THE DEMAND FOR SLEEP

by

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ABSTRACT

I retest Biddle and Hamermesh's 1990 model of sleep allocation from the Journal of Political Economy using new data. Biddle and Hamermesh's model of sleep allocation is an application of Becker's model of time allocation. In the case of sleep, an individual receives utility from performing activities during her waking leisure time and from sleeping. The demand for sleep and the demand for waking leisure time are derived from the model. An increase in the wage rate should induce a person to sleep less, while an increase in non-labour income should induce a person to sleep more. Estimation of the model does not provide evidence of a negative wage effect on sleep.

DEDICATION

For my Mom and Dad.

Without their patience and financial aid, I would never have graduated.

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I. INTRODUCTION

In Nancy Kress' science fiction novel *Beggars in Spain* (1993) by the year 2000 it is common for prospective parents to have their children genetically modified before they are born. People routinely choose traits like eye colour and height. Scientists develop a new genetic modification that allows children to not have to sleep and the first sleepless child is born in 2008. The sleepless use all of their extra time towards studying and pursuing their careers and they quickly end up dominating the academic and business worlds (the main character becomes the editor of the Harvard Law Review). As the sleepless become more and more economically powerful, the sleepers become more resentful of the sleepless and society devolves into an apartheid system, where the two groups are socially isolated from one another. One of the many interesting economic aspects of the story is that the sleepless use all of their extra time to work and acquire human capital. It seems quite plausible that if people did not have to sleep that they might devote a very large part of their extra time to leisure activities and not working. In this paper, issues like these will be explored—peoples' trade-offs among time spent sleeping, time spent working, and time spent doing leisure activities will be examined.

More specifically, I retest Biddle and Hamermesh's 1990 model of sleep allocation from the Journal of Political Economy using new data. Biddle and Hamermesh test the idea that time spent sleeping can be modelled like the demand for leisure using Becker's model of time allocation. In this framework, changes in variables like the wage rate and income change the individual's time allocation between working and leisure.

The amount of time a person sleeps for could be a biologically determined constant where different people have different inherent needs for sleep. Biddle and Hamermesh argue using the results form previous time use studies and their own results that there is variability in time spent sleeping and that this variability is not all due to random fluctuations. Previous studies have uncovered correlations between time spent sleeping and other variables like time spent working, the day of the week, and the presence of a child in the home. These correlations suggest that some portion of the time spent sleeping is consciously controllable and thus possibly subject to economic incentives. The reason time spent sleeping may be economically important is that if time spent sleeping is sensitive to incentives then it could possibly be a significant addition to the bank of time a person has available to devote to the labour supply decision; previous studies on labour supply have ignored this aspect of the labour-leisure trade-off.

The purpose of this paper is to test a model in which a change in the amount of time a person sleeps is determined by changes in her wage and non-labour income; i.e., a demand equation for sleep where the wage rate and non-labour income are exogenous variables. Before getting to this point, I present a summary of the psychological research on sleep followed by statistical evidence of sleeping patterns.

II. SLEEP SCIENCE RESEARCH

A good analogue to sleeping is eating. People differ in the amounts of food they need to eat and the amount an individual eats can be different at different times in his life depending on the circumstances. There is interpersonal variation in food intake and there is variation in individual food intake. Time spent sleeping is also like this. And just as people can harm themselves from severely under-eating and from severely over-eating, people can harm themselves from severely under-sleeping and to a much less degree from over-sleeping. And just as over time, people can learn to alter their eating habits somewhat, over time and slowly, it appears that people can learn to change their sleeping habits somewhat.

In his book *Sleep and Dreaming* (2002) Jacob Empson comprehensively reviews the scientific literature on sleep. The sleep literature indicates that adults sleep on average 7½ hours a day with a standard deviation of around one hour. Sixteen percent of the population sleeps for less than 6½ hours and there is a sizable minority of people who only need to sleep for 5 hours. Variation in sleep time for an individual seems to depend on how the person's sleep schedule is organized. If a person is severely sleep deprived, then on subsequent nights he will sleep longer and reach deeper states of sleep faster and stay in those deeper states for longer periods of time. In most experiments on sleep deprivation, the subjects are severely deprived of sleep for a short time period. Under conditions of severe sleep deprivation (the low end seems to be sleeping for around 2 hours a night) people's reaction times slow down, they start to have difficulty concentrating, and they start to miss events in their environment. Sleep researchers use lapse theory to explain these effects. It appears that under circumstances of severe sleep deprivation, people start to take microsleeps. A person either falls asleep for a very short period or his level of arousal declines a great amount and the person starts to miss things going on in the environment—there is a lapse in the person's attention level. When most people think of sleep deprivation, they are thinking of situations like these.

As opposed to severe sleep deprivation in the short term that has to be rectified, people can also deprive themselves of little bits of sleep over a long time. There is much less scientific literature on this kind of sleep deprivation and the results seem to indicate that this type of sleep deprivation may not be harmful. Sleep researchers study the mal-effects of sleep deprivation by having people perform simple tests that measure abilities like reaction time and memory. People's performance on tests like these does not decline from having decreased their normal sleep time by small amounts. In one experiment people were trained to slowly lower their sleep time by 30 minutes every 2 weeks over an 8 month period and then the subjects were followed up on one year after the study had ended and the average sleep time for these individuals was now lower than it had been before the study. So, it seems there is a habitual aspect to some part of sleep time. Just as people can go on diets for long periods and change their weight somewhat, people may be able to change their sleeping times permanently by small amounts. Researchers have also tested people for the effects of oversleeping. In the short term, people who sleep more than they normally do have the same sorts of declines in their performance on psychological tests as those who are sleep deprived. This is known as the Rip Van Winkle effect—after the character in Washing Irving's short story who falls asleep for 20 years and wakes up to find himself unsuited to dealing with his new environment.

The type of sleep deprivation that seems relevant in studying the demand for sleep is low levels of sleep deprivation over long periods of time. The evidence suggests that it is possible for people to permanently alter their sleep time in this way. The other issue is whether a person who changes her sleep schedule in this way decreases her performance. The little evidence there is suggests that a person's performance will not decrease, and at the same time the evidence also suggests that sleeping more will not improve a person's performance on psychological tests unless he is already severely sleep deprived. The issue of changes in performance (i.e., productivity) and sleep time is important because if there is a relationship it could manifest itself through changes in wages. If sleep has productivity effects then the wage is endogenous in econometric models of work-time/sleep-time allocation.

III. EVIDENCE OF SLEEPING PATTERNS

Sleep occupies about one-third of our daily time, and yet as Biddle and Hamermesh point out economists have not systematically studied this major use of time. They use data from the following two time diary studies to identify sleeping patterns: (1) the Multinational Time Budget Research Project conducted by Alexander Szalai in the 1960s; and (2) Americans' Use of Time: 1975-1976. To this set of results, I add my results from the Americans' Use of Time: 1985 study. A detailed description of this data set is presented in section VI. The purpose behind trying to identify sleeping patterns is to see if there are any regularities in time spent sleeping that might appear to show that a person's sleep time is "reacting" to outside forces—especially market forces. Evidence of this nature would indicate that to some extent time spent sleeping is endogenous (i.e., something that a person is choosing to do). If sleep time is endogenous then there is a justification for modelling it using consumer theory.

Szalai is a major time diary study of daily time use in 12 countries conducted in the 1960s. Biddle and Hamermesh use a subset of this data and regress minutes spent sleeping per day on minutes spent working and on dummy variables for children, male, marriage status, and whether the diary was kept on a workday. They obtain the following results:

Table 1: Sleep Equation for Szalai Data

Dependent Variable: Sleep in Minutes per Day	
Parameter Estimates & Standard Errors	
Minutes Worked	**-.109**
Standard Error	.01
Children	**-11.18**
Standard Error	4.15
Male	**17.84**
Standard Error	2.66
Married	**-8.52**
Standard Error	5.83
Workday	**-30.28**
Standard Error	5.37
Adjusted R2	.70

In the Szalai data, on average people sleep about 8½ hours a day. The regression results indicate that working one more hour (60 minutes) in one day will lower daily sleep time by almost 7 minutes (60 minutes * -0.109 = 6.54 minutes). The effects for the presence of children, married, and workday are all significant and negative.

Biddle and Hamermesh also cite the study by Webb (1985). Webb uses a different subset of the Szalai data and tests for pair wise correlations between time spent sleeping and variables like whether it is a workday or a day off, and whether the correspondent was male or female. Webb finds that both men and women sleep less on workdays and that men sleep more than women on weekends.

The second set of data used by Biddle and Hamermesh is the 1975-76 time use study. The study measures weekly time use by a 1,519 American Household using time diaries. For each person the data set lists how many minutes the person slept, worked, etc., over one week. The data is cross sectional. It contains the total weekly number of minutes each activity is performed. Biddle and Hamermesh use the 1975-76 cross sectional data to estimate the sleep demand and waking-leisure demand equations that are presented in section VII.

This study separates time use into 86 different categories. Time spent sleeping falls under three different categories—night sleep, napping and resting, and miscellaneous personal activities. The miscellaneous personal activities category includes activities like having sex and other things that people may find uncomfortable writing down in their time diaries. Night sleep is the main category measuring sleep time. Combining categories gives progressively broader definitions of sleep; obviously adding the time spent napping and resting during the day to the time spent sleeping at night will lead to a larger number. Biddle and Hamermesh present the following descriptive statistics:

Table 2: Descriptive Statistics for 1975-76 Time Use

Means and standard deviations for three categories of Sleep, and Work Time in minutes over one week				
	Night Sleep	**Sleep + naps/rest**	**Sleep+ naps+ misc.**	**Work Time**
All Correspondents	3,266	3,383	3,438	2,122
Standard Deviation	444	499	520	947

From this set of data, the average daily time spent sleeping is 7¾ hours. Biddle and Hamermesh point out that over the week the average person spends more time sleeping than working.

For this set of data they regress the three different categories of sleep on work time and various control variables.

Table 3: Sleep/Nap Equation using 1975-76 Time Use Data

Dependent variable is Night Sleep + Naps/Rest in minutes per week			
	All Respondents	Men	Women
Work time	**-.199**	**-.219**	**-.169**
t-statistic	-9.95	-7.3	-5.63
Married	**16.04**	**-43.15**	**92.50**
t-statistic	0.288	-0.52	1.12
Years Married	**-2.59**	**2.43**	**-7.62**
t-statistic	-1.12	0.776	-2.18
Age	**1.86**	**24.52**	**-24.81**
t-statistic	0.145	1.51	-1.17
Age Squared	**.02**	**-.26**	**.35**
t-statistic	0.13	1.36	1.4
Years of Schooling	**-14.30**	**-18.28**	**-9.09**
t-statistic	-2.13	-2.13	-0.839
Male	**99.42**		
t-statistic	2.54		
Excellent or Good Health	**-94.16**	**-123.79**	**-59.66**
t-statistic	-1.59	-1.53	-0.669
Children under 3 years	**-35.42**	**39.03**	**-153.00**
t-statistic	-0.627	0.576	-1.49
Protestant	**86.15**	**90.87**	**93.97**
t-statistic	2.30	1.89	1.56
Black	**-69.17**	**-110.65**	**-43.95**
t-statistic	-0.875	-0.967	-0.380
Adjusted R2	.141	.176	.108

Table 4: Night Sleep Equation using 1975-76 Time Use Data

Dependent variable is Night Sleep in minutes per week			
	All Respondents	Men	Women
Work time	**-.164**	**-.184**	**-.134**
t-statistic	-9.11	-7.36	-4.76
Note: Includes all control variables from Table 3			
Adjusted R2	.116	.146	.082

Table 5: Sleep/Naps + Misc. personal time equation using 1975-76 Time Use Data

Dependent variable is Night Sleep + Naps/Rest + Misc. personal time in minutes per week			
	All Respondents	Men	Women
Work time	**-.214**	**-.239**	**-.179**
t-statistic	-10.19	-8.53	-5.42
Note: Includes all control variables from Table 3			
Adjusted R2	.147	.190	.101

All three sets of regressions on the 1975-76 data show a significant negative coefficient on work time for both males and females. Other interesting results from the data include a significant negative coefficient on the children dummy on women's sleep time but not on men's and also education has a negative significant effect in the all-respondents sample and the men-only sample. Biddle and Hamermesh propose that the negative coefficient on education is an effect that is working through wages, suggesting a substitution effect. In fact, a wage substitution effect is what will be modelled in the main part of the paper. For the all-respondents regression presented in table 3, working for one more hour lowers sleep time by about 12 minutes (60 minutes * -.199 = 11.94 minutes) over a week.

Using data from the 1985 Time Use Study which measures time use over one 24 hour period, I found the following correlations between minutes spent sleeping over one day and work time, and the number of children under age 5:

Table 6: Correlations for Sleep Times using 1985 Time Use Data

Correlations		Total Minutes Spent Working	Number of Children Under Age 5
	Night Sleep	-0.351*	-0.028
	Sleeping/Napping	-0.365*	-0.025
	Sleeping/Napping/Personal Activities	-0.379*	-0.02

*significant at the 0.01 level

These results suggest a strong relationship between time spent sleeping and time spent working. The relationship between time sent sleeping and having young children in the home is weak.

Another illustration of sleeping patterns emerges from looking at the average daily time spent sleeping for each day of the week.

Table 7: Average Daily Sleep Time using 1985 Time Use Data

	Sleeping/Napping
Monday	453.51
Tuesday	460.97
Wednesaday	447.52
Thursday	457.29
Friday	448.64
Saturday	485.26
Sunday	515.53
Time in minutes	

People sleep more on the weekends and perhaps this is the case because they have lower time costs on those days because they are probably not working.

Using the 1985 data, I estimate sleep equations on work hours and demographic data. Different definitions of sleep are regressed on work time, age, age^2, education, marital status, female, number of full time workers in home, number of children under age 5, and the day of the week. The base category for education is elementary school. The base category for marital status is single and the base category for day of the week is Sunday. The very last row in table 8 through 10 lists the p-value for the following hypothesis test:

H_0: The coefficients on the Days of the Week are all equal

Table 8: Sleep Equation for All Respondents using 1985 Data

All Respondents

	Sleep/Nap	Night Sleep	Sleep/Nap/Misc.
Constant	**628.98**	**611.73**	**660.43**
t-statistic	25.97	25.49	26.29
WorkTime	**-0.16**	**-0.15**	**-0.17**
t-statistic	-16.29	-15.57	-17.13
Age	**-4.15**	**-4.39**	**-4.23**
t-statistic	-4.15	-4.40	-4.11
Age2	**0.04**	**0.04**	**0.04**
t-statistic	3.74	3.62	3.65
Less then HS	**-2.52**	**6.84**	**-6.99**
t-statistic	-0.19	0.51	-0.50
H.S. Grad	**-1.53**	**8.20**	**-11.67**
t-statistic	-0.13	0.70	-0.95
Some College	**-0.62**	**10.70**	**-12.20**
t-statistic	-0.05	0.86	-0.94
College Grad	**-2.28**	**6.81**	**-16.05**
t-statistic	-0.18	0.55	-1.24
Post Grad	**-4.14**	**16.06**	**-16.29**
t-statistic	-0.30	1.18	-1.15
Married	**3.11**	**7.47**	**-0.17**
t-statistic	0.48	1.15	-0.02
Divorced	**-22.12**	**-18.46**	**-18.89**
t-statistic	-2.55	-2.15	-2.10
Widowed	**-25.07**	**-24.29**	**-9.78**
t-statistic	-1.75	-1.72	-0.66
Female	**-3.85**	**-3.83**	**-5.87**
t-statistic	-0.88	-0.88	-1.30
#Fulltime Workers in Home	**1.75**	**0.94**	**4.12**
t-statistic	0.67	0.36	1.51
#Children < Age 5	**-11.02**	**-15.12**	**-8.45**
t-statistic	-2.26	-3.13	-1.67
Monday	**-23.38**	**-21.37**	**-27.80**
t-statistic	-2.77	-2.56	-3.18
Tuesday	**-10.06**	**-14.16**	**-15.35**
t-statistic	-1.18	-1.68	-1.74
Wednesday	**-26.25**	**-22.21**	**-31.48**
t-statistic	-3.14	-2.68	-3.63
Thursday	**-14.55**	**-13.17**	**-15.13**
t-statistic	-1.73	-1.58	-1.73
Friday	**-29.11**	**-28.68**	**-34.51**
t-statistic	-3.48	-3.46	-3.98
Saturday	**-25.58**	**-20.44**	**-29.38**
t-statistic	-3.22	-2.59	-3.56
Adjusted R2	**0.15**	**0.15**	**0.16**
H0:=Day effects (p-value)	**0.10**	**0.37**	**0.06**

Table 9: Sleep Equation for Male only Sample using 1985 Data

MALES

	Sleep/Nap	Night Sleep	Sleep/Nap/Misc.
Constant	**631.79**	**609.27**	**669.74**
t-statistic	17.63	17.07	17.92
WorkTime	**-0.17**	**-0.16**	**-0.19**
t-statistic	-12.21	-11.91	-13.03
Age	**-4.37**	**-4.34**	**-4.81**
t-statistic	-2.93	-2.93	-3.10
Age2	**0.05**	**0.04**	**0.05**
t-statistic	2.81	2.40	2.87
Less then HS	**-4.86**	**9.17**	**-9.89**
t-statistic	-0.26	0.50	-0.51
H.S. Grad	**1.57**	**13.72**	**-6.58**
t-statistic	0.10	0.87	-0.40
Some College	**4.85**	**21.48**	**0.19**
t-statistic	0.29	1.28	0.01
College Grad	**3.57**	**17.36**	**-7.62**
t-statistic	0.22	1.05	-0.44
Post Grad	**-0.27**	**24.29**	**-8.41**
t-statistic	-0.01	1.34	-0.44
Married	**10.67**	**12.24**	**8.35**
t-statistic	1.08	1.24	0.81
Divorced	**-29.76**	**-30.38**	**-17.54**
t-statistic	-2.14	-2.20	-1.21
Widowed	**-29.52**	**-15.99**	**1.46**
t-statistic	-0.88	-0.48	0.04
#Fulltime Workers in Home	**2.63**	**1.84**	**2.96**
t-statistic	0.71	0.49	0.76
#Children < Age 5	**-11.08**	**-14.27**	**-9.03**
t-statistic	-1.68	-2.17	-1.31
Monday	**-26.78**	**-23.08**	**-30.90**
t-statistic	-2.15	-1.86	-2.38
Tuesday	**-16.17**	**-16.89**	**-19.39**
t-statistic	-1.28	-1.35	-1.48
Wednesday	**-32.78**	**-27.68**	**-34.56**
t-statistic	-2.67	-2.26	-2.69
Thursday	**-22.37**	**-18.07**	**-19.53**
t-statistic	-1.82	-1.48	-1.52
Friday	**-38.82**	**-37.27**	**-42.20**
t-statistic	-3.15	-3.03	-3.28
Saturday	**-44.79**	**-37.27**	**-44.46**
t-statistic	-3.83	-3.20	-3.64
adj r2.	**0.16**	**0.16**	**0.17**
H0:=Day effects (p-value)	**0.14**	**0.27**	**0.16**

Table 10: Sleep Equation for Female only Sample using 1985 Data

FEMALES	Sleep/Nap	Night Sleep	Sleep/Nap/Misc.
Constant	**627.16**	**620.19**	**649.55**
t-statistic	18.81	18.91	18.90
WorkTime	**-0.16**	**-0.14**	**-0.16**
t-statistic	-10.70	-9.65	-10.60
Age	**-3.95**	**-4.48**	**-3.52**
t-statistic	-2.95	-3.39	-2.55
Age2	**0.04**	**0.04**	**0.03**
t-statistic	2.47	2.71	2.06
Less then HS	**-4.40**	**-4.78**	**-10.35**
t-statistic	-0.21	-0.23	-0.48
H.S. Grad	**-8.93**	**-6.64**	**-23.28**
t-statistic	-0.47	-0.35	-1.19
Some College	**-12.32**	**-11.03**	**-32.55**
t-statistic	-0.62	-0.57	-1.60
College Grad	**-13.04**	**-13.13**	**-31.29**
t-statistic	-0.66	-0.67	-1.52
Post Grad	**-15.33**	**-4.30**	**-35.69**
t-statistic	-0.70	-0.20	-1.58
Married	**-4.61**	**3.49**	**-10.11**
t-statistic	-0.53	0.41	-1.12
Divorced	**-17.09**	**-9.56**	**-21.08**
t-statistic	-1.57	-0.89	-1.88
Widowed	**-24.44**	**-26.77**	**-14.01**
t-statistic	-1.55	-1.72	-0.86
#Fulltime Workers in Home	**1.77**	**0.52**	**6.39**
t-statistic	0.47	0.14	1.64
#Children < Age 5	**-12.00**	**-17.16**	**-7.49**
t-statistic	-1.58	-2.30	-0.96
Monday	**-20.30**	**-20.50**	**-25.36**
t-statistic	-1.78	-1.83	-2.16
Tuesday	**-3.00**	**-11.65**	**-11.07**
t-statistic	-0.26	-1.04	-0.94
Wednesday	**-20.00**	**-17.66**	**-29.95**
t-statistic	-1.76	-1.58	-2.56
Thursday	**-5.09**	**-7.91**	**-10.56**
t-statistic	-0.44	-0.69	-0.88
Friday	**-18.81**	**-19.59**	**-26.69**
t-statistic	-1.66	-1.76	-2.29
Saturday	**-3.89**	**-0.61**	**-12.70**
t-statistic	-0.36	-0.06	-1.15
Adjusted R2	**0.13**	**0.13**	**0.14**
H0:=Day effects (p-value)	**0.26**	**0.39**	**0.26**

The results from the 1985 data all show a significant negative coefficient on work time. For example, in the female-only sample, a one-hour increase in work time leads to about a (60 minutes * -0.16) 9½ minute decrease in daily sleeping/napping time. However, these results generally do not indicate any education effects. The presence of negative education effects is one of Biddle and Hamermesh's main arguments for modelling sleep in an economic framework. The strongest education effects are found in the female only sample when the dependent variable is the broadest measure of sleep (the third column of table 10). In this case, higher education leads to lower sleep time. In contrast, Biddle and Hamermesh failed to find any education effect on women's sleeping time in their data.

The regressions also indicate that having children under age five in the home lowers sleep time. The strongest effects are on night sleep and night sleep + napping. For both of these categories, the negative effect on women's sleep time is larger than in the men-only sample and the all-respondents sample.

The day of the week effects are interesting. The base category is Sunday and clearly the results show that people sleep much less on all of the other days compared to Sunday. In all three tables, the p-value for the hypothesis test that the day effects are equal is the largest for the night sleep category. This result implies that night sleep is the most constrained in terms of people being able to change their sleeping time. The lower p-values for the broader measures of sleep suggest that people use non night-sleep to recoup lost sleeping time.

IV. WHAT IS THE DIRECTION OF CAUSATION?

Biddle and Hamermesh's 1975-76 time use data is over one week. This is the data used to generate the results presented in tables 2 through 5. For each person there is one observation—the weekly number of minutes each activity is performed. This is the data that they use to estimate the sleep model (i.e., the demand for sleep and the demand for waking leisure).

In the data that I use, there is also only one observation for each person. The observation lists how the person spent his time over a 24-hour period. This data was used to generate tables 6 through 10 and it will be used to estimate the demand equations for sleep and waking leisure.

So far all of the regression results have indicated a negative relationship between time spent sleeping and time spent working. The theory that we are working towards is that higher wages induce people to work more and sleep less. But the regression results so far cannot be used to infer causation between sleep time and work time. It may be the case that there are people who do not need to sleep very much and so can use this extra time to work. For example, suppose we have three individuals who have different but fixed needs for sleep and assume they spend ½ of their waking hours working. The daily data on these individuals could look something as follows:

Table 11: Plausible Causation Data

	Time spent Sleeping	Awake Time	Work Time
Albert	8	16	8
Bob	7	17	8.5
Charlie	6	18	9

From this made-up data we would get a negative correlation between time spent sleeping and time spent working but in this case sleep is determining the work time. Thus, regressions from cross-sectional data do not allow us to distinguish whether people first decide their work time and then allocate their sleep time accordingly or whether they simply sleep for some amount of time and then decide how long to work. The negative parameter estimates found in the cross-sectional data could be due to people who in Biddle and Hamermesh's words are "innately sleepless workaholics."

To test for this possibility they use panel data from the 1975-81 panel data study. This is the only part of their paper where Biddle and Hamermesh use panel data. They estimate an equation to determine if there is a "need for sleep." In this data set, some of the individuals from the 1975 study were followed up on in 1981. For these individuals there are two observations, one data point from 1975 for one week and one data point for 1981. Again, the data is total number of minutes spent doing an activity over one week.

In the above regressions on sleep time with work time as an independent variable, we have been estimating the following equation over one time period:

$T_{sit} = \alpha T_{wit} + \beta X_{it} + \varepsilon_{it}$

Where

T_{sit} = the time spent sleeping by person i at time t

T_{wit} = the time spent working by person i at time t

X = a vector of control variables

α and β are the vectors of the parameter estimates. ε_{it} is the random term that varies across individuals and time

But, it might really be the case that the actual equation is:

$$T_{sit} = \alpha T_{wit} + \beta X_{it} + v_i + \varepsilon_{it}$$

Where the v_i term is an individual's inherent need for sleep and it is unobservable. The v_i term will be negatively correlated with the T_{wit} term—people who have a high need for sleep will have a low work time. If we have been missing the v_i term in our regressions then the variation due to it will bias the T_{wit} term. Biddle and Hamermesh test for this possibility by taking the above equation and differencing it over time to get the following new equation:

$$\Delta T_{si} = \alpha \Delta T_{wi} + \beta \Delta X_i + \Delta \varepsilon_I$$

Estimating this new equation with first differences over the same individuals gets rid of the bias caused by an unobservable individual effect. Since there are two observations for each person in the panel data, first differencing produces the same estimation results as the fixed effects estimator. Biddle and Hamermesh estimate this equation using the "within estimator" and once again get a negative coefficient on time spent working. From the panel data, Biddle and Hamermesh conclude that a 1-hour increase in work time leads to a 13-minute decrease in sleep time.

The regression on panel data does not conclusively prove that the causation is from time spent working to time spent sleeping. It is still possible that changes in people's time spent sleeping determine the change in their time spent working, but Biddle and Hamermesh argue that for this to be plausible, changes in sleep time would have to be predictable and persistent enough for people to be able to organize their life (i.e., their labour supply decision) around these changes. They further argue that evidence such as people sleeping more on weekends and sleeping less in the presence of children suggest that people can alter some of their time spent sleeping when circumstances change. Also, they argue that the negative effect of education on sleep time is evidence that wages are indirectly affecting the time spent sleeping.

The point of examining sleeping patterns and trying to determine the causation of the sleep/work relationship is to be able to justify modelling time spent sleeping as an endogenous variable. The evidence presented indicates that sleep time is somewhat controllable and that people alter their sleep time in accordance with how much they want to work. The data show a negative relationship between time spent sleeping and time spent working and in the formal model it will be shown how this relationship can arise from people adjusting their optimal time allocation due to exogenous changes in the wage rate and non-labour income. For example, an increase in the wage rate will increase the cost of time and induce a person to sleep less. So, when the model is estimated in section VII using cross sectional data, a negative relationship between time spent sleeping and the wage rate is expected. Biddle and Hamermesh do not consider the possibility that the negative relationship between the *wage rate* and *time spent sleeping* in cross sectional data could arise because sleep time may effect educational attainment and educational attainment in turn may effect the wage rate. People who need less sleep are able to devote more of their time to acquiring human capital or acquiring better quality human capital (e.g., natural short sleepers have more time to study which in turn helps them to earn higher marks in school) and thus leading to a higher wage.

So, there actually are two different causation issues—the causation between time spent working and time spent sleeping, and the causation between time spent sleeping and the wage rate. In their panel data estimation, Biddle and Hamermesh only test for the causation between sleep time and work time. Using their panel data, Biddle and Hamermesh could have used their estimate of the "need for sleep" as an explanatory variable in an equation on educational attainment and then estimated a wage equation using educational attainment as an explanatory variable. The system of equations would have looked something as follows:

Education = $\alpha_1 + \alpha_2$*(need for sleep) + controls

Wage = $\alpha_1 + \alpha_2$*(Education) + controls

Biddle and Hamermesh do not address this issue—that sleep effects wages through education. In fact, when they estimated sleep equations with work time as an independent variable and found significant negative effects on education, they argued that education was affecting the wage and the wage was in turn was affecting sleep time.

In the formal model they leave open the possibility of sleep affecting wages through a productivity enhancing effect (i.e., rested workers are more productive). In the wage-rate/sleep-time relationship, it is important to distinguish between the causation issue and the productivity issue since both are manifested through changes in the wage rate.

There are two issues related to an increase in the wage:

1) Suppose sleep does not make people more productive but it is a leisure activity like all other leisure activities. Then when the wage increases, the person will substitute away from sleep and towards work. This is the substitution effect.

2) If sleep enhances productivity, then the time cost of sleep is lower than in case one. When a person is sleeping he is becoming more productive (i.e., helping to raise his wage). In this case, when the wage increases, the substitution away from sleeping will be less than in the case if sleep has no productivity effect.

V. THE FORMAL MODEL

Biddle and Hamermesh's model of sleep allocation is an application of Becker's time allocation model. In a model of this type, the individual or the household receives utility from the activities they perform and these activities in turn have a time component and a goods component.

The individual purchases goods from the market and then transforms them in someway using her time and labour to produce the activity and it is this activity that gives utility. For example, a person would get utility from eating dinner and this dinner would have been produced using food purchased from the market and combining it with labour and time.

In the model of sleep allocation, the person gets utility from activities (like eating dinner or watching television) produced during their leisure time and the person gets utility from sleeping.

A. The Notation

The utility function that is going to be maximized is the following:

$U = U(Z, T_s)$ $\qquad$ $U_i>0$ and $U_{ii}<0$ $\qquad$ **(1)**

Where

Z = the activity that the person gets utility from

(e.g., Z would be watching television)

T_s = Time spent sleeping

Marginal utility increases at a diminishing rate for both activities.

There are several different constraints that we have to take into account.

$$T_z = bZ \qquad \textbf{(2)}$$

T_z is the time needed to produce/consume the activity Z (e.g., it would be the time spent watching television)

$$X = aZ \qquad \textbf{(3)}$$

X is the market good that is needed in order to experience Z (e.g., a television set)

The budget constraint for purchasing good X will be as follows:

$$PX = W_m T_w + I \qquad \textbf{(4)}$$

Where,

P = Price of the market good X

W_m = Market Wage

T_w = Time spent working

I = Non-labour Income

The person has a fixed amount of time to divide between sleeping (T_s), waking leisure time (T_z), and working (T_w). So, there is a total time constraint:

$$T^* = T_s + T_z + T_w \qquad \textbf{(5)}$$

Where,

T^* = the total amount of time in given period (e.g., a day)

The last step is to let sleep have a productivity enhancing effect on the market wage. Let W_m have two separate components, the "regular part", W_1 and the part due to sleep making the person more productive, W2.

$Wm = W_1 + W_2Ts$ $\quad$ W_1 and $W_2 > 0$ **(6)**

W_1 and W_2 are parameters

B. The Maximization Problem

Max $\quad U = (Z, T_s)$
$\{Z, T_s\}$

Subject to:

$T_z = bZ$	activity time component	**(2)**
$X = aZ$	activity goods component	**(3)**
$PX = W_mT_w + I$	budget constraint	**(4)**
$T^* = T_s + T_z + T_w$	Total time constraint	**(5)**

If we substitute (3) in to (4) we get:

$$PaZ = W_mT_w + I \qquad \textbf{(A)}$$

If we solve (5) for T_w we get:

$$T_w = T^* - T_z - T_s \qquad \textbf{(B)}$$

We can substitute (B) in to (A) to get:

$$PaZ = W_m(T^* - T_z - T_s) + I \qquad \textbf{(C)}$$

We can now substitute (6) in to (C) to get:

$$PaZ = (W_1 + W_2T_s)(T^* - T_z - T_s) + I \qquad \textbf{(D)}$$

And finally we can substitute (2) in to (D) to get:

$$PaZ = (W_1 + W_2T_s)(T^* - bZ - T_s) + I \qquad \textbf{(7)}$$

(7) is the constraint that combines all of the other constraints, so we can now write our maximization problem in the usual lagrangian form with one constraint:

$$\underset{\{Z,\, T_s,\, \lambda\}}{\text{Max}}\ \mathscr{L} = U(Z, T_s) + \lambda\,(PaZ - [(W_1 + W_2T_s)(T^* - bZ - T_s) + I])$$

From this maximization problem, we can now derive the tangency conditions:

$$\frac{U_z}{U_{T_s}} = \frac{aP + bW_m}{W_m - W_2 T_w}$$

The benefit from consuming an additional unit of good Z is equal to the cost of producing it in terms of its price plus the cost of time where the price of time is the wage rate. The benefit from consuming one additional unit of sleep is equal to the cost of time (the wage rate) adjusted down for any increase in productivity that might be brought about by sleeping.

The second order conditions for the maximization problem are:

$$H = \begin{pmatrix} U_{11} & U_{12} + \lambda W_2 b & aP + W_m b \\ U_{12} + \lambda W_2 b & U_{22} + 2\lambda W_2 & W_1 + W_2(T_s - T_w) \\ aP + W_m b & W_1 + W_2(T_s - T_w) & 0 \end{pmatrix} < 0$$

We want to derive predictions on how changes in the wage rate and non-labour income will change sleep time and waking leisure time.

The comparative static result for the change in non-labour income on sleep time is:

$$\frac{\partial T_s}{\partial I} = \frac{U_{11}[w_1 + w_2(T_s - T_w)]}{H} - \frac{U_{12}(ap + bw_m)}{H} + \frac{bU_1 w_2}{H}$$

The first two terms in the above equation are positive while the last term is negative. This is the income effect on sleep and it should be positive provided the third term is not too large.

The comparative static result for the change in the wage on sleep time is:

$$\frac{\partial T_S}{\partial W_1} = \frac{[(U_1 - bU_2)(aP + bW_m)]}{H} + T_w \frac{\partial T_S}{\partial I}$$

The comparative static result for the change in the wage on sleep time if sleeping increases the workers productivity is:

$$\frac{\partial T_s}{\partial W_2} = T_s \frac{\partial T_s}{\partial W_1} - \frac{(aP + bW_m)U_1 T_w}{H}$$

The above two results are substitution effects and both are negative.

To get the comparative static results for waking leisure time, we would re-solve the maximization problem with T_z replacing time spent sleeping as the second choice variable.

The comparative static result for the change in the wage on waking leisure time is:

$$\frac{\partial T_z}{\partial W_1} > 0$$

This substitution effect is positive, but the substitution effect for total non-market time is negative:

$$\frac{\partial (T_s + T_z)}{\partial W_1} < 0$$

The comparative static result for the change in non-labour income on waking leisure time is positive:

$$\frac{\partial T_z}{\partial I} > 0$$

To summarize, the following are the expected predictions for the sleep allocation model.

	Wage	**Non-labour income**
Sleep Time	-	+
Waking leisure	+	+

VI. A DESCRIPTION OF THE DATA

Biddle and Hamermesh use data from the 1975 Americans' Use of Time Study. This study measured time use over a one-week period. In this paper I am using the 1985 Americans' Use of Time Study. This data is available from the Research Data Library at SFU. One can access it from the SFU library website by clicking on the RDL link and then going through to The Inter-university Consortium for Political and Social Research (The ICPSR) page. The study is officially called Study No. 9875, Americans' Use of Time, 1985. The study is conducted by the Survey Research Center at the University of Maryland, College Park. The Americans' Use of Time project started in 1965 and has conducted several different studies of time use since then (Robinson 1999).

There are 4939 cases in the data set. The study contains information obtained from time diaries on all aspects of work and leisure in the United States. People kept track of everything they did over a 24-hour period. Anyone who was over 18 years old, owned a telephone, and lived in the Continental United States was eligible to be chosen. There were three different information-gathering processes. Some individuals were telephoned and asked questions on how they spent their time, others were interviewed in person, and the third group responded by mail. Of the 4,939 time diaries, 2,921 are mail-in diaries, 1,210 are telephone diaries, and 808 are personal in-home interview diaries. The data set includes weights that are used to make the sample equivalent to the US census estimates of the male/female ratio. The data set contains over 100 hundred different categories on time use. There are also some demographic variables for things like age, wage rate, marital status, and gender.

Once observations with missing values for the control variables and those not in the labour force were dropped from the data set, there were 2435 cases left.

Each case contains how many *minutes* a person conducted an activity for in one 24-hour period. The data is cross sectional. There is only information for one day.

The following chart contains all of the variables that I use in the various regressions.

Table 12: Descriptive Statistics for 1985 Time Use Study

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Work Time	2435	.00	1335.00	304.3341	242.69580
Night Sleep	2435	.00	885.00	452.6737	108.91255
Sleeping And Napping	2435	.00	1010.00	466.4518	110.12598
Sleep/naps and other personal activities	2435	.00	1010.00	480.3909	114.94937
Waking Leisure Time	2435	.00	1440.00	603.8638	254.72171
Estimated Wage Rate	2435	3.35	50.00	11.0154	7.70207
Labour Income	2435	1382.88	49635.07	20630.28	12131.00148
Household Income	2435	15000.00	50000.00	35327.22	12519.91608
Non-labour Income	2435	306.90	47234.24	14696.94	10484.38152
Age	2435	18	85	38.93	13.127
Female	2435	.00	1.00	.4568	.49823
# Children in Home Under Age 5	2435	0	3	.17	.474
# Children in Home Under Age 18	2435	0	7	.74	1.064
Not Finished High School	2435	.00	1.00	.0789	.26963
HS School Grad	2435	.00	1.00	.4463	.49721
Some College	2435	.00	1.00	.1785	.38298
College Grad	2435	.00	1.00	.1810	.38513
Post Grad	2435	.00	1.00	.0799	.27119
Married	2435	.00	1.00	.6589	.47416
Divorced	2435	.00	1.00	.1046	.30604
Widowed	2435	.00	1.00	.0315	.17463
Student	2435	0	1	.02	.139
Retired	2435	0	1	.01	.100
Fulltime Woker	2435	0	1	.87	.341
Part-time Worker	2435	0	1	.13	.341
Homemaker	2435	0	1	.01	.091
# Fulltime Workers in Home	2435	0	6	1.72	.840
Valid N (listwise)	2435				

The authors of the study calculated the estimated wage rate variable, but the data dictionary does not discuss the details of how it was calculated.

The daily time measurements (i.e., work time, night sleep, waking leisure time etc.) are calculated across individuals in minutes. For each individual there is data on how many minutes that individual performed each activity for in a one-day period. So, the mean column is the average number of minutes that the individuals performed the activity for in a 24-hour period. At most, an activity could be 1440 minutes long which is equivalent to 24 hours.

According to the data, the average person sleeps for 7¾ hours, and when naps and other rest are added to the sleep category, the average rises to 8 hours.

The most serious problem with the data is that the income variable is coded as category variable with four levels—there is no continuous household income variable. The income categories are (1) less than $15,000; (2) $15,000 - $24,999; (3) $25,000 - $34,999; (4) more than $35,000. Other than the wage rate, the other parameter of interest is the effect of non-labour income on the demand for sleep and leisure. So, I created a continuous non-labour income variable. First, I turned the household income categories into a cardinal variable by assigning to each data point the upper bound of the income range that it fell into. I know this practice is frowned upon. Then I took the wage rate, multiplied it by the individuals weekly work hours, and then multiplied this by 50 weeks to get a yearly labour income. Then I subtracted this labour income from the total household income variable to get a measure for the non-labour income. A regrettable effect of this decision is that when I derived the non-labour income some of the values came out negative. These observations were dropped from the data set.

VII. ESTIMATION AND RESULTS

The empirical version of the sleep allocation model consists of the following three demand equations:

$T_s = \alpha_{1s} + \alpha_{2s}W_m + \alpha_{3s}NLI + \beta_s X$	The demand for sleep
$T_z = \alpha_{1z} + \alpha_{2z}W_m + \alpha_{3z}NLI + \beta_z X$	The demand for waking leisure time
$T_w = \alpha_{1w} + \alpha_{2w}W_m + \alpha_{3s}NLI + \beta_w X$	The demand for work time

where,

W_m	=	log of market wage
NLI	=	log of non-labour income
X	=	control variables

In estimating systems of demand share equations, one of the equations has to be dropped to prevent the variance-covariance matrix from becoming singular and thus making the system impossible to estimate. Therefore, the demand for sleep and the demand for waking leisure will be estimated.

A. Biddle and Hamermesh's Estimation with 1975-76 Data

With the data that they have available, Biddle and Hamermesh have to derive values for the hourly wage rate and non-labour income. They compute these two variables as follows:

W_m = LOG [(respondent's self-reported monthly earnings) / (4.3*weekly hours)]

NLI = LOG [Respondent's 1974 household income – (spouses expected 1975 income + 12 * respondent's self-reported monthly earnings)]

Biddle and Hamermesh believe that the wage rate variable that they calculate may suffer from measurement error. They argue, "Any bias induced by error in this wage measure due to errors in the reported hours is removed by the use of instrumental variables." Later they state, "By using instrumental variables for W_m, we are implicitly viewing the equations for T_s and T_z as part of a simultaneous system, because of the possible impact of both T_s and T_z on the wage rate." It seems to me that the main reason Biddle and Hamermesh use an instrumental variable is to correct for possible measurement error, because in the above system W_m is treated as an exogenous variable.

Having decided to use an instrumental variable for the wage rate, they try estimating the demand equations using two different methods—on their first attempt, they try to correct for sample selection bias, but in the end they decide to estimate the equations for only the working population.

In their attempted sample selection corrected model, they run a probit regression over the entire population where the dependent variable is a dummy for labour force participation (LFP). The equation they estimate is:

$$LFP = \alpha + \alpha_1*(age) + \alpha_2*(age^2) + \alpha_3*(religion) + \alpha_4*(health\ status) + \alpha_5*(sex) + \alpha_6*(marriage) + \alpha_7*(children)$$

From the results of this equation they use the inverse mills ratio as an independent variable in their wage equation. The wage equation is used to generate the instrumental variable for the wage that will then be used in the time demand equations. They run the following wage equation over all of the people with wages:

Log Wage = α + α_1*(education) + α_2*(experience) + α_3*(sex) + α_4*(marriage) + α_5*(union) + α_6*(health) + α_7*(race) + α_8*(metropolitan area) + α_9*(region) + α_{10}*(occupation code) + α_{11}*(inverse mills ration)

Biddle and Hamermesh find the inverse mills ratio is insignificant in the wage equation and so they decide to estimate the wage and demand equations for only those who are working.

Correcting for selection bias does not seem relevant in the sleep/work trade-off. The wage rate is the person's opportunity cost of time, but in order to make the trade-off a person needs to have actual work time that they can trade-off with sleep time. The "demand for sleep" really should be called "workers demand for sleep".

The wage equation Biddle and Hamermesh estimate is:

Log Wage = α + α_1*(education) + α_2*(experience) + α_3*(sex) + α_4*(marriage) + α_5*(union) + α_6*(health) + α_7*(race) + α_8*(metropolitan area) + α_9*(region) + α_{10}*(occupation code)

From the estimated equation, they use the predicted wage as the instrumental variable. They then estimate the two demand equations. Since there are three equations being estimated together (i.e., the wage equation and the two time demand equations), they have to make sure that the equations are identified. Biddle and Hamermesh place zero restrictions on some of the control variables in the time demand equations. They drop the variables for education, experience, union status, metropolitan area, region, and occupation code. So, they estimate the following time demand equations:

$T_s = \alpha + \alpha_1$*(predicted LOG wage) + α_2* (LOG non-labour income) + α_3*(Married) + α_4*(Years Married) + α_5*(Age) + α_6*(Age^2) + α_7*(sex) + α_8*(health) + α_9*(race) + α_{10}*(Children under age 3) + α_{11}*(religion)

$T_z = \alpha + \alpha_1$*(predicted LOG wage) + α_2* (LOG non-labour income) + α_3*(Married) + α_4*(Years Married) + α_5*(Age) + α_6*(Age^2) + α_7*(sex) + α_8*(health) + α_9*(race) + α_{10}*(Children under age 3) + α_{11}*(religion)

They also mention that they tried to test for the possibility that sleep enhances productivity by using sleep time as an independent variable in the wage equation. Recall that Biddle and Hamermesh have sleep affecting productivity in the formal model. Unfortunately for this system of simultaneous equations, Biddle and Hamermesh do not have enough variables in their data that can be used in the time demand equations but dropped from the wage equation in order to identify the system. So, they abandon testing for the productivity effect.

B. Estimation with the 1985 Data

The data dictionary for the data set that I use simply states that the wage rate is "imputed from both household and respondent information." So, it is not obvious whether an instrumental variable should be used for the wage rate. I estimate the time demand equations in two different ways. First, I estimate the two demand equations with the SURE estimator. In the second case, I estimate an instrumental variable for the wage rate as Biddle and Hamermesh did and then estimate the demand equations substituting in the instrumental variable. The system is estimating using an IV systems estimation routine in the LimDep computer program. All of the estimations are only for the model where sleep does not have a productivity enhancing effect because there are not enough variables in the data set to be able to identify all of the equations.

Before testing for the income and substitution effects on sleep and waking leisure time individually, I estimated equations on total non-working time (i.e., sleep time + waking-leisure time). Standard models of leisure demand measure the trade-off between work time and total leisure. In the sleep allocation model, total leisure has been split into two components—waking leisure and sleep. Estimating the demand for total leisure will let us see if the data is consistent with the existing research. According to the formal model, there should be a negative wage effect on total leisure time and a positive income effect on total leisure time. I estimated twice, once using OLS and then using an instrumental variable for the wage rate.

OLS Equation

Total Leisure = α_1 +α_2*(LOG Wage) + α_3*(LOG non-labour income) + α_4*(age)+ α_5*(age^2)+ α_6*(female) +α_7*(children under age 5) + α_8*(married) + α_9*(divorced) + α_{10}*(widowed) + α_{11}*(Sunday)

IV Equations

LOG Wage = α_1 + α_2*(age) + α_3*(age^2) + α_4*(female) + α_5*(under high school) + α_6*(some college) + α_7*(post grad) + α_8*(retired) + α_9*(student) +α_{10}*(homemaker) +α_{11}*(fulltime)

$$\text{Total Leisure} = \alpha_1 + \alpha_2\text{*(predicted LOG Wage)} + \alpha_3\text{*(LOG non-labour income)} + \alpha_4\text{*(age)} + \alpha_5\text{*(age}^2) + \alpha_6\text{*(female)} + \alpha_7\text{*(children under age 5)} + \alpha_8\text{*(married)} + \alpha_9\text{*(divorced)} + \alpha_{10}\text{*(widowed)} + \alpha_{11}\text{*(Sunday)}$$

For the wage equation, I used human capital variables (i.e. education) and labour force status variables (i.e., whether the person is a, student, homemaker, or retired—in the data there are retired people who have work hours). These variables were then dropped from the leisure demand equation in order to make sure that the equation is identified. The human capital and labour force status variables should probably be uncorrelated with error term in the total leisure equation. When the sleep and waking-leisure time equations are separately estimated (e.g., table 15), they have the same variables as the Total Leisure equations in the OLS and IV estimations.

Table 13: Dependent Variable is Total Leisure in minutes/day

	OLS				IV		
	All				All		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Total Leisure	22.02	19.61	0.13	**Total Leisure**	108.84	16.19	0.13
Std. Error	9.55	4.99		Std. Error	28.11	4.75	
t-stat	2.31	3.93		t-stat	3.87	3.41	
	Men				**Men**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Total Leisure	38.20	14.60	0.10	**Total Leisure**	132.31	6.59	0.11
Std. Error	14.70	6.80		Std. Error	38.56	6.30	
t-stat	2.60	2.15		t-stat	3.43	1.05	
	Women				**Women**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Total Leisure	3.50	22.99	0.13	**Total Leisure**	49.50	23.69	0.13
Std. Error	12.20	7.84		Std. Error	32.14	7.68	
t-stat	0.29	2.93		t-stat	1.54	3.08	

For the above results, all of the adjusted R^2 values vary between 0.10 and 0.13. Having the dummy variable for Sunday as one of the regressors changes the value of R^2 by a large amount. Without controlling for the day of the week, the R^2 values ranged from between 0.01 to 0.04. For the entire sample and men-only sample, the wage effect is positive for both the OLS and IV estimations. This positive coefficient on the wage implies that as wages increase, people will work less. This result contradicts both theory and estimation results from other studies that have measured the substitution effect between working and total leisure.

Here are the above results in the form of elasticities:

Table 14: Elasticities for Total Leisure Time Equation

	OLS				IV		
	All				**All**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Total Leisure	0.02	0.02	0.12	**Total Leisure**	0.10	0.02	0.12
Std. Error	0.01	0.00		Std. Error	0.03	0.00	
t-stat	2.47	3.91		t-stat	3.77	3.35	
	Men				**Men**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Total Leisure	0.04	0.01	0.09	**Total Leisure**	0.14	0.01	0.10
Std. Error	0.01	0.01		Std. Error	0.04	0.01	
t-stat	2.68	2.12		t-stat	3.51	0.95	
	Women				**Women**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Total Leisure	0.00	0.02	0.12	**Total Leisure**	0.04	0.02	0.12
Std. Error	0.01	0.01		Std. Error	0.03	0.01	
t-stat	0.36	3.14		t-stat	1.37	3.25	

Looking at the estimation results for total leisure in terms of elasticities shows that the effects are small. For example, for the all-respondents sample under OLS estimations, 1% increase in the wage leads to a 0.02% increase in the daily time devoted to leisure. Under IV estimation, a 1% increase in the wage rate leads to a 0.10% increase in the total time devoted to leisure. The income effects have the predicted positive signs and they are also small. I also estimated total leisure over the entire sample of workers and non-workers and found a negative coefficient on the wage, but it was essentially zero (see tables 19 and 20 in the appendix for the results)

The next table presents Biddle and Hamermesh's results alongside the results from my estimations. In this set of results, the dependent variable is night sleep +

napping during the day. The table shows both OLS and IV estimation results. Also, it shows the *p-values* for the following three hypothesis tests:

H_0: The coefficient on wage rate is equal for both the sleep and waking leisure equations

H_0: The coefficient on non-labour income is equal for both the sleep and waking leisure equations

H_0: All of the coefficients for both the sleep and waking leisure equations are equal.

In their paper, Biddle and Hamermesh do not provide the value of all of their test statistics—for some of the tests, they only mention if it is rejected or not.

Table 15: Sleep Demand and Waking Leisure Demand Equations

	B&H Results					*SURE*			
	All					**All**			
	Wage	Income	Adj. R2	rho		Wage	Income	Adj. R2	rho
Sleep/Naps	-141.44	-1.78	0.02		**Sleep/Naps**	1.65	5.71	0.05	
Std. Error	77.35	4.80			Std. Error	3.98	2.00		
t-stat	-1.83	-0.37			t-stat	0.41	2.86		
				-0.24					-0.05
Waking Leisure	132.18	-1.71	0.16		**Waking Leisure**	23.03	13.72	0.10	
Std. Error	129.37	8.09			Std. Error	8.96	4.68		
t-stat	1.02	-0.21			t-stat	2.57	2.93		
H0: = wage effect	0.24				H0:wage effect	0.03			
H0: = Income effect	fail to reject				H0:Income effect	0.13			
H0: = all coefficients	reject				H0:all coefficients	0.83			
	Men					**Men**			
	Wage	Income	Adj. R2	rho		Wage	Income	Adj. R2	rho
Sleep/Naps	-181.68	-2.88	0.04		**Sleep/Naps**	6.81	5.88	0.05	
Std. Error	120.88	5.77			Std. Error	6.12	2.82		
t-stat	-1.50	-0.50			t-stat	1.11	2.09		
				-0.23					-0.05
Waking Leisure	233.34	-6.69	0.05		**Waking Leisure**	32.86	9.11	0.06	
Std. Error	193.67	9.30			Std. Error	13.73	6.34		
t-stat	1.20	-0.72			t-stat	2.39	1.44		
H0: = wage effect	0.22				H0:wage effect	0.08			
H0: = Income effect	fail to reject				H0:Income effect	0.64			
H0: = all coefficients	reject				H0:all coefficients	0.39			
	Women					**Women**			
	Wage	Income	Adj. R2	rho		Wage	Income	Adj. R2	rho
Sleep/Naps	-64.30	1.55	0.02		**Sleep/Naps**	-2.85	6.42	0.04	
Std. Error	93.44	8.43			Std. Error	5.20	3.32		
t-stat	-0.69	0.18			t-stat	-0.55	1.93		
				-0.27					-0.07
Waking Leisure	-262.42	14.44	0.05		**Waking Leisure**	11.51	15.80	0.10	
Std. Error	166.99	14.80			Std. Error	11.68	7.40		
t-stat	-1.57	0.98			t-stat	0.99	2.14		
H0: = wage effect	0.43				H0: = wage effect	0.27			
H0: = Income effect	fail to reject				H0:Income effect	0.26			
H0: = all coefficients	reject				H0:all coefficients	0.11			

IV				
All	Wage	Income	Adj. R2	rho
Sleep/Naps	-3.84	5.47	0.05	
Std. Error	11.73	1.98		
t-stat	-0.33	2.76		
				-0.05
Waking Leisure	118.65	10.16	0.10	
Std. Error	26.36	4.46		
t-stat	4.50	2.28		
H0: = wage effect	0.00			
H0: = Income effect	0.34			
H0: = all coefficients	0.11			
Men	Wage	Income	Adj. R2	rho
Sleep/Naps	4.57	4.66	0.05	
Std. Error	16.03	2.62		
t-stat	0.29	1.78		
				-0.05
Waking Leisure	125.88	2.05	0.07	
Std. Error	35.85	5.86		
t-stat	3.51	0.35		
H0: = wage effect	0.00			
H0: = Income effect	0.68			
H0: = all coefficients	0.41			
Women	Wage	Income	Adj. R2	rho
Sleep/Naps	-11.50	6.57	0.04	
Std. Error	13.65	3.26		
t-stat	-0.84	2.02		
				-0.07
Waking Leisure	71.29	16.02	0.10	
Std. Error	30.59	7.31		
t-stat	2.33	2.19		
H0: = wage effect	0.02			
H0: = Income effect	0.25			
H0: = all coefficients	0.20			

When looking at the above results, the first thing to keep in mind is that the dependent variable for Biddle and Hamermesh's equations is total minutes spent sleeping or engaged in waking leisure *over the week,* whereas in my results the dependent variables is total minutes spent sleeping or engaged in waking leisure *over the day.*

For Biddle and Hamermesh's estimations, the value for the adjusted R^2 is small except for the equation estimating waking leisure time for all respondents, where it is 0.16. For all of their other equations, the adjusted R^2 never exceeds 0.05. All of the coefficients on the income variables are insignificant. The hypothesis that the wage coefficients are equal in both equations and the hypothesis that the income coefficients are equal in both equations cannot be rejected. The hypothesis that all of the coefficients are equal for both equations can be rejected. In the above chart, the rho column lists the contemporaneous correlations between the error terms in the sleep and waking leisure equations. The negative signs on these correlations indicate that people substitute between waking leisure and sleeping.

The three most significant effects that Biddle and Hamermesh find are a negative coefficient on the wage rate for the sleep equations for both the all-respondents sample and the men-only sample and a negative coefficient on wage rate for the waking leisure equation in the women-only sample. For these three results, the t-statistic ranges from negative 1.50 to negative 1.83, so the results are not very strong. The results indicate that when wages increase, there is substitution away from sleep. Biddle and Hamermesh calculate a sleep-wage elasticity of negative 0.042. In the women-only sample, the coefficient on the wage rate for the sleep equation is insignificant, but the negative coefficient on waking leisure indicates that women will substitute out of leisure and towards work if their wage increases.

In my estimation results, the adjusted R^2 values are also small; they are comparable to Biddle and Hamermesh's results. The values for the contemporaneous correlation between the error terms for the sleep and waking leisure equations average around negative 0.05 whereas Biddle and Hamermesh found the correlation to be in the negative 20% range.

The coefficient on the wage rate for the sleep demand equation is insignificant for all of the sub-samples under both SURE estimation and IV estimation. There are significant positive wage effects on waking leisure for the all-respondents sample and the men-only sample under both SURE and IV estimation and for the women-only sample under IV estimation. This is consistent with the formal model of sleep allocation which predicts a positive substitution effect for waking leisure, however the fact that the sleep substitution effects are zero implies that the total substitution effect is positive which contradicts the formal model. This result is consistent with the estimation results for total leisure time presented in table 11 above. The hypothesis test that the wage effects are equal for the sleep equation and waking leisure equations are strongly rejected under IV estimation.

In the all-respondents sample and the female-only sample, there are positive significant income effects on both sleep and waking leisure for SURE and IV estimation.

I also estimated a sleep demand and waking leisure demand equation over the entire sample and found a negative sign on the wage for the sleep and waking leisure equations, but the coefficient was essentially zero (see tables 21 and 22 in the appendix). In the appendix there are also results for a two stage least squares estimate of the sleep and waking leisure equations (table 23). In the 2SLS estimation, the instrument for the wage rate was created using all of the exogenous variables. The 2SLS estimate does not indicate any significant wage effect on sleep either.

The following chart presents all of the results of table 15 in the form of elasticities:

Table 16: Elasticities for Sleep Demand and Waking Leisure

	SURE				IV		
	All				All		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Sleep/Naps	0.01	0.01	0.03	**Sleep/Naps**	-0.02	0.01	0.03
Std. Error	0.01	0.01		Std. Error	0.03	0.01	
t-stat	1.02	2.55		t-stat	-0.57	2.40	
Waking Leisure	0.05	0.03	0.09	**Waking Leisure**	0.23	0.02	0.09
Std. Error	0.02	0.01		Std. Error	0.05	0.01	
t-stat	3.08	3.40		t-stat	4.52	2.56	
	Men				Men		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Sleep/Naps	0.02	0.02	0.03	**Sleep/Naps**	0.03	0.01	0.03
Std. Error	0.02	0.01		Std. Error	0.04	0.01	
t-stat	1.44	2.00		t-stat	0.64	1.51	
Waking Leisure	0.08	0.03	0.05	**Waking Leisure**	0.25	0.01	0.06
Std. Error	0.03	0.01		Std. Error	0.07	0.01	
t-stat	2.84	2.11		t-stat	3.40	0.90	
	Women				Women		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Sleep/Naps	0.00	0.01	0.02	**Sleep/Naps**	-0.04	0.01	0.02
Std. Error	0.01	0.01		Std. Error	0.03	0.01	
t-stat	-0.12	1.65		t-stat	-1.03	1.61	
Waking Leisure	0.02	0.03	0.09	**Waking Leisure**	0.12	0.03	0.09
Std. Error	0.02	0.01		Std. Error	0.05	0.01	
t-stat	1.21	2.10		t-stat	2.21	2.11	

When viewed in terms of elasticities, the significant positive effects of wages on waking leisure seem small. For example, for the all-respondents sample under IV estimation, a 1% increase in the wage rate leads to a 0.23% increase in time spent on waking leisure time over a day.

The main difference between the results obtained from SURE estimation and IV estimation is the size of the coefficient on the wage rate variable in the waking leisure equations— 23.03 versus 118.65 in the all-respondents sample, 32.86 versus 125.88 in the men-only sample and 11.51 versus 71.29 in the female-only sample.

In the above equations sleep is defined as night sleep plus naps, but sleep can be defined more narrowly as night sleep alone or more broadly as night sleep + naps + miscellaneous personal time. Here are the estimation results for these two definitions of sleep:

Table 17: Demand for Night Sleep

	OLS				IV		
	All				**All**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Night Sleep	3.75	5.62	0.05	**Night Sleep**	9.25	5.05	0.05
Std. Error	3.92	2.05		Std. Error	11.58	1.96	
t-stat	0.96	2.74		t-stat	0.80	2.58	
	Men				**Men**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Night Sleep	9.02	5.57	0.05	**Night Sleep**	23.56	3.76	0.05
Std. Error	6.09	2.81		Std. Error	15.95	2.61	
t-stat	1.48	1.98		t-stat	1.48	1.44	
	Women				**Women**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Night Sleep	-0.07	6.64	0.05	**Night Sleep**	-4.99	6.61	0.05
Std. Error	5.07	3.24		Std. Error	13.30	3.18	
t-stat	-0.01	2.05		t-stat	-0.38	2.08	

Table 18: Demand for Sleep/Naps/Miscellaneous Personal Time

	OLS				IV		
	All				**All**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Sleep/Naps/Misc.	-1.00	5.87	0.05	**Sleep/Naps/Misc.**	-9.81	6.03	0.05
Std. Error	4.45	2.17		Std. Error	12.23	2.07	
t-stat	-0.22	2.71		t-stat	-0.80	2.91	
	Men				**Men**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Sleep/Naps/Misc.	5.43	5.54	0.05	**Sleep/Naps/Misc.**	6.42	4.54	0.05
Std. Error	6.44	2.97		Std. Error	16.87	2.75	
t-stat	0.84	1.87		t-stat	0.38	1.65	
	Women				**Women**		
	Wage	Income	Adj. R2		Wage	Income	Adj. R2
Sleep/Naps/Misc.	-7.95	7.19	0.04	**Sleep/Naps/Misc.**	-21.79	7.67	0.04
Std. Error	5.36	3.42		Std. Error	14.07	3.36	
t-stat	-1.48	2.10		t-stat	-1.55	2.28	

In both of these estimates again there is not any strong evidence of a wage effect on sleep time. The most significant effect is found on women's sleep time for the broadest definition of sleep under both OLS and IV estimation (table 18), where the wage effect is negative.

Overall the estimation results indicate that people do not adjust their daily sleep time to increases in the wage rate. There are small adjustments in waking leisure time to increases in the wage. And there are also small income effects on sleep time and waking leisure time.

APPENDICES

Running Total Leisure over the entire sample:

Table 19: Estimating Total Leisure using the Entire Sample

	IV		
	All		
	Wage	Income	Adj. R2
Total Leisure	-0.34	17.66	0.36
Std. Error	0.01	3.99	
t-stat	-27.72	4.43	

Table 20: Total Leisure in Elasticity form for the entire sample

	IV		
	All		
	Wage	Income	Adj. R2
Total Leisure	-0.000302	0.016000	0.330000
Std. Error	0.000012	0.003840	
t-stat	-26.034483	4.166667	

Table 21: Sleep Demand and Waking-leisure Demand over the entire sample

	IV All Wage	Income	Adj. R2	rho
Sleep/Naps	-0.03	6.98	0.05	
Std. Error	0.01	2.02		
t-stat	-4.78	3.46		
				-0.21
Waking Leisure	-0.29	11.20	0.28	
Std. Error	0.01	3.93		
t-stat	-24.71	2.85		
H0: = wage effect	0.00			
H0: = Income effec	0.38			
H0: = all coefficien	0.45			

The above table in elasticity form:

Table 22: Sleep and waking-leisure demands in elasticity form

	IV All Wage	Income	Adj. R2
Sleep/Naps	-0.00006	0.01590	0.04000
Std. Error	0.00001	0.00493	
t-stat	-4.20134	3.22515	
Waking Leisure	-0.00047	0.02470	0.25400
Std. Error	0.00002	0.00726	
t-stat	-21.36364	3.40220	

Estimating the sleep demand and waking-leisure demand using a 2SLS set up:

Table 23: 2SLS estimates of Time Demand Equations (Sample is workers)

	2SLS			
	All			
	Wage	Income	Adj. R2	rho
Sleep/Naps	-3.58	5.47	0.05	
Std. Error	11.71	1.98		
t-stat	-0.31	2.76		
				-0.05
Waking Leisure	118.06	10.14	0.10	
Std. Error	26.31	4.46		
t-stat	4.49	2.27		
	Men			
	Wage	Income	Adj. R2	rho
Sleep/Naps	5.00	4.66	0.05	
Std. Error	16.00	2.62		
t-stat	0.31	1.78		
				-0.05
Waking Leisure	125.66	2.04	0.07	
Std. Error	35.80	5.86		
t-stat	3.51	0.35		
	Women			
	Wage	Income	Adj. R2	rho
Sleep/Naps	-11.77	6.55	0.03	
Std. Error	13.63	3.26		
t-stat	-0.86	2.01		
				-0.07
Waking Leisure	69.78	16.05	0.10	
Std. Error	30.55	7.32		
t-stat	2.28	2.19		

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