IMPEDANCE, EQUIVALENT CIRCUIT AND CAPACITANCE OF DOUBLE BARRIER RESONANT TUNNELING DIODE

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ABSTRACT

By employing scattering parameter (S parameter) measurements with on-wafer probing techniques, we have measured accurate small signal impedances for two different double barrier quantum well resonant tunneling diodes (RTDs) up to 40GHz. It was found that the impedances can be very well described by an equivalent circuit that consists of a parallel admittance-capacitance (G-C) and a series resistance-inductance (R-L) circuit. The inductance was found to have resulted from the on-wafer connecting lines and the airbridge structure. The quantum well itself is represented by the parallel G-C circuit. With the measured impedances and the equivalent circuit representation, capacitance-voltage (C-V) and conductance-voltage (G-V) characteristics have been obtained.

Our experiment shows that the G-V characteristic obtained from the high frequency measurement is comparable to that obtained from dc current-voltage (I-V)characteristic. Using this comparison, the I-V characteristic that was distorted by selfoscillations in the negative differential resistance (NDR) region of the dc measurement has been recovered from the stable high frequency rf G-V characteristic. A hysteresis that is not affected by oscillations was observed in the S parameter measurements. Our study shows that this hysteresis is not due to the charge storage effect predicted by theory, but to the load-line bistability of the internal series resistance.

We have proposed a new capacitance calculation scheme and calculated the capacitances for RTDs, based on the charge distributions obtained from self-consistent calculations. The calculated C-V characteristics agree qualitatively with experimental

results. Both the theoretical and experimental results show a capacitance peak in the NDR region. Calculation indicates that the peak is caused by electrons discharging from the quantum well, rather than from the charge accumulations in the quantum well as was formerly believed. Also observed was an additional smaller capacitance peak corresponding to electrons discharging from the accumulation region adjacent to the emitter barrier.

We have theoretically studied the effects of the spacer layers on the capacitance. Analysis indicates that the capacitance peak in the NDR region is very sensitive to variations in the cathode spacer layer. The anode spacer layer has a relatively limited effect on the peak. Increasing the anode spacer layer can decrease the capacitance in general and increase the cutoff frequency of the RTD. The effects of the fixed interface space charge on I-V and C-V characteristics have also been considered. The device performance is found to be degraded slightly by the presence of a fixed space charge of normal density.

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CHAPTER 1. INTRODUCTION

1.1 Background

Progress in semiconductor heterostructure growth technology, particularly in the epitaxial technology, has permitted the fabrication of nanometer structures whose behavior is dominated by quantum interference effects. The quantum well resonant tunneling diode (RTD) is the direct result of such technological progress. In 1973, Tsu and Esaki (Tsu and Esaki, 1973) suggested that the tunneling mechanism of the supperlattice structure should lead to a negative differential resistance (NDR) in the current-voltage (I-V) characteristic. In 1974, the first double barrier quantum well resonant tunneling diode was fabricated (Chang *et al.*, 1974) and NDR was observed at low temperatures.

The most important component of a RTD is the double barrier quantum well structure (Fig.1.1.1a). Usually the quantum well consists of a narrow bandgap material and the barriers consist of wide bandgap materials (referred to as type I heterostructure). A GaAs well and $Al_xGa_{1-x}As$ barriers represent one example of a type I heterostructure quantum well. Ideally, when the kinetic energy of an electron E_x is in resonance with the quasi-bound state E_w in the well, the transmission coefficient attains its maximum (for an symmetric quantum well it becomes unity). As the diode is biased and E_w is lowered to the Fermi level, current through the device increases. When E_w and the bottom of the emitter conduction band are aligned, current through the device is at a maximum (Fig.1.1.1b).



Biasing the diode beyond this point causes the current to decrease, giving rise to a NDR (Fig.1.1.1c). Associated with the change in the current is a change in electron density distribution. As indicated in Fig.1.1.1, both electron density in the well and current approach their maximum levels simultaneously at the peak voltage V_{peak} . Both decrease when the RTD is biased into the NDR region and reach their minimum levels at the valley voltage V_{valley} .

Due to the simplicity of its structure, the RTD is ideally suited for basic research on physics phenomena of nanostructure quantum systems, especially with respect to the transport properties,. Early RTD studies focused primarily on fundamental physics. In 1983, Sollner *et al.* (Sollner *et al.*, 1983) demonstrated the detection capabilities of RTDs up to 2.5THz. Their findings quickly stimulated further study on the application aspects of the diode (for examples, see Capasso *et al.*, 1986; articles in Sze, 1990 and Capasso, 1990). To date, many high speed analog and digital applications have been found for the device. Examples of these applications are high frequency oscillators (Sollner *et al.*, 1987; Brown *et al.*, 1987 and 1989), mixers (Mortazawi *et al.*, 1989; Millington *et al.*, 1991, Krishnamurthi and Conn, 1991), multipliers (Batelaan and Frerking, 1987; Sollner *et al.*, 1988; Sen *et al.*, 1988), high speed analog-to-digital converters (Capasso, 1990) and multi-state memory devices (Capasso *et al.*, 1986; Raychaudhuri and Deen, 1993).

For circuit design purposes, especially for ac and rf applications, an equivalent circuit representation is essential. For testing an equivalent circuit under experimental conditions, impedance measurement is the most direct technique. Since the invention of

the diode, a number of equivalent circuits have been proposed and tested (Gering *et al.*, 1987; Sheard and Toombs, 1987; Kesan *et al.*, 1988 and 1989; Lippens and Mounaix, 1988; Brown *et al.*, 1989; Zarea *et al.*, 1990; Genoe *et al.*, 1991; Miles *et al.*, 1991; Schemmann *et al.*, 1991; Whitson *et al.*, 1991; Vanbesien *et al.*, 1992). Except for a few cases (Sheard and Toombs, 1987; Miles *et al.*, 1991; Genoe *et al.*, 1991), most of these models featured the equivalent circuit used for the Esaki diode. The later consists a parallel (lumped) resistance-capacitance circuit, a series resistance and a series inductance (Fig.1.1.2). In some cases, only the Esaki diode equivalent circuit was employed (Gering *et al.*, 1987; Lippens and Mounaix, 1988; Zarea *et al.*, 1990; Schemmann *et al.*, 1991).



Fig.1.1.2 Equivalent circuit used for the resonant tunneling diode. Parallel R-C circuit represents the quantum well structure. The series R-L circuit accounts for the remaining structures.

However, all of these circuits were tested either over a limited frequency range or under very restricted bias conditions. The main reason for these approaches was that spurious self-oscillations in the NDR region make a complete impedance measurement exceedingly difficult. In addition to oscillations, the packaging effect is also considered a problem. Usually, the capacitance of a fabricated RTD is very small. At high frequencies the reactance of the parasitic capacitance and inductance from very short bond wires in a packaged diode are comparable in value to the device capacitance. This makes it very difficult to de-embed the measured impedance values, a prerequisite for meaningful analysis.

Associated with the equivalent circuit is an important diode parameter: capacitance. In the last decade, there have been extensive theoretical and experimental studies on the I-V characteristic of RTD. Much effort has been invested in improving its I-V characteristic. One widely used method involves adding undoped spacer layers to both sides of the quantum well (for example, see Sze, 1990). Compared to studies of I-V characteristics, relatively little effort has been made to understand the capacitance. The reason for this was that determining the capacitance experimentally encounters the same difficulties associated with impedance measurements. In general, determining capacitance requires impedance measurements. To obtain the capacitance value, an appropriate equivalent circuit has to be used and the impedance needs to be measured. As mentioned earlier, before a meaningful impedance result can be obtained, oscillations must be suppressed and the packaging effects de-embedded. Few attempts have been made to measure capacitance. The Esaki equivalent circuit has been used (Eaves et al., 1989; Zarea et al., 1990; Boric et al., 1992). Near the NDR region a peak was found in capacitance-

voltage (C-V) characteristics. Schubert *et al.* (1990) attributed the peak to the charge accumulation in the quantum well. According to their interpretation the capacitance peak should coincide with the current peaks in the *I*-V characteristics. However, most of the measurements failed to provide accurate results. Moreover, Boric *et al.* (1992) pointed out that the peak might in fact shift toward higher voltages in the NDR region. Employing theoretically physical models, some researchers have calculated the capacitance of quantum well structures; however, no peak feature was found (Yokoyama, 1989; Sun *et al.*, 1992). Understanding of the capacitance behavior of the RTD will require more careful measurement and more comprehensive theoretical studies.

1.2 Overview of Thesis Work

Our experimental work involved measuring the microwave frequency small signal scattering parameters (S parameters) of RTDs. Diode impedances were obtained from the S parameters. To eliminate packaging effects, we have used the on-wafer probing technique. Two different quantum well structures were studied. One of them consists of the $Al_{0.4}Ga_{0.6}As$ -GaAs barrier-well structure, the other consists of the AlAs-GaAs structure. We first measured S parameters for $Al_{0.4}Ga_{0.6}As$ -GaAs diodes up to 18GHz at SFU (Wei, Stapleton and Berolo, 1993), and later extended the frequency range to 40GHz at the Communications Research Centre (CRC) in Ottawa (Wei, Stapleton and Berolo, 1995a). We also measured AlAs-GaAs diodes up to 40GHz at CRC (Wei, Stapleton and Berolo, 1995b). We found that, part of the oscillation power can couple into the network

analyzer and distort the S parameter signal. However, in most cases, oscillation frequencies were below 2GHz. It was also discovered that, if the high frequency components were not very strong, the oscillations would affect low frequency signals only. Thus, analysis could be carried out on reliable high frequency data. It was further discovered that, except for distorted low frequency signals, impedances obtained over the whole frequency range and under various biases could be reproduced quite successfully by the Esaki equivalent circuit (shown in Fig.1.1.2). In the equivalent circuit the parallel admittance-capacitance (G-C) structure represents the active quantum well region of the RTD, the series resistance R_s is associated with the contact and spreading resistance in the semiconductor and the inductance L_s is related to the connecting wires. With the equivalent circuit, capacitance C, conductance (admittance) G and series resistance R_s were extracted from the measured impedance (hereafter G and dynamic resistance $R_d=1/G$ will be used interchangeably). It was found that the G-V characteristic is directly comparable to the derivative of the dc I-V characteristic. Peaks were observed in C-Vcharacteristics for all diodes. These peaks were found not in the peak voltages as formerly predicted, but, rather, in the NDR regions. With respect to AlAs-GaAs diodes, a smaller capacitance peak at a lower voltage was also observed.

Oscillations in the NDR region are usually very difficult to suppress. They have been known to distort dc and low frequency ac measurements. Simulations have shown that the oscillations can distort the dc *I-V* measurements and produce some jumps and hysteresis in the NDR regions (Sollner, 1987; Liu, 1988; Belhadj *et al.*, 1990). These

jumps and hysteresis were found in some of our I-V measurements. Our S parameter measurements indicated that, however, if the oscillations are weak and the frequencies are not very high, the impedance obtained at higher frequencies will be affected very little and a smoothly varied G-V characteristic can be extracted. Based on the fact that conductance G is directly related to the derivative of I-V curve, we were able to recover an undistorted I-V curve by integrating the G-V characteristic.

A hysteresis in the NDR region was observed in the S parameter measurement as well as the dc I-V measurements for AlAs-GaAs diodes. The hysteresis phenomenon was originally observed in dc I-V measurements and attributed to the charge storage effect (Goldman *et al.*, 1987; Kluksdahl *et al.*, 1989). However, studies indicate that the hysteresis may result from oscillations in the NDR regions (Sollner, 1987; Liu, 1988; Belhadj *et al.*, 1990). Simulations have shown that oscillations will distort the dc I-Vcharacteristic and produce a hysteresis effect very similar to that observed in the dc I-Vmeasurements (Liu, 1988; Belhadj *et al.*, 1990). In this study it will be demonstrated that (i) the hysteresis observed in the S parameter measurements is intrinsic and not affected by oscillations; (ii) the hysteresis does not necessarily result from charge storage effects, but from load-line effects caused by a relatively large series resistance (Okean, 1971; Jogai, and Koenig, 1991; Chen *et al.*, 1991).

To simulate C-V characteristics, we have calculated the capacitances of RTDs, on the basis of electron density distributions obtained from quasi-static self-consistent calculations (Stapleton and Wei, 1994; Wei and Stapleton, 1995c). The self-consistent

calculations were performed by iteratively solving the time-independent Schrödinger equation and the Poisson equation. Theoretical C-V characteristics agree qualitatively with experimental results. Our results show that the peak in the C-V curve is attributable to the quantum well discharging process, which occurs in the NDR region, rather than the charge accumulation in the well as was formerly believed (Schubert *et al.*, 1990). Calculation also shows that the additional peak observed in the AlAs-GaAs diodes results from electrons discharging from the accumulation region between the emitter barrier and cathode spacer layer.

The calculated capacitance peaks were found to be higher than experimental ones, owing to scattering effects. The later, while considered important with respect to transport mechanism (Kluksdahl *et al.*, 1989; Frensley, 1990; Hu and Stapleton, 1991) were excluded from our treatment in order to simplify calculations. Notwithstanding this limitation, the results still provide a useful guideline for optimizing the diode structure for high speed applications. We have calculated the capacitance of four Al_{0.3}Ga_{0.7}As-GaAs RTDs with identical quantum well structure but different spacer layers (Wei and Stapleton, 1994). The results show that, capacitance generally decreases with an increasing spacer layer. When the cathode spacer layer is increased, the capacitance peak in the NDR region increases very rapidly and then decreases; when the anode spacer layer is increased, the peak only broadens and decreases slightly. Compared to the change (of the peak) resulting from variation in the cathode spacer layer, the effect of varying the anode spacer layer on the peak is relatively small. When we compare the cutoff

frequencies of RTDs with different spacer layers, it is found that the high frequency performance of a RTD can be improved by increasing the anode spacer layer.

By including the fixed space charge in the self-consistent calculation, we were able to take into account the effect of the fixed interface space charge on the I-V and C-Vcharacteristics (Wei and Stapleton, 1995d). It was found that the fixed space charge does not significantly affect the current peak to valley ratio; but it does increase the capacitance, and thereby, degrades the high frequency performance of the device. However, for a typical fixed space charge density resulting from normal interface disorders, this degradation was not found to be significant.

1.3 Thesis Outline

The experimental design and methodology are described in Chapter 2. The theoretical model is delineated in Chapter 3.

Chapter 4 includes S parameter measurement results for two $Al_{0.4}Ga_{0.6}As$ -GaAs RTDs, that range from 0 to 18GHz and 40GHz, respectively (4.1 and 4.2). These two diodes feature identical quantum well structures, but due to the non-uniformity of the wafer, their *I-V* characteristics are slightly different. We show that both devices produce similar *C-V* and *G-V* results. However, with respect to oscillations in the NDR region, the RTDs fall into two different categories. The RTD discussed in 4.1 had a relatively low peak to valley ratio, the oscillations observed in the NDR regions were also very weak; in the NDR regions the *I-V* characteristic was very smooth and remained virtually unaffected

by oscillations. The RTD discussed in 4.2 had a higher peak to valley ratio and stronger oscillations in the NDR regions: in the NDR regions, the measured I-V characteristic was distorted and some irregular jumps were observed. The first RTD is used to demonstrate that the dc G-V characteristic is comparable to that obtained from rf measurements. The second RTD is used to demonstrate that this comparableness makes it possible to restore the distorted I-V characteristic from the almost oscillation-free rf results. In these two sections we also compare the measured C-V results with the calculated results. In section 4.3, the experimental results for an AlAs-GaAs diode are presented and compared to the theoretical results. Section 4.4 includes theoretical C-V characteristics for four Alo ₃Gao ₇As-GaAs diodes featuring different spacer layers. By comparing these results it is possible to study the effect of different spacer layers on capacitance. We incorporated the fixed interface space charge into the self-consistent calculation and calculated the I-Vand C-V characteristics. In section 4.5 the results are compared to that obtained without the fixed space charge and the effects of the space charge on I-V and C-V characteristics are estimated.

The conclusions are presented in Chapter 5.

CHAPTER 2. EXPERIMENTAL

2.1 Device Structures, Layouts and Fabrications

We have studied two different types of diodes, one consisting of an $Al_{0.4}Ga_{0.6}As$ -GaAs quantum well structure, the other of an AlAs-GaAs structure. The double barrier quantum well wafers were grown at Bell-Northern Research Inc. in Ottawa, Ontario and fabricated at the Communications Research Centre (CRC), also in Ottawa.

All double barrier quantum well structures were grown on semi-insulating GaAs substrates employing molecular beam epitaxy (MBE). The Al_{0.4}Ga_{0.6}As-GaAs structure was grown in the following series (Fig.2.1.1): *i*) 500Å undoped GaAs, *ii*) 9000Å n⁺-GaAs contact layer (N_D =5×10¹⁸ cm⁻³), *iii*) 300Å undoped GaAs spacer layer, *iv*) 40Å undoped Al_{0.4}Ga_{0.6}As barrier, *v*) 50Å undoped GaAs quantum well, *vi*) 40Å undoped Al_{0.4}Ga_{0.6}As barrier, *vii*) 100Å undoped GaAs spacer layer, and *viii*) 5000Å n⁺-GaAs contact layer (N_D =5×10¹⁸ cm⁻³). The AlAs-GaAs structure was grown in a similar manner as described in Fig.2.1.2. Note that both devices consist of symmetric quantum well structures but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs and *viii* as the structure was grown in a similar manner as described in Fig.2.1.2. Note that both devices consist of symmetric quantum well structures but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs and *viii* as the structure was grown in a similar manner as described in Fig.2.1.2. Note that both devices consist of symmetric quantum well structures but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs and *viii* and *i* as the structure was grown in a similar manner for the spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs are the structure but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs are the structure but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs are the structure but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs are the structure but feature asymmetric undoped spacer layers. Four identical Al_{0.4}Ga_{0.6}As-GaAs are the structure but feature asymmetric undoped spacer layers and two identical Al_{0.4}Ga_{0.6}As-GaAs are the structure but feature asymmetric undoped spacer layers are the structure but feature asymmetric undoped spacer layers are the structur

A typical RTD layout is illustrated in Fig.2.1.3(a). The schematic cross section view of a fabricated diode is displayed in Fig.2.1.3(b). In the device fabrication, wet chemical etching was used to isolate devices (dashed line in Fig.2.1.3(a) and slope area in



Fig.2.1.1 Schematic cross sectional view of the Al_{0.4}Ga_{0.6}As-GaAs double barrier resonant tunneling diode structure. Diode structure was grown by MBE, on a semi-insulating GaAs wafer.

viii)	5000 Å	GaAs 1X10 ¹⁸ cm ⁻³
vii)	100 Å	GaAs undoped
vi)	17 Å	AIAs undoped
V)	45 Å	GaAs undoped
iv)	17 Å	AIAs undoped
iii)	700 Å	GaAs undoped
ii)	9000 Å	GaAs 1X10 ¹⁸ cm ⁻³
i)	500 Å	GaAs undoped
		Semi-insulating GaAs

Fig.2.1.2 Schematic cross section view for the AlAs-GaAs diode structure.







Schematic cross sectional view of a fabricated diode. Only important structures are shown. Fig.2.1.3b) Fig.2.1.3(b)). Reactive ion etching (RIE) was used to define diode areas (the smallest square in Fig.2.1.3(a)) and to expose the underlying contact layer *ii*). With RIE, the active region of a diode is vertically etched and the diode area is as laid-out (Fig.2.1.3(b)). Diodes with areas ranging from $5\times5 \ \mu\text{m}^2$ to $46\times80 \ \mu\text{m}^2$ were fabricated. After etching, the wafer was covered by a layer of Si₃N₄ to passivate the active diode region. Standard Au/Ge/Ni ohmic contacts were fabricated on the contact layers *viii*) and *ii*). Each diode was connected to a standard on-wafer microprobe ground-signal-ground pad configuration. The signal pad was connected to layer *viii*) via a 50µm long low parasitic capacitance air-bridge, the ground pads were connected to the first level metal contact layer *ii*) (Fig.2.1.3(b)). To study the rf effects of the contact geometries alone, devices of identical geometries but without active layers present were also fabricated (that is, contact was made directly to the bottom contact layer *ii*)). In the following sections we shall refer to them as short-circuited reference devices.

2.2 I-V and S Parameter Measurements

Some of the wafers under study were probed with standard Cascade micro-probes (less than 26GHz) on a Cascade probing station at SFU; others with Picoprober probes (less than 60GHz) on a CK1000 probing station at CRC. The probing stations were mounted on air tables to be free of shocks and vibrations. The dc *I-V* characteristics were measured with either a HP4145B semiconductor parameter analyzer or a HP4142B modular dc source/monitor (Fig.2.2.1). To monitor oscillations in the NDR regions, an oscilloscope or a spectrum analyzer was connected across the diode (Fig.2.2.1). It was



found that if the oscillations were weak, as for some small area $Al_{0.4}Ga_{0.6}As$ -GaAs diodes, smooth *I-V* characteristics were obtained. On the other hand, when the oscillations were strong and could not be suppressed, the *I-V* characteristics showed some small jumps and hysteresis, as predicted by simulations.

The S parameter data was collected by a HP8510B (up to 18GHz at SFU) and HP8510C (up to 40GHz at CRC) network analyzers. To measure S parameters at various voltages, we biased the diode by a low internal resistance dc power source, through the built-in bias line (Fig.2.2.2). The total load line resistance was measured to be 1.5Ω . In some cases a bias tee connected to the probes was used and the load line resistance was reduced to 0.6 Ω (because of a short connecting line). For measurements up to 18GHz, the signal power level of the network analyzer was set at -10dBm. The real power delivered to the probes was, however, attenuated by the connecting cables and consequently was smaller. The signal power at the probes measured -23dBm (corresponding to ~45mV peak-to-peak in magnitude) at 0.5GHz and -32dBm (~15mV_{pp}) at 18GHz, with the signal at intermediate frequencies decreasing monotonically with increasing frequency. As the S parameters were obtained by measuring the ratio of the reflected electromagnetic wave to the incident wave, the non-uniformity of output power did not affect the accuracy of the measurements. For measurements up to 40GHz, the signal power level was set at -5dBm.

The impedance Z can be calculated from the S parameter through the following equation (Roddy, 1986):



in bias line or a bias-tee connected to the probe (dashed lines). The oscillations are studied by replacing the network analyzer by a spectrum analyzer or an oscilloscope (we assume that the oscillation conditions remain the same as the input impedance remains 50Ω). Fig.2.2.2

$$Z = \frac{1 + S_{11}}{1 - S_{11}} Z_0 \tag{1}$$

where S_{11} is the measured one port S parameter and Z_0 the 50 Ω characteristic impedance of the transmission lines. Ohmic measurements of the short-circuited reference devices indicated that the total resistance of the air-bridge and the probe contacts was less than 0.2 Ω . The inductance of the air-bridges was found to be between 50pH to 70pH when the probes were positioned in the middle of the probe pads and between 10pH and 40pH when positioned very close to the device under test. The pads were 100×100µm² in size (Fig.2.1.3(a)). A slight shift (~10-20µm) in probe positions was found to result in a 10-20pH change in the measured inductance. Consequently, the probe positions remained stationary during the whole measurement procedure. All measurements were made at room temperature.

It is known that oscillations in the NDR region can significantly affect measurement results (Sollner, 1987; Liu, 1988; Belhadj *et al.*, 1990). As it was usually very difficult to suppress the oscillations, we decided to study smaller diodes where the magnitudes (powers) of the oscillations were smaller. Before the measurements, the network analyzer was replaced by a spectrum analyzer (or an oscilloscope), the bias was adjusted and oscillation magnitudes were measured (Fig.2.2.1). Since the input impedance of the spectrum analyzer (or oscilloscope) was the same as that of the network analyzer (50 Ω), we assumed that the oscillation conditions were identical to that during the *S* parameter measurements. During the *S* parameter measurement an oscilloscope was

connected to the bias circuit and used as an oscillation indicator (Fig.2.2.2). Since only very low frequency (near dc) signals could couple into the dc bias circuit and the signal observed on the oscilloscope was different from the signal that entered the network analyzer, it was used only as a complementary oscillation indicator. Generally, the variations of the oscillation magnitudes observed on the oscilloscope agreed with what we had predicted from the oscillation measurements (before the S parameter measurements). This observation supported our assumption that the oscillation signals observed before the S parameter measurements were the same as those present during the S parameter measurements. Our oscillation measurements showed that usually the oscillation frequencies were relatively low (<2GHz). In the NDR region, especially in the middle NDR area, the measured S parameters exhibited some irregular patterns at low frequencies due to the oscillations (not smoothly varied in respect to the frequency, see Fig.4.1.2b, for example). However the higher frequency signal was clean and smooth. When smooth lower frequency data was not available, measurements were extrapolated from the more reliable results obtained at high frequencies. We have examined almost all of the small diodes on all wafers. On each wafer at least 4 diodes were carefully studied. For brevity, only results obtained from some representative diodes are discussed in the following.

CHAPTER 3. THEORETICAL CALCULATIONS

3.1 The Schrödinger Equation Solution for an One-Dimensional Quasi-Static RTD Model

A RTD under applied voltage is a complex non-equilibrium many-body system. For theoretical purposes, we shall treat it by making some approximations. In particular, we start by considering the RTD as an one-dimensional system. We adopt the Hartree approximation that assumes each electron moves in an average potential produced by all other electrons (Thomas-Fermi screening potential), neglecting many-body interactions. We use the effective-mass approximation to smooth out the microscopic structure of the semiconductor. We assume that the system is stationary (equilibrium) under bias. To reduce the amount of calculations involved, we also ignore all scattering effects. Previous studies have shown that, without scattering effects, qualitatively correct I-V characteristic can still be obtained, although the obtained valley current can be significantly large than the reality (Cahay et al., 1986). So, with all these approximations, we expect that our calculation would not provide us a quantitatively accurate result, but a qualitatively correct picture. We can used this calculation to analyze our experimental results, to study the tunneling mechanism, and to optimize the device structure for high speed applications.

The time-independent envelope function $\psi(x)$ for an electron of energy E is assumed to be given by the Schrödinger equation (Stern and Sarma, 1984):

$$-\frac{\hbar^2}{2}\frac{d}{dx}\left(\frac{1}{m^*(x)}\frac{d\psi(x)}{dx}\right) + V(x)\psi(x) = E\psi(x)$$
(3.1.1)

where \hbar is the Planck constant and $m^*(x)$ the position-dependent effective mass. V(x) is the effective potential in which the electron moves:

$$V(x) = -e\phi(x) + V_h(x) \tag{3.1.2}$$

variable *e* is the electron charge, $\phi(x)$ the electrostatic potential, and $V_h(x)$ the effective potential associated with the heterojunction discontinuity. Note that the local exchange-correlation energy has been neglected.

For the purpose of numerical calculation, we discretise a continuously varied potential into a multi-step function of N segments (Fig.3.1.1). In each segment the potential and effective mass are regarded as constants:

$$V_j = V(x_j) \tag{3.1.3a}$$

$$m_j^* = m^*(x_j), \qquad x_j < x < x_{j+1}$$
 (3.1.3b)

Variables x_0 and x_N are located at the far ends of the device. Since the potential varies greatly in the device, we used unequal segment widths, that is, smaller segment widths were used when V changes rapidly with the coordinate x.

Given this approximation, the wavefunction between x_j and x_{j+1} can be written as:

$$\Psi_j(x) = A_j e^{ik_j(x-x_j)} + B_j e^{-ik_j(x-x_j)}$$
(3.1.4)

where $k_j = \sqrt{2m^*_j (E - V_j) / \hbar^2}$ is the longitudinal wave vector (kinetic momentum); A_j and B_j are complex constants, they represent the forward and backward waves, respectively.

and



Discretisation of a continuously varied conduction band potential. T is the transmission coefficient for an electron of energy E injected from the left. R is the reflection coefficient. V_{app} is the voltage applied across the diode. Fig.3.1.1

The continuities of $\psi(x)$ and $\psi'(x)/m^*$ (BenDaniel and Duke, 1966) at $x=x_{j+1}$ require that:

$$A_{j}e^{ik_{j}(x_{j+1}-x_{j})} + B_{j}e^{-ik_{j}(x_{j+1}-x_{j})} = A_{j+1} + B_{j+1}$$
(3.1.5)

and

$$\frac{ik_j}{m_j^*} (A_j e^{ik_j(x_{j+1} - x_j)} - B_j e^{-ik_j(x_{j+1} - x_j)}) = \frac{ik_{j+1}}{m_{j+1}^*} (A_{j+1} + B_{j+1}) (3.1.6)$$

The solutions for Eqs.(3.1.5) and (3.1.6) are:

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = D_j \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix}$$
(3.1.7)

where D_j is the transfer matrix:

$$D_{j} = \frac{1}{2} \begin{pmatrix} e^{-ik_{j}(x_{j+1} - x_{j})} (1 + \frac{k_{j+1}m^{*}j}{k_{j}m^{*}j+1}) & e^{-ik_{j}(x_{j+1} - x_{j})} (1 - \frac{k_{j+1}m^{*}j}{k_{j}m^{*}j+1}) \\ e^{ik_{j}(x_{j+1} - x_{j})} (1 - \frac{k_{j+1}m^{*}j}{k_{j}m^{*}j+1}) & e^{ik_{j}(x_{j+1} - x_{j})} (1 + \frac{k_{j+1}m^{*}j}{k_{j}m^{*}j+1}) \end{pmatrix}$$
(3.1.8)

Eq.(3.1.7) shows that (A_j, B_j) and (A_k, B_k) $(k \neq j)$ are closely related. To obtain all A_j s and B_j s, and a complete solution of $\psi(x)$, the boundary conditions at both ends of the device have to be considered.

Consider an electron of energy E and magnitude of unity injected from the far left hand side of the device (Fig.3.1.1). At the far left hand side the wavefunctions of the electron is described as below:

$$\Psi_0(x) = e^{ik_0(x-x_0)} + R \ e^{-ik_0(x-x_0)}$$
(3.1.9)

where R is the reflection coefficient. Since there is not reflection wave at the far right hand side, at the far right hand side we have
$$\Psi_{N}(x) = Te^{ik_{N}(x-x_{N})}$$
(3.1.10)

where T is the transmission coefficient.

Now we define a new matrix

$$\mathcal{D}_j = \prod_{j=s}^{N-1} D_s \tag{3.1.11}$$

Applying Eq.(3.1.7) sequentially to A_j 's and B_j 's gives rise to

$$\begin{pmatrix} 1 \\ R \end{pmatrix} = \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \mathcal{B}_0 \begin{pmatrix} A_N \\ B_N \end{pmatrix} = \mathcal{B}_0 \begin{pmatrix} T \\ 0 \end{pmatrix}$$
(3.1.12)

From Eq.(3.1.12) we have the transmission and reflection coefficients

$$T = \frac{1}{\mathcal{B}_{0,11}} \tag{3.1.13}$$

$$R = \frac{\mathcal{B}_{0,21}}{\mathcal{B}_{0,11}} \tag{3.1.14}$$

Knowing T and R, all A_i 's and B_i 's can be obtained from the following formulas

$$A_j = \mathcal{D}_{j,11}T$$
 (3.1.15)

$$B_{j} = \mathcal{O}_{j,21} T$$
 (3.1.16)

With A_j and B_j , the wavefunction, including magnitude and phase at all coordinates, for an electron traveling from left to right is fully determined by Eq.(3.1.4). Especially, at $x=x_j$, $\psi(x)$ assumes the more simple form of

$$\Psi(x_j) = A_j + B_j \tag{3.1.17}$$

The wavefunction from right to left may be calculated in a similar manner.

and

Knowing the transmission coefficient and thus the tunneling probability $|T|^2$, it is possible to calculate the current in the RTD. Here, we use Landauer's model (Landauer, 1970; Büttiker, 1986) in which the emitter and collector are assumed to be electron reservoirs, that is, they can provide unlimited electrons and remain undisturbed by changes in the device. Given these assumptions, the current from left to right can be obtained by integrating the charge density times the tunneling probability:

$$J_{lr} = e \int_{0}^{+\infty} |T_{lr}(k)|^2 f(k) dk$$
 (3.1.18)

where $|T_{lr}(k)|^2$ is the tunneling probability from left to right and *e* the electron charge. f(k) is the one-dimensional Fermi-Dirac distribution function (Cahay *et al.*, 1986; Hu, 1991):

$$f(k) = \frac{m^* k_B \Theta}{2\pi\hbar^2} \ln(1 + \exp(\frac{E_F - E(k)}{k_B \Theta}))$$
(3.1.19)

 k_B is the Boltzmann constant, Θ the temperature and E_F the Fermi level.

The integral in Eq.(3.1.18) can be replaced by a summation in the numerical calculation:

$$J_{lr} = e \sum_{k=0}^{+\infty} |T_{lr}(k)|^2 f(k) \Delta k$$
 (3.1.20)

A FORTRAN program has been written to perform this summation. In our numerical calculations we used unequal Δk . As |T| is very large at $E \sim E_w$ and f decreases almost exponentially with increasing energy E, the most important contributions come from E<0.2eV and $E\sim E_w$, in these regions we used smaller Δk . Also, since f(k) decreases

rapidly (almost exponentially) with increasing energy E, to implement numerical calculations we truncated the upper limit to a certain k_m . We chose a k_m which corresponds to E_m =4eV in all calculations, because the contribution from k_m to infinity is very small:

$$\int_{k_{m}}^{+\infty} f(k)dk < 10^{-7} \int_{0}^{+\infty} f(k)dk$$
(3.1.21)

For the right to left current J_{rl} , an analogous second equation can be obtained. However, in this case E in f(k) of Eq.(3.1.19) should be replaced by $E \cdot V_N$. The total current is the difference between the current flowing from left to right and that flowing from right to left. The current is a function of the applied voltage, which is the difference in the Fermi energies of the two sides.

With wavefunctions at various energies (or momenta) known, electron density n(x) can also be calculated. n(x) is obtained by summing the contributions of the various momenta k from both contacts (Cahay *et al.*, 1986; Frensley, 1989):

$$n(x) = \int_{0}^{+\infty} |\psi(k,x)|^2 f(k) dk$$
 (3.1.22)

As in Eq.(3.1.18), the above integration is approximated by a summation in numerical calculations. We have written a FORTRAN program for such a calculation. Similar to Eq.(3.1.20), smaller Δk is used for E<0.2eV and $E\sim E_w$, and the summation is truncated at $E_m=4$ eV.

3.2 Solution of Poisson Equation

In the Thomas-Fermi screening approximation, the electrostatic potential satisfies the Poisson's equation (Jackson, 1975; Stern and Sarma, 1984):

$$\frac{d}{dx}\left(\kappa(x)\frac{d\phi(x)}{dx}\right) = e(n(x) - N_D(x))$$
(3.2.1)

where $\kappa(x)$ is the position (material)-dependent dielectric constant and $N_D(x)$ the doping density profile in the device.

Numerically, Poisson equation can be solved by employing the finite difference method. The formula for the special case of equal-segment-width is readily available (Jacoboni and Lugli, 1989). It is generalized below to apply to the non-equal-width case. The left hand side of Eq.(3.2.1) at $x=x_j$ (2 < j < N-1) can be written discretely as

$$\frac{2}{x_{j+1} - x_{j-1}} \left(\frac{\phi_{j+1} - \phi_j}{x_{j+1} - x_j} (\frac{\kappa_{j+1} + \kappa_j}{2}) - \frac{\phi_j - \phi_{j-1}}{x_j - x_{j-1}} (\frac{\kappa_j + \kappa_{j-1}}{2}) \right)$$

= $a_{j,j-1} \phi_{j-1} + a_{j,j} \phi_j + a_{j,j+1} \phi_{j+1}$ (3.2.3)

where

$$a_{j,j-1} = \frac{(x_{j+1} - x_j)(\kappa_j + \kappa_{j-1})}{(x_{j+1} - x_{j-1})(x_{j+1} - x_j)(x_j - x_{j-1})}$$
(3.2.4)

$$a_{j,j} = -\frac{(x_j - x_{j-1})(\kappa_{j+1} + \kappa_j) + (x_{j+1} - x_j)(\kappa_j + \kappa_{j-1})}{(x_{j+1} - x_{j-1})(x_{j+1} - x_j)(x_j - x_{j-1})}$$
(3.2.5)

$$a_{j,j+1} = \frac{(x_j - x_{j-1})(\kappa_{j+1} + \kappa_j)}{(x_{j+1} - x_{j-1})(x_{j+1} - x_j)(x_j - x_{j-1})}$$
(3.2.6)

At both ends of the device, the electrostatic potentials are constants. Assuming that the left end is connected to the cathode and the right to the anode, that is, at the left hand side the electrostatic potential equals zero and at the right it equals the applied voltage, the boundary conditions are

$$\phi_0 = 0, \qquad \phi_N = V_{app} \tag{3.2.7}$$

and

$$\frac{\partial \phi(x = x_0)}{\partial x} = 0, \qquad \frac{\partial \phi(x = x_N)}{\partial x} = 0$$
(3.2.8)

The boundary conditions in Eq.(3.2.8) also require that

$$\phi_1 = 0, \quad \phi_{N-1} = V_{app}$$
 (3.2.9)

Writing the right hand side of Eq.(3.2.1) as ep(x), together with the boundary conditions in Eqs.(3.2.7-9), the Poisson equation can be written into a matrix form:



(3.2.10a)

If we simplify Eq.(3.2.10) to read

$$\widetilde{P}\widetilde{\phi} = \widetilde{\rho} \tag{3.2.10b}$$

the solution for the electrostatic potential $\phi(x)$ will be

$$\tilde{\phi} = \tilde{P} \quad \tilde{\rho} \tag{3.2.11}$$

In our calculations, we used a MATLAB program to conduct this matrix calculation.

3.3 Self-Consistent Calculation

The Poisson and Schrödinger equations are interrelated, therefore they must be solved self-consistently. From the Schrödinger equation one can see that the electron wavefunction $\psi(x)$ and density n(x) depend on the potential distribution V(x); in turn, V(x)depends on the electron density n(x). We carried out the self-consistent solution by iteratively solving the Schrödinger and Poisson equations. We used the partial iteration scheme suggested by Stern (1970):

- 1) Starting with an initial estimate for the electrostatic potential $\phi(x)$, calculate the electron density n(x) with Eq.(3.1.22).
- 2) With this n(x) a new $\phi_n(x)$ is calculated (by solving the Poisson equation with Eq.(3.2.11)).
- 3) The electrostatic potential is then updated by

$$\phi'(x) = (1+\alpha)\phi(x) + \phi_n(x)$$
 (0<\alpha<1) (3.3.1)

and a new n(x) calculated.

4) Repeat processes 2) and 3) until the calculation converges. The convergence is accomplished when $|\phi_n(x)-\phi(x)|$ is sufficiently small.

A MATLAB program has been written to perform the self-consistent calculation procedures 1) to 4). In this program, the FORTRAN program for Eq.(3.1.22) and the MATLAB program for Eq.(3.2.11) are used as subroutines. In our calculations, we partitioned the area of interest into unequal mesh points, the spaces between two points (segment widths) ranging from 2Å in rapid varying areas (around the quantum well) to 500Å in slowly changing contact areas. Generally, convergence of $|\phi_n(x_j)-\phi(x_j)|<0.1$ mV at every point x_i was required for a reasonably good *C-V* curve.

Once convergence takes place, the electron density distribution and the potential profile may be obtained. With the potential known, the tunneling probability $|T(k)|^2$ and the current density J can be calculated and the current-voltage characteristic obtained. With the electron density distributions obtained at various biases, the capacitance can be calculated (see section 3.4).

For the self-consistent calculation of a similar classical system (in which $|\psi(x)|^2$ in Eq.(3.1.22) is replaced by 1), for a given coordinate x, if the starting potential is lower than the final solution, Eq.(3.1.22) will give rise to a higher electron density, which in turn, gives rise to a higher electrostatic potential for the next iteration; if the starting potential is high, the resulting electrostatic potential will be low. This compensating ability usually prevents the self-consistent calculation from diverging and makes the calculation converge faster. However, for a quantum system, when the starting potential becomes too low, the calculated electron density for the next iteration can still be small due to the wave nature of the system. This results in a calculation that will eventually diverge. Thus the self-

consistent calculation can diverge if the initial potential is too far away from the final solution or if α used is too large. However, when α is too small the calculation will progress very slowly. So, to avoid divergence and to make the calculation converge quickly, α must be carefully chosen.

In our calculations, at V=0V, the classical solution $\phi(x)$ was used as the initial estimate for the iteration. At $V\neq 0V$, we used $\phi(x)$ calculated from n(x) at the next closest voltage V. To avoid divergence, we usually set $\alpha < 0.1$. If the starting electrostatic potential was not very good, a even smaller α (~0.005-0.01) had to be used. In some cases, the number of iterations for a single voltage point could be as high as 1000.

In the following calculations we deal with three electron contact concentrations: 1 $\times 10^{18}$ cm⁻³, 2×10^{18} cm⁻³ and 5×10^{18} cm⁻³. At room temperature they corresponded to Fermi levels of 0.042eV, 0.079eV and 0.156eV above the conduction bands, respectively. As an example, Fig.3.3.1 shows the calculated electron density and conduction band potential of the AlGaAs-GaAs RTD at *V*=0.17V. Except for the calculations shown in section 4.5, the effects of the fixed interface space charge will not be considered. When the effects of the fixed interface space charge are taken into account, a fixed space charge density σ is introduced into the doping profile N_D . We use a typical $\sigma=5\times 10^{10}$ cm⁻² (Kroemer *et al.*, 1980) in these calculations. It is assumed that σ will not change with the applied voltage.



line ---) for the $Al_{0.4}Ga_{0.6}As$ -GaAs RTD at V=0.17V. Also shown is the doping density profile N_D (dotted line -----). Note that only results from the quantum well area are shown. Fig.3.3.1

3.4 Capacitance Calculation

Once the charge distributions at various voltages are known, the capacitance can be calculated. The capacitance is defined as dQ/dV, where dV is the change in the bias voltage and dQ the charge injected into or extracted from the system resulting from dV(Gummel and Scharfetter, 1967). In a parallel plate capacitor (or a semiconductor device such as a p-n junction diode under depletion approximation), the positive charge and negative charge are clearly separated. The charge injected (dQ_i) into or the charge extracted (dQ_e) from the device is just the variation in the positive or negative charge in the device (Sze, 1981). In the system, charge neutrality requires that $dQ_i=dQ_e=dQ$.

In a RTD there is no boundary that separates the device into two distinct space charge regions, it is a unipolar system and the space charge distribution is positive, negative and positive (Fig.3.4.1). So dQ should be calculated with a different approach. We know that dQ relates directly to the displacement current in the system (Kennedy *et* al., 1968; Jackson, 1975). The displacement current $I_D = \kappa \frac{\partial E}{\partial t}$. Assuming that at time $t=t_0$, the applied voltage V=V, and at $t=t_1$, V=V+dV, the space charge moves across the cross section at x between t_0 and t_1 is

$$Q_{s}(x) = A \int_{t_{0}}^{t_{1}} I_{D}(x) dt$$
(3.4.1)

where A is the area of the device. Q_s is not uniform along the x-axis. In a segment $x \rightarrow x+dx$ some charge will accumulate (or disappear) if $Q_s(x) > Q_s(x+dx)$ ($Q_s(x) < Q_s(x+dx)$). If we count the accumulation (or disappearance) in the whole system, as with the p-n



Calculated space charge distribution ($N_D(x)$ -n(x)) of the Al_{0.4}Ga_{0.6}As-GaAs RTD at V=0.17V. Fig.3.4.1

junction diode, we obtain the charge injected (or extracted). From the Poisson equation we can find this is just the positive change in the space charge density $N_D(x)$ -n(x), or the negative change in electron density n(x) (since $N_D(x)$ does not change with V) (Fig.3.4.2):

$$dQ_i = eA \int_{-\infty}^{+\infty} \Delta^{-}(n(x,V))dx \qquad (3.4.2)$$

where Δ^{-} denotes the decreasing components:

$$\Delta^{-}(n(x,V)) = \begin{cases} 0 & n(x,V) > 0\\ |n(x,V)| & n(x,V) \le 0 \end{cases}$$
(3.4.2a)

Similarly, the total positive contributions correspond to the extracted charge dQ_e :

$$dQ_e = eA \int_{-\infty}^{+\infty} \Delta^+(n(x,V))dx \qquad (3.4.3)$$

 Δ^+ denotes the increasing components:

$$\Delta^{+}(n(x,V)) = \begin{cases} n(x,V) & n(x,V) > 0\\ 0 & n(x,V) \le 0 \end{cases}$$
(3.4.3a)

From Eqs.(3.4.2) and (3.4.3) we have

$$dQ_i - dQ_e = eA \int_{-\infty}^{+\infty} (n(x, V + dV) - n(x, V))dx$$
 (3.4.4)

Applying the neutrality condition $(N_D(x)$ is the doping profile):

$$\int_{-\infty}^{+\infty} (n(x) - N_D(x))dx = 0$$
(3.4.5)

to Eq.(3.4.4), we obtain Eq.(3.4.4)=0, or $dQ_i = dQ_e$.

Combining Eqs.(3.4.2) and (3.4.3), we obtain





$$dQ_i + dQ_e = eA \int_{-\infty}^{+\infty} (n(x, V + dV) - n(x, V)) dx$$
(3.4.6)

With $dQ_i = dQ_e$, we finally have

$$dQ = \frac{1}{2} eA \int_{-\infty}^{+\infty} (n(x, V + dV) - n(x, V)) dx$$
(3.4.7)

In practical calculations, the integral limits can be replaced by -D and +D (D>> the active area dimension). At greater distances from the quantum well the change in n(x) is negligible ($n(x)\approx N_D(x)$).

Knowing dQ, the capacitance

$$C = \frac{dQ}{dV} \tag{3.4.8}$$

can be calculated. Eq.(3.4.7) represents a general formula; it applies not only to relatively complex systems like the RTD, but also to simple systems such as p-n junction diodes and parallel plate capacitors. Eq.(3.4.7) shows that capacitance is always positive, it represents the fact that any change in charge distribution is associated with a certain delay in time.

Our self-consistent calculation is quasi-static, owing to the fact that the equilibrium Fermi-Dirac statistics has been used. It is known that this system operates in a nonequilibrium state. In addition, the scattering and other localization effects, which are considered very important in the transport of the RTD (Kluksdahl *et al.*, 1989; Frensley, 1990) are not included in our treatment. Studies have shown that, the *I-V* characteristics obtained from coherent (non-scattering) quasi-static self-consistent calculations give rise to negative resistance phenomena, but with much higher peak to valley ratios compared with experimental ones. Thus, it is expected that our calculations provide qualitatively correct I-V and C-V pictures, but not quantitatively accurate results. By comparing theoretical and experimental results, the calculations can provide insights into the current and capacitance behaviors of the RTD. By ascertaining how the capacitance depends on the barriers, quantum well and spacer layers, we can optimize the RTD structure for high speed applications.

CHAPTER 4. RESULTS AND DISCUSSIONS

4.1 Impedance, Equivalent Circuit and C-V Characteristics: Al_{0.4}Ga_{0.6}As-GaAs Diode, up to 18GHz

The structure of this $Al_{0.4}Ga_{0.6}As$ -GaAs RTD is illustrated in Fig.2.1.1. With the top contact layer (layer *viii*)) of the device functioning as an anode, the dc *I-V* characteristic for this $5\times10\mu$ m² diode was measured (Fig.4.1.1). The NDR region was found to be very smooth and no hysteresis was observed. This was because oscillations in the NDR region were very weak. Due to the fact that the cathode undoped spacer layer was longer (300Å) than its anode counterpart (100Å), at positive bias V_{peak} =0.39V and at negative bias V_{peak} =-0.45V. The NDR begins at a lower voltage when positively biased than it does when negatively biased. The peak current is about 2.8mA, corresponding to a current density of 5.6×10^3 A-cm⁻².

With the signal power level of the network analyzer set at -10dBm, the impedances of the diode were measured between bias voltages -1V to 1V. Examples of real and imaginary impedance components are shown in Figs.4.1.2(a-d). The repeatability of the experiment was very good. We have measured the S parameters under a few bias points in a 10 hours interval, it was found that the differences were smaller than the measurement uncertainty. One can see that the measured Re(Z) and Im(Z) are generally smooth, except for the low frequency portion at V=0.411V which is at the onset of the NDR region (Fig.4.1.2(b)). We have observed that S_{11} at the onset of the NDR region is relatively erratic at lower frequencies. This is because of the presence of lower frequency oscillations in the bias circuit. The spectrum analyzer showed that, at the start of the NDR region oscillations could generate signals as strong as -38dBm (~8mV_{pp}) at 1.85GHz and





below -50dBm at its harmonic components. With such signals the generally smooth S_{11} was slightly distorted, especially at lower frequencies. However the higher frequency signals were stable and smooth. Because the $8mV_{pp}$ oscillation at 1.85GHz was very small compared to the incident signal, and its harmonic components were far more less, results obtained at higher frequencies (above 4-5GHz) were considered not significantly affected by the oscillations. In the very beginning of the NDR region, where smooth lower frequency data was not available, analysis was carried out on the more reliable data obtained at higher frequencies.

Equivalent Circuit and Impedance

The impedance of the equivalent circuit shown in Fig.1.1.2 is:

$$Z = \frac{1}{1/R_{d} + j\omega C} + R_{s} + j\omega L_{s}$$
(4.1.1)

where ω is the frequency, R_s the series resistance and L_s the series inductance. $R_d=1/G$ is the dynamic resistance and C the capacitance for the RTD. R_d and C are directly related to the quantum well structure and are functions of voltage.

In Fig.4.1.2, results from Eq.(4.1.1) with $R_s=5.2\Omega$ and $L_s=50$ pH are also shown. Eq.(4.1.1) suggests that if $R_d >> (\omega C)^{-1}$, Re(Z)= R_s . $R_s=5.2\Omega$ is obtained from the experimental value of Re(Z) at the highest frequency (f~18GHz) and at the valley voltage V_{valley} where $R_d = (dI/dV)^{-1}$ is theoretically infinity. The inductance L_s is so chosen that, Re(Z- R_s - $j\omega L_s$)⁻¹ and Im(Z- R_s - $j\omega L_s$)⁻¹/ ω , which are R_d^{-1} and C, respectively from Eq.(4.1.1), could be approximated as constants for all frequencies. It is found that with $R_s=5.2\Omega$ and $L_s=50$ pH, frequency independent Re(Z- R_s - $j\omega L_s$)⁻¹ and Im(Z- R_s - $j\omega L_s$)⁻¹/ ω can be easily obtained for almost all voltages, except at the initial onsets of the NDR regions and at lower frequencies where Z is found to be relatively erratic. Having determined the series resistance R_s and inductance L_s , the dynamic resistance R_d and

Examples of measured and fitted (from Esaki circuit) impedance Z of the Al_{0.4}Ga_{0.6}As-GaAs RTD. This figure shows results for larger positive R_d : 1) V=0, 2) 0.554V and 3) V=-0.205V. Measured and fitted real parts Re(Z) are shown in solid (-----) and dotted chain (-----) lines. Measured and fitted imaginary 1222 b) ມ 2 Frequency (GHz) 10 Re(Z) lm(Z) -----ഹ XXX -100 300 200 100 0 Fig.4.1.2a) (Ω) (Z)ml ,(Z)ອA

parts Im(Z) are shown in dashed (---) and dotted $(\dots \dots)$ lines, respectively.



Measured and fitted impedance Z for a negative R_d : V=0.411V. Measured Re(Z): -----; fitted Re(Z): ---- \cdots ; measured Im(Z): --- and fitted Im(Z): Fig.4.1.2b)









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quantum capacitance C can be obtained. Impedance curves for various voltages are fitted with Eq.(4.1.1) and the resulting $1/(R_d+R_s)$ and C values plotted in Fig.4.1.3 and Fig.4.1.4, respectively. The value of $1/(R_d+R_s)$ is used instead of R_d is because $1/(R_d+R_s)$ can be directly compared to the experimental conductance dI/dV (see below). It is evident from Fig.4.1.3 that the calculated impedances agree with experimental results, indicating that the equivalent circuit used here is a good model for describing the RTD's high frequency electrical properties.

It was suggested that the transit time effect from the depletion region should be included in the impedance of the RTD (Kesan *et al.*, 1988a, 1988b; Whitson *et al.*, 1991). Here we have found that, the measured impedance can be reproduced without it. This may be because the undoped layers in our samples were not very long. With much longer undoped spacer layers (for example longer than 1000Å, which was used in Kesan *et al.*, 1988b) the transit time effect may become more prominent. There were also suggestions that an inductance in series with the capacitance should be included to account for the quasi-bound state lifetime effect at very high frequency (Brown *et al.*, 1989; Whitson *et al.*, 1991; Vanbesien *et al.*, 1992). Our result shows that at this frequency range, to reproduce the experimental results the Esaki equivalent circuit is enough. Miles *et al.* (1991) have proposed a modified equivalent circuit that includes more components. Compared to their measurements, our experiments provide a more accurate result, as we used the on-wafer probing technique and packing effects that were present in their measurement has been excluded. Therefore for frequencies below 18GHz, the Esaki diode equivalent circuit is a good representation for the RTD.

In these measurements, the probes were positioned in the middle of the probe pads. The extracted series inductance $L_s=50$ pH for this diode is comparable to the measured inductance of short-circuited reference devices (with probes positioned in the









middle of the probe pads), ranging from 50pH to 70pH. The inductances for other diodes with areas ranging from 49 to $400\mu m^2$ were found to be between 45 and 60 pH. Since the inductance obtained from the quantum well structure is expected to change with area, we have attributed the observed inductance to the air-bridge as well as the contact pads leading to the device. We expect the inductance of the quantum well structure alone to be less than the uncertainty of our calculation, which is about 10pH.

From the equivalent circuit, it is expected that R_d+R_s is the same as the differential resistance of the *I-V* characteristic, that is, $dI/dV=1/(R_d+R_s)$. dI/dV obtained from the measured *I-V* curve together with the calculated $1/(R_d+R_s)$ are shown in Fig.4.1.3. One can see that, although $1/(R_d+R_s)$ is slightly smaller than dI/dV in magnitude, the calculated $1/(R_d+R_s)$ agrees reasonably well with the experimental dI/dV. With the voltage increasing through the NDR region, $1/(R_d+R_s)$ decreases from a positive value to a negative value and then increases back to a positive value. This represents the fact observed in the measurement: $|S_{11}|$ increased from $|S_{11}|<1$ to $|S_{11}|>1$ (exhibited gain) and then decreased back to $|S_{11}|<1$.

C-V Characteristic: Compared to Theoretical Result

Fig.4.1.4 shows the measured capacitance as a function of applied voltage. The capacitance decreases with increased bias at lower voltages, followed by a well defined peak structure and then finally declines. Determining the RTD capacitance from wideband high frequency impedance measurements has some advantages over conventional C-V measurement techniques. In conventional C-V measurements a fixed and relatively low frequency ($f \le 100$ MHz) signal is used. Since the capacitance of the RTD is usually very small, the inductance of even very short connecting wires can significantly change the

result. With the exception of $V \sim V_{peak}$ and V_{valley} , where R_d is very large, at low frequencies we have $\omega R_d C \ll 1$ and Eq.(4.1.1) can be approximately written as

$$Z \approx R_d j \omega (R_d^2 C - L_s) + R_s \tag{4.1.1a}$$

For typical values of $C\sim0.1\text{pF}$ and $R_d\sim100\Omega$ (Fig.4.1.3 and 4), an inductance $L_s\sim1\text{nH}$ (which corresponds to the self-inductance of a 1mm long straight rectangular $10\times10\mu\text{m}^2$ metal line (Terman, 1943)), is comparable to R_d^2C . That is, the capacitance obtained is significantly changed if L_s is not taken into account. Using a fixed frequency, as is the case in conventional measurements, it is very difficult to determine the value of such an inductance and to extract the correct capacitance.

Under positive (negative) bias, the C-V curve in Fig.4.1.4 has a clear structure which begins at 0.32V (-0.37V) and ends at 0.50V (-0.56V). The structure peaks at V=0.41V (-0.49V), 0.02V (0.04V) higher than the peak voltage V_{peak} of the I-V characteristic. Similar features were also observed in fixed-frequency C-V measurements for RTDs with flat wells (Eaves et al., 1989; Zarea et al., 1990) and parabolic wells (Schubert et al., 1990). They were attributed to the charge accumulations in the quantum well during the resonant tunneling (Schubert et al., 1990). As the charge accumulation approaches its maximum at V_{peak} , it was expected that the peaks in the C-V characteristic would coincide with the peaks in the I-V characteristic. However, if we plot the C-V and G-V characteristics together (Fig.4.1.5), we will see that, these peaks are in fact located at the middle of the NDR regions. We will see in sections 4.2 and 4.3 (Fig.4.2.3 and Fig.4.3.3, respectively) that it is also true for other RTDs. Boric et al. (1992) has also shown that their capacitance peaks were located in the NDR regions. Thus the capacitance features may be associated with the discharging processes which occur in the NDR regions (during which tunneling diminishes), instead of the charge accumulations in the peak regions.





To simulate the C-V characteristic and to study the peaks that were not obtained in previous simplified model theoretical calculations (Yokoyama, 1989; Sun et al., 1992), we have calculated the capacitance of this diode. The calculation is based on the electron density distributions at various voltages obtained from quasi-static self-consistent calculations (Chapter 3). In the calculation, the barrier height was assumed to be 0.4V (Batey and Wright, 1986; Langer et al., 1988). The RTD was partitioned into 149 segments, the segment widths varied from 2Å in the active quantum well area to 200Å in heavily doped contact areas. The calculated C-V characteristic is shown in Fig.4.1.6 and the I-V characteristic is shown in the insert. Shown in Fig. 4.1.7 are examples of electron distributions and potential profiles obtained from self-consistent calculations (at V=0 and 0.174V). Since C and G are derivative values, to obtain good and smooth C-V and G-Vcurves we need very good results from self-consistent calculations. The calculation was very time consuming. In this calculation we stopped the calculation before the convergence reached our targeted limit. Because of this the calculated C-V and G-Vcurves in Fig.4.1.6 are relatively rugged. However, they are good enough to show clear peak features.

As mentioned in Chapter 3, our treatment does not include the scattering and localization effects; also, the RTD operates far from thermal equilibrium but is considered quasi-static. Similar to the *I-V* characteristics obtained from other quasi-static, non-scattering self-consistent calculations, our result shows negative resistances; but when compared to the experimental ones, the peak to valley ratios are higher. It was expected that this calculation would provide only a qualitatively correct C-V picture. From Figs.4.1.5 and 4.1.6 one can see that, except for the peak at the lower forward bias (at V=0.1V and denoted by an arrow), the theoretically calculated C-V and G-V characteristics agree qualitatively with the experimental results.









The theoretical C-V characteristic in Fig.4.1.6 shows that the capacitance peaks are in fact located in the NDR regions. It is found that the NDR produces the most dramatic change in charge distribution. When the RTD is pushed into the NDR region, tunneling current decreases drastically and a large quantity of electrons transfer out of the quantum well. The electron density in the well decreases sharply within a small increase in applied voltage. Charge neutrality (the model used here is quasi-static) ensures that there is also sharp increases in areas outside the quantum well. This rapid change in the electron density distribution results in the peak (increase) in capacitance.

The capacitance peak is directly related to how many electrons are discharged within a certain increase in voltage, associated with this discharging process is the negative conductance. Typically the lower is the negative conductance peak the higher is the capacitance peak. This can be seen by comparing the experimental and theoretical results for the forward and reverse biases (Figs.4.1.5 and 4.1.6). Calculation shows that under forward bias, electrons discharge from the quantum well much more drastic than that under the reverse bias (that is, the number of electrons transferred out of the well within a given change of bias is larger). Reflected in the I-V characteristic is that the negative conductance in forward bias is much larger (Fig.4.1.6). Because the discharging process in the forward bias is far more drastic, the forward bias capacitance peak is much larger than the reverse bias one.

Under forward bias, there is an additional peak in the calculated C-V curve at V=0.1V (Fig.4.1.6, denoted by an arrow). It occurs during the quantum well charging process and coincides with a positive conductance peak. It is much smaller than the discharging one. In reverse bias no similar feature is observed. Our calculation indicates that discharging processes are much faster than charging processes. Reflected in the *I*-V characteristic is that the negative conductance peak (in NDR region) is much larger than

the positive one. In the reverse bias the change in charge distribution is relatively small, and the capacitance peak induced by the charging process is so small that it is not visible in Fig.4.1.6. In the experiment, both charging peaks are very shallow (see dI/dV in Fig.4.1.5), and the charging induced capacitance increases for the forward and reverse biases are invisible.

Compared to the forward bias, the calculated C-V characteristic at the reverse bias is closer to the experimental one, both in shape and value. Calculation indicates that, under forward bias electrons charge and discharge from the quantum well much faster than that under the reverse bias. Capacitance peaks in the forward bias are thus much larger than the reverse bias ones, especially the discharging one. Quantitatively, the calculated capacitance at zero bias (0.19pF) is about twice the measured value (0.09pF). The experimental capacitance peak in the reverse bias is 0.11pF, about one third of the theoretical 0.33pF. The peak in the forward bias is 0.12pF, much smaller than the theoretical 8.6pF. The charging peak in the forward bias is not observed in experiment. For the *I-V* curve, theoretical peak voltages (V_{peak}) are smaller than the experimental ones (-0.43V and 0.17V compared to -0.45V and 0.39V). Peak to valley ratios are larger and negative differential conductances are much larger. These discrepancies are believed to have resulted from the quasi-static and non-scattering approximations in our calculation. It is known that the RTD operates far from equilibrium and scattering is always a very important factor in device transport (Kluksdahl et al., 1989; Frensley, 1990). Considering the RTD as non-equilibrium and including the scattering effects may raise the potentials on undoped spacer layers, and increase the calculated NDR (or V_{peak}) to a more reasonable voltage. Studies on I-V characteristics have suggested that scattering can drastically reduce the peak-to-valley ratio and negative conductance in the NDR region (Frensley, 1990; Hu and Stapleton, 1991). If scattering and localization effects are taken into consideration (which will very much complicate the calculation), we expect that electrons will move less freely and the change in electron density will be less significant, consequently dQ and capacitance will be smaller and more appropriate peak values will be obtained. With scattering effects we also expect the charging peak in the forward bias to be reduced, just as that observed in the experiment.

Cutoff Frequency

There have been many high speed applications for RTDs. Among them are high frequency mixers and multipliers which utilize the non-linearity of the conductance or capacitance. For mixers and multipliers that utilize the voltage variable capacitance property (varactor property), a rapid change in capacitance is desired, as it can result in a stronger nonlinearity, and subsequently in higher conversion efficiency and lower loss (MacPherson, 1957; Uenohara and Gewartowski, 1969). Our result shows that generally a rapid change in capacitance can be obtained by increasing the negative resistance or the peak to valley ratio, which has been the objective pursued by many researchers since the invention of the RTD. There have been many successful efforts in increasing the peak to valley ratio (for example, see Sze, 1990). Certainly these methods are also applicable to optimize the structure for varactor mixer and multiplier applications.

The cutoff frequency f_c has always been considered one of the criteria for high frequency performance. It is defined as the maximum frequency the device can produce gain. Below f_c the real part of the impedance Re(Z) is negative and above f_c Re(Z) is positive. For the Esaki equivalent circuit in Fig.1.1.2, the cutoff frequency is obtained from Re($Z(f_c)$)=0:

$$f_c = \frac{1}{2\pi C} \sqrt{-G(\frac{1}{R_s} + G)}$$
(4.1.2)

Eq.(4.1.2) suggests that, to obtain a high f_c , we need a large G (negative value) and a small C. For most high frequency applications, a higher cutoff frequency is desired and a lower capacitance is wanted. In most RTD applications, the device is operated in the NDR region. Our result shows that capacitance increases in the NDR region, therefore to obtain a higher cutoff frequency, the capacitance peak must be reduced.

Since the results from the small signal equivalent circuit in Fig.1.1.2 agree very well with experiment up to 18GHz, it is expected to be applicable at higher frequencies. f_c under reverse bias has been calculated with Eq.(4.1.2) and the maximum f_c is found to be 49GHz for the diode. The maximum f_c is obtained at the conductance and capacitance peak voltage (V=-0.49V), at which we have a minimum $G=1/R_d=-0.0051\Omega$ and a maximum C=0.11 pF. The corresponding forward bias maximum f_c is 35GHz, obtained at V=0.41V (G=-0.004 Ω and C=0.12pF). The cutoff frequency is higher in the reverse bias, as the negative conductance is larger and the capacitance is lower. Furthermore, if we consider the theoretical result in Fig.4.1.6, we will see that even though the negative conductance is higher under forward bias, the cutoff frequency is still lower. The increase in capacitance has offset the advantage of the higher negative conductance. This will be more clearly seen in our experimental result from AlAs-GaAs diodes with a similar structure (section 4.3).

Obviously, further reductions in capacitance and parasitic resistance are necessary to increase the cutoff frequency. Our result shows that in the reverse bias, the capacitance is lower. If we compare the diode structures in two different biases, we will see that, under reverse bias the diode has a longer anode spacer layer (due to the asymmetry of spacer layer). So increasing the anode spacer layer could be one of the possible solutions for obtaining a high cutoff frequency. We will see that, our following experimental and theoretical results all support this suggestion.

4.2 Impedance, Equivalent Circuit and C-V Characteristics: Al_{0.4}Ga_{0.6}As-GaAs Diode, up to 40GHz

The structure of this diode is the same as the above one, however the diode area is $7 \times 7 \mu m^2$. The dc *I-V* characteristic for the diode is shown in Fig.4.2.1. In the NDR region some irregular jumps were observed. The diode had a larger peak to valley ratio compared to the diode in section 4.1. It exhibited much stronger oscillations due to the larger negative conductance. Oscillations in the NDR region were believed to be responsible for the irregular jumps in the *I-V* curve which were not observed in the diode in section 4.1.

In S parameter measurements, to reduce the parasitic effects, we positioned the probes as close as possible to the diode. Compared to the *I-V* measurement, S parameter measurements give much more stable results in the NDR region, especially at the higher frequencies. Stable S parameter measurements could be obtained because of two reasons: 1) The 50 Ω external impedance presented to the diode by the network analyzer significantly decreases the oscillation amplitude. 2) At higher frequencies the input signals period is a few orders of magnitude smaller than the oscillation period. As the oscillation magnitude was not very large (~15mV_{pp}), its effect on the S parameter is considered not significant.

As examples, Figs.4.2.2(a) and (b) show the measured impedances at V=0V and -0.51V. Also shown are values calculated from the Esaki equivalent circuit description in Fig.1.1.2. Over the entire bias range (-0.7 to 0.55V), with $L_s=21pH$ and $R_s=6.1\Omega$, good agreements can be obtained between calculated and experimental results. In this case L_s is comparable to that of the short-circuited reference devices, indicating that it is a result of the connecting air-bridge and probe pads. This again enforces the argument that the active RTD structure is represented by the G-C plus R_s structure. By fitting the measured results












with the equivalent circuit, C-V and G-V characteristics for the diode are obtained and plotted in Fig.4.2.3.

The agreements seen in Fig.4.2.2 between experimental and predicted curves demonstrate the validity of the Esaki equivalent circuit model up to 40GHz. Since the agreements between predicted and experimental results are good, it is expected that the equivalent circuit model is valid to frequencies in excess of 40GHz, and can be reliably used for microwave circuit designs and simulations.

No hysteresis but some irregular jumps were observed in the I-V characteristic (Fig.4.2.1). We believe that they were the results of oscillations that were present in the NDR region. Simulations have suggested that in a dc I-V measurement, due to the rectified oscillatory current, the measurement can exhibit small jumps or hysteresis (Okean, 1971; Liu, 1988; Belhadj et al., 1990). In this diode, low frequency oscillations were observed in NDR regions during the measurement and jumps were found in the I-Vcharacteristic. On the other hand, the conductance G (which is $(dV/dI-R_s)^{-1}$) obtained from S parameter measurements was smooth, and no jump was observed (Fig. 4.2.1). By integrating the G-V curve in Fig.4.2.3 (R_s is much smaller than dV/dI and is neglected here), a smooth I-V curve has been obtained and is shown as the dashed line in Fig.4.2.1. The integrations are performed only over the distorted portions, that is, the integrals start at the ends of the smooth measured curve. Since the smooth dashed curves are obtained without severe oscillation interference, we believe they represent the intrinsic I-Vcharacteristic in the NDR regions. The recovery of the I-V curve demonstrates that with high frequency impedance measurements, low frequency-low power oscillations can be ignored and meaningful electrical characteristics can be extracted.

The C-V characteristic of the diode is shown in Fig.4.2.3. It is very similar to that obtained from the diode in section 4.1. It shows two electron discharging peaks in two





NDR regions. One can see that, similar to the above diode, the capacitance in the reverse bias is lower than that in the forward bias. The highest cutoff frequencies obtained for forward and reverse bias directions are 54GHz (at V=-0.51V) and 41GHz (at V=0.36V), respectively.

4.3 Impedance, Equivalent Circuit and C-V Characteristics: AlAs-GaAs Diode: up to 40GHz

The area of this diode is $7 \times 7 \mu m^2$ and its structure has been illustrated in Fig.2.1.2. The *I-V* characteristic for the diode is shown in Fig.4.3.1. A strong hysteresis was observed in the reverse *I-V* characteristics, in the forward bias a very weak hysteresis was also observed. Due to the asymmetry in spacer layers, the obtained *I-V* characteristic was asymmetric. Onset of the NDR region (V_{peak}) occurred at a lower voltage in the forward direction. This was due to the fact that the anode undoped spacer layer is 100Å and the cathode spacer layer is 700Å (Fig.2.1.2). An inflection which was not observed in Al_{0.4}Ga_{0.6}As-GaAs diodes was found in the forward bias at 0.6V. This structure has been observed in some other RTDs with long cathode spacer layers (Paulus *et al.*, 1990; Mounaix *et al.*, 1990). It was attributed to the quasi-bound state formed in the triangular well in the cathode spacer layer adjacent to the barrier.

During microwave measurements, under forward bias, strong oscillations were detected in the NDR region and data at lower frequencies was erratic. On the other hand, the magnitude and phase of higher frequency data was stable and smooth. As with the above diodes, when lower frequency data was not available because of the poor phase locking of the network analyzer, measurements were extrapolated from more reliable results obtained at higher frequencies. Under reverse bias conditions, the oscillations were found to be much weaker. Contrast to that in the forward bias, in reverse bias, S





(Am) trient (MA)

-10.0

0.0

AlAs/GaAs

10.0

20.0

parameter measurements were almost unaffected by oscillations. A hysteresis was observed in these clean S parameter measurements in the reverse bias. For this diode which had a wide bias range (-3.5V to 2.5V), it was found that, different from the above RTDs, to obtain a good fit between measured impedances and the equivalent circuit in Fig.1.1.2, series resistance R_s had to be made a function of voltage. With inductance $L_s=5pH$, R_s was found to increase from about 6Ω near V=-3V to about 16Ω around V=1.5V. Shown in Fig.4.3.2 are examples of measured and fitted impedances (V=0V and -2.64V). Obviously, the agreement between experimental and modeled results was quite good, demonstrating the validity of the equivalent circuit up to 40GHz. Fig.4.3.3 shows the modeled C-V and G-V characteristics.

Hysteresis

The hysteresis observed in S parameter measurements under reverse bias conditions can also be seen in the C-V and G-V characteristics in Fig.4.3.3. The hysteresis was originally observed in I-V measurements and attributed to the charge storage mechanism (Goldman *et al.*, 1987). However, some authors pointed out that the self-oscillations could also lead to a similar phenomenon (Sollner, 1987; Liu, 1988; Belhadj *et al.*, 1990). The hysteresis observed in our impedance measurement was different from the one resulting from oscillations, which showed a two step process like that in the I-V characteristic in Fig.4.3.1. Since the impedance data in the reverse bias was obtained without the interference of oscillation, we believe the hysteresis was intrinsic and not from the oscillation.

There were studies that suggested the hysteresis might be a result of the internal or external resistances of the device (Chow, 1964; Okean, 1971; Jogai, and Koenig, 1991;













Chen *et al.*, 1991). Due to the fact that the peak current of the RTD is higher than the valley current, the voltage drop on the series resistance at the peak voltage V_{peak} is higher than that at the valley voltage V_{valley} . If the resistance is not very small, the measured peak voltage V'_{peak} , which is the "intrinsic" RTD voltage V_{peak} plus the resistance voltage, can be higher than the measured valley voltage V'_{valley} (Fig.4.3.4), resulting in a bistability (load-line bistability). From Fig.4.3.4, one can see that the range of such a bistability is

$$V'_{1}-V'_{2}=V_{1}+I_{1}(R_{s}+R_{load})-V_{2}-I_{2}(R_{s}+R_{load})$$
(4.3.1)

where R_{load} is the external load-line resistance. Although V_1 , V_2 , I_1 and I_2 are unknown parameters, we know that V_1 - V_2 <0, $I_1 \sim I_{peak}$ and $I_2 \sim I_{valley}$. If we approximate I_1 and I_2 , respectively, by I_{peak} and I_{valley} , and taking the fact that the series resistance R_s is voltage-dependent, we have

$$V'_{1}-V'_{2} < I_{peak}(R_{s,peak}+R_{load}) - I_{valley}(R_{s,valley}+R_{load})$$
(4.3.2)

For this AlAs-GaAs diode, we have $R_{s,peak}=12\Omega$ and $R_{s,valley}=8\Omega$ from impedance measurements. With the measurement result of $R_{load}=0.8\Omega$, the right hand side of Eq.(4.3.2) gives rise to 290mV, which is slightly larger than the 260mV hysteresis observed. So the hysteresis has been a result of the load-line bistability. Here R_{load} is very small, the hysteresis thus is mainly due to the internal series resistance R_s . Similar hysteresis phenomena were also observed in other AlAs-GaAs diodes, however, the hysteresis and series resistance were all smaller than that of this one. We do not exclude the possibility that there exists charge storage hysteresis, since the existence of such a hysteresis has been supported by theoretical calculations (Kluksdahl *et al.*, 1989). However, to distinguish it from the load-line bistability, great care must be taken.



C-V Characteristic and Cutoff Frequency

To simulate the C-V characteristic, we calculated the diode capacitance through self-consistent calculations; but this time only under the forward bias (because the self-consistent calculations were very time consuming). In the calculation, the barrier height was assumed 1.1V (Batey and Wright, 1986). The calculated C-V and G-V (G=dI/dV) characteristics for the diode in forward bias are plotted in Fig.4.3.5. In the insert the *I-V* characteristic is also shown. The *I-V* curve shows a NDR and an inflection, very similar to that observed in the experiment. However, the peak to valley ratio and bias magnitudes are quite different. Fig.4.3.6 shows the calculated electron density distribution and conduction band potential at V=0.09V, just before the inflection of the *I-V* characteristic.

The C-V characteristic obtained from rf measurements (Fig.4.3.3) shows that, under forward bias, the capacitance increases with increasing voltage and then decreases, followed by a peak that coincides with the inflection in the *I*-V characteristic and a larger peak in the NDR region. The calculated C-V characteristic also demonstrates the same behavior (Fig.4.3.5). However, both capacitance magnitudes and bias voltages are quite different. The calculated inflection in the *I*-V characteristic is at 0.09V, and the NDR begins at 0.14V, both are much lower than the experimental 0.9V and 1.14V. The peak to valley ratio is also higher: 10 versus 3. Around V=0V, the calculated capacitance is about twice the measured value. All capacitance and conductance peaks are higher than experimental ones. The reason is that scattering was not included in the self-consistent calculation. Except for the magnitudes, the calculation agrees qualitatively with the experiment.

Here, the capacitance peak in the NDR region is again observed (Fig.4.3.3). We know that the NDR peak is directly associated with the discharging process and the negative conductance peak, typically the lower is the negative conductance peak the









higher is the capacitance peak. This can be seen quite clearly by comparing the experimental results for forward and reverse biases (Fig.4.3.3). Under forward bias, the negative conductance peak is lower and the capacitance peak is larger.

The capacitance peak at lower voltage is related to the inflection in the *I-V* characteristic. Calculation shows that, band bending in the cathode spacer layer forms a triangular well outside the emitter barrier. Like in the quantum well, a certain quasi-bound state E_l is formed (Fig.4.3.6): at E_l tunneling probability attains one maximum. E_l is lower in energy than the quasi-bound state E_w in the well which is responsible for the usual tunneling and the NDR. Due to the presence of E_l , there is an electron accumulation near the emitter barrier outside the quantum well. The inflection in current is due to the tunneling through E_l (Paulus *et al.*, 1990; Mounaix *et al.*, 1990). Associated with the diminish of this tunneling is an electron discharging process that causes a rapid change in charge distribution and the capacitance peak. In the reverse bias condition, because the emitter spacer layer is shorter and the triangular well is very shallow, no similar capacitance peak were observed.

Although the calculated capacitance agrees qualitatively with experimental results, the differences in magnitudes are quite large. We again attribute these discrepancies to the quasi-static and non-scattering approximations in our self-consistent calculations.

Experimentally, for this diode, the highest cutoff frequencies obtained for the two bias directions are 60GHz (at V=-2.6V) and 41GHz (at V=1.31V). The highest f_c we obtained for same wafer was 105GHz, from another $7\times7\mu m^2$ diode and also in the reverse bias (this diode exhibited much stronger oscillations and was less stable). The cutoff frequency is higher under the reverse bias, because the capacitance is lower (Fig.4.3.3). The reason the capacitance is lower is that, under the reverse bias the anode spacer layer is longer. It is interesting to note that, for this diode, while the negative conductance in

forward bias is slightly higher, the cutoff frequency is still lower than that of reverse bias. The same result was also found for other diodes with the same structure. In the forward bias, although some of these diodes had a negative conductance peak that was almost twice that in the reverse bias, f_c obtained from the reverse bias was still higher than that from the forward bias. The increase in capacitance and resistance have offset the advantage of the larger negative conductance. As has been mentioned in sections 4.1 and 4.2 (and will be stipulated in section 4.4), for a RTD with similar asymmetric spacer layer structure (with a longer anode spacer layer under reverse bias), the theoretical cutoff frequency in the forward bias is lower than that in the reverse bias because of the higher capacitance. The experimental results certainly support this argument. The reason that the capacitance is lower under reverse bias is that the anode spacer layer is longer. Thus, to increase the cutoff frequency for high speed applications, a longer anode spacer layer should be used (assuming transit time effect is still negligible).

4.4 Effects of Spacer Layers on C-V Characteristic: Theoretical Considerations

We have investigated the effects of spacer layers on capacitance, by studying four different RTDs with unit areas. The central parts of all these RTDs are an identical 40Å-50Å-40Å barrier-well-barrier quantum well structure. The spacer layers on one side of the quantum well are 100Å, on the other side they are 100Å, 300Å, 700Å and 1000Å, respectively. The barrier height is 0.22V, resembling that of an Al_{0.3}Ga_{0.7}As-GaAs RTD. The electron concentrations of both contact layers are 2×10^{18} cm⁻³. The temperature is 300K. For convenience, we define the 100Å side as positive. That is, under forward bias, the anode spacer layer is fixed as 100Å and the cathode spacer layer varies from 100Å, 300Å, 700Å to 1000Å; under reverse bias it is the opposite, the cathode spacer layer is fixed and the anode spacer layer varies. The effects of cathode and anode spacer layers on

the capacitance can be respectively studied by studying the capacitance under two different directions.

As was pointed out in sections 4.1 and 4.3, due to the lack of scattering, calculated C-V characteristics agree only qualitatively with experimental ones. Despite the limitation, we expect that the calculations to still provide a correct dependency on how the capacitance varies with spacer layer. We believe this tendency will not be altered by the introduction of scattering. That is, our results can be used as a guideline for optimizing the structure of a RTD.

The calculated current density-voltage characteristics for four diodes are shown in Fig.4.4.1. Under forward bias, magnitudes vary greatly, peak to valley ratios are 3.1:1 for the 100Å diode and about 10:1 for three others. In the reverse bias the peak voltage increases with increasing anode spacer layer. Although the current magnitude for the 100Å is slightly lower, the peak to valley ratios are quite close for all four (~3:1).

The specific capacitances for forward and reverse biases are shown in Figs.4.4.2 and 4.4.3, respectively; also shown are the conductances which are the derivatives of the I-V curves. The common feature for all these diodes is the electron discharging peak in the NDR region, in both positive and negative biases.

Under forward bias, the discharging capacitance peaks are very high for the 300Å, 700Å and 1000Å diodes. It is interesting to see that, the peak increases greatly when the cathode spacer layer is increased from 100Å to 300Å, when the spacer layer is increased further to 1000Å it decreases (Figs.4.4.2 and 4.4.3). The calculations show that the capacitance peak height depends strongly on the discharging process, which in turn, depends partially on the negative conductance peak in the NDR region. The 300Å diode has the largest negative conductance peak and it has the highest capacitance peak. It is found that, changes in I-V characteristic and electron density in the well are primarily





Calculated reverse bias current density-voltage characteristics for RTDs with unit areas. The cathode —), 300Å (dashed line, ---), spacer layer is 100Å. The anode spacer layers are 100Å (solid line, — 700Å (dotted line, ----) and 1000Å (dot line, -----).



Fig.4.4.2 a) Calculated specific C-V characteristics for three diodes under forward bias. The undoped cathode spacer layers are 300Å (dashed line, ----), 700Å (dash-dotted line, ----) and 1000Å (dotted line, ----). b) The corresponding conductances for the diodes. The results for the 100Å diode is symmetric and shown in Fig.4.4.3.



Fig.4.4.3 a) Calculated specific C-V characteristics for diodes under reverse bias. The undoped anode spacer layers are 100Å (solid line, ____), 300Å (dashed line, ____), 300Å (dashed line, ____) and 1000Å (dotted line, ____).
b) The corresponding conductances for the diodes.

determined by the charge distribution in the emitter (cathode) area. Increasing the spacer layer from 100Å to 300Å reduces the cathode electron density. When the cathode density is low and if the diode is biased into the NDR region, quantum well density and current decreases sharply. Increasing the spacer layer further to 700Å and 1000Å, the electron density in the cathode spacer layer increases, bending the potential near the upper barrier (Fig.4.4.4). The result is that, decreases in current and quantum well electron density are slightly less significant. Consequently the negative conductance and capacitance peaks are smaller.

Under reverse bias, the capacitance peak decreases and broadens slightly with increasing anode spacer layer. The change in the peak is relatively small compared to that under the forward bias. This is because the charge distribution in the cathode, by which the discharging process is mainly determined, is affected relatively little by the variation in anode spacer layer. On the other hand, in both forward and reverse biases, the capacitance decreases as a whole with increasing spacer layer (this can be seen more clearly in Fig.4.4.3 for the reverse bias), as the undoped space between the two contacts increases.

Except for the increases in NDR regions, there is an additional capacitance peak in the forward bias at lower voltage (Fig.4.4.2). This peak is quite high for the 1000Å diode, it is smaller in the 700Å diode, and in the 300Å diode it is invisible. This peak is a result of the quantum well charging process which occurs during tunneling. It coincides with a positive peak in the conductance. During the tunneling process, electron density in the quantum well increases rapidly with increasing applied voltage, associated with this increase is a rapid variation in electron density distribution and a peak in the capacitance. The charging processes in the 1000Å and 700Å diodes are more prominent than that in the 100Å and 300Å diodes, consequently larger changes in the electron distributions and higher capacitance peaks are resulted. In the reverse bias the charging process is relatively





slow compared to the forward bias, therefore no such increase is observed. This peak has never been observed in experiments. Similar to the peaks in the NDR regions, in a real device it is believed to be reduced and broadened by scattering effects.

The calculations for the reverse bias show that, changing the anode spacer layer does not significantly affect the peak in the NDR region. But in general, increasing the anode spacer layer results in a decreasing capacitance. However, the peak is quite sensitive to the variation in cathode spacer layer. Increasing the cathode spacer layer could cause the capacitance peak to increase.

For varactor mixer and multiplier applications, to obtain high conversion efficiency and low loss, a rapid variation which results in a strong nonlinearity in capacitance is desired (MacPherson, 1957; Uenohara and Gewartowski, 1969; Roddy, 1986). In these cases a longer cathode spacer layer should be used. For oscillators and logic circuits, lower capacitance is usually needed as higher cutoff frequency is normally desired. To obtain a high cutoff frequency a larger negative admittance and a lower capacitance are desirable. Assuming that the series resistivity has a typical value of $5 \times 10^{-8} \Omega \text{cm}^{-2}$, one can show that for the 700Å diode, the maximum cutoff frequencies (obtained at the minimum negative conductances) are 260GHz and 1.1THz for forward bias and reverse bias, respectively. It is again interesting to see that although the negative conductance in forward bias is larger, the cutoff frequency is in fact lower. Similar results can be also obtained for three other diodes. This agrees with our experimental results for diodes of similar spacer layer structures shown in sections 4.1, 4.2 and 4.3. The advantage of the increasing negative conductance has been reduced by the higher capacitance. This also suggests that the spacer layer does play an important role in diode microwave characteristics and applications.

Since the invention of the RTD, numerous researches have been conducted to obtain higher negative conductance, or higher peak to valley ratio (for example, see Sze, 1990). Our result indicates that, the effect of the increase in capacitance brought on by increasing the negative conductance can be negative for some applications (for example oscillators), as the cutoff frequency is reduced. There is a tradeoff between capacitance and negative conductance in obtaining a higher cutoff frequency. If we consider the reverse bias cutoff frequencies for the 100Å, 300Å and 1000Å diodes, we will see that they are 940GHz, 1.1THz and 1.4THz, respectively. So, as in sections 4.1, 4.2 and 4.3, we find that, one choice for achieving a high cutoff frequency is to increase the anode spacer layer.

4.5 Effects of Fixed Space Charge on *I-V* and *C-V* Characteristics: Theoretical Considerations

The device under study is a Al_{0.3}Ga_{0.7}As-GaAs RTD. The central part of the RTD is an undoped 40Å-50Å-40Å barrier-quantum well-barrier structure. The barrier height is 0.22V. On both sides of the well-barrier structure are undoped spacer layers of 100Å. The whole structure is sandwiched by two n⁺-GaAs ($N_D=2\times10^{18}$ cm⁻³) contact layers.

Assuming a fixed interface space charge density of $\sigma=5\times10^{10}$ cm⁻² (Kroemer *et al.*, 1980) at four heterojunctions (σ is assumed not to change with voltage), the current density and specific capacitance have been calculated. They are plotted in Figs.4.5.1 and 4.5.2, respectively. For comparison, results for $\sigma=0$ are also shown. Fig.4.5.1 clearly indicates that the fixed space charge shifts the location of the current peak. However, the peak to valley ratio is virtually unaffected and remains roughly 3.1:1. Due the existence of the fixed space charges at the interfaces, the potential around the quantum well area decreases, along with this decrease is the quasi-bound state in the quantum well. The









decrease in the quasi-bound state results in a decrease in peak voltage V_{peak} . The decrease in the quasi-bound state also lowers the tunneling energy; thereby increasing the number of electrons that can tunnel through the barriers, which in turn increases the peak current. The peak to valley ratio is determined primarily by the width of the quasi-bound state, the calculation shows that it is not substantially altered by the fixed space charge. For a fixed interface space charge density which has resulted from defects in heterojunctions, the value of 5×10^{10} cm⁻² is very typical (Kroemer *et al.*, 1980). Our results thus show that, like the unintentional background impurities (Wolak *et al.*, 1988), the interfacial impurities in a normal RTD are not sufficient to significantly degrade the *I-V* characteristic.

The effect of the fixed space charge on capacitance is more significant. Our calculation shows that by including a fixed space charge of $\sigma=5\times10^{10}$ cm⁻², the capacitance peak in the NDR region increases by about 10% (Fig.4.5.2).

Usually, when the capacitance increases, the performance of a device degrades. To consider this degradation we calculate the difference in cutoff frequencies. The highest cutoff frequency $f_{c,max}$ is obtained when -G and C are at their maximums. For this RTD, with a resistivity of $5\times10^{-7}\Omega$ cm⁻², $f_{c,max}$ can be found to decrease from 233GHz to 219GHz when we include the fixed interface charge. The decrease is about 6% and less than the 10% change in capacitance. The reason is that the negative admittance value is also slightly increased. Generally speaking, the degradation in the performance due to fixed space charge is not very significant.

CHAPTER 5. CONCLUSIONS

Through on-wafer probing S parameter measurements, accurate small signal impedances for RTDs up to 40GHz have been obtained. It has been found that the impedances can be described by the Esaki equivalent circuit that consists of a parallel G-C and a series R-L circuit. In the equivalent circuit, the inductance has been found to result from the on-wafer connecting lines and the air-bridge structure. The quantum well itself can be represented by the parallel G-C circuit alone. The series resistance is associated with the contact and spreading resistance in the semiconductor. With measured impedances, G-V and C-V characteristics have been obtained from the equivalent circuit representation. The G-V characteristic is found to be comparable to that obtained from dc I-V characteristic. With this oscillation-free rf G-V characteristic, portions of the I-V curve that were distorted in dc measurements have been restored. A hysteresis that is not affected by oscillations has been observed in impedance measurements. It is believed to be intrinsic and not from oscillations. But it is due to the load-line bistability of internal series resistance, rather than the charge storage effects predicted by theories.

Based on the charge distributions obtained from self-consistent calculations, capacitances of RTDs have been calculated and compared to experimental results. The calculated C-V characteristics are found to agree qualitatively with experimental ones. Both the theoretical and experimental results exhibit a capacitance peak in the NDR region. Calculations show that it is a result of the process of electrons discharging from the quantum well, instead of the charge accumulation in the quantum well as was previously believed. Also found is a smaller capacitance peak corresponding to electrons

discharging from the accumulation region in the undoped region adjacent to the emitter barrier.

We have calculated the capacitance for RTDs with different spacer layers. The effects of spacer layers on capacitance have been discussed. Calculations show that the capacitance value decreases in general with increasing spacer layer. The anode spacer layer has a relatively limited effect on the peak in NDR region compared to the cathode spacer layer. The increase in the negative conductance is usually accompanied by an increase in capacitance. Usually a high peak to valley ratio, or a larger negative conductance, is desirable. Our studies indicate that, by varying the cathode spacer layer, the negative conductance can be increased. However, increasing the negative conductance also brings up the capacitance, and the net effect may be negative, as the cutoff frequency is reduced. On the other hand, increasing the anode spacer layer can decrease the capacitance in the NDR region and thereby increase the cutoff frequency of the RTD.

The effects of fixed interface space charge on I-V and C-V characteristics are discussed. It is found that high frequency performance of the device will be degraded by the fixed space charge. However, for a fixed space charge density that has resulted from a normal interface disorder, the effect is not very significant.

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