

**INCREASED CAPACITY FOR WIRELESS MULTITIER  
SYSTEMS THROUGH MAXIMUM LIKELIHOOD  
DETECTION**

by

Shirin Karimifar  
B.Sc. Sharif University of Technology, 2002

THESIS SUBMITTED IN PARTIAL FULFILMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

In the School of Engineering Science

© Shirin Karimifar 2004

SIMON FRASER UNIVERSITY

Fall 2004

All rights reserved. This work may not be  
reproduced in whole or in part, by photocopy  
or other means, without permission of the author.

# Approval

**Name:** Shirin Karimifar  
**Degree:** Master of Applied Science  
**Title of Thesis:** Increased Capacity for Multitier Wireless Systems Through Maximum Likelihood Detection

**Examining Committee:**

**Chair:** Dr Mehrdad Saif  
Professor and Director of the School of Engineering Science

---

**Dr James K. Cavers**  
Senior Supervisor  
Professor of the School of Engineering Science

---

**Dr Rodney Vaughan**  
Supervisor  
Professor of the School of Engineering Science

---

**Dr Karim S. Karim**  
Internal Examiner  
Assistant Professor of the School of Engineering Science

**Date Defended:**

---

December 1<sup>st</sup> 2004

# SIMON FRASER UNIVERSITY



## Partial Copyright Licence

The author, whose copyright is declared on the title page of this work, has granted to Simon Fraser University the right to lend this thesis, project or extended essay to users of the Simon Fraser University Library, and to make partial or single copies only for such users or in response to a request from the library of any other university, or other educational institution, on its own behalf or for one of its users.

The author has further granted permission to Simon Fraser University to keep or make a digital copy for circulation via the Library's website.

The author has further agreed that permission for multiple copying of this work for scholarly purposes may be granted by either the author or the Dean of Graduate Studies.

It is understood that copying or publication of this work for financial gain shall not be allowed without the author's written permission.

Permission for public performance, or limited permission for private scholarly use, of any multimedia materials forming part of this work, may have been granted by the author. This information may be found on the separately catalogued multimedia material.

The original Partial Copyright Licence attesting to these terms, and signed by this author, may be found in the original bound copy of this work, retained in the Simon Fraser University Archive.

Bennett Library  
Simon Fraser University  
Burnaby, BC, Canada

# Abstract

We have proposed a novel detection technique, which allows capacity increase in a narrowband system to *seven times* that of the conventional seven-cell cluster imposed by TDMA/FDMA. The key to achieving a single cell cluster size is that instead of treating users in other cells as cochannel interference, we included them in joint maximum likelihood detection with the desired user. With less interference in the system, especially since stronger users are commonly chosen as participants in the joint detection, cluster size reduction and capacity increase are achieved. Semi-analytical bounds on the BERs of joint detection for various user groupings are presented and computation optimization techniques based on minimizing the number of users included in the detection while maintaining acceptable performance are studied. It is also shown that at the cost of additional antennas the proposed selection scheme allows for additional users in the system while maintaining the same performance.

# Dedication

*To my dear parents, who taught me determination and gave me a reason to be strong; even though they live so far away, I know I can always depend on them for love and support.*

*To my wonderful brother, with whom I share the excitements of studying and living.*

*To my affectionate sister, who has always been inspiration to me.*

*To my beloved friends, near and far.*

# Acknowledgements

I would like to thank my supervisor, *Professor Cavers*, for what he taught me, for his support, and for his patience. I would also like to thank *Professor Vaughan* and *my wonderful friends* at the MTF wireless communications lab.

# Table of Contents

<b>Approval .....</b>	<b>ii</b>
<b>Abstract.....</b>	<b>iii</b>
<b>Dedication .....</b>	<b>iv</b>
<b>Acknowledgements .....</b>	<b>v</b>
<b>Table of Contents .....</b>	<b>vi</b>
<b>List of Figures.....</b>	<b>viii</b>
<b>List of Acronyms .....</b>	<b>x</b>
<b>Chapter One : Introduction .....</b>	<b>1</b>
Motivation .....	3
Thesis Contribution and Organization .....	6
<b>Chapter Two : Background .....</b>	<b>8</b>
2.1 Overview of Wireless Communication Systems .....	9
2.2 Single User Detection Techniques.....	16
2.3 Multiuser Detection Techniques .....	18
2.3.1 Joint Maximum Likelihood Multiuser Detection .....	21
2.4 Channel Estimation.....	23
2.5 Error Rate.....	23
2.6 Diversity.....	23
2.6.1 Microdiversity.....	25
2.6.2 Macrodiversity .....	25
2.7 Outage in a Wireless System .....	26
<b>Chapter Three : ML in Multitier Systems.....</b>	<b>28</b>
3.1 System Model .....	30
3.2 Multitier Performance Analysis.....	34
3.2.1 Union Bound on Pairwise Error Probability .....	35
3.2.2 Semi – Analytical Results for the Union Bound.....	35
3.2.3 Upper and Lower Performance Limits on JML with First Tier Users.....	38
3.2.4 Joint Detection with First Tier Users and Perfect Channel State Information .....	39
3.2.4.1 Joint Detection with First Tier Users and Imperfect Channel State Information .....	41
3.2.5 Selecting the Jointly Detected Users from the First Tier.....	43
3.2.6 Comparisons .....	47

<b>Chapter Four : Optimizations .....</b>	<b>51</b>
4.1 Interference Mitigation .....	51
4.1.1 Directionality .....	52
4.1.2 Is it Worth Looking at JML detection with the Users from the Second Tier? .....	53
4.1.3 How Can We Include Second Tier Users in the Joint Detection? .....	54
4.1.4 What Can Be Done in the Case of Additional Users? .....	59
4.2 Computation Reduction .....	62
4.2.1 How Many Users Should Be Included in the Joint Detection? .....	62
<b>Chapter Five : Conclusions.....</b>	<b>68</b>
<b>Bibliography .....</b>	<b>73</b>



# List of Figures

Figure 1-1	Layout of a wireless system.....	2
Figure 2-1	Wireless communication system.....	9
Figure 2-2	Block diagram of the wireless communication system.....	10
Figure 2-3	Shadowing, attenuation from obstacles .....	11
Figure 2-4	Multipath reception.....	12
Figure 2-5	Rayleigh fading.....	15
Figure 2-6	Autocorrelation of in phase/quadrature components of channel gains .....	16
Figure 2-7	Union bound on BER of JML detection for two users with two antennas and perfect CSI (5 dB difference in signal strength) .....	22
Figure 3-1	System model – 61 hexagonal cells .....	31
Figure 3-2	Upper and lower limits on JML detection in conjunction with first tier users and perfect CSI.....	38
Figure 3-3	Union bound on JML detection with first tier users and perfect CSI .....	40
Figure 3-4	Union bound on JML detection with first tier users and no interference from outer tiers.....	41
Figure 3-5	Union bound on JML detection with first tier users and imperfect CSI ( $L = 6$ antennas) .....	42
Figure 3-6	Union bound on JML detection with selected users from the first tier and perfect CSI ( $L = 6$ antennas).....	44
Figure 3-7	Proposed JML with a selection of first tier users vs. the union bound with six antennas and SNR = 0 dB ( $10^5$ simulations for the system) .....	46
Figure 3-8	Union bound on JML detection with first tier users with/without selection and imperfect CSI ( $L = 6$ antennas, $k_m = .1$ ).....	47
Figure 3-9	Union bound on BER for JML detection with three first tier users vs. actual BER of conventional seven – cell cluster .....	48
Figure 3-10	JML with three selected first tier users and six microdiversity antennas vs. the conventional seven-cell cluster with two antennas ( $10^6$ simulations for the JML system).....	49
Figure 4-1	$120^\circ$ directional antennas .....	52
Figure 4-2	Union bound on JML detection with first tier users with/without CCI from the second tier.....	54

Figure 4-3	Union bound on JML detection with 4 strongest users from the first two tiers – perfect CSI.....	55
Figure 4-4	Selection of 4 strongest users from the first two tiers.....	56
Figure 4-5	Union bound on selection technique (detection set = 5 strongest) vs. actual BER of conventional TDMA/FDMA .....	57
Figure 4-6	Union bound on JML detection with 4 strongest users from the first two tiers and imperfect CSI ( $L = 6$ antennas).....	58
Figure 4-7	Union bound on JML detection with 4 and 6 strongest users and two users in each cell .....	59
Figure 4-8	Selection of one of the users in the cell (detection set = 7 strongest) with two users in each cell .....	60
Figure 4-9	Union bound on selection of 6 strongest users with 2 users per cell vs. TDMA/FDMA with one user per cluster .....	61
Figure 4-10	How the number of selected users affects the union bound BER ( $L = 6$ antennas) .....	62
Figure 4-11	Selection of 3 strongest users from the first two tiers ( $L = 6$ antennas) .....	64
Figure 4-12	Effect of the number of users in the detection set on the union bound for JML .....	65
Figure 4-13	Proposed JML selection technique vs. the union bound with six antennas and SNR = 0 dB ( $10^6$ simulations for the actual system) .....	66
Figure 4-14	Selection technique (detection set = 5 strongest) vs. actual BER of conventional TDMA/FDMA .....	67

# List of Acronyms

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CCI	Cochannel Interference
CDMA	Code Division Multiple Access
CSI	Channel State Information
dB	Decibel
FDMA	Frequency Division Multiple Access
JML	Joint Maximum Likelihood
LOS	Line of Sight
LS	Least Squares
ML	Maximum Likelihood
MMSE	Minimum Mean Squared Error
MRC	Maximal Ratio Combining
MUD	Multiuser Detection
SIC	Successive Interference Cancellation
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
ZF	Zero Forcing

*“The only lifelong, reliable **motivations** are those that come from within, and one of the strongest of those is the joy and pride that grow from knowing that you've just done something as well as you can do it.”*

Lloyd Dobens, *Thinking About Quality*

# **Chapter One :**

## **Introduction**

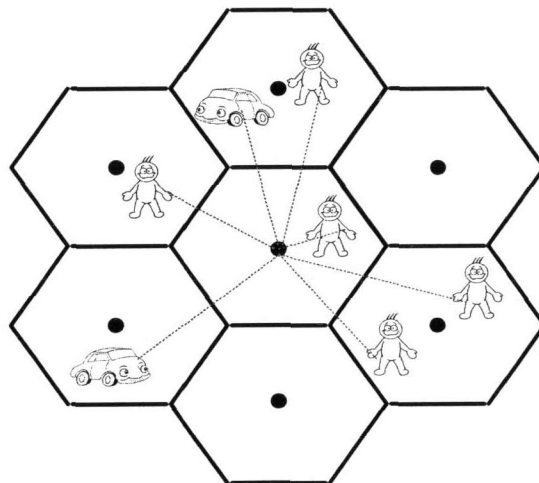
The subject of wireless communications involves transmission of signals from a mobile user to a base and vice versa. For the first time, in 1928, a mobile broadcasting system was employed, of course only for use in police vehicles. The connection became a two-way radio voice system by 1933. However, much has changed since that time; nowadays, wireless devices are an inseparable part of people's lives.

Acknowledging the vast, ever growing interest in wireless devices and knowing that the number of users that can concurrently access wireless services translates directly to profit gain, much of the ongoing research in this field is dedicated to providing service to more simultaneous

users. Since bandwidth resources have been exploited to their limits, in order to accommodate more users, better signal processing techniques are required.

A typical layout for a wireless system can be seen in Figure 1-1. The honeycomb structure represents the *cells* and the center dots represent the *base stations*. There is no strict definition for either the cell or the base station. In brief, the transceiver and controlling unit that operate the wireless system in an area are known as the base station. It is usually assumed that *all* reception antennas used by the base station are located at the base station. The base also provides the connection between the wireless and wired network.

The cell is an illustration of the coverage area of the base station. Subdivision of the service area into cells limits the distance between the mobile user and the nearest base station, ensuring adequate signal strength for the desired user at its base. Consider the central cell in Figure 1-1; as indicated, signals from *all* the users in the system are received at the base station with various power levels. Among those users, some are not of interest to the base station; therefore, their signal is not detected. These unwanted signals form the interference. Our goal in any wireless system would be to overcome the interference in order to detect the desired users, which we consider to be the ones in the central cell.



**Figure 1-1** Layout of a wireless system

This thesis proposes a way to alleviate the interference and detect the desired user. Throughout this work, we deal with three main problems of the wireless system: low signal strength at reception, mutual interference among simultaneous transmitted signals and fading, which will be introduced in more detail in the following chapter.

## Motivation

Cellular organization of wireless systems is the ubiquitous solution to poor signal strength and interference. To eliminate the interference by controlling user access to the wireless channel, various *multiple access techniques* have been developed. Code division multiple access (CDMA), time division multiple access (TDMA) and frequency division multiple access (FDMA) are the three widely recognized multiple access techniques.

In CDMA systems, which are not the subject of this thesis, mutual interference is reduced by assigning each user a different pulse shape. This approach requires an expanded bandwidth to accommodate all the required pulse shapes; therefore, CDMA is known as a *wideband* system. In this multiple access technique, all the cells use the same expanded bandwidth.

Contrary to CDMA, FDMA and TDMA are *narrowband*<sup>1</sup> access techniques. In a narrowband system, the available time/frequency resource is divided and dedicated to individual users. Each of these slots is known as a *channel*. A conventional narrowband multiple access technique operates with the notion of placing one user per time or frequency slot. However, this means that only a small number of users can exploit the limited time/frequency resource.

In FDMA and TDMA all users have the same pulse shape and no bandwidth expansion is required. Although these techniques do not require a wide bandwidth, adopting the same pulse shape for all users leads to extensive mutual interference in the system. Signals from cochannel users operating in neighbouring cells are received as interference at the desired user's base

---

<sup>1</sup> A system is referred to as narrowband when the transmitted signal only occupies a small bandwidth. For voice communications, the bandwidth of the transmitted signal is about 30 KHz.

station. In addition, signals from the base stations of cochannel cells interfere with the signal the mobile user receives from its base station. Systems separate such cochannel cells by enough distance that mutual interference is too weak to cause problems. In technical terms, choosing the distance between cells that occupy the same time/frequency division is called *cochannel spacing*. It should be noted that for the time being we are assuming one user per channel per cell; hence, the terms *user* and *cell* can be used interchangeably for one time/frequency slot.

The group of neighbouring cells that are not cochannel are accumulated as the number of cells in a *cluster*. Defining the cluster size helps us define the *capacity* of the system, which is a measure of how efficiently the channel is utilized. The capacity  $\eta$ , also known as the reuse efficiency, is the share each user has from the time/frequency resource. If the total number of channels in the system is assumed to be  $n$  each user in the cluster can use  $n\eta$  channels. Therefore, for one user per channel per cluster the capacity is

$$\eta = \frac{1}{C} \tag{1-1}$$

where  $C$  is the cluster size. Hence, in order to allow concurrent transmission for one user within each cell in the cluster, we would need a minimum of  $n = \frac{1}{\eta}$  channels.

If  $M$  users are assumed to operate cochannel in the cluster, the capacity is increased to

$$\eta = \frac{M}{C} \tag{1-2}$$

Regarding the definitions, it is obvious that spacing out the cochannel cells, we would increase the cluster size and hence reduce the available capacity. This is the main drawback in narrowband systems. We seek to address this problem and introduce a solution for capacity increase.

One of the recognized techniques applied to overcome interference is *multiuser detection* (MUD). Multiuser detection employs signal processing algorithms in conjunction with multiple antennas to detect signals despite severe mutual interference. Since the ability to tolerate more interference allows more cochannel users to operate within a cluster, MUD can be employed as a means to restore capacity in a narrowband system.

Previously conducted research has always been targeted towards the application of MUD to increasing the number of users within a single cell. Due to the common notion that wideband systems are superior in providing capacity compared to narrowband systems [1], very little effort has been made to investigate the effect of MUD on system-wide capacity.

The work presented in [2] is one of the earliest to study MUD techniques. This work investigates the application of *minimum mean square error* (MMSE)<sup>2</sup> detection to MUD. With a maximum of two interferers in the cell, the authors study the effect of varying the number of antennas on the improvement achieved by MMSE compared to *maximal ratio combining* (MRC), and conclude that MMSE is a practical detection technique to provide capacity increase in a narrowband system.

Later work [3] demonstrates a comparison of *maximum likelihood* (ML) detection and MMSE for MUD and reveals the fact that ML detection provides the system with a noticeable increase in capacity compared to the conventional MMSE.

[4,5] were the first works to consider a multiple cell narrowband system and show that maximum likelihood multiuser detection (ML – MUD) in such a scenario allows capacity increase to a soft limit as users are added. Prior to this work, soft capacity limits were associated with CDMA. Exploiting the potentials of ML – MUD with spatial diversity the authors move on to calculate the net capacity, revealing the fact that with maximum likelihood detection each new user in the system would only cost a few dB loss in the SNR of other users. Comparison and

---

<sup>2</sup> An overview of detection techniques will be presented in Chapter Two.



analysis of *intracell* reuse with CCI from *outer cell* users as a means to provide capacity increase are among the contributions of their work.

[6] explores a successive interference cancellation (SIC) technique in order to exploit the potentials of applying MUD in the narrowband wireless system. Applying a central power control scheme, they study the capacity gain due to the application of MUD with different cluster sizes. Introducing two channel assignment schemes, a capacity increase of about 1.4 is achieved compared to a system that assumes a single user per channel with a cluster size of one. Again, their work addresses the application of a MUD technique to the users within *one* cell.

Since research on the capacity of narrowband systems is still in its early stages, there are yet many unanswered questions.

- How can signal detection be enhanced in order to improve the overall system capacity?
- Can the current seven-cell cluster be reduced to a cluster size of one as in CDMA systems?
- How does the system capacity change in case of imperfect channel estimates?
- Can we reduce the amount of computational effort that is invested in signal detection and retain acceptable system performance?

## **Thesis Contribution and Organization**

In this thesis, we are taking a new approach to the multitier problem. In [3,4] although a multitier layout was assumed, no effort was made to study the net system-wide capacity of a wireless system and provide means by which the overall capacity can be increased. Since MUD is shown to be an effective tool in providing additional capacity in a wireless system, we intend to utilize its latent power.

The idea of joint detection with cochannel interferers from neighbouring cells is also a feature of macrodiversity systems. In such systems, the detection is performed jointly with outer

cell users and information sharing between cells is exercised. Although the outcome is attractive, the computational load and the infrastructure required to pool the signals received by different base stations has prevented wide application of such systems.

The novelty of the work presented in this thesis is in the idea of *performing joint detection* as known from macrodiversity systems, and *omitting* the information sharing in order to reduce the computational burden. In other words, rather than increase the number of users per cell, we use joint detection to shrink the cluster size and thereby increase system capacity.

Acknowledging the properties of maximum likelihood detection and employing joint detection combined with microdiversity, rather than macrodiversity, this work will be the first to address the system wide capacity computation in a multitier narrowband system by providing solutions for cluster size reduction. Performance comparisons to the conventional seven-cell cluster are made and it is shown that utilizing this technique we may increase the capacity of the current narrowband system seven-fold. We explain how the implementation of a simple selection scheme based on signal strengths can focus on computational reduction at little cost in BER performance and overcome mutual interference due to additional users. Otherwise stated, we show how the adoption of a powerful signal detection method in a narrowband system can allow wireless service providers to increase the number of simultaneous users. Doing so would bring significant benefits, both in increased revenue to service providers and in increased availability of the service to their customers.

In the subsequent chapter, a more detailed description of the wireless channel, applicable detection techniques and essential background information will be presented. The proposed detection technique for the multitier system is revealed in Chapter Three; and in Chapter Four optimization methods applicable to this work are reviewed, error rates are illustrated and performance comparisons are made. The conclusions are presented in the final chapter.

*“One’s mind has a way of making itself up in the **background**,  
and it suddenly becomes clear what one means to do.”*

A. C. Benson

## **Chapter Two :**

# **Background**

Although ideas of wireless communication had been around for many years, it was not until the late 1880s that it drew attention and serious research began in this field. The experimental verification of Maxwell’s theory in 1887 played a big role in this movement. In the same year the first wireless system, which had coverage over a radius of about a hundred meters, was developed. In 1895, Marconi demonstrated the transmission of radio signals at a distance of approximately 2 kilometres, and two years later patented a radiotelegraph system and established the Wireless Telegraph and Signal Company [7]. Today, only about a century away from the birth of the first wireless devices with their heavy power consumption, we enjoy wireless

communications in a very convenient style with lightweight, high speed, small size devices that barely have distance limitations, and in addition to voice, transmit data and media.

This opening chapter gives an overview of the principles of wireless communication and presents insight to detection techniques and performance analysis.

## 2.1 Overview of Wireless Communication Systems

A typical wireless communication environment would be similar to that of Figure 2-1. In order to model and analyse such a system, it is represented by the general block diagram of Figure 2-2. As illustrated, the common model consists of three main parts: the transmitter, the wireless channel and the receiver. Signals are generated at the transmitter, conveyed over the wireless channel and collected at the receiver. The channel causes various undesirable alterations on the transmitted signal. Hence, at the receiver end, signal-processing techniques are employed to retrieving the original signal. To enhance our understanding of the effects of the wireless medium, we will elaborate on the channel in the following section.

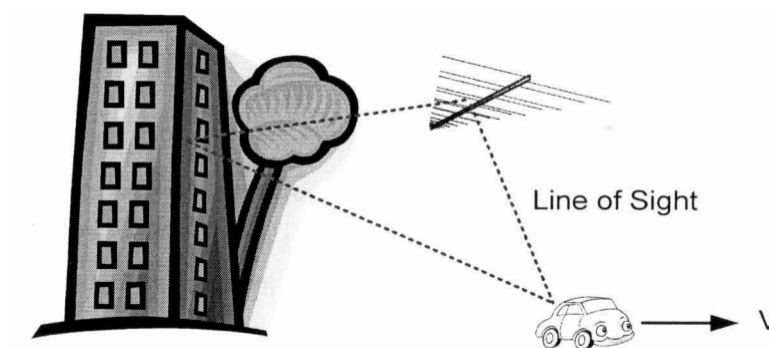
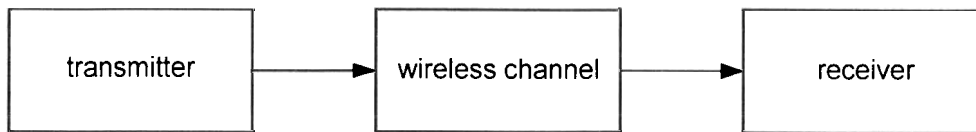


Figure 2-1 Wireless communication system



**Figure 2-2** Block diagram of the wireless communication system

In order to investigate the different physical features of the wireless channel, engineers have developed various mathematical models. Each characteristic of the physical channel is represented by a parameterized mathematical expression and a combination of all of these expressions yields a complete model that can simulate real world phenomena. In the following, we will give a brief introduction to the mathematical model used throughout this thesis.

### **Additive White Gaussian Noise**

In a communication system, noise is defined as *additive noise* reinforcing the notion that it is added to the signal as it travels to the receiver. For analytical purposes, all the noise is combined into one source. Since the largest noise source in the communication system is the thermal noise added at the front end of the receiver, the combined noise source is modelled at the input of the receiver.

This noise is generally modelled as Gaussian noise with zero mean and a known standard deviation, and recognized as additive white Gaussian noise (AWGN). AWGN is mostly defined by its uniform power spectrum over frequency. For further details, the reader is referred to [8].

It is assumed that frequency mixing is performed to a stage where the  $1/f$  (where  $f$  defines the frequency) noise is negligible.

## Path Loss

As the signal propagates through the medium, it gradually loses its power. The more distance it travels the weaker it becomes. This phenomenon is translated to *path loss* in technical terms and is mathematically modelled by the distance raised to an exponent of varying degree.

For near range communications, the loss is assumed to follow the inverse second power law; but as the distance grows, the inverse fourth power becomes more realistic. Real world measurements show that for urban and suburban areas the exponent is usually between 3 and 4, often closer to 4 [9].

## Shadowing

If the signal could travel on a virtual line connecting the transmitter to the receiver, we would have a line of sight (LOS) path as seen in Figure 2-1. However, in real life, this is barely possible; trees, tall buildings and many other obstacles block the way. This leads to a phenomenon recognized as *shadowing* very briefly illustrated in Figure 2-3. Shadowing is modelled as a random perturbation of the value predicted by the path loss model above, and represented as a multiplicative power factor. Its distribution is usually described as *lognormal* with unit mean and standard deviation of about 6 dB to 8 dB. Lognormal distributions are those for which the local-mean power expressed in logarithmic values has a normal (Gaussian) distribution.

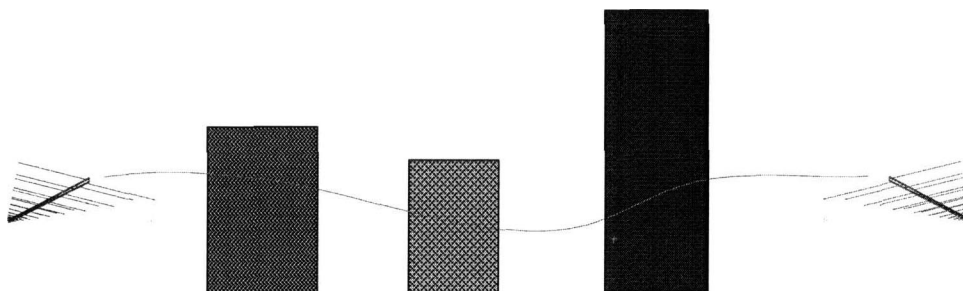


Figure 2-3 Shadowing, attenuation from obstacles

## Multipath Fading

The obstacles that cause shadowing in an environment also generate many reflections of the signal in various directions. Therefore, the transmitted signal arrivals at the receiver from various directions over many paths. This phenomenon is known as *multipath*. Figure 2-4 demonstrates a typical multipath environment.

With multiple copies of the original signal impinging on the receiver, each with an individual phase, degree of attenuation, Doppler shift and delay<sup>3</sup>, there is a chance that the combination will degrade the signal quality and cause a loss in power. This is described as *multipath fading* (see Figure 2-5). Also considering the wireless environment, it is obvious that at each time instant there will be a random number of multipath replicas present at the receiver. Multipath fading is another multiplicative factor affecting the signal; it is mathematically modelled as the impulse response

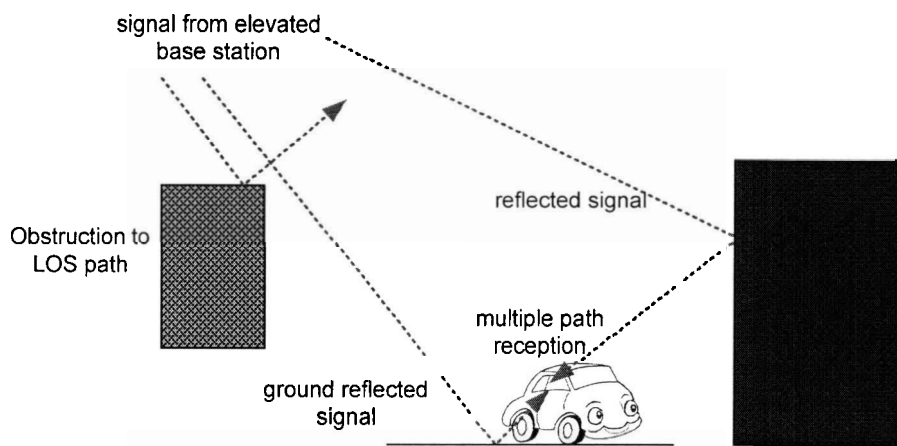


Figure 2-4 Multipath reception

<sup>3</sup> Different delays in the arrival of the signal from various paths may cause intersymbol interference, i.e. bits crashing into each other. This time dispersion is known as *delay spread*.

$$g(t, \tau_n(t)) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j2\pi(f_c + f_{D_n})\tau_n(t)} \delta(t - \tau_n(t)) \quad (2-1)$$

The so-called channel gain  $g$ , which is truly the impulse response of the channel, is shown as a function of both time  $t$  and delay  $\tau_n(t)$ . Expressing the delay as  $\tau_n(t)$  emphasises the fact that each path has its specific delay that is again time variant.  $\alpha_n(t)$  is the attenuation factor associated with path  $n$  at time  $t$  and  $e^{-j2\pi(f_c + f_{D_n})\tau_n(t)}$  shows the relative phase change where  $f_c$  is the carrier frequency and  $f_{D_n}$  is the Doppler shift associated with path  $n$ .

Unlike the amplitude of the channel gain that changes slowly with time, the phase may change very suddenly. Considering the phase  $(-j2\pi f_c \tau_n(t))$ , the reason becomes clear. A change of  $1/f_c$  in the delay can flip the phase by  $2\pi$  and since  $f_c$  is large  $1/f_c$  would be a small change in delay time corresponding to a small displacement in space.

Now that we have seen the effect of phase, time delay and attenuation in the system, we will have a brief overview of how Doppler shift stimulates fading. Assume the mobile user is moving towards an obstacle with a velocity of  $V$  at an angle of  $\varphi$  between the velocity vector and the LOS path. The movement will cause a Doppler shift of  $f_D = \left(\frac{V}{\lambda}\right) \cos \varphi$  in the received signal where  $\lambda$  is the carrier wavelength. Consider the extreme case where the user moves towards or away from the obstacle in a straight line, it will experience the maximum Doppler shift of  $f_{\max}$  or  $-f_{\max}$  respectively where  $f_{\max} = \frac{V}{\lambda}$ . Therefore, in the case of a moving mobile user, a single carrier frequency can be received within a frequency range of  $[-f_{\max} + f_{\max}]$  depending on the angle  $\varphi$ . Assuming omni-directional reception, each additional scatterer in the environment will



introduce at least one new frequency shift from the range  $[-f_{\max} \ f_{\max}]$  to the mix of signals at the receiver.

At this point, we have seen how each of the previously named factors affects the channel gain. To facilitate the modelling of the channel gain, we will now investigate its distributional properties.

The scale of temporal variations for narrowband signals is often much greater than the range of delays in (2-1); in which case, the effective impulse response collapses to a single impulse with complex gain equal to the sum of the complex gains of the paths. That gain is denoted

$$g = \sum_{n=1}^{N(t)} (g_{In} + jg_{Qn}) = g_I + jg_Q \quad (2-2)$$

Again, the number of multipath replicas  $N(t)$  is assumed time variant.

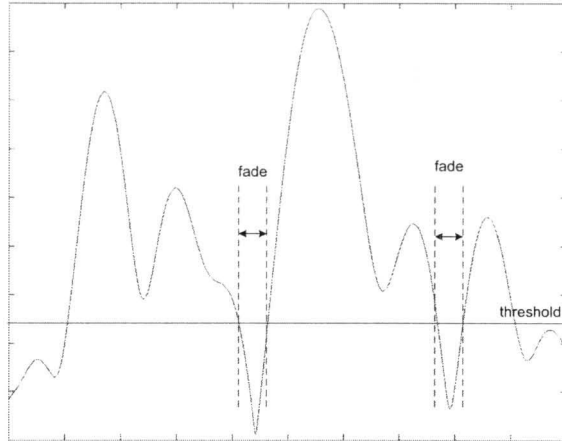
As the number of paths increases,  $g_I$  and  $g_Q$  become the sum of a large number of independent and identically distributed terms; hence, the central limit theorem can be applied.  $g_I$  and  $g_Q$  convert to joint, independent Gaussian random variables with the same variance.

When  $g_I$  and  $g_Q$ , hence  $g$ , is assumed zero mean the channel is described as a *Rayleigh fading channel* and the distribution of the channel gain amplitude is known as *Rayleigh*

$$p_g = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|g|^2}{2\sigma^2}\right) \quad \text{where} \quad \sigma^2 = E\{g_I^2\} + E\{g_Q^2\} \quad (2-3)$$

In other words, *Rayleigh fading* occurs when there is no LOS component so that the mean gain is zero [8,10]. Figure 2-5 illustrates the amplitude of a signal undergoing Rayleigh fading. The threshold depicts the minimum signal level so that it can be distinguished from noise.

If there is a LOS component among the reception signals, i.e. the mean channel gain is no longer zero, we have a *Rician* model for the channel gains; see [8] for more mathematical details.



**Figure 2-5 Rayleigh fading**

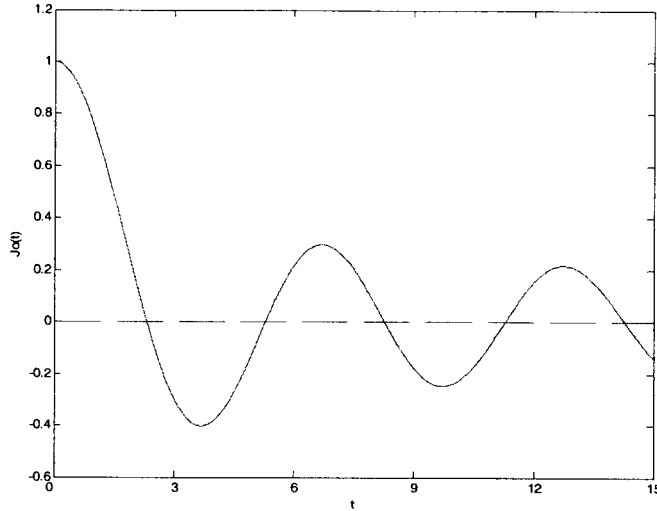
Finally, with the assumption of uniform scattering in azimuth, the normalized autocorrelation function of the in phase/quadrature component of channel gain is

$$R_g(\tau) = J_0(2\pi f_D \tau) \quad (2-4)$$

where  $J_0$  is the zero order Bessel function and  $f_D$  is the Doppler shift previously defined. Figure 2-6 illustrates the autocorrelation function; the Fourier transform of this autocorrelation function is known as the Jakes spectrum.

To conclude the modelling of the wireless channel, we review the application of above-mentioned characteristics to a transmitted signal. Assume  $x$  to be the complex envelope of the transmitted signal; for convenience time dependency is not shown. After undergoing fading, shadowing and path loss the complex envelope of the reception signal will be

$$y = 10^{s/20} d^{-\gamma/2} g \cdot x \quad (2-5)$$



**Figure 2-6 Autocorrelation of in phase/quadrature components of channel gains**

The first term  $10^{s/20}$  represents shadowing. Since shadowing is assumed lognormal,  $s$  is a Gaussian random variable with zero mean and known standard deviation.  $d$  shows the distance from the transmitter; accordingly,  $d^{-\gamma/2}$  denotes path loss and  $\gamma$  is the path loss exponent. Finally,  $g$  is the fading coefficient. Noise is also be added to the received signal so

$$r = 10^{s/20} d^{-\gamma/2} g \cdot x + n \quad (2-6)$$

In the next section, detection techniques applied at the receiver are studied.

## 2.2 Single User Detection Techniques

The simplest scenario for a communication system would be a single user transmitting to a reception antenna with no interference from other users. To facilitate the understanding of the detection techniques, in this section we assume a fading channel with symbol synchronous transmission and zero delay spread (so-called flat fading); thus, the only unwanted signal in the system will be noise. Assuming a simple channel model with no path loss or shadowing, from

(2-6) we model the received signal as  $r$  in (2-7); where  $x$  is the transmitted signal,  $g$  is the channel gain and  $n$  is the additive Gaussian noise.

$$r = g \cdot x + n \quad (2-7)$$

The *minimum mean square error* (MMSE) technique is based on minimizing the error term  $E\{|\varepsilon|^2\}$  where  $\varepsilon = x - \hat{x}$  and  $\hat{x}$  is a hypothesis for the transmitted signal. Otherwise stated, MMSE is based on minimizing the difference between the variance of the estimate and the signal. The estimate is linked to the decision made for the received signal by a gain vector  $w$  where we have

$$\hat{x} = w\hat{r} \quad \text{and} \quad w = (g^*g + N_o)^{-1}g^* \quad (2-8)$$

The asterisk represents conjugation and  $N_o$  is the noise variance. The detection symbol is the signal constellation point closest to  $\hat{x}$ .

Another way to retrieve the desired signal from the reception signal is to use the *maximum likelihood* (ML) method. In this detection method, the hypothesis  $\hat{x}$  is chosen so that the metric  $|r - g \cdot \hat{x}|^2$  is minimized. Substituting  $r$  from (2-7) we will be minimizing  $|g \cdot (x - \hat{x}) + n|^2$ . This detection method depends on the finite alphabet property of the signal space. Employing ML detection, perfect CSI is often assumed which means  $g$  is known to the receiver. However, the real-world situation is imperfect CSI, which causes an increase in BER.

In the more general case where we assume that intersymbol interference is present in the system, (i.e. the transmission is not synchronous) decision feedback methods, block detection methods, detection within a window of received data or any trellis type detection may be performed. More information on detection techniques can be found in [9].

## 2.3 Multiuser Detection Techniques

Detecting the desired signal when there are other users present in the system is a more complicated procedure and usually requires more than one reception antenna. There are various approaches to how one can perform detection on a group of user. Some techniques perform sequential detection of users while others detect the users jointly.

In most cases, there are more users in the system than we *would like to* or we *can* detect. Depending on the applied technique, the relative strength of the users and the number of antennas, a limitation is set on the number of users that can be detected. Hence, a criterion is needed in order to pick the users that should be included in the multiuser detection process. With sequential detection, other than specifying the users that are detected, the order in which the detection is performed is also important; especially since with many of the serial detectors the detection quality degrades successively as errors propagate from stage to stage.

In the following, we will briefly review some of the *linear* and *non-linear* multiuser detection techniques and consider their strengths and weaknesses. The term linear implies that the output of the detection method is a *linear function* of the input i.e. the received signal. Hence, a linear filter can be implemented to perform the desired calculations. As for the non-linear techniques, non-linear filters are required.

Assume the received signal is in the form of

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{2-9}$$

where  $\mathbf{H}$  is the matrix of the channel gains,  $\mathbf{r}$  is the vector of received signals with a component from each antenna and  $\mathbf{n}$  is the noise vector, also with one component from each antenna. The dimension of both of these vectors is  $M \times 1$  where  $M$  is the total number of reception antennas.  $\mathbf{H}$

is an  $M \times K$  matrix where  $K$  is the number of users in the system, and  $\mathbf{x}$  is the  $K \times 1$  vector of transmitted signals.

The first method we will consider is *zero forcing* (ZF). Zero forcing is very similar to the *least squares* (LS) solution for an equation. We multiply the received signal  $\mathbf{r}$  by the pseudo-inverse  $(\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger$ , where the dagger stands for Hermitian transpose. This, results in

$$\hat{\mathbf{x}} = (\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{r} = \mathbf{x} + (\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger \mathbf{n} \quad (2-10)$$

Since the estimate  $\hat{\mathbf{x}}$  equals the transmitted  $\mathbf{x}$  when the noise is zero, this method eliminates interference among the users. Each component of  $\hat{\mathbf{x}}$  contains its user signal, with the others nulled out, hence the term zero forcing. The symbol decision is the closest constellation point to  $\hat{\mathbf{x}}$ .

Regarding the fact that zero forcing requires one order of diversity to null the interference impinging on each user, the diversity order for this system is the number of extra antennas we have, compared to users plus one (i.e.  $M - K + 1$ ). The concept of diversity will be explained in section 2.6.

Each row of the pseudo-inverse acts as a weight vector for detection of a different symbol. To remove interference the row is orthogonal to all columns of  $\mathbf{H}$  but the one corresponding to the desired symbol; depending on the relation of the desired column to the space spanned by the others, the gain of the desired symbol or its corresponding noise component may be adversely affected.

From what we know about the LS solution,  $\mathbf{H}$  is constrained to have no more columns than rows (i.e.  $M \geq K$ ) which means that the number of users must be no more than the number of antennas. Thus, ZF is not suitable for an overloaded system where the number of users is more than the number of antennas. Although it cannot support an overloaded scenario, ZF has the

desirable property of infinite near-far resistance. Near-far resistance is an indication of how well the detection technique can operate in the absence of noise with a high level of interference compared to the case where the interference is low. In a wireless system, there may be times when the interference is strong compared to the desired user, that is when the near-far resistance becomes an important measure of performance for the detection technique. The higher the resistance the better the detection technique can handle such cases.

ZF achieves perfect near-far resistance when perfect CSI is assumed because it eliminates all interference. On the other hand, this means it does not account for noise. Multiplying the noise vector by  $(\mathbf{H}^{\dagger}\mathbf{H})^{-1}\mathbf{H}^{\dagger}$ , considering the random nature of  $\mathbf{H}$  and hence arbitrarily small eigenvalues, an unknown amount of noise enhancement is produced.

Another method that has been used in multiuser detection is the *minimum mean square error* (MMSE) [2]. As in the case of single user detection, the final estimate for the received signal is the hypothesis that minimizes

$$E\{|\boldsymbol{\varepsilon}|^2\} = E\{|\mathbf{x} - \hat{\mathbf{x}}|^2\} \quad (2-11)$$

With MMSE, the desired signal is decided upon by obtaining the appropriate weight vector  $(\mathbf{H}^{\dagger}\mathbf{H} + N_0\mathbf{I})^{-1}\mathbf{H}^{\dagger}$  that multiplies the received signal vector  $\mathbf{r}$ , where  $\mathbf{I}$  is the  $K \times K$  identity matrix and  $N_0$  is the noise variance. As can be seen in the multiplication term, with MMSE the presence of noise is also taken into account so there will be less noise enhancement in this technique. Again, the symbol decision is the closest constellation point to  $\hat{\mathbf{x}}$ .

*Maximum Likelihood* is a nonlinear detection technique introduced in the framework of single user detection, which is extendable to the multiuser case. The metric that is minimized for ML detection is  $|\mathbf{r} - \mathbf{H}\hat{\mathbf{x}}|^2$ . For ML there is no restriction on the number of users, so even one antenna alone can efficiently detect more than one user [3].

Since use of the matrix  $\mathbf{H}$  implicitly assumes perfect CSI, detection techniques are sensitive to the accuracy of channel state information. [11] investigates the sensitivity of detection techniques to CSI and concludes that ML type detection techniques demonstrate desirable performance only when perfect or near perfect CSI is assumed.

### 2.3.1 Joint Maximum Likelihood Multiuser Detection

One of the well-known multiuser detection techniques is the *joint maximum likelihood* (JML) method. In this method, the calculated ML metric is accumulated over all the users (or at least a group of them) and if the channel possesses memory, over a period of time. Thus, the final decision is based on all the data related to the users, gathered from the specific antennas, in a certain time window.

Assuming  $\Lambda(\hat{\mathbf{x}})$  to be the JML metric, it can be calculated as in (2-12), where  $\lambda_{n,l,m}$  is the metric related to user  $m$  at microdiversity antenna  $l$  at discrete time  $n$  and is dependent on the hypothesis vector  $\hat{\mathbf{x}}$ . The hypothesis  $\hat{\mathbf{x}}$  that minimizes the sum is chosen as the detected signal [12].

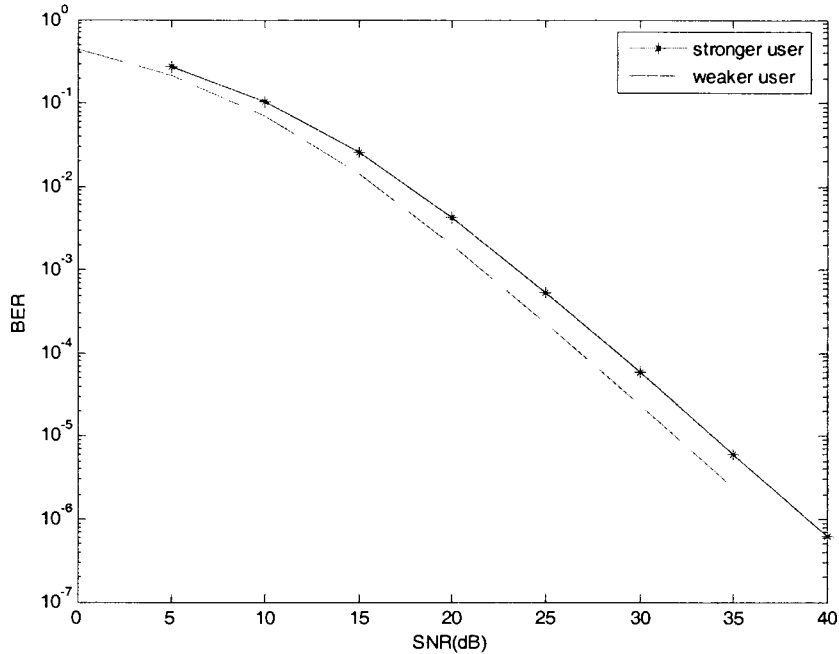
$$\Lambda(\hat{\mathbf{x}}) = \sum_{n=1}^N \sum_{l=1}^L \sum_{m=1}^M \lambda_{n,l,m}(\hat{\mathbf{x}}) \quad (2-12)$$

The main advantage of doing so is the gain achieved for weaker users as a result of joint detection with stronger users. As can be seen in Figure 2-7, the weak user benefits a lot from the presence of the stronger user. This figure represents semi-analytical<sup>4</sup> results for the union bound on the BER with perfect CSI for each user plotted against its own SNR [13]. The stronger user's signal power is assumed 5 dB higher than the weaker user.

---

<sup>4</sup> The BERs are obtained applying the semi-analytical technique introduced in Chapter Three.





**Figure 2-7 Union bound on BER of JML detection for two users with two antennas and perfect CSI (5 dB difference in signal strength)**

In addition, when involving other users in the detection process, those users will be subtracted from the interference present in the system. [14] demonstrates the benefits of multiuser detection in comparison with single user detection.

In contrast to nulling and MMSE combining, where the diversity order is reduced by one each time a new user is added and therefore the maximum number of users the system can support depends on the number of antennas, JML preserves the order of diversity regardless of the number of users [13]. As can be seen from the slope of the curves, each user in the system will enjoy  $L$ -fold diversity if  $L$  is defined as the number of diversity antennas. For more on the comparison of joint detection using MMSE and ML the reader is referred to [4].

JML is the detection technique used through out this thesis. Hence, more mathematical details are explained in subsequent sections.

## 2.4 Channel Estimation

Acknowledging the importance of channel estimates in the performance of ML detection, we will concisely review a few notes in this regard.

Channel estimators are usually optimized to produce the minimum error, with whatever criteria they are based on, with the least computation possible. There are many different categories of estimators; some are pilot-based while others use the actual transmitted data to approximate the channel.

[15] considers simultaneous signal detection and channel estimation in a spread spectrum case for  $k$  users with complexity order  $O(k^2)$ , and acknowledges that the ML estimator is exponentially complex and does not represent a practical solution. In this thesis, we will not explore alternative methods of estimating a channel, but we will use the pilot-based multiuser channel estimation technique introduced in [16].

## 2.5 Error Rate

The rate at which errors occur is one of the most important performance characteristics of a wireless system. There are various definitions for the number of errors in a system but the one applied in this thesis is the well-known *bit error rate* (BER). The BER is defined as the fraction of erroneous bits or, in an ergodic system, as the probability that a bit is detected erroneously. The BER can be calculated conditioned on the amplitude of the received signal; thus, the outcome is an averaged conditional error rate for each SNR.

## 2.6 Diversity

When transmitting a signal, there are many degrees of freedom available to all users in order to avoid being forced to share the channel with others. Data can be sent in a specific time slot (time diversity) as in TDMA, on an individual frequency (frequency diversity) as in FDMA,

from a certain antenna in the set of antennas (space diversity) as in [17] or even with a particular polarization (polarization diversity). In each of these cases, the signals that are transmitted are orthogonal in one dimension and hopefully distinguishable when they arrive at the receiver.

When a given signal affects two or more of the aforementioned diversity channels, so that it is received with different gains in each slot, *transmit* diversity is exercised. In the subsequent text, we will see forms of *reception* diversity. However, before proceeding to the next section, we will explain why diversity is so appealing in a wireless environment.

In addition to providing the possibility to separate the users and reduce cochannel interference, the available diversity can be exploited to combat fading, which is the main drawback in a wireless system. The basic idea of diversity is to take two or more channels and combine them such that a more reliable channel is realized [18]. An error in detection usually occurs when the signal suffers a deep fade. Therefore, when the receiver acquires  $M$  individual copies of the signal, each having experienced independent fading, the probability that all have undergone a deep fade is  $P^M$  where  $P$  is the probability of a deep fade for each channel. Hence, the effect of fading is eliminated to a great degree.

Equation (14.4-15) in [8] depicts the average bit error rate of a BPSK signal  $\bar{P}_e$  transmitted over  $L$  Rayleigh fading channels as

$$\bar{P}_e = \left(\frac{1-\mu}{2}\right)^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left(\frac{1+\mu}{2}\right)^l \quad (2-13)$$

where

$$\mu = \sqrt{\frac{\Gamma_b}{1+\Gamma_b}}$$

and  $\Gamma_b$  is the average SNR per channel, which is assumed to be identical for all channels. It can be seen that the error rate decreases, roughly speaking, inversely with the  $L^{\text{th}}$  power of the SNR.

### 2.6.1 Microdiversity

If the mean signal power impinging on a group of closely spaced antennas is equal but each antenna receives the signal with an independent channel gain, the system possesses reception diversity. To achieve the maximum possible diversity the distance between the antennas should roughly be more than half the wavelength so that each one receives the signal with an independent channel gain. This form of diversity is known as *microdiversity*. In other words, multiple copies of the same transmitted signal are received where each of them has experienced a slightly different set of amplitudes and/or phases in its transit to the receiver. In this case, shadowing is assumed to be the same for all signals.

The idea of how we should combine these signals in order to make the final decision on the data depends on how much computational effort we are willing to invest in detecting the signal. Simple methods such as “pick the best SNR”, also known as selection diversity, to more computationally complex methods such as “weighted combining” (for example maximal ratio combining) have been introduced. In [2] adopting MMSE, the impinging signals are weighted and combined in order to achieve the maximum *signal to interference and noise ratio* (SINR). The weighted combining is implicit in the MMSE and ZF matrices and in the ML metric, all discussed earlier. In all these methods, the available diversity is exploited in order to reduce the BER of the overall system.

### 2.6.2 Macrodiversity

Sometimes antennas are placed so far apart that the signal received at each has a distinct mean power, where the mean is taken over a fading ensemble. This form of reception diversity is known as *macrodiversity*. In a macrodiversity system, the signal received at each antenna experiences a separate channel. Thus, the shadowing and fading effects are unrelated.

Usually in order to operate a wireless system laid out in a vast area, instead of installing one central control unit, the area is divided into cells and each is managed separately. The amount of computation and the level of decisions that each cell is responsible for varies depending on design parameters. Such multi-cell systems are usually designed so that each cell surrounds a macrodiversity antenna.

Obviously, partitioning will not prevent the signals from users within one cell from travelling to neighbouring cells. Therefore, each cell will be able to provide information to neighbouring cells about their users, and sharing information between macrodiversity antennas will provide a boost in the overall BER performance. However, whether macrodiversity will be exploited in a multi-cell system or not is a design issue that depends mostly on the available resources. Unfortunately, pooling data from macrodiversity antennas and processing them jointly is very costly. A great amount of effort has been invested in computation reduction techniques in order to provide more efficient algorithms for information sharing in these systems [12].

Although we will acknowledge all the users present in a macrodiversity system and perform JML with a significant group of outer cell users, due to the high cost of applying macrodiversity methods we do not intend to apply any of these techniques throughout this work.

## 2.7 Outage in a Wireless System

In order to evaluate the functionality of a wireless system, we will need a performance measure. It would seem reasonable to determine the percentage of time that the error rate is above a certain threshold as a measure of how well the system is performing. In the literature, this is known as the *outage probability*. Mathematically, the outage probability for user  $m$  is defined as

$$P_{out} = \Pr\{P_m > P_{th}\} \quad (2-14)$$

where  $P_{th}$  is the threshold error rate. The error rate is an average over fading and is still affected by path loss and shadowing. An outage occurs when either or both of the latter two components cause the fading-average BER to rise above a threshold of acceptability.

In order to maximize the capacity while setting a standard for the outage of the system, the cluster size is chosen as small as possible. In other words, while maintaining a minimum level of performance in the system, cochannel cells are brought in as close as possible. Of course, the cluster size will depend on factors such as the number of users per cell, the number of microdiversity antennas and the detection technique applied. Therefore, with a certain detection technique, the available capacity is a trade off between the size of the cluster and the number of users in the system; as the number of users is increased, the spacing of cochannel cells must expand farther apart to avoid the mutual interference from obscuring the detection of cochannel users.

*“As a doctor, as a man of science, I can tell you there is no such thing as curses! Everything just happens as a question of probability. The statistical **likelihood** of a specific event.”*

Diane Frolov et al. , *Northern Exposure, Slow Dance* (1991)

## **Chapter Three :**

# **ML in Multitier Systems**

In order to meet the ever-growing demand for wireless systems to support more and more users and considering the limited bandwidth resources, much effort has been targeted towards increasing the operating capacity in a wireless system. CDMA systems have been known to outperform narrowband communications by far when the comparison is based on capacity [1]. In this work, we intend to explore and improve the capacity limits in a narrowband system.

As argued in the previous chapter, in order to increase the capacity, it would be our goal to operate cochannel cells as closely as possible. Due to the properties of CDMA systems, operating neighbouring cochannel cells and achieving a cluster size of one is practical. However, in a narrowband system, interference levels in a cluster size of one are too high for conventionally

designed base stations. In the following, we will review the classical approaches to interference received from users outside the cell boundaries.

In the past, there have been two main approaches to outer cell interference. The first approach is to consider the signal from outer cell users as unwanted interference. In this case, we may use one of the very many serial interference suppression or cancellation techniques to remove the effect of cochannel users [2,6,22]. While suppressing or cancelling users, there is a good chance that the process will not remove the effect of the user completely. In the case of strong interferers, even small errors in cancellation will cause significant degradation in the performance of the desired user.

As will be explained in the subsequent sections, although we do treat users from outside the cell as interference, we intend to employ joint detection and utilize the power received from these users to aid the detection of the desired user. Since doing so requires that we estimate their channel states and know their signal constellations, these interferers are referred to as *modelled interferers* as opposed to the remaining users that are known as *un-modelled interferers*.

Another approach to handling signals from outside the cell is to exploit the macrodiversity available in the system. Although due to the benefits of diversity this would seem a very convincing approach, the computational power and infrastructure required for pooling and sharing the received information in a macrodiversity cluster even with the application of computation reduction techniques [23] is overwhelming. Therefore, unless a big improvement is achieved, this solution is far from reality.

In this work, we are investigating a new viewpoint to the multitier problem. Instead of cancelling or suppressing outer cell users, we intend to include a group of them in joint detection and discard all detected bits but the one for the desired user. Performing the detection jointly, especially with strong users, reduces the level of interference for the desired user and in fades enhances the detection. However, we do not intend to share the data with neighbouring cells, as in



a macrodiversity system. This approach, while less powerful than macrodiversity, offers the prospect of retrofits at individual base stations without requiring the infrastructure for pooling signals from different base stations.

In [4] it is shown that JML provides a significant increase in capacity compared to MMSE and consequently ZF. Since our major interest is capacity increase, ML is chosen as the joint detection technique through out this work.

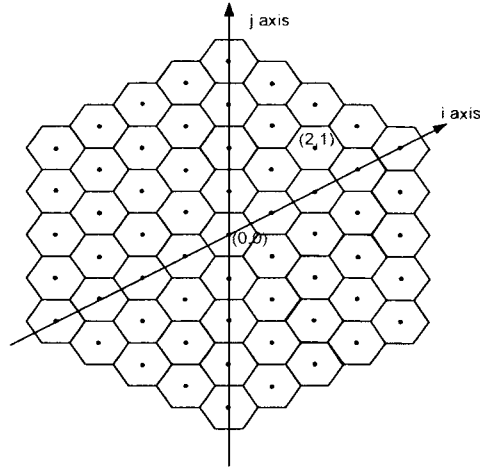
### 3.1 System Model

For the system layout, we employ the 61 cell hexagonal layout in Figure 3-1. With this four-tier layout, there are enough surrounding cells as to ensure that no edge effects are seen at the central cell that we are investigating. In addition, a cluster size of one is assumed; thus, all cells are cochannel. By this approach, we hope to achieve the greatest increase in capacity compared with conventional systems. Consequently, our goal is to determine under what detection conditions we can achieve acceptable performance.

Each cell in the system is characterized by a pair of integers  $(i,j)$  that represent the displacement along the axes shown in the figure; thus cell(2,1) is at  $i = 2$  and  $j = 1$  with respect to cell(0,0) at the center. The angular spacing between the axes is  $60^\circ$  and the distance from one cell to the central cell is calculated as

$$d = \sqrt{i^2 + j^2 + ij}\sqrt{3}R \tag{3-1}$$

where  $R$  is the corner radius of the hexagonal cell. The distance to the nearest cochannel cell is known as the *reuse distance*.



**Figure 3-1 System model – 61 hexagonal cells**

The users in the system are assumed to be uniformly scattered in the cells and to change position each time the simulation is run. Also for convenience, it is assumed that each user communicates with the closest base station. In some cases, this may underestimate the capacity of the system since users are usually allocated to the base station which receives them at the highest signal strength.

The symbols are drawn from a BPSK constellation with unit radius and slow power control is assumed. This indicates that each user is received at its base station with a constant average signal power. This average is obtained over fading; hence, by slow power control, path loss and shadowing effects are eliminated. Due to power control, the transmitted signal from the desired user is received at the central base station in the absence of noise as

$$r = \sqrt{E_b} gb \tag{3-2}$$

$E_b$  is the average transmitted energy per symbol,  $b$  is the transmitted symbol and the effect of the channel gain is demonstrated by the term  $g$  with a variance of one (2-3). However, the signal

received at the central cell from user  $m$  outside the cell would reflect path loss and shadowing, i.e. the received signal, again with no noise, would be of the form

$$r_m = \sqrt{E_b} 10^{(s_{m0} - s_m)/20} \left( \frac{d_{m0}}{d_m} \right)^{-\gamma/2} g_m b_m \quad (3-3)$$

where  $d_m$  is the distance of the user to its base station and  $s_m$  represents the corresponding shadowing along that path. The user adjusts its transmission power according to these factors so that at its base station the reception signal is received free of shadowing and path loss effects.  $d_{m0}$  is the distance of user  $m$  to the central base station and  $s_{m0}$  reflects the shadowing related to this path. The  $s$  exponents are random variables with a standard deviation of 8 dB and  $\gamma = 4$ .

To be able to investigate the proposed detection technique, we have eliminated the complications of less interesting effects. Although with asynchronous arrivals or delays in arrival, the signals would be more distinguishable, we have made the assumption of symbol synchronous transmission with no delay spread. While these simplifying assumptions reduce the computational load, they degrade the performance; therefore, the performance results are somewhat conservative.

In addition, since it is assumed that symbols are transmitted synchronously and the channel is assumed memoryless, no specific pulse shaping is required. With all the received data aligned and no pulse shaping applied, the worst case scenario is set for the detection of the desired user.

Aside from the shadowing and path loss parameters set above, a voice activity factor is also employed. It is presumed that users are active only 35% to 40% of the time; hence, a voice activity factor equal to  $\frac{3}{8}$  is applied to each user.

The fading rate in the system is assumed slow. Fading is considered to be slow or fast compared with the symbol rate. If the maximum Doppler shift is much smaller than the inverse of the symbol period  $f_D \ll 1/T$ , the fading is recognized as slow fading and if  $f_D > 1/T$  the signal is said to experience fast fading. Considering a slow fading channel allows the error in channel gain estimation to be assumed relatively small.

Upon reception, white Gaussian noise with a single-sided power spectral density normalized to  $N_o = 1$ , i.e. the variance of the real and imaginary components are each equal to 0.5, is added to the signal impinging on the antenna (3-4).  $r_l$  is the signal received after match filtering at antenna  $l$ .  $C_l$  is a row vector of the estimates of all the users' complex gains which include shadowing, fading and path loss factors (3-5), and  $\mathbf{b}$  is a column vector of transmitted symbols which in our case is assumed a column of ones.  $v_l$  is the match filtered noise.

$$r_l = C_l \mathbf{b} + v_l \quad (3-4)$$

where

$$C_l = \sqrt{\frac{E_b}{N_o}} \left[ g_{l0}, \dots, 10^{(s_{m0} - s_m)/20} \left( \frac{d_{m0}}{d_m} \right)^{-\gamma/2} g_{lm}, \dots \right] \quad (3-5)$$

in which  $g_{lm}$  is the complex gain at antenna  $l$  from user  $m$ , and all such gains have variance 1 (2-3).

The simplified log likelihood metric for each *microdiversity* antenna  $l$  with the hypothesis vector  $\hat{\mathbf{b}}$  would be

$$\Lambda_l(\hat{\mathbf{b}}) = |r_l - C_l \hat{\mathbf{b}}|^2 \quad (3-6)$$

And the overall metric for the antenna array would be the sum of the metrics from all the  $L$  individual microdiversity antennas

$$\Lambda(\hat{\mathbf{b}}) = \sum_{l=1}^L \Lambda_l(\hat{\mathbf{b}}) = \|\mathbf{r} - \mathbf{C}\hat{\mathbf{b}}\|^2 \quad (3-7)$$

where  $\mathbf{r}$  is a vector with components  $r_l$  and  $\mathbf{C}$  is a matrix with rows  $\mathbf{C}_l$ .

The final decision vector  $\mathbf{b}$  is the hypothesis vector which minimizes the log likelihood metric

$$\mathbf{b} = \text{argmin} (\Lambda(\hat{\mathbf{b}})) \quad (3-8)$$

However, obtaining the BER in this system by averaging the error rate over shadowing, fading and path loss in performed simulations, especially for the case of low BERs, requires many trials and is very time consuming. Therefore, in section 3.2 the results presented in [5,13] are extended to obtain a union bound for the conditional BER in our proposed scenario, which is defined as the fading-averaged BER conditioned on the set of amplitudes that include power control, shadowing and path loss.

It should be noted that *all the BERs* presented throughout this work are associated with the user present in the central cell and are the result of  $10^3$  Monte Carlo simulations over one bit, unless stated otherwise. In addition, since we are investigating the error rate for the central cell user it is assumed active all the time and is not affected by the voice activity factor.

### 3.2 Multitier Performance Analysis

In this chapter, we approach the proposed method of performing joint detection for a group of users to improve the performance of the intended user. For the time being, this group is set to the first tier cells i.e. the six cells that surround the central cell. In this chapter and the following, we investigate the best choice of users to be included in the joint detection. As

previously noted, to facilitate the computation of BERs, in this section we intend to attain a union bound on the probability of error averaged over fading and conditioned on users' local mean powers, as determined by their path loss and shadowing.

### 3.2.1 Union Bound on Pairwise Error Probability

The pairwise error probability between two sequences is the probability that the detector detects one sequence when the other one is transmitted. To determine the union bound on the pairwise error probability for a given user  $m$ , we sum over all the pairwise error probabilities that the chosen sequence  $\mathbf{b}_j$  is erroneous for the symbol  $\mathbf{b}_{im}$  related to the individual user  $m$ . Since it does not affect the generality of the calculations, the data bits transmitted from all users are assumed to be one at all times, i.e.  $\mathbf{b}_i$  is a sequence of ones. Therefore the union bound for error probability of user 0  $P_0$ , i.e. the user in the central cell, is the sum of all pairwise error probabilities  $P_{2i,j}$  related to the sequences that have not detected the symbol related to this user to be one, i.e.

$$P_0 = \sum_{j \in C_{i_0}} P_{2i,j} \quad (3-9)$$

where  $C_{i_0}$  is the set of signal vectors where the bit related to the central cell user is not equal to  $\mathbf{b}_{i_0} = 1$ .

### 3.2.2 Semi – Analytical Results for the Union Bound

Most of the analytical results presented in this section are extensions from the work introduced in [5,13]. The notation is kept consistent with the original text as far as possible.

The resulting pairwise error probability for two sequences  $(\mathbf{b}_i, \mathbf{b}_j)$  for joint detection of cochannel users irrespective of the fact that they may or may not be in the same cell, is

$$P_{2i,j} = \frac{1}{(1-r_{i,j})^{2L-1}} \sum_{k=0}^{L-1} \binom{2L-1}{k} (-r_{i,j})^k \quad (3-10)$$

where  $L$  is the number of microdiversity antennas present at the base station<sup>5</sup>.  $r_{i,j}$  is the ratio of the two non-zero eigenvalues of the matrix  $\mathbf{R}\mathbf{F}_{ij}$  and negative at all times. Therefore  $r_{i,j} = \lambda_{ij1}/\lambda_{ij2}$  where it is assumed that  $\lambda_{ij1}$  is positive and  $\lambda_{ij2}$  is negative [5].  $\mathbf{R}$  is the covariance matrix of the vector consisting of the channel gain estimates and the received signal; and  $\mathbf{F}_{ij}$  is a Hermitian matrix defined as

$$\mathbf{F}_{i,j} = (\mathbf{u}_i \mathbf{u}_i^\dagger - \mathbf{u}_j \mathbf{u}_j^\dagger) \quad (3-11)$$

The vector  $\mathbf{u}_i$  consists of an element equal to one and the set of signal amplitude estimates related to the sequence  $\mathbf{b}_i$ ; the same relation exists for  $\mathbf{u}_j$  and  $\mathbf{b}_j$ . Since the dimension of the  $\mathbf{u}$  vectors<sup>6</sup>, is  $(M+1) \times 1$ , it is explained in [5] that  $\mathbf{F}_{ij}$  is a rank two matrix. And because  $\mathbf{R}$  (being a covariance matrix of received signals) is full rank, the product  $\mathbf{R}\mathbf{F}_{ij}$  is also rank two. This explains why this product only has two non-zero eigenvalues. Complete mathematical details and references can be found in the original work.

As mentioned in [5], in order for all of the mathematical derivations to hold, independence of the users' signals is presumed. Since the users in the system we are observing are located randomly and experience independent fading and individual reception gains, this requirement is met.

The union bound will be the sum of all the pairwise error probabilities which are erroneous for the symbol related to the desired user. The only variant parameter in (3-10) is the

---

<sup>5</sup> Refer to (2-13) for the average BER with  $L$  diversity paths.

<sup>6</sup>  $M$  user estimates and one element equal to one.

eigenvalue ratio. Therefore, we will take a closer look at the derivation of this term. Assuming the SNR for user  $m$  to be  $K_m \Gamma$ , since  $\Gamma = E_b/N_o$  is the SNR of the desired user,  $K_m$  represents the shadowing, average fading and path loss associated with user  $m$ . For the desired user, which is numbered 0,

$$K_0 = 1 \quad (3-12)$$

However, for user  $m$  from outside the cell,

$$K_m = 10^{(s_{m0} - s_m)/10} \left( \frac{d_{m0}}{d_m} \right)^{-\gamma} \quad (3-13)$$

Referring to (3-3), it should be noted that  $s_{m0} - s_m$  is now a Gaussian random variable with standard deviation equal to  $\sqrt{2} \times 8$  dB. Denoting the channel estimate correlation coefficient by  $\rho_m$ , from [13] we have the eigenvalue ratio as

$$r_{i,j} = \frac{\alpha_{i,j} \Gamma + \sqrt{\beta_{i,j} \Gamma^2 + 2\alpha_{i,j} \Gamma}}{\alpha_{i,j} \Gamma - \sqrt{\beta_{i,j} \Gamma^2 + 2\alpha_{i,j} \Gamma}} \quad (3-14)$$

where

$$\begin{aligned} \alpha_{i,j} &= \sum_{m=1}^M |\rho_m|^2 K_m (1 - \text{Re}(b_{jm})) \\ \beta_{i,j} &= 2 \sum_{n=1}^M \sum_{m=1}^M |\rho_m|^2 K_m K_n (1 - \text{Re}(b_{jm})) + \left( \sum_{m=1}^M |\rho_m|^2 K_m \text{Re}(b_{jm}) \right)^2 - \left( \sum_{m=1}^M |\rho_m|^2 K_m \right)^2 \end{aligned} \quad (3-15)$$

and as stated before  $b_{jm}$  is the  $m^{\text{th}}$  bit from the erroneous transmitted sequence  $\mathbf{b}_j$ .

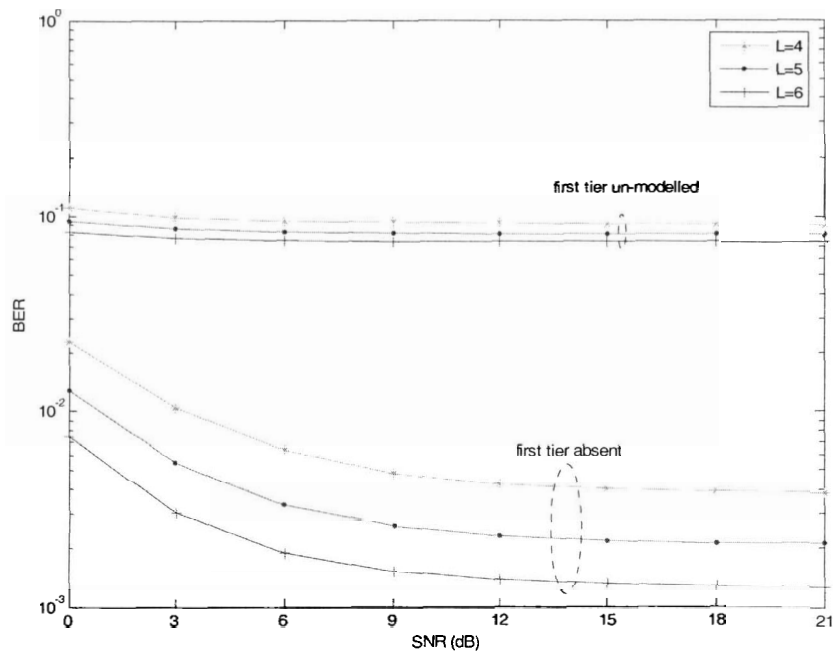


In the preceding section, the analytical derivation for the pairwise error probability was reviewed. In order to calculate the union bound, the eigenvalue ratio is numerically evaluated and the sum of the pairwise error probabilities, i.e. the union bound, is averaged over many simulations, hence the term semi-analytical bound. Results are presented in subsequent sections.

### 3.2.3 Upper and Lower Performance Limits on JML with First Tier Users

In this section, we intend to set the upper and lower limits on the performance of the central cell user when jointly detected with first tier users. The actual performance will be investigated later with the union bounds described in the previous section.

Figure 3-2 illustrates two sets of curves. The lower set represent a single user in the central cell with no interference from the users in the first tier. Obviously since second tier users



**Figure 3-2** Upper and lower limits on JML detection in conjunction with first tier users and perfect CSI

and beyond are generally received at low signal levels, eliminating first tier users removes a significant amount of the interference in the system. The improvement is demonstrated when comparing the abovementioned case with the upper set of curves in Figure 3-2, which demonstrate the case where first tier users are also present in the system. These curves provide us with limits on the performance of the desired user when jointly detected with first tier users.

In the above cases, the base station confronts all outer cell users as un-modelled interference and performs maximum likelihood detection, which in the case of one user is equivalent to maximal ratio combining (MRC), on the signal from the single user located within the central cell.

### 3.2.4 Joint Detection with First Tier Users and Perfect Channel State Information

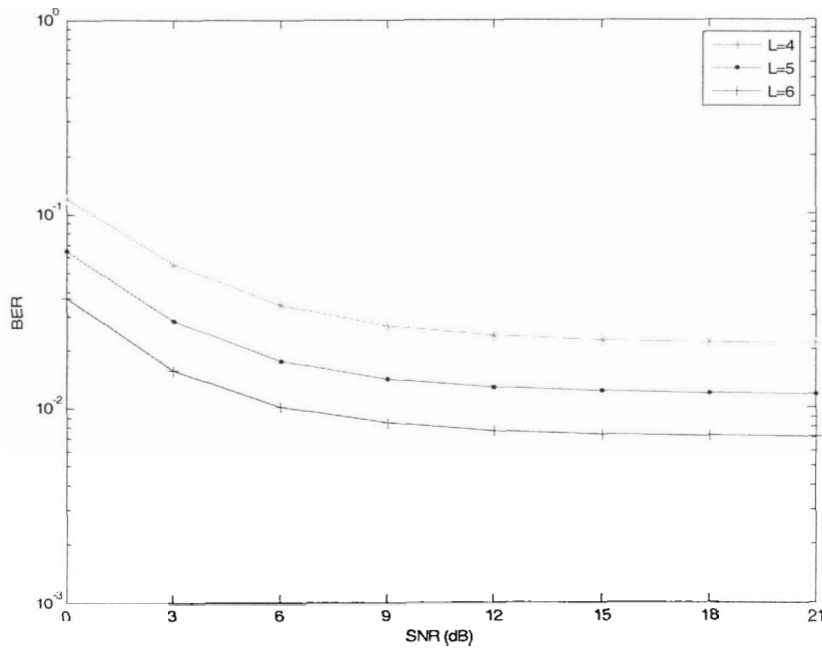
Having set an upper and lower limit on the performance of the desired user, we can now investigate the proposed detection scheme. In this section, we will be looking at the union bounds on JML detection of the central cell user and all the users available in the first tier.

Assuming perfect CSI is equivalent to  $\rho_m = 1$  and in this case according to the definitions in (3-15) we have  $\beta_i = \alpha_i^2$ . Figure 3-3 illustrates the semi-analytical union bound on the BER for the joint detection of the central cell user with the assumption of perfect CSI for all the users. Comparing this figure with the lower bound exhibited in Figure 3-2, it is evident that by performing JML detection with the users present in the first tier much of the degradation due to the presence of these users is eliminated. It should be noted that the presented curves are union bounds and are increasingly loose as we add weak users. The actual performance of the system will be better than that presented in this graph.

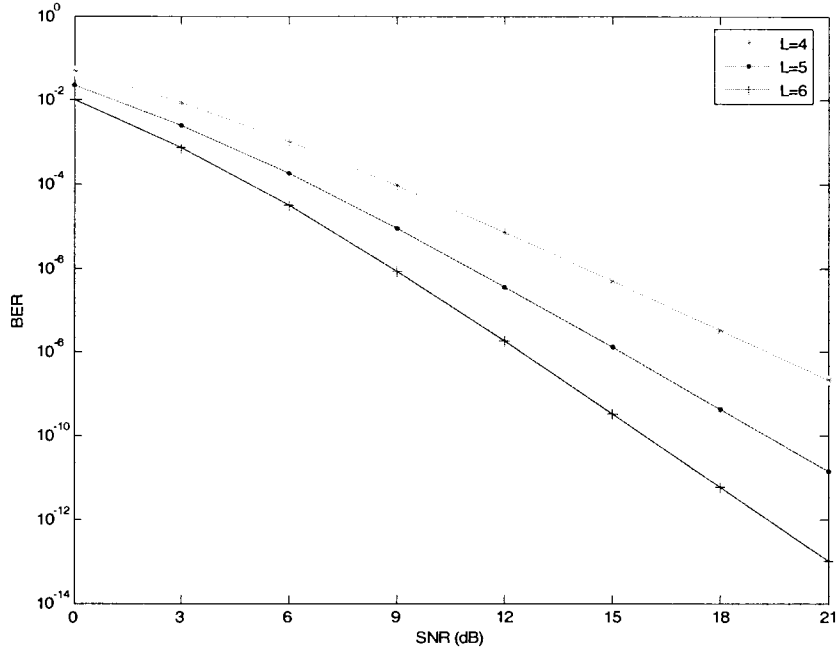
Since the BER curves presented in Figure 3-2 are obtained by performing MRC on the signal received from the central cell user, they are exact results not bounds. However, the curves in Figure 3-3 are bounds that are significantly loose in low SNR regions.

In addition, since including the users in the joint detection is similar to removing them as interferers, observing the increasing gap between the two sets of curves in Figure 3-2 we may conclude that with bounds the benefits of JML detection become more evident at higher SNR values where the additive disturbances are primarily interference rather than noise.

The effect of interferers from the second tier and beyond can be seen when the below curves are compared to those of Figure 3-4. The investigated scenario, where all interference from the users in the second tier and beyond is eliminated, depicts a seven-cell system with joint detection performed on the active users. Although this is an unrealistic assumption, as before, it serves as a guideline for further improvements in the joint detection scheme.



**Figure 3-3 Union bound on JML detection with first tier users and perfect CSI**



**Figure 3-4 Union bound on JML detection with first tier users and no interference from outer tiers**

From the comparison of the two previous graphs, it becomes evident that the error floor exhibited in Figure 3-3 is due to the presence of users in the second tier and beyond. This observation also brings us to the conclusion that the system is more noise limited in the lower SNR region and as the SNR increases, the effect of interferers is more noticeable.

### 3.2.4.1 Joint Detection with First Tier Users and Imperfect Channel State Information

Imperfect CSI is the result of  $\rho_m$  being smaller than one. Real world observations show the fact that channel estimates improve as the SNR increases in a system, so assuming a constant channel estimate quality would be understating the performance. In the current work, we have assumed that CSI is obtained by joint pilot based channel estimation and related to the SNR at which the user is operating. In [16] it is concluded that the normalized channel estimation error variance for user  $m$  is  $1 - |\rho_m|^2$  and is inversely proportional to user SNR i.e.

$$|\rho_m|^2 = 1 - (\kappa_m / K_m) \Gamma^{-1} \quad (3-16)$$

where  $\kappa_m$  is the proportionality constant that is derived according to the number of users in the joint channel estimate and the channel estimation scheme. In [4] it is suggested that typical values for  $\kappa_m$  are in the order of  $10^{-1}$ .

Figure 3-5 shows the BER of the user in the central cell with varying channel estimate quality. In this simulation, the number of microdiversity antennas is set to six.  $\rho_m$ , the correlation coefficient, is dependent on the relative SNR for each user as denoted in (3-16). Since the users are located far from the receiver and at times may be received at relatively low SNR values, the expression on the right hand side of the equation may become negative. This is not meaningful, therefore in these cases  $\rho_m = 0$  is substituted which implies that there is no correlation between the channel gain and the estimate for user  $m$ .

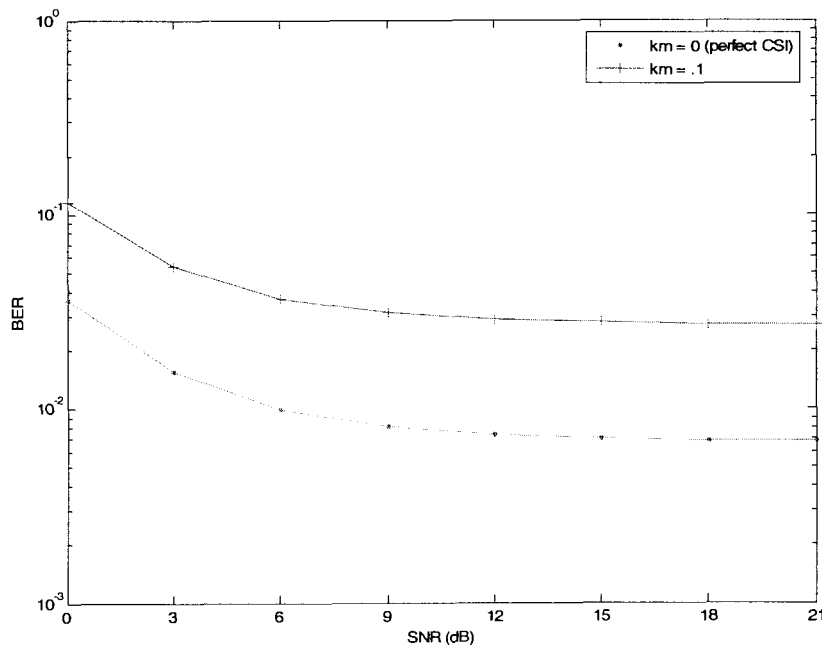


Figure 3-5 Union bound on JML detection with first tier users and imperfect CSI ( $L = 6$  antennas)

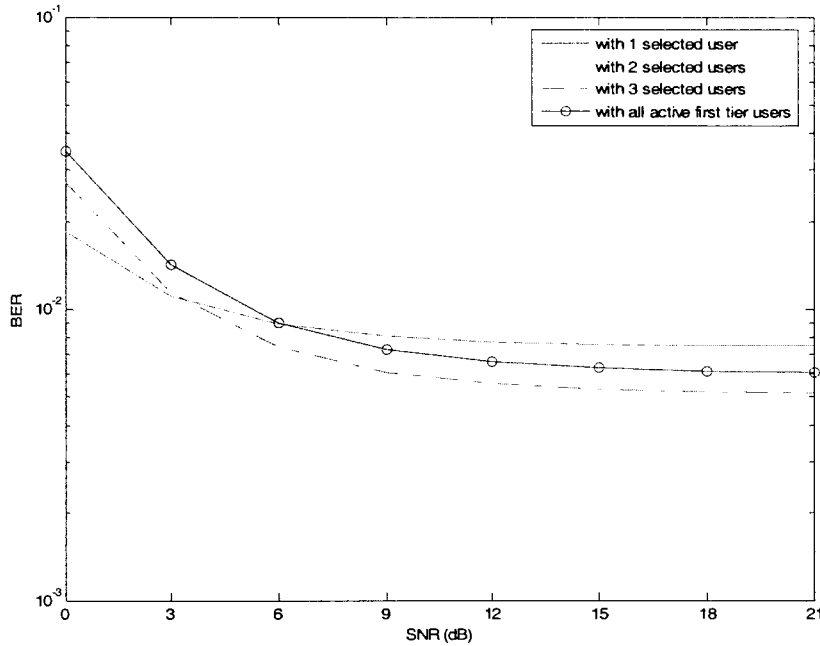
As is evident from the figure above, the change in the accuracy of CSI causes less than an order of magnitude degradation in the BER. It is important to restate that the presented error rates are upper bounds and the actual performance is superior to that presented by these curves. However, the percentage increase in BER due to imperfect CSI on performance should be similar in the true BER and in the bounds of Figure 3-5, particularly at high SNR, since all the pairwise error probabilities are affected similarly.

### 3.2.5 Selecting the Jointly Detected Users from the First Tier

Up to this point, the users that were included in the JML detection were fixed. In other words, the joint detection was always performed on the central cell user and any of the active users in the first tier of cells. Obviously by blindly picking a group, especially knowing that the user's SNR is the key factor that determines how helpful it is in the detection process, we are not exploiting all the available resources. In addition, it should be noted that due to the voice activity factor not all of the six neighbouring users are active all the time. With the voice activity set to  $\frac{3}{8}$  as in our case, there will be approximately 2 to 3 users present at each time interval.

Therefore, at this stage we are proposing to limit the actual number of users that participate in the joint detection technique. It is shown that performing joint detection, only by selecting the right ones, we can reduce the computation with little effect on BER.

There is still one question left to answer, *how do we pick the users we are including in the detection set?* From Figure 2-7 it is evident that a user will benefit from joint detection with a stronger user. Therefore, signal strength would be an appropriate criterion by which users can be chosen. Figure 3-6 shows the outcome of performing detection on a set of varying number of users selected from the active users in the first tier compared to the previously studied case where all the active users in the first tier were included. In each case,  $n$  of the strongest users are chosen



**Figure 3-6 Union bound on JML detection with selected users from the first tier and perfect CSI ( $L = 6$  antennas)**

and joint detection is performed on these users and the desired user in the central cell; thus the detection set consists of  $n + 1$  users. For example, for the curve with one selected user the strongest user in the first tier is jointly detected with the user in the central cell. The case for  $n = 1, 2, 3$  is studied; larger user groups are not investigated since due to the voice activity factor there is an average of 2 to 3 users active in the first tier. It should be noted that the selection of users is based on the average signal strength over fading.

Because a user in the first tier is usually weaker than the desired user and since the benefits of JML are more evident at higher SNR values, performing joint detection with only one or two users in the interference dominant region does not enhance the performance bound. Joint detection with three users on the other hand provides us with a significant improvement compared to one or two users. The actual improvements are greater than these values, because the union bound becomes increasingly loose as we add users to the detection set. Looser bounds due to

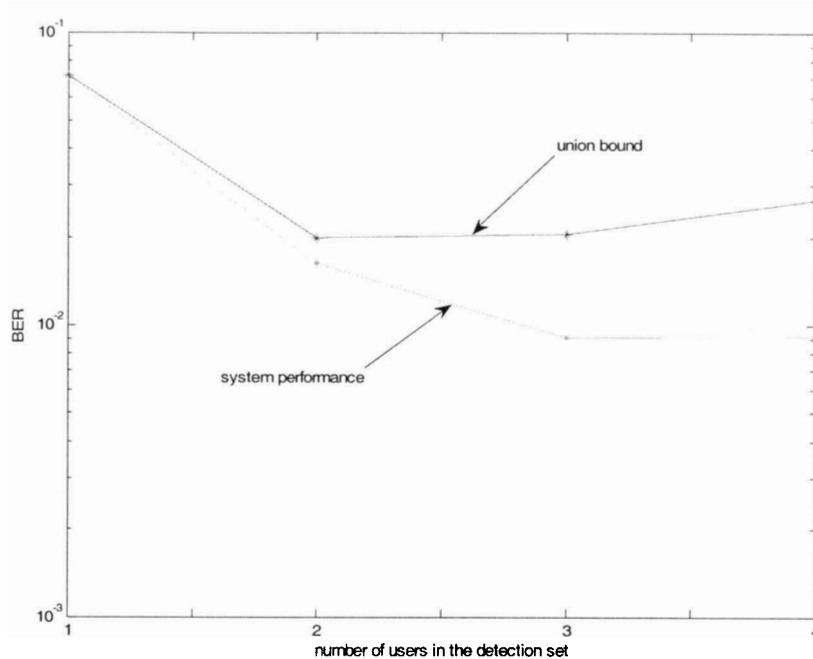
additional weak users may also be the explanation to the similarity in including one and two users in the detection.

Although we can argue that two to three users is about the average number of users active in the first tier and therefore the performance in these cases should be very close to that of JML with all the users, it is important to note that the presented curves are union bounds. Since including additional weak users in the bounds loosens the proximity to actual BER performance, the bound is looser with all the active users than it is with two or three users. This fact also brings us to the conclusion that the improvement gained by the selection of three users compared to the case where all active users are engaged is at least partially due to the change in tightness of bounds. Nevertheless, performance of the real system is always better than the presented bounds, so irrespective of the tightness of bounds we may conclude that the selection of three users provides us with reasonable performance.

To understand the true effect of the number of users on the BER performance and determine the precision of the bounds, a simulation of the actual system is compared to the results obtained from the bounds. To keep the computation required for the actual system performance manageable, the scenario with six antennas at an SNR of 0 dB is studied.

It is important to note the number of users selected from the system is always one less than the number of users in the detection set since the desired users is also included in the detection set. The actual system performance demonstrates the fact that at 0 dB selection of two users from the system will provide us with approximately the same results as the selection of three users. In addition, it is evident that the gap between the actual system performance and the



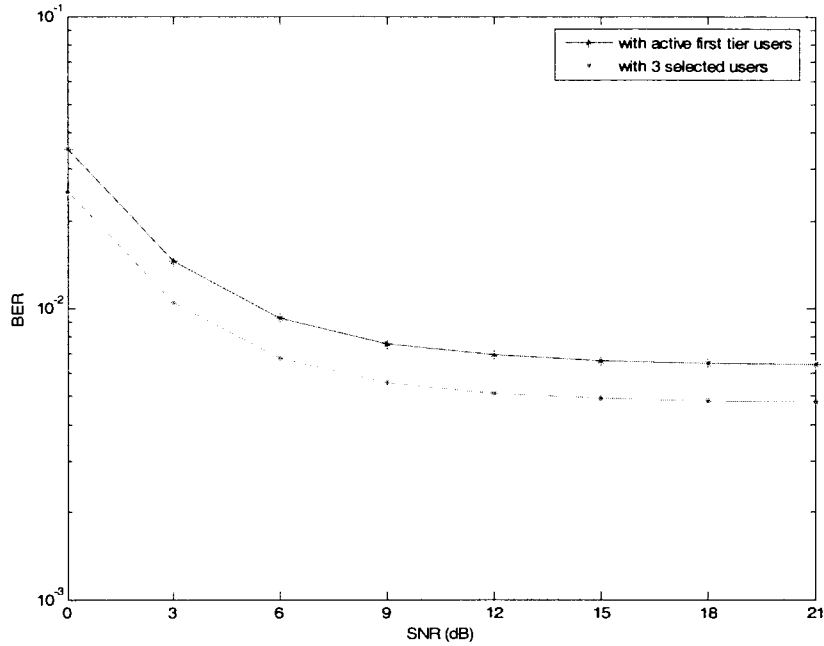


**Figure 3-7 Proposed JML with a selection of first tier users vs. the union bound with six antennas and SNR = 0 dB ( $10^5$  simulations for the system)**

bound increases with the increase in the number of users. These conclusions also strengthen the results we had previously obtained while applying six antennas with a varying number of users (Figure 3-6).

One might argue that since the users included in the detection set change, acquiring perfect CSI for all of them would be an unrealistic assumption. Henceforth, the same scenario is simulated in Figure 3-8 with the assumption of imperfect CSI with  $\kappa_m = .1$ . The comparison is made against the case of imperfect CSI with no selection of users.

With three selected users participating in the joint detection despite the fact that the CSI is imperfect, a reasonable improvement compared to using the fixed group of users is visible.

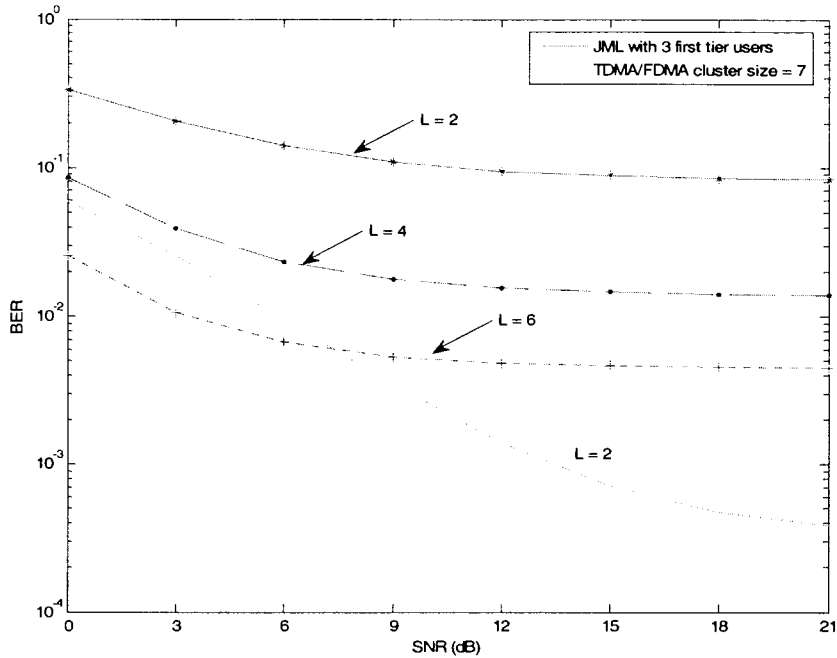


**Figure 3-8 Union bound on JML detection with first tier users with/without selection and imperfect CSI ( $L = 6$  antennas,  $k_m = .1$ )**

Since the case of one or two users selected from the first tier did not illustrate improvement compared to the case with all the active users participating in the detection, we did not investigate the relative performance of these cases with imperfect CSI.

### 3.2.6 Comparisons

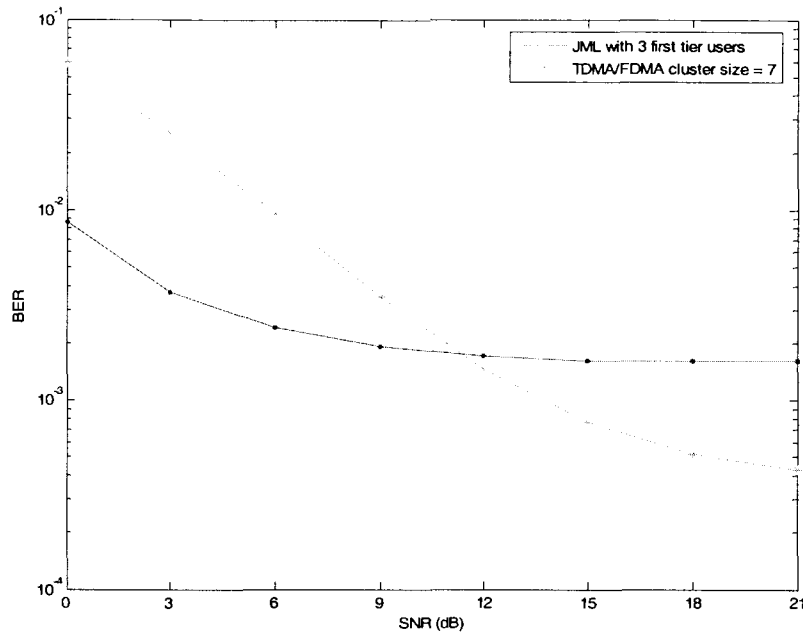
Union bounds on BER curves related to the performance of the proposed scheme were introduced in this chapter. However, we still need to determine how well the single cell cluster introduced in this work performs in comparison with the conventional seven-cell cluster. Figure 3-9 illustrates a comparison between these two systems, with the selection of three users for the JML scheme.



**Figure 3-9 Union bound on BER for JML detection with three first tier users vs. actual BER of conventional seven – cell cluster**

As the figure shows with additional antennas the performance of the proposed joint detection scheme approaches the conventional system. It is important to note that the union bounds on the detection of JML are compared to the actual system performance of the conventional system. In addition, it is important to emphasise that with the JML, the cluster size is reduced to  $\frac{1}{7}$  of the conventional system and the capacity is increased seven-fold.

The fact that in order to match the performance bounds to the actual performance of the conventional TDMA/FDMA system the JML scheme requires a few additional antennas, encourages the idea that with small improvements in JML it may be possible to meet the performance standards of the seven-cell cluster. The observation that the BER bounds on the JML scheme are close to those of the conventional system inspires the thought that the *actual performance* of the system might match that of the conventional seven-cell cluster.



**Figure 3-10 JML with three selected first tier users and six microdiversity antennas vs. the conventional seven-cell cluster with two antennas ( $10^6$  simulations for the JML system)**

The figure above illustrates simulation results for the selection of three users from the first tier compared to the conventional seven-cell cluster. Achieving *seven times* the capacity at the cost of a few additional antennas, the JML scheme outperforms the conventional system up to about 11 dB. Therefore, applying fewer antennas at lower SNR values will also provide us with performance as good as the conventional system.

For SNR values above 11 dB, where the interference from other tiers is higher, choosing three users from the first tier is not sufficient to match the performance of the conventional system. Solutions for this range of SNR values will be introduced in the next chapter.

From the above we may conclude that in the low SNR region, applying JML can provide us with additional capacity while maintaining the performance levels of the conventional system. As the SNR grows to a certain point, we will be able to match the performance of the

conventional system at the cost of additional microdiversity antennas. In order to meet the performance of the TDMA/FDMA system in the interference dominant region, i.e. higher SNR values, other means should be applied, since increasing the number of microdiversity antennas will provide us with limited improvements in performance.

In addition, from the comparison of the figure above with Figure 3-9, the looseness of the union bound is evident. However, we may conclude that the bounds are reliable guidelines for further investigation.

*“Engineers participate in the activities which make the resources of nature available in a form beneficial to man and provide systems which will perform **optimally** and economically.”*

L. M. K. Boelter (1957)

## **Chapter Four :**

# **Optimizations**

In the preceding chapter, we introduced a new approach to detection in the narrowband multitier system. The outage was calculated and comparisons with the conventional seven-cell cluster system were presented. In this chapter, our elaborations are related to optimizing the performance of the system and the amount of calculation it requires.

### **4.1 Interference Mitigation**

Since capacity is interference limited, it would be in our best interest to reduce the interference as far as possible. Resource allocation, especially if it is applied dynamically in the system, is an effective means in interference alleviation. TDMA and FDMA, introduced earlier,

are two of the widely used resource allocation schemes in conventional narrowband systems. In addition, power control which was also presented in Chapter Three, is a means by which we may reduce the perplexity unwanted users cause in the detection process. The above are some of the transmitting techniques practiced in order to reduce interference. In the subsequent sections, interference mitigation at the receiver will be studied.

### 4.1.1 Directionality

On the receiver side, antennas can be designed more effectively to pick up less interference. Use of antennas with a smaller beamwidth is one of the available options. In the literature, the application of directional antennas is referred to as *sectorization*. Usually when applying directional antennas it is assumed that the interference is reduced relative to the coverage area of the antenna. For example in Figure 4-1 where  $120^\circ$  antennas are employed, the interference is assumed to drop to one third compared to the omni-directional antenna since

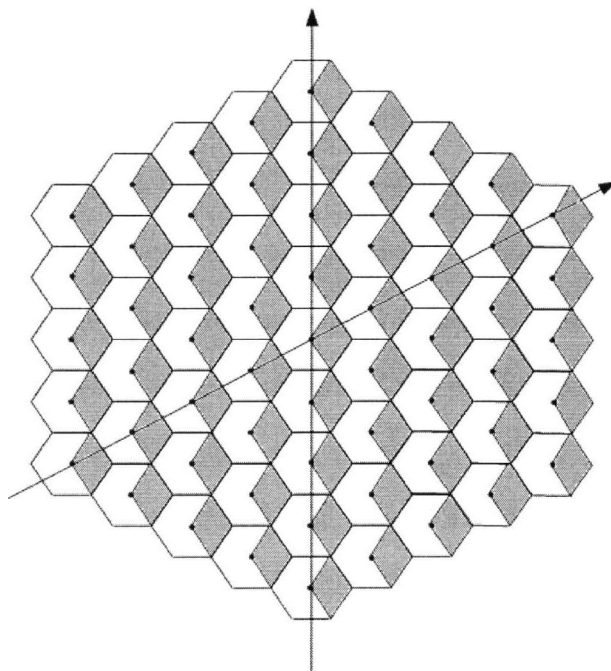


Figure 4-1  $120^\circ$  directional antennas

only one third of the cochannel area is within the beamwidth of the directional antenna.

In other words, the microdiversity antennas that receive signals from the users in the colored section of the central cell only receive interference from users present in the colored section of each cell. Hence, roughly only one third of the users in the system will interfere with the desired user's signal.

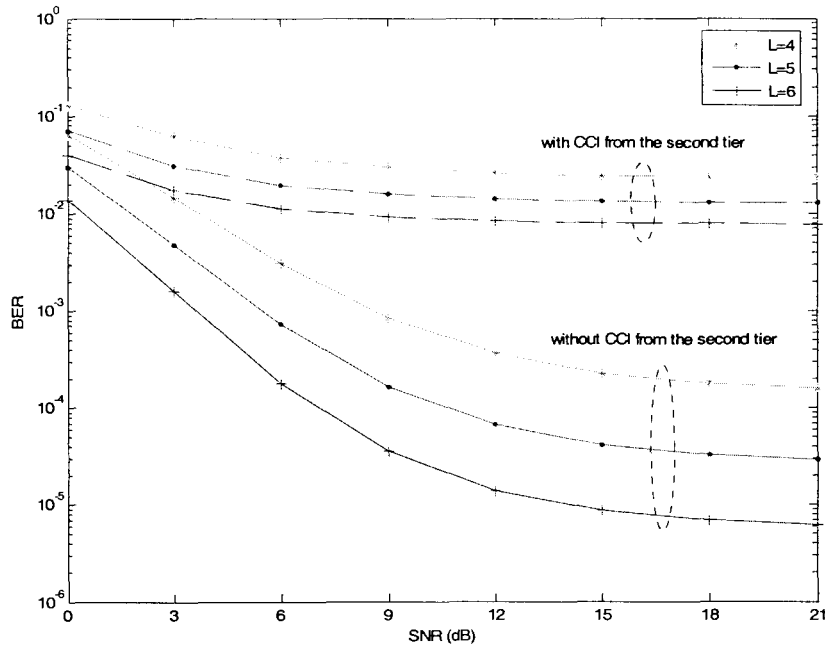
#### **4.1.2 Is it Worth Looking at JML detection with the Users from the Second Tier?**

To determine whether we should investigate including second tier users in the joint detection, in this section we will make the assumption that modelling the interference from these users would be equivalent to removing them from the system. Although including these users in the joint detection set does not enhance the performance as much as if they were eliminated, studying this case will provide us with a lower bound for performance. It should be noted that by eliminating the users in the second tier we have 12 users less in the overall system; applying the voice activity factor, we are troubled by 4 or 5 fewer un-modelled interferers in each time slot.

To determine the effect of removing the CCI from second tier users we should compare this scenario with the results obtained previously for joint detection of the desired user with the users in the first tier and CCI from users in the second tier and beyond.

Figure 4-2 illustrates both of these cases. As can be concluded from the graphs, the effect of second tier users in the high SNR region is quite significant hence studying techniques for modelling these users is valuable. As previously stated, this graph also provides us with a limit on the improvement due to including second tier users in the joint detection. The *relative* performances with and without the 2nd tier in Figure 4-2 should be accurate, despite the looseness of the bound, since the number of users in the detection set is the same for all curves.





**Figure 4-2 Union bound on JML detection with first tier users with/without CCI from the second tier**

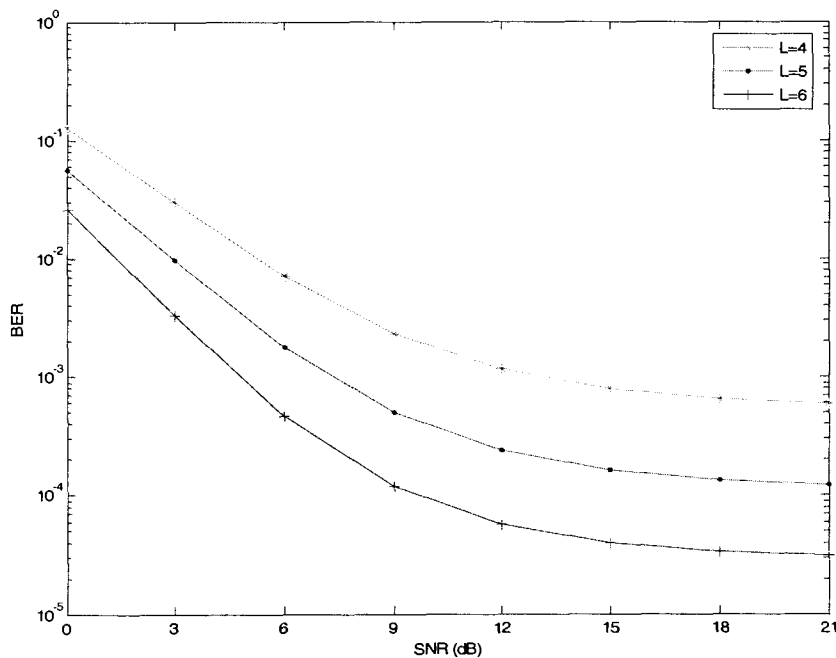
Comparing this graph to that of Figure 3-4, as expected, we can see that the presence of second tier users is far more evident in the performance of the desired user than that of the third and fourth tier users. This strengthens the fact that the users in the third tier and beyond are received at the base station with signal powers near thermal noise, therefore they can barely ever aid the detection of the desired user.

### 4.1.3 How Can We Include Second Tier Users in the Joint Detection?

As seen in Chapter Three, shadowing for users from outside the cell boundaries is described as a random variable with standard deviation  $\sqrt{2} \times 8$  dB; therefore second tier users on the side of their cell nearest the central cell can occasionally be received at the central base station with high signal strengths. Acknowledging this and also noting that due to the voice activity factor at each time instant there are only a few users active in the first tier, it may be reasonable to

consider second tier users as candidates for joint detection. Therefore, in this section we propose that instead of repeatedly engaging the first tier cells in the JML detection, a set of the strongest users in the first two tiers be selected and used for the detection process.

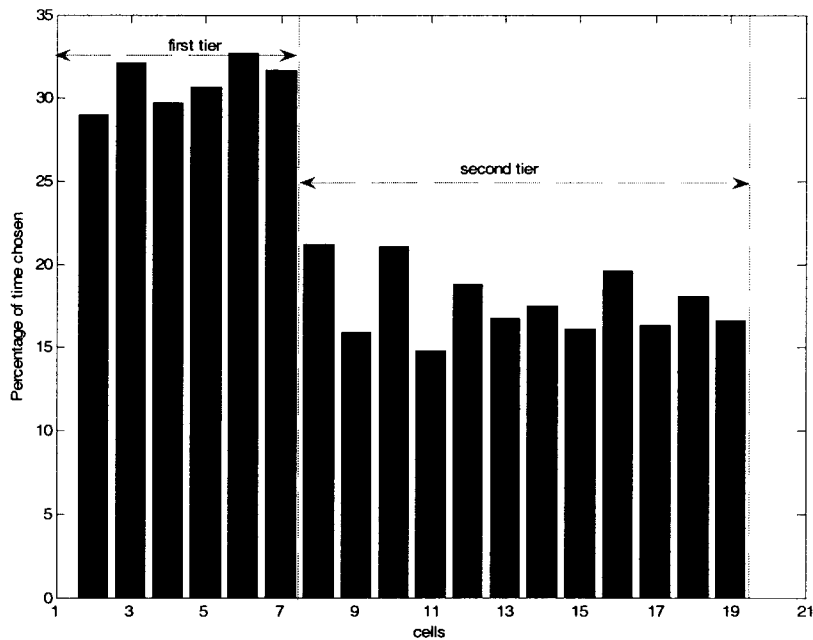
Previously we discussed the fact that joint detection with a selection of users enhances the performance of the system and hence we seek the advantages of including the strong users from the second tier in the detection process. Figure 4-3 illustrates the BER of the central user when joint detection is performed on a wider selection of users – i.e. 4 strongest users from the first or second tier, which results in a total of 5 users in the detection set. The reason for selecting 4 users to participate in the detection is that as concluded from Figure 3-6 there are usually 3 users active in the first tier; thus if we were to select 2 or 3 users there would be very little chance for second tier users to participate in the detection. It should be recalled that possible variations in



**Figure 4-3 Union bound on JML detection with 4 strongest users from the first two tiers – perfect CSI**

the number of users in the detection set were investigated in Chapter Three where the selection was performed only on users from the first tier. Due to the small number of active users in the first tier, simultaneously joint detection with a selection of 4 users was often not possible.

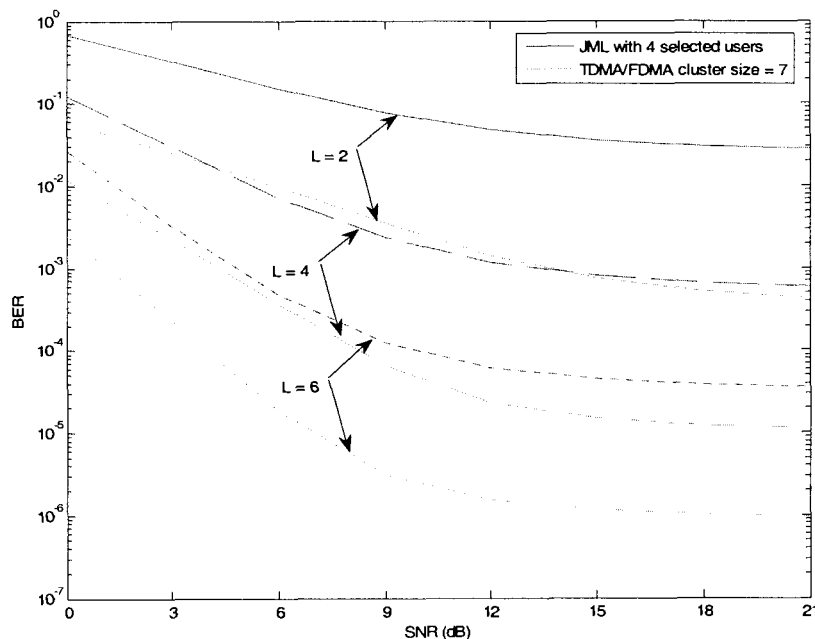
Comparing this figure with Figure 4-2, shows that even selecting the occasionally strong users from the second tier, the performance outcome is very similar to that of totally eliminating these users. Therefore, we may conclude that by increasing the computational power by a small amount we can eliminate the effect of the 12 additional cochannel users in the second tier. In order to provide us with enough information to decide whether we need to investigate users in the second tier or not, a bar chart of the percentage of time users from the first and second tier are picked to participate in the joint detection is presented in Figure 4-4. This chart includes the speech activity factor so no user is selected more than 37.5 % of the time.



**Figure 4-4 Selection of 4 strongest users from the first two tiers**

As can be seen in the figure above on average second tier users are included in the detection 18 % of the time compared to the 31 % related to the first tier users. Acknowledging the fact that each user is active about 37 % of the time, it can roughly be estimated that a second tier user is included in the joint detection half the time it is active. Depending on the available computational power and of course the proximity of the system CSI to perfect channel state information, we may decide whether we should take second tier users into account or not.

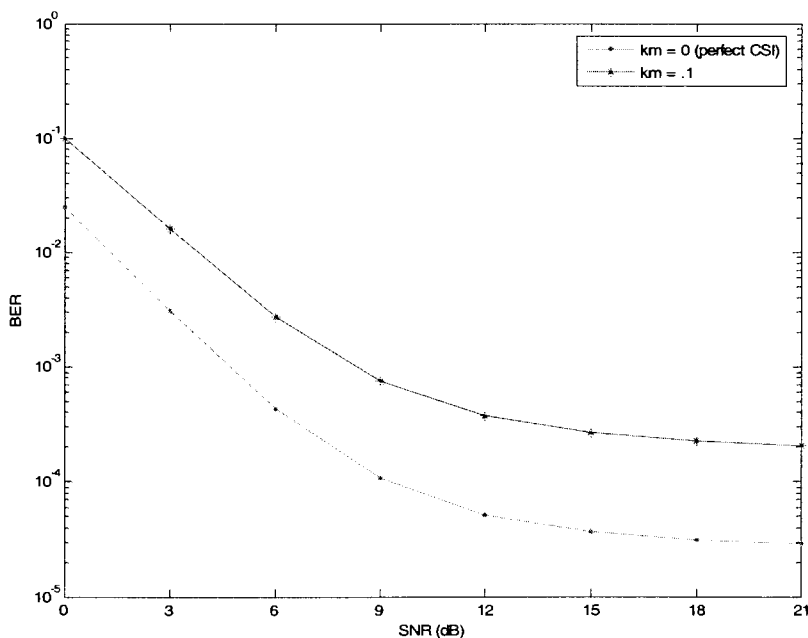
In order to obtain how well this detection technique is performing, we will compare it against that of the conventional seven-cell cluster. Figure 4-5 illustrates this comparison. In comparing the curves, it is important to remember that the JML curves are loose upper bounds, whereas the conventional system curves are exact. Even so, the performance bound of the selection technique with four antennas is not far from the actual performance of the TDMA/FDMA system with two antennas. Also, the union bound on JML with six antennas is



**Figure 4-5** Union bound on selection technique (detection set = 5 strongest) vs. actual BER of conventional TDMA/FDMA

very close to the performance of the conventional system with four antennas. We may conclude that by applying the joint detection to a selection of the strongest users in the system, the upper bound estimate for the performance of this system with two additional antennas approximates the performance of the seven-cell cluster and a seven-fold increase in capacity is achieved. Clearly, comparing union bounds on the performance of JML with exact BER performances of the conventional system is an underestimation of the performance of JML. For a fair comparison, exact BER results for the JML technique are represented further below.

Figure 4-6 demonstrates the abovementioned selection method in comparison with the imperfect CSI scenario. As seen before, imperfect channel estimates only cause a slight shift in the BER curve. Hence, we would predict the outcome for the choice in users to be very similar to that of Figure 4-4. Note that as the SNR grows the gap between the two curves increases. This is mostly due to the fact that second tier users are usually received at lower signal strengths at the central cell compared to first tier users, hence, have poorer channel estimates.



**Figure 4-6** Union bound on JML detection with 4 strongest users from the first two tiers and imperfect CSI ( $L = 6$  antennas)

As we may recall, comparison of Figure 3-2 and Figure 3-3, the beneficial effect of joint detection is more evident at higher SNR values, where the inherent mutual interference cancellation in JML is more free of errors. Therefore, performing JML with imperfect CSI has more effect on the BER at higher signal powers.

#### 4.1.4 What Can Be Done in the Case of Additional Users?

Our ultimate goal in the current work is to push the capacity of the narrowband system to its limits. To study the effect of additional users in the system, we will investigate the system performance in a highly loaded scenario. Figure 4-7 illustrates the BER of the central cell user when there is an additional user present in each cell. In this graph, two sets of curves represent performance when the strongest 4 and 6 users are selected to participate in the JML detection i.e. detection sets of 5 and 7 users respectively. These users are chosen from the additional user in the central cell – when active – and the users in tiers one and two.

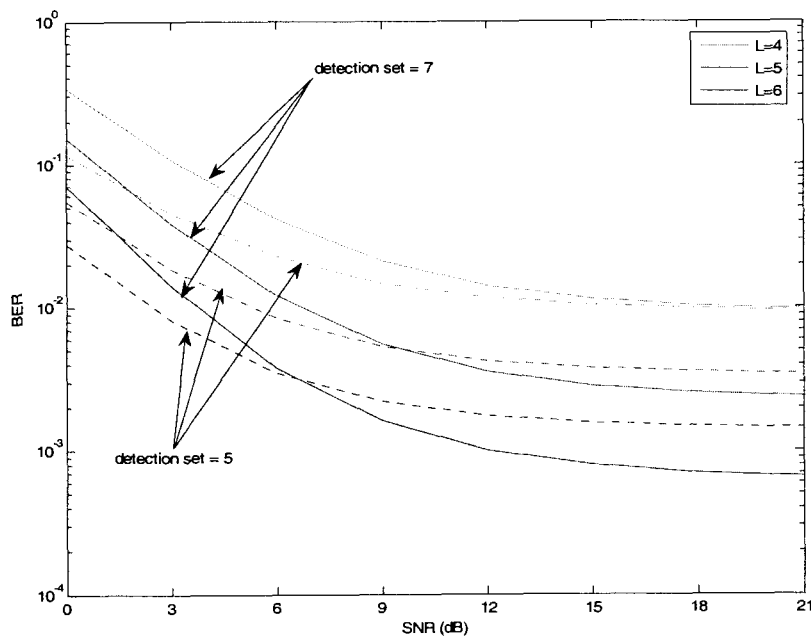
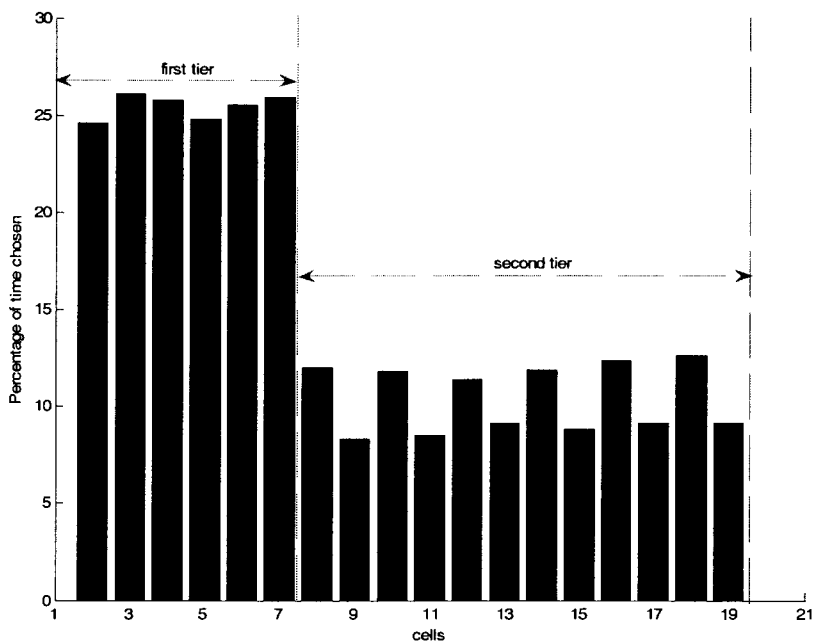


Figure 4-7 Union bound on JML detection with 4 and 6 strongest users and two users in each cell

It is shown in this graph that the increase in the size of the detection set does not necessarily enhance the performance of the system. Of course, it should be reminded that these are upper bounds and in the case of six selected users are looser than for four users. Irrespective of the tightness of the bound, the actual performance of the system is better than that described by this graph. The effect of the size of the detection set will be studied further in this chapter.

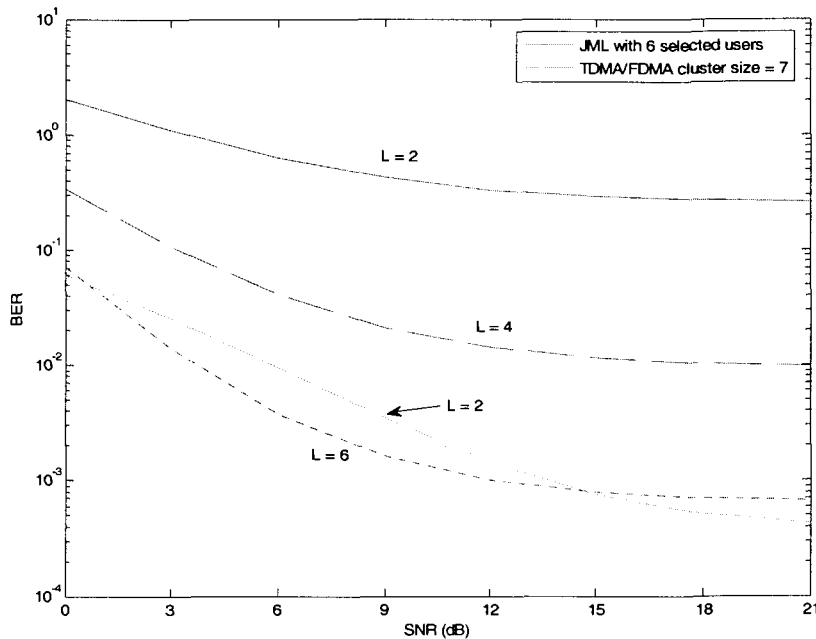
Figure 4-8 is a bar chart of the number of times any of the users in a cell is included in the joint detection. As it is obvious from the graph compared to Figure 4-4 with two users in each cell and a detection set of only seven users, it is less probable that a user from the second tier is included in the detection.



**Figure 4-8** Selection of one of the users in the cell (detection set = 7 strongest) with two users in each cell

Again, the performance of this system is compared to that of TDMA/FDMA. It should be noted that we are estimating the performance of the joint detection scheme with an upper bound and the proposed systems conveys a fourteen-fold increase in capacity compared to FDMA/TDMA.

It is very interesting to see that the bounds on the performance of JML in this case, with six antennas, exceed the performance of the conventional seven-cell cluster. With four antennas, the performance is relatively close and parallel to the conventional system. The addition of extra antennas to the system enhances the performance of the conventional system more than the bounds on the proposed scheme; hence, with more antennas in the system the seven-cell cluster illustrates lower BERs. It is still to be determined whether the actual performance of the proposed system it truly worse than the conventional system when the number of antennas are increased.



**Figure 4-9 Union bound on selection of 6 strongest users with 2 users per cell vs. TDMA/FDMA with one user per cluster**



## 4.2 Computation Reduction

An important factor in optimization is minimizing the computational burden for detection. In this final section, we investigate the possible options to reduce the required computation.

### 4.2.1 How Many Users Should Be Included in the Joint Detection?

If it were possible to attain the best performance while including the minimum number of users in the joint detection, we would be approaching our goal of reducing required calculations while maintaining the capacity of the system. Figure 4-10 shows the BER of the desired user when the number of diversity antennas is set to six and the number of users included in the detection set is changed.

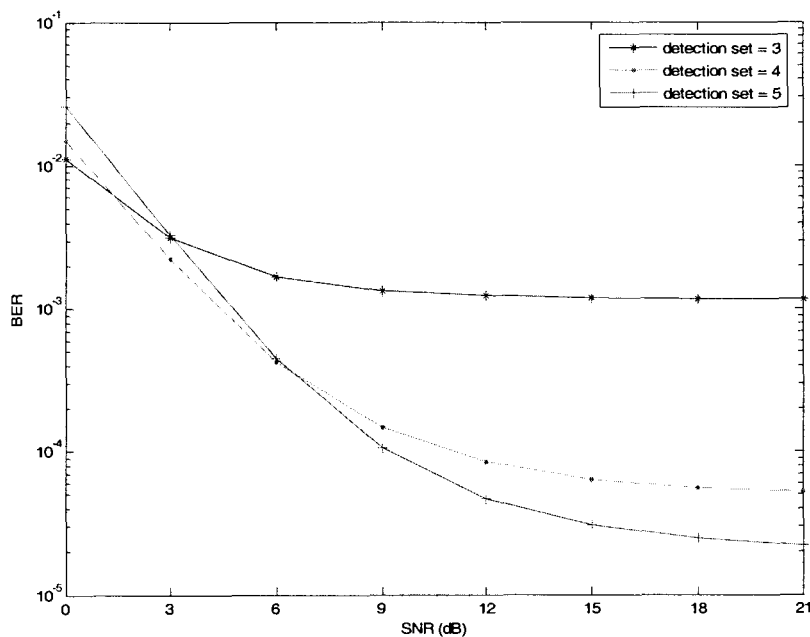


Figure 4-10 How the number of selected users affects the union bound BER ( $L = 6$  antennas)

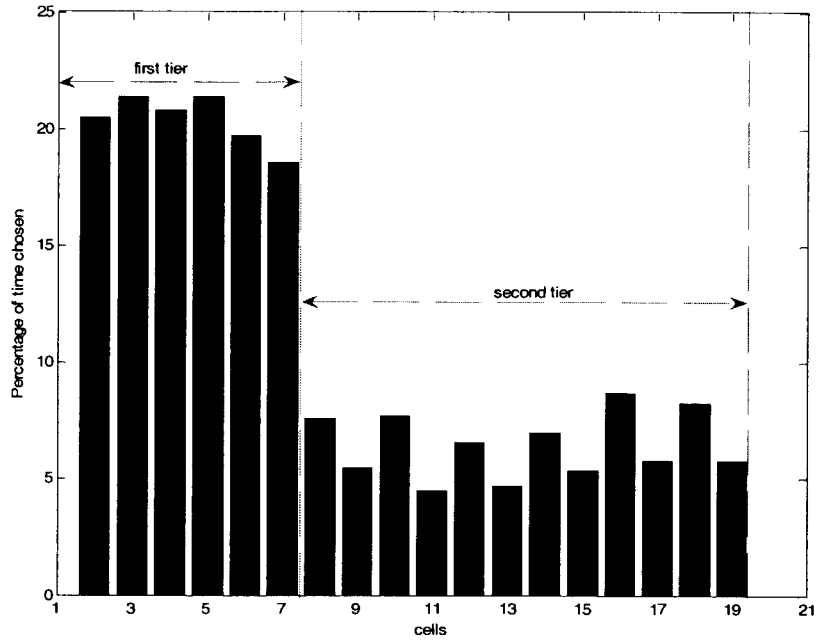
Regarding the fact that the more weak users that are included in the detection set the bounds become looser, it is evident that the curves for 5 selected users are the loosest representation for the actual performance curve. Nevertheless, the curves related to the selection of 3 users are also upper bounds on the actual performance with this number of users; therefore, in the regions where acceptable performance is exhibited it is guaranteed that the actual system will demonstrate a satisfying outcome. Actual (not bound) values are shown later in this chapter.

As stated earlier, as the SNR increases bounds become tighter therefore this may be an explanation to the crossover exhibited in this graph. Whilst the bound for the selection of 5 users is very loose at low SNR values, with the increase in SNR it approximates the actual performance with more accuracy; hence, at higher SNR values the advantages of including more users in the detection are better pronounced.

With the increase in SNR, there is also a change to the dominant cause of performance degradation. In the sloping region on the left side of the graph, users are noise limited; hence, including more users in the detection set amplifies the effect of noise. However, in the noise floor region, the performance limitation is mostly due to the effect of third tier users and beyond, so users in the first tiers have better SNR. Therefore, the change in signal strengths may also be a contributing factor to the crossover exhibited in Figure 4-10.

By adapting the number of users in the detection procedure relative to the SNR of the users, aside from performing the least computation required at each power level, we will be able to experience the best performance possible within each SNR range.

Including additional users in the detection set necessitates extra calculations; in order to keep the required computation manageable, techniques such as sphere decoding [21] can be applied. With the application of sphere decoding, the complexity of the detection will be

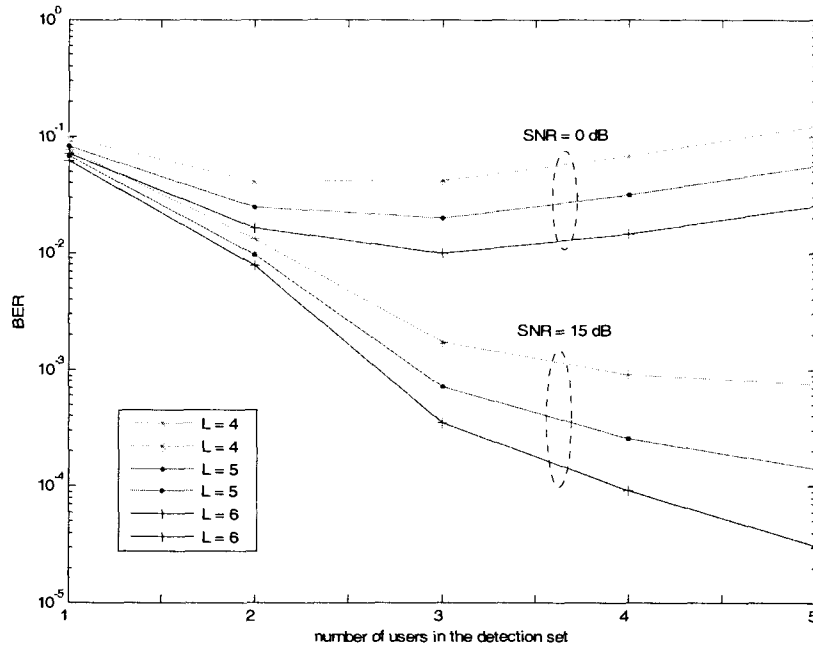


**Figure 4-11 Selection of 3 strongest users from the first two tiers ( $L = 6$  antennas)**

exponential with the excess number of users compared to antennas, i.e. the number of users in the detection set minus the number of antennas, which is a satisfying reduction in computational load.

The above figure shows the percentage of time each user is chosen for the detection set when there are 3 users chosen from the whole system – the detection set consists of 4 users. In this scenario, the number of microdiversity antennas is set to six. In Figure 4-4 the number of users in the detection set is set to five and six antennas are employed. The comparison of these figures reflects that, as expected, it is more likely that the users in the first tier be chosen prior the ones in the second tier.

Inspired by the effect of the size of the detection set on the performance of the system, setting an SNR value we determine to find the best number of users to include in the detection. Figure 4-12 illustrates the BER of various user groups for two SNR values as the number of antennas is increased from 4 to 6. As previously seen in Figure 4-10, consulting bounds, in lower



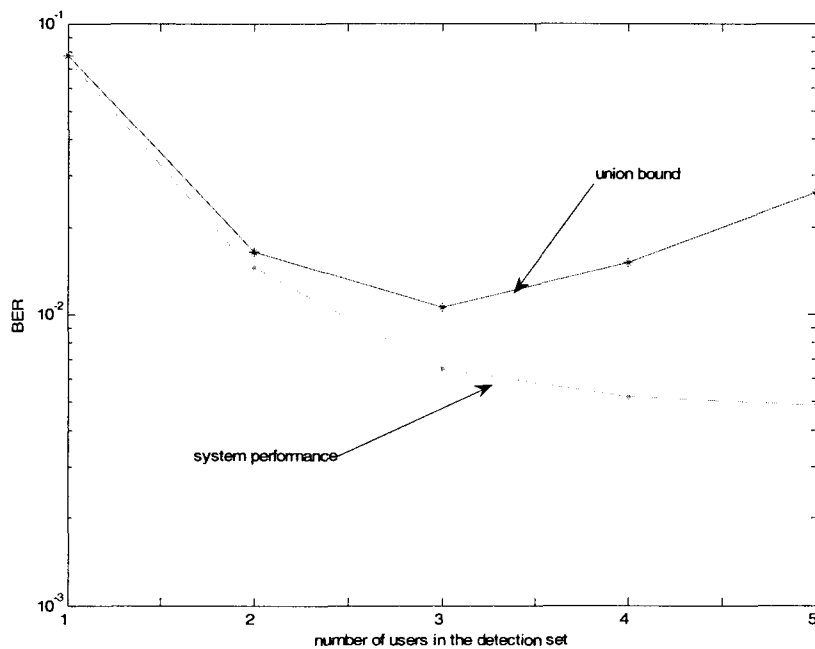
**Figure 4-12** Effect of the number of users in the detection set on the union bound for JML

SNR values (0 dB) smaller detection sets are preferable; in this case only choosing one user from the first or second tier exhibits an acceptable performance. With higher SNR values (15 dB) the improvement due to increasing the number of users in the detection set is illustrated. Aside from tighter bounds at higher signal strength levels, this is due to the fact that the participating users are stronger – lower noise levels – so they will be more effective in the detection and this region of the graph is more interference dominated so there is more to be achieved by joint detection. It should be noted that the bounds are looser in the case of 5 selected users than they are for any other case and as the number of users decreases the bound becomes tighter.

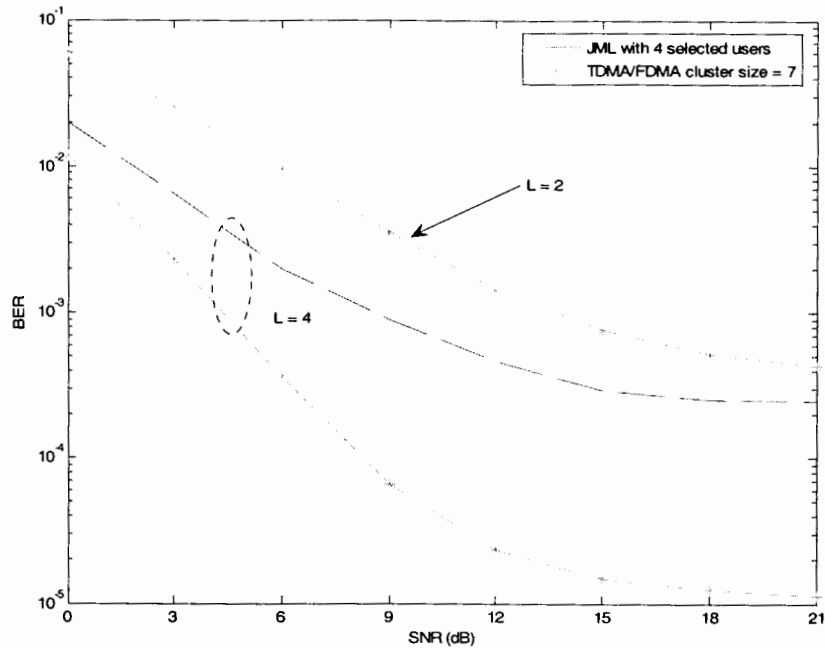
Prior investigations were all based on union bounds and thus serve as a guideline for the genuine performance of the proposed scheme. In this final section, we intend to study the actual performance of the system in order to validate our previous observations.

Figure 4-13 represents the actual performance of the proposed system and the union bound with six antennas and an SNR = 0 dB where the size of the detection set is varied. As the figure illustrates, a detection set of four users is ideal for this number of antennas and this signal power. The fact that bounds are tighter with smaller user groups can be observed in this graph. It should be noted that since the case of one user in the detection set depicts the actual performance of the system not the bound, it was not simulated but adopted from the bound curve – shown by a dotted line.

In Figure 4-14 the actual performance of the JML selection technique with five users in the detection set, i.e. four users selected from the first and second tier, is compared the conventional seven-cell cluster.



**Figure 4-13 Proposed JML selection technique vs. the union bound with six antennas and SNR = 0 dB ( $10^6$  simulations for the actual system)**



**Figure 4-14 Selection technique (detection set = 5 strongest) vs. actual BER of conventional TDMA/FDMA**

As the figure shows, the JML technique with four antennas out performs the conventional TDMA/FDMA system with two antennas, whereas the union bounds in Figure 4-5 illustrate the same performance for both cases. Hence, it can be estimate that with three antennas i.e. an additional antenna compared to the conventional system we can maintain the performance of the conventional system while achieving *seven times* the capacity.

*“A **conclusion** is the place where you got tired of thinking.”*

Martin H. Fischer

*“People do not like to think. If one thinks, one must reach conclusions. **Conclusions** are not always pleasant.”*

Helen Keller (1880 - 1968)

## **Chapter Five :**

# **Conclusions**

This thesis has focused on capacity increase in wireless narrowband systems via employing joint detection with diversity arrays. To conclude, the novel contributions of this work are summarized below.

### ❖ System Structure

The thesis investigated an unusual way to employ joint maximum likelihood (JML) detection to increase the capacity of TDMA or FDMA systems. Rather than increase the number of users per cell, the approach was to reduce the cluster size by including users in the first, and possibly the second, tier of cochannel cells in the powerful JML

detection. In fact, a cluster size of one – giving a seven-fold capacity increase compared with conventional systems – was adopted, and the thesis was directed to ensuring that BER performance was adequate and that the computation load was not excessive.

#### ❖ Performance

The performance of the proposed joint detection scheme was compared to that of the conventional seven-cell cluster known from TDMA and FDMA systems where one user occupies each channel.

- Comparisons of the union bound on joint detection performed including a set of users and the results obtained via removing them, indicate the notion that including users from the first or second tier in the joint detection has about the same effect as removing them from the system. Hence, JML is a powerful detection means by which interference due to the participating users can be eliminated to a great extent.
- Applying the simple joint ML detection with the active users in the first tier, it was shown that by adopting six antennas, the union bound curves for the JML detection demonstrate performance close to that of the conventional system with two antennas. It is very important to note that at the cost of additional antennas a capacity increase of *seven-fold* is achieved, which is a significant improvement.
- By applying selection to the users in the system, the union bounds approach the performance curves of the conventional system. In this case applying four antennas illustrates improved performance compared to the TDMA or FDMA system with two antennas. From the viewpoint of



capacity, that is not a high cost to the seven-fold gain. In addition, the performance of the selection technique was shown to be superior to that of no selection; this improvement is pronounced stronger in the interference dominant region.

- Allowing selection, we may include additional users in the system and increase the capacity to *fourteen times* that of the benchmark system. With additional users, there is a higher probability for the presence of strong users in the first tier; hence, due to joint detection with these users the performance even improves in some cases compared to the case with one user per cell. It is very interesting to realize that with this scenario and six diversity antennas the upper bounds on performance meet the performance of the conventional system while providing such a great increase in capacity.
- Comparisons of the true BER performance of the system show that the proposed JML technique with user selection maintains the performance of the conventional system, at most at the cost of two additional antennas, while allowing for higher capacity. In other words, applying the joint detection seven times the capacity is achieved with *no need* to invest additional power. Our comparisons with the bounds reinstate the looseness of the bounds, which is inherently due to the summation over pairwise error probabilities.

#### ❖ Effect of the Number of Users

Obviously altering the number of users present in the detection set will affect the performance of the system. Since performing joint detection is a costly process, it would be in our best interest to perform detection with the least number of users. Application of complexity reduction methods such as sphere decoding would be beneficiary when the computational load is tedious.

- When the selection technique is applied to the users present in the first tier, consulting bounds, it is shown that with the selection of three users the best performance is experienced.
- For a broader selection of users, the case of choosing users from the second tier in addition to those from the first tier is also studied. In this case, a set of the strongest users from the first two tiers participate in the detection. Comparison of the case with no joint detection and the joint detection applied show that the advantages of adopting selection to JML detection are more visible at higher SNR values where the interference is dominant. By adopting union bounds it is concluded that with lower SNR values (0 dB) a detection set consisting of three users will provide us with the best performance and as the SNR increases, larger groups are favoured. Simulations of the actual performance for the case of six antennas restate the fact that engaging more users in the detection set does not necessarily improve the performance.
- In conjunction with the previous section, it would be desirable to adapt the best number of users so the performance is always as close as possible to the conventional system. As the results indicate, starting with the selection of fewer users and changing the number of users in the

group according to the SNR, we can match the performance of the benchmark system throughout the spectrum.

❖ Effect of Imperfect Channel State Information

Making the assumption of perfect channel state information is somewhat unrealistic. Therefore, the performance of the system with degradation in the quality of CSI has also been investigated.

- It was demonstrated that performing detection with imperfect CSI only causes a small shift in the performance curves. However, it is also observed that the distance between the perfect and imperfect CSI curves increases with SNR increase. This is mainly due to the fact that the JML detection influences the performance more in higher SNR values and therefore the consequences of imperfect CSI are magnified.

# Bibliography

- [1] K. S. Gilhousen, I. M. Jacobs, R. Padvani, A. J. Viterbi, L. A. Weaver and C. E. Wheatly, "On the Capacity of a Cellular CDMA System," *IEEE Trans. on Vehicular Technology*, vol. 40, no. 2, pp 303 – 312, May 1991.
- [2] J. H. Winters, "Optimum Combining in Digital Mobile Radio with Cochannel Interference," *IEEE Journal on Selected Areas in Communications*, vol. SAC-2, no. 4, pp 1222 – 1230, July 1984.
- [3] S. J. Grant and J. K. Cavers, "System-wide Capacity Increase for Narrowband Cellular Systems through Multiuser Detection and Base Station Diversity Arrays," *IEEE Trans. on Wireless Communications*, vol. 3, no. 6, November 2004.
- [4] S. J. Grant, "Increased Capacity for TDMA Cellular Systems Through Joint Detection with Diversity Arrays," Ph.D. Thesis, Simon Fraser University, August 2000.
- [5] S. J. Grant and J. K. Cavers, "Performance Enhancement Through Joint Detection of Cochannel Signals Using Diversity Arrays," *IEEE Trans. on Communications*, vol. 46, no. 8, pp 1038 – 1049, August 1998.
- [6] G. J. M. Janssen and J. Zander, "Capacity Assessment of a Cellular Radio System Using a Narrowband Multiuser Detector," *IEEE Trans. on Wireless Communications*, vol. 2, no. 4, pp 703 – 713, July 2003.
- [7] J. G. Proakis and M. Salehi, *Communication Systems Engineering*, Prentice-Hall, 1994.
- [8] J. G. Proakis, *Digital Communications*, Fourth Edition, McGraw-Hill, 2000.
- [9] J. K. Cavers, *Mobile Channel Characteristics*, Second Edition, Shady Island Press, 2003.
- [10] [http://users.ece.gatech.edu/~mai/tutorial\\_multipath.htm](http://users.ece.gatech.edu/~mai/tutorial_multipath.htm)
- [11] S. D. Gray, M. Kocic and D. Brady, "Multiuser Detection in Mismatched Multiple-Access Channels," *IEEE Trans. on Communications*, vol. 43, no. 12, pp 3080 – 3089, December 1995.
- [12] L. Welburn, J. K. Cavers and K. W. Sowerby, "A Computational Paradigm for Space-Time Multiuser Detection," *IEEE Trans. on Communications*, vol. 52, no. 9, pp 1595 – 1604, September 2004.
- [13] S. J. Grant and J. K. Cavers, "Further Analytical Results on the Joint Detection of Cochannel Signals Using Diversity Arrays," *IEEE Trans. on Communications*, vol. 48, no. 11, pp 1788 – 1792, November 2000.

- [14] S. Verdu, "Minimum Probability of Error for Asynchronous Gaussian Multiple-Access Channels," *IEEE Trans. on Information Theory*, vol. IT-32, no. 1, pp 85 – 96, January 1986.
- [15] Z. Xie, C. K. Rushforth, R. T. Short and T. K. Moon, "Joint Signal Detection and Parameter Estimation in Multiuser Communications," *IEEE Trans. on Communications*, vol. 41, no. 7, pp 1208 – 1216, August 1993.
- [16] S. J. Grant and J. K. Cavers, "Multiuser Channel Estimation for Detection of Cochannel Signals," *IEEE Trans. on Communications*, vol. 49, no. 10, pp 1845 – 1855, October 2001.
- [17] J. K. Cavers, "Single-user and Multiuser Adaptive Maximal Ratio Transmission for Rayleigh Channels," *IEEE Trans. on Vehicular Technology*, vol. 49, no. 6, November 2000.
- [18] R. Vaughan and J. B. Andersen, *Channels, Propagation and Antennas for Mobile Communications*, IEE, 2003.
- [19] <http://www.ensc.sfu.ca/people/faculty/ho/ENSC832/>
- [20] <http://www.stanford.edu/class/ee359/>
- [21] B. Hassibi and H. Vikalo, "On the Expected Complexity of Sphere Decoding," *Conference Record of the Thirty-fifth Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1051 – 1055, November 2001.
- [22] P. W. Wolniansky, G. J. Foschini, G. D Golden and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," *URSI International Symposium on Signals, Systems and Electronics*, pp 295 – 300, September 1998.
- [23] L. R. Welburn, "*Macrodiversity and Multiuser Detection: High Performance Tools in a DS – CDMA System*," Ph.D. Thesis, Simon Fraser University, May 2001.