CALCULATOR LITERACY

by

Jimmy Y. K. Wu
B.Sc. University of British Columbia, 1995
B.Ed. University of British Columbia, 1996

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NAME Yu Keung (Jimmy) Wu

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EXAMINING COMMITTEE:

Chair Stephen Campbell

Thomas O'Shea, Associate Professor
Senior Supervisor

Rina Zazkis, Professor
Member

David Lidstone, Mathematics and Statistics Department,
Langara College, Vancouver BC
Examiner

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Calculator Literacy

Author:

(Signature)

Mr. Yu Keung (Jimmy) Wu

(Name)

(Date)

ABSTRACT

Life without a calculator is unimaginable for the generation of today's teenagers. They carry it, they rely on it, and they trust it. But is their use of the calculator appropriate or effective? Over the years the educational question has shifted from whether calculators should be allowed to how they should be used, but little attention has been given to teaching proper strategies of calculator use.

In this study, I examined what a calculator literate person is and how to measure the calculator literacy level of a person. I started by examining the historical development of both the scientific and graphing calculator noting who the major manufacturers are in the educational market as well as the different logic entry systems. From reviewing the NCTM Principles and Standards as well as the British Columbia Ministry of Education - Integrated Resource Package, I formulated a working definition of what a calculator literate person should be. From there I turned my attention to the Graphing Calculator Resource Package produced by the British Columbia Ministry of Education to see how each component of the definition can be assessed.

From this information I designed and administered a Calculator Literacy Test to two groups of students. One group participated in the non-graphing calculator dependent section while the other group participated in the graphing calculator dependent section. The results of the tests were analyzed and common errors were noted. The most prevalent mistakes involved improper bracket placement in the non-graphing calculator dependent

group and the lack of understanding of the limitations of the graphing calculator in the graphing calculator dependent group.

On a personal level, the calculator literacy test proved to be a valuable resource in helping me gain a deeper insight into the students' usage of the calculator and it will serve to guide me and others in the development of future teaching resources. From an educational perspective, I believe the test has provided a starting point for further research into the area of calculator literacy and how it should be taught and assessed in the classroom.

DEDICATION

To my wife Evelyn, for her love, support, understanding and encouragement

To my son Jared, for his patience

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INTRODUCTION

Statement of Problem

Cathy Lord (2003) of CanWest News Service reported in the Edmonton Journal that many parents, employers and trustees in the Edmonton school district have concerns that students are becoming too reliant on calculators. They believe that students have lost the ability to do pencil-and-paper or mental calculations and that the calculator has become a crutch rather than a tool. Cal Jensen, an assistant principal and Grade 9 mathematics teacher in Edmonton, was quoted to say that even for the simplest of computations students were relying on the calculator, and if the teacher did not allow the students to use the calculator then many of them were lost and unable to proceed with a question. In an attempt to rectify this problem, at least 10 public elementary and junior high schools, in the Edmonton area, will limit the use of calculators in the classroom in the 2004 school year.

This is one side of the long-standing debate that asks to what degree calculators should be used in schools, if at all, and if so, at what grade calculators should be introduced. In the April 2001 National Council of Teachers of Mathematics (NCTM) News Bulletin, Lee V. Stiff, then the president of the NCTM, stated the following, which perhaps better illustrates the actual problem with students' use of calculators:

> A recent newspaper article critical of calculator use reported that calculators harm students' ability to learn mathematics. An example similar to the following was cited: A fifth grader, Tamika, age 10, says she likes to buy potato chips for 60 cents and chocolate chip cookies for $1.15. When asked to find the sum, she enters the numbers in her calculator--but forgets a decimal point. "Sixty-one dollars and 15 cents," she says. The

> article concludes that this and similar examples demonstrate that calculator use among elementary school students is bad. (p. 1)

Stiff went on to say that:

> What is bad are classrooms in which students do not employ their estimation skills in concert with their calculator use to decide about the reasonableness of answers. The rote use of calculators is no more appropriate than the rote memorization of basic facts. In each situation, students should acquire an understanding of the tools and concepts they employ. (p. 1)

What Stiff is suggesting is that Tamika is calculator illiterate. Indeed, Tamika knows how to push the buttons on her calculator, but she does not know how to check, with the use of her estimation skills, if the answer she obtained on her calculator is reasonable.

Knowing how to do something does not necessarily mean proficiency in the subject. Does Tamika represent the majority of students today? Do the students in our classes merely press buttons on their calculators or do they have some understanding as to what is happening? The development of handheld calculating technologies has come a long way, but whether the development of calculator literacy has made as much progress is yet to be determined. In fact, the term calculator literacy is foreign to most people. There has been little literature written on the matter and little data has been gathered on the calculator literacy level of students in British Columbia. The purpose of this study is to tackle this very problem of what calculator literacy is and how we measure it.

Definitions

According to the Integrated Learning Resource Package (IRP) by the Ministry of Education in British Columbia (British Columbia Ministry of Education, 2001, p. 3) the term "appropriate technology" refers to the use of calculators and computers. The term "graphing calculator" refers to any scientific calculator with graphing capabilities (NCTM, 1998b). In this study, the term graphing calculator refers to any scientific calculator that has graphing capabilities but not computer algebra system (CAS) capabilities. The term "non-graphing calculator" refers to features on the calculator that are not graphing calculator dependent. Questions requiring a non-graphing calculator can be done with a scientific or a graphing calculator as no advantages would be gained by using a graphing calculator for such questions. The term "graphing calculator dependent" refers to questions which require the use of the graphing feature of the graphing calculator. The term "non-graphing calculator dependent" refers to questions which do not require the use of the graphing feature of the calculator. Both of these terms refer to questions which do not require the use of a graphing calculator.

The term Integrated Resource Packages (IRP) refers to the documents that contain the mandated curriculum for British Columbia. The IRP contains some of the basic information required to implement the curriculum (British Columbia Ministry of Education, 2000, p. iii).

Significance of the Problem

Why is it so important that students be literate in the use of calculator technology? I believe that the answers to the importance of calculator literacy are the same as the answers to why numeracy is important.

We know it is important to be literate, to be able to read and write and to communicate effectively. It is readily apparent to us how important literacy is in the work place. Being calculator literate is important as well, though not often regarded as vital for an educated person in our society. In fact, the opposite is often true. Many Canadians regard their inability to deal with numbers as acceptable, even fashionable. So often we hear people, even professionals, make remarks such as "Math was always my worst subject" or "I can do a lot of things, but when it comes to numbers I am totally lost". We mistakenly lump our discomfort with simple arithmetic into a notion that dealing well with numbers is irrelevant, and even a wasteful preoccupation or a boring skill. Many feel that using a calculator is cheating and that calculator use is for the less intelligent (Wait & Demana, 1997a). We need to emphasize to our students the importance of calculator literacy, which is a subset of numeracy, as a foundation for careers in science, technology and managerial jobs.

In his paper *Improving Numeracy in Canada*; John Dingwall (2000) stated that:

> In a modern, knowledge- and technology-based economy and society in which networked computers are as common as VCRs, numeracy is increasingly important, and indeed essential both at the more basic and the more advanced levels. (p. 8)

The numeracy that Dingwall refers to includes the ability to use and manipulate calculators as well as other forms of technologies to solve everyday life problems.

One of the greatest obstacles to achieving calculator literacy in school is the notion that somehow calculator literacy and numeracy are disconnected. The common misconception is that one does not need to be calculator literate in order to be numerate or that one does not need to be numerate in order to be considered calculator literate. The common argument is that *any* student can press a few buttons on a calculator and come up with an answer. But that does not mean he/she knows or understands the mathematics required to solve the problem. On the other hand, some believe that a student who has learned how to solve a problem mathematically would surely be able to do it on the calculator because he/she has already understood the mathematical concepts required and it is a matter of pressing a few buttons on the calculator to execute the calculations.

Some mathematics educators feel students should either be allowed to use calculators in all that they do in class or not be allowed to use them in the classroom at all. Many teachers feel that students cannot achieve true numeracy with the integration of regular calculator use; that proficiency in one must come at the cost of the other (Pomerantz, 1997). It has been dubbed the Great Debate (Cowdery, 1999). Should calculators be allowed in the classroom? Some educators believe that extensive calculator use leads to a decline in students' basic arithmetic skills as well as to a poor understanding of the mathematics taught (Pomerantz, 1997).

In reality, to be calculator literate one has to be numerate (Center for Mathematics and Science Education (CMSE), 2003). It is not true that student can press a few buttons on his/her calculator and come up with the right answer. In order for students to arrive at the right answer, they need to understand how to use their calculator and to use it well (CMSE, 2003). As with any other skills in mathematics, calculator literacy must be taught and learned. In order to be considered fully numerate in the twenty-first century one has to be calculator literate as well. An innumerate person can get no further with the aid of a calculator than if he or she does not have one. He/she would be able to add, subtract, multiply and divide numbers but would have no understanding to verify his/her answers. By the same token, a person who is capable of doing mathematics mentally or on paper but has no knowledge of calculators and computers can only progress to a limited extent in today's world. It is no longer about one or the other. Being calculator literate is not an option for a numerate person anymore.

Calculator literacy is as important in the twenty-first century as computer literacy. "The calculator and the computer are tools for learning and doing mathematics in ways that were not possible a few years ago" (Alexander & Kelley, 1998a, p. xii). We must strive to gain a better understanding of this new literacy to aid in the designing of appropriate methods of delivery to our students. However, gaining an understanding of this literacy is no small feat. Calculator Literacy is a new field of study and there are more aspects of this literacy to be examined than can be examined within the scope of this study. Therefore, the goals of this study are to:

- establish a working definition of what a calculator literate person is,
- examine what to test and how to test for this new literacy –that is, to examine the logistics of testing for calculator literacy as well as its problems,
- devise a test to assess calculator literacy for high school students,
- administer this test, collect data and observe patterns and trends, and
- provide grounds for further study to investigate if students today are indeed leaving our high school system calculator literate.

Structure of the Thesis

This thesis consists of five chapters. Chapter One states the problem and the goals of the study. This section also includes relevant definitions used in the study.

Chapter Two is split into three main sections. Section One provides a historical review of the development of handheld computing technology, namely scientific and graphing calculators. Section Two focuses on analyzing documents pertinent to the study. Section Three examines the definition of what a calculator literate person should be.

Chapter Three addresses the setting and data collection in the study. This chapter includes description of the Calculator Literacy Test (CLT), the rationale behind the questions and the results obtained.

Chapter Four analyzes the data collected in the CLT. As well, it contains my own personal observations from the study.

Chapter Five describes what we observed from the CLT. It is divided into three sections. The first section is an overview of the results from the calculator literacy test. It attempts to explain the results, relate them to the literature, and put the results of the study into perspective. The second section describes what changes can be made to improve the CLT. The final section contains my concluding remarks.

CHAPTER TWO
HISTORCIAL REVIEW, DOCUMENT ANALYSIS, AND DEFINITIONS OF CALCULATOR LITERACY

This chapter is presented in three sections, each playing a role in understanding what lies behind the concept of calculator literacy. The first section, the historical review will show pertinent background into the development of handheld computing technology with emphasis on the development of the scientific and the graphing calculator over the last thirty years. This section answers the following questions: When did handheld computing technology emerge and what impact it has made? Who are the major players in the educational calculator market and what are the technologies capable of? And finally, what are the different entry systems are that are found in today's calculators. The second section examines some of the documents related to helping define calculator literacy. The Principles and Standards by the NCTM (2000) as well as various documents from the British Columbia Ministry of Education and other sources will be examined. Lastly, the third section will take an in-depth look at the Graphing Calculator Resource Package produced in 1998 by the British Columbia Ministry of Education.

A Brief History of the Scientific Calculator

It is easy to take for granted the machines we use almost daily – like the calculator. Some people consider the calculator a blessing while others consider it a crutch (Pomerantz, 1997, p. 4). Many people do not realize that the hand-held scientific calculator is over 30 years old (The Museum of Hewlett-Packard Calculator, 2003), and that it has its roots in ancient machines that are still used today. The scientific calculator has forever changed the way we compute numbers.

The need to perform arithmetic calculations has been around for a long time. Rocks and notches in wood and pebbles served as prehistoric forerunners to memory-aid counting devices. One of the best-known devices, the abacus, is an ancient counting instrument of unknown origin (The Abacus, 2003). The Chinese used the *suan-pan,* the best-known form of abacus, to help keep count. The *suan-pan* is a series of beads strung on parallel wires, two above a bar, five below, in a rectangular frame. Each bead has an assigned value and a place, for instance, 1's, 10's and 100's. As the beads are counted, they are shifted in one direction. Since the location of the bead gives its value, one can use a few beads to represent large numbers. To erase a value, one can simply shift the abacus to reset the bead counters. The abacus is still in use today.

One of the earliest machines used to add numbers without someone having to know how to add, or count, was a series of gear wheels and worm gears used by the ancient Greeks to measure how far a carriage travelled. These machines date from the 2nd century AD. We still use machines of this type in the form of gas and water meters and odometers.

By the mid-17th century, Blaise Pascal, a French mathematician, had converted the odometer into an adding machine that could add a column of up to eight figures (Calculating Machines, 2003). The German mathematician Von Leibniz expanded on Pascal's adding machine in order to produce a multiplying calculator. Later, the mechanics in these machines were the basis for the hand-cranked adding machines of the 18th and 19th centuries, which were replaced by electromechanical adding machines of the mid 20th century. These were eventually replaced by today's electronic and digital technologies.

In 1972, Hewlett Packard (HP) created the HP-35, the first scientific calculator to be sold to the general public (Waits & Demana, 1998, p. 1). It had a 10-digit mantissa and a 2-digit exponent on a red light emitting diode (LED) display. The world of handheld computing took a giant step forward. Though it cost $395 and weighed 8.7 ounces and is not what we would consider to be user friendly nowadays, with its rigid press down style keypad, the public's reaction to this product was spectacular. However, Hewlett Packard's niche in this technology did not last long. Other companies soon followed and the calculator war was on. The flood of handheld or "pocket" scientific calculators that followed not only propelled the technology to new heights but it also drove the price of this new technology down to the point where every household could afford to have one. Companies such as Aristo, Compucorp, Victor, Canon, Wang, Casio, Texas Instruments, Hayakawa (Sharp), and many others competed with Hewlett Packard in the handheld calculating market, each trying to establish itself as the dominant manufacturer of this popular technology. But the competition was fierce, prices started to drop dramatically, and many companies started to leave the field. By 1990, the war of the pocket scientific calculator was over (Redin, 2002). Only a few companies survived, among them were Hewlett-Packard and Texas Instruments in the USA, and Sharp Electronics and Casio, Inc. in Japan. Other companies that make handheld scientific calculators today are Canon Inc. and Victor Technologies, which make Algebraic Left-to-right scientific calculators (Graham & Tyler, 1985, p. 33), such as the Canon F-502 and

Victor Technologies' 930-2, but they do not have any significant share of the market nor are they leading the field in handheld computing technology.

The advancement in calculator technology gave birth to the graphing calculator in 1985. Other than the top four companies mentioned above no other company has ventured into this realm. Of those four companies, Texas Instruments has clearly dominated the graphing calculator field, both in market share as well as development.

From an educational standpoint, two of the most important legacies of the great calculator war that have had great educational impact are the advancement of the technology and the reduced cost of scientific calculators. These two factors made the scientific calculator a useful and affordable option for mathematics educators and students. The introduction of the Complementary Metal-Oxide Semiconductor (CMOS), low energy consumption chips, and the Liquid Crystal Display (LCD) that replaced the power hungry LED's in the second half of the 1970's, made the calculators more efficient, cheaper, and smaller (Waits & Demana, 1997b). Students across the United States and Canada could afford to own one of these devices for a fraction of the cost of the original HP-35. The scientific calculator was a much more user-friendly device helping students compute transcendental functions with ease. Tables and slide rules were a thing of the past and more time was freed up in class to deal with mathematical concepts instead of tedious computations and data interpolations.

Recent technological improvement in scientific calculators has made the scientific calculator a useful and intuitive learning tool for mathematics students of all levels (Kaput, 1992). Sharp, formerly known as Hayakawa Electric Industry Co. Ltd., introduced its Direct Algebraic Logic (DAL) system for data input in the mid 1990's.

This "Hierarchy Algebraic" style of keystroke entries, made calculator use much more intuitive with its "what you see is what you type" approach. Before the arrival of the hierarchy algebraic entry system, calculators operated on either the Reverse Polish Logic System (RPL) or the left-to-right Algebraic entry system, both of which were awkward to use (Goldberg, 1982, p. 7). Systems similar to Sharp's DAL, such as Casio's Visually Perfect Algebraic Method (VPAM) and Texas Instruments' Equation Operation System (EOS) followed soon after. Today, the four major manufacturers of scientific and graphing calculators all have some form of their own "hierarchy algebraic" entry system. The traditional left-to-right calculator has always been awkward and not intuitive in its input format. Weaker mathematics students struggled with its use and never realized the full potential of their calculators. For example, on a left-to-right calculator $\sin 30^{o}$ must be entered as 3 0 sin, while an Algebraic system would allow you to type it in as you see it. However, more than the issue of the order in which functions needed to be entered into the calculator, what most left-to-right calculators lacked was a clear and intuitive set of bracket keys. The "what you see is what you type" approach of the "Hierarchy Algebraic" (Goldberg, 1982, p. 7) systems allowed for a more intuitive approach to lengthy and complicated computations. The "what you see is what you type" system allows students to make easy connections between what they see on paper in the mathematic textbooks and what they see on the calculator display. Rather than concentrating on the syntax system of the calculator the DAL system allows students to concentrate more on the mathematics.

For example, given the following calculation: $(-5.3+2.1)^2-4(6.92\div 2)$

on a Left-to-right calculator one would enter:

(5 • 3 (−) + 2 • 1) x^2 − 4 ×
(6 • 9 2 ÷ 2) ENTRY

On a left-to right calculator the screen shows an answer of *3.46* after the last bracket. If the user missed the ENTRY then he/she would mistakenly accept *3.46* as the answer instead of -3.6 . However, on an algebraic system one would simply enter:

((−) 5 • 3 + 2 • 1) x^2 − 4 (
6 • 9 2 ÷ 2) ENTRY

and obtain the correct answer of -3.6 .

The difference between the two systems is that with the left-to-right system, students use the calculator as a supplementary tool after they have learned how to perform order of operations, while a direct algebraic system calculator allows student to integrate the calculator as part of their learning tool as they learn about the order of operations. Furthermore, the introduction of the 2-lines display feature as well as the cursor keys found on many scientific calculators today allow students to see what they have entered into their calculator and editing incorrect keystroke entries has become much easier.

Over the thirty years since the early days of January 1971 when the first HP-35s were produced the scientific calculator has evolved and finally reached a user-friendly status. Not only is it a great technological advancement in the field of handheld computing, but it is also a great advancement in the field of mathematics education. However, as good and as user-friendly as the scientific calculator is, it still leaves the user in the dark when it comes to functions. The scientific calculator would not help a student see or visualize the shape and behaviour of a function without performing a lot of the tedious calculations, time that can be better spent on exploring the mathematical properties of the function instead. This brings the discussion to the introduction of the graphing calculator. In the next section we will take a brief look at the development of graphing calculator technology that has shed new light on the way we can teach mathematics.

A Brief History of the Graphing Calculator

"A little more than 15 years ago calculator manufacturers took a giant evolutionary step forward and added new software functionality in Read Only Memory (ROM) found only on desktop personal computers" (Waits & Demana, 1997b, p. 3). Casio Japan invented the graphing calculator, the Casio FX-7000, in 1985 (Demana-Waits Math Education, 2003). The lead engineer on the Casio 7000 development team was Hideshi Fukaya. As with the Hewlett Packard, Casio's niche in the graphing calculator market did not last for long. The other three major manufacturers, in the scientific calculator market also progressed into the manufacturing of graphing calculators. Close behind Casio's FX-7000 was Hewlett Packard's HP-28C in 1987, Sharp's EL-5200 in 1988, and Texas Instruments came out with its TI-81 in 1990. This last machine took the educational community by storm.

Graphing calculators started a revolution in the teaching and learning of mathematics in the United States and Canada as well as in many other countries. Graphing calculators were looked upon as hand-held computers with built-in graphing software that were available to all students because of their relative ease of use and portability. But like its predecessor, the scientific calculator, it is the affordability aspect that really allowed the graphing calculator to have an edge over technologies such as laptop computers and handheld PCs. Graphing calculator prices, like the scientific calculator, have dropped steadily since its inception and are now considerably less than prices for laptop computers and Personal Digital Assistant (PDA) devices.

Since Casio's FX-7000, graphing calculator technology has continued to make considerable developments. Texas Instruments' first graphing calculator, the TI-81, was widely accepted even though it could not solve simultaneous equations by either finding their zeros or by finding their intersections. The TI-82, which improved upon the TI-81, was equally well received. However, it was the TI-83 that made its mark in the secondary school system. By far one of the most successful graphing calculators of the late 1990's, the TI-83 took a stranglehold on the graphing calculator market. More learning resources have been written for the TI-83 than any other graphing calculator. Casio's CFX-9850 with its colour display, Sharp's EL-9650 with its built-in Equation Editor and pen-touch operation system and Hewlett Packard's RPL, a much more efficient entry system, and its educational applets feature, could not match the popularity of the TI-83 and its successor, the TI-83Plus.

Graphing calculators today have spreadsheet and animation capabilities. They link easily with a PC or one another to easily transfer of data and programs. They have superior projection facilities and are able to link with data loggers, PCs or the Internet. They have increased memory and improved ease with which programs can be transferred from PC to calculator and calculator-to-calculator. This makes the technology so much more powerful than the scientific calculator. With its new technological advancement in upgradeable FLASH memory, increased internal storage space and faster processors, the graphing calculator has become an invaluable tool in the field of mathematics education (Waits & Demana, 1997b).

Analysis of Related Documents

In this section the Principles of Mathematics 12 Calculator Resource Package will be analyzed along with various supporting documentation. These documents are:

- NCTM Position Statement: Calculators and the Education of Youth (NCTM, 1998a)
- NCTM Position Statement: The Use of Technology in the Learning and Teaching of Mathematics (NCTM, 1998b)
- Principles and Standards for School Mathematics (NCTM, 2000)
- Integrated Resource Package: Mathematics K TO 7 (British Columbia Ministry of Education, 1995)
- Integrated Resource Package: Mathematics 8 TO 9 (British Columbia Ministry of Education, 2001)
- Integrated Resource Package: Mathematics 10 TO 12 (British Columbia Ministry of Education, 2000)
- Principles of Mathematics 12 Calculator Resource Package (British Columbia Ministry of Education, 1998b)
- Applications of Mathematics 12 Calculator Resource Package (British Columbia Ministry of Education, 1998a)

A Brief Examination of NCTM's Two Position Statements

In July 1998, the NCTM released two position statements, The Use of Technology in the Learning and Teaching of Mathematics (NCTM, 1998b) and The Calculators and the Education of Youth (NCTM, 1998a). These two statements will help to provide an understanding of the importance of calculator literacy as well as a foundation to understand and define what calculator literacy is.

The position statement titled The Use of Technology in the Learning and Teaching of Mathematics states that it is essential that teachers continue to explore the impact of instructional technology and the perspectives it provides on an expanding array of mathematics concepts, skills, and applications (NCTM, 1998b).

One such impact is the concept of calculator literacy. The calculator has changed how we fundamentally deal with numbers in today's world. From the calculation of percentage discounts to the reconciliation of bank statements, the use of the calculator has become an integral part of society and our lives. It does not change or replace our need to have a firm understanding of the mathematics behind such applications nor does it change or replace our need to have an intuitive sense for numbers, but rather it has enhanced and expedited the way we handle numbers by allowing us to execute computations quickly and to analyze, visually as well as numerically, multiple scenarios rapidly. Technology has changed the ways in which mathematics is used and has led to the creation of both new and expanded fields of mathematics study. People today need to extend their number sense to include the ability to evaluate calculator produced results.

One of the arguments of opponents to the integration of calculators in the high school environment is the so-called "black-box experience" (Lesser, 2001, p. 1). This is like our earlier example of Tamika, who was seen simply entering numerical computations into her calculator. She copied down whatever the calculator displayed on the screen without any thought or understanding as to how the calculator arrived at the answer or whether the answer presented made any sense at all. Educators in the early stages of the ever evolving integration of technology into the mathematics classroom treated the calculator as nothing more than a fancy adding machine that can perform numerical computations (Waits & Demana, 2000). Students had no intuitive understanding of how the wires and transistors within the calculator arrived at an answer and often had no intuitive sense to determine if the answer in a given computation was reasonable or not. Hence, the term "black-box experience" (p. 1) came about to describe how students were completely in the dark as to how the calculator operates. All they had to do was to punch in some numbers and the little box would provide an answer. Early mathematics educators viewed the calculator as the last resort, only to be used when the numbers within a particular problem were too awkward to compute by hand. This, of course, compounded the problem as students had difficulties grasping the magnitude of the results from such calculations to have an intuitive sense of their answer, thus perpetuating the black-box experience.

However, the integration of handheld calculating technology is evolving beyond that. Today, with features such as Graphing and CAS, calculator use has been shown to enhance cognitive gains in areas that include number sense, conceptual development, and visualization (NCTM, 1998b, p. 1). It is true that many of today's students know no more of how the compact circuits within their calculators arrive at the answer displayed on their calculator screens than students before, but they are gaining a much deeper understanding of its application in exploring and solving mathematical problems. After all, understanding the mechanics and circuitries of a calculator was never the main focus of mathematics education. Its primary focus is educating students in solving mathematical problems. The NCTM states that teachers must develop students' ability to know how and when to use a calculator. Skill in estimation, both numerical and graphical, and the ability to determine if a solution is reasonable are essential elements for the effective use of calculators (p. 2). In other words, mathematics educators must ensure students are calculator literate. In addition, the NCTM goes on to say that instruments designed to assess students' mathematical understanding and application must acknowledge students' access to, and use of, calculators (p. 2).

A Brief Examination of the BC IRP Rationales

The Integrated Resource Package (IRP) provides some of the basic information that teachers in British Columbia are required to implement from kindergarten to grade twelve (K-12). The IRP for mathematics is divided into three documents with each document containing the intended learning outcomes (LOs), illustrative examples, learning resource lists as well as the rationale behind the British Columbia mathematics curriculum. The kindergarten to grade seven (K-7) package was released in 1995 while the grade eight to nine (8-9) package and the ten to twelve (10-12) package were released in 2000 and 2001, respectively. In the three packages the IRP's introduction provides general information about the Grade 1 to 12 mathematics curriculum, including special features and requirements. It also provides a rationale for the subject—why mathematics is taught in BC schools—and an explanation of the curriculum organizers. Under the Rationale section of each of the three packages, the IRP provides clarification on the use of technology in mathematics.

The K-7 package of the IRP echoes the philosophy found in the two NCTM position statements on technologies and calculator use. It states that new technology has changed the kind of mathematics problems encountered today, as well as the methods that mathematicians use to investigate them (British Columbia Ministry of Education, 1995, p. 3). It acknowledges the fact that computers and calculators are powerful problem solving tools (p. 3) and their use should be integrated into the curriculum. In fact, the 8-9 package of the IRP go on to state that "students [are required to] be proficient in the use of a [calculator] as a problem solving tool" (British Columbia Ministry of Education, 2001, p. 3).

However, the IRP also clarifies the appropriate use of the calculator by stating that:

> It is important to recognize that calculators and computers are tools that simplify, but do not accomplish, the work at hand. The availability of calculators does not eliminate the need for students to learn basic facts and algorithms. Students must have access to and be able to select and use the most appropriate tool or method for a calculation (p. 11).

The IRP recognizes the role of the calculator within the overall framework of mathematics education, acknowledging its benefits and possible drawbacks. It reinforces the student's need to master basic facts and algorithms while encouraging students to select and use the most appropriate tool or method for a calculation. This is important because at the heart of calculator literacy is the ability to know when to use the calculator and when not to use it, as well as having mastered enough mathematics skills to be able to determine if the answer provided from the calculator is reasonable or not.

The IRP goes on to deal with the concepts of Estimation and Mental Math in the next section of the rationale. It states that mathematics involves more than exactness. Estimation strategies help students deal with everyday quantitative situations. Estimation skills also help them gain confidence and enable them to determine if something is mathematically reasonable (British Columbia Ministry of Education, 2000, p. 4). In the early stages of integrating handheld computing technology into the mathematics curriculum, many educators struggled to achieve a balance between the incorporation of estimation, mental math, and the use of the calculator. Some educators viewed the calculator as nothing more than a computational machine and only made use of it when the numbers in a problem were too awkward, making it difficult for students to apply their estimation and mental math skills. Other educators overemphasized the role of the calculator at the cost of developing proper estimation and mental math skills. As we will see later on in calculator literacy, estimation and mental math are interdependent and that a high degree of estimation and mental math must be employed when we are solving mathematical problems with a calculator. Although students may have access to calculators from Kindergarten to Grade 12, they need to use reasoning, judgment, and decision making strategies when estimating. Instruction should therefore emphasize the role that these strategies play (p. 4).

The Principles and Standards for School Mathematics and the BC Mathematics IRP

In the Principles and Standards (NCTM, 2000) as well as the IRP (Ministry of Education, 1995, 2000, 2001), the importance of proper calculator use and its benefits in producing mathematically literate students were stated. These two documents helped guide us to visualize what it means by the phrase *appropriately applied, the calculator is a very useful tool for investigating, exploring, conjecturing, and computing a variety of quantitative and mathematical problems.* However, the reader of these two documents is left to search out what set of calculator skills is expected of the student.

Table 1 summarizes selected expectations relevant to this study as outlined in the Principles and Standards.

Table 1 Summary of Selected Expectations from the NCTM Principles and Standard

NCTM Standard		
Standards	*Expectations*	*Expectations*
Number and Operations		
	In prekindergarten through grade 2 all students should—	**In Grades 3-5 all students should—**
Compute fluently and make reasonable estimate	Use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.	Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tool.
	In Grades 6-8 all students should—	**In Grades 9-12 all students should—**
Compute fluently and make reasonable estimates	Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;	Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more- complicated cases. Judge the reasonableness of numerical computations and their results.
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;	Develop a deeper understanding of very large and very small numbers and of various representations of them;

NCTM Standard		
Standards	*Expectations*	*Expectations*
	Algebra	
	In Grades 6-8 all students should—	**In Grades 9-12 all students should—**
Understand patterns, relations, and functions		Intercepts, zeros, asymptotes, and local and global behaviour. Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
Represent and analyze mathematical situations and structures using algebraic symbols		Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases
Use mathematical models to represent and understand quantitative relationships	Model and solve contextualized problems using various representations, such as graphs, tables, and equations.	Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships. Draw reasonable conclusions about a situation being modelled.
Analyze change in various contexts	Use graphs to analyze the nature of changes in quantities in linear relationships.	Approximate and interpret rates of change from graphical and numerical data.

NCTM Standard		
Standards	*Expectations*	*Expectations*
	Geometry	
	In Grades 6-8 all students should—	**In Grades 9-12 all students should—**
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships		Use trigonometric relationships to determine lengths and angle measures.
	Measurement	
	In Grades 6-8 all students should—	**In Grades 9-12 all students should—**
Understand measurable attributes of objects and the units, systems, and processes of measurement	Understand relationships among units and convert from one unit to another within the same system;	
Apply appropriate techniques, tools, and formulas to determine measurements		Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
	Data Analysis and Probability	
	In Grades 6-8 all students should—	**In Grades 9-12 all students should—**
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them		Compute basic statistics and understand the distinction between a statistic and a parameter.
Select and use appropriate statistical methods to analyze data		For univariate data, be able to display the distribution, describe its shape, and select and calculate summary statistics; For bivariate data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools;
Develop and evaluate inferences and predictions that are based on data		Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions;

Table 2 presents a brief summary of the selected expected calculator skills found in the K-7 IRP, indicating the range of mathematical situations students should be able to solve with the aid of a calculator as well as be able to manipulate the mathematics be it represented in a numerical, spatial, graphical, statistical and algebraic manner. Not all of the LOs in the IRP are included in the table, only those that were addressed in the Calculator Literacy Test developed in this study.

Table 2 Summary of Selected LOs from the K-7 IRP Learning Outcomes

Prescribed Learning Outcomes Mathematics K to 7		
Number and Operations		
Grade K to 1	**Grade 2 to 3**	**Grade 4**
Use a calculator or computer to explore and represent numbers up to 100	Explore, represent, and describe numbers to 1000 in a variety of ways, including the use of calculators and computers Calculate and justify the methods they used to find sums, differences, products, and quotients using estimation strategies, mental math techniques, manipulative, algorithms, and calculators Verify their solutions to problems by using inverse operations, estimation, and calculators	Verify solutions to multiplication and division problems by using estimation and calculators

Prescribed Learning Outcomes Mathematics K to 7		
Grade 5	**Grade 6**	**Grade 7**
Estimate, mentally calculate, or compute and verify the product (three-digit by two digit numbers) and quotient (three-digit by one-digit numbers) of the multiplication and division of whole numbers		Add, subtract, multiply, and divide decimal fractions (using technology for more than two digit divisors or multipliers) Demonstrate an understanding of the order of operations, using paper and pencil and a calculator Estimate and calculate percentages
	Patterns and Relations	
Grades K to 1	**Grades 2 to 3**	**Grades 4**
		Identify and explain mathematical relationships and patterns through the use of grids, tables, charts, or calculators
	Statistics and Probability	
Statistics and Probability	Display data by hand or by computer in a variety of ways, including histograms, double bar graphs, and stem and leaf plots Estimate, measure, and calculate the volume of composite 3-D objects Display data by hand or by computer in a variety of ways Use various data-collection techniques (including computers) to simulate and solve probability problems	Display data by hand or by computer in a variety of ways, including circle graphs Use simulation or experimentation to solve probability problems

In the 8-9 IRP, the Ministry of Education states that the Grades 8 and 9 Mathematics curriculum requires students to be proficient in using technology as a problem-solving tool (British Columbia Ministry of Education, 2000, p.3). Table 3 summarizes the different learning outcomes and their calculator requirements.

Table 3 Summary of Selected LOs from the 8-9 IRP Learning Outcomes

Prescribed Learning Outcomes Mathematics 8 to 9		
Topic	**Grade 8**	**Grade 9**
	Number and Operations	
Problem Solving	Use appropriate technology to assist in problem solving	Use appropriate technology to assist in problem solving
Number Concept	Distinguish between a square root and its decimal approximation as it appears on a calculator Represent any number in scientific notation Estimate, compute (using a calculator), and verify approximate square roots of whole numbers	Give examples of situations where answers would involve the positive (principal) square root or both positive and negative square roots of a number Document and explain the calculator keying sequences used to perform calculations involving rational numbers
Suggested Extensions	Estimate, compute (using a calculator), and verify approximate square roots of decimals	Use a calculator to perform calculations involving scientific notation and exponent laws
Shape and Space	Use the Pythagorean relationship to calculate the measure of the third side of a right triangle, given the other two sides in 2-D applications Estimate and calculate the area of composite figures	Demonstrate the use of trigonometric ratios (sine, cosine, and tangent) in solving right triangles Calculate an unknown side or an unknown angle in a right triangle, using appropriate technology

Prescribed Learning Outcomes Mathematics 8 to 9		
Topic	**Grade 8**	**Grade 9**
Suggested Extensions	Estimate, measure, and calculate the surface area and volume of any right prism or cylinder Estimate, measure, and calculate the surface areas of composite 3-D objects Estimate, measure, and calculate the volume of composite 3-D objects	Calculate and apply the ratio of area to perimeter to solve design problems in two dimensions Calculate and apply the ratio of volume to surface area to solve design problems in three dimensions
Statistics and Probability	Display data by hand or by computer in a variety of ways Use various data-collection techniques (including computers) to simulate and solve probability problems	

Starting in Grade 10 the BC mathematics curriculum offers students a choice of three different pathways through the different mathematics courses offered. However, no matter which pathway a student chooses, the Grade 10 to 12 Mathematics curriculum requires students to be proficient in using technology as a problem-solving tool (British Columbia Ministry of Education, 2000, p. 3). Table 4 summarizes some of the main Learning Outcomes and the calculator skills required.

Table 4 Summary of Selected LOs from the 10-12 IRP Learning Outcomes

Prescribed Learning Outcomes Mathematics 10 to 12	
Applications of Mathematics 10	
Problem Solving	Use appropriate technology to assist in problem solving
Pattern and Relations	Use a graphing tool to draw the graph of a function from its equation
Statistics and Probability	Use technological devices to determine the correlation coefficient *r*
Application of Mathematics 11	
Problem Solving	Use appropriate technology to assist in problem solving It is expected that students will use a variety of methods to solve real-life, practical, technical, and theoretical problems.
Patterns and Relations	Solve non-linear equations, using a graphing tool. Solve systems of linear inequalities in two variables using graphing technology Apply linear programming to find optimal solutions to decision making problems
Applications of Mathematics 12	
Problem Solving	Use appropriate technology to assist in problem solving
Number	Model and solve problems, including those solved previously, using technology to perform matrix operations of addition, subtraction, and scalar multiplication as required Model and solve consumer and network problems using technology to perform matrix multiplication as required
Patterns and Relations	Collect sinusoidal data; graph the graph using technology, and, represent the data with a best fit equation of the form: • $y = a \sin (bx + c) + d$ Use technology to generate and graph sequences that model real-life phenomena Use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure
Statistics and Probability	Find the population standard deviation of a data set or a probability distribution, using technology Use the normal approximation to the binomial distribution to solve problems involving probability calculations for large samples (where npq>10)

Prescribed Learning Outcomes Mathematics 10 to 12	
Essential of Mathematics 10	
Problem Solving	Use appropriate technology to assist in problem solving
Trigonometry	Use the trigonometric ratios sine, cosine, and tangent in solving right triangles
Probability and Sampling	Use suitable graph types to display data (by hand or using technology)
Essential of Mathematics 11	
Problem Solving	Use appropriate technology to assist in problem solving
Income and Debt	Use simple and compound interest calculations to solve problems
Essential of Mathematics 12	
Problem Solving	Use appropriate technology to assist in problem solving
Principles of Mathematics 10	
Problem Solving	Use appropriate technology to assist in problem solving
Numbers	Perform arithmetic operations on irrational numbers, using appropriate decimal approximations
Principles of Mathematics 11	
Problem Solving	Use appropriate technology to assist in problem solving
Pattern and Relations	Solve systems of linear equations, in three variables with technology Determine the solution to a system of non-linear equations, using technology as appropriate
Principles of Mathematics 12	
Problem Solving	Use appropriate technology to assist in problem solving
Patterns and Relations	Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0. Solve first and second degree trigonometric equations over a domain of length 2π: Analyse trigonometric identities graphically Draw (using technology), sketch, and analyse the graphs of sine, cosine, and tangent functions, for: • amplitude, if defined • period • domain and range • asymptotes, if any • behaviour under transformations
Statistics and Probability	Find the standard deviation of a data set or a probability distribution, using technology

The Principles and Standards document is an easy document to read as it provides an overall picture of how the K-12 mathematics curriculum should look in the Table of

Standards and Expectations. Separating out the intended calculator skills that students at each grade level are expected to learn was not difficult. The IRP however, provided more of a challenge as students in British Columbia are split into three different pathways starting in their grade ten year. Skills acquired and expected to be mastered from one pathway are not necessarily the same as those found in a different pathway even though the grade levels are the same. The result of this split in pathways means that students leaving high school in British Columbia are expected to have mastered a core set of calculator skills that all students should have mastered by grade nine while different sets of additional calculator skills are expected of students in the various pathways. Both the Application and Essential pathways are more calculator driven whereas the Principle pathway integrates the use of the calculator while retaining a significant amount of traditional algebraic manipulation. Furthermore, with courses like Essentials of Mathematics 10, 11 and 12 as well as the Applications of Mathematics 10, 11 and 12 where regular calculator use is assumed and expected the LOs often do not explicitly state the use of a calculator as required or recommended for the completion of the outcomes; thus the LOs do not always acknowledge the required calculator skills. For example, in the Income and Debt section of the Essentials of Mathematics 11 course, one of the learning Outcomes is to use simple and compound interest calculations to solve problems (Baron, Bradford, Kaisser, Sufrin, Tambellini & Wunderlich, 2002, p. 34). Though it is possible to solve some of these problems by hand, in the Application of Mathematics 11 textbook by Addison-Wesley (Alexander & Kelley, 1998c), the official learning resource for the course, compound interest is computed exclusively on a calculator and not by hand. Students are expected to be competent in their use of the bracket keys as well as the exponent function on their calculator as they work through this learning outcome. Hence, even though the term "with technology" is not explicitly written in the LO, it is implied that students are expected to master the calculator skill associated with the LO. Therefore, even though the list of calculator skills expected to be learned by the students in grade 10-12 seems much shorter than the list for K-9 it does not mean that there are fewer calculator skills required in the senior courses. In fact, the opposite is true. Calculator use is an integral part of these courses but the calculator skills required for these courses are simply not listed explicitly.

To further complicate matters, in the IRP for 10-12, the term "graphically" (British Columbia Ministry of Education, 2000, p. 178) carries a slightly different meaning than the other two packages. In the K-7 and the 8-10 packages (British Columbia Ministry of Education, 1995, p. 13), to solve or analyse a mathematical situation "graphically" would imply that students should graph the situation out by hand. If the LO intended students to use the aid of a graphing calculator then it would state so by using the terms "with technology" or "with appropriate technology" (British Columbia Ministry of Education, 2000, p. 178). However, in the 10-12 package, to analyse a mathematical situation graphically can sometimes imply the use of technology. For example, in the Pattern and Relations section of the Principles of Mathematics 12 course, one intended LO is to analyse trigonometric identities graphically (p. 178). In both textbooks, Mathpower 12 (Knill, 1999) by McGraw Hill and Mathematics 12 (Alexander & Kelley, 1998b) by Addison-Wesley, the analysis of trigonometric identities is done exclusively on the graphing calculator. In fact, for complex trigonometric identities, analysis by hand is impractical and time consuming. Thus, it is not as easy to summarize all the intended calculator skills in a concise manner from the reading of the IRP as it is with the Principles and Standards. However, the two documents together provide us with a foundation to visualize what the first part of our definition of calculator literacy should look like.

Though both the Standards and the IRPs provide general philosophies on the responsible use of calculators, a concrete definition on the concept of calculator literacy is missing from both documents. Neither the Standards nor the IRP attempted to make a distinction between novice calculator users and advanced users. The difference between competencies and literacy is missing and the reader is left to insert his or her own definition of what calculator literacy would look like. For this we will have to turn to other sources. But first we will need to examine and clarify the difference between calculator competency and calculator literacy and why it is important to distinguish the two.

In addition to examining the Standards and the IRP, another way of looking at what calculator skills are essential in order for a student to be considered calculator literate can be found in two other, more informal, sources. The Internet and the Calculator Resource Packages produced by the Ministry of Education in 1998. A quick search of the internet under the heading of calculator skills will yield a list of a number of post secondary schools' science department web pages outlining the basic calculator skills required to be successful in Chemistry or Physics courses. These are skills, which these various departments viewed as necessary for their students to master in order to be considered calculator competent.

A quick search on the Internet under the terms "chemistry" and "calculator" will yield a screen full of results linking to various university and college chemistry websites discussing the importance of being "calculator competent" before embarking on first or second year chemistry courses. Similar requirements can be found for other subjects.

Below is an excerpt from the Chemistry Learning Center, CLC, of the Virginia Tech Chemistry department. It outlines the importance of calculator mastery in the study of chemistry.

> A recent Peanuts cartoon displays Snoopy holding up a calculator with the expression, "I don't need to think, I have batteries "This statement is far from reality in any study that requires the use of mathematics. Calculators are invaluable tools when working with long equations and involved calculations. What you can do in a few seconds on your calculator now would have taken a chemistry student 20 years ago several minutes to do by hand. Even with this greater potential speed in problem solving, you have to set up the equations before you use the calculator. Besides writing the equations down on paper, you may have to rearrange them so that you can use your calculator's functions to solve the problem. You will need to become acquainted with your calculator so you will know how to properly set up the problem. The mastery of the equations and your calculator's operation will allow you to get these "speedy" answers. In your general chemistry course you will encounter problems using arithmetic operations (addition, subtraction, multiplication, and division), trigonometric operations (cos, sin, $\cos^{-1}$, and $\sin^{-1}$), and logarithmic operations (log, ln, 10^x, e^x, powers and roots). You will also deal with exponential notation in many of these problems. In the following sections, we will discuss these functions and how to use your calculator to solve problems in General Chemistry. (Virginia Tech Chemistry Learning Centre, 2003)

Table 5 below lists of some of the basic features found on most scientific calculators that many chemistry and physics departments require students to know how to use.

Table 5 Summary of Commonly Used Calculator Functions

Square Root	$\sqrt{\ }$
Square	x^2
Fraction	$a\,{}^{b}\!/\!{}_{c}$
Percent	%
Inverse	x^{-1}
Brackets	( )
Logs and Anti logs	$\log/10^x$
Basic Trig Functions	$\sin/\cos/\tan$
Arcsine, Arccosine, and Arctangent	$\sin^{-1}/\cos^{-1}/\tan^{-1}$
Constants such as Pi and e	π / e
Scientific Notation	EE / EXP
Exponents	$\wedge$ / y^x / x^y

Many science departments consider the above set of calculator skills essential to the success of their science students. Without a good mastery of the above set of calculator skills students would have a hard time dealing with the calculations required in most post secondary first year science courses.

The Graphing Calculator Resource Package

An examination of the two Calculator Resource Packages produced by the Policy, Evaluation and Analysis Branch of the Ministry of Education in May of 1998 for the old Principles of Mathematics 12 course and the Applications of Mathematics 12 course would yield a more concrete picture of what calculator literacy looks like and how we might go about evaluating it.

These two documents, especially the Graphing Calculator Resource Package for the then Principles of Mathematics 12, contains many examples which will help illustrate the following concepts and ideas in our working definition of Calculator Literacy.

- Knowing both the advantages and limitations of the calculator and knowing when the calculator should be used as well as the use of the calculator to develop one's own analysis of situations and adapt and utilize different methods and or strategies to work around the limitations.
- One should know the calculator's limitations and recognize when it is unnecessary and even awkward to use it.
- One should apply his/her estimation and mental math skills to determine the reasonableness of solution provided by the calculator.
- Having calculator literacy also means that one is able to grow and adapt to the changing technology as well as being able to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators.

Within these two documents we can see the difference between various levels of calculator literacy from straightforward calculations to a more advanced understanding of the workings of the graphing calculator. The *Graphing Calculator Resource Package* provides a general philosophy for the use of a graphing calculator as well as sample questions with comments to illustrate when the use of a graphing calculator may be a suitable method of solution, when it may be the best method of solution, and when it may in fact hinder the ease of the solution or be inappropriate (British Columbia Ministry of Education, 1998b, p. 3). The main focus of this section will be on the Principles of Mathematics 12 Graphing Calculator Resource Package, as it has been established longer and is offered at more schools to a greater percentage of the student population, but I will refer to the Application of Mathematics 12 Graphing Calculator Resource Package for a few selected items.

The Principles of Mathematics 12 Graphing Calculator Resource Package is divided into seven main sections. Section I contains the general philosophy of proper calculator use (p. 4), section II explicitly states the specific graphing calculator skills required for the course which can be expanded for general use (p. 5), sections III, IV, and V are exclusively related to the Principles of Mathematics 12 Provincial Examination (pp. 5-6), section VI consists of eight topic areas with sample questions and solutions (pp. 7-37). Section VI helps us to complete our definition of calculator literacy.

In the General Philosophy section of the Principles of Mathematics 12 Graphing Calculator Resource Package, it states that effective use of a graphing calculator presumes a new set of skills such as zooming in or out and setting a desirable viewing window by anticipating suitable domain and range values (p. 4). A black box experience approach to using a graphing calculator simply would not do because students must anticipate suitable domains and ranges for their solutions. The intelligent use of a graphing calculator (p. 5) is needed at all times. Students need to think about the best method of solution before they reach for their calculators (p. 5). This higher level of understanding and thinking is a hallmark difference between competency and literacy.

Section II of both calculator resource packages specified the specific graphing calculator skills required for the two courses. Table 6 below summarizes the skills required for the two courses (p. 5).

Table 6 Specific Graphing Calculator Skills

Specific Graphing Calculator Skills Required	
Principles of Mathematics 12	*Application of Mathematics 12*
Using a graphing calculator, a student should be able to: a) produce a graph within a specified viewing window b) determine an appropriate viewing window to view a graph and change the window dimensions c) use the zoom features of the calculator d) find zeros and intersection points (usually to 2 decimal point accuracy) e) find maximum or minimum points	Graphing calculator skills that will be helpful for students include the following: a) producing a graph within a specified viewing window b) determining an appropriate viewing window to view a graph and changing the window dimensions c) using the zoom features of the calculator d) finding zeros and intersection points e) finding maximum or minimum points f) using the table feature g) using matrix operations h) using the statistical capabilities including: regression equations, correlation coefficients, confidence intervals, list operations, standard deviation, etc. i) using the finance feature (if available) j) using the normal and binomial distribution features.

Section VI in the graphing calculator resource package contains questions by Topics. It consists of eight topics with various sample questions. An analysis of the sample questions found in sections C to G is given below.

Section C Patterns and Relations (Variables and Equations)

Part I

Q1 Solve: $x^3+3x^2-x-3=0$

Q2 Solve: $2x^3-5x^2-x+6=0$

Q3 Solve: $2x^3 - 59x^2 + 125x + 5100 = 0$

Q4 Solve: $x^3 - 5x + 7 = 0$

Q5 Solve: $x^3 - 5x + 2 = 0$

Q6 Solve: $x^3 - 12x^2 + x - 10 = 0$

Q7 Solve: $x^3 - 12x^2 + x + 50 = 0$

Q8 Solve: $x^4 - 2x^3 - 77x^2 - 6x - 100 = 0$

Q9 Solve: $x^5 - 10x^3 + 10 = 0$

Q10 Solve: $x^3 - 4x^2 - 128x + 770 = 0$

Examples of knowing when the calculator should be used can be seen in Q1, Q2 and Q5. All three questions can be done by hand using the traditional method of using the Rational Root Theorem and synthetic division to produce the depressed equation (p. 7). In fact, the first question can be factored by grouping. The graphing calculator could be used for these questions, although it would be more time-consuming. It is important that students be very familiar with traditional approaches (p. 7) and be able to decide when it is better to pick up the calculator and when to solve a problem by a different method. The roots to Q3 are rational; thus, it is possible to use the Rational Root Theorem to find the solutions. However, since the coefficients are so large, the graphing calculator could be a more appropriate method for solving this question.

Questions 4, 6, 7, 8, 9 and 10 however, are calculator dependent. Their roots are not rational. Part of being calculator literate means one is able to recognize when the solution to a problem is calculator-dependent. In addition, in Q10, students must be familiar with the general shapes of polynomial functions of different degrees, and they must have the skill of using a graphing calculator to obtain the proper solution as there appears to be a double positive zero. It is important to zoom into this apparent zero to discover that there is only one zero for this function (p. 8). This is an excellent example of where mere competency is not sufficient. A student who is able to operate the calculator and perform a series of previously learned procedures might not be able solve this problem correctly.

Part II

Q1 Solve: $2x^3 - x^2 - 6x + 3 \geq 0$

Q2 Solve: $x^3 + 3x > x^2 + 8$

Q3 Solve: $x^4 + 5x^2 < 6x^3 - 4x + 12$

Q4 For what values of x does the parabola $y = 2x^2 - 8x + 5$ lie above the cubic function $y = x^3 - 4x^2 + 3x - 2$?

Q4 For what values of x does the curve $y = 2x^3 - 6x^2$ lie below the curve function $y = x^4 + x^3 - 4x^2 - 20$?

Questions involving Inequalities are excellent questions for distinguishing between competent users and literate users, as the graphing calculator does not provide the students with the solution explicitly. Students must interpret the graph presented to them on the calculator and decide what the solution should be. This higher level of understanding represents knowing the limitations of the calculator and being able to work around them.

Section D Patterns and Relations (Relations and Functions I)

Q1 Solve: $\log_2(x+3) + \log_2(x+1) = 3$

Q2 Solve: $\log_2(x-2) + \log_4 x = 2$

Q3 Solve: $\log_3(x+2) + \log_4(x-1) = 5$

Q4 Solve: $3\log x = 4^{5-x}$

As in the previous section Q1 can be done on the graphing calculator but it is much easier and less time-consuming to do it by hand. Also, rewriting the question in order to be read by the calculator is a common skill that students must possess. The use of a graphing calculator to solve logarithmic equations presumes a new set of skills that must be practised. For example in Q2 and Q3, the change of base rule must be used to rewrite the given equation.

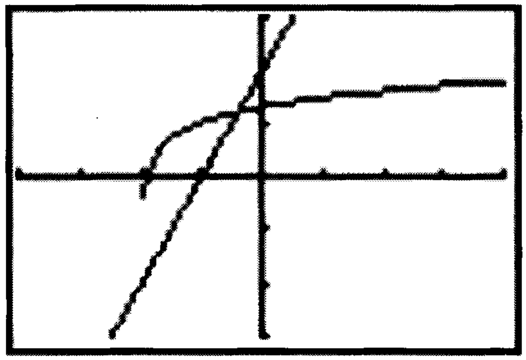
$$
\begin{aligned}
Q2\quad & Y_1 = \log(x-2)/\log(2) + \log(x)/\log(4) \\
& Y_2 = 2 \\
Q3\quad & Y_1 = \log(x+2)/\log(3) + \log(x-1)/\log(4) \\
& Y_2 = 5
\end{aligned}
$$

Therefore, proficiency in the use of the bracket keys on the calculator is a must.

Furthermore the Principle of Mathematics 12 Calculator Resource Package states that:

> When using [the graphing calculator], it is important that students be aware that there may be certain limitations. For example, when using a graphing calculator to solve a system involving a logarithmic function, students should remember their graphing techniques for logarithmic functions. A logarithmic graph is asymptotic and when drawn on graph paper the asymptote is a guide for drawing the curve. The graphing calculator does not show this asymptotic line and often the graph does not appear complete on the calculator screen near the asymptote. Students need to be aware that the graph continues even though their screen does not indicate this fact. (p. 22)

This is certainly true for the extra example question given on p. 22 as students need to be aware of the asymptotic nature of the logarithmic function, if not then they may mistakenly conclude that there is only one solution to this system.



$Y_1 = \log(x+2)+1$

$Y_2 = 2x+2$

x [−4, 4] y [−3, 3]

Figure 1 Graph of a Logarithmic Function

Even though the screen image (See Figure 1 above) does not show that these graphs intersect in two places, it is clear that if the logarithmic graph was extended further, there are two solutions, one at (–0.40, 1.21) and another at (−1.9988995, −1.997991). Similarly, Q4 is a calculator dependent question as it is much easier to find the solution using a graphing calculator.

Section E Patterns and Relations (Relations and Functions II)

Part I

Q1 The following table gives values for $Y_1 = x - 2$ and $Y_2 = (x-2)^2 - 2$. From the table, determine an intersection point of the graphs of Y_1 and Y_2.

X	Y1	Y2
-3	-5	23
-2	-4	14
-1	-3	7
0	-2	2
1	-1	-1
2	0	-2
3	1	-1

X=-3

Q2 Determine the real solution(s) for the following system.

$$y = -x^2 - 12x - 40$$
$$y = 3x - 10$$

Q3 Determine the real solution(s) for the following system.

$$xy = 12$$
$$y = 0.7x^2 - 0.5x + 2$$

There is an increased emphasis on the use of graphing technology to verify and illustrate solutions to systems of quadratic equations in recent years. Therefore, students should be familiar with solving quadratic systems algebraically, graphically, and numerically. For both Q2 and Q3, students could simply be asked for the number of different solutions; thus, setting an appropriate viewing window is one of the skills that students must practise when a graphical solution is chosen. This is one of the skills that make a student more calculator literate.

Part II

Q1 Determine the real solution(s) for the following system.

$$x = -5y^2$$
$$y = \cos x$$

Q2 Determine the real solution(s) for the following system.

$$x^2 - y^2 = 16$$
$$y = 4\log x$$

Once again, the above two questions test if the student knows the calculator's limitations. Since most handheld graphing calculators are only capable of graphing functions, students must rewrite the equations as functions in the form of "$y =$" instead. Alternately, students can also enter the system of equations as a system of parametric equations. Either way, students must understand that they are no longer dealing with a system of two equations but rather a system of three equations. The Principles of Mathematics 12 Calculator Resource Package states that there are a number of graphing calculator skills that students must practise for answering a question such as Q2, including:

- solving the equation of the hyperbola for y and entering the two implied functions,
- setting an appropriate viewing window, and
- finding intersection points.

When using the calculate feature to find the intersection point, students must be aware that they are finding the intersection of the graphs of the functions Y_1 (the top half of the hyperbola) and Y_3, therefore they must move the cursor using the down arrow to select these particular functions. Also, since there are domain restrictions involved, it is necessary to move the cursor over to an x-value that has a y-value before pressing enter.

Section G Shape and Space (Measurement)

Part I

Q1 $3\sin x + 5 = 4, \quad 0 \leq x < 2\pi$ Q2 $\sin 2x + \cos x = 0, \quad 0 \leq x < 2\pi$

Q3 $\cos x + 2 = \csc x, \quad 0 \leq x < 2\pi$

Trigonometric equations require students to understand the difference between radian modes and degree modes. Entering appropriate domain and range values within the specified period as well as setting appropriate scale factors are all calculator skills that students must practice in order to be proficient in the use of their calculators. Reciprocal trigonometric functions are not built-in features in the graphing mode on most graphing calculators and manipulation of such functions is required before entering into the calculator. Q1 and Q2 can be entered into the calculator in its usual manner by setting the left hand side of the equation as Y_1 and the right hand side of the equation as Y_2. However, Q3 requires the user to manipulate $\csc x$ as $(\sin x)^{-1}$ prior to entering the left hand side into the calculator (See figure 2).

$$Y_1 = \cos(x) + 2$$
$$Y_2 = (\sin x)^{-1}$$

Figure 2 Solving Trigonometric Equations

In addition, in the Applications of Mathematics 12 Graphing Calculator Resource Package it states that trigonometric functions should be graphed using radian mode and that students should be able to solve trigonometric equations by either the ZERO function or the INTERCEPT function. (British Columbia Ministry of Education, 1998a, p. 20), and that

> Students can use their graphing calculators to solve trigonometric equations, either by setting the equation equal to zero and finding the zeros of the related function, or by graphing each side of the equation as a function and finding points of intersection.

Being able to find zeros by various methods demonstrates a higher level of understanding than simply reproducing a set of previously learned procedures required for the given problem. In fact, sometimes one method of finding zeros to a problem can be more efficient than another and students who are literate in the various methods would be able to solve the problem more easily. For example, Q5 in the Patterns and Relations (Relations and Functions), section F, in the Application of Mathematics 12 Graphing Calculator Resource Package, is a good example to show when the intersection method is preferable (p. 15).

> Sam purchased $5000 worth of Skeema Resources stocks, which are expected to yield 6% compounded semi-annually and $6000 worth of Bru-X shares that are expected to yield 5% compounded quarterly. Determine when the two investments have equal value. What is this value?

Solution(s)

Method I By Intersections

Graph $Y_1 = 5000\left(1+\frac{0.06}{2}\right)^{2x}$ and $Y_2 = 6000\left(1+\frac{0.05}{4}\right)^{4x}$ and find where the two graphs intersect. The intersection point of Y_1 and Y_2 is the solution to the system.

$$(19.339282, 15685.394)$$

Answer: The solution to the problem is t = 19.34 years and A = $15 685.39

Method II By Setting the Equations to zero

Graph $Y_1 = 5000\left(1+\frac{0.06}{2}\right)^{2x} - 6000\left(1+\frac{0.05}{4}\right)^{4x}$ and find the zeros of the equation.

$$y = 5000\left(1+\frac{0.06}{2}\right)^{4(19.339282)}$$

The zero of Y_1 is 19.339282. Substituting this value gives:

or

$$y = 6000\left(1+\frac{0.05}{4}\right)^{4(19.339282)}$$

$$y = 15685.39$$

Answer: The solution to the system is $t = 19.34$ years, and $\therefore A = \$15\ 685.39$

Though this is not a trigonometric equation, it demonstrates the different way in which equations can be solved on the graphing calculator. In this case, method I gives both parts of the answer without additional substitution (p. 16).

Furthermore, it is important for students to understand that while the intersection points have coordinates in the form *(x, y)*; the original question was not presented as a system. Therefore students would only present the x-values of the intersection points in their response.

Part II

Q1 $\quad \begin{aligned} y &= 5\cos x \\ y &= \tan x \end{aligned}$ $\qquad$ Q2 $\quad \begin{aligned} y &= \sin\frac{\pi}{5}x \\ y &= 0.5x \end{aligned}$

Q3 $\quad \begin{aligned} y &= \sin \pi x \\ y &= \log_3(x+5) \end{aligned}$

Sometimes, a system of equations does not need to be solved. We might only be interested in how many solutions the system has. The Principal of mathematics 12 graphing calculator resource package states that:

Questions 1, 2 and 3 could have asked for the **number of solutions** over a specified interval. While this interval is usually $0 \le x < 2\pi$, the domain could be altered to, for example, $-4\pi \le x \le 4\pi$. For these types of questions, it is possible that students would also need to determine if the graph at a particular place cuts the x-axis in two places, is above the x-axis, or tangent to it. In the case of tangency, the wording of the question would instruct students to count different or distinct solutions only, thus a tangency (or double root) would count as a single solution.

In Q1, Q2 and Q3 a quick sketch of two functions over the proper interval would allow students to count the number of intersections easily. However, graphing calculators tend to draw in asymptotes, leading some students to a possible miscount of 8 solutions. Being able to recognize asymptote lines is and important skill for students to develop.

Part III

Q1 Graph at least one period of $y = -3\sin 2\left(x - \frac{\pi}{2}\right)$ on the grid provided.

Q2 a) What is the amplitude of $y = -3\sin 4x$

b) What is the amplitude of $y = 4 - 8\sin^2 x$

c) What is the amplitude of $y = \sin x + \cos x$

The Principles of Mathematics 12 Graphing Calculator Resource Package included two examples of graphing trigonometric functions in part III of section G. Here the skills of finding maximums and minimums are tested. All skills covered in these sample questions in the resources package are calculator skills that students need to learn in order to be successful in using their graphing calculator. The degree to which these skills are mastered will distinguish a user as either competent or literate.

Lastly, in view of the number of brands of graphing calculators and their various models, students and teachers utilizing these resource packages as a guide to decide what calculator skills must be mastered for the course would have to be able to grow and adapt to the changing technology as well as being to be able to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators. This can be seen in the Applications of Mathematics 12 Graphing Calculator Resource Package in the Statistics and Probability (Chance and Uncertainty), section J, where students are asked to refer to their calculator manual of their brand and/or model for the proper syntax in activating certain statistical functions.

Thus, from examining the various documents we can get a better picture as to what a calculator literate person should be like. The description below will serve as a working definition for this study.

Why is Calculator Literacy Important?

In the Principles and Standards, released in 2000, the NCTM listed the following as examples for the need to understand and be able to use mathematics in everyday life and in the workplace.

- *Mathematics for life.* Knowing mathematics can be personally satisfying and empowering. The underpinnings of everyday life are increasingly mathematical and technological. For instance, making purchasing decisions, choosing insurance or health plans, and voting knowledgeably all call for quantitative sophistication.

- *Mathematics as a part of cultural heritage.* Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects.

- *Mathematics for the workplace.* Just as the level of mathematics needed for intelligent citizenship has increased dramatically, so too has the level of mathematical thinking and problem solving needed in the workplace, in professional areas ranging from health care to graphic design.

- *Mathematics for the scientific and technical community.* Although all careers require a foundation of mathematical knowledge, some are mathematics intensive. More students must pursue an educational path that will prepare them for lifelong work as mathematicians, statisticians, engineers, and scientists (NCTM, 2000, p. 4)

In fact, in its July 1998 position statement regarding calculator use in elementary and secondary schools the NCTM stated that:

> Appropriate instruction that includes calculators can extend students' understanding of mathematics and will allow all students access to rich problem-solving experiences. Such instruction must develop students' ability to know how and when to use a calculator. Skills in estimation, both numerical and graphical, and the ability to determine if a solution is reasonable are essential elements for the effective use of calculators (NCTM, 1998a, p. 2).

Assessment and evaluation must be aligned with classroom uses of calculators. Instruments designed to assess students' mathematical understanding and application must acknowledge students' access to, and use of, calculators.

The NCTM, then recommended that mathematics teachers at all levels should promote the appropriate use of calculators to enhance instruction by modelling calculator applications, by using calculators in instructional settings, by integrating calculator use in assessment and evaluation, by remaining current with state-of-the-art calculator technology, and by considering new applications of calculators to enhance the study and the learning of mathematics (p. 2).

Defining Calculator Literacy

The difficulty of writing and talking about calculator literacy is that no definition for it exists. In our attempt to ensure that students do not abuse the use of the calculator, the mathematics education community has neglected to define what proper calculator usage should look like. A great deal of time and energy has been invested in explaining how calculator should not be used to replace traditional pencil-and-paper methods that we have not stopped to define what calculator literacy should be. The concept of calculator literacy is new and no formal definition of it has been agreed on by the mathematics community. This section examines the word *numeracy*, in general and as it pertains specifically to calculator literacy. This is important because calculator literacy is a subset of numeracy, and a working definition of calculator literacy is then suggested based on the colloquial use of the term *numeracy* as well as on the rationale for technology use in the IRP. A working definition of calculator literacy is then proposed. Finally, a case is made for why students should develop calculator literacy in which a distinction between calculator competencies and calculator literacy is made.

The word "numeracy" is a neologism, a newly invented word that has come into use among specialist communities in Britain, Australia, Canada and the United States. "Numeracy" is the quantitative, mathematical counterpart of literacy. The terms *mathematical literacy*, *numeracy*, and *quantitative literacy* are used either synonymously or as components of one another, depending on who is using them. John Allen Paulos, who brought considerable popular attention to the issue, uses *innumeracy* and *mathematical illiteracy* synonymously in *Innumeracy: Mathematical Illiteracy and Its Consequences* (Paulos, 1988). He provides an array of examples in which the public can be and often is duped due to inadequate number sense and unfamiliarity with basic ideas of probability. Others, for example, Gal (1997) and Forman (1997), write of innumeracy and quantitative literacy as pertaining to the strands of mathematics that are primarily numeric and computational, thus making quantitative literacy a part of mathematical literacy.

Several useful definitions of numeracy have been put forward by various groups, but in this chapter we will focus mainly on the definition put forth by the British Columbia Association of Mathematics Teachers (BCAMT) as well as various citations found in the paper Improving Numeracy in Canada written for the National Literacy Secretariat of Canada by John Dingwall (2000).

In his paper Dingwall cited the International Life Skills Survey (ILSS) Numeracy Framework to define numeracy as:

- The mathematics for effective functioning in one's group and community, and the capacity to use these skills to further one's development and that of one's community.

- The aggregate of skills, knowledge, and dispositions that enable and support independent and effective management of diverse types of quantitative situations

- Numeracy is a critical awareness, which builds bridges between mathematics and the real world, with all its diversity (Dingwall, 2000, p. 4).

The Australian Association of Mathematics Teachers (AAMT) defines numeracy in its Policy and Numeracy Education in Schools as a fundamental component of learning, discourse and critique across all areas of the curriculum and that numeracy involves the disposition to use, in context, a combination of:

- Underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);

- Mathematical thinking and strategies;

- General thinking skills;
- Rounded appreciation of context. (AAMT, 1998, p. 2)

In 1998, the BCAMT released a numeracy pamphlet stating its definition of numeracy as:

> Numeracy can be defined as the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. It includes the ability to apply various aspects of mathematics to understand, predict, and control routine events in people's lives. Numeracy is more than knowing about numbers and number operations. It requires an ability and inclination to solve problems, including those involving money or measures... Numeracy also demands familiarity with the ways in which numerical information is gathered by counting and measuring, presented in graphs, charts and databases, and finally analyzed (BCAMT, 1998).

As we can see, numeracy has no single definition that encompasses the whole concept and the same can be said of the term calculator literacy. However, the above definitions as well as the position statements released by the NCTM, and the Rationales from the BC IRP for mathematics will serve as building blocks for a working definition of what calculator literacy means. This is because without at least a working definition of what calculator literacy is, it would be impossible to examine it and to design an assessment for it.

The mathematics community can find solace in the fact that the language arts community cannot firmly define *literacy* in its conventional sense. Literacy and illiteracy are often thought of as corresponding to the ability or inability to read and write. This is limiting in that such a definition gives no consideration to the ability to read and write *well.* The notion of reading and writing well prompts examination of levels of literacy and odd questions like "How literate is literate?" (Venezky, 1990, p. 9). This leads to notions of *basic literacy* and *inert literacy,* both of which refer to reading and writing at a most rudimentary level, and *functional literacy* and *required literacy,* which suggest reading and writing competently enough to cope in a society. Thus, the definitions of literacy can be extended beyond the ability to read and write well to the ability to read and write well enough for some purpose, which can vary considerably. Costa (1988) provides a sampling of some of the varied definitions and examples of the term literacy for those who wish "to pursue the thorny pathway of definition" (p. 46).

When literacy is considered as that which is needed to function in social contexts, many realize that reading and writing text are not sufficient. The National Assessment of Educational Progress, in its quest to measure students' abilities to process printed and written information, extends the notion of what it means to be literate by dividing literacy into three components: *prose literacy, document literacy,* and *quantitative literacy* (Kirsch & Jungeblut, 1986). Prose literacy is needed to understand and use information from texts like news stories and poems; document literacy enables one to locate and use information from instruments like job applications and maps; and quantitative literacy is applied to solve computational problems that are embedded in printed materials, like completing an order form and determining an interest rate from an advertisement.

Similarly, calculator literacy can be broken down into two components: basic entry level *Calculator Competency* and a high understanding level which we will call *Calculator Literacy.* Traditional assessment in mathematics does not focus on distinguishing between calculator competency and calculator literacy. Calculator competency is the ability to operate the calculator and perform a series of previously learned procedures to solve a problem. The student mimics what he/she has been taught to do by the teacher and is said to be competent when the student can recognize which set of previously learned procedures is required in a given problem to produce the correct answer. Calculator literacy, on the other hand, is the ability to understand the calculator, to know its advantages and disadvantages as well as the ability to adapt and change one's strategy in the use of the calculator from one problem to another. Just as reading and writing text are not sufficient, being calculator competent on a calculator is not always enough. The intelligent use of a calculator is essential in our ever-changing technological world (British Columbia Ministry of Education, 2003).

To be calculator literate one should be able to:

A) Manage or solve a range of mathematical situations with the aid of a calculator.

B) Manipulate the mathematics be it represented in a numerical, spatial, graphical, statistical or algebraic manner to effectively communicate with the calculator.

C) Know when the calculator should be used.

D) Know the calculator's limitations.

E) Apply his/her estimation and mental math skills to determine the reasonableness of the solution provided by the calculator.

F) Grow and adapt to the changing technology as well as to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators.

CHAPTER THREE
THE CALCULATOR LITERACY TEST

It is important for us to design an assessment tool to measure calculator literacy so that we can gain a better understanding of what calculator literacy is. However, testing for calculator literacy is not a simple task. One cannot simply cut and paste together a variety of traditional mathematical questions and administer it to a group of students with calculators and let the results decide if they are calculator literate or not. Most traditional assessments in mathematics are designed to be calculator independent or calculator neutral (Lokar & Lokar, 1998). Their focus is on assessing students' algebraic and problem solving skills, not calculator literacy. Furthermore, traditional assessments are not able to distinguish between calculator competency and calculator literacy. As mentioned above, calculator competency is the ability to operate the calculator and to perform a series of learned procedures to solve a problem. A student need only mimic what his/her teacher has previously taught. The student is said to be competent when he/she can recognize which set of previously learned procedures is required in a given problem to produce the correct answer. Calculator literacy, on the other hand, is having a deeper understanding of the calculator to know its advantages and disadvantages as well as the ability to adapt and change one's strategy in the use of the calculator from one problem to another. Assessing for calculator literacy requires an assessment designed specifically with the calculator in mind.

Design of the Study

As the term "Calculator Literacy" is not a common term, neither is a calculator literacy test a common resource. No standard calculator literacy test can be found in the NCTM's Principles and Standards for School Mathematics. Assessments specifically designed to assess for the calculator literacy level of students can hardly be found at all. Assessments such the Calculators Self-Review Sheets for Year 5 and Year 6 (The National Numeracy Strategy in Cumbria, 2002a, 2002b) can be found in the National Numeracy Strategy's framework for teaching mathematics in Cumbria of the United Kingdom. However, these Self-Review Sheets and other such assessments focus mainly on elementary school level mathematics, emphasizing basic number computations rather than a more comprehensive assessment of the full definition of calculator literacy. In fact, I was not able to find *any* standardized assessment of calculator literacy for students at the middle and secondary school levels.

Thus, one of the goals of this study was to design a calculator literacy test to assess for the calculator literacy level of middle and high school students. It was my hope that the calculator literacy test used in this study would form the basic framework in the design of other calculator literacy assessments in future studies. The problems encountered in the development and administration of the Calculator Literacy Test, its effectiveness, as well as any modifications and improvements made to the test were the main focus of this study. As the actual calculator literacy level of the students in the study was of secondary focus in the study the results were not conveyed to the subjects, their parents, their teachers or schools.

In this study I designed a test to assess calculator literacy as described in the working definition. It consisted of two components: graphing calculator active and non-graphing calculator active (See Appendix B and C). The first part is a graphing calculator dependent section that is meant for students who have had extensive experience with the graphing calculator. The second part is non-graphing calculator dependent in nature and is geared towards students using a scientific calculator. However, the use of a graphing calculator was permitted in this section, as the graphing features were not of any additional help to the students.

A pilot test was designed and given to two small sample groups, one for each component of the test, prior to the finalizing of the actual Calculator Literacy Test. The sample groups for the pilot consisted of 20 students each. A copy of the pilot test is included (See Appendix D). No major changes were made from the pilot test, but the following modifications were included in the CLT in response to feedback from students (See Table 7 below).

Table 7 Changes Made After the Pilot Test

	CHANGES
1.	The questions in the graphing calculator component were left unchanged. The number of questions on the scientific calculator component was reduced by two questions. Questions 13 and 18 were removed. In addition, students were only allotted an hour during the pilot test for both the graphing calculator active and the scientific calculator components, while in the final trial students were allotted one-and-a-half hours for the test.
2.	Additional verbal instructions were given to emphasize that not all questions required the use of a calculator.
3.	The Sharp's DAL entry system notation was adopted.
4.	Wording of a few questions was changed to make the questions a little more clear.

The number of questions on the graphing calculator component was left unchanged because feedback from students after the pilot test indicated that time was not a major factor in finishing the test. Students indicated that questions left unanswered were those that they had either forgotten how to do or they were questions on concepts unfamiliar to them. Questions 13 and 18 on the scientific calculator section were omitted to make the test shorter. Question 18 in particular was omitted because few students knew how to use the Cosine Law. The time allotment for both tests was extended so that the time would not be a factor influencing students' performance.

Additional verbal instructions were added to the final CLT to emphasize to the students that not every question on the test would require them to use the calculator. Students were instructed to perform any of the calculations mentally or manually if it was easier to do so.

Feedback from the pilot test indicated that the Sharp's DAL entry system presented no major problems to most of the students; therefore, I decided to use it for the final version of the CLT.

The Sample

In the study, the majority of the students who participated in the non-graphing calculator dependent component of the test were Grade 9 and 10 students with the exception of a small group of Applications of Mathematics 11 students. The non-graphing calculator active component was designed to assess calculator literacy on the scientific calculator using mathematical concepts taught within the middle school range. The graphing calculator active component was designed to assess calculator literacy on the graphing calculator using mathematical concepts taught within the senior secondary school range. The reason for having two separate components is that in recent years the graphing calculator has permeated into many of the senior level mathematics courses while the scientific calculator is still the calculator of choice in the junior level mathematics courses. I felt it was important to assess for graphing calculator literacy as well as scientific calculator literacy because both types of calculators are used extensively in schools.

The study was conducted close to the end of the second semester in April at both high schools. Items tested in the Calculator Literacy Test were based on the curriculum developed by the British Columbia Ministry of Education with emphasis on Number Concept, Algebraic Manipulation, Functions, Statistics and Trigonometry. The Geometry component in the curriculum was left out. Emphasis in this study was placed on the Principle of Mathematics pathway while the Applications of Mathematics and Essential of Mathematics pathways are referred to but not examined in detail. The total population of the graphing calculator dependent group was 45 students. It consisted of a small group of Principles of Mathematics 11 Honours students as well as one class of Principles of Mathematics 12 students. The total population of graphing calculator independent group was 65 students. It consisted of one class of Mathematics 9 students, one class of Principles of Mathematics 10 students as well as a small class of Applications of Mathematics 11 students.

Problems Encountered in Designing the Calculator Literacy Test

A close examination of the definition of a calculator literate person shows some of the problems encountered in designing an assessment tool for calculator literacy.

A. *To be calculator literate one should be able to manage or solve a range of mathematical situations with the aid of a calculator*

The range of mathematical situations and the level of mathematics the students have been exposed to in school so far limited the range of mathematical situations in which we were able to test in this study. Though many standard features on the calculator such as trigonometric functions, logarithmic functions, and factorial calculations are considered routine low level skills requiring minimal understanding of the advantages, disadvantages and the limitations of the calculator, most students would not have been exposed to the mathematical concepts associated with these functions until much later on in their high school career.

Different expectations are based on the level of schooling of the students. Calculator literacy takes into account of age and learning experience. A calculator literate 5^{th} grader has presumably different skills than a calculator literate Grade 10 student. The questions in the CLT were designed with this in mind and I tried to assess students at the appropriate calculator literacy level based on their schooling.

Within the graphing calculator active component of the test there were further limitations as well. Due to recent changes in the British Columbia Mathematics curriculum, solving systems of quadratic equations is no longer required to be covered in high school. Of the two ministry-authorized textbooks for Principles of Mathematic 11, only one, Addison-Wesley's Mathematics 11 (Alexander & Kelley, 1998a, p. 324), includes a section on solving systems of quadratic equations with the focus on quadratic-linear systems. Solving systems of quadratic equations has been dropped from the Principles of Mathematics 12 curriculum altogether. Thus, this limited the range of mathematical situations that I was actually able to test in the study.

B. *One should be able to manipulate the mathematics be it represented in a numerical, spatial, graphical, statistical or algebraic manner to effectively communicate with the calculator.*

Testing for students' abilities to manipulate the mathematics when it was represented spatially, numerically, graphically and algebraically was not a difficult task; however, it was difficult to assess if students were able to manipulate the mathematics when it was represented statistically. As referred to earlier, the range of mathematics I could assess for in the study was limited by the scope of mathematics the students had been exposed to. Statistics, traditionally, has not been a topic that has been extensively covered in the high school curriculum in British Columbia. Basic one-variable statistical concepts such as calculations and interpretation of standard deviations, z-scores, normal curve distributions, and binomial distributions are often not covered until the senior level and even then not many high school courses will go beyond one variable statistics. So though students' calculators possess extensive features to deal with Statistical questions, I had to limit the questions to basic one-variable Statistical concepts.

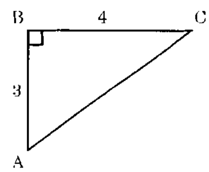
C. *Being calculator literate also involves knowing when the calculator should be used.*

The calculator is helpful in solving many different types of problems but sometimes it is much faster and more convenient to solve a problem by other methods.

For example,

Do the question below and verify if the answer given is correct. ***If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.***

Find the segment AC



Solution: $x = \boxed{\sqrt{\ }}\ \boxed{3}\ \boxed{x^2}\ \boxed{+}\ \boxed{4}\ \boxed{x^2}$ ENTRY Ans. 19

Any student who is familiar with their Pythagorean triples would instantly recognize that 19 is not the correct solution. However, getting students to communicate clearly that they chose to use some other methods of solving the problem or that they solved the problem without the use of their calculator but used their calculator to check their answers afterwards was difficult as not every student indicated that on their test.

After the pilot test some students felt that because this was a calculator test, they were required to use the calculator for every question. Many mathematics teachers in their classrooms have observed this phenomenon. The instructions on the CLT clearly stated that the use of a calculator might not be required for every question on the test and that if the student felt the use of a calculator on a particular question was unnecessary then he/she was to record that on his/her response form. During the actual trial of the test explicit verbal instructions were given to emphasize that not every question required the use of a calculator, before the start of the test.

D. *One should know the calculator's limitations.*

At the heart of this study is the concept of calculator competency and literacy. Many people do not make a distinction between competency and literacy when it comes to the use of calculators. The CLT, especially the graphing calculator active component, was designed to address this. It consists of a number of routine calculations to test the competency level of the students, but it also contains questions that tested the students beyond the basics.

For example,

Example *Solve the following system:*

$$y = \log(x+4)+4$$
$$y = \frac{1}{2}x+4$$

Graph the above functions in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will provide enough of the graph so that recognizable characteristics of each function are shown and all intersection points are found.

This question was designed to test if the user is aware of the disadvantages and limitations of his/her calculator. A calculator competent student would be able to find the obvious intersection between these two functions. However, a calculator literate student understands that the second intersection between these two functions is not shown because the calculator cannot display the vertical part of the logarithmic function (or any other functions where the vertical part of the graph tends toward an asymptote line) properly due to the pixel limitation of the graphing calculator screen.

E. *One should apply his/her estimation and mental math skills to determine the reasonableness of solution provided by the calculator.*

Determining the reasonableness of solutions by applying their estimation and mental skills are required throughout the entire test. For example,

Do the question below and verify if the answer given is correct. ***If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.***

Find the $\sin 45^o$ ***to 2 decimal places***

Solution: sin 4 5 ENTRY *Ans. -0.99*

Here students need to apply their estimation and mental skill to determine that the solution is not reasonable and that the calculator is set to the wrong trigonometric mode.

F. *Having calculator literacy also means that one is able to grow and adapt to the changing technology as well as being able to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators.*

One of the problems with testing for calculator literacy by administering a written test was with the range of syntax and notation found on different brands calculators. Even among calculators that use the same entry system, certain functions are labelled differently. For example, on calculators using an algebraic entry system the exponent key has 3 different representations, $\wedge$ / y^x / x^y, while the scientific notation function has at least two different representations, EE / EXP. It was difficult deciding which notation was to be used on the CLT. However, from the sample data collected from the pilot test, I found that majority of the students used calculators made by Sharp. Therefore, Sharp's DAL notation of y^x and EXP were used to represent the exponent key and the scientific notation key in the sample questions on the CLT as well as the actual test.

For example, consider the following question in the non-graphing calculator component of a calculator literacy test.

> Try the questions below and verify if the answer given is correct. If the given answer is incorrect, then make any necessary corrections or modifications as needed to obtain the correct answer.
>
> Evaluate: $(-2)^2 + (-3)^4$
>
> Solution: $\boxed{(-)}\ \boxed{2}\ \boxed{x^2}\ \boxed{+}\ \boxed{(-)}\ \boxed{3}\ \boxed{y^x}\ \boxed{4}\ \boxed{\text{ENTRY}}$
>
> *Ans. -85*

The pilot test showed that students who used a Sharp calculator had no problem identifying with the symbols portrayed in the solution because all Sharp calculators used the $\boxed{y^x}$ symbol to represent the exponent function. However, a few students who used a HP or a Casio calculator did not identify with the solution set as easily. The same can be said about the $\boxed{\text{ENTER}}$ and the $\boxed{\text{EE}}$ keys. The $\boxed{\text{ENTRY}}$ and the $\boxed{\text{Exp}}$ keys are used by Sharp to represent the execute function and the scientific notation function while Casio uses $\boxed{=}$ and $\boxed{\text{EXP}}$ to represent the same functions. However, most students did not have much trouble switching from one set of notation to another.

The Types of Questions

For this study I classified the questions in the CLT into three categories: Basic knowledge, Understanding, and Higher Understanding. Questions classified as Basic were designed to test for basic competency with the calculator. These questions allowed students to obtain the correct answer by demonstrating a minimal amount of calculator literacy. These questions require minimal manipulation of numbers or interpolation of results. An example of this type of question can be found in both the non-graphing and the graphing calculator components of the CLT.

Evaluate: $6.23 + 5.24 \times 2.1 - 6.25 \div 2.5$

Here students are simply asked to evaluate the above expression. The student only has to enter the expression as it is written in the question from left to right to obtain the correct answer. No higher level of understanding is required. This is the most basic level, or competency, in our assessment for calculator literacy. The same can be said of the question below.

Solve the following equation using a graphing calculator:

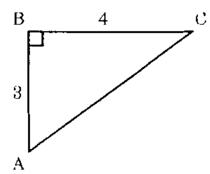
$x^5 - 10x^3 + 10 = 0$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

Once again, this question requires students to simply enter the functions $Y_1 = x^5 - 10x^3 + 10$, $Y_2 = 0$ and find their intersection point using the intersect function on their graphing calculator or alternately the students can enter the equation $x^5 - 10x^3 + 10$ into Y_1 and solve for the zeros of the function. Compared to the non-graphing calculator dependent component, this question requires a much higher level of mathematical knowledge and understanding. However, relative to graphing calculator literacy, solving an equation by either finding the intersection between the expression on the left hand side of the equation and the expression on the right side or by obtaining the zeros of the given equation is a standard procedure that all students using the graphing calculator would have been introduced to at a very early stage. This, therefore, requires students to demonstrate a very low level of understanding of the advantages and limitations of the graphing calculator.

The following question however, gives us a nice illustration of a question which allows students to demonstrate Understanding level of calculator literacy.

> Do the question below and verify if the answer given is correct. If the given answer is incorrect, then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.
>
> *Find the length of segment AC.*



Solution: $x =$ 

Ans. 19

The user must possess some understanding of both the mathematics in this question as well as some understanding of the calculator in order to recognize that the correct answer cannot be 19 and that the solution has left brackets out. In addition, this question was designed in such a way that it also tested whether the student recognized when it was unnecessary and even awkward to use the calculator for this question as this is a standard 3-4-5 right-angle triangle.

Similarly, the example below illustrates a question which allows students to demonstrate an Understanding level of calculator competency for the graphing calculator component.

Solve the following system:

$$y = \log(x+4)+4$$
$$y = \frac{1}{2}x+4$$

Graph the functions in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will provide enough of the graph so that recognizable characteristics of each function are shown and all intersection points are found.

Here the user must possess some understanding of both the logarithmic function as well as an understanding of the limitations of the graphing calculator in order to recognize that there are two solutions instead of just one. The students need to remember that a logarithmic graph is asymptotic and when drawn on graph paper the asymptote is a guide for drawing the curve. However, due to the pixel limitation of the graphing calculator, it does not show this asymptotic line and often the graph does not appear complete on the graphing calculator screen near the asymptote. Students need to be aware that the graph continues even though their screen does not indicate this fact and that a second solution exists at $(-3.989883, 2)$.

An example of a question allowing students to demonstrate a Higher Understanding level question for the non-graphing calculator dependent component would be:

If the fraction key on your calculator is broken, how would you evaluate the following?

$$R = \left(\frac{1}{\frac{1}{33} + \frac{1}{99} + \frac{1}{11}} \right)$$

In this question, we would be able to clearly see the differences between users who are calculator competent and those who are literate. Due to the limitation of the "broken" fraction key on the calculator the calculator competent users would be forced to resort to using a series of decimal values to evaluate this expression. Below are some possible solutions to this question.

Solution A

$$\frac{1}{33} = 0.030303$$
$$\frac{1}{99} = 0.010101$$
$$\frac{1}{11} = 0.090909$$

$$\rightarrow \; 0.030303 + 0.010101 + 0.090909 = 0.131313$$

$$\rightarrow \; \frac{1}{0.131313} \approx 7.6154$$

Solution B

$$1/((1/33) + (1/99) + (1/11)) \approx 7.6154$$

Calculator literate users would be able to draw on their numeracy skill and realize that they can make use of the Reciprocal key to help them in this question.

Solution C

$$\left(33^{-1}+99^{-1}+11^{-1}\right)^{-1}\approx 7.6154$$

The students are forced to integrate and demonstrate their level of mathematical understanding along with their understanding of their calculator. Hence we see that being calculator literate also means that one has to be numerate.

Similarly, the following question is classified as another Understanding level question. It allows students to demonstrate how well they understand their graphing calculator.

Evaluate t_n if $x=5$, given: $t_n=\sum_{k=1}^{n}\left(\frac{1}{x}\right)^{k-1}+\sum_{k=1}^{n}\left(-\frac{1}{x}\right)^{k-1}$

In this question, students are asked to combine a series of different functions that are built into the graphing calculator. This question requires the students to have a firm understanding of Sigma notation as well as an understanding of how to combine a set of built-in functions of the graphing calculator to obtain the correct answer. Here is the solution:

Since x is given to be equal to 5, we evaluate the above expression as:

$$sum\left(seq\left(\left(5^{-1}\right)^\wedge(k-1),k,1,99\right)\right)+sum\left(seq\left(\left(-5^{-1}\right)^\wedge(k-1),k,1,99\right)\right)\approx 2.08\overline{3}$$

The following question can be classified as a question allowing students to demonstrate a basic level of calculator literacy rather than being classified as an Understanding or Higher Understanding literacy level question on the calculator literacy test because the amount of calculator knowledge required to evaluate the expression is of a low level even though secondary trigonometric functions are not covered until the Grade 12 level.

Find $\csc\frac{3\pi}{5}$ to 2 decimal places

Similarly, the following example is classified as an Understanding literacy level question because the question allows students to demonstrate an understanding required to enter a set of data into a scientific calculator.

Number of Students	***2***	***4***	***5***	***7***	***12***	***10***	***8***	***2***
Score on the Test	***30***	***40***	***50***	***60***	***70***	***80***	***90***	***100***

Above is a table of examination results from Mr. Smith's Math class. Find the average mark of the class.

Analysis of the Questions

In this section I will present each question on the non-graphing calculator active and the graphing calculator active components of the CLT. I will explain why each question was included in the CLT and how it will help to assess calculator literacy. Each question will also be ranked on the level of competency or literacy it demonstrates. I have divided the working definition of calculator literacy into six parts. This will help to identify what part or parts of the definition each question on the CLT was testing. A brief rationale is provided with each question as well as which section of the definition it pertains to.

A. To be calculator literate one should be able to manage or solve a range of mathematical situations with the aid of a calculator.

B. One should be able to manipulate the mathematics be it represented in a numerical, spatial, graphical, statistical or algebraic manner to effectively communicate with the calculator.

C. Being calculator literate also involves knowing when the calculator should be used.

D. One should know the calculator's limitations.

E. One should apply his/her estimation and mental math skills to determine the reasonableness of the solutions provided by the calculator.

F. Having calculator literacy also means that one is able to grow and adapt to the changing technology as well as being able to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators.

Scientific Calculator Questions

Question 1: $6.23+5.24\times 2.1-6.25\div 2.5$

This is a basic computational question that tests if the student understands how the calculator handles order of operations. One of the benefits of a scientific calculator over a simple four-function calculator is that a scientific calculator will perform order of operations for the user. A user who does not realize this might choose to multiply 5.24 by 2.1 and divide 6.25 by 2.5 first before entering the respective answers into the calculator to evaluate the final answer. This question allows students to demonstrate a Basic level of calculator literacy. It assesses sections A of the working definition.

Question 2: $-3-14\times -2+16\div -8$

This is another basic computational question. The emphasis here is on the use of the negative sign. The use of the negative sign and the ability to distinguish between the negative key and the subtraction key is particularly important because symbolically there is no difference between a negative sign and a subtraction sign when using the pencil and paper method. However, when a calculator is used students must be able to distinguish within the question which designation is required. Once again this question serves to illustrate the point that a person cannot be calculator literate without being numerate, as the student must understand what the question is asking. Weaker or inexperienced mathematics students may misinterpret the -3 as subtraction and try to enter the expression as "subtraction 3 minus 14". This would yield an incorrect answer of 23 on a LEFT-TO-RIGHT calculator and an error message on an algebraic entry-system logic calculator. The converse can happen as well. The student might misinterpret the "minus 14" as a "negative 14" and try to punch in negative 3 negative 14. This question allows students to demonstrate a Basic level of calculator literacy. It assesses sections A of the working definition.

Question 3: $\dfrac{4\times 2+6}{5-3}$

This question examines the students' understanding of how the scientific calculator handles order of operations. It is true that the scientific calculator will perform order of operations automatically, but it cannot distinguish between the numerator and the denominator within a fractional expression. If a student were to enter the question as it is written without inserting additional sets of brackets in both the numerator and the denominator, then the calculator would not be able to distinguish when the numerator ends and when the denominator begins. Thus, it will take four times two and add it to six divided by five and subtract three, yielding an answer of 6.2 instead of 7. Furthermore, the student must also realize that it is not sufficient just to insert a set of brackets for the numerator but that the denominator must be grouped together by a set of brackets as well. A common error that inexperienced users may make is to enter the numerator as "four times two plus six" and hit the enter key to communicate to the calculator that all that was entered is the numerator, but will forget to insert a set of brackets for the denominator as well causing the calculator to divide the entire numerator by five and then subtract three from the result. Once again, this will yield an incorrect answer of -0.2 instead of 7. This question represents a question that allows students to demonstrate a Basic level of calculator literacy. It assesses sections A, B, C, D and E of the working definition.

Question 4: $-2\left[3+\left(3+4\right)\right]-1$

This question was designed to assess the student's knowledge of the bracket keys. The bracket keys are very important as they allow the user to carry out complex and lengthy calculations in one single entry rather than breaking it down into several parts. One of the key distinctions between a user who is competent and one who is literate is the depth of understanding he/she has of what the calculator can do and utilize the full power of the calculator to simplify the task. Unlike the basic four functions calculator which does not have the bracket feature, this question can be entered as it is written, providing the student understands that all brackets, regardless of whether they are round, square or curled brackets, are represented as round brackets. Thus, instead of having to add three to four first to obtain seven and then add it to three again yielding 10, which then we will multiply by a negative two and then subtract one, we can simply enter negative two, open brackets, three, plus, open bracket, three, plus four, close brackets, close bracket minus 1 and get the correct answer of -21. This is only one use of the bracket keys. Bracket keys are also used to communicate with the calculator when we are entering functions. The calculator will treat anything within the brackets as the argument for the function. This question allows students to demonstrate a Basic level of calculator literacy. It assesses sections A, B, C, D and E of the working definition.

Question 5: $$\frac{\left(6.11\times10^{-11}\right)\left(5.98\times10^{24}\right)(3200)}{\left(6.4\times10^{6}+4400\right)^{2}}$$

The primary goal of this question was to test if students knew how to use scientific notation on their calculators, and the $\boxed{x^2}$ function on their calculators. The question also tests if the student has an advanced understanding of order of operations on the calculator. One of the biggest complaints of senior level science teachers, myself included, is their students' inability to effectively use scientific notation. Often extensive amounts of time at the beginning of each semester must be spent reviewing basic calculator skills. Chemistry and Physics deal with large and small numbers that are expressed in scientific notation on a regular basis. If a student cannot work with scientific notation on the calculator -- in the interpretation of an answer given by the calculator or in entering values in scientific notation into the calculator to perform computations -- then he/she will not be able to perform most calculations required in these courses. Scientific notation is an excellent example of how indispensable calculator technology is. It is almost unheard of today to expect students to multiply numbers to the orders of ten to the power of negative eleven and ten to the power of twenty-four manually. It is not because students cannot do such calculations by pencil and paper nor is it because it is not important for students to develop and master good number sense by working with numbers of such magnitude by hand, but rather it is simply too tedious and too educationally unbeneficial to carry out these calculations question after question. Educators realized that the time spent on performing such calculations can be better spent on teaching students the concepts instead. The same can be said for the x^2 function. The x^2 key is used extensively in both mathematics courses and in science courses. The ability to square numbers quickly is one of the great features of having a calculator.

The more subtle skill tested by this question is order of operations. Once again, as in the previous questions, it is important to understand the order of operations on the calculator. This is not just because it will help the student to obtain the correct answer, but it will also save him some time by eliminating redundant keystrokes. Many inexperienced users who are only competent in their calculator skill or their understanding of order of operations would enter the question with a set of brackets around each value, thus requiring eight more keystroke entries, instead of realizing that the entire expression can be entered into the calculator with no brackets at all. One of the most critical things for calculator literacy is to realize the advantages and the limitations of the calculator. A student who is more calculator literate would be able to perform this computation in keystrokes entry.

[(] [6] [•] [1] [1] [Exp] [(-)] [1] [1] [×] [5] [•] [9] [8]
[Exp] [2] [4] [×] [3] [2] [0] [0] [)] [÷] [(] [6] [•] [4] [Exp]
[6] [+] [4] [4] [0] [0] [)] [x^2] [ENTRY]

This question assesses sections A, B, C, D and E of the working definition and it represents an Understanding level of calculator literacy.

Question 6: Evaluate and **leave answer as an <u>improper fraction</u>**:

$$\frac{1}{2}+1\frac{3}{7}\times-2\frac{3}{5}$$

The main objective of this question is to assess students' knowledge of the fraction keys on the calculator. The ab/c key is found on most current models of scientific calculators, and it is a great feature to help students deal with complex fractional calculations, especially mixed fractions. Novice users may not be aware of how to enter mixed fractions into their calculators, as many scientific calculators make no distinction between the whole number component of the fixed fraction and the fractional component. For example, many calculators would show $2\frac{3}{5}$ as 2 ⌟ 3 ⌟ 5. Where the 2 stands for 2 the whole number component and the 3 ⌟ 5 stands for the fractional component. Similarly, a student must understand the syntax of the calculator and enter mixed fractions into his/her calculator accordingly. Many recent scientific calculators models have improved upon this and they now show the mixed fraction $2\frac{3}{5}$ as 2 ⌟ 3 ⌟ 5. However, the syntax for entering a mixed fraction remains the same. This question allows students to demonstrate a Basic level of calculator literacy. It assesses sections A and B of the working definition.

Question 7: Evaluate and **leave answer as an <u>improper fraction</u>**:

$$\frac{45}{32}+\frac{1}{4}\left(\frac{1}{3}-\frac{5}{6}\right)$$

The main objective of this question is to assess students' knowledge of the fraction keys on the calculator. An often-missed feature of the scientific calculator is the

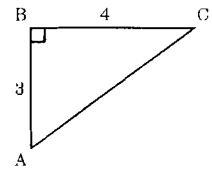
2ndF a b/c feature, which will change a mixed fraction into an improper fraction and an improper fraction into a mixed number. This is an important feature because as students advance to senior level mathematics courses, more and more answers are left as an exact value. Most exact value answers are left in the form of an improper fraction instead of a mixed number. In this particular question the initial answer the student would get would be $1\frac{9}{32}$ which needs to be converted into $\frac{41}{32}$. Often students will neglect to convert $1\frac{9}{32}$ into $\frac{41}{32}$ because they are not aware of this useful feature of a scientific calculator. This question allows students to demonstrate an Understanding level of calculator literacy. It assesses sections A and B of the working definition.

Question 8: Find the area given: $Area = \pi r^2 \quad r = 1.26$

This question is designed to assess basic understanding of various basic functions of the calculator. Most scientific calculators would have the π function built-in. Also, being able to compute various formulas by substituting in appropriate values is a common and useful application of the scientific calculator. This question allows students to demonstrate a Basic level of calculator literacy. It assesses sections A and B of the working definition.

Question 9: Do the question below and verify if the answer given is correct. If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.

Find the segment AC



Solution: $x =$ [√] [3] [x^2] [+] [4] [x^2] [ENTRY] Ans. 19

This question is an example of how a student must be numerate in order to be calculator literate. When the student looks at the answer 19, he/she should be able to reason out in his/her head that the value 19 is far too large a value for a triangle with legs of lengths three and four. In fact, most students would not require a calculator to find the length of the third side, as they would easily recognize the above is a three-four-five triangle. However, this question also assesses the student's ability to edit incorrect keystroke entries as well as to test for proper understanding of how the square root function operates on the calculator. Most scientific calculators treat the number immediately to the right of the function symbol as the argument. If the argument contains more than one value, the user must communicate this to the calculator by inserting a set of brackets at the beginning and at the end of the argument. Failing to do so results in an incorrect answer. This question allows students to demonstrate a Higher Understanding of calculator literacy. It assesses sections A, B, E and F of the working definition.

Question 10: Do the question below and verify if the answer given is correct. If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.

Find the $\sin 45^0$ **to 2 decimal places**

Solution: sin 4 5 ENTRY Ans. -0.99

This question is designed to assess the student's understanding in the use of trigonometric functions. The incorrect answer of -0.99 is obtained when the calculator is mistakenly set in radian mode rather than degree mode. Normally this would be considered a lower calculator literacy level question. However, because this question requires students to identify what mistake was made it allows students to demonstrate a Higher Understanding calculator literacy level question. It assesses sections A, B, E, and F of the working definition.

Question 11: Do the question below and verify if the answer given is correct. If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.

Evaluate: $\sqrt{(2-3)^2+(3-1)^2}$

Solution:

√ ((2 − 3) x^2 + (3 − 1) x^2) ENTRY Ans. 2.24

The solution to this question is correct (to 2 decimal places). In order for a student to be considered calculator literate, he/she must be able to recognize when the calculator has performed an incorrect computation and when it has performed it correctly. This question tests the student's ability to use the square root function properly by inserting appropriate brackets at the beginning and the end of the argument as well as the proper use of the $\boxed{x^2}$ function. A total of three sets of brackets must be entered correctly in order to get the right answer. This question allows students to demonstrate a Higher Understanding level of calculator literacy. It assesses sections A, B, and F of the working definition.

Question 12: Do the question below and verify if the answer given is correct. If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.

Evaluate: $(-2)^2+(-3)^4$

Solution: $\boxed{(-)}\ \boxed{2}\ \boxed{x^2}\ \boxed{+}\ \boxed{(-)}\ \boxed{3}\ \boxed{y^x}\ \boxed{4}$ ENTRY

Ans. -85

This is a very important question in assessing the student's understanding of the calculator as well as their understanding of exponents. The calculator feature tested here is the proper use of the $\boxed{y^x}$ key; the mathematics concepts tested here relate to exponents. $(-2)^2$ is not the same as -2^2. The former means $(-2)(-2)=4$ while the latter means $-1\times 2\times 2=-4$. This is especially important, as many inexperienced users will forget to enter the brackets thus resulting in an incorrect value of -85 instead of 85. This question allows students to demonstrate a Higher Understanding level of calculator literacy. It assesses sections A, B, E and F of the working definition.

Question 13: If the fraction key on your calculator is broken, how would you evaluate the following?

$$R=\left(\frac{1}{\frac{1}{33}+\frac{1}{99}+\frac{1}{11}}\right)$$

This question is designed to test advanced users of the calculator. A calculator literate student is a student who is also numerate. This is an excellent question to demonstrate that with the knowledge of exponent rules and the power of the calculator, students can accomplish more. Instead of literally dividing each one by 33, 99, and 11, add up the bottom three calculations and then divide one by it; students can simply input the question as $\left(33^{-1}+99^{-1}+11^{-1}\right)^{-1}$, saving time. There is no doubt that the former method would have arrived at the exact same answer, but part of being calculator literate, and in fact part of the reason for using the calculator at all, is to do things better and more efficiently. The calculator allows us to do that. It allows us to go beyond what we can do manually. In this case it allows us to work more quickly. This question assesses all sections of the working definition.

Question 14: If the square root, $\boxed{\sqrt{\ }}$, and the nth root key, $\overset{\sqrt[x]{\ }}{\boxed{\ }}$, keys on your calculator are broken, how would you evaluate the following?

$$d=\sqrt{(2-12)^2+(15-35)^2}$$

This question serves to assess the same concepts as the previous question, except this time students would raise the entire expression to the power of 0.5 instead. If indeed both the square root and the nth root key are broken on the calculator it would be very difficult if not impossible to obtain an accurate answer to the question without realizing that the square root of an expression is the same as raising the expression to the power of ½. This shows maturity in both calculator literacy and numeracy. It assesses sections A, B, C, E and F of the working definition.

Question 15: Find θ to the nearest tenth of a degree:

$$\cos\theta = \frac{\sqrt{5}}{6} \quad 90^{o} > \theta \geq 0^{o}$$

This question assessed if students are comfortable working with numbers other than decimals or whole values when dealing with trigonometric functions. This question allows students to demonstrate an Understanding level of calculator literacy. It assesses sections A, B, and E of the working definition.

Question 16: A shirt at Costco costs \$19.89 and a pair of gloves costs \$12.95. If Jack buys 2 shirts and 1 pair of gloves, then how much would Jack have to pay if he has to pay 7.5% PST and 7% GST?

This question is another basic level computational question that tested one of the most commonly used features of the calculator -- percent. Whether it is error analysis for a chemistry lab or finding out how much tax is needed to purchase a new DVD player, percentages are used everywhere, and the calculator is a great device to help us perform such computations. When working with simple percentages like 10%, 20%, 25%, 50%, the use of the calculator is not needed. However, if other percentages are given, it is nice to have the calculator do the calculations or at least have it to check the answers. As most students have been taught to calculate percent manually, some might not know how to use their calculators properly. Some students might take \$52.73 and multiply by 0.14 or 1.14 to calculate the tax or the final amount respectively. However, the scientific calculator is equipped with the percent function and it can calculate the percent without manually converting the percent into a decimal first. \$52.73 times 114% on the calculator will yield an answer of $60.11 right away. It is a much easier and cleaner way of dealing with large numbers of calculations involving percentages. Some students however, are not taught how to use this feature on the calculator. Often educators feel that students overuse or misuse their calculators, but students must be taught to use this technology properly. This question assesses sections A and B of the working definition.

Questions 17 and 18:

Number of Students	***2***	***4***	***5***	***7***	***12***	***10***	***8***	***2***
Score on the Test	***30***	***40***	***50***	***60***	***70***	***80***	***90***	***100***

Question #17

Above is a table of examination results from Mr. Smith's Math class. Find the average mark of the class.

Question #18

Calculate the standard deviation of the above scores.

These two questions are designed to test how well students can work in the statistics mode in their scientific calculators. Mean and standard deviation are standard measures used in most basic statistical analysis. Students were taught how to compute these values by hand in their junior mathematics courses and should have a good understanding of the two values. Once understood though, there is very little educational value in performing these tedious calculations by hand on a regular basis. Students should be able to input the set of data into their calculator and have the calculator perform these calculations for them. However, many calculator illiterate students either continue to compute these calculations by hand because they do not know how to input such data into their calculators or they mistakenly believe that only graphing calculators can perform basic statistical operations. In fact, during the early days in the implementation of the new Principles of Mathematics 12 many teachers were amazed that the TI-83Plus was able to compute the mean and standard deviation of a set of data that has been entered as a list into the calculator. Many of these teachers thought such calculations could only be done by hand. They were not aware of the fact that most, if not all, scientific calculators have similar capabilities. In Dorren Schmitt's Evaluation of Mathematics Reform, (September 1996) it was pointed out that the level of competency with the calculator of the teachers directly affects the competency of the students and their views on how this technology should and could be used. These two questions assess sections A, B and E of the working definition.

Graphing Calculator Questions

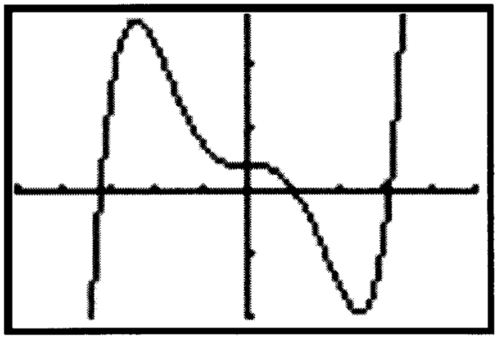
Question 1

Solve the following equation using a graphing calculator:

$$x^5 - 10x^3 + 10 = 0$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

This is a basic computational question that tests if the student understands how to find zeros of a polynomial function on the graphing calculator. In this question, although three zeros can be seen if the x-values in the viewing window are set to [–5, 5], it is necessary to change the range to discover that those are the only three zeros of the function (See Figure 2).



x [−5, 5] y [−50, 70]

Figure 3 Solving Polynomial Equations

This is a 5th degree polynomial equation with a maximum of five possible real solutions. However, there are only three real roots to the equation, as there are only three x-intercepts. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition.

Question 2

Solve the following inequality using a graphing calculator:

$$x^4 + 5x^2 < 6x^3 - 4x + 12$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

This is a more advanced computational question that tests if the student understands how to interpret inequalities functions displayed on the graphing calculator. Most graphing calculators will not explicitly provide user with the solution to an inequalities. Students must interpret the information given. In this question, although the viewing window above does not show the other intersection point, the coordinates (4.92, 706.48) can still be found with this window since the calculator feature used to find intersection points only requires that the x-values be within the domain given. This question allows students to demonstrate a higher skill level of graphing calculator literacy. It assesses sections A, B and C of the working definition.

Question 3

Solve the following system:

$$y = \log(x+4)+4$$
$$y = \frac{1}{2}x+4$$

Graph the functions in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will provide enough of the graph so that recognizable characteristics of each function are shown and all intersection points are found.

This is a more advanced computational question that tests if the student understands the display limitations of the graphing calculator. The graphing calculator will not explicitly provide the user with both solutions to this system of equations. Students must be aware of the asymptotic nature of the logarithmic function, or they may think that there is only one solution to this system. Even though the screen image does not show that these graphs intersect in two places, it is clear that if the logarithmic graph was extended further, there are two solutions. This question assesses sections A, B and C of the working definition.

Question 4

Solve the following equation using a graphing calculator:

$$|x-2| = |1.1x+4|$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

This is a computational question that tests if the student understands how to find the intersections to a system of functions on the graphing calculator. In this question, although one of the intersections can be seen if the x-values in the viewing window are set to [–5, 5], it is necessary to change both the range and domain to discover that there exists another intersection at (-60, 62). Students who compute this question without thinking about the nature of an absolute value function would miss the second intersection. This level of thinking represents a higher level of literacy skill. This question allows students to demonstrate an Understanding level of graphing calculator literacy. It assesses sections A of the working definition.

Question 5

$$\text{Find } \csc\frac{3\pi}{5} \text{ to 2 decimal places}$$

This is a basic computational question that tests if the student understands how to evaluate reciprocal trigonometric functions on the graphing calculator. Though, this can be done on a scientific calculator, students are not introduced to reciprocal trigonometric functions until their senior year. Proper placement of brackets is essential to the solution of this problem. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition.

Question 6

Solve the following equation using a graphing calculator:

$$\cos x + 2 = \csc x, \quad 0 \leq x < 2\pi$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Explain clearly how your solution is derived from the graph.

This is a more advanced computational question that tests if the student understands how to solve trigonometric equations on the graphing calculator. Graphing calculators tend to draw in asymptotes for the trigonometric functions, leading to a possible miscount of solutions. Furthermore, it is important for students to understand that while the intersection points have coordinates in the form *(x, y)*, the original question was not presented as a system. Therefore students may incorrectly present the y-values of the intersection points in their response. This question allows students to demonstrate a higher skill level of graphing calculator literacy. It assesses sections A, B, and C of the working definition.

Question 7

What is the amplitude of

$$y = \sin x + \cos x$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived.

This is a computational question that tests if the student understands how to find the maximum and minimum values on the graphing calculator. Although the question looks simple, the use of a graphing calculator would likely be necessary for most students. Either tracing to a maximum or minimum point or using the calculator's maximum or minimum feature to find these values could find the amplitude. The amplitude is equal to $\frac{Max - Min}{2}$. This question allows students to demonstrate an Understanding level of graphing calculator literacy. It assesses sections A and B of the working definition of calculator literacy.

Question 8

Both the INTERSECT and the ZERO functions on your calculator are not working, but you need to find Kristi's temperature, in degrees Fahrenheit. Over a two-week period of illness her temperature fluctuated according to the Formula:

$$T(t) = 3\sin\left(\frac{\pi}{8}t\right) + 103.6$$

Where t is the day of the illness. On what day did her temperature first reach 106.6°F?

This is a more advanced computational question that tests if the student understands how to use the table of values feature on the graphing calculator as the "intersect" and the "zero" functions are not working. Students would need to use the calculator to construct the graph and to produce the table to find the appropriate values in order to read off on which day Kristi's temperature first reached 106.6°F. This question allows students to demonstrate a higher skill level of graphing calculator literacy. It assesses sections A, B and C of the working definition of calculator literacy.

Question 9

Solve the following system for y only.

$$\begin{aligned} x + y + z &= 2 \\ 3x - 2y - 2z &= 1 \\ 5x + 4y + 6z &= 15 \end{aligned}$$

In reality, this is a computational question that tests if the student understands how to use the matrix features on the graphing calculator. However, many Principles of Mathematics 11 teachers in this province do not teach the use of the matrix features. It was of interest to me to determine how many students know how to utilize this feature. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition of calculator literacy.

Question 10

Evaluate: $\log_2 35$

Like the above question, this is a basic computational question. It tests if the student understands how to enter non-base ten logarithmic expressions on the graphing calculator. However, many students will not have encountered this topic until they enrol in the Principles of Mathematics 12 course, some of the students taking the CLT may not be able to answer this question. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition of calculator literacy.

Question 11

Evaluate t_n if $x = 5$, given: $t_n = \sum_{k=1}^{n} \left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{n} \left(-\frac{1}{x}\right)^{k-1}$

This question is designed to assess three main skills. The first skill is the proper understanding of the Sigma notation. The second skill is the proper use of the sum(and seq(functions on the graphing calculator. But third and more importantly, this question is designed to assess if students can apply their knowledge of these functions to compose the proper calculator expression to utilize the two functions to evaluate this complex mathematical expression. This demonstrates a higher level of competency as students are asked to use the calculator to develop one's own analysis of the situation and adapt and utilize difference methods and or strategies to work around any limitations. It assesses sections A, B and C of the working definition of calculator literacy.

Question 12

Given: $\begin{array}{l} h(x)=2x^3-1 \\ g(x)=2x^7-4x-1 \end{array}$ find $2g(-2)+3g(h(-3))$

This is a basic computational question that tests if the student understands how to use the VARS|Y-VARS|FUNCTION on the graphing calculator to evaluate function notations. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition of calculator literacy.

Question 13

Evaluate: $\dfrac{8!}{3!(8-3)!}$

This is a basic computational question that tests if the student understands how to use the factorial function on the calculator. Furthermore, this question is included in the graphing calculator component as the factorial notation is often hidden within a couple levels of menu screens on most graphing calculator and it is important to test to students are competent at accessing various functions on their graphing calculators. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition of calculator literacy.

Question 14

Evaluate: $\left({}_{13}C_1\right)\left({}_4C_3\right)$

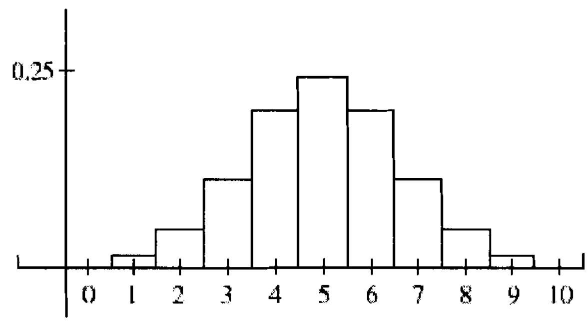
This is a basic computational question that tests if the student understands how to use the ${}_nC_r$ function on the graphing calculator. This question allows students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A of the working definition of calculator literacy.

Question 15

A multiple-choice test has 12 questions. Each question has 4 choices, only one of which is correct. If a student answers each question by guessing randomly, find the probability that the student gets at least 7 questions correct?

Question 16

The following graph shows the theoretical probability distribution for the number of heads obtained when a fair coin is tossed 10 times.



Determine the standard deviation of this binomial distribution.

Question 17

The weights of ball bearings at a manufacturing plant have a normal distribution with a mean of 125 grams and a standard deviation of 0.1 grams. Any ball bearing with a weight of 125.28 grams or more is rejected. If the manufacturing plant makes 2 000 000 ball bearings, how many would you expect to be rejected?

Question 18

The volumes of pop in cans are normally distributed with a mean of 355 mL and a standard deviation of 5 mL. Determine the range, to the nearest mL, of the volumes of the central 90% of the pop cans.

Questions 15 to 18 are computational questions that test if the student understands how to use various statistical functions on their graphing calculators. Functions such as the *binomcdf*, *List*, *1-Var Stats*, *normalcdf*, and the *invNorm* are tested. These questions allow students to demonstrate a Basic level of graphing calculator literacy. It assesses sections A and B of the working definition of calculator literacy.

The table below summarizes the different how each of the questions in the graphing calculator component and the non-graphing calculator component provide students to demonstrate the different levels of calculator literacy.

Table 8 Components of the definition

Non-Graphing Calculator Component			
Question	Basic	Understanding	Higher Level
1	X		
2	X		
3	X		
4	X		
5		X	
6	X		
7		X	
8	X		
9			X
10		X	
11			X
12			X
13			X
14			X
15		X	
16	X		
17		X	
18		X	

Graphing Calculator Component			
Question	**Basic**	**Understanding**	**Higher Level**
1	**X**		
2			**X**
3			**X**
4			**X**
5	**X**		
6			**X**
7		**X**	
8			**X**
9	**X**		
10	**X**		
11			**X**
12	**X**		
13	**X**		
14	**X**		
15	**X**		
16	**X**		
17	**X**		
18	**X**		

Below is a summary of how each question in the two components of the CLT fits into the working definition of what a calculator person.

Table 9 Collation between the CLT and the Workin1g Definition

Non-Graphing Calculator Component							
Question	Definition of Calculator Components						
	A.	**B.**	**C.**	**D.**	**E.**	**F.**	
1	X						
2	X						
3	X	X	X	X	X		
4	X	X	X	X	X		
5	X	X	X	X	X		
6	X	X					
7	X	X					
8	X	X					
9	X	X			X	X	
10	X	X			X	X	
11	X	X				X	
12	X	X			X	X	
13	X	X	X	X	X	X	
14	X	X	X		X	X	
15	X	X			X		
16	X	X					
17	X	X			X		
18	X	X			X		

Question	***Graphing Calculator Component***						
1	X						
2	X	X	X				
3	X	X	X				
4	X						
5	X						
6	X	X	X				
7	X	X					
8	X	X	X				
9	X						
10	X						
11	X	X	X			X	
12	X						
13	X						
14	X						
15	X						
16	X	X					
17	X	X					
18	X	X					

CHAPTER FOUR
DATA ANALYSIS

In this section I will analyze the data collected in the two components of the Calculator Literacy Test. My objective in this examination was to determine how effectively the CLT met the goals I set in this study. My goals was not to assess how well the sample group of students performed on the CLT and hence how calculator literate these students were but rather how to design an assessment to evaluate students' calculator literacy level.

In this analysis I first reviewed if the CLT met the goals set forth in the study, which were:

1) to examine how and what to test for this new literacy, that is, to examine the logistics of testing for calculator literacy as well as its problems,

2) to devise a test to assess calculator literacy for high school students,

3) to administer this test, collect data and look for patterns and trends, and

4) to provide grounds for further study to investigate if students today are leaving our high school system calculator literate.

Goals 1 and 2 were addressed in the previous sections while Goal 3 will be the focus of this analysis and Goal 4 will be examined in Chapter Five.

This analysis will be separated into three parts. In Part One, I will focus on the general results of the test. In Part Two, I will examine how well the CLT addressed and assessed the different calculator skills needed for basic calculator competency as outlined by the Chemistry Learning Center, CLC, of the Virginia Tech Chemistry department. In

Part Three, I will examine how well the CLT assessed the various components of the working definition of calculator literacy defined in this study.

Part I-General Results of the Test

In general, students did reasonably well on the CLT. However, many of the students ran out of time and were not able to complete all of the questions on the test in the hour and a half allotted. This was partially due to the fact that students were unfamiliar with the format of the test. Comments from a few of the students afterwards revealed that some of them found it to be time consuming to have to show their calculator keystroke entries. This was however imperative to the study. I felt that even though some students were not able to complete every question it was an acceptable sacrifice. It is better to have richer and higher quality information for observation from the questions they did answer than to have the students rush through the test and not show their work. Some students were able to finish the entire test while others concentrated on the questions they knew well. Others simply did as many as they could in the order the questions appeared on the test. Some students in each class chose not to participate in the study and they simply returned a blank test to me at the end. In total 25 students decided not to participate in the study. Most of the students who participated, 65 students on the non-graphing calculator test and 45 in the graphing calculator test, took the test seriously and did the best that they could. A number of students told me that they had forgotten how to do some of the questions because they had not encountered that topic recently. This was especially true in the graphing calculator section of the test where many of the grade twelve students informed me afterwards that they had forgotten how to work with three by three systems of equations, how to use their TI-83 to deal with function notations, and how to deal with systems of inequalities. Table 9 below gives a quick summary of the number of students who provide the correct answer to the question regardless of the sophistication of the response. On the whole the questions on Statistics were poorly attempted. Many students either did not know how to solve them or simply never got to them.

Table 10 General Results

General Results of the CLT					
Non-Graphing Calculator Section			Graphing Calculator Section		
Question Number	Number of Students Attempted	Number of Students Who Responded with A Correct Solution	Question Number	Number of Students Attempted	Number of Students Responded with A Correct Solution
1	65	64	1	42	39
2	65	65	2	35	26
3	63	50	3	37	15
4	65	65	4	30	18
5	39	19	5	31	25
6	63	46	6	42	37
7	64	47	7	31	25
8	60	58	8	18	9
9	43	28	9	17	10
10	58	42	10	28	28
11	57	48	11	11	8
12	50	22	12	12	7
13	30	6	13	36	34
14	42	13	14	33	33
15	46	32	15	18	12
16	45	43	16	11	7
17	50	48	17	12	7
18	0	0	18	9	6
[illegible]	[illegible]	[illegible]			

Part II-Calculator Skills Needed for Calculator Competency

The set of basic calculator skills as listed by the Chemistry Learning Center, CLC, of the Virginia Tech Chemistry department as the basic calculator skills needed for most senior science course are listed below in Table 10

Table 11 Calculator Skills

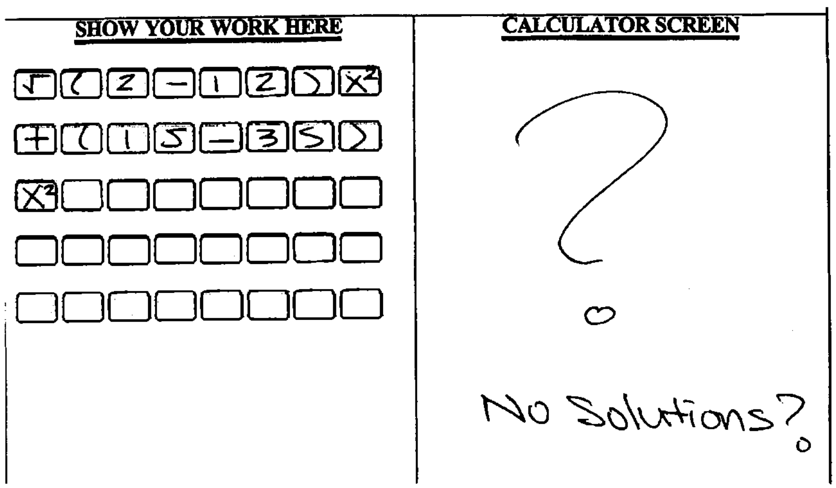
Square Root
Square
Fraction
Percent
Inverse
Brackets
Logs and Anti logs
Basic Trig Functions
Arcsine, Arccosine, and Arctangent
Constants such as Pi and e
Scientific Notation
Exponents

In this part of the analysis I will examine how well each one of these skills has been assessed by the CLT and what patterns of calculator use or common errors I observed. Examples of some of the students' work has been scanned in and included.

The square root, square, fractions, percent, inverse, brackets, basic trigonometric functions, constants, scientific notation and exponents features are all addressed in the non-graphing calculator part of the CLT. The graphing calculator component addresses the log feature in Question 10.

The square root feature was incorporated into various questions in the CLT, but I feel that Question 11 of the non-graphing calculator section of the CLT assesses this feature best. Here students had to be able to take the square root of $(2-3)^2+(3-2)^2$. I found that students who had mastered the square root feature on their calculator were able to determine that the answer provided in the question was correct. Work shown in the CALCULATOR SCREEN section of the test indicated to me that students knew of the square root function on their calculator and most students were familiar with the necessary syntax required to execute the feature.

However, I also observed a common mistake students made in this question. Students who placed their brackets incorrectly often came up with an answer of no solution if they failed to include a set of brackets around the entire argument (See Figure 3). This shows a lack of understanding on the student's part as to what constitutes the argument of the square root function.

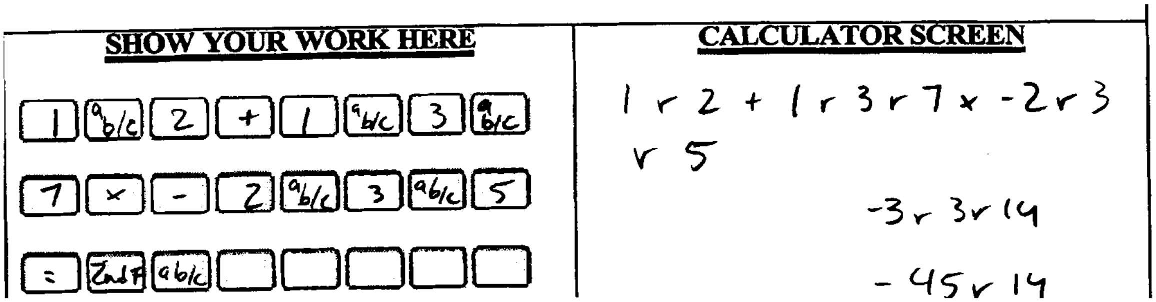


The student did not understand that neglecting to place an additional set of brackets around the entire expression of $(2-12)^2+(15-35)^2$ *made the calculator treat the* $(2-12)$ *part as the argument thus giving the student an error message as the calculator cannot take the square root of a negative quantity*

Figure 4 Square Root Function

The x-square feature was incorporated into various questions in the CLT, but Question 12 of the non-graphing calculator section of the CLT assessed this feature best. Here students were asked to raise -2 to the second power as well as to know that all negative values must be encompassed within a set of brackets when raised to an even power. I found students who understood and mastered the square feature on their calculator were able to determine that the answer provided was incorrect. Work shown in the CALCULATOR SCREEN section of the test indicated that students knew of the x-square feature on their calculator and most students were familiar with the necessary syntax required to execute the feature. However, I observed that some of the students failed to place a set of brackets around the -2 thus giving them the wrong answer.

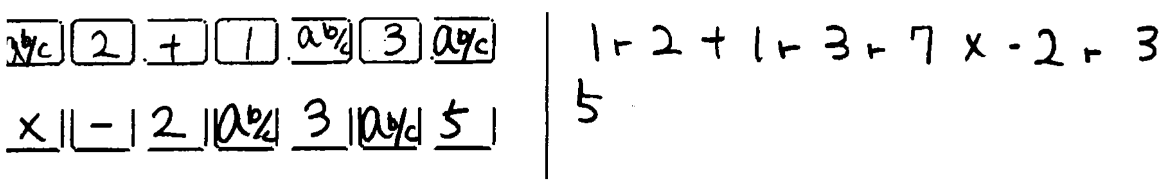
The fraction features were incorporated into Questions 6 and 7 in the non-graphing calculator section of the CLT. Here students were asked to use the fraction feature on their calculator to perform the computations and to use the 2^{nd} fraction feature to convert their answer into an improper fraction. I found that most students who had mastered the fraction feature on their calculator were able to determine the answer correctly in a mixed fraction form, but few converted it into an improper fraction. I believe this was due more to the students not knowing how to activate the conversions feature on their calculators than the students' not reading the question carefully. Feedback from students after the pilot test indicated that many of the students were not even aware of such this feature. I observed that of the students who did convert the final answer into an improper fraction, the work in the CALCULATOR SCREEN area indicated that they performed the calculations manually. Figures 4-7 show 4 examples found in the sample group.



The student entered the calculation correctly into his/her calculator and then by activating the 2^{nd} $a\,{}^{b}\!/_{c}$ feature was able to convert the answer into its improper form.

Figure 5 Proper Use of Fraction Features

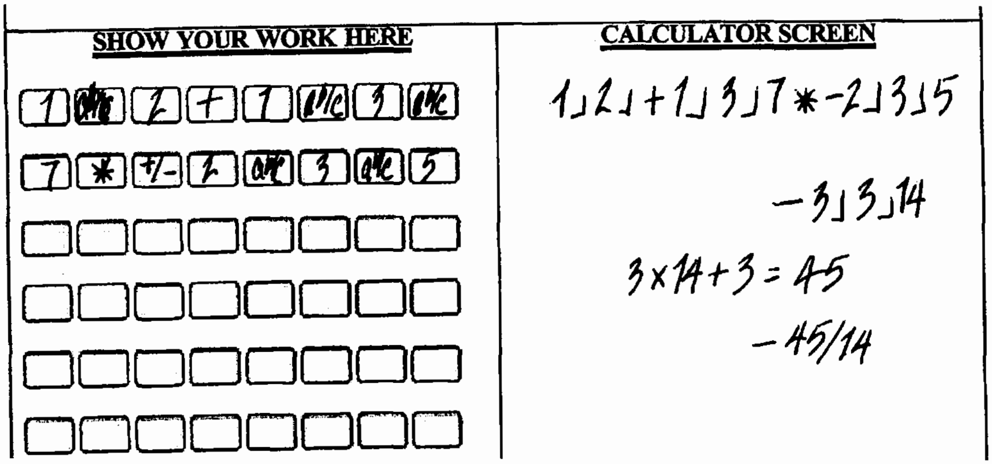
Some students were able to do the question but failed to covert the final answer into an improper fraction (See Figure 5 below).



The student entered the calculation correctly into his/her calculator but failed to convert the answer into its improper form by utilizing the 2^{nd} $a\,{}^{b}\!/_{c}$ feature on his/her calculator.

Figure 6 Improper Fraction Conversion Feature on the Calculator

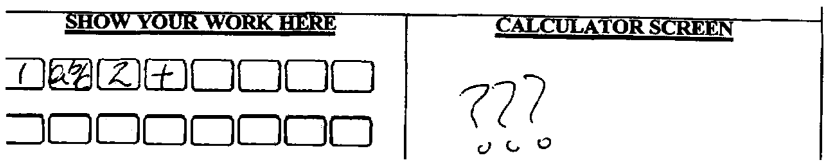
Other students did the conversion manually with pencil-and-paper (See Figure 6).



The student entered the calculation correctly into his/her calculator but failed to convert the answer into its improper form by utilizing the 2nd a b/c feature on his/her calculator. However, he/she realized that the answer is supposed to be left as an improper fraction and performed the conversion manually.

Figure 7 Manual Conversion to Improper Fractions

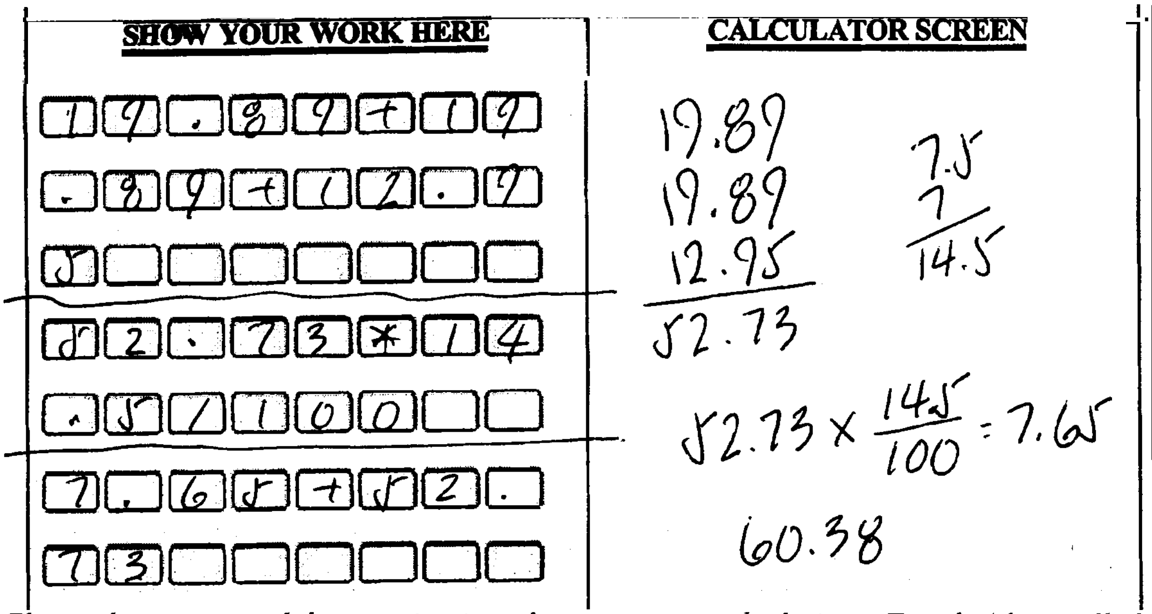
However, I did observe that there was a small population of the sample group who had a very limited understanding of how the fraction feature worked. They were unable to enter the mixed fractions $1\frac{3}{7}$ and $2\frac{3}{5}$ properly into their calculator. They started the question but abandoned it when they encountered the mixed fractions or they tried to convert the improper fraction manually (See Figure 7).



The student didn't really feel comfortable with doing fractions on the calculator. He/she realized that the $a\,{}^{b}\!/_{c}$ *key was needed to perform calculations involving fractions but he/she did not know how to enter the mixed fractions of the question into the calculator and so the question was abandoned.*

Figure 8 Student Showing Unfamiliarity with the Fraction Features

Question16 of the non-graphing calculator section of the CLT assessed the percent feature. Here students needed to calculate the total cost of the purchase including tax. As expected, very few students in the sample group utilized the percent feature on their calculator for this calculation. Of the 69% of students who attempted this question, 96% performed their calculations by treating the 14.5% tax as a number out of a hundred. Figure 8 and 9 show two examples of the typical calculations seen in the sample group.

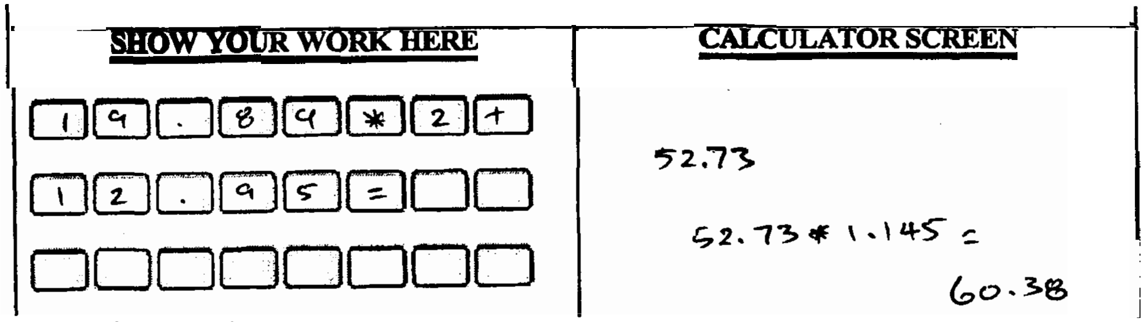


The student separated the question into three separate calculations. First he/she totalled up the price of the purchase before tax and then multiplied the result by $\frac{14.5}{100}$ to find the total amount of tax. Finally, he/she added the purchase price to the total amount of tax to obtain the total price of the purchase.

Figure 9 Percent as a Number Out of 100

Figure 9 shows how some students simply moved the decimal point mentally prior to performing the calculation on the calculator. Once again, the % function on the calculator was not used. Understanding that a percent is a number out of 100 is excellent but students should also know how to use the percent function on their calculators as well to make more effective way of computing percentages. A calculator literate student could have performed the computation as:

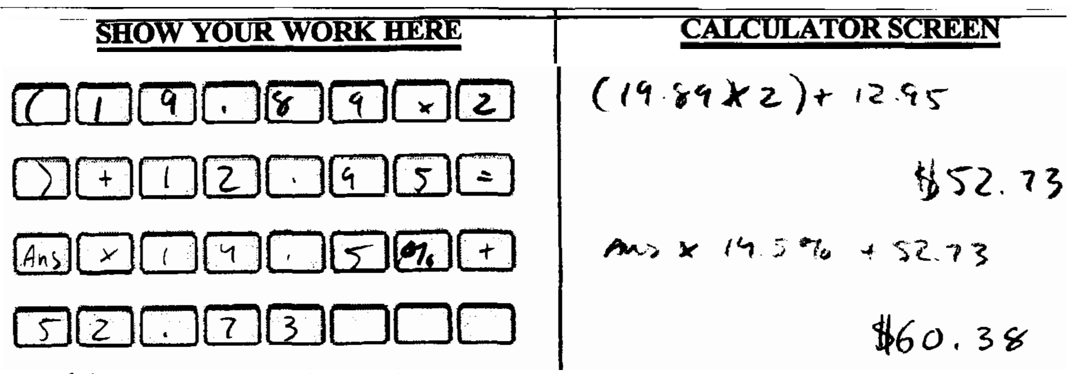
(2 × 1 9 • 9 8 + 1 2 • 9 5) × 1 1 4 • 5 %



The student simply moved the decimals to the left by two places mentally to convert the 14.5% into a decimal. He/she then realized that the final purchase price including tax could be obtained simply by multiplying the total cost of the shirt and gloves by 1.145.

Figure 10 Mental Conversion of Percentages

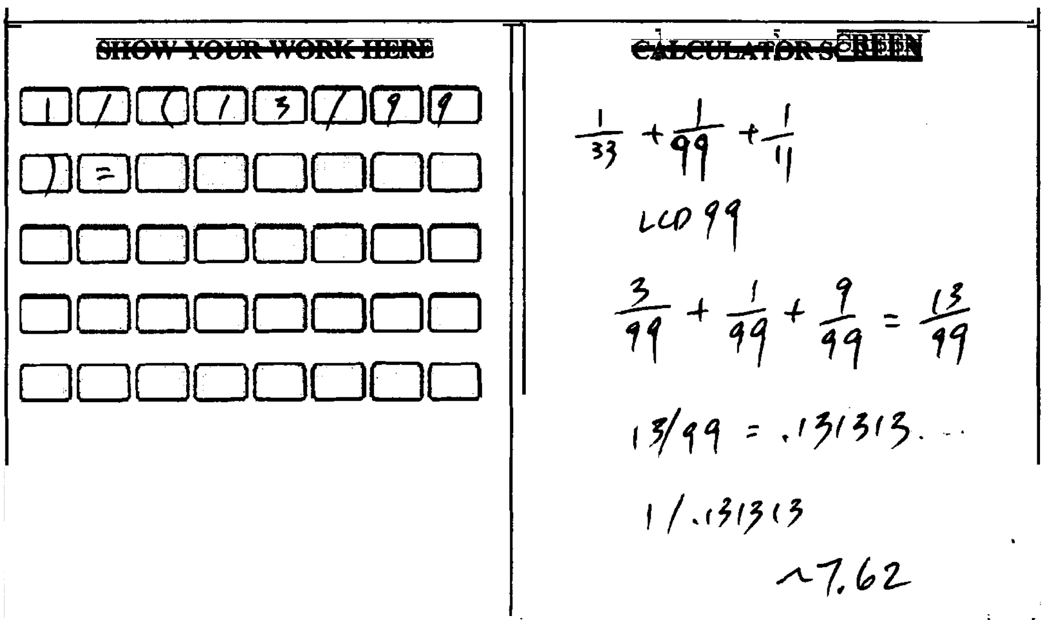
Some students simply mentally shifted the decimal to the left by two places while others treated 14.5% as $\frac{14.5}{100}$ (See Figure 10). Either way, students demonstrated their mental math skill or their knowledge of what a percent is adequately with both methods. However, it also demonstrated that students in high school some students rarely use the percent feature.



Three of the sixty-nine students who attempted this question utilized the percent feature on their calculator. Here the student simply multiplied the cost by 14.5% and added it back to $52.73.

Figure 11 The Percent Feature

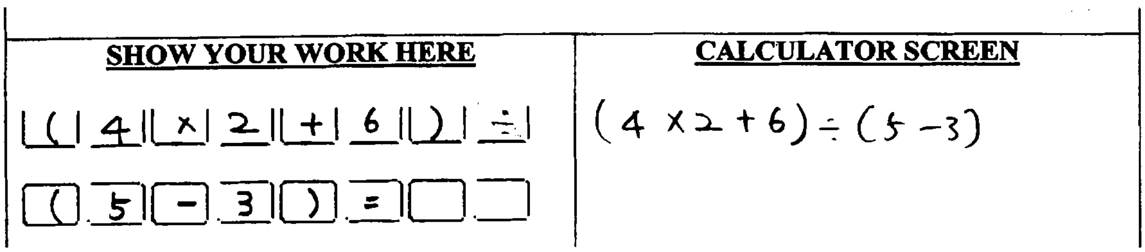
The inverse features were incorporated into Question 13 in the non-graphing calculator section of the CLT. Here students needed to remember and to utilize the exponent law $\frac{1}{x} = x^{-1}$. None of students in the sample group utilized the inverse function to help them compensate for the malfunctioned fraction keys. This is not to say that students were not able to answer this question, but none in the sample group did so by using the inverse function. I observed that of the students who were able to answer this question correctly, they did so by performing piecewise calculations for each part of the question rather than performing the calculation in one single calculation by using the inverse function (See Figure 11). This suggests they were not able to or did not realize how to apply the necessary exponent law to this calculation.



The few students who attempted Question 13 and got it right did not utilize the inverse feature on their calculators, but they were able to complete the calculation correctly.

Figure 12 Inverse Function

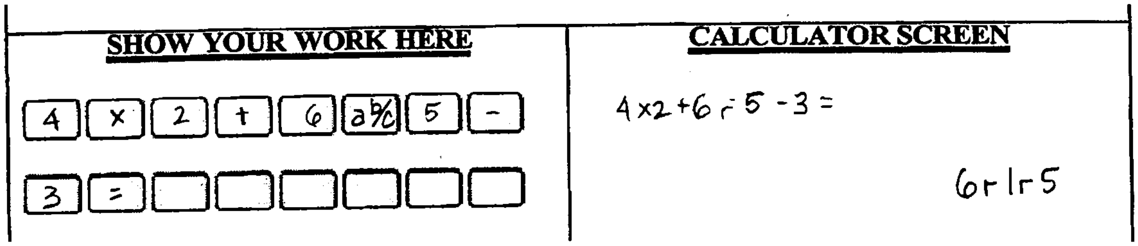
The proper use of brackets was incorporated into various questions in the CLT. However, I focused my observations on Questions 3, 4, 5, 7, 9, 11 and 12 of the non-graphing calculator section. Question 3 addressed students' understanding on how and when to insert appropriate brackets in a calculation. Proper placement of brackets is needed to preserve order of operation. Students must insert brackets to indicate to the calculator which part of the computation entered represents the numerator and which part represents the denominator. Over 80% of the sample group demonstrated the proper placement of brackets for this question. On the other hand, of the 20% of students who were not able to obtain the correct answer, 100% of them failed to insert the necessary brackets (See Figure 12).



The majority of the students answered Question 3. Brackets were placed appropriately around the numerator as well as the denominator.

Figure 13 Proper Bracket Placement

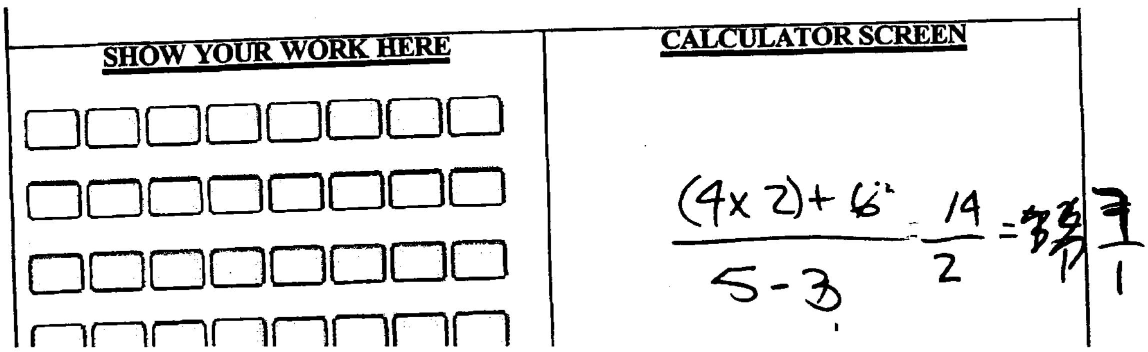
Figure 13 shows how some students did not understand the need to insert additional brackets into an expression in order to help the calculator distinguish between the numerator and the denominator of a rational expression.



The student failed to place a set of brackets around the numerator and an additional set of brackets around the denominator, thus, misleading the calculator to perform the division of six by five instead of fourteen divided by two.

Figure 14 Missing Brackets

Some students were able to evaluate Question 3 manually, as the calculations required to evaluate Question 3 were simple (See Figure 14).

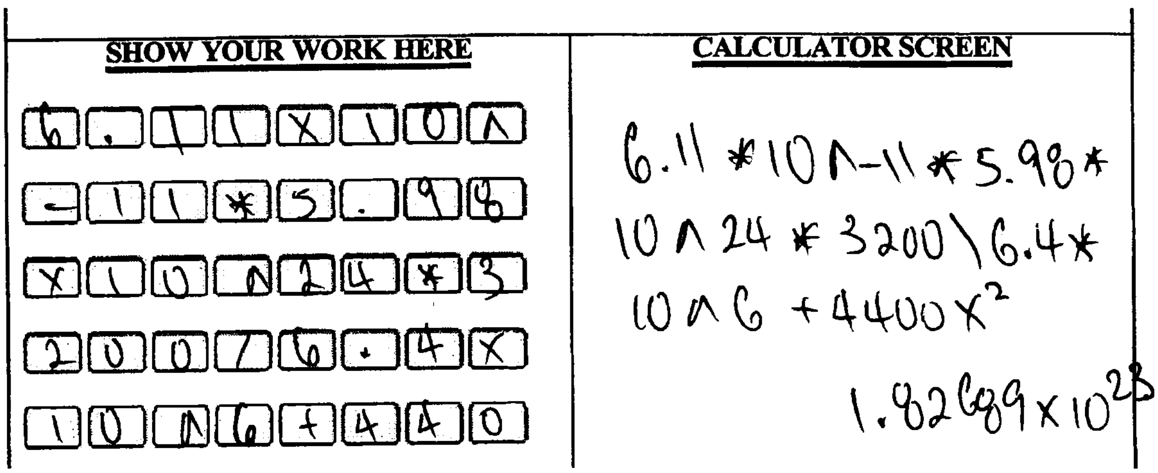


The student indicated that he/she required no calculator to perform this calculation.

Figure 15 Knowing When Not to Use a Calculator

Question 4 assessed if students understand that the calculator does not make the same distinction between square brackets and round brackets as one would when performing calculations manually. Square brackets are reserved for statistical operations by the calculator unlike the traditional pen and paper method where the square bracket is reserved to represent sequence of operations in a calculation. On the calculator, only round brackets are used. I found none of the students in the sample group had a problem making this adjustment between the two representations. Question 4 was extremely well done with 100% of the student who attempted the question answering the question correctly.

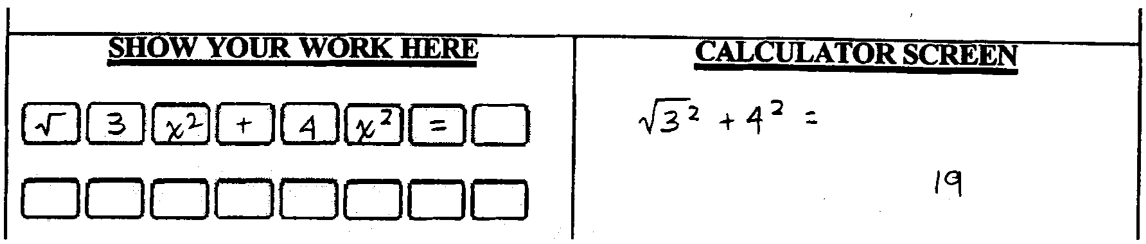
Question 5 focused more on scientific notation than the proper use of brackets. However, the proper placement of brackets was imperative in the calculation of this question. Here, because the denominator was a binomial, brackets must be utilized to help the calculator recognize the entire denominator. I found of the students who knew how to enter scientific notations properly on their calculator the main reason for answering this question incorrectly was because they neglected to place brackets around the entire expression in the denominator (See Figure 15). However this error was not prevalent in the sample group. This can be said of Question 7 as well, where the proper usage of brackets is imperative to the calculation.



Few students forgot to include brackets in this question, but there was a small group of them that neglected to include a set of brackets around the denominator.

Figure 16 Scientific Notation

Questions 9 and 11 addressed the issue of proper bracket placement around a function's argument (See Figure 16). Functions such as square roots require the user to insert an open bracket at the start of the argument as well as a closing bracket at the end of it. In the case of Question 9, students who failed to insert a set of brackets around the argument of $3^2 + 4^2$ got an incorrect answer. About 65% of the students who attempted the question in the sample group were able to identify this mistake.

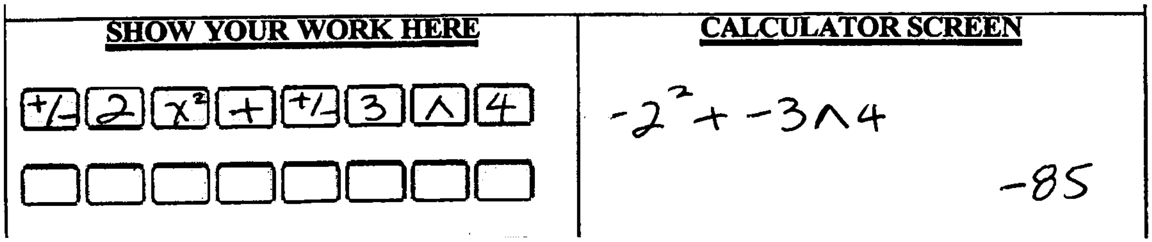


This Student neglected to place a set of brackets around the argument of the square root function. Using the Sharp EL-531V, this student mistook the square root of 3^2 *and added it to* 4^2 *to give her the wrong conclusion that 19 is the correct answer.*

Figure 17 Arguments of a Function

Question 12 addressed the issue of proper bracket placement around bases that are of a negative value when raising a base to an even power. Most students have been taught in grade 8 the rule of inserting a set of brackets around any negative base whenever it is being raised to an even power. Over the years, I have found this to be a common mistake in the traditional pen and paper method of simplifying exponential expressions. Some students do not make the distinction between 2^2 and $(-2)^2$ I have found this to be a common mistake when students are using their calculators as well. Fifty-seven percent of the students who attempted the question in the sample group identified -85 as the incorrect answer while 43% of them concluded that -85 was correct. I observed from the

CALCULATOR SCREEN section, that most students who got this question wrong either completely forgot this rule or were inconsistent in applying the rule (See Figure 17).

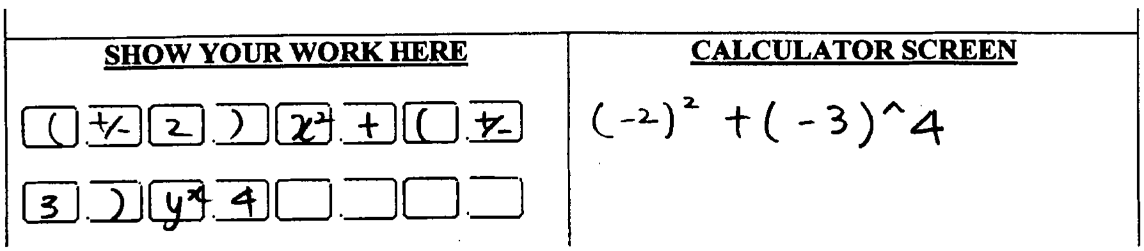


SHOW YOUR WORK HERE	CALCULATOR SCREEN
+/- 2 x^2 + +/- 3 ∧ 4	$-2^2 + -3 \wedge 4$
	-85

The student neglected to include brackets around the -2 and the -3 resulting in an incorrect answer of -85 instead of 85. However, the student demonstrated that he/she knew how to activate the x^2 key as well the $\wedge$ key on his/her calculator.

Figure 18 Negative Bases

Some students remembered that they must insert brackets around any negative bases when raising to a power, thus allowing them to recognize that 85 should be the correct answer (See Figure 18).

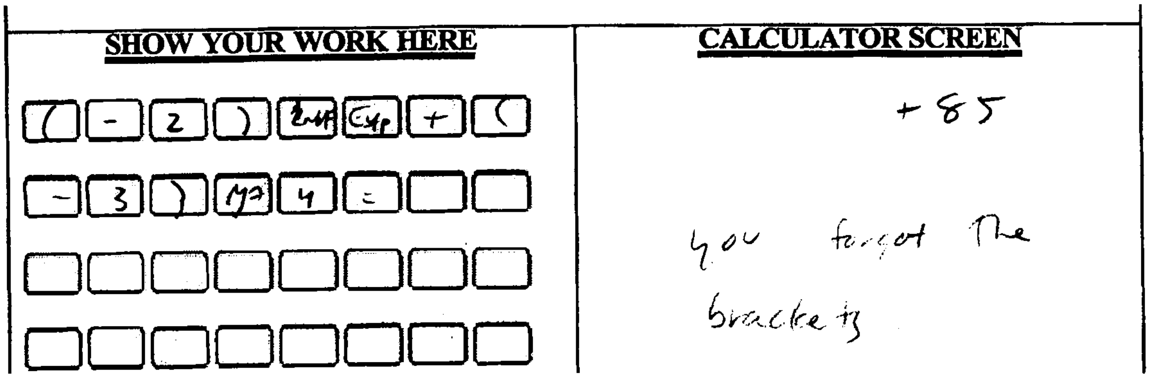


SHOW YOUR WORK HERE	CALCULATOR SCREEN
(+/- 2) x^2 + (+/- 3) y^x 4	$(-2)^2 + (-3)\text{^}4$

The student properly included brackets around the -2 and the -3 giving him/her the correct answer of 85.

Figure 19 Exponents and Brackets

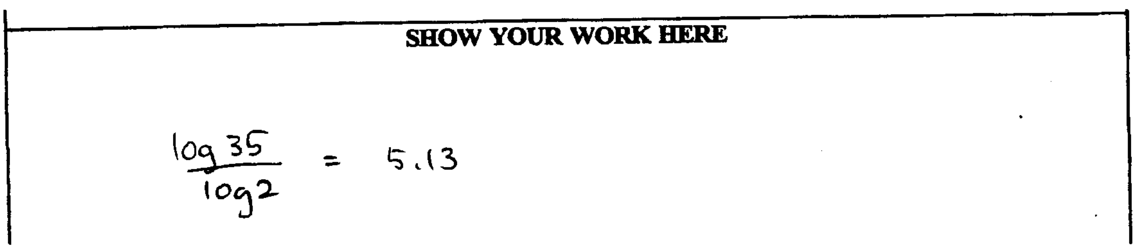
However, very few students actually understood where the mistake was made and pointed that out in the answer (See Figure 19).



The student properly included brackets around the -2 and the -3 giving him/her the correct answer of 85. In addition, he/she was able to identify, with the statement " you forgot the brackets", that the solution provided was incorrect because the brackets have been left out.

Figure 20 Student Showing Higher Level of Understanding

The logarithmic feature was assessed by Question 10 of the graphing calculator section (See Figure 20). Here students needed to apply the change of base rule to calculate the $\log_2 35$. Grade 12 students in the sample group got this question correct, while the other students did not attempt the question.

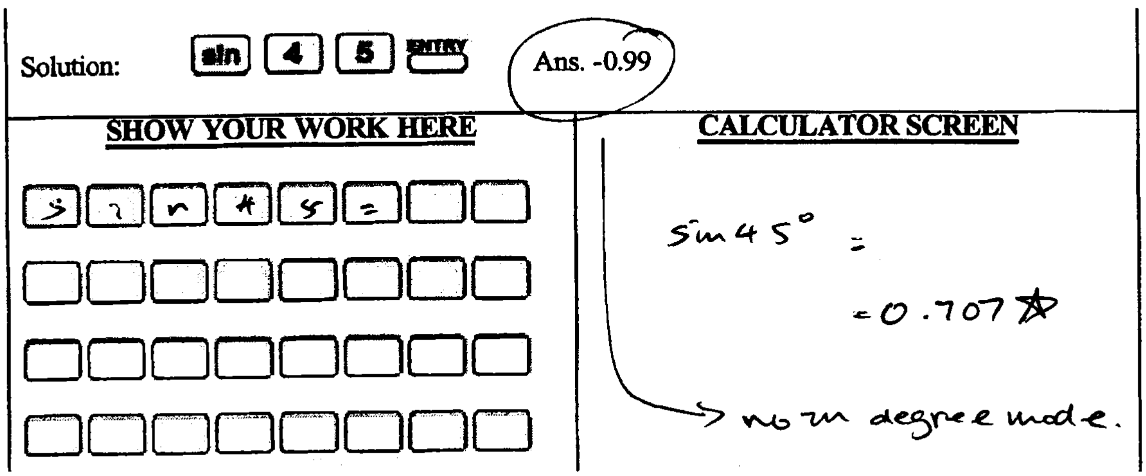


Example of the typical solution presented for this question.

Figure 21 Logarithms

The computation of basic trigonometric ratios was addressed in Questions 10 and 15 of the non-graphing calculator section as well as the Question 5 in the graphing calculator section. Questions 10 and 15 were generally done well in the sample group. Of the 69% of students who attempted Question 10 and of the 77% of students who attempted the Question 15 an average of over 70% got the questions right. Students on the whole knew they needed to set their calculators in degree mode and they knew the basic syntax needed to compute trigonometric ratios (See Figure 21). Question 5 in the non-graphing calculator section was equally well done. Of the 69% of the sample population that attempted this question over 80% of them got the question right. Of those who answered incorrectly, the main source of error was their misinterpretation of

$\csc\frac{3\pi}{5}$ as $\frac{1}{\cos\frac{3\pi}{5}}$ rather then as $\frac{1}{\sin\frac{3\pi}{5}}$

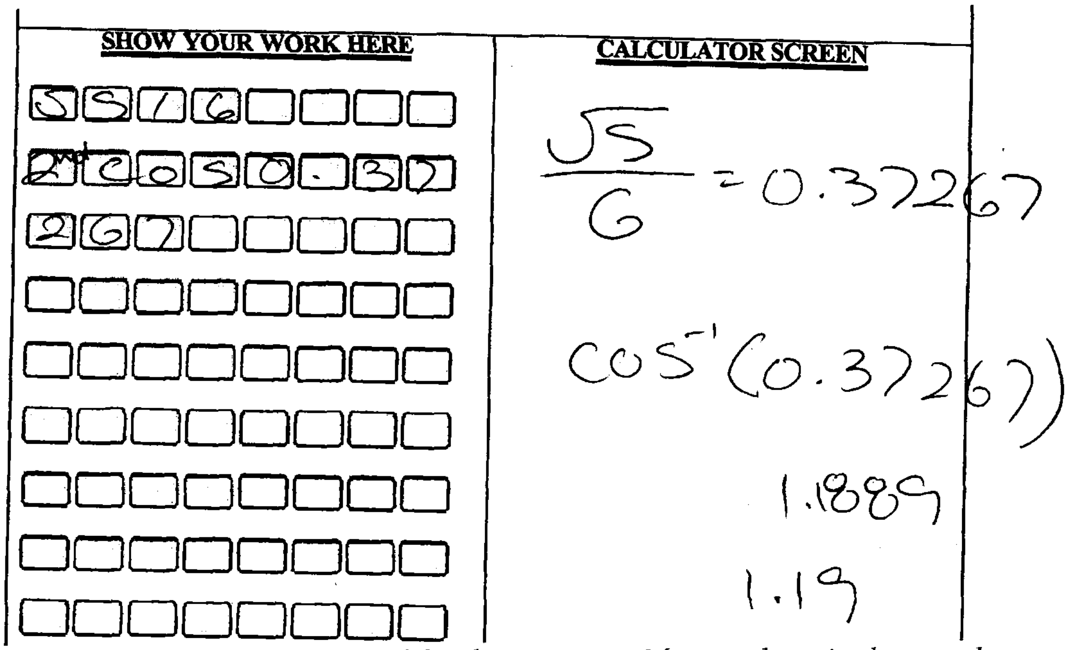


The student identified that the solution given was not in degree mode. Since none of the sample group in the non-graphing calculator section had been exposed to the concept of radian measures, all of their trigonometric calculations should have been done in degree mode.

Figure 22 Degree Mode

The trigonometric ratio question was generally well done on the test. Many students converted the $\frac{\sqrt{5}}{6}$ into a decimal first before taking the arc cosine of the decimal value (See Figure 22).

Find θ to the nearest tenth of a degree: $\cos\theta = \frac{\sqrt{5}}{6} \quad 90^o \geq \theta > 0^o$



This is a typical solution presented for this question. Most students in the sample group broke this question down into two separate calculations: One to turn $\frac{\sqrt{5}}{6}$ into a decimal and the second to find the arccosine of 0.3726779962.

Figure 23 Trigonometric Ratios

A small number of students mixed up the cosecant and the secant ratios, thus giving them an incorrect answer even though they knew how to execute the computation on their calculator (See Figure 23).

Find $csc\frac{3\pi}{5}$ to 2 decimal places

SHOW YOUR WORK HERE

$$\frac{1}{\cos\frac{3\pi}{5}} = 1/(\cos(3\pi/5))$$

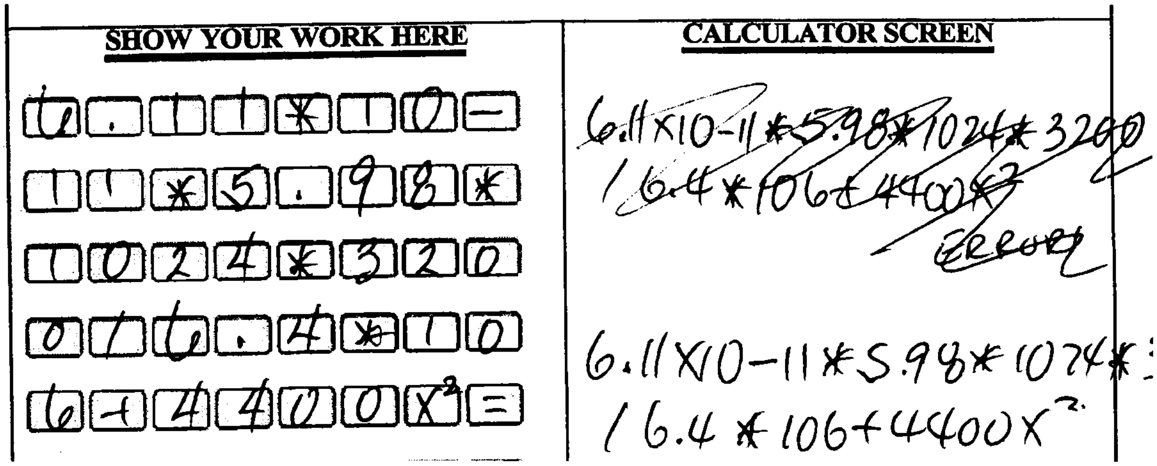
$$-3.236$$

Example of a typical error found in the sample group for this question. The student was able to perform the required calculator syntax to answer the question, but used the wrong reciprocal trigonometric ratio.

Figure 24 Inverse Trigonometric Ratios

The constant π was assessed in Question 8 of the non-graphing calculator section of the CLT. Ninety-two percent of the sample population attempted this question. I observed from the CALCULATOR SCREEN section of the question that over 75% of the students utilized the π constant found on their calculator for this question. Of the remaining students who did not use the π constant, the value of 3.14 was used instead.

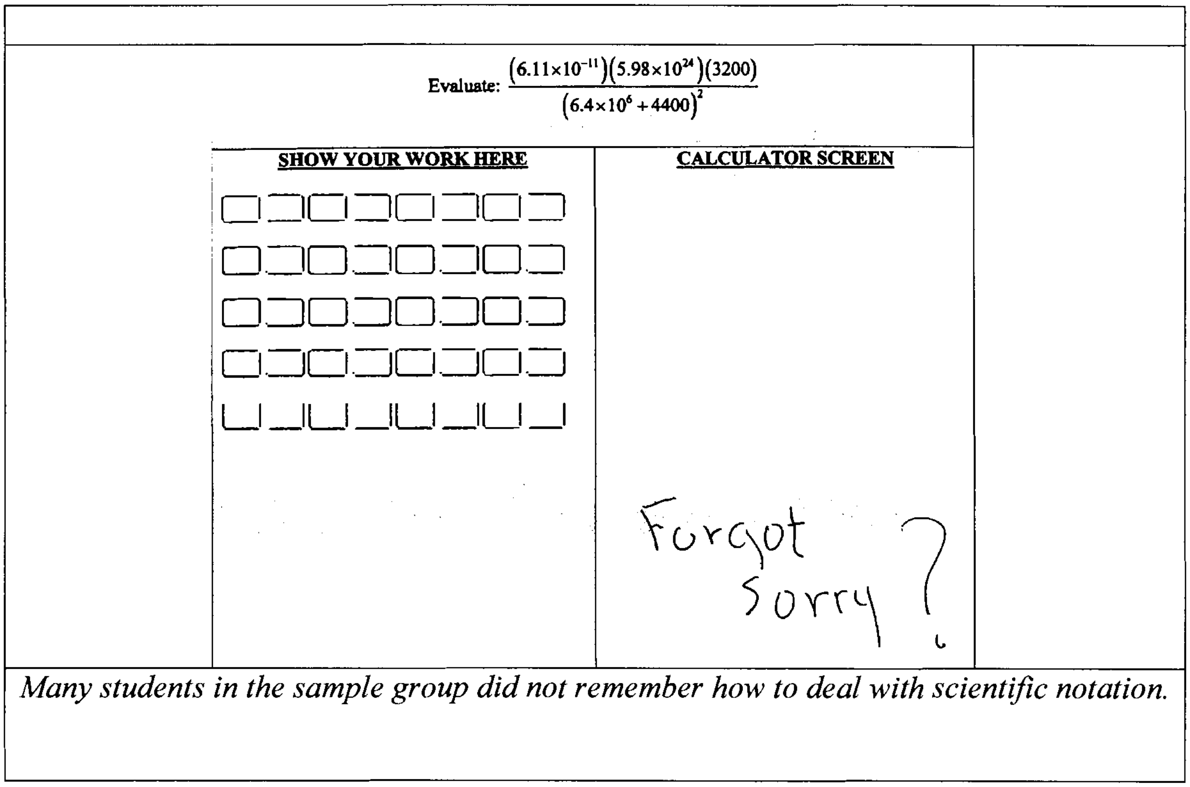
Question 5 of the non-graphing calculator section of the CLT was poorly done (See Figure 24). Of the 60% of students who attempted the question less than 50% knew how to activate the scientific notation feature on their calculator. In fact, in the CALCULATOR SCREEN area some students indicated that they had forgotten what scientific notation was.



The student clearly did not recognize how to activate the scientific notation feature on the calculator. He/She started out by calculating 6.11 times 10 negative 11 and got an error message on his/her calculator. Realizing the mistake, he/she then redid the calculation using 6.11 times 10 subtract 11 and obtained an answer of -3550652099.

Figure 25 Incorrect Use of Scientific Notation

Some students did not remember how to work with scientific notation at all. This is a common complaint among senior level science teachers as they find that a portion of the students entering into their first senior level science courses cannot even evaluate an expression written in scientific notation (See Figure 25).

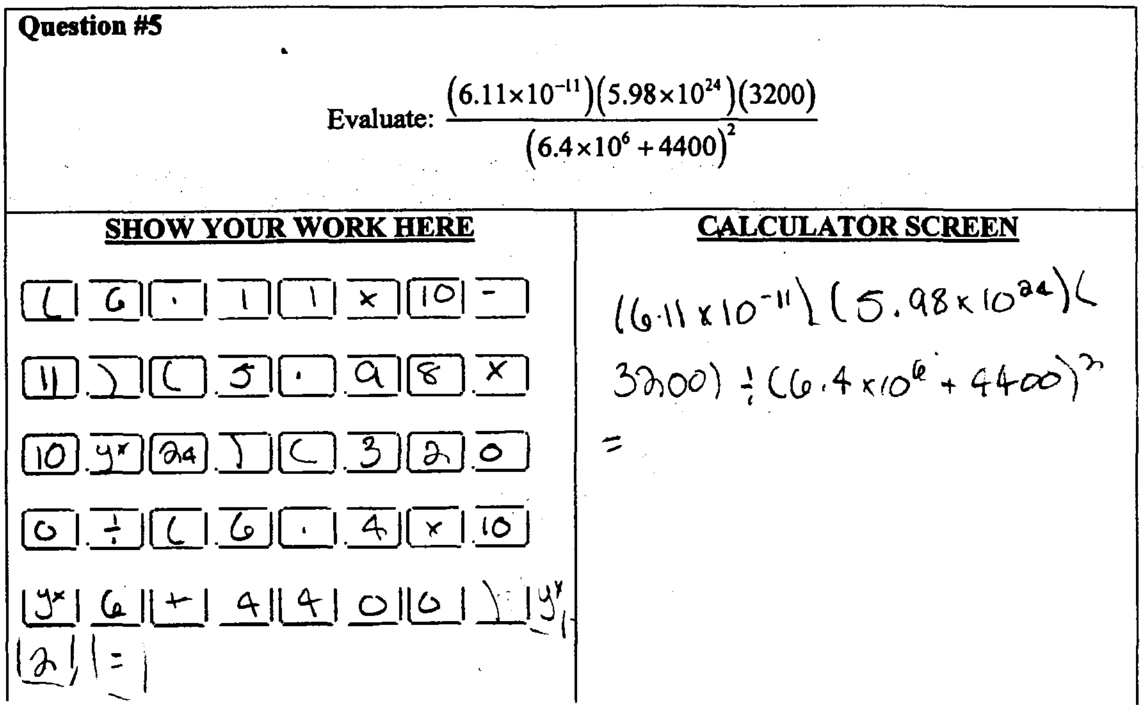

Evaluate: $\frac{(6.11\times10^{-11})(5.98\times10^{24})(3200)}{(6.4\times10^{6}+4400)^{2}}$

SHOW YOUR WORK HERE	CALCULATOR SCREEN
	Forgot Sorry ?

Many students in the sample group did not remember how to deal with scientific notation.

Figure 26 Student Showing Unfamiliarity with Scientific Notation Feature

Some students tried to enter the scientific notation as exponents rather than utilizing the calculator's built-in feature to handle scientific notation (See Figure 26).



Question #5

Evaluate: $\dfrac{(6.11\times10^{-11})(5.98\times10^{24})(3200)}{(6.4\times10^{6}+4400)^{2}}$

SHOW YOUR WORK HERE	CALCULATOR SCREEN
(6 . 1 1 × 10 - 11) (5 . 9 8 × 10 y^x 24) (3 2 0 0 ÷ (6 . 4 × 10 y^x 6 + 4 4 0 0) y^x 2 =	(6.11 × 10^{-11}) (5.98 × 10^{24}) (3200) ÷ (6.4 × 10^{6} + 4400)^{2} =

The student used the y^x function rather than the scientific notation function on his/her calculator. This is a typical pattern observed in the sample group. Many of the students who got this question correct utilized the exponent feature of their calculator rather than the scientific notation feature.

Figure 27 Incorrect Use of the Exponent Feature

Lastly, Question 12 of the non-graphing calculator section of the CLT addressed the exponent feature. Here students need to raise -3 to the fourth power as well as to place brackets around both the -3 and the -2 within a set of brackets before raising them to even powers. Of the 77% of students who attempted this question only 43% got the question right. I observed from the CALCULATOR SCREEN area that many students either completely forgot about this rule or were inconsistent in applying the rule.

In summary, the CLT fulfilled the goal in assessing for basic calculator skills. It provided students with various mathematical questions that tested their calculator skills. It also provided a wealth of information into some of the practices and sources of errors of the sample group.

Part III-The CLT and Definition of Calculator Literacy

In this section, I will examine how effectively the CLT addressed the various components of the working definition of calculator literacy defined in this study.

> *To be calculator literate one should be able to manage or solve a range of mathematical situations with the aid of a calculator*

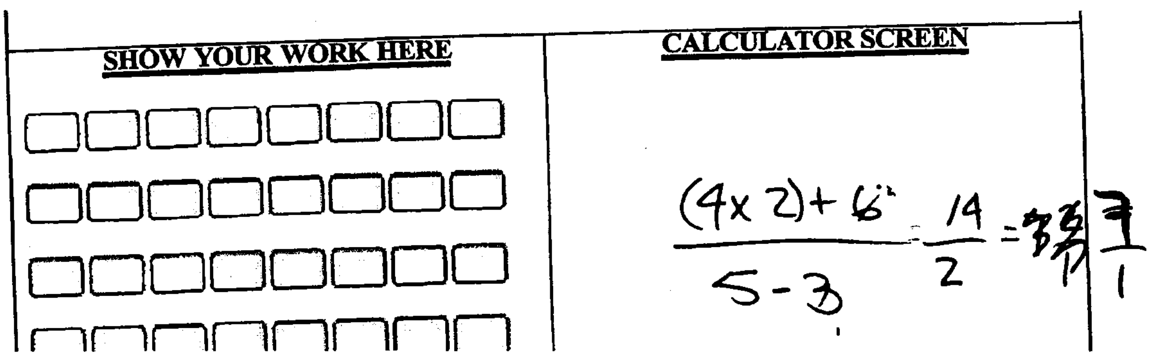
In both the graphing and non-graphing calculator sections of the CLT, students were presented with a variety of mathematical questions to solve by utilizing their calculators. Questions ranged from figuring out the total cost of a purchase including tax to finding the amplitude of a trigonometric function. Based on the sample data collected, I feel that I was able to observe students using their calculator to solve a wide range of mathematical situations in the two components of the CLT.

One should be able to manipulate the mathematics be it represented in a numerical, spatial, graphical, statistical, or algebraic manner to effectively communicate with the calculator.

This criterion is not easily assessable by the CLT. I was only able to observe mainly algebraic manipulations and limited graphical, statistical, and numerical manipulations. Questions 5, 6, 8, 9, 10, 11, 12 and 16 of the non-graphing calculator section required the user to algebraically manipulate and rewrite the question in some way prior to entering the question into the calculator. In fact, Question 11 required a high degree of manipulation by requiring students to first translate the sigma notation in the question into the calculator's own notation and syntax system before finding the answer. From the data collected I observed how well students were able to manipulate the mathematics.

> *Being calculator literate also involves knowing when the calculator should be used.*

Question 3 in the non-graphing calculator section is an example of how the CLT provided students with the opportunity to demonstrate their understanding of when it is easier to pick up a calculator for a solution and when it is not. The computations involved in question 3 are relatively simple and can be performed mentally. However, a few students in the group indicated that it was easier for them to answer Question 3 in their head than to use their calculator (See Figure 28).

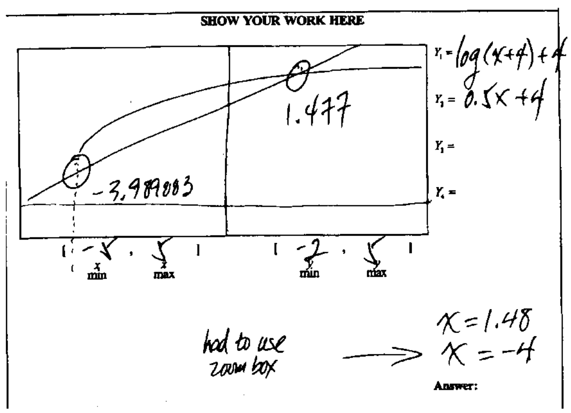


The students indicated that he/she required no calculator to perform this calculation.

Figure 28 Knowing When Not to Use a Calculator

One should know the calculator's limitations.

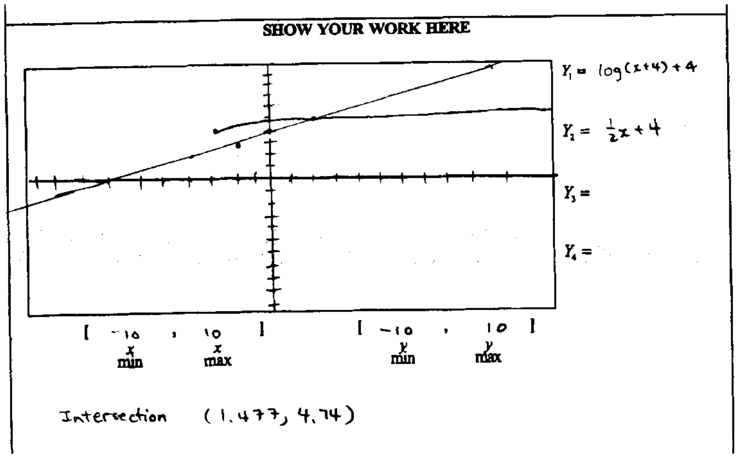
Questions 2 and 3 of the graphing calculator section addressed the limitations of the graphing calculator. Students had to be aware that they needed to extend the graph of the logarithmic function in order to obtain the second solution. Question 3 required students to be able to interpret the graphs plotted by the calculator in Question 2 to determine the proper solution to the given system of inequalities as the graphing calculator does not provide users with the solution directly. From the data collected I was able to observe how well students were able to work around these limitations (See Figure 29).



The student who is aware of the pixel limitations of the graphing calculator when producing logarithmic graphs would make the appropriate compensation to find the second root. This student indicated that the second solution was obtained by setting a zoom box to zoom in on the missing root. However, the student rounded off his/her answer too early and mistakenly took x to equal to -4.

Figure 29 Limitations of the Graphing Calculator

Over 50% of the students were unable to recognize the graphing calculator's limitation in representing a logarithmic function. Students failed to understand that even though their screen did not display two intersection points there were actually two solutions to Question 3 (See Figure 30).

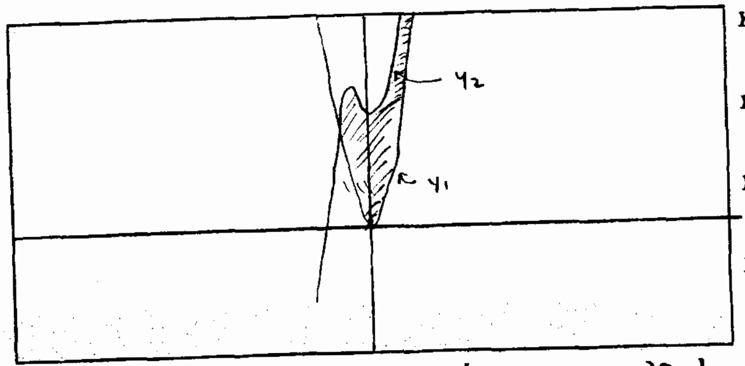


Typical solution found in the sample group. Because the logarithmic graph does not seem to intersect with the line at $x = -3.989883$ *on the screen, most students concluded that there was only one solution at* $x = 1.477$.

Figure 30 Student Showing a Lack of Understanding of the Graphing Calculator.

However, students were able to recognize some limitations of the graphing calculator and were able to work around these limitations. Many students were able to obtain the correct solution for Question 2 in the graphing calculator component even though the graphing calculator was not able to "give" them the solution set directly (See Figure 31).

SHOW YOUR WORK HERE



$Y_1 = x^4 + 5x^2$

$Y_2 = 6x^3 - 4x + 12$

$Y_3 =$

$Y_4 =$

[-10 , 10] [-10 , 20]

x_{min} x_{max} y_{min} y_{max}

intersection (4.9, 706), (-1.125, 7.44)

$x^4 + 5x^2 < 6x^3 - 4x + 12$

when x is between (-1.125 - 4.9)

Answer: $-1.125 < x < 4.9$

The student was able to interpret the graphs produced on his/her calculator and from there was able to utilize the information to solve the system of inequalities. This is an excellent example of how a calculator literate student can combine both calculator and pen and paper method to solve a problem.

Figure 31 Inequalities

One should apply his/her estimation and mental math skills to determine the reasonableness of the solution provided.

Question 3 was easy enough for students to apply their mental calculation skill to estimate an answer to the question. Although it is not possible to observe mental calculations based on what students recorded in the space provided on the CLT, some estimation and or mental calculation skills must have been used as I observed many incidences where work was erased or crossed out and replaced with a more reasonable solution.

Similarly Question 9 in the non-graphing calculator section served the same purpose. Any student who remembers their Pythagorean triple would surely recognize the commonly used 3-4-5 triangle, thus helping them determine that the answer provided was incorrect. In the data collected, some students did indeed recognize the triangle as a Pythagorean triples and they indicated that no calculator was needed in answering the question and they correctly identified that the solution provided was incorrect.

> *Having calculator literacy also means that one is able to grow and adapt to the changing technology as well as being able to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators.*

Though the above criterion was not explicitly tested in any of the questions in the CLT, it was tested indirectly in Questions 9 to 12 in the non-graphing calculator section. Most students were able to adapt to the Sharp calculator notation as well as its entry system in Questions 9 to 12. They were asked to determine if the indicated solutions were correct and to make any necessary corrections to the solutions. Even students who did not use a Sharp calculator, and were therefore unfamiliar with the Sharp system, adapted quite well.

From the CALCULATOR SCREEN section I found that students were able to adapt to a different notation system relatively easily as most who attempted the questions were able to reproduce keystroke entries similar to those used by the Sharp. In the graphing calculator section however, this criterion is not as easily determined as all of the sample population used either a TI83 or a TI83 Plus. From the data collected in the CLT, I developed a list of common mistakes observed in the sample group (See Table 12). Many of these mistakes concur with what I have observed and encountered in my teaching. This list will be helpful in any future development of calculator literacy assessments as well as for my own teaching where it can help me to develop and improve teaching strategies to better enhance my students' calculator skills.

Table 12 Common Errors Observed on the CLT.

- Poor usage/placement of brackets to preserve order of operations in calculations.
- Poor usage/placement of brackets when dealing with negative bases
- Poor usage/placement of brackets around argument of functions such as the square root function.
- Not recognizing/understanding the use of the Scientific Notation feature
- Poor understanding of Advance Fraction features to convert between improper fractions and mixed numbers
- A complete lack of understanding of the most basic statistical features of a scientific calculator.
- Improper interpretations of the solution presented by the graphing calculator in System of Inequalities
- Inappropriate window settings displaying only parts of the function
- Inappropriate window settings failing to display enough of the graph so that recognizable characteristics of the function can be seen.
- Not recognizing the limitations of the graphing calculator in displaying logarithmic functions.
- Poor usage/placement of brackets to preserve order of operations in complex calculations
- Reciprocal Trigonometric Ratios were entered incorrectly
- Not recognizing/understanding the use of the Table of Values feature
- Not recognizing/understanding the use of the Matrix feature
- Not recognizing/understanding the use of the Sum(Seq (features
- Not recognizing/understanding the use of recalling a function from the VARS menu
- Incorrect use of statistical features of the calculator.

I felt the CLT was fair and met the goals set forth by this study. It provided me with an opportunity to observe how various students met the basic criteria of calculator literacy as well an opportunity to observe some common mistakes made by students. The range of student abilities made it difficult to tell whether or not some of these questions were too difficult and the lack of work shown by some students made it difficult to accurately judge their calculator literacy level but on the whole the sample group provided me with an overall picture of the students' knowledge. It also gave me a better perspective of how calculator literate students are.

CHAPTER FIVE
CONCLUSIONS

This chapter of the thesis is divided into three sections. The first section deals with the conclusions reached as a result of the Calculator Literacy Test while the second section deals with what I would change about the CLT. The final section deals with my concluding remarks.

Overview of the Results from the CLT

Testing for calculator literacy is not a simple task. I cannot cut and paste together a variety of traditional mathematical questions and administer it to a group of students with calculators and let the results decide if they are calculator literate or not. Traditional assessments in mathematics are designed to be calculator independent or calculator neutral (Lokar, & Lokar, 2000. p. 1). Their focus is on assessing students' algebraic and problem solving skills, not calculator literacy. Assessing for calculator literacy requires an assessment designed specifically with the calculator in mind. The range of mathematics that the students in the study knew further limited this study. Many standard features such as the trigonometric functions, logarithmic functions, and factorial calculations are considered routine, requiring minimal understanding of the advantages, disadvantages and the limitations of the calculator. However, most of the students in the study were not exposed to these mathematical concepts at the time of the study.

However, despite these limitations, the data collected in the CLT provided a wealth of new and old information to me. I observed new and different mistakes. Various approaches and solutions to correctly answering the questions of the CLT were presented and they opened my eyes to the varying degrees of calculator literacy these students possessed. Some were clearly stronger than others but on the whole most were at least competent in the basic features of their calculator. The improper usage and placement of brackets by many of the students in the sample groups remains one of my biggest concerns. The lack of understanding, shown by some of the students in the sample group, with regard to the importance of order of operations tells me that more time needs to be spent and better resources need to be developed to further help students master and apply this concept.

As with most topics in mathematics, calculator literacy is interrelated with other areas and topics in mathematics and the results from the calculator literacy test do not only reflect students' understanding of their calculator but also their understanding of other aspects of their mathematics studies. Looking at the data obtained, some students who struggled in the more traditional pen and paper environment with fractions did well on the questions dealing with fractions on the CLT. Though many of them did not know how to utilize the conversion feature on their calculator to switch their final answer into its improper fraction form, they at least attempted to solve the problem and were able to get the correct answer into its mixed fraction form. In a traditional pen and paper environment many of these students may have simply given up on the question. This affirms my belief that integration of the calculator in the teaching and learning of mathematics is beneficial to students and that the improvement and assessment of students' calculator literacy level are essential.

Question 8 of the graphing calculator section of the CLT showed me how creative the students in the sample could be in solving a problem. I was fully expecting students to have to utilize the table of values feature on their calculator to solve this problem but I was surprised to observe how many of them utilized the Trace feature or some other means to estimate the correct answer. This showed me that indeed when *appropriately applied, the calculator is a very useful tool for investigating, exploring, conjecturing, and computing a variety of quantitative and mathematical problem.* This was true for Question 13 in the non-graphing calculator section as well.

With the compilation of the working definition of calculator literacy and the data collected from the CLT, I have gained a much deeper and wider insight into students' usage of their calculator and this information will help me in my future design, and development of learning resources as well as the way I assess students.

Future Improvements to the CLT

Writing items for achievement tests such as the CLT is a challenging enterprise (British Columbia Ministry of Education, 2003, p. 5). However, I see the value of the CLT in helping to determine students' strengths and weaknesses in their use of calculators in class. Therefore, I would refine the CLT in the future for regular use in my teaching practice. A couple of workshops I attended recently showed me the possibility of asking some of the questions on the CLT differently to more effectively evaluate calculator literacy. To make the test less labour intensive for students I would like to experiment with incorporating some multiple-choice (See Figure 32), matching (See Figure 33), or survey style (See Figure 34) questions. Multiple-choice items allow for more versatility, objectivity, discrimination achievement levels and reduces guessing (British Columbia Ministry of Education, 2003), p. 13) while Matching items are easy to construct, quick and reliable to score, conserve reading time, and can be used to measure students' ability to discriminate among and apply concepts (British Columbia Ministry of Education, 2003, p. 20). Figure 32 shows an example of how a question that is designed to assess for the proper bracket placement would look like a multiple-choice question.

Evaluate: $$\frac{4.11\times 2.34+6.56}{5.7-3.3}$$

a. $4.11\times 2.34+6.56/5.7-3.3$
b. $(4.11\times 2.34+6.56)/5.7-3.3$
c. $(4.11\times 2.34+6.56)/(5.7-3.3)$
d. $4.11\times 2.34+6.56/(5.7-3.3)$

Figure 32 Multiple Choice Questions

Matching questions are useful as they can cover a variety of skills in one question. Figure 33 is an example of how a matching style question can be used to asses for calculator literacy.

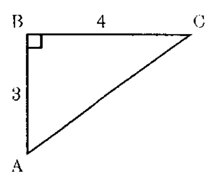
For each question in Column I, choose the appropriate keystroke entry in Column II that would provide the correct answer.	
Column I	Column II
1. $\frac{1}{2}+1\frac{3}{7}\times-2\frac{3}{5}$ leave answer as an improper fraction. 2. Find the area given: $Area = \pi r^2$ $r=1.26$ 3. $(-2)^2+(-3)^4$	A. $3.14\times1.26\times1.26$ B. $-2\ \boxed{x^2}\ +-3\ \boxed{y^x}\ 4$ C. $3.14\times1.26\ \boxed{x^2}$ D. $1\ \boxed{a\,b/c}\ 2+1\ \boxed{a\,b/c}\ 3\ \boxed{a\,b/c}\ 7\times-2\ \boxed{a\,b/c}\ 3\ \boxed{a\,b/c}\ 5\ \boxed{2ndF}\ \boxed{a\,b/c}$ E. $\pi\times1.26\ \boxed{x^2}$ F. $1\ \boxed{a\,b/c}\ 2+1\ \boxed{a\,b/c}\ 3\ \boxed{a\,b/c}\ 7\times-2\ \boxed{a\,b/c}\ 3\ \boxed{a\,b/c}\ 5$ G. $(-2)\ \boxed{x^2}\ +(-3)\ \boxed{y^x}\ 4$ H. $1\ \boxed{a\,b/c}\ 2+13\ \boxed{a\,b/c}\ 7\times-2\ \boxed{a\,b/c}\ 3\ \boxed{a\,b/c}\ 5$

Figure 33 Matching Style Question

Survey type of questions can provide us with a different way to observe students' calculator literacy level. Instead of focusing on the numerical answer alone, survey style questions can reveal more about how effectively students are using their calculators. In Figure 34 below, we can see how many students feel it is necessary to use their calculator to answer this question.

Indicate in the bubbles below if you feel you will need a calculator to solve the following question.

Find $\overline{AC}$



☐ Needed ☐ Undecided ☐ Not Needed

Figure 34 Survey Style Question

It takes quite a bit of time to create and mark the CLT and it is hard to make up new versions of the test each year; therefore, a machine score-able version of the CLT would be much more practical for future classroom use. At present, neither my district, my school, nor I, have a calculator literacy testing policy but I am certainly considering one.

Also, in light of the result from the non-graphing calculator component of the CLT, I would consider further reducing the number of questions on the CLT. I would focus more on basic skills such as proper use of the bracket keys, the fraction key, and the scientific exponent key. For the graphing active component of the CLT, I would omit Question 9 as not enough students in the Principles of Mathematics pathway have been exposed to the matrix feature on the calculator.

Finally, I would explore other means of delivering the CLT, instead of the traditional paper-and-pencil method. I would explore the possibility of delivering the CLT electronically. Software such as LXR•Test 6.0 by Logic eXtension Resources with its Interactive Extension allows teachers to create electronic tests, which promise to make the delivery of the CLT easier. In addition, I would explore the possible use of a virtual calculator or some other means to record students' actual calculator keystrokes more accurately. A virtual calculator would allow me to capture every keystroke students make electronically, thus giving me a more complete picture of their thinking process.

Modification to the Working Definition

One of the goals of this study was to establish a working definition of what a calculator literate person should be. The working definition that I have set forth at the beginning of the study served as a guide for the designing of the CLT as well as for assessing how effectively the CLT assessed students' calculator literacy level. However upon reflection, I would like to make the following modifications to the definition.

To be calculator literate one should be able to:

A) Manage or solve a range of mathematical situations *which are relevant to the person's age or mathematical background* with the aid of a calculator.

B) Manipulate *and interpret* the mathematics be it represented in a numerical, spatial, graphical, statistical or algebraic manner to effectively communicate with the calculator.

C) Know when the calculator should be used.

D) Know the calculator's limitations *and know when it is more appropriate to use other methods to solve a problem*.

E) Apply his/her estimation and mental math skills to determine the reasonableness of the solution provided by the calculator.

F) Grow and adapt to the changing technology as well as to utilize, with minimal difficulty, the technology in its various notations and entry-system logics as it appears in different brands of calculators.

G) *Understand concepts, terminology and operations that relate to general calculator use such that he/she can function independently with a calculator. This functionality includes the ability to solve and avoid problems, and grow with the ever changing technology in order to adapt to new situations*.

Calculator literacy needs to take into account the age and the prior learning of the student. The range of mathematical situations that a 5th grader should be able to solve is different than that of a 10th grader. The range of mathematical situations used to assess for a student's calculator literacy should be relative to mathematics that is relevant to him/her. There is not a fixed list of things that a calculator literate person should be able to do but rather a core group of concepts that he/she should understand. For example, in part B of the definition I added interpretation of the mathematics as an important part of being calculator literate because I feel that the understanding that the calculator might not always provide the solution in a form that is immediately useful in solving the problem is critical in using a calculator effectively. This was clearly seen on Question 2 of the Graphing Calculator component of the CLT as many of the students were not able to interpret how the graph of the two functions can help them determine the solution to the system of inequalities.

Furthermore, it is important for students to understand that the calculator is one method to solve a problem and that sometimes, due to the limitations of the calculator, it is more appropriate to solve a problem by some other method. For example,

Solve $\sqrt{x+2} = -x-2$

If we try to solve this equation using a graphing calculator, we would encounter an error message on the calculator informing us that there was "no sign change". A calculator literate person would see that it would be more appropriate to use some other method to solve this problem instead of using the calculator. In fact, this question can be easily solved algebraically. The working definition as it was did not allow for a calculator literate person to demonstrate this understanding.

Lastly, I added part G to the working definition to address two other points of concern. The first part of part G deals with the general operations of the calculator such as parameter settings (degree mode and radian mode, fixed decimal point calculations or floating decimal point calculations, and Function, Parametric or Polar Coordinates mode). I feel that it is important for a calculator literate person to have an understanding of the basic operation of the calculator so that he/she can avoid or solve problems related to the operation of the calculator should they arise. The latter part of part G deals with how a calculator literate person should be able to grow and adapt to the changing technology. New models of calculators loaded with various new features are being released onto the market every year. A calculator literate person should be able to grow and adapt to new technologies based on his/her understanding of the current technology. As well, the person should be literate enough in the concepts and terminology of calculators to access additional resources in the form of calculator manuals or technical support on the internet to fully utilize the new technology as it becomes available.

Contribution of the Study

One of the goals of this study is to lay the foundation for further research in the study of Calculator Literacy and I feel this study has achieved this goal. I have

- developed and refined a definition for what a calculator literate person should be,
- designed a question format which allows students to communicate the use of their calculator in a paper-and-pencil manner, and
- identified and designed questions which are specifically created to assess for students' calculator literacy level.

Concluding Remarks

The term calculator literacy is not a well recognized term. Neither the NCTM nor the British Columbia Ministry of Education has formally defined it. Various expectations and general guidelines for the appropriate use of a calculator are scattered throughout the

Principles and Standards as well as the IRP. No explicit definition of what it means to be calculator literate has been stated and certainly no formal assessment to test the calculator literacy level of our students has been developed and adopted. Although guidelines and suggestions to test various calculator skills can be seen throughout the different Learning Outcomes in the IRP, no explicit guidelines for testing how well a student can use his/her calculator can be found.

Appropriate instruction that includes calculator use can extend students' understanding of mathematics and will allow all students access to rich problem-solving experiences. Such instructions help students develop their calculator literacy level (AAMT, 1996, p. 5). Students will learn to know how and when to use a calculator to develop the skill of estimation, both numerical and graphical, and learn the ability to determine if a solution is reasonable are essential elements for the effective use of calculators. Calculator Literacy means the appropriate use of paper-and-pencil *and* calculator techniques on a regular basis. Used properly, paper and pencil and calculators can complement each other. It is important to know how to estimate an answer before doing a computation using either a calculator or paper and pencil. It is important that students have enough number sense to recognize when answers are correct and that they know methods of checking answers without doing the problem over. And it is important for students to understand, at least on an intuitive level, why procedures work and when they are applicable. Balance does not mean that we quit teaching such skills as long division or factoring.

Time must be provided in the curriculum and the classroom for *appropriate* practice of calculator skills. Being calculator literate requires students to be more than competent with their calculators. Simply reproducing what had been taught in the classroom and being able to recognize which set of previously learned procedures is required in a given problem to produce the correct answer is not enough. Students should be able to solve problems using paper and pencil and then *support* the results using a calculator to check their solutions. They should be able to solve problems using their calculator and then *confirm* the results using paper and pencil, estimation or mental calculations. Also, students should be able to solve problems for which they choose whether it is most appropriate to use paper-and-pencil techniques, calculator techniques, or a combination of both.

If we want to see both the basic skills and the problem-solving skills of our students improve in contexts that allow the regular use of calculators, then we must continue to develop methods that we might agree to call "appropriate uses," not only for calculators but for paper-and-pencil techniques as well. In this regard, I have come to the conclusion that a balanced approach of paper-and-pencil techniques and technology in the teaching and learning of mathematics is essential. We also need to develop and implement assessments that would determine students' calculator literacy level and from these assessments learn how we can help them improve their calculator skills. Recognizing common mistakes and modelling the proper usage of the calculator in class are vital in the improvement of student's calculator literacy development.

Recommendations

From this study I would strongly recommend that the Ministry of Education and the NCTM put forth packages similar to the two graphing calculator resource packages for the Principles of Mathematics 12 and the Applications of Mathematics 12 with a focus on what learning outcomes students are expected to master at various stages in K-12 framework. Rather than focusing on what will appear on an exam, these calculator resource packages should outline learning outcomes that students should be able to master on their calculator at the elementary, the middle school and the senior high level. It should provide examples that would illustrate the advantages as well as the limitations of the scientific calculator as well as the graphing calculator. A core list of calculator skills should be identified by the Grade 10 level to ensure that students entering senior level science courses are calculator literate. Though the two graphing calculator resource packages produced by the Ministry of Education are very useful, they do not define and illustrate what calculator skills and literacy students are expected to master. With the new Principles of Mathematics 12 curriculum, the Principle of Mathematics 12 Graphing Calculator Resource Package is outdated and not widely used anymore. This leaves the Applications of Mathematics Graphing Calculator Resource Package as the only document in British Columbia that clearly defines and illustrates what calculator skills are expected of our students and what level of calculator literacy they should master. Considering only a small population of the students in British Columbia are enrolled in the Applications of Mathematics 12 program, the Applications of Mathematics 12 Graphing Calculator Resources Package does not address the needs of the students in the province. Also, calculator literacy should be classified as a separate standard within the IRP by the British Columbia Ministry of Education. More attention and study needs to be done on this topic on national level. Furthermore, high stakes examinations such as the future Grade 10 provincial examinations in British Columbia should include a calculator specific section or have calculator specific questions integrated throughout the exam rather than be calculator neutral. A calculator neutral exam fails to test for specific calculator skills as well as intelligence in the use of the technology, thus leaving educators in the dark as to students' ability to appropriately use their calculator. Students' calculator literacy level must be tested, and proper and effective assessment must be developed.

APPENDIX A: ETHICS APPROVAL

SIMON FRASER UNIVERSITY

OFFICE OF RESEARCH ETHICS

BURNABY, BRITISH COLUMBIA
CANADA V5A 1S6
Telephone: 604-291-3447
FAX: 604-268-6785

June 10, 2003

Mr. Jimmy Wu
Graduate Student
Faculty of Education
Simon Fraser University

Dear Mr. Wu:

Re: Calculator Literacy

I am pleased to inform you that the above referenced Request for Ethical Approval of Research has been approved on behalf of the Research Ethics Board. This approval is in effect for twenty-four months from the above date. Any changes in the procedures affecting interaction with human subjects should be reported to the Research Ethics Board. Significant changes will require the submission of a revised Request for Ethical Approval of Research. This approval is in effect only while you are a registered SFU student.

Your application has been categorized as 'minimal risk" and approved by the Director, Office of Research Ethics, on behalf of the Research Ethics Board in accordance with University policy R20.0, http://www.sfu.ca/policies/research/r20-01.htm. The Board reviews and may amend decisions or subsequent amendments made independently by the Director, Chair or Deputy Chair at its regular monthly meetings.

"Minimal risk" occurs when potential subjects can reasonably be expected to regard the probability and magnitude of possible harms incurred by participating in the research to be no greater than those encountered by the subject in those aspects of his or her everyday life that relate to the research.

.../2

Please note that it is the responsibility of the researcher, or the responsibility of the Student Supervisor if the researcher is a graduate student or undergraduate student, to maintain written or other forms of documented consent for a period of 1 year after the research has been completed.

Best wishes for success in this research.

Sincerely,

Dr. Hal Weinberg, Director
Office of Research Ethics

c: Dr. Tom O'shea, Supervisor

/jmy

APPENDIX B: CALCULATOR LITERACY TEST (NON-GRAPHING CALCULATOR SECTION)

Calculator

Literacy Test

Date: ______________

Grade: ______________

Calculator Brand: ______________ *e.g. Casio*

Calculator Model: ________ *e.g. fx-260*

Calculator Literacy Test

Participant Consent Form

As partial fulfilment of a Master of Science degree in Secondary School Mathematics Education at Simon Fraser University, I am conducting a study on Calculator Literacy. As part of my study, I am asking you to participate in a calculator literacy test. The purpose of my study is to design a test that would assess the calculator literacy level of a student. This is important because as our world advance technologically; people must also advance in their technological skills. The calculator literacy test will take no more than 1 ½ hours of your time and no personal information will be required from you. Your name and identity will remain anonymous and this test will in no way affect your grade at school. The information obtained from this test will help me with my study and it will be destroyed when my study is completed.

You are not required to take this test and you may stop anytime during the testing period. Furthermore, any concern or complaints about the study may be registered with Dr. Ian Andrews, acting Dean of the Faculty of Education at Simon Fraser University, or myself. Please follow the instructions for the test carefully and provide as much information as possible as to how you arrived at each answer. Not every question requires the use of a calculator, if you feel the question can be answered better without a calculator feel free to do so. The purpose of this test is not to examine if you know how to answer the question on the test, but rather I am interested in how good of a test this is for assessing calculator literacy. Do the best that you can and do not be discouraged if you cannot answer some of the questions.

Please indicate below your consent to participate in this study.

I hereby consent to participate in the study described above for Mr. Wu's study.

____________________________ ____________________________

Student's Name Student's Signature

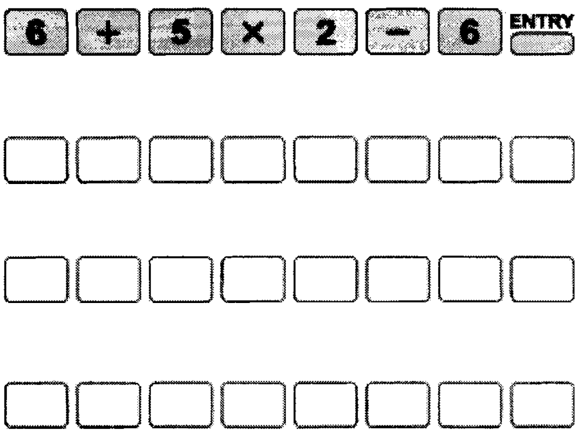
Instructions: Please clearly show **all** calculator keystroke entries and **all** information produced by the calculator in your response. In addition, any manual calculations performed by you must be indicated in the response area as well. Once again, not every question requires the use of a calculator, if you feel the question can be answered better without a calculator feel free to do so. Below are a few sample questions including screen images to illustrate what is required.

Important: Please note that different calculators use different symbols to represent the same function. For this test the following are equivalent and acceptable.

Power	^	x^y	y^x	
Scientific Notations	EE	EXP		
Inverse	$\frac{1}{x}$	x^{-1}		
Negative	(-)	$+/-$		
Equal	EXE	=	ENTRY	ENTER

Sample Question #1

$$6+5\times 2-6$$

SHOW YOUR WORK HERE	CALCULATOR SCREEN
	$6 + 5 \times 2 - 6$ 10

Answer: 10

Sample Question #2

$$\frac{1}{2}+\frac{1}{6}+1$$

SHOW YOUR WORK HERE	CALCULATOR SCREEN
1 ab/c 2 + 1 ab/c 6 +	1 ⌟2 + 1 ⌟6 + 1
1 ENTRY	1 ⌟2 ⌟3

Answer: $1\frac{2}{3}$

Question #1

Evaluate: $6.23 + 5.24 \times 2.1 - 6.25 \div 2.5$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #2

Evaluate: $-3-14\times-2+16\div-8$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #3

Evaluate: $\dfrac{4\times 2+6}{5-3}$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #4

Evaluate: $-2\left[3+(3+4)\right]-1$

SHOW YOUR WORK HERE	CALCULATOR SCREEN

Answer:

Question #5

Evaluate: $$\frac{\left(6.11\times10^{-11}\right)\left(5.98\times10^{24}\right)(3200)}{\left(6.4\times10^{6}+4400\right)^{2}}$$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #6

Evaluate and **leave answer as an <u>improper fraction</u>**: $\frac{1}{2}+1\frac{3}{7}\times-2\frac{3}{5}$

<u>**SHOW YOUR WORK HERE**</u>	<u>**CALCULATOR SCREEN**</u>

Answer:

Question #7

Evaluate and **leave answer as an <u>improper fraction</u>**: $\frac{45}{32}+\frac{1}{4}\left(\frac{1}{3}-\frac{5}{6}\right)$

<u>SHOW YOUR WORK HERE</u>	**<u>CALCULATOR SCREEN</u>**

Answer:

Question #8

Find the area given: $Area = \pi r^2 \quad r = 1.26$

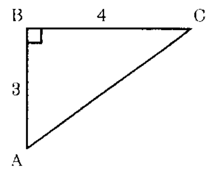
SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #9

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Find the segment AC



Solution: $x =$ [$\sqrt{\ }$] [3] [x^2] [+] [4] [x^2] [ENTRY] Ans. 19

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #10

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Find the $\sin 45^{o}$ **to 2 decimal places**

Solution: [sin] [4] [5] [ENTRY] Ans. -0.99

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #11

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Evaluate: $\sqrt{(2-3)^2+(3-1)^2}$

Solution:

[√] [(] [(] [2] [−] [3] [)] [x^2] [+] [(] [3] [−] [1] [)] [x^2] [)] [ENTRY]

Ans. 2.24

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #12

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Evaluate: $(-2)^2 + (-3)^4$

Solution: $\boxed{(-)}\ \boxed{2}\ \boxed{x^2}\ \boxed{+}\ \boxed{(-)}\ \boxed{3}\ \boxed{y^x}\ \boxed{4}$ ENTRY Ans. -85

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #13

If the fraction key on your calculator is broken, how would evaluate the following?

$$R = \left(\frac{1}{\frac{1}{33} + \frac{1}{99} + \frac{1}{11}} \right)$$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #14

If the square root, $\boxed{\sqrt{\ }}$**, and the** ***nth*** **root key,** $\boxed{\sqrt[x]{\ }}$ **, keys on your calculator are broken**, how would you evaluate the following?

$$d=\sqrt{(2-12)^2+(15-35)^2}$$

<u>**SHOW YOUR WORK HERE**</u>	<u>**CALCULATOR SCREEN**</u>

Answer:

Question #15

Find θ to the nearest tenth of a degree: $\cos\theta = \dfrac{\sqrt{5}}{6} \quad 90^{o} < \theta \le 0^{o}$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #16

A shirt at Costco costs $19.89 and a pair of gloves costs $12.95. If Jack buys 2 shirts and 1 pair of gloves, then how much would Jack have to pay if he has to pay 7.5% PST and 7% GST?

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Use the following set of data to answer Question 17 and 18

Number of Students	*2*	*4*	*5*	*7*	*12*	*10*	*8*	*2*
Score on the Test	*30*	*40*	*50*	*60*	*70*	*80*	*90*	*100*

Question #17

Above is a table of examination results from Mr. Smith's Math class. Find the average mark of the class.

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #18

Calculate the standard deviation of the above scores.

SHOW YOUR WORK HERE	CALCULATOR SCREEN

Answer:

APPENDIX C: CALCULATOR LITERACY TEST (GRAPHING CALCULATOR SECTION)

Graphing Calculator

Literacy Test

Date: ____________________

Grade: ____________________

Calculator

Model: ____________________

Instructions:

Please clearly show any relevant calculator entries and all information produced by the graphing calculator in your response. In addition, any manual calculations performed by you must be indicated in the response area as well. For example, if a graph is used in the solution of the problem it is important to sketch the graph, showing its general shape and indicating the appropriate values and equations entered into the calculator. If the statistical features of the calculator are used, it is important to show the function with the substitution of the relevant numbers. For example: in part of the solution it is acceptable to show normalcdf (40, 40, 47, 10) or the equivalent syntax for the calculator used. Below are a few sample questions including screen images to illustrate what is required.

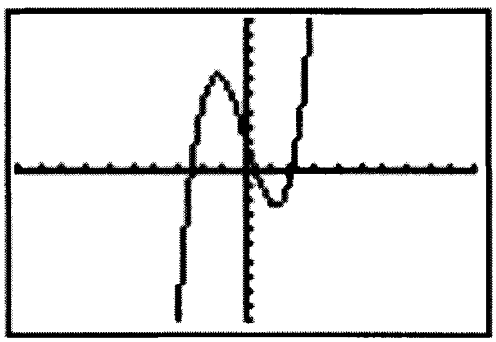
Sample Question #1

Solve the following equation using a graphing calculator:

$$x^3 - 5x + 2 = 0$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

SHOW YOUR WORK HERE



[−10 , 10] [−10 , 10]

x_{min}, x_{max}, y_{min}, y_{max}

$Y_1 =$ $x^3 - 5x + 2$

$Y_2 =$

$Y_3 =$

$Y_4 =$

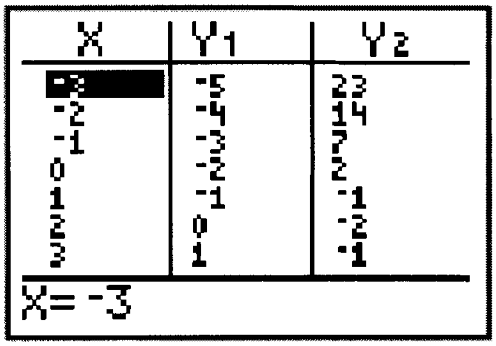
Answer: -2.41, 0.41, 2.00

Sample Question #2

Solve the following equation using a graphing calculator:

$$y = x - 2$$
$$y = (x-2)^2 - 2$$

SHOW YOUR WORK HERE



X	Y1	Y2
-3	-5	23
-2	-4	14
-1	-3	7
0	-2	2
1	-1	-1
2	0	-2
3	1	-1

X= -3

$Y_1 = x - 2$

$Y_2 =$
$(x-2)^2 - 2$

[,] [,]

x min x max y min y max

$Y_3 =$

$Y_4 =$

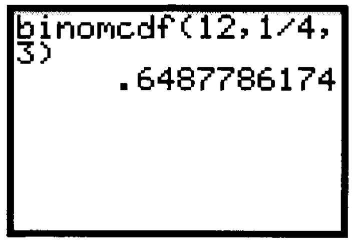
Answer: $(-1,1)$

Sample Question #3

A multiple-choice test has 12 questions. Each question has 4 choices, only one of which is correct. If a student answers each question by guessing randomly, find the probability that the student gets at most 3 questions correct?

SHOW YOUR WORK HERE

Many calculators have a *normal cumulative distribution function* ***normalcdf(***, many calculators have a *binomial cumulative distribution function* ***binomcdf(***. Screen images and key-strokes are provided below



Press [2nd] [DISTR] and [A] to obtain binomcdf(, then enter n, p, x, where

n = the number of trials

p = the probability of "success"

x = the maximum number of "successes" desired

Answer: 0.65

Question #1

Solve the following equation using a graphing calculator:

$$x^5 - 10x^3 + 10 = 0$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

$Y_3 =$

[$x_{\min}$, $x_{\max}$] [$y_{\min}$, $y_{\max}$] $Y_4 =$

Answer:

Question #2

Solve the following inequality using a graphing calculator:

$$x^4 + 5x^2 < 6x^3 - 4x + 12$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

$Y_3 =$

[x_{min} , x_{max}] [y_{min} , y_{max}] $Y_4 =$

Answer:

Question #3

Solve the following system:

$$y = \log(x+4)+4$$
$$y = \frac{1}{2}x+4$$

Graph the functions in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will provide enough of the graph so that recognizable characteristics of each function are shown and all intersection points are found.

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

$Y_3 =$

$[\quad x_{min} \quad , \quad x_{max} \quad] \quad [\quad y_{min} \quad , \quad y_{max} \quad]$ $Y_4 =$

Answer:

Question #4

Solve the following equation using a graphing calculator:

$$|x-2| = |1.1x+4|$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived from the graph.

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

$Y_3 =$

$[\quad x_{min} \quad , \quad x_{max} \quad] \quad [\quad y_{min} \quad , \quad y_{max} \quad]$ $\quad Y_4 =$

Answer:

Question #5

Find $\csc\frac{3\pi}{5}$ to 2 decimal places

SHOW YOUR WORK HERE

Answer:

Question #6

Solve the following equation using a graphing calculator:

$$\cos x + 2 = \csc x, \quad 0 \le x < 2\pi$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Explain clearly how your solution is derived from the graph.

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

$Y_3 =$

[x_{min} , x_{max}] [y_{min} , y_{max}] $Y_4 =$

Answer:

Question #7

What is the amplitude of

$$y = \sin x + \cos x$$

Sketch the graph, indicate appropriate window dimensions, and state the function(s) used in your graph. Ensure that the relative maximum and relative minimum values of the function(s) are shown within your viewing window. Explain clearly how your solution is derived.

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

$Y_3 =$

[,] [,] $Y_4 =$

x_{min} x_{max} y_{min} y_{max}

Answer:

Question #8

Both the INTERSECT and the ZERO functions on your calculator are not working, but you need to find Kristi's temperature, in degrees Fahrenheit. Over a two-weeks period of illness her temperature fluctuated according to the Formula:

$$T(t) = 3\sin\left(\frac{\pi}{8}t\right) + 103.6$$

Where t is the day of the illness. On what day did her temperature first reach 106.6°F?

SHOW YOUR WORK HERE

$Y_1 =$

$Y_2 =$

Answer:

Question #9

Solve the following system for y only.

$$\begin{aligned} x+y+z&=2\\ 3x-2y-2z&=1\\ 5x+4y+6z&=15 \end{aligned}$$

SHOW YOUR WORK HERE

Answer:

Question #10

Evaluate: $\log_2 35$

SHOW YOUR WORK HERE

Answer:

Question #11

Evaluate t_n if $x = 5$, given: $t_n = \sum_{k=1}^{n}\left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{n}\left(-\frac{1}{x}\right)^{k-1}$

SHOW YOUR WORK HERE

Answer:

Question #12

Given: $\begin{array}{l} h(x) = 2x^3 - 1 \\ g(x) = 2x^7 - 4x - 1 \end{array}$ find $2g(-2) + 3g(h(-3))$

SHOW YOUR WORK HERE

Answer:

Question #13

$$\text{Evaluate: } \frac{8!}{3!(8-3)!}$$

SHOW YOUR WORK HERE

Answer:

Question #14

Evaluate: $\left({}_{13}C_1\right)\left({}_{4}C_3\right)$

SHOW YOUR WORK HERE

Answer:

Question #15

A multiple-choice test has 12 questions. Each question has 4 choices, only one of which is correct.

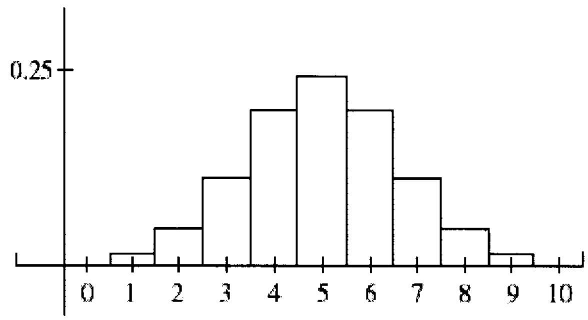
If a student answers each question by guessing randomly, find the probability that the student gets at least 7 questions correct?

SHOW YOUR WORK HERE

Answer:

Question #16

The following graph shows the theoretical probability distribution for the number of heads obtained when a fair coin is tossed 10 times.



Determine the standard deviation of this binomial distribution.

SHOW YOUR WORK HERE

Answer

Question #17

The weights of ball bearings at a manufacturing plant have a normal distribution with a mean of 125 grams and a standard deviation of 0.1 grams. Any ball bearing with a weight of 125.28 grams or more is rejected. If the manufacturing plant makes 2 000 000 ball bearings, how many would you expect to be rejected?

SHOW YOUR WORK HERE

Answer:

Question #18

The volumes of pop in cans are normally distributed with a mean of 355 mL and a standard deviation of 5 mL. Determine the range, to the nearest mL, of the volumes of the central 90% of the pop cans.

SHOW YOUR WORK HERE

Answer:

APPENDIX D: THE PILOT TEST

Calculator

Literacy Test

Date: ________________

Grade: ________________

Calculator Brand: ________________ *e.g. Casio*

Calculator Model: ________ *e.g. fx-260*

Calculator Literacy Test

Participant Consent Form

As partial fulfilment of a Master of Science degree in Secondary School Mathematics Education at Simon Fraser University, I am conducting a study on Calculator Literacy. As part of my study, I am asking you to participate in a calculator literacy test. The purpose of my study is to design a test that would assess the calculator literacy level of a student. This is important because as our world advance technologically; people must also advance in their technological skills. The calculator literacy test will take no more than 1 ½ hours of your time and no personal information will be required from you. Your name and identity will remain anonymous and this test will in no way affect your grade at school. The information obtained from this test will help me with my study and it will be destroyed when my study is completed.

You are not required to take this test and you may stop anytime during the testing period. Furthermore, any concern or complaints about the study may be registered with Dr. Ian Andrews, acting Dean of the Faculty of Education at Simon Fraser University, or myself. Please follow the instructions for the test carefully and provide as much information as possible as to how you arrived at each answer. Not every question requires the use of a calculator, if you feel the question can be answered better without a calculator feel free to do so. The purpose of this test is not to examine if you know how to answer the question on the test, but rather I am interested in how good of a test this is for assessing calculator literacy. Do the best that you can and do not be discouraged if you cannot answer some of the questions.

Please indicate below your consent to participate in this study.

I hereby consent to participate in the study described above for Mr. Wu's study.

____________________________ ____________________________

Student's Name Student's Signature

Instructions: Please clearly show **all** calculator keystroke entries and **all** information produced by the calculator in your response. In addition, any manual calculations performed by you must be indicated in the response area as well. Once again, not every question requires the use of a calculator, if you feel the question can be answered better without a calculator feel free to do so. Below are a few sample questions including screen images to illustrate what is required.

Important: Please note that different calculators use different symbols to represent the same function. For this test the following are equivalent and acceptable.

<table>
<tr><td>Power</td><td colspan="2">^</td><td>x^y</td><td>y^x</td></tr>
<tr><td>Scientific Notations</td><td colspan="2">EE</td><td colspan="2">EXP</td></tr>
<tr><td>Inverse</td><td colspan="2">$\frac{1}{x}$</td><td colspan="2">x^{-1}</td></tr>
<tr><td>Negative</td><td colspan="2">(-)</td><td colspan="2">$+/-$</td></tr>
<tr><td>Equal</td><td>EXE</td><td>=</td><td>ENTRY</td><td>ENTER</td></tr>
</table>

Sample Question #1

$$6+5\times 2-6$$

SHOW YOUR WORK HERE	CALCULATOR SCREEN
6 + 5 × 2 − 6 ENTRY	6 + 5 × 2 - 6 10

Answer: 10

Sample Question #2

$$\frac{1}{2}+\frac{1}{6}+1$$

SHOW YOUR WORK HERE	CALCULATOR SCREEN
1 ab/c 2 + 1 ab/c 6 + 1 ENTRY	1 ⌟2 + 1 ⌟6 + 1 1 ⌟2 ⌟3

Answer: $1\frac{2}{3}$

Question #1	
Evaluate: $6.23+5.24\times2.1-6.25\div2.5$	
SHOW YOUR WORK HERE	**CALCULATOR SCREEN**
Answer:	

Question #2

Evaluate: $-3-14\times-2+16\div-8$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #3

Evaluate: $\frac{4 \times 2 + 6}{5 - 3}$

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #4

Evaluate: $-2\left[3+(3+4)\right]-1$

SHOW YOUR WORK HERE	CALCULATOR SCREEN

Answer:

Question #5	
Evaluate: $\frac{(6.11\times10^{-11})(5.98\times10^{24})(3200)}{(6.4\times10^{6}+4400)^{2}}$	
SHOW YOUR WORK HERE	**CALCULATOR SCREEN**
Answer:	

Question #6

Evaluate and **leave answer as an <u>improper fraction</u>**: $\frac{1}{2}+1\frac{3}{7}\times-2\frac{3}{5}$

<u>SHOW YOUR WORK HERE</u>	**<u>CALCULATOR SCREEN</u>**

Answer:

Question #7

Evaluate and **leave answer as an <u>improper fraction</u>**: $\frac{45}{32}+\frac{1}{4}\left(\frac{1}{3}-\frac{5}{6}\right)$

<u>SHOW YOUR WORK HERE</u>	**<u>CALCULATOR SCREEN</u>**

Answer:

Question #8

Find the area given: $Area = \pi r^2 \quad r = 1.26$

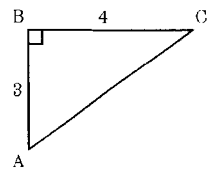
SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #9

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Find the segment AC



Solution: $x =$ [√] [3] [x^2] [+] [4] [x^2] ENTRY Ans. 19

SHOW YOUR WORK HERE	CALCULATOR SCREEN

Answer:

Question #10

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Find the $\sin 45^{o}$ **to 2 decimal places**

Solution: [sin] [4] [5] ENTRY Ans. -0.99

<u>**SHOW YOUR WORK HERE**</u>	<u>**CALCULATOR SCREEN**</u>

Answer:

Question #11

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Evaluate: $\sqrt{(2-3)^2+(3-1)^2}$

Solution:

[√] [(] [(] [2] [−] [3] [)] [x^2] [+] [(] [3] [−] [1] [)] [x^2] [)] [ENTRY]

Ans. 2.24

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

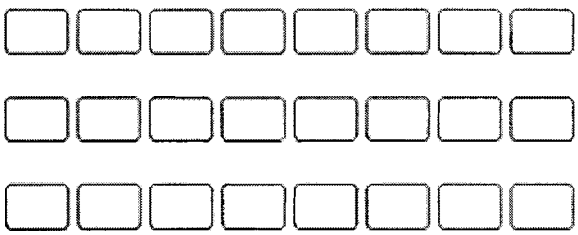
Question #12

Do the question below and verify if the answer given is correct. **If the given answer is incorrect, explain why it is incorrect and then make any necessary corrections or modifications to it as needed to obtain the correct answer. State the correct answer.**

Evaluate: $(-2)^2 + (-3)^4$

Solution: [(-)] [2] [x^2] [+] [(-)] [3] [y^x] [4] ENTRY Ans. -85

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**



Answer:

Question #13	
Evaluate to 2 decimal places: $2\sqrt{3}+4\sqrt{2}-2\sqrt{8}$	
SHOW YOUR WORK HERE	**CALCULATOR SCREEN**
Answer:	

Question #14

<u>If the fraction key on your calculator is broken</u>, how would evaluate the following?

$$R = \left(\frac{1}{\frac{1}{33} + \frac{1}{99} + \frac{1}{11}} \right)$$

<u>SHOW YOUR WORK HERE</u>	**<u>CALCULATOR SCREEN</u>**

Answer:

Question #15

If the square root, [$\sqrt{\ }$]**, and the *nth* root key,** [$\sqrt[x]{\ }$] **, keys on your calculator are broken**, how would you evaluate the following?

$$d = \sqrt{(2-12)^2 + (15-35)^2}$$

SHOW YOUR WORK HERE	CALCULATOR SCREEN

Answer:

Question #16

Find θ to the nearest tenth of a degree: $\cos\theta = \dfrac{\sqrt{5}}{6} \quad 90^o < \theta \leq 0^o$

<u>SHOW YOUR WORK HERE</u>	**<u>CALCULATOR SCREEN</u>**

Answer:

Question #17

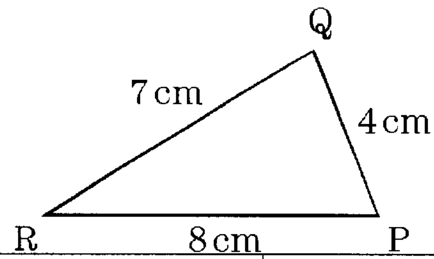
A shirt at Costco costs $19.89 and a pair of gloves costs $12.95. If Jack buys 2 shirts and 1 pair of gloves, then how much would Jack have to pay if he has to pay 7.5% PST and 7% GST?

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #18

To the nearest degree, what is the measure of the largest angle?



SHOW YOUR WORK HERE

CALCULATOR SCREEN

Answer:

Use the following set of data to answer Question 19 and 20								
Number of Students	***2***	***4***	***5***	***7***	***12***	***10***	***8***	***2***
Score on the Test	***30***	***40***	***50***	***60***	***70***	***80***	***90***	***100***

Question #19

Above is a table of examination results from Mr. Smith's Math class. Find the average mark of the class.

SHOW YOUR WORK HERE	**CALCULATOR SCREEN**

Answer:

Question #20

Calculate the standard deviation of the above scores.

SHOW YOUR WORK HERE	CALCULATOR SCREEN

Answer:

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