The Formation of Political Parties:

A Coalition Formation Approach

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Abstract

In this model the level of a public good is determined by majority voting. A set of individuals is free to form political parties. Majority political parties are formed to acquire power and maximize the payoffs of their members. Using a three-individual model, this paper shows that although the grand party maximizes the total payoff, it fails as an equilibrium outcome. Given that, individuals with similar preferences for the public good might be expected to form a party. However, in the three-individual model, parties of individuals with different preferences form equilibrium parties.

To my parents

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1 INTRODUCTION

Economists in many fields have begun to use formal models of coalition formation to model the formation of groups. The formation of governments or trading blocs by countries such as the EU and NAFTA are examples of group formation, both of which are fundamentally important to economics[1]. There are, of course, many other examples across a broad spectrum of economic disciplines.

Consider a set of individuals who have preferences defined over a jointly consumed public good and a private good. Imagine that the level of provision of the public good is determined by majority voting. The economy in this model will be assumed sufficiently simple (a single issue and single-peaked preferences) so that the standard voting equilibrium of the most preferred public good level of the median voter is a Condorcet winner. In such a circumstance it is well understood that unless the mean willingness to pay for the public good equals the median voter's willingness to pay at the median voter's most preferred level then the voting equilibrium is Pareto inefficient. This paper will focus on such Pareto inefficient voting outcomes.

In such a circumstance it is, of course, interesting to allow for changes in the institutional structure which would allow the individuals to exploit the gains from trade (Pareto improvements). Various authors such as Varian (1992) and Tullock (1998) have considered vote buying, bribery, vote trading and log rolling. In this paper individuals are allowed to fully exploit the gains from trade by forming a

[1] Burbidge, DePater, Myers, and Sengupta (1997)

political party. Joining a political party will be defined as a binding agreement to vote for the public good level which maximizes the total utility of the party. This can be thought of as the party discipline as in the Canadian Parliamentary system. The party will then be assumed free to divide that total utility among party members through side-payments.

Only a handful of papers explore endogenous formation of parties. In related literature Levy (2002) finds that people with similar preferences are likely to form parties in a model features a single major social cleavage. His model has two phases of analysis. The first phase assumes that the players are organized in parties and choose platforms on which the voters vote sincerely. The second phase determines which are the platform equilibria and the parties structures that are stable. This procedure allows to find which parties arise endogenously in equilibria and to compare the political outcomes in stable party structures to the ones in the structure in which parties do not exist, i.e. singleton. Poutvaara (2002) finds that increased participation by extremist potential party activists leads to more divergent platforms in a model where party members derive utility from party platforms and not from final policy outcome.

A non-cooperative model of coalition formation borrowed from Myers and Sengupta (2002) is used to determine an equilibrium political party structure. Following Myers and Sengupta (2002), there are three stages to the game of coalition (party) formation. The first stage is the party constitution-setting stage where party founders propose a division of resources amongst party members in trying to establish the political party. The second stage is the party-formation stage where individuals, given the constitutional proposals from the first stage, choose parties. In the third stage, the voting stage, existing parties vote for public good provision levels.

The first result is that the grand party (party of the whole) is not an equilibrium party structure with more than two individuals. The grand party uniquely generates the efficient amount of the public good and, thus, the highest aggregate payoff. In addition, the founder of the grand party is free to divide those larger aggregate resources freely. Though favorable results occur, it fails to yield the equilibrium outcome. "*The logic is that while there are allocations in the grand federation (party) which strictly Pareto dominate the payoffs in any single alternative federation (party) structure, there may be no allocation that constitutes a simultaneous Pareto improvement over allocations associated with every alternative federation (party) structure*[2]".

Now imagine a simple environment where there are three individuals, with individuals 1 and 2 identical in terms of their preferences for the provision of the public good. Given this environment the second result is that a party of two *heterogeneous* individuals (e.g. individuals 1 and 3) with the third person (e.g. individual 2) excluded from the party is an equilibrium outcome. The result is initially surprising, but the intuition can be explained. In the singleton party structure (where everyone is alone in their own party where all decisions are independent) one of the identical individuals is the median voter. Thus, there is no surplus (gains from trade) for

2 See Myers and Sengupta (2002) pp. 7

individuals 1 and 2 in forming a party. Because there is no surplus for a party of 1 and 2, the founder of this party has no degrees of freedom in dividing the resources of the party amongst the individuals. On the other hand, there are gains from trade for individuals i and 3 for $i = 1, 2$ in forming a party of i and 3 and moving the level of public good closer to the most preferred level for individual 3. These gains from trade and the ability of the party founder to divide those gains freely is what facilitates the formation of a party of unlike individuals in equilibrium.

The paper is organized as follows. Section 2.1 outlines the three-individual model and section 2.2 briefly explains the three stages leading to coalition formation. Section 3.1 shows that the grand party cannot be supported as a coalition proof Nash equilibrium, and section 3.2 finds that there exists at least one constitutional plan that leads to a coalition proof Nash equilibrium structure of two unlike individuals. Section 4 concludes.

2 THE MODEL

2.1 The Underlying Economy

There is a set of three individuals $N = \{1, 2, 3\}$ with an individual indexed by i. Each individual has a quasi-linear utility function over a non-depletable and non-excludable publicly provided good G, and a private good x_i given by $U_i = x_i + b_i \ln G$. The marginal rate of substitution is $MRS_{GX} = b_i/G$. Note that preferences are single-peaked, i.e. $\frac{\partial MRS_{GX}}{\partial G} < 0$. The parameter b_i can be interpreted as the strength of preference of the individual for the public good. Let $0 < b_1 \leq b_2 \leq b_3$ without loss of generality.

Each individual is initially endowed with an amount of private good w_i. Assume one unit of private good can be transformed into one unit of public good or $MRT_{Gx} = 1$. Using the Samuelson condition the Pareto efficient level of public good provision, $G^* = b_1 + b_2 + b_3$.

The level of provision of the public good is determined by majority voting. The cost of providing the good is equally divided among all individuals. In other words, each individual pays a tax equal to $1/3$ for each unit of public good in this 3-individual model. The most preferred level of public good provision for individual i is determined by the G where the individual's marginal rate of substitution equals $1/3$ or $G_i = 3b_i$. In this simple environment, where there is a single issue and preferences are single-peaked, the Condorcet winner is the most preferred G of the median voter or $G^e = 3b_2$[3]. If the mean strength of preference does not equal the median, $(b_1 + b_2 + b_3)/3 \neq b_2$, then the voting equilibrium is Pareto inefficient. I shall focus on inefficient cases.

In such a circumstance it is, of course, interesting to allow for changes in the institutional structure which would allow the individuals to exploit the gains from trade (Pareto improvements) by any means such as vote buying, bribery, vote trading and log rolling. In this paper individuals are allowed to fully exploit the gains from trade by forming a political party. Joining a political party will be defined as a binding agreement to vote for the G which maximizes the total utility of the party.

[3] On a pairwise vote between G^e and any other level of G a strict majority would prefer G^e. I will allow for coordination on votes amongst individuals below.

This can be thought of as the party discipline as in the Canadian Parliamentary system. Parties of more than one individual are majority parties here because there are only three individuals. The party will then be free to divide that total utility among party members. A non-cooperative model of group or coalition formation is used to determine an equilibrium political party structure with side-payments.

The first step in modeling this is to determine payoffs for each individual within each possible party. Some essential terminologies are defined here.

A party or coalition S is a nonempty subset of the set of individuals N. A party structure or coalition structure $B = \{S_1 ... S_m\}$ is a partition of the set of individuals N into parties. With three individuals there are five possible coalition structures:

$$\{\{1\},\{2\},\{3\}\};\{\{1,2\},\{3\}\};\{\{1,3\},\{2\}\};\{\{2,3\},\{1\}\};\{\{1,2,3\}\}$$

The voting equilibrium in the singleton structure is as shown above.

Notice that in all other party structures there will be a unique majority party which will determine the voting equilibrium G by maximizing the total utility of the party. The voting equilibrium G in a party structure B is denoted $G^e(\{B\})$. The total utility of a party S in a party structure B is denoted $U_{\{S\}}(\{B\})$. It is straightforward to prove

$$\begin{aligned} G^e(\{\{1\},\{3\},\{2\}\}) &= 3b_2 \\ G^e(\{\{i,j\},\{h\}\}) &= 3(b_i + b_j)/2 \ \forall\{i,j\} \\ G^e(\{\{1,2,3\}\}) &= b_1 + b_2 + b_3 = G^* \end{aligned}$$

Notice that when there is a grand party, the outcome G^* is Pareto efficient.

Within each of these party structures, individuals pay $G^e(\{B\})/3$ in taxes of private good and receive $G(\{B\})$ in public good[4]. This then allows us to work out total utilities.

$$U_{\{i\}}(\{\{i\},\{j\},\{h\}\}) = w_i - b_2 + b_i \ln(3b_2)\ \forall i \tag{1}$$

$$\begin{aligned} U_{\{i,j\}}(\{\{i,j\},\{h\}\}) &= w_i + w_j - (b_i + b_j) + (b_i + b_j)\ln(3(b_i + b_j)/2) \text{ and} \\ U_{\{h\}}(\{\{i,j\},\{h\}\}) &= w_h - (b_i + b_j)/2 + b_h \ln(3(b_i + b_j)/2)\ \forall\{i,j\} \text{ and } \{h\} \end{aligned} \tag{2}$$

$$U_N(\{N\}) = w_1 + w_2 + w_3 - (b_1 + b_2 + b_3) + (b_1 + b_2 + b_3)\ln(b_1 + b_2 + b_3) \tag{3}$$

Equation (1) to (3) are the partition function and are labelled $U_{\{S\}}(\{B\})$. If individuals are to make rational decisions about joining parties, they need to know more than $U_{\{S\}}(\{B\})$. They need to know individual utility, rather than total utility, when they are members of a party $i \in S \in B$ to have complete preference ordering. In other words, they would need to fill in the following table

	$\{\{1\}\{2\}\{3\}\}$	$\{\{1,2\}\{3\}\}$	$\{\{1,3\}\{2\}\}$	$\{\{2,3\}\{1\}\}$	$\{\{123\}\}$
$U_1(B)$	$U_1(\cdot)$	?	?	$U_1(\cdot)$	?
$U_2(B)$	$U_2(\cdot)$	?	$U_2(\cdot)$	?	?
$U_3(B)$	$U_3(\cdot)$	$U_3(\cdot)$	?	?	?

In the model there will party founders who attempt to establish their party by sharing resources across individuals within a party.

[4] I assume throughout that $w_i > G^e(\{B\})/3$

2.2 The Game of Party Formation

Myers and Sengupta (2002)[5] assume three stages in the game of coalition (party) formation: the party constitution-setting stage where potential party founders propose a division of resources amongst party members (shares of $U_{\{S\}}(\{B\})$ to $i \in S$) to achieve the formation of the party; the party-formation stage where individuals given the constitutional proposals choose parties; and the voting stage where existing parties vote for public goods. The last stage has been described above and generates $U_{\{S\}}(\{B\})$.

In the second stage, the individuals take the shares from the first stage as given and looking ahead to the third stage where $U_{\{S\}}(\{B\})$ is determined. Each will have a set of preferences (payoffs) over all possible party structures (the table above with all the question marks answered). Based on these preferences a self–interested individual forms a partnership plan —a set of individuals with whom that individual wishes to form a party. This is denoted by $i \in S_i$. The set of all possible partnership plans for individual i in the three-individual model is denoted by $\mathbf{S}_i$: $\mathbf{S}_i = (\{i\}, \{i,j\}, \{i,k\}, \{i,j,k\})$. A profile of partnership plans is $\sigma = (S_1, S_2, S_3)$ and the set of all such profiles is denoted $\mathbf{S}$. The rule that maps profiles of partnership plans into party structures is called the *coalition structure rule,* $\psi : \mathbf{S} \to B$. In this paper a rule ψ^* is used. It is labelled the **strict consensus rule**[6] by Myers

[5] The model is borrowed from Myers and Sengupta (2002). That model is based on an underlying model of federation formation but the translation to political party formation is straightforward. Here I will only present an informal outline of the model. For details on the model see Myers and Sengupta (2002).

[6] This party formation rule is chosen exogenously. There are many other party formation rules that may lead to different results.

and Sengupta (2002). In words it is the rule where if and only if each individual in the group wishes to join precisely the others in that group then that group forms a party. More precisely define $\psi_i^*(\sigma)$ as the coalition to which i belongs under ψ^*. Call the coalition structure rule $\psi^* : S \to B$, the **strict consensus rule** if for any $\sigma = (S_1, ..., S_n) \in \mathbf{S}$ and any $i \in N$

$$\psi_i^*(\sigma) = \begin{cases} S_i \text{ if } S_j = S_i \text{ for every } j \in S_i \\ \{i\} \text{ otherwise.} \end{cases}$$

This was a rule employed by Hart and Kurz (1983). It has the characteristic that an individual either ends up in the coalition for which it planned or alone as it began. Also notice that there is a unique partnership profile which leads to grand party, $\sigma^N = (N, N, N)$.

In a model of coalition formation such as this, it is natural to use solution concepts which require that the equilibrium profiles are immune to unilateral as well as multilateral coordinated deviations. **Coalition proof Nash equilibrium (CPE)** will be employed in this paper. A profile of partnership plans is a **CPE** if no individual or group of individuals can fashion a profitable deviation for each of its members that is itself immune to further deviations by subsets of the deviating coalition. Note that if a profile is CPE it is also a Nash equilibrium[7].

In the first stage, there exists a **founder** of each party which assigns shares of total utility to party members, i.e. to divide the total utility among its members. The founder of coalition $S \in B$ chooses a **constitution profile** $C_S(B)$, which assigns

[7] See Bernheim, Peleg, and Whinston for the formal definition (1987)

shares of the total utility pie, $U_s(B)$ to the members in its coalition. The share assigned to i is denoted $C_{S,i}(B)$ and

$$\sum_{i \in S} C_{S,i}(B) = 1$$

As an example, if $C_{\{1,2\}}(\{\{1,2\},\{3\}\}) = (1/2, 1/2)$ then

$$U_1(\{\{1,2\},\{3\}\}) = U_{\{1,2\}}(\{\{1,2\},\{3\}\}/2$$

There is one founder for each $S \in B$. The founder of $S \in B$'s sole objective is the establishment of its party and, thus, it receives a positive payoff if its party forms and zero otherwise. Founders will, therefore, engage in a strategic form game where each founder's objective is the establishment of its party and its strategy is a feasible constitution. The solution concept at this stage is Nash equilibrium.

3 RESULTS

3.1 Is the Grand Party an Equilibrium Outcome?

The natural place to start, in looking for an equilibrium coalition structure, is with the grand party $\{N\}$. The reason is that if $(b_1 + b_2 + b_3)/3 \neq b_2$ then the grand party will be unique in providing a Pareto efficient allocation. In other words, the total utility in the grand party will be larger than the sum of the resources in any other party structure. The founder of the grand party may divide its resources among party members freely and costlessly. So the founder of the grand party will be able to strictly Pareto dominate the payoffs for all individuals in any other party structure. It will turn out, however, that the grand party is not an equilibrium outcome. Myers and Sengupta (2002) have a result which will be useful here. First, it is necessary to define **superadditivity** and a specific **core** for this particular environment.

For any coalition structure $B \in \mathbf{B}$ and any two coalitions S and S' in B, if

$$U_{S \cup S'}(B \backslash \{S, S'\} \cup \{S \cup S'\}) \geq U_S(B) + U_{S'}(B) \tag{4}$$

then the partition function is **superadditive**.

To define the core, one must have a game in coalition form. That is, there must be an unique worth for each coalition. However, in the partition function in present model, there may exist more than one worth of a coalition. For example, $U_{\{1\}}(\{\{1\},\{2\},\{3\}\})$ and $U_{\{1\}}(\{\{2,3\},\{1\}\})$. To deal with this problem, Myers and Sengupta (2002) define the ψ^*-coalitional form. The analysis starts with σ^N. Now consider the partnership profile in which $S_i = S$ for every $i \in S$ and $S_i = N$ for every $i \in N \setminus S$. This is interpreted as a profile in which a federation S has made a *unanimous joint deviation* from the profile σ^N. Denote such a profile by d^S.

Hence the ψ^**-coalitional form* is a pair (N, v_{ψ^*}) where ψ^* is the strict consensus coalition structure rule, N is the set of players and v_ψ is a function that associates with each coalition S. The real number $v_{\psi^*}(S)$ defined by

$$v_{\psi^*}(S) = U_s(\psi^*(d^S)).$$

For example $v_{\psi^*}(\{1\}) = U_{\{1\}}(\{\{1\},\{2\},\{3\}\})$. In this way a game in partition form is translated into a game in coalition form and usual concepts such as the core can be applied.

Recall that for any game in coalition form (in our case (N, v_{ψ^*})) the *core* of the game is the set of all payoff vectors for which no coalition can improve upon, that is, $(U_1(N), \ldots, U_n(N)) \in \Re^n$ such that $\sum_{i \in S} U_i(N) \geq v_{\psi^*}(S)$ for every coalition S.

Myers and Sengupta: Proposition 5: *Let the coalition structure rule ψ be the strict consensus rule, ψ^*, and let the underlying game generate a partition function that satisfies superadditivity. The grand federation structure $\{N\}$ can be supported as a coalition proof equilibrium structure if and only if the core of the game in ψ-*coalitional form*, (N, v_{ψ^*}), is nonempty. Moreover, if $\{N\}$ is a coalition proof equilibrium structure, then the equilibrium constitutional profile, c_N^* generates an allocation $U(N)$ which belongs to the core of (N, v_ψ).*

In order to utilize this proposition, one must first check whether the partition function $U_S(B)$ is indeed superadditive. If so,then by checking the emptiness of this particular core, one can determine whether the grand party is an equilibrium outcome.

Result 1: The partition function for the majority voting model is superadditive.

Proof: Consider the general case of n individuals in the majority voting model above[8]. Consider coalition structures $\widehat{B}$ and $\widehat{\widehat{B}}$ which differ only by the union of coalitions S' and S'' in $\widehat{B}$. If the new coalition is not the largest in cardinality (majority) party in $\widehat{B}$, then neither of coalitions S' and S'' were majority parties

[8] The proof of superadditivity is extended to the general form. The model only consists of 3 individuals for all other sections in this paper.

before union (in $\widehat{\widehat{B}}$). Hence $(G_{\widehat{B}} - G_{\widehat{\widehat{B}}}) = 0$ and $U_{S' \cup S''}(\widehat{B}) - U_{S'}(\widehat{\widehat{B}}) - U_{S''}(\widehat{\widehat{B}}) = 0$ or all superadditivity conditions are satisfied by equality to zero. If the new coalition $S' \cup S''$ is the majority party then

$$U_{S' \cup S''}(\widehat{B}) - U_{S'}(\widehat{\widehat{B}}) - U_{S''}(\widehat{\widehat{B}}) = \frac{\mid S' \cup S'' \mid}{\mid N \mid}\left(G_{\widehat{\widehat{B}}} - G_{\widehat{B}}\right) + \sum_{i \in S' \cup S''} b_i \ln\left(\frac{G_{\widehat{B}}}{G_{\widehat{\widehat{B}}}}\right) \quad (5)$$

where $G_{\widehat{B}} = \frac{|N|}{|S' \cup S''|} \sum_{i \in S' \cup S''} b_i$.[9] With manipulation,

$$\begin{aligned}
&U_{S' \cup S''}(\widehat{B}) - U_{S'}(\widehat{\widehat{B}}) - U_{S''}(\widehat{\widehat{B}}) \\
&= \tfrac{|S' \cup S''|}{|N|}\left(G_{\widehat{\widehat{B}}} - G_{\widehat{B}}\right) + \textstyle\sum_{i \in S' \cup S''} b_i \ln\left(\frac{G_{\widehat{B}}}{G_{\widehat{\widehat{B}}}}\right) \\
&= \left(\textstyle\sum_{i \in S' \cup S''} b_i\right)\left(\frac{|S' \cup S''| G_{\widehat{\widehat{B}}}}{|N| \sum_{i \in S' \cup S''} b_i} - 1 + \ln\left(\frac{G_{\widehat{B}}}{G_{\widehat{\widehat{B}}}}\right)\right) \text{ using } G_{\widehat{B}} = \tfrac{|N|}{|S' \cup S''|} \textstyle\sum_{i \in S' \cup S''} b_i \\
&= \left(\textstyle\sum_{i \in S' \cup S''} b_i\right)\left(\frac{G_{\widehat{\widehat{B}}}}{G_{\widehat{B}}} - 1 + \ln\left(\frac{G_{\widehat{B}}}{G_{\widehat{\widehat{B}}}}\right)\right) \\
&= \left(\textstyle\sum_{i \in S' \cup S''} b_i\right)(x - 1 + \ln(1/x)) \text{ where } x \equiv \frac{G_{\widehat{\widehat{B}}}}{G_{\widehat{B}}}
\end{aligned}$$

Define $f(x) = (x - 1 + \ln(1/x))$. Since $\frac{df(x)}{dx} = \frac{x-1}{x}$ and $\frac{d^2 f(x)}{dx^2} = \frac{1}{x^2} > 0$, the unique minimizer is $x = 1$ where $f(x) = 0$. Therefore $f(x) \geq 0$ and the structure is superadditive. *QED*

This result allows the application of proposition 5 where the word *federation* is replaced by *party*.

Result 2: The grand party N is not a equilibrium outcome.

Proof: Using substitution on the list of inequality constraints required for the non-emptiness of the core, one finds that a necessary condition for the non-emptiness of the core of the ψ-coalitional form, (N, v_{ψ^*}) is

$$2U_{\{1,2,3\}}(\{\{1,2,3\}\}) \geq U_{\{1,2\}}(\{\{1,2\},\{3\}\}) + U_{\{1,3\}}(\{\{1,3\},\{2\}\}) + U_{\{2,3\}}(\{\{1\},\{2,3\}\})$$

[9] This comes from working out the most preferred G for the coalition (total utility maximizing).

Using the partition function this can be written as

$$\begin{aligned} & 2\left(b_1+b_2+b_3\right)\ln\left(b_1+b_2+b_3\right) \\ \geq \quad & \left(b_1+b_2\right)\ln\left(\frac{3}{2}b_1+\frac{3}{2}b_2\right)+\left(b_1+b_3\right)\ln\left(\frac{3}{2}b_1+\frac{3}{2}b_3\right)+\left(b_2+b_3\right)\ln\left(\frac{3}{2}b_2+\frac{3}{2}b_3\right) \end{aligned}$$

Define a strictly convex function of x where $f(x) = x\ln x$. Let a value of the variable x be $x_{\{ij\}} \equiv \frac{3}{2}\left(b_i + b_j\right) \; \forall \; i$ and j. The inequality can then be written

$$\left(b_1+b_2+b_3\right)\ln\left(b_1+b_2+b_3\right) \geq \frac{1}{3}\sum_{\{i,j\}\in N} f(x_{\{i,j\}})$$

Now note that $\frac{1}{3}\sum x_{\{i,j\}} = \frac{1}{3}(\frac{3}{2}\left(b_1+b_2\right)+\frac{3}{2}\left(b_1+b_3\right)+\frac{3}{2}\left(b_2+b_3\right)) = b_1+b_2+b_3$, so that the inequality can be written

$$f\left(\sum_{\{i,j\}\in N} x_{\{i,j\}}/3\right) \geq \frac{1}{3}\sum_{\{i,j\}\in N} f(x_{\{i,j\}})$$

A property of convex functions[10] is:

$$\frac{1}{K}\sum f\left(x_i\right) \geq f\left(\frac{\sum x_i}{K}\right)$$

that leads to a conclusion

$$U_{\{1,2\}}(\{\{1,2\},\{3\}\})+U_{\{1,3\}}(\{\{1,3\},\{2\}\})+U_{\{2,3\}}(\{\{1\},\{2,3\}\}) > 2U_{\{1,2,3\}}(\{\{1,2,3\}\}) \quad (6)$$

Therefore, the core is empty and the grand party is not an equilibrium outcome.

The logic here is that even though the founder of the grand coalition can Pareto dominate any allocation in any other given party structure, there is no way to divide

[10]See Alpha C. Chiang, Fundamental Methods of Mathematical Economics, 3rd ed., p342. For n >3, one may prove this property by induction.

the resources to *simultaneously* Pareto dominate allocations in all alternative party structures. With more than 2 people, there is more than one alternative to the grand party structure (with 3 players there are 4 alternatives).

3.2 What is the Equilibrium Coalition Structure?

In this subsection, assume $0 < b_1 = b_2 < b_3$.

Define c as the following list of constitutional profiles

$$C_{\{1,2\}} = (\tfrac{1}{2}, \tfrac{1}{2})$$
$$C_{\{i,3\}} = \left(\frac{U_{\{i\}}(\{\{i,\}\{j\},\{3\}\})}{U_{\{i,3\}}(\{\{i,3\}\{j\}\})}, 1 - \frac{U_{\{i\}}(\{\{i,\}\{j\},\{3\}\})}{U_{\{i,3\}}(\{\{i,3\}\{j\}\})} \right) \forall i = 1,2$$
$$C_{\{1,2,3\}} = (C_{1,\{1,2,3\}}, C_{2,\{1,2,3\}}, C_{3,\{1,2,3\}})$$

Then denote $s = \{\{1\}\{2\}\{3\}\}$. These lead to

	$\{\{1\}\{2\}\{3\}\}$	$\{\{1,2\}\{3\}\}$	$\{\{1,3\}\{2\}\}$
$U_1(B)$	$U_{\{1\}}(s)$	$U_{\{1\}}(s)$	$U_{\{1\}}(s)$
$U_2(B)$	$U_{\{2\}}(s)$	$U_{\{2\}}(s)$	$U_{\{2\}}(\{\{1,3\}\{2\}\})$
$U_3(B)$	$U_{\{3\}}(s)$	$U_{\{3\}}(\{\{1,2\}\{3\}\})$	$U_{13} - U_{\{1\}}(s)$

	$\{\{2,3\}\{1\}\}$	$\{\{123\}\}$
$U_1(B)$	$U_{\{1\}}(\{\{2,3\}\{1\}\})$	$C_{1,\{1,2,3\}}U_N$
$U_2(B)$	$U_{\{2\}}(s)$	$C_{2,\{1,2,3\}}U_N$
$U_3(B)$	$U_{23} - U_{\{2\}}(s)$	$C_{3,\{1,2,3\}}U_N$

where U_S has been used instead of $U_S(B)$ when there is no possible ambiguity.

Notice that

$$U_{13} - U_{\{1\}}(s) = U_{23} - U_{\{2\}}(s) \tag{7}$$

Result 3: The constitution profile c is a Nash equilibrium of the constitution setting stage and $\widehat{\sigma} = \{\{1,3\},\{2\},\{1,3\}\}$ is a CPE of the party formation stage. Therefore since $\psi^*(\widehat{\sigma}) = (\{\{1,3\},\{2\}\})$ a party of heterogeneous individuals is a CPE outcome.

Proof: Consider the second stage or take c as given. Given $\widehat{\sigma}$ a joint deviation by all individuals is required to go to the grand party σ^N. If this is to be individually profitable, individual 3 must receive more than $U_{13} - U_{\{1\}}(s)$. If this is to be immune to further unilateral deviations by 1 and 2 each must receive more than $U_{\{1\}}(s)$ and $U_{\{2\}}(s)$ respectively. It is because these are what they can achieve with a unilateral deviation from σ^N. However $U_{13} - U_{\{1\}}(s) + U_{\{1\}}(s) + U_{\{2\}}(s) = U_{13} + U_{\{2\}}(s) > U_{123}$. From equations (6) and (7)

$$\begin{aligned} U_{\{1\}}(s) + U_{\{2\}}(s) + U_{13} + U_{23} &> 2U_{123} \\ 2(U_{13} + U_{\{2\}}(s)) &> 2U_{123} \end{aligned}$$

Given $\widehat{\sigma}$ there is no profitable unilateral deviation by 3 because $U_{13} - U_{\{1\}}(s) > U_{\{3\}}(s)$ by superadditivity. There is no profitable unilateral deviation by 2 because no deviation has consequence for the coalition structure and there is no profitable unilateral deviation for 1 because any deviation would lead to the singleton structure. There is no joint deviation leading to a doubleton party which is profitable to 3. Joint deviation by 1 and 2 to achieve $\{1,2\},\{3\}$ is not profitable for 1. Therefore given c, $\widehat{\sigma} = \{\{1,3\},\{2\},\{1,3\}\}$ is a CPE partnership profile which leads to an equilibrium party of $\{1,3\}$.

Now consider whether there exists unilaterally profitable deviations from c at the first stage. Obviously there is no profitable deviation by a founder of a singleton coalition or the founder of $\{1,3\}$. There is no way for the founder of the grand party to divide its resources to induce a profitable and stable deviation as discussed above. The founder of $\{1,2\}$ cannot change its constitution without making any deviation by 1 and 2 at the second stage unstable (further deviations by 1 or 2 would lead to the singleton structure). This leaves the founder $\{2,3\}$. To make the joint deviation $S_2 = S_3 = \{2,3\}$ by 3 individually profitable, 3 must achieve more than $U_{13} - U_{\{1\}}(s)$; however, its founder must give 2 less than $U_{\{2\}}(s)$. Thus it cannot induce deviations that would be profitable to 3 and stable to further deviations by 2. *QED*

Notice that for the case $0 < b_1 = b_2 < b_3$, the constitution profile c and $\widehat{\sigma} = \{\{1\}, \{2,3\}, \{2,3\}\}$ is also a CPE of the party formation stage. Therefore, since $\psi^*(\widehat{\sigma}) = (\{\{1\}, \{2,3\}\})$, any party of heterogeneous individuals, is a CPE outcome.

It is interesting to find that individuals with identical preferences do not form party to vote against the individuals having different preferences[11]. The following section is intended to explain this by examining the surpluses generated by each coalition formation.

Gains From Coalition Formation: Surplus Approach

Calculating the surpluses generated from all possible coalitions can help provide

[11]This finding does not imply an inconsistency with Levy(2002). One may test by allowing marginal difference between b_1 and b_2.

intuition for this result. The baseline payoff is the singleton payoff. The surplus of a party formation is calculated by subtracting the sum of the baseline payoffs of all members, as if they were not forming a party, from the total payoff generated by the existence of the coalition:

$$surplus_{\{S\}} = U_{\{S\}}(S) - \sum_{i \in S} U_i(\{i\}) \tag{8}$$

In the three-individual model where $0 < b_1 = b_2 < b_3$, the surpluses are calculated as follow:

$surplus_{\{1,2\}} = U_{\{1,2\}}(\{1,2\}) - U_1(\{1\}) - U_2(\{2\}) = 0$

$surplus_{\{i,3\}} = U_{\{i,3\}}(\{i,3\}) - U_i(\{i\}) - U_3(\{3\}) > 0$, by superadditivity (4), where i =1 or 2

$surplus_{\{1,2,3\}} = U_{\{1,2,3\}}(\{1,2,3\}) - U_1(\{1\}) - U_2(\{2\}) - U_3(\{3\}) > 0$, by superadditivity (4).

recall the emptiness of the core (6):

$U_{\{1,2\}}(\{\{1,2\},\{3\}\}) + U_{\{1,3\}}(\{\{1,3\},\{2\}\}) + U_{\{2,3\}}(\{\{1\},\{2,3\}\})$

$> 2U_{\{1,2,3\}}(\{\{1,2,3\}\})$

where $\forall i,j = 1,2,\ i \neq j$ and use (7):

$U_{\{i,3\}}(\{i,3\}) - U_i(\{i\}) + U_{\{j,3\}}(\{j,3\}) - U_j(\{j\}) > 2U_{\{1,2,3\}}(\{1,2,3\}) - 2U_i(\{i\}) - 2U_j(\{j\})$

$2(U_{\{i,3\}}(\{i,3\}) - U_i(\{i\})) > 2U_{\{1,2,3\}}(\{1,2,3\}) - 2U_i(\{i\}) - 2U_j(\{j\})$

therefore

$U_{\{i,3\}}(\{i,3\}) - U_i(\{i\}) - U_3(\{3\}) > U_{\{1,2,3\}}(\{1,2,3\}) - U_i(\{i\}) - U_j(\{j\}) - U_3(\{3\})$

$$surplus_{\{i,3\}} > surplus_{\{1,2,3\}} > surplus_{\{1,2\}} = 0$$

The coalition structure$\{i, 3\}$ is capable of extracting the largest gain whereas individuals with identical preferences toward the provision of a public good do not gain from forming coalition among themselves. Therefore, the possibility of the formation of$\{i, 3\}$ is higher than any other possible coalition structure.

A drawback of the surplus approach is that the constitution profile has not been considered. The individual payoffs of members consequently remain undetermined.

4 CONCLUDING REMARKS

In this paper, I consider a set of individuals, whose preferences are defined over a public good and a private good, voting for the provision of the public good. Individuals are allowed to fully exploit the gains from trade by forming a political party which is assumed free to divide that total utility among party members through side-payments. With the aid of the non-cooperative model of coalition formation in Myers and Sengupta (2002), I am able to find the following results. The first result reveals that the grand party is, in fact, not an equilibrium party structure when more than two individuals are involved. Although the grand party generates the highest aggregate payoff for its members, it cannot simultaneously maximize their individual payoffs. While there are allocations in the grand party which strictly Pareto dominate the payoffs in any single alternative party structure, there may be no allocation that constitutes a simultaneous Pareto improvement over allocations associated with every alternative party structure.

The second result indicates that when two individuals have the same preferences over the provision of a public good, a party of two heterogeneous individuals with the third person excluded from the party is an equilibrium outcome. The party of the two identical individuals cannot be formed because there is no surplus for the founder of this party to divide among the members. On the other hand, there are gains from trade for the heterogeneous individuals in forming a party and moving the level of public good closer to the most preferred level for the individual demanding a high level. These gains from trade and the ability of the party founder to divide those gains freely is what facilitates the formation of a party of unlike individuals in equilibrium.

One major drawback of the model is that the model here only consists of three individuals. A more general model with n individuals capable of more coalition structures may lead to different results. Further, the equilibrium structure found in this model is based on a given constitution profile. There may exist other constitution profiles that lead to other equilibrium structures. In addition to increasing the number of individuals involved in the model, allowing all individuals for different preferences, may lead to different equilibrium structures.

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