FIXED INCOME ATTRIBUTION: ANALYZING SOURCES OF RETURN

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ABSTRACT

This paper investigates several methods of analyzing performance of bond portfolios and presents an empirical framework for conducting fixed income attribution calibrated to a particular portfolio. First, we discuss characteristic of fixed income portfolio management and explain some of the challenges for attribution reporting. Our primary focus is on depicting deficiencies in methodologies when measuring shift, twist, butterfly movements, and credit spread changes in a non-smooth yield curve environment. In our empirical example, we present a systematic approach to fixed income performance measurement. We also show that attribution results are consistent with manager's strategy and changes in the interest rate environment.

Keywords: fixed income; performance measurement; return attribution

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1: INTRODUCTION

Performance attribution helps investors and managers understand sources of portfolio absolute and relative returns. It answers questions such as if the portfolio beat the market, whether the risks a manager took paid off, or did manager add value and was it due to skill or chance. Performance attribution is an important tool for both investors and fund managers. Investors use attribution to evaluate fund managers, their concern is largely the return on initial investment and income received. Investment managers use performance attribution to assess strategies and determine if those need to be reinforced or rethought.

Unlike for the equity attribution based on Brinson-Fachler (1986) model there is no one-size-fits-all, standardized approach to fixed income attribution. The literature on this subject has been diverse. General understanding is that bonds are unlike stocks, consequently sources of risk and decision-making processes differ, therefore the traditional equity-style attribution introduced by Brinson-Fachler (1986) is not suitable for fixed income portfolios. The appropriate method should be representative, reflecting the decision making process within the fixed income portfolio.

Major determinants of fixed income performance are income and changes in the treasury (default-free) yield curve such as shift, twist, and credit spreads. Furthermore, the spreads between the benchmark and portfolio returns are minute relative to equities and require additional precision when calculating. Models that use a reference curve, may interpolate using different methods, introducing a dose of bias. Additionally, as risks and models become more complex, the process becomes more tedious. Lastly, complexities introduced by different models may have intricate data feed requirements which can introduce significant expenditures, but offer little improvement in accuracy.

Our approach takes a real world bond portfolio, and presents a step-by-step example of performance attribution based primarily on Campisi's framework (2000). We show that this particular method is consistent with management's decision-making process and therefore appropriate for reporting attribution.

Following is the organization of this paper. Section 2 discusses some aspects of fixed income asset management. We compare and contrast that to equity asset management and show why it is that traditional equity frameworks are not appropriate for fixed income attribution. We also list some of the challenges in designing and implementing a fixed income attribution tool. Finally, we describe some of the characteristics of a good fixed income attribution framework. Section 2 also provides a literature review of the existing frameworks and their analysis. Section 3 outlines data and details of Stephen Campisi's framework that we have adopted. Section 4 presents and summarizes results. Section 5 concludes with a discussion of concerns that a fixed income manager may have, and presents several suggestions for improving the model in the future.

2: FIXED INCOME PERFORMANCE MEASUREMENT

2.1 Overview of Fixed Income Management Process

Factors that drive the performance of bonds are fundamentally different from those of stocks. As financial instruments, bonds and stocks differ in structure, pricing, potential upside returns and market in which they trade. Investment managers consider this when valuing bonds and making investment decisions, thus it is important for performance measurement to account for the same difference.

Yield curves are essential to fixed income management as changes in the curve have immediate impact on prices of fixed income securities. At any point in time, a yield curve shows market consensus of where the interest rates are expected to be in the future across different maturities (Colin, 2005). Moreover, yield curves carry an embedded view on future inflation, economic growth, exchange rates, perceived default probability of the issuer, and much more. Fixed income managers form investment strategies with respect to their expectation of the movement in the yield curve (up, down, steepen, flatter, etc). This is different from the equity approach where managers assess the growth potential of a particular stock or sector and implement selection and allocation strategies accordingly. Because of the large variety of fixed income securities, different sources of risk, and wide range of scenarios in terms of yield structure movements, fixed income attribution should go beyond simple selection and allocation approach.

One of the major challenges of fixed income attribution is as mentioned earlier the lack of uniformed approach. One of the reasons for this is that market pricing and risk factors are not as straight forward as for equity securities. Another reason involves data flow issues, mathematical background and computing knowledge. Lastly, sophisticated fixed income models may be costly and out of reach for many institutions or managers.

2.2 Fixed Income Performance Attribution Models

We have established earlier that in order for attribution to prove meaningful it needs to account for the decision making process in portfolio management. According to Campisi (2000), following are some of the characteristics of a comprehensive fixed income attribution framework.

- Representative: consistent with the investment and decision-making process, demonstrating attribution of return for taking on systematic risk;

- Rigorous: tells a story of what happened during the holding period; accurately explains sources of over and under performance;

- Reasonable: offers a balance between rigor and cost;

- Responsive: provides ability to meet client needs by customizing benchmark to match manager's strategy;

When applied to fixed income portfolios, many equity-focused attribution frameworks fail to address systematic risk drivers of bond returns, and ignore some or all drivers of manager's decision making process. For example, a manager may feel optimistic about an economic outlook and hence overweighs the portfolio towards corporate bond sector. A "Sector Allocation" Brinson-Fachler (1985) attribution model may show a positive excess return due to the sector allocation decision. Unfortunately, we cannot be exactly sure that excess returns were derived solely from the sector overweighting. Such approach would ignore manager's decision about the maturity of the bonds. Perhaps the manager invested in long term corporate bonds, which delivered a positive return because of a downward shift in long term interest rates. In this case, a “Sector Allocation” model would provide misleading results.

The goal of fixed income attribution models is to show a link between changes in the yield curve environment and portfolio performance. The appropriate models should explain how the return was generated and distinguish skill from luck. Our research shows that Campisi’s (2000) attribution methodology fits best with fixed income portfolios. Our implementation of the framework, which we will further present in a practical example, has been executed with enough rigor to tell a story of how the value added was created.

Brinson and Fachler (1985) and Brinson et al (1986) commonly known as the Brinson model has set a foundation for performance attribution. This approach is widely used and generally expected in equity-style attribution. Often times Brinson model is used for fixed income, however as discussed earlier this may not be the most suitable technique.

Wagner and Tito (1977) use a duration approach to fixed income attribution based on Fama (1972) where the duration was used as a measure of systematic risk as opposed to beta in the original Fama framework. Duration alone, however, is not sufficient to explain non parallel yield curve movements.

In explaining how actual portfolio returns were achieved Fong, Gifford, Pearson and Vasicek (1983) framework decomposes return first on a macro level, and drills down to more of a micro analysis. In simplest terms total return on the fixed income portfolio can be contributed to the external changes in the interest rate environment and management contribution. The change in the interest rate environment is that one that management has no control over and can be partitioned into an expected return on the Treasury portfolio and an unexpected return. The management contribution can be further decomposed into three categories: return from maturity management, return from spread/quality management and return from selection of selected securities.

Kahn (1991) introduced a multi-factor single and multi period fixed income attribution models. His multi-factor analysis is developed in great detail, while multi-period performance offers a useful tool to distinguish skill from luck. The framework identifies six different sources of fixed income return: portfolio moving closer to maturity, default-free term structure has changes (sovereign curve moves), sector and quality spreads have changes, unexpected cash flows, unexpected changes in quality ratings, bond specific price changes.

Van Breukelen (2000) combines Wagner and Tito duration-based approach with Brinson equity-style model. This "weighted duration approach" first calculates duration contribution to the total return and then computes allocation and selection components.

Campisi's (2000) framework on macro level decomposes total returns into income return and price change. The price change can be further partitioned into duration and yield change. Where yield change is composed of treasury change and spread change. Campisi model is easy to implement and requires minimum inputs, while at the same time considers the management process and provides meaningful decomposition of the total return.

Silva Jr. et al (2009) uses a simple combination of duration-based attribution with asset selection. First, a sovereign yield curve is fitted using the Nelson-Segal (1987) approach. Second, three hypothetical portfolios are created so that the returns may be classified according to appropriate factors.

From what we found, Campisi's (2000) framework is the only one to account explicitly for portfolio's income component. Many investors choose bond portfolios because they provide a predictable stream of cash flows, therefore, we feel that it is important to make sure that attribution results account for income return. In North America, it is the market convention to quote bond prices in terms of "clean price", which is the price that is most often used in attribution calculations. If attribution professionals take the extra step to incorporate accrued interest in price calculations ("dirty price"), then other models may provide results that account for income returns.

3: DATA AND METHODOLOGY

3.1 Data

Our study focuses on building an instrument suitable for the use at SIAS fixed income portfolio. SIAS fixed income portfolio is benchmarked against DEX Universe Bond Index. Accordingly, SIAS portfolio data is obtained from BNY Mellon Workbench platform and DEX data is collected from the PC Bond application. Selected data covers a period between March 31st, 2010 and June 30, 2010. Following are inputs that went into our model:

- Total return - calculated as a percentage price change over the holding period, plus an income component.

- Weight - this model takes in the beginning/ending weight and assumes that the weights were held constant over the period

- Coupon - annualized coupon rate

- Price – price at the beginning/end of the period

- Duration - modified duration extracted out of PC bond

- Key rate duration (KRD) – sourced from PC Bond

- Treasury yield curve at the beginning and end of the period.

Below we present data for the benchmark and portfolio respectively.

Table 1: Benchmark Data

Sectors	Sector Weights	Total Return	Coupon	Duration	Prices at t-1
Federal	47.27%	0.0193	3.84	5.04	$ 105.75
Prov	24.46%	0.0165	5.53	8.23	$ 110.48
Muni	1.42%	0.0128	5.32	6.18	$ 106.16
Corp	26.85%	0.0152	5.67	5.37	$ 107.34
Total:	**100%**	**1.0637**	**4.77**	**5.92**	**$ 107.33**

Table 2: Portfolio Data

Sectors	Sector Weights	Total Return	Coupon	Duration	Prices at t-1
Federal	17.73%	0.0111	4.55	2.14	$ 105.66
Prov	46.41%	0.0224	5.60	8.05	$ 110.51
Muni	3.55%	0.0132	5.54	5.06	$ 109.90
Corp	32.31%	0.0153	5.57	4.70	$ 106.46
Total:	**100%**	**1.0621**	**5.40**	**5.81**	**$ 108.32**

Below, sector weight graphs show that portfolio is overweight credit risk in provincial, corporate and municipal sectors. We may infer that portfolio manager's allocation strategy revolves around spreads narrowing. In a flight-to-quality scenario, however, we would expect spreads to widen and portfolio to underperform as a result.

Figure 1: Benchmark Sector Allocation

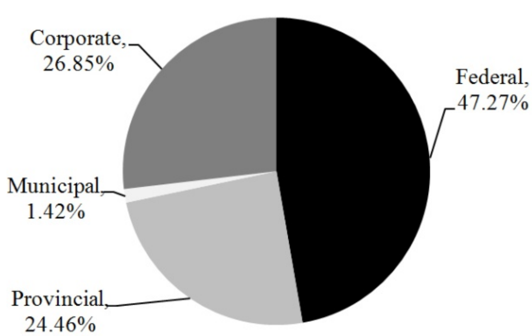
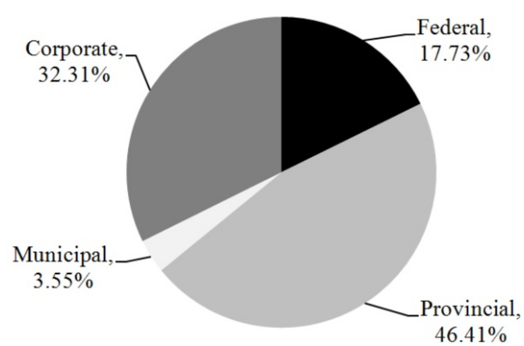


Figure 2: Portfolio Sector Allocation



In terms of measuring parallel yield curve shift, and twist effect, we require portfolio and benchmark sensitivity to changes in 5-year rate. We pick 5-year point as the key rate because both portfolio's and benchmark's durations are near the 5 year mark. This part of the attribution process offers flexibility. For portfolios that are heavily invested in long maturity bonds, a 10-year point may show a more meaningful result. Below we present KRD (key rate duration) data for both benchmark and portfolio.

Table 3: Five-Year Key Rate Durations

	Benchmark			**Portfolio**		
	Beginning	Ending	**ΔKRD**	Beginning	Ending	**ΔKRD**
Federal	4.98	5.08	0.1	2.14	1.92	-0.22
Provincial	8.07	8.26	0.19	7.91	7.88	-0.03
Municipal	6.11	6.79	0.68	5.03	4.85	-0.18
Corporate	5.29	5.38	0.09	4.63	4.55	-0.08
Total	**5.84**	**5.97**	**0.13**	**5.72**	**5.66**	**-0.06**

Key rate durations measure sensitivity of the portfolio and the benchmark to changes in five-year yields, holding all other maturities constant.

Next, we present the interest rate environment at the beginning and at the end of the attribution period. Figure 4 shows a scenario that includes an upward shift in short term interest rates and downward move in long end of the curve. As we will see further, this move will be decomposed into a shift and twist components. What follows is a detailed description methodology for fixed income attribution analysis.

Table 4: Treasury Yield Curve Change

Years	Yield (t-1)	Yield (t)	Δ Yield
0.08	0.21	0.31	0.1
0.16	0.24	0.4	0.16
0.25	0.29	0.51	0.22
0.5	0.47	0.75	0.28
1	0.94	1.04	0.1
2	1.73	1.44	-0.29
3	2.27	1.89	-0.38
4	2.8	2.3	-0.5
5	2.91	2.36	-0.55
7	3.11	2.78	-0.33
10	3.57	3.1	-0.47
15	3.82	3.36	-0.46
20	4.08	3.61	-0.47
25	4.11	3.67	-0.44
30	4.07	3.65	-0.42
40	4.07	3.65	-0.42
41	4.07	3.65	-0.42

Figure 3: Treasury Yield Curve Movement

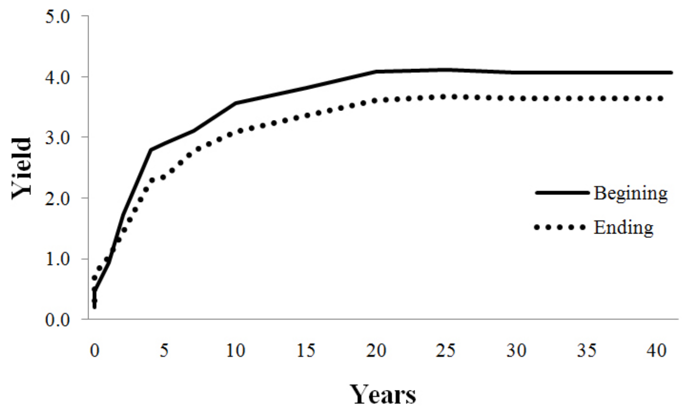
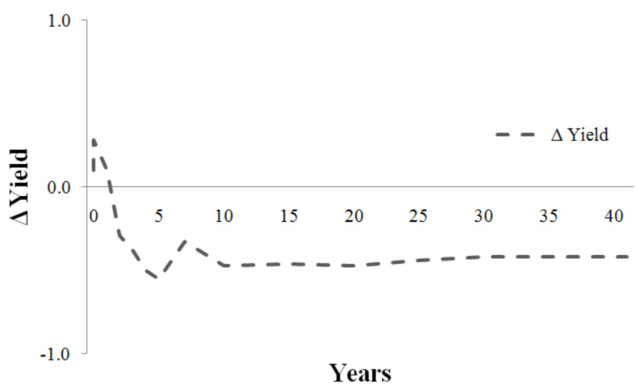


Figure 4: Treasury Yield Curve Change



3.2 Methodology

In our glossary section we offer details on some of the terms and formulas used in the framework. Detailed formulas are outlined in appendices. As we stated earlier our methodology closely follows that outlined by Campisi (2000). After collecting the necessary data and importing it into our model, we define total return as the price change effect and income effect over the attribution period. Appendix 1 provides detailed formulas for calculating total return.

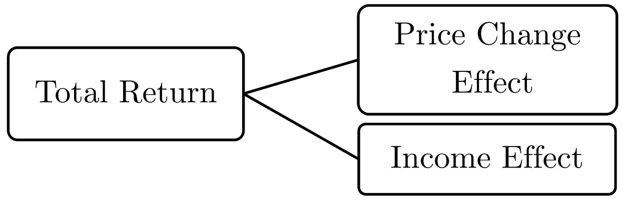


Figure 5: Total Return Decomposition

We first calculate the Income effect by dividing the coupon rate by the ending price. This is equivalent to Current Yield, not to be confused with, Coupon Yield or Yield To Maturity (YTM). Unlike YTM, Current yield does not reflect reinvestment risk or total return over the life of the bond. Moreover, current yield fluctuates with changes in bond prices, and doesn't assume a constant reinvestment rate. Another component of total return - Price change effect is calculated as time weighted return for the period. We take the change in bond's clean price over the time period and divide it by the price at the end of the period.

We can further decompose Price Change Effect it into four categories: shift, twist, spread and selection effects.

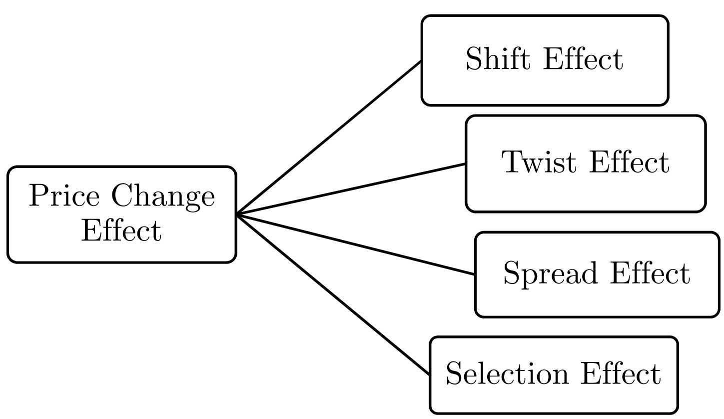


Figure 6: Price Change Effect Decomposition

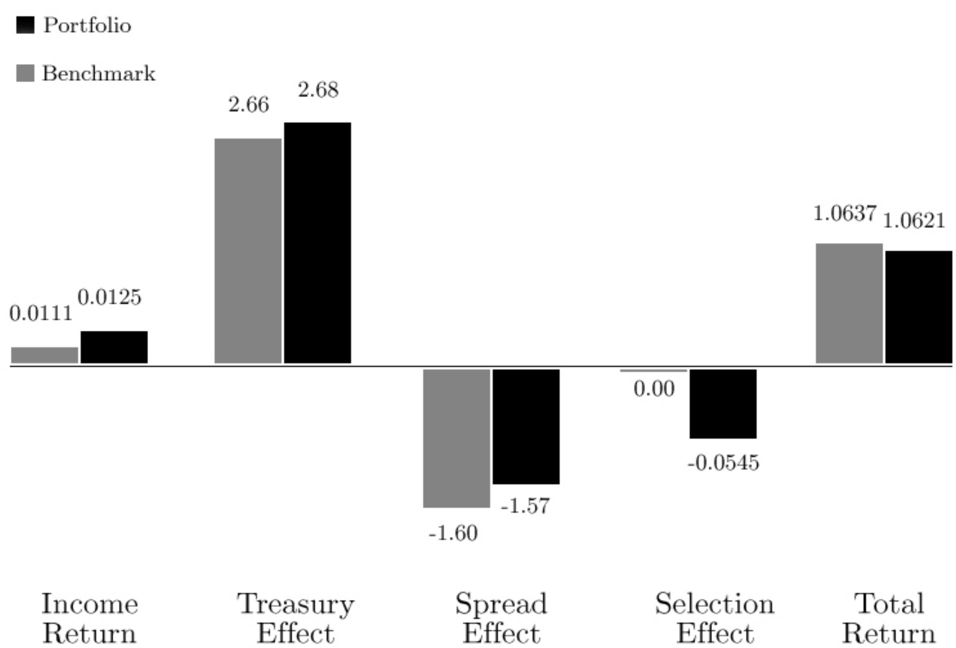
Shift and twist effects are derived from change in the reference curve (usually a risk-free treasury curve) and portfolio's sensitivity to curve movements. This can be further broken down into sector level analysis. For detailed calculations refer to Appendix 2.

We then multiply the change in treasury curve with negative modified duration to get the total treasury return. We further decompose the treasury effect into shift and twist. To help us clarify shift and twist effects we first calculate the change in key rate durations for both portfolio and a benchmark. This is accomplished by taking the difference between ending and beginning key rate duration values. Shift effect is a product of the change in key rate duration and negative modified duration. Twist effect is obtained by multiplying the difference in changes in the yield curve and key rate duration with negative modified duration. Spread effect is calculated by subtracting income and treasury effects from the total return. Selection effect is the amount remaining once income, treasury, and spread effects are subtracted from the total return. Consequently for the benchmark there is no selection effect, however for the portfolio there will be a selection effect relative to the benchmark. Moreover, selection effect may incorporate the difference in convexities. This will be addressed further in the conclusion.

4: RESULTS

In our empirical example, the fixed income portfolio has underperformed relative to DEX Universe benchmark by 16 bps. We attempt to explain where this different-from-the-benchmark performance came from. To do so we look at both benchmark and portfolio performance during the attribution period. We can clearly see that the portfolio has outperformed the benchmark on income, treasury and spread elements. However, underperformance was due to the negative selection effect.

Figure 7: Contribution to Return for Portfolio and Benchmark



First, we look at the benchmark in more detail. We explain the total return by decomposing it into income, treasury and spread. There is no selection component to the benchmark return because the assumption is that the benchmark includes the entire universe of securities. Selection effect is relevant for portfolio management as it demonstrates the skill of actively managing the portfolio. Income return for the benchmark represents the income earned during the attribution period. Treasury return is further decomposed into parallel effect (shift) and non-parallel effect (twist). Spread return shows how much credit exposure the portfolio had and how much spread return was generated as a result of the spread changes.

Table 5: Analysis of Benchmark Return

Bench	Income	Treasury Return	Shift	Twist	Spread Return	Selection	Return (Yield)
Total	**0.011**	**2.656**	**-0.770**	**3.426**	**-1.603**	**0.000**	**1.064**
Federal	0.009	2.750	-0.504	2.029	-2.740	0.000	0.019
Provincial	0.013	3.188	-1.564	4.683	-3.184	0.000	0.016
Municipal	0.013	2.597	-4.202	7.561	-2.597	0.000	0.013
Corporate	0.013	2.735	-0.483	3.356	-2.733	0.000	0.015

Portfolio strategy was to generate more income by underweighting federal bonds and overweighting corporate and provincials bonds, which delivered higher income return. The portfolio income return was positive, as was the treasury return. Spread returns for benchmark and portfolio were negative as a result of widening in spreads, however the spread excess return was positive. Selection for the quarter was negative mainly driven by municipal sector.

Table 6: Analysis of Portfolio Return

Port	Income	Treasury Return	Shift	Twist	Spread Return	Selection	Return (Yield)
Total	**0.012**	**2.677**	**0.349**	**2.328**	**-1.573**	**-0.054**	**1.062**
Federal	0.011	0.648	0.471	0.177	-1.163	0.516	0.011
Provincial	0.013	3.051	0.242	2.809	-3.115	0.073	0.022
Municipal	0.013	2.750	0.911	1.839	-2.126	-0.623	0.013
Corporate	0.013	2.515	0.376	2.139	-2.392	-0.120	0.015

As we can see from attribution results, portfolio underperformed from the twist effect due to being underweight federal long-maturity bonds. During the attribution period, the yield curve has twisted resulting in a decline in long-term yields at the same time driving the price of long-term bonds up. This performance was direct consequents of management's decision to underweight long-term bonds by remaining short duration.

Table 7: Detailed Excess Portfolio Attribution Analysis by Sector

	Income	Shift	Twist	Spread	Selection	Total
Federal	0.0017	0.9748	-1.8523	1.5763	0.5160	-0.0082
Provincial	0.0002	1.8052	-1.8734	0.0696	0.0735	0.0059
Municipal	0.0001	5.1132	-5.7218	0.4706	-0.6230	0.0005
Corporate	-0.0001	0.8593	-1.2178	0.3410	-0.1203	0.0002
Total	**0.0014**	**1.1190**	**-1.0977**	**0.0301**	**-0.0545**	**-0.0016**

In summary, the portfolio has done better then the benchmark in three out of the five categories. Non-parallel changes in the yield curve have contributed to underperformance, as did poor selection. However, the underperformance was not significant, and it can very well be described by the management's strategy. Most fixed income portfolios are managed for long term, thus small deviations in the short run are not uncommon.

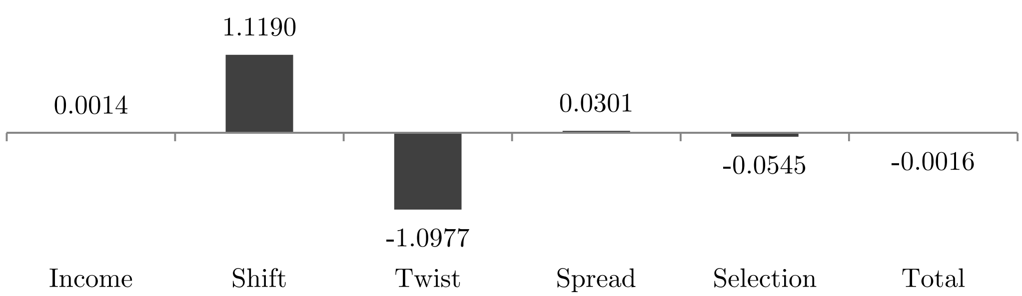


Figure 8: Summary of Excess Returns

5: DISCUSSION AND CONCLUSION

5.1 Discussion

Our model uses a buy-and-hold approach to attribution as oppose to transaction-based approach. The buy-and-hold method assumes that portfolio weighting is constant over the attribution period. It captures a "snapshot" of the portfolio weights and holdings. The buy-and-hold approach assumes that there are no transaction costs and that all transactions happen at the end of the holding period. Consequently, the shortcomings of this approach are that it ignores transaction costs and change in weights of individual holdings. (Spaulding 2003). The buy-and-hold approach is quite common in fixed income analysis and given the infrequent activity in the portfolio used in our empirical section, we believe this approach to be relevant for our analysis.

Additionally, it is important to note that our model does not include convexity. Bond price change is approximated by duration and spread or yield change. When we add convexity, we move away from a linear model to a quadratic one. While linear model allow for straightforward calculations of various return effects, quadratic model offers no mathematical equivalence for **(Duration + Convexity) * Treasury Change, and (Duration + Convexity) * Spread Change** formulas. The two equations would not be mathematically equivalent. We also know that convexity tends to have a very small impact on excess basis between benchmark and portfolio. However, some managers do take active convexity bets, for example through asset/mortgage backed securities. In that case, convexity effect can be calculated at the total yield level. We would calculate portfolio's and benchmark's total yield change by using respective durations and convexity. Further we could infer from both results the return component generated by an active bet in convexity. Although we don't explicitly break out convexity effect in the existing model, it is aggregated in the Selection effect with other factors.

The introduction of fixed income attribution model has provided useful insight into the nature of the portfolio returns. The goal was to assist fund management in forming strategies and help client better understand sources of return. We have adopted Campisi framework to fixed income attribution and calibrated it according to the needs of this portfolio. In our review, we found Campisi's method to be most compatible with the management process.

5.2 Conclusion

We have showed a number of different approaches to performance attribution. We explained how unique fixed income environment is and why it requires a special approach to attribution. Furthermore, we implemented an empirical example using the Campisi (2000) method. Empirical calculations used a Canadian fixed income portfolio. Our performance attribution model is parsimonious yet it provides useful insight into the sources of return. More importantly, this model is unique because it includes income return in addition to price return. Generally, fixed income models focus on price return only and ignore income return. This is incorrect, because bonds are primarily income instruments, and over time price returns tend to revert to zero so that most of the total long-term effect is generated through income. Our model also reflects the management's decision-making process.

Additional improvements can be done to our model. First, we could introduce a transaction-based approach to accounting for returns. Another improvement is to use a more sophisticated method such as Nelson-Segal to interpolate the curve when determining changes in Duration Matched Treasuries (DMT). Furthermore, our approach is limited to sector level attribution; possible enhancement would introduce attribution down to the security level.

The attribution model that we have presented in the paper decomposes total return into components related to portfolio income, and yield curve movements, however it can also be applied to portfolio volatility. To better understand portfolio volatility and greatest sources of tracking error when compared to benchmark, total portfolio volatility can also be decomposed using this model.

APPENDICES

Appendix 1: Total Return Calculation

$$Total\ Return = \frac{\text{V(t) +Income −V(t−t)}}{\text{V(t−1)}} \qquad (1)$$

Income Component is dependent on selected attribution period. For annual attribution period use (2) but for quarterly use (3).

$$Income = \frac{Coupon}{Price} \qquad (2)$$

$$Income = \frac{Coupon}{Price} * \frac{1}{4} \qquad (3)$$

Example:

Benchmark

Sector	Price Mar-31	Price Jun-30	Price Return	Coupon	Coupon Return (quarterly)	Total
Federal	105.75	108.04	0.01024	0.0384	0.0091	0.01932
Provincial	110.48	112.26	0.00394	0.0553	0.0125	0.01646
Municipal	106.16	107.88	0.00023	0.0532	0.0125	0.01276
Corporate	107.33	108.13	0.00200	0.0567	0.0132	0.01520
Total	**107.33**	**109.11**	**0.01658**	**0.0477**	**0.00011**	**0.01637**

$$Total\ Return = \frac{109.11\ + 0.0444\ - 107.33}{107.33} * \frac{1}{4} = 0.0167 \text{ or } 1.67\%$$

Sector Prices were obtained by taking a weighted average of individual security prices within each sector.

Appendix 2: Treasury Return decomposed into Shift and Twist

Changes in Key Rate Durations, as well as changes in Duration Matched Treasury ("DMT") are needed for calculating shift and twist effect. Key Rate Durations can be manually calculated or obtained from the PC Bond application. DMT however requires its own calculation using the method that we have adopted. ΔDMT stands for change in treasury rate corresponding to each sector duration. In the data that is available at sector level, we are given duration for the total portfolio and each sector individually.

Example:

Portfolio Sectors	**Durations**	**DMT at t-1**	**DMT at t**	**ΔDMT**
Federal	2.14	1.806	1.503	-0.303
Provincial	8.05	3.271	2.892	-0.379
Municipal	5.06	2.916	2.373	-0.543
Corporate	4.70	2.877	2.342	-0.535
Total	5.81	2.99	2.53	-0.460

DMT (at t-1) is a treasury rate on a 2.14 year treasury bill at the beginning of the attribution period. DMT (at t) is a treasury rate on a 2.14 year treasury bill at the end of the attribution period. It is unlikely that we are going to find 2.14 year treasury bill trading in the market at any given point in time. As such, we will be required to interpolate it's yield from a standard treasury yield curve. There are several choices available for interpolation, with the simplest one being linear interpolation. Models that are more complex may apply quadratic, cubic interpolation, or Nelson-Siegel (1987) approach. As long as interpolation approach is consistent for both benchmark and portfolio, the bias is kept to minimum.

Appendix 3: Comprehensive list of formulas used in attribution calculations

$$Income = \frac{Coupon}{Beginning\ Price}$$

$$Treasury\ Return\ (shift\ and\ twist) = -Duration * \Delta DMT$$

$$Shift\ Return = -Duration * \ \Delta\ KRD^{1}$$

[1] Changes in 5 year or 10 year key rate duration

$$Twist\ Return = -Duration * (\Delta DMT - \ \Delta KRD)$$

$$Spread\ Return = -Duration * Benchmark\ Spread\ Change$$

$$Benchmark\ Spread\ Change = \frac{Total\ Return - Income - Treasury\ Return}{Duration}$$

$$Selection = Total\ Return - Income - Treasury\ Return - Spread\ Return$$

Appendix 4: Excel model inputs

Data Inputs (all at the beginning of the period) — Attribution from Mar 31, 2010 to June 30, 2010

Sectors	Sector Weights		Total Return		Coupons		Price Chng Ret		Durations		Change in DMT		Beginning Prices	
	Bench	Port	Bench	Port	Bech	Port	Bench	Port	Bech	Port	Bech	Port	Bech	Port
Federal	0.4727	0.1773	0.01932	0.01110	3.84	4.55	0.0102	0.0003	5.04	2.14	-0.5456	-0.3026	105.746	105.656
Provincial	0.2446	0.4641	0.01649	0.02242	5.53	5.60	0.0040	0.0098	8.23	8.05	-0.3874	-0.3790	110.480	110.513
Municipal	0.0142	0.0355	0.01276	0.01322	5.32	5.54	0.0002	0.0006	6.18	5.06	-0.4202	-0.5434	106.160	109.895
Corporate	0.2685	0.3231	0.01519	0.01534	5.67	5.57	0.0020	0.0023	5.37	4.70	-0.5093	-0.5350	107.335	106.462
Total	1.0000	1.0000	**1.0637**	**1.0621**	**4.77**	**5.40**	**0.0164**	**0.0130**	**5.92**	**5.81**	**-0.4483**	**-0.4605**	**107.334**	**108.32**

Attribution: 4 (4 for Quarterly; 1 for annualized)

Twist Point	KEY RATE DURATIONS	Bench Begining	Bench Ending	Change (2.91)	Port Beginning (2.36)	Port Ending (-0.55)	Change
	Total	5.84	5.97	0.13	5.72	5.66	-0.06
5yr	Federal	4.98	5.08	0.10	2.14	1.92	-0.22
	Provincial	8.07	8.26	0.19	7.91	7.88	-0.03
	Municipal	6.11	6.79	0.68	5.03	4.85	-0.18
	Corporate	5.29	5.38	0.09	4.63	4.55	-0.08

Data In (all at the beginning o — Attribution

Sectors	Sector Weights		Total Return		Coupons		Price Chng Ret		Durations		Change in DMT		Beginning Prices	
	Bench	Port	Bench	Port	Bech	Port	Bench	Port	Bech	Port	Bech	Port	Bech	Port
Federal	0.4727311012844	0.177314467512	=I6+(G6/O6/$C$17)	=J6+(H6/$C$17/P6)	3.84	4.55	0.01024108	0.00033647	5.04	2.14	=AE8	=AE18	105.746	105.656
Provincial	0.244577805853	0.464068010049	=I7+(G7/O7/$C$17)	=J7+(H7/$C$17/P7)	5.53	5.6	0.00397191	0.00975378	8.23	8.05	=AE9	=AE19	110.48	110.513
Municipal	0.0141924615708	0.03553487676	=I8+(G8/O8/$C$17)	=J8+(H8/$C$17/P8)	5.32	5.54	0.00022914	0.00061332	6.18	5.06	=AE10	=AE20	106.16	109.895
Corporate	0.268477574226	0.32308223006	=I9+(G9/O9/$C$17)	=J9+(H9/$C$17/P9)	5.67	5.57	0.00198322	0.00226449	5.37	4.7	=AE11	=AE21	107.335	106.462
Total	=SUM(C6:C11)	=SUM(D6:D11)	**=SUM(E6:E11,C6:**	**=SUM(F6:F11,D6:D**	**=SUMPRODUCT(C6:C**	**=SUMI**	**=SUM(I6:**	**=SUM(J6**	**=SUMPR**	**=SUMPI**	**=AE14**	**=AE24**	**=SUMF**	**=SUMPRC**

Attribution: 4 (4 for Quarterly;

Twist Point	KEY RATE DURATIONS	Bench Begining	Bench Ending	Change (=C36)	Port Beginning (=D36)	Port Ending (=N14-M1)	Change
	Total	5.84	5.97	=L17-K17	5.72	5.66	=O17-N17
5yr	=B6	4.98	5.08	=L18-K18	2.14	1.92	=O18-N18
	=B7	8.07	8.26	=L19-K19	7.91	7.88	=O19-N19
	=B8	6.11	6.79	=L20-K20	5.03	4.85	=O20-N20
	=B9	5.29	5.38	=L21-K21	4.63	4.55	=O21-N21

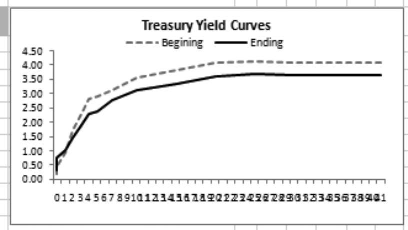
Appendix 5: Duration Matched Treasury calculations – Part1

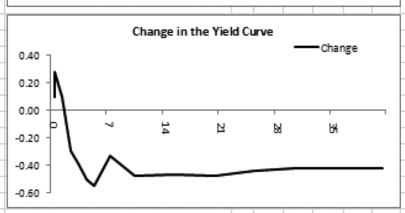
In order for the excel MATCH() function to find the upper and lower bounds for the duration matched treasury to interpolate from, Yield curve rates need to be sorted in both Ascending and Descending order.

Yield Curve Inputs

Treasury Curve Changes

Time	Begining	Ending	Change
0.08	0.21	0.31	0.10
0.16	0.24	0.40	0.16
0.25	0.29	0.51	0.22
0.5	0.47	0.75	0.28
1	0.94	1.04	0.10
2	1.73	1.44	-0.29
3	2.27	1.89	-0.38
4	2.80	2.30	-0.50
5	2.91	2.36	-0.55
7	3.11	2.78	-0.33
10	3.57	3.10	-0.47
15	3.82	3.36	-0.46
20	4.08	3.61	-0.47
25	4.11	3.67	-0.44
30	4.07	3.65	-0.42
40	4.07	3.65	-0.42
41	4.07	3.65	-0.42





Yields

This is created specifically for MATCH() function, don't modifiy anything here

Ascending				Descending			
0.08	0.21	0.31	0.1	41	4.07	3.65	-0.42
0.16	0.24	0.4	0.16	40	4.07	3.65	-0.42
0.25	0.29	0.51	0.22	30	4.07	3.65	-0.42
0.5	0.47	0.75	0.28	25	4.11	3.67	-0.44
1	0.94	1.04	0.1	20	4.08	3.61	-0.47
2	1.73	1.44	-0.29	15	3.82	3.36	-0.46
3	2.27	1.89	-0.38	10	3.57	3.1	-0.47
4	2.8	2.3	-0.5	7	3.11	2.78	-0.33
5	2.91	2.36	-0.55	5	2.91	2.36	-0.55
7	3.11	2.78	-0.33	4	2.8	2.3	-0.5
10	3.57	3.1	-0.47	3	2.27	1.89	-0.38
15	3.82	3.36	-0.46	2	1.73	1.44	-0.29
20	4.08	3.61	-0.47	1	0.94	1.04	0.1
25	4.11	3.67	-0.44	0.5	0.47	0.75	0.28
30	4.07	3.65	-0.42	0.25	0.29	0.51	0.22
40	4.07	3.65	-0.42	0.16	0.24	0.4	0.16
41	4.07	3.65	-0.42	0.08	0.21	0.31	0.1

Yields

This is created specifically for MATCH() function, don't modifiy anything here

Ascending				Descending			
=B28	=C28	=D28	=E28	=U47	=V47	=W47	=X47
=B29	=C29	=D29	=E29	=U46	=V46	=W46	=X46
=B30	=C30	=D30	=E30	=U45	=V45	=W45	=X45
=B31	=C31	=D31	=E31	=U44	=V44	=W44	=X44
=B32	=C32	=D32	=E32	=U43	=V43	=W43	=X43
=B33	=C33	=D33	=E33	=U42	=V42	=W42	=X42
=B34	=C34	=D34	=E34	=U41	=V41	=W41	=X41
=B35	=C35	=D35	=E35	=U40	=V40	=W40	=X40
=B36	=C36	=D36	=E36	=U39	=V39	=W39	=X39
=B37	=C37	=D37	=E37	=U38	=V38	=W38	=X38
=B38	=C38	=D38	=E38	=U37	=V37	=W37	=X37
=B39	=C39	=D39	=E39	=U36	=V36	=W36	=X36
=B40	=C40	=D40	=E40	=U35	=V35	=W35	=X35
=B41	=C41	=D41	=E41	=U34	=V34	=W34	=X34
=B42	=C42	=D42	=E42	=U33	=V33	=W33	=X33
=B43	=C43	=D43	=E43	=U32	=V32	=W32	=X32
=B44	=C44	=D44	=E44	=U31	=V31	=W31	=X31

Appendix 6: Duration Matched Treasury calculations – Part2

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
2	**Duration Matched Treasury Calculations**										
4	Finding Duration Matched Treasuries										
6		Sector Durations			Begining	Ending	Beging	Ending	DMT	DMT	DMT
7		**Benchmark**	Biggerthan	SmallerThan	BiggerRate	BiggerRate	SmallerThan	SmallerThan	Beginning	Ending	Change
8	Federal	5.04	5	7	2.91	2.36	3.11	2.78	2.914	2.368	-0.546
9	Provincial	8.23	7	10	3.11	2.78	3.57	3.1	3.299	2.911	-0.387
10	Municipal	6.18	5	7	2.91	2.36	3.11	2.78	3.028	2.608	-0.420
11	Corporate	5.37	5	7	2.91	2.36	3.11	2.78	2.947	2.438	-0.509
14	**Total**	**5.92**	**5**	**7**	**2.91**	**2.36**	**3.11**	**2.78**	**3.0025**	**2.554**	**-0.4483**
16		Sector Durations			Begining	Ending	Beging	Ending	DMT	DMT	DMT
17		**Portfolio**	Biggerthan	SmallerThan	BiggerRate	BiggerRate	SmallerThan	SmallerThan	Beginning	Ending	Change
18	Federal	2.14	2	3	1.73	1.44	2.27	1.89	1.806	1.503	-0.303
19	Provincial	8.05	7	10	3.11	2.78	3.57	3.1	3.271	2.892	-0.379
20	Municipal	5.06	5	7	2.91	2.36	3.11	2.78	2.916	2.373	-0.543
21	Corporate	4.70	4	5	2.8	2.3	2.91	2.36	2.877	2.342	-0.535
24	**Total**	**5.81**	**5**	**7**	**2.91**	**2.36**	**3.11**	**2.78**	**2.9913**	**2.531**	**-0.4605**

Match() and VLOOKUP() functions are used extensively to sort data for easy yield curve rate interpolation. The interpolation of the Duration Matched Treasury is calculated in columns AC and AD. Change in DMT which is then used in the model is calculated in column AE.

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
2	**Dur**										
5	Findin										
6		Sector Duratic			Begining	Ending	Beging	Ending			
7		**Benchmark**	Biggerthan	SmallerThan	BiggerRate	BiggerRate	SmallerThan	SmallerThan	Beginning	Ending	Change
8	=B6	=K6	=INDEX($U$31:$U$47,MATCH(V8,$U$31:$U$47,1),1)	=INDEX($AA$31:AA47,MATCH(V8,$AA$31:AA47,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X8-V8)/(X8-W8))*Y8+((V8-W8)/(X8-W8))*AA8	=((X8-V8)/(X8-W8)	=AD8-AC8
9	=B7	=K7	=INDEX($U$31:$U$47,MATCH(V9,$U$31:$U$47,1),1)	=INDEX($AA$31:AA48,MATCH(V9,$AA$31:AA48,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X9-V9)/(X9-W9))*Y9+((V9-W9)/(X9-W9))*AA9	=((X9-V9)/(X9-W9)	=AD9-AC9
10	=B8	=K8	=INDEX($U$31:$U$47,MATCH(V10,$U$31:$U$47,1),1)	=INDEX($AA$31:AA49,MATCH(V10,$AA$31:AA49,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X10-V10)/(X10-W10))*Y10+((V10-W10)/(X10-W10))*AA10	=((X10-V10)/(X10-W	=AD10-AC10
11	=B9	=K9	=INDEX($U$31:$U$47,MATCH(V11,$U$31:$U$47,1),1)	=INDEX($AA$31:AA50,MATCH(V11,$AA$31:AA50,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X11-V11)/(X11-W11))*Y11+((V11-W11)/(X11-W11))*AA11	=((X11-V11)/(X11-W11	=AD11-AC11
14	=B12	=K12	=INDEX($U$31:$U$47,MATCH(V14,$U$31:$U$47,1),1)	=INDEX($AA$31:AA53,MATCH(V14,$AA$31:AA53,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X14-V14)/(X14-W14))*Y14+((V14-W14)/(X14-W14))*AA14	=((X14-V14)/(X14-W	=AD14-AC14
16		Sector Duratic			Begining	Ending	Beging	Ending			
17		**Portfolio**	Biggerthan	SmallerThan	BiggerRate	BiggerRate	SmallerThan	SmallerThan	Beginning	Ending	Change
18	=B6	=L6	=INDEX($U$31:$U$47,MATCH(V18,$U$31:$U$47,1),1)	=INDEX($AA$31:AA58,MATCH(V18,$AA$31:AA58,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X18-V18)/(X18-W18))*Y18+((V18-W18)/(X18-W18))*AA18	=((X18-V18)/(X18-W	=AD18-AC18
19	=B7	=L7	=INDEX($U$31:$U$47,MATCH(V19,$U$31:$U$47,1),1)	=INDEX($AA$31:AA59,MATCH(V19,$AA$31:AA59,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X19-V19)/(X19-W19))*Y19+((V19-W19)/(X19-W19))*AA19	=((X19-V19)/(X19-W	=AD19-AC19
20	=B8	=L8	=INDEX($U$31:$U$47,MATCH(V20,$U$31:$U$47,1),1)	=INDEX($AA$31:AA60,MATCH(V20,$AA$31:AA60,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X20-V20)/(X20-W20))*Y20+((V20-W20)/(X20-W20))*A	=((X20-V20)/(X20-	=AD20-AC20
21	=B9	=L9	=INDEX($U$31:$U$47,MATCH(V21,$U$31:$U$47,1),1)	=INDEX($AA$31:AA61,MATCH(V21,$AA$31:AA61,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X21-V21)/(X21-W21))*Y21+((V21-W21)/(X21-W21))*AA21	=((X21-V21)/(X21-W	=AD21-AC21
24	=B12	=L12	=INDEX($U$31:$U$47,MATCH(V24,$U$31:$U$47,1),1)	=INDEX($AA$31:AA64,MATCH(V24,$AA$31:AA64,-1),1)	=INDEX($U$3	=INDEX(U	=INDEX($AA	=INDEX($AA	=((X24-V24)/(X24-W24))*Y24+((V24-W24)/(X24-W24))*A	=((X24-V24)/(X24-	=AD24-AC24

Appendix 7: Excel model attribution calculations – Part 1

Bench Sector Data

		Weight	Coupon	Price	Duration	Treasury Change	Total Return	Treasury Return	Spread Change	Spread Return
Bench	Total	1.0	4.77	107.33	5.92	-0.45	1.06	2.66	0.27	-1.60
	Federal	0.473	3.84	105.75	5.04	-0.55	0.02	2.75	0.54	-2.74
	Provincial	0.245	5.53	110.48	8.23	-0.39	0.02	3.19	0.39	-3.18
	Municipal	0.014	5.32	106.16	6.18	-0.42	0.01	2.60	0.42	-2.60
	Corporate	0.268	5.67	107.34	5.37	-0.51	0.02	2.73	0.51	-2.73

Portfolio Sector Data

		Weight	Coupon	Price	Duration	Treasury Change	Total Return	Treasury Return	Spread Change	Spread Return
Portfolio	Total	1.0	5.40	108.32	5.81	-0.46	1.06	2.68	0.28	-1.57
	Federal	0.177	4.55	105.66	2.14	-0.30	0.01	0.65	0.30	-1.16
	Provincial	0.464	5.6	110.51	8.05	-0.38	0.02	3.05	0.38	-3.11
	Municipal	0.036	5.54	109.90	5.06	-0.54	0.01	2.75	0.54	-2.13
	Corporate	0.323	5.57	106.46	4.7	-0.54	0.02	2.51	0.53	-2.39

Attribution Calculations

Bench	Income	Treasury Return	Shift	Twist	Spread Return	Selection	Return (Yield)
Total	0.0111	2.66	-0.77	3.43	-1.60	0.00	1.0637
Federal	0.0091	2.75	-0.50	2.03	-2.74	0.00	0.0193
Provincial	0.0125	3.19	-1.56	4.68	-3.18	0.00	0.0165
Municipal	0.0125	2.60	-4.20	7.56	-2.60	0.00	0.0128
Corporate	0.0132	2.73	-0.48	3.36	-2.73	0.00	0.0152

Port	Income	Treasury Return	Shift	Twist	Spread Return	Selection	Return (Yield)
Total	0.0125	2.68	0.35	2.33	-1.57	-0.05	1.0621
Federal	0.0108	0.65	0.47	0.18	-1.16	0.52	0.0111
Provincial	0.0127	3.05	0.24	2.81	-3.11	0.07	0.0224
Municipal	0.0126	2.75	0.91	1.84	-2.13	-0.62	0.0132
Corporate	0.0131	2.51	0.38	2.14	-2.39	-0.12	0.0153

Attributuion of Excess Return

	Income	Shift	Twist	Spread	Selection	Total
Portfolio	0.0014	1.1190	-1.0977	0.0301	-0.0545	-0.0016

Detailed Attribution Analysis by Sector

	Income	Shift	Twist	Spread	Selection	Total
Total	0.00	1.12	-1.10	0.03	-0.05	0.00
Federal	0.00	0.97	-1.85	1.58	0.52	-0.01
Provincial	0.00	1.81	-1.87	0.07	0.07	0.01
Municipal	0.00	5.11	-5.72	0.47	-0.62	0.00
Corporate	0.00	0.86	-1.22	0.34	-0.12	0.00

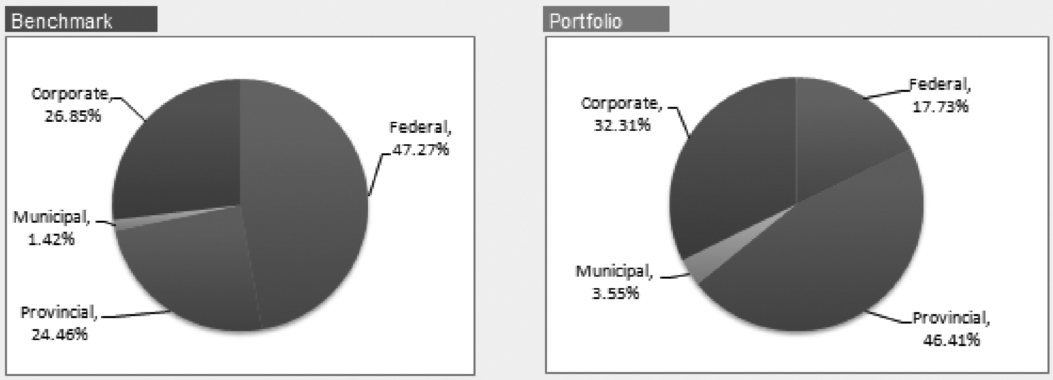
Appendix 8: Excel model attribution calculations – Part 2

Formula view of Appendix 7 shows how formulas in Appendix 3 are used to perform calculations.

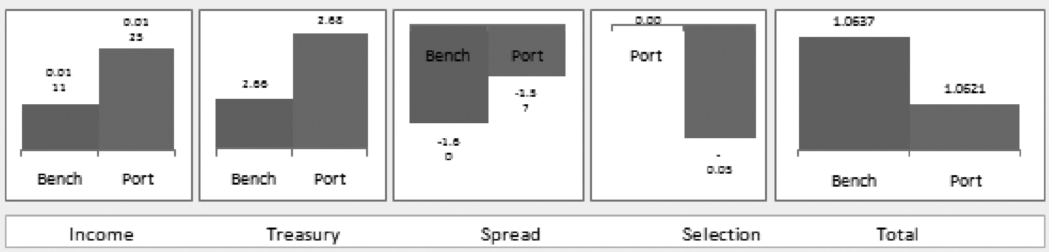
	A	B	C	D	E	F	G	H	I	J	K	L	M
50													
51		**Bench Se**											
52													
53					**Weight**	**Coupon**	**Price**	**Duration**	**Treasur y**	**Total Return**	**Treasury Return**	**Spread Change**	**Spread Return**
54			**Bench**	Total	=SUM(E55:E60)	=G12	=O12	=K12	=M12	=E12	=-H54*I54	=M54/-H54	=J54-E74-K54
55				=B6	=C6	=G6	=O6	=K6	=M6	=E6	=-H55*I55	=M55/-H55	=J55-E75-K55
56				=B7	=C7	=G7	=O7	=K7	=M7	=E7	=-H56*I56	=M56/-H56	=J56-E76-K56
57				=B8	=C8	=G8	=O8	=K8	=M8	=E8	=-H57*I57	=M57/-H57	=J57-E77-K57
58				=B9	=C9	=G9	=O9	=K9	=M9	=E9	=-H58*I58	=M58/-H58	=J58-E78-K58
59													
60													
61													
62		**Portfolio S**											
63													
64				**Weigh**	**Coupon**	**Price**	**Duration**	**Treasury Change**	**Total Return**	**Treasury Return**	**Spread Change**	**Spread Return**	
65			**ortfolio**	Total	=SUM(E66:E70)	=H12	=P12	=L12	=N12	=F12	=-H65*I65	=(J65-E81-K65)/-H65	=-H65*L54
66				=B6	=D6	=H6	=P6	=L6	=N6	=F6	=-H66*I66	=(J66-E82-K66)/-H66	=-H66*L55
67				=B7	=D7	=H7	=P7	=L7	=N7	=F7	=-H67*I67	=(J67-E83-K67)/-H67	=-H67*L56
68				=B8	=D8	=H8	=P8	=L8	=N8	=F8	=-H68*I68	=(J68-E84-K68)/-H68	=-H68*L57
69				=B9	=D9	=H9	=P9	=L9	=N9	=F9	=-H69*I69	=(J69-E85-K69)/-H69	=-H69*L58
70													
71		**Attribution**											
72													
73			**Bench**		**Income**	**Treasury Return**	**Shift**	**Twist**	**Spread Return**	**Selection**	**Return (Yield)**		
74			**Total**		=(F54/$C$17/G54)	=K54	=-H54*M17	=-H54*(I54-M17)	=M54	=K74-E74-F74-I74	=J54		
75			**=B6**		=(F55/$C$17/G55)	=K55	=-H55*M18	=-H55*(I66-M18)	=M55	=K75-E75-F75-I75	=J55		
76			**=B7**		=(F56/$C$17/G56)	=K56	=-H56*M19	=-H56*(I67-M19)	=M56	=K76-E76-F76-I76	=J56		
77			**=B8**		=(F57/$C$17/G57)	=K57	=-H57*M20	=-H57*(I68-M20)	=M57	=K77-E77-F77-I77	=J57		
78			**=B9**		=(F58/$C$17/G58)	=K58	=-H58*M21	=-H58*(I69-M21)	=M58	=K78-E78-F78-I78	=J58		
79													
80			**Port**		**Income**	**Treasury Return**	**Shift**	**Twist**	**Spread Return**	**Selection**	**Return (Yield)**		
81			**Total**		=(F65/$C$17/G65)	=K65	=-H65*P17	=-H65*(I65-P17)	=M65	=K81-E81-F81-I81	=J65		
82			**=B6**		=(F66/$C$17/G66)	=K66	=-H66*P18	=-H66*(I66-P18)	=M66	=K82-E82-F82-I82	=J66		
83			**=B7**		=(F67/$C$17/G67)	=K67	=-H67*P19	=-H67*(I67-P19)	=M67	=K83-E83-F83-I83	=J67		
84			**=B8**		=(F68/$C$17/G68)	=K68	=-H68*P20	=-H68*(I68-P20)	=M68	=K84-E84-F84-I84	=J68		
85			**=B9**		=(F69/$C$17/G69)	=K69	=-H69*P21	=-H69*(I69-P21)	=M69	=K85-E85-F85-I85	=J69		
86													
87		**Attributuic**											
88													
89					**Income**	**Shift**	**Twist**	**Spread**	**Selection**	**Total**			
90			**ortfolio**		=E81-E74	=G81-G74	=H81-H74	=I81-I74	=J81-J74	=K81-K74			
91													
92													
93		**Detailed A**											
94													
95					**Income**	**Shift**	**Twist**	**Spread**	**Selection**	**Total**			
96			**=B12**		=E81-E74	=G81-G74	=H81-H74	=I81-I74	=J81	=K81-K74			
97			**=B6**		=E82-E75	=G82-G75	=H82-H75	=I82-I75	=J82	=K82-K75			
98			**=B7**		=E83-E76	=G83-G76	=H83-H76	=I83-I76	=J83	=K83-K76			
99			**=B8**		=E84-E77	=G84-G77	=H84-H77	=I84-I77	=J84	=K84-K77			
100			**=B9**		=E85-E78	=G85-G78	=H85-H78	=I85-I78	=J85	=K85-K78			
101													

Appendix 9: Excel model graphical output

Benchmark vs Portfolio Weights



Total Return Decomposition



Attribution of Excess Return

Detailed Attribution Analysis by Sector

Analysis of Each Sector's Contribution to Excess Return

	Income	Shift	Twist	Spread	Selection	Total
Federal	0.00	0.97	-1.85	1.58	0.52	-0.0082
Provincial	0.00	1.81	-1.87	0.07	0.07	0.0059
Municipal	0.00	5.11	-5.72	0.47	-0.62	0.0005
Corporate	0.00	0.86	-1.22	0.34	-0.12	0.0002
Total	**0.0014**	**1.1190**	**-1.0977**	**0.0301**	**-0.0545**	**-0.0016**

Glossary

Duration (modified): a linear measure of the sensitivity of the bond's price to interest rate changes.

Key Rate Duration (KRD): measures the sensitivity of a security or the value of a portfolio to a 1% change in yield for a given maturity, holding all other maturities constant.

Duration Matched Treasury (DMT): a point on the treasury yield curve that corresponds to a specific duration number. i.e. 2.14 duration would correspond to the yield on a 2.14 year treasury bond.

Convexity: a measure of the curvature of how the price of a bond changes as the interest rate changes. Second derivative, that measures how the duration of a bond changes as the interest rate changes.

Current Yield: coupon rate divided by the price of the security. It represents the return an investor would expect if they purchased the bond and held it for a year/quarter/month/day.

Yield to Maturity (YTM): return anticipated on a bond if it is held until the maturity date. Assumes that coupons can be reinvested at YTM rate.

Spread (credit): difference in yield between securities with different credit quality i.e. difference in yields on a Corporate and a Treasury bond.

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