A COMPARISON BETWEEN TWO APPROACHES TO LIFE-CYCLE INVESTING

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ABSTRACT

This paper two approaches to changing asset mix as the maturity of an investment nears. We examine two types of mutual funds to determine if they are consistent with academic work on maximizing return for a given risk tolerance. One class of funds (date-targeted) changes the asset mix through time without independently evaluating the investor's tolerance for risk while the other class of funds (risk-targeted) attempts to match the client's risk tolerance but it does not automatically adjust the asset mix as the liability associated with the investment nears.

Both fund classes are evaluated using the Markowitz mean variance framework and Kahneman and Tversky's prospect theory. In both cases, we consider human capital in the portfolios. We find it difficult to justify the prescribed asset mix for investors with less than a ten year investment horizon and propose areas for further research.

ACKNOWLEDGEMENTS

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1 INTRODUCTION

The Life-cycle fund, one of the newest types of mutual funds, is intended to simplify investment decisions after investors have evaluated their attitudes toward and capacities for risk. Investors select life-cycle funds on the basis of dates for their specific savings goals. For example, those saving for retirement would select funds targeted for the date of their expected retirement; those saving for their children's education would select funds targeted for the dates the children will need money for college. The farther into the future a target goal lies, the greater an investor's capacity for risk. The discussion in this paper has focused on retirement as the target of the investor's time horizon.

As Zvi Bodie (1992) stated "Investing for retirement is today a matter of intense concern to millions, perhaps billions, of people around the world. Today, many diverse retirement income systems coexist around the world, each relying in varying proportions on one or more of the following institutional forms:

1. Support from family or community

2. Pension plans sponsored by employers and/or labor unions

3. Social insurance program run by governments; and

4. Personal savings in the form of real and financial assets- equity in one's home or business, saving accounts, insurance contracts, mutual finds, and so on."

Many experts agree, however, that the mix of these institutional forms will change significantly in the next few years. In May 2006, HSBC has published a comprehensive study on

global attitudes to aging and retirement. Entitled 'The Future of Retirement', the study examines attitudes in countries and territories, which contain over half of the world's people and combine to give a representative sample of the global population. There are two major points uncovered by the research:

1. A growing desire to redefine how we traditionally think about later life. The research uncovered shows a major shift away from the notion that later life is dominated by traditional passive retirement. For example, people are expecting that alternating work and leisure is the ideal life-style in later life.

2. People in many countries are unsure of where to go to find appropriate retirement investment products and advice.

The new way of thinking highlighted by the HSBC research should change the way governments, companies and financial institutions deal with aging and retirement. A truly blended life plan where people can cycle through periods of work, leisure and education creates a clear need for better, smarter advisory services combined with more flexible, more transparent saving and investment products.

One purpose of this study is to examine the asset allocations of the existing 2 types of Life-Cycle Funds, characterized as either passive or active in their asset allocations, and ascertain whether the products of both types of funds match 'The Future of Retirement' research conducted by HSBC.

The most common approach is the passive fund, which is actually a series of funds with specific asset allocations in equity securities, bonds, and cash that depends on the fund's risk classification as aggressive, moderate, or conservative. Investors are expected to move among the funds according to their own risk tolerances and financial goals; that is, they would move into the

more conservative funds as their financial target dates, i.e. retirement, approach. In contrast, in an active life-cycle fund, the portfolio manager adjusts the asset allocation over time, with the portfolio becoming more conservative as the target date nears. Investors do not have to move among funds over time. The primary purpose of both types of funds is to provide investors with a simple solution to the problem of how to allocate a portfolio appropriately among the various asset classes with their differing risks.

The second purpose of the study is to suggest ways to align the practice of life-cycle fund investing with the latest scientific knowledge. Among the important insights of modern financial science are the following from Zvi Bodie's (2003):

- A person's welfare depends not only on his/her end-of period wealth but also on the consumption of goods and leisure over his/her entire lifetime.

- The value, risk, and flexibility of a person's labour earning are of first-order importance in optimal portfolio selection at each stage of their life-cycle.

- Because of transaction costs, agency problems, and limited knowledge on the parts of consumers, dynamic asset allocation will and should become an activity performed by financial intermediaries, rather than by their retail customers.

1.1 Our Approach to Evaluation

It is the objective of this paper to compare the two main approaches to life-cycle investing by drawing on the theories of choice under uncertainty. The first theory is that of Markowitz. The mean-variance framework provides a domain to find the intersection between the universe of available investments and an investor's indifference curve. The indifference curve plots the trade off between expected return and expected variance in that return. This paper will evaluate industry funds in the Markowitz framework with a quadratic utility function. We

will attempt to model decreasing human capital by reducing its relative weight in the portfolio. Investor's have limited choice to apply an asset weighting to their human capital; whether a recent graduate with no credit history or an employee approaching mandatory retirement. If human capital is included in the portfolio, it has the affect of moving the efficient frontier and therefore impacting the optimal asset weight for an investor.

Figure 1-1 Markowitz Mean Variance Framework

μ

Optimal Portfolios

Human Capital

σ
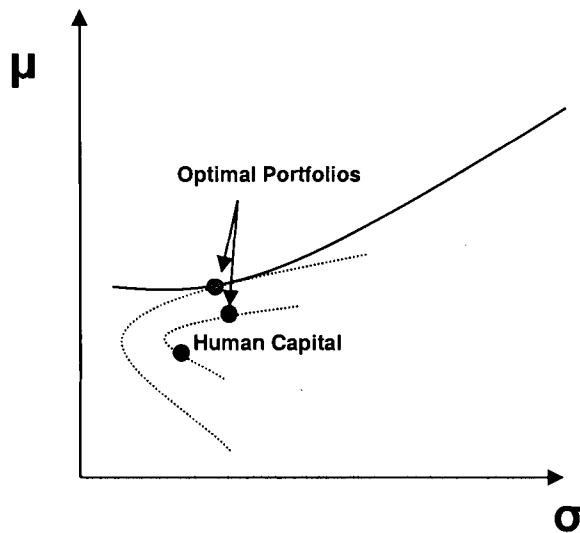
An alternate theory to choice under uncertainty was developed by Kahneman and Tversky: Prospect Theory. This theory is rooted in psychology and has been developed with the use of controlled experiments. Prospect Theory suggests that investors evaluate possible outcomes and weigh them in a certain way to determine which investment is more attractive. Because final states of wealth are log-normally distributed, prospect theory includes the dimension of time horizon in choosing the optimal asset mix (even without limiting the human capital weight over time).

We have created binomial trees that represent possible outcomes of each of the funds we are evaluating. These distribution trees have a little over 12,000 nodes. Some of the trees used represent monthly returns for 10 years while others represent quarterly returns for 30 years. We

validate that changes in risk tolerance will lead one portfolio to dominate another in the mean-variance framework. Rather, time impacts the percentage of human capital in the portfolio. Under the prospect theory framework we show how risk tolerance combined with time allow one portfolio to dominate another regardless of human capital. We employ both these frameworks to determine if they are used in the construction of industry date-targeted and risk targeted funds.

2 CHOICE UNDER UNCERTAINTY

The oldest views on money at risk were presented in 1738 by Nicolas Bernoulli. He concluded most people were risk-averse most of the time. Von Neumann and Morgenstern quantified utility theory further in 1952 and developed a model of risk tolerance that was dependent on the current wealth of an individual. Kahneman and Tversky (1979) proposed ,and backed up with empirical evidence, that decision making under uncertainty did not abide by the axioms of cardinal utility and that decision making under uncertainty was a complex function where probabilities of outcomes required weighting. These ideas presented by Kahneman and Tversky have been developed further and the models expand as anomalies are identified.

A broad survey of the wealth management business reveals a common pattern for the discovery of a client's risk tolerance so that an appropriate portfolio may be constructed. The "Know your client" or KYC survey is often a very simple questionnaire to help determine a client's tolerance toward receiving portfolio statements showing negative returns or returns below an established benchmark. "Assets under management" is the name of the wealth management business and it is imperative that a wealth manager expecting to grow a business through retention ensures that the return volatility of a client's portfolio is acceptable to a client. Because this risk tolerance is so important, it is surprising that more sophisticated tools are not employed. Modelling investor's risk tolerance and decision making under uncertainty is a domain with significant debate.

Client expectations must be managed to ensure minimal fund withdrawal. Risk modelling is a key to understanding client's expectations of their portfolio's performance so that these expectations can be set and met throughout the investor's life. This paper will review the

most common methods of modelling investor's risk tolerance and discuss the strengths and weaknesses of the models.

2.1   Main Areas of research

In 1947 Von Neumann and Morgenstern set out to "find mathematically complete principals which define rational behaviour for the participants in a social economy and derive from them the general characteristics of that behaviour". In order to develop this theory, assumptions about an individual's behaviour were made. These assumptions are known as the *axioms of cardinal utility* and they provide the rules followed by rational actors in the economy. The basic results of utility theory (marginal utility, decreasing absolute risk aversion) conform to economists models and have been used to constrain investment decisions by finance practitioners.

There was little testing of the axioms until 1979 when Kahneman and Tversky proposed a different model for decision making under uncertainty. They showed that people don't necessarily make rational decisions and several of the axioms proposed by Von Neumann and Morgenstern are broken. They proposed an alternate theory of choice under uncertainty, *Prospect Theory* but acknowledged it did not address some of the violations of axioms they had observed. Particularly, framing the same choice differently influenced decision makers significantly.

If individual decision making is not described by the axioms of cardinal utility then it becomes necessary to revisit the validity of quadratic utility in choosing an optimal portfolio. It is safe to say that the theoretical underpinnings of utility theory have been rocked by experimental evidence (Kahneman and Tversky, 1992). Work continues to be done to model investor choice in situations of uncertainty. Though psychologists offer up explanations of each anomaly to rational decision making, it has not yet been unified under one theory.

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2.2 The Axioms of Choice under Uncertainty

There are five axioms that define rational behaviour. They are the basis upon which utility theory is founded.

2.2.1 Axiom 1: Comparability

For the entire set, S, of uncertain alternatives, an individual can decide which outcome is preferred. Notations are as follows: $x \prec y$ (y is preferred to x) or x ~ y (indifference between x and y)

2.2.2 Axiom 2: Transitivity

If an individual prefers x over y and y over z then the individual should prefer x over z. Similarly, if x ~ y and y ~ z then x ~z.

2.2.3 Axiom 3: Strong Independence

An individual will also be indifferent to a gamble between where x and y are the outcomes if pitted against a similar outcome z.

If x ~ y, then G(x,z: α) ~ G(y,z: α)

2.2.4 Axiom 4: Measurability

If outcome y is preferred more than z but less than x, then there is a unique probability (α) where an individual will be indifferent between y and a gamble with the probability of x equal to α and the probability of z as (1- α).

If $x \prec y \prec z$ or $x \succ y \succ z$ then there exists a unique α such that $y \sim G(x,z : \alpha)$

2.2.5 Axiom 4: Ranking

If four outcomes exist in order of preference: w, x, y, z then gambles can be constructed

between w and z such that an individual is indifferent between x and a gamble (x ~ $G(w{:}z{:}\alpha_1)$)

and a separate gamble can be constructed such that the individual is indifferent between y and a

gamble (y ~ $G(w{:}z{:}\alpha_2)$). It follows that if $\alpha_1 > \alpha_2$ then y is preferred to x.

Though these axioms are reasonable at first read, people are assumed to make these

rational choices among 1000s of alternatives in the market. This is not an easy task. To

compound the problem further, many of these axioms have been shown to be violated even

among a very few alternatives.

2.3 Modelling Risk: Utility Theory

Utility theory was Von Neumann and Morgenstern's attempt at modelling an individual's

preference between uncertain options. The function of utility is a function of wealth, so that for a

different level of wealth, an individual may have different preferences. It bears repeating that

utility is a function of wealth and the possible outcomes are evaluated in this backdrop. Utility is

not a function of the change in wealth.

Figure 2-1 - Risk Neutral U(W) is constant
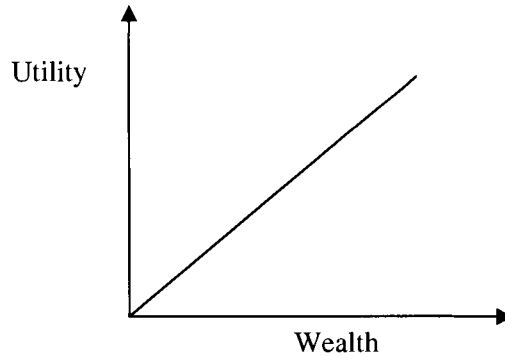
Figure 2-2 - Risk Adverse (dotted) and Risk Seeking (solid)
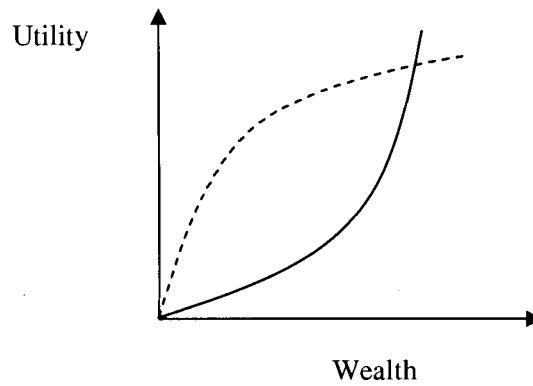
In the case of the risk-averse individual, the utility of more wealth decreases with wealth. The opposite is true for the risk-seeker. The risk neutral individual makes actuarially sound choices. It should be noted that the more frequent the gamble, the more likely the expected value will be the mean value of the gambles. This consideration of frequency of the gamble is not discussed in this paper. Being presented with G(x,y:α) with an initial wealth of W_0, the utility of the gamble is represented by:

$$U(G(x, y : \alpha)) = \alpha U(W_0 + x) + (1 - \alpha)U(W_0 + y) \qquad \textbf{Equation 1}$$

If x or y is negative the gamble may provide negative utility even if the expected value of wealth is greater than zero. The beauty of utility theory was that it allowed economists to model this seemingly irrational decision making behavior as risk tolerance. If only one gamble is presented, this is considered rational and explained as risk aversion. If an infinite number of gambles are presented, this seems irrational as the expected value of the gamble is greater than zero.

2.4 Challenges to the Axioms

In expected utility theory, the utilities of the gamble are a weighted sum of the individual outcomes' utilities. Kahneman and Tversky proposed a series of choice problems where people's preferences violate this principle. Most of these violations are best illustrated with examples.

2.4.1 Certainty Effect

Problem 1:

Gamble A		Gamble B	
Outcome	Likelihood	Outcome	Likelihood
$2500	.33	$2400	1
$2400	.66		
$0	.01		
18% of Respondents choose A		82% of Respondents choose B	

Problem 2:

Gamble C		Gamble D	
Outcome	Likelihood	Outcome	Likelihood
$2500	.33	$2400	.34
$0	.67	$0	.66
83% of Respondents choose C		17% of Respondents choose D	

Both these preferences were shown to be significant at the .01 level. The violation of cardinal utility is clearly expressed in equation form. From Problem one:

$$u(2400) > .33u(2500) + .66u(2400) \text{ or } .34u(2400) > .33u(2500)$$

But, from Problem 2, recalling u(0)=0, we find the opposite inequality:

$$.33u(2500) + .67u(0) > .34u(2400) + .66u(0) \text{ or } .34u(2400) < .33u(2500)$$

The substitution (transitivity) axiom of utility theory is violated. Strong independence asserts that if X is preferred to Y then the any gamble involving X and Y should always have a preference of (X, α) over (Y, α).

2.4.2 Reflection Effect

Problem 3:

Gamble A		Gamble B	
Outcome	Likelihood	Outcome	Likelihood
$4000	.8	$3000	1
$0	.2		
20% of Respondents choose A		80% of Respondents choose B	

Reflecting this problem in the negative domain yields opposite results and is referred to as the reflection effect.

Problem 3':

Gamble A		Gamble B	
Outcome	Likelihood	Outcome	Likelihood
-$4000	.8	-$3000	1
$0	.2		
65% of Respondents choose A		35% of Respondents choose B	

This effect implies that risk aversion in the positive domain is accompanied by risk aversion in the negative domain. One resolution to the overweighting of certainty in the positive domain is discussed by Allais, Markowitz ant Tobin. That is to say, individuals prefer prospects with high expected value and lower variance.

2.4.3 Isolation Effect

Problem 4: Consider a two stage game where in the first stage there is a 75% chance of ending with nothing and a 25% chance of getting to the second stage. If you reach the second stage you have the following choice:

Gamble A		Gamble B	
Outcome	Likelihood	Outcome	Likelihood
$4000	.8	$3000	1
$0	.2		

The choice must be made before the first stage starts. The second choice (B) is dominant in Kahneman and Tversky's testing of this problem. However if one follows the probabilities through .25*.8 = .2 and .25*1=.25, one can reconstruct the problem as follows.

Gamble A		Gamble B	
Outcome	Likelihood	Outcome	Likelihood
$4000	.2	$3000	.25
$0	.8	$0	.75

Empirical testing of choice under this problem shows a dominance of choice A. This significant finding shakes the rational decision making framework as evidence that choices between prospects are made based on the probability of their final states. This anomaly is not addressed but not captured in the theory proposed by Kahneman and Tversky.

2.5 Prospect Theory

Prospect theory attempted to provide a model for decision making under uncertainty that would consider most of the effects discussed above. Prospect theory distinguishes two phases in the choice process: an early phase of editing and a subsequent phase of evaluation. The editing phase is broken down as follows:

Coding – people perceive outcomes as gains or losses rather than final states of wealth.

Combination – prospects can be simplified by combining similar outcomes.

Segregation – Some prospects contain a riskless component that is segregated from the risky component in the editing phase.

Cancellation – Isolation effects described above in problem 4 cause individuals to disregard common parts of the choices.

Following the editing phase, the individual evaluates each edited outcome and chooses the one with the highest value as defined by the following:

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y) \quad \text{Equation 2}$$

The prospect function is similar to the utility function except that the probabilities are not used to weight the outcomes directly. They are adjusted with the π function. The π function accounts for overweighting of outcomes with small probabilities. It can also be seen from equation 2 that the prospect theory value is not a function of wealth but rather a function of the gains or losses presented to the individual. The following diagram shows how the π function helps to weight small probability events.
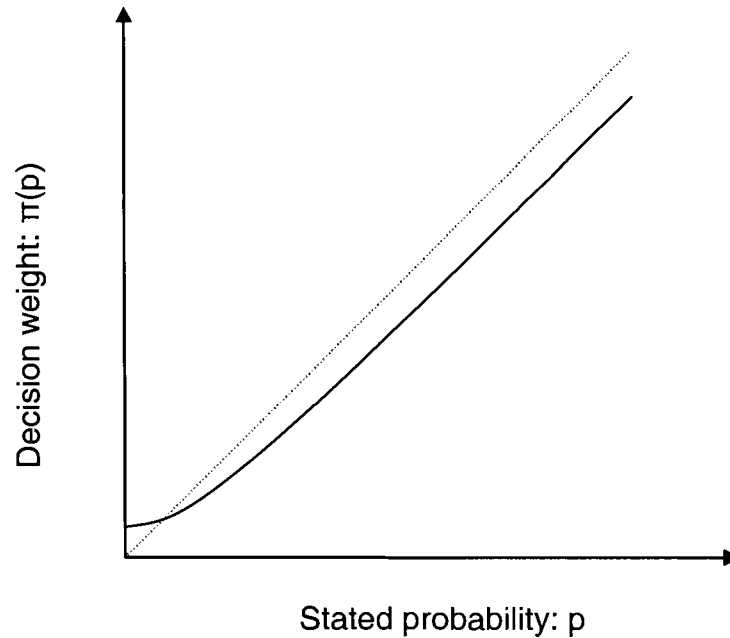
Figure 2-3 – A hypothetical weighting function

The assumption is that π(p) ≠ p. Zeckhauser gives an illustration. Suppose you are compelled to play Russian roulette. What would you pay to have a bullet removed if there are three bullets in the gun? What would you pay to have the bullet removed if there was only one bullet in the gun? A serious objection to the non-linearity of π involves the potential violations of dominance. Direct violations of dominance are addressed by the theory by the assumption that dominated alternatives are detected and eliminated prior to the evaluation of prospects.

Kahneman and Tversky conclude that individuals are typically risk averse in the positive domain and risk seeking in the negative domain as illustrated in Problems 1 and 2 earlier. The following diagram illustrates this phenomenon. Not only is the value function concave in the negative domain, it also has a steeper slope in the negative domain. The value function is not

dependent on the initial wealth of the investor. This is a key distinction from the utility theory

developed by Von Neumann and Morgenstern.
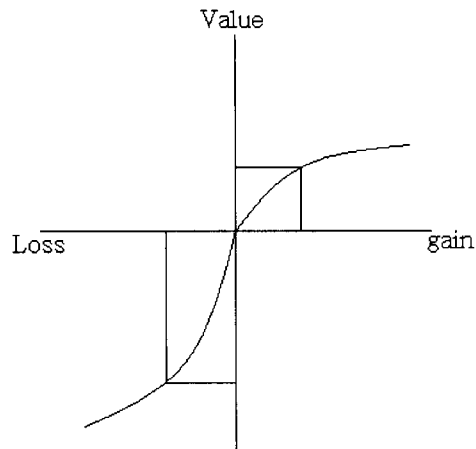
Figure 2-4 - Value of Outcomes in Prospect Theory

2.6 Cumulative Prospect Theory

After much debate and clarification regarding their original paper, Kahneman and

Tversky addressed some of the behavior they had identified but did not model in their original

proposal of prospect theory. In 1992, they published "Advances in Prospect Theory: Cumulative

Representation of Uncertainty" where they develop a model of prospect theory that employs

cumulative decision weights. They further break down risk attitudes into a four fold pattern: risk

aversion for gains and risk seeking for losses of high probability; risk seeking for gains and risk

aversion for losses of low probability.

They set out that there exist 5 phenomena that must be addressed by any comprehensive

theory of choice.

Framing Effects – there should exist no descriptive variance yet Kahneman and Tversky,

1986 show significant effects.

Nonlinear Preferences – According to utility theory, the utility of a prospect is a linear combination of the probabilities of the individual outcome utilities. However, Allais (1953) showed that the difference between probabilities .99 and 1 and .10 and .11 are significant.

Source Dependence – People are more inclined to bet on uncertain events where they are familiar with the event rather than considering only the probability. Ellsberg (1961) and Heath and Tversky (1991) indicate people prefer to bet on an event in their area of competence over a matched chance event even if the competent bet has a more certain probability and the other is vague.

Risk Seeking – Risk aversion is generally assumed in economic analyses of decision under uncertainty. It has been shown that risk seeking is prevalent when people must choose between a sure loss and a substantial probability of a larger loss.

Loss Aversion – Losses loom larger than gains in both utility theory and prospect theory. However, Kahneman and Tversky assert that the asymmetry between gains and losses is far too extreme to be explained by income effects or by decreasing risk aversion.

Cumulative prospect theory attempts to model all these phenomena. It asserts that there exists a strictly increasing value function V and weights W^+ and W- that are applied for different values of π (the probability weighting function)

$$U(G) = \sum \pi(p_i)v(x_i)$$

where

$v(x) = x^{\alpha}$ if $x \geq 0$

$v(x) = -\lambda(-x)^{\beta}$ if $x < 0$

and

$\pi(p) = w^+(P) - w^+(P^*)$ if $x \geq 0$

$\pi(p) = w^-(P) - w^-(P^*)$ if $x \geq 0$

where

P^* is the probability that an outcome is strictly greater than x_i

P is the probability that an outcome greater than or equal to x_i

Figure 2-5 - Statement of Cumulative Prospect Value by Kahneman and Tversky

Recalling that distinct domains of probability weighting exist, the weights applied to the π

function are described as follows:

$$w^+(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}}, \quad w^-(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}}.$$

Equation 3 - Weights for the probability weighting in positive and negative domains

The weighting of the probability weighting can be viewed graphically. These results are

very interesting and show a consistency to overweight small probabilities but to distinguish

between the cumulative effects in the positive and negative domains.
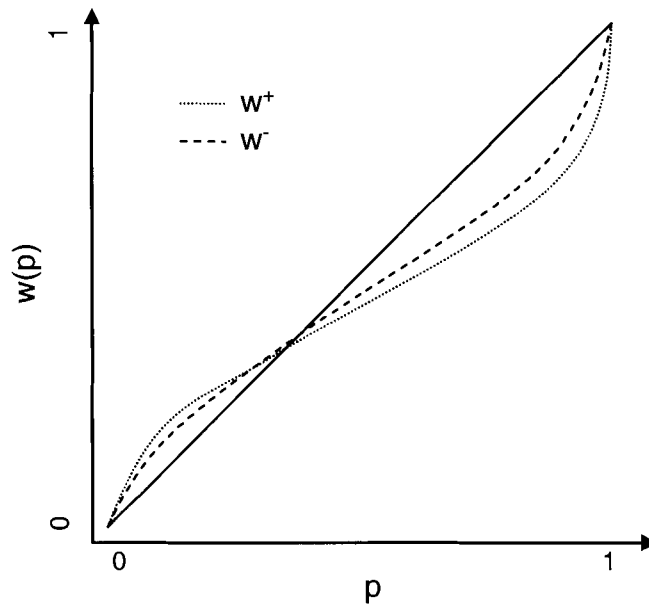
Figure 2-6 - Weighting functions for gains (w+) and losses (w-) based on gamma and delta estimates by Kahneman and Tversky

Kahneman and Tversky showed that a sure gain of $100 is equally attractive as a 71%

chance to win $200 or nothing, and a sure loss of $100 is equally as averse as a 64% chance to

lose of $200 or nothing. They point out that the standard applications of expected utility theory

include probability distributions around a given wealth, the valuation rule is expected utility and

that utility is a concave function of wealth. The empirical evidence they present requires major

revisions of all three elements.

3 ANALYSIS OF RISK-TARGETED FUNDS

We analysed three risk targeted funds from HSBC. We first looked at their risk-reward

trade off in the single period mean-variance framework. This confirmed that one portfolio will

dominate depending on the risk tolerance of the investor. We then looked at the same portfolios

using prospect theory to evaluate the utility of the portfolios over time. This confirmed that

portfolios that may dominate for short investment horizons do not necessarily dominate for longer

time horizons. Finally, we considered human capital and adjusted its weight in the portfolio to

simulate the depleting earning power of an individual as they near retirement. This human capital

adjustment was made in both the mean variance and prospect theory setting.

Table 1 HSBC Risk Targeted Funds

	Fund Name		
	Moderate Conservative	Balanced	Growth
Domestic Equity	20%	27.5%	35%
Foreign Equity	15%	27.5%	40%
Bonds	57.5%	45%	25%
Cash	7.5%	0%	0%

Source: HSBC Asset Management.

The expected rate of return for each asset class was ranked cash, bonds, domestic equity

and foreign equity. The expected standard deviation of each asset class followed the reverse

order as would be expected. When these assets are combined in a portfolio, the following returns

and standard deviations can be expected.

Table 2 Risk-targeted expected returns and standard deviations (monthly)

	Fund Name		
	Moderate Conservative	Balanced	Growth
Expected Return	.0079	.00825	.00845
Std. Deviation	.016492	.023148	.030622

These returns were generated using QOS software and historic data. Specifically, we used SCUBI index for bond data, the TSX 60 for the domestic equity data, the 90 day t-bill rate for cash data and the MSCI EAFE index for foreign returns. The time horizon for the data analyzed was from January 1985 to January 2004. All returns were calculated in Canadian dollars.

3.1 Mean-Variance Analysis of HSBC Funds without Human Capital

As would be expected, the different funds appeal to different investors with different appetites for risk. We have based our single period utility function on the function used by the QOS optimization software.

Equation 4 - Utility function for single period mean-variance optimization

$$U = E(r_P) - \frac{\lambda}{2}\sigma_P^2$$

Table 3.2   Values of λ for each of HSBC's funds

	Fund Name		
	Moderate Conservative	Balanced	Growth
Lambda	1.85	1.65	1.48

Kritzman shows that asset mix should not change with time horizon with simple utility functions. He provides the following reasons as the only justification for changing asset mix through time:

1. The investor believes in mean reversion

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2. The so-called "riskless" asset is exposed to the disasters that would cause extreme returns

3. Even if an investor believes returns are random, they may still choose to accept more risk over the longer horizons than short horizons because they have more discretion to adjust consumption and work habits.

4. An investor has a discontinuous utility function.

5. An investor is irrational. Kritzman says that this does not make the investor a bad person rather that the investor simply behaves inconsistently.

Based on the work of Kritzman, an investor should remain in the same asset mix until the liabilities consume the investment. Kritzman would suggest that once the advisor has established the client's tolerance for risk, he would be serving his client well to maintain the same asset mix throughout the investor's life.

We created a binomial tree where each step in the tree represented 1 month. Using the utility function in equation 4 and calculating the utility at each step in the tree, we observe that the utility at each step is constant. By varying the value of lambda in equation 4 any portfolio can be made to dominate any other portfolio for all time horizons.

3.2 Prospect Theory Analysis of HSBC Funds without Human Capital

Prospect Theory would qualify as a discontinuous utility function as described by Kritzman. We use the same binomial tree that represents the possible outcomes of each of HSBC's funds and evaluate the utility under prospect theory at each step.

Figure 3-1 First 6 months of tree representing possible returns for HSBC Balanced fund

```
                                                                        114.8995
                                                        112.27039
                                        109.7014                        109.7014
                        107.1912032                     107.1912
            104.7384                    104.7384                        104.7384
102.3417994             102.3417994                     102.3418
100             100                     100                             100
97.71178596             97.71178596                     97.711786
            95.47593                    95.47593                        95.47593
            93.2912375                  93.291238
                                        91.15653                        91.15653
                                        89.070678
                                                                        87.03255
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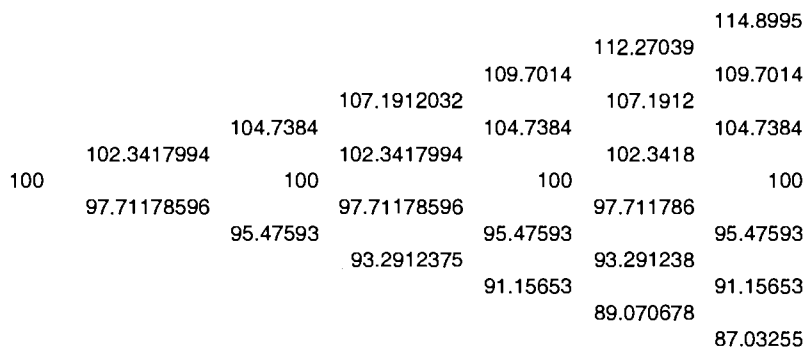
Step (month)	1	2	3	4	5	6
Utility Under Prospect Theory	-0.920993355	-0.42686	-0.070630281	0.714483	1.2648662	2.056607

There are several variables in prospect theory (Please refer to figure 2.5). Our initial valuation of utility for each step in the binomial tree was based on the coefficients Kahneman and Tversky had produced in their experiments. These coefficients represent the average investor's decision making under uncertainty. The following figure (3-2) shows how the utility functions of the three portfolios between step 6 and 18 (from month 6 to month 18). It can be seen that the conservative portfolio has a higher utility when the investment horizon is less than 12 months. However, as the investment horizon goes beyond about 14 months, the growth portfolio dominates. This leaves a very small space for the balanced portfolio between 12 and 14 months. Since prospect theory considers framing effects, gambles are based on predicted gains or losses relative to the wealth at the time of evaluating the gambles. As an example, an investor with an initial horizon of 25 years may evaluate their portfolio annually. It would not be until the final year (year 24 with 12 months remaining) that the investor would choose the balanced portfolio over the growth portfolio. However, if the investor where to re-evaluate their choice again at the 9 month mark, they would elect to switch to the conservative portfolio.

Figure 3-2 Utility of each of HSBC's funds when Lambda = 2.25, Alpha and Beta = .88, Gamma = .61 and Delta =.69

6-18 months Utility of HSBC Portfolios under Prospect Theory - Average Risk Aversion
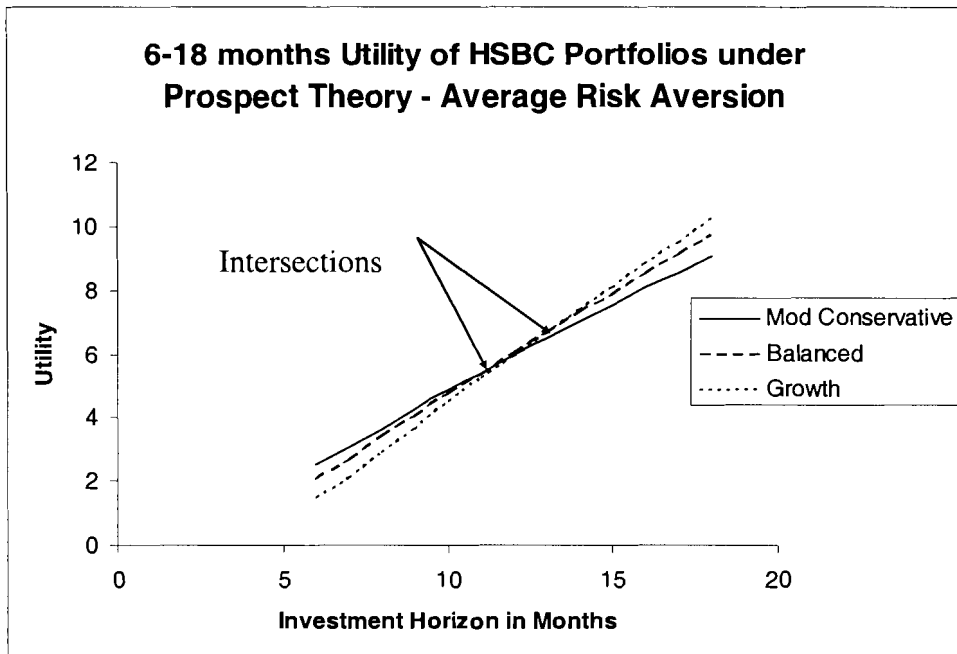
We are assuming that risk tolerance is constant over time. The dominance of one portfolio over the other under prospect theory stems from the log-normal distribution of final values that serves to devalue the weight of negative returns over longer time horizons. We continued with this experiment and adjusted the downside risk aversion co-efficient (λ) to see the affect on the intersection of the portfolios. The results follow. The returns and standard deviation of the HSBC portfolios are relatively clustered. Increasing risk aversion pushes the intersection of the portfolios to further time horizons. It is peculiar that these portfolios intersect between 19 and 23 months in a risk adverse situation. The returns on the funds are very tightly clustered based on the index data we choose to model their returns. We suspect that having funds with significantly different return profiles would cause these intersection points to spread out to longer time horizons.

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Figure 3-3 Utility of each of HSBC's funds when Lambda = 1, Alpha and Beta = .88, Gamma = .61 and Delta =.69
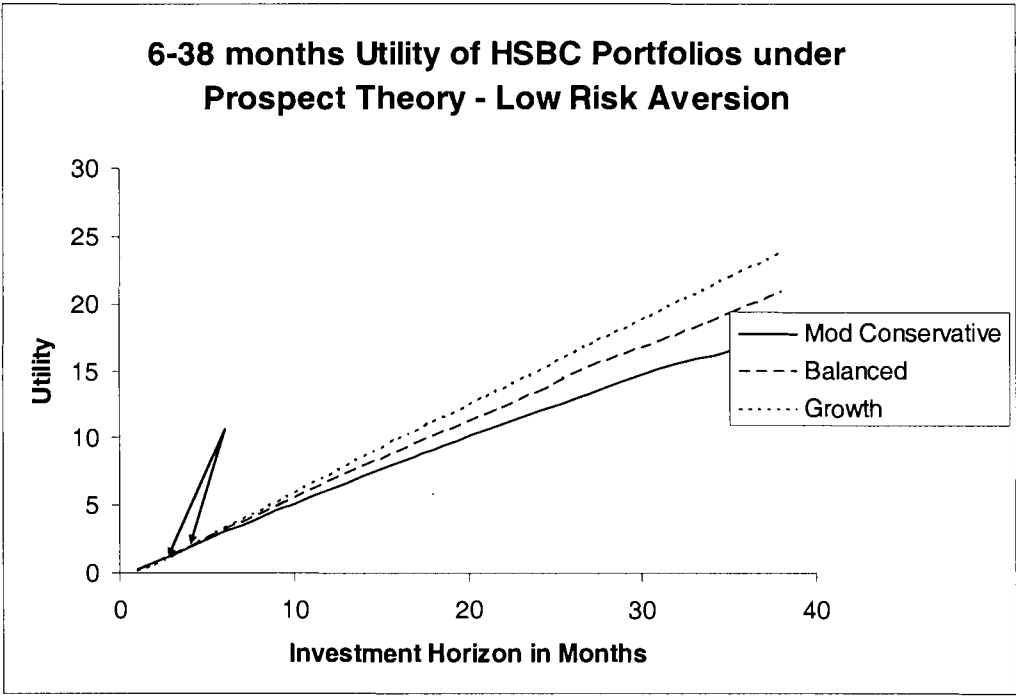
Figure 3-4 Utility of each of HSBC's funds when Lambda = 5, Alpha and Beta = .88, Gamma = .61 and Delta =.69
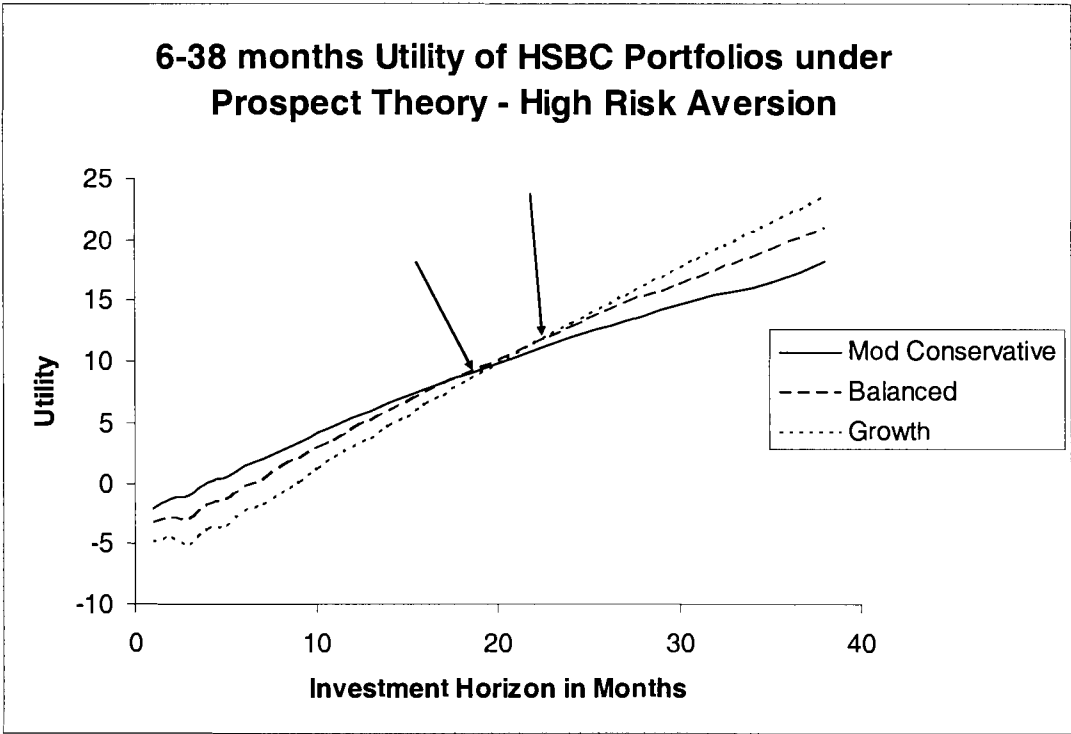
3.3 Mean-Variance Analysis of HSBC Funds considering Human Capital

Human capital is defined as earning potential of the investor. One can picture investors with significant human capital but otherwise small portfolios (i.e. recently graduated physicians). For our evaluation, we assume that the weight of human capital in most investor's portfolio is not a choice after the completion of formal education and work habits have developed. We assume that human capital decreases over time and that total savings increase. The investor has a choice regarding the asset mix of her savings but not regarding the percentage of human capital in her portfolio.

Table 3 Hypothetical investor starting with only human capital and a 10% savings rate

Year	Discounted future earnings	Discounted Future savings potential	Portfolio	Percentage in Human Capital	Available for Allocation
1	$2,425,427.39	$242,542.74	$10,000	96.04%	3.96%
2	$2,398,190.21	$239,819.02	$20,800	92.02%	7.98%
3	$2,370,135.92	$237,013.59	$32,464	87.95%	12.05%
4	$2,341,240.00	$234,124.00	$45,061	83.86%	16.14%
5	$2,311,477.20	$231,147.72	$58,666	79.76%	20.24%
6	$2,280,821.51	$228,082.15	$73,359	75.66%	24.34%
7	$2,249,246.16	$224,924.62	$89,228	71.60%	28.40%
8	$2,216,723.54	$221,672.35	$106,366	67.58%	32.42%
9	$2,183,225.25	$218,322.52	$124,876	63.61%	36.39%
10	$2,148,722.01	$214,872.20	$144,866	59.73%	40.27%
12	$2,076,579.18	$207,657.92	$189,771	52.25%	47.75%
14	$2,000,042.85	$200,004.28	$242,149	45.23%	54.77%
16	$1,918,845.46	$191,884.55	$303,243	38.75%	61.25%
18	$1,832,703.15	$183,270.31	$374,502	32.86%	67.14%
20	$1,741,314.77	$174,131.48	$457,620	27.56%	72.44%
25	$1,487,747.49	$148,774.75	$731,059	16.91%	83.09%
30	$1,193,793.51	$119,379.35	$1,132,832	9.53%	90.47%
35	$853,020.28	$85,302.03	$1,723,168	4.72%	95.28%
40	$457,970.72	$45,797.07	$2,590,565	1.74%	98.26%
45	$0.00	$0.00	$3,865,056	0.00%	100.00%

The values in this table were generated for an individual earning $100,000 per year indexed to inflation at 3%. We assumed a 10% annual savings rate and that the portfolio would grow at 8%.

Table 2 illustrates how human capital can dominate the portfolio of an investor at the beginning of her career. If we add the human capital asset to the single period optimization problem, we find that risky assets dominate the portfolio when a human capital has a significant weight in the portfolio. The quantitative analysis of which assets dominate follows in the analysis of target-date funds. In the context of HSBC funds, the assets that dominate the optimization are closest to those of the growth portfolio (for which we simulated results) for all but the last year of the investment horizon.

3.4 Prospect Theory Analysis of HSBC funds considering Human Capital

For this analysis, we reproduced the binomial tree for our theoretical investor saving $10,000 / year. Recalling that under prospect theory, the investor frames his options based on the changes in wealth from a reference point, we use the expected value of a step as the starting node in the evaluation of future.

Figure 3-3 First 6 months of binomial tree representing possible returns for HSBC Balanced fund with Savings – first year's initial savings of $10,000

							17404.99
						16072.06	
					14780.22		16440.53
				13528.2614		15185.113	
			12315		13968.4		15579.79
		11139.28076		12789.25075		14397.289	
10000			11646.54		13251.23		14815.25
		10539.19075		12140.54954		13701.315	
			11064.19		12621.71		14139.85
				11575.40606		13090.391	
					12073.26		13546.98
						12558.159	
							13030.48

Investment Horizon (months)	1	2	3	4	5	6
Utility under Prospec Theory	357.8545012	649.5036	916.8899546	1169.272	1410.6013	1642.881

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Figure 3-4 First 6 months of binomial tree of HSBC Balanced fund with Savings being evaluated in the 20th year's initial value with an estimated wealth of $457,600 and the savings rate is indexed to inflation

```
                                                                          557682.9
                                                         539722.5
                                            522296                        525294.7
                              505387.5759               508380.04
                 488981.9                  491968.2                       494878.5
    473064.2288              476044.2582               478948.55
457620           460593.8                  463492.1                       466316.9
    445602.9101              448495.1422               451314.06
                 433944.1                  436757.2                       439499.1
                              422633.0461               425369.28
                                            411659.4                      414321.1
                                                         401013.27
                                                                          390684.9
```

Investment Horizon (months)	1	2	3	4	5	6
Utility under Prospec Theory	-1625.452121	-544.9561	407.1366535	2044.383	3474.3424	5084.899

Recalling that the utility of the initial wealth is 0, we can see in figures 3-3 and 3-4 that for short investment horizons investors will behave differently depending on the relationship between the current size of their portfolio and their savings rate. There is an opportunity to develop an indifference plane incorporating savings rate and human capital into the prospect theory evaluation of asset mix. The simplest framework to evaluate the utility of investment options would assume a particular savings rate as a percentage of earnings. Initially, we expect the efficient frontier to be significantly smaller because of the dominance of one asset, human capital. As the liability approaches, the ratio of the periodic savings to the reference wealth used for evaluation will also decrease. This has the effect of making short term investments in risky portfolios less appealing as the time horizon decreases.

3.5 Operational Considerations of Risk-Targeted Funds

Target Risk Funds feature an asset mix determined by the level of risk and return that is appropriate for an individual investor. Many of the considerations that we try to separate in our analysis can be combined to determine an appropriate asset mix for a forecast liability. Factors that determine this mix include an investor's age, level of risk aversion, the investment's purpose and the length of time until the principal will be withdrawn. These funds are usually

focus more on investor's risk tolerance and split into three groups: aggressive, moderate, and conservative. It is up to the investor to decide when they want to switch from one to the other, so for retirement, someone might start with the aggressive fund, then switch to moderate halfway towards retirement and then conservative when they are a few years from retirement.

Target Risk Funds do offer some flexibility to the investors to switch between portfolios at any time. They do not assume all investors at the same age will have the same retirement needs and expectations. In this sense the target risk funds seems to be a more tailored solution for individuals who have different risk tolerance and different rations of human capital to the rest of their portfolio.

However, Target Risk Funds require more involvement from individual investors in the sense of deciding when to switch portfolio and to what they will switch. People generally believe they are made better off when offered more choices or more flexibility. But when people do not have the knowledge to make choices that are in their best interests, increasing the flexibility does not necessarily make them better off. So Target Risk Funds is more suitable to more sophisticated investors or investors who have a very good Investment Advisor;

Target Risk Funds require knowledgeable Investment Advisors and/or Financial Planners who understands the implication of the prospect theory utility function, the events and periods of time to be most sensitive to changing risk tolerance. Investors could be vulnerable to well-intentioned but unqualified advisors.

4 Analysis of Target date funds

Target date funds are designed on the basis of dates for the investor's specific savings

goals, such as year 2020, 2025, or 2030. Those saving for their children's education would select

funds targeted for the dates the children will need money for college; those saving for retirement

would select funds targeted for the date of their expected retirement date.

In this part of the report, we evaluate the appropriateness of time-dated funds. We looked

at the asset mix of Fidelity's Clear Path products. The following table was published in 2005 so

the ClearPath 2005 fund is the terminal asset mix. Fidelity will keep these funds around after the

date stamp and they will continue to adjust the asset mix until it is essentially the same fund as

the ClearPath Income fund. We evaluate the effect of risk tolerance, time horizon and human

capital on the optimal asset mix under mean-variance and prospect theory.

Table 4 - Asset Mix of Fidelity ClearPath Portfolios

Portfolios	Equity	Fixed Income	Cash & Short-Term
Fidelity ClearPath 2005	36%	35%	29%
Fidelity ClearPath 2010	44%	44%	12%
Fidelity ClearPath 2015	50%	46%	4%
Fidelity ClearPath 2020	63%	37%	*
Fidelity ClearPath 2025	69%	31%	*
Fidelity ClearPath 2030	80%	20%	*
Fidelity ClearPath 2035	81%	19%	*
Fidelity ClearPath 2040	84%	16%	*
Fidelity ClearPath 2045	85%	15%	*
Fidelity ClearPath Income	30%	35%	35%

Based on the same returns series used to evaluate the HSBC funds, we determine the expected return and standard deviation of each of these funds.

Table 5 – Expected Returns of Fidelity ClearPath Portfolios

Portfolios	Expected Monthly Return	Monthly Standard Deviation
Fidelity ClearPath 2005	.005274	.076956
Fidelity ClearPath 2010	.006529	.09498
Fidelity ClearPath 2015	.007147	.10496
Fidelity ClearPath 2020	.007619	.12301
Fidelity ClearPath 2025	.007649	.12598
Fidelity ClearPath 2030	.007785	.1410
Fidelity ClearPath 2035	.007810	.14463
Fidelity ClearPath 2040	.007835	.14642
Fidelity ClearPath 2045	.007843	.14823

4.1 Mean-variance Analysis of Fidelity Funds without Human Capital

The obvious question regarding these portfolios in a single period setting is whether or not these portfolios are mean-variance efficient. From 2020 to 2045, the portfolios are mean-variance efficient and they plot on an efficient frontier matching different levels of risk aversion. However, from the 2015 portfolio through to the 2005, one can observe that the ratio of equity to fixed income changes. Assuming the cash and short term instruments are a proxy for the risk free asset, one would expect the ratio of equity to bonds to remain constant while adjusting the weight of the risk free asset. The change in this ratio is not consistent with the capital asset pricing model.

4.2 Prospect Theory Analysis of Fidelity Funds without Human Capital

We attempted to simulate the preference of one asset mix over another based on the investment horizon. To accomplish this, we generated binomial trees for a pair of funds and calculated their utilities at each step of the process. We were not able to demonstrate the preference of one asset mix over another simply by adjusting the risk variables of the utility function. Our evaluation was not exhaustive but it provided us confidence that prospect theory without considering human capital is not the justification for the changing asset mix exhibited by the Fidelity funds. As an example, it was required to set lambda to 213 to establish a dominant utility of the 2010 fund over the 2015 fund for investment horizons less than 5 years. Recall that lambda is the multiplier effect the weighs losses with greater significance than gains. In Kahneman and Tversky experiments, lambda for the experimental population was a 2.25. This is a far cry from the values we required to establish the dominance of the 2010 portfolio over the 2015 portfolio for 5 year investment horizons. It must be concluded that the asset mix adjustments made by these funds are not consistent with applying prospect theory to an investor that represents the median risk tolerance.

Table 6 – Required Risk tolerance to explain switch in asset classes at the 5 year mark

Portfolios	Risk aversion required for transition to next fund (λ)
Fidelity ClearPath 2010	213
Fidelity ClearPath 2015	782
Fidelity ClearPath 2025	

4.3 Mean-variance Analysis of Fidelity Funds with Human Capital

Can the depleting human capital resource explain the change in asset mix we observe in the Fidelity ClearPath funds? To answer this question, we used the values from Table 2 combined with the asset class returns inside the QOS optimizer. Even with a consistent risk tolerance, we saw a dominance of the riskiest asset when human capital made up a large part of

the portfolio and a gradual diversification into other assets as the human capital component of the portfolio depleted.

Table 7 Asset Allocation Optimized including Human Capital

			Asset Allocation of Investible Portfolio		
Years Employed	% HC	%Portfolio	Stocks	Bonds	Risk Free
1	96.04	3.96	100.00	0.00	0.00
10	59.73	40.27	100.00	0.00	0.00
20	27.56	72.44	90.28	9.72	0.00
30	9.53	90.47	72.83	27.17	0.00
40	1.75	98.25	66.83	33.17	0.00
45	0.00	100.00	63.26	36.74	0.00

This table was created from optimizations run in QOS. We assume that the variance on the human capital portion of the portfolio is zero and the return on the portfolio is the risk free rate. Years employed would pair up with fidelity funds forecasting retirement so 1 year of employment may match up two ClearPath 2040.

It is worth discussing the very simple approach we took to modelling human capital. In the QOS optimizer we assumed that human capital had a variance of zero and a return equal to the risk-free rate. Human capital is likely to be an appreciating asset in the early years of one's career as raises and promotions outpace the reduction in length of the annuity and then a depleting asset as one approaches then last half of one's career. There is also variance associated with human capital. The security of some jobs is more volatile than others. Because human capital under the mean variance framework can be used to justify the target-dated asset mix change, modelling it to properly reflect the client's human capital becomes a critical focus of the advisor. If the client is an undertaking an entrepreneurial venture with a probability of failure and therefore high variance, one may be prudent to have assets in cash.

4.4 Prospect Theory Analysis of Fidelity Funds with Human Capital

We established earlier (in Section 3.4) that prospect theory does not provide the justification for changing asset mixes significantly for time periods over 2 years. We attempt to explain the asset mix from years 1 to 10 using prospect theory. As mentioned previously, the

ratio of stocks to bonds should not change as the percentage of the portfolio in the risk-free asset

changes under the mean variance framework. We created the binomial trees for each of the three

portfolios (2005, 2010 and 2015) including the contributions of human capital. The starting node

is relatively large compared with the monthly contribution from human capital. The starting node

of wealth based on table 2 is $1,723,168 with a projected monthly savings $2027.

Figure 4-1 Utility of three Date-Target funds under Prospect Theory with Average Risk Aversion
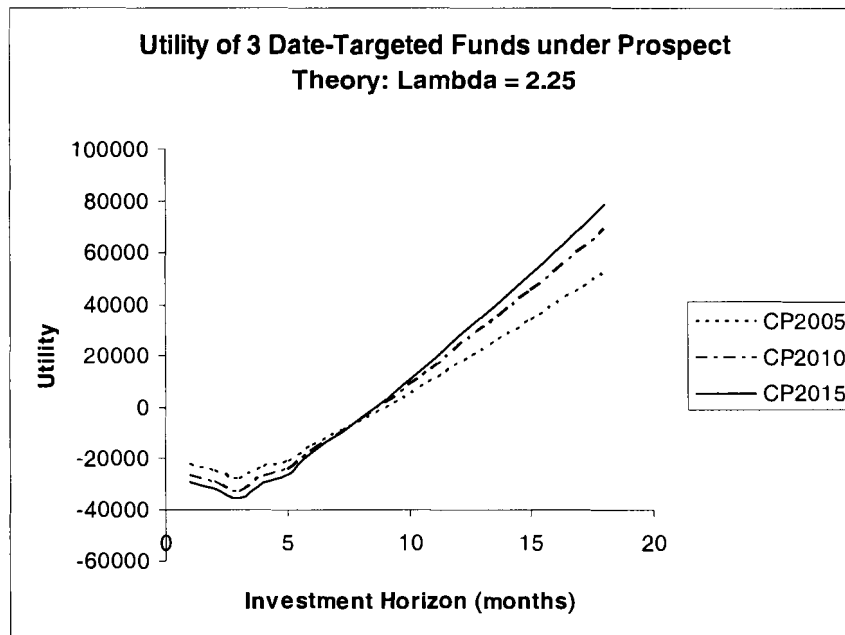

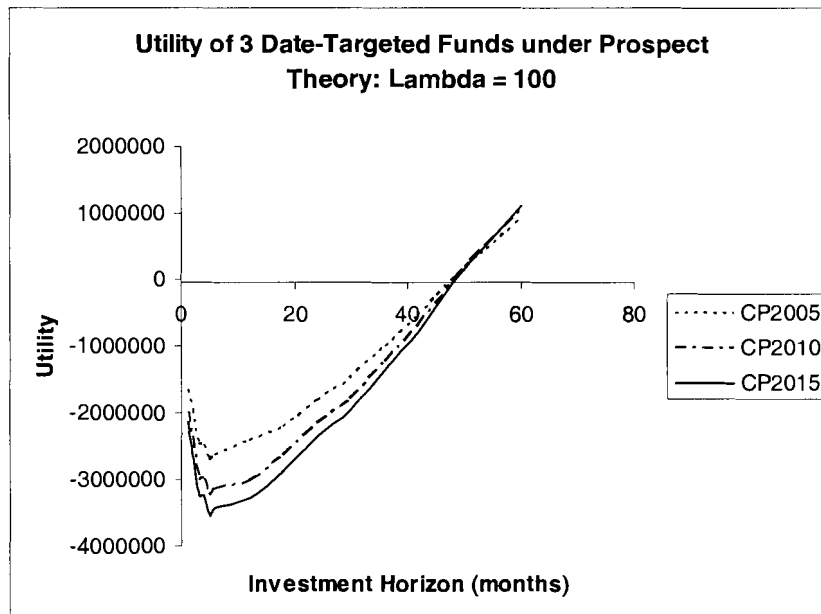
Utility of 3 Date-Targeted Funds under Prospect Theory: Lambda = 2.25

Figure 4-2 Utility of three Date-Target funds under Prospect Theory with High Risk Aversion

Utility of 3 Date-Targeted Funds under Prospect Theory: Lambda = 100

From the perspective of prospect theory, an investor would require a very sever risk aversion (lambda = 100) to choose the 2015 portfolio over the 2010 and 2005 portfolios with more than 5 years left in his investment horizon. As illustrated in 4-2, ClearPath 2015 can be shown to have a higher utility at the 5 year point but any horizon shorter than that is dominated by ClearPath 2005 leaving no room for the ClearPath 2010 portfolio.

It appears unlikely that prospect theory can justify the asset mix shift in the short horizon date-targeted funds.

4.5 Discussion of Date-targeted Funds

The target date funds have gained popularity for their sensibility and simplicity - something that can be tough to find from some investment advisors. Many retail investors are overwhelmed by the responsibility of managing their retirement portfolio and by the bewildering number of investing options facing them. Target Date Funds bring sanity to all of this with a solution that delivers simplicity, focus, and peace of mind.

While target date funds are an ideal option for the uninformed, unsophisticated retail investors who don't want to take an active role in managing their investments, the target date funds are not a solution for everyone. Particularly obvious is the justification of asset mix change based on human capital. Unfortunately, this target date funds are not only used for retirement. They may be used for a child's education or a second home. It is inappropriate to have a 60 year old saving for retirement and a 35 year old saving for a child's education to have the same asset mix based on human capital. Similarly, an inheritance can distort the percentage of human capital in a portfolio.

It is our opinion that target date funds, though appealing from a simplicity standpoint, are not well positioned from a theoretical standpoint. For an individual with little more than human capital, the asset mix changes can be justified until about 10 years before retirement based on steady employment income. It is not possible under either mean-variance analysis or prospect theory to justify the asset mixes suggested for investment horizons of less than 10 years.

Target Date Funds may no longer match investor's new growing desire for their later life. Some comprehensive research on global attitudes to aging and retirement, for example, 'The Future of Retirement' studied by HSBC, shows a major shift away from the notion that later life is dominated by traditional passive retirement. For example, people are expecting a later retirement, and people are expecting that alternating work and leisure is the ideal life-style in later life. If retirement date is becoming more flexible, then it is difficult to predict an accurate future date.

From a theoretical basis, these time-dated products should only be advised for people with a human capital portfolio similar to table 2 with very steady employment. And even in this situation, investment horizons of less than 10 years should be treated differently. We feel an advisor would be better serving his client if he were to estimate his client's risk tolerance for any expenditure forecast to occur within 10 years and used a mean-variance approach to determining the asset mix. The future growth of target-date funds will really depend on their acceptance by the investment advisory community. Given their simplicity, many advisors might feel threatened because their clients may perceive that they are no longer adding any value. Their clients might ask themselves, "Why do I need a pilot for my retirement portfolio, when I can put it on autopilot?" Investors may be more likely to neglect a fund's performance because everything is automatic.

5 DISCUSSION AND CONCLUSION

The exercise of attempting to explain industry practise with academic theory has yielded interesting results. The mean variance framework can explain the risk targeted funds but it relies on advisors to re-evaluate a client's risk aversion as a liability nears. The data we used to simulate the returns and standard deviation of HSBC's funds resulted in funds that were tightly clustered from a return perspective (within 1% of each other) and this may have effected our conclusions regarding the impact of prospect theory to identify dominant portfolios for different investment horizons.

The inclusion of human capital in the portfolios seems to provide an excellent explanation of date-targeted funds and a solid justification of altering the preferred choice of risk-based funds at different investment horizons while still maintaining the same risk tolerance. We can simulate the changing asset mix of Fidelity's funds for investment horizons of 4o years down to 10 years in the mean variance framework. The asset mix changes in the last 10 years however, cannot be explained in this framework. If we look to prospect theory for an explanation of this change, we are still unable to explain this approach to asset allocation.

We propose that the appropriate model for determining the optimal asset mix through time is based on a portfolio including human capital and a utility function determined by prospect theory. This paper validates that this is not a method used by Fidelity's ClearPath funds. This paper does not present a solution to the time horizon asset mix problem but it does provide a framework that would be well served with further research.

Figure 5-1 Indifference Plane Intersecting Portfolio that includes Human Capital
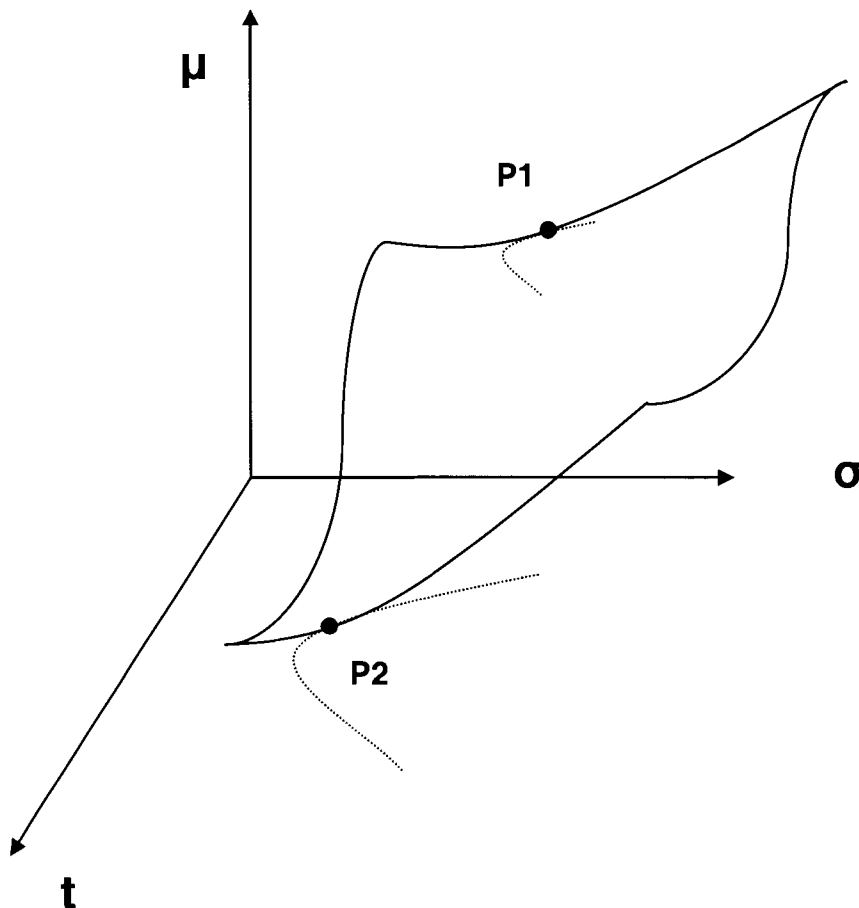
In the above figure, the efficient frontier grows wider through time as the fixed weight of

human capital decreases. At the same time, the indifference curve takes on a different shape

depending on the horizon being evaluated. The result is an indifference plane. We have

demonstrated that the significant inputs into the development of such a model include an accurate

model of an investor's human capital coupled with a realistic utility function such as that

presented by prospect theory.

REFERENCE LIST

Bodie, Zvi. (2003). "Thoughts on the future: Life-cycle investing in theory and practice." *Financial Analysis Journal*, vol. 59, no. 4 (Jan/Feb): 24-29

Bodie, Zvi, R.C. Merton, and W.Samuelson (1992). "Labor Supply Flexibility and Portfolio Choice in a Lifecycle Model. " *Journal of Economic Dynamics and Control*, vol. 16, nos.3/4 (July-October):427-449

Merton, Robert C. (1969). "Lifetime Portfolio Selection under Uncertainty: The continuous-Time Case." *Review of Economics and Statistics*, vol. 51, no. 3 (August): 247-257

Kent Hickman, Hugh Hunter, John Byrd, John Beck and Will Terpening (2001) "Life-Cycle Investing, Holding Periods, and Risk – Following the standard advice impose huge penalties." *Journal of Portfolio Management*, Winter 2001; 27, 2

J. Von Neumann and O. Morgenstern, "Theory of Games and Economic Behavior", Princeton University Press, Princeton, 1947, p 31

Pratt, John, Zeckhauser, Richard J, "Willingness to Pay and the distribution of Risk and Wealth", *The Journal of Political Economy*; Aug 1996; 104, 4;

Kahneman, Daniel and Amos Tversky, 1979, "Prospect Theory: An analysis of decision under Risk." *Econometrica* 47, 263-291

Kahneman, Daniel and Amos Tversky, 1992, "Advances in Prospect Theory: Cumulative Representation of Uncertainty", *Journal of Risk and Uncertainty*, 5: 297-323