A DYNAMIC GENERAL EQUILIBRIUM MODEL OF THE TERM STRUCTURE OF INTEREST RATES

by

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ABSTRACT

The study of the term structure interest rates is an important area of research by academics and practitioners because it contains valuable information about the future expected growth of an economy. Most studies have modelled the term structure of interest rates using static equilibrium or econometric models. Thus, the purpose of this paper will be to replicate the term structure of interest rates using a dynamic general equilibrium model. The paper will also compare the accuracy of the yield curve from the simulated economy with the real world.

DEDICATION

To my family and Amy,

Thank you for supporting me through the good and the bad.

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1. INTRODUCTION

The term structure of interest rates analyzes the relationship between the yields on risk-free securities that differ only by maturity. This relationship has been an important area of research for economist because the term structure contains important information about expected economic growth. For instance, over a 25 year period, from 1969-1994, the term structure of interest rates correctly predicted all economic booms and recessions in the United States (Harvey, 1993). In addition, the term structure also provided information about the severity of each economic downturn, or upturn, through the magnitude of its inversion.

In most writings, the term structure of interest rates has been modelled primarily through static equilibrium or econometric forecasting models. Research utilizing the dynamic general equilibrium framework has not been thoroughly explored. Thus, the purpose of this paper will be to investigate how a general equilibrium model of the term structure of interest rates performs in a simulated economy compared to the real world Canadian economy. Furthermore, this paper will analyze how changes in the technology parameter can influence the term structure over the business cycle.

1.1 Term Structure of Interest Rates

The term structure of interest rate analyzes how the differences in long and short-term interest can provide insight into future economic market conditions. On average, short-term interest rates are lower than long-term interest rates (Bank of Canada, 2000). In this specific case, the yield curve is upward sloping. The yield curve, however, has been shown to change or invert over time. These changes have been linked to different stages of the business cycle.

The term structure theory is driven by the agents' expected consumption patterns. The theory also assumes that individuals like consumption smoothing and bonds because they allow trade between current consumption and future consumption. Therefore, if individuals receive news that future economic growth is going to be positive, they will have incentive to sell long-term bonds and buy short-term bonds. The resulting demand change will bid up (down) the price of the short (long) – term bond and decrease (increase) its yield. To signal an economic recovery, the yield curve's slope should be positive or become steeper. Likewise, an economic recession could also be forecasted by the yield curve. If agents believe that expected economic growth is low, they will demand long-term bonds and sell short-term bonds. This action allows them to transfer consumption into the recessionary period. The demand change will bid up (down) the price of the long (short) – term bond and decrease (increase) its yield. Thus, the yield curve should signal a period of recession by inverting or flattening out.

2. MODEL

The model used is a modified version of Lucas' (1978) asset pricing model and Ljungqvist and Sargent's (2001) term structure model. In the model, time is assumed to be discrete and the horizon is infinite. In addition, the model consists of an economy with a group of identical individuals with preferences represented by the following expression:

$$(1) \qquad E_t \ \Sigma \beta^t \ U(C_{t+j})$$

where C_t is the individuals consumption at time t, β is the individuals rate of time preference, and E_t represents the mathematical expectation conditional on information at time t. The utility function is strictly increasing, concave, and twice differentiable. Also, β is assumed to lie between zero and one.

Production in the economy will be generated through a Lucas tree. Each individual starts off at time zero with one tree, which generates output of y_t at the beginning of each period t. The tree is assumed to be durable good. The output generated by the tree, however, will be non-storable. The sequence of output generated by the tree will be regulated by a Markov process with a time-invariant transition probability distribution function. This process is represented as:

$$prob \{ y_{t+1} < y' \mid y_t = y \} = F(y', y)$$

In the model, individuals' choices will be restricted to either consuming current output (yt) at time t or investing in two different risk-free bonds with different maturity dates. One bond will have a maturity date of one period and the other will have a maturity date of two periods. Due to the fact that agents prefer consumption smoothing, individuals will invest in bonds when output is high in order to compensate for low consumption when output is low. Therefore, an individual's budget constraint and "the law of motion for wealth" is the following (Ljungqvist and Sargent, 2001):

$$(2) \qquad C_t + B_{1t} / R_{1t} + B_{2t} / R_{2t} \leq W_t$$

$$(3) \qquad W_{t+1} = B_{1t} + B_{2t} / R_{1t+1} + y_{t+1}$$

where Bjt is gross earnings payout on all zero coupon bond, Rjt is the risk-free real gross return between periods t and t+j, and Wt denotes the wealth that and individual has at time t. The subscript j represents the date of maturity and subscript t represents an agent's decision to hold the security between period t and t+j for both B and R.

It is important to note that even though the bonds guarantee a certain return at its maturity date, there is still uncertainty in their price before maturity. For instance, a two period bond B_{2t} is sold at price R_{1t-1} in period t+1. If an individual plans to buy this bond at time t and sell it at time t+1, the gain from this transaction will be uncertain because R_{1t-1} is unknown at time t (Ljungqvist and Sargent, 2001).

2.1 Equilibrium

An agent's choice problem will be to maximize his utility (equation 1) subject to equations (2) and (3) and the stochastic output process. If we let $V(W_t, y_t)$ be the optimal value of maximizing the individual's utility function subject to its constraints, then the following Bellman equation can be written:

$$(4)\ \underset{B_{1t},\, B_{2t}}{Max}\ U \quad \{(W_t - B_{1t}/R_{1t} - B_{2t}/R_{2t}) + \beta E_t\, V(B_{1t} + B_{2t}/R_{1t+1} + y_{t+1},\, y_{t+1})\}$$

If $\phi_S = 1 / R_{1t}$ (the price of a short bond) and $\phi_L = 1 / R_{2t}$ (the price of a long bond). Then, the first order conditions with respect to B_{1t} and B_{2t} are:

$$(5) \qquad \phi_s\, U'(C_t) = \beta E_t\, V_1(W_{t+1},\, y_{t+1})$$

$$(6) \qquad \phi_L\, U'(C_t) = \beta E_t\, V_1(W_{t+1},\, y_{t+1})$$

Next, the market clearing conditions ($y_t = C_t$) will be invoked. (Note, the market clearing conditions imply $B_1 = B_2 = 0$) In addition, a logarithmic utility function will be used. Both of these conditions allow us to derive the equilibrium rates of return for both the long and short bonds:

$$(7) \qquad \phi_S^*\ (y_t) = \beta E_t\, [(y_t)(y_{t+1})^{-1}]$$

$$(8) \qquad \phi_L^*\ (y_t) = \beta^2 E_t\, [(y_t)(y_{t+2})^{-1}]$$

The equilibrium interest rates of each bond will be the inverse of equations (7) and (8).

3. CALIBRATION

The model is calibrated in order to closely replicate the Canadian economy from 1961-2000. (All data sources used to calibrate the model can be found in Appendix I) The parameter β is set to 0.96, which is concurrent with most literature that utilizes the representative agent model (Scarth, 1996).

The sequence of output that an individual receives over his life can be shown by:

$$y_{t+1} = (1-\rho)\, y_{avg} + \rho y_t + \varepsilon_{t+1}$$

where ε_{t+1} is normally distributed with mean zero and variance σ^2. The parameter row measures the persistence of output over time. In the model, the parameter rho is set to 0.85, which is the same value observed in the real world data. The rho parameter estimate is feasible because it avoids a random walk process, while allowing for some persistence in output. The probabilities of the distinct set of output values are each set to 50 percent.

In empirical literature, the term structure of interest rates is also used to forecast economic growth (Harvey, 1997). Papers on the term structure written by Campbell Harvey have utilized the following forecasting model:

$$(9) \qquad Growth_{t+1,t+5} = \alpha + b(YIELDSPREAD_t) + u_t$$

where growth is the annual real growth in GDP, $YIELDSPREAD_t$ is the difference between the long and short term yields to maturity, and u_t is the error term. The YIELDSPREAD is calculated by the following formula: ln ((1 + long i-rate) / (1 + short i-rate)). The coefficient α refers to the average annual GDP growth in the economy when the slope of the yield curve is zero and coefficient b denotes the average risk tolerance in the economy. Harvey found that with his model, the term structure provided a perfect forecast of annual real economic growth in the United States and Canada. More than 40 percent of the variation in economic growth between 1976 and 1989 could be explained by the yield curve (Harvey, 1997). The same forecasting model is estimated using the data obtained from the simulated economy.

4. EQUILIBRIUM VERSUS FACTUAL ANALYSIS

This section analyzes how equilibrium bond prices and yields fluctuate over the business cycle in the simulated economy and the Canadian economy. (from 1961 to 2000) The variations in output are viewed as technology shocks to the production process. For instance, when output increases from one period to the next, it indicates a positive technology shock. The results for the simulated economy are shown in Figure 1:

FIGURE 1

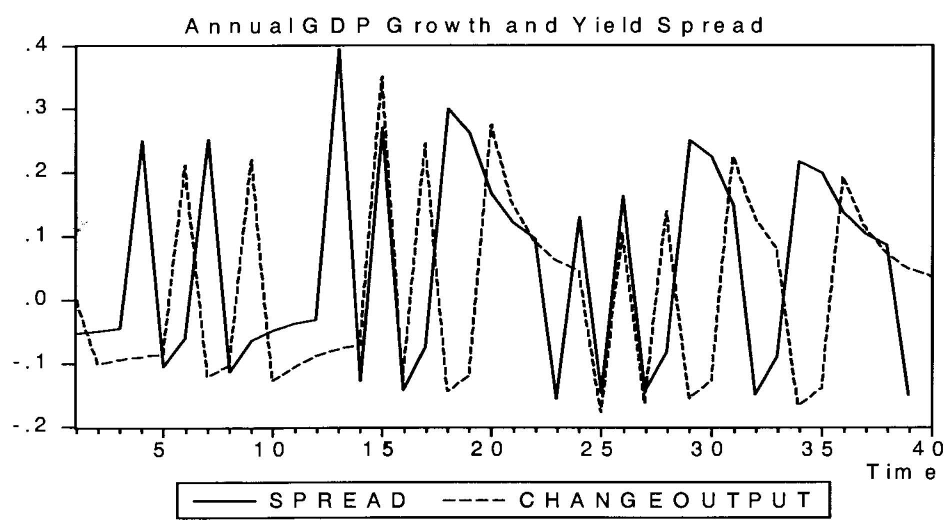
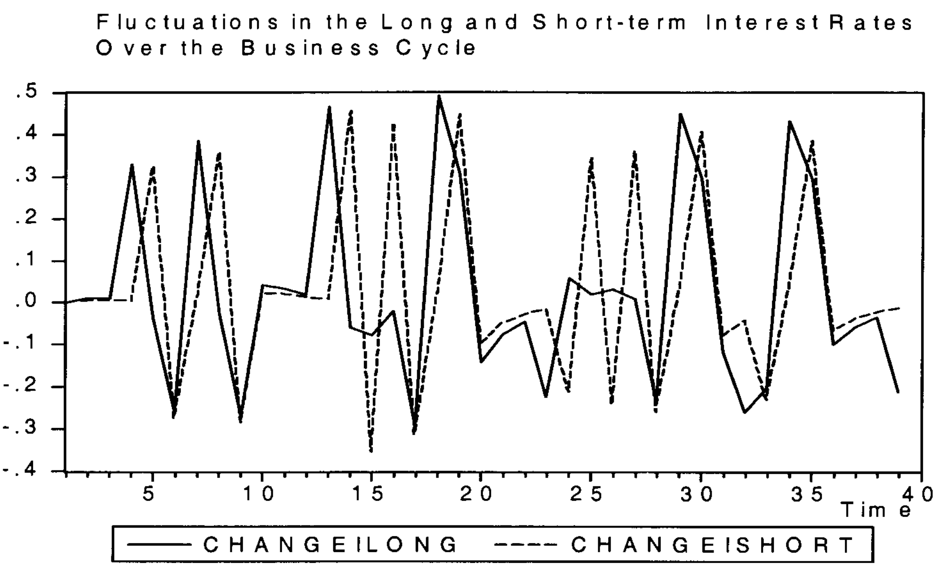


Figure 1 shows that the yield spread forecasts economic growth. It is quite clear that the yield curve leads and moves with GDP growth. On average, the yield spread correctly forecasts economic growth one to two periods in advance. Based on these results, it can be stated that a negative technology shock will cause a decrease in GDP; this in turn causes an inversion of the yield curve. The process can be shown through analyzing fluctuations in the short and long-term interest rates throughout the business cycle. The fluctuations are depicted in Figure 2.

FIGURE 2



As mentioned previously, a negative technology shock causes an inversion of the yield spread. The inversion occurs because a negative productivity shock causes a decrease in current output, which induces individuals to revise their expectations about the future. Thus, if future output is expected to be lower, individuals will revise their financial portfolios to include more long-term bonds and less short-term bonds. The positive (negative) demand shock for long (short) term bonds will bid up (down) the price and decrease (increase) its yield. As a result, the yield curve will invert and forecast an economic recession. The opposite process occurs when there is a positive technology shock to the production process.

Another important feature of the Figure 2 is that the short-term interest rates exhibit more variation over the business cycle then long-term interest rates. The volatility in the short-term interest rates is most apparent over periods 14 to 17 and, periods 24 to 28, where the short-term interest rate exhibits large fluctuations over the long-term interest rates. The increased volatility is likely due to the high rate of time preference that individuals possess. For instance, a lower rate of time preference corresponds to a higher preference toward long-term securities. In this situation, the demand for long-term securities is more stable because individuals will not switch away from them as much as short-term securities.

This paper will now analyze the term structure exhibited in Canadian economy between 1961-2000. Figure 3 summarizes the yield spread and annual real economic growth in the Canadian economy during the years 1961 to 2000. It is important to note that the ex-post interest rates are used.

FIGURE 3

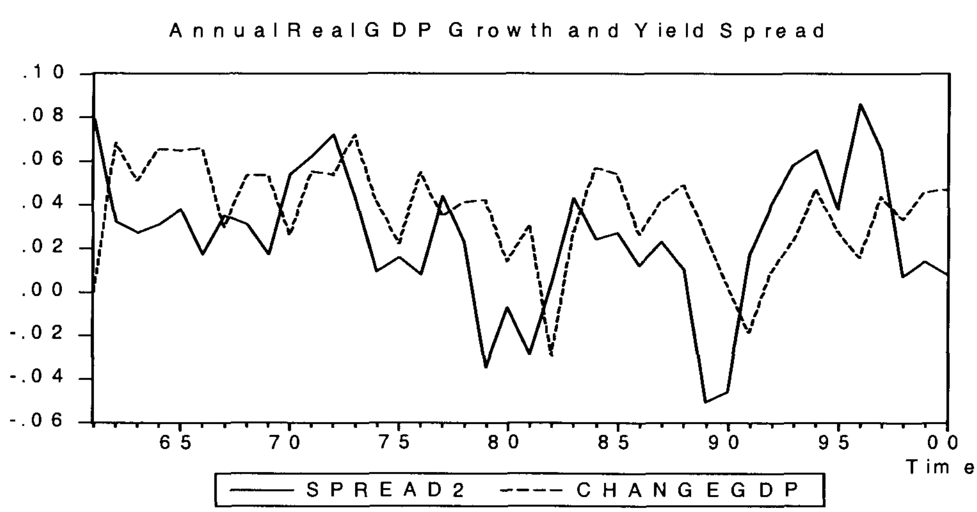


Figure 3 shows that the yield spread in the Canadian economy demonstrates many similar characteristics to the simulated economy. Like the simulated economy, the yield spread in the Canadian economy leads and moves with GDP growth. Each economic boom or recession is correctly predicted by the yield spread one to two periods in advance. The interest rate fluctuations throughout the Canadian economy are shown by Figure 4:

FIGURE 4

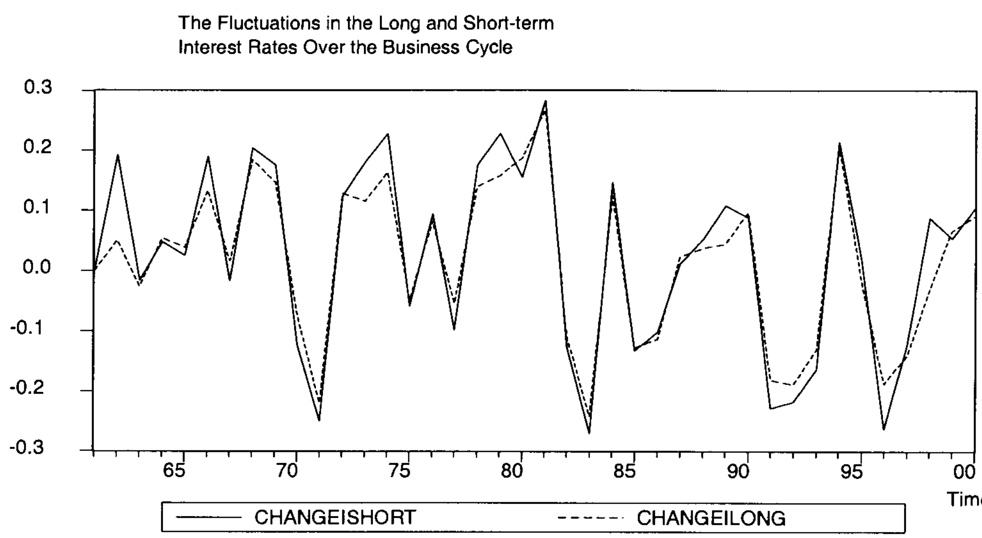


Figure 4 shows that both the long and short-term interest rates move together over business cycles. The two interest rates share a correlation coefficient of 0.97 between them. There exists a positive correlation between the two interest rates because even though the bonds differ in maturity dates, they both offer a way to transfer consumption between the good and bad states of the world. Therefore, an increase in demand for future consumption, resulting from low expected future output, will affect both securities. In addition, the Canadian economy also demonstrates higher volatility in its short-term interest rate.

There are many similarities between the results from the simulated economy and the Canadian economy. The most important characteristic analyzed, however, was the predictive power of the yield spread for each respective economy. Based on our results, we cannot state that one yield spread dominates the other in performance. Both yield spreads were successful in predicting the expected economic conditions within one to two periods, on average.

5. FORECASTING MODEL

This section of the paper will analyze how Harvey's (1993) econometric forecasting model performs when applied to our simulated economy. Table 1 displays the estimation results by running Harvey's forecast model on the simulated data:

TABLE 1: Simulated Estimation Results

Dependent Variable: GROWTH Method: Least Squares				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
α	0.033	0.167	1.99	0.054
SPREAD	0.297	0.077	3.87	0.000
R-squared	0.288			
Adjusted R-squared	0.269			

The estimation results show that 28.8 percent of the variation in annual GDP growth can be explained by the variation in the yield spread. The spread variable has significant effect on GDP at a five percent significance level. These results are similar to the one's Harvey (1997) obtained in his study. (shown in TABLE 2)

TABLE 2: Harvey's Estimation Results

Dependent Variable: GROWTH Method: Least Squares				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
A	0.020	0.004	5.17	N/a
SPREAD	1.294	0.224	5.76	N/a
R-squared	0.304			
Adjusted R-squared	0.294			

Using Harvey' model, approximately 30 percent of annual real GDP growth can be explained by the yield curve. Therefore, the simulated model contains roughly as much explanatory power as Harvey's model.

The estimation results from the simulated economy can be used to create a forecast of GDP growth with the yield spread. The forecasting can be accomplished by substituting the yield spread observed in the real world into the estimated equation. (outlined by equation 9) For instance if the yield spread is negative 0.001, then the predicted increase in GDP growth is 3.33 percent (shown by equation 10).

$$(10) \quad 0.0333393 + (0.297190*-0.001) = 0.0333$$

The out of sample forecast results are shown in Figure 5.

FIGURE 5

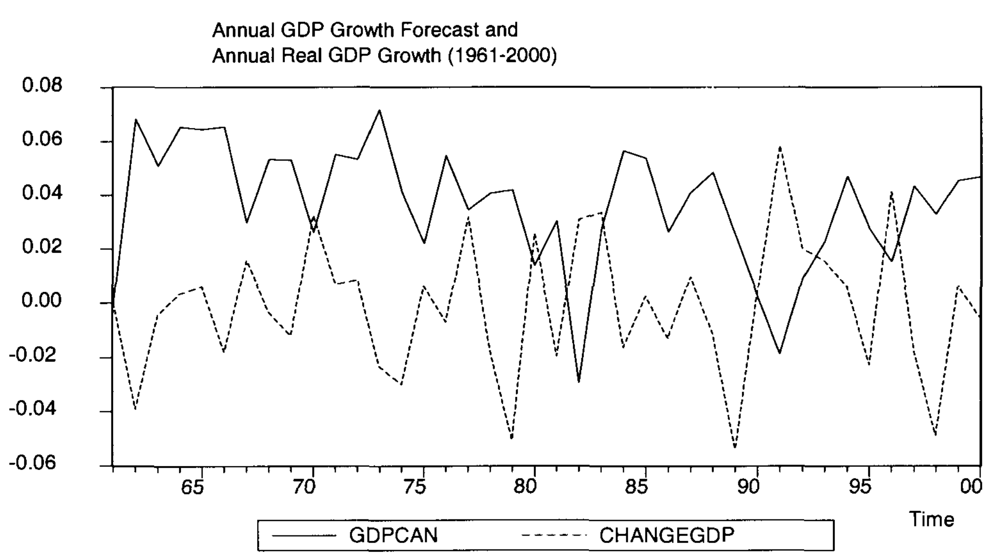


Figure 5 shows that the GDP growth predicted by the forecasting model is not very accurate. The GDP growth predicted by the model appears to be a step ahead of actual GDP growth. The simulated GDP forecast obtained a root mean squared error of 0.03, while Harvey's model obtained a root mean square error of 0.02. This suggests that Harvey's model was more accurate. Possible reasons that my forecast performed more poorly than Harvey's include the exclusion of certain variables that affect GDP growth such as: capital accumulation, monetary policy, inflation and fiscal policy. Nevertheless, in spite of this error, the yield curve still provides valuable insight into the future economic conditions because of its systematic and accurate predictions of future GDP growth.

6. CONCLUSION

In this paper, a general equilibrium model of the term structure of interest rates was constructed to analyze the behaviour of the yield curve over the business cycle. The model was calibrated to replicate the Canadian economy from 1961-2000. When simulated, the model generated results, which were consistent with the term structure theory and quite similar to the real world data. The simulated yield curve was able to predict economic growth on average 1 to 2 periods in advance. Other results include an out of sample forecast of GDP through a forecasting model. The results of the forecast were not very accurate. Possible explanations for the inaccurate forecast may be the omission of other variables related to GDP growth in the forecasting model such as monetary or fiscal policy.

Therefore, analyzing the effects of the yield curve over an economy's business cycle has provided valuable insight into expected economic growth. This information about future economic conditions can be extremely useful for firms, governments, and individuals within the economy. The changes in the yield curve may help them in making better decisions about future plans or policies.

APPENDIX: DATA SOURCE (CANSIM)

Label	B14009
Title	Selected Cda.Bond Yields & Other Int.R. / Govt. of Canada Bond Yield Average - 1 To 3 Years
Subtitle	Selected Canadian Bond Yields & Other Interest Rates, Percent (Series 63, 64, 65, 66: Millions of Dollars)
Factor	Unscaled
Unit	Percent
Source	Bank of Canada
Update	01 June, 2002
Period	1949 - 2002
Frequency	Annual

Label	D15689
Title	G.D.P., Expenditure-Based / Gross Domestic Product at Market Prices
Subtitle	Gross Domestic Product, Expenditure-Based, In Millions of Current Dollars
Factor	Million
Unit	Dollars
Source	Sdds 1901 Stc 13-001
Update	01 June, 2002
Period	1961 – 2002
Frequency	Annual

Label	B14010
Title	Selected Cda.Bond Yields & Other Int.R. / Govt. of Canada Bond Yield Average - 3 To 5 Years
Subtitle	Selected Canadian Bond Yields & Other Interest Rates, Percent (Series 63, 64, 65, 66: Millions Of Dollars)
Factor	Unscaled
Unit	Percent
Source	Bank Of Canada
Update	01 June, 2002
Period	1951 – 2002
Frequency	Annual

Label	P100000 CPI, 1996 Class, Cda, 1992=100, Monthly /
Title	All-Items
Subtitle	Consumer Price Indexes for Canada, Monthly, 1996 Classification, (1992=100 Unless Otherwise Specified)
Factor	Unscaled
Unit	(1992=100)
Source	Sdds 2301 Stc (62-001;62-010)
Update	01 June, 2002
Period	1914 – 2002
Frequency	Annual

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