THE IMPACT OF NETTING AND PORTFOLIO EFFECTS ON THE PRICE OF VULNERABLE OPTIONS

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ABSTRACT

This thesis extends the model of Klein and Inglis (2001) by taking into account the effect of a netting agreement, e.g. an ISDA Master Agreement, and the effect of portfolio diversification on the price of vulnerable European options. The model considers options traded mutually between two option writers one of which may default. Based on this model the credit-risk adjusted price of an option is a conditional price with respect to the portfolio of options to which it is added. Using a numerical approximation (Monte-Carlo simulation), netting and portfolio effects are shown to increase the credit-risk adjusted value of a trading position. The paper shows that the price which a counterparty is willing to receive (pay) for selling (buying) an option, is less (more) than the usual price if the option has a credit risk mitigation effect on the existent portfolio.

Keywords: Vulnerable Options, Netting, Default, Credit Risk, Pricing.

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CHAPTER 1: INTRODUCTION

Over the last decade, the largest banks in the world have developed sophisticated models in an attempt to evaluate the credit risk arising from important aspects of their business lines. These models are intended to help banks in quantifying, aggregating and managing risks. As a result of the banks' individual efforts, models lead to differences when designing and pricing financial instruments. These differences constitute a competitive advantage for those who are able to develop and apply pricing models.

The tremendous growth in the unregulated markets that trade these financial instruments, specifically the over-the-counter markets, creates increasing requirements to mitigate credit risk. One of the most efficient and common strategies in the market is to agree on collateral or guarantees by third counterparties. Nevertheless, those instruments are not always feasible or even desirable, allowing for other solutions such as the aggregation of two or more obligations to achieve a reduced net obligation. This netting process clearly reduces the credit risk of the contract and thus leads to a modification of the overall risk of the transaction which should be included in the model when pricing those instruments. If the risk mitigation due to the netting can be modelled, there is a huge competitive advantage to be exploited.

Various models have been developed to calculate the price of options that may be affected by a credit default (vulnerable options). This paper deals with the stream of literature that arose from the structural default models of Black and Scholes (1973) and Merton (1974). Specifically, this paper adopts the pricing model produced by Klein and Inglis (2001) in which important real-world considerations have been taken into account: stochastic interest rates, option payout linked to the firm value, possibility of other liabilities and default before and at maturity. Additionally, no independency assumption is made, allowing for correlation between the assets of the option writer and the option underlying.

The contribution of this paper is to model the impact of netting and of portfolio diversification when valuing vulnerable European options. The remaining of the paper is organized as follows: Chapter 2 gives a description of the existing pricing models and relevant netting methodologies. Chapter 3 values vulnerable options on a stand-alone basis, replicating the findings of Klein and Inglis (2001). Chapter 4 specifies a model to price a portfolio of options with a netting agreement in place and provides some numerical examples. The credit-risk adjusted price of an option is shown to be a conditional price with respect to the portfolio of options to which it is added. Chapter 5 demonstrates the effects of portfolio diversification on the price of vulnerable options. Chapter 6 gives a summary of the conclusions and provides directions for future research.

CHAPTER 2: LITERATURE REVIEW

The structure of this section is linked to the main objective of the paper: Defining the impact of netting and of portfolio diversification when valuing vulnerable European options. Consequently, the first part tracks back the different models used to price European options when credit risk of the option writer is considered (vulnerable options). The second part reviews available literature on netting agreements and specifies the definitions commonly used in these type contracts, with a focus on standards developed by the International Swaps and Derivatives Association (ISDA). Finally, the last part reviews existent pricing models that consider netting or portfolio effects when finding the price for derivative instruments.

Pricing models on vulnerable options

The impact of credit risk on the price of financial instruments, such as derivatives, can be traced back to 1973 and 1974, when Black & Scholes and Merton created the basis for valuing corporate liabilities. This model is considered as the basis for structural default models, which have been extended to price different types of derivatives under various assumptions.

Fourteen years later Johnson and Stulz (1987) develop the first model to price options taking into account credit risk, in this case, the default of the option writer. The model has some important considerations. It allows for correlation between the value of the assets of the option writer and the underlying. And more important, it conditions the default event on the value of the written option which results in a recovery amount that is related to the value of the firm. Nevertheless, the model has several restrictions that are not realistic. For instance, the model does not consider other liabilities of the option writer and defines default as an event only possible at maturity.

Improvements to this model were made by Hull and White (1995) by including other liabilities into the financial structure of the option writer. Additionally, default is not restricted to take place only at maturity but during the life of the option. Default happens if the value of the assets falls below a defined fixed boundary. However, the recovery rate is now considered as a ratio of the claim and not directly dependent on the value of the assets.

At the same time, Jarrow and Turnbull (1995) develop a pricing model that not only considers default of the option writer but also of the underlying asset. Also, the model assumes independency between the value of the option and the default boundary which creates a pay out ratio that is exogenous to the model.

Klein (1996) extends Johnson and Stulz (1987) by considering other liabilities of the option writer and including dead weights costs in the payoff, thus

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bankruptcy costs are reflected in this model. The pay-off ratio in case of default is linked to the assets of the option writer. Nevertheless, the model does not recognize the effects of the option payoff on the default boundary which restricts it to situations where the option liabilities do not constitute an important portion of the ongoing liabilities.

This problem was solved by Klein and Inglis (2001). Their model maintains most of the relevant attributes of Klein (1996) and makes significant improvements, the most important being the incorporation a default boundary with a stochastic component that corresponds to the value of the option at maturity.

This paper selects the Klein and Inglis (2001) model as the most advanced and flexible in this family of models and develops it further to allow for netting of claims and portfolio diversification effects from mutually traded options.

Netting agreements for derivatives trading

The process of netting combines multiple off-setting claims into a single claim (Bergman, Bliss & Johnson, 2003). The rights to receive amounts and the obligation to pay amounts under different transactions are summed up to a single net amount which, in the course of normal business as well as in the event of an insolvency, is alone payable or receivable, depending on the sign.

Netting can substantially reduce credit risk for the counterparties that enter into a netting agreement. In case of insolvency the non-defaulted counterparty does not risk to lose the sum of its individual claims but only the net claim after considering all obligations to the defaulted counterparty.

With regard to derivatives, netting implies the summation of all market values of the transactions under consideration. Consequently, the negotiation of a netting agreement can considerably free up existing credit lines and allow for more business to be done at the same level of credit risk. In a survey performed by Deloitte Development LLC (2007), one out of two financial institutions acknowledged to use on-balance/off-balance sheet netting to reduce credit risk. As such, the netting agreement is and has been a catalyst for the tremendous growth of derivatives markets (Bergman, Bliss & Johnson, 2003).

An agreement to net mutual claims usually comes along with a provision to close out all transactions in case of insolvency ("close-out netting"). This implies that in case of insolvency all derivatives are settled at their current market value and there is no further uncertainty about the magnitude of the claim to the defaulted counterparty.

Bergmann, Bliss & Johnson (2003) state that netting puts creditors, other than the netting party, at a disadvantage. The assets of the defaulted counterparty are distributed preferably to counterparties with netting agreements, thus a netting agreement can be seen as creating an unpublicized security. This

paper does not deal with the economic implications to creditors other than the transaction counterparties under consideration.

One particular market standard for netting agreements which grew popular over the last 20 years is the template provided by the International Swaps and Derivatives Association (ISDA). The ISDA standard enables counterparties to negotiate in a very succinct way a netting agreement by completing a schedule to an otherwise unchanged template (Master Agreement). The ISDA Master Agreement is also popular for its legal enforceability under the jurisdiction of most financially developed countries[1].

The impact of netting on prices of derivative instruments

Netting can impact the prices of options, swaps and other financial instruments. Duffie and Huang (1996) show that the value of a portfolio of swaps is equal or greater with netting in place than without netting.

Numerical examples for the impact of netting provisions on swaps are also given by Duffie and Huang (1996). In their example, the fixed-coupon rate that is agreed when entering into a swap is a function of a previously existing swap with reversed fixed and floating payments. The hedge ratio, i.e. ratio of notional amounts of existing and new swap, determined the new fixed-coupon rate. The

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[1] ISDA publishes the legal opinions it obtained from law firms at www.isda.org

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rate is a linear function of the hedge ratio for values between zero and one, and assumes a constant value for hedge ratios above one.

Hull (2006) describes the payoffs of derivatives as being contingent on the ability of the counterparty to pay. In this sense the expected loss from default is represented as an option payoff, regardless of the nature of the claims. With netting, the payoff to the non-defaulted counterparty is those of an option on a portfolio of contracts, while without netting it is the payoff of a portfolio of options, which has a smaller value.

Cooper and Mello (1999) show that a bank will offer better terms when entering into a forward contract with a corporate customer if the forward is motivated by hedging purposes and the bank holds some of the outstanding debt of the customer. The favourable change to the forward rate is due to a reduction in credit risk. However, the risk reduction is not a reward for netting the claims under the debt and the forward, but acknowledges that the hedge increases the value of the firm to the debt holders.

There is no literature dealing specifically with the impact of netting and portfolio diversification effects on the prices of vulnerable options. The main contribution of this paper will be to develop a pricing model for options, taking into account netting and portfolio effects, and to provide numerical solutions based on the model.

CHAPTER 3: REPRODUCTION OF VULNERABLE OPTION PRICES OF KLEIN AND INGLIS (2001)

Model Setting

The assumptions made by Klein and Inglis (2001) will be valid for the model developed in this paper and are reproduced and extended below.

Assumption 1: V is the market value of the assets of the option writer. V follows a geometric Brownian motion given by:

(1)
$$\frac{dV}{V} = \mu_V dt + \sigma_V dz_V$$

Where μ_V is the instantaneous expected return, σ_V is the instantaneous standard deviation and z_V is the standard Wiener process.

Assumption 2: S is the market value of the asset underlying the option. S follows a geometric Brownian motion given by:

(2)
$$\frac{dS}{S} = \mu_S dt + \sigma_S dz_S$$

Where μ_S is the instantaneous expected return, σ_S is the instantaneous standard deviation and z_S is a standard Wiener process. The correlation between z_V and z_S is ρ_{VS}.

Assumption 3: Markets are perfect and frictionless. There are no taxes, transaction costs or information asymmetries. Securities can be traded in continuous time.

Assumption 4: Default occurs at the expiration of the option, T, only if the value of the option writer's assets V_T is less than a threshold value D^*+L. D^* denotes the value of the other liabilities of the option writer. It can be seen as the face value of a zero coupon bond that has the same maturity as the option under consideration[2]. Klein and Inglis (2001) define L as the payoff under the call option ($C_T = \max(S_T - K,0)$) where S_T represents the price of the underlying asset at maturity date of the option and K is the strike price of the option. For the model developed in this paper, L can denote the payoff of a portfolio of options and, in case of a netting agreement in place, represent a net claim against the defaulted counterparty.

Assumption 5: The nominal claim of the option holder is the intrinsic value of the option at maturity.

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[2] Other liabilities of the option writer have a fixed value D^*. There are no bond covenants restricting future derivatives transactions.

Assumption 6: In case of financial distress, the option holder receives only a proportion $(1-\alpha)V_T/(D^* + L)$ of the nominal claim, where α are the deadweight costs of the financial distress.

The value for a vulnerable call option ($\hat{C}$), defined in Klein and Inglis (2001), can be expressed in two stages. The first corresponds to the payoff at maturity:

(3)
$$\hat{C}_T = \begin{cases} S_T - K & S_T \geq K & V_T \geq D^* + S_T - K \\ (1-\alpha)V_T \dfrac{S_T - K}{D^* + S_T - K} & S_T \geq K & V_T < D^* + S_T - K \\ 0 & Otherwise \end{cases}$$

The second corresponds to the time-t value of the vulnerable option, $\hat{C}_t$, which is the risk-neutral expectation of the option's payoff at maturity, $\hat{C}_T$.

(4)
$$\hat{C}_t = e^{-r(T-t)}E^*\left[\hat{C}_T\right]$$

Re-estimation with Monte-Carlo simulation

The payoff of a vulnerable European call option, as defined in the model of Klein and Inglis (2001), is reproduced using a Monte-Carlo simulation. Equation (3) can be expressed in a computationally convenient way as:

(5)

$$\hat{C}_T = \left[\left(S_{T,i} - K\right)[1]_{\{S_{T,i} \geq K\}}[1]_{\{V_{T,i} \geq D^* + S_{T,i} - K\}} + \cdots\right.$$

$$\left.\left(S_{T,i} - K\right)[1]_{\{S_{T,i} \geq K\}}[1]_{\{V_{T,i} < D^* + S_{T,i} - K\}} \frac{(1-\alpha)V_{T,i}}{D^* + S_{T,i} - K}\right] \qquad i = 1,\ldots,n$$

Where n is the number of simulations and $[1]_{\{\}}$ denotes a digital option that pays a value of one if the condition is fulfilled and zero otherwise.

The model assumes path independency for the underlying asset (S_T), which is non-problematic for a European option. Since default is only observed at maturity date of the option, path independency is assumed for the market value of the firm (V_T) as well. A correlation between asset and underlying, ρ_{VS}, is taken into account by generating correlated random variables for dz_V and dz_S, employing Cholesky factorization.

Table 1 shows the results for vulnerable European calls, calculated by Klein and Inglis (2001), using a binomial tree approach and alternatively an analytical approximation, and compares those with the Monte-Carlo simulation results which are slightly higher than the prices calculated using the binomial tree approach. Black-Scholes prices are given as the credit-risk-free reference price.

Table 1: Prices of European calls

	Numerical Solution (Binomial Tree)	Analytical Approximation	Numerical Solution (Monte-Carlo Simulation)	Black-Scholes
Base Case	6.24	6.28	6.25	8.37
S = 30	2.01	2.03	2.02	2.56
S = 50	11.59	11.74	11.62	16.59
V = 90	5.71	5.76	5.72	8.37
V = 110	6.65	6.73	6.69	8.37
$\rho = 0.5$	7.35	7.39	7.36	8.37
$\rho = -0.5$	5.23	5.28	5.24	8.37
$\sigma_S = 0.15$	5.66	5.67	5.68	7.25
$\sigma_S = 0.25$	6.70	6.87	6.74	9.54
$\sigma_V = 0.15$	6.47	6.58	6.48	8.37
$\sigma_V = 0.25$	5.97	5.99	6.01	8.37
T-t = 2	4.79	5.02	5.00	6.45
T-t = 4	7.27	7.36	7.30	10.09
$\alpha = 0$	7.11	7.16	7.13	8.37
$\alpha = 0.5$	5.36	5.41	5.36	8.37
r = 3%	5.16	5.20	5.17	7.16
r = 7%	7.40	7.47	7.42	9.64

Calculations of vulnerable call option prices $\hat{C}_t$ are based on the following parameters values: S=40, K=40, V=100, D^*=90, T-t=3, α=0.25, σ_V=0.20, σ_S=0.20, ρ=0, r=5% unless otherwise noted. Binomial Tree values and analytical approximation taken from Klein and Inglis (2001). Monte-Carlo simulation performed with 60.000.000 runs. Prices converge to two decimal digits.

CHAPTER 4: MODELING OF NETTING EFFECTS

This section computes the price of a vulnerable European call with a netting agreement in place. Netting can significantly reduce the credit exposure if the credit risk of one trade is offset by the risk of another trade. The price of a vulnerable option can then be seen as a conditional price with respect to the portfolio that existed prior to the transaction[3]. However, to get started the term netting must be specified. In order to do this, the following model is defined:

Model Setting I: Adding a short call to a long call

To illustrate the impact of netting on the option price, the following example is set up. Counterparty A buys from counterparty B a European call option on the underlying asset S_1. A is assumed to have no credit risk while B is risky. Consequently, the call is vulnerable and has values at maturity of $\hat{C}_{1,T}$ and during the life time of the option of $\hat{C}_{1,t}$, equal to those defined in equations (3) and (4) respectively.

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[3] In this paper the conception of price is the compensation for the change in the value of a portfolio, and it varies with the characteristics of the portfolio and the counterparty. While this conception may seem to contradict the traditional economic meaning of a price, it will be shown that the conditional price can be expressed as a tradable price.

An instant later counterparty A sells to B a European call option on the underlying asset S_2. This option has no credit risk and the same exercise date as the call before. Its payoff at maturity is defined by $C_{2,T} = \max(S_{2,T} - K_2, 0)$. The correlation between the two underlying assets is given by $\rho_{S1,S2}$. Since the model developed in this paper deals with the interaction between different options in a portfolio, it is rather this correlation, $\rho_{S1,S2}$ than the correlation between the underlying asset and the assets of the option writer, $\rho_{V,S1}$ and $\rho_{V,S2}$ that is of interest.

The goal for this chapter is to set up a matrix that shows the payoff at maturity to counterparty A in all possible states of nature. It is sufficient to analyze the options' payoff at maturity because time-t values are expressed as risk-neutral expectations of the payoff at maturity and consequently default is only observed at that moment in time. The possible states of nature are:

- The moneyness of the options: To allow for an easier and more generic notation, let $C_{1,T}$ be the claim at maturity under the option bought by A, $C_{1,T} = \max(S_{1,T} - K_1, 0)$, and $C_{2,T}$ be the claim at maturity under the option sold by A, $C_{2,T} = \max(S_{2,T} - K_2, 0)$. For the following analysis, $C_{1,T}$ and $C_{2,T}$ might as well be the claims at maturity under any plain-vanilla option. For the above example, the four possible states are:
  - $C_{1,T} \geq 0, C_{2,T} = 0$: The long call is in the money and the short call is not.
  - $C_{1,T} = 0, C_{2,T} > 0$: The short call is in the money and the long call is not.

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- $C_{1,T} \geq C_{2,T} > 0$: Both options are in the money, the value of the long call is greater.

- $C_{2,T} > C_{1,T} > 0$: Both options are in the money, the value of the long call is smaller.

- The existence of a netting agreement.

- The ability of counterparty B to meet its obligations: As stated in assumption 4, the occurence of a default at maturity depends on the value of the assets of the options writer and the liabilities. With no netting in place, B reports the value of the options it holds as part of its current assets, and the value of the options written as part of its liabilities. In this case the default is given when:

(6)
$$V_T + C_{2,T} < D^* + C_{1,T}$$

With netting in place, B will report the net value of the options, where two cases must be distinguished:

- If $C_{1,T} < C_{2,T}$, the net position is a current asset for B. Then, the default is given by:

(7)
$$V_T - (C_{1,T} - C_{2,T}) < D^*$$

However, in this situation A will not suffer a loss if B defaults: A pays $(C_{2,T} - C_{1,T})$ to B, the netting agreement shields A from a loss on its long position $C_{1,T}$.

- If $C_{1,T} > C_{2,T}$, the net position is a current liability of B to A. The default is given when:

(8) $$V_T < D^* + (C_{1,T} - C_{2,T})$$

Examination of equations (6), (7) and (8) reveals that the default barriers are equal, with our without netting in place. Table 2 shows the cash flows at maturity to counterparty A with this default specification, the netting definition as in Chapter 2 and the vulnerable European option payoff defined by Klein and Inglis (2001).

If B can meet it's obligation to pay at the maturity of the option, the netting agreement does not change the payoffs, consequently these cases are not distinguished in Table 2. However, if B is in default, the netting agreement makes a difference. The payoff to A at maturity is, when both long call and short call mature in the money and with a netting agreement in place, greater than

without netting agreement[4]. This shows that the value of the option portfolio is greater for counterparty A when a netting agreement is in place, similar to the findings of Duffie and Huang (1996) and Hull (2006).

Table 2: Cash flows to counterparty A at maturity.

Possible values for $C_{1,T}$ and $C_{2,T}$	Counterparty B's ability to pay		
	Solvent	Insolvent	
		No Netting	Netting
$C_{1,T} \geq 0, C_{2,T} = 0$	$C_{1,T}$	$\dfrac{(1-\alpha)V_T C_{1,T}}{D^* + C_{1,T}}$	$\dfrac{(1-\alpha)V_T C_{1,T}}{D^* + C_{1,T}}$
$C_{1,T} = 0, C_{2,T} > 0$	$-C_{2,T}$	$-C_{2,T}$	$-C_{2,T}$
$C_{1,T} \geq C_{2,T} > 0$	$C_{1,T} - C_{2,T}$	$\dfrac{(1-\alpha)(V_T + C_{2,T})C_{1,T}}{D^* + C_{1,T}} - C_{2,T}$	$\dfrac{(1-\alpha)V_T(C_{1,T} - C_{2,T})}{D^* + (C_{1,T} - C_{2,T})}$
$C_{2,T} > C_{1,T} > 0$	$C_{1,T} - C_{2,T}$	$\dfrac{(1-\alpha)(V_T + C_{2,T})C_{1,T}}{D^* + C_{1,T}} - C_{2,T}$	$C_{1,T} - C_{2,T}$

Based on Table 2, let $\hat{P}_T(.)$ denote the cash flow at maturity of a vulnerable position to counterparty A and let $\hat{P}_T^*(.)$ denote the cash flow of the

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[4] For $C_{1,T} \geq 0, C_{2,T} = 0$ and $C_{1,T} = 0, C_{2,T} > 0$ the payoffs are equal. Using the constraint $V_T < D^* + (C_{1,T} - C_{2,T})$ that is imposed by the default, it can be shown that the payoff for the cases $C_{1,T} \geq C_{2,T} > 0$ and $C_{2,T} > C_{1,T} > 0$ is greater with a netting agreement.

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same vulnerable position with a netting agreement in place. The previous analysis has shown that:

(9)
$$\hat{P}_T^* \left(\hat{C}_{1,T} - C_{2,T}\right) > \hat{P}_T \left(\hat{C}_{1,T} - C_{2,T}\right)$$

The risk-neutral expectation of the cash-flow for the portfolio with netting is given by:

(10)
$$\hat{P}_t^* = e^{-r(T-t)} E^* \left[\hat{P}_T^*\right]$$

Similarly, without netting the risk-neutral expectation is given by:

(11)
$$\hat{P}_t = e^{-r(T-t)} E^* \left[\hat{P}_T\right]$$

It follows also that for any time-t:

(12)
$$\hat{P}_t^* \left(\hat{C}_{1,T} - C_{2,T}\right) > \hat{P}_t \left(\hat{C}_{1,T} - C_{2,T}\right)$$

Let $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ denote the conditional price of the short call, i.e. the change in the value of the portfolio for A when adding the short call to the long call if no netting is in place:

(13)
$$P_t(-C_{2,T})_{|\hat{C}_{1,T}} = \hat{P}_t\left(\hat{C}_{1,T} - C_{2,T}\right) - \hat{P}_t\left(\hat{C}_{1,T}\right)$$

The conditional price can be seen as the time-t value of a swap that exchanges the cash flows $\hat{P}_T\left(\hat{C}_{1,T} - C_{2,T}\right)$ and $\hat{P}_T\left(\hat{C}_{1,T}\right)$ at maturity. Then it can be considered as a price with a market-wide meaning which any counterparty would agree on, regardless of a previously existing portfolio. Equivalently, for the case with netting the conditional price is given by:

(14)
$$P_t^{\bullet}(-C_{2,T})_{|\hat{C}_{1,T}} = \hat{P}_t^{\bullet}\left(\hat{C}_{1,T} - C_{2,T}\right) - \hat{P}_t^{\bullet}\left(\hat{C}_{1,T}\right)$$

The swap argument holds here as well. Furthermore,

(15)
$$\hat{P}_t^{\bullet}\left(\hat{C}_{1,T}\right) = \hat{P}_t\left(\hat{C}_{1,T}\right)$$

because the long call is the first transaction and there is no other transaction in place that can mitigate the risk even with a netting agreement in place. The price of this long option is equivalent to the vulnerable option price computed with the model of Klein and Inglis (2001).

From (9) and (15) follows that $P_t^{\bullet}(-C_{2,T})_{|\hat{C}_{1,T}} > P_t(-C_{2,T})_{|\hat{C}_{1,T}}$: the conditional price of the short call, when added to the portfolio that consists so far of the long call only, is greater with a netting agreement in place. Furthermore, $P_t^{\bullet}(-C_{2,T})_{|\hat{C}_{1,T}}$,

which is negative, cannot be lower than the value of a Black-Scholes non-vulnerable short option, $-C_{2,t}$, because A cannot default. From:

(16) $$-C_{2,t} \le P_t(-C_{2,T})_{|\hat{C}_{1,T}} < P_t^*(-C_{2,T})_{|\hat{C}_{1,T}} < 0$$

follows that A would be ready to write the call for a premium less than the non-vulnerable (credit-risk-free) price of this option. The option portfolio as it existed prior to writing the call justifies a lower price for this additional transaction because the latter reduces the credit risk of A due to a default of B.

Contractual Assumptions

The effects of a netting agreement as it is defined in Table 2 can be facilitated by making the following contractual assumptions about the specification of the netting agreement:

- The term "netting agreement" refers to a 2002 ISDA Master Agreement.
- The netting agreement is legally enforceable.
- The ISDA schedule contains the following provisions:
  - The counterparties opt to close out the transactions immediately after an event of default occurred (automatic early termination).
  - Bankruptcy (failure to pay the due debt) is the only chosen event of default.

With these assumptions, the default under the ISDA agreement corresponds to an event defined as:

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$$V_T < D^* + L$$

where L is, as specified in assumption 4, the net liability of the defaulted counterparty to the claimant under consideration.

The following section will find a value for the conditional price of the short call option, $P_t^*(-C_{2,T})_{|\hat{C}_{1,T}}$, in different scenarios, based on the netting agreement as it is defined in Table 2 and in line with the ISDA specifications shown above.

Estimation of conditional option prices with and without netting

The initial portfolio is defined as $\hat{P}_t(\hat{C}_{1,T})$, containing a long position in a vulnerable European call with the base case parameters in Table 1. Then a short position in a non-vulnerable European call option is added $(-C_{2,t})$, with parameters that are initially chosen to equal those of the base case. Parameters of both options are later equally changed as defined in Table 3, which shows sensitivity of conditional prices.

The conditional price (base case) with a netting agreement in place, $P_t^*(-C_{2,T})_{|\hat{C}_{1,T}}$, is approximated with -7.66. Obviously, writing an option reduces the value of any given position because it creates the possibility of a cash-outflow at maturity. Consequently, 7.66 is the fair price that counterparty A can demand as a premium paid by B in exchange for obtaining the option.

Table 3: Prices/Conditional prices for different parameters values (Model I). With and without netting.

	$\hat{C}_{1,t}$ (Monte-Carlo)	$-C_{2,t}$ (Black-Scholes)	$\hat{P}_t(\hat{C}_{1,T} - C_{2,T})$	$P_t(-C_{2,T})\big|_{\hat{C}_{1,T}}$	$\hat{P}_t^*(\hat{C}_{1,T} - C_{2,T})$	$P_t^*(-C_{2,T})\big|_{\hat{C}_{1,T}}$
Base Case	+6.25	-8.37	-1.72	-7.97	-1.41	-7.66
$S_i = 30$	+2.02	-2.56	-0.50	-2.52	-0.46	-2.48
$S_i = 50$	+11.62	-16.59	-3.42	-15.04	-2.54	-14.16
$V = 90$	+5.72	-8.37	-2.18	-7.90	-1.77	-7.49
$V = 110$	+6.69	-8.37	-1.33	-8.02	-1.11	-7.80
$\rho_{V,S_i} = 0.5$	+7.36	-8.37	-0.90	-8.26	-0.85	-8.21
$\rho_{V,S_i} = -0.5$	+5.24	-8.37	-2.55	-7.79	-1.89	-7.13
$\sigma_{S_i} = 0.15$	+5.68	-7.25	-1.25	-6.93	-0.95	-6.63
$\sigma_{S_i} = 0.25$	+6.74	-9.54	-2.30	-9.04	-1.99	-8.73
$\sigma_V = 0.15$	+6.48	-8.37	-1.47	-7.95	-1.25	-7.73
$\sigma_V = 0.25$	+6.01	-8.37	-1.95	-7.96	-1.57	-7.58
$T-t = 2$	+5.00	-6.45	-1.19	-6.19	-0.97	-5.97
$T-t = 4$	+7.30	-10.09	-2.22	-9.52	-1.84	-9.14
$\alpha = 0$	+7.13	-8.37	-0.95	-8.08	-0.82	-7.95
$\alpha = 0.5$	+5.36	-8.37	-2.48	-7.84	-2.00	-7.36
$r = 3\%$	+5.17	-7.16	-1.68	-6.85	-1.41	-6.58
$r = 7\%$	+7.42	-9.64	-1.70	-9.12	-1.38	-8.80

Base case: $S_i=S_1=S_2=40$, $K_1=K_2=40$, $\sigma_{S_i}=\sigma_{S1}=\sigma_{S2}=0.20$, $\rho_{V,S_i}=\rho_{V,S1}=\rho_{V,S2}=0$, $\rho_{S1,S2}=0$, $V=100$, $D^*=90$, $T-t=3$, $\alpha=0.25$, $\sigma_V=0.20$, $r=5\%$ unless otherwise noted. Monte-Carlo simulation performed with 60.000.000 runs. Prices converge to two decimal digits.

Since A is assumed to be credit-risk-free, a benchmark for the price for this option is the Black-Scholes price (8.37). The absolute value of the

conditional price $P_t^*(-C_{2,T})_{|\hat{C}_{1,T}}$ is significantly below 8.37, acknowledging the reduction of credit risk of the option portfolio.

It can be seen from Table 3 that for all the cases the conditional price of the short call is, in absolute terms, lower than the Black-Scholes price. For the case with netting, the result confirms the fact that counterparty A will be willing to receive less for writing the option. For counterparty B the result implies the willingness to pay less than the Black-Scholes price for the option. This corresponds to an increment of risk for B when buying an option from a writer that has a claim against B that can potentially be netted. For the case without netting, the result does not follow from the methodology developed so far. It will be explained in the next section. As in Klein and Inglis (2001), the conditional price of the short call increases with the value of the assets and decreases with volatility of the assets of the option writer and the deadweight cost. Special consideration will now be given to effects of the correlation between the two underlying assets on the conditional prices.

Calls on the same underlying (perfect positive correlation)

For a more detailed analysis the base case is set up now for perfectly positively correlated underlying assets. As shown in Table 4, with a netting agreement the conditional price of the sold call $P_t^*(-C_{2,T})_{|\hat{C}_{1,T}}$ exactly equals the cost of the long call. Thus the net value of the portfolio is zero. This is intuitive

because the position is flat with respect to market and credit risk, irrespective of the chosen asset value and asset volatility.

Table 4: Prices/Conditional prices for perfect positive correlation

	$\hat{C}_{1,t}$ (Monte-Carlo)	$-C_{2,t}$ (Black-Scholes)	$P_t(-C_{2,T})_{\mid \hat{C}_{1,T}}$	$P_t^{*}(-C_{2,T})_{\mid \hat{C}_1}$	$P_t(C_{2,T})_{\mid \hat{C}_{1,T}} - C_{2,t}$	$P_t^{*}(C_{2,T})_{\mid \hat{C}_{1,T}} - P_t(C_{2,T})_{\mid \hat{C}_{1,T}}$
Base Case[+]	+6.25	-8.37	-7.11	-6.25	+1.26	+0.86
$\sigma_V = 0$	+7.03	-8.37	-7.03	-7.03	+1.34	+0.00
V=10	+0.63	-8.37	-6.96	-0.63	+1.41	+6.33
V=1000	+8.37	-8.37	-8.37	-8.37	+0.00	+0.00
R=3%	+5.17	-7.16	-6.08	-5.17	+1.08	+0.91
R=7%	+7.42	-9.64	-8.19	-7.42	+1.45	+0.77

Base case[+] for calls on identical underlyings (perfect positive correlation): $S_1=S_2=40$, $K_1= K_2=40$, $\sigma_{S1}=\sigma_{S2}=0.20$, $\rho_{V,S1}=\rho_{V,S2}=0$, $\rho_{S1,S2}=1$, V=100, D[*]=90, T-t=3, $\alpha=0.25$, $\sigma_V=0.20$, r=5% unless otherwise noted. Monte-Carlo simulation performed with 60.000.000 runs. Prices converge to two decimal digits.

For the no-netting case the results look different. The conditional price $P_t(-C_{2,T})_{\mid \hat{C}_{1,T}}$ is not off-setting the premium of the long call, because counterparty A is exposed to credit risk. This portfolio can never generate a positive payoff to A because:

- If the option payoffs are positive and B does not default, the net gain is zero $(C_{1,T} - C_{2,T})$, but

- If the option payoffs are positive and B defaults, A has a negative net gain because it pays $C_{2,T}$ but does not receive the full value of $C_{1,T}$.

Consequently $\hat{P}_T(\hat{C}_{1,T} - C_{2,T})$ is either zero or negative. Then, $\hat{P}_t(\hat{C}_{1,T} - C_{2,T})$ must be negative, and thus the absolute value of $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ be greater than the price of the call $\hat{P}_t(\hat{C}_{1,T})$. Now, should $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ equal the credit-risk-free price $-C_{2,t}$? One might argue that this must be the case because without netting the short call cannot have a credit-risk mitigating effect on the long call. However, the conclusion is wrong because A contributes to the solvency of B by paying the option payoff $C_{2,T}$ to B. The conditional price $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ is given by -7.11, compared to the Black-Scholes price of -8.37. Table 4 shows that $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ approaches the credit-risk-free price if the asset value of B becomes very high and default consequently unlikely.

It is interesting to note the effect of decreasing the asset volatility, in the limit to zero. In the shown example, the conditional price $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ offsets the price of the long call. This is the case because $V_T > D^*$ and, with $C_{1,T} = C_{2,T}$, $V_T + C_{1,T} > D^* + C_{2,T}$. The conditional price $P_t(-C_{2,T})_{|\hat{C}_{1,T}}$ will be (in absolute terms) higher than $\hat{P}_t(\hat{C}_{1,T})$ if $V_T < D^*$.

Calls on underlyings with perfect negative correlation

In this section the conditional price of the short call is calculated assuming perfect negative correlation between the underlying assets. Again the netting case is considered first. Other than with positively correlated underlyings, the position $\hat{P}_t^*(\hat{C}_{1,T} - C_{2,T})$ is not flat with regard to market and credit risk.

Unfortunately, perfectly negatively correlated Wiener processes for the underlyings do not imply perfect negative correlation of the option payoffs. The asymmetry of option payoffs by definition rules out that $C_{1,T} = -C_{2,T}$, apart from the trivial solution $C_{1,T} = C_{2,T} = 0$. One also cannot conclude that the asymmetry imposes a boundary of the form $C_{2,T} = 0$ for $C_{1,T} > 0$ or $C_{1,T} = 0$ for $C_{2,T} > 0$, even not if the strikes are equal: S_1 and S_2 grow at a drift of $(r - \sigma^2/2)$, hence both underlyings, though negatively correlated, may trade at maturity above the strike, and are more likely to do so if the drift increases. Consequently, netting can improve the position of counterparty A, but to a much lesser degree than it would do with positively correlated underlying assets. Table 5 confirms the hypothesis. The conditional price of the call with netting is, when compared with the no-netting case, 0.04 lower, while with positive correlation the credit risk reduction was rewarded with 0.86. Figure 1 illustrates this effect. Credit risk reduction also

changes when varying interest rates and it is zero for an interest rate of $r = \sigma^2/2$, chosen to set the drift to zero.

Table 5: Prices/Conditional prices for perfect negative correlation

	$\hat{C}_{1,t}$ (Monte-Carlo)	$-C_{2,t}$ (Black-Scholes)	$P_t(-C_{2,T})|_{\hat{c}_{1,T}}$	$P_t^*(-C_{2,T})|_{\hat{c}_{1,T}}$	$P_t(C_{2,T})|_{\hat{c}_{1,T}} - C_{2,t}$	$P_t^*(C_{2,T})|_{\hat{c}_{1,T}} - P_t(C_{2,T})|_{\hat{c}_{1,T}}$
Base Case¯	+6.25	-8.37	-8.37	-8.33	+0.00	+0.04
$\sigma_V = 0$	+7.03	-8.37	-8.36	-8.36	+0.01	+0.00
V=10	+0.63	-8.37	-8.36	-8.07	+0.01	+0.29
V=1000	+8.37	-8.37	-8.37	-8.37	+0.00	+0.00
$r=\sigma^2/2=2\%$	+4.67	-6.58	-6.61	-6.61	-0.03	+0.00
R=3%	+5.17	-7.16	-7.16	-7.16	+0.00	+0.00
R=7%	+7.42	-9.64	-9.61	-9.53	+0.03	+0.08

Base case¯ for calls on underlyings (perfect negative correlation): $S_1=S_2=40$, $K_1=K_2=40$, $\sigma_{S1}=\sigma_{S2}=0.20$, $\rho_{V,S1}=\rho_{V,S2}=0$, $\rho_{S1,S2}=-1$, V=100, $D^*=90$, T-t=3, $\alpha=0.25$, $\sigma_V=0.20$, r=5% unless otherwise noted. Monte-Carlo simulation performed with 60.000.000 runs. Prices converge to two decimal digits.

Although in reality it will be difficult to find equity underlyings that are negatively correlated, the argument developed in this section is not irrelevant because negative correlation can be created by a call and a put on positively correlated assets.

Figure 1: Conditional prices, with netting and no netting effects, for different correlations
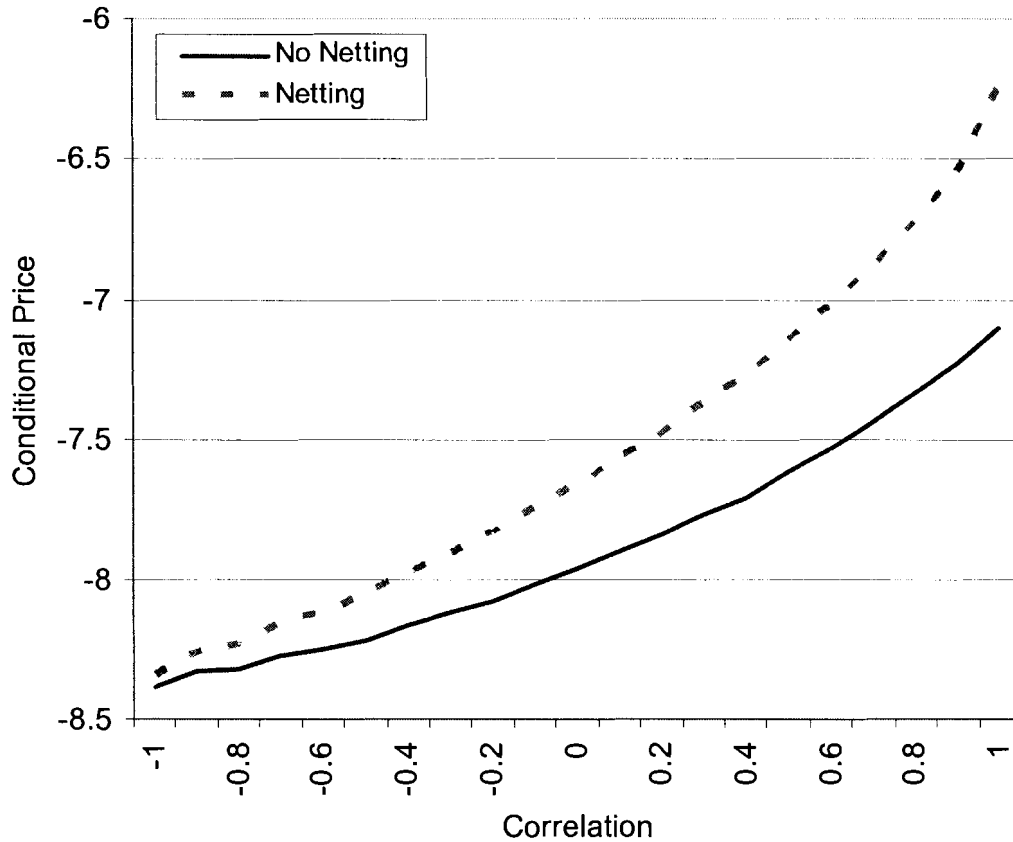
Base case for calls on underlyings with varying correlation: $S_1=S_2=40$, $K_1=K_2=40$, $\sigma_{S1}=\sigma_{S2}=0.20$, $\rho_{V,S1}=\rho_{V,S2}=0$, $V=100$, $D^*=90$, $T-t=3$, $\alpha=0.25$, $\sigma_V=0.20$, $r=5\%$. $\rho_{S1,S2}$ is changed on increments of 0.1, from -1 to 1. Each value is based on simulations of 4.000.000 runs.

Model Setting II: Adding a long call to a short call

In this scenario, counterparty A sells to counterparty B a European call option on the underlying asset S_2. A is again assumed to have no credit risk while B is risky. Consequently, the call is non-vulnerable. Its payoff at maturity is again defined by $C_{2,T} = \max(S_{2,T} - K_2, 0)$.

An instant later counterparty A buys from B a European call option on the underlying asset S_1. Consequently, the call is vulnerable and has values at maturity of $\hat{C}_{1,T}$ and during the life time of the option of $\hat{C}_{1,t}$, equal to those defined in equations (3) and (4), respectively. This option has the same exercise date as the call before. The correlation between the two underlying assets is given by $\rho_{S1,S2}$. All other specification made for model setting I remain unchanged. Consequently, the payoff to counterparty A from the portfolio of options at maturity is that one specified in Table 2.

Similar to Equation (16), but with the model setting described above, the following inequality holds:

(18)
$$0 < P_t(\hat{C}_{1,T})\big|_{C_{2,T}} < P_t^*(\hat{C}_{1,T})\big|_{C_{2,T}} \le C_{1,t}$$

Consequently, counterparty A will be willing to pay more than the price of the vulnerable price of the Klein and Inglis (2001) model. The following section will find the conditional price for a long call in different scenarios.

Estimation of conditional option prices with and without netting

The initial portfolio is defined as $P_t(-C_{2,T})$, containing a short position in a non-vulnerable European call with the base case parameters in Table 1. Then a long position in a vulnerable European call option $(\hat{C}_{1,t})$ is added, with

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parameters that are initially chosen to equal those of the base case. Parameters of both options are later equally changed as defined in Table 6.

Table 6: Prices/Conditional prices for different parameters values (Model II). With and without netting.

	$-\hat{C}_{2,t}$ (Black-Scholes)	$\hat{C}_{1,t}$ (Monte-Carlo)	$\hat{P}_t(\hat{C}_{1,T} - C_{2,T})$	$P_t(-\hat{C}_{1,T})_{|-C_2}$	$\hat{P}_t^*(\hat{C}_{1,T} - C_{2,T})$	$P_t^*(-\hat{C}_{1,T})_{|-C_2}$
Base Case	-8.37	+6.25	-1.72	+6.65	-1.41	+6.96
$S_i = 30$	-2.56	+2.02	-0.50	+2.06	-0.46	+2.10
$S_i = 50$	-16.59	+11.62	-3.42	+13.17	-2.54	+14.05
V = 90	-8.37	+5.72	-2.18	+6.19	-1.77	+6.60
V = 110	-8.37	+6.69	-1.33	+7.04	-1.11	+7.26
$\rho_{V,S_i} = 0.5$	-8.37	+7.36	-0.90	+7.47	-0.85	+7.52
$\rho_{V,S_i} = -0.5$	-8.37	+5.24	-2.55	+5.82	-1.89	+6.48
$\sigma_{S_i} = 0.15$	-7.25	+5.68	-1.25	+6.00	-0.95	+6.30
$\sigma_{S_i} = 0.25$	-9.54	+6.74	-2.30	+7.24	-1.99	+7.55
$\sigma_V = 0.15$	-8.37	+6.48	-1.47	+6.90	-1.25	+7.12
$\sigma_V = 0.25$	-8.37	+6.01	-1.95	+6.42	-1.57	+6.80
T-t = 2	-6.45	+5.00	-1.19	+5.26	-0.97	+5.48
T-t = 4	-10.09	+7.30	-2.22	+7.87	-1.84	+8.25
$\alpha = 0$	-8.37	+7.13	-0.95	+7.42	-0.82	+7.55
$\alpha = 0.5$	-8.37	+5.36	-2.48	+5.89	-2.00	+6.37
r = 3%	-7.16	+5.17	-1.68	+5.48	-1.41	+5.75
r = 7%	-9.64	+7.42	-1.70	+7.94	-1.38	+8.26

Base case: $S_i = S_1 = S_2 = 40$, $K_1 = K_2 = 40$, $\sigma_{S_i} = \sigma_{S1} = \sigma_{S2} = 0.20$, $\rho_{V,S_i} = \rho_{V,S1} = \rho_{V,S2} = 0$, $\rho_{S1,S2} = 0$, V=100, $D^* = 90$, T-t=3, $\alpha = 0.25$, $\sigma_V = 0.20$, r=5% unless otherwise noted. Monte-Carlo simulation performed with 60.000.000 runs. Prices converge to two decimal digits.

Reconciliation of Model I and Model II

Unsurprisingly, the value of the portfolio $\hat{P}_t(\hat{C}_{1,T} - C_{2,T})$ is the same for model I and model II, regardless if the short call is added to the long call or vice versa. The same statement is valid for the netting case $\hat{P}_t^*(\hat{C}_{1,T} - C_{2,T})$:

(19)
$$\hat{P}_t^*(\hat{C}_{1,T} - C_{2,T}) = P_t(-C_{2,T}) + \hat{P}_t^*(\hat{C}_{1,T})\big|_{-C_{2,T}} = \hat{P}_t(\hat{C}_{1,T}) + P_t^*(-C_{2,T})\big|_{\hat{C}_{1,T}}$$

This can be rearranged to:

(20)
$$\hat{P}_t(\hat{C}_{1,T}) - \hat{P}_t^*(\hat{C}_{1,T})\big|_{-C_{2,T}} = P_t(-C_{2,T}) - P_t^*(-C_{2,T})\big|_{\hat{C}_{1,T}}$$

The amount that A is willing to pay more for the long call, when compared to the Klein and Inglis (2001) stand-alone price for that option, is equal to the amount that A is willing to receive less for the short call, when compared to the credit-risk-free stand-alone Black-Scholes price. This relationship holds with or without netting, and can be verified observing the results in Table 3 and Table 6.

CHAPTER 5: MODELING OF PORTFOLIO DIVERSIFICATION EFFECTS

This section computes the price of vulnerable European calls considering portfolio diversification effects. Diversification is here understood by the effect of correlation between the two underlying assets on the credit-risk adjusted price of the portfolio of options. Pricing options individually charges a premium for the probability and severity of default for each option. Nevertheless, when considering a portfolio of options, the probability and severity of default of the portfolio is not equivalent to the sum of those of each of the options.

Model Setting: Adding another long call to a long call

To illustrate the impact of portfolio diversification effects on the option price, the following example is set up. Counterparty A buys from counterparty B a European call option on the underlying asset S_1. Again A is assumed to have no credit risk while B is risky. Consequently, the call is vulnerable and has values at maturity of $\hat{C}_{1,T}$ and during the life time of the option of $\hat{C}_{1,t}$, equal to those defined in equations (3) and (4), respectively.

An instant later counterparty A buys from B another European call option on the underlying asset S_2. This option has the same exercise date as the call before. The correlation between the two underlying assets is given by $\rho_{S1,S2}$.

As it was mentioned before, the first option $\hat{C}_{1,t}$ can be correctly priced as a vulnerable call using Klein and Inglis' (2001) model. However, the second call $\hat{C}_{2,t}$ cannot be priced using the same model because, among other reasons, there is no longer a fixed default boundary. It also should be apparent that the credit risk of the second call cannot be less than the credit risk of the first call.

Similar to the methodology used for the netting analysis, the value of the second call can be found as a conditional price,

(21)
$$P_t(\hat{C}_{2,T})_{|\hat{C}_{1,T}} = \hat{P}_t(\hat{C}_{1,T} + \hat{C}_{2,T}) - \hat{P}_t(\hat{C}_{1,T})$$

Where $\hat{P}_t(\hat{C}_{1,T})$ is the Klein and Inglis (2001) price and $\hat{P}_t(\hat{C}_{1,T} + \hat{C}_{2,T})$ is given by:

(22)
$$\hat{P}_T(\hat{C}_{1,T} + \hat{C}_{2,T}) = \left[(C_{1,T} + C_{2,T})[1]_{V_T \geq D^* + C_{1,T} + C_{2,T}} + \dots \right.$$
$$\left. \left(\frac{(1-\alpha)V_T(C_{1,T} + C_{2,T})}{D^* + C_{1,T} + C_{2,T}} \right)[1]_{V_T < D^* + C_{1,T} + C_{2,T}} \right]$$

Again, the risk-neutral expectation at time t is given by:

(23)
$$\hat{P}_t = e^{-r(T-t)} E^* \left[\hat{P}_T \right]$$

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It can be shown that for a portfolio consisting of two long calls equation (23) can be written as follows:

(24)
$$\hat{P}_t\left(\hat{C}_{1,T}+\hat{C}_{2,T}\right)=\hat{P}_t\left(\hat{C}_{1,T}\right)+\hat{P}_t\left(\hat{C}_{2,T}\right)+I_t$$

Where $\hat{P}_t\left(\hat{C}_{1,T}\right)$ and $\hat{P}_t\left(\hat{C}_{2,T}\right)$ are the unconditional vulnerable option prices and I is a risk increment given by:

(25)
$$I_T = \underbrace{-\left(C_{1,T}+C_{2,T}\right)[1]_{V_T=D^*+C_{1,T}+C_{2,T}}}_{\leq 0} + \underbrace{\left(\frac{(1-\alpha)V_T C_{1,T}}{D^*+C_{1,T}+C_{2,T}}-C_{1,T}\right)[1]_{D^*+C_{1,T}<V_T<D^*+C_{1,T}+C_{2,T}}}_{\leq 0}$$

$$\cdots + \underbrace{\left(\frac{(1-\alpha)V_T C_{2,T}}{D^*+C_{1,T}+C_{2,T}}-C_{2,T}\right)[1]_{D^*+C_{2,T}<V_T<D^*+C_{1,T}+C_{2,T}}}_{\leq 0}$$

$$\cdots + \underbrace{\left(\frac{D^*+C_{1,T}}{D^*+C_{1,T}+C_{2,T}}-1\right)\left(\frac{(1-\alpha)V_T C_{1,T}}{D^*+C_{1,T}}\right)[1]_{V_T<D^*+C_{1,T}}}_{\leq 0}$$

$$\cdots + \underbrace{\left(\frac{D^*+C_{2,T}}{D^*+C_{1,T}+C_{2,T}}-1\right)\left(\frac{(1-\alpha)V_T C_{2,T}}{D^*+C_{2,T}}\right)[1]_{V_T<D^*+C_{2,T}}}_{\leq 0}$$

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The risk-neutral expectation of the risk increment is given by:

(26)
$$I_t = e^{-r(T-t)} E^* [I_T]$$

The increment I_t is always negative, representing the amount by which the portfolio of two options is riskier than the two options priced on a stand-alone basis. In other words, A will pay less for the portfolio of options than the sum of the option prices when priced as if they were the first and only one to be traded. The portfolio diversification determines the magnitude of I_t. Numerical analysis will show that if the option payoffs are negatively correlated, I_t will be low.

Estimation of conditional option prices with portfolio effects

The initial portfolio is defined as $\hat{P}_t(\hat{C}_{1,T})$, containing a long position in a vulnerable European call with the base case parameters in Table 1. Then another long position in a vulnerable European call option $\hat{C}_{2,t}$ is added, with parameters that are initially chosen to equal those of the base case. Parameters of both options are later equally changed as defined in Table 7.

Table 7: Prices/Conditional prices for different parameters values. With portfolio effects.

	$\hat{C}_{1,t}$ (Monte-Carlo)	$\hat{C}_{2,t}$ (Monte-Carlo)	$\hat{P}_t\left(\hat{C}_{1,T}+\hat{C}_{2,T}\right)$	I_t	$P_t\left(\hat{C}_{2,T}\right)_{|\hat{C}_{1,T}}$
Base Case	+6.25	+6.25	+11.74	-0.76	+5.49
$S_i = 30$	+2.02	+2.02	+3.96	-0.08	+1.94
$S_i = 50$	+11.62	+11.62	+20.61	-2.63	+8.99
$V = 90$	+5.72	+5.72	+10.70	-0.74	+4.98
$V = 110$	+6.69	+6.69	+12.66	-0.72	+5.97
$\rho_{V,S_i} = 0.5$	+7.36	+7.36	+14.24	-0.48	+6.88
$\rho_{V,S_i} = -0.5$	+5.24	+5.24	+9.86	-0.62	+4.62
$\sigma_{S_i} = 0.15$	+5.68	+5.68	+10.76	-0.60	+5.08
$\sigma_{S_i} = 0.25$	+6.74	+6.74	+12.58	-0.90	+5.84
$\sigma_V = 0.15$	+6.48	+6.48	+12.10	-0.86	+5.62
$\sigma_V = 0.25$	+6.01	+6.01	+11.35	-0.67	+5.34
$T-t = 2$	+5.00	+5.00	+9.48	-0.52	+4.48
$T-t = 4$	+7.30	+7.30	+13.63	-0.97	+6.33
$\alpha = 0$	+7.13	+7.13	+13.67	-0.59	+6.54
$\alpha = 0.5$	+5.36	+5.36	+9.82	-0.90	+4.46
$r = 3\%$	+5.17	+5.17	+9.80	-0.54	+4.63
$r = 7\%$	+7.42	+7.42	+13.82	-1.02	+6.40

Base case: $S_i = S_1 = S_2 = 40$, $K_1 = K_2 = 40$, $\sigma_{S_i} = \sigma_{S1} = \sigma_{S2} = 0.20$, $\rho_{V,S_i} = \rho_{V,S1} = \rho_{V,S2} = 0$, $\rho_{S1,S2} = 0$, $V = 100$, $D^* = 90$, $T-t = 3$, $\alpha = 0.25$, $\sigma_V = 0.20$, $r = 5\%$ unless otherwise noted. Monte-Carlo simulation performed with 60.000.000 runs. Prices converge to two decimal digits.

Table 7 shows that the risk increment is always negative, i.e. $\hat{C}_{2,t}$ is priced

below the Klein and Inglis (2001) model price. Similarly to what was done when

Modeling netting effects, Figure 2 shows the effect of correlation, $\rho_{S1,S2}$, on the

price of the conditional call $P_t\left(\hat{C}_{2,T}\right)_{\Big|\hat{C}_{1,T}}$ when considering portfolio effects.

Figure 2: Conditional prices, with portfolio effects for different correlations
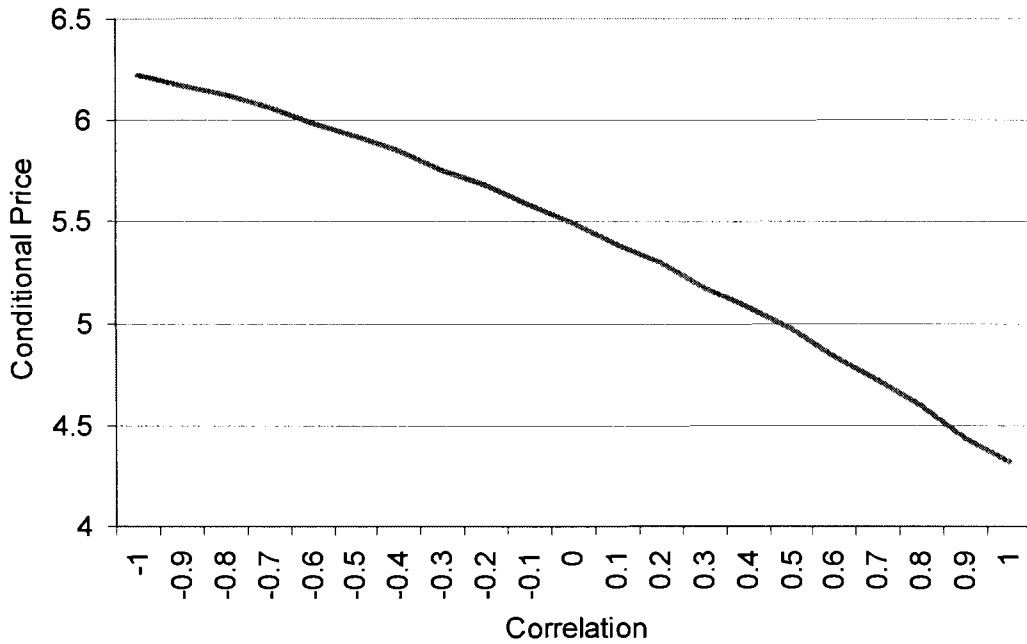
Base case for calls on underlyings with varying correlation: $S_1=S_2=40$, $K_1=K_2=40$, $\sigma_{S1}=\sigma_{S2}=0.20$, $\rho_{V,S1}=\rho_{V,S2}=0$, $V=100$, $D^*=90$, $T\text{-}t=3$, $\alpha=0.25$, $\sigma_V=0.20$, $r=5\%$. $\rho_{S1,S2}$ is changed on increments of 0.1, from -1 to 1. Each value is based on simulations of 4.000.000 runs.

With perfect negative correlation, the probability of default slightly more

than doubles but the severity is similar to that of the default under a single option.

Consequently, credit risk is appropriately charged when pricing the portfolio at

the price of two stand-alone vulnerable options and the conditional price

$P_t\left(\hat{C}_{2,T}\right)_{\Big|\hat{C}_{1,T}}$ will be close to $\hat{P}_t(\hat{C}_{2,T})$, i.e. the increment I_t will be low.

At a contrast, positive correlation increases the severity of a default and also increases the probability of default. V_T hits the default barrier $D^* + C_{1,T} + C_{2,T}$ sooner than it hits the stand-alone barrier $D^* + C_{1,T}$. Apparent from Figure 3 , the combined effect of increasing probability and severity of default in case of positive correlation is stronger than the effect of increasing the probability in case of negative correlation. Consequently, A will pay less for the second option if the correlation between the options underlyings increases.

CHAPTER 6: CONCLUSIONS

Credit risk of the option writer can significantly lower the value of a portfolio of options. It is well known that a market participant will pay less for a vulnerable option than he would for a non-vulnerable option. Pricing models have been developed by Johnson and Stulz (1987), Hull and White (1995), Jarrow and Turnbull (1995), Klein (1996) and Klein and Inglis (2001). This paper builds on the model of Klein and Inglis (2001) which employs a variable default boundary, links option payouts to the firm value, allows deadweight costs, other liabilities of the option writer and correlation between the assets of the option writer and option underlying.

The next level of appropriate credit-risk adjustment acknowledges the previously existing option portfolio and measures the impact of the new transaction on the credit exposure. Consideration of netting and portfolio effects will change the value of an option portfolio, the price of a transaction is then a conditional or incremental price with regard to the prior existing portfolio. A netting agreement increases the value of a portfolio that consists of bought and sold options. Even without netting, the valuation of vulnerable options in a portfolio, as opposed to an individual valuation, influences the value and consequently the conditional price of a transaction. Portfolio effects, i.e. effects

of correlation between the option underlyings on credit risk, are present in a portfolio consisting of bought options only.

Positive correlation between option underlyings increases the credit-risk adjusted value of a portfolio of long and short options, but decreases the value of a portfolio of long options only. This influences the price at which a counterparty is willing to enter in an additional transaction and also implies that the value of a static portfolio of vulnerable options is sensitive to changes in correlation between underlyings.

Figure 3 shows what could be called an evolution of credit-risk adjusted option pricing. A market participant that does not price the default risk of an option writer will pay the credit-risk-free price for a long option, in this case, the Black-Scholes price for a plain-vanilla call. Advancement to the recognition of credit risk significantly reduces the willingness to pay for the option. Furthermore, acknowledging the positive impact of portfolio valuation in a portfolio of long and short options leads to a value closer to the fair price of the long option. Finally, recognition of netting effects yields the fair price, conditional on the prior existing option portfolio.

Figure 3: Prices/Conditional prices of a long call for different pricing models, when added to a short call.
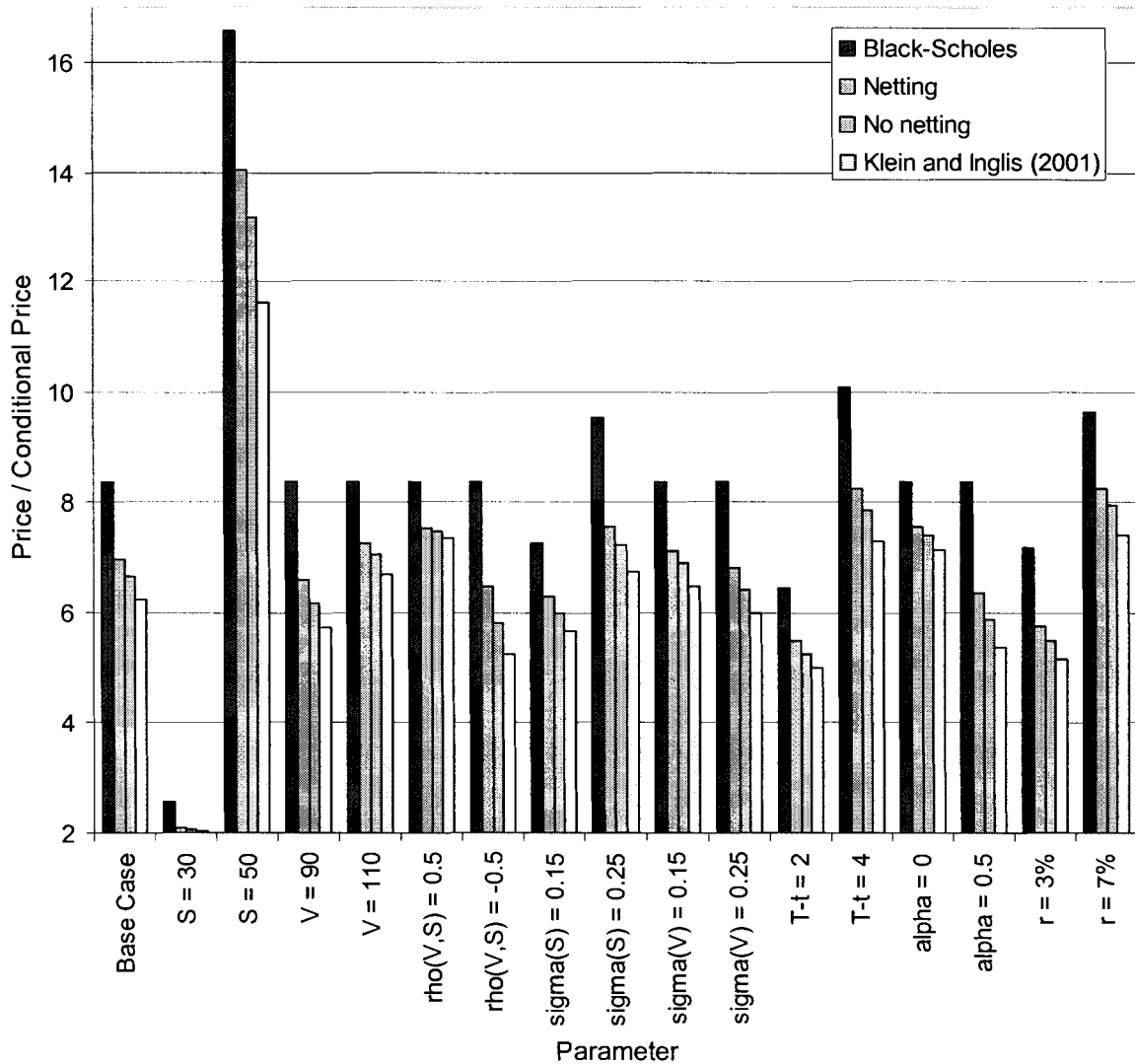
Base case: S=S₁=S₂=40, K₁= K₂=40, sigma(S)=σ$_{S1}$=σ$_{S2}$=0.20, rho(V,S)=ρ$_{V,S1}$=ρ$_{V,S2}$=0, ρ$_{S1,S2}$=0, V=100, D*=90, T-t=3, α=0.25, σ$_V$=0.20, r=5% unless otherwise noted. Monte-Carlo simulation performed with 60.000.000 runs.

A market participant that correctly models netting and portfolio effects may accept to write an option for a premium below the credit-risk-free price and buy

an option for a premium above the stand-alone price of a vulnerable option. In both cases the seemingly unfavourable deal offsets exactly the beneficial change to the credit risk of the option portfolio. In real life, a trader may not have the incentive to price an option according to this model, because profit & loss calculation for options is predominantly based on credit-risk-free prices. Also, as long as financial institutions do not compete on the basis of fair credit-risk adjusted prices; there may be no incentive to reward a customer for a reduction of credit risk by adjusting the transaction price accordingly.

Price transparency and consequently competitiveness is by definition limited because the option prices derived in this paper depend on the existing option portfolio. This may give the opportunity to a financial institution to generate excess profits when quoting transactions differently from the "fair" conditional price. Furthermore, if not all competing financial institutions price sensitive with regard to netting and portfolio effects, the one that acknowledges credit risk reduction of a trade will have a competitive advantage and will be able to offer a comparatively lower price.

APPENDICES: MATLAB CODE

Appendix A: Re-estimation of Klein and Inglis (2001) prices

```
% Calculation of the Option Price for a Vanilla Call Option
% Uses Monte-Carlo Simulation's approach
%
% Author: Martin Hammer and Patricia Restrepo. 05/25/2007.
%
% This function calculates price of a vulnerable option,
% using the pricing model of Klein and Inglis (2001),
% based on the followings parameters:
%
% Inputs:
% SO= Asset Price
% X= Strike Price
% r=Interest rate (i.e. 0.05)
% sUnder= Standard Deviation of Underlying (i.e. 0.1)
% sAsset= Standard Deviation of Counterparty Asset (i.e. 0.1)
% V0= Asset value of the option writer
% T= Time to maturity [years]
% runs= Numbers of runs (simulations)
% rho= Correlation between asset of the option writer and underlying
% alpha= deadweight costs
% FDB = fixed default boundary, value of other liabilites of option writer
% Type = 'Call' or 'Put'

% Outputs:
% OPTION_KI= Price of the vulnerable European option,
% Note: where Klein and Inglis (2001) choose a threedimensional tree
% for the numerical approximation, this function employs
% a Monte-Carlo-Simulation

% When using the function, use the following notation:
% MCoption_Proj_KI(SO, X, r, sUnder, sAsset, T, FDB, alpha, V0, rho, runs, type)


function OPTION_KI=MCoption_Proj(SO, X, r, sUnder, sAsset, T, FDB, alpha, V0, rho, runs, type)
switch type
    case 'Call'
    type_flag=1;
    case 'Put'
    type_flag=-1;
end

%Error checking:
if nargin~=12
    disp=('Incorrect number of inputs, check again')
    return
end
if r>1
    disp=('Interest Rate should be input in decimals, the value is too high')
    return
end
if r<0
```

```matlab
    disp=('Interest Rate should be positive, please check again')
    return
end
if sUnder<0
    disp=('Volatility should be positive, please check again')
    return
end
if sAsset<0
    disp=('Volatility should be positive, please check again')
    return
end
if SO<0
    disp=('Stock prices should be positive, please check again')
    return
end

%Creation of uncorrelated random variables
RandomNumbers=randn(runs,2);
%Cholesky Factorization
c=[1 rho;rho 1];
R=chol(c);
RandomCorrelated=RandomNumbers*R;
RandomUnder=RandomCorrelated(:,1);
RandomAsset=RandomCorrelated(:,2);

%Create vector of asset returns
ReturnAsset=(r-sAsset^2/2)*T+RandomAsset*sAsset*sqrt(T);
%Create vector of correlated underlying returns
ReturnUnder=(r-sUnder^2/2)*T+RandomUnder*sUnder*sqrt(T);

OPTION_KI=(SO*exp(ReturnUnder)*type_flag>type_flag*X)...        %Condition: Option in the money
    .*(V0*exp(ReturnAsset)>(FDB+(SO*exp(ReturnUnder)-X)*type_flag))...  %Condition: Not bankrupt
    .*(SO*exp(ReturnUnder)-X)*type_flag;                         %Option Pay-off

OPTION_KI=OPTION_KI ...                                          %Add to above option pay-off
    +(SO*exp(ReturnUnder)*type_flag>X*type_flag)...             %Condition: Again, option in the money
    .*(V0*exp(ReturnAsset)<(FDB+(SO*exp(ReturnUnder)-X)*type_flag))...  %Condition: writer bankrupt
    .*(SO*exp(ReturnUnder)-X)*type_flag...                      %Option Pay-off
    .*(1-alpha)*V0.*exp(ReturnAsset)./(FDB+(SO*exp(ReturnUnder)-X)*type_flag); %reduced by recovery

OPTION_KI=mean(OPTION_KI)*exp(-r*T);                            %Average and discount
```

Appendix B: Estimation of portfolio prices with/without netting

```
% Calculation of the portfolio value (vulnerable Vanilla Call Options)
% Uses Monte-Carlo Simulation's approach
%
% Author: Martin Hammer and Patricia Restrepo. 05/25/2007.
%
% This function calculates the value of a portfolio of
% two vulnerable options, one long (Index 1) and one short (Index 2),
% extending the pricing model of Klein and Inglis (2001),
% based on the followings parameters::
%
% Inputs:
% S1,S2= Asset Prices
% K1,K2= Strike Prices
% r=Interest rate (i.e. 0.05)
% sUnder1,sUnder2= Standard Deviations of Underlyings (i.e. 0.1)
% sAsset= Standard Deviation of Counterparty Asset (i.e. 0.1)
% V0=Asset value of the option writer
% T= Time to maturity [years]
% nsims= Numbers of runs (simulations)
% rhoAsset1, rhoAsset2= Correlation between asset of the option writer and underlying
% alpha= deadweight costs
% rho12= Correlation between underlyings
% FDB = fixed default boundary, value of other liabilites of option writer
% Type1, Type2= 'Call' or 'Put'

% Outputs:
% [OPTION_NoN,OPTION_N]= Portfolio value without and with considering netting effects.

% When using the function, use the following notation:
% MCoption_Proj_HaRe(T,V0,FDB, sAsset, rho12,alpha,r, nsims,...
%             S1,K1,sUnder1,rhoAsset1,type1,S2,K2,sUnder2,rhoAsset2,type2)


function [OPTION_NoN, OPTION_N]=MCoption_Proj_HaRe(T,V0,FDB, sAsset, rho12,alpha,r, nsims,...
    S1,K1,sUnder1,rhoAsset1,type1,...
    S2,K2,sUnder2,rhoAsset2,type2)
switch type1
    case 'Call'
    type1_flag=1;
    case 'Put'
    type1_flag=-1;
end
switch type2
    case 'Call'
    type2_flag=1;
    case 'Put'
    type2_flag=-1;
end


%Error checking:
if nargin~=18
    disp=('Incorrect number of inputs, check again')
    return
end

if abs(rho12)==1
    if rhoAsset1~=rhoAsset2
    disp=('If assets are perfectly correlated, the rhoAsset1 and rhoAsset2 must be equal')
```

```
        return
    end
end


%Creation of uncorrelated random variables

%CHOLESKY FACTORIZATION
if abs(rho12)<1
%3x3 correlation matrix if rho12 does not equal 1 or -1
RandomNumbers=randn(nsims,3);
c=[1 rho12 rhoAsset1;rho12 1 rhoAsset2; rhoAsset1 rhoAsset2 1];
R=chol(c);
RandomCorrelated=RandomNumbers*R;
RandomAsset=RandomCorrelated(:,3);
RandomUnder1=RandomCorrelated(:,1);
RandomUnder2=RandomCorrelated(:,2);
else
%2x2 correlation matrix if rho12 equals 1 or -1
%error checking above ensures that rhoAsset1=rhoAsset2
RandomNumbers=randn(nsims,2);
c=[1 rhoAsset1;rhoAsset1 1];
R=chol(c);
RandomCorrelated=RandomNumbers*R;
RandomAsset=RandomCorrelated(:,2);
RandomUnder1=RandomCorrelated(:,1);
RandomUnder2=RandomCorrelated(:,1)*rho12;
end


%create vector of asset returns
ReturnAsset=(r-sAsset^2/2)*T+RandomAsset*sAsset*sqrt(T);
%create vector of correlated underlying returns
ReturnUnder1=(r-sUnder1^2/2)*T+RandomUnder1*sUnder1*sqrt(T);
ReturnUnder2=(r-sUnder2^2/2)*T+RandomUnder2*sUnder2*sqrt(T);


%Define payoff on first option
X=max((S1*exp(ReturnUnder1)-K1)*type1_flag,0);
%Define payoff on second option
Y=max((S2*exp(ReturnUnder2)-K2)*type2_flag,0);
%Define asset value at maturity
VT=V0*exp(ReturnAsset);


%Price component for the case that option writer is solvent
OPTION_SOLVENT=(VT>=(FDB+X-Y)).*(X-Y);
%Price component for the case that option writer is insolvent,
%NO NETTING
OPTION_NoN=(VT<(FDB+X-Y)).*(Y==0).*(X>0)*(1-alpha).*VT.*X./(FDB+X);
OPTION_NoN=OPTION_NoN+(VT<(FDB+X-Y)).*(X==0).*(Y>0).*(-Y);
OPTION_NoN=OPTION_NoN+(VT<(FDB+X-Y)).*(X>=Y).*(Y>0).*((1-alpha).*(VT+Y).*X./(FDB+X)-Y);
OPTION_NoN=OPTION_NoN+(VT<(FDB+X-Y)).*(Y>X).*(X>0).*((1-alpha).*(VT+Y).*X./(FDB+X)-Y);
OPTION_NoN=OPTION_NoN+OPTION_SOLVENT;

OPTION_NoN=mean(OPTION_NoN)*exp(-r*T);%Average and discount


%Price component for the case that option writer is insolvent,
%WITH NETTING
OPTION_N=(VT<(FDB+X-Y)).*(Y==0).*(X>0)*(1-alpha).*VT.*X./(FDB+X);
OPTION_N=OPTION_N+(VT<(FDB+X-Y)).*(X==0).*(Y>0).*(-Y);
OPTION_N=OPTION_N+(VT<(FDB+X-Y)).*(X>=Y).*(Y>0).*(1-alpha).*VT.*(X-Y)./(FDB+X-Y);
OPTION_N=OPTION_N+(VT<(FDB+X-Y)).*(Y>X).*(X>0).*(X-Y);
OPTION_N=OPTION_N+OPTION_SOLVENT;
OPTION_N=mean(OPTION_N)*exp(-r*T);%Average and discount
```

Appendix C: Estimation of portfolio prices with diversification effects

```
% Calculation of the Option Price for a vulnerable Vanilla Call Option
% Uses Monte-Carlo Simulation's approach
%
% Author: Martin Hammer and Patricia Restrepo. 05/25/2007.
%
% This function calculates the value of a portfolio of
% two vulnerable long options,
% extending the pricing model of Klein and Inglis (2001),
% based on the followings parameters:
%
% Inputs:
% S1,S2= Asset Prices
% K1,K2= Strike Prices
% r=Interest rate (i.e. 0.05)
% sUnder1,sUnder2= Standard Deviations of Underlyings (i.e. 0.1)
% sAsset= Standard Deviation of Counterparty Asset (i.e. 0.1)
% V0=Asset value of the option writer
% T= Time to maturity [years]
% nsims= Numbers of runs (simulations)
% rhoAsset1, rhoAsset2= Correlation between asset of the option writer and underlying
% alpha= deadweight costs
% rho12= Correlation between underlyings
% FDB = fixed default boundary, value of other liabilites of option writer
% Type1, Type2= 'Call' or 'Put'

% Outputs:
% OPTION_PF= Portfolio value considering portfolio diversification effects


% When using the function, use the following notation:
% MCoption_Proj_PF(T,V0,FDB, sAsset, rho12,alpha,r, nsims,...
%              S1,K1,sUnder1,rhoAsset1,type1,S2,K2,sUnder2,rhoAsset2,type2)

function [OPTION_PF]=MCoption_Proj_HaRe(T,V0,FDB, sAsset, rho12,alpha,r, nsims,...
    S1,K1,sUnder1,rhoAsset1,type1,...
    S2,K2,sUnder2,rhoAsset2,type2)
switch type1
    case 'Call'
    type1_flag=1;
    case 'Put'
    type1_flag=-1;
end
switch type2
    case 'Call'
    type2_flag=1;
    case 'Put'
    type2_flag=-1;
end


%Error checking:
if nargin~=18
    disp=('Incorrect number of inputs, check again')
    return
end
```

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```
if abs(rho12)==1
    if rhoAsset1~=rhoAsset2
    disp=('If assets are perfectly correlated, the rhoAsset1 and rhoAsset2 must be equal')
    return
    end
end
```

%Creation of uncorrelated random variables

%CHOLESKY FACTORIZATION

```
if abs(rho12)<1
```
%3x3 correlation matrix if rho12 does not equal 1 or -1
```
RandomNumbers=randn(nsims,3);
c=[1 rho12 rhoAsset1;rho12 1 rhoAsset2; rhoAsset1 rhoAsset2 1];
R=chol(c);
RandomCorrelated=RandomNumbers*R;
RandomAsset=RandomCorrelated(:,3);
RandomUnder1=RandomCorrelated(:,1);
RandomUnder2=RandomCorrelated(:,2);
else
```
%2x2 correlation matrix if rho12 equals 1 or -1
%error checking above ensures that rhoAsset1=rhoAsset2
```
RandomNumbers=randn(nsims,2);
c=[1 rhoAsset1;rhoAsset1 1];
R=chol(c);
RandomCorrelated=RandomNumbers*R;
RandomAsset=RandomCorrelated(:,2);

RandomUnder1=RandomCorrelated(:,1);
RandomUnder2=RandomCorrelated(:,1)*rho12;
end
```

%create vector of asset returns
```
ReturnAsset=(r-sAsset^2/2)*T+RandomAsset*sAsset*sqrt(T);
```
%create vector of correlated underlying returns
```
ReturnUnder1=(r-sUnder1^2/2)*T+RandomUnder1*sUnder1*sqrt(T);
ReturnUnder2=(r-sUnder2^2/2)*T+RandomUnder2*sUnder2*sqrt(T);
```

%Define payoff on first option
```
X=max((S1*exp(ReturnUnder1)-K1)*type1_flag,0);
```
%Define payoff on second option
```
Y=max((S2*exp(ReturnUnder2)-K2)*type2_flag,0);
```
%Define asset value at maturity
```
VT=V0*exp(ReturnAsset);
```

%Price component for the case that option writer is solvent
```
OPTION_SOLVENT=(VT>(FDB+X+Y)).*(X+Y)+(VT==(FDB+X+Y)).*(X+Y);
```
%Price component for the case that option writer is insolvent,
```
OPTION_INS=(VT<(FDB+X+Y)).*(1-alpha).*VT.*(X+Y)./(FDB+X+Y);
OPTION_PF=OPTION_INS+OPTION_SOLVENT;
OPTION_PF=mean(OPTION_PF)*exp(-r*T);%Average and discount
```

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