

ON THE OPTIMAL USE OF INFLATIONARY TAX

by

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Title of Project *On The Optimal Use Of An Inflation Tax*

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Abstract:

Many of the previous studies have addressed the validity of inflationary tax as a way to finance government deficit and efforts has been done to determine the optimal rate of inflation for the seigniorage revenue. The conclusion of previous literature is that in the world where only distortionary taxes are available, the inflationary taxes are justified and the optimal rate of inflation depends on various other monetary and fiscal policy parameters. However previous studies didn't go further to see what's the optimal proportion of seigniorage in government budgeting and how this optimal proportion is affected by government spending as a percentage of GDP, g/y .

In my paper, I presented an exploratory analysis on this topic by developing an overlapping generations model and arrive at the conclusion that as g/y increases, the optimal inflation rate increases; however, as g/y increases, the optimal proportion of inflationary tax in total government budget decreases. Also this paper looks at the implication of the model on the adoption of currency union in Western European countries.

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Table of Content:

Approval	ii
Abstract:	iii
Acknowledgement:	iv
Table of Content:	v
List of Tables.....	vi
List of Figures.....	vi
Introduction:.....	1
Section 1: The Model.....	3
Individuals:	3
Government and the market:	6
Section 2: The Numerical Example.....	9
Section 3: The Implication	17
The Welfare Cost of a Currency Union.....	17
One Possible Way to Avoid this Cost.....	19
Conclusion:	21
Appendix:	22
References:.....	27

List of Tables

Table 1	10
Table 2	11
Table 3	12
Table 4	14
Table 5	15
Table 6	19
Table 7	25
Table 8	26

List of Figures

Figure 1	10
Figure 2	11
Figure 3	12
Figure 4	14
Figure 5	15
Figure 6	25
Figure 7	26

Introduction:

When looking at the inflation indices of different countries, we observe positive inflation in almost every country considering a fairly long horizon, say, a decade; This means most governments are raising some positive revenue from inflation tax. Also we note that the use of seigniorage as a source of government revenue varies from time to time and from country to country.

This leaves us an open question: when only distortionary tax is available, should a benevolent government ever use the inflation tax as a partial source of revenue? If so, other questions arise: what is the optimal composition of government revenue? How much percent of government revenue should come from inflation tax? More importantly, how does the optimal composition change with change of the nation's fiscal policy, namely, the government budget as a percentage of endowment for the young, g/y ?

In this paper I investigate the optimal financing of the government's budget when only distortionary taxes are available. In particular, I consider taxation and money creation as alternative or complementary means of financing in an attempt to answer the questions above.

In section I I develop a model of overlapping generations in which money serves as a store of value. I examine the consumption and saving choice problem of individuals where, taking the government fiscal and monetary policy parameters, they maximize their lifetime utility subject to their budget constraints. Then I analyze the optimal steady state financing of the government's budget for this model where government, taking individuals' "policy reaction", chooses the optimal composition of revenue to maximize the lifetime welfare of individuals.

In section II, I provide a numerical example and study in detail the optimal use of inflation tax for different g/y . Then I draw the conclusion from this numerical example that as g/y , the government spending as a percentage of endowment for the young increases, optimal inflation rate increases and the optimal proportion of the seigniorage revenue decreases. Then I provide some intuitive explanations to support the result.

In section III, the implication part of the paper, I incorporate the model in the study of European currency union. This model addresses one of the costs of Europe forming a monetary union and the model suggests one way to reduce this cost is to coordinate the fiscal policy among the nations.

Section 1: The Model

Individuals:

I developed an overlapping-generations model of identical individuals in which every individual lives for two periods. In the first period of his life an individual consumes and saves. In the second period he consumes the fruit of what he has saved. All individuals have identical preferences over their young-age and old-age consumption summarized by a time-separable utility function:

$$U(c_{1t}, c_{2t}) = u(c_{1t}) + v(c_{2t})$$

In this model, young agents receive a fixed endowment, y , of the consumption good; the old receive nothing. Savings are primarily held in the form of money and capital. Capital saving can generate future consumption by an exponential production function $zf(k_t) = zk_t^\alpha$, α stands for the capital's share of income. The return on money saving is the inverse of inflation and we assume that money saving is always return dominated by capital¹. Therefore, agents hold money solely to satisfy an irremovable legal restriction (namely, reserve requirement). According to this legal restriction, following Bhattacharya and

¹ This assumption is tested when the optimal inflation rate is applied and the test results says

this assumption always holds, $\Pi^{-1} < \frac{zk^\alpha}{k} = zk^{\alpha-1}$.

Haslag (2001), minimum money holding depends on capital saving given the required reserve ratio. If the required reserve ratio is γ , then minimum money

holding should satisfy $q^D = \frac{\gamma}{1-\gamma} k_t = k_t \sigma$.²

In the model, there is inflation rate Π and there is income tax τ on the old. Note here we don't have income tax on the young because the income of the young is a lump-sum endowment. If we impose income tax on that lump-sum endowment, we are levying a lump-sum tax, which is not allowed in this model. Then the individual's consumption, capital saving and money saving are restricted by the following first and second period budget constraints:

Individual's budget constraints:

$$P_t c_{1t} + P_t k_t + P_t q^D = P_t y \longrightarrow c_{1t} = y - k_t - q^D$$

$$P_{t+1} c_2 = P_{t+1} (1-\tau)(z f(k_t)) + P_t q^D \longrightarrow c_2 = z(1-\tau) f(k_t) + \frac{P_t}{P_{t+1}} q^D$$

Plug in the linear production function and the legal restriction I obtain:

$$P_t c_{1t} + P_t k_t + P_t k_t \sigma = P_t y \longrightarrow c_{1t} = y - k_t (1 + \sigma)$$

$$P_{t+1} c_2 = P_{t+1} (1-\tau)(z k_t^\alpha) + P_t k_t \sigma \longrightarrow c_{2t} = z k_t^\alpha (1-\tau) + \Pi_{t+1}^{-1} \sigma k$$

where Π_{t+1}^{-1} is the inverse of gross inflation; τ is the income tax on the old.

² See Bhattacharya, Joydeep, and Haslag, Joseph (2001), "On the use of the inflation tax when non-distortionary taxes are available" for the derivation.

In a steady state, a young individual, taking the policies of the government as given, i.e. taking the parameters $\tau; \Pi$ as given, computes his optimal saving and consuming choice to maximize his lifetime utility subject to his budget constraints:

Utility Maximization:

$$\text{Max } W = u(c_{1t}) + v(c_{2t}) \quad \text{s.t. budget constraints}$$

$$\text{Max } W^1 = u(y - k_t(1 + \sigma)) + v(zk_t^\alpha(1 - \tau) + \Pi_{t+1}^{-1}\sigma k)$$

$$F.O.C : -u'_{c_{1t}}(y - k_t(1 + \sigma))(1 + \sigma) + v'_{c_{2t}}(zk_t^\alpha(1 - \tau) + \Pi_{t+1}^{-1}\sigma k)((1 - \tau)\alpha zk_t^{\alpha-1} + \Pi_{t+1}^{-1}\sigma) = 0 \longleftrightarrow k_t^*$$

..... I

The solution to the above *F.O.C* yields the optimal capital investment k_t^* and the ordinary consumption demand functions.

k_t^* can be given by a function of “policy parameters” i.e. $\tau; \Pi^{-1}$, and “economic parameters” i.e. z, σ, α .

$$k_t^* = k(\tau; \Pi^{-1}; z, \sigma, \alpha) \quad \text{..... II}$$

the ordinary consumption demand functions:

$$c_{1t}^* = y - k_t^*(1 + \sigma) \quad \text{..... III}$$

$$c_{2t}^* = zk_t^{*\alpha}(1 - \tau) + \Pi_{t+1}^{-1}k_t^*\sigma \quad \text{.....IV}$$

Using (II), we can further reduce these expressions to:

$$c_1^* = c_1(\tau; \Pi^{-1}; z, \sigma, \alpha); \quad c_2^* = c_2(\tau; \Pi^{-1}; z, \sigma, \alpha)$$

Government and the market:

In this model the government is assumed to purchase in each period a required and fixed quantity, g , of the useless consumption good. By useless I mean that the goods the government purchases from its revenue are used in such a way as not to affect an individual's consumption bundle choice. We might think of such expenditure as foreign aid or defense expenditure. Government purchase is financed by a combination of (a) income taxes and (b) inflation tax by monetary expansion, i.e. the government increases the money supply at a fixed rate of α .

MKT clearing condition:

Since the government increases the money supply at a fixed rate of α , we have:

$$M_t = \alpha M_{t-1} \quad \Delta M = (1 - \frac{1}{\alpha})M_t \quad \dots\dots\dots(a)$$

The money market clearing condition: $M^D = M^S$:

$$v_t M_t = N_t(k_t \sigma) \quad v_t = \frac{N_t(k_t \sigma)}{M_t} \quad \dots\dots\dots(b)$$

Because we assume constant population ($N_t = N$ for every period t) and because the money supply increases at the same rate in each period, we looked at the stationary solution where $k_{t+1} = k_t$ for all t . We have:

$$\frac{v_{t+1}}{v_t} = \frac{P_t}{P_{t+1}} \frac{\frac{N_{t+1}(k_{t+1}\sigma)}{M_{t+1}}}{\frac{N_t(k_t\sigma)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{\alpha} = \frac{1}{\Pi} = \Pi^{-1} \quad \dots\dots\dots(c)$$

Again, in steady states we look at the stationary solutions, therefore all the subscript t will be dropped.

Government budget constraint:

Government revenue from inflation tax:

$$g_{\text{inf}} = v_t \Delta M / N_t$$

plug in (a), (b) and (c) we get:

$$g_{\text{inf}} = (1 - \Pi^{-1})k\sigma$$

Therefore, government's budget constraint can be given:

$$g = (1 - \Pi^{-1})k^* \sigma + \tau k^{*\alpha}$$

The proportion of inflationary tax is:

$$(1 - \Pi^{-1})\sigma k^* / g$$

The government takes individuals "policy reaction functions", namely, equation II, III, IV as given, and chooses the mix of the inflation tax and the distortionary tax subject to its budget constraint, so as to maximize the welfare of current and future generations in a stationary setting.

The maximizing problem becomes:

$$\text{Max } W = u(c_1(\tau, \Pi^{-1})) + v(c_2(\tau, \Pi^{-1})) \quad \text{s.t. budget constraints}$$

Using the budget constraint, we can write τ as a function of Π^{-1} .

$$\tau = \tau(\Pi^{-1}) \quad \dots\dots\dots(1)$$

substitute (1) into the welfare function. Thus, the welfare function is given by:

$$W(\Pi^{-1}) = u(c_1(\Pi^{-1})) + v(c_2(\Pi^{-1}))$$

differentiate W with respect to Π^{-1} , yielding:

$$\frac{\partial W}{\partial \Pi^{-1}} = u_{c_1} c_{1\Pi^{-1}} + v_{c_2} c_{2\Pi^{-1}} = 0 \rightarrow \Pi^{-1*} = \Pi^{-1}(z, g, \sigma, g/y, \alpha) \dots\dots\dots(2)$$

the optimal inverse of inflation and thus the optimal inflation rate is given explicitly by equation (2). The optimal proportion of inflationary tax can be written as:

$$\text{proportion} = (1 - \Pi^{*-1}) \sigma k^* (\Pi^{*-1}) / g$$

Section 2: The Numerical Example

Let the lifetime utility of an individual be represented by:

$$U(c_1, c_2) = \ln(c_1) + \ln(c_2)$$

Let the parameters of the economy be as follows:

The reserve requirement $r=0.173$ and thus the money holding requirement

coefficient $\sigma = \frac{r}{1-r}$.

Overall productivity $z=1.15$.

The capital share of income $\alpha = 0.3$.³

Government spending as a percentage of endowment for the young g/y range

from $[0.01\sim 0.4]$.

Run the model in GAUSS⁴, we get the following results:

³ I've done a sensitivity test for empirically plausible α , $\alpha \in (0.1, 0.4)$, the conclusion for them is the same as for $\alpha = 0.3$. Data and graphs are available in appendix.

⁴ Code is available in Appendix for reference.

Table 1

optimal inflation rate for different g/y

g/y	0.01	0.03	0.05	0.1	0.15
optimal inflation	0.075269	0.123596	0.176471	0.315789	0.515152
g/y	0.2	0.25	0.3	0.35	0.4
optimal inflation	0.785714	1.12766	1.857143	2.571429	6.142857

Figure 1

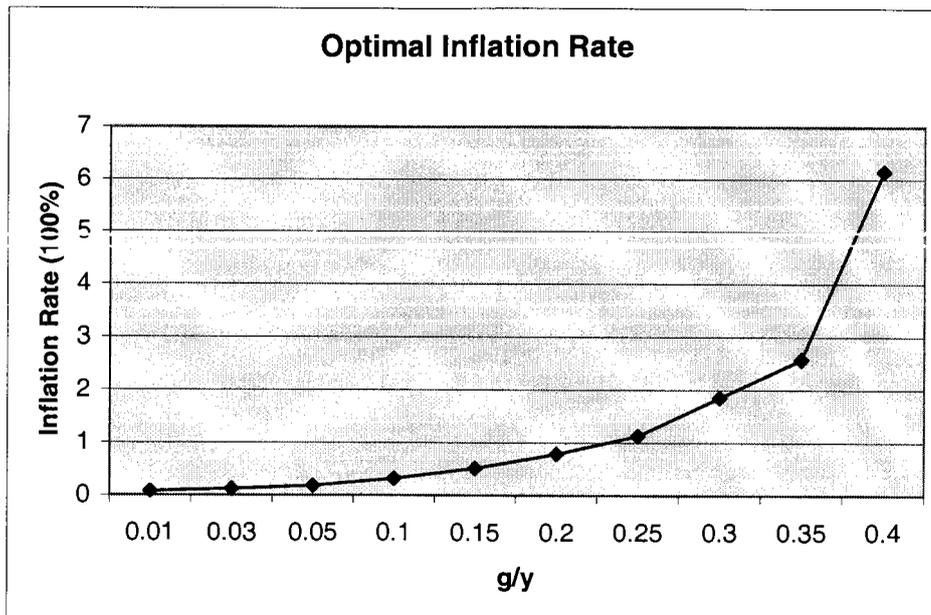


Table 2

Optimal tax rate for different g/y

g/y	0.01	0.03	0.05	0.1	0.15
optimal tax	0.015075	0.055497	0.095929	0.197933	0.299051
g/y	0.2	0.25	0.3	0.35	0.4
optimal tax	0.400273	0.502116	0.603141	0.705024	0.81119

Figure 2

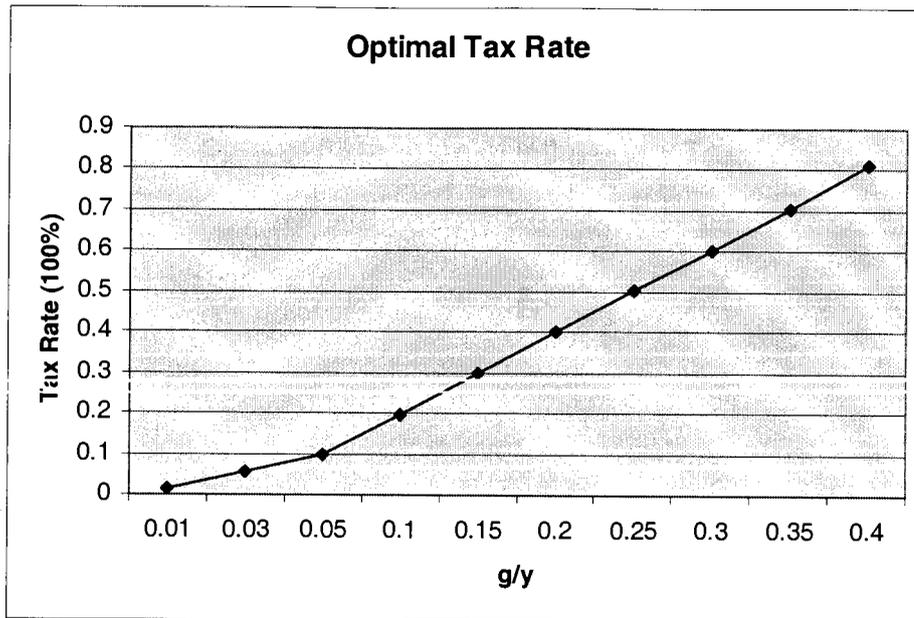
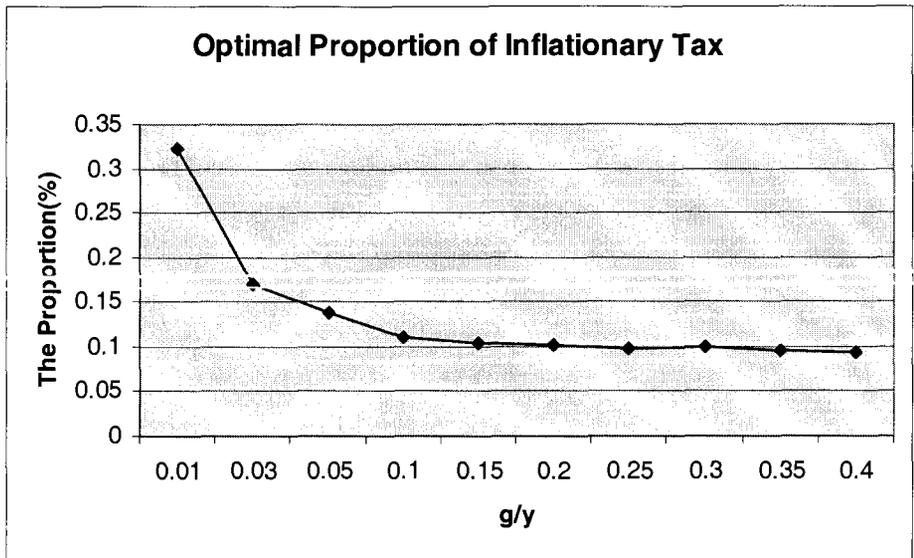


Table 3

Optimal proportion of inflationary tax for different g/y

g/y	0.01	0.03	0.05	0.1	0.15
proportion	0.322802	0.169043	0.13827	0.110723	0.10449
g/y	0.2	0.25	0.3	0.35	0.4
proportion	0.101312	0.09776	0.099091	0.094927	0.093105

Figure 3



The figure suggests that as g/y increases, it's always optimal to increase income tax and at the same time increase the inflation rate. For example, when g/y increases from 0.03 to 0.05, optimal income tax rate increases from 0.055497 to 0.095929 and at the same time, the optimal inflation rate should increase from 0.123596, i.e. 12.3596% to 0.176471, i.e. 17.6471%.

Also we can see from the above figure that, as g/y increases, although optimal inflation rate should increase accordingly, the optimal proportion of inflationary tax is decreasing. This means, as government increases its spending, it should rely more on the income tax revenue than the inflationary tax revenue. While a benevolent government should increase both inflationary tax revenue and income tax revenue to finance its increasing deficit, it should increase the income tax revenue more than the inflation revenue. The proportion of income tax revenue out of the total tax revenue should increase as g/y increases.

One way to explain this is to look at the tax collection of government as a production and look at the welfare cost of inflation and income tax as the cost of the production. Then the optimal choice of resources of production is to equate the ratio of marginal product over marginal cost of two resources. In other words, the marginal welfare cost of the inflation tax per unit of revenue collected should equal that of the income tax per unit of revenue collected.

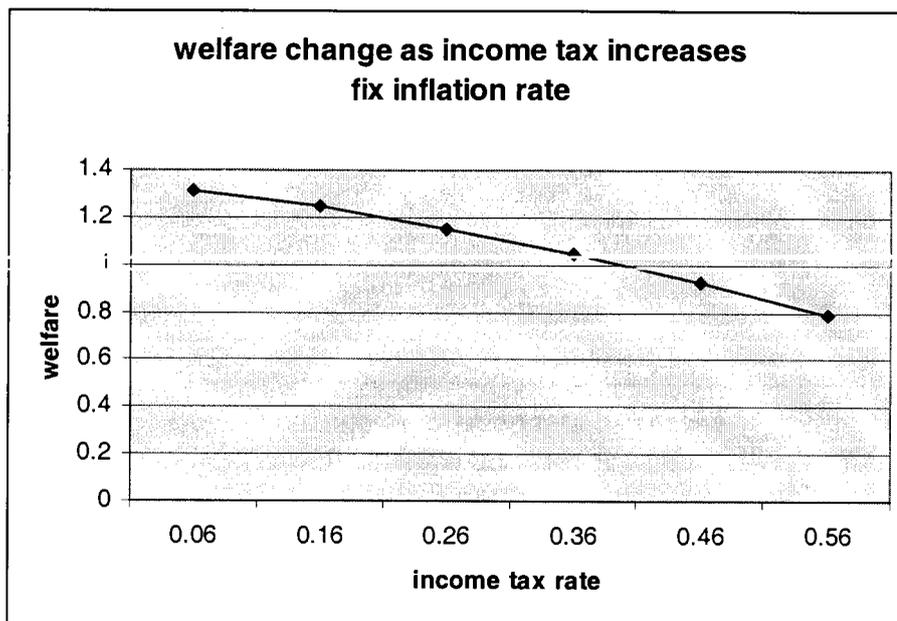
Now instead of looking at the ratio of marginal product over marginal cost, I provide some analysis of marginal cost from this numerical example. We will see that the marginal cost of collecting inflation tax increases as we increase the inflation rate and the same result applies to income tax too.

First we hold inflation rate at 0.17647, i.e. 17.647%, and let the income tax rate increase from 0.06, i.e. 6%.⁵ We look at the corresponding decrease in welfare.

Table 4

Income tax rate	welfare
0.06	1.310914
0.16	1.250242
0.26	1.148203
0.36	1.043245
0.46	0.925025
0.56	0.791403

Figure 4



⁵ These initial rates are chosen randomly from the set of data of optimal combination of inflation rate and income tax rate. Here the inflation rate is the optimal inflation rate when g/y is 0.05, and the initial income tax rate is the corresponding optimal income tax rate when g/y is 0.05. The result holds for every g/y in this model.

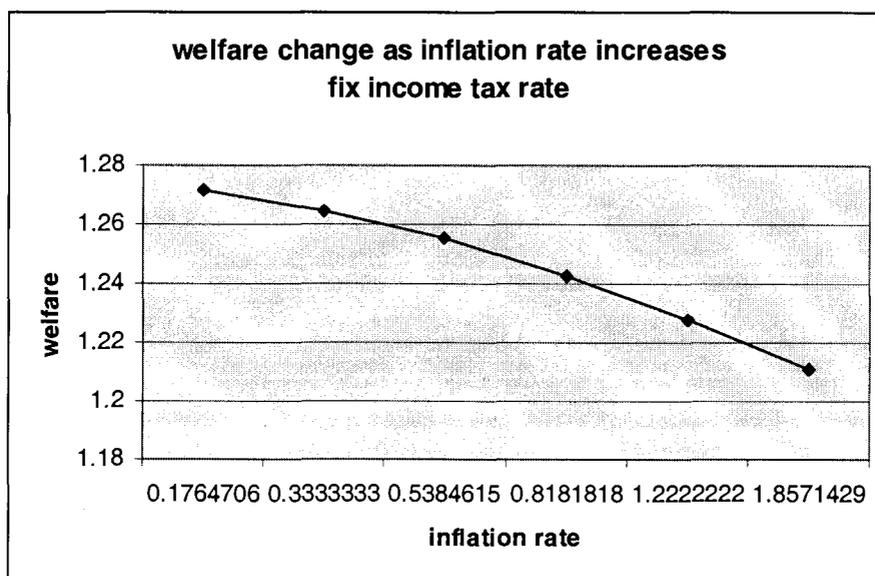
The concavity of the welfare curve indicates that as income tax rate increases, the welfare decreases at an increasing rate. This implies that the marginal cost of collecting income tax is increasing.

First we hold income tax rate at 0.06, i.e. 6%, and let the inflation rate increase from 0.17647, i.e. 17.647%. We look at the corresponding decrease in welfare.

Table 5

Inflation rate	welfare
0.176471	1.271295
0.333333	1.264778
0.538462	1.255279
0.818182	1.242398
1.222222	1.227538
1.857143	1.211099

Figure 5



The concavity of the welfare curve indicates that as inflation rate increases, the welfare decreases at an increasing rate. This implies that the marginal cost of collecting inflationary tax is increasing.

Since the marginal cost of collecting both the inflationary tax and income tax are increasing, as government increases g/y , it should increase the income tax revenue and at the same time increase the inflationary tax revenue so that the marginal cost of inflationary tax per unit g collected equals that of the income tax, hence maximizing tax collection while minimizing the cost.

One possible explanation of the decreasing proportion of inflationary tax is that the marginal cost of the per unit g collected from inflationary tax increases faster than that of the income tax. Therefore, government should rely more on income tax collection as it increase its budget.

Section 3: The Implication

This model sheds light on the adoption of common currency in Western European countries. While the model doesn't address the benefit of common currency, it does include the cost of forming a currency union.

The Welfare Cost of a Currency Union

Generally countries differ in their economic policy needs. For our discussion, let's assume there are two countries in the currency union. They have the same capital's share of income α ; the same reserve requirement γ ; the same productivity parameter z ; but different ratio of government budget over endowment for the young. (g/y).

According to our model, other things equal, countries with different g/y level should implement different inflation rate so as to maximize the overall welfare of its citizens. However, in a monetary union, the sole inflation rate is decided by a central bank maximizing a weighted sum of the utility of each country's citizens. In other words, there can be only one inflation rate in the monetary union and it's highly likely to deviate from the particular optimal inflation rate of any one country. Therefore, inevitably, this one inflation would hurt the two countries or at least one of them.

The following computation gives a better illustration:

Let $\alpha = 0.3$; $\gamma = 0.173 \Rightarrow \sigma = 0.21$; $z = 1.15$; country A with $g/y = 0.1$; country B with $g/y = 0.3$. Then we know the optimal inflation rate for country A is 0.31579 or 31.579%; the optimal inflation rate for country B is 1.85714 or 185.714%. Obviously, the two countries want to implement different inflation rate to maximize their own country's welfare. However, in a currency union, there is only one common inflation rate set by the central bank. This common inflation rate, however much it is, is going to hurt at least one of the two countries.

For example, if the central bank set the common inflation rate at the unweighted arithmetic average of the two respective optimal inflation rates, in this case, at $(0.31579 + 1.85714)/2 = 1.086465$, 108.6465%, both country A and country B are going to suffer from this common inflation rate.

We can compute the welfare loss by computing how much welfare decreases after participating in a monetary union⁶.

Run the model in GAUSS, comparing the welfare levels of individuals over their lifetime under different inflation regime, namely, for A: $\Pi = 1.31579$ and

⁶ For simplicity, we don't consider the transition period here. But we know that if we take into consideration of transition period, the welfare loss is going to be a bit less that what we find here.

$\Pi = 2.086465$; for B: $\Pi = 2.85714$ and $\Pi = 2.086465$. We get the following results:

Table 6

For individuals in country A:	For individual in country B:
The amount decrease of welfare: 0.00038	The amount decrease of welfare: 0.00392
The percentage decrease of welfare: 0.033%	The percentage decrease of welfare: 0.636%

The welfare loss indicated above is the cost of joining in a currency union in this model. The decision rule here is if the benefit of joining the currency union outweigh the cost of it, a benevolent government should by all means join the currency union, on the other hand, if the cost of it outweigh the benefit, then the government should by all means avoid it.

One Possible Way to Avoid this Cost

We know that the welfare cost of common currency arise from the different optimal inflation rates in the two countries, and the different optimal inflation rate results from the diverse needs of government budget. Therefore, one possible way to avoid the welfare cost is to coordinate the needs of government budgets of the two countries.

For example in this case, country A could increase its g/y level to 0.2 and at the same time, country B decrease its g/y level to 0.2. Thus they will be expecting the same optimal inflation rate which can be guaranteed by the sole inflation rate in the monetary union and hence no welfare cost associated with the common currency. In one word, if the two countries can coordinate their fiscal policies optimally, they can eliminate the welfare cost of participating in a monetary union. This result extends to cases of more than two countries.

Conclusion:

In this paper, I examine welfare-maximizing inflation rate and the optimal percentage of inflationary tax out of government budget financing in a world with reserve requirement and fixed proportion of endowment for the young as government budget. I draw the conclusion that a benevolent government will increase inflation rate while decrease the proportion of inflationary tax, i.e. increase the proportion of income tax as it increases its budget deficit.

The implication of this analysis on the European currency union is that when evaluating the benefit and cost of a common currency, one should consider the welfare cost of adopting a common inflation rate and one way to avoid this welfare cost is to coordinate the fiscal policy in the countries.

Appendix:

1. GAUSS Code for optimal inflation rate and income tax rate:

```
library nlsys;
y=2;
segma=0.21;
z=1.15;
govt=0.6;
step=-0.01;
inverflation=seqa(1,step,100);
Nt=rows(inverflation);
k=zeros(Nt,1);
tao=zeros(Nt,1);
c1=zeros(Nt,1);
c2=zeros(Nt,1);
w=zeros(Nt,1);
alpha=0.3;
i=1;
do while i le Nt;
    _output=0;
    x0=0.05|0.1;
    {x,f,g,h}=nlsys(&foc,x0);
    k[i]=x[1];
    tao[i]=x[2];
    c1[i]=y-(1+segma)*k[i];
    c2[i]=(1-tao[i])*z*k[i]^alpha+inverflation[i]*segma*k[i];
    w[i]=ln(c1[i])+c2[i];
    i=1+i;
endo;
print (1/inverflation-1)~w~c1~c2;

proc foc(x);
local f1,f2;
f1=(inverflation[i]*segma+z*(1-x[2]))*alpha*x[1]^(alpha-1)
/((1-x[2])*z*x[1]^alpha+
segma*inverflation[i]*x[1])-(1+segma)/(y-(1+segma)*x[1]);
f2=x[2]*z*x[1]^alpha+(1-inverflation[i])*segma*x[1]-govt;
retp (f1|f2);
endp;
```

(2) GAUSS Code for the welfare change as income tax rate

changes, holding inflation rate constant:

```
library nlsys;
y=2;
segma=0.21;
z=1.15;
step=0.1;
tao=sega(0.06,step,6);
inverflation=0.89;
Nt=rows(tao);
k=zeros(Nt,1);
c1=zeros(Nt,1);
c2=zeros(Nt,1);
w=zeros(Nt,1);
alpha=0.3;
i=1;
do while i le Nt;
    _output=0;
    x0=0.05;
    {x,f,g,h}=nlsys(&foc,x0);
    k[i]=x[1];
    c1[i]=y-(1+segma)*k[i];
    c2[i]=(1-tao[i])*z*k[i]^alpha+inverflation*segma*k[i];
    w[i]=ln(c1[i])+c2[i];
    i=1+i;
endo;
print Nt;
print tao~w;

proc foc(x);
local f1;
f1=(inverflation*segma+z*(1-tao[i])*alpha*x[1]^(alpha-1))/
((1-tao[i])*z*x[1]^alpha+
segma*inverflation*x[1])-(1+segma)/(y-(1+segma)*x[1]);
retp (f1);
endp;
```

(3) GAUSS Code for the welfare change as inflation rate changes, holding income tax rate constant:

```
library nlsys;
y=2;
segma=0.21;
z=1.15;
step=-0.14;
inverflation=sega(0.85,step,6);
Nt=rows(inverflation);
k=zeros(Nt,1);
tao=0.1;
c1=zeros(Nt,1);
c2=zeros(Nt,1);
w=zeros(Nt,1);
alpha=0.3;
i=1;
do while i le Nt;
    _output=0;
    x0=0.05;
    {x,f,g,h}=nlsys(&foc,x0);
    k[i]=x[1];
    c1[i]=y-(1+segma)*k[i];
    c2[i]=(1-tao)*z*k[i]^alpha+inverflation[i]*segma*k[i];
    w[i]=ln(c1[i])+c2[i];
    i=1+i;
endo;
print Nt;
print (1/inverflation-1)~w;

proc foc(x);
local f1;
f1=(inverflation[i]*segma+z*(1-tao)*alpha*x[1]^(alpha-1))/
((1-tao)*z*x[1]^alpha+
segma*inverflation[i]*x[1])-(1+segma)/(y-(1+segma)*x[1]);
retp (f1);
endp;
```

(4) Result for sensitivity analysis:

$\alpha=0.15$ and $\alpha=0.2$;

Table 7

optimal inflation rate under different alpha for different g/y.

g/y	0.01	0.03	0.05	0.1	0.15
alpha=0.15	0.020408	0.06383	0.111111	0.25	0.428571
alpha=0.2	0.041667	0.086957	0.136364	0.282051	0.470588
g/y	0.2	0.25	0.3	0.35	0.4
Alpha=0.15	0.666667	1	1.5	2.333333	5.666667
Alpha=0.2	0.694915	1.040816	1.564103	2.571429	6.142857

Figure 6

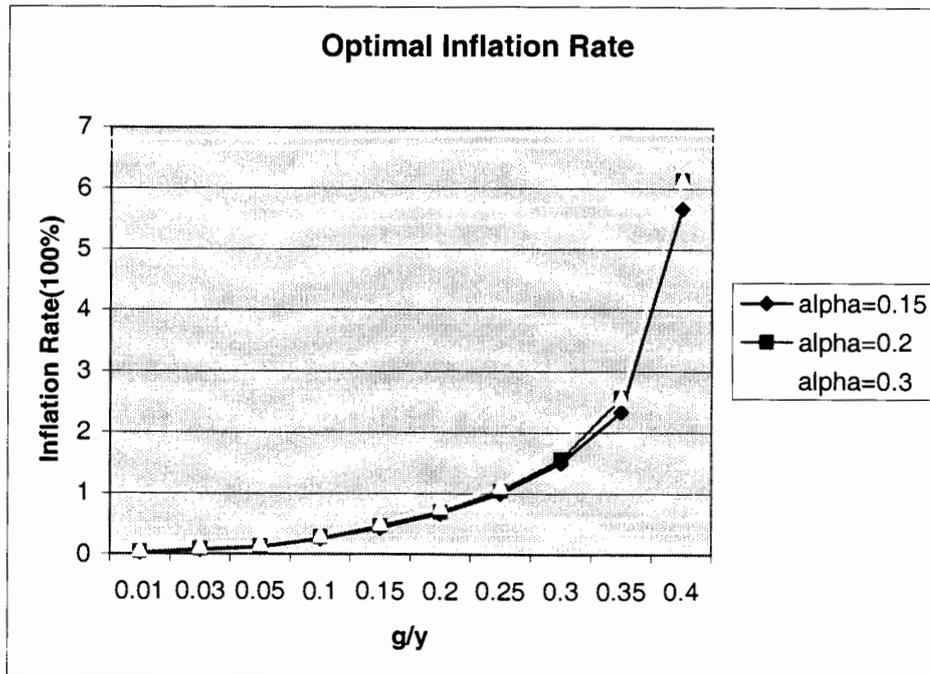
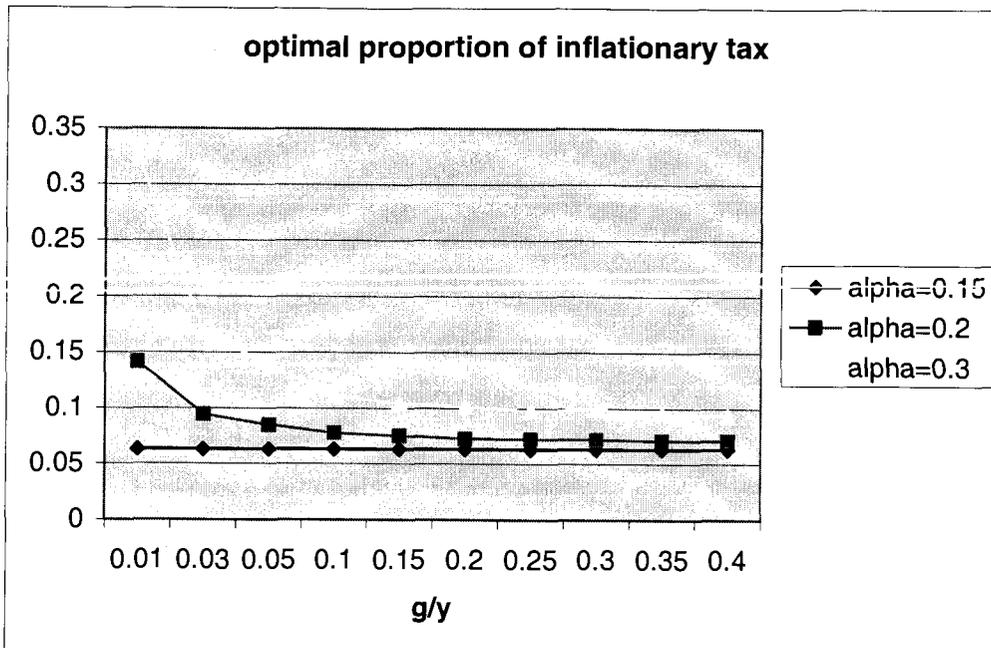


Table 8

optimal proportion of inflationary tax under different alpha
for different g/y

g/y	0.01	0.03	0.05	0.1	0.15
alpha=0.15	0.063146	0.06314	0.063135	0.063132	0.063129
alpha=0.2	0.141333	0.094209	0.084776	0.077677	0.075282
g/y	0.2	0.25	0.3	0.35	0.4
alpha=0.15	0.063127	0.063125	0.063124	0.063123	0.063123
alpha=0.2	0.072527	0.072145	0.07187	0.071032	0.07103

Figure 7



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