ROBUST FAULT DIAGNOSIS AND COMPENSATION IN NONLINEAR SYSTEMS VIA SLIDING MODE AND ITERATIVE LEARNING OBSERVERS

by

Wen Chen

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
in the School
of
Engineering Science

SIMON FRASER UNIVERSITY

September 2003

APPROVAL

Name: Wen Chen

Degree: Doctor of Philosophy

Title of Thesis: Robust Fault Diagnosis and Compensation in Nonlinear Systems via Sliding Mode and Iterative Learning Observers

Examining Committee:

Dr. Albert M. Leung, Chair
School of Engineering Science
Simon Fraser University

Dr. Mehrdad Saif, Senior Supervisor
School of Engineering Science
Simon Fraser University

Dr. William A. Gruver, Supervisor
School of Engineering Science
Simon Fraser University

Dr. John Jones, Supervisor
School of Engineering Science
Simon Fraser University

Dr. Roya Rahbari, Internal Examiner
NRC Institute for Fuel Cell Innovation
National Research Council Canada (NRC)

Dr. Bahram Shafai, External Examiner
Department of Electrical and Computer Engineering
Northeastern University, MA, USA

Date Approved: Sept 29, 2003

PARTIAL COPYRIGHT LICENCE

Title of Thesis/Project/Extended Essays:

Robust Fault Diagnosis and Compensation in Nonlinear Systems via Sliding Mode and Iterative Learning Observers

Author:

(Signature)

Sept. 29, 2003

(Date Signed)

Abstract

This thesis deals with issues of robust Fault Detection and Isolation (FDI) and compensation in uncertain nonlinear systems using second order sliding mode and iterative learning observers.

The problem of detecting and diagnosing actuator faults using a variable structure adaptive observer (VSAO) is first discussed. The VSAO is constructed directly based on the uncertain nonlinear system itself. The VSAO-based FDI can achieve robust fault detection and estimation. Furthermore, a second order sliding mode observer (SOSMO)-based robust fault detection in uncertain nonlinear systems is addressed. The SOSMO has the property of sharply filtering unwanted high frequency signals due to unmodelled dynamics, as the sliding surface dynamics forms a low-pass filter. The SOSMO is then extended to an uncertain constrained nonlinear system (UCNS) for fault detection and estimation, where the SOSMO can directly supply fault estimation. This makes fault isolation become easier.

An Iterative Learning Observer (ILO), which is updated online by immediate past system output errors as well as inputs, is constructed for the purpose of fault diagnosis. An automatic control reconfiguration scheme for fault accommodation using iterative learning strategy is then suggested. It is shown that the effects of disturbances can be attenuated by ILO inputs. The ILO is applied to excite an adaptive law in order to generate an additional control input to the nonlinear system. The additional control input can annihilate the effect of faults on system dynamics. ILO-based adaptive fault compensation strategy is independent from any existing strategies. It can supply fault detection, estimation, and compensation at the same time, and does not need a fault detection and isolation subsystem.

The last chapter is concerned with the design of a sliding mode observer (SMO) for a class of uncertain nonlinear differential-algebraic systems (DAS). An algorithm is developed to reconstruct the algebraic variables with a singular distribution matrix. An SMO is then designed based on the reconstructed algebraic variables to compensate the effect of disturbances on estimation error dynamics such that the estimated states including both the differential and algebraic variables can track the actual ones.

Dedication

To my wife Honglei, my lovely daughter Teresa and my parents.

Acknowledgements

With my deepest appreciation, I acknowledge my senior supervisor Dr. Mehrdad Saif for his support, guidance, understanding, inspiration, enthusiastic help and invaluable suggestions. I am grateful that he allowed me considerable freedom in conducting this research and provided me with all the equipment necessary for my work.

I would like to give special thanks to Professor William Gruver and Professor John Jones for serving as my Supervisory Committee members. Thanks are also due to Professor Bahram Shafai for accepting to act as the External Examiner, and Dr. Roya Rahbari for accepting to be my Internal Examiner. I am very grateful to them for taking their valuable time to review my thesis.

I am thankful for many colleagues whom I have been fortunate to get to know and had the pleasure to work with during the past years.

Last but not least I am particularly indebted to my wife Honglei for her understanding and stable emotional support during these years. Our daughter Teresa has been a source of inspiration and motivation. I would also like to thank my parents and parents-in-law for their encouragement and love.

Table of Contents

Approval ii

Abstract iii

Dedication v

Acknowledgements vi

Table of Contents vii

List of Figures xi

Table of Acronyms xiii

Nomenclature xiv

1 Introduction 1

1.1 Basic Concept of Fault Diagnosis and Compensation 2

1.2 Fault Diagnosis Methodologies 4

1.3 Model-Based Fault Diagnosis 6

1.4 Observer-Based Robust Fault Diagnosis and Compensation : an Overview 7

1.5 Thesis Outline 12

2 Robust FDI via a VSAO 14

2.1 Introduction 14

2.2 Preliminaries and Problem Statement 16
2.3 Main Results 18
2.4 An Example 26
2.5 Conclusions 28

3 Robust Fault Detection via a SOSMO **29**
3.1 Introduction 29
3.2 Preliminaries and Problem Statement 31
3.3 Main Results 32
3.3.1 Stability Analysis of the SOSMO with a Single Output 37
3.3.2 Stability Analysis of the SOSMO with Multiple Outputs. 40
3.4 Simulation Results 45
3.4.1 Fault Detection by a SOSMO 46
3.4.2 Fault Detection by a First Order SMO 46
3.5 Conclusions 47

4 Robust FDI in UCNS via a SOSMO **53**
4.1 Introduction 54
4.2 Preliminaries 54
4.3 Main Results 58
4.3.1 Stability Analysis of the SOSMO with a Single Output 63
4.3.2 Stability Analysis of the SOSMO with Multiple Outputs. 68
4.3.3 Fault Isolation Strategy 75
4.4 An Illustrative Example 76
4.4.1 Fault Detection and Estimation by a SOSMO 76
4.4.2 Fault Detection by a First Order SMO 78
4.5 Conclusions 79

5 An ILO-Based Fault Detection and IL-Based Accommodation in Nonlinear Systems **85**
5.1 Introduction 86

5.2 Problem Statement and Preliminaries 87
5.3 ILO-Based Fault Detection Approach 89
5.4 Fault Accommodation by the IL Approach 94
5.5 An Illustrative Example 98
5.5.1 Fault Detection 98
5.5.2 Fault Accommodation 99
5.6 Conclusions 99

6 An Iterative Learning Observer-Based Fault Diagnosis Strategy 102
6.1 Introduction 103
6.2 Problem Statement 104
6.3 Main Results 106
6.3.1 The ILO and Disturbance Estimation 106
6.3.2 Output Disturbance Attenuation by the ILO 113
6.4 Application to Automotive Engine Fault Diagnosis 120
6.4.1 An Automotive Engine Model 121
6.4.2 Fault Detection and Estimation for Automotive Engine 123
6.4.3 Output Disturbance Attenuation By the ILO 125
6.5 Conclusions 125

7 Fault Diagnosis and Compensation by an ILO 134
7.1 Introduction 134
7.2 Problem Statement and System Formulation 136
7.3 Analysis of the ILO regarding Disturbance and/or Fault Estimation . 138
7.4 Main Results 144
7.4.1 ILO-Based Adaptive Fault Detection, Estimation, and Compensation 144
7.4.2 Stability Analysis 147
7.5 Application to Automotive Engine Fault Diagnosis and Compensation 153
7.6 Conclusions 155

8 **An SMO in Nonlinear DAS** **164**

8.1 Introduction 164

8.2 Problem Statement and System Formulation 166

8.3 Main Result 168

8.3.1 An Algorithm for the Reconstruction of Algebraic Variables $z(t)$ 168

8.3.2 An SMO for the uncertain DAS with Singular $l(x)$ 176

8.3.3 Stability Analysis 177

8.4 An Illustrative Example 179

8.5 Conclusions 181

9 **Conclusions** **185**

Bibliography **190**

List of Figures

2.1 Fault Diagnosis by a VSAO. 27

3.1 No faults occur-1. 48
3.2 No faults occur-2: S and $\dot{S}$. 49
3.3 A fault occurs: S and $\dot{S}$. 49
3.4 An abrupt actuator fault occurs. 50
3.5 Gain d is chosen too big. 51
3.6 The first order SMO with a fault. 52

4.1 Fault detection and estimation by a SOSMO. 80
4.2 State Estimation Error by the SOSMO. 81
4.3 The S and $\dot{S}$ of the SOSMO. 82
4.4 State Estimation Error by First Order Sliding Mode Observer. 83
4.5 Output Error and Sliding Surface by First Order Sliding Mode Observer. 84

5.1 ILO-based fault detection. 100
5.2 Output Tracking Error with IL Control. 101
5.3 Output Tracking Error without IL Control. 101

6.1 A Simulink block diagram description of the ILO. 107
6.2 System states and their estimations. 127
6.3 Disturbance estimation by ILO input. 128
6.4 Post-fault system states. 129
6.5 The fault and its estimate. 130
6.6 The fault and its estimate. 131

6.7 Estimation error dynamics between system and Luenberger observer. 132
6.8 Estimation error dynamics between the system and the ILO. 133

7.1 Dynamic process of fault compensation. 145
7.2 Disturbance estimation by the ILO– a sinusoid disturbance. 157
7.3 Disturbance estimation by the ILO– a random disturbance. 158
7.4 The additional control input and ILO input. 159
7.5 Engine speed and pressure in the manifold and their estimations . . . 160
7.6 The $w(t)$ and $v(t)$ under multiple faults in order. 161
7.7 Engine speed and pressure in the manifold and their estimations under multiple faults in order. 162
7.8 The $w(t)$ and $v(t)$ under simultaneous multiple faults. 163

8.1 System state estimation errors. 183
8.2 Algebraic variable estimation errors. 184

Table of Acronyms

DAS:	Differential-Algebraic Systems
FDI:	Fault Detection and Isolation
IL:	Iterative Learning
ILO:	Iterative Learning Observer
SMO:	Sliding Mode Observer
SOSMO:	Second Order Sliding Mode Observer
UCNS:	Uncertain Constrained Nonlinear Systems
UIO:	Unknown Input Observer
VSAO:	Variable Structure Adaptive Observer

Nomenclature

A^T:	Transpose of matrix A
A^{-1}:	Inverse of matrix A
$\lambda_{max}(A)$:	Maximum eigenvalue of matrix A
$\lambda_{min}(A)$:	Minimum eigenvalue of matrix A
$\|x\| = (x^T x)^{\frac{1}{2}}$:	Euclidean norm of vector x
$\mathbf{0}$:	The bold face denotes a zero vector or matrix
$I_{\alpha\times\alpha}$:	An $\alpha \times \alpha$ identity matrix
$\|A\| = [\lambda_{max}(A^T A)]^{\frac{1}{2}}$:	Matrix norm

Chapter 1

Introduction

Increased productivity requirements and stringent performance specifications have led to more demanding operating conditions in many modern engineering systems. Such conditions increase the possibility of system faults which are characterized by critical, unpredictable changes in the system dynamics.

In general, feedback control algorithms, which are designed to handle small system perturbations that may arise under "normal" operating conditions, can not accommodate abnormal behavior due to faults [35]. Automated maintenance for early detection of worn equipment is becoming a crucial problem in many practical applications.

System faults can result in off-specification production, increased operating costs, and the possibility of detrimental environmental impacts. More importantly, from the safety point of view, a single fault can develop into multiple faults, which can further lead to catastrophe. Therefore, in order to satisfy the needs for safety, reliability, and performance in the industrial processes, it is important to promptly detect system component faults, actuator faults, and sensor faults, and to accurately diagnose the source and severity of each malfunction so that corrective actions can be taken.

Motivated by achieving high levels of reliability, maintainability and performance in faulty systems, more and more attention has recently been devoted to fault accommodation. On April 26th, 1986, for example, the worst nuclear accident in history took place in the small town of Chernobyl, Ukraine. Thirty one people were claimed dead. Total casualties are unknown and estimates run into the thousands. The reason for this incident was that the reactor's design made it unstable at low power. Sophisticated monitoring and fault management systems could have prevented this and similar accidents.

Developing new design and analysis methods for health monitoring and fault diagnosis, as well as fault accommodation are the main tasks in this thesis

1.1 Basic Concept of Fault Diagnosis and Compensation

Faults can occur in both hardware and software of the controlled systems. This thesis is concerned with hardware faults.

A *fault* that tends to degrade overall system performance represents an undesired change in a system of interest, while a *failure* denotes a complete breakdown of a system component or function. In this thesis, *fault* rather than *failure* is used to indicate a tolerable malfunction, rather than a catastrophe. Typical faults are [48]

- Construction defects such as cracks, ruptures, fractures, leaks, and loose parts.
- Actuator faults such as damage in the bearings, deficiencies in force or momentum, defects in the gears, and aging effects.
- Sensor faults, including scaling errors, hysteresis, drift, dead zone, short cuts, and contact failures.

- Abnormal parameter variations in the systems.
- External obstacles such as collisions and clogging of outflows.

Due to the fact that no consistent terminology in the fault diagnosis field existed, the SAFEPROCESS Technical Committee discussed this matter and tried to find some commonly accepted definitions. Based on the discussions within the committee, a fault was defined as: *"an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard conditions."* Meanwhile, a failure is *" a permanent interruption of a system's ability to perform a required function under specified operating conditions "* [62].

The role of a *fault diagnosis system* [14] is to detect faults and to diagnose their locations and significance in a system of interest. Such a system normally consists of three tasks: fault detection, fault isolation, and fault identification. A fault in a dynamic system can take on many forms, such as actuator faults, sensor faults, unexpected abrupt changes of some parameters, or even unexpected structural changes [124].

The purpose of fault detection is to generate an alarm which informs the operations that there is at least one fault in the system. This can be achieved from either the direct observation of system inputs and outputs, or the use of certain types of redundant relations (i.e. the model-based fault detection and diagnosis or analytical redundancy methods). Fault isolation is to determine the locations of faults, e.g. which sensor or actuator has become faulty. Identification, however, is not an easy task, as it requires that after an alarm has been set, an estimation of the location, size and nature of the fault should be made [52, 124]. The isolation and identification tasks together are referred to as *fault diagnosis.* Most practical systems contain only fault detection and isolation stages (FDI). In many cases, *diagnosis* is used simply as a synonym for *isolation.*

Over the last two decades, fault diagnosis has attracted a great deal of attention. The majority of this work has been to design and analyze fault detection and isolation issues [15, 17, 68, 70, 76, 129, 134]. Fault accommodation or fault-tolerant control, in particular, is becoming more and more interesting to researchers [6, 16, 65, 92, 99, 115, 139]. A fault-tolerant control is defined as a control system with fault-tolerant capability. There may be some performance degradation under the operation of a fault-tolerant control system. The main objective of a fault-tolerant control is to maintain the specified operations of a system under consideration, and give operators (or automatic monitoring systems) enough time to repair the damage or take alternative measures to avoid catastrophe [14]. In the case of flight control systems, for example, safety is the greatest priority. This implies that even in the presence of failed components, the aircraft must still be able to land safely.

The aim of fault-tolerant control is to adjust or modify the system control inputs in order to maintain the safety and reliability of the system so that the controlled system can still continue according to its original specifications [65, 92, 139]. Some fault-tolerance measures will be suggested in this thesis.

1.2 Fault Diagnosis Methodologies

A traditional approach for fault diagnosis is a hardware-based method, where a particular variable is measured using multiple sensors, actuators, computers and software. Several problems that hardware redundancy based fault diagnosis encounters are the extra equipment, cost, and additional space required to accommodate this equipment [14].

Analytical redundancy is potentially more reliable than hardware redundancy

[119]. A wide range of analytical redundancy fault diagnosis approaches can be broadly divided into model-based techniques, knowledge-based methodologies, and signal-based techniques [95].

- There are two classes of model-based approaches, i.e. quantitative models based approach (differential equations, state space methods, transfer functions, etc.), which generally utilizes results from control theories, such as parameter estimation, state estimation or parity space concepts; and qualitative models based approach, where qualitative models of the process are used to predict the behavior of the process, and fault detection is achieved by comparing the actual observations to the predicted behavior. Model-based fault diagnosis is defined as *"the determination of faults of a system from the comparison of available system measurements with a priori information represented by the systems's mathematical model, through generation of residual quantities and their analysis"* [14].

- A great deal of knowledge is required to develop knowledge-based fault diagnosis systems (the process structures, process unit functions etc.).

- In signal-based FDI, signals or symptoms that carry as much fault information as possible must be extracted from systems. The limitation of signal-based FDI is its lack of efficiency, especially for early fault detection.

The main advantages of model-based approaches are that no additional hardware components are required to realize the fault diagnosis algorithms, and that the existing measurements for process control are sufficient to implement the fault diagnosis strategies. Model-based fault diagnosis approaches will be discussed in this thesis. More specially, observer-based fault diagnosis will be the main concern.

1.3 Model-Based Fault Diagnosis

In contrast to physical redundancy, where measurements from parallel sensors are compared to each other, the sensor measurements in model-based FDI are compared with analytically computed values of the respective variables. The resulting differences are called *residuals*, which are indications of the presence of faults in systems of interest [52]. The following is a list of some generally used approaches to model-based residual generation.

- *Observer-Based Approach* The basic idea behind this approach is to use an observer or a filter in FDI to estimate system outputs from measurements by employing a Luenberger observer in a deterministic setting or a Kalman filter in a stochastic setting. In this case, output estimation errors or innovations can be taken as residuals, respectively.

- *Parity Space Approach* The parity space approach is based on a check of the parity (consistency) of parity equations that are properly modified system equations by system measurements. The purpose of modifying system equations is to decouple residuals from system states and to decouple among different faults. An inconsistency demonstrates the presence of faults [48].

- *Parameter Estimation Approach* Model-based FDI can also be achieved using system identification techniques in an input-output system model. Usually, faults are reflected in system parameters, such as friction, mass, viscosity, resistance, inductance, etc. The parameters of the actual process can be repeatedly estimated using online parameter estimation methods [14, 60, 63]. The estimated parameters are then compared with those of the reference model. Any substantial discrepancies indicate a fault.

It is well known that a perfectly accurate mathematical model of a practical system never exists because some parameters of the considered system may vary in an uncertain manner, and the characteristics of disturbances are unknown so that they can not be modelled accurately. In the process of fault diagnosis design, model uncertainties and disturbances have to be taken into account, as they constitute a source of false and missed alarms of the fault detection system. They may corrupt FDI performance to such an extent that the FDI system is totally useless [14]. Therefore, when designing a fault diagnosis system, one has to consider the effect of model uncertainties and disturbances so that the FDI system will be robust to them, i.e. insensitive or even invariant, while still being sensitive to real faults. An FDI scheme designed to provide satisfactory sensitivity to faults, associating with the necessary robustness with respect to uncertainties and disturbances, is called *a robust FDI scheme* [14]. During the last ten years, numerous approaches towards the solution of robust fault diagnosis have been developed, such as observer-based robust FDI [2, 93, 105, 106, 113, 124], unknown input observers [15, 96, 107, 108, 125], eigenstructure assignment [33, 97, 98], etc. In the next section, the observer-based robust fault detection strategies will be reviewed.

1.4 Observer-Based Robust Fault Diagnosis and Compensation : an Overview

The most widely considered tools for fault detection are observers. The idea behind using of an observer for fault detection is to estimate system outputs from measurements using an observer, and then construct residuals by properly weighted output estimate errors [39]. When the considered systems are subject to unknown disturbances and uncertainties, the effect of them has to be decoupled from the residual signals to avoid false alarms in detection. This problem is well known in the field of FDI as *robust fault detection* [39].

Ever since Beard [5] founded the concept of analytic redundancy for fault detection, model-based fault diagnosis has been in development at various places since the early 1970s. The main idea of the analytic redundancy, which replaces the hardware redundancy, is to generate directional residuals by failure detection filters. Different fault effects can be mapped into different directions or planes in the residual vector space so that fault isolation can be achieved [14, 48]. Beard's approach was then redefined in a geometric setting by Jones [66] and Massoumnia [86], leading to the so-called Beard-Jones Fault Detection Filter. Its design issue was later discussed by White and Speyer [126], Liu and Si [81], and Chung and Speyer [30].

Clark and co-workers are believed the first to apply the Luenberger observer for fault detection [32], Leininger [78] first pointed out the impact of modelling errors on FDI performance, and the first work to tackle robustness of observer-based FDI approach was contributed by Frank and Keller [44].

The most important task in model-based fault diagnosis is the generation of robust residuals. Unknown input observers can achieve this task [12, 13, 59, 105, 125, 127]. The uncertain factors in system modelling are considered to act via unknown inputs (disturbances). Based on the known distribution matrix of unknown inputs, they can be decoupled from output estimation errors that are defined as residuals. Therefore, residuals are decoupled from unknown inputs. The definition of an unknown input observer in [15] is as follows

> *An observer is defined as an unknown input observer for the considered system if its state estimation error approaches zero asymptotically, regardless of the presence of the unknown input (disturbance) in the system.*

The work of [15], which is widely cited, combines the unknown input observer and fault detection filter to form a new approach that ensures that the residual vector, generated by the filter, has both robust and directional properties.

An unknown input observer FDI strategy for linear systems can not be applied to nonlinear system FDI [46]. When a fault occurs, the nonlinear system will run out of the operating point. The fault detection subsystem may enhance the modelling errors. To tackle this problem, Seliger and Frank [107] extended linear unknown input observer theory to nonlinear systems. In their contribution, a new concept of nonlinear unknown input observers is used for component and actuator FDI in a class of nonlinear dynamic systems with disturbances and uncertainties that are expressed as unknown input signals. Under some conditions, the system model can be transformed into a form that remains unaffected by unknown inputs, but still reflects the occurrence of component or actuator faults. The nonlinear unknown input observer is designed based on the transformed model. The observer outputs can be used for state estimation and residual generation.

Yang and Saif [133] designed a novel nonlinear unknown input observer for a class of nonlinear systems whose states and outputs can be decomposed into two parts. The first part is affected only by actuator faults, whereas the other is decoupled from them. The subsystem that is decoupled from faults is then used to design the nonlinear unknown input observer. The estimates are used for FDI purposes.

In recent years, the SMO-based FDI strategy that originated from sliding mode control has been attracting researchers' attention [96, 97, 113, 130, 131]. The main characteristic of the SMO is that, despite disturbances and uncertainties, the output estimation errors between the system and the SMO can be forced to and maintained at zero while sliding. Once a fault occurs, sliding will cease to exist, and based on this a fault alarm signal can be generated. Therefore, SMOs are qualified candidates for robust FDI.

The pioneering work on SMO design which can be found in [112] examines the potential uses of the SMO and introduces a particular observer structure that includes switching terms. The analysis shows that SMOs have promising properties in the presence of modeling errors and sensor noise. Another earlier work [118] also describes an observer with discontinuous switched components. Edwards et al. [39] considered the application of a particular SMO to fault detection and isolation problems. The novelty of their research lies in the reconstruction of fault signals by the equivalent injection concept. Also, the SMO gain is chosen to maintain the sliding of system output errors even after a fault occurs.

A novel SMO design for both linear and nonlinear systems is proposed by Xiong and Saif [130, 131]. The new SMO has the advantages of working under much less conservative conditions than Wallcot and Zak's observers [122], and of estimating a state function when estimating all states is impossible.

Authors of [55, 68, 71, 76, 113, 123, 130, 131] presented the robust fault detection of a subset of sensor, actuator, and process faults using SMOs. The performance of SMO-based FDI techniques is shown to be robust to parameter uncertainties in system models.

A generic observer cannot efficiently detect faults with slow time constants because the enhancement of robustness is associated with a decrease of the sensitivity to faults with slow time constants [4, 46, 85]. Adaptive observers are proposed to tackle this problem because they can estimate both system states and the slowly varying unknown parameters of the observed systems [45, 124, 132, 135]. In order to apply the adaptive observer scheme to nonlinear system FDI, the considered nonlinear system usually has to be transformed to form a so-called adaptive observer canonical form, as shown in [36, 83].

Yang and Saif [135] consider a class of special nonlinear systems for FDI purposes. The nonlinear system under consideration can be transformed into two different subsystems. One takes the adaptive observer canonical form on which an adaptive observer design, under certain conditions, is based. The other subsystem is affected only by actuator faults. With the aid of the estimations of states as well as uncertain parameters, the faults are approximated using discretization technique. The approximated faults can be used for fault detection and isolation.

Recently, some researchers [6, 16, 65, 92] have sought the solution of the fault accommodation problem. The typical approach is based on a set of fault detection and isolation subsystems. An additional control input resulting from fault detection and isolation subsystem is added to the original control inputs in order to reduce or compensate the effects of faults [65, 92, 139]. As a matter of fact, the fault detection and isolation subsystem is not always necessary for the purpose of fault accommodation [6].

The work of [94] is very interesting: a controller embedding an internal fault model is able to not only automatically offset the effect of an incipient fault in an induction motor, but also reconstruct the fault whose effect has been offset. The authors of [67] introduce an extra input to the nonlinear observer used as a filter to directly estimate the time-varying faults. This estimation of faults is then employed to establish a fault-tolerant controller to guarantee the stability of the closed-loop system. From the nonlinear robust control viewpoint, a set of robust fault-tolerant control is proposed in [101]. The resulting closed-loop system has the properties that the stability and performance can be guaranteed in the presence of uncertainties and, when there is a sensor fault, stability can still be maintained.

Fault accommodation will be another issue raised in this thesis.

1.5 Thesis Outline

Chapter 2 of this thesis is concerned with the problem of detecting and diagnosing actuator faults using a variable structure adaptive observer (VSAO). The observer construction in the existing approaches for FDI is based on the observer canonical form transformed from nonlinear systems. The transformation conditions, however, are not always satisfied in the practical systems. Motivated by this, a VSAO will be constructed directly based on the uncertain nonlinear system itself for diagnosing actuator faults.

The property of filtering unwanted high frequency signals, which is due to the fact that the second order sliding surface dynamics forms a low-pass filter, makes it possible for a SOSMO be applied to uncertain nonlinear systems for fault detection in Chapter 3. Another SOSMO with fault estimation, as proposed in Chapter 4, can achieve fault detection and estimation at the same time, making fault isolation easier. Compared with the first order SMO, the SOSMO has a better performance especially when the magnitude of a fault is relatively small.

Chapter 5 presents a general framework for fault detection and accommodation using the IL strategy. An iterative learning observer (ILO), which is updated online by immediate past system output errors as well as inputs, is constructed for the purpose of fault detection. Further, using the IL strategy, an automatic control reconfiguration scheme for fault accommodation is also described. One of the main features of the proposed scheme is that the control reconfiguration is achieved automatically based only on the response of the overall system. This IL controller does not require a fault detection and isolation subsystem.

Chapters 6 and 7 further explore the properties of the ILO, discovering that it can estimate and compensate disturbances and /or actuator faults. More specifically, it can be alert to any variation of the considered system. This attractive feature makes it possible for the ILO to be used to excite an adaptive law. An additional input generated from the adaptive law is added to the nominal system inputs for the purpose of fault accommodation. The simulation example in Chapter 7 shows that the ILO-based fault accommodation is very effective.

The last chapter is concerned with the design of an SMO in a class of uncertain nonlinear differential-algebraic systems (DAS) described by so-called semi-explicit forms with the differential variables being coupled with algebraic variables. An algorithm is developed, using serial elementary matrices followed by differentiation, to transform the singular distribution matrix into a nonsingular matrix such that the algebraic variables can be expressed as a function of system state variables and inputs. An SMO is then designed based on the reconstructed overall system equation. The stability of the proposed observer is also proved and an illustrative example is given in simulation to describe the design of the SMO.

Chapter 2

Robust FDI via a VSAO

This chapter is concerned with the problem of detecting and diagnosing actuator faults using a variable structure adaptive observer (VSAO). A VSAO which uses only inputs and outputs for diagnosing actuator faults by a learning method will be constructed. The construction of this observer is directly based on the uncertain nonlinear system under consideration. Coordinate changes are not employed to transform the nonlinear system into a linear one. In addition, stability condition of the proposed observer is relaxed by introducing the expansions of the nonlinear terms into power series. This makes the calculation of the observer gain matrix easier. It will be shown that the proposed approach is robust in the sense that the residual will only produce an alarm only after a fault occurs. No alarm signal is generated even when the system of interest is subject to some parameter uncertainties or disturbances.

2.1 Introduction

Many efforts have been made towards observer-based approaches to fault diagnosis in nonlinear systems [35, 40, 47, 48, 99, 124]. Especially, Demetriou and Polycarpou

[35] developed a general framework for model-based fault detection and diagnosis for a class of systems with incipient faults. The changes in the system dynamics due to a fault are modelled as nonlinear functions of the state and input variables, while the time profile of the fault is assumed to be exponentially developing. An automated fault diagnosis architecture using nonlinear online approximations with an adaption scheme is designed and analyzed. To implement their approach, system states are assumed to be measurable, which is not always true in practice. Wang and Daley [124] presented a novel approach for the fault diagnosis of actuators in known deterministic dynamic systems using an adaptive observer technique. A system without model uncertainties is initially considered, followed by a discussion of a general situation where the system is subject to either model uncertainties or external disturbances. The adaptive diagnostic algorithm is then developed to diagnose faults. Vemuri and Polycarpou [120] proposed a fault diagnosis algorithm for a class of nonlinear systems with modelling uncertainties, where not all states of the systems are measurable. The main idea behind this approach is to monitor the plant for any off-nominal system behavior utilizing a nonlinear online approximator with adjustable parameters. Under some assumptions, the nonlinear systems are first transformed into linear systems on which the proposed estimation model is based.

As for observer design, Rjamani [102] presented a systematic design methodology and some fundamental insight into observer design for a class of Lipschitz nonlinear systems. It is pointed out that the stability conditions of the observer proposed by Thau [117] are only useful to check the stability of the observer once it has been designed. Choosing observer gain matrix so as to satisfy the stability conditions, however, is not straightforward.

In this chapter, a nonlinear VSAO for the purpose of actuator fault diagnosis is proposed. This kind of observer is based directly on the uncertain nonlinear system under consideration. Coordinate changes are not employed to transform the considered nonlinear system into a linear one. Additionally, using the stability conditions we can easily design the observer parameters.

2.2 Preliminaries and Problem Statement

Consider a class of dynamic systems described by the following differential equation:

$$\begin{aligned} \dot{x}(t) &= f(x) + F\Delta f(x,u,t) + g(x)\theta(t)u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

where: $x(t) \in \mathbb{R}^n$ is immeasurable state vector, $u(t) \in \mathbb{R}^m$ is measurable input vector, $y(t) \in \mathbb{R}^m$ is measurable output vector, $C \in \mathbb{R}^{m\times n}$ and $F \in \mathbb{R}^{n\times m}$ are constant matrices, $f(x) : \mathbb{R}^n \to \mathbb{R}^n$, $g(x) : \mathbb{R}^n \to \mathbb{R}^{n\times m}$, $\Delta f(x,u,t) : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+ \to \mathbb{R}^m$, the system uncertainty, $\theta(t)$, is an $m \times m$ time-varying matrix, representing the gain of actuators connected to input $u(t)$. It is through this matrix that we model the effects of the actuator faults. When the actuators are healthy, $\theta(t) = \theta_H$, a known matrix. However, any $\theta(t) \neq \theta_H$ would indicate the presence of actuator faults. It is assumed that $g(x) = F\bar{g}(x)$. Lastly, we assume that the system is observable and, in this chapter, only actuator faults are considered. The model used here is not a general nonlinear system because there is no a unified model, such as $\dot{x} = f(x,u,\theta)$, can be used in this thesis. Actuator fault is expressed by a gain matrix $\theta(t)$ of control inputs. This supplies readers with an intuition of the actuator faults.

The purpose of the actuator fault detection and diagnosis is to generate an alarm signal when a fault occurs and produce an accurate estimate of the matrix $\theta(t)$ which defines the actuator fault behavior.

For convenience, we have following abbreviations

$$A(t) = \frac{\partial f}{\partial x}(\hat{x}),$$

$$B_i(t) = \frac{\partial g_i}{\partial x}(\hat{x}) \qquad i = 1, \ldots, m.$$

The expansions of $f(x)$ and $g(x)$ into power series lead to

$$f(\hat{x}) - f(x) = A(t)(\hat{x} - x) + \phi(\hat{x}, x), \tag{2.2}$$

$$g(\hat{x}) - g(x) = [B_1\tilde{x}, B_2\tilde{x}, \ldots, B_m\tilde{x}] + \psi(\hat{x}, x) \tag{2.3}$$

where ϕ, ψ are the terms of second and higher order in $\tilde{x} = \hat{x} - x$. $A(t), B_i(t)$ are $n \times n$ matrices, $i = 1, \ldots, m$.

Throughout this chapter we will make the following assumptions:

Assumption 2.1 *Actuator gain $\theta(t)$ is first-order differentiable, and $\|\theta(t)\| \leq \gamma_\theta$, where $\|\cdot\|$ refers to the Euclidean norm.*

Assumption 2.2 *System input is bounded via $\|u\| \leq \gamma_u$.*

Assumption 2.3 *The uncertainty term $\Delta f(x, u, t)$ is unknown but bounded, so that*

$$\|\Delta f(x, u, t)\| \leq \rho_{\Delta f} < +\infty, \forall x \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, t \geq 0.$$

Assumption 2.4 *Matrix $B(t) = [B_1, B_2, \cdots, B_m]$ is bounded with γ_B and $\|\bar{g}(\cdot)\| \leq \gamma_{\bar{g}}$.*

Assumption 2.5 *There are positive real numbers $k_\phi, k_\psi > 0$ such that the ϕ, ψ are bounded via $\|\phi(\hat{x}, x)\| \leq k_\phi \|\hat{x} - x\|$, and $\|\psi(\hat{x}, x)\| \leq k_\psi \|\hat{x} - x\|$.*

Remark 2.2.1 It should be noted that the assumptions presented above are all regarding norm bounds; no special or rigorous requirements are claimed. Especially, the Lipschitz condition, a very common and often employed condition is not needed. This relaxes the stability condition to be derived in the following theorem.

2.3 Main Results

In this section, a VSAO-based fault diagnosis approach for uncertain nonlinear systems will be considered.

In practice, it is not always possible to have the state measurements. In this case, only input and output information can be used to construct the observer for the purpose of actuator fault diagnosis.

Recall the nonlinear systems (2.1):

$$\begin{array}{rcl} \dot{x}(t) & = & f(x) + F\Delta f(x,u,t) + g(x)\theta(t)u(t) \\ y(t) & = & Cx(t). \end{array} \tag{2.4}$$

Note that, based on assumption 2.3:

$$\|\Delta f(x,u,t)\| \leq \rho_{\Delta f} < +\infty, \forall x \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, t \geq 0.$$

In the fault diagnosis literature, it is well known that the presence of modelling errors, in general, increases the probability of false alarms. During last few years, the designs of so-called robust fault diagnosis schemes have resulted in a variety of tools for dealing with such modelling uncertainties. In this chapter, we consider the fault diagnosis issue for the uncertain nonlinear system using a VSAO. In some work, for example [120], the observer is constructed based on a linear system transformed from the considered nonlinear system. To obtain the diffeomorphism that transforms the nonlinear system into a linear one, a set of general observability like conditions are needed [84]. Also, with the addition of actuator faults, an additional restriction will have to be placed on them in that they will have to depend on the known signals (i.e. inputs and outputs) in the new coordinates. Generally, the difficulty in obtaining the diffeomorphism, as well as the restriction placed on the type of faults allowed, will limit the applicability of the linearization type of approaches. Here, we directly construct the VSAO based on the nonlinear system of interest itself. No coordinate changes are employed to transform the considered nonlinear system into a linear one.

Consider a VSAO of the form

$$\begin{aligned}\dot{\hat{x}}(t) &= f(\hat{x}) + g(\hat{x})\hat{\theta}(t)u(t) + L(t)(C\hat{x}(t) - y(t)) + P^{-1}C^T v(t) \\ \hat{y}(t) &= C\hat{x}(t) \\ \dot{\hat{\theta}}(t) &= -G^T \bar{g}(\hat{x})^T e_y u^T \qquad G > 0\end{aligned} \tag{2.5}$$

where:

$$v(t) = \begin{cases} -\rho \dfrac{e_y(t)}{\|e_y(t)\|}, & \text{if } e_y(t) \neq 0, \\ 0, & \text{otherwise,} \end{cases} \tag{2.6}$$

and $\hat{x}(t) \in \mathbb{R}^n$ is the observer state vector. The parameter $\hat{\theta}(t)$ is the estimate of $\theta(t)$ and $\tilde{\theta} = \hat{\theta} - \theta$. The positive definite matrix P is symmetric, and $L(t)$ is the gain matrix to be determined. Matrix $G \in \mathbb{R}^{m \times m}$ is nonsingular. The vector $v(t)$ is the switching term defined as above. Finally, ρ is a positive constant to be derived.

Remark 2.3.1 The variable structure term $v(t)$ can guarantee the robustness of the residual due to its capability of disturbance rejection. The adaptive law is used to estimate actuator faults. This makes the VSAO possess the capability of both robust fault detection and estimation.

The main task of designing the VSAO is to choose the gain matrix $L(t)$ and the switching gain ρ. In what follows, we will discuss the design of $L(t)$ and ρ, and will give a proof of the stability and convergence of the above proposed observer.

If the state estimate errors are defined as $\tilde{x}(t) = \hat{x}(t) - x(t)$, and output estimation errors $e_y(t) = \hat{y}(t) - y(t)$, then it is straightforward to show

$$\dot{\tilde{x}}(t) = f(\hat{x}) - f(x) + [g(\hat{x})\hat{\theta}(t)u(t) - g(x)\theta(t)u(t)] + L(t)e_y(t) - F\Delta f(x, u, t) + P^{-1}C^T v(t). \tag{2.7}$$

The value of $\hat{\theta}(t)$ is set to θ_H until a fault is detected. It is assumed that after a fault occurs, $\theta(t) = \theta = constant \neq \theta_H$.

Output estimation error e_y is selected as the residual vector and is used for the purpose of monitoring for the actuator fault detection as follows:

$$\begin{cases} \|e_y(T)\|_\lambda \leq \|C\|\epsilon; & \text{Healthy} \\ \|e_y(T)\|_\lambda > \|C\|\epsilon; & \text{Faulty} \end{cases} \tag{2.8}$$

where $\|e_y(T)\|_\lambda$ is λ-norm, defined as $\|e_y(t)\|_\lambda = \sup_{t\in[0,t']} e^{-\lambda t}\|e_y\|, \lambda > 0$. $\|C\|\epsilon$ is a prespecified threshold and T is the time when a fault occurs.

As a result, the purpose of the actuator fault diagnosis is to find a diagnosis algorithm for $\hat{\theta}(t)$ such that

$$\lim_{t\to\infty} \tilde{x}(t) = 0; \qquad \lim_{t\to\infty} \tilde{\theta}(t) = 0.$$

To avoid false alarms generated by $\Delta f(x, u, t)$, we could simply increase the value of ϵ. However, an arbitrary increase may lead to an insensitivity of the observer to faults of small magnitude. So, the following proofs of sensitivity and robustness actually supply a method to determine the minimum threshold.

Theorem 2.1 (Robustness) *The robust variable structure fault diagnosis described by equations (2.5) and (2.6) guarantees that*

$$\|e_y\|_\lambda \leq \|C\|\epsilon$$

for time $t < T$ prior to the occurrence of the fault.

Proof:

It is straightforward to show that

$$\|e_y\| \leq \|C\|\|\tilde{x}\|. \tag{2.9}$$

Multiplying both sides of the above equation by $e^{-\lambda t}$ and considering the definition of λ-norm, we have

$$\|e_y\|_\lambda \leq \|C\|\|\tilde{x}\|_\lambda. \tag{2.10}$$

On the other hand

$$\begin{aligned}
\tilde{x}(t) &= \int_0^t [f(\hat{x}) - f(x) + L(t)e_y - F\Delta f + P^{-1}C^T v]d\tau + \tilde{x}(0) \\
\|\tilde{x}\| &\leq \int_0^t [(k_1 + k_2)\|\tilde{x}\| + \rho_{\Delta f}\|F\| + l_1]d\tau + \|\tilde{x}(0)\| \qquad (2.11) \\
&= \int_0^t (k_3\|\tilde{x}\| + l)d\tau + \|\tilde{x}(0)\|
\end{aligned}$$

where

$$k_1 = \|A(t)\| + k_\phi$$

$$k_2 = \|LC\|$$

$$l_1 = \rho\|P^{-1}C^T\|$$

and

$$k_3 = k_1 + k_2$$

$$l = l_1 + \rho_{\Delta f}\|F\|.$$

Using basic integral inequality (see [43], p. 96), we have that

$$\|\tilde{x}\| \leq \|\tilde{x}(0)\|e^{k_3 t} + l\int_0^t e^{k_3(t-\tau)}d\tau. \tag{2.12}$$

Multiplying the above equation by $e^{-\lambda t}$, and assuming that $\lambda > k_3$, we have

$$e^{-\lambda t}\|\tilde{x}\| \leq \|\tilde{x}(0)\|e^{(k_3-\lambda)t} + l\int_0^t e^{-\lambda\tau}e^{(k_3-\lambda)(t-\tau)}d\tau. \tag{2.13}$$

Taking supremum on both sides of the above equation, we obtain

$$\begin{aligned} \|\tilde{x}\|_\lambda &\leq \|\tilde{x}(0)\| + \frac{l}{\lambda - k_3}(1 - e^{(k_3-\lambda)t}) \\ &\leq \|\tilde{x}(0)\| + \frac{l}{\lambda - k_3} = \epsilon. \end{aligned} \tag{2.14}$$

So,

$$\|e_y\|_\lambda \leq \|C\|\epsilon. \tag{2.15}$$

∎

Remark 2.3.2 Theorem 1 states that despite the presence of the uncertainties, the fault detection logic will not produce a false alarm.

In what follows, time t of matrix $A(t)$ is omitted for the convenience of derivation.

Theorem 2.2 (Sensitivity) *If the fault matrix θ is such that*

$$\left\| \int_T^{T+t_d} Ce^{(A+LC)(T+t_d-\tau)}[g(\hat{x})\hat{\theta}u(t) - g(x)\theta u(t)]d\tau \right\|_\lambda > \|C\|(1+\mu)\epsilon + b_2$$

for some $t_d > 0$, then

$$\|e_y(T+t_d)\|_\lambda > \|C\|\epsilon.$$

Proof:

Rewrite error dynamics equation (2.7) as

$$\dot{\tilde{x}} = (A+LC)\tilde{x} + \phi + [g(\hat{x})\hat{\theta}(t)u(t) - g(x)\theta(t)u(t)] - F\Delta f(x,u,t) + P^{-1}C^T v. \tag{2.16}$$

The solution of the above equation is

$$\begin{aligned} \tilde{x}(T+t) &= e^{(A+LC)t}\tilde{x}(T) + \int_T^{T+t} e^{(A+LC)(T+t-\tau)}[g(\hat{x})\hat{\theta}(t)u(t) - g(x)\theta(t)u(t)]d\tau \\ &\quad + \int_T^{T+t} e^{(A+LC)(T+t-\tau)}[\phi - F\Delta f + P^{-1}C^T v]d\tau. \end{aligned} \tag{2.17}$$

Considering output error expression and taking norms, we obtain

$$\begin{aligned}\|e_y(T+t)\| = \|C\tilde{x}(T+t)\| \;&\geq\; \left\|\int_T^{T+t} Ce^{(A+LC)(T+t-\tau)}[g(\hat{x})\hat{\theta}u(t) - g(x)\theta u(t)]d\tau\right\| \\ &-\|C\| \left\|\int_T^{T+t} e^{(A+LC)(T+t-\tau)}[\phi - F\Delta f + P^{-1}C^T v]d\tau\right\| \\ &-\mu\|C\|\|\tilde{x}(T)\|e^{-\lambda_0 t}.\end{aligned} \tag{2.18}$$

In the above derivation, we assume that $A + LC$ is a stable matrix and λ_0 and μ could be selected such that inequality $\|e^{(A+LC)t}\| \leq \mu e^{-\lambda_0 t}$ holds.

Furthermore

$$\begin{aligned}\|C\tilde{x}(T+t)\| \;&\geq\; \left\|\int_T^{T+t} Ce^{(A+LC)(T+t-\tau)}[g(\hat{x})\hat{\theta}u(t) - g(x)\theta u(t)]d\tau\right\| \\ &-\mu\|C\|\|\tilde{x}(T)\|e^{-\lambda_0 t} - \frac{\mu}{\lambda_0}\|C\|(\phi_0 + l)(1 - e^{-\lambda_0 t})\end{aligned} \tag{2.19}$$

where ϕ_0 is the norm bound of ϕ; $l = \rho_{\Delta f}\|F\| + \rho\|P^{-1}C^T\|$.

Multiplying both sides of the above equations by $e^{-\lambda t}$, we have that

$$\begin{aligned}e^{-\lambda t}\|C\tilde{x}(T+t)\| \;&\geq\; e^{-\lambda t}\left\|\int_T^{T+t} Ce^{(A+LC)(T+t-\tau)}[g(\hat{x})\hat{\theta}u(t) - g(x)\theta u(t)]d\tau\right\| \\ &-\mu\|C\|\|\tilde{x}(T)\|e^{-\lambda_0 t}e^{-\lambda t} - e^{-\lambda t}\frac{\mu}{\lambda_0}\|C\|(\phi_0 + l)(1 - e^{-\lambda_0 t}).\end{aligned} \tag{2.20}$$

Taking supremum on both sides of the above equation, we obtain

$$\|C\tilde{x}(T+t)\|_\lambda \geq \left\|\int_T^{T+t} Ce^{(A+LC)(T+t-\tau)}[g(\hat{x})\hat{\theta}u(t) - g(x)\theta u(t)]d\tau\right\|_\lambda - \mu\|C\|\epsilon - b_2 \tag{2.21}$$

where $b_2 = \dfrac{\mu}{\lambda_0}\|C\|(\phi_0 + l)$.

If

$$\left\|\int_T^{T+t} Ce^{(A+LC)(T+t-\tau)}[g(\hat{x})\hat{\theta}u(t) - g(x)\theta u(t)]d\tau\right\|_\lambda > \|C\|(1+\mu)\epsilon + b_2 \tag{2.22}$$

then,

$$\|e_y\|_\lambda > \|C\|\epsilon. \tag{2.23}$$

∎

The following theorem provides sufficient conditions for convergence of the state estimates and the stability of the observer proposed above.

Theorem 2.3 (Stability) *Consider an uncertain nonlinear system (2.1). If there exist positive definite symmetric matrix $P(t)$ and gain matrix $L(t)$ such that $(A(t) + L(t)C)^T P(t) + P(t)(A(t) + L(t)C) + \dot{P}(t) = -Q(t)$, where $Q(t)$ is a positive definite symmetric matrix, then the state estimation error dynamics (2.7) is stable.*

Proof:

Consider the Lyapunov function candidate

$$V = \tilde{x}^T P \tilde{x} + tr(\tilde{\theta}^T G^{-1} \tilde{\theta}), \tag{2.24}$$

then

$$\begin{aligned}
\dot{V} &= \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + \tilde{x}^T \dot{P} \tilde{x} + 2tr(\dot{\tilde{\theta}}^T G^{-1} \tilde{\theta}) \\
&= \tilde{x}^T [(A + LC)^T P + P(A + LC) + \dot{P}]\tilde{x} + 2\tilde{x}^T P \phi + 2tr(\dot{\tilde{\theta}}^T G^{-1} \tilde{\theta}) + 2\tilde{x}^T P g(\hat{x}) \tilde{\theta} u \\
&\quad + 2\tilde{x}^T P [g(\hat{x})\theta(t)u(t) - g(x)\theta(t)u(t)] - 2\tilde{x}^T P F \Delta f(x, u, t) + 2\tilde{x}^T C^T v \\
&\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\lambda_{max}(P) k_\phi \|\tilde{x}\|^2 + 2\tilde{x}^T P B \tilde{x} \theta u + 2\tilde{x}^T P \psi \theta u \\
&\quad + 2e_y^T \bar{g}(\hat{x}) \tilde{\theta} u - 2tr(u e_y^T \bar{g}(\hat{x}) G G^{-1} \tilde{\theta}) - 2e_y^T \Delta f + 2e_y^T v
\end{aligned} \tag{2.25}$$

where $PF = C^T$. In the above derivation, equation (2.2) is used and the adaptive law has been inserted.

Considering Assumptions 2.4 and 2.5, and substituting $v(t)$ from equation (2.6) into equation (2.25), we have the following further extended equation:

$$\begin{aligned}\dot{V} \leq & -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\lambda_{max}(P)k_\phi\|\tilde{x}\|^2 + 2\lambda_{max}(P)\gamma_B\gamma_\theta\gamma_u\|\tilde{x}\|^2 \\ & +2\lambda_{max}(P)k_\psi\gamma_\theta\gamma_u\|\tilde{x}\|^2 + 2e_y^T\bar{g}(\hat{x})\tilde{\theta}u - 2e_y^T\bar{g}(\hat{x})\tilde{\theta}u \\ & +2\rho_{\Delta f}\|e_y\| - 2\rho\|e_y\|.\end{aligned} \quad (2.26)$$

Let $\lambda_{min}(Q) \geq b = 2\lambda_{max}(P)k_\phi + 2\lambda_{max}(P)\gamma_B\gamma_\theta\gamma_u + 2\lambda_{max}(P)k_\psi\gamma_\theta\gamma_u$ and $\rho = \rho_{\Delta f}$, then the above equation has the form of

$$\dot{V} \leq -(\lambda_{min}(Q) - b)\|\tilde{x}\|^2 \leq 0. \quad (2.27)$$

So, the proposed VSAO is stable. ∎

From the stability derivation of the VSAO, we know that the main reason why the Lipschitz condition is not required is the introduction of the power series of $f(x)$ and $g(x)$.

Remark 2.3.3 From adaptive theory viewpoint, a learning methodology for actuator fault diagnosis has been constructed. By using the adaptivity capabilities of this observer, it can be used not only to detect the occurrence of the actuator failures, but also to provide an online estimate of the fault characteristic.

Remark 2.3.4 From the derivation of the stability of the proposed VSAO, it is known that the modelling uncertainties can be dynamically compensated by the discontinuous term $v(t)$. So, introducing this discontinuous term into the observer has improved the robustness of the fault diagnosis algorithm for uncertain nonlinear systems (2.1).

2.4 An Example

In this section, the proposed VSAO-based approach will be tested by detecting and diagnosing actuator faults in a simple nonlinear system as follows

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)\theta(t)u(t) + F\Delta f(x,u,t) \\ y(t) &= x_2 \end{aligned} \tag{2.28}$$

where

$$f(x) = \begin{bmatrix} -2x_1 + x_2^2 \\ 0 \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad F = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \Delta f = 0.01cos(x_2).$$

The VSAO is constructed as follows

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}) + g(\hat{x})\hat{\theta}(t)u(t) + L(t)\tilde{x}_2 + P^{-1}C^T v(t) \\ \hat{y}(t) &= \hat{x}_2(t). \end{aligned} \tag{2.29}$$

where

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & -2\hat{x}_2 \\ 0 & -4.5 \end{bmatrix} \tag{2.30}$$

Assume that healthy actuator gain $\theta_H = 1$, and a fault occurs as follows:

$$\theta(t) = \begin{cases} 1; & t < 10(sec) \\ -2; & 10 \leq t \leq 15(sec) \\ 2; & t \geq 15(sec). \end{cases} \tag{2.31}$$

The simulation results are shown in subplots 1, 2, and 3 of Figure 2.1. It can be seen that $\hat{\theta}$ tracks the faulty actuator behavior in a desired manner. After an actuator fault occurs at 10 sec and 15 sec, residual e_y rapidly jumps to a value indicating a fault has occurred. Therefore, e_y provides a good measurement for detecting actuator faults. It should be noted that the initial nonzero value of the residual is due to the observer's initial conditions mismatch.

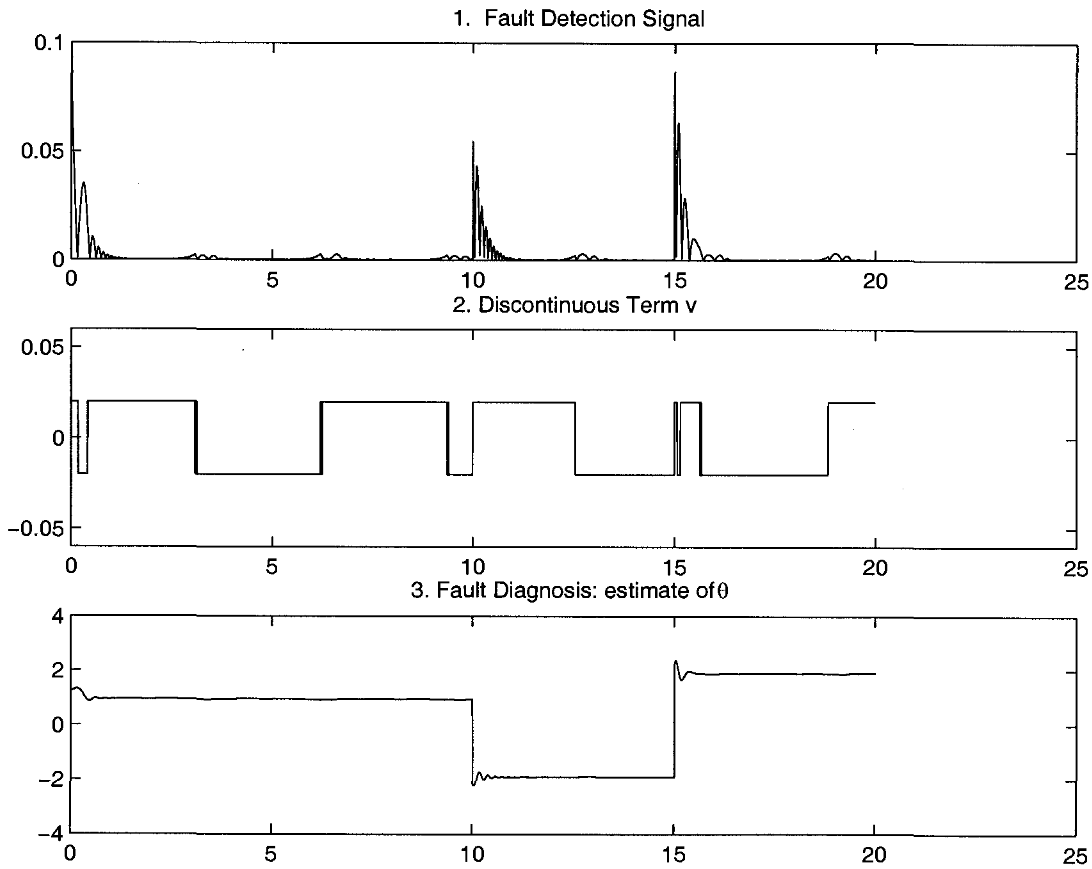

Figure 2.1: Fault Diagnosis by a VSAO.

2.5 Conclusions

In this chapter, a VSAO for actuator fault diagnosis in uncertain nonlinear systems has been constructed based directly on the nonlinear systems under consideration. Coordinate changes are not used. This makes the VSAO easier be applied to industrial processes. The effects of model uncertainties on estimation error dynamics can be dynamically attenuated by the variable structure term, while the estimated fault compensates the effects of the actuator fault. This is why the VSAO can still accurately estimate the post-faulty system states. The simulation results show that this kind of VSAO-based fault diagnosis strategy can work efficiently.

Chapter 3

Robust Fault Detection via a SOSMO

In this chapter, a second order sliding mode observer (SOSMO)-based robust fault detection in uncertain nonlinear systems is discussed. The reason why the SOSMO is used for fault detection is that the second-order $S(t)$ (sliding surface) dynamics can sharply filter unwanted high frequency signals due to unmodelled dynamics. The sliding condition will be first derived such that the observer switching gain can be selected. The stability of the reduced sliding mode observer is then proved by assuming that the considered uncertain nonlinear system has a single output and two outputs, respectively. An example will be employed to show that the proposed sliding mode observer can work very effectively.

3.1 Introduction

The growing needs of FDI in complex systems, such as the automotive, manufacturing autonomous vehicles, and robots, have attracted a lot of attention [28, 50, 61, 106].

It is well known that the core element of model-based fault detection in industrial systems is the generation of residual signals that act as indicators of faults. Various design approaches for residual generation have been proposed. Fault detections using parity space approaches and detection filters are discussed in [50, 96, 97]. Besides these strategies, the most widely considered tools for fault detection are observers. The basic idea behind the utilization of observers for fault detection is to estimate the outputs of the systems from the measurements by using some type of observer. After that, one can construct the residuals by properly weighted output estimate errors [39]. When the considered system exhibits unknown disturbances and uncertainties, their effects have to be de-coupled from the residual signals to avoid false alarms in detection. This problem is commonly known in the literature as robust fault detection [39].

In recent years, SMOs that originate from sliding mode control have been provoking researchers' attention [96, 97, 113, 130, 131]. The main characteristic is that the output estimation errors between the considered system and the SMO can be forced to and maintained at zero, which is called sliding, despite the disturbances and uncertainties. Therefore, the SMOs can be applied to robust FDI. The pioneering work on SMOs can be found in [112, 118].

Additionally, Hermans et al. [55] introduced an SMO on the basis of transforming the considered system into a canonical form. After analyzing the structure of the uncertainties, an SMO is constructed. Edwards et al. [39] considered the application of a particular SMO to FDI problems. The novelty of their paper lies in the reconstruction of the fault signals by the equivalent injection concept. The SMO gain is chosen to maintain the sliding of system output estimation errors even after a fault occurs. The equivalent control concept is also used in [123] to prove the convergence of the proposed SMO that is used to estimate the states of nonlinear systems of interest. Sreedhar et al. [113] presented a robust detection of a subset of sensor, actuator, and process faults using the SMO. The performance of the SMO-based FDI technique is shown to be robust to parameter uncertainties of the system model.

In this chapter, a SOSMO based robust fault detection will be discussed, which is motivated from the second order sliding mode control [11, 41]. The reason why we adopt the second order sliding mode concept is that the second order sliding surface dynamics forms a low-pass filter that can sharply filter unwanted high frequency signals caused by disturbances and uncertainties. This makes the SOSMO robust to disturbances, while being sensitive to the low frequency faults. As long as the fault is not a high frequency signal, it can be successfully detected.

3.2 Preliminaries and Problem Statement

Consider a general uncertain nonlinear system as follows:

$$\begin{aligned} \dot{x}(t) &= \xi(x) + \delta f(x,t) + \sum_{i=1}^{m} g_i(x) u_i(t) \\ &= \xi(x) + \delta f(x,t) + G(x)u(t) \\ y_j(t) &= h_j(x), \qquad j = 1, \cdots, m \end{aligned} \tag{3.1}$$

where: $x(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ is the system state vector; $u(\cdot), y(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^m$ are system input and output vectors, respectively; $\xi(\cdot), g_i(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n, i = 1, \cdots, m$ are smooth vector fields and $h_i(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}, i = 1, \cdots, m$ is a smooth function; $\delta f(x,t) \in \mathbb{R}^n$ represents the disturbances and uncertainties.

The goal of this chapter is to design a SOSMO, which is named after the second order sliding condition, to detect system faults. Fault detection could be accomplished by measuring the deviation of the system trajectories from the sliding surfaces. The second order sliding condition will be used to design the SOSMO.

Throughout this chapter, the following assumptions are required:

Assumption 3.1 *The system output vector* $h(x) = [h_1(x), \cdots, h_m(x)]^T$ *is differentiable.*

Assumption 3.2 *The system inputs* $u = [u_1, \cdots, u_m]^T$ *are bounded.*

Assumption 3.3 *Matrix* $\dfrac{\partial h}{\partial x}$ *is bounded.*

Assumption 3.4 *Uncertainties have upper bounds in the following forms:*

$$|\delta f_i(x)| \leq \eta_i(x), \quad i = 1, \cdots, n,$$

where $\eta_i(x)$ *is the known upper bound of unknown disturbances.*

Assumption 3.5 *Matrix* $\dfrac{\partial h(\hat{x})}{\partial \hat{x}} L(t) \in \mathbb{R}^{m \times m}$ *is nonsingular, where* $L(t)$ *is observer gain to be determined.*

3.3 Main Results

In this section, the SOSMO design issues and its stability will be discussed.

The key point here is that if the deviation of the SOSMO outputs and the real outputs is within an acceptable range, the residual should not generate an alarm signal. In the sliding mode context, one says that the observer is sliding [55]. A fault will destroy the sliding.

To achieve robust fault detection for the uncertain nonlinear systems despite the existence of uncertainties and disturbances, the following SOSMO is proposed:

$$\begin{aligned} \dot{\hat{x}}(t) &= \xi(\hat{x}) + G(\hat{x})u(t) - L(t)\left[\frac{\partial h(\hat{x})}{\partial \hat{x}} L(t)\right]^{-1} \cdot v(t) \\ \hat{y}_j(t) &= h_j(\hat{x}), \qquad j = 1, \cdots, m \end{aligned} \tag{3.2}$$

where $L(t) \in \mathbb{R}^{n \times m}$ is observer gain to be determined; $v(t)$ is the switching term to be derived which includes sliding surface vectors.

The expressions of the first order derivatives of observer output vector $\hat{y}$ and system output vector y can be obtained from system equation (3.1) and observer equation (3.2)

$$\dot{\hat{y}}(t) = \bar{B}(\hat{x}) + \bar{A}(\hat{x})u(t) - v(t) \tag{3.3}$$

$$\dot{y}(t) = \bar{B}(x) + \bar{A}(x)u(t) + \Delta\bar{F}(x) \tag{3.4}$$

where:

$$\bar{B}(\hat{x}) = \begin{bmatrix} \frac{\partial h_1}{\partial x}(\hat{x})\xi \\ \vdots \\ \frac{\partial h_m}{\partial x}(\hat{x})\xi \end{bmatrix} \qquad \bar{B}(x) = \begin{bmatrix} \frac{\partial h_1}{\partial x}(x)\xi \\ \vdots \\ \frac{\partial h_m}{\partial x}(x)\xi \end{bmatrix} \qquad \Delta\bar{F}(x) = \begin{bmatrix} \frac{\partial h_1}{\partial x}(x)\delta f \\ \vdots \\ \frac{\partial h_m}{\partial x}(x)\delta f \end{bmatrix}$$

$$\bar{A}(\hat{x}) = \begin{bmatrix} \frac{\partial h_1}{\partial x}(\hat{x})g_1 & \cdots & \frac{\partial h_1}{\partial x}(\hat{x})g_m \\ \vdots & \cdots & \vdots \\ \frac{\partial h_m}{\partial x}(\hat{x})g_1 & \cdots & \frac{\partial h_m}{\partial x}(\hat{x})g_m \end{bmatrix} \qquad \bar{A}(x) = \begin{bmatrix} \frac{\partial h_1}{\partial x}(x)g_1 & \cdots & \frac{\partial h_1}{\partial x}(x)g_m \\ \vdots & \cdots & \vdots \\ \frac{\partial h_m}{\partial x}(x)g_1 & \cdots & \frac{\partial h_m}{\partial x}(x)g_m \end{bmatrix}.$$

Remark 3.3.1 From the viewpoint of nonlinear system theory [64], either equation (3.3) or (3.4) implies that the considered system has relative degree one because $\bar{A}(\cdot) \neq \mathbf{0}$.

If output estimate errors are defined as $e(t) = y(t) - \hat{y}(t)$, then it is straightforward to show that

$$\dot{e}(t) = (\bar{B}(x) - \bar{B}(\hat{x})) + (\bar{A}(x) - \bar{A}(\hat{x}))u(t) + \Delta\bar{F}(x) + v(t). \tag{3.5}$$

In what follows, the switching term $v(t)$ will be derived. To this end, the relationship between sliding surface dynamics and output errors can be selected as a second order differential equation for which the so-called SOSMO is named:

$$\ddot{S} + z_0\dot{S} = \dot{e} + ce \tag{3.6}$$

where $S = col[s_1, s_2, \cdots, s_m]$, the sliding surface. Parameters c and z_0 are two constant coefficients.

Substituting equation (3.5) into (3.6), we have:

$$\begin{aligned} \ddot{S} + z_0\dot{S} &= \dot{e} + ce \\ &= (\bar{B}(x) - \bar{B}(\hat{x})) + (\bar{A}(x) - \bar{A}(\hat{x}))u + \Delta\bar{F}(x) + v + ce \\ &= \delta\bar{B} + \delta\bar{A}u + \Delta\bar{F} + v + ce \end{aligned} \tag{3.7}$$

where

$$\delta\bar{B} = \bar{B}(x) - \bar{B}(\hat{x}), \qquad \delta\bar{A} = \bar{A}(x) - \bar{A}(\hat{x}).$$

A Lyapunov function candidate is taken to create the attractivity condition as follows:

$$V = \frac{1}{2}(\dot{S}^T\dot{S} + S^T\Omega S), \quad \Omega = Diag(w) \tag{3.8}$$

where w is a positive constant.

Differentiating equation (3.8), we obtain:

$$\dot{V} = \dot{S}^T(\ddot{S} + wS). \tag{3.9}$$

To constitute the attractivity condition for the sliding mode towards $dS/dt = S = 0$, the following inequality should hold [41]:

$$\dot{S}^T(\ddot{S} + wS) \leq 0. \tag{3.10}$$

Substituting equation (3.7) into the above equation, we have:

$$\dot{S}^T[\delta\bar{B} + \delta\bar{A}u + \Delta\bar{F} + v + ce - z_0\dot{S} + wS] \leq 0. \tag{3.11}$$

If the switching term $v(t)$ is chosen as $v = z_0\dot{S} - ce - wS - d \cdot sgn(\dot{S})$, equation (3.11) can be further simplified as

$$\dot{S}^T[\delta\bar{B} + \delta\bar{A}u + \Delta\bar{F} - d \cdot sgn(\dot{S})] \leq 0. \tag{3.12}$$

Noticing that $\dot{S}^T sgn(\dot{S}) \geq \|\dot{S}\|$ and consequently $-d\dot{S}^T sgn(\dot{S}) \leq -d\|\dot{S}\|$, therefore using this inequality, equation (3.12) can be further derived by using vector norms

$$\dot{V} \leq \|\dot{S}\|(\|\delta\bar{B}\| + \|\delta\bar{A}\|\|u\| + \|\Delta\bar{F}\| - d) \leq 0, \tag{3.13}$$

therefore, if

$$d \geq \|\delta\bar{B}\| + \|\delta\bar{A}\|\|u\| + \gamma_{\delta F}\left\|\frac{\partial h(x)}{\partial x}\right\| \tag{3.14}$$

where $\gamma_{\delta F}$ is the norm bound of known function $\eta(x)$, then,

$$\dot{V} \leq 0, \tag{3.15}$$

which means that output errors are kept sliding on the sliding surface. □

The faulty uncertain nonlinear systems can be in the following form:

$$\begin{aligned} \dot{x}(t) &= \xi(x) + \delta f(x,t) + G(x)u(t) + \zeta(t) \\ y_j(t) &= h_j(x), \qquad j = 1, \cdots, m \end{aligned} \tag{3.16}$$

where $\zeta(t)$ represents system faults such as actuator faults or aged components.

We could take $e(t)$, the output estimation error between the SOSMO and the considered uncertain nonlinear system, as a residual.

If there are no faults in the uncertain nonlinear system, and a proper gain d has been selected, then the SOSMO should be sliding and the residual should be zero. Once there exists a fault, if we have chosen a proper gain d such that the SOSMO is robust to disturbances and uncertainties while sensitive to faults, the residual will then generate a fault alarm signal.

As we can see from equation (3.7), $v(t)$ is discontinuous across $\dot{S} = 0$, and $\dot{S}$ is accordingly discontinuous, which leads to chattering. If we use $\dot{S}$ as a residual, the chattering is not desirable. To smooth out the discontinuity, a boundary layer ϕ neighboring the switching surface $\dot{S}$ is introduced [11]. A saturation function (sat) is used to replace signum function. The saturation function is defined as

$$sat(\frac{\dot{S}}{\phi}) = \left[sat(\frac{\dot{s}_1}{\phi}), sat(\frac{\dot{s}_2}{\phi}), \cdots, sat(\frac{\dot{s}_m}{\phi})\right]^T \tag{3.17}$$

and

$$sat(\frac{\dot{s}_i}{\phi}) = \begin{cases} sgn(\frac{\dot{s}_i}{\phi}), & \text{when } |\dot{s}_i| \geq \phi \\ \frac{\dot{s}_i}{\phi}, & \text{when } |\dot{s}_i| < \phi. \end{cases} \tag{3.18}$$

The S dynamics outside the boundary layer can be obtained by substituting $v(t)$ into equation (3.7) as

$$\ddot{S} + wS + dsgn(\dot{S}) = \delta\bar{B} + \delta\bar{A}u + \Delta\bar{F}. \tag{3.19}$$

Within the boundary layer, the S dynamics has the form of

$$\ddot{S} + wS + d\frac{\dot{S}}{\phi} = \delta\bar{B} + \delta\bar{A}u + \Delta\bar{F}. \tag{3.20}$$

Equation (3.20) represents a low-pass filter [11, 27] that can block high-frequency signals while letting all low-frequency signals pass. So, as long as the fault is not a high-frequency signal, it will have an impact on S dynamics. Therefore, S can also be selected as a residual. In fact, $\dot{S}$ can also be selected as a residual because once a fault occurs, the $\dot{S}$ will stop sliding, thereby enabling us to detect faults, which will be seen in the simulations.

Remark 3.3.2 According to equation (3.14), we know that if the chosen d is big enough, then the system output errors can be guaranteed to slide. On the other hand, if d is too big, the chattering will become an especially important issue and, in addition, the sliding mode observer may become robust to faults as well. That is, when a fault occurs, the residual may not be able to produce an alarm signal. Therefore, the selection of gain d needs careful consideration.

3.3.1 Stability Analysis of the SOSMO with a Single Output

Under some conditions, a nonlinear system can be transformed to a nonlinear system with linear outputs [123]. Therefore, assume that the considered nonlinear system has an output variable $y = x_1$. Based on this assumption, the stability of the proposed SOSMO is to be proved.

Expanding the SOSMO (3.2) according to each observer state variable leads to the following:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{f}_1 - k_1(t)v \\ \dot{\hat{x}}_2 &= \hat{f}_2 - k_2(t)v \\ &\vdots \\ \dot{\hat{x}}_n &= \hat{f}_n - k_n(t)v \end{aligned} \tag{3.21}$$

where $[k_1(t), k_2(t), \cdots, k_n(t)]^T = L(t)\left[\frac{\partial h(\hat{x})}{\partial \hat{x}} L(t)\right]^{-1}$; v is the discontinuous term in the SOSMO; and $[\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_n]^T = \xi(\hat{x}) + G(\hat{x})u$.

Let $\tilde{x}_i = x_i - \hat{x}_i, i = 1, \cdots, n$ and subtract equation (3.21) from system equation (3.1), we have

$$\begin{aligned} \dot{\tilde{x}}_1 &= \Delta f_1 + \delta f_1 + k_1(t)v \\ \dot{\tilde{x}}_2 &= \Delta f_2 + \delta f_2 + k_2(t)v \\ &\vdots \\ \dot{\tilde{x}}_n &= \Delta f_n + \delta f_n + k_n(t)v \end{aligned} \tag{3.22}$$

where $[\Delta f_1, \Delta f_2, \cdots, \Delta f_n]^T = [f_1, f_2, \cdots, f_n]^T - [\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_n]^T = (\xi(x) - \xi(\hat{x})) + (G(x)u - G(\hat{x})u)$, and δf_i is the disturbance, $i = 1, \cdots, n$.

Based on the following lemma, Theorem 1 will be presented.

Lemma 3.3.1 The coefficient $k_1(t)$ in estimation error dynamics equation (3.22) is equal to 1.

Proof: The proof is straightforward.

Because the output is $x_1(t)$, we have the following calculation:

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} = L(t)\left[\frac{\partial h}{\partial x}L(t)\right]^{-1} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix}(l_1)^{-1} = \begin{bmatrix} 1 \\ l_2/l_1 \\ \vdots \\ l_n/l_1 \end{bmatrix}$$

where $l_1 \neq 0$. ∎

Recall that inequality (3.14) can guarantee the output errors to reach the sliding surface $S(t)$ and be kept sliding on it, i.e. $\tilde{x}_1$ is zero on this surface. Applying the concept of equivalent dynamics in accordance with [118], we have the reduced estimation error dynamics in the form of

$$\begin{aligned} \dot{\tilde{x}}_2 &= \Delta f_2 + \delta f_2 - \tfrac{l_2}{l_1}(t)(\Delta f_1 + \delta f_1) \\ \dot{\tilde{x}}_3 &= \Delta f_3 + \delta f_3 - \tfrac{l_3}{l_1}(t)(\Delta f_1 + \delta f_1) \\ &\vdots \\ \dot{\tilde{x}}_n &= \Delta f_n + \delta f_n - \tfrac{l_n}{l_1}(t)(\Delta f_1 + \delta f_1). \end{aligned} \tag{3.23}$$

By expanding $\Delta f_1, \Delta f_2, \cdots, \Delta f_n$ into power series, we get the following differential form of the above equation:

$$\begin{aligned} \dot{\tilde{x}}_2 &= [\tfrac{\partial f_2}{\partial x_2} - \tfrac{l_2}{l_1}\tfrac{\partial f_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\tfrac{\partial f_2}{\partial x_n} - \tfrac{l_2}{l_1}\tfrac{\partial f_1}{\partial x_n}]\tilde{x}_n + \phi_2 + (\delta f_2 - \tfrac{l_2}{l_1}\delta f_1) \\ \dot{\tilde{x}}_3 &= [\tfrac{\partial f_3}{\partial x_2} - \tfrac{l_3}{l_1}\tfrac{\partial f_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\tfrac{\partial f_3}{\partial x_n} - \tfrac{l_3}{l_1}\tfrac{\partial f_1}{\partial x_n}]\tilde{x}_n + \phi_3 + (\delta f_3 - \tfrac{l_3}{l_1}\delta f_1) \\ &\vdots \\ \dot{\tilde{x}}_n &= [\tfrac{\partial f_n}{\partial x_2} - \tfrac{l_n}{l_1}\tfrac{\partial f_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\tfrac{\partial f_n}{\partial x_n} - \tfrac{l_n}{l_1}\tfrac{\partial f_1}{\partial x_n}]\tilde{x}_n + \phi_n + (\delta f_n - \tfrac{l_n}{l_1}\delta f_1) \end{aligned} \tag{3.24}$$

where $\phi_i, i = 2, \cdots, n$, are the terms of second and higher order in $(x_i - \hat{x}_i)$.

Let $\tilde{x} = [\tilde{x}_2, \tilde{x}_3, \cdots, \tilde{x}_n]^T$, we have:

$$\dot{\tilde{x}} = A(t)\tilde{x} + \Phi + \delta F \tag{3.25}$$

where $\delta F = [(\delta f_2 - \frac{l_2}{l_1}\delta f_1), (\delta f_3 - \frac{l_3}{l_1}\delta f_1), \cdots, (\delta f_n - \frac{l_n}{l_1}\delta f_1)]^T$, $\Phi = [\phi_2, \cdots, \phi_n]^T$, and

$$A(t) = \begin{bmatrix} \frac{\partial f_2}{\partial x_2} - \frac{l_2}{l_1}\frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} - \frac{l_2}{l_1}\frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_3}{\partial x_2} - \frac{l_3}{l_1}\frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_3}{\partial x_n} - \frac{l_3}{l_1}\frac{\partial f_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_2} - \frac{l_n}{l_1}\frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} - \frac{l_n}{l_1}\frac{\partial f_1}{\partial x_n} \end{bmatrix}$$

$$= \begin{pmatrix} \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \cdots & \frac{\partial f_3}{\partial x_n} \\ & & \cdots & \\ \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} - \begin{pmatrix} \frac{l_2}{l_1} \\ \frac{l_3}{l_1} \\ \vdots \\ \frac{l_n}{l_1} \end{pmatrix} \left(\frac{\partial f_1}{\partial x_2} \quad \frac{\partial f_1}{\partial x_3} \cdots \frac{\partial f_1}{\partial x_n} \right).$$

Assume that gains $l_i(t), i = 1, \cdots, n$ can be chosen such that matrix $A(t)$ is a stability matrix and there exists a positive definite symmetric matrix P such that

$$PA(t) + A^T(t)P = -Q \tag{3.26}$$

where Q is a positive definite matrix.

Let us further consider a Lyapunov function candidate

$$V = \tilde{x}^T P \tilde{x}. \tag{3.27}$$

Differentiating it, we get:

$$
\begin{aligned}
\dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} \\
&= (\tilde{x}^T A^T + \Phi^T + \delta F^T)P\tilde{x} + \tilde{x}^T P(A\tilde{x} + \Phi + \delta F) \\
&= \tilde{x}^T A^T P\tilde{x} + \Phi^T P\tilde{x} + \delta F^T P\tilde{x} + \tilde{x}^T PA\tilde{x} + \tilde{x}^T P\Phi + \tilde{x}^T P\delta F \\
&= \tilde{x}^T(A^T P + PA)\tilde{x} + \Phi^T P\tilde{x} + \delta F^T P\tilde{x} + \tilde{x}^T P\Phi + \tilde{x}^T P\delta F.
\end{aligned}
\tag{3.28}
$$

Considering equation (3.26), the above equation can be further extended as

$$
\begin{aligned}
\dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\gamma_\phi\|P\|\|\tilde{x}\|^2 + 2\gamma_{\delta F}\|P\|\|\tilde{x}\| \\
&= ((-\lambda_{min}(Q) + 2\gamma_\phi\|P\|)\|\tilde{x}\| + 2\gamma_{\delta F}\|P\|)\|\tilde{x}\|
\end{aligned}
\tag{3.29}
$$

where $\gamma_{\delta F}$ is the norm bound of known function $\eta(x)$. In the derivation process above, inequality $\|\Phi\| \leq \gamma_\phi\|\tilde{x}\|$ is used. Therefore, if $\forall\|\tilde{x}\| \geq \frac{2\gamma_{\delta F}\|P\|}{(\lambda_{min}(Q)-2\gamma_\phi\|P\|)}$ holds, then the reduced state estimation error is bounded.

The above results can now be summarized in the following theorem.

Theorem 3.1 *Consider uncertain nonlinear system (3.1) with a single output and its SOSMO defined in equation (3.2). If inequality (3.14) and equation (3.26) hold, then the system state estimation error is bounded.*

3.3.2 Stability Analysis of the SOSMO with Multiple Outputs.

For a multi-output nonlinear system, the SOSMO has the same construction as that in equation (3.2). Nevertheless, in this case the observer gain matrix $L(t)$ is an $n \times m$ matrix, where m is the number of outputs. In addition, output estimation error vector $e(t)$, discontinuous term v, and sliding surface $S(t)$ are m dimension vectors.

For simplicity of stability derivation, assume that the system outputs $y = [x_1,\ x_2]^T$. Under this circumstance, rewrite SOSMO (3.2) as follows

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{f}_1 - k_{11}v_1 - k_{12}v_2 \\ \dot{\hat{x}}_2 &= \hat{f}_2 - k_{21}v_1 - k_{22}v_2 \\ &\vdots \\ \dot{\hat{x}}_n &= \hat{f}_n - k_{n1}v_1 - k_{n2}v_2 \end{aligned} \tag{3.30}$$

where $[\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_n]^T = \xi(\hat{x}) + G(\hat{x})u(t)$ and $u \in \mathbb{R}^2$.

Subtracting equation (3.30) from systems (3.1), we have

$$\begin{aligned} \dot{\tilde{x}}_1 &= \Delta f_1 + \delta f_1 + k_{11}v_1 + k_{12}v_2 \\ \dot{\tilde{x}}_2 &= \Delta f_2 + \delta f_2 + k_{21}v_1 + k_{22}v_2 \\ &\vdots \\ \dot{\tilde{x}}_n &= \Delta f_n + \delta f_n + k_{n1}v_1 + k_{n2}v_2 \end{aligned} \tag{3.31}$$

where $[\Delta f_1, \Delta f_2, \cdots, \Delta f_n]^T = [f_1, f_2, \cdots, f_n]^T - [\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_n]^T = (\xi(x) - \xi(\hat{x})) + (G(x)u - G(\hat{x})u)$, and δf_i is the disturbance, $i = 1, \cdots, n$.

In the following lemma, the coefficients k_{ij} in equation (3.31) are about to be derived according to each component of matrix $L(t)$. Also, based on the following lemma, Theorem 2 will be stated.

Lemma 3.3.2 In equation (3.31), the coefficients $k_{11} = k_{22} = 1$ and $k_{12} = k_{21} = 0$.

Proof: The proof process is straightforward. Starting with

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \\ \vdots & \vdots \\ k_{n1} & k_{n2} \end{bmatrix} = L(t)[\frac{\partial h}{\partial x}L(t)]^{-1} = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \\ l_{31} & l_{32} \\ \vdots & \vdots \\ l_{n1} & l_{n2} \end{bmatrix} \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}^{-1} \tag{3.32}$$

and considering

$$\begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{l_{22}}{\Delta} & -\frac{l_{12}}{\Delta} \\ -\frac{l_{21}}{\Delta} & \frac{l_{11}}{\Delta} \end{bmatrix} \tag{3.33}$$

where $\Delta = l_{11}l_{22} - l_{12}l_{21} \neq 0$,

we have

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \\ \vdots & \vdots \\ k_{n1} & k_{n2} \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \\ l_{31} & l_{32} \\ \vdots & \vdots \\ l_{n1} & l_{n2} \end{bmatrix} \begin{bmatrix} \frac{l_{22}}{\Delta} & -\frac{l_{12}}{\Delta} \\ -\frac{l_{21}}{\Delta} & \frac{l_{11}}{\Delta} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{l_{31}l_{22}-l_{32}l_{21}}{\Delta} & \frac{l_{32}l_{11}-l_{31}l_{12}}{\Delta} \\ \vdots & \vdots \\ \frac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta} & \frac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta} \end{bmatrix}. \tag{3.34}$$

∎

The dynamics of state errors x_1 and x_2 is

$$\begin{aligned} \dot{\tilde{x}}_1 &= \Delta f_1 + \delta f_1 + v_1 \\ \dot{\tilde{x}}_2 &= \Delta f_2 + \delta f_2 + v_2. \end{aligned} \tag{3.35}$$

From the above equation, the equivalent controls can be obtained:

$$(v_1)_{eq} = -(\Delta f_1 + \delta f_1), \qquad (v_2)_{eq} = -(\Delta f_2 + \delta f_2). \tag{3.36}$$

Redefine $\tilde{x} = [\tilde{x}_3, \tilde{x}_4, \cdots, \tilde{x}_n]^T$ and its dynamics is:

$$\begin{aligned} \dot{\tilde{x}}_3 &= \Delta f_3 + \delta f_3 + \tfrac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}[-(\Delta f_1 + \delta f_1)] + \tfrac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}[-(\Delta f_2 + \delta f_2)] \\ \dot{\tilde{x}}_4 &= \Delta f_4 + \delta f_4 + \tfrac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}[-(\Delta f_1 + \delta f_1)] + \tfrac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}[-(\Delta f_2 + \delta f_2)] \\ &\vdots \\ \dot{\tilde{x}}_n &= \Delta f_n + \delta f_n + \tfrac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}[-(\Delta f_1 + \delta f_1)] + \tfrac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}[-(\Delta f_2 + \delta f_2)]. \end{aligned} \tag{3.37}$$

The expansions of $\Delta f_1, \Delta f_2, \cdots, \Delta f_n$ into power series lead to:

$$\begin{aligned}
\dot{\tilde{x}}_3 &= [\tfrac{\partial f_3}{\partial x_3} - \tfrac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}\tfrac{\partial f_1}{\partial x_3} - \tfrac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}\tfrac{\partial f_2}{\partial x_3}]\tilde{x}_3 + \cdots \\
&\quad +[\tfrac{\partial f_3}{\partial x_n} - \tfrac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}\tfrac{\partial f_1}{\partial x_n} - \tfrac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}\tfrac{\partial f_2}{\partial x_n}]\tilde{x}_n + \bar{\phi}_3 \\
&\quad +\delta f_3 - \tfrac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}\delta f_1 - \tfrac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}\delta f_2 \\
\dot{\tilde{x}}_4 &= [\tfrac{\partial f_4}{\partial x_3} - \tfrac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}\tfrac{\partial f_1}{\partial x_3} - \tfrac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}\tfrac{\partial f_2}{\partial x_3}]\tilde{x}_3 + \cdots \\
&\quad +[\tfrac{\partial f_4}{\partial x_n} - \tfrac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}\tfrac{\partial f_1}{\partial x_n} - \tfrac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}\tfrac{\partial f_2}{\partial x_n}]\tilde{x}_n + \bar{\phi}_4 \qquad (3.38) \\
&\quad +\delta f_4 - \tfrac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}\delta f_1 - \tfrac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}\delta f_2 \\
&\vdots \qquad\qquad \vdots \\
\dot{\tilde{x}}_n &= [\tfrac{\partial f_n}{\partial x_3} - \tfrac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}\tfrac{\partial f_1}{\partial x_3} - \tfrac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}\tfrac{\partial f_2}{\partial x_3}]\tilde{x}_3 + \cdots \\
&\quad +[\tfrac{\partial f_n}{\partial x_n} - \tfrac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}\tfrac{\partial f_1}{\partial x_n} - \tfrac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}\tfrac{\partial f_2}{\partial x_n}]\tilde{x}_n + \bar{\phi}_n \\
&\quad +\delta f_n - \tfrac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}\delta f_1 - \tfrac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}\delta f_2
\end{aligned}$$

where $\phi_i, i = 3, \cdots, n$ is the term of second and higher order in $(x_i - \hat{x}_i)$.

Let

$$B(t) \triangleq \begin{bmatrix}
\frac{\partial f_3}{\partial x_3} - \frac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}\frac{\partial f_1}{\partial x_3} - \frac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}\frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_3}{\partial x_n} - \frac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}\frac{\partial f_1}{\partial x_n} - \frac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}\frac{\partial f_2}{\partial x_n} \\
\frac{\partial f_4}{\partial x_3} - \frac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}\frac{\partial f_1}{\partial x_3} - \frac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}\frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_4}{\partial x_n} - \frac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}\frac{\partial f_1}{\partial x_n} - \frac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}\frac{\partial f_2}{\partial x_n} \\
\cdots & \cdots & \cdots \\
\frac{\partial f_n}{\partial x_3} - \frac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}\frac{\partial f_1}{\partial x_3} - \frac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}\frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_n}{\partial x_n} - \frac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}\frac{\partial f_1}{\partial x_n} - \frac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}\frac{\partial f_2}{\partial x_n}
\end{bmatrix}$$

$\in \mathbf{R}^{(n-2)\times(n-2)}$.

The reduced error dynamics can be as follows

$$\dot{\tilde{x}} = B(t)\tilde{x} + \bar{\Phi} + \delta D \qquad (3.39)$$

where

$$\bar{\Phi} = [\bar{\phi_3}, \cdots, \bar{\phi_n}]^T$$

and

$$\delta D = \begin{bmatrix} \delta f_3 - \frac{l_{31}l_{22}-l_{32}l_{21}}{\Delta}\delta f_1 - \frac{l_{32}l_{11}-l_{31}l_{12}}{\Delta}\delta f_2 \\ \delta f_4 - \frac{l_{41}l_{22}-l_{42}l_{21}}{\Delta}\delta f_1 - \frac{l_{42}l_{11}-l_{41}l_{12}}{\Delta}\delta f_2 \\ \vdots \\ \delta f_n - \frac{l_{n1}l_{22}-l_{n2}l_{21}}{\Delta}\delta f_1 - \frac{l_{n2}l_{11}-l_{n1}l_{12}}{\Delta}\delta f_2 \end{bmatrix}.$$

In the following, assume that gains $l_{ij}, i = 1, \cdots, n, j = 1, 2$ can be chosen such that matrix $B(t)$ is a stability matrix and there exists a positive definite symmetric matrix $\Pi(t)$ such that

$$\Pi B(t) + B^T(t)\Pi = -R \tag{3.40}$$

where R is a positive definite matrix.

Considering a Lyapunov function candidate $V = \tilde{x}^T\Pi\tilde{x}$, it follows that:

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(R)\|\tilde{x}\|^2 + 2\gamma_{\bar{\phi}}\|\Pi\|\|\tilde{x}\|^2 + 2\gamma_{\delta D}\|\Pi\|\|\tilde{x}\| \\ &= ((-\lambda_{min}(R) + 2\gamma_{\bar{\phi}}\|\Pi\|)\|\tilde{x}\| + 2\gamma_{\delta D}\|\Pi\|)\|\tilde{x}\| \end{aligned} \tag{3.41}$$

where $\gamma_{\delta D}, \gamma_{\bar{\phi}}$ are the norm bounds of δD and $\bar{\Phi}$.

Furthermore, take a value $0 < \alpha < 1$, provided that the gains l_{ij} are such that

$$\lambda_{min}(R) - 2\gamma_{\bar{\phi}}\|\Pi\| - \alpha \geq 0 \tag{3.42}$$

and using the inequality (3.41), it is easy to show that

$$\dot{V} \leq -\alpha\|\tilde{x}\| \tag{3.43}$$

if

$$\|\tilde{x}\| \geq \varsigma \tag{3.44}$$

with

$$\varsigma = \frac{2\gamma_{\delta D}\|\Pi\|}{(\lambda_{min}(R) - 2\gamma_{\bar{\phi}}\|\Pi\| - \alpha)}.$$

From Khalil [69], this means that $\tilde{x}$ is convergent to the ball

$$\|\tilde{x}\| \leq \sqrt{\frac{\lambda_{max}(\Pi)}{\lambda_{min}(\Pi)}}\varsigma. \tag{3.45}$$

Therefore, given any $\varsigma^\star > \sqrt{\frac{\lambda_{max}(\Pi)}{\lambda_{min}(\Pi)}}\varsigma$ there exists a finite time T such that for all $t > T$ we have

$$\|\tilde{x}\| < \varsigma^\star.$$

Remark 3.3.3 Actually, the equivalent control $v_{eq} = [dsgn(\dot{S})]_{eq}$. Because, on the sliding surfaces, $S(t) = \dot{S}(t) = 0$ and $e = y - \hat{y} = 0$.

Theorem 3.2 *Assume that assumptions 3.1-3.5 and inequality (3.14) hold. If gains $l_{ij}, i = 3, \cdots, n, j = 1, 2$ are chosen such that (3.40) and (3.42) are satisfied, then the state estimation errors of the reduced sliding mode observer converge to the ball*

$$\|\tilde{x}\| < \varsigma^\star, \quad \forall \varsigma^\star > \sqrt{\frac{\lambda_{max}(\Pi)}{\lambda_{min}(\Pi)}}\varsigma. \tag{3.46}$$

3.4 Simulation Results

In this section, the above presented SOSMO for fault detection will be illustrated on a simple nonlinear system. These simulation results confirm that the SOSMO can be used to robustly detect system faults. A first order SMO based fault detection will be also presented for the purpose of comparison.

3.4.1 Fault Detection by a SOSMO

Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -2x_1 + x_1^2 x_2 \\ 0 \end{bmatrix} + \delta f(x,t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + f_a \\ y &= x_2 \end{aligned} \tag{3.47}$$

where f_a is an actuator fault with a size 0.5, occurring at time 10sec.

Uncertainties and disturbances are taken as

$$\delta f = \begin{bmatrix} 0.01(x_1 + x_2) + 0.02\mathrm{cos}x_1\mathrm{sin}t \\ 0.1x_1x_2 + 0.04\mathrm{sin}x_2\mathrm{sin}t \end{bmatrix}.$$

The SOSMO has the following form:

$$\begin{aligned} \dot{\hat{x}} &= \begin{bmatrix} -2\hat{x}_1 + \hat{x}_1^2 \hat{x}_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u - \begin{bmatrix} 1 \\ 1 \end{bmatrix} v \\ \hat{y} &= \hat{x}_2 \\ v &= 2\dot{S} + e - S - 0.35sgn(\dot{S}). \end{aligned} \tag{3.48}$$

Output estimate error $e(t) = x_2(t) - \hat{x}_2(t)$ and sliding surface $\dot{S}$ can be selected as residuals. The simulation results are shown in Figures 3.1-3.5.

Figures 3.1 and 3.2 describe that, when there exists no fault, this SOSMO can converge to the system model very accurately. In Figures 3.3 and 3.4, the residuals produce alarm signals after a fault occurs, i.e. the sliding mode observer stops sliding after the occurrence of a fault. The residuals fly to infinity, at which point the fault detection can be achieved. When the gain d is increased largely, the chattering of discontinuous term $v(t)$ becomes large. Meanwhile, see output error/residual in Figure 3.5, the residual cannot work anymore because the sliding observer can still slide even if a fault appears.

3.4.2 Fault Detection by a First Order SMO

A first order SMO is constructed according to [3, 123]

$$\begin{aligned}\dot{\hat{x}} &= \begin{bmatrix} -2\hat{x}_1 + \hat{x}_1^2\hat{x}_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u - \begin{bmatrix} k_1 sgn(y-\hat{y}) \\ k_2 sgn(y-\hat{y}) \end{bmatrix} \\ \hat{y} &= \hat{x}_2. \end{aligned} \tag{3.49}$$

The sliding surface is selected as a residual. As shown in Figure 3.6, the same fault as that used in the SOSMO can not be efficiently detected due to the chattering, even though this observer can converge to the system state equation.

3.5 Conclusions

This chapter has explored a SOSMO for the purpose of fault detection. It is the fact that the stability proof is hard work, especially the stability proof for nonlinear systems with multiple outputs since it needs a much greater workload. To guarantee the sliding of output errors, parameter d should be selected according to an inequality derived in Section 3. The key issue here is the selection of the observer gain d because it is the measure of uncertainties and disturbances. It must be taken properly. If it is too big, it muffles faults; if too small, it cannot guarantee the sliding of output estimation errors. This SOSMO is sensitive to small-sized faults because the S dynamics within the boundary layer is a continuous signal. That is, there is no chattering in there. In addition, the sliding surface dynamics forms a low-pass filter that can block high frequency signals caused by disturbances or modelling uncertainties, while letting all low frequency signals pass. As long as the faults are not high frequency signals, they will be successfully detected. The simple example has verified these results.

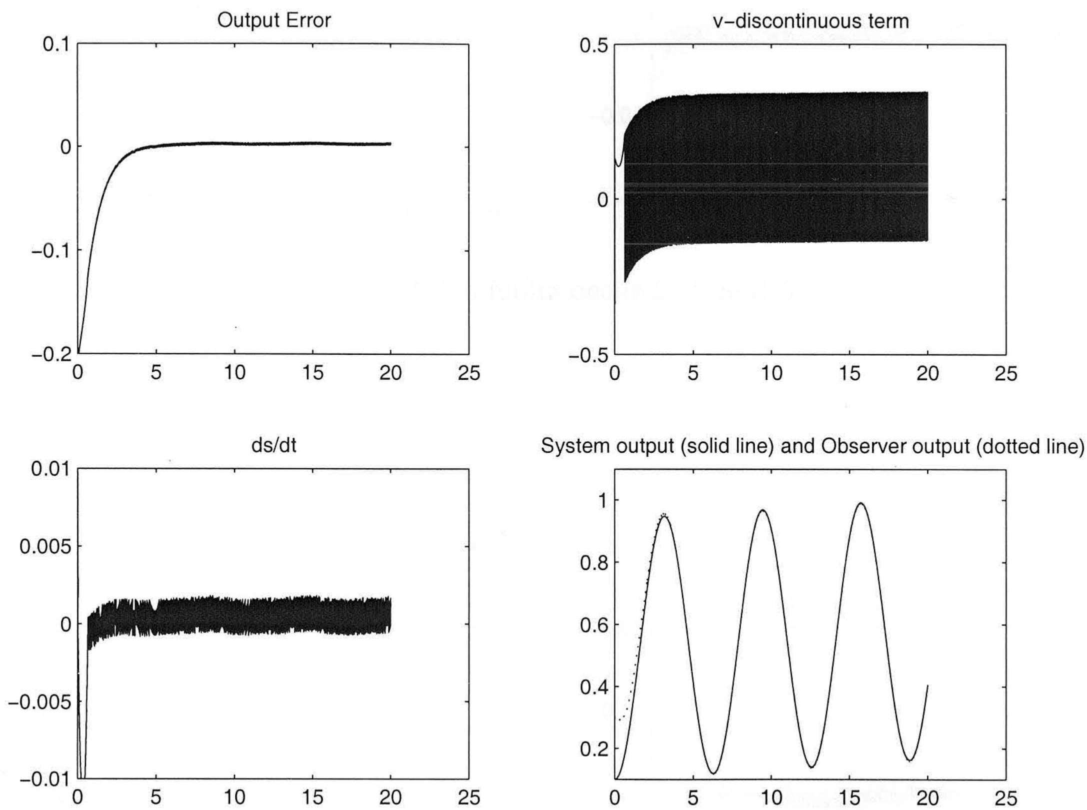

Figure 3.1: No faults occur-1.

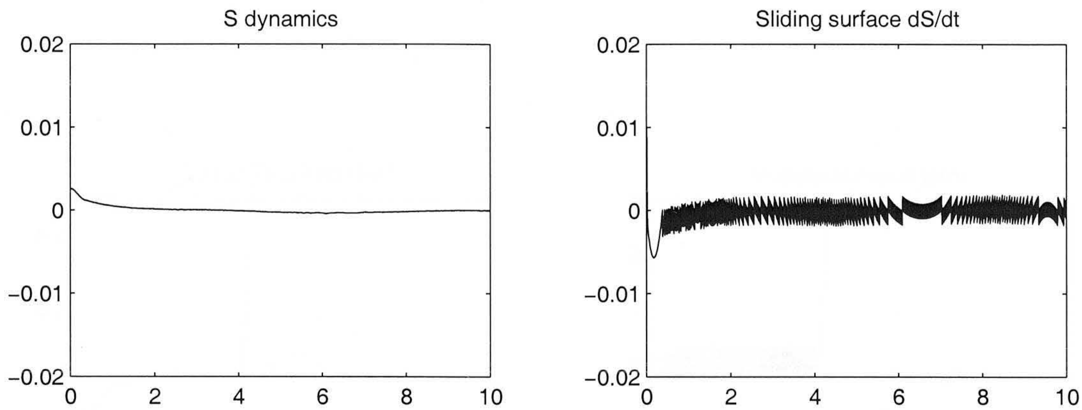

Figure 3.2: No faults occur-2: S and $\dot{S}$.

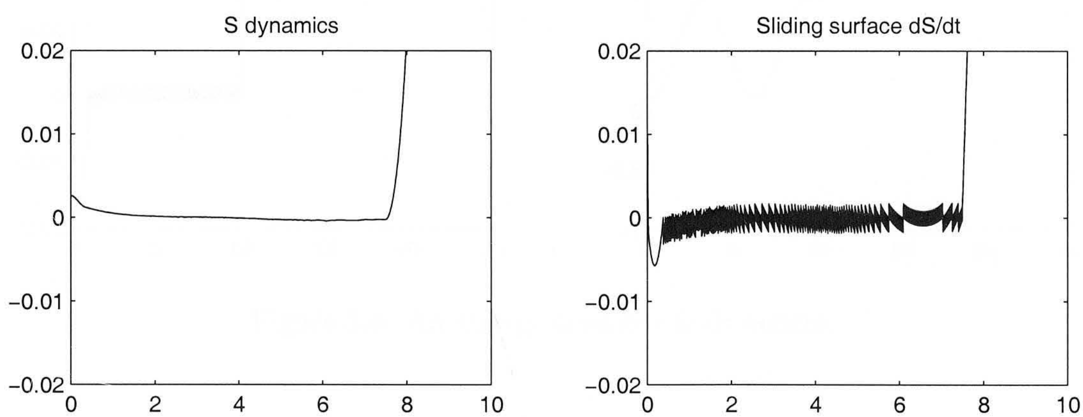

Figure 3.3: A fault occurs: S and $\dot{S}$.

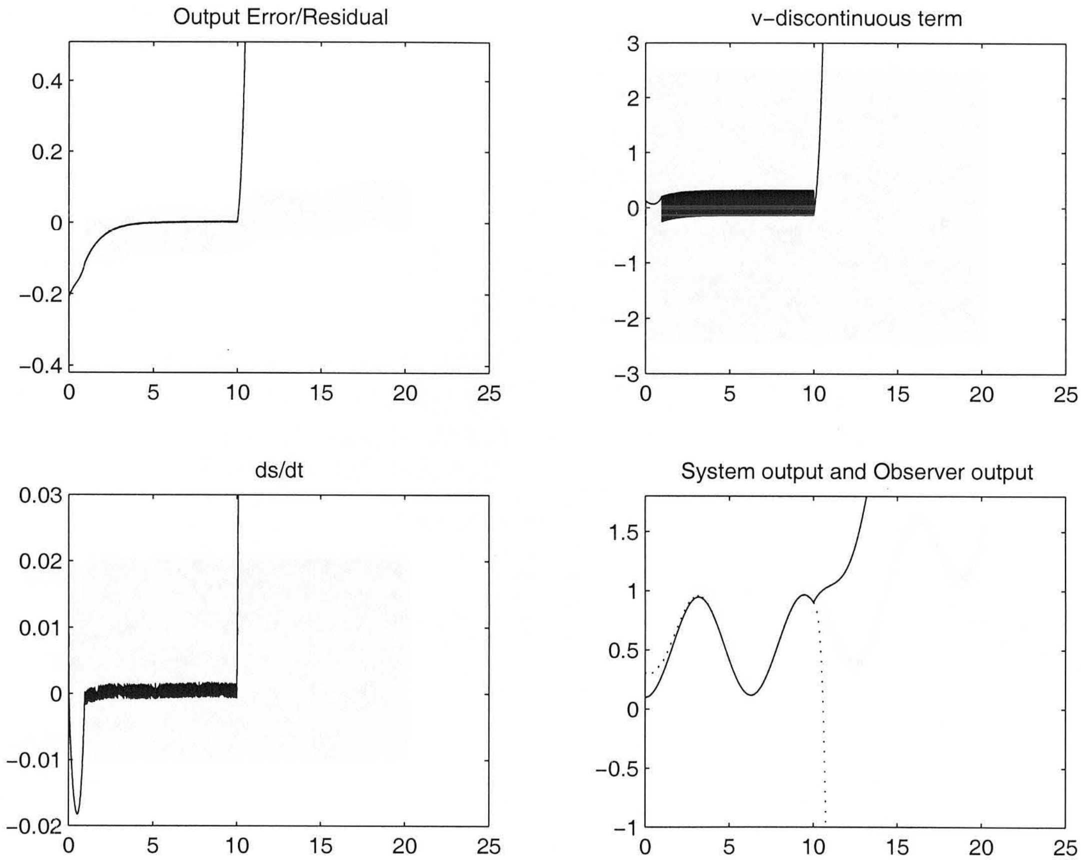

Figure 3.4: An abrupt actuator fault occurs.

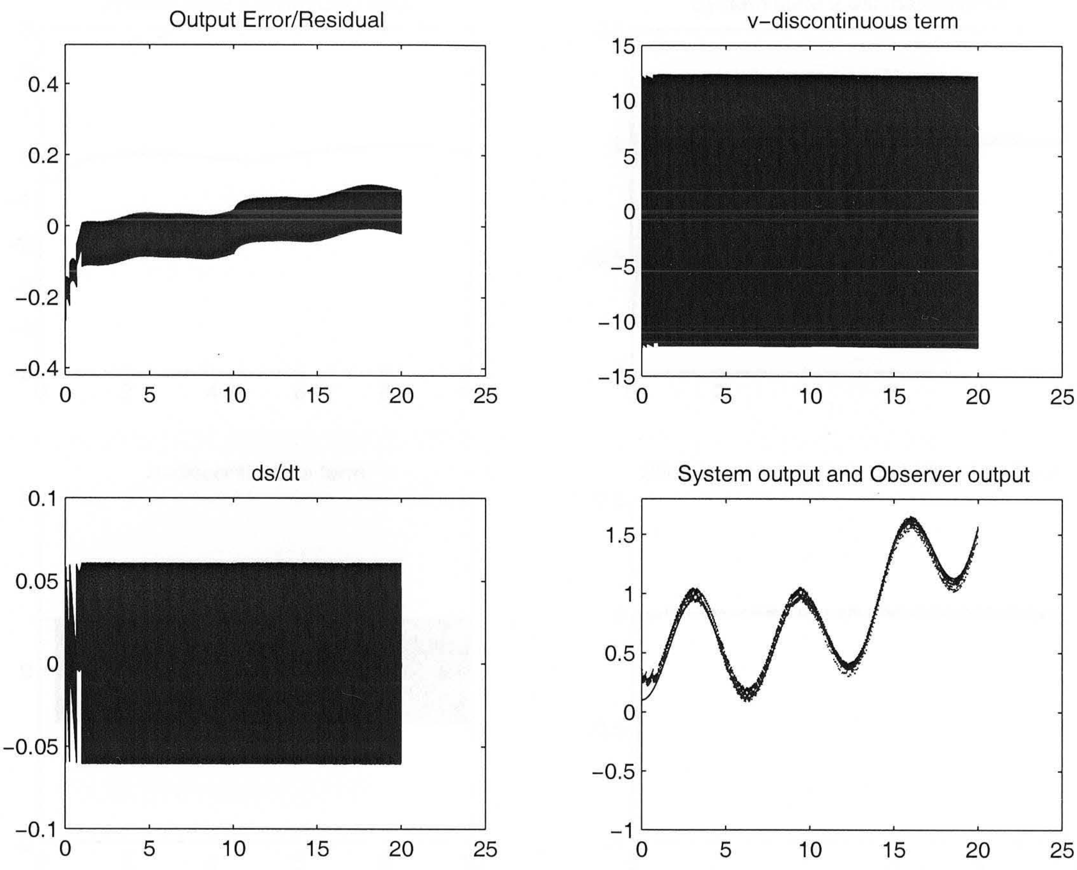

Figure 3.5: Gain d is chosen too big.

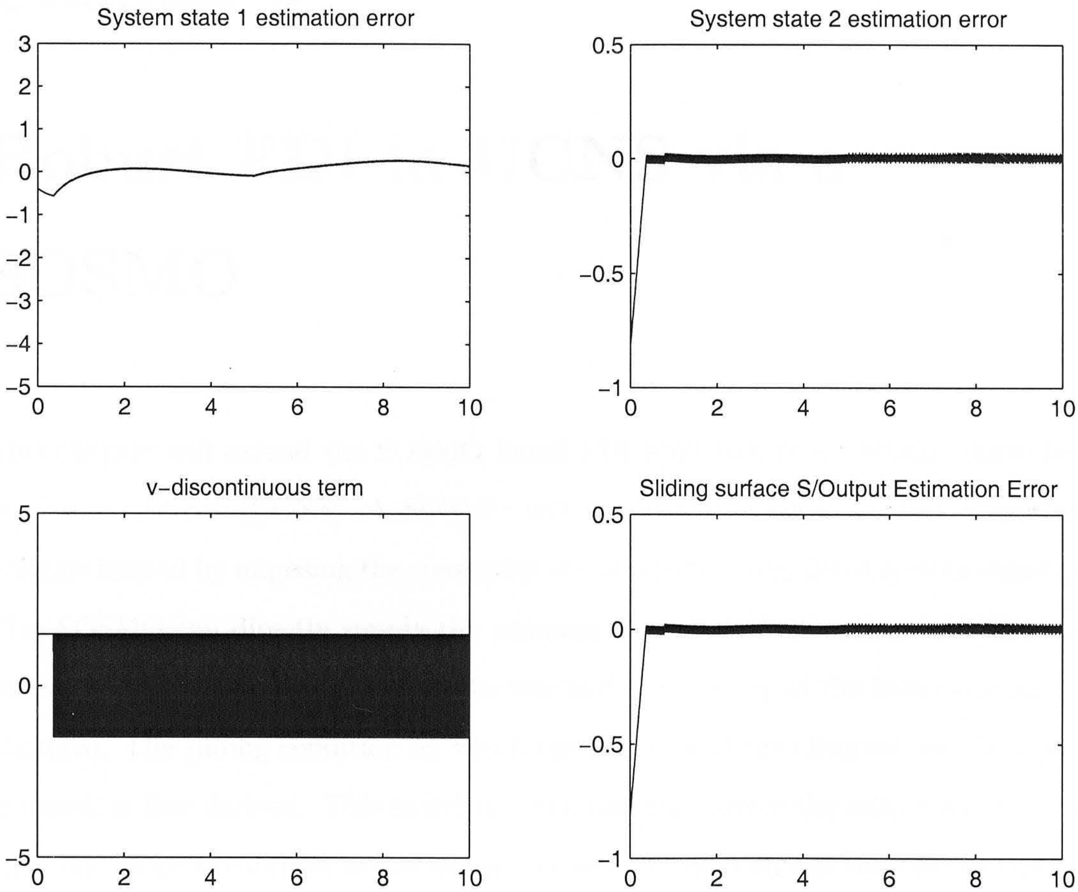

Figure 3.6: The first order SMO with a fault.

Chapter 4

Robust FDI in UCNS via a SOSMO

This chapter will extend the SOSMO based FDI approach to uncertain constrained nonlinear systems (UCNS). A SOSMO will be presented based on the closed-loop systems formed by imposing the constraint terms into the considered system equation. This SOSMO can directly supply the estimate of faults, which makes fault isolation easier. From the estimate of faults, the size and the severity of the faults can also be obtained. The sliding condition on which the selection of the observer switching gain is based, is first derived. This switching gain not only forces the output errors to be zero, but keeps the output errors at zero as well. To smooth out the discontinuity of the sliding surface, a saturation function is introduced to replace the signum function so that the sliding surface dynamics forms a low-pass filter. The stability of the reduced SOSMO resulting from the use of equivalent control concept is then proved by assuming that the considered UCNS has a single output and then two outputs, respectively. An example is employed to show that the proposed SOSMO can work very effectively. Especially for a relatively small-sized fault, this SOSMO can still efficiently detect the fault. In comparison to the SOSMO, the first order SMO cannot realize this task because of the chattering.

4.1 Introduction

Many practical systems can be modelled as constrained nonlinear systems, such as the mechanical systems with holonomic or nonholonomic constraints [75, 88], power systems [56], chemical processes [77], etc. A mobile robot moving on a surface [26], for example, is a typical constrained nonlinear system whose constraint is the specified surface. Little FDI work has been done in the constrained nonlinear systems. Therefore, research on fault diagnosis in constrained nonlinear systems is necessary, especially research on fault diagnosis in UCNS.

In this chapter, a SOSMO with fault estimation is to be constructed in a class of UCNS. The proposed SOSMO can achieve three tasks: first, it can achieve fault detection; second, fault estimation can be realized by the observer itself; and finally, fault isolation can also be performed through fault estimation.

4.2 Preliminaries

Consider a class of UCNS defined by the following differential equations

$$\begin{aligned} \dot{x}(t) &= f(x) + \sum_{j=1}^{m} g_j(x)u_j(t) + \omega(x)d(t) + E(x)f_a(t) \quad x(t) \in U \subset \mathbb{R}^n, \\ z_i(t) &= k_i(x) = 0, \quad i = 1, \ldots, l; \\ y_i(t) &= h_i(x), \qquad i = 1, \ldots, p; \end{aligned} \tag{4.1}$$

where $f(x), g_i(x), k_i(x)$ and $h_i(x)$ are analytic functions, $d(t) \in \mathbb{R}^n$ and $f_a(t) \in \mathbb{R}^\rho$ are disturbance and actuator fault, respectively. It is assumed that $f_a = constant$.

For convenience, we denote $G(x) = [g_1(x), \ldots, g_m(x)]$, $k(x) = [k_1(x), \ldots, k_l(x)]^T$, $u(t) = [u_1, \ldots, u_m]^T$ and $h(x) = [h_1(x), \ldots, h_p(x)]^T$. Suppose that for all $x(t) \in U$, vector fields $g_1(x), \ldots, g_m(x)$ are linearly independent, $[dk_1(x), \ldots, dk_l(x)]$ and $[dh_1(x), \ldots, dh_p(x)]$ are each linearly independent sets of covector fields and $m = l+p$. Further, $\omega(x) \in \mathbb{R}^{n\times n}$ and $E(x) \in \mathbb{R}^{n\times \rho}$ are distribution matrices of disturbances and faults, and $E(x)$ is full rank such that all faults can be detected.

Some notations to be used later are stated in the following:

The derivative of a scalar function $\phi(x)$ along a vector $f(x) = [f_1(x), \ldots, f_n(x)]^T$ can be expressed as

$$L_f\phi(x) = \frac{\partial \phi}{\partial x} f(x) = \sum_{i=1}^{n} \frac{\partial \phi}{\partial x_i} f_i(x) \tag{4.2}$$

where $x = [x_1, \ldots, x_n]^T$. The derivative of $\phi(x)$ taken first along $f(x)$ and then along a vector $g(x)$ is

$$L_g L_f \phi(x) = \frac{\partial (L_f \phi)}{\partial x} g(x). \tag{4.3}$$

If $\phi(x)$ is differentiated j times along $f(x)$, the notation $L_f^j\phi(x)$ is used with $L_f^0\phi(x) = \phi(x)$.

In reference to Z-H Li [79], we have the following definitions.

Definition 4.2.1 The constrained characteristic index r_i^ω of disturbances $d(t)$ is defined to be the least positive integer such that $L_{\omega_j} L_f^{r_i^\omega - 1} k_i(x) \neq 0$ for some $x \in U \subset \mathbb{R}^n$, $i = 1, \cdots, l, j = 1, \cdots n$.

Definition 4.2.2 The constrained characteristic index r_i^a of fault $f_a(t)$ is defined to be the least positive integer such that $L_{E_j} L_f^{r_i^a - 1} k_i(x) \neq 0$ for some $x \in U \subset \mathbb{R}^n$, $i = 1, \cdots, l, j = 1, \cdots \rho$.

Definition 4.2.3 The constrained characteristic r_i^c of system input $u(t)$ is defined to be the least positive integer such that $L_{g_j} L_f^{r_i^c - 1} k_i(x) \neq 0$, $i = 1, \cdots, l, j = 1, \cdots m$.

Throughout this chapter, system (4.1) satisfies the following assumptions:

Assumption 4.1 *The considered uncertain constrained nonlinear system output vector* $h(x) = [h_1(x), \cdots, h_p(x)]^T$ *and constraint vector* $k(x) = [k_1(x), \cdots, k_l(x)]^T$ *are* C^∞.

Assumption 4.2 *The system input vector* $u(t) = [u_1, \cdots, u_m]^T$ *is bounded.*

Assumption 4.3 *Matrix* $\frac{\partial h}{\partial x}$ *is bounded with* b_h*, and* $\frac{\partial h}{\partial x}E(x) \neq \mathbf{0}$.

Assumption 4.4 *The constrained characteristic indices satisfy* $r_i^\omega > r_i^c$ *and* $r_i^a > r_i^c$.

Assumption 4.5 *Matrices* $\frac{\partial h}{\partial x}(\hat{x})L_1(t)$ *and* $\frac{\partial h}{\partial x}(\hat{x})L_2(t)$ *are both nonsingular. Matrices* $L_1(t)$ *and* $L_2(t)$ *are SOSMO gains to be determined later.*

Assumption 4.6 *Both disturbance* $d(t)$ *and system actuator fault* $f_a(t)$ *are bounded with* b_d *and* b_f*, respectively.*

Assumption 4.4 explains that disturbance and fault terms do not affect the derivative of the constraint term, which assures that the $d(t)$ and $f_a(t)$ terms do not appear in the derivatives of k_i of order r_i^c.

Using assumption 4.4 and differentiating constraint term $k_i(x)$ as in [26], we can obtain the following equations

$$\begin{aligned}
\frac{dz_i}{dt} &= L_f k_i(x) = 0 \\
&\vdots \\
\frac{d^{r_i^c-1} z_i}{dt^{r_i^c-1}} &= L_f^{r_i^c-1} k_i(x) = 0 \\
\frac{d^{r_i^c} z_i}{dt^{r_i^c}} &= L_f^{r_i^c} k_i(x) + \sum_{j=1}^{m} L_{g_j} L_f^{r_i^c-1} k_i(x) u_j = 0
\end{aligned} \tag{4.4}$$

i.e.

$$\begin{aligned} k_i(x) &= L_f k_i(x) = \ldots = L_f^{r_i^c - 1} k_i(x) = 0 \\ b(x) &+ A(x)u(t) = 0 \end{aligned} \tag{4.5}$$

where $b(x) = [L_f^{r_1^c} k_1(x), \ldots, L_f^{r_l^c} k_l(x)]^T$, $A(x) = [L_{g_j} L_f^{r_i^c - 1} k_i(x)]_{l \times m}$. The solution of (4.5) can be written as a feedback law

$$u(t) = -A^+(x)b(x) + (I - A^+(x)A(x))\bar{u}(t) \qquad \bar{u}(t) \in \mathbb{R}^m \tag{4.6}$$

where $A^+(x) = A^T(x)(A(x)A^T(x))^{-1}$ is the pseudo-inverse of $A(x)$, I is an identity matrix and $\bar{u}(t)$ is a reference input.

Substituting feedback law (4.6) into UCNS (4.1) to combine the constraint into considered UCNS, we form a closed loop system

$$\dot{x} = (f(x) - G(x)A^+(x)b(x)) + G(x)(I - A^+(x)A(x))\bar{u} + \omega(x)d(t) + E(x)f_a(t) \tag{4.7}$$

on which our SOSMO will be based.

Remark 4.2.1 In assumption 4.4, the condition $r_i^a > r_i^c$ can guarantee that faults do not appear in the feedback law (4.6) such that one can combine the constraint term into considered UCNS system equation. In fact, this condition can be relaxed when the constrained characteristic index $r_i^c = 1$. Under this condition, feedback law (4.6) becomes

$$u = -A^+(x)(b(x) + \frac{\partial k}{\partial x}E(x)f_a) + (I - A^+(x)A(x))\bar{u} \tag{4.8}$$

where $A(x) = [L_{g_j} k_i(x)]_{l \times m}$ and $b(x) = [L_f k_1(x), \cdots, L_f k_l(x)]^T$.

Consequently, the closed-loop system equation (4.7) has a new form

$$\begin{aligned}\dot{x}(t) &= (f(x) - G(x)A^{+}(x)b(x)) + G(x)(I - A^{+}(x)A(x))\bar{u} + \omega(x)d(t) + \\ &\qquad (I - G(x)A^{+}\frac{\partial k}{\partial x})E(x)f_a(t).\end{aligned} \tag{4.9}$$

4.3 Main Results

In this section, based on system equation (4.7), we shall design a SOSMO with fault estimation for the purpose of FDI.

There is a clear intuitive link between sliding surfaces and fault detection. In the case of Sliding Mode, the sliding surfaces are made insensitive to internal or/and external disturbances (actually, the so-called sliding surface is just a differential equation that has a unique solution $e(t) = y - \hat{y} = \dot{e}(t) = 0$). This coincides with the requirement of a robust FDI observer where if there is no fault, the system output errors are kept sliding on the surface. This means $e(t) = \dot{e}(t) = 0$. When a fault occurs, the system output errors will stop sliding. This implies that the sliding has been destroyed.

The goal of this chapter is to design a SOSMO with the capability of fault estimation for a class of UCNS to diagnose system actuator faults. This SOSMO based FDI approach assures that the residual is robust to disturbances and modelling uncertainties, while remaining sensitive to system faults. The second order sliding condition will be used to design the SOSMO that is named after the second order sliding mode dynamics, a low-pass filter that can sharply reject unwanted high frequency signals due to unmodelled dynamics [11, 41].

To achieve robust fault detection and estimation for the uncertain constrained nonlinear systems despite the existence of uncertainties and disturbances, a SOSMO in reference to Chang [11] and Elmali et al. [41] is proposed as follows:

$$
\begin{aligned}
\dot{\hat{x}}(t) &= (f(\hat{x}) - G(\hat{x})A^{+}(\hat{x})b(\hat{x})) + G(\hat{x})(I - A^{+}(\hat{x})A(\hat{x}))\bar{u} \\
&\quad -L_1(t)(\frac{\partial h(\hat{x})}{\partial \hat{x}}L_1(t))^{-1}v(t) + L_2(t)(\frac{\partial h(\hat{x})}{\partial \hat{x}}L_2(t))^{-1}D\hat{f}_a(t) \\
\hat{y}(t) &= h(\hat{x}) \\
v(t) &= (D + \Gamma^T)\hat{f}_a(t) - ce(t) + z_0\dot{S} - wS - \kappa sgn(\dot{S}) \\
\dot{\hat{f}}_a(t) &= \Gamma\dot{S}
\end{aligned}
\tag{4.10}
$$

where D and Γ are $p \times \rho$ and $\rho \times p$ matrices, respectively. Matrices $L_1(t)$ and $L_2(t)$ are $n \times p$ observer gains. Parameters w, c and z_0 are some constants, $\dot{S}(t) \in \mathbb{R}^p$ is the sliding surface vector, sgn is a signum function and κ is a positive switching gain to be determined. The term $\hat{f}_a(t)$ is the estimate of actuator faults and $v(t)$ is the discontinuous term. Finally, $e(t) = y - \hat{y}$ is the output estimation error vector.

This SOSMO can directly estimate faults. The estimated faults, which can provide a direct estimate of the size and severity of the faults, can be used directly to isolate all faults.

The output estimation error dynamics can be obtained by differentiating system equation (4.7) and observer equation (4.10):

$$
\begin{aligned}
\dot{e}(t) &= (\frac{\partial h}{\partial x}f(x) - \frac{\partial h}{\partial x}G(x)A^{+}(x)b(x)) + \frac{\partial h}{\partial x}G(x)(I - A^{+}(x)A(x))\bar{u}(t) \\
&\quad +\frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) - (\frac{\partial h}{\partial \hat{x}}f(\hat{x}) - \frac{\partial h}{\partial \hat{x}}G(\hat{x})A^{+}(\hat{x})b(\hat{x})) - \\
&\quad \frac{\partial h}{\partial \hat{x}}G(\hat{x})(I - A^{+}(\hat{x})A(\hat{x}))\bar{u} + v(t) - D\hat{f}_a(t) \\
&= \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + v(t) - D\hat{f}_a(t)
\end{aligned}
\tag{4.11}
$$

where

$$\frac{\partial h}{\partial x} \triangleq \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \\ \frac{\partial h_p}{\partial x_1} & \frac{\partial h_p}{\partial x_2} & \cdots & \frac{\partial h_p}{\partial x_n} \end{bmatrix} \tag{4.12}$$

and

$$\begin{aligned} \Delta A &= (\frac{\partial h}{\partial x} f(x) - \frac{\partial h}{\partial x} G(x) A^{+}(x) b(x)) + \frac{\partial h}{\partial x} G(x)(I - A^{+}(x)A(x))\bar{u}(t) \\ &\quad -(\frac{\partial h}{\partial \hat{x}} f(\hat{x}) - \frac{\partial h}{\partial \hat{x}} G(\hat{x}) A^{+}(\hat{x}) b(\hat{x})) \\ &\quad -\frac{\partial h}{\partial \hat{x}} G(\hat{x})(I - A^{+}(\hat{x})A(\hat{x}))\bar{u}. \end{aligned} \tag{4.13}$$

Remark 4.3.1 The assumption of $\frac{\partial h}{\partial x} E(x) \neq \mathbf{0}$ in assumption 4.3 guarantees that fault $f_a(t)$ has an impact on output estimation errors for the purpose of fault detection and estimation.

The relationship between sliding surface dynamics and output errors can be taken as:

$$\ddot{S} + z_0 \dot{S} = \dot{e} + ce \tag{4.14}$$

where $S = col[s_1, s_2, \cdots, s_p]$.

Substituting equation (4.11) into (4.14), we have:

$$\begin{aligned} \ddot{S} + z_0 \dot{S} &= \dot{e} + ce \\ &= \Delta A + \frac{\partial h}{\partial x}\omega(x) d(t) + \frac{\partial h}{\partial x} E(x) f_a(t) + v(t) - D\hat{f}_a + ce. \end{aligned} \tag{4.15}$$

A Lyapunov function candidate is chosen to create the attractivity condition as follows:

$$V = \frac{1}{2}(\dot{S}^T \dot{S} + S^T \Omega S) + \frac{1}{2}\tilde{f}_a^{\,T} \tilde{f}_a, \quad \Omega = Diag(w). \tag{4.16}$$

The time derivative of Lyapunov function candidate is as follows

$$\dot{V} = \dot{S}^T(\ddot{S} + wS) + \dot{\tilde{f}}_a^T \tilde{f}_a. \tag{4.17}$$

To constitute the attractivity condition, the following inequality should hold:

$$\dot{S}^T(\ddot{S} + wS) + \dot{\tilde{f}}_a^T \tilde{f}_a \leq 0. \tag{4.18}$$

Substituting equation (4.15) into above equation, we have:

$$\dot{S}^T[\Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + v - D\hat{f}_a + ce - z_0\dot{S} + wS + \Gamma^T\tilde{f}_a] \leq 0. \tag{4.19}$$

Substituting discontinuous term $v(t)$ from equation (4.10) into equation (4.19) yields

$$\dot{S}^T[\Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + \Gamma^T f_a - \kappa sgn(\dot{S})] \leq 0. \tag{4.20}$$

Noticing that $\dot{S}^T sgn(\dot{S}) \geq \|\dot{S}\|$ and consequently $-\kappa\dot{S}^T sgn(\dot{S}) \leq -\kappa\|\dot{S}\|$, and using this inequality, equation (4.20) can be further derived by using vector norms

$$\dot{V} \leq \|\dot{S}\|(\|\Delta A\| + \|\frac{\partial h}{\partial x}\omega(x)\|\|d(t)\| + \|\frac{\partial h}{\partial x}E(x)\|\|f_a(t)\| + \|\Gamma\|\|f_a(t)\| - \kappa) \leq 0 \tag{4.21}$$

therefore, if

$$\kappa \geq \|\Delta A\| + b_h\|\omega(x)\|b_d + \|E(x)\|b_h b_f + \|\Gamma\|b_f, \tag{4.22}$$

then, $\dot{V} \leq 0$, which means that output estimation errors are kept sliding on the sliding surface if switching gain κ is selected according to equation (4.22).

As shown in equation (4.15), $v(t)$ is discontinuous across $\dot{S} = 0$, and $\dot{S}$ is accordingly discontinuous, which leads to chattering. If we use $\dot{S}$ as a residual, the chattering is not desirable. To smooth out the discontinuity, a boundary layer ϕ neighboring the switching surface $\dot{S}$ is introduced [11]. A Saturation function (sat) is used to replace the signum function. The saturation function is defined as

$$sat(\frac{\dot{S}}{\phi}) = \left[sat(\frac{\dot{s}_1}{\phi}), sat(\frac{\dot{s}_2}{\phi}), \cdots, sat(\frac{\dot{s}_p}{\phi})\right]^T \tag{4.23}$$

and

$$sat(\frac{\dot{s}_i}{\phi}) = \begin{cases} sgn(\frac{\dot{s}_i}{\phi}), & \text{when } |\dot{s}_i| \geq \phi \\ \frac{\dot{s}_i}{\phi}, & \text{when } |\dot{s}_i| < \phi. \end{cases} \tag{4.24}$$

The S dynamics outside the boundary layer can be obtained by substituting v into equation (4.15) as

$$\ddot{S} + wS + \kappa sgn(\dot{S}) = \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + \Gamma^T \hat{f}_a. \tag{4.25}$$

Within the boundary layer, the S dynamics has the form of

$$\ddot{S} + wS + \kappa\frac{\dot{S}}{\phi} = \Delta A + \frac{\partial h}{\partial x}\omega(x)d(t) + \frac{\partial h}{\partial x}E(x)f_a(t) + \Gamma^T \hat{f}_a. \tag{4.26}$$

Equation (4.26) represents a low-pass filter [11, 27]; that means it can filter high-frequency signals, while letting all low-frequency signals pass. So, as long as the fault is not a high-frequency signal, it will have impact on S dynamics. Therefore, S can be selected as a residual. As a matter of fact, $\dot{S}$ can also be selected as a residual because, once a fault occurs, the $\dot{S}$ will temporarily stop sliding, then recover sliding. This way we could detect faults, as will be seen in the simulations.

□

Remark 4.3.2 The observer is designed to maintain a sliding motion even in the presence of faults, which is quite different from the approaches in [50, 55, 113] and the approach proposed in Chapter 3 where a fault will destroy the sliding of the sliding mode observers.

Remark 4.3.3 From equation (4.22), one can know that the selection of switching gain κ is related only to some norm bounds. Therefore, one can claim that if these norm bounds are known, then κ can be chosen easily without knowing all system states for calculation of norm bounds.

4.3.1 Stability Analysis of the SOSMO with a Single Output

In this section, the stability analysis is based on creating a sliding mode on the first one or several state equations (it depends on how many outputs the considered system has), which leads to a reduced estimation error equivalent dynamics. The asymptotic convergence of the reduced estimation error dynamics is then proved.

Assume that system output $y = Cx(t)$, where C is a constant matrix. For the simplicity of proof, let $y = x_1(t)$.

Stability analysis begins by expanding the sliding observer (4.10) according to each observer state variable

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{\zeta}_1 - k_{11}(t)v + k_{12}D\hat{f}_a \\
\dot{\hat{x}}_2 &= \hat{\zeta}_2 - k_{21}(t)v + k_{22}D\hat{f}_a \\
&\vdots \\
\dot{\hat{x}}_n &= \hat{\zeta}_n - k_{n1}(t)v(t) + k_{n2}D\hat{f}_a
\end{aligned} \tag{4.27}$$

where

$[k_{11}(t), k_{21}(t), \cdots, k_{n1}(t)]^T = L_1(t)\left[\frac{\partial h}{\partial \hat{x}}L_1(t)\right]^{-1}$; $v(t)$ is the discontinuous term in the SOSMO; $[k_{12}(t), k_{22}(t), \cdots, k_{n2}(t)]^T = L_2(t)\left[\frac{\partial h}{\partial \hat{x}}L_2(t)\right]^{-1}$; and

$$[\hat{\zeta}_1, \hat{\zeta}_2, \cdots, \hat{\zeta}_n]^T = (f(\hat{x}) - G(\hat{x})A^+(\hat{x})b(\hat{x})) + G(\hat{x})(I - A^+(\hat{x})A(\hat{x}))\bar{u}. \tag{4.28}$$

Let $\tilde{x}_i = x_i - \hat{x}_i, i = 1, \cdots, n$, subtracting equation (4.27) from system equation (4.7), we have

$$\begin{aligned}
\dot{\tilde{x}}_1 &= \Delta\zeta_1 + \omega_1 d(t) + E_1 f_a + k_{11}(t)v - k_{12}D\hat{f}_a \\
\dot{\tilde{x}}_2 &= \Delta\zeta_2 + \omega_2 d(t) + E_2 f_a + k_{21}(t)v - k_{22}D\hat{f}_a \\
&\vdots \\
\dot{\tilde{x}}_n &= \Delta\zeta_n + \omega_n d(t) + E_n f_a + k_{n1}(t)v - k_{n2}D\hat{f}_a
\end{aligned} \tag{4.29}$$

where

$$\begin{aligned}
[\Delta\zeta_1, \Delta\zeta_2, \cdots \Delta\zeta_n]^T &= [\zeta_1, \zeta_2, \cdots, \zeta_n]^T - [\hat{\zeta}_1, \hat{\zeta}_2, \cdots, \hat{\zeta}_n]^T \\
&= (f(x) - G(x)A^+(x)b(x)) - (f(\hat{x}) - G(\hat{x})A^+(\hat{x})b(\hat{x})) \\
&\quad + G(x)(I - A^+(x)A(x))\bar{u} - G(\hat{x})(I - A^+(\hat{x})A(\hat{x}))\bar{u},
\end{aligned} \tag{4.30}$$

and ω_i and E_i are row vectors of matrices ω and E, respectively.

On the basis of the following lemma, Theorem 4.1 will be introduced.

Lemma 4.3.1 The coefficients $k_{11}(t)$ and k_{12} in state estimation error dynamics (4.29) are both equal to 1.

Proof: Because the output is $x_1(t)$, we have the following straightforward calculation:

$$\begin{bmatrix} k_{11} \\ k_{21} \\ \vdots \\ k_{n1} \end{bmatrix} = L_1(t)\left[\frac{\partial h}{\partial \hat{x}} L_1(t)\right]^{-1} = \begin{bmatrix} l_1^1 \\ l_2^1 \\ \vdots \\ l_n^1 \end{bmatrix} (l_1^1)^{-1} = \begin{bmatrix} 1 \\ l_2^1/l_1^1 \\ \vdots \\ l_n^1/l_1^1 \end{bmatrix}$$

and

$$\begin{bmatrix} k_{12} \\ k_{22} \\ \vdots \\ k_{n2} \end{bmatrix} = L_2(t)\left[\frac{\partial h}{\partial \hat{x}} L_2(t)\right]^{-1} = \begin{bmatrix} l_1^2 \\ l_2^2 \\ \vdots \\ l_n^2 \end{bmatrix} (l_1^2)^{-1} = \begin{bmatrix} 1 \\ l_2^2/l_1^2 \\ \vdots \\ l_n^2/l_1^2 \end{bmatrix}$$

where $l_1^1 \neq 0$, $l_1^2 \neq 0$. ∎

Recall that inequality (4.22) guarantees that output errors will reach the sliding surface $\dot{S}(t)$ and will be kept there while sliding on it, i.e. the output estimation error $\tilde{x}_1$ is zero on this surface. Applying the concept of equivalent dynamics in accordance with [118, 123], we have the reduced SOSMO error dynamics in the form of

$$\begin{aligned}
\dot{\tilde{x}}_2 &= \Delta\zeta_2 + \omega_2 d(t) + E_2 f_a - \tfrac{l_2^1}{l_1^1}(\Delta\zeta_1 + \omega_1 d(t) + E_1 f_a) + (\tfrac{l_2^1}{l_1^1} - \tfrac{l_2^2}{l_1^2})D\hat{f}_a \\
\dot{\tilde{x}}_3 &= \Delta\zeta_3 + \omega_3 d(t) + E_3 f_a - \tfrac{l_3^1}{l_1^1}(\Delta\zeta_1 + \omega_1 d(t) + E_1 f_a) + (\tfrac{l_3^1}{l_1^1} - \tfrac{l_3^2}{l_1^2})D\hat{f}_a \\
&\vdots \\
\dot{\tilde{x}}_n &= \Delta\zeta_n + \omega_n d(t) + E_n f_a - \tfrac{l_n^1}{l_1^1}(\Delta\zeta_1 + \omega_1 d(t) + E_1 f_a) + (\tfrac{l_n^1}{l_1^1} - \tfrac{l_n^2}{l_1^2})D\hat{f}_a.
\end{aligned} \tag{4.31}$$

By expanding $\Delta\zeta_1, \Delta\zeta_2, \cdots, \Delta\zeta_n$ into power series, we have the following differential form of the above equation:

$$\begin{aligned}
\dot{\tilde{x}}_2 &= [\frac{\partial\zeta_2}{\partial x_2} - \frac{l_2^1}{l_1^1}\frac{\partial\zeta_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\frac{\partial\zeta_2}{\partial x_n} - \frac{l_2^1}{l_1^1}\frac{\partial\zeta_1}{\partial x_n}]\tilde{x}_n + \phi_2 + (\frac{l_2^1}{l_1^1} - \frac{l_2^2}{l_1^2})D\hat{f}_a \\
&\quad +((\omega_2 - \frac{l_2^1}{l_1^1}\omega_1)d(t) + (E_2 - \frac{l_2^1}{l_1^1}E_1)f_a) \\
\dot{\tilde{x}}_3 &= [\frac{\partial\zeta_3}{\partial x_2} - \frac{l_3^1}{l_1^1}\frac{\partial\zeta_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\frac{\partial\zeta_3}{\partial x_n} - \frac{l_3^1}{l_1^1}\frac{\partial\zeta_1}{\partial x_n}]\tilde{x}_n + \phi_3 + (\frac{l_3^1}{l_1^1} - \frac{l_3^2}{l_1^2})D\hat{f}_a \\
&\quad +((\omega_3 - \frac{l_3^1}{l_1^1}\omega_1)d(t) + (E_3 - \frac{l_3^1}{l_1^1}E_1)f_a) \\
&\vdots \\
\dot{\tilde{x}}_n &= [\frac{\partial\zeta_n}{\partial x_2} - \frac{l_n^1}{l_1^1}\frac{\partial\zeta_1}{\partial x_2}]\tilde{x}_2 + \cdots + [\frac{\partial\zeta_n}{\partial x_n} - \frac{l_n^1}{l_1^1}\frac{\partial\zeta_1}{\partial x_n}]\tilde{x}_n + \phi_n + (\frac{l_n^1}{l_1^1} - \frac{l_n^2}{l_1^2})D\hat{f}_a \\
&\quad +((\omega_n - \frac{l_n^1}{l_1^1}\omega_1)d(t) + (E_n - \frac{l_n^1}{l_1^1}E_1)f_a)
\end{aligned} \tag{4.32}$$

where $\phi_i, i = 2, \cdots, n$, are the second and higher order terms in $(x_i - \hat{x}_i)$.

Let $\tilde{x} = [\tilde{x}_2, \tilde{x}_3, \cdots, \tilde{x}_n]^T$, we have:

$$\dot{\tilde{x}} = A(t)\tilde{x} + \Phi + \delta F + \Lambda D\hat{f}_a \tag{4.33}$$

where

$$\delta F = \begin{bmatrix} (\omega_2 - \frac{l_2^1}{l_1^1}\omega_1)d(t) + (E_2 - \frac{l_2^1}{l_1^1}E_1)f_a \\ (\omega_3 - \frac{l_3^1}{l_1^1}\omega_1)d(t) + (E_3 - \frac{l_3^1}{l_1^1}E_1)f_a \\ \vdots \\ (\omega_n - \frac{l_n^1}{l_1^1}\omega_1)d(t) + (E_n - \frac{l_n^1}{l_1^1}E_1)f_a \end{bmatrix} \qquad \Lambda = \begin{bmatrix} \frac{l_2^1}{l_1^1} - \frac{l_2^2}{l_1^2} \\ \frac{l_3^1}{l_1^1} - \frac{l_3^2}{l_1^2} \\ \vdots \\ \frac{l_n^1}{l_1^1} - \frac{l_n^2}{l_1^2} \end{bmatrix} \tag{4.34}$$

and

$$A(t) = \begin{bmatrix} \frac{\partial \zeta_2}{\partial x_2} - \frac{l_2^1}{l_1^1}\frac{\partial \zeta_1}{\partial x_2} & \cdots & \frac{\partial \zeta_2}{\partial x_n} - \frac{l_2^1}{l_1^1}\frac{\partial \zeta_1}{\partial x_n} \\ \frac{\partial \zeta_3}{\partial x_2} - \frac{l_3^1}{l_1^1}\frac{\partial \zeta_1}{\partial x_2} & \cdots & \frac{\partial \zeta_3}{\partial x_n} - \frac{l_3^1}{l_1^1}\frac{\partial \zeta_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial \zeta_n}{\partial x_2} - \frac{l_n^1}{l_1^1}\frac{\partial \zeta_1}{\partial x_2} & \cdots & \frac{\partial \zeta_n}{\partial x_n} - \frac{l_n^1}{l_1^1}\frac{\partial \zeta_1}{\partial x_n} \end{bmatrix}.$$

Choose gains $l_i^1(t), i = 1, \cdots, n$ such that the matrix $A(t)$ is a stability matrix and there exists a positive define symmetric matrix $P(t)$ such that

$$P(t)A(t) + A^T(t)P(t) + \dot{P}(t) = -Q \tag{4.35}$$

where Q is a positive define matrix. If gain matrix $L_2(t) = L_1(t)$, then equation (4.33) can be reduced to

$$\dot{\tilde{x}} = A(t)\tilde{x} + \Phi + \delta F \tag{4.36}$$

Consider a Lyapunov candidate function of the form

$$V = \tilde{x}^T P \tilde{x}. \tag{4.37}$$

Differentiating it, we have:

$$\begin{aligned}\dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T \dot{P}\tilde{x} \\ &= \tilde{x}^T(A^T P + PA + \dot{P})\tilde{x} + \Phi^T P\tilde{x} + \delta F^T P\tilde{x} + \tilde{x}^T P\Phi + \tilde{x}^T P\delta F.\end{aligned} \tag{4.38}$$

Considering equation (4.35), the above equation can be further extended as

$$\begin{aligned}\dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\gamma_\phi\|P\|\|\tilde{x}\|^2 + 2\gamma_{\delta F}\|P\|\|\tilde{x}\| \\ &\leq ((-\lambda_{min}(Q) + 2\gamma_\phi\lambda_{max}(P))\|\tilde{x}\| + 2\gamma_{\delta F}\lambda_{max}(P))\|\tilde{x}\|\end{aligned} \tag{4.39}$$

where $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ are minimum and maximum eigenvalues, respectively, and $\gamma_{\delta F}$ is the norm bound of δF.

In the above derivation process, inequality $\|\Phi\| \leq \gamma_\phi\|\tilde{x}\|$ is used.

Therefore, if the following inequality holds for all $\tilde{x}$

$$\|\tilde{x}\| \geq \frac{2\gamma_{\delta F}\lambda_{max}(P)}{(\lambda_{min}(Q) - 2\gamma_\phi\lambda_{max}(P))} \tag{4.40}$$

then $\dot{V} \leq 0$, which implies that the reduced estimation error is bounded.

Following [123], SOSMO gain $L_1(t)$ can be directly calculated.

Finally, the following theorem summarizes the above results.

Theorem 4.1 *Consider UCNS (4.7) with a single output and its SOSMO defined in equation (4.10). If inequality (4.22) and equation (4.35) hold, then system state estimation error is bounded.*

Remark 4.3.4 In this subsection, we discussed the stability of UCNS with a single output $y = x_1(t)$. The coefficient of $v(t)$ in the first estimation error equation is 1 due to the special structure of the proposed observer $L_1(t)\left[\frac{\partial h}{\partial \hat{x}}L_1(t)\right]^{-1}$. From this special structure, we can also derive that if $y = x_i(t), i = 1, \cdots, n$, then the coefficient of v in the i-th estimation error equation is 1.

Remark 4.3.5 It is worth noting that the linear output $y = Cx$ has been used to prove the stability of the UCNS. If there is a single output of the form of $y = h(x)$, from equation (4.11), $v(t)$ can be calculated as

$$v(t) = D\hat{f}_a(t) - \Delta A - \frac{\partial h}{\partial x}\omega(x)d(t) - \frac{\partial h}{\partial x}E(x)f_a(t). \tag{4.41}$$

Substituting the above expression into equation (4.29) will lead to a similar form as equation (4.31), and similarly, the stability of the UCNS can be proved.

4.3.2 Stability Analysis of the SOSMO with Multiple Outputs.

For multi-output nonlinear systems, one has the same construction of the SOSMO as that in equation (4.10). Nevertheless, observer gain matrices $L_1(t)$ and $L_2(t)$ are $n \times p$ matrices, where p is the number of outputs. In addition, output estimation error $e(t)$, discontinuous term $v(t)$ and sliding surface $S(t)$ are dimension-p vectors. For simplicity of stability derivation, assume that $y = [x_1,\ x_2]^T$. Under this condition, rewrite SOSMO (4.10) as follows:

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{\zeta}_1 - k_{11}v_1 - k_{12}v_2 + m_{11}D_1\hat{f}_a + m_{12}D_2\hat{f}_a \\
\dot{\hat{x}}_2 &= \hat{\zeta}_2 - k_{21}v_1 - k_{22}v_2 + m_{21}D_1\hat{f}_a + m_{22}D_2\hat{f}_a \\
&\vdots \\
\dot{\hat{x}}_n &= \hat{\zeta}_n - k_{n1}v_1 - k_{n2}v_2 + m_{n1}D_1\hat{f}_a + m_{n2}D_2\hat{f}_a
\end{aligned} \tag{4.42}$$

where D_1 and D_2 are row vectors of D.

Subtracting (4.42) from system (4.7), we have

$$\begin{aligned}
\dot{\tilde{x}}_1 &= \Delta\zeta_1 + \omega_1 d(t) + E_1 f_a + k_{11} v_1 + k_{12} v_2 - m_{11} D_1 \hat{f}_a - m_{12} D_2 \hat{f}_a \\
\dot{\tilde{x}}_2 &= \Delta\zeta_2 + \omega_2 d(t) + E_2 f_a + k_{21} v_1 + k_{22} v_2 - m_{21} D_1 \hat{f}_a - m_{22} D_2 \hat{f}_a \\
&\vdots \\
\dot{\tilde{x}}_n &= \Delta\zeta_n + \omega_n d(t) + E_n f_a + k_{n1} v_1 + k_{n2} v_2 - m_{n1} D_1 \hat{f}_a - m_{n2} D_2 \hat{f}_a.
\end{aligned} \tag{4.43}$$

Before stating Theorem 4.2, Lemma 4.3.2 is introduced.

Lemma 4.3.2 In equation (4.43), the coefficients $k_{11} = k_{22} = m_{11} = m_{22} = 1$ and $k_{12} = k_{21} = m_{12} = m_{21} = 0$.

Proof:

The proof process is straightforward as follows:

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \\ \vdots & \vdots \\ k_{n1} & k_{n2} \end{bmatrix} = L_1(t)\left[\frac{\partial h}{\partial x} L_1(t)\right]^{-1} = \begin{bmatrix} l^1_{11} & l^1_{12} \\ l^1_{21} & l^1_{22} \\ l^1_{31} & l^1_{32} \\ \vdots & \vdots \\ l^1_{n1} & l^1_{n2} \end{bmatrix} \begin{bmatrix} l^1_{11} & l^1_{12} \\ l^1_{21} & l^1_{22} \end{bmatrix}^{-1} \tag{4.44}$$

$$= \begin{bmatrix} l^1_{11} & l^1_{12} \\ l^1_{21} & l^1_{22} \\ l^1_{31} & l^1_{32} \\ \vdots & \vdots \\ l^1_{n1} & l^1_{n2} \end{bmatrix} \begin{bmatrix} \frac{l^1_{22}}{\Delta} & -\frac{l^1_{12}}{\Delta} \\ -\frac{l^1_{21}}{\Delta} & \frac{l^1_{11}}{\Delta} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{l^1_{31} l^1_{22} - l^1_{32} l^1_{21}}{\Delta} & \frac{l^1_{32} l^1_{11} - l^1_{31} l^1_{12}}{\Delta} \\ \vdots & \vdots \\ \frac{l^1_{n1} l^1_{22} - l^1_{n2} l^1_{21}}{\Delta} & \frac{l^1_{n2} l^1_{11} - l^1_{n1} l^1_{12}}{\Delta} \end{bmatrix} \tag{4.45}$$

where $\Delta = l^1_{11} l^1_{22} - l^1_{12} l^1_{21} \neq 0$.

Similarly

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \\ \vdots & \vdots \\ m_{n1} & m_{n2} \end{bmatrix} = L_2(t)\left[\frac{\partial h}{\partial x}L_2(t)\right]^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{l^2_{31}l^2_{22} - l^2_{32}l^2_{21}}{\Delta} & \frac{l^2_{32}l^2_{11} - l^2_{31}l^2_{12}}{\Delta} \\ \vdots & \vdots \\ \frac{l^2_{n1}l^2_{22} - l^2_{n2}l^2_{21}}{\Delta} & \frac{l^2_{n2}l^2_{11} - l^2_{n1}l^2_{12}}{\Delta} \end{bmatrix} \tag{4.46}$$

where $\Delta = l^2_{11}l^2_{22} - l^2_{12}l^2_{21} \neq 0$.

Therefore, the dynamics of the state estimation errors $\tilde{x}_1$ and $\tilde{x}_2$ are

$$\begin{aligned} \dot{\tilde{x}}_1 &= \Delta\zeta_1 + \omega_1 d(t) + E_1 f_a + v_1 - D_1\hat{f}_a \\ \dot{\tilde{x}}_2 &= \Delta\zeta_2 + \omega_2 d(t) + E_2 f_a + v_2 - D_2\hat{f}_a. \end{aligned} \tag{4.47}$$

∎

From equation (4.47) we have the equivalent control:

$$(v_1)_{eq} = D_1\hat{f}_a - \Delta\zeta_1 - \omega_1 d(t) - E_1 f_a \quad \text{and} \quad (v_2)_{eq} = D_2\hat{f}_a - \Delta\zeta_2 - \omega_2 d(t) - E_2 f_a. \tag{4.48}$$

These two equivalent control laws will be used in the stability proof.

Redefine $\tilde{x} = [\tilde{x}_3, \tilde{x}_4, \cdots, \tilde{x}_n]^T$ and its dynamics:

$$\begin{aligned} \dot{\tilde{x}}_3 = \ & \Delta\zeta_3 + \omega_3 d(t) + E_3 f_a + \tfrac{l^1_{31}l^1_{22} - l^1_{32}l^1_{21}}{\Delta}(D_1\hat{f}_a - \Delta\zeta_1 - \omega_1 d(t) - E_1 f_a) \\ & + \tfrac{l^1_{32}l^1_{11} - l^1_{31}l^1_{12}}{\Delta}(D_2\hat{f}_a - \Delta\zeta_2 - \omega_2 d(t) - E_2 f_a) - \tfrac{l^2_{31}l^2_{22} - l^2_{32}l^2_{21}}{\Delta}D_1\hat{f}_a \\ & - \tfrac{l^2_{32}l^2_{11} - l^2_{31}l^2_{12}}{\Delta}D_2\hat{f}_a \end{aligned}$$

$$
\begin{aligned}
\dot{\tilde{x}}_4 &= \Delta\zeta_4 + \omega_4 d(t) + E_4 f_a + \tfrac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}(D_1\hat{f}_a - \Delta\zeta_1 - \omega_1 d(t) - E_1 f_a) \\
&\quad + \tfrac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}(D_2\hat{f}_a - \Delta\zeta_2 - \omega_2 d(t) - E_2 f_a) - \tfrac{l^2_{41}l^2_{22}-l^2_{42}l^2_{21}}{\Delta}D_1\hat{f}_a \\
&\quad - \tfrac{l^2_{42}l^2_{11}-l^2_{41}l^2_{12}}{\Delta}D_2\hat{f}_a \\
&\ \ \vdots \qquad\qquad\qquad \vdots \\
\dot{\tilde{x}}_n &= \Delta\zeta_n + \omega_n d(t) + E_n f_a + \tfrac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}(D_1\hat{f}_a - \Delta\zeta_1 - \omega_1 d(t) - E_1 f_a) \\
&\quad + \tfrac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}(D_2\hat{f}_a - \Delta\zeta_2 - \omega_2 d(t) - E_2 f_a) - \tfrac{l^2_{n1}l^2_{22}-l^2_{n2}l^2_{21}}{\Delta}D_1\hat{f}_a \\
&\quad - \tfrac{l^2_{n2}l^2_{11}-l^2_{n1}l^2_{12}}{\Delta}D_2\hat{f}_a.
\end{aligned}
\tag{4.49}
$$

The expansions of $\Delta\zeta_1, \Delta\zeta_2, \cdots, \Delta\zeta_n$ into power series lead to:

$$
\begin{aligned}
\dot{\tilde{x}}_3 &= [\tfrac{\partial\zeta_3}{\partial x_3} - \tfrac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta}\tfrac{\partial\zeta_1}{\partial x_3} - \tfrac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta}\tfrac{\partial\zeta_2}{\partial x_3}]\tilde{x}_3 + \cdots + \\
&\quad [\tfrac{\partial\zeta_3}{\partial x_n} - \tfrac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta}\tfrac{\partial\zeta_1}{\partial x_n} - \tfrac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta}\tfrac{\partial\zeta_2}{\partial x_n}]\tilde{x}_n \\
&\quad + \bar{\phi}_3 + \tfrac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta}D_1\hat{f}_a + \tfrac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta}D_2\hat{f}_a - \tfrac{l^2_{31}l^2_{22}-l^2_{32}l^2_{21}}{\Delta}D_1\hat{f}_a - \tfrac{l^2_{32}l^2_{11}-l^2_{31}l^2_{12}}{\Delta}D_2\hat{f}_a + \\
&\quad \omega_3 d(t) + E_3 f_a - \tfrac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta}(\omega_1 d(t) + E_1 f_a) - \tfrac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta}(\omega_2 d(t) + E_2 f_a)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\tilde{x}}_4 &= [\tfrac{\partial\zeta_4}{\partial x_3} - \tfrac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}\tfrac{\partial\zeta_1}{\partial x_3} - \tfrac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}\tfrac{\partial\zeta_2}{\partial x_3}]\tilde{x}_3 + \cdots + \\
&\quad [\tfrac{\partial\zeta_4}{\partial x_n} - \tfrac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}\tfrac{\partial\zeta_1}{\partial x_n} - \tfrac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}\tfrac{\partial\zeta_2}{\partial x_n}]\tilde{x}_n \\
&\quad + \bar{\phi}_4 + \tfrac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}D_1\hat{f}_a + \tfrac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}D_2\hat{f}_a - \tfrac{l^2_{41}l^2_{22}-l^2_{42}l^2_{21}}{\Delta}D_1\hat{f}_a - \tfrac{l^2_{42}l^2_{11}-l^2_{41}l^2_{12}}{\Delta}D_2\hat{f}_a + \\
&\quad \omega_4 d(t) + E_4 f_a - \tfrac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}(\omega_1 d(t) + E_1 f_a) - \tfrac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}(\omega_2 d(t) + E_2 f_a) \\
&\ \ \vdots \qquad\qquad\qquad \vdots
\end{aligned}
$$

$$\begin{aligned}\dot{\tilde{x}}_n = & [\frac{\partial \zeta_n}{\partial x_3} - \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_3} - \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_3}]\tilde{x}_3 + \cdots + \\ & [\frac{\partial \zeta_n}{\partial x_n} - \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_n} - \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_n}]\tilde{x}_n \\ & +\bar{\phi}_n + \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}D_1\hat{f}_a + \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}D_2\hat{f}_a - \frac{l^2_{n1}l^2_{22}-l^2_{n2}l^2_{21}}{\Delta}D_1\hat{f}_a - \frac{l^2_{n2}l^2_{11}-l^2_{n1}l^2_{12}}{\Delta}D_2\hat{f}_a + \\ & \omega_n d(t) + E_n f_a - \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}(\omega_1 d(t) + E_1 f_a) - \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}(\omega_2 d(t) + E_2 f_a)\end{aligned} \tag{4.50}$$

where $\bar{\phi}_i, i = 3, \cdots, n$ are the second and higher order terms in $(x_i - \hat{x}_i)$.

Let

B(t)$\triangleq$

$$\begin{bmatrix} \frac{\partial \zeta_3}{\partial x_3} - \frac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_3} - \frac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_3} & \cdots & \frac{\partial \zeta_3}{\partial x_n} - \frac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_n} - \frac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_n} \\ \frac{\partial \zeta_4}{\partial x_3} - \frac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_3} - \frac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_3} & \cdots & \frac{\partial \zeta_4}{\partial x_n} - \frac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_n} - \frac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial \zeta_n}{\partial x_3} - \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_3} - \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_3} & \cdots & \frac{\partial \zeta_n}{\partial x_n} - \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta}\frac{\partial \zeta_1}{\partial x_n} - \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta}\frac{\partial \zeta_2}{\partial x_n} \end{bmatrix}$$

$\in \mathbf{R}^{(n-2)\times(n-2)}$.

The reduced error dynamics can be described as follows

$$\dot{\tilde{x}} = B(t)\tilde{x} + \bar{\Phi} + \delta N + HD\hat{f}_a \tag{4.51}$$

where $D\hat{f}_a = [D_1\hat{f}_a, D_2\hat{f}_a]^T$, $\bar{\Phi} = [\bar{\phi}_3, \cdots, \bar{\phi}_n]^T$.

Moreover

$$H = \begin{bmatrix} \frac{l^1_{31}l^1_{22}-l^1_{32}l^1_{21}}{\Delta} - \frac{l^2_{31}l^2_{22}-l^2_{32}l^2_{21}}{\Delta} & \frac{l^1_{32}l^1_{11}-l^1_{31}l^1_{12}}{\Delta} - \frac{l^2_{32}l^2_{11}-l^2_{31}l^2_{12}}{\Delta} \\ \frac{l^1_{41}l^1_{22}-l^1_{42}l^1_{21}}{\Delta} - \frac{l^2_{41}l^2_{22}-l^2_{42}l^2_{21}}{\Delta} & \frac{l^1_{42}l^1_{11}-l^1_{41}l^1_{12}}{\Delta} - \frac{l^2_{42}l^2_{11}-l^2_{41}l^2_{12}}{\Delta} \\ \vdots & \vdots \\ \frac{l^1_{n1}l^1_{22}-l^1_{n2}l^1_{21}}{\Delta} - \frac{l^2_{n1}l^2_{22}-l^2_{n2}l^2_{21}}{\Delta} & \frac{l^1_{n2}l^1_{11}-l^1_{n1}l^1_{12}}{\Delta} - \frac{l^2_{n2}l^2_{11}-l^2_{n1}l^2_{12}}{\Delta} \end{bmatrix} \in \mathbb{R}^{n\times 2} \tag{4.52}$$

and

$$\delta N = \begin{bmatrix} \omega_3 d(t) + E_3 f_a - \frac{l^1_{31}l^1_{22} - l^1_{32}l^1_{21}}{\Delta}(\omega_1 d(t) + E_1 f_a) - \frac{l^1_{32}l^1_{11} - l^1_{31}l^1_{12}}{\Delta}(\omega_2 d(t) + E_2 f_a) \\ \omega_4 d(t) + E_4 f_a - \frac{l^1_{41}l^1_{22} - l^1_{42}l^1_{21}}{\Delta}(\omega_1 d(t) + E_1 f_a) - \frac{l^1_{42}l^1_{11} - l^1_{41}l^1_{12}}{\Delta}(\omega_2 d(t) + E_2 f_a) \\ \vdots \\ \omega_n d(t) + E_n f_a - \frac{l^1_{n1}l^1_{22} - l^1_{n2}l^1_{21}}{\Delta}(\omega_1 d(t) + E_1 f_a) - \frac{l^1_{n2}l^1_{11} - l^1_{n1}l^1_{12}}{\Delta}(\omega_2 d(t) + E_2 f_a) \end{bmatrix}$$

$\in \mathbb{R}^{n\times 1}$.

In the following, we choose the gains l^1_{ij}, $i = 1, \cdots, n, j = 1, 2$ such that matrix $B(t)$ is a stability matrix and there exists a positive definite symmetric matrix $\Pi(t)$ such that

$$\Pi(t)B + B^T\Pi(t) + \dot{\Pi}(t) = -R \tag{4.53}$$

where R is a positive definite matrix. If gain matrix $L_2(t) = L_1(t)$, then equation (4.51) can be reduced as

$$\dot{\tilde{x}} = B(t)\tilde{x} + \bar{\Phi} + \delta N \tag{4.54}$$

Considering a Lyapunov function candidate $V = \tilde{x}^T\Pi\tilde{x}$, it follows that:

$$\begin{aligned} \dot{V} &\leq -\lambda_{min}(R)\|\tilde{x}\|^2 + 2\gamma_{\bar{\phi}}\|\Pi\|\|\tilde{x}\|^2 + 2\gamma_{\delta N}\|\Pi\|\|\tilde{x}\| \\ &\leq ((-\lambda_{min}(R) + 2\gamma_{\bar{\phi}}\lambda_{max}(\Pi))\|\tilde{x}\| + 2\gamma_{\delta N}\lambda_{max}(\Pi))\|\tilde{x}\|. \end{aligned} \tag{4.55}$$

where $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ are minimum and maximum eigenvalues, respectively, and $\gamma_{\delta N}, \gamma_{\bar{\phi}}$ are the norm bounds of δN and $\bar{\Phi}$.

Furthermore, take a value $0 < \alpha < 1$, provided that the gains l^1_{ij} are such that

$$\lambda_{min}(R) - 2\gamma_{\bar{\phi}}\|\Pi\| - \alpha \geq 0 \tag{4.56}$$

and using the inequality (4.55), it is easy to show that

$$\dot{V} \leq -\alpha\|\tilde{x}\|^2 \tag{4.57}$$

if

$$\|\tilde{x}\| \geq \varsigma \tag{4.58}$$

with

$$\varsigma = \frac{2\gamma_{\delta N}\|\Pi\|}{(\lambda_{min}(R) - 2\gamma_{\bar{\phi}}\|\Pi\| - \alpha)}.$$

From Khalil [69], this means that $\tilde{x}$ is convergent to the ball

$$\|\tilde{x}\| \leq \sqrt{\frac{\lambda_{max}(\Pi)}{\lambda_{min}(\Pi)}}\varsigma. \tag{4.59}$$

Therefore, given any $\varsigma^\star > \sqrt{\frac{\lambda_{max}(\Pi)}{\lambda_{min}(\Pi)}}\varsigma$ there exists a finite time T such that for all $t > T$ we have

$$\|\tilde{x}\| < \varsigma^\star.$$

Theorem 4.2 *Consider constrained uncertain nonlinear system (4.7) with multiple outputs and its SOSMO defined in equation (4.10). If inequality (4.22) and equations (4.53) and (4.56) hold, then the state estimation error converges to the ball*

$$\|\tilde{x}\| < \varsigma^\star, \quad \forall\varsigma^\star > \sqrt{\frac{\lambda_{max}(\Pi)}{\lambda_{min}(\Pi)}}\varsigma. \tag{4.60}$$

Remark 4.3.6 The equivalent control v_{eq} represents the average behavior of the discontinuous term v and indicates the effort necessary to maintain the output errors sliding on the sliding surface [39]. The usual way to recover the equivalent control signals is to use a low-pass filter [72, 118, 131]. Therefore, if the equivalent control signals can be recovered, then disturbances $d(t)$ can be roughly estimated from equation (4.48) if the UCNS has no faults.

4.3.3 Fault Isolation Strategy

For fault detection and isolation purpose both output estimation errors and sliding surface $\dot{S}(t)$ can be selected as residuals. When a fault occurs, the sliding surface is no longer zero. Nevertheless, the observer switching gain κ has been designed to keep the system in sliding mode. Therefore the sliding surface will recover to zero, and by this process the fault can be detected.

In order to solve the fault isolation problem one can employ the following strategy [107]:

- Partition the set of all possible faults into α disjoint subsets that are to be isolated.
- An observer is designed corresponding to each subset, which results in a bank of α fault detection observers generating α residuals where the i^{th} residual is sensitive only to the i^{th} subset of faults while being robust to other all faults as well as to disturbances and uncertainties.

In addition, H. Yang and M. Saif [135] use an adaptive observer to isolate faults, where the disturbances are estimated, then by some calculation, faults can also be estimated by examining each component of the fault vector. Thus, fault isolation can be accomplished.

Here, faults are directly estimated in the SOSMO. As soon as any of the components of the estimate of faults is greater than zero, then the alarm for the corresponding fault component will be activated. The following algorithm summarizes the fault diagnosis process by the SOSMO.

Step 1: Impose constraint into system equation by differentiating constraint term $k(x)$ under assumption 4.4 to form a closed-loop system.

Step 2: Construct a SOSMO for this closed-loop system.

step 2.1 Select switching gain κ from equation (4.22) under assumptions 4.5 and 4.6.

step 2.2 Choose the gain matrices $L_1(t)$ and $L_2(t)$, where $L_1(t)$ must make $A(t)$ or $B(t)$ stable. Then solve equation (4.35) or (4.53) to get P or Π. If equation (4.40) or (4.58) holds, then go to step 3, otherwise, reselect gain matrices $L_1(t), L_2(t)$ and Q or R to solve P or Π.

Step 3: Fault detection and isolation is now accomplished based on the estimate of faults. In addition, one can also know the size and severity of the faults.

4.4 An Illustrative Example

In this section the above presented SOSMO for fault detection and estimation will be illustrated on a simple nonlinear system. These simulation results confirm that the SOSMO can be used to robustly detect and estimate actuator faults. A first order SMO will be also presented for the purpose of comparison.

4.4.1 Fault Detection and Estimation by a SOSMO

Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0.1x_3^2 - x_2 \\ x_3 \\ x_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 0.5d_1(t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_a^1 \\ f_a^2 \end{bmatrix} \\ y &= x_1 \\ k &= x_2 + x_3 - const. = 0 \end{aligned} \tag{4.61}$$

where *const.* is a constant, disturbance $d_1(t)$ is designated as a random function.

Obviously, the constrained characteristic index r^c=1, after calculation of $b(x)$ and $A(x)$, we form a feedback law as in the equation (4.8)

$$u = -\begin{pmatrix} \frac{1}{2}(x_1+x_3) \\ \frac{1}{2}(x_1+x_3) \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} f_a^2. \tag{4.62}$$

Substituting equation (4.62) into system equation (4.61), we obtain the closed-loop system with the effect of constraint

$$\dot{x} = \begin{pmatrix} 0.1x_3^2 - x_2 \\ \frac{1}{2}x_3 - \frac{1}{2}x_1 + \frac{1}{2}\bar{u}_1 - \frac{1}{2}\bar{u}_2 \\ -\frac{1}{2}x_3 + \frac{1}{2}x_1 - \frac{1}{2}\bar{u}_1 + \frac{1}{2}\bar{u}_2 \end{pmatrix} + \begin{pmatrix} 0.5d_1(t) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -f_a^1 \\ \frac{1}{2}f_a^2 \\ -\frac{1}{2}f_a^2 \end{pmatrix}. \tag{4.63}$$

Based on equation (4.63), the SOSMO is constructed as follows:

$$\begin{aligned} \dot{\hat{x}} &= \begin{bmatrix} 0.1\hat{x}_3^2 - \hat{x}_2 \\ \frac{1}{2}\hat{x}_3 - \frac{1}{2}\hat{x}_1 + \frac{1}{2}\bar{u}_1 - \frac{1}{2}\bar{u}_2 \\ -\frac{1}{2}\hat{x}_3 + \frac{1}{2}\hat{x}_1 - \frac{1}{2}\bar{u}_1 + \frac{1}{2}\bar{u}_2 \end{bmatrix} - L_1(t)(\tfrac{\partial h}{\partial \hat{x}}L_1(t))^{-1}v + L_2(t)(\tfrac{\partial h}{\partial \hat{x}}L_2(t))^{-1}D\hat{f}_a \\ \hat{y} &= \hat{x}_1 \\ v &= (D+\Gamma^T)\hat{f}_a - ce + z_0\dot{S} - wS - \kappa sat(\dot{S}) \\ \dot{\hat{f}}_a &= \Gamma\dot{S}. \end{aligned} \tag{4.64}$$

The SOSMO will detect and estimate the actuator fault when f_a^1 occurs.

We take output estimation error and sliding surface dynamics S and $\dot{S}(t)$ as residuals. When a fault occurs, $\dot{S}(t)$ will deviate from zero and recover to zero as described in Figure 4.1. The S can also generate an alarm as shown in figure 4.3. Meanwhile, fault estimation as demonstrated in Figure 4.1, is accurately achieved. Hence, the estimated fault is also a perfect residual candidate. Figure 4.2 demonstrates state estimation errors produced by the SOSMO. It is shown that all three state estimation errors can converge to zero, which implies that this kind of SOSMO can reconstruct the system states very accurately.

In Figure 4.1, for the sake of comparison with the first order SMO, a small-sized fault is employed to test the SOSMO. It turns out that the SOSMO works very well.

4.4.2 Fault Detection by a First Order SMO

A first order SMO is constructed according to [3, 123]

$$\dot{\hat{x}} = \begin{bmatrix} 0.1\hat{x}_3^2 - \hat{x}_2 \\ \frac{1}{2}\hat{x}_3 - \frac{1}{2}\hat{x}_1 + \frac{1}{2}\bar{u}_1 - \frac{1}{2}\bar{u}_2 \\ -\frac{1}{2}\hat{x}_3 + \frac{1}{2}\hat{x}_1 - \frac{1}{2}\bar{u}_1 + \frac{1}{2}\bar{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 sign(y - \hat{y}) \\ k_2 sign(y - \hat{y}) \\ k_3 sign(y - \hat{y}) \end{bmatrix}. \tag{4.65}$$

This observer can achieve state estimation efficiently as can be seen in Figure 4.4, but when the magnitude of the fault is small it is not appropriate for fault detection. This indicates that when the sliding surface is selected as a residual, small-sized faults can not be efficiently detected due to the chattering as shown in Figure 4.5. This is true even though this observer can converge to the system state equation very accurately. In Figure 4.5, the same fault as that used in last subsection occurs. Neither the sliding surface nor the output estimation error can efficiently produce alarm signals to indicate the occurrence of a fault. The switching gain k_i in observer equation (4.65) must be selected to guarantee the stability of the observer [3] while suppressing the disturbances. Therefore, it is not possible to make it small and the chattering is unavoidable. Compared with the first order SMO, the stability of the proposed SOSMO is guaranteed by both L_1 and L_2. Consequently, it is possible to choose the switching gain κ small and, accordingly, chattering is not big. In addition, the residuals S and $\dot{S}$ are continuous signals within the boundary layer, which makes it more sensitive to faults. In summary, the proposed SOSMO is appropriate for fault diagnosis.

4.5 Conclusions

This chapter explored a SOSMO with a fault estimation scheme for the purpose of FDI in uncertain constrained nonlinear systems. The crucial problem for imposing the constraint term into the uncertain constrained nonlinear system equation is that Assumption 4.4 can guarantee to form a feedback equation with system inputs and constraints without involving disturbances and uncertainties. To guarantee the sliding of the output estimation errors on the sliding surface, parameter κ should be selected according to inequality (4.22). This switching gain κ is the measurement of both uncertainties and/or disturbances, and faults. We can also conclude that the direct estimation of faults can supply one with both a fault isolation method as well as an indication of the size and severity of the fault. The proposed SOSMO can achieve three tasks at the same time: 1) fault detection by the second order sliding surface or the output estimation errors; 2) fault estimation supplied by the observer itself; and 3) fault isolation. One of the main properties of this SOSMO is that it is very sensitive to low-frequency faults while robust to high-frequency disturbances. The first order SMO, because of the chattering problem, cannot efficiently detect a small-sized fault that can be successfully detected by the SOSMO. The simple example has verified this result.

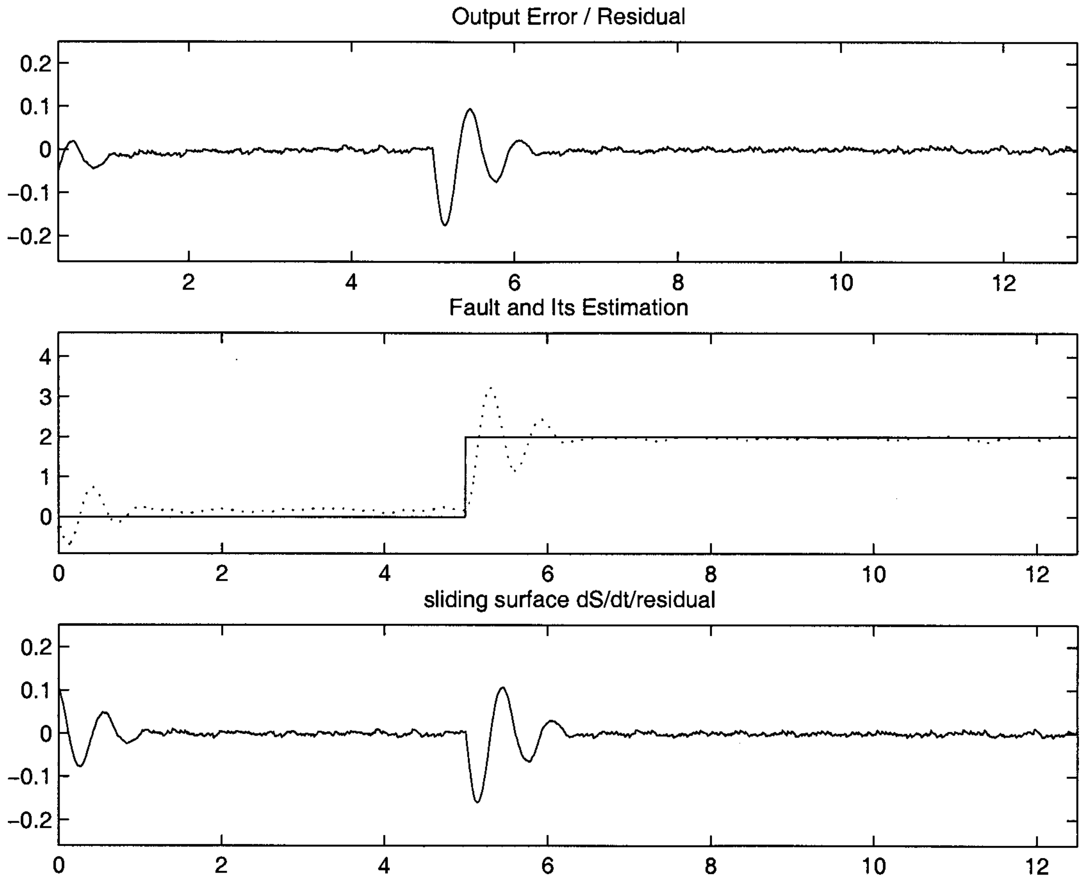

Figure 4.1: Fault detection and estimation by a SOSMO.

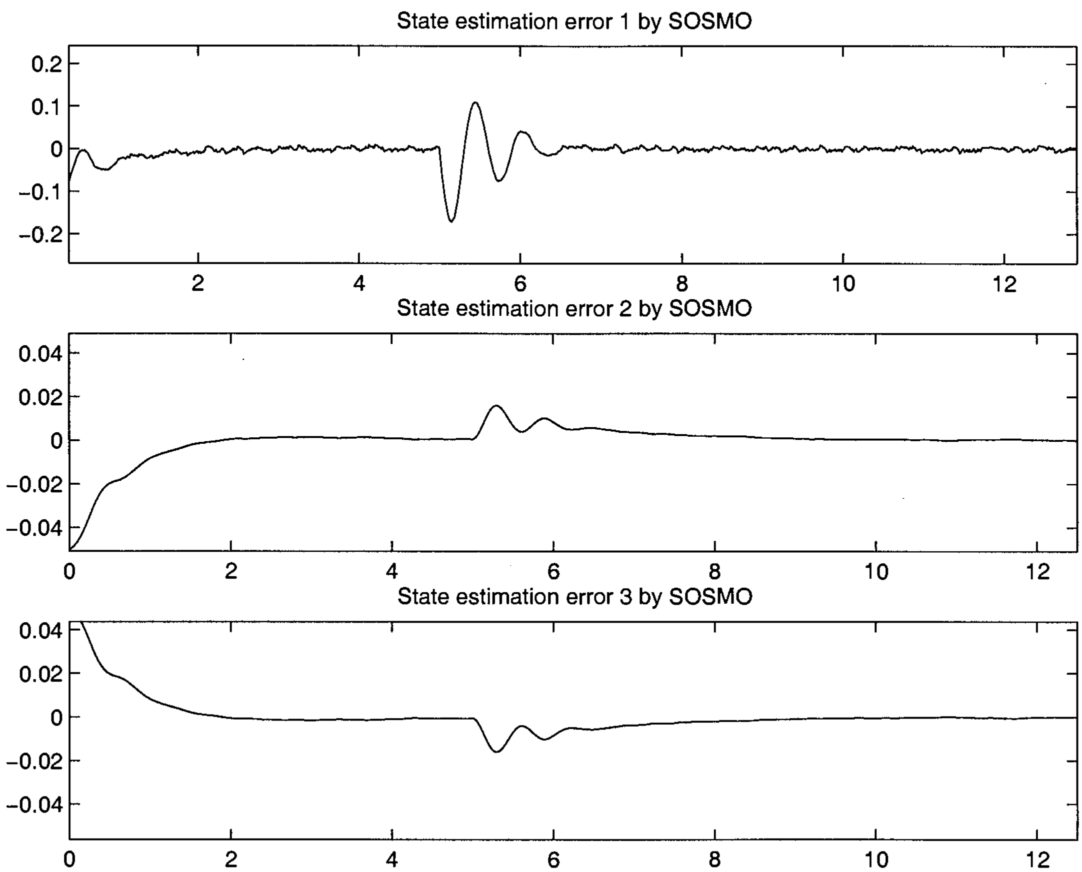

Figure 4.2: State Estimation Error by the SOSMO.

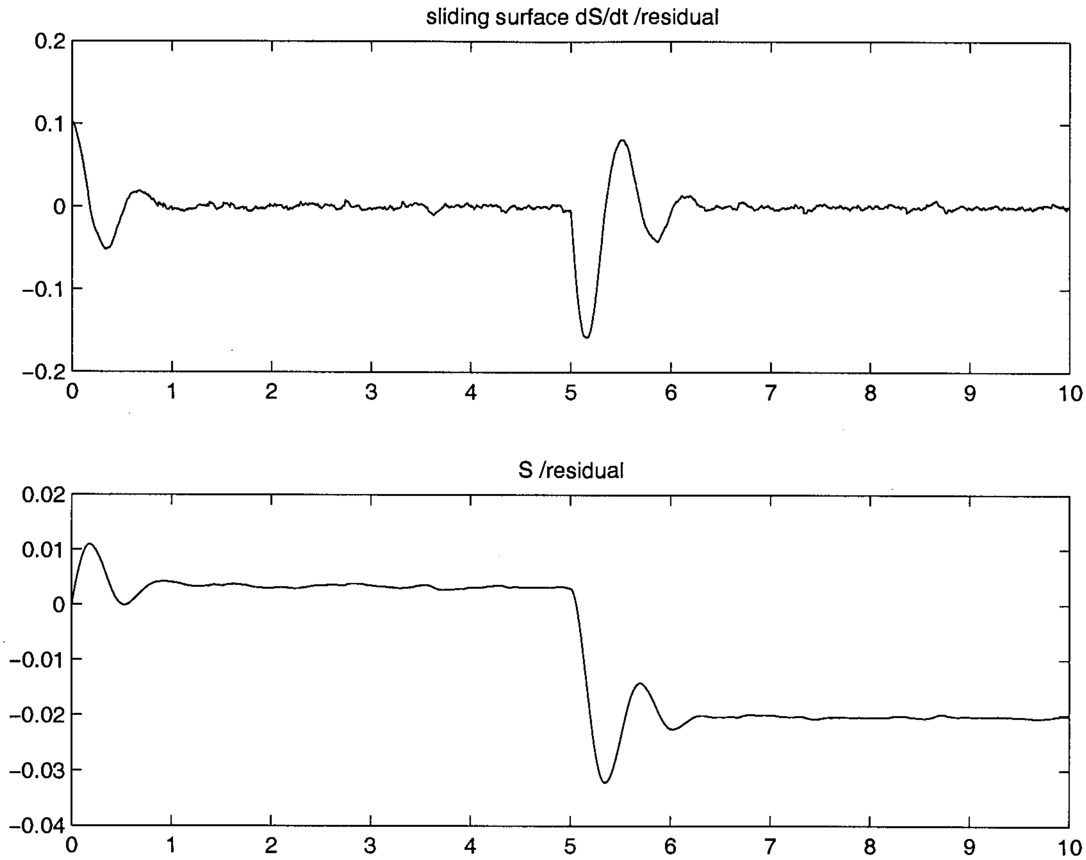

Figure 4.3: The S and $\dot{S}$ of the SOSMO.

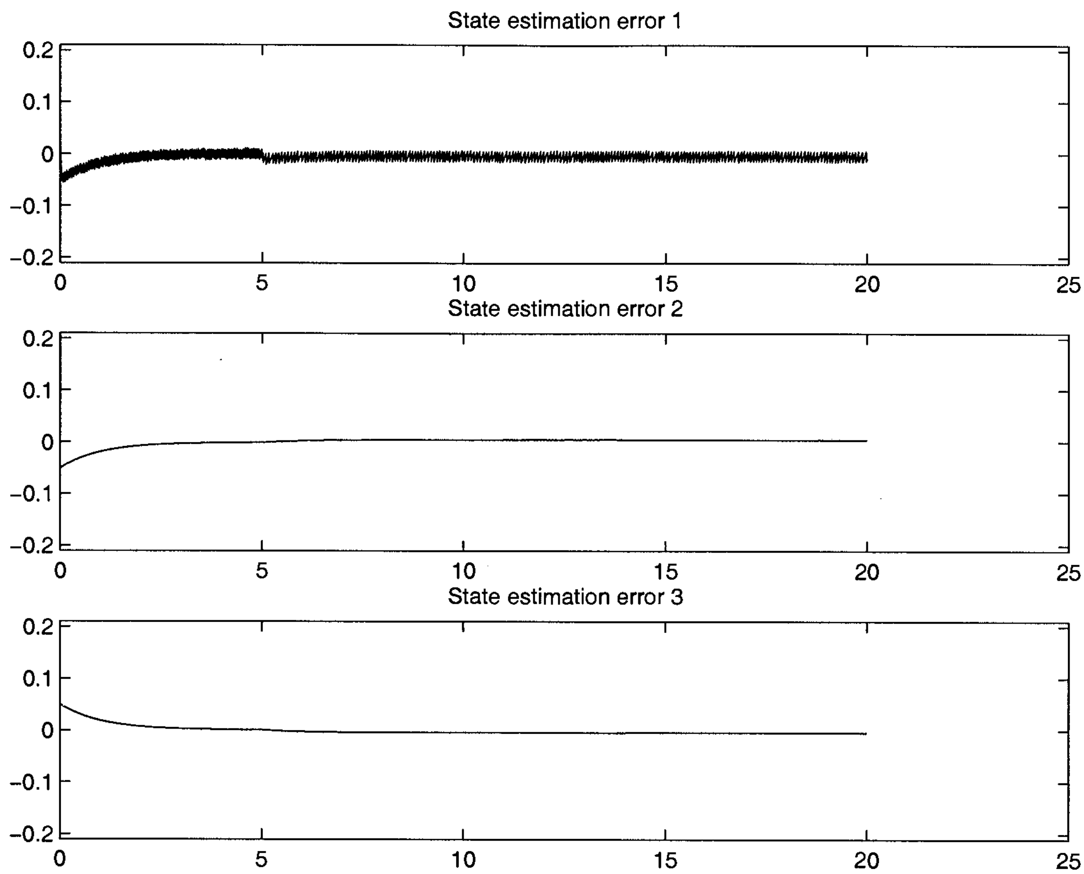

Figure 4.4: State Estimation Error by First Order Sliding Mode Observer.

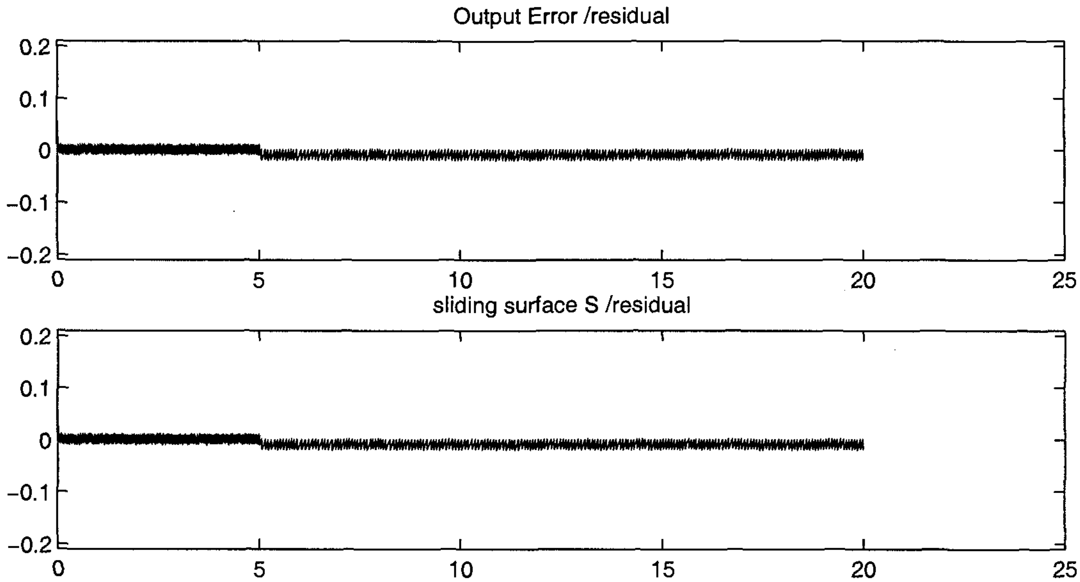

Figure 4.5: Output Error and Sliding Surface by First Order Sliding Mode Observer.

Chapter 5

An ILO-Based Fault Detection and IL-Based Accommodation in Nonlinear Systems

This chapter presents a general framework for fault detection and accommodation using an Iterative Learning (IL) strategy. An Iterative Learning Observer (ILO), which is updated online by immediate past system output errors as well as inputs, is constructed for the purpose of fault detection. This ILO involving previous system information is different from the conventional Luenberger Observer whose states are only a function of the current inputs, outputs, and the estimation errors. Furthermore, using IL strategy, an automatic control reconfiguration scheme for fault accommodation is also described. One of the main features of the proposed scheme is that the control reconfiguration is achieved automatically based only on the response of the overall systems. The IL controller does not require a fault detection and isolation subsystem. An example is employed to verify the effectiveness of the ILO-based fault detection and IL fault accommodation scheme.

5.1 Introduction

A commonly utilized approach for fault detection is through the use of an observer. The well known Luenberger (or Luenberger-like) observers are adopted broadly to detect the faults in linear, time delay, as well as certain classes of nonlinear systems [47, 105, 124, 128, 136, 137].

Fault diagnosis in nonlinear systems has not received as much attention as its linear counterpart partly due to the fact that, in general, nonlinear control and observer design themselves are not as mature of fields as the linear ones. References [40, 120, 133, 136] are examples of certain works in this direction.

Nonlinear diagnostic observer design is at the heart of observer based FDI for nonlinear systems. In this regard, [102] presents some fundamental insights into observer design for a class of Lipschitz nonlinear systems. The work of [31], which is an extension of the well known Luenberger observer, proposes a state observer for nonlinear continuous time systems. In particular, the construction of the observer proposed in their work does not require a preliminary nonlinear change of coordinates, and the observer convergence can be proved under very general conditions. References [37, 131] propose a class of sliding mode observer for nonlinear systems. The observer is based on the equivalent control concept. In recent works [9, 58], observers based on some ideas from the high gain approach, whose gain could easily be designed, were proposed for multivariable nonlinear systems.

As for accommodation of faults, [6] describes the design of an automatic control reconfiguration scheme for accommodation of actuator faults in a class of plants where the number of control inputs is larger than the number of controlled outputs. The method is developed for a particular type of fault and the information about the fault is not available to the controller. In addition, [42, 99] proposed accommodation approaches from the neural network viewpoint.

In this chapter a new observer design methodology using IL concept for fault detection is proposed. In addition, the IL strategy is also used to reconfigure the control system structure for fault accommodation.

5.2 Problem Statement and Preliminaries

Consider a nonlinear system described by the following equation

$$\begin{aligned} \dot{x}(t) &= \xi(x(t)) + g(x(t))u + f_a(x(t), u) \\ y(t) &= h(x(t)) \end{aligned} \tag{5.1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $y(t) \in \mathbb{R}^m$ is the system's measurement, $\xi(x) : \mathbb{R}^n \to \mathbb{R}^n$, $f_a(x, u) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $g(x) : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ and $h(x) : \mathbb{R}^n \to \mathbb{R}^m$ are smooth vector fields. The function $f_a(x, u)$ characterizes the change in the system due to a fault. A system fault typically results in changes in the parameters of the system or even in changes in the dynamics of the system. In the most general case, such changes are represented by a nonlinear function of the system states and inputs [120]. It should be noted that the fault representation given by (5.1) does not address sensor faults. The description of a sensor fault requires an additional term in the output equation.

The objective of this chapter is to develop an IL fault detection and accommodation strategy based on the dynamic system model in (5.1). An ILO, which is updated or driven successively by the ILO input, will be constructed. The ILO possess the capability of detecting any changes in the system dynamics. Once a fault is detected, self-correction based on IL approach is achieved through the reconfiguration of the control system. This reconfiguration of the control system is based on the response of the overall system, and not based on the information about faults.

For convenience, the following notation is introduced

$$A(t) = \frac{\partial \xi}{\partial x}(\hat{x}).$$

The expansion of $\xi(x)$ into power series leads to

$$\xi(x) - \xi(\hat{x}) = A(t)e(t) + \phi(\hat{x}, x) \tag{5.2}$$

where $\phi(\hat{x}, x)$ is the second and higher order terms in $e(t) = x(t) - \hat{x}(t)$. In addition, $A(t)$ is an $n \times n$ matrix. The expansion of $\xi(x)$ into power series makes it convenient to discuss the stability of the proposed ILO.

Throughout this chapter we will make the following assumptions

Assumption 5.1 *Functions $g(x)$ and $h(x)$ are Lipschitz in x with Lipschitz constants k_g and k_h*

$$\|g(x) - g(\hat{x})\| \leq k_g \|e(t)\|$$
$$\|h(x) - h(\hat{x})\| \leq k_h \|e(t)\|.$$

Assumption 5.2 *System input vector is bounded by k_u, and fault $f_a(x, u)$ is bounded by f_m.*

Assumption 5.3 *There is a real number $k_\phi > 0$ such that ϕ is bounded via*

$$\|\phi(\hat{x}, x)\| \leq k_\phi \|e(t)\|.$$

Assumption 5.4 *Symmetric matrix $P(t)$ has the following property:*

$$\beta_1 I \leq P(t) \leq \beta_2 I$$

where β_1, β_2 are positive real numbers, and $P(t)$ is the solution of the following Riccati equation:

$$A^T(t)P(t) + P(t)A(t) + \dot{P}(t) + P(t)P(t) = -Q, \quad Q = Q^T > 0. \tag{5.3}$$

In [99], a learning paradigm was proposed for fault detection and accommodation based on neural network. Here, we shall propose another approach for fault detection and accommodation based on IL concept. The main characteristic of this IL approach is the fact that the observer inputs are updated online by the past inputs and previous information which drives the observer to follow the system model and to detect any changes in the system dynamics once this ILO is constructed.

5.3 ILO-Based Fault Detection Approach

In this section, the ILO is discussed at some length for the fault detection in a class of nonlinear systems. Recall the nonlinear system:

$$\begin{aligned} \dot{x}(t) &= \xi(x(t)) + g(x(t))u + f_a(x(t), u) \\ y(t) &= h(x(t)). \end{aligned} \tag{5.4}$$

In practice, it is not always possible to measure the state vector. Therefore, only inputs and outputs can be used to construct the observer for the purpose of fault detection. Thus, an ILO is constructed using only the system inputs and outputs:

$$\begin{aligned} \dot{\hat{x}}(t) &= \xi(\hat{x}(t)) + g(\hat{x}(t))u(t) + m(t) \\ \hat{y}(t) &= h(\hat{x}(t)) \\ m(t) &= Km(t-\tau) + Le_y(t-\tau) \end{aligned} \tag{5.5}$$

where τ denotes the sampling time interval, $\hat{x}(t)$ is the observer state vector at time t, K and L are ILO gain matrices. The term $m(t)$, which is called *ILO input*, is updated at each time instant t, and output estimation error $e_y(t-\tau) = h(x(t-\tau)) - h(\hat{x}(t-\tau))$. It is used to estimate fault f_a. This ILO has a similar structure with PI observers [109, 110, 111]. The difference between them lies in the fact that the ILO uses previous system information to estimate faults. The PI observer employs an additional differential equation to achieve fault estimation.

Subtracting observer equation (5.5) from system equation (5.1) and considering equation (5.2), we have

$$\dot{e}(t) = A(t)e(t) + \phi + (g(x) - g(\hat{x}))u(t) + f_a(x, u) - m(t). \tag{5.6}$$

Note that the above observer differs from a typical Luenberger observer which is driven by the inputs, outputs and the output errors at the current sampling time described as:

$$\dot{\hat{x}}(t) = \xi(\hat{x}(t)) + g(\hat{x}(t))u(t) + Le_y(t). \tag{5.7}$$

In the proposed ILO, observer states are updated by previous system output errors and previous observer input $m(t - \tau)$ as can be seen in equation (5.5).

Remark 5.3.1 To gain an understanding of this ILO, we can regard the nonlinear systems (5.1) as a reference model where the observer model tracks it driven by the iterative input $m(t)$.

Remark 5.3.2 The gain matrix of ILO input $m(t)$ is designated as an identity matrix, which coincides with that of the fault in the considered system equation. If the fault is described by $\Gamma f_a(x(t), u)$, where Γ is a constant gain matrix, then observer input will accordingly be expressed as $\Gamma m(t)$. The identical gain matrices make discussion convenient.

Remark 5.3.3 A similar concept as to the ILO input update law presented in equation (5.5) was referred to Time Delay Control by [138]. In this chapter, we shall refer to it as IL update law.

Lemma 5.3.1 describes a norm feature of the ILO input update law on which the proof of Theorem 5.1 will be based.

Lemma 5.3.1 If ILO input $m(t)$ is defined as $m(t) = Km(t-\tau) + Le_y(t-\tau)$, then following inequality holds

$$m^T(t)m(t) \leq 2m^T(t-\tau)K^TKm(t-\tau) + 2e_y^T(t-\tau)L^TLe_y(t-\tau). \tag{5.8}$$

Proof:

Substituting ILO input $m(t)$ into $2m^T(t)m(t)$, we have:

$$\begin{aligned} 2m^T(t)m(t) &= 2m^T(t-\tau)K^TKm(t-\tau) + 2m^T(t-\tau)K^TLe_y(t-\tau) \\ &+2e_y^T(t-\tau)L^TKm(t-\tau) \\ &+2e_y^T(t-\tau)L^TLe_y(t-\tau). \end{aligned} \tag{5.9}$$

By applying the following inequality

$$2a^Tb \leq a^Ta + b^Tb, \qquad \forall\ a, b \in \mathbb{R}^n, \tag{5.10}$$

we have:

$$\begin{aligned} 2m^T(t)m(t) &\leq 2m^T(t-\tau)K^TKm(t-\tau) + m^T(t-\tau)K^TKm(t-\tau) \\ &+e_y^T(t-\tau)L^TLe_y(t-\tau) + m^T(t-\tau)K^TKm(t-\tau) \\ &+e_y^T(t-\tau)L^TLe_y(t-\tau) + 2e_y^T(t-\tau)L^TLe_y(t-\tau). \end{aligned} \tag{5.11}$$

Therefore,

$$m^T(t)m(t) \leq 2m^T(t-\tau)K^TKm(t-\tau) + 2e_y^T(t-\tau)L^TLe_y(t-\tau). \tag{5.12}$$

This completes the proof. ∎

Theorem 5.1 *Consider the nonlinear systems (5.1) and its ILO given in equation (5.5). Let assumptions 5.1-5.4 hold. If inequalities (5.21) and (5.22) hold, then estimation error* $e(t)$ *is bounded.*

Proof: Consider the following Lyapunov function candidate

$$V = e^T(t)P(t)e(t) + \int_{t-\tau}^{t} e_y^T(\theta)Re_y(\theta)d\theta + \int_{t-\tau}^{t} m^T(\sigma)m(\sigma)d\sigma \tag{5.13}$$

where R is a symmetric positive definite matrix and $P(t)$ satisfies assumption 5.4.

The time derivative of the Lyapunov function candidate can be obtained as follows:

$$\begin{aligned} \dot{V}(e(t),t) &= \dot{e}(t)^T P(t)e(t) + e^T(t)\dot{P}(t)e(t) + e^T(t)P(t)\dot{e}(t) \\ &\quad + m^T(t)m(t) - m^T(t-\tau)m(t-\tau) + e_y^T(t)Re_y(t) \\ &\quad - e_y^T(t-\tau)Re_y(t-\tau). \end{aligned} \tag{5.14}$$

Substituting estimation error equation (5.6) into above equation, we have:

$$\begin{aligned} \dot{V}(e(t),t) &\leq e^T(t)(A^T(t)P(t) + P(t)A(t) + \dot{P}(t))e(t) + 2e^T(t)P(t)\phi \\ &\quad + 2e^T(t)P(t)(g(x)u(t) - g(\hat{x})u(t)) \\ &\quad + 2\|e(t)\|\|P(t)\|f_m + 2\|e^T(t)P(t)\|\|m(t)\| \\ &\quad + m^T(t)m(t) - m^T(t-\tau)m(t-\tau) + e_y^T(t)Re_y(t) \\ &\quad - e_y^T(t-\tau)Re_y(t-\tau). \end{aligned} \tag{5.15}$$

For any two vectors, following inequality holds

$$2\|X\|\|Y\| \leq X^T X + Y^T Y, \quad \forall X, Y \in \mathbb{R}^n, \tag{5.16}$$

therefore,

$$2\|e^T(t)P(t)\|\|m(t)\| \leq e^T(t)P(t)P(t)e(t) + m^T(t)m(t). \tag{5.17}$$

Equation (5.15) can be further extended by combining equation (5.17) into it

$$\begin{aligned}\dot{V}(e(t),t) \leq & -\lambda_{min}(Q)\|e(t)\|^2 + 2k_\phi\|P(t)\|\|e(t)\|^2 + 2k_g k_u\|P(t)\|\|e(t)\|^2 \\ & +2\|e(t)\|\|P\|f_m + (2+\sigma)m^T(t)m(t) - m^T(t-\tau)m(t-\tau) \\ & +e_y^T(t)Re_y(t) - e_y^T(t-\tau)Re_y(t-\tau) - \sigma m^T(t)m(t).\end{aligned} \quad (5.18)$$

where σ is a positive constant.

Considering Lemma 5.3.1, the above equation can be further derived

$$\begin{aligned}\dot{V}(e(t),t) \leq & -\lambda_{min}(Q)\|e(t)\|^2 + 2k_\phi\|P(t)\|\|e(t)\|^2 + 2k_g k_u\|P(t)\|\|e(t)\|^2 \\ & +2\|e(t)\|\|P\|f_m + m^T(t-\tau)((4+2\sigma)K^TK - I)m(t-\tau) \\ & k_h^2\lambda_{max}(R)\|e(t)\|^2 + e_y^T(t-\tau)((4+2\sigma)L^TL - R)e_y(t-\tau) \\ & -\sigma m^T(t)m(t)\end{aligned} \quad (5.19)$$

where

$$A^T(t)P(t) + P(t)A(t) + \dot{P}(t) + P(t)P(t) = -Q. \quad (5.20)$$

Therefore, if

$$\lambda_{min}(Q) \geq 2k_\phi\beta_2 + 2k_g k_u\beta_2 + k_h^2\lambda_{max}(R)\beta_2 \quad (5.21)$$

and select K and L such that

$$0 < (4+2\sigma)K^TK \leq I; \quad 0 < (4+2\sigma)L^TL \leq R, \quad (5.22)$$

then

$$\begin{aligned}\dot{V} & \leq -\gamma\|e(t)\|^2 + 2\|e(t)\|\|P\|f_m - \sigma m^T(t)m(t), \quad \gamma > 0 \\ & \leq -\gamma/2\|e(t)\|^2 - \sigma m^T(t)m(t) + k_a, \quad k_a > 0\end{aligned}$$

where

$$-\lambda_{min}(Q) + 2k_\phi\beta_2 + 2k_g k_u\beta_2 + k_h^2\lambda_{max}(R)\beta_2 = -\gamma. \quad (5.23)$$

This completes the proof of Theorem 5.1. ∎

Remark 5.3.4 The residual e_y can be used for fault monitoring and detection purposes. When there are no faults in the system, e_y should be zero. On the other hand, an e_y greater than a certain threshold value would point to the occurrence of a fault. It is the property of this observer that, some time after the occurrence of the fault, it would learn about the fault and e_y will be again driven to zero. This means that this ILO can learn and follow the post-fault system model driven by $m(t)$. Simulation studies to be presented later will confirm this.

Remark 5.3.5 It should be mentioned that the above observer can be used for the purpose of fault detection. Actually, fault isolation can be also accomplished without additional work, in that if estimation error $e(t)$ is bounded, then $f_a - m(t)$ can be proved to be bounded as well. Therefore, $m(t)$ is an estimation of system fault f_a, and accordingly, fault isolation can be achieved.

5.4 Fault Accommodation by the IL Approach

Fault accommodation, if possible, is typically achieved through reconfiguration of the control systems. In this section, a fault accommodation scheme by an IL approach similar to the one used for fault detection will be proposed.

Note that most of the theoretical studies on FDI in control systems focus on detection and isolation of sensor or actuator faults. In these cases, typically accommodation may take place once a fault of the sensors or actuators is detected and the faulty device is isolated. Accommodation then often amounts to compensating for the loss of the faulty instrument through estimation in case of a sensor, or controller reconfiguration, in case of an actuator. However, if the likely sources of the faults are no longer confined to sensor or actuators, it may no longer be possible to isolate and accommodate the fault in the manner described previously. In these cases, the goal will be to determine appropriate control actions to offset the effects of faults. Development of such a methodology is the primary focus of this section.

In the case of a fault, the control objective is to adjust the control input u such that system outputs track the outputs of a reference model given by

$$\begin{aligned} \dot{x}_m &= \xi(x_m) + g(x_m)u_m \\ y_m &= h(x_m) \end{aligned} \tag{5.24}$$

In this IL fault accommodation approach, the control law is updated online by its previous control inputs and the previous tracking errors. The iterative learning law is given by

$$u(t) = u_m + u(t-\tau) + M(t)\zeta_y(t-\tau) \tag{5.25}$$

where $u(\cdot)$ is the control input, $M(t)$ is a gain matrix, $\zeta_y(t-\tau) = y_m(t-\tau) - y(t-\tau)$ is system output error at time $t-\tau$.

One of the main features of the proposed scheme is that the control reconfiguration is achieved automatically based only on the response of the system. Hence, the controller does not require a fault detection and isolation subsystem. Therefore, no information about the fault is available to the controller, and the fault can occur at any unknown time.

The error dynamic equation can be obtained by subtracting equation (5.24) from the system equation

$$\dot{\zeta}(t) = [\xi(x) - \xi(x_m)] + [g(x)u - g(x_m)u_m] + f_a. \tag{5.26}$$

Before stating Theorem 5.2, we make two assumptions.

Assumption 5.5 *Symmetric matrix* $\Pi(t)$ *has following property:*

$$\alpha_1 I \leq \Pi(t) \leq \alpha_2 I$$

where α_1, α_2 *are positive real numbers, and* $\Pi(t)$ *is the solution of the following Lyapunov equation:*

$$A^T(t)\Pi(t) + \Pi(t)A(t) + \dot{\Pi}(t) = -R, \quad R = R^T > 0. \tag{5.27}$$

Assumption 5.6 *The function* $g(x)$ *is bounded with* b_g.

Assumption 5.7 *Assume that* $W(\zeta(t)) = \zeta^T(t)\Pi(t)\zeta(t)$ *is a positive definite function, where* $\Pi(t)$ *satisfies equation (5.27). If there exists* $W(\zeta(t-\tau)) \leq c^2 W(\zeta(t)), c > 1$, *then* $\|\zeta(t-\tau)\| \leq c\epsilon\|\zeta(t)\|$, *where* $\tau > 0$ *and* $\epsilon = (\alpha_2/\alpha_1)^{1/2}$.

Theorem 5.2 *Consider the faulty system (5.1), and let Assumptions 5.1-5.3, 5.5 - 5.7 hold. If*

$$\lambda_{min}(R) - 2k_\phi\alpha_2 - 2\epsilon c k_h b_g b_M \alpha_2 - 2k_g k_{u_m}\alpha_2 - \alpha \geq 0 \tag{5.28}$$

where $0 < \alpha < 1$, *then control law (5.25) guarantees both* $\zeta(t)$ *and* $\zeta_y(t)$ *are bounded.*

Proof: The Lyapunov function candidate is taken as :

$$V(\zeta(t), t) = \zeta^T(t)\Pi(t)\zeta(t). \tag{5.29}$$

This Lyapunov function possesses definiteness and decrescence as follows:

$$\alpha_1\|\zeta(t)\|^2 \leq V(\zeta(t), t) \leq \alpha_2\|\zeta(t)\|^2. \tag{5.30}$$

The time derivative of the Lyapunov function is:

$$\dot{V}(\zeta(t), t) = \dot{\zeta}^T(t)\Pi(t)\zeta(t) + \zeta^T(t)\dot{\Pi}(t)\zeta(t) + \zeta^T(t)\Pi(t)\dot{\zeta}(t). \tag{5.31}$$

Substituting error dynamic equation (5.26) into the above equation, we have

$$\begin{aligned} \dot{V}(\zeta(t), t) &= \zeta^T(t)(A^T(t)\Pi(t) + \Pi(t)A(t) + \dot{\Pi}(t))\zeta(t) + 2\zeta^T(t)\Pi(t)\phi \\ &\quad -2\zeta^T(t)\Pi(t)g_m(x)u_m(t) + 2\zeta^T(t)\Pi(t)g(x)u(t) \\ &\quad +2\zeta^T(t)\Pi(t)f_a(x, u). \end{aligned} \tag{5.32}$$

Considering assumptions 5.5 and 5.7 and substituting control law (5.25), we have:

$$\begin{aligned}\dot{V}(\zeta(t),t) &= -\zeta^T(t)R\zeta(t) + 2\zeta^T(t)\Pi(t)\phi - 2\zeta^T(t)\Pi(t)g_m(x)u_m(t) \\ &\quad +2\zeta^T(t)\Pi(t)g(x)u_m + 2\zeta^T(t)\Pi(t)g(x)M(t)\zeta_y(t-\tau) \\ &\quad +2\zeta^T(t)\Pi(t)f_a(x,u) + 2\zeta^T(t)\Pi(t)g(x)u(t-\tau) \\ &\leq -\lambda_{min}(R)\|\zeta(t)\|^2 + 2k_\phi\|\Pi(t)\|\|\zeta(t)\|^2 + 2f_m\|\Pi(t)\|\|\zeta(t)\| \\ &\quad +2k_g k_{u_m}\|\Pi(t)\|\|\zeta(t)\|^2 + 2b_g k_u\|\Pi(t)\|\|\zeta(t)\| \\ &\quad +2\epsilon c k_h\|g(x)\|\|M(t)\|\|\Pi(t)\|\|\zeta(t)\|^2\end{aligned} \tag{5.33}$$

where f_m is the bound of fault f_a.

Furthermore, take a value $0 < \alpha < 1$, provided that

$$\lambda_{min}(R) - 2k_\phi\alpha_2 - 2\epsilon c k_h b_g b_M\alpha_2 - 2k_g k_{u_m}\alpha_2 - \alpha \geq 0 \tag{5.34}$$

where b_g and b_M are the norm bounds of $g(x)$ and $M(t)$, respectively, it is easy to show that

$$\dot{V} \leq -\alpha\|\zeta(t)\|^2 \tag{5.35}$$

if $\|\zeta(t)\| \geq \varsigma$ with

$$\varsigma = \frac{2f_m\alpha_2 + 2b_g k_u\alpha_2}{\lambda_{min}(R) - 2k_\phi\alpha_2 - 2\epsilon c k_h b_g b_M\alpha_2 - 2k_g k_{u_m}\alpha_2 - \alpha}.$$

From Khalil [69], this means that ζ is convergent to the ball

$$\|\zeta(t)\| \leq \sqrt{\frac{\alpha_2}{\alpha_1}}\varsigma. \tag{5.36}$$

Therefore, given any $\varsigma^\star > \sqrt{\frac{\alpha_2}{\alpha_1}}\varsigma$, there exists a finite time T such that for all $t > T$ we have

$$\|\zeta(t)\| < \varsigma^\star.$$

Furthermore,

$$\|\zeta_y(t)\| \leq k_h \|\zeta(t)\| < k_h \varsigma^\star. \tag{5.37}$$

This completes the proof. ∎

5.5 An Illustrative Example

In this section, the proposed IL approach will be used to detect and accommodate faults in a simple nonlinear system described as follows:

$$\begin{aligned} \dot{x}(t) &= \xi(x(t)) + g(x(t))u(t) + f_a(x,u) \\ y(t) &= x_1(t) \end{aligned} \tag{5.38}$$

where

$$\xi(x) = \begin{bmatrix} -2x_1 + 0.5x_2 \\ -2x_2 \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 1 \\ 0.3x_1x_2 \end{bmatrix}$$

and $f_a(x,u)$ is a fault. Using the methodology described in Sections 5.3 and 5.4, the ILO is constructed as follows:

$$\begin{aligned} \dot{\hat{x}} &= \xi(\hat{x}) + g(\hat{x})u(t) + m(t) \\ \hat{y} &= \hat{x}_1 \\ m(t) &= Km(t-\tau) + Le_y(t-\tau) \end{aligned} \tag{5.39}$$

where

$$\xi(\hat{x}) = \begin{bmatrix} -2\hat{x}_1 + 0.5\hat{x}_2 \\ -2\hat{x}_2 \end{bmatrix} \qquad g(\hat{x}) = \begin{bmatrix} 1 \\ 0.3\hat{x}_1\hat{x}_2 \end{bmatrix}$$

5.5.1 Fault Detection

The norm of output estimation error is chosen to be the monitoring function:

$$\|e_y\| = \|x_1 - \hat{x}_1\|. \tag{5.40}$$

Assume that a fault occurs at $t = 5sec$. The simulation results are shown in Figure 5.1. It can be seen that the size of residual signal suddenly increases after the occurrence of a fault. Additionally, it can be observed that the residual converges to zero within very short time, which explains that the ILO can still follow system after the fault. Note as well that the initial nonzero value of the residual is due to the initial condition mismatch between the system and the ILO.

5.5.2 Fault Accommodation

In this experiment, we consider the problem of fault accommodation by the proposed approach of control system reconfiguration. Using the same example as in fault detection, the control objective is to force the system output to follow the desired output (the output of a desired model). Assume that the desired output is $sin(t)$ and that a fault occurs at $t = 5sec$. The simulation results are shown in Figures 5.2 and 5.3. It can be seen that the reconfiguration of the control system results in a very quick reduction in the tracking errors subsequent to the occurrence of the fault. The tracking error becomes huge and can not be reduced after the occurrence of a fault if there is no IL control reconfiguration in the control system.

5.6 Conclusions

In this chapter, an ILO was presented for fault detection and an IL reconfiguration for fault accommodation. The proposed ILO, driven by the ILO inputs and updated online by previous observer inputs and previous output errors between system outputs and the observer outputs, can track the system and detect any changes in the system due to faults. Also, as shown in the simulation results, the ILO can still follow the post-fault system. The proposed IL control law can automatically reconfigure the system inputs based only on the system response (output errors) once a fault occurs.

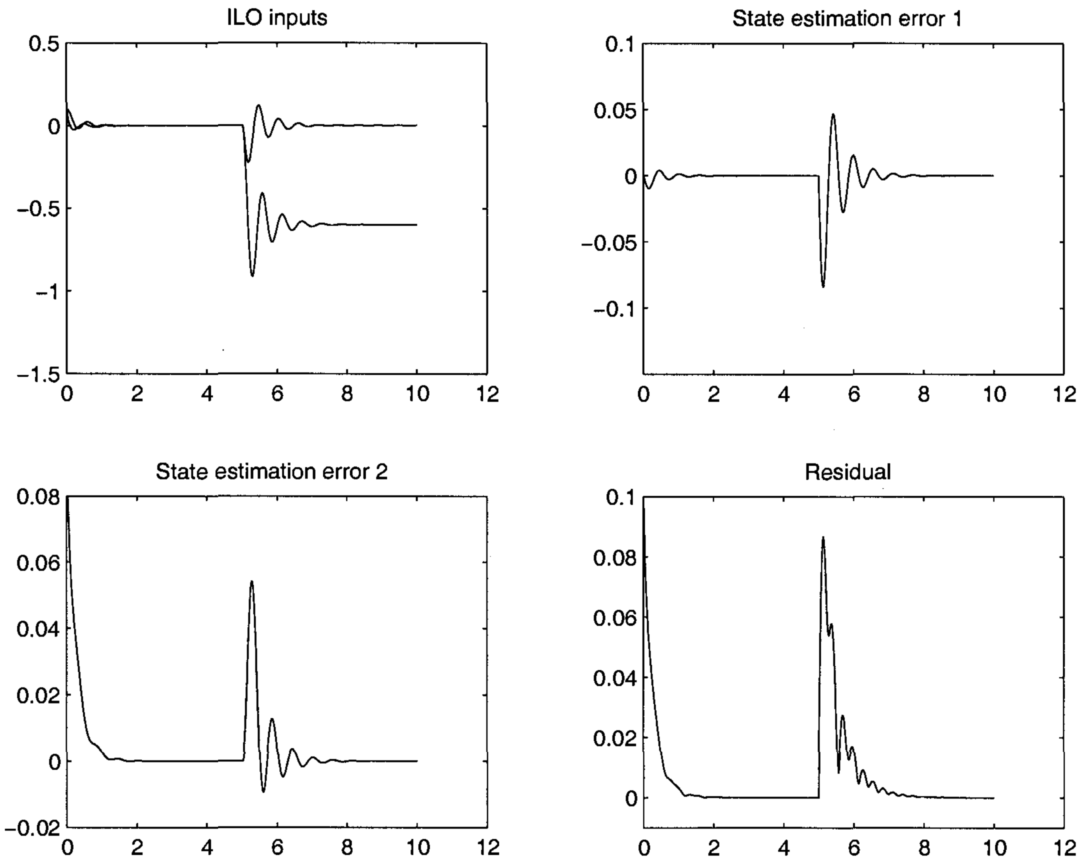

Figure 5.1: ILO-based fault detection.

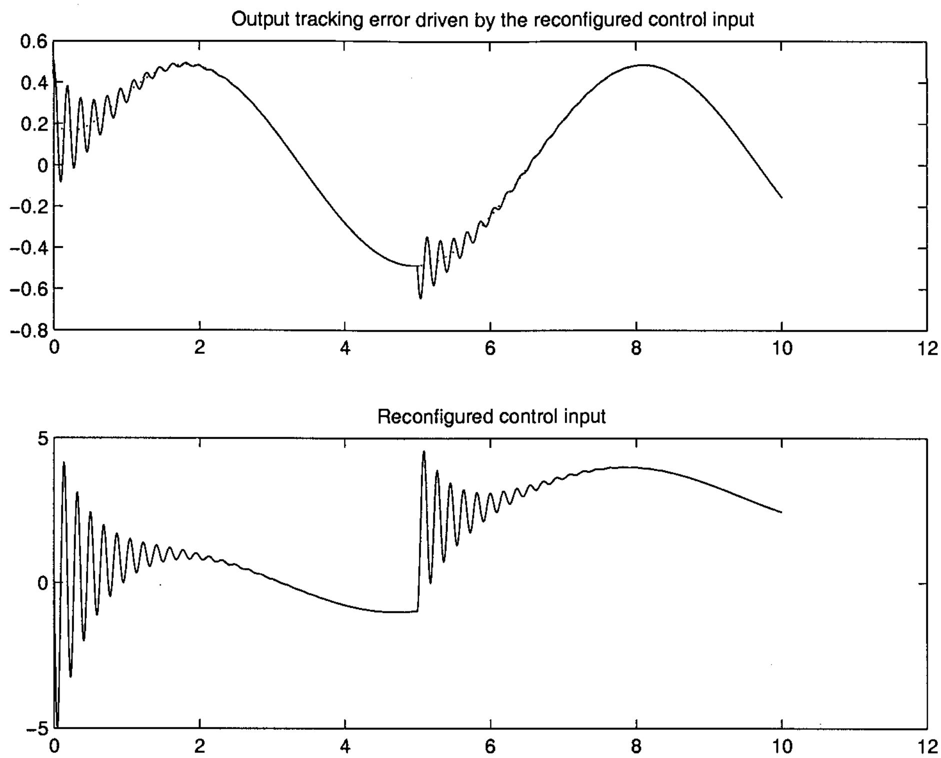

Figure 5.2: Output Tracking Error with IL Control.

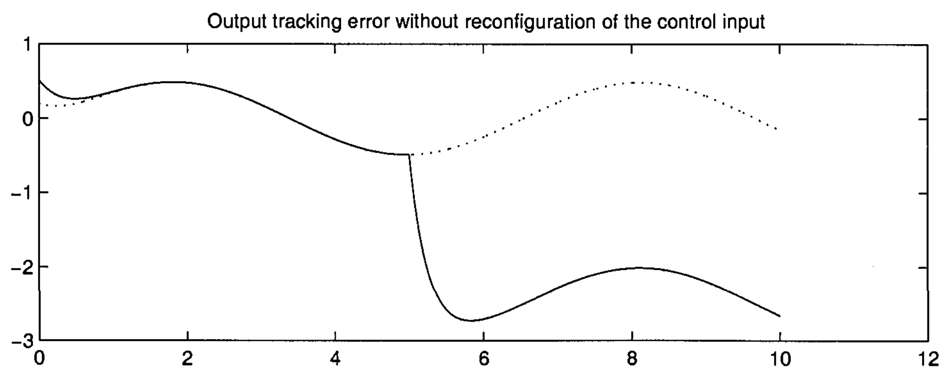

Figure 5.3: Output Tracking Error without IL Control.

Chapter 6

An Iterative Learning Observer-Based Fault Diagnosis Strategy

In this chapter, an Iterative Learning Observer (ILO) updated or driven successively and iteratively by immediate past system output errors and ILO inputs is to be proposed in a class of time-delay nonlinear systems for the purpose of robust fault diagnosis. Its main characteristic is that it can estimate not only system states but also disturbances and actuator faults such that the ILO can still track post-fault system model. It will be shown that the output disturbances are attenuated by the last sampling values. The differences between current and immediate past sampling values of disturbances are further compensated by the ILO input. Therefore, this ILO can attenuate output measurement disturbances. Moreover, the implementation of this ILO approach consumes less on-line calculation time in practice. The ILO fault diagnosis approach will be then applied to automotive engine fault detection and estimation in order to test its effectiveness.

6.1 Introduction

Some practical processes, for example, biology, mechanical and chemical engineering, involve delays that may cause instability or affect the performance of the control systems [2]. The fault diagnosis issues in this kind of time-delay systems have been attracting researchers' attention [73, 134]. Still, there exist limited results on fault diagnosis for time-delay systems, especially for nonlinear time-delay systems. Yang and Saif [134] proposed a robust observer for state estimation in a class of state-delayed dynamic systems. The existence condition of the proposed observer and the convergence proof are derived based on the Razumikhin theory. This observer is then used to detect and isolate actuator and sensor faults in a class of time-delay systems. An alternative parity space approach is used to synthesize a residual generator for time-delay systems in [73]. The application of a modern symbolic computation system like MAPLE to the development and the design of fault detection systems for time-delay systems is investigated.

An automotive powertrain is a typical time-delay nonlinear system [28]. Some research results on fault diagnosis were presented in [28, 54, 68, 70, 76, 93, 114], in all of which the main strategy for fault detection and isolation is based on the first order SMOs that can be seen in [68, 70, 71, 76]. Nevertheless, a crucial drawback of the first order SMOs is the chattering issue that stems from the switching term, which in most cases is undesirable because it may cause false fault alarm.

The ILO is first proposed in [18]. In this chapter, an ILO-based robust fault diagnosis strategy using the immediate past output errors and ILO inputs is presented for fault detection and estimation in the class of time-delay nonlinear systems. This ILO approach is then applied to fault detection and estimation of an automotive engine that is employed as an application example. The main property of this ILO is that it can compensate both system disturbances and actuator faults, which makes ILO so robust that it can still follow the post-fault model after the occurrence of an actuator fault. Compared with the SMO strategy for fault diagnosis, this ILO approach is robust while not having the chattering problem that is unavoidably met in the SMO. Additionally, the output measurement disturbances that are usually amplified by a classical Luenberger observer can be attenuated by the ILO.

6.2 Problem Statement

Consider a time-delay nonlinear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Phi(x,u) + Bx(t-t_h) + d(t) \\ y(t) &= Cx(t) \end{aligned} \tag{6.1}$$

where $x(t) \in \mathbb{R}^n$ is unmeasurable system state vector; $y(t)$ is measurable output vector; $d(t)$ is unmeasurable disturbance vector; t_h is a fixed delay; $\Phi(x,u)$ is a Lipschitz nonlinearity.

In this chapter, we shall construct an ILO whose states are updated by the previous system output errors and the previous ILO inputs. This differs from a classical Luenberger observer that is driven by system inputs and output errors of the current sampling time as described below:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \Phi(\hat{x},u) + B\hat{x}(t-t_h) + Le_y(t) \tag{6.2}$$

where $e_y(t) = y(t) - \hat{y}(t)$, and L is observer gain matrix.

By subtracting the equation above from the system equation (6.1), estimation error dynamics can be obtained

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + [\Phi(x,u) - \Phi(\hat{x},u)] + B\tilde{x}(t-t_h) + d(t) \tag{6.3}$$

where $\tilde{x}(\cdot) = x(\cdot) - \hat{x}(\cdot)$.

Obviously, disturbance $d(t)$ has impacts on error dynamics. Thus, the main drawback of the classical Luenberger observer is the lack of robustness. By contrast, the ILO to be proposed is so robust that the effects of disturbance $d(t)$ on error dynamics can be compensated by ILO input $v(t)$, which can be seen in the following derivation of the stability proof.

Output measurement disturbances are usually met in practice. The other issue to be discussed in this chapter is output disturbance attenuation. A Luenberger observer usually amplifies the effects of output disturbances on error dynamics. In what follows, the details are analyzed.

Rewrite time-delay nonlinear system (6.1) with output disturbances as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Phi(x, u) + Bx(t - t_h) \\ y(t) &= Cx(t) + d(t). \end{aligned} \tag{6.4}$$

The estimation error dynamics can be obtained by subtracting the Luenberger observer equation (6.2) from system equation (6.4)

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + [\Phi(x, u) - \Phi(\hat{x}, u)] + B\tilde{x}(t - t_h) - Ld(t). \tag{6.5}$$

From the estimation error equation above, we can know that the disturbance $d(t)$ is amplified by gain L. That means that the measurement disturbances are increased. On the other hand, if gain L is chosen smaller to attenuate the effects of the disturbances, then the stability of the observer will be affected. Busawon et al. [8] proposed a new PI observer to attenuate the effects of disturbances. Though the disturbance $d(t)$ is not amplified by PI observer gain, the disturbance itself still has influence on the estimation error dynamics.

In this chapter, an ILO is presented to attenuate output disturbances. More importantly, the implementation of this ILO approach uses less calculation time. The

PI observer consumes much more precious on-line calculation time in practice. Because, in the PI observer, an extra differential equation is added into the $n-$dimension system such that the original system is extended to an $(n+1)$-dimension one that will consume much more calculation time in practice. Accordingly, this PI observer probably could not be implemented if the sampling time interval is small.

6.3 Main Results

First of all, we construct a robust ILO with the property of disturbance compensation and estimation. Furthermore, output disturbance attenuation by this ILO will be discussed. The application of this ILO to robust fault diagnosis issues will be further introduced in section 6.4.

6.3.1 The ILO and Disturbance Estimation

In this investigation, the following assumptions are required.

Assumption 6.1 *Disturbance $d(t)$ and its derivative $\dot{d}(t)$ are bounded with known bounds*

$$\|d(t)\| \leq b_d, \quad \|\dot{d}(t)\| \leq b_{du}. \tag{6.6}$$

Assumption 6.2 *System is bounded input bounded state stable, and the derivative of system input vector u is bounded.*

Assumption 6.3 *Functions $\Phi(t)$, $\dfrac{\partial \Phi}{\partial x}$, and $\dfrac{\partial \Phi}{\partial u}$ are bounded and satisfy Lipschitz conditions as follows:*

$$\|\Phi(x,u) - \Phi(\hat{x},u)\| \leq \eta_1 \|x - \hat{x}\|, \tag{6.7}$$

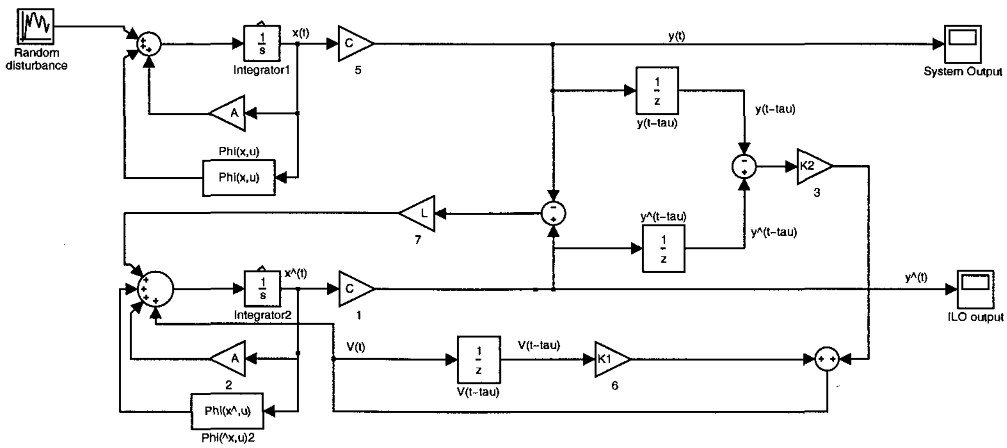

Figure 6.1: A Simulink block diagram description of the ILO.

$$\left\| \frac{\partial \Phi}{\partial x}(x,u) - \frac{\partial \Phi}{\partial x}(\hat{x},u) \right\| \leq \eta_2 \|x - \hat{x}\|, \tag{6.8}$$

and

$$\left\| \frac{\partial \Phi}{\partial u}(x,u)\dot{u} - \frac{\partial \Phi}{\partial u}(\hat{x},u)\dot{u} \right\| \leq \eta_3 \|x - \hat{x}\|. \tag{6.9}$$

Based on system equation (6.1), the ILO is proposed as follows

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + \Phi(\hat{x},u) + B\hat{x}(t-t_h) + L(y(t) - \hat{y}(t)) + v(t) \\ v(t) &= K_1 v(t-\tau) + K_2[y(t-\tau) - \hat{y}(t-\tau)] \end{aligned} \tag{6.10}$$

where $\hat{x}(\cdot)$ is estimated system state vector; τ is sampling time interval; $y(t-\tau)$ is the immediate past measurable output vector, i.e. the output at time $t-\tau$; $v(t)$ is called ILO input; L and $K_i's$ are some gain matrices to be determined. A Simulink block diagram describing the structure of the ILO is demonstrated in Figure 6.1 in order to obtain an insight of it, where tau:=τ, x$\hat{}$(t):=$\hat{x}(t)$, y$\hat{}$(t):=$\hat{y}(t)$, and Phi(x,u):=$\Phi(x,u)$.

The main feature of this ILO is that its states are updated by previous system output errors and previous ILO input $v(t-\tau)$, as can be seen in equation (6.10). It can detect and estimate any changes in the system dynamics. In the simulation, we shall see that this ILO can be used to robustly detect and estimate actuator faults. In fact, it is so robust that it can still track the post-fault system model driven by ILO input $v(t)$.

Subtracting observer equation (6.10) from system equation (6.1), we have:

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + [\Phi(x,u) - \Phi(\hat{x},u)] + B\tilde{x}(t - t_h) + d(t) - v(t) \qquad (6.11)$$

where $\tilde{x}(\cdot) = x(\cdot) - \hat{x}(\cdot)$ is state estimation error, matrix $(A - LC)$ can be a stable matrix by selecting an appropriate gain matrix L.

Remark 6.3.1 To obtain an understanding of this ILO, we can regard the nonlinear systems (6.1) as a reference model and the ILO driven by its input $v(t)$ can track it .

The following lemma will be helpful for the proof of Theorem 6.1.

Lemma 6.3.1 If ILO input $v(t)$ is defined in equation (6.10), then following inequality holds

$$v^T(t)v(t) \le 2v^T(t-\tau)K_1^T K_1 v(t-\tau) + 2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \quad (6.12)$$

Proof: Substituting ILO input $v(t)$ in equation (6.10) into $2v^T(t)v(t)$, we have:

$$\begin{aligned} 2v^T(t)v(t) &= 2v^T(t-\tau)K_1^T K_1 v(t-\tau) + 2v^T(t-\tau)K_1^T K_2 C\tilde{x}(t-\tau) \\ &\quad +2\tilde{x}^T(t-\tau)(K_2C)^T K_1 v(t-\tau) \qquad (6.13) \\ &\quad +2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \end{aligned}$$

By applying the following inequality

$$2a^T b \le a^T a + b^T b \qquad \forall\ a, b \in \mathbb{R}^n, \qquad (6.14)$$

we have:

$$
\begin{aligned}
2v^T(t)v(t) \;\; \leq \;\; & 2v^T(t-\tau)K_1^TK_1v(t-\tau) + v^T(t-\tau)K_1^TK_1v(t-\tau) \\
& +\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) + v^T(t-\tau)K_1^TK_1v(t-\tau) \\
& +\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \\
& +2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau).
\end{aligned} \tag{6.15}
$$

Therefore,

$$v^T(t)v(t) \leq 2v^T(t-\tau)K_1^TK_1v(t-\tau) + 2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \tag{6.16}$$

This completes the proof. ∎

Theorem 6.1 *Consider a time delay nonlinear system (6.1) satisfying Assumptions 6.1-6.3, and having an ILO given in equation (6.10). If equation (6.23), and inequalities (6.24) and (6.26), hold, then state estimate error is bounded.*

Proof:

Consider following Lyapunov function candidate:

$$V = \tilde{x}^TP\tilde{x} + \int_{t-\tau}^{t} \tilde{x}^T(\theta)R\tilde{x}(\theta)d\theta + \int_{t-t_h}^{t} \tilde{x}^T(\beta)\Gamma\tilde{x}(\beta)d\beta + \int_{t-\tau}^{t} v^T(\alpha)v(\alpha)d\alpha \tag{6.17}$$

where P, R and Γ are symmetric positive definite matrices.

Substituting estimation error equation (6.11) into the derivative of Lyapunov function candidate V, we have

$$
\begin{aligned}
\dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T(t)R\tilde{x}(t) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) + \tilde{x}^T(t)\Gamma\tilde{x}(t) \\
&\quad -\tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) + v^T(t)v(t) - v^T(t-\tau)v(t-\tau) \\
&= \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma)\tilde{x} + 2\tilde{x}^T PB\tilde{x}(t-t_h) \\
&\quad +2\tilde{x}^T P(\Phi(x,u) - \Phi(\hat{x},u)) + 2\tilde{x}^T Pd(t) - 2\tilde{x}^T Pv(t) \\
&\quad -\tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) - \tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) \\
&\quad +v^T(t)v(t) - v^T(t-\tau)v(t-\tau).
\end{aligned}
\tag{6.18}
$$

Combining inequalities

$$
2\|\tilde{x}^T P\|\|v(t)\| \leq \tilde{x}^T PP\tilde{x} + v^T(t)v(t) \tag{6.19}
$$

$$
2\tilde{x}^T PB\tilde{x}(t-t_h) \leq \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-t_h)B^T B\tilde{x}(t-t_h) \tag{6.20}
$$

into equation (6.18) leads to

$$
\begin{aligned}
\dot{V} &\leq \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + 2PP)\tilde{x} + 2b_d\|P\|\|\tilde{x}\| \\
&\quad +2\tilde{x}^T P(\Phi(x,u) - \Phi(\hat{x},u)) + \tilde{x}^T(t-t_h)B^T B\tilde{x}(t-t_h) + 2v^T(t)v(t) \\
&\quad -\tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) - \tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) - v^T(t-\tau)v(t-\tau).
\end{aligned}
\tag{6.21}
$$

Considering equation (6.7) of Assumption 6.3 and Lemma 6.3.1, equation (6.21) can be further extended as:

$$
\begin{aligned}
\dot{V} \leq\ & \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + 2PP)\tilde{x} + 2\eta_1 \|P\| \|\tilde{x}\|^2 \\
& + \tilde{x}^T(t-t_h) B^T B \tilde{x}(t-t_h) + (4+2\sigma) v^T(t-\tau) K_1^T K_1 v(t-\tau) \\
& - \tilde{x}^T(t-\tau) R \tilde{x}(t-\tau) - \sigma v^T(t) v(t) \\
& + (4+2\sigma)\tilde{x}^T(t-\tau)(K_2 C)^T (K_2 C)\tilde{x}(t-\tau) \\
& - \tilde{x}^T(t-t_h)\Gamma \tilde{x}(t-t_h) - v^T(t-\tau) v(t-\tau) + 2b_d \|P\| \|\tilde{x}\| \qquad (6.22) \\
\leq\ & \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + 2PP)\tilde{x} + 2\eta_1 \lambda_{max}(P) \|\tilde{x}\|^2 \\
& + \tilde{x}^T(t-t_h)(B^T B - \Gamma)\tilde{x}(t-t_h) - \sigma v^T(t) v(t) \\
& + \tilde{x}^T(t-\tau)((4+2\sigma)(K_2 C)^T (K_2 C) - R)\tilde{x}(t-\tau) + 2b_d \lambda_{max}(P) \|\tilde{x}\| \\
& + v^T(t-\tau)((4+2\sigma) K_1^T K_1 - I) v(t-\tau)
\end{aligned}
$$

where $I \in \mathbb{R}^{n \times n}$ is an identity matrix, and σ is a positive constant.

For any $Q = Q^T > 0$, there exists a $P = P^T > 0$ satisfying the following Ricatti equation

$$(A-LC)^T P + P(A-LC) + R + \Gamma + 2PP = -Q, \qquad (6.23)$$

and let

$$B^T B \leq \Gamma, \quad 0 < (4+2\sigma) K_1^T K_1 \leq I, \quad 0 < (4+2\sigma)(K_2 C)^T (K_2 C) \leq R, \qquad (6.24)$$

then equation (6.22) can be simplified as

$$
\begin{aligned}
\dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\| - \sigma v^T(t)v(t) \\
&= -\mu\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\| - \sigma v^T(t)v(t) \qquad (6.25) \\
&\leq -\mu/2\|\tilde{x}\|^2 - \sigma v^T(t)v(t) + 2k_a b_d, \qquad k_a > 0,
\end{aligned}
$$

where

$$
\mu = \lambda_{min}(Q) - 2\eta_1\lambda_{max}(P) > 0. \qquad (6.26)
$$

The proof is complete. ∎

Remark 6.3.2 In fact, $\dot{\tilde{x}}$ can also be proved bounded. To this end, let $z := \dot{\tilde{x}}$, and differentiate state estimation error equation (6.11) to obtain

$$
\dot{z} = (A - LC)z + s + Bz(t - t_h) + \dot{d}(t) - \dot{v}(t) \qquad (6.27)
$$

where $\dot{v}(t) = K_1\dot{v}(t-\tau) + K_2Cz(t-\tau)$,

and

$$
\begin{aligned}
s := \frac{d}{dt}(\Phi(x,u) - \Phi(\hat{x},u)) &= \left(\frac{\partial\Phi}{\partial x}(x,u)\dot{x} - \frac{\partial\Phi}{\partial x}(\hat{x},u)\dot{\hat{x}}\right) \\
&+ \left(\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u}\right).
\end{aligned}
$$

Assumptions 6.1, 6.2, and 6.3 can guarantee the boundedness of $\dot{x}$ and

$$
\begin{aligned}
\|s\| &\leq \left\|\frac{\partial\Phi}{\partial x}(x,u)\dot{x} - \frac{\partial\Phi}{\partial x}(\hat{x},u)\dot{\hat{x}}\right\| + \left\|\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u}\right\| \\
&\leq \left\|\frac{\partial\Phi}{\partial x}(x,u) - \frac{\partial\Phi}{\partial x}(\hat{x},u)\right\| \|\dot{x}\| + \left\|\frac{\partial\Phi}{\partial x}(\hat{x},u)\right\| \|z\| + \eta_3\|\tilde{x}\| \qquad (6.28) \\
&\leq r_1 + r_2\|z\|
\end{aligned}
$$

where r_1 and r_2 are two positive constants.

Using an analysis similar to that used in the analysis of the estimation error dynamics, one can know that $\|z\|$ is bounded.

Remark 6.3.3 Observing estimation error equation (6.11), if it is stable, then estimation error $\tilde{x}$ is bounded, and $\dot{\tilde{x}}$ is also bounded from remark 6.3.2. Accordingly, $-v(t)+d(t)$ is bounded. Therefore, we could say that the ILO input $v(t)$ can estimate or reconstruct disturbance $d(t)$. This will be seen in the simulation. In addition, the boundness of $-v(t)+d(t)$ also explains that the robustness of ILO results from ILO input $v(t)$. It is $v(t)$ that compensates the effects of disturbance $d(t)$ on estimation error dynamics.

6.3.2 Output Disturbance Attenuation by the ILO

In this subsection, output disturbance attenuation issue will be discussed by considering equation (6.4). To this end, the following assumptions are required:

Assumption 6.4 *The variation of $d(t)$ is bounded with a known bound*

$$\|d(t) - d(t-\tau)\| \leq l_d \tag{6.29}$$

where τ is the sampling interval in a sampled-data system.

Assumption 6.5 *Consider that $W(\tilde{x}) = \tilde{x}^T(t)P\tilde{x}(t)$ is a positive definite function, where $P = P^T > 0$ satisfies equation (6.48) and $\tilde{x}(t) = x(t) - \hat{x}(t)$. Assume that $W(\tilde{x}(t-\tau)) \leq q^2 W(\tilde{x}(t)), q > 1$, then $\|\tilde{x}(t-\tau)\| \leq q\rho\|\tilde{x}(t)\|$, where $\tau > 0$, the sampling time interval and $\rho = (\lambda_{max}(P)/\lambda_{min}(P))^{1/2}$.*

Remark 6.3.4 Assumption 6.5 is based on the stability theorem of Razumikhin [53].

To attenuate output disturbances, an ILO that is a little bit different from that in equation (6.10) is constructed as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \Phi(\hat{x}, u) + B\hat{x}(t - t_h) + K_1(y - \hat{y}) + v(t) \\ v(t) &= K_2 v(t - \tau) - K_1[y(t - \tau) - \hat{y}(t - \tau)]. \end{aligned} \tag{6.30}$$

The ILO (6.30) is almost the same as (6.10). The only difference between them is that the ILO (6.30) has two identical gain matrices $K_1's$.

The estimation error dynamics between system equation (6.4) and observer equation (6.30) is then given by

$$\begin{aligned} \dot{\tilde{x}} = & (A - K_1 C)\tilde{x} + [\Phi(x, u) - \Phi(\hat{x}, u)] + B\tilde{x}(t - t_h) + K_1 C\tilde{x}(t - \tau) \\ & -K_1[d(t) - d(t - \tau)] - K_2 v(t - \tau). \end{aligned} \tag{6.31}$$

Remark 6.3.5 We can see, from the estimation error dynamics (6.31), that the effect of disturbance $d(t)$ is attenuated by its immediate past sampling value that results from measurable output $y(t-\tau)$ in the ILO input $v(t)$. And, $K_1(d(t)-d(t-\tau))$ can be further attenuated by $v(t)$ if the estimation error dynamics (6.31) is stable. Also, this approach of disturbance attenuation consumes less on-line calculation time compared with PI observer where an extra differential equation is added into the $n-$dimension system, such that the original system is extended to an $(n + 1)$-dimension one that will consume much more calculation time in practice.

Remark 6.3.6 The observer gain K in [8] is guaranteed not to amplify the effect of $d(t)$. But, disturbance $d(t)$ itself still has an impact on error dynamics. In this ILO approach, the effect of disturbance $d(t)$ on estimation error dynamics is further reduced regardless of $(d(t) - d(t - \tau))$ being multiplied by K_1. This is important in fault detection because the reduction of disturbance effect can improve the robustness of fault detection.

Before stating Theorem 6.2, Lemma 6.3.2 is first introduced as follows.

Lemma 6.3.2 Consider ILO update law $v(t) = K_2 v(t-\tau) - K_1(y(t-\tau) - \hat{y}(t-\tau))$. If Assumption 6.5 holds, then $\|v(t)\| \leq l_n \|\tilde{x}(t-\tau)\| + b_n$, where l_n and b_n are two positive constants.

Proof:

For the initial $v(t_0)$, we could select it such that $\|v(t_0)\| \leq l_0 \|\tilde{x}(t_0)\|, t_0 \in [0, \tau]$, and for any $t > 0$, there exists $t = n\tau + t_0$, where n is non-negative. So, we have

$$\begin{aligned} \|v(\tau + t_0)\| &\leq \|K_2\|\|v(t_0)\| + \|K_1\|\|C\|\|\tilde{x}(t_0)\| + \|K_1\|b_d \\ &\leq l_0\|K_2\|\|\tilde{x}(t_0)\| + \|K_1\|\|C\|\|\tilde{x}(t_0)\| + \|K_1\|b_d \\ &= l_1\|\tilde{x}(t_0)\| + b_1 \end{aligned} \tag{6.32}$$

where $l_1 = l_0\|K_2\| + \|K_1\|\|C\|$, and $b_1 = \|K_1\|b_d$.

Next, we consider $v(2\tau + t_0)$

$$\begin{aligned} \|v(2\tau + t_0)\| &\leq \|K_2\|\|v(\tau + t_0)\| + \|K_1\|\|C\|\|\tilde{x}(\tau + t_0)\| + \|K_1\|b_d \\ &\leq l_1\|K_2\|\|\tilde{x}(t_0)\| + b_1\|K_2\| + \|K_1\|\|C\|\|\tilde{x}(t_0 + \tau)\| + \|K_1\|b_d \\ &\leq l_1 q\rho\|K_2\|\|\tilde{x}(\tau + t_0)\| + \|K_1\|\|C\|\|\tilde{x}(\tau + t_0)\| + b_2 \\ &= l_2\|\tilde{x}(t_0 + \tau)\| + b_2 \end{aligned} \tag{6.33}$$

where $l_2 = l_1 q\rho\|K_2\| + \|K_1\|\|C\|$, $b_2 = b_1 + b_1\|K_2\|$.

Assumption 6.5 is used in the derivation above. It will be also considered in equations (6.34) and (6.35).

Continuously, consider $v(3\tau + t_0)$, we have

$$\begin{aligned}
\|v(3\tau + t_0)\| &\leq \|K_2\|\|v(2\tau + t_0)\| + \|K_1\|\|C\|\|\tilde{x}(2\tau + t_0)\| + b_d\|K_1\| \\
&\leq l_2\|K_2\|\|\tilde{x}(\tau + t_0)\| + b_2\|K_2\| + \|K_1\|\|C\|\|\tilde{x}(2\tau + t_0)\| + b_d\|K_1\| \\
&\leq l_2 q\rho\|K_2\|\|\tilde{x}(2\tau + t_0)\| + \|K_1\|\|C\|\|\tilde{x}(2\tau + t_0)\| + b_3 \\
&= l_3\|\tilde{x}(2\tau + t_0)\| + b_3
\end{aligned} \tag{6.34}$$

where $l_3 = l_2 q\rho\|K_2\| + \|K_1\|\|C\|$, $b_3 = b_2\|K_2\| + b_1$

$$\vdots$$

$$\vdots$$

Finally,

$$\begin{aligned}
\|v(n\tau + t_0)\| &\leq \|K_2\|\|v((n-1)\tau + t_0)\| + \|K_1\|\|C\|\|\tilde{x}((n-1)\tau + t_0)\| + b_d\|K_1\| \\
&\leq l_{n-1}\|K_2\|\|\tilde{x}((n-2)\tau + t_0)\| + b_{n-1}\|K_2\| + b_d\|K_1\| \\
&\quad +\|K_1\|\|C\|\|\tilde{x}((n-1)\tau + t_0)\| \\
&\leq l_{n-1} q\rho\|K_2\|\|\tilde{x}((n-1)\tau + t_0)\| + \|K_1\|\|C\|\|\tilde{x}((n-1)\tau + t_0)\| \\
&\quad +b_{n-1}\|K_2\| + b_1 \\
&= l_n\|\tilde{x}((n-1)\tau + t_0)\| + b_n
\end{aligned} \tag{6.35}$$

where

$$l_n = l_{n-1} q\rho\|K_2\| + \|K_1\|\|C\|, \quad b_n = b_{n-1}\|K_2\| + b_1. \tag{6.36}$$

Therefore, $\|v(t)\| \leq l_n\|\tilde{x}(t-\tau)\| + b_n$. The proof is complete. ∎

Remark 6.3.7 The positive constant l_n has an explicit expression by substituting l_{i-1} into l_i, repetitively, where, $i = 1, \cdots, n$, i.e.

$$\begin{aligned} l_n &= l_0 q^{n-1}\rho^{n-1}\|K_2\|^n + q^{n-1}\rho^{n-1}\|K_2\|^{n-1}\|K_1\|\|C\| \\ &\quad + q^{n-2}\rho^{n-2}\|K_2\|^{n-2}\|K_1\|\|C\| + \cdots + q\rho\|K_1\|\|K_2\|\|C\| + \|K_1\|\|C\|. \end{aligned} \tag{6.37}$$

Meanwhile, the expression of b_n has the form of

$$b_n = b_1\|K_2\|^{n-1} + b_1\|K_2\|^{n-2} + \cdots + b_1\|K_2\| + b_1. \tag{6.38}$$

To guarantee the convergence of equations (6.37) and (6.38), K_2 must be selected such that $\|K_2\| < 1$ and $q\rho\|K_2\| < 1$. It is worth noting that once the significant digits after the decimal points of l_n and b_n are designated, l_n and b_n converge to constants, respectively, as n increases. It is easy to calculate them by using Matlab.

To derive the stability condition of estimation error equation (6.31), we choose a Lyapunov function candidate as follows

$$V = \tilde{x}^T P\tilde{x} + \int_{t-\tau}^{t} \tilde{x}^T(\theta)R\tilde{x}(\theta)d\theta + \int_{t-2\tau}^{t} \tilde{x}^T(\gamma)S\tilde{x}(\gamma)d\gamma + \int_{t-t_h}^{t} \tilde{x}^T(\beta)\Gamma\tilde{x}(\beta)d\beta \tag{6.39}$$

where P, R, S and Γ are symmetric positive definite matrices.

Substituting estimation error equation (6.31) into the derivative of Lyapunov function V, we have

$$
\begin{aligned}
\dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} + \tilde{x}^T(t)R\tilde{x}(t) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) + \tilde{x}^T(t)\Gamma\tilde{x}(t) \\
&\quad -\tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) + \tilde{x}^T(t)S\tilde{x}(t) - \tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau) \\
&= \tilde{x}^T((A-K_1C)^TP + P(A-K_1C) + R + \Gamma + S)\tilde{x} - 2\tilde{x}^T PK_2 v(t-\tau) \\
&\quad +2\tilde{x}^T PB\tilde{x}(t-t_h) + 2\tilde{x}^T PK_1C\tilde{x}(t-\tau) + 2\tilde{x}^T P(\Phi(x,u) - \Phi(\hat{x},u)) \\
&\quad -2\tilde{x}^T PK_1(d(t) - d(t-\tau)) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) - \tilde{x}^T(t-t_h)\Gamma\tilde{x}(t-t_h) \\
&\quad -\tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau).
\end{aligned}
\tag{6.40}
$$

By applying inequality (6.14), we have:

$$
2\tilde{x}^T PK_1C\tilde{x}(t-\tau) \le \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-\tau)(K_1C)^T(K_1C)\tilde{x}(t-\tau), \tag{6.41}
$$

$$
2\tilde{x}^T PB\tilde{x}(t-t_h) \le \tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-t_h)B^TB\tilde{x}(t-t_h). \tag{6.42}
$$

Considering equation (6.7) of Assumption 6.3 and equations (6.41) and (6.42), equation (6.40) can be further extended as

$$
\begin{aligned}
\dot{V} &\le \tilde{x}^T((A-K_1C)^TP + P(A-K_1C) + R + \Gamma + S + 2PP)\tilde{x} + 2\eta_1\|P\|\|\tilde{x}\|^2 \\
&\quad +2\|K_2\|\|\tilde{x}^T P\|\|v(t-\tau)\| + \tilde{x}^T(t-\tau)((K_1C)^T(K_1C) - R)\tilde{x}(t-\tau) \\
&\quad +\tilde{x}^T(t-t_h)(B^TB - \Gamma)\tilde{x}(t-t_h) + 2l_d\|K_1\|\|P\|\|\tilde{x}\| \\
&\quad -\tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau).
\end{aligned}
\tag{6.43}
$$

Applying Lemma 6.3.2 to the equation above, we obtain

$$\begin{aligned}
\dot{V} \leq\ & \tilde{x}^T((A - K_1C)^TP + P(A - K_1C) + R + \Gamma + S + 2PP)\tilde{x} \\
& + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + \tilde{x}^T(t - t_h)(B^TB - \Gamma)\tilde{x}(t - t_h) \\
& + \tilde{x}^T(t-\tau)((K_1C)^T(K_1C) - R)\tilde{x}(t-\tau) + 2l_{n-1}\|K_2\|\|\tilde{x}^TP\|\|\tilde{x}(t-2\tau)\| \\
& + 2b_{n-1}\lambda_{max}(P)\|K_2\|\|\tilde{x}\| + 2l_d\lambda_{max}(P)\|K_1\|\|\tilde{x}\| - \tilde{x}^T(t-2\tau)S\tilde{x}(t-2\tau).
\end{aligned} \tag{6.44}$$

Since

$$2\|\tilde{x}^TP\|\|\tilde{x}(t-2\tau)\| \leq \tilde{x}^TPP\tilde{x} + \tilde{x}^T(t-2\tau)\tilde{x}(t-2\tau), \tag{6.45}$$

then

$$\begin{aligned}
\dot{V} \leq\ & \tilde{x}^T((A - K_1C)^TP + P(A - K_1C) + R + \Gamma + S + 2PP \\
& + 2l_{n-1}\|K_2\|PP)\tilde{x} \\
& + \tilde{x}^T(t - t_h)(B^TB - \Gamma)\tilde{x}(t - t_h) + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 \\
& + \tilde{x}^T(t-\tau)((K_1C)^T(K_1C) - R)\tilde{x}(t-\tau) \\
& + 2b_{n-1}\lambda_{max}(P)\|K_2\|\|\tilde{x}\| + 2l_d\lambda_{max}(P)\|K_1\|\|\tilde{x}\| \\
& + \tilde{x}^T(t-2\tau)(2l_{n-1}\|K_2\|I - S)\tilde{x}(t-2\tau).
\end{aligned} \tag{6.46}$$

Let

$$0 < (K_1C)^T(K_1C) - R \leq 0, \quad (B)^T(B) - \Gamma \leq 0, \quad 2l_{n-1}\|K_2\|I - S \leq 0, \tag{6.47}$$

and for a positive definite symmetric matrix Q there exists a positive definite symmetric matrix P in the following equation

$$(A - K_1C)^TP + P(A - K_1C) + R + \Gamma + S + 2PP + 2l_{n-1}\|K_2\|PP = -Q, \tag{6.48}$$

then

$$\begin{aligned}\dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + 2\lambda_{max}(P)\|K_1\|\|\tilde{x}\|l_d \\ &\quad +2b_{n-1}\lambda_{max}(P)\|K_2\|\|\tilde{x}\| \\ &\leq -(\lambda_{min}(Q) - 2\eta_1\lambda_{max}(P))\|\tilde{x}\|^2 \\ &\quad +2(\lambda_{max}(P)\|K_1\|l_d + b_{n-1}\lambda_{max}(P)\|K_2\|)\|\tilde{x}\|\end{aligned} \tag{6.49}$$

where $\lambda_{min}(Q) > 2\eta_1\lambda_{max}(P)$.

Therefore, $\forall\ \|\tilde{x}\| \geq \dfrac{2\lambda_{max}(P)\|K_1\|l_d + 2b_{n-1}\lambda_{max}(P)\|K_2\|}{\lambda_{min}(Q) - 2\eta_1\lambda_{max}(P)}$, $\dot{V} \leq 0$.

Theorem 6.2 *Consider system equation (6.4) satisfying Assumptions 6.3, 6.4, 6.5, and Lemma 6.3.2. If inequality (6.47) and equation (6.48) hold, then, system state estimate error is bounded.*

Remark 6.3.8 We can say, from Theorem 6.2, that $-K_1(d(t)-d(t-\tau))-K_2v(t-\tau)$ is bounded because equation (6.31) is stable. By a similar operation, one can prove that $\dot{\tilde{x}}$ is also bounded. On one hand, disturbance $d(t)$ is attenuated by its immediate past value; on the other hand, the boundedness of $-K_1(d(t)-d(t-\tau))-K_2v(t-\tau)$ implies that ILO input $v(t)$ can further compensate the effect of $(d(t)-d(t-\tau))$ on estimation error dynamics. This demonstrates the effectiveness of output disturbance attenuation by the ILO.

6.4 Application to Automotive Engine Fault Diagnosis

In this section, the above proposed ILO will be applied to detect and estimate actuator faults in an automotive engine described by a second-order nonlinear engine model that involves intake to torque production delay and unmeasurable time varying disturbances. This delay is due to the fact that the engine torque production is a discrete process, but it is modelled as a continuous time domain. Therefore, the delay must be introduced [114].

6.4.1 An Automotive Engine Model

In what follows, we introduce an engine model based on [114]. We start with pressure p in the intake manifold that satisfies the following dynamic equation:

$$\dot{p} = k_1(\dot{m}_{maf} - \dot{m}_{cyl}) \tag{6.50}$$

where $k_1 = 180$. $\dot{m}_{maf}$ is the mass rate of air entering the manifold and $\dot{m}_{cyl}$ is the mass rate of air leaving the manifold and entering the combustion chamber. The mass rate of air entering the manifold is modelled as

$$\dot{m}_{maf} = a_1 u_1 p_r \tag{6.51}$$

where $a_1 = 0.3861 kg/s$ for the engine of interest. u_1 is normalized throttle characteristic $[0:1]$, taken as system input 1. p_r is normalized pressure influence that has following expression

$$p_r = \begin{cases} \sqrt{(\frac{p}{p_0})^{1.428} - (\frac{p}{p_0})^{1.714}} & \text{if } \frac{p}{p_0} > 0.528, \\ 0.259, & \text{otherwise,} \end{cases} \tag{6.52}$$

where p_0 is atmospheric pressure, $p_0 = 1 Bar$.

The mass flow rate entering the combustion chamber has the following expression

$$\dot{m}_{cyl} = kw\frac{p}{p_0} \tag{6.53}$$

where w is the engine speed (rad/s). $k = 1.73 \times 10^{-4}$.

The second nonlinear equation of engine model is the rotational dynamics

$$J_e\dot{w} = T_{ind} - T_f - T_h - T_p \tag{6.54}$$

where T_{ind} is indicated torque, T_f is friction torque, T_p is pump torque and T_h is disturbance torque. $J_e = 0.255kgm^2$ is the inertia moment of the engine.

Indicated Engine Torque can be modelled as

$$T_{ind} = a_2 k \frac{p(t-t_h)}{p_0} f_s(t-t_s) \tag{6.55}$$

where $t_h = 5.48/w$ is the intake to torque production delay, $t_s = 1.3/w$ is spark to torque production delay that will be neglected in this chapter. $a_2 = 8.51 \times 10^5 Nm/kg/rad$ represents the maximum torque and

$$f_s(t-t_s) = (cos(-b+u_2))^{2.875} \tag{6.56}$$

is the spark influence, where b is the distance from MBT (minimum spark advance for the best torque). Distance b can be fixed or adjusted within the interval $[0, 15°]$, thereby, control input u_2 varies within the interval $[-b, b]$.

Engine Friction Torque:

$$T_f = (97 + 0.1432w + 2.74 \times 10^{-4}w^2)\frac{V_{1cyl}1000z}{4\pi} \tag{6.57}$$

where $V_{1cyl} = 0.5 \times 10^{-3}m^3$ is the volume of one cylinder, $z = 5$ is the number of cylinders.

Engine Pump Torque:

$$T_p = (p_0 - p)\frac{V_{1cyl}z}{4\pi} \tag{6.58}$$

Engine Disturbance Torque T_d is bounded with $\|T_d(t)\| \leq c$, where c is a positive constant.

Finally, we have the nonlinear two state engine model as follows:

$$\begin{aligned} J_e\dot{w} &= a_2k\frac{p(t-t_h)}{p_0}(cos(-b+u_2))^{2.875} - T_f - T_d - T_p \\ \frac{\dot{p}}{p_0} &= k_1(a_1p_ru_1 - kw\frac{p}{p_0}). \end{aligned} \tag{6.59}$$

Based on this time delay two state engine model, the ILO fault detection strategy is to be implemented.

6.4.2 Fault Detection and Estimation for Automotive Engine

For convenience, letting $x_1(t) = w, x_2(t) = p$, equation (6.59) can be written as

$$\begin{aligned} \dot{x}_1 &= 576.65x_2(t-t_h) - 76 - 0.112x_1 - 2.148\times10^{-4}x_1^2 - 7.84\times10^{-4}(1-x_2) \\ &\quad +f_{actuator} \\ \dot{x}_2 &= 69.498p_ru_1 - 3.114\times10^{-2}x_1x_2 \\ y &= [x_1 \quad x_2]^T. \end{aligned} \tag{6.60}$$

Based on the equation above, the ILO is constructed according to equation (6.10) as follows:

$$\begin{aligned} \dot{\hat{x}}_1 &= 576.65\hat{x}_2(t-t_h) - 76 - 0.112\hat{x}_1 - 2.148\times10^{-4}\hat{x}_1^2 - 7.84\times10^{-4}(1-\hat{x}_2) \\ &\quad +2e_{y1}(t) - 0.0001e_{y2}(t) + v(t) \\ \dot{\hat{x}}_2 &= 69.498\hat{p}_ru_1 - 3.114\times10^{-2}\hat{x}_1\hat{x}_2 - 0.0001e_{y1}(t) + 3e_{y2}(t) \\ v(t) &= 0.49v(t-\tau) + 4.8e_{y1}(t-\tau) - 0.0001e_{y2}(t-\tau) \\ \hat{y} &= [\hat{x}_1 \quad \hat{x}_2]^T. \end{aligned} \tag{6.61}$$

The ILO will detect and estimate actuator fault $f_{actuator}$. It is assumed in the following fault detection that the healthy system has a fault $f_{actuator} = 0$, that sampling time interval $\tau = 0.01$, that $t_h \simeq \tau$, and that both system disturbance and output disturbance $d(t) = 0.15sin(5t)$.

Some comments on simulation are listed in the follows.

- Figures 6.2 and 6.3 show system and observer trajectories without actuator faults. Under this condition, the ILO input $v(t)$ can compensate and estimate system disturbance $d(t)$ in equation (6.1). In Figure 6.2, the states of the ILO asymptotically converge to system states after starting from the initial points. That is, the ILO can track system model very accurately. Figure 6.3 demonstrates the system disturbance estimation by ILO input $v(t)$. The zoom-in of $v(t)$ and $d(t)$ shows that, after some oscillations of the ILO input $v(t)$, the $v(t)$ can reconstruct $d(t)$ very accurately.

- Figures 6.4 and 6.5 indicate system and observer trajectories under an actuator fault. An actuator fault occurs at 15 seconds, taking system states off their original tracks. However, by observing engine speed diagram in Figure 6.4, the ILO states can still track the varied engine speeds. In Figure 6.5, the ILO input $v(t)$ can be chosen as a residual because it can estimate the actuator fault very accurately. Actually, the $v(t)$ has been estimating system disturbance $d(t)$. After the occurrence of an actuator fault, the $v(t)$ jumps to another higher value, such as 4 which is the real fault value. Meanwhile, the $v(t)$ varies according to disturbance $d(t)$. Therefore, the $v(t)$ can estimate both the actuator fault and system disturbance at the same time (see Figure 6.6). This is the main reason why the ILO can track the post-fault system model.

6.4.3 Output Disturbance Attenuation By the ILO

The ILO input $v(t)$ has the following form

$$v(t) = 0.05v(t-\tau) + 0.4e_{y1}(t-\tau) - 0.0001e_{y2}(t-\tau) \tag{6.62}$$

for output disturbance attenuation. Other ILO parameters are the same as those in last subsection.

Figures 6.7 and 6.8 describe output disturbance attenuation. Figure 6.7 shows state estimation errors between the considered system and a Luenberger observer with state estimation error 1 equal to 0.0262 at time 40 seconds. Figure 6.8 describes the state estimation errors between the considered system and an ILO with state estimation error 1 equal to -0.0017 at time 40 seconds. Both figures have the same coordinate scales. The absolute value of state error 1 in Figure 6.8 is about 15 times less than that in Figure 6.7. Furthermore, the absolute value of state error 2 at time 40 seconds in Figure 6.8 is about 20 times less than that in Figure 6.7. This demonstrates the effectiveness of the ILO-based disturbance attenuation strategy.

6.5 Conclusions

In this chapter, an ILO has been presented for fault detection, estimation, and output disturbance attenuation and has been applied to the automotive engine fault detection and estimation. It has been shown that the ILO can not only compensate the effects of disturbances and actuator faults, but also attenuate measurement disturbances. The convergence of ILO input vector to disturbances or actuator faults enables this kind of ILO to compensate the effects of disturbances or faults. Accordingly, it is robust. The principle of attenuating output disturbances by the ILO is due to both the immediate past output errors that raise an immediate past disturbance $d(t-\tau)$, and the ILO input $v(t)$. In the application example, the ILO input $v(t)$ is an ideal residual candidate because it can produce a "Jump" after an actuator fault occurs. Meanwhile, it can successfully estimate disturbances and faults, which enables the ILO to follow the post-fault system model.

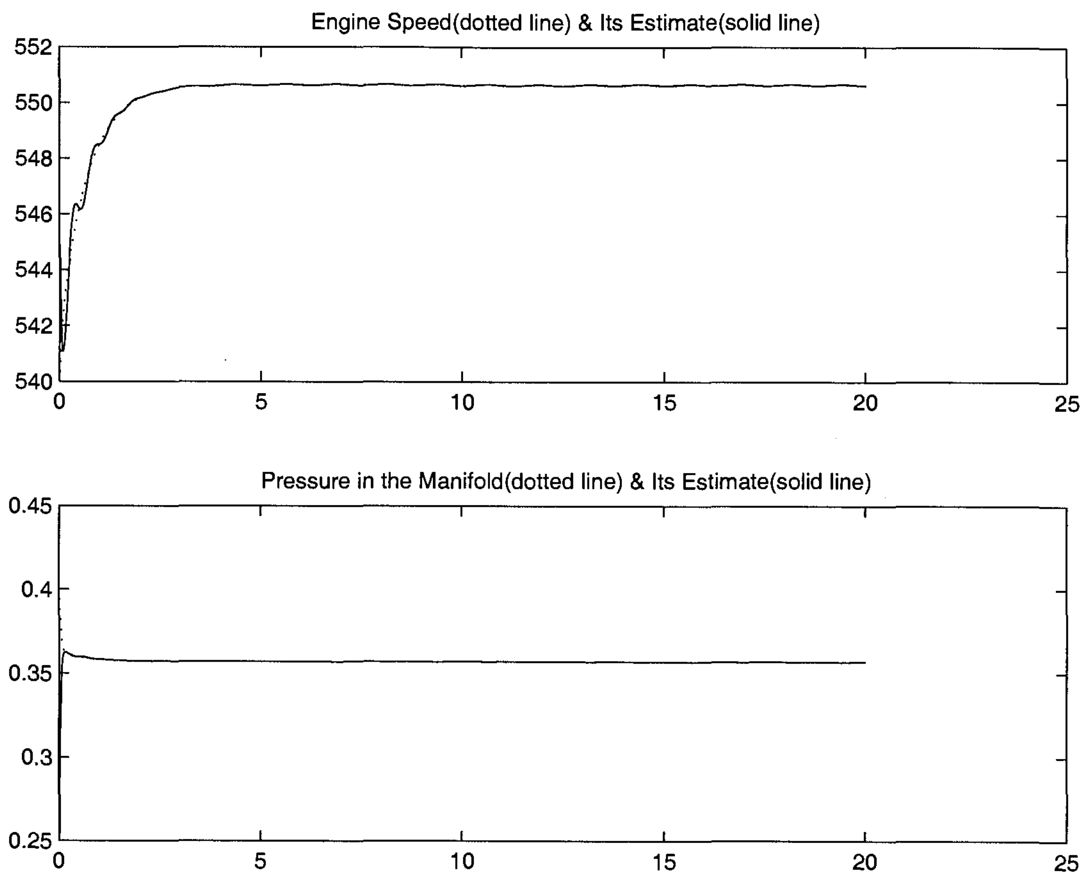

Figure 6.2: System states and their estimations.

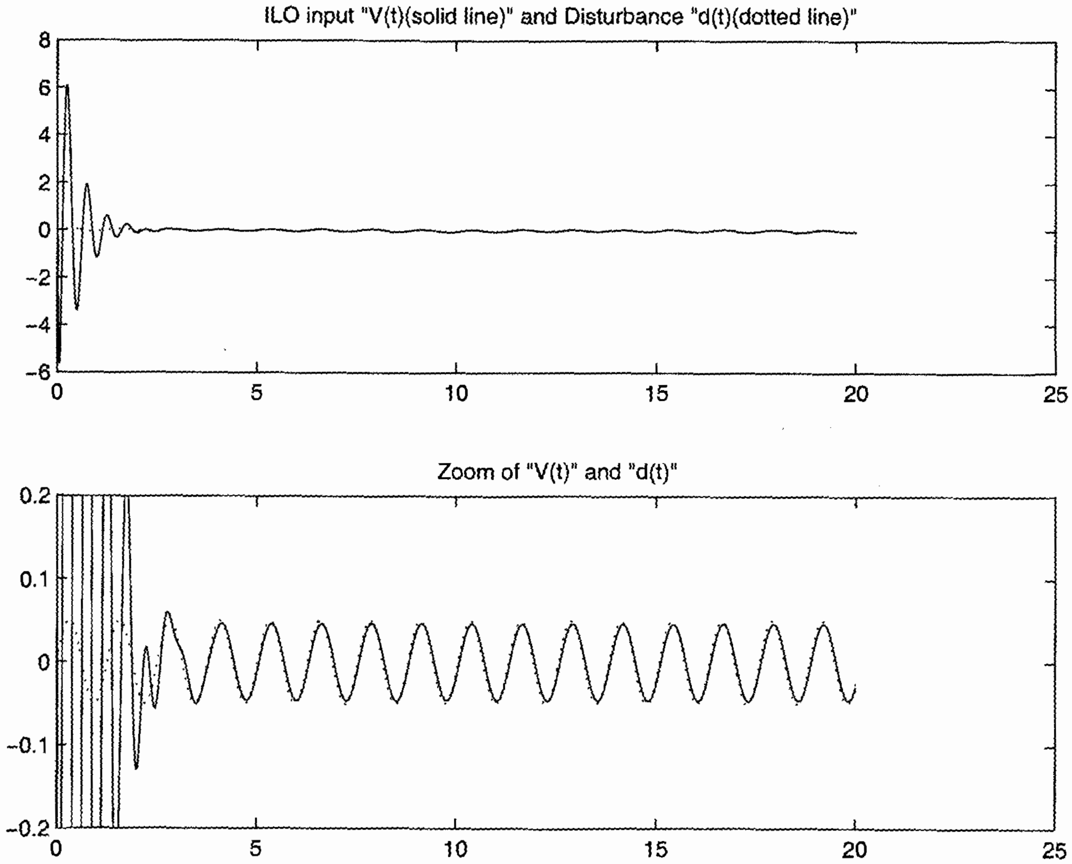

Figure 6.3: Disturbance estimation by ILO input.

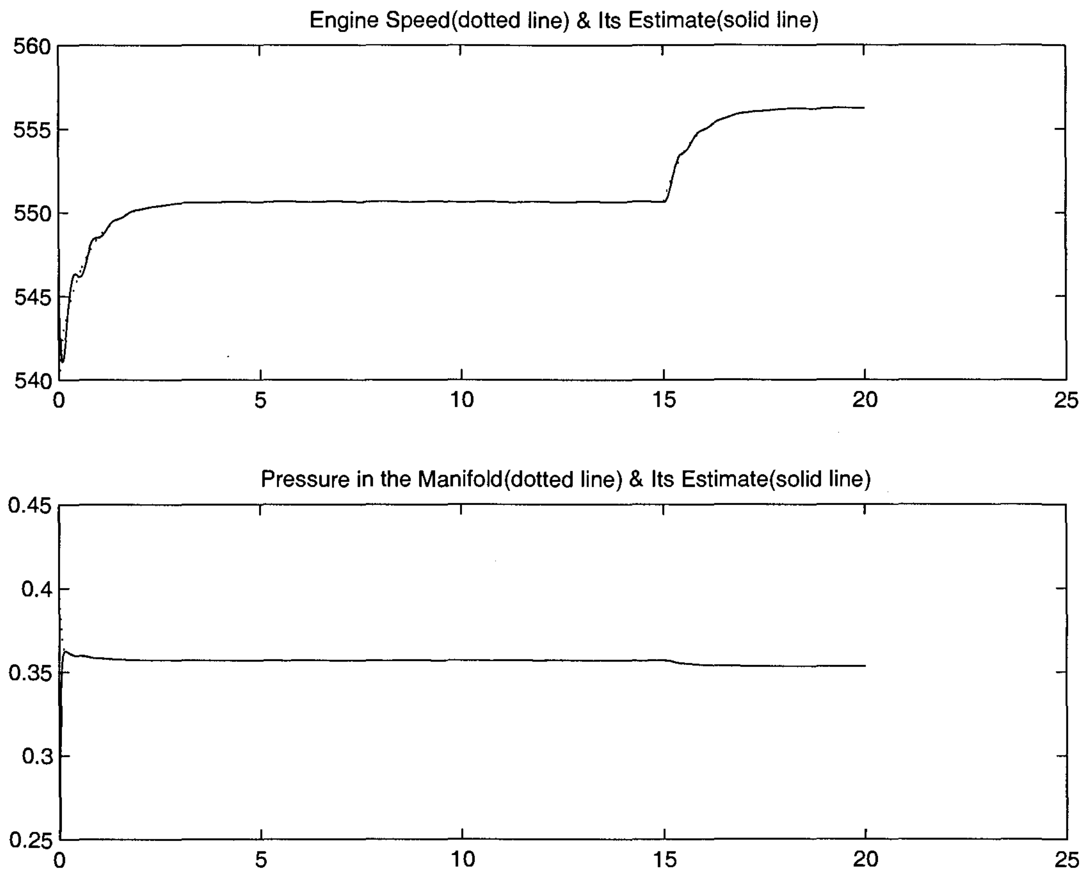

Figure 6.4: Post-fault system states.

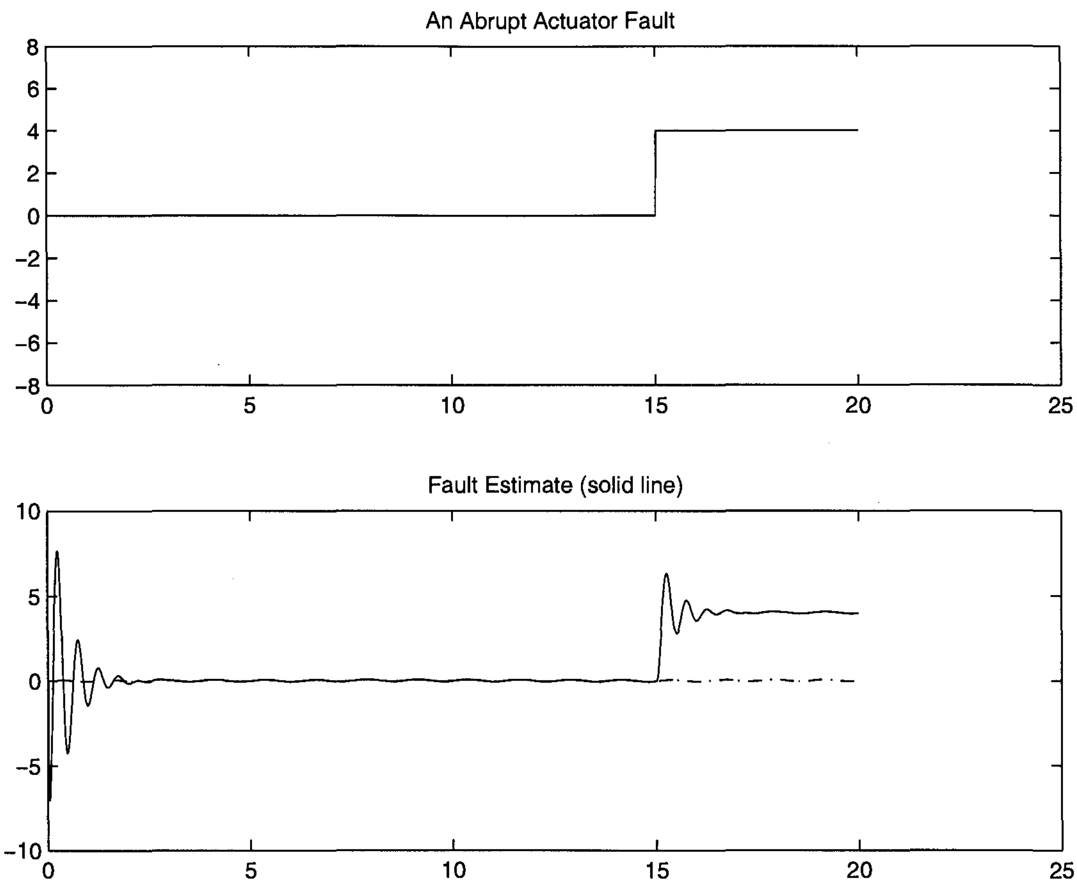

Figure 6.5: The fault and its estimate.

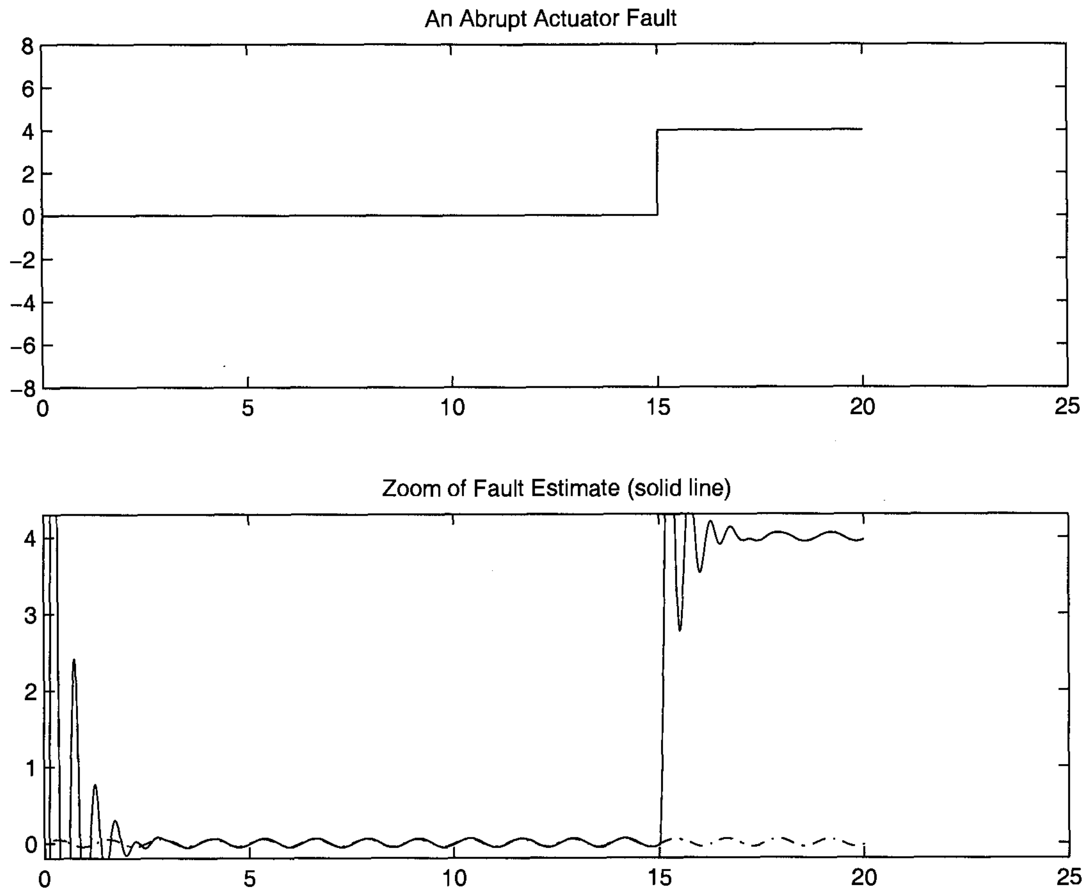

Figure 6.6: The fault and its estimate.

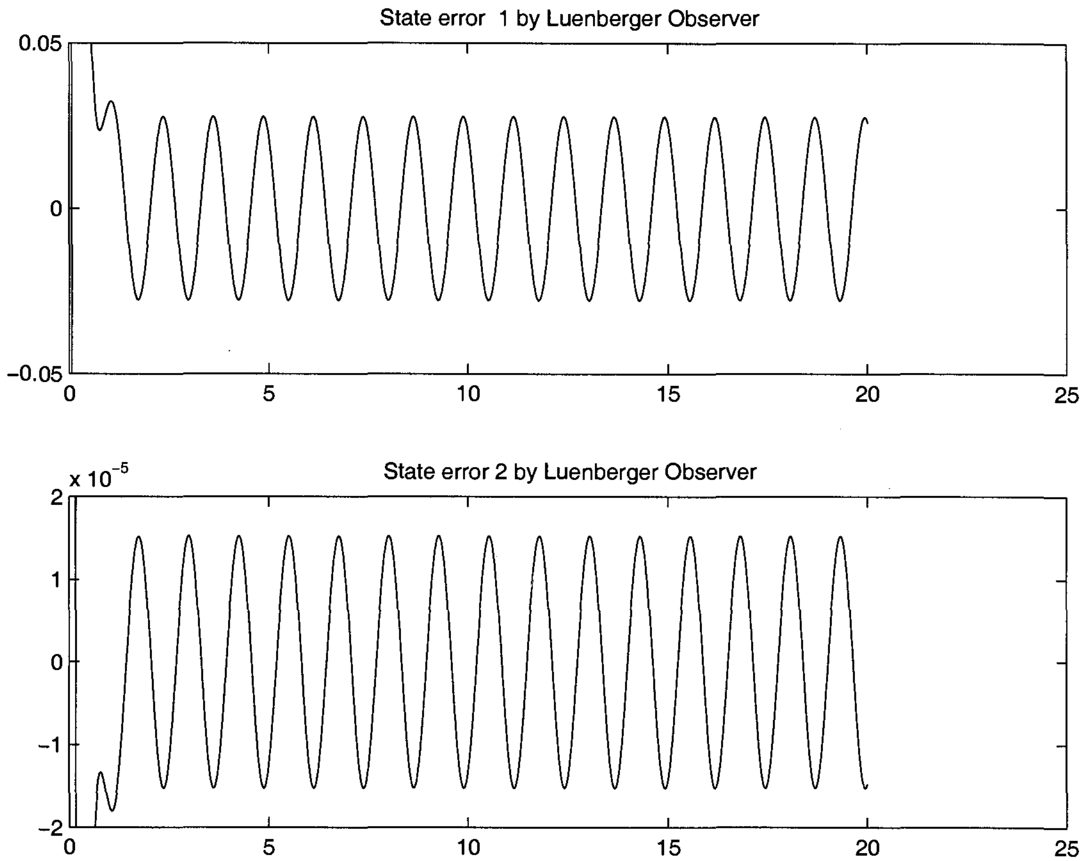

Figure 6.7: Estimation error dynamics between system and Luenberger observer.

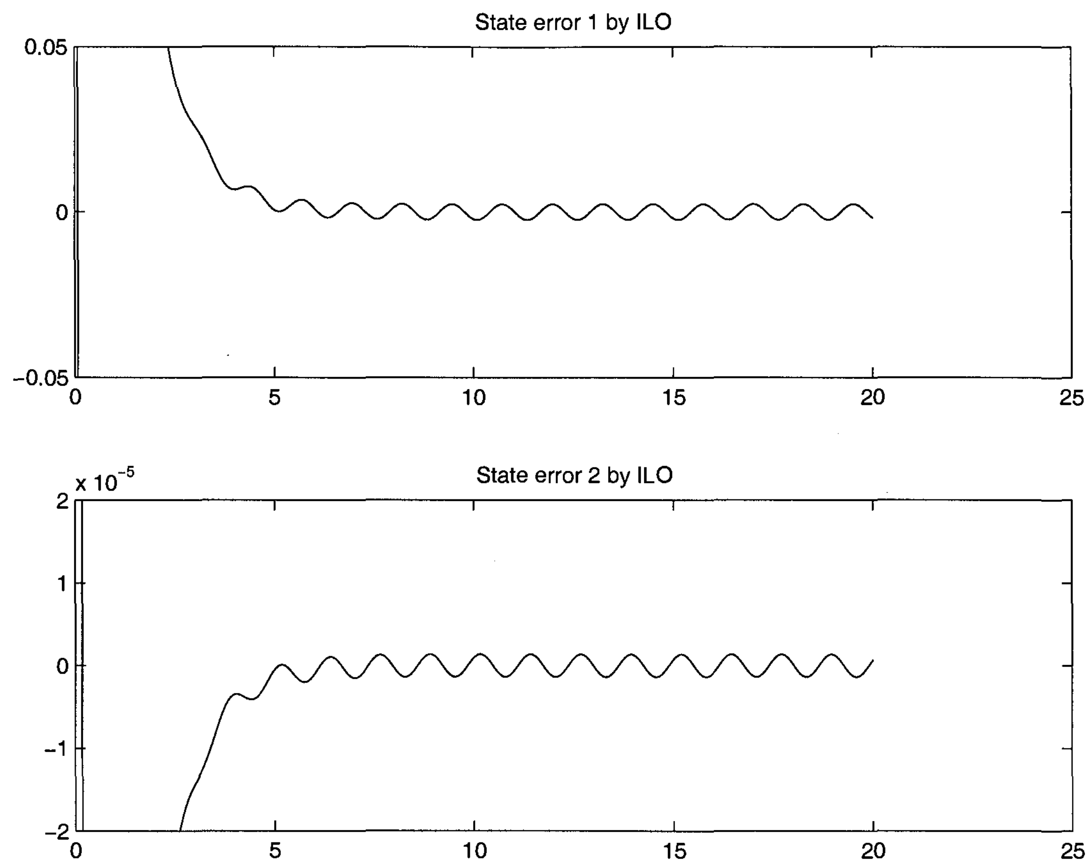

Figure 6.8: Estimation error dynamics between the system and the ILO.

Chapter 7

Fault Diagnosis and Compensation by an ILO

Fault detection, estimation, and compensation problem for a class of disturbance driven time delay nonlinear systems is addressed. The proposed approach relies on an ILO for fault detection and estimation. Under no faults, the ILO supplies accurate disturbance estimation where the effect of disturbances on estimation error dynamics can be accordingly attenuated. The proposed ILO can detect sudden changes in the nonlinear system. Thus the ILO is used to excite an adaptive control law in order to offset the effect of faults on the system dynamics. In addition, the ILO-based adaptive fault compensation strategy can handle multiple faults. The fault detection and compensation strategy is demonstrated in simulation on an automotive engine model.

7.1 Introduction

Issues dealing with the health of dynamical systems have attracted a great deal of attention in recent years. The majority of the research has dealt with the fault detection and isolation issue [17, 15, 70, 76, 68, 129, 134]. However, fault accommodation has also been subject of many studies [6, 16, 65, 92, 99, 115, 139]. Typical approach for fault accommodation is based on a fault detection and isolation subsystem where an additional control input resulting from this subsystem is added to the original control input to reduce or compensate the effect of faults [65, 92, 139]. It should be noted however that the fault detection and isolation subsystem is not always necessary for fault compensation [6].

The work in [94] reports a controller embedding an internal model of the fault, where in addition to reconstructing the fault, it is able to automatically offset the effect of it in induction motors. Another work related to fault tolerant control is [67], where an extra input to a nonlinear observer is used as a filter to directly estimate time-varying faults. Further, the estimate of faults is employed to establish a fault tolerant controller to guarantee the stability of the closed-loop system. From the nonlinear robust control viewpoint, a robust fault tolerant control is proposed in [101]. Stability and performance of the closed-loop system can be guaranteed in presence of uncertainties and when there are sensor faults.

In this work we propose a fault detection, estimation and compensation approach without resorting to the fault detection and isolation subsystem. The proposed methodology is based on the design of an ILO that monitors system dynamics variations caused by faults or/and disturbances. Additionally, once a fault is detected, the ILO will excite an adaptive control law that is used to generate an extra control input in order to offset the effect of faults on system dynamics such that system outputs can be maintained at their prescribed values. In addition to control adaptation, this process will also provide an estimate of the faults.

In recent years, environmental factors have been a driving push behind the interest in fault diagnosis in automotive engines [76]. It is now necessary to constantly monitor vehicle's operation in order to detect and compensate any abnormal behavior. An automotive powertrain is a typical time-delay nonlinear system [28]. Some research results on automotive fault diagnosis were presented in [28, 70, 76, 68, 93, 114], among which, the main strategy for fault diagnosis is based on sliding mode observers [70, 71, 76, 68]. In this work, the proposed ILO-based fault detection, estimation and compensation approach is suggested for an automotive engine.

7.2 Problem Statement and System Formulation

Consider a time-delay nonlinear system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Phi(x(t), x(t-t_d)) + Bu(t) + Ed(t) + Ef_a(x,t) \\ y(t) &= Cx(t) \end{aligned} \tag{7.1}$$

where $x(t) \in \mathbb{R}^n$ is unmeasurable system state vector; $y(t) \in \mathbb{R}^p$ is measurable output; $u(t) \in \mathbb{R}^m$ is system control input; $d(t) \in \mathbb{R}^q$ is unmeasurable disturbance; t_d is a fixed delay; $\Phi(x(t), x(t-t_d))$ is a Lipschitz nonlinearity; $f_a(x,t)$ represents system faults, such as aged components; A, B, C and E are constant matrices with appropriate dimensions, respectively. Distribution matrices of both disturbance $d(t)$ and fault $f_a(x,t)$ are designated as the same matrix E for the convenience of discussion. In this chapter, all actuators are assumed to be free from any faults.

The aim of the fault compensation or fault tolerance control is to adjust or modify the system control input in order to maintain the safety and reliability of the considered nonlinear system so that controlled system can still continue its original specifications [65, 92, 139].

In this chapter, we deal with fault detection, estimation, and compensation issues for the class of time delay nonlinear systems described above. Fault detection, estimation, and compensation will be achieved all at the same time by using an ILO together with an adaptive law, such that the formed closed-loop time delay nonlinear system has the property of fault rejection.

Throughout this chapter, following assumptions are required.

Assumption 7.1 *System control input $u(t)$ is bounded by b_u, and both disturbance $d(t)$ and fault $f_a(x,t)$ are bounded with $\|d(t)\| \leq b_d$ and $\|f_a(x,t)\| \leq b_f, \quad \forall t \geq 0$.*

Assumption 7.2 *[65, 92] Vector space spanned by the columns of E is a subset of the space spanned by the column vectors of B, that is $span(E) \subseteq span(B)$.*

Assumption 7.3 *Function $\Phi(x(t), x(t-t_d))$ satisfies Lipschitz condition with Lipschitz constants η_1 and η_2 i.e.*

$$\|\Phi(x(t), x(t-t_d)) - \Phi(\hat{x}(t), \hat{x}(t-t_d))\| \leq \eta_1\|\tilde{x}(t)\| + \eta_2\|\tilde{x}(t-t_d)\| \tag{7.2}$$

where $\tilde{x}(\cdot) = x(\cdot) - \hat{x}(\cdot)$.

Assumption 7.4 *Matrix A is Hurwitz and system (7.1) is bounded input-bounded state stable.*

Assumption 7.5 *Consider that $W(\tilde{x}) = \tilde{x}^T(t)P\tilde{x}(t)$ is a positive definite function, where $P = P^T > 0$ satisfies equation (7.37) and $\tilde{x}(t) := x(t) - \hat{x}(t)$. Assume that $W(\tilde{x}(t-\tau)) \leq q^2W(\tilde{x}(t)), q > 1$, then $\|\tilde{x}(t-\tau)\| \leq q\rho\|\tilde{x}(t)\|$, where $\tau > 0$, the sampling time interval and $\rho = (\lambda_{max}(P)/\lambda_{min}(P))^{1/2}$.*

Remark 7.2.1 Assumption 7.5 is based on the stability theorem of Razumikhin [53].

Remark 7.2.2 This chapter concerns only fault detection, estimation and compensation issues. Stabilization of the nonlinear system is not focused here. Therefore, the considered nonlinear time-delay system is assumed stable.

Remark 7.2.3 Assumption 7.2 guarantees that it is always possible to find a matrix M such that $BM = E$ for the purpose of fault compensation. Intuitively speaking, fault compensation can be achieved if we have an additional control input $w(t)$ that is adjusted on-line, satisfying

$$BMw(t) + Ef_a(x,t) = 0. \tag{7.3}$$

In this chapter, we first of all analyze a robust ILO regarding the properties of fault or/and disturbance estimation. The discussion regarding ILO based adaptive fault compensation issue is then followed where we shall show that this ILO, together with an adaptive law, can achieve fault detection, estimation and compensation at the same time. The application of this ILO plus the adaptive law to an automotive engine for robust fault diagnosis and compensation will be further demonstrated in section 7.5.

7.3 Analysis of the ILO regarding Disturbance and/or Fault Estimation

The ILO that was first proposed in [18] is to be discussed with respect to its main features. Considering its main feature, the ILO-based adaptive fault detection, estimation, and compensation issues will be introduced.

For the time being, temporarily ignore matrix E and the fault term $f_a(x,u,t)$ in system equation (7.1) for the convenience of discussion. In this case the following system will be of interest

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Phi(x(t), x(t-t_d)) + Bu(t) + d(t) \\ y(t) &= Cx(t). \end{aligned} \tag{7.4}$$

It is reasonable to rewrite system equation as (7.4) because one could regard $d(t)$ as either a disturbance or a fault.

According to [18, 20], the ILO has the following form

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + \Phi(\hat{x}(t), \hat{x}(t-t_d)) + Bu(t) + L(y(t) - \hat{y}(t)) + v(t) \\ v(t) &= K_1 v(t-\tau) + K_2[y(t-\tau) - \hat{y}(t-\tau)] \end{aligned} \tag{7.5}$$

where $\hat{x}(\cdot)$ is the estimated system state; $\hat{y}(\cdot)$ is the estimated system output; τ is sampling time interval; $y(t-\tau)$ is the immediate past measurable output, i.e. the output at time $t-\tau$; $v(t)$ is called *ILO input*; L and $K_i's$ are gain matrices with appropriate dimensions to be determined, where $i = 1, 2$.

The main characteristic of this ILO is that its states are updated or driven successively by the previous system output errors and the previous ILO input $v(t-\tau)$, as can be seen in equation (7.5). The term "*iterative*" indicates that ILO repeats the same operation, i.e. the ILO input being always updated by the previous information.

Subtracting observer equation (7.5) from system equation (7.4), we have:

$$\dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + [\Phi(x(t), x(t-t_d)) - \Phi(\hat{x}(t), \hat{x}(t-t_d)] + d(t) - v(t) \tag{7.6}$$

where $\tilde{x}(t) = x(t) - \hat{x}(t)$ is system state estimate error, matrix $(A - LC)$ can be a stable matrix by selecting an appropriate gain matrix L.

To prove theorem 7.1, the following lemma is needed.

Lemma 7.3.1 If ILO input $v(t)$ is defined in equation (7.5), then following inequality holds

$$v^T(t)v(t) \leq 2v^T(t-\tau)K_1^T K_1 v(t-\tau) + 2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \tag{7.7}$$

Proof:

Substituting expression of ILO input $v(t)$ in equation (7.5) into $2v^T(t)v(t)$, we have:

$$\begin{aligned} 2v^T(t)v(t) &= 2v^T(t-\tau)K_1^TK_1v(t-\tau) + 2v^T(t-\tau)K_1^TK_2C\tilde{x}(t-\tau) \\ &+2\tilde{x}^T(t-\tau)(K_2C)^TK_1v(t-\tau) \\ &+2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \end{aligned} \tag{7.8}$$

By applying the following inequality

$$2a^Tb \leq a^Ta + b^Tb \qquad \forall\ a, b \in \mathbb{R}^n, \tag{7.9}$$

we have:

$$\begin{aligned} 2v^T(t)v(t) &\leq 2v^T(t-\tau)K_1^TK_1v(t-\tau) + v^T(t-\tau)K_1^TK_1v(t-\tau) \\ &+\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) + v^T(t-\tau)K_1^TK_1v(t-\tau) \\ &+\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) \\ &+2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \end{aligned} \tag{7.10}$$

Therefore,

$$v^T(t)v(t) \leq 2v^T(t-\tau)K_1^TK_1v(t-\tau) + 2\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau). \tag{7.11}$$

This completes the proof. ∎

Theorem 7.1 *[20] Consider time delay nonlinear systems (7.4) satisfying Assumptions 7.1, 7.3, and 7.4, and having an ILO given in equation (7.5). If equation (7.17) and inequalities (7.18) and (7.20) hold, then state estimate error is bounded.*

Proof:

In what follows, time t of each variable will be omitted for the convenience of statement.

Consider following Lyapunov function candidate:

$$V = \tilde{x}^T P \tilde{x} + \int_{t-\tau}^{t} \tilde{x}^T(\theta) R \tilde{x}(\theta) d\theta + \int_{t-t_d}^{t} \tilde{x}^T(\beta) \Gamma \tilde{x}(\beta) d\beta + \int_{t-\tau}^{t} v^T(\alpha) v(\alpha) d\alpha \quad (7.12)$$

where P, R and Γ are symmetric positive definite matrices.

Substituting estimation error equation (7.6) into the derivative of Lyapunov function candidate V, we have

$$\begin{aligned} \dot{V} &= \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + \tilde{x}^T(t) R \tilde{x}(t) - \tilde{x}^T(t-\tau) R \tilde{x}(t-\tau) + \tilde{x}^T(t) \Gamma \tilde{x}(t) \\ &\quad - \tilde{x}^T(t-t_d) \Gamma \tilde{x}(t-t_d) + v^T(t) v(t) - v^T(t-\tau) v(t-\tau) \\ &= \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma)\tilde{x} \\ &\quad + 2\tilde{x}^T P(\Phi(x, x(t-t_d)) - \Phi(\hat{x}, \hat{x}(t-t_d))) + 2\tilde{x}^T P d(t) - 2\tilde{x}^T P v(t) \\ &\quad - \tilde{x}^T(t-\tau) R \tilde{x}(t-\tau) - \tilde{x}^T(t-t_d) \Gamma \tilde{x}(t-t_d) \\ &\quad + v^T(t) v(t) - v^T(t-\tau) v(t-\tau). \end{aligned} \quad (7.13)$$

Combining inequality

$$2\|\tilde{x}^T P\| \|v(t)\| \leq \tilde{x}^T P P \tilde{x} + v^T(t) v(t) \quad (7.14)$$

into equation (7.13) leads

$$\begin{aligned} \dot{V} &\leq \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \Gamma + PP)\tilde{x} \\ &\quad + 2\tilde{x}^T P(\Phi(x, x(t-t_d)) - \Phi(\hat{x}, \hat{x}(t-t_d))) + 2v^T(t) v(t) + 2b_d \|P\| \|\tilde{x}\| \\ &\quad - \tilde{x}^T(t-\tau) R \tilde{x}(t-\tau) - \tilde{x}^T(t-t_d) \Gamma \tilde{x}(t-t_d) - v^T(t-\tau) v(t-\tau) \end{aligned} \quad (7.15)$$

where b_d is the upper bound of disturbance $d(t)$.

Considering Assumption 7.3 and Lemma 7.3.1, equation (7.15) can be further extended as:

$$\begin{aligned}
\dot{V} \leq\ & \tilde{x}^T((A-LC)^TP + P(A-LC) + R + \Gamma + PP)\tilde{x} \\
& +2\eta_2\|\tilde{x}^TP\|\|\tilde{x}(t-t_d)\| + 2\eta_1\|P\|\|\tilde{x}\|^2 + (4+2\sigma)v^T(t-\tau)K_1^TK_1v(t-\tau) \\
& +(4+2\sigma)\tilde{x}^T(t-\tau)(K_2C)^T(K_2C)\tilde{x}(t-\tau) - \sigma v^T(t)v(t) \\
& +2b_d\|P\|\|\tilde{x}\| - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) - \tilde{x}^T(t-t_d)\Gamma\tilde{x}(t-t_d) \\
& -v^T(t-\tau)v(t-\tau) \qquad (7.16) \\
\leq\ & \tilde{x}^T((A-LC)^TP + P(A-LC) + R + \Gamma + PP + \eta_2 PP)\tilde{x} \\
& +\tilde{x}^T(t-t_d)(\eta_2 I - \Gamma)\tilde{x}(t-t_d) + v^T(t-\tau)((4+2\sigma)K_1^TK_1 - I)v(t-\tau) \\
& +\tilde{x}^T(t-\tau)((4+2\sigma)(K_2C)^T(K_2C) - R)\tilde{x}(t-\tau) + 2b_d\lambda_{max}(P)\|\tilde{x}\| \\
& +2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 - \sigma v^T(t)v(t)
\end{aligned}$$

where $I \in \mathbb{R}^{n\times n}$ is an identity matrix, and σ is a positive constant.

For any $Q = Q^T > 0$, there exists a $P = P^T > 0$ satisfying the following Riccati equation

$$(A-LC)^TP + P(A-LC) + R + \Gamma + PP + \eta_2 PP = -Q \qquad (7.17)$$

and let

$$\eta_2 I \leq \Gamma, \quad 0 < (4+2\sigma)K_1^TK_1 \leq I, \quad 0 < (4+2\sigma)(K_2C)^T(K_2C) \leq R, \qquad (7.18)$$

then equation (7.16) can be simplified as

$$\begin{aligned}\dot{V} &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\eta_1\lambda_{max}(P)\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\| - \sigma v^T(t)v(t) \\ &= -\mu\|\tilde{x}\|^2 + 2b_d\lambda_{max}(P)\|\tilde{x}\| - \sigma v^T(t)v(t) \qquad (7.19) \\ &\leq -\mu/2\|\tilde{x}\|^2 - \sigma v^T(t)v(t) + k_a b_d, \quad k_a > 0\end{aligned}$$

where

$$\mu = \lambda_{min}(Q) - 2\eta_1\lambda_{max}(P) > 0. \qquad (7.20)$$

The proof is complete. ∎

Remark 7.3.1 In fact, $\dot{\tilde{x}}$ can be proved bounded, to this end, let $z := \dot{\tilde{x}}$, and differentiate state estimation error equation (7.6) to obtain

$$\dot{z} = (A - LC)z + s + Bz(t - t_h) + \dot{d}(t) - \dot{v}(t) \qquad (7.21)$$

where $\dot{v}(t) = K_1\dot{v}(t-\tau) + K_2Cz(t-\tau)$

and $s := \frac{d}{dt}(\Phi(x,u) - \Phi(\hat{x},u)) = (\frac{\partial\Phi}{\partial x}(x,u)\dot{x} - \frac{\partial\Phi}{\partial x}(\hat{x},u)\dot{\hat{x}}) + (\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u})$.

Assumptions 7.1, 7.2, and 7.3 can guarantee the boundedness of $\dot{x}$ and

$$\begin{aligned}\|s\| &\leq \|(\frac{\partial\Phi}{\partial x}(x,u)\dot{x} - \frac{\partial\Phi}{\partial x}(\hat{x},u)\dot{\hat{x}})\| + \|\frac{\partial\Phi}{\partial u}(x,u)\dot{u} - \frac{\partial\Phi}{\partial u}(\hat{x},u)\dot{u}\| \\ &\leq \|\frac{\partial\Phi}{\partial x}(x,u) - \frac{\partial\Phi}{\partial x}(\hat{x},u)\|\|\dot{x}\| + \|\frac{\partial\Phi}{\partial x}(\hat{x},u)\|\|z\| + \eta_3\|\tilde{x}\| \qquad (7.22) \\ &\leq r_1 + r_2\|z\|\end{aligned}$$

where r_1 and r_2 are two positive constants. Using an analysis similar to that used in the analysis of the estimation error dynamics, one can know that $\|z\|$ is bounded.

Remark 7.3.2 From the proof of Theorem 7.1 and Remark 7.3.1, it is clear that both estimation error $\tilde{x}(t)$ and its derivative $\dot{\tilde{x}}(t)$ are bounded. Accordingly, $-v(t) + d(t)$ is also bounded. Thereby, the ILO input $v(t)$ can detect, estimate or reconstruct disturbance or fault $d(t)$, which will be seen in the simulation. If both disturbances and faults exist in the system, the ILO input $v(t)$ will estimate the composition of them. Therefore, given that the ILO input $v(t)$ can monitor variations of system dynamics. The ILO-based adaptive fault detection, estimation and compensation can be designed based on it. In addition, the boundedness of $-v(t)+d(t)$ also explains that the robustness of the ILO results from ILO input $v(t)$. It is $v(t)$ that compensates the effects of disturbance $d(t)$ on estimation error dynamics. The capability to estimate disturbances and faults will be demonstrated in the simulation study later.

7.4 Main Results

In this section, an ILO-based adaptive fault detection, estimation and compensation strategy, with proof of its stability will be presented.

7.4.1 ILO-Based Adaptive Fault Detection, Estimation, and Compensation

The ILO, together with an adaptive law, can achieve the fault detection, estimation, and compensation task. The task is accomplished in three stages: first fault detection is achieved by the ILO input $v(t)$ or the additional control input $w(t)$; second, the additional control input can estimate faults during fault compensation; and third, this estimation will offset the effect of faults on the system.

The fault compensation issue is about to be emphasized in this work because the fault detection and estimation can be achieved at the same time. The main goal

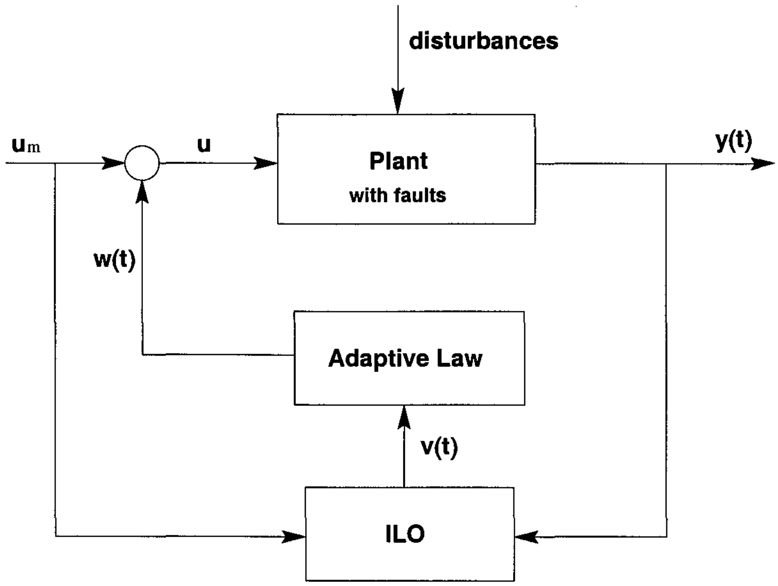

Figure 7.1: Dynamic process of fault compensation.

of this chapter is to annihilate the effect of faults on system performance and to maintain system outputs at their nominal values, while realizing fault detection and estimation. This will be achieved by the addition of a new control input, generated from an adaptive law excited by ILO input $v(t)$, to the nominal input.

The dynamic fault compensation process is described in Figure 7.1. That is, first of all, an ILO is constructed to monitor any system variations. Second of all, the ILO input $v(t)$ will be used to excite the adaptive law in order to generate an additional system input $w(t)$ for the purpose of eliminating the effect of faults. The transition phase of fault estimation and elimination will be successively monitored by the ILO. The adaptive law will then be successively updated stimulated by ILO input $v(t)$ until there exists no variation in the considered system. That means faults have been completely compensated. The current $w(t)$ is the estimations of the faults. Under the circumstance of no faults in the systems, the additional input $w(t)$ can estimate and eliminate the effect of disturbances.

We are in the position to present the ILO-based adaptive fault compensation strategy. Rewrite system equation (7.1) with faults as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Phi(x(t), x(t-t_d)) + Bu_m(t) + BMw(t) + Ed(t) + Ef_a(x,t) \\ y(t) &= Cx(t) \end{aligned} \tag{7.23}$$

where $w(t)$ is the additional system input for fault estimation and compensation; $u_m(t)$ is system nominal control input. According to Assumption 7.2, M can be selected such that $BM = E$. Vector $f_a(x,t)$ is the faults, such as component faults other than actuator or sensor faults.

The following ILO and adaptive law consist of the fault detection, estimation, and compensation methodology

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + \Phi(\hat{x}(t), \hat{x}(t-t_d)) + Bu_m(t) + L(y(t) - \hat{y}(t)) - Ev(t) \\ v(t) &= K_1 v(t-\tau) + K_2[y(t-\tau) - \hat{y}(t-\tau)] \\ \dot{w}(t) &= Fv(t) \end{aligned} \tag{7.24}$$

where L, K_1, K_2 and F are constant matrices to be determined.

As stated above, $v(t)$ is used to excite the adaptive law due to its sensitivity to variations in the system.

Subtracting equation (7.24) from system equation (7.23) leads to estimation error dynamics for the need of stability proof of the ILO-based adaptive fault compensation approach in the following subsection:

$$\begin{aligned} \dot{\tilde{x}}(t) = &\ (A - LC)\tilde{x}(t) + [\Phi(x(t), x(t-t_d)) - \Phi(\hat{x}(t), \hat{x}(t-t_d))] \\ &+ E[f_a(x,t) + d(t) + w(t)] + Ev(t). \end{aligned} \tag{7.25}$$

In the following subsection, stability of equation (7.25) is first analyzed. Some comments will then follow.

7.4.2 Stability Analysis

Theorem 7.2 states the stability conditions of the proposed ILO-based adaptive fault compensation strategy. To prove Theorem 7.2, the following lemma is needed.

Lemma 7.4.1 Consider ILO update law $v(t) = K_1 v(t-\tau) + K_2 C\tilde{x}(t-\tau)$. If Assumption 7.5 holds, then $\|v(t)\| \leq l_n \|\tilde{x}(t-\tau)\|$, where l_n is a positive constant to be derived.

Proof: For the initial $v(t_0)$, we could select it such that $\|v(t_0)\| \leq l_0 \|\tilde{x}(t_0)\|, t_0 \in [0, \tau]$, and for any $t > 0$, there exists $t = n\tau + t_0$, where n is non-negative, so, we have

$$\begin{aligned} \|v(\tau + t_0)\| &\leq \|K_1\|\|v(t_0)\| + \|K_2\|\|C\|\|\tilde{x}(t_0)\| \\ &\leq l_0\|K_1\|\|\tilde{x}(t_0)\| + \|K_2\|\|C\|\|\tilde{x}(t_0)\| \\ &= l_1\|\tilde{x}(t_0)\| \end{aligned} \tag{7.26}$$

where $l_1 = l_0\|K_1\| + \|K_2\|\|C\|$.

Next, we consider $v(2\tau + t_0)$

$$\begin{aligned} \|v(2\tau + t_0)\| &\leq \|K_1\|\|v(\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}(\tau + t_0)\| \\ &\leq l_1\|K_1\|\|\tilde{x}(t_0)\| + \|K_2\|\|C\|\|\tilde{x}(t_0 + \tau)\| \\ &\leq l_1 q\rho\|K_1\|\|\tilde{x}(\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}(\tau + t_0)\| \\ &= l_2\|\tilde{x}(t_0 + \tau)\| \end{aligned} \tag{7.27}$$

where $l_2 = l_1 q\rho\|K_1\| + \|K_2\|\|C\|$.

Assumption 7.5 is used in the derivation above, it will be also considered in equations (7.28) and (7.29).

Continuously, consider $v(3\tau + t_0)$, we have

$$\begin{aligned} \|v(3\tau + t_0)\| &\leq \|K_1\|\|v(2\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}(2\tau + t_0)\| \\ &\leq l_2\|K_1\|\|\tilde{x}(\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}(2\tau + t_0)\| \\ &\leq l_2 q\rho\|K_1\|\|\tilde{x}(2\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}(2\tau + t_0)\| \\ &= l_3\|\tilde{x}(2\tau + t_0)\| \end{aligned} \tag{7.28}$$

where $l_3 = l_2 q\rho\|K_1\| + \|K_2\|\|C\|$.

$$\vdots$$

Finally,

$$\begin{aligned} \|v(n\tau + t_0)\| &\leq \|K_1\|\|v((n-1)\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}((n-1)\tau + t_0)\| \\ &\leq l_{n-1}\|K_1\|\|\tilde{x}((n-2)\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}((n-1)\tau + t_0)\| \\ &\leq l_{n-1} q\rho\|K_1\|\|\tilde{x}((n-1)\tau + t_0)\| + \|K_2\|\|C\|\|\tilde{x}((n-1)\tau + t_0)\| \\ &= l_n\|\tilde{x}((n-1)\tau + t_0)\| \end{aligned} \tag{7.29}$$

where $l_n = l_{n-1} q\rho\|K_1\| + \|K_2\|\|C\|$. Therefore, $\|v(t)\| \leq l_n\|\tilde{x}(t - \tau)\|$.

This completes the proof. ∎

Remark 7.4.1 l_n can have an explicit expression by substituting l_{i-1} into l_i, repetitively, where, $i = 1, \cdots, n$, i.e.

$$\begin{aligned} l_n &= l_0 q^{n-1}\rho^{n-1}\|K_1\|^n + q^{n-1}\rho^{n-1}\|K_1\|^{n-1}\|K_2\|\|C\| \\ &\quad + q^{n-2}\rho^{n-2}\|K_1\|^{n-2}\|K_2\|\|C\| + \cdots + q\rho\|K_1\|\|K_2\|\|C\| + \|K_2\|\|C\|. \end{aligned} \tag{7.30}$$

To guarantee the convergence of equation (7.30), K_1 must be selected such that $q\rho\|K_1\| < 1$. It is worth noting that once the significant digits after the decimal point of l_n are designated, l_n will converge to a constant, as n increases. It is easy to calculate it using Matlab.

Theorem 7.2 *Consider time delay nonlinear system (7.23) satisfying Assumptions 7.1-7.5. If the proposed ILO input satisfies Lemmas 7.3.1 and 7.4.1 along with conditions (7.37) and (7.38), then fault compensation control can be achieved by an ILO plus an adaptive law proposed in equation (7.24).*

Proof: Consider the following Lyapunov function candidate:

$$\begin{aligned} V(t) &= \tilde{x}^T P\tilde{x} + \int_{t-\tau}^{t} \tilde{x}^T(\theta)R\tilde{x}(\theta)d\theta + \int_{t-t_d}^{t} \tilde{x}^T(\beta)\tilde{x}(\beta)d\beta + w^T(t)w(t) \\ &+ \int_{t-\tau}^{t} v^T(\alpha)v(\alpha)d\alpha \end{aligned} \tag{7.31}$$

where $P = P^T > 0$ and $R = R^T > 0$.

Substituting estimation error equation (7.25) into the derivative of Lyapunov function candidate V leads to

$$\begin{aligned} \dot{V} &= \dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}} - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) + \tilde{x}^T(t)\tilde{x}(t) - \tilde{x}^T(t-t_d)\tilde{x}(t-t_d) \\ &+\tilde{x}^T(t)R\tilde{x}(t) + 2w^T(t)\dot{w}(t) + v^T(t)v(t) - v^T(t-\tau)v(t-\tau) \\ &= \tilde{x}^T((A-LC)^T P + P(A-LC) + R + I)\tilde{x} + 2\tilde{x}^T PEv(t) \\ &+2\tilde{x}^T P(\Phi(x, x(t-t_d)) - \Phi(\hat{x}, \hat{x}(t-t_d))) + 2\tilde{x}^T PE(f_a + d(t) + w(t)) \\ &-\tilde{x}^T(t-\tau)R\tilde{x}(t-\tau) - \tilde{x}^T(t-t_d)\tilde{x}(t-t_d) \\ &+v^T(t)v(t) - v^T(t-\tau)v(t-\tau) + 2w^T(t)\dot{w}(t) \end{aligned} \tag{7.32}$$

where $I \in \mathbb{R}^{n\times n}$ is an identity matrix.

By applying inequality (7.9), the following inequality holds

$$2\tilde{x}^T PEv(t) \leq \tilde{x}^T PEE^T P\tilde{x} + v^T(t)v(t). \tag{7.33}$$

With the aid of Assumption 7.3, the following extension can be obtained

$$\begin{aligned}
&2\tilde{x}^T P(\Phi(x, x(t-t_d)) - \Phi(\hat{x}, \hat{x}(t-t_d))) \\
&\leq 2\|P\tilde{x}\|\|\Phi(x, x(t-t_d)) - \Phi(\hat{x}, \hat{x}(t-t_d))\| \\
&\leq 2\eta_1\|P\tilde{x}\|\|\tilde{x}\| + 2\eta_2\|\tilde{x}(t-t_d)\|\|P\tilde{x}\| \\
&\leq \eta_1^2\tilde{x}^T PP\tilde{x} + \tilde{x}^T\tilde{x} + \eta_2^2\tilde{x}^T PP\tilde{x} + \tilde{x}^T(t-t_d)\tilde{x}(t-t_d).
\end{aligned} \tag{7.34}$$

Substituting the adaptive law in equation (7.24) and combining equations (7.33) and (7.34) into equation (7.32), we have

$$\begin{aligned}
\dot{V} \leq\ & \tilde{x}^T((A-LC)^T P + P(A-LC) + R + \eta_1^2 PP + \eta_2^2 PP + 2I + PEE^T P)\tilde{x} \\
&+2\tilde{x}^T PE(f_a + d(t) + w(t)) + 2w^T(t)Fv(t) + 2v^T(t)v(t) \\
&-v^T(t-\tau)v(t-\tau) - \tilde{x}^T(t-\tau)R\tilde{x}(t-\tau).
\end{aligned} \tag{7.35}$$

Using Lemma 7.3.1, the above equation can be further extended

$$\begin{aligned}
\dot{V} \leq\ & -\tilde{x}^T((A-LC)^T P + P(A-LC) + R + \eta_1^2 PP + \eta_2^2 PP + 2I + PEE^T P)\tilde{x} \\
&+2\tilde{x}^T PE(f_a + d(t) + w(t)) + 2w^T(t)Fv(t) \\
&+v^T(t-\tau)((4+2\sigma)K_1^T K_1 - I)v(t-\tau) - \sigma v^T(t)v(t) \\
&+\tilde{x}^T(t-\tau)((4+2\sigma)(K_2C)^T(K_2C) - R)\tilde{x}(t-\tau).
\end{aligned} \tag{7.36}$$

For any $Q = Q^T > 0$, there exists a $P = P^T > 0$ satisfying the following equation

$$(A - LC)^T P + P(A - LC) + R + \eta_1^2 PP + \eta_2^2 PP + 2I + PEE^T P = -Q. \quad (7.37)$$

Consider Assumption 7.5 and Lemma 7.4.1 and let

$$0 < (4 + 2\sigma)K_1^T K_1 \leq I; \quad 0 < (4 + 2\sigma)(K_2 C)^T (K_2 C) \leq R, \quad (7.38)$$

equation (7.36) can be simplified as

$$\begin{aligned} \dot{V} \leq & -\lambda_{min}(Q)\|\tilde{x}\|^2 - \sigma v^T(t)v(t) + 2\lambda_{max}(P)\|E\|(b_f + b_d + b_u)\|\tilde{x}\| \\ & +2b_u q\rho l_n \|F\| \|\tilde{x}(t)\|. \end{aligned} \quad (7.39)$$

According to [69, 100], the above inequality has following form by some operation

$$\begin{aligned} \dot{V} & \leq -\lambda_{min}(Q)\|\tilde{x}\|^2 - \sigma v^T(t)v(t) + c\|\tilde{x}\| \\ & \leq -\lambda_{min}(Q)/2\|\tilde{x}\|^2 - \sigma v^T(t)v(t) + k_a c, \quad k_a > 0 \end{aligned} \quad (7.40)$$

where $c = 2\lambda_{max}(P)\|E\|(b_f + b_d + b_u) + 2b_u q\rho l_n \|F\|$, then ILO-based adaptive fault compensation control is achieved. ∎

Remark 7.4.2 It is worth noting that this ILO-based adaptive reconfiguration of the control input is a feedback dynamic control process, as ILO input $v(t)$ can reflect the control transition phase aroused by the additional adaptive input $w(t)$ and fault $f_a(x, t)$. So, we can say that $v(t)$ is an indicator of the variations of the considered systems. Then this indicator is employed as the lasting stimulation of the adaptive law such that $w(t)$ is updated on-line. Furthermore, this $w(t)$ is added back to the system input to further attenuate the effect of faults. This process forms a dynamic control loop. This dynamic adjusting process will end if the indicator $v(t)$ approaches a steady small value or even zero if there exist no disturbances in the systems, which means the faults have been compensated. In addition, this indicator keeps its “eyes” on system dynamics. Once a new fault occurs, it will inform $w(t)$ immediately, then some counteraction will be taken again.

Remark 7.4.3 This ILO-based adaptive fault compensation strategy can offset multiple faults that occur simultaneously or in order because the additional control input is reconfigured according only to the system's response.

Remark 7.4.4 In the course of the proof of Theorem 7.2, a method has been derived to select each parameter of the ILO, as shown in equations (7.37) and (7.38).

Remark 7.4.5 One of the main features of this ILO-based adaptive fault compensation strategy is that the additional input can be in running fault compensation from the beginning of the system operation because the additional input $w(t)$ is zero in the fault free case if there exist no disturbances in the systems. It will be the estimation of disturbances in the fault free case if there exist disturbances. This property makes systems more robust to disturbances because the effect of disturbances can be eliminated by the additional input. The fault compensation process starts only after a fault occurs.

Remark 7.4.6 Though there is no fault detection and estimation subsystem in our fault compensation system, fault detection, compensation, and estimation can be completed at the same time by the additional input $w(t)$. It is not hard to notice that once a fault occurs, ILO input $v(t)$ will immediately detect it, feeding itself to the adaptive law so that the additional control input $w(t)$ is produced. Therefore, either $v(t)$ or $w(t)$ can be regarded as a residual. If ILO input $v(t)$ is not varying (if no disturbances exist), then faults have been completely compensated by the $w(t)$. The current $w(t)$ is the estimation of the fault, which can be seen in the application example. If there exist no faults in the nonlinear systems, $w(t)$ is the estimation of the disturbance.

7.5 Application to Automotive Engine Fault Diagnosis and Compensation

In this section, we apply the above proposed ILO-based adaptive strategy to detect, estimate, and compensate system faults in an automotive engine described by a second-order nonlinear engine model that involves intake to torque production delay and unmeasurable time varying disturbances. This delay is due to the fact that the engine torque production is a discrete process. But it is modelled as a continuous time domain. Therefore, the delay must be introduced [114]. The automotive engine model discussed in Chapter 6 will be used here to demonstrate the effectiveness of the above proposed fault detection, estimation, and compensation approach.

Rewrite the nonlinear two-state engine model as follows:

$$\begin{aligned} J_e\dot{w} &= a_2 k \frac{p(t-t_d)}{p_0}(cos(-b+u_2))^{2.875} - T_f - T_d - T_p \\ \frac{\dot{p}}{p_0} &= k_1(a_1 p_r u_1 - kw\frac{p}{p_0}). \end{aligned} \tag{7.41}$$

Based on this time delay two-state engine model, the ILO-based fault detection, estimation, and compensation strategy is to be implemented.

For convenience, letting $x_1(t) = w, x_2(t) = p$, equation (7.41) can be written as

$$\begin{aligned} \dot{x}_1 &= 576.65x_2(t-t_d) - 76 - 0.112x_1 - 2.148\times 10^{-4}x_1^2 - 7.84\times 10^{-4}(1-x_2) \\ \dot{x}_2 &= 69.498p_r u_1 - 3.114\times 10^{-2}x_1x_2 + f_a(x,t) \\ y &= [x_1 \quad x_2]^T. \end{aligned} \tag{7.42}$$

Based on the equation above, the ILO and the adaptive law are constructed according to equation (7.24) as follows:

$$\begin{aligned}
\dot{\hat{x}}_1 &= 576.65\hat{x}_2(t-t_d) - 76 - 0.112\hat{x}_1 - 2.148\times10^{-4}\hat{x}_1^2 - 7.84\times10^{-4}(1-\hat{x}_2) \\
&\quad +10e_{y1}(t) + 7.84e_{y2}(t) \\
\dot{\hat{x}}_2 &= 69.498\hat{p}_r u_1 - 3.114\times10^{-2}\hat{x}_1\hat{x}_2 + 13e_{y2}(t) - v(t) \\
v(t) &= 0.35v(t-\tau) + 0.12e_{y1}(t-\tau) - 0.01e_{y2}(t-\tau) \\
\hat{y} &= [\hat{x}_1 \quad \hat{x}_2]^T. \\
\dot{w}(t) &= -10v(t).
\end{aligned} \tag{7.43}$$

In the following simulations, the ILO will estimate disturbances if no faults exist in the system. If a fault occurs, ILO-based adaptive method will achieve fault detection, estimation and compensation. We assume that the healthy system has a fault $f_a(x,t) = 0$, and that sampling time interval $\tau = 0.01$ and $t_d \simeq \tau$.

1. *Disturbance Estimation by the ILO*

   The disturbance $d(t)$ is designated as a random function and a sinusoid. In Figure 7.2, a slowly varying disturbance (sinusoid) is accurately estimated by the ILO. Figure 7.3 shows that the ILO can also supply an accurate estimation of a random disturbance. This implies that the ILO can be used as a very effective robust control tool.

2. *ILO-Based Adaptive fault Detection, Estimation, and Compensation*

   The main result of this work is verified in Figures 7.4 and 7.5 in which an abrupt fault occurs at time 10 seconds. ILO input $v(t)$ immediately detects this fault and the additional control input $w(t)$ becomes greater than zero to offset the fault. The steady value of $w(t)$ is the estimation with a negative sign of the fault. By this way, the effects of the fault on system dynamics are compensated. Under the circumstance of counteraction, automotive engine speed and the pressure in manifold recover to their original values after a transition phase as shown in Figure 7.5. These two figures also reveal that both $w(t)$ and $v(t)$ can be selected as residuals.

3. *Multiple faults case*

   If there exist multiple faults in the system, the ILO-based adaptive fault compensation strategy can still work. Figure 7.6 describes the responses of both $w(t)$ and $v(t)$ after two faults occur in order. The $w(t)$ can retrieve its original value after compensating the first fault. After the second fault appears, it immediately generates another estimation to counteract the second fault. It is $w(t)$ that makes engine speed and pressure robust to the two faults demonstrated in Figure 7.7. Figure 7.8 shows the counteraction of $w(t)$ to two faults occurring simultaneously.

7.6 Conclusions

Fault detection, estimation, and compensation issues have been achieved by the ILO plus an adaptive control law. The main property of the proposed approach is that the fault compensation is based totally on system response, without resorting to a fault detection and isolation subsystem. This avoids a degraded performance due to inaccurate fault estimation. Though fault estimation is also attained in the process of fault compensation, it is not employed to offset faults. The crucial point here is the ILO input $v(t)$ that is on the alert against any variations of the dynamic system, and that excites an adaptive law to generate an additional control input for the needs of fault compensation. From simulations, it is easy to notice that both $w(t)$ and $v(t)$ can be treated as residuals. In addition, a method has been derived to select observer parameters such that ILO can be easily constructed in practice.

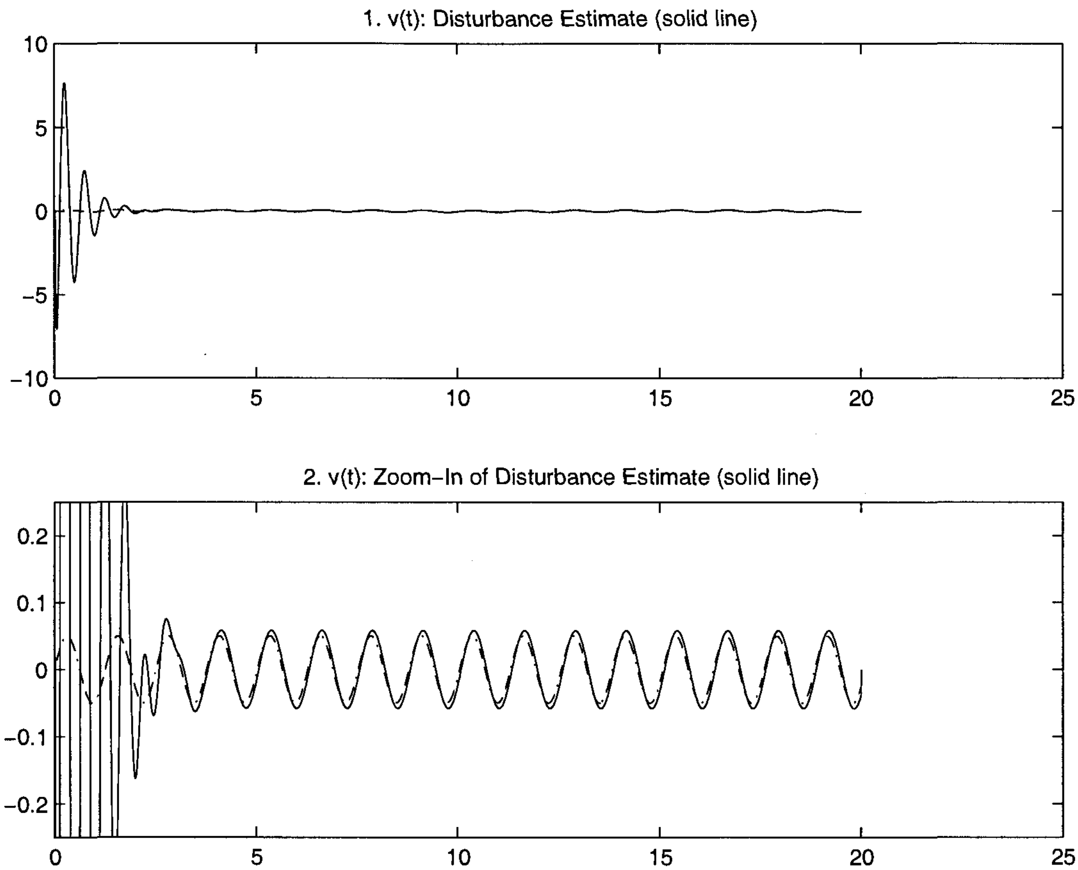

Figure 7.2: Disturbance estimation by the ILO– a sinusoid disturbance.

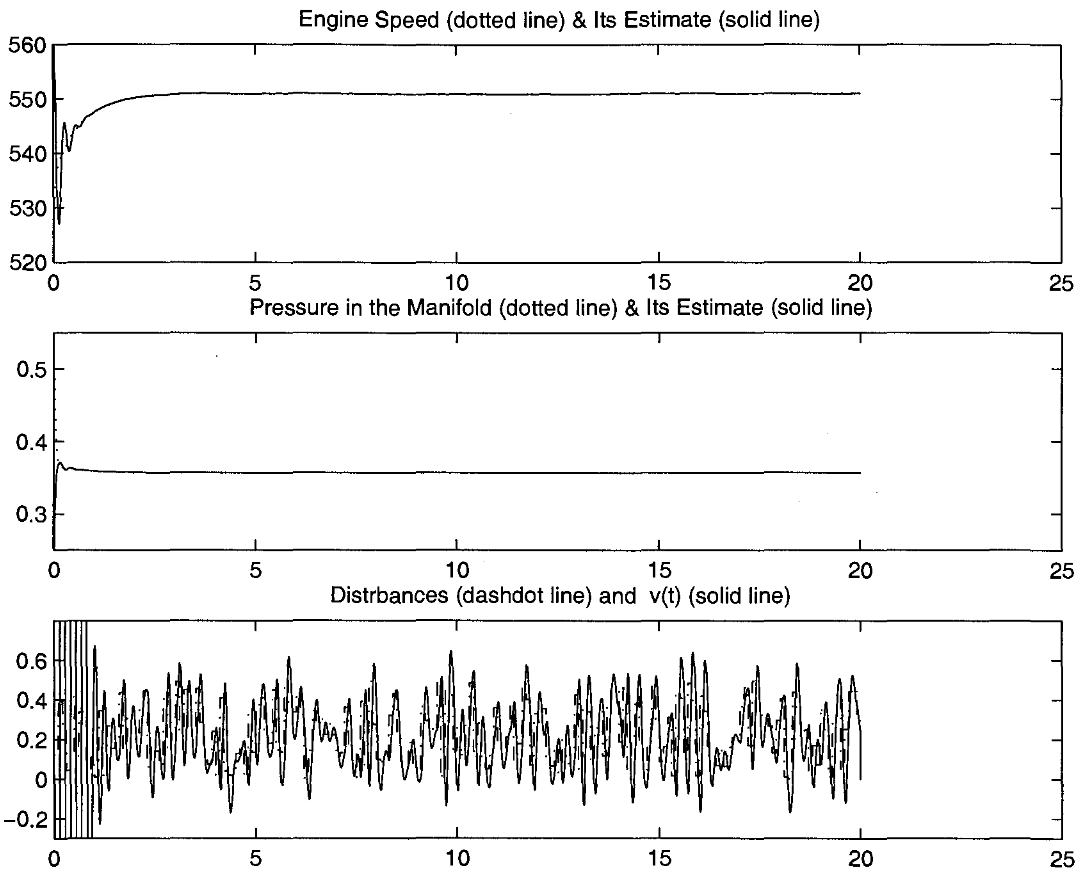

Figure 7.3: Disturbance estimation by the ILO– a random disturbance.

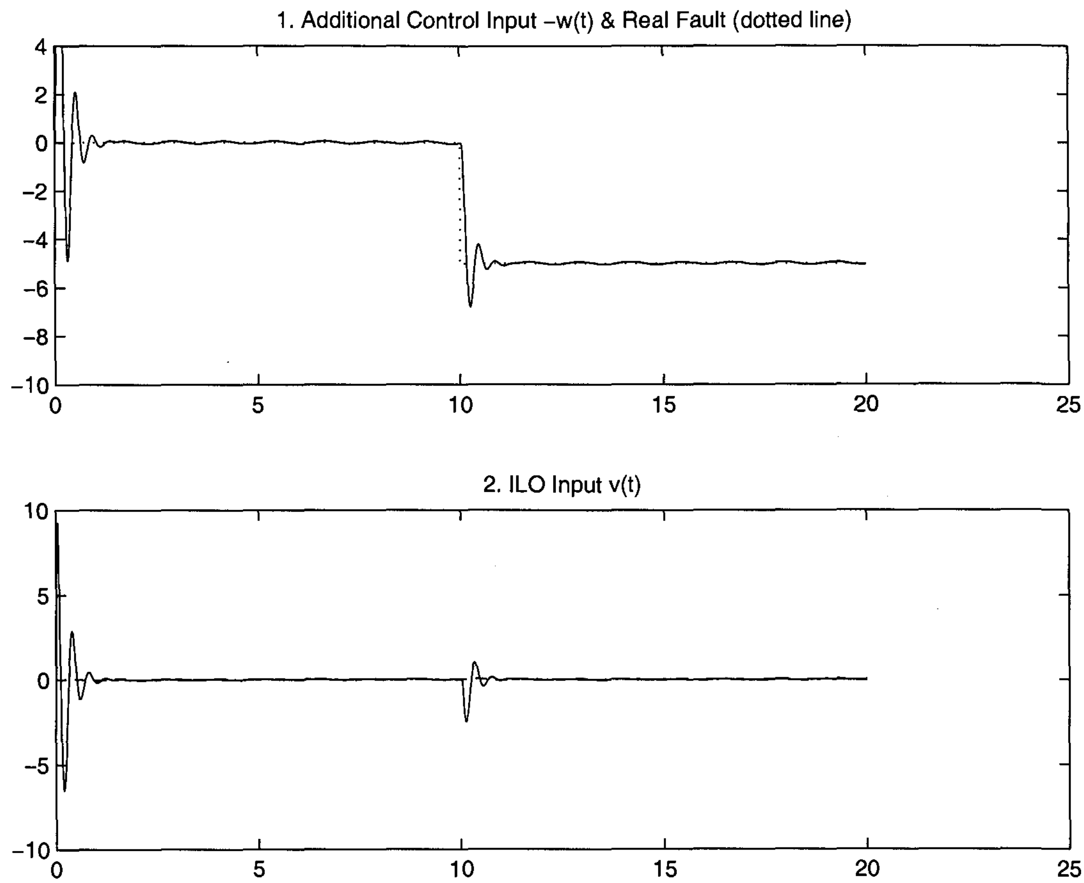

Figure 7.4: The additional control input and ILO input.

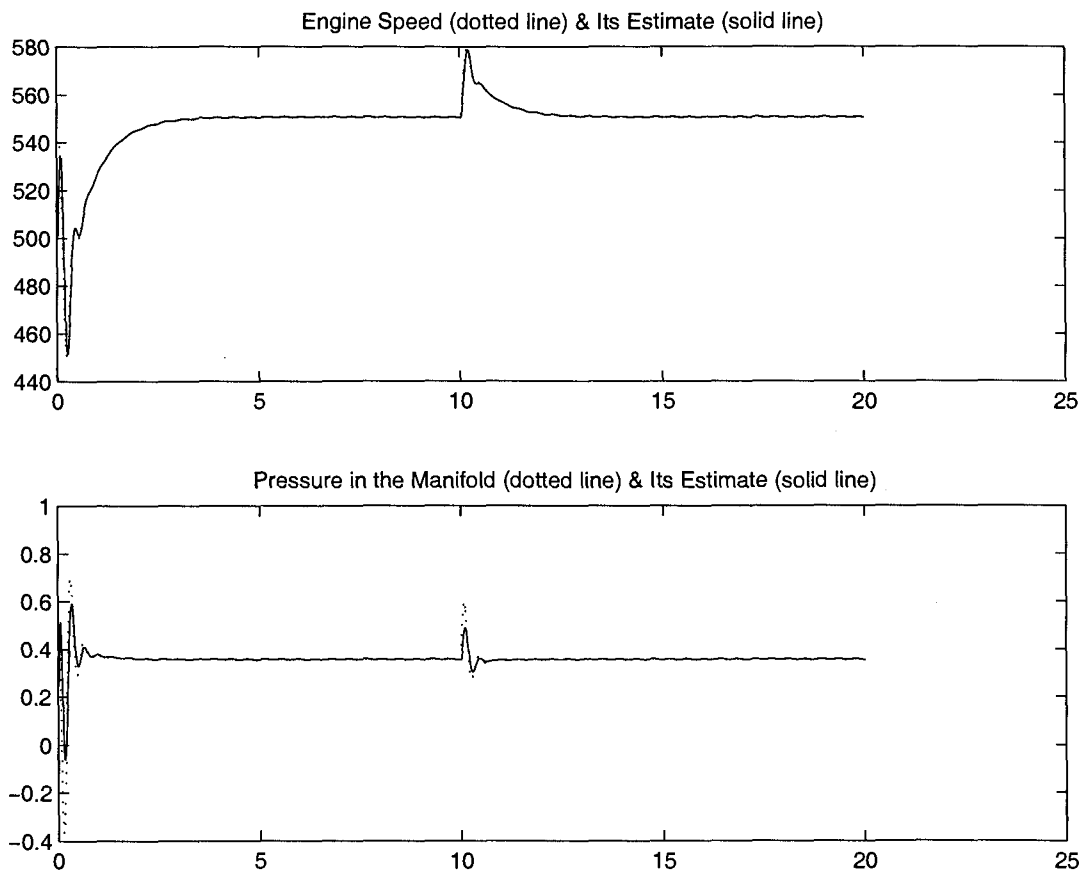

Figure 7.5: Engine speed and pressure in the manifold and their estimations .

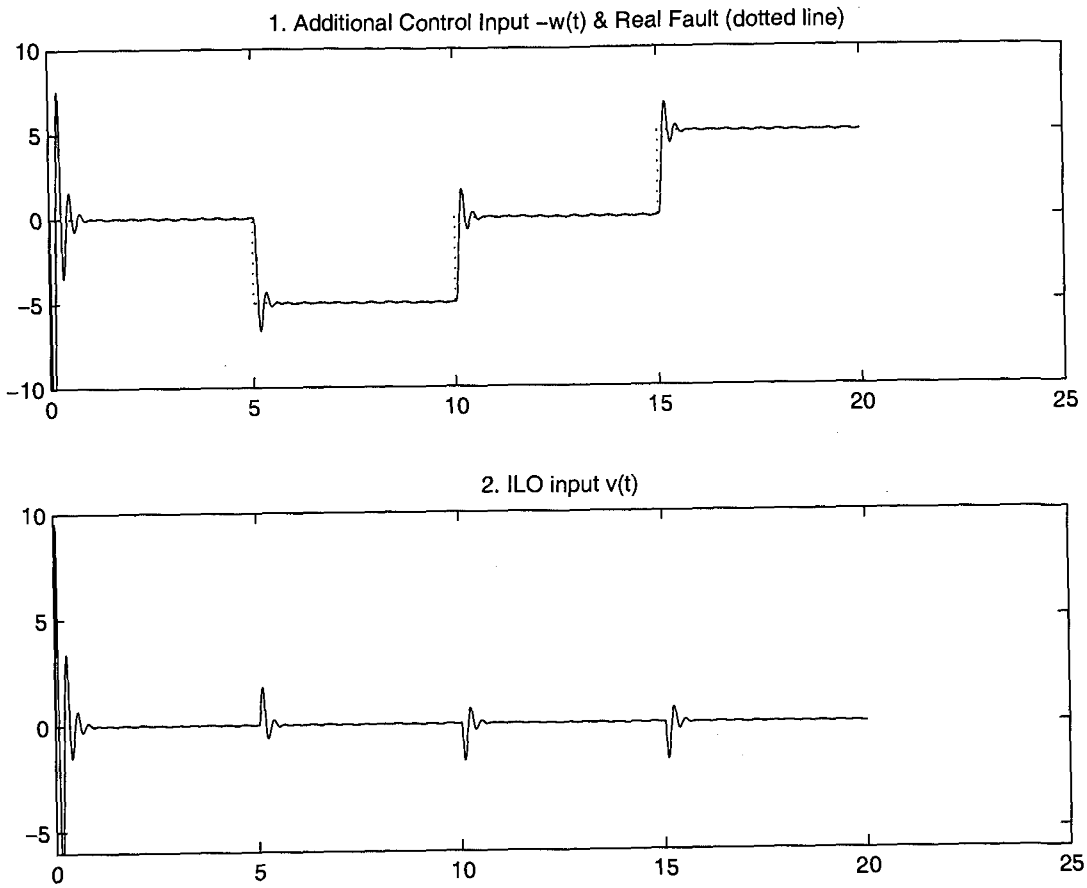

Figure 7.6: The $w(t)$ and $v(t)$ under multiple faults in order.

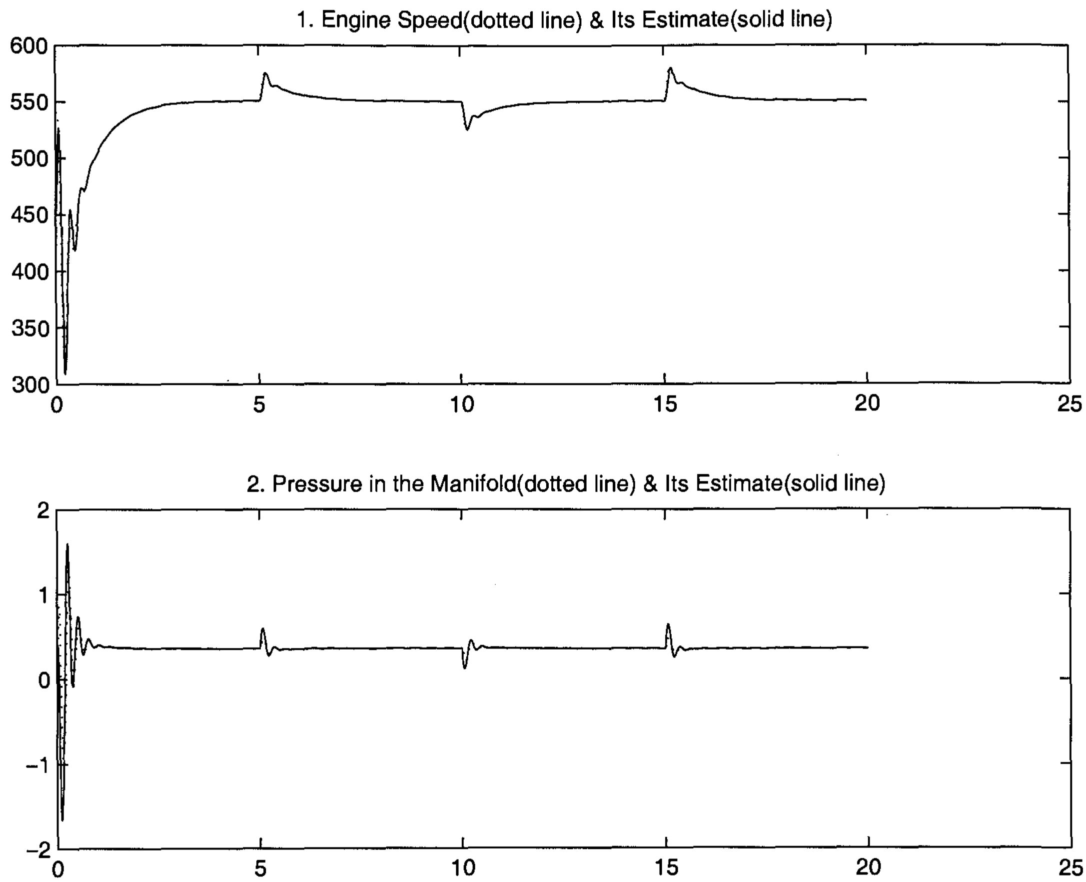

Figure 7.7: Engine speed and pressure in the manifold and their estimations under multiple faults in order.

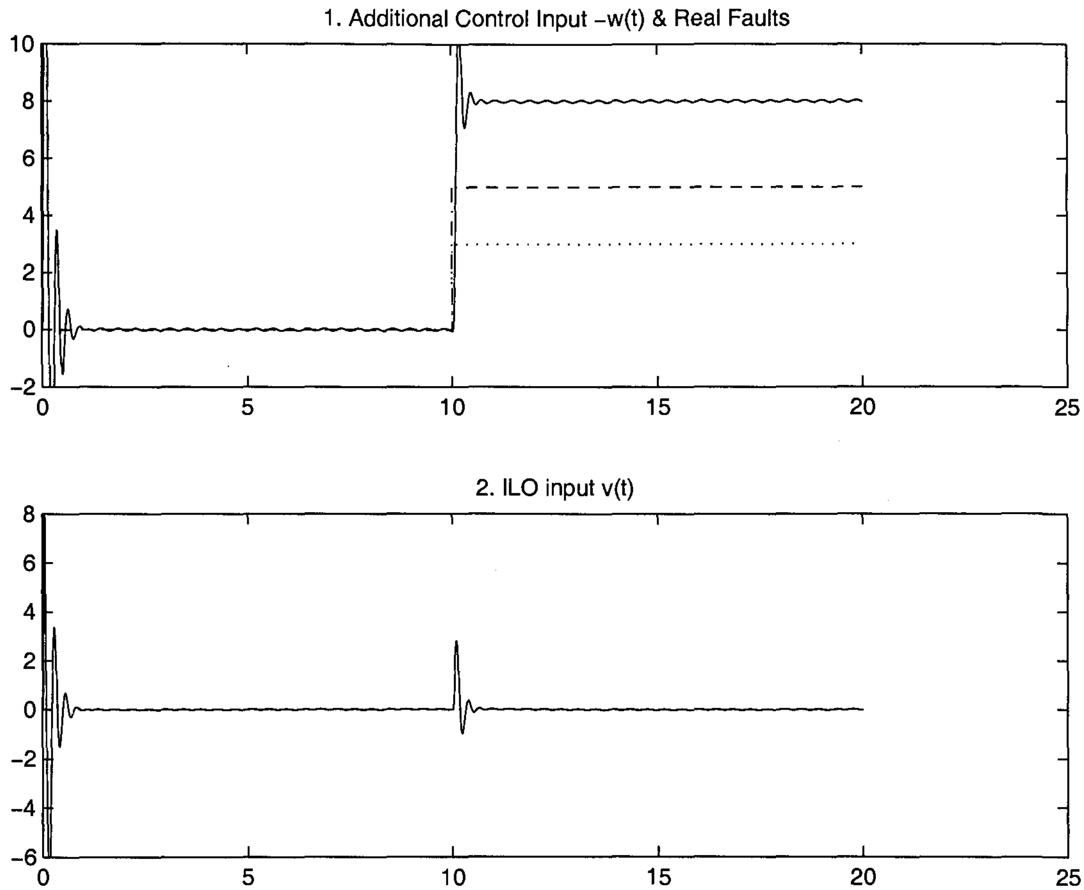

Figure 7.8: The $w(t)$ and $v(t)$ under simultaneous multiple faults.

Chapter 8

An SMO in Nonlinear DAS

This chapter is concerned with the design of an SMO in a class of uncertain nonlinear differential-algebraic systems (DAS) described by so-called semi-explicit forms with the differential variables being coupled with algebraic variables. In order to estimate the algebraic variables directly, an algorithm is developed to reconstruct the algebraic variables whose distribution matrix is singular, using serial elementary matrices followed by differentiation. An SMO is then designed based on the reconstructed algebraic variables. The estimated states, including both the differential and algebraic variables, can converge to the actual ones. The stability of the proposed observer is proved and an illustrative example is given in simulation to describe the design of the SMO.

8.1 Introduction

An important research area is concerned with a large class of engineering systems that are described by both ordinary differential equations and algebraic equations, such as robotic systems with kinematic constraints [75, 88]. Power systems and electric circuits also fall into this category [56, 116, 121]. In the chemical process, for example, the differential equations stem from dynamic conservation equations, while the algebraic equations commonly arise from thermodynamic equilibrium relations, empirical correlations, pseudo-steady-state assumptions, and so on.

The majority of the research on nonlinear DAS has focused on solvability and numerical solutions [7, 10]. The problem of feedback controller synthesis has been addressed only for restricted classes of DAS that mainly arise from mechanical systems [74, 87]. A framework for the study of Lyapunov stability of equilibria in DAS is presented in [56]. [77] addresses the output feedback control problem for nonlinear multi-variable high-index DAS in semi-explicit form. A local disturbance decoupling issue is considered by [82] in uncertain DAS where an algorithm is first developed such that the system can be expressed in a simple form. Based on this simple form, a feedback control law is then constructed to ensure that the closed loop system has a unique solution without impulses and its output is not affected by disturbances.

Only a few papers have been published in uncertain nonlinear DAS regarding observer design although this turns out to be very important in many applications, such as control issues or fault diagnostics [20, 129, 134]. In [116], an SMO is constructed in a class of linear DAS where the DAS is first converted into an equivalent control problem via the singularly perturbed sliding manifold approach. A robust sliding observer is then designed ensuring asymptotic stability in the presence of disturbances. Reference [121] is concerned with designing and analyzing a numerically feasible learning scheme for robust and stable fault diagnosis of DAS. The proposed fault diagnosis architecture monitors the physical system for any off-nominal behavior by estimators. As a matter of fact, observers are ideal tools for fault diagnosis [20, 129, 134] under the existence of unmeasurable differential and algebraic variables.

Therefore, it is necessary to consider observer design issues in uncertain nonlinear DAS no matter whether it is for the control application or fault diagnosis. In this work, an SMO will be designed based on a class of uncertain nonlinear DAS with singular distribution matrix of the algebraic variables.

8.2 Problem Statement and System Formulation

Consider a class of uncertain nonlinear DAS described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b(x)z(t) + g(x)u(t) + Wd(x, z, u, t) \\ \mathbf{0} &= k(x) + l(x)z(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8.1}$$

with compatible initial conditions, where $x(t) \in \mathbb{R}^n$ is unmeasurable system state vector; $z(t) \in \mathbb{R}^p$ denotes algebraic variable vector; $y(t) \in \mathbb{R}^q$ is measurable outputs; $u(t) \in \mathbb{R}^m$ is system control inputs; $A \in \mathbb{R}^{n\times n}$, $W \in \mathbb{R}^{n\times\theta}$, and $C \in \mathbb{R}^{q\times n}$ are constant matrices; $d(x, z, u, t) : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^+ \to \mathbb{R}^\theta$ represents disturbance or uncertainty vector; $k(x) : \mathbb{R}^n \to \mathbb{R}^p$; $l(x) : \mathbb{R}^n \to \mathbb{R}^{p\times p}$; $b(x) : \mathbb{R}^n \to \mathbb{R}^{n\times p}$; $g(x) : \mathbb{R}^n \to \mathbb{R}^{n\times m}$.

Remark 8.2.1 Incompatible initial conditions will typically lead to jumps at constant x to the constraint [56].

Remark 8.2.2 The above description of DAS is in the so-called semi-explicit form with the algebraic variables $z(t)$ appearing linearly [51]. The semi-explicit differential-algebraic system model is motivated by some practical applications, such as chemical processes. Moreover, the linear form of the algebraic variables $z(t)$ is also typical in chemical processes.

For a vector-valued smooth function $f(x) = [f_1(x), f_2(x), \cdots, f_n(x)]^T$, a matrix-valued smooth function $g(x) = [g_1(x), g_2(x), \cdots, g_m(x)]$ with $g_i(x) = [g_i^1(x), g_i^2(x), \cdots, g_i^n(x)]^T$, and a smooth function $h(x)$, the following notations will be used

$$dh(x) = \left[\frac{\partial h(x)}{\partial x_1}, \frac{\partial h(x)}{\partial x_2}, \cdots, \frac{\partial h(x)}{\partial x_n}\right],$$

$$L_f h(x) = dh(x) f(x) = \left[\frac{\partial h(x)}{\partial x_1}, \frac{\partial h(x)}{\partial x_2}, \cdots, \frac{\partial h(x)}{\partial x_n}\right] \cdot \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix},$$

$$L_g h(x) = [dh(x)g_1(x), dh(x)g_2(x), \cdots, dh(x)g_m(x)].$$

The problem in question is to construct an SMO for the uncertain DAS of interest with a singular distribution matrix $l(x)$.

In system equation (8.1), the algebraic variables are coupled with differential variables. If they can be decoupled and expressed by system states $x(t)$ and system input $u(t)$, then the algebraic variables can be directly estimated by differential variables. Nevertheless, matrix $l(x)$ in the algebraic constraint equation is singular. This makes algebraic variables $z(t)$ impossible to be directly solved, and causes additional underlying constraints among differential variables to be present. If singular matrix $l(x)$ can be transformed to a nonsingular matrix by some operations, consistent with the algebraic constraints, then the algebraic variables can be expressed in terms of differential variables. Motivated by these considerations, in what follows an algorithm will be developed to reconstruct the algebraic variables. An SMO for the uncertain DAS of interest with reconstructed algebraic variables will be constructed for the purpose of estimating both the differential and algebraic variables.

8.3 Main Result

In this section, we shall design an SMO for the uncertain nonlinear DAS. To this end, we first propose an algorithm to reconstruct $z(t)$ in terms of state $x(t)$ and system input $u(t)$, consistent with the algebraic constraints. After that, an SMO will be designed based on the reconstructed algebraic variables.

8.3.1 An Algorithm for the Reconstruction of Algebraic Variables $z(t)$

Based on [57], [77] proposes an algorithm to reconstruct algebraic variables $z(t)$ because of the singular distribution matrix $l(x)$ for the purpose of state-space realization. On that basis, an improved algorithm, which is motivated by [57] for the reconstruction of algebraic variables $z(t)$, is about to be presented. This algorithm consists of elementary row operations and differentiation with respect to time t.

Prior to stating the algorithm, the following assumptions are introduced:

Assumption 8.1 $[L_{b_1}\hat{k}_i^s(x), L_{b_2}\hat{k}_i^s(x), \cdots, L_{b_p}\hat{k}_i^s(x)]$, $i = 1, \cdots, p - p_s$, $s = 1, \cdots, r$, *can not be a zero vector at every iteration, where r is the number of iterations for the reconstruction of algebraic variables, i.e. at rth iteration, $l(x)$ can be developed to be a nonsingular matrix; b_j denotes the jth column of matrix $b(x)$; $\hat{k}_i^s(x)$ is the ith component of vector $\hat{k}^s(x)$ as shown in equation (8.5) or (8.18).*

Assumption 8.2 $L_{g_j}\hat{k}_i^s = 0$ *for* $s < r$, $i = 1, \cdots, p - p_s$; $j = 1, \cdots, m$; *where g_j is the jth column of matrix $g(x)$.*

Assumption 8.3 $L_{W_j}\hat{k}_i^s = 0$ *for* $s \leq r$, $i = 1, \cdots, p - p_s$; $j = 1, \cdots, \theta$; *where W_j is the jth column of matrix W.*

Remark 8.3.1 Assumption 8.1 can guarantee that singular matrix $l(x)$ can be surely developed to a nonsingular matrix because, if a vector in assumption 8.1 is always a zero vector at every iteration, $l(x)$ would forever be a singular matrix.

Remark 8.3.2 Assumption 8.2 assures that system input u will not appear until the last iteration. This prevents us from differentiating system input u. Meanwhile, Assumption 8.3 guarantees that the disturbance d has no influence on algebraic constraints when they are differentiated.

Remark 8.3.3 Assumption 8.1 implies that no component of vector $\hat{k}^s$ can always be a constant for $s \leq r$, as the derivative of a constant is zero. This makes the vector in Assumption 8.1 be a zero vector.

We are in the position to present the procedure for the reconstruction of algebraic variables $z(t)$.

Iteration 1: Consider the algebraic constraint in equation (8.1) with $p_1 = \max_{x \in M}$ $\{\text{rank } l(x)\} < p$, $M = \mathbb{R}^n$.

1. Pre-multiply $l(x)$ by E_1^1, an elementary matrix that reorders the rows of $l(x)$ such that the first p_1 rows of $E_1^1 l(x)$ are linearly independent for some $x \in M$, i.e.

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \check{k}^1(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \check{l}^1(x) \end{bmatrix} z \tag{8.2}$$

where $\bar{l}^1(x) \in \mathbb{R}^{p_1 \times p}$ and $\bar{k}^1(x) \in \mathbb{R}^{p_1}$.

Now, reduce the last $p - p_1$ rows of $E_1^1 l(x)$ to zeros by pre-multiplying a $p \times p$ elementary matrix

$$E_1^2(x) = \left[\begin{array}{c:c} I_{p_1 \times p_1} & \mathbf{0} \\ \hdashline F_1(x) & I_{(p-p_1)\times(p-p_1)} \end{array} \right] \tag{8.3}$$

with the property of

$$E_1^2(x)E_1^1 l(x) = \begin{bmatrix} \bar{l}^1(x) \\ \mathbf{0} \end{bmatrix} \quad \text{for } \ x \in M_1 \tag{8.4}$$

where the entries of $F_1(x)$ are real analytic function on $M_1 = \{x \in M |$ rank $\bar{l}^1(x) = p_1\}$.

By the two operations above, we have

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \hat{k}^1(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \mathbf{0} \end{bmatrix} z \tag{8.5}$$

where $\bar{l}^1(x) \in \mathbb{R}^{p_1 \times p}$ has full row rank p_1 for all $x \in M_1$. $\hat{k}^1(x) = [\hat{k}_1^1(x), \cdots, \hat{k}_{p-p_1}^1(x)]^T = F_1(x)\bar{k}^1(x) + \check{k}^1(x)$ and $\mathbf{0} = F_1(x)\bar{l}^1(x) + \check{l}^1(x)$ are of dimensions $(p - p_1)$ and $(p - p_1) \times p$, respectively.

2. Differentiating $\hat{k}^1(x)$ once with respect to t leads to following algebraic equation

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \tilde{k}^2(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \tilde{l}^2(x) \end{bmatrix} z + \begin{bmatrix} \mathbf{0} \\ \tilde{c}_1^2(x) \end{bmatrix} u + \begin{bmatrix} \mathbf{0} \\ \tilde{c}_2^2(x) \end{bmatrix} d \tag{8.6}$$

where

$$\tilde{k}^2(x) = [\, L_{Ax}\hat{k}_1^1(x) \quad \cdots \quad L_{Ax}\hat{k}_{p-p_1}^1(x)\,]^T, \tag{8.7}$$

$$\tilde{l}^2(x) = \begin{bmatrix} L_{b_1}\hat{k}_1^1(x) & \cdots & L_{b_p}\hat{k}_1^1(x) \\ \vdots & \vdots & \vdots \\ L_{b_1}\hat{k}_{p-p_1}^1(x) & \cdots & L_{b_p}\hat{k}_{p-p_1}^1(x) \end{bmatrix}, \tag{8.8}$$

$$\tilde{c}_1^2(x) = \begin{bmatrix} L_{g_1}\hat{k}_1^1(x) & \cdots & L_{g_m}\hat{k}_1^1(x) \\ \vdots & \vdots & \vdots \\ L_{g_1}\hat{k}_{p-p_1}^1(x) & \cdots & L_{g_m}\hat{k}_{p-p_1}^1(x) \end{bmatrix}, \tag{8.9}$$

and

$$\tilde{c}_2^2(x) = \begin{bmatrix} L_{W_1}\hat{k}_1^1(x) & \cdots & L_{W_q}\hat{k}_1^1(x) \\ \vdots & \vdots & \vdots \\ L_{W_1}\hat{k}_{p-p_1}^1(x) & \cdots & L_{W_q}\hat{k}_{p-p_1}^1(x) \end{bmatrix} \tag{8.10}$$

where b_i, g_i, and W_i denote the ith columns of matrices $b(x)$, $g(x)$, and W, respectively.

Assumptions 8.2 and 8.3 suggest that $\tilde{c}_1^2(x) = \mathbf{0}$ and $\tilde{c}_2^2(x) = \mathbf{0}$. Therefore, equation (8.6) can be simplified as

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \tilde{k}^2(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \tilde{l}^2(x) \end{bmatrix} z. \tag{8.11}$$

3. Evaluate the rank of

$$D_1(x) := \begin{bmatrix} \bar{l}^1(x) \\ \tilde{l}^2(x) \end{bmatrix}.$$

Let $p_2 = \max_{x \in M_1}\{\text{rank } D_1(x)\}$. If $p_2 = p$, then stop, otherwise, proceed to the next iteration, beginning with equation (8.11).

Iteration s**:** After $s-1$ iterations, following algebraic equation can be obtained

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \bar{k}^2(x) \\ \vdots \\ \tilde{k}^s(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \tilde{l}^s(x) \end{bmatrix} z \tag{8.12}$$

with

$$p_s = \max_{x \in M_{k-1}} \{\text{rank } D_{s-1}(x)\} \tag{8.13}$$

where

$$D_{s-1}(x) = \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \tilde{l}^s(x) \end{bmatrix}. \tag{8.14}$$

1. Pre-multiply $D_{s-1}(x)$ by E_s^1, an elementary matrix that reorders the rows of $D_{s-1}(x)$ such that the first p_s rows of $E_s^1 D_{s-1}(x)$ are linearly independent for some $x \in M_{k-1}$, i.e.

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \bar{k}^2(x) \\ \vdots \\ \bar{k}^s(x) \\ \check{k}^s(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^s(x) \\ \check{l}^s(x) \end{bmatrix} z \tag{8.15}$$

where $\check{k}^s(x)$ and $\check{l}^s(x)$ are of dimensions $p-p_s$ and $(p-p_s)\times p$, respectively. Reduce the last $p-p_s$ rows of $E_s^1 D_{s-1}(x)$ to zeros by pre-multiplying a $p \times p$ elementary matrix

$$E_s^2(x) = \left[\begin{array}{c:c} I_{p_s \times p_s} & \mathbf{0} \\ \hdashline F_s(x) & I_{(p-p_s)\times(p-p_s)} \end{array}\right] \tag{8.16}$$

with the property of

$$E_s^2(x)E_s^1 D_{s-1}(x) = \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^s(x) \\ \mathbf{0} \end{bmatrix} \quad \text{for} \quad x \in M_k \tag{8.17}$$

where the entries of $F_k(x)$ are real analytic functions on $M_k = \{x \in M_{k-1} |$ rank $[\bar{l}^1(x), \bar{l}^2(x), \cdots, \bar{l}^s(x)]^T = p_s\}$.

By the two operations above, we have

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \bar{k}^2(x) \\ \vdots \\ \bar{k}^s(x) \\ \hat{k}^s(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^s(x) \\ \mathbf{0} \end{bmatrix} z \tag{8.18}$$

where $l^s(x) = [\bar{l}^1(x), \bar{l}^2(x), \cdots, \bar{l}^s(x)]^T \in \mathbb{R}^{p_s \times p}$ has full row rank p_s for all $x \in M_k$, $\hat{k}^s(x) = [\hat{k}_1^s(x), \cdots, \hat{k}_{p-p_s}^s(x)]^T = F_s(x)k^s(x) + \check{k}^s(x)$, $\mathbf{0} = F_s(x)l^s(x) + \check{l}^s(x) \in \mathbb{R}^{(p-p_s)\times p}$, and $k^s(x) = [\bar{k}^1(x), \bar{k}^2(x), \cdots, \bar{k}^s(x)]^T \in \mathbb{R}^{p_s}$.

2. Differentiating $\hat{k}^s(x)$ once with respect to t and considering Assumptions 8.2 and 8.3, the following algebraic equation can be obtained

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \bar{k}^2(x) \\ \vdots \\ \bar{k}^s(x) \\ \tilde{k}^{s+1}(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^s(x) \\ \tilde{l}^{s+1} \end{bmatrix} z \tag{8.19}$$

where

$$\tilde{k}^{s+1}(x) = [\, L_{Ax}\hat{k}_1^s(x) \quad \cdots \quad L_{Ax}\hat{k}_{p-p_s}^s(x)\,]^T \tag{8.20}$$

and

$$\tilde{l}^s(x) = \begin{bmatrix} L_{b_1}\hat{k}_1^s(x) & \cdots & L_{b_p}\hat{k}_1^s(x) \\ \vdots & \vdots & \vdots \\ L_{b_1}\hat{k}_{p-p_s}^s(x) & \cdots & L_{b_p}\hat{k}_{p-p_s}^s(x) \end{bmatrix}. \tag{8.21}$$

3. Evaluate the rank of

$$D_s(x) := \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^s(x) \\ \tilde{l}^{s+1} \end{bmatrix}. \tag{8.22}$$

Let $p_{s+1} = \max_{x \in M_k}\{\text{ rank } D_s(x)\}$. If $p_{s+1} = p$, then stop, otherwise, proceed to the next iteration, beginning with equation (8.19).

By construction, the procedure can converge after a finite number of iterations r, then the final algebraic equation can be obtained as follows

$$\mathbf{0} = \begin{bmatrix} \bar{k}^1(x) \\ \bar{k}^2(x) \\ \vdots \\ \bar{k}^r(x) \\ \tilde{k}^{r+1}(x) \end{bmatrix} + \begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^r(x) \\ \tilde{l}^{r+1} \end{bmatrix} z + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \tilde{c}_1^{r+1} \end{bmatrix} u \tag{8.23}$$

with matrix $[\bar{l}^1(x), \bar{l}^2(x), \cdots, \bar{l}^r(x), \tilde{l}^{r+1}]^T$ having the full rank p, and

$$\tilde{c}_1^{r+1} = \begin{bmatrix} L_{g_1}\hat{k}_1^r(x) & \cdots & L_{g_m}\hat{k}_1^r(x) \\ \vdots & \vdots & \vdots \\ L_{g_1}\hat{k}_{p-p_r}^r(x) & \cdots & L_{g_m}\hat{k}_{p-p_r}^r(x) \end{bmatrix}. \tag{8.24}$$

Therefore, algebraic variables z can be reconstructed based on equation (8.23) as follows:

$$z := f(x,u) = -\begin{bmatrix} \bar{l}^1(x) \\ \bar{l}^2(x) \\ \vdots \\ \bar{l}^r(x) \\ \tilde{l}^{r+1} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \bar{k}^1(x) \\ \bar{k}^2(x) \\ \vdots \\ \bar{k}^r(x) \\ \tilde{k}^{r+1}(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \tilde{c}_1^{r+1} \end{bmatrix} u \right\}. \tag{8.25}$$

The above expression of z can be directly used for observer design.

In the course of the reconstruction of the algebraic variables, a set of new constraints among the differential variables is defined as

$$\begin{aligned} \hat{k}_1^1(x) &= 0 \\ &\vdots \\ \hat{k}_{p-p_1}^1(x) &= 0 \\ &\vdots \\ \hat{k}_1^s(x) &= 0 \\ &\vdots \\ \hat{k}_{p-p_s}^s(x) &= 0 \end{aligned} \tag{8.26}$$

which leads to a set $N = \{x \in \mathbb{R}^n \mid \hat{k}_j^i(x) = 0, i = 1, \cdots, s; j = 1, \cdots, p - p_i\}$. So, the differential variables $x \in N \cap M_{k+1}$ after the reconstruction of the algebraic variables.

Remark 8.3.4 It is Assumption 8.3 that makes equation (8.23) not include the disturbance term d.

Remark 8.3.5 Substituting equation (8.25) into system equation leads to an ordinary differential equation of dimension n which can be dealt with by some existing approaches for some needs, such as stabilization, linearization, etc.

Remark 8.3.6 $F_s(x)$ can be obtained by $F_s(x) = -\check{l}^s(x)l^s(x)^+$, where $l^s(x)^+ = l^s(x)^T(l^s(x)l^s(x)^T)^{-1}$ is the generalized inverse of $l^s(x)$.

Remark 8.3.7 In the algorithm proposed above, it is not necessary for the elementary matrix to render the input distribution matrix to be zeros, making the proposed algorithm easier to realize. In the algorithm of [77], it is a hard task to find a matrix $E(x)$ that not only relocates rows of matrix $l(x)$, and sets the last $p-p_s$ rows to zero, but also brings the last $p-p_s$ rows of the system input distribution matrix to be zero. In addition, the rank of matrix $l(x)$ at each iteration in [77] cannot be evaluated because there is no given domain of x.

8.3.2 An SMO for the uncertain DAS with Singular $l(x)$

Based on the expression of z in equation (8.25), an SMO is proposed as follows

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + b(\hat{x})\hat{z} + g(\hat{x})u + L(y(t) - \hat{y}(t)) + Wv(t) \\ \hat{z}(t) &= f(\hat{x}, u) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{8.27}$$

where

$$v(t) = \begin{cases} \alpha \dfrac{F\tilde{y}}{\|F\tilde{y}\|}, & \text{if } \tilde{y} \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{8.28}$$

where $\hat{x}(t) \in N \cap M_{k+1}$ is the estimated system states; $\hat{z}(t) \in \mathbb{R}^p$ is the estimated algebraic variables; $\hat{y}(t) \in \mathbb{R}^q$ is the estimated system outputs; $\tilde{y}(t) = y(t) - \hat{y}(t)$;

$F \in \mathbb{R}^{\theta \times q}$ is a constant matrix; $L \in \mathbb{R}^{n \times q}$ is a gain matrix to be determined; α is a positive constant.

Subtracting observer equation (8.27) from system equation (8.1) leads to the following estimation error equation:

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A - LC)\tilde{x}(t) + [b(x)z - b(\hat{x})\hat{z}] + [g(x) - g(\hat{x})]u - Wv(t) + Wd(x, z, u, t) \\ \tilde{z}(t) &= f(x, u) - f(\hat{x}, u) \\ \tilde{y}(t) &= C\tilde{x}(t) \end{aligned} \tag{8.29}$$

where $\tilde{x}(t) = x(t) - \hat{x}(t)$ is system state estimation errors; $\tilde{z}$ is algebraic variable estimation errors; matrix $(A-LC)$ can be a stable matrix by selecting an appropriate gain matrix L.

8.3.3 Stability Analysis

To prove the stability of the estimation error equation (8.29), the following assumptions are required.

Assumption 8.4 *Both $b(x)$ and $f(x, u)$ are bounded with b_b and b_f and satisfy Lipschitz condition with Lipschitz constants b_1 and b_2 i.e.*

$$\|b(x) - b(\hat{x})\| \leq b_1 \|x(t) - \hat{x}(t)\|, \tag{8.30}$$

$$\|f(x, u) - f(\hat{x}, u)\| \leq b_2 \|x(t) - \hat{x}(t)\|. \tag{8.31}$$

Assumption 8.5 *Function $g(x)$ satisfies Lipschitz condition with Lipschitz constant b_3 i.e.*

$$\|g(x) - g(\hat{x})\| \leq b_3 \|x(t) - \hat{x}(t)\|. \tag{8.32}$$

Assumption 8.6 *System control inputs and disturbances are bounded by*

$$\|u(t)\| \leq b_u \quad \text{and} \quad \|d(x, z, u, t)\| \leq b_d, \quad \text{respectively.}$$

The following theorem addresses stability conditions of estimation error equation (8.29).

Theorem 8.1 *Consider DAS (8.1) satisfying assumptions 8.4-8.6, and the algebraic variables $z(t)$ is reconstructed in equation (8.25). If both equation (8.35) and inequality (8.37) hold, then estimate error dynamics (8.29) is stable.*

Proof:

A Lyapunov function candidate $V = \tilde{x}^T P \tilde{x}$ is chosen for the proof of the stability of estimation error equation, where $P = P^T > 0$.

Substituting estimation error equation (8.29) into the derivative of Lyapunov function candidate V leads to:

$$\begin{aligned} \dot{V} &= \tilde{x}^T((A - LC)^T P + P(A - LC))\tilde{x} + 2\tilde{x}^T P[b(x)z - b(\hat{x})\hat{z}] + \\ &\quad 2\tilde{x}^T P[g(x) - g(\hat{x})]u - 2\tilde{x}^T PWv(t) + 2\tilde{x}^T PWd \\ &= \tilde{x}^T((A - LC)^T P + P(A - LC))\tilde{x} + 2\tilde{x}^T P[b(x) - b(\hat{x})]f(x, u) \\ &\quad +2\tilde{x}^T Pb(\hat{x})[f(x, u) - f(\hat{x}, u)] + 2\tilde{x}^T P[g(x) - g(\hat{x})]u \\ &\quad -2\tilde{x}^T PWv(t) + 2\tilde{x}^T PWd. \end{aligned} \tag{8.33}$$

With the aid of assumptions 8.4, 8.5, 8.6, and letting $PW = (FC)^T$, equation (8.33) has the form of

$$\begin{aligned} \dot{V} &\leq \tilde{x}^T((A - LC)^T P + P(A - LC))\tilde{x} + 2b_1 b_f \lambda_{max}(P)\|\tilde{x}\|^2 \\ &\quad +2b_2 b_b \lambda_{max}(P)\|\tilde{x}\|^2 + 2b_3 b_u \lambda_{max}(P)\|\tilde{x}\|^2 - 2\alpha\|FC\tilde{x}\| + 2\|FC\tilde{x}\|b_d. \end{aligned} \tag{8.34}$$

Letting $\alpha = b_d$, and for any $Q = Q^T > 0$, there exists a unique $P = P^T > 0$ satisfying following equation

$$(A - LC)^T P + P(A - LC) = -Q, \tag{8.35}$$

then, equation (8.34) can be simplified as

$$\begin{aligned}\dot{V} \leq & -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2b_1 b_f \lambda_{max}(P)\|\tilde{x}\|^2 + 2b_2 b_b \lambda_{max}(P)\|\tilde{x}\|^2 \\ & +2b_3 b_u \lambda_{max}(P)\|\tilde{x}\|^2.\end{aligned} \tag{8.36}$$

If

$$(\lambda_{min}(Q) - 2b_1 b_f \lambda_{max}(P) - 2b_2 b_b \lambda_{max}(P) - 2b_3 b_u \lambda_{max}(P)) > 0, \tag{8.37}$$

then estimation error equation (8.29) is stable. Furthermore, $\tilde{z}$ is also bounded according to assumption 8.4. The proof is complete. ∎

8.4 An Illustrative Example

To illustrate the construction of an SMO in a class of DAS described in equation (8.1), we consider the following system given by

$$\begin{aligned}\dot{x} &= Ax + b(x)z + g(x)u + Wd(x, z, u, t) \\ \mathbf{0} &= k(x) + l(x)z \\ y &= Cx\end{aligned} \tag{8.38}$$

with

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & -3 \\ 0 & 1 & -5 \end{bmatrix}, \qquad b(x) = \begin{bmatrix} -0.2x_1 & 0.3 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix},$$

$$k(x) = \begin{bmatrix} x_2 x_1 \\ x_2^2 - x_2 + x_3 \end{bmatrix}, \quad l(x) = \begin{bmatrix} x_1 & 0 \\ x_2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0.5 \\ 0 & 2 & 1 \end{bmatrix}, \quad W = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

The maximum rank p_1 of $l(x)$ is 1. Pre-multiplying algebraic equation by $E_1^1 = I$, where I is a 2×2 identity matrix, and reducing the last row of $E_1^1 l(x)$ to zero by pre-multiplying a matrix

$$E_1^2(x) = \begin{bmatrix} 1 & 0 \\ F_1(x) & 1 \end{bmatrix} \tag{8.39}$$

where $F_1(x) = -x_2/x_1$ on M_1, we obtain

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 x_1 \\ x_3 - x_2 \end{bmatrix} + \begin{bmatrix} x_1 & 0 \\ 0 & 0 \end{bmatrix} z, \quad x \in M_1 \tag{8.40}$$

where $M_1 = \{x \in \mathbb{R}^3 | x_1 \neq 0\}$.

Differentiating $\hat{k}^1(x) = x_3 - x_2$ leads to the following form

$$\begin{aligned} &\frac{\partial \hat{k}^1(x)}{\partial x}\dot{x} \\ &= [0 \quad -1 \quad 1] \left\{ \begin{bmatrix} -3x_1 \\ -3x_3 - x_2 \\ -5x_3 + x_2 \end{bmatrix} + \begin{bmatrix} -0.2x_1 & 0.3 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix} u \right\} \\ &= -2x_3 + 2x_2 + z_2 + 0.2u. \end{aligned} \tag{8.41}$$

where $\dfrac{\partial \hat{k}^1(x)}{\partial x} W = 0$.

Therefore, algebraic equation (8.40) has a new form of

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_2 x_1 \\ -2x_3 + 2x_2 \end{bmatrix} + \begin{bmatrix} x_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} u, \quad x \in M_2 = M_1 \tag{8.42}$$

where matrix $\begin{bmatrix} x_1 & 0 \\ 0 & 1 \end{bmatrix}$ has rank 2 for all $x \in M_2$.

The algebraic variables z can be solved from equation (8.42)

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = -\begin{bmatrix} 1/x_1 & 0 \\ 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} x_2 x_1 \\ -2x_3 + 2x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} u \right\} = \begin{bmatrix} -x_2 \\ 2x_3 - 2x_2 - 0.2u \end{bmatrix}. \quad (8.43)$$

Based on the derivation above, we have following observer

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + b(\hat{x})\hat{z} + g(\hat{x})u + L(y - \hat{y}) + Wv(t) \\ \hat{z} &= \begin{bmatrix} -\hat{x}_2 \\ 2\hat{x}_3 - 2\hat{x}_2 - 0.2u \end{bmatrix} \\ \hat{y} &= C\hat{x}. \end{aligned} \quad (8.44)$$

where

$$v(t) = \begin{cases} \alpha \frac{F\tilde{y}}{\|F\tilde{y}\|}, & \text{if } \tilde{y} \neq 0 \\ 0, & \text{otherwise} \end{cases}, \quad (8.45)$$

$$F = [1 \quad -0.5], \quad L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (8.46)$$

A random function is taken as disturbance d, and the system states and algebraic variables are initialized to consistent values.

Figures 8.1 and 8.2 show that both the estimated algebraic variables and system states can quickly converge to the actual values of the considered differential-algebraic system with reconstructed algebraic variables.

8.5 Conclusions

An SMO has been proposed in a class of uncertain nonlinear DAS with a singular distribution matrix of the algebraic variables. The selling point of this work is the improved algorithm for the reconstruction of the singular matrix $l(x)$. It is simpler and more rigorous because this algorithm only requires serial elementary matrices that can be obtained easily, and differentiation to transform the singular $l(x)$ into a nonsingular matrix. The transformation enables the algebraic variables z to be expressed as a function of system state variables and inputs. Based on the reconstructed algebraic variables, an SMO that can attenuate the effects of disturbances on system estimation error dynamics is suggested using the direct estimation of the algebraic variables $z(t)$. The illustrative example has shown both the reconstruction of the algebraic variables and the design of the SMO.

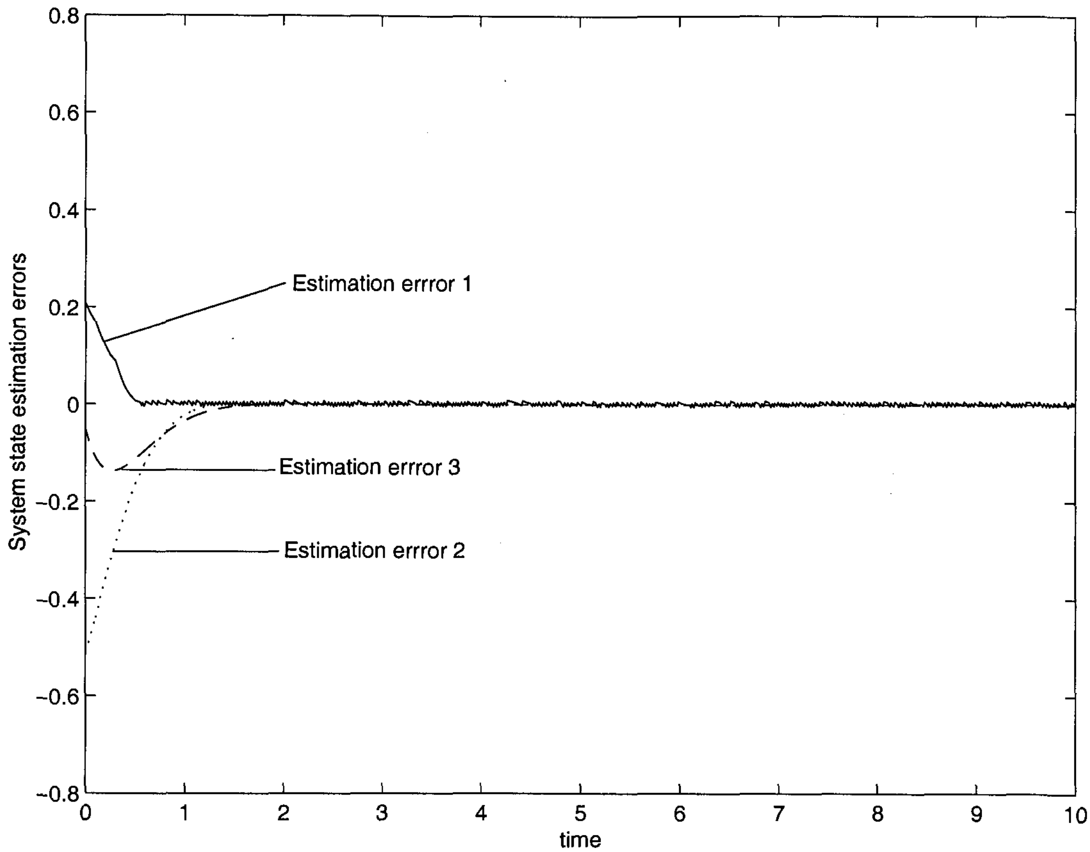

Figure 8.1: System state estimation errors.

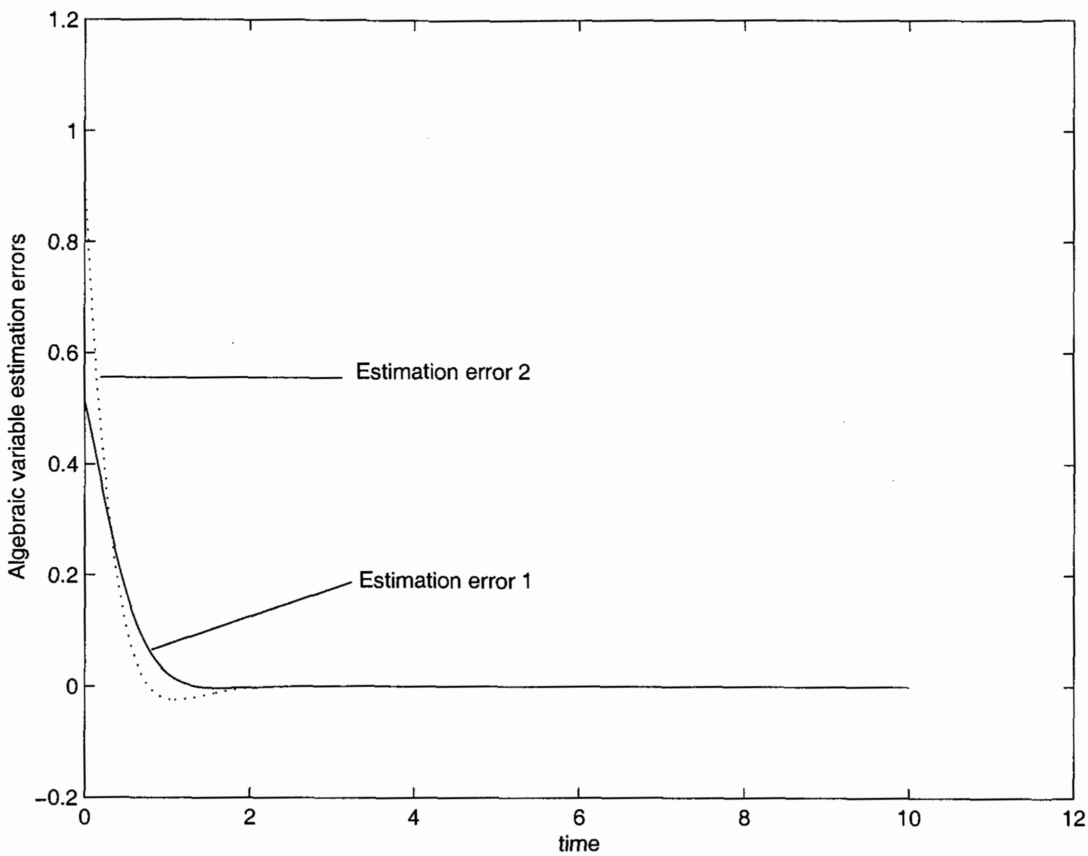

Figure 8.2: Algebraic variable estimation errors.

Chapter 9

Conclusions

Sliding mode observers, including both SMOs and SOSMOs, and the ILO have been designed and applied to uncertain nonlinear systems for the purpose of fault diagnosis. The main contribution of this thesis is the design of the SOSMO and the ILO that have been used to detect and estimate actuator faults.

The most widely used tools for fault diagnosis are observers as demonstrated in the thesis. The basic idea behind the utilization of observers for fault diagnosis is to estimate system outputs from measurements by using some types of observers, and then residuals can be constructed by weighted output estimation errors.

The classical Luenberger observer cannot efficiently detect faults when disturbances exist in the system. The SMO, a robust observer that can eliminate the effects of disturbances on the estimation error dynamics, can tackle this problem. That is, it can detect the occurrence of a fault by suppressing the disturbance. If an adaptive law is used together with the SMO, fault estimation can also be achieved as reported in Chapter 2. However, the disadvantage of the SMO is the chattering problem that may cause the residuals to be robust to real faults. Given that a SOSMO is addressed to detect small-sized faults. The attractive feature of the SOSMO is that it can not only detect relatively small faults, but also supply the operation with fault estimations so that one can know the size and severity of the faults. The sliding surface in the SOSMO design can generate an alarm signal after a fault occurs because this sliding surface is so sensitive that a small fault can destroy its sliding property.

An important issue in FDI is the fault estimation that can be used for fault accommodation, fault isolation, or even fault detection. The adaptive strategy has been widely used for fault estimation, and a version is reported in Chapter 2. The VSAO that is constructed according to the nonlinear system itself can achieve both fault detection and fault estimation despite the disturbances. The simulation example verifies that the VSAO can efficiently detect and estimate actuator faults in a class of uncertain nonlinear systems.

Additionally, as addressed in Chapters 5 and 6, the ILO can accomplish fault estimation. It can estimate and compensate faults and/or disturbances by monitoring the system's dynamic variations caused by the faults and/or disturbances. The estimation and compensation enable the residual, which may be defined as system estimation errors or ILO inputs, to be robust to disturbances, and make the ILO itself follow the considered system after the occurrence of a fault.

Fault accommodation is a crucial issue in FDI. The existing strategies for fault compensation control are based on adding an additional control input, resulting from the fault detection and isolation subsystem, to the original control input in order to reduce or compensate the effects of faults. The IL approach addressed in Chapter 5 can accomplish fault accommodation without utilizing a fault detection and isolation subsystem. The main idea behind the IL approach for fault accommodation is that the controller can be reconfigured automatically by the system output estimation errors. As a result, the fault detection and isolation subsystem is not required in the

IL approach.

An alternative approach for fault accommodation is to combine the ILO and the adaptive law. The ILO plays an important role in fault accommodation because it monitors any of the system's variations, while feeding its inputs to the adaptive law for producing an additional control input for the purpose of fault elimination. This fault accommodation strategy is a dynamic process because the transition phase of fault estimation and compensation will be successively monitored by the ILO. The adaptive law will be successively updated till there exists no variation in the considered system. This implies that faults have been completely compensated. The current additional control input is the estimation of the fault.

The necessity to consider observer design issues in uncertain nonlinear DAS, either for the control applications or for fault diagnosis, has prompted the author to address the design and analysis of an SMO for the DAS that exists in many industrial processes. The key to constructing the SMO for the DAS is the reconstruction of the algebraic variables because of the singular distribution matrix. An SMO other than a Luenberger observer is used to estimate both the differential and algebraic variables due to its robustness to disturbances. Therefore, the SMO is a potential tool for fault diagnosis in the DAS.

The author wishes to emphasize that output estimation errors are not the only residual candidates for fault diagnosis. Accurate fault estimation makes it possible for the ILO input to be selected as a residual as well, as revealed in Chapters 6 and 7. In addition, the additional control input produced from the adaptive law proposed in Chapter 7 can be another residual candidate because it is the estimate of the fault. The third potential residual candidate is the sliding surface of the SOSMO which can generate an alarm signal when the sliding is destroyed by a fault as presented in

Chapters 3 and 4.

Time delays in system states exist in many industrial systems. The FDI issue for these systems, however, has not been discussed extensively. The time-delay nonlinear system considered in this thesis is concerned only with the fixed and known time delays. Nevertheless, variable and unknown time delays also exist. The FDI for these systems needs a great deal of work because the FDI approach and fault detection observers must be reconsidered.

This thesis is mainly concerned with the FDI issue for actuator and component faults. Sensor faults have not been considered. The ILO, proposed in some chapters, may be applied to sensor fault diagnosis. This needs more exploration for future.

The ILO, as shown in this thesis, is a powerful tool for estimating disturbances or faults. As a matter of fact, the ILO requires more than a single sampling time interval for estimation. Disturbances that vary at every sampling interval can not be accurately estimated by the ILO. In addition, the problem of how many sampling time intervals are enough for the ILO to estimate a disturbance requires more research.

The majority of the research on nonlinear DAS has focused on control issues [7, 10], such as the problem of feedback controller synthesis [74, 87], Lyapunov stability of equilibria in DAS [56], and the output feedback control problem for nonlinear multivariable high-index DAS [77]. Few papers have been published on FDI in uncertain nonlinear DAS. The reference [121] is concerned with designing and analyzing a numerically feasible learning scheme for robust and stable fault diagnosis of DAS. The existence of unmeasurable differential and algebraic variables usually makes this approach useless in practice. Thus, the fault diagnosis problem in DAS needs further consideration. The decoupling problem of algebraic variables from the differential variables will be the first challenge. Chapter 8 discussed the class of DAS with disturbance term appearing only in the differential equation. If the disturbance exists in both the differential and algebraic equations, the decoupling problem will be more difficult. This raises an attractive research direction. Besides the SMO discussed in Chapter 8, the ILO may be further explored in DAS for fault detection and estimation. Actuator and sensor faults will be mainly considered in DAS for fault diagnosis purpose. However, the component faults resulting from the algebraic variables will also be an issue that needs much more research effort. In summary, the fault diagnosis in DAS will be a promising research direction.

Bibliography

[1] Adi, B. I. and Greville, T. N. E. (1974). *Generalized Inverses theory and applications.* Willey-Intersceince Publication.

[2] Aggoune, W., Boutayeb, M. and Darouach, M. (1999). Observers design for a class of nonlinear systems with time-varying delay. *Proceedings of the 38th Conference on Decision and Control*, Phoenix, Arizona, USA, December (pp. 2912-2913).

[3] Ali, T. A. and Lamnabhi-Lagarrigue, F. (1999). Sliding observer-controller design for uncertain triangular nonlinear systems. *IEEE Transaction on Automatic Control, 44*, 1244-1249.

[4] Basseville, M. (1998). Detecting changes in signals and systems - A survey. *Automatica, 24*, 309-326.

[5] Beard, R V. (1971). *Failure accommodation in linear systems through self-reorganization.* PhD thesis, Department of Aeronautics and Astronautics, MIT, Cambridge, MA.

[6] Boskovic, J. D., Yu, S. H. and Mehra, R. K. (1998). A stable scheme for automatic control reconfiguration in the presence of actuator failures. *Proceedings of American Control Conference*, Philadelphia, Pennsylvania, June (pp. 2455-2459).

[7] Brenan, K. E. at al. (1989). *Numerical solution of initial-value problems in differential-algebraic equations.* Elsevier, New York.

[8] Busawon, K. K. and Kabore, P. (2001). Disturbance attenuation using proportional integral observers. *International Journal of Control, 74*, 618-627.

[9] Busawon, K. and Saif, M. (1999). A state observer for nonlinear systems. *IEEE Transactions on Automatic Control, 44*, 2098-2103.

[10] Campbell, S. L. and Griepentrog, E. (1995). Solvability of general differential-algebraic equations. *SIAM J. Sci. Comput., 16*, 257-270.

[11] Chang, L. W. (1991). A MIMO sliding control with a first-order plus integral sliding condition. *Automatica, 27*, 853-858.

[12] Chang, S. and Hsu, P. L. (1993). Fault detection observer design for linear systems with unknown inputs. *Proceedings of the 2nd European Control Conference*, Groningen, Holland (pp. 1975-1799).

[13] Chen, J. and Zhang, H. Y. (1991). Robust detection of faulty actuators via unknown input observers. *International Journal of Systems Science, 22*, 1829-1839.

[14] Chen, J. and Patton, R. J. (1999). *Robust model-based fault diagnosis for dynamic systems.* Kluwer Academic Publishers.

[15] Chen, J. Patton, R. J. and Zhang, H. Y. (1996). Design of unknown input observers and robust fault detection filters. *International Journal of Control, 63*, 85-105.

[16] Chen, S., Tao, G. and Joshi, S. M. (2001). Adaptive actuator failure compensation for a transport aircraft model. *Proceedings of American Control Conference*, Arlington, VA, USA (pp. 1827-1832).

[17] Chen, W. and Saif, M. (2000). A variable structure adaptive observer approach for actuator fault detection and diagnosis in uncertain nonlinear systems. *Proceedings of American Control Conference*, Chicago, USA (pp. 2674-2678).

[18] Chen, W. and Saif, M (2001). An iterative learning observer-based approach to fault detection and accommodation in nonlinear systems . *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, USA (pp. 4469-4474).

[19] Chen, W. and Saif, M (2001). Robust fault detection in uncertain nonlinear systems via a second order sliding mode observer. *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, USA (pp. 573-578).

[20] Chen, W. and Saif, M. (2002). Robust fault diagnosis in time-delay nonlinear systems via an ILO with application to automotive engine. *Proceedings of IFAC Congress'02*, Barcelona, Spain.

[21] Chen, W. and Saif, M. (2002). Robust fault detection and isolation in constrained nonlinear systems via a second order sliding mode observer. *Proceedings of IFAC Congress'02*, Barcelona, Spain.

[22] Chen, W. and Saif, M. (2003). Fault detection and accommodation in nonlinear time-delay systems. *Proceedings of American Control Conference'03*, Denver, USA (pp. 4255-4260).

[23] Chen, W. and Saif, M. (2003). Disturbance attenuation observer in time-delay nonlinear systems with application to automotive engine fault detection. *Accepted for publication by IEEE Conference on Control and Automation*, Montreal, Canada.

[24] Chen, W. and Saif, M. (2002). A variable structure adaptive observer for robust fault diagnosis in nonlinear systems. *Submitted to International Journal of Control.*

[25] Chen, X. and Fukuda, T. (1998). VSS theory-based disturbance estimation scheme for MIMO systems and its applications. *International Journal of Control, 69*, 733-752.

[26] Chen, X. and Shayman, M.A. (1992). Dynamics and control of constrained nonlinear systems with application to robotics. *Proceedings of American Control Conference*, (pp. 2962-2966).

[27] Chirlian, P. M. (1994). *Signals and filters.* Van Nostrand Reinhold, New York.

[28] Cho, D. and Paolella, P. (1990). Model-based failure detection and isolation of automotive powertrain systems. *Proceedings of American Control Conference*, (pp. 2898-2905).

[29] Cho, Y. M. and Gajamani, R. (1997). A systematic approach to adaptive observer synthesis for nonlinear systems. *IEEE Transaction on Automatic Control, 42*, 534-537.

[30] Chung, W. H. and Speyer, J. L. (1998). A game theoretic fault detection filter. *IEEE Transaction on Automatic Control, 43*, 143-161.

[31] Ciccarella, G., et al (1993). A Luenberger-like observer for nonlinear systems. *International Journal of Control, 57*, 537-556.

[32] Clark, R. N., Fosth, D. C. and Walton, V. M. (1975). Detecting instrument malfunctions in control systems. *IEEE Transaction on Aero. & Electron. Syst., 11*, 465-473.

[33] Daley, S. and Wang, H. (1992). On the generation of a optimally robust residual signal for systems with structured, model uncertainties. *Proceedings of American Control Conference*, USA (pp. 2104-2108).

[34] Daoutidis, P. and Kravaris, C. (1993). Dynamic compensation of measurable disturbances in nonlinear multivariable systems. *International Journal of Control, 58*, 1279-1301.

[35] Demetriou, M. A. and Polycarpou, M.M. (1998). Incipient fault diagnosis of dynamical systems using online approximators. *IEEE Transactions on Automatic Control, 43*, 1612-1617.

[36] Ding, X., Frank, P. M. and Guo, L. (1990). Nonlinear observer design via an extended observer canonical form. *Systems & Control Letters, 15*, 313-322.

[37] Drakunov, S.V. (1992). Sliding-mode observers based on equivalent control method. *Proceedings of the 31st IEEE Conference on Decision and Control,* Tucson, Arizona, December (pp. 2368-2369).

[38] Edwards, C. and Spurgeon, S. (1994). On the development of discontinuous observers. *International Journal of Control, 59*, 1211-1229.

[39] Edwards, C., Spurgeon, S. K. and Patton, R.J. (2000). Sliding mode observers for fault detection and isolation. *Automatica, 36*, 541-553.

[40] Edwin, E. Y. and Azemi, A. (1998). Actuator fault detection in nonlinear systems using LMI's and LME's. *Proceedings of American Control Conference*, Philadelphia, Pennsylvania, June (pp. 1590-1593).

[41] Elmali, H. and Olgac, N. (1992). Robust output tracking control of nonlinear MIMO systems via sliding mode technique. *Automatica, 28*, 145-151.

[42] Farrell, J., et al. (1993). Using learning techniques to accommodate unanticipated faults. *IEEE Control Systems, June*, 40-49.

[43] Flett, T. M. (1980). *Differential Analysis.* Cambridge University Press.

[44] Frank, P. M. and Keller, L. (1981). Sensitivity discriminating observer design for instrument failure detection. *IEEE Trans. Aero. & Electron. Syst., 16*, 460-467.

[45] Frank, P. M., Ding, X. and Wochnik, J. (1991). Model-based fault detection in diesel hydraulically driven industrial trucks. *Proceedings of American Control Conference*, Boston (pp. 1528-1533).

[46] Frank, P. M. (1993). Advances in observer-based fault diagnosis. *International Conference on Fault Diagnosis*, Toulouse, April.

[47] Frank, P. M. (1994). Enhancement of robustness in observer-based fault detection. *International Journal of Control, 59*, 955-981.

[48] Frank, P. M. (1996). Analytical and qualitative model-based fault diagnosis–A survey and some new results. *European Journal of Control, 2*, 6-28.

[49] Garg, V. (1995). *Fault detection in nonlinear systems: An application to automated highway systems.* PhD Thesis, Mechanical Engineering, University of California, Berkeley.

[50] Ge, W. and Fang, Z. (1988). Detection of faulty components via robust observation. *International Journal of Control, 47*, 581-599.

[51] Gear, C. W. (1988). Differential-algebraic equation index transformations. *SIAM Journal on Scientific and Statistical Computing*, 9.

[52] Gertler, J. J. (1998). *Fault detection and diagnosis in engineering systems.* Marcel Dekker Inc, New York.

[53] Hale, J. (1977). *Theory of functional differential equations.* Springer-Verlag, New York.

[54] Hamidi-Hashemi, H. and Leondes, C. T. (1979). Observer theory for systems with time-delay. *International Journal of Systems Science, 10*, 797-806.

[55] Hermans, F. J. J. and Zarrop, M. B. (1996). Sliding mode observers for robust sensor monitoring. *Proceedings of 13th IFAC World Congress*, San Francisco, USA (pp. 211-216).

[56] Hill, D. J. and Mareels, I. M. Y. (1990). Stability theory for differential/algebraic systems with application to power systems. *IEEE Transaction on Circuits and Systems, 37*, 1416-1422.

[57] Hirschorn, R. M. (1979). Invertibility of multivariable nonlinear systems. *IEEE Transaction on Automatic Control, 24*, 855-865.

[58] Hou, M., Busawon, K.K. and Saif, M. (2000). Observer design for nonlinear systems via injective mapping. *To appear in IEEE Transactions on Automatic Control.*

[59] Hou, M. and Muller, P. C. (1994). Fault detection and isolation observers. *International Journal of Control, 60*, 827-846.

[60] Isermann, R. (1984). Process fault detection based on modelling and estimation methods: A survey. *Automatica, 20*, 387-404.

[61] Isermann, R. (1995). Model based fault detection and diagnosis methods and their applications. *Proceedings of American Control Conference*, Seattle, WA (pp. 1605-1609).

[62] Isermann, R. and Balle, P. (1997). Trends in the application of model-based fault detection and diagnosis of the technical processes. *Control Engineering Practice, 5*, 709-719.

[63] Isermann, R. (1997). Supvision, fault detection and fault diagnosis methods - an introduction. *Control Engineering Practice, 5*, 639-652.

[64] Isidori, A. (1985). *Nonlinear control systems, An Introduction.* Springer, New York.

[65] Jakubek, S. and Jorgl, H.P. (2000). Fault-diagnosis and fault-compensation. *Proceedings of American Control Conference*, Chicago, Illinois, June (pp. 3198-3202).

[66] Jones, H. L. (1973). *Failure detection in linear systems through self - reorganization.* PhD thesis, Department of Aeronautics and Astronautics, MIT, Cambridge, MA.

[67] Kabore, P. and Wang, H. (2001). Design of fault diagnosis filters and fault-tolerant control for a class of nonlinear systems. *IEEE Transaction on Automatic Control, 46*, 1805-1810.

[68] Kao, M. and Moskwa, J. J. (1994). Engine load and equivalence ratio estimation for control and diagnostics via nonlinear sliding observer. *Proceedings of American Control Conference*, Baltimore, Maryland, June (pp. 1574-1578).

[69] Khalil, H. K. (1996). *Nonlinear systems.* Upper Saddle River. NJ: Prentice-Hall.

[70] Kim, Y. W., Rizzoni, G. and Utkin, V. (1998). Automotive engine diagnosis and control via nonlinear estimation. *IEEE Control Systems*, October, 84-99.

[71] Kim, Y. W., Rizzoni, G. et al. (1997). Powertrain diagnostics using nonlinear sliding mode observer. *Proceedings of IFAC Fault Detection, Supervision and Safety for Technical Processes*, Kingston Upon Hull, UK (pp. 821-825).

[72] Korondi, P., Young, D. and Hashimoto, H. (1998). Sliding mode based disturbance observer for motion control. *Proceedings of 37th IEEE Conference on Decision and Control*, Tampa, Florida.

[73] Kratz, F., Nuninger, W. and Ploix, S. (1998). Fault detection for time-delay systems: a parity space approach. *Proceedings of American Control Conference*, Philadelphia, Pennsylvania, June (pp. 2009-2011).

[74] Krishnan, H. and McClamroch, N. H. (1993). Tracking in control systems described by nonlinear differential-algebraic equations with applications to constrained robot systems. *Proceedings of American Control Conference, 1993*, San Francisco, USA (pp. 837).

[75] Krishnan, H. and McClamroch, N. H. (1994). Tracking in nonlinear differential-algebraic control systems with applications to constrained robot systems. *Automatica, 30*, 1885-1897.

[76] Krishnaswami, V. et al. (1996). Application of sliding mode observers to automobile powertrain diagnostics. *Proceedings of IEEE International Conference on Control Applications*, Dearborn, MI, September (pp. 355-360).

[77] Kumar, A. and Daoutidis, P. (1995). Feedback control of nonlinear differential-algebraic equation systems. *AIChE Journal, 41*, 619-636.

[78] Leininger, G. G. (1981). Model degradation effects on sensor failure detection. *Proceedings of the 1981 Joint American Control Conference*, Charlottesville, VA (Paper FP-3A) (Vol. 3).

[79] Li, Z. H., Chai, T. Y., Wen, C. and Soh, C. B. (1995). Robust output tracking for nonlinear uncertain systems. *Systems & Control Letters, 25*, 53-61.

[80] Liao, T. L., Fu, L. C. and Hsu, C. F. (1992). Output tracking control of nonlinear systems with mismatched uncertainties. *Systems & Control Letters, 18*, 39-47.

[81] Liu, B. F. and Si, J. N. (1997). Fault isolation filter design for linear time-invariant systems. *IEEE Transaction on Automatic Control, 42*, 704-707.

[82] Liu, X. P. (1998). Local disturbance decoupling of nonlinear singular systems. *International Journal of Control, 70*, 685-702.

[83] Marino, R. (1990). Adaptive observer for single output nonlinear systems. *IEEE Transaction on Automatic Control, 35*, 1054-1058.

[84] Marino, R. and Tomei, P. (1995). *Nonlinear systems design : Geometric, Adaptive and Robust.* Prentice Hall International.

[85] Marquez, H. Z. and Diduch, C. P. (1990). Sensitivity robustness in failure detection: A frequency domain approach. *Proceedings of 29th IEEE Conference on Decision and Control*, (pp. 839-846).

[86] Massoumnia, M. A. (1986). A geometric approach to the synthesis of failure detection filters. *IEEE Transaction on Automatic Control, 31*, 839-846.

[87] McClamroch, N. H. (1990). Feedback stabilization of control systems described by a class of nonlinear differential-algebraic equations. *Systems & Control Letters, 15*, 53-60.

[88] McClamroch, N. H. and Wang, D. W. (1988). Feedback stabilization and tracking of constrained robots. *IEEE Transactions on Automatic Control, 33*, 419-426.

[89] Narendra, K. S. and Annaswamy, A. (1989). *Stable adaptive systems.* Englewood Cliffs, NJ: Prentice-Hall.

[90] Nijmeijer, H. and Van Der Schaft, A. J. (1983). The disturbance decoupling problem for nonlinear systems. *IEEE Transactions on Automatic Control, 28*, 621-623.

[91] Nikoukhah, R. (1998). A new methodology for observer design and implementation. *IEEE Transaction on Automatic Control, 43*, 229-234.

[92] Noura, H., Theilliol, D. and Sauter, D. (2000). Actuator fault-tolerance control design: demonstration on a three-tank-system. *International Journal of Systems Science, 31*, 1143-1155.

[93] Paolella, P. and Cho, D. (1991). A robust failure detection and isolation method for automotive powertrain sensors. *Proceedings of IFAC Fault Detection, Supervision and Safety for Technical Processes*, Baden-Baden, Germany (pp. 509-515).

[94] Paoli, A., Marconi, L. and Bonivento, C. (2001). A fault-tolerant strategy for induction motors. *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, USA (pp. 179-184).

[95] Patton, R. J., Frank, P. M. and Clark, R. N. (2000). *Issues of fault diagnosis for dynamic systems.* Springer.

[96] Patton, R., Frank, P. and Clark, R. (1989). *Fault diagnosis in dynamic systems: theory and application.* New York: Prentice Hall.

[97] Patton, R. and Chen, J. (1991a). Robust fault detection using eigenstructure assignment: a tutorial consideration and some new results. *Proceedings of the 30th IEEE Conference on Decision and Control* , Brighton (pp. 2242-2247).

[98] Patton, R. and Chen, J. (1991b). A robust parity space approach to fault diagnosis based on optimal eigenstructure assignment. *Proceedings of IEE Int. Conference: Control'91*, Edinburgh (pp. 1056-1061).

[99] Polycarpou, M. M. and Helmicki, A. J. (1995). Automated fault detection and accommodation: A learning systems approach. *IEEE Transactions on Systems, Man, and Cybernetics, 25*, 1447-1458.

[100] Qu, Z. (1998). *Robust control of nonlinear uncertain systems.* John Wiley & Sons, Inc.

[101] Qu, Z., Ihlefeld, C.M., Jin, Y. and Saengdeejing, A. (2001). Robust control of a class of nonlinear uncertain systems– fault tolerance against sensor failures and subsequent self recovery. *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, USA (pp. 1472-1478).

[102] Rajamani, R. (1998). Observers for Lipschitz nonlinear systems. *IEEE Transaction on Automatic Control, 43*, 397-400.

[103] Rajaman, R. and Cho, Y. M. (1998). Existence and design of observers for nonlinear systems: relation to distance to unobservability. *International Journal of Control, 69*, 717-731.

[104] Roozbeh, I. Z. (1999). *Fault-tolerant supervisory control-system analysis and logic design.* PhD Thesis, Department of Control Engineering, Aalborg University, Denmark.

[105] Saif, M. and Guan, Y. (1993). A new approach to robust fault detection and identification. *IEEE Trans. on Aerospace and Electronic Systems, 29*, 685-695.

[106] Schneider, H. and Frank, P. M. (1993). Observer-based supervision and fault detection for robots. *TOOLDIAG'93, International conference on fault diagnosis*, Toulouse, France.

[107] Seliger, R. and Frank, P. M. (1991a). Fault-diagnosis by disturbances decoupled nonlinear observers. *Proceedings of the 30th IEEE Conference on Decision and Control*, Brighton, England (pp. 2248-2253).

[108] Seliger, R. and Frank, P. M. (1991b). Robust component fault detection and isolation in nonlinear dynamic systems using nonlinear unknown input observers.

Preprints of IFAC/IMACS Symp.: SAFEPROCESS'91, Badenbaden (pp. 313-318) (Vol.1).

[109] Shafai, B. and Nork, S. (2002). Simultaneous disturbance attenuation and fault detection using Proportional Integral observers. *Proceedings of American Control Conference*, Anchorage, AK (pp. 1647-1649).

[110] Shafai, B., Pi, C. T., Nork, S. and Linder, S. P. (2002). Proportional Integral observer adaptive observer for parameter and disturbance estimations. *Proceedings of the 41th IEEE Conference on Decision and Control*, Las Vegas, Nevada (pp. 4694-4699).

[111] Shafai, B. and Oloomi, H. M. (2003). Output derivative estimation and disturbance attenuation using PI observer with application to a class of nonlinear systems. *Proceedings of the 35th Southeastern Symposium on System Theory*, Morgantown, Virginia (pp. 196-200).

[112] Slotine, J.-J., Hedrick, E. J. K. and Misawa, E.A. (1987). On sliding observers for nonlinear systems. *Journal of Dynamic Systems, Measurement, and Control, 109*, 245-252.

[113] Sreedhar, R., Fernandez, B. and Masada, G. Y. (1993). Robust fault detection in nonlinear systems using sliding mode observers. *Proceedings of IEEE Conference on Control Applications*, Vancouver, BC. September 13-16,

[114] Stotsky, A. E. and Eriksson, S. (2000). Variable structure control of engine idle speed with estimation of unmeasurable disturbances. *Journal of Dynamic Systems, Measurement and Control, 122*, 599-603.

[115] Tao, G. and Ma, X. (2000). Adaptive actuator compensation control for multivariable nonlinear systems. *Proceedings of American Control Conference*, Chicago, USA (pp. 2664-2668).

[116] Tarraf, D. C. and Asada, H. H. (2001). A sliding observer for differential-algebraic (DAE) systems. *Proceedings of American Control Conference, 2001*, Arlington, USA (pp. 25-27).

[117] Thau, F.E. (1973). Observing the state of nonlinear dynamic systems. *International Journal of Control, 17*, 471-479, .

[118] Utkin, V. I. (1992). *Sliding modes in control optimization.* Springer-Verlag.

[119] Van Schrick, D. (1991). Investigations of reliability for instrument fault detection state-estimator schemes. *European Journal of Diagnosis and Safety in Automation, 1*, 63-78.

[120] Vemuri, A.T., and Polycarpou, M. M. (1997). Robust nonlinear fault diagnosis in input-output systems. *International Journal of Control, 68*, 343-360.

[121] Vemuri, A. T., Polycarpou, M. M. and Ciric, A. R. (2001). Fault diagnosis of differential-algebraic systems. *IEEE Transactions on Systems, Man and Cybernetics-Part A, 31*, 143-152.

[122] Wallot, B. L. and Zak, S. H. (1988). Combined observer-controller synthesis for uncertain dynamical systems with application. *IEEE Transactions on Systems, Man and Cybernetics, 18*, 88-104.

[123] Wang, G. B., Peng, S. S. and Huang, H. P. (1997). A sliding observer for nonlinear process control. *Chemical Engineering Science, 52*, 787-805.

[124] Wang, H. and Daley, S. (1996). Actuator fault diagnosis: an adaptive observer-based technique. *IEEE Transactions on Automatic Control, 41*, 1073-1078.

[125] Watanabe, K. and Himmelblau, D. M. (1982). Instrument fault detection in systems with uncertainties. *International Journal of Systems Science, 13*, 137-158.

[126] White, J. E. and Speyer, J. L. (1987). Detection filter design: spectral theory and algorithm. *IEEE Transactions on Automatic Control, 32*, 593-603.

[127] Xiong, Y. (2001). *Robust fault diagnosis in linear and nonlinear systems based on unknown input and sliding mode functional observer methodologies.* PhD thesis, School of Engineering Science, Simon Fraser University, Canada.

[128] Xiong, Y. and Saif, M. (1998). A novel design for robust fault diagnostic observer. *Proceedings of 37th IEEE Conference on Decision and Control*, Tampa, FL (pp. 592-597).

[129] Xiong, Y. and Saif, M. (2000a). Robust fault detection and isolation via a diagnostic observer. *International Journal of Robust and Nonlinear Control, 11*, 1175-1192.

[130] Xiong, Y. and Saif, M. (2000b). Sliding-mode observer for uncertain systems Part I: Linear systems case. *Proceedings of the 39th IEEE Conference on Decision and Control*, Sydney, Australia (pp. 316-321).

[131] Xiong, Y. and Saif, M. (2000c). Sliding-mode observer for uncertain systems Part II: Nonlinear systems case. *Proceedings of the 39th IEEE Conference on Decision and Control*, Sydney, Australia (pp. 322-327).

[132] Yang, H. and Saif, M. (1995). Nonlinear adaptive observer design for fault detection. *Proceedings of American Control Conference*, Seattle, Washington, June (pp. 1136-1139).

[133] Yang, H. and Saif, M. (1996). Monitoring and diagnostics of a class of nonlinear systems using a nonlinear unknown input observer. *Proceedings of IEEE CCA*, (pp. 1006-1011).

[134] Yang, H. and Saif, M. (1997a). Robust observation and fault diagnosis in a class of time-delay control systems. *Proceedings of American Control Conference*, Albuquerque, New Mexico, June (pp. 478-482).

[135] Yang, H. and Saif, M. (1997b). Fault detection and isolation for a class of nonlinear systems using an adaptive observer. *Proceedings of American Control Conference*, Albuquerque, New Mexico (pp. 463–467).

[136] Yang, H. and Saif, M. (1997c). State observation, fault detection and isolation (FDI) in bilinear systems. *International Journal of Control, 67*, 901-920.

[137] Yang, H. and Saif, M. (1998). Observer design and fault diagnosis for state retarded dynamical systems. *Automatica, 34*, 217-227.

[138] Youcef-Toumi, K. (1992). Input/output linearization using time delay control. *Journal of Dynamic Systems, Measurement, and Control, 114*, 10-19.

[139] Zhang, X., Polycarpou, M. M. and Parisini, T. (2001). Integrated design of fault diagnosis and accommodation schemes for a class of nonlinear systems. *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando, USA (pp. 1448-1453).