

ESTIMATING CANADA'S NAIRU

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Title of Project
Estimating Canada's NAIRU

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Abstract

It's difficult to find estimates of Canada's "Non-accelerating inflation rate of unemployment" (NAIRU). In this paper, the productivity growth trend, both in levels and in changes, is included in a state-space model to estimate a time-varying NAIRU (TV-NAIRU). Using the Kalman filter, the results show it does not significantly improve the model performance for the period 1978-2003. The small variation in the productivity trend may be the source of this ambiguity. Finally, an estimate of Canada's NAIRU is also obtained in the model without productivity growth. It is quite stable and robust to different measures of inflation rate and unemployment rate, which suggests that the major determinants of Canada's NAIRU didn't change much in this period.

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1 Introduction

Debates on monetary policy often focus on the level of unemployment and, especially, whether the unemployment rate is approaching its natural rate. As the empirical counterpart of the natural rate of unemployment, the non-accelerating inflation rate of unemployment (NAIRU) concept was first developed by Modigliani and Papademos (1975) and is defined as the rate of unemployment at which there is no tendency for inflation to increase or decrease. But the problem is that the NAIRU is not directly observable. So some combination of economic and statistical reasoning must be used to estimate it from observable data. This paper is devoted to estimating Canada's NAIRU using standard estimation methods.

The remainder of the paper is organized as follows. The next section provides a brief review of alternative methods of estimating the NAIRU. Section 3 deals with the question of whether adding productivity can improve estimates of the NAIRU significantly. Section 4 estimates the NAIRU in the sample period. Finally, the conclusion is drawn in Section 5.

2 Literature Review

2.1 General framework

The relationship between inflation and unemployment is an important issue in macroeconomics and has inspired many works on it. For example, Ireland (1999) stated that Barro and Gordon's Time-Consistency Model implies long-run trends in the natural rate of unemployment will introduce similar trends into the inflation rate when the central bank cannot commit to a monetary policy rule, and confirmed this by 10-year centered moving average for inflation rate and unemployment rate during 1960-1997 in US.

Another example is the famous Phillips curve, which states that in the short run monetary neutrality breaks down and aggregate demand pushes inflation and unemployment in opposite direction. Once this short run trade-off is admitted, there must be some level of unemployment consistent with stable inflation. In most standard theories, we can write this short-run trade-off between inflation and unemployment as:

$$\pi = \pi^e - a(U - U^*) \quad (1)$$

where π and π^e are actual and expected inflation rates; U is unemployment rate; and U^* is NAIRU.

Equation (1) can be rewritten as:

$$\pi - \pi^e = k - aU \quad (a > 0) \quad (2)$$

Comparing these two equations, it's obvious that the NAIRU can be easily got:

$$U^* = \frac{k}{a} \quad (3)$$

According to Gordon's (1997) "Triangle" Model of inflation, inflation rate depends on three basic determinants: inertia, demand and supply. Thus a general framework to measure NAIRU becomes:

$$\pi_t = a(L)\pi_{t-1} - b(L)(U_t - U^*) + c(L)X_t + \varepsilon_t \quad (4)$$

Here π_t is current inflation rate. Inertia is conveyed by lagged inflation rate π_{t-1} . It also stands for expected inflation. U_t and U^* are current unemployment rate and NAIRU, respectively. And they represent the demand shock (so if $U_t = U^*$, there is no excess demand). X_t is a vector of current supply shock variables. It's assumed to be contemporaneously uncorrelated with unemployment. ε_t is a serially uncorrelated error term. L is the lag operator, and $a(L)$, $b(L)$ and $c(L)$ are polynomials in it. Note here that U^* is "no-supply-shock" NAIRU, that is, the unemployment rate that is consistent with stable inflation in the absence of supply shocks. "Without this qualification, NAIRU would jump around as supply shocks arrived and departed, which is not what most economists are trying to convey when they speak of the natural rate of unemployment." (Gordon, 1997)

Recently a prevailing view is the NAIRU has changed over the postwar period. If a time-varying NAIRU (TV-NAIRU) is allowed, U^* is replaced by U_t^* and the latter is described to be a random walk:

$$\pi_t = a(L)\pi_{t-1} - b(L)(U_t - U_t^*) + c(L)X_t + \varepsilon_t \quad (5)$$

$$U_t^* = U_{t-1}^* + \eta_t \quad (6)$$

Here the error term η_t is well behaved, with a mean of zero and a standard deviation of σ_η . If the standard deviation is zero, then NAIRU is constant.

Despite all the endeavors above, there has been a consensus that NAIRU is hard to be measured because of the uncertainty around the model specification. The estimate of NAIRU is imprecise with large confidence intervals. For example, the 95 percent confidence interval for the value of US's NAIRU in 1997 based on the GDP deflator inflation is 4.3 percent to 7.3 percent (Staiger, Stock, Watson, 1997a). Fortunately, the declining NAIRU in America in the late 1990's inspired a lot of work on the relationship between productivity growth and the NAIRU. For example, Ball and Moffitt's (2001) "wage aspiration" story suggests that workers' real wage targets depend on aspirations, a weighted average of past real wages. Because of limited information, a mismatch between wage aspirations and productivity may happen, which worsens (improves) the inflation-unemployment trade-off and thus raises (decreases) the NAIRU. Another example exists in the theoretical job search literature (Aghion and Howitt, 1994, Mortensen and Pissarides, 1998). Productivity has two competing effects. First, higher labour productivity growth increases the value of a worker to the firm, and thus stimulates the creation of job vacancies, which in turn decreases unemployment (capitalization effect). Second, higher productivity growth is often accompanied by structural change. Old jobs are destroyed and replaced by new ones (creative destruction effect). So increase in productivity growth shortens the employment duration and raises the natural rate. The relationship between productivity and the natural rate depends on which effect dominates.

Based on the above theories, Slacalek (2003) proposes a productivity-augmented model, productivity entering into the random-walk model to explain the variation in NAIRU:

$$\pi_t = a(L)\pi_{t-1} - b(L)(U_t - U_t^*) + c(L)X_t + \varepsilon_t \quad (7)$$

$$U_t^* = U_{t-1}^* + \beta\Delta Z_t + \eta_t \quad (8)$$

where Z_t is productivity trend. Using 1960-2002 US quarterly data, he argues that this specification significantly improves the efficiency of the estimate of NAIRU. He also compares the estimation results when the level and the change of productivity growth enter this specification separately and finds the level does a much better job for his case.

The empirical analysis in the literature that was closest to examining the influence of productivity on the NAIRU for Canadian data is Gruber (2003). The entire error in the Phillips curve estimation is assigned to the variation in the NAIRU. That is, if Equation (1) is rewritten as:

$$\pi_t = \pi_t^e - a(U_t - U^*) + \varepsilon_t \quad (9)$$

combining with Equation (2), the TV-NAIRU is:

$$U_t^* = \frac{k + \varepsilon_t}{a} \quad (10)$$

If the TV-NAIRU is relatively constant, the errors are small or offsetting. Then the productivity term¹ is put directly into the Phillips curve and the NAIRU with and without productivity are compared. If productivity can explain the NAIRU significantly, addition of it will flatten the path of NAIRU significantly.

¹ It's a productivity aspiration term following Ball and Moffitt (2001).

2.2 Estimation Methods

There are four approaches to measuring a varying NAIRU.

a). Detect breakpoints in the data and assume each time period has a constant NAIRU.

(Staiger et al.,1997a)

b). Approximate the NAIRU by a spline function in time. (Staiger et al.,1997b)

c). For $a(L)=1$, Equation (4) can be transformed as:

$$U_t^* + \frac{c(L)}{b(L)} X_t = U_t + \frac{1}{b(L)} \Delta \pi_t - \frac{\varepsilon_t}{b(L)} \quad (11)$$

Neglecting ε_t , the right hand side can be computed from the data if the value of $b(L)$ is known. This yields an estimate of left hand side. Here U_t^* represents the longer-term trends, and the other term is proportional to the shorter-term supply shocks. Then HP filter is used to extract U_t^* from left hand side. To compute $b(L)$, first assume constant NAIRU, then use Equation (4) to get the estimate of it. The rationale behind it is “reasonable variation in the assumed coefficient has little effect on our conclusion” (Ball, Mankiw, 2002).

d). Use Kalman filter technique (Franz, 2003). Equations (5) and (6) serve as state-space model, where Equation (5) is the “observation equation” and equation (6) is the “transition equation”.

2.3 General Results

Despite the fact that the NAIRU is very imprecisely measured, at least there is a consensus that, based on US data, the NAIRU has exhibited pronounced cycles over the postwar period and particularly, it has fallen in the late 1990s from its peak in the early 1980s (Ball, Mankiw, 2002; Gordon, 1997). More employable labour force, new economy and

productivity acceleration may explain part of this phenomenon (Ball, Mankiw, 2002). As a comparison, (West) Germany experienced an increasing NAIRU until the end of last century and a fairly modest declining after that (Franz, 2003). For Canada, it experiences fairly persistent NAIRU in recent years and a slightly declining tendency (Richardson et al., 2000). Gruber(2003) finds the addition of productivity into the Phillips curve cannot flatten the TV-NAIRU to any significant degree.

Faced with the uncertainty about the NAIRU, it's not surprising that forecasts of inflation based on the Phillips curve are insensitive to different assumptions about NAIRU: using NAIRU ranging from 4.5 to 6.5 percent would have produced similar forecasts of inflation over the next year. Seen in another light, it explains why it's so difficult to measure NAIRU: if NAIRU is more closely interrelated with inflation, it would be more precisely estimated (Staiger, Stock, Watson,1997a).

3 Does productivity matter?

This section uses the Kalman filter to figure out whether productivity plays an important role in estimating NAIRU in Canada in the sample period. Both the level and the change of productivity growth rate are considered. The results suggest that productivity cannot significantly improve the model for the sample.

3.1 Data Description

Here quarterly data for Canada are used. The regressions are run over the period 1978Q4—2003Q3².

In the empirical work, typically there are three measures for the inflation rate: GDP deflator, CPI and core inflation rate. According to Staiger et al(1997a), the results are fairly robust to these measures. Following the literature, the all-item CPI is used to generate quarter-to-quarter inflation rates (CPI) as baseline. GDP deflator (DGDP) and core inflation rate (CORE) are two alternatives.

The unemployment rate for total above 15-year-old is the unemployment rate in the specification (U). The unemployment rate for males in the age group 25-54 (MU) is also a good candidate because it controls for potential demographic shifts that could affect the stability of coefficients (Stock, Watson, 1999).

² Except for CPI for Food and Energy, all data are seasonally adjusted. The data source is CANSIM and CANSIM II.

Following Staiger et al. (1997b), all regressions control for the demeaned difference between the inflation rate of food and energy and CPI inflation as the supply shock (SUPPLY). So in the steady state they do not impact the rate of inflation.

Productivity is defined as labor productivity in the business sector, which is measured as output per person-hour. For the problem that only annual data is available during the period 1978Q4—1986Q4, I deal with it in two different ways: (a) linearly approximating the annual data into quarterly data; (b) constructing the data myself, that is, dividing the real GDP by the total actual hours worked. But here only the data for all industries are available. Both measures are used in the analysis of productivity (PRODA and PRODB). Productivity growth is constructed as quarter-to-quarter growth rate (GPRODA and GPRODB). Following Ball and Moffitt (2001), the change of productivity growth (CHGPRODA and CHGPRODB) is calculated as the difference between current productivity growth rate and a moving average of past growth rate, that is,

$$chgprod_t = gprod_t - \frac{1-b}{b} \sum_{i=1}^{\infty} b^i gprod_{t-i} \quad (12)$$

where the parameter b gives the rate of decline in the weights and is set at 0.95.

Especially, the second term expresses wage aspiration. If we denote it as A , then

$$A_t = bA_{t-1} + (1-b)gprod_{t-1} \quad (13)$$

Here I use the HP-filtered trend value of 1950's productivity growth rate as initial value of A .

Table 1 Summary measures of inflation rates, unemployment rates, supply shock and productivity

Average over interval	Inflation rate			Unemployment rate		Supply Shock	Growth rate of productivity		Change in the growth rate of productivity	
	CPI	DGDP	CORE	U	MU	SUPPLY	A	B	A	B
1978Q1-1980Q4	2.5	2.7	2.4	7.6	5.0	0.6	0.1	0.0	-0.7	-0.8
1981Q1-1985Q4	1.6	1.3	1.4	10.5	8.4	0.1	0.4	0.5	-0.1	-0.1
1986Q1-1990Q4	1.1	1.0	1.1	8.4	7.0	-0.3	0.0	0.0	-0.4	-0.4
1991Q1-1995Q4	0.5	0.4	0.6	10.5	9.8	-0.2	0.5	0.5	0.2	0.2
1996Q1-2000Q4	0.5	0.4	0.4	8.3	7.3	0.2	0.4	0.4	0.0	0.0
2001Q1-2003Q3	0.5	0.5	0.5	7.5	6.5	-0.0	0.5	0.5	0.1	0.1
Total	1.0	0.9	1.0	9.0	7.7	0.0	0.3	0.4	-0.1	-0.1

In Table 1, it's obvious that all alternative series for the same variable have a similar pattern during this period. The average level of inflation rate before 1990's is above 1 in every interval and is much higher than that after it, which is around 0.5. According to the literature, it's consistent with the adoption of inflation targeting in 1991. The unemployment rate has double peaks at the beginning of 1980's and 1990's, and this phenomenon is obvious for both U and MU, which suggests that this variation in unemployment may be caused by factors other than demographic change in the labor market. As expected, the average of the supply shock is near zero. Two measures of productivity give a similar pattern in the level and in the change of productivity growth and the value difference is minor, which implies the estimation results may be robust to these two measures. Additionally, except during 1978-1980 and the late of 1980's, productivity has almost the same average growth rate in most of the sample. The change in productivity growth has very modest increasing tendency in this period.

3.2 Estimation Analysis

3.2.1 Correlation Analysis

Before estimation, the correlation between productivity and the NAIRU is checked using the HP filter to create a crude approximation to the NAIRU. Figure 1 and 2 shows the relationship between productivity trend and this HP estimate of NAIRU. The HP filter is used to estimate all of trends³. Different measures for productivity suggest the same trend pattern and their correlations with the unemployment trend are ambiguous. Table 1 summarizes the correlation between the NAIRU and the level and the change of productivity. These relationships suggest that the addition of productivity may not improve the estimate of Canada's NAIRU to any significant degree during the sample period.

Table 2 Correlation between productivity and unemployment trend

	Correlation coeff. with HPU
HPGPRODA	0.17
HPGPRODB	0.14
HPCHGPRODA	0.34
HPCHGPRODB	0.29

3.2.2 Estimating productivity trend

Now the Kalman filter algorithm, an alternative to HP filter, is used to extract the trends. The advantage of it is that the algorithm produces an optimal estimator of the trend (the minimum mean squared error linear estimator), see e.g. Harvey (1989). For both cases: the level of productivity and the change of productivity, the trend of them θ_t^* is estimated by the random walk plus noise model,

$$\theta_t = \theta_t^* + z_{T_t}, \quad \theta_t^* = \theta_{t-1}^* + z_{P_t}, \quad \text{var}(z_{P_t}) = \lambda_\theta \text{var}(z_{T_t}) \quad (14)$$

³ The smoothing parameter is 1600 for quarterly data.

where θ_t is the observed, measured productivity data, θ_t^* is the unobserved trend to be estimated and z_{T_t} and z_{P_t} are the temporary and permanent shocks to productivity, respectively. Both shocks are assumed to be i.i.d. normal $N(0, \text{var}(z_{T_t}))$ and $N(0, \text{var}(z_{P_t}))$, respectively, and uncorrelated. The coefficient $\text{var}(z_{T_t})$ is estimated by MLE. Following the literature, I use 0.005 as the signal-to-noise ratio λ_θ . Figure 3 and 5 show the estimated trends.

Whether the level or the change of productivity as regressor, unsurprisingly, (CH)GPRODA is much less volatile than (CH)GPRODB for period 1978Q4—1986Q4 because of linear approximation. And, compared with the remaining period, (CH)GPRODA is flatter whereas (CH)GPRODB is more volatile, which can be caused by imprecise proxy for labor productivity in business sector.⁴ But, on the other hand, the two productivity growth paths have similar trend, which is slightly higher around the middle of 1980's and slightly lower around 1990. Figure 4 confirms this observation more clearly. Figure 6 shows the change in productivity is increasing. During 1978Q4—1986Q4, the two trends have similar pattern, whether in levels or in changes. Considering the interest focuses on the correlation between the pattern of this trend and NAIRU, the small discrepancy should make little difference for the final estimation of NAIRU.

⁴ I compare the data constructed according to (b) during 1978Q4—1986Q4 with the available data for labor productivity in business sector, and find the former is more volatile than the latter, which supports my tentative explanation to some extent.

3.2.3 Estimating NAIRU in the productivity-augmented Model

Now I follow much of the empirical literature to assume that inflation expectations follow random walk, $\pi_t^e = \pi_{t-1}$. So the Kalman filter specification becomes:

$$\Delta\pi_t = a(L)\Delta\pi_{t-1} - b(L)(U_t - U_t^*) + c(L)X_t + \varepsilon_t \quad (15)$$

$$U_t^* = U_{t-1}^* + \beta\Delta Z_t + \eta_t \quad (16)$$

$$\text{var}(\eta_t) = \lambda \text{var}(\varepsilon_t) \quad (17)$$

where $\Delta\pi$ is first difference in inflation rate. ΔZ is the first difference in the trend of the level and the change of productivity growth, separately. As in the productivity model, the two error terms, ε_t and η_t , are assumed to be i.i.d. normal and uncorrelated.

In this state-space model, the amount of time variation in U^* is governed by the signal-to-noise ratio λ . According to Slacalek (2003), imposing a reasonable value for it is preferred to estimating it for the following reasons: (a) Since NAIRU varies slowly over time, the variance of η_t is usually very small. As a result, the estimate of $\text{var}(\eta_t)$ has bad small-sample properties—it's estimated very imprecisely with a downward bias; (b) The distribution of the signal-to-noise ratio λ has a non-zero probability mass at zero in the small sample—so called small pile-up problem. Thus I pick some reasonable values for it and estimate the other parameters in Equation (15)-(17) with MLE. The regression results are quite robust to the selection of λ .

Obviously, the sign of the four estimates of the coefficient on productivity is ambiguous and none of them is significantly different from zero. This suggests that productivity can be

excluded from the model, despite in what form it enters. The sign of the coefficient on unemployment is right, but it's not significantly different from zero, too.

Table 3 Estimation results, change vs. level in productivity⁵

	Using GPRODA		Using GPRODB	
	Level Model	Change Model	Level Model	Change Model
Sum of coeffs. On diff. between Unemployment and NAIRU	-0.019	-0.022	-0.017	-0.021
P-value on sum of diff. between Unemployment and NAIRU	0.44	0.44	0.48	0.46
P-value on lags of first diff. in Inflation	0.00	0.00	0.00	0.00
P-value on Supply Shock	0.00	0.00	0.00	0.00
Coeff. on Productivity	-1.734	0.763	-8.97	0.027
P-value on Productivity	0.94	0.94	0.81	1.00

3.3 Sensitivity Analysis

In the above subsection, productivity is shown very insignificant in estimating time-varying NAIRU. Now a series of sensitivity tests are done to check the robustness of this conclusion. The estimation results are very consistent with each other and all suggest productivity is not significant in this model.

First, the signal-to-noise ratio in equation (14), λ_θ , is changed to 0.01. Table 4 shows the estimation results are similar as in Table 3. All the coefficients on productivity are very insignificant, although they all suggest the negative sign.

⁵ The lags of inflation, unemployment and supply shock are 3, 1 and 0, respectively.

Table 4 Estimation results, change vs. level in productivity, $\lambda_\theta = 0.01$

	Using GPRODA		Using GPRODB	
	Level Model	Change Model	Level Model	Change Model
Sum of coeffs. On diff. between Unemployment and NAIRU	-0.018	-0.022	-0.018	-0.018
P-value on sum of diff. between Unemployment and NAIRU	0.48	0.49	0.55	0.53
P-value on lags of first diff. in Inflation	0.00	0.00	0.00	0.00
P-value on Supply Shock	0.00	0.00	0.00	0.00
Coeff. on Productivity	-2.584	-0.075	-11.09	-1.142
P-value on Productivity	0.91	0.99	0.77	0.93

If the productivity change is defined as a naïve one: the difference in the productivity growth between the current period and the previous period⁶, the estimation results are shown in Table 5. They are quite robust to the current conclusion.

Table 5 Estimation results in the change model with alternative measure of change in productivity

	Using GPRODA	Using GPRODB
Sum of coeffs. On diff. between Unemployment and NAIRU	-0.036	-0.042
P-value on sum of diff. between Unemployment and NAIRU	0.12	0.15
P-value on lags of first diff. in Inflation	0.00	0.00
P-value on Supply Shock	0.00	0.00
Coeff. on Productivity	-456.30	-47.59
P-value on Productivity	0.30	0.58

A notable thing is the p-value of the coefficient on productivity is much lower when GPRODA is used, and the magnitude is very large. Inspecting this estimation further, an incredible NAIRU path is generated with near zero value in some periods. It's not the case for GPRODB.

⁶ According to Equation (12) and (13), it implies that the wage aspiration and productivity growth are equal in last period.

Now different time series for inflation and unemployment, the core inflation rate (as defined by the Bank of Canada, it's the inflation rate for all-items excluding 8 most volatile components) and unemployment rate for the male 25-54 year-old, are used. (GPRODB is the measure for productivity.) As expected, when using core inflation rate, the estimate of the coefficient on the supply shock is marginally insignificant. The coefficients on productivity are still insignificant, in spite of the different signs and magnitude.

Table 6 Estimation results of alternative time series, level vs. change in productivity

	Core inflation rate		Unemployment for men 25-54	
	Level Model	Change Model	Level Model	Change Model
Sum of coeffs. On diff. between Unemployment and NAIRU	-0.02	-0.022	-0.012	-0.011
P-value on sum of diff. between Unemployment and NAIRU	0.31	0.37	0.65	0.70
P-value on lags of first diff. in Inflation	0.00	0.00	0.00	0.00
P-value on Supply Shock	0.25	0.25	0.00	0.00
Coeff. on Productivity	2.786	2.014	-11.046	-4.907
P-value on Productivity	0.91	0.88	0.83	0.86

If different lags for differenced inflation rate, unemployment rate and supply shocks are used, the coefficient on the productivity keeps insignificant⁷.

Table 7 Estimation results for different lags

Number of lags			Level Model		Change Model	
$\Delta\pi$	$U - U^*$	X	Coeff. On productivity	p-value	Coeff. On productivity	p-value
3	1	1	-4.225	0.88	0.78	0.94
4	1	0	-4.266	0.85	1.25	0.88
3	2	0	-40.32	0.41	-41.39	0.48
3	2	1	-41.94	0.52	-39.61	0.5
3	1	N/A ⁸	0.228	0.99	1.33	0.87

⁷ When the lag of unemployment is 2, negative or near zero NAIRU path is created.

⁸ It means the supply shock variable is excluded from the model now.

In a word, the result is consistent with that in the last subsection: none of the coefficients on productivity are significantly different from zero, which confirms that productivity should be excluded from the model. One possible explanation may be that during the sample period the productivity trend doesn't show any apparent variation, hence cannot significantly explain the time variation of the NAIRU. In other words, the relationship between the NAIRU and productivity is so vague that it is kind of impossible to inspire the initial research on it, just as in US, if we only look at the data in Canada. Crawford (2002) compares the labor productivity growth in Canada with US and reports from 1996-2000 that Canada's annual rate is 1.6 and US's is 2.6, whereas from 1975-1995 the former is 1.3 and the latter is 1.5. The sharp increase in the growth rate in the late 1990's didn't happen in Canada. To some degree this probably explains the poor explanatory power of productivity growth in Canada during the sample period. And to some extent, it is consistent with Gruber (2003), who finds directly adding productivity aspiration term (just like the change of productivity growth in this paper) into the Phillips curve does not seem to influence the NAIRU significantly and concludes low realized productivity growth over the 1990s prevent it from being a significant part of Canada's low inflation story. But the final conclusion is too far to reach.

4 Estimating NAIRU without productivity

Based on the conclusion in the last section, NAIRU is estimated according to Equation (13)-(15) but without productivity. Here alternative series for inflation and unemployment are used. Table 8 reports the MLE estimation results.

Table 8 Estimation results of alternative time series, without productivity

	CPI inflation	GDP deflator	Core inflation	UM 25-54
Sum of coeffs. On diff. between Unemployment and NAIRU	-0.04	-0.01	-0.02	-0.034
P-value on sum of diff. between Unemployment and NAIRU	0.21	0.75	0.42	0.16
P-value on lags of first diff. in Inflation	0.00	0.00	0.00	0.00
P-value on Supply Shock	0.00	0.00	0.92	0.00
RMSE for final state estimate	1.37	3.07	1.84	1.51

The first column is the baseline model, with CPI inflation and unemployment rate for all above 15-year-old being used. The second column uses GDP deflator as inflation rate. The third column uses core inflation rate. Figure 7 plots the three time paths of the NAIRU. All of them are flat, which suggest that Canada's NAIRU doesn't change much during 1979-2003. It's different from the experience of US's NAIRU, which is obviously declining in the late 1990's according to the literature. This further suggests that in recent years the evolutions of main factors determining the NAIRU are different in the two countries, although they are neighbors in geography. Thus this explains why productivity cannot improve the random-walk estimation results significantly whereas the US data suggest an opposite story. Especially, the NAIRU estimated by GDP deflator is the highest and its average value is about 8.1. Core inflation produces the lowest estimates of NAIRU,

the average of which is about 7.2. The average of the result estimated by CPI inflation is about 7.6. As expected, the supply shock, as defined in Section 3.1, becomes insignificant when core inflation is used.

The last column uses unemployment rate for all male with age 25-54 year-old. The coefficient estimation results are quite robust to different measures of unemployment. These two paths of NAIRU are plotted in Figure 8. Consistent with the intuition, the NAIRU produced by UM is apparently lower than that produced by U and the average difference is 0.54 percent.

Another notable thing is the Root Mean Square Error is quite higher when using GDP deflator than using other measures, which suggest GDP deflator may not be a good alternative for inflation rate when estimating the NAIRU.

The estimated NAIRU is also robust to the choice of signal-to-noise ratio. Figure 9 shows it makes little difference to choose 0.01, 0.05, 0.1 or 0.2 as λ . Generally speaking, the NAIRUs don't change much during the sample period, although higher signal-to-noise ratio gives more variation and to some extent suggests modest decline tendency recently compared with the beginning of the sample period.

5 Conclusion

Productivity growth, whether in levels or in changes, cannot improve the estimate of Canada's NAIRU to any significant degree during the sample period 1978Q4-2003Q3. A tentative explanation is this may be caused by difference experience in the development of productivity from US. The variation of the productivity growth in Canada provides little power to explain the time variation in NAIRU. Furthermore, the estimate is quite stable, between 7-8 percent, no matter which inflation rate measure, unemployment rate measure and signal-to-noise ratio are used. Seen in another light, these imply the main determining factors of NAIRU, such as demographic composition in the labor market, productivity growth and so on, did not change much during this period. The high variation in the unemployment rate is temporary. This may help explain why few published works on the determination of Canada's NAIRU exist. Additionally, the most recent data suggest the actual unemployment is near the natural rate.

References

- Aghion, Philippe and Peter Howitt (1994), Growth and Unemployment, *The Review of Economic Studies*, Vol. 61, No.3, pp. 477-494
- Atkeson, Andrew and Lee E. Ohanian (2001), Are Phillips Curves useful for Forecasting Inflation? *Federal Reserve Bank of Minneapolis Quarterly Review* 25, pp. 2-11
- Ball, Laurence and N. Gregory Mankiw (2002), The NAIRU in Theory and Practice, NBER Working Paper 8940, available at: <http://www.nber.org/papers/w8940>, accessed: October 05, 2003
- Ball, Laurence and Robert Moffitt (2001), Productivity Growth and the Phillips Curve, in *The Roaring Nineties: Can Full Employment be sustained?* Edited by Alan B. Krueger and Robert Solow, New York: The Russell Sage Foundation and The Century Foundation Press
- Beaudry, Paul and Matthew Doyle (2000), What happened to the Phillips Curve in the 1990s in Canada? Available at: <http://www.bankofcanada.ca/publications/working.papers/2000/beaudry-final.pdf>, accessed: October 23, 2003
- Crawford, Allan (2002), Trends in Productivity Growth in Canada, *Bank of Canada Review (Spring 2002)*, pp. 19-32
- Franz, Wolfgang (2003), Will the (German) NAIRU Please Stand up? Discussion Paper No. 03-35, Center for European Economic Research, available at <http://www.eabcn.org/research/documents/franz03.pdf>, accessed: September 20, 2003
- Gordon, Robert J. (1997), The Time-Varying NAIRU and its Implications for Economic Policy, *The Journal of Economic Perspectives* 11, pp. 11-32
- Gruber, Joseph W. (2003), Productivity Growth and the Phillips Curve in Canada, International Finance Discussion Papers, Board of Governors of the Federal Reserve System, available at <http://www.federalreserve.gov/pubs/ifdp/2003/787/ifdp787.pdf>, accessed: January 11, 2004
- Harvey, Andrew C. (1989), Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press
- Ireland, Peter N. (1999), Does the Time-Consistency Problem Explain the Behavior of Inflation in the U.S.? *Journal of Monetary Economics* 44, pp. 279-291
- Longworth, David (2002), Inflation and the Macroeconomy: Changes from the 1980s to the 1990s, *Bank of Canada Review (Spring 2002)*, pp. 3-18

- Maddala, G.S. and In-Moo Kim (1998), Unit Roots, Cointegration and Structural Change, Cambridge University Press
- Modigliani, Franco and Lucas Papademos (1975), "Targets for Monetary Policy in the Coming Year." *Brookings Papers on Economic Activity* 1, pp. 141-63.
- Mortensen, Dale T. and Christopher A. Pissarides (1997), Technological Progress, Job Creation, and Job Destruction, *Review of Economic Dynamics* 1, pp. 733-753
- Richardson, Pete, Laurence Boone, Claude Giorno, Mara Meacci, David Rae and David Turner (2000), The Concept, Policy Use and Measurement of Structural Unemployment: Estimating A Time Varying NAIRU Across 21 OECD Countries, *OECD Economics Department Working Papers No. 250*, available at: www.oecd.org/eco/eco, accessed: September 26, 2003
- Slacalek, Jiri (2003), Productivity and the Natural Rate of Unemployment, available at <http://www.econ.jhu.edu/people/slacalek/research/prodNairu.pdf>, accessed: November 20, 2003
- Staiger, Douglas; James H. Stock and Mark W. Watson (1997a), The NAIRU, Unemployment and Monetary Policy, *The Journal of Economic Perspectives* 11, pp. 33-49
- Staiger, Douglas; James H. Stock and Mark W. Watson (1997b), How Precise Are Estimates of the Natural Rate of Unemployment? In *Reducing Inflation: Motivation and Strategy*, edited by Christina D. Romer and David H. Romer, Chicago: University of Chicago Press, pp. 195-242
- Stock, James H. and Mark W. Watson (1999), Forecasting Inflation, *Journal of Monetary Economics* 44, pp. 293-335

Figures

Figure 1 The level of productivity growth and the unemployment rate, HP filter

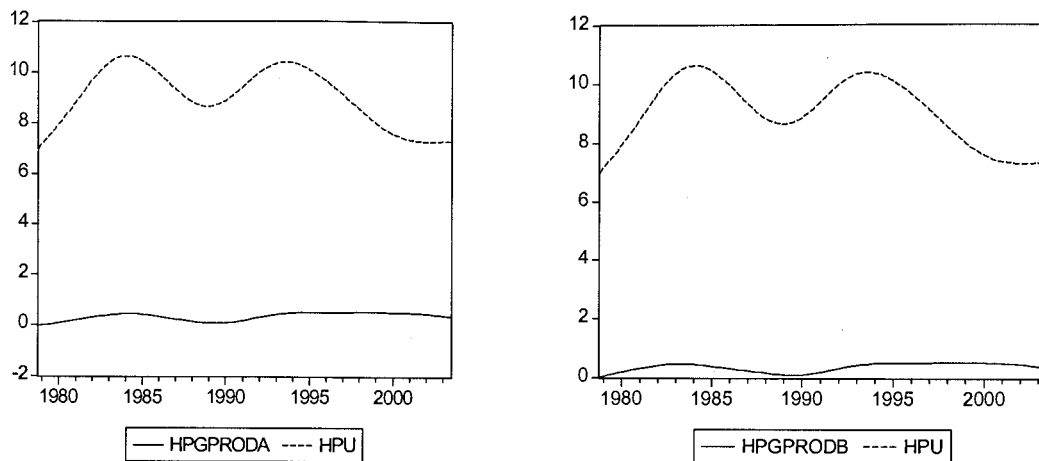


Figure 2 The change of productivity growth and the unemployment rate, HP filter

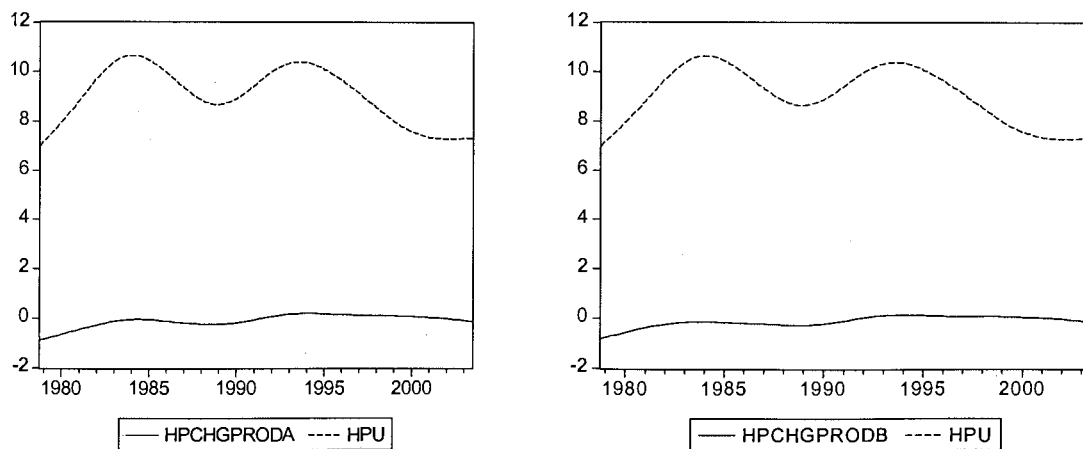


Figure 3 Productivity growth trend, Kalman filter

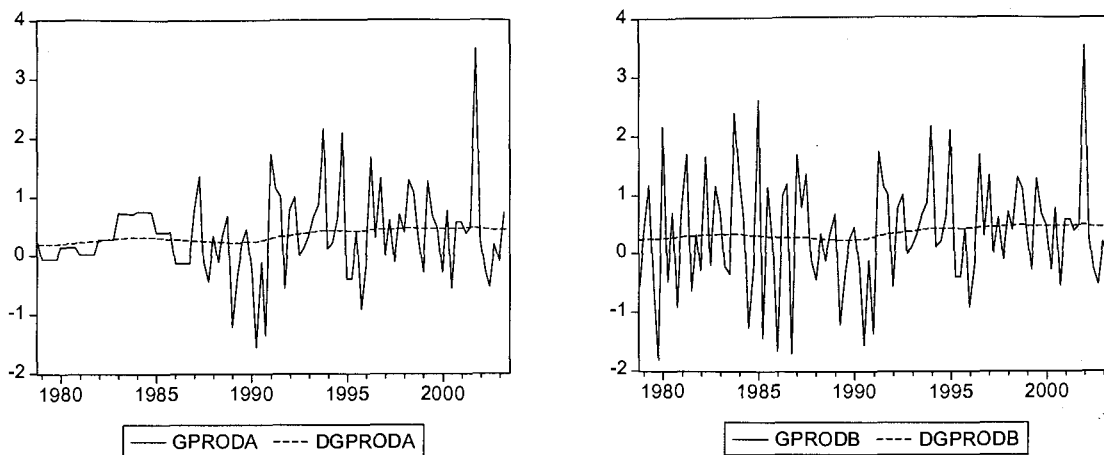


Figure 4 Comparison of two productivity growth trends, Kalman filter

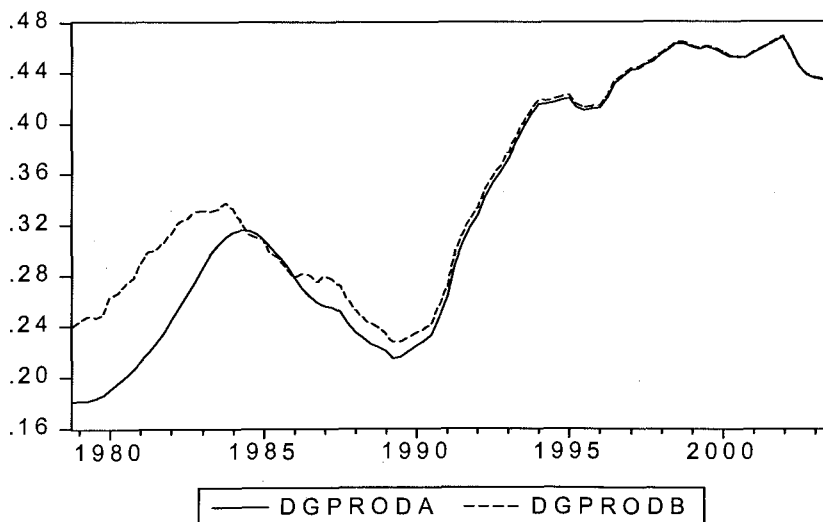


Figure 5 The trend in the change of productivity growth, Kalman filter

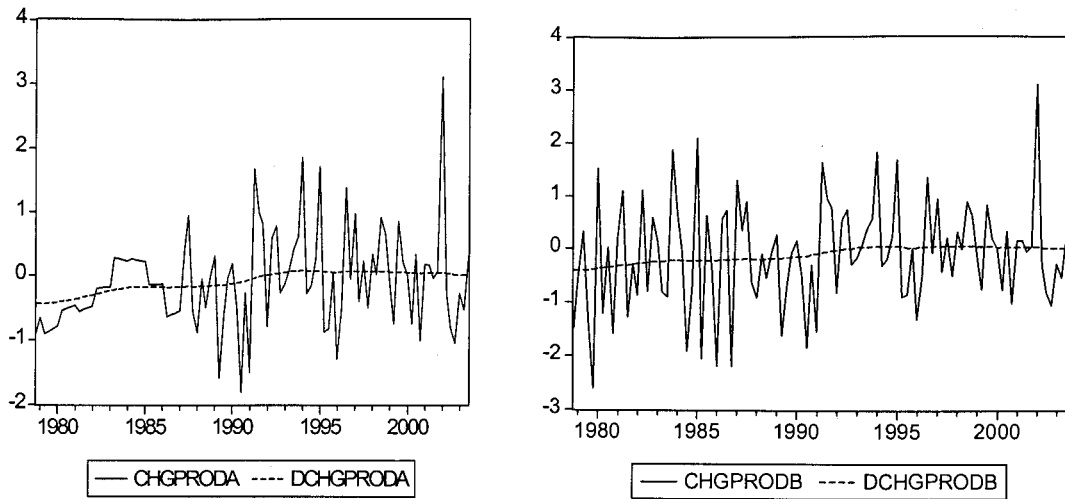


Figure 6 Comparison of two trends in the change of productivity growth, Kalman filter

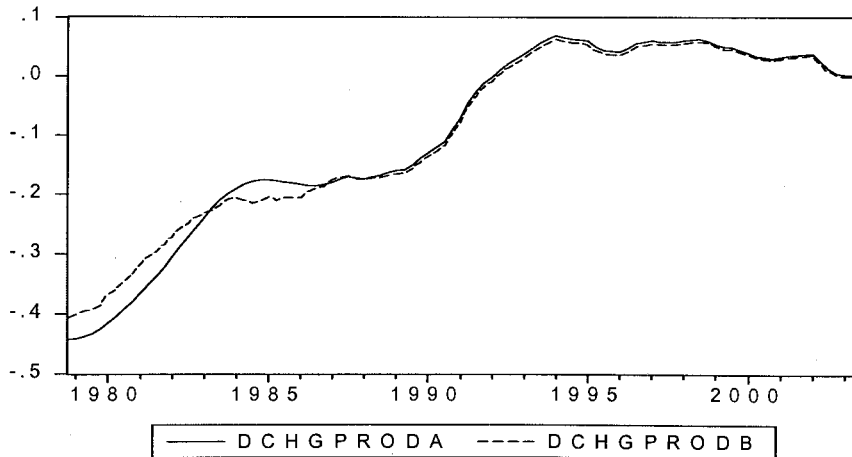


Figure 7 Time paths of NAIRU using different inflation rates

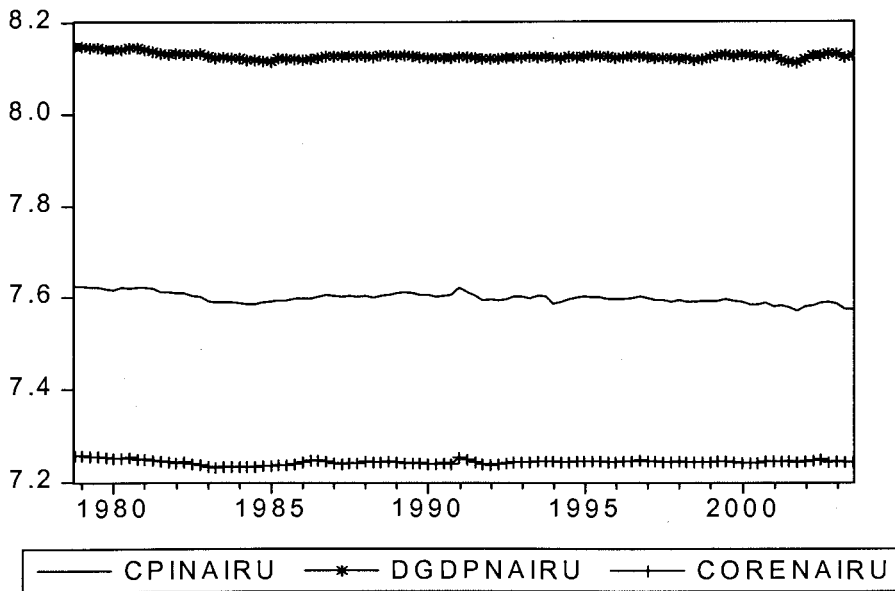
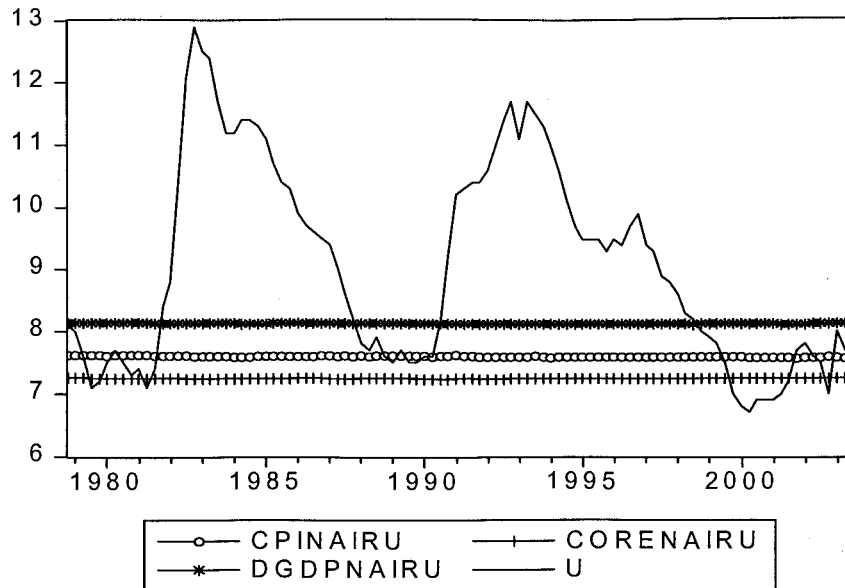


Figure 8 Time paths of NAIRU using different unemployment rates

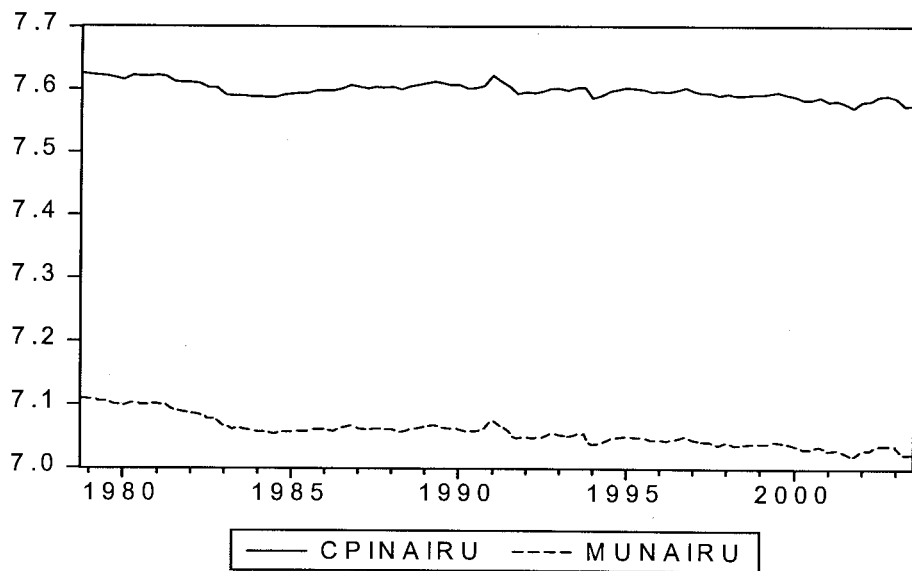
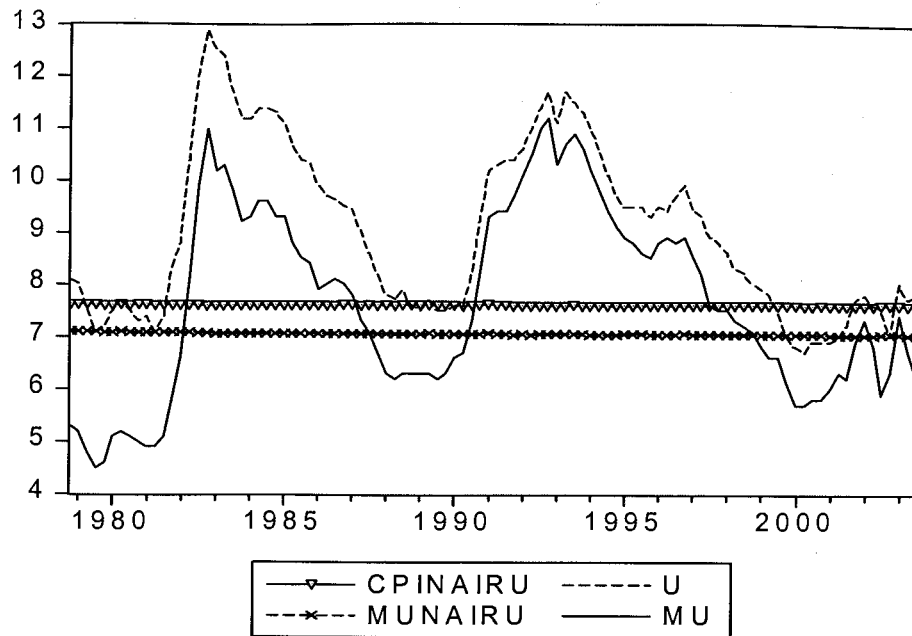


Figure 9 Time paths of NAIRU using different signal-to-noise ratio

