APPLICATION OF DEFAULT BARRIER AND THE SCRUTINY OF DEFAULT DECISION UNDER SECURITIZATION

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Abstract

The rates of default and foreclosure have increased sharply due to the subprime mortgage meltdown back in late 2006. To a certain extent, the crisis is related to the popularity of securitization that has increased the supply of credit, especially in areas in which rejections of mortgage application were prevalent before. Many believe the notion that the crisis is mainly caused by a large increase of low-quality borrowers. Using a sample of foreclosed loans under securitization, our purpose is to provide a groundwork for determining whether the aforementioned notion can be substantiated through the use of actual data. Specifically, if securitization is not the key to the crisis, a large number of rational default should exist. Contrary to such idea, the statistical result from our dataset shows that a large number of defaults are caused by life-events, and therefore, providing grounds to believe that securitization is a likely suspect for the collapse of the housing market.

Keywords: foreclosure, default barrier, rational default, negative equity, life-event, subprime, LTV, FICO

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1: Introduction

Default, at the option of borrowers, can be triggered by two major reasons - life-events and depreciation of house prices[1]. Life-event is relatively straight-forward, as events such as divorce, unemployment, death of loved ones can lead to an immediate shortage of cash, and thus ultimately lead to default. The depreciation of house prices would need to be scrutinized. For a person without much knowledge in mortgage, one may determine his/her default decision based on the house price relative to the outstanding loan balance, i.e. the loan-to-value ratios (LTV). For instance, a LTV of higher than one would put a borrower at an advantageous position to exercise her option to default. However, this is a naïve decision rule as there is a possibility for the house price to appreciate during the remaining life of the term. In fact, Jones (1995) stated that a loan must be considerably "under water" for a borrower to rationally default. Foote, Gerardi and Willen (2008) further concluded the economic rationale that negative equity is a necessary but not a sufficient condition for foreclosure. At any time before the termination or maturity of the loan, there exists a certain amount of time value (in addition to intrinsic value) to the borrower's claim.

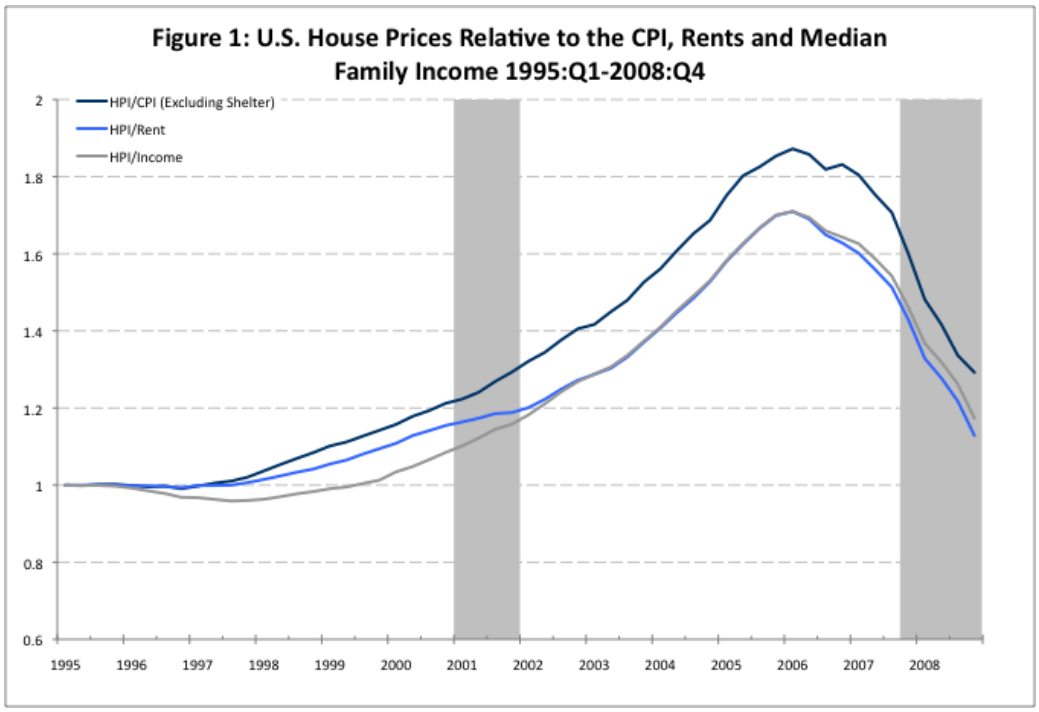
Due to the put-like feature of the borrower's behavior, Jones (1995) and Theunissen (2009) employed numerical methods to model the borrower's default decision (i.e. the prepayment and rational default region). A similar framework is also adopted in this paper, albeit the existence of other default types (positive equity default and life-event default) that we will be exploring as the paper unfolds.

Traditionally, academics and practitioners (i.e. lenders) use a number of variables as indicators of default rate. Demyanyk (2009) identified four major factors that seem to most affect the probability of default two years after origination: post-origination house price appreciation, FICO score, Combined LTV (CLTV[2]) and mortgage rate. Due to the limitation of our dataset, we can only utilize the mortgage-specific LTV, estimated house price, and static FICO[3] to analyze the default behavior of different groups of borrowers.

[1] In the paper "Reducing Foreclosures", Foote et. al suggested that one of the important sources of default is the interaction of falling prices and adverse life events.
[2] Combined LTV refers to aggregated loans (such as second mortgage, car loans, education loans etc).
[3] FICO (Fair Isaac Corporation) is one of the most used credit score for measuring credit risk. Static FICO refers to the FICO of the borrower as at the date of loan origination.

2: Overview of the US Mortgage Market

The overall US mortgage market was an attractive pool from 2001 to 2005. **Figure 1**[4] shows the level of US house price (in terms of HPI) relative to three different measures – the CPI, rents and median family income. As shown, HPI has an ongoing increase from 1997 to 2006. In a study of house prices between 2001 and 2005, Mian and Sufi (2008) suggests that securitization may have increased the supply of credit, especially in areas in which rejections of mortgage application were prevalent before. The increase in supply of credit allowed for more home purchases and thus led to rapid increase in house prices. Under such an increasing trend, bubble of the overall housing market comes into shape as borrowing was strongly encouraged in both the prime and sub-prime market.



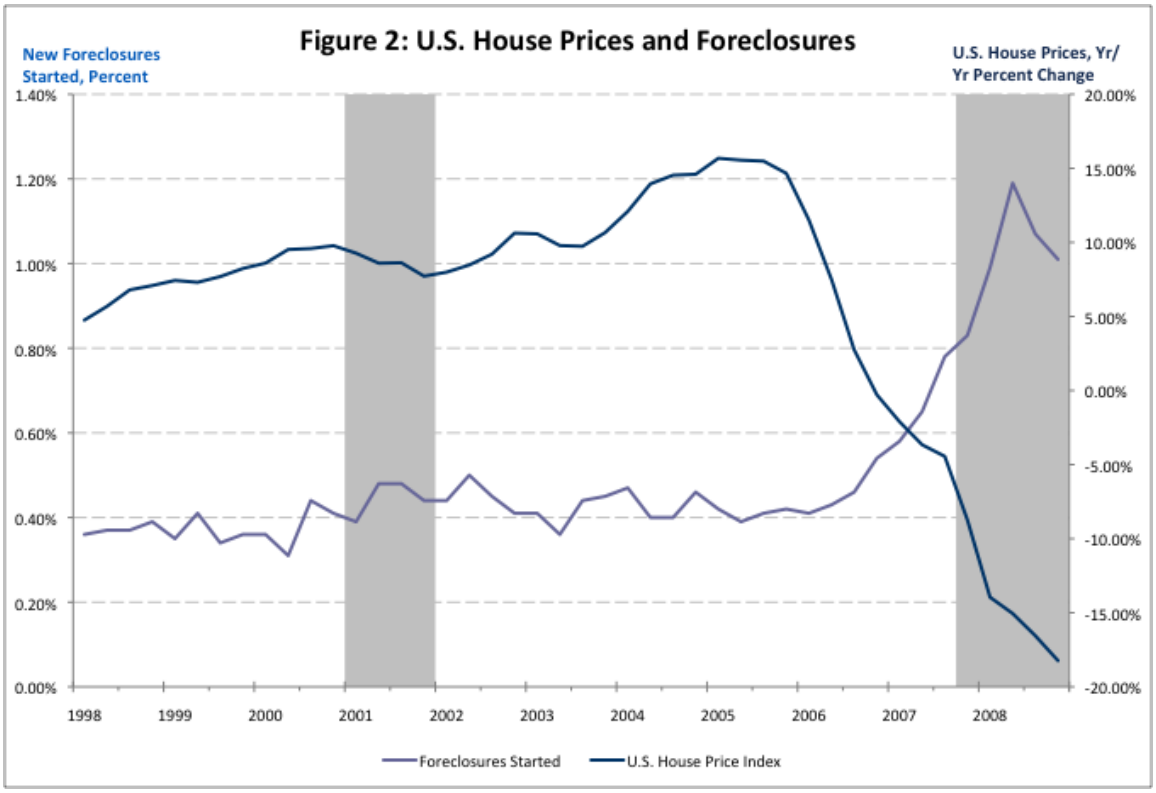
Source: "Systemic Risk and the Financial Crisis: A Primer" – James Bullard, Christopher J. Neely, and David C. Wheelock (September 2009)

For a typical borrower, ownership of a house takes precedence. From the standpoint of a person who is looking for purchasing her first home, an upward move of house price

[4] The house price index (HPI) shown is the S&P/Case-Shiller National HPI; consumer price index (CPI) data exclude the shelter component; the rent index is a separate component of the CPI; median family income is an aggregated monthly series from the National Association of Realtors; and recession dates (vertical gray bars) are from the National Bureau of Economic Research.

would be unfavorable. Furthermore, U.S. interest rate was at a relatively low level prior to 2006. Therefore, when house prices are increasing, one would want to obtain a mortgage as soon as possible in order to lock in a reasonable price and rate. However, it is such a lock-in of house price during the booming period that brings in losses to borrowers when the housing market collapses subsequently. In general, the buy-side of the US housing market are mainly comprised of three groups of different intentions – those looking for a primary residence; those who purchase for long-term investments; and those who want to profit from speculation in the short-run.

The sub-prime crisis caused by the declined house price crushed the financial market and job market. Consequently, a lot of borrowers, regardless of their original intent, began to exit the market. Figure 2 clearly shows that the sharp decline in house prices in late 2006 was followed by the dramatic increase in the number of foreclosure.



Source: "Systemic Risk & the Financial Crisis: A Primer" – J. Bullard, Christopher J. Neely, & David C. Wheelock (Sep 09)

Demyanyk (2009) defined the term "subprime" as (i) borrowers with a low credit score, history of delinquency or bankruptcy, or poor employment history; (ii) lenders specializing in high-cost loans and selling fewer loans to government-sponsored

enterprises; (iii) securities that encompass a subprime loan; and (iv) certain mortgage generally not available in the prime market. Borrowers who have less capability to meet the debt obligations could borrow from subprime market. Besides, many of these loans came in the form of adjustable-rate which offers a teaser rate in the first two or three years, after which the resetting mechanism kicks in and thus exposes borrowers to the risk of increasing interest rates.

3: Exiting the Market

With the assumption that lender always forecloses if the borrower defaults[5], prepayment and foreclosure are the two ways borrowers can exit the market. However, they are costly. Some mortgage loans contain a prepayment penalty that requires borrowers to pay a certain amount should they choose to prepay in favorable conditions, one of which is when the market value of house price exceeds the outstanding loan balance.

3.1: The Foreclosure Process and Cost

Foreclosure is defined as the process through which the lender (or the contracted loan servicer) reclaims the property when the borrower defaults by means of consecutive non-payment (i.e. delinquencies and defaults). When exactly foreclosure occurs depends on the specification of the loan contract. As soon as the loan has entered the foreclosure process, it is governed by the foreclosure law that is dictated by the state in which the loan is originated. Therefore, foreclosure law varies from state to state. However, in general, the foreclosure process is lengthy. A typical foreclosure process, counting from the date the loan has entered the foreclosure process, takes about a year.

The lengthy process produces a large cost shared by the lenders, servicers and borrowers. According to a Financial Times article "*America needs a way to stem foreclosure*" (2008), Summers stated that a typical foreclosure runs at one-third or more of a home's value.

[5] In other words, strategic default is ruled out here

For borrowers, the cost mainly comes in the form of worsened credit report, which affects their ability to rent or secure a future loan. A few intangible impacts include changing lifestyle and lowered self-esteem. These costs are offset by the benefit of "free rent" that is usually enjoyed by the borrowers during the lengthy foreclosure process. As such, lenders and servicers bear the remaining bulk of foreclosure costs which include lost principal and interest payments, tax and insurance payments incurred, cost of maintaining (or in some cases restoring) the underlying property, legal costs and other administrative costs. Due to the time-dependent nature of these costs, the longer the foreclosure process, the higher the cost will be.

3.2: Factors Affecting Prepayment and Foreclosure

Demyanyk (2009) summarized the largest impact on the probability of a borrower to prepay or refinance a loan. Most foreclosures happened within the first two or three years of origination. The main factors affecting the probability of prepayments and foreclosures are shown in the following table, with the highlighted ones being the most important.

Factors	Prepayment	Foreclosure
House price appreciation (pre-origination and post-origination)	√	
The presence of prepayment penalties	√	
The resetting structure of mortgage rates	√	
CLTV (Combined Loan-to-Value ratio)	√	√
FICO credit score		√
The mortgage rate		√
Post-origination house price appreciation	√	√

4: Data

Our data are obtained from the Wells Fargo's CTSLink that collects loan-level data from pools of mortgage-backed securities issuers. The static information about individual loans such as FICO score at origination, purchase price, Loan-to-Value (LTV) ratio at origination, city, state and zip code are collected. The loan-level data also provides useful information about the loan's current status, for e.g. the number of months delinquent and whether the loan has entered the foreclosure process. The database contains information on over 330 mortgage-backed securities issuers. Readers should

be reminded that, due to the limited scale of the sample, our dataset is unlikely to represent the whole US mortgage market, but instead represents only a small securitized portion of the particular states and period being studied.

For estimation, our sample contains information on approximately 12,000 individual loans originated between 2006 Q2 and 2007 Q2, and followed through 2010 Q2. The data set is limited to 30-year adjustable-rate mortgage (ARM) issued by three private-labels including Citigroup, Morgan Stanley ABS Capital Inc., and Merrill Lynch Mortgage Loans Inc. For the purpose of our report, we are only interested in loans that have entered the foreclosure process.

Among all the country-wide loans, we further limited our scope to the states that contain the highest proportion of negative and near-negative equity ("negative share"). Arizona, California, Florida, and Nevada are the four states with a significant proportion of negative share (see appendix 1). By focusing on the major cities of these four states, the sample was narrowed to contain 412 individual foreclosed loans. For each loan, we applied the respective city HPI[6] to estimate the house price at the quarter of foreclosure. Using the estimated house price and the already-known outstanding debt, the LTV at the foreclosure quarter can be calculated with ease.

Based on our sample, the mean and standard deviation of the FICO at origination are 620.59 and 57.35, respectively (see appendix 2). In the study of mortgage "affordability" by Foote, Gerardi, Goette and Willen (2009), their sampled data has a FICO mean of 714.1 for prime market and 609.0 for sub-prime market based on the loans originated from 2005-2008 (see appendix 3). Therefore, we can reasonably conclude that most of the loans in our sample are more weighted to the sub-prime market side.

[6] The HPI (House Price Index) is a broad measure of the movement of single-family house prices. It is a weighted, repeat-sales index, i.e. a measure of average price changes in repeat sales or refinancing on the same properties.

5: Modelling the Borrower's Optimal Behavior

To determine the borrower's optimal behavior along the loan horizon, it is best to start off by introducing some notations. The borrower's loan can be represented by the following equation:

$$1 = p\int_0^t e^{-c\tau}d\tau + b(t)e^{-ct}$$

, where 1 represents the total amount of loan, *p* is the yearly rate of payment flow over the time horizon [0,T], and *c* is the contractual loan rate.

The loan contract confers the right, but not the obligation, to the lender to foreclose the property should the borrower be delinquent or offers a rate of p^* that is less than *p* at maturity. The lender will have to incur the foreclosure cost if he chooses to foreclose. Since our emphasis of this paper is on the borrower's behavior, the lender's behavior will not be examined.

In general, a borrower defaults (and thus leading to foreclosure) when one of the following occurs: (i) a life-event that leads to shortage of cash to make up the contractual loan payment. For the purpose of this paper, we define this type as "life-event default"; (ii) the house price declines to a level that is much lower than the outstanding loan balance. As such, the probability of recovery is remote. In this case, it would be rational for the borrower to "put" back the loan to the lender. We define this as "rational default".

At any point in time, by ruling out the option of loan renegotiation, the rational default decision is mainly driven by the fluctuation of house price, which is assumed to follow a stochastic process. As long as the loan is alive, the property generates a continuous flow of housing services, *dh(t)* favoring the borrowers. Taking the housing service flow into account, the stochastic process of the house price can be represented by the following equation:

$$dh(t) = (rh(t) - d(h,t))dt + \sigma h(t)d\tilde{z}(t)$$

where *r* is the constant instantaneous rate, and $\tilde{z}(t)$ is a standard Brownian motion.

To expand our analysis of the borrower's optimal strategy before time T, we have to introduce a new level of debt service flow, $\underline{p} \leq p$. At this level, the borrower is indifferent between servicing the loan or defaulting. Therefore, default and thus foreclosure occur when the borrower offers a debt service flow at $p^* < \underline{p}$. Logically, we refer to $\underline{p}$ as the default barrier along the loan horizon. By the same token, there also exists a prepayment barrier $\overline{p}$. However, our analysis will not be covering the prepayment side due to the sharp decline of the general housing market in the period covered.

Based on the sample of data we obtained, we model the typical mortgage as a short-term 3-year loan. Due to its short-term nature, the modeled mortgage can also be assumed to be fixed-rate. In this 3-year term loan, we have to embrace all possible states of the borrower. We do this by setting up a state space H x T, where $H \equiv [0,\infty)$ represents the vertical scope for which the house price *h* can take on at any given time. Likewise, $T \equiv [0,T]$ denotes the 3-year term horizon.

5.1: Borrower's Value

In the state space of H x T, borrowers make decisions that maximize their own payoff. In our simple framework, borrowers either continue to service their debt or default, whichever maximizes the value of their own position (again prepayment is out of consideration given the declining nature of house price in the period covered). A subset *D* is contained inside the H x T grid. We refer to it as the default region, where borrowers would default should their states fall within it. To make this a feasible model, we assume that borrowers can continuously observe the value of *h*. In short, the loan is terminated when it falls within the default region or when it reaches the maturity T.

5.2: Value to Borrower

In the H x T grid, there exists two types of values to the borrower – the continuation value and the termination value, represented by $B(h,t)$ and $\Omega_B(h,t)$, respectively. From the arbitrage/replication argument of the option pricing theory, it can be shown that *B* satisfies the following differential equation in the open region:

$$\frac{1}{2}\sigma^2 h^2 B_{hh} + [rh - d]B_h + B_t + d - p^* = rB$$

Together with the proper specifications of the boundary conditions, we can solve for the borrower's (and lender's) optimal policies. Theunissen (2008) and Dixit (1993) provide more details for the boundary conditions.

5.3: Borrower's Strategies

Recall that the lender is conferred the right to foreclose should the borrower makes an offer at $p^* < p$. For securitized mortgage, we make a solid assumption that the servicer is obligated to foreclose for any offer at $p^* < p$ at or any time before maturity.

i. Strategies at Maturity

At the maturity of the loan, depending on the relative house price *h(T)* and outstanding loan balance *P*, the borrower makes his/her rational offer as follows:

$$P^* = 0 \text{ for } h(T) \le P$$
$$P^* = P \text{ for } h(T) > P$$

Accordingly, the lender's and borrower's values can be represented by:

$$L(h,T) = \begin{cases} \max\{0, h(T) - l(h,T)\} & \text{if } h(T) \le P \\ P & \text{if } h(T) > P \end{cases}$$
$$B(h,T) = \max\{0, h(T) - P\}$$

ii. Strategies prior to Maturity

Recall the obligated foreclosure assumption we made earlier, default and thus foreclosure occur whenever the offer $p^* < p$ is made by the borrower. Therefore, along the border of the subset *D*, the borrower's continuation value *B(h,T)* is driven to be nil, same as the termination value $\Omega_B(h,t)$. Consequently, the termination value of both the lender and borrower can be represented by:

$$\Omega_L(h,t) = \max\{0, h(t) - l(h,t)\}^7$$
$$\Omega_B(h,t) = 0$$

The default barrier (or subset *D*) defines the severity of the LTV ratio that encourages the borrower to put the loan back to the lender. Therefore, the borrower's default decision (i.e. optimal policies) tracing back from T to the loan initiation period is comparable to an American put option. Let $\underline{h}(t)$ be the house price that makes the LTV fall on the default barrier. For $\underline{h}(t) < h(t) < b(t)$[8], the borrower would incur a negative equity and thus the resulting intrinsic value is zero. However, in this "sandwiched" section, instead of defaulting, the borrower would wait for recovery of the house in the hope of future positive equity. Therefore, the time value would be positive.

To complete the picture of our analysis, there also exists an upper termination barrier (or prepayment barrier) $\overline{D}$, where the borrower would terminate the loan by prepayment should the loan falls into this subset. Similarly, let $\overline{h}(t)$ be the house price that makes the LTV fall on the prepayment barrier. Again, such region will not be elaborated on this paper due to the declining nature of the house price in the period covered.

Figure 2 provides a quick snapshot of the time grid H x T. The three regions can be defined by the following ranges:

Symbol	Region	Range	Decision
$\overline{D}$	Prepayment region (not covered)	$h(t) > \overline{h}(t)$	Prepayment
C	Continuation region	$\underline{h}(t) < h(t) < b(t)$	Debt Servicing
D	Default region	$h(t) < \underline{h}(t)$	Default/Foreclosure

[7] *l* represents the foreclosure cost which is assumed to bear a linear relationship (see appendix 6)
[8] *b(t)* denotes the outstanding loan balance

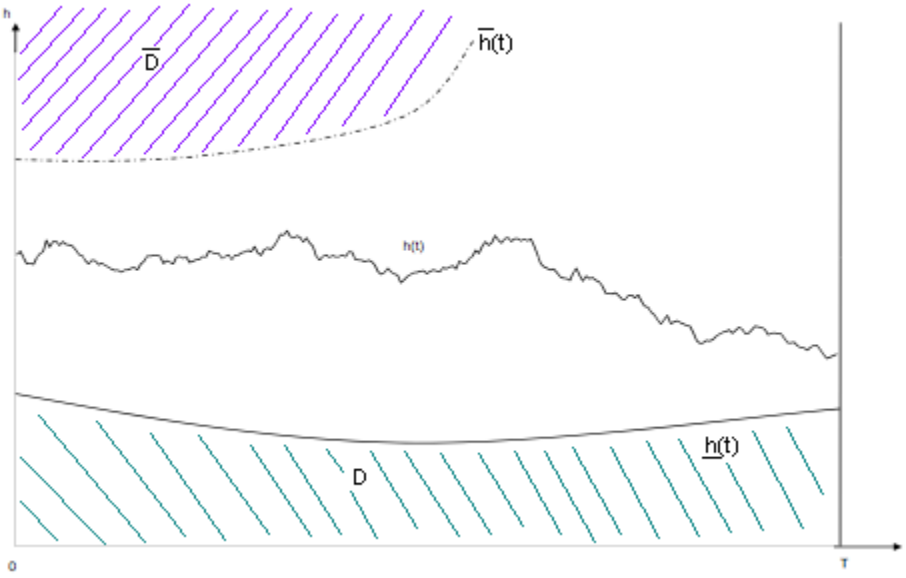


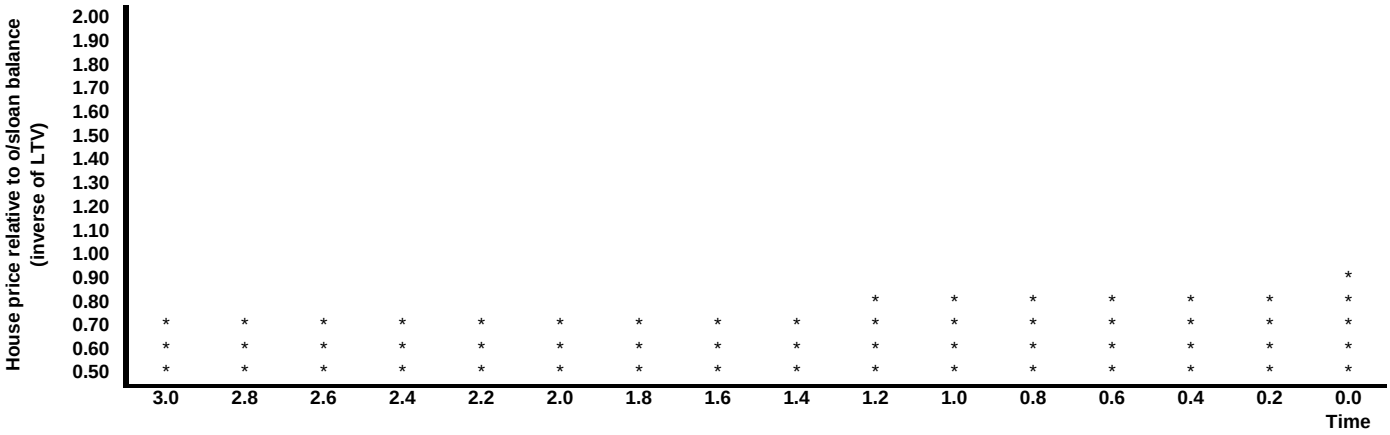
Figure 2: Strategy space for securitized mortgages

6: Theoretical Results and Interpretations

Recall the three groups being analyzed are rational default, life-event default and positive equity default.

Figure 3 displays the theoretical defaullt barrier model based on a set of inputs including the risk-free rate, housing service flow, contractual loan rate, payment flow rate, loan term, foreclosure cost, etc.

Our dataset contains an average duration of 2.63 years. Somewhat, this conincides with the study by Demyanyk (2009) who found that 80% of the subprime loans originated in 2006 and 2007 defaulted within three years after origination.
*(**Appendix 6** shows how the other inputs for the default barrier model were obtained, for e.g. contractual interest rate, housing service flow, volatility).*



** denotes default status*

Figure 3: Securitized mortgage: Borrower's strategies

Based on the result, for time t ≥ 1.2 (time to maturity), the default barrier is modeled to be 0.70, which is the house price relative to the outstanding loan. Similarly, for t < 1.2, the default barrier is modeled to be 0.80. By converting the y-axis of the grid to loan-to-value ratio (by taking the reciprocals), we obtain the following conclusion:

$$\begin{cases} LTV < 1 \; Positive\ Equity\ Foreclosure \\ LTV > 1 \; Negative\ Equity\ Foreclosure \begin{cases} t < 1.2 \begin{cases} LTV > 1.25 & -Rational Foreclosure \\ 1 < LTV < 1.25 & -Life-Event\ Foreclosure \end{cases} \\ t \geq 1.2 \begin{cases} LTV > 1.43 & -Rational Foreclosure \\ 1 < LTV < 1.43 & -Life-Event\ Foreclosure \end{cases} \end{cases} \end{cases}$$

Theoretically, positive equity foreclosure should not exist as it is deemed irrational to do so. If the market is efficient, even borrowers with low credit scores should be able to refinance their loans as long as the loans have positive equity.

For life-event foreclosure, the borrower is more likely to encounter life events such as loss of employment and divorce that lead to shortage of cash. To a certain extent, the FICO score can be used by the lender as an indication of the probability of default. Historically, borrowers with FICO scores below 620 ("low FICO") have a much higer default rate than those with FICO scores above 700 ("high FICO"). Besides, low FICO borrowers tend to have a lower capability to meet the debit obligation. Therefore, in this default group, we would expect to see more low-credit (i.e. low FICO) foreclosure.

LTV increases as the house price continues to drop. When the LTV is excessively high, i.e. when the borrower is way underwater, it would be rational (theoretically under all circumstances) for him to default and put the loan back to the lender. As a result, FICO should be a irrelevant variable in this group.

As mentioned earlier, securization may have increased the supply of credit, i.e. granting the ability to more low-quality borrowers to enter the mortgage market. Theoretically, if securitization is a major cause of the housing market crisis, we would expect to see a dominating number of life-event defaults. This is because, by intuition, life-events are considered to have a more profound impact to the low-quality (low-FICO) borrowers. On the other hand, if there exists a large number of rational defaults, one may suggest that securitization is not a critical factor of the crisis based on the idea that high-quality borrowers are considered to have the ability to persist longer even in the presence of life-events and economic hardships.

7: Analytical Results

We want to compare the number of default/foreclosure between low FICO and high FICO borrowers for the three groups – positive equity foreclosure, life-event foreclosure and rational foreclosure. The statistical result is summarized below.

	≤620	20<FICO<70	≥700	Total
Positive Equity Default	49	45	0	94
Life-Event Default	121	129	24	274
Rational Default	20	19	5	44
Total	190	193	29	412

Out of the 412 loans, 94 belong to the positive equity group. The lack of high FICO foreclosure in this group indicates that life-event is potentially the cause of default. In our opinion, there are several likely causes of positive equity default. Firstly, the reset mechanism of the adjustable-rate mortgage (ARM), to a certain extent, catalyses homeowners towards deliquency. This reason can be substantiated by the paper written by Christopher L. Cagan (2006), who studied the sensitivity of reset on the number of foreclosures for ARMs originated from 2004 to 2006. Secondly, in general, the latest appraised value of a property does not necessarily equate to its actual current

market value. For e.g., the last appraisal was performed six months ago, and that a negative economic shock can adversely impact the house price in the mean time. In fact, this is consistent with the view that banks may be reluctant to write down further (by reassessing the house value) in the hope of disposing the property at a more favorable price. Thirdly, there may be a natural tendancy for banks to move faster to foreclosure in homes that have positive equity. In our dataset, Miami contains the most positive equity default (50 out of 94).

Another way to explain the existence of positive equity default is more applicable to a specific subset of the pool – loans with outstanding balances that are marginally less than the nominal value of the house, for e.g. a LTV of 0.95. Borrowers have to pay for closing costs, such as lawyer's fees and commissions when they are to sell the house. On top of that, the advertised price would have to be discounted if the borrowers want to complete the deal as soon as possible. After taking into account all of these closing costs, the borrower may no longer be classified as a positive equity status and thus it may be more beneficial for them (especially for those who do not worry much about further deterioration of credit quality) to simply default on the loan. Using the range between 0.95 and 1.0 for the LTV as our definition of "marginally positive equity", we found that there are 22 out of 412 loans (equivalent to 5%) that fall into this category. Based on such logic, we tried using 0.90 in lieu of 1.0 as the border of positive equity. The result turns out that there are only 65 defaults, a significant drop of almost thirty cases.

In the life-event default group, there exists more low FICO borrowers. During the period from which the samples are taken, house prices decline drastically. The adverse impact to the financial market brought by subprime mortgage crisis had also crushed the job market. As a result, many borrowers bore their losses from their financial investments and job layoffs. In turn, these losses highly affect their cash flow and thus the ability to meet the mortgage payments. By intuition, it is the low-FICO group that absorbs a bigger piece of these losses. Therefore, relative to to high-FICO borrowers who tend to have a more stable cash flow, a larger proportion of low-FICO foreclosure is plausible.

Finally for rational foreclosure, the number of foreclosure units is lower than what we expected. This is attributable to the fact that some borrowers may not be able to persist long enough to reach the default barrier (life-events and decreasing time to

maturity can lower the "tolerance level" of borrowers). Secondly, since default can adversely affect the borrower's creditability for future house purchase, rental and insurance, those with a sound capital base and stable cash flow may choose to continue to service the debt rather than default.

Apparently, with 2/3 of the total defaults falling in the life-event group and only 44 in the rational default group, we suggest that securitizatoin is one of the major causes of the subprime market crisis.

8: Hypothesis Testing

So far in the development of our paper, besides LTV, FICO was used as the variable for analyzing default decision. However, one may question the actual practicality of FICO when foreclosure is classified into the three groups. Furthermore, as mentioned, FICO may not be a relevant variable when LTV breaches the default barrier, i.e. if it is a rational default. As a way to confirm this, we formulated a null and alternative hypothesis about the equality of two sample means, one being the mean of the rational default group, and the other of the other extreme case, i.e. the mean of the positive equity group.

In order to apply such hypothesis testing, we have to be reasonably certain that the samples are independent and normally distributed. Instead of performing statistical testing, we simply make an assumption here that the samples are independent.

For the setup of the hypothesis testing, we divided the samples into two groups. The first group is comprised of loans with LTV that breaches the default barrier (i.e. either larger than 1.25 or 1.43, depending on the duration of the loan). We defined this group as "Rational Default". The second group consists of loans with LTV < 1, and thus is termed "Positive Equity".

In order to test for normality, which is one of the requirements for applying the hypothesis testing of sample means, we employed the Anderson-Darling test, which is commonly used to detect departure from normality. The detail of the test is explained in appendix 4. The samples in the rational default group yield a statistical number that

is less than the critical value of 0.751 at 5% significance level. Therefore, we can reasonably assume that the samples in this group are normally distributed. On the other hand, the samples in the positive default group yield a statistical number that is larger than the same critical value, thus preventing us from concluding that the samples are normally distributed. Despite such contradictory result, we believe that a different testing conclusion will be produced by having a larger sample size.

Since the objective is to test for the equality of the two sample means. The null and alternative hypotheses are set up as follow:

$$\text{Ho: } \mu_r = \mu_p \text{ or } \mu_r - \mu_p = 0$$

$$\text{Ha: } \mu_r \neq \mu_p \text{ or } \mu_r - \mu_p \neq 0$$

, where μ_r and μ_p represent the FICO means of the rational default group and positive equity group, respectively.

Rejecting the null hypothesis means that the FICO means of the two groups are significantly different, and thus FICO is not a relevant variable for explaining the default behavior.

The t-statistic can be computed by the following equation:

$$t-stat = \frac{\mu_p - \mu_r}{\sqrt{\frac{\sigma_p^2}{n_p} + \frac{\sigma_r^2}{n_r}}}$$

Whereas the degrees of freedom (DOF) for determining the critical value can be computed as follows:

$$DOF = \frac{\left(\frac{\sigma_p^2}{n_p} + \frac{\sigma_r^2}{n_r}\right)^2}{\frac{\left(\frac{\sigma_p^2}{n_p}\right)^2}{n_p} + \frac{\left(\frac{\sigma_r^2}{n_r}\right)^2}{n_r}}$$

The detail of the hypothesis test is contained in appendix 5. The calculated t-statistic is approximately 2.85, which is greater than the t-critical value of 2.0 at 5% significance

level. Therefore, we reject the null hypothesis and conclude that FICO is not a relevant variable for explaining defaults in the two groups.

9: Conclusion

Applying to our dataset, the default barrier model shows that optimal/rational default occurs when borrowers are well underwater, i.e. with LTV significantly larger than one. In fact, our model also shows that such barrier varies slightly across time due to the changing intrinsic value and time value across the loan horizon.

One step further into the analysis of the foreclosed loans, a large number of positive equity default was discovered. Among other possibilities, a few rationales were came up to explain such abnormality – the resetting mechanism of ARM, the potential inequality between the observed value and true value of housing, and the natural tendency for banks to foreclose on loans with positive equity, as well as the inclusion of closing costs that may turn borrowers into being the negative equity status.

Our statistical result clearly shows that the number of life-event defaults is way over the number of rational defaults (274 vs. 44). To sharpen the key point, low-quality borrowers are more sensitive to life-events leading to immediate cash shortages more than they do to depreciation of house prices. Therefore, we suggest that securitization that increased the supply of credit, particularly to the low-quality borrowers, is a major cause that of the subprime market crisis.

While default barrier is a theoretical way for determining the optimal point to exercise the default option, most borrowers from our sample are unable to endure the period of negative equity. This can be attributable to the adverse economic impact brought by the subprime market meltdown. In turn, a massive wave of life-events was followed by, and thus reducing the tolerance level of borrowers in terms of LTV. On the other end of the spectrum, the dataset contains some LTV that are significantly higher than the default barrier LTV, thus suggesting that these borrowers may have good cash flow standing that encourages them to persist a bit longer.

10: Leeway for Future Improvement

Our paper came up with some plausible reasons as an attempt to explain some of the anomalies, such as the significant number of positive equity default and less-than-expected number of rational default, as observed from our statistical result. However, these results are likely to contain unintentional biases caused by input errors or representativeness of the dataset. For improvement on future research on a similar topic, employing the combined LTV (CLTV, as opposed to mortgage-specific LTV), as well as tracking the FICO scores would contribute to the analysis of the borrower behavior. The term "default" can be further defined in a narrower scope. For instance, any loans (foreclosed or not) with a delinquency status of at least 270 days would be deemed to have defaulted. In our dataset, the average number of months delinquent is a staggering high of 18. Therefore, among the other "continuing" loans, it is likely that those have been delinquent for 270 days or more will end up defaulting and thus be in the foreclosure process. Furthermore, adding the debt-to-income ratio (DTI)[9] can help better analyze the actual cash flow of the borrowers. Not to mention increasing the sample size in order to enhance the overall representativeness of the result.

[9] DTI is the ratio of mortgage-related payments (i.e. excluding other debts) to the borrower's income

Appendix

Appendix 1: Negative Equity by States (Q4 2009)

	Q4 2009 Negative Equity by State*					
State	Mortgages	Negative Equity Mortgages	Near** Negative Equity Mortgages	Negative Equity Share	Near** Negative Equity Share	
Alabama	326,957	28,016	15,828	8.6%	4.8%	13.4%
Alaska	84,834	7,923	4,670	9.3%	5.5%	14.8%
Arizona	1,361,551	698,645	56,189	51.3%	4.1%	55.4%
Arkansas	232,151	29,217	14,286	12.6%	6.2%	18.7%
California	6,928,821	2,430,688	270,378	35.1%	3.9%	39.0%
Colorado	1,126,955	227,184	89,345	20.2%	7.9%	28.1%
Connecticut	806,856	93,559	31,102	11.6%	3.9%	15.5%
Delaware	177,876	25,514	8,716	14.3%	4.9%	19.2%
Florida	4,559,245	2,178,822	171,710	47.8%	3.8%	51.6%
Georgia	1,585,718	441,533	126,713	27.8%	8.0%	35.8%
Hawaii	230,089	21,493	7,587	9.3%	3.3%	12.6%
Idaho	236,729	53,663	11,835	22.7%	5.0%	27.7%
Illinois	2,228,544	464,811	113,502	20.9%	5.1%	26.0%
Indiana	569,082	60,947	26,378	10.7%	4.6%	15.3%
Iowa	311,869	27,733	14,053	8.9%	4.5%	13.4%
Kansas	289,582	30,978	15,906	10.7%	5.5%	16.2%
Kentucky	267,186	23,989	14,158	9.0%	5.3%	14.3%
Louisiana	NA	NA	NA	NA	NA	0.0%
Maine	NA	NA	NA	NA	NA	0.0%
Maryland	1,359,820	311,321	66,045	22.9%	4.9%	27.8%
Massachusetts	1,485,309	234,458	53,709	15.8%	3.6%	19.4%
Michigan	1,384,172	532,774	81,253	38.5%	5.9%	44.4%
Minnesota	527,010	87,517	25,660	16.6%	4.9%	21.5%
Mississippi	NA	NA	NA	NA	NA	0.0%
Missouri	773,035	120,190	43,339	15.5%	5.6%	21.2%
Montana	108,370	7,457	3,602	6.9%	3.3%	10.2%
Nebraska	218,936	19,190	10,510	8.8%	4.8%	13.6%
Nevada	602,847	421,165	18,528	69.9%	3.1%	72.9%
New Hampshire	206,081	39,437	11,707	19.1%	5.7%	24.8%
New Jersey	1,887,445	303,745	80,808	16.1%	4.3%	20.4%
New Mexico	229,312	28,282	10,580	12.3%	4.6%	16.9%
New York	1,811,769	114,024	39,772	6.3%	2.2%	8.5%
North Carolina	1,476,042	151,028	94,698	10.2%	6.4%	16.6%
North Dakota	43,796	3,341	1,296	7.6%	3.0%	10.6%
Ohio	2,204,936	437,519	140,943	19.8%	6.4%	26.2%
Oklahoma	399,438	23,830	12,361	6.0%	3.1%	9.1%
Oregon	699,889	111,489	38,814	15.9%	5.5%	21.5%
Pennsylvania	1,777,917	133,866	56,124	7.5%	3.2%	10.7%
Rhode Island	225,737	38,032	7,003	16.8%	3.1%	20.0%
South Carolina	577,251	77,723	33,661	13.5%	5.8%	19.3%
South Dakota	NA	NA	NA	NA	NA	0.0%
Tennessee	940,026	130,952	65,201	13.9%	6.9%	20.9%
Texas	3,225,118	383,974	200,065	11.9%	6.2%	18.1%
Utah	470,518	99,250	29,642	21.1%	6.3%	27.4%
Vermont	NA	NA	NA	NA	NA	0.0%
Virginia	1,241,757	302,153	70,774	24.3%	5.7%	30.0%
Washington	1,403,098	223,793	76,143	15.9%	5.4%	21.4%
Washington, DC	100,080	16,049	4,544	16.0%	4.5%	20.6%
West Virginia	NA	NA	NA	NA	NA	0.0%
Wisconsin	582,270	85,741	30,407	14.7%	5.2%	19.9%
Wyoming	NA	NA	NA	NA	NA	0.0%
Nation	47,595,942	11,321,676	2,312,334	23.8%	4.9%	28.6%

* This data only includes properties with a mortgage. Non-mortgaged properties are by definition not included.
** Defined as properties in negative equity or within 5% of being in a negative equity position.

Source: First American CoreLogic (http://www.facorelogic.com)

Appendix 2: Summary Statistics from Sample

	Mean	Std Dev
FICO Score	620.59	57.35
LTV Ratio	78.82	14.13
Number of Loans		412

Appendix 3: Summary Statistics: Loan Originated from 2005-2008

	Prime		Subprime	
	Mean	**Std Dev**	**Mean**	**Std Dev**
DTI Ratio	35.1	13.8	40.0	11.1
FICO Ratio	714.1	61.6	609.0	54.9
LTV Ratio	73.4	18.2	79.2	12.5
Adjustable Rate Dummy	0.21	0.40	0.56	0.50
Number of Loans	501,317		41,132	

Source: "Reducing Foreclosures" – Christopher L.Foote, Kristopher S.Gerardi, Lorenz Goette, and Paul S.Willen (April 2009)

Appendix 4: Testing for Normality – Anderson-Darling Test

Steps (for the two groups: Rational Default and Positive Equity)

1) Sort the data X_i = 1, 2, …., n

2) Calculate the mean and standard deviation

3) Standardize X_i to make Y_i

4) Calculate the "A-statistics" A^2 using the equation:

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)(\ln \Phi(Y_i) + \ln(1 - \Phi(Y_{n+1-i})))$$

Group I: Rational Default

Loans	Rational Default	Yi = (Xi - mu)/s	A = (2i - 1)	B= ln(phi(Yi))	C = ln(1 - phi(Y_{n+1-1}))	A * (B+C)
1	466	-2.58304356	1	- 5.319	-5.167931866	- 10.49
2	498	-2.068481402	3	- 3.948	-4.015732607	- 23.89
3	510	-1.875520593	5	- 3.495	-2.708452925	- 31.02
4	547	-1.280558098	7	- 2.301	-2.378570335	- 32.76
5	561	-1.055437154	9	- 1.927	-2.293157159	- 37.98
6	566	-0.975036816	11	- 1.803	-1.894848672	- 40.68
7	567	-0.958956749	13	- 1.779	-1.543348274	- 43.19
8	567	-0.958956749	15	- 1.779	-1.499927835	- 49.19
9	572	-0.878556412	17	- 1.662	-1.41547356	- 52.31
10	594	-0.524794928	19	- 1.204	-1.314332135	- 47.86
11	597	-0.476554726	21	- 1.149	-1.275237068	- 50.92
12	600	-0.428314524	23	- 1.096	-1.144437427	- 51.53
13	607	-0.315754051	25	- 0.978	-1.12650904	- 52.61
14	607	-0.315754051	27	- 0.978	-1.056661223	- 54.93
15	607	-0.315754051	29	- 0.978	-1.039662156	- 58.51
16	609	-0.283593917	31	- 0.946	-1.022846801	- 61.03
17	612	-0.235353714	33	- 0.899	-0.973495626	- 61.79
18	614	-0.203193579	35	- 0.869	-0.941499746	- 63.36
19	615	-0.187113512	37	- 0.854	-0.820620865	- 61.95
20	620	-0.106713175	39	- 0.782	-0.806297034	- 61.94
21	621	-0.090633107	41	- 0.768	-0.723934528	- 61.17
22	623	-0.058472972	43	- 0.741	-0.685015902	- 61.31
23	626	-0.01023277	45	- 0.701	-0.647573508	- 60.70
24	629	0.038007432	47	- 0.663	-0.623419762	- 60.47
25	635	0.134487837	49	- 0.592	-0.611582429	- 58.95
26	636	0.150567904	51	- 0.580	-0.554752811	- 57.88
27	644	0.279208444	53	- 0.494	-0.543851465	- 55.03
28	646	0.311368579	55	- 0.474	-0.522505407	- 54.83
29	649	0.359608781	57	- 0.446	-0.491612712	- 53.42
30	650	0.375688848	59	- 0.436	-0.471756522	- 53.58
31	651	0.391768916	61	- 0.427	-0.471756522	- 54.83
32	655	0.456089185	63	- 0.392	-0.471756522	- 54.40
33	656	0.472169253	65	- 0.383	-0.406782585	- 51.36
34	663	0.584729725	67	- 0.328	-0.381025622	- 47.48
35	665	0.61688986	69	- 0.313	-0.356479076	- 46.18
36	670	0.697290197	71	- 0.278	-0.210499968	- 34.69
37	674	0.761610467	73	- 0.253	-0.18487313	- 31.93
38	676	0.793770602	75	- 0.240	-0.18487313	- 31.89
39	691	1.034971613	77	- 0.163	-0.180049339	- 26.41
40	706	1.276172625	79	- 0.106	-0.157370697	- 20.84
41	709	1.324412827	81	- 0.097	-0.105554382	- 16.43
42	720	1.501293569	83	- 0.069	-0.030830998	- 8.28
43	757	2.096256064	85	- 0.018	-0.019486014	- 3.20
44	784	2.530417885	87	- 0.006	-0.004908676	- 0.92
	44					
					S = sum of G / n	- 44.41
Org Mean	626.6363636					
Std Dev	62.18879392				-n - s	0.41
					critical value at 5%	0.751
					Reject?	**No Reject**
					note: we are rejecting normality thus, we don't want rejection	

Group II: Positive Equity Default

Loans	Positive Equity	Yi = (Xi - mu)/s	A = (2i - 1)	B= ln(phi(Yi))	C = ln(1 - phi(Yn+1-1))	A * (B+C)
1	492	-2.605481519	1	-5.384472134	-2.846074143	-8.23054628
2	501	-2.411719444	3	-4.835999832	-2.718719884	-22.6641591
3	506	-2.304073847	5	-4.546029084	-2.554432336	-35.5023071
4	509	-2.239486488	7	-4.37706777	-2.554432336	-48.5205007
5	515	-2.110311772	9	-4.050380014	-2.514340199	-59.0824819
6	517	-2.067253533	11	-3.944797513	-2.474638215	-70.613793
7	519	-2.024195294	13	-3.840864377	-2.43532549	-81.5904683
8	526	-1.873491458	15	-3.490019559	-2.134725052	-84.3711692
9	540	-1.572083786	17	-2.847906227	-1.959250056	-81.7216568
10	542	-1.529025547	19	-2.762574149	-1.858476441	-87.7999612
11	542	-1.529025547	21	-2.762574149	-1.761049243	-94.9960912
12	545	-1.464438188	23	-2.637538661	-1.761049243	-101.167522
13	559	-1.163030516	25	-2.100390971	-1.761049243	-96.5360053
14	562	-1.098443158	27	-1.995059859	-1.729311675	-100.558031
15	569	-0.947739322	29	-1.762407938	-1.697941173	-100.350124
16	569	-0.947739322	31	-1.762407938	-1.666936501	-106.309678
17	573	-0.861622844	33	-1.637592828	-1.666936501	-109.049468
18	574	-0.840093725	35	-1.607300602	-1.636296405	-113.525895
19	575	-0.818564605	37	-1.577370444	-1.636296405	-118.905673
20	582	-0.667860769	39	-1.377884877	-1.606019612	-116.372275
21	586	-0.581744291	41	-1.271647022	-1.546550754	-115.546109
22	586	-0.581744291	43	-1.271647022	-1.517356051	-119.927132
23	590	-0.495627814	45	-1.170929746	-1.460039358	-118.39361
24	591	-0.474098694	47	-1.146601264	-1.431914613	-121.190246
25	592	-0.452569575	49	-1.122609874	-1.431914613	-125.1717
26	594	-0.409511336	51	-1.075631702	-1.431914613	-127.884862
27	594	-0.409511336	53	-1.075631702	-1.404143735	-131.428098
28	598	-0.323394858	55	-0.98564587	-1.376725295	-129.930414
29	598	-0.323394858	57	-0.98564587	-1.376725295	-134.655156
30	600	-0.280336619	59	-0.942609992	-1.349657849	-135.243803
31	601	-0.2588075	61	-0.921574955	-1.322939927	-136.915408
32	601	-0.2588075	63	-0.921574955	-1.296570042	-139.743135
33	601	-0.2588075	65	-0.921574955	-1.296570042	-144.179425
34	602	-0.23727838	67	-0.900859369	-1.270546684	-145.484206
35	603	-0.215749261	69	-0.880461339	-1.270546684	-148.419554
36	606	-0.151161902	71	-0.821153292	-1.270546684	-148.510698
37	607	-0.129632783	73	-0.80200607	-1.270546684	-151.296351
38	608	-0.108103664	75	-0.783166573	-1.244868324	-152.102617
39	608	-0.108103664	77	-0.783166573	-1.21953341	-154.207899
40	609	-0.086574544	79	-0.764632763	-1.194540368	-154.774677
41	609	-0.086574544	81	-0.764632763	-1.074646645	-148.981632
42	610	-0.065045425	83	-0.746402574	-1.051670567	-149.240071
43	615	0.042600173	85	-0.659731997	-1.006707373	-141.647346
44	615	0.042600173	87	-0.659731997	-0.963050997	-141.18212
45	616	0.064129292	89	-0.643278815	-0.85953135	-133.750105
46	616	0.064129292	91	-0.643278815	-0.820332221	-133.188604
47	617	0.085658411	93	-0.627114118	-0.782371688	-131.08218
48	618	0.107187531	95	-0.611235642	-0.763850839	-130.633216
49	620	0.15024577	97	-0.580328194	-0.745633525	-128.618287
50	622	0.193304009	99	-0.550537893	-0.745633525	-128.32097
51	627	0.300949606	101	-0.4808243	-0.727717655	-122.062737
52	629	0.344007845	103	-0.454792221	-0.727717655	-121.798517
53	631	0.387066084	105	-0.429789015	-0.642585095	-112.599281
54	632	0.408595203	107	-0.417666811	-0.626432623	-111.718639
55	637	0.5162408	109	-0.360744813	-0.626432623	-107.602341
56	638	0.53776992	111	-0.350079528	-0.610566277	-106.631684
57	639	0.559299039	113	-0.339647629	-0.610566277	-107.374171
58	640	0.580828159	115	-0.329446368	-0.594983771	-106.309466
59	640	0.580828159	117	-0.329446368	-0.579682792	-106.368112
60	640	0.580828159	119	-0.329446368	-0.535445556	-102.922139

	61	640	0.580828159	121	-0.329446368	-0.5212471	-102.93391
	62	641	0.602357278	123	-0.319472985	-0.507318258	-101.695323
	63	641	0.602357278	125	-0.319472985	-0.507318258	-103.348905
	64	642	0.623886397	127	-0.309724699	-0.507318258	-103.764455
	65	643	0.645415517	129	-0.300198713	-0.493656575	-102.407332
	66	644	0.666944636	131	-0.290892218	-0.467124756	-99.3002236
	67	644	0.666944636	133	-0.290892218	-0.467124756	-100.816258
	68	645	0.688473756	135	-0.281802386	-0.417156524	-94.3594529
	69	646	0.710002875	137	-0.272926377	-0.417156524	-94.5413575
	70	646	0.710002875	139	-0.272926377	-0.393678873	-92.6581298
	71	646	0.710002875	141	-0.272926377	-0.382307475	-92.3879732
	72	647	0.731531995	143	-0.264261338	-0.371177475	-90.8677502
	73	649	0.774590234	145	-0.247552689	-0.329017347	-83.6026552
	74	650	0.796119353	147	-0.239503312	-0.329017347	-83.5725368
	75	652	0.839177592	149	-0.223999955	-0.290501023	-76.6606458
	76	653	0.860706711	151	-0.216540149	-0.23132371	-67.6274428
	77	653	0.860706711	153	-0.216540149	-0.223678593	-67.3534676
	78	654	0.882235831	155	-0.209271027	-0.216226961	-65.9521882
	79	654	0.882235831	157	-0.209271027	-0.188296679	-62.4181298
	80	655	0.90376495	159	-0.202189657	-0.188296679	-62.0873274
	81	656	0.92529407	161	-0.195293102	-0.146188891	-54.9786009
	82	657	0.946823189	163	-0.18857842	-0.130574125	-52.0218648
	83	657	0.946823189	165	-0.18857842	-0.074224888	-43.3625459
	84	657	0.946823189	167	-0.18857842	-0.065209739	-42.3826226
	85	660	1.011410547	169	-0.16949614	-0.065209739	-39.6652936
	86	663	1.075997906	171	-0.151944561	-0.059713445	-36.1935191
	87	668	1.183643503	173	-0.125877451	-0.030975089	-27.1354894
	88	676	1.355876459	175	-0.091643075	-0.021708975	-19.8366087
	89	677	1.377405578	177	-0.087950121	-0.019544898	-19.0266184
	90	678	1.398934697	179	-0.084378065	-0.017569194	-18.2485593
	91	679	1.420463817	181	-0.080924139	-0.012641711	-16.9354188
	92	679	1.420463817	183	-0.080924139	-0.010665929	-16.7609824
	93	683	1.506580295	185	-0.06823509	-0.007970427	-14.0980205
	94	686	1.571167653	187	-0.059826287	-0.004597815	-12.0473071
		94					
Mean		613.0212766				S = sum of O / n	-95.8080153
Std Dev		46.4487181					
						-n - s	1.80801533
						critical value at 5%	0.751
						Reject?	**Reject**
						note: we are rejecting normality thus, we don't want rejection	

Appendix 5: Hypothesis Testing

Objective: Investigate whether the mean FICO differ in the two LTV groups

$$\begin{cases} \mu_p: & Positive\ Equity\ Foreclosure.....LTV<1 \\ \mu_r: & Rational Foreclosure... \begin{cases} LTV>1.25\ if\ t<1.2 \\ LTV>1.43\ if\ t\geq 1.2 \end{cases} \end{cases}$$

Null and Alternative Hypotheses:

$$H_0: \quad \mu_p - \mu_r = 0$$
$$H_a: \quad \mu_p - \mu_r \neq 0$$

Decision Rule: Rejection means that the distributions of the two groups are significantly different. Thus FICO is not a relevant variable for foreclosures.

The summarized statistics of data are shown below, while the raw data and the distributions are shown in the next page.

	Positive Equity	**Rational Default**
Mean	610.00	640.00
Std Dev	46.45	62.19
Sample Size	94	44

Test-Statistics: T-test (justified by the number of samples used >30)

Test Requirement:

(i) Samples are independent (assumed in this case)

(ii) Samples are normal distributed (confirmed by Anderson-Darling test – see **appendix 6**)

Significant Level: 5%

Solution Table:

	t-value for our test	**Critical t-value (absolute)**
t-stat	-2.8494	2
Reject/Fail to Reject	**Reject**	

Conclusion: Therefore, we **REJECT** the null hypothesis and conclude that the FICO distributions of the two groups are **NOT** significantly different.

Data for Hypotheses testing

Positive Equity	Rational Default	Bins	Frequency Positive Equity
492	466	500	1
501	498	510	3
506	510	520	3
509	547	530	1
515	561	540	1
517	566	550	3
519	567	560	1
526	567	570	3
540	572	580	3
542	594	590	4
542	597	600	7
545	600	610	12
559	607	620	7
562	607	630	3
569	607	640	9
569	609	650	13
573	612	660	11
574	614	670	2
575	615	680	5
582	620	690	2
586	621		94
586	623		
590	626		Frequency
591	629	Bins	Rational Default
592	635	470	1
594	636	480	0
594	644	490	0
598	646	500	1
598	649	510	1
600	650	520	0
601	651	530	0
601	655	540	0
601	656	550	1
602	663	560	0
603	665	570	4
606	670	580	1
607	674	590	0
608	676	600	3
608	691	610	4
609	706	620	4
609	709	630	4
610	720	640	2
615	757	650	4
615	784	660	3
616		670	3
616		680	2
617		690	0
618		700	1
620		710	2
622		720	1
627		730	0
629		740	0
631		750	0
632		760	1
637		770	0
638		780	0
639		790	1
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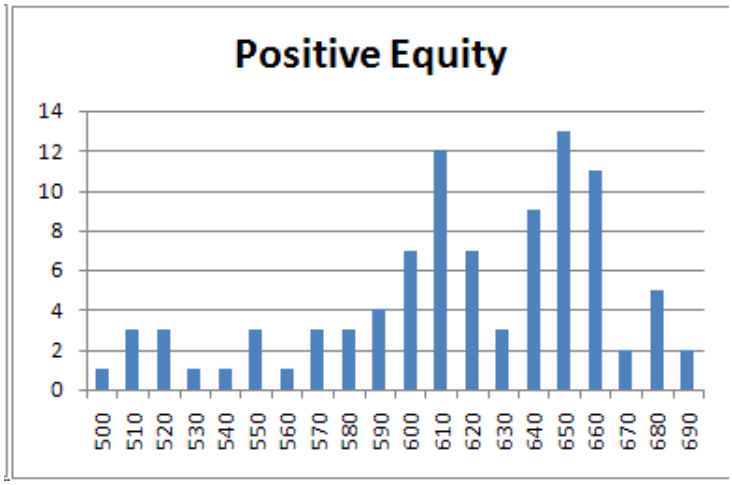

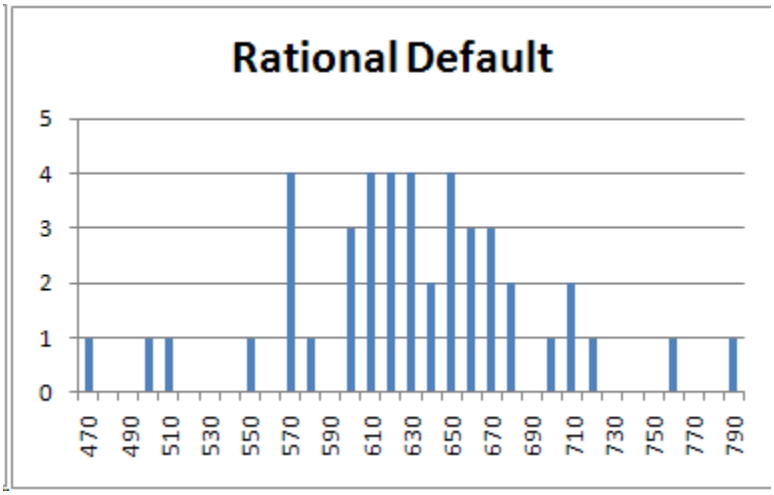

Appendix 6: Inputs (contractual interest rate, housing service flow, foreclosure cost and volatility)

Contractual Interest Rate, *c*:

Date	Contract Interest Rate (%)	Effective Interest Rate (%)
July 2006	6.53	6.58
August 2006	6.66	6.70
September 2006	6.30	6.37
October 2006	6.30	6.36
November 2006	6.31	6.37
December 2006	6.29	6.35
January 2007	6.25	6.31
February 2007	6.27	6.35
March 2007	6.15	6.22
April 2007	6.10	6.16
May 2007	6.17	6.24
June 2007	6.35	6.42
Average	6.31	

Source: National average contract mortgage rate (http://www.fhfa.gov)

Housing Service Flow, *d*:

Date	Average annual rent (imputed)	Case-Shiller after 2000 Average house price	rent-price ratio
Q4 2006	$9,643.69	$297,692.65	3.24%
Q1 2007	$9,755.61	$295,121.93	3.31%
Q2 2007	$9,844.34	$292,471.37	3.37%
Q3 2007	$9,924.76	$287,425.73	3.45%
Q4 2007	$10,030.23	$272,640.10	3.68%
		Average	3.41%

Source: "Rent-Price Ratio" (http://www.lincolninst.edu/subcenters/land-values/rent-price-ratio.asp)

Rent-to-price ratio is calculated by dividing the estimated average national rents by the house price based on the Case-Shiller House Price Index

Foreclosure Cost, *f*

We assume that the foreclosure cost follows a linear relationship as follows:

$$f(h,t) = f_0 + f_1 h(t)$$

For illustration purpose, f_0 and f_1 are set to be 0.10 and 0.15, respectively.

Volatility, σ, of house price from December 2006 to December 2009

Date	AZ-Phoenix PHXR	CA-Los Angeles LXXR	FL-Miami MIXR	NV-Las Vegas LVXR	Composite-20 SPCS20R
December 2006	221.50	270.03	280.87	231.57	203.33
January 2007	220.20	268.68	279.42	230.52	202.31
February 2007	218.07	266.63	279.43	229.64	201.57
March 2007	216.86	264.58	276.89	228.55	201.01
April 2007	215.04	263.37	273.53	226.65	200.54
May 2007	213.94	263.19	269.52	224.79	200.12
June 2007	212.52	262.12	264.89	221.86	199.44
July 2007	210.78	260.84	260.39	219.91	198.72
August 2007	208.86	258.07	255.29	216.83	197.37
September 2007	205.28	254.79	249.61	213.47	195.69
October 2007	200.72	249.50	244.35	208.68	192.98
November 2007	194.45	240.43	237.99	201.95	188.94
December 2007	187.67	233.03	231.71	196.05	184.97
January 2008	180.06	224.41	225.40	186.05	180.68
February 2008	172.72	214.83	218.74	177.18	175.96
March 2008	166.97	207.11	208.88	169.31	172.20
April 2008	161.33	202.45	200.42	165.71	169.98
May 2008	157.32	198.54	193.19	161.10	168.60
June 2008	153.19	195.70	189.87	158.80	167.78
July 2008	149.09	192.55	186.84	154.64	166.36
August 2008	144.83	189.18	183.48	150.93	164.64
September 2008	139.79	184.54	178.72	146.86	161.64
October 2008	135.18	179.82	173.42	142.74	158.09
November 2008	130.54	175.85	169.62	138.04	154.50
December 2008	123.93	171.40	165.01	131.40	150.54
January 2009	117.11	166.54	159.04	125.64	146.34
February 2009	111.89	163.16	154.28	121.06	143.11
March 2009	106.83	160.88	148.77	116.44	140.05
April 2009	104.45	159.37	145.78	112.39	139.25
May 2009	103.56	159.18	144.59	109.49	139.98
June 2009	104.73	160.90	145.38	107.31	141.97
July 2009	106.66	163.86	147.27	106.08	144.35
August 2009	108.41	166.52	148.91	105.78	146.16
September 2009	109.26	167.93	149.69	104.82	146.71
October 2009	110.71	168.43	149.09	104.70	146.59
November 2009	111.96	169.72	149.08	104.22	146.25
December 2009	112.53	171.40	148.66	104.39	145.90
Volatility from Dec 2006 to Dec 2009 (based on logarithmic change)	**0.2451**	**0.1876**	**0.1789**	**0.1853**	
			Average:	**0.1992**	

Source: S&P/Case-Shiller Home Price Indices (http://www.standardandpoors.com/indices/sp-case-shiller-home-price-indices)

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