THE IMPACT ON PORTFOLIO CREDIT RISK WITH DIFFERENT CORRELATION ASSUMPTIONS

by

Ru (Jessie) Jia
B.A., Tianjin University of Finance and Economics, 2008

Han (Dabria) Guo
B.A., Dongbei University of Finance and Economics, 2009

PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF FINANCIAL RISK MANAGEMENT

In the Faculty of Business Administration

SIMON FRASER UNIVERSITY
Summer 2010

Approval

Name:	**Ru (Jessie) Jia**
	Han (Dabria) Guo
Degree:	**Master of Financial Risk Management**
Title of Project:	**The Impact on Portfolio Credit Risk with Different Correlation Assumptions**

Supervisory Committee:

____

Anton Theunissen
Senior Supervisor
Academic Director

____

Graeme Fattedad
Second Reader
MA

Date Approved: ____

Abstract

The main idea of this paper is to apply default analysis to the Student Investment Advisory Service (SIAS) fixed income portfolio, which contains 19 bonds.

The portfolio credit risk analysis includes default probability, simulation of default time by using Gaussian copula and t copula, Economic Capital, Credit Value at Risk (VaR) and Expected Tail Loss (ETL).

Keywords: default probability; copula; credit risk; credit VaR; expected tail loss; SIAS

Acknowledgements

We would like to extend our sincere gratitude to our senior supervisor, Anton Theunissen, for his instructive advice and useful suggestions. We are also deeply indebted to Graeme Fattedad for his support and practical advice on model testing.

Without their patient instruction, insightful criticism and expert guidance, the completion of this thesis would not have been possible.

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1: Introduction

The purpose of this paper is to analyze how the portfolio credit risk behaves under different default correlation assumptions. Both credit Value at Risk (VaR) and Expected Tail Loss (ETL) are used to measure the portfolio credit risk.

Credit VaR is defined as the maximum unexpected credit loss of a portfolio at a specific confidence level over a given time horizon. It is an important and widely used methodology to measure credit risk. Implementation of using credit VaR to quantify portfolio credit risk forces institutions to not only consider their exposure to financial risks but also to set up a proper risk management function. ETL, also known as conditional VaR or expected shortfall, is defined as the average of the tail losses in a portfolio loss distribution, which is larger than the VaR for a given confidence level. It is a more conservative way to evaluate the risk since it focuses on the extreme portfolio loss.

2: Literature Review

During the early development of credit risk modelling, structural model is the first and only valuation methodology, which is originated from Black/Scholes (1973) and Merton (1974). Merton model is the foundation for structural models. Structural model focuses on the capital structure of the issuer to model default probability. It is also known as firm value model, because actual firm values can be measured based on firm financials. Therefore, structure model is considered to be more appropriate to analyze default probabilities of corporate issuers. As for sovereign issuers, it is hard to find specific financial information. However, some other literatures have presented the implementation of using the structural model for sovereign credit by using national stock indices as proxies for firm values, C.F. LEHRBASS(2000).

Adopting the idea from actuarial sciences, reduced-form model became more popular since it was initiated by Jarrow/Turnbull (1995). The reduced-form model ignores the reasons behind a default event, but pays more attention to the default event itself.

Based on the analysis above, we choose to use reduced-form model instead of structural model due to that a large proportion of the bonds in our portfolio are sovereign bonds and it is very cumbersome to model government value based on public available financial information.

The different treatments for the recovery value in the event of a default between structural model and reduced-form model is one of the major differences between the modes. When using the structural model approach the recovery amount for a bond holder in the event of a default is simply the value of the firm's assets minus liquidation cost at the bond's maturity. Within the reduced-form approach, however, we could make various assumptions regarding the recovery amounts as different recovery assumptions could be arbitrarily assigned. Currently, three popular methodologies are used to model the recovery process.

The first assumption is equivalent recovery, which is introduced by Jarrow/Turnbull (1995). Under this assumption, the recovery amount is the discounted value of the recovered amount of par value at maturity.

The second one is the fractional recovery assumption, which is introduced by Duffie/Singleton (1999) and later extended to multiple defaults by Schonbucher (1998). Under this assumption, the value recovered is calculated as a fraction of the bond's market value when default occurs.

The last one is recovery of par value. Under this assumption, the recovery amount is just a fraction of par value. This assumption is based on the hypothesis that bonds are not likely to be traded below expected par recovery. Unlike this assumption, the equivalent and the fractional recovery assumptions do not correspond to market conventions, therefore the recovery of par value assumption is used in our reduced form model.

3: Data

The Student Investment Advisory Service (SIAS) portfolio is one of North America's largest student-run endowment funds, which has over $9 million in assets. We have chosen to analyze the fix income portion of the portfolio. Our portfolio consists of 19 bonds from a total of 16 issuers. Most of the bonds are highly rated. Eight bonds are issued by Canadian government and the rest is issued by corporations. Detail information regarding all the bonds is shown in **Table 1**.

To calculate default probability under reduced-form method, we obtained bond prices and coupon rates through Bloomberg. We used bonds with different maturities issued by the same issuer to generate the term structure of default probabilities. Because not enough bonds with different maturities were issued by INDUSTRIAL ALLIANCE CAP TR and TORONTO DOMINION, we used bonds issued by their peer groups with same rating and coupon payment method to generate the term structure of defaults.

As stated earlier, the bonds in our portfolio are issued either by the government or by corporations; different recovery rates are applied respectively. For the recovery rate of the government bonds, we choose to use 50% based on Recovery Rates on Defaulted Sovereign Bond Issuer, which is established by Moody's Global Credit Policy. (**Table 2**). In this table, the recovery rates are evaluated based on both percentage of par value and percentage of cash flow. Based on our recovery rate assumption, recovery rate evaluated as percentage of par value is chosen. Since the collateral types of most of the bonds in our portfolio are Senior Secured, Senior Unsecured and Senior Subordinated, we chose to use the recovery rates of these bonds to calculate the recovery rate for the corporate bond in our portfolio. According to Average Annual Bond and Loan Recovery Rates table (**Table 3**), which is established in Moody's Global Credit Policy, the recovery rate of corporation bonds is 45.4%.

4: Methodology

4.1 Default Probability

In order to simulate default time, we calculated default probability under both risk neutral and objective measures. Under risk neutral measure, the default probabilities are implied by current bond prices and coupon rates; while the default probabilities are obtained from historical default events under objective measure.

In this paper, we obtained average cumulative issuer-weighted global default rates for the time period 1983-2008 based on Moody's rating under objective measure, which is shown in **Table 4**.

For reduced-form model, after all necessary data is collected, spot λ, which is the average number of credit events per unit time, can be calculated based on the formula below.

$$P = \frac{c}{n}\sum_{i=1}^{nT} e^{-\lambda(0,t_i)t_i} \cdot b(0,t_i) + FV \cdot e^{-\lambda(0,T)T} \cdot b(0,T) + R\sum_{i=1}^{nT}(e^{-\lambda(0,t_{i-1})t_{i-1}} - e^{-\lambda(0,t_i)t_i}) \cdot b(0,t_i)$$

P: market price of the bond

c: coupon payment per year

n: coupon payment frequency per year

FV: par value of the bond

R: recovery rate

$b(t_m, t_n)$: discount factor

Canadian LIBOR swap rates, which can be used as a good prediction for future interest rates, are converted to continuously compounded rates in order to perform discount factor calculation.

All the bonds in our portfolio pay coupons semi-annually, so n equals to 2 in the equation above.

Under piece-wise constant λ assumption, the following formula is derived to calculate forward λ:

$$b(t_1,t_2)\left[R+(1-R)e^{-\lambda(t_1,t_2)(t_2-t_1)}\right]=\frac{b(0,t_2)\left[R+(1-R)e^{-\lambda(0,t_2)t_2}\right]}{b(0,t_1)\left[R+(1-R)e^{-\lambda(0,t_1)t_1}\right]}$$

$$\lambda(t_1,t_2)=-\frac{1}{t_2-t_1}\ln\left(\frac{R+e^{-\lambda(0,t_2)t_2}-Re^{-\lambda(0,t_1)t_1}}{R+(1-R)e^{-\lambda(0,t_1)t_1}}\right)$$

4.2 Default Time

Default dependence structure can be modelled by copulas. For multivariable models, Gaussian and t-copula are wildly used because of the easy implementation. Therefore, the default time is modelled for each issuer based on both Gaussian and t-copula, using the following procedures:

a) Gaussian copula $C_{\Sigma}^{G}(u_1,...,u_n)=\Phi_{\Sigma}\left(\Phi^{-1}(u_1),...,\Phi^{-1}(u_n)\right)$

(1) Specify or estimate the correlation matrix **Σ**.

(2) Determine **A** by performing a Cholesky-decompositon $\Sigma = AA^T$

(3) Generate a series of iid (independent and identically distributed) standard normal random variables **Z** = (z1,… zn)'.

(4) Bring in the dependence structure by calculating **X** = **AZ**

(5) Set Ui = ϕ(Xi), where ϕ is the standard normal cumulative distribution function. Then the Ui have a Gaussian Copula dependence structure.

(6) Calculate default time **τ** from **U**. Since we assume piece-wise constant λs, $\tau_i = t_n + \frac{-\ln U_i - \lambda(0,t_n)\times t_n}{\lambda(t_n,t_{n+1})}$ if $t_n < \tau < t_{n+1}$.

b) T copula $C^t_{\upsilon,\Sigma}(u_1,...,u_n) = t_{\upsilon,\Sigma}\left(t_\upsilon^{-1}(u_1),...,t_\upsilon^{-1}(u_n)\right)$

(1) Specify or estimate the correlation matrix **Σ**.

(2) Generate correlated Xi as above.

(3) Generate an independent $\xi \sim \chi^2_\upsilon$ via $\xi = \sum_{i=1}^{\upsilon} Y_i^2$, where Y_i are iid standard normal random variables.

(4) Set $U_i = t_\upsilon\left(\frac{X_i}{\sqrt{\xi/\upsilon}}\right)$, where t_υ is the cumulative distribution function of an univariate student-t distribution with υ degrees of freedom.

(5) Calculate default time **τ** from **U** as above.

4.3 Loss distribution

After modelling default time for each issuer, we compare each default time τ_i to the time horizon T. If $\tau_i < T$, the issuer is considered to be in default. To compute the portfolio loss, we assume constant Loss Given Default (LGD), which is 50% for government bonds and 54.5% for corporate bonds. This is consistent with the recovery rate assumptions used for default probability calculation under risk neutral measure. The time horizon is set to be one year based on industry convention. Portfolio loss distributions are generated using Monte Carlo Method with 2 million simulations.

To measure the credit risk of the portfolio, credit Value at Risk (credit VaR) and Expected Tail Loss (ETL) are calculated based on portfolio loss distributions. The confidence level is chosen to be 99.9%, because most of the bonds in our portfolio are highly rated.

5: Results

Case 1: A constant correlation of 0.2 is applied to the entire portfolio and is considered to be the base case.

We compared credit VaR and ETL based on four loss distributions, which are generated by using Gaussian and t-copulas with 6 degrees of freedom under risk neutral and objective measures.

As shown in **Table 5 and 6**, under risk natural measures, credit VaR and ETL are bigger when using t-copula than Gaussian copula. This is expected as student-t distribution has fatter tail than normal distribution. Under objective measure, the credit VaR are the same when using different copula functions, while the ETL behave the same as under risk neutral measure.

Using the same copula, the risk neutral credit VaR and ETL are larger than objective ones, because the default probability is higher under risk neutral measure.

Case 2: In order to observe how Credit VaR and ETL change with respect to the change of correlations, we run a series of correlations from 0.1 to 0.5. The results are shown in **Table 7 and 8**.

As the correlation increases, credit VaR stays the same under objective measure. Under risk neutral measure, using Gaussian copula, credit VaR increases until the correlation reaches 0.3 and stays afterwards; using t-copula with 6 degrees of freedom, credit VaR is monotonically increasing. The ETL based on all the four distributions are increasing as the default correlation increases.

In order to take the tail dependence into consideration, we also calculated credit VaR and ETL of the portfolio using t-copula with 2 degrees of freedom. A t-copula's degrees of freedom determine the level of tail dependence. Smaller degrees of freedom correspond to higher tail dependence, in other words, a higher probability to have the extreme losses. As the results shown in **Table 7 and 8**, credit VaR do not change under objective measure as the default correlation changes, while the risk neutral VaR as well as ETL increase.

Since no change is observed on credit VaR under objective measure, we increase the confidence level to 99.99%. As the result shown in **Table 9 and 10**, ETL reacts the same way to the change of correlation. The change of credit VaR is more obvious when default correlation is high.

As the default correlation increases, we expect the portfolio loss distribution to have both a higher head and a fatter tail. In other words, higher default correlation indicates higher probability of having both no losses and extreme losses. In general, the portfolio's credit risk should increase as default correlation increases. However, due to limited numbers of bonds in our portfolio, the loss distribution is discrete. This causes the VaR to be unchanged as correlation increases in some situations, i.e. default correlation goes up from 0.1 to 0.3 under objective measurement.

As stated above, credit VaR is unable to capture the impact on the portfolio's credit risk in certain situations, since it focus on the body part of the loss distribution. On the other hand, ETL is observed to be able to better capture the impacts from changes in default correlation, because it pays more attention on the tail of the loss distribution. It is a great complement to credit VaR as a measure of portfolio credit risk.

Case 3: The bonds in the portfolio are divided into two groups, government bonds and corporate bonds. To be more realistic, instead of using a constant correlation throughout the entire portfolio, we assigned different correlations for each group. The correlation between government bonds, corporate bonds as well as between government and corporate bonds are set to be 0.3, 0.26 and 0.13, respectively. To be comparable with our base case, the simple average correlation is kept to be 0.2.

As the results shown in **Table 9 and 10**, at the 99.9% confidence level over one year horizon, the portfolio VaR do not change except under risk neutral measure with Gaussian copula, while ETL based on all four distributions increase.

To further analyze the impact from different correlations, we increased the correlation within the same group from 0.3 to 0.38 and from 0.26 to 0.33 respectively. In order to maintain the average correlation 0.2, we lowered the correlation between two groups to 0.06.

Compare to the base case, the portfolio VaR remains the same under objective measure and increased under risk neutral measure. However, the changes on ETL are noticeable.

6: Conclusion

In general, an increase in the default correlation will lead to an increase in the portfolio credit risk. However, in our portfolio, VaR is unable to reflect this relationship perfectly because of other impacts, such as portfolio's size and composition. With more bonds having higher default probabilities in the portfolio, the positive relationship between default correlation and credit VaR of the portfolio will be more properly captured.

The current market value of our portfolio is $2,404,682.00. Under a very conservative assumption, which has a constant correlation of 0.5 under objective measure with t-copula with 2 degrees of freedom, our portfolio's VaR is $187,154 and the ETL is $220,605 at a 99.99% confidence level, which is 7.78% and 9.17% of the current portfolio value, respectively. This shows the SIAS fixed-income portfolio has a very low credit risk, which is in accord with the SIAS Investment Policy Statement.

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Appendix

Table 1 SIAS Fixed Income Securities

ISSUER	CUSIP	COUPON	MATURITY DATE	PRICE	RATINGS(Moody's)
BMO CAPITAL TRUST	05560HAC7	6.685	12.31.2011	100.000	A3
PROV OF BRITISH COLUMBIA	110709DF6	7.5	6.9.2014	97.900	Aaa
	110709FJ6	5.4	6.18.2035	99.619	Aaa
GOVERNMENT OF CANADA	135087YC2	3.75	9.1.2011	98.970	Aaa
CANADA MORTGAGE & HOUSING CORP	135143AS4	5.5	6.1.2012	99.050	Aaa
CANADIAN IMPERIAL BK OF COMM	13591Z3L2	3.05	6.3.2013	99.947	Aa2
EXPORT DEVELOPMENT CANADA	30215ZNR7	5.1	6.2.2014	99.992	Aaa
407 INTL INC	35085ZAD8	5.96	12.3.2035	99.930	A2
GE CAPITAL CANADA FUNDING CO	36158ZBH8	5.53	8.17.2017	99.954	Aa2
GREATER TORONTO AIRPORTS AUTH	39191ZAD1	6.25	1.30.2012	99.948	A2
INDUSTRIAL ALLIANCE CAP TR/CALLABLE	455869AA5	5.714	12.31.2013	100.000	Baa2
LONDON ONTARIO	541908BZ1	5.875	8.6.2017	99.720	Aaa
MUNICIPAL FINANCE AUTH OF B C	626209GW4	4.9	12.3.2013	99.848	Aaa
PROV OF ONTARIO	683234SL3	5.85	3.8.2033	99.444	Aa1
	683234UV8	5.35	6.2.2019	102.000	Aa1
	683234WM6	4.5	3.8.2015	99.195	Aa1
PROV OF QUEBEC	748148RK1	5.25	10.1.2013	100.423	Aa2
SHAW COMMUNICATIONS INC	82028KAL5	6.15	5.9.2016	98.052	Baa3
TORONTO DOMINION BANK	891145DH4	5.141	11.19.2012	100.000	Aaa

Table 2 Recovery Rates on Defaulted Sovereign Bond Issuer

Year of Default	Defaulting Country	Average Trading Price (% of par)	PV Ratio of Cash Flows (ratio in %)
1998	Russia	18	50
1999	Pakistan	52	65
1999	Ecuador	44	60
2000	Ukraine	69	60
2000	Ivory Coast	18	NA
2001	Argentina	27	30
2002	Moldova	60	95
2003	Uruguay	66	85
2004	Grenada	65	NA
2005	Dominican Republic	95	95
2006	Belize	76	NA
2008	Seychelles	29	NA
2008	Ecuador	26	NA
Recovery Rates		50	68

Table 3 Average Annual Bond and Loan Recovery Rates

Year	Sr. Sec.	Sr. Unsec.	Sr. Sub.	All Bonds
1989	46.54%	43.81%	34.57%	41.64%
1990	33.81%	37.01%	25.64%	32.15%
1991	48.39%	36.66%	41.82%	42.29%
1992	62.05%	49.19%	49.40%	53.55%
1993	n.a.	37.13%	51.91%	44.52%
1994	69.25%	53.73%	29.61%	50.86%
1995	62.02%	47.60%	34.30%	47.97%
1996	47.58%	62.75%	43.75%	51.36%
1997	75.50%	56.10%	44.73%	58.78%
1998	46.82%	41.63%	44.99%	44.48%
1999	43.00%	38.04%	28.01%	36.35%
2000	39.23%	23.81%	20.75%	27.93%
2001	37.98%	21.45%	19.82%	26.42%
2002	48.37%	29.69%	21.36%	33.14%
2003	63.46%	41.87%	37.18%	47.50%
2004	73.25%	52.09%	42.33%	55.89%
2005	71.93%	54.88%	26.06%	50.96%
2006	74.63%	55.02%	41.41%	57.02%
2007	80.54%	53.25%	54.47%	62.75%
2008	57.98%	33.80%	23.02%	38.27%
Avg.	56.96%	43.48%	35.76%	45.40%

Table 4 Average Cumulative Issuer-Weighted Global Default Rates , 1983-2008

Rating	1 Year	2 Year	3 Year	4 Year	5 Year	6 Year	7 Year	8 Year	9 Year	10 Year	11 Year	12 Year	13 Year	14 Year	15 Year	16 Year	17 Year	18 Year	19 Year	20 Year
Aaa	0.000	0.016	0.016	0.049	0.088	0.136	0.188	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193	0.193
Aa1	0.000	0.000	0.000	0.094	0.141	0.159	0.159	0.159	0.159	0.159	0.159	0.159	0.323	0.526	0.759	0.849	0.849	0.849	0.849	0.849
Aa2	0.000	0.010	0.042	0.104	0.201	0.245	0.294	0.350	0.412	0.483	0.565	0.658	0.702	0.702	0.702	0.810	0.998	1.217	1.470	1.495
Aa3	0.038	0.118	0.174	0.246	0.319	0.370	0.402	0.417	0.420	0.468	0.527	0.684	0.835	0.933	1.030	1.210	1.466	1.826	2.417	3.139
A1	0.018	0.154	0.366	0.544	0.692	0.793	0.868	0.935	0.997	1.076	1.202	1.322	1.470	1.673	1.910	2.188	2.333	2.568	2.568	2.568
A2	0.026	0.092	0.244	0.445	0.639	0.891	1.230	1.615	1.955	2.209	2.401	2.532	2.684	2.880	3.091	3.372	4.061	4.639	5.192	5.609
A3	0.032	0.151	0.318	0.463	0.714	0.997	1.203	1.432	1.660	1.799	2.020	2.309	2.661	3.035	3.665	4.379	5.077	6.074	6.785	7.420
Baa1	0.135	0.357	0.622	0.867	1.091	1.289	1.547	1.730	1.859	2.088	2.382	2.827	3.469	4.281	5.209	6.332	6.940	7.192	7.192	7.192
Baa2	0.139	0.426	0.796	1.367	1.850	2.317	2.756	3.178	3.666	4.292	5.140	6.002	6.686	7.339	7.980	8.667	9.527	10.568	11.574	11.940
Baa3	0.291	0.816	1.459	2.129	2.926	3.741	4.463	5.189	5.859	6.520	6.998	7.452	8.517	9.589	10.181	10.820	11.865	12.986	13.985	14.855
Ba1	0.682	1.862	3.363	4.857	6.280	7.789	8.889	9.649	10.346	11.120	11.938	13.222	14.161	15.087	16.669	17.886	19.306	21.066	23.830	25.925
Ba2	0.728	2.066	3.760	5.608	7.230	8.425	9.661	11.006	12.330	13.365	14.746	16.342	17.864	19.321	21.354	23.013	24.599	25.112	25.499	25.499
Ba3	1.791	4.954	8.873	12.932	16.209	19.227	22.017	24.755	27.188	29.601	31.635	33.344	35.421	38.298	39.905	41.745	43.470	45.411	47.196	49.403
B1	2.450	6.800	11.358	15.361	19.513	23.576	27.853	31.305	34.187	36.717	38.998	41.585	44.132	46.736	48.110	48.850	49.644	50.649	51.792	53.186
B2	3.827	9.116	14.386	19.204	23.232	27.013	30.514	33.495	36.607	39.110	41.376	43.339	45.820	48.886	53.355	56.778	58.350	59.986	59.986	59.986
B3	7.666	15.138	22.336	28.744	34.261	39.643	44.081	48.016	50.948	53.684	56.398	59.278	60.552	61.298	61.546	61.546	61.999	62.735	62.735	62.735
Caa1	9.150	18.763	28.028	35.629	42.389	46.914	49.140	51.686	57.028	62.344	74.557	83.038								
Caa22	16.388	25.807	32.990	38.799	41.983	45.823	48.900	51.959	55.997	61.737	65.577	65.577	68.127	70.995	73.226	77.097	78.154	78.154	78.154	78.154
Caa3	24.806	36.604	43.417	49.310	55.959	57.672	60.527	64.744	70.661	82.018	82.018									
Ca-C	32.949	44.297	53.255	58.406	63.932	66.489	70.337	74.990	74.990	74.990	74.990	74.990	74.990	74.990	74.990	74.990	74.990	74.990		
Investment-Grade	0.072	0.229	0.436	0.673	0.917	1.154	1.381	1.599	1.803	2.008	2.230	2.470	2.766	3.080	3.408	3.793	4.232	4.707	5.116	5.433
Speculative-Grade	4.351	8.917	13.373	17.316	20.686	23.696	26.388	28.687	30.708	32.516	34.180	35.865	37.452	39.176	40.810	42.171	43.389	44.609	45.983	47.195
All Rated	1.565	3.192	4.726	6.037	7.118	8.037	8.824	9.482	10.045	10.544	11.012	11.478	11.960	12.467	12.963	13.461	13.981	14.530	15.046	15.467

Table 5 VaR at 99.9% confidence level over 1 year time horizon, ρ=0.2

	Risk Neutral	Objective
Gaussian	81,984	58,328
T (v = 6)	127,254	58,327
T (v = 2)	185,808	58,322

Table 6 ETL at 99.9% confidence level over 1 year time horizon, ρ=0.2

	Risk Neutral	Objective
Gaussian	112,957	59,924
T (v = 6)	173,477	72,859
T (v = 2)	245,489	95,844

Table 7 VaR at 99.9% confidence level over 1 year time horizon

ρ		*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
	Gaussian	56,988	81,984	115,548	115,550	115,556
Risk Neutral	**t (v = 6)**	115,552	127,254	140,559	152,266	174,106
	t (v = 2)	174,108	185,808	199,101	199,113	210,827
	Gaussian	58,323	58,328	58,328	58,321	58,327
Objective	**t (v = 6)**	58,323	58,327	58,324	58,324	58,323
	t (v = 2)	58,323	58,322	58,320	58,326	58,324

Table 8 ETL at 99.9% confidence level over 1 year time horizon

ρ		*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
	Gaussian	90,447	112,957	126,501	140,550	163,843
Risk Neutral	**t (v = 6)**	150,982	173,477	193,833	214,750	238,131
	t (v = 2)	225,160	245,489	262,280	276,188	295,649
	Gaussian	59,268	59,924	61,668	64,799	69,277
Objective	**t (v = 6)**	70,085	72,859	78,537	83,834	91,612
	t (v = 2)	91,443	95,844	98,478	99,947	107,210

Table 9 VaR at 99.99% confidence level over 1 year time horizon

ρ		*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
Risk Neutral	**Gaussian**	115,541	115,543	152,264	174,118	232,675
	t (ν = 6)	199,098	232,679	257,680	294,380	327,948
	t (ν = 2)	294,387	316,229	327,933	377,937	386,508
Objective	**Gaussian**	58,320	58,325	58,327	70,043	116,886
	t (ν = 6)	116,883	116,890	116,887	128,595	128,601
	t (ν = 2)	128,595	175,444	175,444	187,154	187,154

Table 10 ETL at 99.99% confidence level over 1 year time horizon

ρ		*0.1*	*0.2*	*0.3*	*0.4*	*0.5*
Risk Neutral	**Gaussian**	122,622	144,885	182,448	216,516	268,004
	t (ν = 6)	241,352	267,771	301,880	340,324	369,136
	t (ν = 2)	335,264	352,733	384,838	405,233	418,708
Objective	**Gaussian**	65,028	71,962	86,354	105,291	129,007
	t (ν = 6)	129,706	135,766	155,577	166,532	180,575
	t (ν = 2)	175,214	193,161	199,337	210,408	220,605

Table 11 VaR at 99.9% confidence level over 1 year time horizon, with different correlation between groups

Default Correlation		Constant $\rho = 0.20$	$\rho_{GG} = 0.30$ $\rho_{CC} = 0.26$ $\rho_{GC} = 0.13$	$\rho_{GG} = 0.38$ $\rho_{CC} = 0.33$ $\rho_{GC} = 0.06$
Risk Neutral	**Gaussian**	81,984	115,545	115,544
	t (ν = 6)	127,254	127,243	131,974
Objective	**Gaussian**	58,328	58,326	58,324
	t (ν = 6)	58,327	58,321	58,320

Table 12 ETL at 99.9% confidence level over 1 year time horizon, with different correlation between groups

Default Correlation		Constant $\rho = 0.20$	$\rho_{GG} = 0.30$ $\rho_{CC} = 0.26$ $\rho_{GC} = 0.13$	$\rho_{GG} = 0.38$ $\rho_{CC} = 0.33$ $\rho_{GC} = 0.06$
Risk Neutral	**Gaussian**	112,956	123,511	126,039
	t (ν = 6)	173,477	187,510	194,265
Objective	**Gaussian**	59,923	62,089	62,768
	t (ν = 6)	72,859	79,104	80,473

Matlab Code

Code 1: spot λ for issuer GOVERNMENT OF CANADA

```
% Converting Excel date format to Matlab date format
date(:,1) = x2mdate(can(:,1));
date(:,2) = x2mdate(can(:,2));
date(:,3) = can(:,3);
C=can(:,4);
P=can(:,5);
% Calculating time factor for each cashflow of bonds
%Calculating discounted cashflow for each bonds
C = C/100;
L = length(C);
C = C';
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(C, date(:,1), date(:,2));
%Calculating discount factor Z(t)
TFactors = TFactors';
U = length(TFactors);
TFactors = TFactors';
R = R(1:U,1);
R = R';
A = nan(L,U);

for v = 1:L
   for w = 1:U
A(v,w) = exp(-(TFactors(v,w)*R(w)));
   end
end
Z = A(:,2:end);
CF = CFlowAmounts(:,2:end);
TF = TFactors(:,2:end);
Z = Z';
W = length(Z);
Z = Z';
b=nan(L,W);
for i=1:L
   for j= 1:W
      if isnan(Z(i,j)) == 1
   b(i,j) = 0;
      else
   b(i,j) = 1;
      end
   end
```

```
end
b = b';
c = sum(b);
c = c';
RecRate = 0.5;
FV = 100;
Rec = RecRate*FV;
Coef = nan(L,W);
for i = 1:L
    n = c(i,1);
    if n == 1
    Coef(i,n) = (CF(i,n)-Rec)*Z(i,n);
    elseif n > 1
        for k = 1:n-1
        Coef(i,k) = (CF(i,k) - Rec)*Z(i,k)+Rec*Z(i,k+1);
        end
        Coef(i,n) = (CF(i, n) - Rec)*Z(i,n);
        for m = 1+n:W
          Coef(i,m) = 0;
        end
    end
end
%calculating credit spread lamda and survival probability Q
AITime = -(TF(:,1)-1);
C = C';
AI = (AITime.*C/2)*100;
P0 = P0+AI;
LHS = P0 - Rec*Z(:,1);

Q(1,1) = LHS(1)/Coef(1,1);
Lambda(1,1) = -log(Q(1,1))/TF(1,1);
for p = 2:L
   n = c(p,1);
   d = c(p-1,1);
   u = n - d;
   if u == 1
   for q = 1:n-1
      Q(p,q) = exp(-Lambda(p-1,q)*TF(p,q));
      if Q(p,q)>1
         Q(p,q) = 1;
      end
      Lambda(p,q) = -log(Q(p,q))/TF(p,q);
      J(p,q) = Q(p,q)*Coef(p,q);
   end
      RTerm(p) = sum(J(p,1:n-1));
      Q(p,n) = (LHS(p)-RTerm(p))/Coef(p,n);
      if Q(p,n)>1
         Q(p,n) = 1;
      end
      Lambda(p,n) = -log(Q(p,n))/TF(p,n);
```

```
    elseif u > 1
        Lambda(p,1:d) = Lambda(p-1,1:d);
        Lambda(p,d+1:n) = Lambda(p,d);
        for q = 1:n-1
            Q(p,q) = exp(-Lambda(p,q)*TF(p,q));
            if Q(p,q)>1
            Q(p,q) = 1;
        end
            J(p,q) = Q(p,q)*Coef(p,q);
        end
        RTerm(p) = sum(J(p,1:n-1));
        Q(p,n) = (LHS(p)-RTerm(p))/Coef(p,n);
        if Q(p,n)>1
            Q(p,n) = 1;
        end
        Lambda(p,n) = -log(Q(p,n))/TF(p,n);
    end
end
```

Code 2: default simulations for VaR and ETL calculations

```
% Import data from Excel
[input, Ctype] = xlsread('Default.xlsx', 'Portfolio', 'B2:B10');
Ctype = char(Ctype);
n =  input(1);  % Number of issuers in the portfolio
correlation = input(2);  % Default correaltion, assume constant throughout the portfolio
if correlation == 0
    rho = xlsread('Default.xlsx', 'Correlation Matrix', 'C6:Z36');
else
    rho = correlation*ones(n)+(1-correlation)*eye(n);  % correlation matrics
end
T = input(3);  % Time to maturity
dof = input(7);
sim = input(9);  % Number of simulations
c = input(8); % Confidence level

portinfo = xlsread('Default.xlsx', 'Portfolio', 'D12:H62');
port =portinfo(:,1); % Portfolio information
FV =portinfo(:,2);
R = portinfo(:,4);  % Recovery rate

lam = xlsread('Default.xlsx', 'Lambda', 'C2:L53');
lamda = lam(2:end, 1:5);
flamda = lam(2:end, 6:10);
time = lam(1,1:5);

def_probA = xlsread('Default.xlsx', 'Def_prob', 'H3:L53');
def_prob = def_probA * (time == T)';
```

```
% Simulation
U = nan(n, sim);
if Ctype == 'G'
    U = copularnd('Gaussian',rho,sim)';
    disp('Gaussian Copula')
elseif Ctype == 'T'
    U = copularnd('t',rho,dof,sim)';  % T Copula with dof
    disp (['T-copula with dof  ' , num2str(dof)])
end

%%%%%%%%%%%%%%%%% RN %%%%%%%%%%%%%%%%%%
% convert Ui to default time tao
S = -log(ones(size(U)) - U);
tao = nan(n,sim);
for j = 1:sim
    for i = 1:n
        if S(i,j) > lamda(i,5)*time(5)
            tao(i,j) = S(i,j) / lamda(i,5);
        elseif S(i,j) > lamda(i,4)*time(4)
            tao(i,j) = time(4) + (S(i,j) - lamda(i,4)*time(4)) / flamda(i,5);
        elseif S(i,j) > lamda(i,3)*time(3)
            tao(i,j) = time(3) + (S(i,j) - lamda(i,3)*time(3)) / flamda(i,4);
        elseif S(i,j) > lamda(i,2)*time(2)
            tao(i,j) = time(2) + (S(i,j) - lamda(i,2)*time(2)) / flamda(i,3);
        elseif S(i,j) > lamda(i,1)*time(1)
            tao(i,j) = time(1) + (S(i,j) - lamda(i,1)*time(1)) / flamda(i,2);
        else
            tao(i,j) =S(i,j) / flamda(i,1);
        end
    end
end

Def = tao < T;
N_Def = sum(Def);
% Loss Distribution
 La = (diag(ones(n,1) - R) * FV)' * Def;
 ELa = mean(La);     % Expected loss
 Lax = sort(La);
 VaRa = Lax(sim*c) - ELa;  % VaR
 ETLa = mean(Lax(sim*c:end));   % Expected short fall

%%%%%%%%%%%%%% Rating %%%%%%%%%%%%%%%%%%%
Def_R = U < def_prob*ones(1,sim);
N_Def_R = sum(Def_R);
% Loss Distribution
 La_R = (diag(ones(n,1) - R) * FV)' * Def_R;     % loss = (1-R)*FV
 ELa_R = mean(La_R);     % Expected loss
 Lax_R = sort(La_R);
 VaR_R = Lax_R(sim*c) - ELa_R;                    % VaR
 ETL_R = mean(Lax_R(sim*c:end));   % Expected short fall
```