

FISCAL POLICY AND THE CANADIAN BUSINESS CYCLE

by

Stephen Cheung
BBA (Honours), Simon Fraser University 2002

PROJECT
SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

In the Department
of
Economics

© Stephen Cheung 2004

SIMON FRASER UNIVERSITY

March 2004

All rights reserved. This work may not be
reproduced in whole or in part, by photocopy
or other means, without permission of the author.

Approval

Name: **Stephen Cheung**

Degree: **M.A. (Economics)**

Title of Thesis/Project: **Fiscal Policy and the Canadian Business Cycle.**

Examining Committee:

Chair: Gordon Myers

David Andolfatto
Senior Supervisor

Ken Kasa
Supervisor

Brian Krauth
Internal Examiner

Date Approved:

March 24, 2004

SIMON FRASER UNIVERSITY



Partial Copyright Licence

The author, whose copyright is declared on the title page of this work, has granted to Simon Fraser University the right to lend this thesis, project or extended essay to users of the Simon Fraser University Library, and to make partial or single copies only for such users or in response to a request from the library of any other university, or other educational institution, on its own behalf or for one of its users.

The author has further agreed that permission for multiple copying of this work for scholarly purposes may be granted by either the author or the Dean of Graduate Studies.

It is understood that copying or publication of this work for financial gain shall not be allowed without the author's written permission.

The original Partial Copyright Licence attesting to these terms, and signed by this author, may be found in the original bound copy of this work, retained in the Simon Fraser University Archive.

Bennett Library
Simon Fraser University
Burnaby, BC, Canada

ABSTRACT

In this paper I document basic facts about Canadian business cycle fluctuations for the period 1971 to 1996 and assess the ability of simple real business cycle models to reproduce the qualitative and quantitative characteristics of these business cycle fluctuations. I develop a dynamic general equilibrium model with government spending that is financed by either lump-sum tax or labour-income tax. The inclusion of a government sector in the model weakly improves the model's ability to reproduce basic facts about Canadian business cycle fluctuations in the aforementioned period. When government spending is financed by a lump-sum tax, the model predicts a weak negative correlation between hours of work and productivity that is close to the value measured in the actual data. Inclusion of government also improves the model's predictability of the relative volatility of consumption to output.

DEDICATION

To my mother and sisters.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank Dr. David Andolfatto, my senior supervisor, Dr. Kenneth Kasa, my junior supervisor, and Dr. Brian Krauth, my internal examiner for their very kind support and guidance on my project. I especially owe a great deal of gratitude to Dr. Andolfatto who has spent a great deal of time answering my questions and checking my work. I would also like to thank Dr. Andolfatto for offering a great course in Macroeconomic Theory which equips me with the knowledge and skills to finish this project.

Finally, I wish to thank my mother for her love, care, and support. I could not have reached this stage of my life without my mother.

TABLE OF CONTENTS

Approval.....	ii
Abstract.....	iii
Dedication.....	iv
Acknowledgements.....	v
Table of Contents.....	vi
List of Figures.....	vii
List of Tables.....	viii
1 Introduction.....	1
2 General Equilibrium Model with Government Expenditure.....	5
3 General Equilibrium Model with Labour-Income Tax Regime.....	8
3.1 Household's Problem.....	8
3.2 Firm's Problem.....	9
3.3 Government.....	10
3.4 General Equilibrium.....	11
4 General Equilibrium Model with Lump-Sum Tax Regime.....	13
5 A Linear Representation of the General Equilibrium Model.....	15
6 Data and Summary Statistics of the Canadian Economy.....	19
7 Model Calibrations.....	21
8 Evaluations of General Equilibrium Models.....	26
9 Impulse Response Analyses.....	35
10 Conclusion.....	40
Reference.....	41
Appendix A: Data Descriptions.....	43
Appendix B: Solving for the Dynamics of the Models.....	50
Appendix C: Gauss Code.....	57

LIST OF FIGURES

Figure 1 Responses to a temporary 1% increase in government spending when government spending is financed with labour-income tax.....	36
Figure 2 Responses to a temporary 1% increase in government spending when government spending is financed with lump-sum tax.	38
Figure 3 Logarithms and HP Trends of the Data	47

LIST OF TABLES

Table 1 Coefficients of Law of Recursive System with Labour-Income Tax	17
Table 2 Coefficients of Law of Recursive System with Lump-Sum Tax	18
Table 3 Business Cycle Statistics for the Canadian Economy	20
Table 4 Parameters of Models	25
Table 5 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$: (Percentage Standard Deviations)	29
Table 6 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$. (Relative Percentage Standard Deviations).....	29
Table 7 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$. (Contemporaneous Correlations with Output.)	30
Table 8 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$. (Contemporaneous Correlations with Hours.).....	30
Table 9 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$. (Percentage Standard Deviations.)	31
Table 10 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$. (Relative Percentage Standard Deviations.).....	31
Table 11 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$. (Contemporaneous Correlations with Output.)	32
Table 12 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$. (Contemporaneous Correlations with Hours.).....	32
Table 13 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$. (Percentage Standard Deviations.)	33
Table 14 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$. (Relative Standard Deviations to Output.)	33
Table 15 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$. (Contemporaneous Correlations with Output.)	34
Table 16 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$. (Contemporaneous Correlations with Hours.).....	34

1 INTRODUCTION

I perform two major tasks in this paper. The first task of this paper is to document the characteristics of the time-series of the Canadian macroeconomic variables over the period of 1971 to 1996. The characteristics being documented include percentage standard deviations, relative percentage standard deviations, and contemporaneous correlations among the major macroeconomic variables. The second task is the assessment of whether standard neoclassical models with a government sector can generate time-series that have the same characteristics as suggested by the actual data.

The motivation of the quantitative exercise of the second task is induced by the methodological recommendations of Lucas (1980) and by Plosser (1989). Lucas (1980), in his paper "Methods and Problems in Business Cycle Theory", explained the importance of building artificial model economies and the need for computational experiments in business cycle research. He said, "...our task as I see it is to write a FORTRAN program that will accept specific economic policy rules as input and will generate as output statistics describing the operating characteristics of time series we care about" (p. 709). Plosser (1989) demonstrates a computational experiment using a very simple RBC model. He shows how even a very simple, unrealistic model can have considerable explaining power in terms of mimicking the observed fluctuations of a real economy¹. Using a dynamic stochastic general equilibrium model with certain types of disturbances, which may include productivity, fiscal policy, or monetary shocks, one can

¹ Plosser (1989) uses the statistics of the US economy from 1954 to 1985.

potentially understanding some of the characteristics of the time series of an actual economy.

The basic framework of a benchmark RBC model is built upon a set of primitive assumptions where individuals have preferences over consumption and leisure and individuals are utility maximizers. The constraint that these utility maximizers face with is often the production technology from which they generate their income. Given this framework of the RBC models, fluctuations in economic activities such as consumption, investment, and work hours initiated by some shocks is Pareto optimal. Following from this Pareto efficiency argument, an important insight provided by the RBC literature is that an increase in employment and decrease in output do not automatically call for government intervention to boost the economy².

On the other hand, there could be many other reasons for government interventions, such as provision of national defense, social welfare programs (for distributive reasons), and other public goods. In fact, government tax and spending policies are present in virtually every economic system. One of the goals of this paper is to incorporate government into the baseline real business cycle model and see how this added component affects the ability of general equilibrium models to explain fluctuations in an actual economy. The usefulness of the model in explaining economic fluctuations will be judged on its ability to generate time series with the same patterns of persistence, comovement, and volatility as those of actual economies.

² Long and Plosser (1983) develop a general equilibrium model in which there is no technological change, no long-lived commodities, no frictions or adjustment costs, no government, and no money. They argue that the major features of observed business cycles will be found in this kind of model. In other words, they believe that business-cycle fluctuations should not be viewed entirely as welfare-reducing deviations from efficient Walrasian economy.

The main advantage of adding government to the model is that it introduces shift in labour supply. As explained in Hansen and Wright (1992), shocks to government spending shift the labour supply curve because of the negative wealth effect brought about by the increase in taxes that is associated with an increase in government spending. At the same time, technology shocks shift the labour demand curve along the supply curve. The first effect produces a negative relationship between hours and productivity, while the second effect produces a positive relationship. The net effect on the correlation between hours and productivity in the model depends on the size of the government spending shocks as well as the response of people's actions to those shocks. Braun (1994) finds that when fluctuating tax rates are combined with shocks to productivity in a real business cycle model economy, the model predicts a weak negative correlation between average productivity and hours that is close to the value measured in the postwar U.S. data. Lopez-Pinto (2001) has similar conclusions in his study of fiscal policy in the Spanish business cycle. He finds that when shocks to tax rates are included, the model predicts a negative correlation between productivity and hours of work. McGrattan (1994) finds that if changes in both technology and fiscal variables occur, the model predicts a correlation close to zero. Christiano and Eichenbaum (1992) also claim that allowing government consumption shocks to influence labour-market dynamics bring the models into closer conformity with the U.S. data. More recent works include Guo (2004) and Devereux et al. (1996). They show that a real business cycle model with increasing returns-to-scale is able to produce quantitative realistic business cycles driven solely by disturbances to government purchases.

The main objective of this paper is to assess whether incorporation of government spending in the baseline RBC model helps improve the performance of the model in predicting the characteristics of the time series of the Canadian economy. To

do this, I develop a stochastic general equilibrium model with government spending and compare its quantitative and qualitative implications of several macroeconomic variables to those of the baseline model that includes only technology shocks. I assume that government purchases are financed by either a lump-sum tax or a labour-income tax in my models. Consistent with Baxter and King (1993), the general equilibrium model with government purchases financed by lump-sum tax predicts a rise in hours worked and a decrease in after-tax real wages from a positive government spending shock. On the other hand, when government purchases are financed by labour-income tax, both hours worked and after-tax real wages fall in response to a positive government spending shock. A major deficiencies of the baseline real business cycle with only technology shocks is that it predict a strong positive correlation between hours of work and productivity. The major improvement in the performance of the model brought about by the incorporation of a government sector is the ability to generate a weak negative correlation between hours of work and productivity and a much higher relative volatility of consumption to output.

This paper is structured as follows: Section 2, 3 and 4 set up general equilibrium models with one type of government expenditure and two different tax regimes, namely lump-sum tax and labour-income tax; Section 5 describes a linear representation of the models; Section 6 describes data and statistics on the Canadian economy; Section 7 describes in detail the estimation and calibration strategies; Section 8 presents results of the simulations; Section 9 provides some concluding remarks.

2 GENERAL EQUILIBRIUM MODEL WITH GOVERNMENT EXPENDITURE

In this section I provide a general discussion of a stochastic general equilibrium model in which there is one type of government expenditure. In Section 3, I present a model where government purchase is financed by a labour-income tax and in Section 4, I present the model where government purchase is financed by a lump-sum tax. In both cases, I assume that government consumption is not a substitute for private consumption. Given this assumption, an increase in government spending financed by a lump-sum tax has a negative wealth effect on individuals, which induces them to work more if leisure is a normal good. Under the case with labour-income tax, an increase in tax rate implies a decrease in the marginal benefit of working, which reduces people's incentive to work. This substitution effect is working in the opposite direction to the negative wealth effect. Therefore, how people respond to government spending shocks depends on the magnitudes of the wealth and substitution effects.

Assume there is an infinite number of infinitely lived, homogeneous individuals who have preferences over consumption and leisure. Each household maximizes its utility for its whole lifetime with a discount factor, β , on future utilities derived from future consumption and leisure. Preferences are then given by:

$$E \sum_{t=0}^{\infty} \beta^t U(C_t + \lambda G_t, l_t), \quad 0 < \beta < 1 \quad (2.1)$$

where C_t denotes private consumption and G_t denotes government consumption. The utility function in (2.1) is assumed to be concave so that it implies a preference for smooth profiles of consumption and leisure. The parameter, λ , governs how government purchase affects the individual's utility. If $\lambda > 0$, the marginal utility of consumption decreases with an increase in G . If $\lambda < 0$, the opposite is true. Throughout the analysis in this paper, I will assume λ is equal to 0. Individuals' available time is distributed between hours of work and hours of leisure

$$T = N_t + l_t, \quad (2.2)$$

where T is the individuals' endowment of time that is normalized to 1, N_t is the fraction of the time endowment which is supplied to the labour market, and l_t is the amount of time devoted to leisure. In section 3, I develop the model with labour-income tax regime. In section 4, I develop the model with lump-sum tax regime.

The output of the economy is determined by some function of capital stock and effective labour input. Formally,

$$\tilde{Y}_t = A_t F(\tilde{K}_t, X_t L_t N_t), \quad (2.3)$$

where A_t is technology shock, \tilde{Y}_t is total output of the economy, X_t is the deterministic level of labour-augmenting productivity in the economy, and L_t is the size of the labour force³. It is assumed in my model that the deterministic gross growth rate of X_t is η .

³ L_t can be normalized to 1 and so be excluded from the model without affecting the quantitative and qualitative implications of the models. However, L_t needs to be included to estimate the Solow residual.

So $X_t = \eta X_{t-1}$. The parameter η will be set at the average growth rate of per capita GDP over my sample period. The production function can be transformed into per effective labour basis that renders a stationary system in the general equilibrium models by replacing \tilde{Y}_t and \tilde{K}_t with variables of the form $Y_t = \tilde{Y}_t / (X_t L_t)$ and $K_t = \tilde{K}_t / (X_t L_t)$. Output per effective labour is written as $Y_t = A_t F(K_t, N_t)$.

3 GENERAL EQUILIBRIUM MODEL WITH LABOUR-INCOME TAX REGIME

In this section, I develop a general equilibrium model with labour-income tax regime. I describe the household optimization problem in section 3.1, the firm's optimization problem in section 3.2, government budget constraint in section 3.3, and the general equilibrium in section 3.4.

3.1 Household's Problem

The household generates income through renting capital and labour service.

Under labour-income tax regime, the household's budget constraint is:

$$C_t + \eta K_{t+1} - (1 - \tau_{w_t})W_t N_t + R_t K_t \quad (3.1.1)$$

where τ_{w_t} is the tax rate on labour income at time t , w_t is the wage rate per hour, and r_t the rental rate of one unit of capital. The household choice problem can be stated as

$$MAXE_t \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \frac{\theta(1 - N_t)^{(1-\gamma)}}{1-\gamma} \right) \right] \quad (3.1.2)$$

subject to equation (3.1.1)

The first order conditions that characterize the household's optimal decision rule are:

$$\frac{1}{C_t}[(1-\tau_w)W_t] - \theta(1-N_t)^{-\gamma} = 0 \quad (3.1.3)$$

$$\frac{-\eta}{C_t} + \beta E_t \left[\frac{1}{C_{t+1}} R_{t+1} \right] = 0$$

OR

$$1 = \beta E_t \left[\frac{C_t}{\eta C_{t+1}} R_{t+1} \right] \quad (3.1.4)$$

Given the specification of the utility function of (3.1.2), labour supply elasticity is

$$\frac{\gamma^{-1}(1-N_t)}{N_t}$$

3.2 Firm's Problem

Assume there are a large number of identical firms operating in a competitive market and each of them owns the following Cobb-Douglas production function

$$Y_t = A_t F(K_t, N_t) = A_t K_t^\alpha N_t^{1-\alpha}$$

Then, the firm's problem can be stated as

$$\underset{K_t, N_t}{\text{MAX}} [A_t F(K_t, N_t) + (1-\delta)K_t - W_t N_t - R_t K_t]$$

Subject to the production technology and productivity shocks

$$\ln A_t = (1-\phi) \ln \bar{A} + \phi \ln A_{t-1} + \varepsilon_{A,t}, \quad \varepsilon_{A,t} \sim i.i.d.N(0; \sigma_A^2)$$

The first order conditions of this optimisation problem are

$$A_t \alpha K_t^{\alpha-1} N_t^{1-\alpha} + (1-\delta) = R_t \quad (3.2.1)$$

$$(1-\alpha)A_t K_t^\alpha N_t^{-\alpha} = W_t \quad (3.2.2)$$

3.3 Government

In this economy, it is assumed that government is able to balance its budget in every period. Under the case with distortionary labour-income tax regime, the government budget constraint is the following:

$$G_t = \tau_{wt} W_t N_t$$

$$\tau_{wt} = \frac{G_t}{W_t N_t}$$

The stochastic government spending, G_t , is assumed to be governed by

$$\ln G_t = (1-\rho) \ln \bar{G} + \rho \ln G_{t-1} + \varepsilon_{G,t}^4 \quad \varepsilon_{G,t} \sim i.i.d.N(0; \sigma_G^2)$$

⁴ This specification of stochastic government spending process follows the one specified in Christiano and Eichenbaum (1992).

3.4 General Equilibrium

Now I can substitute the firm's first order condition into the individual's budget constraint to characterize the general equilibrium. The individual's budget constraint becomes

$$C_t = A_t K_t^\alpha N_t^{1-\alpha} + (1-\delta)K_t - \eta K_{t+1} - G_t \quad (3.4.1)$$

Here is the summary of the set of conditions that characterize the general equilibrium:

$$\theta(1-N_t)^{-\gamma} = (1-\tau_t) \frac{W_t}{C_t} \quad (3.4.2)$$

$$1 = \beta E_t \left[\frac{C_t}{\eta C_{t+1}} R_{t+1} \right] \quad (3.4.3)$$

$$R_t = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} + (1-\delta) \quad (3.4.4)$$

$$W_t = (1-\alpha) A_t K_t^\alpha N_t^{-\alpha} \quad (3.4.5)$$

$$1-\tau_t = 1 - \frac{G_t}{W_t N_t} \quad (3.4.6)$$

$$\ln A_t = (1-\varphi) \ln \bar{A} + \varphi \ln A_{t-1} + \varepsilon_{A,t} \quad (3.4.7)$$

$$\ln G_t = (1-\rho) \ln \bar{G} + \rho \ln G_{t-1} + \varepsilon_{G,t} \quad (3.4.8)$$

This completes the description of the stochastic general equilibrium model under labour-income tax regime. The parameter values of the model are presented in Table 1 of Section 7. I will perform sensitivity analysis by choosing different values for labour

supply elasticities. That is, different values for γ . Calibration strategy is discussed in Section 7.

Using the above conditions and the fact the average tax rate and N are 21.7% and 0.227⁵ respectively over my sample period, I can derive the steady-state values of real interest rate (\bar{R}), capital stock (\bar{K}), output (\bar{Y}), real wage (\bar{W}), government spending (\bar{G}), and consumption (\bar{C}). Note that steady-state level of technology (\bar{A}) is normalized to 1. The following set of equations summarizes these steady-state values:

$$\bar{R} = \frac{\eta}{\beta} \quad (3.4.9)$$

$$\bar{K} = \left[\frac{\alpha \bar{A} \bar{N}^{(1-\alpha)}}{\bar{R} - 1 + \delta} \right]^{\frac{1}{1-\alpha}} \quad (3.4.10)$$

$$\bar{Y} = \bar{A} \bar{K}^{\alpha} \bar{N}^{(1-\alpha)} \quad (3.4.11)$$

$$\bar{G} = \tau \bar{W} \bar{N} \quad (3.4.11)$$

$$\bar{W} = (1-\alpha) \bar{K}^{\alpha} \bar{N}^{(-\alpha)} \quad (3.4.13)$$

$$\bar{C} = \bar{Y} + (1-\delta-\eta) \bar{K} - \bar{G} \quad (3.4.14)$$

⁵ The estimations of average hours of work and tax rate over my sample period are explained in Section 7.

4 GENERAL EQUILIBRIUM MODEL WITH LUMP-SUM TAX REGIME

Mathematically, the model with lump-sum tax is similar to the one with labour-income tax, except the household's budget constraint becomes

$$C_t = (1 - \delta)k_t + w_t n_t + r_t k_t - \eta k_{t+1} - T_t, \quad (4.1)$$

The first order conditions of the household's optimization problem are:

$$\frac{w_t}{C_t} - (1 - N_t)^{-\gamma} = 0 \quad (4.2)$$

$$\frac{-1}{C_t} + E_t \left\{ \beta \left[\frac{1}{\eta C_{t+1}} R_{t+1} \right] \right\} = 0 \quad (4.3)$$

The firm's problem is the same as in the previous model. On the other hand, government budget constraint is now simple $G_t = T_t$. The general equilibrium with lump-sum tax are characterized by the following set of equations:

$$C_t = (1 - \delta)k_t + w_t n_t + r_t k_t - \eta k_{t+1} - G_t \quad (4.4)$$

$$(1 - N_t)^{-\gamma} = \frac{W_t}{C_t} \quad (4.5)$$

$$1 = \beta E_t \left[\frac{C_t}{\eta C_{t+1}} R_{t+1} \right] \quad (4.6)$$

R_t , W_t , $\ln A_t$, and $\ln G_t$ are the same as in the model with labour-income tax. To derive the baseline model, that is the model with only technology shocks, one simply sets the steady state tax rate and government spending to zero.

5 A LINEAR REPRESENTATION OF THE GENERAL EQUILIBRIUM MODEL

I can now turn to the problem of solving for the dynamics in the stochastic general equilibrium models presented in sections 3 and 4. I first log-linearize the constraints and the first-order conditions and then solve for the recursive equilibrium law of motion via the method of undetermined coefficients. I sketch the procedures for the log-linearization and method of undetermined coefficients here and provide the details in Appendix B. Interested readers can see Campbell (1994) and Christiano (1998) for a more general and advanced discussion on solving dynamic equilibrium models by a method of undetermined coefficients.

I define $c_t, n_t, r_t, w_t, \tau_t, y_t, a_t, g_t$ and i_t as the logarithmic deviations of $C_t, N_t, R_t, W_t, \tau_t, Y_t, A_t, G_t$ and I_t respectively from their respective steady-state values. For example, $c_t = \log C_t - \log \bar{C}$ and $C_t = \bar{C}e^{c_t} \approx \bar{C}(1 + c_t)$. The log-linearized constraints and first-order conditions of the model with labour-income tax are

$$c_t \approx \frac{\bar{K}}{\bar{C}} \bar{R} k_t + \frac{\bar{Y}}{\bar{C}} a_t + \frac{\bar{Y}}{\bar{C}} (1 - \alpha) n_t - \frac{\bar{K}}{\bar{C}} k_{t+1} - \frac{\bar{G}}{\bar{C}} g_t \quad (5.1)$$

$$n_t \approx \left(\frac{1 - \bar{N}}{\bar{N}} \right) \frac{1}{\gamma} (\tau_t + w_t - c_t) \quad (5.2)$$

$$0 \approx E_t [c_t - c_{t+1} + r_{t+1}] \quad (5.3)$$

$$r_t \approx (1 - \beta + \beta\delta) [a_t + (1 - \alpha)n_t - (1 - \alpha)k_t] \quad (5.4)$$

$$w_t \approx \alpha k_t + a_t - \alpha n_t \quad (5.5)$$

$$\tau_t \approx \frac{\bar{\tau}}{(1-\bar{\tau})} (w_t + n_t - g_t) \quad (5.6)$$

$$y_t \approx \alpha k_t + a_t + (1-\alpha)n_t \quad (5.7)$$

$$i_t \approx \frac{1}{\delta} k_{t+1} + (1-\frac{1}{\delta})k_t \quad (5.8)$$

$$a_t \approx \varphi a_{t-1} + \varepsilon_{A,t} \quad (5.9)$$

$$g_t \approx \rho g_{t-1} + \varepsilon_{G,t} \quad (5.10)$$

The postulated linear recursive laws of motion are

$$k_{t+1} = v_{kk} k_t + v_{ka} a_t + v_{kg} g_t \quad (5.11)$$

$$c_t = v_{ck} k_t + v_{ca} a_t + v_{cg} g_t \quad (5.12)$$

$$\bar{r}_t = v_{rk} k_t + v_{ra} a_t + v_{rg} g_t \quad (5.13)$$

$$n_t = v_{nk} k_t + v_{na} a_t + v_{ng} g_t \quad (5.14)$$

$$w_t = v_{wk} k_t + v_{wa} a_t + v_{wg} g_t \quad (5.15)$$

$$\tau_t = v_{\tau k} k_t + v_{\tau a} a_t + v_{\tau g} g_t \quad (5.16)$$

$$y_t = v_{yk} k_t + v_{ya} a_t + v_{yg} g_t \quad (5.17)$$

$$i_t = v_{ik} k_t + v_{ia} a_t + v_{ig} g_t \quad (5.18)$$

What is given at time t are the state variables k_t , a_t , and g_t . The details of solving the above system of linear recursive laws of motion are explained in Appendix B. The values of the coefficients of the state variables of the above eight recursive laws of

motions under labour-income tax and lump-sum tax regimes are presented in Tables 1 and 2. These coefficients can be interpreted as elasticities. For example, the value of v_{kk} under labour-income tax regime is 0.98. It means that if k_t is 1 percent above its steady state level, C_t would be 0.98 percent above its steady state level. On the other hand, the signs of the coefficients of the state variables a_t , and g_t show the directions of responses to technology or government spending shocks. For example, positive v_{ng} under lump-sum tax regime implies that people will respond to a government spending shock by increasing labour supply. (Once again, this is due to the negative wealth effect that is brought about by an increase in tax) On the other hand, negative v_{ng} under labour-income tax regime implies that people will respond to a government spending shock by decreasing labour supply.

Table 1 Coefficients of Law of Recursive System with Labour-Income Tax

Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$	Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$
v_{kk}	0.9818	0.9821	0.9823	v_{ck}	0.8252	0.8301	0.8335
v_{ka}	0.1117	0.1362	0.1570	v_{ca}	0.0967	0.1186	0.1372
v_{kg}	-0.0064	-0.0074	-0.0081	v_{cg}	-0.5333	-0.5562	-0.5175
Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$	Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$
v_{rk}	-0.0153	-0.0151	-0.0150	v_{nk}	0.0181	0.0293	0.0370
v_{ra}	0.0433	0.0528	0.0608	v_{na}	0.7367	1.3463	1.8631
v_{rg}	-0.0012	-0.0018	-0.0023	v_{ng}	-0.0744	-0.1185	-0.1477
Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$	Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$
v_{wk}	0.5008	0.4951	0.4911	$v_{\tau k}$	0.3606	0.3644	0.3670
v_{wa}	0.6243	0.3134	0.0498	$v_{\tau a}$	0.9458	1.1533	1.3293
v_{wg}	0.0380	0.0604	0.0753	$v_{\tau g}$	-0.7203	-0.7353	-0.7452
Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$	Coefficient	$\gamma=6.81$	$\gamma=3.41$	$\gamma=2.27$
v_{yk}	0.5189	0.5244	0.5281	v_{ik}	0.2719	0.2842	0.2926
v_{ya}	1.3610	1.6597	1.9130	v_{ia}	4.4683	5.4481	6.2787
v_{yg}	-0.0365	-0.0581	-0.0724	v_{ig}	-0.2578	-0.2972	-0.3231

Table 2 Coefficients of Law of Recursive System with Lump-Sum Tax

Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$	Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$
v_{kk}	0.9791	0.9773	0.9761	v_{ck}	0.8222	0.7994	0.7855
v_{ka}	0.0960	0.1056	0.1123	v_{ca}	0.0830	0.0888	0.0930
v_{kg}	-0.0007	0.0008	0.0017	v_{cg}	-0.3066	-0.2788	-0.2623
Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$	Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$
v_{rk}	-0.0175	-0.0186	-0.0192	v_{nk}	-0.1244	-0.1914	-0.2342
v_{ra}	0.0375	0.0412	0.0438	v_{na}	0.3654	0.6029	0.7709
v_{rg}	0.0019	0.0029	0.0035	v_{ng}	0.1222	0.1845	0.2229
Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$	Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$
v_{wk}	0.5734	0.6076	0.6294	$v_{\tau k}$	N/A	N/A	N/A
v_{wa}	0.8137	0.6925	0.6069	$v_{\tau a}$	N/A	N/A	N/A
v_{wg}	-0.0623	-0.0941	-0.1137	$v_{\tau g}$	N/A	N/A	N/A
Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$	Coefficient	$\gamma = 6.81$	$\gamma = 3.41$	$\gamma = 2.27$
v_{yk}	0.4490	0.4162	0.3953	v_{ik}	0.1655	0.0921	0.0442
v_{ya}	1.1790	1.2954	1.3778	v_{ia}	3.8402	4.2231	4.4939
v_{yg}	0.0599	0.0904	0.1092	v_{ig}	-0.0269	0.0304	0.0670

6 DATA AND SUMMARY STATISTICS OF THE CANADIAN ECONOMY

In this section, I describe the relevant business cycle facts of the Canadian economy from the first quarter of 1971 to the last quarter of 1996. The variables of interest include output, private consumption, investment, hours of work, real wage and government expenditures. It is important for the purpose of this paper to understand the volatilities of these variables in question and their comovement with output. The sample moments documented in this section include percentage standard deviations, the ratio of standard deviations of each variable to that of output and their contemporaneous correlation with output, and first order serial correlation. These are the empirical features of the time series of an actual economy that have been extensively discussed in the research on real business cycles. For any given data series, I first take logarithms and then use the Hodrick-Prescott filter to remove their trends. All of the data used in this paper come from the CANSIM data series. The detailed definitions and sources of data are listed in Appendix A. Table 3 contains the summary statistics for the quarterly Canadian data from that are of interest in this paper. Y denotes real GDP per capita, C denote real consumption per capita, I denotes real investment per capita, N denotes total hours worked per capita, W denotes real wage, and G denotes real government purchase per capita.

Note that hours worked are defined as the average quarterly hours per worker in manufacturing times employment. This series is far from perfect since it uses hours per worker in manufacturing rather than in the total private sector. Unfortunately, I can find no other series that is available at a quarterly frequency. The best next alternative is to

use manufacturing hours. Cansim series I601001 only provides average hours worked per worker at an annual frequency.

Table 3 Business Cycle Statistics for the Canadian Economy

	Percentage Standard Deviation	Relative Standard Deviation with Output	Firs Order Auto-Correlation	Contemporaneous Correlation with Output
Y	1.82	1.000	0.898	1.000
C	1.48	0.779	0.887	0.916
I	5.26	2.213	0.892	0.832
N	1.85	1.016	0.686	0.116
W	2.62	1.093	0.850	0.703
G	4.39	1.514	0.898	0.340

The research of real business cycle focuses in understanding the volatilities, comovement, and persistence of key macroeconomic aggregates. Figures from table two indicate that investment is more than two times as volatile as output, while consumption is just about 78% as volatile as output. The last column of table two shows that all of the macroeconomic variables are procyclical, that is, they exhibit a positive contemporaneous correlation with output. All macroeconomic aggregates display substantial persistence. The first order serial correlation for their detrended quarterly variables is on the order of close to 0.9, where the first order serial correlation of employment hours is slightly lower at 0.686.

7 MODEL CALIBRATIONS

In this section I describe the way I calibrated the model's parameters. The average labour share of real GDP, which corresponds to $1-\alpha$ in my model economy, is 0.49 over my sample period. Therefore, I will set α at 0.51. The average value of G/Y over my sample period is 0.203, where G is total government expenditure, including defense spending. This is the steady-state tax rate in the model with lump-sum tax. In the model with labour-income tax, the average tax rate over my sample period is 0.41, which is calculated by dividing G by total wage bill. I set the fraction of time devoted to work, N , at 0.227. To calculate this fraction, I assume that there are 90 days per quarter 24 hours per day. Thus, the total hours per quarter are 2160. I then take the average of the division of average hours worked per quarter by total hours per quarter over my sample period to get 0.227.

I estimate the parameter of the process for government spending shocks according to equation (3.4.8). The value of θ is calculated as 1.58⁶. Following the suggestion from King and Rebelo (2000), I choose the discount factor, β , so that the steady state real interest rate coincides with the average return to capital in the economy. This is 7.2% per annum, if I equate it with the annualised return on the TSX

⁶ I use the parameter values of the model and the steady conditions to get the value of θ . First note that $\frac{\bar{w}\bar{N}}{\bar{Y}} = (1-\alpha)$. From the first order condition specified in equation (3.4.2), $w = \frac{\theta\bar{C}}{(1-\bar{N})^\gamma(1-\bar{\tau})}$.

Thus, by substitution, $(1-\alpha) = \frac{\theta\bar{C}}{(1-\bar{N})^\gamma(1-\bar{\tau})} \frac{\bar{N}}{\bar{Y}}$, which gives $\theta = \frac{(1-\alpha)(1-\bar{N})^\gamma(1-\bar{\tau})\bar{Y}}{\bar{C}\bar{N}}$. This

is an unimportant parameter in terms of calculating the recursive law of motion because it drops out in the calculation.

300 index over the period 1971-1996. Since I am interested in a quarterly model, I choose the discount factor so that the quarterly real interest rate is 1.8%. Using the fact that the discount factor equals η/R in the steady state β is set at 0.993.

Next is to estimate the stochastic processes of government spending and theology. I regress the logarithm of government spending on its one-period lagged value. The estimated value of ρ is 0.992 and σ_{ε_G} is 0.019.

Since direct measures of productivity, A_t , are not available, I choose the parameters of the process for technology shocks on the basis of the empirical behaviour of the Solow residual. Traditional measure of the Solow residual often assumes full and constant utilization of capital. King and Rebelo (2000) pointed out three reasons to distrust the Solow residual as a measure of technology shocks. First, the Solow residual can be forecasted using variables such as military spending, which are unlikely to cause changes in total factor productivity. Second, the conventional Solow residual implies probabilities of technological regress that are implausibly large. Third, cyclical variations in labour effort and capital utilization can significantly contaminate the Solow residual. It is also important to ensure that productivity shocks are exogenous to the state of the economy. Paquet and Robidoux (2000) argue that since the utilization of capital is likely to be procyclical, the assumption of constant utilization of capital could have important implications for the interpretation of the procyclical behaviour and exogeneity of productivity shocks. Using Canadian data for the period 1971 to 1993, Paquet and Robidoux (2000) find that when the capital stock is adjusted for variations in utilization rates, Canadian productivity shocks are exogenous to real and monetary variables. Given the above concerns and considerations, I will adjust the capital stock for utilization rates in estimating the Solow residual.

Recall that output is produced according to the Cobb-Douglas production function⁷. Assuming that a fraction, μ_t , of capital is utilized at date t , the production function becomes

$$\tilde{Y}_t = A_t (\mu \tilde{K}_t)^\alpha X_t^{1-\alpha} (L_t N_t)^{1-\alpha}$$

$$\ln \tilde{Y}_t = \ln A_t + \alpha \ln \mu \tilde{K}_t + (1-\alpha) \ln X_t + (1-\alpha) \ln(L_t N_t)$$

$$\ln SR_t = \ln \tilde{Y}_t - \alpha \ln \mu \tilde{K}_t - (1-\alpha) \ln(L_t N_t) = \ln A_t + (1-\alpha) \ln X_t \quad (7.1)$$

Using the assumptions that the deterministic level of technology, X_t , have a gross growth rate of η ⁸, and $\ln A_t$ follows an AR(1) process, it is possible to estimate the stochastic process for productivity and the persistence parameter φ and the standard deviation of the innovation $\varepsilon_{A,t}$. To do this, one needs to do the following substitutions:

$$\ln X_t - \ln X_{t-1} = \ln \eta$$

$$(1-\alpha) \ln X_t = (1-\alpha) \ln X_{t-1} + (1-\alpha) \ln \eta \quad (7.2)$$

Note that from (7.1),

$$\ln SR_t = \ln A_t + (1-\alpha) \ln X_t$$

⁷ Readers can refer to Section 2 and equation (2.3) for the discussion of the properties of the production function and its variables.

⁸ η is set to be equal to 1.09%. This is the average quarterly growth rate of per capita income over my sample period. I set η at the average growth rate of per capita income because in neoclassical models, per capita income growth rate is equal to technological process.

Thus,

$$(1 - \alpha) \ln X_{t-1} = \ln SR_{t-1} - \ln A_{t-1} \quad (7.3)$$

Substituting equations (7.2) and (7.3) into the Solow Residual and get

$$\ln SR_t = \ln A_t + \ln SR_{t-1} - \ln A_{t-1} + (1 - \alpha) \ln \eta$$

$$\ln A_t - \ln A_{t-1} = \ln SR_t - \ln SR_{t-1} - (1 - \alpha) \ln \eta \quad (7.4)$$

Since it is assumed that $\ln A_t = \varphi \ln A_{t-1} + \varepsilon_{A,t}$, the Solow Residual becomes

$$\ln SR_t = \varphi \ln A_{t-1} + (1 - \alpha) \ln X_t + \varepsilon_{A,t} \quad (7.5)$$

$$\ln SR_{t-1} = \varphi \ln A_{t-2} + (1 - \alpha) \ln X_{t-1} + \varepsilon_{A,t-1} \quad (7.6)$$

Subtracting (7.6) from (7.5) to get

$$\ln SR_t - \ln SR_{t-1} = \varphi (\ln A_{t-1} - \ln A_{t-2}) + (1 - \alpha) (\ln X_t - \ln X_{t-1}) + (\varepsilon_{A,t} - \varepsilon_{A,t-1})$$

$$\ln SR_t - \ln SR_{t-1} - (1 - \alpha) \ln \eta = \varphi (\ln SR_{t-1} - \ln SR_{t-2} - (1 - \alpha) \ln \eta) + (\varepsilon_{A,t} - \varepsilon_{A,t-1}) \quad (7.7)$$

One can use equation (7.7) to estimate the persistence parameter and the standard deviation of the innovation in the stochastic process of productivity. For my

quarterly data set the resulting point estimates are 0.0583 for φ and 0.0227 for the standard deviation of $\varepsilon_{A,t}$ ⁹. Table 4 summarizes the parameter values of my models.

Table 4 Parameters of Models

$\bar{\tau}$	δ	β	α	ρ	σ_{ε_G}	φ	σ_{ε_A}	\bar{N}	θ
0.217/ 0.41	0.025	0.993	0.51	0.992	0.019	0.0523	0.0227	0.227	1.58

⁹ In the regression of equation (7.7), the error term is $\varepsilon_{A,t} - \varepsilon_{A,t-1}$. Given the assumption that $\varepsilon_{A,t} \sim i.i.d.N(0, \sigma_A^2)$, $Var(\varepsilon_{A,t}) = Var(\varepsilon_{A,t} - \varepsilon_{A,t-1})/2$.

8 EVALUATIONS OF GENERAL EQUILIBRIUM MODELS

In this section, I present the simulation results for the three models at hand. The success of the models is judged on the basis of three dimensions. The first dimension is percentage standard deviations. The second dimension is relative standard deviations. The third dimension is contemporaneous correlations. Tables 5, 6, 7 and 8 display these moments of the actual data and the simulated-economies when labour supply elasticity is set at 1. Sensitivity analysis is done by assuming different labour supply elasticities. The results are presented in Tables 9 to 16. Increase in labour supply elasticities increases the fluctuations of output, investment, and hours worked and decrease the fluctuations of consumption and real wage predicted by model 1. On the other hand, fluctuations of all variables predicted by model 2 increase as labour supply elasticities increase, except for real wage where the opposite is true. In addition, in both models 1 and 2, relative volatility of hours of work and real wage to output increases as labour supply elasticity increase. On the other hand, both models 1 and 2 predict lower relative volatilities of real wage and consumption to output as labour supply elasticity goes up.

Table 5 shows that the baseline model and both models with government spending generate more fluctuations in output and investment than what the actual data suggests. However, the baseline model's prediction in fluctuation in consumption is much less than what the actual data shows. The incorporation of government in the models helps generate a much higher fluctuation in consumption. Model 2 generate

more fluctuations in hours worked, which is closer in line with the data, than the other models. The model in which government spending is financed by labour-income tax manages to generate fluctuation in consumption that is very close to the actual data. Except for consumption and hours worked, the model with lump-sum tax generates fluctuations that are closer to what actual data suggest than the model with labour-income tax does. Thus, on the basis of generation of absolute percentage standard deviations, the model in which government spending is financed with lump-sum tax outperforms the model in which government spending is financed with labour-income tax.

Table 6 shows relative standard deviations of consumption, investment, hours, and real wage to output. All three models can account for the relative standard deviations qualitatively, except for real wage. However, all of them fail to predict the right magnitudes of relative volatilities of the variables in the models to output. Both models 1 and 2 greatly underestimate the relative volatilities of consumption and real wage to output, but overestimate the relative volatilities of investment to output. The baseline model greatly underestimates the relative volatilities of consumption to output. It predicts that consumption is only around 10 to 20 percent as volatile as output, where the data suggests it is 78 percent as volatile as output.

Model 2 performs reasonably well in terms of predicting contemporaneous correlations between output and other variables. Model 2 predicts the correct signs of correlations of all of the variables to output. However, it greatly overestimates the correlations of hours worked, real wage, and investment with output and slightly

underestimates the correlation of consumption with output. On the other hand, model 1 performs poorly in predicting contemporaneous correlations among the variables with output. Its major failure is that it incorrectly predicts that consumption is negatively correlated with output. Similar to model 2, model 1 overestimates the correlation between hours worked, real wage and investment with output and underestimates the correlation of consumption with output.

As an additional check for the performance of the models, contemporaneous correlations of key variables with hours of work were calculated. They are presented in Tables 8, 12, and 16. All three models perform badly in generating this moment. Broadly speaking, all models greatly overestimate the correlations between the variables and hours of work. The only exception is that model 2 and the baseline model underestimate the correlation between hours of work and consumption. Both model 2 and baseline model fail to generate the negative correlation between hours of work and real wage as suggested by the data. Model 1 successfully predicts a weak negative correlation between real wage and hours of work as suggested by the real world data. As explained in the introduction, technology shocks shift the labour demand curve along the supply curve. This effect produces a positive relationship between hours and productivity. Since the baseline model includes only technology shocks, it predicts strong positive correlation between hours and productivity. On the other hand, government-spending shocks shift the labour supply curve that produces negative correlation between hours and productivity. The net effect on correlation between the two depends partly on the response of variables to government spending shocks. It appears that the response of variables to government spending shocks is greater in the model where government spending is financed by lump-sum tax than in the model where

government spending is financed by labour-income tax. Lastly, the model with lump-sum tax predicts a negative correlation between hours of work and consumption, which contradicts what the data shows.

Table 5 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81^{10}$: (Percentage Standard Deviations)

	Percentage Standard Deviation with (data)	Percentage Standard Deviation (model 1)	Percentage Standard Deviation (model 2)	Percentage Standard Deviation (baseline model)
Y	1.817	2.5357	2.9226	2.5429
C	1.481	0.7913	1.3573	0.1728
I	5.26	8.3782	9.7148	8.4925
N	1.853	0.8666	1.6241	0.8252
W	2.62	1.7355	1.3216	1.7210

Table 6 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$. (Relative Percentage Standard Deviations)

	Relative Standard Deviation with Output (data)	Relative Standard Deviation with Output (model 1)	Relative Standard Deviation with Output (model 2)	Relative Standard Deviation with Output (baseline model)
C	0.779	0.3121	0.4644	0.06795
I	2.213	3.3041	3.3240	3.3397
N	1.016	0.3418	0.5557	0.3245
W	1.093	0.6844	0.4522	0.6768

¹⁰ Setting γ at 6.81 implies a labour supply elasticity of 0.5.

Table 7 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$.
(Contemporaneous Correlations with Output.)

	Contemporaneous Correlation with Output (data)	Contemporaneous Correlation with Output (model 1)	Contemporaneous Correlation with Output (model 2)	Contemporaneous Correlation with Output (baseline model)
C	0.7874	-0.1764	0.6528	0.4003
I	0.7807	0.9380	0.9364	0.9833
N	0.116	0.6143	0.9454	0.9718
W	0.5789	0.7139	0.9596	0.9946

Table 8 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 6.81$.
(Contemporaneous Correlations with Hours.)

	Contemporaneous Correlation with Hours (data)	Contemporaneous Correlation with Output (model 1)	Contemporaneous Correlation with Output (model 2)	Contemporaneous Correlation with Output (baseline model)
Y	0.116	0.6143	0.9454	0.9718
C	0.78736	-0.8850	0.5794	0.1731
I	0.41837	0.3689	0.9854	0.9985
W	-0.16403	-0.1140	0.8154	0.9420

Table 9 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41^{11}$. (Percentage Standard Deviations.)

	Percentage Standard Deviation (data)	Percentage Standard Deviation (model 1)	Percentage Standard Deviation (model 2)	Percentage Standard Deviation (baseline model)
Y	1.817	2.7938	3.5658	2.8011
C	1.481	0.7465	1.4269	0.2006
I	5.26	9.2323	11.8964	9.2671
N	1.853	1.4192	2.9542	1.3611
W	2.62	1.4810	0.6644	1.4467

Table 10 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$. (Relative Percentage Standard Deviations.)

	Relative Standard Deviation with Output (data)	Relative Standard Deviation with Output (model 1)	Relative Standard Deviation with Output (model 2)	Relative Standard Deviation with Output (baseline model)
C	0.779	0.2672	0.4002	0.07161
I	2.213	3.3046	3.3362	3.3084
N	1.016	0.5080	0.8285	0.4859
W	1.093	0.5301	0.1863	0.5165

¹¹ Recall that labour supply elasticity at the steady state is equal to $\frac{\gamma^{-1}(1-\bar{N})}{\bar{N}}$. Setting γ at 3.41 implies a labour supply elasticity of 1.

Table 11 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$.
 (Contemporaneous Correlations with Output.)

	Contemporaneous Correlation with Output (data)	Contemporaneous Correlation with Output (Model 1)	Contemporaneous Correlation with Output (Model 2)	Contemporaneous Correlation with Output (Baseline Model)
C	0.7874	-0.3818	0.6701	0.3949
I	0.7807	0.8962	0.9313	0.9844
N	0.116	0.7926	0.9413	0.9738
W	0.5789	0.4292	0.8273	0.9816

Table 12 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 3.41$.
 (Contemporaneous Correlations with Hours.)

	Contemporaneous Correlation with Hours (data)	Contemporaneous Correlation with Hours (Model 1)	Contemporaneous Correlation with Hours (Model 2)	Contemporaneous Correlation with Hours (Baseline Model)
Y	0.116	0.7926	0.9413	0.9738
C	0.78736	-0.8861	0.5367	0.1756
I	0.41837	0.4867	0.9909	0.9986
W	-0.16403	-0.2105	0.5891	0.9125

Table 13 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27^{12}$.
(Percentage Standard Deviations.)

	Percentage Standard Deviation (data)	Percentage Standard Deviation (model1)	Percentage Standard Deviation (model 2)	Percentage Standard Deviation (baseline model)
Y	1.817	2.9812	4.0941	2.9900
C	1.481	0.6990	1.4684	0.2020
I	5.26	9.7820	13.6213	9.9403
N	1.853	1.7999	4.0844	1.7476
W	2.62	1.3085	0.2931	1.2581

Table 14 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$. (Relative Standard Deviations to Output.)

	Relative Standard Deviation with Output (data)	Relative Standard Deviation with Output (model 1)	Relative Standard Deviation with Output (model 2)	Relative Standard Deviation with Output (baseline model)
C	0.779	0.2345	0.3587	0.06756
I	2.213	3.2812	3.3271	3.3245
N	1.016	0.6038	0.9976	0.5845
W	1.093	0.4389	0.0716	0.4208

¹² Recall that labour supply elasticity at the steady state is equal to $\frac{\gamma^{-1}(1-\bar{N})}{\bar{N}}$. Setting γ at 3.41 implies a labour supply elasticity of 1.

Table 15 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$.
 (Contemporaneous Correlations with Output.)

	Contemporaneous Correlation with Output (data)	Contemporaneous Correlation with Output (Model 1)	Contemporaneous Correlation with Output (Model 2)	Contemporaneous Correlation with Output (Baseline Model)
C	0.7874	-0.3504	0.6889	0.4059
I	0.7807	0.9077	0.9270	0.9822
N	0.116	0.8464	0.9363	0.9702
W	0.5789	0.3997	0.6496	0.9596

Table 16 A Calibrated Real-Business-Cycle Model versus Actual Data with $\gamma = 2.27$.
 (Contemporaneous Correlations with Hours.)

	Contemporaneous Correlation with Hours (data)	Contemporaneous Correlation with Hours (Model 1)	Contemporaneous Correlation with Hours (Model 2)	Contemporaneous Correlation with Hours (Baseline Model)
Y	0.116	0.8464	0.9363	0.9702
C	0.78736	-0.7954	0.5157	0.1723
I	0.41837	0.5978	0.9937	0.9984
W	-0.16403	-0.1498	0.3412	0.8627

9 IMPULSE RESPONSE ANALYSES

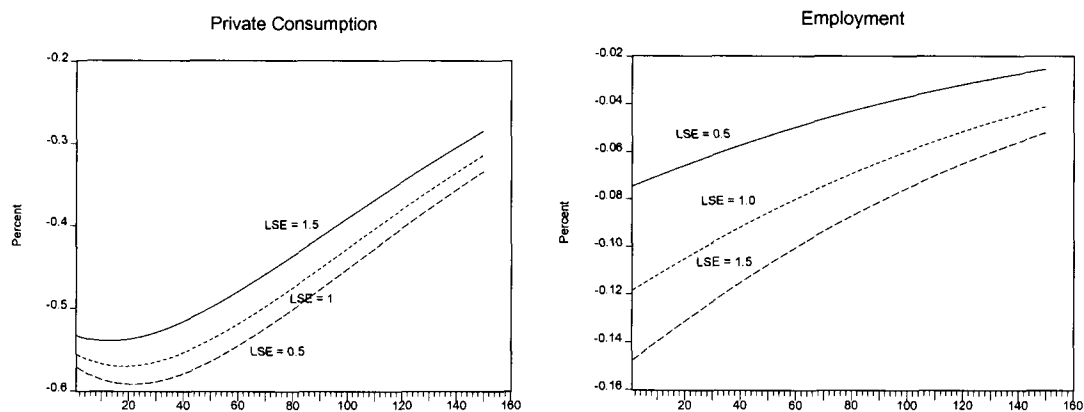
The purpose of this section is to study how the key variables response to government shocks. Figure 1 and 2 present the responses of some of the variables in models 1 and 2 to a 1% increase in government spending, respectively. We see that the response of the variables to government spending shocks are bigger the higher the labour supply elasticities. The only exception is the response of private consumption to government spending shock when the spending is financed by lump-sum tax. Changes in labour supply elasticity do not affect the basic dynamics of responses to government spending shocks.

Note that the shocks to government spending have different qualitative and quantitative effects on the variables of the models. When government spending is financed by labour-income tax, employment decreases in response to an increase in government spending. The reason is that the increase in tax associated with an increase in government spending reduces the marginal benefit of work. As a result, people will chose to work less and enjoy more leisure. One also observes that with labour-income tax, capital stock decreases in response to an increase in government spending. Since both hours of work and capital stock decrease, output also decreases in response to an increase in government spending. Real wage increases initially because hours worked decreases, which increases the marginal productivity of hours worked. Over time, when hours worked start to increase and capital stock continues to decrease, real wage moves back to its steady state value. There is a period where real wage falls below its steady state value. This is because within this period of time, the downward pressure on real wage from a reduction in capital is greater than the upward

pressure on real wage from a reduction of hours worked. Investment decreases after a government spending shocks because of the motive of consumption smoothing.

On the other hand, when government spending is financed with lump-sum tax, an increase in tax implies a pure negative wealth effect, which induces people to work more, provided that leisure is a normal good. From the graph, one can observe that capital stock increases slightly when labour supply elasticities are 1 or 1.5. Since both hours of work and capital stocks increase, output increases as a consequence of an increase in government spending. Once again, investment falls because of the motive to smooth consumption over time. Real wage decreases because the increase in hours worked and reduction in capital stock reduce marginal productivity of hours worked. Real wage moves back to its steady state level when employment starts to fall and capital stock starts to accumulate when people increase their investment level.

Figure 1 Responses to a temporary 1% increase in government spending when government spending is financed with labour-income tax¹³.



¹³ LSE stands for labour supply elasticity.

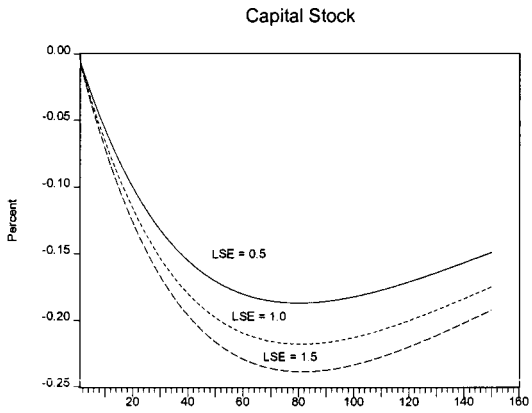
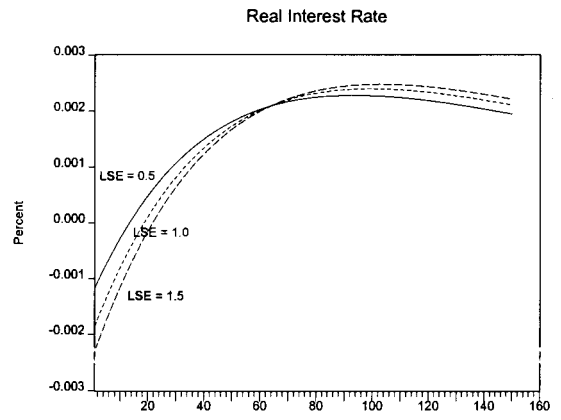
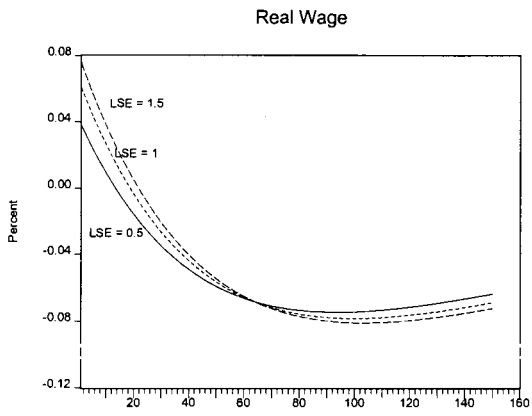
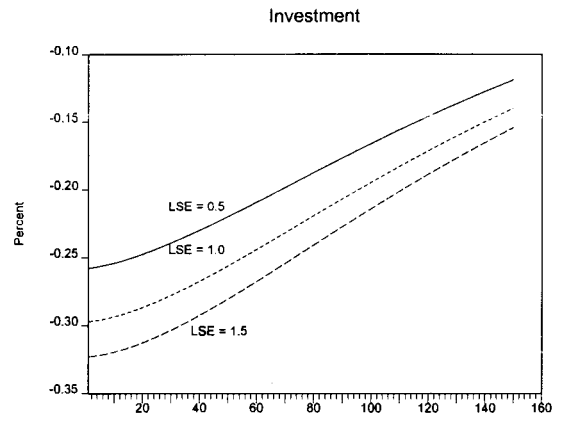
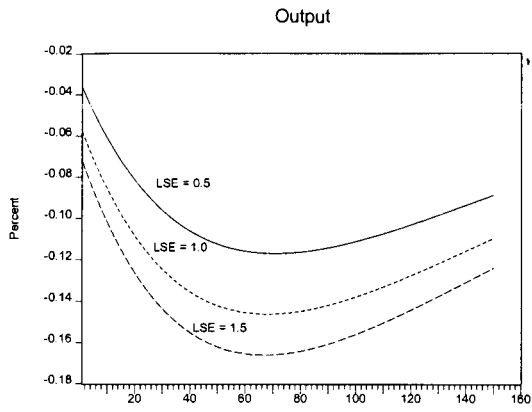
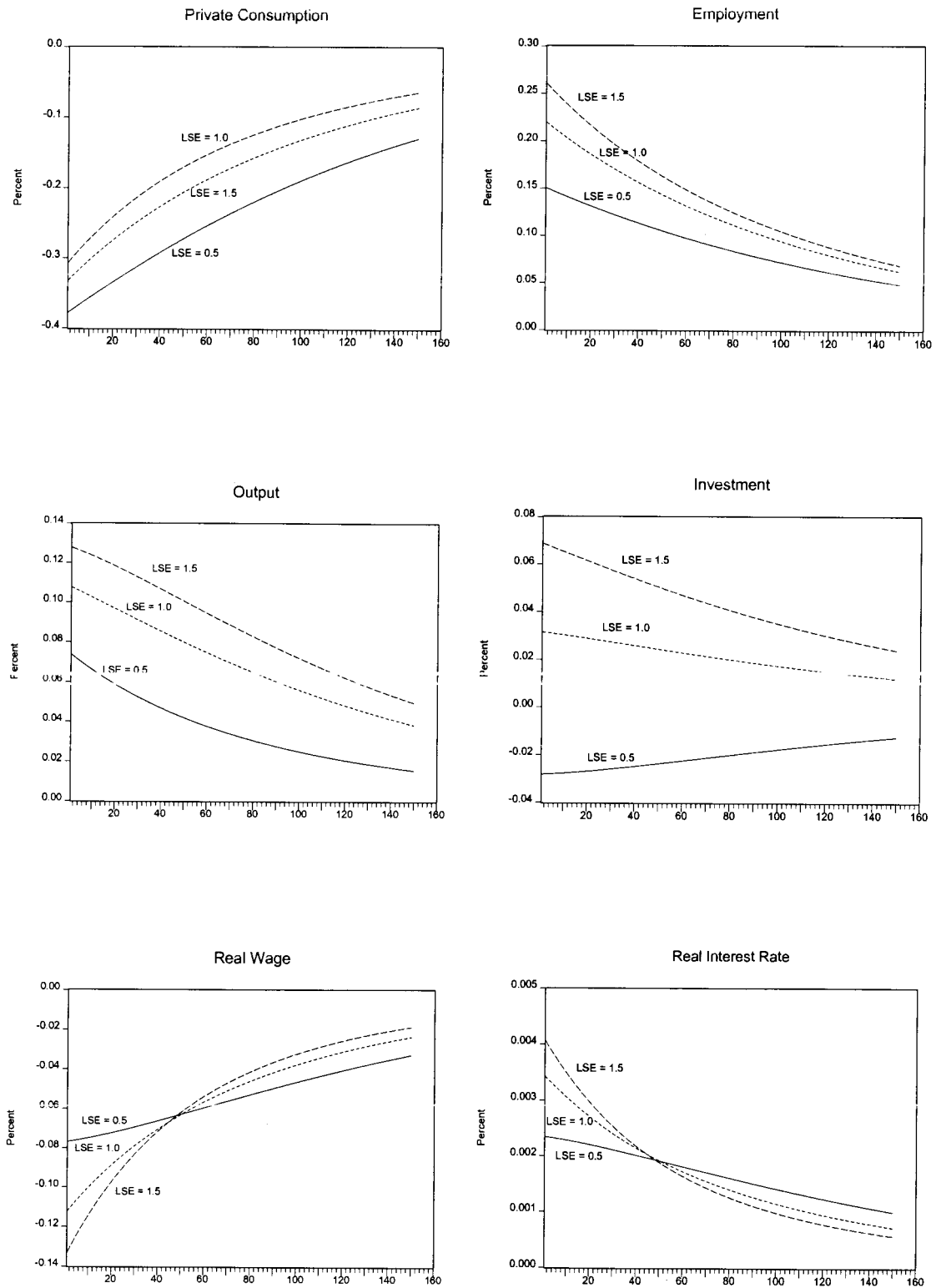
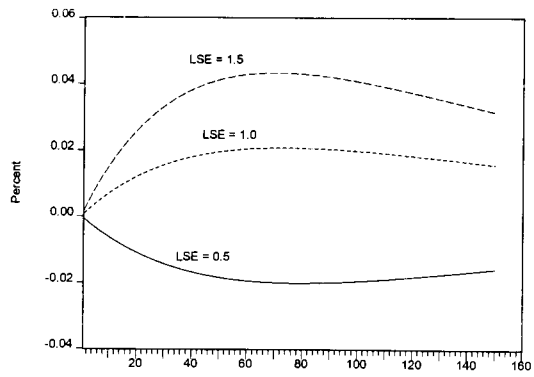


Figure 2 Responses to a temporary 1% increase in government spending when government spending is financed with lump-sum tax.



Capital Stock



10 CONCLUSION

In this paper I show that incorporation of a government sector in a stochastic general equilibrium model mildly improves the model's ability to reproduce basic facts about Canadian business cycle fluctuations during the period 1971 to 1996. The major improvement of the performance of the model is that when government spending is financed by lump-sum tax, it predicts a weak negative correlation between hours of work and productivity that is close to what the actual data suggest. Model with government spending shocks are also able to produce a much higher relative volatility of consumption to output than the baseline model with only technology shocks, although the model with government still underestimates this relative volatility. The major deficiencies of the model with government spending shocks is that, same as the model with only technology shocks, overestimates the relative volatilities of investment and hours of work to output, but underestimates the relative volatility of real wage to output. Base on the results of this paper, it is clear that the models with government outperform the model with only technology shocks in replicating the characteristics of the Canadian economy. However, it is not clear whether the model with a lump-sum tax is better than the model with a labour-income tax or vice versa.

REFERENCE

- Baxter, Marianne and King, Robert G. (1993). "Fiscal Policy in General Equilibrium." *American Economic Review*. Volume 83 No.3: 315-334.
- Braun, R. Anton (1994). "Tax Disturbances and Real Economics Activity in the Post-war United States." *Journal of Monetary Economics*. 33: 441-462.
- Campbell, J. (1994). "Inspecting the Mechanism: An Analytical Approach to the Stochastic Growth Model," *Journal of Monetary Economics*, 33: 463-506.
- Christiano, Lawrence J. (1998). "Solving Dynamic Models by a Method of Undetermined Coefficients." NBER Working Paper No. t0225.
- Christiano, Lawrence J. and Eichenbaum, Martin (1992). "Current Real-Business-Cycle Theories and Aggregate Labour-Market Fluctuations." *American Economic Review*. Volume 82 No.3: 430-450.
- Devereux, M.B., Head, A.C., Lapham, B.J. (1996). "Monopolistic Competition, increasing returns, and the effects of government spending." *Journal of Money, Credit, and Banking*. 28: 233-254.
- Guo, Jang-Ting (2004). "Increasing Returns, Capital Utilization, and the Effects of Government Spending." *Journal of Economic Dynamics and Control*. 28: 1059-1078.
- Hansen, Gary and Wright, Randall (1992). "The Labor Market in Real Business Cycle Theory," *Federal Reserve Bank of Minneapolis Quarterly Review*. Vol. 16 No. 2
- King, Robert G. and Rebelo Sergio T. (2000). "Resuscitating Real Business Cycles." Working Paper 467. Rochester Center for Economic Research. University of Rochester.
- Long, John B. and Plosser, Charles I. (1983). "Real Business Cycles." *Journal of Political Economy*. Volume 91 No.1: 36-69.
- Lopez-Pinto Juan Sole (2001). "Fiscal Policy and the Spanish Business Cycle." *Spanish Economic Review*. 3: 295-311.
- Lucas, Robert E. (1980). "Methods and Problems in Business Cycle Theory." *Journal of Money, Credit, and Banking*. Volume 12 No. 4: 696-715.
- McGrattan, Ellen R. (1994). "The Macroeconomic Effects of Distortionary Taxation." *Journal of Monetary Economics*. 33: 573-601.

Paquet, Alain and Robidoux, Benoit (2001). "Issues on the Measurement of the Solow Residual and the Testing of its Exogeneity: Evidence for Canada." *Journal of Monetary Economics*. 47: 595-612.

Plosser, Charles I. (1989). "Understanding Real Business Cycles." *Journal of Economic Perspectives*. Volume 3, No. 3: 51-77.

APPENDIX A: DATA DESCRIPTIONS

Definitions, units, and sources of data:

Population:

Source: Cansim Number: D1

Units: Unscaled persons

Frequency: Quarterly

Employment:

Source: Cansim Number: D44949

Units: Thousands

Frequency: Annual

Nominal GDP:

Source: Cansim Number: D14840

Units: Millions

Frequency: Quarterly

Nominal Business Gross Fixed Capital Formation: (Non-Residential Structures & Equipment)

Source: Cansim Number: D14827

Units: Millions

Frequency: Quarterly

Nominal Personal Expenditure on Consumer Goods & Services:

Source: Cansim Number: D14817

Units: Millions

Frequency: Quarterly

Total Nominal Wage Bill:

Source: Cansim Number: D17023

Units: Thousands

Frequency: Quarterly

Average Hours Worked per Year:

Source: Cansim Number: I601001

Units: Unscaled Hours

Frequency: Annual

Nominal Government Defense Spending:

Source: Cansim Number: D18134

Units: Millions

Frequency: Quarterly

Nominal Government Non-Defense Spending:

Source: Cansim Number: D18133

Units: Millions

Frequency: Quarterly

Capital Stock at 1992 prices

Source: D990167

Units: Millions

Frequency: Annual

Price Index for Nominal GDP:

Source: D15652

Base Year: 1992 = 100

Frequency: Quarterly

Toronto Stock Exchange, Stock Price Index

Source: D4237

Units: Unscaled 1975=1000

Frequency: Monthly

Real GDP:

Derived by: $\text{Nominal GDP} \times \text{Price Index}/100$

Units: Millions

Frequency: Quarterly

Real per capita GDP:

Derived by: $(\text{Real GDP} \times \$1000000)/\text{Population}$

Units: Unscaled Dollars

Frequency: Quarterly

Real Business Gross Fixed Capital Formation:

Derived by: Nominal Business Gross Fixed Capital Formation \times Price Index/100

Units: Millions

Frequency: Quarterly

Real Personal Expenditure on Consumer Goods & Services:

Derived by: Nominal Personal Expenditure on Consumer Goods & Services \times Price Index/100

Units: Millions

Frequency: Quarterly

Total Real Wage Bill:

Derived by: Total Nominal Wage Bill \times Price Index/100

Units: Thousands

Frequency: Quarterly

Total Hours Worked Per Capita

Derived by: (Average Hours Worked per Quarter \times Employment) /Population

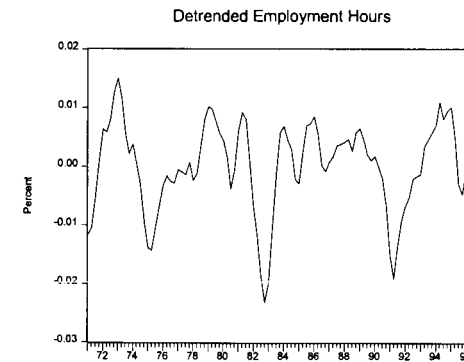
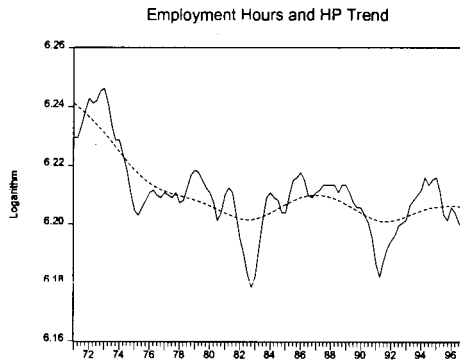
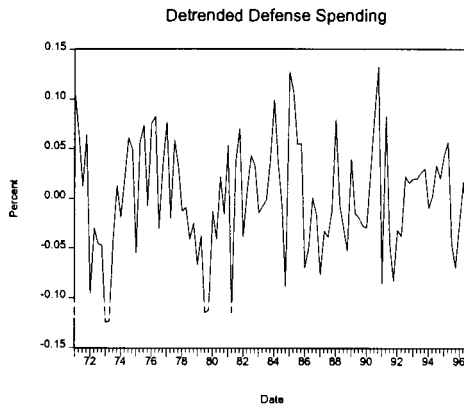
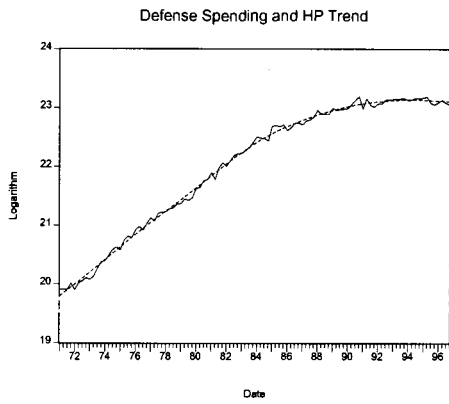
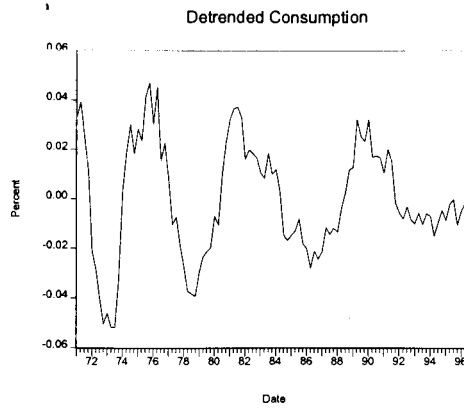
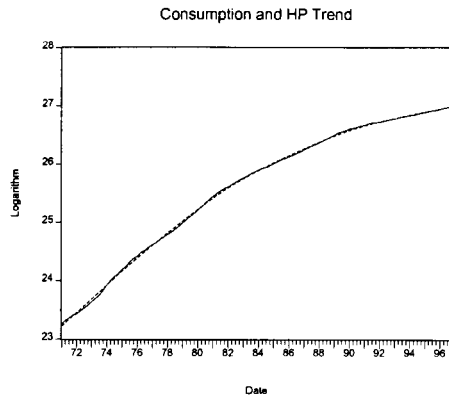
Real Wage per Hour:

Derived by: Total Real Wage Bill/(Employment \times 1000 \times Average Hours Worked per Quarter)

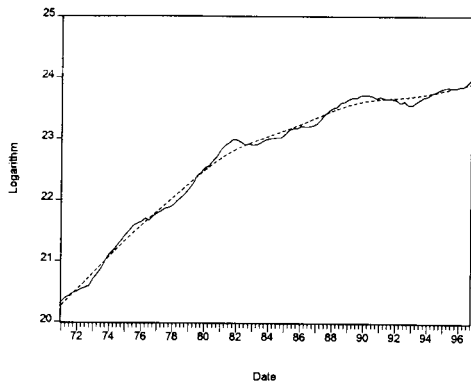
Units: Thousands

Frequency: Quarterly

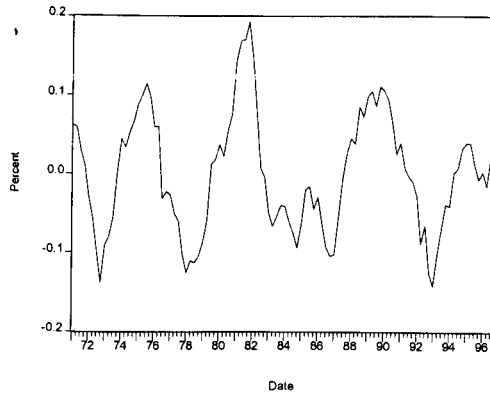
Figure 3 Logarithms and HP Trends of the Data



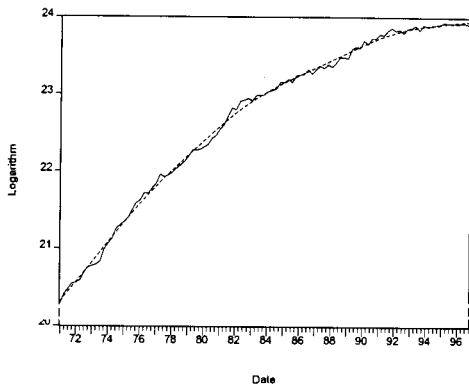
Investment and HP Trend



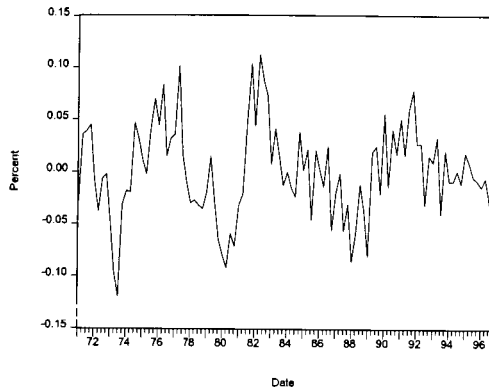
Detrended Investment



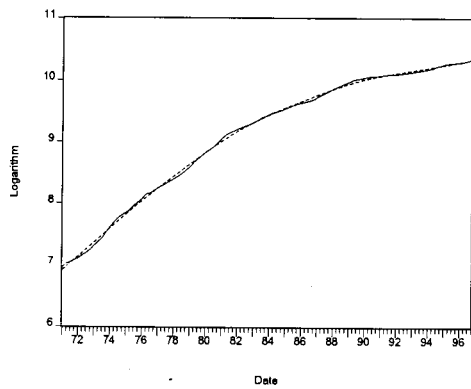
Non-Defense Spending and HP Trend



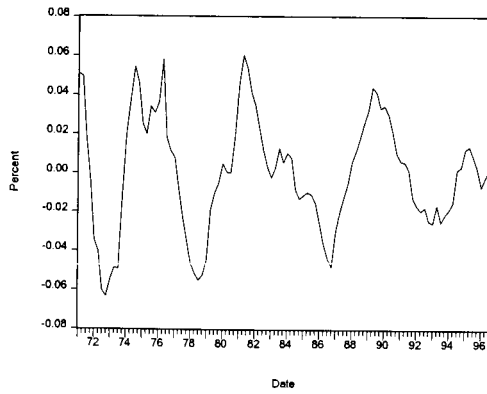
Detrended Non-Defense Spending



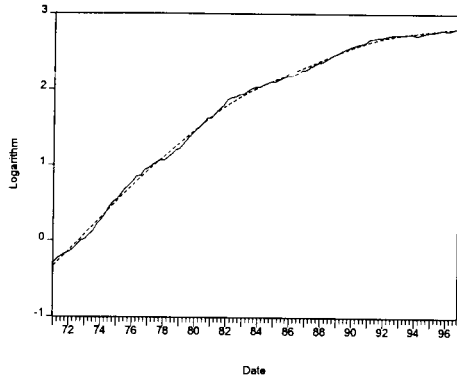
GDP and HP Trend



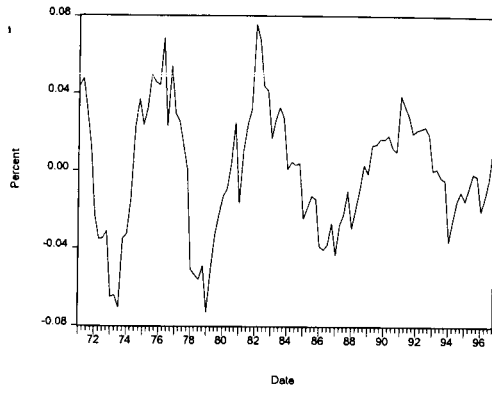
Detrended GDP



Wage and HP Trend



Detrended Wage



APPENDIX B: SOLVING FOR THE DYNAMICS OF THE MODELS

The purpose of this appendix is to explain how to solve the recursive law of motions in general equilibrium models via the method of undetermined coefficients. To do this, one needs to first find the first order conditions of the optimization problem, constraints within the model, and the steady state conditions of the model economy. Recall that the first order conditions and constraints are given by equations (3.4.1) to (3.4.8) and the steady state are given by equations (3.4.9) to (3.4.13). The next step is to log-linearize the constraints and the first order conditions. The final step is to solve for the recursive law of motions via the method of undetermined coefficients.

Recall that I define $c_t, n_t, r_t, w_t, \tau_t, y_t, a_t, g_t$ and i_t as the logarithmic deviations of $C_t, N_t, R_t, W_t, \tau_t, Y_t, A_t, G_t$ and I_t respectively from their respective steady-state values. For example, $c_t = \log C_t - \log \bar{C}$ and so $C_t = \bar{C}e^{c_t} \approx \bar{C}(1 + c_t)$. The next step is to log-linearize equations (3.4.9) to (3.4.13) one by one. For the first equation, the budget constraint of an individual, one obtains:

$$C_t = A_t K_t^\alpha N_t^{1-\alpha} + (1-\delta)K_t - K_{t+1} - G_t$$

$$\bar{C}e^{c_t} = \bar{A}e^{a_t} \bar{K}^\alpha e^{\alpha k_t} \bar{N}^{(1-\alpha)} e^{(1-\alpha)n_t} + (1-\delta)\bar{K}e^{k_t} - \bar{K}e^{k_{t+1}} - \bar{G}e^{g_t}$$

$$\bar{C}(1 + c_t) \approx \bar{A}\bar{K}^\alpha \bar{N}^{(1-\alpha)}(1 + a_t + \alpha k_t + (1-\alpha)n_t) + (1-\delta)\bar{K}(1 + k_t) - \bar{K}(1 + k_{t+1}) - \bar{G}(1 + g_t)$$

$$\bar{C} + \bar{C}c_t \approx \bar{Y} + \bar{Y}(a_t + \alpha k_t + (1-\alpha)n_t) + (1-\delta)\bar{K} + (1-\delta)\bar{K}k_t - \bar{K} - \bar{K}k_{t+1} - \bar{G} - \bar{G}g_t$$

Using the steady state relationships, $\bar{C} = \bar{Y} - \delta\bar{K} - \bar{G}$, $\left[\frac{\bar{Y}}{\bar{K}}\alpha + (1-\delta)\right] = \bar{R}$, and

$\bar{R} = \frac{1}{\beta}$, the above expression can be simplified to

$$\bar{C}c_t \approx \bar{Y}(a_t + \alpha k_t + (1-\alpha)n_t) + (1-\alpha)\bar{K}k_t - \bar{K}k_{t+1} - \bar{G}g_t$$

$$\bar{C}c_t \approx \bar{K}\left[\frac{\bar{Y}}{\bar{K}}\alpha + (1-\delta)\right]k_t + \bar{Y}a_t + \bar{Y}(1-\alpha)n_t - \bar{K}k_{t+1} - \bar{G}g_t,$$

$$c_t \approx \frac{\bar{K}}{\beta\bar{C}}k_t + \frac{\bar{Y}}{\bar{C}}a_t + \frac{\bar{Y}}{\bar{C}}(1-\alpha)n_t - \frac{\bar{K}}{\bar{C}}k_{t+1} - \frac{\bar{G}}{\bar{C}}g_t,$$

For the second equation, the intertemporal substitution between labour supply and consumption, one obtains

$$\theta(1-N_t)^{-\gamma} = (1-\tau_t)\frac{W_t}{C_t}, \quad \sigma \equiv \frac{1}{\gamma}$$

$$1-N_t = \frac{1}{\theta}(1-\tau_t)^{-\sigma}W_t^{-\sigma}C_t^\sigma$$

$$1-\bar{N} - \bar{N}n_t \approx \frac{1}{\theta}(1-\bar{\tau})^{-\sigma}e^{-\sigma\tau_t}\bar{W}^{-\sigma}e^{-\sigma w_t}\bar{C}^\sigma e^{\sigma c_t}$$

$$1-\bar{N} - \bar{N}n_t \approx \frac{1}{\theta}(1-\bar{\tau})^{-\sigma}\bar{W}^{-\sigma}\bar{C}^\sigma(1-\sigma\tau_t - \sigma w_t + \sigma c_t)$$

$$1-\bar{N} - \bar{N}n_t \approx 1-\bar{N} + (1-\bar{N})(-\sigma\tau_t - \sigma w_t + \sigma c_t)$$

$$-\bar{N}n_t \approx -(1-\bar{N})\sigma(\tau_t + w_t - c_t)$$

$$n_t \approx \left(\frac{1 - \bar{N}}{\bar{N}} \right) \sigma (\tau_t + w_t - c_t)$$

I will do one more example and then the reader just need to apply the same technique to get the other log-linearized equations. For the third equation, the intertemporal substitution between current and future consumption, one obtains

$$1 = \beta E_t \left[\frac{C_t}{C_{t+1}} R_{t+1} \right]$$

$$1 = \beta E_t \left[\frac{\bar{C} e^{c_t}}{\bar{C} e^{c_{t+1}}} \bar{R}_{t+1} \right]$$

$$1 \approx \beta E_t \left[\bar{R} (1 + c_t - c_{t+1} + r_{t+1}) \right]$$

$$1 \approx E_t [1 + c_t - c_{t+1} + r_{t+1}]$$

$$0 \approx E_t [c_t - c_{t+1} + r_{t+1}]$$

Applying the same technique, one can obtain the other log-linearized equations presented from equations (5.3) to (5.10) in Section 5.

What is given at time t are the state variables k_t , a_t , and g_t . What we need to find are k_{t+1} , c_t , r_t , n_t , w_t , τ_t , y_t , and i_t . We can postulate a linear recursive law of motion presented from equations (5.11) to (5.18) in Section 5. The next step is to solve for the coefficients of the state variables in each of the recursive law of motion. To solve

for these coefficients, we can start with substituting the postulate recursive law of motion of c_t into the log-linearized budget constraint:

$$v_{ck}k_t + v_{ca}a_t + v_{cg}g_t = \frac{\bar{K}}{\beta\bar{C}}k_t + \frac{\bar{Y}}{\bar{C}}a_t + \frac{\bar{Y}}{\bar{C}}(1-\alpha)n_t - \frac{\bar{K}}{\bar{C}}k_{t+1} - \frac{\bar{G}}{\bar{C}}g_t$$

$$v_{ck}k_t + v_{ca}a_t + v_{cg}g_t = \frac{\bar{K}}{\beta\bar{C}}k_t + \frac{\bar{Y}}{\bar{C}}a_t + \frac{\bar{Y}}{\bar{C}}(1-\alpha)(v_{nk}k_t + v_{na}a_t + v_{ng}g_t) - \frac{\bar{K}}{\bar{C}}(v_{kk}k_t + v_{ka}a_t + v_{kg}g_t) - \frac{\bar{G}}{\bar{C}}g_t$$

Collecting terms to get

$$v_{ck}k_t + v_{ca}a_t + v_{cg}g_t = \left[\frac{\bar{K}}{\beta\bar{C}} + \frac{\bar{Y}}{\bar{C}}(1-\alpha)v_{nk} - \frac{\bar{K}}{\bar{C}}v_{kk} \right] k_t + \left[\frac{\bar{Y}}{\bar{C}}(1-\alpha) + \frac{\bar{Y}}{\bar{C}}(1-\alpha)v_{na} - \frac{\bar{K}}{\bar{C}}v_{ka} \right] a_t + \left[\frac{\bar{Y}}{\bar{C}}(1-\alpha)v_{ng} - \frac{\bar{G}}{\bar{C}} - \frac{\bar{K}}{\bar{C}}v_{kg} \right] g_t$$

Since this needs to be satisfied for any value of k_t , a_t , and g_t , we must have

$$v_{ck} = \frac{\bar{K}}{\beta\bar{C}} + \frac{\bar{Y}}{\bar{C}}(1-\alpha)v_{nk} - \frac{\bar{K}}{\bar{C}}v_{kk}$$

$$v_{ca} = \frac{\bar{Y}}{\bar{C}}(1-\alpha) + \frac{\bar{Y}}{\bar{C}}(1-\alpha)v_{na} - \frac{\bar{K}}{\bar{C}}v_{ka}$$

$$v_{cg} = \frac{\bar{Y}}{\bar{C}}(1-\alpha)v_{ng} - \frac{\bar{G}}{\bar{C}} - \frac{\bar{K}}{\bar{C}}v_{kg}$$

I will do one more example using the log-linearized intertemporal substitution between current and future consumption.

$$0 = E_t [c_t - c_{t+1} + r_{t+1}]$$

$$0 = E_t [v_{ck} k_t + v_{ca} a_t + v_{cg} g_t - v_{ck} k_{t+1} - v_{ca} a_{t+1} - v_{cg} g_{t+1} + v_{rk} k_{t+1} + v_{ra} a_{t+1} + v_{rg} g_{t+1}]$$

Since $E_t [g_{t+1}] = \rho g_t$ and $E_t [a_{t+1}] = \varphi a_t$, we have

$$0 = k_t (v_{ck} - v_{ck} v_{kk} + v_{rk} v_{kk}) + a_t (v_{ca} - v_{ck} v_{ka} - \varphi v_{ca} + v_{rk} v_{ka} + \varphi v_{ra}) + g_t (v_{cg} - v_{rk} v_{kg} - \rho v_{cg} + v_{rk} v_{kg} + \rho v_{rg})$$

Thus,

$$0 = v_{ck} - v_{ck} v_{kk} + v_{rk} v_{kk}$$

$$0 = v_{ca} - v_{ck} v_{ka} - \varphi v_{ca} + v_{rk} v_{ka} + \varphi v_{ra}$$

$$0 = v_{cg} - v_{ck} v_{kg} - \rho v_{cg} + v_{rk} v_{kg} + \rho v_{rg}$$

Applying the above substitution technique to each of the log-linearized equation, one obtains

$$v_{nk} = \left(\frac{1 - \bar{N}}{\bar{N}} \right) \sigma (v_{\pi k} + v_{wk} - v_{ck})$$

$$v_{na} = \left(\frac{1 - \bar{N}}{\bar{N}} \right) \sigma (v_{\tau a} + v_{wa} - v_{ca})$$

$$v_{ng} = \left(\frac{1 - \bar{N}}{\bar{N}} \right) \sigma (v_{\tau g} + v_{wg} - v_{cg})$$

$$v_{rk} = B(1 - \alpha)v_{nk} - B(1 - \alpha)$$

$$v_{ra} = B + B(1 - \alpha)v_{na}$$

$$v_{rg} = B(1 - \alpha)v_{ng}$$

$$v_{wk} = \alpha - \alpha v_{nk}$$

$$v_{wa} = 1 - \alpha v_{na}$$

$$v_{wg} = -\alpha v_{ng}$$

$$v_{\tau k} = \frac{\bar{\tau}}{1 - \bar{\tau}} (v_{wk} + v_{nk})$$

$$v_{\tau a} = \frac{\bar{\tau}}{1 - \bar{\tau}} (v_{wa} + v_{na})$$

$$v_{\tau g} = \frac{\bar{\tau}}{1 - \bar{\tau}} (v_{wg} + v_{ng} - 1)$$

$$v_{yk} = \alpha + (1 - \alpha)v_{nk}$$

$$v_{ya} = 1 + (1 - \alpha)v_{na}$$

$$v_{yg} = (1 - \alpha)v_{ng}$$

One can then collect all the coefficients on the k , a , and the g state variables together and solve for the system simultaneously.

APPENDIX C: GAUSS CODE

The following is the GAUSS code that I use to do the simulations of the models.

```
beta = 0.993;
alpha = 0.51;
delta = 0.025;
rho = 0.992;
rho2 = 0.052167;
taobar = 0.41;
psi = 3.41;
psi2=rho2;
sigma = 0.019;
sigma2 = 0.02273;
rbar = 1/beta;
nbar = 0.227;
kbar = ((alpha*nbar^(1-alpha))/(rbar-1+delta))^(1/(1-alpha));
ybar = (kbar^alpha)*(nbar^(1-alpha));
wbar = (1-alpha)*(kbar^alpha)*(nbar^(-alpha));
gbar = taobar*wbar*nbar;
cbar = ybar - delta*kbar - gbar;

"Simulating the Model..."; print;
```

```
s=1;
sim = 1000;
sdkk2 = zeros(sim,1);
sdnn = zeros(sim,1);
sdcc = zeros(sim,1);
sdrr = zeros(sim,1);
sdww = zeros(sim,1);
sdt = zeros(sim,1);
sdy = zeros(sim,1);
sdii = zeros(sim,1);
do while s le sim;
kk1 = 0;
gg0 = 0;
aa0 = 0;
t = 1;
length = 104;
kpath = zeros(length,1);
npath = zeros(length,1);
cpath = zeros(length,1);
rpath = zeros(length,1);
wpath = zeros(length,1);
tpath = zeros(length,1);
ypath = zeros(length,1);
ipath = zeros(length,1);
```

```
do while t le length;

gg = rho*gg0 + sigma*rndn(1,1);
ashock = rho2*aa0 + sigma2*rndn(1,1);

kk2 = vkk*kk1 + vkg*gg +vka*ashock;
nn = vnk*kk1 + vng*gg +vna*ashock;
cc = vck*kk1 + vcg*gg +vca*ashock;
rr = vrk*kk1 + vrg*gg +vra*ashock;
ww = vwk*kk1 + vwg*gg +vwa*ashock;
tt = vtk*kk1 + vtg*gg +vta*ashock;
yy = vyk*kk1 + vyg*gg + vya*ashock;
ii = vik*kk1 + vig*gg + via*ashock;
```

```
kpath[t] = kk2;
```

```
npath[t]= nn;
```

```
cpath[t] = cc;
```

```
rpath[t] = rr;
```

```
wpath[t] = ww;
```

```
tpath[t] =tt;
```

```
ypath[t] = yy;
```

```
ipath[t] = ii;
```

```
gg0 = gg;
```

```
aa0 = ashock;
```

```
kk1 = kk2;
```

```
t = t+1;

        endo;

sdkk2[s] = stdc(kpath);
sdnn[s] = stdc(npath);
sdcc[s] = stdc(cpath);
sdr[r] = stdc(rpath);
sdww[s] = stdc(wpath);
sdtf[s] = stdc(tpath);
sdy[y] = stdc(ypath);
sdii[s] = stdc(ipath);
```

```
s = s+1;

endo;
```

```
mnkk2 = meanc(sdkk2)*100;
```

```
"Average percentage standard deviation of capital stock:"; print mnkk2;
```

```
mnnn = meanc(sdnn)*100;
```

```
"Average percentage standard deviation of employment:"; print mnnn;
```

```
mncc = meanc(sdcc)*100;
```

```
"Average percentage standard deviation of consumption:"; print mncc;
```

```
mnrr = meanc(sdr)*100;
```

```
"Average percentage standard deviation of real interest rate:"; print mnrr;
```

```

mnww = meanc(sdww)*100;

"Average percentage standard deviation of real wage:"; print mnww;

mntt = meanc(sdtt)*100;

"Average percentage standard deviation of tax rate:"; print mntt;

mnyy = meanc(sdyy)*100;

"Average percentage standard deviation of output:"; print mnyy;

mnii = meanc(sdii)*100;

"Average percentage standard deviation of investment:"; print mnii;

```

The following is the GAUSS code to generate the impulse response functions:

```

"Simulating the Model..."; print;

kk1 = 0;

gg = 1;

t = 1;

length = 150;

kpath = zeros(length,1);

npath = -zeros(length,1);

cpath = zeros(length,1);

rpath = zeros(length,1);

ypath = zeros(length,1);

ipath = zeros(length,1);

wpath = zeros(length,1);

```

```

do while t le length;

    kk2 = vkk*kk1 + vkg*gg;
nn = vnk*kk1 + vng*gg;
cc = vck*kk1 + vcg*gg;
rr = vrk*kk1 + vrg*gg;

    yy = vyk*kk1 + vyg*gg;
    ii = vik*kk1+ vig*gg;
    ww = vwk*kk1 + vwg*gg;

    kpath[t,1] = kk2;
    npath[t,1] = nn;
    cpath[t,1] = cc;
    rpath[t,1] = rr;
    ypath[t,1] = yy;
    ipath[t,1] = ii;
    wpath[t,1] = ww;

gg = rho*gg;

    kk1 = kk2;

t = t+1;

endo;

```

"The path of Capital Stock:"; print kpath;
"The path of Labour:"; print npath;
"The path of Consumption:"; print cpath;
"The path of Real Interest Rate:"; print rpath;
"The path of Output:"; print ypath;
"The path of Investment:"; print ipath;
"The path of Real Wage:"; print wpath;