MODELING AND FORECASTING CANADIAN YIELD CURVE WITH MACROECONOMIC DETERMINANTS

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PROJECT SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS IN FINANCIAL RISK MANAGEMENT

In the
Faculty
of
Business Administration

SIMON FRASER UNIVERSITY

Summer 2007

APPROVAL

Name: **Di Huo and Fang Lu**

Degree: **Master of Arts**

Title of Project: **Modeling and Forecasting Canadian Yield Curve with Macroeconomic Determinants**

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Date Approved: August 7, 2007

ABSTRACT

Term structure of interest rates is crucial for pricing bonds and managing financial risks. The yield curve of zero-coupon bonds can typically be used to measure the term structure of interest rates. In this paper, we use the popular Nelson-Siegel three-factor framework to model the entire Canadian yield curve. The empirical results show that the model fits the Canadian yield curve well. We estimate vector autoregressive models for the three factors in order to produce out-of-sample forecasts, and also employ seven natural competitors for comparison. Our forecast results are encouraging. Our model is superior to most competitors, especially at longer horizons. We further incorporate macro variables into the yield-only model. From the results of forecast comparison test between the yield-only model and yield-macro model, we conclude that a joint dynamic term structure model incorporating macro variables contributes to sharpening our ability of forecasting yields accurately out of sample.

ACKNOWLEDGEMENTS

We are indebted to researchers who shared their invaluable research on modeling and forecasting yield curve. Special thanks to Dr. Francis X. Diebold for the profound study of Nelson-Siegel yield-macro model which provides the guidance for our project.

Most importantly, we would like to acknowledge Dr. Daniel Smith and Dr. Chris Perignon who helped us through all the phases of this project with challenging questions and encouragement.

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1 INTRODUCTION

Modeling and forecasting yield curve are important in both pricing and risk management aspect. Following the trend of financial literature related to modeling the dynamics of the term structure through time-varying factors, this paper aim to formulate a model that is consistent with finance theory from a macroeconomic perspective to produce accurate out-of-sample forecasts for Canadian zero-coupon yields. In this paper, we use the popular Nelson-Siegel three-factor model with extension proposed by Diebold and Li (2006). We present that the Nelson-Siegel model fits the Canadian yield curve well and the interpretation of the three time-varying parameters as factors is appropriate. We estimate vector autoregressive models for the three factors in order to produce out-of-sample forecasts, and also employ seven natural competitors for comparison. Our forecast results are encouraging. Our model is superior to most competitors, especially at longer horizons.

In order to explore whether macroeconomic determinants have an effect on the Canadian yield curve, we further incorporate several macro variables into the yield-only model. In addition to the domestic macro factors, such as inflation and monetary policy, the feature of U.S. term structure is introduced to encompass more comprehensive macroeconomic for the objective of improving out-of-sample forecast accuracy. Particularly, we apply Vector Autoregressive (VAR) models to model how yields directly respond to macroeconomic variables.

A comparison of yields-only and yields-macro model shows the improvements of forecasting accuracy is noticeable. We also analyse how the three factors individually respond to main macroeconomic variables. The movements in yield curve have been attributed primarily to U.S. term structure and somewhat to shocks of monetary policy. By employ the Diebold-Mariano forecast accuracy test, our yield-macro model has produced noticeable improvement in longer horizon forecast. This conclusion provides evidence of the effects of macro variables on future movements in the yield curve that are insufficiently encompassed in yield-only Nelson-Siegel three-factor model.

We proceed as follows. In Section 2 we do a literature review mainly on some important term structure models. In section 3 we describe the extended Nelson-Siegel model and provide a detailed interpretation of the three factors. In section 4 use empirical data to test our model's in-sample fitting as well as out-of-sample forecasting. In section 5, we further incorporate macro variables into our Nelson-Siegel with VAR(1) model and explore the effects of macro factors on the yield curve. Last, in section 6 we end the paper with some concluding remarks.

2 LITERATURE REVIEW

Modeling term-structure dynamics is an important component in bond portfolio management, and interest rate forecasting is important for both derivatives pricing and risk management. The last decades have produced major advances in theoretical models of the term structure as well as their econometric estimation, yet the resulting models vary in form and fit. Two popular approaches to term structure modeling are no-arbitrage models and equilibrium models. A number of recent papers have introduced into question the ability of some popular models to adequately describe yield curve dynamics. The arbitrage-free term structure literature has little to say about dynamics or forecasting, as it concentrates primarily with fitting the term structure at a point in time. The affine equilibrium term structure literature is concerned with dynamics driven by the short rate, and so is potentially linked to forecasting.

Despite powerful advances in dynamic yield curve modeling in the more recent work, little attention has been paid to the key practical problem of forecasting the yield curve. While most traditional models focus only on in-sample fit as oppose to out-of-sample forecasting, those who do focus on out-of-sample forecasting, Duffee (2002), conclude that both in-sample forecasts and out-of-sample forecasts produced with the standard class of affine models are typically worse than forecasts produced by simply assuming yields follow random walks.

Rather than the traditional approaches mentioned above, a number of diverse extension of Nelson-Siegel model is applied to produce out-of-sample forecasts with factors evolving dynamically. Diebold and Li (2006) use neither the no-arbitrage approach nor the equilibrium approach to model the yield curve. Instead, they use variations on the Nelson–Siegel exponential components framework to model the entire yield curve. Previous work shows that the three time-varying parameters may be interpreted as factors corresponding to "level", "slope" and "curvature", which are explored to perform out-of-sample yield curve forecasting. Diebold and Li (2006) find that although the 1-month-ahead forecasting results are not notably better than those of random walk and other leading competitors, the forecasting results in longer time horizon are more accurate to other standard benchmarks. Pooter (2007) draw to the similar conclusion that amongst various extensions of the Nelson-Siegel model, the more flexible model leads to a better in-sample fit of the term structure as well as out-of-sample predictability superior to competitor models. The results also show that this outperformance is consistent across maturities and forecast horizon.

Still, an apparent large gap between the predominant yield curve models and macroeconomy neglects the role of expectations of inflation and future real economic activity in the determination of yields. Foremost among these are the popular factor models in which only a handful of unobserved factors explain the entire set of yields. Most existing factor models of term structure are unsatisfactory, for these factors do not depict explicitly how yields respond to macroeconomic variables. There are some paper take a step toward bridging the joint dynamics of macroeconomic and bond prices in a factor model of the term structure. More recent work has been seeking the direct linkage between macroeconomic variables and yields forecasting. Some have constructed the yield curve with Nelson-Siegel extension models dominant in the finance literature, which possess the feature allowing macroeconomic variables. Although this class of models is not linked explicitly to macroeconomic variables, its state-space representation facilitates the extraction of latent yield-curve factors, and the incorporation of dynamic macroeconomic variables.

Related work includes Wu (2002) and Hördahl and Tristani (2004). Wu (2002) examines the empirical relationship between the movement of the slope factor in term structure and exogenous monetary-policy shocks in the U.S. Results from the correlation study support the strong correlation between the slope factor and the exogenous monetary-policy shocks. Moreover, monetary-policy shocks account for a large part of variability of the slope factor. Taking one step further, Hördahl *et al.* (2002) constructs and estimates a joint model of macroeconomic and yield curve dynamics. In an application to German data, Hördahl shows that their out-of-sample forecasts beats the predictions of the random walk benchmark in almost all cases, and outperform the alternatives for all maturities, at least beyond the very shortest forecast horizon. Studies most related to our analysis are Ang and Piazzesi (2003) and Diebold *et al.* (2006). Ang and Piazzesi (2003) describe the joint dynamics of bond yields and macroeconomic variables in a vector autoregression, and find that models with macro factors forecast better than models with only unobservable factors. In particular, macro factors primarily explain movements at the short end and middle of the yield curve while unobservable factors still explain most of the movement at the long end of the yield curve and the effects of inflation shocks are strongest at the short end of the yield curve. Diebold *et al.* (2006) presents a Nelson-Siegel extension model of the yield curve with observable macroeconomic variables and traditional latent yield factors. He finds strong evidence of macroeconomic effects on the future yield curve and in particular, Diebold's forecasts incorporating macroeconomic factors appear much more accurate at long horizons than other competitive models.

This paper, in attempting to find a relatively explicit term-structure model that provides a reasonable description and forecasts of Canadian interest rate dynamics for risk management purposes, introduces a dynamic extension to the Nelson-Siegel models incorporated both yield factors (level, slope, and curvature) and macroeconomic variables (U.S. term structure, inflation, and the monetary policy shocks). This yield-macro model, built on the recent work of Diebold *et al.* (2006), involves variations of the Canadian specific macro factors as well as U.S. term structure.

3 A THEORETICAL MODEL

3.1 Term Structure Estimation Method

The yield curve of zero-coupon bonds can typically be used to measure the term structure of interest rates. However, we can only observe short maturities zero-coupon yields, usually one year or less. Therefore, we can not directly obtain the entire yield curve and need to use approximation methods. Three main theoretical representations of the term structure are: the forward curve, the yield curve and the discounted curve. Once we get a presentation of one of the three, the relationships among them enable us to derive the other two. Let $f_t(\tau)$ denote the instantaneous forward rate for a forward contract initiated τ periods in the future, let $y_t(\tau)$ denote the yield on a τ-period zero-coupon bond, and let $P_t(\tau)$ denote the present value of a zero-coupon bond with \$1 receivable after τ periods . Given the forward curve, we can derive the yield curve by taking the equally-weighted average of the forward rates,

$$y_t(\tau) = \frac{1}{\tau}\int_0^\tau f_t(s)ds \tag{1}$$

The discount curve can be derived from the yield curve by,

$$P_t(\tau) = \exp\left(-\tau y_t(\tau)\right) \tag{2}$$

And in turn we obtain the instantaneous forward rate curve by,

$$f_t(\tau) = -\frac{P'_t(\tau)}{P_t(\tau)} \tag{3}$$

A variety of methods have been proposed to estimate forward curves, yield curves and discounted curves from observed bond prices. Fama and Bliss (1987) suggest constructing yields via estimated forward rates at the observed maturities. They assume that the forward rate between observed maturities is constant and construct forward rate to price successively longer maturities. The yields constructed are unsmoothed. There are other term structure estimation methods to construct yields by estimating a smooth discount curve. For example, McCulloch (1975) models the discount curve with cubic splines, Vasicek and Fong (1982) propose to fit exponential splines to discount curve and Chambers *et al.* (1984) use polynomials functions.

3.2 Three-factor Model

Nelson and Siegel (1987) suggest using a mathematical approximating function to fit the forward rate curve at a given date. The functional form can be viewed as a constant plus a Laguerre function which consists of the product between a polynomial and an exponential decay term. The resulting Nelson-Siegel forward rate curve is,

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t}\exp(-\lambda_t\tau) + \beta_{3,t}\lambda_t\exp(-\lambda_t\tau) \quad (4)$$

As presented the in above section, the zero-coupon bond yield is an equally weighted average over the forward rates, therefore, we obtain the corresponding yield curve by taking the integral of the forward rate,

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau}\right] + \beta_{3,t}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} - \exp(-\lambda_t\tau)\right] \quad (5)$$

The Nelson-Siegel model is popular among theoretical models of the term structure estimation because it provides a parsimonious approximation of the yield curve. First let us interpret the parameters in the model.

The exponential decay rate is determined by the parameter λ_t. Smaller value of λ_t results in a slow decay to zero and fits long maturity yield curves better, while larger value of λ_t results in a fast decay to zero and fits short maturity yield curves better. λ_t also governs at which maturity the loading on $\beta_{3,t}$ reaches its maximum.[1]

The three loadings on $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are,

$$l_1(\tau) = 1 \tag{6}$$

$$l_2(\tau) = \frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} \tag{7}$$

$$l_3(\tau) = \frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} - \exp(-\lambda_t\tau) \tag{8}$$

Diebold and Li (2006) suggest that $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ can be interpreted as three latent factors in term of level, slope and curvature. The interpretation is based on the following reasons. Firstly, $l_1(\tau)$, the loading on $\beta_{1,t}$, is a constant which does not decay to zero, as a result, $\beta_{1,t}$ is the infinite-maturity value, which can be regarded as the long-term factor governing the yield curve level. As the loading is identical for all maturities, it is easy to note that an increase in $\beta_{1,t}$ will increase the level of yield curve.

Secondly, $l_2(\tau)$, the loading on $\beta_{2,t}$, starts at 1 but quickly decays to zero, therefore, $\beta_{2,t}$ can be regarded as the short-term factor which is closely related to the yield curve slope. Alternatively, an increase in $\beta_{2,t}$ will increase short maturity yields more than long term yields, by that means changing the slope. We define the slope as the ten-year yield minus the three-month yield[2], as Diebold and Li (2006) do. Some papers define the slope as the infinite-maturity yield minus the zero-maturity yield, the two extreme cases, which is exactly equal to $-\beta_{2,t}$.[3] It is

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[1] Throughout this paper, we follow Diebold and Li (2006) and set λ_t equal to 0.0609.
[2] In particular, if λ_t=0.0609, slope $= y_t(120) - y_t(3) = -0.78\beta_{2,t} + 0.06\beta_{3,t}$.
[3] See, for example, Frankel and Lown (1994).

also worth noting that yield curves start from an instantaneous short-maturity value of $\beta_{1,t} + \beta_{2,t}$ and approach a finite infinite-maturity value of $\beta_{1,t}$.[4]

Lastly, $l_3(\tau)$, the loading on $\beta_{3,t}$, starts at zero, increases for medium maturities and decays to zero again, hence, $\beta_{3,t}$ can be regarded as the medium-term factor which is closely related to the yield curve curvature. we can see that an increase in $\beta_{3,t}$ will have strong effect on the medium-maturity yields but weak effect on the very short or long maturity yields, leading the increase of the yield curve curvature. We still follow Diebold and Li (2006) and define the curvature as twice the two-year yield minus the sum of the ten-year and three-month yields.[5] The three factor loadings are plotted in Figure 1, which gives us a direct vision. They are in the shape as the above discussion.

- Insert Figure 1 here -

Furthermore, Diebold and Li (2006) explain that the three-factor model could replicate some of the most important stylized facts of the term structure of yields over time: the average yield curve is upward sloping and concave; the yield curve is capable of assuming different shapes, such as upward sloping, downward sloping, humped and inverted humped; yield dynamics are strongly persistent, and spread dynamics are less persistent; yields of longer maturities are more persistent but less volatile than that of shorter maturities. We try to explore whether the three-factor model fits the Canadian yield curve and could reproduce the main historical stylized facts.

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[4] $\lim_{\tau\to 0} y_t(\tau) = \beta_{1,t} + \beta_{2,t}$; $\lim_{\tau\to\infty} y_t(\tau) = \beta_{1,t}$.

[5] In particular, if λ_t=0.0609, curvature $= 2y_t(24) - y_t(120) - y_t(3) = 0.00053\beta_{2,t} + 0.37\beta_{3,t}$

4 EMPIRICAL RESULTS

4.1 Data

The database we use in this paper consists of end-of-month zero-coupon bond yields in Canada, from January 1986 to February 2007, with 254 observations, taken from the website of Bank of Canada. Unfortunately, some end-of-month data are unavailable; instead we use the last available data in that month. In the estimation we choose 17 fixed maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months, covering most of the short term, medium term and long term bonds.

Figure 2 shows a three-dimensional plot of yield curve of all the maturities, indicating how the yield level, slope and curvature vary during the sample period. It is clear from the figure that the yields experienced a more fluctuating period during the first ten years from 1986 to 1995, while from 1996 onwards the yields were less volatile, almost around 4%. We can also observe that the variation in the slope and curvature is less apparent than that in the level.

- Insert Figure 2 and Table 1 here -

Table 1 presents the summary statistics for yields for all maturities. The table indicates that the average yield curve is upward sloping, that long term yields are less volatile than short term yields, and there are very high autocorrelations. In terms of level, slope and curvature, the curvature the least persistent and the most volatile factor relative to its mean.

4.2 In-sample Fitting Yield Curve

In this section, we use the Nelson-Siegel three-factor model to fit the yield curve.

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau}] + \beta_{3,t}[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} - \exp(-\lambda_t\tau)]$$

We could use Maximum Likelihood Estimation (MLE) to estimate the four unknown parameters, $\beta_{1,t}$, $\beta_{2,t}$, $\beta_{3,t}$ and λ_t for each month. However, in order to simplify the computation of the two factor loadings $l_2(\tau)$ and $l_3(\tau)$, and use Ordinary Least Squares (OLS) to estimate the betas, Diebold and Li (2006) suggest fixing λ_t to a pre-specified value. Fabozzi *et al.* (2005) and Dolan (1999) also fix λ_t and then progress with the model. In this way, the nonlinear measurement become linear which could be estimated by the simple OLS in stead of the challenging complicated MLE. As discussed in the section, the parameter λ_t governs the medium term maturity at which the loading on the curvature factor $\beta_{3,t}$, reaches its maximum. Diebold and Li (2006) simply choose 30-month maturity as this medium term and obtain the value of 0.0609. We try different values of λ_t including the value of 0.12 suggested by Bolder (2006), who also models the Canadian term structure dynamics. It turns out that the value of 0.0609 produces the best results of the three factors $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ in our database. Therefore, throughout this paper, we follow Diebold and Li (2006) and set λ_t equal to 0.0609.

Subsequently, cross-sectional OLS estimation is used for each month separately. We obtain time series of estimates of $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$, and consequently seventeen sets of residuals. Fitted yield curves for all the maturities are plotted in Figure 3. Compare with Figure 2, we can not detect any large difference between estimated yield curves and historical ones. In Figure 4 we plot the average estimated yield curve against the average actual yield curve. It is clear that the fitted curve and the actual curve are very close. Both figures suggest that the model fits the yield curves quite well.

- Insert Figure 3, 4 and Table 2 here -

The detailed statistics describing yield curve residuals are summarized in Table 2. Main standard criteria, such as the mean, standard deviation, minimum, maximum, mean absolute error (MAE) and root mean square error (RMSE), indicate a good in-sample fit. However, the residual autocorrelations suggest that pricing errors are persistent.

The three time-series factor estimates $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ are plotted against the data-based level, slope and curvature in Figure 5. We can see that the three estimated factors $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$and $\hat{\beta}_{3,t}$ are closely related to the yield curve level, slope and curvature, respectively. Table 3 presents detailed statistics for the three estimated factors. Comparing Table 3 with the last three rows in Table 1, we can see that the third estimated factor is the least persistent, and presents less correlation with curvature than the first estimated factor with level and the second estimated factor with slope.

- Insert Figure 5 and Table 3 here –

4.3 Modeling and Forecasting Yield Curves

Since our interest is not only in fitting the term structure of yield curve, but also in the out-of-sample forecasting, we need a model for the factor dynamics. We follow the dynamic framework of Diebold, Rudebusch, and Aruoba (2006) by modeling and forecasting the Nelson-Siegel three factors as multivariate VAR(1) process. The yield forecasts are therefore,

$$\hat{y}_{t+h}(\tau) = \hat{\beta}_{1,t+h} + \hat{\beta}_{2,t+h}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau}\right] + \hat{\beta}_{3,t+h}\left[\frac{1-\exp(-\lambda_t\tau)}{\lambda_t\tau} - \exp(-\lambda_t\tau)\right] \qquad (9)$$

Where

$$\hat{\beta}_{t+h} = \hat{C} + \hat{\Gamma}\hat{\beta}_t \qquad (10)$$

We directly regress β_t on β_{t-h}, which produces $\hat{C}$ and $\hat{\Gamma}$, then obtain $\hat{\beta}_{t+h}$ h-period ahead.

For comparison, we also employ seven popular models for yield curves. In the following part, we briefly describe those competitors about their forecasts processes

(1) Random walk

Random walk is a benchmark model. Many interest rate forecasting studies demonstrate that consistently outperforming the random walk is difficult because random walk usually well performs in short-horizon forecasting. The h-period-ahead forecasted yield is,

$$\hat{y}_{t+h}(\tau) = y_t(\tau) \tag{11}$$

(2) AR(1) model on yield levels

AR(1) model on yield levels is a model which allows for mean reversion, and the h-period ahead forecasted yield is,

$$\hat{y}_{t+h}(\tau) = \hat{c}(\tau) + \hat{\gamma}(\tau) y_t(\tau) \tag{12}$$

(3) VAR(1) model on yield levels

There is a little difference between AR(1) and VAR(1), that in VAR(1) we regress the yield at time *t* on yields for all maturities at time *t-h*. The h-period-ahead forecasted yield is,

$$\hat{Y}_{t+h}(\tau) = \hat{C}(\tau) + \hat{\Gamma}(\tau) Y_t(\tau) \tag{13}$$

Where,

$$Y_t(\tau) = [y_t(\tau_1) \ \cdots \ y_t(\tau_i)]'$$

(4) VAR(1) model on yield changes

Instead of yield levels, VAR(1) on yield changes regresses yield changes at time *t* on corresponding yield changes at time *t-h*. The h-period-ahead forecasted yield change is,

$$\hat{Z}_{t+h}(\tau) = \hat{C}(\tau) + \hat{\Gamma}(\tau) Z_t(\tau) \quad (14)$$

Where,

$$Z_t(\tau) = [y_t(\tau_1) - y_{t-1}(\tau_1) \cdots y_t(\tau_i) - y_{t-1}(\tau_i)]'$$

(5) Slope regression

In the model of slope regression, we derive the forecasted yield change from a regression of historical yield changes of yield curve slopes.

$$\hat{y}_{t+h}(\tau) - y_t(\tau) = \hat{c}(\tau) + \hat{\gamma}(\tau)(y_t(\tau) - y_t(3)) \quad (15)$$

(6) Fama-Bliss forward rate regression

Fama and Bliss (1987) suggest a famous forward rate regression, from which we obtain the forecasted yield change from a regression of historical yield changes on forward spreads.

$$\hat{y}_{t+h}(\tau) - y_t(\tau) = \hat{c}(\tau) + \hat{\gamma}(\tau)(f_{t+h,t}(\tau) - y_t(\tau)) \quad (16)$$

Where $f_{t+h,t}(\tau)$ is the interest rate of a forward contract at time *t* which is initiated at time $t+h$ and matures at $t+h+\tau$.

(7) Cochrane and Piazzesi (2002) forward rate regression

Cochrane and Piazzesi (2002) run regressions of yield changes on all forward rates, which is a generalized Fama-Bliss regression. The forecasted yield change is obtained by regressing the historical yield changes on forward rates.

$$\hat{y}_{t+h}(\tau) - y_t(\tau) = \hat{c}(\tau) + \hat{\gamma}_0(\tau)y_t(12) + \sum_{j=1}^{k} \hat{\gamma}_j(\tau) f_{t+j/12,t}(12) \quad (17)$$

Where $f_{t+j/12,t}(12)$ is the interest rate of a forward contract at time t which is initiated at time $t+(j/12)$ with a maturity of 12 months.

4.4 Out-of-sample Forecasting

In this section we employ Nelson-Siegel with VAR(1) factor dynamics as well as seven competitor models introduced in the last section to forecast the yield curve. We take recursive forecasts, using data from January 1986 to the time that the forecast is made, starting from January 2000 to February 2007. Forecast error at time $t+h$ is defined as $y_{t+h}(\tau) - \hat{y}_{t+h(\tau)}$. We use some main forecast error evaluation criteria, including mean, standard deviation, RMSE and autocorrelations at different displacements, to examine the quality of forecasts.

The results for the h-month-ahead out-of-sample forecasting are presented in Table 4 – 6. In particular, we choose yield curves for maturities 3 months, 1 year, 3 years, 5 years and 10 years, and horizons of 1 month, 6 months and 12 months.

- Insert Table 4-6 here -

Let us now interpret those forecast results. As we expected, the absolute values of error mean, standard deviation, RMSE increase as the forecasting horizon increases for all maturities in the eight models. The RMSE comparison reveals, though better than VAR(1) on yield levels, Fama-Bliss and Cochrane-Piazzesi forward rate regression, our model's the 1-month-ahead forecast, does not outperform the rest models. The Diebold-Mariano forecast accuracy test[6] in Table 9 Panel A provides further proof. It indicates that only at 3-month maturity our model is significantly better than random walk, but at the other maturities it is inferior to random walk.

[6] See Diebold and Mariano (1995).

However, results improve as the forecast horizon increases. For the 6-month-ahead forecasts, our model's forecast results as reported in Table 5 become stronger than those in the 1-month-ahead forecasts. The superiority over the competitors is apparent which can be reinforced by the Diebold-Mariano test. Most test results are negative, implying that our model is better than random walk. The Nelson-Siegel with VAR(1) model presents an overwhelming majority when the forecast horizon extends to 12 months. As we can see that the RMSE of our model at various maturities are smaller than those from random walk, especially at longer maturities. Furthermore, all of Diebold-Mariano tests are negative and three of the five statistics are significant, indicating the great superiority of our forecasts at the 12-month horizon.

In summary, the Nelson-Siegel with VAR(1) model provides an encouraging forecast results, especially at longer horizons. It is worth noting that our results are comparable to Diebold and Li (2006), which prove that the Nelson-Siegel with AR(1) model produces more accurate forecast results at long horizons than short horizons. Our results further demonstrate that the Nelson-Siegel three factor model not only in-sample fits Canadian yield curves, but also produces satisfying out-of-sample forecasts.

However, there is still a problem puzzling us. When we observe the Canadian yields at different maturities from January 1986 to February 2007, we find that yield curves are quite volatile in the first ten years and more stable in the recent ten years. Bolder, Johnson and Metzler (2004) explain that high and volatile inflation, large government borrowing requirements and a large amount of relatively small, illiquid issues are the main factors resulting in the sharp changes over the first ten years. Yields over the recent ten years were less volatile due to low and stable inflations, efficient government regulations and the improvement of the bond market. From this point of view, we believe Canadian yield curves are affected by macroconomy factors, such as inflation and monetary policy. In addition, as a relative small economy closely related to U.S., we want to explore whether U.S. yield curves have an effect on the Canadian yield curves. Therefore, in the following section, we incorporate those macro-factors into our model to explore their effects on yield curves.

5 YIELD-MACRO MODEL

5.1 Macro Variables

In this section, we formulate and examine the dynamic impact of the macroeconomic on the shape of Canadian yield curve using an extended version of Nelson-Siegel three-factor model integrated with the observable macroeconomic variables. Our measures of the macroeconomy include three key variables: U.S. yield curve; Canada Consumer Price Index; and Canada Target for Overnight Rate. The three variables represent the movements in U.S. yield curve, the inflation rate and the monetary policy instrument in Canada, respectively, which are considered to be the fundamental information set needed to capture basic macroeconomic dynamics.

Since the Nelson-Siegel "level", "slope", and "curvature" factors provide a good representation of the yield curve, we can explicitly incorporate the U.S. term structure into Canadian yield curve factors. To extract the movement of U.S. yield curve, we interpret the U.S. observable zero-coupon yields into level (USL), slope (USS), and curvature (USC):

$$USL_t = y_t(120) \tag{18}$$

$$USS_t = y_t(120) - y_t(3) \tag{19}$$

$$USC_t = 2y_t(24) - [y_t(120) + y_t(3)] \tag{20}$$

where $y_t(\tau)$ denotes yields at τ-month maturity.

The above extraction of U.S. yields enables us to analyze the direct effect of actual movements in U.S. yield curve level, slope and curvature on Canadian term structure.

In regard of inflation rate (IFR), the consumer price index (CPI) is the appropriate measure of domestic inflation. We calculate the monthly logarithmic change of consumer price index from January 1986 to February 2007:

$$IFR_{t+1} = \text{Log}(\frac{CPI_{t+1}}{CPI_t}) \tag{21}$$

The reason why we choose Target for Overnight Rate (TOR) as one essential variable to forecast yield curve is that Bank of Canada carries out monetary policy mainly by raising and lowering the target for the overnight rate in the purpose of influencing short-term interest rate as well as other interest rates, including mortgage rates and prime rates charged by commercial banks. Also, the target for overnight rate is directly comparable with the U.S. Federal Reserve's Target for the federal funds rate[7]. Since Canada Target for overnight rate is a discrete series, we interpret it as monetary policy shocks treated as a dummy variable by calculation the changes in individual time point and fixing it at zero for the rest of the months when there is no change announced by Bank of Canada. By adding this dummy variable to our yield-macro model, we are able to capture the impact of monetary policy shocks to yield curve.

5.2 Yield-macro Model

We formulate a model that construct the yield curve using latent factors (level, slope, and curvature) and also includes observable macroeconomic variables (U.S. yield curve, inflation, and the monetary policy instrument). Our goal is to produce out-of-sample forecasts evolving the dynamic impact of the macroeconomy. A straightforward extension of the Nelson-Siegel three-factor model adds the three macroeconomic variables to the set of factors:

$$y_{t+h}(\tau) = a + \Lambda f_{t+h} + \varepsilon \tag{22}$$

[7] Diebold (2005) includes federal funds rate (FFR) as a measurement of the monetary policy instrument.

$$f_{t+h} = \gamma + Af_t + \upsilon \quad (23)$$

where $f'_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, USL_t, USS_t, USC_t, IFR_t, TOR_t)$, and Λ is a 1x3 matrix, A is a 8x8 matrix.

The above equation forms our yields-macro model, with which we will compare our earlier yields-only model. The advantage of our joint treatment of macroeconomics and term-structure dynamics is that we are able to derive the direct response of yield curve to macroeconomy. And than we examine how the latent factors change when macro variables are incorporated into out model.

We use the VAR(1) to forecast the latent three factors. We generate our out-of-sample forecasts by placing estimated future factors into equation h-month-ahead forecasting equation. At last we apply Diebold-Mariano[8] tests in order to examine the significance of improvements in our yield-macro model forecasts compare with those from the yield-only model.

5.3 Out-of-sample Forecasting Results

- Insert Figure 6 here -

In Figure 6, we plot the estimated facts from yield-macro model and yield-only model at 12-month-ahead forecast horizon. Comparing the Nelson-Siegel three-factor model of term structure, the "slope" factor survives almost intact when macro variables are incorporated, meanwhile the "level" and "curvature" estimates vary noticeably from those obtained from yield-only model.

- Insert Table 7-8 here -

[8] Diebold and Mariano (1995) propose explicit tests to compare the accuracy of forecasts between two competing models.

In Table 7 we display the estimates of the parameters of the yields-macro VAR(1) model. And we present our results from examining the correlation between Nelson-Siegel three factors and macro variables in Table 8.

Individually, many of the coefficients appear insignificant; however, as we discuss below, key blocks of coefficients appear significant. Generally speaking, the long-established fact about treasury yields holds that the current term structure itself contains information about future term structures. More specifically, in our data set, the historical yields explain most part of future term structure, yet a significant proportion of the three factors response to movements in U.S. term structure. Furthermore, the target for overnight rate shows more important influence on yield curve than the inflation rate.

The coefficients of U.S. term structure level, slope and curvature factors shown in Table 7 are significant for their corresponding Canadian factors, which indicate that the U.S. term structure is positively related to Canadian yield curve. By regressing U.S. factors solely against h-month-ahead Canadian factor, the three U.S. yield curve factors can explain almost half of the future Canadian level, slope and curvature respectively. However, when combined with VAR(1) model of individual factor, the U.S. factors contains much less information rather than past Canadian factors itself to predict the future movements in Canadian yield curve, especially for short horizon. Combined with correlation coefficient presented in Table 8, these results demonstrate high positive correlation between U.S. and Canadian factors. The reason is that both the U.S. and Canadian yield curve are driven by the same U.S. macroeconomic shocks. Our results are consequences of recent working paper by Fousseni et al (2007)[9] supporting that the US macroeconomic shocks explain a majority of the unconditional variations in Canadian yields. Thus, U.S. term structure assists the forecast of Canadian yields in providing additional

[9] Fousseni Chabi-Yo and Jun Yang, (2007) conclude that all three US macro shocks contribute around 50% of the variations in the expected excess returns of holding Canadian bonds for one quarter at various forecast horizons.

predictability since the underlying driving forces from U.S. macroeconomy are incorporated in Canadian yield curve. Therefore, we expect that adding the US curvature factor contributes to better estimation of $\hat{\beta}_{i,t+h}$ at h-month-ahead forecasts.

Compared with other macro variables used in our model, inflation is not an essential element to explain neither factor of the term structure. And the coefficients in Table 7 indicate insignificant effects on yield curve. Although we find significant correlation between inflation rate and level and slope factors, due to the relatively stable inflation rate over past ten years, the effect of changes in inflation rate on the movements of latent factors is minor. Thus it suggests that the change of inflation rate does not have apparent impact on the latent factors or contains little information about predictability of future yields in our sample period.

The Target for Overnight Rate which represents the monetary policy shocks has relatively significant impact on all three factors. The level factor of yield curve responds adversely to the changes in TOR, while the slope and curvature factors have positive correlation coefficient with TOR. Moreover, we find the three factors of Canadian yield curve response more actively to the target for overnight rate comparing with little responds to the other macro variable: inflation. We expect that incorporating TOR may lead to better prediction in yields with short maturities given its function as the signal sent by Bank of Canada about the direction in which it wants short-term interest rates to go. Hence it should have an impact greater in short maturities than longer maturities. Also, our findings are similar to the results in Wu (2001a)'s general-equilibrium based simulation study. The monetary policy shocks have most significant impact on slope factor, while the level and curvature factors indicate less response to the same monetary policy shocks. However, our monetary policy shock – target for overnight rate has a positive correlation with slope factor[10]. As the short-term yields increase with arising target for overnight rate, the expectations for future yields increase further, and as a result the slope of the term structure tends to be steeper.

The out-of-sample forecasts in our yields-macro model at the same horizon are somehow different to those obtained in the yields-only model. Furthermore, we employ Diebold-Mariano forecast accuracy comparison tests to our yield-macro model forecasts against those from the yield-only model in Section 4.

- Insert Table 9 here -

The Diebold-Mariano statistics reported in Table 9 Panel B indicate superiority of our yield-macro model to the yield-only model in mean square errors from out-of-sample forecasting in 12-month-ahead forecasting, our yield-macro model performs significantly more accurate forecasts than those from yield-only model at longer forecast horizon. This result is consistent with Diebold *et al.* (2006)[11]. Our findings suggest that the three factors explain most of the short-term variation in the yield curve while incorporating macro variables increases estimates errors. This over-parameterization may be the cause of its poor out-of-sample performance. However, at longer horizon, the macro factors become more influential to yield curve. As a result, macroeconomic variables conduce to generating more accurate forecast at longer horizon. The constant positive D-M statistics for 10-year yields imply that the macro factors introduced into our model account for short end and middle of the yield curve while latent factors still explain most of the movement at the long end of the yield curve. To summarize, incorporating macro variables helps to explain a significant portion of the yield curve at short and medium maturities.

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[10] Wu (2001a) find negative relationship between slope factor and monetary policy shocks. The inconsistent results may simply caused by the different macro variables used as well as the size of information set.

[11] Diebold *et al.* (2006) suggest a large amount of short-term idiosyncratic variation in the yield curve that is unrelated to macroeconomic fundamentals

And our yield-macro model does produce more accurate forecasts than yield-only model especially for longer horizon at which the macro factors are more influential leading to further improvement in forecasts.

5.4 Limitation

Our modeling and forecasting yield curve confront the same questions arising from existing macro VAR studies. From a finance perspective, our analysis is inadequate in the absence of no-arbitrage restrictions. Ang and Piazzesi (2003)[12] state that the explicit macro VAR model may not rule out arbitrage opportunities when the cross-equation restrictions implied by this assumption are not imposed in the estimation. Many other studies suggest that such restriction improves the forecast of term structure. Another advantage of no-arbitrage models is their effectiveness in shrinking the dimensionality of the parameter space when supplemented with large macro information set.

However, no-arbitrage factor models often appear to fit the cross-section of yields at a particular point in time. On the contrary, unrestricted VAR models are more successful in explaining the dynamics of the yield curve via the macro fundamentals because of the flexible lag specification. Such a dynamic feature is crucial to our goal of relating the evolution of the yield curve over time to movements in macroeconomic variables, so there is a trade off between potential accuracy in out-of-sample forecasting and loss in flexibility when imposing the no-arbitrage restrictions.

[12] Ang and Piazzesi (2003) also find that imposing the cross-equation restrictions from no- arbitrage helps in out-of-sample forecasts.

6 CONCLUSION

We have interpreted the Nelson-Siegel three factor model extended by Diebold and Li (2006) and demonstrated that the model not only in-sample fits Canadian yield curves, but also produces satisfying out-of-sample forecasts, especially at longer horizons. Our results are slightly different form Diebold and Li (2006) which may simply be caused by the variation of sample data. The Canadian yield curves in our sample period are usually lower and more volatile than U.S. yield curves. The larger variance of short and medium maturities yields leads to less accurate modeling and forecasting of the short-end and middle part of Canadian yield curve. We have further incorporated macro variables into the model which proves to be more accurate.

It is worth emphasising that those time-varying latent factors and macro variables have an intuitive explanation for the yield curve dynamics. Given the empirical results of our analysis from Canadian data, we therefore conclude that the inclusion of macroeconomic variables within a Nelson-Siegel tree-factor framework contributes to sharpening our ability of forecasting yields accurately out of sample. The improvement is due both to the inclusion of additional macroeconomic information in the model, and to the U.S. term structure movements. Our analysis presents evidence to support that a joint dynamic yield-macro term structure model, from both a macroeconomic perspective and from a finance perspective, provides the more comprehensive description at the short and middle of the term structure of interest rate and also more accurate out-of-sample forecasts especially at long horizon. Nevertheless, in future work, we hope to derive the no-arbitrage condition in our framework and explore whether its imposition is indeed helpful for forecasting.

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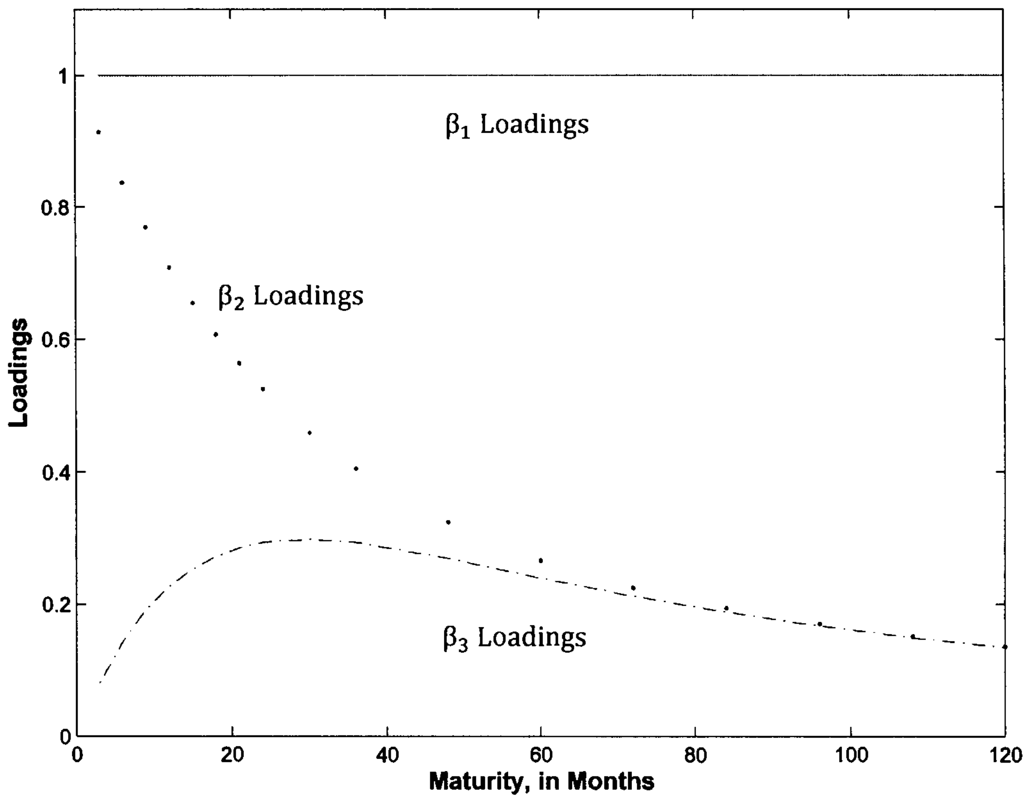
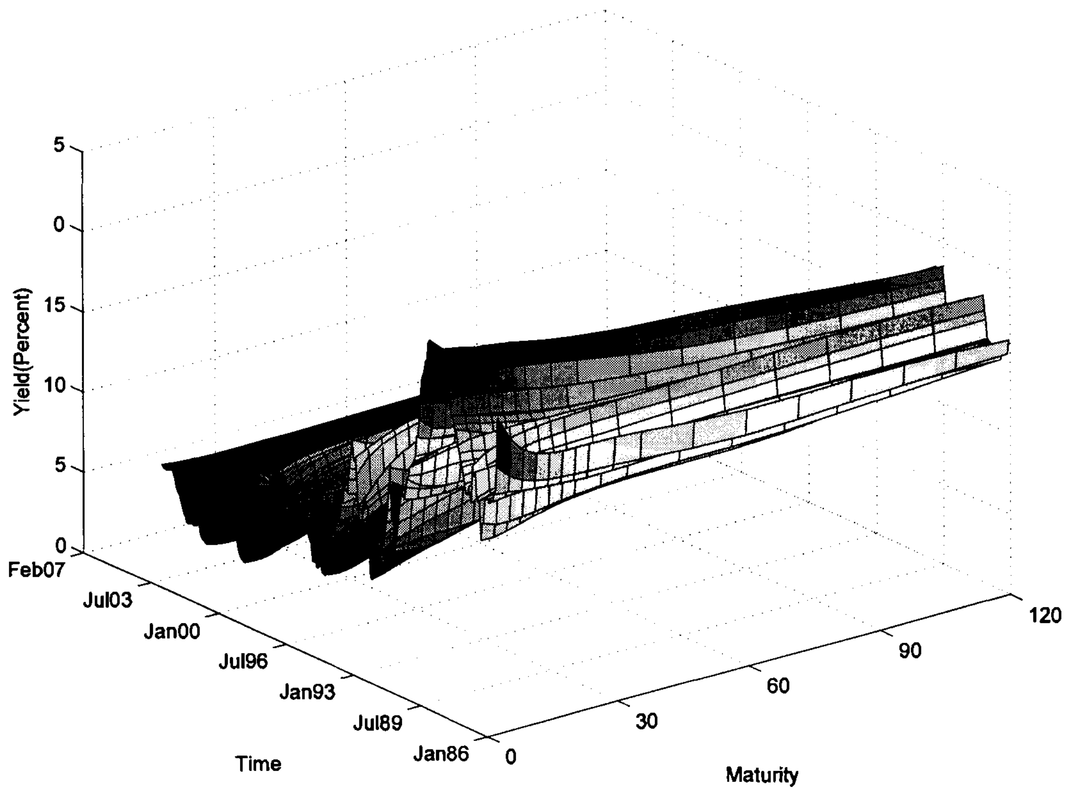
Figure 1 Nelson-Siegel Factor Loadings



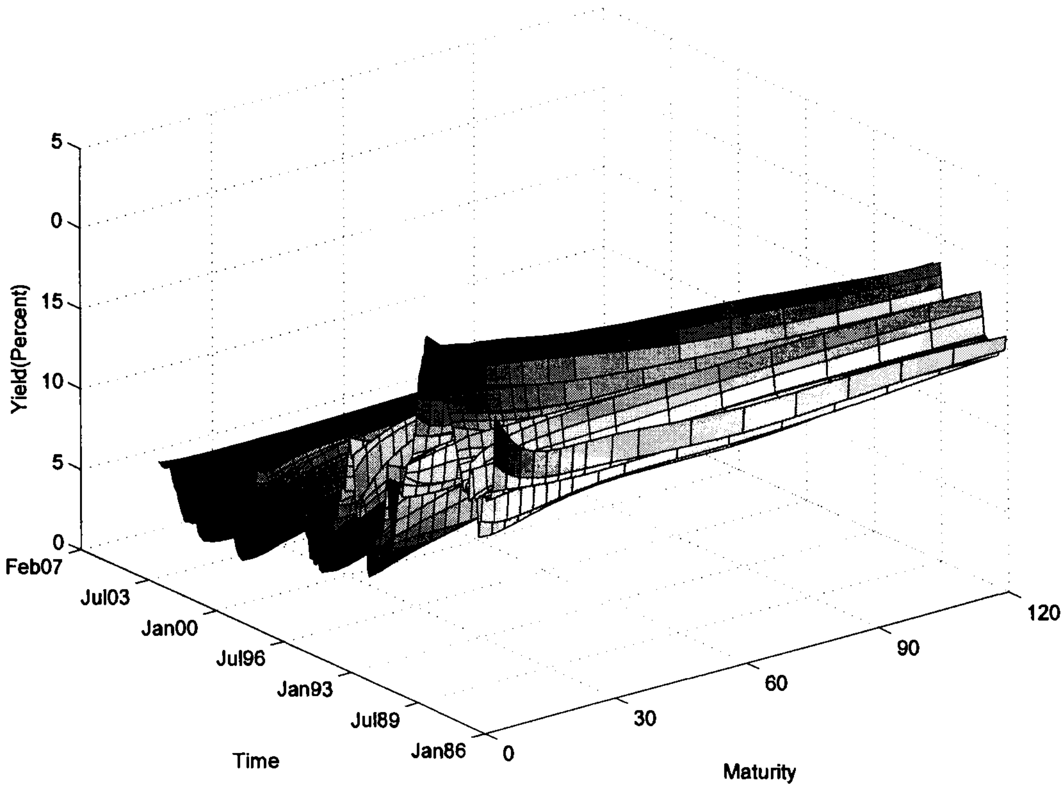
Notes: The figure depicts the three factor loadings for $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$. The factor loadings are plotted using λ_t=0.0609.

Figure 2 Yield Curves



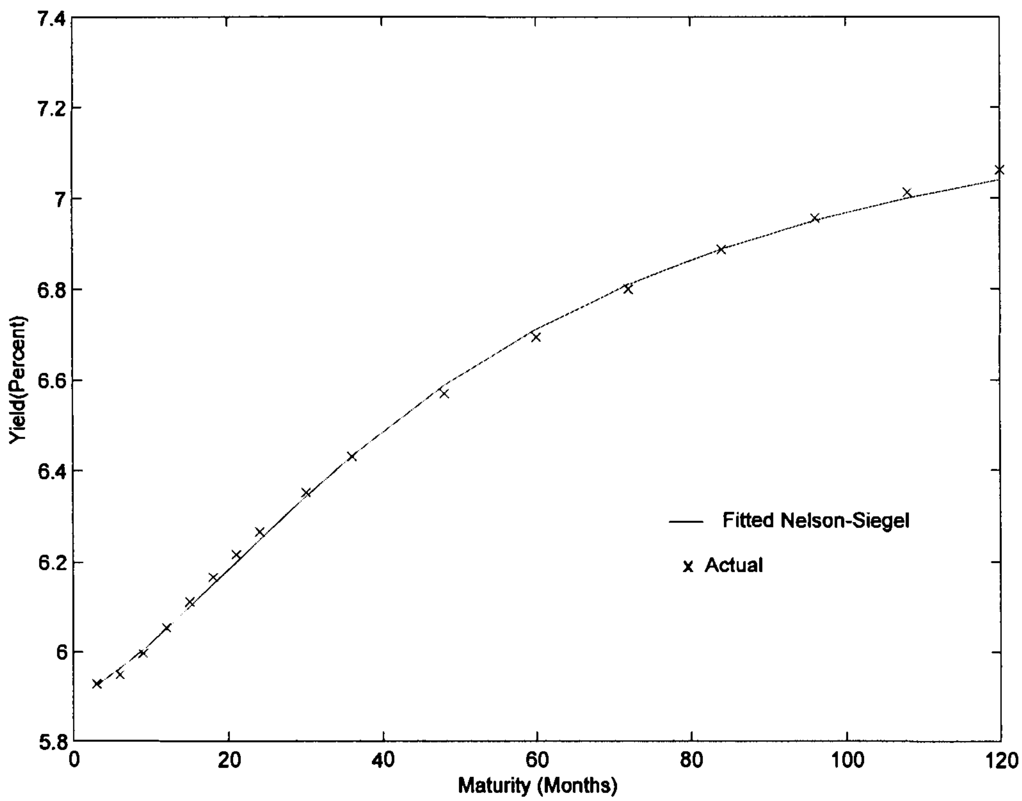
Notes: The figure shows a three-dimension plot of the monthly zero-coupon yields from January 1986 to Febuary 2007(254 observations) for maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

Figure 3 Fitted Yield Curves



Notes: The figure shows the fitted yield curves constructed by Nelson-Siegel three-factor model, from January 1986 to Febuary 2007 for maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 months.

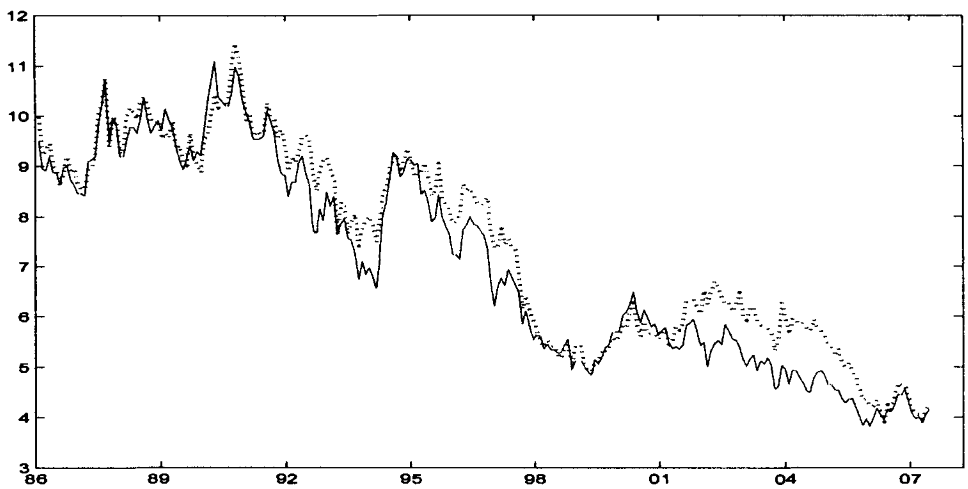
Figure 4 Fitted and Actual Average Yield Curve



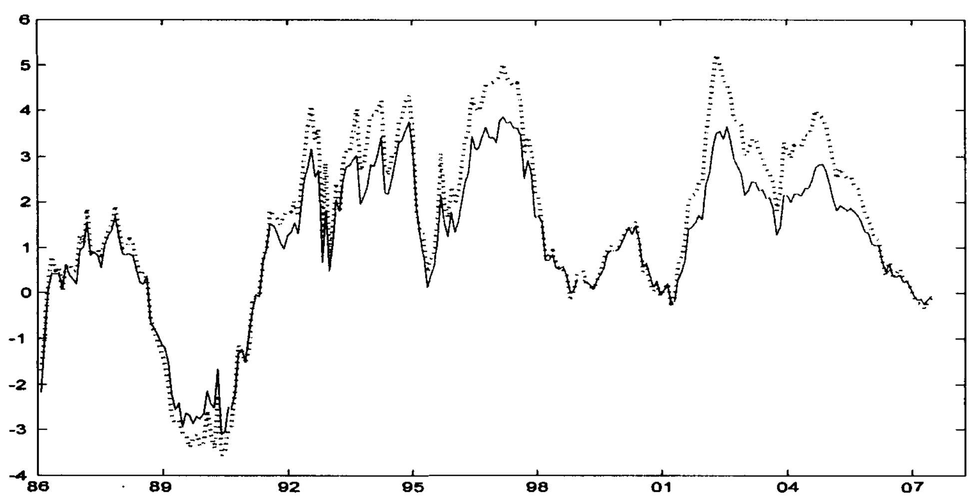
Notes: The figure shows the average fitted curve obtained by the Nelson-Siegel three-factor model at the mean values of $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$. The dots in the figure are the actual data-based averages.

Figure 5 Model-based vs. Data-based Level, Slope and Curvature

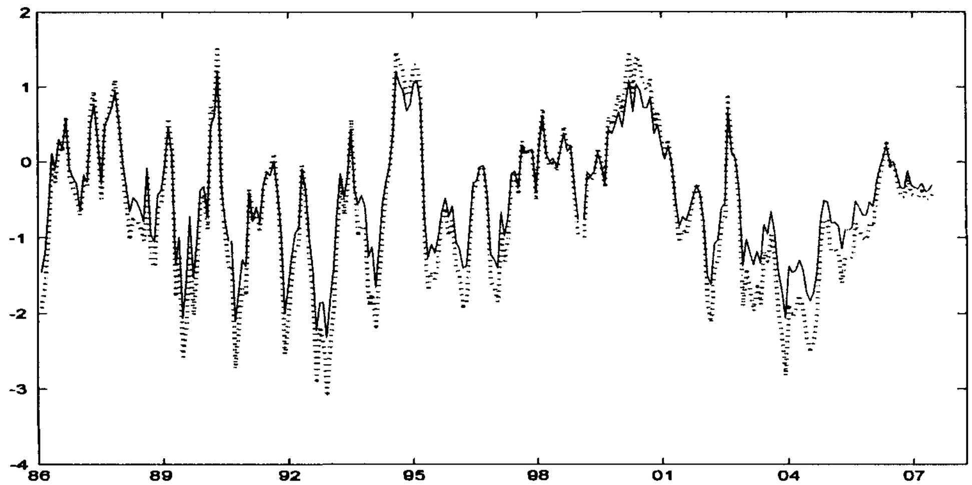
Dotted line: $\widehat{\beta}_{1,t}$ Solid line: Level



Dotted line: - $\widehat{\beta}_{2,t}$ Solid line: Slope



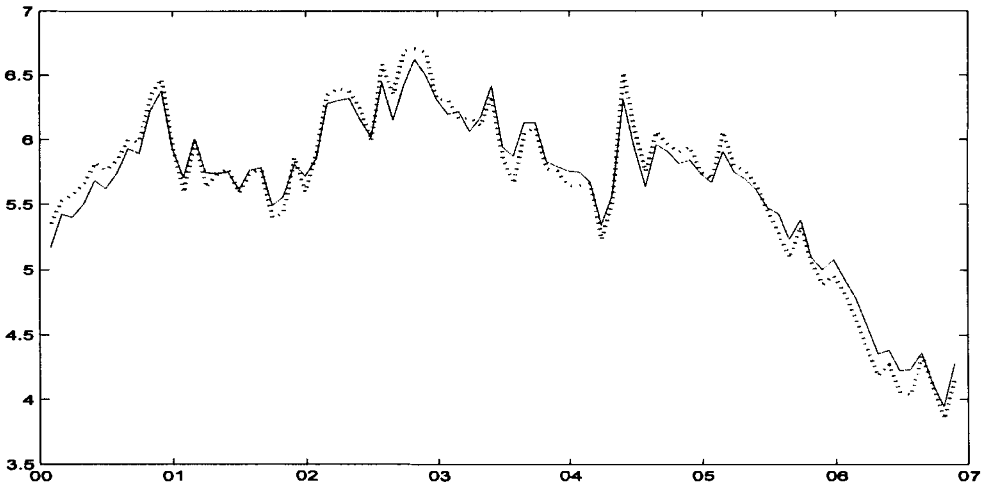
Dotted line: $\hat{\beta}_{3,t}$ Solid line: Curvature



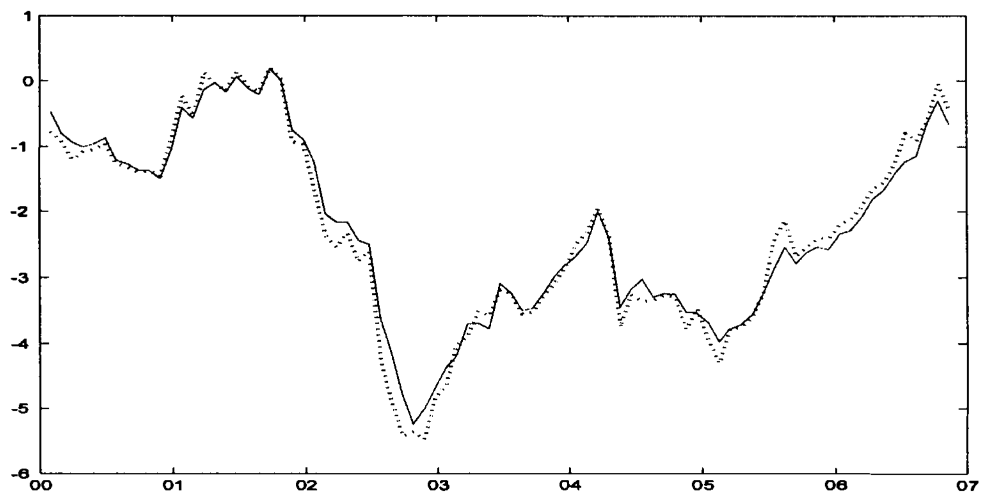
Notes: $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ are plotted separately against level, slope and curvature, respectively. We define the level as the ten-year yield, the slope as the ten-year yield minus the three-month yield and the curvature as twice the two-yield minus the sum of the three-month yield and ten-year yield.

Figure 6 Yield-only vs. Yield-macro Factor estimates

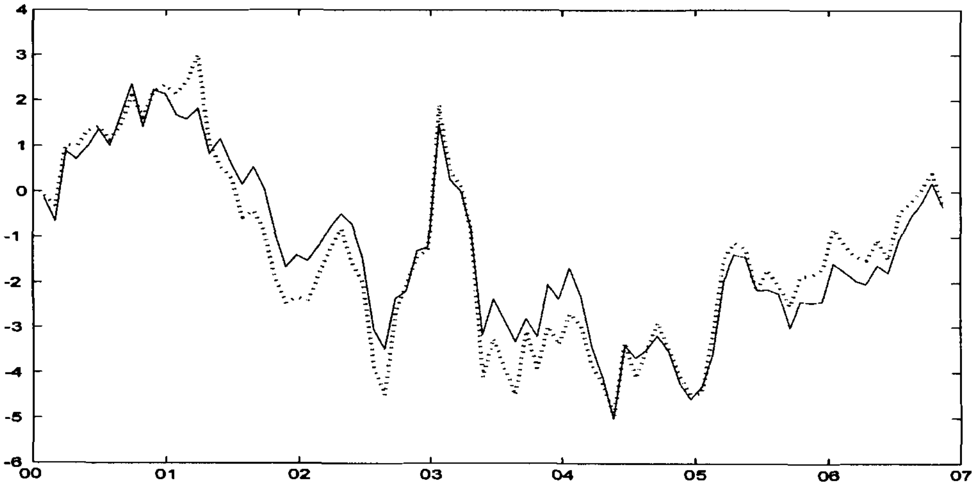
Dotted line: level estimates from Y-M Solid line: level estimates from Y-O



Dotted line: slope estimates from Y-M Solid line: slope estimates from Y-O



Dotted line: curvature estimates from Y-M Solid line: curvature estimates from Y-O



Notes: we plot the estimated facts from yield-macro model and yield-only model at 1-month-ahead forecast horizon. Comparing the Nelson-Siegel three-factor model of term structure, the "slope" factor remains almost intact when macro variables are incorporated, while the "level" and "curvature" varies from yield-only model estimates.

LIST OF TABLES

Table 1 Descriptive statistics, yield curves

Maturity (Months)	Mean	Std.dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	5.9407	3.0069	1.9790	13.4840	0.9828	0.7844	0.4915
6	5.9592	2.9000	1.9366	13.0730	0.9832	0.7948	0.5028
9	6.0060	2.8137	1.9257	12.9790	0.9831	0.8012	0.5107
12	6.0606	2.7362	1.9806	12.8580	0.9826	0.8045	0.5174
15	6.1163	2.6662	2.0722	12.7860	0.9820	0.8064	0.5237
18	6.1703	2.6038	2.1814	12.7150	0.9815	0.8077	0.5297
21	6.2214	2.5489	2.2968	12.6280	0.9810	0.8088	0.5356
24	6.2692	2.5010	2.4121	12.5340	0.9806	0.8100	0.5412
30	6.3558	2.4231	2.6328	12.3520	0.9800	0.8127	0.5514
36	6.4332	2.3632	2.8379	12.1990	0.9797	0.8156	0.5604
48	6.5713	2.2750	3.1192	11.9920	0.9794	0.8205	0.5747
60	6.6938	2.2081	3.2984	11.8610	0.9793	0.8233	0.5857
72	6.7997	2.1550	3.4703	11.7290	0.9793	0.8248	0.5947
84	6.8866	2.1155	3.6102	11.5630	0.9796	0.8263	0.6023
96	6.9558	2.0890	3.6974	11.3760	0.9801	0.8286	0.6087
108	7.0124	2.0738	3.7629	11.2050	0.9807	0.8315	0.6141
120(level)	7.0626	2.0675	3.8168	11.0870	0.9813	0.8347	0.6186
Slope	1.1220	1.5861	-3.1198	3.8399	0.9595	0.4618	0.0701
Curvature	-0.4650	0.7477	-2.3210	1.2080	0.8282	0.2103	-0.0783

Notes: The table presents summary statistics for monthly zero-coupon yields from 1986.01 to 2007.02 at different maturities. Reported are the mean, standard deviation, minimum, maximum and sample autocorrelations at displacements of 1, 12 and 30 months for those yields. In addition, the level defined as the ten-year yield, slope defined as the ten-year minus three-month yield and the curvature defined as twice the two-year yield minus the sum of the three-month and ten-year yields are presented in the last three rows.

Table 2 Descriptive statistics, yield curve residual from model estimates

Maturity (months)	Mean	Std. Dev.	Min.	Max.	MAE	RMSE	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	0.003	0.084	-0.227	0.453	0.062	0.084	0.604	0.068	-0.042
6	-0.014	0.026	-0.085	0.074	0.024	0.030	0.397	0.242	-0.100
9	-0.009	0.045	-0.277	0.108	0.033	0.046	0.539	0.097	-0.014
12	0.000	0.053	-0.295	0.140	0.040	0.053	0.568	0.071	0.041
15	0.007	0.047	-0.218	0.141	0.037	0.048	0.584	0.070	0.032
18	0.012	0.037	-0.103	0.122	0.030	0.039	0.603	0.119	-0.031
21	0.014	0.029	-0.082	0.115	0.025	0.032	0.600	0.256	-0.124
24	0.013	0.027	-0.074	0.110	0.023	0.030	0.534	0.332	-0.101
30	0.005	0.038	-0.077	0.222	0.028	0.039	0.477	0.124	0.054
36	-0.005	0.050	-0.132	0.232	0.039	0.050	0.524	0.042	0.030
48	-0.020	0.062	-0.189	0.174	0.051	0.065	0.632	0.152	-0.108
60	-0.021	0.061	-0.245	0.196	0.048	0.065	0.658	0.278	-0.153
72	-0.013	0.053	-0.216	0.167	0.038	0.055	0.635	0.275	-0.083
84	-0.004	0.041	-0.194	0.109	0.031	0.041	0.643	0.215	0.009
96	0.003	0.029	-0.111	0.089	0.022	0.029	0.683	0.252	-0.101
108	0.010	0.041	-0.133	0.159	0.031	0.042	0.672	0.253	-0.132
120	0.019	0.076	-0.192	0.308	0.058	0.078	0.705	0.218	0.010

Notes: We fit the Nelson-Siegel three-factor model using monthly yield data from 1986.01 to 2007.02 for different maturities, with λ_t fixed at 0.0609. The table presents in-sample fit error statistics including mean, standard deviation, minimum, maximum, mean absolute error (MAE) and root mean square error (RMSE). The last three columns show the residual autocorrelations at displacements of 1, 12 and 30 months.

Table 3 Estimated factor statistics

Factor	Mean	Std. Dev.	Min.	Max.	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
$\hat{\beta}_{1t}$	7.4224	1.9822	3.9007	11.4620	0.9771	0.8166	0.5764
$\hat{\beta}_{2t}$	-1.5132	2.0100	-5.2270	3.5661	0.9642	0.4784	0.0519
$\hat{\beta}_{3t}$	-1.2634	1.9631	-6.1144	3.1051	0.8322	0.2271	-0.0803

Notes: The table shows descriptive statistics of the three estimated factors for the Nelson-Siegel three-factor model. We fix λ_t at 0.0609. The last three columns show the sample autocorrelations at displacements of 1, 12 and 30 months.

Table 4 Out-of-sample forecasting statistics, 1-month forecast horizon

Maturity	Mean	Std. Dev.	RMSE	$\hat{\rho}(1)$	$\hat{\rho}(12)$
Nelson-Siegel with VAR(1) factor dynamics					
3 months	0.0324	0.1594	0.1617	0.3786	0.0373
1 year	-0.0513	0.2503	0.2540	0.3757	0.0813
3 years	-0.0625	0.2443	0.2507	0.1307	-0.0618
5 years	-0.0703	0.2171	0.2269	0.1024	-0.1014
10 years	-0.0599	0.1846	0.1930	-0.0477	0.0055
Random Walk					
3 months	-0.0096	0.1913	0.1904	0.5658	-0.0047
1 years	-0.0214	0.2325	0.2321	0.2714	0.0314
3 years	-0.0298	0.2415	0.2419	0.0962	-0.0296
5 years	-0.0306	0.2145	0.2154	0.0589	-0.0792
10 years	-0.0291	0.1883	0.1895	-0.0577	-0.0367
AR(1) on yield levels					
3 months	-0.0320	0.1903	0.1919	0.5628	-0.0394
1 years	-0.0493	0.2306	0.2345	0.2605	0.0153
3 years	-0.0616	0.2394	0.2458	0.0860	-0.0328
5 years	-0.0567	0.2133	0.2195	0.0540	-0.0795
10 years	-0.0418	0.1886	0.1921	-0.0569	-0.0344
VAR(1) on yield levels					
3 months	0.0125	0.1642	0.1637	0.3617	0.0017
1 years	-0.0355	0.2417	0.2429	0.2959	0.0302
3 years	-0.0486	0.2517	0.2549	0.1951	-0.0934
5 years	-0.0376	0.2260	0.2278	0.1709	-0.1305
10 years	-0.0428	0.1937	0.1972	0.0048	-0.0442
VAR(1) on yield changes					
3 months	0.0284	0.1991	0.1999	0.9665	0.1618
1 year	0.0139	0.2403	0.2392	0.9449	0.2382
3 years	0.0010	0.2458	0.2444	0.9159	0.3914
5 years	-0.0012	0.2195	0.2182	0.9078	0.4596
10 years	-0.0020	0.1911	0.1899	0.9137	0.5501

Table 4 continued

Maturity	Mean	Std. Dev.	RMSE	$\hat{\rho}(1)$	$\hat{\rho}(12)$
Slope Regression					
3 months	NA	NA	NA	NA	NA
1 year	-0.1749	0.2369	0.2934	0.9471	0.2055
3 years	-0.0058	0.2395	0.2382	0.9229	0.3720
5 years	0.0128	0.2141	0.2132	0.9166	0.4491
10 years	0.0446	0.1882	0.1923	0.9178	0.5475
Fama-Bliss forward rate regression					
3 months	0.0334	0.2108	0.2122	0.5713	0.0554
1 years	0.0378	0.1758	0.1788	0.4740	0.1180
3 years	0.0176	0.2385	0.2377	0.2414	0.0536
5 years	0.0020	0.2422	0.2407	0.1010	-0.0300
10 years	-0.0020	0.2143	0.2130	0.0599	-0.0804
Cochrane-Piazzesi forward curve regression					
3 months	-0.0089	0.1793	0.1785	0.5178	0.0319
1 years	-0.0595	0.2513	0.2568	0.4221	0.0849
3 years	-0.0786	0.2460	0.2569	0.2411	-0.0022
5 years	-0.0687	0.2178	0.2271	0.2022	-0.0402
10 years	-0.0517	0.1952	0.2008	0.0659	-0.0049

Notes: The out-of-sample with 1-month horizon forecast statistics by eight models are presented in the table. We take recursive forecasts, using data from 1986.01 to the time that the forecast is made, starting from 2000.01 to 2007.02. Forecast error at time $t+h$ is defined as $y_{t+h}(\tau) - \hat{y}_{t+h(\tau)}$. In this case, h=1. Error statistics including mean, standard deviation, minimum, root mean square error (RMSE) are reported. The last two columns show the 1st and 12th autocorrelations.

Table 5 Out-of-sample forecasting statistics, 6-month forecast horizon

Maturity	Mean	Std. Dev.	RMSE	$\hat{\rho}(6)$	$\hat{\rho}(18)$
Nelson-Siegel with VAR(1) factor dynamics					
3 months	0.1778	0.7637	0.7797	0.1605	-0.2832
1 year	0.0542	0.7679	0.7652	0.0850	-0.1563
3 years	0.0081	0.5640	0.5606	-0.2585	0.0337
5 years	-0.0025	0.4501	0.4474	-0.3550	0.0379
10 years	0.0140	0.3385	0.3367	-0.3407	-0.0313
Random Walk					
3 months	-0.0424	0.7990	0.7954	0.2131	-0.3050
1 years	-0.1005	0.7511	0.7534	0.0484	-0.1544
3 years	-0.1434	0.5692	0.5837	-0.2493	0.0606
5 years	-0.1479	0.4586	0.4793	-0.3458	0.0659
10 years	-0.1426	0.3395	0.3664	-0.3907	-0.0156
AR(1) on yield levels					
3 months	-0.2405	0.7560	0.7891	0.1972	-0.4107
1 years	-0.2977	0.6998	0.7567	0.0063	-0.2471
3 years	-0.3440	0.5234	0.6237	-0.2945	0.0066
5 years	-0.2958	0.4273	0.5176	-0.3709	0.0267
10 years	-0.2155	0.3263	0.3895	-0.3770	-0.0384
VAR(1) on yield levels					
3 months	-0.0546	0.6509	0.6493	-0.0001	-0.2122
1 years	-0.3079	0.7085	0.7687	0.0277	-0.2248
3 years	-0.3906	0.5461	0.6688	-0.0190	-0.1499
5 years	-0.3393	0.4327	0.5478	-0.0955	-0.1328
10 years	-0.2694	0.3266	0.4219	-0.1339	-0.1078
VAR(1) on yield changes					
3 months	0.2257	0.7367	0.7663	0.6100	-0.1106
1 year	0.1443	0.7502	0.7596	0.6799	-0.0292
3 years	0.0656	0.5699	0.5703	0.7481	0.1938
5 years	0.0558	0.4640	0.4646	0.7595	0.2849
10 years	0.0479	0.3445	0.3458	0.7640	0.3341

Table 5 (continued)

Maturity	Mean	Std. Dev.	RMSE	$\hat{\rho}(6)$	$\hat{\rho}(18)$
Slope Regression					
3 months	NA	NA	NA	NA	NA
1 year	-0.8651	0.8415	1.2034	0.6087	-0.1247
3 years	0.2151	0.5511	0.5885	0.6851	0.2098
5 years	0.2353	0.4414	0.4979	0.6974	0.2858
10 years	0.3076	0.3281	0.4483	0.7070	0.3078
Fama-Bliss forward rate regression					
3 months	0.2513	0.7754	0.8107	0.3061	-0.0135
1 years	0.1174	0.7678	0.7723	0.0845	-0.0659
3 years	0.0271	0.5778	0.5750	-0.2397	0.0595
5 years	0.0056	0.4633	0.4606	-0.3508	0.0584
10 years	0.0069	0.3500	0.3480	-0.3918	0.0110
Cochrane-Piazzesi forward curve regression					
3 months	-0.0743	0.5992	0.6003	0.0555	-0.2698
1 years	-0.3363	0.7143	0.7857	0.2498	-0.1985
3 years	-0.4177	0.5856	0.7165	0.2212	-0.1069
5 years	-0.3722	0.4766	0.6026	0.1185	-0.0883
10 years	-0.2649	0.3689	0.4524	0.0137	-0.1451

Notes: The out-of-sample with 6-month horizon forecast statistics by eight models are presented in the table. We take recursive forecasts, using data from 1986.01 to the time that the forecast is made, starting from 2000.01 to 2007.02. Forecast error at time *t+h* is defined as $y_{t+h}(\tau) - \hat{y}_{t+h(\tau)}$. In this case, h=6. Error statistics including mean, standard deviation, minimum, root mean square error (RMSE) are reported. The last two columns show the 6th and 18th autocorrelations.

Table 6 Out-of-sample forecasting statistics, 12-month forecast horizon

Maturity	Mean	Std. Dev.	RMSE	$\hat{\rho}(12)$	$\hat{\rho}(24)$
Nelson-Siegel with VAR(1) factor dynamics					
3 months	0.1250	1.2427	1.2415	-0.2058	-0.0419
1 year	-0.0154	1.1317	1.1250	-0.0633	-0.0430
3 years	-0.0740	0.7472	0.7463	-0.0727	0.0103
5 years	-0.0835	0.5646	0.5674	-0.2321	0.0879
10 years	-0.0614	0.4231	0.4250	-0.3500	0.1623
Random Walk					
3 months	-0.0787	1.2537	1.2488	-0.1884	-0.0519
1 years	-0.1558	1.1216	1.1258	-0.0686	-0.0498
3 years	-0.2151	0.7568	0.7825	-0.0550	-0.0048
5 years	-0.2186	0.5906	0.6265	-0.2272	0.0851
10 years	-0.2108	0.4323	0.4787	-0.3565	0.1628
AR(1) on yield levels					
3 months	-0.5869	1.0727	1.2172	-0.2930	-0.0700
1 years	-0.6049	0.9697	1.1380	-0.1566	-0.0613
3 years	-0.5477	0.6588	0.8538	-0.0875	-0.0152
5 years	-0.4517	0.5195	0.6861	-0.2284	0.0775
10 years	-0.3141	0.3904	0.4993	-0.3419	0.1573
VAR(1) on yield levels					
3 months	-0.3547	1.0017	1.0571	-0.2997	0.0168
1 years	-0.5859	0.9706	1.1288	-0.2529	0.0187
3 years	-0.6161	0.6547	0.8962	-0.2556	-0.0155
5 years	-0.5172	0.4912	0.7113	-0.3183	-0.0507
10 years	-0.3755	0.3623	0.5203	-0.2471	-0.1572
VAR(1) on yield changes					
3 months	0.6375	1.0567	1.2288	0.0956	-0.1532
1 year	0.5380	0.9391	1.0775	0.1930	-0.1320
3 years	0.3834	0.7037	0.7978	0.3193	-0.0588
5 years	0.3338	0.5611	0.6500	0.3478	0.0161
10 years	0.2923	0.4387	0.5250	0.3061	0.1240

Table 6 (continued)

Maturity	Mean	Std. Dev.	RMSE	$\hat{\rho}(12)$	$\hat{\rho}(24)$
Slope Regression					
3 months	NA	NA	NA	NA	NA
1 year	-2.3082	1.6409	2.8264	0.2495	-0.0797
3 years	-0.0735	0.8034	0.8020	0.4741	0.0343
5 years	0.2697	0.6023	0.6567	0.4908	0.0839
10 years	0.6376	0.4161	0.7601	0.4417	0.0723
Fama-Bliss forward rate regression					
3 months	0.3665	1.2104	1.2578	0.1212	-0.0229
1 years	0.1822	1.1013	1.1099	0.0660	-0.0159
3 years	0.0822	0.7413	0.7415	-0.0671	0.0351
5 years	0.0704	0.5715	0.5725	-0.2692	0.1152
10 years	0.0939	0.4041	0.4126	-0.4523	0.1405
Cochrane-Piazzesi forward curve regression					
3 months	-0.3520	1.0401	1.0923	-0.3319	0.0824
1 years	-0.5607	1.0355	1.1722	-0.2383	0.0993
3 years	-0.6050	0.7274	0.9428	-0.2125	0.1139
5 years	-0.5375	0.5380	0.7583	-0.2988	0.1572
10 years	-0.3612	0.3735	0.5180	-0.3629	0.1391

Notes: The out-of-sample with 12-month horizon forecast statistics by eight models are presented in the table. We take recursive forecasts, using data from 1986.01 to the time that the forecast is made, starting from 2000.01 to 2007.02. Forecast error at time $t+h$ is defined as $y_{t+h}(\tau) - \hat{y}_{t+h(\tau)}$. In this case, h=12. Error statistics including mean, standard deviation, minimum, root mean square error (RMSE) are reported. The last two columns show the 12th and 24th autocorrelations.

Table 7 Estimates of the parameters of the yields-macro VAR(1) model

Yield-macro model parameter estimates

	γ	$\beta_{1,t-1}$	$\beta_{2,t-1}$	$\beta_{3,t-1}$	USL_{t-1}	USS_{t-1}	USC_{t-1}	IFR_{t-1}	TOR_{t-1}
$\beta_{1,t}$	-0.2213	0.7540	0.0129	-0.0077	0.3013	0.0924	-0.1190	-0.0319	-0.0110
	(-0.9693)	(17.0698)	(0.6013)	(-0.4528)	(4.2203)	(2.2921)	(-1.3937)	(-0.3888)	(-1.0774)
$\beta_{2,t}$	-0.7301	-0.0175	0.8968	0.0141	0.1374	-0.1432	0.0060	-0.0402	0.0317
	(-1.8293)	(-0.2267)	(23.9980)	(0.4717)	(1.1006)	(-2.0318)	(0.0401)	(-0.2800)	(1.7775)
$\beta_{3,t}$	1.3612	-0.1755	0.1242	0.5707	-0.1186	0.4266	1.2038	-0.1628	0.0392
	(1.7050)	(-1.1367)	(1.6617)	(9.5471)	(-0.4751)	(3.0256)	(4.0334)	(-0.5674)	(1.0985)

Notes: we display the estimates of the parameters of the yields-macro VAR(1) model. The target for overnight rate has more significant impact on latent factors than inflation rate. The U.S. level, slope and curvature suggest much important effects on the Canada. The impact of inflation rate on three factors is neither consistent nor significant.

Table 8 Correlation between latent factors and macro variables

Correlation Coefficient

	β_1	β_2	β_3	USL	USS	USC	IFR	TOR
β_1	1	0.2313*	-0.0752	0.8946*	0.2806*	0.1715*	0.1908*	-0.1063
β_2		1	0.1233	0.4468*	-0.6181*	0.3017*	0.2172*	0.0952
β_3			1	0.2277*	-0.2442*	0.6903*	0.0325	0.2257*
USL				1	0.0039	0.5128*	0.2298*	-0.0094
USS					1	-0.413*	-0.0353	-0.1498*
USC						1	0.1035	0.175*
IFR							1	0.1192
TOR								1

*indicates significant correlation at 95% confidence level.

Notes: we present our results from examining the correlation between Nelson-Siegel three factors and macro variables. The correlation coefficients indicate that U.S. term structure factors are highly related to Canadian yield curve. And target for over night rate and inflation rate appear less or insignificantly correlated with three factors.

Table 9 Diebold-Mariano Tests

Panel A

	Maturity	Random Walk h=1	h=6	h=12
Yield-only Model	3-month	-1.8025*	-0.7901	-0.8086
	1-year	2.2547*	0.2341	-0.6561
	3-year	1.4350	-1.3844	-2.2458*
	5-year	1.8635*	-1.8682*	-3.4477*
	10-year	1.2921	-1.7124*	-2.7100*

Panel B

	Maturity	Yield-Macro Model h=1	h=6	h=12
Yield-only Model	3-month	3.2739*	-2.3193*	-2.1238*
	1-year	0.6506	-1.8566*	-3.2588*
	3-year	2.1857*	0.6400	-2.4673*
	5-year	2.2571*	2.3260*	-0.9380
	10-year	2.8171*	4.0335	2.0342*

*denote significance relative to the asymptotic null distribution at the 10 percent level

Note: We present Diebold–Mariano forecast accuracy comparison tests. In Panel A, the Nelson-Siegel VAR(1) model forecasts against random walk forecasts; In Panel B, yield-macro model out-of-sample forecasts against those of the yield-only model mentioned in Section 4. The null hypothesis is that the two forecasts have the same mean squared error. Negative values indicate superiority of Nelson-Siegel VAR(1) forecasts in Panel A and our yield-macro model forecasts in Panel B. The results indicate that incorporation of macro variables contributes to produce more accurate out-of-sample forecasts for all maturities less than 10-year in 12-month-ahead forecasting and also demonstrate minor or no improvement for 6-month and 1-month-ahead horizon.