BENEFITS OF INTERNATIONAL DIVERSIFICATION USING EQUITY AND BONDS

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Abstract

Much academic research has been completed on the use of portfolio optimizers in an international setting. Many optimizers use historical data and almost all allow for short sales. Some researchers have begun to question the output from such strategies. While some papers have tackled the problem of finding reasonable estimates for risk, the focus in this paper is on the inherent risk of investing in the tangency portfolio while allowing short sales. To test this theory, I first used historical data to test the validity of the theory that diversifying into international stock and bond index investments can provide gains to a US investor. Second, I performed the same test but with quarterly portfolio revision based on four separate strategies, two with short sales and two without. Last, I compared the results to those of an investor who followed two passive strategies. I found that while there was evidence that international diversification did provide US investors with an opportunity to increase returns, the risk adjusted return was not superior to that of the passive strategies. I also found that the use of short sales greatly increased the overall risk of the portfolio.

Dedication

To my family for all their love and support through the years and especially during the completion of this program. A thank you as well to my classmates and the T & C gang for their friendship, encouragement and group therapy sessions.

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1 Introduction

Eun and Resnick (1994) examined the "potential" gains to be made from international diversification. They gathered return data for stock and bond indices from seven countries and input this data into a mean-variance portfolio optimizer. After performing various tests, they found that "potential" gains could be realized through international investing. "Potential" is used here, because these results were based purely on historical data rather than the expected returns, variances, and covariances. Grauer (2004) looked at many of the same issues, but from the perspective of diversification among US industries. He found that in many instances the tangency portfolio resulted in bankruptcy due to the use of short sales. This paper will attempt to reconcile these differences by taking Eun and Resnick's data and applying it to Grauer's testing methods. The research that provided a foundation for these two papers is summarized below in the Literature Review section.

Here, I provide a brief synopsis of both papers. Eun and Resnick tested the ability of international portfolios to generate "potential" gains for US and Japanese investors. They took monthly data for bond and stock indices from seven countries from 1978-89; converted the data to US and Japanese currencies, and then calculated the tangency portfolios based on maximizing the Sharpe ratio. They then assumed that an investor had invested in 1978 according to the results and calculated the returns, standard deviations and Sharpe ratios. Comparing those results to domestic investment only, they found that international diversification was a valid strategy.

Grauer tested over a longer time period (1934-1999) and used quarterly data from US stocks categorized by industry. He assumed the investor revised the portfolio quarterly (after a 32-period estimation period) and calculated the end values for the minimum variance portfolio, tangency portfolio, and according to risk tolerance. His findings differed significantly from those of Eun and Resnick in that he found the tangency portfolio led to bankruptcy in many cases.

Several reasons can be given to account for the difference in the results of Grauer compared to those of Eun and Resnick:

1) Eun and Resnick used international indices vs. Grauer's US industries.
2) Grauer's testing method revised portfolio weights quarterly.
3) Eun and Resnick used international bond indices, which allowed greater diversification than was possible under Grauer's approach.
4) Eun and Resnick assumed a risk-free rate of zero. A positive risk-free rate would place the tangency portfolio higher on the efficient frontier, which would increase the chance of bankruptcy.

To reconcile these differences, I begin by replicating the first part of Eun and Resnick's work with up to date data. I do this to check if their conclusions still hold; i.e., to confirm that "potential" gains can still be realized through international diversification. Assuming their conclusions hold, I move on to replicate a portion of Grauer's paper. Using the data from part one, but converted from monthly to quarterly data, I create four portfolios, a minimum variance portfolio and a mean-variance (MV) efficient tangency portfolio, each with and without short sales. I then calculate the optimal portfolio weights for the 32-month estimation period and then revise weighting quarterly. Results are tabulated and the process is repeated every quarter until the end of my testing period. As

Eun and Resnick assume that short sales are allowed, I allow them in two of the portfolios to enable a useful comparison.

Last, I take the results from the second test and compare them to the results from owning equally weighted portfolios. This is done to enable comparison with Eun and Resnick, who also consider equally weighted portfolios, and to provide a benchmark for the four portfolios created in this work.

This paper is organized as follows. In section 3, I replicate part one of Eun and Resnick's 1994 paper. Section 4 replicates a portion of Grauer's paper: I determine the tangency and mean variance portfolios with and without short sales. In section 5, I compare results from section 4 with two equally weighted portfolios. Section 6 contains my summary and conclusions.

2 Literature Review

When Harry Markowitz published his article "Portfolio Selection" in 1952, little was know about concepts such as portfolio optimization and the efficient frontier. Now, 50 years later, it is hard to find any financial model that does not incorporate at least some of the concepts of what is now called modern portfolio theory (MPT). In the following, I first review the research leading up to Eun and Resnick's paper and then the work that led to Grauer's paper.

The study of international investing within an MPT framework again begins with Markowitz. His initial studies, though, are primarily concerned with domestic investments. Over the years, as international markets developed and granted greater access to foreign investors, a large amount of research literature focused on the gains to be made from international investments. The first such work was Grubel (1968), who shows that MPT can be applied to international investing and there are gains to be realized through international investing. His work led to further studies by Solnik (1974) and Lessard (1974) who collectively establish a convincing case for diversifying into international equity markets. More recently, studies have looked at international diversification from the perspective of the fixed income markets. Jorion (1987) concludes that using world government bond indices rather than US government bond indices can lead to lower volatility while keeping returns constant. Hunter and Simon (2004) look at the growth in correlations between international bonds, but conclude that the correlations are not high enough to nullify the gains that can be made from diversifying internationally.

Simons (1999) looks at international portfolios that include international fixed income along with stocks. She concludes that the inclusion of bonds in an international portfolio produced desirable results.

Grauer (2004) was based on research being done to find a better estimate of expected returns, variances, and covariances for input into a mean-variance optimizer. This research produced estimators such as the James-Stein and Bayes-Stein and a Capital Asset Pricing Model (CAPM)-based estimator. Jorion (1991) and Jobson, Korkie and Ratti (1979) found that using these estimators can improve investment performance. Grauer and Hakansson (1995) found that these estimators yield mixed results – while they may work in some settings the results are not consistent in others.

Grauer (2004) also focuses on the use of unconstrained short sales and their effect on the weighting assigned by mean-variance optimizers. Black and Litterman (1992), suggest that unconstrained short sales could be a contributing factor in the unusual composition of portfolios constructed using historical data. To overcome this problem, they suggest placing bounds on portfolio weights when short sales are allowed. Frost and Savarino (1988) do this and find that imposing upper bounds can both reduce estimation bias and improve performance. On the other hand, Grauer and Shen (2000), find that if constraints are imposed, most of the time, the increase in return in is not worth the reduction in risk.

3 Gains from International Diversification: An Ex Post Analysis from a US Perspective

In this section, I replicate Eun and Resnick's analysis of the gains from international diversification. I determine the optimal portfolios and compare the results from these portfolios with those of portfolios consisting of only US stocks and bonds. This analysis considers markets in seven countries: Canada (CA), Japan (JA), Germany (GE), France (FR), Switzerland (SW), the United Kingdom (UK), and the United States (US). I take monthly data from stock market indices and bond indices and solve for the optimal portfolio. Data for stock indices is taken from Morgan Stanley Capital International indices[1] denoted in US dollars. The bond indices are provided by Scotia Capital and converted to US dollars using data from Pacific Exchange rate services.[2]

The following maximization formula was used to solve for the tangency portfolio:

$$\underset{\{x_i\}}{\text{Max}}\ \theta \equiv \frac{\check{R}_p - R_f}{\sigma_p}$$

$$= \sum_{i=1}^{N} x_i (\check{R}_t - R_f) \Bigg/ \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i\, x_j\, \sigma_{ij} \right]^{1/2} \qquad (1)$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1.0, \text{ where}$$

θ = the Sharpe ratio
x_i = fraction of wealth invested in the *i*th asset
$\check{R}_i$ = the expected return of the *i*th asset
$\check{R}_p$ = the expected return of the portfolio
R_f = the risk-free interest rate
σ_{ij} = the covariance of returns between the *i*th and *j*th securities
σ_p = the standard deviation of returns on the portfolio

[1] Morgan Stanley Capital International, http://www.msci.com/
[2] The University Of British Columbia, http://fx.sauder.ubc.ca/

In normal practice, the determination of the optimal portfolio is a combination of the risk-free rate and the tangency portfolio. Using an investor's risk tolerance (desired risk and return), an optimal set of assets for that investor is calculated. To simplify the analysis, Eun and Resnick assumed that the risk-free rate is zero. To calculate the optimal portfolio, I determine the portfolio that maximizes the Sharpe ratio, as denoted by θ above.

Modern portfolio theory first quantified the concept of diversification by introducing the notion of covariance or correlation between assets. In the following comments, I consider the correlations among different indices; this is done knowing that the correlations between investments directly affect their weightings in a portfolio.

From the historical data, I first calculated the correlation matrix, returns, and standard deviation for each data set. The results are summarized in Table 1.

Before proceeding with this analysis, I should mention a few points. First, US stocks had the highest Sharpe ratio (0.2917); this was due in large part to the rapid rise in US share prices during the 1990s. Second, there were much higher correlations between stock indices than between bond indices. The largest correlation was between US and Canadian stocks at 0.76 with the lowest correlation being between US and Japanese stocks at 0.27. Conversely, the lowest correlation among bond indices (between Germany and Switzerland) was -0.10.

Moving forward, using the data from Table 1, I computed optimal international portfolios from three different perspectives: From a stock-only perspective, from a bond-only perspective, and from a combined stock and bond perspective. These results are summarized in Table 2.

Table 1 Summary statistics of correlations, means, standard deviations and Sharpe ratios of equity and bonds indices: 1984.1-2000.1.

	Stocks							Bonds							ME	SD	SHP
	CA	JA	GE	FR	SW	UK	US	CA	JA	GE	FR	SW	UK	US	(%)	(%)	
CA		0.30	0.40	0.46	0.46	0.55	0.76	0.42	0.12	0.22	0.04	0.03	0.17	0.13	0.75	4.95	.1506
JA	0.30		0.31	0.44	0.39	0.43	0.27	0.13	0.40	0.30	0.20	0.26	0.31	0.08	1.15	7.42	.1553
GE	0.40	0.31		0.72	0.66	0.53	0.43	0.18	0.14	0.09	0.23	0.10	0.16	0.10	1.40	6.38	.2201
FR	0.46	0.44	0.72		0.62	0.60	0.51	0.20	0.27	0.22	0.32	0.19	0.23	0.18	1.59	6.13	.2585
SW	0.46	0.39	0.66	0.62		0.62	0.50	0.20	0.23	0.09	0.25	0.36	0.30	0.18	1.41	5.42	.2597
UK	0.55	0.43	0.53	0.60	0.62		0.59	0.24	0.24	0.15	0.28	0.26	0.49	0.20	1.22	5.34	.2285
US	0.76	0.27	0.43	0.51	0.50	0.59		0.29	0.05	0.17	0.05	0.01	0.13	0.26	1.26	4.31	.2917
CA	0.42	0.13	0.18	0.20	0.20	0.24	0.29		0.12	0.15	0.23	0.13	0.31	0.51	0.05	2.73	.0186
JA	0.12	0.40	0.14	0.27	0.23	0.24	0.05	0.12		0.47	0.50	0.58	0.48	0.20	0.59	3.96	.1485
GE	0.22	0.30	0.09	0.22	0.09	0.15	0.17	0.15	0.47		-0.09	-0.10	0.02	0.16	0.33	2.91	.1149
FR	0.04	0.20	0.23	0.32	0.25	0.28	0.05	0.23	0.50	-0.09		0.75	0.56	0.28	0.34	3.33	.1035
SW	0.03	0.26	0.10	0.19	0.36	0.26	0.01	0.13	0.58	-0.10	0.75		0.63	0.22	0.26	3.36	.0761
UK	0.17	0.31	0.16	0.23	0.30	0.49	0.13	0.31	0.48	0.02	0.56	0.63		0.25	0.23	3.65	.0630
US	0.13	0.08	0.10	0.18	0.18	0.20	0.26	0.51	0.20	0.16	0.28	0.22	0.25		0.10	2.10	.0496

ME – Mean Return
SD- Standard Deviation
SHP – Sharpe Ratio

Table 2 Descriptive Statistics for Optimal Tangency Portfolios and Domestic Strategies. Weights in decimals points and sum to 1.00. Data from 1984.1-2000.1

Market	Stock Portfolio	Bond Portfolio	Bond/Stock Portfolio
Canada	-0.5467	-0.1515	-0.6207/-0.0757
Japan	0.0825	0.3167	0.0155/0.4338
Germany	0.0265	0.4464	0.0156/0.1470
France	0.1991	0.4687	0.1118/0.3067
Switzerland	0.2665	-0.1532	0.3424/-0.2405
UK	-0.0130	-0.0335	0.0355/-0.1856
US	0.9936	.1063	1.184/-0.4692
Total	1.00	1.00	1.00
ME	1.64%	0.45%	2.03%
SD	4.56%	2.68%	5.29%
SHP	0.3590	0.1719	0.3844
Domestic Strategy[a]			
ME	1.26%	0.10%	0.68%
SD	4.13%	2.10%	2.62%
SHP	0.2938	0.0496	0.2595

[a] For the domestic strategy, 50% weights were respectively assigned to the stock and bond indices in the stock/bond portfolio.
ME – Mean Return
SD- Standard Deviation
SHP – Sharpe Ratio

The results of the stock-only optimization are examined first. Because of the exceptional returns of the US equity market in the 1990s, the weight for the US index was close to 100% (99.36%). Canada with the lowest mean return (0.75%) and highest correlation with the US index (0.76) was given the largest negative weight (-54.67%). In our bond portfolio, France had the highest weighting (46.81%), although it did not have as high a Sharpe ratio as Japan (0.1485 vs. 0.1035); the high weighting for France was due to its high correlations with Switzerland and the UK, both of which had negative weightings.

Moving on to the combined portfolio, Switzerland was assigned the highest stock holding (34.24%). Switzerland had one of the highest correlations with all indices; combined with its mean return, this led to its high weighting. An interesting observation on the stock side is the drop of US stocks from 99.36% to 18.40%. Although the US stock index alone had the highest Sharpe ratio, its lack of correlation with other markets led to the drop in its weighting. In the bond portion, Japan had the highest weighting (43.38%) with the US having the lowest (-46.92%).

In terms of gains from diversification, in the stock-only portfolios the Sharpe ratio of the internationally diversified portfolio was 0.0653 higher than that of the domestic strategy. This gain was due to an increase of 0.38% on the return side that more than compensated for an increase in the standard deviation (the risk factor) of 0.27%. In the bond portfolio, international diversification again increased the mean return (by 0.35%) while standard deviation also rose (by 0.59%); as a result, the Sharpe ratio increased by 0.1223. Last, in the combined portfolio, again there are gains from diversifying internationally. The mean return greatly increased (by 1.35%), as did the Sharpe ratio (by 0.1249) despite an increase of 2.66% in standard deviation.

Overall, this analysis indicates that there are "potential" gains to be made from diversifying into international investments. We say "potential" because our results are based purely on historical data rather than the expected returns, variances, and covariances.

4 Portfolio Strategies

In the first part of this paper, my analysis suggested that, in theory, gains could be made from diversifying internationally. In this part, I take the analysis further by back testing these assumptions. I look at four different strategies by calculating:

1. The tangency portfolio that maximizes the Sharpe ratio with no short sale constraints
2. The tangency portfolio that maximizes the Sharpe ratio with short sale constraints
3. The minimum variance portfolio with no short sale constraints
4. The minimum variance portfolio with short sale constraints

For the first two strategies, we are again concerned with the optimal portfolio based on the Sharpe ratio (i.e., according to Eq. 1). For the latter two strategies, we want to calculate the minimum variance efficient portfolio. To do this, we use

$$\min_{\{x_i\}} \lambda \equiv \sigma_p^2$$

$$= \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}\right] \quad (2)$$

Subject to $\sum_{i=1}^{N} x_i = 1.0$, where

x_i = fraction of wealth invested in the *i*th asset.
σ_{ij} = the covariance of returns between the *i*th and *j*th securities.
σ_p^2 = the variance of returns on the portfolio.
λ = the calculation on the minimum variance portfolio.

4.1 Test Structure and Data

Once again, stock indices data taken from the Morgan Stanley Capital International indices denoted in US dollars was used. The bond indices data was provided by Scotia Capital and converted to US dollars using data from Pacific Exchange rate services. The data was then converted into quarterly data beginning in Q1 1983 and ending in Q4 1999.

To begin, I began with a 32-quarter estimation period using data from Q1 1983 to Q4 1990 to calculate the optimal portfolios for the four strategies. The resulting weights were tabulated and multiplied by the beginning index value. The index was then invested according to the results of the four strategies. These investments were then held until the beginning of the next quarter. The total return for each strategy and the ending index value was then calculated and recorded. At this point, another set of optimizations were done using data from the beginning of the data set to the end of the first holding period. The resulting weights were again tabulated and multiplied by the ending index weight. The index was again invested according to our results. This process was repeated for a total of thirty-six times.

The results of the four tests are summarized in tables 3-6. For ease of interpretation, a series of index values is shown for each investment strategy (second column from the left) as well as the quarterly returns.

Table 3 Summary statistics of optimal portfolio weights based on optimization of the Sharpe ratio; quarterly revision after a 32 month estimation period, short sales permitted: 1983 Q1 – 1999 Q4. Index value shows cumulative value of hypothetical portfolio with beginning value of 100. Returns reported in decimal form.

		Quarterly	Stocks							Bonds						
Date	Index	Returns	CA	JA	GE	FR	SW	UK	US	CA	JA	GE	FR	SW	UK	US
	100															
91 Q1	111	0.1131	-1.10	0.05	0.09	0.00	-0.15	0.78	0.88	1.61	0.07	-0.18	1.02	0.22	-0.90	-1.38
91 Q2	94	-0.1571	-1.25	0.05	0.04	-0.02	-0.12	0.75	1.08	1.78	0.01	-0.08	1.09	0.24	-0.92	-1.65
91 Q3	102	0.0917	-0.54	0.01	0.22	-0.04	-0.23	0.37	0.60	0.76	0.01	0.43	0.41	0.34	-0.54	-0.79
91 Q4	107	0.0422	-0.58	0.00	0.21	-0.01	-0.23	0.41	0.57	0.84	0.01	0.43	0.41	0.31	-0.55	-0.82
92 Q1	100	-0.0594	-0.60	0.00	0.19	-0.07	-0.22	0.35	0.74	0.77	0.07	0.33	0.48	0.31	-0.56	-0.78
92 Q2	105	0.0468	-0.71	-0.09	0.26	0.04	-0.24	0.35	0.79	0.80	0.14	0.54	0.52	0.19	-0.51	-1.08
92 Q3	105	-0.0029	-0.68	-0.13	0.23	0.05	-0.16	0.39	0.66	0.82	0.11	0.55	0.55	0.10	-0.44	-1.05
92 Q4	112	0.0698	-0.74	-0.09	0.11	0.00	-0.05	0.33	0.81	0.71	0.09	0.47	0.63	0.20	-0.52	-0.94
93 Q1	121	0.0787	-0.68	-0.10	0.11	-0.02	-0.08	0.32	0.79	0.62	0.09	0.52	0.59	0.24	-0.52	-0.87
93 Q2	119	-0.0142	-0.61	-0.09	0.12	-0.02	-0.08	0.29	0.69	0.58	0.06	0.55	0.53	0.24	-0.48	-0.79
93 Q3	126	0.0537	-0.41	-0.06	0.06	-0.07	0.05	0.24	0.52	0.29	0.05	0.56	0.57	0.12	-0.40	-0.53
93 Q4	128	0.0182	-0.42	-0.06	0.09	-0.06	0.03	0.23	0.51	0.26	0.03	0.58	0.50	0.16	-0.36	-0.48
94 Q1	124	-0.0336	-0.30	-0.09	0.07	-0.05	0.07	0.25	0.36	0.25	0.00	0.60	0.49	0.16	-0.34	-0.46
94 Q2	123	-0.0055	-0.30	-0.05	0.07	-0.08	0.13	0.24	0.42	0.06	0.05	0.65	0.60	0.12	-0.40	-0.51
94 Q3	123	0.0004	-0.41	-0.02	0.08	-0.13	0.16	0.27	0.57	0.02	0.16	0.62	0.63	0.08	-0.49	-0.54
94 Q4	122	-0.0051	-0.37	-0.06	0.09	-0.16	0.15	0.31	0.61	0.18	0.24	0.57	0.74	0.13	-0.62	-0.79
95 Q1	142	0.1573	-0.41	-0.06	0.12	-0.15	0.12	0.30	0.65	0.07	0.23	0.60	0.64	0.16	-0.55	-0.72
95 Q2	158	0.1129	-0.43	-0.04	0.12	-0.16	0.13	0.31	0.65	0.08	0.22	0.61	0.66	0.13	-0.55	-0.74
95 Q3	155	-0.0134	-0.41	-0.05	0.12	-0.16	0.12	0.30	0.64	0.08	0.23	0.60	0.65	0.13	-0.54	-0.71
95 Q4	162	0.0440	-0.54	-0.02	0.11	-0.23	0.18	0.32	0.80	0.26	0.13	0.58	0.80	0.13	-0.59	-0.91
96 Q1	167	0.0298	-0.44	-0.02	0.08	-0.21	0.18	0.26	0.71	0.16	0.03	0.58	0.77	0.14	-0.50	-0.75
96 Q2	163	-0.0270	-0.50	-0.02	0.07	-0.25	0.26	0.25	0.84	0.23	0.07	0.60	0.99	0.04	-0.57	-1.01
96 Q3	171	0.0492	-0.57	-0.01	0.09	-0.25	0.26	0.20	1.00	0.21	0.08	0.62	1.06	-0.06	-0.49	-1.14
96 Q4	168	-0.0160	-0.51	-0.03	0.10	-0.25	0.21	0.22	0.92	0.22	0.04	0.63	1.01	0.01	-0.48	-1.09
97 Q1	168	0.0022	-0.40	-0.06	0.11	-0.21	0.15	0.22	0.82	0.12	0.01	0.60	0.92	-0.02	-0.37	-0.90
97 Q2	193	0.1480	-0.49	-0.07	0.13	-0.19	0.20	0.23	0.89	0.18	0.02	0.66	0.93	-0.06	-0.37	-1.06
97 Q3	196	0.0140	-0.50	-0.08	0.13	-0.19	0.21	0.24	0.90	0.20	0.03	0.66	0.96	-0.07	-0.39	-1.08
97 Q4	196	0.0003	-0.46	-0.10	0.13	-0.20	0.18	0.29	0.86	0.14	0.05	0.59	0.90	-0.06	-0.40	-0.94
98 Q1	218	0.1121	-0.47	-0.12	0.11	-0.19	0.20	0.27	0.91	0.04	0.02	0.56	0.87	-0.03	-0.34	-0.83
98 Q2	220	0.0082	-0.46	-0.12	0.11	-0.18	0.20	0.27	0.89	0.04	0.01	0.57	0.86	-0.02	-0.33	-0.82
98 Q3	228	0.0373	-0.49	-0.12	0.15	-0.17	0.17	0.22	0.98	-0.04	0.00	0.52	0.85	-0.04	-0.29	-0.77
98 Q4	281	0.2346	-0.57	-0.13	0.15	-0.17	0.13	0.21	1.04	0.00	-0.01	0.56	0.87	-0.03	-0.25	-0.79
99 Q1	280	-0.0033	-0.65	-0.14	0.16	-0.20	0.15	0.27	1.09	0.05	0.07	0.47	0.95	-0.10	-0.33	-0.80
99 Q2	265	-0.0546	-0.79	-0.14	0.16	-0.23	0.14	0.33	1.31	0.17	0.14	0.50	1.02	-0.14	-0.41	-1.06
99 Q3	243	-0.0831	-0.76	-0.12	0.27	-0.22	-0.02	0.25	1.46	0.16	0.21	0.46	0.85	-0.06	-0.38	-1.12
99 Q4	263	0.0824	-0.75	-0.11	0.24	-0.22	0.11	0.29	1.42	0.02	0.46	0.44	0.95	-0.31	-0.44	-1.10

ME - Mean Return
SD- Standard Deviation
SHP – Sharpe Ratio

Table 4 Summary statistics of optimal portfolio weights based on optimization of the Sharpe ratio; quarterly revision after a 32 month estimation period, short sales precluded: 1983 Q1 – 1999 Q4. Index value shows cumulative value of hypothetical portfolio with beginning value of 100. Returns reported in decimal form

		Quarterly	Stocks							Bonds						
Date	Index	Returns	CA	JA	GE	FR	SW	UK	US	CA	JA	GE	FR	SW	UK	US
	100															
91 Q1	100	0.0043	0.00	0.06	0.15	0.04	0.00	0.23	0.10	0.00	0.00	0.00	0.42	0.00	0.00	0.00
91 Q2	94	-0.0642	0.00	0.08	0.11	0.02	0.00	0.14	0.26	0.00	0.00	0.00	0.39	0.00	0.00	0.00
91 Q3	99	0.0543	0.00	0.05	0.16	0.00	0.00	0.09	0.17	0.00	0.00	0.30	0.22	0.00	0.00	0.00
91 Q4	102	0.0311	0.00	0.02	0.15	0.02	0.00	0.15	0.12	0.00	0.00	0.32	0.22	0.00	0.00	0.00
92 Q1	99	-0.0318	0.00	0.02	0.14	0.00	0.00	0.09	0.21	0.00	0.00	0.27	0.29	0.00	0.00	0.00
92 Q2	103	0.0387	0.00	0.00	0.16	0.06	0.00	0.08	0.14	0.00	0.00	0.35	0.21	0.00	0.00	0.00
92 Q3	102	-0.0088	0.00	0.00	0.16	0.05	0.00	0.12	0.11	0.00	0.00	0.35	0.21	0.00	0.00	0.00
92 Q4	102	0.0014	0.00	0.00	0.09	0.03	0.06	0.04	0.20	0.00	0.00	0.27	0.31	0.00	0.00	0.00
93 Q1	109	0.0652	0.00	0.00	0.09	0.00	0.03	0.02	0.22	0.00	0.00	0.35	0.28	0.00	0.00	0.00
93 Q2	112	0.0350	0.00	0.00	0.10	0.01	0.02	0.02	0.21	0.00	0.00	0.37	0.27	0.00	0.00	0.00
93 Q3	117	0.0387	0.00	0.00	0.08	0.00	0.06	0.03	0.18	0.00	0.00	0.40	0.26	0.00	0.00	0.00
93 Q4	121	0.0327	0.00	0.00	0.09	0.00	0.06	0.03	0.16	0.00	0.00	0.41	0.25	0.00	0.00	0.00
94 Q1	118	-0.0226	0.00	0.00	0.09	0.00	0.08	0.05	0.12	0.00	0.00	0.43	0.23	0.00	0.00	0.00
94 Q2	115	-0.0213	0.00	0.00	0.09	0.01	0.11	0.04	0.10	0.00	0.00	0.44	0.22	0.00	0.00	0.00
94 Q3	115	0.0002	0.00	0.03	0.09	0.00	0.12	0.02	0.12	0.00	0.00	0.41	0.20	0.00	0.00	0.00
94 Q4	115	-0.0033	0.00	0.02	0.09	0.00	0.11	0.03	0.15	0.00	0.00	0.38	0.23	0.00	0.00	0.00
95 Q1	121	0.0519	0.00	0.02	0.11	0.00	0.10	0.03	0.14	0.00	0.00	0.39	0.18	0.02	0.00	0.00
95 Q2	129	0.0639	0.00	0.01	0.10	0.00	0.09	0.03	0.16	0.00	0.01	0.38	0.17	0.05	0.00	0.00
95 Q3	126	-0.0205	0.00	0.00	0.10	0.00	0.09	0.02	0.17	0.00	0.02	0.38	0.16	0.05	0.00	0.00
95 Q4	131	0.0393	0.00	0.01	0.07	0.00	0.13	0.01	0.22	0.00	0.00	0.31	0.20	0.05	0.00	0.00
96 Q1	133	0.0118	0.00	0.01	0.06	0.00	0.15	0.00	0.23	0.00	0.00	0.30	0.22	0.03	0.00	0.00
96 Q2	132	-0.0054	0.00	0.01	0.05	0.00	0.17	0.00	0.24	0.00	0.00	0.28	0.25	0.00	0.00	0.00
96 Q3	134	0.0180	0.00	0.02	0.06	0.00	0.15	0.00	0.27	0.00	0.00	0.27	0.23	0.00	0.00	0.00
96 Q4	137	0.0219	0.00	0.00	0.06	0.00	0.13	0.00	0.28	0.00	0.00	0.27	0.25	0.00	0.00	0.00
97 Q1	138	0.0025	0.00	0.00	0.07	0.00	0.10	0.02	0.29	0.00	0.00	0.27	0.24	0.00	0.00	0.00
97 Q2	152	0.1058	0.00	0.00	0.09	0.00	0.13	0.03	0.28	0.00	0.00	0.28	0.20	0.00	0.00	0.00
97 Q3	155	0.0173	0.00	0.00	0.09	0.00	0.13	0.03	0.28	0.00	0.00	0.28	0.20	0.00	0.00	0.00
97 Q4	155	-0.0002	0.00	0.00	0.10	0.00	0.11	0.05	0.29	0.00	0.00	0.26	0.19	0.00	0.00	0.00
98 Q1	169	0.0936	0.00	0.00	0.08	0.00	0.15	0.04	0.32	0.00	0.00	0.21	0.20	0.00	0.00	0.00
98 Q2	171	0.0100	0.00	0.00	0.08	0.00	0.15	0.04	0.32	0.00	0.00	0.21	0.20	0.00	0.00	0.00
98 Q3	162	-0.0524	0.00	0.00	0.11	0.00	0.13	0.01	0.36	0.00	0.00	0.17	0.21	0.00	0.00	0.00
98 Q4	184	0.1368	0.00	0.00	0.10	0.00	0.08	0.00	0.36	0.00	0.00	0.19	0.28	0.00	0.00	0.00
99 Q1	182	-0.0088	0.00	0.00	0.10	0.00	0.08	0.00	0.36	0.00	0.00	0.17	0.29	0.00	0.00	0.00
99 Q2	183	0.0052	0.00	0.00	0.10	0.00	0.06	0.00	0.39	0.00	0.00	0.18	0.27	0.00	0.00	0.00
99 Q3	180	-0.0198	0.00	0.00	0.13	0.00	0.01	0.00	0.43	0.00	0.00	0.19	0.24	0.00	0.00	0.00
99 Q4	197	0.0988	0.00	0.00	0.13	0.00	0.03	0.00	0.37	0.00	0.04	0.22	0.20	0.00	0.00	0.00

ME – Mean Return

SD- Standard Deviation

SHP – Sharpe Ratio

Table 5 Summary statistics of optimal portfolio weights based on variance minimization optimization; quarterly revision after a 32 month estimation period, short sales precluded: 1983 Q1 – 1999 Q4. Index value shows cumulative value of hypothetical portfolio with beginning value of 100. Returns reported in decimal form. Optimal portfolio weights based on with no short sale constraints.

		Quarterly	Stocks							Bonds						
Date	**Index**	**Returns**	**CA**	**JA**	**GE**	**FR**	**SW**	**UK**	**US**	**CA**	**JA**	**GE**	**FR**	**SW**	**UK**	**US**
	100															
91 Q1	101	0.0058	0.14	-0.08	0.11	-0.03	-0.08	0.08	-0.02	-0.10	-0.23	0.59	0.05	0.35	-0.04	0.27
91 Q2	104	0.0343	0.14	-0.08	0.11	-0.03	-0.08	0.08	-0.03	-0.11	-0.23	0.59	0.05	0.35	-0.04	0.28
91 Q3	105	0.0051	0.11	-0.08	0.09	-0.03	-0.07	0.10	-0.02	-0.07	-0.24	0.55	0.09	0.34	-0.05	0.26
91 Q4	105	0.0063	0.11	-0.08	0.09	-0.03	-0.07	0.10	-0.02	-0.06	-0.24	0.55	0.09	0.34	-0.05	0.26
92 Q1	104	-0.0154	0.12	-0.08	0.09	-0.03	-0.07	0.10	-0.02	-0.06	-0.24	0.56	0.08	0.34	-0.05	0.26
92 Q2	105	0.0135	0.12	-0.09	0.09	-0.02	-0.07	0.09	-0.03	-0.08	-0.24	0.58	0.08	0.33	-0.03	0.26
92 Q3	105	0.0018	0.12	-0.08	0.10	-0.02	-0.07	0.09	-0.02	-0.09	-0.24	0.58	0.07	0.34	-0.04	0.26
92 Q4	96	-0.0866	0.00	-0.08	0.10	-0.02	-0.07	0.09	-0.02	-0.09	-0.24	0.58	0.08	0.34	-0.04	0.26
93 Q1	101	0.0530	0.13	-0.08	0.09	-0.01	-0.06	0.08	-0.05	-0.08	-0.26	0.56	0.07	0.33	-0.01	0.28
93 Q2	103	0.0214	0.13	-0.09	0.08	-0.02	-0.05	0.09	-0.02	-0.10	-0.26	0.51	0.09	0.34	0.00	0.32
93 Q3	106	0.0254	0.11	-0.09	0.09	-0.01	-0.07	0.09	0.00	-0.07	-0.26	0.51	0.08	0.36	-0.01	0.29
93 Q4	110	0.0398	0.12	-0.09	0.08	-0.02	-0.06	0.09	-0.01	-0.07	-0.26	0.50	0.10	0.35	-0.02	0.29
94 Q1	105	-0.0426	0.09	-0.08	0.08	-0.02	-0.09	0.08	0.04	-0.07	-0.25	0.49	0.08	0.36	-0.02	0.31
94 Q2	102	-0.0294	0.13	-0.06	0.09	-0.03	-0.08	0.05	0.03	-0.17	-0.26	0.50	0.07	0.37	0.00	0.37
94 Q3	102	-0.0053	0.13	-0.05	0.09	-0.04	-0.09	0.05	0.05	-0.20	-0.25	0.48	0.05	0.37	0.00	0.42
94 Q4	102	-0.0006	0.13	-0.06	0.09	-0.04	-0.09	0.05	0.05	-0.19	-0.25	0.47	0.05	0.38	-0.01	0.41
95 Q1	106	0.0384	0.13	-0.06	0.09	-0.04	-0.10	0.05	0.05	-0.19	-0.25	0.47	0.05	0.38	0.00	0.41
95 Q2	110	0.0379	0.13	-0.05	0.10	-0.04	-0.09	0.04	0.02	-0.19	-0.27	0.49	0.05	0.35	0.03	0.43
95 Q3	109	-0.0107	0.13	-0.04	0.10	-0.02	-0.10	0.04	0.01	-0.20	-0.31	0.50	0.05	0.37	0.04	0.43
95 Q4	113	0.0393	0.12	-0.04	0.10	-0.03	-0.09	0.04	0.02	-0.18	-0.32	0.49	0.06	0.37	0.04	0.42
96 Q1	109	-0.0336	0.11	-0.04	0.11	-0.03	-0.11	0.05	0.02	-0.16	-0.30	0.49	0.03	0.38	0.03	0.41
96 Q2	107	-0.0136	0.12	-0.04	0.10	-0.03	-0.10	0.04	0.03	-0.15	-0.31	0.50	0.08	0.35	0.03	0.37
96 Q3	111	0.0290	0.13	-0.04	0.11	-0.03	-0.10	0.03	0.04	-0.16	-0.31	0.50	0.07	0.34	0.05	0.38
96 Q4	113	0.0202	0.13	-0.03	0.10	-0.03	-0.09	0.02	0.04	-0.18	-0.31	0.49	0.07	0.33	0.06	0.40
97 Q1	111	-0.0198	0.12	-0.03	0.10	-0.03	-0.09	0.02	0.05	-0.17	-0.31	0.50	0.07	0.34	0.05	0.39
97 Q2	117	0.0598	0.11	-0.03	0.11	-0.02	-0.09	0.01	0.04	-0.16	-0.31	0.51	0.05	0.34	0.06	0.39
97 Q3	118	0.0081	0.10	-0.04	0.11	-0.01	-0.10	0.04	0.00	-0.12	-0.31	0.49	0.06	0.35	0.03	0.39
97 Q4	119	0.0043	0.10	-0.04	0.11	-0.01	-0.10	0.04	0.00	-0.12	-0.31	0.50	0.06	0.35	0.03	0.39
98 Q1	122	0.0261	0.10	-0.04	0.11	-0.01	-0.10	0.04	0.00	-0.12	-0.31	0.50	0.06	0.35	0.03	0.39
98 Q2	121	-0.0084	0.10	-0.04	0.11	-0.02	-0.10	0.04	0.01	-0.12	-0.31	0.49	0.06	0.35	0.03	0.40
98 Q3	123	0.0207	0.10	-0.04	0.11	-0.01	-0.11	0.04	0.01	-0.13	-0.31	0.48	0.06	0.35	0.03	0.42
98 Q4	130	0.0549	0.12	-0.04	0.11	-0.01	-0.10	0.04	0.00	-0.13	-0.31	0.48	0.05	0.35	0.03	0.43
99 Q1	127	-0.0250	0.12	-0.04	0.11	-0.02	-0.11	0.05	-0.04	-0.12	-0.29	0.44	0.03	0.34	0.02	0.50
99 Q2	126	-0.0085	0.12	-0.03	0.11	-0.02	-0.12	0.06	-0.03	-0.10	-0.29	0.44	0.02	0.35	0.01	0.49
99 Q3	123	-0.0228	0.12	-0.03	0.12	-0.02	-0.13	0.05	-0.03	-0.10	-0.29	0.44	0.01	0.35	0.01	0.49
99 Q4	128	0.0451	0.14	-0.03	0.11	-0.01	-0.12	0.05	-0.06	-0.12	-0.28	0.44	0.00	0.34	0.01	0.52

ME – Mean Return
SD- Standard Deviation
SHP – Sharpe Ratio

Table 6 Summary statistics of optimal portfolio weights based on variance minimization optimization; quarterly revision after a 32 month estimation period, short sales precluded: 1983 Q1 – 1999 Q4. Index value shows cumulative value of hypothetical portfolio with beginning value of 100. Returns reported in decimal form. Optimal portfolio weights based on with no short sale constraints.

		Quarterly	Stocks							Bonds						
Date	Index	Returns	CA	JA	GE	FR	SW	UK	US	CA	JA	GE	FR	SW	UK	US
	100															
91 Q1	101	0.0132	0.05	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00	0.10	0.00	0.37
91 Q2	104	0.0277	0.05	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00	0.11	0.00	0.37
91 Q3	107	0.0237	0.05	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.39	0.00	0.11	0.00	0.39
91 Q4	109	0.0202	0.06	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00	0.12	0.00	0.38
92 Q1	106	-0.0216	0.06	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.12	0.00	0.37
92 Q2	108	0.0129	0.05	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.11	0.00	0.35
92 Q3	109	0.0125	0.05	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.11	0.00	0.35
92 Q4	112	0.0260	0.05	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.10	0.00	0.35
93 Q1	120	0.0706	0.06	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00	0.11	0.00	0.36
93 Q2	125	0.0409	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00	0.12	0.00	0.41
93 Q3	128	0.0252	0.06	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.34	0.00	0.11	0.00	0.43
93 Q4	130	0.0147	0.07	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.33	0.00	0.11	0.00	0.43
94 Q1	125	-0.0349	0.07	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.33	0.00	0.11	0.00	0.43
94 Q2	123	-0.0210	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00	0.13	0.00	0.40
94 Q3	122	-0.0069	0.06	0.00	0.05	0.00	0.00	0.00	0.01	0.00	0.00	0.35	0.00	0.13	0.00	0.41
94 Q4	121	-0.0079	0.08	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00	0.13	0.00	0.41
95 Q1	126	0.0392	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00	0.13	0.00	0.41
95 Q2	133	0.0570	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.00	0.11	0.00	0.42
95 Q3	128	-0.0332	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00	0.12	0.00	0.42
95 Q4	131	0.0227	0.07	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.31	0.00	0.12	0.00	0.45
96 Q1	128	-0.0268	0.07	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.31	0.00	0.11	0.00	0.45
96 Q2	126	-0.0138	0.09	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.12	0.00	0.44
96 Q3	128	0.0128	0.09	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.11	0.00	0.45
96 Q4	129	0.0129	0.09	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.11	0.00	0.45
97 Q1	127	-0.0186	0.08	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.31	0.00	0.11	0.00	0.45
97 Q2	135	0.0613	0.08	0.00	0.05	0.00	0.00	0.00	0.01	0.00	0.00	0.32	0.00	0.10	0.00	0.45
97 Q3	135	0.0044	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00	0.10	0.00	0.49
97 Q4	133	-0.0136	0.07	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.29	0.00	0.09	0.00	0.49
98 Q1	137	0.0247	0.07	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.28	0.00	0.09	0.00	0.51
98 Q2	135	-0.0151	0.07	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00	0.10	0.00	0.51
98 Q3	136	0.0097	0.06	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.09	0.00	0.53
98 Q4	142	0.0481	0.06	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.09	0.00	0.53
99 Q1	140	-0.0157	0.06	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.08	0.00	0.58
99 Q2	139	-0.0086	0.06	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.08	0.00	0.57
99 Q3	141	0.0119	0.07	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.08	0.00	0.57
99 Q4	144	0.0270	0.07	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.08	0.00	0.58

ME – Mean Return
SD- Standard Deviation
SHP – Sharpe Ratio

4.2 Portfolio Weights and Observations

The differences in the returns, standard deviations, and Sharpe ratios among the indices over the testing period are shown in Table 7. The average Sharpe ratios over the two periods are also shown. These values will be useful for analyzing the results of each strategy.

Table 7 Means, standard deviations and Sharpe ratios for time periods 1983 Q1 to 1990 Q4 and 1983 Q1 to 1999 Q4: quarterly numbers. Average SHP reported is an average of both periods ratios, used for ease of comparison.

To Q1 90	ME (%)	SD (%)	SHP	To Q4 99	ME (%)	SD (%)	SHP	Average SHP
Canada	1.5900	8.80	0.1809	Canada	2.2600	8.83	0.2555	0.2182
Japan	6.3000	15.73	0.4004	Japan	3.6400	13.15	0.2769	0.3387
Germany	5.1300	13.57	0.3781	Germany	4.2400	10.18	0.4164	0.3973
France	6.1200	13.90	0.4405	France	4.8400	9.66	0.5010	0.4708
Switzerland	3.9200	11.81	0.3318	Switzerland	4.3100	9.25	0.4655	0.3986
UK	4.5500	10.69	0.4261	UK	3.6400	7.81	0.4658	0.4460
US	2.8300	8.87	0.3189	US	3.8200	7.44	0.5133	0.4161
Canada	0.6900	5.12	0.1352	Canada	0.1400	4.49	0.0313	0.0833
Japan	2.3200	9.65	0.2405	Japan	1.8400	7.99	0.2310	0.2357
Germany	-0.1500	4.41	-0.0329	Germany	1.0100	5.23	0.1940	0.0806
France	2.6300	7.19	0.0719	France	1.0600	6.37	0.0637	0.0678
Switzerland	1.9000	7.77	0.2448	Switzerland	0.8300	6.89	0.1205	0.1827
UK	1.3500	8.95	0.1512	UK	0.7400	7.14	0.1031	0.1272
US	0.5900	3.90	0.1514	US	0.3000	3.27	0.0915	0.1214

Country names in shaded areas indicate bond indices, while those in unshaded areas indicate stock indices
ME – Mean Return
SD- Standard Deviation
SHP – Sharpe Ratio

For the first strategy, testing the tangency portfolio with no short sale constraints, we found the largest holding was in US equities. Although the UK and French equity indices each had a higher average Sharpe ratio over the period, the high correlation of the US index with the Canadian and Swiss indices led to a high weighting in US equities. The largest and most consistently negative weightings were for the Canadian equity index and US bond index. The Canadian index had the lowest returns among the equity indices both at the beginning and the end of the time period. The US bond index had the largest negative weight of all indices; this is not surprising since it provided the lowest returns in the portfolio and consequently had the third-lowest average Sharpe ratio. The biggest change in the portfolio over the testing period occurred in the Canadian bond index, whose weight dropped from a high of 178% in Q2 91 to 2.13% in our last observation. The largest overall change for a country was for the UK – its stock weight decreased 49% and its bond weight increased 46%. The most unexpected result was a weighting close to 100% for the French bond index – this index had the lowest Sharpe ratio of all. Its high correlations and superior returns relative to the UK and US bond indices apparently compensated for its higher volatility. This led to large over-weighting in the French bond index and large under-weighting in the UK and US bond indices.

When short sales were constrained in calculating the tangency portfolio (the second strategy), two distinct patterns emerged. The first strategy, which allowed short sales, led to a wide dispersion of assets, both positively and negatively, and the "noise" of short selling obscured these two patterns. First, among the equity indices, the largest holdings were in the indices with high Sharpe ratios. Second, among the bond indices, the opposite is true: the largest holdings were in the indices with low Sharpe ratios. The effect of the stock indices on the Sharpe ratio seems to be primarily felt in the numerator; i.e., excess returns. Apparently, the higher returns of the stock indices, relative to those of the bond indices, cause our optimization to include these indices in the portfolio. On the bond side, the primary effect of including the German and French indices seems to be felt by the denominator of the Sharpe ratio; i.e., standard deviation. Bond indices have low returns and standard deviations compared to stock indices, so inclusion of any of these indices in the optimal portfolio would be based on their contribution to lower volatility of the entire portfolio. Thus, all the indices work dynamically to provide the optimal portfolio.

The third strategy was to invest in the minimum variance portfolio with no short sale constraints. The results in this case were strikingly different from our tangency portfolios because our goal was to minimize variance rather than to optimize the risk-reward trade-off. US equities, which had a weighting of over 100% with the first strategy, had a negative weighting in this case. In contrast, US bonds moved from a negative to a positive weighting. The overall weighting of the bond indices was much higher than that of the stock indices because of their relatively low standard deviations. France's bond index had the second lowest average volatility of all indices, yet because of its low returns it had a low weighting in the portfolio. Germany, with the lowest volatility, had the highest average weight because of its higher average returns and low correlation with all other indices. Switzerland, with the sixth-highest average volatility, had a high weighting due to its high correlation with France and superior returns. On the equity side, the only positive weightings were given to the Canadian, German, and UK indices.

For the fourth strategy, there was no shift from one asset class to the other, around 90% of the weightings remained in bonds. The two largest holdings remained in US and German bonds with Canadian and German Stocks being the only equity holdings. The only change in equities on the positive side was the drop of UK equities from 5% to zero.

5 Observations with the Addition of Equally Weighted Portfolios

Table 8 shows the main results from the four strategies discussed above. In addition, the results from two equal-weighting strategies are shown. The first of these latter strategies was to assign equal weights to each of our fourteen indices, and the second was to assign equal weights to the US bond and equity indices.

Table 8 Summary of the performance of alternative strategies for 1983 Q1 – 2000 Q1: quarterly results

A. Short Sales Permitted			
Tangency portfolio	ME (%)		2.98
	SD (%)		7.35
	SHP		0.41
	Final index value	263	
Minimum variance	ME (%)		0.74
	SD (%)		3.11
	SHP		0.24
	Final index value	128	
B. Short Sales Precluded			
Tangency portfolio	ME (%)		2.00
	SD (%)		4.38
	SHP		0.45
	Final index value	197	
Minimum variance	ME (%)		1.06
	SD (%)		2.67
	SHP		0.39
	Final index value	144	
1.1 C. Equally Weighted			
All indices	ME (%)		1.94
(7.1429% each)	SD (%)		3.94
	SHP		0.49
	Final index value	195	
US only	ME (%)		2.33
(50% each)	SD (%)		3.58
	SHP		0.65
	Final index value	224	

ME – Mean Return, SD- Standard Deviation, SHP – Sharpe Ratio

As Table 8 shows, the portfolio with the highest Sharpe ratio was the equally weighted US strategy (0.6515). While this can be attributed to the excellent returns in US equities during our testing period, it is worth noting that the second highest Sharpe ratio belongs to the other equal-weighting strategy (0.4917). While these outcomes could call into question the validity of performing optimizations, they were probably due to our holding period. The superior returns of our tangency portfolio with no short sale constraints suggest that using optimizations to improve returns is a valid strategy. In addition, performing these optimizations on a monthly, weekly, or even daily basis could greatly improve performance.

When evaluating the performance of the minimum variance portfolios, using the Sharpe ratio as a performance measurement tool seems unfair. The minimum variance portfolios achieved the lowest volatility of all the portfolios and thus performed as they were designed to. We should note that by permitting short sales there

Allowing short selling greatly increased the returns and volatility of the tangency portfolio, and this affected the Sharpe ratio. For the minimum variance portfolio, allowing short sales slightly increased volatility and reduced returns leading to lowest Sharpe ratio of all of our portfolios.

The most striking outcome of our analysis was that none of the portfolios became bankrupt, which is not consistent with the results of Grauer (2004). Perhaps the inclusion of the bond indices provided better diversification, thus lowering the probability of bankruptcy.

Also, the absence of the risk-free asset would have created a tangency portfolio further down the efficient frontier. Certainly, the use of short sales did greatly increase the risk (standard deviation) of our tangency portfolio (7.35% vs. 4.38%). The choice of time period might also have made bankruptcy unlikely. Conceivably, I could have picked a time period that would only allow for positive returns regardless of the allocation.

6 Conclusion

In the first part of this paper, I have shown by using ex post data to determine optimal portfolios, with the Sharpe ratio as a performance measure, that there are "potential" gains to be made from international diversification. This held true in all three cases I considered: stock indices, bond indices, and a combination of the two. The gains from international diversification came from an increase in overall returns, albeit at the expense of increased volatility.

In the second part, I back-tested quarterly data to determine the end value, volatility, and returns of four different investment strategies. The optimal (i.e., tangency) portfolios held equity indices with high Sharpe ratios and bond indices with low Sharpe ratios. The equity indices primarily affected the numerator of the Sharpe ratio, while the bond indices primarily affected the denominator. In the minimum variance portfolios, a large overall weighting was assigned to the bond indices. The strategy with the highest ending index value was the tangency portfolio that allowed short sales. Conversely the strategy with the highest Sharpe ratio was the tangency portfolio that precluded short sales.

In part three, I showed that equally weighted portfolios out-performed the four optimized portfolios on a risk-adjusted basis (i.e., in terms of the Sharpe ratio). This may have been because a quarterly holding period was excessive, and a shorter holding period might result in better performance. As expected, our minimum variance portfolios exhibited the lowest volatility.

In addition, allowing short sales greatly increased the return and volatility of the tangency and minimum variance portfolios. I also found that those portfolios that permitted short sales exhibited lower Sharpe ratios than those that did not, the use of short sales greatly increased the volatility of those portfolios. Short sales contributed to an increase in returns but they also resulted in greatly increased volatility. In fact, the minimum variance portfolio, that allowed short sales, exhibited a lower end index value than the minimum variance portfolio, which precluded short sales. While a tangency portfolio strategy that allows short sales seems likely to greatly enhance returns, a note of caution is in order. As described in Grauer (2004), there is still the possibility of bankruptcy when using this strategy. The analysis discussed here yielded enticing results based on historical data, but the extrapolation of this data into the future is fraught with uncertainty.

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