Labor Markets and Internal Migration in Developing Countries

by

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Abstract

Low and middle income countries frequently have a substantial informal sector within large cities. In the tradition of the Harris-Todaro classical model of rural-urban migration I develop a two-sector general equilibrium model with matching frictions in the urban labor market which allows for migration between sectors. Having the wage as the driving force of migration I show that without further assumptions the Todaro paradox - a productivity increase in the urban sector that can cause an increase in unemployment - can be attained. The model also answers the problem - previously unexplained by Harris and Todaro - of why unemployed workers in the urban sector do not migrate back to the agricultural sector. Furthermore, I demonstrate that, in contrast to the existing literature, an efficient equilibrium will naturally result without government intervention.

Keywords matching frictions; informal sector; efficiency; rural urban migration; Todaro paradox

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1 Introduction

During the last century in many developing countries the source of a large fraction of GDP shifted from the agricultural to the urban sector. This lead to a large decline in employment in the agricultural sector and let to enormous migration from the countryside to the cities. Examples for this are Mexico City, Manila or Sao Paulo where population exploded during this time. Due to the lack of social security systems this created a phenomenon of two economies in the cities: modern industry/business coexists with a huge informal sector - i.e. people that are not included in any unemployment statistic and live basically on a self sufficiency level. On the other side there still exists an agricultural sector in the rural areas which is usually declining in size since productivity increases are larger in cities and attract more and more workers.

There have been quite a few studies on the existence of an informal sector in the cities because many of the migrants were actually worse of in the informal sector than they were in an agricultural job which made economists wonder why this phenomenon exists. Recent contributions to this literature are Fugazza and Jacques (2004) and Dessy and Pallage (2003) which both study the positive and negative externalities created by the informal sector. Another paper describing possible reasons for the existence of the informal sector is Rauch (1991) but the most famous papers are certainly Harris and Todaro (1970) and Todaro (1969). Their model uses a high urban minimum wage in combination with migration as causes for the existence of an informal sector.

The motivation for this paper is a small existing literature focussing on the effects of growth[1] on labor markets in developing countries and my observation that there is still much space for improvement on previous research. While there has been done much empirical work on the effects of growth on e.g. income (pro poor growth), not much attention had been paid to its effects on the labor market in developing countries. An exception is Lucas (2004) who uses a growth model with human capital in combination with a Harris-Todaro model to model the transition of developing countries from rural to urban. In opposite to the approach undertaken in this paper

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[1]In the form of total factor productivity shocks

Lucas (2004) has an endogenous growth model to explain the diminishing agricultural sector because individuals can earn higher wages in cities because by accumulating human capital. Despite the elegance of his results there is a crucial assumption made that does not seem realistic in a developing country context: Lucas assumes that all unemployed workers in the city are devoting all time to studying with no income.

This paper differs from Lucas (2004) in the sense that I am looking at a labor search model[2]. Growth only occurs in the form of exogenous productivity shocks. So far there exist only a few models of developing countries in a search model context. Examples are Satchi and Temple (2006), Sato (2004) or Zenou (2008). Common to all papers that are using search models in the context is that they are based on the Mortensen-Pissarides model, which was developed by Diamond (1982), Mortensen (1982) and Pissarides (1990.). [3] In contrast I will be using a competitive search model introduced by Moen (1997).

This paper will extent the existing literature in the following dimensions: Following the criticism by Lucas that a successful theory of the urbanization process must answer why migrants to the city that do not find a job in the formal sector in the Harris-Todaro model do not go back to the agricultural sector (see Lucas (2004), p. S31) this paper will show that as well as in Satchi and Temple (2006) the competitive search approach provides reasons for individuals to stay in the informal sector instead of migrating back to the agricultural sector. In contrast to Satchi and Temple (2006) the approach of using the Moen (1997) model avoids using ex post bargaining and introduces a setting in which all markets are competitive. [4] This has the consequence that opposite to all models using the Mortensen-Pissarides model the outcome will be socially efficient. Contradicting Sato (2004) I will show that this implies that there is no need for government intervention such that actual policy recommendation for developing countries might be frequently flawed. Finally this paper does not need an exogenous minimum wage for explaining high city wages and might well explain that urban wages are considerably higher than the minimum wages. Harris-Todaro models were criticized for this by Stiglitz (see Stiglitz (2002), p. 464).

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[2] A survey of recent developments in search models in labor markets can be found in Rogerson et al. (2005)

[3] One reason for its popularity is its tractability.

[4] Competitive is used in the sense here that all agents are price takers.

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Common to all search model approaches a limitation of using a search model is that it just looks at steady state changes. Transitional dynamics will be completely absent in this paper although they are certainly of great interest in predicting what would happen in the short run after productivity changes for example.

Before I actually start to set up the search model in a developing country context I briefly want to outline the differences between developed and developing countries in order to justify a modified approach using several sectors. Some general differences are (see Harris and Todaro (1970), p. 126 and Satchi and Temple (2006)):

- During the last decades there has been massive migration from the rural areas of many developing countries to the big cities.

- This migration happened although unemployment in the cities is quite high and migrants often do not find a job in the city.

- In many developing countries there is no unemployment insurance unlike most developed countries but there exists a large informal sector, i.e. people being self-employed (low wage, low productivity).

This paper will first briefly explore the results achieved by Harris and Todaro (1970) and Satchi and Temple (2006). Later on I will develop a model which is closely related to the model developed by Satchi and Temple (2006) but instead of using the Mortensen-Pissarides model as a basis I will use the competitive search approach developed by Moen (1997). First the equilibrium conditions for a very simple model will be derived and then I will look at some comparative statics and compare the predictions of this model to those of Harris and Todaro (1970) and Satchi and Temple (2006). Then I will propose some extensions of my model which will be subject of further research and for which simulations will be necessary because of the complexity of the model. Finally I will discuss the question of what can be done to improve social welfare in developing countries based on the results of this model.

1.1 The Harris-Todaro model

Harris and Todaro (1970) was a path breaking paper when it was published and still is the benchmark model for looking at labor markets in developing countries. Harris-

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Todaro are using a two sector internal trade model (rural and urban). The rural sector can either fully employ rural labor and sell the output to the urban sector or use the labor only partly and export the rest of the labor to the urban sector (migration). The crucial assumption is that migration will occur until the *expected* urban income equals marginal agricultural product. The *expected* urban income will be equal to the existing minimum wage multiplied by the probability of getting a job. This means that not all the workers who actually go to the urban sector will get a job. Although there is perfect competition in both sectors they obtain the result that in equilibrium there will be unemployment (see Harris and Todaro (1970) p. 131). Building upon this work Todaro (1976) shows that an autonomous increase in urban job creation can in fact increase unemployment. This is the famous Todaro paradox, which will be encountered later on in the paper again.

1.2 The Satchi-Temple model

Satchi and Temple (2006) use the basic Mortensen-Pissarides model (see Ljungqvist and Sargent (2004), p. 946 for a brief overview) and extend it to a developing country context. The idea of the two sector model is the following. Workers live either in the countryside and work in the agricultural sector (perfectly competitive) or they live in the city and are either unemployed/selfemployed or work in the formal sector. They can migrate from one sector to the other but face unemployment for the first period that they are in the urban sector. Then they literally wander around and in case they meet a firm, the worker and firm bargain over the wage following an exogenously given Nash-bargaining rule. Satchi and Temple use this model to look at some comparative statics which in general have results similar to Harris and Todaro (1970). Finally they calibrate it to the data of Mexico, closely following assumptions introduced to search theory by Andolfatto (1996).

2 A Simple Competitive Search Model

In this section I will first describe the structure of the model. Generally this means that the features of the search model by Moen (1997) will be merged with some of the features used by Satchi and Temple (2006). At this point I will use a simpli-

fied/degenerate version of the model of Moen to keep the math as simple as possible and for being able to look at some comparative statics without using simulations. It is a general equilibrium model, which looks like the following:

It is a small open economy with two sectors. An agricultural sector and a urban sector.[5] The outputs of those can be traded at an exogenous relative price.[6] Capital is assumed to be exogenous for the agricultural sector, i.e. the capital stock is fixed and perfectly immobile. A different interpretation for this would be that the only capital that is used in the agricultural sector is land. The capital stock in the urban sector is assumed to be endogenously determined. Capital can flow in and out of the urban sector from the rest of the world. This implies that capital in the urban sector is perfectly mobile. The reason is that an open capital account means that the marginal product of the formal sector in the city must equal the interest rate in the world. These assumptions are fairly unrestrictive and they are used to simplify the model and it's comparative statics. Nonetheless I will maintain this assumption for determining the equilibrium analytically but it certainly can be relaxed when using computational methods.

In the following model the individuals have three 'choices':

- They can stay in the rural area and work in the agricultural sector and will never be unemployment since this sector is perfectly competitive

- They can decide to move to the city and be unemployed[7] in the city for at least one period. This reflects information problems. Workers are not able to look for a job and go to the city when they found one. In order to find the job they have to move to the city first and then can look for a job in the formal sector.

- If they already are in the city they can either move back to the countryside and find a job there immediately, due to perfect competition in the agricultural sector, or they can search for a job in the city which might result in finding a job or being unemployed.

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[5] The subscript a will further stand for the agricultural sector while the urban sector will have the subscript m

[6] Following Satchi and Temple, the agricultural goods price is the numeraire. The unit for urban outputs is selected such that it is also possible to normalize it to one. This makes it simple to handle.

[7] I will often use the term unemployed instead of self employed or work in the informal sector.

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It is simple to introduce fixed cost of migration or congestion cost if moving to the urban sector which I will show later but for simplicity I will set those to zero. Let L_i, K_i, k_i for $i = a, m$ be the mass of workers, the capital stock and the capital stock per worker in the agricultural and urban sector respectively. Labor supply is normalized such that $L_a + L_m = 1$ or $L_a = 1 - L_m$. The wage is w_i and the productivity factor is A_i for $i = a, m$. The model takes place in a continuous time setting and the production functions are assumed to be constant return to scale. All workers are risk neutral and infinitely lived.[8]

2.1   The Agricultural Sector

The agricultural sector is assumed to be perfectly competitive and workers can not be self employed.[9] This means that this market assigns labor efficiently and that there exists no unemployment. The production function in the intensive form for each worker is $g(k_a)$, with $k_a = \frac{K_a}{L_a}$, and the wage w_a is determined of profit maximization of the firm. In addition each worker receives $o_a \geq 0$, which stands for a preference for living in the countryside.[10] The total utility stream of a worker will therefore be $w_a + o_a$. Firms maximize their profits Π over choosing the amount of labor they want to employ since capital (i.e. land) is assumed to be exogenous.

$$\Pi = \max_{L_a} A_a g(k_a) L_a - w_a L_a - r_a K_a = \max_{L_a} A_a g(\frac{K_a}{L_a}) L_a - w_a L_a - r_a K_a$$

First order condition with respect to L_a:

$$
\begin{aligned}
w_a &= A_a g(k_a) - A_a \underbrace{g'(k_a)}_{\equiv r_a} k_a \\
w_a &= A_a \left[g(k_a) - r_a k_a \right]
\end{aligned}
\tag{2.1}
$$

r_a is the rental cost of capital in the agricultural sector. This shows that the wage is positive dependent on L_m, which is endogenously determined as will be shown later,

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[8]The assumption of risk neutrality is an important assumption for the search model but will not be discussed. Other models like Lucas' and Prescott's search island model assume risk averse workers (Lucas and Prescott (1974)).

[9]This seems to be a common assumption in this literature, see Satchi and Temple (2006), p. 8.

[10]This can be seen as a benefit from cleaner air and water etc.

and K_a, which is exogenous. An increase in productivity A_m increases the wage as can easily be seen in equation 2.1. Before turning to the migration between the two sectors it is necessary to introduce the labor market model of the urban sector first.

2.2 The Urban Sector

This sectoral labor market is based on the competitive search idea introduced by Moen (1997). The crucial difference between this model and the Mortensen-Pissarides model used by Satchi and Temple (2006) is the following:

In Mortensen-Pissarides models workers and firms meet randomly and then bargain over the wages according to an exogenously given Nash bargaining rule. This implies that wages are determined ex post. In contrast to this a competitive search model determines wages ex ante. Firms post wages[11] and workers try to locate the firms with attractive offer, i.e. search is directed.[12] This is what Moen calls competitive search because all agents are price takers and maximize their income subject to some constraints. If you assume heterogeneous firms (in terms of productivity) there will exist different submarkets across which the wages differ. Workers and firms choose which submarket to enter. In submarkets with higher wages it will be harder to find a job for the worker (easier to fill a vacancy for a firm)..

Here I will assume for simplicity that there are homogenous firms, i.e. firms do not differ in productivity. I will introduce heterogeneous firms later on which can potentially alter comparative statics.

2.2.1 The Matching Function

In order to describe the equilibrium it needs to be defined how firms and workers meet. This is done through the introduction of a matching function $m(u, v)$, with u being the unemployment rate and v being the vacancy rate, which depends on number of unemployed and the number of vacancies.[13]The matching function is assumed to

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[11]It is assumed that firms are committed to their wage announcement, otherwise there could be Nash bargaining later on which would eliminate all the efficiency properties.

[12]Despite the existence of many interpretations of these models this is one interpretation also used by Rogerson et al. (2005) that I find most intuitive.

[13]You get the number of unemployed if you multiply the rate by the number of people in the sector, the same holds of course for the number of vacancies. This term logically cancels out.

be of a Cobb-Douglas type which is a common assumption in this setting:

$$m(u,v) = u^\alpha v^{1-\alpha} \tag{2.2}$$

Furthermore let $\theta \equiv \frac{v}{u}$ be the labor market tightness. This means you can write the probability of filing a vacancy as:

$$q(\theta) \equiv \frac{m}{v} = \theta^{-\alpha} \tag{2.3}$$

It is obvious that this equation is decreasing in θ. Intuition: If there are relatively many vacancies per unemployed worker then it is less likely to find an unemployed to fill this vacancy.
Similarly you can derive the probability for a worker of finding a job, which is:

$$p(\theta) \equiv \frac{m}{u} = \theta^{1-\alpha} = \theta q(\theta) \tag{2.4}$$

Consequently this probability is increasing in θ, i.e. it is more likely to find a job the more vacancies there are.
It is necessary to assume that there is not only matching but also separation of workers and firm since otherwise all workers would be employed at some point. Here it is assumed that workers and firms separate with the rate s, s being an exogenous Poisson rate.

2.2.2 Workers

The workers are assumed to be homogenous. This assumption can certainly be relaxed in different ways and some possibilities will be discussed later on.
The Hamiltonian-Jacobian-Bellman (HJB) Equations of the workers represent the payoff/utility a worker gets when he is employed or unemployed per period. The payoff of an unemployed worker is:

$$rU = z + p(\theta)(W - U) \tag{2.5}$$

r is the interest rate which is exogenous. The workers benefit is composed of two parts: z is the income from being self employed/working in the informal sector and the last term in 2.5 is the option value looking for a job, i.e. the gain of getting a job

times the probability of getting a job.

The payoff of an employed worker is:

$$rW = w_m - s(W - U) \tag{2.6}$$

This benefit is also composed of two parts: The first is the wage the worker receives. This wage is the same for all workers since I assume firms to be homogenous.[14]. The last part is the loss a worker occurs in case he looses his job, which happens with probability s.

2.2.3 Firms

There is free entry of firms in the market. Firm incur a sunk cost k when they enter the market and all firms have the same productivity.[15] The output of a firm is defined as

$$Y = A_m f(k_m) L_m \qquad k_m = \frac{K_m}{(1 - u) L_m}$$

The marginal product of each worker is

$$y(k_m) = A_m f(k_m) - A_m \underbrace{f'(k_m)}_{=r} k_m = A_m f(k_m) - A_m r k_m \tag{2.7}$$

Given constant returns to scale and free capital flow to the city, k_m is endogenous. The world interest rate is exogenous such that $f'(k_m) = r$, which will pin down the capital labor ratio and changes only if r changes, which is assumed to be constant for now. You can see that this will dramatically simplify the comparative statics when looking at the definition of k_m. The unemployment rate is incorporated here and having k_m as an endogenous variable would most likely make it necessary to compute any comparative statics.

The Hamiltonian-Jacobian-Bellman (HJB) Equations represent the payoff of a firm when it is vacant and when it is producing. The equation for a vacant firm is:

$$rV(y, w, \theta) = -c + q(\theta)(J(y, w) - V(y, w, \theta) \tag{2.8}$$

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[14]There would be different wages for different submarkets in the case of heterogeneous firms.

[15]Therefore all subscripts i can be dropped from here on.

When a firm is vacant it occurs cost c that is looses plus the benefit from filling the vacancy times the probability that there is a match between a worker and that firm. It is the easiest to think about this model if you assume that each firm employs only one worker or has one vacancy.[16]The payoff for a producing firm is:

$$rJ(y, w) = y(k_m) - w_m - sJ(y, w) \tag{2.9}$$

The operating firm produces output $y(k_m)$, it has to pay w_m to the worker and looses with probability s that match with the worker in which case it would loose J.

2.3 Migration Between Agricultural and Urban Sector

Before being able to finally look at the equilibrium it is necessary to introduce the conditions for migration of individuals from the countryside to the city for work in the urban sector. Migration is solely economically motivated in this model. People migrate from the rural areas to the city when the payoff they get in the city is higher then the payoff they get in the countryside. They move into the other direction if the payoff in the agricultural sector is higher. As mentioned before workers from the countryside move to the city and are unemployed first. This means that only the payoff of being unemployed in the city matters for the decision of the worker. Furthermore it is assumed that there is a flow f of workers, which can be positive or negative, between the two sectors and that there are congestion cost $b \geq 0$ that occur to each worker who moves to the city. [17]

Given this the migration condition for the worker is the following:

$$\underbrace{w_a + o_a}_{\text{Payoff living in the countryside}} = \underbrace{rU - rbf}_{\text{Payoff living in the City}} \tag{2.10}$$

It can easily be seen that through this equation it will be the case that workers move from one sector to the other in case on sector is hit by a productivity shock and if institutional changes occur. Having laid out the model it is now possible to look at the equilibrium.

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[16]This is not a necessary assumption but makes it easier to understand the firms payoffs

[17]There can also be fixed cost $X \geq 0$ of moving but I will not include them here. The reason is that fixed cost would make the analysis of the comparative statics more complicated since a productivity shock might not necessarily result in migration (see Satchi and Temple (2006))

2.4 Equilibrium

Before discussing the formal conditions for an equilibrium I first want to visualize the equations of the urban sector model to make the equilibrium and the comparative statics more intuitive. I will illustrate the workers and firms HJB equation in the θ, w_m space.

2.4.1 Worker's and Firm's Indifference Curves

The indifference curve for a worker is easily derived by solving equation 2.6 for W_i and plugging this into equation 2.5. This equation can then be solved for $p(\theta)$. This gives the following expression:

$$p(\theta) = \frac{rU - z}{w_m - rU}(r + s) \tag{2.11}$$

As was shown before, $p(\theta)$ is positively dependent on θ. For a given U one can plot an indifference curve which is falling in w_m.[18] It is also obvious that

$$\lim_{w_m \to rU} p(\theta) = \infty$$

Intuitively this is because no worker will ever accept a wage lower then what he gets when being unemployed. Workers are better off away from the origin, the higher the wage for a given labor market tightness θ the better it is for the worker. The workers indifference curve is downward sloping for two reasons: They are obviously better off with a higher wage given a labor market tightness and given a wage they are better off the higher the labor market tightness.

The indifference curve for the firms can be derived in a similar manner. Solving 2.9 for J and plugging this into 2.8 gives:

$$(r + q(\theta))V(y, w, \theta) = -c + q(\theta)\frac{y(k_m) - w_m}{r + s} \tag{2.12}$$

This is also a curve falling for a given V in the θ, w_m space. Firms are better off the lower the labor market tightness, i.e. the easier it is to fill a vacancy and the lower

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[18]It has to be noted that the restrictions on the matching function are not sufficient to ensure that the curve is in fact convex but this is of no importance of the equilibrium conditions and in accordance with Moen (1997), p. 392
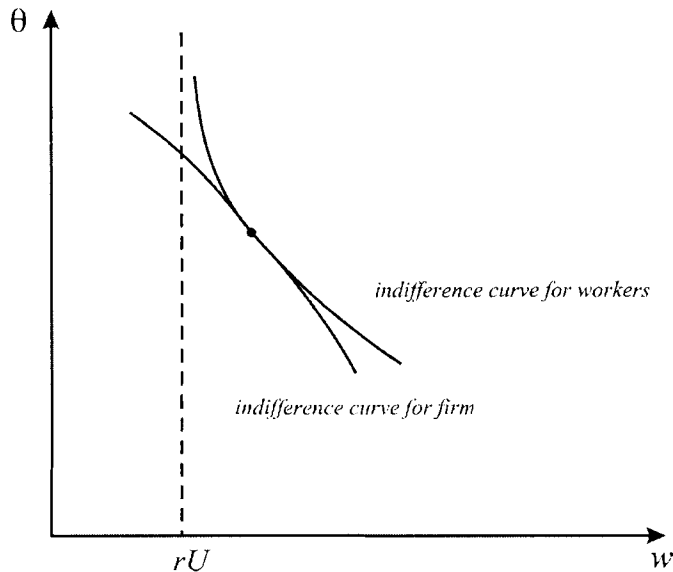
Figure 1: Indifference Curves for Workers and Firms

the wage. These observations result in a downward sloping indifference curve[19] In this model firms make zero profit because of the free entry condition in equilibrium. For illustration and later for the comparative statics it is convenient to draw a graph as seen in Figure 1. The equilibrium is the tangency point of the two curves.

In the next part I will derive the formal equilibrium conditions which are needed to ensure that there exists a tangency point for the urban sector and a optimal migration between the sectors.

2.4.2 Formal Equilibrium Conditions

In this section I will derive five conditions which need to be fulfilled in the steady state of the model. They are basically the same as in Moen (1997) - with the difference that they are somewhat simpler due to the assumptions of homogenous firms - plus an additional condition needed for the migration which is the same as in Satchi and Temple (2006). The equilibrium conditions for the agricultural sector are trivial. They are determined of the migration condition. In the steady state there is no

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[19]Here it has to be noted again that the matching function does not suffice to ensure the concavity of this indifference curve, see Moen (1997), p. 392

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migration between the countryside and the city, i.e. $f = 0$, such that the necessary condition is:

$$w_a + o_a = rU \tag{2.13}$$

The other equilibrium condition are as in Moen (1997). Firms will enter the market as long as the payoff from opening a vacancy is at least as high as the entry costs k. Formally:

$$\overline{V}(U) \equiv \max_w V(y, w, \theta(w, U))$$
$$= \frac{1}{r + q(\theta)} \left(-c + q(\theta) \frac{y(k_m) - w_m^*}{r + s} \right) \geq k \quad \rightarrow \quad U^* \tag{2.14}$$

In equilibrium this equation holds with equality. This determines a unique value of U Given this U firms determine the optimal wage rate w_m^* by maximizing the value of a vacancy as

$$w = \arg\max_w V = \frac{1}{r + q(\theta)} \left(-c + q(\theta) \frac{y(k_m) - w_m}{r + s} \right) \quad \rightarrow \quad w_m^*(y, U^*) \tag{2.15}$$

Given the optimal wage w_m^* the workers indifference curve will pin down the value for θ

$$\theta^{1-\alpha} = \frac{rU - z}{w_m - rU}(r + s) \quad \rightarrow \quad \theta(w_m^*, U^*) \tag{2.16}$$

Firms maximize the value of a vacancy to determine the optimal w_m^*. As can be seen this is in principle a recursive problem. The only thing missing is the equilibrium condition for the unemployment rate in the urban sector. In the state steady unemployment has to remain constant which means that flow into unemployment and flow out of unemployment have to be the same. Put differently

$$\underbrace{p(\theta)u}_{\text{Flow out of unemployment}} = \underbrace{(L_m - u)s}_{\text{Flow into unemployment}} \tag{2.17}$$

Now the equilibrium completely determined by equations 2.13, 2.16, 2.15, 2.14 and 2.17. Given initial conditions these equations determine the equilibrium which is unique in the case of homogeneous firms. Note that this is not true anymore in the

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case of heterogeneous firms.

2.4.3 Efficiency of the Equilibrium

The more interesting part is that this model, as well as the original Moen (1997) model, in equilibrium is always efficient in contrast to the Mortensen-Pissarides model because it endogenously incorporates the Hosios condition[20], developed by Hosios (1990). While the Mortensen Pissarides model is only efficient if the negative elasticity of the probability of filling a vacation with respect to θ is equal to the bargaining power of the worker, i.e. the Hosios condition is fulfilled (see Ljungqvist and Sargent (2004), p. 950), the Moen model is always efficient.[21] Since the equilibrium in the agricultural market is always efficient due to perfect competition it is clear that both labor markets in the model I am using are efficient. Since every other part of the model is the same as in Satchi and Temple (2006) and in addition all other models using the Mortensen-Pissarides migration framework like Sato (2004) show that the Hosios condition is necessary and sufficient for an efficient outcome, it is sufficient to show that the Hosios condition is endogenously incorporated in my model. For a proof of this see the Appendix. The introduction of another frictionless sector does not alter these results.

In the next section I will have a look at some comparative statics to see what happens to the labor market in developing countries of this type if there are different shocks.

2.5 Comparative Statics

In this section I will look at what happens to the economy if there is a (positive) productivity shock in the urban sector/the agricultural sector. Briefly I will look at the changes one would expect in the case of changes in self-employed income z, separation rate s and vacancy cost c. In any case I will assume no costs of migration for simplification.

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[20] This condition generally says that efficiency is assured if private and social return to search coincide

[21] This of course holds only under the assumption that there is no Nash bargaining after firms and workers meet and firms have committed to their wage announcements.

2.5.1 Productivity Shock in the Urban Sector

A productivity shocks means that y increases due to the definition of the production function of the firm because A_m goes up. This has the consequence that firms profits increase and firms want to create more vacancies because the value of a vacancy V from 2.15 increases. Obviously this increases the labor market tightness $\theta = \frac{v}{u}$ and therefore the probability of finding a job increases while the probability of filling a vacancy decreases. In order to fill vacancies more easily firms are willing to pay a higher wage which is proven in Appendix B. Both - an increase in the probability of finding a job and an increase in the wage rate - has the implication that the expected value of being unemployed U also goes up. Looking at the migration condition in equation 2.13 shows that workers will migrate from the agricultural sector to the urban sector. Migration increases the number of unemployed in the city which will then decrease θ. In addition as shown in Appendix B an increase in U also results in an increased θ at all wages, which is in line with Moen (1997). In the new equilibrium all equilibrium conditions are going to hold with equality again. Migration will result in an increased wage in the agricultural sector and eventually cease when the migration condition 2.13 holds again.

In summary: An increase in productivity in the urban sector leads to higher wages in both sectors, people from the countryside migrate to the city. Note that it is not trivial to say anything about the unemployment in the city. There are two opposing effect on θ - which includes the unemployment rate - and it uncertain whether θ increases or decreases. This leaves room in this model for the Todaro Paradox:[22] In their model an increased productivity in the urban sector can lead to an increased unemployment. The same is theoretically possible here. It should be possible to show this in calibrations. It is clear though that it is rational for workers to move from the countryside to the city because of the increased payoff rU. They do this although the unemployment might end up being higher then before. Moving from work into unemployment apparently can be economically rational. Graphically: Given the new wage there can be two situations: One in which the new θ is higher then in the original equilibrium and one in which it is lower as illustrated in Figure 2. What the actual

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[22]Satchi and Temple (2006) obtain a similar result for the Mortensen Pissarides model.
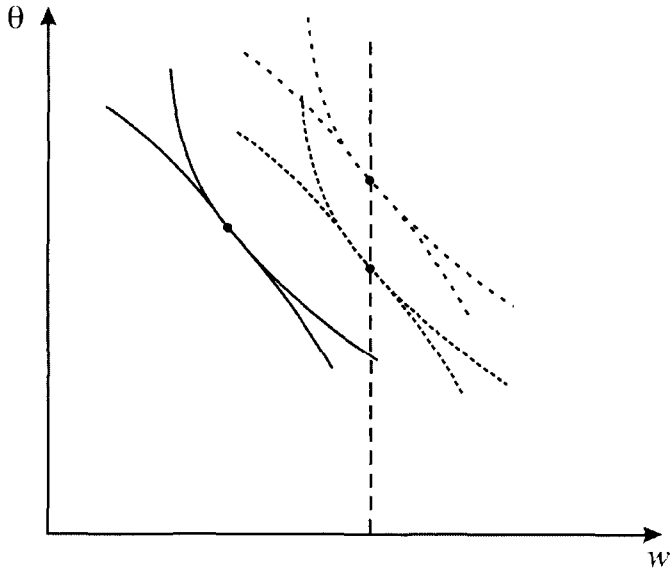
Figure 2: Equilibrium Changes Caused by a Productivity Increase

outcome would be for θ has to be computationally determined.

This extents the existing literature in that sense that in contrast to the Harris Todaro model this model provides a good reason for migrants to stay. While in Harris-Todaro Migrants that loose the lottery would be better of migrating back, this does not apply in a search model setting. Since workers here are indifferent between living on the countryside and being unemployed in the city they have no reason to go back. In Harris-Todaro they were indifferent between the wage in the agricultural sector and the expected wage in the city. Furthermore my model improves on the Mortensen-Pissarides modeling in the sense that it was shown that even a efficient equilibrium can incorporate the Todaro Paradox.

2.5.2 Productivity Shock in the Agricultural Sector

A productivity shock in the agricultural sector leads to an increased A_a and the production function to higher wages in the agricultural sector. Through equation 2.13 it is obvious that unemployed workers move from the city to the countryside which will make it harder for firms to find workers, i.e. θ increases. This should result in increased wages in the city and firm will decrease vacancies (decrease θ)

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which will result in an increased value for U and at the same time migration drives wages down again in the agricultural sector. In the new equilibrium 2.13 will hold again with equality. Wages in both sectors will be higher and it remains unclear how θ changes. It is desirable to look at a model where capital in the city is less mobile and the effects in such a model of a productivity shock in agriculture.This will be subject of further research.

2.5.3 Increase in Self-Employed Income z, Separation Rate s or Vacancy Cost c

In case of an increase in z it is expected that U increases, i.e. the benefit of being unemployed/self employed it higher. This would also cause θ to increase through equation 2.16 and w^* to increase over equation 2.15. The increases would adjust that much that in equilibrium equation 2.14 holds with equality. Certainly workers would move from the countryside to the urban sector. What happens to unemployment is also unclear.

The effect of an increase in the separation rate or in c are not as simple to see will be subject of further research.

3 Extensions

3.1 Closed Capital Account and Heterogeneous Firms

The most obvious extension as I already mentioned before is to drop the assumption of free capital flow from and to the world in the urban sector and/or to drop the assumption of fixed capital (land) in agriculture, i.e. make capital able to flow between agriculture and the urban sector. That probably would create some interesting results. Since it can be assumed that in general capital is not perfectly mobile the model could have different policy implications for developing countries.

In addition one could drop the assumption of homogenous firms and include heterogeneous firms in the model as in Moen (1997). The difference would be that firms would draw their productivity from a distribution and then decide to enter the market or not. In essence what this would change is simple that in equilibrium there

would be different wages for different submarkets.[23] Firms with a higher productivity would offer higher wages, get a low labor market tightness and low productivity firms would get higher labor market tightness and lower wages. Workers would be exactly indifferent of which submarket to enter: If the wage is high it is harder to find a job, if it is low it is easier to find a job. The comparative statics could change.

3.2 Endogenous Search Intensity and Firing Cost

Another interesting extension would be to include all the specifications Satchi and Temple (2006) use in their paper in this model and look at some comparative statics. The new model would look like the following:

$$\text{Worker unemployed} \quad rU_i = z + p(\theta_i)(W_i - U_i) - \sigma \tag{3.1}$$

$$\text{Worker employed} \quad rW_i = w_{m,i} - s(W_i - U_i - P) \tag{3.2}$$

$$\text{Firm vacant} \quad rV = -c + q(\theta_i)(J - V) \tag{3.3}$$

$$\text{Firm producing} \quad rJ = y_i(k_m) - w_{m,i} - s(J + F + P) \tag{3.4}$$

Where the newly introduced variables are: F- firing cost, P - severance payment from the firm to the worker in case of separation, σ - search cost. Workers would be able to choose their search intensity and influence the probability of finding a job in doing so. Therefore it would also be necessary to introduce a new matching function which takes care of the reasonable assumption that higher search effort is rewarded with a higher probability of finding a job. An example for such a matching function is: $m(u, v) = (\lambda u)^{\alpha} v^{1-\alpha}$. It would be interesting to see more comparative statics and see if the equilibrium still would be efficient. The last point looks especially interesting since search cost in combination with discounting seem to make the result inefficient in a model of Nash bargaining even if the Hosios condition is satisfied (see Mortensen and Wright (2002)).

Heterogeneous firms result in having a subscript i in every of the HJB equations because wages will differ across submarkets. Note that $U_i = U$ since workers will only enter a submarket with the highest expected payoff U. Since all submarkets that nobody enters are closed, it must hold that in all other submarkets the expected

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[23]A submarket is formed by all firms which offer the same wage.

payoff is the same.

4 Conclusions

This paper has shown that it is possible to transform the model introduced by Moen (1997) into a setting applicable to developing countries. This was done combining the work of Moen (1997) and Satchi and Temple (2006). Because of wage determination in the model the equilibrium is a lot more complicated then the one found by Satchi and Temple (2006) where the wages where ex post determined through Nash bargaining. The complexity certainly is a disadvantage of this model and it needs to be seen how this model fits actual data and what results simulations would predict for changes in the parameters.

The advantage of the model is that the number of jobs created in this economy is efficient since the Moen model endogenously incorporates the Hosios condition, which still holds in this setting. Despite the efficiency it was shown that there is still room for the Todaro Paradox, which means in this model that an increased labor demand due to higher productivity might lead to an increased unemployment. The comparative statics results were mostly in line with Satchi and Temple (2006).

As mentioned earlier, this paper extents the existing literature in the sense that even an efficient equilibrium can incorporate the phenomenon of job creation in the city that eventually leads to higher unemployment. It was also shown that policy implications derived from a Mortensen-Pissarides setting e.g. by Sato (2004), do not hold if the real world works more like the Moen's competitive search approach, rather than the Mortensen-Pissarides approach. This could make an important difference for policy implications in developing countries.

Another desirable extension would be to verify if it is feasible to incorporate a search environment into a Lucas (2004) framework.

The model introduced in this paper offers a wide range of possible extensions as discussed in the previous section and leaves many interesting questions for further research.

Appendix

A Efficiency of the Moen Model

This proof follows closely Moen (1997), p. 408f. Taking the derivative of 2.16 with respect to w and keeping in mind that $p(\theta) = \theta q(\theta)$ gives:

$$\frac{\partial \theta}{\partial w} q + \theta \frac{\partial q}{\partial w} = -\frac{rU - z}{(w - rU)^2}(r + s)$$

$$q \frac{\partial \theta}{\partial w}(1 - \Psi) = -\frac{rU - z}{(w - rU)^2}(r + s) \qquad \Psi \equiv -\frac{\theta}{q}\frac{\partial q}{\theta}$$

$$(1 - \Psi)\frac{\partial \theta}{\partial w} = \frac{-\theta}{w - rU} \tag{A.1}$$

where the last line uses equation 2.11 and the definition of $p(\theta)$. Ψ is obviously the elasticity of probability of finding a job with respect to θ. The firm maximizes the value of a vacancy in equation 2.15. Multiplying this equation with $r + q$ and taking the derivative of this equation with respect to w yields:[24]

$$\frac{\partial q}{\partial \theta}\frac{\partial \theta}{\partial w}\left[\frac{y - w}{r + s} - V\right] = \frac{q}{r + s} \tag{A.2}$$

Finally one can plug in A.1 into A.2 to get

$$\frac{\Psi}{1 - \Psi}\left[\frac{y - w - (r + s)V}{w - rU}\right] = 1 \tag{A.3}$$

From equation 2.6 you can derive that $w - rU = (r + s)W - U$. Putting this in the equation above yields

$$\frac{\Psi}{1 - \Psi}\left[\frac{J - V}{W - U}\right] = 1 \tag{A.4}$$

Define the matching surplus as $S = W - U + J - V$, split the above equation into two parts and you get the Hosios condition:

$$\Psi S = W - U \tag{A.5}$$

$$(1 - \Psi)S = J - V \qquad\qquad \blacksquare \tag{A.6}$$

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[24]You have to take into account that according to the envelope theorem the derivative of V with respect to w is zero.

B Proofs for the Comparative Statics

Dependence of the lhs of $\overline{V}(U)$ on $q(\theta)$

Simply take the derivative of equation 2.14 with respect to q and show that this derivative is positive.

$$\frac{-1}{(r+q)^2}\left[\frac{y-w}{r+s}q-c\right]+\frac{-1}{r+q}\left[\frac{y-w}{r+s}\right] \ > \ 0$$

$$(y-w)\frac{r}{r+q} \ > \ -c \tag{B.1}$$

This obviously holds under the assumption the $y > w$. ∎

Dependence of θ on U

Taking the derivative of 2.16 with respect to U yields:

$$\frac{\partial p(\theta)}{\partial U} = \frac{r}{w-rU}+\frac{rU-z}{(w-rU)^2} > 0 \tag{B.2}$$

This is obviously larger then zero when looking at 2.16, keeping in mind that the lhs has the interpretation of probability of finding a job. ∎

Dependence of w on y

It needs to be proven that w increases with a productivity shock:
To derive this it is necessary to take the first derivative of 2.15 with respect to w because from this equation it can be shown by the implicit function theorem how w depends on y:

$$-\frac{1}{r+q(\theta(w,U))}\frac{1}{r+s}q(\theta(w,U))+\frac{1}{r+q(\theta(w,U))}\frac{y-w}{r+s}\frac{\partial q}{\partial w} \tag{B.3}$$

$$-\frac{1}{[r+q(\theta(w,U))]^2}\frac{\partial q}{\partial w}\left[\frac{y-w}{r+s}q(\theta(w,U))-c\right]=0$$

Lets define this function as $F(w,y)$. The implicit function theorem says that

$$\frac{\partial w}{\partial y}=-\frac{\frac{\partial F}{y}}{\frac{\partial F}{\partial w}}$$

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The arguments of the functions will often be dropped for conciseness from here on. The easy part is now to calculate $\frac{\partial F}{y}$, which is

$$\frac{\partial F}{y} = \frac{\partial q}{\partial w} \frac{1}{r+s} \left[\frac{r}{r+q} \right] > 0 \quad \text{since} \quad \frac{\partial q}{\partial w} > 0 \tag{B.4}$$

The last follows from the definition of q and equation 2.16.

$$\frac{\partial q}{\partial w} = \frac{\alpha}{1-\alpha} \left[\frac{rU-z}{w-rU} \right]^{\frac{-\alpha}{1-\alpha}-1} (r+s)^{\frac{\alpha}{1-\alpha}} \frac{rU-z}{(w-rU)^2} > 0 \tag{B.5}$$

Multiplying the derivative of 2.15 by $r+s$ gets

$$-q + (y-w)\frac{\partial q}{\partial w} - \frac{1}{r+q}\frac{\partial q}{\partial w} \left[(y-w)q - c(r+s) \right]$$

Taking the derivative with respect to w gives for $\frac{\partial F}{\partial w} =$

$$\frac{\partial F}{\partial w} = -\frac{\partial q}{\partial w} + (y-w)\frac{\partial^2 q}{\partial w^2} \underbrace{- \frac{\partial q}{\partial w} - \frac{1}{r+q}\frac{\partial^2 q}{\partial w^2} \left[(y-w)q - (r+s)c \right]}_{\text{positive element}}$$

$$\underbrace{+ \frac{1}{(r+q)^2}\frac{\partial q}{\partial w}\frac{\partial q}{\partial w} \left[(y-w)q - (r+s)c \right]}_{\text{positive element}} - \frac{1}{r+q}\frac{\partial q}{\partial w}\frac{\partial q}{\partial w}(y-w) < 0 \tag{B.6}$$

It can be shown that this is negative because $\frac{\partial^2 q}{\partial w^2}$ is negative as can easily be seen when taking the derivative of equation B.5 and the fact that the two positive terms are smaller then the negative terms. The proof is done in separately looking at the two positive elements of the equation and showing that with taking some of the negative parts of the equation those new components are negative. Since all remaining elements will be negative this also holds for the whole equation. For both positive elements you can ignore the $-(r+s)c$ part because it decreases the positive part. Proof:

$$(y-w)\frac{\partial^2 q}{\partial w^2} - \frac{1}{r+q}\frac{\partial^2 q}{\partial w^2}(y-w)q = (y-w)\frac{\partial^2 q}{\partial w^2}\frac{r}{r+q} < 0$$

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and

$$\frac{1}{(r+q)^2}\frac{\partial q}{\partial w}\frac{\partial q}{\partial w}(y-w)q - \frac{1}{r+q}\frac{\partial q}{\partial w}\frac{\partial q}{\partial w}(y-w) = \frac{1}{r+q}\frac{\partial q}{\partial w}\frac{\partial q}{\partial w}(y-w)\frac{-r}{r+q} < 0$$

All other remaining terms are obviously negative which makes the whole expression negative. This means for the derivative we were looking for that

$$\frac{\partial w}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial w}} = -\frac{>0}{<0} > 0$$

which implies that a productivity shock in the urban sector increases the wage in this sector. ∎

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