THE USE OF CULTURAL PERSPECTIVES IN THE ELEMENTARY SCHOOL MATHEMATICS CLASSROOM

by

Irene Percival

B.A., Oxford University, 1970
Certificate in Education, Oxford University, 1971
M.A., Oxford University, 1976
M.Sc., Simon Fraser University, 1999

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APPROVAL

NAME Irene Percival

DEGREE Doctor of Philosophy

TITLE The Use of Cultural Perspectives in the Elementary School Mathematics Classroom

EXAMINING COMMITTEE:

Chair Cheryl Amundsen, Associate Professor

Thomas O'Shea, Associate Professor
Senior Supervisor

Rina Zazkis, Professor
Member

Allan MacKinnon, Associate Professor
Member

Stephen Campbell, Assistant Professor
External Examiner

Elaine Simmt, Associate Professor, Faculty of Education, University of Alberta
Examiner

Date January 26, 2004

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Author: ______________________
(Signature)

Ms. Irene Percival
(Name)

January 26, 2004
(Date)

ABSTRACT

The use of historical perspectives in the mathematics classroom has long been promoted in the literature, and current mathematics curricula call for activities to be presented to help students understand the cultural background of the subject. However, no specific suggestions are provided in the curriculum documents to indicate how this can be done at the elementary level, and since few elementary teachers are themselves aware of the cultural dimension of mathematics, this aspect of the subject is frequently ignored.

This research investigated the feasibility of using cultural perspectives in elementary mathematics classrooms. A case study approach was adopted to document the work of six teachers as they incorporated material from the historical and multicultural background of mathematics into their teaching. However, since most research participants had limited knowledge of cultural mathematics, elements of the "teacher development experiment" methodology were also included.

The data are presented in two ways. A few lessons from each case study are described, painting a series of narrative pictures to indicate the variety of approaches which teachers used to incorporate cultural perspectives. An analysis of teachers' views and actions is also presented, summarizing the benefits they discovered from such work, both for their students and for themselves, and noting the implementation issues which arose.

It was found that teachers who have access to suitable resources *can* teach mathematics in a way which promotes its cultural background, and that they are aware of the advantages which arise from doing so. Many of the benefits mentioned by the teachers are consistent with those found in the literature, but the teachers put a greater emphasis on the affective value of introducing cultural perspectives, and on the effect that such work had on their teaching. However, it was also apparent that only those teachers who have a particular interest in the cultural dimension of mathematics *will* teach the subject in this way. The many other commitments upon teachers' time, both in and out of the classroom, provide ready excuses to postpone an exploration of such ideas.

DEDICATION

To my husband Paul,

who was always there to help at moments of despair.

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and my long-suffering family, who have supported me throughout, and are now aware of far more facts concerning historical and multicultural aspects of mathematics than they ever wished to know.

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List of Abbreviations and Acronyms

NCTM	National Council of Teachers of Mathematics
IRP	Integrated Resource Package (the curriculum guide used in British Columbia)
ICMI	International Commission on Mathematics Instruction
HMN	Humanistic Mathematics Network
AIMS	Activities Integrating Mathematics and Science
G4	Group of four (four teachers in the group case study)
TDE	Teacher development experiment
MMC	*Mathematics from Many Cultures* (book title)

CHAPTER 1
INTRODUCTION

For over two hundred years, there have been calls from both mathematicians and mathematics educators to situate mathematics in its historical, and more recently, its multicultural setting. In 1795, the mathematician Lagrange told his class of trainee school-teachers that a knowledge of the development of mathematics "is not a matter of idle curiosity. It can afford us guidance in similar inquiries and sheds an increased light on the subjects with which we are employed" (quoted in Fauvel & van Maanen, 2000, p. 35). Many others have echoed this sentiment, and the focus has now broadened to include other cultural issues, a recognition that "students in our 'global village' must learn to respect and appreciate the contributions of peoples in all parts of the world" (Zaslavsky, 1991, p. 32).

In 1989, these views were included in the *Curriculum and Evaluation Standards for School Mathematics* produced by the National Council of Teachers of Mathematics (NCTM):

> Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities. (p. 5)

This paragraph appears to have galvanized the publishing world into producing materials designed to allow teachers in both elementary and secondary schools easy access to cultural perspectives on mathematics. It also affected curricula elsewhere: In British Columbia the value of understanding the cultural roots of mathematics was identified in the *Mathematics K to 7 Integrated Resource Package* (IRP), the document which sets out the basic curriculum requirements to be followed by elementary school teachers (British Columbia Ministry of Education, 1995). However, no specific suggestions are provided to indicate how teachers could achieve this goal, and my experiences in presenting workshops on these cultural ideas have suggested that, despite the introductory recommendation in the IRP, few elementary teachers are aware of this dimension of mathematics or the materials available to introduce it to their students.

My own interest in the historical dimension of mathematics was aroused by a lecture I attended in which the Egyptian method of solving equations was explained. This made me realize how little I knew of the cultural background of mathematics, a recognition which led me to delve into literature on the subject and eventually to take a university course in the history of mathematics. I became fascinated with the topic, and started to introduce historical material into the mathematics enrichment classes which I taught at my sons' elementary school. A desire to better understand the success of these classes led me to conduct an action research project for my masters' thesis based on this approach to teaching mathematics (Percival, 1999).

This M.Sc. research confirmed the pedagogical benefits of looking at mathematics from an historical viewpoint, and led me to promote this idea through Professional Development workshops. However, as my study had involved only a small group of "gifted" children, it was not clear whether my conclusions could be extrapolated to regular size classes. A further complication, from a research perspective, was that I had taught the students myself, and the specialized knowledge of the history of mathematics which I brought to the classroom is rarely possessed by elementary school teachers. My obvious interest in the subject matter also raised the question of potential researcher bias, an issue which is equally relevant to the present study and is addressed on page 6.

A literature review revealed that other research in this area had similarly been carried out by researchers who had doubled as teachers (Ofir, 1991; Ofir and Arcavi, 1992), and even articles reporting the use of historical aspects of mathematics by elementary teachers were scarce (Bohan & Bohan, 1993; Gardner, 1991; van Mannen, 1992; Voolich, 1993). Although these articles were gradually supplemented with others focussing on multicultural aspects of mathematics (Barta & Schaelling, 1998; Dolinko, 1996; Kliman & Janssen, 1996; Smith, 1995), evidence for the positive effects of using cultural perspectives existed largely in anecdotal reports from a few teachers who had a personal interest in this area. Research data to show whether generalist elementary teachers could successfully implement the curriculum recommendation on cultural mathematics was lacking. It seemed that an investigation into this issue was long overdue.

Defining "cultural mathematics"

In this dissertation, the term "cultural mathematics" is often used as a shorthand for the phrase "history of mathematics and/or multicultural mathematics". The distinction often made between these two ways of looking at the cultural background of mathematics is in many cases a false dichotomy, as it is now acknowledged that the early development of the subject was influenced by many cultural groups (Joseph, 1991). Nevertheless, published materials often suggest a split, with the choice of terminology being justified by the major focus of the material: Texts labelled as "historical" often refer to specific mathematicians and their work, whereas those labelled "multicultural" focus on the mathematical achievement and problem-solving practices of particular cultural groups, often comparing one with another. This approach to multiculturalism is the third of the five distinguished by Grant and Sleeter (1988), and is popular because it allows students insight into the activities of other cultures without getting involved in the political issues which arise in some of the other approaches to this topic.

The terms "cultural dimension", "cultural perspectives" and "cultural aspects" of mathematics are used interchangeably, and cover a wide spectrum of activities relating to historical and multicultural perspectives of mathematics. These ranged from practical endeavours, such as manipulating ancient calculating devices or presenting skits about famous mathematicians, to the more theoretical pursuit of studying primary historical sources. The phrase "cultural approaches to mathematics" is used similarly, for while it could be argued that an "historical approach to mathematics" should be one in which topics are presented chronologically, the recapitulation approach implied by such an organizational strategy has been attacked during the last century (Radford, 1997). Arguments for and against the possibility or advisability of using such an approach to curriculum planning are discussed in Chapter 2.

There are of course many other cultural issues which are relevant to students: "youth culture", "pop culture", "consumer culture" and "computer culture" are some of the terms currently found in educational literature. While these concepts are at least as important as those described above, my interest in historical issues and the availability of classroom-ready resources for historical and "multicultural" mathematics led me to focus on these two aspects of the cultural background of mathematics.

Research objectives and significance of the study

My overall objective in this study was to investigate the feasibility of including cultural perspectives of mathematics into elementary classrooms taught by non-specialist teachers. This goal has two major components: first, to determine whether teachers see value in such work, and second, to investigate the implementation issues which arise, particularly those which may deter teachers from employing such perspectives. These objectives reflect the "suggestions for future research" given in the ICMI study, History in Mathematics Education (Fauvel & van Maanen, 2000, p. 90).

Much has been written from a theoretical perspective on the value of cultural approaches to mathematics (Barbin, 1990; Jones, 1969; Swetz, 1989; Zaslavsky, 1996), but there is little other than anecdotal evidence to show that these values are recognised by practising teachers, particularly those working at the elementary level. While both the existing theory and the anecdotes are obviously relevant background material, my research used an ethnographic approach to case study (Eisenhart, 1988; Goetz & LeCompte, 1984) in which conclusions are based on first-hand experience of the words and actions of the teachers themselves. Teachers were selected to include both those who had some experience with cultural approaches to mathematics and those to whom it was quite new. To overcome the difficulty of finding teachers willing to take part in my research, particularly those in the latter category, I offered to assist them to whatever extent they requested, thus gaining some insight into what they hoped to achieve. The lessons and conversations which followed allowed me to explore two specific objectives concerning the extent to which teachers valued this work: the benefits which they considered that they themselves received from teaching mathematics from a cultural perspective, and those which they felt accrued to their students.

For my second goal, that of investigating the practical aspects of implementing cultural approaches to mathematics, I determined five specific issues to explore, based on the literature on educational reform (Fullan, 2001; Hall & Hord, 1987; House and Lapan, 1978; Huberman & Miles, 1984). The first question is that of time commitment: How much time is required to plan lessons, particularly when the mathematical content appears new, and how much classroom time do teachers choose to spend on cultural aspects of mathematics. The second concerns the relevance of the cultural work to the curriculum guidelines to which teachers are expected to conform: Are teachers able to see the connections between the culturally-oriented material and the specified learning outcomes; do they make these connections clear to their students; and do they consider that such connections are necessary to justify this work. The third practical issue deals with the resources needed to teach from a cultural perspective: Do teachers know how to locate these, and how easily can they be used in the classroom. The fourth implementation issue concerns the unfamiliarity of the cultural material: How much of the material is new to the teachers, and do they have sufficient mathematical ability to assimilate new concepts or teach old ones in new ways. I was also interested to discover the extend to which these issues are inter-related, and felt that other issues might arise which I had not foreseen.

The final implementation issue concerned the lasting effects of teachers' exposure to cultural mathematics: Would teachers continue to include historical or multicultural topics in mathematics after their first, supervised year of implementing the material, and what factors might affect their decision to continue or end this practice. An analysis of the relative importance of these factors could provide useful knowledge to those wishing to encourage the wider use of cultural aspects of mathematics, and should also reveal whether the presence of a mentor is necessary.

The Rationale section which opens the British Columbia mathematics curriculum document clearly supports the inclusion of historical and multicultural mathematics. This might suggest that it is a "faît accompli" that such approaches to mathematics will take place in the classroom, but these recommendations have been made many times before, with singular lack of success (Fauvel, 1991). My research should provide some insight as to why so few elementary teachers are teaching in ways which promote one of the goals of the curriculum document which they are intended to follow.

Limitations of the study

Although I have discussed the teaching of cultural aspects of mathematics with many teachers in workshops and in their classrooms, the case-study methodology chosen for this research study demanded that only a small number of research participants be formally included (McMillan & Schumacher, 1997). This raises the question invoked by all such studies as to how representative of the wider community are those involved in the research programme. I have tried to minimise this problem by selecting teachers with a variety of background experiences, but my choice of research subjects was restricted to those with whom I had some prior connection, albeit very slight in several cases. This non-random selection of participants can itself be considered a limitation of the study, since it could be argued that I selected teachers who would provide the sort of results I wished to achieve: My only response to such criticism is to note that if this were the case, I might have chosen a different set of participants at the onset.

My own interest in the teaching of cultural mathematics could clearly lead to bias in reporting this research. Similarly, my belief in the efficacy of this teaching approach, based on my earlier action research with small groups of "gifted" students, could prejudice me towards noticing data which confirmed my own views. However, the focus of this research is one step removed from the question of whether teaching from a cultural perspective is effective. Unlike my previous study, which focussed on the children's work, this research probes teachers' views on the value of such a teaching approach and the problems encountered in using it. I have attempted to minimise my bias in favour of this teaching approach by reporting teachers' words and actions to illustrate the complete range of their views.

A further potential problem arises from the apparent conflict between my dual role of teacher and researcher. However, by limiting my teaching to providing help only when it was requested, I felt that this duality gave me added insight into the teachers' comfort level with the material. All the teachers who agreed to take part in my research did so as volunteers, and received no reward other than assistance from someone experienced in the use of cultural dimension of mathematics who was willing to help them introduce such material into their classes. The scheduling of these cultural mathematics sessions was therefore arranged for the teachers' convenience, and this sometimes meant that I was unable to observe the classes. However, in such cases, the teachers usually recorded their lessons for me, and we discussed them after I had listened to the tapes.

The study was further limited by its duration. As Fullan notes (2001), the implementation period for most curriculum changes is at least two years, with the total time "from initiation to institutionalization" (p. 52) often being considerably longer. Although the two year window during which I tracked most of the participants in this research was enough to give some indication of trends, it was not sufficient to determine the long-term outcome.

Organization of the dissertation

Chapter 2 reviews the literature concerning the role which cultural perspectives of mathematics can play in mathematics education, and thus presents the "voice of the expert" upon this matter. The chapter starts by summarizing how the philosophical stance on the nature of mathematics changed during the last century, as it is the emergent humanistic viewpoint which justifies the introduction of cultural material into the mathematics classroom. The pedagogical implications of a cultural approach to mathematics are explored in the following section, prior to being situated within a framework for mathematics education: These points provide the theoretical background to which teachers' views are later compared. The final section of this chapter looks at evidence for the use of cultural perspectives in the elementary school.

Whereas Chapter 2 considers the research literature, the next chapter reviews material of a different kind. One of the major practical problems discussed in the literature is that of finding suitable resource materials (Lingard, 1997; Michalowicz, 2000; Rogers, 1991), and Chapter 3 surveys resources from a variety of media which can be used by elementary teachers wishing to include cultural dimensions into their mathematics teaching. Although the borderline between "historical" and "multicultural" aspects of mathematics is rather vague, publications are often identified as including one or the other approaches, so this chapter analyzes the materials separately, based upon the authors' classification where possible, and the major focus of the material where no such categorization is made.

The fourth chapter explains the methodology employed for this research, and introduces the teachers who are the subjects of the case studies. Their work is described in the next chapter, which explains the different strategies employed by these teachers in order to include cultural perspectives into their mathematics classes, and describes individual lessons to show how they implemented their ideas. Although the overall format of these class descriptions is anecdotal, each account includes the voice of the teacher as he or she explains the rationale behind the choice and presentation of material and comments on the progress of the lesson.

Chapter 6 takes a more objective look at the teachers' words and actions, and although the teachers' voice still predominates, it is focussed by the research lens. The data examined here includes some of that introduced in the previous chapter, but those lessons were only one part of the interaction I had with the teachers involved in my research. The questionnaires completed by these teachers and the many conversations we had before and after classes gave me further insight into their views. This chapter analyses information gained both in and out of the classroom to determine why elementary school teachers might decide to implement cultural approaches to mathematics and the practical issues they face in doing so.

The final chapter compares the results of this analysis with the expert opinions reviewed in Chapter 2, and suggests reasons for the discrepancies. Conclusions are then presented as to the expediency of recommending that generalist elementary teachers include cultural perspectives in their mathematics classes, and the teachers' work is assessed to determine whether it is really promoting an appreciation of the cultural background of mathematics or merely presenting a set of "curiosities", to use D'Ambrosio's term (2001). The chapter closes with implications drawn from this study and suggestions for further research into this issue.

CHAPTER 2
LITERATURE REVIEW

There is an apparent contradiction between cultural approaches to mathematics and the traditional, absolutist philosophies of the subject which held sway until the second half of the last century. In order to validate the use of cultural dimensions of mathematics in the classroom, this chapter starts by reviewing the paradigm shift that occurred in the philosophy of mathematics during that period. The next section discusses the integration of historical and multicultural perspectives into mathematics education, and is followed by a brief summary of the ideas discussed as they relate to a philosophical framework for the field. The final section summarizes evidence for the use of cultural dimensions of mathematics in the elementary school, looking briefly at the extent to which mathematics curricula around the world promote such ideas, but focussing mainly on articles documenting classroom experiences of such work.

A paradigm shift in the philosophy of mathematics

The momentum for replacing the absolutist philosophies of mathematics came from events in the wider sphere of the philosophy of science. Kuhn's idea of "scientific revolutions" and Popper's view of the fallible nature of scientific theories both led to reappraisals of the nature of the subject. The Hungarian mathematician Lakatos was greatly influenced by Popper's work, and in a series of journal articles in 1963-4 he put forward the idea that mathematical proofs need not be, in fact cannot be, rigorously logical and final. Thirteen years later these articles appeared in his book *Proofs and Refutations*, in which he used the dramatic setting of a classroom discussion between teacher and students to put forward his thesis that "informal, quasi-empirical mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations" (1976, p. 5). Although one of the student characters calls for "not only certainly but finality" (p. 63), Lakatos' makes the distinction between these two concepts clear, and has the teacher respond that the student should accept finality as "a pleasant bonus but not obligatory" (p. 64).

His ideas must have come as a shock to the mathematical community. No longer could mathematics to be seen as "a body of infallible and objective truth" (Ernest, 1991, p. xi). In its place we have a subject in which conjectures are made and reviewed by members of the community of mathematicians. Their role is to challenge any conjecture, modifying it until a consensus is reached. However, the result is no longer regarded as a final absolute proof, merely a result with a high probability of truth. Although this sounds extreme, it is in fact an acknowledgement of the actual state of affairs. As Davis and Hersh pointed out:

> The geometry of Euclid was studied intensively for 2,000 years, yet it had major logical gaps that were first detected in the 1880s. How could we ever be sure that we are also not blind to some flaw in our reasoning. (1986, p. 68)

Lakatos' ideas on the fallibility of mathematics and the realisation of the role played by the community of mathematicians, rather than merely by individuals, are two of the predominant themes in the various modern philosophies of mathematics. The third is the nature of mathematical objects themselves. Lakatos himself wrote little about this, but those who followed him produced a new concept that did not rely on a Platonic realm to house the objects of their discussions. A clear explanation of this idea is given by Hersh (1994), who discarded the traditional assumption of Western philosophy that any object is either mental or physical, pointing out that there are many abstract objects ("sonatas, poems, ..., academies of science") which fit into neither category. His argument that a third type of object exists, which he called a "social-cultural-historical entity", is based on the work of Karl Popper who introduced three "Worlds" in order to distinguish between three levels of reality: World 1 contains physical objects, World 2 is the mental world, and World 3 is the home of "all the nonmaterial culture of mankind ...(which is) of course, ... the world where mathematics is located" (Davis & Hersh, 1981, p. 410).

The concept of mathematics which developed from the work of Lakatos and others in the 1970s and 80s became more widely known after the foundation of the Humanistic Mathematics Network (HMN) in 1986. This group originated in a meeting of university mathematicians, mathematics educators and philosophers, who were called together by Alvin White to discuss the relationship between mathematics and the humanities, and to find some solution to the general dissatisfaction with the way that mathematics was portrayed at all three levels of education. Brown listed some of the tenets created at that meeting, of which the following seem to have particular relevance for the inclusion of an historical or multicultural approach to the learning of mathematics:

> a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished version to be 'merely technical'.
>
> b) An appreciation for the human dimensions that motivate discovery – competition, cooperation, the urge for holistic pictures.
>
> c) An understanding of the value judgements implied in the growth of any discipline. Logic alone never completely accounts for what is investigated, how it is investigated and why it is investigated.
>
> d) A need for teaching-learning formats that will help wean our students from a view of knowledge as certain, to-be-received.
>
> e) The chance for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems and to participate in controversy over mathematical issues. (1996, p. 1302)

Hersh (1979, 1994; Davis & Hersh, 1981) wrote extensively on the philosophy of mathematics, putting forward a humanist viewpoint which showed mathematical knowledge to be as fallible, corrigible, tentative and evolving as any other branch of human knowledge, and concluding that mathematics has to be understood as a human activity, situated in a social context. In his book *What is mathematics, really?* (1997), Hersh also discussed the work of others who promoted this view of mathematics, such as Wilder, Ernest and Tymoczko, each of whom focused on some aspect of the role that people played in the development of mathematics: its history and applications, its place in human culture and the everyday practices of mathematicians. Thus an historical and multicultural awareness of mathematics is built into modern philosophic views of the subject.

The influence of cultural perspectives on mathematics pedagogy

Absolutist philosophies of mathematics portrayed the discipline as decontextualized, depersonalised and detemporalized and fostered a pedagogy which stressed the importance of learning rules, theorems and proofs with almost no mention of their human origins (Davis & Hersh, 1981). When these philosophies were replaced by others based on humanist and constructivist principles, it became possible to view mathematics through historical or multicultural lenses without conflict. Much has been written about these perspectives, and the importance of an historical approach to mathematics was acknowledged in the decision of the International Commission on Mathematics Instruction (ICMI) to study this issue (Fauvel & van Maanen, 2000). This section reviews the many arguments which have been presented to justify or dispute the inclusion of cultural perspectives on mathematics.

Changing teacher and student perceptions of mathematics

The literature on students' beliefs about mathematics, summarized by McLeod (1992), includes work on the importance, relevance, difficulty and rule-driven nature of mathematics, the uniqueness of algorithms and answers, and the speed with which problems can be solved, but makes no mention of students beliefs about the cultural background of the subject. On the other hand, texts specifically concerned with historical or multicultural topics commonly note that students are unaware of the origins of mathematics (Swetz, 1994; Zaslavsky, 1996). Thompson (1984) reported that only a small number of teachers have any knowledge of the history of mathematics, and noted that most teachers equate mathematics to the mathematics taught in school. These beliefs contradict modern philosophies of the discipline, and the following paragraphs examine how the use of cultural dimensions in the mathematics classroom can help change these perceptions.

Mathematics as a human endeavour

The idea that history gives mathematics a human face is one of the most frequent justifications for the use of historical approaches to mathematics (Bidwell, 1993; Fauvel, 1991, Lingard, 2000; Reimer & Reimer, 1995c), and exemplifies modern humanistic philosophies of the subject. The realization that mathematics is actually created by people stands in stark contrast to the commonly observed belief of students in the traditional mathematics class that mathematics merely "springs full-blown from the textbook or the teacher's head" (Zaslavsky, 1996, p. 29).

Many ways have been found to dispel this myth, with the practice of providing anecdotal information about famous mathematicians often being the first to be mentioned (Fauvel, 1991; Swetz, 1994; Voolich, 1993). Stories of specific instances of men and women creating mathematics help students accept mathematics as a human endeavour, and, by extension, one in which they can also participate (Reimer & Reimer, 1995c). Such stories not only show the human origin of mathematics, but also provide students with a name, place or event which can help them recall the material (Kelly, 2000). However, academia seems split on the benefits of the story-telling strategy. Fowler (1991) sounded a warning note that narratives "may be creating a mythical and misleadingly simple past, and not doing justice to history" (p. 15), a view shared by Radford (1997). Führer (1991) took the opposing view, noting that this problem was less serious than it might appear: first, because historical data, by its very nature, cannot be considered as certain truth; second, because students forget much of what they learn; and third, because "good stories have the overwhelmingly beneficial effect of opening the student's mind and attention towards studying mathematical details" (p. 25). Führer's third point is supported by Egan's work (1986) on the importance of story-telling as a means of engaging a child's imagination, which he considers to be a very powerful, and often under-utilized, learning tool.

Students can become even more involved in the human side of mathematics through dramatic activities, varying from simple skits portraying episodes in mathematicians' lives (Ponza, 1998; Reimer & Reimer, 1995c), to those explaining mathematical concepts (Hitchcock, 1992, 1997; Hoechsmann & Galay, 2000). Voolich (2001) also recommended other formats, such as the celebration of mathematicians' birthdays or the adoption of a TV talk show scenario, in which students role-play mathematicians and answer questions about their life and work. Van Maanen admitted that these activities, and others such as watching films, constructing models and doing projects, can be time consuming, but noted that "involving the students affectively ... had a possibly lifelong benefit ... in securing their engagement with the idea of developing mathematical strengths" (Fauvel & van Maanen, 2000, p. 332).

Mathematics of and for all cultures

Looking at the work of individual mathematicians is an easy way to humanize mathematics, but the basic concepts of mathematics evolved gradually rather than being the brainchild of one person. Bishop (1988) suggested six major areas in which mathematics developed, all of them part of the everyday life of early peoples: counting, measuring, locating, designing, playing and explaining. His book, *Mathematical Enculturation*, outlines an approach to mathematics which shows how the subject has developed in response to the needs of people from all over the world, and suggests a way of teaching it which is radically different from that suggested by traditional topic-based curricula. He argues that an awareness of the cultural background of mathematics is a pre-requisite for any multicultural work in mathematics, noting that "in order to multi-culturalise a curriculum, one must first culturalise it" (p. 96).

In the past two decades, ideas of enculturation, ethnomathematics, and polemics against the "myth of neutrality" of mathematics have often appeared (D'Ambrosio, 2001; Fasheh, 1982; Gerdes, 1997; Joseph, 1991; Powell & Frankenstein, 1997; Zaslavsky, 1996). The authors noted that although many different cultures have contributed to the evolution of mathematics, the mathematics presented in schools is that of a Eurocentric tradition. This ignores the indigenous mathematics of other parts of the world, much of which shows mathematics in a very different light to that of the structured, logical, European methods. These points have important implications in the classroom.

It is claimed that the use of Eurocentric curricula may be counterproductive to the goal of recruiting students from minority groups into mathematics, as the lack of an ethnic role model could cause them to believe that people from non-European cultures are incapable of working in mathematics. Introducing these students to mathematics developed by people of their own ethnic group not only gives them pride in their background, but builds their confidence (D'Ambrosio, 2001; Shirley, 1995; Wiest, 2002; Zaslavsky, 1996).

However, the goal of cultural pluralism is often considered to be an even more important justification for including the mathematics of many cultures into the classroom (D'Ambrosio, 2001; Zaslavsky, 1991, 1996). Joseph (1993) suggested that such material should be included in the curriculum "not primarily to enhance the self-image of minority children but to help all children in the future to negotiate more effectively in a multicultural environment" (p. 6). D'Ambrosio considered that multicultural mathematics activities are important not only for inculcating an appreciation of the mathematics of other cultures, but also to help students to "develop a greater respect for those who are different from themselves" (2001, p. 308). However he noted that many present practices are inappropriate, claiming that teachers "often engage their students in multicultural activities merely as a curiosity". Zaslavksy gave a similar warning against introducing cultural material as "quaint customs" or "primitive practices", adding "It is easy to trivialize the concept of multicultural education by throwing in a few examples as holidays approach. Better not to do it at all!" (1993b, p. 53).

Educators in favour of including non-European mathematics have suggested that activities such as tracing complex networks in the sand (Zaslavsky, 1973) or exploring the Euclidean geometry hidden in Mozambican woven buttons (Gerdes, 1997) shed new light upon the traditional curriculum topics. Pimm (1995) sounded a note of warning about this practice, querying whether the members of these cultures actually believed that they were doing mathematics in such circumstances. However, he admitted that "by including, exploring, and genuinely valuing the diversity of mathematical practices of different peoples and cultures, mathematics education can make a contribution to keeping intellectual diversity alive and productive" (p. 135).

Early attempts to introduce culturally related mathematics in England were opposed by the 1987 Campaign for Real Education, which claimed that social issues have no place in mathematics: In her address to a Conservative Party Conference the same year, Margaret Thatcher complained that "children who needed to count and multiply were learning anti-racist mathematics – whatever that might be" (cited in Ernest, 1991). The call for social justice has been taken up by the Criticalmathematics Educators Group (Frankenstein, 1997), but several years earlier, a strong attack on the traditional school mathematics curriculum had been made by Shan and Bailey (1991) in their book *Multiple Factors: Classroom Mathematics for Equality and Justice*. This promoted the idea that the mathematics classroom can be used as a forum to reveal the social inequalities and prejudices which still prevail in much of the world, and suggested the comparison of statistics from "First World" and "Third World" countries as a way of achieving this goal. However, Shan admitted that although she found many colleagues willing to multiculturalize mathematics, none were willing to use their mathematics classes to explore issues of equality and justice. Tate (1995) suggested two reasons for this reluctance. He noted that mathematics teachers may be uncomfortable talking about racial or social issues in their classes, partly because this is a departure from the traditional approach to mathematics, but also because they are "socialized not to use race as a heuristic to guide their pedagogy" (p. 349). He also suggested that teachers consider mathematics to be "at risk" when lessons focus on social or political issues, although in his analyses of such lessons he has shown that the curriculum content is still covered.

The ideas discussed so far have focussed mainly on the ethnic dimension of culture, but Zaslavsky (1996) noted that there are actually two groups whose contributions to mathematics are frequently ignored: "females and people of colour". This was acknowledged by the NCTM in their 1997 yearbook, *Multicultural and Gender Equity in the Mathematics Classroom* (Trentacosta & Kenney, 1997). The concept of gender as a basis of discrimination in mathematics has often been noted (Campbell, 1995; Leder, 1992; Secada, 1992), and Ernest (1991) discussed this issue in relation to the theory of moral frameworks proposed by Gilligan (1982). He pointed to the reliance of traditional mathematics pedagogy upon the "separated" framework often considered part of the cultural definition of masculinity, and noted that such methods are consistent with absolutist philosophies of mathematics. The recent humanist and constructivist philosophies of mathematics fit with the "connected" framework, which focuses on the caring, emphathic human aspects of situations generally considered to be feminine, attributes which are also found in cultural perspectives on mathematics (Elliott, Lingard & Povey, 2001). Many elementary-level texts for these historical and multicultural approaches highlight the contribution of women, although Anglin (1992) warns that overemphasizing women's role in mathematics can not only distort history but also patronize them. Joseph (1993) issued a similar caution against adopting a condescending attitude when discussing the mathematics of ethnic minorities, noting that cultural issues have to be dealt with sensitively.

Beliefs about mathematical problem-solving

It has often been noted that people react with surprise when they learn of the existence of computational algorithms which differ from those they know: Their own experience of having learnt only a single algorithm for any specific operation leads them to assume that no others exist (Mason, 1998; Sgroi, 1998; Zaslavsky, 1996). Students' belief that "there is only one correct way to solve any mathematics problem" (Schoenfeld, 1992, p. 359) can be challenged by exposing them to the diversity of techniques revealed by cultural approaches. This not only dispels the myth that algorithms are both universal and permanent, but can also help students to accept mathematics as a way of solving problems rather than simply following rules (Morrow, 1998).

Hirigoyen (1997) discussed not only different algorithms but different number representations and geometrical conventions. He noted that a student's apparent lack of understanding of mathematics may actually be merely a lack of understanding of the "mathematical dialect" used in the school, and stressed that children from other cultures should be allowed to continue using methods they already know. To privilege a specific set of algorithms discriminates against students who have learnt other techniques while living in different geographical regions, possibly confusing them and jeopardising their belief in their own abilities. Allowing students to use their own methods and to share these with other students validates their culture, whereas to insist on a change is demeaning. Ron (1998) emphasized that teachers need to be aware of the existence of algorithms from other parts of the world, as these techniques are often used by parents trying to help their children.

Cultural approaches to mathematics also reveal that even the acceptable answers to a question may vary over time and place. This negates the student belief that "mathematics problems have one and only one right answer" (Schoenfeld, 1992, p. 359). Classic examples from the history of mathematics are the reluctance to accept zero, negative or imaginary numbers as solutions to equations. Bishop gave an example from his work with students in Papua New Guinea, relating how he drew two rectangles of different sizes and asked, "If these were gardens which would you rather have?" The students' answer, "It depends on many things, I cannot say. The soil, the shade..." (1979, p. 144), later led him to reflect:

> It was clear that my so-called mathematical education had made me look only at the relationship between the numerical sizes of the two gardens. For [the student], the size of the garden was in many ways its least important feature. (1988, p. 38)

Such problems are rarely mentioned in traditional math texts, yet they give students a glimpse of what Borasi calls the "problematic situations" encountered by today's applied mathematicians (1992, p. 159).

Another student belief reported by Schoenfeld is that all mathematics questions can be answered within a few minutes, a view which leads to a lack of persistence in problem-solving. The history of mathematics provides many examples which show the value of persevering with problems and Swetz (1994) considered that students are encouraged and reassured by knowing that "frequently it was not so much the individual's genius that resulted in accomplishments and discoveries, but rather that person's persistence" (p. 2). A related issue is that of the role of errors in problem-solving. In current pedagogy these are regarded as "springboards for inquiry" (Borasi, 1994), and a study of the history of mathematics can reinforce this view. Paola noted that history "[gives] dignity to the mistakes made by students: it was not a trivial mistake if a mathematician made it' (p. 33, quoted in Furinghetti and Radford, 2002), and others consider that students will be reassured by the knowledge that presently renowned mathematicians had their doubts and made mistakes (Jones, 1969; Ofir, 1991).

Schoenfeld also noted that students consider mathematics to be "a solitary activity, done by individuals in isolation" (1992, p. 359). Although history may at first appear to confirm this through the way theorems are named, a closer look reveals that mathematicians often communicated with each other, albeit by mail in some cases, forming a community. This validates the concept of group work which is now gaining popularity in the mathematics classroom (Lingard, 2000). History also validates the use of intuition in mathematics (Davitt, 2000; Kleiner, 1988), reinforcing the modern focus on discovery learning.

A common student complaint about school mathematics is that it has "little or nothing to do with the real world" (Schoenfeld, 1992, p. 359). Cultural approaches to mathematics show that the subject influenced and was influenced by the society in which it existed (Bishop, 1988; Kline, 1953; Wilder, 1981). The introduction of ancient instruments, such as those for calculation or measuring angles, can help to show the connection between "school mathematics" and "real-world mathematics" (Bartolini Bussi, 2000; Charbonneau, 2002).

Increasing affective and cognitive development

Many years ago Barzun (1945) suggested that the lack of historical context, and by implication, apparent lack of human involvement, was a major cause for students' dislike of at least one branch of mathematics:

> I have more than an impression – it amounts to a certainty –that algebra is made repellent by the unwillingness or inability of teachers to explain why ... There is no sense of history behind the teaching, so the feeling is given that the whole system dropped down ready-made from the skies, to be used only by born jugglers. (p. 82)

Modern educators have echoed his comment, noting that historical explanations help to overcome students' aversion to a subject which they see as a set of arbitrary rules. Such explanations can include word origins, answers to factual questions such as "Why do we have sixty minutes in an hour?", or the origins of symbolic conventions, all of which have their basis in human decisions (Bidwell, 1993; Jones, 1969; Reimer & Reimer, 1995c). Cultural perspectives on mathematics have also been credited with a decrease in math-phobia and increased motivation for learning mathematics (Ernest, 1998; Fauvel & van Maanen, 2000; Lingard, 2000; Perkins 1991). This increased motivation is often presented as sufficient reason for claiming a corresponding increase in understanding, although, as Barbin (2000) noted, two underlying assumptions about the nature of learning are often made by those advocating cultural approaches to mathematics education: that the interested student will work harder, and that increased learning and understanding will result. However, McLeod (1992) noted that Leder and others observed that "attitudes toward mathematics are not a unidimensional factor: there are many different kinds of mathematics, as well as a variety of feelings about each type" (p. 581).

Cultural approaches to mathematics often introduce students to a variety of problem-solving methods. Carroll and Porter (1998) noted the affective benefits of this practice, observing that students who had studied several different algorithms seemed more confident in approaching problems, knowing that if one approach failed, they would have another procedure to use. More generally, the opportunity to compare and contrast techniques and solutions from different times and places is recognized as a valuable training in critical thinking (Ernest, 1991; Ransom, 1991). Similarly, the study of other number systems is generally felt to improve students' understanding of the system taught today (Ofir, 1991; Percival, 1999; Uy, 2003), although Nelson (1993) warns that such experience could confuse students.

Further opportunities for critical thinking arise from playing strategy games (Barta & Schaelling, 1998; Gorman, 1997). This practise is ideally suited to cultural approaches to mathematics, as games have been played by people all over the world, often before the development of written records stating their rules (Bishop, 1988). This can lead to uncertainly as to what the rules "should be", and students who have to negotiate rules and look ahead to see the consequences of their decisions gain valuable practice in developing their communication and logical thinking skills. Szendrei (1996) pointed out that the strict rules of games give students familiarity with axiom system, a valuable training in mathematical thinking.

The use of primary sources from the history of mathematics can encourage intuitive, non-linear thinking, and help students to "think like a mathematician" (Jahnke, 2000). Unlike modern student textbooks, in which the mathematics has undergone an "antididactical inversion" (Freudenthal, 1983), original texts often show the chronological development of a topic, rather than hiding the motivation for the work behind a set of logically deduced statements. Making the process apparent not only helps students understand this motivation, but also validates what Lakatos called "the zig-zag of discovery" (1976). However some educators claim that the details of history can confuse rather than clarify mathematics concepts (Fauvel, 1991; Fowler, 1991), and even Freudenthal, an advocate of integrating history into the mathematics class, considers that "the dead ends (of history) ... are only interesting as curiosities" (1981, p. 30). Nevertheless, the practice of reading original mathematical texts is a learning strategy endorsed by many historians of mathematics for the depth of understanding it can provide (Barbin, 2000; Calinger, 1995; Fauvel, 1990; Fauvel & Gray, 1987; Jahnke, 2000), and is one which has been the subject of considerable research by the Institutes for Research on the Teaching of Mathematics (IREMs) in France (Barbin, 1990, 1991).

Swetz (1989) recommended the use of historical problems, and considered that students can attain "a certain thrill and satisfaction in solving problems that originated several centuries ago" (p. 371). He wrote eloquently about the mathematical and social insights which can be gained through such questions, comparing them to works of art in a museum:

> [the mathematical problems of history] ... are intellectual and pedagogical works of art that testify to an expression of human genius. But, unlike the museum pieces, these pieces can actually be possessed by the viewers through a participation in the solution processes. (p. 376)

Ancient problems have also been valued for the way that they lead students to understand "the economy and the power of present mathematical symbols and processes" (Grugnetti & Rogers, 2000, p. 78). Although the study of primary documents is particularly applicable to the mathematics taught in secondary and tertiary levels of education, some original texts are simple enough to use at the elementary level (Arcavi, 1987; Bruckheimer, Ofir & Arcavi, 1995; Gardner, 1991; Ofir, 1991; Percival, 1999, 2001).

Material from the history of mathematics can be of particular value for gifted children, for whom the routine exercises of the regular classroom provide little to excite their curiosity or stimulate their thinking. Not only does such material provide a source of enrichment activities, but the variety of approaches to mathematics which are revealed by studying its history provide gifted children with a validation of the unconventional problem-solving strategies which they often employ (Daniel, 2000). However, questions about the historical background of mathematics can motivate investigations for children of all ability levels (Bidwell, 1993; Ponza, 1998).

Making connections

The introduction of cultural dimensions of mathematics has been justified by noting both the intra- and inter-disciplinary opportunities which such perspectives offer. (Ernest, 1998; Furinghetti & Somaglia, 1998; Grugnetti & Rogers, 2000, Wilson & Chavot, 2000). This is consistent with the belief of the Humanistic Mathematics Network (HMN) that mathematics should be viewed holistically, but is hardly a new idea in mathematics education: Almost a century ago, Branford exhorted teachers to "stimulate the free interchange of ideas in the pupil's varied departments of school studies ... lest we finally fashion a being whose intellect is as a house with many chambers lacking doors and windows alike" (1908, pp. 262-263). The notion of "making connections" has become a cornerstone of modern mathematics curricula, such as those of the NCTM (1989, 2000), the British Columbia Ministry of Education (1995) and the Western Canadian Protocol (1995).

Grugnetti and Rogers (2000) wrote that the intradisciplinary connections made apparent by a study of the history of mathematics help students develop a deeper understanding of the mathematics. They mentioned several links, ranging from those which can be demonstrated with a relatively elementary knowledge of mathematics, such as Descartes' breakthrough in relating algebra and geometry, to more recent, advanced connections such as those between algebraic group structure and the classification of different types of geometrical transformations. However, it is the interdisciplinary connections which are more commonly the focus of curricular attention, as is evident in the NCTM statement that students "should have many opportunities to observe the interaction of mathematics with other school subjects" (1989, p. 84). Grugnetti and Rogers discussed such connections, giving examples from science, geography, economics, art, music, religion and philosophy. They noted that such cross-curricular links enrich students' understanding of the individual disciplines: For example, in the case of perspective in Renaissance art, a study of the paintings can enhance the mathematical study of projective geometry, while an awareness of the mathematics underlying the artist's use of perspective can lead to increased appreciation of the picture. The cross-curricular advantages of studying ancient number systems have often been discussed (Ofir, 1991; Percival, 1999; Zaslavsky, 2001), and further descriptions of interdisciplinary connections are found in *The Mathematical Experience* (Davis & Hersh, 1981).

Barbin (1990) argued that the history of mathematics demonstrated why interdisciplinary teaching is essential, but she admitted that some mathematics teachers might need assistance from teachers of other disciplines, which could be problematic in the subject-oriented scheduling of secondary schools. Michalowicz (2000) noted the opposite situation in primary schools, where most teachers are generalists, many of whom have little interest in mathematics. She claimed that mathematics became more meaningful for such teachers once they were able to connect it to their social studies or literature curricula, but admitted that there is "little if any research" (p. 173) to support her assertion, which was based on a large amount of anecdotal evidence. Interdisciplinary connections are also promoted by multicultural perspectives on mathematics (D'Ambrosio, 2001; Shirley, 1995; Silvermann, Strawser, Strohauer, & Marzano, 2001; Taylor, 1997; Zaslavksy, 1996). The social studies links which can be forged by such work are obvious, and the arts and crafts of cultural groups from all over the world provide excellent material for discussions of symmetry, transformations and tessellations, although Zaslavsky (1991) notes that teachers often include such activities without being aware of the mathematics inherent in them.

A criticism sometimes levelled again the use of historical mathematics in the elementary school is that young children lack a sense of history (Fauvel, 1991; Grattan-Guiness, 1973). Piaget considered that this sense did not develop until well into secondary school, but others placed it earlier, around the age of eleven (Jahoda, 1963). More recently, history educators have claimed that even primary age children can start to develop the concept of historical time with the help of stories, art work, models and time charts (Blyth, 1982). Lingard (2000) considered that the history of mathematics fits into the curriculum of primary children just as well as into that of older students. Charbonneau (2002) echoed this comment but acknowledged the importance of finding ways to evoke an historical period if mathematics history is to be meaningful to young students. He discussed how the above methods could be used, also mentioning the use of architecture, music, local history, and the history of a subject of particularly interest to individual students, and gave examples to show how these ideas could be related to specific mathematical topics.

Implementation issues

The general literature on implementing educational reform is sufficiently well known that it need not be included in this dissertation (Fullan, 2001; Hall & Hord, 1987; House and Lapan, 1978; Huberman & Miles, 1984). However, several issues of specific relevance to the introduction of cultural approaches to mathematics have been mentioned elsewhere, and these are reviewed in the following paragraphs.

Barbin (2000) noted that the introduction of an historical dimension into the mathematics classroom proceeds in stages, starting with the modification of the teachers' own understanding and perception of mathematics. This new appreciation of the subject can then influence their teaching of mathematics, which in turn affects the way their students regard mathematics. However, this is not an easy process to start. Michalowicz observed that "when one finds a primary or secondary teacher using mathematics history in a pedagogical way, it is usually ... because the teacher is an amateur mathematics historian" (2000, p. 171). She noted two prerequisites for the introduction of historical perspectives of mathematics into the elementary level classroom: teacher education and resources.

Commenting that traditional history of mathematics courses are inappropriate for elementary teachers, many of whom are not confident mathematicians, Michalowicz (2000) suggested that these teachers need courses focussing specifically on the connection between the cultural material and their regular curriculum. In addition to discussing the cultural background of this work, such sessions could acquaint them with issues which held up the development of mathematical concepts, many of which act as epistemological obstacles to students today (Hefendehl-Hebeker, 1991; Radford, 1995, 2000; Sfard, 1995). The ICMI study included a survey of history of mathematics courses for trainee teachers (Schubring, 2000), and revealed that only a few countries organise such courses for those intending to teach at the elementary level. The examples given suggest that these courses have been arranged along traditional lines and not been completely successful, but Lingard's (1996, 1997) accounts of his courses in England, which include a selection of student evaluations, demonstrate that such programmes can be viewed positively. His courses involved a variety of ways of approaching the material, and he reported that students were so enthusiastic about it that many of them are using historical perspectives in their own elementary mathematics classes.

Michalowicz (2000) also commented on the expense and lack of global availability of resources, and recommended that teachers search the Internet for suitable material. Although resources on the history of mathematics do exist (see Chapter 3), Lingard criticized those aimed at the elementary level, claiming that they contain "spurious [materials] of the most patronising kind, which will have some of the mathematicians concerned turning in their graves" (1997).

A third problem is the lack of time (Bühler, 1990), both that required for the teachers' initial knowledge acquisition and that spent in the classroom. However, the former can be justified by the arguments that a teacher's private study provides considerable personal enrichment (Barbin, 1990), and that once learnt the knowledge can be used every year (Gulikers & Blom, 2001). Time spent in the classroom can be defended on the grounds that historical material can often be presented so as to reinforce the goals of the regular curriculum (Tzanakis & Arcavi, 2000).

A further objection to the inclusion of cultural perspectives on mathematics is the difficulty of assessing students' work in this area, with the corollary that work which is not assessed will not be considered of value to the students (Tzanakis & Arcavi, 2000). However, Zaslavsky (1996) noted the suitability of performance-based assessment for a multicultural curriculum, suggesting that this material allows students to "focus on the knowledge that [they] have acquired rather than penalizing them for what they do not know" (p. 18). Lingard (1996) has also reported the success of research papers and presentations as a means of evaluation, and although his students were preservice teachers, these ideas could be adapted for elementary school children.

The history of mathematics as a curriculum guide

The acceptance of an historical dimension of mathematics opens up the "ontogeny recapitulates phylogeny" argument, which is frequently implied, if not specifically mentioned, in the literature linking the history and pedagogy of mathematics (Ernest, 1998; Fauvel, 1991; Jones, 1969) This was postulated as a biogenetic law by Haeckel in 1866, and asserts that the biological growth of an individual retraces the evolution of mankind. However, ideas of genetics were present earlier in the century, and influenced Herbert Spencer to include the following passage in his 1854 article for the North British Review:

> The education of the child must accord both in mode and arrangement with the education of mankind, considered historically; or in other words, the genesis of knowledge in the individual must follow the same course as the genesis of knowledge in the race. (reprinted in Spencer, 1896, p. 122)

Mathematicians such as Poincaré and Klein supported this law, but in both cases, its application was seen as a way to avoid the excessive rigour advocated by their colleagues, rather than as direction to be followed literally (Furinghetti & Radford, 2002). However, Branford (1908) used it as a strategy for curriculum design, and produced a detailed account of the relationship between the historical growth of mathematical ideas and the educational levels at which they should be learnt. Nevertheless, even he admitted that the parallels between the development of the individual and the race lay in the overall structure, rather than the details, a point frequently echoed in more recent times (Byers, 1982; Charbonneau, 2002; Ernest, 1998), and both Branford and others have pointed out the futility of repeating all the errors of the past (Freudenthal, 1981; Jones, 1969).

Another early pedagogical use of the history of mathematics was the so-called "genetic approach to teaching" which appeared in the 1920s in the work of Izvolsky and Toeplitz (cited in Furinghetti & Radford, 2002). This can be viewed as a teaching compromise between a strictly logical approach and an historical approach which explains the rationale for a topic's development. As Freudenthal explains:

> Urging that ideas are taught genetically does not mean that they should be presented in the order in which they arose ... It is not the historical footprints of the inventor we should follow but an improved and better guided course of history. (1973, pp. 101, 103)

Recapitulation theory went out of favour when the importance of cultural factors in cognitive growth became apparent. The assumption of a unique phylogeny, necessary for this theory, is seen to be false when different cultural backgrounds are considered, and this led Werner and Vygotsky to reject recapitulationism. Piaget and Garcia chose to reinterpret the law in terms of the transition mechanisms between stages of development, rather than the mathematical content acquired, and referred to their work as "genetic epistemology" (cited in Radford, 1997).

However, as noted earlier, recapitulation theory still has a place in mathematics education literature: Ernest (1998) noted that the history of mathematics can both suggest a logical order of development and can also point out steps at which difficulties may occur, thus interpreting Haeckel's Law for both large scale curriculum design and for small scale planning of individual topics. It has also become the basis for the concept of "epistemological obstacles": An historical approach to mathematics provides "a perfect lens for detecting invisible pitfalls (and also) a valuable source of ideas about how these pitfalls might be overcome" (Sfard, 1994, p. 269). If teachers are aware of these pitfalls, not only can they develop strategies to help their students avoid or overcome them, but they can also allow more time for their students to become familiar with difficult ideas and reassure worried students that these problems are common, knowledge which relieves anxiety for many students (Jones, 1969)

Hefendehl-Hebeker (1991) used this idea in her article on negative numbers, in which she detailed problems which occurred prior to their ultimate acceptance, and discussed how teachers with knowledge of such problems could help their students to avoid these difficulties. However, recapitulationism is often invoked to condemn present methods of teaching: Many aspects of calculus are now taught in a logically sound but non-intuitive fashion which completely reverses the historical order in which they were developed, with consequent difficulties for students (Grabiner, 1983). At much earlier educational level Byers (1982) questioned the introduction of the decimal-place notation to first grade students, noting that this took thousands of years to develop, and Pimm (1987) advocated teaching the simpler additive system of the Egyptians first. However, curriculum planners have not taken up this suggestion.

The points explored in the above paragraphs suggest that Haekel's Law has some relevance to mathematical pedagogy. However Wheeler (1981) sounded a note of caution:

> To get at the true relation between the pedagogy and the history of mathematics requires going deeper. The common ground between pedagogy and history is that both are concerned with mathematics in the making, in the process of construction, in human minds. (p. 3)

Although he draws back from admitting a direct correlation between the history of mathematics and its pedagogy, his comment reinforces the view that historical and multicultural approaches to mathematics are consistent with the current humanist, constructivist philosophies of the subject.

Situating cultural mathematics in a framework for mathematics education

Thom noted that "all mathematics pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (1973, p. 204), so this chapter opened by summarizing the development of the present humanistic view of the nature of mathematics. Expert opinions on the impact of these views were then discussed, organized according to the various justifications given for implementing cultural approaches. However, a different structuring of data can often shed new light upon it, and the aim of this section is to provide an alternative way of viewing the material presented in the previous pages.

The benefits of constructing an analytic framework for mathematics education were first noted by Higginson (1980), and he proposed the "MAPS tetrahedral model" named for "M-Mathematics, A-Philosophy (arbitrary?), P-Psychology, S-Sociology" (p. 5). A decade later, Ernest (1991) produced a more detailed framework for the philosophy of mathematics education, listing fourteen elements which combined to form a model of educational ideology based upon epistemological and ethical perspectives. However, in his introduction to this text, he suggested four basic issues which need to be addressed by a philosophy of mathematics education: the philosophy of mathematics, the aims of mathematics education, the nature of learning and the nature of teaching.

These four themes form the framework within which the points discussed in this chapter are summarized. However since many of the issues raised have relevance for more than one of these categories, I have constructed a Venn diagram to show this composite nature (see Figure 1). For example, the idea that mathematics is created by "a community of mathematicians" reveals an important philosophic detail about the way in which mathematical concepts develop, yet its validation of group learning makes it important for both teachers and learners. Similarly, the philosophical idea that cultural perspectives of mathematics show students what it means to 'think like a mathematician' can also be considered as a worthy aim of mathematics education, as well as having implications for how students learn. It can be argued that this issue also has implications concerning how teachers teach, but decisions of priority had to be made in fitting a four-dimensional array onto the two-dimensional page.

As the diagram includes all the issues discussed in the literature, some redundancy occurs: There is considerable overlap between the idea of "thinking like a mathematician" and the three points shown to its right on the diagram, those focussing on the fallible nature of mathematics and the importance of intuition and persistence in problem-solving. However, these issues are often raised separately, and this duplication provided a concise labelling in the Aims section of these three aspects of students' approaches to learning. Similarly, the philosophical view that mathematics is not a culturally neutral subject combines the aims of social justice and cultural pluralism, the latter term being inclusive of all definitions of culture, whether determined by race, class, gender or other labels.

On the other hand, although most of the points mentioned in this chapter will impact both the teaching and learning of mathematics, some issues have more specific relevance than others to those who are doing the teaching or learning. For example, the idea that cultural approaches to mathematics make mathematics less daunting is primarily of importance to learners, whereas the role of cultural mathematics in highlighting epistemological obstacles benefits teachers as they prepare their lessons.

As Wheeler reminds us in the title of the journal he founded ("For the Learning of Mathematics"), mathematics education is about students' learning, so it should not be surprising that many of the points mentioned in this chapter can be situated in the Learning section of the chart. However, it should be noted that the issues addressed in this section can apply to both students and teachers, since the latter often continue to learn about the cultural background of mathematics as they teach it.

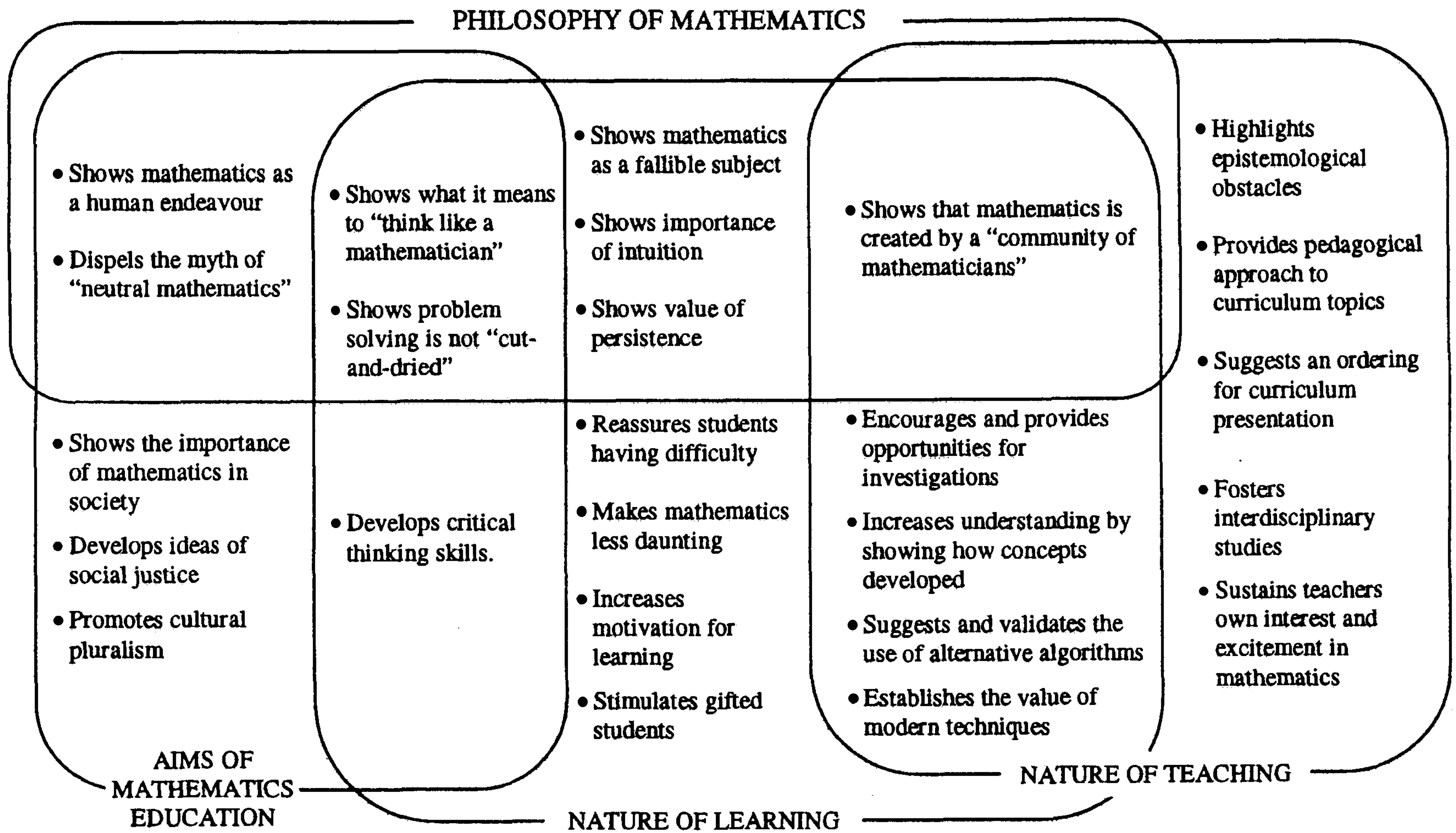

Fig. 1: "Expert" views on the role of cultural perspectives in mathematics education

Seeing the data in this framework provides a quick summary of the humanist philosophy underlying mathematics education. It highlights the considerable impact which cultural perspectives can have upon student learning, and clarifies the aims of mathematics education which can be addressed in this fashion. It also draws attention to ways in which a knowledge of cultural aspects of mathematics can provide guidance to teachers, both in structuring the introduction of curriculum material, and in suggesting ways for students to explore this information.

The use of cultural mathematics in the elementary classroom

The ICMI study includes a summary of the role that the history of mathematics currently occupies in national curricula (Fasanelli, 2000). This reveals that most countries include reference to the cultural background of mathematics in their overall goals of mathematics education, but that few mandate attention to individual topics, particularly at the elementary level. An interesting exception is the use of a unified series of mathematics textbooks in China which include sixteen items related to the history of mathematics, half of which refer to the role played by Chinese mathematicians. However, Fasanelli reports that few teachers use this material, other than sections which can be directly employed in education for patriotism. A more thorough treatment of cultural issues is given in the *Mathematics 2001* project in Poland, one of several proposals for a new mathematics curriculum in that country. Even at the elementary level, this project includes learning outcomes which specifically mention cultural topics, such as the Chinese abacus, Aztec numeral system and Eratosthenes' Sieve.

Given the general lack of curriculum guidance on cultural perspectives of mathematics, mathematics journals have played an important role in disseminating these ideas. During the inaugural year of *Arithmetic Teacher*, the first journal specifically devoted to mathematics in the elementary school, two articles on the history of mathematics appeared. The first (Willerding, 1954) was a theoretical article promoting the use of the history of mathematics in teaching arithmetic, whereas the later article (Jenkins, 1954) used the story format to provide information about the abacus and several counting systems from ancient times. Although articles on the use of the history of mathematics continued to appear in later issues, they were infrequent, and as classroom experiences with the work were not reported, it is unclear what impact such articles had upon their audience of elementary teachers. The NCTM acknowledgement that attention should be given to the cultural background of mathematics (NCTM, 1989) encouraged the publication of several articles which either promoted this approach (Bidwell, 1993; D'Ambrosio, 1997; Fauvel, 1991; Shirley, 1995; Zaslavsky, 1991) or explained how specific topics could be taught (Ascher, 2001; Naylor and Naylor, 2001; Phillip, 1996; Swetz, 1989; Zepp, 1992). However articles reporting the classroom use of such perspectives still remained scarce: Those relevant to elementary school teaching are surveyed in the following paragraphs.

The importance attached to the history of mathematics by educators has been demonstrated by the fact that four English language mathematics education journals have devoted complete issues to a discussion of the relationship between the history of mathematics and the teaching and learning of the subject: *For the Learning of Mathematics* (1991, 11 (2)), *Mathematical Gazette* (1992, 76 (475)), *Mathematics in School* (1997, 26(3) and 1998, 27(4)), and *Mathematics Teacher* (2000, 93(8)). The first of these special issues focussed on classroom experiences, and covered the academic spectrum from elementary school to university level. The other journals included a mixture of theoretical and practical articles.

The special issue of *For the Learning of Mathematics* included three elementary level papers. Despite its acknowledged focus on "History in Mathematics Education", it contained Zaslavsky's article "World cultures in the mathematics class" (1991), showing that the borderline between "historical mathematics" and "multicultural mathematics" is indeed ill-defined. The author wrote mainly about what *can* be done with young students, rather than what *has* been done, but she first did include a short description of a session on finger counting with a class of eight-year-olds, inspired by her book *Count on your fingers African style* (1980). The other two articles concerning elementary level students both included material on Egyptian mathematics (Gardner, 1991; Ofir, 1991). This has been a very popular topic, and work in this area has also been reported by Bohan and Bohan (1993), Michalowicz (1996), Moldavan (2001), Montgomery (1995), and Percival (1999, 2001). The article by Bohan and Bohan is particularly interesting as it describes how students reconstructed the place-value structure of the modern number system through their attempts to "improve" the tally-like system used by the Egyptians. The articles by Michalowicz, Percival and Ofir show how elementary school students made use of primary sources for Egyptian mathematics: This involved translating and working with passages from the Rhind Papyrus. Ofir and Percival designed similar activities using sources from several other ancient civilizations, with the goal of helping children gain a better understanding of our modern number system.

The *Mathematical Gazette* history issue included van Maanen's article "Teaching geometry to 11 year old 'medieval lawyers'" (1992), which is particularly fascinating in that the Euclidean constructions generated by the work described are no longer required learning in the Dutch curriculum. The other article of relevance to elementary classrooms is that in which Ofir and Arcavi (1992) described a unit on early methods of algebra. Although the material goes beyond the elementary curriculum in British Columbia, some of their ideas are of general interest. They note that alternative methods of solving equations are discussed to give students the "flavour" of various techniques and to "let them do mathematics differently" (p. 84), and mention their practice of showing old documents to "create the 'historical atmosphere'" (p.73). These points can easily be incorporated into elementary work on alternative numerical algorithms.

The 1998 *Mathematics in School* special issue includes an article in which Ponza described how her group of thirteen-year-old students in Argentina wrote and performed a play about Galois. The author/teacher wrote that "the history of mathematics exercised what can only be described as a magical effect ... [on a class in which] indiscipline was rife" (1998, p. 11). Another of the articles, *Counting in Cuneiform* (Robson, 1998), contained activity sheets on Babylonian mathematics which the author had used in a series of workshops for middle-school students, but she gave no indication of the students' reaction to the work.

Even though the *Mathematics Teacher* now concentrates on high school mathematics, the three lessons described in its 2000 focus issue could easily be adapted for elementary classes. Barry (2000) wrote about his Grade 12 students' enthusiastic attempts to decode an ancient mathematical tablet, an approach also used with Grade 7 students by Percival (2001). Shirley (2000) described a lesson in which he dressed as Pythagoras to instil some notions of Greek geometry into his class, and Horn, Zamierowski and Barger (2000) explained a project they developed called "Correspondence from mathematicians". This required each student to investigate the life and work of a specific mathematician, and then compose a letter from the mathematician to an appropriate person from his or her time period. Although the authors admitted that their project was not completely successful, they noted that several students who normally showed no interest in mathematics did valuable research on their mathematician.

A few articles concerning elementary school experiences with the history of mathematics have also appeared elsewhere. In 1993, Voolich wrote about the ways she used biographical information about mathematicians with her students, ideas which she later elaborated in her book *A Peek into Math of the Past* (2001). Stemn and Collins (2001) described their Grade 6 students' work on the Vedic square, which provides an unusual link between number and geometry. Dobler and Klein (2002) took the popular idea of using children's literature as a starting point for mathematics, and developed the concept of the Cartesian coordinate system with their first grade students after reading *A fly on the ceiling* (Glass, 1998). Percival (2003) wrote about a series of "cross-curricular adventures in mathematics", in which a Grade 3 teacher made considerable use of topics from the history of mathematics in a series of "Time-Travel Days"[1].

Articles discussing multicultural perspectives on mathematics in the classroom are more recent than those dealing with its history, reflecting the later acknowledgement of this aspect of the subject: With a few rare exceptions, such as early papers by Zaslavsky (1973, 1981), this material did not appear in journals until the 1990s. The first to include a description of students' work was *Threading mathematics into social studies* (Smith, 1995), in which third-grade children not only explored the spatial concepts found in traditional quilts, but also investigated the mathematical patterns connected with square numbers by trying to predict how many small squares would be needed for large quilt blocks. The use of cultural quilting in the elementary mathematics class has also been promoted by Paznokas (2003), and for middle school students by McCoy and Shaw (2003).

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[1] This material was taken from the second case-study described in this dissertation.

This perspective has seen a rapid growth in popularity in the past few years, and in 2001 "Mathematics and Culture" was the selected topic for the focus issue of the NCTM elementary level journal *Teaching Children Mathematics*. As with the historical material, articles promoting this approach and explaining how it can be used outnumber those reporting actual classroom usage. For example, Gorman (1997) relates her fifth-grade class' experiences with the Sumerian game of kalah, pointing out how such game-playing satisfies the NCTM standards (1989), but she then proceeds to explain several other games and indicate how they could be turned into classroom investigations, rather than giving direct feedback from her students. However, articles which do refer to classroom experience frequently mention the enthusiasm of the students, and provide useful anecdotal evidence for the success of this teaching approach.

Schaelling's combined first- and second-grade class constructed and played a Native American dice-stick game, and both aspects of this activity gave rise to genuine problem posing and solving situations: One student's use of negative numbers in her invented scoring system gave Schaelling insight into a level of knowledge and mathematical ability which might not have surfaced in regular curriculum work (Barta & Schaelling, 1998). Uy (2003) wrote about a class of fourth-grade Hispanic students who reinforced ideas of place-value through their work with the Chinese numeration system, and a sixth-grade class compared number words in a variety of languages (Kliman & Janssen, 1996). Some activities also included aspects of space and shape: A fifth-grade class in New York designed a new flag for their country after investigating the structure of those from other parts of the world (Dolinko, 1996). These descriptions of culturally based work generally received enthusiastic responses from readers of the journal, as evidenced by the letters printed in the "Readers Exchange" and similar sections.

CHAPTER 3
CLASSROOM RESOURCES

Until the last decade of the twentieth century a major drawback to the introduction of cultural perspectives into the elementary classroom was the scarcity of material which could be used directly with students (Rogers, 1991). However, the situation has now improved, and this chapter surveys the resources for historical and multicultural mathematics currently available to elementary school teachers. These materials are listed in Appendix B and include posters, videos, internet sites and more than sixty books, over half of which were written specifically for elementary school students. A list of more advanced reference books is also given, as two of the teachers in this study found such material useful.

Material for teaching the history of mathematics

The first history of mathematics text accessible to elementary school students was Smith's classic book *Number Stories of Long Ago*, first published in 1919 but reprinted many times throughout the century, most recently in 1995. This "little series of human incidents" (p. x) illuminates the history of arithmetic by telling stories featuring children from different times and places learning about the numbers used by their elders. Smith justified his text as follows:

> The story of our numbers, of the world's attempts to count, of the many experiments in writing numerals, and of the difficulties encountered through the ages in performing our everyday computations – all this is so interwoven with the history of humanity as to have an interest for every thinking person. ... The history of mathematics is no small part of the history of civilization. (p. ix)

Over thirty years were to pass before another children's historical mathematics text appeared: *The Wonderful World of Mathematics* (Hogben, 1955) gives an overview of mathematics that can be appreciated by students in the upper levels of elementary school. A similar text, *The Giant Golden Book of Mathematics* (Adler, 1960), appeared shortly afterwards, but was less focused on the historical approach than Hogben's book.

In 1969 the National Council of Teachers of Mathematics recognized the importance of the history of mathematics by making it the subject of their thirty-first yearbook. This text, and others from a similar period (Bunt, Jones & Bedient, 1976; Dedron & Itard, 1959/1973; Popp, 1968/1975; Wilder, 1968), provided useful information for teachers wishing to incorporate some historical mathematics into their classroom, but the material was rarely presented in a way that enabled it to be used directly with elementary students.

Eves' book *In Mathematical Circles* (1969) and its four successors provided an early source of mathematical stories and anecdotes, many of which shed light on the cultural background of mathematics. Pappas' *Joy of Mathematics* (1989) can perhaps be viewed as the heir to Eves' tradition, as her book also presents a large number of short articles, often involving historical aspects of mathematics. The material given in this text and her many subsequent publications can be easily adapted for classroom presentation, and could also provide source material for student projects at the upper elementary school level.

A variety of approaches have been adopted by recent books designed to help elementary students situate mathematics in its historical setting. Several follow Smith's example of concentrating on some specific aspect of mathematics. Schmandt-Besserat (1999) gives an account of the history of counting which is simple enough for young children to read, and Burnett's *Sights, sounds and symbols* (1999) has a similar focus, but includes exercises for the students. Wahl's *Mathematical Mystery Tour* (1988) takes the students on an exploration of many aspects of Fibonacci numbers and the Golden Ratio, encompassing many regular curriculum concepts and higher-level thinking skills along the way, and similar material is covered in Garland's *Fascinating Fibonaccis* (1987).

A few books focus on the mathematics of a particular civilization, with Egypt being the most popular topic at the elementary level (Brading, 1994; Burnett and Irons, 1996). More commonly though, books include mathematics from several cultures, sometimes as separate chapters (Eagle, 1995), but frequently as different approaches to specific concepts. In the latter texts, the distinction between "historical mathematics" and "multicultural mathematics" becomes blurred, and these texts will be discussed later in this chapter.

Several books give biographical information about mathematicians. The most famous of these is Bell's *Men of Mathematics* (1937), but this was written for a general audience, rather than specifically for elementary students: The most appropriate books for children are the two volumes of *Mathematicians are people, too: stories from the lives of great mathematicians* (Reimer & Reimer, 1990, 1995b). Like Smith, the authors point out that these stories "are an important part of our heritage, a vital link to our past" (1990, p. iv), and comment that "almost nothing captures the interest and attention of students like a good story" (1995b, p. iv). Their stories are both short enough to be read aloud to a class, and easy enough for children to read for themselves. Other books focus on the life and work of a single mathematician. Some give a reasonably accurate account, such as Ipsen's books about Newton (1995) and Archimedes (1988), whereas others, such as Lasky's story of Eratosthenes (1994), include a great deal of imaginative detail for which there is little or no evidence. Several authors have taken a gender-specific focus, writing books to highlight the fact that women have also had a role in the development of mathematics (Cooney, 1996; Osen, 1975/2003; Perl, 1978, 1993). Perl's books are designed to be used in the elementary school, and encourage girls to "dream big and keep your options wide open" (1993, Preface). There are also books which take a specific anecdote about a mathematician, and build it into a children's story, complete with entertaining illustrations: *Mr. Archimedes' Bath* (Allen, 1994) and *The Fly on the Ceiling* (Glass, 1998) are examples of this genre.

In addition to their specifically biographical texts, Reimer and Reimer also wrote a series of books, *Historical Connections in Mathematics* (1992, 1993, 1995a), which include student activity sheets. These are grouped together following biographical information about the mathematician to whose work they are related, and although they sometimes involve drastic simplifications in order to make the mathematics accessible to young students, they stimulate many valuable problem-solving experiences.. Although they can be used without reference to the preceding anecdotes and quotations, the comments in these introductions help to place the work in its cultural setting and often intrigue the students. Many activities encourage a hands-on approach, as Reimer and Reimer consider that "students will become more involved and remember concepts better if they can participate in the process" (1992, p. 92). In format these books are similar to

Mitchell's early text, *Mathematical history: Activities, puzzles, stories and games* (1978/2001), in which most of the activities, puzzles and stories related to specific mathematicians. However, unlike the Reimer and Reimer books, the methods needed to solve Mitchell's puzzles often have no specific connection to the work of the mathematician considered, relying instead upon very traditional arithmetic or even language arts skills such as word searches.

A growing interest in the history of mathematics is indicated by the fact that the *Historical Connections* texts are included in the Learning Resources section of the Integrated Resource Packages (IRP) (British Columbia Ministry of Education, 1995), where they are recommended for use by students in Grades 2 to 9. For children in Grades 7 to 12, the IRP recommends the text *Classic Math: History Topics for the Classroom* (Johnson, 1994). This aims to "convince students of the humanity of [mathematics]" and does so by providing a wealth of material that can be used to generate class discussions. Although much of this material is too advanced for elementary school students, the two sections on the history of mathematics symbols and terms are an excellent resource for their teachers. Similar texts for high school students have been written by Swetz (1994), Smith (1996) and Voolich (2001) and experienced teachers can adapt these materials for younger students.

A list of web-sites on the history of mathematics is given in Barrow-Green (1998), but the majority of the sites referenced there are too advanced for elementary students. One of the sites mentioned was specifically created for students by the students of a Grade 4 class, but unfortunately their "Moldy Oldies Collection" site is no longer available. However Heller, the students' teacher, reported on its creation at the INET96 conference, <http://www.isoc.org/inet96/proceedings/c7/c7_3.htm>, noting that although the initial student assignment had merely been to research famous mathematicians using the internet:

> It quickly became apparent that the material was "over the head" of most of the students. ... The students were frustrated by difficult vocabulary, and the teacher was frustrated by having to "translate" what they found. ... When someone commented, "If it had only been written with kids in mind!" we knew we had a way to correct the problem.

Although Heller admitted she had to do a considerable amount of preliminary work to find sites for her students to visit, she noted that the children were very proud of the web-pages they produced, and she considered that they had acquired a good knowledge of the lives and work of the mathematicians they researched. Unfortunately, the ephemeral nature of this web-site is typical of many produced by teachers and students. However, some web-sites suitable for young students can still be found.

"Mathematicians are People Too" is on the web-site of the *National Center for Education Statistics* (<http://nces.ed.gov/nceskids/>) which gives students short, simply presented information about nine mathematicians, including two twentieth century women. Another site designed for students, *Everything you always wanted to know about Maths (but were afraid to ask)*, is found at <http://www.liz.richards.btinternet.co.uk/> and contains two relevant sections: "The History of Math" and "Who's Who in Math". Apart from biographical information, there are also some simple web-sites about the mathematics of early civilizations, such as <http://www.eyelid.co.uk/numbers.htm> which explains the Egyptian number system, and contains some interactive activities. Students also enjoy exploring ancient number systems through the use of Java applets: Examples include the *Cuneiform Calculator* (<http://it.stlawu.edu/~dmelvill/mesomath/calculator/scalc.html>), and the various types of abacus found at the site <http://www.ee.ryerson.ca:8080/~elf/abacus/>.

For teachers needing background material, the *Math Forum* site, <http://mathforum.org/library/topics/history/branch.html>, provides links to many history of mathematics web-pages, including Miller's sites on the origins of words and symbols, a *Mathematical Quotations Server* from Furman University, and several pages of *Famous Problems in the History of Mathematics*. It also lists the web-site it calls "the premier site on the Web for math history": The *MacTutor History of Mathematics archive*, contains information on many different aspects of the history of mathematics, including time-lines, biographies and posters which can be used to motivate students to learn more about the people who contributed to the development of mathematics.

Videos concerning the history of mathematics at an elementary level are rather scarce, but in 1998, Channel 4 Television in the United Kingdom produced three videos from its broadcasts for schools and home learners. These 15 minute programmes (*Maths* *from History: The Egyptians,* also *The Greeks* and *The Romans*) are complemented by a set of student texts, together with teachers' notes and reproducible masters (Brading, 1994; Bibby, 1994). However, these written materials can also be used independently.

A much earlier film is *Donald in Mathmagic Land,* which was produced by Walt Disney Educational Media in 1959. Donald Duck takes a trip to Ancient Greece and discovers that "you find mathematics in the darndest places!" as he explores the relationship of mathematics to architecture, biology and music, particularly with reference to the golden rectangle. This 27 minute film is recommended for students in Grades 6 to 8, but is also enjoyed by younger students, and generates many interesting mathematical activities in the hands of a skilful teacher.

Another video which students find very entertaining is *The Platonic Solids* (Key Curriculum Press, 1991), a 17 minute show in which the various solids twirl around the screen in computer animation. Although the historical background is not a major feature of the video, it helps the students see that such mathematical objects have been around for thousands of years, and have generated considerable interest during this time.

Posters can also generate interest in the history of mathematics. *The Colorful Characters of Mathematics, Historic Women of Mathematics* and *Speaking of Math* are available from the on-line teachers store, <http://www.mathteacherstore.com>, which also offers two containing humorous cartoons, *Great Ideas of Mathematics* and *Isaac Asimov's History of Mathematics.* Another poster, *Great Moments in Math* (McDougal Littel, 2001), lists developments in mathematics from 6000 B.C.E. to the present, although the selection of the "great moments" is rather curious, including such events as the invention of the first piano, a reference to the conjectured connection between music and mathematical performance known as the "Mozart effect".

Material for teaching multicultural mathematics

The recognition of multicultural mathematics as a topic of interest in the elementary school lagged behind that of the history of mathematics. The first publications easily accessible to young children were written by Zaslavsky. *Count on your Fingers African style* (1980) deals with the variety of finger counting systems found in the African continent, and *Tic Tac Toe and Other Three-in-a-row Games* (1982) is one of the many multicultural game books which have been published (Bell, 1979; Bell and Cornelius, 1990; Culin, 1975; Orlando, 1999). However Zaslavky's text includes a strong mathematical focus which is not found in some of the others.

Krause's *Multicultural Mathematics Materials* (1983) was the first book using this approach specifically designed for classroom use. It provides information about games and activities from many countries and includes game boards and numerical or geometrical activity sheets for elementary students. Although it lacks the colourful illustrations of its successors, it was a valuable introduction to multicultural perspectives on mathematics, and its second printing in 2000 suggests that it had become a popular resource. In 1987 this book was followed by Zaslavsky's *Math comes alive: Activities from many countries*.

The first half of the 1990s saw the publication of two books which not only described classroom activities suitable for elementary level students, but also raised important issues about multicultural mathematics. Shan and Bailey (1991) and Nelson, Joseph and Williams (1993) both aimed to refute the Eurocentric view of mathematics found in the majority of mathematics texts, and offered historical examples to show how non-European cultures contributed to the development of mathematics. However, Shan and Bailey's book, subtitled *Classroom Mathematics for Equality and Justice* went much further in creating "antiracist approaches to the teaching of mathematics" (p. v). This book stressed the misleading picture of the world given by traditional mathematics texts, and suggested some "current affairs" mathematical activities to replace them.

The 1990s also saw the publication of more classroom texts by Zaslavsky. In 1993, the middle-school book *Multicultural mathematics: Interdisciplinary co-operative learning activities*(1993a) appeared. This presented a variety of activities grouped by topic, a format which was also followed in her next book, *Multicultural math: hands-on activities from around the world* (1994), written for younger children. The later text also included background information and pedagogical suggestions to help teachers introduce the material. In both cases, indications of content area are given which assist teachers to select activities that relate to specific learning outcomes in their curriculum, a feature which Zaslavsky maintained in the book *Math Games and Activities from Around the World* (1998).

The British Columbia Ministry of Education included Zaslavsky's book *The Multicultural Math Classroom* (1996) in their 1999 list of recommended teacher resources. This book describes many class activities but lacks the reproducible sheets found in the texts described in the previous paragraph. However, it compensates by providing considerably more background material and teaching suggestions than are given in the earlier texts, in addition to including several preliminary chapters which present a rationale for multicultural approaches to mathematics.

The most extensive material for multicultural mathematics is *Mathematics from Many Cultures* (Irons & Burnett, 1995), a resource which is listed in the British Columbia IRP (British Columbia Ministry of Education, 1995). There are six packs in the series, each of which includes a *Big-Book*, six large wall posters which duplicate the Big Book material, and a teachers' guide. This *Teachers' Notes* book contains blackline masters for activities, in addition to providing explanations and extensions for the material given on the posters. It is also valuable for the many interdisciplinary links mentioned, as well as for its clear indication of the mathematical topics to which each section relates. Each pack contains information on six topics taken from a variety of curriculum area: for example, the most advanced pack has chapters on geometrical designs, number triangles, measurement, "never-ending paths" (focussing on problem-solving), number systems and concludes, as do all the packs, with a chapter describing games of a specific type. The packs gradually increase in difficulty, and are recommended for students in all elementary school grades. Burnett and Irons have produced several other packs of mathematical material for elementary students, two of which explore the mathematics of other cultures: Mathematics of the Americas (1998) and Egyptian Genius (1996). As with the earlier material, the mathematics of each civilisation is related to the social context in which it occurred, and colourful illustrations stimulate the children's imagination.

Other texts on multicultural mathematics are less attractively produced, but find different ways to interest the students. *Multicultural Math Fun: Holidays around the World* (Bock, Guengerich & Martin, 1997) takes the students through the school year, giving activities related to the various holidays that arise. Although the social context of this work is interesting, the mathematics is very traditionally focussed on the regular curriculum material. Lumpkin and Strong's *Multicultural Science and Math Connections* (1995) intersperses mathematical activities among readings and activities related to broader scientific areas. The first part of the book highlights some of the cultural groups whose achievements are underrepresented in traditional curricula, and this is followed by chapters related to individuals from these cultures, whom the authors propose as role models for the students. This text is written for students in Grades 5 to 9, and includes many hands-on projects in both mathematics and science. Lumpkin's *Algebra Activities from Many Cultures* (1997) and the corresponding geometry book contain a large number of interesting student activities, but only the simpler ones are accessible to elementary level students. However, her story book *Senefer: A Young Genius in Old Egypt* (1992) is aimed at younger children, and introduces them to the number system of Ancient Egypt, in addition to showing them something about everyday life there.

There are several other popular story-books for primary age children which place mathematics in a cultural setting. *The Rajah's Rice* (Barry, 1994) and *One Grain of Rice* (Demi, 1997) are both set in India, and are among the many books which explore the concept of exponential growth. *Grandfather Tang's Story* (Tompert, 1990) introduces the Chinese custom of playing with tangrams, an activity which can develop children's spatial sense, and *The Village of Round and Square Houses* (Grifalconi, 1986) gives an African perspective on geometric solids and shapes. For older children, *The Man Who Counted* (Tahan, 1972/1993) presents a series of puzzles woven into the travels of an Arabian mathematician and gives a cultural context which a creative teacher can exploit. Costain's story *Mathematics around the World* (1995) tells of a family's journey around the world and of the mathematics they noticed along the way, and provides a cultural link between mathematics and many other curricular areas.

Brief introductions to cultural aspects of mathematics are given at the *Ethnomathematics* site, <http://www.cs.uidaho.edu/~casey931/seminar/ethno.html>, and at <http://people.clarityconnect.com/webpages/terri/multiculturalideas.html>. The latter *Multicultural Ideas for your Math Class* website also includes suggestions for bringing these ideas into the classroom. Other practical suggestions for teachers are given in the *Multicultural Math Fair* site, <http://www.rialto.k12.ca.us/frisbie/mathfair/about.html>. This describes an event organized by middle school students which received enthusiastic reviews from visiting elementary students, and provides a good model for similar activities that could be created by younger students.

There are some specialist sites on the mathematics of particular cultures, such as <http://www.niti.org/mayan/lesson.htm>, which explains the Mayan number system and gives simple conversion and addition problems for students to solve. A more general site, *Cultural Math* at <http://everyschool.org/u/logan/culturalmath/index.htm> contains simple information about the mathematics of four regions, Asia, Middle East, Africa and Americas, and would be a good first resource for elementary student projects. It also contains many links to more advanced web-sites, as well as several which give instructions about playing games from different cultures. There are also web-sites about the games of particular cultural groups such as the *First Nations' Games of Chance* site (<http://web.uvic.ca/~tpelton/fn-math/index.html>). Many web-sites are devoted to women mathematicians, such as the *Alphabetical Index of Women Mathematicians* (<http://www.agnesscott.edu/lriddle/women/alpha.htm>), although they may be too advanced for elementary school students..

Videos promoting multicultural mathematics are still rare, but *Second Voyage of the Mimi* (Bank Street College of Education, 1988) includes a segment about the Mayan number system. However, posters are more common. The most extensive set is that published by Key Curriculum Press in 1988: *Multicultural Math Classroom Posters* is a collection of sixteen posters, each one devoted to the mathematics of a particular geographic region or culture In the same year, J. Weston Walch published a set of three posters, *Math Around the World*, and another poster of the same name can be found at the on-line teachers store mentioned above (page 41). The latter poster actually takes an historical approach rather than multicultural ones, but its large world map emphasizes that mathematics has appeared in many different forms around the world over the past millenia. Many years earlier NCTM had published a set of *Multicultural Posters and Activities* (Seattle Public Schools, 1984), and although these lack the colour and sophistication of those mentioned above, the presence of activities related to the posters makes them a very useful resource.

CHAPTER 4
METHODOLOGY

This chapter starts by setting the scene for the research: The participants and their environments are introduced, and the time schedule for the study is explained. The research design and data collection procedures are then described, and the chapter closes with an account of the methods used to analyze the data.

The Research Participants

My research programme used case study methods, so the careful selection of research participants was one of the first issues to be addressed. I used the purposeful sampling approach to this process, a method which aims to increase the value of information obtained by choosing "information-rich key informants" (McMillan & Schumacher, 1997, p. 397). Such sampling needs some prior knowledge of the possible research participants, a requirement which fitted well with my practice of working with anyone who expresses interest in introducing cultural ideas to the mathematics classroom. Before starting my formal research, I had worked with ten such teachers, but I chose to focus my research on three case studies: two individual and one group study of four teachers. Nevertheless, I stayed in intermittent contact with the other teachers, and a few of their comments have been included in my analysis of teachers' reactions to teaching culturally focused mathematics. To maintain anonymity, all names of teachers, students and schools have been changed.

The first two individuals I selected, Ruth and Sarah, had already explored some aspects of cultural mathematics in their classrooms. The third case study, an example of "snowball sampling" (Bogdan & Bicklen, 1998, p. 64), concerns a group of four teachers from Highbury Elementary school where Sarah taught. Although these teachers were intrigued by Sarah's work, none of them had tried to include cultural perspectives into their own teaching of mathematics before taking part in this study. This choice of research participants gave me access to classes from Grade 3 to 6, and included both male and female teachers with a range of teaching experience from five to over thirty years.

My first acquaintance with Ruth came through an e-mail message forwarded to me by my supervisor, in which she wrote:

> One of my great recent finds is the AIMS books "Mathematicians are People Too - Stories from the Lives of Great Mathematicians" and "Historical Connections in Mathematics Vol. 1". The kids at school loved it, to be able to go around and talk about Euler (William loved that one) and Archimedes, they thought it was great to talk about the ancient Greek geek (in a respectful friendly tone). Why not math stories too???!!!!!

Ruth was clearly receptive to the idea of an historical approach to mathematics, yet the tone of the message suggested that such perspectives were relatively new to her. When I met her the following month, she enthusiastically agreed to take part in my research.

Sarah was a student in the course *Math Around the World and Across the Ages* which I taught in 2000 as part of Simon Fraser University's Post-Baccalaureate Diploma programme. She showed great interest in the material I presented, often trying it out in her Grade 3 classroom in the days following each class. For the final project of the Diploma, she created a programme of ten "Time-Travel Days", which integrated mathematics with other aspects of the elementary school curriculum, and planned to implement this during the following academic year. Given her obvious interest in using historical ideas in the mathematics classroom, and her well-defined outline of how to put this into effect, Sarah was an obvious choice for my research project. She readily agreed to my request that I observe all her Time-Travel Days and discuss them with her, and commented that it would be useful to have another expert in the classroom.

Sarah's Time-Travel Days soon captured the imagination of the other teachers at her small school, as recess and lunch time conversations often involved discussions of what was happening in her classroom. This led Sarah to present some historical mathematics topics in Joan's Grade 4 classroom. Joan was fascinated by the material, and also appreciated the attraction it held for her students, so decided that she would also implement an historical approach to mathematics the following year.

At the start of the next academic year, 2001/2002, I visited the school to discuss my research project with Joan, but was waylaid in the staff-room by Barbara, a Grade 6 teacher, who also expressed interest in this novel approach to mathematics. I enthusiastically accepted her participation in my study, and to my surprise and delight, the snowballing continued. Joan invited Maureen, the Grade 5/6 teacher, to join the research, and during my conversation with these three teachers, Barbara suggested that it would be good to include all the Intermediate level teachers. Mike, the Grade 4/5 teacher, subsequently agreed to their plan, so all the students in Grades 4 to 6 were involved in my research. The Grade 7 teacher was also invited, but he declined, claiming that his position as Vice-Principal left him little time to explore new approaches to mathematics. As these four Intermediate teachers took a common approach to teaching historical and multicultural mathematics in their classroom, they have been grouped together as the "Group of Four" (G4), and will treated as a group case study, rather than that of four individuals.

The Research Schedule

The fieldwork for two of the case studies, those of Ruth and the G4 group, took place during the 2001/2002 academic year. However, I had worked with six teachers the previous year, including Ruth and Sarah. While most of this early work was of a preliminary, exploratory nature, it was clear from my earliest conversations with Sarah that she was well prepared to teach mathematics from an historical perspective, and her willingness to videotape her classes and discuss them with me led me to take her contributions to my research very seriously from the start. In retrospect, this was a fortunate decision, as the students she had in 2000/2001 were a remarkably astute, perceptive and inquiring group, whereas her class the following year lay at the opposite end of the academic spectrum.

Sarah ran her programme for a third time in 2002/2003, and although I already had a considerable amount of data about her work, I could not resist taking the opportunity to visit her classes whenever possible, as the children's reaction to the Time-Travel Days was always a delight to watch. However, these trips to her school also allowed me to talk to the G4 teachers about their use, or lack of use, of the cultural perspectives they had taught the previous year. I made it clear that I was willing to help them again whenever they wished to do more work in this area, but left the onus on them to contact me, although I did arrange one formal meeting half way through the year. I also remained in contact with Ruth during this year, but medical problems took her out of the classroom during the period in which she had planned to repeat her focus on cultural aspects of mathematics. At the end of this year, I contacted each teacher to ask for their reflections on the cultural mathematics work they had done since the start of the study.

During the fieldwork, the research schedule was determined by the teaching plans of my participants. Ruth's seventeen culturally oriented classes were unevenly spread over her regular math classes for a two-month period. Sarah taught ten Time-Travel Days, each originally planned for one complete day per month throughout the year, although the material usually carried over into the following days and weeks. Each member of the G4 group planned five single-class sessions of "Multicultural Math", but like Sarah, they often found that extra sessions were needed to complete the topics selected.

I talked to all the teachers before and after their culturally focussed classes, but conflicts between the schedules of the Highbury teachers and Ruth meant that I was unable to attend every session. The compromise I reached was to visit the first cultural mathematics class of each Highbury teacher every month, and rely upon audio-tapes or teachers' written or verbal reports to learn about the follow-up sessions. As Ruth's schedule was less predictable, frequent e-mail exchanges kept me informed about suitable classes to visit, and I was able to attend ten of the seventeen lessons in which she planned to include cultural perspectives on mathematics, with the remaining classes being audio-taped.

The Research Environment

My study is not only multi-case, but also involves sites in two different school districts. However, both schools follow the same British Columbia curriculum guidelines. Ruth's school was in an area classified as having low socio-economic status, and she commented that she often had a relatively large number of special needs students in her classes. The school in which Sarah and the G4 group taught is in a predominantly middle class area, although Sarah noted that there was a considerable range of affluence among her students' families. However, she felt that cultural approaches to mathematics helped to put her students on an equal footing, as the material was new to all of them.

The observed classes took place in the teachers' own classrooms, except on the rare occasions when the activities took students to other parts of the school. These rooms varied considerably in both the arrangement of furniture and the material displayed on the walls. Most rooms were formally organized with the students' desks in neat rows, but a few had the desks grouped together. Sarah was the only teacher who changed the room set-up according to the material she was to teach. Although her students were usually seated in the traditional rows and columns, on Time-Travel Days their desks were often pushed together to form castles, schools, miniature villages and so on, or pushed to the side of the room so that children could work together in groups on the floor. For these occasions the usual wall decorations were augmented by posters relating to the culture to which the mathematics was connected. Both Sarah and Ruth regularly displayed art-work and charts connected with mathematics on the classroom walls, but the other teachers gave mathematics less emphasis.

The conversations I had with the teachers before and after their classes were a very important part of the research. These interviews took place in any undisturbed setting: the staffroom during teacher's "free periods", an empty classroom, or even at my home or that of the teacher concerned. Occasionally, a teacher's other commitments required these lesson-planning or lesson-feedback discussions to take place by phone or through an exchange of e-mail messages. In addition to these "semi-structured interviews", I often spent time chatting to these teachers, and others at their school, during morning recess or lunchtime, and found it useful to hear what my research participants told the others teachers about their work.

Bogdan and Biklen (1998) note that "qualitative researchers go to the particular setting under study because they are concerned with *context*. They feel that action can best be understood when it is observed in the setting in which it occurs" (p. 5, italics in original). As Ernest wrote, "teachers' personal philosophies of mathematics ... have a powerful impact on the way mathematics is taught" (1991, p. xiv), and the lessons I observed gave me the opportunity to check the validity of the views which the teachers professed in interview.

Research Design

The value of the qualitative paradigm for researching historical approaches to mathematics was noted by Barbin (2000), who claimed that quantitative attempts to determine the impact of such work have failed, since "the attainment of objectives claimed for using history cannot be measured by assessments" (p. 66). In summarizing historical approaches to mathematics in the secondary school, she wrote:

> The question of judging the effectiveness of integrating historical resources into mathematics teaching may not be susceptible to the research techniques of the quantitative experimental scientist. It is better handled through qualitative research paradigms such as those developed by anthropologists. (p. 63)

My research used case study analysis, this being "appropriate for intensive, in-depth examination of one or a few instances of some phenomena" (Goetz & LeCompte, 1984, p. 46). Each of my three case studies is what Denzin and Lincoln (2000) call an "instrumental case study", in that individual teachers are being studied to provide insight into an issue, rather than being the subject of interest themselves. Taken together they form the basis of a "collective case study", which will also include insights gained by talking to other teachers who have explored mathematics from this different viewpoint or who are interested in doing so.

As I was actively helping my research participants discover how to use cultural perspectives in mathematics in addition to investigating their reactions towards this unfamiliar approach to teaching mathematics, my research design was influenced by the teacher development experiment (TDE). Simon described this as:

> A methodology for studying the development of teachers (which) builds on the central principle of the constructivist teaching experiment ..., that is, that a team of knowledgeable and skilful researchers can study development by fostering development as part of a continuous cycle of analysis and intervention. (2000, p. 336)

Here, the term "teacher development" is used to denote "the changes in knowledge, beliefs, dispositions, and skills that support teachers' increased ability to implement successfully the principles of the current mathematics education reform" (p. 335). Such a methodology would clearly support my research, but Simon's description emphasized both the need for a team of researchers and access to a group of teachers willing to participate in development activities led by a member of this team. Nevertheless, I took the TDE as a starting point from which I could extract some useful ideas, while using more traditional qualitative methodology.

In the TDE, the researchers participate in teacher development by leading classes designed to promote specific reforms in classroom teaching. Individual teachers, the subjects of case studies, are observed during these sessions, and also as they take their new ideas back to their own classrooms. The teachers' work, both as students themselves and as teachers of their own students, is then analyzed for evidence of both mathematical and pedagogical development. I modified this design in several ways. In the absence of a complete class to instruct, I worked with individual elementary school teachers, starting with some preliminary studies with several teachers whom I already knew to have an interest in novel approaches to mathematics. Analysis of these early studies helped me to formulate research questions and procedures for data collection and analysis. My work with of all these teachers was recorded, although this dissertation focuses on only three complete case studies.

An approximation to the class aspect of the TDE was achieved by giving workshops to teachers, both at conferences and on "professional development" days. Although these events did not allow the detailed surveillance which could be gained by one member of a team observing the effects of the teaching of another, discussions with participants during or after sessions sometimes revealed how the presentations changed their view of mathematics and the ways in which it can be presented to their students. Another source of contact with elementary teachers was through the historically-oriented mathematics enrichment sessions which I gave for my local school board. The students were accompanied by teachers or parents, and these adults often discussed the material with me while the students were engrossed with their activities. While these experiences are not a formal part of my research, they provide useful background information.

My commitment to help my research participants understand the new ideas they encountered is an integral part of the TDE research paradigm, yet Lesh and Kelly pose the following dilemma:

> If the research is designed to find ways to think about the nature of the [teachers'] mathematical knowledge (or abilities), then how can the

> researchers also be inducing changes in that knowledge? ... How can researchers avoid simply observing what they themselves created? (2000, p. 201)

However, they speculated that it is possible to create situations in which development will occur without having its path determined by the researchers. They suggested that environments should be created which involve conflict with previously held conceptions, thus forcing teachers to develop new frameworks.

Data Collection

Data collection is necessarily preceded by gaining permission to do so. I found Bodgan and Bicklen's advice, "first court potential subjects" (1998, p. 75), to be very valuable. Having gained the co-operation of individual teachers through earlier work with them, I obtained consent from district school boards, school principals, the research participants (teachers) themselves, the students involved in those teachers' classes, and the parents of these students. An example of the letter format of the consent form sent to these individuals in given in Appendix C.

I used several different methods of data collection: questionnaires and interviews, involving both teachers, students and parents, classroom observations documented by field notes, photographs, audio-tapes and occasionally video-tapes; and journal reflections, both my own and those of teachers and students. Copies of classroom activities and examples of students work were also collected.

At workshops I gave before starting my formal research, I had occasionally asked the participants to complete a short questionnaire concerning their views on cultural aspects of mathematics (see Appendix D). This questionnaire, and another concerning more personal information, were completed by my research participants as the first step in my data collection. I then discussed their answers with them in a very informal interview. This, and all the others with my research participants, took the form of a "purposeful conversation ... that is directed by one in order to get information from the other" (Morgan, quoted in Bogdan & Bicklen, 1998, p. 93). However, the amount of "direction" involved varied considerably. Although I prepared questions to use if necessary, I encouraged my research participants simply to talk about their teaching, as this provided insight into their beliefs about mathematics and pedagogy which may not have emerged from direct questioning. The students of my case-study teachers also completed a short questionnaire designed to discover what they knew about the mathematics of other places and times, and to determine their attitude towards mathematics. If the children were too young for such a written procedure, the teacher's aide wrote their responses.

Classroom observation was the other main strategy used in my data collection. Although the teachers were the main focus of my attention, I also observed and talked to the students during their classes, and interviewed some of them at the end of the year about their experiences, as their reactions to the work not only reflected their teachers' attitudes but highlighted factors stressed during the classes. At the end of the research period, I gave all my research participants one further set of questions, in which I asked them to reflect upon their experiences in teaching cultural perspectives of mathematics and predict their future use of these ideas (see Appendix D).

During classroom periods my role along the participant/observer continuum varied somewhat. At the start of each class, I assumed a completely observational role, remaining seated at the back of the room, writing fieldnotes, and audio-taping the teacher's comments to the class and the students' responses. However, once the class started on individual or group work, the written notes were either replaced or augmented by whispered comments to my tape-recorder, as I shadowed the teacher to record individual conversations he or she held with the students. However, I was mindful of the danger, noted by Borg and Gall, that "the observer often becomes an active participant in the environment being studied. This can lead to role conflicts and emotional involvement, which can reduce the validity of the data being collected" (1989, p. 391). As an enthusiastic teacher of cultural approaches to mathematics myself, it was sometimes hard to avoid discussing the mathematics with the students, but careful recording of all that happened in the classroom allowed me to take account of any effects that such interactions might have caused. On the other hand, there were occasions on which I took the opportunity to discuss the mathematics with the teacher, and I feel that this was an important part of my dual role as "the teacher's teacher" in addition to "the researcher".

I considered a tape-recorder to be a vital "third party" in any interview, and always obtained my research participants' permission to record our conversations. The audio-tapes not only provided an accurate account of what was said, but also allowed me to give my full attention to the interviewee, thereby being better prepared to draw out further comments when necessary. In addition to the initial, questionnaire-based interview, I tried to meet with each of my participants before and after each of their teaching sessions although this was not always possible. The pre-class conversations focussed on the teacher's lesson plans, as this allowed us to discuss how the lesson could best be organized, whereas those following a class were also based on the fieldnotes I wrote. Occasionally, a teacher's other commitments required these conversations to take place by phone, in which case I took notes and typed up the discussion as soon as possible. All audio-tapes were summarized so that they could be reviewed if a line of inquiry arose that made the data contained particularly relevant, and those of the case study teachers were also transcribed.

Audio-tapes provide a good basis for validity checks, but subjectivity enters even here, as the researcher has to decide which conversation to record if, as often happens in classroom situations, there are many occurring simultaneously. Video-tapes were made of Sarah's classes, but as I had no experience in using a video-recorder, my practice was merely to set it up at one side of the classroom, and leave it running as a general record of time spent on tasks and the atmosphere in the classroom, rather than as a precise means of recording student-teacher interactions. Since the other teachers were reluctant to have their classes recorded in this way, I took photographs instead, as I find that some visual record is very useful: In qualitative research it is often true that "a picture is worth a thousand words". In addition to the obvious subjects of teacher and students, I made use of my camera to "take inventories of objects in a setting" (Bogdan & Bicklen, 1998, p. 101). Pictures of the chalk-board, during and at the end of classes, often provided a clear record of the path of the lesson, the arrangement of the furniture gave a sense of the formality of the lesson, and the contents of notice-boards provided information about the sort of atmosphere which the teacher wished to create in his or her classroom. Modern digital photography increased the benefit of this method of recording information, as my pictures were immediately accessible, and available to augment my fieldnotes.

One problem with both audio-taping and photography is that children can be distracted from their activities when they see a tape-recorder or camera focussed upon them. Bogdan and Biklen comment on the "extinction time" required for people to ignore the camera and become "themselves" (1998, p. 102). The elementary school students with whom I came into contact soon become accustomed to my presence, and the attention I initially received surfaced as curiosity about my activities rather than as objections to them.

Fieldnotes are a very important tool in data collection, and my written comments served two very different purposes. In the descriptive mode they concentrated on factual portrayals of classroom events: I used them mainly to focus on activities and comments which would not have been caught on audio-tape, such as facial expressions, gestures and quiet teacher-student or student-student interactions. However, fieldnotes can also be used for "researcher introspection" (Eisenhart, 1988, p. 106), a mode in which the researcher writes reflective responses to the activities taking place. In addition to including comments about similarities or contrasts between the activities in one class and another, or how the actual activities compared with the teacher's plan for the class, I included reflections about my own frame of mind, ideas and concerns during the session. Both modes provided a useful source of questions for my post-lesson conversation with the teacher.

After each class my fieldnotes were typed up and combined with transcripts of selected portions of the audio-tape and a summary of its contents. These computer records also noted the existence of any additional data, including both extra electronic documents, such as summaries or transcripts of conversations with the teachers before and after the class, and "hard-copy" records: video-tapes, photographs, copies of teachers' lesson plans and students' classwork, as well as documents written later, such as teacher or student journals or student homework. The combined data provided a relatively complete picture of any particular lesson.

However, the question of "whose picture" is one which had to be faced. Bishop writes that "Ethnographers admit – sometimes even celebrate – the subjective nature of their inquiry" (1999, p. 2), yet given that an ethnographer's task is to preserve the "participant perspectives" (Erickson, 1986), the picture should clearly be that of each individual teacher. To confirm that I had correctly presented their beliefs, I used "member checks" (Lincoln & Guba, 1985). Each research participants was given a copy of what I had written about his or her class and asked to comment upon its accuracy. I did not do this until after the first year of observation was completed, as Carspecken (1996) warned that using such checks too soon can provoke misleading Hawthorne effects.

In addition to all the material collected "in the field", I kept a journal in which I not only included factual records of all relevant contacts I had with elementary school teachers, but also wrote down my thoughts about what I had learnt and comments about the research methods I was using. I also encouraged my research participants to keep journals to reflect upon their experiences. Sarah was the only teacher to do so, although several teachers asked their students to write journal entries about their cultural mathematics classes. However, most teachers responded to the request I made at the end of the research period for their written reflections on their work.

Data Analysis

The accumulation of large amounts of data is a natural consequence of the nature of qualitative research. As Wolcott warns, the critical task "is not to accumulate all the data you can, but to 'can' (i.e. get rid of) most of the data you accumulate" (1990, p. 35). The interview and lesson summaries I wrote helped me keep track of the data I collected, and an on-going analysis of the transcripts helped me to highlight the most revealing parts of the data for future consideration.

I did most of the transcription myself, taking the opportunity to review the material, perform some preliminary analysis, and add "observer's comments" to the script, pointing out significant statements, or speculating upon the exact meaning and possible consequences of certain remarks (Carspecken, 1996). The features of the modern word-processor are invaluable in this respect: My annotated transcripts incorporated different colours and fonts to highlight statements I considered important for various reasons, and formed a preliminary, easily visible, coding device, which proved useful later when I began formal analysis. Transcribing the tapes also alerted me to comments which needed to be followed up in future conversations. When time pressure and medical problems forced me to pay a third party to do the transcribing, I compared the transcripts with the tapes, corrected punctuation and spelling mistakes, and added my colour-coding analysis.

Although the conventional description of qualitative research is one of "constructing a picture as you collect and examine the parts" (Bodgan & Biklen, 1998, p. 6-7), the theoretical framework described in Chapter 2, together with the literature on implementing educational reform, provided an outline as to what features this picture might contain. However, my data was analysed inductively, and the results were not compared to the views presented in the literature until the analysis was complete.

I started my formal analysis with a quick overview of all my computer files of collected data, using the previously colour-coded sections to determine the major categories for a preliminary coding scheme. Later, I went through this data more carefully, coding each "unit", a term used by Lincoln and Guba to designate "some understanding or some action that the inquirer needs to have or to take. ...It must be the smallest piece of information about something that can stand by itself" (1985, p. 345). This process generated several more codes: A complete list is given in Appendix E. The coded data was then grouped into separate files.

The sorted information allowed me to focus on specific aspects of using cultural mathematics, and the results of this analysis appear in Chapter 6. However, a complementary method of illustrating the feasibility of such work is to present a few examples of lessons in progress. This approach is taken in Chapter 5, and required a different analytic focus. The transcripts of these lessons were subjected to a closer analysis than the others, and the teacher's rationale for their choice of material and method of presentation was woven into each classroom account. In doing so, I addressed what van Maanen calls "the underrated criteria of apparency and verisimilitude", thus adhering to his advice that "ethnography must explicitly consider ... the role of the reader engaged in the active reconstruction of the tale" (1988, p. xi).

CHAPTER 5
CULTURAL MATHEMATICS IN ACTION

The teachers in the case studies at the heart of this research took very different approaches to using cultural perspectives of mathematics. This chapter explains their various strategies, and describes a few lessons in detail to demonstrate how the teachers implemented their ideas. Although reported in narrative style, each description includes the teacher's voice, as he or she explains the rationale behind the choice and presentation of material, and comments on the progress of the lesson.

Ruth's approach is the first to be described. She took the curriculum guide (IRP) as her starting point, but used several non-textbook resources to enable her to address the various topics through culturally oriented activities. Sarah took the opposite approach in developing a series of activities she called "Time-Travel Days". She started by selecting various historical periods and then chose mathematical topics from those times and places which provided appropriate enrichment material for the Learning Outcomes listed in the IRP. The "Group of Four" (G4) took one particular resource as their guide, and each teacher selected four chapters from the six given in the pack appropriate for their grade. They made some attempt to relate these activities to the IRP, but overall seemed to view Multicultural Math as a separate subject: The class schedule written on the board often included "Math" as well as "Multicultural Math".

Ruth's approach:
Cultural perspectives selected for specific curriculum topics

Ruth's first comprehensive attempt to introduce cultural perspectives of mathematics took place in 2002, during her work on the "Space and Shape" strand of the IRP. At that time, her class contained twenty-seven students from Grades 4 and 5, several of whom had recently arrived from Korea. However, she had used some cultural ideas in her mathematics classes before taking part in my research, following personal visits to Egypt in 1999 and Jordan in 2000. Since Ruth was always eager to relate her teaching to her own life-experiences and those of her students, her first trip provoked a discussion of the properties of pyramids. I first observed her teach one year later, just after she had returned from Amman. On this occasion, her lessons focussed on the number symbols and geometrical patterns she had seen there, although she supplemented her own experiences with material taken from two of my history of mathematics texts: Smith (1996) and Ifrah (1985).

However, both these excursions into cultural mathematics were presented as "special events", rather than as part of the regular curriculum, although Ruth was careful to note connections to the IRP requirements. Despite her intention to include further cultural mathematics in her 2000/2001 programme, circumstances prevented this, and it was not until the following January that she had the time to "multiculturalize" her mathematics programme. In response to my enquiry, she wrote:

> Yes, I would be interested in doing some more multicultural math, I do have to start teaching Shape and Space soon, so there would be a lot of stuff in there that would naturally lend itself to multicultural math.

It was clear that this time Ruth's approach to cultural mathematics was to fit it into the regular curriculum wherever she could, taking the IRP as her starting point.

IRP Topic: Three Dimensional Objects

Illness prevented me from attending the first class of the unit on Space and Shape, but Ruth wrote to me, telling me what occurred.

> I introduced the new unit directly from a multicultural point of view by saying that we were going to study Shape and Space but that we were starting off looking at different shapes and their use from different cultures. I gave them two pages, Blackline Masters from the info you had given me showing the four different houses. We used the different cultural houses and our large map of the world and looked where we could find each of them in the world. We talked about the Indian temple because I have a student who is going off to India at the end of the month for 6 weeks. The students were fascinated.

When I arrived at the class the following day, I was introduced to the students as being "very, very interested in historical math and multicultural math". Before the class started, Ruth had told me that she wanted to discover what her students understood by this latter term, so this provided a good preface to the following discussion:

> Ruth: So what do you think we mean by "multicultural math"
>
> Richard: Not just one culture.

Ruth: Right, and we've got lots of students in this classroom who are connected to more than one culture, haven't we.

Daniel: "Multi" means more than one, so more than one culture's math.

Ruth: So math from more than one culture. Anybody else like to add something to that.

Lesley: Multi means like different kinds of things.

Edward: Multi sounds like multiplication, so multicultural math is math from many countries.

Vicky: I think it's like looking at what other people do.

Ruth was clearly pleased by these responses, but took the opportunity to note that "I just want to first of all reassure you that what we're doing is regular math, part of the regular math that we would do, we're just going to look at it from a different angle".

The lesson continued with a review of the geographical location and geometrical shapes of houses examined the previous day. Ruth personalized this discussion by directing questions about the Taj Mahal to her Indian student. Earlier, Ruth had commented to me that "one of the main reasons I do the multicultural stuff is because of the connections we can make with the kids in the class, and the more ways you can make it relevant and they can connect with it, then the better". Later she brought out a poster which a previous class had made, commenting "you'll see lots of different shapes, particularly in the pictures from Eastern European countries". In her favourite "let's make connections" teaching mode, she pointed to the photograph of Oxford, mentioning that my son Colin was studying there. The students identified the shapes in many of the pictures, using the names for two-dimensional shapes as well as those for three-dimensional objects, the latter having been the focus of the "shape-hunt" of the previous night's homework. Later Ruth explained that she spent a long time on this part of the lesson because she was trying to encourage her students to distinguish between the terminology for flat shapes and solid objects.

She also focussed on the properties of various shapes and the meaning of geometrical words. The cultural context sometimes led to rather unusual ways of introducing some of these ideas:

Ruth: I think it would be interesting to live in different shapes of homes. If you lived in a building that was like that Kenyan

Round House, you couldn't be sent into the corner, could you? There isn't a corner to be sent into, is there!

David: Yes there is – on the roof!

Ruth: Is that a corner?

James: No, it's a point, there's not two sides connected.

Ruth praised the student's thinking, then talked about the pictures again, but Richard suddenly interrupted to ask "how many sides are there to a pyramid?". Ruth replied "Good question. The pyramid is a pyramid because all the sides meet at one point. Now there's something called a triangular pyramid, is that its proper name?" She turned to me, so I suggested that she was thinking of "tetrahedron", so she commented that it would be a good word to include in the week's spelling list, which were all mathematical terms. She led a class discussion about various types of pyramids, again commenting on her own experience of visiting the square pyramids in Egypt, and closing with the comment that "I really like it when you talk about these things, because the more you talk, the more these words will become part of your language, and I want you to use a lot of math language". In our conversation after the class, Ruth noted that "one of the things we are supposed to be promoting is the use of language".

At the end of the class, Ruth displayed the *Great Moments in Math* poster which I had given her, and talked about several of the mathematicians featured there. Fibonacci was mentioned, as the students had investigated his series recently, and Euler, since they had also enjoyed the Bridges of Königsberg problem. She also took great delight in pointing out the name Colin Percival to her class, asking them to guess who that might be, so I had to rise out of my observational role and say a few words about my son's work on Pi. Ruth commented that "knowing who actually did the math, even if it's only a name and a picture, seems to help the math come alive for the students". This poster was then fastened to the door, and on several occasions I saw the students reading it.

IRP Topic: Two Dimensional Shapes

The next three classes did not include any multicultural activities, instead taking exercises from the *Quest 2000* (Wortzman et al., 1997) text books for Grades 4 and 5 as their starting points. This textbook work again focussed on naming two-dimensional shapes, and Ruth wrote:

> These could be used to do a spring board into more multicultural math. I'll get them to talk about the types of shapes and properties that are being used, and hope to ask them to think about why they think certain people chose certain patterns.

Unfortunately this last idea was not followed through, but in the sixth class, Ruth returned to cultural dimensions of mathematics again, admitting that the lesson had been inspired by the book *Underground to Canada* which one of her students was reading. She explained the significance of the "underground railroad", the network developed to help slaves escape from the American South to freedom in Canada, and talked in some detail about Harriet Tubman and the quilts she made, pointing out that these had "two sorts of math in them – not only is it patterns and shapes, but this is a code" She explained that the quilts were sewn to give information about the slaves' escape route, and gave examples such as "the flying geese pattern was symbolic because slaves, like geese, travelled north".

The students then looked at two activity sheets [Zaslavsky, 1994, pp. 77, 78] based on patchwork quilts made by the American women of early Colonial days. The first page, assigned for homework, showed different arrangements of 3x3 quilt blocks, some of which were split into two triangles, others left as complete squares, and the text asked questions about the total areas involved. The second activity showed a particular quilt pattern, the "Ohio Star", and focused on the symmetry of the design. This activity was left until the next day, and was completed after a review of the homework assignment. Ruth gave me the following report:

> I asked them to follow the directions and to design a quilt (it was the one that was based on symmetry and the Ohio Star) pattern. We then followed part of the directions on the page which asked them to write a description of their design so carefully that they could give it to someone else, without the picture and get them to draw their quilt. They have been assigned to complete this for homework, and then we will swap tomorrow, just like a game!!!!
>
> I think we will have a higher rate of completion for this homework than with some others. Everyone also got to have a close look today at the quilt poster (which is a large page out of an old National Post newspaper).

> It was an excellent multicultural and historical and mathematics lesson today. I enjoyed it a lot too.

Before handing out these two activity sheets Ruth had commented "this is an example of stuff where in some respects I don't have to be totally independently creative all the time, but it allows me with relative ease to do the multicultural stuff with them." In her reflections at the end of the year she wrote:

> I would most definitely use the h/m [historical and multicultural] stuff and quilting instead of the regular textbook because I think it is overall much more relevant to the students. I want to make the Math as relevant and connected to the students as I possibly can because I think they then have a far greater chance of learning it and becoming more engaged with the topic.

The following day Ruth let the students build quilt designs with pattern blocks. She talked about tessellations, reminding them of the *Tesselmania* computer programme they had used, and mentioning both the tilings done in ancient times and Escher's work. Later in the week, she reinforced this idea by giving the students an art project involving tiling, relating this to her own experiences by telling the class about the tiling patterns she had seen when visiting a big tiling centre in Jordan.

Despite the above quotation Ruth spent the next two lessons on textbook activities concerning angle and side properties of triangles and quadrilaterals. She justified this by saying some students seemed to need the "security blanket" of working from the book occasionally, but noted that these exercises reinforced the work they had done on house designs. She continued this topic in her next class, reminding them about the shapes they'd seen in quilts and houses, and then went on to tell them about tangrams "which are very common in some cultures around the world". Two of her Korean students then talked about the tangram sets they had played with at home, and that night's homework assignment was to cut out a set of tangram pieces from a plan Ruth supplied.

The next day Ruth read *Grandfather Tang's Story* (Tompert, 1990) in which two fox fairies, standard characters in Chinese folklore, transform themselves into many different creatures. While listening to the story, the children manipulated their tangram pieces to form these creatures, guided by overheads which Ruth had made from the book's illustrations. After she'd finished the story, Ruth read the information at the back of the book, which explained how tangrams are regularly used in story telling in China.

At the end of the class, Ruth invited the students to comment on what they'd learnt, and Vicky produced what Ruth called "a lovely idea":

Vicky: In the Underground Railroad, why didn't they just use the tangram pieces to show which way to go, instead of taking all that time to make the quilts.

Ruth: That's an interesting connection you've made with that, Vicky. I don't know if they knew about tangrams. So you're thinking that if they used them instead of the quilts ...

Vicky: Yes, because it would be less time to wait, say faster, because they'd have to wait a long time to make the quilt.

Ruth: What a neat idea, I wouldn't have thought of that one. Good for you, well done Vicky.

Ruth was clearly delighted with Vicky's comment, and commented to me that it showed that the student was thinking about the fact that people were using specific shapes to convey particular meaning.

In her next mathematics session, Ruth approached triangle properties through the concept of balance, relating it to work done by "Mr. Archimedes", as she wanted to show that concepts we use today were developed a long time ago. After the students had cut out circles and triangles they were asked to balance the circle on a pencil point. They did this easily, explaining that they needed to find "the exact centre of the circle" "so that both sides on each side of the centre of the circle should be the same weight, so one side doesn't pull it over." However, when challenged to do the same for the triangle there was an general outcry, which gradually gave way to shouts of satisfaction:

Robert: It's too big.

David: You can't balance a triangle on a pencil point.

Daniel: It hasn't equal sides. There's more on this side than there is on this side.

Lesley: It doesn't have a centre.

James: If you find a certain point, maybe exactly in the middle of the triangle...

Richard: Hey, I've done it. Hooray!

Ruth: What do you think Richard has found?

Richard: I just put it further on this side than that side.

Ruth: What has Richard found?

Richard: That it's true.

Ruth: What has he found?

Vicky: Balance.

Ruth: Right, he's found the balance. How has he found that balance?

Richard: I was just holding it and then it stayed.

Jenny: He found a spot that equally spread out the weight.

Ruth repeated that comment and told the class that "this is what Mr. Archimedes was interested in doing". She then explained how to find the balance point of a triangle, an exercise which involved measuring its sides, dividing the lengths by two to find the "centre points" then joining these points to the opposite vertices. Although this was done in a slow, step-by-step fashion, many difficulties arose in the process, some of which led to useful mathematical discussions: how accurately should the lines be measured? how can 25.5 cm be halved? which is the "opposite vertex"? Even when measurements were restricted to integers, some students still had difficulty halving 25. However, the delight the children experienced as they successfully balanced the triangle on the point they'd constructed quickly replaced the feelings of frustration. Amazed shouts of "I did it and it works!" echoed round the room, and the students thanked "Mr. Archimedes" for discovering how to find the balance point.

Ruth then showed them a drawing that "supposedly shows what Mr. Archimedes looked like", and talked about where and when he lived. This led her to explain how the present calendar is organised, and the students were able to calculate that Archimedes (born 287 B.C.) would have been 2089 years old if he were alive today [2002]. She closed the lesson with the comment that "you've played around with your hands today with triangles, and you've been finding out things that were discovered a couple of thousand years ago!".

IRP Topic: Area and perimeter

Ruth started this section by discussing the rectangle to demonstrate the concepts or perimeter and area, and to derive a method by which these values could be calculated for that particular shape. She then asked students to explore the circle, and challenged them to produce a way to calculate its circumference and area. However, she couldn't resist telling them that there was a "magic potion" for doing this, and promised to tell them about it the next day. An interesting cultural episode occurred when she was questioning whether anyone knew the mathematical language associated with circles. There were many wrong answers for the term "circumference", but finally John, one of the Korean students, said that he could write it in his own language, and was invited to do so on the board. He wrote down the Korean words for all three important terms (radius, diameter and circumference) and Ruth encouraged all her students to copy these down. To John's question "why do we learn about Korean", Ruth shouted enthusiastically "why not!"

To finish the lesson, Ruth read the book *Sir Cumference and the First Round Table* (Neuschwander, 1997). As usual, story-time was greeted with great enthusiasm, and although this story has little historical value from a mathematical viewpoint, some of the word play can lead to interesting historical ideas. In this class, the name of the carpenter, "Geo of metry", encouraged Ruth to prompt the students into recognizing that "geometry" actually means "earth measurement".

The students' ideas about methods of finding the area of a circle were discussed at the start of the next class, ending with the following exchange:

James: I did the circle, and I cut it into quarters, and then I thought "there's four right angles in it", and then I tried to use like the outside bits in the quarters and I tried to make little straight lines to make them like triangles, and then I measured the triangles, and I put like squares in and then I did the loop around it, and then I tried to add like the half squares and all the other stuff, and then just add it onto the other triangles that I had.

Ruth: Do you have a middle name?

James: Robert. (in questioning tone, as if to say, "Why do you want to know?")

Ruth: It doesn't begin with A?

James: No (still sounding bewildered)

Ruth: Did you say Robert? Are you sure it's not Archimedes. The very way that you're talking about was the way that Mr. Archimedes, that Ancient Greek guy did it. He used to draw lines and he did exactly what you just described to find a way of measuring the circle.

She went on to tell them that the problem they'd been trying to solve, that of finding the area of a circle, had been:

> puzzled on for years and years and years, even before the Ancient Greeks got to it, way back, because people wanted to know how much they needed to cover an area so Mr. Archimedes explored this, and there is a magic formula, a magic function to do that.

However, rather than telling them the formula, she again chose to use the story format and read *Sir Cumference and the Dragon of Pi* (Neuschwander, 1999), the second book in the series she had started the previous day. In *Dragon of Pi*, Radius (the child hero of the first book) figures out the value of "the circle's measure" by using a pie to answer a riddle[2]. Ruth told me that she really liked the way that the story introduced a way to find Pi, and admitted that she didn't mind that it related to the circumference formula rather than that for area.

The third lesson in this section again started by reviewing students' ideas about circle area and how to calculate it, but was soon side-tracked by Robert's comment that "You'd need to know the area of soccer balls so that you could make them in different sizes". Ruth told him that he'd raised a very good point, and went on to explain the difference between two-dimensional and three-dimensional objects:

> If we were to try to buy leather or plastic to cover this [soccer ball], it wouldn't be round, but if we sliced the earth in two, maybe at the equator, that would be round, but this, how am I going to work out how to put this onto a two dimensional sheet of paper?

One student talked about a map "cut into long ovals", and another talked about "peeling the skin off an orange". Ruth praised these suggestions, then led the class back to their exploration of circle area. David finally asked "Why don't we just use the formula for Pi, that 3.14 something number". Ruth's reply sums up her philosophy of teaching:

> I'll tell you exactly why, it's because Mr. Peter van Hiele, a Dutch mathematician believes, and research proves, that if you don't lay the foundation for your knowledge, if you don't experiment, if you don't discover, your knowledge will only go up a little amount. But if you discover things and try things out for yourselves, your knowledge could be

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[2] "Measure the middle and circle around. Divide and the number can be found.
Every circle great and small, the number is the same for all."

> limitless. And this is what the Ancient Greeks were very, very good at, especially Mr. Archimedes who we are going to talk about just before lunchtime. So, yes, the important thing is to experiment and discover. Imagine that you're going back in time, and this is a classroom in Ancient Greece, we're in the Parthenon or the Acropolis [shouts of "cool" from the class] and when you go out for lunch, read that expression that's written above the door – "Let no one ignorant of some mathematics leave this classroom". That was on Mr. Plato's door in Ancient Greece, or on Mr. Archimedes', I can't remember which one. So nobody is going to leave this classroom without having learnt some mathematics, so that is exactly why you are being asked to experiment, to discover, to play. [more cries of "cool"] Because of Mr. Pierre van Hiele, and Mr. Archimedes. So you're now in Ancient Greece. Go and experiment!

Ruth knew about Sarah's Time-Travel Days, and had told me that she was planning to try something similar with her students. They all became quite involved with the idea of being in Ancient Greece, so much so that when Steve called out "I've found some staples", Ruth replied "They didn't have staples in Ancient Greece, go away." All the students laughed, but the setting seemed to motivate them towards making mathematical discoveries, and they produced some interesting ideas. In response to John's question "How can we make a circle come to an octagon", Ruth showed him how to use a protractor to measure the central angles of a regular octagon.

In the next mathematics session, Ruth read about Archimedes from the book *Mathematicians are People too: Stories from the Lives of Great Mathematicians* (Reimer and Reimer, 1990). However, before she started on the Archimedes story, she commented that "They even have your problem in here, David" and John immediately called out "That one with the bridges?". There was a buzz of enthusiastic chatter from the class, and Ruth reminded them that the mathematician involved with that problem was Euler, "pronounced Oiler". She took the inevitable Canadian question "Did he come from Edmonton[3]" at face value, and replied that he came from Switzerland and was an interesting figure because he continued doing mathematics even after he went blind. Then she turned back to the topic at hand, and suggested that the students might want to make some notes about the story, particularly regarding what "Mr. Archimedes did about the circle". The Sand Reckoner was another of Archimedes' ideas mentioned in the

[3] A reference to the "Edmonton Oilers", a Canadian hockey team.

story, so she took time out to discuss exponents so that the students would understand the meaning of "10 to the 63rd power". Unfortunately, this was a short class, and she was unable to complete the story.

At the start of the next class, Ruth told her students that they were to play around with Mr. Archimedes' ideas of finding the area by drawing polygons inside and outside the circle. John again suggested using an octagon, but Ruth replied that they could try different ones, "and I won't tell you the number of sides of the polygon that was the last one that Mr. Archimedes experimented with, but you will be amazed". Making connections yet again, Ruth reminded her students about my son's work on Pi, and ended the class by commenting:

> If Archimedes was playing around with numbers, let's say about the year 250, then it's 2250 years later that Colin was playing around with Pi, so all these things that you're doing had to start somewhere.

She continued with the Archimedes story in the next class, and after reading that "Archimedes announced that the ratio of circumference to diameter was less than 3 1/7 and greater than 3 10/71" she wrote these numbers on the board, and asked the students to copy them. In the discussion following the story, some of the students were able to relate these numbers to their own discovery that the diameter "fitted into" the circumference about three times. As the IRP only calls for an understanding of simple fractions at the Grade 4/5 level, I later asked Ruth whether she thought that her students would understand the significance of 10/71. She replied:

> I put up [the fractions] as an intrigue, as a point of possibility that fractions aren't just 1 over 2 and 1 over 4, ... to look at the true meaning of what a fraction is as a part of something, so that was the idea. And I'd rather go to those moments of possible intrigue, to get the kids to think "what is she doing that for", than just keep to the rule-bound stuff.

Despite Ruth's earlier comments that the students should focus on the circle aspects of Archimedes work, they were naturally amused by the idea of him jumping out of the bath, and so the lesson turned briefly towards the scientific ideas of buoyancy. Ruth showed them the book *Mr. Archimedes Bath* (Allen, 1980) and outlined the story for them. Stephanie asked if this really happened, so Ruth replied "well there is that story from history about him jumping out of the bath and shouting 'Eureka' but we don't actually know".

She finally brought the discussion back to Archimedes' method of finding the area of a circle, and the following exchange took place, based on what the students had heard in the story:

Vicky: He drew a polygon inside the circle.

John: The circumference of a circle, it adds up to 3.1415.

Ruth: Etc, etc.

David: He also put polygons around it.

Ruth: What do we mean by "polygon".

Alex: A closed figure.

Ruth: Right, so we've got a polygon inside the circle and another one outside the circle, and he came up with a drawing of a 96-sided polygon inside the circle.

Students: WOW!!

Ruth then asked them to explore this method of finding the area of a circle for themselves, commenting "I'll give you one little clue: I think it would be very helpful to you to look at the little centre dot". She confirmed that they should use regular polygons, and noted that "you have the advantage of a protractor, which Archimedes didn't." After a few minutes, she stopped their explorations to reinforce that "in a regular polygon, all the sides must be equal", and suggested that "to find out how much space is inside that circle, make your polygon as big as possible", a response to her observation that some students were drawing small shapes that did not reach to the circumference of the circle. Most students then started to draw "the spokes of a wheel"[4] and made triangles by "cutting off the crusts of the pie". After they'd all made a good attempt at this, Ruth stopped them and used the overhead to explain how to estimate the area of the circle by adding the area of all the triangles. Unfortunately, few of the students knew how the calculate the area of a triangle, so Ruth had to go into that process in detail. However, the previous day John had invented this method of finding the circle's area by himself, and in the middle of Ruth's explanation he suddenly called out "Is that my idea, Archimedes did my idea also?" Ruth replied "You must have been talking with him in a

[4] An etymologically sound analogy, since the word 'radius' derives from the Latin word for the spokes of a wheel, although Ruth was unaware of this fact at the time.

former life" and everyone laughed. Richard then asked "what was the most regular measurement that Archimedes did?", and when Ruth looked puzzled, he explained "the way that we use centimetres now". Ruth restated his question as "What units of measurement did he use?", congratulated him for asking a good question, and looked across at me for assistance. Since I didn't know either, she promised that she'd find out for him[5]. The homework assignment for that night was to estimate the area of two circles using the triangle method, and then to calculate it using the πr^2 formula which Ruth explained to her students very quickly at the end of the lesson.

The mandated curriculum for Grade 4 contains no mention of either area or circumference, and that for Grade 5 only a brief mention of the latter, so I asked Ruth why she had spent so much time on these topics. She replied:

> I thought it was something that the students would have some sense about. I purposefully wanted to stretch that as far as it would go AND I wanted to give the students the historical background on it as far as I thought they could go with that aspect. I have to confess that I also had a relatively ready supply of materials which I thought could get the things across in a child-friendly manner without hindering later development. I sort of regard it a bit like Shakespeare or Charles Dickens or for that matter George Orwell, where one reads some of the works at a certain time in one's life, and then goes back to them at a later date, to find that the perception and enjoyment can be quite different. Nothing wrong in my opinion with an early informal introduction which it would be hoped might lead to a heightened sense of enjoyment and development down the road.

IRP Topic: Nets

Ruth started her unit on nets using material from the *Quest 2000* textbooks (Wortzman et al., 1997), but then decided to give the lessons an historical twist using some materials I lent her [*Platonics with Frameworks* (Baker & Harris, 1998) and *The Platonic Solids* video (Key Curriculum Press, 1991)]. The first class along these lines started with a quick review of what nets are, but soon moved onto some historical scene setting. Once again, Ruth and her students went back to a classroom in Ancient Greece,

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[5] After an internet search, I sent Ruth the information that the Greeks measured small distances in feet, a unit slightly longer than the modern foot.

and the students were delighted to hear that they would be constructing solids from nets. However, before they travelled back in time to "Academus", Ruth's name for their Ancient Greek school, she tested their powers of observation by asking "who can remember what it says above the doorway, outside our classroom?". Vicky correctly answered, "Let no one ignorant of mathematics leave this classroom", and Ruth talked about the quotation from Plato's Academy on which her sign was based. She read some information about Plato from an encyclopaedia, and showed her students the Greek words of the original quotation. She also showed them pictures of some Neolithic stones which have been identified as early models of the five regular solids, as she wanted her students to realize that "the thing that you're going to work on goes back to very, very ancient times". Some of the students wanted to know how big the stones were, and as the book didn't give that information, Ruth promised to make some enquiries[6].

Ruth then handed out six nets to the students: one for each of the five regular solids and also that of a cylinder, which she included to provide an example of a very different type of object. Motivation was high, and the students commented on the nets as they were constructing the objects: John, working on the net for the cylinder remarked that "this seems to be different to all the others", and other students noticed the three different shapes used as faces for the Platonic solids. When most models were finished, Ruth asked her students to write descriptions all six solids, adding "please make sure that you use mathematical vocabulary to describe the properties – don't just say 'corner', talk about angles, lines, vertices and faces".

Before starting the next class, Ruth updated the students on her research into the Neolithic "Platonic Stones". This had not yet produced any satisfactory results, despite several e-mail messages, but she told them that Colin would soon visit the Ashmolean Museum where they were kept and would send a description of the stones to her. She then focussed on the properties of the six solids, and reinforced the meaning of various mathematical terms. Ruth's description of the faces of a solid as "in a sense what you sit

[6] She was eventually able to report that they were about 10 cm in diameter.

on", was more accurate than she realized[7], but another comment showed some confusion: Pointing to the intersection of the planes on a cube, she told the students "this is a 'vertice', what you used to call 'corner' but a vertice is somewhere where all the points meet, it is not one vertex, it is several. On this cube I actually have four vertex coming to this one point." However, the students seemed to make sense of this, and completed the chart Ruth had handed out, which had columns labelled "SOLID, number of faces (F), number of vertices (V), number of edges (E)". This proved a useful organiser, but Ruth did not discuss the relation between F, V and E, although this was mentioned in the video they watched a few days later. In fact, a class discussion revealed that some students were having difficulty counting the vertices and edges, so it is unlikely that the data collected would have allowed them to deduce Euler's famous equation. However, an interesting point in this discussion was the comparison between the title of the chart, "Euler and the five Platonic Solids", and the chart itself, which in the version handed out by Ruth contained a sixth line for the cylinder. Students commented that the cylinder didn't have any vertices, and there was a short argument as to whether it had two faces or three. Eventually Vicky pointed out that the chart claimed that there was a rule that was "true for all simple polydedral solids" and since they couldn't agree upon the numbers to put into the chart, then "maybe the cylinder isn't one". Ruth congratulated her, adding "So we cannot include this one, this is your stumper, this is the rebel. This one really should be thrown out, because it's not a Platonic solid".

As some of the students were looking rather confused at the end of this discussion, Ruth changed topic, and told the students that they were about to return to Academus, a statement that was greeted with cheers by the class. This time, rather than doing mathematical constructions, they were to practise writing, and she asked them to copy the Greek quotation that had appeared above the entrance to Plato's Academy. The students were quite fascinated by the shapes of the Greek letters, and although they found them difficult to copy at first, Ruth explained how to think of each letter in terms of modern shapes that they knew.

[7] Several weeks later I discovered that "hedron" was an Indo-European word meaning "seat", so "polyhedron" is literally "many seats".

She then turned to an aspect of the Platonic solids which intrigued her, that of Plato's identification of the solids with the four "elements" of his time (earth, air, fire and water). This really captured the imagination of some of the students, and they wanted more details than Ruth was able to supply as to why the octahedron represented water and so on. One boy finally commented that Plato had done it "to see how life was related to shapes", and Ruth praised this remark, noting that "everything's got a shape, how would you describe us? Well, we're all sort of shapes all stuck together and mixed up". Robert took up the challenge, noting that "our arms are like cylinders and our feet are almost rectangles".

IRP Topic: Units of length

Ruth started the new unit by asking the students to write down their individual thoughts about three questions: "What do you know about measurement?; What is it?, Why do we use it?". After several minutes, she added another question, "Where did it come from?". At first the only suggestion was David's remark that "It came from Mr. Measure", so Ruth read them an introductory passage on measurement from the *Teachers' Notes* for *Mathematics from Many Cultures (MMC)* (Irons and Burnett, pack 6, 1995). After some discussion of the metric system, Ruth asked "Who came up with the first unit of measurement that was written down?". Vicky answered "I think it was the Egyptians because I read in a book that said the Egyptians used a measurement from their fingertips to their elbows". Ruth identified this unit as the cubit, and then read more information about it from the *MMC* text. She then pointed out that "we went to Academus in Ancient Greece and now we've jumped over to Ancient Egypt. We could imagine that we're helping the royal guys to build pyramids and how do we know what size to cut these gigantic stones, and how far do we have to haul them, and how much do they weigh, and how much water are we going to need to drink, so we don't get too tired." The students then spent the rest of the class working in groups to brainstorm examples of different types of measurement, and when they would be used.

After three textbook oriented classes, Ruth returned to the historical perspective, handing out copies of two pages of pictures and information on the history of measuring units, taken from the book *Everything you need to know about Math homework* (Zeman

& Kelly, 1994). The class discussed the use of "hands" to measure horses, as well as units from the Vikings (fathom), the Romans (foot, inch, mile) and the British (yard). Napoleon's role in the development of the Metric system was mentioned, as was the 1790s definition of the metre as one ten-millionth of the distance between the North Pole and the Equator. Ruth even mentioned the new standard for the metre, 1/299,792,548 of the distance that light travels in a vacuum in one second. As she commented "Isn't that incredible trivia!" She posed the questions "How do we get from some of this old stuff to where we are now?" and "Why do you think people needed to make the change?" as topics for them to explore in their next journal entry, and as a further homework assignment, asked them to measure their digit and fathom. The following day these measurements were graphed, together with the students' height, and the students were fascinated to discover that their fathom (arm span) and height were almost equal.

The Measurement chapter in the *Quest 2000* Grade 5 textbook includes a description of the Egyptian method of measuring with knotted ropes. This motivated Ruth to take her students on a trip to Ancient Egypt, and she justified this to me as follows:

> I would hope that it was interesting, and that, if I taught in that way, the students might remember that, plus more of the formal stuff that I was supposed to teach them. I wanted to engage them and hopefully interest them, and we had to study this anyway, so why not do it in a way that was engaging and interesting, plus I also found it interesting myself.

To start the class, she read the *Quest* pages with her students, then told them that they'd be taking the skipping ropes outside to do some Egyptian-style measurement, a statement that was greeted with great enthusiasm. She left it to them to decide how far apart the knots should be. Some students took the rather anachronistic approach of measuring equal distances with their rulers, but others entered into the spirit of the lesson and marked out equal distances using their feet or their cubit.

Once outside, she declared that she was Queen Hapshetsut and that the students were her subjects, and for the rest of the class, all instructions were given in that context. When the students had tied knots in their ropes, they started measuring. One boy climbed up a column supporting the school roof and announced that it was six and a half knots high, whereas other students measured distances along the ground. Finally Ruth asked them to line up all the ropes, giving a total length of about fifty metres. They counted the knots, getting a variety of numbers just above a hundred. Since the knots were not consistently spaced, the purpose of this activity was unclear, but as a scene-setting activity it was good: These students will never forget that ropes preceded rulers as tools of measurement. Although Ruth had intended to use the ropes again the following day to introduce the 3-4-5 right triangle used by the Egyptians, illness kept her from school, and on her return, she moved on to more traditional textbook activities.

Ruth brought her geometry unit to a close a week later, treating her class to a whole day of mathematics activities, an affair which proved very popular with the students. Although the original purpose of this event was to investigate van Hiele's theories of developmental levels, the scheduled time was so close to Pi Day[8], that Ruth decided to hold it on that date. The Pi activities included a list of web-sites to explore, several of which gave historical accounts of the various methods of evaluating Pi, and a class discussion in which many of Archimedes' ideas about Pi were reviewed.

Ruth introduced some cultural ideas into other mathematics topics throughout the school year, but relied on her previous knowledge rather than specifically searching for historical or multicultural connections. However she also organized two "Multicultural Objects" mathematics classes, for which students brought items related to their ethnic background. In keeping with her philosophy that "There's math in everything", she asked her students to compose mathematics questions based upon these objects. They were fascinated by the artefacts from other cultures and produced an interesting collection of questions on such topics as symmetry, estimation and exchange rates.

Sarah's approach: Mathematics selected related to specific cultures

Sarah's enthusiasm for history permeates all her teaching, including her mathematics classes, but nowhere is it more evident than in her "Time-Travel Days". However, unlike other imaginary trips taken by elementary students, the main purpose of these Days is to learn about the mathematics and mathematicians of previous ages, and

[8] March 14, which appears as 3-14 in the American dating system.

though the students "journeyed through time and back" in a single day, the mathematical ideas generated often spilled over into the following days or weeks: When she started he Time-Travel programme Sarah noted, "I've got to find some way to reinforce this so it's not just a one shot affair - whatever I do in those Time-Travel Days, I have to find ways to reinforce that right up until the next time travel".

There were twenty-two students in Sarah's Grade 3 class when she first implemented her series of Time-Travel Days in the 2000/2001 acedemic year. In the years that followed she modified the material to suit the abilities of successive groups of students and made improvements to some of the activities as she increased her understanding of the mathematics and its place in the social setting of the time. She described her motivation for the course as follows:

> What I'm hoping with these kids is that they'll see that mathematics comes from real people, out of what was going on at the time in society. I'm trying to show them how the mathematics of the past connects to what we're doing today, and the more ways I can do that, the more they'll understand that math is alive.

She took her students on a series of ten "trips", starting in the Stone Ages, progressing through thousands of years of civilization, and ending with a questioning look at "Math of the Future". She noted that an important goal of this series was to show that:

> Math was going on and developing in one place while other math was developing in another. It wasn't like there was some nice little plan where one person developed something and then everybody did it. All this stuff was happening all over the place.

To help promote this idea, Sarah often referred to the mathematics of one civilization while actually "visiting" another.

The structure of the Days varied considerably, but most began with a short discussion of the history and geography of the place to which the students were about to travel, sometimes followed by a short video. At this point, Sarah usually asked her students, "What do you suppose these people would need math for?", hoping that by seeing the importance of mathematics in the past, the students would come to share her view that "Math is absolutely essential to get by in our society". Then the children took their imaginary journey. As they held hands with eyes tightly shut, electronic music set a "time-travelling" mood, and Sarah's voice quickly led them back through the highlights of hundreds or thousands of years of history until they arrived at their destination. In the first year of her programme, Sarah also helped her students grasp the passage of time by rolling a long scroll from one spindle to another, marking the civilizations they visited at appropriate points on this time-line. However, this was rather unwieldy and very time-consuming, so in later years she resorted to a much simpler, stationary wall-chart, a small copy of which the children kept in their binders.

Some Days focussed on the work of individual mathematicians from a particular period, whereas others looked more generally at the mathematics used in a specific civilization. Several activities contained material not specifically mentioned in the Grade 3 curriculum, but Sarah noted that these were included for their value in developing skills in communication, problem solving, and reasoning: "all that stuff that's at the beginning of the IRPs about how to teach math". She also commented:

> It's incredible how much I'm learning. One of the best parts of this whole thing is that my math learning is still going on, and I'm learning along with the kids. I think it's important for kids to know that teachers are always learning.

In this section, I give detailed accounts of the third and fourth journeys, those to Egypt and Greece respectively. Each is based upon Sarah's first presentation of the material, but includes reference to later years if changes were made. The other eight Days will be summarised briefly at the end of the section to provide an overall picture of her programme.

Visit to Ancient Egypt

Although the first two Time Travel destinations had been a closely guarded secret until the Day itself, the students had guessed the location of the third Time-Travel Day from the materials that Sarah had assembled in the classroom, so she took the opportunity to do some preparatory art work, giving them amulets, necklaces and hair ornaments to colour, which they then wore during the day. Another preview tactic, one which was common to all the Days, concerned the class calendar: All the dates for the month were written in the number system of the civilization to be visited. Sarah commented on the problem-solving that this provoked:

> [The students] have been looking at the symbols and talking about them. They recognise that there's a pattern, and I heard one kid say today "I know what she's putting up tomorrow". But they haven't come to me, they're just having their own little discussions.

Sarah credited this idea to Pappas' book *Math-a-day* (1999), a collection of 366 puzzles, one for each day of the year, in which the date is given in a different number system each month.

On arrival in Egypt, the time-travellers were eager to share their knowledge of the country and its most famous monuments, the pyramids. Sarah had planned for this by exploring three-dimensional objects with her students the previous week, so this session provided a useful review of the vocabulary. After making her own contribution to the discussion by noting that some pyramids were forty storeys high, and comparing this height with buildings that the children knew, Sarah handed out small paper models of pyramids. She also gave each child an activity sheet containing information about King Khufu's pyramid, and asked them to work in groups to solve the questions asked, using the model and the facts provided.

The students had little difficulty answering the first few questions, which required them to name the two-dimensional shapes used in the pyramid and the number of faces, edges and vertices, but the word "perimeter" was new to them. Sarah had purposely left this unexplained, and told me later, "I want the kids to get used to using a dictionary". With its help, the students decided that they needed to measure one side (5 cm.) and add it four times, although a few students said they had calculated this as 4x5 instead, a connection which Sarah reviewed with the class. They were also able to calculate the perimeter of King Khufu's tomb, 230 metres along each side, although all students used addition for this larger number. The final question, concerning the weight of all the blocks in the pyramid took them considerably beyond the Grade 3 curriculum, as there were two millions blocks of average weight 2,500 kilograms. David was able to correctly multiply the two numbers together, and explained that "two times twenty five is fifty, then I added six zeros for the million and then two more from the weight". Sarah went through this slowly on the board and the students giggled mischievously at her command, "Don't tell anyone you're doing such big numbers".

Returning to the geometry of the pyramid, Sarah focussed on the shape of the base asking, "what sort of angle did they need in the corner to make sure the pyramid was straight and even?" James answered "right angle", and added, "you'd get a rhombus if you don't get it quite right". Discussing this part of the lesson later, Sarah stressed the importance of developing an accurate mathematical vocabulary and commented, "That's why I put all the words up on the word-wall – but it's also for me so that I can look up there if I forget a word".

The class discussion turned to practical matters involved in building the pyramids: where and when they were constructed and the various jobs that needed to be done. The students were led to realize that pyramid construction took place during the flooding of the Nile, which caused Tara to ask, "how did they know when it was going to flood". Sarah seized the opportunity to talk about the Egyptian calendar, noting that "The Egyptians were the first people to know that there were three hundred sixty five and a bit days in the year", and stressing the importance of mathematics for their astronomical calculations.

The students were then given small blocks to build their own pyramids, but were first asked to estimate how many blocks would be needed to build a five level step pyramid with a square base with ten blocks along each side. Most of the estimates ranged from twenty blocks to fifty, so as each level was completed, Sarah asked her students to count how many blocks it contained, and gave them a chance to revise their initial estimate. She noted the informal assessment opportunities that arose during this task: "There are some kids that can't even count the number of blocks correctly, but Tara and Edward recognised the pattern of 10x10, 8x8 and so on and predicted how many blocks would be needed for the next layer". This project also gave valuable insight into the students' understanding of three-dimensional shapes. The first year this activity took place the students were able to describe a square pyramid, yet few of the groups actually made a square base on which to build their monument, using rectangular approximations instead. In later years, Sarah reinforced the equal side property of a square before the students started to construct the base of the pyramid. After doing this geometrical work with her students she commented that it was "so interesting that I'm going to carry on doing a whole bunch of geometry stuff. Usually I don't get to that till later in the year".

At this point in the first year of the Time-Travel programme, Sarah asked me to talk about the Egyptian method of papyrus making, based on my own experiences helping students to make a sheet of papyrus from a set of dried reeds. However, in future years, she explained the method herself, and showed her students a commercially produced sheet of papyrus. Each year, the student asked if they could make their own papyrus, and responded very enthusiastically when Sarah agreed. The first year, they glued together strips of construction paper, but in later years, the "fake papyruses" were make out of newsprint paper. They took about half an hour to make, but the students were so keen to be able to write on them in the afternoon that those who had not already finished willingly stayed in the classroom during the lunch break to complete the task. Sarah had often spoken of her wish to have real papyrus for her students to use, so after the first year of her programme she bought herself a papyrus plant for the pond at her home. Although this did not produce enough for construction purposes, she was able to show her students the stems' unusual triangular cross-section.

Sarah planned to have her students write mathematical sentences on their papyrus, but rather than simply tell them the value of the Egyptian number symbols, she handed out "a little puzzle" she had constructed, saying:

> You are children in the school of scribes and you're going to learn about the numbers you'll be required to use. You're very lucky to be able to go to scribe school – it's your chance to get ahead in the world.

Most of the clues in the puzzle were based on the information sheet used earlier, and gave the same facts using Egyptian number symbols: for example, students compared the sentences "The total number of stones in King Khufu's pyramid would be written like this 𓁨𓁨" and "King Khufu's pyramid contained two million stones" to deduce that 𓁨 stood for one million. During the class discussion on the value of each symbol, Sarah also focussed on their origins in everyday life: for example, when talking about the hundred symbol (𓍢) she asked, "If you were an Egyptian, what would this spiral be in your environment". After suggestions of a question mark and a snake, Sarah asked, "what else do you coil up", eventually receiving the suggestion of a rope, which she acknowledged to be the experts' opinion.

Sarah then wrote some numbers in modern numerals and asked how they would have been written by the Egyptians. She started with 32, and Tony told her it would need "three hoops and two ones", so Sarah wrote this on the board as II∩∩∩. Alison objected that it should be the other way around, so Sarah put up ∩∩∩II, but then Tara pointed out that the calendar numbers used the first version. Sarah explained that it could be either way, adding that "the Ancient Egyptians didn't have a place-value system like the Babylonians or like we do – they didn't have to put the ones in a certain place. They could have written ∩I∩I∩". When she asked the students why they thought this wasn't done, they recognized that it made it harder to read. Sarah then gave them a much larger number to translate: 30,142. Tony managed the first three digits with no difficulty, but then asked his friend "what's four?" and they both wrote down IIII. It was not until he came to write down the symbols for the 2 at the end of the number that he realized his mistake, and changed his representation for forty into ∩∩∩∩. Similarly, for the number 100,319, one group immediately said "one hundred thousand, so that's the tadpole", but another group said "a hundred, so we need the curly thing", and wrote down the symbol for one hundred rather than one hundred thousand. Sarah considered that this work was very useful for the students because "if you don't understand place-value, you don't understand numbers", but also noted its value as an assessment tool for her. She finished this section of the class by asking the students to translate "a really easy number" (999), which led to a discussion of the respective advantages and disadvantages of the Egyptian and modern number systems.

Next Sarah handed out an activity sheet which used simple Egyptian number symbols in mathematical sentences: These equations are shown in Figure 2. She asked students to work together in their groups to figure out what the three unknown symbols meant. Although all the students had difficulty at first, a few eventually recognized that the combination of the numbers 3, 2, and 5 suggested an addition sentence, whereas 3, 2 and 1 suggested subtraction. They shared

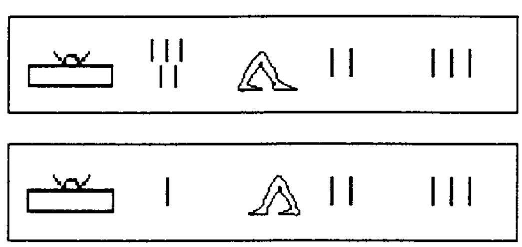


Fig. 2: Egyptian addition (above) and subtraction (below)

their insight with the rest of the class, and everyone was delighted by the meaning of the "legs": One of the students explained "If you walk forward, it's like pushing the number together". To reinforce the significance of this symbol and the operations it could represent, Sarah invited five children to the front of the class, put them into groups of two and three, and then pushed the groups together, with a similar exercise for subtraction.

Although the students realized that the other unknown symbol had to mean "equals", they did not understand why until Sarah rolled up a sheet of paper to demonstrate that the symbol represented a rolled papyrus, and pointed out that this was "where the Egyptians wrote the answers". Sarah also pointed out that "we wouldn't write [the sentence] in this order – we could do 5=3+2 or 3+2=5", demonstrating to her students that there is more than one correct way to write mathematics.

The children then worked individually to compose five equations of their own. Most students used small numbers, although a few delighted in using larger numbers than are commonly encountered at their grade level. Sarah's requirement that they translate their work into modern symbols reinforced place-value concepts, and the Egyptian tally-like representation of numbers helped the students understand the regrouping process that is often necessary in adding and subtracting. Edward brought up an interesting point:

Edward: What did they write for zero?

Sarah: You know what? They didn't have a zero.

Edward: So it's different from our numbers then?

Sarah: Yes, we have a zero symbol, but it wasn't invented yet. Remember, the Babylonians didn't have a zero either.

Edward: So how would they write the answer for one million take away one million?

Sarah: I don't know – let's ask Mrs. Percival.

She called me across, and I suggested that they would just leave it blank, but added that the question never arose in Ancient Egypt. Sarah then told Edward "put the little answer symbol, but don't put anything beside it".

Although the students had recognized the physical significance of the "walking legs", the activity sheet had led the children to deduce that  meant addition, so they used this symbol even when they wrote their equations starting from the left. However, as soon as Sarah copied one of their incorrect sentences on the board, they realized their mistake, saying, "Oh yes, we wrote it like English". After their success with addition and subtraction, one of the students asked, "How do we do times?" Sarah replied, "Let's not go there today. After all, we don't know how to do times in English yet".

However, in her third year of Time-Travelling, Sarah did introduce her students to the Egyptian method of multiplication. In a telephone conversation with her the evening before her class I had explained the additive procedure (see Appendix G), and she was so impressed by its simplicity that she decided to try it with her students. She did this by just writing up a pattern of numbers on the board (shown in Figure 3) and asking the students to work out what was going on. She told me:

> I'm going to explain the mathematics as little as possible, because I want them to struggle with it. I don't care if they ever get the right answer, but I want to see them thinking and working as a group and trying to solve it on their own. And I'll start off by telling them that the answer's not nearly as important as their thinking when they try it.

The students quickly recognized the doubling pattern in the left hand column, then saw a similar pattern in the right column, and finally realized that the bottom number was the sum of the two "checked" numbers above it. Sarah then led them through the pattern step by step, helping them to see that 38 was "two groups of 19" and so on, and the students were eventually able to tell her that 323, "the answer" which they identified from the papyrus symbol next to it, was 17 (16 plus 1) groups of 19. Sarah worked through one more example with them, and seemed convinced that they understood the method. The three student teachers who had come to observe this Time-Travel Day seemed amazed that a method existed by which Grade 3 students could understand how to perform the multiplication of two two-digit numbers.

1	19	/
2	38	
4	76	
8	152	
16	304	/
	323	

Fig. 3: Egyptian multiplication (17 x 19)

In the first year of this programme, Sarah ended the Egyptian Day with two stories about the Egyptian gods, the second of which told of Horus, whose eye was torn into six pieces in battle, but was united by Thoth, the god of wisdom. She then explained that the Egyptians had taken the various part of the Horus-eye symbol to represent a series of fractions, commenting to me later that she had wanted to show her students that "math is in literature, math is in art, math is everywhere". As she started to write up the modern representations of these fractions on the board, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, the children started calling out that they recognized the pattern.

Janice: It's counting by twos.

Sarah: No, but you're close.

Tony: Two plus two is four, four plus four is eight...

Edward (interrupting): It's doubling.

Sarah: Can you tell me what the next one will be?

Breanne: One over sixty-four.

Sarah then showed them the other way to write Egyptian fractions, telling them that "the whole eye meant one, so if they wanted to write one part of thirty-two, they could also put it as II∩∩∩ ". She gave them several examples to work out, in each case stressing that the fraction meant "one part of", and commented to me, "This fits in perfectly with how I do fractions. A fraction has some kind of meaning then, it's not just some abstract thing with a line in the middle".

She ended the class with a set of questions using Egyptian units of capacity, taken from *Egyptian Genius* (Burnett & Irons, 1996). These started with the information that one khar was equal to 16 hekats, and required students to write 8 hekats, 4 hekats and 2 hekats as fractions of a khar. Sarah led them through this by drawing a "one khar basket" divided into 16 strips and counting up the appropriate number of strips to identify the fractions involved. Most students were jumping up from their seats in their eagerness to answer, although a few had lost interest by this time. Although Sarah admitted that this work went beyond the Grade 3 curriculum, she commented, "This group seem to be so excited by everything we're doing. It's as if, for the first time in their educational career, they're doing things that are challenging and they're rising to the occasion".

In the second and third years of the programme, Sarah omitted the fractions work, although she still told the Horus story and explained its connection to mathematics. This left time at the end of the day for the students time to write some mathematical sentences on their papyrus. Each student was given ink and a "stylus" (skewer), and after they had written their mathematics, Sarah gave them a list of the Egyptian hieroglyphics corresponding to the sounds of the modern alphabet, so they were also able to write their names in the cartouche form which she had demonstrated to them.

Later in the week the students learnt about the Egyptian units of length, which Sarah considered to be the "obvious stuff to focus on with these Grade 3 kids". She pointed out that:

> The cubit and so on may sound like standard measurements but they're not really. I've got this story about somebody buying cloth in the market, and asking for ten cubits of cloth and then being furious when they got home because they didn't have enough for whatever they were making, and of course, when they got back to the market, they discovered that their cubit was longer than the merchant's. So this helps the kids see the need for standardization.

Unfortunately, I could not be there when Sarah taught this material, but she told me that the students learnt about digits, palms and cubits and role-played the story. She also planned to have them actually do some measurement using knotted ropes, but this was one scheme which was not successful. She commented "What a disaster! Do you know there's a whole bunch of eight year-olds that cannot tie a knot!".

The students also explored *Senet*, a board-game seen in the wall paintings of ancient tombs. The precursor of many modern games, Senet involves two players who move their counters around a board, with the winner being the first to get all his or her pieces to the final square. The students constructed their own boards and had many contests, developing strategies to improve their chances of winning. The introduction of cultural games was a regular feature of the early Time-Travel Days, and Sarah told me how a Babylonian game had played a large part in her student-led conferences[9]:

> It was really interesting watching the kids trying to explain how to play the Royal Game of Ur to their parents. You could see the parents' eyes glazing over because every throw of the dice has a different value and you got to do different things, and the parents are saying "this is a really complicated game" and the kids say "oh, yes, it's really fun though". I was pleased to hear a couple of parents say to the kids "so what are you learning in this game" and the kid had an explanation. I've been doing a

[9] Meetings early in the school year at which student, teacher and parents discuss the student's work.

> lot of this with the kids – "yes, this was fun, but what have we learnt, how does this relate to the other stuff we're doing".

Discussing her Egyptian unit with me later, Sarah pointed out the many ways in which it linked to the learning outcomes listed in the curriculum guide for Grade 3, mentioning such issues as place-value, regrouping, properties and vocabulary of two-dimensional shapes and three-dimensional objects, estimation, fraction sense and "lots of problem-solving". She also reported the motivational value of this work, as evidenced by a conversation with Tara:

> She came to me today and she said, "Why are we learning Egyptian math, because we don't use these Egyptian numbers anymore." I said, "Well why do you think we're doing it?" "Well," she said, "it really makes math exciting and fun."

Earlier in the year, Sarah had told me:

> I don't really care if kids can do all the math they're supposed to do by the end of Grade 3. What's really important is that they leave Grade 3 feeling that they really like math, that's it's fun and that it's something they can do. You just have to hope that every kid, somewhere in the thirteen years they're in school, has one experience where they feel the joy of math.

Tara's comment suggests that Sarah's teaching was having the desired effect.

Visit to Ancient Greece

Sarah had borrowed several books from her school library about Ancient Greece, and noted indignantly that "There's not one listing for math". However, she suggested that "A lot of elementary teachers are scared of math, so the social studies authors leave it out". Nevertheless, her own study of the history of mathematics had convinced her of the importance of the subject to the Ancient Greeks.

This fourth Time-Travel Day differed from the previous three in two respects. The most obvious difference was the use of costume. Sarah had sewn herself an Ancient Greek outfit, and a few days before the trip, she gave the students simple instructions for turning a sheet into a Greek toga: When the journey started, most of the travellers were appropriately dressed for their destination.

An academically more important distinction between this Day and those before it was the change of focus from the mathematics of a cultural group to that of specific mathematicians. Sarah noted, "I switched to studying the individual mathematicians because I had material on them, and because I wanted to show a woman mathematician!" To facilitate this approach, Sarah divided the class into six three- or four-person groups, each of which worked independently on a particular mathematician. She spoke enthusiastically about the advantages of small-group work:

> It forces them to do the learning by themselves because I'm not available. By not having somebody leading them through it step by step, it forces them to think for themselves and to make decisions, and to struggle with things more on their own, so it isn't until they hit something that they really can't deal with that I'll get a call.

Each group was given activity sheets and an information envelope, containing a fact sheet and story about a Greek mathematician, and "some pictures of temples and so on to set the mood". The stories about Archimedes, Euclid, Hypatia, Pythagoras and Thales were copied from the two volumes of *Mathematicians are People, Too* (Reimer & Reimer, 1990, 1995b), but since these texts did not include a chapter on Eratosthenes, Sarah used Lasky's book *The Librarian who measured the Earth* (1994). The information sheets for all six mathematicians were copied from the three volumes of *Historical Connections in Mathematics* (Reimer & Reimer, 1992, 1993, 1995a), and most of the activities were also taken from that resource. Sarah said that she had been through the books:

> pulling out the activities I think they can do. I'm not sure that they're all simple enough, but I've got a couple of kids that are gifted, so I thought "let's put a challenge in there. They'll be able to figure it out, and then they can teach the other kids". But for the guy who did stuff on logic [Thales], I went to some other logic stuff I had and picked a simpler version of that sort of puzzle, because they've never seen anything like that before.

She admitted that she didn't know how long it would take the students to complete the activities, or how long they would be able to stay focussed, adding "we'll just have to go with the flow. If there's enough time, we'll change packages, so they get to do a second, maybe even a third, and then we'll do the rest over the next few days".

In fact, the students remained extremely focussed on their work. After the preliminary introduction to Greek civilization, they spent the rest of the morning working on one mathematician and the whole afternoon on another, and although Sarah had originally planned to have each group give a presentation about their mathematician, they were so involved with the mathematical activities that she decided not to interrupt them. However, Sarah remarked that this class included many strong, independent workers, and with a much weaker group the following year, she used whole class teaching for this Day, selecting activities from those discussed below.

The students' first task was to read the story about their mathematician. All groups chose to read aloud, sometimes in unison, sometimes taking turns, and Sarah was often called over to help with the pronunciation of the Greek names. Next each student had to write down "ten interesting facts", based on the story and the fact sheets: Sarah noted that "having to write something down helps them to focus". They were then allowed to select which activities they wanted to explore. Most students had time to work on three activities, but this varied according to both the difficulty of the tasks and the children's ability.

The authors of *Historical Connections* had chosen activities related to the particular interests of each mathematician, but this often involved considerable simplification of their ideas. For example, Hypatia's work on Diophantine equations was addressed by two activities requiring students to find combinations of stamps or coins to give a certain total. Although the correct mathematical terms were mentioned in the text, they were ignored in the classroom. Sarah noted later, "We didn't need to know what Diophantine equations were to be able to solve the problem", although she added "but it's interesting to know why they were called that".

The "Hypatia group" tried the "Stamp Stumper" puzzle and managed to produce several combinations of stamps giving the correct total. The following year, Sarah tried this activity with the whole class, noting "We've done similar sorts of activities before, so they have some familiarity with it". At first it generated many "guess and test" solutions, for which the testing process proved quite challenging for these Grade 3 students, even with the help of a calculator. However, some of the children eventually started to look for patterns and were able to predict combinations, rather than simply guessing.

For her first Greek Time-Travel Day, Sarah had selected a paper-folding activity for the Hypatia group, related to their mathematician's interest in conic sections (see Appendix G). However, neither Sarah nor the students had been able to understand what was required, and at the end of the day, Sarah admitted that she had not had time to try the activity beforehand, and asked me to explain it to her. The students who had attempted this activity stayed to listen, and as they saw what was happening Tara said "it's making sort of a circle, no, it's making one of those things that we made earlier, what are they called?", and Breanne supplied "ellipses". As the curve became clearer Tara expressed doubts about this, so I provided the name "parabola". When the envelope of the curve was clear, I cut out the parabola and rotated it around its axis: Tara visualized this as producing a cup, which led to a discussion of parabolic reflectors in flashlights and satellite dishes. Then I drew a cone on the board and we talked about the shapes which are formed by cutting across it. My suggestion that they could try this with a carrot was enthusiastically received, but unfortunately not followed through.

The Pythagoras group enjoyed the challenge of fitting together four sets of styrofoam balls to form a pyramid, although they were more successful with the two-dimensional task of setting out counters to illustrate various figurate numbers. However, the colouring and construction of Platonic solids proved to be the most popular activity, and several students begged to be allowed to stay inside at lunchtime to continue working on their models.

The first group to study the Archimedes package enjoyed reading the skit about King Hieron's crown, and enthusiastically accepted Sarah's suggestion that they should perform it later in the week. A few days later she told me:

> They've conscripted people to be the stage-hands and extras – it's gone from three kids to being at least ten at last count, and I thought, "Who am I to interfere", so they put on their Greek costumes again and stayed in and practised at lunch time. They've got the whole thing set up. We're going to turn the table upside down to be a bath tub. I asked how long it would take, and one kid said, "oh, a long time", then another said, "it takes about five minutes with everything included".

The other activities all concerned Archimedes' interest in balance. The questions about the position of children on a teeter-totter caused Emily to remark, "I wish we could test this out on the playground", but their yard did not have suitable equipment. The activity titled "Archimedes Mobiles" required students to realize that the two sides of a simple balanced mobile must contain equal weight, but the way the assignment was constructed also gave them practise in adding, doubling and halving numbers, so Sarah chose this activity for all her students to explore in later years. The other Archimedes activity to be repeated was "A Balancing Act", which required students to find the balance point of a circle and a triangle. This was the same activity that Ruth had explained to her Grade 4/5 students (see pages 65 and 66), so both Sarah and I were surprised when one of her Grade 3 students not only found the centre of gravity of the circle, but also that of the triangle. At Sarah's request, he told the other students that they had to "draw the line from the point to the middle of the other side, and then see where the lines meet". When some students had difficulty finding the midpoint of the sides by measurement and division, Sarah suggested they just fold the side instead.

The Eratosthenes group explored prime numbers, starting by using the Sieve method to find those below one hundred. Sarah commented, "David, the gifted kid, saw what was happening and took the short cut, but the rest of them just went through circling every second number, every third number and so on. Some of these kids can't count by twos yet – we need to do more of that". The other activities involved Goldbach's conjecture, and magic squares using only prime numbers. This last puzzle included the hint that the "magic total" is always three times the number in the centre square. As Sarah and I discussed this activity the evening after the class, Sarah admitted:

> When I read that I thought "Oh, if only I'd known that before", and of course my next question was, "I wonder why?" Obviously it's because they're three by three, but I don't understand why that happens.

I led her through an algebraic proof of the statement (see Appendix G), which she appeared to understand, although she commented humorously, "I don't think I should try this with my Grade 3 kids!"

Most of the Euclid activities in *Historical Connections* involved concepts beyond the Grade 3 level, so Sarah only gave this group the "White Faced Cubes" problem, that of determining the colouring of small cubes which make up a bigger cube painted on the outside. She commented that "Once they had done the first one, they were off to the races. They didn't need to paint anything, they would just look at each cube they'd made, maybe argue a bit, and then write down the numbers". The students were able to spot patterns in the table they were completing, but when the edge length jumped from six blocks to twenty, they were about to give up. Sarah told them, "you're not done yet", and when they protested, "but twenty by twenty!", she just told them to think about it and then walked away. When they started building with the blocks again, Sarah said to me, "so much for patterns!", but added:

> You know, I do the same thing, I trust a pattern to a certain point, and then something will make me say, "Well, I'm not sure if I want to trust this pattern" and then I go back to the concrete situation, whether it's counting on my fingers or whatever, and that's exactly what they did.

The Thales activities in *Historical Connections* were all logic puzzles, and as they were not particularly easy, Sarah had also selected some simpler ones, taken from the book *Playing with Logic* (Schoenfield & Rosenblatt, 1985). She commented:

> I wasn't sure how these would work out. I've had them sitting there and I haven't been using them, thinking they were too difficult. But Mary, who had always hated math and been really bad at it, solved them way better than the others. So this gave me the opportunity to say, "Wow, did you do all this by yourself?" and she said, "Yes, I did", so I told her, "You're a pretty smart mathematician, aren't you". And she left that room just floating.

Although none of the students had seen puzzles like these before, they all enjoyed working on them, so Sarah decided that she would use them much more in the future.

The students were so interested in the activities during the Time-Travel Day that Sarah had no time to discuss where the mathematicians' names should appear on the time-chart. She did this the following morning, and reported to me that "The kids remembered the names pretty well, except for Eristophanes (*sic*). They just knew that their guy was a librarian". However, none of the students could recall the birthdates, so Sarah handed out the packages again. When she wrote Pythagoras' dates as 560 – 480 B.C., Janice said, "oh, you've put that up wrong", but other students were able to convince her that the dates had been written correctly. Sarah noted that "because it came before the birth of Christ, they saw it as negative numbers, so they were counting down until they hit zero".

As expected, the students had not had the opportunity to try all the activities, so Sarah read the Pythagoras story to the whole class. When she asked the Pythagorean group what mathematics they particularly connected to their mathematician, they immediately suggested "making those nets", a reaction which Sarah had anticipated. She had the video about the Platonic solids (Key Curriculum Press, 1991) and noted:

> It fitted in perfectly ... and they loved the big words. "Dodecahedron", they thought that was the most wonderful word. I asked them to look at the part of the word that's the same, so they said "hedron". When I asked what they thought that meant they suggested "three-dimensional" so I said, "Could be. That would make sense". They caught on right away that the shapes of the faces were regular, and when the film showed the ones that weren't, they said, "Oh yes, that one can't be right because it looks different there."

She handed out all five nets to the students, and also gave them a summary sheet I had made, which explained how Plato had connected the solids to the four elements of his time. She noted, "They really liked that idea, and asked if they could illustrate the faces with pictures of fire and so on". She was also delighted that Breanne had spotted a mistake on my summary, pointing out that this showed that she was really thinking about what she read. Sarah admitted that she spent longer on this topic than she usually would "but they enjoy colouring, and it's Christmas". Later the students made mobiles from their solids, reviewing the ideas which some of them had encountered in the Archimedes balance activities.

One aspect of Sarah's plan that did not get addressed in detail at this time was the Greek number systems. However, Sarah had put both the "old" (Attic) and "new" (Ionic) numbers on her calendar, and told me that some of her students had figured out both systems "although they used a few post-it notes before they got it right on the calendar!" She returned to this material at the start of the Roman Day, handing out a list of the Ionic (alphabetic) numerals. Somewhat to her surprise, Brenda commented, "I know them without reading them ...A is alpha and B is bet, so that's why we have 'alphabet'". Sarah wrote several Greek numbers on the board for them to translate, including ΩH, which they correctly read as eight hundred eight. She then compared the Greek system with the modern one, discussing the modern need for zero, and Maria noted, "it's the same as the Chinese system we learnt about".

The other eight time-travel days

The two Days described above demonstrate the two fundamentally different organizational strategies which Sarah used in her Time-Travel Days. The "Renaissance" and "Women in Mathematics" Days followed the Greek Day format of dividing the class into small groups, each working on their own mathematician, whereas the "Stone Age", "Mesopotamia" "China", "Rome and India", "Middle Ages" and "Math of the Future" Days used the Egyptian model of having all the students investigate the same mathematical topics. Sarah had commented, "I keep trying to come up with new ways to do things: I don't want to do all Days the same", so she introduced a considerable amount of variety into these latter days, with a constant interplay between individual work, pair work, small group work and whole class discussions.

Several of these "whole class" days allowed Sarah to invoke the children's imagination by assigning them a role appropriate to the period studied. During the Stone Age Day they were groups of archaeologists, exploring the "notched bones" which give some clues about the mathematics known at that time. While in Mesopotamia, they first role-played the inhabitants of six villages, bartering goods with the help of an unscrupulous trader, played by Sarah. This led them to realize the need for a written record of transactions, and Sarah's skilful questioning helped them retrace the history of numbers from the early Sumerian number tokens to the cuneiform symbols of the Babylonians. The Day ended with the children becoming students in a Babylonian scribe school, as they constructed their own mathematical clay tablets, which were then fired in a kiln to provide a permanent record of their visit to the Fertile Crescent.

The trip to China, which was planned to coincide with the Chinese New Year, saw the children becoming students in yet another country, as they worked in pairs to discover how to use an abacus. Later they were each given their own set of "counting rods", and investigated how these were used to display, add and subtract numbers. In the third year of her programme, Sarah took note of the Chinese custom of colouring the rods differently for positive and negative numbers, and the students were soon able to manipulate them to add and subtract integers. In the afternoon they followed paper-folding instructions to construct their own set of tangrams, and then used these to make a variety of geometrical figures in addition to the creatures mentioned in *Grandfather*

Tang's Story (Tompert, 1990), which Sarah read to them. She also recounted the "Lo Shu" legend of the magic square, and the children competed to be "the child who had saved the village" by realizing the significance of the numbers on the turtle's back. Later in the week, they also learnt the Chinese number system which is still in use today.

The sixth Time-Travel Day included both Roman and Indian mathematics. After exploring Roman numerals with her students, Sarah used the story "The Rise and Fall of Roman Numerals", taken from Pappas' book *Math Stories for Kids and Other People Too* (1997), to change the scene to India. The mathematics of this civilization was first represented by the "Vedic Square" based on the 9x9 multiplication chart: The students filled in the square, and looked at the number patterns it contained. Later, Sarah used *One Grain of Rice* (Demi, 1977), a story set in India, to explore the effect of constantly doubling a number.

The following month, the students became the Lords and Ladies of six medieval castles, as they experimented with the geometrical constructions suggested in *Sir Cumference and the First Round Table* (Neuschwander, 1997), which Sarah read to them. Later in the day, they explored the Fibonacci series and several magic tricks based upon it.

The next two Days reverted to the "mathematician groups" format. Forewarned by the Greek Day, Sarah allowed each group to study only one mathematician, which left them time to give a presentation about their mathematician and his or her work at the end of the day. Six mathematicians were chosen from the Renaissance, but as these were all male, Sarah decided to address the balance the following month by studying female mathematicians from the eighteenth century to the present. An ardent feminist, Sarah values an historical approach to mathematics as a way to "highlight another area where women were written out of history", and noted, "girls need role models in all areas".

The final Time-Travel Day, "Math of the Future", took place right at the end of the school year, so the "Day" only lasted for the morning, as other activities had to be fitted into the schedule. Sarah invited a high school teacher to talk about graphing calculators. He first asked the children to be the points in a Cartesian coordinate system indicated on the floor. With his help, they derived the equations of some simple lines, and he then showing them how these could be displayed on a calculator. Later, the children explored binary numbers, which Sarah referred to as "the numbers used by computers". She then showed them some computer-generated fractal pictures, but unfortunately there was insufficient time for students to try constructing their own fractals.

"Group of Four" approach: Resource driven selection of topics

This section gives detailed accounts of three lessons taught by the G4 group of teachers, closing with a brief summary of their other sessions. These four teachers had little or no knowledge of the cultural background of mathematics, and although they were all happy to accept any help I offered, they rarely sought it. Instead, they relied almost exclusively upon the material in the *Mathematics from Many Cultures* (*MMC*) series (Irons and Burnett, 1995), with each teacher selecting the pack most suited to the grade level taught.

The first class described below started by making considerable use of the *Big Book* from the *MMC* materials, a feature which was typical of many of the classes given by these teachers. However, the other two descriptions show how teachers sometimes used the ideas given in these texts and adapted them to their own teaching style. Nevertheless, the questions they asked were frequently inspired by those suggested in the *MMC Teachers' Notes*. Although these teachers did provide some overall justification for their work, their comments suggested that once they had decided to teach a particular chapter, they basically followed the material provided without questioning its purpose.

During the first four months of their "Multicultural Math" sessions, each teacher selected one of the six chapters from his or her *MMC* pack. However, they all devoted their final class to an investigation of the properties of various polygons through paper-folding, inspired by a workshop which Sarah had presented. These constructions involved some excellent problem-solving activities, but the link with multicultural work was tenuous, often comprising nothing more than a reference to the Japanese use of origami and the cultural origin of the word itself. The teachers' comments suggested that they had simply used the time set aside for Multicultural Math to try another non-traditional approach to mathematics.

MMC chapter: "A Number Triangle"

Barbara's class was a group of twenty-six Grade 6 students. When I first met her to discuss her Multicultural Math classes, she was planning a field trip to the exhibition on Islam at the Museum of Anthropology at the University of British Columbia, and was therefore immediately attracted to the "Elegant Art" chapter in the *MMC* pack, which contains several Islamic designs. However, after looking further through the book she decided:

> I'm going to do the Number Triangle chapter, because that is what we are doing in class right now, number patterns, and I'll have them do some advance research. Quite a few topics are suggested: maths from these three different cultures here, Iran, China, France, and then there are different people that they can look up for their presentations.

Unfortunately, her ideas for student research did not materialise, and her own preparation was limited to reading the *Big Book* and the *Teachers' Notes* for the chapter she had selected. She did not request any help from me, and as we went into the class she seemed quite confident.

Like Ruth, Barbara was interested to learn how her students would react to the idea of mathematics as a cultural subject, so at the start of the first session Barbara announced that "In these classes we'll be studying cultural and historical math", and then asked her students what this might entail. Student suggestions were "dates", "size of populations" and "percentages of cultural groups, like, one percent Muslims in Canada". Barbara accepted all these responses, then continued, "in math though, these words are describing the kind of math that we're going to do. How do you think these two words might be related to math in another way?" After a long pause she prompted the class towards the path she had planned, asking, "Can you think of other systems of math, different ways of writing numbers?" After another pause, one student suggested Roman numerals, and after a brief discussion of these, Barbara told the class that "math systems" had originated in three different countries, Iran, China, and France, and asked individual students to show these countries on the world map.

Returning to the mathematics, she told the class:

> We'll see how these countries came up with some systems that had some similarities and some differences. We'll be looking at the different

> number systems and looking for patterns – that's what we're doing in *Interactions* right now. Everybody looks for patterns in the world. What can you think of where there are patterns?

The students mentioned several places where artistic patterns were displayed, such as T-shirts, walls and floors, but neither they nor Barbara mentioned the mathematical concepts involved in any of these patterns, nor were there any references to numerical patterns. They then watched as Barbara held up the *Big Book* at the front of the class and listened as she read the following paragraph, pausing twice, first to ask students for the modern name for Persia, and again to question whether anyone had heard of Omar Khayyam:

> People have always been fascinated by number patterns. They have found special ways of putting numbers together, such as magic squares. Mathematicians from Persia, India, China, and France have all experimented with creating triangular arrangements of numbers. The earliest number triangle we know of was made in 1070 by Omar Khayyam. Omar Khayyam was a famous poet who lived in Persia, which is now called Iran. His original number triangle used Persian numerals.

Barbara admitted later that she liked to start by reading the information in the *Big Book* because she didn't have much of this background knowledge herself, adding, "and the kids like to see the pictures". After reading the text, she asked the students, "What are Persian numerals – are they like ours?", to which the students all replied "no", since this was obviously the expected answer. Unfortunately, the picture in the *Big Book* gave the modern version of Khayyam's triangle, and Barbara did not explain what these numerals looked like, merely noting that "the numbers we use today are called Hindu-Arabic numbers".

Having been introduced to the triangle, the students were given a copy of its first nine rows (Blackline Master 8, from the *Teachers' Notes,* shown in Figure 4). Following a suggestion in the *Notes*, Barbara asked the students to look for number patterns in the

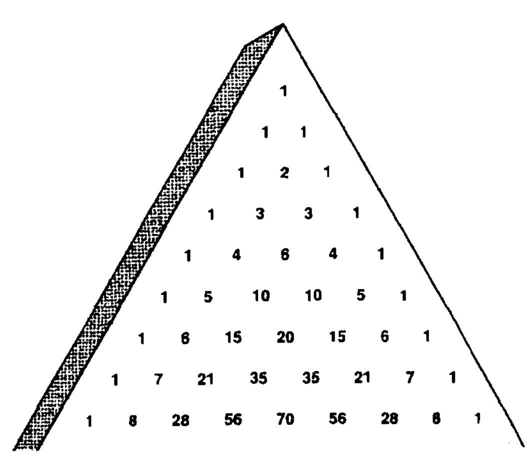


Fig. 4: Blackline Master 8
(used by permission of Creative Publications, Wright Group/McGraw- Hill)

triangle, and also suggested that they try to determine how the triangle had been constructed. Although some students managed to do so, this caused some confusion later as Barbara had jumped ahead of the learning sequence outlined in the *Notes,* yet continued to ask the questions as ordered in the *Big Book*, which only introduced the triangle's construction method after looking at Pascal's version of the triangle.

While the students were working, Barbara talked to me about the questions suggested in the *Teachers Notes*, commenting, "I'm sure [the students] will be able to figure things out better than me. I'm not particularly on top of this stuff, to tell you the truth. I highlight things and write things in my notes." She added, "I was wishing you were here to talk to earlier when I was reading this stuff", and admitted that she was not familiar with the concept of "triangle numbers". After I explained the Greek idea of figurate numbers she decided that it would be a good thing to do next. She also asked me "why bother counting numbers?", interpreting the word "counting" as a verb, and seemed surprised to learn that in this context it is an adjective describing the numbers in the sequence 1, 2, 3 ... .

Barbara then addressed the class again: "Now first let me show you something really interesting here. You can physically represent the numbers with the counters." She drew circles on the board to show the students how to arrange their counters to make the triangular numbers. At first the students were rather confused, but after a while they were able to predict how many more counters would be needed for the next row of the pattern, at which stage Barbara told them, "This pattern was first recorded about two and a half thousand years ago in Greece".

As soon as the students understood how to form the sequence of triangular numbers, Barbara drew their attention back to the Number Triangle, and asked them to identify the patterns it contains. However, when Adam told the class that "this sequence keeps increasing by one, so it goes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10", she did not name the sequence he had described, whereas David announced that the next diagonal gave "the triangle numbers".

The discussion then moved on to the next version of the triangle shown, and Barbara again held up the *Big Book* so that the students could look at its picture (shown in Figure 5) while she paraphrased the text for them:

This French version was written down 600 years after the death of Omar Khayyam, in 1654. Pascal arranged the triangular pattern of numbers where each number is the sum of the two numbers above it, and often used it to help in calculations of probability.

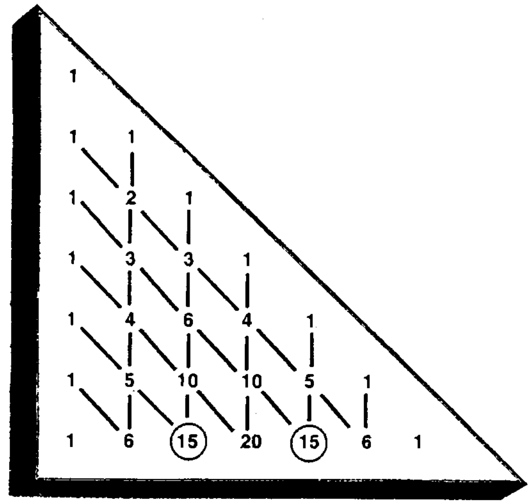


Fig. 5: Pascal's Triangle.
(used by permission of Creative Publications, Wright Group/McGraw- Hill)

She asked the questions given in the *Big Book*, the first being "which two numbers of the triangle do you add to get each 15". A student eventually answered "10 and 5", so she moved on to more general questions about the construction of the triangle, even though these repeated the students' earlier work: As Barbara noted later, she'd been "getting ahead of herself" in her reading.

Later she held up the *Big Book* and asked the class to identify differences between the two triangles. The students didn't suggest anything, so she talked about the fact that one triangle had all its sides the same length, whereas the other only had two sides the same, and used this to remind the students of some geometrical vocabulary.

By this stage of the lesson, which occurred at the end of the school day, the students were very restless and playing with their counters, so Barbara decided to leave the Chinese triangle for the next class, and asked the students to do some "silent reading" instead. She then spoke to me, noting:

> They were really fooling around, which is unusual for this group, usually it's a very focussed group, but I guess we were moving too slowly. I can think of lots of excuses, but the bottom line is that it wasn't the greatest, and some of that could be my fault too, because my planning wasn't the best for the end of the day perhaps. Anyway, as we go along, we'll find this more interesting, I know that we shall. The book is interesting, I've really been getting ahead of myself at one point.

We continued this conversation later in the staff room. I helped her to explore some of the properties of "Pascal's triangle" which could be investigated by students, and also suggested that she might ask her students why this set of numbers is named for Pascal when there were two similar arrays which pre-dated it. She didn't answer the implied question, other than to say "Well, it's not really his originally", but after I talked about the Eurocentric perspective which is often applied to the history of mathematics, she said "the students will like that because they've just learned about Ethnocentricity, so we've been using that word quite a bit recently". Barbara said that she wanted to continue the lesson later in the week, and we agreed that we would discuss her lesson plan beforehand.

In her reflections about this class, Barbara wrote, "I wasn't pleased with the lesson because I didn't feel the students were really engaged. I felt the activities didn't move along smoothly and quickly enough". However, Barbara had asked her students to write about the lesson in their Math Journals, and she sent a message to me, reporting "All of them were positive, in general, about it, and all appeared to want to do more of it! I found their Journals heartening and was very pleased that we were planning on proceeding!" Barbara and I talked for about fifteen minutes before the second class, discussing both the mathematics and the teaching strategies. She had decided that she could improve the lesson both by setting a faster pace, and also by giving the students closer access to the pictures in the *Big Book.* I showed her three activities that she could use instead of those from the *Teachers' Notes*: one of mine and two from the *Historical Connections* book. She decided to use these instead of the blackline masters from the other text, as she thought they gave a more structured approach.

Barbara started her second session by asking one of her students to read aloud from the *Big Book*, as she thought that this might encourage the other students to listen more carefully than when she read to the class. The text concerned the Chinese version of Omar Khayyam's triangle, and after the reading, another student carried the book round the room so that all the students could see the picture clearly. The students first worked on the activity sheet which I had constructed for this triangle, which required them to decode the symbols in the Chinese triangle into their Hindu-Arabic equivalents. They enjoyed this task, and were able to complete it without looking at the copy of the triangle they had been given in the first class. When one of the quicker students had finished his assigned task, he decided to add the next row, in both modern symbols and Chinese. Barbara thought that this was a good challenge, and encouraged everyone to try it. Later she invited two students to put their solutions on the board. This led to some class discussion, as the two versions differed in their representation of the number thirty-six, and the students seemed to gain confidence in their ability to express an opinion when they realized that Barbara was not going to act as "expert" since ancient Chinese symbols were also new to her.

During the written work, Qiao, one of the Chinese students, had told Barbara that the symbols used in this activity were no longer in use, so Barbara asked her to write the modern symbols on the board. Much to Barbara's amazement, the student wrote down the numbers for one to ten using three different sets of symbols. The other students thought it was "really cool" to have so many different ways to write the numbers. Barbara asked them to find similarities between the symbols, which elicited the comments that "the first three are just turned sideways", and "some bits of the symbols on that side [pointing to the third column] are the same as those over there [pointing to the first column]". Derek wanted to know what the other parts meant, but Qiao could not tell him. Alison asked whether the first two columns had a zero symbol. Barbara complemented her on the question, then asked, "Do you remember what we said about the zero the other day, that zero hadn't been named until just a few hundred years ago?", a comment which elicited a sudden gasp of remembrance from several students. As there were no further comments, Barbara thanked Qiao, saying how wonderful it was to see things from other cultures.

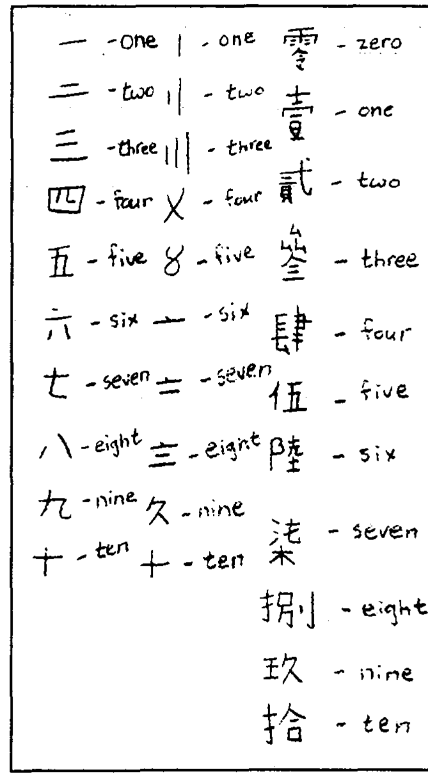


Fig. 6: Qiao's Chinese number symbols

The activities from *Historical Connections* (Vol. I, pp. 51, 52) were then handed out, and the students enjoyed investigating some of the number patterns connected with Pascal's triangle. However, Barbara seemed rather lost in this, and referred students to me if they asked questions. Later she invited students to the board to explain their work to the rest of the class.

Barbara had to attend a meeting immediately after the class, so she phoned me that evening. She had obviously enjoyed this class, and talked about it being "the start of something new and big". She seemed particularly impressed that Qiao had agreed to write the Chinese numbers on the board, explaining that she was usually rather a shy person. In her written comments on this class she noted "They're ahead of me! Does that bother me? Well, not too much but a little bit – not enough to make me stop but enough to make it a real challenge for me."

MMC chapter: "Shells, Sticks, and Pebbles"

Joan taught a group of twenty-six Grade 4 students. She had selected the chapter on Magic Squares for her first Multicultural Math class, but afterwards had commented, "I felt I talked too much, although they were coming up with ideas. But I did feel that I was too much at the front of the room". This led her to chose the "Shells, Sticks and Pebbles" section of her *MMC* pack for her next class. This features various forms of dice from around the world, and the games in which they are used, and Joan noted that "It's the hands-on stuff that appeals to kids. You get far more sparkle out of them when you provide them with manipulatives". She also observed, "I haven't done much game-playing this year, but kids love that, and actually that's where you really see the critical thinking skills".

In discussing this topic prior to her class, I commented on the connection between "throwing games" and probability, and asked if she taught the Statistics and Probability strand of the IRP.

Joan: It's one of those units that you leave until the end, and if you get time you throw it in, so this might be a good time to do that. I was thinking about that with the dice, with the probability of throwing a three.

Irene: It would be nice to do it with something other than the regular six-sided dice, 'cause that crops up all the time, and they know what to expect, but the throwing sticks are different.

Joan: Well, it's interesting because which number shows up all the time, if you do it enough times, you do find they're weighted a bit. Especially when you buy the cheap ones.

Irene: Yes, and you can also tie it in with the shells that other countries use.

I felt that we were talking at cross-purposes here, but as will be seen from the following description, Joan incorporated some of the ideas I had mentioned.

Regarding the cultural value of the work, she commented, "I wanted them to do some thinking about how dice might have occurred, that people didn't just go buy them in the store, they used what was in their environment". She also planned to ask the students to locate the countries in which the games were played on a world map, but later admitted that she "tripped over that part of the notes".

She started her second Multicultural Math class with them by asking how many of them had played games during the weekend, and was pleased to discover that at least half the class had done so. Using the information from the *Big Book* and *Teachers' Notes*, she talked generally about the different sorts of dice that have been used around the world. She then focussed on the "dice sticks" used in ancient Egypt, illustrating them with Popsicle sticks which she had coloured black on one side. However, she did not display the *Big Book* during this class, and she told me later that she had done this on purpose, "because I didn't want them reading ahead".

To stimulate some problem-solving, Joan asked her students to suggest several ways in which various combinations of these sticks could be used to generate a score. The first idea was that "white meant one and black meant ten". Joan accepted this, pointing out to the class that this suggestion was probably influenced by a discussion they'd had earlier that day about place-value. Ben then suggested that blacks could be one and whites nothing. When Joan asked "Why would you have a nothing one", he explained "You're just going to count the blacks". Another suggestion was that one side represented two and the other side one.

Joan then challenged the students to try to find a way to get all the numbers from 1 to 6 using four of the sticks Since only five combinations of rods are possible, this was perhaps an unfair task, but Joan admitted later that she had not thought through the consequences of her request. Nevertheless, it was very productive. Several students spontaneously made tables to keep track of their results, and some of the strategies were quite ingenious. One student suggested, "[The white side] could be a minus, so if the black stick was worth two, three blacks and one white would be five". After several minutes of exploration and sharing results, Joan explained the Egyptian method of scoring with four sticks ("count the number of white sticks, but score 6 if they're all black"), so that they could compare this with their own systems. Later she told me:

> I was really impressed with the different ideas they had for what they could possibly be worth. Actually that lesson kind of developed as I did it; it didn't go the same train I'd been thinking about for the last couple of weeks. I thought we would sort of quickly get into the games, and yet as I started talking about it, I thought "I'm not just going to tell them what these dice mean, what are some different possibilities", so I was really pleased with that part; they were really getting into that.

She then gave groups of four sticks to the students, saying that they would be doing some probability with them: "Let's see how many times we get all black – does it happen very often? Everybody throw the sticks once." This caused great excitement, and she had to warn the students, "Don't throw them more than once – if we're going to do probability, we don't just keep doing it until you get all black, you have to keep count of how many times you have to throw it until you get black." After each throw, she asked the class how many students had got all black sticks, but the numbers were not recorded. Despite Joan's instructions, many of the students still seemed to think that the goal was to *try* to drop four blacks sticks, and there were many shouts of "I got it" when this happened.

Discussing this part of the lesson later, Joan commented that she had been concerned that she was "getting off track". She agreed that this "could be a whole other lesson", although she was concerned that "probability's not really a multicultural thing, is it?". However, by the end of our discussion, she was convinced that a follow-up lesson was the right strategy, and started to write down a few ideas about organizing a probability lesson based on the various types of dice.

After the dice sticks activity, Joan went on to talk about cowrie shells, the next scoring object mentioned in the *Big Book*, telling the class, "These were how the Africans and Indian people did their dice". However, her decision not to use the *Big Book* lost her the opportunity to show a picture of these shells, which she later regretted. She gave each pair of students six pasta shells, these being an easily obtainable approximation to cowrie shells, and again asked the class to find a way of scoring. After about ten minutes exploration, a large part of which was spent dropping the shells to see which way they landed, the various strategies were discussed. Most students had simply counted the shells which fell "mouth up", but Eric suggested, "Open is one and closed is zero, but when they are all open you get another turn, and if they're all closed you miss a turn." Joan congratulated him, and told the class that the strategy of extra turns and missing turns is common in traditional games, illustrating her remark by discussing the use of shells in determining moves in the Indian game Pachisi. She summarised the scoring system on the board ("Open – 1 each (2-6); Only one open – 10 & another throw; None open – 25 & another throw"), but did not stop to compare this to the students' suggestions, explaining later that she was feeling "pushed for time". Rather than playing the game itself, she suggested a modified version in which the goal was simply to be the first to reach exactly twenty-five. While playing, many of the students realized that "if you get to twenty-four, you can't win", a situation which Joan discussed with the whole class after the game.

To end the class, Joan introduced them to the Hyena game from North Africa. This is played with regular dice, and earlier she had told me that she planned to get the students to make their own dice by putting dots on wooden blocks, "and then, depending on what kind of wood it is, they could analyze the different results and see which is the most reliable". However, towards the end of the class, she admitted that she had "way too much planned for the morning", so she modified her plan and simply encouraged her students to identify which numbers were on opposite faces of a regular dice. Joan wrote these number pairs on the board, and a student volunteered the information that "they all equal seven". The rules of the Hyena game were then read to the students and they played the game for the remaining ten minutes of the lesson, which was fun although without any explicit mathematical connection.

Joan talked to me while the students played, admitting that "so far this year we've concentrated on the stuff from the *Interactions* book, focussing on multiplying and so on, 'cause they have to know it". However, she commented upon the problem solving that had occurred during this class, and felt that it had "worked out really well". She said that she had "pretty much followed the lesson in that package. It's a Grade 4 level, and I'm finding that it's taking me longer than I think, because you can easily go off on all sorts of tangents".

The follow-up probability experiments took place the following Friday afternoon. Joan reported that the students threw the same number of shells, sticks and regular dice as they had in the first class, for a total of thirty trials with each, and then looked to see if there were any patterns. Each group then wrote down their conclusions, but there was not sufficient time for a whole-class analysis of the findings.

MMC chapter: "Easy Ways to Multiply"

There were twenty-four students from Grades 4 and 5 in Mike's class. He decided to teach the unit on Lattice multiplication (see Appendix G) for his third Multicultural Math session, as he considered that:

> It might be a way to help some of the kids who just aren't grasping the ways we normally do it, who just can't see any rhyme or reason to it, who just put this here, and that there, and that's it. It might make some of them, you know, the little light bulb might go on, some might realize that "I can do that and I'll get the right answer". They might even see why on [the lattice method] better than on the other one. And there's lots of stuff that you don't have to remember, like putting the zero down. It does the stuff automatically for you.

I asked him if the light bulb had gone on for him when he first looked at the method. He replied:

> At first no, because I just followed the rules, and then as I looked at it some more, I suddenly thought, "I can see why that works". It took me a little while, but I realized that, of course, that's just moving this over one space.

He also commented on the benefits of Napier's Rods for students who were still uncertain about their multiplication facts, and planned to have every student construct his or her own sets of Rods and learn how to use them.

Mike started his session by reminding the class of the "different things we've been doing that come from other parts of the world, from other cultures, some from a long time ago like Stonehenge, others more recent, like that symmetry stuff", then went on to explain that in this class they'd be learning about something they already knew how to do. He told them:

> When I was in Grade 4 or whatever, a long time ago, right, I learnt how to multiply two-digit numbers together, but the other day, I looked at a page

in this Multicultural Math booklet, and I found a completely different method of multiplying. And this shocked me. And the thing that really shocked me was that people in other parts of the world multiply different than we do, still.

He went on to admit that some of the methods mentioned in the *MMC* book came from a long time ago, but he thought that it was still interesting to learn about them. He reminded the students of the rote method by which they usually multiply:

> You know, you just write down your numbers, and you draw another line and you add them all up, and away you go, and that's the way that everybody in the world does it.

By this stage, the students were beginning to look rather bored, but when he shouted out, "Wrong, I found out. There's other ways of doing it, and you come up with the right answer", they suddenly snapped to attention. One of the boys asked, "Is it the same answer", to which Mike replied, "Yes, the same answer, which is good", adding "and some of the ways that other people have worked out may be easier than the way we do it!". He went on to talk about "a second part to this that was like a computer before they had computers", and promised that they'd get to that later [Napier's Rods].

As Mike's class included a few immigrants from Asia and the Middle East, he asked if any of them had learnt to multiply in different ways. Unfortunately they had all left their previous homes too long ago for this question to be really relevant, so he simply told his students that the Lattice method of multiplication they were about to learn came from Iran and Iraq. Almost as an aside he asked, "Does anybody know what the word 'lattice' means?", and after the quick exchange of "something you eat", "No, that's lettuce", the students suggested that it was "like a grid" or "trellis". With this image in the students' mind, Mike then drew a framework for the Lattice method of multiplication, using the version shown in *MMC* pack 5. Mike explained how to use it, using the example of 23 x 19, admitting that, "You still need to know your times tables, and that's where this other person, that we'll talk about more next time, kind of helps you out there". As he filled in the grid, he explained the lattice method as follows:

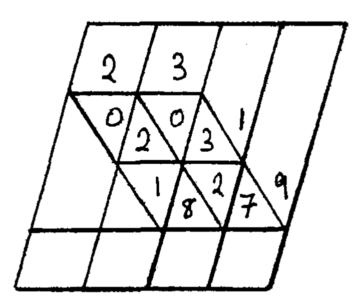


Fig. 7: Partial products of Lattice multiplication

> What you do is to go 1x3 is 3, 1x2 is 2, and you just fill up those spots with zero. Then we go 9x3, 27,

> so we write 7, 2. 27, you just put one number in each of these sections. 9x2, 18, 1, 8. OK so far? Really you're writing down differently, but you're still thinking in the same way as in our regular way.

After he'd finished going through these steps, Mike wrote the regular method of multiplication on the board, chanting as he went: "9x3 is 7 carry 2, 9x2 is 18 plus 2 makes 20, 3, 2 and we get that thing", indicating the layout shown in Figure 8. Comparing the two partial solutions, he noted that in the Lattice Method, "All you do is write down the numbers", implying, but not stating explicitly, that the Lattice methods avoids the "carry 2" step of the traditional method. He then finished the Lattice calculation, pointing out:

$$\begin{array}{r} 2\ 3 \\ \times\ \ 1\ 9 \\ \hline 2\ 0\ 7 \\ 2\ 3\ \ \\ \hline \end{array}$$

Fig. 8: Partial products of regular multiplication

> The addition step's a bit different too, because the numbers you're adding are between these diagonal lines. See, the 7's on its own, so you just write that again underneath, but then you have to add the 8, the 3 and the 2 together to get the next digit in the answer.

By the time he had finished the last step, the students were very enthusiastic about the Lattice method, calling out "that's so easy, and so fast", so he gave them some questions taken from the blackline master in the *Teachers Notes* of *MMC*. Most students managed these really quickly, although a few seemed rather confused as to where the digits of each partial product should be written. Mike went through the solutions with them, stressing the zeros that had to be included when a product was a single digit, and that addition was done diagonally, although at this stage he did not explain either of these points.

While the students were working on these questions, I showed Mike the alternative, rectangular grid sometimes used for Lattice multiplication (see Appendix G). He agreed that this would be much simpler for anyone having to construct their own grid, and later drew it on the board, so that the students could compare the two versions.

However, he first led a discussion comparing Lattice Multiplication to the method regularly taught. About two thirds of the students put up their hands to indicate that they thought the Lattice method was easier, causing Mike to comment in surprise "and that's even without much practice". He asked the students why they thought it was easier.

Kevin: It sorts the numbers for you.

Mike: That's a good way of putting it, I think, because one of the things I see when we're doing multiplying is people aren't putting the things in the right column. ... By having these lines in here, you've just got to fill all the spaces, it automatically does that for you. What else makes it easier for you?

Jenny: It's easier to check, to see the mistakes.

Carson: You don't have to put a zero down.

Mike: How come they don't need a zero in there? When we do it, I know we have lots of people forgetting to put the zero in or putting the zero in over here instead of underneath the other one [referring to the calculation on the board], and that was a problem that you worked your way through when you were first learning to multiply with two numbers. How come they don't need that? There isn't anywhere in here where you have to add a zero on the second row, they don't do that. (pause) Hmm.

[A few students muttered something, but their remarks were inaudible to me.]

Mike: Well it works, they've made it easier somehow. What have they done so that we don't have to add that zero in? What have they done so that we don't have to remember to do that?

Kevin: They made it sort of like this [waving his arms in the air] so it's sort of like the zero would have gone here.

Mike: OK, by drawing it at the angle like that, yes. That this number here (pointing to the 1), which is your tens number, which in our multiplying we would do second, right? - if we were doing this question we would multiply by the 9 first and then put down a zero and then multiply by the tens - they're multiplying by the tens first, then by the ones row but because the lines are drawn at an angle it automatically moves it over one spot. They don't have to remember to do that. And just think of all the times when you got something wrong because you forgot to do that. I think whoever designed this, a long, long time ago, it was somebody who was a mathematician who said "how can I make this simple and easy for the people, so that they don't have to remember all this stuff". 'Cause the people of a few hundred years ago, they had enough things to try to remember without trying to remember all those kinds of things. If they could remember their times tables to fill in those spaces, they were doing pretty well. ... So it's nicely laid out so that they can check to see that they've done their nine times table right and so on. It's just very elegantly designed. It automatically puts the things in the right rows for you, and you don't have to remember.

Kevin asked if there was also a method of Lattice division. Mike admitted that he didn't know and looked towards me for guidance, so I said that the multiplication method could be modified for division, but that it was much harder than the method used now. When Edward asked why we didn't do multiplication this way now, Mike admitted that I'd told him the answer but turned the question back to the students:

Mike: Why should we do it our way, when the majority of students in here seem to think it's a little bit harder. Do you think people sat down and said "Well, we have this way of doing it, and we have this way of doing it, let's do it the harder way"? I don't think anybody would think that, would they?

Ben: Well, it takes a long time to write down the chart.

Jenny: But it's so easy.

Mike: Well, maybe not so easy. You saw me trying to draw it up here, and I had one to copy. Mrs. Percival told me that the main reason that we don't use this method any more is basically what you're saying. When they were trying to print books, it was really hard to do. If you have a nice piece of paper already printed out with all these lines on it, then it's nice and easy, but it was hard to print out those pages, on the kinds of printing machines they had then. So because they couldn't print calculations like this, people stopped teaching them.

Terry suggested that the Lattice Method took up more space, and Mike admitted that "it might be a big bigger" and promised that in the next lesson they'd try a calculation by both methods so that they could compare both the amount of paper needed and the time taken to complete the calculation. He then showed them the rectangular version of the grid, commenting that this version was much easier to draw. However, he then started to compare the old and new methods of calculating 19x23 which were still on the board, and seemed very surprised when his count showed that the Lattice method only used four lines, whereas the regular school method used five.

Jenny then asked him where the method came from, so he went over to the world map to indicate "Iran, Iraq and all the areas around here". After all the other students had left at the end of the class, Jenny carried on talking to Mike about the fact that people in other parts of the world do things differently to the way we do them, but unfortunately when I got near with my tape-recorder she refused to say anything else.

Mike told me he was intending to do more examples of Lattice Multiplication the following day and that he was also planning to have the students make their own set of Napier's Rods. I missed that lesson, but a few days later I visited the school during the lunch hour and six students from his class volunteered to show me how to use the Rods. They clearly knew that each Rod represented a multiplication table, and were able to set them up to multiply two- or three-digit numbers by a single digit. Three of the girls were able to explain the connection between the Rods and the Lattice method of multiplication, but even with this knowledge they lost track of which digits had to be added when numbers with more than three digits were involved. The Rods were certainly popular, particularly among the Grade 4 students who were still rather unsure about their multiplication facts. Mike reported that several students had asked to use them all the time, but that he had vetoed this request.

For comparison, I include here a very brief description of an alternative method of teaching Lattice multiplication which was used in a Grade 4/5 split class in another school. As a challenge to her class, Marcia had written up the calculation for 27 times 46 on her classroom board, and challenged her students to "figure out what is going on". Within a few minutes, several students had recognised that that the numbers inside the grid were the two-digit products of the individual digits around the edge, but it took some careful questioning to reveal that the final product was obtained by adding diagonally. Once this was apparent to them, they also exclaimed how easy it was, and most selected this method when given a choice of "old or new" for the rest of the class. However, in all future work, neither these students nor those in Mike's class used the Lattice method when faced with the need to multiply in their regular mathematics class. Historical methods were seen as fun, but definitely as something "different", and only to be used on special occasions.

Other MMC chapters selected

For her second class, Barbara selected the chapter on "Elegant Art" which had attracted her earlier, as it now fitted well with the geometrical work the class was studying. This lesson, and the two which followed, used only the materials from the *MMC* pack. Her third topic was the non-traditional concept of "Never Ending Paths" which generated some heated debate over the Bridges of Königsberg problem. In this case the motivation was not mathematical but literary: The students' novel study concerned mazes, so she made the cross-curricular link to mathematical networks. Her fourth set of classes, "Cubits to Meters", introduced the students to some ancient measurements as well as giving a brief history of the metric system supposedly in use today. Barbara started by asking the students to write down their weight and height and was surprised to find that everyone used pounds, feet and inches. This led to an interesting discussion as to why they used "old units" rather than those which they were taught in school. These lessons all followed a pattern similar to that described earlier. Students were always called to the map to indicate the origin of the mathematics to be studied, and the *Big Book* was always prominently displayed and discussed. Most of the activities were taken from the blackline masters found at the back of the *Teachers Notes*, and the students were often left to work through these with little guidance.

Unlike the games lesson outlined above, most of Joan's classes also gave a prominent role to the pictures in the *Big Book*, and students were asked to point out the countries discussed. Her first class, that on magic squares, was notable for the amount of problem-solving it entailed. The children became fascinated by the fact that there are a number of apparently different versions of the three by three square, and wanted to find them all. Jeff realized that one square could be obtained from another by rotating and reflecting the arrangement of the numbers, and explained his method to the class.

The third chapter Joan selected was "Perfect Balance", which discussed the symmetry of designs found in Native American, Japanese and West African artefacts. For this class, she also used Zaslavsky's *Multicultural Math: Hands-on Math Activities from Around the World* (1994), as she felt that these activities connected with the learning outcomes of the IRP better than those in the *MMC* pack. Shortly after this session, she and Barbara spent some time decorating Ukrainian Easter Eggs, an activity which reinforced the idea that symmetry is an aspect of mathematics which is present in the artistic designs of all cultures. Her final session used the "Cones, Domes and Rectangles" chapter and showed students the connection between the three-dimensional shapes they learn about in the classroom and buildings constructed around the world.

Mike's first class, "Sun, Seasons, and Solstice", looked at methods of measuring time and seasons in several parts of the world, and he connected this material to both the geographical regions and to the students' work in their science classes. The chapter on "Jumping Games" introduced students to games from around the world, and encouraged the development of strategic thinking. His fourth topic, "Captured Designs", looked at the geometrical transformations found in Persian, Japanese and Native American artefacts, and was closely related to some of the students' recent work in mathematics.

Maureen, the fourth teacher in this group of Intermediate grade teachers, had a class of twenty-seven students from Grades 5 and 6. She made considerable use of the *Big Book* at the start of each class, and sometimes even made overhead transparencies of some of the pictures so that her students could see them more clearly. She selected the "Thinking Games" chapter for her first session, noting that some of the playing strategies seemed to rely on factors and multiples, which had been the recent focus of the students' regular curriculum work. She also commented that playing games "has some logical thinking which will fit in with what we're been talking about – and it's a good way to get them excited about math".

She chose the chapter on Egyptian numbers for her second lesson, and had intended to use the activities from the *MMC* pack. However, in our discussion before the class I mentioned some problem-solving activities that I had constructed for this topic, and she decided to use these instead. She also followed Sarah's example by asking her students to construct their own mathematical papyrus. Her third topic, "Warps and Wefts", involved the students in analyzing and reproducing a weaving pattern, and proved to be extremely motivational for several members of the class who usually had difficulty focussing on their work. The final chapter she selected from *MMC* was "Slides and Turns", which she chose for its close connection to the Transformations strand of the IRP which the class was studying at the time.

CHAPTER 6
ANALYSIS OF QUESTIONNAIRES, CONVERSATIONS AND LESSONS

The lessons I observed, a few of which were described in the previous chapter, illustrated some of the ways in which non-specialist elementary school teachers can implement cultural approaches to mathematics. However, these class observations were complemented by the views expressed by both teachers and students in questionnaires and informal interviews.

This chapter contains an analysis of the information gained from all these sources, categorizing teachers' justifications for bringing a cultural dimension into the mathematics classroom and the implementation issues they encountered. Most of the comments were made by the case study participants, but occasional remarks have also been included from other teachers: Wendy, Marcia, Ingrid and Ken were teachers with whom I worked during my preliminary studies prior to the formal field-work year and Brian was a student teacher working in Sarah's classroom. Parents had also been asked to comment on their children's experiences in their culturally oriented mathematics classes, but the few responses received merely conveyed a general appreciation of the work, and so have not been included.

Teachers' views on the benefits of using cultural approaches to mathematics

The research participants produced many arguments supporting the value of using cultural approaches to mathematics. These included issues which benefited the students, in addition to those which teachers saw as adding to their own mathematical and professional development.

Benefits to students

Teachers' views on the advantages for students covered issues of motivation, changing beliefs about the nature of mathematics, ways in which cultural material could be used to enrich the regular mathematics curriculum, interdisciplinary connections, and the special role that culturally oriented material plays in the ethnically mixed classrooms commonly found in the geographical area in which this research took place. Their comments are reviewed, with reference to the situations giving rise to them where necessary.

Motivation

All the teachers remarked on the motivational aspects of including cultural dimensions into their mathematics classes: "The students are really enjoying this" was the comment I heard most often. The question "Why do we have to learn this?", which several teachers reported as being quite common in their regular mathematics classes, rarely arose during these lessons, whereas the student request, "Can we carry on over the lunch hour?" was frequently heard in the culturally oriented sessions. In the pre-study questionnaire completed by the students, only a third claimed that they "always liked" mathematics, whereas all students said that they enjoyed the days in which cultural perspectives were employed.

Several teachers commented on specific instances in which the cultural mathematics sessions generated student interest beyond what was expected of them. Ruth introduced her students to the number system now used in Iran, and was surprised and delighted when one of her students arrived the following day with a list he had compiled of the numerals used in seven ancient cultures. Joan reported in an amazed tone that "a number of [the students] have carried on with the magic square stuff at home, so they're doing math strictly voluntarily". Sarah recalled the motivational effect of her Time-Travel Days upon a student who was always behind in his work:

> Tim had to make up old work instead of taking part in the Greek Time-Travel activities, but he just wants to do this so badly. We negotiated: He said, "Can I just make one of [the Platonic solids], then I'll go back to my old homework?", so I agreed and he was happy.

Wendy noted that after she had introduced her students to addition and subtraction with Egyptian symbols, they were "really keen to know how [the Egyptians] did multiplication, if there was a sign for it, and they also wanted to know how to do division".

Teachers noted that the overall classroom noise level rose during the Multicultural Math lessons, but considered that this was an indication of student enthusiasm. Wendy commented "I don't mind the chatting, because it's new and so exciting and [the students] want to talk about it." She went on to add that her students enjoyed this work because "it was really non-threatening ... even the ones that might not venture answers at other times were willing to try. When we worked with the Egyptian symbols with pipe-cleaners, even the non-math students enjoyed that".

Cultural approaches to mathematics often involved hands-on work, and several other teachers noted the motivational value of this, particularly for those students who struggled with traditional approaches. In one of my discussion sessions with the G4 group, Maureen talked about her experience using the *MMC* weaving activity:

Maureen: Ted gets into a lot of difficulties in math, well in all subject areas, so when he did this, he just sat there at first ... I had to cut the strips for him, and then I didn't hear another word from him for about 45 minutes, and he sat there and really concentrated, and it was really good.

Barbara: I'd like to see it, then I can say something to him about it, because he's definitely struggling with his French, and it would be good to be able to compliment him on something.

Maureen: Well he needs that. When I showed it to the class, he really looked pleased, 'cause for him any activity is difficult, and this was something where he'd really succeeded.

Teachers also commented on the motivational aspect of cultural mathematics for students who had a strong family background from another country. Ruth noted "I've had more students come to me asking, 'Can we do something about so-and-so', almost as if it's a validation of their own culture, their own connections". She pointed out that making associations between their lives and their school work was often done in Social Studies and Personal Planning, "but why should it not be done in mathematics?". Similarly, Mike had a Chinese student ask if they could learn about the abacus, and was astounded at this expression of interest from one of his least academically inclined students.

Changing perceptions of mathematics

When asked whether their students knew anything about cultural aspects of mathematics, the only teachers who answered positively were those whose students had previously been taught by Sarah or myself. In general the student questionnaires confirmed this, although students from other countries were sometimes aware of different ways of writing numbers or calculating.

A few teachers led into cultural mathematics by discovering what their students already knew of "Multicultural Mathematics". Ingrid started her class with a general discussion of multiculturalism, and after the mention of the more obvious issues of religion, language, food, clothes, and games, Luke raised his hand and asked "Can math be multicultural?" in a tone which implied that he clearly did not think this was possible. The rest of the class laughed but looked equally doubtful. However, the brain-storming session that followed this question elicited examples of mathematics from several different times and places: Roman and Chinese number symbols, the Arabic origin of "Canadian numbers", calculating devices from the Chinese abacus to the slide rule, and the suggestion that the sundials from various civilisations all around the world had something to do with math. Both the students and their teacher were surprised and pleased that they had been able to find examples of mathematics from other cultures, given their original reaction to the idea. In Ruth's class, a discussion of multicultural mathematics turned to written and spoken numbers, and her students were similarly amazed to discover that members of the class were able to write four different sets of number symbols in use today: Hindu-Arabic, Punjabi, Persian and Chinese.

These students were brought to realize that "mathematics" has a broader cultural background than is usually supposed. The following paragraphs document teachers' views on how cultural approaches to mathematics can correct other student misconceptions about the subject. Brief descriptions of the relevant class experiences are sometimes given to set these remarks in context.

Myth 1: Mathematics "just is" and people played no part in its development.

The "human endeavour" aspect of mathematics history was considered very important. Wendy noted that "kids don't seem to consider that real people actually thought up this stuff", and several teachers admitted that they themselves had never given any thought as to who created the mathematics that they regularly teach. Being aware that her students liked history, Sarah rationalized, "If knowing about the people in the past makes the learning so exciting in other subject areas, why wouldn't knowing about people in the past make math interesting, it would make it human". She went on to reflect:

> I think that's why math is so deadly dull for most of them – it doesn't seem to be connected to human activity. You know, it's just numbers and it's abstract. It's not real people solving real problems, struggling with real issues, running through bean fields and doing whatever.

Ruth commented:

> Someone had to start these things at some point, so why not look at the person who did start an idea. If the students can go round talking about Archimedes as the "Greek Dude", enjoy what they are doing and learn more background, then probably their level of learning is greater.

To support her view, Ruth even produced the following paragraph written by van Hiele:

> The fifth form of motivation consists in bringing pupils into contact with the thinking and striving of others by means of the subject matter. It is possible, for instance, that someone who admires a very old vase will wonder what went on in the mind of its maker. In mathematics one can ask oneself about the world of thinking of the Greeks, which led them to such important results in geometry and such poor results in algebra. The subjective value of this motivation may be described as development of appreciation of fellow man. (1986, p. 192)

The story aspect of the "human endeavour" approach was very popular with all teachers, who noted that "we do problems, but we don't often have a story behind them". Sarah noted the positive effects that a story about Fibonacci had on her students:

> We looked at his life and some of the things he did, and we looked at how his patterns occurred. Every kid remembers that, it sort of put it into some kind of reality, it makes sense. It's a real person doing real stuff.

She often required her students to read simple biographies of the mathematicians they were studying, usually taken from the *Mathematicians are People Too* books (Reimer & Reimer, 1990, 1995b). During one of her early Time-Travel Days she also gave a group of students a skit based on the famous story of Archimedes and King Hieron's crown. She was amazed by their enthusiasm, and admitted to me that she had never done anything like that before. After the success of this skit, several others were performed, all taken from the *Historical Connections* texts (Reimer & Reimer, 1992, 1993, 1995a). The audience watched with rapt attention, and Sarah felt that such play-acting helped to "bring home the fact that mathematicians are real people".

Myth 2: Mathematics is, and always has been, the same all over the world

Sarah remarked that "Kids have this idea that math is cut-and-dried, it's right, it's wrong, it's always been that way and it's always going to be that way, and what I want them to understand is that that's not true, that math has evolved, math has changed". She used her Time-Travel Days to dispel this myth, often pointing out how the development of mathematics was influenced by the needs of the people. She reported that:

> [The students] were fascinated with the fact that some cultures only needed numbers like one, two, many. And they said, "They must have been really stupid". And I said, "No, I don't think they were really stupid, I just don't think they needed anything else. Like if you don't need a big number, why would you learn a big number?".

On another occasion Sarah and her students role-played trading situations in a pre-literate society, exploring how written numbers were developed to label transactions and hence prevent fraud. Later they discussed how the addition of a zero symbol to the original Babylonian number system helped to avoid confusion, and noted the role which zero plays today in differentiating between 302 and 32.

Other teachers also felt that it was valuable to investigate the contributions made by various cultures to the development of mathematics. Ruth noted, "Everything has to start somewhere, so why not think about where it starts, in terms of culture and locale". Ingrid used this approach very successfully in the Fall of 2001 to help counteract the "bad press" that the Iranian students were suffering after the September 11th terrorist attack on New York. She told her students about the important contributions that the Arabs had made to the development of mathematics, and reported that "the kids were very interested to know that the numbers they use are Arabic – and they seemed to have a better opinion of the Arabs afterwards". Later she commented that she'd also like to teach Roman numerals "because that's stuff they actually come into contact with".

Several teachers used the Chinese counting rods (see Appendix G) to reinforce place-value concepts or teach integer operations, and commented that they liked doing so because "this is real, this is how it really happened in the past". Sarah contrasted these historical manipulatives to those commonly used in the classroom, noting that "[the students] are told to put a bean here and a bean there and put the beans together, and it never makes any sense to them cause they're just moving around beans, and who cares about beans!".

Myth 3: The mathematics learned in school has little or nothing to do with the real world.

Joan noted that her students often seemed to view mathematics as "just a sterile thing that comes out of a textbook", and made a point of telling them that "math has been coming along for a very long time and it's been changed by the people that come into contact with it". While modern textbooks often try to make mathematics appear relevant to students by pointing out applications of the topics studied to real-world problem, Ruth and Sarah commented on the parallel approach of looking at how mathematics had developed to solve problems in the past. Ruth felt very strongly about the need to connect mathematics to the "real world", and although she often did this through contexts familiar to the students, she commented:

> I find it more interesting, more enjoyable to think of those multicultural connections with the wider world, and I think the kids really do gain from it. I want kids to find something which fascinates them, and they might explore more. You don't know when it's going to be, and it's often the most unexpected. I think [a cultural approach] can help demystify math and make it a really wonderful, enjoyable subject.

For some of her students, this fascination came from the mathematical aspects of quilting, whereas others were intrigued by Archimedes' ideas on circle measurement.

Myth 4: There is only one correct way to solve any mathematics problem, and only one correct answer.

Most teachers showed their students at least one of the many different arithmetic algorithms which have been produced in the last five thousand years. They considered that these alternative algorithms not only demonstrated that mathematics problems can be solved in more than one way, but also provided options for students having difficulty with the regular techniques. Ingrid commented:

> In my view, it is always good to have more than one way of doing things.
> As a teacher, the more ways something can be done, the more accessible a

> subject such as math becomes to students, and the likelihood of reaching them all increases. My kids have always been interested in how things are done somewhere else, so if they find a new, cool way to do something, they're usually into that, specially if it's quicker.

Marcia shared this view and commented, "Did you see how interested [the students] were when I showed them that English method of subtraction?". She added that "Learning these ancient methods of calculating makes them so much more receptive to learning how to do things differently", and also commented that it helped with that "classic struggle" between the methods used at school and those the parents knew, as it "opened up their ideas of what was acceptable".

Ruth talked about the advantages of this approach with respect to Gardner's theories of multiple intelligences, noting that "Other methods help [the students] link into mathematics in some way ... and they can engage more, that's the important thing". She asked her students to write about learning different algorithms, and several shared the pragmatic view that it was useful to learn a different approach "because you can sometimes use it when you are stuck".

After a lesson on strategy games, Mike commented that the cultural mathematics lessons helped the students to realize that mathematics isn't always "either right or wrong, because in some of this stuff, there really aren't any 'right answers'; very often there's different ways of getting it right".

Increased understanding and enrichment of regular curriculum material

Several teachers claimed that learning about mathematics from other periods enhanced their students' understanding. Ruth commented:

> Since I have given an historical context to many units when I introduce them, the elementary grade students that I teach seem to have found it much more enjoyable to proceed with a unit and, I am sure at times, easier to grasp the concept in question.

However, specific examples of such increased understanding were rare. Wendy was convinced that doing multicultural mathematics had helped her students "lose their fear of the decimal", but she admitted:

> I don't know how that happened, but when I was talking to them, they weren't afraid of it, and they said it was "because we saw all the other

> different kinds of math" ... I guess it just made math come more alive for them, or they were more familiar with it – I just don't know.

She also noted, "They really like it when we do place-value using the Mayan math, the Egyptian and the Chinese", and both she and the four other teachers who had worked with ancient number systems considered that "The more [students] understand about how other number systems are structured, the more they will understand about their own number system". Sarah specifically noted that "When they see systems that didn't have a zero, it helps them realize why we need one as a place-holder today".

Multiplication is another curriculum area which teachers considered to be enhanced by material from the history of mathematics, both as regards gaining a conceptual understanding of the process, and simply as a way of reinforcing the multiplication facts. Marcia's Grade 4/5 class included many students who were struggling with these facts, so she introduced the Lattice method of multiplication to give these students extra practice, while using a format that provided some variety from conventional worksheets. The students appreciated how simple it was to check their work compared with that in the more traditional method in which individual products are obscured by "carry-figures" from previous partial products. This led some of the more advanced students to explore products of three and four digit numbers, which they declared to be "just as easy". Mike's class explored both the Lattice method of multiplication and Napier's Rods. Discussing the latter with me he commented, "The kids could see that each strip was a multiplication table", and when I visited the students the following week, they were able to explain to me that this was why the method worked. He also felt that his students were more successful with these methods than with the traditional algorithms, and both he and Marcia noted that students were enthusiastic about the new methods: Class votes showed that about two thirds of the students preferred them. However, the students seemed to regard the work in their Multicultural Math sessions as something special, and reverted to the "tried and true" algorithms in their regular mathematics classes, even when these were less successful.

A few teachers found that word origins present a simple, but effective way to historically reinforce the meaning of mathematical terms. Sarah wrote:

> I like to do whatever I can with word origins so that students learn how the English language is structured from other older languages such as Latin. If they learn the meaning of prefixes (bi means two, tri means three, quad means four, etc.) they find it easier to remember new, bigger words (bilateral symmetry springs to mind). When we learnt about isosceles triangles, they were really excited by the ancient Greek skeletos (for legs) and how that created skeleton in modern English. I guess it's the whole business of scaffolding again, where they connect bits of learning about language as well as math. A few even tried to create new words by using what little knowledge they had about prefixes, suffixes and roots (ex. triskeletos for a three legged beast).

Ken was very enthusiastic about the enrichment opportunities provided by cultural approaches, and made the following comment about his work on Egyptian mathematics

> Grade 7 math is a repeat of Grade 6 is a repeat of 5 and so on, so being able to take a couple of weeks and do something like this that isn't in that repeat zone is fun and exciting and it breathes life into math, and for the kids it makes it a fun thing, and that's what it should be.

Sarah's Time-Travel Days sometimes allowed considerable scope for students to choose their own activities, so discoveries were sometimes made which took them beyond the learning outcomes required for Grade 3. Her favourite example was of a group of girls whose work had led them to the square root concept: She related how Helen had "very clearly explained that it was the opposite of when we use exponents".

Interdisciplinary connections

All teachers were excited by the cross-curricular connections which cultural mathematics encouraged. Ken spent several lessons on mathematics during his Social Studies unit about Egypt, and commented:

> I think it brings excitement to learning, it allows them to bring this whole Egyptian thing together, that looking at Egypt isn't just Social Studies. You start to erase some of those labels that this is Socials and this is Math, and you get to "This is learning". It's learning about the people and the systems they use, and why they did things, and that's what life is. We don't have compartment lives, little areas that we live, it's all close together.

When Ruth first introduced "Multicultural Math" to her students, she told them that they would have "a wonderful time combining math, socials, art and maybe language arts". Her cultural mathematics classes often led from one curriculum area to another, and she commented to me:

> One of the things that fascinated me, once I start to get interested [in cultural approaches to mathematics] is suddenly realizing that when you start looking at math it takes you on all sorts of journeys – you can go back in history, or you can go around the world with geography.

A lesson which started by looking at the Arabic connection to our modern numerals moved on to mention the Persian mathematician Al-Khwarizmi and his role in the development of algebra, and as a cartographer. This in turn led to a discussion of mapping, flat-earth beliefs and Captain Cook's voyages (which she traced on her world map), before returning to Islamic mathematics and the cultural and intellectual significance of the historic city of Baghdad. Her treatment of tessellations included a description of mosaic tiling in Rome and her visit to a tiling factory in Maddaba, in addition to Escher's work, and also developed into an art project.

Several teachers made connections to Language Arts through literature, drama and, as previously noted, word origins. Ruth often read children's literature which connected culturally to mathematical topics. Books such as *Grandfather Tang's Story* and *One Grain of Rice* were selected to tie in with the mathematical topic being studied, and the students enjoyed both listening to the story and discussing the mathematics. Sarah also made frequent use of such literature, and encouraged her students to watch out for mathematical connections in everything they read. She told me that, as a result:

> Almost every week some kid will come up and say, "Look, there's the name of a mathematician" and show me the novel they're reading. Even if they can't remember what that mathematician did, the name is there and that's really exciting for them.

She reported that the names Fibonacci, Pythagoras and Archimedes often occurred, and that the children seemed to feel that this gave them extra insight into the stories. Tara told her that "If you hadn't introduced us to all this math, we'd be missing a lot of the meaning in the stuff that we've been reading". Sarah recalled reading a story about Merlin, in which Archimedes, his pet owl, was sitting on his shoulder:

> They all looked at me and said "Now we know why that name was chosen – it's because Archimedes had all this knowledge and he was a real

thinker and an owl has wisdom and knowledge and is a thinker too". I thought it was amazing that they could see that connection.

Sarah also asked her students to read biographical information about mathematicians and encouraged them act in skits based on details of the mathematicians' life.

While discussing the connections between mathematics and the other curriculum areas, Sarah noted, "It's amazing how easy it is to tie in", and an example arose almost immediately afterwards. Each year her Social Studies lessons involve a Heritage Theme, in which the students are divided to represent four immigrant groups: the Chinese, Japanese, Ukrainian and Mennonites. She remarked, "This year, I'll find a way to do math for all of them, and tie that into the experience. If nothing else, we could learn to count in four languages". However it didn't take long before she recalled other mathematical topics related to each culture: the Chinese abacus (or any of the activities from the Time-Travel Day to China), Japanese origami and the geometrical shapes that can be made through paper-folding, the symmetrical patterns drawn on Ukrainian Easter Eggs, and the transformations used in Mennonite quilting.

Teachers also noted that students started to make their own connections between mathematics and other subjects. Sarah reported a dialogue with Tara, one of the girls who had explored conic sections during the Greek Time-Travel Day. Tara had answered Sarah's question "Why is there less daylight today than any other time of the year?" with the comment, "It probably has to do with the elliptical orbit of the planets". The nature of planetary orbits had been mentioned briefly in the students' ellipse-plotting activity, but Sarah was amazed that Tara had remembered this fact and had connected it to the amount of light on earth. When telling me about this incident, Sarah added, "as soon as Tara said 'elliptical', [the other members of the group] all grabbed their books from their desks and held up the things they'd done with the string".

A useful history/mathematics link which several teachers remarked upon is the dating system used for ancient times. Some of Sarah's students were confused by Pythagoras' birth-date being a larger number than that of his death, and this gave her the opportunity to review both the dating system and the abbreviations B.C., B.C.E. and A.D.

Other teachers made fewer connections than Ruth and Sarah, but usually pulled down a world map to indicate the origin of each mathematical object or idea they were discussing. Apart from this geographical link, and some mention of the historical period in which mathematical discoveries were made, the most popular connection was between mathematics and art. Many teachers generated art projects from their cultural work on symmetry and transformations, and this connection also worked in reverse. For several years two of the teachers in the G4 group had decorated eggs in Ukrainian style at Easter, but had given little consideration to the symmetry of the designs. After they and their students had explored the symmetrical patterns found in weavings and pottery around the world, the teachers took care to made the mathematics/art link more apparent.

Value in ethnically mixed classes

All the classes I observed had an ethnic mix of students, and my student questionnaires revealed that in some groups almost half the children spoke a language other than English in their homes. Asian backgrounds predominated, but cultural roots from many other countries were also found. Although all teachers claimed that the presence of these students increased the importance of cultural approaches to mathematics, five of the ten did little to acknowledge their students' cultural heritage during their classes.

Ruth's students made posters celebrating the diversity in their classroom: These included the words and symbols for the numbers one to ten from the sixteen different countries represented. Sarah selected six children who could count in non-English languages, and made them teachers of the other students. She commented:

> All kinds of those languages do one and ten, two and ten, or ten and one, ten and two, and the kids picked up the sound of that repeating part, so we had a great discussion about how different cultures say numbers once you go past nine, and we discovered these patterns, and then we looked at our own "teen" pattern.

By creating a class atmosphere in which the mathematics of other countries was explored and valued, teachers considered that it was easier for the new arrivals to share their own way of doing mathematics, an activity which gave them "something to feel special about".

The activities mentioned above were planned to provide opportunities for students to share their cultural knowledge, but teachers also took advantage of opportunities that presented themselves. Ruth told me:

> I never suggested to the Korean kids that they should bring in their tangrams, but that has come about as a result of starting off in a multicultural sense, and I really think if we hadn't done that, they wouldn't feel as valued as they do – or at least I think they feel valued.

She also noticed one of her recent immigrants using his Chinese number symbols in his math journal, and invited him to the board to demonstrate the symbols to the other students. She commented that "It was the first time I'd seen him smile in a math class". Later in the week, Ruth asked him to teach the class more about his way of writing numbers, and he produced two activity sheets for his classmates. The first dealt with the symbols for numbers to twenty, and the other required students to complete number sentences involving the four basic arithmetic operations. Ruth commented that this was not only good for his self-esteem, but also for the other students because "they got to see the development of a different sort of number system, and they didn't all get it right away ...but I saw a real sense of pride and accomplishment from them once they'd understood it". She also pointed out the social benefit that derived from this episode:

> It gave them a chance to see how difficult it is for a student who doesn't speak the language to learn anything. Here they were; they almost certainly had never seen the Chinese number system before, and then they had to get to do it very quickly, and learn, from a multicultural point of view, what it's like to be so overwhelmed. So that was a bit of an eye-opener.

The Chinese number symbols were the most common topic of cultural mathematics to be presented by students. Almost every class I observed included at least one student who knew them, and was prepared to share with the rest of the class. Some students also knew different arithmetic algorithms, although these were usually only minor variants upon those regularly taught. Teachers encouraged them to share these with the other students, but rarely grasped the opportunities that such algorithms offered.

Sometimes the ethnic background of a student in the class determined which cultural mathematics the teacher decided to include. Marcia chose to discuss the origin of the Hindu-Arabic numerals because she had a student from India in her class. Amit listened intently as Marcia talked about the role that her ancestors had played in the development of these numerals, and although usually a reserved child, she was very eager to answer questions about the symbols when the class reviewed them the following day.

Although cultural discrepancies can sometime cause difficulty for teachers if they are not aware of them, teachers who make the effort to understand the differences find that additional learning for themselves and their students emerges. Sarah told me about a problem that she encountered:

> We've been doing stuff with money recently and [the students] went on a shopping spree and found pictures of things they wanted to buy. And Michael kept getting his stuff wrong and he kept insisting on rounding up the number. If it was $19.99 he kept saying it was $20.00 and I said "no it's not, it's $19.99". And we went through days of frustration about that and neither one of us understanding what was wrong. But it turns out there are no pennies in New Zealand. And so, yes you can still buy something that's $19.99 but everybody just rounds it up, you have to pay $20.00. And he didn't realize that that doesn't happen in Canada, and I didn't realize that's what happens in New Zealand.

This problem was finally resolved after Sarah sent a note to Michael's mother who explained the situation to her son. He relayed the explanation to Sarah and his classmates, and this led to an interesting discussion on the cost of making money.

Games were often included as part of historical or multicultural approaches to mathematics, and Mike mentioned that he would like to "find a Korean game that Jee-Hoon would know and could take the lead in" but unfortunately he did not follow through on this idea. However, in other classes, Korean and Chinese students often took the lead when tangrams were introduced, as many of these Asian students had played with them from an early age.

Benefits to teachers

Several of the teachers with whom I worked noted that learning about the cultural background of mathematics had changed their perception of the subject, and helped them to lose their fear of it. They also noted that using cultural perspectives increased their enthusiasm for teaching mathematics, and some felt that this work gave them a better understanding of the subject. Pedagogical advantages of this approach were also noted, and the following paragraphs illustrate their views on all these issues, as revealed by their words and actions. However, it should also be noted that many of the features which they recognised in their students' changing perceptions of mathematics mirrored changes which they themselves had experienced when they first encountered historical or multicultural aspects of mathematics.

Changing perceptions of mathematics

Only three of the ten elementary teachers with whom I worked claimed to have enjoyed mathematics when young. Sarah's explanation of her dislike of the subject was typical: "I was really bad at it. When I went to school, math was all memorizing and it was page after page of boring stuff, and it never made any sense." The questionnaires completed by teachers also showed that they had learnt nothing about the cultural background of mathematics in their pre-service courses, but that they would be interested in finding out about it, both for personal interest and because they felt that this material would interest their students. The student teachers I have met in elementary classrooms in the past few years have echoed this interest, but have also told me that cultural issues had not been discussed in their mathematics education courses.

Five of the ten teachers did claim to have heard something about the origins of mathematics, but most had little knowledge beyond an acquaintance with Roman numerals. Only Sarah had studied the history of mathematics in detail. She had first encountered "snapshots of math people" in her Post Baccalaureate Diploma programme. She became so intrigued by the historical ideas being presented that she visited her local bookstore, and found *The Joy of Math* (Pappas, 1986). She recalled, "I was hooked. The minute I read it, I said 'I have to have more of these'". Since then she has purchased many of the materials discussed in Chapter 3, and has become a keen advocate of the use of cultural approaches to mathematics. Although my other research participants had attended mathematics in-service courses or workshops, cultural issues had rarely been mentioned.

The remaining five teachers echoed Marcia's candid remark: "Mathematics as a cultural subject? No, it didn't cross my mind". Maureen commented that "I always do a web about math on the first day of [the school year], but [the origins of the subject] didn't come up, even for me", and Barbara added "math just seems to be in its own little slot".

However, once they were introduced to the cultural background of mathematics, all ten teachers felt that this was an important dimension of the subject, and were enthusiastic about sharing it with their students.

Motivation

As noted earlier, the teachers I observed had different overall levels of awareness in cultural aspects of mathematics, but they all commented on how interesting they found the material they had selected to teach their students. Their interest in the material was clear in the lessons they taught, and it was quite inspiring to see how this novel approach to mathematics animated them. This was particularly apparent in the Time-Travel Day which took place on the Monday morning following the weekend that Sarah's father had died. She started the morning understandably tired and irritable but by the afternoon her usual cheerful manner had returned. In conversation after the class she admitted that "I was absolutely bagged before we started, but by the end of the day I was all hyped up".

The Time-Travel Days fascinated Sarah's student teacher. Sarah commented that "Brian is quite excited about it, and he's told his group what we're doing, and a number of them said they would like to come and be part of it too". Although he initially knew nothing about the history of mathematics, Brian became very enthusiastic, and planned to use some of Sarah's ideas in his future teaching.

Wendy commented on the stimulation provided by new concepts: "When you've been teaching for thirty years, you've got to do something different once in a while or it just gets to be dull, and I think maybe the kids are picking up on my excitement", a sentiment echoed by several other veteran teachers. Joan pointed out that "[cultural mathematics] is something you don't normally do – you get in the humdrum 'follow the book' and 'model the curriculum', and yet it's all curriculum, it's just a different way of looking at it". She added that "I know my history background isn't strong, it's never really been an interest of mine, but math is, so ... I'm beginning to think that I don't dislike history quite so much as I did".

Increased understanding of mathematics

Teachers acknowledged that historical information sometimes helped them understand why the modern "rules" worked. The following example is taken from my journal, and describes a conversation which took place in my first interview with Wendy (an explanation of the Chinese counting rods is given in Appendix G).

> The discussion turned to operations with integers, and Wendy mentioned that her text book used red and black checkers to explain addition and subtraction. I commented that this approach was similar to that used by the Chinese two thousand years ago, and after expressing surprise she said that it would be much nicer to use an authentic method, rather than something invented for school children. However, as soon as I started to explain how the Chinese would have dealt with $(^{+}3) - (^{-}2)$, she interrupted with "Well here, what you do is you change that to an add and change that sign". I agreed that that was the rule, but when I asked her how she explained it to her students she said that she just talked a bit about "losses and gains on a football field, and then you basically give them the rule". I showed her how a group of Grade 6 students had discovered the rule for themselves during a lesson on Chinese counting rods. At first she looked a little puzzled and asked me to explain it again, but the second time round, she suddenly interrupted with an excited "OK, now I understand – you have to add the red ones before you can take the black ones away", and went on to say that "it would have been much easier to teach it in that [hands-on] way." Clearly the context of the Chinese rods had captured her imagination, and made sense of the operations far more effectively than the checkers drawn in the textbook, even though the two methods are mathematically equivalent.

By the end of this conversation I felt sure that Wendy had not only understood, perhaps for the first time, why the "two minuses make a plus" rule worked when subtracting negative integers, but that she also had a method she could use with her students to help them understand.

Two years later I mentioned the Chinese use of positive and negative numbers to Sarah, who immediately said that her Grade 3 students would enjoy working with the red and black rods. To discover what she already knew, I asked her to calculate $(^{+}3) - (^{-}2)$. She looked puzzled, and was unable to tell me the result, admitting, "I have absolutely no recollection of ever have done anything like that before, but I guess I must have learnt some rule in high school". I led her through a few examples using coloured strips of paper and within five minutes she was able to use them to perform any integer addition or subtraction. She noted the cultural discrepancy between the ancient Chinese adoption of red for positive numbers and the modern consternation at being "in the red", but rationalized that "red is lucky for the Chinese, so obviously the red rods will be the positive numbers". After her class she admitted that she hadn't been quite sure how to explain that adding a positive one to a negative one made zero, but told me, "I just listened to the kids' words, and when Alan said 'the black rod covers up the red one', I knew that was a phrase the kids could understand". With Alan's explanation in mind, the students had no difficulty adding integers, and once they had grasped the idea of "adding nothings" (a pair of rods, one red and one black) they were also able to manage subtraction. Sarah was delighted with the success of this lesson, and planned to ask the Grade 7 teacher's permission to demonstrate the method to the older students, showing a great confidence in her own understanding of a topic she had known for less than twenty-four hours.

Sometimes the insights gained related more to the cultural mathematics being studied than to its modern academic counterpart, but comparison with modern methods reinforced understanding of the latter for both students and teachers. Mike's discussion of Lattice multiplication (quoted on pages 111 and 112) exemplifies this, particularly his final comment that "because the lines are drawn at an angle it automatically moves it over one spot". Discussing this algorithm before the class, Mike had told me "I know why that works – that's where you carry", but his later comment suggested a clearer understanding of the place-value aspect of the calculation.

Teachers' understanding of the modern method of multiplication also seemed to be increased by study of the Egyptian process, which required only repeated doubling followed by an addition (see Appendix G). At first, the ancient algorithm seemed like a magic trick to the elementary teachers to whom I showed it, but with a little help they were able to see how the distributive property of numbers was being used. They were then able to relate this property to the regular method of multiplication, in which a multiplication by 29 involves adding the ninth and twentieth multiples of a number. Although no one said so explicitly, I had the impression that this was the first time that they had actually realized what they were doing in the regular school algorithm. When I had previously asked them to justify the method of multiplication they usually taught, their explanations were procedural: "You put a zero down there because the 2 is in the tens place" was the highest level of conceptual knowledge demonstrated.

The new material sometimes included mathematical terms which were unfamiliar to the teachers. A particularly interesting cultural example is that of the word "trapezium". On encountering this in an activity taken from an Australian text, Barbara had consulted her class dictionary (Barnhart & Barnhart, 1976), and then explained to her students, "A trapezium is a quadrilateral having no two sides parallel". This was clearly not the definition that the activity's designer had in mind, so together we explored the English and American definitions of "trapezium" and "trapezoid", and discovered to our surprise that they are exact opposites. This led me to explain the meaning of "billion" on either side of the Atlantic, and Barbara eagerly shared these facts with her students as examples of the cultural differences that occur in mathematics.

New teaching methods

Teachers discussed several pedagogical issues that arose from their use of culturally oriented mathematics. The most obvious difference between these classes and their regular mathematics sessions was the presence of material focussing on the role that individuals or cultural groups had played in the development of mathematics. However, several teachers noted differences in the way they organized their lessons, and commented on some additional assessment opportunities that arose during the cultural work. They also discussed the impact that teaching Multicultural Math had upon their regular teaching.

Humanistic focus of lessons

All the teachers considered that showing the human involvement in the development of mathematics helped to make the subject more interesting because "students like learning about other people and other times". One approach was to refer to the people in other civilizations. Rather than telling her students they would be learning "the Egyptian method of multiplication", Marcia said, "The Egyptians did their times-ing simply by adding, so they didn't need to memorise their times tables for ever like you do now". Referring to the artefacts created by other cultures was another humanizing technique. Joan noted, "You think of Navaho blankets and all that ... and you could spend some time on it, and build up a collection of that stuff", adding, "I'm sure [the students] can see stuff like that in their own homes". She contrasted this approach with her previous teaching of symmetry, in which she merely used the figures in the textbook. Ruth's "Multicultural Objects" classes (described on page 77) carried Joan's idea of artefacts in the home one step further. Her students brought objects related to their ethnic background to show to their classmates, and these provided the basis for some thoughtful mathematical discussion.

Culturally oriented lessons often included humanizing elements of imagination and role-playing which were rarely present in the regular mathematics classes. Sarah's concept of Time-Travel is the most obvious example, and Ruth adopted this approach for her explorations of Greek geometry and Egyptian measurement. However, Sarah also noted a more specific variant from her usual teaching style:

> I've never been one to do much in the way of drama or plays, it's just not my thing, but it's been interesting for me to see what kids can do with a script. They're not just reading it, they're really attempting to act it out, and there's facial expressions happening and so on. And the kids watching are absolutely enthralled – there's not a sound in the room while they watch.

Organization of lessons

Although most of the elementary teachers I observed usually relied heavily on the material in the standard student text, they all commented that it was good to have a change from the regular textbook. Maureen, who regularly used the *Quest 2000* text (Wortzmann et al., 1997), observed that "If there's no text book, it's like a different setting for [the students]". She commented that her teaching style in the Multicultural Math lessons differed from her regular approach "because I'm not totally confident with teaching it". She noted, rather apologetically, that "[In the multicultural math sessions] I don't spend as much time up there, doing the pre-teaching and then getting them to do exercises. ... I get them to try things a lot more and I don't give them the reinforcement, I just pick different activities".

This lack of direct teaching was common in the cultural mathematics lessons I observed. Only Wendy maintained her transmission style of teaching, but even she encouraged the students to discuss the material she presented more than in her regular classes. With the exception of Maureen, teachers seemed positive about this change in their teaching style. Mike, who often used *Journeys in Mathematics* (Connelly et al., 1987), noted that his Multicultural Math lessons were "more open-ended, and the activities are not so much trying to teach them things, but trying to get them to investigate things. I have them question more, rather than trying to absorb information". However, like most of his colleagues, he still considered drill to be very important "because [students] have to know all those facts", and he used transmission methods to teach algorithms, noting that "borrowing in subtraction, it's just something where you show them how to do it, and then you can get on with the other stuff". Drill played no part in the cultural approaches to mathematics though. Instead lessons focussed on "brain-storming and 'try your own discovery' type of thing". Mike considered that both styles of learning were important, and that "each has its place".

The culturally oriented mathematics lessons often required the students to work with materials used in other times or places. Sarah's Time-Travel Days contained the greatest concentration of such activities, but all teachers who used cultural approaches to mathematics included a hands-on component. Several teachers shared Joan's view that "It's the hands-on stuff that appeals to kids", although my classroom observations suggested that the students were equally intrigued by written pursuits such as those involved in the "Magic Squares" or "Number Triangles" activities. However, when the students wrote about their Multicultural Math sessions in their math journals, the hands-on activities were mentioned most frequently, and they seemed pleased to have something tangible to display in the classroom. Teachers noted that these constructs and the frequent use of pictures in the multicultural mathematics materials also helped them to establish links with Social Studies.

Joan commented that her culturally based lessons "went off on different tangents from what I thought they were going to go. Because they're broad topics, they can go anywhere". Several other teachers shared this view, but I heard no complaints about this. On the contrary, teachers felt that such lessons were "more fun" than the regular ones, for themselves as well as for the students. Joan went on to note the value of these diversions:

> [At the start of the year] I found these kids were very hesitant to go off the beaten trail, ... they wouldn't even try anything that wasn't just like something they'd done before, so these kinds of things are good, because they get to try different ways of getting to the same place. Usually, if you ask them to explain their answers they have a tough time, whereas this

> way, because they see different ways of doing it, and different kids taking a risk, it's working. So I like it, I find they're opening their minds a little bit more.

Teachers enjoyed being able to call upon their students to make contributions from their own background. This was particularly true for the children who had recently arrived from other countries, but most children had some cultural knowledge to share, based upon information from their parents or grandparents. Teachers felt that this took some of the pressure off them, echoing Mike's comment that "It's nice not to have to be 'the expert' all the time".

Marcia combined the interest of a new way of doing mathematics with the thrill of competition. She commented, "I would take the Indian way of multiplying and the North American way, and I'd have races - Which can you do faster?". She also assigned the students the homework task of teaching the new material to their parents, and noted that "They really enjoyed teaching their parents something that they knew their parents would not know how to do, and I thought it added a really nice dimension to my teaching". Other teachers also employed this learning-through-teaching technique and commented on its popularity.

Assessment opportunities

The discovery nature of many of the cultural mathematics classes often provided teachers with unexpected assessment opportunities, as the novelty of the work took away the students' preconceptions as to what they "should" be doing. Sarah noticed that when her Grade 3 students were calculating the sum of the terms of the Fibonacci sequence "Some of the kids would start fresh each time, ...but kids like Amanda would just add on to what they'd already found. It's the equivalent of kids that go back and start counting the beans all over each time". Although all her students had moved past this level in traditional arithmetic exercises, the concept was clearly not sufficiently grasped to allow them to adapt it to other situations. Similarly, when she asked her students to construct a square pyramid using small cubes, only two of the five groups started by laying out a square base, even though the class discussion immediately prior to this activity had seemed to suggest that they all had a good understanding of the properties of this three-dimensional object.

Not all teachers used "math journals" on a regular basis, but most asked their students to write something about their cultural mathematics classes. The students' comments often focussed on the hands-on activities, with comments such as "Measuring with ropes was really neat" and "I enjoyed writing Babylonian numbers on the clay" showing an approval of this practical side of this work. However, there were also more thoughtful comments, such as "It's important to find out how things were happening years ago because we might find new ways to use things", and teachers felt that these statements suggested that the students were "making connections between the culture and the math", and "thinking about the origins of mathematics".

Although most teachers introduced the same material to all the students in their class, Sarah sometimes assigned each group of students a different mathematician to study, and asked them to report on their investigation at the end of the day. She felt that these presentations gave her insight into the students' grasp of the mathematics they had studied, in addition to allowing extra assessment opportunities for their speaking skills. Some of the students' descriptions generated so much interest that Sarah agreed to let the whole class try them. Referring to one such case, Sarah said, "I'm going to use those kids to be my little teacher helpers. I can partner them up with the other kids ... I've got six willing teachers who are just wild about Hypatia". This teaching experience was not only valuable for the students, but also provided Sarah with further informal assessment opportunities.

Impact on regular mathematics teaching

Ruth commented, "One of the things that I am trying to get better at is my introductions to my lessons, and it's here that I think the multicultural stuff is incredibly useful because it gives a depth to something". By way of justifying her view, she added, "Coincidentally in my readings of van Hiele this weekend, he talks about the importance of context". Ruth and Sarah integrated cultural aspects of mathematics into their regular curriculum work whenever possible. Sarah commented that:

> I really like to integrate things but I'd never been able to integrate math and that's always bugged me. You could integrate measurement a little bit, and money a little bit, but how do you integrate the other stuff? And it never crossed my mind to integrate it with art and literature and all that stuff. But the pieces are all falling together now.

However, other teachers seemed to set the cultural mathematics apart by including "Multicultural Math" as a item on the day's schedule, often in addition to "Math". This led me to ask these teachers if they considered the two as separate topics. The teachers denied this, commenting that they knew that "some of the concepts carry over", but admitted that they sometimes did not make the students aware of these connections. Maureen noted that "Next year it may be easier. When you teach something a second time, it becomes a lot easier to make connections".

Joan occasionally revealed a rather paradoxical view of multicultural mathematics. In our discussion after her class on games from other countries, she remarked that she had actually cut short the spontaneous discussion on the probability of having four sticks land in a particular orientation because she had felt that she was "getting off track". Although she claimed to appreciate the relationship between "multicultural math" and "regular math", in this case she acted as though she thought they should be taught separately. However, she decided to do a follow-up lesson in which throwing the Egyptian dice sticks would provide a novel probability experiment, commenting that it would be a useful activity because "Probability's a unit we can offer, but most teachers end up leaving it out because they put it at the end of the year and then run out of time. Doing it now [students] can hear the terminology and they get to try some of it."

The teachers who specifically identified their Multicultural Math sessions seemed uncertain as to whether the students thought of this work as "real" mathematics. Maureen considered that "They knew it was math but it was a different type of math, a lot more creative for some of them", adding that another difference was that "They didn't have to complete something by the end of class, and it wasn't something they had to take home and worry about what their mark was going to be".

Mike, Joan and Barbara used the *Interactions* textbook (Hope & Small, 1994) for at least part of the time, and noted that the discovery approaches they were using in their Multicultural Math classes were similar to those required by the activities in this text. Barbara commented that "Doing this [multicultural math] stuff is making me more confident using *Interactions*", and Mike felt that the sorts of questions suggested by the *Teachers' Notes* from *MMC* (Irons & Burnett, 1995) were changing his focus in his regular classes. He noted that "I'm doing a lot more of that 'What do you think is going to happen?', 'Why do you think that?', 'What are some other ways you could look at this?', so it's great for the problem-solving focus". The success of the more open-ended approach he used in his Multicultural Math sessions led him to tell me, "I should do it more often. The kids are more involved, they're thinking in more different ways, and they're not so passive". He also noted that, whereas in his regular teaching he tended to push ahead, in an "I've got to teach this today" mode, in the Multicultural Math sessions, he slowed down and gave his students time to explore. Like the other teachers in the G4 group, he planned to integrate much of the new material into his regular classes the following year, although these intentions were not followed through.

Several teachers commented on the fact that some of the material they met in the cultural mathematics texts was new to them. This let them appreciate mathematics from a student's perspective again, and Barbara noted "it's a good reminder to know what it's like to be a child who has difficulty – it's humbling".

Situating teachers' views in a framework for mathematics education

The views discussed on the previous pages have been summarised in a Venn diagram (see Figure 9), using the framework constructed to display the expert opinions reviewed in Chapter 2 (page 30). To facilitate comparison, the previous text has been kept when the teachers' views were similar to those in the literature, and expert opinions indirectly related to the teachers' views have retained, but written in italic script. Points made by teachers which were not specifically mentioned in the earlier chapter have been included in bold type, a feature which highlights one obvious difference between the two diagrams, namely the teachers' focus on the nature of teaching. Expert opinions on other issues are conspicuous only by their absence. A comparison of these two sets of opinions on the role of cultural mathematics is made in Chapter 7.

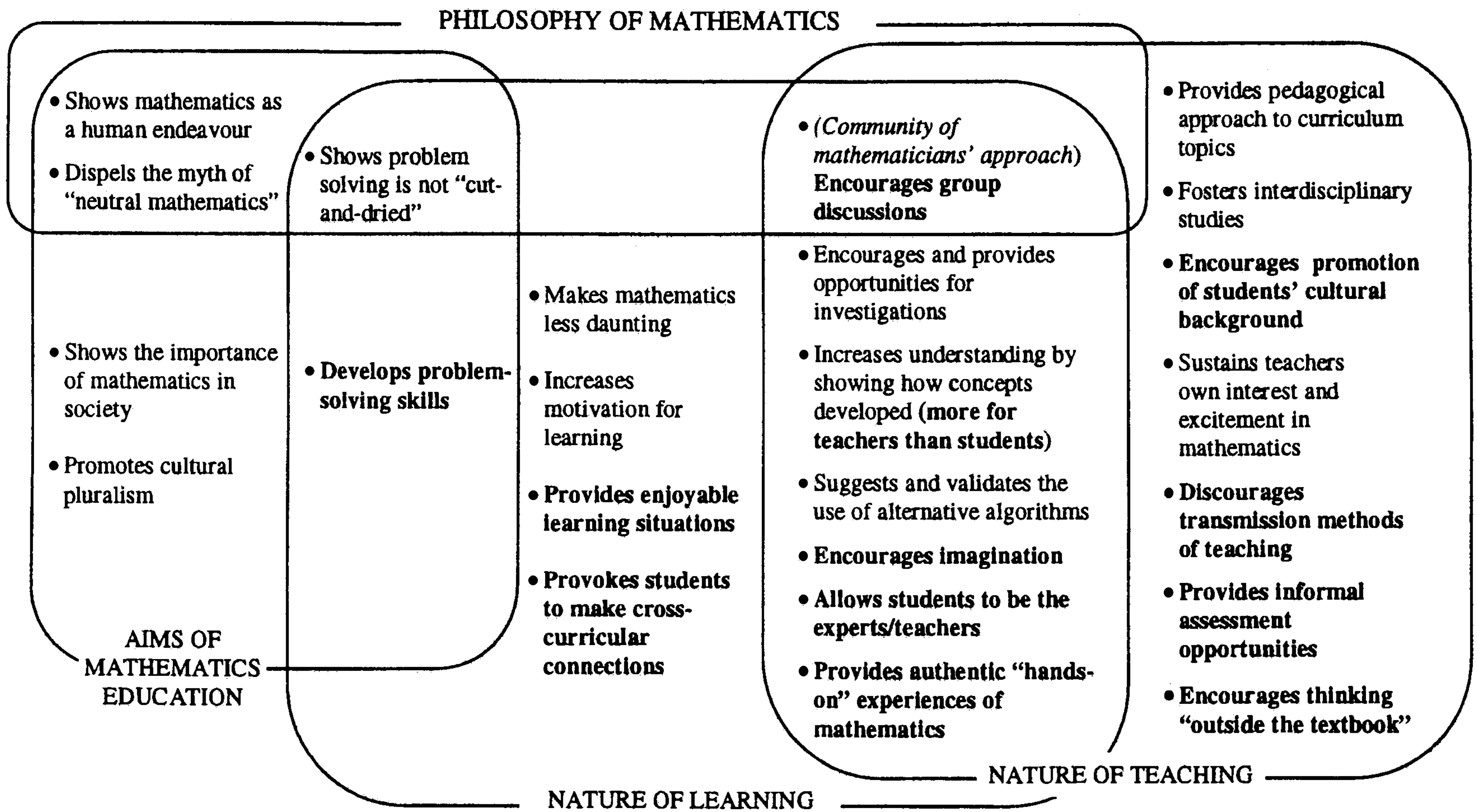


Fig. 9: Teachers' views on the role of cultural perspectives in mathematics education

Implementation issues

Although teachers appreciated the benefits of introducing a cultural dimension to their mathematics teaching, they noted several practical problems. These included the time commitment involved in teaching new material, its relevance to the IRP, and the availability of classroom resources. This section reviews these concerns and also assesses the amount of guidance which teachers need in implementing cultural approaches to mathematics. It closes by noting teachers' views on their long-term commitment to such approaches.

Time commitment

There are two major issues of time commitment: the preparation time teachers need to familiarize themselves with the material and plan their lessons, and the classroom time spent introducing this material to their students. Although the second of these can be seen as a philosophical decision, based upon the view of mathematics that a teacher wishes to instil, it is clearly influenced by the first point.

Time needed for lesson planning

A cultural approach to mathematics often involves new concepts, requiring teachers to become students again in order to understand and select activities relevant to the information they wish to share with their students. These tasks obviously require extra expenditure of time and effort, but the amount of time spent planning lessons varied considerably among the teachers with whom I worked. For Sarah, discovering how mathematics was done in the past soon became a very time-consuming hobby, and she spent many hours researching the information and constructing hands-on materials. She commented that "I'm learning as much as the kids are 'cause I have to do so much more reading and be sure I really understand things". She constructed clearly defined lesson plans, but was also willing to follow any unexpected direction that arose during her classes. Although she did not usually have time to work through all the activities she selected, she considered them carefully to be sure they were appropriate for her students. When her class contained particularly gifted children, she would give them problems she did not know how to solve, confident that my presence in the classroom would ensure that the children were not left without help.

Ruth spent much less time than Sarah in class preparation as she had many other commitments during the period she worked with me. She often asked me to provide material for her to look through, but whereas Sarah's planning was done days or weeks ahead, Ruth was more likely to make lesson decisions at the last minute, which gave her lessons a serendipitous quality. Discussing her use of the book *One Grain of Rice*, Ruth commented, "As I was starting to look at things this morning, I thought 'yes, I'll pull that book out, just in case'": Her knowledge of the role that India played in the development of our present number system allowed her to capitalize upon resources already available in her room. However, this spontaneous approach to lessons occasionally led her to go deeper into a topic than was perhaps appropriate for the majority of the students in her class.

Despite their enthusiasm, both these highly motivated teachers acknowledged that planning for such lessons requires more effort than teaching from a textbook. Sarah noted that "It's a lot of work in terms of researching and digging stuff up and finding resources". Ruth remarked, "Sometimes I don't feel in a creative mood, and I feel 'Oh, we'll just to do basic stuff'", but added that "Sometimes something will surprise you, and I just like to bring in the multicultural stuff. It makes it interesting for me, but I don't want to feel that I have to be creative all the time."

Although few teachers had the high level of interest in cultural aspects of mathematics displayed by Ruth and Sarah, most said that they enjoyed reading the historical or multicultural material. However, the G4 group admitted that their preparation was usually limited to the appropriate chapter of the *Teachers' Notes* for *MMC*, often the only resource they used. I felt that these texts sometimes gave the teachers a false sense of security. When I met the G4 group a few days before each of their Multicultural Math sessions, they had rarely looked at the material in detail, and when I asked if I could see their lesson plans, Joan joked, "Some of us might need to fax it to you!" and Barbara added "That's right, it's the first time I've seen this".

Maureen discovered the need for more preparation the hard way. Her first session was on "Thinking Games", and she commented:

> I brought [the book] home and said to my husband, 'We have to play these games this weekend', and then [the book] sat there, and I looked over it last night, and I kind of thought 'Oh, yes, I understand', but when the kids started to ask questions, I realized I didn't. So I learnt my lesson. I think I need to give myself a bit more time, and for the next one I'll probably plan now, and then go back to it again just before I teach it.

However she added, "but it's difficult fitting it in", and the other G4 teachers all nodded agreement. She went on to contrast the cultural mathematics material with the regular curriculum:

> When I teach things now, I know the content so well, and I remember what happened last year and the year before, and what worked and what didn't, and if a problem does come up that I forget about, I'm OK with it because I know the content so well. So I thought I could do the same with this.

Joan specified that it took her "maybe an hour or two" to read through the text and decide what to do, but noted that she was fortunate in having a teacher's aide who would then photocopy material. She said that she listed the main ideas she wanted to include, but shared the experience of many of the teachers that "the kids really get into these lessons, so that they take them over". In discussing the amount of time required to plan lessons, she pointed out how lucky her school was to have the *MMC* packs, and claimed that without them she would have had to spend "a whole lot of time on the Internet".

Class time allocated

The amount of class time devoted to cultural approaches to mathematics also varied considerably, and was influenced by two factors: the level of importance which teachers attached to this humanistic way of looking at mathematics, and the pressure they felt to comply with the prescribed learning outcomes of the IRP. Ruth and Sarah were the most enthusiastic teachers, and as they were aware of the cultural aspects of many of the curriculum topics they introduced these as appropriate, in addition to having specific days in which the main focus was on the cultural perspective. Among the teachers for whom cultural mathematics was a new idea, the material was either taught in a specific time-slot each week or month, or a week was set aside during which all the mathematics sessions were devoted to exploring either the cultural connections of a specific mathematical topic or the mathematics from a specific culture.

The teachers in the G4 group initially scheduled Multicultural Math as a series of independent monthly sessions, but they had difficulty fitting in all the material they had prepared. Joan commented:

> I pretty much followed the lesson in that package. I find more than enough for this [Grade 4] level ... but it's taking me longer than I think because you can easily go off on all sorts of tangents.

Her sentiment was echoed by many other teachers, but their solutions to the time problem varied. At first Joan, Maureen and Mike kept to their initial plan of spending only one hour-long session per month on Multicultural Math, and accepted that they would not cover all their material. However, as the year progressed, they allowed the material to extend over the mathematics periods of two or three consecutive days, as the enthusiasm generated by the work convinced them that this was worth spending time on. Barbara decided immediately after her first, rather unsuccessful, lesson that she wanted to spend more time on the material, and continued to work on it for several days, a pattern which she maintained for each of her five topics.

Wendy decided to spend one session each week on Multicultural Math, and specific topics often stretched over several weeks. Although lessons were occasionally missed for assemblies and other school events, she kept to this schedule from October to March.

Three teachers took the alternative approach of spending all the scheduled mathematics time for a week on a particular historical or multicultural topic. As Ken rationalized:

> We teach math almost every day, and there's about two hundred teaching days in a year, so if you have seven or even ten days to a unit, that's only one twentieth, five percent of your time. Yes, I could easily spend that much of my time on historical math.

He chose to introduce historical mathematics to complement his social studies unit on Egypt. However, the other two teachers made no specific connection between cultural mathematics and other curricular activities, and this lack of purpose led them to repeatedly delay their implementation of this new approach. Marcia had planned to use cultural ideas early in the school year, yet it was March before a suitable opportunity arose, as events such as field trips, extra assemblies, and the presence of a student teacher kept pushing the promised activities later and later. Ingrid discussed the scheduling of her "Multicultural Math week" with me at Christmas, and suggested that early March would be an opportune time because "they have the Canadian tests of basic skills then, which sort of wipes their brains out, and it would be kind of nice to do some totally different kind of math." When this fell through, the project was postponed until the end of the year. She explained, "By then we've made it through the curriculum, that's often the time when I experiment, just have some fun and do all sorts of different things". Although she seemed fascinated by the various historical tidbits I related to her, she clearly felt that her duty lay in teaching to the IRP, and that the cultural material was a rather frivolous "extra", to be slotted in only if time permitted. There were some hints of this attitude in other teachers' remarks: Joan commented that she'd use some of the multicultural games on Valentine's Day, as "[the students] play games in the afternoon, so they see that as a treat".

Relevance of cultural mathematics to the curriculum

The curriculum guidelines for elementary school students in British Columbia are given in the *Mathematics K to 7 Integrated Research Package* (British Columbia Ministry of Education, 1995). Mention of this text, commonly known as "the IRP", brought out different reactions in teachers. Only Ruth and Sarah had read the whole document, and Sarah commented:

> I read all the introductory stuff that comes before the actual 'teach them how to add, and...', and there's that little section, that I'm sure everybody ignores, on the history of mathematics and how it evolved, and I thought, 'Of course, we try to teach kids social studies and social skills and the understanding of the world through history, but we don't do that through math' and I thought, 'Hey, if it makes the other stuff I do exciting, it's got to make the math exciting too', and it has!

Both these teachers felt that they had an obligation to introduce cultural material. However, whereas Sarah was prepared to do so regardless of its explicit connection to the Learning Outcomes for her grade, Ruth said apologetically that "We have some formal responsibilities [to the Learning Outcomes], so the cultural emphasis will be there, but I don't think it will always be as strong [as in the lessons which focus on this aspect]".

Nevertheless, she was convinced that any topic could be "tweaked" to show its cultural background.

Most teachers admitted that the only parts of the IRP they looked at were the Learning Outcomes for the grade they taught. For them, the question of the relevance of cultural material came down to how easily they could relate it to the particular outcomes they were expected to cover. Barbara commented that, "If we had the freedom of the Primary curriculum, like Sarah has, [the cultural approach] would be more fun and more beneficial", but Joan disagreed with this remark, noting, "But this material is curriculum-based – look at the history behind some of the lessons we teach anyway", and Brian pointed out, "The nice thing about our IRPs is that they are so general. They don't go into the specifics of what teachers should do, so they're open to interpretation". Consequently, teachers were usually able to forge good connections between the cultural material they were using and the Learning Outcomes for their grade.

They approached this link in a variety of ways. Those who used historical approaches to mathematics usually decided which particular period of history they wished to explore and found material about mathematics or mathematicians from that era. Sarah claimed that "just about everything we've done I can relate directly to some part of the IRPs", and her lesson plans indicated these connections. However, other teachers were less explicit about these links, and even when the relationships were clear in their minds, the teachers did not always make the students aware of how the historical work connected to the mathematics they were required to cover.

Other teachers took the IRP as their starting point, and looked for culturally oriented mathematics activities relating to a specific Learning Outcomes for the grade level they taught. In such cases, those teachers who had little prior experience in cultural approaches to mathematics usually asked me for suggestions, rather than search through a variety of recommended resources for themselves.

A third group just wanted to "do something along cultural lines", and these teachers were happy to browse through the books I lent them until something caught their eye. Ingrid, a Grade 5 teacher, looked through one of the *MMC* packs and commented "this stuff with slides and turns, I have to teach that because of the IRPs, and so that just leapt out at me". Marcia, who taught a Grade 4/5 class, noted that "Anything you can do together when you're teaching a split is great, and anything to do with operations or patterns, I jump on wholeheartedly".

Some teachers seemed to feel that they had to justify a cultural approach to mathematics. Before starting to use the multicultural materials, Ruth had reassured her class they would still be doing "regular math ... we're just going to look at if from a different angle". Such comments were sometimes made as a result of questions from parents as to the relevance of the work, but in general, teachers merely wanted the students to feel that this work was as important as that which came from their text books.

Topics from all five strands of the IRP were addressed in cultural ways: A summary is given in Appendix F, but a few examples are mentioned here for illustration. Students from Grades 3 to 7 were introduced to the Egyptian number system and a comparison of this with the modern system not only reinforced place value concepts, but also made the regrouping processes for addition and subtraction more transparent. Students in Grades 6 and 7 explored the Egyptian method of multiplication and division, which sometimes led to discussions of the distributive property of multiplication over addition, useful preparation for future work in algebra. Both the Problem Solving and Pattern strands were addressed in many of the activities in the *Historical Connections* books. Several teachers used examples of the tessellations found in Islamic art to illustrate transformations to students from Grades 3 to 7, and the body-based units of length used in earlier times provided excellent examples of "non-standard units" for the Measurement strand. Investigations based on some of these measuring units provided data for statistical displays, and the Probability strand was addressed in a lesson on throwing games, a topic in the third *MMC* pack.

The focus on historical and multicultural approaches to mathematics sometimes took the students to levels of mathematics which were more advanced than required by the IRP. This can have a very positive influence, as when a group of Grade 3 students were led to discover the square root concept through an activity concerned with number patterns. They delighted in "that hooky thing" (named after the calculator key), and were able to explain the connection between the new idea and that of squaring a number, an operation which they already knew.

On the other hand, I sometimes felt that the historical approach took the students out of their depth. The fractions Archimedes determined as bounds for the ratio of the circumference of a circle to its diameter go substantially beyond the set of fractions mentioned in the IRP for the Grade 4 level, and while some members of Ruth's Grade 4/5 class looked confident when the numbers 1/7 and 10/71 were written on the board, I felt that others could have benefited from some discussion of their relative size. While I agree that taking students beyond the Learning Outcomes for their grade certainly has value, particularly for those at the top end of the academic spectrum, the expressions on some students' faces showed that this has to be handled with care.

Resource Materials

The availability of good resources is obviously crucial to the success of an exploration of the cultural background of mathematics. Ruth noted that:

> If you can get your hands on the resources with which to do it, so that you don't need to have a whole bunch of extra knowledge of these things, then it's easy to do. ... It allows me to do multicultural stuff with [the students] with relative ease, without having to be independently creative all the time.

This section details how teachers found such resources, and the ease with which they were able to use them.

Locating resources

The first phrase of Ruth's comment highlights the major difficulty involved in including cultural perspectives into the mathematics classroom. Although the Resources section of the Mathematics IRP lists two useful sets of culturally oriented publications, very few teachers read the entire IRP, and most react with surprise when I mention that it includes references for books on historical and multicultural mathematics. The teachers who owned their own historical or multicultural mathematics texts had discovered these independently of the IRP. Sarah first explored such texts as a result of her Post Baccalaureate Diploma course, whereas Ruth started her collection with books discovered at Mathematics education conferences. In general; personal recommendation seemed to be the most successful route to resources. Although many ideas for cultural approaches to mathematics are given in the journals produced by mathematics associations, teachers reported that they rarely read such journals. Similarly, the Internet is less useful than might be imagined. Although it includes some cultural material suitable for elementary mathematics classes, many elementary teachers reported that they "wouldn't know where to start looking".

The G4 teachers commented on how fortunate they were to have the complete set of *MMC* packs in the school. These packs are usually very expensive, and several teachers who enjoyed using my copies regretted the lack of funding to purchase their own. By chance, Sarah had been able to buy them at a very reduced rate, and the Intermediate teachers at her school were able to borrow her copies. Joan commented "I think it's nice that we have these lovely new materials. They are real motivators." On rare occasions, this material was augmented by other books borrowed from Sarah, or by information from the Internet.

I felt that a breakthrough occurred when I showed Zaslavsky's book *Multicultural Math* to Barbara, causing her to exclaim excitedly "Oh, it's a Scholastic book". The possibility of ordering materials for cultural mathematics from the catalogues regularly sent to schools increases the likelihood that teachers will become aware of them, and both the accessibility and the relatively low cost makes them attractive. Four of the teachers working with me had bought a few cultural math texts, but only Ruth and Sarah owned extensive libraries of such material.

Most of the other teachers who explored cultural approaches to mathematics relied upon whatever texts I lent them, chosen to relate to the IRP strand they wished to teach. However, on several occasions the students introduced their teachers to material on the history of mathematics which they had discovered in their school library. This was rarely in books specifically labelled as such, yet texts such as *Mathemagic* (1978), *How to Count like a Martian* (St. John, 1975) and a *Mathematics Encyclopaedia* (Foster, 1985) include much information about the development of mathematics which fascinated these young readers.

Except for Sarah, the teachers involved in my research based their regular teaching on the students' mathematics textbook. Some of these books include a little cultural information, but teachers' reactions to these sections varied. Ruth commented that "[the *Quest 2000* texts] put those little headings there, 'People, Society and

Mathematics', but I wish there was more of that, because you'll see the odd thing, but they don't really do anything with it." She used these pages as starting points, and expanded upon the material given. Other teachers admitted that they used to "just pass over to the next page", but claimed that "Now I'll start to use those things".

Several teachers had purchased Multicultural Mathematics posters, and were willing to share these with others. They felt that posters were valuable, not only to create discussion points during lessons, but also for students to look at in their free time. Marcia commented "They give the whole flavour: it's like eating Mexican food on a Mexican plate, it just makes things taste better".

Ruth and Sarah borrowed my copy of the *Platonic Solids* video, and Sarah's students were delighted by *Donald in Mathmagic Land* which she had obtained from her district Resource Centre. Sarah also used several social studies videos to give her students an overview of the civilizations to which they were time-travelling, some of which mentioned the mathematics of the period, and Wendy frequently used the *Voyage of the Mimi* video as enrichment for her Mayan unit.

User-Friendliness

In general, teachers considered the published materials easy to use. The *MMC* series was the most popular resource (Irons & Burnett, 1995), and Joan commented that "All the information is in here, and I just have to learn it". Teachers using these packs often focussed their class discussions on the questions asked in the *Big Book.* These refer to pictures shown there, and teachers' fears about leading a discussion on new material were allayed by the provision of answers to these questions in the *Teachers' Notes*. The use of the *Big Book* worked particularly well in Marcia's classroom, as she and the children gathered together on the carpeted area of the room, which allowed all the students to see the book clearly. However, when other teachers used this book, they usually held it at the front of the class, which made it difficult for the students at the back of the room to see the pictures to which the questions related. Barbara often asked a student to carry the book around the class so that everyone could have a close look at the pictures, but this sometimes led to unruly behaviour from the students to whom the book was not being shown. Ingrid commented that a set of transparencies for the overhead projector would have made this book considerably more user-friendly. Nevertheless, Ruth said that "When I'm doing something I want to show the kids, those *Big Books* are perfect".

In general the teachers appreciated the background information included in the *Notes*, but sometimes felt that more was needed. After her first lesson ("Thinking Games" from pack 4) Maureen complained that some of the directions weren't good enough, although she admitted:

> I guess I wasn't quite where I thought I was, and I had [the students] start playing and when they asked questions I realized that. I'd say that the book needs to maybe clarify it a bit more for the children, and maybe it would be helpful if they gave a couple of strategies that they could try.

Joan agreed with her, commenting "That's similar to [my experience] with the 45 on the magic square. That wasn't in that book at all, I got that from Irene". They admitted that their difficulties arose from the fact that they had not taken the time to try out and reflect upon the activities first, but as noted earlier, time commitment becomes an important issue in implementing any new approach to mathematics, particularly when the teachers' mathematical background is rather weak.

The blackline masters included in the *Notes* were very popular. However, in some cases I felt that too much importance was attached to these hand-outs. These merely became the new textbook, with students working through the exercises without any consideration of the cultural aspects of the material. Class discussions after these activities usually focussed on the new mathematical ideas that the students had discovered, and while this, and the problem-solving that it had entailed, are important activities in their own right, reinforcing the cultural links would have made the exercise even more valuable.

Two other texts also proved very popular. *Multicultural Math: Hands on Math Activities from around the World* (Zaslavsky, 1994) was used by those teachers who did not have access to the *MMC* series, and sometimes to provide additional activities by those who did. Teachers found Zaslavsky's choice of material interesting, and liked her clear identification of the curriculum content, as this allowed them to find material to enrich a particular topic very easily. Those teachers taking an historical approach to mathematics liked the *Historical Connections* series, although they found some of the activities rather challenging. Sarah wrote in her journal "I haven't decided if it is best to know the solutions to these pages or discover it along with the kids", but admitted that it was usually time that determined which route she chose.

These elementary texts occasionally confused the reader in their desire to either simplify the material or to make a point. Wendy used Zaslavsky's *Multicultural Math* text for her work on Egyptian numbers, and was worried by the apparent contradiction between two phrases in the same paragraph: "The position of the symbols is not important" and "The smallest value goes on the left" (p. 24). When I looked at the text I realized that these characteristics were listed to contrast with those of the "Indo-Arabic" numerals ("has a place value notation" and "the smallest value goes on the right"), but this was not obvious to a teacher unfamiliar with the material.

Although most of the published texts of historical or multicultural mathematics activities give some indication of the grade level for which they are appropriate, this was usually rather broad, and teachers sometimes found it difficult to judge which material best suited their class. Initially Barbara tended to underestimate the ability of many of her students. After her first class, described in Chapter 5, she apologized that her students were not concentrating, and reflected that this could have been because the pace she set was too slow. I felt that the problem lay in the rather open-ended instructions she was giving the students, such as "I'd like you to look at this triangle, talk about it with your partner, and try to find out how this triangle is constructed." Being unfamiliar with the material herself, she did not realize that some students would find the triangle's structure very obvious, so while she helped those who had not grasped it, she left the others without anything to focus upon. Later in the year, she addressed this problem by handing out a booklet of activities, allowing the students to work at their own pace.

On the other hand, after one session with a rather weak class, Ruth commented that she "should not have been so challenging". Again, I believe the trouble arose through the use of an activity which she had not tried herself, in this case, one which had been adapted from materials intended for more advanced students. She and Sarah often used this strategy, and although it sometimes caused problems, it gave them a much greater flexibility in the topics they could cover.

Guidance required

Implementing cultural approaches to mathematics often requires teachers to learn new mathematics, or new methods of tackling familiar topics, and for many teachers this can be a huge deterrent. To help my research participants overcome this I offered to advise them as much as they required, but left each individual to decide how much guidance was needed. In the sections below, I describe their reactions to teaching this new material, and give some examples to show where assistance was needed. However, it should be noted that the majority of lessons worked well with little direct teaching from me.

Teachers Confidence Level

The majority of teachers with whom I worked admitted to having disliked or had difficulty with mathematics during their own education, so having to learn new mathematics presented a challenge, and brought back feelings of insecurity to some. However, most seemed to thrive on the excitement of a new challenge, and even when confidence was low, enthusiasm remained high. Nevertheless, Marcia admitted to me that "It's always my big fear that I'm going to teach them something bogus".

Before starting to teach her first "Historical Math" session, Wendy, a teacher of thirty years experience, commented, "I like this idea. Can I actually teach it wrong?", and her desire to try new ideas always seemed tempered by a fear of the unknown. After her first class she commented "Well, I did remember some of the stuff. When you get up there, you think 'Oh gosh, am I doing this right'". Lack of confidence led her to write information about the material on filing cards, and she frequently referred to these cards while talking to the students, behaviour very different from her usual assured, spontaneous method of teaching. She often looked across at me, as if she needed my nod of approval to be sure she was explaining the material correctly, and echoed the sentiment expressed by Barbara, who remarked "I'm sure [the students] will be able to figure things out better than me". Neither teacher admitted to being worried by this possibility, but there was a defensiveness about their frequent comments to this effect that suggested an underlying concern. Although Wendy did once confess "I was feeling really upset that I don't know it all", she looked on the bright side and added "but it doesn't hurt [the students] to see I'm learning too".

Even those teachers who seemed confident with the new material frequently noted that "This is not my expertise, it's yours", and requested, "If I'm going in the wrong direction, or if I'm making a mistake, please do feel free to step in or make suggestions." Some teachers handled their anxiety by having long discussions with me about each topic before their classes, but others seemed to shy away from the problem, producing excuses to explain why lessons had not gone well, yet still not finding the time to discuss future lessons with me before they took place.

Situations in which help was needed

Although most lessons were successfully planned and delivered without my assistance, all the teachers needed my help occasionally. In some cases they had difficulty understanding the material while preparing their lessons, and asked me specific questions about the mathematical content of the material. At other times they claimed that they had grasped this, yet pre-class discussions showed that their understanding was either flawed or incomplete. Sometimes a combination of over-confidence and lack of time resulted in teachers going into the classroom without such prior discussions, and this occasionally caused them to run into difficulties. There were also occasions on which teachers missed opportunities to include cultural or mathematical material related to the topic studied. Examples of each category are given in the following sections.

Difficulties noted during lesson preparation

Teachers rarely encountered problems when using materials specifically designed for elementary students, but Barbara, who was using the most advanced of the six *MMC* packs, sometimes asked me to explain the text. Not surprisingly, difficulties occurred most often when the topics were very different from those regularly taught. One such topic was "Never Ending Paths": Barbara admitted that this idea was new to her, and that she did not understand what was required for the exercise on the Bushoong Sand Network (Figure 10) given in the *Teachers' Notes* (p. 25). The text reads "Start at the top of the path. Move your finger along the path and try to get to the finish. Make sure you go over every part only once". Barbara said that this was too easy, suggesting that "You can just go here and here", indicating a path vertically down 5 units, then horizontally across, "or zigzag along the edge". When I explained that you had to trace along every edge she realized it was a lot harder than she had thought, and decided to photocopy it for the class to try. Unfortunately, she left the numbers around the outside, and a few of her students quickly discovered that these gave the solution: Barbara had not realized their significance.

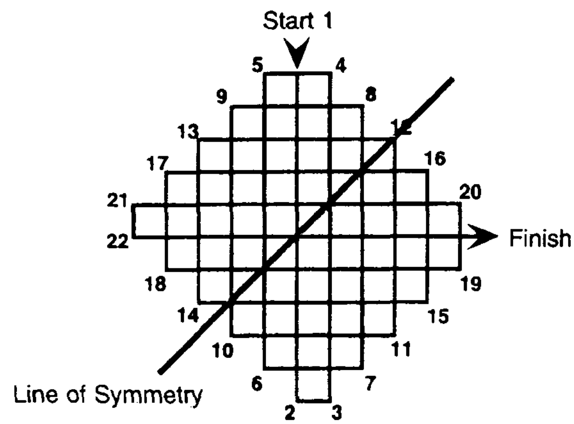


Fig. 10: Bushoong Sand Network.
(used by permission of Creative Publications, Wright Group/McGraw- Hill)

Although such conversations usually took place before classes, teachers were sometimes rather poorly prepared, and took advantage of periods while the students were working to question me about the material for the next part of the lesson. This was the case with Barbara's work on Pascal's Triangle. She drew my attention to page 12 in the *Teachers' Notes* which gave the mathematical descriptions of the counting number and triangular number sequences contained in the Triangle.

Barbara: I don't understand this section which makes some suggestions about giving the name for each of the number sequences.

Irene: Do you know why these are called triangular numbers (pointing to a diagonal in the Triangle).

Barbara: No.

Irene: (I explained how the Greeks had noticed that 3, 6, 10 ... counters could be placed in a triangular array.)

Barbara: OK, so we can use the counters for that.

Irene: Yes. You've heard of square numbers, 4, 9, 16 and so on?

Barbara: Yes, I'm OK with that, but here it talks about adding counting numbers, and I didn't really get that, why bother counting numbers?

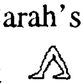
Irene: They're called "counting numbers", they go 1, 2, 3, 4,...

Barbara: Oh, so that's what it meant, I see, OK.

Having sorted out these concepts, she was then able to help her students explore them during the class that followed.

Those teachers who used more advanced texts predictably needed the most help in interpreting the material and finding ways to present it to their students. Sarah often used Ifrah's very complex book on the history of numbers (1985), and would write questions on post-it notes stuck to her lesson plans, so that when we met before her classes I could help her understand the material. The Chinese method of calculating with counting rods was one such topic (see Appendix G). Since this is a "hands-on" approach to calculation the text description is hard to follow. However, once I had shown her how to manipulate the rods she became very enthusiastic about their value in reinforcing place-value concepts, and has used them every year since. In this case, and many others, a conversation sufficed to clarify the problems, but occasionally the material needed so much simplification that I volunteered to redesign activity sheets for her. This was the case when she wanted her students to plot the "witch of Agnesi" curve referred to in the biography of Maria Agnesi they had been given (see Appendix G). Sarah commented that the mathematical formula presented in the *Historical Connections* text was "like a foreign language", but after I had produced a table of coordinates from it, her Grade 3 students had little difficulty plotting the set of points.

Difficulties emerging during pre-class discussion

Even when teachers claimed to have grasped the material, it sometimes became apparent in our pre-class discussions that they had either misunderstood parts of it or not fully appreciated the opportunities it offered. The most common example of misunderstanding was that of Egyptian addition and subtraction. Several teachers investigated this from the same activity sheet that I had given Sarah, and they all made the same mistake as the majority of Sarah's students (see page 84): They were able to deduce that  meant "add", and that  was used for subtraction, but did not grasp the significance of the direction in which the "legs" were walking. However, making this mistake in discussion with me helped them become aware of the potential problem, and better able to spot it in their students' work.

An example of incomplete appreciation of a topic occurred when Sarah selected the activity in which the pieces of a dissected square are arranged to form a rectangle of greater area (see Appendix G). Although she knew the geometrical fallacy at the heart of this puzzle, she was unaware that the brainteaser was connected to the Fibonacci series. She was delighted when I explained this, and showed her how to construct other squares/rectangles with the same "magic" property.

Difficulties arising during lessons

On some occasions, teachers felt sufficiently confident about their knowledge of the material to go into the classroom without discussing the details of their lesson with me. Although this usually turned out satisfactorily, teachers occasionally discovered that words or concepts arose which they did not completely understand themselves. This happened to Ruth when she handed out a page titled "The Unknown Origins of Counting" taken from *Agnesi To Zeno* (Smith, 1996, p. 1), a text designed for secondary school students. This contained many "complicated" words, and after Ruth had read it aloud, with her students following the text, she admitted to them:

> To be honest, I don't know what ideograms are. I know what hieroglyphics are, that's the writing, but I don't know what cuneiform is, so we should highlight 'Ancient Mesopotamian cuneiform', 'hieroglyphics' and 'Chinese ideograms', and what we'll do, we'll put question marks by those and we'll investigate.

Ruth always followed through on such commitments, one of which involved sending e-mail messages to universities and museums around the world. If such problems occurred while I was in the room, Ruth would draw me into the conversation, a reaction which was common to all the teachers whose lessons I observed.

The novelty of the cultural mathematics resulted in a large number of student questions on both cultural and mathematical issues, some of which the teachers were unable to answer. Requests for the pronunciation of mathematicians' names (e.g. Hypatia, Eratosthenes) or the meaning of technical terms (e.g. nodes, trapezium, order of rotation) could have been anticipated and prepared for, but there were also many questions which could not have been predicted. John asked, "In English, we say Archimedes. Was it something different in Ancient Greece?". Ruth clarified the question as "Would it be pronounced differently?", but directed the inquiry to me, rather than attempting to answer it herself. Similarly, Mike directed a question about the existence of Lattice Division to me, since no such algorithm was mentioned in his *Teachers' Notes.*

Occasionally a teacher's handling of the material defeated its purpose. At first, Barbara asked her students to use their fingers to trace the network shown in Figure 10, an excellent way to mimic the drawing technique of the Bushoong children. However, she then asked them to draw it for themselves using a ruler. After I pointed out that the mathematical significance of these sand pictures was that they could be drawn in one continuous line, she realized the inherent contradiction in the instruction she had given and asked her students to draw it freehand instead, but by then the die was cast: There were several groans of objection and many of the students continued to use their rulers. This episode seems to suggest that Barbara had not really understood the significance of the sand drawings.

When Barbara taught this topic the following year, she did not provide the students with copies of the network to trace, but merely showed them the picture in the *Big Book* and asked them to draw it for themselves. This is quite a difficult task until the right method is discovered, and it was clear than Barbara had not tried it for herself before giving the task to her students. Later she asked them to find the line of symmetry, but having made this request she came to me to ask why it couldn't simply be vertical. As I was explaining this to her, we heard one student telling her friend that "it's because it's got two squares at the top and only one at the bottom", and another girl called out "it's got to be diagonal", so Barbara let these students explain the symmetry to the class.

Missed opportunities

Although the cultural mathematics texts indicate some of the curriculum connections, their limited size prevented explanation of all the possibilities. Since most teachers shared the attitude that "all the information is in here, and I just have to learn it", they sometimes missed excellent opportunities to enhance their teaching of the Learning Outcomes in the IRP. This was particularly noticeable with the games activities. Most teachers introduced some multicultural games to their students, but few considered the link between these and probability. Even those teachers who were aware of this connection tended to think in terms of the traditional examples, as was evident from my conversation with Joan before she taught her games unit (see pages 104 and 105).

Other opportunities arose which could have led to interesting discussions. Barbara asked her class whether the Persian numbers were "like ours", but other than noting the name "Hindu-Arabic", she did not explain the origin of the numerals and number system we use today. Even if she herself did not have this knowledge, she could have suggested this as a research project for the many Internet enthusiasts among her students. She did grasp the opportunity provided by her Chinese student to introduce other number symbols to the class, but a comparison between our number system and "traditional" Chinese systems shown by Qiao could also have provided a useful conceptual review of place value had Barbara encouraged her student to write the symbols for numbers beyond ten.

Sometimes it was the interdisciplinary connections which were missed. Mike's explanation that people stopped using the Lattice methods of multiplication due to the difficulties of printing the grid could have led to a discussion of the invention of the Gutenberg press and other implications that the printing press had upon society in general and the development of mathematics in particular. Likewise, Ruth's comment that "geometry" means "measurement of the earth" could have been extended into a description of the Egyptian surveying practices that gave rise to the Greek term. However, such digressions need a good grasp of both general history and that of mathematics, so it was only those teachers who had done extensive reading in both areas that were able to expand upon the material provided in the *Teachers' Notes*.

On other occasions cultural connections came up spontaneously in the class, but teachers did not grasp the opportunity to reinforce the curricular material. Ingrid told me that a student had demonstrated the method of multiplication she had learnt in Japan, in which "Instead of putting the zero in the one's column when you go to the next one, you skip that and just move over". She admitted that she had not spent time comparing this technique with the "regular method", and realized that such a discussion about the role of zero might have increased the understanding of both Asian and Western students.

Long-term commitment

The major field-work for this research occurred during the 2001/2002 academic year. However I remained in contact with my research participants during the following year, and this gave me some indication of their long term commitment to introducing a cultural dimension into their mathematics classes. In the paragraphs below I first review the comments made in June, 2002, which indicated that teachers were eager to continue with this work. I then report the situation during the following year, which suggests that enthusiasm alone does not always provide sufficient motivation to persevere with a new approach to teaching mathematics.

Teachers' predictions after one year of implementation

All the teachers I worked with were quick to assure me that they would continue working with the cultural material in future years. Mike even commented:

> I'll be a bit better next year. You've got to get your feet wet first, and this is basically what I did this year, but now I can see how it would fit right in with all the usual stuff.

The other G4 teachers agreed that they would try to integrate the material into their regular lessons, rather than treat it separately, although Joan specified that she would probably use it at the beginning of a unit. Those who intended to teach the same grade in the following year said that they would use the same material again, but Mike commented that he would also search out new material, rather than just relying on the one text: "I'd use Sarah as a resource, and I'd check some of the old textbooks I've got, 'cause some of those have pages about games and bits of history". He added that he would "maybe look at trying to add a topic a year, to build up the knowledge and the activities".

Ruth said that she would like to develop cultural approaches to all the topics in the IRP, but admitted "It's going to take me a few years to get there". However, she intended to repeat her "Space and Shape" unit the following year, and felt that the cultural background she herself had acquired from her first session of teaching this material would allow her to integrate the historical material more easily.

Sarah reported that she planned to repeat her Time-Travel Days programme every year until she retires in 2005, and talked enthusiastically about some of the changes that she would make. However, during the year she had admitted that "If I hadn't had that programme laid out ahead of time, I don't know how much of this stuff I would have done. It's a lot of work, but once the basic idea was there, it was easy to add to it".

Teachers follow-up work and reflections during the following year

I contacted all my research participants early in the 2002/2003 academic year and most indicated that they were still interested in carrying out the plans they had proposed the previous June. The two exceptions were Maureen, who was about to go on maternity leave, and Joan, who had agreed to pilot a new mathematics programme and so felt that she had to focus on that. The other four teachers agreed to let me know when they planned to include cultural material in their mathematics class. However, despite the enthusiasm and good intentions declared at this time, Ruth and Sarah were the only teachers who contacted me before Christmas.

In January, 2003, I arranged to meet Barbara, Mike and Joan. As noted above, Joan had been occupied with a different mathematics programme, but Barbara and Mike said apologetically that they "really were going to do [Multicultural Math] ... but we started to feel crowded by the curriculum". Mike commented:

> I seem to be one of those guys that's on the Ed Sullivan show with all the plates on those stands. It seemed like it was just one more plate that you're trying to get to work in, and I just was not getting around to that plate.

However, I felt that Barbara's comment, "We understood that you weren't going to be involved any more" was key to their decision not to pursue the work. This seemed to be confirmed by the fact that while talking to me they gained enthusiasm again. They recalled the advantages of the cultural approach, and reminded each other that they still had the materials they'd used the previous year. They all agreed to teach at least one more lesson for me to observe, but Barbara was the only one whose enthusiasm lasted long enough to see this through.

Sarah continued her Time-Travel Days with some modifications to accommodate the lower overall ability level of her class. I visited her classes whenever possible, but it was clear that she was sufficiently motivated to carry on with her programme regardless of whether I was there. Ruth also started to repeat her previous year's work, but a long period of sick leave meant that she was unable to complete more than the first two sessions of this unit. However, she assured me that she would try again the following year.

Of the other four teachers I had worked with previously, two had not repeated their cultural mathematics experiences, but Wendy and Marcia, who had started using historical material in September 2000, had continued to use it every year, even though I had little contact with them after December 2001. When we met at the end of their third year of implementation they were still enthusiastic about the work. Marcia expected to continue with it in the future, while Wendy, who was about to retire, explained that she was donating all her resources to her successor, and seemed certain that she would want to use them.

CHAPTER 7
CONCLUSIONS AND IMPLICATIONS

A comparison of Chapters 2 and 6 of this dissertation shows that the research participants' opinions on the advantages of using cultural perspectives in their mathematics classes generally parallel those found in the literature. However some significant differences exist, and after noting the similarities, this chapter looks at these discrepancies and suggests possible explanations for them. The feasibility of including cultural approaches to mathematics in classes taught by generalist teachers is then considered. This question has two distinct parts: whether teachers *can* teach in this way, and whether they *will* do so. The first part of this discussion includes the issue of whether the work reported here is really promoting historical and multicultural dimensions of mathematics, or merely presenting a set of interesting digressions to provide relief from what is often seen as the boredom of regular mathematics classes. The chapter closes by noting the implications of this study and making suggestions for future research into the issues it raises.

The rationale for including cultural perspectives

The teachers in this study were aware of many of the benefits mentioned in the literature. The role which material from the multicultural background of mathematics plays in showing the subject as a human endeavour was felt by all to be particularly important as a way to make mathematics more approachable and was also considered to help students value the achievements of cultural groups from all over the world. Teachers also echoed the belief that an understanding of why particular aspects of mathematics had developed in the past could help students gain a greater appreciation of the importance of the subject in present society. Concerning more practical issues, the research participants agreed with the opinions stated in the literature that the different approaches used in teaching cultural aspects of mathematics helped students develop their problem-solving and critical thinking skills. Teachers also agreed with the notion that such work provides opportunities for group work and investigations.

While teachers and experts alike noted the motivational aspects of culturally oriented mathematics classes, there was considerable disparity in the different emphasis given to value of this work for students' affective and cognitive development. The teachers' comments showed that they considered the most important advantage of the cultural work to be the impact it had on their students' attitude towards studying mathematics, whereas much of the literature focuses on the increased understanding which is claimed to come from study of such material.

In many cases the teachers' viewpoint may have arisen from their own unhappy experiences with mathematics, but even those teachers who reported having enjoyed mathematics in their childhood still regarded the affective component as a major reason to implement such work. Teachers noted their students' engagement with the culturally oriented material in several ways: their willingness to do extra work, their enthusiastic and work-related chatter during lessons, and their requests to study mathematical topics specifically related to their own ethnic backgrounds. These observations pointed to a higher level of affective development than has been suggested in the literature. This advantage of the cultural work was particularly noted after the many classes which included a "hands-on" component. For several teachers, the opportunities which cultural mathematics offers for practical approaches to the subject gradually became the determining factor in selecting which topics to study, whereas this issue was rarely mentioned in the literature. Teachers felt that the positive attitude generated by culturally oriented mathematics carried over into their regular mathematics classes, a view supported by many comments in the literature. However a few of the students' comments seemed to reflect the opposing view, also found occasionally in the literature, that favourable feelings about one area of mathematics do not necessarily transfer to another.

Although the "experts" obviously acknowledged the affective benefits of cultural mathematics, many considered that the major role of multicultural mathematics was to promote cultural pluralism and develop ideas of social justice. Teachers recognized the former goal and were particularly eager to implement it in connection with the cultural background of students in their classes. However, the only issue of social justice to be addressed was that of the role of women, and that was discussed only in Sarah's class.

Although Ruth had indicated to me that her own experiences of racial prejudice were an important reason for her decision to include cultural dimensions of mathematics in her teaching, her ideas on this issue rarely found an outlet in the material she presented in her classroom, confirming the view reluctantly acknowledged by Shan (1991) that teachers are unwilling to use their mathematics classes to explore issues of equality and justice.

The literature concerning the use of the history of mathematics as a pedagogical tool strongly emphasized the advantages that such work provides for increasing students' understanding of mathematical concepts. Expert opinions frequently focused on the use of primary sources, noting that these not only help students to see the intuitive thinking processes that have led to the development of mathematical ideas, but also provide a good source of interesting questions to explore. However, much of this literature focuses on secondary or tertiary level students, and even though there are reports of primary sources being used very successfully in the elementary classroom (see page 21), my research participants did not have access to suitable materials.

A few of my research participants shared the view that culturally oriented work helped their students to achieve a better understanding of the regular curriculum material, but they provided little justification for their belief. Nevertheless the evidence presented in Chapter 6 (pages 132 - 135) suggests that the discussions I had with the teachers about alternative algorithms sometimes moved them to a higher conceptual understanding of both the new algorithms and those they used regularly. However it was apparent that teachers rarely thought sufficiently about the material they read to achieve this increased understanding by themselves. In most cases, the new algorithms they learnt merely added to their repertoire of instrumentally grasped techniques, suggesting that teachers need guidance if maximum benefit is to be achieved from the use of cultural material. Although the literature focuses on students' cognitive development, this is unlikely to occur unless teachers themselves are aware of the possibilities that cultural materials offer in this regard.

On some occasions teachers' stated opinions agreed with those found in the literature, but their actions were inconsistent with these views. The use of alternative algorithms illustrates this point. Teachers shared the experts' opinion that introducing these algorithms gives students confidence by providing them with a variety of techniques to use, yet they rarely encouraged students to try these alternate methods if difficulties arose when using the regular techniques. This suggests that, contrary to their declared beliefs, teachers regarded Multicultural Math as something to be kept separate from their regular curriculum work. This viewpoint is encouraged by the appearance of texts including such wording in their title, and more needs to be done to help teachers integrate this material into their regular classes.

The literature contains mixed views as to the viability of teaching history of any sort to elementary school children. However, most of my research participants felt that even young students can acquire a rudimentary time-frame for historical events, and Sarah's work shows how this can be achieved through the use of time charts, stories, movies, and frequent sequential comparisons between the various civilizations. Although none of the teachers specifically noted the role of mathematics in this task, considerations of how long ago events happened, how many years elapsed between two events, or how long a particular civilization lasted can play an important part in developing a sense for historical time periods.

Assessment was another issue on which the experts were divided. Although some felt that the inappropriateness of traditional methods of assessment might prevent teachers from using this material, this did not appear to be the case in practice. On the contrary, teachers valued class activities which students could enjoy for their own sake, rather than worrying about how they were to be graded, although this factor perhaps added to the work's status as "special" in the students' eyes. Several teachers appreciated the informal, on-going assessment activities which the cultural work provided. They felt that it often gave them new insight into their students' thinking processes, and allowed them to see both strengths and weaknesses not revealed by the regular textbook exercises, a view expressed by some educators promoting this work.

Teachers also mentioned several other ways in which the use of cultural mathematics had influenced their teaching style. Some of these changes were motivated by suggestions made in the resources used, whereas others arose from the nature of the subject matter itself. This stimulus to teachers' professional development is rarely noted in the literature, but it is a very valuable side-effect of implementing cultural approaches to mathematics.

Although the literature stresses the value of cultural mathematics in making cross-curricular connections, teachers took this issue further, mentioning the importance of such work in provoking students to make their own links between mathematics and other curriculum topics. Other issues they emphasized more include the role which cultural approaches play in stimulating children's imagination, and the social benefits of allowing students who have particular cultural information to assume the role of teacher.

The "ontogeny recapitulates phylogeny" argument is prominent in the literature on the pedagogical implications of the history of mathematics, but its implications for large-scale curriculum planning are irrelevant to classroom teachers constrained to teach specific learning outcomes each year. Some teachers occasionally used the related concept of epistemological obstacles, but only if they were already aware of the appropriate historical facts. They did not search for historical solutions to explain tricky mathematical concepts, and only rarely did they refer to conceptual difficulties in the past to reassure students struggling to assimilate new ideas.

A comparison of the Venn diagrams on pages 30 and 142 indicates that teachers also paid far less attention to philosophical views on the nature of mathematics than is seen in the literature. Although the mathematical texts supporting these views are not accessible to elementary level students, anecdotes about famous mathematicians can be used to illustrate the role which intuition and persistence have played in mathematical discoveries of the past. In the lessons I observed, teachers sometimes read such stories to their students, but in discussing them afterwards little attention was paid to these characteristics of a mathematicians' work.

Although expert opinion was divided on the merits of anecdotal material, the elementary teachers had no doubts as to the motivational value of story-telling. Similarly, teachers were far less critical of the resource material than were the experts, perhaps because they were not so aware of the drastic simplifications which are sometimes made in the cultural texts. However, even when these were pointed out, the consensus was that the details were unimportant compared with opportunity to show mathematics as a human activity.

The feasibility of including cultural mathematics

There are two distinct issues of feasibility to be considered: what teachers are *able* to do, and what they are *willing* to do. I believe that the work reported in this dissertation suggests that all teachers can teach mathematics from a cultural perspective, but that only those who find such work particularly interesting will do so.

Although the previous section shows that teachers are unaware of some of the benefits noted by the experts, the results from the first year of my study were encouraging. In many cases, participation in my research represented teachers' first attempts to address the IRP recommendation that "students [should] understand that mathematics is a changing and evolving domain, to which many cultural groups have contributed" (British Columbia Ministry of Education, 1995). They clearly found the new approach both interesting and useful, and all teachers had ideas as to how they could improve their programmes in the following year. The practical problems of incorporating cultural material into their teaching were overcome, and it was apparent that, under the conditions of this research, cultural perspectives can be incorporated into the mathematics classroom without too much difficulty.

Nevertheless, it must be admitted that my dual role of teacher/researcher facilitated the introduction of this approach, particularly in the location of resource materials. Most teachers are unaware of the many books, posters and videos which are now available to help promote cultural perspectives. When given these resources, teachers generally found them well-explained and easy to use in the classroom, but without initial assistance in locating them, the majority of my research participants would have been neither willing nor able to include cultural dimensions of mathematics into their teaching.

A variety of approaches were developed to introduce cultural perspectives into their mathematics classes, and these can be seen as lying along a continuum (see Figure 11). At one end, teachers took the IRP as their point of departure and found cultural material to illustrate the learning outcomes they wished to teach. At the other extreme, they started by considering specific cultural groups, and searched for information about the role that mathematics played within these societies. Intermediate situations occurred when teachers selected IRP topics to which ethnic groups represented in their classes had made major contributions, a strategy which stresses the importance of both ends of the continuum. On the other hand, some teachers simply selected topics from a predetermined text, in which both the IRP connection and the specific cultures are determined for them.

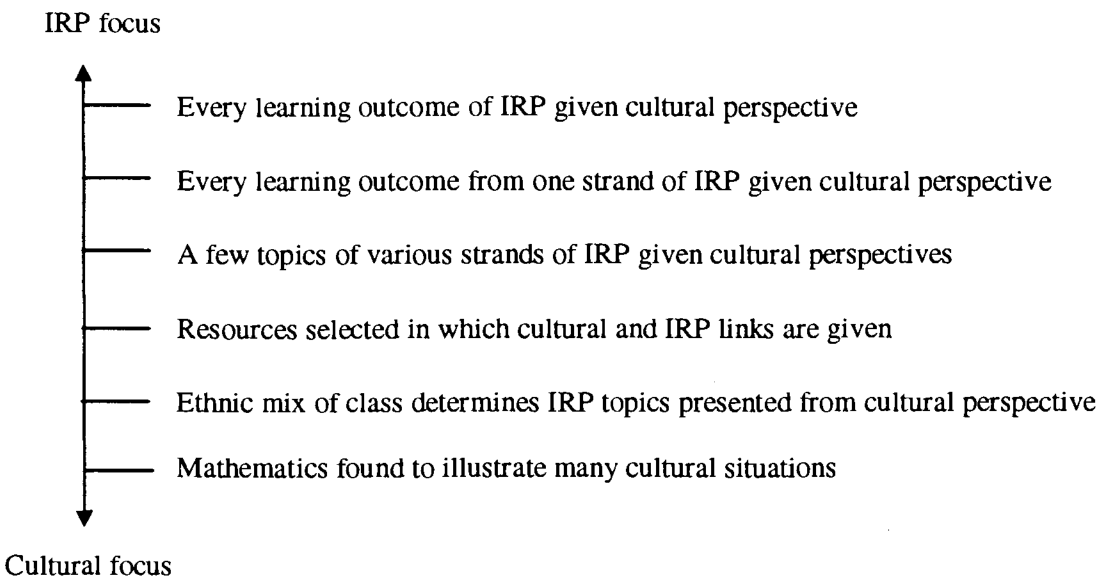


Fig. 11: Range of approaches used in implementing cultural perspectives of mathematics

The upper category is perhaps an unrealistic goal at present. There is no single resource giving teachers sufficient information to make all the required connections, so an extensive knowledge of the cultural background of mathematics would be needed. However, this is the target towards which Ruth is working, and she has already made a good start on using cultural connections for one complete strand of the IRP. On the whole, the teachers with whom I worked preferred to stay towards the IRP end of the spectrum, as this enabled them to situate the cultural work within their overall plan to cover the learning outcomes. However, since teachers do not usually have an "expert" on call to advise them of suitable materials for specific topics, the easiest route to start exploring cultural aspects of mathematics is that of using a single resource and drawing upon the mathematical and cultural links which are listed. Teachers who pursued this path for several years would gradually build up the expertise to integrate this material into their regular curriculum work.

Several teachers chose to illustrate their social studies classes with examples of the mathematics studied in other times and places, as they felt that mathematics was an important, but usually neglected, part of their study of ancient civilizations. Sarah's Time-Travel Days took this cultural focus for the choice of material to its limit, but it should be noted that her major motivation for these Days was to enhance her students' concept of mathematics. I consider that the work done in these culturally focused lessons provided a more valuable problem-solving environment than those chosen to enrich material from the mathematics curriculum, as the children had no preconceptions as to which area of mathematics would be appropriate for the activities that arose.

Although my study showed that all the research participants managed to convey the fundamental notion of mathematics as a human endeavour, it was apparent that some teachers were more adept at implementing cultural perspectives than others. A good knowledge of history was obviously a major asset, but confidence in mathematics appeared to be a more important criterion for success. This seemed to hold true even when this confidence was perhaps misplaced. Neither Ruth nor Sarah were strong mathematicians, but they were prepared to take risks by teaching the new material which the inclusion of cultural dimension sometimes required. Less confident teachers found this situation problematic.

On the other hand, the cultural information required to teach historical or multicultural aspects of mathematics was easily assimilated, at least at the level required to teach elementary students. Although confidence in mathematics thus appears more important than a love of history for teachers to maintain a cultural programme, this study also suggested that an enthusiasm for history is a necessary prerequisite for elementary teachers to initiate its implementation, except in those cases where some external motivation is present. As several of my research participants commented, the notion that mathematics has any cultural background simply does not occur to most people, confirming Michalowicz's (2000) observation that teachers using historical perspectives in their mathematics class are usually amateur historians of the subject.

The final issue to be addressed in considering whether teachers can usefully implement cultural perspective arises from the concerns of D'Ambrosio and Zaslavsky that some teachers trivialize this work. I felt that the way cultural material was used in some classes, particularly those of the G4 group, could justifiably be assigned to D'Ambrosio's (2001) category of "curiosities", particularly when little or no explicit connection was made to the regular curriculum. The idea of teaching one session per month promotes such a designation, as the work is then seen as something extra. However, I partially condone this use of cultural material. If these sessions can help change students' perceptions of mathematics, then I feel that they are valuable, and do not share Zaslavksy's view that it is "better not to do them". They will have served a useful affective purpose.

Regardless of how the G4 work is viewed, I consider that the other two case studies provide excellent demonstrations that genuine cultural approaches to mathematics can be achieved at the elementary school level. Ruth and Sarah chose different starting points, but they both placed mathematics in a social setting, and while their work did not demonstrate a "cultural approach" in the sense described by Bishop (1988), it helped the students to appreciate the contributions which various cultural groups have made to mathematics.

Although the above comments show that teachers can be successful in implementing cultural perspectives into their teaching, the research suggests that they will not necessarily be prepared to follow this path. The G4 group's reluctance to continue promoting this approach after one year reveals that teachers' enthusiasm can quickly dissipate unless there is external encouragement to continue. Despite their claims that they could reuse the activities they had already prepared, their failure to do so suggests that one year was insufficient for them to have become completely comfortable with the material. In such circumstances, it was easier to bow to the pressure of the many other classroom commitments, and omit the Multicultural Math sessions. Nevertheless, Ruth, Sarah and two of the other teachers with whom I worked have persevered with their programmes.

In the present political climate it is important for teachers to be able to justify their work to students, parents and administrators, but this caused no problem for my research participants. They found it easy to relate their cultural work to either the specific learning outcomes of the mathematics IRP, or to its more general goals, such as developing positive attitudes, making connections, problem solving, and of course to the cultural goal quoted above. However, while Ruth and Sarah replaced traditional textbook exercises with cultural material, others teachers seemed not to realize how this could be done. This contributed to the neglect of the cultural mathematics sessions as they were often seen as time-consuming "extras". Even when the material covered the same content area as that found in the students' textbook, it was often used to supplement the regular work, rather than replace it. While this provided valuable reinforcement, teachers who felt pressured by the number of learning outcomes in the IRP took the obvious step of omitting cultural material. On the other hand, those teachers who were able to use cultural resources to implement the learning outcomes validated the importance of this dimension of mathematics.

A related issue is the preparation time required to be able to teach this material effectively. My research participants acknowledged that some extra work had been required, although they rarely gave the impression that this had been disagreeably time-consuming. The G4 teachers, who only taught their Multicultural Math classes for one year, blamed other pressures in the school system for their decision to stop. However, their reluctance to repeat their lessons suggested that they had not entirely absorbed the material. As with any topic, repeated exposure to the material may be necessary to create a feeling of security, and the research suggests that only those teachers who gain considerable personal satisfaction by learning about cultural mathematics will devote sufficient time to such study.

Implications

This study focussed on the teaching of only six individuals, but their words and actions have confirmed that cultural mathematics can have many positive influences on students' work, that teachers become aware of these advantages when they explore the relevant material in their own classrooms, and that the implementation problems can be overcome. These encouraging conclusions suggest that the cultural goal stated in the IRP and similar curricula can be achieved under the conditions of this research, namely, that an expert is available to help teachers locate and understand the available resources. Nevertheless, the study also suggests several less optimistic conclusions.

The most pessimistic finding is that it seems likely that the majority of elementary school teachers will never start to implement the cultural perspectives called for in the curriculum. The notion that mathematics has a cultural background is not one which comes readily to the mind of such teachers, and the very limited references to this idea, in sections of the IRP often ignored by teachers, are unlikely to encourage them to explore this issue. More connection needs to be made between the specific learning outcomes listed in curriculum documents and the texts which can be used to provide cultural enrichment for these topics. The research shows that, in general, teachers do not have the appropriate background either to find resources or to decide for themselves which aspects of historical or multicultural work can be used to complement particular learning outcomes. If government authorities are sincere in their desire to have students acquire an appreciation of the cultural background of mathematics, then more explicit guidance is required.

Teacher education also has an important role to play, and this research suggests that cultural dimensions of mathematics have received little attention during pre-service or in-service courses. Since much of the required curriculum can be covered using materials which provide cultural perspectives of mathematics, courses for elementary teachers need to include information about these resources. However, it appears that information alone will not provide sufficient motivation to provoke implementation, so such courses should also require their participants to explore some cultural dimensions of mathematics in detail, and, whenever possible, have some practical experience in teaching the work they have investigated. In the absence of both prior experience and easy location of relevant resource material, it is not surprising that so few teachers implement the desired cultural goal.

The other discouraging conclusion comes from the second year of the research programme, during which I made fewer demands upon my research participants. Although they were all initially very enthusiastic about the novel ideas being presented, only those who had a particular interest in such an approach continued to pursue it in my absence. It appears that even teachers who have had successful experiences with the work, and have prepared material with which it could be taught on future occasions, may abandon this approach in the face of other pressures. More research is needed to determine ways to motivate teachers to continue their efforts in this field.

The general lack of knowledge of cultural mathematics suggests that any studies in this area should include a component of teacher education. A more formal teacher development experiment is indicated, in which classes require participants to develop lesson plans for specific learning outcomes, based upon material showing the cultural background of mathematics. Such research should include long-term studies of a group of teachers, with relatively close contact between the researchers and teachers being maintained over a period of years to provide the external motivation which appears to be a important component of continued implementation. The larger scale of this experiment should determine whether teachers who are supported in their endeavours become sufficiently confident with these ideas to continue using them on a regular basis, and to encourage their colleagues to do likewise.

APPENDIX A
ETHICS APPROVAL

SIMON FRASER UNIVERSITY

OFFICE OF RESEARCH ETHICS

BURNABY, BRITISH COLUMBIA
CANADA V5A 1S6
Telephone: 604-291-3447
FAX: 604-268-6785

September 26, 2003

Ms. Irene Percival
Graduate Student
Faculty of Education
Simon Fraser University

Dear Ms. Percival:

Re: The Use of Cultural Perspectives in the Elementary School Mathematics Classroom

The above-titled ethics application has been granted approval by the Simon Fraser Research Ethics Board, in accordance with Policy R 20.01, "Ethics Review of Research Involving Human Subjects".

Sincerely,

Dr. Hal Weinberg, Director
Office of Research Ethics

APPENDIX B
CLASSROOM RESOURCES

† denotes resources used by teachers taking part in this study.

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Arabic Numerals, <http://www.islamicity.org/mosque/ihame/Ref6.htm>

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Egyptian mathematics, <http://www.eyelid.co.uk/numbers.htm>.

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APPENDIX C
CONSENT FORMS

Consent forms were signed by all the teachers involved in my research, and also by their principals, their students and the students' parents. The following letter shows the general structure of these forms.

Dear [teacher's name],

I am presently working for a Ph.D. degree in Mathematics Education at Simon Fraser University, and will be conducting some of my research at [name of school] Elementary School. The title of my thesis is "The use of cultural perspectives in the elementary school mathematics classroom." My goals are to discover the present level of knowledge and interest in historical and multicultural mathematics amongst elementary school teachers, to determine how easily such teachers can use the material which has been published on these topics and to assist them as required, and finally to determine whether such material changes the attitude of teachers and/or students to mathematics.

I was delighted that you agreed to take part in my research. You will be required to complete some short questionnaires, be interviewed about your mathematical and educational background, and hold short discussions with me before and after any teaching of historical and/or multicultural mathematics. I would also like to observe these lessons. All conversations will be audio-taped, and the lessons will be audio- or video-recorded providing that your students and their parents give permission. At the end of each session, a short questionnaire will be given to the students to discover their reaction to the multicultural material, and towards the end of the study I would like to interview a few of the students.

You and your students will be free to leave the project if you wish, and any information acquired during the research will be completely confidential. The results of my work will be published in my thesis, a copy of which will be filed in SFU library, but neither the teachers, students nor their school will be identified by name. If you have any questions about the project, feel free to contact me [tel: 299-5430; e-mail:

iperciva@sfu.ca] or address more serious concerns to Dr. Barrow, the Dean of Education at S.F.U.

I would be grateful if you would complete the form enclosed, indicating your consent to take part in this study.

Yours sincerely,

I agree/do not agree (please delete) to take part in the research project of Mrs. Irene Percival. I understand that all material is completely confidential and that I may withdraw from the research at any time.

NAME (please type or print legibly): ______________________________

ADDRESS: ______________________________

____

SIGNATURE: ____________________ **WITNESS:** ____________________

DATE: ____________________

(A signed copy of this consent form and a subject feedback form will be returned to you.)

APPENDIX D
QUESTIONNAIRES

1. Questionnaire given to all teachers before taking part in the research

Many of the people I talk to seemed surprised at the idea that mathematics has a history, or that it is a multicultural subject. This questionnaire is designed to find out how much you knew about these approaches to mathematics before today ("h/m" is used as an abbreviation for "history of math and/or multicultural math").

Name (or pseudonym) ____________________ School ______________________

Grade level taught _________

1. Was h/m ever discussed during your teaching training sessions?
   Yes ○ (please specify on back of page) No ○ N/A ○

2. Have you read anything about h/m:
   (a) in elementary school text-books? .... Yes ○ (please specify) No ○
   (b) in other books ? .... Yes ○ (please specify) No ○
   (c) in professional journals (e.g. *Arithmetic Teacher*)? ...... Yes ○ (please specify) No ○

3. Have you ever taught h/m? .... Yes ○ (please specify) No ○
   If yes, where did you find the information ? ______________________________

4. Would you like to learn (more) about h/m:
   (a) for personal interest? .... Yes ○ No ○
   (b) to teach to your students? .... Yes ○ No ○

5. Do you think your students would be interested in learning h/m? .... Yes ○ No ○
   Why do you think they would/would not be interested? ______________________
   ____

6. Is there anything in h/m you would particularly like information about?
   ____
   ____

7. Any other comments? ____
   ____

Thank-you for your co-operation.

Irene Percival (iperciva@sfu.ca)

2. Additional questionnaire completed by "case study" teachers

As you have volunteered to be the subject of a "case study" for my research into the use of historical and/or multicultural mathematics ("h/m"), I need to collect some information on your academic background. This information will be only be used to give me a clearer picture of your use of h/m and will be kept completely confidential.

Name ______________________ School ______________

Grade level taught ________

1. Did you enjoy math when you were at school? .... Yes O No O

2. Were you good at math as a child? .... Yes O No O

3. Please list any academic qualifications beyond high school.

   ____

   ____

4. Have you had any full-time employment other than as an elementary school teacher?

   Yes O (please specify on back of page) No O

5. How long have you been teaching at the elementary school level? ____________

6. How long have you been teaching at your present school? ______________

7. Why are you interested in teaching h/m to your students?

   ____

   ____

   ____

8. What do you think is (are) the most important reason(s) for teaching math at school?

   ____

   ____

9. How do you think that h/m can contribute to this rationale?

   ____

   ____

Thank-you for your co-operation.

Irene Percival

3. Questionnaire completed by students of "case study" teachers

Name ______________________________ Grade __________

1. Fill in the circle that finishes in the sentence best.

   I like math .... always O, usually O, sometimes O, never O.

2. Write down something that you like about math.

   ____

3. Write down something that you DO NOT like about math.

   ____

4. Have you learnt any math at home that is different from what you learn at school? If you have, please tell me about it.

   ____

   ____

5. What language do you speak at home? ______________________________

   Did your parents use this language when they were children? .... Yes O No O

6. Have you learnt anything at school about how math is done in other countries. If you have, please tell me about it.

   ____

   ____

7. Would you like to find out about the sort of math that children do in other parts of the world? .... Yes O No O

   Are there any particular countries that you are interested in?

   ____

8. Would you like to find out about the sort of math that children did a long time ago? .... Yes O No O

   Are there any particular historical times that you are interested in?

   ____

Thanks for your help.

Irene Percival

4. Reflections by teachers after using cultural perspectives of mathematics

Reflections on teaching "Multicultural Math" classes

Please comment upon how (or if) the use of multicultural mathematics materials has changed the way you view mathematics, understand mathematics and teach mathematics. The following questions raise some specific issues to consider, but they are only suggestions.

- What did you like/not like and why? (general comments as well as topic-specific).
- What did your students like/not like and why? (general comments as well as topic-specific)
- Did this work change your attitude towards mathematics?
- Did it change your students' attitudes towards mathematics?
- Did any of the multicultural topics increase your understanding of a topic in the IRP (at any grade level)? [give examples if possible]
- Did the multicultural work increase the students' understanding? [give examples if possible]
- How did your teaching style for the multicultural math compare with your regular teaching style?
- Did you find any connections between the multicultural material and the learning outcomes of the IRP? If so, did you make these apparent to the students. [give examples if possible]
- Did you find the published resources easy to use, and did they give you sufficient information?
- Did you teach any "Multicultural Math" classes during the 2002/2003 academic year. Please explain why you did or did not do so.
- Did you include any of the ideas from your "Multicultural Math" classes in your regular math class.
- Would you use multicultural material again in future years? Please explain your reasons for and/or against.

APPENDIX E
CODING SCHEME

TEACHERS' RATIONALE for teaching from cultural perspectives

Student benefits

SM Motivation

SP Changes perceptions

- SPH Human origins of mathematics
- SPM Multicultural nature of mathematics
- SPU Non-uniqueness of methods
- SPC "Real-world" connections

SE Enrichment opportunities

SU Increases understanding

IC Interdisciplinary connections

EM Ethnic mix

SF Student feedback

Professional Development benefits

ET Enthusiasm for teaching

TM Learning new mathematics

TV View of mathematics

TU Increases understanding of mathematics.

Impact on teaching

RT Role of teacher and students

CO Classroom organization

HF Humanistic focus

II Invoke imagination

AO Assessment opportunities

SPECIFIC TOPICS

ST"X" Material taught by "X"

IMPLEMENTATION ISSUES

Time commitment

TL Lesson planning

TA Allocation of class time

Long term commitment

CI Interest

CC Conflict with other duties

Relevance to curriculum

LO Learning outcomes

CR General rationale

Resources

RL Locating

RF User-friendliness

Guidance needed by teachers

TC Teacher confidence

LP Lesson planning

GC During class

MO Missed opportunities

EH "Expert" help

TEACHERS' PERSONAL BACKGROUND

TP Turning points

TE Teaching experience

IH Interest in history

IM Interest in mathematics

ICM Interest in cultural mathematics

ITM Interest in teaching mathematics

APPENDIX F
CONNECTING CULTURAL MATHEMATICS TO THE CURRICULUM

The chart shows the connections which teachers made between the topic areas of the British Columbia IRP and the cultural material they taught during my work with them.

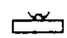
IRP strand	Historical and multicultural context
Problem Solving	Specific examples include strategies for games from a variety of countries, networks (bridges of Königsberg, sand drawings), and determining the meaning of Egyptian number and operation symbols. However, problem solving was promoted by many of the culturally oriented activities, as these were not presented in the context of a particular curriculum topic.
Number Concepts	Multicultural origins of present day number system. Historical time sense (Mathematicians' dates, relative time periods of various civilisations). Dates B.C. and A.D. and time span between them. Number systems from several ancient civilisations (tally-like systems, named place-value, and place-value). Comparison with modern system. Conversion of numbers into different bases (Binary numbers, Mayan base 20, Babylonian base 60). Egyptian unit fractions and Horus eye fractions. Role of zero. Magic squares. Estimation. Approximation. Complements of 5 and 10 using abacus. Exponents (Archimedes' Sand Reckoner and in *One Grain of Rice* and other stories). Prime numbers (on Ishango Bones; representation as single line of pebbles; Eratosthenes Sieve; Goldbach conjecture). Chinese counting rods (integer representation).
Number Operations	Origin of operation symbols. Alternative algorithms from a variety of countries (Lattice multiplication; Napier's Rods; Russian Peasant multiplication; "Japanese" multiplication; Egyptian multiplication and division; "English" subtraction). "Regrouping" using Egyptian, Babylonian, Chinese, Mayan or Roman symbols. Distributive law of multiplication over addition (Egyptian multiplication). Chinese counting rods (Integer addition and subtraction). Square roots (from pattern recognition activity in *Historical Connections*)

Patterns	Many pattern recognition activities in connection with specific mathematicians in the *Historical Connections* texts. Suggest meaning for patterns of notches in pre-historic bones. Doubling pattern in Egyptian multiplication. Arithmetic Triangle (Omar Khayyam, Chu Shih-Chieh, Pascal). Fibonacci sequence. Vedic Square. Magic Squares. Repeating patterns in art work from around the world.. Weaving patterns.
Variables and Equations	Point and curve plotting (plotting picture of "Descartes' Fly", "Witch of Agnesi" curve). Generalize patterns arising from problem-solving context (many examples from *Historical Connections*).
Measurement	Ancient units and origins of modern units (feet & inches, mile, minutes & seconds, metric units). Measuring with knotted ropes. Historical necessity for standardization. Time (calendars and seasons).
3-D Objects and 2-D Shapes	Etymology of geometric terms. Identification of 3-D objects used in buildings around the world. Nets (Platonic solids). Euler's formula for polyhedra. Archimedes' work on circles (brings in use of protractor, triangle area formula, circle area formula, regular polygons, fractions). Properties of shapes and the relationships between them, similar and congruent shapes, regular and irregular polygons as shown in art and artefacts from around the world. Tangrams. Conic sections (paper-folding parabola and drawing ellipse). "Witch of Agnesi" curve.
Transformations	Analysis of art and artefacts from around the world for symmetry (e.g. Ukrainian egg decoration, North American masks) and tessellations employing translations, reflections and rotations (e.g. Islamic wall decorations, Roman mosaic floors, quilting patterns from around the world, Escher drawings). Perspective (Renaissance art). Magic squares. Cartesian coordinates ("Descartes' fly").
Data Analysis	Bar charts to compare measured "fathom" and "height".
Chance and Uncertainty	Determining probability of specific throws using alternative forms of "dice". Connection with scores for particular throws in games from various countries.

APPENDIX G
EXPLANATION OF MATHEMATICAL TOPICS DISCUSSED

Egyptian multiplication

The Egyptians made use of the distributive property of multiplication over addition, and the fact that any number can be expressed as the sum of powers of 2.

1	19	/
2	38	
4	76	
8	152	
16	304	/
	323	

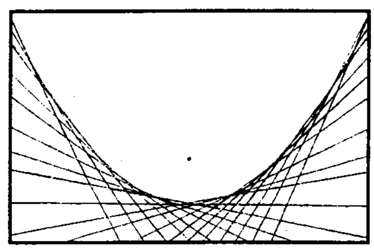
$$17 \times 19 = (16 + 1) \times 19$$

$$= (16 \times 19) + (1 \times 19)$$

$$= 304 + 19$$

$$= 323$$

Paper-folding a parabola



A dot is drawn towards the bottom of a sheet of paper, and the bottom edge of the paper is then repeatedly folded to pass through this point. In the illustration, the fold lines are drawn in to show how the envelope of the parabola develops.

Proof that the "magic total" of a Magic Square is three times the centre number

In a magic square, all rows, columns and diagonals have the same "magic total". If N is this sum, then:

a	b	c
d	e	f
g	h	i

$d + e + f = N$ (middle row)

$a + e + i = N$ (diagonal)

$g + e + c + N$ (diagonal)

Adding these three equations gives:

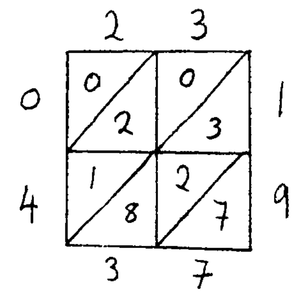
$$(a + d + g) + 3e + (c + f + i) = 3N$$

But $a + d + g = N$ (left column) and $c + f + i = N$ (right column)

So $N + 3e + N = 3N$, giving $3e = N$.

Lattice Multiplication (rectangular grid)

The grid shows 23 x 19 = 437. The two factors are placed above and to the right of the grid. Each interior square cell shows the product of the outer numbers above it and to the right, with the diagonal line dividing the tens digit from the ones digit. The complete product is found by adding along the diagonals, and appears at the left and bottom sides of the grid.



Chinese Counting Rods

In ancient China, calculations were performed on a counting board (similar to a chess board with large squares) using counting rods (bamboo sticks similar to present day toothpicks) to represent numbers.

The rods were placed as follows:

	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
Odd powers of ten	1	2	3	4	5	6	7	8	9
	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱
Even powers of ten	10	20	30	40	50	60	70	80	90

The rods were coloured either red or black, representing positive and negative numbers respectively. Once students have grasped that a pair of red and black rods have zero sum, they can use the rods to gain an understanding of the addition and subtraction of integers as follows.

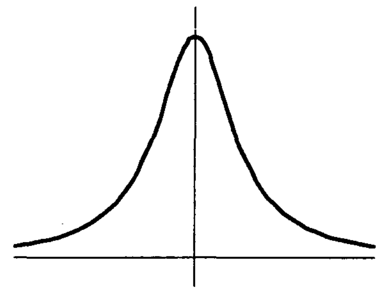
(+3) + (-4) = (-1). When three red rods and four black rods are put together, the red rods can be paired with three of the black rods, leaving one black rod.

(+3) – (-4) = (+7). Four black rods cannot be subtracted from three red rods, so students have to add four "zero sum" pairs. The four black rods can then be literally "taken away" leaving seven red rods.

"Witch of Agnesi" curve

The activity given in *Historical Connections* (Reimer & Reimer, 1995a), states that a special case of this curve is given by the equation

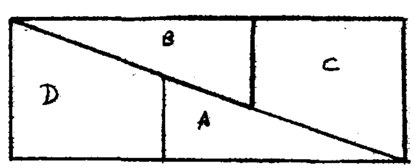
$$Y = \frac{64}{x^2 + 16}.$$

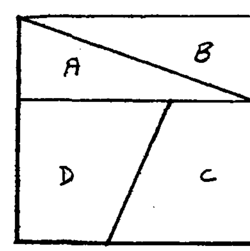


To make this curve accessible to Grade 3 students, I constructed a table of values in which all the function values were multiplied by 10, then rounded to the nearest whole number or simple mixed number, such as 4½. The points were identified by two numbers labelled "Horizontal (left or right)" and "Vertical". The horizontal values increased by 1 from 0 to 16, and the students were able to use this table to plot the basic shape of the "Witch of Agnesi" curve, thereby attaching some meaning to the words they had read.

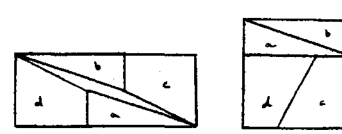
The "missing square" puzzle

8 x 8 square and 13 x 5 rectangle





Students calculated the area of the 8x8 square, then cut it into pieces as shown. These pieces were then rearranged to form a 13x5 rectangle, which was found to have an area one square unit more. They were unable to explain the fallacy from these pieces, but a similar situation, using the smaller Fibonacci numbers 3, 2 and 5, revealed the missing area. The trick derives from the fact that the square of a Fibonacci number is one more or less than the product of the numbers on either side.



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