

**THE IMPACT OF CALCULUS REFORM ON THE TEACHING OF  
CALCULUS IN BRITISH COLUMBIA SECONDARY SCHOOLS**

by

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## **ABSTRACT**

The teaching of calculus in secondary schools in British Columbia, apart from International Baccalaureate (IB) and Advanced Placement (AP) Calculus, has taken many forms throughout the years. Calculus started as a locally developed course, then changed to an introductory chapter in Mathematics 12, and now is a Ministry-approved course with a provincial curriculum. These changes came at a time when universities in the United States were reexamining the calculus courses being offered in their institutions. This review in the mid 1980's started a movement now known as calculus reform. The major themes of the reform movement were in alignment with the common themes in mathematics education: changing pedagogy to a student-centred approach, making students accountable for understanding mathematics and not relying on rote memorization, giving mathematics a context student can relate to, and incorporating technology into the learning of mathematics.

This investigation was designed to analyze how the calculus reform movement has influenced the teaching of calculus in secondary schools in British Columbia. The analysis included the Calculus 12 curriculum, the textbooks commonly used by secondary school teachers, the university challenge examinations, and professional development opportunities. To form a basis for this comparison an analysis was also carried out on one of the first calculus textbook designed using a reform model (the Harvard Book). Evidence was found that the British Columbia Calculus 12 curriculum shared many common themes, as found in the literature review and the Harvard book, with calculus reform. It appears that calculus reform has influenced the intended calculus

course. However the intended calculus course is often different than that implemented in classrooms.

The responses by teachers to a survey on teaching practices revealed that many of the calculus reform themes are present in secondary schools. No definitive statement can be made to suggest that calculus reform is the main reason for this. However, with smaller classrooms, availability of technology and teachers using current educational practices, secondary schools may be an ideal setting to what reformers had in mind when they envisioned a calculus reform course.

*This work is dedicated to my entire family:*

*My mom and dad who made me the man that I am,*

*My loving wife who gives me the strength to be the man that I am,*

*And my three wonderful children who inspire me to be the man I should be.*

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# CHAPTER 1

## INTRODUCTION

### **Personal Motivation:**

I can recall my first calculus class back in high school, a locally developed course that offered little in terms of substance and understanding. It was the first time that the school had offered this course and it was evident that the teacher lacked confidence. This introduction is one that I would not want any student to experience. After completing the course and writing the final examination, my feelings about calculus were frustration and emptiness. This sense of vacuity came from the realization that calculus was a powerful concept that I did not understand or appreciate.

The following year I entered university, majoring in Physics. As part of my chemistry and physics studies, concepts of calculus were required to solve complex problems. It was at this point that calculus began to change from a course in symbolic manipulation to a dynamic concept allowing comprehension of infinitesimally small changes as well as sums of infinite terms. The equations for topics such as planetary motion, energy conservation, kinematics, forces and electromagnetism, were derived using calculus. The power of calculus became apparent to me as I began to construct my own personal knowledge of this subject. This newfound understanding would prove to be fundamental in developing a full appreciation of calculus.

My personal battle with calculus has now come full circle as I taught Calculus 12 in the 2000-2001 school year and am currently teaching Advanced Placement (AP) Calculus. The AP Calculus course follows a very demanding syllabus with an

examination in May written by students around the globe. Of all our students who are enrolled in AP Calculus approximately 65% of them score either 4 or 5 out of a possible 5 on the examination. Motivation is not an issue, as these capable students aspire to do well on the examination. These students are usually very successful in their first year as most enter university majoring in the sciences or engineering faculties.

From my experience, the students that register for the British Columbia Calculus 12 course tend to lack the strength in mathematics or the discipline and desire that is required for successful completion of the AP Calculus program. As a result of working with the Calculus 12 students, I realized that the introduction to calculus they received resembled my own experience as a student. These students could not fully appreciate the elegance and power of calculus as they had not yet been given an opportunity to experience it for themselves. I found myself focusing my lessons on complex algebraic manipulations and, as a result, many students were giving up halfway through the course. Additionally, I had a strong sense that the students who completed the course with a reasonable mark relied more on rote memory than on an understanding of the difficult concepts. This was evident in the varied discrepancies between some students' final examinations and their unit tests. That is, students who had solved a particular question on a unit test were not able to begin a similar question on the final examination. Discussing these matters with other colleagues throughout the province, I sensed similar frustrations in the teaching of Calculus 12. It made me reflect on how effectively we were preparing students for their futures. Is it in the student's best interest to preview calculus in high school in a similar way to what they will see in university?

It was also at this time that I heard about Calculus Reform. It appeared that many reform ideas could be found within the Calculus 12 curriculum, including the use of technology to assist in the exploration of mathematics, a focus on “hands on” learning that engages students to understand calculus and a break from the traditional lecture style teaching. Teachers in the province may be using methods of instruction consistent with that of Calculus Reform. Is Calculus Reform present in British Columbia without its formal title? The issue that is the forum of my investigation is the impact of Calculus Reform on our province.

**Statement of the problem:**

The purpose of this investigation is to make a critical analysis of the impact that Calculus Reform is having on the way that calculus is being taught in the province of British Columbia. This analysis will involve three elements that are the major components of this investigation in British Columbia’s secondary schools: the Calculus 12 curriculum, the teachers currently teaching calculus in secondary schools, and the universities of British Columbia.

The first task will be to look at the current Calculus 12 curriculum in British Columbia, as outlined by the Integrated Resource Package (IRP) (British Columbia Ministry of Education, 2000). Some of the questions that will lead my investigation include:

- Does the current curriculum have any ideas in common with those of Calculus Reform?
- Are the instructional strategies given in the IRPs consistent with methods used by the colleges and universities involved with Calculus Reform?

- How do the textbooks, given in the prescribed learning resources of the IRPs, compare to the textbooks designed for Calculus Reform, with respect to questions, activities and content?

The second stage of this analysis will be to look at the teachers in British Columbia currently teaching calculus:

- Are the ideas of Calculus Reform present in the secondary classrooms of British Columbia?
- What methods teachers are using to teach Calculus are similar to the methods used by the colleges and universities involved with Calculus Reform?
- Are there opportunities for professional development, at both a provincial and district level, for teachers to incorporate the methods of Calculus Reform?

The third consideration will be to look at the universities in B. C. by examining the University Challenge Examinations:

- What influence do the Challenge Examinations offered by the universities have on the teaching of calculus in the secondary schools?
- Do the Challenge Examinations reflect a more traditional method of evaluating calculus or do they incorporate the ideas of Calculus Reform?

### **Definition of Terms:**

Some of the concepts used throughout this investigation refer to common terms used within the mathematics community of British Columbia.

Algebra 12- The pre-calculus mathematics course in British Columbia taken by secondary students in the 1980's. This course was later changed to Mathematics 12 in 1990 with an introductory calculus unit added to the new curriculum.

AP Calculus AB- A course offered at secondary schools across Canada and the United States with a detailed syllabus prescribed by the College Board. Successful



completion of the examination gives students advanced placement for one semester of calculus, although colleges differ on what constitutes a passing grade.

AP Calculus BC- A course offered at some secondary schools across Canada and the United States with a detailed syllabus given by the College Board. Successful completion of the examination gives students advanced placement for the first two semesters of calculus, although colleges differ on what constitutes a passing grade.

British Columbia Association of Mathematics Teachers (BCAMT)- An association of mathematics teachers in British Columbia, with the objective to promote excellence in mathematics teaching. They offer many professional development opportunities with an annual fall conference in October.

Calculus 12- A ministry approved course with a provincial curriculum that began in September 2000. This replaced locally developed calculus courses for which teachers had developed their own curricula. This course has no final provincial examination but the curriculum is used as a guideline for the Challenge Examination.

Challenge Examination- An examination used by the universities to give students in secondary schools an opportunity to gain credit for first semester calculus. The University of British Columbia and Simon Fraser University alternate each year in creating the examination and is based on the new Calculus 12 curriculum. All four of the universities in British Columbia will accept this credit.

Integrated Resource Package (IRP)- A ministry document that outlines the curriculum for each course. Along with the prescribed learning outcomes this document offers instructional and assessment strategies as well as prescribed learning resources.

International Baccalaureate (IB) Calculus- A calculus course that follows a detailed IB syllabus and is offered at schools having the International Baccalaureate program. Successful completion of the course and final examination gives students advanced placement for the first semesters of calculus, although colleges differ on what constitutes a passing grade.

Mathematics 12- This course replaced Algebra 12 in 1990 with an introductory calculus unit added to the new curriculum. Principles of Mathematics 12, in which the introductory calculus unit was dropped, replaced Mathematics 12 in 1999.

National Council of Teachers of Mathematics (NCTM) Standards- A document written by the NCTM in 1989 to establish high quality in mathematics education in the United States. This document would influence curriculum and assessment in many parts of the United States and Canada.

Principles of Mathematics 12- This course replaced Mathematics 12 in 1999. Along with the removal of the introductory calculus unit, this course focused on the use of graphing calculators. Students were expected to graph and solve equations using the graphing calculator in the course and provincial final examination.

Western Canadian Protocol- A document based on collaboration in basic education among the western provinces for a common curriculum for mathematics (K-12). The British Columbia curriculum is based on this protocol.

## Chapter 2

### LITERATURE REVIEW: CALCULUS REFORM

#### The Beginning of Calculus Reform

Although the actual birth date of Calculus Reform is unknown, Schoenfeld (1995) set it to be January 2, 1986, the day that the Tulane “Lean and Lively” calculus conference began. During the late 1970s and early 1980s, there was increasing discontent with calculus instruction across the United States. This set the stage for the Tulane Conference in 1986 to revitalize calculus and maintain its importance as a central course in college mathematics (Crocker, 1990). Crocker states that the conference recognized the following problems with calculus instruction:

- (a) The curriculum had not been rethought for over two decades and contains too many topics;
- (b) the mix of calculus students is different from when the curriculum was devised;
- (c) expectations are too low for current students; and
- (d) enrolment in calculus has grown and classes are too large for interaction between students and instructors. (p.62)

Crocker continues by stating that, “the issues of too many topics in the typical calculus course and the methods for teaching calculus became central themes in the reform movement” (p. 62).

The Tulane Conference sparked an interest in Calculus Reform among many mathematicians, scientists and educators. The National Academy of Sciences and the National Academy of Engineering sponsored the Calculus for a New Century colloquium that was held in Washington, D. C. in October of 1987. Some of the major themes to come out of the conference as stated by Crocker (1990) were the importance of

instruction stressing understanding of calculus and the integration of technology within the course. Tucker (1988) writes of his hope that the new calculus “uses calculators and computers, not for demonstrations but as tools, tools that raise as many questions as they answer” (p.16). Regarding instruction that promotes student understanding of calculus, White (1988) states:

As I look at it, what we need to do now is to teach calculus in a way that provides a body of understanding which contributes to the flexibility and adaptability... The reason why flexibility is important is that in an era of very rapid technological change, with newly emerging fields of all kinds, what we need to build into our students (and eventually into the people in our work force) is an ability to move from field to field. To do that you need the kind of understanding that comes from an appreciation of calculus. (p. 9)

Steen (1988), Starr (1988) and Lovelock & Newell (1988) make similar comments regarding instruction stressing understanding of calculus.

### **Major themes of Calculus Reform**

The Calculus for a New Century colloquium raised awareness for Calculus Reform as evident by the many new projects that began. The National Science Foundation (NSF) began to fund projects dealing with Calculus Reform. Some of the those projects as stated by Judson (1997) include: Calculus in Context (The Five Colleges, 1988), The Calculus Consortium (Harvard University, 1989), Calculus as a Laboratory Course: Project CALC (Duke University, 1989), Calculus & Mathematica (University of Illinois, 1990), The Oregon State Calculus Connections Project (Oregon State University, 1989), Calculus from Graphical, Numerical and Algebraic Points of View (St. Olafs College, 1990), Calculus and Concepts, Computers and Cooperative Learning (Purdue University,

1990). These projects incorporated many of the themes that had come out of the Tulane Conference and the Calculus for a New Century colloquium.

One of those themes is the integration of technology into the instruction of calculus. Schoenfeld (1995) states that, “Of the eighteen calculus course implementation projects I reviewed for this report, sixteen explicitly mentioned the use of technology—and they invoked its use in fundamental rather than incidental ways” (p. 4). Another common theme that many of these projects shared was the engagement of the student in the learning of calculus. Schoenfeld (1995) comments that “reform versions of calculus are much more “hands on” than their antecedents, and they often engage students in working on much more extended problems (more typically, “projects”) than the previous courses” (p. 4). These include courses that are “problem driven” as questions involve mathematical modelling of real world situations. Other courses use laboratory data to allow students to investigate, conjecture and make conclusions based on their results. Some courses are activity-based as the student is expected to discover the mathematics by making mental constructions as they perform the activities.

A third theme as stated by Schoenfeld (1995), was that the universities expected the students to be “held accountable for understanding complex mathematical concepts in more and more connected ways than in the past” (p. 4). This included what one project calls the ‘Rule of Three’, as students are constantly confronted with three aspects of calculus-graphical, numerical and analytical. Students are encouraged to understand the concepts with respect to these three approaches. Other institutions extended this ‘Rule of Three’ and suggested that it should be changed to a ‘Rule of Four’ as verbal explanation

should be included in the student's ability to understand the concepts of calculus (Schoenfeld, 1995).

The final theme that many of the projects dealing with Calculus Reform shared was the change in pedagogy used by the universities as stated by Schoenfeld (1995). As the courses became more "hands on", many of the universities used cooperative learning to help students engage in the learning of mathematics. Schoenfeld writes that one of the projects stated that "such engagement can actually be more profitable for students than having a steady diet of wisdom dispensed from the front of the classroom"(p. 5). However, to allow for this style of teaching, the size of the classes had to become smaller and universities started to form sections of approximately 30 students. This adoption of smaller classes would then allow for better interaction between the teacher and students.

### **The National Council of Teachers of Mathematics (NCTM)**

At the same time that many universities were dissatisfied with the current state of calculus, there was a growing concern with the teaching of mathematics in elementary and secondary schools. Steen (1999) states that the response to these growing concerns include the beginning of Calculus Reform at the undergraduate level and the NCTM Standards at the school level. The NCTM Standards (NCTM, 1989) were designed, "(1) to ensure quality, (2) to indicate goals, and (3) to promote change" (p. 2) in the mathematics education community. The high quality of mathematics education set by the Standards would influence curriculum and evaluation practices throughout the United States and Canada. The new goals stated in the Standards can be summarized by, "students will become mathematically literate. This term denotes an individual's ability

to explore, to conjecture, and to reason logically, as well as to use a variety of mathematical methods effectively to solve problems” (p.6). Along with this focus on problem solving that engages the student, the Standards mention the importance of using technology in a classroom. Technology would be used to enhance the teaching and learning of mathematics without making students dependent on them for simple calculations. Finally with regards to change, the NCTM states that the Standards’ “intent is to improve or update practices when necessary” (p. 2).

The NCTM Standards (1989) and the major themes of Calculus Reform share many common ideas. The first idea is to explore mathematics and allow technology to be a tool to make such explorations feasible. Secondly, there is a focus on problem solving that gets away from rote memorization. This includes the use of real life problems, experimental activities and more project-based work. The NCTM Standards stress the importance of making students more “mathematically literate”, and this is consistent with the Calculus Reform idea of communicating concepts of Calculus verbally as part of the “Rule of Four”. Finally, the use of cooperative learning to assist in these changes is highly recommended in the NCTM Standards and is used extensively in many of the reform models.

With so many similarities between the NCTM Standards and Calculus Reform it is important to consider the NCTM’s (NCTM, 1989) stance on the teaching of Calculus in secondary schools.

This standard does not advocate the formal study of calculus in high school for all students or even for college-intending students. Rather, it calls for opportunities for students to systematically, but informally, investigate the central ideas of calculus... that contribute to a deepening of their understanding of function and its utility in representing and answering questions about real-world phenomena” (p. 180).

This statement reflects the NCTM's stance on the importance of allowing students to understand mathematics and not rely on rote memorization. With the time constraints in secondary schools, the teaching of calculus to all students would become a course of superficial knowledge in symbolic manipulations. Instead the NCTM recommends instruction that is highly exploratory with the aid of technology to allow a development of the conceptual underpinnings of calculus. This is also consistent with the ideas in Calculus Reform where students are encouraged to construct their own understanding through exploration.

### **Current State of Calculus Reform**

In the mid 1990's Calculus Reform had reached many parts of the United States. Wilson (1997) states that in 1994 nearly 70 % of 1048 institutions had made "modest" or "major" changes in the teaching of calculus. Also it was estimated that about a third of all students taking calculus in the United States during the spring of 1994 were in a "reform" course (p. A12). Some of these courses relied on computers and graphing calculators to allow for investigation. To further promote an exploratory style course, new textbooks were developed that were more in keeping with this method of learning. One of the first and most popular reform textbooks was *Calculus* (John Wiley & Sons, 1994) by two Harvard professors, Andrew M. Gleason and Deborah Hughes-Hallett. Often called the "Harvard Book", Wilson states that this book contains "intricate calculus questions" (p. A12). These textbooks were being used by approximately 500 institutions in the United States and led the way for other publishers to develop similar textbooks.



With change also came many critics. Wilson (1997) writes that the critics claimed that the reform courses did not teach students how to solve difficult calculations or give them the skills that they needed. Rosen and Klein (1996) heavily criticize the reform movement, describing it as a watered down course and listing many deficiencies in the Harvard Calculus Textbook. Similar debates are seen throughout the United States with battles between the “Traditionalists” and the “Reformers” occurring at many universities. Wilson describes these battles as “deep as those between two different religious groups” (p. A12). With many opposed to the reform movement, many institutions have started to return to more traditional methods of teaching calculus. Wilson mentions the University of Iowa, Stanford and Purdue University as larger institutions that have reverted back to a lecture style teaching. Rosen and Klein also cite UCLA and USC as other schools that have rejected a reform approach to teaching calculus.

The Calculus Reform has had a definite impact on how undergraduates are learning calculus in the United States, although its future is uncertain. Some are pessimistic about its future fearing that the movement will fade away in the coming years (Wilson, 1997). Others, like Schoenfeld (1995), stress the need for proper nurturing and support and that “such efforts will be necessary if calculus reform is to make it into its teens, and flower into adult maturity” (p. 5). Regardless of its future, Calculus Reform will continue to transform the traditional methods of calculus. Its influence on teaching practices and textbooks will ensure that the traditional methods of calculus cannot be returned to.

## Calculus in British Columbia

Calculus instruction in secondary schools has taken on many forms in British Columbia. It evolved from a locally developed course in schools and then became a chapter within the new Mathematics 12 curriculum in 1990. This chapter gave an introduction to calculus that included limits and derivatives and applications such as optimization, curve sketching and rates of change. Although calculus was contained in the new mathematics curriculum, it was also offered separately as a locally developed course in many schools.

Around the same time that the curriculum in British Columbia changed from Algebra 12 to Mathematics 12, there was considerable growth in the number of students taking AP Calculus in this province. Massel (1992) shows that the number of students enrolled in AP Calculus in British Columbia had grown from 267 in 1987 to approximately 3400 in 1992. These numbers continued to rise throughout the 1990's as more schools offered the AP Calculus course.

There are many possible reasons for this rise in popularity. One contributing factor may have been the alarming failure rate of first year calculus students. Hearing stories of such a high failure rate many students desire to experience the rigor of university mathematics while in high school. The AP Calculus program was highly respected and schools offering this program are considered to be elite institutions. Finally, a possible reason could be the fact that the AP Calculus program has credibility, with a detailed syllabus, whereas the locally developed courses do not. This credibility is further reinforced by the fact that colleges and universities of the United States and Canada award credit for a passing mark on the final examination.

The rise of interest in AP Calculus in British Columbia can also be attributed to the University of British Columbia's (UBC) change in position with regards to giving students credit for first year mathematics. In 1988, UBC changed its policy and gave students who scored a 4 or 5 on the AP examination advanced placement. In 1991, they changed this policy again, not only giving students advanced placement but also credit toward degree completion. About the same time Simon Fraser University and University of Victoria also decided to give credit toward degree completion. Success in the AP Calculus AB program is worth 3 credits whilst the AP Calculus BC program is worth 6 credits. This change in policy further enhanced the popularity of AP Calculus as it gave another incentive to take this program in high school.

In September of 1999, the Mathematics 12 program was renamed Principles of Mathematics 12 as the curriculum changed again. Two of the major changes included the deletion of the calculus unit as well as the introduction of graphing calculators to the course and final examinations. This dropping of the calculus unit in Mathematics 12 also saw the beginning of a curriculum for a Calculus 12 course. The ministry developed a Calculus 12 Integrated Resource Package with a provincial curriculum (British Columbia Ministry of Education, 2000). Calculus 12 was no longer a locally developed program but instead a ministry issued course that could be used for admission into British Columbia universities.

The Calculus 12 course is now offered in many high schools in British Columbia, as more students accept the credibility of a provincial curriculum. Additionally, the four universities in British Columbia offer a Challenge Examination by which students can gain first semester calculus credit with the mark attained on the examination. The four

universities participating are the University of British Columbia, Simon Fraser University, the University of Victoria and the University of Northern British Columbia. These Challenge Examinations are prepared by the universities but are based on the Calculus 12 curriculum. It is certain that with a provincial curriculum and the possible benefits of writing the Challenge Examination, calculus will continue to increase in popularity among students in British Columbia.

Calculus Reform has affected parts of the United States and Canada; however it is not clear how it has affected the secondary Calculus curriculum in British Columbia. With the advent of the new Calculus 12 curriculum, one that has laid a foundation for the instruction of calculus in this province and the rise in the number of students taking calculus in British Columbia secondary schools, it may prove to be a valuable time to assess where we are. Many groups have assessed Calculus Reform, for example Purdue and Stanford, but no definitive conclusions have been made about the effectiveness of this movement. Some universities have reverted back to more traditional methods of calculus instruction after attempting methods of Calculus Reform. With the future of Calculus Reform uncertain, this analysis may prove to be valuable to teachers and universities of British Columbia, as well as to the Ministry of Education, in order to further shape the way in which Calculus is taught.

## Chapter 3

### DESCRIPTION OF THE STUDY

This chapter will deal with the specific details of this study. This includes an explanation of the documents that will be examined, the textbooks that will be compared and other key components of this investigation.

#### Calculus Reform

##### A Calculus Reform Textbook

The textbook that the teacher uses will have a major influence on the way in which calculus is taught. It is often the case that the textbook will be the teacher's primary resource when preparing lessons for their students. Therefore, an examination of the textbooks which are on the recommended learning resource list is an essential part of this analysis. The focus of this examination will be to find evidence of the major themes of Calculus Reform within the textbook. However, in order to have a baseline with which to compare it to, an analysis of *Calculus-Single Variable 2<sup>nd</sup> ed.* by Hughes-Hallet, Gleason et al. (1998) (the first edition of which in 1994 was one of the first books to be developed under Calculus Reform and often called the Harvard Book) will be made.

The Harvard Book was designed and written with all the elements of Calculus Reform embedded within the textbook. The authors state:

The first edition of this book was the work of faculty at a consortium of eleven institutions, generously supported by the National Science Foundation. It represents the first consensus between such a diverse group of research

mathematicians and instructors to have shaped a mainstream calculus text.  
(p.v)

This consortium of research mathematicians and instructors were strong advocates of Calculus Reform and felt that the curriculum needed to be re-examined. The authors write, “When we designed this curriculum we started with a clean slate” (p.v). Along with the change in curriculum, the book made major changes to the presentation of material that focused on conceptual understanding with a student-centred approach and the incorporation of technology.

To highlight the changes made in the Harvard textbook, the analysis will include a comparison to a traditional Calculus textbook, *Calculus (first edition)* by James Stewart (1983). This traditional textbook was chosen, since it was pre-reform and used by many institutions throughout the 1980s. This traditional textbook will not be analyzed in depth, but instead, as mentioned earlier, will be used to highlight the changes made in the Harvard Book.

It is also important to note that the Harvard Book that will be examined is the second edition, as some changes were made following the first. The authors state, “We were offered many valuable suggestions, which we have tried to incorporate into this second edition of the text, while maintaining our original commitment to a focused treatment of a limited number of topics” (p.viii). These changes include:

- A new section titled Focus on Theory with theoretical treatment of selected topics like epsilon-delta definition of limits, differentiability, and the definite integral.
- Treatment of l’Hospital’s Rule.
- A new section called Focus on Practice in which drill problems are given.

These changes offer the instructor more choices in the way in which they present their course, while still maintaining a focus on Calculus Reform. However, this investigation will not focus on the changes made in the second edition and they will not be discussed in Chapter 4. The reason this book is being analyzed and not the first edition is because this book reflects the current state of Calculus Reform and is comparable to the publication date of the two textbooks analyzed in the next section.

### **Calculus Textbooks Commonly Used in British Columbia Secondary Schools**

With the completion of the analysis of the Harvard Book, a baseline is then established to examine the textbooks commonly used in secondary schools in British Columbia. The two textbooks examined are *Single Variable Calculus Early Transcendentals (Fourth Edition)* by James Stewart (1999) and *Calculus-Graphical, Numerical, Algebraic* by Finney et al. (1999). Both of these books are on the recommended learning resource list of the IRP's. The analysis of these two books compares and contrasts with the Harvard book in terms of the major themes of Calculus Reform. These major themes are, once again, the engagement of students using extended problems and real-world situations, making conceptual connections between graphical, numerical and analytical as part of the rule of four, changing pedagogy to a student-centred approach and the incorporation of technology throughout the course. Evidence of these major themes are looked for in the instructional portion of the book, problems assigned at the end of each topic and extra sections found in the book.

## **Intended Calculus Course**

### **Calculus Twelve Integrated Resource Package**

The Calculus 12 curriculum is given as part of the Integrated Resource Package (IRP's) published by the Ministry. This package went into effect September 2000, the first year that Calculus 12 existed as a Ministry-approved course. Similar to the IRP's of the other Mathematics courses, this package is broken down into different topics:

- Problem Solving
- Overview and History of Calculus (Overview of Calculus)
- Overview and History of Calculus (Historical Development of Calculus)
- Functions, Graphs, and Limits (Functions and their Graphs)
- Functions, Graphs, and Limits (Limits)
- The Derivative (Concept and Interpretations)
- The Derivative (Computing Derivatives)
- Applications of Derivatives (Derivatives and the Graph of the Function)
- Applications of Derivatives (Applied Problems)
- Antidifferentiation (Recovering Functions from their Derivatives)
- Antidifferentiation (Applications of Antidifferentiation)

Although no specific order is given for the topics to be covered, they do have a suggested time of how long each topic should take. These instructional time guidelines assume one hundred hours allotted to the teaching of the course.

For each topic the IRP's are broken down into four main sections: Prescribed Learning Outcomes (PLO's), Suggested Instructional Strategies (SIS), Suggested Assessment Strategies (SAS) and Recommend Learning Resources. The PLO's make up the curriculum, as each topic is broken down into the different tasks that the student is expected to understand and perform. The PLO's dictate what topics are taught in secondary schools in British Columbia and therefore will be examined for their consistency with the major themes of Calculus Reform.



Along with the topics covered, Calculus Reform is also about the method in which Calculus is taught. Therefore examining the method in which Calculus is taught is as important as examining what is being taught. The Suggested Instructional Strategies (SIS), as the name suggests, provide methods for teachers to introduce many of the topics to be covered and are explicit as to the approach being recommended. The SIS will also be examined to see whether these suggestions share any similarities with instructional strategies recommended by Calculus Reform.

With the analysis of possible changes to the instructional component of the Calculus 12 curriculum, it is also imperative to examine how the student will be assessed. The IRP's include Suggested Assessment Strategies (SAS) that are once again very explicit as to the approach recommended. Teachers looking at the IRP's for suggestions on instruction may also examine the suggestions on assessment. The SAS will be examined to see if these assessment suggestions are in line with Calculus Reform recommendations.

To assist in the assessment of students, the types of questions students are expected to answer are given in Appendix G of the IRP's. Appendix G contains the *Illustrative Examples* for each PLO, including a list of questions relating to that topic. These questions are similar to problems found in the textbooks and are good indicators of what students should be able to solve. These will be examined to see if they have any parallel to the kinds of questions one might see under Calculus Reform.

## **Implemented Calculus Course**

### **Questionnaire for Secondary School Calculus Teachers of British Columbia**

Although the textbooks and the curriculum shape the way calculus is taught, the ultimate decision comes from the teachers. Therefore, this investigation will attempt to attain a snapshot of what is currently happening in the classroom by asking teachers to complete a survey (Appendix C). This survey will be completed on a voluntary basis by a sample of secondary school mathematics teachers who have taught Calculus. The first set of volunteers is from teachers on the British Columbia Association of Mathematics Teachers (BCAMT) listserve. As a member of this listserve, I sent a message asking for participants in a Calculus survey as part of a research investigation. Fifteen teachers responded and the survey was electronically mailed to them. Some of the participants electronically mailed their responses while others sent it by mail. Twelve completed surveys were returned. The next set of volunteers came from graders of the June Principles of Mathematics 12 Final Examination in July of 2003. Volunteers were asked on the final marking day and most completed the survey after the marking session. In total eight teachers completed the survey from the marking session.

The first part of the survey deals with what influences the teacher when deciding what and how calculus will be taught. After indicating which calculus course is being taught (Calculus 12, AP Calculus, IB (International Baccalaureate) Calculus), the survey inquires as to which textbook is being used in their classroom. This question has a dual purpose: to examine the diversity in the number of textbooks used in the province and secondly and more important to determine how many teachers use a textbook on the recommended learning resource list of the IRP's. The textbook will undoubtedly

influence the student's calculus experience and therefore will be an important focus of this investigation.

The next question in this part of the survey inquires as to which resources the teacher uses when deciding how Calculus will be taught in their classroom. The intent of this question is to confirm the assumptions of this investigation that the IRP's and textbooks greatly influence how calculus is being taught in this province.

The second part of this survey examines course content as four topics were chosen as a focus. These topics were chosen upon examining the 'Report of the Content Workshop' in the MAA document *Toward a lean and lively calculus* (Douglas, 1986) in which recommendations are given on course content. The first topic to be examined is the precise epsilon-delta definition of a limit. The report states, "We recommend a precise english definition of ' $\lim f(x) = L$ ' rather than the mathematically professional  $\epsilon - \delta$  definition" (p. ix). It is also important to note that the IRP's do not make any reference to this topic.

The second topic examined is the use of the second derivative. This topic is traditionally related to curve sketching as its expression is at times difficult to determine for rational and radical functions. The report makes reference to the use of the second derivative in a qualitative manner for curve sketching without a focus on the algebra involved in determining it. The report also makes reference to the use of the second derivative in determining the errors involved with linear approximations to linear differentials and integral approximations.

The third topic in this section inquires about the introduction of integration. Although the report suggests that the concept of integration be introduced before connecting it to

differential calculus, the Calculus 12 curriculum does not make reference to this. Many of the reform textbooks make strong references to the word Antiderivative, as do the IRPs. Therefore, this question will probe what teachers use to introduce the concept of an Integral.

The last topic examined in this section of the survey is The Fundamental Theorem of Calculus. The 'Report of Content Workshop' indicates that it should be covered as an essential topic of the calculus course; the IRP's do not. Although the concept of relating the differential component of calculus to integration is mentioned, there is no mention of this theorem. This question will determine to what degree the teacher stresses this concept in their classroom.

The third part of this survey examines the teaching methods used in the classroom. The questions reflect teaching practices that are consistent with the methods under calculus reform. The first question covers lessons that use graphing calculators in fundamental ways. The term fundamental refers to a lesson in which the graphing calculator is used to deepen the understanding of a concept and not just as a time saver. The next question deals with lessons that use a student-centred investigative approach, as this is consistent with one of the major themes of calculus reform. The third question asks about the use of real-world examples in their class. Traditionally, this approach is used with topics such as related rates, optimization and differential equations, but this question will seek to see if others are used. The final question relates to the opportunities for teachers to acquire ideas to incorporate the types of lesson mentioned above. This is important, and is part of the final component of this investigation on the opportunities for professional development involving calculus reform ideas.

## **University Challenge Examinations**

Another possible influence on what and how calculus will be taught is the University Challenge Examination. The next part of this analysis deals with the 2001, 2002 and 2003 University Challenge examinations as they are based on the Calculus 12 curriculum. This analysis focuses on questions that support the views of calculus reform mentioned previously in this investigation. These include questions that are similar to ones found in calculus reform textbooks and topics mentioned as a focus in the teacher questionnaire.

## **Opportunities for Professional Development**

Opportunities for teachers to develop ideas in changing their way of teaching often occur at math conferences and workshops. In British Columbia, the biggest conference for secondary mathematics teachers is the annual fall conference held by the British Columbia Association of Mathematics Teachers. This conference offers workshops by fellow mathematics teachers in hopes of sharing ideas and information to their colleagues.

In this investigation the final component is to determine whether there are opportunities for professional development conducive to reform ideas. This is asked as part of the teacher's questionnaire, but the programs from large conferences are also examined. The conferences examined are the 2002 BCAMT fall conference held at North Surrey Secondary School, the 42<sup>nd</sup> Northwest Mathematics Conference held October 23-25, 2003 in Whistler British Columbia and the NCTM's Canadian Regional Conference held November 20-22, 2003 in Edmonton, Alberta. The programs are

reviewed to see if workshops are made available that promote calculus reform methods, including workshops for senior mathematics and not solely for calculus.

## **The Framework for this Investigation**

Chapter 4 of this investigation will deal with the analysis of the topics mentioned above. Chapter 5 will deal with assessing how Calculus Reform has influenced the teaching of calculus in secondary schools of British Columbia. To do this a model will be used similar to the one used in the Third International Mathematics Study (TIMMS) as a comparison will be made between the intended and implemented curriculum (Martin & Kelly, 1996). For this investigation the intended curriculum will consist of the intended calculus course and this will be developed in two stages. First, the analysis of the reform textbook, *Calculus Single Variable 2<sup>nd</sup> ed.*, by Hughes-Hallett and Gleason will give an indication of the intended reform calculus course. This will be followed up with a look at the curriculum as given in the IRP's for Calculus 12 as this is the intended calculus course for secondary schools in British Columbia. The implemented curriculum consists of the implemented calculus course and will be examined by the responses in the teacher's survey as well as a look at some of the major influences on how teachers choose to teach their course: textbooks, the University Challenge Examinations and opportunities for professional development.

## Chapter 4

### ANALYSIS OF RESULTS

This chapter deals with the analysis of the different components that may influence how Calculus is taught in secondary schools in British Columbia. This includes the Integrated Resource Package, the textbooks used in secondary schools of British Columbia, the teachers of calculus in secondary schools of British Columbia and the UBC/SFU Challenge examinations.

#### **An Analysis of a Calculus Reform Textbook**

To establish the intention of a calculus reform course, I have analyzed a textbook developed under this reform model. *Calculus- Single Variable (2<sup>nd</sup> ed.)* by Hughes-Hallett, Gleason, et al. (1998) is known as the Harvard Book and was one of the first books developed with Calculus Reform in mind. This book will be compared to a more traditional textbook, *Calculus (1<sup>st</sup> ed.)* by James Stewart (1983).

#### **Trimming the Content**

One of the major differences between the two textbooks is in the amount of content. The Reform textbook contains fewer topics in the main part of the book, and traditional topics, including the “Precise epsilon-delta Definition of a Limit”, “Properties of Limits” and “Continuity” are included in optional sections called “Focus on Theory”. It is important to note that the Reform textbook does introduce the idea of a limit early in the

textbook, but only in using it to find the derivative. There is no treatment of the limit as a separate topic as found in many traditional textbooks. Further examples of fewer topics in the Reform textbook can also be found in the chapter “Applications of Derivatives”. The Stewart text contains ten sections in this chapter, six related to the idea of curve sketching. These include Maximum and Minimum Values, First Derivative Test, Concavity, Limits at Infinity (Horizontal and Vertical Asymptotes) and Curve Sketching. The Reform textbook contains only six sections under the Applications of Derivatives and four of them are related to practical applications. The other two deal with the relationship between the first and second derivative to the curve without an emphasis on curve sketching as in the traditional book. Instead, the Reform book deals with the interpretation of the first and second derivative with respect to the curve. This difference will be further discussed under the consideration of the de-emphasis on ‘plug-and-chug’.

At this point, it should be noted that many of the omitted topics mentioned above can be found at the end of the chapter in a section called “Focus on Theory”. This section is in place to allow teachers wanting to teach a more rigorous, theoretical course to do so. However, because it is in a separate section at the end of the chapter, it does not appear to be intended as one of the core topics of the course. This trimming of the number of topics is consistent with the “Lean and Lively” approach to Reform Calculus. The authors of the Reform textbook make note of this in the preface of the book, “We focused on a small number of key concepts, emphasizing depth of understanding rather than breadth of coverage” (p. v).



### Assumption of a Graphing Tool

Another major difference evident in the Reform textbook is an emphasis on using a graphing calculator or computer-graphing program. In the preface, the authors state, “This book assumes that you have access to a calculator or computer that can graph functions, find (approximate) roots of equations and compute integrals numerically.” (p. xii). This emphasis is evident in the examples and questions given for many of the topics throughout the book. One example of this is in determining the  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ . In the traditional textbook, this is done by using a scientific calculator to evaluate smaller and smaller values of  $x$  and numerically determining that the limit approaches a value of 1. However, in the Reform textbook this limit is determined by having the student graph  $y = \frac{\sin x}{x}$  using a graphing calculator and determining that the value is 1. Another example of this is found in the *optimization* section of the Reform textbook. The question reads,

Suppose an object on a spring oscillates about its equilibrium position at  $y=0$ . Its distance from equilibrium is given as a function of time,  $t$ , by  $y = e^{-t} \cos t$ . Find the greatest distance the object goes above and below the equilibrium for  $t \geq 0$ . (p. 258)

To answer this question, the authors immediately refer to the graph of this function without justification as to why the graph looks the way it does. The answer to this example is clearly explained and justified but the graph, using the graphing tool, is integral to the understanding of the solution. Similar questions are found in the examples given in the traditional textbook, but none of the solutions use a graph as an integral part of the solution.

The assumption that students should have a graphing calculator is also evident in the exercises given at the end of each topic. An example of this is found on page 248 of the Reform textbook, as the directions read:

Using a calculator or computer, sketch the graph of each of the functions in Problems 8-15. Describe briefly in words the interesting features of the graph including the location of the critical points and where the function is increasing/decreasing. Then use the derivative and algebra to explain the shape of the graph.

The list of functions given is very similar to those listed under Curve Sketching found in the traditional book. The reform textbook views the interpretation of the graph and its relationship to the first and second derivative as being more significant than sketching the curve by finding the first and second derivative, critical points, inflection points and asymptotes. This emphasis toward using a graphing calculator or computer program is consistent with the Reform model of teaching calculus.

### **De-emphasis on “plug-and-chug”**

In the preface of the reform textbook, the authors state, “there is much less emphasis on ‘plug-and-chug’ and using formulas, and much more emphasis on the interpretation of these formulas than you may expect” (p. xii). As part of the de-emphasis of the ‘plug-and-chug’, the amount of algebra involved in solving complex problems has been reduced. This also appears to be a major difference between the reform and traditional textbooks.

The first example of this is found when dealing with limits. The exercises on limits, in the traditional textbook, often require algebraic manipulation to determine their value.

For example questions like  $\lim_{x \rightarrow -3} \frac{x^2 - x + 12}{x + 3}$  (which requires factoring and cancelling),

$\lim_{t \rightarrow 9} \frac{9-t}{3-\sqrt{t}}$  (which requires multiplication by the conjugate of the denominator before

cancelling),  $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 5}{4x^2 + 4 - 2x^3}$  (which require algebraic manipulation) and more complex

limit questions like  $\lim_{x \rightarrow 0} \frac{\sin x}{5x}$  and  $\lim_{x \rightarrow 0^+} (1 + \sin 4x)^{\cot x}$  (which at times requires

L'Hospital's rule) are not dealt with in an algebraic approach in the Reform textbook.

Many of these limit questions required guided instruction on the algebraic manipulation and practice for a student to solve them. The Reform textbook does not have questions of this sort, but instead limit values are determined by taking smaller and smaller values of 'h' (distance from where the limit is) both numerically and graphically. An example of this is found on page 95 of the Reform textbook as the question asks, "Estimate the limits in Problems 9-12 by substituting smaller and smaller values of h. Give answers to one decimal place. 9)  $\lim_{h \rightarrow 0} \frac{(3+h)^3 - 27}{h}$ ". This approach is consistent with the reduced emphasis on the 'plug-and-chug' as no algebraic manipulation is required. The focus is on interpreting the limit and not on the algebra required to solve the problem.

It should also be noted that L'Hospital's rule is covered in the Reform textbook, but its approach is once again different from that of a traditional textbook. The first difference is the omission of treating 'Indeterminate Powers', of the form  $0^0$ ,  $\infty^0$  and  $1^\infty$  in the reform textbook as these questions require detailed instruction, difficult algebra (including dealing with logarithmic manipulation) and practice in order to solve.

Secondly the exercise questions in the reform textbook that deals with L'Hospital's rule takes a more interpretive approach. Problem number 11 on page 232 of the reform textbook asks, "Based on your knowledge of the behavior of the numerator and denominator, first predict the value of the following limits, and then find each limit using l'Hopital's rule. a)  $\lim_{x \rightarrow 0} \frac{\sin x}{x^2}$ ". Instead of just taking the derivative of the top and bottom to determine the limit value as asked in most traditional textbooks, the reform textbook asks the student to predict the value by thinking about which function, the numerator or denominator, is dominant before applying L'Hospitals rule. Once again, both of these differences are consistent with an emphasis on interpretation and a de-emphasis on 'plug-and-chug'.

A second example of this reduced emphasis can be found in using the first and second derivative for curve sketching. This topic was mentioned earlier in both the 'Trimming of Content' and 'Assumption of a Graphing Tool' sections as they both pertain to this de-emphasis. The Reform textbook does not ask questions that require the difficult algebra often found in dealing with second derivatives of rational and radical functions. Instead, the emphasis of the Reform textbook is on the interpretation of positive and negative values of the first and second derivative. This is evident in the problems assigned in the textbook. On page 228 of the traditional textbook, an example is given in which the graph  $f(x) = \frac{x^2}{\sqrt{x+1}}$  is to be sketched. The author then continues to go through all eight stages required for what he feels is a complete analysis of the graph. This includes finding all vertical and horizontal asymptotes by using limits, finding the first derivative using the quotient rule and finding the second derivative using the quotient rule coupled

with the chain rule. Further convoluted algebraic manipulation is required to simplify these expressions in order to find both critical and inflection points. This is typical of questions the students are asked to solve as they ‘chug’ along the eight stages in order to sketch a curve.

In the Reform textbook, questions of this sort are not asked. Instead the student is asked to sketch a graph based on the information given. On page 249 of the Reform textbook, the question reads, “For Problems 27-32, sketch a possible graph of  $y = f(x)$ , using the given information about the derivatives  $y' = f'(x)$  and  $y'' = f''(x)$ . Assume that the function is defined and continuous for all real  $x$ ”. Problems 27-32 then gives for what  $x$  is  $y' = 0, y' > 0, y' < 0, y'' = 0, y'' > 0$  and  $y'' < 0$ . Thus the emphasis is on the actual curve sketching and not on the algebraic manipulation required to determine when the function is increasing, decreasing, concave up and concave down.

Both of these examples of the de-emphasis on ‘plug-and-chug’ are once again consistent with the reform model of teaching Calculus. The emphasis is placed on the interpretation and meaning of the concepts that are important in learning Calculus and not on the involved algebra that often interferes this process.

### **Further Evidence of Calculus Reform in the Textbook**

Along with the major differences discussed above, there is further evidence that highlights the major themes of Calculus Reform. These include:

1. Engaging students by using problems that are more extended, require mathematical modelling of real world situations and/or allow students to investigate concepts in calculus.
2. Making students more accountable for understanding complex mathematical concepts using the rule of four (graphical, numerical, analytical and verbal) and the connections between them.
3. Promoting a change in the pedagogy in teaching calculus from a teacher-centred lecture style to a student-centred approach.
4. Incorporating technology to help in the three themes mentioned above.

### **The Engagement of Students in the Learning of Calculus**

The engagement of students, which is the one of the major themes of Calculus Reform, is evident throughout the Reform textbook. The first example of this is in the use of velocity when introducing the idea of a derivative. The traditional textbook uses the idea of a limit at a point of a graph when introducing the derivative. However, the Reform textbook uses the concept of instantaneous velocity to develop the concept of a derivative before moving to the graphical representation. This use of velocity is an attempt to engage students by using a real-world example. This continues throughout the Reform textbook as many real-world situations are used. These examples include the speed at which the air comes out of your windpipe as you cough (p. 260), the number of birds that survive relative to the number of eggs laid (p. 261), minimizing fuel consumption due to drag forces with on an aircraft (p. 262) and the use of filters to reduce the pollution of carbon tetrachloride in Sioux Lake in eastern South Dakota (p. 304).

This is just a small list of many intricate problems that attempt to engage students by giving them a context in which they are applied.

To continue with this theme, it should also be noted that modeling of real-world examples is also used extensively. The authors indicate in the Preface that, “The *Focus on Modeling* sections take the time to explore selected applications of calculus in depth” (p.vi). This includes an in-depth look at Newton’s equations of motion and its relation to calculus (p. 317), distribution functions (p.409) and disease spreading and population change as related to differential equations (p. 563). Along with the topics mentioned above, there is further evidence of the emphasis on modeling real-world examples. These include separate topics like “More Optimization: Introduction to Modeling” (p. 270), “Applications and Modeling” (p. 520) and “Models of Population Growth” (p. 530).

Further evidence that the Reform textbook has an emphasis on modeling of real-world examples can be found in the order of the topics. The traditional textbook introduces the exponential and logarithmic functions and their applications much later in the textbook. However, the Reform textbook introduces the exponential function in the first chapter as the authors’ write, “they are fundamental to the understanding of real-world processes” (p. vi). This change allows the use of modeling in each of the chapters that follow.

### **Making Connections and Understanding the Concepts**

To make students accountable for understanding the concepts of Calculus, the authors write, “we have found that multiple representations encourage students to reflect on the meaning... Where appropriate, topics should be presented, geometrically, numerically,

analytically and verbally” (p. v). This is known as the “Rule of Four” as “geometrically” is often used interchangeably with “graphically”. Evidence of this is found throughout the book as many of the explanations give a graphical, numerical and analytical representation. One such example of this is found in the interpretation of the derivative function. The authors start with a graphical look at the derivative function as the rate of change of the tangent line at any point. They continue this graphical representation by drawing the derivative function and the original graph on the same grid and explaining the relationship between them. Then, the derivative function is examined numerically as an example is given with only a table of function values. Thus, the derivative is approximated using the table of values and the rate of change concept is reinforced. Finally, the book makes a final examination of the derivative using the algebraic formula of Newton’s quotient. This is done analytically as a formula is generated to determine the derivative for polynomial functions.

Along with the explanations in the textbook that use multiple representations, the problems given in the book are also consistent with this Rule of Four. One such example is question 15 on page 308 which reads:

Suppose a car going at 30ft/sec decelerates at a constant rate of  $5 \text{ ft/sec}^2$ .

- a) Draw a table showing the velocity of the car every half second. When does the car come to rest?
- b) Using your table, find left and right sums which estimate the total distance traveled before the car comes to rest. Which is an overestimate, and which is an underestimate?
- c) Sketch a graph of velocity against time. On the graph, show an area representing the distance traveled before the car comes to rest. Use the graph to calculate this distance.
- d) Now find a formula for the velocity of the car as a function of time. And then find the total distance traveled by antidifferentiation. What is the



relationship between your answer to parts (c) and (d) and your estimates in part (b)?

This one question requires all four representations to be examined. Part a) and b) require a numerical response as students are asked to calculate the distance traveled by making a table of values. In part c) a graph must be drawn to show this car decelerating as the student is also expected to interpret the distance traveled from this graph. Finally, an analytical approach is taken in part d) as the student is asked to develop a formula that relates the motion of this car. To tie it all together, part d) also asks the student to verbally explain the relationship between parts a, b, c and d. Once again, all four aspects given in the Rule of Four are present in this one example. Many other questions can be found throughout the book that require this type of understanding between the multiple representations.

### **Changing Pedagogy-Student Centred Approach**

The third major theme that will be examined is the shift from a teacher-centred lecture style teaching of calculus to a more student-centred approach. Evidence of this is limited as the book is often superseded by the manner which the instructor chooses to teach the course. Also, the University often dictates how a class will be taught by determining the class size and the resources available. However, the textbook does contain evidence that would support the de-emphasis of a lecture style course. In the preface, the authors write, “Students using this book have found discussing these problems in small groups helpful...If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on” (p.xii). Also each chapter ends with a section called “Projects”. These questions are

quite extended and require students to pursue an investigation. Including such questions in the book may encourage instructors to allow for these investigations, leading to a more student-centred approach to learning.

The last major theme, the use of technology, was touched upon in the discussion of the major differences between a reform and traditional textbook.

### **An Analysis of Two Textbooks Listed in the Recommended Learning Resource of the Integrated Resource Package.**

Two textbooks given in the list of recommended learning resources that will be looked at are *Single Variable Calculus Early Transcendentals (Fourth Edition)* by James Stewart (1999) and *Calculus-Graphical, Numerical, Algebraic* by Finney et al. (1999). Both of these books will be compared to *Calculus- Single Variable* (the Harvard Book) by Hughes-Hallet, Gleason, et al. (1998). The focus of the comparisons will be limited to the major themes of Calculus Reform as mentioned earlier.

#### **The Engagement of Students in the Learning of Calculus**

Like the Harvard book, both Stewart and Finney et al. incorporate real-world examples throughout the text. Stewart writes in the preface, “My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate and illustrate the concepts of calculus” (p.viii). Finney et al. also write:

The text includes a rich array of interesting applications in biology, business, chemistry, economics, engineering, finance, physics, the social sciences, and statistics. Some applications are based on real data from cited sources. Students are exposed to functions as mechanisms for modeling data and learn about how various functions can model real-life problems. They learn to analyze and model data, represent data graphically, interpret from graphs, and fit curves. (p.viii)

Both books clearly indicate that an attempt is being made to engage students by using real-life problems. Some of the examples used by Finney et al. include: tree growth rings (p. 123), optics (p.151), blood flow through arteries (p.228) and computing cardiac output using dye concentrations (p. 252). Stewart also has many examples of real-world data with some of his more interesting ones including seismograms from the Northridge earthquake (p.11), San Francisco power consumption (p.406), smoking rates among high school seniors (p. 172), the velocity of the space shuttle Endeavour (p. 377) and the Consumer Price Index (p.408). This focus is present throughout this book as there are other extensive problems that have a real-world context.

Along with the use of real-world examples, there is further evidence that Finney et al. have tried to engage students in the learning of calculus. Throughout the book there are sections called 'Explorations' where students are expected to perform a guided investigation of a concept. This exploration is designed to, "help build problem-solving ability by guiding students to develop a mathematical model of a problem, solve the mathematical model, support or confirm the solution, and interpret the solution" (p. viii). These student investigations are consistent with the Calculus Reform goal of trying to engage students in the learning of calculus.

Although Stewart does not have these student explorations within the instructional part of the book, he does include some before or after a particular topic. He calls them Discovery Projects as students are asked to perform an investigation and become engaged in the learning of a concept in calculus. An example of this can be found on page 390 in an exploration that deals with determining the area under a curve, sketching its graph and then taking the derivative. This is a prelude to the Fundamental Theorem of Calculus that relates integration to differentiation.

Another similarity that these two textbooks share with the Harvard Book is the location of exponential functions. Like the Harvard book, exponential functions are introduced in the first chapter. Finney et al. and Stewart both discuss the exponential and logarithmic functions in chapter 1 and deal with their derivatives in chapter 3. It should also be mentioned that the Stewart book is an alternate edition to his original 4<sup>th</sup> edition, the only change being that exponential and logarithmic functions are dealt with early in the text. As mentioned earlier this early introduction of exponential functions is imperative in the modeling of real-world processes.

### **Making Connections and Understanding the Concepts**

The use of multiple representations to help in the understanding of the concepts of calculus is evident in both of the textbooks. Both Finney et al. and Stewart make reference to the 'Rule of Four' in the preface of their respective books. Finney et al. change this 'Rule of Four' to a 'Rule of Five' as they separate algebraic from analytic treatments. To compare the two books to the Harvard book, the introduction of the derivative will be examined.

In the textbook by Finney et al. the introduction of the derivative is given by its definition (Newton's Quotient). They immediately follow this up with an example in which the derivative for  $y = x^3$  is determined algebraically using this definition. This is followed by a graphical look at a derivative at a point as an alternative definition is given in which  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ . Continuing with the graphical interpretation, a graph of  $f'$  is drawn for a function  $f$ . Finally, to examine the derivative numerically the slope is calculated to confirm that  $f'$  is a rate of change. This use of multiple representations is similar to that of the Harvard book as discussed earlier.

Stewart also introduces the derivative in a similar manner as the definition is given first, followed by a graphical representation and then numerical. However Stewart, like the Harvard book, seems to show a better connection between the three different approaches. This is done by showing multiple representations in the solutions to each example and not having examples for each representation as in Finney et al.

The problems given in the textbooks also reflect this focus on multiple representations. In the textbook written by Finney et al., the preface states, "The different types of exercises included are: algebraic and analytic manipulation, interpretation of graphs, graphical representation, numerical representations, explorations, writing to learn..." (p. ix). Stewart also reflects this focus on page 172 under the heading "The Derivative as a Function", where question 18 reads:

$$\text{Let } f(x) = x^3.$$

- a) Estimate the values of  $f'(0)$ ,  $f'(\frac{1}{2})$ ,  $f'(1)$ ,  $f'(2)$  and  $f'(3)$  by using a graphing device to zoom in on the graph of  $f$ .
- b) Use symmetry to deduce the values of  $f'(-\frac{1}{2})$ ,  $f'(-1)$ ,  $f'(-2)$  and  $f'(-3)$ .
- c) Use the values from parts (a) and (b) to graph  $f'$ .
- d) Guess the formula for  $f'(x)$
- e) Use the definition of a derivative to prove that your guess in part (d) is correct.

This question starts with numerically estimating the values of the derivative at different points in parts a) and b) from the graph of  $f(x)$ . Dealing with the graphical interpretation of the derivative, part (c) asks the student to sketch  $f'(x)$ . Finally in parts d) and e) the connection between the graphical, numerical and analytical is confirmed by algebraically finding the function  $f'(x)$ . There are other examples throughout the book that stress the use of multiple representations to answer the question.

The fourth component of the “Rule of Four” refers to the verbal explanation to a solution. Both textbooks have evidence of this in many of the problems asked. In the textbook by Finney et al. almost every topic has at least one problem in the exercises called “Writing to Learn”. These questions require a verbal explanation or argument to show an understanding of a particular concept. One such example is found on page 299. Question 35 states, “Your friend knows how to compute integrals but never could understand what difference the “dx” makes, claiming that it is irrelevant. How would you explain to your friend why it is necessary?”. In a similar manner, Stewart also has questions in the exercises that ask the students to verbally explain a solution or a concept of calculus. In fact, Stewart takes it a step further by having what he calls, “Writing Projects” that provide opportunities for an extensive explanation to a particular topic.

One such project is “Early Methods for Finding Tangents” in which students are asked to compare other methods of finding limits used by mathematicians like Pierre Fermat and Isaac Barrow to that of Sir Isaac Newton.

### **Changing Pedagogy to a Student-Centred Approach**

The third major theme of Calculus Reform to be examined is the change in the instruction of calculus to provide opportunities for a student-centred approach in the learning of particular concepts. Stewart uses the idea of projects to allow for this as he states that, “One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed” (p. viii). These projects are termed ‘Applied Projects’, ‘Laboratory Projects’, ‘Writing Projects’, and ‘Discovery Projects’. By having these projects available in the textbook, instructors may be more inclined to try some of these student-centred activities. These activities allow students to construct their own knowledge of particular concepts of calculus using cooperative learning. This approach is consistent with the methods of Calculus Reform as many institutions use this model in the teaching of calculus.

Although Stewart offers projects in an attempt to promote student-centred learning, many instructors may be inclined to ignore them because they are separate from the main topics of the course. However in Finney et al., the promotion of student-centred learning is found throughout the book. As mentioned earlier, students are expected to perform the student explorations found in the instructional part of the book. These unanswered explorations are guided so students can construct their own knowledge of the concepts.

This is noticeably different than traditional textbooks in which the author provides examples with a detailed solution to show the concept being used. Another example of a student-centred approach to learning is the suggestion to solve problems in groups. This is evident by many of the problems starting with the instruction, “*Work in groups of two or three...*”. This can be found throughout the book and this promotion of group work is consistent with the student-centred approach of Calculus Reform.

### **The Use of Technology**

The focus on technology is also evident in both books as it is assumed that a graphing tool is available to the student. In the preface of the textbook by Finney et al. they state:

The book assumes familiarity with a graphing utility that will produce the graph of a function within an arbitrary viewing window, find the zeros of a function, compute the derivative of a function numerically, and compute definite integrals numerically. ... This is one of the first calculus textbooks to take full advantage of graphing calculators, philosophically restructuring the course to teach new things in new ways to achieve new understanding, while (courageously) abandoning some old things and old ways that are no longer serving a purpose. (p. ix)

These statements not only stress the focus of a graphing tool in using this particular textbook but also the authors’ feelings of its importance in the learning of calculus.

Similar to the Harvard book, many examples and problems refer to the graph of a function, using the graphing tool, as part of the solution. This is done without dwelling on producing the graph but instead using the graph and its interpretation to solve a problem.

Stewart also addresses this focus on technology, although his approach is more conservative. He writes in his preface that:



This textbook can be used either with or without technology and I use two special symbols to indicate clearly when a particular type of machine is required...technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate (p.viii).

This more conservative approach is also evident in the examples given, questions assigned and the topics given in the book. Similar to the traditional textbooks, the sketching of curves using critical points, inflection points and asymptotes is heavily stressed in the *Applications of Derivatives* section of the book. This is followed up with a topic *Graphing with Calculus and Calculators* which is an extension of an earlier topic called *Graphing Calculators and Computers*. Both of these sections deal with the role of the graphing calculator in the learning of calculus and how it can be used appropriately. However, because it is a separate topic and not incorporated within the textbook, the importance of technology is less evident in this book. Many instructors may choose to ignore these sections and the exercises when using this book, not taking advantage of the technology available.

### **Summary**

It is quite evident that both textbooks being examined have many similarities with the Harvard book. The textbook written by Finney et al. seems to share more similarities as all the major themes of Calculus Reform are present in the book. Stewart also contains elements of Calculus Reform in his textbook, with evidence of the use of multiple representations and the engagement of students using real-world situations. However, the change in pedagogy and the use of technology is not emphasised as they are found as separate topics in the book. Because they are separate topics and not integral components

of the book, instructors may choose not to incorporate these major themes into the teaching of their course. This appears to be a distinct difference from the Harvard book.

## **An Analysis of the Calculus Twelve Integrated Resource Package**

### **The Engagement of Students in the Learning of Calculus**

The Calculus 12 Integrated Resource Package (IRP) (2000) contains many terms and themes that are consistent with the ideas of Calculus Reform. The first theme to be examined will be the engagement of students in the learning of Calculus. This is achieved by giving extended problems or projects which require multi-step solutions, as well as questions with context or that involve mathematical modelling of real-world situations. By using examples with which the student can identify, it is hoped that the student will become engaged in learning the concepts of calculus.

The first example is the use of displacement, velocity and acceleration in introducing key concepts of calculus. As stated in the Suggested Instructional Strategies (SIS) under the heading *Overview and History of Calculus*:

The concepts of average and instantaneous velocity are a good place to start. These concepts could be related to the motion of a car. Ask students if the displacement of the car is given by  $f(t)$ , what is the average velocity of the car between times  $t = t_1$  and  $t = t_2$ , and the instantaneous velocity at  $t = t_1$ ? Point out that calculus is not needed to determine the average velocity  $\frac{f(t_2) - f(t_1)}{t_2 - t_1}$ , but is needed to determine the velocity at  $t_1$  (the speedometer reading). (p. 194)

This continues in the Prescribed Learning Outcomes under the heading of *Applications of Derivatives (Applied Problems)*, “It is expected that students will: solve problems involving displacement, velocity and acceleration” (p. 208). In the SIS of the same section, it notes, “Discuss average velocity over a specified interval, instantaneous velocity at specified times. Have students in pairs create a graph and have their partner perform the motion.” (p.208). Furthermore, in the Suggested Assessment Strategies (SAS) it states:

Provide a number of problems where students are required to use average velocity or instantaneous velocity. Observe the extent to which students are able to:

- determine whether the situation calls for the calculation of average velocity or instantaneous velocity
- explain the differences between the two
- provide other examples (p. 209)

Finally, in the PLO and SIS under the heading of *Antidifferentiation (Application of Antidifferentiation)* it states:

It is expected that students will:

- use antidifferentiation to solve problems about motion of a particle along a line that involve:
  - computing the displacement given initial position and velocity as a function of time
  - computing velocity and/or displacement given suitable initial conditions and acceleration as a function of time
- Demonstrate how physics formulae about motion under constant acceleration can be justified using antidifferentiation. If  $a(t) = -g$ , then  $v(t) = -gt + C$  for some  $C$ , and therefore  $s(t) = -\frac{1}{2}gt^2 + Ct + D$  for some  $D$ . The two constants of integration can be found by using initial conditions. Given  $g = 9.81$  and a rock thrown upward at a given speed from a 100 m tower, it is possible to calculate the maximum height reached and the time it takes for the rock to hit the ground. There are many variations on this problem. (p. 212)

All of these statements suggest that by using familiar ideas such as velocity and acceleration, students will be able to identify with the some of the calculus concepts taught.

Calculus Reform also suggests the use of mathematical modelling of real-life examples to further engage students in the learning of Calculus. We find an example of this in the SIS under the heading of *Applications of Derivatives (Applied Problems)* as it suggests:

Have students brainstorm examples of the need for calculus in the real world:

- population growths of bacteria
- the optimum shape of a container
- water draining out of a tank
- the path that requires the least time to travel
- marginal cost and profit (p. 208)

This continues in the SIS under the heading of *Antidifferentiation (Applications of Antidifferentiation)* as it states:

- Illustrate the wide applicability of the concepts by using examples that are not from the physical sciences. For example, let  $C(x)$  be the cost of producing  $x$  tons of a certain fertilizer. Suppose that for reasonable  $x$ ,  $C(x)$  is about  $30 - 0.02xV(t)$  [sic]. The cost of producing 2 tons is \$5000. What is the cost of producing 100 tons?
- Have students generate and maintain a list of phenomena other than radioactive decay that are described by the same differential equation. For example, the illumination  $I(x)$  that reaches  $x$  metres below the surface of the water can be described by  $\frac{dI}{dx} = kx$ . Under constant inflation, the buying power  $V(t)$  of a dollar  $t$  years of now can be described by  $\frac{dV}{dt} = kV$  (p. 212)

The use of real life examples is further evident in Appendix G – *Illustrative examples of the IRP's*. Some of the examples given include:

The diameter of a ball bearing is found to be 0.48 cm, with possible error  $\pm 0.005$  cm. Use the tangent line approximation to approximate:

- a) the largest possible error in computing the volume of the ball bearing,
- b) the maximum percentage error (p. G-236)

*Boyle's Law* states that if gas is compressed but kept at constant temperature, then the pressure  $P$  and the volume  $V$  of the gas are related by the equation  $PV=C$ , where  $C$  is a constant that depends on temperature and the quantity of gas present. At a certain instant, the volume of gas inside a pressure chamber is 2000 cubic centimetres, the pressure is 100 kilopascals, and the pressure is increasing at 15 kilopascals per minute. How fast is the volume of the gas decreasing at this instant? (p.G237)

Another method by which to engage students in the learning of Calculus is to provide opportunities for students to investigate a topic. This can be done by giving students projects and allowing them to construct their own knowledge of a particular concept. This is also evident in the IRP's as many examples of potential projects are given in the Suggested Instructional and Assessment Strategies under the different headings. These include:

- Have students research the historical context of the applications of antidifferentiation. For example:
  - The area above the  $x$  axis, under  $y = x^2$ , from  $x = 0$  to  $x = b$  is  $\frac{b^3}{3}$ . Archimedes was able to show this without calculus, using a difficult argument.
  - The area problem for  $y = \frac{1}{x}$  was still unsolved in the early 17th century. Using antiderivatives, the area under the curve  $y = \frac{1}{t}$  from  $t = 1$  to  $t = x$  is  $\ln x$ .
  - The area under  $y = \sin x$  from  $x = 0$  to  $x = \frac{\pi}{2}$ , was calculated by Roberval, without calculus, using a difficult argument. It can be solved easily using antiderivatives. (p. 196)
- Pose the problem, "In how many different ways can we find the area under  $y = x^2$ , above the  $x$  axis, from  $x = 0$  to  $x = 1$ , exactly or approximately?" Students can write a report on this. Students should be able to identify
  - Archimedes' method

- standard antiderivative method
- approximation techniques of their own devising (p. 213)

These are just a few examples of suggestions that will engage students in the study of mathematics, an approach that is consistent with the ideas of Calculus Reform.

### **Making Connections and Understanding the Concepts**

A second theme of Calculus Reform evident in the IRP's is the goal of making students accountable for understanding the complex mathematical concepts involved in the study of calculus. This involves making connections between the graphical, numerical and analytical aspects as well as expressing them through verbal explanation.

We find the first example of this under the heading of *Functions, Graphs and Limits (Limits)* in the PLO requiring students to, “evaluate the limit of a function analytically, graphically and numerically” (p. 200). This continues in the SIS under the same heading:

- Give students limit problems that call for analytic, graphical, and/or numerical evaluation, as in the following examples:
  - $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$  (evaluate analytically)
  - $\lim_{x \rightarrow 0} \frac{\sin x}{x}$  (evaluate numerically, geometrically, and using technology)
  - $\lim_{x \rightarrow 0} \frac{3x^2 + 5}{4 - x^2}$  (evaluate analytically and numerically)
  - $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$  (draw conclusions numerically) (p. 200)

Finally, the third example of this is found in the SIS of the *Antidifferentiation (Recovering Functions from their Derivatives)* topic as it mentions to, “Use a variety of methods to explain the need for the constant, "C": ... geometrical: if we draw curves  $y =$

$F(x)$ ,  $y = F(x) + C$ , one curve is obtained from the other by simply lifting, so slopes of tangent lines match” (p.210).

Another manner in which students can be made accountable for understanding the concepts is assessing through verbal explanation. Under the heading *Problem Solving* it states to “clearly communicate a solution to a problem and justify the process used to solve it” (p. 192). This continues in the SAS of *Functions, Graphs and Limits (Limits)* where it mentions “To check students’ abilities to reason mathematically, have them describe orally the characteristics of limits in relation to one-sided limits...To what extent do they give explanations that are mathematically correct, logical, and clearly presented” (p. 201). Finally in the SAS, under the heading of *Antidifferentiation (Applications of Antidifferentiation)* it suggests to “Take a problem (e.g., an exponential decay problem with several parts) and ask them (the student) for the solution to be written up as a prose report, in complete sentences, with the reasons for each step clearly explained” (p.213).

### **Changing Pedagogy-Student Centred Approach**

A third theme of Calculus Reform found in the IRP’s is the suggested change in pedagogy. This change includes the use of a student-centred approach that departs from the teacher-centred lecture often found in Calculus courses. The first example of this is found in the PLO’s under the heading of *Problem Solving* as it is expected that students should “demonstrate the ability to work individually and co-operatively to solve problems” (p. 192). This theme continues in the SIS section, under the heading of *The Derivative (Concept and Interpretations)* as it states to:

Have students work in groups to prepare a presentation on tangent lines and the use of mathematics by the ancient Greeks (circle, ellipse, parabola).

Ask them to include an explanation of how the initial work of the Greeks was surpassed by the work of Fermat and Descartes and how previously difficult arguments were made into essentially routine calculations. (p. 202)

Along with promoting cooperative learning, the student-centred approach to teaching is also evident in the manner in which the SIS and SAS is worded. Almost every SIS and SAS starts with the phrase: “Allow students to...”, “Have students...”, “Give students...”, “Ask students to...”, “Challenge students to...” and “Discuss with students...”. This use of words promotes the idea of having the students perform the tasks themselves, which is consistent with the student-centred model suggested under Calculus Reform.

### **The Use of Technology**

A final example of a Calculus Reform theme found in the IRP’s is the use of technology within the teaching of Calculus, in fundamental ways that would allow students to gain further insight into the many difficult concepts of Calculus. An example of this is found in the PLO’s under the heading of *Problem Solving* as it states that students are expected to, “use appropriate technology to assist in problem solving” (p.192). This continues in the SIS as it states:

Have students use technology (e.g., graphing calculators) to compare the graphs of functions such as:

$$y = e^x \text{ and } y = \ln x$$

$$y = e^{\ln x} \text{ and } y = \ln e^x$$

$$y = 3^x \text{ and } y = e^{x \ln 3} \text{ (p. 198)}$$

Ask students to use technology to:

- reinforce the result found when using the definition to calculate the derivative of a function
- verify that the tangent line as calculated, appears to be the tangent to the curve. As an extension, have students zoom in at the point of



tangency and describe the relationship between the tangent line and the curve (p. 202)

Although full proof of the chain rule is uninformative, a graphing calculator can be used to show quite persuasively that for example  $\frac{d}{dx}(\sin 3x)$  ought to be  $3\cos 3x$  at any particular place  $x$  [given that  $\frac{d}{dx}(\sin x) = \cos x$ ] (p. 204)

Have students use technology to explore features of functions such as the following:

- when a function has maximum/minimum points or neither
- where the inflection points occur
- where curves are concave up or down
- vertical or horizontal tangent lines
- the relationship among graphs of the 1st, 2nd derivatives of a function and the function
- the impact that endpoints have on the maximum/minimum (p. 206)

All of these examples suggest that technology should have its place in the calculus classroom.

## **Results of the Survey Completed by Secondary Calculus Teachers**

An examination of calculus in secondary schools in British Columbia would not be complete without looking at what the teachers are doing. Although the IRP's and the textbook will influence what and how calculus is taught, the teachers will make the ultimate decision. This section offers a qualitative look at the twenty surveys completed by secondary calculus teachers in the hope of obtaining a snapshot of what is happening in the classrooms.

The first part of the survey dealt with which course is being taught, the textbooks being used and a teacher's resource preference. The majority of teachers (14 of 20)

indicated that they taught Calculus 12. Of the teachers that indicated they taught Calculus 12, almost half did not use a textbook given in the recommended learning resource. It should be noted that a couple of the teachers not using a recommended textbook were using an earlier edition of a textbook that is. Also, of the teachers not teaching Calculus 12 but instead AP Calculus or IB Calculus, half used books on the Calculus 12 recommended list. Finally, in looking at the resources used by teachers to decide what will be taught in the classroom, almost all teachers that teach Calculus 12 indicated that the IRP's, textbook and University Challenge exam were their primary source. However, some of the other responses given by teachers include university and college course outlines, other textbooks, discussions with colleagues and "my own ideas on the balance between rigour, intuition, theory and practical uses". The teachers teaching AP Calculus all listed the AP syllabus as well as the resources listed above by Calculus 12 teachers.

### **Course Content**

The next part of the survey dealt with some of the topics traditionally taught in Calculus courses. The first question asked if the precise epsilon-delta definition was covered in their classroom. As shown in Table 1, four teachers indicated that they teach this definition of a limit. Of the four, one teacher did not evaluate this concept during tests and quizzes. It should be pointed out that none of the AP Calculus teachers are included in this group of teachers who teach the epsilon-delta definition of a limit.

Table 1

Results of Teacher Survey Indicating How Certain Topics are Taught

	Teachers Did	Teachers Did Not
Taught epsilon delta definition of limit	4	16
Uses of second derivative other than curve sketching	16	4
Introduction of integration by the antiderivative	14	6
Used the term “Fundamental Theorem of Calculus” moderately or heavily	16	4

The next question asked whether teachers show uses of the second derivative other than curve sketching. Only four teachers indicated that they show it only for curve sketching. Of the teachers indicating they show other uses the most common example was for acceleration in connection with motion and as a test of whether a critical point is a maximum or minimum. Another use mentioned was using the second derivative to determine whether a linear approximation using differentials is smaller or larger than the actual value.

The third question in this section asked the teachers how they introduced the concept of integration. The majority of respondents, as shown in Table 1, stated that they introduced integration using the idea of the Antiderivative: that is recognizing a function

and its derivative. A few indicated they used Riemann's Sum to introduce integration while a couple stated they use a combination of Riemann's Sum, Antiderivative and the solution to the differential equation. No one stated that they used solely the solution to the differential equation to introduce integration.

Finally, the last question in this section dealt with the Fundamental Theorem of Calculus. Of the fourteen teachers teaching Calculus 12, ten indicated that they use this term at a moderate or heavy level. Moderately is defined to be at a level in which the teacher expects their students to answer questions that relate to it. It should also be noted that three of the teachers responded that the Fundamental Theorem of Calculus is not mentioned in their classroom.

### **Methodology**

The third part of the survey dealt with some of the methods used by the teachers in teaching calculus. The first question inquired about the use of graphing calculators in fundamental ways during a lesson. Although some teachers stated that they did not do so in fundamental ways, most indicated that they did use them. The most common response was that graphing calculators were used when dealing with limits. Some of the responses include:

Building concept of limit, especially when function has a hollow point or horizontal asymptote.

When exploring limits of different kinds of functions at particular values, we graphed them and saw limits visually, as well as using input-output tables to verify limits from the graph. When a function had a hole at a particular value, the input-output table was used to demonstrate that the function could be evaluated at a point arbitrarily close to the discontinuity without actually inputting the restricted value. The students were thus able to see that the

function returned values closer and closer to some specific value we called the limit.

One teacher indicated that rigorous geometrical proofs like  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$  could be avoided

with the assistance of a graphing calculator. This preference to avoid rigorous situations was also seen in other topics. Some of the comments include:

They (graphing calculators) are powerful tools that can enhance one's understanding of many concepts by taking the drudgery out of tedious calculations.

I use the graphics calculator as a tool in investigating curves and as a check on sketches and/or calculations.

Technology was used mainly to support or investigate a concept along with discussion and other analysis.

We certainly use graphing calculators as a tool to explore more in-depth situation or to do more advanced questions. Also to illustrate more quickly concepts such as graphing.

Working with graphing techniques, for them to check their algebraic solutions.

Another topic mentioned several times was the use of the graphing calculator in dealing with the area under a curve. One teacher stated that they used the graphing calculator to "sweep out" the area under a curve. Another stated that the graphing calculator allowed for the evaluation of an integral that is normally very difficult or impossible algebraically. This, in one teacher's opinion, "helps develop intuition regarding Riemann sums".

The next question in this part of the survey asked whether lessons were prepared in which students performed an investigation to construct their own knowledge of a topic. Many teachers stated that they did not do this, and a couple gave reasons for it. One stated that they had no time (it should be noted that this person teaches AP Calculus).

Another stated, “Not yet but would like to (This is only my third year of teaching calculus and am still in the process of acquiring ideas)”. However, a number of teachers gave examples of topics in which they allowed their students to perform an investigation into a topic. Some of the examples include:

Riemann sums and area under a curve. Students calculate areas under a curve as number of rectangles approach infinity.

Evaluate limits numerically and graphically and using this to see relationships to asymptotes.

Derivatives of basic trig functions – from graph of  $\sin x$ , plot a graph of slopes at key points to discover its derivative is  $\cos x$ .

Slope field programs used with a function to get a sense of the family of curves of its antiderivative.

Concept of derivative (limits of slope of a secant).

Finding the value of ‘e’.

There were other notable responses. One was a history project in which students were to research a mathematician and then write a letter to someone alive at their time. This history project was mentioned earlier (Analysis of IRP’s) as it is part of the Calculus 12 curriculum, under the heading of Overview and History of Calculus. The other notable response was the use of the projects given in the Stewart textbook. These projects were mentioned earlier in the analysis of textbooks given in the Calculus 12 recommended textbook list. This teacher uses the *Single Variable Calculus Early Transcendentals (Fourth Edition)* by James Stewart and makes use of the projects given in them.

The third question asked whether real-world examples were used in the classroom. All the teachers surveyed indicated they used them; the common responses related to

optimization, related rates, population growth and motion. Many of the responses also indicated that the textbook gave ample questions giving real-world situations. One teacher noted that very good applications could be found on previous AP examination, e.g. the modelling of the rate of people entering/leaving an amusement park.

Finally, the last question on the survey asked teachers if they had attended workshops or conferences that had helped in preparing lessons that use the graphing calculator, investigations or real-world situations. Most responded with a 'no' but many AP Calculus teachers indicated that the Fall AP Conference was very helpful. A couple of teachers indicated that the BCAMT fall conference had some workshops that related to this.

## **University Challenge Examinations**

### **Making Connections and Understanding the Concepts**

The University Challenge examinations seem to reflect some of the ideas of Calculus Reform as evidenced by the questions asked. The first example of this is the de-emphasis of the 'plug-and-chug' and an emphasis on conceptual understanding. This can be found on the 2001 Challenge examination as question #10 asks:

A certain function  $f$  obeys  $f'' < 0$  for all  $x$ , and  $f(2) = 2$ . In seeking a zero of  $f$  with Newton's method, the starting point  $x_0 = 2$  gives the next guess  $x_1 = 1$ .

a) Find  $f'(2)$

b) With the aid of a suitable sketch, explain why  $f$  must have a zero at some point  $x$  satisfying  $1 < x < 2$ .

Your answer should apply to every function  $f$  with the properties described above.

This question requires that the connections between the analytical, numerical and graphical components of Newton's method be made and not just using the procedure to find a root. Part a) requires analytical and numerical work while part b) requires the understanding that the next approximation,  $x_1$ , is a root of the tangent line at  $x_0$ .

Furthermore the student must also understand that because  $f'' < 0$ , the tangent line will always be above the curve and hence the root will be between  $x_0$  and  $x_1$ . Thus, the connection to the graphical component of Newton's method must be applied. Finally, as part of the solution the student is expected to use a written explanation with a graphical sketch to receive full marks. Therefore this one question requires students to use the Rule of Four to completely answer it.

Further evidence of the emphasis on conceptual understanding can be seen regarding the use of the second derivative. This is evident in a number of questions found in the recent challenge examinations. The first example is found on Question 12 of the 2001 challenge examination:

A certain function  $f$  is given. Its second derivative,  $f''(x)$ , is defined and continuous for all real  $x$ . Furthermore,  $f$  satisfies  $f(-2) = 0, f'(-2) = 0, f(0) = -2, f(4) = 0$ . For  $y = f(x)$ ,

$$x < -2 \Rightarrow y' < 0, y'' > 0$$

$$-2 < x < 0 \Rightarrow y' < 0, y'' < 0$$

$$0 < x < 2 \Rightarrow y' < 0, y'' > 0$$

$$2 < x \Rightarrow y' > 0, y'' > 0$$



Sketch the curve  $y = f(x)$ , paying particular attention to slope and concavity. Label any local maxima and minima and points of inflection on your sketch.

This question requires students to be able to use the signs of the first and second derivative as they relate to curve sketching without dwelling on the process of figuring this out.

The concept of the second derivative in relation to the shape of the curve was seen in question 10 of the 2001 challenge examination as mentioned earlier and question 14 of the 2002 challenge examination which asks:

$$\text{Let } f(x) = 2x^4 - 3x^3 + 5x^2 - 3x + 2 \dots$$

d) Let a real number  $a$  be given as well as the exact value of  $f(a)$ . Now suppose that a linear approximation is used to estimate  $f(a + 0.01)$ . Show that the estimate will be an underestimate whatever the value of  $a$ .

A similar question is asked in the 2003 challenge examination and all three questions require the understanding that the second derivative will dictate whether the curve lies above or below the tangent line, once again emphasizing the concept of the second derivative and its connection between the graphical, numerical and analytical.

The final example of the de-emphasis of the 'plug-and-chug' and an emphasis on the ability to apply conceptual understanding can be found in question #8 of the 2001 challenge examination:

Suppose the function  $y = y(t)$  satisfies this differential equation for some  $c \geq 0$ :

$$y''(t) + cy'(t) + y(t) = 0.$$

Use  $y$  to define  $E(t) = (y(t))^2 + (y'(t))^2$ . Prove that whenever  $t_1 < t_2$  we have  $E(t_1) \geq E(t_2)$ . (Note: It is possible to present a convincing proof without solving the differential equation.)

Furthermore question #13 on the 2003 examination asks:

Suppose that the function  $f(x)$  is defined on an interval  $(a, b)$  where  $a < 3 < b$ .

- i) Express  $f'(3)$  as a limit
- ii) Prove that if  $f(3) = 9$  and  $f'(3) = 4$  then  $\lim_{x \rightarrow 3} \frac{\sqrt{f(x)} - 3}{x - 3} = \frac{2}{3}$

Note: Assuming that  $f'(x) \rightarrow 4$  as  $x \rightarrow 3$  is not allowed, since this behaviour is not shared by all  $f$  with the given properties. Find a way to use part (i).

Both of these questions are not slight variations on a question seen in a textbook. These questions require the student to prove something using their understanding of the calculus concepts. The first question requires students to use differential equations to determine an expression for  $E'$  as a basis for their proof. The second question requires the student to know the definition of a derivative coupled with limit properties to justify their answer. Both questions make students accountable for understanding the concept and not simply requiring that they remember the steps.

### **The Engagement of Students in the Learning of Calculus**

In an attempt to engage the student, textbooks make use of real-world examples and the modelling of them, and the challenge examinations seem to share this approach. Examples of this includes question #13 on the 2002 challenge examination which deals with the population of a bacteria-infested swimming pool, question #5 on the 2003 challenge examination which deals with the volume change of a stalagmite that models a perfect circular cone and question 11 of the same examination which gives a differential

equation of a person's mass as a function of calorie intake. Furthermore, there is a question on each of the three examinations regarding velocity and acceleration. Question #10 of the 2002 challenge examination asks:

A particle moves along the x-axis with velocity  $\frac{1}{1+t^2}$  at time  $t$ . If it passes the point  $\frac{\pi}{6}$  at time  $t = 1$ , what is its acceleration when it passes the point  $\frac{\pi}{4}$ .

This question requires the knowledge that the derivative of velocity is acceleration while its antiderivative is the position function, thus applying the concepts of calculus while giving it a context with which the student can identify.

### **Use of Technology**

The graphing calculator is permitted on the challenge examination, although there seem to be no questions that place an emphasis on it. Evidence of this is also reinforced by the fact that in the Rules and Instructions to the students on the examination, it clearly states that calculators are optional.

### **Other Comments Regarding the Examination**

The last significant point to be made about the challenge exam regards the use of the word Antiderivative. Question #4 on the 2002 examination asks:

Find the general Antiderivative of  $(9 - 4x^2)^{-1/2}$ .

This is in place of the more traditional wording, 'find the integral'. This is also found on question #9 on the 2003 examination where they once again use the word Antiderivative.

## **Professional Development Opportunities**

### **BCAMT Fall Conference 2002- North Surrey**

The 2002 fall conference held by the British Columbia Association of Mathematics Teachers (BCAMT) offered a number of workshops for calculus teachers. These included a workshop that was titled ‘Real World Applications of Calculus in Technical Professions’, and examples were given that related calculus to geophysics, computer graphics and mechanical engineering. Other workshops significant to this study include “Hands on Computer Graphing from Lines to Conics to Trig to Numerals...” and “Transformations using TI-83”, which pertain to the use of technology for senior mathematics.

### **42<sup>nd</sup> Northwest Mathematics Conference 2003 – Whistler, British Columbia**

The program for the 2003 Northwest Mathematics Conference indicates a number of workshops for senior mathematics teachers that are significant to this study. The first set of workshops that reflect an alternative approach to the teaching of calculus include:

To Eat Or Not To Eat: Solids For Understanding Volume In Calculus

Teaching Calculus: A Graphic, Numerical, Algebraic and Verbal Approach.

The second set of workshops listed deals with the engagement of students using modeling of real-world situations and extended problems. Although these workshops are not designed solely for Calculus, the ideas given in this workshop are intended for senior mathematics. These workshops include:

Data Collection and Modeling- Finding that “Best Fit”

Mathematics Modeling in Algebra and Beyond

Building Mathematical Power Using Mathematical Modeling

Mathematical Modeling with the CBR and CBL

The third set of workshops relate to the emphasis on technology and how technology can be incorporated into a classroom. They include:

Tackling Transformations on the TI-83

Assessment & the TI-83: How to Design Assessment Tasks Effectively for Principles of Math 12

Using the TI-83+ to Teach Outcomes in Principles of Mathematics 10 and 11

Beyond Geometry: From Algebra to Calculus with Sketchpad Version4

Using Spreadsheets to Investigate Probabilities

Enriching Mathematics – Computer Aided (Polyhedra-making, Tessellating and Graphing)

Mathematics Activities for the TI83+

Finally the last set of workshops significant to this study is ones that relate to a student-centred approach to teaching and they include:

Student Centered Problem-Solving Techniques with Student Generated scoring Criteria

And They Said It Couldn't Be Done? – Successful hands-on projects in upper level high school mathematics courses

## **NCTM's Canadian Regional Conference 2003- Edmonton, Alberta**

Similarly to the Northwest Mathematics Conference, the NCTM's Canadian Regional Conference also offers many workshops that share common themes with Calculus Reform. Once again, some of the workshops listed are not intended solely for Calculus but also senior mathematics. The list of workshops significant to this study include:

Mathematical Modeling in Algebra

Discovering Mathematics through Nature

Mathematics and Science: Connected through Link Cables

Technology: The Good, the Bad, and the Ugly

Using Cellsheet on TI-83 to Perform Spreadsheet Operations

Cooperative Learning in College Algebra, Precalculus, and Calculus 1

Algebraic and Real-World Links

Modeling Mathematics with the TI-83 Plus Parametric and Sequence Modes

## **Chapter 5**

### **DISCUSSION AND CONCLUSION**

The discussion and conclusions made in this chapter are based on the analysis of the results in Chapter 4 as well as the results from the literature review in Chapter 2. The discussion will incorporate the influences in the teaching of calculus and a comparison will be made between the intended and implemented calculus course. The intended calculus course, as given by the Calculus 12 Integrated Resource Package, will be compared with the literature review which states the main themes of calculus reform and a calculus reform textbook. This will be followed by a look at the implemented calculus course in which the focus of the examination will be on the teacher and the major influences in how they choose to teach their class. Conclusions will then be drawn as to how calculus reform has influenced and will continue to influence the teaching of calculus in secondary schools of British Columbia.

#### **Evidence of Calculus Reform in the Intended Calculus 12 Course**

The major themes of calculus reform have been well documented since the beginning of the movement in the late 1980's. Although no set curriculum was established for all institutions wanting to be involved with reform, the major themes set guidelines to do so. Many reformers wanted a trimming of the content, coupled with change in how the first year calculus course was to be instructed. The focus of the instructional changes can be summarized by the four major themes of calculus reform:

1. Engaging students in the learning of calculus by using problems set in a context, are more extended, and require mathematical modeling of real world situations.
2. Making students more accountable for understanding complex mathematical concepts by using the rule of four (graphical, numerical, analytical and verbal) and the connections between them.
3. Promoting a change in pedagogy in teaching calculus from a teacher-centered lecture style to a student-centered approach.
4. Using technology to help facilitate the three themes mentioned above.

These changes to the calculus course were coupled with new textbooks designed to reflect the reform model. One of the first reform calculus textbooks was written by Gleason and Hughes-Hallet and is commonly called the Harvard book. An analysis of this book, in Chapter 4, gave an indication of the major changes from a traditional calculus textbook as well as what reformers had in mind for the new calculus course. Thus the Harvard book can be used as to establish a baseline for the intended calculus course. This can then be compared to the Calculus 12 Integrated Resource Package (IRP's) to determine the intended calculus course for secondary schools of British Columbia.

### **The Engagement of Students in the Learning of Calculus**

The Harvard Book makes many attempts to use problems that have a real world context. The theme of using displacement, velocity and acceleration helps students



establish a basis for the concepts being learned. In the Harvard book the notion of a derivative is developed using the idea of velocity rather than the secant line turning into a tangent line for a curve on a graph. This is consistent with the IRP's suggestion of using instantaneous velocity to start the section entitled *Overview and History of Calculus*. The IRP's also suggest the development of the physics formulae for motion under constant gravitational acceleration. This same development can be found in the Harvard book as an extended problem in *The Focus on Modeling* which examines the development of Newton's equations of motion.

Further similarities between the Harvard book and the IRP's can be found in the emphasis on modeling of real-world situations. In both the IRP's and the Harvard book examples are given to show unique real-world situations in which calculus is used. Furthermore, to assist in the use of modeling the IRP's suggest, early in the course, the use of exponential functions. This is also found in the Harvard book as exponential functions are found in the first chapter along with the traditional functions like polynomial, rational, and trigonometric.

### **Making Connections and Understanding the Concepts**

The Harvard book states in the preface that an emphasis is placed on the "Rule of Four". This use of multiple-representation can be found in the instructions, examples and problems. One such example is in developing the concept of a derivative. It is initially examined with a graph but then followed up with a numerical and an analytical approach. Similarly the IRP's suggest that the concept of limits should be examined analytically,

numerically and geometrically and although the IRP's deal with limits and not the derivative, the concept of the derivative is based on the limit.

The fourth component of the rule of four addresses the use of verbal explanation in assessing concepts of calculus. The Harvard book contains problems that require students to explain their thoughts to support their calculations, graphs or analysis. The use of verbal explanation is also mentioned in the IRP's as many assessment strategies suggest this to allow students to communicate their understanding.

### **Changing Pedagogy-Student Centred Approach**

The third major theme of calculus reform is the change in pedagogy from the teacher-centred lecture to a more hands-on student-centred course. Evidence for this theme is hard to establish by examining the Harvard book as the book cannot dictate how the teacher chooses to teach the course. However from the literature review it was found, in the past, that many institutions attempted to incorporate this idea when developing reform-oriented calculus courses. Some of the techniques employed to make a hands-on course include the use of laboratories, investigations, smaller classes, and cooperative learning. This intention of a student-centred course is also evident in the IRP's. The use of cooperative learning and having students investigate are common techniques given in the suggested instructional strategies. As well, many of the suggested assessment and instructional strategies in the IRP's were phrased in a student-centred manner. These phrases include: Allow students to..., Have students..., Give students..., Ask students to..., Challenge students to... and Discuss with students....

## **The Use of Technology**

As stated in the literature review, reformers strongly advocated for the use of technology in the new calculus course. Many of the institutions using the reform model made attempts to incorporate graphing tools to assist with the other major changes mentioned above. For instance the graphing tool can be used to produce graphs that model a real-world situation. Also, students had the opportunity to make further connections between the graphical, analytical and numerical by focusing on the concept rather than how the graph is drawn. Finally, to be consistent with the student-centred approach, lessons could be designed in which students could investigate concepts, with the aid of a graphing calculator, and develop their own understanding.

The Harvard book also showed this emphasis on technology as evident by the many instructional examples and problems that required the use of a graphing tool. The IRP's in a similar manner suggest the use of technology to further develop key concepts of calculus. This includes the use of technology to verify that the derivative of a function is the tangent line at a given point, to explore features of functions such as critical and inflection points, concavity, vertical and horizontal tangents and the relationship between the first and second derivative.

## **Other Features of Calculus Reform**

Along with the major themes of calculus reform mentioned in the literature review more specific differences exist between the traditional and reform calculus courses. An analysis of the Harvard book shows a reduction in the number of topics, with emphasis on topics dealing with real-world applications. Although the IRP's do not show this

trimming in the number of topics it is evident that there is an emphasis toward real-world situations.

Another notable difference between the traditional and reform calculus course is the de-emphasis of routine problems in which the understanding of the concept is compromised. The Harvard book calls this the de-emphasis of the 'plug-and-chug' approach where students rely on rote memorization of typical problems instead of focusing on the concepts behind the solution. An example of this would be curve sketching. Traditionally this topic is done in a series of steps, the first and second derivative is determined to find the critical and inflection points, vertical and horizontal asymptotes are found and then an analysis is performed to determine concavity. For difficult rational and radical functions this analysis requires extensive algebraic manipulation to sketch the curve. Although the Harvard book does show curve sketching it takes a different approach as the focus is more on the relationship between the curve and the first and second derivative rather than on the process of determining them. This is also evident in the IRP's as it suggests the use of a graphing calculator to explore the features of a graph like critical and inflection points. Hence the focus is on the interpretation and analysis and not on the many steps required to sketch the curve.

Another example of this de-emphasis on 'plug-and-chug' is found in regards to L'Hospital's rule. The Harvard book mentions L'Hospital but does not treat all cases as in the traditional book. The IRP's make no mention of L'Hospital rule in finding limits. Instead the IRP's suggest using technology to determine limits of complex functions by examining the graph and taking closer and closer values. Once again this is consistent

with an emphasis toward understanding rather than the rote-memorization of the many steps often required solving difficult limit questions.

## **Summary**

Many similarities can be found in comparing the major themes of calculus reform that were presented in the literature review and the analysis of the Harvard book with the Calculus 12 IRP's. Evidence was found to indicate that the IRP's are consistent with the major themes of calculus reform. This evidence supports the claim that the Calculus 12 curriculum has been influenced by the reform model. Therefore a claim can be made that the intended calculus course for secondary schools in British Columbia shares many similarities to the calculus course that reformers had in mind.

## **Implemented Calculus Course**

Although there is evidence that calculus reform has influenced the intended calculus course, a complete analysis requires a look at how it has influenced the implemented calculus course. The implemented calculus course consists of what is happening in the classroom and is related to the question: How has calculus reform influenced the teaching of calculus in secondary schools in British Columbia? The ultimate decision on how the student will experience calculus, the implemented calculus course, is determined by the teacher. However many contributing factors come into play when teachers decides how they will teach the calculus course. Along with curriculum, the other factors include the textbooks used, personal experience, professional development opportunities and, in British Columbia, the University Challenge Examinations. These major factors along

with the teacher will determine the implemented calculus course in secondary schools of British Columbia.

The analysis of the implemented curriculum is based on the teacher surveys, the textbooks commonly used in secondary schools of British Columbia, the university challenge examinations and opportunities for professional development.

### **The Engagement of Students**

The results of the survey indicate that teachers made attempts to use real-world examples whenever possible. Traditional topics such as related rates, optimization, population growth, motion and applications of integration were commonly mentioned. This is consistent with the calculus books commonly used in secondary schools of British Columbia as both books which I examined, like the Harvard book, had an emphasis on using real-world situations. The university challenge examinations also had questions using a real-world context, including situations such as the population of a bacteria infested pool and the rate of change of a stalagmite growing.

It appears that the implemented calculus course attempts to engage student by using real-world examples as evidenced in both the textbooks and the challenge examinations. However, this does not guarantee that teachers will use this approach on a regular basis in an attempt to engage students. Many of the mentioned applications to the real-world were also part of the traditional calculus course. Furthermore, when reformers envisioned this theme of using real-world situations, it was intended for all topics and not just a select few.

Although no definitive statement can be made in regards to teachers using real-world situations to engage students, it appears that the stage is set to move in this direction. With the textbook and challenge examinations promoting this theme, teachers may follow. Also many teachers indicated on the survey an attempt to use real-world situation whenever possible. This acceptance of the idea coupled with opportunities for professional development may prove to be fruitful in regards to this theme. All three conferences examined had professional development opportunities related to using real-world situations. Although many of the workshops were not designed specifically for calculus, they are for senior mathematics and many of these teachers also teach calculus.

### **Making Connections and Understanding the Concepts**

The teacher survey did not ask questions regarding lessons that incorporated the rule of four. However the textbooks examined, similar to the Harvard book, used multiple representations in developing key concepts of calculus. Moreover many of the problems in the book encouraged students to examine the solution by using the rule of four. Evidence was also found in the challenge examination as questions required students to approach a solution graphically, numerically or analytically and explain their solution verbally. Furthermore, many questions were asked that emphasized conceptual understanding rather than rote memorization as the questions were quite unique. An example of this is the use of the second derivative to justify if a linear approximation is an underestimate or overestimate.

In regards to how teachers teach their class the survey inquired about the instruction of certain topics: the precise mathematical epsilon-delta definition of a limit, other uses

for second derivative, introduction to integration, and the Fundamental Theorem of Calculus. As expected most teachers did not teach the epsilon-delta definition of a limit as the proof of a limit's existence often hinders the student's ability to understand the concept of a limit. This is consistent with the reform model as understanding the concept of a limit as a hollow dot on a curve and an approaching numerical value as you get closer and closer to the point in question is more significant than the proof.

In dealing with the introduction to integration, many teachers responded that this was done using the concept of the Antiderivative. Once again, this is consistent with the calculus reform model as the conceptual connection between the derivative and the integral is more important than the algebraic manipulation required in solving many integrals. It is worthwhile to note that the challenge examination also used the term "antiderivative" instead of the traditional wording "integration". Finally it should be noted that many teachers do not stress the term Fundamental Theorem of Calculus. Perhaps teachers feel that telling students about the connection between the derivative and integral is more important than giving it an official name. This would be consistent with the reform model of teaching calculus in which understanding the concept is stressed over the detail of its formal title.

### **Changing Pedagogy to a Student Centred Approach**

When asked on the survey about designing lessons in which students performed an investigation, most teachers indicated a desire to do so. For those that said they did not, time seemed to be a major constraint. AP Calculus teachers felt the time constraint of covering all material before the AP examination. Calculus 12 teachers felt the time



constraint of adequately preparing lessons of this sort. One teacher noted that they used the investigations given in the textbook. Both textbooks examined have sections in which students can perform investigations to construct their own understanding of calculus concepts. Some of these investigations are quite extensive and suggest the use of cooperative learning. Both student investigation and cooperative learning are consistent with the reform model of a student-centred approach to teaching of calculus.

It would be an unfair statement to say that calculus reform is the reason that many calculus teachers strive for a student-centred classroom. Most secondary school teachers are aware of the importance of this approach as this has been a common theme in mathematics education. Also cooperative learning is a common technique used by teachers, not just reform calculus teachers. Therefore it would be a fair statement that secondary school teachers may be in a better position to employ this method of instruction than post-secondary institutions. With smaller classes, more interaction and opportunities to develop cooperative learning strategies the secondary classroom may be an ideal situation for a calculus course that the reformers had envisioned. Also an examination of the three conferences showed opportunities to develop strategies that focus on cooperative and student-centred learning.

### **Use of Technology**

All teachers surveyed indicated that technology was used in their classroom. Although most could not say that technology was used in fundamental ways, most stated that technology was incorporated into their lessons. The most common use of technology was to enhance the understanding of difficult concepts by avoiding rigorous and tedious

calculations. Both textbooks also share this theme as many solutions do not spend time in justifying a graph used in the explanation. By using a graphing tool the difficulty often found in producing graphs is avoided as the focus can be placed on the concept being examined.

When calculus reformers envisioned the use of technology in the calculus course, it was to facilitate student investigations and enhance understanding by making connections between the graphical, numerical and analytical concepts. Although the avoidance of rigorous situations was a benefit in using technology, reformers envisioned the graphing tool to have a bigger role. As found in the literature review, technology was to be used in fundamental not incidental ways. The results of the teacher survey suggest that technology is being used in incidental ways to avoid rigorous and tedious situations. A couple of teachers gave examples in which technology was being used in an investigation but most did not. The Stewart textbook seems to share this approach as this textbook could be used without the use of a graphing tool. However the Finney textbook, more like the Harvard book, does have evidence to suggest that technology is a key component in using that book. Many of the topics include a student-investigation in which a graphing tool must be used. Also many of the problems in this book require the use of technology in order to solve the question.

Although most mathematics teachers would agree that technology has its place in the classroom, most can not agree about the extent to which it should be used. The survey indicated that technology was being used by most teachers but not to the degree that was in the minds of the reformers. There are many possibilities as to why the use of technology in the implemented course is different than the intended. The first possibility

is that teachers in secondary schools have not had the opportunity to develop lessons in which technology can be used to enhance the understanding of a concept. An examination of the conferences gives a limited number of workshops that incorporate technology into a calculus course in fundamental ways. Second, many teachers may find it difficult to effectively incorporate technology without losing the integrity of a concept. This often comes into play during assessment because substantial analysis, that a teacher may feel is important in a solution, is not needed with the aid of a graphing calculator. The next possible reason is the secondary school teacher's limited familiarity with the graphing calculator. Although most secondary school calculus teachers use it because the Principles of Mathematics 12 requires it, it is not to the extent that reformers had in mind. Due to the limited time in a Principles of Mathematics 12 course, rarely will teachers incorporate investigations that use the graphing calculator. Finally, careful thought and consideration is placed in designing the university challenge examinations to ensure that the integrity of a question is not lost because of the graphing calculator. Teachers of Calculus 12, preparing students for the challenge examinations, may not find the need to use the graphing calculator.

## Conclusion

In attempting to answer the question of how calculus reform has influenced the teaching of calculus in secondary schools of British Columbia a distinction had to be made between the implemented and intended curriculum. The intended curriculum as stated by the IRP's show a strong correlation with the literature findings on what reformers had in mind for a new calculus course. All the major themes were examined and evidence was found to suggest that calculus reform has influenced the philosophy of the intended Calculus 12 course.

Although it has been shown that calculus reform has influenced the intended calculus course, the implemented calculus course is more related to the primary question of this research study. The implemented course is often different than the intended calculus course because the teacher will make the ultimate decision on how calculus will be taught. From the results of the analysis it is evident that some of the major themes of calculus reform are present in calculus classrooms. No conclusion can be made to suggest that calculus reform is the reason for this, as mathematics education shares many of the same themes. The NCTM makes strong comments to suggest the use of technology and student-centred teaching to make students accountable for understanding the material. However, by the mere fact that the major themes of calculus reform are present in the textbooks, university challenge examinations and professional development opportunities, secondary calculus classes will be influenced by calculus reform.

As stated earlier, many institutions in the United States are opposed to calculus reform and they are moving back to a traditional model. However with this said, the envisioned calculus course under calculus reform seems to be well suited for secondary

schools. Mathematics teachers in secondary schools are familiar with the major themes of calculus reform as they are consistent with those of mathematics education. Smaller classrooms will allow for student-centred lessons in which students can investigate and work cooperatively while the teacher has the opportunity to interact. With the Principles of Mathematics 12 requiring a graphing calculator, all students will have access to technology that can be used in the classroom. With the number of hours available in a secondary course, more time can be used to reinforce key concepts and make connections between the graphical, analytical and numerical aspects of the course. Also the extra time can be used to perform investigations that will engage students in the power and beauty of calculus.

The only drawback to teaching calculus in secondary schools is the teachers' lack of expertise. Many secondary calculus teachers do not have the knowledge or time to prepare lessons in which key concepts of calculus can be investigated with the aid of technology. However this issue can be addressed with more workshops for secondary calculus teachers that reflect the major themes of calculus reform. By doing so, secondary schools of British Columbia may offer a calculus course true to the intentions of calculus reformers.

It was not the intent of this investigation to make conclusions regarding the merits of calculus reform. However it is my opinion that the major themes of calculus reform are consistent with sound teaching methods. Therefore to effectively teach calculus in secondary schools, teachers will incorporate many of these ideas. I am reluctant to suggest that all secondary school calculus teachers should fully adopt calculus reform methods as some of the rigour required to fully understand calculus may be lost.

However teachers should be made aware that there are alternative methods to teaching calculus that may better suit their students. A course heavy in rigour that stresses formal mathematics may be suited for students entering engineering, mathematics or the sciences. For students wishing to get an appreciation for calculus a reform course, with a focus on the main ideas, may be more beneficial. Therefore the teacher will need to find a balance between the two to best meet the needs of their students.

## **APPENDIX A**

### **Ethics Approval**

## APPENDIX A\*

# SIMON FRASER UNIVERSITY

OFFICE OF RESEARCH ETHICS



BURNABY, BRITISH COLUMBIA  
CANADA V5A 1S6  
Telephone: 604-291-3447  
FAX: 604-268-6785

January 21, 2004

Mr. James Ahn  
Graduate Student  
Faculty of Education  
Simon Fraser University

Dear Mr. Ahn:

**Re: The Impact of Calculus Reform in the Teaching of Calculus  
in British Columbia Secondary Schools**

The above-titled ethics application has been granted approval by the Simon Fraser Research Ethics Board, in accordance with Policy R 20.01, "Ethics Review of Research Involving Human Subjects".

Sincerely,

Dr. Hal Weinberg, Director  
Office of Research Ethics

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\* For inclusion in thesis/dissertation/extended essays/research project report, as submitted to the university library in fulfillment of final requirements for graduation. Note: correct page number required.



## **APPENDIX B**

### **Informed Consent by Subjects to Participate in a Research Project**

Simon Fraser University Faculty of Education

Informed consent by subject to participate in a research project

Dear Colleague:

I am currently writing a thesis towards the completion of a Master's Degree at Simon Fraser University. The purpose of this letter is to ask for your consent to participate in this project. This project has been carefully reviewed and approved by Dr. Tom O'Shea and examined by the ethics board at the University.

The topic of the thesis will be The Impact of Calculus Reform in the Teaching of Calculus in Secondary Schools in British Columbia. The survey that I will be asking you to complete will deal with the teaching of Calculus by secondary mathematics teachers. The information collected will be confidential as you may withdraw from this project at any time. Any concerns may be directed to Ian Andrews the Dean of the Faculty of Education at 604-291-3148.

Your participation will be very much appreciated and if you would like to have any more information please do not hesitate to ask me about it. The final results of this project are going to be made available to you on request.

Sincerely,

James Ahn

---

I agree to participate in this project and allow the researcher to use the result from the survey.

---

Name

---

Date

---

Signature

## **APPENDIX C**

### **A Copy of the Survey**

Thank you in advance for completing this survey. I am trying to attain a snapshot of how Calculus is currently taught in British Columbia as part of a research investigation. Please answer the questions in a way that best reflect your current teaching practice.

1. Do you teach (check one of the following):

A.P. Calculus     Calculus 12     IB Calculus

(If you currently teach more than one please choose one, to which your answers to the survey apply. If you wish please fill out a separate survey for each course.)

2. Which textbook is being used in your class (please include the Author, Title and edition number)?

Title	Author	Edition
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3. What resources do you use to decide how Calculus will be taught in your course? Please list in order of importance. (This could include IRP's, textbooks, AP Syllabus, University challenge exams, University or College course outlines...)

Course Content

4. Do you teach the precise epsilon-delta definition of a limit in class? If so, do you evaluate students on this topic?

5. Do you teach uses of the second derivative other than for curve sketching? If so please give an example of a question that you would expect your students to be able to do that would test their knowledge of this.

6. When introducing integration, which approach is used in your classroom? If you use more than one, please indicate the order in which they are discussed.

Riemann sums  
 Antiderivative (recognition of a function and its derivative)  
 Solution to a differential equation  
 Other (please give a brief explanation)

7. To what degree is the term "Fundamental Theorem of Calculus" used in your classroom? (Please check off one of the following)

Heavily- students are expected to know it, use it and explain it  
 Moderately- students know of it and can answer questions that relate to it  
 Slightly- students have heard of it but that is all  
 Not mentioned

### Methodology

8. Did you set up any lessons in which the graphing calculator (or other forms of technology) was used in a fundamental way in teaching of calculus? Please explain why the graphing calculator was fundamental in the teaching of that lesson.
9. Did you set up any lessons in which the students performed an investigation to construct their own knowledge of a topic? Please explain the form of investigation and the topic covered.
10. Did you present questions in your classroom that deal with real world examples? This can be a question asked on a test, questions from a textbook, questions asked on a worksheet or question regarding an investigation. Please summarize the question.
11. Have you attended any workshops or conferences that have helped you prepare lessons that deal with the questions asked in 8,9 and 10 above? If so please list them and explain how they helped.

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