EPISTEMIC STYLES AND MATHEMATICS PROBLEM SOLVING: EXAMINING RELATIONS IN THE CONTEXT OF SELF-REGULATED LEARNING

by

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ABSTRACT

This dissertation examines relations between personal epistemology and facets of self-regulated learning, moves away from correlational designs, and adopts a more process-oriented methodology. For this study, a philosophical conceptualization of epistemology was integrated with conceptualizations from educational psychology and mathematics education. The primary purpose of this study was to examine relations between approaches to knowing, mathematics problem solving, and regulation of cognition. A secondary purpose was to examine whether mathematics students become more rational in their approaches to knowing and whether their epistemic beliefs change through higher levels of education.

One hundred twenty-seven students were sampled from undergraduate university mathematics and statistics courses. Students completed self-report measures to reflect epistemic styles, epistemic beliefs, and dispositions regarding elements of self-regulated learning. Students were profiled as predominantly rational, predominantly empirical, or both rational and empirical in their approaches to knowing. Seventeen students were chosen to participate in two problem-solving sessions. Problem-solving episodes were coded for evidence of planning, monitoring, control, use of empirical and rational argumentation, and justification for solutions.

Differences in self-reported metacognitive self-regulation were found between students profiled as high on rationalism and empiricism and students profiled as predominantly empirical. No other self-reported differences were found. When problem solving, students profiled as predominantly rational had the highest frequency of

planning, monitoring, and control. These differences were attributed to patterns identified in students' self-efficacy. No differences in rationalism scores were found between lower- and upper-year university students but differences were found in their beliefs about the structure of knowledge and the source of knowledge. Differences were also found in the quality of rational arguments between lower- and upper-year university students when solving problems.

Students profiled as predominantly rational in their approaches to knowing were predominantly rational in their approaches to problems solving. Similarly, students profiled as predominantly empirical in their approaches to knowing were predominantly empirical in their approaches to problem solving. Finally, students profiled as both rational and empirical in their approaches to knowing were predominantly rational in their approaches to problem solving. Results are discussed in the context of various theoretical frameworks.

DEDICATION

I dedicate this thesis to my parents, Paul and Sandra Muis, who have supported and encouraged me throughout my education.

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CHAPTER 1

INTRODUCTION

Epistemology is a branch of philosophy concerned with the nature of knowledge and justification of belief. According to Arner (1972), epistemology is divided into three broad areas of inquiry by the following three general questions: What are the limits of human knowledge? What are the sources of human knowledge? What is the nature of human knowledge? The first question addresses whether there are questions upon which it is impossible for humans to acquire grounds, gather evidence, or accumulate reasons so as to be rationally justified in taking a position. The second question addresses what genuine sources of knowledge are; that is, whether sources of knowledge are, for example, derived from sense experience or from pure reason. An examination of sources of knowledge includes an analysis of how knowledge is acquired and how knowledge is represented. Finally, the third question concerns the analysis of concepts that are prominent in discussions of knowledge. The most common of these concepts are knowledge and truth. What does it mean for a person to know something? What is it for a proposition to be true? The concern with what one may be justified in believing and with what gives one justification for believing presents a central concern for the nature of justification itself (Arner, 1972).

According to Hofer and Pintrich (1997), over the past decade, educational psychologists have become increasingly interested in personal epistemological development and epistemic beliefs: how individuals acquire knowledge, the theories and beliefs they hold about knowing, and how these beliefs are a part of and influence

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cognitive processes, especially thinking and reasoning. Within educational psychology, a number of research programs have investigated students' beliefs about the nature of knowledge and knowing, including what knowledge is, how knowledge is constructed, and how it is evaluated (for reviews, see Hofer & Pintrich, 1997; Schommer, 1994b). These various research programs have used divergent definitions, labels, and theoretical frameworks, and have applied different methodologies to explore students' epistemic beliefs (Hofer & Pintrich, 1997). Examples of various labels include: epistemic beliefs (Schraw, Bendixen, & Dunkle, 2002), epistemological beliefs (Schommer, 1990), epistemological meta-knowing (Kuhn, Cheney, & Weinstock, 2000) and, epistemic theories (Hofer, 2004). The label espoused here is epistemic beliefs (see Hofer, 2004, for a discussion on terminology).

Studies of personal epistemology have not been the sole interest of educational psychologists. Other disciplines, including developmental and instructional psychology, higher education, science and mathematics education, reading and literacy, and teacher education have also been interested in the study of personal epistemology (Hofer, 2002). Each discipline has used various research traditions and paradigms and, as a result, research in this area has materialized in numerous locations and has been labeled under different constructs (Hofer, 2002). Although a number of models within these disciplines have been developed and various lines of research have been pursued, researchers in educational psychology and mathematics education have found that personal epistemology is related to cognition, motivation, learning, and achievement (Muis, in press).

Epistemic beliefs are theorized to affect how students approach learning tasks (Schoenfeld, 1985), monitor comprehension (Schommer, Crouse, & Rhodes, 1992), plan for solving problems and carry out those plans, and are theorized to directly and indirectly affect achievement (Schommer, 1990). Empirical studies support these hypotheses (Muis, in press). For example, Ryan (1984) found a relationship between epistemic beliefs and standards students use to monitor comprehension. Students holding dualistic conceptions of knowledge reported using low-level, fact-oriented standards to learn from a textbook chapter. In contrast, students holding relativistic epistemic beliefs about knowledge reported using more fruitful, context-oriented standards.

In a study of mathematical text comprehension, Schommer et al. (1992) found that epistemic beliefs correlated with achievement and with students' self-assessments of comprehension. The more students believed, for example, that the structure of knowledge is simple, the lower their achievement and the less accurate their self-assessments of comprehension. These findings support Schommer's (1998) and Hofer and Pintrich's (1997) hypothesis that epistemic beliefs affect achievement mediated through self-regulated learning, a model that accounts for how students design their approaches to learning and adapt those approaches as feedback about progress becomes available.

Theory and research investigating self-regulated learning are also prominent in educational psychology. Self-regulated learners manage their learning, engage in more metacognitive monitoring and control, are more intrinsically motivated (Zimmerman, 1990), and are more strategic and perform better than less self-regulated learners (Pressley & Ghatala, 1990). Students who self-regulate are theoretically more aware of task demands, can accurately estimate whether they are able to meet those demands, are

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more efficacious in learning, attribute outcomes to facets of learning under their control, and have a repertoire of learning strategies that they use appropriately under various learning situations (e.g., see Winne, 2001).

In the field of educational psychology, although studies have been conducted to examine relations between personal epistemology and cognition, motivation, and learning, Pintrich (2002) has made a call for more empirical studies to advance theoretical specifications of how and why epistemic beliefs can facilitate or constrain cognition, motivation, and learning. For example, Pintrich recommended researchers examine more precisely how epistemic beliefs are related to facets of self-regulation. To set the stage for future research, Hofer and Pintrich (1997) proposed that epistemic beliefs can generate particular types of goals for learning and that these goals can serve as guides for self-regulatory cognition and behavior. These goals, in turn, can influence the types of learning and metacognitive strategies learners use when learning and problem solving. The types of learning strategies and metacognitive strategies students use subsequently influence academic performance and achievement. To date, this hypothesis has not been empirically tested.

Pintrich (2002) also proposed there is a critical need to move beyond correlational designs and the reliance of self-report measures to examine relations. Specifically, he argued that to improve understanding of how beliefs are related to cognition, motivation, and learning, studies should use more dynamic process-oriented research designs. Moreover, some of the conceptualizations that have been offered within educational psychology have been criticized for a lack of philosophical grounding (see Hofer & Pintrich, 1997). Research is needed that examines relations between personal

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epistemology and cognition, motivation, and learning in the context of actual learning; research that includes conceptualizations grounded in philosophy. Philosophical conceptualizations, therefore, need to be integrated into new developments in this area.

In my review of research (Muis, in press), with a focus on mathematics learning and problem solving, I located only two studies that connected models of personal epistemology to models of self-regulated learning (e.g., Lester & Garofalo, 1987; Schoenfeld, 1983). I also found, in the field of educational psychology, the majority of studies on personal epistemology relied solely on self-report measures. As Winne and Perry (2000) and I (Muis, Winne, & Jamieson-Noel, 2004; Winne, Jamieson-Noel, & Muis, 2002a) demonstrated, there are key technical and conceptual issues that limit the contribution of studies that use only self-report measures of these constructs. For example, after studying "on the fly," students may answer self-report items by constructing an answer grounded in a schema rather than by retrieving facts about events. Winne, Jamieson-Noel, and Muis (2002b) found students err in self-reports about actual study events. To address these issues, Winne et al. suggested researchers use traces, data about actual studying events recorded while learners study and solve problems.

My dissertation research responds to Pintrich's (2002) calls for studies linking epistemic beliefs to facets within models of self-regulated learning, to move away from one-point-in-time correlational designs and reliance on self-report measures, and to adopt a more process-oriented methodology. Specifically, my research addresses shortcomings that we (Muis et al., 2004; Winne et al., 2002a) identified in uses of self-report measures. Moreover, my research integrates a more philosophical conceptualization of epistemology with the conceptualizations currently used in educational psychology and

mathematics education. The primary objective of my research is to investigate relations among students' personal epistemologies, metacognitive strategies they report using in mathematics and statistics classes, metacognitive strategies they actually use as they solve mathematics and statistics problems, and how they justify the veracity of their solutions.

To address the first call to link epistemic beliefs to self-regulated learning, I adopt Schoenfeld's (1983) self-regulated learning model of mathematics problem solving. Using this model, Schoenfeld examined the influence of students' epistemic beliefs on problem solving behavior. Based on his observations of mathematics students and an expert, Schoenfeld classified students' epistemic beliefs as empiricist, whereby knowledge is derived through observation. When solving problems, students did not plan a course of action, tested hypotheses one by one until they found a correct method, and spent most of their time on "wild goose chases" to find information that would help them solve the problems. Moreover, they typically did not monitor problem-solving behaviors, a key facet of self-regulated learning (Winne, 2001). When they produced solutions to the problems, their justifications were rooted in empirical evidence; that is, students argued that solutions were correct because "they worked" or "looked correct." In contrast, the mathematician's epistemic beliefs were classified as rationalist, whereby mathematical knowledge is derived through logic and reason. When solving problems, the mathematician derived necessary information through proof-like procedures prior to a verification process. Unlike students, the expert planned a course of action and closely monitored his progress. When a solution was produced, justification was rooted in the rational and logical evidence that was derived through the problem-solving episode.

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Although Schoenfeld (1983) links epistemic beliefs and self-regulated learning, two aspects of his research need to be addressed. First, he identified relations between epistemic belief systems and regulation of cognition based on his observations of how students approached problems compared to an expert mathematician. Given that research on expert-novice differences in mathematics problem solving has established that experts engage in more regulation of cognition (e.g., Bookman, 1993) and that mathematics experts are more rational in their approaches to problem solving (Pólya, 1957), it is not surprising that there were differences in approaches to problem solving between students and the expert. Differences in self-regulation should be examined between individuals with similar levels of expertise in mathematics.

Second, Schoenfeld's sample consisted mostly of freshman. Developmental research on personal epistemology (e.g., Kitchener & King, 1981; Perry, 1970; Schommer, 1993a), which predominantly focuses on post-secondary students, has found that students' epistemic beliefs change over time. For example, senior college students are more likely to believe there are multiple possibilities for knowledge whereas junior college students are more likely to believe that knowledge is dualistic. Although I do not take the position that a rational world view is superior to an empirical world view, research has supported the notion that students' majoring in mathematics-related fields are more rational in their approaches to knowing than students in other fields such as the social sciences (e.g., Royce & Mos, 1980). Consequently, one may question whether students in mathematics-related fields become more rational in their approaches to knowing over the course of their undergraduate experience. This has not been empirically examined. A second objective of my research is to examine this.

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To address the second call Pintrich (2002) made, I have utilized a more dynamic process-oriented approach to examine relations between personal epistemology and self-regulated learning in the context of mathematics problem solving. As Winne et al. (2002a) suggested, researchers should trace learners' behaviors as they engage in problem solving. Traces can be recorded using a think aloud protocol. That is, aspects of learners' thought processes can be captured as learners think aloud while problem solving (Ericsson & Simon, 1993). I adopt this methodology to examine relations between epistemic beliefs and processes of self-regulated learning during problem solving. Three processes I focus on are planning, metacognitive monitoring, and metacognitive control. In general, metacognition refers to knowledge of one's own cognitive processes, that is, knowledge of how one monitors cognitive processes and how one regulates those processes (Flavell, 1976). According to Brown, Bransford, Ferrara, and Campione (1983) metacognition can be divided into two components: knowledge of cognition and regulation of cognition. Knowledge of cognition refers to the relatively stable information that learners have about their own cognitive processes including knowledge of how they store and retrieve information (Brown et al., 1983). Regulation of cognition refers to processes of planning activities prior to engaging in a task, monitoring activities during learning, and checking outcomes against set goals. These processes are assumed to be unstable, task and situation dependent (Brown et al., 1983). It is these processes of regulation of cognition that are the focus of this study.

Finally, to address the third issue reported here on research in personal epistemology, I integrate a model of epistemology from theoretical psychology. The model chosen is Royce's (1983) model of psychological epistemology that is grounded in

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psychology and philosophy. This model was chosen because of the theoretical links made between epistemological stances and cognitive processes. Royce proposed that an empirical understanding of the knowing process is necessary to comprehend epistemological issues; that is, the psychologist's empirical knowledge of cognitive processes complements the philosopher's rational examination of epistemological issues.

Royce and Mos (1980) developed the Psycho-Epistemological Profile (PEP) to delineate an individual's epistemological hierarchy. The epistemological dimensions the PEP measures reflect three basics ways of knowing: rationalism, empiricism, and metaphorism. These three ways of knowing are considered to be three different epistemic styles which determine one's world view and that depend on a particular sub-hierarchy of psychological processes. These include how one processes sensory information and, of particular interest, what cognitive processes are preferred when acquiring knowledge (e.g., learning). This model was used to categorize a priori learners along the two dimensions Schoenfeld (1983) identified in his research. By a priori categorizing learners' approaches to knowing as predominantly rational, predominantly empirical, or a combination of both, Schoenfeld's model of relations between epistemic beliefs and problem-solving behavior can be tested.

Although the PEP can be used to profile individuals along three epistemic dimensions, only two of these dimensions, rationalism and empiricism, are examined. Since this research evaluates Schoenfeld's model and his model includes only rationalism and empiricism, metaphorism is not considered in the analyses. Metaphorism is measured, however, to conduct psychometric assessments on the entire scale. Consequently, all three dimensions of Royce's model are described in detail.

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General Purpose And Description

The general purpose of this study is to examine whether individuals' epistemic profiles predict differences in how they engage in mathematics problem solving. One hundred twenty-seven mathematics and statistics students were sampled from first-through fourth-year undergraduate university mathematics and statistics courses. Students were asked to complete self-report measures to reflect epistemic beliefs and dispositions regarding elements of self-regulated learning. Schraw et al.'s (2002) Epistemic Belief Inventory (EBI) was used to categorize students along dimensions of Schommer's (1990) epistemic beliefs model. Royce and Mos's (1980) PEP was used to categorize students a priori as predominantly empirical, rational, or a combination of both in their approaches to knowing. To predict differences in metacognitive strategy use, students filled out the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, Smith, Garcia, & McKeachie, 1991).

Using data from the PEP, students were epistemologically profiled as predominantly empirical, rational, or both. Seventeen students were then chosen to participate in two problem-solving sessions. Based on students' epistemic profiles, Schoenfeld's (1983) model was used to predict differences in behavior between the three profiles. Because of the influence self-efficacy is theorized to have on learning and problem solving behavior (Bandura, 1997), self-efficacy was measured to ensure students were confident in successfully carrying out the problems. Problem-solving episodes were coded for evidence of planning, metacognitive monitoring, metacognitive control, use of empirical and rational argumentation, and justification for solutions. These were examined across two different problem-solving contexts. In the first context, students

were given three problems to solve but no additional information other than the problems was provided. In the second problem-solving session, students were first asked to study a short chapter on the binomial distribution after which they were given three problems related to material covered in the short chapter. During this problem-solving session, students could use the information presented in the chapter as an aid to solve the three problems. For both problem-solving sessions, students were asked to think aloud and problem-solving sessions were audio-recorded. Data from the self-report questionnaires were used to examine relations between epistemic profiles and metacognitive self-regulation. Data from the problem-solving sessions were transcribed verbatim and analyzed to examine differences in approaches to problem solving.

A secondary purpose of this study is to examine whether there are differences in students' epistemic profiles and beliefs across the four years of undergraduate school. Students' average scores on each dimension of the PEP and EBI were compared across the four years.

Overview of the Chapters

This study advances theory about how epistemic beliefs affect cognition in the context of mathematics problem solving. Previous studies in educational psychology that have used self-report questionnaires have measured personal epistemology out of context. This study advances theory by moving away from measuring personal epistemology as a decontextualized set of beliefs. It measures personal epistemology as an activated, situated facet of cognition that is theorized to influence problem-solving processes (Hofer, 2004). Second, it establishes links to self-regulated learning that are absent in the current literature. Third, it integrates a more philosophically grounded model of personal

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epistemology with models in educational psychology and mathematics education. Integration of various models across these three fields of study has not, heretofore, been attempted. Consequently, it is critical to describe the conceptual framework used for this synthesis.

The first section of Chapter 2 details Royce's (1983) model of psychological epistemology from theoretical psychology. To contextualize the literature review, the prominent theoretical models of personal epistemology in educational psychology are then briefly reviewed. Since Schommer's (1990) model of personal epistemology is one of the theoretical models used in this study, more attention is given to this model than the others. This is followed by a description of three prominent and current theoretical perspectives on approaches to knowing and relevant constructs and definitions in mathematics education. The synthesized definition used to guide the literature review is then presented.

The second section of Chapter 2 reviews the relevant literature. The literature review is divided into three major sections: students' epistemic profiles and beliefs about mathematics, the development of epistemic profiles and beliefs, and relations between epistemic profiles and beliefs and learning. Each major section is further broken down into two subsections, one for studies in theoretical psychology (e.g., those that examined epistemic profiles) and one for studies in educational psychology and mathematics education (e.g., those that examined epistemic beliefs). For each section, a brief introduction to the topic area is provided followed by a detailed review of the literature. Each subsection is summarized and a critique is presented. In the section that reviews literature on the effects of epistemic beliefs on learning, prior to the literature review, an

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extended discussion of Schommer's (1990) model is presented to provide a theoretical basis for examining relations between epistemic beliefs and learning. Schoenfeld's (1983) model of self-regulated learning in problem solving in mathematics is then presented followed by relevant literature. Chapter 2 concludes with a discussion of the specific research questions and hypotheses examined in this dissertation.

Chapter 3 presents the methodology used for this research and is divided into two sections. The first section details the first component of the study, the large-scale data collection component. It includes a description of the sample, the questionnaires used, and the procedure by which participants filled out the questionnaires. The second section details how a sub-sample of participants was selected to participate in the problem-solving sessions, materials used, procedures, and how the data were coded. Issues of reliability and validity also are addressed. Chapter 4 presents the results, again divided into two sections. Finally, Chapter 5 presents a discussion of the results in the context of the theoretical models, issues, and future directions.

In sum, by tackling issues in research about personal epistemology that leaders in the field have pointed to as critical for advancing theory, my research will generate new knowledge that is more broadly grounded than previous research. Findings from my study will also bear on educational practice because better understandings about relations among epistemic beliefs and cognition will provide a platform for designing improved instruction.

CHAPTER 2

THEORETICAL MODELS

AND LITERATURE REVIEW

Theoretical Psychology and Epistemology

The establishment of Division 24 of the American Psychological Association brought together two disciplines that had long ago separated, philosophy and psychology. A major result of the two fields rejoining was the development of the interdisciplinary subject of psychological epistemology, whose primary concern is to answer, "How do we know?" (Royce, Coward, Egan, Kessel, & Mos, 1978). Since this was a topic of psychology in the early years, many people questioned the reappearance of this issue and asked, "What's new?" and, "Why now?" The answer to this had to do with an increasing awareness of some philosophers and psychologists of the limitations of a linguistic-conceptual analysis of epistemology. Thus, philosophers who were agreeable to empirical evidence joined with cognitive psychologists and began a line of empirical inquiry into theories of knowledge. What was new was an empirical approach to such problems as knowing via the senses, knowing via thought, knowing via intuition, and the developmental aspects of knowledge acquisition (Royce et al., 1978). The thesis of psychological epistemology is that an empirical understanding of the knowing process is necessary for a complete comprehension of epistemological issues (Royce, 1983). Knowledge, then, is defined as those cognitions of an organism's cognitive structure

(psychological perspective) that are epistemologically justifiable (philosophical perspective) (Royce, 1978).

Royce (1959) proposed that knowing involves several modes. For example, knowing in the arts is not the same as knowing in the sciences. They might differ philosophically as well as psychologically. Philosophically speaking, there may be different truth criteria. For example, one's truth criteria for evaluating science claims may be quite different for one's truth criteria for evaluating truth claims in literature. Psychologically, one might expect diverse involvement of the cognitive processes. He further argued that if it can be shown that there are different ways of knowing then it is reasonable to anticipate that people will combine these different ways in a particular preference order which can be described as a hierarchical structure. These hierarchies may account for differences in Weltanschauung, or people's world view.

Subsequent analyses (e.g., Royce, 1967, 1974; Royce & Rozeboom, 1972) and empirical research on how people know (e.g., Mos, Wardell, & Royce, 1974; Royce & Smith, 1964; Smith, Royce, Ayers, & Jones, 1967) led Royce to the conclusion that there are three basic and valid ways of knowing: rationalism, empiricism, and metaphorism (initially called intuitionism). He contended that although there have been numerous theories of knowledge proposed in the history of philosophical thought, these three ways of knowing are basic because of their direct dependence on psychological cognition on one hand, and their epistemological testability on the other hand. Thus, people can know only in terms of the three cognitive processes that underlie these three ways of knowing (Royce, 1978).

Rationalism is considered to be primarily dependent on logical consistency. For example, one will accept something as true if it is logically consistent but will reject something as false if it is illogical. Moreover, rationalism requires critical thinking and conceptualizing and a rational analysis and synthesis of ideas. This includes cognitive processing which focuses on concepts, their formation, elaboration, and functional significance to the person. Conceptualizing is more deductive than inductive and the primary focus of conceptualizing is on the logical consequences of information currently available (Royce, 1978). This way of knowing does, however, include other psychological components such as sensing and intuiting, but the primary psychological processes which one uses in knowing rationally are critical thinking and conceptualizing (Royce, 1959).

Empiricism maintains that we know to the extent that we perceive correctly and, consequently, empiricism requires cognitive processes that focus on observables. This requires an analysis of sensory inputs and their meanings, an analysis that relies on valid and reliable information. Perceiving is more inductive than deductive and the primary focus is on the processing of sensory information. Like rationalism, empiricism includes other psychological components like conceptualizing and intuiting, but the main cognitive process one relies on is active perception and other sensory experiences (Royce, 1978).

Metaphorism claims that knowledge is dependent on the degree to which symbolic cognitions lead to universal rather than distinctive awareness. This type of knowing requires symbolizing, or a focus on the formation of symbols. Constructed productions are offered as representations of reality. In contrast to rationalism and

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empiricism, metaphorism is neither deductive nor inductive; instead, it is analogical. Thus, the focus is primarily on the processing of "new-formation" (e.g., internally generated forms) rather than "information." Like rationalism and empiricism, metaphorism includes other psychological components like conceptualizing and perceiving, but the primary psychological process which one uses in knowing metaphorically is symbolizing (Royce, 1978).

Based on these three ways of knowing, Royce and his colleagues (e.g., Royce & Smith, 1964; Royce & Mos, 1980; Smith et al., 1967) developed a standardized inventory, called the psycho-epistemological profile (PEP), as a way to assess empirically a person's epistemological hierarchy. A person's profile on the PEP consists of three scores, one for each of the three ways of knowing. The highest score signifies the dominant epistemology for that person. Thus, each person can be labeled according to a particular epistemic style and cognitive style. Epistemic style is defined as a construct that simultaneously elicits a valid truth criterion, which then leads to a justifiable knowledge claim in addition to being a characteristic way of interacting with the environment. Cognitive style is a construct that is limited primarily to the three cognitive processes (Royce, 1978). The different ways of knowing are due to a difference in the underlying profile in terms of both cognitive style and cognitive abilities. Since each person can be labeled in terms of his or her epistemic hierarchy, it follows that his or her label can be used to explain differences that exist between each person's beliefs about knowledge and beliefs about reality (Royce, 1983). While Royce acknowledged these cognitive processes do not function independently and that, for a comprehensive understanding of the world, all three ways of knowing should be invoked, a person is

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partial to one of the cognitive processes that reflects his or her predominant epistemology.

Finally, Royce (1978) hypothesized that since people develop more specialized forms of knowledge as they progress through their education, it follows that specialized forms of knowledge are also dependent on the three epistemologies. Thus, all three epistemologies are involved in each discipline of knowledge, but each discipline gives greater credence to one or more of the three ways of knowing. For example, scientific, social scientific, and mathematical knowledge involve all three epistemologies but the epistemologies for each are differentially weighted. For science, empiricism is given the most weight followed by rationalism, then metaphorism. For social science, metaphorism is given the most weight followed by empiricism, then rationalism. The high degree of metaphorism for social science reflects a continuous search for the "right" paradigm, and the low rationalism weight signifies that the right paradigm has not yet been identified. The proportion of empiricism reflects the concern for facts as a basis on which to build a more mature science of social phenomena. Finally, for mathematics, rationalism is given the most weight followed by empiricism and metaphorism. Each discipline can be similarly construed and, consequently, each discipline's beliefs about how we know can be identified.

Consequently, in response to the question "How do we know?" Royce (1983) suggested the answer involves both epistemological and psychological considerations. His major thesis is that all knowledge claims are differentially dependent on three epistemologies, rationalism, empiricism, and metaphorism, and while each of these three ways of knowing may lead to truth, they may also lead to error. Although each cognitive

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mode contributes to the advancement of knowledge, including those based on a combination of all three ways of knowing, these approaches lead to a limited view of reality.

Educational Psychology and Epistemology

Two cornerstones of research on personal epistemology in educational psychology can be traced to Piaget's consideration of genetic epistemology and to Perry's (1968, 1970) work on epistemological development among college students (Hofer, 2002). Piaget's work on cognitive development was guided by one prevalent conception, that "[the] problem of knowledge, the so-called epistemological problem, cannot be considered separately from the development of intelligence" (Piaget, 1970a, p. 704). On the philosophical questions, "What is the source of knowledge, from sensory experience (empiricism), reasoning capacity (rationalism), or innate attributes (nativism)?" and, "What is the relationship between the knower and the known? Does knowledge depend on the individual (idealism), is it completely independent of the individual (realism), or is truth somewhere between these two extremes (constructivism)?" Piaget took a constructivist position. He claimed the development of intelligence is one aspect of biological growth characterized by the processes of assimilation and accommodation. Piaget (1970b) believed the acquisition of knowledge to be active rather than passive; individuals construct their knowledge by stimulating biological structures in an active social environment. To Piaget, knowing meant transforming reality to understand how a certain state is brought about. Piaget's position on epistemology has influenced various researchers whose models reflect a developmental sequence and, for example, who claim

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developmental change occurs through assimilation and accommodation (e.g., Kitchener & King, 1981).

Perry (1968, 1970) was interested in differences in students' responses to the diverse intellectual and social environment of the university. Based on two decades of interviews with predominantly male college students, Perry proposed that students progress serially through nine intellectual and ethical positions. In the early stages, students view knowledge as either right or wrong and believe that authority figures have all of the answers. After being exposed to more conflicting paradigms and models, students eventually conclude that one point of view is as good as another. As they progress through higher levels of education, students begin to perceive knowledge as correct relative to various contexts. Finally, near the end of their undergraduate careers, students realize there are multiple possibilities for knowledge and there are times when they must make a strong yet tentative commitment to some ideas.

Dissatisfied with the sex bias in Perry's (1968, 1970) samples, Belenky, Clinchy, Goldberger, and Tarule (1986) used Perry's framework to interview women from diverse educational settings to gain a better understanding of "women's ways of knowing" and how these developed over time. Based on extensive interviews with women, Belenky et al. devised five epistemological perspectives. In the beginning stages, women perceive themselves as mindless and voiceless, unable to generate knowledge that is held by authority. As women develop, they begin to see knowledge as subjective, personal and private. In the final stages, women see knowledge as obtained through both objective and subjective processes.

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Another model of personal epistemology that focuses on intellectual development is Kitchener and King's (1981). They refined Perry's (1968, 1970) intellectual development model by focusing on how individuals cope with ill-structured problems. Their Reflective Judgment Model includes seven stages of beliefs about knowledge and reality (King, Kitchener, Davison, Parker, & Wood, 1983; Kitchener, 1983). Each stage is related to individuals' justification of their claims. In the beginning stages of this model, they argue that individuals see knowledge as absolute; but, as individuals progress through the stages, their beliefs evolve into understanding knowledge as being temporarily uncertain. In later stages, individuals begin to see multiple perspectives of knowledge and conclude that knowledge is subjective. In the final stage, individuals believe that knowledge is an ongoing process of inquiry and only approximates reality.

In contrast to a developmental approach, Schommer (1990) developed a model of personal epistemology that attempted to capture the multidimensionality of epistemic beliefs. She argued that epistemic beliefs are a multidimensional system of more or less independent beliefs. The development of those beliefs is not necessarily coordinated. Personal epistemology is too complex to be captured in a single dimension. Further, while this set of beliefs may function in unison at certain times during one's life, this is not universal. Thus, she suggests that beliefs can be characterized as a profile that reflects a person's belief on each dimension. The most prevalent belief guides a learner's general approach to learning and interpreting information. Moreover, Schommer contends the development of epistemic beliefs may be recursive, whereby beliefs are revisited, revised, and honed throughout life, and that development and change is influenced by experience. These experiences include formal educational experiences (e.g., engagement in problem

solving and learning, teacher influences, peer influences, etcetera) and life experiences (e.g., home environment).

Schommer (1990) proposed three dimensions of knowledge: (1) the *certainty of knowledge*, ranging from knowledge is unchanging to knowledge is evolving; (2) the *source of knowledge*, ranging from knowledge is handed down by authority to knowledge is acquired through reason or logic; and, (3) the *structure of knowledge*, ranging from knowledge is organized as isolated bits and pieces to knowledge is organized as highly interrelated concepts. She proposed two further dimensions relating to knowledge acquisition. The fourth dimension of her model is (4) the *control of knowledge acquisition*, ranging from the ability to learn is inherited and unchangeable to the ability to learn can improve over time. The last dimension is (5) the *speed of knowledge acquisition*, ranging from learning is quick or not at all to learning is gradual.

Over the past twelve years, Schommer (1990, 1994a, 1994b, 1998, 2002, 2004) has refined and elaborated her conceptualization of epistemic beliefs. Initially, Schommer (1990) defined epistemic beliefs as students' beliefs about the nature of knowledge and learning. More recently, however, Schommer-Aikins (2004) acknowledged that beliefs about learning are not epistemic beliefs per se but are important to consider along with epistemic beliefs. One underlying assumption of her model is that individuals have an unconscious system of beliefs about what knowledge is and how it is acquired. Further, these beliefs have subtle yet important effects on how individuals comprehend, monitor their comprehension, solve problems, and persist in the face of challenging tasks. Moreover, these beliefs are likely to affect reasoning, learning, and decision making with

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both direct and indirect effects on learning. Therefore, beliefs based on each dimension are likely to affect students' behaviors.

Mathematics Education and Epistemology

Over the past two decades, three predominant perspectives on approaches to knowing have influenced research in mathematics education: the cognitive constructivist perspective, the symbolic interactionist perspective, and the socio-cultural perspective (Cobb, 1996). According to Cobb, the cognitive constructivist perspective views students as actively constructing individual ways of knowing by establishing coherence across various personal experiences. Cognitive constructivism is typically regarded as being influenced by Piaget's (1970b) genetic epistemology or ethnomethodology (Mehan & Wood, 1975). For example, von Glasersfeld (1989a) developed a theoretical foundation for a psychological perspective (Cobb & Yackel, 1998) that incorporates Piagetian notions of assimilation and accommodation and the cybernetic notion of viability. von Glasersfeld (1992) uses the term "knowledge" to refer to sensory-motor and conceptual operations that have been demonstrated viable in the knower's experience. According to his model, truth is related to the effective or viable organization of activity; truths are replaced by viable models of experience where viability is always relative to a chosen goal (von Glasersfeld, 1992). Thus, the driving force of development is perturbations a person produces relative to a purpose or goal. Consequently, learning is described as a process of self-organization whereby a person reorganizes activity to eliminate perturbations (von Glasersfeld, 1989b). Because this activity occurs most often as a person interacts with other members of a community (von Glasersfeld, 1992), the most frequent source of perturbations is interaction with others (von Glasersfeld, 1989b).

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Cobb and Yackel (1998) report that Bauersfeld's interactionist perspective complements von Glasersfeld's psychological perspective: both view communication as a process of shared adaptation wherein individuals negotiate meanings by transforming their interpretations (Bauersfeld, 1980). In contrast to von Glasersfeld, Bauersfeld (1988) describes learning as the "subjective reconstruction of societal means and models through negotiation of meaning in social interaction" (p.39). Specifically, Bauersfeld focuses on the teacher and students' interactions in a classroom microculture. Moreover, he claims that perturbations not only occur when communications are believed to have broken down and individuals begin to negotiate meaning, but also occur implicitly when subtle shifts of meaning occur outside of participants' awareness. Thus, Bauersfeld focuses on processes by which the teacher and students constitute mathematical practices and social norms through classroom interactions. As Cobb (1989) suggests, this focus proposes that individual students' mathematical activities and the classroom microculture are reflexively related. Consequently, students are viewed as actively contributing to developing classroom mathematical practices that enable and constrain individual mathematical activities (Cobb, 1996).

In contrast to these two perspectives, the socio-cultural perspective emphasizes the socially and culturally situated nature of activity (Cobb, 1996). The theoretical foundations of the socio-cultural perspective were inspired by Vygotsky's work as well as the work of activity theorists such as Davydov, Leont'ev, and Galperin (Nunes, 1992). Sociocultural theorists typically assume that cognitive processes are subsumed by social and cultural processes (Cobb & Yackel, 1998), and have found empirical evidence to support their claims that individuals' mathematical activities are constituted by their

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participation in surrounding cultural practices (e.g., Lave, 1988a). Given the assumption

that social and cultural processes are of primary importance, the individual-in-social-

action is regarded as the as the fundamental unit of analysis (Minick, 1989).

Consequently, the principal concern is to explain psychological development in reference

to participating in social interactions and culturally organized activities (Cobb & Yackel,

1998).

According to Cobb and Yackel (1998), theorists have addressed this principal

concern in a number of ways. For example, Vygotsky (1978) stressed the zone of

proximal development, whereby a learner is engaged in social interaction with another

more knowledgeable person, and culturally developed symbols and signs as

psychological tools for thinking. Leont'ev (1981), in contrast, proposed that thought

develops from practical, object-oriented labor or activity while other theorists have

identified cognitive apprenticeship (Brown, Collins, & Duguid, 1989), legitimate

peripheral participation (Lave & Wenger, 1991), and the negotiation of meaning in the

construction zone (Newman, Griffin, & Cole, 1989). For each of these theorists, learning

is situated in co-participation in cultural practices. Negotiation, from a socio-cultural

perspective, is viewed as a process of mutual appropriation by which a teacher and

students (or peers) constantly use each other's contributions. Thus, a teacher's role is

viewed as mediating between students' meanings and culturally established mathematical

meanings of the larger society (e.g., not just the microculture of the classroom; Cobb,

1996).

Research on students' beliefs about the nature and acquisition of knowledge in

mathematics education has not typically been labeled as "personal epistemology" or

"epistemic beliefs." Instead, the literature has examined this line of inquiry under the construct of "beliefs" and has usually assessed how beliefs develop, how they influence engagement in mathematical learning and problem solving, and how beliefs may change. Research on beliefs in mathematics education has become an important line of inquiry but, much like the field of educational psychology, there is no single consistent theoretical framework from which to examine students' beliefs about mathematics (McLeod, 1992).

Within the literature on mathematics beliefs, some scholars have defined beliefs as a metacognitive construct (e.g., Garofalo & Lester, 1985) whereas others have defined it as an affective construct (e.g., McLeod, 1992). Scholars have categorized beliefs as beliefs about the nature of mathematics, mathematical learning, and problem solving (e.g., Schoenfeld, 1985); beliefs about the self in the context of mathematics learning and problem solving (e.g., Kloosterman, Raymond, & Emenaker, 1996), beliefs about mathematics teaching (Thompson, 1984), and beliefs about the social context (Cobb, Yackel, & Wood, 1989). Finally, some approaches are even broader. For example, Lester, Garofalo, and Kroll (1989) describe beliefs in terms of students' subjective knowledge regarding mathematics, self, and problem solving activities. Underhill (1988) describes beliefs within a two by two dimensional framework. The first dimension divides students' beliefs about mathematics into rule-oriented versus concept-oriented. For the second dimension, students' beliefs are divided into whether mathematics is learned by knowledge transmission versus construction. (For overviews of the varying conceptualizations, see McLeod, 1992; De Corte, Op 't Eynde, & Verschaffel, 2002.)

A Synthesized Definition of Epistemology

Among theoretical models of personal epistemology in theoretical psychology, educational psychology, and mathematics education, there is at least one common thread: all three have examined one or more facets of epistemology. Accordingly, the definition selected for this review is derived from the more philosophical notion of epistemology. The Cambridge Dictionary of Philosophy (Audi, 1999) defines epistemology as "the study of the nature of knowledge and justification: specifically, the study of (a) the defining features, (b) the substantive conditions or sources, and (c) the limits of knowledge and justification" (p. 273). Thus, an examination of personal epistemology and epistemic beliefs includes exploration of the nature of knowledge, justification of knowledge, sources of knowledge, and developmental aspects of knowledge acquisition (Royce et al., 1978).

Although beliefs about learning are not treated as an epistemological issue in traditional debates, I have included them in the literature review since these have been shown to influence various facets of cognition, motivation, and learning (Muis, in press). For simplicity, for the literature review, I have adopted the broader label of beliefs to refer to both epistemic and learning beliefs. This definition includes the several models in the discipline of educational psychology, narrows the definition in mathematics education to beliefs about the nature of mathematical knowledge and mathematical learning, and includes the model presented from theoretical psychology. Given this broadened yet more illuminating conception of personal epistemology, the research chosen for this review is described next.

Criteria for Inclusion

The literature included in this review is empirical research on personal epistemology and epistemic beliefs of students of all ages from studies that used Royce's (1983) model and studies from the domains of educational psychology and mathematics education. Studies were selected if they examined mathematics students' personal epistemologies or students' beliefs about mathematics (either as a main focus of the investigation, a secondary focus, or minor focus) that could be identified as satisfying one or more of the components of the definition. These components included the nature of knowledge in mathematics, justifications of knowledge in mathematics, and sources of knowledge in mathematics, including beliefs about learning mathematics.

Students' Epistemic Profiles and Beliefs about Mathematics

Studies from Theoretical Psychology

Royce (1978) proposed that people who have different specialized forms of knowledge have different epistemic profiles. Specifically, he argued that as people progress through formal educational experiences, their knowledge in their field of study would become more specialized. He further proposed that specialized forms of knowledge are also dependent on the three types of epistemologies. Consequently, he predicted that science majors and professionals in science, such as biology or chemistry, would most likely be profiled as predominantly empirical in their approaches to knowing. Students or professionals in the arts, such as music, would most likely be committed to a metaphoric epistemology. Finally, students or professionals in mathematics, theoretical physics, or philosophy, would most likely be profiled as predominantly rational in their

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approaches to knowing. Studies that have been conducted with samples of professionals have found support for Royce's hypothesis (e.g., Royce & Mos, 1980; Smith et al., 1967).

To examine whether these results could be generalized to students specializing in different fields, Kearsley (no reference reported; cited and reported in Royce & Mos, 1980) administered the PEP (Royce & Mos, 1980) to various groups of graduate students. Ninety-seven graduate students from diverse disciplines, including life sciences (botany and zoology), analytic sciences (theoretical physics, chemistry, and mathematics), and humanities (classics, fine arts, English, and philosophy), completed the PEP. Kearsley found that for students in the life sciences, their average score on empiricism was statistically detectably different than their average scores on rationalism and metaphorism. Specifically, their score on the empiricism scale was greater and, consequently, students from life sciences were epistemologically profiled as predominantly empirical. For students in the analytic sciences, their average score on rationalism was statistically detectably greater than their scores on empiricism and metaphorism. Analytic sciences students were epistemologically profiled as predominantly rational. Finally, with the exception of graduate students in philosophy who were epistemologically profiled as predominantly rational, for students in the humanities, their average score on metaphorism was statistically detectably greater than their average scores on empiricism and rationalism. These students were epistemologically profiled as predominantly metaphorical. Kearsley concluded that, consistent with studies that examined professionals' profiles, graduate students' field of study can predict students' predominant epistemology.

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Summary and Critique

The one study reported here on students' epistemic profiles supports Royce's (1978) hypothesis that people who have different specialized forms of knowledge have different epistemic profiles. Of specific interest, students specializing in mathematics were profiled as predominantly rational in their approaches to knowing. Although other studies also support this hypothesis (e.g., Royce & Mos, 1980; Smith et al., 1967), what remains to be examined is whether this hypothesis is supported with undergraduate students. Royce speculated that by gradual socialization through formal education in the epistemic patterns of their discipline, students' epistemic profiles become consistent with the predominant epistemology of their major field of study. Are undergraduate mathematics students also predominantly rational in their approaches to knowing? One of the objectives of my dissertation is to address this question.

Studies from Educational Psychology and Mathematics Education

Over the past two decades, researchers and educators have been more and more concerned with students' lack of comprehension of mathematics (National Council of Teachers of Mathematics [NCTM], 1989). Recently, researchers have turned their attention to beliefs, in the context of the affective domain, to examine how beliefs influence learning (McLeod, 1992). Beliefs, in general, constitute an individual's knowledge about the world. An individual's worldview, or Weltanschauung, is composed of an overall perspective on life that entails all that one knows about the world, how to evaluate the world emotionally, and how to respond to it volitionally (Audi, 1999).

In the context of mathematics, epistemic beliefs include perspectives on the nature of mathematics knowledge, justifications of mathematics knowledge, sources of

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mathematics knowledge, and acquisition of mathematics knowledge. These epistemic

beliefs serve to establish a psychological context for learning (Schoenfeld, 1985).

Schoenfeld (1985) refers to beliefs as an individual's mathematics worldview; the

perspective one takes to approach mathematics and mathematical tasks. As Cobb (1986)

states, beliefs are critical components that help to create meaning and establish overall

goals that define the contexts for learning mathematics. The act of devising a goal

delimits possible actions; the goal, as an expression of beliefs, consists of anticipations

and expectations about how a situation will unfold. If beliefs are argued to have such an

influence on the way students engage in learning and problem solving, then the first

question that needs to be addressed is: What are students epistemic beliefs about

mathematics?

In 1986, the fourth mathematics assessment of the National Assessment of

Educational Progress (NAEP) was administered to secondary school students in the

United States. Included in the assessment were a number of questions about students'

beliefs and attitudes about mathematics. Specifically, four categories were included:

mathematics in school, mathematics and one's self, mathematics and society, and

mathematics as a discipline. Questions that addressed students' beliefs about mathematics

as a discipline dealt with perceptions of mathematics as a process-oriented versus rule-

oriented subject or as a dynamic rather than a static subject, and with perceptions of

mathematicians and mathematics as a formal discipline.

Brown, Carpenter, Kouba, Lindquist, Silver, and Swafford (1988) reported on

results from the seventh and eleventh grades. When asked whether they agreed or

disagreed with the following statements, the majority of seventh-grade and eleventh-

grade students reported that they agreed that mathematics problems always have a rule to follow and that doing mathematics requires lots of practice in following rules. Approximately half believed that learning mathematics meant mostly memorizing. Moreover, 36% of the students in grade seven and eleven agreed that mathematicians work with symbols rather than ideas, approximately 20% agreed that mathematics is made up of unrelated topics, and approximately 30% believed that new discoveries are seldom made.

Garofalo (1989a) also assessed secondary school students' beliefs about mathematics. From his experiences as a secondary school mathematics teacher, observations of secondary school mathematics classrooms, and discussions with students, he found that students at various levels in secondary school held similar beliefs. These beliefs included beliefs about the nature of mathematics and beliefs about oneself and others as "doers" of mathematics. Garofalo found that students typically believe that almost all mathematics problems can be solved by applying facts, rules, formulas, and procedures the teacher has taught or as presented in the textbook. When it comes to learning mathematics, students believe that memorizing facts and formulas and practicing procedures is sufficient. Garofalo also found that students believe mathematics textbook exercises can be solved only by the methods presented in the textbook in the section in which they appear. Specifically, students viewed mathematics as a highly fragmented set of rules and procedures rather than a complex, highly interrelated conceptual discipline. Finally, Garofalo found that students believe very prodigious and creative people create mathematics, and that the source of knowledge for everyone else is some authority figure. It follows that students believe teachers and textbooks are the authorities and dispensers

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of mathematical knowledge and that students readily accept that knowledge without challenge (Schoenfeld, 1985). Students rarely question what they are told and view themselves as copiers of others' mathematical knowledge.

Similar to the two previous studies, Diaz-Obando, Plasencia-Cruz, and Solano-Alvarado (2003) also found that two secondary school students from two different contexts, Spain and Costa Rica, believe that school mathematics is based on rules and memorization and mostly driven by procedures rather than concepts. Using an interpretive approach, they examined two secondary school students' beliefs about mathematics. The first student, Kevin, attended a secondary school in a rural area, considered to be of middle to low social class, in Tenerife, Spain. The second student, Sam, attended a school located in an urban marginal area in the province of Heredia, Costa Rica.

Based on classroom observations and semi-structured interviews with the two students, Diaz-Obando et al. (2003) constructed an image of these students' beliefs about mathematics and learning. From Kevin's interview, they interpreted that he believed all types of knowledge are uniformly gathered and learned by the same method. Learning mathematics included learning how to add, subtract, multiply, divide, find square roots, and all other procedures they practiced in the mathematics classroom. Moreover, Kevin was perceived to believe that learning school mathematics included rote memorization using fixed procedures. In the classroom, procedures are explained and taught and students are expected to follow.

The interview with Sam was interpreted to express beliefs similar to Kevin's. Diaz-Obando et al. (2003) interpreted that Sam believed it is important for students to

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solve the task the way the teacher requested. The structure of Sam's class was such that the teacher typically explained how to formulate and solve problems. Thus, Sam was interpreted to believe that school mathematics is a subject that needs to be practiced and that going home and mimicking the procedures the teacher taught is beneficial. Moreover, the researchers interpreted that Sam believed learning school mathematics required memorizing the procedures taught in class. Diaz-Obando et al. inferred that when these students are successful in mimicking the procedures they believe they understand mathematics.

Like secondary school students, elementary school students also appear to have similar beliefs about mathematics. Kloosterman and Cougan (1994) examined mathematical beliefs of students of varying ability in grades 1 through 6, most of whom were from lower and lower-middle socioeconomic backgrounds. Students were interviewed about various aspects of mathematics that included whether they believed all children had the ability to learn mathematics. One question specifically addressed a belief about learning, whether ability is fixed or incremental. Kloosterman and Cougan found that only 4 of 14 first-grade students and 2 of 11 second-grade students believed all students could learn mathematics. In contrast, by grades 3 and 4, most students indicated that all students who tried hard enough could learn mathematics. Of those third and fourth grade students who did not believe all students could learn mathematics through effort, some stated that, "Some [students] just weren't born to do math" (p. 383). Finally, of the fifth- and sixth-grade students who were interviewed, all said that anyone can learn mathematics and most indicated that effort was a key component of learning.

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Based on Kloosterman and Cougan's (1994) study, it appears that very young students believe that ability is fixed but, as they develop, at least until grade six, students believe ability is incremental and effort plays a major role in learning mathematics. These results, however, may not generalize to most elementary students because the study took place in a school where teachers were participating in a project to improve mathematics teaching and only a small sample of students from one school participated.

Frank (1988) examined beliefs of middle school students who were considered to be mathematically talented (based on results of a standardized achievement test). Twenty-seven students enrolled in a course in mathematics problem solving with computers filled out a survey on mathematical beliefs. Fifteen of the 27 students were observed daily, and 4 of these 15 students were interviewed at least four times over 2 weeks. Based on her analysis of the survey, observational, and interview data, Frank devised a list of five commonly held beliefs about mathematics. First, students believed mathematics is computation and learning mathematics involves memorizing arithmetic facts and algorithms. Second, students believed mathematics problems should be solved quickly and in just a few steps. If a problem took longer than 5 to 10 minutes to solve, students believed something was wrong with them or with the problem. Third, students believed the goal of doing mathematics is to obtain one right answer. Students tended to view mathematics as dichotomized into right or wrong answers and that the teacher was the only source for determining whether their answers were right or wrong. Fourth, students believed their role was to receive mathematical knowledge by paying attention in class and to demonstrate that it has been received by producing correct answers. Finally, students believed the mathematics teacher's role was to transmit mathematical knowledge

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and to verify that students have received that knowledge. If teachers explain the material well, then students should be able to produce correct answers and produce them quickly.

Fleener (1996) also examined beliefs of students who were considered to be mathematically and scientifically talented (based on ratings given by teachers and counselors). Twenty sophomore and junior high school students enrolled in a four-week summer residential program participated in the study. The program was designed to present ideas and concepts distinct from traditional high school mathematics and science curricula. At the beginning of the program, students completed a 46-item questionnaire assembled from various science and mathematics scales (these were not identified). Field notes, class discussions, and information from social gatherings were used to validate and clarify survey data.

Based on students' responses to the questionnaire, Fleener (1996) found that, for mathematics, students strongly agreed that "2 + 2 always equals 4" (percentage of students strongly agreeing to this was not given), that mathematics is slowly revealing truths about reality, and that changes in knowledge about mathematics are a result of scientific, empirical investigations which reveal truths about reality. Fleener also found that students strongly believe there are some mathematical truths that will never be proven wrong. Based on these beliefs, Fleener concluded that students' strong agreement to these items suggest that they believe mathematics knowledge consists of given truths which may be revealed empirically. In contrast, Fleener found that students strongly disagreed that only geniuses have what it takes to be successful in mathematics and were mixed in their beliefs that there is often more than one solution to a mathematics problem and that mathematics is changing.

Elementary, middle, and secondary school students are not the only students who hold these beliefs about mathematics. Spangler (1992a) presented a variety of open-ended questions to elementary, junior high and senior high school students, and graduate students in mathematics education to assess their beliefs about mathematics. Surprisingly, the responses across all levels of education were strikingly similar. Students believed mathematics involves searching for one correct answer. When faced with two different answers to the same question, students would rework the problem or accept the "smarter" student's answer. Rarely did students indicate that both answers might be correct. Regarding learning mathematics, most students held the belief that memorization is key. These students preferred to have only one method for solving a problem because it would reduce the amount of memorizing they would have to do. Only a few students preferred to have several methods to choose from because some methods were more efficient and other methods could help to check answers. Similar to the previous studies, one major focus for these students was to obtain the correct answer. Finally, many students at the elementary level believed mathematics problems should be done quickly and that mathematically talented people could do them quickly. Older students typically indicated that mathematically talented people were logical and could work problems in their head.

Schoenfeld (1988) gathered further support for the typical nature of students' mathematical beliefs. He conducted a year-long intensive study in a suburban school district to examine the presence and robustness of four beliefs about mathematics typically found among students and to seek the possible origins of those beliefs. He selected one of 12 secondary geometry classes to observe at least once a week over the year. Two weeks of instruction in this focal grade 10 geometry class were videotaped and

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analyzed in detail. The 20 students in the chosen class filled out an 81-item questionnaire, as did the 210 other students in the remaining 11 classes. The other 11 classes were observed periodically to determine whether the students and instruction in the target class could be considered typical.

Schoenfeld (1988) found strong support among all students for the four beliefs he identified: 1) The processes of formal mathematics (e.g., "proof") have little or nothing to do with discovery or invention. 2) Students who understand mathematics can solve assigned problems in five minutes or less. 3) Only geniuses are capable of discovering, creating, or really understanding mathematics. And, 4) one succeeds in school mathematics by performing the tasks, to the letter, as described by the teacher. At all levels of education, it appears that students hold similar beliefs about mathematics.

Summary and Critique

The majority of research that has examined students' beliefs about mathematics suggests that students at all levels hold similar beliefs. In general, when asked about the certainty of mathematical knowledge, students believe knowledge is unchanging. Mathematics proofs support this notion and the goal in mathematics problem solving is to find *the* right answer. Students also believe mathematics knowledge is passively handed to them by some authority figure, typically the teacher or textbook, and they are incapable of learning mathematics through logic or reason. Moreover, they believe those who are capable of doing mathematics were born with a "mathematics gene" (a belief in innate ability).

Another common belief is that various components of mathematical knowledge are unrelated; the structure consists of isolated bits and pieces of information. Students do

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not typically perceive relationships among concepts, and thus rely on the teacher and textbook to tell them what they need to know for each type of problem they encounter. Students do not believe they are capable of constructing mathematical knowledge and solving problems on their own. Finally, students typically believe learning mathematics should be quick, within 5 to 10 minutes. If they have not solved the problem or come up with the correct answer in that time period, students believe they will never be able to figure it out because they are incapable of understanding the problem or something is wrong with the problem itself.

These studies provide a clear picture of what students commonly believe about mathematics. They are not, however, without methodological flaws. In a chapter on methodological issues and advances in researching self-regulated learning, Winne et al. (2002a) recapitulated four concerns about measuring self-regulated learning that Winne and Perry (2000) addressed and advanced a number of other concerns they identified in a review of the literature on self-regulated learning. Although the chapter specifically addresses research on self-regulated learning, a number of issues are relevant to any research that uses self-report measures (e.g., questionnaires, interviews, surveys, etcetera). One technical issue of measurement that Winne et al. reiterated from Winne and Perry was the reliability and dependability of self-report measures. Reliability refers to the consistency of a measuring instrument in repeatedly providing the same result for a given person. In research that uses self-report inventories with researcher-provided response formats, reliability is typically reported as a coefficient of internal consistency. Only one of the studies reviewed in this section that used researcher-provided response formats reported reliability estimates. This poses a challenge in determining the potential

for consistency of students' responses. One fundamental criterion for defining a category of beliefs is that it has adequate internal consistency. Not reporting reliability estimates poses a challenge for readers to determine the reliability of the scales used and, thereby, the defined category or categories which the scale attempts to measure.

Other technical issues Winne et al. (2002a) advanced include the influence(s) response formats, situational factors, and other methodological features (e.g., instructions) may have on students' responses. Since the studies reviewed used several response-generating formats – Likert-type or dichotomized scales that measure the frequency of an event or agreement with a statement, structured interviews with probes, open-ended questions, etcetera – one may question what influence(s) these various formats have on students' responses. Second, if response data are aggregated into various factors or dimensions that represent students' beliefs about mathematics, to what extent are these dimensions similar or different across studies? No examination of these issues has been conducted.

Winne and Perry (2000) also noted that most measurements are designed to engage the person to generate or recall a specific kind of response. Self-report inventories include instructions that establish a context, a response scale, and items that are assumed a priori to affect people to respond in particular ways. For example, in Schoenfeld's (1989) survey, students are instructed to "circle the number under the answer that best describes how you think or feel" using a 4-point rating scale anchored by "very true" (recorded as 1), "sort of true" (recorded as 2), "not very true" (recorded as 3), or "not at all true" (recorded as 4), or for some items anchored by "always" (1), "usually" (2), "occasionally" (3), or "never" (4).

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As Winne et al. (2002a) indicate, it is well known that memory searches more often entail a construction rather than a retrieval of the requested information (Bartlett, 1932). When asked to indicate the frequency of an event, people may select a less cognitively taxing process of heuristic development to answer rather than engage in an exhaustive search through memory. Moreover, Tourangeau, Rips, and Rasinski (2000) reported that, when asked to estimate the frequency of events, rare events were underestimated and common events were overestimated. Thus, if students use a less taxing strategy and therefore under- or over-estimate the frequency of an event, the result may be a less accurate portrait of what their beliefs are or how their beliefs may influence learning behaviors. This hypothesis, however, has not yet been investigated in research on epistemic beliefs. Future research on this issue is needed.

Although burdened with a number of methodological issues, the studies that have examined what students' beliefs are about mathematics have found similar results at all levels of education. The question to address next is: How do these beliefs develop?

The Development of Epistemic Profiles and Beliefs

Studies from Theoretical Psychology

In discussions of epistemological development, Royce (e.g., Royce et al., 1978) centered his hypotheses around Piaget's and Chomsky's positions on epistemological development but also proposed that formal educational experiences may play an important role in students' personal epistemologies. No empirical studies have been conducted using Royce's model to examine whether students' epistemic profiles change as a function of formal educational experience. The question I pose is whether

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mathematics students, profiled as predominantly rational in their approaches to knowing, become more rational in their approaches to knowing as a function of more educational experience in mathematics. Ideally, to answer this question requires a longitudinal study. Unfortunately, I was not able to collect such data. Instead, I address this question using a cross-sectional design. The specific question I address is whether more experienced students (measured by year of study) are more rational in their approaches to knowing than students with less experience.

Studies from Educational Psychology and Mathematics Education

Perry (1968, 1970) set the stage for examining the development of epistemic beliefs in educational psychology. His work and that which followed (e.g., Belenky et al., 1986; Kitchener, 1983; King et al., 1983; Kitchener & King, 1981) focused specifically on what students' beliefs were and how those beliefs developed sequentially over time. In contrast, based on her multidimensional model of epistemic beliefs, Schommer (1994a) argued the development of epistemic beliefs may be recursive rather than sequential. This recursive process of revisiting, revising, and honing beliefs throughout life is influenced by experience, such as formal educational experiences and other life experiences. For example, Schommer and others have found that epistemic beliefs are related to early home environment (Schommer, 1993a), pre-college schooling experiences (Schommer & Dunnell, 1994), and the level and nature of postsecondary educational experiences (Jehng, Johnson, & Anderson, 1993; Schommer, 1993a).

When investigating the development of epistemic beliefs, researchers have assessed how beliefs develop sequentially (e.g., how beliefs develop naturally over time, typically viewed from a "development of epistemic cognition" perspective), or have

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examined what environmental factors, such as classroom cultures, influence beliefs. Typically, researchers in educational psychology have concentrated on how beliefs develop sequentially (e.g., Perry, 1970). Some research, however, has been conducted to examine sources of influence on beliefs (e.g., Jehng et al., 1993; Schommer, 1993a). In contrast, researchers in mathematics education have predominantly explored relationships between classroom environments and students' beliefs (e.g., Schoenfeld, 1988), while very few researchers have examined sequential development of beliefs (e.g., Kloosterman et al., 1996). Research in this area can be divided into two types: studies that address sequential development of beliefs, and studies that assess elements of classroom environments that may influence the development of beliefs. Since classroom environments were not examined in my research, I present here only studies that examined sequential development of beliefs. For a full review, see Muis (in press).

Sequential Development

Research in education that has focused on sequential development of epistemic beliefs has typically focused on college students' development (e.g., Ryan, 1984; Stonewater, Stonewater, & Hadley, 1986). There has been some research, however, that focused on secondary school students' beliefs over time (e.g., Schommer, 1993b; Schommer, Calvert, Gariglietti, & Bajaj, 1997) and one study that focused on elementary students' beliefs over time (Kloosterman et al., 1996).

In relation to sequential development of students' beliefs about mathematics according to Perry's (1970) conceptualization, only one study was found that examined development using a longitudinal design. Two other studies were found that examined developmental differences using a cross-sectional design. Kloosterman et al. (1996)

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interviewed students in first through fourth grades at one school each spring over 3

successive years (Kloosterman & Cougan [1994] also used data from this study). As part

of this longitudinal study, Kloosterman et al. designed interview questions to determine

developmental trends and to examine the temporal stability of students' beliefs. One

belief they addressed was whether students believed that learning was innate or

incremental. They found students' beliefs about the control of learning were fairly

consistent over the three years of the study. Those that did change, however, changed

from innate beliefs to incremental beliefs. Of the eight students who progressed through

grades 1, 2, and 3 during the study, three of the first grade students who said not everyone

can learn mathematics reported by grades 2 and 3 that anyone who tried could learn. Only

4 of the 48 students above second grade in the first year reported that learning

mathematics was innate. Kloosterman et al. did not indicate whether these students'

beliefs changed over time, but concluded that almost all of the students they studied

thought that anyone who wanted to could learn mathematics.

Mason (2003) examined high school students' beliefs about mathematics,

problem solving, and their achievement. Two of four purposes of Mason's study were to

examine differences in beliefs across five high school grades and to assess the

relationship between beliefs and mathematics achievement (to be addressed in the next

section). Five hundred ninety-nine students in Italian high schools participated in the

study. An Italian translation and revision of Kloosterman and Stage's (1992) Indiana

Mathematics Belief Scale and the Fennema-Sherman Usefulness Scale were used to

measure students' beliefs. Six dimensions of students' beliefs were measured and two of

those six fit the definition for this review. Specifically, one dimension measured students'

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belief that they can solve time-consuming problems (belief in quick learning was measured under this dimension) and a second dimension measured a belief that effort can increase mathematical ability (a belief in incremental ability).

A multivariate analysis of variance using a 5 (grade level) by 2 (sex) design with grade and sex as the between-subjects variables and scores on the six dimensions of the questionnaire as the dependent variables revealed a main effect of grade, a main effect of gender, but no interaction. For the two dimensions of interest, using a post hoc analysis, Mason (2003) found that students' belief that they can solve time-consuming problems changed over the five years of high school. Specifically, average scores across the grades increased from first to second year high school but then were lowest in the third and fifth years. Thus, change occurred but not in a linear fashion. No differences across the five grades were found on belief in incremental ability and no gender differences were found for either of the two dimensions.

King, Wood, and Mines (1990) also examined developmental differences using a cross-sectional design. Two of four purposes of their study were to examine whether graduate students score higher on tests of critical thinking than undergraduate seniors, and whether there are differences in critical thinking scores between students majoring in mathematics versus the social sciences. Forty college seniors and 40 graduate students, in their second year or beyond, participated in the study. An equal number of students from each discipline were chosen. King et al. examined students' critical thinking ability using the Cornell Critical Thinking Test, which measures students' ability to solve well-structured problems (Ennis & Millman, 1971), the Watson-Glaser Critical Thinking Appraisal, which uses both well-structured and ill-structured problems (Watson & Glaser,

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1964), and the Reflective Judgment Interview, which uses ill-structured problems (Kitchener & King, 1981). Students' ACT, SAT or GRE scores were also obtained.

To examine differences in critical thinking scores between college seniors and graduate students, King et al. (1990) conducted three three-way (educational level by discipline by gender) analyses of variance, one for each measure of critical thinking, and an analysis of covariance for each measure partialling out the effects of the ACT/GRE scores. They found a significant main effect for educational level for the Cornell Critical Thinking Test, the Watson-Glaser Critical Thinking Appraisal, and the Reflective Judgment Interview. Specifically, graduate students scored higher on all three measures than undergraduate students. When academic aptitude was controlled for, difference by educational level remained significant only for the Reflective Judgment Interview. They argued that the differences found in the other two critical thinking tests might reflect differences in academic ability, but not for the Reflective Judgment Interview.

Summary and Critique

The results of these three studies suggest that, regardless of level of education, in general, as students progress through higher levels of education, beliefs change. While these studies document what students' beliefs are and how they develop over time, they have not identified external factors that influence beliefs. Why do students hold the beliefs they do? What factors influence the development of students' beliefs about mathematics? While beliefs people have are determined in part by their experiences (e.g., Schommer, 1990), what exactly is the nature of those experiences? Moreover, if we assume that beliefs are multidimensional, do the various dimensions develop in synchrony? Is development recursive? The three studies presented here do not address

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these questions. More studies are needed, particularly studies that examine the development of beliefs using longitudinal designs.

It appears, however, that one plausible hypothesis is that formal mathematics education plays a major role in the development of students' beliefs about the nature of mathematical knowledge and learning. One question that remains to be addressed at this point is why researchers and educators should be so concerned about students' beliefs. Why are beliefs important? What influence do beliefs have on learning and achievement? The answers to these questions are addressed in the next section of this literature review.

Relations Between Epistemic Profiles and Beliefs and Learning

Studies from Theoretical Psychology

Diamond and Royce (1980) proposed that one's epistemic and cognitive styles are subsumed under a larger personality system called the supra-system, defined as a hierarchical organization of systems, subsystems, and traits which transform and integrate information. This supra-system includes a learning adaptive layer that relies on the cognitive system and an integrative layer that relies on the epistemic system. More specifically, they theorized that one's cognitive style determines what preferences a person has when acquiring information and this cognitive style determines one's epistemic style. Thus, by identifying a person's epistemological hierarchy one can determine what learning preferences and strengths a learner will have.

For example, a person profiled as predominantly rational in his or her approach to knowing may, theoretically, prefer conceptualizing as a means of learning. Researchers who have conducted factor analytic work to examine what constitutes conceptualizing

have found a general verbal factor and a reasoning factor (Botzum, 1951; Cattell, 1963; Horn & Cattell, 1966a). The verbal factor included verbal comprehension (e.g., knowledge of the meaning of words), syllogistic reasoning (e.g., formal reasoning from stated premises) and numerical ability (e.g., speed and accuracy in basic arithmetic operations). The reasoning factor included inductive reasoning (e.g., discovery of a rule which characterizes some sequence), deductive reasoning (application of an abstract rule to solve a problem) and spontaneous flexibility (e.g., the formation of an array of logical groupings). Based on this analytic work, Wardell and Royce (1975) hypothesized that rationalists would preferentially focus on elaborating concepts and developing a network of concepts through critical thinking, and would score higher on tests of general verbal ability and reasoning than those profiled as empirical or metaphorical in their approaches to knowing.

A person profiled as predominantly empirical in his or her approach to knowing may, theoretically, rely on perceptual processes as a means of learning. Researchers found that perceptual ability was comprised of a spatio-visual factor and a memorization factor (Cattell, 1971; Horn & Bramble, 1967; Horn & Cattell, 1966b). The spatio-visual factor included: 1) visualization, to recognize an object rotated in space, 2) spatial relations, to identify arrangements or elements out of their usual place, 3) flexibility of closure, to remember a visual configuration, 4) speed of closure, to combine visual components into a whole, 5) figural adaptive flexibility, to flexibly organize figure material, 6) spatial scanning, to solve spatial mazes, and 7) perceptual speed, to quickly identify visual elements. The memorization factor was comprised of measures of rote commitment to memory, which included memory span (e.g., the number of elements that

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can be held in working memory), memory for designs (e.g., the ability to reproduce a design) and associative memory (e.g., paired associates or serial learning). Based on these results, Wardell and Royce (1975) proposed that empiricists, whose primary approach to learning is perceptual in nature, are fact oriented and would perform better on spatio-visual and memorization tasks than individuals profiled as predominantly rational or metaphorical.

A person profiled as predominantly metaphoric in his or her approach to knowing may, theoretically, rely on symbolizing for learning. Royce (1964) proposed that symbolism refers to the representation of numerous objects or notions within a single unit and refers to cultural structures expressed in artistic or literary productions. Two factors that loaded on to symbolizing included fluency and imaginativeness (Horn & Bramble, 1967; Horn & Cattell, 1966b; Rossman & Horn, 1972). Fluency consisted of ideational fluency (e.g., the ability to quickly produce ideas about an object or condition), expressional fluency (e.g., the ability to quickly find an expression that satisfies some structural constraint), associational fluency (e.g., facility in producing words with a particular meaning) and, word fluency (e.g., facility in producing words that fit particular structural limits). The imaginativeness factor consisted of originality, measured by the ability to produce clever plot titles and remote consequences of hypotheses. One key distinction between conceptualizing and symbolizing is the reliance on the suggestive rather than denotative aspects of concepts when symbolizing. Thus, Wardell and Royce (1975) proposed that metaphorists, whose primary approach to knowing is symbolizing, focus more on meaningful symbols than other information and would perform better on tests of fluency than individuals profiled as rational or empirical. (For a detailed

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interpretation and explanation of relations between these elements, see Royce & Mos, 1980.)

In sum, Diamond and Royce (1980) argued, theoretically, one could identify an individual's epistemic profile and predict what cognitive processes that individual may prefer when acquiring information and how one would justify the veracity of that information. To test this hypothesis, Kearsley (1976) compared eight students' performance on a relational ordering task to the performance of a computer simulation model[1]. The computer simulation model was designed to behave according to Royce's (1983) psycho-epistemological profile model. Each epistemic style, rationalism, empiricism, and metaphorism, was designed to correspond to a different cognitive rule for acquiring new knowledge. The rational program accepted new information only if it was logically consistent with previous knowledge; the empirical program accepted new information only if it confirmed prior knowledge; and the metaphorical program accepted new information if it was similar to previous known facts (more explicit information on what prior knowledge the program possessed was not stated). The different processing orders of the three programs were presumed to correspond to the three different epistemologies.

Students were epistemologically profiled and then given a task that involved relational ordering of sets of nonsense sentences. It was predicted that there would be similar ordering patterns between the students and the computer programs based on epistemic profiles. Patterns of ordering between the students and computer programs

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[1] Students' majors were not identified. I chose, however, to include this study in the review since no other studies were found that examined preferences for learning or justification.

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were then compared. In general, Kearsley (1976) found similar patterns of performance between the students and computer programs and concluded that the cognitive processes examined underlie the three epistemic styles.

Summary and Critique

The one study reported here claimed to find support for Diamond and Royce's (1980) hypothesis that epistemic styles predict cognitive styles. I argue the only support this study found was for the ability of a computer program to successfully predict human sorting behavior on nonsense sentences. Although the computer program was designed to mimic human behavior based on epistemic and cognitive styles, the task given to the computer program and students was not meaningful. Students were not required to learn new meaningful material and, consequently, this study cannot be generalized to meaningful learning. Certainly, there are numerous learning behaviors that could be measured to assess relations between epistemic styles and cognitive styles. Diamond and Royce proposed a number of relations. These have yet to be tested empirically.

Second, this study did not directly measure the underlying processes students used to sort the nonsense sentences. Although the simulation program was moderately successful at predicting students' sorting behaviors, the study could have directly measured the cognitive processes students used to complete the task using a think aloud protocol. A more powerful approach to assess relations between epistemic styles and cognitive styles is to have students think aloud as they engage in a particular activity. This would allow for a more in depth analysis of relations between epistemic styles and cognitive styles.

Studies from Educational Psychology and Mathematics Education

Since its inception in the 1960s, epistemological research focused on how

students' beliefs matured over time. By the early 1980s and into the 1990s, research

began to focus more specifically on how these beliefs mediated students' behavior,

precisely, how students' beliefs mediated cognitive and motivational factors that underlie

learning and performance. Based on empirical research, scholars in educational

psychology and mathematics education have stressed the importance of students' beliefs

(e.g., Garofalo, 1989a; Kloosterman, 1996; McLeod, 1992; Schommer et al., 1992;

Dweck & Leggett, 1988). The research in this area can be divided into two types: studies

that focus on ways that beliefs shape students' behavior as they engage in learning, and

studies that focus on how beliefs relate to other cognitive and motivational factors and

how this constellation is related to achievement. It is important to note that, for the

second line of inquiry, researchers are also interested in how beliefs shape people's

behavior when learning. Typically, however, the outcome of interest is how beliefs

indirectly influence academic achievement via their direct influence on learning

behaviors. These two lines of inquiry are reviewed next. (Note: Two of the studies from

the first section will be reviewed in the section on *Schoenfeld's Model of Mathematics

Problem Solving* since they specifically test Schoenfeld's (1983) hypotheses of how

epistemic beliefs influence facets of self-regulation in the context of mathematics

problem solving.)

Learning

Garofalo (1989a) examined how students' beliefs influenced their engagement in

learning mathematics. (Details of this study have been previously described in the first

section of this review.) By first measuring what students believed about mathematics and subsequently assessing how students learned mathematics, he found that students who believe almost all mathematics problems can be solved by applying facts, rules, formulas, and procedures relied on memorization as the main avenue for learning. Students who believed mathematics textbook exercises could be solved only by the methods presented in the textbook relied on trying to remember the method given in the book rather than attempting the problem through reason. Finally, Garofalo found that students who believed the source of mathematical knowledge is some authority figure did not attempt to derive knowledge on their own; they relied on memorizing formulas and procedures but did not engage in attempting to understand the nature of the question.

Academic Achievement

To describe how epistemic beliefs might affect students' academic achievement, Schommer (1998) proposed that epistemic beliefs have direct effects on intellectual performance as well as indirect effects mediated through other facets of cognition. Specifically, she argues that if individuals hold particular conceptions of what it means "to know," their standards for comprehension would direct them to believe they have understood when in reality, they had not. For example, if a student believes that knowledge consists of isolated facts, he or she may think that memorizing lists of definitions constitutes a strategy for understanding. This student is therefore less likely to engage in transfer since he or she would not consider relationships among facts. In contrast, a student who believes knowledge is organized as highly interwoven concepts may be more able to transfer and apply information (Schommer, 1998).

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Second, strong believers in simple and certain knowledge are more likely to search for a single answer to a question, an answer they may believe is written in stone. Thus, when presented with mathematics problems they may expect there is only one path toward the solution. In areas such as statistics, this can become quite problematic as these students may fail to recognize alternative solutions and tentative interpretations of results. In contrast, strong believers of complex and tentative knowledge may search for more complex answers and may anticipate various solutions (Schommer, 1998).

Third, belief in the speed of knowledge acquisition may influence the time students devote to studying and solving problems. Strong believers in quick learning may set a maximum time they will engage in a particular task without regard for the complexity or difficulty of the task. Further, these individuals may believe very little time is required for studying, that a session or two is adequate for full understanding of material and "extra" time spent studying is a waste. In contrast, students who strongly believe in gradual learning are more likely to examine the material or problem and then decide how much time is needed. Time invested will be estimated but will be modified during the studying process depending on progress toward understanding. Thus, a much more planful, strategic approach toward learning may be used (Schommer, 1998).

Finally, beliefs in the control of learning are likely to influence interpretations of mistakes and persistence in the face of difficulty while learning and problem solving. Strong believers in fixed ability are more likely to believe that mistakes reflect their inadequacy. Thus, they may feel more frustrated and may be more likely to quit in the face of difficulty. Strong believers in incremental ability are more likely to see mistakes as opportunities to learn. They may experience an increased intensity of interest in

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studying or problem solving and different study strategies may be attempted rather than simply giving up in the face of difficulty (Schommer, 1998).

Several correlational studies support Schommer's (1998) hypotheses that these beliefs predict academic achievement (e.g., Schommer, 1990; Schommer et al., 1992) and are related to motivational and cognitive factors (e.g., Schutz, Pintrich, & Young, 1993). To assess the relationship between differences in beliefs in mathematics, learning, and academic achievement, Schommer et al. (1992) examined students' beliefs and mathematical text comprehension. Specifically, they examined whether belief in simple knowledge predicted mathematical text comprehension, assessed whether task demands influenced epistemological effects, and investigated whether effects of beliefs on learning were mediated by study strategies. Four hundred twenty four undergraduate and graduate students were given one of two sets of instructions. Prior to reading the passage, students were told to assess whether the passage would be considered clearly written and understandable by the average college freshman, or were told they would be teaching the material to another student. To assess metacomprehension, after students read the passage they were asked to rate their confidence in understanding the passage. Students also filled out the Learning and Study Strategies Inventory (LASSI; Weinstein, Palmer, & Schulte, 1987), which includes questions involving the integration of knowledge and test preparation. After reading the passage, students were given a mastery test.

Schommer et al. (1992) found that, after removing the effects of age and students' grade point average in a regression analysis, belief in simple knowledge was negatively related to comprehension and metacomprehension. The more students believed in simple knowledge, the worse they did on the comprehension test, and the more overconfident

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they were in their understanding of the passage. Moreover, using a path analysis, they found that the influence of simple knowledge on comprehension might be mediated by study strategies. In other words, students who believed the structure of mathematical knowledge is complex had better comprehension of the passage and were more accurate in their estimations of their understanding of the passage (better calibrated). The types of strategies students used to study the information may have mediated these differences.

Hofer (1999) also found that students' beliefs were related to cognitive, motivational, and achievement factors. She examined relationships among students' beliefs and motivation, learning strategies, and academic performance in two different instructional contexts in introductory calculus. One instructional context used traditional methods; instructors used a standard calculus text that proceeded sequentially, and were expected to cover a required amount of material primarily by lectures and demonstrations of problem sets. The alternative instructional context, called the "New Wave" approach, used more social-constructivist approaches where collaborative learning was emphasized, students engaged in active learning and were expected to work situated problems with potentially multiple approaches and more complex solutions.

Four hundred thirty-eight students filled out the Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich et al., 1991) that was used to assess their motivational orientations and their use of different learning strategies. To measure students' beliefs, Hofer adapted six items from Lampert's (1990) and Schoenfeld's (1992) work with a specific focus on students' beliefs about simple knowledge (e.g., "Mathematics problems have one and only one right answer") and isolated beliefs (e.g., "Mathematics is a solitary activity done by individuals in isolation.") Hofer (1999) found that these beliefs were

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significantly negatively correlated with intrinsic motivation, self-efficacy, and self-regulation, as well as with course grades. Moreover, at the end of the term, based on group means, students enrolled in the "New Wave" sections agreed less with these beliefs than students enrolled in the traditional style instruction sections; that is, students in classes that emphasized active learning were less likely to believe that the structure of mathematics knowledge was simple.

Similar to Hofer's (1999) methodology, Schoenfeld (1989) examined whether students' beliefs were predictive of academic achievement. (Data from Schoenfeld [1988] was used for this study.) Two hundred thirty students in grades 10 through 12, who were enrolled in either a year-long grade 10 geometry course, grade 11 algebra-trigonometry precalculus course, or a grade 12 calculus or problem solving course, filled out a questionnaire that examined various beliefs about mathematics. Questionnaires were distributed during the last two weeks of the school year. The questionnaire, which included 70 closed items and 11 open items, dealt with students' attributions of success or failure, their comparative perceptions of mathematics, English, and social studies, their view of mathematics as a discipline, and their attitudes towards mathematics which included beliefs about mathematics.

Students were asked to rate their agreement to statements on a 4-point Likert-type scale. A series of questions also asked students to report what grade they had received on their last report card, to predict what they expected to get at the end of the term, and how they rated themselves in terms of their mathematical ability in comparison to other students. Schoenfeld (1989) found that students who reported higher grades were less

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likely to believe that mathematics is mostly memorizing, that success depends on memorization, or that problems get worked from the top down in step-by step procedures.

Based on Schommer's (1990, 1993b; Schommer et al., 1992), Ryan's (1984), Perry's (1970), and Schoenfeld's (1983) work, Koller (2001) devised a four-dimension questionnaire to measure students' beliefs about mathematics, comparable to constructs proposed by Schommer, Ryan, and Perry. These include certain knowledge (Schommer), simple knowledge (Schommer), constructivism (Ryan), and relevance (similar to Perry's relativism). He was interested in examining whether students' beliefs about mathematics were mediated by learning strategies, interest in mathematics, and motivation. Two thousand one hundred thirty-eight upper secondary academic track students participated in his study. They rated their agreement to statements measuring the four dimensions on a 4-point Likert-type scale. Students' interest in mathematics was also measured by a short Likert-type rating scale, and students' learning strategies were measured using the Learning and Study Strategies Inventory (LASSI; Weinstein et al., 1987). Finally, achievement in advanced mathematics was measured using students' scores on 65 items from TIMSS.

Using path analysis, Koller (2001) found that all four dimensions of beliefs were significant predictors of mathematics achievement. Specifically, certain knowledge and relativism were directly related to achievement. Simple knowledge, constructivism and relativism were indirectly related to achievement through interest and rehearsal for simple knowledge, interest for constructivism, and interest for relativism. Certain knowledge and simple knowledge were negatively related to achievement whereas constructivism and relativism were positively related to achievement. Thus, students who

believed that mathematical knowledge was unchanging had lower achievement scores

than students who believed mathematical knowledge to be evolving. Students who

believed mathematical knowledge to be isolated bits of information had lower

achievement scores than students who believed mathematical knowledge to be

interrelated. Finally, students who held more constructivist and relative beliefs about

mathematical knowledge had higher achievement scores than those who believed

mathematical knowledge to be dualistic (either right or wrong).

Similar to Koller (2001), Lin (2002) examined relations of four dimensions of

beliefs on students' learning of mathematical concepts. More specifically, one of three

purposes of Lin's study was to investigate the relations of epistemic beliefs and various

computer graphics types on students' performance in a computer-based learning

environment. Similar to dimensions Schommer (1990) proposed, Lin examined beliefs

about: 1) First Time Learning, 2) Omniscient Authority, 3) Quick Learning and, 4)

Simple Learning. To measure students' beliefs, Lin translated and revised Jacobson and

Jehng's (1998) questionnaire to make it suitable for Taiwanese elementary students. A

preliminary exploratory factor analysis (N = 1240) replicated the four dimensions.

Computer graphics types had three levels: static graphics, computer animation, and video

clips. One hundred sixty-seven grade four students were randomly assigned to one of the

computer graphics types. Instruction was self-paced and students learned about the

concept of volume. At the end of the lesson, students were given an achievement test

designed to assess students' learning of the concept.

According to the four epistemic beliefs dimensions, students' responses were

grouped and then divided into "agree" versus "disagree" based on the mean score for that

particular dimension. Four two-way analyses of variance (one for each dimension) were then computed. Lin (2002) found no main effects or interactions between First Time Learning and computer graphics types, between Quick Learning and computer graphics types, or between Simple Learning and computer graphics types. There was, however, a significant main effect for different computer types when coupled with Omniscient Authority. In sum, this study did not find any significant effects of students' beliefs on learning. One could argue, however, that by dividing the scores into "agree" versus "disagree" by a mean split, there was not enough power to detect a difference if a difference did exist. A regression analysis, similar to the one Mason (2003) conducted, would have been a more appropriate test of the relations of students' beliefs to achievement.

Mason (2003) examined relationships between six dimensions of beliefs and academic achievement. (Details of this study have been previously described in the second section of this review.) Using stepwise regression, students' grades in mathematics were regressed onto students' scores on the dimensions of beliefs that were measured. To recap, two of the beliefs of interest for this review were students' belief that they can solve time-consuming problems (gradual learning) and their belief that effort can increase mathematical ability (incremental learning). Order of entry was determined by the variable accounting for the most variance entered at each step. All beliefs, except for the belief that effort can increase mathematical ability, predicted achievement. The belief that accounted for the most variance was the belief that they can solve time-consuming problems (R = .31).

Like Koller (2001) and Lin (2002), Kloosterman (1991) examined relationships between four facets of students' beliefs and mathematics achievement. He examined four general beliefs about how mathematics is learned: (1) self-confidence in learning mathematics, (2) attributional style in mathematics, (3) failure as an acceptable phase of learning mathematics, and (4) effort as a mediator of mathematical ability. Of specific interest, a high score on the effort scale indicated a strong belief in an incremental view of ability and a low score indicated an innate or static view of ability.

Four hundred twenty-nine students in grade 7 from three lower-middle to upper-middle classes from small schools filled out questionnaires designed to address the four dimensions listed above (from previously derived valid and reliable scales). Mathematics achievement was measured using application tests designed to measure students' conceptual knowledge of mathematics and ability to apply those concepts. Students completed the test one week after they filled out the questionnaires. In contrast to the previous studies, Kloosterman (1991) did not find a significant correlation between effort and achievement. The other three factors, however, were significantly related to achievement. Moreover, when the four dimensions were combined into a single latent variable, *beliefs*, he found the correlation to be significantly larger than any of the single belief indicators to achievement.

Summary and Critique

The research study that examined the influence of students' beliefs on behavior found consistent patterns between students' beliefs and their learning behaviors. Specifically, Garofalo (1989a) observed that students' beliefs appear to influence their

engagement in learning with respect to the strategies they use to learn and solve problems and their justifications as to what constitutes a correct response.

While Garofalo (1989a) observed students during problem solving, the quantitatively oriented studies have relied on self-report measures to assess the types of strategies students use, their motivational orientations, and their confidence in being able to carry out a task (self-efficacy). These studies have also found a significant relationship between students' beliefs and types of behaviors they engage in while learning, and how those behaviors relate to achievement. The focus has been on how learning behaviors may mediate beliefs and achievement.

Taken together, both approaches provide convincing evidence of the relationship beliefs have with learning and performance. Neither approach, however, can provide strong evidence of relations between epistemic beliefs and learning and achievement. A more powerful approach to this line of inquiry would be to include both self-report and observational techniques. Statistical analyses could be combined with an in-depth analysis of how beliefs influence students' behaviors, and self-report measures could be better validated by observational techniques.

Although all studies that used self-report measures reported reliability estimates (e.g., Cronbach's alpha), a large majority of the estimates were low, some quite low (less than .50). This poses a threat to the dependability of the measures used to assess students' beliefs, learning strategies, and motivation, which further poses a threat to the validity of the results. Second, students' memory for specific events may be inaccurate. Consequently, having students fill out self-report measures and observing them as they engage in problem solving may help to reduce the problems researchers face when

relying solely on self-report data. Including a think aloud protocol, for example, may help improve this line of research.

Schoenfeld's Model of Mathematics Problem Solving

Schoenfeld (1983, 1985) proposed that when people engage in mathematics problem solving a number of factors shape behavior. These factors include resources, heuristics, control, and belief systems. Resources are comprised of the prior knowledge an individual has and may access when problem solving. Prior knowledge includes factual, procedural, and propositional knowledge. Specific types of knowledge consist of informal and intuitive knowledge about the domain, facts, definitions, algorithmic procedures, routine procedures, relevant competencies, and knowledge of rules of discourse of the domain. These resources are the foundation on which problem-solving performance is constructed and to know what knowledge an individual possesses, it is necessary to understand what takes place in a problem solving session. If an individual does not use a particular body of knowledge that is relevant to a problem, it is important to assess whether that individual possesses that knowledge; if not, that individual could not have used that information during problem solving. If, however, the individual does possess the relevant knowledge, one needs to assess why that information was not accessed or used during problem solving. Consequently, when examining problem solving behavior, researchers need an inventory of what an individual knows, believes, or suspects to be true to understand his or her problem-solving behavior.

Heuristics are rules of thumb that can be used when problem solving (Schoenfeld, 1985). These include general strategies, such as using analogies or working backward, that an individual may use to make progress on problems. During problem solving, an

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individual may choose a particular strategy to work a problem. That strategy may not, however, be useful for solving the problem. It is through one process of control that an individual identifies that a chosen strategy is not appropriate since progress toward the goal is not being made. Instead, a new strategy or set of strategies should be implemented, a second component of control. One of the most important factors in problem solving is control (Schoenfeld, 1985).

Control refers to processes of management and allocation of resources during problem solving (Schoenfeld, 1985). More broadly defined, Schoenfeld refers to control as one of the two factors of metacognition that Brown et al. (1983) identified: regulation of cognition. Recall that metacognition refers to knowledge of one's own cognitive processes (Flavell, 1976) and regulation of cognition refers to processes of planning activities prior to engaging in a task, monitoring activities during learning, and checking outcomes against set goals (Brown et al., 1983). During problem solving, selecting and pursuing appropriate approaches, recovering from inappropriate choices, and monitoring progress are important components of successful problem solving (Schoenfeld, 1985).

While resources, heuristics, and control shape behavior, Schoenfeld (1983, 1985) argues the most important factor is one's belief system since beliefs establish the context within which resources, heuristics, and control operate. Recall that Schoenfeld refers to beliefs as an individual's mathematics worldview, the perspective one takes to approach mathematics and mathematical tasks that serve to establish a psychological context for learning. An individual's beliefs about mathematics – beliefs about the nature of mathematics knowledge, justifications of mathematics knowledge, sources of mathematics knowledge, and acquisition of mathematics knowledge – shape how one

engages in problem solving. That is, beliefs shape cognition by influencing what and how the three factors are managed and allocated. For example, beliefs can influence how one plans to approach a problem, which heuristics will be implemented, how one monitors and controls behavior, and how much effort is expended to solve a problem.

Two perspectives that Schoenfeld (1983) identified are empiricism and rationalism. Schoenfeld proposed, with respect to problem solving, that there are four empiricist axioms that underlie the empiricist belief system. First, individuals with an empiricist view believe that insight to solve problems comes from accurate drawings or step-by-step procedures. The more accurate the drawings or step-by-step procedures, the more likely useful information will be identified. Second, when generating and rank ordering hypotheses, two factors dominate. These are the intuitive apprehensibility of a solution and the perceptual salience of certain physical features of a problem. For the first factor, the hypothesis that can be most clearly perceived to reach the end of a problem will be ranked highest and tested first. This holds true unless some feature of the problem, that is, a perceptually salient feature of the problem, dominates as essential to solving the problem. The third axiom is that plausible hypotheses are tested sequentially. The first hypothesis is tested until it is accepted or rejected. If rejected, the second hypothesis is tested until accepted or rejected, and so on. The fourth axiom is that verification of a solution is purely empirical. Solutions are tested by implementing them and are correct if and only if they produce the desired result. Finally, individuals with an empiricist view believe that mathematical proof is not relevant to the discovery and verification process. Only in situations where proofs are required (e.g., when a teacher demands that proofs be included) will the empiricist include them.

In contrast, Schoenfeld (1983) argued that individuals with a rationalist perspective believe that mathematical argumentation serves as a form of discovery. Mathematical proof procedures are used to discover a solution and deductive logic is viewed as an important tool for problem solving. When relevant information is not available rationalists derive necessary information through the use of argumentation. Argumentation is necessary throughout problem solving and is not used as a means of after-the-fact verification as it is with empiricists. Empirical verification is not viewed as necessary to validate a solution. In sum, empiricists predominantly rely on empirical means to solve problems whereas rationalists predominantly rely on logical means to solve problems. It is important to note, however, that Schoenfeld does not suggest that empiricists or rationalists rely solely on one form. Empiricists can exploit rational approaches as can rationalists exploit empirical approaches. Moreover, both can use intuition as another tool to solve problems but their predominant belief system largely influences how they engage in mathematics problem solving.

Schoenfeld (1983) further hypothesized that belief systems influence various behaviors throughout the six stages of problem solving that he identified: read, analyze, explore, plan, implement, and verify. The reading stage includes reading a problem statement, considering problem conditions, rereading, and contemplating a problem. If a strategy to solve a problem is readily apparent an individual may immediately begin a plan to approach the problem or may move directly to the implementation stage. When a strategy is not readily identified after the problem is read, the next stage of problem solving is typically the analysis stage. During the analysis stage an individual may attempt to understand a problem, select strategies that he or she thinks are appropriate,

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reformulate a problem, or identify potentially relevant principles or mechanisms. The individual may also simplify the problem. These processes often lead directly into the development of a plan.

The analysis stage of problem solving is generally well structured and is focused on the conditions or goals of a problem (Schoenfeld, 1985). In contrast, the exploration stage is less structured and is further detached from the original problem. Exploration includes a broad tour through a problem space to search for relevant information that can be included in the analysis-plan-implementation sequence. If new information is identified, an individual may return to the analysis stage to attempt to further understand a problem. During exploration, a number of heuristics may be used. These heuristics may include the examination of related problems or the use of analogies. One essential element for successful exploration is monitoring. Because exploration is weakly structured, it is imperative to engage in local and global assessments to identify whether paths taken are too far removed from the original problem or whether any information acquired during exploration can be used to help solve a problem.

The next stage of problem solving is planning (Schoenfeld, 1985). During this phase, an individual may plan a course of action to solve a problem. A set of strategies may be identified that he or she will implement. During implementation, the next stage of problem solving, a plan is carried out. Monitoring during implementation is a key factor in successful problem solving. If monitoring occurs to assess whether a plan is effective, that information can be used as feedback. If a plan is not successful, an individual may discontinue the plan and return to the planning stage, an earlier stage, or may quit. If a plan is successful or a solution is reached during implementation or exploration, the

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individual may evaluate the solution. The process of checking one's work is the final stage of problem solving, the verification stage.

To describe how beliefs influence the stages of problem solving, Schoenfeld (1985) proposed that empiricists spend most of the time in the exploration phase. Since empiricists do not typically use mathematical argumentation, proof-like procedures, or logic, they engage in trial-and-error exploration of the problem space to identify and test hypotheses. Since hypotheses are tested one by one, they engage in little metacognitive monitoring and control. If a solution is reached, it is verified by empirical means. In contrast, rationalists progress through a number of stages and may return to earlier stages when feedback becomes available. Rationalists constantly monitor progress at both the tactical and strategic levels, and plans are continually assessed and acted upon. Since mathematical argumentation, proofs, and logic are important facets of problem solving, rationalists rely on this information during the discovery and verification processes.

Empirical Studies

To test his hypotheses, Schoenfeld (1983) examined post-secondary students' beliefs about mathematics and observed how they engaged in solving geometry problems. The students were mostly freshman who had completed one full year of geometry in high school and had completed at least one semester of calculus. He contrasted their beliefs and behaviors during a problem solving session to a mathematician's, who had not done any geometry problems for almost a decade. Both the students and mathematician were given the same problem.

Based on his observations, Schoenfeld (1983) classified students' beliefs as empiricist in nature. They believed that insight comes from very accurate drawings and

that hypothetical solutions come from dominant perceptual features of the drawings. When engaged in problem solving, solutions typically began with a quick choice of a particular direction with little serious evaluation as to whether the direction would be useful. Students tested their hypotheses in a serial fashion, that is, one hypothesis was tested until accepted or rejected. If rejected, students tested a second hypothesis. Little metacognitive monitoring occurred and, when it did, students typically ignored what information they acquired from monitoring. For example, many students admitted when a strategy was not "getting them anywhere," but in spite of this information they continued to test their hypothesis using the same strategy. When given similar problems, students did not recognize the similarities between the problems and overlooked what relevant information they could have used. Moreover, students felt justified in their answer when their construction "worked." They based their justifications on empirical evidence and did not use rational and logical evidence such as theorems.

In contrast, the mathematician's beliefs were based on a rationalist perspective, that mathematical knowledge is derived through logic and reason. When engaged in problem solving, the mathematician derived the necessary information to solve the problem through proof-like procedures prior to the verification process. Unlike the students, the mathematician had not dealt with geometric problems for a number of years and had less domain-specific knowledge at his disposal. Because of his continuous monitoring and use of mathematical argumentation, however, the mathematician, unlike most students, was able to solve the problem. As Schoenfeld (1983) noted, key to the mathematician's success was the use of mathematical argumentation and metacognitive monitoring throughout the problem-solving process. Rarely did more than a minute pass

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by without the mathematician monitoring his progress. There was constant monitoring of the solution process at the tactical and strategic levels. Moreover, plans and implementation of those plans were continually assessed and then acted upon accordingly. When a solution was achieved, the mathematician verified his solution through logic and reason.

Using a similar framework, Lester and Garofalo (1987) examined two grade 7 mathematics classes of differing ability, one average and one advanced, taught by the same teacher. They observed and interviewed students and examined their metacognitive behaviors and the role of those behaviors on mathematical problem solving over 12 weeks. Moreover, they examined the effect of teaching students how to be more self-aware and how to evaluate those behaviors in a pretest-posttest design. A secondary focus in their study included an examination of how affective factors and students' beliefs about mathematics influenced their metacognitive behaviors.

Based on their observations, Lester and Garofalo (1987) categorized typical problem solving scenarios of students' attitudes and beliefs. Of particular interest, like the students in Schoenfeld's (1983) study, they found that students whose beliefs were empirical in nature (e.g., their justification was rooted in empirical evidence) tried all four arithmetic operations (i.e., +, −, ×, ÷) and then chose the answer that made most sense. That is, students tested hypotheses in a serial manner until a satisfactory solution was identified. Students' belief that performing a series of computations could solve all mathematics problems, they argued, led to a subsequent lack of metacognitive behavior. Moreover, students who believed that problems should be solved quickly spent no time assessing whether their answers made sense. The only solution checking that students did

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was whether their calculations were correct. Thus, even though they attempted to teach students to be metacognitively aware, students' behaviors were indicative of their beliefs.

Summary and Critique

Researchers from both studies argued that students' beliefs were empiricist in nature and that these beliefs influenced how they engaged in problem solving. They proposed that students' empirical approaches to knowing influenced processes of self-regulated learning. They observed students as they engaged in problem solving and found that students' approaches to problem solving were empirical in nature. Specifically, students relied on dominant perceptual features rather than mathematical argumentation, tested hypotheses one by one until a solution was found, and verified solutions based on empirical evidence rather than proofs or logic. The process by which students attempted to solve problems was characterized as trial-and-error. Moreover, students engaged in little planning and metacognitive monitoring. When monitoring did occur, and students identified that their course of action was not useful, students chose to ignore that information and continued to test the hypothesis before moving on to the next one.

As previously mentioned, although Schoenfeld (1983) linked epistemic beliefs and self-regulated learning, two aspects of his research need to be addressed. First, he identified relations between epistemic belief systems and regulation of cognition based on his observations of how students approached problems compared to an expert mathematician. Differences in self-regulation should be examined between individuals with similar levels of expertise in mathematics. Second, Schoenfeld's sample consisted mostly of freshmen. As previously noted, developmental research on personal epistemology (e.g., Kitchener & King, 1981; Perry, 1970; Schommer, 1993a), has

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demonstrated that students' beliefs change over time. Since previous research has found that students majoring in mathematics-related fields are profiled as more rational in their approaches to knowing than students in other fields such as the social sciences (e.g., Royce & Mos, 1980), one may question whether students in mathematics-related fields become more rational in their approaches to knowing over the course of their undergraduate experience.

To extend Schoenfeld's (1983) research, I have sampled a broader range of students, first- through fourth-year undergraduates, to examine whether there are epistemological differences in approaches to knowing. Second, according to Schoenfeld's research, freshman mathematics students' beliefs are empirical in nature as measured by their approaches to problem solving. This categorization is inconsistent with research that Royce and Mos (1980) have conducted. Specifically, Royce and Mos found that mathematics students were predominantly rational in their approaches to knowing. One problem in making a comparison across the studies is that Schoenfeld and Royce and Mos used disparate means by which to epistemologically label students. To address this disparity, using Royce and Mos's psycho-epistemological profile instrument, students are a priori epistemologically categorized to predict how they will engage in problem solving. Students are then observed as they problem solve to assess whether their approaches to problem solving are consistent with their epistemic profiles.

Common Hypothesis Across the Models

All models presented in this chapter have suggested that one's epistemic profile, epistemic beliefs, or belief systems influence cognitive processes when acquiring or processing information. To reiterate, Royce (1978) proposed there are three approaches

to knowing, rationalism, empiricism, and metaphorism, and each of these approaches are related to cognitive processes of conceptualizing, perceiving, and intuiting, respectively. Schommer (1990) proposed there are five epistemological dimensions, the certainty of knowledge, the source of knowledge, the structure of knowledge, the speed of knowledge acquisition, and the control of knowledge acquisition. Finally, Schoenfeld (1983) identified two approaches to knowing, empiricism and rationalism, in the context of mathematics problem solving. All three models hypothesize that a person's predominant approach to knowing or beliefs about knowing influences how he or she acquires and interprets information and how he or she justifies whether that information can be accepted as true. Research that has been conducted to test this hypothesis was reviewed and a number of issues were identified. My research addresses some of these issues.

Rationale

To improve the examination of how students' personal epistemologies influence processes of self-regulated learning in the context of mathematics problem solving, three extensions of the current research are needed. First, studies are required that a priori categorize students according to their approaches to knowing and that use this categorization to predict how they will approach problem solving. In Schoenfeld's (1983) study, for example, participants were categorized after they were observed problem solving. An a priori categorization would allow for an improved investigation of Schoenfeld's model, one that is theoretically driven. Such a theoretical model should describe the relationships between epistemic style and cognition. Royce's (1978) model does precisely this. Second, studies are needed that examine a broader range of students. Studies that have examined relations between beliefs and mathematics problem solving

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typically focused on only one or two grade levels. Third, studies are needed that include quantitative and more process-oriented approaches to examine relations between epistemic beliefs and learning behavior. As Pintrich (2002) noted, more dynamic process-oriented studies are essential to advance our understanding. Moreover, since the accuracy of self-report measures has been called into question (e.g., Winne et al., 2002a), research is needed that includes both self-report measures and measures of actual learning behaviors.

My dissertation research responds to Pintrich's (2002) calls for studies linking epistemic beliefs to models of self-regulated learning, to move away from one-point-in-time correlational designs and reliance on self-report measures, and to adopt a more process-oriented methodology. It addresses shortcomings identified in uses of self-report measures (see Winne et al. 2002a) and integrates a more philosophical conceptualization of epistemology with the conceptualizations currently used in educational psychology and mathematics education. The primary purpose of my research is to investigate relations among students' personal epistemologies, metacognitive strategies they report using in mathematics and statistics classes, metacognitive strategies they actually use as they solve mathematics and statistics problems, and how they justify their solutions. The secondary purpose is to examine whether there are differences in students' epistemic profiles across the four years of undergraduate school.

Research Questions and Hypotheses

Two broad research questions are examined in this study. Stated in general terms, the questions are: Are there differences in how students engage in mathematics problem solving based on their epistemic profiles? Are there differences in epistemic profiles

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between undergraduate students in first- through fourth-year university? The first question can be broken down into three more specific questions based on the type of data collected. These questions are: Do students who are profiled as predominantly rational in their approaches to knowing self-report using more metacognitive strategies (e.g., planning, monitoring, and control) than students who are profiled as predominantly empirical or predominantly both in their approaches to knowing? While problem solving, do students who are profiled a predominantly rational in their approaches to knowing engage in more planning, metacognitive monitoring, and metacognitive control than students who are profiled as predominantly empirical or predominantly both in their approaches to knowing? Do students who are profiled as predominantly rational in their approaches to knowing use more mathematical argumentation, such as proofs and theorems, to solve problems and justify solutions than students who are profiled as predominantly empirical or predominantly both in their approaches to knowing? The second question can also be broken down into two more specific questions. These are: Are upper-year undergraduate students self-reportedly more rational in their approaches to knowing than lower-year undergraduate students? Do upper-year undergraduate students hold different self-reported epistemic beliefs than lower-year undergraduate students?

Based on Schoenfeld's (1983) and Royce's (1978) models, it is predicted that students profiled as predominantly rational in their approaches to knowing, as measured by the PEP (Royce & Mos, 1980), will have a higher average rating of self-reported metacognitive strategy use, measured by the MSLQ (Pintrich et al., 1991). This difference is expected to be statistically detectable with students profiled as

predominantly empirical reporting the lowest use of metacognitive strategies followed by students profiled as both. Similarly, while problem solving, it is predicted that students profiled as predominantly rational in their approaches to knowing will engage in more planning, metacognitive monitoring, and metacognitive control than students profiled as predominantly both or predominantly empirical, with students profiled as empirical having the lowest frequency of these behaviors.

This prediction is made on the theoretical assumption that, since rationalists are theorized to engage in more critical thinking (Royce, 1978) they are more likely to be more metacognitively active in their learning. According to Cacioppo and Petty (1982), individuals high in need for cognition, defined as a tendency to engage in and enjoy effortful cognitive endeavors (i.e., critical thinking), tend to seek, acquire, think about, and reflect back on information to make sense of the world around them. Thus, individuals profiled as predominantly rational are expected to be higher in need for cognition, as measured by the Need For Cognition Scale (NFC; Cacioppo, Petty, Feinstein, & Jarvis, 1996) than individuals profiled as predominantly empirical or both. Consequently, individuals profiled as predominantly rational in their approaches to knowing are predicted to self-report and use more metacognitive strategies.

Again, based on Schoenfeld's (1983) and Royce's (1978) models, it is predicted that students who are profiled as more rational in their approaches to knowing will use more mathematical argumentation during problem-solving attempts and will use more proofs and logic to justify their solutions than the other two groups. Students profiled as predominantly empirical in their approaches to knowing will engage in more trial-and-error exploration of the problem space (e.g., go off-task), will more likely focus on

perceptual features of problems, and will test hypotheses one by one until a solution is found. Moreover, students profiled as both rational and empirical will use both rational and empirical approaches to solve problems. Finally, since two problems in each problem-solving session are isomorphic in nature (e.g., same structural features but different surface features), it is predicted that students who are profiled as predominantly rational in their approaches to knowing are more likely to identify similarities between the isomorphic problems than students in the other two groups. Accordingly, students profiled as predominantly rational are predicted to more often use information from the previous problem to help solve a similar problem than students in the other two groups. This is predicated on the previous prediction that students profiled as rational engage in more metacognitive monitoring. If students engage in more monitoring, theoretically, they may be more aware of specific facets of the problems than students who do not similarly monitor (Schoenfeld, 1983).

For the second general question, although research that has examined students' beliefs about mathematics has typically found similar beliefs across all levels of education, I base my predictions on results from developmental research that has found that as students progress through higher levels of education, their beliefs change (e.g., Kitchener & King, 1981; Perry, 1970). It is predicted that there will be statistically detectable differences in self-reported rationalism between lower- to upper-year students based on each group's average rationalism score on the PEP. More specifically, it is predicted that average rationalism scores will increase as a function of progressively higher year of study. Finally, it is predicted that upper-year students' average scores on epistemic beliefs across the five dimensions, as measured by the EBI (Schraw et al.,

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2002), will be statistically detectably different from lower-year students' average scores. Specifically, it is predicted that upper-year students' average scores will be lower (lower scores represent greater disagreement with the items).

CHAPTER 3

METHODOLOGY

FIRST COMPONENT OF THE STUDY

Participants

A sample of 127 undergraduate students at Simon Fraser University volunteered to participate in the study. Participants were selected from various undergraduate mathematics and statistics courses being offered during the Fall and Spring semesters of 2003-2004. All students who agreed to participate signed a university-approved consent form (the university ethics approval form is presented in Appendix A and a sample consent form is provided in Appendix B). Sixty students were enrolled in their first year of university (N = 36 females, 60% of 60), 15 were enrolled in their second year (N = 8 females, 53.3%), 21 in their third year (N = 8 females, 38.1%), 24 in their fourth year (N = 10 females, 41.7%), and 7 had graduated but were enrolled in fourth-year courses to improve their grade point averages prior to entering graduate school (N = 3 females, 42.9%). The mean age was 20.80 (SD = 4.12), the average self-reported cumulative general grade point average was 3.19 (SD = .53), and the average self-reported cumulative mathematics grade point average was 3.12 (SD = .77). Of the 127 students, 52 (40.9%) had reported they learned English as a second language. The average reported age these students learned to speak English was 9.25 (SD = 4.59). Of the 127 students, 57 had declared their major or minor in mathematics or statistics. The other 54 students had not yet officially declared their major or minor but were enrolled in mathematics and/or

statistics courses required for a major or minor in one or both of the disciplines. To compensate students for their participation in the first component of the study, all students were entered into a draw to win $25 (odds of winning were 25:1).

Materials

Demographics Questionnaire

A demographics questionnaire was designed to measure various characteristics of the sample. Students were asked to report information on their age, sex, cumulative grade point average in all post-secondary courses, cumulative grade point average in all mathematics / statistics courses, academic major, and academic minor. The questionnaire also asked students to report the number of courses in which they were enrolled for the current semester, total number of courses taken, year of study, average hours worked per week, average hours studying per week, whether English was their first spoken and written language and, if not, at what age they learned to speak and write English. Finally, the questionnaire asked students to report what they would like to improve about how they study for their mathematics / statistics courses and to list the names of all mathematics and statistics courses they have taken. A sample questionnaire is presented in Appendix C.

Epistemic Profile

The Psycho-Epistemological Profile (PEP; Royce & Mos, 1980) was used to assess students' epistemic profiles. The PEP is presented in Appendix D and is used by permission of L. Mos. The letter granting copyright permission is included in Appendix E. (Although all three dimensions were measured, since Schoenfeld's (1983) model

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includes only rationalism and empiricism, the metaphorism component of Royce's (1978) model was not examined. Metaphorism was measured, however, to conduct a confirmatory factor analysis and to assess other psychometric properties of the entire scale for future research. Thus, a full description of the metaphorism scale is included.) The PEP is a 90-item self-report measure designed to reflect three approaches to knowing: empiricism, rationalism, and metaphorism. The assumption is that different people stress each of these approaches to knowing in different ways that can be hierarchically profiled. Students rate each item on a five-point Likert scale ranging from "completely disagree" (a rating of 1) to "completely agree" (a rating of 5). Each scale is comprised of 30 items. A sample item reflecting an empiricist approach to knowing is "When people are arguing a question from two different points of view, I would say that the argument should be resolved by actual observation of the debated situation." A sample item from the rationalism subscale is "Higher education should place a greater emphasis on mathematics and logic." A sample item from the metaphorism subscale is "Our understanding of the meaning of life has been furthered most by art and literature." For all three subscales, a higher score reflects a greater agreement and the highest score of the three subscales represents a person's predominant epistemology.

Previous research that has used the PEP has found high test-retest reliability coefficients ranging from .80 to .90 (e.g., Royce & Smith, 1964; Smith et al., 1967), and factor analyses indicate there are three independent scales composed of rational, empirical, and metaphorical content (e.g., Schopflocher & Royce, 1978). The PEP has also been standardized on a junior college sample of 925 male and 417 female students between 19 to 24 years of age (Royce & Mos, 1980). All students were enrolled in a first-

year arts or science program. The mean scores for the male and female participants were similar on all three approaches to knowing and no extreme outliers were found. Scores ranged from 70 to 122 for the empiricism scale, 66 to 130 for the rationalism scale, and 66 to 136 for the metaphorism scale, with a possible range of scores from 30 to 150. Finally, correlations between the three approaches to knowing were .63 (empiricism and rationalism), .51 (empiricism and metaphorism), and .63 (rationalism and metaphorism). Royce and Mos argued that, although the correlations between the three approaches to knowing indicate considerable dependence, variance explained for the highest correlation was only 39.69% which suggests there is still a relative degree of independence.

Critical Thinking

The 18-item short form of the Need for Cognition Scale (NFC; Cacioppo et al., 1996) was used to measures students' preferences to engage in critical thinking. Cacioppo and Petty (1982) developed the NFC scale to measure what they described as "the (enduring) tendency for an individual to engage in and enjoy effortful analytic activity" (p. 116). According to Caccioppo and Petty, individuals high in need for cognition are proposed to naturally seek, acquire, think about, and reflect back on information to make sense of their world. In contrast, individuals low in need for cognition are proposed to more likely rely on others (e.g., authority figures), cognitive heuristics, or social comparison processes to provide this structure. Of particular interest, Berzonsky and Sullivan (1992) found that need for cognition was statistically significantly positively related to individuals' tendencies to seek out, scrutinize, and use relevant information when solving problems. Leary, Sheppard, McNeil, Jenkins, and Barnes (1986) found that need for cognition was statistically significantly positively

related to a measure of basing beliefs and judgments on rational considerations. The NFC scale was included as a measure of concurrent validity for the rationalism scale on the PEP.

Students are asked to rate each statement on a five-point Likert scale ranging from "extremely uncharacteristic" (a rating of 1) to "extremely characteristic" (a rating of 5). A sample item from the scale is "I really enjoy a task that involves coming up with new solutions to problems." For this scale, higher agreement indicates higher need for cognition. Previous research has reported reliability estimates typically greater than .85 (Caccioppo et al., 1996). As ancillary analyses, it was expected that NFC, as a measure of critical thinking, would be moderately positively related to rationalism and weakly positively related to empiricism. Moreover, it was expected that students profiled as predominantly rational in their approach to knowing would have a higher average NFC score than students in the other two groups.

Learning Strategies

The Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich et al., 1991) was used to assess the learning strategies students use to study for their mathematics and / or statistics courses. The MSLQ is presented in Appendix F and is used by permission of W. J. McKeachie. The letter granting copyright permission is presented in Appendix G. The MSLQ is a widely used 81-item self-report measure designed to assess undergraduate students' use of varying learning strategies and motivational orientations for an undergraduate course. It consists of statements that are designed to measure students' self-reported qualities of their approaches to learning and

to self-regulating that learning. Students rate each item on a seven-point Likert scale ranging from "not at all true of me" (a rating of 1) to "very true of me" (a rating of 7).

The MSLQ is hierarchically organized and consists of two main scales, the motivation scale (31 items) and the learning strategies scale (50 items). These two scales are further divided into three subscales for the motivation scale and two subscales for the learning strategies scale. The two learning strategies subscales are the cognitive and metacognitive strategies subscale and the resource management strategies subscale. Of particular interest, the cognitive and metacognitive strategies subscale measures five strategies that include: rehearsal, elaboration, organization, critical thinking, and metacognitive self-regulation. The rehearsal subscale consists of four items that measure reciting or naming items from a list to be learned. An example item is "When I study for this class, I practice saying the material to myself over and over." The elaboration subscale consists of six items and includes strategies such as paraphrasing, summarizing, creating analogies, and generative note taking. An example item is "When I study, I pull together information from different sources, such as lectures, readings, and discussions." The organization subscale consists of four items that includes strategies such as clustering, outlining, and selecting main ideas in passages. An example item is "I make simple charts, diagrams, or tables to help me organize course material." The critical thinking subscale consists of five items and measures the degree to which students report applying previous knowledge to new situations to solve problems, make decisions, and critical evaluations. An example item is "I treat the course material as a starting point and try to develop my own ideas about it." Finally, the metacognitive self-regulation subscale includes twelve items that measure processes of planning, monitoring and regulating

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cognitive activities. An example item is "When studying for this course I try to determine which concepts I don't understand well." For all of these subscales, higher scores indicate greater agreement and thus greater strategy use. As reported in the MSLQ manual, the instrument has demonstrated acceptable factor validity, and Cronbach alphas range from .64 to .80.

Epistemic Beliefs

The Epistemic Belief Inventory (EBI; Schraw et al., 2002) was used to measure students' beliefs about knowledge, knowing, and learning. The EBI is presented in Appendix H, used by permission of G. Schraw and Lawrence Erlbaum Associates, Inc. Both letters of copyright permission are included in Appendix I. This inventory includes 28 self-report items designed to measure students' beliefs on the five dimensions Schommer (1990) proposed in her model. These dimensions include the certainty of knowledge (7 items), the source of knowledge (4 items), the structure of knowledge (7 items), the control of knowledge acquisition (6 items), and the speed of knowledge acquisition (4 items). Students rate each item on a five-point rating scale ranging from "strongly disagree" (a rating of 1) to "strongly agree" (a rating of 5). A sample item from the certainty of knowledge subscale is "What is true today will be true tomorrow." A sample item from the source of knowledge subscale is "People shouldn't question authority." A sample item from the structure of knowledge subscale is "The best ideas are often the most simple." A sample item from the control of knowledge acquisition subscale is "People's intellectual potential is fixed at birth." Finally, a sample item from the speed of knowledge acquisition is "If you don't learn something quickly, you won't

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ever learn it." For all of these subscales, a lower score reflects stronger disagreement to items.

Research that has assessed the reliability and validity of the EBI has found support for the five proposed dimensions and better reliability estimates (both internal consistency and test-retest) and construct and predictive validity than Schommer's (1990) Epistemological Questionnaire (SEQ) (Schraw et al., 2002). Internal consistency coefficients for the five subscales range from .58 to .68, and test-retest reliability estimates range from .62 to .81.

Procedure

Participants spent 30 to 60 minutes completing all four inventories. Order of inventories was always PEP, NFC, MSLQ, and EBI. Counterbalancing was not used as the inventories measured different constructs and previous research using similar inventories has not found order effects (e.g., Sinatra, Southerland, & McConaughy, 2001).

SECOND COMPONENT OF THE STUDY

Participants

From the larger sample of 127 students, a sub-sample of students was selected to participate in the second component of the study. Students were chosen based on a number of factors. These included: students' epistemic profiles, type of major and minor, types and number of courses they had taken, and if they agreed to volunteer for the second component of the study (see Consent Form, Appendix B).

Based on their summed subscale scores, all 127 students were profiled along two of the dimensions of the PEP, rationalism and empiricism. Rationalism and empiricism scores were computed by summing all 30 items for a total subscale score for each dimension (the minimum score possible was 30 and the maximum score possible was 150). Students were categorized as high on rationalism, moderate on rationalism, or low on rationalism. Similarly, they were categorized as high on empiricism, moderate on empiricism, or low on empiricism. Thus, students were labeled along both dimensions for a total of nine possible categorization types.

Since the lowest and highest scores on the rationalism and empiricism subscales were 53 and 130, and 60 and 132, respectively, "perfect"[2] subscale scores of 60 were considered low, subscale scores of 90 were considered moderate, and subscale scores of 120 were considered high. A standard error was computed for each subscale to create a range around each of the "perfect" scores that could be used to categorize students as low, moderate or high along the two dimensions. Initially, the range was computed as plus or minus one standard error around each perfect score, but too few participants' scores fell within the selected ranges. To increase the number of possible candidates to participate in the problem-solving sessions, the range was increased to plus or minus two standard errors around each perfect score. Specifically, individuals who scored between 48.48 to 71.52, 78.48 to 101.52, or 108.48 to 131.52 were categorized as low, moderate, or high on rationalism, respectively. Similarly, individuals who scored between 48.26 to

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[2] Hypothetically, in error-free measurement, a person who is highly rational in his or her approach to knowing would select a rating of 5 for all 30 items for a total subscale score of 150.

71.74, 78.26 to 101.74, or 108.26 to 132 (the highest score) were categorized as low, moderate, or high on empiricism, respectively.

An attempt was made to select three participants from each of the nine possible categorizations (e.g., three from low rationalism and low empiricism, three from moderate rationalism and low empiricism, etcetera) for a total of 27 students. Only one student was categorized as low on rationalism (this participant was selected for the second component of the study). Consequently, it was not possible to sample three participants within each cell. Thus, an attempt was made to select as many students as possible across all cells, while satisfying each of the constraints. Due to the number of constraints used to select participants for the second component of the study (described below), I was able to select only 17 students (N = 4 females, 23.5%). All 17 students were declared mathematics majors. Students doing a major or minor in statistics, or students who had taken any geometry courses or more than one statistics course at the undergraduate level were not selected. Mathematics minors were not selected since I felt it pertinent to sample students with similar characteristics (e.g., mathematics majors must take the same required courses). No statistics majors or minors were sampled since the problems used for the second problem-solving session would not have been a challenge for those students (see description of problems section).

Six students were enrolled in their first year of university (N = 2 females, 33.33%), 3 were enrolled in their second year (N = 1 female, 33.33%), 4 were enrolled in their third year (N = 0 females), and 4 were in their fourth year (N = 1, 25%). The mean age was 20.18 (SD = 2.04), and the average self-reported cumulative mathematics grade-point average was 3.19 (SD = .69). Finally, of the 17 students, 9 (52.9%) reported

learning English as a second language. The average reported age these students learned to speak English was 8.78 (SD = 3.67). Of the 17 students, 1 was profiled as moderate on rationalism and low on empiricism, 1 was profiled as low on rationalism and moderate on empiricism, 5 were profiled as moderate on both rationalism and empiricism, 1 was profiled as moderate on rationalism and high on empiricism, 4 were profiled as high on rationalism and moderate on empiricism, and 5 were profiled as high on both rationalism and empiricism. In total, 5 were profiled as predominantly rational, 10 were profiled as both rational and empirical, and 2 were profiled as predominantly empirical. To compensate students for their participation in the second component of the study, all students were paid $25 and were offered two follow-up sessions to help them improve their problem-solving techniques.

Materials

Self-Efficacy

According to Bandura (1997), self-efficacy refers to "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments." (p. 3) One's task-specific self-efficacy, or confidence in being able to carry out a particular task, is said to influence a variety of cognitive and motivational factors. For example, Bandura argues that self-efficacy can influence the courses of action people choose to pursue, how much effort they will expend, how long they will persist in the face of difficulty and failure, and the level of accomplishment they realize. In the context of mathematics problem solving, Pajares and Miller (1994) found that, using path analysis, mathematics self-efficacy was more predictive of problem solving than mathematics self-

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concept, perceived usefulness of mathematics, prior experience with mathematics, or sex. Moreover, self-efficacy mediated relations between sex and prior experience on self-concept, perceived usefulness, and problem solving.

Self-efficacy has typically been measured on two levels, one that examines self-efficacy at a micro-analytic level for performance on a specific task (Pajares, 1996), and one that measures self-efficacy at a more macro level for learning and performance (Pintrich et al., 1991). Because of the influence self-efficacy is theorized to have on learning and problem solving behavior, both levels of self-efficacy were measured. Task-specific measures were included to assess whether students felt similarly capable of solving the problems. A more macro-analytic assessment was also included to assess whether students were similarly confident in learning mathematics concepts since, for the second problem-solving session, students were given a short chapter to learn.

Using the guidelines Pajares (1996) specified for measuring task-specific self-efficacy, prior to solving any problems students rated their confidence in being able to successfully complete similar problems. Students were given enough time to read each problem but not enough time was available for them to begin to solve the problems. Students rated their confidence using a 7-point Likert scale ranging from "Not confident at all" (a rating of 1) to "Very confident" (a rating of 7). The self-efficacy measure is presented in Appendix J. The six problems used to measure task-specific self-efficacy for both problem-solving sessions are presented in Appendix K.

The Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich et al., 1991) was used to measure self-efficacy for learning and performance. The self-efficacy for learning and performance subscale of the MSLQ is comprised of 8 items, and assesses

expectancy for success and self-efficacy. Expectancy for success refers specifically to task performance and self-efficacy includes judgments of one's ability to successfully complete a task as well as one's confidence in one's skills to perform that task. As previously mentioned, students rate each statement on a 7-point Likert scale ranging from "not at all true of me" (a rating of 1) to "very true of me" (a rating of 7). An example item is "I'm confident I can understand the most complex material presented by the instructor in this course."

Problem Set for the First Session

Three problems were chosen for the first problem-solving session. These are presented in Appendix L. The problems chosen were those Schoenfeld (1983) used in his research. These were chosen since, as Schoenfeld described, they are non-standard problems in that they are not typically covered in high school geometry courses. Schoenfeld claimed, however, that both high school and university students have the prior knowledge needed to solve the problems but that students are not able to solve them by simply recalling and applying familiar solution patterns. Moreover, no additional information is presented to the students that would contextualize the problems to help orient students toward "appropriate" solution methods. These problems were also chosen given that, based on years of research, Schoenfeld argued these problems do not bias individuals toward one particular method to solve the problems (e.g., an empirical versus a rational approach to solving the problems).

Two of the problems chosen were geometry problems and the third problem was an algebra problem. The two geometry problems were chosen because of their isomorphic nature. That is, both problems are structurally identical; the elements of the

problems are the same. The difference between these two problems is what specific facet of the general problem the students are asked to solve.

Prior Knowledge Test

As Schoenfeld (1983, 1985) noted, to develop a better understanding of students' problem-solving attempts it is necessary to assess students' prior mathematical knowledge that they are capable of accessing and using during a problem-solving attempt. To understand what an individual does while problem solving, one needs an inventory of what that individual knows, believes, or suspects to be true (from a genetic epistemology rather than a mathematical epistemology). For each problem attempted, whether an individual has the necessary prior knowledge may greatly influence how that problem-solving attempt proceeds. For example, if an individual has no prior knowledge of a particular topic, that individual may proceed in a trial-and-error fashion to attempt to solve the problem, may not be capable of deriving the necessary mathematical argumentation needed to solve the problem, or may simply give up. If, on the other hand, an individual does have the required prior knowledge but does not use that knowledge during a problem-solving attempt, one needs to assess why.

Consequently, a prior knowledge test was devised to measure students' prior knowledge of facts, theorems, and proofs necessary to solve each of the three problems. A total of 10 statements were included and are presented in Appendix M. The statements used for the prior knowledge test originated from end-of-chapter tests found in various undergraduate introductory geometry and algebra textbooks. Five of the statements were written as false and five as true, presented in a random order. For each statement, students were asked to indicate whether it was true or false. If they were uncertain whether a

statement was true or false, students were asked to make their best guess. If they

indicated a statement was false, they were required to provide a correct statement. For

each of their true / false answers and their corrected statements, students were asked to

rate how sure they were their answer was correct on a five-point Likert scale ranging

from "absolutely sure it is incorrect" (a rating of 1) to "absolutely sure it is correct" (a

rating of 5). The maximum possible score on the prior knowledge test was 15 (e.g., 1

score for each correct true-false statement and 1 score for each corrected statement).

Short Chapter on Binomial Distribution

A 1,135-word chapter on the binomial distribution was written for students to

study. Content, equations, layout, examples, and methods of explanation for each

component of the chapter were designed based on an examination of 8 different

introductory statistics textbooks. The chapter was written to ensure all elements described

were clearly explained and simple problems were presented to provide students the

opportunity to learn the material without difficulty. Each concept was defined under the

assumption that students had no prior knowledge of the material.

Once the chapter was written, two members from the Faculty of Education at

Simon Fraser University provided feedback on the chapter. One faculty member had self-

reported little prior knowledge while the other taught advanced statistics courses on a

regular basis. Moreover, two graduate students with self-reported little prior knowledge,

two undergraduate statistics students with self-reported advanced knowledge, and one

"math phobic" adult were given the chapter to provide feedback on the clarity and ease

with which the material could be learned. The goal was to present a chapter that was clear

and simple to understand. Based on initial feedback, changes were made to improve the

quality of the chapter and the same people provided additional feedback. The final version of the chapter is presented in Appendix N.

Problem Set for Second Session

To change the problem-solving context, for the second problem-solving session students were given three problems that could be solved using information presented in the short chapter. The three problems are presented in Appendix O. As judged by two faculty members in the Statistics department at Simon Fraser University and my search through eight different introductory statistics textbooks and searches through course assignments posted on the Word Wide Web, the structure of the three problems given for the second session was considered typical of problems students solved in introductory statistics courses. For two of the problems, there was a variation in what students are typically asked to solve. Most statistics problems on probability specifically ask students to compute the probability of a particular event. Two of the problems chosen for this study did not directly ask this of students. Instead, students were required to make judgments indirectly related to the probability of specific events and to justify their judgments. This was done to increase the difficulty level of the problems and to ensure that specific information in the problem statement was not provided to the students to lead them toward appropriate solution methods. The two faculty members agreed these variations achieved these goals.

Similar to two of the problems in the first problem-solving session, two problems from this session were isomorphic in nature. The two problems presented identical numerical information (e.g., the given probabilities were the same as were the number of

trials and the number of successes), but presented a different "story" and posed a slightly different question for students to answer.

Procedure

Think Aloud Protocol

For each problem, participants were asked to think aloud (Ericsson & Simon, 1993). Before the first problem-solving session began, using Ericsson and Simon's guidelines for concurrent verbalization, participants were trained to think aloud. The customary method to get participants to think aloud is to instruct them to verbalize their thoughts as they enter attention. It is important to note that participants were not asked to explain what they were doing but rather to simply verbalize the information they attended to while generating an answer. Using instructions similar to those that Ericsson and Simon recommend, students were given a brief introduction to the study and were then instructed on how to think aloud. The introduction and instructions are presented in Appendix P.

Participants were instructed to think aloud after which I provided a brief example of what thinking aloud entails. The demonstrated task was to compute the trace of a matrix. After the demonstration, participants practiced thinking aloud on two problems similar to the one demonstrated. Once participants felt comfortable in thinking aloud, the first problem-solving session began. Participants were told the sessions would be audio-recorded and timed, that they could quit working a problem at any point, and that I was more interested in how they solved the problems rather than whether or not they got an answer (see the instructions).

First Problem-Solving Session

Participants were first asked to rate their self-efficacy for three similar problems. The presentation order of the self-efficacy problems was the same counterbalanced order as the actual problems. For the self-efficacy problems, participants were given 25 seconds to read the first problem, 15 seconds to read the second problem, and 15 seconds to read the third problem. After students rated their self-efficacy they were given three problems to solve. Pencils, a ruler, a compass, a calculator and extra paper were provided for students to use. Students were told they could work the problems on the problem sheet. To ensure no order effects occurred, problems were presented in a counterbalanced order. There were six possible orders of presentation for the three problems and the counterbalanced order was randomized within each of the cells used to profile participants along the two epistemic dimensions. An attempt was also made to ensure an equal number of participants received the same order.

To minimize social interactions between the participants and myself, I sat behind them to the right but with a full view of their actions to take notes as participants solved problems. It was important that I did not interrupt their attempts and allowed the sessions to run their course. Ericsson and Simon (1993) suggest that, to ensure participants' sequences of thought are not altered, an attempt should be made to avoid intervening as much as possible. If more than 30 seconds passed without any verbalizations I reminded them to think aloud. On the rare occasion that a participant asked a question, I responded. For example, if a participant asked me how to turn on the calculator, I gave them the directions. If a question was a clarification about a problem, I indicated that I could not clarify anything for them with the exception of one of the problems. The only

clarification I made was when students asked if the "C" on one of the diagrams represented the center of the circle (that information was not presented in the problem statement). Only two participants asked for this clarification.

After participants completed the three problems they were given the prior knowledge test. The prior knowledge test was given after students completed the problems to ensure statements in the test would not cue participants toward appropriate solution methods or to cue them to use particular theorems, proofs, or facts to solve the problems. Once participants completed the test the second problem-solving session was scheduled. All but two participants scheduled the second session 2 days later. The other two participants scheduled the second session the following week.

Second Problem-Solving Session

For the second problem-solving session the think aloud instructions were repeated. Participants were also told that they would be studying a short chapter prior to being given the problems. Participants were asked to read the chapter aloud and to think aloud any thoughts that came to mind as they read the chapter. Participants were told to study the chapter as they would for an actual course, but that they would not be tested on the material. Instead, they would be given three problems to solve based on the content covered in the chapter and that they could refer back to the content in the chapter to help them solve the problems.

Prior to studying the chapter, participants were asked to rate their self-efficacy on three similar problems, which were presented in the same counterbalanced order as the actual problems. For the self-efficacy problems, participants were given 25 seconds to read the first problem, 15 seconds to read the second problem, and 25 seconds to read the

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third problem. After rating their self-efficacy on each of the three problems participants were given the short chapter on the binomial distribution. The study session was audio-recorded and timed, and participants were told they could take as long as they needed to learn the chapter. Once participants had completed their study session they were asked to rate their self-efficacy on the same three problems given prior to studying the chapter. This was done to assess whether, after studying the chapter, participants felt more confident working probability problems. Each problem was then given to participants in the same counterbalanced order as in the first session.

Retrospective Accounts, Feedback, and Follow-Up Interview

At the end of the second session and prior to feedback being given, participants were asked to give a retrospective account of each of their problem-solving attempts. Written materials from their attempts were provided to guide the discussion. To begin the retrospective accounts, all participants were asked, "What was the first thing that came to mind when you saw this question?" A discussion ensued and typical general questions I posed included, "What were you thinking about when you did this?" "Why did you do this?" "Why did you use this information?" and, "Do you typically take this approach when problem solving?" The purpose of the retrospective accounts was to obtain further information about the types of approaches participants took to solve problems, whether they planned approaches to solve problems, what they did when they were at an impasse, and whether the episodes were representative of their problem-solving approaches in the context of their undergraduate mathematics and/or statistics courses.

Once retrospective accounts for each of the problems were given, participants were asked to characterize their attempts at each problem as more rational, more

empirical, or a combination of both. Each participant was given a written description for each type of approach. The written descriptions are included in Appendix Q. The description was derived from the protocol I used to categorize each attempt. A rational approach was described as one where a person might use mathematical argumentation, such as proofs or theorems, during a problem-solving attempt to derive necessary information to solve the problem. This information might then be used to justify the solution. An empirical approach was described as one where a person does not use proofs or theorems but rather explores a problem space in a trial-and-error fashion, where hypotheses are tested serially until accepted or rejected, and/or dominant perceptual features of a problem are the focus of exploration rather than concepts or theorems underlying a problem. The answer may then be justified by testing the solution to see if it "works" (e.g., substituting a value into an equation to assess whether the solution was correct, or measuring elements of a construction to see if the construction was correct).

Participants were told that one approach was not better than another and that both could be used during the same problem-solving attempt. This was stated to reduce possible bias in answers. Moreover, participants were asked to justify the characterization of each of their problem-solving attempts using information from their written work. Once students characterized their attempts, feedback was given on each question. I then proceeded to describe how I characterized each of their problem-solving attempts using written materials and notes I had taken during the attempts. Participants were invited to correct any misperceptions they felt I had in characterizing each episode. Finally, participants were debriefed as to the purpose of the study, were told how they were

profiled, and asked whether their profile was consistent with how they viewed themselves and their general approaches to problem solving.

Protocol Coding Schemes

The sequences of overt actions that participants take in the process of solving problems can be traced or identified by creating protocol coding schemes (Schoenfeld, 1985). Three overt actions were coded for each problem-solving attempt: planning, metacognitive monitoring, and metacognitive control. To characterize approaches as rational, empirical, or both, the types of information participants derived during the problem attempts, how participants explored the problem space, and justifications of solutions were examined. Schoenfeld's (1983, 1985) coding scheme was used to develop the protocol coding schemes.

Planning

A plan is a course of action an individual decides to implement prior to solving a problem. In general, an individual may devise a plan to approach a task using a tactic or set of tactics or strategies to carry out that task (Winne & Hadwin, 1998). Evidence of a plan was noted if a heuristic, tactic, or strategy was overtly stated as a plan prior to being implemented. Examples of overt plans included: "I'll try isolating this term." "I'll implement this strategy." "I'm going to try proof by contradiction." "I'll try drawing the circle first to see if my hypothesis is correct." When a participant made a decision to return to an earlier tactic or strategy, this was counted as another plan.

Metacognitive Monitoring and Control

Metacognition refers to knowledge about one's own thinking processes, or awareness of one's own cognitive processes and how they work (Flavell, 1976). This knowledge is used to monitor and regulate cognitive processes. When learning or problem solving, metacognitive monitoring establishes opportunities to change tactics if the products created during learning or problem solving do not meet standards set for a task. Metacognitive monitoring activities include, for example, examining whether progress is being made toward a goal, checking whether errors are being made, and self-testing and questioning. Metacognitive monitoring sets the stage for metacognitive control such that when products fall short of standards, one can change the tactics or strategies for working on a task (Winne & Hadwin, 1998). Metacognitive control refers to regulating or fine-tuning and adjusting activities such as changing a course of action by implementing a new tactic or strategy. Examples of overt metacognitive monitoring included: "Is this working?" "Let's check this again." "This looks like the right direction. Is it?" Examples of metacognitive control included both overt statements and actions taken during problem solving. These included: actions made to abandon an unsuccessful approach or change a course of action when a previous course appeared unfruitful, and statements such as "This approach isn't working [from monitoring]. I'll quit and try something else [control]."

Rational Approaches and Justifications

Approaches to problem solving were coded as rational if participants used mathematical argumentation or derived proofs, theorems, and/or facts during the problem-solving attempt. Examples included the use of the Pythagorean Theorem to

prove two triangles are congruent, properties of congruent triangles such as side-angle-side, the proof that if $(a-b)^2 = 0$ then a and b must equal 0, and the binomial expansion. Justifications of solutions were coded as rational if the justifications included information as described above. An example statement included: "I know this is right because I have proven that these two triangles are congruent by side-angle-side."

Empirical Approaches and Justifications

Approaches to problem solving were coded as empirical if participants engaged in trial-and-error exploration of the problem space, tested hypotheses in a serial fashion, and/or used perceptual information to work the problem. An example of trial-and-error exploration included attempts to find information to help solve the problem by working another problem not directly related to the given problem. An example of testing hypotheses in a serial fashion included implementing one equation to solve the problem followed by another equation and continuing until an answer was perceived to make sense. Examples of perceptual information included: testing a construction and making adjustments to the compass setting until a construction worked (e.g., the circle was tangent to both lines), measuring distances on lines to find the center point of a circle, and measuring the angle of a triangle. Justifications of solutions were coded as empirical if participants tested their solutions using perceptual information (e.g., the construction worked), by substituting a solution into an equation to test whether the solution made sense, or by claiming the solution made sense without providing logical information to support that claim.

Problem attempts were coded for both rational and empirical approaches. If an attempt was comprised of more elements of a rational approach, that attempt was

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characterized as rational. If, however, an attempt was comprised of more empirical elements it was characterized as more empirical. If an attempt was comprised of both to an equivalent degree, that attempt was characterized as both rational and empirical. To examine whether participants' problem-solving approaches were consistent with their approaches to knowing, I selected the predominant approach taken for all six problems to make an overall characterization of participants' problem-solving attempts. The two were then compared.

Addressing Issues of Validity and Reliability

The use of verbal data to study various cognitive processes has been criticized for the effects verbal reports may have on participants' cognitive processes and on the validity and reliability of such reports (Ericsson & Simon, 1993). First, questions have been posed that challenge whether think aloud problem-solving sessions provide accurate reflections of various processes individuals use as they solve problems or of the processes they may have used had they not been asked to think aloud. Second, issues have been raised as to the validity and reliability of the analyses of such protocols. These issues are addressed in this section followed by a discussion of general issues of validity and reliability for the entire study.

The Effects of Think Aloud Protocols

In a review of how verbal reports affect participants' cognitive processes, and the validity and completeness of such reports, Ericsson and Simon (1993) found that Type 1 and Type 2 think aloud verbalization and retrospective reports do not influence the sequence of thoughts. Type 1 and 2 think aloud protocols require participants to talk or

think aloud while engaging in a task whereas Type 3 think aloud protocols require

participants to provide explanations of what they are doing as they think aloud. The only

consistent effect that Ericsson and Simon found in studies that reported using Type 1 and

Type 2 think aloud protocols was an increase in time required to complete a task; the

cause of which they claimed was the time required to produce verbalizations. Requiring

participants to think aloud, using Type 1 or Type 2 instructions, did not affect

performance when compared to participants who were not asked to think aloud. Type 3

verbalizations, in contrast, were found to change participants thought sequences to

generate the explanations required and were also found to improve performance on tasks.

For retrospective reports, participants are asked to recall the sequence of events

that occurred after a task has been completed. In their review of studies that used

retrospective accounts, Ericsson and Simon (1993) found that when tasks took very little

time to complete (e.g., less than 10 seconds), these accounts were more accurate than

concurrent accounts but when tasks took a longer time to complete, concurrent accounts

were more accurate. In this study, a Type 2 think aloud protocol and a retrospective

account was employed (see instructions).

Validity

In general, validity refers to the accuracy and trustworthiness of inferences made

based on an outcome measure (Eisenhart & Howe, 1992; Suter, 1998). Traditional

conceptions of validity in educational research were derived from Campbell and Stanley

(1963). They identified two types of validity, internal validity and external validity.

According to Campbell and Stanley, internal validity refers to the trustworthiness of

inferences that experimental treatments cause effects under specific well-defined

circumstances. Campbell (1963a,b) listed eight factors that threaten internal validity. These are: history, maturation, participant selection, interaction of maturation and selection, mortality, testing effects, changes in instrumentation, and statistical regression.

External validity refers to the generalizability of the results or the extent to which results can be compared across samples (Campbell & Stanley, 1963). These two types of external validity are labeled as ecological validity, the degree to which results can be generalized to other situations or conditions, and population validity, the degree to which results can be generalized to other populations.

Because definitions and threats to internal and external validity that Campbell and Stanley (1963) proposed are more appropriate for quantitative research designs, questions have been raised as to what standards researchers should set for more qualitative approaches. For example, qualitative methodologists proposed that not all types of research are concerned with generalizing findings to populations but are instead concerned with understanding people, events or situations. Consequently, advocates of more qualitative methodologies began to develop their own conceptions that evolved from traditional notions of internal and external validity (Eisenhart & Howe, 1992). The integration of research methodologies and development of more eclectic and radical methodological approaches to research posed even more challenges for researchers addressing issues of validity. Accordingly, Eisenhart and Howe (1992) proposed five general standards of validity that all researchers should address. The five include: 1) a fit between research questions, data collection procedures, and analysis techniques, 2) effective application of specific data collection and analysis techniques, 3) alertness to

and coherence of prior knowledge, 4) internal and external value constraints, and 5) comprehensiveness. Each of these five issues is addressed in turn.

A Fit Between Research Questions, Data Collection Procedures,

and Analysis Techniques

For this study, I apply multiple methodologies to examine relations between personal epistemology and mathematics problem solving in the context of self-regulated learning. Each specific methodology I employ complements each research question that I pose. Each question that I pose is more fine-grained than the previous question and the method by which each question is examined reflects that change. I examine relations between each of the constructs by using self-reports intended to measure those constructs. I also examine differences in problem solving approaches in the context of students actually solving problems. The analyses conducted for each question are appropriate given the type of data collected.

Effective Application of Specific Data Collection and Analysis Techniques

According to Eisenhart and Howe (1992), how data are collected and analyzed cannot be validated without credible reasons for a specific choice in participants, data-gathering procedures, and analysis techniques. A number of principles and systematic procedures have been developed to conduct and assess numerous research designs. For my dissertation, I have adhered to each of these principles. I provided reasons why specific participants were chosen and used self-report instruments that have been established as valid and reliable. Although these instruments have been shown to be valid and reliable, they are not without methodological flaw. Consequently, I employed another method, which has also been systematically developed. Specifically, I used the think

aloud methodology that Ericsson and Simon (1993) have suggested and adopted

Schoenfeld's (1983, 1985) coding scheme for think aloud protocols in the context of

mathematics problem solving.

Alertness to and Coherence of Prior Knowledge

Eisenhart and Howe (1992) proposed that studies must be judged against and built

on existing theoretical, substantive, or explicit practical knowledge. Moreover, a

researcher's biases should be made explicit. In this dissertation, I integrate three

theoretical frameworks and develop my research questions based on the integration of

these frameworks. My biases are emphasized in the framing of Chapter 2 and the specific

facets I define and included in the protocol analysis.

Internal and External Value Constraints

Internal value constraints refer to research ethics and external value constraints

refer to whether a study is valuable for informing and improving educational practice

(Eisenhart & Howe, 1992). The internal value of a study is judged by the ethical

treatment of participants and the ethical application of the data collection and analysis

procedures. Internal value was achieved by obtaining informed consent and abiding by

the ethical standards upheld by Simon Fraser University and specified by the Tri-Council

of Canada for the ethical conduct for research involving humans.

To address external value, in Chapter 1 and Chapter 2 I describe how this research

advances theory and knowledge and the educational implications this research may have

for providing a platform for designing educational practice. This is elaborated in Chapter

5. To reiterate, by gaining a better understanding about relations between personal

epistemology and cognition, researchers and educators can begin to design improved

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instruction based on implications from this research. Naturalistic studies are needed, however, to further our understanding. Future studies I plan will attempt to accomplish this.

Comprehensiveness

In any study, participants' behaviors may be shaped by a wide variety of subtle factors. When participants are asked to fill out self-report measures or provide verbal reports as they solve problems, they may respond to pressures of having to fill out questionnaires or being recorded. Consequently, participants may produce something "just for the record" (Schoenfeld, 1985). In clinical trials, where participants are asked to solve problems out of the context of an actual course, they may alter their problem solving approaches because they are being observed. Despite all that experimenters may do to safeguard against this, there are no guarantees that participants are approaching problems in ways they typically would when not being observed. Consequently, evidence from the problem-solving protocols should be compared with evidence from as many other sources as possible. This process is known as triangulation, and is the last standard Eisenhart and Howe (1992) proposed.

Triangulation refers to the cross validation of data sources and data collection or cross-checking procedures, or cross validation of analytic methods used on the same set of data (Suter, 1998). Triangulation is best used when the strengths of each data collection or analysis method help compensate for the weaknesses of the other methods (McKnight, Magid, Murphy, & McKnight, 2000). Triangulation is achieved in this study from data collected from multiple sources. First, participants' personal epistemologies were measured using two different self-report measures, the PEP (Royce & Mos, 1980)

and the EBI (Schraw et al., 2002). The rationalism component of the PEP was tested for concurrent validity using the NFC (Cacioppo et al., 1996). Second, facets of self-regulated learning were measured using a self-report instrument, the MSLQ (Pintrich et al., 1991), and overt instances of these facets were observed using a think aloud protocol as participants engaged in mathematics problem solving. Third, participants took part in two problem-solving sessions, each of which was designed to reflect a different problem-solving context. Moreover, participants were asked to solve three problems in each session to provide data to compare their problem-solving behaviors within each session and between each context. Fourth, once participants had completed the problem-solving sessions, they were asked to provide a retrospective account of their thought processes as they worked each problem. Fifth, participants were invited to characterize each of their problem-solving attempts as rational or empirical using definitions I provided. Finally, I provided participants the opportunity to clarify any misperceptions or challenge my characterizations of their problem-solving attempts.

Reliability

Reliability refers to the consistency of an outcome measure or ability to reproduce an outcome independent of a specific researcher (Suter, 1998). To address reliability for the self-report measures, Cronbach's alpha, an internal consistency coefficient, for each subscale on each instrument is reported in Chapter 4. To address reliability of the think aloud protocol analysis, a graduate student at Simon Fraser University was trained to use the coding scheme reported in this chapter. Once the student was trained, he and I conducted a trial run on one of the protocols to identify any issues or uncertainties in the coding scheme. No problems were identified. The student was then given the remaining

protocols to code independently. Inter-rater reliability estimates, measures of the

consistency of observations, are reported in Chapter 4.

CHAPTER 4

RESULTS

The general purpose of this study was to examine relations between individuals' epistemic profiles, metacognitive strategies they reported using in their mathematics and statistics classes, metacognitive strategies they actually used as they solve mathematics and statistics problems, and the types of argumentation students used as they solved problems and justified their solutions. A secondary purpose was to examine whether there were differences in students' epistemic beliefs and profiles across the four years of undergraduate school. Results are presented in two main sections. The first section reports results from the first component of the study. Relations and differences between students' personal epistemologies and self-reported metacognitive self-regulation are presented followed by an examination of differences in beliefs and profiles as a function of year of university. The next section reports results from the second component of the study. Students' self-efficacy, frequency of planning, metacognitive monitoring and metacognitive control, duration of problem-solving attempts, and performance on the problems are reported followed by a description of how each student approached the problems and justified solutions. Information from follow-up interviews is also included in the descriptions of students' problem-solving attempts. It is important to note that results for both components of the study are not interpreted or discussed in this chapter. Discussion and interpretation of results are presented in Chapter 5.

Data were screened for outliers, normality, linearity, homoscedasticity, multicollinearity, and singularity. All assumptions were met. For example, Kline (1998)

suggested using absolute cut-off values of 3.0 for skewness and 8.0 for kurtosis. All skewness and kurtosis values were near zero with the exception of kurtosis values for three variables: rationalism, need for cognition, and collaboration. Kurtosis values were 2.14, 1.09, and −1.01, respectively. These values indicate that all variables were normally distributed. Two participants failed to complete an entire page on one of the questionnaires which resulted in missing data. One page was on the PEP and the other on the MSLQ. Because of the large number of items not completed, the two participants were not included in analyses that required data from the PEP or MSLQ. Number of participants, descriptive statistics, and p values are reported for all analyses.

FIRST COMPONENT OF THE STUDY

Preliminary Analyses

First, scores on negatively worded items were reversed before creating subscale scores according to each inventory's manual or as described in articles authored by the inventory's first author. Means, standard deviations, and reliability coefficients are presented in Table 1 for all subscales for the PEP, NFC (only one scale), EBI, and MSLQ. A correlation matrix of the variables examined in this study is presented in Table 2. Reliability estimates for the PEP, NFC, and MSLQ were considered moderately reliable with internal consistency α coefficients ranging from .58 to .94 with a median of .76. Reliability estimates for the EBI were low to moderate, ranging from .42 to .68 with a median of .56. These estimates are consistent with estimates reported in previous research (e.g., Cacioppo et al., 1996; Pintrich et al., 1990; Royce & Mos, 1980; Schraw et al., 2002). Also consistent with previous research (e.g., Royce & Mos, 1980),

mathematics students had a higher average rationalism score than empiricism and metaphorism scores. The difference in means between rationalism and empiricism was statistically detectable, t (125) = 7.06, p < .001.

Table 1.

Descriptive Statistics and Reliability Coefficients for the Four Inventories.

Inventory Subscale	Mean	SD	α
Psycho-Epistemological Profile [a]			
Rationalism	106.06	11.94	.77
Empiricism	99.58	12.90	.80
Metaphorism	99.23	14.87	.86
Need for Cognition [b]	3.49	.61	.87
Epistemic Belief Inventory [b]			
Source of Knowledge	2.84	.71	.56
Certainty of Knowledge	2.40	.56	.42
Adjusted Certainty of Knowledge [c]	2.40	.68	.50
Structure of Knowledge	2.84	.69	.68
Adjusted Structure of Knowledge [c]	2.69	.77	.72
Speed of Acquisition	2.03	.59	.49
Control of Acquisition	3.19	.71	.68

Note: SD = standard deviation, [a] 30-150 range, [b] 1-5 point scale, [c] adjusted subscale estimates are reported based on results of the next section.

Table 1 Continued.

Descriptive Statistics and Reliability Coefficients for the Four Inventories.

Inventory Subscale	Mean	SD	α
Motivated Strategies for Learning – Motivation [d]			
Intrinsic Goals	4.69	1.31	.77
Extrinsic Goals	5.17	1.26	.67
Task Value	5.03	1.37	.90
Control	5.58	1.07	.73
Self-Efficacy	4.86	1.40	.94
Anxiety	4.06	1.37	.76
Motivated Strategies for Learning – Cognitive Strategies [d]			
Rehearsal	3.99	1.36	.72
Elaboration	4.90	1.08	.73
Organization	4.32	1.38	.70
Critical Thinking	3.97	1.26	.81
Metacognitive Self-Regulation	4.42	.85	.74
Time Management	4.82	.99	.72
Effort	5.01	1.32	.77
Collaboration	3.53	1.46	.66
Help Seeking	4.21	1.29	.58

Note: [d] 1-7point scale.

Table 2.

Correlations Between Select Variables.

	1	2	3	4	5	6	7	8	9	10	11
Rationalism$_1$											
Empiricism$_2$	66^b										
Metaphorism$_3$	45^b	52^b									
Need for Cognition$_4$	39^b	12	20^a								
Source of Knowledge$_5$	05	15	-11	-19^a							
Certainty of Knowledge$_6$ ^{c}	14	18	-18^a	00	18^a						
Structure of Knowledge$_7$ ^{c}	-13	15	02	-34^b	20^a	-05					
Speed of Knowledge Acquisition$_8$	-07	14	-08	-31^b	13	04	35^b				
Control of Knowledge Acquisition$_9$	-04	-01	-12	-20^a	-03	23^a	01	36^b			
Self-Efficacy$_{10}$	31^b	11	12	49^b	-05	19^a	-06	-25	-11		
Critical Thinking$_{11}$	40^b	25^b	26^b	58^b	-18^a	06	-11	-21^a	-18^a	51^b	
Metacognitive Self-Regulation$_{12}$	48^b	32^b	38^b	44^b	-14	-01	-11	-22^a	-19^a	47^b	55^b

Note: Decimals were removed. ^{a} p < .05, ^{b} p < .01. ^{c} Adjusted mean.

Similar to Royce and Mos's (1980) results, correlations between the three

epistemic profiles ranged from .45 to .66. Other correlations should be highlighted.

Specifically, the correlation between rationalism and need for cognition was statistically

detectably positive, r = .39, as was the correlation between rationalism and critical

thinking, r = .40. Empiricism was not statistically detectably related to need for cognition,

r = .12, but was statistically positively correlated with critical thinking, r = .25. Moreover,

both rationalism and empiricism were statistically positively related to metacognitive

self-regulation, r = .48 and r = .32, respectively. Finally, self-efficacy was statistically

detectably positively related to metacognitive self-regulation, r = .47.

Second, Byrne (1998) suggests that confirmatory factor analysis (CFA) of a

measuring instrument is appropriate when it has been fully developed and its factor

structure has been validated. Each inventory met this requirement. Accordingly, four

separate CFAs were conducted using the EQS software (Bentler & Wu, 1995). CFAs

were conducted to assess how well items on each inventory fit the facets the authors

identified on each inventory. Indicators of fit for the CFAs were the comparative fit index

(CFI), which is more appropriate for small sample sizes (Tabachnick & Fidell, 2001), and

the root mean squared error of approximation (RMSEA). Values for the CFI greater than

or equal to .80, and values for the RMSEA less than .08 were interpreted as confirming a

good fit. Values for the CFI greater than or equal to .90 and values for the RMSEA less

than or equal to .05 were interpreted as confirming a very good fit.

An item-level CFA model demonstrated moderate fit to the three facets of the

PEP, χ^2 / df = 1.72, CFI = .75, RMSEA = .07. Item loadings ranged from .31 to .78 and,

consequently, all original items were retained for subsequent analyses. For the NFC, an

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item-level CFA model demonstrated good fit, χ^2 / df = 1.67, p < .01, CFI = .85, and RMSEA = .07. Item loadings ranged from .34 to .77 and, consequently, all original items were retained for subsequent analyses. For the MSLQ – Motivation scale, an item-level CFA model resulted in a good fit, χ^2 / df = 2.16, p < .01, CFI = .80, and RMSEA = .09. Item loadings ranged from .31 to .92 and, accordingly, all items were retained for subsequent analyses. For the MSLQ – Cognitive Strategies scale, an item-level CFA resulted in a poor fit, χ^2 / df = 2.02, p < .01, CFI = .57, RMSEA = .09. Examination of the misspecification of the model revealed that all items loaded high onto their respective factors. Specifically, all loadings were within an acceptable range, .32 to .93. Moreover, changing variable loadings or removing variables would not have significantly improved the fit of the model. Loadings between factors were high, however, and were the largest source of misfit in the model. Since reliability coefficients and loadings for the variables were high, all original items were retained for subsequent analyses.

For the EBI, an item-level CFA model resulted in a fair fit, χ^2 / df = 1.55, p < .01, CFI = .66, and RMSEA = .07. Three items, one of the seven items from the Structure of Knowledge subscale and two of the seven items from the Certainty of Knowledge subscale, had near zero loadings. A subsequent CFA was conducted with these items removed. The new model resulted in a better fit, χ^2 / df = 1.27, p < .01, CFI = .76 and RMSEA = .06. (Since items were removed, factor loadings for both CFIs for the EBI are included in Appendix R.) Because of the significant increase in fit with the adjusted model, participants' average scores on these two subscales were computed with the three items removed. Correlations between the original and adjusted subscales were .97 and .90 for the Structure of Knowledge and Certainty of Knowledge subscales, respectively. The

adjusted subscale preserved the construct reflected by the original subscale; modifying

these subscales by removing items did not alter the scales' reflections of constructs.

Consequently, subsequent analyses with the EBI were conducted with the adjusted

subscales (as recommended by Tabachnick, personal communication, May, 2004).

Adjusted means, standard deviations, and reliability estimates are presented in Table 1.

Finally, of the 127 students who participated in the study, based on the method by

which students were epistemologically profiled as predominantly rational, both rational

and empirical, or predominantly empirical, 11 students did not fit any of the three

categories. A close examination of these students' scores revealed that for 10 of the

students only one of their scores (rationalism or empiricism) did not fall within the ranges

used. Moreover, of these 10 scores, all were within less than 1.5 points of one of the

ranges. Consequently, all 10 students were epistemologically profiled to boost the

number of participants per group and were included in subsequent analyses. In total, 39

were profiled as predominantly rational in their approaches to knowing (36 were high on

rationalism and moderate on empiricism), 75 were profiled as both rational and empirical

in their approaches to knowing (31 were high on both), and 12 were profiled as

predominantly empirical in their approaches to knowing (11 were high on empiricism and

moderate on rationalism).

Relations Between Epistemic Profiles and Metacognition

Analyses were conducted to examine whether there were differences in self-

reported metacognitive self-regulation and need for cognition between students profiled

as predominantly rational, both rational and empirical, and predominantly empirical in

their approaches to knowing. Since 36 of the 39 individuals profiled as predominantly rational in their approaches to knowing were profiled as high on rationalism and moderate on empiricism, this group was kept intact. This judgment was similarly made for individuals profiled as predominantly empirical (e.g., only one individual was profiled as low on rationalism and moderate on empiricism while the others were profiled as moderate on rationalism and high on empiricism). However, for individuals profiled as both rational and empirical in their approaches to knowing, since the group was divided by students profiled as high on both rationalism and empiricism (N = 31) and students profiled as moderate on both rationalism and empiricism (N = 44), means were compared between these two groups on self-reports of metacognitive self-regulation and of need for cognition to assess whether the two groups could be combined as one.

An independent samples t-test revealed statistically detectable differences in metacognitive self-regulation between participants profiled as high on both rationalism and empiricism and participants profiled as moderate on both rationalism and empiricism, t (73) = 2.41, p = .02, d = .09. In contrast, an independent samples t-test revealed no statistically detectable differences on need for cognition between the two groups, t (73) = 1.84, p = .07. Since differences were found between the two groups on self-reported metacognitive self-regulation, these two groups were not combined for subsequent analyses on self-reported metacognitive self-regulation. They were, however, combined for subsequent analyses on need for cognition. Means and standard deviations for each group for self-reported metacognitive self-regulation and need for cognition are presented in Table 3.

Table 3.

Means and Standard Deviations for Metacognitive Self-Regulation and Need for Cognition as a Function of Epistemic Profile.

	Metacognitive Self-Regulation (MSR) [a]		Need for Cognition (NFC) [b]	
Profile	Mean	SD	Mean	SD
Predominantly Rational	4.58	1.02	3.73	.58
High on Rationalism and Empiricism	4.65	.71	3.58	.53
Moderate on Rationalism and Empiricism	4.24	.71	3.36	.51
Predominantly Empirical	4.02	.83	2.95	.79

Note: [a] 1-7 point scale, [b] 1-5 point scale.

A univariate analysis of variance (ANOVA) revealed statistically detectable differences among the four groups, $F (3, 121) = 2.75$, $p = .04$, $\eta^2 = .06$. As presented in Table 3, participants profiled as high on both rationalism and empiricism had the highest overall average, followed by participants profiled as predominantly rational, moderate on both rationalism and empiricism and, finally, predominantly empirical. Post hoc independent samples t-tests revealed no statistically detectable differences between participants profiled as predominantly rational and participants profiled as moderate on both rationalism and empiricism, $t (81) = 1.76$, $p = .08$, or between participants profiled as predominantly rational and predominantly empirical, $t (48) = 1.66$, $p = .10$. Statistically detectable differences were found, however, between individuals profiled as

high on both rationalism and empiricism and participants profiled as predominantly empirical, t (40) = 2.40, p = .02.

Students profiled as predominantly rational in their approaches to knowing had a higher average need for cognition than students in the other two groups. This difference was statistically detectable, $F (2, 123) = 8.79$, $p < .001$, $\eta^2 = .13$. A priori independent-samples t-tests revealed statistically detectable differences on need for cognition between students profiled as both rational and empirical and students profiled as predominantly rational, t (112) = -2.54, p = .01, and between students profiled as predominantly empirical and students profiled as both rational and empirical, t (85) = -2.83, p = .01. That is, students who were profiled as predominantly rational had a higher average need for cognition score than students profiled as predominantly both. Moreover, students profiled as both rational and empirical had a higher average need for cognition score than students profiled as predominantly empirical.

Differences in Epistemic Profiles and Beliefs

A secondary purpose of this study was to examine whether there were differences in average rationalism scores and average epistemic beliefs scores between lower- and upper-year university students. Table 4 presents the means and standard deviations for rationalism, source of knowledge, certainty of knowledge, structure of knowledge, speed of knowledge acquisition, and control of knowledge acquisition as a function of year of university. Since the sample sizes for second-year and third-year students were small, years one and two were merged as were years three and four for both analyses. Means and standard deviations for the merged years are also presented in Table 4.

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An independent samples t-test revealed no statistically detectable differences in average rationalism scores between lower- and upper-year university students, t (124) = -1.47, p = .14. For the five dimensions on the EBI, statistically detectable differences were found for two of the dimensions, source of knowledge, t (125) = 2.04, p = .04, d = .03, and structure of knowledge, t (125) = 4.05, p < .001, d = .12. Specifically, upper-year university students had lower average scores on both dimensions than lower-year students. Although the same trend resulted for the other three dimensions, no statistically detectable differences were found for certainty of knowledge, t (125) = 1.28, p = .21, speed of knowledge acquisition, t (125) = 1.39, p = .17, and control of knowledge acquisition, t (125) = .71, p = .48.

Table 4.

Means and Standard Deviations for Rationalism and the Five Dimensions of the EBI as a
Function of Year of University.

Year	1	2	3	4	1-2	3-4
Scale						
Rationalism						
Mean	104.38	106.33	108.45	107.61	104.77	107.94
SD	12.58	11.45	10.50	11.81	12.31	11.22
Source of Knowledge						
Mean	2.99	2.78	2.57	2.77	2.95	2.69[a]
SD	.74	.62	.68	.68	.72	.68
Certainty of Knowledge						
Mean	2.47	2.45	2.40	2.24	2.46	2.31
SD	.66	.53	.86	.67	.63	.74
Structure of Knowledge						
Mean	3.00	2.49	2.40	2.34	2.90	2.37[b]
SD	.68	.84	.67	.75	.73	.71
Speed of Acquisition						
Mean	2.01	2.03	1.82	2.03	2.09	1.95
SD	.60	.72	.42	.58	.62	.53
Control of Acquisition						
Mean	3.27	3.06	3.39	2.96	3.23	3.13
SD	.68	.92	.54	.73	.93	.69
N	60	15	21	31	75	52[c]

Note: [a] Statistically detectable difference, $p < .05$. [b] Statistically detectable difference, $p <$ .01. [c] N = 51 for rationalism. Rationalism scale ranges from 30 to 150, EBI based on 1-5 point scale.

SECOND COMPONENT OF THE STUDY

Descriptive Statistics

The general purpose of the second component of the study was to examine whether there were differences in the use of metacognitive strategies and mathematical argumentation during problem solving between participants who were profiled as predominantly rational, both rational and empirical, and predominantly empirical. Participants' self-efficacy, prior knowledge of facts and theorems, duration of problem-solving attempts, and performance on problems were measured. Transcriptions of participants' problem-solving attempts were coded for evidence of planning, metacognitive monitoring, and metacognitive control according to the protocol coding schemes described in Chapter 3. Within each of the three groups, averages for each of the variables between participants profiled as high, moderate, or low were similar. For example, participants profiled as high on both rationalism and empiricism and participants profiled as moderate on both rationalism and empiricism had similar average scores on all variables (e.g., for self-efficacy, M = 5.09, SD = .35, and M = 4.80, SD = 1.21, respectively; for monitoring, M = 21.60, SD = 18.50, and M = 21.00, SD = 10.93, respectively, etcetera). Consequently, the groups profiled as high on both rationalism and empiricism and moderate on both rationalism and empiricism were merged. Means and standard deviations for self-efficacy, prior knowledge, duration of attempts, performance, planning, monitoring, and control are presented in Table 5.

Since the number of participants for each presentation order of the problems was small, a test for order effects was not feasible. Mean number correct per order ranged

from 0 to 2 for the first session and 1.33 to 2.33 for the second session. Three participants solved the first problem, 7 solved the second problem, 11 solved the third problem, 11 solved the fourth problem, 8 solved the fifth problem, and 7 solved the sixth problem. Inter-rater agreement was calculated for planning, metacognitive monitoring, and metacognitive control. Both raters scored each participant's problem solving attempts by underlining verbal evidence of each instance of these behaviors. Agreement was calculated by counting the frequency with which raters agreed and disagreed on instances of these behaviors. For example, if both raters coded a particular sentence as an instance of planning, that was counted as an agreement. If, however, one rater coded a particular sentence as a plan and the other rater did not, that was coded as a disagreement. Inter-rater agreement was 93% for planning, 96% for metacognitive monitoring, and 83% for metacognitive control.

Table 5.

Means and Standard Deviations for Self-Efficacy, Prior Knowledge, Time, Performance, Planning, Metacognitive Monitoring, and Metacognitive Control.

Measure	Session 1		Session 2		Overall	
	Mean	SD	Mean	SD	Mean	SD
Self-Efficacy [a]	4.69	1.06	4.90	1.35		
Self-Efficacy [b]			5.71	.87	5.10 [c]	.88
Prior Knowledge	11.88 (79.2%)	2.03				
Time (minutes)	31.76	16.81	23.09	10.65	54.85	24.94
Performance	1.29 (43%)	.92	1.59 (53%)	1.00	2.88 (48%)	1.58
Planning	6.94	3.88	1.29	1.31	8.24	4.71
Metacognitive Monitoring	13.77	10.40	7.71	6.21	21.47	15.38
Metacognitive Control	2.53	2.12	.88	1.17	3.41	2.45

Note: [a] Prior to studying. [b] After studying. [c] Average across all three measures of self-efficacy. Self-efficacy based on 1-7 point scale. N = 17 for all measures.

As previously stated, it was important to measure participants' general self-efficacy for learning and performance in mathematics and statistics classes and task-specific self-efficacy for successfully completing specific problems. It was also crucial to measure participants' prior knowledge of various facts, proofs, and theorems that could be used to solve the problems. The sample's average score for self-efficacy for learning

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and performance, measured using the MSLQ, was 4.86 on a 7-point scale. This was interpreted to indicate that, in general, participants were somewhat confident in their ability to learn material from their mathematics and statistics courses and apply that material when problem solving. For task-specific self-efficacy, participants' overall average self-efficacy score was 4.69. Although all 17 participants reported that they had not done any geometry since high school and were seldom required to work algebra proofs, I considered them to be somewhat confident they could successfully complete the problems in the first session. For the second set of problems, prior to studying, participants' average self-efficacy ($M = 4.90$, $SD = 1.35$) was slightly higher than for the first set of problems. After participants studied the short chapter on the binomial distribution, all participants' confidence in being able to solve the problems increased. Based on their average self-efficacy score of 5.71 ($SD = .87$), I considered participants to be quite confident they could successfully solve the second set of problems.

Finally, all participants were considered to have sufficient prior knowledge of proofs, facts, and theorems that could be used to solve the problems in the first session ($M = 11.88$, $SD = 2.03$, maximum 15). Moreover, participants were considered highly accurate (e.g., calibrated) in predicting whether their answers on the prior knowledge test were correct (gamma could not be computed due to near perfect scores). Two participants were perfectly calibrated and the remaining 15 participants were correct for at least 12 of the responses (4 could not be calculated due to guessing – a score of 3). Although three participants scored low on the prior knowledge test (one scored 8 and the other two scored 9), they used appropriate facts and theorems in their attempts but provided incorrect answers on the prior knowledge test. Since these students correctly used facts

and theorems during their problem attempts, they were considered to have sufficient prior knowledge.

Differences in Metacognitive Strategy Use

To examine whether there were differences in planning, metacognitive monitoring, and metacognitive control during problem solving as a function of epistemic profile, transcriptions of participants' problem-solving attempts were coded according to the coding scheme previously described in Chapter 3. Means, standard deviations, and medians for planning, monitoring, and control averaged across both sessions are reported in Table 6. Since the duration of the problem attempts ranged from 1 minute to 26 minutes, average rate per minute for monitoring is also reported in parentheses.

Participants profiled as predominantly rational in their approaches to knowing engaged in more planning, metacognitive monitoring, and metacognitive control than those in the other two groups. Participants profiled as predominantly empirical had the lowest occurrence of these behaviors. Due to the small number of participants, however, statistical tests could not be conducted to examine whether these differences were statistically detectable. Moreover, because of the small samples size and large standard deviations, these results must be interpreted with caution. A detailed description of participants' problem-solving attempts may help clarify differences between the groups.

Table 6.

Means and Standard Deviations for Planning, Metacognitive Monitoring, and Metacognitive Control as a Function of Epistemic Profile.

	Planning			Monitoring			Control			
Profile	Mean	SD	Med	Mean	SD	Med	Mean	SD	Med	N
Predominantly Rational	9.80	5.72	11	27.20 (1.96)	18.29 (.67)	32	4.40	3.21	3	5 (4)[a]
Both Rational and Empirical	8.20	4.21	9	21.30 (4.13)	14.33 (2.84)	17.5	3.40	2.01	3	10
Predominantly Empirical	4.50	4.95	4.5	8.00 (7.24)	8.46 (6.70)	8	1.00	1.41	.5	2

Note: Med = median. [a] One participant, AC, was removed due to zero monitoring (see below). Including this participant would have biased the estimate.

Participants' Problem-Solving Attempts

Participants' approaches to problem solving were evaluated to examine whether they were consistent with their approaches to knowing. Transcriptions of participants' attempts were coded for use of mathematical argumentation, trial-and-error exploration of the problem spaces, serial testing of hypotheses, use of perceptual information to solve problems, and rational and empirical justifications of solutions as described in Chapter 3. I and another independent rater used the coding schemes to characterize each participant's problem-solving attempt for each problem. Based on the characterization of each attempt, participants were labeled according to the most frequent approach they used for solving the problems. For example, if the majority of a participant's approaches and justifications were characterized as predominantly rational, that person was profiled

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as predominantly rational in his or her approach to problem solving. Each coder's evaluations of the problem-solving attempts were then compared.

Inter-rater agreement was calculated by counting the frequency with which the raters agreed on each participant's overall characterization and by the characterization of each problem-solving attempt. For example, if both raters characterized participant X as rational, that was coded as an agreement. If, however, one rater coded participant X as rational and the other rater coded participant X as empirical, it was counted as a disagreement. Comparisons of overall categorization of participants' approaches to problem solving yielded a 100% agreement. Characterization of each problem-solving attempt yielded an 89% agreement. Differences in characterizations of the individual problems occurred in instances where participants' descriptions of what they were doing were difficult to comprehend (e.g., sections of three participants' tapes were difficult to hear and a number of "unclear" sections of the tapes were noted). Because I was present during problem solving attempts and took detailed notes as participants solved problems, I had the advantage of having more information about what participants were doing as they solved problems[3]. To clarify differences, the second rater was given my notes taken during the problem-solving attempts. Based on information from the notes, the second rater agreed with my categorizations.

After the second problem-solving session, participants were asked to give a retrospective account and to characterize each of their problem-solving attempts. A follow-up interview was also conducted. During the follow-up interview, participants had the opportunity to correct any misperceptions they thought I had when recounting their

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[3] Arguably, this made me not blind to coding, which could be construed as one methodological issue.

attempts and were asked to describe how they generally approached problem solving. Information gathered from the various sessions was used to evaluate the episodes.

I summarize the problem-solving attempts of all 17 participants. Based on how they approached problems and justified their solutions, I characterized them as predominantly rational, both rational and empirical, or predominantly empirical in their approaches to problem solving. Comparisons were then made between their approaches to knowing and approaches to problem solving. Participants profiled as predominantly rational in their approaches to knowing (N =5) are presented first. Participants profiled as both rational and empirical (N = 10) are described next, followed by participants who were profiled as predominantly empirical (N = 2). A summary of variables examined for each participant's problem-solving attempts is presented in Table 7.

Table 7.

Summary of Each Participant's Problem-Solving Attempts.

Epistemic Profile	# of Plans	# of Monitors	# of Controls	# of ISI	# of Problems Correct	SE	Approach
Predominantly Rational							
AC [a]	1	0	0	2	6	5.33	R
RC [d]	8	47	4	0	1	6.67	R
BR [c]	11	19	4	2	2	5.22	R
GS [c]	16	38	9	2	4	6.11	R
AA [d]	13	32	5	0	3	4.89	R
High on Rationalism and Empiricism							
SG [a]	2	36	7	0	4	5.44	R
SQ [b]	8	4	2	1	5	5.00	R
MC [b]	4	7	1	0	4	4.89	R
EF [d]	9	46	3	1	1	4.67	R
AL [d]	11	15	6	2	3	5.44	R
Moderate on Rationalism and Empiricism							
BC [a]	2	8	1	0	2	2.89	R
JS [a]	9	20	4	0	3	4.67	R
AM [b]	11	15	4	0	3	4.78	R
AF [c]	12	37	4	0	3	6.00	R
PB [d]	14	25	2	2	2	5.67	R
Predominantly Empirical							
KF [a]	1	2	0	0	0	3.67	E
PC [a]	8	14	2	0	1	5.33	E

Note: ISI = isomorphic identifications, R = rational, E = empirical. [a] First-year university student. [b] Second-year university student. [c] Third-year university student. [d] Fourth-year university student. SE = self-efficacy, based on 1-7 point scale.

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Predominantly Rational

AC

AC, a first-year student, was profiled as high on rationalism and moderate on empiricism (scores were 118 and 101, respectively). On the prior knowledge test, AC scored 15 out of 15. Based on his overall average self-efficacy score of 5.33, I considered AC to be somewhat self-confident in being able to successfully complete all six problems. Within less than 3 minutes for each problem, AC successfully solved all six. Each attempt was coded as rational as were the justifications he provided. Consistent with his approaches to knowing, overall, AC was characterized as predominantly rational in his approaches to problem solving.

When given a problem, AC immediately identified one or more theorems or proofs that could be used to solve the problem. In total, only one plan was made and no monitoring or control occurred for any of the problem attempts. Because I was perplexed by the speed with which AC identified a useful theorem or proof and subsequently solved each problem, after one of the problem solving attempts I asked AC two follow-up questions. The following is the complete excerpt from this episode.

AC reads: Show that for all sets of real numbers a, b, c, and d, $a^2 + b^2 + c^2 + d^2 = ab + bc + cd + da$ implies $a = b = c = d$. Okay, I am just going to multiply everything by 2. Bring everything over to the other side. Turn them all into squares. Since every square in a difference is zero each of these have to be zero. Done.

Krista: So, what made you multiply everything by 2? What made you decide that?

AC: Well, that is simple. I know that this A^2 plus B^2 is greater than or equal to

2AB. That is true, so therefore I multiply them.

Krista: So, you just saw that as a strategy to do?

AC: Well, I did a contest before. A lot of them I solved. I do have quite a large

arsenal of strategies.

Krista: Okay. So you have seen this question before or something similar?

AC: Simpler versions of it, but I thought that I could prove that, so.

As Schoenfeld (1985) argued, when experts solve problems in domains with

which they are highly familiar, they immediately move to the implementation stage of

problem solving. No planning or exploration is needed and little monitoring or control is

required since the expert knows precisely what argumentation to use to solve the

problems. Although AC was only in his first year of university, I considered his behavior

to be expert-like. In the interview, AC revealed that he recalled similar problems he had

solved in the past and used information from those problems to solve the ones given.

Moreover, for both problem-solving sessions, AC quickly identified the similarities

between the isomorphic problems. For example, after reading a problem, AC stated,

"Okay, this question is basically the same as the first. I see that already." He subsequently

used the information from the previous problem to help solve the next problem.

AC's evaluations of his problem-solving attempts were consistent with how I had

characterized them. Specifically, AC characterized all attempts and justifications as

rational. When asked how he typically approaches problems, AC believed he was "quite

rational." He indicated he had competed in mathematics competitions since he was 15

and could typically identify some underlying theorem or proof that would be useful in

solving the problems. He revealed that he had a lot of experience doing problems and the more problems he did, the more he saw connections between them. At the end of the interview, AC admitted, "I am probably not your typical student."

RC

RC, a student in his third year of university, was also profiled as high on rationalism and moderate on empiricism (scores were 113 and 100, respectively). RC scored 11 on the prior knowledge test and his overall average self-efficacy score was 6.67. I interpreted his score as reflecting that he was very confident in being able to solve the problems. Out of the six problems, RC solved one. For four of the problems, RC spent 12 to 20 minutes to complete each of them. For the *Geometry 1* and *Multiple Choice Exam* problems, after spending over 12 minutes on each problem, RC quit working the problems before coming to a solution. Because RC did not complete two of the problems, he was not able to provide justifications for those problems. Five of RC's problem-solving attempts were coded as predominantly rational while one was coded as predominantly empirical. Of the four problems he did complete, two justifications were rational, one was empirical, and one was based on intuition. Overall, RC was characterized as predominantly rational in his approaches to problem solving. This was consistent with his profile on the PEP.

Throughout RC's attempts, he continuously monitored his progress a total of 47 times during 83 minutes of problem solving (an average of one monitor event every 1.77 minutes). Eight plans were made and, on two occasions when strategies were not going according to plan, he switched strategies. The following excerpt from *A Little Algebra* illustrates the frequency of his metacognitive behaviors.

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If ABCD, uh, ABCD minus ABD squared plus AC squared D plus ACD squared minus AB squared, A squared minus B squared plus B squared D squared. Hold on, did I just, I think I'm still trying to, hold on, I'm not doing this right. Uh, four terms and then, okay I missed the term. This is later so I'll just, mmm, so at first I will go plus A squared, BD, oh yeah, at the rate I'm going it's not going to show anything that I want I think. Oh maybe it will. All these terms have all cancelled so it's not gonna work which has ended with 0. So I'll just stop now and try a different, try something else. ...

Even as RC worked a problem empirically, he continued to monitor. The following is an excerpt from *Geometry 1*.

Okay, so let me think, uh, this short side is 2.7 centimetres so minus the radius from the center to R is, is equal to this other side. Um, or is equal to R, is equal to, to, um, some ratio. So some alpha to this other length, which is, which is, uh, or 4.3. So I know these. Well this doesn't make sense. 2.7 centimetres minus R equals same proportional to 4.3. But it happens to also equal R. So that doesn't make sense. If I subtract, mmm, what the equation is kind of telling me that, seems like the equation is telling me that 2.7 minus R equals R. If that's true so R equals, or I mean 2R equals 2.7 and R equals, um, oh that's 1.35 centimetres. That seems kind of funny. Uh, some kind of a like a funny solution. I don't think it's right. Let's just check. 1.35, 1.35 so it's about here and if going down it's 1.35 then I'm done. 1.3, no it's not. Mmm. Then why does the equation tell me that it is?

Unlike AC, RC did not see the similarities between the isomorphic problems. When asked during the follow-up interview if he noted anything of interest about the problems, he was not able to identify them as similar. Not until I pointed out the similarities did he notice they were isomorphic.

With the exception of one of the problems, RC's evaluations of his attempts were consistent with how I characterized them. For *A Little Algebra*, RC felt his attempt was both rational and empirical. He justified his characterization on how he tried a number of different strategies, which he thought was "a bit trial-and-error," but at the same time he was trying to use proof-like information, such as "if the sum of two squares is equal to zero, the numbers must be equal to zero." I characterized his attempt as predominantly rational since his choices in strategies were logically sound (e.g., isolating variables and working backwards from the problem). Moreover, RC worked directly with the problem given and did not go off on "wild goose chases."

In comparing his problem-solving attempts, I noticed RC solved the two geometry problems using different approaches. Specifically, he solved *Geometry 1* empirically and *Geometry 2* rationally. Theorems that he could have used to solve *Geometry 1* were used to solve *Geometry 2* (the problem he correctly solved). When asked why he thought he solved them in distinct ways, he revealed that he had taken a drafting course that required him to construct objects such as circles in figures. He believed that he approached *Geometry 1* in an empirical way because of his drafting experience. He explained:

> As soon as I saw the word 'construct' I thought of how we constructed circles in drafting – with compass and ruler in hand. Everything was measured and you had

to prove your construction using numbers even though everything had a theory behind it.

When asked how he typically approaches problems, RC believed he was predominantly rational, although, as he described, "Sometimes I do some trial-and-error when I have no idea how to solve the problem. Like, when I can't think of something that will help, I play around to see if I can remember something."

BR

BR, also a third-year university student, was profiled as high on rationalism and moderate on empiricism (scores were 118 and 87, respectively). On the prior knowledge test, BR scored 13 out of 15. Based on his overall self-efficacy score of 5.22, I interpreted BR to be somewhat confident in being able to successfully complete the problems. Of the six problems, BR successfully completed four. The two problems he did not solved were *A Little Algebra* and *Rolling the Dice*. For *A Little Algebra*, BR had selected a proof that would have helped him solve the problem. Since he believed the solution would not be a sufficient proof, he switched strategies. For *Rolling the Dice*, BR had made a calculation error.

Five of BR's problem-solving attempts were coded as predominantly rational and one was coded as a mix of rational and empirical. All six of BR's justifications were coded as rational. Consistent with his epistemic profile, BR was profiled as predominantly rational in his approaches to problem solving. Throughout his problem-solving attempts, BR made 11 plans, monitored his progress a total of 19 times over a period of 56 minutes (an average of one monitor event every 3 minutes), and changed strategies a total of four times when he perceived that progress was not being made. For

all six problems, BR immediately identified which theorems or proofs could be used to solve the problems. Each time he solved a problem he noted that he could "see" how the solution worked but he needed to prove it "logically." For one of his attempts, BR tested his solution empirically to ensure his answer was correct. After BR solved *Geometry 1* using theorems, he decided to try the construction to assess whether it worked. When his construction did not work, he reasoned that his original solution must be right. By working backwards from the circle, he proved why his original solution was correct. The following excerpt from his attempt illustrates this.

> *BR*: So my guess would be is that you have to construct something that looks somewhat like the last picture that I saw. (BR proceeds to solve the problem using the same theorems from *Geometry 2*.) ...I guess the justification would be pretty much identical to the justification in the last problem. Which is, the same terms. Do you want me to actually draw the circle?
>
> *Krista*: It's not –
>
> *BR*: I will anyways, just for the heck of it. (Begins to draw.) This may not come out looking so good. (Drawing.) And that's no good is it? (Long pause.) It doesn't appear to be working. I am hoping it's mechanical. ...But – I am just trying to, to think of whether I can prove that I must be right. (Begins to draw another circle.) And now I have the opposite problem, the circle is too small. Hm. I guess my only explanation is either that it is mechanical or it seems not so likely. (Rechecks his drawing.)...Well, I guess we can start by working the other way around. ... So, I'm kind of going backwards, if I were given a circle I would know that this line, these two lines are the same and from that these two are right angles. Given

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two sides of a right angle triangle you can also determine that you have two sides

to be the same. So my only explanation is that there must be something

mechanical in my drawing.

As revealed in his problem-solving attempt, after reading the problem BR immediately

noticed the similarity between the two geometry problems. He also identified the

isomorphic problems in the second problem-solving session and, like the first session,

used that information to help solve the second problem.

BR's evaluations of his problem-solving attempts were consistent with how I

characterized them. When asked how he typically approaches problems, he felt he was

predominantly rational. He admitted, however, that when he was uncertain of how to

solve a problem and could not recall "anything useful," he engaged in "brute force."

GS

As a third-year university student, GS was profiled as moderate on rationalism

and low on empiricism (scores were 101 and 64, respectively). On the prior knowledge

test, GS scored 11 out of 15. His overall self-efficacy score of 6.11 was interpreted to

suggest that GS was quite confident he could solve all six problems. GS successfully

solved four of the problems, each of which typically took him less than 7 minutes to

complete. For the two questions GS did not correctly solve, he spent a total of over 40

minutes trying to solve them. Four of his problem-solving attempts were coded as

predominantly rational and two were coded as a mix of rational and empirical. All six of

his justifications were coded as rational. Consistent with his epistemic profile, GS was

profiled as predominantly rational in his approaches to problem solving.

For episodes that were coded as rational, GS quickly stated what theorem could

be used and what relations or properties he needed to prove to solve the problem. For one of the episodes coded as both rational and empirical, GS used proof-like information throughout the problem but also engaged in trial-and-error exploration of the problem space. For the other attempt coded as both rational and empirical, GS first solved the problem empirically and then worked backwards to solve the problem rationally using proofs and theorems. The following is an excerpt from *Geometry 1* to illustrate this transition from empirical to rational.

> Uh, now I think if I take an arc. No, that can't be right. If I took an arc it's the length of P, okay on one end if I take an arc point P off this and use that as the center of my circle it will, uh, work. But somehow I feel doubtful of this anymore. So I can make it easily pass through P by, there it is, perfect, cool. ...I'm claiming that should be correct. So why is it correct? ... Probably because I can create a similar triangle. If I draw a line from O to their crossing that I don't know, I know they both share a radius. The radius is one side. I know they both share this angle of 90 degrees and they both share the other side. So it's two angles, two sides, so that's probably not enough. Uh, I think I need side angle, angle side side, angle side side, okay. Does that make sense? Um, 90, is it possible for them not to have an equal side? No, because they're right angle triangles so by Pythagorean identity I know the last two sides are the same, which is what I used to construct it. So I, I'm certain my system works because I can go backwards from it.

Throughout his problem-solving attempts, GS continuously planned and monitored his progress. In total, GS made 16 plans and monitored a total of 38 times over the duration of 64 minutes of problem solving (an average of one monitor event every

1.68 minutes). When he identified that a particular strategy was not going according to plan, he changed his course of action a total of 9 times. Moreover, GS immediately identified the similarities between the isomorphic problems for both problem-solving sessions. After reading *Geometry 2*, GS stated, "I think that's what I just did in the last question." GS then proceeded to use information from the first problem to solve the next problem.

GS's evaluations of his problem-solving attempts and justifications were consistent with my characterizations with the exception of one of the attempts. For one attempt I had coded as both rational and empirical (*A Little Algebra*), GS had difficulty deciding whether the attempt was more rational or a combination of both. GS did not make a final decision on that particular attempt and, consequently, I could not code the characterization of that attempt as being consistent with mine. When asked how he generally approaches problems, GS believed that he was generally rational. He admitted, "I am empirical at times and other times you just know something intuitively, especially when it comes to statistics. But, like, you have to prove it because intuition just isn't going to cut it on an assignment or exam."

AA

AA, a fourth-year undergraduate student, was profiled as high on rationalism and moderate on empiricism (scores were 109 and 86, respectively). On the prior knowledge test, AA scored 13 and had an overall average self-efficacy score of 4.89. Based on her self-efficacy score, I considered her to be somewhat confident in being able to correctly solve the problems. Of the six problems, AA successfully completed three. For two of the problems, she spent more than 13 minutes working each problem and less than 7 minutes

for the other four. Four of her problem-solving attempts and justifications were coded as predominantly rational and the other two attempts and justifications were coded as predominantly empirical. Consistent with her epistemic profile, AA was profiled as predominantly rational in her approaches to problem solving.

Like RC, AA's attempt at *Geometry 1* was predominantly empirical, which contrasted with her attempt at *Geometry 2*, coded as rational. The following excerpt from her attempt at *Geometry 1* illustrates the empirical nature of the attempt and the justification for her solution.

> So that's probably it, fit it in between. I take the ruler and from V, I would draw a straight line out between the two intersecting lines to give me a mid point. And then I would draw, actually I need to draw a line from P too I think between the two lines so I get my circle the right size. So my radius would be from the point of intersection between the two to point P between the two lines. (Draws circle.) Okay. I can see that even if it wasn't shifted, I think I'm right. I think it's tangent to both just about but I'm not using this compass properly. So intersecting through lines point P a circle that is tangent to both lines and has the point P as a point of tangency to one of the lines. And so that's my circle. Justification? So I drew a line from the intersection of the two lines of V marked on the figure. I drew that out between the two lines. And then I drew a line down through P through my other line so I could get a mid point and then from that mid point I used that and point P as the radius from the compass and then I just drew the circle around so it's tangent to both lines.

This excerpt contrasted her attempt and justification for *Geometry 2*, as illustrated in the

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following segment of her problem-solving attempt.

I think C's the center of the circle, line segment C. So I know that for any

tangent, anything line tangent to a circle to the radius, gonna mark the radius

myself, just mark it R, is a right angle. So I remember that. So on either side of

the circle I'm gonna label this point A and B I think, so it's gonna go A, EAF and

then G, BF. At those two points that's where the circle's tangent to the line.

They're right angles. ...So I can't remember all my rules of geometry but two

right triangles of two sides are the same sides and I think the third one, this one

has to be Pythagoras. ... Because these are two right triangles, [Coughs] two right

triangles with three equal sides that the two right angles would have to be the

same, so AFR and BFR are the same. So if those two are the same, then the line

going through them, then cause they're equal then the line would bisect the big

angle of EFG.

Throughout AA's problem-solving attempts, she continuously monitored her

progress and checked to see whether her approaches were logical. During the 46.5

minutes of problem solving, AA monitored her progress a total of 32 times (an average of

one monitor event every 1.45 minutes). Thirteen plans were made and, when AA thought

strategies were not useful, she changed her approaches a total of five times. Like RC, AA

did not see the relationships between the similar problems. Not until the similarities were

explicitly stated did she realize the problems were isomorphic.

Consistent with my characterizations, AA labelled four of her attempts and

justifications as rational and the other two as empirical. When she evaluated her *Rolling*

the Dice attempt, she began to laugh and revealed that she could not believe she "plugged

in the numbers from formula to formula." She admitted, "I even tried to count out all the possible combinations by actually trying them!" When asked why she approached two problems empirically, she had difficulty explaining her approach for *Rolling the Dice* but did reveal, "I don't really like statistics and I think when I'm stuck, I just rely on something I know I can do. Like, I can count out the possible ways." For *Geometry 1*, however, AA admitted that she believed she had to construct a circle using a straightedge and compass and did not think to use proofs or theorems. She said, "I just thought that I had to actually draw it. But I guess that doesn't explain why my justification was not rational. I know the theory. Like, it was probably in the back of my mind or something."

Both Rational and Empirical

SG

SG, a first-year university student, was profiled as high on rationalism and empiricism (scores were 130 and 116, respectively). On the prior knowledge test, SG scored 12 out of 15. Based on his overall self-efficacy score of 5.44, I considered him to be somewhat confident in being able to solve all six problems. Of the six problems, SG successfully completed four and times to complete the problems ranged from 5 minutes to 27 minutes. Five of SG's attempts and justifications were coded as predominantly rational while the other was coded as a mix of rational and empirical. Inconsistent with his epistemic profile, SG was characterized as predominantly rational in his approaches to problem solving.

For each attempt, SG identified the givens in the problem and, consequently, selected what theorems would help solve the problem. An excerpt from his attempt at the *Heart Transplant* problem is provided as an illustration.

We'll call this event A and P of event A is .75. Okay. The proportion of patients who do not experience any difficulty after a heart transplant is 16 people. Okay. N is 16 and .75 is the probability of success. You take 8 patients to interview and they have difficulties so 8. X is number of hits is 7 or 8. Um. Okay. Then binomial is best. N! (N-X!)X! P to the N, Q to the N-X. ...

For the problem *A Little Algebra*, coded as rational and empirical, SG attempted to solve it using proofs he could recall but was convinced his solution was not sufficient. After he perceived his attempt had failed, SG solved the problem by substituting numbers into both sides of the equation to "prove" the statement must be correct. After showing the equality, SG stated, "So the theory holds." Ironically, when SG initially began the problem he admitted that, "It's not like I can use a context and prove it." SG revealed that substituting in numbers to prove the equality would not suffice as a logical proof. In the end, however, when SG believed he could not solve the problem logically, he used substitution.

Throughout his problem solving attempts, SG occasionally monitored his progress. During the 89.5 minutes of problem solving, SG monitored a total of 36 times (an average of one monitor event every 2.49 minutes). Only two plans were made but, in all seven occasions when he felt a particular strategy was not working, he changed strategies. Finally, SG did not identify the similarities between the problems. Once the similarities were described to him, SG recognized the problems as isomorphic.

SG's characterizations of his problem-solving attempts were consistent with my characterizations. When asked why he solved *A Little Algebra* using substitution, SG admitted, "I was stuck. I knew I could see the pattern in the problem and I could see why

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it would hold true, but I couldn't think of how to prove it." When asked how he typically approaches problems, SG believed he was predominantly rational but that he was not entirely confident that the proofs or theorems he used were correct applications. At the end of the interview, SG revealed that, "I still need to build confidence but I think that will come over time and with practice. I'm only in my first year, so, I still have a lot to learn."

SQ

SQ, a second-year undergraduate student, was profiled as high on rationalism and empiricism (scores were 109 and 109, respectively). His prior knowledge score was 11 and his overall self-efficacy score was 5.00. I considered him to be somewhat confident in being able to solve the six problems. SQ successfully solved five problems, each of which took him less than 14 minutes to complete. Five of his problem-solving attempts and justifications were coded as predominantly rational and the other problem attempt was coded as predominantly empirical but no justification was given. Overall, inconsistent with his epistemic profile, SQ was characterized as predominantly rational in his approaches to problem solving. Like SG, SQ identified the givens in the problems and chose which theorems and formulas could be used to solve the problems. To illustrate, an excerpt from the *Heart Transplant* problem is presented.

So, the result of the interview shows that 8, 8 out of 16 patients, uh, are having difficulties. That's 50%. And, the proportion of patients who do not experience any difficulties after heart transplant per person is 75%. So, p is 75%, q is 1 – p, is 25%. And, I select 16 patients. If the selection is random, then this should be independent event. (Flipping through chapter.) Independent events. The

proportion of patients who do not experience any difficulties is 75%. If I select 16

patients, then the probability 16 patients, 8 of them have difficulties. P is .75.

And, I need to calculate p that X = 8. I'll need the formula (he points to the

binomial), and that's, that's 16! Over 8! 8! P is .75, Q is .25. Times Q, so that is

0.25 to the 8. ...

During his problem-solving attempts, SQ infrequently monitored his progress. For

two of the problems, SQ did not immediately identify how they could be solved. During

those attempts, SQ monitored a total of 4 times in 40.25 minutes (an average of one

monitor event every 10.06 minutes). He did, however, make a plan for each problem. In

total, eight plans were made. When his attempts were not going according to plan, he

made another plan and changed strategies. SQ changed strategies a total of 2 times during

his problem solving attempts. Moreover, SQ identified the similarities between the

Multiple Choice Exam problem and the *Heart Transplant* problem but did not identify the

similarities between the two geometry problems.

SQ's evaluations of his problem-solving attempts were consistent with my

characterizations. When questioned why he approached *Geometry 1* empirically and

Geometry 2 rationally, he revealed that he thought he had to construct the circle. When

giving a retrospective account of the problem, SQ identified what theorem he used for

each element he constructed. When asked whether those theorems came to mind, SQ

admitted they had not, but he knew his construction was based on theorems. Overall, SQ

believed that he was predominantly rational when problem solving. He explained, "I'm

good at figuring out what formula to use and what theorems are relevant, but sometimes I

struggle and have to explore a problem a bit to figure it out. When I figure out how to

solve it, I just do it."

MC

As a second-year undergraduate student, MC was profiled as high on rationalism

and empiricism (scores were 120 and 109, respectively). Her score on the prior

knowledge test was 15 and her overall self-efficacy score was 4.89. I considered her to be

somewhat confident in being able to solve all six problems. MC successfully solved four

of the problems, all of which took 15 minutes or less to complete. Five of MC's problem-

solving attempts and justifications were coded as predominantly rational and the other

was coded as both rational and empirical. For the problem coded as both, after 14

minutes of working the problem, MC chose to quit. Consequently, no justification was

provided. Overall, inconsistent with her epistemic profile, MC's problem-solving

attempts were coded as predominantly rational.

During her problem-solving attempts, although MC could not recall proper names

of theorems, she acknowledged that certain properties could be proven and used those

properties to solve the problems. The following is an excerpt from her attempt at

Geometry 2 to illustrate this.

> Okay. So, uh, since F is a point outside of the circle, these two points A and B, by
>
> drawing two lines AF or BF from the point F that are both tangent to the circle of
>
> AF equals to BF, I don't know what that thing is called, by theorem. Anyways,
>
> construct radii CA, CB which are perpendicular to EF and GF, respectively. Since
>
> CF and CF is the same line, triangle ACF is congruent to triangle BCF by side
>
> side side. From the congruency, angle of EFC is equal to angle GFC, which

implies that CF bisects angle EFG.

For *A Little Algebra*, coded as both rational and empirical, MC began with a logical

argument to prove the equations were equal. After two failed efforts, MC chose to prove

they were equal by substitution. Below is an excerpt from this problem-solving attempt to

illustrate this.

In most of the school of thumb it has like if this equals to that, like there's so

many if and only if that you have to work both ways. So you have to first prove

the A part and then you prove over this part. ...I'm thinking if I substitute all the

same numbers then I'll try to substitute some real numbers into A, B, C, or D.

Right. Um, because, uh, if I substitute it will show that they, first I'm gonna

substitute all the same numbers for A, B, C and D. So if that turns out to be

correct, um, then I will say that this case implies that A equals to B equals to C

equals to D.

Over the course of MC's attempts, she engaged in very little planning (a total of 4

plans were made), monitoring, and control (she quit working a problem). In total, over a

period of 58 minutes, she monitored her problem-solving attempts 7 times (an average of

one monitor event every 8.29 minutes). Moreover, MC did not identify the similarities

between the problems. In the follow-up interview, when asked if she noticed anything in

particular about the problems, she recognized the similarities without being explicitly

told.

MC's characterizations of her problem-solving attempts were consistent with

mine. Initially, however, MC believed her attempt at the algebra problem was more

empirical but, after describing the attempt to me, she felt the attempt was both rational

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and empirical. As she described the attempt, she noted that she should not have solved the problem using substitution. When asked why she took that approach, MC revealed that she was at an impasse and did not know how to prove that the two equations were equal.

EF

EF, a fourth-year university student, was profiled as high on rationalism and empiricism (scores were 112 and 109, respectively). On the prior knowledge test, EF scored 11 out of 15 and, based on his overall self-efficacy score of 4.67, I considered him somewhat confident he could solve all six problems. Of the six problems, EF successfully completed one. For two of the problems, after working one for 15 minutes and the other for over 25 minutes, EF quit. For one problem, he believed he needed to prove the theorems he was using to solve the problem. Since he did not know how to prove them, he decided to quit. For the other problem, EF judged he had spent too much time working the problem and, in a typical situation, he would leave the problem and come back to it at a later point in time. Of the six problem-solving attempts, five were coded as predominantly rational and one was coded as predominantly empirical. All four of his justifications were rational. Inconsistent with his epistemic profile, EF was characterized as predominantly rational in his approaches to problem solving.

For each attempt, EF labelled the nature of his approach as he proceeded. For example, for *Geometry 1*, coded as empirical, EF described his approach as trial-and-error. An excerpt from *Geometry 1* illustrates this.

> So this is 7 centimetres. Yeah. Okay. So both these sides better be the same. So why do I put a mark there centimetres? So I connect these two triangles, okay.

And the line I just draw the little point, so how, length of this line I'm gonna draw 3.9 point centimetres. ...So what I did was, um, compass, right, I made sure this tip end so it touches point P, okay. And then I tried to see, I just tried to see which, which point, uh, which point on dash line it, so I do trial-and-error. ...

Throughout his problem-solving attempts, EF continuously monitored his progress. During the 90 minutes of problem solving, EF monitored a total of 46 times (an average of one monitor event every 1.96 minutes). EF also made nine plans and on one occasion when EF identified that a strategy was not working, he switched strategies. Of the three instances of control, two were occasions when he quit. Finally, EF identified the isomorphic relationship between the two geometry problems but did not use information from the first problem to solve the second. EF did not identify the isomorphic relationship between the two statistics problems.

EF's evaluations of his problem-solving attempts were consistent with my characterizations. EF believed he was predominantly rational in his approaches to problem solving but admitted that sometimes he would solve problems using trial-and-error. For statistics problems, he would first answer the problem based on intuition and then try to solve the problem. In the follow-up interview, I asked EF why he had not approached *Geometry 1* in a rational way, given that he identified the similarities between the two geometry problems. EF admitted, "I knew I could not prove the theorem, Krista, so I, um, I knew I could show they were equal with ruler and then draw circle."

AL

In his fourth year of university, AL was profiled as highly rational and empirical (scores were 115 and 113, respectively). AL scored 13 on the prior knowledge test and,

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based on his overall self-efficacy score of 5.44, I perceived AL to be somewhat confident he could solve all six problems. Of the six problems, AL successfully completed three. On one problem, AL quit after working it for over 23 minutes. He explained that when he was at an impasse he would typically stop and return to the problem at a later time. Consequently, for this problem, a justification could not be provided. All six of AL's problem-solving attempts and all five of the justifications provided were coded as predominantly rational. Inconsistent with his epistemic profile, AL was characterized as predominantly rational in his approaches to problem solving.

Theorems, proofs, and properties were used to solve the problems and justify answers. An example of the rational nature of his problem-solving attempts is provided in the excerpt from *A Little Algebra*.

Um, what's standing out right now is, uh, when you're independence of, uh, of polynomials. But if you have the sum, if you have some polynomial ANX to the N is equal to some other polynomial, BNX, so some, BNX to the N, then you have to have that AN equals BN by linear independence. If it's true for, if it's true for all X. ... So by linear independence, um, that doesn't work. ... Okay, so I guess we can try contradiction because I pose that the left hand side of the implication is true and yet the right hand side somehow manages to get lost. So without loss of generality, let's say, let's assume the left hand side and also say A greater than B equals C equals D. So let's just say one of the variables is not equal to the other three. ...

During his problem-solving attempts, AL occasionally monitored his progress. In total, during the 56 minutes of problem solving, AL monitored 15 times (an average of

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one monitor event every 3.73 minutes). On each occasion that AL revealed a particular strategy was not going according to plan, he changed strategies. This occurred a total of six times. In total, he made 11 plans. AL also identified the similarities between the isomorphic problems in both problem-solving sessions and used the information from one problem to solve the other.

AL's assessments of his problem-solving attempts were consistent with mine. When asked how he typically approaches problems, AL responded that he was "definitely rational." When I revealed to AL that he was profiled as high on both rationalism and empiricism, AL considered this and then said:

> You know, I would say that's pretty accurate. I mean, when it comes to mathematics, I am definitely rational. We have to be. We've been trained for so long to think like that. Whenever we do assignments or exams, we have to be logical in our thinking, or, at least, we have to support our answers in a logical way – I guess what you would call rational. Arguments have to be sound otherwise our answers are not acceptable. I'm not saying that's the only way to do problems, no, but we always have to support our work with a logical argument, depending, of course, on the nature of the material. Take geometry, for example, I would love to take a course that does construction because I had a lot of fun with it in elementary school. It's the, it's the noblest of math, really it's –
>
> Krista: Yeah.
>
> AL: The truth, the noblest.
>
> Krista: Not only is the logic there, [AL:] you can actually see it.
>
> AL: Exactly. And it doesn't, I mean yeah there are numerical values, like the

lengths of things, but you're never actually working directly with numbers

[*Krista:* With numbers] which is sort of, you know, it's dirty, it's dirty work – but

not only can you see it empirically, combined with the logic, it's the shits. But, in

life in general, I would agree that I am a combination of both.

When asked whether he thought he became more rational over the course of his

undergraduate career, AL admitted that he believed that was true. He noted that his

problem solving techniques improved over the course of his undergraduate education and

that he learned to be more logical in his thinking and how he justified his answers.

BC

As a first-year university student, BC was profiled as moderate on rationalism and

empiricism (scores were 97 and 82, respectively). On the prior knowledge test, BC scored

8 out of 15. Based on her overall self-efficacy score of 2.89, I considered BC to be

somewhat unconfident in being able to successfully solve the problems. BC successfully

completed two, all of which she spent less than 10 minutes to complete. Three of her

problem-solving attempts and four of her justifications were coded as predominantly

rational, two of her attempts and justifications were coded as predominantly empirical,

and one problem-solving attempt was coded as both rational and empirical.

Because of the variety of approaches she used to solve the problems, it was

difficult to characterize BC's overall approach. After discussing the attempts with her, I

gained a better understanding of why she approached certain problems empirically. Given

her low prior knowledge of theorems that could be used to solve the geometry problems

(the two problems she solved empirically), I characterized her as more rational than

empirical. As Schoenfeld (1985) noted, if a person does not have the prior knowledge of

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theorems and proofs that can be used to solve the problems, that person may be more likely to approach a problem empirically or use other theorems that are not related to the problem as given. Consequently, inconsistent with her epistemic profile, BC was characterized as predominantly rational in her approaches to problem solving.

To illustrate her lack of prior knowledge and the empirical nature of her problem-solving attempt at *Geometry 2*, the episode is presented.

So, this is a circle which is tangent to E F G, so, it touches the segment E F and F G and then, and then so. Then this circle touches segment C F and G F and the line segment C F and line segments C F cuts the circle in half that is why it is the, it bisects the angle E F G, but is that a good proof? So, the circle inside the triangle E F G and then the circle is tangent to E F and G F. So, a line that passes through the circle C is also a bisector of EFG.

For the three problems coded as rational, BC identified the givens in each problem and assessed which theorem was needed to solve the problem. For *Rolling the Dice*, the problem coded as both rational and empirical, BC solved the first component of the problem using the binomial expansion. For the second component of the problem, however, BC proceeded to determine the probability by counting the number of ways the dice could be thrown. An excerpt from the attempt illustrates this shift in her approach.

So, at least one dot, so, it's either an independent event or like the binomial expansion. So at least one dot, at least one dot in 4 throws. So it's this binomial thing. ... So, the possibility, the probability of getting a pair in 24 throws with 2 dice is 1 over 7 times, 1over, no 1 over 6 times 1 over 6 equals 1 over 36. So, for example, you have 1 dice with one 1, one 2, one 3, one 4, one 5, one 6, two 1s,

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two 2s, two 3s, two 4s, two 5s, two 6s, 3 ones. Oops. One 1, one 2, one 3, one 4, one 5, one 6. Two 1, two 2, two 3, two 4, two 5, two 6. You have 3 1, 3 2, 3 3, 3 4, 3 5, 3 6, we have 4 4, 4 5, 4 6, 5 5, 5 6, and 6 6. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21. So, you have your p is 1/21. (This was not a correct method.)

During her problem-solving attempts, BC occasionally monitored her progress. In total, over the course of 35.75 minutes of problem solving, BC monitored 8 times (an average of one monitor event every 4.47 minutes). Only two plans were made and one change in strategy occurred. BC did not identify the similarities between the problems and recognized them only when they were explicitly described.

BC's characterizations of her problem-solving attempts were consistent with mine. When asked how she typically approaches problems, she revealed that in the context of a course she could generally identify what theorem to apply to solve a problem. She believed that made her more rational. Out of context, however, she explained that it was much more difficult to solve problems. She described her difficulty in recalling theorems and, when she did not know how to approach a problem or could not recall a theorem, she relied on trial-and-error. In those situations, she revealed that she was more empirical.

JS

JS, a first-year undergraduate student, was profiled as moderate on rationalism and empiricism (scores were 101 and 85, respectively). JS's score on the prior knowledge test was 14 and her overall self-efficacy score was 4.67. I considered JS to be somewhat confident in being able to solve the problems. JS successfully completed three problems,

each of which were solved in less than 5 minutes. For one problem, *A Little Algebra*, after 12 minutes of working the problem JS decided to quit. Consequently, only five justifications were provided. Five of her problem-solving attempts were coded as predominantly rational and one was coded as predominantly empirical. All five of her justifications were coded as rational.

Like other students' rational attempts, JS identified the given properties in each problem and selected theorems that could be used to solve the problem. Similarly, theorems and rational argumentation were used to justify her solutions. An excerpt from the *Multiple Choice Exam* problem is provided as an illustration of a rational argument she used to justify her solution.

> Well if she guessed each, each of the questions, the probability of getting eight of
> them correct is 1.966%. I am not too sure if this is really true because I don't
> really believe her because, because her, because of the probability of getting 8 out
> of 16 right is pretty small.

Throughout her problem attempts, JS continuously monitored her progress and checked her calculations. Over the 30.5 minutes of problem solving, JS monitored a total of 20 times (an average of one monitor event every 1.53 minutes). Nine plans were made and, when she was at an impasse, she changed her strategy a total of four times. Finally, JS did not detect the similarities between the isomorphic problems but did recognize the similarities once they were explicitly stated.

JS's characterizations of her problem-solving attempts were consistent with mine. Overall, JS viewed herself as predominantly rational in her approaches to problem solving. In the follow-up interview, I asked JS why she had approached *Geometry 1*

empirically and *Geometry 2* rationally. She explained that she was applying theorems to solve the problem. She knew that the distances from the point of intersection to the tangent points were equal by some theorem and then subsequently measured the distance from one point to derive the second point. Each element of the problem was similarly derived. She further revealed that the names of the theorems never came to mind but that she did apply them.

AM

AM, a second-year undergraduate student, was profiled as moderate on rationalism and empiricism (scores were 96 and 94, respectively). On the prior knowledge test, AM's score was 13 and, based on his overall self-efficacy score of 4.78, I considered him to be somewhat self-confident he could successfully solve all six problems. AM correctly solved three of the problems. Of the six problems, five were solved within 10 minutes each. Five of his problem-solving attempts were coded as predominantly rational and one was coded as predominantly empirical. Since AM quit working one of the problems after a period of 25 minutes, he did not provide a justification for that attempt. For the other five attempts, two justifications were coded as rational, one was coded as empirical, and the other two were coded as irrational. Specifically, for two of the attempts, although the probabilities that AM calculated were extremely small, he based his final answers and justifications on information that contradicted the probabilities he calculated and on information not relevant to the problems. Inconsistent with his epistemic profile, AM was profiled as predominantly rational in his approaches to problem solving.

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To illustrate the irrational nature of his answers and justifications for the *Multiple Choice Exam* and *Heart Transplant* problems, excerpts are presented.

...equals .0000015. That's the chance of getting 8 right. Seems kinda low, but. Um. If I did this right, which I guess I did, it seems a lot lower than it should but, I would think that she probably guessed but kinda educated guesses on some of them, most likely. (Completed.)

Uh, interview on the 16 people was independent with the transplants so the group was random. Unless you did something, unless before that the selection was chosen. But, or something but that is not even said in the question. Yah. I guess it's just random because they interviewed them and the fact that it was something before that and you interviewed people and then chose and that's how you made the selection after knowing what their eating habits were.

Occasionally, during his problem attempts, AM monitored his progress. Over the period of 58 minutes of problem solving, AM monitored a total of 15 times (an average of one monitor event every 3.87 minutes). On seven of the occasions that AM did monitor, he revealed that his problem attempt was not working but decided to continue with his chosen course of action. For three of those decisions, however, AM eventually decided to quit and made a new plan. In total, 11 plans were made.

AM's evaluations of his problem-solving attempts were consistent with mine. When AM had the opportunity to assess his justifications, for the two irrational justifications, AM immediately recognized them as irrational. He acknowledged,

Now what was I thinking here? That wasn't too good, was it? I guess it didn't help that I had only four hours of sleep the night before. Can that be my excuse?

That would never pass on an exam. (Begins laughing.) My answers totally

contradict the probability of these events occurring.

Overall, AM believed himself to be predominantly rational is his approaches to problem

solving. He felt, however, that he still had much to learn and believed that with

experience, he would improve his ability to present arguments in logical form. He also

admitted he needed to check his work more frequently, particularly to catch errors in

logic such as the ones he missed on the probability problems.

AF

AF, a third-year university student, was profiled as moderate on rationalism and

empiricism (scores were 94 and 94, respectively). On the prior knowledge test, AF scored

13 out of 15. Based on his overall self-efficacy score of 6.00, I considered AF to be quite

confident that he could solve the problems. Of the six problems, AF successfully

completed three and spent more than 13 minutes each to complete five of the six

problems. Four of AF's attempts and justifications were coded as predominantly rational

and two were coded as predominantly empirical. Inconsistent with his epistemic profile,

AF was characterized as predominantly rational in his approaches to problem solving.

For one of the attempts coded as empirical, once AF had completed the problem

and was justifying his solution, he admitted his attempt was informal and that he should

have proved his construction more rigorously. An excerpt from *Geometry 2* illustrates

this.

I am assuming that this is, that this line FA bisects the triangle going downwards,

and it is probably. Oh geeze, what should I do with this? Oh, because um. It

seems fairly obvious. I should probably prove this rigorously. This is pretty

informal. Okay. (AF takes the ruler and begins measuring his construction.) So we've got two equal line segments and two equal angles, um, and I think that's probably. Actually, the line, in one of these cases, yeah, you could use good old Pythagoras theorem or law of sines, law of cosines. But it is quite obvious that the triangle FAG and FAE are similar triangles and so, I think that it has been shown the angle EFA and GFA are both going to be the same. So, CF does seem to bisect the angle EFG. Well, actually you can just see that from the line segments.

During his problem-solving attempts, AF occasionally monitored progress and checked his work. Over the 99.5 minutes of problem solving, AF monitored a total of 37 times (an average rate of one monitor event every 2.69 minutes). On seven occasions, AF judged that his approach was not useful. Based on his judgment, he changed his strategy a total of four times. AF also made a total of 12 plans over the course of his problem-solving episodes. Finally, AF did not identify the isomorphic problems in either session. When asked whether he noticed anything in particular about the problems, AF recognized the similarities without being prompted.

AF's characterizations of his problem-solving attempts were similar to mine with the exception of one of the episodes. For the *Multiple Choice Exam* problem, which I had coded as predominantly rational, AF initially felt his approach was not rational or empirical. He argued that since he first guessed the answer to the problem and then proceeded to calculate the probability of the event, he felt his attempt was more intuitive. He believed he had solved the problem using more intuition than reason. He chose the binomial formula because he believed it would give him the answer he intuitively guessed. When asked why he specifically chose the binomial formula, he revealed that

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the properties of the problem corresponded with the elements necessary to apply the binomial distribution. After contemplating this, AF changed his characterization to both intuitive and rational. Overall, AF believed that he was predominantly rational in his approaches to problem solving but that this was something he had learned over the course of his education.

PB

As a fourth-year undergraduate student, PB was profiled as moderate on rationalism and empiricism (scores were 94 and 84, respectively). His score on the prior knowledge test was nine and his overall self-efficacy score was 5.67. I considered PB to be quite confident in being able to successfully complete all six problems. PB correctly solved two of the problems, each of which he worked for less than 14 minutes. For one problem, after 13.75 minutes, PB quit. For *Geometry 2*, PB was unable to recall a particular property of triangles and, after an attempt to reconstruct the proof, PB believed he would not be able to complete the problem. He revealed that in a typical context, he would set the problem aside and return to it at a later point in time. Consequently, only five justifications could be coded. Five of PB's problem attempts and justifications were coded as predominantly rational and one was coded as predominantly empirical. Inconsistent with his epistemic profile, PB was profiled as predominantly rational in his approaches to problem solving.

For the problems coded as rational, PB used theorems and proofs to solve the problems and justify his solutions. For the second session, PB immediately identified the properties of the problems, referred back to the chapter to ensure the properties he identified satisfied specific theorems, and then proceeded to solve the problems. For the

Heart Transplant problem, PB made one minor calculation error resulting in an incorrect answer. For the *Multiple Choice* problem, PB saw the similarities between the two problems but decided he would recalculate the probability of the event to double-check his work on the previous problem. PB discovered the error in his calculation from the previous problem but made another calculation error resulting in another incorrect answer.

Throughout his problem-solving attempts, PB monitored his progress and checked his work. Over a period of 56.75 minutes of problem solving, PB monitored a total of 25 times (an average of one monitor event every 2.27 minutes). In total, 14 plans were made. An excerpt from *A Little Algebra* illustrates how PB made plans to solve problems.

So I'm thinking about substitutions now and whether I can eliminate things, eliminate possibilities. Sometimes it's good to work from the answer back so like, assume the answer is true and then show it's impossible otherwise. In other words like, let A not be B and then see what happens. So maybe I'll try that. ...

PB's evaluations of his problem-solving attempts were consistent with mine. PB also saw the similarities between the two geometry problems. Despite having stated the similarities between two problems during one of the attempts, he proceeded to approach the second problem empirically when the first was approached rationally. When asked why he had proceeded to solve *Geometry 1* empirically, PB admitted that since he was at an impasse with the other problem, he believed he would not be able to similarly solve the second problem. Consequently, he deemed it necessary to attempt the second problem using an empirical approach. He admitted that he believed it occasionally helped him to recall theoretical information. He acknowledged, "It actually helped me to identify the

missing piece of the puzzle but I didn't think I could go back to the other question so I just left it."

Predominantly Empirical

KF

KF, a first-year university student, was profiled as moderate on empiricism and low on rationalism (scores were 79 and 53, respectively). On the prior knowledge test, KF scored 9 out of 15 and, based on his overall self-efficacy score of 3.67, I considered KF to be uncertain of his confidence in being able to correctly solve the problems. KF did not successfully solve any of the problems and spent less than 7 minutes to complete each problem. KF monitored his attempts a total of two times over the period of 23.95 minutes (an average of one monitor event every 11.98 minutes). In total, only one plan was made. Moreover, KF did not identify the similarities between the isomorphic problems until they were explicitly stated. Five of KF's problem-solving attempts were coded as predominantly empirical and one was coded as predominantly rational. For his justifications, two were coded as rational, one was coded as empirical, and the other three were coded as illogical. Consistent with his epistemic profile, KF was characterized as predominantly empirical in his approaches to problem solving.

For the problems coded as empirical, KF's attempts were similar. KF began each problem by performing some operation, such as multiplication, on the values or variables provided in the problem. On three occasions, KF explained that he could no longer perform any other operations; consequently, he provided an answer. For another problem, KF began adding new variables and assigned values to those variables.

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Like AM, KF's answer to the *Multiple Choice Exam* problem contradicted the probability he had computed. KF acknowledged that the probability he computed, 0.000001, was an extremely low probability and it was highly unlikely that a person would correctly guess 8 of the 16 questions. KF argued, however, that the person "got really lucky" and decided that he believed she correctly guessed all eight questions. For the *Heart Transplant* problem, to make his decision, KF used information that was not relevant to the problem. Although he had computed the probability of the event occurring, he ignored that information and based his answer on information not related to the problem. A passage from that problem is provided to illustrate the nature of KF's answer.

> I think random can't be related to the type of people, the type of patient you select. Also, if they are patient, they might have problems in other areas in the body so which could affect the result of the study. So, I think selection of the group is random but is, the randomness doesn't really depend on whether there is 8 patient or 16 patient. It depends on the type of patient and whether the patient has disease in other area of them.

KF's evaluations of his problem-solving attempts for the first problem-solving session were consistent with mine. For the second session, however, only one of his characterizations was similar. Specifically, KF believed two of his attempts were rational whereas I had characterized them as empirical. When asked to justify his classification, KF's reason was that he had used information from the problem to compute the probabilities. He believed his application of various operations to values given in the problems was a rational way to solve them. To better understand KF's attempts, I asked

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KF to explain to me why he had multiplied certain values, added others, and added new variables. I attempted to have him explain his logic. KF's explanations were reiterations of the steps he had taken to solve the problems. I did not succeed in understanding KF's attempts. When asked how he typically approaches problem, KF believed he was a mix of both empirical and rational but he admitted he found it very difficult to assess.

PC

As a first-year university student, PC was profiled as high on empiricism and moderate on rationalism (scores were 109 and 99, respectively). On the prior knowledge test, PC scored 11 out of 15 and, based on his overall self-efficacy score of 5.33, I considered him to be somewhat confident he could successfully solve the problems. PC correctly solved one. For two of problems, after spending over 10 minutes on each problem, PC quit. The two problems were *A Little Algebra* and *Rolling the Dice*. Consequently, only four justifications were provided. Five of PC's problem attempts were coded as predominantly empirical and one was coded as predominantly rational. For his justifications, one was coded as empirical, one was coded as illogical, and two were coded as rational. Overall, consistent with his epistemic profile, PC was profiled as predominantly empirical in his approaches to problem solving.

For the problem coded as rational, although PC could not recall the names of the theorems, he used the properties of those theorems to solve *Geometry 2*. For the other problems, PC's attempts were similar to KF's. Specifically, like KF, PC applied various operations to the numbers given in the problems to find solutions. PC continued to multiple, divide, or subtract numbers until a satisfactory solution resulted. If PC was

satisfied with the solution, he provided his answer; otherwise, PC quit. An excerpt from *Rolling the Dice* illustrates this.

> Okay, 5 and 6 times 5 and 6 plus the probability of one dot, one throw, at least one throw with one dot. So, it will be five in six and times 1 over 6 plus... Okay. Five in six times. Okay, which will be five in six times 24 roles times the other role and that will give me the probability of no dots or one dot in 0 throws. Plus 24 again, 24 roles times 5 over 6. .... So, you get zero double dots it would be zero double dot minus that so the probability of zero double dots would be, in 24 throws. So, it would be the probability, now the probability of getting this number times the probability of getting this number times 24 which would give you... which is wrong. One double dot in 24 throws, which is five or six times one over six which is five over 36 minus 5 over 36 would give you at least one double dot, which is 31 over 36, Okay. Wrong. Four throws, four throws, at least one dot in for throws I cannot figure this one out.

PC's justification that was coded as illogical was also similar to KF's. Although PC calculated the probability of a specific event occurring, he ignored that information when making his decision. PC acknowledged his answer was not consistent with the data, as illustrated below.

> Therefore we will say it is random even though it is not based on this probability because if, well they said that if the proportion of patients who do not experience any difficulty after a heart transplant is 0.75 so 75 percent of the people do not experience difficulties but that leaves 25 percent of the people with difficulties. They feel, and they experience difficulties after a heart transplant and when you

have 16 of them, 8 of them said they did experience difficulties and that is 50

percent, since you are basically asking, well I think it is random because, well it's

not consistent with the data from, it is not consistent with the data from before

when, where you said, where the question said 25 percent experience, would

experience difficulties rather than 50 percent of the people experience difficulties.

I don't make sense.

During his problem-solving attempts, PC continuously monitored his work by

checking his answers and questioning his approach. Over the period of 35 minutes of

problem solving, PC monitored a total of 14 times (an average of one monitor event

every 2.5 minutes). For four of the problems, PC made at least one plan but in instances

when PC assessed that plans were not going accordingly, he did not switch strategies. A

total of eight plans were made. Finally, PC did not identify the similarities between the

isomorphic problems. When asked whether he saw any similarities, PC was able to

identify them.

PC's characterizations of his problem-solving attempts were consistent with mine.

When asked how he typically approaches problems, PC admitted that he was probably

more empirical than rational. He acknowledged that for many problems he would try

various operations or formulas until an answer made sense. He revealed that he was

struggling in his mathematics courses and found it difficult to understand the theorems

and how to apply them. He explained that he had succeeded in high school mathematics

and had not found it challenging since he was able to memorize formulas and how to

solve specific types of problems. He further admitted that he put little effort into solving

problems and was not one to retry a problem even when he believed his answer was

illogical. At the end of his interview, PC confessed that he was likely going to drop his mathematics courses and pursue his undergraduate degree in Biology.

Summary

Seventeen undergraduate university mathematics students participated in the second component of this study. Using the PEP, 5 were profiled as predominantly rational, 10 were profiled as both rational and empirical, and 2 were profiled as predominantly empirical in their approaches to knowing. Students were given six problems to solve. Each problem attempt was coded as predominantly rational, predominantly empirical, or a combination of both. Based on their approaches and justifications for their solutions, participants were profiled as predominantly rational, both rational and empirical, or predominantly empirical in their approaches to problem solving. Consistent with their profiles, all 5 participants profiled as predominantly rational in their approaches to knowing were profiled as predominantly rational in their approaches to problem solving. Inconsistent with their profiles on the PEP, all 10 participants profiled as both rational and empirical in their approaches to knowing were profiled as predominantly rational in their approaches to problem solving. Finally, consistent with their profiles on the PEP, both participants who were profiled as predominantly empirical in their approaches to knowing were profiled as predominantly empirical in their approaches to problem solving.

Of the 5 participants profiled as predominantly rational in their approaches to knowing, 60% identified both isomorphic problem sets and 20% identified one isomorphic problem set, for a total of 80% that identified at least one isomorphic problem set. On average, these students monitored their problem attempts every 1.96 minutes. For

the 10 participants profiled as both rational and empirical in their approaches to knowing, 20% identified both sets of isomorphic problems and 30% identified one of the isomorphic sets, for a total of 50% that identified at least one isomorphic problem set. On average, these students monitored their problem attempts every 4.13 minutes. Finally, the two students profiled as predominantly empirical in their approaches to knowing did not identify either of the two isomorphic problem sets. Their average monitoring behavior was one monitor every 7.24 minutes.

Interpretations of the results from both components of the study are discussed in the following chapter. The discussion begins with a brief summary of the purpose of the study. Results of both components of the study are combined for an overall interpretation and discussion of this research.

CHAPTER 5

DISCUSSION

For this study, relations were examined between individuals' epistemic profiles, self-reported metacognitive strategy use, and actual metacognitive strategy use as individuals engaged in problem solving. Relations were also examined between epistemic profiles and the types of mathematics argumentation individuals used to solve problems and justify solutions. Finally, differences in epistemic profiles and epistemic beliefs were assessed across the four years of undergraduate school. For the first component of the study, it was predicted that individuals profiled as predominantly rational in their approaches to knowing would self-report using more metacognitive strategies than individuals profiled as both rational and empirical and predominantly empirical in their approaches to knowing. Second, it was anticipated that individuals profiled as high on rationalism would report higher need for cognition scores than individuals in the other two groups. Finally, it was hypothesized that upper-year university students would have high rationalism scores and lower epistemic beliefs scores than lower-year university students.

For the second component of the study, it was expected that when problem solving, individuals profiled as predominantly rational in their approaches to knowing would engage in more planning, metacognitive monitoring and metacognitive control than individuals in the other two groups. Second, it was hypothesized that individuals profiled as predominantly rational in their approaches to knowing would be more rational in their approaches to problem solving; they would use more rational argumentation, such

as proofs and theorems, to solve problems and justify solutions than individuals in the other two groups. Individuals profiled as predominantly empirical in their approaches to knowing were predicted to engage in more trial-and-error exploration of the problems, to test hypotheses serially until a solution was found, and rely more on perceptual features to solve the problems. For individuals profiled as both rational and empirical in their approaches to knowing, it was anticipated they would be both rational and empirical in their approaches to problem solving. Finally, since individuals profiled as predominantly rational in their approaches to knowing were expected to engage in more metacognitive monitoring, it was hypothesized these individuals would more likely identify relations between the isomorphic problems when compared to individuals in the other two groups.

Results for both components of the study are jointly interpreted to present an overall analysis of the results. Specifically, results are divided into sections according to each question examined. First, a discussion of relations between epistemic profiles, critical thinking, and metacognition is presented followed by a discussion of differences in epistemic profiles and beliefs. Finally, relations between epistemic profiles and approaches to problem solving are evaluated. The chapter ends with a discussion of limitations of this research.

Relations Between Epistemic Profiles, Critical Thinking, and Metacognition

Royce (1978) proposed that individuals profiled as predominantly rational in their approaches to knowing acquire knowledge through logic and reason. Through critical thinking, ideas are evaluated for logical consistency and, if judged to be logical, are

synthesized with prior knowledge. As Wardell and Royce (1975) hypothesized, when learning, these individuals preferentially rely on critical thinking and reasoning. Similarly, Schoenfeld (1983) hypothesized that individuals with a rationalist belief system use deductive logic and reasoning when problem solving. These individuals are theorized to plan how to approach problems, continuously assess whether progress is being made, and alter plans when goals are not being achieved.

In contrast to a rationalist epistemic style, Royce (1978) theorized that individuals profiled as empirical in their approaches to knowing acquire knowledge through perceptual experience. Information processed through sensory inputs is evaluated for reliability and validity. If information is consistent with prior knowledge, it is accepted as true. Wardell and Royce (1975) proposed that, when learning, these individuals preferentially rely on perceptual information and memorization of facts. Schoenfeld (1983) also theorized that individuals with an empiricist belief system focus on perceptual features of problems rather than mathematical argumentation, explore problems in a trial-and-error fashion, and serially test hypotheses until a satisfactory solution is found. Accordingly, Schoenfeld proposed that these individuals engage in very little planning, monitoring, and control when problem solving.

The present study provides some support for these hypotheses. First, to test the concurrent validity of the rationalism scale, both need for cognition and critical thinking were measured. Consistent with Royce's (1978) hypothesis that rationalism is associated with critical thinking, a positive correlation was found between rationalism and need for cognition. These results are also consistent with Leary et al.'s (1986) findings that need for cognition was positively related to rational beliefs and judgments. Moreover, a

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positive correlation was found between rationalism and critical thinking as measured by the MSLQ.

In contrast, empiricism was not related to need for cognition but was positively related to critical thinking. The relationship between empiricism and critical thinking was, however, weaker than the relationship between rationalism and critical thinking. The positive correlation between empiricism and critical thinking supports one of Royce's (1978) hypotheses. He proposed that individuals do not rely solely on the cognitive processes associated with their predominant profile; other processes may be used when acquiring knowledge but to a lesser extent. This result supports his hypothesis that individuals profiled as predominantly empirical may also critically evaluate information but rely less on critical thinking than individuals profiled as predominantly rational.

To further test this hypothesis, differences in need for cognition were examined. As predicted, individuals profiled as predominantly rational had the highest need for cognition compared to the other two groups. More specifically, individuals profiled as predominantly rational had a higher need for cognition than individuals profiled as both rational and empirical. Moreover, individuals profiled as both rational and empirical had a higher need for cognition than individuals profiled as predominantly empirical. These results support Royce's (1978) hypothesis that individuals profiled as predominantly rational in their approaches to knowing preferentially engage in critical thinking in comparison to individuals with other epistemic profiles.

Based on differences in critical thinking, it was further predicted that individuals profiled as predominantly rational in their approaches to knowing would engage in more regulation of cognition. To examine whether individuals profiled as predominantly

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rational engaged in more metacognitive self-regulation than individuals in the other three groups, average scores between groups were compared. Although differences in means were found between the groups, this hypothesis was not supported statistically. The only difference found was between individuals profiled as high on both rationalism and empiricism and individuals profiled as predominantly empirical. Moreover, both rationalism and empiricism were positively related to self-reported metacognitive self-regulation.

Taken together, these results suggest that individuals profiled as predominantly rational in their approaches to knowing have a higher need for cognition but do not engage in more regulation of cognition compared to individuals with different profiles. Moreover, since positive correlations were found between rationalism and self-reported metacognitive self-regulation and between empiricism and self-reported metacognitive self-regulation, these results challenge Schoenfeld's (1983) theory that individuals who hold empiricist belief systems engage in little regulation of cognition.

One could argue, however, that these results are not accurate reflections of how students metacognitively self-regulate learning. Specifically, self-report measures have been criticized on a number of technical and methodological issues (Winne et al., 2002a) and researchers have found that self-reports are not intrinsically accurate measures of how students behave as they engage in learning and problem solving (e.g., Winne et al., 2002b). The second component of this study addressed this issue and compared differences in planning, metacognitive monitoring, and metacognitive control between participants profiled as predominantly rational, both rational and empirical, and predominantly empirical.

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Results from the second component of the study revealed that individuals profiled as predominantly rational in their approaches to knowing engaged in more planning, monitoring, and control than the other two groups. Moreover, individuals profiled as predominantly empirical had the lowest occurrence of these behaviors. Although these results support the hypothesis that individuals profiled as predominantly rational in their approaches to knowing engage in more regulation of cognition, they must be interpreted with caution. First, a small number of students participated in the second component of the study. Of most concern, only two of the twelve students who were profiled as predominantly empirical participated in the problem solving sessions. This limited my ability to examine relations between epistemic profiles and regulation of cognition and to assess differences across the groups. Second, within each group, there were large individual differences in planning, monitoring, and control. Thus, results from the second component of the study provide only weak evidence to support this hypothesis. Taken as a whole, both components of the study provide conflicting evidence for relations between epistemic profiles and regulation of cognition. Given the lack of evidence from the first component of the study and weak evidence from the second component, additional studies with a larger sample of participants are needed to examine for the existence and nature of relations among epistemic profiles and regulation of cognition.

Furthermore, in evaluating students' problem-solving attempts, I identified a pattern in students' regulation of cognition that opposes Schoenfeld's (1983) theory. Students' frequency of regulating cognition was similar from one problem attempt to the next. That is, if individuals did not frequently check their work during one problem attempt, they typically did not check their work on a subsequent attempt. Alternatively,

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individuals who frequently planned and monitored one problem attempt continued to engage in these behaviors throughout other problem attempts. This pattern was found across all epistemic profiles.

In the broader context of self-regulation, self-regulated learning theorists have defined self-regulation as both an aptitude (Snow, 1996) and an event (Winne & Hadwin, 1998). As an aptitude, theorists propose that self-regulation is more trait-like rather than state-like. In contrast, viewed as an event, theorists propose that how individuals regulate learning and problem solving depends on the specific task and the manner in which it unfolds with engagement. Although individuals selected different strategies and approaches to solve the problems, there was some consistency in their regulation of cognition. One could argue, however, that individuals defined each of the tasks similarly; that is, they defined them as mathematics problems.

Whether one adopts a trait or state view of self-regulation, the pattern I identified was that, regardless of whether students solved one problem empirically and another problem rationally, their frequency of regulation of cognition was stable from one problem to the next. More specifically, when students approached problems empirically, they continued to engage in planning, monitoring, and control. This finding opposes Schoenfeld's (1983) hypothesis that students who solve problems empirically, by testing hypotheses in a serial manner, by exploring a problem space in a trial-and-error fashion, or by focusing on perceptual features of a problem, engage in little planning, monitoring, and control. I interpret the frequency of students' regulation of cognition as not being indicative of an empirical or rational approach to problem solving. Rather, other factors, such as their self-efficacy, may have influenced the frequency of their self-regulatory

behaviors. This was a second pattern that I identified in my evaluation of students' problem-solving attempts.

Prior research has found that students' motivational beliefs about mathematics affect their use of learning strategies such as time management, study strategies, self-monitoring, and self-evaluation (e.g., Hanlon & Schneider, 1999). Of particular interest, higher levels of self-efficacy are associated with higher levels of self-regulation, such as self-monitoring (Zimmerman, 2000). Bandura's (1997) social cognitive theory predicts these positive relations between self-efficacy and self-monitoring. Consistent with these hypotheses and the research that supports them, a positive relationship was found between self-efficacy for learning and performance and metacognitive self-regulation in the first component of this study. Moreover, for the second component, I found that students who had higher overall self-efficacy scores engaged in more regulation of cognition than individuals who were not as efficacious.

Overall, given that no differences were found in frequency of self-reported metacognitive self-regulation between individuals profiled as predominantly rational and predominantly empirical, the positive relations between the epistemic profiles and self-reported metacognitive self-regulation, weak evidence from the second component of the study, and the relations found between regulation of cognition and self-efficacy, I argue there is not sufficient evidence to suggest that one's approaches to knowing or approaches to problem solving influence the extent to which one regulates problem solving. Evidence from both components of the study suggests that other factors, such as motivational beliefs or variables not directly examined in this study, may be more predictive of students' regulation of cognition.

Differences in Epistemic Profiles and Beliefs

Royce (1978) proposed that specialized forms of knowledge are dependent on the three types of epistemologies. As individuals progress through formal educational experiences, they are gradually socialized into the epistemic patterns of their specialized discipline. Thus, individuals' epistemic profiles become comparable to that of their discipline. Similarly, educational psychologists have conjectured that individuals' beliefs about knowledge (e.g., Belenky et al., 1986; Kitchener & King, 1981; Perry, 1970; Schommer, 1990) and beliefs about learning (e.g., Schommer, 1990) develop over the course of their educational experiences. For example, first-year university students may initially believe that knowledge is dualistic but, by their fourth year, may believe that there are multiple possibilities for knowledge (Perry, 1970).

Results from my research support these hypotheses. Consistent with Royce's hypothesis and previous research (e.g., Kearsley, as cited in Royce & Mos, 1980; Royce and Mos, 1980; Smith et al., 1967), undergraduate mathematics students had a higher average rationalism score than their scores for empiricism and metaphorism. This implies that, in general, undergraduate mathematics students are more rational in their approaches to knowing. To examine whether undergraduate university students become more rational in their approaches to knowing as they progress through formal mathematics education, differences in means were compared between lower-year and upper-year university students. Although upper-year university students had a higher average rationalism score, this difference was not statistically detectable. Perhaps if graduate-level students and more second- through fourth-year students were sampled, differences may have been observed. Moreover, this study used a cross-sectional design. Longitudinal research is

needed to examine whether mathematics students become more rational as they proceed through higher levels of education, particularly graduate school.

In contrast, differences in epistemic beliefs were supported for two of the five dimensions examined. As in Schommer's (1993a) study, upper-year university students more strongly disagreed that knowledge is handed down by some authority figure and more strongly disagreed that knowledge consists of isolated bits and pieces of information. No differences were found, however, among beliefs about the certainty of knowledge, the speed of knowledge acquisition, or the control of knowledge acquisition. In general, the mathematics students sampled for this study disagreed that knowledge is certain and more strongly disagreed that learning should be quick. These results contrast with the typical beliefs reported in other samples of students (e.g., Fleener, 1996; Schoenfeld, 1988; Spangler, 1992a). Conversely, consistent with previous research, the sample of students in my study slightly agreed that the ability to learn is innate.

It is important to note, however, that for this study students' beliefs were measured using the Epistemic Beliefs Inventory (Schraw et al., 2002), which is designed to measure individuals' general beliefs about knowledge and not mathematics-specific beliefs. Had students' mathematics-specific beliefs been measured, responses may have differed. Specifically, research examining domain differences has predominantly found that students' beliefs in one domain are dissimilar to their beliefs in other domains. For example, Buehl, Alexander, and Murphy (2002) examined whether students held different beliefs across domains and also examined whether their beliefs about general knowledge were similar to their beliefs about domain-specific knowledge. In general, Buehl et al. (2002) found that students held domain-specific beliefs about knowledge.

They also found, however, a significant moderate relationship between domain-specific beliefs and domain-general beliefs, which they argued provides some evidence of domain-generality in undergraduate students' beliefs.

In my study, it is not possible to determine whether students contextualized items (e.g., thought about a specific domain) on the questionnaire as they responded or considered knowledge in general. Future studies are needed that examine the development of beliefs for domain-specific and domain-general knowledge. Moreover, longitudinal studies are required to examine more precisely whether students' epistemic beliefs and epistemic profiles change as a function of educational experiences.

Relations Between Epistemic Profiles and Approaches to Problem Solving

Royce (1978), Schommer (1990) and Schoenfeld (1983) theorized that one's epistemic profile, epistemic beliefs, and beliefs systems establish a psychological context for learning, and that context influences how one acquires knowledge and how one justifies whether information can be accepted as true. This study examined two specific epistemic styles, rationalism and empiricism. Royce theorized that rationalism depends on logical consistency and individuals who are predominantly rational in their approaches to knowing rely on critical thinking, conceptualizing, and a rational analysis and synthesis of ideas. Based on his theory, one could hypothesize that when rationalists work mathematics problems they focus on conceptual information rather than perceptual information to solve a problem. When a solution is achieved, the answer is accepted if it can be logically justified. Schoenfeld (1983) similarly theorized that individuals with a

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rationalist belief system use mathematical argumentation, such as proofs and theorems, as a form of discovery when working problems. When solutions are generated, argumentation is also used as a means of justification.

In contrast to a rationalist perspective, Royce (1978) proposed that empiricism depends on the extent to which perceptual information is valid and reliable, and individuals who are predominantly empirical in their approaches to knowing rely on sensory information. Using his theory, one could reason that when empiricists work mathematics problems they focus on perceptual information rather than conceptual information. Similarly, Schoenfeld (1983) proposed that when problem solving, empiricists focus on the perceptual salience of certain physical features of a problem. If perceptual features are not salient, they test hypotheses that can be most clearly perceived to solve a problem. If the first hypothesis tested does not produce a desired result, the next plausible hypothesis is attempted. Once an acceptable solution is achieved, the answer is verified by empirical means unless a rational justification is required (e.g., when a teacher requests that information).

The results of the second component of the study, to a certain extent, support these hypotheses but also challenge facets of Royce's (1978) and Schoenfeld's (1983) theories and the research that supports them. Moreover, the results of my study raise questions that should be addressed in future research. To guide the discussion of the evaluation of the problem-solving attempts, this section is divided into two subsections, *Epistemic Profiles and Approaches to Problem Solving*, and *Trends in Problem-Solving Attempts.*

Epistemic Profiles and Approaches to Problem Solving

All five students profiled as predominantly rational in their approaches to knowing were predominantly rational in their approaches to problem solving. Similarly, both students profiled as predominantly empirical in their approaches to knowing were predominantly empirical in their approaches to problem solving. Inconsistent with predictions, all ten students profiled as both rational and empirical in their approaches to knowing were predominantly rational in their approaches to problem solving. Students who were predominantly rational in their approaches to problem solving would more frequently use theorems and proofs to solve problems, identify the givens in the problems to select appropriate formulas, and would justify answers based on relevant information and rational argumentation. In contrast, students KF and PC more frequently focused on dominant perceptual features of problems, engaged in more trial-and-error exploration of the problem spaces, and continued to test hypotheses until a satisfactory solution was found. Moreover, when justifying solutions, KF and PC more frequently based their justifications on empirical evidence or on information not relevant to the problems.

In comparing students profiled as high versus moderate on both rationalism and empiricism, I did not identify any differences in the use or quality of argumentation or justifications as they solved problems. Furthermore, I did not distinguish differences in approaches to problem solving between students profiled as predominantly rational in their approaches to knowing and students profiled as both rational and empirical in their approaches to knowing.

I explain these results from two different perspectives. First, Royce (1978) proposed that an individual can be hierarchically profiled along three dimensions based

on scores on each dimension of the PEP. An individual's highest score represents his or her predominant epistemic and cognitive styles. Royce did not discuss differences in levels of profiles, such as highly rational or moderately rational, nor did he suggest that individuals could be profiled simultaneously as rational and empirical. For this study, however, I chose to profile individuals along three levels – high, moderate and low – and selected a range of scores that represented each of those levels. With the exception of two students, SQ and AF, all students profiled as both rational and empirical in their approaches to problem solving had higher rationalism scores. Consequently, one could argue that these students were predominantly rational in their approaches to knowing and, consistent with Royce's theory, were predominantly rational in their approaches to problem solving. Based on this line of reasoning, these results corroborate Royce's theory.

One concern with this argument is that for any variable that is measured, there will be some measurement error. Of the ten students profiled as both rational and empirical, two had equal scores and three had rationalism scores that were two to three points higher than their empiricism scores. Since students completed the PEP only once, test-retest reliability could not be measured. Consequently, one cannot assess whether students would be similarly profiled at another time. Moreover, one could argue that scores that differed by only a few points might not differ at all; they are within the confidence interval (calculated using the standard error of measurement reported in Chapter 3). Thus, consideration of individuals' absolute scores does not provide a sufficient rationale to explain why students profiled as both rational and empirical in their

approaches to knowing were predominantly rational in their approaches to problem solving.

Conversely, one could theorize that individuals may have more than one predominant epistemic style and cognitive style, which is how I chose to profile students for this study. In profiling students, however, I arbitrarily selected specific ranges within which they could be profiled. Other ranges or methods could have been selected to profile individuals, methods that are arguably as valid as the one I chose to use. If one allows for more than one predominant epistemic and cognitive style, how should individuals be profiled? This is not a question that can be answered easily by relying solely on a number system. Instead, individuals could be interviewed to gain a better understanding of how they interpret their approaches to knowing. For example, as AL revealed, he considered himself to be both rational and empirical in his approaches to knowing. When problem solving, he believed he was more rational, but for other facets of life, he believed he was more empirical. As he suggested, he had learned to become more rational through experience. SG and AM also believed they would learn to be more rational with experience.

This was corroborated by the distinct variations I discovered in the quality of upper- versus lower-year university students' problem-solving attempts. This pattern was not found for all students, however (e.g., both AC's and SQ's approaches were similar in quality to AL's, PB's, BR's, and EF's), but the trend I identified was that as students gained more experience in mathematics, they were more logical in their approaches. Thus, one plausible interpretation of the results is that since mathematics is considered to

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be a rational discipline (Royce, 1978), students learn to be rational in their approaches to problem solving.

This interpretation is consistent with an interactionist view that accounts for how students' beliefs and methods of problem solving develop. Scholars in mathematics education generally agree that the formal mathematics education students receive influences the development of their beliefs and approaches to problem solving in mathematics. Without excluding the importance of the general cultural environment and home environment, researchers have concentrated on sociomathematical norms (Yackel & Cobb, 1996) to account for how students develop specific mathematics beliefs and approaches to problem solving. This interactionist view assumes that cultural and social processes are integral to mathematical activity (Voigt, 1995). As Bauersfeld (1993) stated:

> Participating in the process of a mathematics classroom is participating in a culture of using mathematics, or better: a culture of mathematizing as a practice. The many skills, which an observer can identify and will take as the main performance of the culture, form the procedural surface only. These are the bricks for the building, but the design for the house for mathematizing is processed on another level. As it is with cultures, the core of what is learned through participation is *when* to do what and *how* to do it. (p. 4)

From this view, the development of individuals' analytic and logical processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematics meanings. Individuals, therefore, are believed to develop personal understandings and beliefs and approaches to mathematics as they participate in

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negotiating classroom norms specific to mathematics (Yackel & Cobb, 1996).

Accordingly, future research I plan will extend this research by situating it in the mathematics classrooms.

Trends in Problem-Solving Attempts

Problems Approached Empirically

Compared to KF and PC, students who were profiled as predominantly rational or both rational and empirical in their approaches to knowing more frequently identified one or more theorems or proofs that could be used to solve a problem. When a theorem or proof could not be identified or recalled, properties of theorems were recollected or derived through an analysis of the problem space. However, not all problem attempts were approached rationally. On several occasions students would approach a problem rationally but solve the related problem empirically even, in some cases, when they had identified the isomorphic relationship between the problems.

In the follow-up interviews, nine of the students revealed they approached problems empirically when they were uncertain how to approach a problem, were unable to recall the formal logic, or did not have the prior knowledge of proofs or theorems that could be used to solve the problems. I interpret this in the context of van Hiele's (1976) theory of the acquisition of mathematics concepts and processes. According to van Hiele, individuals pass through five qualitatively different levels of thought when learning mathematics: recognition, analysis, ordering, deduction, and rigor. First, individuals learn mathematics definitions and the primitive patterns associated with those definitions. Specifically, individuals learn the definitions by empirical exploration and manipulation. By understanding definitions empirically, individuals begin to develop intuitions about

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them and, eventually, learn more formal approaches. The pedagogical suggestion based on his theory is that students must have an empirical understanding of mathematics first before formal properties can be learned. An empirical understanding provides a bridge to a more formal understanding; without an empirical understanding, formal knowledge cannot be achieved. Of particular relevance, van Hiele's research has shown that when individuals are unable to access more formal properties of mathematics, they rely on empirical approaches since an empirical base, he argued, had been established.

Failing to recall formal logic, being at an impasse, or not having prior knowledge to solve problems are not the only explanations of why some problems were approached empirically. Two students revealed that they believed they were required to construct a circle for the *Geometry 1* problem rather than explain theoretically how the circle could be constructed. When provided the opportunity, these students recounted the logic they would have used to solve the problem. Moreover, when asked why they approached certain problems empirically, three students revealed they had used properties of theorems and proofs to derive the empirical information they used to solve a problem. For example, for the *Geometry 1* problem, students revealed they knew that the points of tangency were equidistant from the point of intersection of the two lines. Given this property, they calculated the distance from the point of intersection to P, the tangent point given, to derive the distance for the second point of tangency.

Given that students had the tendency to resort to empirical methods when they were unable to recall the formal logic, did not have the prior knowledge, or did not articulate the theoretical information they used to solve the problems, I question Schoenfeld's (1983) method of assessing individuals' mathematics belief systems.

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Although Schoenfeld evaluated students' prior knowledge, he did not ascertain other

possibilities that could explain why students solved problems empirically. Like the

protocol I used to assess students' thought processes, students in Schoenfeld's study were

asked to think aloud but were directed not to explain what they were doing and why. As

Ericsson and Simon (1993) caution, think aloud data do not capture all major decisions

that occur or information that may used when problem solving. Consequently, relying

solely on this type of data restricts one's capacity to assess, in depth, the information

individuals may use when they solve problems. Individuals may use rational

mathematical argumentation to solve problems but their actions and verbalizations may

be representative of an empirical approach. Limited assessments of individuals' belief

systems based solely on how they solve problems may not accurately reflect their

underlying beliefs. Studies should use several sources of information to assess the nature

of individuals' beliefs.

Identifying Isomorphic Relationships

Royce (1978) proposed that individuals profiled as predominantly rational in their

approaches to knowing preferentially engage in critical thinking when learning.

Contemporary research has demonstrated that critical thinking, as measured by need for

cognition, is positively related to processes of regulation of cognition such as

metacognitive monitoring (Cacioppo et al., 1996). Consequently, it was predicted that

individuals profiled as predominantly rational would be more likely to identify

isomorphic relationships between the problems since they were more likely to monitor as

they solved problems.

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Although a higher percentage of students profiled as predominantly rational in their approaches to knowing identified the isomorphic relationships between the problems, I argue that this result was not a function of differences in metacognitive monitoring. As previously discussed, there was weak evidence to suggest differences in monitoring across the three groups. Instead, a pattern I identified was that upper-year university students more often recognized relationships between similar problems than lower-year students. Since eighty percent of the students profiled as predominantly rational were upper-year students and sixty percent profiled as both rational and empirical were lower-year students, this discrepancy may account for the differences found between the groups. Specifically, one may expect that individuals' ability to identify similarities in problems would improve with expertise (Schoenfeld, 1985).

A number of studies support this notion that expert problem solvers are more likely to identify structural similarities in problems than novices (e.g., Chi, Feltovich, & Glaser, 1981, Simon & Simon, 1978). With students, Shavelson (1972, 1974) found that as their knowledge of a domain developed, their perceptions of that knowledge became more expert-like. That is, as students learned a discipline, their knowledge of structural relationships among various components of the discipline became more similar to that of experts. Accordingly, I interpret the pattern found in identifying isomorphic problems as reflecting a difference in expertise between lower-year and upper-year university students.

LIMITATIONS

Four general limitations of this study should be addressed. First, a small number

of students participated in the second component of the study. This limited the capacity to

test certain hypotheses from a more quantitative perspective and restricted the ability to

evaluate aspects of the theories tested. Of particular concern, only twelve students were

profiled as predominantly empirical in their approaches to knowing and only two of those

students participated in the second component of the study. Although both cases

supported Royce's (1978) theory, it is necessary to evaluate more cases. Since both

students profiled as predominantly empirical were in their first year of university, one

may question whether students in their second, third or fourth year of university who are

profiled as predominantly empirical in their approaches to knowing, would also approach

problems empirically. As Royce (1978) theorized, individuals become more socialized in

the epistemic patterns of their discipline. One may question whether individuals who are

profiled as predominantly empirical in their approaches to knowing would learn to be

rational in their approaches to problem solving through experience.

Second, although students' characterizations of their problem-solving attempts

were highly consistent with mine, I provided students the definitions with which to

characterize them. These definitions focused on processes by which students solved the

problems. Consequently, instances when students may have applied rational

mathematical argumentation to derive empirical evidence were not captured. Moreover,

one student, KF, held conceptualizations of rationalism and empiricism that differed from

mine. I did not assess other students' conceptualizations of these constructs. Had

discourse occurred that provided students the opportunity to convey their conceptualizations of rationalism and empiricism, their characterizations of their problem attempts may have differed, although this can only be speculated. Future research I plan will assess students' conceptualizations of these constructs.

Third, interpretations of this study are limited by the operational definitions of the constructs and theoretical frameworks used to make predictions of relations between constructs. For example, Schoenfeld (1983) hypothesized that an empirical approach to problem solving would result in little or no regulation of cognition. He identified relations between these constructs based on his observations of how students approached problems compared to an expert mathematician. Establishing relations among constructs based on observed differences in behavior between novices and experts seems arbitrary. Given that research on expert-novice differences in mathematics problem solving has established that experts engage in more regulation of cognition (e.g., Bookman, 1993) and that mathematics experts are more rational in their approaches to problem solving (Pólya, 1957), it is not surprising that there were differences in approaches to problem solving between students and the expert. If theorists propose relations between epistemic profiles and regulation of cognition, then a more coherent theoretical framework is needed.

Finally, since Royce's (1978) definitions of rationalism and empiricism are founded on philosophical notions of these constructs, his theory is broadly defined. Theoretical specifications of relations between epistemic profiles and learning have been proposed but little research has been conducted to assess whether the three epistemic styles are dependent on the three cognitive styles. Future research is needed to examine

relations between epistemic styles and cognitive styles and more precise theoretical

specifications of these relations are essential.

CONCLUSIONS

The beliefs that students have about mathematics have been widely studied over

the past two decades (e.g., Diaz-Obando et al., 2003; Fleener, 1996; Frank, 1988;

Schoenfeld, 1983). There is much agreement within the mathematics education

community that students' commonly held beliefs negatively influence their learning and

performance. The National Council of Teachers of Mathematics (NCTM; 1980, 1989,

1993) and the National Research Council (NRC; 1989) have called for a radical shift in

school mathematics instruction, particularly at the elementary level. The current view is

that elementary school mathematics curricula overemphasize efficient computational skill

at the expense of understanding. This type of teaching and learning is not what was

envisioned in the *Curriculum and Evaluation Standards for School Mathematics*

[*Standards*] (NCTM, 1989).

Because of growing concerns among mathematics educators regarding students'

beliefs and how they influence learning, the *Standards* suggest that the assessment of

students' beliefs about mathematics is a crucial component of the general assessment of

students' knowledge of mathematics. In the field of educational psychology, the

assessment of students' epistemic beliefs certainly has not influenced any reform in

education. There is, however, growing agreement that students' beliefs about the nature

of knowledge and learning is an important line of research in education and an important

factor to consider in terms of the influence of beliefs on cognition and motivation.

The purpose of this dissertation was to respond to Pintrich's (2002) call for research linking personal epistemology to facets of self-regulated learning and to implement a more process-oriented methodology to examine these relations. He argued that more empirical studies are needed to advance theoretical specifications of how and why epistemic beliefs can facilitate or constrain cognition, motivation, and learning. The focus of much previous research in educational psychology has been on individuals' beliefs about knowledge and knowing and beliefs about learning. This study introduced another facet of epistemology that has not received much attention in the literature, approaches to knowing. I integrated Royce's (1978) model of psychological epistemology with current conceptualizations in educational psychology and mathematics education. Although this model is not a philosophical model in the traditional sense, it is more grounded in philosophy than current conceptualizations. Accordingly, I claim that a more philosophical conceptualization of epistemology has been integrated. My primary purpose was to examine relations among approaches to knowing, mathematics problem solving, and regulation of cognition. A secondary purpose was to examine whether mathematics students become more rational in their approaches to knowing and whether their epistemic beliefs change as they progress through higher levels of education.

Differences in self-reported metacognitive self-regulation were found for students with differing epistemic profiles. In particular, inequalities were found between students profiled as high on rationalism and empiricism and students profiled as predominantly empirical. Students profiled as high on both rationalism and empiricism had the highest self-reported metacognitive self-regulation. Inconsistent with predictions, no differences were confirmed between students profiled as predominantly rational and the other three

groups. In contrast, when problem solving, students profiled as predominantly rational had the highest frequency of planning, monitoring, and control. Differences were explained by the pattern found between metacognitive strategy use and students' motivational beliefs, specifically, their self-efficacy. Students who were more self-efficacious had higher rates of planning, monitoring, and control than students who were less confident in their ability to solve the problems. This pattern is consistent with Bandura's (1997) social cognitive theory that predicts positive relations between self-efficacy and self-monitoring and supports current research that has found that individuals' motivational beliefs are positively related to self-monitoring when learning mathematics (e.g., Zimmerman, 2004).

Results of the second component of the study support Royce's (1978) theory. Students profiled as predominantly rational in their approaches to knowing were predominantly rational in their approaches and justifications when problem solving. Individuals profiled as empirical in their approaches to knowing were also empirical in their approaches to problem solving but provided both rational and empirical justifications of their solutions. Students profiled as both rational and empirical in their approaches to knowing were predominantly rational in their approaches and justifications when problem solving. As Royce (1978) proposed, individuals' epistemic profiles become more comparable to the epistemic patterns of experts in the discipline. This is consistent with an interactionist view which suggests that the formal mathematics education students receive influences the development of their beliefs and approaches to problem solving in mathematics (Bauersfeld, 1993).

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Two patterns in students' problem-solving attempts were found. First, when students were not certain how to approach a problem, were at an impasse, or did not have prior theoretical knowledge that could be used to solve a problem, they resorted to an empirical approach. These results corroborate van Hiele's (1976) theory that describes how individuals learn mathematical concepts and why they resort to empirical approaches when they are incapable of accessing more formal mathematical argumentation. Second, upper-year university students more often identified the relations between the isomorphic problems than lower-year university students. As Schoenfeld (1985) proposed, identifying patterns in problems is learned through experience and as individuals develop their conceptual knowledge, more connections between problems are made.

Finally, no differences in rationalism scores were found between lower- and upper-year university students. Differences were found, however, in the quality of rational arguments between lower- and upper-year university students when solving problems. These results lend support to Royce's (1978) hypothesis that, through experience, individuals' epistemic styles and cognitive styles become more comparable to the epistemic patterns of their discipline. Future research is needed to examine more precisely what facets of mathematics cultures facilitate the development of individuals' epistemic and cognitive styles.

In sum, although previous research has found that epistemic beliefs influence cognition (e.g., Hofer, 1999), there is no substantial evidence from this study to suggest that epistemic styles influence regulation of cognition. Perhaps a plausible hypothesis is that as individuals studying mathematics learn to be more rational in their approaches,

they become more confident as they succeed in learning mathematics and solving problems. As recent research has shown, motivational beliefs influence regulation of cognition (e.g., Hanlon & Schneider, 1999). This argument can be similarly applied to individuals in the sciences and the arts. Future research is needed to test this hypothesis and to further our understanding of relations among epistemic styles, cognitive styles, and self-regulated learning. Improving our understanding of how students become more rational in their approaches to knowing and problem solving may help to inform instructional techniques that focus on developing students' conceptual understanding of mathematics.

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APPENDIX A[*]

SIMON FRASER UNIVERSITY

OFFICE OF RESEARCH ETHICS

BURNABY, BRITISH COLUMBIA
CANADA V5A 1S6
Telephone: 604-291-3447
FAX: 604-268-6785

November 20, 2003

Ms. Krista Muis
Graduate Student
Faculty of Education
Simon Fraser University

Dear Ms. Muis:

Re: Personal epistemology and mathematics: examining the impact of beliefs on problem solving behaviour

The above-titled ethics application has been granted approval by the
Simon Fraser Research Ethics Board, at its meeting on November 17, 2003
in accordance with Policy R 20.01, "Ethics Review of Research Involving
Human Subjects".

Sincerely,

Dr. Hal Weinberg, Director
Office of Research Ethics

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[*] For inclusion in thesis/dissertation/extended essays/research project report, as submitted to the university library in fulfillment of final requirements for graduation. Note: correct page number required.

APPENDIX B

CONSENT FORM

I am investigating students' perceptions about knowledge and learning. If you would like to participate in the first component of the study, I will ask you for some basic information about yourself (e.g., your age, sex, major, GPA, courses taken) then you will respond to four different questionnaires that address various facets of knowledge and learning. For each statement on the questionnaires, you will be asked to estimate your agreement or disagreement, or whether the statement is true or not true of you. Filling out the four questionnaires should take you approximately 45 minutes to 1 hour. If you do participate in the first component of the study, you will be entered into a draw to **win** **$25**. The chances of winning are 1 in 25!

If you decide to participate in the first component of the study, you may eligible to participate in the second component. For the second part of the study, you will be asked to attend two problem-solving sessions, approximately one hour each. For the first session, I will ask you to solve three problems. During this session, you will be asked to "think out loud" while you problem solve. This session will be tape-recorded. For the second session, you will study a short paragraph on a particular math topic. After you study, you will be asked to solve three math problems. Again, you will be asked to "think out loud" while you study and problem solve. For this component of the study, you will be **paid $25**. Please check (√) the box below if you would like to participate in the second component after completing the first.

None of the information from this study will be known to your professor or your TA, and it will have absolutely no effect whatsoever on your scores on assignments, on tests, or on your grade in the course. Only I, Krista Muis, will see your answers. There are no risks in participating in this research. The benefits of participating in this study include gaining helpful information on improving learning and problem solving strategies in mathematics and statistics courses.

The University and Krista Muis conducting this project subscribe to the ethical conduct of research and to the protection at all times of the interests, comfort, and safety of participants. This research is being conducted under permission of the Simon Fraser Research Ethics Board. The chief concern of the board is for the health, safety and psychological well-being of research participants.

Should you wish to obtain information about your rights as a participant in research, or about the responsibilities of researchers, or if you have any questions, concerns or complaints about the manner in which you were treated in this study, please contact the Director, Office of Research Ethics by e-mail at hweinber@sfu.ca or phone at 604-268-6593.

Your participation is completely voluntary. As soon as all information for the research has been gathered, your personal information (e.g., name) will be erased in the research files and replaced with a random number to insure all information about you remains anonymous. If you decide at any time that you don't want to continue participating in this research, tell Krista Muis and all information about you will be eliminated from the research files.

Any information that is obtained during this study will be kept confidential to the full extent permitted by the law. Knowledge of your identity is not required. You will not be required to write your name on any other identifying information on research materials. Materials will be maintained in a secure location.

If you want to participate in this research, please sign below to indicate that you understand the voluntary nature of your participation. Your signature on this form will signify that you have received information describing the procedures, possible risks, and benefits of this project, and that you have received an adequate opportunity to consider the information in the description.

Please bring this form and your completed questionnaires to the next class, or return to Krista Muis' office located in the Education Building, room 8645.

If you would like to receive a brief report on this research after it is completed, please provide an address (below) to which it can be mailed. If at any time you have questions about this project, please contact Krista Muis at 604-291-4548 or e-mail krmuis@sfu.ca.

Having been asked to participate in a research study, I certify that I have read the procedures specified in the paragraphs above, describing the project. I understand the procedures to be used in this experiment and the personal risks to me in taking part in the project.

I understand that I may withdraw my participation at any time. I also understand that I may register any complaint with the Director of the Office of Research Ethics, Krista Muis, or with the Dean of Education, Dr. Paul Shaker, 8888 University Drive, Simon Fraser University, Burnaby, BC, V5A 1S6.

Thank you!
Your participation is *greatly* appreciated.
Krista R. Muis

Consent for first component of study:

Signature _____

Name (please print) _____

optional
Mailing address _____

I would like to participate in the second component of the study. You may contact me by e-mail _____ or phone _____ to set up a time.

Consent for the second component of the study.

Signature _____

Name (please print) _____

APPENDIX C

DEMOGRAPHICS QUESTIONNAIRE

I am interested in your views on studying and how you study. Please answer the following questions. All responses are completely confidential.

_____ Age (in years)

_____ Sex (F or M)

_____ Grade Point Average in all your post-secondary studies (0-4.33, or %)

_____ Grade Point Average in your post-secondary *math/statistics* courses (0-4.33, or %)

_____ Academic major

_____ Academic minor

_____ Number of courses enrolled in this semester

_____ Number of courses taken at SFU, including this semester

_____ Year of study (e.g., 1st, 2nd, 3rd, or 4th year of study)

_____ Average hours worked per week

_____ Average hours studying per week

_____ Was English the first language you learned to speak? (Yes or No).
If no, how old were you when you learned to speak English? _____

_____ Was English the first language you learned to write? (Yes or No).
If no, how old were you when you learned to write in English? _____

What would you like to improve about how you study for math/statistics courses?

List here the names of the math courses you have taken (e.g., math 100, stat 370, etc...).

APPENDIX D

PSYCHO-EPISTEMOLOGICAL PROFILE SCALE

For each of the following statements, you are to indicate your personal agreement or disagreement on the scale provided next to each statement. If you **completely disagree** with the statement, please circle **(1)** next to the statement. If you **completely agree** with the statement, please circle a **(5)** next to the statement. If you neither completely disagree or completely agree with the statement, circle the number in between 1 and 5 that best describes your agreement. Use the following scale to rate your agreement: (Note: Number column has been removed.)

1 = Completely Disagree
2 = Moderately Disagree
3 = Neutral
4 = Moderately Agree
5 = Completely Agree

1. A good teacher is primarily one who has a sparking entertaining delivery.

2. The thing most responsible for a child's fear of the dark is thinking of all sorts of things that could be "out there".

3. Most people who read a lot, know a lot because they come to know of the nature and function of the world around them.

4. Higher education should place a greater emphasis on fine arts and literature.

5. I would like to be a philosopher.

6. A subject I would like to study is biology.

7. In choosing a job I would look for one which offered opportunity for experimentation and observation.

8. The Bible is still a best seller today because it provides meaningful accounts of several important eras in religious history.

9. Our understanding of the meaning of life has been furthered most by art and literature.

10. More people are in church today than ever before because they want to see and hear for themselves what ministers have to say.

11. It is of primary importance for parents to be consistent in their ideas and plans regarding their children.

12. I would choose the following topic for an essay: The Artist in an Age of Science.

13. I feel most at home in a culture in which people can freely discuss their philosophy of life.

14. Responsibility among people requires an honest appraisal of situations where irresponsibility has transpired.

15. A good driver is observant.

16. When people are arguing a question from two different points of view, I would say that the argument should be resolved by actual observation of the debated situation.

17. I would like to visit a library.

18. If I were visiting India, I would primarily be interested in understanding the basis for their way of life.

19. Human morality is molded primarily by an individual's conscious analysis of right and wrong.

20. A good indicator of decay in a nation is a decline of interest in the arts.

21. My intellect has been developed most by learning methods of observation and experimentation.

22. The prime function of a university is to teach principles of research and discovery.

23. A good driver is even tempered.

24. If I am in a contest, I try to win by following a pre-determined plan.

25. I would like to have been Shakespeare.

26. Our understanding of the meaning of life has been furthered most by mathematics.

27. I like to think of myself as a considerate person.

28. I would very much like to have written Darwin's "The Origin of Species".

29. When visiting a new area, I first try to see as much as I possibly can.

30. My intellect has been developed most by gaining insightful self-knowledge.

31. I would be very disturbed if accused of being insensitive to the needs of others.

32. The kind of reading which interests me most is that which creates new insights.

33. The greatest evil inherent in a totalitarian regime is alienation of human relationships.

34. Most atheists are disturbed by the absence of factual proof of the existence of God.

35. In choosing a job I would look for one which offered the opportunity to use imagination.

36. In my leisure I would most often like to enjoy some form of art, music, or literature.

37. The kind of reading which interests me most is that which stimulates critical thought.

38. I prefer to associate with people who are spontaneous.

39. In my leisure I would like to play chess or bridge.

40. Most people who read a lot, know a lot because they develop an awareness and sensitivity through their reading.

41. When visiting a new area, I first pause to try to get a "feel" for the place.

42. Many TV programs lack sensitivity.

43. I like to think of myself as observant.

44. Happiness is largely due to sensitivity.

45. I would be very disturbed if accused of being inaccurate or biased in my observations.

46. A good teacher is primarily one who helps his or her students develop their powers of reasoning.

47. I would like to be a novelist.

48. The greatest evils inherent in a totalitarian regime are restrictions of thought and criticism.

49. More people are in church today than ever before because theologians are beginning to meet the minds of the educated people.

50. The most valuable person on a scientific research team is one who is gifted at critical analysis.

51. Many TV programs lack organization and coherence.

52. I like country living because it gives you a chance to see nature first hand.

53. Upon election to Parliament I would endorse steps to encourage an interest in the arts.

54. It is important for parents to be familiar with theories of child psychology.

55. The prime function of a university is to train the minds of the capable.

56. I would like to have written Hamlet.

57. Higher education should place a greater emphasis on mathematics and logic.

58. The kind of reading which interests me most is that which is essentially true to life.

59. A subject I would like to study is art.

60. I feel most at home in a culture in which realism and objectivity are highly valued.

61. The prime function of a university is to develop a sensitivity to life.

62. When playing bridge or similar games I try to think my strategy through before playing.

63. If I were visiting India, I would be primarily interested in noting the actual evidence of cultural change.

64. When buying new clothes I look for the best possible buy.

65. I would like to visit an art gallery.

66. When a child is seriously ill, a good parent will remain calm and reasonable.

67. I prefer to associate with people who stay in close contact with the facts of life.

68. Many TV programs are based on inadequate background research.

69. Higher education should place greater emphasis on natural science.

70. I like to think of myself as logical.

71. When people are arguing a question from two different points of view, I would say that each should endeavor to assess honestly his or her own attitude and bias before arguing further.

72. When reading an historical novel, I am most interested in the factual accuracy found in the novel.

73. The greatest evil inherent in a totalitarian regime is distortion of the facts.

74. A good driver is considerate.

75. Our understanding of the meaning of life has been furthered most by biology.

76. I would have liked to be Galileo.

77. My children must posses the characteristics of sensitivity.

78. I would like to be a Geologist.

79. A good indicator of decay in a nation is an increase in the sale of movie magazines over news publications.

80. I would be very disturbed if accused of being illogical in my beliefs.

81. Most great scientific discoveries came about by thinking about a phenomenon in a new way.

82. I feel most at home in a culture in which the expression of creative talent is encouraged.

83. In choosing a job I would look for one which offered a specific intellectual challenge.

84. When visiting a new area, I first plan a course of action to guide my visit.

85. A good teacher is primarily one who is able to discover what works in class and is able to use it.

86. Most great scientific discoveries come about by careful observation of the phenomena in question.

87. Most people who read a lot, know a lot because they acquire an intellectual proficiency through sifting of ideas.

88. I would like to visit a botanical garden or zoo.

89. When reading an historical novel, I am most interested in the subtleties of the personalities described.

90. When playing bridge or similar games I play the game by following spontaneous cues.

APPENDIX E

COPYRIGHT PERMISSION FOR THE PEP

From: Leo Mos <lmos@ualberta.ca>
Date: Sat, 24 Jul 2004 15:55:30 -0600
Subject: Re: Psycho-Epistemological Profile scale

Dear Krista, Congratulations. Of course, you have my permission. Best
regards on successful employment. Leo

Thursday, July 22, 2004.

Dear Dr. Mos.

I have completed my doctoral thesis in Educational Psychology at Simon Fraser
University. The title of my thesis is "Epistemic Styles and Mathematics Problem Solving:
Examining Relations in the Context of Self-Regulated Learning.

For my thesis research, I obtained a copy and used your Psycho-Epistemological Profile
Scale (PEP; Royce & Mos, 1980) to measure mathematics students' epistemic styles.

I am requesting your permission to reprint the entire scale in one of the appendices of my
thesis.

The requested permission extends to any future revisions and editions of my thesis,
including a partial copyright license to my university for circulating and archival copies
permitting personal photocopying, and non-exclusive licenses, which I may give to the
National Library of Canada, and its agents to circulate my work. These rights will in no
way restrict re-publication of the material in any other form by you or by your assigns.

Your reply to this e-mail will also confirm that Dr. L. P. Mos, University of Alberta,
owns the copyright to the above-described material.

If the above is acceptable to you, may I ask you to reply to this e-mail using your reply
button to include the full text of my request? You may include at the top of the e-mail
that you agree to this request, and include your name and date at the bottom. Thank you
very much.

Yours truly,

Krista R. Muis, PhD

Permission for the use outlined about is hereby granted.

(Your name in full) Leendert P. Mos

Date of approval: July 24, 2004

Leendert P. Mos
Professor
Departments of Psychology and Linguistics
University of Alberta
Edmonton, AB T6G 2E9
Canada
Tel.: 780-492-5216 (O)
780-436-1539 (H)
Fax: 780-492-1768 (0)
email: lmos@ualberta.ca

APPENDIX F

MOTIVATED STRATEGIES

FOR LEARNING QUESTIONAIRE

The following questions ask about your study habits in your math/statistics course(s). Remember, there are no right or wrong answers. Just answer as accurately as possible for you. Use the scale below to answer the questions.

If you think the statement is very true of you, circle 7.

If a statement is not at all true of you, circle 1.

If the statement is more or less true of you, circle the number between 1 and 7 that best describes you. (Note: Number column has been removed.)

1. In a class like this, I prefer course material that really challenges me so I can learn new things.

2. If I study in appropriate ways, then I will be able to learn the material in this course.

3. When I take a test I think about how poorly I am doing compared with other students.

4. I think I will be able to use what I learn in this course in other courses.

5. I believe I will receive an excellent grade in this class.

6. I'm certain I can understand the most difficult material presented in the readings for this course.

7. Getting a good grade in this class is the most satisfying thing for me right now.

8. When I take a test I think about items on other parts of the test I can't answer.

9. It is my own fault if I don't learn the material in this course.

10. It is important for me to learn the material in this class.

11. The most important thing for me right now is improving my overall grade point average so my main concern in this class is getting a good grade.

12. I'm confident I can learn the basic concepts taught in this course.

13. If I can, I want to get better grades in this class than most of the other students.

14. When I take tests I think of the consequences of failing.

15. I'm confident I can understand the most complex material presented by the instructor in this course.

16. In a class like this, I prefer course material that arouses my curiosity, even if it is difficult to learn.

17. I am very interested in the content area of this course.

18. If I try hard enough, then I will understand the course material.

19. I have an uneasy, upset feeling when I take an exam.

20. I'm confident I can do an excellent job on the assignments and tests in this course.

21. I expect to do well in this class.

22. The most satisfying thing for me in this course is trying to understand the content as thoroughly as possible.

23. I think the course material in this class is useful for me to learn.

24. When I have the opportunity in this class, I choose course assignments that I can learn from even if they don't guarantee a good grade.

25. If I don't understand the course material, it is because I didn't try hard enough.

26. I like the subject matter of this course.

27. Understanding the subject matter of this course is very important to me.

28. I feel my heart beating fast when I take an exam.

29. I'm certain I can master the skills being taught in this class.

30. I want to do well in this class because it is important to show my ability to my family, friends, employer, or others.

31. Considering the difficulty of this course, the teacher, and my skills, I think I will do well in this class.

32. When I study the readings for this course, I outline the material to help me organize my thoughts.

33. During class time I often miss important points because I'm thinking of other things.

34. When studying for this course, I often try to explain the material to a classmate or friend.

35. I usually study in a place where I can concentrate on my course work.

36. When reading for this course, I make up questions to help focus my reading.

37. I often feel so lazy or bored when I study for this class that I quit before I finish what I planned to do.

38. I often find myself questioning things I hear or read in this course to decide if I find them convincing.

39. When I study for this class, I practice saying the material to myself over and over.

40. Even if I have trouble learning the material in this class, I try to do the work on my own, without help from anyone.

41. When I become confused about something I'm reading for in this class, I go back and try to figure it out.

42. When I study for this course, I go though the readings and my class notes and try to find the most important ideas.

43. I make good use of my study time for this course.

44. If course readings are difficult to understand, I change the way I read the material.

45. I try to work with other students from this class to complete the course assignments.

46. When studying for this course, I read my class notes and the course readings over and over again.

47. When a theory, interpretation, or conclusion is presented in class or in the readings, I try to decide if there is good supporting evidence.

48. I work hard to do well in this class even if I don't like what we are doing.

49. I make simple charts, diagrams, or tables to help me organize course material.

50. When studying for this course, I often set aside time to discuss course material with a group of students from the class.

51. I treat the course material as a starting point and try to develop my own ideas about it.

52. I find it hard to stick to a study schedule.

53. When I study for this class, I pull together information from different sources, such as lectures, readings and discussions.

54. Before I study new course material thoroughly, I often skim it to see how it is organized.

55. I ask myself questions to make sure I understand the material I have been studying in this class.

56. I try to change the way I study in order to fit the course requirements and instructor's teaching style.

57. I often find that I have been reading for this class but don't know what it was all about.

58. I ask the instructor to clarify concepts I don't understand well.

59. I memorize key words to remind me of important concepts in this class.

60. When course work is difficult, I either give up or only study the easy parts.

61. I try to think through a topic and decide what I am supposed to learn from it rather than just reading it over when studying for this course.

62. I try to relate ideas in this subject to those in other courses whenever possible.

63. When I study for this course, I go over my class notes and make an outline of important concepts.

64. When reading for this class, I try to relate the material to what I already know.

65. I have a regular place set aside for studying.

66. I try to play around with ideas of my own and relate them to what I am learning in this course.

67. When I study for this course, I write brief summaries of the main ideas from the readings and my class notes.

68. When I can't understand the material in this course, I ask another student in this class for help.

69. I try to understand the material in this class by making connections between the readings and the concepts from the lectures.

70. I make sure that I keep up with the weekly readings and assignments for this course.

71. Whenever I read or hear an assertion or conclusion in this class, I think about possible alternatives.

72. I make lists of important terms for this course and memorize the lists.

73. I attend this class regularly.

74. Even when the course materials are dull and uninteresting, I manage to keep working until I finish.

75. I try to identify students in this class whom I can ask for help if necessary.

76. When studying for this course, I try to determine which concepts I don't understand well.

77. I often find that I don't spend very much time on this course because of other activities.

78. When I study for this class, I set goals for myself in order to direct my activities in each study period.

79. If I get confused taking notes in class, I make sure I sort it out afterwards.

80. I rarely find time to review my notes or readings before an exam.

81. I try to apply ideas from course readings in other class activities such as lecture and discussion.

APPENDIX G

COPYRIGHT PERMISSION FOR THE MSLQ

From: Bill McKeachie <billmck@umich.edu>
Date: July 23, 2004 8:37:13 AM PDT
To: Krista Muis <krmuis@sfu.ca>
Subject: Re: Motivated Strategies for Learning Questionnaire

You have my permission to reprint the scale. I'd appreciate a copy of the dissertation abstract.

Bill McKeachie

Thursday, July 22, 2004.

Dear Dr. McKeachie.

I have completed my doctoral thesis in Educational Psychology at Simon Fraser University under the supervision of Dr. Philip Winne. The title of my thesis is "Epistemic Styles and Mathematics Problem Solving: Examining Relations in the Context of Self-Regulated Learning."

For my thesis research, I obtained a copy and used your Motivated Strategies for Learning Questionnaire (MSLQ; Pintrich, Smith, Garcia, & McKeachie, 1991). I am requesting your permission to reprint the entire scale in one of the appendices of my thesis.

The requested permission extends to any future revisions and editions of my thesis, including a partial copyright license to my university for circulating and archival copies permitting personal photocopying, and non-exclusive licenses which I may give to the National Library of Canada, and its agents to circulate my work. These rights will in no way restrict re-publication of the material in any other form by you or by your assigns. Your reply to this e-mail will also confirm that *Dr. W. J. McKeachie, University of Michigan*, owns the copyright to the above-described material.

If the above is acceptable to you, may I ask you to reply to this e-mail using your reply button to include the full text of my request? You may include at the top of the e-mail that you agree to this request, and include your name and date at the bottom. Thank you very much.

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Yours truly,

Krista R. Muis, PhD

Permission for the use outlined about is hereby granted.

(Your name in full) _Wilbert J. McKeachie_____

Date of approval: July 2 3, 2004_____

W.J. McKeachie   billmck@umich.edu
University of Michigan   Phone: 734-763-0218
Dept of Psychology   Fax: 734-764-3520
525 E. University
Ann Arbor, MI 48109-1109

APPENDIX H

EPISTEMIC BELIEFS INVENTORY

For each of the following statements, indicate your personal agreement or disagreement by circling a number on the rating scale that most closely reflects your agreement. Remember, there are no right or wrong answers. Use the scale below to rate each statement. (Note: Number column has been removed.)

If you strongly disagree with the statement, circle 1.
If you strongly agree with the statement, circle 5.
If you more or less agree or disagree, circle the number between 1 and 5 that best describes your agreement.

1. Most things worth knowing are easy to understand.

2. What is true is a matter of opinion.

3. Students who learn things quickly are the most successful.

4. People should always obey the law.

5. People's intellectual potential is fixed at birth.

6. Absolute moral truth does not exist.

7. Parents should teach their children all there is to know about life.

8. Really smart students don't have to work as hard to do well in school.

9. If a person tries too hard to understand a problem, they will most likely end up being confused.

10. Too many theories just complicate things.

11. The best ideas are often the most simple.

12. Instructors should focus on facts instead of theories.

13. Some people are born with special gifts and talents.

14. How well you do in school depends on how smart you are.

15. If you don't learn something quickly, you won't ever learn it.

16. Some people just have a knack for learning and others don't.

17. Things are simpler than most professors would have you believe.

18. If two people are arguing about something, at least one of them must be wrong.

19. Children should be allowed to question their parents' authority.

20. If you haven't understood a chapter the first time through, going back over it won't help.

21. Science is easy to understand because it contains so many facts.

22. The more you know about a topic, the more there is to know.

23. What is true today will be true tomorrow.

24. Smart people are born that way.

25. When someone in authority tells me what to do, I usually do it.

26. People shouldn't question authority.

27. Working on a problem with no quick solution is a waste of time.

28. Sometimes there are no right answers to life's big problems.

APPENDIX I

COPYRIGHT PERMISSION FOR THE EBI

From: Greg Schraw <gschraw@unlv.nevada.edu>
Date: July 23, 2004 12:45:08 PM PDT
To: Krista Muis <krmuis@sfu.ca>, gs <gschraw@unlv.nevada.edu>
Subject: Re: copyright permission for the EBI

Krista,
Yes, you have permission to use the EBI.

Gregory Schraw

Krista Muis wrote:

Dear Dr. Schraw.

I have completed my doctoral thesis in Educational Psychology at Simon Fraser University under the supervision of Dr. Philip Winne. The title of my thesis is "Epistemic Styles and Mathematics Problem Solving: Examining Relations in the Context of Self-Regulated Learning."

For my thesis research, I obtained a copy and used your Epistemic Beliefs Inventory (EBI; Schraw, Bendixen, & Dunkle, 2002).

I am requesting your permission to reprint the entire scale in one of the appendices of my thesis.

The requested permission extends to any future revisions and editions of my thesis, including a partial copyright license to my university for circulating and archival copies permitting personal photocopying, and non-exclusive licenses which I may give to the National Library of Canada, and its agents to circulate my work. These rights will in no way restrict re-publication of the material in any other form by you or by your assigns. Your reply to this e-mail will also confirm that Dr. G. Schraw owns the copyright to the above-described material. A similar letter has been sent to Lawrence Erlbaum Associates.

If the above is acceptable to you, may I ask you to reply to this e-mail using your reply button to include the full text of my request? You may include at the top of the e-mail that you agree to this request, and include your name and date at the bottom. Thank you very much.

Yours truly,

Krista R. Muis, PhD

Permission for the use outlined about is hereby granted.

(Your name in full) _____ Dr. Gregory Schraw

Date of approval: _____ July 23, 2004

From: "Bonita D'Amil" <Bonita.D'Amil@erlbaum.com>
Date: Mon, 26 Jul 2004 10:57:49 -0400
Subject: RE: Permission Request from Web

Hello Dr. Muis, In view of your request below:

PERMISSION GRANTED provided that material has appeared in our work without credit to another source; you obtain the consent of the author(s); you credit the original publication; and reproduction is confined to the purpose for which permission is hereby given.

This is an original email document; no other document will be forthcoming. Should you have any questions, please don't hesitate to contact me.

Regards,
Bonita R. D'Amil

Bonita R. D'Amil
Executive Assistant/Office Manager
Permissions and Translations Manager
Office of Rights and Permissions
Lawrence Erlbaum Associates
10 Industrial Avenue
Mahwah, NJ 07430
E-mail: Bonita.D'Amil@erlbaum.com
Phone: (201) 258-2211
Fax: (201) 236-0072

For more information on LEA visit our website at: www.erlbaum.com
-----Original Message-----
From: Krista Muis [mailto:krmuis@sfu.ca]
Sent: Friday, July 23, 2004 1:13 AM
To: Bonita D'Amil
Subject: Permission Request from Web

Thursday, July 22, 2004.

Dear Lawrence Erlbaum Associates Representative:

I have completed my doctoral thesis in Educational Psychology at Simon Fraser University under the supervision of Dr. Philip Winne. The title of my thesis is "Epistemic Styles and Mathematics Problem Solving: Examining Relations in the Context of Self-Regulated Learning."

For my thesis research, I obtained a copy and used the Epistemic Beliefs Inventory (EBI; Schraw, Bendixen, & Dunkle, 2002), published in the book entitled "Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing," authored by B. K. Hofer and P. R. Pintrich in 2002.

I am requesting your permission to reprint the entire scale (Appendix A, which includes 28 items, on page 275) in one of the appendices of my thesis.

The requested permission extends to any future revisions and editions of my thesis, including a partial copyright license to my university for circulating and archival copies permitting personal photocopying, and non-exclusive licenses which I may give to the National Library of Canada, and its agents to circulate my work. These rights will in no way restrict re-publication of the material in any other form by you or by your assigns. Your reply to this e-mail will also confirm that *Lawrence Erlbaum Associates* owns the copyright to the above-described material. A similar letter has been sent to Dr. Gregory Schraw.

If the above is acceptable to you, may I ask you to reply to this e-mail using your reply button to include the full text of my request? You may include at the top of the e-mail that you agree to this request, and include your name and date at the bottom. Thank you very much.

Yours truly,

Krista R. Muis, PhD

Permission for the use outlined about is hereby granted.

(Your name in full) _____

Date of approval: _____

APPENIDIX J

RATING SELF-EFFICACY

---

1. How confident are you that you could correctly do **Problem 1?**

    1   2   3   4   5   6   7

    Not   Not sure   Very
    confident   confident
    at all

---

2. How confident are you that you could correctly do **Problem 2?**

    1   2   3   4   5   6   7

    Not   Not sure   Very
    confident   confident
    at all

---

3. How confident are you that you could correctly do **Problem 3?**

    1   2   3   4   5   6   7

    Not   Not sure   Very
    confident   confident
    at all

APPENIDIX K

SELF-EFFICACY PROBLEMS

First Problem-Solving Session

A Little Algebra:

Show that for all sets of real numbers w, x, y, and z,

$w^3 + x^3 + y^3 + z^3 = wxy + xyz + yzw + zwx$ implies $w = x = y = z$.

Geometry 1:

Let two circles be tangent to point A. Two lines have been drawn through A that meet the circles at further points B, C, D, and E. Show that BC is parallel to DE.
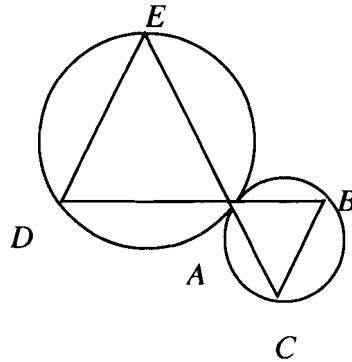
Geometry 2:

Show that the three angle bisectors of a triangle meet in a point.
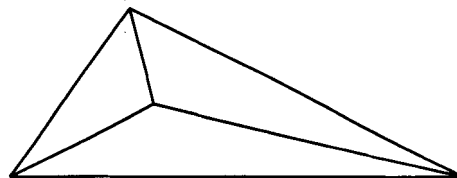
Juvenile Delinquents:

The proportion of juvenile delinquents who wear glasses is known to be 0.2 whereas the proportion of non-delinquents wearing glasses is 0.6. A researcher plans to randomly select 15 delinquents from a database. Calculate the exact probability that two delinquents wear glasses.

Rolling the Dice:

A pair of dice is thrown. Assuming the dice are fair, what is the exact probability of rolling a 2 on one die and a 4 on the other die?

Telephones:

Twenty percent of all telephones are submitted for service while under warranty. Of these, 60% can be repaired whereas the other 40% must be replaced with a new phone. If a company purchases 10 phones, what is the exact probability that exactly 2 will end up being replaced under warranty?

APPENDIX L

PROBLEM SET FOR FIRST SESSION

A Little Algebra:

Show that for all sets of real numbers a, b, c, and d,

$a^2 + b^2 + c^2 + d^2 = ab + bc + cd + da$ implies $a = b = c = d$.

Geometry 1:

You are given two intersecting straight lines and a point P marked on one of them, as in the Figure below. Show how to construct, using straightedge and compass, a circle that is tangent to both lines and that has the point P as its point of tangency to one of the lines. Justify your answer.
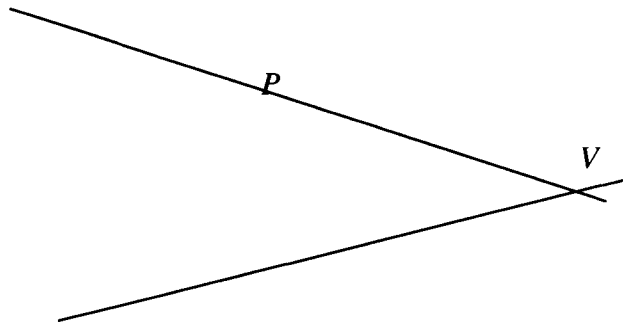
Geometry 2:

The circle in the triangle in the Figure below is tangent to sides *EF* and *GF*, respectively. Show that the line segment *CF* bisects angle *EFG*.
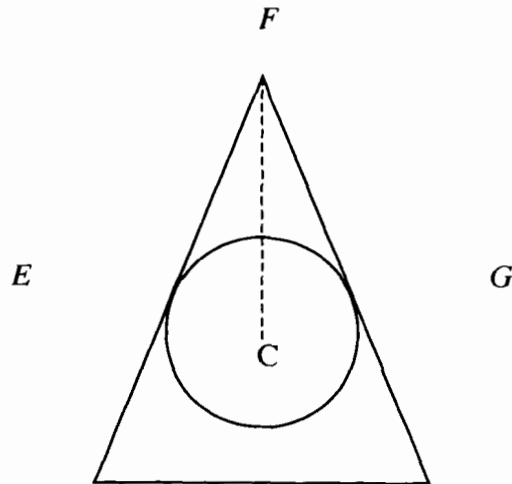
APPENDIX M

PRIOR KNOWLEDGE TEST

For the following 10 statements, please indicate whether the statement is true or false by circling **T** for true or **F** for false. If you indicate the answer is false, please provide the correct statement in the space provided. If you do not know the answer, make your best guess.

Then, or both your T/F answer and your corrected statement (where appropriate) circle your rating of how sure you are your answer is correct. Use this rating scale:

        5 = absolutely sure it is correct
        4 = sort of sure it is correct
        3 = no idea whether it is correct, I guessed
        2 = sort of sure it is incorrect
        1 = absolutely sure it is incorrect

1. The sum of 3 angles of a triangle is 360 degrees.   T / F   1 2 3 4 5

                                                             1 2 3 4 5

_____

2. If a sum of squares is equal to zero, each term must be equal   T / F   1 2 3 4 5
    to one.

                                                             1 2 3 4 5

_____

3. Two tangents drawn from a point to a circle are of equal   T / F   1 2 3 4 5
    length.

                                                             1 2 3 4 5

_____

4. The tangent to a circle is perpendicular to the radius drawn   T / F   1 2 3 4 5
    to the point of tangency.

                                                             1 2 3 4 5

_____

5. A ray which separates an angle into 2 congruent halves   T / F   1 2 3 4 5
   bisects the angle.
                                                                       1 2 3 4 5

_____

_____

6. The centre of a circle inscribed in a triangle lies at the   T / F   1 2 3 4 5
   intersection of the triangle's medians.
                                                                         1 2 3 4 5

_____

_____

7. If $(a - b)^2 = 0$, then $a = b$.   T / F   1 2 3 4 5

                                                 1 2 3 4 5

_____

_____

8. Two triangles are congruent by angle-side-side.   T / F   1 2 3 4 5

                                                               1 2 3 4 5

_____

_____

9. The circumference of a circle is πr^2.   T / F   1 2 3 4 5

                                                           1 2 3 4 5

_____

_____

10. The side of a triangle opposite a greater angle is the greater   T / F   1 2 3 4 5
    side.
                                                                               1 2 3 4 5

_____

_____

APPENDIX N

SHORT CHAPTER ON BINOMIAL DISTRIBUTION

BINOMIAL DISTRIBUTION

When a coin is flipped, the outcome is either a head or a tail; when a person guesses the card selected from a deck, the person can either be correct or incorrect; when a baby is born, the baby is either born in the month of March or is not. In each of these examples, an event has only two possible outcomes. If one outcome occurs, the other did not occur. For convenience, one of the outcomes can be labeled a "hit" and the other outcome a "miss."

Suppose that we toss a coin or a die repeatedly. Each toss is called a trial. In any single trial there will be a probability associated with a particular event such as a head on the coin or 4 dots on the die. This probability will not change from one trial to the next. Such trials are said to be independent.

Let p be the probability that an event will happen in any single trial (the probability of a hit).

Then, the probability that the event will fail to happen in a particular trial, the probability of a miss or q, can be described by this equation:

$$q = 1 - p \qquad \text{(Equation 1)}$$

For example, let's say you entered into a draw to win $100. Only 4 people entered the draw. You want to win. A win would be considered a "hit." What is the probability (or chance) you will win? The probability of a hit or **p** is equal to 1 in 4. You have a 25% chance of winning. So, the probability you will win is

$$p = 0.25.$$

What is the probability you will not win? What is **q**, the probability of a miss? Since **p** = 0.25, Equation 1 shows how to calculate **q**:

$$q = 1 - p$$
and $p = 0.25$, so
$$q = 1 - 0.25$$
$$q = 0.75$$

The probability the event will happen exactly **x** times in **n** trials—that there will be **x** hits and **n** - **x** misses—is given by a probability function. In this function,

p is the probability of a hit
q (or 1 – **p**) is the probability of a miss
n is the number of trials
X (upper case x) is the number of hits in n trials
x (lower case x) is a specific number that can range from 0, 1,... up to **n**
! is a factorial defined for any positive integer

$$f(X) = P(X = x) = \frac{n!}{x!(n\text{-}x)!} \; p^x \, q^{n\text{-}x}$$

(Equation 2)

This formula assumes that the events:

(a) fall into only two categories, a hit or a miss: that is, the events are dichotomous.
(b) cannot occur at the same time: that is, a flip of the coin can have only one outcome.
(c) are independent: that is, the outcome on a particular trial has no influence whatsoever on the outcome of any other trial.
(d) are randomly selected.

The general factorial $n!$ is defined for a positive integer n as

$$n! \equiv \begin{cases} n \cdot (n-1) \cdots 2 \cdot 1 & n = 1, 2, \ldots \\ 1 & n = 0. \end{cases}$$

(Equation 3)

So, for example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

The factorial gives the number of ways in which n objects can be put in different sequences or permutations. For example,

$3! = 3 \cdot 2 \cdot 1 = 6$

The six possible permutations of $\{1, 2, 3\}$ are

$\{1, 2, 3\}, \{1, 3, 2\}, \{2, 1, 3\}, \{2, 3, 1\}, \{3, 1, 2\},$ and $\{3, 2, 1\}$.

Since there is a single permutation of zero elements (the empty set $\varnothing$),

$0! = 1$.

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Example Problem:

What is the exact probability of getting exactly 2 heads in 6 tosses of a fair coin?

How to solve it:

When a coin is a fair coin, the probability of a head is .5, or a 50% chance of flipping a head. So:

$p = 0.5$, and

$q = 1 - 0.5 = 0.5$

We want exactly 2 heads in six tosses, so

$X = 2$, and

$n = 6$.

We substitute these values into Equation 2 and solve:

$$f(X) = P(X = x) = \frac{n!}{x!(n-x)!} \; p^x q^{n-x}$$

$P(X = 2) = [6! / (2! \, 4!)] \, (.5)^2(.5)^{6-2} = [(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)/(2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1)] \, (.5)^2 \, (.5)^{6-2}$

$= 15/64 \cong 0.23.$

The probability function f(X) given in equation 2 is often called the binomial distribution because for

$x = 0, 1, 2,..., n,$

it corresponds to successive terms in the binomial expansion.

Binomial Expansion

Where:

p is the probability of a hit
q (or $1 - p$) is the probability of a miss
n is the number of trials
X (upper case x) is the number of hits in **n** trials
x (lower case x) is a specific number that can range from 0, 1,... up to **n**
! is a factorial defined for any positive integer

$$(q + p)^n = q^n + \frac{n!}{1!(n-1)!} q^{n-1} p + \frac{n!}{2!(n-2)!} q^{n-2} p^2$$

$$= \sum_{x=0} \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

(Equation 4)

What do we do if we want to know the probability of obtaining, say, 3 **or more** heads with $n = 6$ and $p = 0.5$? We add together the separate probabilities for 3 heads (hits) plus for 4 hits plus for 5 hits plus for 6 hits. Or, to do it in one step, we use what is called the cumulative form of the binomial distribution (Equation 4).

$P(3) + P(4) + P(5) + P(6)$.

This is equal to

$0.3125 + 0.2344 + 0.0938 + 0.0156 = 0.6564$.

Independent Events

Two events, let's call them event A and event B, are independent if the fact that A occurs has no effect the probability of B occurring.

Some examples of independent events are:

- Tossing a coin and landing on heads, then tossing the coin again. Either heads or tails can come up.
- Rolling a die and getting a 5, then rolling it again. Any number can come up.
- Choosing a card from a deck of cards and getting a 4 of hearts, replacing the card, then choosing another card. Any other card, including the 4 of hearts, might be chosen.

To find the probability that two independent events will occur one after the other, you have to calculate the probability of each event occurring separately. Then multiply the answers.

When two events, A and B, are independent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$ (Equation 5)

Example:

A dresser drawer contains 5 pairs of socks: one pair of blue socks, one pair of brown socks, one pair of red socks, one pair of white socks, and one pair of black socks. Each pair of socks is folded together so the colors match. You reach into the sock drawer and choose a pair of socks without looking, that is, randomly. The first pair you pull out is red, but you don't want to wear red socks. You replace that pair and, without looking, choose another pair. What is the probability you will get the red pair of socks twice in a row?

Solution:

$$P(\text{red}) = \frac{1}{5}$$

$$P(\text{red and red}) = \frac{1}{5} \cdot \frac{1}{5}$$

$$= \frac{1}{25}$$

$$= 0.04$$

APPENDIX O

PROBLEM SET FOR SECOND SESSION

Multiple Choice Exam:

A multiple choice exam has 16 questions with four possible responses to each question. A student takes the test whereby each question is answered independently. The student gets 8 questions correct and claims that she guessed the answer to each question. Do you believe her? Justify your answer.

Rolling the Dice:

Which is more likely: at least one dot with 4 throws of a fair die or at least one double dot (i.e., a pair of ones) in 24 throws of two fair dice?

Heart Transplant:

The proportion of patients who do not experience any difficulties after a heart transplant operation is .75. You select sixteen patients from a national database who are waiting for a transplant to interview them on their eating habits. After their transplants, eight of the patients you interviewed experienced difficulties. Was the selection of your group random? Justify your answer.

APPENDIX P

INSTRUCTIONS TO PARTICIPANTS

"I am interested in how people solve mathematics problems. What I am going to do is give you three problems to solve. For each problem, I will ask you to begin by reading the problem statement out loud. Because I want to know what you are thinking as you solve the problem, I would like you to think aloud as you work on the problem. What I mean by think aloud is that I want you to tell me everything you are thinking from the time you first see the question until you give me an answer or decide to quit. I would like you to talk aloud constantly from the time I present each problem until you have given your final answer to the question. I don't want you to plan out what you say or try to explain to me what you are saying. It is important that you keep talking. If there is a period of time that goes by and you have not said something, I will ask you to keep talking. Do you have any question at this point?"

"OK. What I am going to do now is demonstrate to you what thinking aloud looks like." [I then gave the demonstration.]

"Now I would like you to try it on a couple of simple problems. This is just a practice run before I give you the problems for the main experiment. I want you to do the same thing for each of these problems. I want you to read the problem out loud and then say everything as it comes to you." [Participants were then given a couple of simple problems to solve, similar to the one I demonstrated.]

"You can use any of the materials on this desk to help you solve the problems and you can write things down on the problem sheet if you would like to do so. For each problem, if you write things down please do not erase anything. If you make a mistake or would like to make another attempt, please put brackets around the work and begin on a new line. There is no time limit to solving these problems but I am going to keep a record of how long it takes to solve the problems. I am not concerned about whether you get the answer correct. What I am interested in is what you are thinking as you solve the problem. You can quit at any point during the problem-solving attempt. Again, I am not interested in whether you solve the problem. I am interested in what goes on in your head as you solve the problem. Once you have completed the problem I will not provide any feedback to you until the end of the second session. I am not proving feedback at this point because I don't want to influence how you solve the other problems. At the end of the second session we will go through, in detail, each of the problems at which point I will provide full feedback."

APPENDIX Q

INSTRUCTIONS FOR CODING PROBLEMS

"Approaches to problem solving should be coded as rational if you used mathematical argumentation or derived proofs, theorems, and/or facts during the problem-solving attempt. Examples include the use of the Pythagorean Theorem to prove two triangles are congruent, properties of congruent triangles such as side-angle-side, the proof that if $(a-b)^2 = 0$ then a and b must equal 0, and the binomial expansion. Justifications of solutions should be coded as rational if the justifications include information as described above.

Approaches to problem solving should be coded as empirical if you engaged in trial-and-error exploration of the problem space, tested hypotheses in a serial fashion, and/or used perceptual information to work the problem. An example of trial-and-error exploration includes attempts to find information to help solve the problem by working another problem not directly related to the given problem. An example of testing hypotheses in a serial fashion includes implementing one equation to solve the problem followed by another equation and continuing until an answer is perceived to make sense. Examples of perceptual information includes: testing a construction and making adjustments to the compass setting until a construction, measuring distances on lines to find the center point of a circle, and measuring the angle of a triangle. Justifications of solutions should be coded as empirical if you tested your solutions using perceptual information, by substituting a solution into an equation to test whether the solution made sense, or by claiming the solution made sense without providing proof-like information to support that claim.

APPENDIX R

FACTOR LOADINGS FOR THE EBI

Dimension	First Loading	Second Loading
Structure of Knowledge		
Item 1	.49	.49
Item 9	.62	.62
Item 10	.62	.63
Item 11	.07	X
Item 12	.65	.65
Item 17	.56	.55
Item 21	.37	.37
Certainty of Knowledge		
Item 2	.41	.43
Item 6	.00	X
Item 7	.32	.31
Item 18	.29	.29
Item 22	.07	X
Item 23	.55	.54
Item 28	.47	.49
Source of Knowledge		
Item 4	.35	.34
Item 19	.32	.32
Item 25	.33	.33
Item 26	.99	.99

Loadings Continued.

Dimension	First Loading	Second Loading
Control of Knowledge Acquisition		
Item 5	.52	.52
Item 8	.45	.45
Item 13	.33	.33
Item 14	.60	.60
Item 16	.46	.46
Item 24	.72	.72
Speed of Knowledge Acquisition		
Item 3	.30	.30
Item 15	.59	.59
Item 20	.55	.55
Item 27	.51	.51

Note: X = item removed. Loadings are standardized.