21%, 23% and 27% respectively (Maddison, 1991:app.E).'

will be forced to liquidate even positive net-present-value investments (at fire-sale prices) to make good on its obligations. With a sequential service constraint in place, depositors who are slow to act may be left with nothing. It is this fear that justifies 'running' (on the expectation that others will do) and which renders a bank-run a self-fulfilling prophesy.

Diamond and Dybvig (1983) were the first to attempt formalizing the concept of a bank-run as an equilibrium phenomenon (see also: Waldo, 1985, Cooper and Ross, 1998, Loewy, 1991, and Peck and Shell, 2003). I choose not to go this route for a number of reasons. First, most of these papers (with the exception of Loewy, 1991) feature models that are static in nature with no role for money. Second, the existence of multiple equilibria in these environments appears to be an artifact of exogenously imposed sub-optimal bank contracts (see Green and Lin, 1996, 2003). Third, it is by no means clear that actual bank crises are the product of expectations-driven shocks or other events related to a change in underlying fundamentals; see, for example, Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988), Allen and Gale (1998), Morris and Shin (2000), and Goldstein and Pauzner (2005).

Thus, the approach I take here is that 'crisis' events are triggered by fundamentals; or, to be more precise, shocks to information relating to fundamentals (along the lines of Allen and Gale, 1998). But I also need a dynamic model and, in particular, a model that features money and banks. The basic framework I adopt here is based on Champ, Smith and Williamson (1996) and Smith (2003), who introduce money via an overlapping generations structure where banks exist to insure agents against random needs for liquidity. As with these authors, I do not focus on modelling a 'crisis' as a bank-run *per se*; rather, a 'crisis' is defined by particular behavior that maps into real-world phenomena that are commonly associated with crisis events (as in the widespread demand for liquidity by agents that would not normally desire it, along with what - on the surface at least - looks like a partial suspension of payments). Unlike these authors (see also Loewy, 2003), however, I choose to view the shocks precipitating crisis events as 'technology' shocks that alter the real value of a bank's assets, rather than an exogenous shock to the aggregate demand for liquidity. My approach is also related in many ways to that of Loewy (1998), whose monetary Diamond-Dybvig model features 'information-based' shocks along the lines of Jacklin and Bhattacharya (1988); but whose analysis is focussed primarily on understanding a specific episode in U.S. banking history (1929-33).

The interpretative setup I choose is therefore based on a combination of Smith (2003) and Allen and Gale (1998). As in Smith (2003), there is a friction that allows money to coexist with capital, even in the absence of aggregate uncertainty, and despite money being dominated in rate of return. Banks exist in the model to provide agents with insurance

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against idiosyncratic shocks to their liquidity needs. An optimal banking arrangement requires that banks take deposits, invest them in a portfolio that consists of capital investment and reserves of cash. The liabilities that they issue are made demandable for cash. Following Allen and Gale (1998) (and in contrast to Smith, 2003), I introduce an aggregate technology shock (in the form of 'news' that reveals the future return on the bank's capital investment). The bank's portfolio decision (cash versus capital) must be made before the arrival of this information. The presence of aggregate uncertainty generates an additional demand for money in the form of 'precautionary balances,' since the rate of return on money (the inverse of the inflation rate) is stable relative to the risky return on capital. The inflation rate is dictated solely by monetary policy.

In this environment, *ex ante* behavior (prior to the arrival of information about the productivity shock) all looks the same (a by-product of the fact that technology shocks are *i.i.d.* and that agents live for two periods only): the bank always chooses the same currency/deposit ratio. But *ex post* behavior falls into one of two classes, depending on whether the realization of the shock falls above or below some critical value (determined endogenously by the rate of return on money and the bank's currency-to-deposit ratio). In 'normal' times (associated with news that the return on a bank's capital is above the critical value), only depositors who are subject to a liquidity shock hold cash and in doing so, forgo the higher yield on capital (their interest-bearing deposits at the bank). Those who do not require liquidity enjoy the high return on their bank deposits and do not wish to hold any cash. Occasionally, however, people receive bad news that the return on the bank's investment on capital will be abnormally low (below the critical value). In this event, even depositors without pressing liquidity needs request some cash to offset the low return on capital. In this event too, all depositors experience a lower-than-normal return on their deposits - an event I associate with a partial suspension of payments. Admittedly, this does not capture all of the features that one would normally associate with a banking crisis, it does appear to generate behavior that along some dimensions, at least, resembles observed behavior in many crisis episodes. As in Allen and Gale (1998), given the existence of liquidity and productivity shocks, banking crises are just part of an optimal risk sharing scheme: it allows depositors to share liquidity risk in the face of an unfavorable productivity shock.

I then ask, within the context of this model, how the probability of a crisis (so defined) is related to the conduct of monetary policy (inflation). The key thing to recognize here is that the frequency with which the economy experiences 'good' or 'bad' news depends on the frequency with which the *ex post* return to capital falls above or below the above-

mentioned critical value. This critical value serves as a sort of 'hurdle' that the return to capital must exceed if the economy is to avoid an *ex post* allocation that I have associated with a 'crisis' event. Holding fixed all other parameters (in particular, those governing the realization of technology shocks), I find that higher inflation reduces the probability of a crisis; in particular, it decreases the above-mentioned critical value. There are two effects associated with a higher inflation or lower return on cash: a direct and an indirect effect. The direct effect is that a decrease in the rate of return on money makes it more likely that the return to capital will exceed it. The indirect effect is that a lower rate of return on money induces a substitution in the bank's portfolio away from cash and into capital, making it more likely now that the residual claimants (people that hold their bank deposits for longer periods of time) will receive a higher payoff. As a result, depositors without liquidity needs are more likely to prefer holding on to their bank deposits rather than cashing out, which in turn, means a lower probability of crises. This captures the downward part of the U-shaped relationship between banking crises and inflation.

To account for the whole U-shaped relationship between banking crises and inflation, I follow Antinolfi, Landeo and Nikitin (2007) to introduce a third asset - a stable foreign currency called 'dollars' - that can also provide insurance against liquidity and productivity shocks.[3] With legal restriction on the domestic currency/capital ratio, the model is able to generate the threshold effect of inflation on banking crises. In particular, when domestic inflation is below a threshold, higher domestic money growth rate and inflation reduce the likelihood of banking crises, with the reverse happening when domestic inflation is above the threshold. The threshold is determined by the real rate of return of the foreign currency. When domestic inflation is low, domestic currency dominates foreign currency in rate of return. In this case, only domestic currency is used to insure against liquidity and productivity shocks, generating behavior that was described earlier. When domestic inflation is high, dollars dominate domestic currency in rate of return and are used to insure against liquidity and productivity shocks along with the domestic currency. The legal restriction on domestic currency/capital ratio binds, and dollars compete with domestic assets. In this case, higher domestic inflation induces the bank to substitute away from domestic capital and into dollars making it more likely now that the residual claimants will receive a lower payoff. As a result, depositors without liquidity needs are less likely to prefer holding on to their bank deposits rather than cashing out, which in turn, means a higher probability of crises.

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[3]Antinolfi, Landeo and Nikitin (2007) use a similar framework to explain the threshold effect of inflation on capital investment and output.

The paper is organized as follows. Section 2 presents the basic model in which only domestic assets are available and investigates the effect of inflation on banking crises. Section 3 extends the basic model to include a stable foreign currency and re-investigates the relationship between inflation and banking crises. Section 4 concludes.

1.2 The Basic Model

In this section, I study banking crises in a model with only domestic assets. I first describe the physical environment and talk about the role of money and banking in such an environment. I then characterize banking crises and examine the effect of inflation on banking crises.

1.2.1 Physical Environment

The economy consists of overlapping generations people who live for two periods and the initial old generation that lives for one period only. People inhabit on two locations, which I refer to as 'islands'. At each date $t = 1, 2, 3...\infty$, a new generation is born on each location consisting of continuum of *ex ante* identical young agents with unit mass. For simplicity, assume that people care only for consumption when old (this renders the saving decision trivial and allows me to focus on portfolio allocation). Each person is subject to an idiosyncratic relocation shock that is realized at the end of the first period of life. Let $0 < \pi < 1$ denote the probability of being relocated (applying the law of large numbers, this also represents the fraction of young agents who transit from one location to the other - note that these flows are symmetric across locations). The expected utility of a representative young agent is given by:

$$U = E\left[\pi u(c_m) + (1 - \pi)u(c_n)\right];$$

where c_m and c_n denote the consumption of movers and non-movers respectively, and $u(c) = \ln c.$[4] Note that I suppress time subscripts as I focus on stationary allocations.

Each young agent is endowed with $y > 0$ units of output. In addition, there is a storage technology where k units of investment (which must be made at the beginning of the agent's first period of life) yields xk units of future output. Assume that x (an aggregate

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[4]The conclusions of the paper continue to hold with CRRA utility function $u(c) = c^{1-\sigma}/(1 - \sigma)$, $0 < \sigma < 1$.

shock determining the realized return on capital investment) follows an exogenous stochastic process with cumulative distribution function $F(a) \equiv \Pr[x \leq a]$. The distribution of x is *i.i.d.* over time. Assume further that x is realized when agents are young - at the same time as the realization of their idiosyncratic relocation shock. Hence, x takes the form of 'news' concerning the future return to contemporaneous capital expenditure.[5] Finally, assume that capital depreciates fully after it is used in production and that goods are not transportable across locations.

Following Smith (2003), I also assume that private liabilities issued in one location cannot be used in the other location. As in Arouba, Waller, and Wright (2006), one can suppose, for example, that private liabilities can be costlessly counterfeited outside the location in which they were issued.

If young agents (who are *ex ante* identical) cannot communicate with each other across locations (assume that this is so), then the resulting allocation is essentially autarkic. That is, young agents can do no better than invest their entire endowment and then hope for the best (in particular, hope that they do not experience a relocation shock, and hope for a high return on their capital investment). It is, however, possible for young agents to attain a superior allocation if they have access to a public record-keeping technology or, absent this, fiat money (with fiat money serving as an imperfect substitute for an absent record-keeping technology; see Kocherlakota, 1998). I explain how in the following subsection.

1.2.2 Money and Banking

Let the initial old (in each location) be endowed with M_0 units of fiat money and assume that the government expands the supply of money at an exogenous (gross) rate z, so that $M_t = z M_{t-1}$. New money is used to finance government purchases of output.[6] Fiat money cannot be counterfeited and so (unlike private money) can be used on both islands. In this environment then, fiat money will be valued for two reasons. First, as in Smith (2003), fiat money can provide insurance against idiosyncratic relocation shocks (usefully interpreted now as 'liquidity' shocks). Second, fiat money (offering more stable rate of return) can function as 'precautionary balances' to insure agents against low realizations of return to capital. These insurance properties of fiat money ensure that it can be valued even if it is dominated in expected rate of return by capital investment (which is assumed to be the case throughout the paper).

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[5] Refer to Beaudry and Portier (2003) for a detailed discussion of 'news' shocks. For simplicity, I assume that the 'news' contains accurate information about the future return on capital.

[6] Alternatively, one could assume that new money is distributed to agents as a lump-sum transfer.

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As in Smith (2003), I view a 'bank' as a local coalition of young agents. The bank (in each location) takes as a deposit y units of output from the young. Prior to the realization of the 'news' shock, the bank must make a portfolio decision that allocates deposits across 'reserves' of fiat money (which it purchases from the old) and capital expenditure. In return for their deposit, each young agent is issued a bank liability (private money) that is made redeemable for fiat on demand. This redemption option is necessarily exercised by those agents that experience the relocation shock. These agents take their fiat money to the other location where they can use it in the next period to purchase output (from the new generation bank that demands cash reserves). Those agents that do not move take their bank money into the next period where it is then redeemed for the output that is produced with the maturing capital project. Depending on the realization of the productivity shock, however, even non-movers may find it optimal to redeem a part of their bank money for cash.

With money and banking, the sequence of events involving generation t agents are described by Figure 1:

Figure 1.1: Timeline

t	$t+1$
A. Young generation t agents are born;	F. Movers move;
B. Generation t bank is formed;	G. Bank's capital project matures;
C. Bank makes portfolio decision (q,k);	H. Non-movers share return from the bank's asset;
D. Relocation shock is realized; 'news' about x is received;	I. Old generation t agents exchange money for output with generation $t+1$ bank;
E. Movers withdraw cash;	J. Old generation t agents consume.

Formally, the choice problem facing a local bank can be written as follows:

$$\max_{q,k,c_m,c_n} \int [\pi u(c_m) + (1 - \pi)u(c_n)]dF(x); \qquad (P1)$$

subject to:

$$q + k = y;$$
$$\pi c_m(x) \leq R(x)q;$$
$$\pi c_m(x) + (1 - \pi)c_n(x) = R(x)q + xk;$$

where q and k represent investment in real cash balances and capital respectively. In formulating the problem above, note that the portfolio choice cannot depend on x (as this

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decision must be made prior to the arrival of news). Consumption allocations, on the other hand, can be made conditional on the realized return to capital. $R(x)$ denotes the (gross) real return on fiat money (the inverse of inflation rate $\gamma(x)$, which is potentially a function of the productivity shock). The second constraint above asserts that the (locally) aggregate level of future consumption for movers must be financed entirely out of bank reserves, the return of which depends on realized inflation. The last constraint simply reflects a (local) resource constraint.

At this stage, it is possible to deduce the equilibrium inflation rate (and the rate of return on money). To do so, I exploit the fact that x follows an *i.i.d.* process and that I am focussing on a stationary allocation (so that q remains constant over time). In this case, market-clearing at every date $t \geq 1$ can be expressed as:

$$M_t = p_t q;$$

where p_t denotes the period price-level. It follows then that $\gamma^*(x) = z$ and $R^*(x) = 1/z$. In other words, the equilibrium inflation rate (and rate of return on money) depends only on the rate of money expansion (and not on the realized technology shock).

Let me now characterize optimal behavior conditional on knowing that, in equilibrium, it must be the case that $R^*(x) = 1/z \equiv R$. The problem (P1) can be solved recursively. To do so, let me first take the portfolio decision (q, k) as given and assume that x is now known. Conditional on this, the problem entails choosing an optimal allocation of consumption across movers and non-movers; i.e.,

$$\max_{c_m, c_n} \pi u(c_m) + (1 - \pi)u(c_n); \tag{P2}$$

subject to:

$$\pi c_m(x) \leq Rq;$$
$$\pi c_m(x) + (1 - \pi)c_n(x) = Rq + x(y - q).$$

In the problem above, the first constraint is either slack or it is not. If the constraint binds, then $c_m(x) = Rq/\pi$ and $c_n(x) = x(y - q)/(1 - \pi)$. That is, consumption for movers is independent of x and depends only on the predetermined level of cash reserves (and the rate of inflation). Consumption for non-movers, on the other hand, is an increasing function of x (the realized return on their bank deposits). If, on the other hand, the first constraint is slack, then it is a simple matter to establish that full consumption insurance is desirable;

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i.e., $c_n(x) = c_m(x) = c(x)$, with $c(x) = Rq + x(y - q)$. Whether the first constraint binds or not depends on the configuration of parameters (x, q, R). This result is summarized in the following Lemma.

Lemma 1.1 *For a given* (q, R), *there exists a* $w > 0$ *such that an optimal consumption allocation satisfies:*

$$c_m(x) = c_n(x) = c(x) = Rq + x(y - q) \text{ if } x < w(q, R);$$
$$c_n(x) = x(y - q)/(1 - \pi) \text{ and } c_m(x) = c_m = Rq/\pi \text{ if } x \geq w(q, R)$$

with $w(q, R) = [(1 - \pi)/\pi]Rq/(y - q)$.

The formal proof of Lemma 1.1 can be found in the Appendix. The intuition though is relatively straightforward. *Ex ante*, depositors prefer to smooth their consumption across states of nature (i.e., whether they experience the relocation shock or not). It is desirable to smooth perfectly if the realized return on investment is low enough (i.e., below the critical value w). To take an extreme example, imagine that $x = 0$. In this case, it is optimal for both movers and non-movers to finance their consumption entirely out of cash. Since depositors are *ex ante* identical, full insurance is desirable. On the other hand, imagine that the realized return on capital investment is very high (above the critical value w). In this case, consumption smoothing is still desirable, but is not feasible (remember that goods cannot be transported across locations). The best the bank can do in this case is to let movers have all of the cash (which earns a relatively low rate of return) and let non-movers enjoy the high return on capital. Figure 2 displays this result in the form of a diagram.

The critical value w depends positively on q and R. Higher q (and thus lower k) and R imply higher return to depositors who redeem their bank money for cash and lower return to depositors who choose to hold on to their bank deposits, inducing the latter to require a higher return on capital to bridge the gap.

Let me now return to the problem (P1), given what we know about the nature of the (conditional) solution in (P2). Formally, the choice problem may now be written as:

$$\max_q \int^{w(q,R)} u(c(x))dF(x) + \int_{w(q,R)} [\pi u(c_m) + (1 - \pi)u(c_n(x))]dF(x);$$

where $c(x) = Rq + x(y - q)$, $c_m = Rq/\pi$ and $c_n(x) = x(y - q)/(1 - \pi)$. Differentiation

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Figure 1.2: Solution to (P2)
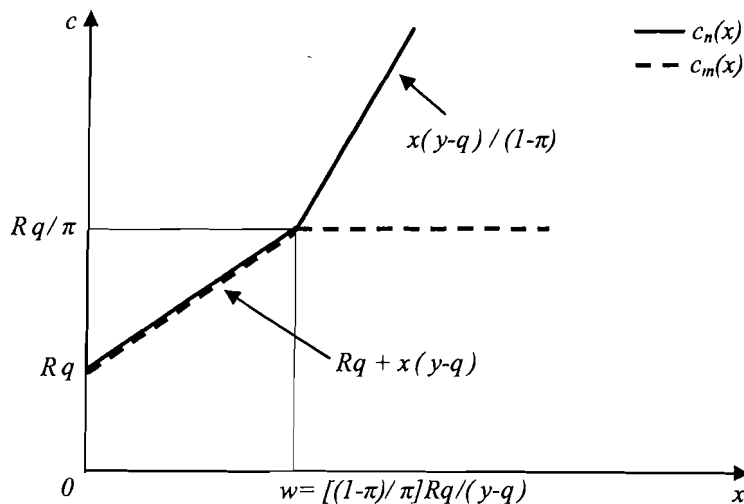
with respect to q yields:[7]

$$\int^{w} (R-x)u'(c(x))dF(x) + \int_{w} [Ru'(c_m) - xu'(c_n(x))]dF(x)$$
$$+\{u(c(w)) - [\pi u(c_m) + (1-\pi)u(c_n(w))]\}f(w)\partial w/\partial q.$$

Since $c(w) = c_m = c_n(w)$ (see Figure 2), the term in the curly brackets equals zero, and the solution $\hat{q}(R)$ is characterized by:[8]

$$\int^{\hat{w}} (R-x)u'(R\hat{q}+x(y-\hat{q}))dF(x) + \int_{\hat{w}} [Ru'(R\hat{q}/\pi) - xu'(x(y-\hat{q})/(1-\pi))]dF(x) = 0;$$
$$(1)$$

where $\hat{w}(R) = w(\hat{q}(R), R) = [(1-\pi)/\pi]R\hat{q}/(y-\hat{q})$.

Equation (1) says that the expected marginal benefit of an extra unit of investment in cash for movers must be equal to the expected marginal benefit of an extra unit of investment on capital for non-movers. With $\hat{q}$ determined in this manner, the equilibrium level of

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[7]$f(x)$ is the population density function corresponding to $F(x)$.

[8]The assumption that $\lim_{c->0} u'(c) = \infty$ implies that the demand for cash is positive (remember that goods are non-tranportable and non-movers' consumption must be financed by cash). The assumption that money is dominated in expected rate of return by capital guarantees a positive demand for capital investment. To see this, let us look at the first-order derivative of bank's objective function with respect to real cash balances q at $q = y$, which is given by $\int u'(Ry)(R-x)dF(x) = u'(Ry)\int(R-x)dF(x)$ and is negative given the assumptions that $u' > 0$ and money is dominated in expected rate of return by capital. The optimal solution requires $q < y$, or $k > 0$.

capital spending is simply $\hat{k} = y - \hat{q}$, and the equilibrium level of government purchases is given by $\hat{g} = (1 - R)\hat{q}$. The equilibrium consumption allocation may be expressed as follows:

$$\hat{c}_m(x) = [1 - I(x, \hat{w})][R\hat{q} + x(y - \hat{q})] + I(x, \hat{w})R\hat{q}/\pi;$$
$$\hat{c}_n(x) = [1 - I(x, \hat{w})][R\hat{q} + x(y - \hat{q})] + I(x, \hat{w})x(y - \hat{q})/(1 - \pi);$$

where $I(b, c) = 1$ if $b \geq c$ and $I(b, c) = 0$ otherwise.

What happens in the equilibrium is that young agents deposit their entire endowment with the bank and receive in return the bank's liability (bank money). The bank makes investment decision to invest $\hat{q}$ on real money holdings, and $\hat{k}$ on capital. After the liquidity shock is realized and the 'news' about future productivity arrives, movers go to the bank to redeem their bank money in government cash and relocate, and non-movers hold on to their bank deposits. At the end of each date, movers exchange cash for consumption goods, and non-movers share the return on the bank's remaining assets. With money and banking, agents are able to insure themselves against both types of risks (though not completely).

1.2.3 Banking Crises

In the equilibrium described above, when x is above $\hat{w}$ movers are paid a fixed amount $R\hat{q}/\pi$ and agents without liquidity needs do not want to hold cash. However, when x is below $\hat{w}$, movers are paid $R\hat{q} + x\hat{k}$ which is only a fraction of the payment when x is above $\hat{w}$; at the same time, non-movers desire to hold cash even in the absence of pressing liquidity needs. I define such situations as banking crises, which happen with probability $F(\hat{w})$.[9]

In the real world, banking crises are relatively rare events, which means that $F(\hat{w})$ is usually a small number. Most of the time, the bank's capital earns a return that is above the critical value $\hat{w}$, only depositors with liquidity needs hold cash and in doing so, forgo the higher yield on capital (their interest-bearing bank deposits). Depositors without liquidity needs enjoy the high return of the bank's capital and do not demand cash. Occasionally, however, depositors receive bad news that the return on the bank's capital will be abnormally low (below the critical value $\hat{w}$). In this event, even depositors without pressing liquidity need request to hold some cash to offset the low return on capital. At the same

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[9]When there is a banking crisis ($x < \hat{w}$), the severity of the crisis can be measured by the fraction of cash requested by non-movers: $[R(\hat{q} + x\hat{k})(1 - \pi) - x\hat{k}]/(R\hat{q}) = (1 - \pi)(1 - x/\hat{w})$.

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time, all depositors experience a lower-than-normal return on their deposits which look like a partial suspension of payments. Admittedly, this does not capture all of the features that one would normally associate with a banking crisis, it does appear to generate behavior that along some dimensions, at least, resembles observed behavior in many crisis episodes. The result is consistent with Rolnick and Weber's (1982) study on banking problems in the free banking era, which concludes that most of the free banking problems were 'caused by capital losses that banks suffered when market forces drastically pushed down the prices of state bonds, a significant part of all free bank portfolios'. It is also consistent with the empirical evidence that banking crises often occur during economic slow-downs.[10]

Note that as in Allen and Gale (1998), banking crises are optimal because they allow depositors to achieve complete risk sharing when the economy is hit by adverse productivity shocks. Depositors know that their payoffs will depend on the realization of economic fundamentals, and getting smaller-than-usual payment in case of an unfavorable productivity shock is just part of the optimal contract. In the model economy, the bank does not explicitly announce suspension of payments to depositors; it simply allocates resources according to the optimal contract which requires lower payment to depositors in case of weak fundamentals.

Note also that the explanation about the cause of banking crises is different from the aggregate liquidity risk models (including Champ, Smith and Williamson, 1996, Smith, 2003, and Loewy, 2003). In my model, banking crises arise when depositors receive negative signals about the return on capital. According to the aggregate liquidity risk models, banking crises occur due to exhaustion of bank reserves in response to high realizations of aggregate liquidity needs. The policy implication is also different. The economy in aggregate liquidity risk models faces a shock to money demand, and a discount window policy that makes money supply contingent on the realization of the aggregate liquidity shock will totally eliminate crises. In my model, since each young generation has the same demand for liquidity (remember that there is no aggregate liquidity risk) and precautionary balances (remember that productivity shocks are $i.i.d.$ across time), the demand for real cash balances stays the same at each date. With a constant demand for real cash balances, a similar discount window policy will only affect the price level, but will not eliminate crises.

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[10]For example, see Gorton (1988) and Demirguc-Kunt and Detragiache (1998, 2005).

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1.2.4 Effect of Inflation on Banking Crises

Using the above developed model, I am now ready to investigate the effect of inflation on banking crises.

Lemma 1.2 *The probability of a crisis, $F(\hat{w})$, is a decreasing function of z.*

Lemma 1.2 states that higher inflation reduces the probability of banking crises. Refer to the appendix for the formal proof. The intuition is as follows. Given the distribution of productivity shocks, the probability of banking crises is determined by the probability of the *ex post* return to capital falling below the critical value $(\hat{w})$. That is, the return to capital must exceed a 'hurdle' if the economy is to avoid an *ex post* allocation that I have associated with a 'crisis' event. A higher inflation or a lower rate of return to cash has an direct and indirect effect on banking crises. The direct effect is that a decrease in the rate of return on money makes it more likely that the return to capital will exceed it. The indirect effect is that a lower rate of return on money induces the bank to substitute away from cash and into capital, making it more likely that the residual claimants (people who hold their bank deposits for longer periods of time, here the non-movers) will receive a higher payoff. The two effects reinforce with each other making it more likely that non-movers will choose to hold on to their bank deposits instead of cashing out, which in turn, reduces the probability of crises.

The basic model described here predicts that lower inflation increases the likelihood of banking crises, which captures the downward part of the U-shaped relationship between inflation and banking crises, and is consistent with the observation in the Great Depression and Japan's lost decade.

However, from Table 1, one can see that many banking crises are associated with high or hyper inflation, which contradicts the model's prediction. The basic model's 'failure' to account for the positive relationship between inflation and probability of banking crises is due to the simplifying assumption that only two assets - domestic fiat money and capital - are available. In section 3, I extend the basic model by introducing a third asset; this extended model features a U-shaped relationship between banking crises and inflation.

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1.3 Threshold Effect of Inflation on Banking Crises

Following Antinolfi, Landeo and Nikitin (2007), I now introduce a third asset into the basic model. Like domestic fiat money, the third asset is also liquid and yields a stable real rate of return. In this paper, I interpret it as a stable foreign currency named 'dollar'. Dollars are in perfectly elastic supply and can be exchanged for goods at a fixed (real) rate. It is assumed that only the bank has access to the foreign exchange market.[11] The (gross) rate of return of dollars is fixed at R_0, and like the domestic currency, dollars are non-counterfeitable and can be used on both islands. The features of dollar enable it to perform similar roles as domestic currency by providing insurance against liquidity and productivity shocks. To ensure positive demand for domestic currency in all situations, I assume that the government imposes a legal restriction that the bank must hold θk ($\theta > 0$) units of real balances of domestic currency.[12] Let d be the real balances of dollars held by the bank. The bank's problem is now described by (P3) as follows:[13]

$$\max_{q,k,d,c_m,c_n} \int \left[\pi u(c_m) + (1 - \pi)u(c_n) \right] dF(x); \tag{P3}$$

subject to:

$$q + k + d = y;$$
$$\pi c_m \leq Rq + R_0 d;$$
$$\pi c_m + (1 - \pi)c_n = Rq + R_0 d + xk;$$
$$\theta k \leq q.$$

When $R \geq R_0$, domestic currency dominates dollars in rate of return so that $d = 0$. (P3) is equivalent to (P1) plus the legal restriction, which binds when R is below a threshold value R_L, with R_L solving $\hat{q}(R_L)/\hat{k}(R_L) = \theta$. For simplicity, I assume that $R_0 > R_L$, i.e., when domestic currency offers a return of R_0 or higher, the legal restriction does not bind,

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[11]Since the bank maximizes the representative depositor's welfare, even if depositors have access to the foreign exchange market themselves, they are willing to deposit their total endowment with the bank and let the bank access the exchange market on their behalf. The assumption that only the bank has access to the foreign exchange market does not change the optimal allocation that can be achieved (compared to the case that only depositors have access to the foreign exchange market), but it does have the advantage of making the identification of banking crises more straightforward.

[12]The assumption that reserve requirement is imposed only on domestic capital is motivated by the practice that higher reserve requirement is imposed on domestic deposits than on foreign currency deposits.

[13]Again, I concentrate on the stationary equilibrium where inflation rate and rate of return on domestic money are dictated solely by the domestic money growth rate.

and (P3) has the same solution as (P1).[14]

When $R < R_0$, dollars offer higher rate return than domestic currency, the legal restriction binds, and (P3) can be rewritten as:[15]

$$\max_{d,k,c_m,c_n} \int [\pi u(c_m) + (1 - \pi)u(c_n)]dF(x); \tag{P4}$$

subject to:

$$(1 + \theta)k + d = y;$$

$$\pi c_m \leq R\theta k + R_0 d;$$

$$\pi c_m + (1 - \pi)c_n \leq R\theta k + R_0 d + xk.$$

One can solve (P4) (which is similar to (P1)) recursively in two steps. The first step takes as given the portfolio decision d (and $k = (y - q)/(1 + \theta), q = \theta k$) and solves payment/consumption schedules to maximize the expected utility of a representative young depositor. The second step determines the optimal portfolio taking into consideration the optimal consumption schedules. As in the basic model, the solution to the problem in the first step involves a threshold strategy. When x is below a critical value $w(d, R)$, the bank retains some of the cash (domestic and foreign currencies) for the non-movers and all depositors get the same consumption. When x is higher than $w(d, R)$, all of the cash is paid to movers, and non-movers divide the output produced from the bank's capital and enjoy higher consumption than movers. The critical value $w(d, R)$ is derived by solving the following equation for x :

$$\frac{R(y - d)\theta/(1 + \theta) + R_0 d}{\pi} = \frac{x(y - d)/(1 + \theta)}{1 - \pi}. \tag{2}$$

---

[14]There are three cases to consider if I make the alternative assumption that $R_0 < R_L$. Case (i): $R > R_L$. In this case, $d = 0$ and the legal restriction does not bind so that (P3) is equivalent to (P1) of the basic model. Case (ii): $R_0 < R < R_L$. In this case, $d = 0$ and the legal restriction binds. The bank's portfolio choice is trivial; the bank invests $\theta y/(1 + \theta)$ on real cash balances and $y/(1 + \theta)$ on capital. The threshold value of x is given by $(1 - \pi)R\theta/\pi$ so that higher inflation (and thus lower rate of return on cash) reduces the probability of crises, which is qualitatively the same as in case (i). Case (iii): $R < R_0$. In this case, $d > 0$ and the legal restriction binds so that (P3) can be reformulated as (P4). The paper's conclusion is thus robust to the alternative assumption that $R_0 < R_L$.

[15]To ensure postive demand for domestic assets, it is assumed that dollars are dominated in expected rate of return by domestic assets:

$$R_0 < \int \left(\frac{\theta}{1 + \theta}R + \frac{1}{1 + \theta}x \right) dF(x).$$

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The left-hand-side and right-hand-side of the equation are the consumption by movers and non-movers respectively when the bank gives all the cash to movers and uses only output from capital to pay non-movers. The solution to equation (2) is:

$$w(d, R) = \frac{1 - \pi}{\pi} \left(\theta R + (1 + \theta) R_0 \frac{d}{y - d} \right).$$

In the second step, one solves the following problem:

$$\max_d \int^{w(d,R)} u(c(x)) dF(x) + \int_{w(d,R)} [\pi u(c_m) + (1 - \pi) u(c_n(x))] dF(x);$$

where $c(x) = \theta R(y-d)/(1+\theta) + R_0 d + x(y-d)/(1+\theta)$, $c_m = [\theta R(y - d)/(1 + \theta) + R_0 d]/\pi$, $c_n(x) = [x(y - d)/(1 + \theta)]/(1 - \pi)$, and $w = [(1 - \pi)/\pi][\theta R + (1 + \theta) R_0 d/(y - d)]$.

Differentiation with respect to d yields:

$$\int^w [-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)] u'(c(x)) dF(x)$$

$$+ \int_w \{[-\theta R/(1 + \theta) + R_0] u'(c_m) - [x/(1 + \theta)] u'(c_n(x))\} dF(w)$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi) u(c_n(w))] f(w) \partial w/\partial d.$$

Since $c(w) = c_m = c_n(w)$, the last term equals zero, and the solution $\tilde{d}(R)$ is characterized by:

$$\int^w [-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)] u'(c(x)) dF(x) \tag{3}$$

$$+ \int_w \{[-\theta R/(1 + \theta) + R_0] u'(c_m) - [x/(1 + \theta)] u'(c_n(x))\} dF(x) = 0.$$

With $\tilde{d}$ determined, the equilibrium level of capital spending $\tilde{k}$ is simply $(y - \tilde{d})/(1+\theta)$, and the equilibrium demand for real domestic currency balances $\tilde{q}$ is $(y - \tilde{d})\theta/(1 + \theta)$. The probability of banking crises can be calculated as $F(\tilde{w})$, where[16]

$$\tilde{w} = w(\tilde{d}(R), R) = [(1 - \pi)/\pi] \left[\theta R + (1 + \theta) R_0 \tilde{d}/(y - \tilde{d}) \right].$$

---

[16]As in the basic model, in the case of a crisis, the severity of the crisis can be measured by $(1 - \pi)(1 - x/\tilde{w})$.

The equilibrium government spending is given by $\tilde{g} = (1 - 1/z)\tilde{q}$. The equilibrium consumption can be expressed as follows:

$$\tilde{c}_m(x) = [1 - I(x, \tilde{w})](R\tilde{q} + R_0\tilde{d} + x\tilde{k}) + I(x, \tilde{w})(R\tilde{q} + R_0\tilde{d})/\pi;$$

$$\tilde{c}_n(x) = [1 - I(x, \tilde{w})](R\tilde{q} + R_0\tilde{d} + x\tilde{k}) + I(x, \tilde{w})x\tilde{k}/(1 - \pi).$$

Proposition 1.1 *When* $z \leq 1/R_0$, *higher inflation reduces the probability of banking crises; when* $z > 1/R_0$, *higher inflation increases the probability of banking crises.*

Refer to the Appendix for the proof. The intuition is as follows.

When domestic inflation is low ($z \leq 1/R_0$), agents use only domestic currency to insure themselves against the two types of risks since it dominates dollars in rate of return. As discussed in the basic model, higher inflation reduces the probability of banking crises.

When domestic inflation is high ($z > 1/R_0$), dollars dominate domestic currency in rate of return and are used to insure against liquidity and productivity shocks along with the domestic currency. The legal restriction on domestic currency/capital ratio binds, and dollars compete with domestic assets (domestic currency plus capital are viewed as a bundle when the legal restriction binds). As in the basic model, a banking crisis occurs when depositors receive news that the future return to the bank's capital will be below the critical value ($\tilde{w}$), and even those without liquidity needs demand to hold cash. Again, there are direct and indirect effects associated with higher domestic inflation and lower return on domestic currency. The direct effect is that lower return to domestic currency lowers the return to cash (domestic plus foreign currency) increasing the probability that the rate of return on capital exceeds it and thus reducing the probability of crises. The indirect effect is that a lower rate of return on domestic currency (and thus domestic asset) induces a substitution in the bank's portfolio away from domestic capital into dollars making it more likely now that the residual claimants will receive a lower payoff. As a result, depositors without liquidity needs are more likely to prefer cashing out instead of holding on to their bank deposits, which in turn, means a higher probability of crises. In the model, the second effect dominates so that higher inflation raises the probability of crises.

The extended model in this section thus captures the empirical observation that when inflation in beyond a threshold, higher inflation is associated with higher probability of banking crises.[17] I provide a numerical example below.

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[17]As a by-product, the model offers an explanation about the 'twin crises' phenomenon; i.e., the tendency for banking and currency crises to occur in tandem (refer to Kaminsky and Reinhart, 1999, and Glick and Hutchison, 2001).

Example. Let $y = 1$, $\pi = 0.1$, $\theta = 1/9$, $R_0 = 0.97$ and x be distributed uniformly over the range $[0.965, 1.155]$. Figure 3 graphs the demand for real balances of domestic currency, capital investment, the demand for real dollar balances and the probability of banking crises against the net domestic inflation rate.

Figure 1.3: Threshold Effect of Inflation on Banking Crises
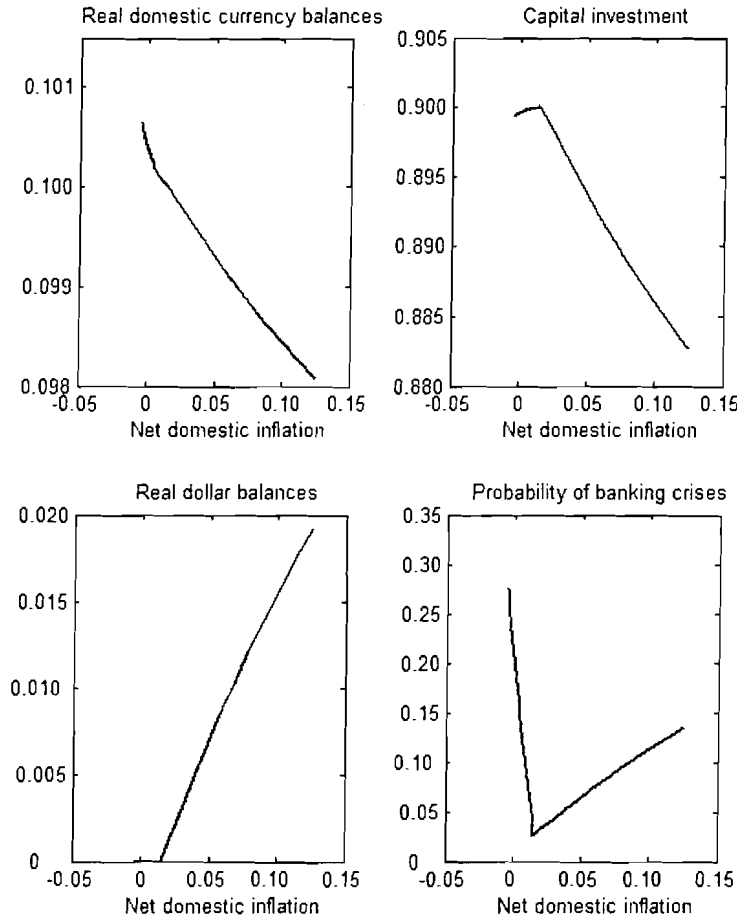
Finally, I would like to point out that adding dollars into aggregate liquidity risk models (including Champ, Smith and Williamson, 1996, Smith, 2003, Loewy, 2003) will not generate the U-shaped relationship between the probability of banking crises and inflation that is supported by the data. When inflation is low, domestic currency dominates dollars in rate of return and this corresponds to the basic model with only domestic assets. Lower inflation causes the bank to hold more cash decreasing the probability of banking crises (remember that in aggregate liquidity risk model, banking crises occur because of exhaus-

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tion of bank reserves in case of high liquidity needs). When inflation is high, domestic currency is dominated in rate of return by dollars. Legal restriction binds, and domestic currency and capital are bundled together as domestic asset. Higher inflation induces the bank to hold more dollars reducing the probability of banking crises (higher dollar holding means that the bank can meet higher liquidity needs). The aggregate liquidity risk models will generate an inverted U-shaped relationship between the probability of banking crises and inflation.

1.4   Conclusion

In this paper, I analyze the effect of inflation on banking crises in an overlapping generations model in which money is valued for its insurance roles against liquidity and productivity risks, and the banking sector acts as a mechanism through which depositors pool their liquidity risk. The model's equilibrium is consistent with some key features of actual banking crises; namely, the partial suspension of payments and depositors demanding cash even in the absence of liquidity needs. According to the model, banking crises are due to adverse information about the future return on the bank's capital, rather than exogenous fluctuations in the demand for liquidity as in Champ, Smith and Williamson (1996), Smith (2003) and Loewy (2003); this result is consistent with the empirical observation that banking crises tend to occur during economic slow-downs.

The model is able to explain the U-shaped relationship between inflation and banking crises. There are three assets: domestic currency, a stable foreign currency called dollars and domestic capital. Both currencies can be used to insure liquidity and productivity shocks. There is legal restriction on the domestic currency/capital ratio. When domestic inflation is low, domestic currency dominates foreign currency in rate of return and only domestic currency is used to insure against liquidity and productivity shocks. Higher inflation reduces the rate of return to domestic currency and induces the bank to invest more on capital; this makes it more likely that the residual claimants prefer holding on to bank deposits than cashing out and reduces the probability of banking crises. When domestic inflation is high, domestic currency is dominated in rate of return by dollars, the legal restriction binds and domestic currency and capital are viewed as a bundle. Higher inflation reduces the return to domestic asset (currency plus capital) inducing the bank to invest more on dollars and less on capital, which makes it less likely that the residual claimants prefer keeping bank deposits instead of cashing out and increases the likelihood of banking crises.

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The model in this paper can be extended in a number of directions. First, I have concentrated on a class of simple monetary policies: fixed money growth to finance government spending. It would be interesting to study more sophisticated monetary policies which make money growth contingent on the realization of productivity shocks and to deduce the welfare implications of such policies. The model is simplistic and the analysis in the paper is qualitative in nature. Investigating the quantitative implications of a suitably developed model should prove a useful endeavor.

1.5 References

Allen, Franklin, and Douglas Gale (1998), 'Optimal Financial Crises,' *Journal of Finance*, 53(4), 1245-1284.

Arouba, S. Boragan, Waller, J. Christopher, and Randall Wright (2006), 'Money and Capital,' manuscript.

Antinolfi, Gaetano, Nikitin, Maxim, and Claudia M. Landeo (2007), 'Dollarization, Financial Intermediation and Real Activity: The Inflation Threshold,' *Canadian Journal of Economics*, 40, 628-649.

Beaudry, Paul, and Franck Portier (2004), 'Stock Prices, News and Economic Fluctuations,' NBER Working Paper No. W10548

Champ, Bruce, Smith, D. Bruce, and Stephen D. Williamson (1996), 'Currency Elasticity and Banking Panics: Theory and Evidence,' *Canadian Journal of Economics*, 29(4), 828-864.

Chari, V.V., and Ravi Jagannathan (1988), 'Banking Panics, information, and Rational Expectations Equilibrium,' *Journal of Finance*, 43, 749-761.

Cooper, Russell, and Thomas W. Ross (1998), 'Bank Runs: Liquidity Costs and investment Distortions,' *Journal of Monetary Economics*, 41, 27-38.

Demirguc-Kunt, Asli, and Enrica Detragiache (1998), 'The Determinants of Banking Crises in Developing and Developed Countries,' *International Monetary Fund Staff Papers*, 45, No. 1, 81-109.

Demirguc-Kunt, Asli, and Enrica Detragiache (2005), 'Cross-Country Empirical Studies of Systemic Bank Distress: A Survey,' IMF working paper 05/96, International Monetary Fund.

Diamond, W. Douglas, and Philip H. Dybvig (1983), 'Bank Runs, Deposit insurance, and Liquidity,' *Journal of Political Economy*, 91, 401-419

Glick, Reuven, and Michael M. Hutchison (2001), 'Banking and Currency Crises: How Common Are Twins?' in *Financial Crises in Emerging Markets*, ed. Reuven Glick, Ramon Moreno and Mark M. Spiegel (Cambridge: Cambridge University).

Goldstein, Italy, and Ady Pauzner (2005), 'Demand Deposit Contracts and the Probability of Bank Runs,' *the Journal of Finance*, 60(3) 1293-1327.

Gorton, Gary (1988), 'Banking Panics and Business Cycles,' *Oxford Economic Papers*, 40,

751-781.

Green, J. Edward, and Ping Lin (2000), 'Diamond and Dybvig's classic theory of financial intermediation : what's missing?' *Federal Reserve Bank of Minneapolis Quarterly Review*, issue Winter, 3-13.

Green, J. Edward, and Ping Lin (2003), 'Implementing efficient allocations in a model of financial intermediation,' *Journal of Economic Theory*, 109, 1-23.

Jacklin, J. Charles, and Sudipto Bhattacharya (1988) 'Distinguishing Panics and information-Based Bank Runs: Welfare and Policy Implications,' *Journal of Political Economy*, 96(3), 568-592.

Jonker, Joost, and Jan Luiten van Zanden (1995), 'Method in the madness? Banking crisis between the Wars, an International comparison,' in *Banking, Currency, & Finance in Europe between the Wars,* ed. Charles H. Feinstein, (Cambridge: Cambridge University), 77-93.

Kaminsky, Graciela, and Carmen M. Reinhart (1999), 'The Twin Crises: The Causes of Banking and Balance-of-Payments Problems,' *American Economic Review*, 89(3), 473-500.

Kocherlakota, R. Narayana (1998), 'Money Is Memory,' *Journal of Economic Theory*, 81(2), 232-251.

Loewy, B. Michael (1991), 'The Macroeconomic Effects of Bank Runs: An Equilibrium Analysis,' *Journal of Financial Intermediation*, 1, 242-256.

Loewy, B. Michael (1998), 'Information-Based Bank Runs in a Monetary Economy,' *Journal of Macroeconomics*, 20, 681-702

Loewy, B. Michael (2003), 'To Furnish an Elastic Currency: Banking, Aggregate Risk, and Welfare,' *Topics in Macroeconomics*, 2003, 3(1), Article 3.

Morris, Stephen, and Hyunsong Shin (2000), 'Rethinking Multiple Equilibria in Macroeconomic Modeling,' *NBER Macro Annuals*, 2000

Peck, James, and Karl Shell (2003) 'Equilibrium Bank Runs,' Journal of Political Economy, 111, 103-123.

Rolnick, J. Arthur, and Warren E. Weber (1982), 'Free Banking, Wildcat Banking, and Shinplasters,' *Federal Reserve Bank of Minneapolis Quarterly Review*, issue Fall, 10-19.

Smith, D. Bruce (2003), 'Taking Intermediation Seriously,' *Journal of Money, Credit, and*

Banking, 35, 1319-1357.

Waldo, G. Douglas (1985), 'Bank Runs, the Deposit-Currency Ratio and the Interest Rate,' *Journal of Monetary Economics*, 15, 269-277.

1.6 Appendix

Proof of Lemma 1.1

The Lagrangian is $\mathcal{L} = (1-\pi)u\left((Rq + xk - \pi c_m)/(1 - \pi)\right) + \pi u(c_m) + \lambda(Rq - \pi c_m)$.

The first-order condition with respect to c_m is given by: $u'(c_n) = u'(c_m) + \lambda$.

There are two cases to be considered.

When $\lambda > 0$, the constraint $\pi c_m \leq Rq$ binds with equality, and I have $c_m = Rq/\pi$, $c_n = x(y - q)/(1 - \pi)$, $u'(c_n) > u'(c_m) \longrightarrow c_n < c_m$, and $Rq/\pi > x(y - q)/(1 - \pi)$, or $x < w$.

When $\lambda = 0$, the constraint $\pi c_m \leq Rq$ is slack, and I have $\pi c_m < Rq$, $u'(c_n) = u'(c_m)$ $\longrightarrow c_n = c_m = c = Rq + x(y - q)$, and $Rq/\pi > x(y - q)/(1 - \pi)$ or $x > w$. $\blacksquare$

Proof of Lemma 1.2

I first show that $\partial \hat{q}/\partial R > 0$. It then follows naturally from $R \equiv 1/z$ and the expression for $\hat{w}$ that $\hat{w}'(z) < 0$ or that higher inflation reduces the probability of banking crises.

$$\text{Let } G(q, R) \equiv \int^w [(R - x)u'(c(x))]dF(x) + \int_w [Ru'(c_m) - xu'(c_n(x))]\,dF(x)$$
$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))]f(w)\partial w/\partial q;$$

where $c(x) = Rq + x(y - q)$, $c_m = Rq/\pi$, $c_n(x) = x(y - q)/(1 - \pi)$ and $w = [(1 - \pi)/\pi]Rq/(y - q)$.

Differentiating $G(q, R)$ with respect to q, one gets:

$$
\begin{aligned}
G_q &= \int^w u''(c)(R - x)^2 dF(x) + \int_w [u''(c_m)R^2/\pi - u''(c_n)x^2/(1 - \pi)]dF(x) \\
&\quad + \{u'(c)(R - w) - [u'(c_n(w))(-w) + u'(c_m)R]\}f(w)\partial w/\partial q \\
&\quad + [u'(c(w)) - u'(c_n(w))](y - q)f(w)(\partial w/\partial q)^2 \\
&\quad + [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))]f'(w)(\partial w/\partial q)^2 \\
&\quad + [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))]f(w)\partial^2 w/\partial q^2.
\end{aligned}
$$

Using $c(w) = c_n(w) = c_m$, I can simplify G_q as follows: $G_q = \int^w (R - x)^2 u''(c)]dF(x) + \int_w [u''(c_m)R^2/\pi - u''(c_n)x^2/(1 - \pi)]dF(x) < 0$.

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Differentiating $G(q, R)$ with respect to R, I get:

$$
\begin{aligned}
G_R &= \int^w [u''(c)(c - xy) + u'(c)] \, dF(x) + \int_w [u''(c_m)c_m + u'(c_m)] \, dF(x) \\
&\quad + \{u'(c(w))(R - w) - [u'(c_n(w))(-w) + u'(c_m)R]\} f(w)\partial w/\partial R \\
&\quad + [u'(c(w)) - u'(c_n(w))] (y - q)f(w)(\partial w/\partial q)(\partial w/\partial R) \\
&\quad + [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f'(w)(\partial w/\partial q)(\partial w/\partial R) \\
&\quad + [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f(w)(\partial^2 w/\partial q\partial R) \\
&= \int^w [u''(c)(R - x)q + u'(c)] \, dF(x) + \int_w [u''(c_m)c_m + u'(c_m)] \, dF(x) \\
&= \int^w [u''(c)(c - xy) + u'(c)] \, dF(x) + \int_w [u''(c_m)c_m + u'(c_m)] \, dF(x).
\end{aligned}
$$

Since $u(c) = \ln c$ has the property that $u''(c)c = -u'(c)$, one can rewrite G_R as: $G_R = -\int^w u''(c)xy\,dF(x) > 0$. It then follows that: $\partial\hat{q}/\partial R = -G_R/G_q > 0$. ∎

Proof of Proposition 1.1

Refer to the proof of Lemma 1.2 for the case when $R \geq R_0$.

Here is the proof of the second part of the Proposition (when $R < R_0$).

$$
\begin{aligned}
\text{Let } H(d, R) &\equiv \int^w u'(c(x)) [-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]dF(x) \\
&\quad + \int_w \{u'(c_m)[-\theta R/(1 + \theta) + R_0] - u'(c_n(x))x/(1 + \theta)\}dF(x) \\
&\quad + [u(c(w)) - (1 - \pi)u(c_n(w)) - \pi u(c_m)]f(w)\partial w/\partial d;
\end{aligned}
$$

where $c(x) = \theta R(y-d)/(1+\theta)+R_0d+x(y-d)/(1+\theta)$, $c_m = [\theta R(y - d)/(1 + \theta) + R_0d] /\pi$, $c_n(x) = [x(y - d)/(1 + \theta)] /(1 - \pi)$, and $w = [(1 - \pi)/\pi] [\theta R + (1 + \theta)R_0d/(y - d)]$.

Differentiating $H(d, R)$ with respect to d, I get:

$$H_d = \int^w u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]^2 dF(x)$$

$$+ \int_w \left\{ u''(c_n) \left[x/(1 + \theta) \right]^2 /(1 - \pi) + u''(c_m)[-\theta R/(1 + \theta) + R_0]^2/\pi \right\} dF(x)$$

$$+ \{ u'(c(w)) \left[-\theta R/(1 + \theta) + R_0 - w/(1 + \theta) \right]$$

$$- [u'(c_m)[-\theta R/(1 + \theta) + R_0] - u'(c_n(w))w/(1 + \theta)] \} f(w)\partial w/\partial d$$

$$+ [u'(c(w)) - u'(c_n(w))] [(y - d)/(1 + \theta)] f(w) (\partial w/\partial d)^2$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f'(w) (\partial w/\partial d)^2$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f(w)(\partial^2 w/\partial d^2).$$

Using $c(w) = c_n(w) = c_m$, one can simplify H_d as follows:

$$H_d = \int^w u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]^2 dF(x)$$

$$+ \int_w \left\{ u''(c_n) \left[x/(1 + \theta) \right]^2 /(1 - \pi) + u''(c_m)[-\theta R/(1 + \theta) + R_0]^2/\pi \right\} dF(x) < 0.$$

Differentiating $H(d, R)$ with respect to R, one gets:

$$H_R = \int^w u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]\theta(y - d)/(1 + \theta) + u'(c)[-\theta/(1 + \theta)]dF(x)$$

$$+ \int_w \{ u''(c_m)[-\theta R/(1 + \theta) + R_0]\pi^{-1}[-\theta d/(1 + \theta)] + u'(c_m)(-\theta/(1 + \theta)) \} dF(x)$$

$$+ \{ u'(c(w))[-\theta R/(1 + \theta) + R_0 - w/(1 + \theta)]$$

$$- [u'(c_n(w))(-w/(1 + \theta)) + u'(c_m)(-\theta R/(1 + \theta) + R_0)] \} f(w)\partial w/\partial R$$

$$+ [1/c(w) - 1/c_n(w)] [(y - d)/(1 + \theta)] f(w)(\partial w/\partial d)(\partial w/\partial R)$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f'(w)(\partial w/\partial d)(\partial w/\partial R)$$

$$+ [u(c(w)) - \pi u(c_m) - (1 - \pi)u(c_n(w))] f(w)(\partial^2 w/\partial d\partial R)$$

$$= [-\theta/(1 + \theta)] \{ \int^w [u''(c)(c - R_0 y) + u'(c)] dF(x)$$

$$+ \int_w [u''(c_m)(c_m - R_0 y/\pi) + u'(c_m)] dF(x) \}.$$

Since $u(c) = \ln c$ has the property that $u''(c)c = -u'(c)$, H_R can be expressed: $H_R = [\theta/(1 + \theta)]R_0 y \left[\int_0^w u''(c)dF(x) + (1/\pi) \int_w u''(c_m)dF(x) \right] < 0.$

Remember that $w = [(1 - \pi)/\pi]\,[R\theta + R_0(1 + \theta)d/(y - d)]$.

$$
\begin{aligned}
\partial w/\partial R &= \theta(1 - \pi)\pi^{-1}\{1 + \theta^{-1}R_0(1 + \theta)y/(y - d)^2\partial d/\partial R\} \\
&= \theta(1 - \pi)\pi^{-1}\{1 - [\theta^{-1}R_0(1 + \theta)yH_R]/[(y - d)^2 H_d]\} \\
&= \theta(1 - \pi)\pi^{-1} \times \\
&\quad \left\{1 - (R_0y)^2/(y - d)^2\left[\int^w u''(c)dF(x) + (1/\pi)\int_w u''(c_m)dF(x)\right]/H_d\right\} \\
&= \theta(1 - \pi)\pi^{-1}(1 - A/B);
\end{aligned}
$$

where $A \equiv (R_0y)^2\left[\int^w u''(c)dF(x) + (1/\pi)\int_w u''(c_m)dF(x)\right]$, and $B \equiv (y - d)^2 H_d$.

$$
\begin{aligned}
B &= \int^w u''(c)[-\theta R/(1 + \theta) + R_0 - x/(1 + \theta)]^2(y - d)^2dF(x) \\
&\quad + \int_w \left\{u''(c_n)\,[x/(1 + \theta)]^2/(1 - \pi) + u''(c_m)[-\theta R/(1 + \theta) + R_0]^2\pi\right\}(y - d)^2dF(x) \\
&= \int^w u''(c)[c - R_0y]^2dF(x) + \int_w \left\{-(1 - \pi) + u''(c_m)[c_m - R_0y/\pi]^2/\pi\right\}dF(x) \\
&= -\left\{\int^w (1 - R_0y/c)^2dF(x) + (1 - \pi)\int_w dF(x) + \int_w (1 - R_0y/\pi/c_m)^2dF(x)\right\} \\
&= -\{1 - \int^w (2R_0y/c)dF(x) - \int_w (2R_0y/c_m)dF(x) \\
&\quad + \int^w (R_0y/c)^2dF(x) + (1/\pi)\int_w (R_0y/c_m)^2dF(x)\} \\
&= -\left\{1 - \int^w (2R_0y/c)dF(x) - \int_w (2R_0y/c_m)dF(x) - A\right\} = -1 + C + A;
\end{aligned}
$$

where $C \equiv \int^w (2R_0y/c)dF(x) + \int_w (2R_0y/c_m)dF(x)$.

From the first order condition $H(d, R) = 0$, one has:

$\int^w u'(c)[R\theta k - R_0(1 + \theta)k + xk]dF(x) + \int_w \{u'(c_n)(xk) + u'(c_m)[R\theta k - R_0(1 + \theta)k]\}\,dF(x) = 0$;

or $\int^w u'(c)(c - R_0y)dF(x) + \int_w [(1 - \pi) + \pi u'(c_m)(c_m - R_0y/k)]\,dF(x) = 0$;

or $\int^w u'(c)(R_0y)dF(x) + \int_w [u'(c_m)(R_0y)]\,dF(x) = 1$;

or $C = 2$.

Now one has: $\partial\tilde{w}/\partial R = [\theta(1 - \pi)/\pi]/(A + 1)$. Remember that $B = -1 + C + A < 0 \longrightarrow 1 + A < 0$. It then follows that $\partial\tilde{w}/\partial R < 0$.∎

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Chapter 2

Learning and Experimental Evidence in the Diamond and Dybvig Model of Bank Runs[1]

This paper studies bank runs in the laboratory that mimics the Diamond and Dybvig (1983) environment in which depositors play a repeated coordination game. We find that whether bank runs can occur as a result of pure coordination failures, but only when coordination is difficult. The difficulty in coordination is measured by the percentage of depositors required to leave money in the bank until assets mature; so that in this manner, a better payoff is achieved rather than withdrawing early. We introduce imitation-based learning algorithm modified from Temzelides (1997) into the same environment, and find that the simulation results are largely consistent with the behavior observed in the laboratory.

2.1 Introduction

One important function of a bank is to pool depositors' resources and invest on profitable (illiquid) long-term assets. At the same time, the bank keep cash reserves (or other liquid assets) and issue interest-bearing demand deposits to meet depositors' liquidity needs. During the process, the bank improves social welfare by providing a type of insurance which allows depositors with liquidity needs to earn interest on their deposits and share the high

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proceeds from long-term investment.

An unappealing feature of the demand deposit contract is that it is associated with multiple self-fulfilling equilibria, and opens the gate to bank runs in which a large number of depositors 'run' to the bank to withdraw money even in the absence of liquidity needs. Whether the optimal risk sharing can be achieved hinges critically on the depositor's expectations about other depositors' actions. In the 'good' equilibrium, only those with liquidity needs (impatient consumers) withdraw early, earning return higher than what liquidating the long-term asset entitles them. Those who do not need liquidity (patient consumers), expecting other patient consumers to do the same, wait until the long-term asset matures, earning return lower than the rate of return of the long-term asset (but higher than the return for impatient consumers). At this equilibrium, optimal risk sharing is achieved by a transfer of consumption from patient consumers to impatient consumers which improves the ex ante welfare of depositors. In the 'bad' equilibrium, however, expecting other patient consumers to do the same, every patient consumer 'runs' to the bank to withdraw money, and the bank is forced to liquidate its profitable long-term investment at fire sale prices to honor the demand deposit contract. In this case, profitable long-term projects are interrupted, risk sharing is destroyed, and the allocation is even worse than that in the autarky where the bank does not exist.

The critical question is then what determines depositors' expectations. The existing theoretical literature can be broadly classified into two categories. The first view is that the coordination device that determines depositors expectations and actions is some extraneous variables unrelated fundamental (the rate of return on the bank's long-term assets) often called 'sunspots'. This view was initially formalized by Diamond and Dybvig (1983), and substantiated later by Waldo (1985), Loewy (1991), Cooper and Ross (1998), and Peck and Shell (2003). An alternative view is that bank runs occur only when depositors receive unfavorable (noisy) signals about economic fundamentals. In Chari and Jagannathan (1988), there is aggregate liquidity risk so that the fraction of impatient depositors is uncertain. At the same time, some patient agents receive information on the payoffs to the risky long-term asset, and will withdraw early upon observing poor fundamentals. Bank runs occur when uninformed patient agents misinterpret liquidity withdrawal shocks as withdrawals caused by pessimistic information about bank assets.[2] In Jacklin and Bhattacharya (1988), some patient agents receive a signal which they use to update their prior assessment of the return of long-term investment. Runs are caused by rational revisions in beliefs about the

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[2]Other papers attributing bank runs to aggregate liquidity risk include Champ, Smith and Williamson (1996), Smith (2003), Loewy (2003), and Gomis-Porqueras and Smith (2006).

bank's portfolio performance. Morris and Shin (2000), and Goldstein and Pauzner (2005) develop a global game theory of bank runs, and demonstrate that allowing agents to have a small amount of idiosyncratic uncertainty about fundamentals pinpoints a unique equilibrium in which depositors initiate a run on the bank while receiving unfavorable information about the fundamentals.[3]

Empirical works offer mixed evidence about the competing theories of bank runs. For example, Gorton (1988), Allen and Gale (1998) and Schumacher (2000) show that bank runs have historically been strongly correlated with deteriorating economic fundamentals. In contrast, Boyd, Gomis, Kwack and Smith (2001) conclude that banking crises may often be the outcome of bad realizations of sunspot equilibria.

In this paper, we take a different route to investigate the formation and evolution of agents' beliefs and actions in the laboratory mimicking the Diamond and Dybvig (1983) environment. The paper offers a thorough experimental investigation about whether bank runs can occur as the result of pure coordination failures in this type of environment, and if yes, under what conditions and what factors affect the severity of bank runs. To do that, we enroll human subjects to play a repeated one-shot 10×2 (there are 10 players and 2 possible actions: withdrawing early or late) game as described in Diamond and Dybvig (1983). The main result is that bank runs can occur as a result of pure coordination failures, but only when coordination is difficult. The difficulty in coordination is measured by the percentage of depositors required to leave money in the bank until assets mature; so that in this manner, a better payoff is achieved rather than withdrawing early. We also find that, when around 70% of patient depositors are required, the experimental economies perform very differently across different sessions. We compare the experimental results and the simulation results from a learning algorithm modified from Temzelides (1997), and find that learning offers a good approximation to the observed lab behavior.

To the best of our knowledge, so far there exist four experimental studies on bank runs - Schotter and Yorulmazer (2005), Garratt and Keister (2006), Madies (2006), and Klos and Sträter (2007). In Schotter and Yorulmazer (2005), bank runs always occur, and the main goal is to study the factors that affect the speed of withdrawals. Garrat and Keister (2006) do two treatments (with different liquidation costs) with the basic Diamond and Dybvig (1983) model and find that in both cases the experimental economies stay close to the non-run equilibrium. They then turn to extensions of the basic model by adding aggregate

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[3]Other papers that attribute bank runs to the deterioration in the quality of the bank's assets include Alonso (1996), Allen and Gale (1998), Loewy (1998), Jiang (2007), Chen (1999), Yorulmazer (2003) and Gu (2007). The last three papers model bank runs in sequential games.

liquidity risk and studying a sequential game. Madies (2006) studies the basic Diamond and Dybvig (1983) model with two difficulty levels of coordination and two payoff differentials between the two equilibria. He finds that full bank runs with all depositors withdrawing early are rare and partial runs are persistent and difficult to prevent. Klos and Sträter (2007) study the Goldstein and Pauzner (2005) model in the laboratory. In our paper, we will study the Diamond and Dybvig (1983) model with various difficulty levels of coordination and investigate in more detail how they affect the extent of bank runs.

The paper also offers new insight into the experimental literature on coordination games featuring multiple Nash equilibria that can be Pareto ranked, by looking at how the difficulty level of coordination (measured by the percentage of people required to take the 'good' action so that the 'good' action offers better payoff than the 'bad' action) affects the result of coordination games. Battalio, Samuelson and Van Huyck (1990, 1991) study $N \times 7$ games in the laboratory and find that which equilibrium the economy converges to is affected by the number of players - it converges to the payoff dominant equilibrium when there are two players and converges to the minimax equilibrium when there are many players.[4] Battalio, Samuelson and Van Huyck (2001) experiment with 2×2 coordination games and find that the economy is more likely to converge to the payoff dominant equilibrium with higher payoff differentials.[5] Cabrales, Nagel and Armenter (2003, working paper) get similar results. Heinemann, Nagel and Ockenfels (2004) study a $N \times 2$ coordination game based on the currency speculative attack model by Morris and Shin (1998) to test the predictions of the global game theory. They compare results with common and private information about random fundamentals, and find that there is not much difference between the two. In an environment featuring changing fundamentals, subjects learn to use the threshold strategy overtime.[6] In our experiment, we fix the value of the fundamental for some time, which we think is more fitted to study pure coordination games.

The paper proceeds as follows. Section one outlines the Diamond and Dybvig (1983) model, section two describes the experimental procedure and presents the experimental results, section three describes the learning algorithm and compares the simulation results to the experimental results, and section four concludes and points out directions for future research.

[4]The group size for 'many players' ranges from 14 from 16 and each game is run played for 10 rounds.

[5]Battalio, Samuelson and Van Huyck (2001) run three games all featuring two Nash equilibria: the payoff dominant equilibrium and the risk dominant equilibrium. The games are played by 8 subjects with random matching, and each game is played for 75 rounds.

[6]Each session of the game consists of 8 rounds where 15 subjects play a coordination game. In each round of the experiment, subjects are presented 10 scenarios corresponding to 10 values of the fundamental randomly drawn from a known distribution, and asked to choose to attack or not attack for each scenario.

2.2 The Model

Here we give a brief discussion of the Diamond and Dybvig (1983) model.

There are three dates (indexed by 0,1,2) and a single homogeneous good. There are M *ex ante* identical agents in the economy. At date 0 (planning period), each agent is endowed with 1 unit of good and faces a preference/liquidity shock that determines their types. Liquidity shocks are realized at the beginning of date 1. N of the agents will be patient agents and are indifferent between consumption in period 1 and 2; $M - N$ agents will be impatient agents and care about only first period consumption. Realization of liquidity shocks is private information.

Preferences are described by the state-dependent utility function:

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{if impatient;} \\ u(c_1 + c_2) & \text{if patient;} \end{cases}$$

where $u'' < 0 < u'$, $\lim_{c \to \infty} u'(c) = 0$ and $\lim_{c \to 0} u'(c) = \infty$. Also, assume that the relative risk aversion coefficient $-cu''(c)/u'(c) > 1$ everywhere.

There is a productive technology that transfers 1 unit of date 0 output into 1 unit of date 1 output or $R > 1$ units of date 2 output.

In this environment, the social planner's problem is as follows:

$$\max_{c_1, c_2} \left(1 - \frac{N}{M}\right) u(c_1) + \frac{N}{M} u(c_2)$$

subject to:

$$\left(1 - \frac{N}{M}\right) c_1 + \frac{N}{M} \frac{c_2}{R} = 1$$

The solution is $1 < c_1^* < c_2^* < R$.

A bank, by offering demand deposit contracts, can support the optimal risk-sharing allocation. The contract requires agents to deposit their endowment with the bank at date 0. In return, agents receive a bank security which can be used to demand consumption at either date 1 or 2. The bank promises to pay $r > 1$ to agents who demand consumption at date 1. If the bank does not have enough money to fulfill its promise, it divides the available resource evenly among depositors who demand consumption.

The deposit contract can thus be described by:

$$c_1 = \min \left\{ r, \frac{M}{M - z} \right\}$$

$$c_2 = \max \left\{ 0, \frac{M - r(M - z)}{z} R \right\}$$

where c_1 and c_2 are the payoffs to early and late withdrawers respectively, and z is the number of depositors who choose to withdraw late.

There are two symmetric Nash equilibria with the demand deposit contract. One is the non-run equilibrium, which occurs when each impatient agent withdraws at date 1, while each patient agent anticipates all other patient agents will withdraw at date 2, and thus also waits until date 2. At this equilibrium, the optimal risk sharing allocation is achieved if r is set to c_1^*. There is also another equilibrium (the run equilibrium) when each patient agent assumes all other patient agents withdraw at date 1, and thus also finds it optimal to withdraw at date 1. In this case, all consumers end up with 1 unit of consumption. The expectation or belief of the depositors thus plays a critical role in determining the equilibrium outcome.

2.3 Experimental Evidence

To carry out the experiments, we create in the laboratory an environment similar to the settings in the Diamond and Dybvig (1983) model. Our main purpose is to examine whether banks runs can be the result of pure coordination failures, and if yes, under what conditions and what factors affect the severity of runs. Our hypothesis is that patient agents' expectations and actions will depend on how difficult the coordination task is. We can determine the difficulty level of the coordination task by asking the question: what fraction of the depositors have to coordinate to withdraw late so that the rate of return of withdrawing late is larger than that of withdrawing early? We call this fraction the coordination parameter and denote it by η. η is solved in two steps:

(i) Solve for the z that equates the payoffs to early and late withdrawers:

$$r = \frac{M - (M - z)r}{z} R$$

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Table 2.1: Parameters in the Experiments

Situation	0	1	2	3	4	5	6	7
r	1.43	1.05	1.11	1.18	1.33	1.54	1.67	1.82
η	0.60	0.10	0.20	0.31	0.50	0.70	0.80	0.90
Period (Session 1 & 2)	-9-0	1-10	11-20	21-30	31-40	41-50	51-60	61-70
Period (Session 3 & 4)	-9-0	61-70	51-60	41-50	31-40	21-30	11-20	1-10

and denote it by z^*. We have

$$z^* = \frac{MR(r-1)}{r(R-1)}.$$

(ii) Divide z^* by N to get η :

$$\eta = \frac{z^*}{N} = \frac{R(r-1)}{r(R-1)}\frac{M}{N}.$$

For the experiments, we set $M = N = 10$, $R = 2$ and try 8 different values of η (or equivalently, r).

The experiments are programmed and conducted in computer labs with the software z-Tree (Fischbacher 2007). In each session, 10 subjects (enrolled from graduate and fourth year undergraduate economics classes) are each assigned a computer terminal through which they can input their decisions. Instructions are handed out at the beginning of the experiment.[7] Subjects are given plenty of time to read the instructions and ask questions. No communication between subjects is allowed during the experiments.

Each session of experiment consists of 8 situations (each characterized by a different value of r or η), each of which lasts for 10 periods. Every subject starts each period with 1 unit of experimental dollar (ED) in the bank, and makes decision to withdraw or leave money in the bank. To facilitate decision making, we provide subjects with payoff tables (for each situation) which list the payoff that the individual will get if he/she chooses to withdraw early or leave money in the bank if $n = 1 \sim 9$ of the other 9 subjects choose to withdraw early. The aim is to liberate the subjects from calculation and concentrate on forming beliefs and playing the coordination game. After all subjects make their withdrawing decisions, the total number of withdrawers and payoff for each subject are calculated as specified in Section 2. Each subject is presented the history of his/her own actions, payoffs, and cumulative payoffs for the current and all previous periods (measured in EDs). Subjects are reminded with a salient message on the computer screen when r is changed so

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[7]Refer to Appendix I for the experimental instructions.

Table 2.2: Experimental Statistics

η	μ_{UIBE1}	μ_{SFU1}	σ_{UIBE1}	σ_{SFU1}	μ_{UIBE2}	μ_{SFU2}	σ_{UIBE2}	σ_{SFU2}
0.1	10.0	9.8	0.0000	0.4000	10.0	10.0	0.0000	0.0000
0.2	9.7	9.7	0.4538	0.4583	9.9	9.9	0.3000	0.3000
0.3	9.9	9.7	0.3000	0.4583	9.7	9.1	0.6403	0.3000
0.5	9.3	9.6	0.6403	0.4899	7.2	8.9	1.2490	0.3000
0.7	7.5	7.5	0.8062	2.5475	0.4	7.6	0.4899	0.9165
0.8	1.2	1.2	0.9798	1.1358	0.5	1.3	0.5000	0.7810
0.9	0.2	0.2	0.4000	0.4000	0.6	2.1	0.4899	0.9434

that they will refer to the correct payoff table.

The first 10 periods are for trial, so that the subjects can get familiar with their task. After the trial, we give subjects time to ask more questions. When there are no more questions, we start the formal experiment. Subjects are paid only for the formal experiment.

After the experiment, the total payoff that each subject earns is converted into cash. The conversion rate at SFU is 1 $ED = 0.2\ CAD$, and UIBE is 1 $ED = 0.8\ RMB$.

We have run four sessions of experiments so far: two with increasing order of the coordination parameter η, two with decreasing order of η. For each ordering, one session was run at Simon Fraser University (SFU), and one at the University of International Business and Economics (UIBE). Table 2 lists the mean and standard deviation of the number of late withdrawals. Figure 1 plots the path of the number of late withdrawals.

Figure 2.1: Experimental Results
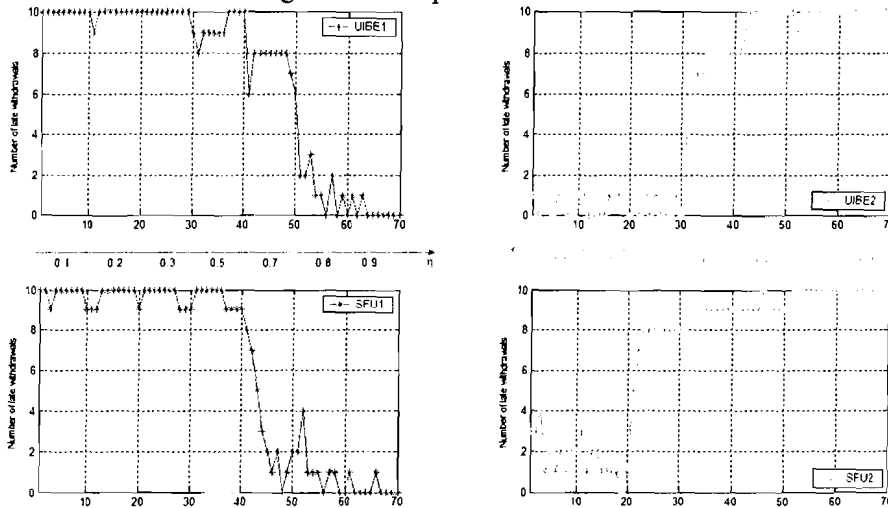
We make the following observations about the results.

1. There is more coordination when the coordination task is easier. Note the downward trend in the two experiments (left panel) with increasing η and the upward trend in the two experiments (right panel) with decreasing η.

2. When the coordination task is easy (when η is equal to 0.1, 0.2, 0.3 and 0.5), the economy converges to the non-run equilibrium; when the coordination task is difficult (when η is 0.8 and 0.9), the economy converges to the run equilibrium.

3. When $\eta = 0.7$, the experimental results are very different across the four sessions. UIBE1 starts with 6 coordinations, and goes up to 8 and back to 7 and 6. SFU1 starts from 8 and goes down to 0, 1, 2. UIBE2 starts from 1 (which might be due to the bad coordination history in the first 20 rounds) and gets stuck at 1 or 2. SFU2 starts with 5 and goes up to 8. It looks like that $\eta = 0.7$ serves as a watershed for coordination. When $\eta < 0.7$, most subjects perceive that it is pretty easy to coordinate and choose to leave money with the bank. When $\eta > 0.7$, most subjects perceive that coordination is difficult and choose to withdraw early. When $\eta = 0.7$, the consensus breaks down and a lot of things can happen.

2.4 Learning

Learning has been introduced into many rational expectation models with multiple equilibria as an equilibrium selecting mechanism, and has greatly contributed to the understanding of the models (refer to Arifovic, 2000 for a survey of the applications of evolutionary algorithms in macroeconomic models). Temzelides (1997) introduces imitation-based learning into a repeated version of the Diamond and Dybvig (1983) model in which depositors play myopic best responses following the action that has performed better in last period (imitation), and experiment with a small exogenous probability to play the two actions with equal probability. Temzelides (1997) shows that when the probability of experimentation approaches zero, the economy stays at the non-run equilibrium with probability one if and only if withdrawing late is risk dominant, or when less than $1/2$ of patient depositors are required to withdraw late so that withdrawing late offers higher return than withdrawing early, or when $\eta < 1/2$.

We modify the learning algorithm specified in Temzelides (1997) by introducing 'directed' experimentation. First, we allow the experimentation rate $\delta \in [0,1]$ to be a func-

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tion of the coordination parameter η, and $\delta(\eta)$ has the properties $\delta(0) = \delta(1) = 0$ and $\partial^2\delta/\partial\eta^2 < 0$. I.e., agents do not experiment if withdrawing early or late is the dominant strategy irrespective of the actions by other agents, and the probability of experimentation is higher for intermediate values of η. Second, instead of modelling agents as playing the two actions with equal probability during experimentation, we allow the probability of playing each action to be a function of the coordination parameter η and the number of late withdrawals in the previous period z_{-1}. Let $\theta(\eta, z_{-1}) \in [0,1]$ be the probability of withdrawing early during experimentation. We assume that $\theta(0, z_{-1}) = 0$, $\theta(1,0) = 1$, $\partial\theta/\partial\eta > 0$, and $\partial\theta/\partial z_{-1} < 0$. I.e., agents always withdraw late when $\eta = 0$, always withdraw early when $\eta = 1$ and everybody has withdrawn early in the previous period, and the probability of withdrawing early increases with the difficult level of the coordination task and the fraction of early withdrawers in the previous period.

2.4.1 Simulations

In this subsection, we present some simulation results from the learning algorithm.

We adopt the following functional forms for $\delta(\eta)$ and $\theta(\eta, z_{-1})$:

$$\delta(\eta) = \frac{\rho_0}{\rho_1^{\rho_1}(1-\rho_1)^{1-\rho_1}}\eta^{\rho_1}(1-\eta)^{1-\rho_1};$$
$$\theta(\eta, z_{-1}) = \eta^{\rho_2}\rho_3^{-z_{-1}/N};$$

where ρ_0, $\rho_1 \in (0,1)$, and ρ_2, $\rho_3 \geq 1$. For the simulations, we use $\rho_0 = 0.4$, $\rho_1 = 0.7$, $\rho_2 = 3$, and $\rho_3 = 2$. Figure 2 plots $\delta(\eta)$ and $\theta(\eta, z_{-1})$.

We use the parameter values as outlined in Table 1 and try 9 situations characterized by 9 η values from 0.1 to 0.9 with step 0.1. For each situation, we try 11 different starts by varying z_1 from 0 to 10, and 10 different seeds to generate random numbers. Each simulation is run for 100 periods.

The main findings are as follows (refer to Table 3 and the simulation figures in Appendix III).

1. The initial condition (represented by z_1) affects the performance of the economy. There is on average more coordination (higher average value of z) with more optimistic start (higher z_1).

2. The difficulty level of the coordination task affects the performance of the economy.

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Figure 2.2: Learning Specification

Learning specification - Prob. of experimentation   δ

Learning specification - Prob. of withdrawing early during experimentation   θ
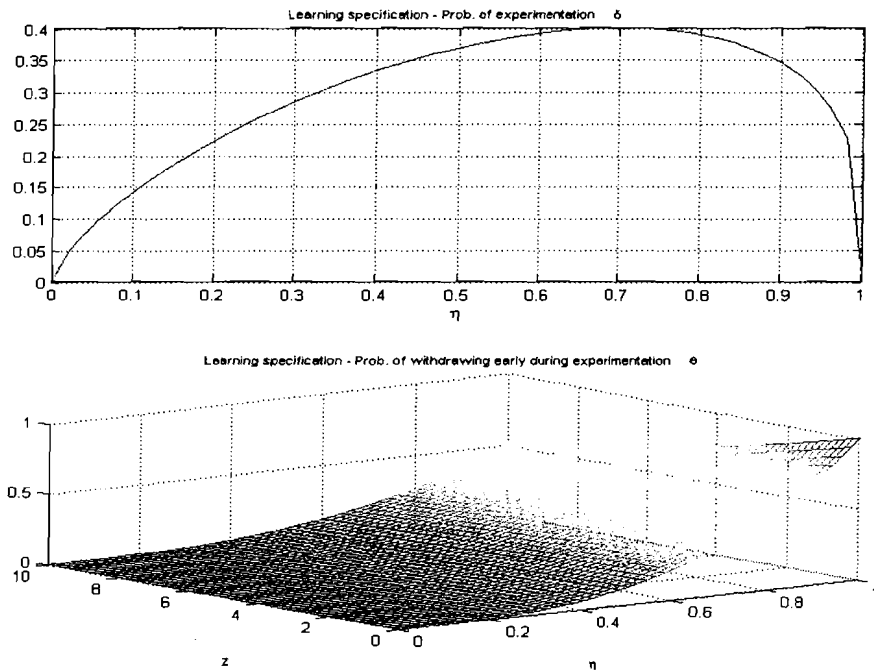
(a) There is on average more coordination (higher average value of z) when the coordination task is easier (lower η).

(b) The economy tends to converge to either the 'bad' or 'good' coordination state, and which state the economy converges to depends on η as follows. For $\eta = 0.1 \sim 0.6$, if the economy has an optimistic start and everyone thinks that withdrawing late will get higher payoff, and thus $z_1 = N = 10$, the pattern will persist into the future. If the economy has a pessimistic start with $z_1 = 0$, it will reverse to the non-run equilibrium. The smaller the value of η, the closer the economy sticks to the non-run state. For $\eta = 0.8 \sim 0.9$, a pessimistic start will sustain itself, while an optimistic start will be reversed. By the end, the economy stays in the neighborhood of the run equilibrium. The higher the value of η, the closer the economy stays to the run equilibrium. For $\eta = 0.7$, very different things can happen. Pessimism or optimism might persist or be reversed. Refer to the Appendix III for the simulation figures.

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Table 2.3: Average Number of Late Withdrawals

z_1 η	0	1	2	3	4	5	6	7	8	9	10
0.1	9.808	9.910	9.920	9.930	9.940	9.950	9.960	9.970	9.980	9.990	10.000
0.2	9.777	9.787	9.914	9.924	9.934	9.944	9.954	9.964	9.974	9.984	9.994
0.3	9.702	9.712	9.722	9.884	9.884	9.904	9.914	9.924	9.934	9.944	9.954
0.4	9.651	9.661	9.671	9.681	9.681	9.840	9.850	9.860	9.870	9.881	9.891
0.5	9.383	9.393	9.404	9.416	9.416	9.705	9.716	9.726	9.736	9.747	9.757
0.6	8.423	8.433	8.443	8.453	8.453	8.474	9.490	9.510	9.522	9.532	9.543
0.7	5.582	5.592	5.602	5.612	5.612	5.636	5.646	8.671	8.681	8.691	8.701
0.8	2.155	2.166	2.176	2.187	2.187	2.208	2.218	2.228	2.795	2.806	2.819
0.9	1.122	1.134	1.144	1.159	1.159	1.183	1.194	1.204	1.216	1.402	1.412

Note: the average is taken over 10 random number generating seeds.

2.4.2 Comparison with the Experimental Results

An comparison between the learning simulations and the experimental results shows that the learning algorithm constitutes a reasonable approximation to the experimental results. For example, the level of coordination is higher when the coordination task is easier. The economy tends to converge to the run equilibrium when the coordination task is more difficult and to the non-run equilibrium when the coordination task is easy. When $\eta = 0.7$, different things can happen. Comparing with the learning algorithm in Temzelides (1997), the learning algorithm with 'directed' experimentation captures the experimental data better. For example, 'directed' experimentation correctly captures 0.7 as the cut-off point.

2.5 Conclusion and Future Research

Experiments with human subjects show that bank runs can happen as the result of pure coordination failures, but only when the coordination task is hard. For intermediate values of the coordination parameter, the performance of the economy is hard to predict. We also find that the simulations from the learning algorithm can closely replicate the experimental results and thus conclude that the learning consists of a reasonable explanation about the behavior of human subjects in the laboratory.

In the near future, we will run more experiments with the basic Diamond and Dybvig (1983) setting. The case with $\eta = 0.7$ needs further investigation. It would be interesting to see if 'sunspots' can play a role in determining which equilibrium will be selected in that case. After that, we will carry out some experiments to examine the effectiveness of

different policies to prevent bank runs such as the suspension of payments and deposit insurance. Finally, we will add a 'sunspot' ingredient to the case with $\eta = 0.7$ and check if subjects will respond to variables unrelated to fundamentals. Duffy and Fisher (2005) provide the first direct evidence of sunspots in the laboratory. They use a very simple market design with two different demand and supply curves that intersect near two prices. Subjects' marginal costs and valuations depend upon the end-of-period median price, which is determined solely by subjects' actions during the trading period. Duffy and Fisher (2005) purposefully make the two equilibria not Pareto comparable because they suspect that if one were Pareto dominant, subjects might coordinate on it as a focal point for their expectations. They show that when information is highly centralized, as in a call market, subjects use realizations of a sunspot variable as a device for coordinating on low or high price equilibria. In the Diamond and Dybvig (1983) model, the non-run equilibrium is always the Pareto dominant equilibrium. Our experiments with the model show that when $\eta = 0.7$, subjects lose consensus whether to withdraw early or late, and there is a possibility that they will rely on sunspots to coordinate their actions. We plan to follow Duffy and Fisher (2005) to introduce a sunspot variable. After several trial periods, an announcement will be made at the beginning of each of the formal periods. The announcement will be either 'the forecast is that many people will withdraw early' or 'the forecast is that only a small number of people will withdraw early.' This forecast will be determined by flipping a coin. Subjects will be told that the forecasts based on the coin flips may be wrong and their payoffs are determined solely by their actions in that period. This research is designed to provide experimental evidence for sunspot behavior in games with multiple equilibria that can be Pareto-ranked.

2.6 References

Allen, Franklin, and Douglas Gale (1998), "Optimal Financial Crises," *Journal of Finance*, 53(4), 1245-1284.

Alonso (1996), Irasema, "On Avoiding Bank Runs," *Journal of Monetary Economics*, 37(1), 73-87.

Arifovic, Jasmina (2000), "Evolutionary Algorithms in Macroeconomic Models," *Macroeconomic Dynamics*, 4, 373-414.

Cabrales, Antonio, Rosemarie Nagel and Roc Armenter (2003), "Equilibrium Selection through Incomplete Information in Coordination Games: an Experimental Study," working paper.

Boyd, John H., Pedro Gomis, Sungkyu Kwak, and Bruce D. Smith (2001), "A User's Guide to Banking Crises," Manuscript.

Champ, Bruce, Smith, D. Bruce, and Stephen D. Williamson (1996), "Currency Elasticity and Banking Panics: Theory and Evidence," *Canadian Journal of Economics*, 29(4), 828-864.

Chari, V.V. and Ravi Jagannathan (1988), "Banking Panics, Information, and Rational Expectations Equilibrium," Journal of Finance, 43, 749-761.

Chen Yehning (1999), "Banking Panics: The Role of the First-Come, First-Served Rule and information Externalities," *Journal of Political Economy*, 107(5), 946-968.

Cooper, Russell, and Thomas W. Ross (1998), "Bank Runs: Liquidity Costs and Investment Distortions," *Journal of Monetary Economics*, 41, 27-38.

Diamond, Douglas W., and Philip H. Dybvig (1983), "Bank Runs, Deposit insurance, and Liquidity," *Journal of Political Economy*, 91, 401-419.

Duffy, John and Eric Fisher (2005), "Sunspots in the Laboratory", *American Economic Review*, 95(3), 510-529.

Fischbacher, Urs (2007), "z-Tree: Zurich Toolbox for Ready-made Economic Experiments," *Experimental Economics*, 10(2), 171-178.

Garratt, Rod and Todd Keister (2006), "Bank run: an experimental study," working paper, University of California.

Goldstein, Italy, and Ady Pauzner (2005), "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 60(3), 1293-1327.

Gomis-Porqueras, Pere and Bruce Smith (2006), "The seasonality of banking failures during the late National Banking Era," *Canadian Journal of Economics*, 39(1), 296-319.

Gorton, Gary (1988), 'Banking Panics and Business Cycles,' *Oxford Economic Papers*, 40, 751-781.

Gu, Chao (2007), "Herding and Bank Runs," working paper, Cornell University.

Heinemann, Frank, Rosemarie Nagel and Peter Ockenfels (2004), "The Theory of Global Games on Test: Experimental Analysis of Coordination Games with Public and Private Information," *Econometrica*, 72, 1583-99.

Jacklin Charles J. and Sudipto Bhattacharya (1988), "Distinguishing Panics and Information-Based Bank Runs: Welfare and Policy Implications," *Journal of Political Economy*, 96, No.3. 568-592.

Klos, Alexander and Norbert Sträter (2007), "Global Games and Demand-Deposit Contracts: an Experimental Study of Bank Runs," working paper, University of Münster.

Loewy, B. Michael (1991), "The Macroeconomic Effects of Bank Runs: An Equilibrium Analysis," *Journal of Financial Intermediation*, 1, 242-256.

Loewy, B. Michael (1998), "Information-Based Bank Runs in a Monetary Economy," *Journal of Macroeconomics*, 20, 681-702

Loewy, B. Michael (2003), "To Furnish an Elastic Currency: Banking, Aggregate Risk, and Welfare," *Topics in Macroeconomics*, 2003, 3(1), Article 3

Jiang, Janet Hua (2007), "Banking Crisis in Monetary Economies," *Canadian Journal of Economics*, forthcoming.

Madies, Philippe (2006), "An Experimental Exploration of Self-fulfilling Bank Panics: Their Occurrence, Persistence and Prevention," *Journal of Business*, 79, 1836-1866.

Morris, Stephen, and Hyunsong Shin (2000), "Rethinking Multiple Equilibria in Macroeconomic Modeling," *NBER Macro Annuals*, 2000.

Peck, James, and Karl Shell (2003), "Equilibrium Bank Runs", *Journal of Political Economy*, 111, 103-123.

Schotter, Andrew and Tanju Yorulmazer (2005), "On the Severity of Bank Runs: an Experimental Study," working paper, New York University.

Schumacher, Liliana (2000), "Bank Runs and Currency Run in a System without a Safety Net: Argentina and the 'Tequila' Shock," *Journal of Monetary Economics*, 46(1), 257-277.

Smith, D. Bruce (2003), "Taking Intermediation Seriously," *Journal of Money, Credit, and Banking*, 35, 1319-1357.

Temzelides, Theodosios (1997), "Evolution, Coordination and Banking Panics," *Journal of Monetary Economics*, 40, 163-183.

Van Huyck, John B., Battalio, Raymond C., and Richard O. Beil (1990), "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure," *American Economic Review*, 180(1), 234-48.

Van Huyck, John B., Battalio, Raymond C., and Richard O. Beil (2001), "Optimization Incentives and Coordination Failure in Laboratory Stag Hunt Games," *Econometrica*, 2001, 69(3), 749-64.

Van Huyck, John B., Battalio, Raymond C., and Richard O. Beil (1991), "Strategic Uncertainty, Equilibrium Selection, and Coordination Failure in Average Opinion Games," *Quarterly Journal of Economics*, 1991, 106(3), 885-910.

Waldo, G. Douglas (1985), "Bank Runs, the Deposit-Currency Ratio and the Interest Rate," *Journal of Monetary Economics*, 15, 269-277.

Yorulmazer, Tanju (2003), "Herd Behavior, Bank Runs and Information Disclosure," working paper, New York University.

2.7 Appendix I: Experimental Instructions

The instructions are designed for experiments featuring an increasing order of the coordination parameter.

This experiment has been designed to study decision-making behavior in groups. During today's session, you will earn income in an experimental currency called experimental dollars or for short ED. At the end of the session, the currency will be converted into dollars. 1 ED corresponds to 0.2 dollar. You will be paid in cash. The participants may earn different amounts of money in this experiments because each participant's earnings are based partly on his/her decisions and partly on the decisions of the other group members. If you follow the instructions carefully and make good decisions, you may earn a considerable amount of money. Therefore, it is important that you do your best.

Please read these instructions very carefully. You will be required to complete a quiz, in order to demonstrate that you have a complete and accurate understanding of these instructions. After you have completed the quiz, the administrator will check your answers and discuss with you any questions that have been answered incorrectly.

Description of the task

You and 9 **other people** (there are 10 of you altogether) have 1 ED deposited in an experimental bank. You must decide whether to withdraw your 1 ED or leave it deposited in the bank. The bank promises to pay $r > 1$ EDs to each withdrawer. The money that remains in the bank will earn interest rate $R > r$. At the end, the bank will divide what it has evenly among people who choose to leave money in the bank. Note that if too many people desire to withdraw, the bank may not be able to fulfill the promise to pay r to each withdrawer(remember that $r > 1$). In that case, the bank will divide the 10 EDs evenly among the withdrawers and those who choose to wait will get nothing.

Your payoffs depend on your own decision and the decisions of the other 9 people in the group. Specifically, how much you receive if you make a withdrawal request or how much you earn by leaving your money deposited depends on how many people in the group place withdrawing requests.

Let e be the number of people who request withdrawals.

The payoff (in ED) to those who withdraw will be:

$$\min\{r, \tfrac{10}{e}\} \text{ or the minimum of } r \text{ and } \tfrac{10}{e}.$$

The payoff (in ED) to those who leave money in the bank will be:

$$\max\{0, \tfrac{10-er}{10-e}R\} \text{ or the maximum of 0 and } \tfrac{10-er}{10-e}R.$$

In the experiment, R will be fixed at 2.0. r will take 8 values 1.43, 1.05, 1.11, 1.18, 1.33, 1.54, 1.67 and 1.82. To facilitate your decision, the payoff tables $0 \sim 7$ list the payoffs if n **of the other 9 subjects** request to withdraw (n is unknown at the time when you make the withdrawing decision - it can be any integer from 1 to 9 - and you have to guess it) for the 8 situations. Table 0 will be used for practice, and table $1 \sim 7$ will be used for the formal experiment. Let's look at two examples:

Example 1. Use table 0 where $r = 1.43$. Suppose that 3 other subjects request withdrawals. Your payoff will be 1.43 if you request to withdraw (the number of withdrawing requests e will be $3 + 1 = 4$, and your payoff $= \min\{r = 1.43, \tfrac{10}{e} = \tfrac{10}{4} = 2.5\} = 1.43$). If you choose to leave money in the bank, your payoff will be 1.63 (the number of withdrawing requests $e = 3$, and your payoff $= \max\{0, \tfrac{10-er}{10-e}R = \tfrac{10-3\times1.43}{10-3}2.0 = 1.63\} = 1.63$).

Example 2. Use table 7 where $r = 1.82$. Suppose that 6 of other subjects request withdrawals. Your payoff for withdrawing will be $\min\{1.82, \tfrac{10}{e} = \tfrac{10}{7} = 1.43\} = 1.43$ and your payoff for leaving money in the bank will be $\max\{0, \tfrac{10-6\times1.82}{10-6}2.0 = -0.46\} = 0$.

Now let's take a closer look at the tables. Notice the following features of tables:

- The payoff to withdrawing is more stable, it is fixed at r for most of the time and is bounded below by 1.

- The payoff to leaving money in the bank is more volatile. When n - the number of other people requesting withdrawals - is small, leaving money in the bank offers higher payoff than withdrawing. The opposite happens when n is large enough. For your convenience, bold face is used to identify the threshold value of n at which withdrawing starts to pay equal to or higher than leaving money in the bank.

- The threshold values of n varies from table to table. The general pattern is that it is smaller when r is bigger.

Note you are not allowed to ask people what they will do and you will not be informed about the other people's decisions. You must guess what other people will do - how many of the other 9 people will withdraw - and act accordingly.

Procedure

You will perform the task described above for 70 times. Each time is called a period. Each period is completely separate. I.e., you start each period with 1 in the experimental bank. You will keep the money you earn in every period. At the end of each round, the computer screen will show you your decision and payment for that period. Information for earlier periods and your cumulative payment is also provided.

Note that the payment scheme changes every 10 periods, so please use the **correct** payoff table:

Table 1 for period 1-10,

Table 2 for period 11-20,

Table 3 for period 21-30,

Table 4 for period 31-40,

Table 5 for period 41-50,

Table 6 for period 51-60, and

Table 7 for period 61-70.

You will be reminded when you need to change to a new table, pay attention to the message.

Beside the 70 paid periods, you will also be given 10 trials to practice and get familiar with your task. You will not be paid for the trials. Please use table 0 for the trials. After the practice periods, you will have a chance to ask more questions before the experiment formally start. You will be paid for each formal period.

Computer instructions

You will see three types of screens: the decision screen, the payoff screen and the waiting screen.

Your withdrawing decisions will be made on the decision screen as shown above. You can choose to withdraw money or leave money in the bank by pushing one of the two buttons. **Note** that your decision will be final once you press the buttons, so be careful when you make the move. The header provides information about what period you are in and the time remaining to make a decision. After the time limit is reached, you will be given a flashing reminder 'please reach a decision!'. The screen also shows which table you should look at.

After all subjects input their decisions, a payoff screen will appear as shown above. You

Figure 2.3: The Decision Screen

trial2   of   10   Remaining time [seconds]:

Now practicing, please use payoff table   0

Do you want to withdraw money or leave money in the bank? | withdraw money |
                                                             | leave money in the bank |
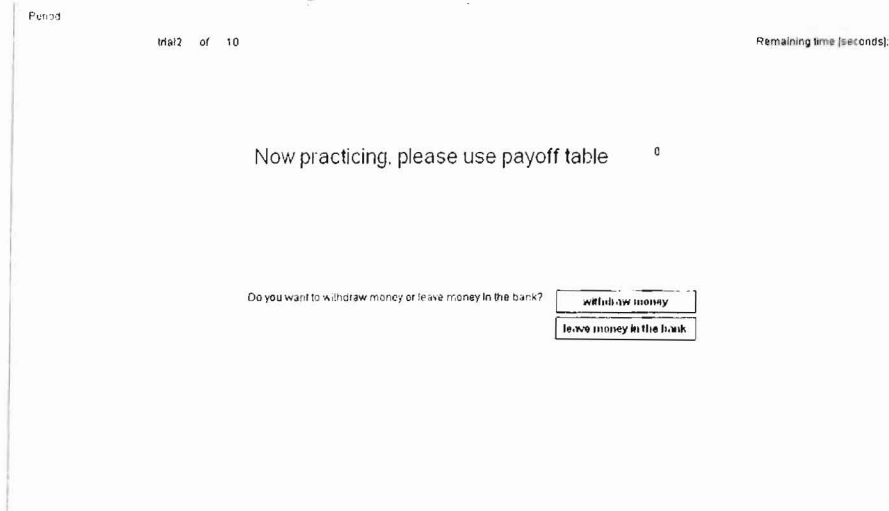
will see your decision and payoff in the current period. The history of your decisions and payoffs as well as your cumulative payoff are also provided. After finishing reading the information, push the 'continue' button to go to the next period. You will have 30 seconds to review the information before a new period starts.

You might see a waiting screen following the decision or payoff screens - this means that other people are still making decisions or reading the information, and you will need to wait until they finish to go to the next step.

Payment

At the end of the entire experiment, the supervisor will pay you in cash. Your earnings in dollars will be:

$$\text{total payoff in } ED \times 0.2.$$

Figure 2.4: The Payoff Screen

Period

trial1 of 10

Remaining time (seconds):

In this period, you decided to: withdraw

And your payment is: 1.11

Continue

Period	Decision	Payoff	Total payoff
-9	withdraw early	1.11	0.00

Figure 2.5: The Waiting Screen

Please wait until the experiment continues.

Table 2.4: Payoff Table 0 (for Practice), r=1.43

n	payoff if withdraw	payoff if leave money in the bank
0	1.43	2.00
1	1.43	1.90
2	1.43	1.79
3	1.43	1.63
4	1.43	1.43
5	1.43	1.14
6	1.43	0.71
7	1.25	0.00
8	1.11	0.00
9	1.00	0.00

Table 2.5: Payoff Table 1, r=1.05

n	payoff if withdraw	payoff if leave money in the bank
0	1.05	2.00
1	1.05	1.99
2	1.05	1.98
3	1.05	1.96
4	1.05	1.93
5	1.05	1.90
6	1.05	1.85
7	1.05	1.77
8	1.05	1.60
9	1.00	1.11

Table 2.6: Payoff Table 2, r=1.11

n	payoff if withdraw	payoff if leave money in the bank
0	1.11	2.00
1	1.11	1.98
2	1.11	1.94
3	1.11	1.91
4	1.11	1.85
5	1.11	1.78
6	1.11	1.67
7	1.11	1.49
8	1.11	1.12
9	1.00	0.02

Table 2.7: Payoff Table 3, r=1.18

n	payoff if withdraw	payoff if leave money in the bank
0	1.18	2.00
1	1.18	1.96
2	1.18	1.91
3	1.18	1.85
4	1.18	1.76
5	1.18	1.64
6	1.18	1.46
7	1.18	1.16
8	1.11	0.56
9	1.00	0.00

Table 2.8: Payoff Table 4, r=1.33

n	payoff if withdraw	payoff if leave money in the bank
0	1.33	2.00
1	1.33	1.93
2	1.33	1.84
3	1.33	1.72
4	1.33	1.56
5	1.33	1.34
6	1.33	1.01
7	1.25	0.46
8	1.11	0.00
9	1.00	0.00

Table 2.9: Payoff Table 5, r=1.54

n	payoff if withdraw	payoff if leave money in the bank
0	1.54	2.00
1	1.54	1.88
2	1.54	1.73
3	1.54	1.54
4	1.54	1.28
5	1.54	0.92
6	1.43	0.38
7	1.25	0.00
8	1.11	0.00
9	1.00	0.00

2.8 Appendix II: Individual Players' Actions

Here we graph each individual's actions. A dot on the upper line represents 'leave money in the bank' and a dot on the lower line represents 'withdraw early'. P1 ~P10 represent the ten players. The two numbers in the brackets represent the total payoff (in experimental dollars) and rank of the total payoff among the group of players.
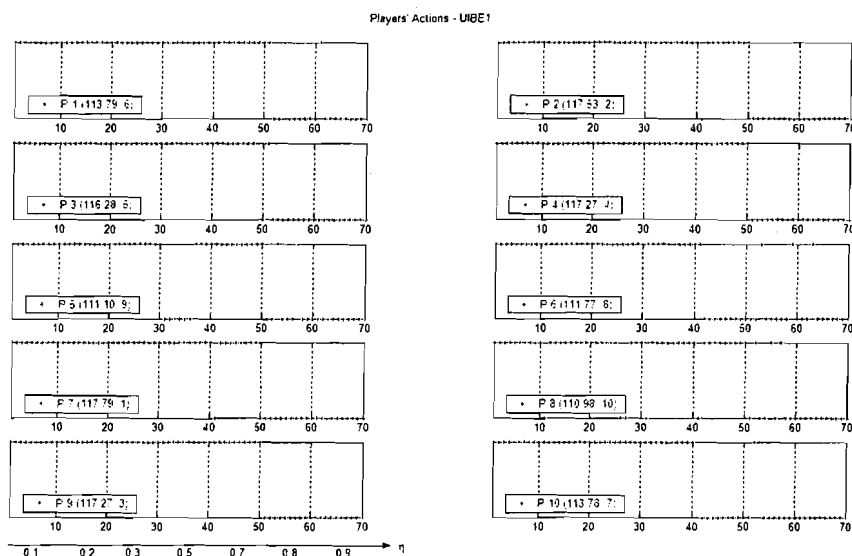
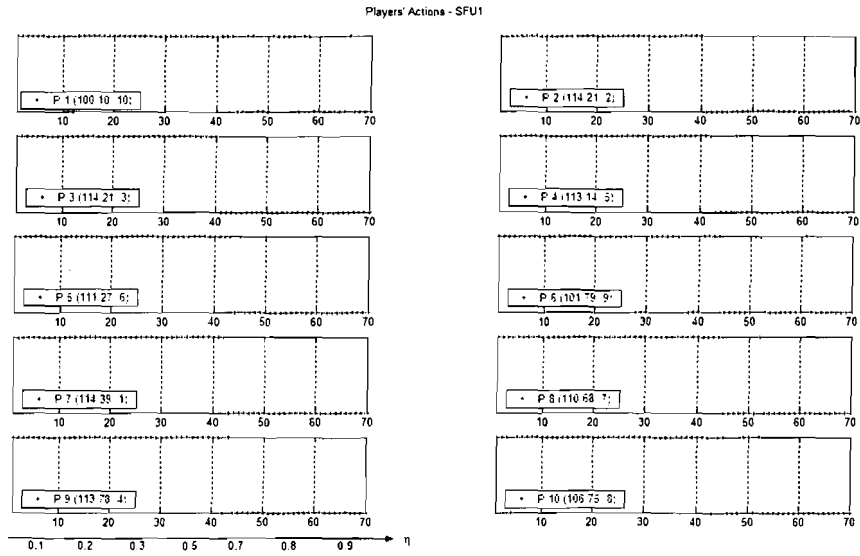Figure 2.6: Players' Actions - UIBE1

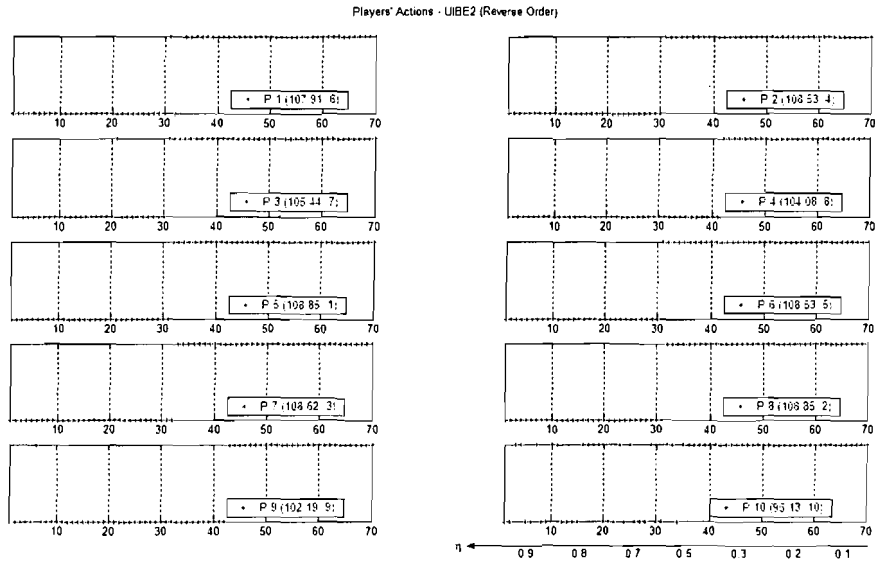Figure 2.7: Player's Actions - SFU1

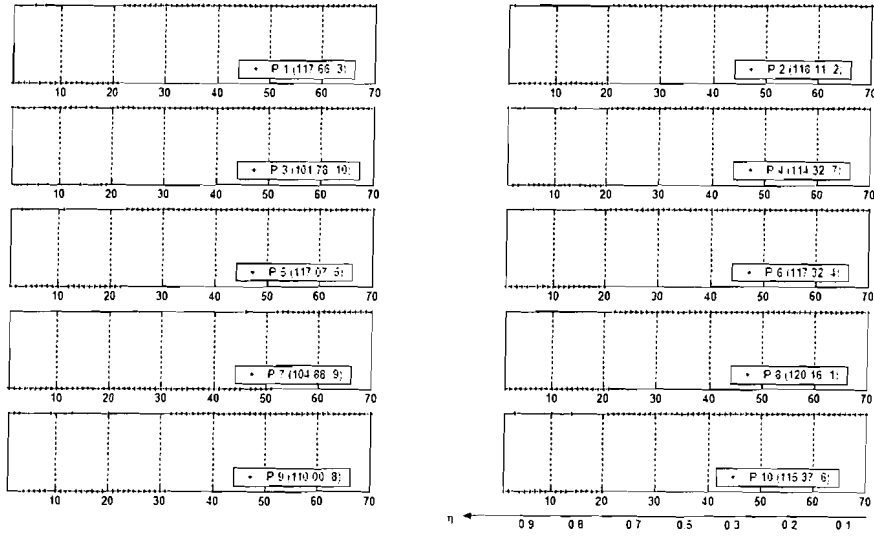Figure 2.8: Players' Actions - UIBE2

Figure 2.9: Players' Actions - SFU2

Players' Actions - SFU2 (Reverse Order)

2.9 Appendix III: Simulation Figures

The simulation results are similar with different seeds when η=0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 0.9, so we only plot the cases with seed=1 (for three starting values of z). When $\eta = 0.7$, different seeds give very different paths, and plot simulations with seed=1, 2, 3.
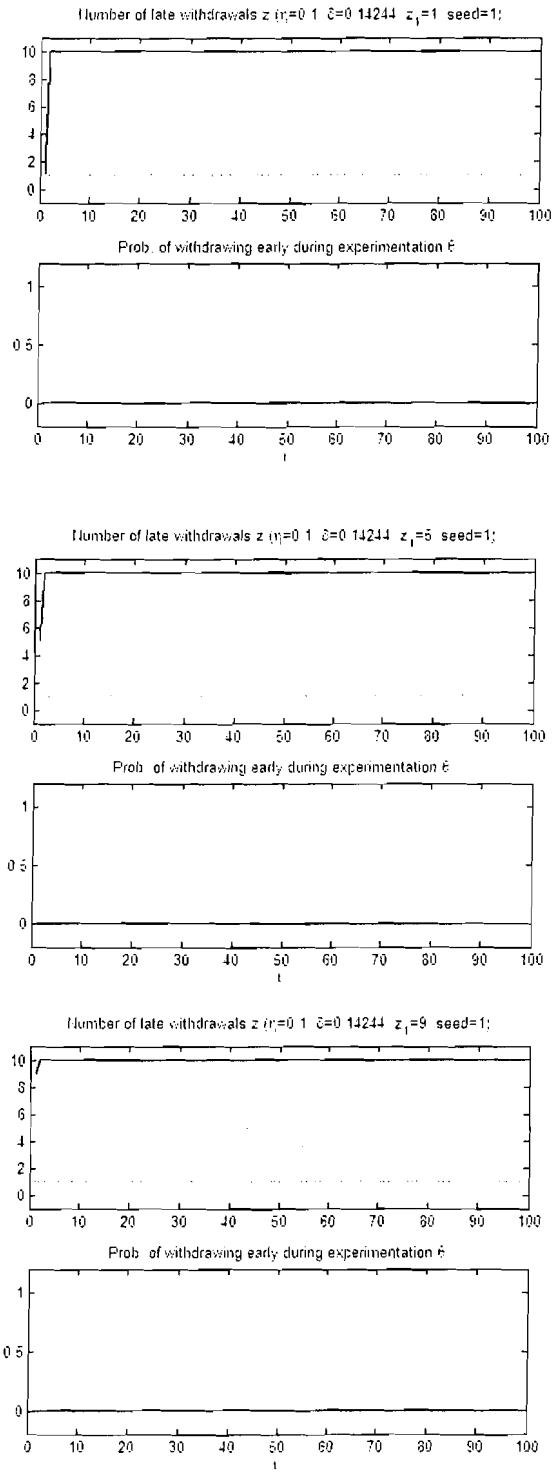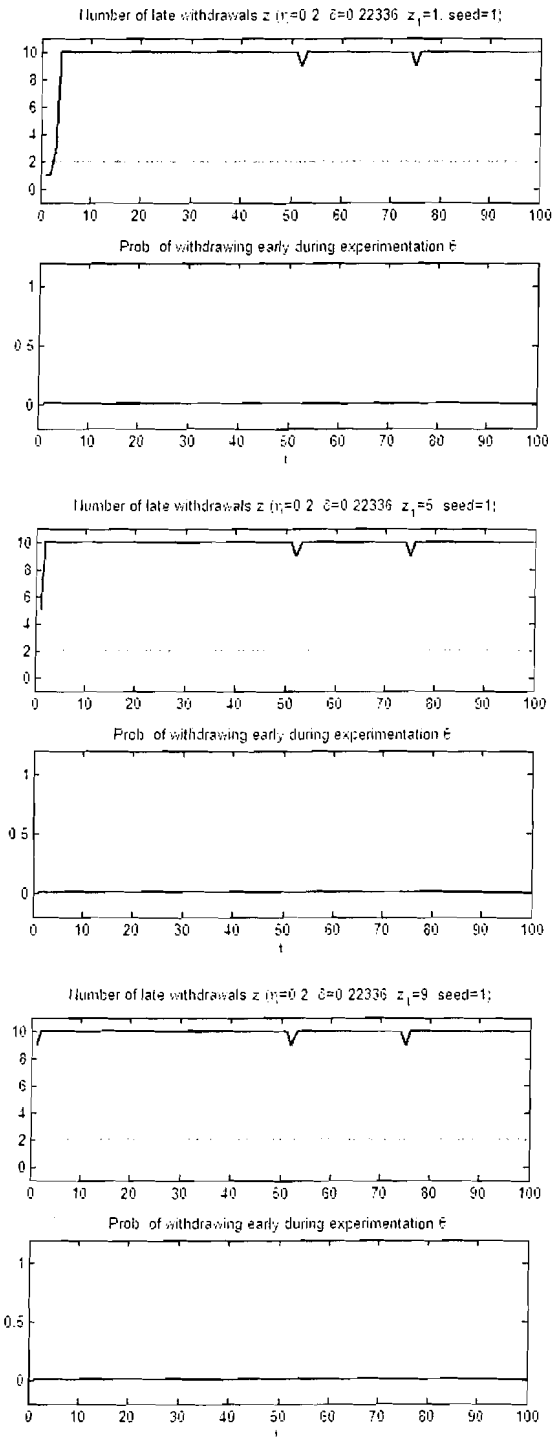
Figure 2.10: Simulation Path - $\eta = 0.1$

Number of late withdrawals z (η=0 1 $\bar{c}$=0 14244 z_1=1 seed=1)

Prob. of withdrawing early during experimentation ξ

Number of late withdrawals z (η=0 1 $\bar{c}$=0 14244 z_1=5 seed=1)

Prob. of withdrawing early during experimentation ξ

Number of late withdrawals z (η=0 1 $\bar{c}$=0 14244 z_1=9 seed=1)

Prob. of withdrawing early during experimentation ξ

56

Figure 2.11: Simulation Path - $\eta = 0.2$

Number of late withdrawals z (η=0.2 $\bar{c}$=0.22336 z_1=1. seed=1)

Prob of withdrawing early during experimentation ϵ

Number of late withdrawals z (η=0.2 $\bar{c}$=0.22336 z_1=5 seed=1)

Prob of withdrawing early during experimentation ϵ

Number of late withdrawals z (η=0.2 $\bar{c}$=0.22336 z_1=9 seed=1)

Prob of withdrawing early during experimentation ϵ
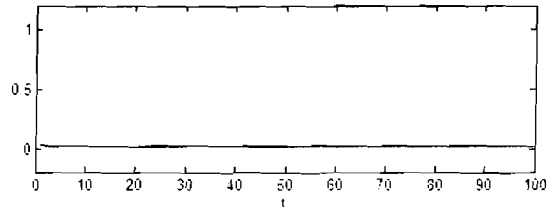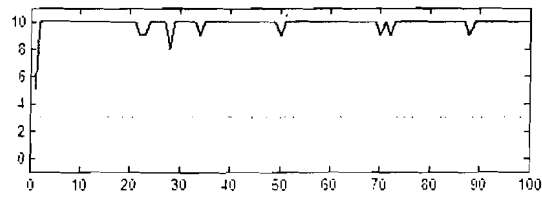
57

Figure 2.12: Simulation Path - $\eta = 0.3$

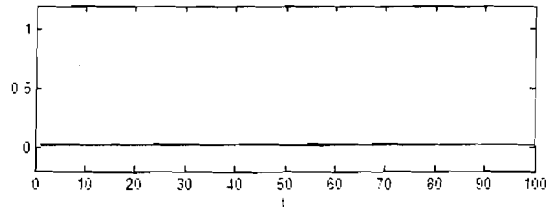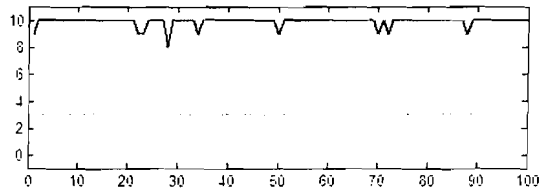Number of late withdrawals z (η=0.3 δ=0.28502 z_1=1 seed=1)
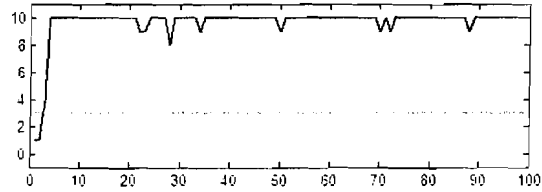
Prob of withdrawing early during experimentation θ

Number of late withdrawals z (η=0.3 δ=0.28502 z_1=5 seed=1)

Prob of withdrawing early during experimentation θ

Number of late withdrawals z (η=0.3 δ=0.28502 z_1=9 seed=1)

Prob of withdrawing early during experimentation θ

58

Figure 2.13: Simulation Path - $\eta = 0.4$

Number of late withdrawals z (η=0.4 $\tilde{c}$=0.33285 z_1=1 seed=1)

Prob. of withdrawing early during experimentation ε

Number of late withdrawals z (η=0.4 $\tilde{c}$=0.33285 z_1=5 seed=1)

Prob. of withdrawing early during experimentation ε
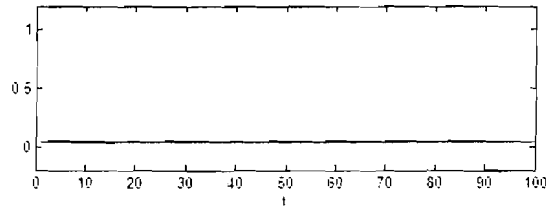
Number of late withdrawals z (η=0.4 $\tilde{c}$=0.33285 z_1=9 seed=1)

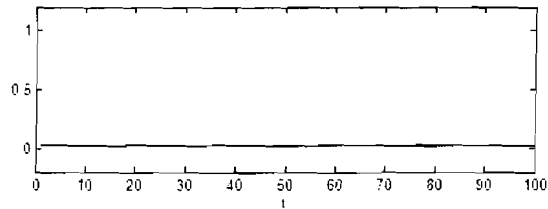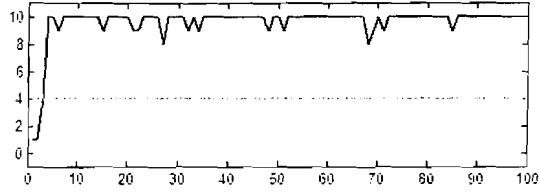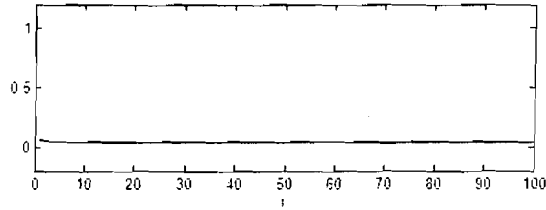Prob. of withdrawing early during experimentation ε

59
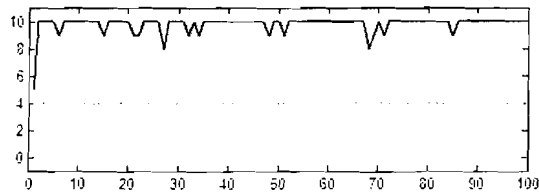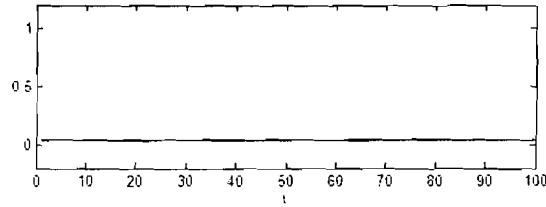
Figure 2.14: Simulation Path - $\eta = 0.5$

Number of late withdrawals z (η=0.5, δ=0.3684, z_1=1, seed=1)
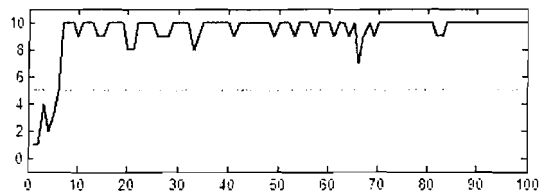
Prob. of withdrawing early during experimentation θ

Number of late withdrawals z (η=0.5, δ=0.3684, z_1=5, seed=1)

Prob. of withdrawing early during experimentation θ

Number of late withdrawals z (η=0.5, δ=0.3684, z_1=9, seed=1)

Prob. of withdrawing early during experimentation θ

60

Figure 2.15: Simulation Path - $\eta = 0.6$
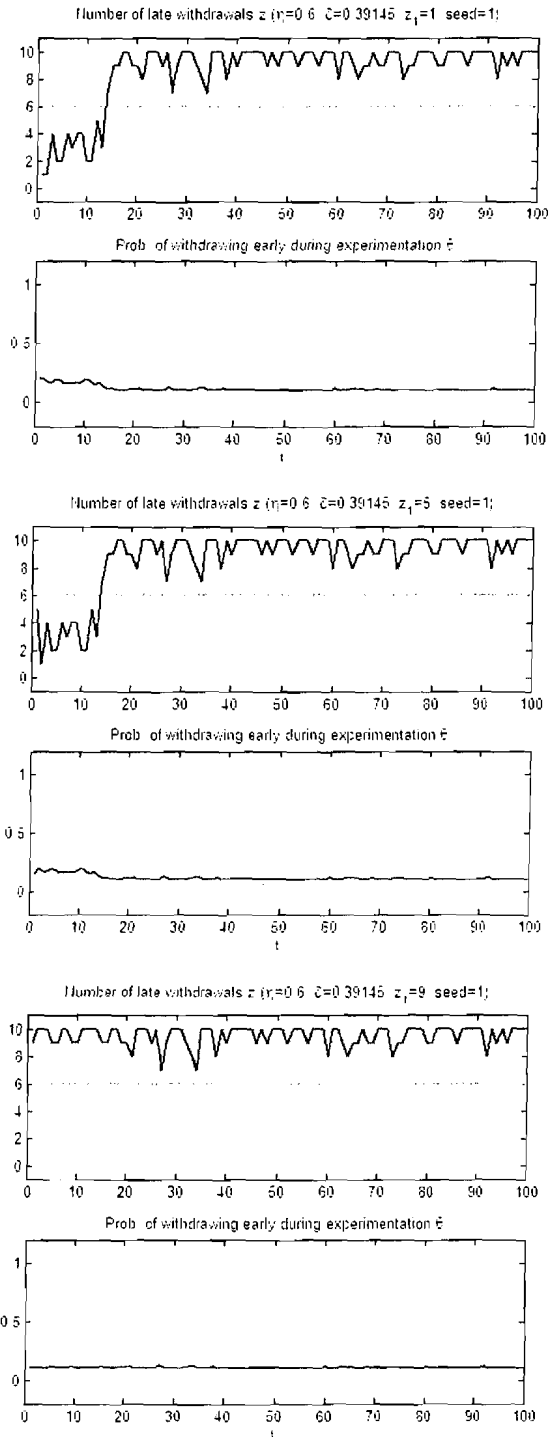
Figure 2.16: Simulation Path - $\eta = 0.7$, $z_1 = 1$

Number of late withdrawals z (η=0.7, $\tilde{c}$=0.4, z_1=1, seed=1)

Prob. of withdrawing early during experimentation $\tilde{\epsilon}$

Number of late withdrawals z (η=0.7, $\tilde{c}$=0.4, z_1=1, seed=2)

Prob. of withdrawing early during experimentation $\tilde{\epsilon}$

Number of late withdrawals z (η=0.7, $\tilde{c}$=0.4, z_1=1, seed=3)

Prob. of withdrawing early during experimentation $\tilde{\epsilon}$
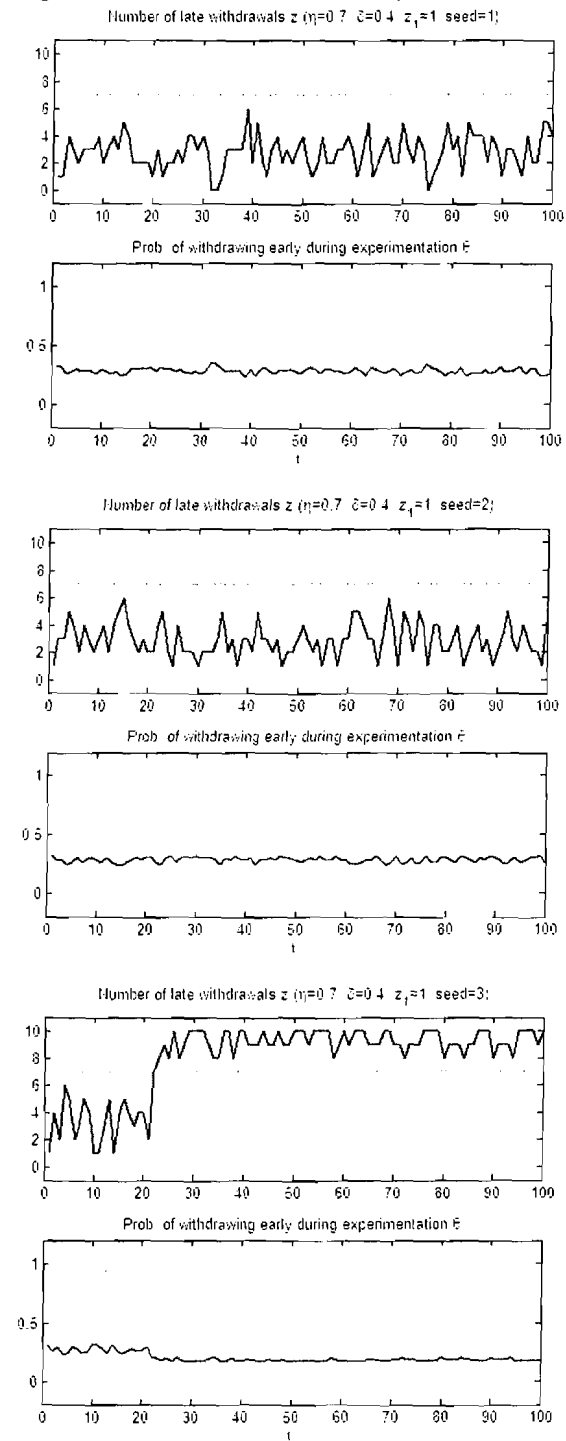
Figure 2.17: Simulation Path - $\eta = 0.7, z_1 = 5$

Number of late withdrawals z (η=0.7, c̃=0.4, z₁=5 seed=1)

Prob. of withdrawing early during experimentation e

Number of late withdrawals z (η=0.7, c̃=0.4, z₁=5 seed=2)

Prob. of withdrawing early during experimentation e

Number of late withdrawals z (η=0.7, c̃=0.4, z₁=5 seed=3)

Prob. of withdrawing early during experimentation e

63
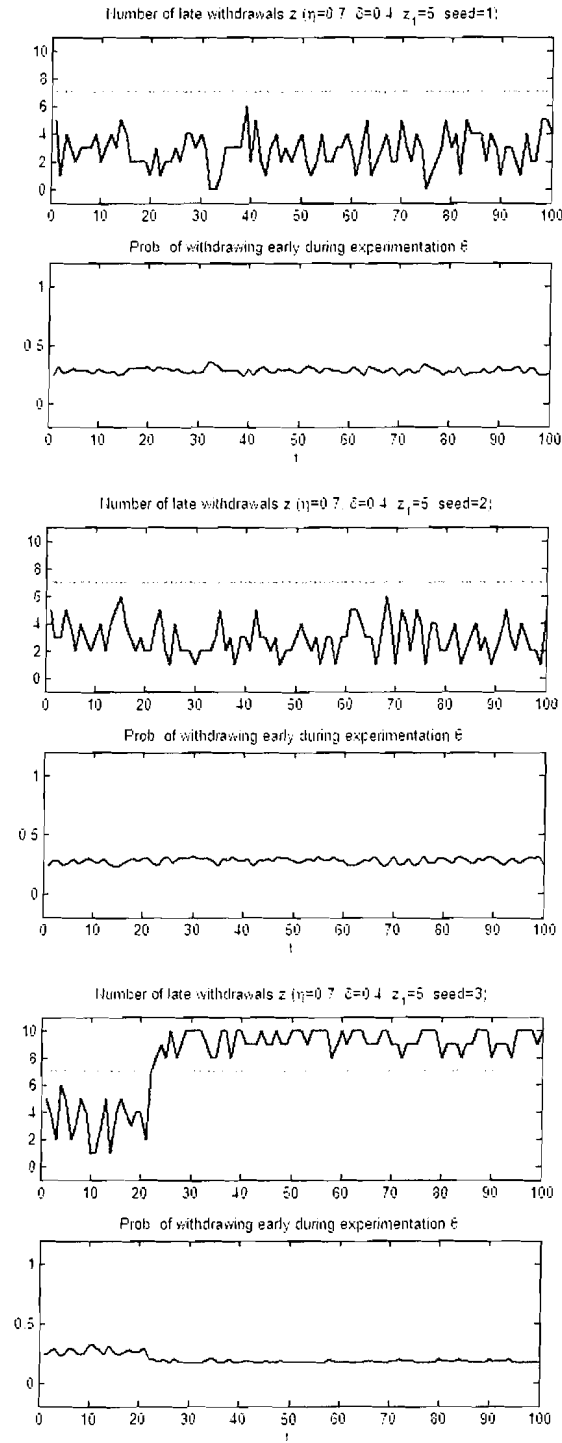
Figure 2.18: Simulation Path - $\eta = 0.7, z_1 = 9$

Number of late withdrawals z (η=0.7 $\hat{c}$=0.4 z_1=9 seed=1)

Prob of withdrawing early during experimentation $\hat{e}$

Number of late withdrawals z (η=0.7 $\hat{c}$=0.4 z_1=9 seed=2)

Prob of withdrawing early during experimentation $\hat{e}$

Number of late withdrawals z (η=0.7 $\hat{c}$=0.4 z_1=9 seed=3)

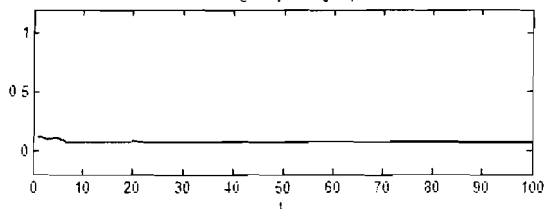Prob of withdrawing early during experimentation $\hat{e}$
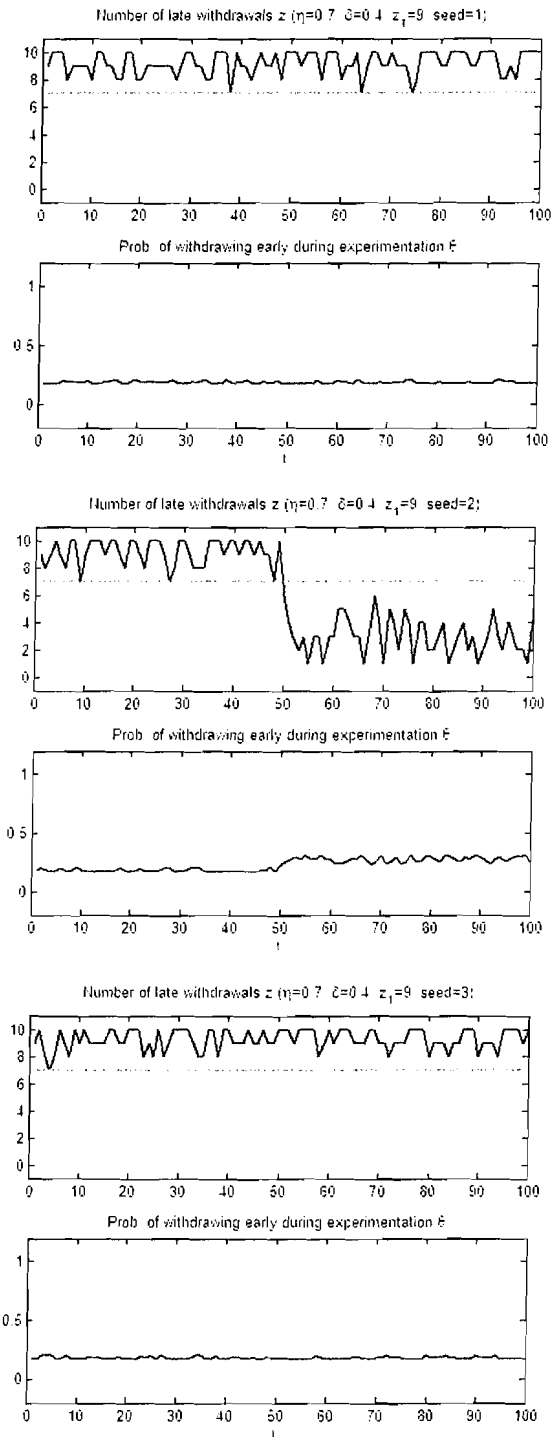
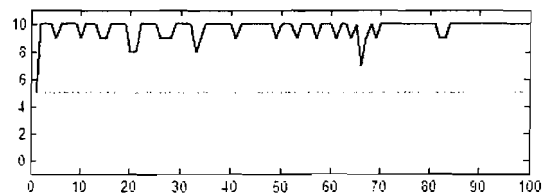# Figure 2.19: Simulation Path - $\eta = 0.8$

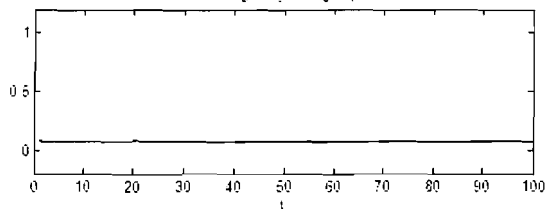Number of late withdrawals z (η=0.8 $\bar{c}$=0.38889 z_1=1 seed=1)
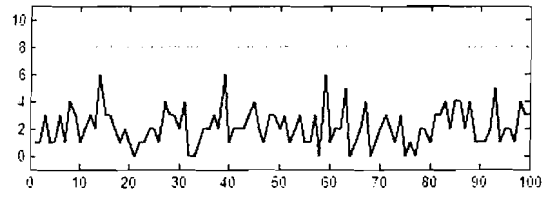
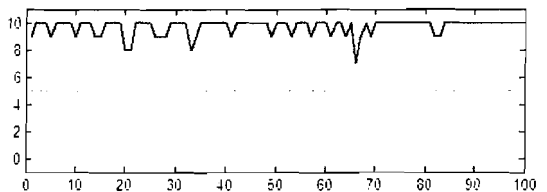Prob of withdrawing early during experimentation $\hat{e}$
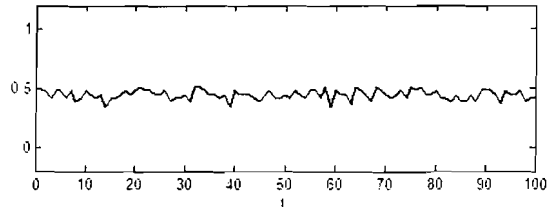
Number of late withdrawals z (η=0.8 $\bar{c}$=0.38889 z_1=5 seed=1)
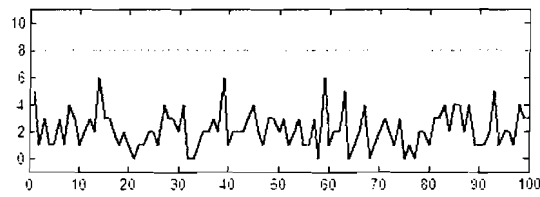
Prob of withdrawing early during experimentation $\hat{e}$
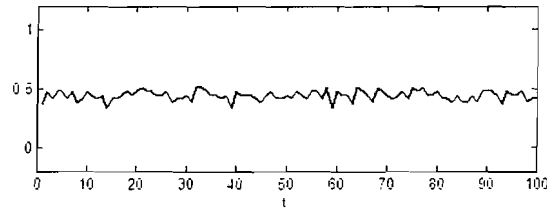
Number of late withdrawals z (η=0.8 $\bar{c}$=0.38889 z_1=9 seed=1)

Prob of withdrawing early during experimentation $\hat{e}$

65
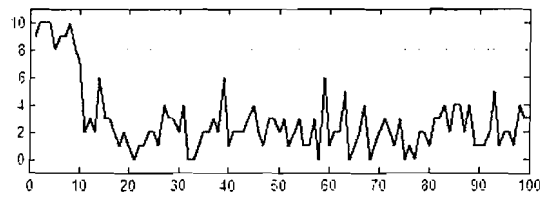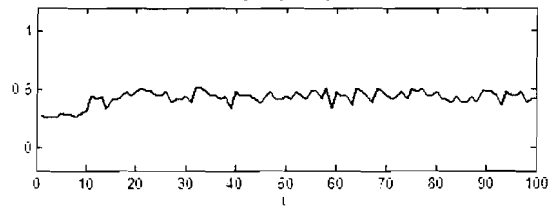
Figure 2.20: Simulation Path - $\eta = 0.9$

Number of late withdrawals z (η=0.9, $\bar{c}$=0.34302, z_1=1, seed=1)

Prob. of withdrawing early during experimentation θ

Number of late withdrawals z (η=0.9, $\bar{c}$=0.34302, z_1=5, seed=1)

Prob. of withdrawing early during experimentation θ

Number of late withdrawals z (η=0.9, $\bar{c}$=0.34302, z_1=9, seed=1)
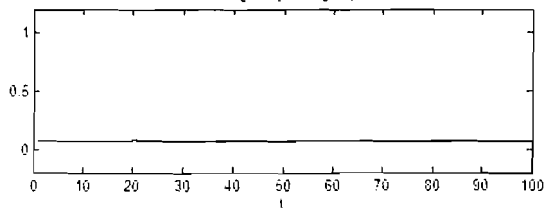
Prob. of withdrawing early during experimentation θ

66
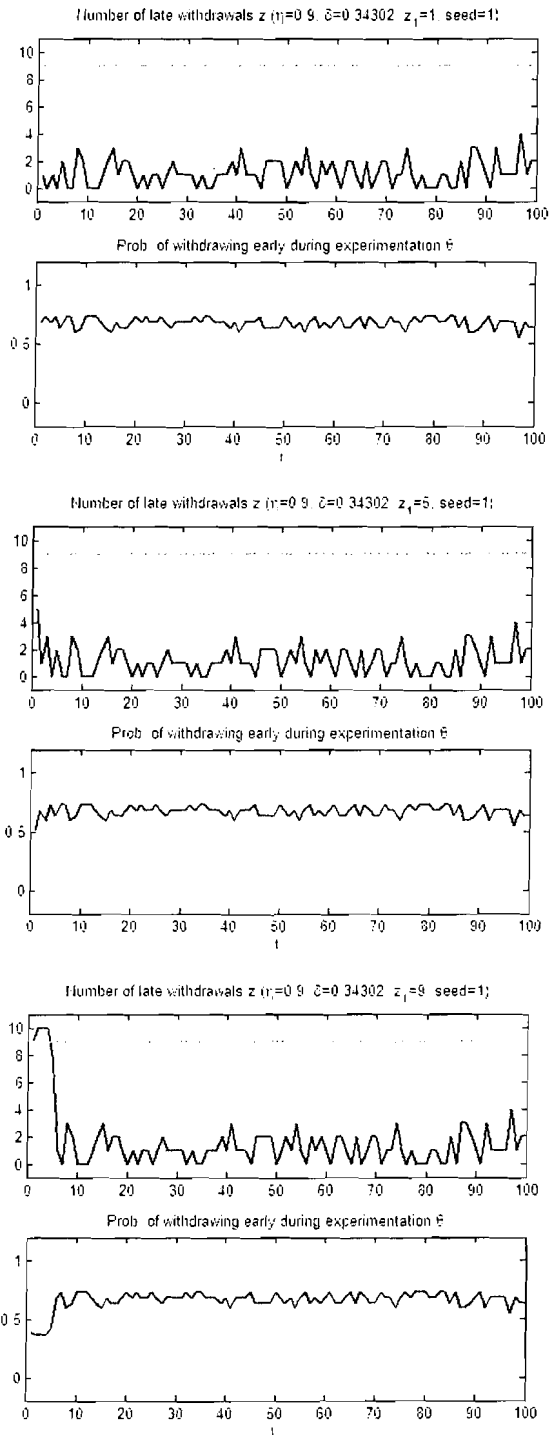
Chapter 3

One or Two Monies?[1]

In this paper, we study the essentiality of a second money. The main features of the model are: alternating day and night stage as in Lagos and Wright (2005), divisible and concealable money in variable supply, private information about preferences, and limited commitment. We adopt the mechanism design approach and solve a social planner's problem subject to the resource constraint and incentive constraints imposed by the existing frictions. We specify the conditions under which a second money is essential and show that two monies are a perfect substitute to the missing record-keeping technology. We investigate how exogenously imposed structure such as indivisibility of money, pair-wise trading or fixed money supply affects the analysis of the essentiality of a second money.

3.1 Introduction

Recent advances in micro-founded monetary theory seem to have reached a consensus that the role of money is to make up for the missing record-keeping technology (see Kocherlakota 1998*a, b*). A natural question to ask is that given that money plays the role of record-keeping, is a single money the optimal choice?

Several decades ago, Mundell (1961) proposed that from the standpoint of money's unit of account and medium of exchange properties, 'the optimum currency area is the world' suggesting that one money is the optimal choice. The intuition behind Mundell's con-

clusion is that using fewer monies reduces the costs associated with valuation and money changing.

Recently, several authors have reinvestigated the optimality of a single currency in environments where money's role as a substitute for the missing record-keeping technology is explicitly modelled. All models feature pair-wise trading and random matching as in Trejos and Wright (1995, 1997). Kocherlakota and Krueger (1999) show that in a two-country model where buyers' nationalities/preferences are private information, two indivisible monies are potentially essential to act as a preference signalling device. Kocherlakota (2002) shows that when agents cannot commit to trading, and when money is divisible, concealable and held in fixed supply, a second money is essential to prevent agents from forging past trading history by concealing money balances. Ravikumar and Wallace (2002) show that in a two-country model, a single money regime is preferred to a two-money regime since the latter is associated with inferior equilibria.[2]

This paper reexamines the essentiality of multiple currencies in a Lagos and Wright (2005) environment where agents' activities occur in two stages - night and day - sequentially in each period. Money is divisible, concealable and in variable supply. There is limited commitment and private information about preferences. We abstract from search friction and pair-wise trading. Following Townsend's (1989) spirit, rather than imposing a certain market structure, we adopt a mechanism design approach and consider a social planner's problem subject to the resource constraints and incentive compatibility constraints implied by the present frictions. The main results are as follows. First, including a second stage allows money balances to serve as an alternative (to the type of money) preference signalling device and a single money is sufficient if agents are patient enough. Second, two monies constitute a perfect substitute for the missing record-keeping technology so that there is no need for a third money. Third, it is the presence of private information about preferences rather than limited commitment that makes a second money essential. Fourth, exogenously imposed structure such as indivisibility of money, pair-wise trading or fixed money supply affects the analysis of the essentiality of a second money.

The paper proceeds as follows. Section 2 lays out the physical environment featuring

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[2]In Ravikumar and Wallace (2002), a country is defined by the trading pattern: the probability of encountering a domestic citizen is higher than the probability of meeting a foreigner. This is opposed to Kocherlakota and Krueger (1999), where a country is defined by preferences: agents from the same country have the same preferences.

There is another strand of literature investigating whether multiple currencies can coexist or circulate at the same time. Examples are Camera and Winkler (2003), Camera et al. (2004), and Craig and Waller (2004). Our paper's goal is to study the welfare enhancing role of multiple currencies, or whether multiple currencies are essential.

two types of agents and characterize the first-best allocation. Section 3 introduces private information and limited commitment and shows the conditions to achieve the first-best allocation when the society has access to a record-keeping technology. Section 4 studies the conditions under which a second money is essential to achieve the first-best allocation. Section 5 extends the result from 2-type-agent model to multiple-type agent model. Section 6 compares our results from previous literature. Section 7 concludes and points out directions for future research.

3.2 The Physical Environment

The framework we adopt is the quasi-linear environment suggested by Lagos and Wright (2005) without the search friction. Time is discrete and runs from date 1 to date ∞. There are two symmetric locations a and b. At date 0, a continuum of measure 1 of infinitely lived agents are born at each location. We call agents type a or type b according to their birth places. Each period consists of two subperiods: day and night. The two locations fuse into one large economy during the day, and become spatially separated economies at the night stage. There is one day good, and two night goods indexed by 1 and 2. Good 1(2) is produced and consumed at location $a(b)$.

During the day, all agents can produce and consume the day good. At night, agents are subject to a technology shock and become either a consumer or a producer with equal probabilities. Producers can produce but do not want to consume, and consumers want to consume but cannot produce. Agents are then subject to a relocation shock and go to each location with the same probabilities. Producers at location $a(b)$ are endowed with a linear technology to produce good 1(2) and the disutility of producing 1 unit of good is 1. Type a consumers want to consume good 1 for which they have high valuation at location a, and want to consume good 2 for which they have low valuation at location b. Type b consumers want to consume good 2 for which they have high valuation at location b, and want to consume good 1 for which they have low valuation at location a. The technology and relocation/preference shocks are $i.i.d.$ across time and individuals originated from the same location. Refer to Figure 1 for a graphical illustration of the environment.

The life-time expected utility of a type a agent $i \in (0, 1)$ is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^a(i) + \frac{1}{4} \left[\delta u(c_{1,t}^a(i)) + u(c_{2,t}^a(i)) - y_{1,t}^a(i) - y_{2,t}^a(i) \right] \right\}.$$

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Figure 3.1: Environment

c_i: consumption of night good 1; c_2: consumption of night good 2;
y_i: production of good 1; y_2: production of good 2;
z: production (consumption if negative) of day good;
Shocks are *i.i.d.* across time and agents (of the same type).
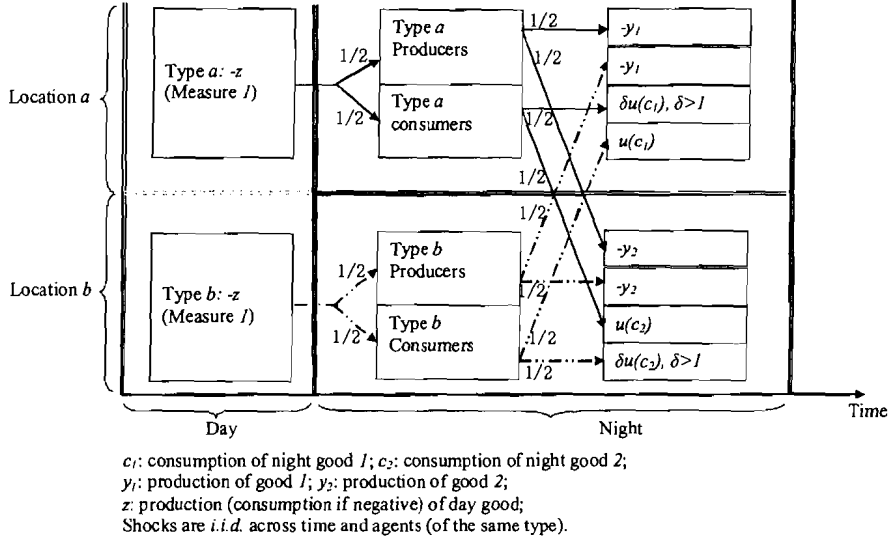
where $0 < \beta < 1$ is the discount factor, $z_t^a(i)$ is the production (consumption if negative) of the day good, $\delta > 1$, $c_{1,t}^a(i)$ and $c_{2,t}^a(i)$ are the consumption of (night) good 1 and 2 respectively, and $y_{1,t}^a(i)$ and $y_{2,t}^a(i)$ are the production of good 1 and 2 respectively. The function $u(\cdot)$ has the properties that $u'' < 0 < u'$ and $u(0) = 0$.

Similarly, the expected utility of a type b agent $j \in (0,1)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -z_t^b(j) + \frac{1}{4} \left[\delta u(c_{2,t}^b(j)) + u(c_{1,t}^b(j)) - y_{1,t}^b(j) - y_{2,t}^b(j) \right] \right\}.$$

The resource constraints are given by:

$$\int_0^1 z_t^a(i)di + \int_0^1 z_t^b(j)dj = 0$$

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at the day stage, and

$$\int_0^1 c_{1,t}^a(i)I_t^a(i)di + \int_0^1 c_{1,t}^b(j)I_t^a(j)dj$$

$$= \int_0^1 y_{1,t}^a(i)I_t^a(i)di + \int_0^1 y_{1,t}^b(j)I_t^a(j)dj \text{ at location } a;$$

$$\int_0^1 c_{2,t}^a(i)I_t^b(i)di + \int_0^1 c_{2,t}^b(j)I_t^b(j)dj$$

$$= \int_0^1 y_{2,t}^a(i)I_t^b(i)di + \int_0^1 y_{2,t}^b(j)I_t^b(j)dj \text{ at location } b;$$

at the night stage for all $t \geq 0$. $I_t^k(\bullet)$ is an indicator function and is equal to 1 if the agent is at location $k \in \{a, b\}$ at date t.

A social planner who treats all agents equally (and the two types of agents symmetrically) chooses $c_{1,t}^a = c_{2,t}^b = c_h$, $c_{2,t}^a = c_{1,t}^b = c_\ell$, $y_{1,t}^a = y_{1,t}^b = y_{2,t}^a = y_{2,t}^b = y$ to maximize the *ex ante* utility:

$$W(c_h, c_\ell, y) = \frac{1}{4}\frac{1}{1-\beta}\left[\delta u(c_h) + u(c_\ell) - 2y\right] \tag{1}$$

subject to:

$$c_h + c_\ell = 2y.$$

The solution is characterized by:

$$\delta u'(c_h^*) = 1;$$
$$u'(c_\ell^*) = 1;$$
$$y^* = (c_h^* + c_\ell^*)/2.$$

I.e., the planner instructs each producer to produce y^* units of night goods, and splits the output among consumers: high valuation (local) consumers get c_h^* and low valuation (non-local) consumers get c_ℓ^*. Note that since $\delta > 1$, $c_h^* > c_\ell^*$. For the day stage allocation, note that since z_t enters linearly in preferences, any $z_t^a(i)$ and $z_t^b(j)$ that satisfy $E_0 z_t^a(i) = E_0 z_t^b(j) = 0$ would satisfy the day stage resource constraint and entail no *ex ante* welfare loss. In the current context, one such allocation is $z_t^a(i) = z_t^b(j) = 0$ for all i and j and $t \geq 0$.

The first-best allocation can be achieved if agents' types and the realization of the technology and preferences are public information, and agents are able to commit to the actions

prescribed by the allocation.

3.3 Limited Commitment and Private Information

Assume that agents cannot commit, and agents' types and the realization of relocation shocks are not publicly observable. The realization of the technology shock or whether an agent is a producer or consumer at the night stage is assumed to be public information.[3] In this case, a record-keeping technology - which allows the planner to keep track of agents' actions - becomes essential to overcome the frictions caused by limited commitment and private information (see Kocherlakota, 1998a, b).

We will focus on symmetric stationary allocations where for all i and $j \in (0,1)$ and $k \in \{a, b\}$,

- $c_{1,t}^a(i) = c_{2,t}^b(j) = c_h, c_{2,t}^a(j) = c_{1,t}^b(i) = c_\ell, y_{1,t}^a(i) = y_{1,t}^b(j) = y_{2,t}^a(i) = y_{2,t}^b(j) = y$
  with $c_h + c_\ell = 2y$ for $t \geq 0$;

- $z_0^a(i) = z_0^b(j) = 0$;

- $z_t^k(\cdot) = \begin{cases} z_h, & \text{if the agent consumed } c_h \text{ at the night stage of time } t-1; \\ z_\ell, & \text{if the agent consumed } c_\ell \text{ at the night stage of time } t-1; \\ z_p, & \text{if the agent produced } y \text{ at the night stage of time } t-1; \end{cases}$
  with $z_p = -(z_h + z_\ell)/2$ for $t \geq 1$.

In the presence of private information and lack of commitment, an implementable allocation must satisfy the incentive constraints (so that individuals have the incentive to truthfully reveal their private information) and individual rationality or participation constraints (so that individuals have the incentive to stick to the actions prescribed by the mechanism).

If a mechanism prescribes higher consumption for high valuation consumers (which is the case at the first-best allocation) at the night stage, low valuation consumers will have the incentive to claim to be high valuation consumers. Private information about types and preferences implies that at the night stage, consumers can potentially lie about their valuation of the night good. Due to the structure of the preference shocks, there are two ways to deal with problems caused by private information. The planner can induce the two types

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[3]We assume this to make the paper more comparable to Kocherlakota and Krueger (1999) and Kocherlakota (2002). The paper's results about the essentiality of multiple currencies go through if the realization of the technology shocks is private information.

of agents to truthfully reveal their types before the realization of the preference shocks and use the information to infer a consumer's valuation after the relocation/preference shock is realized. For example, if an agent reports to be a type a consumer and wants to consume good 2, the planner can infer that the agent is a low valuation consumer. We call this mechanism the early sorting mechanism. To use the early sorting mechanism, the following constraint needs to be satisfied:[4]

$$\frac{1}{1-\beta} \left\{ \frac{\delta u(c_h) - z_h}{4} + \frac{u(c_\ell) - z_\ell}{4} - \frac{y}{2} \right\}$$
$$\geq \frac{1}{1-\beta} \left\{ \frac{\delta u(c_\ell) - z_\ell}{4} + \frac{u(c_h) - z_h}{4} - \frac{y}{2} \right\}; \qquad \text{(ICT)}$$

which holds if $c_h > c_\ell$. Alternatively, the planner can try to induce the consumers to truthfully report their valuation toward the night goods by resorting to variation in the day production. We call this the late sorting mechanism, which is effective if and only if the following two conditions are satisfied:[5]

$$\delta u(c_h) + \beta(-z_h + W) \geq \delta u(c_\ell) + \beta(-z_\ell + W);$$
$$u(c_\ell) + \beta(-z_\ell + W) \geq u(c_h) + \beta(-z_h + W).$$

where W is as defined in equation 1. We can rearrange the two incentive constraints as:

$$z_h \leq \frac{\delta[u(c_h) - u(c_\ell)]}{\beta} + z_\ell; \qquad \text{(ICH)}$$

$$z_h \geq \frac{u(c_h) - u(c_\ell)}{\beta} + z_\ell. \qquad \text{(ICL)}$$

The first constraint ensures that high valuation consumers do not want to imitate low valuation consumers (note that this means that type a agents desiring good 1 do not want to imitate type b agents desiring good 1, and that type b agents desiring good 2 do not want imitate type a agents desiring good 2). The second constraint is that the low valuation consumers do not want to imitate the high valuation consumers (note again that the constraint is for agents wanting to consuming the same good and they are *ex ante* different types). [6]

When agents cannot commit, the allocation $\{c_h, c_\ell, y, z_h, z_\ell, z_p\}$ must respect *ex post* rationality. Assume that the punishment for noncompliance is autarky where the welfare

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[4]We call this 'early sorting' because information used in this mechanism is revealed before agents' valuation of the night goods is realized.

[5]We call this 'early sorting' because information used in this mechanism is revealed after agents' valuation of the night goods is realized.

[6]Strictly speaking, there is a third way to align the consumers' incentives by giving all consumers the same night stage consumption, which is obviously not optimal.

is $W_0 = 0$. At the night stage, there are three individual rationality conditions: one for producers, one for high valuation consumers and one for low valuation consumers:

$$-y + \beta(-z_p + W) \geq 0; \tag{3}$$

$$\delta u(c_h) + \beta(-z_h + W) \geq 0; \tag{4}$$

$$u(c_\ell) + \beta(-z_\ell + W) \geq 0. \tag{5}$$

At the day stage, there are also three individual rationality conditions:

$$-z_p + W \geq 0; \tag{6}$$

$$-z_h + W \geq 0; \tag{7}$$

$$-z_\ell + W \geq 0. \tag{8}$$

Note that for consumers, if the day stage rationality conditions - (7) and (8) - are satisfied, the night stage individual rationality conditions - (4) and (5) - are automatically satisfied. For producers, the night stage individual rationality constraint (3) implies the day stage individual rationality condition (6). We can thus drop conditions (4), (5), and (6). An implementable allocation must satisfy conditions (3), (7) and (8), which we rewrite and label as (IRP), (IRH) and (IRL) respectively.[7]

$$z_h \geq -z_\ell + 2(y/\beta - W); \tag{IRP}$$

$$z_h \leq W; \tag{IRH}$$

$$z_\ell \leq W; \tag{IRL}$$

Lemma 3.1 *When agents' types and the realization of the preference shocks are private information, and when the economy has access to a record-keeping technology, the first-best allocation can be achieved if and only if $\beta \geq \beta_0$ where β_0 is defined as:*

$$\beta_0 = \frac{2y^*}{\delta u(c_h^*) + u(c_\ell^*)}.$$

Proof. We graph IRP, IRL and IRH at the optimal allocation (c_h^*, c_ℓ^*, y^*), at which the *ex ante* welfare is given by

$$W^* = \frac{1}{4}\frac{1}{1-\beta}\left[\delta u(c_h^*) + u(c_\ell^*) - 2y^*\right].$$

[7]Note that (IRP) incorporates the day time resource constraint $z_h/2 + z_\ell/2 + z_p = 0$.

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Figure 3.2: Full Record-keeping Technology
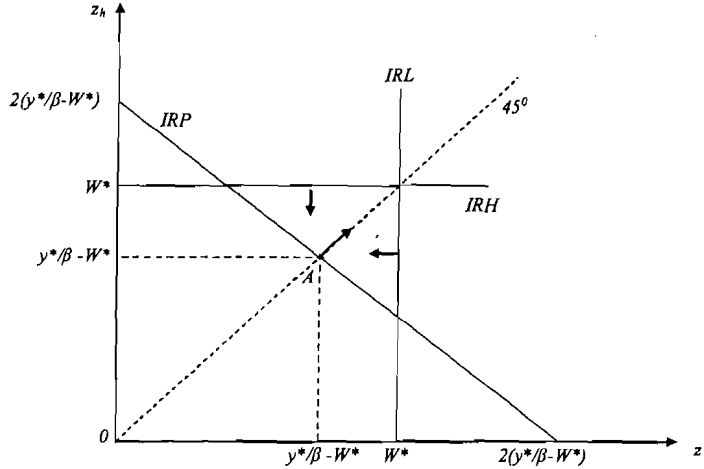
As shown in Figure 2, the first-best allocation can be achieved if and only if the shaded area is non-empty, or

$$y^*/\beta - W^* \leq W^*;$$

which, with some manipulation, can be rewritten as:

$$\beta \geq \frac{2y^*}{\delta u(c_h^*) + u(c_\ell^*)} = \beta_0.$$

Note that we do not need to worry about the incentive constraint for consumers since ICT is satisfied if $c_h > c_\ell$. ∎

3.4 Monetary Mechanisms

Now suppose that the society has no access to a record keeping technology so that it is impossible to directly pass information across time. In this case, the planner uses tokens - which we call money - as a substitute for the missing record-keeping technology to communicate information across stages. [8] Following Kocherlakota (2002), we assume that money is perfectly divisible and money balances are concealable (or private information).

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[8]The society, though, has access to a contemporaneous memory technology which can remember agents' actions *within* a stage.

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3.4.1 Single Money Mechanisms

When there is a single money, it is impossible to use ICT to align consumers' incentive. If the two types of agents hold the same amount of money, after the realization of the relocation shocks, all low valuation consumers can lie to be high valuation consumers. The other possibility is that the two types of agents hold different amounts of money, which, however, still cannot prevent those with higher amount of money from lying to be high valuation consumers when they are in fact low valuation consumers by concealing money balances. To induce consumers to truthfully reveal their valuation, a single money mechanism must rely on the late sorting mechanism and resort to differences in day time production. In this case, we need to replace ICT by ICH and ICL.

We will show that there exists single monetary mechanisms to catch nonparticipants and effectively cast them into perpetual autarky, so that the individual rationality conditions remain the same as in the full record-keeping case. An implementable single money mechanism must satisfy ICH, ICL, IRP, IRH and IRL.

Lemma 3.2 states the condition under which a single money mechanism can achieve the first-best allocation.

Lemma 3.2 *When agents' types and preferences are private information, and in the absence of a record-keeping technology, single money mechanisms can achieve the first-best allocation if and only if $\beta \geq \beta_1$, with β_1 given by:*

$$\beta_1 = \frac{2y^* + u(c_h^*) - u(c_\ell^*)}{(\delta + 1)u(c_h^*)} < \beta_0.$$

Proof. Consider the following mechanism.

At date 0 day stage, the mechanism endows each agent with one unit of money ($).

At date 0 night stage, after the technology and relocation/preference shocks are realized, the mechanism offers agents the following choices:

If consumer, show 1 $, get $\begin{cases} c_h^* \text{ good (1 or 2) and } (1 - \rho_h) \text{ $; or} \\ c_\ell^* \text{ good (1 or 2) and } (1 - \rho_\ell) \text{ $;} \end{cases}$

with $1 > \rho_h > \rho_\ell > 0$;

If producer, show 1 $, use y^* output to exchange for 1 $.

With this mechanism, non-participants leave with 1 $ and participants leave with more than 1 $. Participating consumers leave with $2 - \rho_h$ or $2 - \rho_\ell$ units of money depending on whether they choose to consume c_h^* or c_ℓ^* units of goods. Participating producers produce

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y^* units of output and leave with 2 units of money.[9]

At date 1 day stage, the mechanism offers agents the following options:

Show $(2 - \rho_h)$ \$, use z_h day goods to exchange for ρ_h \$;

Show $(2 - \rho_\ell)$ \$, use z_ℓ day goods to exchange for ρ_ℓ \$;

Show 2 \$, get $z_p = (z_h + z_\ell)/2$ units of day good.

With this mechanism, all participants leave the stage with 2 units of money and non-participating agents who were consumers in the proceeding night stage leave with less than 2 units of money.

At date 1 night stage, the choice are:

If consumer, show 2 \$, choose from
$$\begin{cases} c_h^* \text{ good (1 or 2) and } (1 - \rho_h) \text{ \$; or} \\ c_\ell^* \text{ good (1 or 2) and } (1 - \rho_\ell) \text{ \$} \end{cases}$$

If producer, show 2 \$, use y^* output to exchange for 1 \$.

At date $t \geq 2$ day stage, the choices are:

Show $(t + 1 - \rho_h)$ \$, use z_h day goods to exchange for ρ_h \$;

Show $(t + 1 - \rho_\ell)$ \$, use z_ℓ day goods to exchange for ρ_ℓ \$;

Show $(t + 1)$ \$, get $z_p = (z_h + z_\ell)/2$ units of day goods.

At date $t \geq 2$ night stage, the choices are:

If consumer, show $(t + 1)$ \$, choose from
$$\begin{cases} c_h^* \text{ good (1 or 2) and } (1 - \rho_h) \text{ \$; or} \\ c_\ell^* \text{ good (1 or 2) and } (1 - \rho_\ell) \text{ \$} \end{cases}$$

If producer, show $(t + 1)$ \$, use y^* output to exchange for 1 \$.

Note that under this mechanism, if an agent skip the night stage as a producer, or skip the day stage as a previous consumer, his money balances will fall short of the required balances to participate in the following stages; the mechanism thus effectively catches non-participants and casts them into perpetual autarky. The individual rationality conditions thus remain the same as in the case with the record-keeping technology.

In Figure 3, we graph the incentive compatibility and individual rationality conditions $(ICH, ICL, IRP, IRH$ and $IRL)$ at the optimal allocation (c_h^*, c_ℓ^*, y^*). As is obvious from the graph, any combination (z_ℓ, z_h) in the shaded area can achieve the first-best allocation. The area is non-empty if and only if

$$\frac{y^*}{\beta} - W^* + \frac{[u(c_h^*) - u(c_\ell^*)]}{2\beta} \leq W^*;$$

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[9]Since there is a contemporaneous memory technology, the mechanism can prevent agents from participating more than once.

which can be rearranged as:

$$\beta \geq \frac{2y^* + [u(c_h^*) - u(c_\ell^*)]}{(\delta + 1)u(c_h^*)} = \beta_1.$$

Figure 3.3: Lack of Record-keeping Technology, Single Money
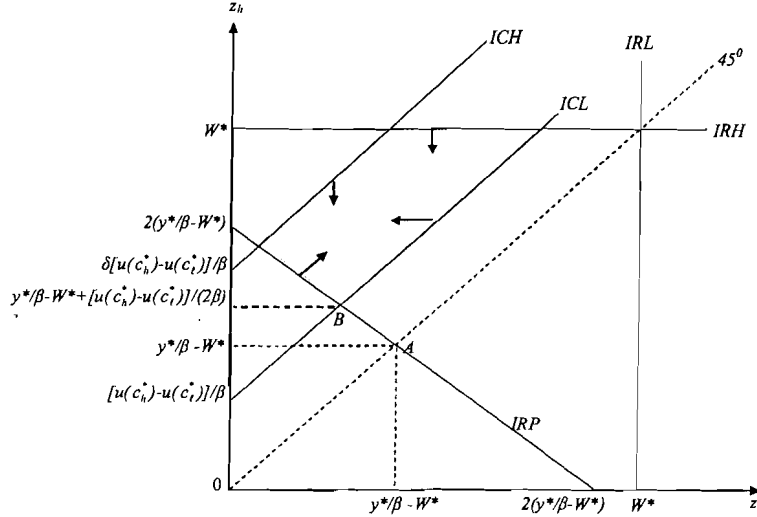
Note that since

$$\frac{1}{\beta_1} = \frac{(\delta + 1)u(c_h^*)}{2y^* + [u(c_h^*) - u(c_\ell^*)]} = \frac{\delta u(c_h^*) + u(c_\ell^*) - 2y^*}{2y^* + [u(c_h^*) - u(c_\ell^*)]} + 1;$$

$$\frac{1}{\beta_0} = \frac{\delta u(c_h^*) + u(c_\ell^*)}{2y^*} = \frac{\delta u(c_h^*) + u(c_\ell^*) - 2y^*}{2y^*} + 1;$$

it follows that $1/\beta_1 < 1/\beta_0$, or $\beta_1 > \beta_0$. ∎

The single monetary mechanism outlined above deals with the friction caused by limited commitment and private information as follows. By rewarding participants with newly issued money and increasing the money balances required for future participation, the mechanism effectively catches non-participants and casts them to perpetual autarky. By requiring previous high valuation consumers work more than previous low valuation consumers in the day stage, the mechanism induces consumers to truthfully reveal privately held information and signal preferences by choosing different money balances while exiting the night stage.

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3.4.2 Two Money Mechanisms

In this subsection we introduce a second money and show that two monies consist of a perfect substitute for the missing record-keeping technology, and improve welfare over one money when $\beta < \beta_1$. Lemma 3.3 states the condition under which two-money mechanisms can achieve the first-best allocation.

Lemma 3.3 *When agents' types and the realization of the preference shocks are private information, in the absence of a record-keeping technology, two-money mechanisms can achieve the first-best allocation if and only if $\beta \geq \beta_0$.*

Proof. Call the two monies red money (R) and green money (G). Consider the following mechanism.

At date 0 day stage, the mechanism asks agents to choose from two monetary portfolios: $1\ R$ and $1\ G$.

At date 0 night stage, after the shocks are realized, the mechanism offers agents the following choices:

At location a,

If consumer and show R, get c_h^* good 1 and $(1 - \rho)\ R$;

If consumer and show G, get c_ℓ^* good 1 and $(1 - \rho)\ R$, where $0 < \rho < 1$;

If producer and show R, use y^* good to exchange for $1\ R$;

At location b,

If consumer and show G, get c_h^* good 2 and $(1 - \rho)\ G$;

If consumer and show R, get c_ℓ^* good 2 and $(1 - \rho)\ G$;

At both locations,

If producer and show R, use y^* good to exchange for $1\ R$;

If producer and show G, use y^* output to exchange for $1\ G$.

With this mechanism, non-participants leave with 1 unit of money and participants leave with more than 1 unit of money. Participating consumers with higher consumption (local consumers) leave the stage with a single type of money; participating consumers with lower consumption (non-local consumers) leave with two types of money; all participating consumers exit with the same total money balances $2 - \rho$. Participating producers produce y^* units of output and leave with 2 units of single-colored money.

At date 1 day stage,

If show $(2 - \rho) R$, use z_h day output to exchange for ρR;

If show $(2 - \rho) G$, use z_h day output to exchange for ρG;

If show $R + (1 - \rho) G$, use z_ℓ day output and $(1 - \rho)G$ to exchange for 1 R;

If show $G + (1 - \rho) R$, use z_ℓ day output and $(1 - \rho)R$ to exchange for 1 G;

If show 2 R or 2 G, get $z_p = (z_h + z_\ell)/2$ units of day goods.

At date 1 night stage,

At location a,

If consumer and show 2 R, get c_h^* good 1 and $(1 - \rho) R$;

If consumer and show 2 G, get c_ℓ^* good 1 and $(1 - \rho) R$;

At location b,

If consumer and show 2 G, get c_h^* good 2 and $(1 - \rho) G$;

If consumer and show 2 R, get c_ℓ^* good 2 and $(1 - \rho) G$;

At both locations,

If producer and show 2 R, use y^* output to exchange for 1 R;

If producer and show 2 G, use y^* output to exchange for 1 G.

At date t $\geq$ 2 day stage,

If show $(t + 1 - \rho) R$, use z_h day output to exchange for ρR;

If show $(t + 1 - \rho) G$, use z_h day output to exchange for ρG;

If show $t R + (1 - \rho) G$, use z_ℓ day output and $(1 - \rho) G$ to exchange for 1 R;

If show $t G + (1 - \rho) R$, use z_ℓ day output and $(1 - \rho) R$ to exchange for 1 G;

If show $(t + 1) R$ or $(t + 1) G$, get $z_p = (z_h + z_\ell)/2$ units of day goods.

At date t $\geq$ 2 night stage,

At location a:

If consumer and show $(t + 1) R$, get c_h^* good 1 and $(1 - \rho) R$;

If consumer and show $(t + 1) G$, get c_ℓ^* good 1 and $(1 - \rho) R$;

At location b:

If consumer and show $(t + 1) G$, get c_h^* good 2 and $(1 - \rho) G$;

If consumer and show $(t + 1) R$, get c_ℓ^* good 2 and $(1 - \rho) G$;

At both locations,

If producer and show $(t+1)$ R, use y^* output to exchange for 1 R;

If producer and show $(t+1)$ G, use y^* output to exchange for 1 G.

Similar to the single monetary mechanism in last subsection, the two money mechanism described here rewards participants with more money balances and effectively catches non-participants and bars them from participating in the mechanism forever. The individual rationality conditions thus stay the same as in the case with the record-keeping technology where defectors are directly caught and forced into perpetual autarky. What is different from the single monetary mechanism is that if the two types of agents can be induced to hold different monetary portfolios with the same total balances, they will not be able to falsely claim to be high valuation consumers in the night stages. For example, suppose that type a choose to hold red money and type b choose to hold green money at date 0. At the following night stage, a type a agent at location b (desiring good 2) is a low valuation consumer cannot claim to have high valuation since he does not have the green money required for higher consumption. We verify in the following that the two types of agents indeed have the incentive to differentiate themselves from each other by choosing different monetary portfolios. Take type a agents as an example. The expected life-time utility from holding the red money is:

$$W_r^a = \frac{1}{4}\frac{1}{1-\beta}[\delta u(c_h^*) + u(c_\ell^*) - 2y^*];$$

and the expected utility from holding the green money is:

$$W_g^a = \frac{1}{4}\frac{1}{1-\beta}[\delta u(c_\ell^*) + u(c_h^*) - 2y^*];$$

and $W_r^a > W_g^a$ so that type a agents prefer holding the red money.

The mechanism outlined above can achieve the first-best allocation if the allocation $(c_h, c_\ell, y, z_h, z_\ell, z_p)$ satisfies ICT, IRP, IRH and IRL at (c_h^*, c_ℓ^*, y^*) as depicted in figure 2. As in the case with the full record keeping technology, the first-best allocation can be achieved if and only if $\beta \geq \beta_0$.■

Two monies induces the two types of agents to hold different monetary portfolios and since the two portfolios feature the same total balances, it is impossible to juggle one portfolio to look like the other. The early sorting mechanism is thus reinstated and two monies provide a perfect substitute for the missing record-keeping technology, and improves wel-

fare over single money mechanisms when $\beta < \beta_0$.[10]

3.5   Extension to Multi-type-agent Models

In this section, we show that as in Townsend (1987), two monies consist of a perfect substitute for the missing record-keeping technology even when there are more than two types of agents.[11] The optimal two money mechanism is to endow different types with different combinations of the red and green monies, with all combinations giving the same total money balances.

There are $N < +\infty$ symmetric locations, N types of agents distinguished by their locations of origin, and N night goods.

During the day, all agents can produce and consume the day good. At night, agents are subject to a technology shock and become either a consumer or a producer with equal probabilities. Producers can produce but do not want to consume and consumers want to consume but cannot produce. Agents are then subject to a relocation/preference shock and go to each location with the same probability $1/N$. Producers at location $n \in \{1, 2, ..., N\}$ are endowed with a technology to produce good n and the disutility of producing 1 unit of output is 1. Type m consumers at location n derive utility from night good n and the utility of consuming c_{mn} is given by $\delta_{mn} u(c_{mn})$ with $\delta_{mn} = \delta_{m-n+I(m<n)N}$, $\delta_0 > \delta_1 > \delta_2 > ... > \delta_{N-1} > 0$, and $I(m < n) = 1$ if $m > n$ and 0 otherwise.

The first-best consumption c_{mn}^* $(m, n = 1, 2, ..., N)$ is characterized by

$$\delta_{mn} u'(c_{mn}^*) = 1.$$

Let c_q^* $(h \in \{0, 1, ..., N - 1\})$ be the solution to

$$\delta_q u'(c_q^*) = 1.$$

We have $c_{mn}^* = c_{m-n+I(m<n)N}^*$.

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[10]Note that when $\beta < \beta_1$, the first-best allocation cannot be achieved even with the record-keeping technology. It can be shown that two monies are still a perfect substitute for the record-keeping technology and strictly improve welfare over single monetary mechanisms.

[11]Kocherlakota and Krueger (1999) also mention that two monies are sufficient in their model which has only two types of agents. With indivisible monies, however, more monies will be needed if there are more than two types of agents.

Table 3.1: Preference/Relocation Shocks

Good Agent type	1	2	...	n	...	N-1	N
1	δ_0	δ_{N-1}	...	δ_{1-n+N}	...	δ_2	δ_1
2	δ_1	δ_0	...	δ_{2-n+N}	...	δ_3	δ_2
...	...	...	...	...	...	...	...
m	δ_{m-1}	δ_{m-2}	...	$\delta_{m-n+I(m<n)N}$	...	δ_{m+1}	δ_m
...	...	...	...	...	...	...	...
N-1	δ_{N-2}	δ_{N-3}	...	δ_{N-1-n}	...	δ_0	δ_{N-1}
N	δ_{N-1}	δ_{N-2}	...	δ_{N-n}	...	δ_1	δ_0

The first-best production per producer is:

$$y^* = \frac{1}{N} \sum_{q=0}^{N-1} c_q^*.$$

The first-best life-time welfare of a representative agent is:

$$W^* = \frac{1}{2N} \frac{1}{1-\beta} \left\{ \left[\sum_{q=0}^{N-1} \delta_q u(c_q^*) \right] + Ny^* \right\}$$

Agents cannot commit and agents' types and the realization of preference shocks are private information. The realization of the technology shock is public information.

We focus on symmetric stationary allocations where for $m, n \in \{1, 2, ..., N\}$,

- $c_{mnt} = c_{m-n+I(m<n)N}$ and $y_{mnt} = y = (1/N) \sum_{q=0}^{N-1} c_q$ for $t \geq 0$;

- $z_{m0} = 0$;

- $z_t = \begin{cases} z_q, & \text{if the agent consumed } c_q \text{ at the night stage of time } t-1; \\ z_p, & \text{if the agent produced } y \text{ at the night stage of time } t-1; \end{cases}$

with $z_p = -(1/N) \sum_{q=0}^{N-1} z_q$ for $t \geq 1$.

Let us first look at the achievable allocation when there is a record-keeping technology. There are two ways to deal with the friction caused by private information. If early sorting

is used, the following (N) constraints must be satisfied:

$$\frac{1}{2N}\frac{1}{1-\beta}\left\{\left[\sum_{n=1}^{N}\delta_{mn}u(c_{mn})\right]+Ny\right\}$$
$$\geq \frac{1}{2N}\frac{1}{1-\beta}\left\{\left[\sum_{n=0}^{N}\delta_{m'n}u(c_{m'n})\right]+Ny\right\};$$

for all $m, m' \in \{1, 2, ..., N\}$ and $m' \neq m$. If late sorting is used, the following ($N^2 - N$) constraints must be satisfied:

$$\delta_q u(c_q) + \beta(-z_{q'} + W) \geq 0 \text{ for all } q, q' \in \{0, 1, ..., N - 1\};$$

where

$$W = \frac{1}{2N}\frac{1}{1-\beta}\left\{\left[\sum_{q=0}^{N-1}\delta_q u(c_q)\right]+Ny\right\}.$$

To deal with limited commitment, the following ($N + 1$) individual rationality conditions must be satisfied:

$$-y + \beta(-z_p + W) \geq 0;$$
$$-z_q + W \geq 0 \text{ for all } q \in \{0, 1, ..., N - 1\}.$$

Following the argument in section 3, it can be shown that ICT holds at the first-best allocation, and the first-best allocation can be achieved if and only if

$$\beta \geq \frac{Ny^*}{\sum_{q=0}^{N-1}\delta_q u(c_q^*)} = \beta_0^N.$$

In the absence of a record-keeping technology, single monetary mechanism must resort to late sorting and the first-best allocation can be achieved if and only if:

$$\beta \geq \beta_1^N = \frac{X^* + Ny^*}{X^* + \sum_{q=0}^{N-1}\delta_q u(c_q^*)} > \beta_0^N;$$

where

$$X^* = \sum_{q=0}^{N-1}(N - 1 - q)\delta_{q+1}[u(c_q^*) - u(c_{q+1}^*)].$$

The following proposed mechanism with two monies (R and G) can act as a perfect substitute for the missing record-keeping technology, and improve welfare over single monetary mechanisms when $\beta < \beta_1^N$.

At the day stage of date 0, the mechanism asks people to choose from among N money portfolios:

$$r_m\, R + (1 - r_m)\, G;$$

with $0 < r_m < 1$ for all $m \in \{1, 2, ..., N\}$ and $r_m \neq r_{m'}$ for all $m \neq m'$.

At the night stage of date 0, after the shocks are realized, the planner offers each agent the following choices:

At location $n \in \{1, 2, ..., N\}$,

If consumer and show $r_m\, R + (1 - r_m)\, G$, get $c_{mn} = c_{m-n+I(m<n)N}$ good n and $\varepsilon[r_n\, R + (1 - r_n)\, G]$ where $0 < \varepsilon < \min(|r_m - r_{m'}|_{m \neq m' \in \{1,2,...,N\}})$;

If producer and show $r_m\, R + (1 - r_m)\, G$, use y^* output to exchange for $r_m\, R + (1 - r_m)\, G$.

The mechanism requires agents show 1 unit of money to participate in the stage, and proposes consumption contingent on the composition of the monetary portfolio held by consumers. The mechanism rewards participating consumers ε units of money, the composition of which differs across different locations. Producers who produce y^* units of output earn 1 unit of money and maintain the same composition as their currently held portfolios. We restrict ε to ensure that consumers of different types and consuming at different locations exit the night stage with different monetary portfolios.

At the day stage of date 1,

If show $r_m\, R + (1 - r_m)\, G + \varepsilon[r_n\, R + (1 - r_n)\, G]$, use $z_{mn} = z_{m-n+I(m<n)N}$ day output and $\varepsilon[r_n\, R + (1 - r_n)\, G]$ to exchange for $[r_m\, R + (1 - r_m)\, G]$;

If show $2[r_m\, R + (1 - r_m)\, G]$, get $z_p = (1/N)\sum_{q=1}^{N-1} z_q$ units of day good.

If an agent has participated in at the date 0 night stage and date 1 day stage, he augments his money balances by 1 unit and maintains the same composition as his monetary portfolio at the date 0 day stage.

At the night stage of date 1,

At location n,

If consumer and show $2[r_m\, R + (1 - r_m)\, G]$, get $c_{mn} = c_{m-n+I(m<n)N}$ good n and $\varepsilon[r_n\, R + (1 - r_n)\, G]$;

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If producer and show $2[r_m \ R+(1-r_m) \ G]$, use y^* output to exchange for $r_m \ R+(1-r_m) \ G$;

At the day stage of the t $\geq$ 2 period,

If show $t[r_m \ R + (1 - r_m) \ G] + \varepsilon[r_n \ R + (1 - r_n) \ G]$, use $z_{mn} = z_{m-n+I(m<n)N}$ day output and $\varepsilon[r_n \ R + (1 - r_n) \ G]$ to exchange for $[r_m \ R + (1 - r_m) \ G]$;

If show $(t + 1)[r_m \ R + (1 - r_m) \ G]$, get $z_p = (1/N) \sum_{q=1}^{N-1} z_q$ units of day output.

At the night stage of the $t \geq 2$ period,

At location n,

If consumer and show $(t + 1)[r_m \ R + (1 - r_m) \ G]$, get $c_{mn} = c_{m-n+I(m<n)N}$ good n and $\varepsilon[r_n \ R + (1 - r_n) \ G]$;

If producer and show $(t + 1)[r_m \ R + (1 - r_m) \ G]$, use y^* output to exchange for $r_m \ R + (1 - r_m) \ G$.

3.6   Comparison with Previous Literature

There are two papers closely related to our paper, Kocherlakota and Krueger (1999) and Kocherlakota (2002).

The setup of our model differs from Kocherlakota and Krueger (1999) in several ways. First, we have a day market. Second, we deal with centralized allocation and abstract from pair-wise trading. Third, we have divisible money. The results we have are also different from Kocherlakota and Krueger (1999).

In Kocherlakota and Krueger (1999), two monies are essential only if agents are patient enough. Kocherlakota and Krueger (1999) get the result because the assumption of pair-wise trading, indivisibility of money and upper unit bound for money holdings. In such a framework, when agents are not patient enough, the production for high and low valuation consumers is the same, or there should be no production for the low valuation consumers, so that there is no need to distinguish the two types of consumers and thus no need to use a second money.[12] In our model, it is always necessary to distinguish the two types of agents, and two monies are not essential if money balances can effectively signal

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[12]Kocherlakota and Krueger (1999) mentioned that two monies are not essential if $c_h^* = c_\ell^*$ or $c_\ell^* = 0$. The first case occurs due to the assumption that people can only engage in pair-wise trading and trading pairs are isolated from each other. The second case occurs due to the assumption that money is indivisible and there is a unit upper bound for money holdings so that it might be optimal for agents to hold onto their 1 unit of domestic money and trade only with a domestic producer.

preferences (which is the case if agents are patient enough).

Kocherlakota and Krueger (1999) also mention that in their framework, two monies are sufficient. The result, however, cannot be extended to multiple-type-agent models if money remains indivisible. In this paper, we show that when money is divisible, two monies serve as a perfect substitute for the missing record-keeping technology so that two monies are sufficient even when there are more than two types of agents.

Our model reduces to Kocherlakota (2002) if $\delta = 1$ and day time production is limited to be zero. Kocherlakota (2002) shows that when there is limited commitment, it is necessary to have two monies. The result is due to the assumption that money supply is kept fixed. When money supply is fixed, the only way to 'record' whether a producer has produced or not is to give the producer some money, which has to be transferred from his trading partner or a consumer; this will result in different money holdings for producers and consumers. However, the first-best allocation specifies that future allocation should not discriminate those with less money balances because they might have been consumers in last period; this makes it possible for producers to defect and claim to have been a consumer in the previous period. The mechanism we propose is to make money supply variable, reward all participants (no matter whether they are producers or consumers) by more money and require ever increasing money balances for future participation to effectively catch non-participants and cast them into perpetual autarky. Limited commitment thus does not justify a role for a second money if we change the restriction of fixed money supply.

3.7   Conclusion and Future Research

In this paper, we show in a Lagos and Wright (2005) type of model that in the face of private information and limited commitment, a second money can potentially improve welfare by providing an efficient way to pass information across time.

In the model, the first-best allocation requires that producers produce for consumers and high valuation consumers consume more than low valuation consumers. When agents' types and the realization of the preference shocks are private information, low valuation consumers have the incentive to claim to be high valuation consumers. There are two options to align consumers' incentives. The first is to induce agents to truthfully report their types and before the realization of preference shocks and use the information later on to infer agents' valuation; we call this early sorting. The second option is to induce agents to report their valuation after the preference shocks, and use the day stage consump-

tion/production to align the incentives; we call this late sorting.

Mechanisms with a single concealable money rule out early sorting. When agents are patient enough, the late sorting mechanism effectively aligns the incentive by inducing agents to leave the night stage with different money balances. When agents are not patient enough, the late sorting mechanism is not powerful enough to align incentives and the introduction of a second money permits the early sorting mechanism allowing agents to signal their preferences by holding different monetary portfolios (with the same total money balances).

Our research in this paper can be carried on in several aspects. In our model's environment, two monies are sufficient to replace the missing record-keeping technology. Townsend (1987) and Kocherlakota (2002) have a similar result. It would be interesting to see if two money is always sufficient as a substitute for record-keeping technology. Our hunch tell us that the answer is 'yes'. We provide an intuitive argument here. If money balances are not concealable, we can always encode different action/characteristic into a number, and the number will carry the relevant information into future periods. When money balances are concealable, the proposed solution does not work since individuals can cut larger numbers to smaller ones; in a sense, the concealability of money balances makes it possible for people to fake past history. The introduction of a second money solves the problem by encoding actions/characteristics into different two coordinates summing to the same number. Two distinct actions/characteristics are thus represented by two distinct pairs of numbers, and it is impossible to change one pair (record) to the other by cutting the components of the couplet (concealing money balances) since the sum will be less than required.

In the paper, we take the mechanism design approach and there are no markets in the mechanisms proposed in the paper. We would like to follow Waller (2007) to see if the allocations can be decentralized with market mechanisms (and with the help of monetary and fiscal policies).

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3.8 References

Camera, Gabriele, Ben Craig and Christopher Waller (2004), "Currency Competition in a Fundamental Model of Money," *Journal of International Economics*, 64, 521-544.

Camera, Gabriele and Johannes Winkler (2003), "International Monetary Trade and the Law of One Price," *Journal of Monetary Economics*, 50, 1531-1553.

Craig, Ben and Christopher J. Waller (2004), "Dollarization and Currency Exchange," *Journal of Monetary Economics*, 51, 671-689.

Kocherlakota, Narayana (2002). "The Two-money Theorem," *International Economic Review*, 43(2), 333-346.

Kocherlakota, Narayana (1998a), "The Technological Role of Fiat Money," *Federal Reserve Bank of Minneapolis Quarterly Review*, 22 (3), 2-10.

Kocherlakota, Narayana (1998b), "Money is Memory," *Journal of Economic Theory*, 81(2), 232-251.

Kocherlakota, Narayana and Thomas Krueger (1999), "A Signaling Model of Multiple Currencies", *Review of Economic Dynamics*, 2, 231-244.

Lagos, Ricardo and Randall Wright (2005), "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113(3), 463-484.

Levine, David K. (1991), "Asset Trading Mechanisms and Expansionary Policy," *Journal of Economic Theory*, 54, 148-164.

Martin, Antoine (2006), "Endogenous multiple currencies", *Journal of Money, Credit, and Banking* 38, 247 – 264.

Mundell, Robert A. (1961), "A Theory of Optimum Currency Areas," *The American Economic Review*, 51(4), 509-517.

Ravikumar, B. and Neil Wallace, (2002), "A Benefit of Uniform Currency", working paper, The Pennsylvania State University.

Townsend, Robert M. (1987), "Economic Organization with Limited Communication", *The American Economic Review*, 77(5), 954-971.

Trejos, Alberto and Randall Wright (1995), "Search, Money, Bargaining, and Prices," *Journal of Political Economy*, 103, 118-141.

Trejos, Alberto and Randall Wright (1997), "Toward a Theory of International Currency: A Step Further," Manuscript.

Waller, Christopher (2007), "Dynamic Taxation, Private Information and Money," Manuscript, University of Notre Dame.