COMPETITION AS A DISCOVERY PROCEDURE: HAYEK IN A MODEL OF INNOVATION

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Pedro Miguel Bento
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Name: **Pedro Miguel Bento**

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Examining Committee:

Chair: Steeve Mongrain
Associate Professor, Department of Economics

____
Alexander Karaivanov
Senior Supervisor
Assistant Professor, Department of Economics

____
David Andolfatto
Supervisor
Professor, Department of Economics

____
Fernando Martin
Supervisor
Assistant Professor, Department of Economics

____
Geoffrey Dunbar
Internal Examiner
Assistant Professor, Department of Economics

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Abstract

I model a mechanism through which competition can encourage innovation and growth. Although the often-cited 'Schumpeterian effect' of competition is to decrease the expected rents from an innovation, the competitive process also acts to uncover better ways of satisfying consumers and lowering costs. When firms are uncertain about the best direction in which to innovate, more competition results in better innovations. By endogenizing the level of competition and introducing this uncertainty into a general equilibrium model of vertical innovation, I show how this Hayekian effect of competition works against the Schumpeterian effect, resulting simultaneously in a positive relationship between competition and growth, and an inverted-U relationship between competition and firm-level innovation.

Keywords: competition; economics; endogenous growth; innovation; Hayek; economic development

Subject Terms: Competition; Technological innovations - Economic aspects; Economic development; Hayek

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Contents

List of Figures

1 Introduction

Schumpeter (1942) argued that the expectation of some temporary market power is a precondition for investment in innovation. Although his views were certainly more nuanced, a simplistic implication is that the expectation of more market power should translate into a greater level of innovation. Recent theoretical models of both vertical and horizontal innovation reinforce this view, sharing the conclusion that competition diminishes the expected rents to innovators and thus dampens innovation and growth.[1] At the same time, though, a burgeoning amount of evidence is accumulating that suggests a *positive* relationship between growth and the level of competition. This evidence suggests that competition is positively correlated with innovation and growth, at least up to some threshold, after which firm-level innovation may decline.[2]

The purpose of this paper is to model a mechanism through which competition can encourage growth, in a general equilibrium setting. I do this by building a simple model of vertical innovation, similar to the simple model developed by Aghion and Howitt (2005). I assume that entrepreneurs are uncertain about the relative value of each potential direction of innovation, until after an innovation has actually been introduced. I also endogenize the level of competition by introducing Bertrand competition. Once firms realize the value of their innovations, the best firm can capture the relevant market by pricing at its closest competitor's marginal cost. Contrary to the negative relationship between competition and growth implied by conventional endogenous growth models, such as Grossman and Helpman (1991), and Aghion and Howitt (1992, 2005), the model developed here generates a positive relationship between competition and growth, as well as an inverted-U relationship between competition and firm-level innovation.

It is the contention of this paper that certainty of the demand for new goods, and of the value of innovations in general, is the source of the consistent negative relationship between competition and growth in endogenous growth theory. Assuming that demand is known assumes away much of the value that competition brings to a society, leaving only the benefit of a higher allocative efficiency at some fixed point in time. A survey of the empirical evidence of the magnitude of the deadweight loss due to market power suggests that any benefit from greater static allocative efficiency is most likely overwhelmed by the supposed ill effects of greater competition on the

[1] For example, see Aghion and Howitt (1992) and Romer (1990).

[2] Nickell (1996), Blundell et al. (1999), and Dutz (2000) all find that growth is positively correlated with various measures of competition. Blundell et al. find that firm-level innovation may be falling with competition while industry-wide growth is increasing. Aghion et al. (2005) provide evidence of an inverted-U relationship between firm-level innovation and competition.

returns to research.[3]

What else of value does competition bring to the table? In Hayek's words, competition is (2002, p.9):

> a procedure for discovering facts which, if the procedure did not exist, would remain unknown or at least would not be used.

In a static setting, the process of competition tends to result in higher profits for more efficient producers, and losses for inefficient producers. Equilibrium prices start to emerge, or are *discovered*, as a result of the competitive process. In a dynamic setting, the value of technical efficiency becomes more ambiguous. Schumpeter's (1942, p.85) "perennial gale of creative destruction" destroys not just the profits of incumbents, but the very information that firms previously relied on to determine their 'optimal' size, product space, and organizational structure. As technology advances and consumer behaviour adapts to new conditions, yesterday's efficient behaviour becomes today's mistakes. Over time, static efficiency becomes less important, and profits tend toward those producers who best anticipate changes in the economy.

Why are entrepreneurs so uncertain about the value of innovations? When firms set out to improve a product, there is no one-dimensional measure of quality that they can progress along. Lancaster (1966) and others have explicitly modeled the demand for a good as being derived from the demand for the various characteristics of that good. A consumer's willingness to pay for a car depends on how comfortable it is, its appearance, level of safety, durability, and a host of other characteristics, not to mention the characteristics of other goods that are available. For example, a consumer may choose safety over style in his car, and then purchase more style through other goods, like clothing and furniture.

Likewise, a new way of producing a good or service may conserve on one factor while increasing reliance on another, making the future prices of inputs important. A more technically efficient assembly line may result in duller work for employees, increasing the wage rate required to hold on to them.

Entrepreneurs, then, are constantly making informed guesses about how to change their product or production process. They may decide to invest in R&D to improve

[3]Harberger (1954) pioneered the empirical measurement of dead weight loss due to various distortions. Jenny (1983) provides a partial survey of the subsequent literature. Estimates of the deadweight loss due to monopoly power in the U.S. economy range from about 0.1% to 8% of G.N.P. To justify my characterization of these magnitudes as inconsequential, imagine that the U.S. stopped enforcing intellectual property rights, effective immediately. Further, imagine this reform would result in an increase in the level of output of 8%, due to efficiency gains, as well as a decrease in subsequent growth from 3% to 2% - an estimated decrease that seems conservative, given most economists' confidence in the efficacy of intellectual property rights in spurring innovation. In such a senario, the reformed economy would perform better for just 15 years before permanently falling below its previous potential.

their product or production process in what they perceive to be the best direction. If this research succeeds, however, firms must still incur further costs before realizing the value of their new product or process. These costs are made necessary by both the uncertainty inherent in a market and any regulations that govern the behaviour of firms.

The market portion of these costs may flow from the necessity of holding an inventory for a length of time, the need to disseminate information about the new product to consumers, or the desire to provide sales training - all before demand is realized. Similarly, new machinery must be purchased and employees trained before a new production process can be implemented.

An obvious example of regulatory costs that must be incurred before introducing a new good is the extensive safety and efficacy testing required by the FDA (in the U.S.) before a new drug is allowed to be marketed.[4] A new restaurant must obtain licenses and a new car requires safety testing. In addition to these costs, new products must often include a plethora of mandatory features, such as airbags, seatbelts and catalytic converters in automobiles, various ingredient and warning labels on food and appliances, and Vitamin D in milk.

Regulatory compliance can be just as burdensome for process innovations. Labour and wage restrictions often increase the transitional cost of changing production processes, and a process innovation that involves a change in the size of a firm may run afoul of competition policy. Again, the value of a new product or process only becomes fully known after all of these costs are incurred.

In the next section, I provide a brief review of related literature. I then present a general equilibrium model of cost-reducing innovations that features this uncertainty. When firms decide to improve their production process, they must first choose the direction in which to innovate. One firm might try to make the process more routine, while another may try to incorporate a new computer network. The mechanics and implications of the model are equivalent to those of a model of quality improving innovations.[5] In such a model, one firm might improve the speed of a car more than its safety, while another firm may eschew both in favour of better handling or a larger carrying capacity.

By combining this uncertainty with an endogenously determined level of competition, I show how a model of vertical innovation can generate a positive relationship between competition and growth.

I start by modeling an economy where the level of investment in each research

[4] Klein and Tabarrok (ongoing) provide a detailed history of U.S. Food and Drug Administration regulations.

[5] In Appendix A.5, I explain how a model of cost-reducing innovations can be adapted to one of quality-improving innovations.

direction is exogenously fixed, isolating and making explicit the Hayekian effect of competition on growth. I then extend the model by endogenizing each firm's research investment decision, thus allowing for both a Hayekian and Schumpeterian effect in the model. I show that the Schumpeterian effect can dominate in a firm's research decision, but the Hayekian effect always dominates at the industry level. Sections 5 and 6 contain comparative statics and a comparison of the model's results with those of the empirical literature, respectively. The last section concludes.

2 Related Literature

Economists have explored two general avenues through which productivity growth can occur as a result of research investments by profit-maximizing firms. The first is through the proliferation of product varieties, where the introduction of a new variety carries a fixed cost, but the addition of new products leads to a lower cost of final output (or utility). This is a model of horizontal innovation, exemplified by Romer (1990). The second is through the introduction of higher quality products, which displace older products of the same type. This is a model of vertical innovation, or a Schumpeterian growth model, developed by Grossman and Helpman (1991) and Aghion and Howitt (1992), among others.

In these models, the ratio of marginal cost over price is exogenously fixed. A comparison of the steady-state growth rates that result as this ratio is increased makes it clear that competition is unambiguously bad for growth in both of these workhorse models of endogenous growth. A number of empirical studies, however, have found evidence that more competition is actually associated with higher growth. Nickell (1996) uses data on 670 U.K. manufacturing firms through the 1970s and 1980s to test the relationship between the level of competitive rivalry in an industry, and its rate of productivity growth. He finds that lower profit margins are associated with higher growth in an industry.

Blundell et al. (1999) use U.K. cross-industry data and firm-level data within the U.K. pharmaceutical industry to test the relationship between the number of citation-weighted patents, and both market-share and concentration. The authors find that firm-level innovation is *positively* related to market-share, but industry-wide innovation is *negatively* correlated with concentration.

Dutz and Hayri (2000) test the relationship between domestic competition and economic growth across countries, using data from over 100 countries from 1986 to 1995. Controlling for the effects of trade liberalization, the authors report a positive relationship between competition and economic growth.

More recently, Aghion et al. (2005) use an unbalanced panel of approximately 300 firms listed on the London Stock Exchange from 1970 to 1994 to test the importance of the Schumpeterian effect in the relationship between firm-level innovation and competition. They use the average number of citation-weighted patents per firm in an industry as the dependent variable, and one minus the Lerner Index (price minus marginal cost over price) as their measure of competition. Allowing and testing for a nonlinear relationship, they report an inverted-U relationship between firm-level innovation and growth. According to their results, an increase in competition is associated with an increase in innovative output per firm when competition is low, but the correlation becomes negative after some threshold level of competition.

To my knowledge, only two attempts to model this inverted-U relationship have been made. The first is by Aghion et al. (2005). They model an economy with a continuum of intermediate sectors, each structured as a duopoly. Each pair of firms is characterized by the leader's level of technology and the gap between its own technology and its competitor's. Competition is measured as the inverse of the ability to collude when the two firms are 'neck-and-neck', with respect to their level of technology. The model implies that less ability to collude results in more research being done by neck-and-neck firms, in an attempt to escape competition. When the duopolists are far from each other, however, less ability to collude results in less research being done by the lagging firm, since there is less reward for catching up. A very low level of collusion results in an industry exhibiting a greater tendency to be in a leader-follower situation, where even less collusion reduces innovation. A relatively high level of collusion, on the other hand, results in a greater tendency for an industry to be in a neck-and-neck situation, where less ability to collude would *increase* the average level of innovation. This is the idea behind the inverted-U result of the model.

The level of collusion in an industry, however, is a troublesome way to define competition. While the effect of differing levels of collusion on innovation is interesting in itself, the question remains as to whether or not the model can explain the connection between firms' innovative activity and the Lerner Index. One might presume that the model is representative of an economy with more firms, where the level of collusion could be redefined more generally as an innovator's markup over marginal cost. But then the assumption that more competition leads to a lower markup for innovating followers, but not for innovating leaders, would no longer seem plausible. Dropping this assumption would remove any positive effect of competition on innovation, as competition would no longer be increasing the relative value of innovating for neck-and-neck firms.

The second attempt to model the inverted-U relationship is by Swann (2007), who builds on a model of vertical innovations where the quality of an intermediate good is assumed to be an increasing function of total industry R&D. She further assumes a parameterized amount of duplication in research, as well as spillovers from simultaneous research. She goes on to show that for certain levels of duplication, the level of growth may first increase, and then decrease, in the number of firms. Swann's model keeps the number of firms exogenous, though, so is unable to determine the relationship between endogenously determined levels of competition and resulting growth rates, as I do here. The beneficial effect of competition through spillovers is also assumed in the model, and so left unexplained.

3 The Simple Model

3.1 Setup

Consider a closed economy where time is discrete, indexed by t=0,1,2... There are L_t individual consumers, each supplying one unit of labour inelastically and allocating their income between consumption and savings in each period. I assume no population growth, so $L_t = L$. A representative final goods firm competitively produces the final good, y, using an intermediate good, x, according to $y = x^{\alpha}$[6], $\alpha \in (0,1)$. y is also the numeraire. The assumption of a competitive final goods market ensures that the marginal product of the intermediate good, $\alpha x^{\alpha-1}$, will be equal to its price, P, in equilibrium.

In each period, e_t intermediate firms can try a new and improved production process in an effort to produce x at a lower cost. Introducing a new process in period t requires an investment of Z_t in period $t-1$. Firms finance this investment by issuing equity to consumers. I set Z_t equal to zy_{t-1}, a constant fraction of the previous period's output of the final good.

Once these new processes are introduced in period t, and the cost of introducing them is sunk, the productivity of each process is revealed. Once productivity is revealed, each firm *can* produce according to $x_{it} = A_{it}L_{it}$, where x_{it} is the quantity of x produced by the i-th intermediate firm, A_{it} is its productivity, and L_{it} is the amount of labour used by the firm. The firm with the highest A, however, can capture the entire intermediate market by pricing at the marginal cost of its closest competitor, $w_t/A_{t[2]}$, where w_t is the wage rate, and $A_{t[2]}$ is the productivity of the second-best firm. In equilibrium, then, only the best firm will produce the intermediate good, although other firms stand ready to enter if the best firm sets too high a price. Henceforth, $A_{t[1]}$ will represent the productivity of the best firm.

The eventual 'winner', then, will earn the following operating profits in period t;

$$\left(P_t - \frac{w_t}{A_{t[1]}}\right) x_t.$$[7]

$w_t/A_{t[1]}$ is the marginal cost of the winning firm, who will set P_t at $w_t/A_{t[2]}$ in order

[6]Implicit in this production function is an input of some fixed factor like land, so that $f(x) = F(x, land)$. With free entry into final good production, as well as constant returns to scale, the value marginal product of each factor will be equal to its respective price. I normalize this fixed factor to 1, in order to simplify the model.

[7]I am implicitly ignoring the possibility that the realized productivity of the second-best firm, or even the best firm, is lower than the productivity of the previous period's best firm, $A_{t-1[1]}$, by imposing a constraint on firms to use only new technologies in production. Although the parameters of the model can be chosen to make the probability of this outcome close to zero, the relaxation of this constraint may be of interest in future research.

to capture the intermediate market. This implies that operating profits in period t will be

$$\left(\frac{1}{A_{t[2]}} - \frac{1}{A_{t[1]}}\right) w_t x_t,$$

or

$$\left(\frac{A_{t[1]}}{A_{t[2]}} - 1\right) w_t L,^8$$

since $x_t = A_{t[1]} L$.

Since profits are linear in labour, the winning firm's labour decision is trivial - the firm will simply employ all the workers in the economy. Before any investment is made, however, firms must decide whether or not to enter the market and try a new process. Each firm faces an expected discounted profits function equal to the eventual winner's expected discounted profits multiplied by the probability of being the winner. I assume each firm is identical *ex ante*, with no experience or reputation giving any one firm an advantage over another. With all firms facing an identical chance of being the winner, the probability of winning is $1/e_t$, where e_t is the number of processes introduced. Expected discounted profits in period $t-1$ for firm-i, then, are equal to

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = (1/e_t) \cdot E_{t-1}\left[\left(\frac{A_{t[1]}}{A_{t[2]}} - 1\right)\frac{w_t L}{R_t}\right] - Z_t,$$

or

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = (1/e_t) \cdot E_{t-1}\left[\left(\frac{A_{t[1]}}{A_{t[2]}} - 1\right)\frac{w_t L}{R_t}\right] - zA^{\alpha}_{t-1[1]}L^{\alpha},$$

where R_t is the endogenous gross rate of interest, and $Z_t = zy_{t-1} = zA^{\alpha}_{t-1[1]}L^{\alpha}$.

Free entry into the intermediate sector implies that firms will introduce new processes until $E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = 0$. As a consequence,

$$E_{t-1}\left[\left(\frac{A_{t[1]}}{A_{t[2]}} - 1\right)\frac{w_t L}{R_t}\right] = e_t z A^{\alpha}_{t-1[1]}L^{\alpha} \text{ in equilibrium.} \qquad (1)$$

Bertrand competition in the intermediate market leads to a price of $P_t = w_t/A_{t[2]}$, as explained above. At the same time, the competitive final good producer will choose x_t such that $P_t = \alpha x_t^{\alpha-1}$. In equilibrium, then, it must be the case that

$$\alpha x_t^{\alpha-1} = w_t/A_{t[2]},$$

or

$$w_t = \alpha A_{t[2]} x_t^{\alpha-1} = \alpha A_{t[2]} A^{\alpha-1}_{t[1]} L^{\alpha-1}.$$

[8] This is a departure from previous endogenous growth models. By endogenizing the productivity of competitors and tying this productivity to the winning firm's markup, I endogenize the measured level of competition.

Substituting the above expression for w_t, equation (1) becomes

$$\alpha L^{\alpha} \cdot E_{t-1}\left[\left(\frac{A_{t[1]}}{A_{t[2]}}-1\right)\frac{A_{t[2]}A_{t[1]}^{\alpha-1}}{R_t}\right] = e_t z A_{t-1[1]}^{\alpha} L^{\alpha},$$

or

$$E_{t-1}\left[\frac{A_{t[1]}-A_{t[2]}}{A_{t[1]}^{1-\alpha}R_t}\right] = \frac{e_t z A_{t-1[1]}^{\alpha}}{\alpha}. \tag{2}$$

All that remains is to use the consumer's problem to pin down R_t, and to define $A_{t[1]}$ and $A_{t[2]}$.

3.2 Innovation

When making their entry decision, all (potential) intermediate firms can costlessly imitate the best firm of the prior period, and so they try to improve on the best. For each firm, i, the implementation of a new production process results in a productivity parameter of $A_{it} = A_{t-1[1]} \cdot h_i$, where h_i is a random draw from $U(0, \lambda)$. λ can be thought of as the level of research invested in developing a new production process, assumed to be constant over time and across firms.[9]

If $h_{[1]}$ and $h_{[2]}$ are defined as the highest and second highest realized values of e draws from $h \sim U(0, \lambda)$, respectively, then the joint density function of $h_{[1]}$ and $h_{[2]}$, conditional on e, is

$$f(h_{[1]} = v, h_{[2]} = u | e) = \frac{e(e-1)u^{e-2}}{\lambda^e}.^{10} \tag{3}$$

Finally, the productivities of the best and second best firms are

$$A_{t[1]}(e_t) = A_{t-1[1]} \cdot h_{t[1]}(e_t), \text{ and} \tag{4}$$

$$A_{t[2]}(e_t) = A_{t-1[1]} \cdot h_{t[2]}(e_t), \text{ respectively.} \tag{5}$$

3.3 Consumer's Problem

Consumers are infinitely lived, value only consumption (c_t), and have a constant discount rate, β. Each consumer is endowed with one unit of labour, all of which is supplied inelastically. The consumer's problem, then, is to take w_t, π_{tk}[11], and R_t as

[9] Research investment will be made endogenous in Section 4.
[10] This is derived in Appendix A.1.1.
[11] π_{tk} denotes income from the rental of the fixed factor to the final good firm. I include it here for completeness.

given, and to choose a stream of savings, $\{s_t\}_{t=0}^{t=\infty}$, to

$$\max_{\{s_t\}_{t=0}^{t=\infty}} \sum_{t=0}^{\infty} \beta^t \cdot E_0 \left[\ln(c_t)\right], \quad \text{s.t. } c_t = w_t + s_{t-1}R_t - s_t + \pi_{tk},$$

where the only vehicle for savings is the purchase of equity in intermediate good firms, earning a gross return of R_t.[12]

The resulting Euler equation is $\frac{1}{c_{t-1}} = \beta \cdot E_0 \left[\frac{R_t}{c_t}\right]$. To solve for a balanced-growth-path equilibrium, I assume $\frac{c_t}{c_{t-1}} = \frac{y_t}{y_{t-1}}$, which is equal to $\frac{A^{\alpha}_{t[1]}L}{A^{\alpha}_{t-1[1]}L} = h^{\alpha}_{t[1]}$. This implies an equilibrium condition of

$$\frac{1}{\beta} = E_0 \left[\frac{R_t}{h^{\alpha}_{t[1]}}\right].$$

Using equation (3), the joint-density function, to expand the expectations operator,[13] this condition becomes

$$\frac{1}{\beta} = \frac{e_t(e_t-1)}{\lambda^{e_t}} \int_0^{\lambda} \int_0^{v} \frac{R_t u^{e_t-2}}{v^{\alpha}} du dv,$$

or

$$\frac{\lambda^{e_t}}{e_t(e_t-1)} = \int_0^{\lambda} \int_0^{v} \frac{\beta R_t u^{e_t-2}}{v^{\alpha}} du dv, \forall t. \tag{6}$$

3.4 Competitive Equilibrium

Using equations (4) and (5), the productivities of the best and second-best firms, the zero-expected-discounted-profits condition from (2) can now be written as

$$E_{t-1} \left[\frac{h_{t[1]} - h_{t[2]}}{h^{1-\alpha}_{t[1]} R_t}\right] = \frac{e_t z}{\alpha}.$$

The expectations operator can be expanded using equation (3), the joint-density function, to get

$$\frac{e_t(e_t-1)}{\lambda^{e_t}} \int_0^{\lambda} \int_0^{v} \frac{u^{e_t-2}(v-u)}{v^{1-\alpha} R_t} du dv = \frac{e_t z}{\alpha},$$

or

$$\frac{\lambda^{e_t}}{e_t(e_t-1)} = \int_0^{\lambda} \int_0^{v} \frac{\alpha u^{e_t-2}(v-u)}{e_t z v^{1-\alpha} R_t} du dv, \forall t. \tag{7}$$

[12]Each consumer diversifies his investment over all intermediate firms, so that R_t is the gross return from the entire investment, equal to the return on the investment in the winning firm divided by the number of firms.

[13]It is necessary to use the *joint*-density function, because R_t will presumably be a function of the winner's markup, which is a function of the highest and second-highest draws from the distribution.

The *ex post* realized value of R_t can now be solved for by equating the right-hand sides of equations (6) and (7).

$$\int_0^\lambda \int_0^v \frac{\beta R_t u^{e_t-2}}{v^\alpha} du dv = \int_0^\lambda \int_0^v \frac{\alpha u^{e_t-2}(v-u)}{e_t z R_t v^{1-\alpha}} du dv,$$

which implies

$$\frac{\beta R_t h_{t[2]}^{e_t-2}}{h_{t[1]}^\alpha} = \frac{\alpha h_{t[2]}^{e_t-2}(h_{t[1]} - h_{t[2]})}{e_t z R_t h_{t[1]}^{1-\alpha}}.$$

Solving for the gross rate of interest,

$$R_t = \left(\frac{\alpha(h_{t[1]} - h_{t[2]})}{\beta e_t z h_{t[1]}^{1-2\alpha}} \right)^{1/2}. \tag{8}$$

Finally, the number of new processes introduced in period t, e_t, is determined by substituting the realized value of R_t from (8), into equation (7);

$$\frac{\lambda^{e_t}}{e_t(e_t-1)} = \int_0^\lambda \int_0^v \frac{\alpha u^{e_t-2}(v-u)}{e_t z v^{1-\alpha}} \left(\frac{\beta e_t z v^{1-2\alpha}}{\alpha(v-u)} \right)^{1/2} du dv,$$

or

$$\frac{\lambda^e}{(e-1)} \left(\frac{z}{\alpha\beta e} \right)^{1/2} = \int_0^\lambda \int_0^v \frac{u^{e-2}(v-u)^{1/2}}{v^{1/2}} du dv.$$ [14]

Integrating the right-hand side results in the competitive equilibrium condition;

$$\left(\frac{ze}{\alpha\beta} \right)^{1/2} = \frac{(e-1)\Gamma(3/2)\Gamma(e-1)}{\Gamma(e+1/2)}, \tag{9}$$

which can be solved for $e(z, \alpha, \beta)$.[15]

Figure 1 plots e, the number of innovations in each period, as a function of z, the cost of introducing an innovation ($\alpha = 3/4$, $\lambda = 2$, $\beta = 0.96$). As expected, the number of innovations introduced each period is decreasing in z. Keeping this in mind, it should be clear from equation (9) that e is increasing in both α and β - when either x becomes more important for final good production or lenders become more patient, the level of investment in innovation should be expected to increase. It is also clear that the choice of e is independent of λ, the upper bound on the distribution of possible values for each innovation. This is due to the endogenous rate of interest, which increases with λ to exactly offset the effect of λ on expected profits. While a higher upper bound on the potential value of innovations encourages higher growth,

[14]Time subscripts have now been dropped, since the choice of e is independent of time.

[15]The Gamma function, $\Gamma(a)$, is defined as $\int_0^\infty exp(-i) \cdot i^{a-1} di, \forall a > 0$. Many of the mathematical calculations and all graphs that follow were produced using Maple 10, copyright (c) Maplesoft, a division of Waterloo Maple Inc. 1981-2007.

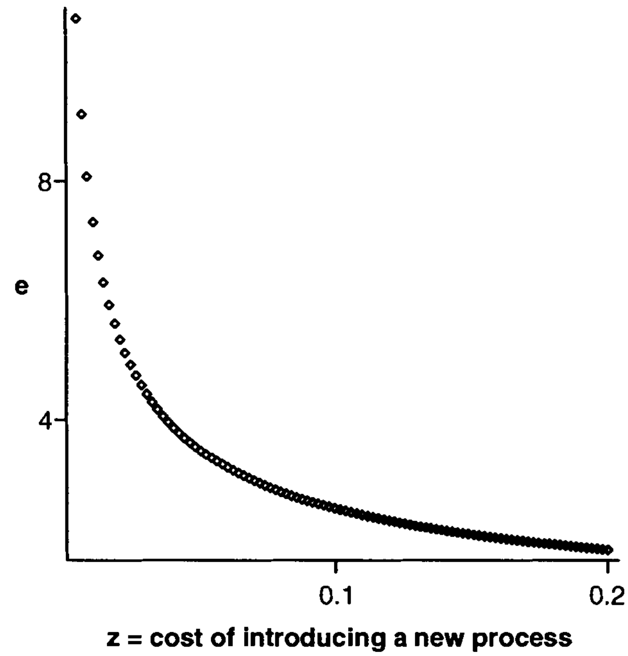


Figure 1: e(z) = Number of Innovations

it does so solely through an increase in the average value of each innovation, and not through a change in the number of innovations introduced.[16]

The expected growth rate of the economy is

$$E[g] = E\left[\frac{y_t}{y_{t-1}} - 1\right] = E\left[h_{[1]}^{\alpha} - 1\right] = \frac{e(e-1)}{\lambda^e}\int_0^{\lambda}\int_0^{v} u^{e-2}v^{\alpha}dudv - 1, \text{ or}$$

$$E[g] = \frac{e\lambda^{\alpha}}{e+\alpha} - 1, \tag{10}$$

which is monotonically increasing in e, the number of innovations each period.

Following Aghion et al. (2005), I use one minus the Lerner Index as the measure of competition when analyzing the relationship between competition and growth.

$$E[1 - Lerner] = E\left[1 - \left(\frac{P - MC}{P}\right)\right] = E\left[\frac{MC}{P}\right].$$

[16]If the interest rate were exogenous, equation (7) could be solved for an equilibrium number of innovations equal to $e = \frac{\lambda^{\alpha}}{Rz}$, which would be increasing in λ.

In this model,

$$E\left[\frac{MC}{P}\right]=E\left[\frac{A_{[2]}}{A_{[1]}}\right]=\frac{e(e-1)}{\lambda^{e}}\int_0^{\lambda}\int_0^{v}\frac{u^{e-1}}{v}dudv,$$

or

$$E\left[1-Lerner\right]=\frac{e-1}{e} \tag{11}$$

Figure 2 plots E [growth rate] against $E\left[1-Lerner\right]$, both as functions of z ($\alpha=3/4$, $\lambda=2$, $\beta=0.96$).

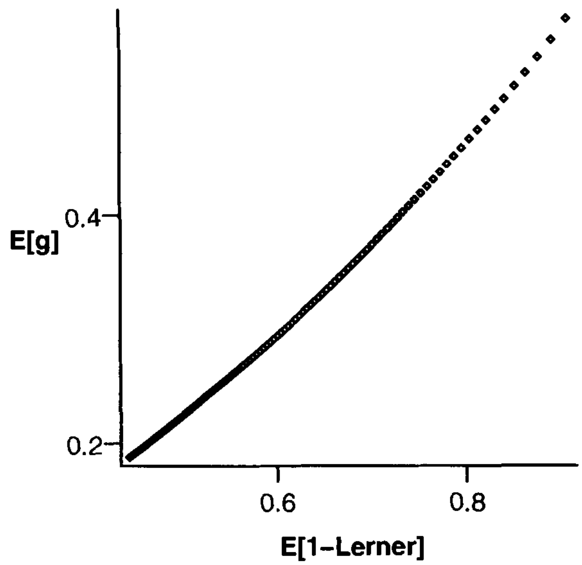


Figure 2: Growth and Competition

This unambiguously positive relationship between competition and growth is a direct result of both the uncertainty of the relative value of innovations, and the assumption of a fixed distribution of values for potential innovations. When this later assumption is relaxed, and the distribution made a function of research investment, more competition could presumably lead to less research by each firm, via the Schumpeterian effect discussed earlier, and potentially a lower expected growth rate. Section 4 will analyze this further.

For this model to produce the *negative* relationship between competition and growth found in current models of both vertical and horizontal innovation[17], the *ex*

[17]For examples, see Aghion and Howitt (1992) and Romer (1990), respectively.

post markup over marginal cost would have to be disassociated with the number of firms and made exogenous, as it is in current models. This would result in a model where new innovations would still be introduced until expected discounted profits were equal to zero, but the number of firms would be indeterminate, and only the number of new innovations could be solved for. The level of innovation would then depend on the *ex post* markup. If competition is measured by this exogenous markup (as is common), then a lower markup would result in less innovation. The result would be a negative relationship between competition and growth. Although such an assumption may seem unrealistic, it might hold for industries with very broad patents. In such an industry, the winning firm could ignore the possibility of its competitors having access to similar levels of technology (or in the case of a quality-improving model of innovation, it could ignore the possibility of its competitors offering close substitutes).

3.5 Social Planner

Schumpeter (1942) and Arrow (1962) both observed that the inability of innovators to appropriate the full benefit of an innovation should tend to result in a lower than optimal rate of innovation. Recent Schumpeterian growth models, however, have shown that equilibrium growth rates can be higher than optimal. To compare the results of this model with the current literature, I derive the optimal rate of growth for this economy.

The optimal rate of growth can be found by solving the social planner's problem. The social planner chooses the number of innovations for each period, $\{e_t\}_{t=0}^{t=\infty}$, to maximize expected utility;

$$\max_{\{e_t\}_{t=0}^{t=\infty}} \sum_{t=0}^{\infty} \beta^t \cdot E_0 \left[\ln(c_t)\right],$$

subject to the resource constraint in each period t;

$$c_t = y_t - e_{t+1}Z_t = y_t(1 - e_{t+1}z),$$

or

$$c_t = A_{t[1]}^{\alpha} L^{\alpha}(1 - e_{t+1}z).$$

This problem is equivalent to

$$\max_{\{e_t\}_{t=0}^{t=\infty}} \sum_{t=0}^{\infty} \beta^t \cdot E_0 \left[\alpha \ln(A_{t[1]}(\{e_s\}_{s=1}^{s=t})) + \alpha \ln(L) + \ln(1 - e_{t+1}z)\right],$$

or

$$\max_{\{e_t\}_{t=0}^{t=\infty}} \sum_{t=0}^{\infty} \beta^t \cdot E_0 \left[\alpha \ln(A_0) + \alpha \left(\sum_{s=1}^{t} \ln(h_{s[1]}(e_s)) \right) + \alpha \ln(L) + \ln(1 - e_{t+1} z) \right].$$

Since the optimal choice of e_t is independent of A_0, L, and $e_s, \forall s \neq t$, the optimal e is independent of time. The maximization problem is thus equivalent to

$$\max_{e} \ \alpha E_0 \left[\sum_{s=1}^{\infty} \beta^s \ln(h_{[1]}(e)) \right] + \ln(1 - ez).$$

Using the joint-density function to expand the expectations operator[18], the problem becomes

$$\max_{e} \ \alpha \left(\sum_{s=1}^{\infty} \beta^s \right) \frac{e(e-1)}{\lambda^e} \int_0^{\lambda} \int_0^{v} u^{e-2} \ln(v) du dv + \ln(1 - ez),$$

or

$$\max_{e} \left(\frac{\alpha\beta}{1-\beta} \right) \left[\ln(\lambda) - \frac{1}{e} \right] + \ln(1 - ez),$$

The resulting first order condition is

$$\frac{\alpha\beta}{e^2(1-\beta)} - \frac{z}{(1-ez)} = 0,$$

or

$$e^2 z(1-\beta) + ez\alpha\beta - \alpha\beta = 0.$$

The optimal number of innovations is

$$e = -\frac{\alpha\beta}{2(1-\beta)} + \frac{\left[(z\alpha\beta)^2 + 4z\alpha\beta(1-\beta)\right]^{1/2}}{2z(1-\beta)}. \tag{12}$$

Figure 3 compares the competitive equilibrium and optimal expected growth rates ($\alpha = 3/4$, $\lambda = 5$, $\beta = 0.96$).

As in Aghion and Howitt's (1992) model, the competitive equilibrium can exhibit higher growth than is optimal. In a Schumpeterian model, this can occur because the benefit to researching firms from capturing a market includes the value of appropriating all rents previously flowing to the incumbent, while the benefit to society from their innovation is only the incremental value of the improvement. There is a similar 'business stealing' effect here, only the effect of entry on the expected rents

[18]One could first integrate the joint-density function over v to get the marginal-density function $f(h_{[1]} = v|e) = \frac{e}{\lambda} \int_0^{\lambda} v^{e-1} dv$. Since the choice of method is irrelevant, I have chosen to avoid introducing another density function.

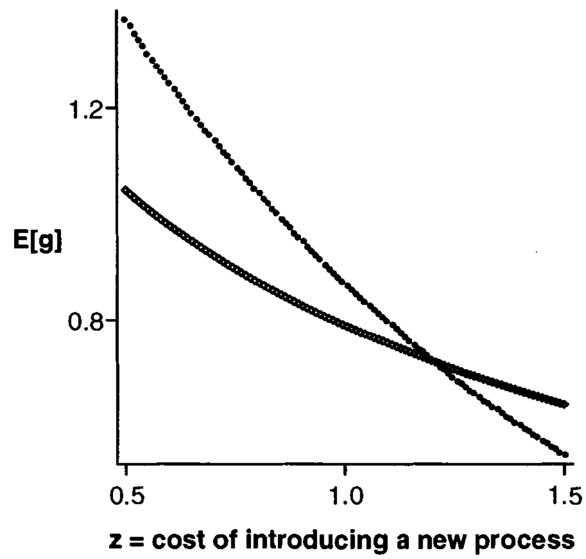


Figure 3: Equilibrium and Optimal Growth

of *simultaneous* entrants are what a potential entrant disregards. As is common in the patent-race literature, firms continue to enter after expected discounted profits are maximized, until these profits are driven to zero.[19] This result, however, holds only at very high levels of z, the cost of introducing an innovation. For any plausible levels of z, the expected level of growth in equilibrium will be less than optimal.

[19]See Reinganum (1989) for a discussion of the patent-race literature.

4 The Extended Model

As discussed in the previous section, the assumption that the distribution of the possible values of an innovation is fixed may be driving the unambiguously positive relationship between competition and growth. I relax this assumption by endogenizing each entrant's research decision, thus allowing for a Schumpeterian effect on firm-level research in the model.

I now assume that each firm can decide how much to invest in research, in order to increase the expected magnitude of its innovation. In terms of the simple model, firms can now choose their own λ. Once the value of a new production processes has been revealed, firm-i can sill produce according to $x_{it} = A_{it}L_{it}$, and A_{it} is still defined as $A_{t-1[1]} \cdot h_{it}$. But now, $h_{it} \sim U(0, n_{it}^{\theta})$, where n_{it} is the level of research purchased by firm-i for a period t innovation, and $\theta \in (0,1)$. I assume no spillovers from simultaneous research, although intertemporal spillovers still exist. The cost of one unit of research is M_t, which I set at my_{t-1}, a fixed fraction of output in period t-1, similar to Z_t. Each entrant must therefore finance an investment equal to $(z + mn_{it})y_{t-1}$ in period t-1 in order to introduce a new production process in period t.

Expected discounted profits in period t-1 for firm-i are now

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = \text{Probability}[i \text{ wins}] \cdot E_{t-1}\left[\left(\frac{A_{t[1]}}{A_{t[2]}} - 1\right)\frac{w_t L}{R_t}\right] - y_{t-1}(z + mn_{it}),$$

which are identical to expected discounted profits in the simple model, except for the change in required financing.

Since $w_t = \alpha A_{t[2]} A_{t[1]}^{\alpha-1} L^{\alpha-1}$, $y_{t-1} = A_{t-1[1]}^{\alpha} L^{\alpha}$, and $A_{it} = A_{t-1[1]} h_{it}$, this function can be rewritten as

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = \Pr[i \text{ wins}] \cdot \alpha A_{t-1[1]}^{\alpha} L^{\alpha} \cdot E_{t-1}\left[\frac{h_{t[1]} - h_{t[2]}}{h_{t[1]}^{1-\alpha} R_t}\right]$$
$$-A_{t-1[1]}^{\alpha} L^{\alpha}(z + mn_{it}), \tag{13}$$

where $\Pr[i \text{ wins}] = \Pr[h_{it} > h_{jt} | e_t, n_{it}, n_{jt}, \forall j \neq i]$, since the probability that firm-i will capture the market is equal to the probability that it has the highest productivity of all entrants. Although the assumption of identical firms will ensure that the probability of winning will still be $1/e_t$ in equilibrium, each firm can improve its chance of winning (as well as increase the expected value of its innovation) by increasing its level of research.

Firm-i's probability of being the most productive firm is

$$\Pr[h_{it} > h_{jt} | e_t, n_{it}, n_{jt}, \forall j \neq i] = \frac{n_{it}^{\theta(e_t-1)}}{e_t \left(\prod_{j \neq i}^{e_t-1} n_{jt}^{\theta} \right)},^{20} \tag{14}$$

and the joint-density function for $h_{t[1]}$ and $h_{t[2]}$ is

$$f(h_{t[1]} = v, h_{t[2]} = u | e_t, n_{t[1]}, n_{t[2]}) = \frac{e_t(e_t-1)u^{e_t-2}}{n_{t[2]}^{\theta(e_t-1)} n_{t[1]}^{\theta}},^{21} \tag{15}$$

where $n_{t[1]}$ is the level of research by the (*ex post*) best firm, and $n_{t[2]}$ the level of the second-best firm.

Using equations (14) and (15), expected discounted profits for firm-i from (13) become

$$\left(\frac{n_{it}^{\theta(e_t-1)}}{e_t \left(\prod_{j \neq i}^{e_t-1} n_{jt}^{\theta} \right)} \right) \alpha A_{t-1[1]}^{\alpha} L^{\alpha} \left(\frac{e_t(e_t-1)}{n_{t[2]}^{\theta(e_t-1)} n_{it}^{\theta}} \right) \int_0^{n_{it}^{\theta}} \int_0^{v} \frac{u^{e_t-2}(v-u)}{v^{1-\alpha} R_t} du dv$$

$$-A_{t-1[1]}^{\alpha} L^{\alpha}(z + m n_{it}),$$

or

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = \frac{\alpha A_{t-1[1]}^{\alpha} L^{\alpha} n_{it}^{\theta(e_t-2)}(e_t-1)}{n_{t[2]}^{\theta(e_t-1)} \left(\prod_{j \neq i}^{e_t-1} n_{jt}^{\theta} \right)} \int_0^{n_{it}^{\theta}} \int_0^{v} \frac{u^{e_t-2}(v-u)}{v^{1-\alpha} R_t} du dv$$

$$-A_{t-1[1]}^{\alpha} L^{\alpha}(z + m n_{it}). \tag{16}$$

To solve the model for a balanced-growth-equilibrium, I first turn to the consumer's problem. As in the simple model, the Euler equation is

$$\frac{1}{c_{t-1}} = \beta \cdot E_0 \left[\frac{R_t}{c_t} \right].$$

From the Euler equation, the following condition emerges;[22]

$$\frac{n_{t[2]}^{\theta(e_t-1)}}{(e_t-1)} = \int_0^{n_{t[1]}^{\theta}} \int_0^{v} \frac{\beta e_t u^{e_t-2} R_t}{n_{t[1]}^{\theta} v^{\alpha}} du dv, \forall t. \tag{17}$$

To get the zero expected-discounted-profits condition, I set equation (16) to zero,

[20]This probability is derived in Appendix A.2.

[21]This function is derived in Appendix A.1.2. Note that the probability of winning is still equal to $1/e_t$ if $n_{it} = n_{jt}, \forall i, j \in [0, e_t]$.

[22]This is derived in Appendix A.3.1.

which results in

$$\frac{\alpha n_{it}^{\theta(e_t-2)}(e_t-1)}{n_{t[2]}^{\theta(e_t-1)}\left(\prod_{j\neq i}^{e_t-1} n_{jt}^{\theta}\right)}\int_0^{n_{it}^{\theta}}\int_0^{v}\frac{u^{e_t-2}(v-u)}{v^{1-\alpha}R_t}dudv = z+mn_{it},$$

or

$$\frac{n_{t[2]}^{\theta(e_t-1)}}{e_t-1}=\int_0^{n_{it}^{\theta}}\int_0^{v}\frac{\alpha n_{it}^{\theta(e_t-2)}u^{e_t-2}(v-u)}{\left(\prod_{j\neq i}^{e_t-1} n_{jt}^{\theta}\right)(z+mn_{it})v^{1-\alpha}R_t}dudv, \forall t. \tag{18}$$

The *ex post* realized value of R_t can now be solved for by equating the right-hand sides of equations (17) and (18). Note that from firm-i's perspective, n_{it} and $n_{t[1]}$ are equivalent, since firm-i cares only about the outcome in which *it* realizes the highest productivity.

$$\int_0^{n_{t[1]}^{\theta}}\int_0^{v}\frac{\beta e_t u^{e_t-2}R_t}{n_{t[1]}^{\theta}v^{\alpha}}dudv=\int_0^{n_{t[1]}^{\theta}}\int_0^{v}\frac{\alpha n_{t[1]}^{\theta(e_t-2)}u^{e_t-2}(v-u)}{\left(\prod_{j\neq[1]}^{e_t-1} n_{jt}^{\theta}\right)(z+mn_{t[1]})v^{1-\alpha}R_t}dudv,$$

which implies

$$\frac{\beta e_t h_{t[2]}^{e_t-2}R_t}{n_{t[1]}^{\theta}h_{t[1]}^{\alpha}}=\frac{\alpha n_{t[1]}^{\theta(e_t-2)}h_{t[2]}^{e_t-2}(h_{t[1]}-h_{t[2]})}{\left(\prod_{j\neq[1]}^{e_t-1} n_{jt}^{\theta}\right)(z+mn_{t[1]})h_{t[1]}^{1-\alpha}R_t},$$

or

$$R_t=\left(\frac{\alpha n_{t[1]}^{\theta(e_t-1)}(h_{t[1]}-h_{t[2]})}{\left(\prod_{j\neq[1]}^{e_t-1} n_{jt}^{\theta}\right)\beta e_t(z+mn_{t[1]})h_{t[1]}^{1-2\alpha}}\right)^{1/2}. \tag{19}$$

Substituting this expression for R_t into equation (16), and again noting that firm-i considers $n_{t[1]}$ as equivalent to n_{it}, expected discounted profits for firm-i, $E_{t-1}\left[\frac{\pi_{it}}{R_t}\right]$, become

$$\left(\frac{\alpha\beta e_t}{\prod_{j\neq[1]}^{e_t-1} n_{jt}^{\theta}}\right)^{1/2}\frac{A_{t-1[1]}^{\alpha}L^{\alpha}(e_t-1)(z+mn_{it})^{1/2}n_{it}^{\frac{1}{2}\theta(e_t-3)}}{n_{t[2]}^{\theta(e_t-1)}}\int_0^{n_{it}^{\theta}}\int_0^{v}\frac{u^{e_t-2}(v-u)^{1/2}}{v^{1/2}}dudv$$
$$-A_{t-1[1]}^{\alpha}L^{\alpha}(z+mn_{it}),$$

or

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right]=\left(\frac{\alpha\beta}{e_t\prod_{j\neq[1]}^{e_t-1} n_{jt}^{\theta}}\right)^{1/2}\frac{A_{t-1[1]}^{\alpha}L^{\alpha}(e_t-1)\Gamma(1.5)\Gamma(e_t-1)}{n_{t[2]}^{\theta(e_t-1)}\Gamma(e_t+1/2)}\cdot n_{it}^{\frac{3}{2}\theta(e_t-1)}(z+mn_{it})^{1/2}$$
$$-A_{t-1[1]}^{\alpha}L^{\alpha}(z+mn_{it}). \tag{20}$$

By setting this expected-profits function to zero, and anticipating the fact that

$n = n_i = n_j$ in an identical-firm equilibrium, the zero-expected-discounted-profits condition becomes

$$\left(\frac{\alpha\beta}{e}\right)^{1/2} \frac{(e-1)\Gamma(1.5)\Gamma(e-1)}{\Gamma(e+1/2)(z+mn)^{1/2}} = 1,^{23} \tag{21}$$

or

$$n = \frac{\alpha\beta(e-1)^2}{em}\left[\frac{\Gamma(1.5)\Gamma(e-1)}{\Gamma(e+1/2)}\right]^2 - \frac{z}{m}. \tag{22}$$

Firm-i's research investment decision can be solved for to get the research investment condition;

$$\frac{\partial}{\partial n_{it}} E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = 0,$$

or

$$\left(\frac{\alpha\beta}{e}\right)^{1/2} \frac{(e-1)\Gamma(1.5)\Gamma(e-1)}{\Gamma(e+1/2)(z+mn)^{1/2}} = \frac{2mn}{3\theta(e-1)(z+mn)+mn},^{24} \tag{23}$$

where $n = n_i = n_j$.

Finally, the right-hand sides of (21) and (23) can be equated to get

$$2mn = 3\theta(e-1)(z+mn) + mn,$$

or

$$mn = \frac{3\theta(e-1)z}{1-3\theta(e-1)}.$$

Combined with equation (22), this yields a condition for e;

$$\frac{3\theta(e-1)z}{1-3\theta(e-1)} = \frac{\alpha\beta(e-1)^2}{e}\left[\frac{\Gamma(1.5)\Gamma(e-1)}{\Gamma(e+1/2)}\right]^2 - z \tag{24}$$

e, the number of innovations introduced each period, and n, the amount of research by each firm, can now be solved for using equations (22) and (24).

[23]Time subscripts have now been dropped, as the choice of e and n are independent of time.
[24]This is derived from the first-order condition in Appendix A.3.2.

5 Comparative Statics

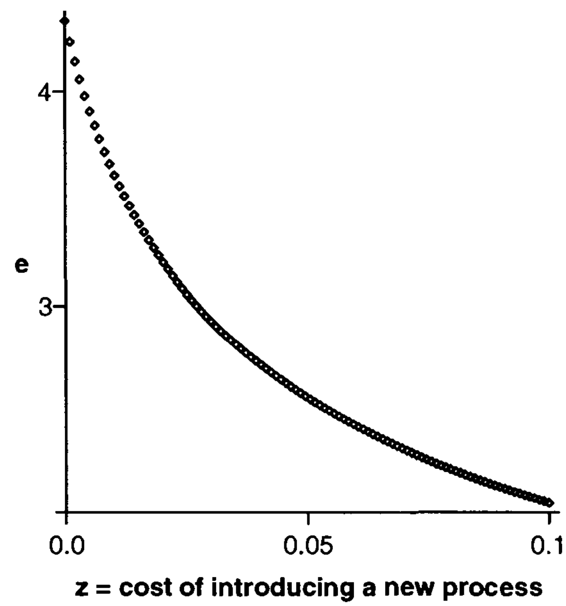
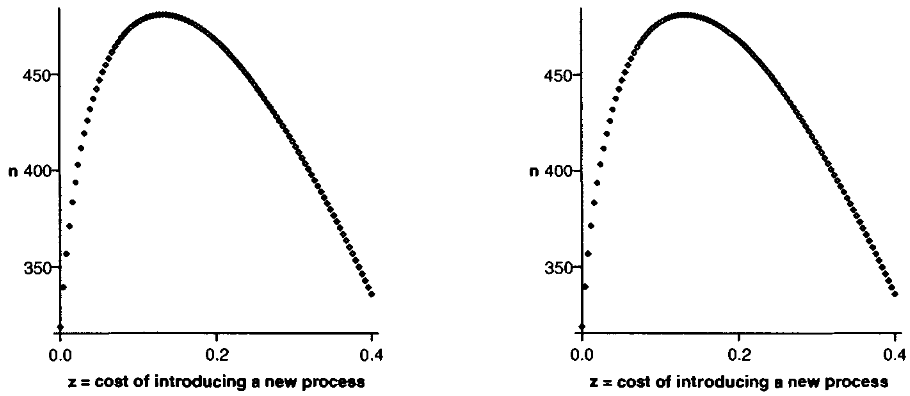


Figure 4: e(z) = Number of Innovations

Figure 4 shows the equilibrium number of innovations as a function of z, the cost of introducing a new process ($\alpha = 3/4$, $\theta = 0.1$, $\beta = 0.96$, $m = 0.0001$). As should be expected, the number of innovations is a decreasing function of the cost of introducing an innovation. It is clear from equation (24) that the equilibrium e is independent of m, the cost of research. This is due to two factors. First, the assumption of identical firms results in identical levels of research across firms, thus keeping the probability of capturing the market independent of the level of research in equilibrium. The second reason for this independence is similar to e's independence of λ (the upper bound on the distribution of possible values) in the simple model. Although research increases the expected operating profits of the eventual winner, this increase is entirely offset by an increase in the rate of interest. While a lower cost of research encourages higher growth, it does so entirely through an increase in the level of investment per innovation, and not through an increase in the number of innovations introduced. As a result, the total level of investment as a portion of output, $e(z+mn)$, is constant in the cost per unit of research. Equation (22) makes it clear that nm is also constant in m.

Figures 5a and 5b show n, the level of research by each firm, as a function of z, for both a low and high value of θ, respectively ($\alpha = 3/4$, $\beta = 0.96$, $m = 0.0001$).



(a) $\theta = 0.1$

(b) $\theta = 0.9$

Figure 5: n(z) = Firm-Level Research

The relationship between firm-level research and the cost of introducing an innovation depends on θ, the elasticity of the upper bound on the potential value of an innovation with respect to the input (the level of research). When this elasticity is low, an increase in the cost of introducing an innovation (z) is associated with first an increase, and then a decrease, in firm-level research. When θ is high, this relationship is always negative. To see why, note that an increase in θ increases the effect of n_i on the probability that firm-i will capture the market, and reduces the rate at which this effect diminishes[25], resulting in a higher level of research per firm. This, in turn, increases the effective cost of entering, given z and m, lowering the number of entrants at each z. A lower number of entrants decreases the effect of firm-i's research on expected operating profits (if firm-i wins)[26]. As a result of these two opposing effects, a higher θ unambiguously leads to higher firm-level research, but this increase is magnified at lower levels of z (and the associated higher number of entrants), so that the relationship between n and z is transformed from an inverted-U shape to a negative relationship as research increases at all z. More intuitively, a higher productivity of research in the innovative process leads to higher levels of research overall, but also to a greater tendency to use more intensive research in each direction of innovation, rather than trying more directions. Because this later effect on the number of firms is more pronounced when that number is already high (as when z is low), an increase in θ increases firm-level research much more at low levels of z.

[25]This is shown in Appendix A.4.1.

[26]This is shown in Appendix A.4.2.

The average level of innovation per firm is closely tied to firm-level research. Each firm in the model is drawing from the same distribution, $h \sim U(0, n^\theta)$, so the average value of innovation per firm can be measured simply as $\frac{n^\theta}{2}$, the expected value of each firm's innovation. The Lerner Index $\left(\frac{P-MC}{P}\right)$ is independent of research when all firms are identical, so the expected value of one minus the Lerner Index is still $\frac{e-1}{e}$, as in the simple model. By varying z, the resulting expected values of firm-level innovation and the associated measures of competition can be compared. Figure 6 shows the relationship between competition and the average value of innovation per firm at low to moderate levels of θ, which is essentially the same as the relationship between firm-level research and competition ($\alpha = 3/4$, $\theta = 0.1$, $\beta = 0.96$, $m = 0.0001$).

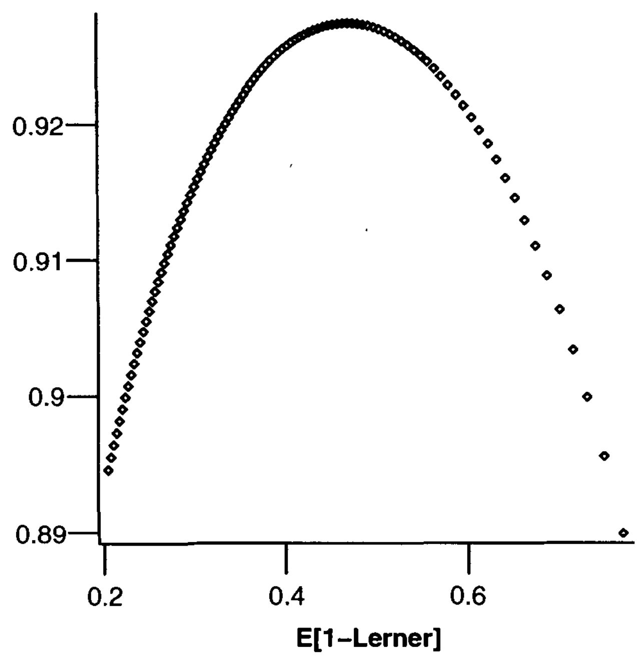


Figure 6: Average Innovation Per Firm

The expected growth rate of the economy is

$$E[g] = E\left[\frac{y_t}{y_{t-1}}\right] = E\left[h_{[1]}^\alpha\right] - 1,$$

as in the simple model. Using the joint-density function;

$$E[g] = \frac{e(e-1)}{n^{e\theta}} \int_0^{n^\theta} \int_0^v u^{e-2} v^\alpha du dv - 1 = \frac{e n^{\alpha\theta}}{e+\alpha} - 1.$$

Since e is independent of the cost of research, and n is linearly decreasing in m, the expected growth rate must be decreasing in m. It also decreases with the cost of introducing an innovation, z.

A number of studies have examined the empirical relationship between growth and 'research intensity', defined as total research expenditure as a fraction of output, or emn. Since $e'(m) = 0$, and $n'(m) = -1$, research intensity is independent of the cost of research. Figure 7 plots the expected growth rate against research intensity, as z changes ($\alpha = 3/4$, $\theta = 0.1$, $\beta = 0.96$, $m = 0.0001$).

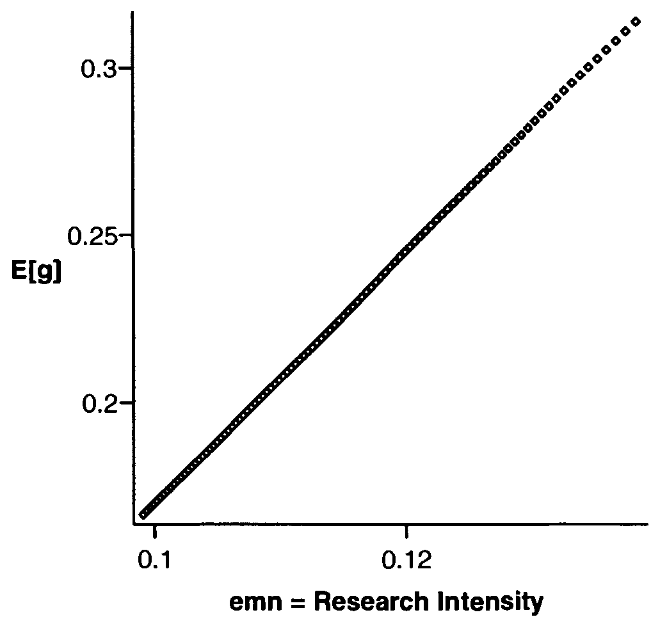


Figure 7: Research Intensity and Growth

As the cost of introducing an innovation increases, both e and n decrease. As a result, research intensity decreases as the cost of introducing an innovation drops, as does the expected growth rate. The result is a positive relationship between growth and research intensity, illustrated in Figure 7.

Finally, the relationship between growth and competition is illustrated in Figure 8 ($\alpha = 3/4$, $\theta = 0.1$, $\beta = 0.96$, $m = 0.0001$).

As in the simple model, growth is unambiguously increasing with competition. The decrease in the expected markup of the winning firm will tend to discourage research, since a drop in the fraction of value captured by the winner means firms will be less interested in winning the market - this is the Schumpeterian effect. Besides

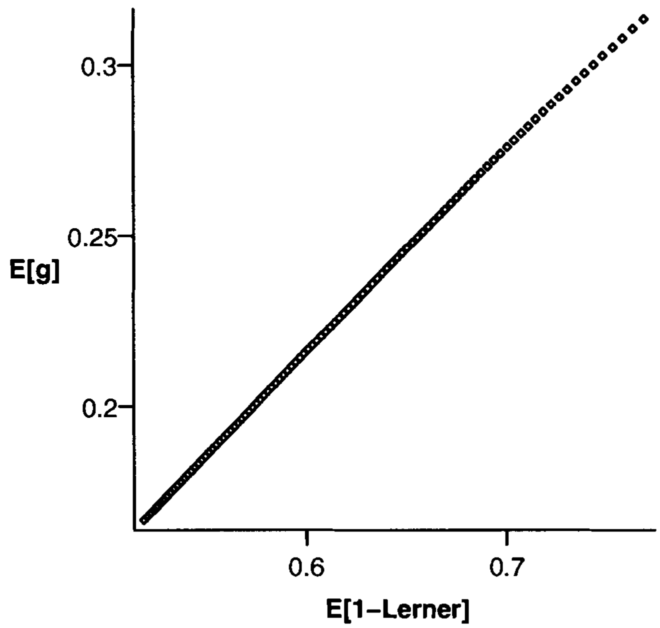


Figure 8: Competition and Growth

the probability of winning, however, research also tends to *increase* the expected value of the best innovation. At low levels of competition, an increase in competition is associated with *more* firm-level research, more innovations, and higher growth. At relatively high levels of competition, where the Schumpeterian effect causes firm-level research and *average* innovation to decline with competition, as in Figure 6, the increase in firms more than compensates, increasing the expected value of the *best* innovation, and thus increasing the expected rate of growth.

The one stipulation for this result is that θ, the productivity parameter for research in the production of a valuable innovation, is kept fixed. If the above exercise is repeated for varying levels of θ while keeping z fixed, the opposite relationship will emerge. At any fixed level of z, a higher θ would decrease competition, but *increase* expected growth.

6 Empirical Support

The primary goal of this Hayekian model of innovation is to show that when the best direction of innovation is uncertain, whether due to uncertain shadow prices for the characteristics of a good or inexperience with a new production process, it is often necessary to just try a new product or process, and hope for the best. Depending on the definition of product failure, anywhere from 50-90% of products fail within one or two years. $8.9 billion was spent on market research in North America in 2006, presumably by firms attempting to mitigate this uncertainty.[27] These casual observations suggest that the assumption of uncertainty is well-grounded.

Introducing uncertainty and endogenous competition into an endogenous growth model reverses the negative relationship between competition and growth generated by previous models, such as Romer (1990), and Aghion and Howitt (1992). The main result of the model, that competition and growth are positively related, is well supported in empirical studies of both across-industry and across-country differences in growth.[28] To my knowledge, no studies have provided evidence to the contrary. The stipulation for this result in the model, however, is that differences in the level of competition are being driven by differences in the cost of introducing an innovation, and not by differences in the productivity of research in producing innovations. As evidence that this condition is satisfied, consider first that both Geroski (1990) and Nickell (1996) make efforts to control for differences across industries in 'technological opportunity'. By controlling for the importance of research across observations, it seems likely that uncontrolled differences such as varying marketing costs and regulatory environments are driving the variation in levels of competition. Secondly, a number of studies have suggested that both higher marketing costs and more burdensome regulations are associated with lower growth, which is consistent with the model.[29]

Aghion et al. (2005) test the relationship between competition and the average level of innovation per firm in an industry. For a measure of the level of innovation by each firm, the authors total the number of patents in the industry, each weighted by the number of citations from other patent applications. They report an inverted-U relationship between the average level of innovation per firm and the level of competition. The present model results in the same relationship between competition and average innovation (Figure 6), except when θ, the parameter on research, is relatively high (> 0.6). Research intensity in North America has remained at approximately

[27]See ESOMAR (2007).

[28]Nickell (1996), Blundell et al. (1999), and Dutz and Hayri (2000) are summarized in the Related Literature section.

[29]See Graham et al. (1983) and Nicoletti and Scarpetta (2003) for examples.

2.1-2.7% of GDP since 1957[30], suggesting that the real equivalent to θ is quite low.

There have been a number of studies that find a positive relationship between research intensity and growth, both across industries and across countries.[31] The model's results are fully consistent with these findings. The model also fails to exhibit any 'scale effect', whereby an increase in the absolute level of expenditure on research results in a higher rate of growth. This is consistent with both recent Schumpeterian models and empirical evidence.[32] This result, however, is driven by the assumption that the cost of introducing an innovation grows at the same rate as total output. This constraint seems justified for the regulatory portion of the cost of introducing an innovation. Different economies seem to set regulatory costs, mandatory features, etc..., in each industry as a fraction of total wealth per capita. As an economy grows, the same policy regime will require a greater number of mandatory features, as well as greater testing requirements for safety and efficacy. The market-driven portion of these costs also presumably increases with output - the higher the productivity of inputs in other uses, the more costly to employ them in an attempt to improve a production process.

[30]National Science Foundation (2007).

[31]OECD (2003) is one such study that also surveys part of the literature.

[32]See Aghion and Howitt (2005) for both a growth model without scale effects and a survey of the evidence. Much ado was made about the scale effect present in early endogenous growth models (see Jones (1995) for an example), but more recent models have sterilized the effect, while retaining all other previous implications.

7 Conclusion

Hayek (2002) argued that the competitive process could be thought of as a procedure for discovering and making use of knowledge that would otherwise not emerge. When firms are uncertain of which direction to innovate in, the best innovation to emerge will tend to be of higher value when more innovations are tried. Although competition can lower the expected rents to innovators, the Hayekian effect can, and in this model does, dominate the Schumpeterian effect. When both the number of innovations and the level of research are endogenized, along with the level of competition, the model presented here mimics the relationships found in the data more closely than current endogenous growth models, where competition is kept exogenous.

The inclusion of Bertrand competition into an endogenous growth model also results in an emphasis on the *best* ideas developed in a market, rather than the average value of innovations, or average levels of research and productivity. When attempting to explain differences in growth across industries or countries, a Hayekian model suggests that empiricists should focus more on determining which policies and institutional structures encourage more experimentation in the market, and as a consequence, better ideas being tried.

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A Appendices

A.1 Joint Density Function

A.1.1 Exogenous Research

The joint density function of the highest and second-highest draws from a distribution is

$$f(h_{[1]} = v, h_{[2]} = u|e) = \frac{e!}{(e-2)!} \{F_h(u)\}^{e-2} f_h(v) f_h(u).$$[33]

Given that $h \sim U(0, \lambda)$, it follows that

$$f_h(v) = f_h(u) = \frac{1}{\lambda},$$

$$\text{and } F_h(u) = \frac{u}{\lambda}.$$

The joint density function in the simple model thus becomes

$$f(h_{[1]} = v, h_{[2]} = u|e) = \frac{e(e-1)u^{e-2}}{\lambda^e}.$$

A.1.2 Endogenous Research

The joint density function of the highest and second-highest draws from a distribution remains

$$f(h_{[1]} = v, h_{[2]} = u|e) = \frac{e!}{(e-2)!} \{F_h(u)\}^{e-2} f_h(v) f_h(u),$$

as in Appendix A.1.1 above. Given that $h_i \sim U(0, n_i^\theta)$, it follows that

$$f_h(v) = \frac{1}{n_{[1]}^\theta},$$

$$f_h(u) = \frac{1}{n_{[2]}^\theta},$$

$$\text{and } F_h(u) = \frac{u}{n_{[2]}^\theta}.$$

The joint density function in the extended model thus becomes

$$f(h_{[1]} = v, h_{[2]} = u|e, n_{[1]}, n_{[2]}) = \frac{e(e-1)u^{e-2}}{n_{[1]}^\theta n_{[2]}^{\theta(e-1)}}.$$

[33]This is derived in Balakrishnan and Cohen (1991, pp. 8-11).

A.2 Probability of Winning

The probability of firm-i realizing the highest productivity is

$$\Pr[h_i > h_j | e, n_i, n_j, \forall j \neq i] = \int_0^{n_i^\theta} \left(\prod_{j \neq i}^{e-1} F_{h_j}(v) \right) f_{h_i}(v) dv.$$

$$F_{h_j}(v) = \frac{v}{n_j^\theta} \text{ and } f_{h_i}(v) = \frac{1}{n_i^\theta}, \text{ so}$$

$$\begin{aligned}\Pr[h_i > h_j | e, n_i, n_j, \forall j \neq i] &= \int_0^{n_i^\theta} \left(\prod_{j \neq i}^{e-1} \frac{v}{n_j^\theta} \right) \frac{1}{n_i^\theta} dv \\ &= \int_0^{n_i^\theta} \frac{v^{e-1}}{n_i^\theta \left(\prod_{j \neq i}^{e-1} n_j^\theta \right)} dv \\ &= \frac{n_i^{\theta(e-1)}}{e \left(\prod_{j \neq i}^{e-1} n_j^\theta \right)}.\end{aligned}$$

A.3 Explicit Derivation of Steps in the Endogenous Research Model

A.3.1 Maximized Expected Utility Condition

Start with the Euler equation;

$$\frac{1}{c_{t-1}} = \beta \cdot E_0 \left[\frac{R_t}{c_t}\right].$$

In a balanced-growth-path equilibrium, the Euler equation becomes

$$\frac{1}{\beta} = E_t \left[\frac{R_t}{h_{t[1]}}\right].$$

The expectations operator can be expanded using equation (15) to get

$$\frac{1}{\beta} = \frac{e_t(e_t-1)}{n_{t[1]}^{\theta} n_{t[2]}^{\theta(e_t-1)}} \int_0^{n_{t[1]}^{\theta}} \int_0^{v} \frac{u^{e_t-2}R_t}{v^{\alpha}} du dv,$$

or

$$\frac{n_{t[2]}^{\theta(e_t-1)}}{(e_t-1)} = \int_0^{n_{t[1]}^{\theta}} \int_0^{v} \frac{\beta e_t u^{e_t-2}R_t}{n_{t[1]}^{\theta} v^{\alpha}} du dv.$$

A.3.2 Research Investment Condition

The expected-discounted-profits function for firm-i, equation (20), is

$$E_{t-1}\left[\frac{\pi_{it}}{R_t}\right] = \Delta \; A_{t-1[1]}^{\alpha} L^{\alpha} n_{it}^{\frac{3}{2}\theta(e_t-1)} (z+mn_{it})^{1/2} - A_{t-1[1]}^{\alpha} L^{\alpha}(z+mn_{it}),$$

$$\text{where } \Delta \equiv \left(\frac{\alpha\beta}{e_t \prod_{j\neq i}^{e_t-1} n_{jt}^{\theta}}\right)^{1/2} \frac{(e_t-1)\Gamma(1.5)\Gamma(e_t-1)}{n_{t[2]}^{\theta(e_t-1)}\Gamma(e_t+1/2)}.$$

Maximizing expected profits with respect to n_{it}, firm-i will choose n_{it} to satisfy the following first-order-condition;

$$\frac{\Delta}{2(z+mn_it)^{1/2}} \left[3\theta(e_t-1)(z+mn_{it})n_{it}^{\frac{3}{2}\theta(e_t-1)-1} + mn_{it}^{\frac{3}{2}\theta(e_t-1)}\right] - m = 0.$$

All firms face an identical research investment decision, so $n_{it} = n_{jt}$ in equilibrium. The above equation thus becomes the research investment condition;

$$\left(\frac{\alpha\beta}{e_t}\right)^{1/2} \frac{(e_t-1)\Gamma(1.5)\Gamma(e_t-1)}{\Gamma(e_t+1/2)(z+mn_t)^{1/2}} = \frac{2mn_t}{3\theta(e_t-1)(z+mn_t)+mn_t}.$$

A.4 Effect of θ On Firm-Level Research

A.4.1 Effect Through Probability of Winning

An increase in θ increases the effect of n_i on the probability that firm-i realizes the highest probability.

$$\frac{\partial^2}{\partial\theta\partial n_i}(\Pr[i \text{ wins}]) = \frac{\partial^2}{\partial\theta\partial n_i}\left(\frac{n_i^{\theta(e-1)}}{en_j^{\theta(e-1)}}\right)$$

$$= \frac{\partial}{\partial\theta}\left(\frac{\theta(e-1)n_i^{\theta(e-1)-1}}{en_j^{\theta(e-1)}}\right)$$

$$= \frac{(e-1)n_i^{\theta(e-1)-1}}{en_j^{\theta(e-1)}}\left(1+\theta(e-1)\left[\ln\left(\frac{n_i}{n_j}\right)\right]\right),$$

where n_j is the level of research by other firms, each assumed to be equal here for simplicity. In equilibrium, $n_i = n_j$, so the effect of a an increase in θ on firm-i's ability to increase its probability of success is

$$\frac{e-1}{en} > 0.$$

A.4.2 Effect Through Expected Operating Profit of Winner

A decrease in θ leads to a higher number of entrants. The following shows that a higher number of entrants increases the effect of firm-i's research on expected operating profits.

$$\frac{\partial^2}{\partial e\partial n_i}\left(E\left[\frac{h_{[1]}-h_{[2]}}{h_{[1]}^{1-\alpha}}\right]\right) = \frac{\partial^2}{\partial e\partial n_i}\left(\frac{n_i^{\theta(e+\alpha-1)}}{n_j^{\theta(e-1)}(e+\alpha)}\right)$$

$$= \frac{\partial}{\partial e}\left(\frac{\theta(e+\alpha-1)n_i^{\theta(e+\alpha-2)}}{(e+\alpha)n_j^{\theta(e-1)}}\right).$$

In equilibrium, $n_i = n_j$, so the effect of higher entry on firm-i's ability to increase expected operating profits is

$$\frac{\partial}{\partial e}\left(\frac{\theta(e+\alpha-1)}{(e+\alpha)n^{\theta(1-\alpha)}}\right)$$

$$= \frac{\theta}{n^{\theta(1-\alpha)}(e+\alpha)^2} > 0.$$

A.5 Hayek in a Model of Quality-Improving Innovations

Both the exogenous-research and endogenous-research models can be adapted to a quality-improving model of vertical innovation. The only difficulty arises from from the assumption of Bertrand competition in the cost-reducing innovation models, since all firms will now face the same marginal cost. The solution proceeds as follows.

The competitive final good firm produces according to

$$y = \left(\sum_{j=1}^{e} A_j x_j \right)^{\alpha},$$

where x_j is the quantity of the intermediate good produced by firm-j, and A_j is the quality of x_j.

Intermediate firm-j *can* produce according to $x_j = L_j$, where L_j is the amount of labour used by firm-j. Once the quality of each intermediate firm's product is realized, however, the winning firm can capture the market by choosing a price, P_t, and wage, w_t, such that

$$\frac{MP_{[1]}}{P_{[1]}} \geq \frac{MP_{[2]}}{P_{[2]}},$$

where $MP_{[1]}$ is the marginal product of the highest-quality x in the production of y. Since the lowest price the second-best firm can charge is its marginal cost, w, the winning firm can capture the market if

$$\frac{MP_{[1]}}{P_{[1]}} \geq \frac{MP_{[2]}}{w}.$$

The final good firm will choose $x_{[1]}$ such that $P_{[1]} = MP_{[1]}$, so the market-capturing condition becomes

$$w \geq MP_{[2]} = \alpha A_{[2]} \left(\sum_{j=1}^{e} A_j x_j \right)^{\alpha-1},$$

or

$$w \geq \alpha A_{[2]} A_{[1]}^{\alpha-1} L^{\alpha-1},$$

since only the winning firm is producing at this wage. Operating profits for the winner will be

$$(P - w)\, x = \left(MP_{[1]} - w \right) L = \left(\alpha A_{[1]}^{\alpha} x^{\alpha-1} - \alpha A_{[2]} A_{[1]}^{\alpha-1} x^{\alpha-1} \right) x,$$

or

$$\alpha L^{\alpha} \left(\frac{A_{[1]} - A_{[2]}}{A_{[1]}^{1-\alpha}} \right).$$

From here, the simple model continues as before from equation (2), and the endogenous-growth model from equation (13). All results from the models continue to hold under this framework.