FORECASTING THE CANADA-US

REAL EXCHANGE RATE

by

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ABSTRACT

The wide fluctuations in the Canadian Dollar exchange rate over the last two decades have led to numerous papers attempting to explain those fluctuations and to develop a reliable forecasting technique. This paper attempts to duplicate recent work finding that an Error Correction Model incorporating commodity prices and the Canada-US interest rate differential performs better in out-of-sample forecasts than a random walk. The replication of the model using data that are more recent yields very similar results.

Extending the model to include a measure of the Canada-US bilateral current account as an additional explanatory variable increases the explanatory power of the model and its out-of-sample forecasting performance.

DEDICATION

This paper is dedicated to my friends, and especially my family, who have supported me in my "folly". To my mother Norma who has always supported me in everything I have done; to my father Denis, and brother Randall whose memories are always with me; to Keith, Linda, and Laura, whose lives in their unique ways have been an inspiration; and to my son Cory, whose life and adventures are a true source of pride.

Most of all, this is dedicated to my wife and best friend, Colleen. I could not have done this without you.

ACKNOWLEDGEMENTS

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1 INTRODUCTION

The Canadian Dollar has floated freely against other currencies in the world since the

mid-twentieth century (except for a brief period in the 1960's), longer than any other

major currency. Most of that time, the exchange rate with the US Dollar has been

relatively stable. The 1990's saw the beginning of a period of steady decline and

increased volatility however, prompting considerable debate about the choice of

exchange rate regime, and given the floating regime, methods of forecasting the exchange

rate. In real terms, the Canadian dollar lost even more value during that time. Rising US

prices relative to Canadian prices resulted in the real exchange falling even more

dramatically than the nominal exchange rate. Figure 1 shows the nominal and real

bilateral C$/US$ exchange rate for the period from 1972 to 2002.

Figure 1:   Canada-US Exchange Rate
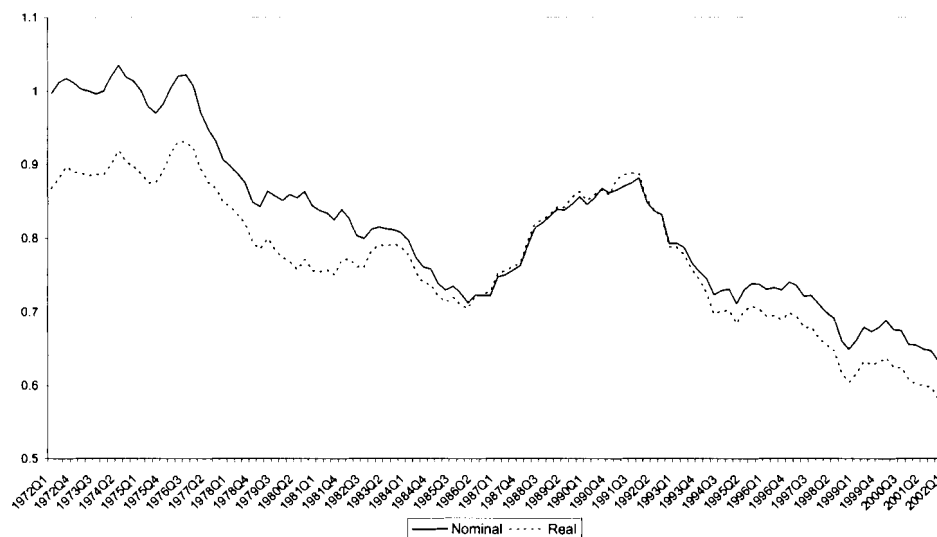
This paper examines some aspects of the forecasting debate, particularly a forecasting

model first proposed by two Bank of Canada economists, attempts to replicate that

model, and to expand on it by incorporating a variable that has attracted some discussion

of late, Canada's Current Account balance with the United States.

The first section discusses some of the background surrounding the development of

exchange rate forecasting models. The second section describes a model first developed

by Robert Amano and Simon van Norden in 1992 that is still used by the Bank of Canada

for internal forecasting.

The third section gives the results of a replication of an updated version of the Amano

and van Norden model that yields very similar results, and the fourth section describes

the inclusion of an additional explanatory variable to the model that increases the

explanatory power of the model.

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2 BACKGROUND

A significant body of research on exchange rate models in the 1980s and 1990s exposed

some significant weaknesses. Monetary exchange rate models that seemed to fit the data

for the period of the 1970s failed when extended to the 1980s. Many competing models

subsequently emerged that fit relatively well in sample, but not out-of-sample.

Meese and Rogoff (1983) find that the exchange rate could be characterized as a random

walk, and that existing monetary and portfolio-balance models could not explain out-of-

sample exchange rate movements better than a random walk. This prompted researchers

to investigate the use of newly developed unit root and cointegration methods for

coefficient estimation and testing and for out-of-sample forecasting. These methods only

confirmed the Meese and Rogoff findings because existing models did not provide a

cointegrating relationship for the exchange rate.[1] That is, monetary shocks did not appear

to explain long-run exchange rate movements.

The exchange rate can be subject to two types of shocks however: monetary and real

shocks. In a recent working paper for the Institute of Policy Analysis, University of

Toronto, Carr and Floyd (2001) found that there was no evidence that observed changes

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[1] Amano and van Norden (1993)

in the Canada/US exchange rate were caused by monetary shocks, [2] so the answer must lie in real shocks.

For real shocks to affect the exchange rate they must be asymmetric; that is they must affect the Canadian and US economies differently to cause a change in the bilateral exchange rate. There is a large amount of econometric research that shows that the two countries are subject to significant asymmetric shocks. Murray (1999) summarizes the research with the following results:

i. "The structural shocks hitting Canada, Mexico and the United States share very few common characteristics...

ii. The structural shocks hitting the nine regions of the United States are all very similar...

iii. The structural shocks hitting the six regions of Canada also share a strong common component with one another, but their contemporaneous correlation with US shocks is very small..." [3]

Given that the two economies are subject to this asymmetry, what is the shock or shocks that have caused the volatility in the Canada-US exchange rate, especially the depreciation of the Canadian Dollar in the 1990s?

The impact of terms-of-trade shocks on a floating exchange rate, especially for a small open economy such as Canada, has been accepted in theoretical models for some time. A fall in the relative world price for a country's exports as a result of a drop in world demand for those exports, will lead to a depreciation of the country's currency, *ceteris paribus*.

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[2] Carr and Floyd (2001)
[3] Murray (1999) p 11

Table 1 shows that there is a significantly large correlation difference in the terms-of-trade and oil and commodity prices between Canada and the USA.

Table 1:   Absolute and relative terms of trade for Canada and the United States

	Canada	United States
Absolute terms of trade variability [a]	7.7	12.1
Absolute terms of trade correlation with G-10 [b]	-0.85	0.63
Relative terms of trade variability [c]	17.5	9.1
Absolute terms of trade correlation with oil price	0.85	-0.89
Absolute terms of trade correlation with non-oil commodity price	0.87	-0.92

a. Standard deviation of the terms of trade for each country
b. Calculated as the correlation against the trade-weighted average terms of trade of the other G-10 countries plus Switzerland
c. Terms of trade relative to a trade relative to a trade-weighted average of the other G-10 countries plus Switzerland
Source: Murray (1999)

Since Canada and the United States represent the largest bilateral trade relationship in the world, it is reasonable to expect that there may be terms of trade effects on the bilateral exchange rate.

Commodities make up a large share of Canada's total exports. While that share has been steadily declining, from 55% in the 1970s to 37% in the 1990s, their share of GDP has remained constant at 11%,[4] while the USA is a net importer of commodities.

Figure 2 graphs Canada's real import price index against the real price index for its commodity exports. As can be seen, the real price of Canada's commodity exports is

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[4] Laidler and Aba (2001) p 8

much more volatile than the price of its imports, more than 60% of which come from the

United States[5]. This volatility should affect the bilateral exchange rate.

Figure 2:   Real Import Price Index and Export Price Index for Canada
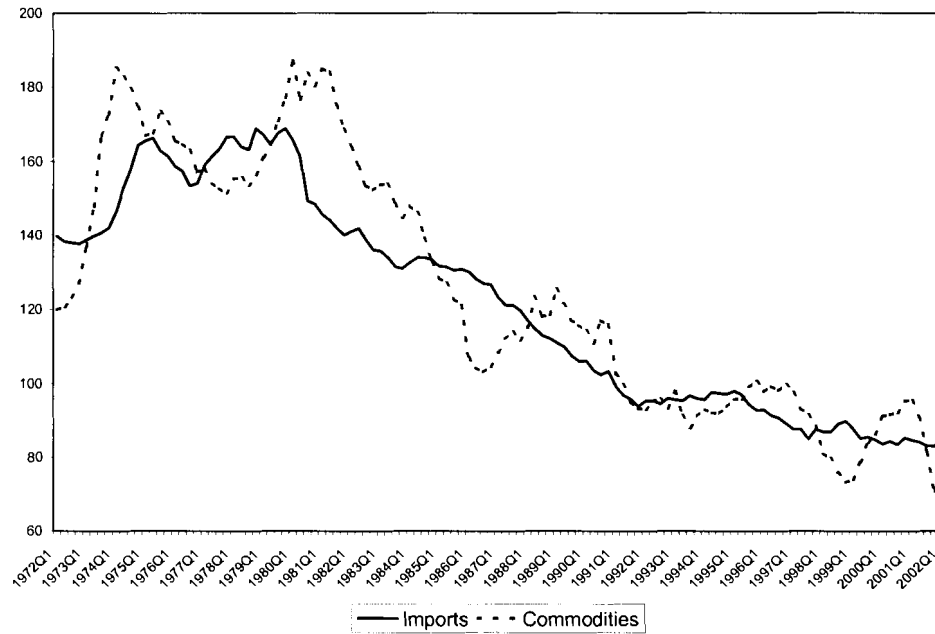
Since Canada is small relative to world markets, its terms of trade might be expected to

behave exogenously.

These facts led some researchers to focus on the impact of commodity prices on the

U.S./Canada bilateral exchange rate - however early attempts to find an empirical link

met with little success.[6]

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[5] CANSIM V212329 & V212571
[6] See Lafrance and Longworth (1987)

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Amano and van Norden (1993) found that energy prices played a significantly different role from non-energy commodity prices and separated them in their analysis. Figure 3 shows the real price index of energy and non-energy commodities against Canada's import price index. "The result is that [they] find a cointegrating relationship for the real exchange rate, coefficients that appear stable over the last decade, and a very simple exchange rate equation that generally forecasts better than a random walk out-of-sample."[7]

Figure 3: Real Energy, Non-Energy and Import Price Indexes
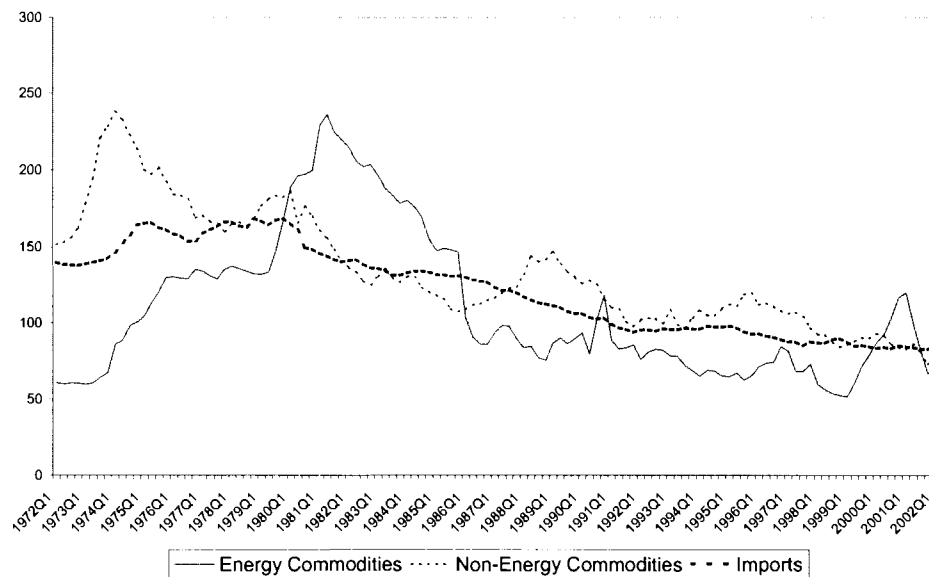
[7] Amano and van Norden (1993) p. 210

3 FORECASTING EQUATIONS

3.1 Amano and van Norden (1993 and 1995)

The presence of a unit-root in the real exchange rate and commodity price data will result in biased estimation using traditional econometric techniques. In a Bank of Canada working paper (a version was published in the Journal of International Money and Finance in 1995) Amano and van Norden (1993) develop a simple exchange rate equation using an Error Correction Model (ECM) that forecasts better than a random walk out-of-sample. Using modern unit-root and cointegration techniques, they use commodity prices to capture the long-term effects on the exchange rate and a measure of the Canada/US interest rate differential to reflect the deviation of the real exchange rate from its expected long-run level.

In their analysis, the authors use monthly data for all variables. As a measure of the real exchange rate (RPFX), they use the bilateral exchange rate with the United States (C$/US$) deflated using consumer price indexes. For the terms-of-trade index, they use two components: the price of exported energy and non-energy commodities, each divided by the price of imported manufactured goods to obtain TOTENERGY and TOTCOMOD. These indexes are obtained by combining several CANSIM series and weighting them according to their volume shares in 1986.[8] Finally, to capture monetary

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[8] Significant problems with their formula for the price indexes were uncovered while researching this paper. These are described in detail in the Appendix.

influences on the short-term deviation from long-run exchange rate values, they include a measure of the Canada-US interest rate differential, RDIFF, defined as:

$$RDIFF = (i\,Canada - I\,Canada) - (i\,US - I\,US)$$

where i is the 30 day prime corporate rate and I is secondary market yields on long-term industrial bonds. The authors note that "this particular formulation (of the rate differential) is not critical to our results"[9] and obtained similar results with different interest rate measures.

All variables except the interest rate differential are expressed in natural logarithms. After extensive testing, they are able to reduce their specification to a single equation estimated by the least squares method:

$$\Delta RPFX_t = \alpha(RPFX_{t-1} - \beta^0 - \beta_C TOTCOMOD_{t-1} - \beta_E TOTENERGY_{t-1}) + \gamma RDIFF_{t-1}$$

The variables inside the parentheses form the error correction term and capture the long-run dynamics of exchange rate movements, and those outside, in this cast RDIFF capture short-term deviations from long-term trends.

The estimates from their error correction model are all statistically significant and withstand several diagnostic tests. They show that a one percent improvement in commodity terms of trade leads to a 0.811 per cent appreciation of the Canada-US real

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[9] Amano and van Norden (1995) p. 87

exchange rate, and a one per cent improvement in energy terms of trade results in a 0.233 per cent *depreciation* in the real exchange rate. This latter result was somewhat surprising given the conventional view of the Canadian Dollar as a *petro-currency*, but is not inconsistent with economic theory. While Canada is a net exporter of energy, it is also one of the world's largest consumers of energy, so a negative supply shock in the form of an energy price increase can have the expected negative impact on productivity. Finally, as monetary models predict, this model estimates that an increase in the interest rate differential leads to an appreciation in the exchange rate by 0.187 per cent.

Perhaps the most important result of their work is how well the equation works in forecasting the exchange rate. After appropriate testing, they compare out-of-sample forecasts for several time horizons produced by their equation to those generated by a random walk and find that the ECM model generates a smaller root-mean square error than a random walk model in all time horizons, with the ECM model's performance improving as the forecasting horizon increases.

3.2 Lafrance and van Norden (1995)

Lafrance and van Norden (1995) use virtually the same specification, but a somewhat different data set. The US$ crude oil price index is used for the energy price series, and non-energy commodity prices are represented by the Bank of Canada's production weighted US$ commodity price index. Both indexes are deflated using the US implicit GDP deflator to convert them to real price variables. These changes in commodity price indexes may have been due in part to the problem with the indexes in Amano and van Norden (1993) mentioned in the previous section and detailed in the Appendix of this paper. They also make changes in commodity prices easier to interpret. In this case a one per cent change in the real price of the commodity (as opposed to a one per cent change in its price relative to import prices) will result in a direct per centage change in the real exchange rate.

The interest rate differential is simply the difference between Canadian and US 90-day commercial paper rates, rather than the differential between long and short rate differences as in Amano and van Norden (1993). This change reflects the finding that the particular interest rate formulation was not critical to the results obtained.

The only change in specification is the inclusion of the change in the real exchange rate in the previous period as an additional exogenous variable outside of the error correction term in an attempt to further capture the short-run dynamics of the exchange rate movements.

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The specification estimated is:

$$RFX_t - RFX_{t-1} = \alpha(RFX_{t-1} - \beta^0 - \beta_C COM_{t-1} - \beta_E ENE_{t-1}) + \gamma INT_{t-1} + \theta(RFX_{t-1} - RFX_{t-2})$$

As with Amano and van Norden (1993), all estimates are statistically significant and the out-of-sample forecast outperforms that of a random walk.

3.3 Djoudad, Murray, Chan and Daw (2000)

Djoudad, Murray, Chan and Daw (2000) use the same ECM approach as Amano and van Norden (1993) and Lafrance and van Norden (1995), updated to include data to 1999 and obtain very similar results: statistically significant estimates that outperform a random walk in out-of-sample forecasts.

The authors use the benefit of continued work on the model over the years in choosing their data. For the real exchange rate (*rfx*) they use the nominal US$/C$ exchange rate deflated by the respective country's CPI. To give a broader measure of energy commodity prices they use the Bank of Canada energy price index (*enetot*), and use the BOC non-energy price index for non-energy prices (*comtot*). Work subsequent to Amano and van Norden (1995) indicated that it does not make much difference whether the GDP deflator or the CPI was used to convert these indexes to a real price variable, so they chose the CPI for this purpose. For the interest rate differential, they use the difference between Canada and US 90 day corporate paper rates.

To test the model for its relevancy over different time periods, they estimate it with quarterly data over two different sample periods, 1973-1990 and 1973-1999. Their estimates show very little movement as the sample period is extended, and maintain their statistical significance. As with the two previously discussed papers, the model outperforms a random walk in out-of-sample forecasts.

4 MODEL REPLICATION

In this section, I present the results of attempts to replicate the Djoudad et al.. version of the model. Similar data sets were used and the same basic procedure in testing and setting up the model as in Amano and van Norden and Djoudad et al.. was followed. The real exchange rate (*rer*) was obtained by using quarterly averages of the average noon spot rate of the CDN$/US$ exchange rate deflated by the respective country's CPI. The commodities terms-of-trade were obtained by deflating the Bank of Canada non-energy terms-of-trade (*comtot*) and energy terms-of-trade (*enetot*) indexes (in US$ terms) by the US CPI to give a real price variable. Finally, the interest rate differential (*rdiff*) is the difference between the Bank of Canada overnight rate, and the US Federal Reserve funds rate. The data series are described in detail in the Appendix.

Before estimating an error correction model, tests for stationarity and cointegration of the variables must be performed. Tests for unit-root are notoriously lacking in power, so three separate tests were performed on each variable: the Augmented Dickey Fuller (ADF) test and the Phillips-Perron (PP) test, both which have the existence of a unit root as their null hypotheses, and the often more reliable Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test whose null is stationarity. The real exchange rate and the terms-of-trade

variables are found to have unit roots, and the interest rate differential is stationary,[10] results that are consistent with the work being replicated. Table 2 summarizes the test results.

Table 2: **Tests for unit roots and stationarity. Sample period 1972Q1 to 2002Q1**

Variable	ADF*	PP*	KPSS**
Rer	0.4613	0.7589	0.129469[a]
Comtot	0.9155	0.7650	1.106969[b]
Enetot	0.4898	0.5432	0.466860[c]
Rdiff	0.0915	0.0491	0.171923[d]

* p-values
** LM statistic
a Reject stationarity at 8% level
b Reject stationarity at 1% level
c Reject stationarity at 5% level
d Fail to reject stationarity at 10% level

The non-stationary series must now be tested for cointegration using the Johansen and Juselius test. Both the trace statistic and maximum eigenvalue show evidence of one cointegrating relationship (see Table 3).

Table 3: **Tests for cointegration**

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.291196	39.92442	25.54	30.34	53.32017	42.44	48.45
At most 1	0.087841	10.66521	18.96	23.65	3.39574	25.32	30.45
At most 2	0.023264	.730538	12.25	16.26	2.730538	12.25	16.26

Trace test indicates 1 cointegrating equation(s) at both 5% and 1% levels
Max-eigenvalue test indicates 1 cointegrating equation(s) at both 5% and 1% levels

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[10] When a trend term was included in the tests on *comtot*, the results were mixed in terms of finding a unit root. Since including the trend term when not appropriate further reduces the power of the tests, it was assumed that *comtot* was non-stationary. For a discussion of this See Kennedy (2003).

Given the above results, an Error Correction Model may be employed with the general specification:

$$\Delta\ln(rer) = \alpha\,(\ln(rer)_{t-1} - \beta_0 - \beta_c \ln(comtot)_{t-1} - \beta_e \ln(enetot)_{t-1}) + \gamma rdiff_{t-1} + \epsilon_t$$

if it can be shown that the variables *comtot* and *enetot* are weakly exogenous, which can be tested using the likelihood-ratio test developed by Johansen and Juselius. This is a simple test of whether the speed of adjustment α is significantly different from zero in the above equation. The test gives a χ^2 (1) statistic of 0.0138 (p-value = 0.906) meaning we fail to reject the null hypothesis of weak exogeneity. This suggests that weak exogeneity is a valid working assumption and the single-equation ECM model will give valid inferences.

The estimates obtained using my data are very close to those obtained by Djoudad et al. and are summarized in Table 4. Note that all coefficients are statistically significant. The speed of adjustment α is -0.107, implying that about 11% of adjustment is completed within one quarter, that is, the system suggests that when the real exchange rate is out of equilibrium, it will move about 11% toward equilibrium each quarter.

Table 4:   Error-correction model for the Canadian Dollar. Djoudad et al. and replication.

Variable	Djoudad et al		Replication	
	Coefficient	p-value	Coefficient	p-value
Adjustment (a)	-0.11	0.00	-0.11	0.00
Constant	-0.43	0.00	-0.82	0.01
comtot	0.41	0.00	0.40	0.00
enetot	-0.09	0.03	-0.09	0.02
rdiff	0.58	0.00	0.40	0.00
R^2	0.16		0.13	

The coefficient estimates suggest that a one percent increase in the real price of non-energy commodities will lead to a 0.4% appreciation in the real exchange rate, and a one percent increase in the price of energy commodities will see the Canadian dollar depreciate by 0.1%. And finally, an increase of 100 basis points in the difference between Canadian and US short-term interest rates will cause the Canadian dollar to appreciate by 0.4%. The R^2 of the regression is 0.131 which tells us that this model explains 13% of the movement in the Canada-US real exchange rate.

The Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH) in the residuals reveals no evidence of heteroskedasticity, however the LM test for serial correlation reveals an autocorrelated error of order one, the latter differing from the results of Amano and van Norden, and Djoudad et al.. When the model was estimated using an AR(1) process, the coefficient estimates were very close to the least squares estimates and maintained their statistical significance.[11] Since the models being replicated showed no evidence of autocorrelation, and since the AR estimates were so close to those from the least squares estimates, the least squares estimates are used for forecasting.

Dynamic simulations of this error correction model show that it performs reasonably well (see Figure 4).

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[11] The coefficient on *enetot* had a p-value of 0.067, the other coefficients all had p-values of 0.016 or less. The results are detailed in the Appendix.

Figure 4:   Dynamic Simulation for the Canadian Dollar (Baseline)

RER
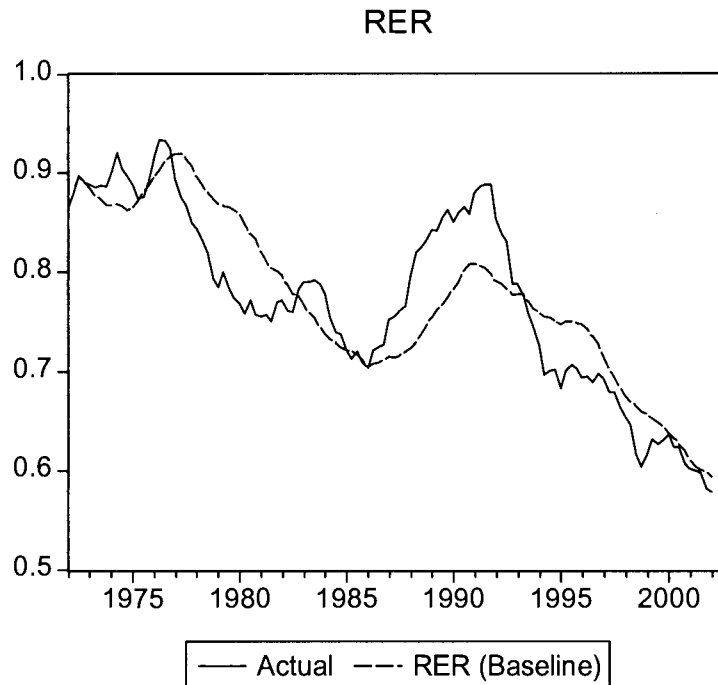
Following Meese and Rogoff (1983), Amano and van Norden use the root mean square

error (RMSE) and the mean absolute error (MAE) to evaluate the out-of-sample

forecasting performance of the error correction model compared to a random walk. The

model estimated here has an MAE and RMSE of 0.039916 and 0.047626 respectively,

while a random walk's are 0.057313 and 0.072605.[12] A lower value for these statistics

means better forecasting performance, suggesting that this error correction model

forecasts better out-of-sample than a random walk.

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[12] The MAE and RMSE were even lower when forecast with the AR model.

5 EXTENSION OF THE MODEL

In this section, the bilateral current account between Canada and the United States is investigated as a possible addition to the error correction model estimated in the previous section.

Large and growing current account imbalances among major economies, particularly the huge U.S. current account deficit, has been continually in the news of late, and has led to substantial academic and public discussion about its sustainability. While not a problem in and of itself, a current account deficit must be financed, generally by international debt. This net liability position cannot grow indefinitely however; it must stabilize at a position that is acceptable to both the debtor country and its international lenders. Any significant change in the portfolio of international lenders as a result of concern about the long-term sustainability of that net international liability position may result in a depreciation of the currency.

The theoretical impact of the current account on real exchange rates is the subject of considerable academic debate. Some international economists use an adjustment model derived from the basic Mundell-Fleming model summarized in Lane and Milesi-Ferretti (2002) as:

$$rer = -\phi tb + \lambda X$$

where *rer* is the real exchange rate, *tb* is the trade balance to GDP ratio, and *X* are other

factors affecting the real exchange rate. In this equation, an increase in *rer* is a real

appreciation. This simply says that for given values of *X*, the real exchange rate will be

more depreciated the bigger the steady-state trade surplus.[13]

Krugman (1991) discusses a slightly more complex version of the above basic model that

he calls the Mass. Ave. model (after the Federal Reserve bank, located a few blocks off

of Massachusetts Avenue), which suggests a similar relationship between the current

account and the real exchange rate.

Some economists dispute these models but recognize the empirical evidence to support

them. Obstfeld and Rogoff (1996) for example concede that "a large empirical literature

uses variants of the Mundell-Fleming-Dornbusch model to analyze how current-account

deficits affect real exchange rates...While we have identified a number of theoretical

failings with such models, it is still very useful to consider this substantial body of

evidence."[14]

In a study of twenty-five episodes of macro-variable adjustments in conjunction with

large current account adjustments, Freund (2000) finds "that a typical current account

reversal...is associated with slowing income growth and a significant real (currency)

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[13] Lane and Milesi-Ferretti (2002) p. 4
[14] Obstfeld and Rogoff (1996) p. 695

depreciation over a period of about three years."[15] Two of the episodes studied were

from Canada, in 1981 and 1993.

Is there evidence of a current account impact on the bilateral exchange rate between

Canada and the United States? Figure 5 shows the Canadian current account as a percent

of Canadian GDP, the US current account over US GDP, and the bilateral current account

between the two countries as a percent of Canadian GDP.

Figure 5:   Canada, US, and Canada-US Bilateral Current Account
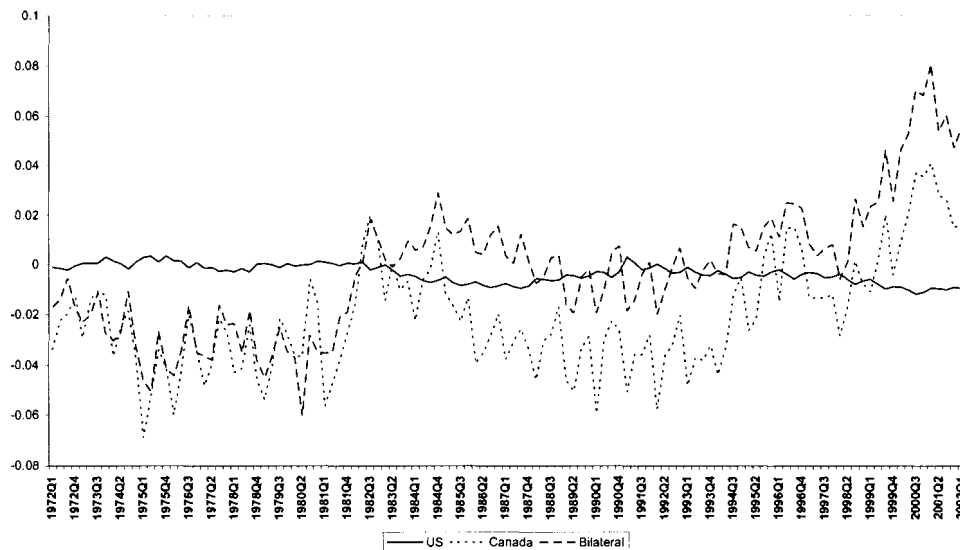
We can see from this graph that the two countries current accounts often move in

opposite directions, especially beginning in the early 1990s when we notice a secular

improvement in Canada's current account, and a secular decline in the US.

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[15] Freund (2000) p. 2

If the current account does have an impact on a country's real exchange rate, it can be expected that when looking at the Canada-US bilateral exchange rate, both countries current accounts will be important.

With respect to the simple forecasting equation presented here, one option would be to include the difference between the Canadian and US current accounts (as a percent of their respective GDPs). Attempts to use this series found evidence of a unit root and two cointegrating relationships, which would complicate the simple one equation-forecasting model. Another option would be to simply use one, or a combination of the three measures discussed above. When all three were included together in the ECM model, only the bilateral current account was statistically significant (although when specified individually, all showed statistical significance, and all yielded very similar coefficient estimates). As a result, the bilateral current account series was used to capture the effect of both current accounts on the bilateral exchange rate.

The data used is the quarterly bilateral current account between Canada and the United States expressed as a percentage of Canadian GDP (*cagdp*).

The same three unit root tests were applied to this series and are summarized in Table 5.

Table 5: Unit root test for cagdp.

Variable	ADF*	PP*	KPSS**
cagdp	0.0948	0.0085	0.092773[a]

* p-values
** LM statistic
a Fail to reject stationarity at 10% level

The conclusion is that *cagdp* is stationary, so will have only a short-term impact on the exchange rate and should be included outside of the error correction term on the right hand side of the equation:

$$\Delta \ln (rer) = \alpha(\ln (rer)_{t-1} - \beta_0 - \beta_c \ln(comtot)_{t-1} - \beta e \ln(enetot)_{t-1}) + \gamma rdiff_{t-1} + \theta cagdp_{t-2} + \epsilon_t$$

The least-squares estimates of this model, and those for the model without *cagdp* are shown in Table 6.

Table 6: Error-correction model for the Canadian Dollar with current account

Variable	Baseline Model		Extended Model	
	Coefficient	p-value	Coefficient	p-value
Adjustment (*a*)	-0.11	0.00	-0.10	0.00
Constant	-0.82	0.01	-1.88	0.01
comtot	0.40	0.00	0.62	0.00
enetot	-0.09	0.02	-0.10	0.03
rdiff	0.40	0.00	0.48	0.00
cagdp			0.27	0.01
R^2	0.13		0.18	

All estimates are statistically significant, and the R^2 is 0.184, an increase of 0.05 over the model without *cagdp*. This model has a speed of adjustment (α) of -0.09, slightly slower

than previously. The model now estimates a 0.62% appreciation in the real exchange rate

as a result of a one per cent increase in the real price of non-energy commodities, a

depreciation of 0.1% associated with an increase in the real price of energy commodities

of one per cent, a 0.48% appreciation as a result of a 100 basis points increase in the

interest rate differential, and a 0.27% appreciation as a result of an improvement in

Canada's bilateral current account of one percentage point (as a percent of GDP). Note

that *cagdp* enters the equation with a 2 period lag. Lags of up to eight periods were

tested, but only the second period lag was significant. It is not surprising that current

account figures will have this delay due to the delay in the publication of this information

by the federal governments.

As was the case with the baseline model, the ARCH LM test revealed no evidence of

heteroskedasticity, but the LM test for serial correlation revealed an AR(1) residual, and

the AR estimation showed similar results. The least squares estimates were used for

forecasting here for consistency.

A dynamic simulation of the model again shows a reasonable fit (Figure 6).

A comparison of the MAE and RMSE from the out-of-sample forecast of this model

reveals that this model not only outperforms a random walk, but outperforms the baseline

model from the previous section. See Table 7 for a summary of these statistics.

Figure 6: **Dynamic Simulation for the Canadian Dollar (with CA variable)**

RER

Actual — — RER (with cagdp)
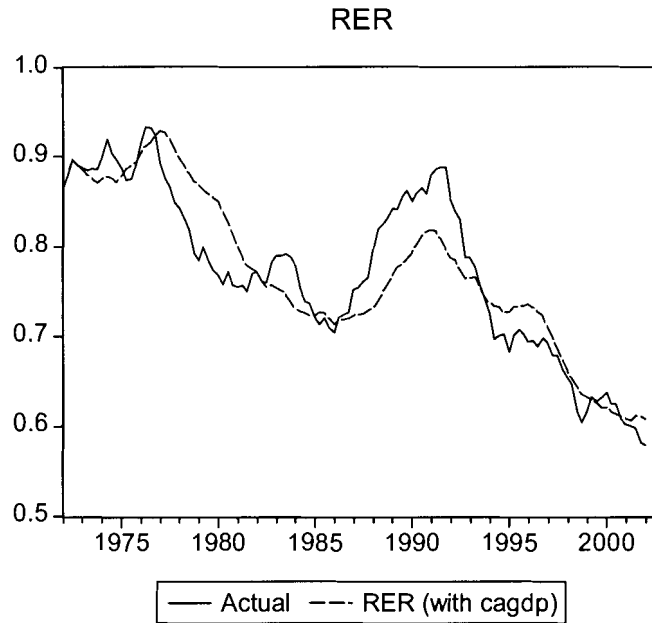
Table 7: **Out-of-sample forecast error statistics.**

Model	MAE	RMSE
Baseline	0.040	0.048
With cagdp	0.034	0.041
Random Walk	0.050	0.058

The above suggests that including a measure of the Canada-US. bilateral current account in the error correction model increases its forecasting accuracy.

6 SUMMARY AND CONCLUSION

This paper has successfully replicated an error correction model for forecasting the Canada-US bilateral real exchange rate first developed by Robert Amano and Simon van Norden of the Bank of Canada in 1992, and introduced an additional explanatory variable that appears to improve the explanatory power of the model and its forecasting ability. The model demonstrates a cointegrating relationship for the real exchange rate that appears to have remained stable for three decades, and seems to have a significant ability to forecast the exchange rate changes out-of-sample better than a random walk.

Using unit-root and cointegration techniques, the original model uses commodity prices to capture the long-term effects on the exchange rate and a measure of the Canada-US interest rate differential to reflect the deviation of the real exchange rate from its expected long-run level. The addition of a measure of the Canada-US bilateral current account reflects additional short-term deviations.

The model suggests that an increase in the price of non-energy commodities, an increase in the gap between the Canada-US short-term interest rate, and an improvement in the Canadian current account with its major trading partner will lead to an appreciation of the real Canada-US exchange rate, while an increase in real energy commodity prices is associated with a depreciation.

APPENDICES

Appendix 1: Amano and van Norden Data

While researching this paper, certain problems with the calculation of the commodity price indexes in Amano and van Norden (1993 and 1995) were found. Canada's energy and commodity terms of trade are defined as:

$$TOTENERGY = (0.06138*B1503 + 0.04104*B1504 + 0.07613*B1505)/PM$$

$$TOTCOMOD = (PX/PM) - TOTENERGY$$

where

$$PX = 0.4664*B1501 + 0.06138*B1503 + 0.04104*B1504 + 0.07613*B1505 +$$

$$0.77484*(B1502 + B1506 + B1507 + B1508 + B1509)$$

$$PM = 0.2986*B1558 + 0.7014*(B1554 + B1555 + B1551 + B1556 + B1557 + B1559)$$

and the B**** expressions represent CANSIM series for commodity price indexes and their coefficients represent trade volume shares (in 1986). The CANSIM series are: B1501 – wheat; B1502 – other farm and fish products; B1503 – crude petroleum; B1504 – natural gas; B1505 – other energy products; B1506 – lumber and sawmill products; B1507 – pulp and paper; B1508 – other metals and minerals; B1509 – chemicals and fertilizer; B1558 – machinery and equipment; B1554 – construction materials; B1555 – industrial materials; B1551 – food; B1556 – motor vehicles and parts from the USA; B1557 – motor vehicles and parts from the rest-of-the world; B1559 – other consumer goods.

The series used seem reasonable enough; however note that in the calculation of PX, the coefficients add up to more than one. It is assumed that the coefficient on B1501 should actually be 0.04664. Further, in the calculations for PX and PM, the coefficient on the series in the parentheses represents the total trade share of all series inside the parentheses, however when adding these price indexes together, they all receive equal weight. This is clearly not reasonable since the commodities represented by the series have vastly different weights in Canada's trade. It is not known if the authors used these calculations in their work of if this is merely an error in representation. The calculation was represented in this manner in both the 1993 Bank of Canada working paper, and the article published in the Journal of International Money and Finance in 1995.

When attempts were made to replicate this paper, the calculations as presented were used, as well as modifying them according to trade weights, both yielding results significantly different from Amano and van Norden.

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Appendix 2:   Data used

The following data were used in the Djoudad et al. model replication and extension:

$$rer = E*CPI(US)/CPI(CANADA)$$

where E is the CDN\$/US\$ exchange average noon spot rate from CANSIM B3400,

CPI(US) in the United States Consumer Price Index (82-84=100) from CANSIM

D139105 re-indexed to 1992 = 100, and CPI(CANADA) is the Canadian Consumer Price

Index (1992 = 100) from CANSIM P100000;

$$comtot = NECPI \ / \ CPI(US)$$

where NECPI is the Bank of Canada non-energy commodity price index (US\$ terms)

from CANSIM B3301 and CPI(US) is as above;

$$enetot = ECPI \ / \ CPI(US)$$

where ECPI is the Bank of Canada energy commodity price index (US\$ terms) from

CANSIM B3302 and CPI(US) is as above;

$$rdiff = CDN \ i - US \ i$$

where CDN i is the Bank of Canada overnight interest rate from CANSIM B14006 and

US i is the United States Federal Funds rate from Fedstats (http://www.federalreserve.

gov/releases/h15/data/m/fedfund.txt);

$$cagdp = \text{CA} / \text{GDP}$$

where CA is the Canada – United States bilateral current account in millions of current

dollars from CANSIM D59132 and GDP is the Canadian nominal Gross Domestic

Product in millions of current dollars from CANSIM D15689.

Appendix 3:   AR estimates for Baseline and Extended model

As discussed in the body of this paper, both the baseline model and my extension using the bilateral current account as an additional exogenous variable showed evidence of serial correlation according to the Lagrange Multiplier test for serial correlation. Further testing showed an AR(1) error. The following table shows the AR(1) regression results using E-Views$^©$ ARMA routine compared to the least squares results used.

Table 8:   AR Regression estimates

Variable	ARMA Coefficient	p-value	Least Squares Coefficient	p-value
Adjustment (a)	-0.14	0.00	-0.10	0.00
Constant	-1.41	0.01	-1.88	0.01
comtot	0.51	0.00	0.62	0.00
enetot	-0.07	0.07	-0.10	0.03
rdiff	0.58	0.00	0.48	0.00
cagdp	0.26	0.02	0.27	0.01
ar(1)	0.28	0.00		
R^2	0.25		0.18	
MAE	0.030		0.034	
RSME	0.037		0.041	

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