THE ROLE OF INFLATIONARY FINANCE WITH DISTORTIONARY TAXES

by

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ABSTRACT

This paper examines the role of inflationary finance together with the presence of distortionary taxes as a welfare-maximizing policy package. I derive an overlapping-generation model with a linear production function and a fixed required reserve ratio for the model economy under various scenarios. The results illustrate that unless the distortionary tax system is not well-formulated, inflation tax is undesirable with other distortionary taxes. The computation also found that income tax and inflation tax are almost perfect substitutes to each other. Furthermore, three comparative static studies are carried out that show, ceteris paribus; government purchases and reserve requirement are negatively correlated with optimal inflation rate, whereas productivity is positively correlated with optimal inflation rate.

DEDICATION

This Project is dedicated to my father, Handel, my mother, Helena, and my sisters, Lily and Susan.

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1 INTRODUCTION

Consider the growth rate of real money supply over a fairly long horizon, such as a decade to half a century. At this frequency, it is remarkable that both the growth rate of the "money base", though exhibited fluctuations in the very short run, are observed to be positive in almost every country. This suggests that most governments are raising some revenue from the inflation tax.[1] It is also true that most countries impose legal restrictions on money holdings. Is it possible that the presence of the very legal restrictions makes it desirable to inflate the money stock? More precisely, if an inflation tax base has been created via a given reserve requirement, would a benevolent government use the inflation tax as a source of revenue? Would government use the inflation tax as a major source of revenue when a distortionary revenue source is available? The use of seigniorage revenue as a source of government revenue varies from time to time and from country to country.[2] This leads us to an interesting question: what is the optimal composition of government revenue? Specifically, what percent of government revenue should come from an inflation tax? More importantly, what is the optimal rate of inflation?

There is already a large literature on inflationary finance and issues of optimal taxation to which these questions are addressed or related.[3] In a stationary setting with infinitely lived agents, the Friedman rule, for example, stipulates that a Pareto efficient allocation of resources in an economy can be supported by a policy that sets the money growth rate equal to the subjective time-rate of preference of its agents. In the presence of discounting, the Friedman rule thus requires that the money supply should contract.[4] Wallace (1980) studies a similar question in an overlapping-generations economy. With money as the only store of value, Wallace demonstrates

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[1] See Click (1998) for some evidence on money growth rates across countries.
[2] See Fisher (1982) for cross-country accounting of revenue from seigniorage.
[3] See Chari, Christiano, and Kehoe (1996) for a treatment of the issues.
[4] See Chari, Christiano, and Kehoe (1996) for details.

that the Pareto efficient allocation can be supported by shrinking the money stock so as to equate the returns to storage and money. The Friedman and the Wallace results hold for economies in which either the government does not need to finance any spending or if it does, non-distortionary taxes and transfers are available for that purpose. One thing is then clear: if non-distortionary taxes are available and if governments followed Pareto efficient monetary policies, we should expect to see non-positive money growth rates. This leads us to the question: why is positive money stock growth still observed among countries over time? Is it because non-distortionary taxation does not exist in reality?[5] If so, what is the optimal role of deficit financing (inflation tax financing) when only distortionary taxes are available? Both Phelps (1973) and later, Helpman and Sadka (1979), derive conditions in which seigniorage is part of an optimal policy package with other distortionary taxes. Specifically, Helpman and Sadka use an overlapping generation framework to present two versions of their model, one in which money is the only store of value and one in which money co-exists with bonds. They successfully showed that if lump-sum taxes were available in their world, then it is apparent that for both versions, the use of seigniorage would not be desirable. Now we are led to the remaining puzzle: what role should inflation tax be taking when only distortionary taxes are available?

In this paper, I try to justify the place of seigniorage in a welfare maximizing fiscal tool kit that excludes non-distortionary taxes. I investigate the optimal financing of government's budget when only distortionary taxes are available. More specifically, I use the tax on consumption and the tax on income. Here, however, due to the specification of the production function, the income tax is in fact a tax on the capital income. This is because the return to capital is obtained from the linear production function and thus, the income tax imposed is directly effecting on the amount of capital invested. Also I consider taxation and money creation as either an alternative or a complementary means of financing in an attempt to address the question.

[5] Phelps (1973) has abandoned the assumption of unavailability of non-distortionary taxes.

Unlike Helpman and Sadka, however, I solve the problem of return-dominance of money by fixing a reserve requirement.

In section 2, I develop a model economy of overlapping generations in which money serves as a store of value. With the government fiscal and monetary policy fully parameterized, I try to solve the consumption and the saving choice problem of individuals by maximizing their lifetime utility subject to their budget constraints. I then derive the optimal steady state financing of the government's budget for this model economy where government, taking individuals' "policy reaction/utility maximization response", chooses the optimal composition of revenue to maximize the lifetime welfare of the individuals. In this particular study, however, I only study the three specific cases for the simplicity of model calibration: (i) consumption tax and inflation tax; (ii) income tax and inflation tax; (iii) inflation tax together with both the consumption and the income tax. In section 3, the results of the model calibration are illustrated. By taking some of the exogenous parameters as given, I use the numerical results from the literature to as given parameters to run the experimentation so as to obtain the relationships between inflation tax with the government fiscal policy positions, and also with the individuals' well-being. Some conclusion can be drawn from the calibration results. As we shall see, inflationary finance is in fact a superfluous policy instrument. With everything else unchanged, given a well developed income tax policy, inflationary finance can be replaced by income taxes, with the welfare position staying constant. On the other hand, if the income tax policy is not so well implemented or if the policy is not well formulated, it might be a Pareto improvement to have positive inflationary taxation implemented as part of the source of the government revenue. Here, the problem of a non-optimal income tax policy might arise when the technology of tax collection is not advanced enough or when the prevention of tax evasion are effective. This optimal inflation rate, however, is positively affected by the productivity parameter and is negatively affected by both the reserve requirement and the proportion of government expenditure. Finally, in section 4, I try to continue with the calibration of the model from section II to perform some comparative statics. Here, the ceteris paribus effect of the changes of real government purchases, reserve requirement and productivity on the optimal inflation rate are investigated separately.

2 THE MODEL

2.1 Individuals

I develop an overlapping-generations model of identical (within generation) individuals where every individual lives for two periods. In the first period, an individual consumes and saves. This is the so-called "young" period of the individual. In the second period the person consumes the fruit of what he or she has saved when "young". All individuals are assumed to have identical preferences over their young-age and old-age consumption summarized by a time-separable utility function:

$$U(c_{1t}, c_{2t}) = u(c_{1t}) + v(c_{2t})$$

In this model, young agents receive a fixed endowment, y, of the consumption good, while the old ones receive nothing. Thus, in order for a person to be able to live when he or she gets old, the person must save some of his or her endowments for the " future". Savings are primarily held in the form of money and capital. Capital savings yield future consumption by a linear production function $zf(k_t) = zk_t$.[6] The return on money savings is the inverse of inflation and we assume that money saving is always return dominated by capital. Thus, agents hold money solely to satisfy an irremovable legal restriction (namely, reserve requirement). According to this legal restriction, following Bhattacharya and Haslag (2001), minimum money holdings depend on capital savings given the required reserve ratio. This required reserve ratio could be viewed as the minimum reserve required to be held by financial institution. If the required reserve

[6] Follow Bhattacharya and Haslag((2001).

ratio is γ, then minimum money holdings should satisfy the restriction: $q^D = \frac{\gamma}{1-\gamma}k_t = k_t\sigma$.[7]

Individual's consumption, capital saving and money saving are then restricted by the following first and second period budget constraints:

$$P_t(1+\tau)c_{1t} + P_t k_t + P_t q^D = P_t y \longrightarrow c_{1t} = \frac{y - k_{1t} - q^D}{(1+\tau)}$$

$$P_{t+1}(1+\tau)c_2 = P_{t+1}(1-\tau_i)(zf(k_t)) + P_t q^D \longrightarrow c_2 = \frac{z(1-\tau_i)f(k_t) + \frac{P_t}{P_{t+1}}q^D}{(1+\tau)}$$

where τ is the consumption tax on both the young and old; τ_i is the income tax on the old. Plugging in the linear production function and the legal restriction yields:

:

Individual's budget constraints:

$$P_t(1+\tau)c_{1t} + P_t k_t + P_t k_t \sigma = P_t y \longrightarrow c_{1t} = \frac{y - k_t(1+\sigma)}{(1-\tau)}$$

$$P_{t+1}(1+\tau)c_2 = P_{t+1}(1-\tau_i)(zk_t) + P_t k_t \sigma \longrightarrow c_{2t} = \frac{k_t(z(1-\tau_i) + \Pi_{t+1}^{-1}\sigma)}{(1-\tau)}$$

where Π_{t+1}^{-1} is the inverse of gross inflation.

Notice that the income tax, τ_i, is a tax on the production formation from the capital investment made during the first period under the given technology, z. this can be interpreted as the tax on capital savings from period one or as the tax on the capital income plus the principal involved. Due to the fact that the model here does not take into account the labour activity, the income tax is not a tax on the labour earnings but solely on the purpose of taxing the capital investment.

One thing important to be noted here, is that we do not have income tax on the young because the income of the young is a lump-sum endowment. This is very true if we think of the

[7] See Bhattacharya, Joydeep, and Haslag, Joseph (2001), "On the use of the inflation tax when non-distortionary taxes are available" for the derivation.

"young" period as the intensive human capital formation period. The idea is that youngsters during this period are earning nothing but acquiring intensively education and training so as to prepare themselves for their career. Hopping into the second period, these youngsters in period are all adults now and they have turned their savings using their human capital into "fruits" through the linear production function. Moreover, if we impose income tax on that lump-sum endowment, we are levying a lump-sum tax, which is not the idea of the model in this paper.

In the steady state, a young individual, taking the government policy as given, i.e. taking the parameters τ, τ_i and Π as given, computes his or her optimal saving and consuming choices so as to maximize his lifetime utility subject to his budget constraints:

Utility Maximization:

Max $W = u(c_{1t}) + v(c_{2t})$, subject to budget constraints, that is,

$$\text{Max } W^1 = u(\frac{y - k_t(1+\sigma)}{(1-\tau)}) + v(\frac{k_t(z(1-\tau_i) + \Pi_{t+1}^{-1}\sigma)}{(1-\tau)})$$

$$F.O.C: -u'(\frac{y - k_t(1+\sigma)}{(1+\tau)})(\frac{1+\sigma}{1+\tau}) + v'(\frac{k_t(z(1-\tau_i) + \Pi_{t+1}^{-1}\sigma)}{(1+\tau)})(\frac{(z(1-\tau_i) + \Pi_{t+1}^{-1}\sigma)}{(1+\tau)} = 0 \longleftrightarrow k_t^*$$

(I)

The solution to the above $F.O.C$ yields the optimal capital investment k_t^* and the thus ordinary consumption demand functions for both periods. k_t^* can be given by a function of "policy parameters", that is, $\tau;\tau_i;\Pi^{-1}$; and "economic parameters", that is z,σ; such that:

$$k_t^* = k(\tau;\tau_i;\Pi^{-1};z,\sigma)$$

(II)

The ordinary consumption demand functions are then given by:

$$c_{1t}^* = \frac{y - k_t^*(1+\sigma)}{(1+\tau)}$$

(III)

$$c_{2t}^{*} = \frac{k_t^{*}(z(1-\tau_i)+\Pi_{t+1}^{-1}\sigma)}{(1+\tau)}$$

(IV)

Using **(II)**, we can further reduce these expressions to:

$$c_1^{*} = c_1(\tau;\tau_i;\Pi^{-1};z,\sigma)\,;\; c_2^{*} = c_2(\tau;\tau_i;\Pi^{-1};z,\sigma)$$

2.2 Government and the Market

In this model the government is assumed to be purchasing in each period a required and fixed quantity, g. The government purchase is not affecting the optimal choice of individuals' consumption and saving behaviours of the representative agent.[8] We might think of such expenditure as foreign aid or national defence. Government expenditure is financed by a combination of (a) consumption taxes, (b) income taxes, and (c) inflation tax by monetary expansion, that is to say that the government increases the money supply at a fixed rate of α - 1, where α is the gross annual growth rate of money supply.

MKT clearing condition:

Since the government increases the money supply at a fixed rate of α - 1, we have:

$$M_t = \alpha M_{t-1}\;;\; \Delta M = (1-\frac{1}{\alpha})M_t$$

(V)

The money market clearing condition, $M^D = M^S$, also yields:

[8] The government expenditure is not affecting the consumption and the saving decision of the individuals because the per capita government purchase, g, is separable in the utility function. Due to this specification, the Marginal Rate of Substitution (MRS) is independent on g.

$$v_t M_t = N_t(k_t \sigma) \qquad v_t = \frac{N_t(k_t \sigma)}{M_t}$$

(VI)

Dividing the money stock of the next period by the current period using **(V)** and **(VI)** gives us:

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(k_{t+1}\sigma)}{M_{t+1}}}{\frac{N_t(k_t \sigma)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{\alpha} = \frac{1}{\Pi} = \Pi^{-1}$$

(VII)

By assuming constant population ($N_t = N$ for every period t) and constant growth rate of money supply in each period, the stationary solution for the steady state can be obtained by setting $k_{t+1} = k_t$ for all t. Thus, all the subscript t is dropped and we have here, the gross rate of money growth, α, equals to the gross rate of inflation, Π.

Government budget constraint:

Per capita government revenue from inflation tax, $g_{\inf}$, is defined as:

$$g_{\inf} = v_t \Delta M / N_t$$

Plugging in **(V)**, **(VI)** and **(VII)** yields:

$$g_{\inf} = (1 - \Pi^{-1}) k\sigma$$

Thus, the government's budget constraint can be given as:

$$g = (1 - \Pi^{-1}) k^* \sigma + \tau(c_1^* + c_2^*) + \tau_i z k^*$$

The government, taking the individuals "policy reaction functions", namely, equation **(I)**, (II), and (III), as given, chooses the optimal combination of the inflationary financing and the distortionary taxation, subjected to its budget constraint, so as to maximize the welfare of the current and future generations in a stationary setting:

$$\text{Max } W = u(c_1(\tau, \tau_i, \Pi^{-1})) + v(c_2(\tau, \tau_i, \Pi^{-1})) \text{ subject to budget constraints}$$

In reality, the costs of administration for different types of taxes vary greatly. Inflation tax is likely to be the one with the lowest cost among the candidates because of its direct impact on the economy. Levying inflation tax requires no enforcement and the only thing that the government basically has to do is to print the notes. On the contrary, administrative cost of income tax is extremely high, especially for those poorer nations that lack the extensive informational infrastructure required to enforce such tax. Besides, a well-formulated income tax system is very difficult to be done.

Here, I simply assume that there are only two types of taxes available in the model. Since the focus of the study is the role and the impact of the inflation tax in the economy, I will consider three cases. To gather intuition, I will first look at (A) Income tax and Inflation Tax and (B) Consumption tax and Inflation tax. These will allow us to understand the nature of the relationship between inflation tax and the two distortionary taxations and more importantly, showing us how they work together. After that, I am going to look at the scenario where all the three taxes are in existence. Here, using the intuition earned from the previous sections, we will try to comprehend the result of the role of inflation tax in this hypothetical economy.

2.3 Income tax and inflation tax

The government budget constraint becomes:

$$g = (1 - \Pi^{-1})k^{*}\sigma + \tau_i z k^{*}$$

The optimization problem becomes:

$$\text{Max } W = u(c_1(\tau_i, \Pi^{-1})) + v(c_2(\tau_i, \Pi^{-1})) \text{ subject to budget constraints}$$

From the budget constraint, we can rewrite τ_i as a function of Π^{-1}:

$$\tau_i = \tau_i(\Pi^{-1})$$

(VIII)

By substituting **(VIII)** into the welfare function, we obtain:

$$W(\Pi^{-1}) = u(c_1(\Pi^{-1})) + v(c_2(\Pi^{-1}))$$

Now, differentiate W with respect to Π^{-1} yields:

$$\frac{\partial W}{\partial \Pi^{-1}} = u' c'_{1\Pi^{-1}} + v' c'_{2\Pi^{-1}} = 0 \rightarrow \Pi^{-1*} = \Pi^{-1}(z, g, \sigma, y)$$

(IX)

Here, we have the optimal inverse of inflation as a function of the four exogenous parameters: technology, z, government expenditure, g, money holdings ratio, σ, and endowment, y. This means that the optimal inflation rate for the economy will be determined by these parameters. Thus, the optimal inflation rate in this case is given explicitly by equation **(XI)**.

2.4 Consumption tax and inflation tax

The government budget constraint becomes:

$$g = (1 - \Pi^{-1})k^* \sigma + \tau(c_1^* + c_2^*)$$

The optimization problem becomes:

Max $W = u(c_1(\tau, \Pi^{-1})) + v(c_2(\tau, \Pi^{-1}))$ subject to budget constraints

From the budget constraint, we can once again rewrite τ as a function of Π^{-1}:

$$\tau = \tau(\Pi^{-1})$$

(X)

By substituting **(X)** into the welfare function we obtain:

$$W(\Pi^{-1}) = u(c_1(\Pi^{-1})) + v(c_2(\Pi^{-1}))$$

Now, differentiate W with respect to Π^{-1}, yields:

$$\frac{\partial W}{\partial \Pi^{-1}} = u' c'_{1\Pi^{-1}} + v' c'_{2\Pi^{-1}} = 0 \rightarrow \Pi^{-1*} = \Pi^{-1}(z, g, \sigma, y)$$

(XI)

Again, we have the optimal inverse of inflation as a function of the four exogenous parameters: technology, z, government expenditure, g, money holdings ratio, σ, and endowment, y; meaning that the optimal inflation rate for the economy will be determined by these parameters. Thus, the optimal inflation rate in this case is given explicitly by equation **(XI)**.

The optimal inflation rate in these two cases might differ even if other parameters, z, g, σ remain constant. This is because the distortionary tax functions **(VII)** and **(X)** are likely to differ from each other. This result is consistent with the idea that monetary policy depends crucially on the nature of fiscal policy. There are many other factors, along with z, g, σ, that affect the determination of the optimal inflation. Such factors might include price and income elasticities of consumption, social stability, and political background. For simplicity, these factors are not considered in the model and I will only focus the roles of z, g, σ on the government's choice problem.

2.5 Inflation tax with both Income and Consumption tax

The setting of the problem in this case is the same as the one derived at the very beginning of **Section I**. Here, we have equation **(I)** as the first order condition for the utility maximization problem; equation **(III)** and **(IV)** addressing the ordinary consumption demands; equation **(V)**, **(VI)** and **(VII)** for structure of the government budget constraint; and equation **(IX)** showing the derivation for the function of the optimal rate of inflation.

The government budget constraint becomes:

$$g = (1 - \Pi^{-1}) k^* \sigma + \tau (c_1^* + c_2^*) + \tau_i z k^*$$

The optimization problem becomes:

Max $W = u(c_1(\tau, \tau_i, \Pi^{-1})) + v(c_2(\tau, \tau_i, \Pi^{-1}))$ subject to budget constraints

From the budget constraint, we can once again rewrite τ as a function of Π^{-1}:

$$\tau = \tau(\Pi^{-1})$$

By substituting the above into the welfare function, we obtain:

$$W(\Pi^{-1}) = u(c_1(\Pi^{-1})) + v(c_2(\Pi^{-1}))$$

Now, differentiate W with respect to Π^{-1}, yields:

$$\frac{\partial W}{\partial \Pi^{-1}} = u' c'_{1\Pi^{-1}} + v' c'_{2\Pi^{-1}} = 0 \rightarrow \Pi^{-1*} = \Pi^{-1}(z, g, \sigma, y)$$

Once again, the inverse optimal inflation rate is obtained explicitly as a function of the four exogenous variables and thus, the optimal rate of inflation is determined.

3 SIMULATION AND RESULTS OF THE MODEL

Let the lifetime utility of an individual be represented by:

$$U(c_1, c_2) = \ln(c_1) + c_2$$

The reason for a quasi-linear utility function to be used is because in our model, the consumer only concern about the utility generated from the consumptions of the first and the second period. A third period does not exist. Thus, diminishing marginal rate of substitution is in effect when the representative agent is consuming more and more today as he or she is also concerned about the consumption of the next period. However, since period 2 is the last period before the end of the world, he or she will always be happier to consume more. This is reflected by the fact that the marginal utility of second period's consumption is 1. The marginal rate of substitution depends only on period 1's consumption. This is the major advantage of using the quasi-linear utility function. With constant marginal utility of period 2's consumption, the relative price for the consumption in the second period is not going to be affected by the volume of consumption demand of period one. With no labour activity, savings is the only mechanism to deal with the trade-off between the consumption in period one and in period two. In fact, the consumer is going to keep on consuming in period 1 until the marginal utility of period 1 equals to the marginal utility of period two equal to 1. Thus, quasi-linear utility function makes it easy to perform analysis.

3.1 Income tax and inflation tax

Individual's budget constraint:

$$c_1 = y - (1 + \sigma)k$$

$$c_2 = ((1-\tau_i)z + \Pi^{-1}\sigma)k$$

Choice problem:

$$W = \ln(c_1) + c_2$$

$$F.O.C: \frac{-(1+\sigma)}{y-(1+\sigma)k} + (1-\tau_i)z + \Pi^{-1}\sigma = 0 \rightarrow k^* = \frac{y}{1+\sigma} - \frac{1}{(1-\tau_i)z + \Pi^{-1}\sigma}$$

(XII)

Government budget constraint:

$$zk\tau_i + (1-\Pi^{-1})k\sigma = g \rightarrow z\tau_i - \Pi^{-1}\sigma = \frac{g}{k} - \sigma$$

(XIII)

Substituting **(XIII)** into **(XII)**, we get:

$$k^* = \frac{y}{1+\sigma} - \frac{1}{z - \frac{g}{k^*} + \sigma}$$

(XIV)

From **(XIV)**, we can see that k^* is only a function of y, g, z, σ, that is, k^* does not depend on τ_i, the income tax rate, nor Π^{-1}, the inflation rate. Given y, g, z, σ, k^* is a constant.[9] The intuition behind this is fairly straightforward. On the one hand, the inflation tax is a tax on the old. The tax base is $k\sigma$. Since σ is exogenous to individuals, σ can be dealt with as part of the rate of the tax. Thus the tax base is actually k^*. On the other hand, income tax is also a tax on the old. The tax base is also k^*. This in a sense gives us the idea that the income tax and the inflation tax are perfect substitutes. In fact, as mentioned in the footnote, the result of the optimal capital saving function would be different if a different utility function is used. However, the two taxes, inflation

[9] This result is due to the functional form of the utility function specified. With the quasi-linear utility function, the marginal rate of substitution between the consumptions of the two periods is independent from the period 2's consumption. Since the income tax is only effective on the yield of the capital investment from period one, the optimal capital saving, k^*, is thus independent from the income tax rate. However, the result would not be the same if a different functional form of the utility function were used.

tax and the income tax, are still substitutes because both taxes are effective only on the second period generation. This lead us to the fact that individuals are indifferent between the two taxes, especially when these two taxes are the only taxation available.[10] As a result, individuals are indifferent between the two. More importantly, individuals are not able to choose the types of tax to be subjected to by choosing his k^* due to the fact that when one unit of k is being taxed by income tax, a fixed proportion σ of it is being taxed by inflation. Thus, whatever k individual chooses to save in the first period, as he or she gets old, the government will take a quantity of goods, g, away from the fruit of k. Therefore, the choice of k^* is not influenced by different compositions of τ_i and Π^{-1}. The consumption c_1 and c_2 are indifferent to government choice of τ_i and Π^{-1}, and so is the welfare. The conclusion here is that when income tax can be well implemented, inflationary finance should be abandoned.

Let's go over a numerical example. Here, I let the parameters of the economy be as follows: y = 2 (the endowment), g = 0.12 (the government expenditure), r = 0.173 (the reserve requirement), z = 1.07 (overall productivity) and the money holding requirement coefficient $\sigma = \frac{r}{1-r}$. Using GAUSS[11], I compare the welfare of individuals at different inflation rate given level of government purchase g. The following table and graph depicts the results:

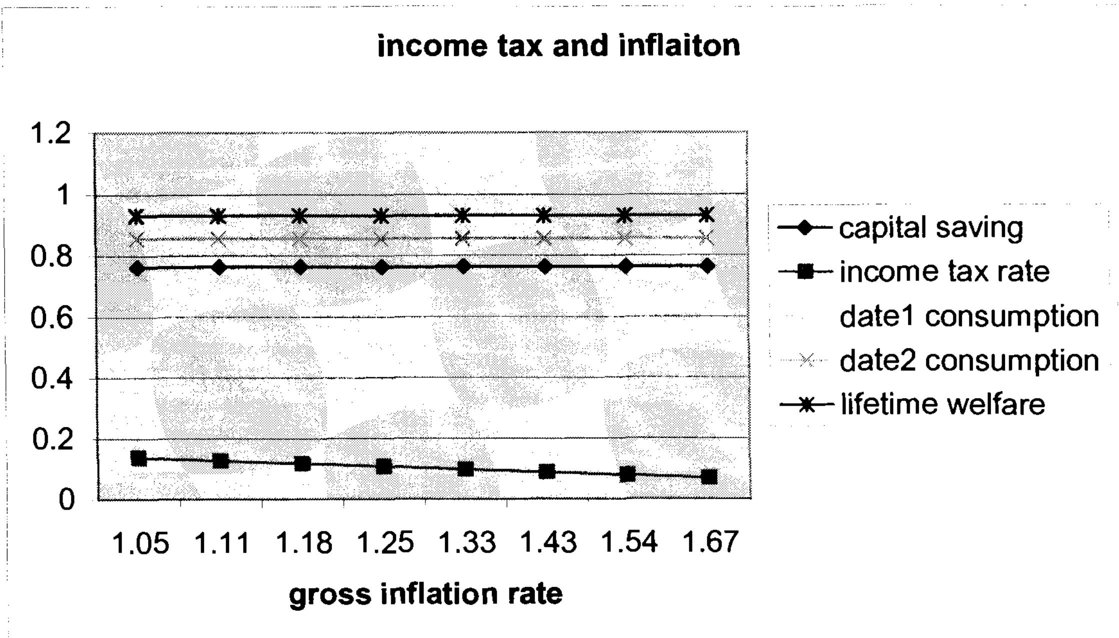
Table 1 Calibration results for Inflation Tax and Income Tax

Inflation	k	τ_i	c_1	c_2	Welfare
1.052632	0.762603	0.137286	1.077868	0.855514	0.930499
1.111111	0.762603	0.127511	1.077868	0.855514	0.930499

[10] In general, as long as the elasticities of taxation are the same for both taxes, the government is still going to use both with the same proportion because the two taxes are going to affect the tax base by the same proportion.
[11] Code available in the Appendix for reference.

1.176471	0.762602	0.117736	1.077869	0.855513	0.930499
1.25	0.762603	0.107961	1.077869	0.855513	0.930499
1.333333	0.762603	0.098185	1.077868	0.855514	0.9305
1.428571	0.762606	0.088408	1.077865	0.855519	0.930501
1.538462	0.762602	0.078635	1.077869	0.855513	0.930499
1.666667	0.762603	0.06886	1.077869	0.855513	0.930499

Figure 1 Calibration results for Inflation Tax and Income Tax



The results of this numerical experiment are consistent with what we have predicted. Since the MRS is independent of the second period's consumption, k^* is independent of the income tax rate and thus the consumption decisions over the two periods are constant. Also, the decisions to save and to invest are also independent from the inflation rate. Thus, with a linearly inverse relationship between the inflation rate and the income tax rate, the welfare levels remain unchanged.

Given g, when the inflation rate changes, k, c_1, c_2 and W remain constant. What changes with inflation rate is the rate of income tax, τ_i. It is observed that there is an inverse relationship between the income tax rate and the inflation rate. This is very true in a sense that when inflation decreases (increases), τ_i has to go up (down) in order to finance the fixed government purchase, g. The conclusion drawn here is consistent with the study by Helpman and Sadka. They concluded that inflationary finance is not optimal in the case where government can impose different rate of consumption tax on the young and the old. Indeed, monetary policy depends greatly on the nature of fiscal policy. When certain taxes are well available, inflation tax is not necessary. Notice that the results are sensitive to the linear production function.

3.2 Consumption tax and inflation tax

Assume that the rates of consumption tax are the same for both young and old. (This is very sensible, in fact.)

Individual's budget constraint:

$$c_1 = \frac{y-(1+\sigma)k}{1+\tau} \qquad c_2 = \frac{(z+\Pi^{-1}\sigma)k}{1+\tau}$$

Choice problem:

$$W = \ln(c_1) + c_2$$

$$F.O.C: \frac{1}{c_1}\left(\frac{-(1+\sigma)}{1+\tau}\right) + \frac{(z+\Pi^{-1}\sigma)}{1+\tau} = 0 \rightarrow k^* = \frac{y}{1+\sigma} - \frac{1+\tau}{z+\Pi^{-1}\sigma}$$

(XV)

We can see from equation **(XV)** that when other things being constant, as τ increases, k^* decreases. The intuition behind this is again, straightforward. Increase in the consumption tax, τ, will lead to a decrease in people's disposable income and they will thus have less to save and spend. Other things being constant, as Π^{-1} increases, k^* increases. The idea is that a decrease in inflation rate is equivalent to a corresponding increase in the return of money holding. People worry less about their money holdings and thus save more.

Different from the previous case is that with the income tax, the (capital) income tax and the inflation tax is taxing on the same tax base, *k*. Nonetheless, now with income tax replaced by the consumption tax, distortion across the two periods appears. The consumption tax, τ, and the inflation tax have different tax base. The existence of the consumption tax distorts the decision to consume over the two periods. That is why with the spread of the distortion across the two tax base, we will be able to find the optimal level of inflation tax rate for maximized utility.

Government budget constraint:

$$\tau(c_1(\tau,\Pi^{-1}))+(c_2(\tau,\Pi^{-1}))+(1-\Pi^{-1})k^*(\tau,\Pi^{-1})\sigma = g$$

Using the government budget constraint, τ can be expressed as a function of z,g,σ,Π^{-1},y:

$$\tau = \tau(z,g,\sigma,\Pi^{-1},y)$$

Thus, we can derive the welfare as a function of z,g,σ,Π^{-1},y:

$$W = w(z,g,\sigma,\Pi^{-1},y)$$

$$F.O.C: \frac{\partial W}{\partial \Pi^{-1}} = \frac{\partial W(z,g,\sigma,\Pi^{-1},y)}{\partial \Pi^{-1}} = 0$$

(XVI)

Here, the optimal inverse of inflation and thus the optimal inflation rate in this case is given explicitly by equation **(XVI)**.

Now, let's do another numerical example with GAUSS[12]. We will then be able to picture the change in welfare at different levels of inflation. We can also observe the relationship between the consumption tax rate and the inflation tax rate. Once again, I set the default values of

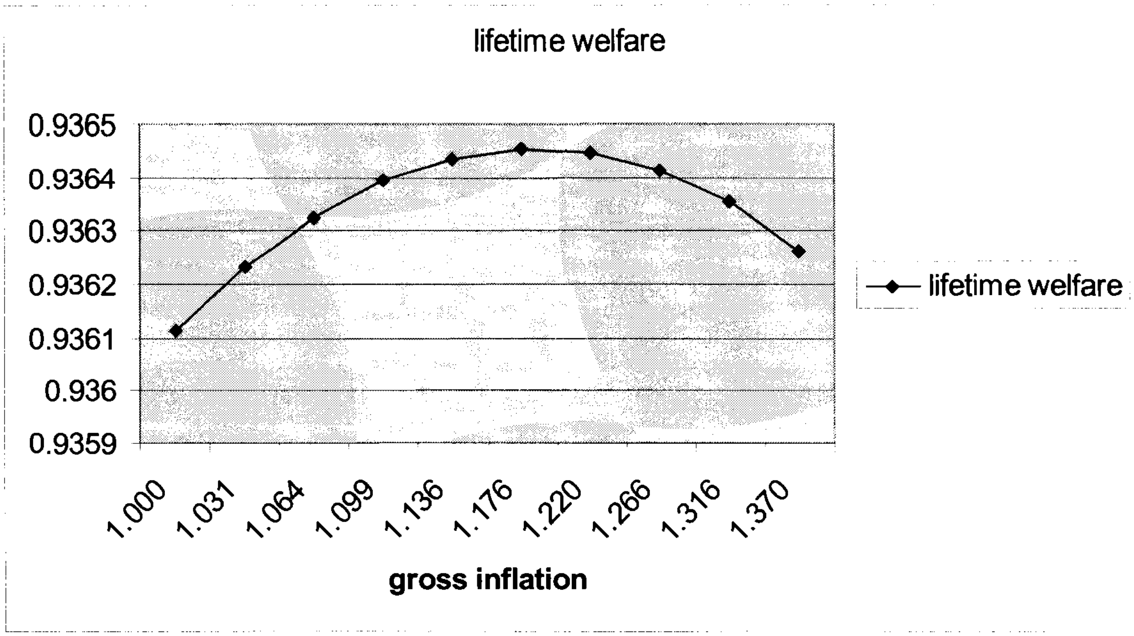
[12] Code available in the Appendix for reference.

the parameters as $y = 2$; $r = 0.173$; $z = 1.07$; and $g = 0.12$. The following table and graph depict the results:

Table 2 Calibration results for Inflation Tax and Consumption Tax

Inflation	k	τ	c_1	c_2	Welfare
1	0.82384181	0.06193008	0.94527772	0.9923912	0.9361147
1.0309278	0.82183667	0.05927262	0.94993812	0.98759045	0.93623201
1.0638298	0.81980151	0.05662802	0.9546447	0.9827414	0.93632536
1.0989011	0.81773564	0.05399658	0.95939815	0.97784334	0.93639423
1.1363636	0.81563836	0.05137859	0.96419917	0.97289551	0.93643812
1.1764706	0.81350896	0.04877438	0.96904849	0.96789714	0.93645651
1.2195122	0.81134667	0.04618426	0.97394683	0.96284744	0.93644887
1.2658228	0.80915074	0.04360856	0.97889494	0.95774561	0.93641466
1.3157895	0.80692036	0.04104763	0.98389359	0.95259084	0.93635331
1.369863	0.80465473	0.03850183	0.98894354	0.94738229	0.93626425

Figure 2 Calibration results for Inflation Tax and Consumption Tax



As seen from the graph, when inflation goes up, the welfare first increases at a decreasing rate. The optimal inflation rate is around 20%. The idea is that if government implements a higher inflation rate, collecting a larger proportion of revenue by inflation tax, the rate of consumption tax can be decreased for the government purchase level to stay constant at g. The decrease in consumption tax will stimulate the current consumption (see table above). However, raising more revenue by inflation tax is simply equivalent to extracting more resources from the old and so the future consumption goes down (see table again). Thus it is clear that by increasing inflation, government trades off individuals' "tomorrow's" consumption for "today's" consumption. Also, since people value their current consumption and their future consumption quasi-linearly (owing to the specification of the utility function at the very beginning), their welfare increases at first. But as future consumption keeps on decreasing due to the increasing rate of inflation, they are forced to consume too little so that their welfare starts to decrease.

3.3 Inflation tax with consumption tax and income tax

Finally, the core of this section. I am going to search for the optimal inflation rate in our model economy, with the presence of both the consumption tax and the income tax. Specifically, I will try to track down the optimal inflation rate under various consumption tax rates and income tax rates.

Individual's budget constraint:

$$c_1 = \frac{y-(1+\sigma)k}{1+\tau} \qquad c_2 = \frac{(z(1-\tau_i)+\Pi^{-1}\sigma)k}{1+\tau}$$

Choice problem:

$$W = \ln(c_1) + c_2$$

$$F.O.C: \frac{1}{c_1}\left(\frac{-(1+\sigma)}{1+\tau}\right) + \frac{(z(1-\tau_i)+\Pi^{-1}\sigma)}{1+\tau} = 0$$

(XVII)

Government's Budget Constraint:

$$\tau(c_1(\tau,\tau_i,\Pi^{-1})) + (c_2(\tau,\tau_i,\Pi^{-1})) + \tau_i z k^* + (1-\Pi^{-1})k^*(\tau,\tau_i,\Pi^{-1})\sigma = g$$

Using the government budget constraint, Π^{-1} can be expressed as a function of $z, g, \sigma, \tau, \tau_i, y$:

$$\Pi^{-1} = \Pi^{-1}(z, g, \sigma, \tau, \tau_i, y)$$

Thus, we can derive the welfare as a function of $z, g, \sigma, \tau, \tau_i, y$:

$$W = W(z, g, \sigma, \tau, \tau_i, y)$$

$$F.O.C: \frac{\partial W}{\partial \Pi^{-1}} = \frac{\partial W(z,g,\sigma,\tau,\tau_i,y)}{\partial \Pi^{-1}} = 0 \longrightarrow \Pi^*$$

Here, indi vidual tries to maximize his o r he r ut ility subject to t he b udget c onstraint and t he government tries to maximize the welfare of the representative agent by adjusting the optimal rate of inflation tax with the presence of income tax and consumption tax. Using the first order condition above, we can solve for the optimal inflation rate at each and every level of consumption and income tax rate.

From the choice problem, we can see that the first fraction term of the *F.O.C.* is negative while the second fraction term is positive. The first term here consists only of the consumption tax. The second fraction consists of two components, the income tax and the inflation tax. We can see from this second term that increasing either the income tax or the inflation tax is going to decrease the utility level of the individual. Thus, with inflation tax and the income tax both taxing on the same tax base, k^*, the substitutability between the two taxes causes the government to focus only on one of them maximizing the welfare of the economy. However, we can tell from here that inflationary finance has no role in this model economy, even without the simulation. This is simply because of the *F.O.C.* that we have derived. With both taxes taxing on the same tax base, we can assert that decreasing the inflation tax rate gives a bigger increase in the welfare level than the same decrease in the income tax rate. This is because with the required reserve ratio smaller than 50% (which is a very realistic assumption), σ will always be smaller than one. But the z parameter is assumed to be greater than zero because we always assume positive technological growth. Thus, some tax is always preferred to the inflation tax.[13]

For the final simulation of the section[14], again, identical default values of the parameters are used: $y = 2$; $r = 0.173$; $z = 1.07$; and $g = 0.12$. This time, instead of setting a grid for the inverse of inflation, I set a grid for the 30 different income tax rates, ranging from 0.15 to 0.295, and a grid for the 30 different consumption tax rates, ranging from 0.07 to 0.215. I try to solve the

[13] Results would not be the same if a different specification of the utility functions used.
[14] Code available in the Appendix for reference.

model by letting GUASS to pick the appropriate levels of capital savings, k^*, and inflation rate, Π^* for welfare maximization.

From the GAUSS output, the result is unsurprising and obvious. Other than one particular observation, the remaining 899 results of the welfare-maximizing optimal inflation rate are all zeros. This means that inflationary finance is certainly not in any favour with the existence of consumption and income tax. Welfare levels (or utility levels) of the representative agent are the highest at the lowest rates of income and consumption tax. With increasing income tax rate, the agent shifts to consume more "today" from "tomorrow". Also, capital savings decrease with it. The results are sensible and are well expected. Most importantly, this simulation answers the question of what role inflation tax should be take with and only with the availability of distortionary taxation. In fact, this is coherent with the study done by Helpman and Sadka: the use of seigniorage revenue for government finance is not desirable under the availability of alternative taxation methods.

The result is also consistent with the study of the two distinct scenarios that we have studied previously. In the one with only income tax and inflation tax, we can see that the two taxes are prefect substitutes. In the second one, with consumption tax and inflation tax, we have the consumption tax distorting the decision to consume across the two periods and thus an optimal inflation tax is obtained. Now, with all three taxes available, the consumption tax breaks the substitutability between the income tax and the inflation tax. With the presence of the inflation tax, the required reserve ratio reduces the utility of the representative agent. This in turn affects the cost of savings, affecting the optimal rate of capital investment and therefore, the welfare of the individual. We can hereby conclude that in the presence of all the three taxation method, inflationary financing has no role in the economy for welfare maximization.

4 COMPARATIVE STATICS

4.1 Optimal Inflation with Government Expenditure

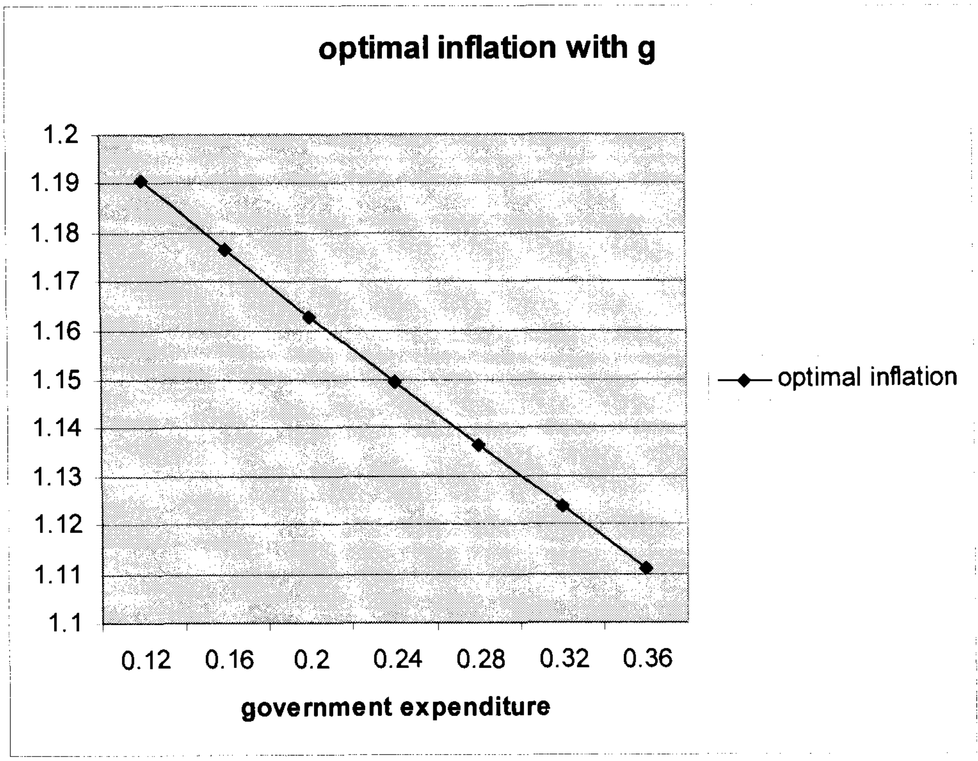
Now we come to the comparative static of the study[15]. I will hereby try to examine how the parameters g, r, z affect the determination of the optimal inflation rate, using the same calibration by GAUSS. First I change the value of g to see what happens to the optimal inflation rate.

Table 3 Comparative Static: Optimal Inflation with Government Expenditure

g	Inflation rate
0.12	1.1904762
0.16	1.1764706
0.2	1.1627907
0.24	1.1494253
0.28	1.1363636
0.32	1.1235955
0.36	1.1111111

[15] Codes available in the Appendix for reference.

Figure 3 Comparative Static: Optimal Inflation with Government Expenditure



As the government expenditure g increases, optimal inflation rate decreases. This is due to the fact that when government increases its expenditure, it is imposing a heavier tax burden on people and this will decrease their disposable income sharply. The decrease in income will lead to a corresponding decrease in capital saving, creating two different effects on the individuals' consumption level: (1) increase current consumption; (2) decrease future consumption. Now suppose that the government raises a higher inflation tax, a tax on the old. This will further decreases the second period consumption. Owing to consumption smoothing, the government will be requested to decrease inflation so as to help people in achieving a smoother consumption path and thus higher lifetime welfare.

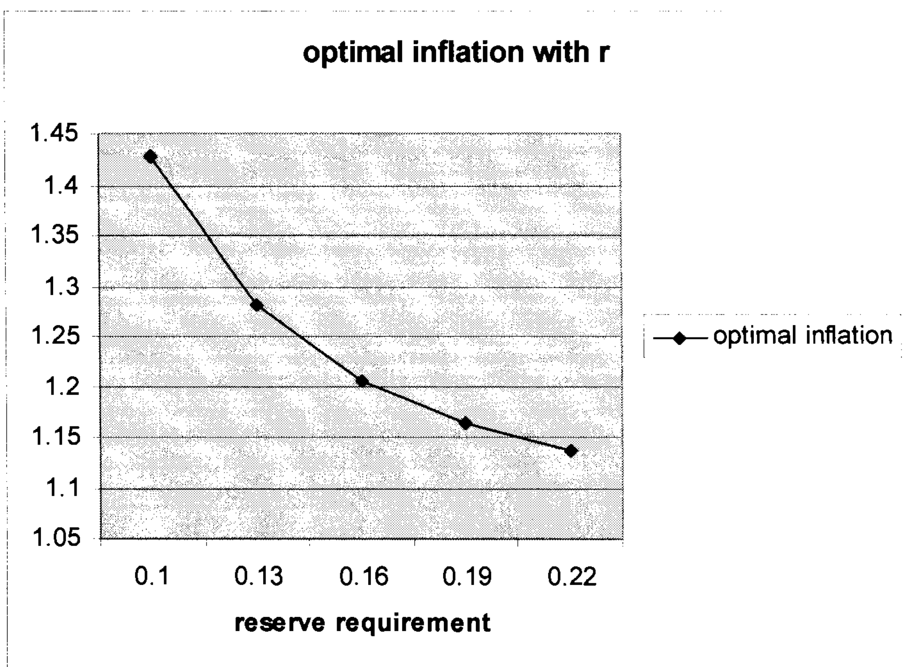
4.2 Optimal Inflation with Required Reserve Ratio

Second I change the value of r to see what happens to the optimal inflation rate.

Table 4 Comparative Static: Optimal Inflation with Required Reserve Ratio

r	Inflation rate
0.1	1.4285714
0.13	1.2820513
0.16	1.2048193
0.19	1.1627907
0.22	1.1363636

Figure 4 Comparative Static: Optimal Inflation with Required Reserve Ratio



As the reserve requirement r increases, the optimal inflation rate decreases. This observation is attributable to the following intuition behind the saving incetive of the individuals. In this model people hold positive amount of return dominated money just to satisfy the legal restriction. In this sense, the reserve requirement r distorts people's incentive to save. Increase in r intensifies this distortion and therefore the government has to decrease inflation so as to mitigate such distortion.

4.3 Optimal Inflation with Productivity

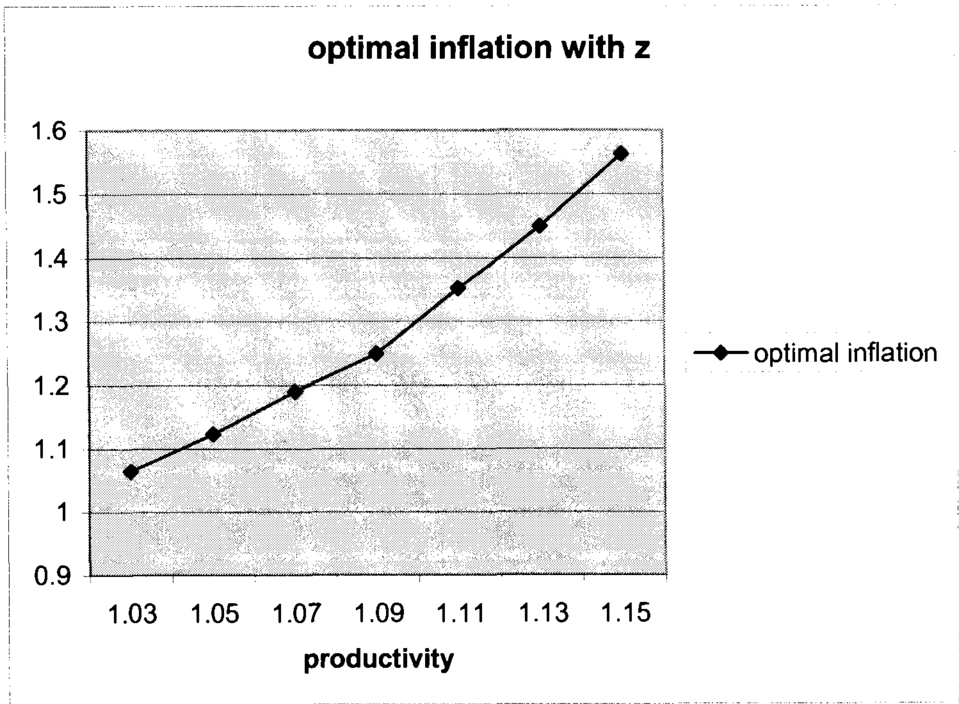
Finally, I change the value of z to see what happens to the optimal inflation rate.

Table 5 Comparative Static: Optimal Inflation with Productivity

Z	Inflation rate
1.03	1.0638298
1.05	1.1235955
1.07	1.1904762
1.09	1.25
1.11	1.3513514
1.13	1.4492754
1.15	1.5625

As the productivity parameter, z, increases, the optimal inflation rate increases. The increase in productivity causes the rate of return on k increases and therefore, capital saving increases more at first period. The consumption in the first period thus falls as people save more for the capital investment. The second period's consumption increases. In this scenario, the government should increase inflation rate to impose less tax on the young and more on the old so as to help people in achieving a smoother consumption path and hence higher lifetime welfare.

Figure 5 Comparative Static: Optimal Inflation with Productivity



5 DISCUSSION AND CONCLUSION

In this paper, I examine welfare-maximizing monetary policy in a world with reserve requirement. I derive results in a simple general equilibrium model with finitely lived agents. In the model, there is a benevolent government that must finance a fixed level of government spending through a package of distortionary taxes (income tax or consumption tax or both) and seigniorage. Interestingly, the inflation tax may be part of the welfare-maximizing policy package depending on whether the distortionary taxes system is well imposed and on whether the young are taxed or not. A tax on the young means less disposable income when young, which could potentially lower lifetime utility more than what an agent would obtain from a higher return when old. I illustrated that I demonstrate the conclusion that a benevolent government will raise a positive fraction of its revenue from the inflation tax when income tax is not available. I have also shown that with a suitable income tax system, income tax and inflation tax are very likely to be perfect substitutes. When only consumption tax is available with inflationary financing, the optimal inflation tax rate is about 20 percent. This is somewhat striking as this implies that inflationary financing has a role in the welfare-maximizing policy package. However, when both income and consumption taxes are available in the fiscal tool kit (which is more realistic), optimal inflation rates are all found to be zeros. The simulation asserts that the role of seigniorage is totally unfavourable in the mix. Why, then, is there still a widespread use of inflationary finance among the countries in the world? This is a very interesting potential research topic.

Nonetheless, I have also carried out comparative static studies to analyze respectively how productivity, official reserve requirement and the amount of government expenditure, influence the determination of the optimal inflation rate. Other things being equal, increase in productivity will increase the optimal rate of inflation; increase in the required reserve ratio will decrease the optimal rate of inflation; and increase in the amount of government expenditure will decrease the optimal rate of inflation. These conclusions can shed light on part of the reason for the widespread application of seigniorage revenue in developing countries, which lack the conditions to implement income tax; and on part of the reason for the different use of seigniorage revenue in different countries at different times.

APPENDIX

GAUSS Code and results for the simulation of Section 3 and section 4

Section 3

(I) Income tax and inflation tax

GAUSS CODE AND RESULTS:

```
» library nlsys;
y=2;
r=0.173;
segma=r/(1-r);
z=1.07;
govt=0.12;
step=-0.05;
inverflation=seqa(0.95,step,8);   /*try different inverse of inflation rate to see if anything
                                          changes accordingly*/
Nt=rows(inverflation);
k=zeros(Nt,1);
tao=zeros(Nt,1);
c1=zeros(Nt,1);
c2=zeros(Nt,1);
w=zeros(Nt,1);
alpha=zeros(Nt,1);                /*the vector for gross inflation rate*/
i=1;
do while i le Nt;
 _output=0;
 x0=0.05|0.1;
 {x,f,g,h}=nlsys(&foc,x0);
 alpha[i]=1/inverflation[i];
 k[i]=x[1];
 tao[i]=x[2];
 c1[i]=y-(1+segma)*k[i];
 c2[i]=((1-tao[i])*z+inverflation[i]*segma)*k[i];
 w[i]=ln(c1[i])+c2[i];
i=1+i;
endo;
print alpha~k~tao~c1~c2~w;

proc foc(x);
```

```
local f1,f2;
f1=x[1]-y/(1+segma)+1/((1-x[2])*z+inverflation[i]*segma);     /*individual's optimal
                                           capital saving given government policy parameters*/
f2=x[2]*z*x[1]+(1-inverflation[i])*segma*x[1]-govt;   /*government budget constraint*/
retp (f1|f2);
endp;

   1.0526316     0.76260304     0.13728592     1.0778681     0.85551433
0.93049949
   1.1111111     0.76260287     0.12751081     1.0778684     0.85551405
0.93049939
   1.1764706     0.76260247     0.11773591     1.0778688     0.85551334
0.93049913
   1.2500000     0.76260253     0.10796064     1.0778688     0.85551344
0.93049916
   1.3333333     0.76260307    0.098184955     1.0778681     0.85551442
0.93049954
   1.4285714     0.76260557    0.088407754     1.0778651     0.85551884
0.93050115
   1.5384615     0.76260248    0.078635005     1.0778688     0.85551334
0.93049913
   1.6666667       0.76260250      0.068859760        1.0778688        0.85551338
0.93049914
```

(II) Consumption tax and inflation tax

GAUSS CODE AND RESULTS:

```
» library nlsys;
y=2;
r=0.173;
segma=r/(1-r);
z=1.07;
govt=0.12;
step=-0.03;
inverflation=seqa(1,step,10);  /*try different inverse of inflation rate to look for the
                                       optimal inflation rate that maximizes the welfare*/
Nt=rows(inverflation);
alpha=zeros(Nt,1);
k=zeros(Nt,1);
tao=zeros(Nt,1);
c1=zeros(Nt,1);
c2=zeros(Nt,1);
w=zeros(Nt,1);
i=1;
do while i le Nt;
```

```
 _output=0;
 x0=0.05|0.1;
 {x,f,g,h}=nlsys(&foc,x0);
 alpha[i]=1/inverflation[i];
 k[i]=x[1];
 tao[i]=x[2];
 c1[i]=(y-(1+segma)*k[i])/(1+tao[i]);
 c2[i]=((z+inverflation[i]*segma)*k[i])/(1+tao[i]);
 w[i]=ln(c1[i])+c2[i];
i=1+i;
endo;
print alpha~k~tao~c1~c2~w;

proc foc(x);
local f1,f2;
f1=-(1+segma)/(y-(1+segma)*x[1])+(z+inverflation[i]*segma)/(1+x[2]);
f2=x[2]*((y-(1+segma)*x[1])/(1+x[2])+(z+inverflation[i]*segma)*x[1]/(1+x[2]))+(1-
inverflation[i])
*segma*x[1]-govt;
retp (f1|f2);
endp;

    1.0000000     0.82384181     0.061930084     0.94527772     0.99239120
0.93611470
    1.0309278     0.82183667     0.059272625     0.94993812     0.98759045
0.93623201
    1.0638298     0.81980151     0.056628026     0.95464470     0.98274140
0.93632536
    1.0989011     0.81773564     0.053996583     0.95939815     0.97784334
0.93639423
    1.1363636     0.81563836     0.051378598     0.96419917     0.97289551
0.93643812
    1.1764706     0.81350896     0.048774383     0.96904849     0.96789714
0.93645651
    1.2195122     0.81134667     0.046184262     0.97394683     0.96284744
0.93644887
    1.2658228     0.80915074     0.043608565     0.97889494     0.95774561
0.93641466
    1.3157895     0.80692036     0.041047637     0.98389359     0.95259084
0.93635331
    1.3698630     0.80465473     0.038501831     0.98894354     0.94738229
0.93626425
```

(III) Inflation tax with the presence of both the consumption tax and the income tax

GAUSS CODE: (Results not shown because of the huge size of output)

```
library nlsys;
y=2;
r=0.173;
segma=r/(1-r);
z=1.07;
step=0.005;
taoc=seqa(0.05,step,30);
Nt=rows(taoc);
govt=0.12;
step=0.005;
taoi=seqa(0.05,step,30);
NZ=rows(taoi);
k=zeros(Nt,NZ);
c1=zeros(Nt,NZ);
c2=zeros(Nt,NZ);
w=zeros(Nt,NZ);
alpha=zeros(Nt,NZ);
optimflation=zeros(Nt,NZ);
inflation=zeros(Nt,NZ);

i=2;
do while i le Nt;
j=2;
do while j le NZ;
  _output=0;
  x0=0.05|0.1;
  {x,f,g,h}=nlsys(&foc,x0);
  alpha[i,j]=1/optimflation[i,j];
  k[i,j]=x[1];
  optimflation[i,j]=x[2];
  c1[i,j]=(y-(1+segma)*k[i,j])/(1+taoc[i]);
  c2[i,j]=((1-taoi[j])*z+optimflation[i,j]*segma)*k[i,j]/(1+taoc[i]);
  w[i,j]=ln(c1[i,j])+c2[i,j];
  if w[i,j]<w[i-1,j-1];
  inflation[i,j]=1/optimflation[i-1,j-1];
i=30;
j=30;
endif;
j=j+1;
endo;
i=1+i;
endo;
```

```
print; "optimal inflation:" inflation;print;
print; "inflation:" alpha; print;
print; "k:" k; print;
print; "income tax:" taoi; print;
print; "con tax:" taoc; print;
print; "welfare:" w; print;
print; "c1:" c1; print;
print; "c2:" c2; print;

proc foc(x);
local f1,f2;
f1=-(1+segma)/(y-x[1]*(1+segma))+((1-taoi[j])*z+x[2]*segma/(1+taoc[i]));
f2=taoi[j]*z*x[1]+(1-x[2])*segma*x[1]+taoc[i]*(c1[i,j]+c2[i,j])-govt;
retp (f1|f2);
endp;
```

Section 4

(1) How the optimal inflation rate changes as government expenditure increases.

GAUSS CODE AND RESULTS:

```
library nlsys;
y=2;
r=0.173;
segma=r/(1-r);
z=1.07;
step=0.04;
govt=seqa(0.12,step,7);          /*change the value of govt from 0.12 to 0.36*/
Nt=rows(govt);
step=-0.01;
inverflation=seqa(1,step,40);    /*try different inverse of inflation rate to look for
the
                                 optimal inflation rate that maximizez the
welfare*/
NZ=rows(inverflation);
k=zeros(Nt,NZ);
tao=zeros(Nt,NZ);
c1=zeros(Nt,NZ);
c2=zeros(Nt,NZ);
w=zeros(Nt,NZ);
optimflation=zeros(Nt,1);        /*the vector for optimal gross inflation rate*/
i=1;
do while i le Nt;
j=2;
do while j le NZ;
```

```
 _output=0;
 x0=0.05|0.1;
 {x,f,g,h}=nlsys(&foc,x0);
 k[i,j]=x[1];
 tao[i,j]=x[2];
 c1[i,j]=(y-(1+segma)*k[i,j])/(1+tao[i,j]);
 c2[i,j]=((z+inverflation[j]*segma)*k[i,j])/(1+tao[i,j]);
 w[i,j]=ln(c1[i,j])+c2[i,j];
 if w[i,j] < w[i,j-1];                    /*w[i,j-1] is the maximum welfare under z[i]*/
 optimflation[i]=1/inverflation[j-1];          /*the optimal inflation rate under z[i]*/
 j=50;                                         /*in order to end the loop*/
 endif;
 j=j+1;
 endo;
i=1+i;
endo;
print optimflation;

proc foc(x);
local f1,f2;
f1=-(1+segma)/(y-(1+segma)*x[1])+(z+inverflation[j]*segma)/(1+x[2]);
f2=x[2]*((y-(1+segma)*x[1])/(1+x[2])+
(z+inverflation[j]*segma)*x[1]/(1+x[2]))+(1-inverflation[j])
*segma*x[1]-govt[i];
retp (f1|f2);
endp;
     1.1904762
     1.1764706
     1.1627907
     1.1494253
     1.1363636
     1.1235955
     1.1111111
```

(2) How the optimal inflation rate changes as reserve requirement increases.

GAUSS CODE AND RESULTS:

```
library nlsys;
y=2;
step=0.03;
r=seqa(0.1,step,5);         /*change the value of r from 0.1 to 0.22*/
Nt=rows(r);
segma=zeros(Nt,1);
z=1.07;
govt=0.12;
step=-0.01;
```

```
inverflation=seqa(1,step,40); /*try different inverse of inflation rate to look for the
                    optimal inflation rate that maximizez the welfare*/
NZ=rows(inverflation);
k=zeros(Nt,NZ);
tao=zeros(Nt,NZ);
c1=zeros(Nt,NZ);
c2=zeros(Nt,NZ);
w=zeros(Nt,NZ);
optimflation=zeros(Nt,1);   /*the vector for optimal gross inflation rate*/
i=1;
do while i le Nt;
segma[i]=r[i]/(1-r[i]);            /*give value to sigma as r changes*/
j=2;
do while j le NZ;
 _output=0;
 x0=0.05|0.1;
 {x,f,g,h}=nlsys(&foc,x0);
 k[i,j]=x[1];
 tao[i,j]=x[2];
 c1[i,j]=(y-(1+segma[i])*k[i,j])/(1+tao[i,j]);
 c2[i,j]=((z+inverflation[j]*segma[i])*k[i,j])/(1+tao[i,j]);
 w[i,j]=ln(c1[i,j])+c2[i,j];
 if w[i,j] < w[i,j-1];      /*w[i,j-1] is the maximum welfare under z[i]*/
 optimflation[i]=1/inverflation[j-1]; /*the optimal inflation rate under z[i]*/
 j=50;                  /*in order to end the loop*/
 endif;
 j=j+1;
 endo;
 i=1+i;
endo;
print optimflation;

proc foc(x);
local f1,f2;
f1=-(1+segma[i])/(y-(1+segma[i])*x[1])+(z+inverflation[j]*segma[i])/(1+x[2]);
f2=x[2]*((y-(1+segma[i])*x[1])/(1+x[2])+
(z+inverflation[j]*segma[i])*x[1]/(1+x[2]))+(1-inverflation[j])
*segma[i]*x[1]-govt;
retp (f1|f2);
endp;

    1.4285714
    1.2820513
    1.2048193
    1.1627907
    1.1363636
```

(3) How the optimal inflation rate changes as productivity increases.

GAUSS CODE AND RESULTS:

```
» library nlsys;
y=2;
r=0.173;
segma=r/(1-r);
step=0.02;
z=seqa(1.03,step,7);                /*change the value of z from 1.03 to 1.15*/
Nt=rows(z);
govt=0.12;
step=-0.01;
inverflation=seqa(1,step,40);          /*try different inverse of inflation rate to look for
the
                                        optimal inflation rate that maximizes the
welfare*/
NZ=rows(inverflation);
k=zeros(Nt,NZ);
tao=zeros(Nt,NZ);
c1=zeros(Nt,NZ);
c2=zeros(Nt,NZ);
w=zeros(Nt,NZ);
optimflation=zeros(Nt,1);              /*the vector for optimal gross inflation rate*/
i=1;
do while i le Nt;
j=2;
do while j le NZ;
  _output=0;
  x0=0.05|0.1;
  {x,f,g,h}=nlsys(&foc,x0);
  k[i,j]=x[1];
  tao[i,j]=x[2];
  c1[i,j]=(y-(1+segma)*k[i,j])/(1+tao[i,j]);
  c2[i,j]=((z[i]+inverflation[j]*segma)*k[i,j])/(1+tao[i,j]);
  w[i,j]=ln(c1[i,j])+c2[i,j];
  if w[i,j] < w[i,j-1];                    /*w[i,j-1] is the maximum welfare under
z[i]*/
  optimflation[i]=1/inverflation[j-1];      /*the optimal inflation rate under z[i]*/
  j=50;                                     /*in order to end the loop*/
  endif;
  j=j+1;
  endo;
 i=1+i;
endo;
print optimflation;
```

```
proc foc(x);
local f1,f2;
f1=-(1+segma)/(y-(1+segma)*x[1])+(z[i]+inverflation[j]*segma)/(1+x[2]);
f2=x[2]*((y-(1+segma)*x[1])/(1+x[2])+
(z[i]+inverflation[j]*segma)*x[1]/(1+x[2]))+(1-inverflation[j])
*segma*x[1]-govt;
retp (f1|f2);
endp;
```

```
    1.0638298
    1.1235955
    1.1904762
    1.2500000
    1.3513514
    1.4492754
    1.5625000
```

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