

**AN EVALUATION OF NEGATIVE INCOME TAXES  
WITH HETEROGENEOUS AGENTS**

by

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## **ABSTRACT**

In this study I evaluate a negative income tax system set in a simple economy populated by different types of individuals, whose only choice is a decision about how to allocate discretionary time between working in the paid labour force and consuming leisure. In particular, I examine how different skill levels and different preferences for leisure, both separately and interactively, affect labour supply, earnings, income and welfare across these different types of individuals. A numerical simulation will predict the immediate response of these individuals to the sudden implementation of the tax system, with no attempt to judge the dynamics of the response. The findings are that there are unambiguously negative labour supply effects for all individuals, which are worse for those people who either have low skill, a high preference for leisure, or a combination of both. The effects on income and welfare vary with the type of individual.

## **DEDICATION**

*Dedicated to my family and friends.*

## **ACKNOWLEDGEMENTS**

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# 1 INTRODUCTION

For decades, economists have been studying the issue of poverty, specifically, how to design a mechanism to reduce the number of people having very low incomes. The most well-known of these mechanisms is what some call the “traditional” welfare system where, individuals are provided with a sum of money, which is decreased dollar for dollar with increases in income.<sup>1</sup> Recognizing the obvious work disincentive of such a policy, economists have studied a variant of the traditional welfare system – the negative income tax.

Although popularized by Milton Friedman in his 1962 book “Capitalism and Freedom,” precursors to negative income taxes (NIT) can actually be dated back a couple of centuries. Since that time this type of policy, commonly referred to as guaranteed annual income, has been subject to much research as a mechanism to alleviate poverty. The majority of these studies have been conducted since the 1960’s. Over these past four decades, there have been many commissions and many governments in Canada and the United States proposing variants of the NIT system as part of their political agendas. Despite this extensive amount of research, there has still not been a consensus among policy-makers and academics as to whether an NIT, or a variant thereof, is an effective policy in alleviating poverty while providing adequate work incentives.

To understand the reason for the lack of consensus, a quick review of the theory is necessary. There are varying designs of the program, but all have the same basic idea:

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<sup>1</sup> Many people refer to a tax-back rate of 100% as a “traditional welfare regime.” There are other schemes with lower tax-back rates, but this paper will follow in that trend.

individuals are guaranteed a basic annual income by the government. If they choose not to work in the paid labour force, then their income is simply the amount of the guarantee. If, however, an individual does choose to do paid work, then that person's income is taxed at a positive rate and subtracted from the basic income guarantee to determine the amount of benefit the individual will receive. For example, if the guaranteed basic annual income was \$5,000, and the individual earned \$1,000 in the paid labour force, and was subject to a 50% tax rate (typically called the reduction rate), then the benefit for this person would be \$4,500. In this example, for every dollar the individual earns, his benefit is reduced by \$0.50.

This benefit will be a positive amount as long as the after-tax earnings are below the annual income guarantee. The point at which they are both equal is normally referred to as the break-even point. Typically, when a person is below this threshold they are eligible for the NIT program, if they are above the break-even point, they are ineligible. Once ineligible, they return to the regular tax system. One can now see that the term "negative income tax" refers to the situation where an individual receives a net benefit from the government. Figure 1 shows a graphical representation of the textbook NIT policy. Line  $LI_1$  represents a traditional welfare regime, while line  $AI_2$  represents an NIT with some tax rate  $\tau$  below 100%, and a basic benefit equal to the distance  $LA$ . Point B is the break-even point in the NIT system. Note that here income and consumption are interchangeable.

Unfortunately, the theory behind NIT suggests some potential drawbacks. The positive work incentives of an NIT only apply in a world where a welfare system already exists; introducing an NIT system in a world where there was no previous welfare system

yields unambiguously negative labour supply effects. First, there is an adverse effect on labour supply of low-income individuals, taking leisure as a normal good. The income effect of this system is to induce the individual to work less, given that income is now higher for this individual at all hours of work below the break-even point. Furthermore, since each extra dollar earned in the labour force is taxed away - i.e. the cost of leisure is lower - the individual now has even more incentive to work less (the substitution effect). These two effects combine to produce a strong disincentive to increase work hours for those individuals who are receiving a net subsidy from the NIT.

The issue of the amount of the guarantee provides some problems also. For the government to promise a high amount of basic annual income could be very costly if there are a large proportion of people eligible for the NIT program. The government could simply reduce the basic benefit in that case, but if the guaranteed income were lowered so much that it became insufficient, then this policy would not achieve its goal of poverty reduction.

Fortunately, it is possible that this work disincentive effect could be very small and insignificant; also, people could use the extra leisure time they are consuming to engage in productive job search activities. Finally, comparing the NIT regime to other welfare designs may actually lead to increased work incentives relative to the alternatives.<sup>2</sup>

The discussion of the theoretical implications provides some important insights into the possible consequences of this poverty-reduction scheme, and begs the question:

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<sup>2</sup> Dwayne Benjamin and Morley Gunderson and W. Craig Riddell, *Labour Market Economics: Theory, Evidence and Policy in Canada*, 5<sup>th</sup> ed. (Toronto: McGraw-Hill Ryerson Limited, 2002), 84.

Would the model's predictions be dissimilar for different preferences and skills? Indeed they are, and the goal of this paper is to examine to what extent this heterogeneity has an effect on the sudden and unanticipated introduction of an NIT.

Within this context, I propose to evaluate a negative income tax system in a simple economy populated by different types of individuals, whose only choice is a decision between working in the paid labour force or consuming leisure. In particular, I will be examining how different skill levels and different preferences for leisure, both separately and interactively, affect labour supply, earnings, income and welfare across these different types of individuals. A numerical simulation will predict the immediate response of these individuals to the sudden implementation of the tax system, with no attempt to judge the dynamics of the response. Intuition suggests that those individuals with the lowest wage and the highest preference for leisure will enjoy the highest amount of welfare from the NIT system. Conversely, individuals with a high wage and a low preference for leisure will like the NIT regime the least compared to the other types of individuals in the model.

The paper will be structured as follows: Section 2 describes the model and provides the equation that characterizes the equilibrium labour supply. Section 3 discusses the calibration of the parameters within the model to some realistic values based on previous studies. Section 4 provides the results of the model using output from gauss. Section 5 discusses some of the implications of the results presented, and Section 6 concludes.

## 2 THE MODEL

The model used in this paper is slightly different than the basic model described above, but still captures the essence of the NIT design. The difference is that instead of having a set benefit rate and an eligibility criterion, this model assumes that all individuals pay a positive amount of income tax, and are subsequently given a subsidy from the government that is equal across all agents. Notice that this still retains the major elements of the NIT system: There is a guaranteed annual income equal to the government subsidy if the individual chooses no hours of work. As individuals increase their work hours above zero, there exists a point at which the income tax paid by individuals will exceed the amount of subsidy they receive – this is the break-even point in this model. In fact, the model used here closely resembles a universal demogrant, which is another type of guaranteed annual income policy.

Individuals maximize utility over two goods, consumption and leisure, with the following utility function<sup>3</sup>:

$$U = \ln c_{ij} + \left[ \frac{\phi_j}{(1-\eta)} \right] l^{(1-\eta)} \quad (1)$$

The parameter  $\phi$  represents the individual's preference for leisure over consumption. Higher values of  $\phi$  represent a higher weight being placed on leisure relative to consumption. The subscript  $j = L, M, H$  placed on this variable refers to its three possible values: low medium and high. Discussion of the quantitative

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<sup>3</sup> This utility function produces smooth, convex indifference curves. We can thus be certain that we have a unique interior solution.

characteristics of these three values follows in the next section. The parameter  $\eta$  is the curvature parameter on leisure. The higher is the value of this parameter, the less willing this individual will be to trade leisure for consumption. In other words, the higher the value of  $\eta$ , the more consumption and leisure are complements, the lower the value, the more they are substitutes. Note that when  $\eta$  is equal to zero, utility is linear in consumption and leisure, and non-linear otherwise.

Individuals can choose to allocate their fixed amount of time only to work or to leisure in this model, thus:

$$n_{ij} + l_{ij} = 1 \quad (2)$$

Where work, in this case, refers to participation in the paid labour market, and leisure refers to any other activity, including activities such as: unpaid work, labour market search, etc. Individuals are endowed with an amount of time equal to 1.

Individuals are subject to a budget constraint:

$$c_{ij} \leq (1 - \tau)w_i n_{ij} + S \quad (3)$$

where  $c$  is consumption,  $\tau$  is a distortionary income tax,  $w$  is the wage rate,  $n$  is employment and  $S$  is a per-person subsidy given to all individuals by the government. This constraint binds the individual to consume an amount equal to or less than his after-tax income. The subscript  $i = L, M, H$  on wages refers to three possible levels, low medium and high, the numerical values of which are discussed in the following section. Notice that since there is no saving in this model, individuals will consume all of their after-tax income.

Finally, the government in this model has only one role, which is to collect tax and redistribute its total collection equally to all individuals. Thus, the government's budget constraint is given by:

$$\tau[\lambda_L(w_L \sum_j n_{Lj}) + \lambda_M(w_M \sum_j n_{Mj}) + \lambda_H(w_H \sum_j n_{Hj})] = S \quad (4)$$

With the weighting parameter  $\lambda$ , the government's budget constraint incorporates that approximately 25% of individuals are high-school dropouts, 50% are high-school graduates and 25% are university graduates.<sup>4</sup> The wage rates in the model will be calibrated to match the earnings differentials among these three educational groups.

The goal of this paper is a) to examine the effects of changes in the preference parameter  $\phi$ , b) to estimate the effects of changes in  $\mathbf{w}$ , and c) to study the changes resulting from an interaction of the changes in those two parameters. To accomplish this end, the labour supply function of each agent must be found. To simplify the maximization, the constraint in equation (3) is substituted into equation (1), the individual's utility function. The resulting equation:

$$U = \ln((1 - \tau)w_i n_{ij}) + \left[ \frac{\phi_j}{(1 - \eta)} \right] n_{ij}^{(1-\eta)} \quad (5)$$

is then maximized with respect to  $\mathbf{n}_{ij}$  to yield the agents' labour supply functions.

The first order conditions of this maximization exercise are:

$$\left( \frac{(1 - \tau)w_i}{(1 - \tau)w_i n_{ij} + S} \right) - \phi_j (1 - n_{ij})^{-\eta} = 0 \quad (6)$$

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<sup>4</sup> David Andolfatto and Christopher Ferrall and Paul Gomme, "Lifecycle Learning, Earning, Income and Wealth," Manuscript: Simon Fraser University (2000), 1.

where the solutions to this system of equations characterize labour supply choices  $n_{ij}^*(\tau)$ . Due to the non-linearity of the utility function, the first-order conditions for this problem are a series of non-linear functions, which cannot be solved analytically, thus a numerical estimation procedure will be used. Depending on the number of different values for the leisure parameter and wage parameter, we will have  $i*j$  different first order conditions, plus the government budget constraint. A natural question to ask is why did I not specify the utility function so that the solution would be easier to characterize? The reasoning is that the labour-leisure decision makes much more sense as a non-linear decision.

It is natural to think that a person's labour-leisure choice has the diminishing MRS property, that is, that given that a person has almost no leisure, he would be willing to trade a lot of income for a little more leisure, and so on. The utility function in this paper generates smooth, convex indifference curves which possess exactly this property. Another nice property is that the solutions to this problem are interior. It would make less sense, for example, to use a linear specification, which would say that a person is equally willing to trade income for leisure regardless of how much of each he already has. Furthermore, a linear specification can only lead to either non-unique or corner solutions.

With the equilibrium so characterized, we can construct the Indirect Utility function, which will be used to calculate changes in welfare associated with changing the tax rate:

$$W_{ij}(\tau) = \ln c_{ij}^* + \left[ \frac{\varphi_j}{(1-\eta)} \right] l_{ij}^{*(1-\eta)} \quad (7)$$

where  $c_{ij}^*$  and  $l_{ij}^*$  are consumption and leisure choices respectively evaluated at their equilibrium levels.

Now that the solution equations have been characterized, it is time to choose values for the parameters to reflect real-world observations.

### 3 CALIBRATION

There are four parameters in the model that need to be calibrated. First,  $\tau$ , the value of the tax rate will be a 10-point grid:  $\tau \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ . The purpose of this is to examine first the difference between having an NIT against a laissez-faire regime, and second to observe the effects of an increase in the tax rate as it gets closer to 1 (the traditional welfare regime).

The value for  $\eta$  is chosen to be 1.5. This is a standard choice for this taste parameter.<sup>5</sup>

The value of  $\phi$  for an average individual is chosen to be 1.48, which reflects the fact that in the data<sup>6</sup>, average individuals spend about 35% of their discretionary time in the paid labour market and 65% of their time in leisure activities. This value will be considered to be the median value for this taste parameter. The low value is chosen to be 0.84 and the high value 3.52. These values are calibrated to match the fact that, according to the distribution of hours worked per week, the first third of individuals spend about 20% of their time working, and the last third spend about 46% of their time working.<sup>7</sup>

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<sup>5</sup> David Andolfatto and Christopher Ferrall and Paul Gomme, "Lifecycle Learning, Earning, Income and Wealth," Manuscript: Simon Fraser University (2000), 9.

<sup>6</sup> The source data for this study is a 10% random sample of the 1996 Canada Census individual file.

<sup>7</sup> To get these values, I took the variable for hours worked per week and sorted it from lowest to highest. I then separated it into three equal groups which I define to be the people with high, median and low preferences for leisure respectively (i.e. low, median and high preferences for work). I then calculate the average hours worked in each group and solve for the value of  $\phi$  using the first order conditions.

Finally,  $w$  is calibrated to match the earnings distribution over three different education types: high-school dropouts, high-school graduates and university graduates. From the data, the values of average earnings were calculated for these three groups.<sup>8</sup> In particular, the earnings values reflect only those individuals aged 24-65 who worked positive hours during the year. Furthermore, observations that were missing were excluded.<sup>9</sup> From this exercise, it was determined that  $w \in \{10.34, 12.63, 17.92\}$ , where the numbers are expressed as hourly wage rates.

Some justification is in order for the groups selected in this calibration. First, the age group was chosen because to compare between groups, it is necessary that most individuals have finished all of their schooling and begun careers. It is reasonable to assume that by the age of 24, the university graduates have completed their (bachelor's) degrees and found employment. Since this paper is concerned more with labour supply at the *intensive* margin (i.e. time spent in the labour force), and less at the *extensive* margin (i.e. the binary choice of being in or out of work), it is better to have calibrated the wage parameter with those people who are actually working. For this reason, only those individuals who worked positive hours were included.

One last note about the calibration of the wage parameter is that there has been almost no disaggregation in the data (except by what has already been noted in the previous paragraph). The reader should keep in mind that these values of the wage

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<sup>8</sup> The variable for highest degree, certificate or diploma (dgreep) was the variable of choice in this calibration. From the Census survey, those who answered "No degree, certificate or diploma" form the high school dropout group; those who answered "Secondary (high) school graduation certificate or equivalent" form the high school graduate group; those who answered "Bachelor's degree(s)" form the university graduate group.

<sup>9</sup> Of course, there is the possibility that there is some systematic reason why certain persons have missing observations and by throwing them away I throw away information. However, there has been no regression done here; simple averages were calculated, so including these observations would have added no useful information.

parameter would be significantly different when comparing females versus males for example. This will be discussed later in section 5.

With the model so calibrated, it is now time to examine first the effects of the NIT on individuals with heterogeneous skills and homogeneous preferences for leisure. Following that, I will study the opposite case where skill is the same across agents, but their preferences for leisure are heterogeneous. Subsequently, I will examine the effects when both wages and preferences for leisure are heterogeneous.

## 4 RESULTS

### *4.1 Heterogeneous Skill*

As mentioned in the last section, there will be three values for the wage, which are calibrated to match earnings differences among high school dropouts, high school graduates and university graduates. The parameter  $\phi$  is set here to its median value, 1.48. The numerical results of this simulation can be found in Table 1.

Figure 2 plots the after-tax income of the three types of people in this model economy. As expected, for all individuals, after-tax income is decreasing in the tax rate.<sup>10</sup> Notice that the steepness of the function is greatest for the high-wage individual, which is to say that marginally, this type of person is most affected by increases in the tax rate. Intuitively this makes sense because with the NIT regime that has been constructed in this paper, the high-wage individual is basically subsidizing the lower-wage individuals. The NIT has a waterfall effect, which is to say that the highest income individuals are providing subsidy to all individuals below them. However, this is not always necessarily true, since it is dependent on the concentration of a certain income group within the population. The population distribution here was calibrated so that high-school graduates represent 50% of the population, so conceivably there could be wage structures where the high school grads would actually be subsidizing both higher income groups and lower income groups.

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<sup>10</sup> Remember that given the budget constraint for the individual, after-tax income is equivalent to consumption in the NIT regime, while consumption at a zero tax rate (i.e. without the NIT regime) is equivalent to earnings. Thus, any comparison of after-tax income to earnings is the same as comparing consumption with and without the NIT regime. This follows from the fact that there is no saving in this model.

What about the effect if the implementation of the regime? One way to gauge the result is to compare the after-tax income to the earnings for each type of individual. Figure 3 plots the earnings of the three types of wage earners. For the low-wage individual, the earnings function lies below the after-tax income function at all levels of the tax. In fact, the more the tax rate increases, the greater is the gap between these two functions. Therefore, in this regard, the low-wage individual benefits greatly from the welfare regime.

For the median-wage individual, the earnings function also lies entirely below the after-tax income function. Both he and the low wage individual are being subsidized by the high wage individual. The gap between these two functions widens as the tax rate rises, but not by much; that is, his labour supply decisions are such that he is paying almost as much into the subsidy fund as he is receiving.

Finally, for the high-wage individual, earnings are always higher than after-tax income, with the gap widening at each level of the tax rate. This individual enjoys consumption most without the NIT system no matter what the tax rate. This result is not surprising since this type of person contributes a disproportionately high amount to the total subsidy given his wage is 41% higher than the level below him and 73% above the lowest wage earner. Thus, what we are seeing here is the waterfall effect that was described above.

In terms of consumption, the model predicts that for most of the population (about 66% of individuals) the NIT regime is beneficial, and for the other 34% (the high wage earners) it is detrimental. To get a quantitative estimate of exactly how much better or

worse off these individuals may be from this NIT regime, a welfare function has been constructed, and is presented in Figure 4.

The first thing to notice is that both the median-wage and low-wage individuals maximize their welfare at positive tax rates. For the low-wage individual, this tax rate is around 35%, and for the median wage individual, it is about 20%. This is not true for the high-wage individual who maximizes his welfare at a zero tax rate. Consequently, using the welfare function, we can reiterate what was discussed previously with the consumption comparisons: The low-wage individual likes a NIT regime with a very heavy tax burden. The reason is that this type of individual contributes relatively little to the government in terms of taxes, so he stands to gain the most from the subsidy. On the other hand, the high-wage individual is exactly the opposite, and the median-wage individual is somewhere in between, leaning slightly towards a positive tax rate.

Although the previous paragraph shed light on exactly how high of a tax rate would be most preferred by each type of individual, it did not tell us exactly how much would be gained or lost by the implementation of the NIT regime, and also with successive increases in the tax rate. For this reason, the compensating variation is used to provide the percentage of consumption that the individual would have to be given to live in the progressively burdensome NIT regime. Figure 5 shows these percentages at increasing levels of the tax rate. It is showing us essentially what the individual would pay to not have the negative income tax redistribution policy imposed on him, that is, where he is indifferent between having the negative income tax or no redistribution policy at all. Intuitively, this should be equal to the losses he expects to generate when he compares this to the world with no redistribution policy. If there are any gains, then it is

also clear that the individual would be willing to pay that amount to have the policy imposed on him.

With that in mind, there are a few interesting things to notice in Figure 5. First, the low-income individual is actually willing to give up a significant amount of consumption to live in the NIT regime up to a tax rate of about 35%. The median-income person is virtually indifferent between a tax rate of 20% and no NIT regime. Finally, the high-wage individual will always need to be compensated a percentage of consumption to live in a world where the NIT exists. This can be most easily seen on the graph where the line crosses the horizontal axis.

One final note about Figure 5 is that the cost of a marginal increase in the tax rate is increasing at an increasing rate. For example, the low-wage individual would have to be compensated 25% of consumption to live with a tax rate of 0.8, but would have to be compensated 61% of consumption to live with a tax rate of 0.9. The results are not surprising since at low levels of tax, the low-wage individual is contributing little and receiving much, so he will enjoy the welfare state. At the high tax level, the low-wage person is contributing nothing, but his subsidy is quickly decreasing, so he will not like the increasing tax burden. As the tax rate increases, all individuals supply less and less labour in equilibrium, the amount of the subsidy declines, and the individuals lose an increasing amount in terms of consumption.

What about the labour supply effects? Judging by the results using this model, there is only a negative incentive effect. Looking at Figure 6, we can see that for all individuals, the equilibrium labour supply function is always decreasing (at an increasing rate) in the tax rate. The low-wage individuals have strongest disincentive effect, and

will actually supply no labour once the tax rate reaches about 90%. The other individuals will supply a positive amount of labour even at very high tax rates, although it is not a very high amount (as low as 0.7% of their available time). See Table 1 for the tax elasticity of labour supply.

The substitution effect here is negative since the increase in the tax rate makes the cost of leisure lower by essentially lowering the wage; hence the incentive is to consume more leisure. There are two income effects. First, the increase in the tax rate reduces income at some given number of hours of work, and since leisure is a normal good, he consumes less of it – and more work. Second, the subsidy provides extra income to individuals at some given number of hours of work, which decreases work incentives. What we are seeing here is either a dominant substitution effect or that the net income effect is working in the same direction as the substitution effect. In any case, individuals unambiguously choose more leisure at higher tax rates.

The implications of the adverse labour supply effect are somewhat discouraging. At positive tax rates, the equilibrium labour supply of all individuals is lower, with the strongest disincentive effect shown by the lowest-wage level. This suggests that the NIT system has strong negative incentives, which has concerned many researchers in the past, with the added problem that *everyone* will supply less labour in equilibrium, not just the low-wage individuals. The NIT will be evaluated for its redistributive qualities in section 5.

Now that the theoretical implications have been considered, it is useful to see how the model's predictions match up with the data. Table 4 presents this comparison for

after-tax income, earnings and subsidy using model results with a flat tax rate of 10%.<sup>11</sup> The comparison shows that the hypothetical NIT system actually does not do a terrible job in replicating the real world data, with the obvious exception of the level of the subsidy.

The general observations are that the model under-predicts the level of earnings and labour supply hours, and over-predicts the after-tax income and the level of the subsidy. It must also be noted here that the hypothetical NIT at the 10% tax level is the best at generating values close to the data. At other tax rates, the subsidy prediction in particular is much higher than is observed in the data. Nevertheless, the most promising part of the model is that it captures the fact that in the real world, people with higher levels of education tend to have higher levels of earnings and after-tax income; In fact, no matter what the tax rate, the model results possess this feature.

#### ***4.2 Heterogeneous Preference for Leisure***

Now that we have seen the results of the NIT system on different skill levels, it is time to switch to a comparison across preferences, specifically that for leisure. Remember that in the section on calibration, it was noted that the average person spends about 1/3 of his time in the labour force, which led to  $\phi = 1.48$ , and that the low and high values are taken to be 0.84 and 3.52 respectively. Results from this simulation are presented in Table 2.

The conclusions are much the same as with the different values of skill, which is to say that the individuals who enjoy leisure the most will have the highest preference for

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<sup>11</sup> It must be noted here that the Census data did not contain a variable for total taxes paid by individuals. Thus, I had to approximate the after-tax income in the data. To accomplish this end, I used the federal marginal tax rates (16% for first \$32,183 and 22% between \$32,184 and \$64,368) for 1996 and applied them to the individual's total income.

the NIT system, and the strongest negative equilibrium labour supply effect. Figure 7 shows the after-tax income of the three types of individuals at increasing values of the tax rate. As expected, the person that enjoys leisure the least (or work the most) has the highest value of after-tax income, while the person who enjoys leisure the most has the lowest after-tax income. So it follows that the person with the low preference for leisure enjoys the most consumption at all levels of the tax rate, with the exception that in the limit as  $\tau \rightarrow 1$  everyone converges to a zero level of consumption. The reason for this is that labour supply goes to zero for all individuals as  $\tau \rightarrow 1$ . Intuitively, this is reasonable: why would anyone work if all of the earnings are taxed away?

Comparing once again the after-tax income functions in Figure 7 to the earnings functions in Figure 8 for each individual, we see nearly the same results as in the previous subsection. The individual with the highest preference for leisure has the earnings function lying below the after-tax income function at all levels of the tax rate, with the gap widening as the tax rate increases. The median-preference-for-leisure individual has an earnings function slightly below the after-tax function and the low-preference-for-leisure individual has his earnings function always above the after-tax income function. The intuition behind this result is that the person who enjoys work the most is subsidizing those who prefer work less, via the waterfall effect described previously.

Note that there is a slight upward-sloping portion to the after-tax income function of the person with the high preference for leisure between a tax rate of 60% and 70%. This results from the fact that after 60%, this individual drops out of the labour force, and

that his subsidy at a tax rate of 70% is higher than his after-tax income at the lower tax rate. This also accounts for the funny shape in the welfare cost function.

The welfare functions for the three types of individuals, shown in Figure 9, are once again maximized at different values of the tax rate. For the person with the highest preference for leisure, welfare is maximized at a tax rate of about 60%, while the person with the median preference for leisure maximizes welfare at a very low tax rate in the range of about 5%. Not surprisingly, the person with the lowest preference for leisure maximizes welfare at a zero tax rate.

Quantifying the welfare costs, Figure 10 shows that it is always costly in terms of consumption to increase the tax rate marginally by 10% for the median and low preference for leisure individuals, while the person with the high preference for leisure would be willing to give up consumption for such marginal increases up to 60%. Yet again, these functions are increasing at an increasing rate so that at very high tax rates, the marginal increases in the tax rate become very costly for individuals. The logic is that everyone is receiving a lower and lower subsidy at high tax levels. Those still supplying labour are paying an increasing share, so their welfare cost is the highest. Those who are not supplying labour are only receiving the subsidy as their income, which is in decline, so their welfare costs are on the rise.

Finally, examining the labour supply effects, Figure 11 shows that indeed there is a decreasing equilibrium labour supply effect of increasing the tax rate. The individual with the highest preference for leisure will stop supplying labour at a tax rate of 60%, while the other two types will always supply a positive amount in equilibrium. The

income and substitution effects are identical to the ones states in the previous section. See Table 2 for the tax elasticity of labour supply.

This has potential negative consequences for the NIT theory. The homogeneity assumption that characterizes the theory of the NIT does not account for the negative equilibrium labour supply effects on different types of individuals. We see now that there are in fact significant differences in equilibrium labour supply effects across individuals with different preferences, which could make the negative effects we saw with the different wages even more negative. In fact, one could predict now that an individual with a low wage and a high preference for leisure will experience very negative labour supply effects of the increasing tax rate. The implications for the NIT policy are quite different from those when we only have skill differences. This will be discussed below in section 5.

The comparison of the NIT at a 10% tax level with the data is presented in Table 5. As with the results from the heterogeneous wage, the hypothetical NIT system does a remarkable job in predicting the earnings and after-tax income, but a fairly poor job in predicting the level of the subsidy and a mediocre job in prediction labour supply hours per year. The second to last point is not terribly surprising given that fact that in the observed data, the level of government transfers to individuals is not uniform across all individuals. Still, if we were to average the level of the real world subsidy across the three leisure-preference groups, the model predicts values that are much too high.

The model does a very good job at mimicking the fact that people with a high preference for leisure tend to earn less and receive less after-tax income than people with

a low preference for leisure, in spite of the fact that it under-predicts earnings and after-tax income levels.

### ***4.3 Heterogeneous Skill and Heterogeneous Preference for Leisure***

Now that differences in skills and preferences for leisure have been examined within an NIT model separately, the focus will now shift to the interactive effect of these differences. Specifically, skill and preference for leisure will differ quantitatively in the same manner as above:  $w \in \{10.34, 12.63, 17.92\}$ ,  $\phi \in \{0.84, 1.48, 3.52\}$ . Thus, there will be nine types of individuals in this simulation. The numerical results are presented in Table 3.

Figure 12 plots the after-tax income functions of the nine different types of individuals. First, as predicted, the person with the lowest after-tax income is the low-wage, high preference for leisure individual, while the highest after-tax income function is exhibited by the high-wage, low leisure preference individual. There are a couple of interesting things to observe in Figure 12. First, it is the type of wage category that has the biggest impact on how much a person earns, while the leisure preference has a much smaller effect. It is easy to see that despite their differing preferences for leisure, the high wage individuals have after-tax income functions that lie a good deal above the after-tax income functions of any other wage category. This reflects the fact that the university graduates do earn much more than the other two categories, so it is expected that this will always have a pronounced effect in this analysis.

Another note is the cluster of lines amongst the low and medium wage earners with their differing preferences for leisure. It shows that the medium-wage, high leisure preference persons and the low-wage, low leisure preference individuals will have almost

identical responses to the redistribution policy. This is intuitively clear: although a person may be earning a low wage, his distaste for leisure may drive him to work so much that he earns almost the equivalent to someone who does not enjoy working so much, but earns more money.

The final observation in Figure 12 is that the high-wage, low preference for leisure individual has the steepest after-tax income function, while the low-wage, high preference for leisure individual has the shallowest after-tax income function. This says that on the margin, the former individual is hurt more by increases in the tax rate, while the latter is hurt the least. The other individuals lie somewhere in between. As before, after-tax income goes to zero as  $\tau \rightarrow 1$ , since as the tax rate approaches unity, no individual will supply labour.

Comparing the after-tax income to the earnings shown in Figure 13, all low-wage individuals have an after-tax income function that lies above the earnings function. Thus no matter what their preference for leisure, low-wage individuals have significant gains in consumption from the imposition of the NIT regime. The median wage individuals experience a very similar difference, with the exception of the person with the low preference for leisure, who actually experiences higher earnings than after-tax income over all values of the tax rate. Finally, the high-wage individuals all display after-tax income functions that lie below their earnings functions.

Once again we can decipher who is a subsidizer and who is a subsidizee from the results of this simulation. What is observed is that the high-wage persons and the medium-wage, low leisure preference person are subsidizing the five types of people below them. That is, we have the four top earners providing assistance to the five lowest

earners. To gain some perspective on the quantitative aspects of the amount of the subsidy, a quick inspection of Figure 14 gives a nice picture of who is really losing the most with this policy.

This graph shows the individual contributions to the government for each of the nine types of people, as well as the per-person subsidy each of them receives (the dashed line). The most striking feature of this picture is the distance between what the high-wage, high leisure preference individual pays in taxes, and what he receives in subsidy, especially at very high tax rates – this is reflective of his strong desire to work and his extra incentive to do so given to him by high wages.

Another prominent feature of this graph is that the median-wage, low leisure preference person has a tax payment function that is almost identical to the subsidy function, except at very high levels of tax. This means that he is at the break-even point and will be indifferent to this policy for the most part.

Finally, the lower wage, higher preference for leisure individuals all have tax functions below the subsidy functions, with some of them not paying any amount into the tax function and reaping the full amount of the subsidy paid by the rest of the people in this simulation economy. The implications of this will be discussed in the following section.

The labour supply effects are as expected. The lower wage, higher preference for leisure individuals begin to work substantially less even with low levels of the tax because they are receiving more and more subsidy, while working less and contributing less to the government. The tax elasticity of labour supply is presented in Table 3. Some even opt to stop supplying labour, as can be seen in Figure 15. The NIT system

falls apart once people stop supplying labour in equilibrium because the high wage individuals are paying a disproportionately high amount to the government, and since the subsidy is distributed equally to all individuals, they receive little in return. This is in fact the major incentive problem facing this type of redistributive policy.

The welfare functions are shown in Figure 16. There are some interesting things to notice here. First, for a wide range of tax rates, the welfare functions for all individuals are fairly flat. That is, not much welfare is lost by any individual by increasing the tax rate at very low levels. Second, at a tax rate of about 55%, welfare starts to drop very rapidly, since nobody in the economy prefers a very high tax rate. The intuition behind this is that once the tax rate is raised to such a high level, the total amount contributed to the subsidy fund declines, and so even those people who are supplying zero hours of labour will dislike a higher tax rate because their total subsidy is declining.

Not surprisingly, the low-wage, high preference for leisure individual maximizes his welfare at a very high tax rate, around 55%. The reason for this is the same as discussed above, which is to say that the redistribution done by the NIT system gives all individuals a greater level of welfare since the amount of the per-person subsidy is very high at low tax levels.

It must be noted here that the results hinge crucially on the assumption that the population is distributed unequally in skill.<sup>12</sup> The same simulation yields much different results when the individuals are distributed equally in the population.

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<sup>12</sup> Remember that the model was calibrated so that 25% of individuals are high school dropouts, 50% are high school grads and 25% are university grads.

The welfare cost functions, shown in Figure 17, exhibit the same shape as they did when heterogeneity was examined separately. Interestingly, at very high tax rates, all individuals experience about the same welfare cost in terms of percentage of consumption. This can be explained by the fact noted above that as  $\tau \rightarrow 1$ , consumption across individuals is getting closer, thus it makes sense that the cost of marginal increases in the tax rate are about the same.

Finally, comparing the model at a 10% tax rate with the data yields some interesting insights.<sup>13</sup> As with the previous two sections, the model captures the ordering of earnings and after-tax income levels well. In this case we see that the person that likes work the most and earns the highest wage has the highest level of after-tax income and earnings, and vice versa for the person who likes work the least and earns the lowest wage. Also like the previous two sections, the level of the subsidy is very much over-predicted while labour supply is underpredicted.

The actual numbers do match fairly well, with the exception of the individuals that have a high wage; the real world observations are much higher than the model predicts.

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<sup>13</sup> It should be noted here that to get the actual data for these groups, I first separated the data into the three educational groups, then within those groups I separated the data into leisure preference categories, to get 9 different types of individuals.

## 5 DISCUSSION

What are the major implications of this study? In this section, I will abstract from making social welfare statements in order to avoid the many thorny issues in specifying a social welfare function, and so this discussion will be limited to positive and negative effects as judged only by the individual himself.

The conclusions of this numerical exercise support this theory that labour supply declines when we impose a negative income tax regime on people. Not only is there no positive labour supply effect in equilibrium, but the low-wage individuals have the *most negative* labour supply effects. Thus, in this regard, the NIT system fails when estimated by the model in this paper. The reason is that if this type of person knows that there are many other individuals in the economy that are much richer in terms of earnings and like to work much more, then he will always receive a large enough subsidy to satisfy his needs.

In the section with only differing skill levels, there are a few implications that must be considered when trying to generalize to the population. Recall that it was the people with the lowest skill level that enjoyed this policy the most, and vice versa for the most skilled. When we think about real world, skill differences are not constant when people are disaggregated by age and by sex. The conclusions of the heterogeneity in skill exercise imply that the NIT policy may not have a pronounced effect on younger individuals, since their earnings differences tend not to be very large. However, since earnings differences by age are the highest at around age 55, older people would be

strongly affected by this policy, specifically the most skilled. When sex is taken into account, one can conclude based on this model's predictions that men will be more adversely affected by the NIT than women since they tend to have lower levels of earnings, despite their steeper age-earnings profiles.

What are the implications for the NIT when people vary along a different dimension, namely in their preference for leisure? The results were as expected, with the person with the lowest preference for leisure disliking the NIT program, and the person with the highest preference for leisure enjoying it. If the government imposes the NIT on people, as the tax rate increases, the hard-working individual will be subsidizing the lazy individual because the lazy person has such strong negative equilibrium labour supply effects. This is not fair to the person who chooses to work hard. It is basically penalizing him for having a strong work ethic, and rewarding the person who is lazy.

When individuals vary across both skill and preferences for leisure, once again the model predicts a steadily declining equilibrium labour supply function for all individuals as the tax rate increases. Judged by their respective Indirect Utility Functions, it appears that many of the people with low skill levels in this economy would enjoy this policy. The higher skill people, on the other hand, would not. This is basically a natural extension of the intuition that was formed in the previous two sections. The fact that the results make such sense is reassurance that the numerical exercise was done correctly.

Given that a large proportion of the population maximizes their welfare at positive tax rates, and also given the distributional characteristics of the population on the basis of skill, it seems likely that if the individuals in the economy were to be given equal votes

on an optimal tax rate, then the NIT would likely be implemented if the government were to take a majority voting type of rule as their decision criterion. That is, if we were to look to the median voter to make the decision on whether or not this policy would be implemented, in all cases it would indeed.

## 6 CONCLUSION

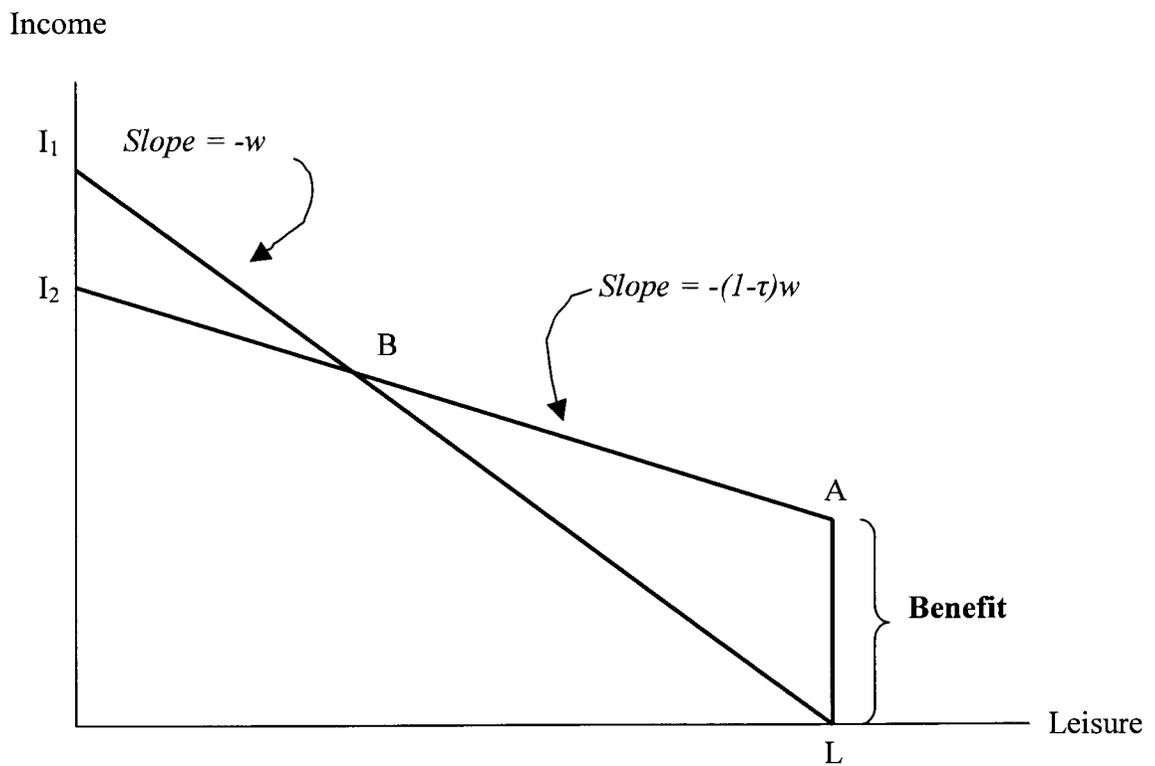
Taking a simple model of individual preferences, this study tested the theory of negative income taxes along varying levels of skill and preferences for leisure. The findings are that there are unambiguous negative labour supply effects for all individuals, which are worse for those people who either have low skill, a high preference for leisure, or a combination of both. Despite these findings, negative income taxes could prove useful in situations where redistribution is the main goal.

Of course, these conclusions have been derived using a simple numerical exercise, where no attempt has been made to take dynamics into consideration. If all people made their decisions in a static manner, perhaps this model would have more predictive power. However, despite the fact that all of the conclusions of this paper are specific to its numerical exercise, they all make intuitive sense and even replicate important features found in real world data. It reaffirms some of the ideas that people have in the back of their minds regarding the problems associated with such a redistributive policy. Even if a static model is used, an honest attempt has been made to calibrate this model to fit the behaviour of people, from the model of preferences, down to the calibration of the model based on actual earnings data. Thus, this paper's conclusions should perhaps be given some credit in its explanatory power.

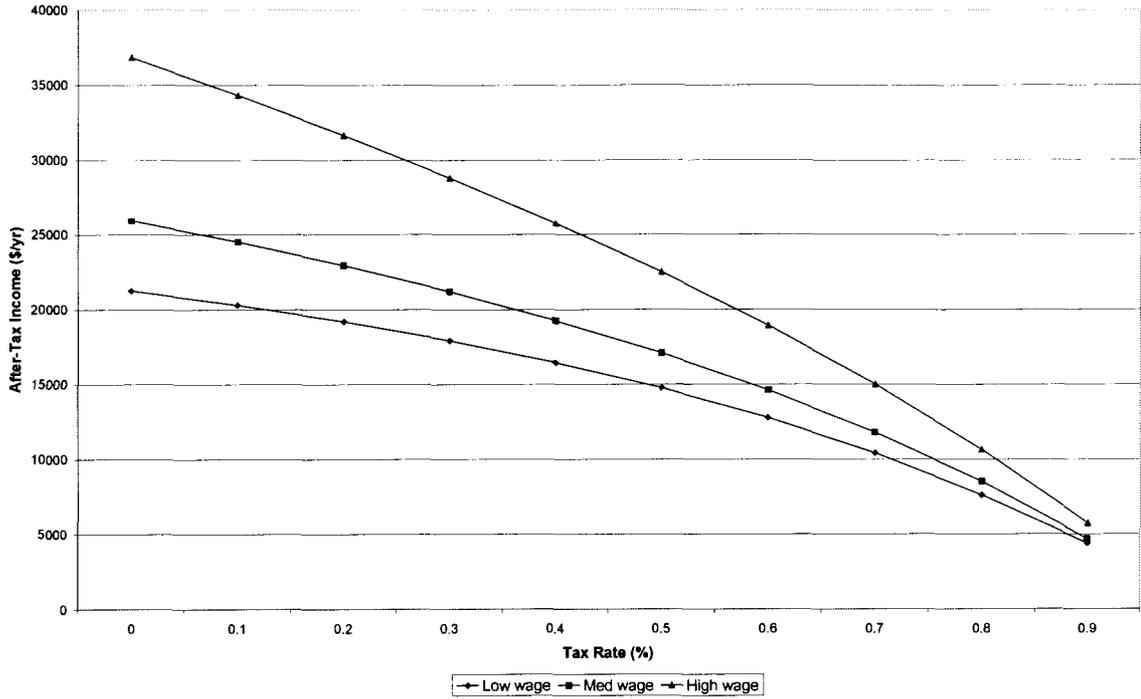
## **APPENDICES**

## Appendix A: Figures

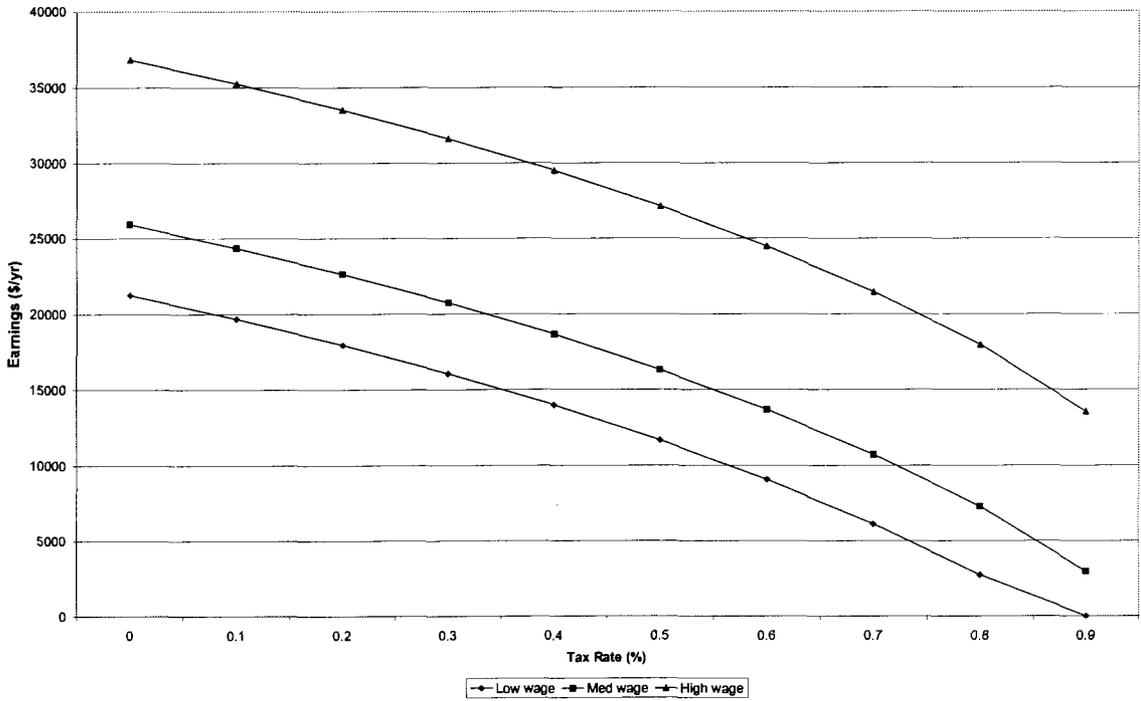
Figure 1 - The Negative Income Tax



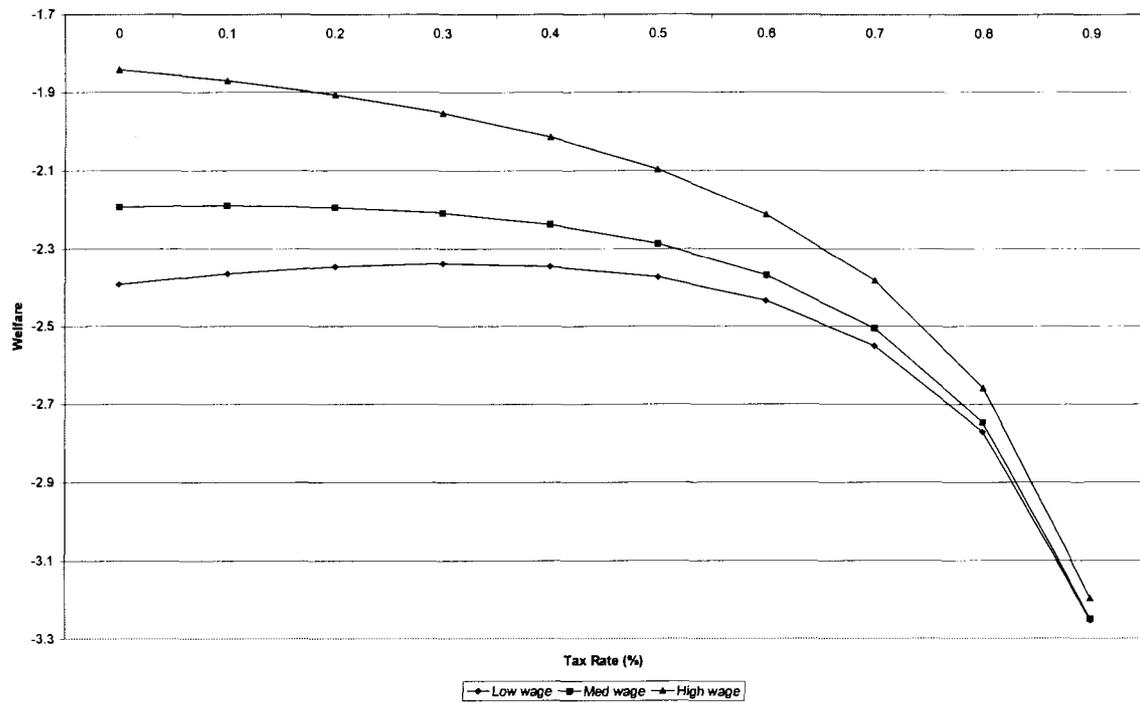
**Figure 2 - After Tax Income with Heterogeneous Wage**



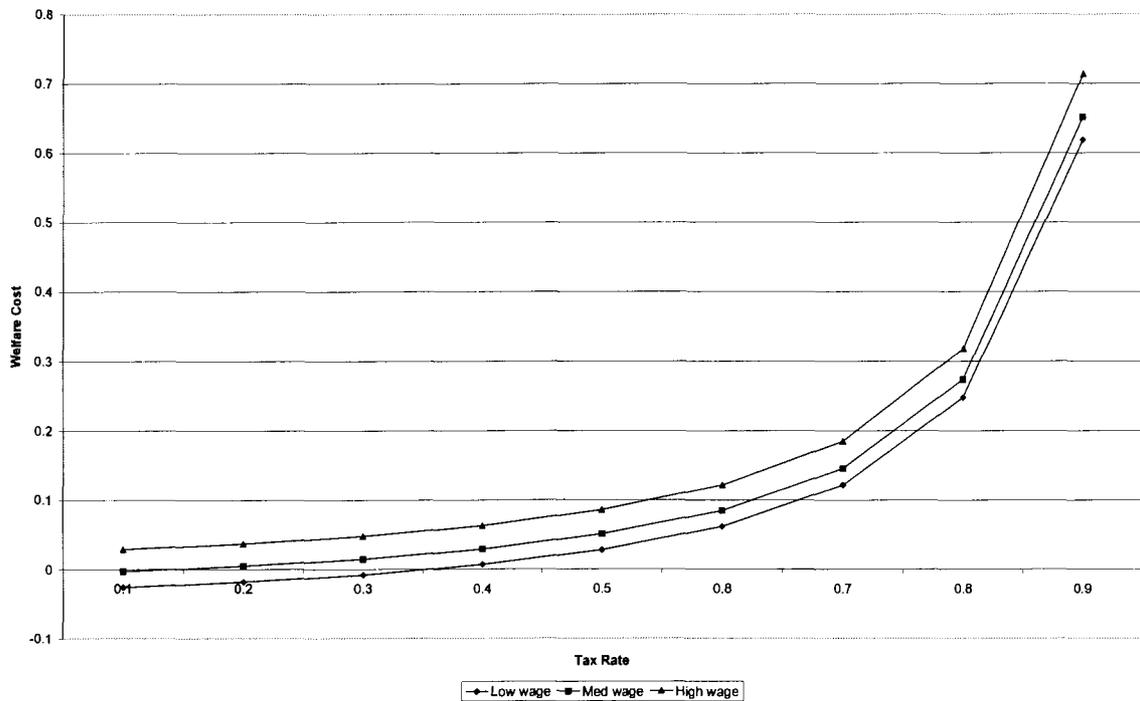
**Figure 3 - Earnings with Heterogeneous Wage**



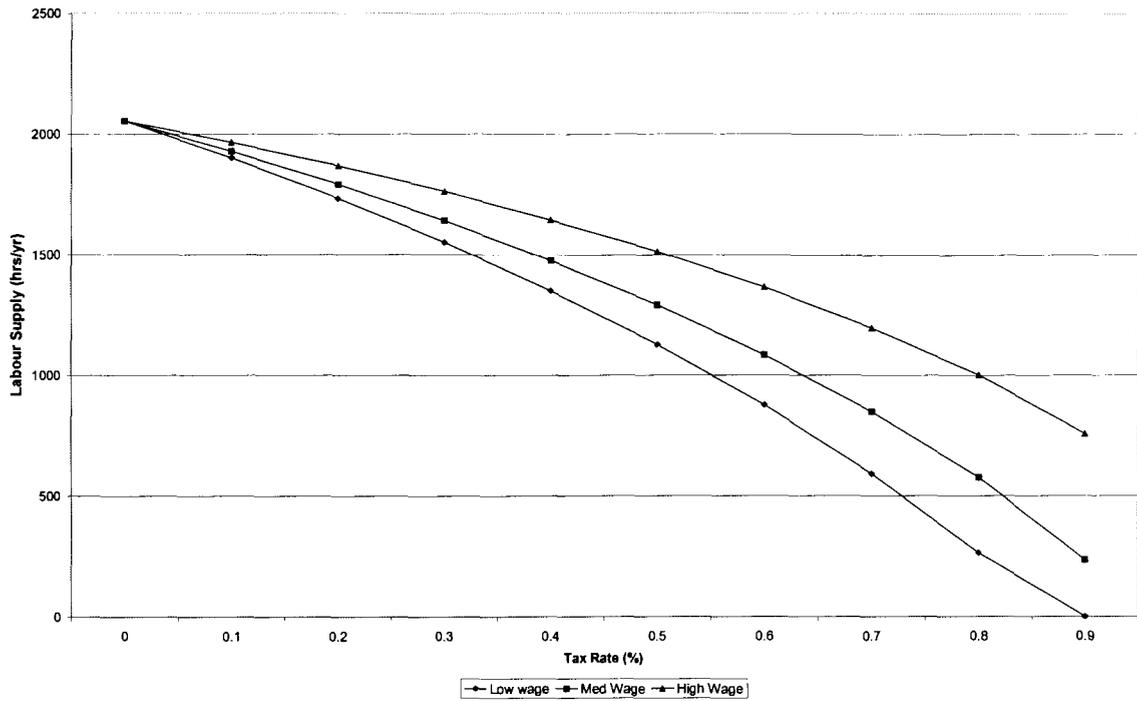
**Figure 4 - Welfare with Heterogeneous Wage**



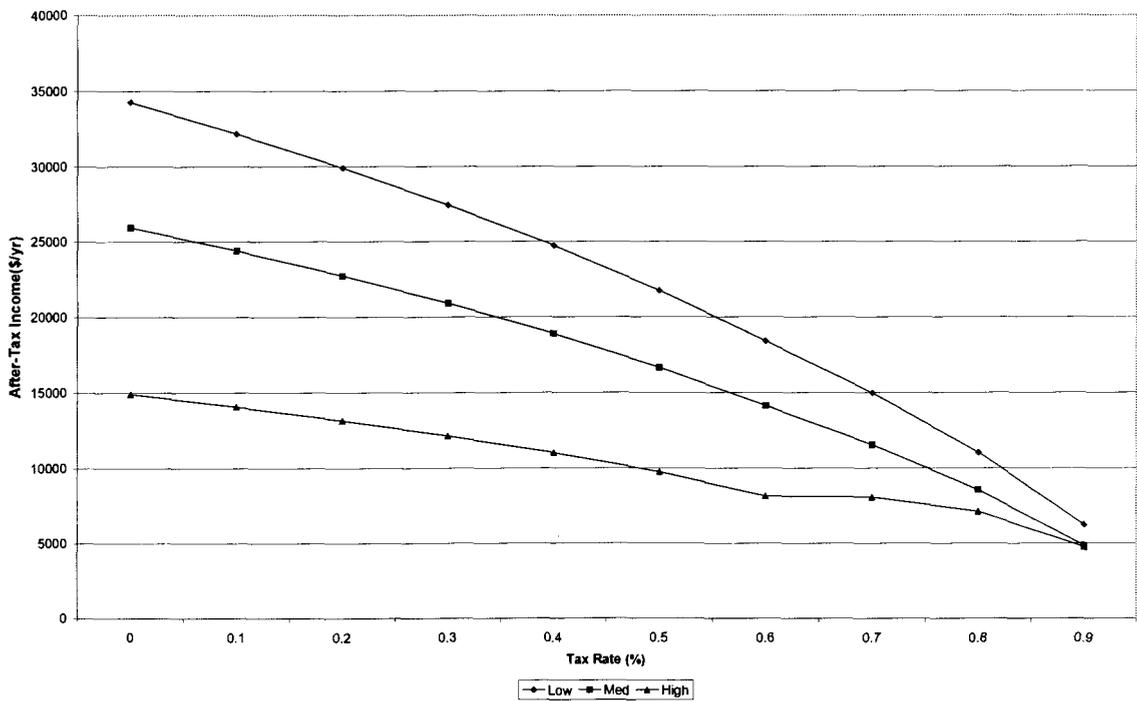
**Figure 5 - Welfare Cost with Heterogeneous Wage**



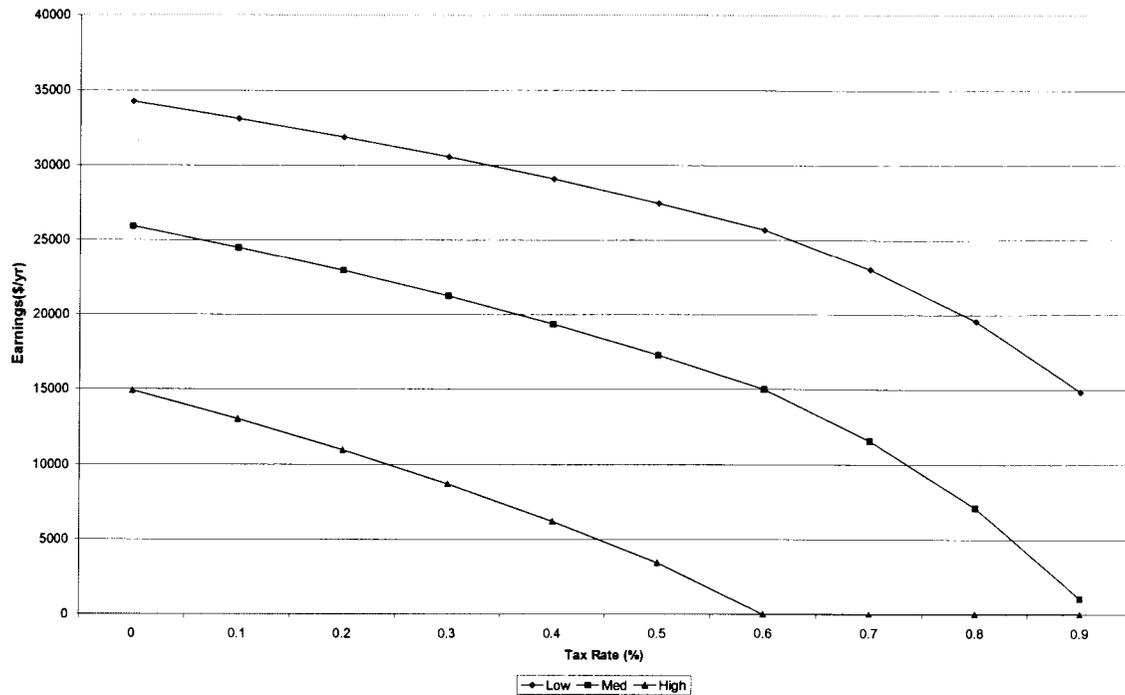
**Figure 6 - Equilibrium Labour Supply with Heterogeneous Wage**



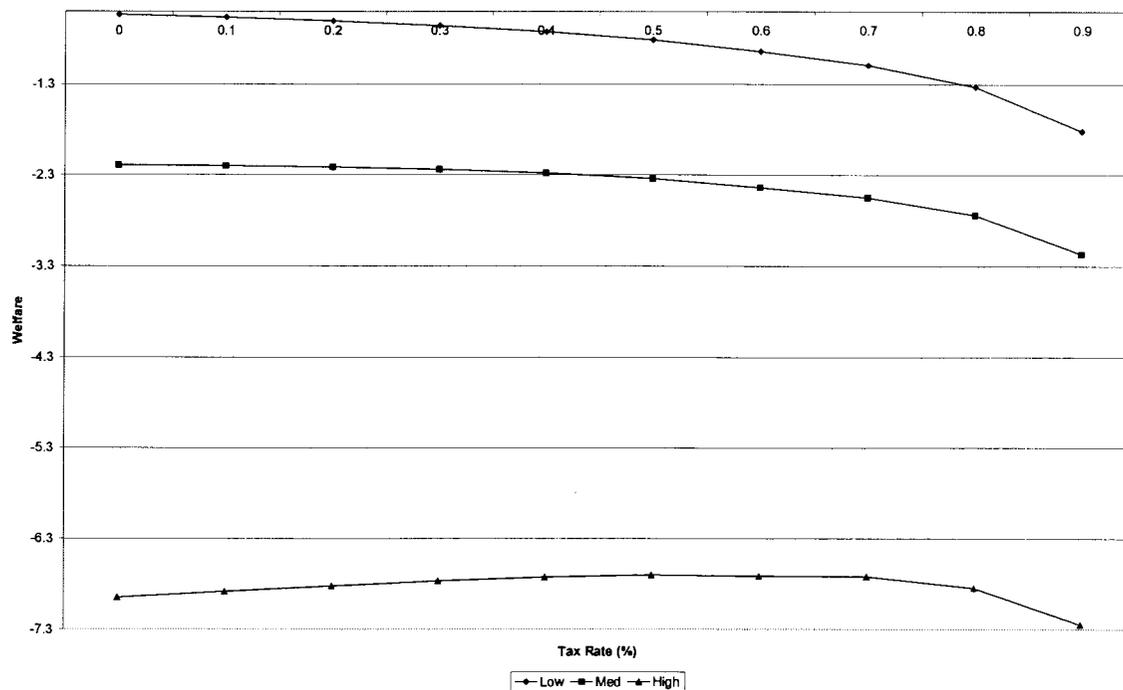
**Figure 7 - After Tax Income with Heterogeneous Leisure Preference**



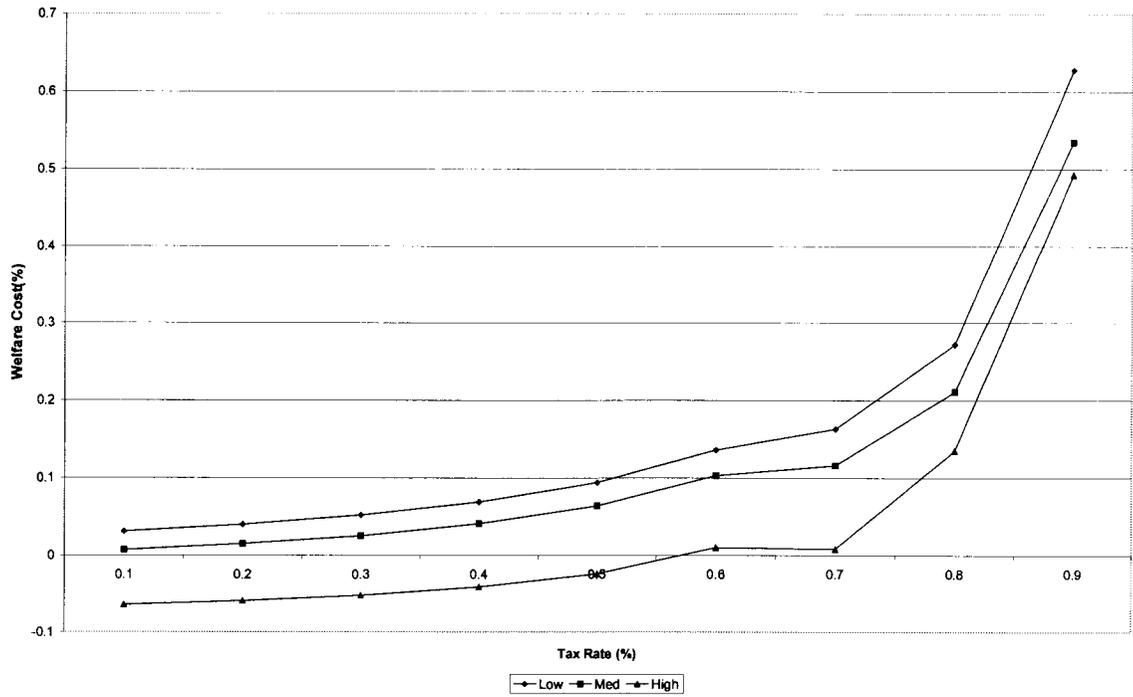
**Figure 8 - Earnings with Heterogeneous Leisure Preference**



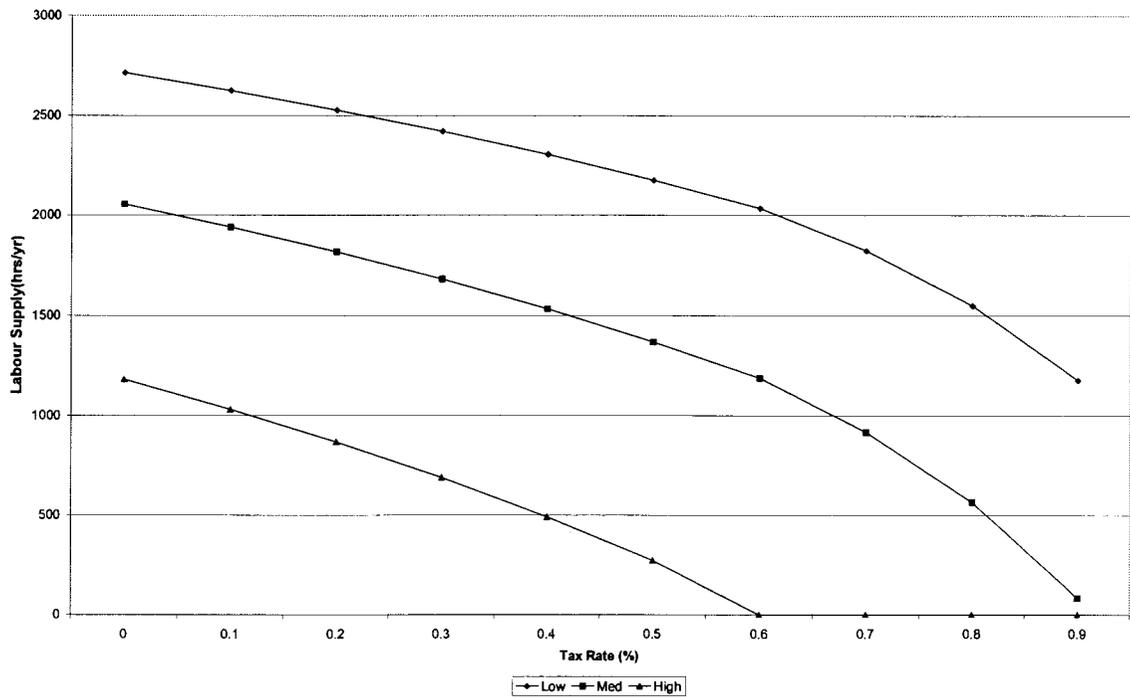
**Figure 9 - Welfare with Heterogeneous Leisure Preference**



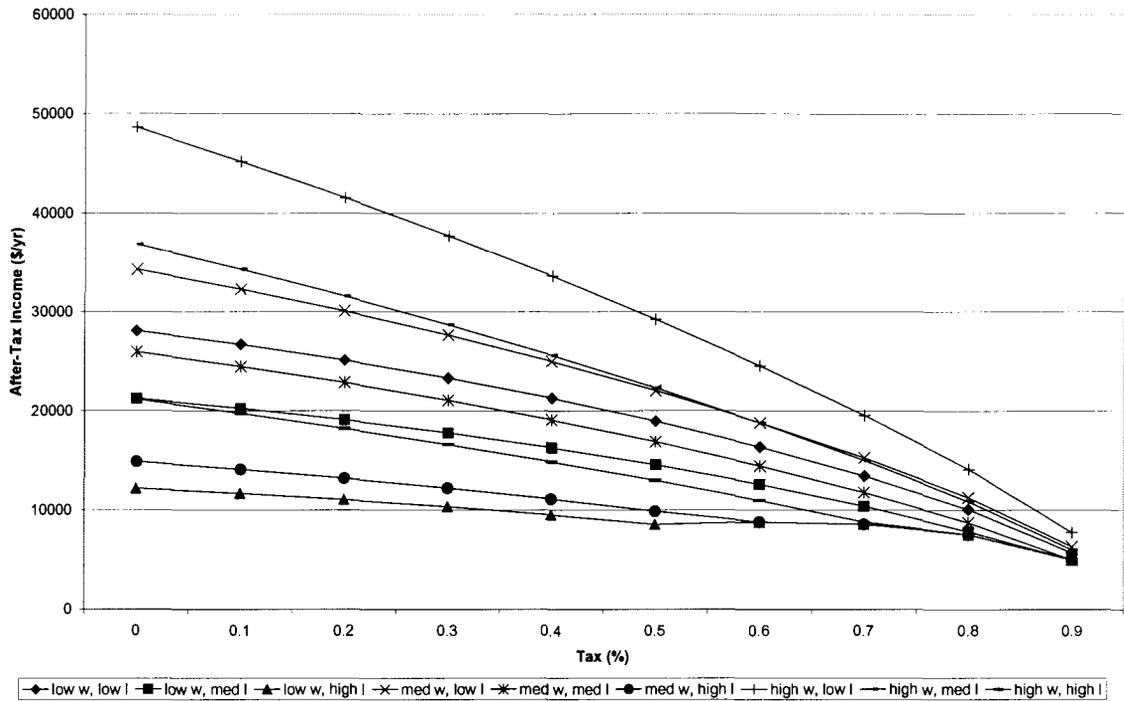
**Figure 10 - Welfare Cost with Heterogeneous Leisure Preference**



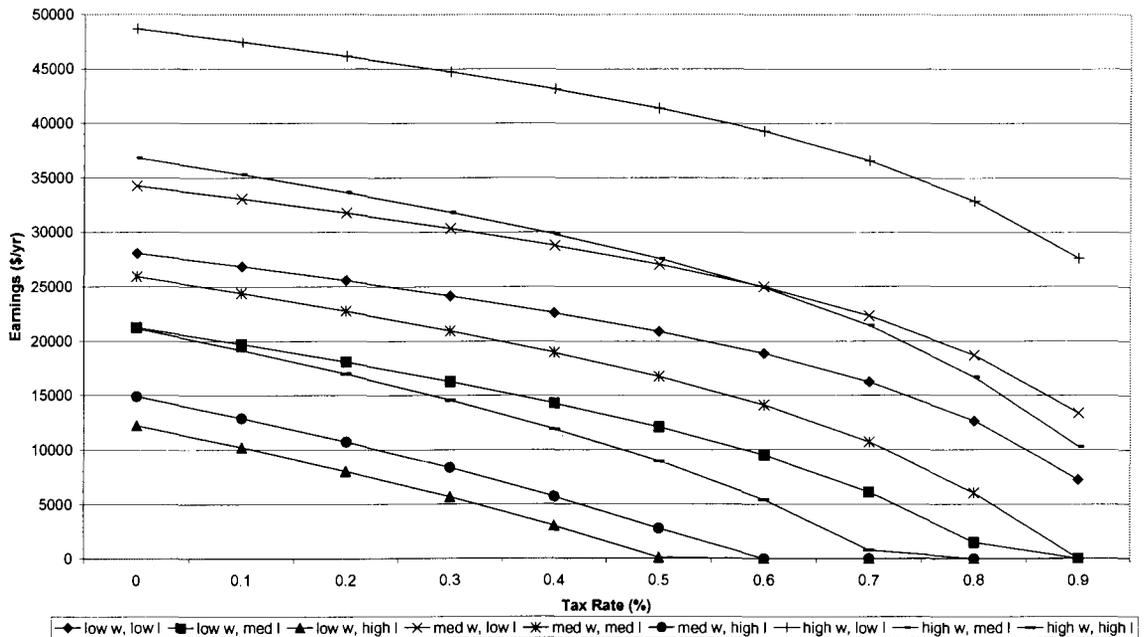
**Figure 11 - Equilibrium Labour Supply with Heterogeneous Leisure Preference**



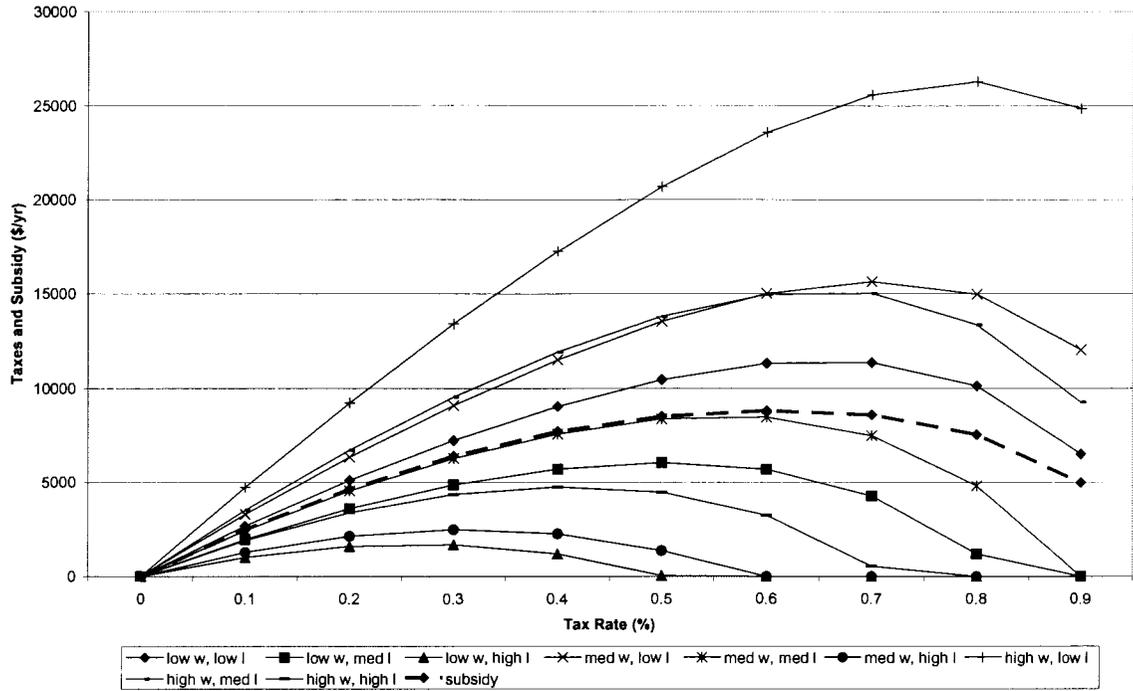
**Figure 12 - After Tax Income with Heterogeneous Wage and Leisure Preference**



**Figure 13 - Earnings with Heterogeneous Wage and Leisure Preference**



**Figure 14 - Total Taxes Paid and Subsidy**



**Figure 15 - Equilibrium Labour Supply with Heterogeneous Wage and Leisure Preference**

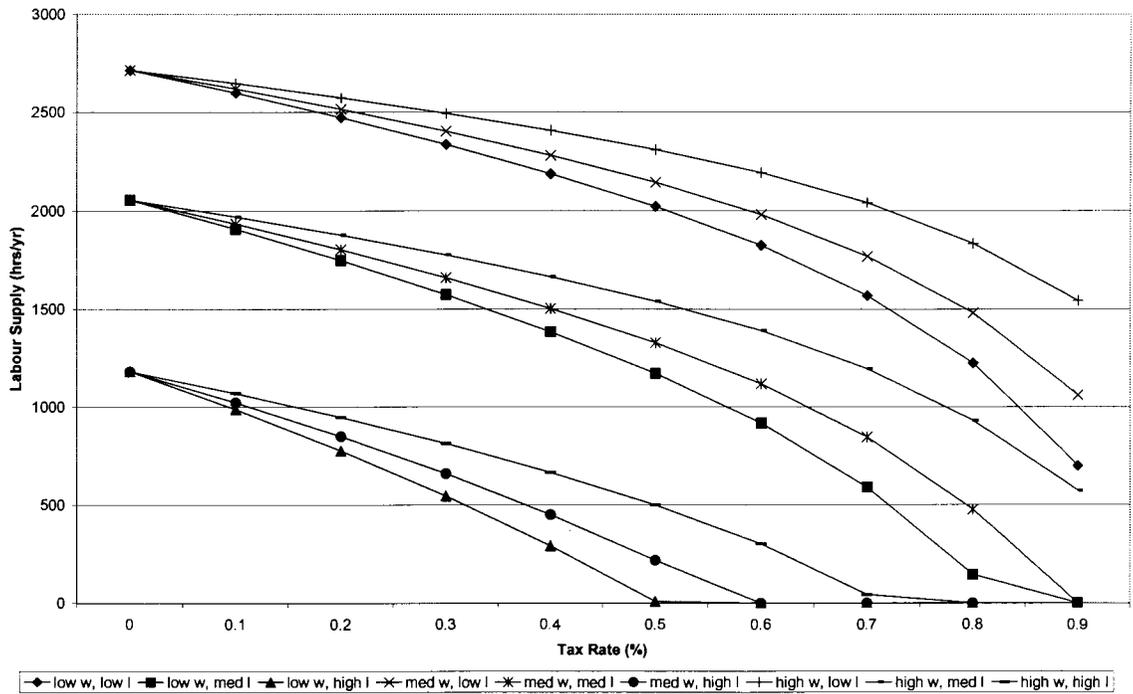


Figure 16 - Welfare with Heterogeneous Wage and Leisure Preference

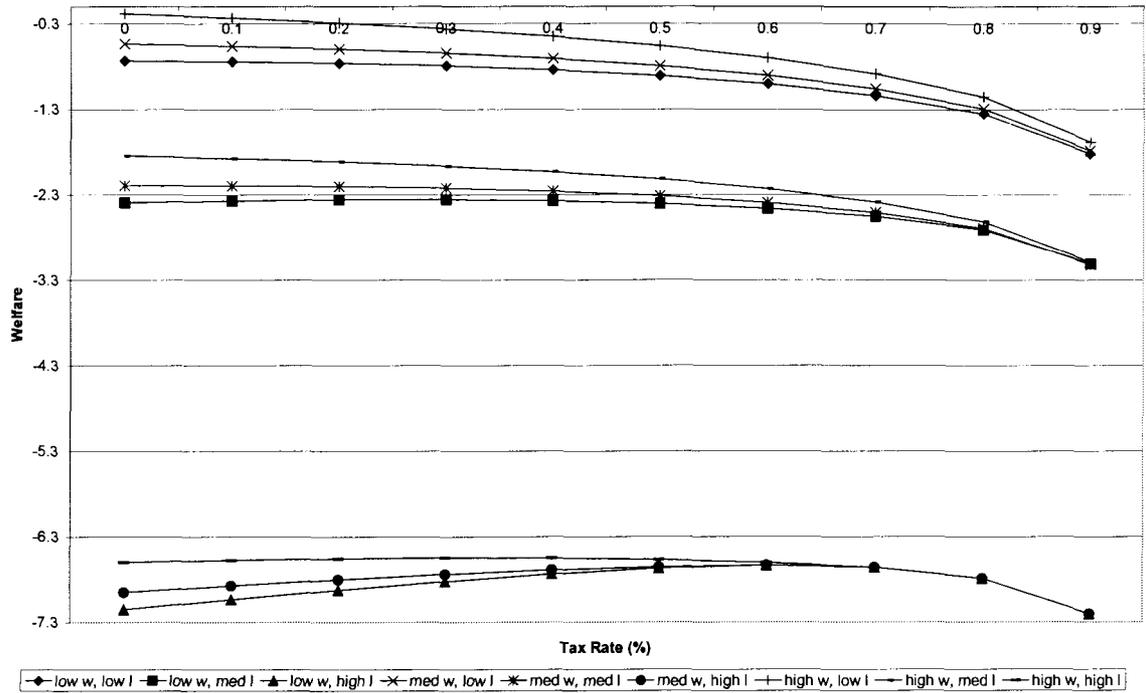
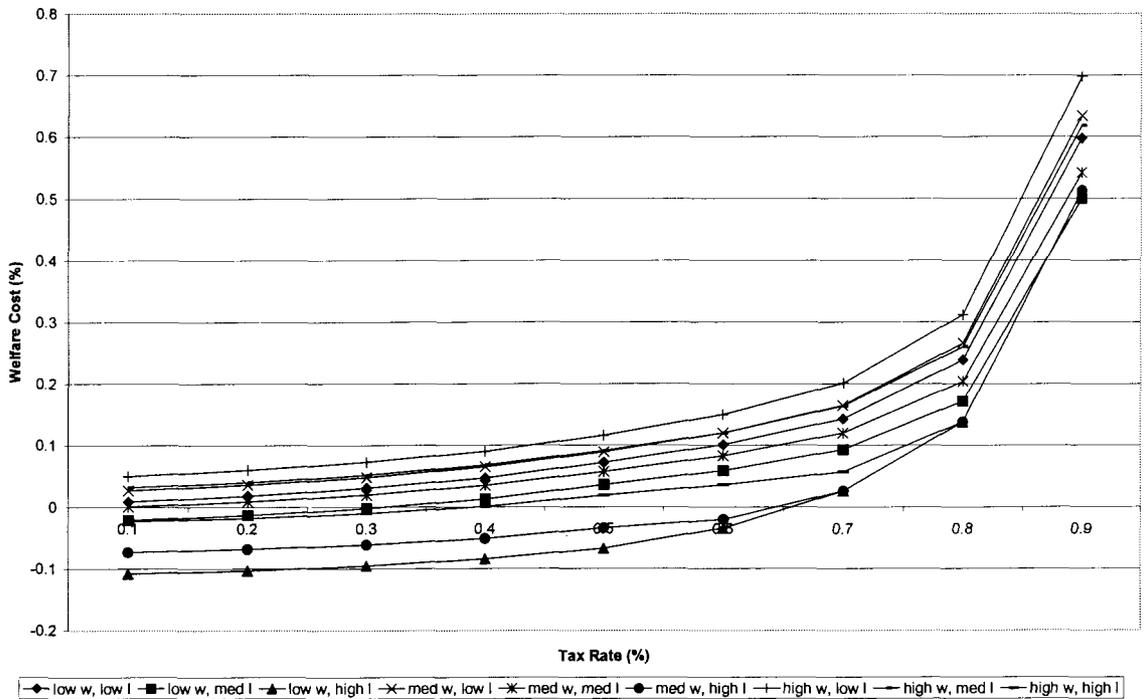


Figure 17 - Welfare Cost with Heterogeneous Wage and Leisure Preference



## Appendix B: Tables

**Table 1 - Model Results with Heterogeneous Wage**

Tax	After-tax income			Earnings			Welfare		
	Low	Medium	High	Low	Medium	High	Low	Medium	High
0	21255.858	25947.642	36831.431	21255.858	25947.642	36831.431	-2.392	-2.192	-1.842
0.1	20299.887	24521.345	34315.227	19676.046	24366.556	35248.646	-2.365	-2.189	-1.870
0.2	19200.796	22949.880	31651.018	17954.022	22640.377	33516.799	-2.347	-2.194	-1.907
0.3	17933.829	21208.892	28814.994	16066.190	20744.852	31610.713	-2.339	-2.208	-1.953
0.4	16467.873	19267.840	25777.324	13982.976	18649.587	29498.728	-2.346	-2.237	-2.014
0.5	14763.116	17087.672	22499.974	11666.623	16315.735	27140.340	-2.374	-2.287	-2.097
0.6	12767.587	14617.471	18933.472	9067.967	13692.677	24482.680	-2.434	-2.369	-2.212
0.7	10411.838	11789.258	15011.924	6121.465	10712.864	21455.085	-2.550	-2.505	-2.382
0.8	7600.377	8509.646	10645.010	2737.207	7283.552	17960.369	-2.771	-2.747	-2.658
0.9	4380.208	4676.468	5734.448	0.000	2962.597	13542.398	-3.253	-3.249	-3.196

Tax	Labour Supply			Welfare Costs			Labour Supply Elasticity		
	Low	Medium	High	Low	Medium	High	Low	Medium	High
0	2055.096	2055.096	2055.096	nil	nil	nil	0.000	0.000	0.000
0.1	1902.354	1929.871	1966.781	-0.026	-0.003	0.029	-0.009	-0.007	-0.005
0.2	1735.862	1793.155	1870.148	-0.018	0.005	0.037	-0.046	-0.036	-0.025
0.3	1553.340	1643.027	1763.794	-0.008	0.015	0.048	-0.137	-0.107	-0.070
0.4	1351.927	1477.078	1645.951	0.007	0.029	0.063	-0.341	-0.257	-0.164
0.5	1127.973	1292.233	1514.359	0.028	0.051	0.086	-0.803	-0.580	-0.353
0.6	876.725	1084.483	1366.069	0.062	0.085	0.122	-1.958	-1.311	-0.744
0.7	591.846	848.476	1197.137	0.122	0.146	0.185	-5.567	-3.221	-1.637
0.8	264.643	576.869	1002.141	0.248	0.274	0.318	-25.926	-9.888	-4.095
0.9	0.000	234.643	755.630	0.619	0.652	0.714	-	-66.852	-14.992

**Table 2 - Model Results with Heterogeneous Leisure Preference**

Tax	After-tax income			Earnings			Welfare		
	Low	Medium	High	Low	Medium	High	Low	Medium	High
0	34264.275	25941.477	14913.205	34264.275	25941.477	14913.205	-0.534	-2.192	-6.949
0.1	32174.396	24408.886	14074.701	33132.366	24504.022	13021.595	-0.564	-2.199	-6.883
0.2	29908.981	22739.040	13153.962	31902.728	22940.302	10958.954	-0.604	-2.214	-6.822
0.3	27440.878	20910.090	12137.304	30560.072	21230.376	8697.824	-0.655	-2.239	-6.768
0.4	24736.469	18894.900	11007.622	29085.451	19349.502	6204.039	-0.722	-2.279	-6.726
0.5	21753.503	16659.247	9743.216	27455.017	17266.506	3434.444	-0.812	-2.340	-6.702
0.6	18411.125	14137.632	8137.189	25684.839	15001.106	0.000	-0.939	-2.438	-6.712
0.7	14975.631	11544.228	8071.261	23014.566	11576.555	0.000	-1.090	-2.548	-6.720
0.8	11023.768	8538.079	7113.399	19551.847	7123.400	0.000	-1.330	-2.740	-6.846
0.9	6246.924	4867.174	4763.185	14837.392	1039.889	0.000	-1.818	-3.169	-7.247

Tax	Labour Supply			Welfare Costs			Labour Supply Elasticity		
	Low	Medium	High	Low	Medium	High	Low	Medium	High
0	2714.432	2055.096	1181.431	nil	nil	nil	0.000	0.000	0.000
0.1	2624.762	1941.220	1031.577	0.031	0.007	-0.064	-0.005	-0.007	-0.014
0.2	2527.349	1817.342	868.174	0.040	0.015	-0.059	-0.022	-0.033	-0.081
0.3	2420.983	1681.880	689.046	0.052	0.025	-0.052	-0.062	-0.094	-0.281
0.4	2304.163	1532.877	491.487	0.069	0.041	-0.041	-0.139	-0.224	-0.877
0.5	2174.999	1367.861	272.078	0.094	0.064	-0.024	-0.287	-0.494	-3.206
0.6	2034.765	1188.395	0.000	0.136	0.103	0.010	-0.570	-1.071	-
0.7	1823.225	917.100	0.000	0.163	0.116	0.008	-1.266	-2.815	-
0.8	1548.907	564.319	0.000	0.272	0.211	0.135	-3.239	-10.190	-
0.9	1175.425	82.381	0.000	0.628	0.535	0.493	-12.177	-205.681	-

**Table 3 – Model Results with Heterogeneous Wage and Preference for Leisure**

After Tax Income									
Tax	Low wage			Med wage			High wage		
	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0	28075.370	21255.858	12219.551	34264.275	25941.477	14913.216	48650.765	36833.485	21174.806
0.1	26685.591	20256.661	11690.605	32254.604	24472.659	14113.985	45201.092	34274.037	19748.169
0.2	25112.097	19114.431	11076.634	30059.428	22858.911	13228.074	41563.464	31567.063	18232.608
0.3	23325.623	17805.565	10362.744	27649.903	21077.035	12240.909	37709.546	28689.864	16613.932
0.4	21289.981	16300.645	9530.260	24990.340	19098.158	11134.275	33604.302	25614.272	14874.517
0.5	18959.539	14562.523	8555.425	22035.796	16885.849	9885.028	29203.711	22304.815	12992.044
0.6	16356.873	12608.831	8811.071	18809.110	14458.010	8811.071	24530.944	18779.737	10977.588
0.7	13453.353	10416.316	8584.560	15282.106	11791.865	8584.560	19558.420	15016.878	8817.289
0.8	10077.119	7845.540	7544.795	11284.811	8749.993	7544.795	14118.823	10881.655	7544.795
0.9	5711.707	4987.666	4987.666	6324.755	4987.666	4987.666	7751.436	6016.137	4987.666

Earnings									
Tax	Low wage			Med wage			High wage		
	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0	28075.370	21255.858	12219.551	34264.275	25941.477	14913.216	48650.765	36833.485	21174.806
0.1	26877.151	19733.896	10216.055	33064.943	24418.337	12908.699	47449.929	35308.757	19168.903
0.2	25578.473	18081.390	8034.144	31762.636	22761.991	10723.444	46142.681	33647.180	16979.111
0.3	24163.641	16277.843	5645.242	30341.184	20951.373	8328.336	44712.102	31826.843	14575.511
0.4	22613.328	14297.769	3013.793	28780.593	18960.290	5687.151	43137.197	29820.480	11920.888
0.5	20903.129	12109.096	94.899	27055.642	16755.748	2754.105	41391.472	27593.680	8968.138
0.6	18864.505	9494.398	0.000	24995.097	14117.347	0.000	39299.682	24921.664	5416.291
0.7	16229.310	6105.857	0.000	22325.156	10691.020	0.000	36579.535	21441.061	775.764
0.8	12661.619	1503.723	0.000	18700.080	6025.990	0.000	32870.139	16684.301	0.000
0.9	7240.412	0.000	0.000	13370.886	0.000	0.000	27637.695	10284.713	0.000

Welfare									
	Low wage			Med wage			High wage		
Tax	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0	-0.733	-2.392	-7.148	-0.534	-2.192	-6.949	-0.183	-1.842	-6.598
0.1	-0.742	-2.370	-7.033	-0.560	-2.193	-6.873	-0.232	-1.873	-6.575
0.2	-0.760	-2.357	-6.924	-0.595	-2.202	-6.802	-0.291	-1.912	-6.557
0.3	-0.790	-2.355	-6.824	-0.641	-2.221	-6.739	-0.361	-1.963	-6.546
0.4	-0.836	-2.368	-6.736	-0.704	-2.256	-6.687	-0.447	-2.028	-6.548
0.5	-0.906	-2.403	-6.667	-0.790	-2.312	-6.653	-0.557	-2.116	-6.567
0.6	-1.002	-2.460	-6.632	-0.903	-2.391	-6.632	-0.697	-2.229	-6.602
0.7	-1.136	-2.549	-6.658	-1.056	-2.504	-6.658	-0.880	-2.381	-6.658
0.8	-1.350	-2.708	-6.787	-1.291	-2.690	-6.787	-1.151	-2.612	-6.787
0.9	-1.818	-3.113	-7.201	-1.782	-3.123	-7.201	-1.681	-3.093	-7.201

Equilibrium Labour Supply									
	Low wage			Med wage			High wage		
Tax	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0	2714.432	2055.096	1181.432	2714.432	2055.096	1181.432	2714.432	2055.096	1181.432
0.1	2598.584	1907.947	987.727	2619.420	1934.432	1022.633	2647.432	1970.025	1069.514
0.2	2473.023	1748.177	776.771	2516.251	1803.216	849.516	2574.495	1877.319	947.336
0.3	2336.231	1573.803	545.803	2403.643	1659.778	659.775	2494.677	1775.754	813.229
0.4	2186.341	1382.362	291.385	2280.012	1502.043	450.539	2406.807	1663.811	665.117
0.5	2020.993	1170.753	9.175	2143.361	1327.398	218.181	2309.405	1539.568	500.370
0.6	1823.891	917.954	0.000	1980.123	1118.383	0.000	2192.696	1390.485	302.198
0.7	1569.111	590.337	0.000	1768.609	846.948	0.000	2040.927	1196.288	43.283
0.8	1224.173	145.386	0.000	1481.429	477.382	0.000	1833.964	930.888	0.000
0.9	700.030	0.000	0.000	1059.248	0.000	0.000	1542.024	573.828	0.000

Welfare Cost									
	Low wage			Med wage			High wage		
Tax	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0.1	0.009	-0.021	-0.108	0.027	0.001	-0.073	0.050	0.032	-0.023
0.2	0.018	-0.013	-0.103	0.036	0.009	-0.068	0.060	0.040	-0.018
0.3	0.031	-0.002	-0.095	0.048	0.020	-0.061	0.073	0.052	-0.010
0.4	0.047	0.013	-0.084	0.065	0.035	-0.051	0.090	0.068	0.001
0.5	0.072	0.036	-0.067	0.089	0.057	-0.034	0.116	0.091	0.019
0.6	0.101	0.059	-0.034	0.120	0.083	-0.020	0.150	0.120	0.036
0.7	0.143	0.093	0.026	0.165	0.120	0.026	0.201	0.164	0.057
0.8	0.239	0.172	0.138	0.266	0.204	0.138	0.312	0.260	0.138
0.9	0.598	0.500	0.513	0.634	0.542	0.513	0.698	0.618	0.513

Total Tax Paid									
	Low wage			Med wage			High wage		
Tax	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.1	2687.715	1973.390	1021.606	3306.494	2441.834	1290.870	4744.993	3530.876	1916.890
0.2	5115.695	3616.278	1606.829	6352.527	4552.398	2144.689	9228.536	6729.436	3395.822
0.3	7249.092	4883.353	1693.573	9102.355	6285.412	2498.501	13413.631	9548.053	4372.653
0.4	9045.331	5719.108	1205.517	11512.237	7584.116	2274.860	17254.879	11928.192	4768.355
0.5	10451.564	6054.548	47.450	13527.821	8377.874	1377.053	20695.736	13796.840	4484.069
0.6	11318.703	5696.639	0.000	14997.058	8470.408	0.000	23579.809	14952.998	3249.775
0.7	11360.517	4274.100	0.000	15627.609	7483.714	0.000	25605.675	15008.742	543.035
0.8	10129.295	1202.979	0.000	14960.064	4820.792	0.000	26296.112	13347.441	0.000
0.9	6516.371	0.000	0.000	12033.797	0.000	0.000	24873.925	9256.242	0.000

Labour Supply Elasticity									
	Low wage			Med wage			High wage		
Tax	Low L	Med L	High L	Low L	Med L	High L	Low L	Med L	High L
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.1	-0.006	-0.009	-0.019	-0.005	-0.007	-0.015	-0.003	-0.005	-0.010
0.2	-0.029	-0.044	-0.118	-0.024	-0.035	-0.088	-0.016	-0.024	-0.055
0.3	-0.082	-0.130	-0.465	-0.066	-0.101	-0.312	-0.045	-0.067	-0.175
0.4	-0.186	-0.319	-1.949	-0.149	-0.242	-1.018	-0.101	-0.155	-0.476
0.5	-0.390	-0.739	-125.657	-0.307	-0.539	-4.257	-0.206	-0.331	-1.275
0.6	-0.813	-1.807	-	-0.629	-1.228	-	-0.414	-0.706	-4.173
0.7	-1.837	-5.587	-	-1.376	-3.231	-	-0.877	-1.640	-60.155
0.8	-5.058	-50.210	-	-3.556	-12.724	-	-2.134	-4.699	-
0.9	-25.519	-	-	-14.318	-	-	-7.319	-22.385	-

**Table 4 - Model Comparison with Data for Heterogeneous Wage**

	Earnings		ATI		Subsidy		Labour Supply	
	Model	Data	Model	Data	Model	Data	Model	Data
Low Wage	19676.05	21989.27	22397.35	21330.81	2591.445	1943.47	1902.354	2071.68
Med Wage	24366.56	25655.59	25068.72	23874.97	2591.445	1547.08	1929.871	2036.32
High Wage	35248.65	35911.32	34315.23	32557.28	2591.445	1103.23	1966.781	2059.21

**Table 5 - Model Comparison with Data for Heterogeneous Preference for Leisure**

	Earnings		ATI		Subsidy		Labour Supply	
	Model	Data	Model	Data	Model	Data	Model	Data
Low Psi	33132.37	33944.01	32174.39	31767.84	2355.26	1444.68	2624.762	2702.44
Med Psi	24504.02	30724.46	24408.89	26809.19	2355.26	1472.29	1941.22	2047.76
High Psi	13021.59	18646.73	14074.71	16466.75	2355.26	1824.77	1031.577	1271.4

**Table 6 - Model Comparison with Data for Heterogeneous Wage and Preference for Leisure**

Earnings							
	Model				Data		
	Leisure Preference				Leisure Preference		
Wage	low	med	high	Wage	low	med	high
low	26877.15	19733.89	10216.05	low	25720.38	24687.25	13231.96
med	33064.94	24418.33	12908.69	med	30807.18	28014.91	16919.29
high	47449.92	35308.75	19168.91	high	43067.78	38514.61	23880.58

ATI							
	Model				Data		
	Leisure Preference				Leisure Preference		
Wage	low	med	high	Wage	low	med	high
low	26685.59	20256.66	11690.6	low	26658.24	23542.68	15210.77
med	32254.61	24472.65	14113.98	med	28156.44	26272.68	17606.61
high	45201.09	34274.03	19748.16	high	39386.15	32909.29	25023.87

Subsidy							
	Model				Data		
	Leisure Preference				Leisure Preference		
Wage	low	med	high	Wage	low	med	high
low	2496.15	2496.15	2496.15	low	1887.47	1783.15	2249.19
med	2496.15	2496.15	2496.15	med	1434.48	1528.11	1698.05
high	2496.15	2496.15	2496.15	high	981.99	912.01	1456.03

Labour Supply							
	Model				Data		
	Leisure Preference				Leisure Preference		
Wage	low	med	high	Wage	low	med	high
low	2598.584	1907.947	987.727	low	2744.560	2059.210	1217.840
med	2619.420	1934.432	1022.633	med	2673.840	2045.160	1297.410
high	2647.432	1970.025	1069.514	high	2659.280	2041.520	1289.080

## Appendix C: Gauss Code

### Code 1 - Program to Calculate Equilibrium Labour Supply with Heterogeneous Wage

```
cls;

library nlsys;

eta = 1.5;

wl=10.343;
wm=12.626;
wh=17.922;

psi=1/(0.3519*(1-0.3519)^(-eta));

length=10;
grid = 0.1;
tao = seqa(0,grid,length);

NZ = rows(tao);
ati = zeros(length, 3);
earnings = zeros(length, 3);
welfare = zeros(length, 3);
lsupplylow = zeros(length,1);
lsupplymed = zeros(length,1);
lsupplyhi = zeros(length,1);
subsid = zeros(length,1);
welfcost = zeros(length,1);
elasticity = zeros(length, 3);

i=1;
do while i le NZ;

x0 = {0.3573, 0.3474, 0.3549, 0};
__output=0;
{x,f,g,h} = nlsys(&foc,x0);

lsupplylow[i] = x[1];
lsupplymed[i] = x[2];
lsupplyhi[i] = x[3];
subsid[i] = x[4];

elasl = wl*x[4]*(-1+x[1])/(wl^2-wl^2*x[1]-2*wl^2*tao[i]
+2*wl^2*tao[i]*x[1]+wl^2*tao[i]^2-wl^2*tao[i]^2*x[1]+eta
*(1-x[1])^(-psi)*psi*wl^2*x[1]^2-2*eta*(1-x[1])^(-psi)*psi
*wl^2*x[1]^2*tao[i]+2*eta*(1-x[1])^(-psi)*psi*wl*x[4]*x[1]
+eta*(1-x[1])^(-psi)*psi*wl^2*x[1]^2*tao[i]^2-2*eta*(1-x[1])^(-psi)
*psi*wl*x[1]*tao[i]*x[4]+eta*(1-x[1])^(-psi)*psi*x[4]^2);

elasm = wm*x[4]*(-1+x[2])/(wm^2-wm^2*x[2]-2*wm^2*tao[i]
+2*wm^2*tao[i]*x[2]+wm^2*tao[i]^2-wm^2*tao[i]^2*x[2]+eta
*(1-x[2])^(-psi)*psi*wm^2*x[2]^2-2*eta*(1-x[2])^(-psi)*psi
*wm^2*x[2]^2*tao[i]+2*eta*(1-x[2])^(-psi)*psi*wm*x[4]*x[2]
+eta*(1-x[2])^(-psi)*psi*wm^2*x[2]^2*tao[i]^2-2*eta*(1-x[2])^(-psi)
*psi*wm*x[2]*tao[i]*x[4]+eta*(1-x[2])^(-psi)*psi*x[4]^2);

elash = wh*x[4]*(-1+x[3])/(wh^2-wh^2*x[3]-2*wh^2*tao[i]
+2*wh^2*tao[i]*x[3]+wh^2*tao[i]^2-wh^2*tao[i]^2*x[3]+eta
*(1-x[3])^(-psi)*psi*wh^2*x[3]^2-2*eta*(1-x[3])^(-psi)*psi
*wh^2*x[3]^2*tao[i]+2*eta*(1-x[3])^(-psi)*psi*wh*x[4]*x[3]
+eta*(1-x[3])^(-psi)*psi*wh^2*x[3]^2*tao[i]^2-2*eta*(1-x[3])^(-psi)
*psi*wh*x[3]*tao[i]*x[4]+eta*(1-x[3])^(-psi)*psi*x[4]^2);
```

```

elasticity[i, .]=elas1*(tao[i]/x[1])~elasm*(tao[i]/x[2])~elash*(tao[i]/x[3]);

earnl = (wl*x[1])*5840;
earnm = (wm*x[2])*5840;
earnh = (wh*x[3])*5840;

earnings[i, .] = earnl-earnm-earnh;

atil = (1-tao[i])*earnl + subsid[i]*5840;
atim = (1-tao[i])*earnm + subsid[i]*5840;
atih = (1-tao[i])*earnh + subsid[i]*5840;

ati[i, .] = atil~atim~atih;

welfl = ln((1-tao[i])*wl*x[1] + subsid[i]) + (psi/(1-eta))*((1-x[1])^(1-eta));
welfm = ln((1-tao[i])*wm*x[2] + subsid[i]) + (psi/(1-eta))*((1-x[2])^(1-eta));
welfh = ln((1-tao[i])*wh*x[3] + subsid[i]) + (psi/(1-eta))*((1-x[3])^(1-eta));

welfare[i, .] = welfl~welfm~welfh;

i=i+1;

endo;

y0= 0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5;
{y,f,g,h} = nlsys(&foc1,y0);

z0= 0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5;
{z,f,g,h} = nlsys(&foc2,z0);

a0= 0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5;
{a,f,g,h} = nlsys(&foc3,a0);
/*=====*/

/*=====FIRST ORDER CONDITIONS=====*/
proc foc(x);
  local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8;

  fn1 = ((1-tao[i])*wl/((1-tao[i])*x[1]*wl + x[4])) - psi*(1-x[1]) ^ (-eta);
  fn2 = ((1-tao[i])*wm/((1-tao[i])*x[2]*wm + x[4])) - psi*(1-x[2]) ^ (-eta);
  fn3 = ((1-tao[i])*wh/((1-tao[i])*x[3]*wh + x[4])) - psi*(1-x[3]) ^ (-eta);
  fn4 = (((tao[i]/4)*wl*x[1]) + ((tao[i]/2)*wm*x[2]) + ((tao[i]/4)*wh*x[3]))-x[4];

  fn5 = x[1];
  fn6 = ((1-tao[i])*wm/((1-tao[i])*x[2]*wm + x[4])) - psi*(1-x[2]) ^ (-eta);
  fn7 = ((1-tao[i])*wh/((1-tao[i])*x[3]*wh + x[4])) - psi*(1-x[3]) ^ (-eta);
  fn8 = ((tao[i]/2)*wm*x[2]) + ((tao[i]/4)*wh*x[3])-x[4];

  if x[1] > 0 and x[2] > 0 and x[3] > 0;
  retp (fn1|fn2|fn3|fn4);
  elseif x[1] le 0 and x[2] > 0 and x[3] > 0;
  retp (fn5|fn6|fn7|fn8);
  endif;
endp;

/*=====CALCULATION OF WELFARE COST=====*/
proc foc1(y);
  local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9;

  fn1 = (ln((1+y[1])*((1-tao[2])*wl*lsupplylow[2] + subsid[2])) + (psi/(1-eta))*
  ((1-lsupplylow[2])^(1-eta))) - welfare[1,1];
  fn2 = (ln((1+y[2])*((1-tao[3])*wl*lsupplylow[3] + subsid[3])) + (psi/(1-eta))*
  ((1-lsupplylow[3])^(1-eta))) - welfare[2,1];

```

```

fn3 = (ln((1+y[3])*((1- $\tau$ [4])*wl*lsupplylow[4] + subsid[4])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[4])^(1- $\eta$ ))) - welfare[3,1];
fn4 = (ln((1+y[4])*((1- $\tau$ [5])*wl*lsupplylow[5] + subsid[5])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[5])^(1- $\eta$ ))) - welfare[4,1];
fn5 = (ln((1+y[5])*((1- $\tau$ [6])*wl*lsupplylow[6] + subsid[6])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[6])^(1- $\eta$ ))) - welfare[5,1];
fn6 = (ln((1+y[6])*((1- $\tau$ [7])*wl*lsupplylow[7] + subsid[7])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[7])^(1- $\eta$ ))) - welfare[6,1];
fn7 = (ln((1+y[7])*((1- $\tau$ [8])*wl*lsupplylow[8] + subsid[8])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[8])^(1- $\eta$ ))) - welfare[7,1];
fn8 = (ln((1+y[8])*((1- $\tau$ [9])*wl*lsupplylow[9] + subsid[9])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[9])^(1- $\eta$ ))) - welfare[8,1];
fn9 = (ln((1+y[9])*((1- $\tau$ [10])*wl*lsupplylow[10] + subsid[10])) + (psi/(1- $\eta$ ))*
((1-lsupplylow[10])^(1- $\eta$ ))) - welfare[9,1];

```

```

retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9);
endp;

```

```

proc foc2(z);
    local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9;

```

```

fn1 = (ln((1+z[1])*((1- $\tau$ [2])*wm*lsupplied[2] + subsid[2])) + (psi/(1- $\eta$ ))*
((1-lsupplied[2])^(1- $\eta$ ))) - welfare[1,2];
fn2 = (ln((1+z[2])*((1- $\tau$ [3])*wm*lsupplied[3] + subsid[3])) + (psi/(1- $\eta$ ))*
((1-lsupplied[3])^(1- $\eta$ ))) - welfare[2,2];
fn3 = (ln((1+z[3])*((1- $\tau$ [4])*wm*lsupplied[4] + subsid[4])) + (psi/(1- $\eta$ ))*
((1-lsupplied[4])^(1- $\eta$ ))) - welfare[3,2];
fn4 = (ln((1+z[4])*((1- $\tau$ [5])*wm*lsupplied[5] + subsid[5])) + (psi/(1- $\eta$ ))*
((1-lsupplied[5])^(1- $\eta$ ))) - welfare[4,2];
fn5 = (ln((1+z[5])*((1- $\tau$ [6])*wm*lsupplied[6] + subsid[6])) + (psi/(1- $\eta$ ))*
((1-lsupplied[6])^(1- $\eta$ ))) - welfare[5,2];
fn6 = (ln((1+z[6])*((1- $\tau$ [7])*wm*lsupplied[7] + subsid[7])) + (psi/(1- $\eta$ ))*
((1-lsupplied[7])^(1- $\eta$ ))) - welfare[6,2];
fn7 = (ln((1+z[7])*((1- $\tau$ [8])*wm*lsupplied[8] + subsid[8])) + (psi/(1- $\eta$ ))*
((1-lsupplied[8])^(1- $\eta$ ))) - welfare[7,2];
fn8 = (ln((1+z[8])*((1- $\tau$ [9])*wm*lsupplied[9] + subsid[9])) + (psi/(1- $\eta$ ))*
((1-lsupplied[9])^(1- $\eta$ ))) - welfare[8,2];
fn9 = (ln((1+z[9])*((1- $\tau$ [10])*wm*lsupplied[10] + subsid[10])) + (psi/(1- $\eta$ ))*
((1-lsupplied[10])^(1- $\eta$ ))) - welfare[9,2];

```

```

retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9);
endp;

```

```

proc foc3(a);
    local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9;

```

```

fn1 = (ln((1+a[1])*((1- $\tau$ [2])*wh*lsupplyhi[2] + subsid[2])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[2])^(1- $\eta$ ))) - welfare[1,3];
fn2 = (ln((1+a[2])*((1- $\tau$ [3])*wh*lsupplyhi[3] + subsid[3])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[3])^(1- $\eta$ ))) - welfare[2,3];
fn3 = (ln((1+a[3])*((1- $\tau$ [4])*wh*lsupplyhi[4] + subsid[4])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[4])^(1- $\eta$ ))) - welfare[3,3];
fn4 = (ln((1+a[4])*((1- $\tau$ [5])*wh*lsupplyhi[5] + subsid[5])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[5])^(1- $\eta$ ))) - welfare[4,3];
fn5 = (ln((1+a[5])*((1- $\tau$ [6])*wh*lsupplyhi[6] + subsid[6])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[6])^(1- $\eta$ ))) - welfare[5,3];
fn6 = (ln((1+a[6])*((1- $\tau$ [7])*wh*lsupplyhi[7] + subsid[7])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[7])^(1- $\eta$ ))) - welfare[6,3];
fn7 = (ln((1+a[7])*((1- $\tau$ [8])*wh*lsupplyhi[8] + subsid[8])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[8])^(1- $\eta$ ))) - welfare[7,3];
fn8 = (ln((1+a[8])*((1- $\tau$ [9])*wh*lsupplyhi[9] + subsid[9])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[9])^(1- $\eta$ ))) - welfare[8,3];
fn9 = (ln((1+a[9])*((1- $\tau$ [10])*wh*lsupplyhi[10] + subsid[10])) + (psi/(1- $\eta$ ))*
((1-lsupplyhi[10])^(1- $\eta$ ))) - welfare[9,3];

```

```

retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9);
endp;

```

```

/*=====PRINT COMMANDS=====*/

"atil      atim      atih";
print ati;

"earnl      earnm      earnh";
print earnings;

"welfl      welfm      welfh";
print welfare;

"lsupplylow  lsupplymed  lsupplyhi";
print lsupplylow*5840~lsupplymed*5840~lsupplyhi*5840;

"welfare cost low";
print y[1]|y[2]|y[3]|y[4]|y[5]|y[6]|y[7]|y[8]|y[9];

"welfare cost Med";
print z[1]|z[2]|z[3]|z[4]|z[5]|z[6]|z[7]|z[8]|z[9];

"welfare cost high";
print a[1]|a[2]|a[3]|a[4]|a[5]|a[6]|a[7]|a[8]|a[9];

"subsidy";
print subsid*5840;

"Labour Supply Elasticity";
print elasticity;

```

## Code 2 - Program to Calculate Equilibrium Labour Supply with Heterogeneous Leisure Preference

```

cls;
library nlsys;

eta = 1.5;

w = 12.623;

psil = 0.84237998;
psim = 1.4826675;
psih = 3.5217904;

length=10;
grid = 0.1;
tao = seqa(0,grid,length);

NZ = rows(tao);
ati = zeros(length, 3);
earnings = zeros(length, 3);
welfare = zeros(length, 3);
lsupplylow = zeros(length,1);
lsupplymed = zeros(length,1);
lsupplyhi = zeros(length,1);
subsid = zeros(length,1);
welfcost = zeros(length,1);
elasticity = zeros(length, 3);

i=1;
do while i le NZ;

x0 = {0.25, 0.4 , 0.5, 0};
__output=0;
{x,f,g,h} = nlsys(&foc,x0);

lsupplylow[i] = x[1];
lsupplymed[i] = x[2];
lsupplyhi[i] = x[3];
subsid[i] = x[4];

elas1 = w*x[4]*(-1+x[1])/(w^2-w^2*x[1]-2*w^2*tao[i]+2*w^2*tao[i]*x[1]
+w^2*tao[i]^2-w^2*tao[i]^2*x[1]+eta*(1-x[1])^(-psil)*psil*w^2*x[1]^2-2
*eta*(1-x[1])^(-psil)*psil*w^2*x[1]^2*tao[i]+2*eta*(1-x[1])^(-psil)*psil
*w*x[4]*x[1]+eta*(1-x[1])^(-psil)*psil*w^2*x[1]^2*tao[i]^2-2*eta*(1-x[1])^(-psil)
*psil*w*x[1]*tao[i]*x[4]+eta*(1-x[1])^(-psil)*psil*x[4]^2);

elasm = w*x[4]*(-1+x[2])/(w^2-w^2*x[2]-2*w^2*tao[i]+2*w^2*tao[i]*x[2]
+w^2*tao[i]^2-w^2*tao[i]^2*x[2]+eta*(1-x[2])^(-psim)*psim*w^2*x[2]^2-2
*eta*(1-x[2])^(-psim)*psim*w^2*x[2]^2*tao[i]+2*eta*(1-x[2])^(-psim)*psim
*w*x[4]*x[2]+eta*(1-x[2])^(-psim)*psim*w^2*x[2]^2*tao[i]^2-2*eta*(1-x[2])^(-psim)
*psim*w*x[2]*tao[i]*x[4]+eta*(1-x[2])^(-psim)*psim*x[4]^2);

elash = w*x[4]*(-1+x[3])/(w^2-w^2*x[3]-2*w^2*tao[i]+2*w^2*tao[i]*x[3]
+w^2*tao[i]^2-w^2*tao[i]^2*x[3]+eta*(1-x[3])^(-psih)*psih*w^2*x[3]^2-2
*eta*(1-x[3])^(-psih)*psih*w^2*x[3]^2*tao[i]+2*eta*(1-x[3])^(-psih)*psih
*w*x[4]*x[3]+eta*(1-x[3])^(-psih)*psih*w^2*x[3]^2*tao[i]^2-2*eta*(1-x[3])^(-psih)
*psih*w*x[3]*tao[i]*x[4]+eta*(1-x[3])^(-psih)*psih*x[4]^2);

elasticity[i, .]=elas1*(tao[i]/x[1])~elasm*(tao[i]/x[2])~elash*(tao[i]/x[3]);

earnl = (w*x[1])*5840;
earnm = (w*x[2])*5840;
earnh = (w*x[3])*5840;

earnings[i, .] = earnl~earnm~earnh;

```

```

atil = (1-tao[i])*earnl + x[4]*5840;
atim = (1-tao[i])*earnm + x[4]*5840;
atih = (1-tao[i])*earnh + x[4]*5840;

ati[i, .] = atil-atim-atih;

welfl = ln((1-tao[i])*w*x[1] + x[4]) + (psil/(1-eta))*((1-x[1])^(1-eta));
welfm = ln((1-tao[i])*w*x[2] + x[4]) + (psim/(1-eta))*((1-x[2])^(1-eta));
welfh = ln((1-tao[i])*w*x[3] + x[4]) + (psih/(1-eta))*((1-x[3])^(1-eta));

welfare[i, .] = welfl-welfm-welfh;

i=i+1;

endo;

y0= 0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5;
{y,f,g,h} = nlsys(&foc1,y0);

z0= 0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5;
{z,f,g,h} = nlsys(&foc2,z0);

a0= 0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5;
{a,f,g,h} = nlsys(&foc3,a0);
/*=====*/

/*=====FIRST ORDER CONDITIONS=====*/

proc foc(x);
    local fn1,fn2,fn3,fn4,fn5,fn6,fn7,fn8;

    fn1 = ((1-tao[i])*w/((1-tao[i])*x[1]*w + x[4])) - psil*(1-x[1]) ^ (-eta);
    fn2 = ((1-tao[i])*w/((1-tao[i])*x[2]*w + x[4])) - psim*(1-x[2]) ^ (-eta);
    fn3 = ((1-tao[i])*w/((1-tao[i])*x[3]*w + x[4])) - psih*(1-x[3]) ^ (-eta);
    fn4 = ((w*(tao[i]/3))*(x[1] + x[2] + x[3]))-x[4];
    fn7 = x[3];
    fn8 = ((w*(tao[i]/3))*(x[1] + x[2]))-x[4];
    if x[1] > 0 and x[2] > 0 and x[3] le 0 and x[4] > 0;
    retp (fn1|fn2|fn7|fn4);
    else;
    retp (fn1|fn2|fn3|fn4);

endif;
endp;

/*=====CALCULATION OF WELFARE COST=====*/
proc foc1(y);
    local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9;

    fn1 = (ln((1+y[1])*((1-tao[2])*w*lsupplylow[2] + subsid[2])) + (psil/(1-eta))*
    ((1-lsupplylow[2])^(1-eta))) - welfare[1,1];
    fn2 = (ln((1+y[2])*((1-tao[3])*w*lsupplylow[3] + subsid[3])) + (psil/(1-eta))*
    ((1-lsupplylow[3])^(1-eta))) - welfare[2,1];
    fn3 = (ln((1+y[3])*((1-tao[4])*w*lsupplylow[4] + subsid[4])) + (psil/(1-eta))*
    ((1-lsupplylow[4])^(1-eta))) - welfare[3,1];
    fn4 = (ln((1+y[4])*((1-tao[5])*w*lsupplylow[5] + subsid[5])) + (psil/(1-eta))*
    ((1-lsupplylow[5])^(1-eta))) - welfare[4,1];
    fn5 = (ln((1+y[5])*((1-tao[6])*w*lsupplylow[6] + subsid[6])) + (psil/(1-eta))*
    ((1-lsupplylow[6])^(1-eta))) - welfare[5,1];
    fn6 = (ln((1+y[6])*((1-tao[7])*w*lsupplylow[7] + subsid[7])) + (psil/(1-eta))*
    ((1-lsupplylow[7])^(1-eta))) - welfare[6,1];
    fn7 = (ln((1+y[7])*((1-tao[8])*w*lsupplylow[8] + subsid[8])) + (psil/(1-eta))*
    ((1-lsupplylow[8])^(1-eta))) - welfare[7,1];
    fn8 = (ln((1+y[8])*((1-tao[9])*w*lsupplylow[9] + subsid[9])) + (psil/(1-eta))*
    ((1-lsupplylow[9])^(1-eta))) - welfare[8,1];
    fn9 = (ln((1+y[9])*((1-tao[10])*w*lsupplylow[10] + subsid[10])) + (psil/(1-eta))*
    ((1-lsupplylow[10])^(1-eta))) - welfare[9,1];

retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9);

```

```

endp;

proc foc2(z);
    local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9;

fn1 = (ln((1+z[1])*((1-tao[2])*w*lsupplymed[2] + subsid[2])) + (psim/(1-eta))*
((1-lsupplymed[2])^(1-eta))) - welfare[1,2];
fn2 = (ln((1+z[2])*((1-tao[3])*w*lsupplymed[3] + subsid[3])) + (psim/(1-eta))*
((1-lsupplymed[3])^(1-eta))) - welfare[2,2];
fn3 = (ln((1+z[3])*((1-tao[4])*w*lsupplymed[4] + subsid[4])) + (psim/(1-eta))*
((1-lsupplymed[4])^(1-eta))) - welfare[3,2];
fn4 = (ln((1+z[4])*((1-tao[5])*w*lsupplymed[5] + subsid[5])) + (psim/(1-eta))*
((1-lsupplymed[5])^(1-eta))) - welfare[4,2];
fn5 = (ln((1+z[5])*((1-tao[6])*w*lsupplymed[6] + subsid[6])) + (psim/(1-eta))*
((1-lsupplymed[6])^(1-eta))) - welfare[5,2];
fn6 = (ln((1+z[6])*((1-tao[7])*w*lsupplymed[7] + subsid[7])) + (psim/(1-eta))*
((1-lsupplymed[7])^(1-eta))) - welfare[6,2];
fn7 = (ln((1+z[7])*((1-tao[8])*w*lsupplymed[8] + subsid[8])) + (psim/(1-eta))*
((1-lsupplymed[8])^(1-eta))) - welfare[7,2];
fn8 = (ln((1+z[8])*((1-tao[9])*w*lsupplymed[9] + subsid[9])) + (psim/(1-eta))*
((1-lsupplymed[9])^(1-eta))) - welfare[8,2];
fn9 = (ln((1+z[9])*((1-tao[10])*w*lsupplymed[10] + subsid[10])) + (psim/(1-eta))*
((1-lsupplymed[10])^(1-eta))) - welfare[9,2];

retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9);
endp;

proc foc3(a);
    local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9;

fn1 = (ln((1+a[1])*((1-tao[2])*w*lsupplyhi[2] + subsid[2])) + (psih/(1-eta))*
((1-lsupplyhi[2])^(1-eta))) - welfare[1,3];
fn2 = (ln((1+a[2])*((1-tao[3])*w*lsupplyhi[3] + subsid[3])) + (psih/(1-eta))*
((1-lsupplyhi[3])^(1-eta))) - welfare[2,3];
fn3 = (ln((1+a[3])*((1-tao[4])*w*lsupplyhi[4] + subsid[4])) + (psih/(1-eta))*
((1-lsupplyhi[4])^(1-eta))) - welfare[3,3];
fn4 = (ln((1+a[4])*((1-tao[5])*w*lsupplyhi[5] + subsid[5])) + (psih/(1-eta))*
((1-lsupplyhi[5])^(1-eta))) - welfare[4,3];
fn5 = (ln((1+a[5])*((1-tao[6])*w*lsupplyhi[6] + subsid[6])) + (psih/(1-eta))*
((1-lsupplyhi[6])^(1-eta))) - welfare[5,3];
fn6 = (ln((1+a[6])*((1-tao[7])*w*lsupplyhi[7] + subsid[7])) + (psih/(1-eta))*
((1-lsupplyhi[7])^(1-eta))) - welfare[6,3];
fn7 = (ln((1+a[7])*((1-tao[8])*w*lsupplyhi[8] + subsid[8])) + (psih/(1-eta))*
((1-lsupplyhi[8])^(1-eta))) - welfare[7,3];
fn8 = (ln((1+a[8])*((1-tao[9])*w*lsupplyhi[9] + subsid[9])) + (psih/(1-eta))*
((1-lsupplyhi[9])^(1-eta))) - welfare[8,3];
fn9 = (ln((1+a[9])*((1-tao[10])*w*lsupplyhi[10] + subsid[10])) + (psih/(1-eta))*
((1-lsupplyhi[10])^(1-eta))) - welfare[9,3];

retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9);
endp;

/*=====PRINT COMMANDS=====*/
"atil      atim      atih";
print ati;

"earnl     earnm     earnh";
print earnings;

"welfl     welfm     welfh";
print welfare;

"lsupplylow  lsupplymed  lsupplyhi";
print lsupplylow*5840~lsupplymed*5840~lsupplyhi*5840;

"welfare cost low";
print y[1]|y[2]|y[3]|y[4]|y[5]|y[6]|y[7]|y[8]|y[9];

"welfare cost Med";
print z[1]|z[2]|z[3]|z[4]|z[5]|z[6]|z[7]|z[8]|z[9];

```

```
"welfare cost high";  
print a[1]|a[2]|a[3]|a[4]|a[5]|a[6]|a[7]|a[8]|a[9];  
  
"subsidy";  
print subsid*5840;  
  
"Labour Supply Elasticity";  
print elasticity;
```

### Code 3 – Program to Calculate Equilibrium Labour Supply with Heterogeneous Wage and Preference for Leisure

```
cls;

library nlsys;

eta = 1.5;

wl=10.343;
wm=12.623;
wh=17.923;

psil = 0.84237998;
psim = 1.4826675;
psih = 3.5217904;

length=10;
grid = 0.1;
tau = seqa(0,grid,length);

NZ = rows(tau);
ati = zeros(length, 9);
earnings = zeros(length, 9);
welfare = zeros(length, 9);
elasticity = zeros(length, 9);
lsupplyllow = zeros(length,1);
lsupplylmed = zeros(length,1);
lsupplylhi = zeros(length,1);
lsupplymlow = zeros(length,1);
lsupplymmed = zeros(length,1);
lsupplymhi = zeros(length,1);
lsupplyhlow = zeros(length,1);
lsupplyhmed = zeros(length,1);
lsupplyhhi = zeros(length,1);

taxll = zeros(length,1);
taxlm = zeros(length,1);
taxlh = zeros(length,1);
taxml = zeros(length,1);
taxmm = zeros(length,1);
taxmh = zeros(length,1);
taxhl = zeros(length,1);
taxhm = zeros(length,1);
taxhh = zeros(length,1);

subsid = zeros(length,1);
welfcost = zeros(length,1);

i=1;
do while i le NZ;

x0 = 0.3|0.3|0.3|0.3|0.3|0.3|0.3|0.3|0.3|0.3|0;
__output=0;
{x,f,g,h} = nlsys(&foc,x0);
```

```

lsupplyllo[i] = x[1];
lsupplylme[i] = x[2];
lsupplylhi[i] = x[3];
lsupplyllo[i] = x[4];
lsupplylme[i] = x[5];
lsupplylhi[i] = x[6];
lsupplyllo[i] = x[7];
lsupplylme[i] = x[8];
lsupplylhi[i] = x[9];

subsid[i] = x[10];

elasl1 = wl*x[10]*(-1+x[1])/(wl^2-wl^2*x[1]-2*wl^2*tau[i]+2*wl^2*tau[i]*x[1]
+wl^2*tau[i]^2-wl^2*tau[i]^2*x[1]+eta*(1-x[1])^(-psil)*psil*wl^2*x[1]^2-2*eta
*(1-x[1])^(-psil)*psil*wl^2*x[1]^2*tau[i]+2*eta*(1-x[1])^(-psil)*psil*wl*x[10]
*x[1]+eta*(1-x[1])^(-psil)*psil*wl^2*x[1]^2*tau[i]^2-2*eta*(1-x[1])^(-psil)*psil
*wl*x[1]*tau[i]*x[10]+eta*(1-x[1])^(-psil)*psil*x[10]^2);

elaslm = wl*x[10]*(-1+x[2])/(wl^2-wl^2*x[2]-2*wl^2*tau[i]+2*wl^2*tau[i]*x[2]
+wl^2*tau[i]^2-wl^2*tau[i]^2*x[2]+eta*(1-x[2])^(-psim)*psim*wl^2*x[2]^2-2*eta
*(1-x[2])^(-psim)*psim*wl^2*x[2]^2*tau[i]+2*eta*(1-x[2])^(-psim)*psim*wl*x[10]
*x[2]+eta*(1-x[2])^(-psim)*psim*wl^2*x[2]^2*tau[i]^2-2*eta*(1-x[2])^(-psim)*psim
*wl*x[2]*tau[i]*x[10]+eta*(1-x[2])^(-psim)*psim*x[10]^2);

elaslh = wl*x[10]*(-1+x[3])/(wl^2-wl^2*x[3]-2*wl^2*tau[i]+2*wl^2*tau[i]*x[3]
+wl^2*tau[i]^2-wl^2*tau[i]^2*x[3]+eta*(1-x[3])^(-psih)*psih*wl^2*x[3]^2-2*eta
*(1-x[3])^(-psih)*psih*wl^2*x[3]^2*tau[i]+2*eta*(1-x[3])^(-psih)*psih*wl*x[10]
*x[3]+eta*(1-x[3])^(-psih)*psih*wl^2*x[3]^2*tau[i]^2-2*eta*(1-x[3])^(-psih)*psih
*wl*x[3]*tau[i]*x[10]+eta*(1-x[3])^(-psih)*psih*x[10]^2);

elasml = wm*x[10]*(-1+x[4])/(wm^2-wm^2*x[4]-2*wm^2*tau[i]+2*wm^2*tau[i]*x[4]
+wm^2*tau[i]^2-wm^2*tau[i]^2*x[4]+eta*(1-x[4])^(-psil)*psil*wm^2*x[4]^2-2*eta
*(1-x[4])^(-psil)*psil*wm^2*x[4]^2*tau[i]+2*eta*(1-x[4])^(-psil)*psil*wm*x[10]
*x[4]+eta*(1-x[4])^(-psil)*psil*wm^2*x[4]^2*tau[i]^2-2*eta*(1-x[4])^(-psil)*psil
*wm*x[4]*tau[i]*x[10]+eta*(1-x[4])^(-psil)*psil*x[10]^2);

elasmm = wm*x[10]*(-1+x[5])/(wm^2-wm^2*x[5]-2*wm^2*tau[i]+2*wm^2*tau[i]*x[5]
+wm^2*tau[i]^2-wm^2*tau[i]^2*x[5]+eta*(1-x[5])^(-psim)*psim*wm^2*x[5]^2-2*eta
*(1-x[5])^(-psim)*psim*wm^2*x[5]^2*tau[i]+2*eta*(1-x[5])^(-psim)*psim*wm*x[10]
*x[5]+eta*(1-x[5])^(-psim)*psim*wm^2*x[5]^2*tau[i]^2-2*eta*(1-x[5])^(-psim)*psim
*wm*x[5]*tau[i]*x[10]+eta*(1-x[5])^(-psim)*psim*x[10]^2);

elasmh = wm*x[10]*(-1+x[6])/(wm^2-wm^2*x[6]-2*wm^2*tau[i]+2*wm^2*tau[i]*x[6]
+wm^2*tau[i]^2-wm^2*tau[i]^2*x[6]+eta*(1-x[6])^(-psih)*psih*wm^2*x[6]^2-2*eta
*(1-x[6])^(-psih)*psih*wm^2*x[6]^2*tau[i]+2*eta*(1-x[6])^(-psih)*psih*wm*x[10]
*x[6]+eta*(1-x[6])^(-psih)*psih*wm^2*x[6]^2*tau[i]^2-2*eta*(1-x[6])^(-psih)*psih
*wm*x[6]*tau[i]*x[10]+eta*(1-x[6])^(-psih)*psih*x[10]^2);

elashl = wh*x[10]*(-1+x[7])/(wh^2-wh^2*x[7]-2*wh^2*tau[i]+2*wh^2*tau[i]*x[7]
+wh^2*tau[i]^2-wh^2*tau[i]^2*x[7]+eta*(1-x[7])^(-psil)*psil*wh^2*x[7]^2-2*eta
*(1-x[7])^(-psil)*psil*wh^2*x[7]^2*tau[i]+2*eta*(1-x[7])^(-psil)*psil*wh*x[10]
*x[7]+eta*(1-x[7])^(-psil)*psil*wh^2*x[7]^2*tau[i]^2-2*eta*(1-x[7])^(-psil)*psil
*wh*x[7]*tau[i]*x[10]+eta*(1-x[7])^(-psil)*psil*x[10]^2);

elashm = wh*x[10]*(-1+x[8])/(wh^2-wh^2*x[8]-2*wh^2*tau[i]+2*wh^2*tau[i]*x[8]
+wh^2*tau[i]^2-wh^2*tau[i]^2*x[8]+eta*(1-x[8])^(-psim)*psim*wh^2*x[8]^2-2*eta
*(1-x[8])^(-psim)*psim*wh^2*x[8]^2*tau[i]+2*eta*(1-x[8])^(-psim)*psim*wh*x[10]
*x[8]+eta*(1-x[8])^(-psim)*psim*wh^2*x[8]^2*tau[i]^2-2*eta*(1-x[8])^(-psim)*psim
*wh*x[8]*tau[i]*x[10]+eta*(1-x[8])^(-psim)*psim*x[10]^2);

elashh = wh*x[10]*(-1+x[9])/(wh^2-wh^2*x[9]-2*wh^2*tau[i]+2*wh^2*tau[i]*x[9]
+wh^2*tau[i]^2-wh^2*tau[i]^2*x[9]+eta*(1-x[9])^(-psih)*psih*wh^2*x[9]^2-2*eta
*(1-x[9])^(-psih)*psih*wh^2*x[9]^2*tau[i]+2*eta*(1-x[9])^(-psih)*psih*wh*x[10]
*x[9]+eta*(1-x[9])^(-psih)*psih*wh^2*x[9]^2*tau[i]^2-2*eta*(1-x[9])^(-psih)*psih
*wh*x[9]*tau[i]*x[10]+eta*(1-x[9])^(-psih)*psih*x[10]^2);

elasticity[i,.] = elasl1*(tau[i]/x[1])~elaslm*(tau[i]/x[2])~elaslh*(tau[i]/x[3])~elasml*
(tau[i]/x[4])~elasmm*(tau[i]/x[5])~elasmh*(tau[i]/x[6])~elashl*(tau[i]/x[7])~elashm*
(tau[i]/x[8])~elashh*(tau[i]/x[9]);

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```

earnll = (wl*x[1])*5840;
earnlm = (wl*x[2])*5840;
earnlh = (wl*x[3])*5840;

earnml = (wm*x[4])*5840;
earnmm = (wm*x[5])*5840;
earnmh = (wm*x[6])*5840;

earnhl = (wh*x[7])*5840;
earnhm = (wh*x[8])*5840;
earnhh = (wh*x[9])*5840;

earnings[i, .] = earnll-earnlm-earnlh-earnml-earnmm-earnmh-earnhl-earnhm-earnhh;

atill = (1-tau[i])*wl*x[1]*5840 + x[10]*5840;
atilm = (1-tau[i])*wl*x[2]*5840 + x[10]*5840;
atilh = (1-tau[i])*wl*x[3]*5840 + x[10]*5840;

atiml = (1-tau[i])*wm*x[4]*5840 + x[10]*5840;
atimm = (1-tau[i])*wm*x[5]*5840 + x[10]*5840;
atimh = (1-tau[i])*wm*x[6]*5840 + x[10]*5840;

atihl = (1-tau[i])*wh*x[7]*5840 + x[10]*5840;
atihm = (1-tau[i])*wh*x[8]*5840 + x[10]*5840;
atihh = (1-tau[i])*wh*x[9]*5840 + x[10]*5840;

ati[i, .] = atill-atilm-atilh-atiml-atimm-atimh-atihl-atihm-atihh;

welfll = ln((1-tau[i])*wl*x[1] + x[10]) + (psil/(1-eta))*((1-x[1])^(1-eta));
welflm = ln((1-tau[i])*wl*x[2] + x[10]) + (psim/(1-eta))*((1-x[2])^(1-eta));
welflh = ln((1-tau[i])*wl*x[3] + x[10]) + (psih/(1-eta))*((1-x[3])^(1-eta));

welfml = ln((1-tau[i])*wm*x[4] + x[10]) + (psil/(1-eta))*((1-x[4])^(1-eta));
welfmm = ln((1-tau[i])*wm*x[5] + x[10]) + (psim/(1-eta))*((1-x[5])^(1-eta));
welfmh = ln((1-tau[i])*wm*x[6] + x[10]) + (psih/(1-eta))*((1-x[6])^(1-eta));

welfhl = ln((1-tau[i])*wh*x[7] + x[10]) + (psil/(1-eta))*((1-x[7])^(1-eta));
welfhm = ln((1-tau[i])*wh*x[8] + x[10]) + (psim/(1-eta))*((1-x[8])^(1-eta));
welfhh = ln((1-tau[i])*wh*x[9] + x[10]) + (psih/(1-eta))*((1-x[9])^(1-eta));

welfare[i, .] = welfll-welflm-welflh-welfml-welfmm-welfmh-welfhl-welfhm-welfhh;

taxll[i] = tau[i]*wl*x[1]*5840;
taxlm[i] = tau[i]*wl*x[2]*5840;
taxlh[i] = tau[i]*wl*x[3]*5840;
taxml[i] = tau[i]*wm*x[4]*5840;
taxmm[i] = tau[i]*wm*x[5]*5840;
taxmh[i] = tau[i]*wm*x[6]*5840;
taxhl[i] = tau[i]*wh*x[7]*5840;
taxhm[i] = tau[i]*wh*x[8]*5840;
taxhh[i] = tau[i]*wh*x[9]*5840;

i=i+1;

endo;

y0=
0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|
0.5|0.5|0.5|0.5|0.5;
{y,f,g,h} = nlsys(&foc1,y0);

z0=
0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|
0.5|0.5|0.5|0.5|0.5;
{z,f,g,h} = nlsys(&foc2,z0);

a0=
0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|0.5|
0.5|0.5|0.5|0.5|0.5;
{a,f,g,h} = nlsys(&foc3,a0);

```

```

/*=====FIRST ORDER CONDITIONS=====*/
proc foc(x);
  local
  fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9, fn10, fn11, fn12, fn13, fn14, fn15, fn16, fn17, fn18, fn19;

  fn1 = (((1-tau[i])*wl)/((1-tau[i])*x[1]*wl + x[10])) - psil*(1-x[1]) ^ (-eta);
  fn2 = (((1-tau[i])*wl)/((1-tau[i])*x[2]*wl + x[10])) - psim*(1-x[2]) ^ (-eta);
  fn3 = (((1-tau[i])*wl)/((1-tau[i])*x[3]*wl + x[10])) - psih*(1-x[3]) ^ (-eta);

  fn4 = (((1-tau[i])*wm)/((1-tau[i])*x[4]*wm + x[10])) - psil*(1-x[4]) ^ (-eta);
  fn5 = (((1-tau[i])*wm)/((1-tau[i])*x[5]*wm + x[10])) - psim*(1-x[5]) ^ (-eta);
  fn6 = (((1-tau[i])*wm)/((1-tau[i])*x[6]*wm + x[10])) - psih*(1-x[6]) ^ (-eta);

  fn7 = (((1-tau[i])*wh)/((1-tau[i])*x[7]*wh + x[10])) - psil*(1-x[7]) ^ (-eta);
  fn8 = (((1-tau[i])*wh)/((1-tau[i])*x[8]*wh + x[10])) - psim*(1-x[8]) ^ (-eta);
  fn9 = (((1-tau[i])*wh)/((1-tau[i])*x[9]*wh + x[10])) - psih*(1-x[9]) ^ (-eta);

  fn10 = ((tau[i])/12)*(wl*(x[1]+x[2]+x[3])) + (2*wm*(x[4]+x[5]+x[6])) +
  (wh*(x[7]+x[8]+x[9]))-x[10];

  fn11=x[1];
  fn12=x[2];
  fn13=x[3];
  fn14=x[4];
  fn15=x[5];
  fn16=x[6];
  fn17=x[7];
  fn18=x[8];
  fn19=x[9];

  if x[1] > 0 and x[2] > 0 and x[3] le 0 and x[4] > 0 and x[5] > 0 and x[6] le 0 and x[7] > 0
  and x[8] > 0 and x[9] > 0;
  retp (fn1|fn2|fn13|fn4|fn5|fn16|fn7|fn8|fn9|fn10);
  elseif x[1] > 0 and x[2] > 0 and x[3] le 0 and x[4] > 0 and x[5] > 0 and x[6] le 0 and x[7]
  > 0 and x[8] > 0 and x[9] le 0;
  retp (fn1|fn2|fn13|fn4|fn5|fn16|fn7|fn8|fn19|fn10);
  elseif x[1] > 0 and x[2] le 0 and x[3] le 0 and x[4] > 0 and x[5] > 0 and x[6] le 0 and
  x[7] > 0 and x[8] > 0 and x[9] le 0;
  retp (fn1|fn12|fn13|fn4|fn5|fn16|fn7|fn8|fn19|fn10);
  elseif x[1] > 0 and x[2] le 0 and x[3] le 0 and x[4] > 0 and x[5] le 0 and x[6] le 0 and
  x[7] > 0 and x[8] > 0 and x[9] le 0;
  retp (fn1|fn12|fn13|fn4|fn15|fn16|fn7|fn8|fn19|fn10);
  else;
  retp (fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9|fn10);
  endif;
endp;

/*=====CALCULATION OF WELFARE COST=====*/
proc foc1(y);
  local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9, fn10, fn11, fn12, fn13, fn14,
  fn15, fn16, fn17, fn18, fn19, fn20, fn21, fn22, fn23, fn24, fn25, fn26, fn27;

  fn1 = (ln((1+y[1])*((1-tau[2])*wl*lsupplyllow[2] + subsid[2])) + (psil/(1-eta)) *
  ((1-lsupplyllow[2])^(1-eta))) - welfare[1,1];
  fn2 = (ln((1+y[2])*((1-tau[3])*wl*lsupplyllow[3] + subsid[3])) + (psil/(1-eta)) *
  ((1-lsupplyllow[3])^(1-eta))) - welfare[2,1];
  fn3 = (ln((1+y[3])*((1-tau[4])*wl*lsupplyllow[4] + subsid[4])) + (psil/(1-eta)) *
  ((1-lsupplyllow[4])^(1-eta))) - welfare[3,1];
  fn4 = (ln((1+y[4])*((1-tau[5])*wl*lsupplyllow[5] + subsid[5])) + (psil/(1-eta)) *
  ((1-lsupplyllow[5])^(1-eta))) - welfare[4,1];
  fn5 = (ln((1+y[5])*((1-tau[6])*wl*lsupplyllow[6] + subsid[6])) + (psil/(1-eta)) *
  ((1-lsupplyllow[6])^(1-eta))) - welfare[5,1];

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fn6 = (ln((1+y[6])*((1-tau[7])*wl*lsupplyllo[7] + subsid[7])) + (psil/(1-eta))*
((1-lsupplyllo[7])^(1-eta))) - welfare[6,1];
fn7 = (ln((1+y[7])*((1-tau[8])*wl*lsupplyllo[8] + subsid[8])) + (psil/(1-eta))*
((1-lsupplyllo[8])^(1-eta))) - welfare[7,1];
fn8 = (ln((1+y[8])*((1-tau[9])*wl*lsupplyllo[9] + subsid[9])) + (psil/(1-eta))*
((1-lsupplyllo[9])^(1-eta))) - welfare[8,1];
fn9 = (ln((1+y[9])*((1-tau[10])*wl*lsupplyllo[10] + subsid[10])) + (psil/(1-eta))*
((1-lsupplyllo[10])^(1-eta))) - welfare[9,1];

fn10 = (ln((1+y[10])*((1-tau[2])*wl*lsupplylmed[2] + subsid[2])) + (psim/(1-eta))*
((1-lsupplylmed[2])^(1-eta))) - welfare[1,2];
fn11 = (ln((1+y[11])*((1-tau[3])*wl*lsupplylmed[3] + subsid[3])) + (psim/(1-eta))*
((1-lsupplylmed[3])^(1-eta))) - welfare[2,2];
fn12 = (ln((1+y[12])*((1-tau[4])*wl*lsupplylmed[4] + subsid[4])) + (psim/(1-eta))*
((1-lsupplylmed[4])^(1-eta))) - welfare[3,2];
fn13 = (ln((1+y[13])*((1-tau[5])*wl*lsupplylmed[5] + subsid[5])) + (psim/(1-eta))*
((1-lsupplylmed[5])^(1-eta))) - welfare[4,2];
fn14 = (ln((1+y[14])*((1-tau[6])*wl*lsupplylmed[6] + subsid[6])) + (psim/(1-eta))*
((1-lsupplylmed[6])^(1-eta))) - welfare[5,2];
fn15 = (ln((1+y[15])*((1-tau[7])*wl*lsupplylmed[7] + subsid[7])) + (psim/(1-eta))*
((1-lsupplylmed[7])^(1-eta))) - welfare[6,2];
fn16 = (ln((1+y[16])*((1-tau[8])*wl*lsupplylmed[8] + subsid[8])) + (psim/(1-eta))*
((1-lsupplylmed[8])^(1-eta))) - welfare[7,2];
fn17 = (ln((1+y[17])*((1-tau[9])*wl*lsupplylmed[9] + subsid[9])) + (psim/(1-eta))*
((1-lsupplylmed[9])^(1-eta))) - welfare[8,2];
fn18 = (ln((1+y[18])*((1-tau[10])*wl*lsupplylmed[10] + subsid[10])) + (psim/(1-eta))*
((1-lsupplylmed[10])^(1-eta))) - welfare[9,2];

fn19 = (ln((1+y[19])*((1-tau[2])*wl*lsupplylhi[2] + subsid[2])) + (psih/(1-eta))*
((1-lsupplylhi[2])^(1-eta))) - welfare[1,3];
fn20 = (ln((1+y[20])*((1-tau[3])*wl*lsupplylhi[3] + subsid[3])) + (psih/(1-eta))*
((1-lsupplylhi[3])^(1-eta))) - welfare[2,3];
fn21 = (ln((1+y[21])*((1-tau[4])*wl*lsupplylhi[4] + subsid[4])) + (psih/(1-eta))*
((1-lsupplylhi[4])^(1-eta))) - welfare[3,3];
fn22 = (ln((1+y[22])*((1-tau[5])*wl*lsupplylhi[5] + subsid[5])) + (psih/(1-eta))*
((1-lsupplylhi[5])^(1-eta))) - welfare[4,3];
fn23 = (ln((1+y[23])*((1-tau[6])*wl*lsupplylhi[6] + subsid[6])) + (psih/(1-eta))*
((1-lsupplylhi[6])^(1-eta))) - welfare[5,3];
fn24 = (ln((1+y[24])*((1-tau[7])*wl*lsupplylhi[7] + subsid[7])) + (psih/(1-eta))*
((1-lsupplylhi[7])^(1-eta))) - welfare[6,3];
fn25 = (ln((1+y[25])*((1-tau[8])*wl*lsupplylhi[8] + subsid[8])) + (psih/(1-eta))*
((1-lsupplylhi[8])^(1-eta))) - welfare[7,3];
fn26 = (ln((1+y[26])*((1-tau[9])*wl*lsupplylhi[9] + subsid[9])) + (psih/(1-eta))*
((1-lsupplylhi[9])^(1-eta))) - welfare[8,3];
fn27 = (ln((1+y[27])*((1-tau[10])*wl*lsupplylhi[10] + subsid[10])) + (psih/(1-eta))*
((1-lsupplylhi[10])^(1-eta))) - welfare[9,3];

retp
(fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9|fn10|fn11|fn12|fn13|fn14|fn15|fn16|fn17|fn18|fn19|
fn20|fn21|fn22|fn23|fn24|fn25|fn26|fn27);
endp;

proc foc2(z);
local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9, fn10, fn11, fn12, fn13, fn14,
fn15, fn16, fn17, fn18, fn19, fn20, fn21, fn22, fn23, fn24, fn25, fn26, fn27;

fn1 = (ln((1+z[1])*((1-tau[2])*wm*lsupplymlow[2] + subsid[2])) + (psil/(1-eta))*
((1-lsupplymlow[2])^(1-eta))) - welfare[1,4];
fn2 = (ln((1+z[2])*((1-tau[3])*wm*lsupplymlow[3] + subsid[3])) + (psil/(1-eta))*
((1-lsupplymlow[3])^(1-eta))) - welfare[2,4];
fn3 = (ln((1+z[3])*((1-tau[4])*wm*lsupplymlow[4] + subsid[4])) + (psil/(1-eta))*
((1-lsupplymlow[4])^(1-eta))) - welfare[3,4];
fn4 = (ln((1+z[4])*((1-tau[5])*wm*lsupplymlow[5] + subsid[5])) + (psil/(1-eta))*
((1-lsupplymlow[5])^(1-eta))) - welfare[4,4];
fn5 = (ln((1+z[5])*((1-tau[6])*wm*lsupplymlow[6] + subsid[6])) + (psil/(1-eta))*
((1-lsupplymlow[6])^(1-eta))) - welfare[5,4];
fn6 = (ln((1+z[6])*((1-tau[7])*wm*lsupplymlow[7] + subsid[7])) + (psil/(1-eta))*
((1-lsupplymlow[7])^(1-eta))) - welfare[6,4];
fn7 = (ln((1+z[7])*((1-tau[8])*wm*lsupplymlow[8] + subsid[8])) + (psil/(1-eta))*
((1-lsupplymlow[8])^(1-eta))) - welfare[7,4];

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fn8 = (ln((1+z[8])*((1-tau[9])*wm*lsupplym[9] + subsid[9])) + (psil/(1-eta))*
((1-lsupplym[9])^(1-eta))) - welfare[8,4];
fn9 = (ln((1+z[9])*((1-tau[10])*wm*lsupplym[10] + subsid[10])) + (psil/(1-eta))*
((1-lsupplym[10])^(1-eta))) - welfare[9,4];

fn10 = (ln((1+z[10])*((1-tau[2])*wm*lsupplymmed[2] + subsid[2])) + (psim/(1-eta))*
((1-lsupplymmed[2])^(1-eta))) - welfare[1,5];
fn11 = (ln((1+z[11])*((1-tau[3])*wm*lsupplymmed[3] + subsid[3])) + (psim/(1-eta))*
((1-lsupplymmed[3])^(1-eta))) - welfare[2,5];
fn12 = (ln((1+z[12])*((1-tau[4])*wm*lsupplymmed[4] + subsid[4])) + (psim/(1-eta))*
((1-lsupplymmed[4])^(1-eta))) - welfare[3,5];
fn13 = (ln((1+z[13])*((1-tau[5])*wm*lsupplymmed[5] + subsid[5])) + (psim/(1-eta))*
((1-lsupplymmed[5])^(1-eta))) - welfare[4,5];
fn14 = (ln((1+z[14])*((1-tau[6])*wm*lsupplymmed[6] + subsid[6])) + (psim/(1-eta))*
((1-lsupplymmed[6])^(1-eta))) - welfare[5,5];
fn15 = (ln((1+z[15])*((1-tau[7])*wm*lsupplymmed[7] + subsid[7])) + (psim/(1-eta))*
((1-lsupplymmed[7])^(1-eta))) - welfare[6,5];
fn16 = (ln((1+z[16])*((1-tau[8])*wm*lsupplymmed[8] + subsid[8])) + (psim/(1-eta))*
((1-lsupplymmed[8])^(1-eta))) - welfare[7,5];
fn17 = (ln((1+z[17])*((1-tau[9])*wm*lsupplymmed[9] + subsid[9])) + (psim/(1-eta))*
((1-lsupplymmed[9])^(1-eta))) - welfare[8,5];
fn18 = (ln((1+z[18])*((1-tau[10])*wm*lsupplymmed[10] + subsid[10])) + (psim/(1-eta))*
((1-lsupplymmed[10])^(1-eta))) - welfare[9,5];

fn19 = (ln((1+z[19])*((1-tau[2])*wm*lsupplymhi[2] + subsid[2])) + (psih/(1-eta))*
((1-lsupplymhi[2])^(1-eta))) - welfare[1,6];
fn20 = (ln((1+z[20])*((1-tau[3])*wm*lsupplymhi[3] + subsid[3])) + (psih/(1-eta))*
((1-lsupplymhi[3])^(1-eta))) - welfare[2,6];
fn21 = (ln((1+z[21])*((1-tau[4])*wm*lsupplymhi[4] + subsid[4])) + (psih/(1-eta))*
((1-lsupplymhi[4])^(1-eta))) - welfare[3,6];
fn22 = (ln((1+z[22])*((1-tau[5])*wm*lsupplymhi[5] + subsid[5])) + (psih/(1-eta))*
((1-lsupplymhi[5])^(1-eta))) - welfare[4,6];
fn23 = (ln((1+z[23])*((1-tau[6])*wm*lsupplymhi[6] + subsid[6])) + (psih/(1-eta))*
((1-lsupplymhi[6])^(1-eta))) - welfare[5,6];
fn24 = (ln((1+z[24])*((1-tau[7])*wm*lsupplymhi[7] + subsid[7])) + (psih/(1-eta))*
((1-lsupplymhi[7])^(1-eta))) - welfare[6,6];
fn25 = (ln((1+z[25])*((1-tau[8])*wm*lsupplymhi[8] + subsid[8])) + (psih/(1-eta))*
((1-lsupplymhi[8])^(1-eta))) - welfare[7,6];
fn26 = (ln((1+z[26])*((1-tau[9])*wm*lsupplymhi[9] + subsid[9])) + (psih/(1-eta))*
((1-lsupplymhi[9])^(1-eta))) - welfare[8,6];
fn27 = (ln((1+z[27])*((1-tau[10])*wm*lsupplymhi[10] + subsid[10])) + (psih/(1-eta))*
((1-lsupplymhi[10])^(1-eta))) - welfare[9,6];

retp
(fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9|fn10|fn11|fn12|fn13|fn14|fn15|fn16|fn17|fn18|fn19|
fn20|fn21|fn22|fn23|fn24|fn25|fn26|fn27);
endp;

proc foc3(a);
    local fn1, fn2, fn3, fn4, fn5, fn6, fn7, fn8, fn9, fn10, fn11, fn12, fn13, fn14,
fn15, fn16, fn17, fn18, fn19, fn20, fn21, fn22, fn23, fn24, fn25, fn26, fn27;

fn1 = (ln((1+a[1])*((1-tau[2])*wh*lsupplyhlow[2] + subsid[2])) + (psil/(1-eta))*
((1-lsupplyhlow[2])^(1-eta))) - welfare[1,7];
fn2 = (ln((1+a[2])*((1-tau[3])*wh*lsupplyhlow[3] + subsid[3])) + (psil/(1-eta))*
((1-lsupplyhlow[3])^(1-eta))) - welfare[2,7];
fn3 = (ln((1+a[3])*((1-tau[4])*wh*lsupplyhlow[4] + subsid[4])) + (psil/(1-eta))*
((1-lsupplyhlow[4])^(1-eta))) - welfare[3,7];
fn4 = (ln((1+a[4])*((1-tau[5])*wh*lsupplyhlow[5] + subsid[5])) + (psil/(1-eta))*
((1-lsupplyhlow[5])^(1-eta))) - welfare[4,7];
fn5 = (ln((1+a[5])*((1-tau[6])*wh*lsupplyhlow[6] + subsid[6])) + (psil/(1-eta))*
((1-lsupplyhlow[6])^(1-eta))) - welfare[5,7];
fn6 = (ln((1+a[6])*((1-tau[7])*wh*lsupplyhlow[7] + subsid[7])) + (psil/(1-eta))*
((1-lsupplyhlow[7])^(1-eta))) - welfare[6,7];
fn7 = (ln((1+a[7])*((1-tau[8])*wh*lsupplyhlow[8] + subsid[8])) + (psil/(1-eta))*
((1-lsupplyhlow[8])^(1-eta))) - welfare[7,7];
fn8 = (ln((1+a[8])*((1-tau[9])*wh*lsupplyhlow[9] + subsid[9])) + (psil/(1-eta))*
((1-lsupplyhlow[9])^(1-eta))) - welfare[8,7];
fn9 = (ln((1+a[9])*((1-tau[10])*wh*lsupplyhlow[10] + subsid[10])) + (psil/(1-eta))*
((1-lsupplyhlow[10])^(1-eta))) - welfare[9,7];

```

```

fn10 = (ln((1+a[10])*((1-tau[2])*wh*lsupplyhmed[2] + subsid[2])) + (psim/(1-eta))*
((1-lsupplyhmed[2])^(1-eta))) - welfare[1,8];
fn11 = (ln((1+a[11])*((1-tau[3])*wh*lsupplyhmed[3] + subsid[3])) + (psim/(1-eta))*
((1-lsupplyhmed[3])^(1-eta))) - welfare[2,8];
fn12 = (ln((1+a[12])*((1-tau[4])*wh*lsupplyhmed[4] + subsid[4])) + (psim/(1-eta))*
((1-lsupplyhmed[4])^(1-eta))) - welfare[3,8];
fn13 = (ln((1+a[13])*((1-tau[5])*wh*lsupplyhmed[5] + subsid[5])) + (psim/(1-eta))*
((1-lsupplyhmed[5])^(1-eta))) - welfare[4,8];
fn14 = (ln((1+a[14])*((1-tau[6])*wh*lsupplyhmed[6] + subsid[6])) + (psim/(1-eta))*
((1-lsupplyhmed[6])^(1-eta))) - welfare[5,8];
fn15 = (ln((1+a[15])*((1-tau[7])*wh*lsupplyhmed[7] + subsid[7])) + (psim/(1-eta))*
((1-lsupplyhmed[7])^(1-eta))) - welfare[6,8];
fn16 = (ln((1+a[16])*((1-tau[8])*wh*lsupplyhmed[8] + subsid[8])) + (psim/(1-eta))*
((1-lsupplyhmed[8])^(1-eta))) - welfare[7,8];
fn17 = (ln((1+a[17])*((1-tau[9])*wh*lsupplyhmed[9] + subsid[9])) + (psim/(1-eta))*
((1-lsupplyhmed[9])^(1-eta))) - welfare[8,8];
fn18 = (ln((1+a[18])*((1-tau[10])*wh*lsupplyhmed[10] + subsid[10])) + (psim/(1-eta))*
((1-lsupplyhmed[10])^(1-eta))) - welfare[9,8];

```

```

fn19 = (ln((1+a[19])*((1-tau[2])*wh*lsupplyhhi[2] + subsid[2])) + (psih/(1-eta))*
((1-lsupplyhhi[2])^(1-eta))) - welfare[1,9];
fn20 = (ln((1+a[20])*((1-tau[3])*wh*lsupplyhhi[3] + subsid[3])) + (psih/(1-eta))*
((1-lsupplyhhi[3])^(1-eta))) - welfare[2,9];
fn21 = (ln((1+a[21])*((1-tau[4])*wh*lsupplyhhi[4] + subsid[4])) + (psih/(1-eta))*
((1-lsupplyhhi[4])^(1-eta))) - welfare[3,9];
fn22 = (ln((1+a[22])*((1-tau[5])*wh*lsupplyhhi[5] + subsid[5])) + (psih/(1-eta))*
((1-lsupplyhhi[5])^(1-eta))) - welfare[4,9];
fn23 = (ln((1+a[23])*((1-tau[6])*wh*lsupplyhhi[6] + subsid[6])) + (psih/(1-eta))*
((1-lsupplyhhi[6])^(1-eta))) - welfare[5,9];
fn24 = (ln((1+a[24])*((1-tau[7])*wh*lsupplyhhi[7] + subsid[7])) + (psih/(1-eta))*
((1-lsupplyhhi[7])^(1-eta))) - welfare[6,9];
fn25 = (ln((1+a[25])*((1-tau[8])*wh*lsupplyhhi[8] + subsid[8])) + (psih/(1-eta))*
((1-lsupplyhhi[8])^(1-eta))) - welfare[7,9];
fn26 = (ln((1+a[26])*((1-tau[9])*wh*lsupplyhhi[9] + subsid[9])) + (psih/(1-eta))*
((1-lsupplyhhi[9])^(1-eta))) - welfare[8,9];
fn27 = (ln((1+a[27])*((1-tau[10])*wh*lsupplyhhi[10] + subsid[10])) + (psih/(1-eta))*
((1-lsupplyhhi[10])^(1-eta))) - welfare[9,9];

```

```

retp
(fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9|fn10|fn11|fn12|fn13|fn14|fn15|fn16|fn17|fn18|fn19|
fn20|fn21|fn22|fn23|fn24|fn25|fn26|fn27);
endp;

```

```

/*=====PRINT COMMANDS=====*/
"atil      atim      atih";
print ati;

"earnl     earnm     earnh";
print earnings;

"welfl     welfm     welfh";
print welfare;

"lsupplyllow  lsupplylmed  lsupplylhi";
print lsupplyllow*5840~lsupplylmed*5840~lsupplylhi*5840;

"lsupplymlow  lsupplymmed  lsupplymhi";
print lsupplymlow*5840~lsupplymmed*5840~lsupplymhi*5840;

"lsupplyhlow  lsupplyhmed  lsupplyhhi";
print lsupplyhlow*5840~lsupplyhmed*5840~lsupplyhhi*5840;

"welfare cost low low";
print y[1]|y[2]|y[3]|y[4]|y[5]|y[6]|y[7]|y[8]|y[9];

```

```

"welfare cost low Med";
print y[10]|y[11]|y[12]|y[13]|y[14]|y[15]|y[16]|y[17]|y[18];

"welfare cost low high";
print y[19]|y[20]|y[21]|y[22]|y[23]|y[24]|y[25]|y[26]|y[27];

"welfare cost med low";
print z[1]|z[2]|z[3]|z[4]|z[5]|z[6]|z[7]|z[8]|z[9];

"welfare cost med Med";
print z[10]|z[11]|z[12]|z[13]|z[14]|z[15]|z[16]|z[17]|z[18];

"welfare cost med high";
print z[19]|z[20]|z[21]|z[22]|z[23]|z[24]|z[25]|z[26]|z[27];

"welfare cost hi low";
print a[1]|a[2]|a[3]|a[4]|a[5]|a[6]|a[7]|a[8]|a[9];

"welfare cost hi Med";
print a[10]|a[11]|a[12]|a[13]|a[14]|a[15]|a[16]|a[17]|a[18];

"welfare cost hi high";
print a[19]|a[20]|a[21]|a[22]|a[23]|a[24]|a[25]|a[26]|a[27];

"subsidy";
print subsid*5840;

"taxes";
print taxl1~taxlm~taxlh~taxml~taxmm~taxmh~taxhl~taxhm~taxhh;

"Labour Supply Elasticity";
print elasticity;

```

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