Strategic Debt Service and the Limits to Lending

by

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Abstract

This thesis develops a framework for studying the design and valuation of collateralized loan contracts in a dynamic setting under complete information and uncertainty. Contingent claims valuation techniques are integrated into a game theoretic setting in which borrowers and lenders behave noncooperatively to maximize the values of their claims as specified by the terms of the loan contract and applicable bankruptcy laws.

The analysis presumes that the market value of the loan collateral follows a diffusion process. The borrower attempts to deviate from the terms of the loan contract to enhance the value of his claim. This behaviour is tempered by contractual provisions which allow the lender to foreclose and seize the collateral in the face of such deviation. Hence the rational borrower engages in 'strategic default', deviating from the terms of the contract without provoking foreclosure. However, certain contractual indentures do yield foreclosure in some states along the equilibrium path of the game analyzed.

Consistent with empirical evidence, foreclosure is assumed to be costly. The incidence of these costs on the contracting parties is state dependent. Also, the level of the market value of the collateral at which foreclosure occurs is determined endogenously.

Results are obtained analytically and by numerical methods. Noteworthy results include: (1) The upper limit on credit extended by a rational lender is a modest fraction of the initial market value of the collateral when foreclosure costs and dividend flows are positive, regardless of the interest rate the borrower offers. (2) The credit supply curve facing a particular borrower may be 'backward bending', with more credit supplied at lower interest rates than higher interest rates. (3) Strategic default by the borrower has a significant negative effect on the quantity of credit supplied for any given contractual interest rate. (4) A contractual indenture which allows the lender to recover prior concessions made to the borrower, at a later date, mitigates this negative effect. (5) The quantity of credit extended

is decreasing in the volatility of the market value of the collateral, the cash flows generated by the collateral and the term to maturity of the loan contract.

For the purposes of this study results (1) and (2) are referred to as 'credit rationing'. Such credit rationing prevails despite the lack of any informational asymmetries between the borrower and lender.

Acknowledgments

Sincere thanks to my examining committee.

Dedication

To my parents.

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Chapter 1

Introduction

Black and Scholes (1973) and Merton (1973, 1974), were the first to recognise that the debt of a firm can be viewed as a contingent claim on its assets. This marked the first significant development in the modeling and pricing of default risky debt. Numerous extensions to this framework have emerged.[1] Black and Cox (1976) incorporated classes of senior and junior debt. Brennan and Schwartz (1977) and Ingersoll (1977) studied convertible bonds and coupon paying debt. Brennan and Schwartz (1980) allowed for stochastic interest rates. Cox, Ingersoll and Ross (1980) modeled variable rate debt. Mason and Bhattacharya (1981) included a jump process for the underlying asset value, while Jones, Mason and Rosenfeld (1984) incorporated callable debt.

While these contributions have been important in their own right, they all seem to be characterized by a common shortcoming. Empirical evidence suggests that the default risk premia on corporate debt significantly exceed those implied by these models. For example, the estimates of Jones, Mason and Rosenfeld (1984) systematically overestimate observed bond prices. Kim, Ramaswamy and Sundaresan (1993) report that the credit spreads on AAA rated corporate bonds ranged from 15 to 215 basis points with an average of 77 basis points, while credit spreads on BAA rated bonds ranged from 51 to 787 basis points with an average of 198 basis points over the period 1926 to 1986. Merton's model, however, is unable to generate credit spreads in excess of 120 basis points, even when excessive debt ratios and volatility parameters are used (Pan, 1995).

Recent contributions to the contingent claims literature on corporate debt (Anderson

[1] Only a small number of noteworthy contributions are listed here. See Cooper and Martin (1996) and Ingersoll (1987, chapter 19) for comprehensive summaries.

and Sundaresan, 1996, Anderson, Sundaresan and Tychon, 1996 and Mella-Barral and Perraudin, 1996) claim that these models fail due to their stylized treatment of financial distress, default and bankruptcy procedures. To illustrate the nature of this problem, consider Merton's original analysis of zero- coupon debt (Merton, 1974). The boundary condition on the value of the bond at maturity T is:

$$B(s,T) = \min(s,P) \tag{1.1}$$

This states that the bond value at maturity is the minimum of the principal P, or the value of the firm s. This condition implies a model of the bankruptcy process. Upon default of the debt contract ($s < P$), the bondholders seize the assets of the firm instantly and costlessly, and then liquidate the assets or continue to operate them without any loss of value. This assumption about the bankruptcy process has an important bearing on the predictions of the models which employ it.

Research on the resolution of default and the implications of bankruptcy procedures has established a number of stylized facts:[2]

1. The formal renegotiation of debt contracts in the face of financial distress, by private 'workouts' or via the bankruptcy courts, is costly, both because of direct costs and because of disruptions of the firm's activities.
2. Bankruptcy procedures allow considerable scope for opportunistic behaviour by the parties to the loan contract
3. Deviations from absolute priority of claims on the assets of the firm are common.[3]
4. Debtholders of firms experiencing financial distress are often persuaded by equityholders to accept concessions prior to formal bankruptcy proceedings.

Based on a sample of 11 retailing firms and 5 industrial firms operating under the protection of Chapter 11 of the US Bankruptcy Reform Act of 1978, Altman (1984) reports

[2] This literature is voluminous. Important contributions include Altman (1984), Franks and Torous (1989, 1993), Warner(1977a, 1977b) and Weiss (1990). See Pan (1995) and Longhofer and Carlstrom (1995) for useful surveys.

[3] The absolute priority rule is the theoretical standard by which financial contracts are resolved when a debtor is insolvent. In short, a debtor shall receive no value from his assets until all creditors have been repaid in full.

that the sum of direct and indirect renegotiation costs amounted to 8.7% of market value one year prior to bankruptcy for the retailing firms and 15% for the industrial firms.

Violations of the absolute priority of claims in Chapter 11 reorganizations are well documented. Betker (1995) and Franks and Torous (1991) find that equityholders of publicly traded companies that go through reorganization receive value approximately 75% of the time, even though their creditors do not receive the full value of their claims. The magnitude of these deviations is not small. Eberhart, Moore and Roenfeldt (1990) find that the firm's equityholders retain approximately 7.6% of the firm's value.

These bankruptcy facts are absent from the contributions cited in the opening remarks to this chapter. They are in large part due to the 'second best' nature of loan contracts (Freixas and Rochet, 1997). In an ideal world, a loan contract would specify, at every date over the term of the loan and for every state of nature:

1. The payment to be made by the borrower to the lender
2. The interest rate to be applied to the outstanding principle
3. A possible adjustment in the collateral required by the lender
4. The actions (in particular investment decisions) to be undertaken by the borrower.

In practice loan contracts are much less complex. Payment obligations (points 1 and 2) and collateral (point 3) are generally specified for the duration of the contract, whereas actions to be taken (point 4) are left to the borrower. Consequently loan contracts typically leave a great deal of scope for opportunistic behaviour by the borrower.

Early attempts to incorporate some of these stylized facts include Bergman and Callen (1991) who study the extraction of concessions from debtholders during financial distress in a static model of capital structure determination. Kim et al. (1993) and Leland (1994) include costly bankruptcy in a contingent claims model of corporate debt, while Longstaff and Schwartz (1995) incorporate departures from absolute priority. These models simply impose the various bankruptcy facts on the underlying analysis. The bankruptcy facts do not emerge as a consequence of the rational behaviour of the contracting parties and the indentures of the loan contract.

Anderson and Sundaresan (1996), Anderson, Sundaresan and Tychon (1996) and Mella-Barral and Perraudin (1996) have incorporated a game theoretic framework into the standard model of contingent claims valuation such that the bankruptcy facts are endogenised in the model. Central to the analysis is a game in which borrowers attempt to deviate from the indentures of the debt contract to enhance equity value. This behaviour is tempered by contractual provisions which allow debtholders to foreclose and seize the collateral in the face of such deviation. Hence borrowers engage in 'strategic default', deviating from the terms of the contract without inducing foreclosure. Deviations from absolute priority and the extraction of concessions from debtholders occur along the equilibrium paths of the games modeled. The threshold value of the collateral at which control thereof is passed from borrower to lender is determined endogenously in these models, as is the compensation to be received by lenders when this default boundary is reached.

The implications for the valuation of default risky debt are significant. These models generate credit spreads consistent with the empirical evidence, without resorting to unrealistically high bankruptcy costs or excessive levels of firm asset volatility. We shall refer to these models as 'strategic debt service models'.[4]

Our objective is to combine the structure of these strategic debt service models with the approach developed by Jones (1995) to develop a framework which explores the implications of default risk and the rational opportunistic behaviour of the contracting parties for the extension of credit by banks, within the context of 'standard' collateralized loan contracts.[5] In particular, we are interested in the implications for credit rationing. For our purposes, credit rationing refers to instances in which the amount of credit which the lender is willing to extend falls short of the financing requirement of the borrower. Variations in the interest rate specified in the loan contract do not remedy this situation.[6] Unlike many attempts to study the phenomenon of credit rationing, we do not rely on assumptions of informational

[4]There exists an extensive body of research pertaining to strategic behaviour and incentive compatibility within the context of debt contracts. See, for example, Bolton and Scharfstein (1990) and Dewatripoint and Maskin (1995). However, this literature does not employ a contingent claims valuation framework.

[5]We claim that the strategic debt service models are in fact better suited to situations like this where a single lender interacts with a borrower. In the case of publicly traded corporate bonds, coordination problems amongst the bondholders are bound to occur as they decide on the appropriate reaction to the opportunistic behaviour of the bond issuer. While Anderson and Sundaresan (1996) couch the propositions of their model in terms of corporate bond yields, the game they develop is one played by a single borrower and lender.

[6]This definition of credit rationing is due to Jaffee and Russell (1976) and Gale and Helwig (1985). Alternative approaches to credit rationing such as that of Baltensperger (1978) would contend that what we describe here is not credit rationing at all. He argues that the lender's refusal to supply additional credit despite the borrower's willingness to pay a higher rate of interest is not a sufficient condition for credit rationing. The borrower must be willing to pay all the 'nonprice' elements of the loan contract as well. One of these elements would be to supply additional collateral. In our framework the initial market value of the collateral is fixed. The borrower cannot add to this. See Freixas and Rochet (1997, chapter 5)for a survey of the credit rationing literature.

asymmetries or costly state verification.[7] Here, as in Jones (1995) and the strategic debt service models, borrowers and lenders have 'full information' at the time a debt contract is negotiated. Neither party can influence the riskiness of the underlying collateral and hence, the subsequent riskiness of their claims as specified in the contract. Our objective is to provide a benchmark analysis of credit relations based on rational strategic behaviour and option value alone. These elements are pervasive, whether information asymmetries exist or not.

Our analysis extends the framework of the strategic debt service models in an important way. We include contractual indentures which allow the lender to extract concessions from the borrower in certain states of nature. Thus, unlike the approach adopted in the work cited above, the ongoing implicit contract renegotiation is not always advantageous to the borrower.

As in the case of the strategic debt service models, costs associated with the renegotiation of loan contracts or the transfer of ownership of assets in the event of default, play a central role in our analysis. While the literature on the costs associated with bankruptcy of companies which issue publicly traded debt is extensive, far less research effort has been directed at establishing the magnitudes of the cost associated with default on bank loans. Asarnow and Edwards (1995) study the losses incurred by Citibank on defaulted bank loans over the period 1970 to 1993. For a portfolio of general commercial and industrial loans they find that the loss incurred in the event of default amounted to 34.79% of the outstanding principal.[8] The part of this loss which may be associated with loan renegotiation and attempts to seize and liquidate collateral amount to at least 10% of the principal for the entire portfolio and amount to 13.68% for loans with principal amounts exceeding $10 million.[9]

Our analysis also sheds light on the design of loan contracts and allows us to draw some conclusions regarding the social efficiency of a variety of contractual arrangements which typify actual bank lending practices. Lenders who are cognizant of the limitations of standard loan contracts to constrain the opportunistic behaviour of borrowers may demand collateral requirements which exceed the fair market value of a project, in order to satisfy a borrower's financing requirements. If the borrower has no additional collateral, the lender

[7] Jaffe and Stiglitz (1990) survey the literature based on these assumptions.
[8] Unfortunately, this portfolio contains a mix of secured and unsecured loans.
[9] Henceforth, we shall refer to these costs as foreclosure costs.

will choose to extend an amount of credit (if any) which falls short of the borrower's financing requirements. Hence, economically viable projects may go unfunded when the borrower has insufficient funds to cover the unfunded balance of the project's value. From a social welfare standpoint this is clearly inefficient.[10]

The analysis developed here is presented as follows. Chapter 2 sets out the parameters of the representative loan contract and describes the environment in which the lender and borrower operate. Chapter 3 describes the games of strategy which may be played out between the contracting parties. Chapter 4 provides analytical solutions and chapter 5 provides numerical solutions to these games and considers the implications for the values of the claims of the borrower and the lender. Chapter 5 also reports the implications for credit rationing and considers some issues in contract design. Chapter 6 concludes.

[10] Harris and Raviv (1991, 1992) survey the incomplete contracting approach to loan contracts and financial structure.

Chapter 2

The contracting environment

In keeping with the contingent claims approach, we develop a continuous time 'arbitrage-free' valuation framework. Risk-free interest rates are assumed constant. By assuming complete markets we afford the agents the opportunity to hedge their respective positions at prevailing market rates for such 'insurance'.

Two features of our environment account for credit rationing. First, the collateral is assumed to generate a service flow or dividend stream over the multiperiod term of the loan contract. These flows contribute to the initial market value of the collateral. However, in the case of default, the lender is unable to recover the value of such flows which have accrued to the borrower. Consequently, the value of the lender's claim is determined, not by the initial market value of the collateral, but by the expected value of the collateral at the unknown date of default, discounted to the present.

Second, the option to default belongs to the borrower. He controls the timing of its exercise and the extent of the default. Default refers to any behaviour by the borrower which is not in compliance with the indentures of the loan contract. In the game theoretic framework developed here we distinguish between two types of default. ***Terminating default*** induces foreclosure as a best response by the lender. Terminating default may be a rational choice of the borrower or it may occur due to binding constraints which make it impossible for the borrower to avoid. ***Strategic default*** does not induce foreclosure as a best response by the lender. Instead the lender allows the loan to continue. Hence, strategic default implies that the borrower is successful at extracting concessions form the lender. Strategic debt service is a particular type of strategic default. Here, the lender accepts debt service payments from the borrower which fall short of the contractual payments. The presence of foreclosure costs increases the scope for strategic default by increasing the 'reluctance' of the lender to foreclose in certain states of nature. The borrower can appropriate value form the lender by following a strategic default policy which amounts to ongoing renegotiation of the loan contract in favour of the borrower.[1]

2.1 The debt contract

We employ a simple multiperiod, specified collateral, non-recourse loan contract similar to the contract described in Jones (1995). The lender advances a sum to a borrower in exchange for the borrower's promise to make a scheduled sequence of payments over some interval of time. For a finite interval, $[0, T]$, the contract may call for a lump-sum payment at T. For an initial sum of \$1 and continuous payments at a rate of p per year over the interval $[0, T]$, with a lump-sum payment, P, at T, the contractual loan rate, c , satisfies:

$$1 = p \int_0^t e^{-c\tau} d\tau + b(t)e^{-ct} \tag{2.1}$$

The outstanding loan balance at $t \in [0, T]$ is:

$$b(t) = e^{ct} - (e^{ct} - 1)p/c \tag{2.2}$$

The lump-sum payment is simply:

$$P = b(T) \tag{2.3}$$

Perpetual loans and pure discount loans (discount notes) are simple special cases of equation 2.1. In the case of a perpetual loan, the contractual rate c is the coupon rate applied to a notional principle P. This implies a continuous stream of payments, $p = cP$ which satisfies:

$$1 = cP \int_0^{\infty} e^{-c\tau} d\tau \tag{2.4}$$

This implies that $P = 1$ and $b(t) = 1$, for $t \in [0, \infty)$. In the case of a pure discount loan, a single contractual payment is specified at T. Equation 2.1 now becomes:

[1]Since we confine our attention to strategic debt service in this paper, we will use the terms strategic default and strategic debt service interchangeably.

$$1 = Pe^{-cT} \tag{2.5}$$

2.1.1 Remedies in the event of default

Loan contracts typically include a number of indentures which specify remedies available to the lender in the event of a breach of the contract (default) by the borrower. We confine our attention to the following remedies:

Foreclosure

A fundamental indenture contained in all loan contracts is meeting the currently scheduled payment of interest and principal. If the borrower fails to make these payments in a timely fashion he is deemed to be in default. Default entitles the lender to foreclose and seize the collateral. If the value of the collateral net of foreclosure costs, exceeds the outstanding loan balance, the lender is obligated to return this surplus to the borrower. Foreclosure always implies the termination of the loan contract.

Penalty rates

Default does not force the lender to foreclose. She may be willing to defer the payment in question to some later time. In such cases, the lender may apply a 'penalty' rate of interest, equal to or perhaps greater than the contractual rate, to any overdue debt service payments until such time as the payments are brought up to date. The penalty rate w to be applied is specified in the contract. Let $k(t)$ represent the balance of outstanding debt service payments at t. Over the term of the loan the change in this balance is

$$\mathrm{d}k(t) = [wk(t) + p - p^*]\,\mathrm{d}t \tag{2.6}$$

where p^* represents the continuous payment stream offered by the borrower in lieu of the contractually specified stream, p.[2] So, if the borrower does not make any payments over the term of the loan, and the loan is not terminated prior to maturity, the outstanding balance at maturity will be:

[2] In the analysis which follows we assume that $k(t) \geq 0$. In other words, if the borrower offers $p^* > p$ when $k(t) = 0$, the contractual balance $b(t)$ is adjusted as if $p^* = p$.

$$b(T) + k(T) = P + p/w(e^{wT} - 1) \tag{2.7}$$

This provides the lender with an alternative remedy to foreclosure when the borrower becomes delinquent in his payments.

Technical default

Loan contracts often afford the lender the option to declare a 'technical default' under circumstances specified in the contract. For example, the contract may specify that the borrower is in default whenever the collateral value falls below some predetermined proportion of the outstanding loan balance even though the borrower has made all contractual payments to date. In the event of such technical default the lender may foreclose. We assume that the technical default provision, if it is included, allows the lender to foreclose whenever the value of the collateral falls below the contractual loan balance, $s(t) < b(t)$.[3] This indenture, if it is present in the contract, may allow the lender to extract a payment flow p^*, from the borrower which *exceeds* the contractual flow p, in certain circumstances.

2.1.2 Additional indentures

Loan contracts often contain indentures which prevent the borrower from undermining the lender's claim on the underlying collateral. For example, the contract may prohibit the issuance of any additional claims on the collateral. In the case where the collateral is the assets of the firm, the loan contract may deny the borrower the option of issuing additional debt or equity.[4] In the analysis which follows we model this indenture by imposing a 'cash flow' constraint on the borrower's debt servicing choices: all debt service payments must be financed by the cash flows generated by the underlying collateral when this indenture is present in the contract.

[3]Technical default provisions are often significantly more onerous from the borrower's point of view. Default ratios in the neighbourhood of 1.5 times the collateral value are common.

[4]In what follows we will demonstrate that restrictive covenants of this nature have far reaching implications for the behaviour of the borrower after the contract is in place, and consequently for the value of the lender's claim. Denying the borrower the option of issuing additional debt or equity to finance scheduled debt service payments on the original loan, in times of financial distress, may not seem to be in the interests of the lender. We will demonstrate that, under certain assumptions, this restriction actually enhances the value of the lender's claim.

2.1.3 The borrower's options

In addition to his default option, the borrower has the option to pay off the loan prior to maturity by making a payment equal to the outstanding balance. If the borrower exercises this option, we assume he incurs transaction costs, of negotiating a new loan to refinance the old $f[b(t)]$, in addition to incurring a new obligation with a market value equal to the outstanding balance $b(t)$.[5] Prepayment is rational in instances where the value of the collateral has risen sufficiently since the origination of the loan to render the loan less (default) risky. The borrower is now paying a premium, $c - r$, over the risk free rate which is consistent with greater default risk. A new lender would be willing to accept a smaller premium. If the benefit of the lower premium over the remaining term of the loan exceeds the refinancing costs, the borrower will exercise this option.

The prepayment option is generally viewed as being detrimental to the lender. In the absence of explicit compensation to the lender in the event of prepayment, one may expect to observe contracts which expressly deny this option to the borrower.[6] However, the legal enforceability of such a provision is not clear (Jones, 1995, p.5f). Unlike the prepayment or refinancing of fixed rate loans that occurs when the general level of interest rates has declined, the lender has a far more onerous burden of proof in claiming that prepayment in the face of an increase in the market value of the collateral is damaging. After all, the lender was charging a premium to compensate for the possibility of default. If default is now less likely, why should the lender continue to receive the risk premium? Thus, whether or not the contract specifically provides for, or prohibits, this option, it may be available to the borrower.

If the borrower is effectively constrained in his prepayment behaviour, he may choose to default in circumstances where he would otherwise prepay. Terminating default may be rational when the credit spread is sufficiently large. Thus, default at 'high' collateral values is a (costly) substitute for prepayment.[7]

[5] We assume a competitive loan market.

[6] One of the surprising results of our analysis is that in the presence of certain indentures, a prepayment option actually *benefits* the lender.

[7] We are assuming that the refinancing costs associated with prepayment are lower than the foreclosure costs which would be imposed on the borrower in the event of terminating default at high collateral values.

2.2 The collateral

Let $s(t)$ be the equilibrium market value of the collateral at time t. Assume this value follows a continuous Markov process over time:

$$ds(t) = \alpha(s,t)dt + \sigma s(t)dz(t) \tag{2.8}$$

where $z(t)$ is a standard Brownian motion, σ is a constant volatility parameter and $\alpha(s,t)$ is the expected instantaneous drift in s.[8] The collateral generates a continuous dividend flow at the rate $d(s,t)$ which accrues to the borrower provided that foreclosure has not occurred. In the event of foreclosure, the lender seizes the collateral, incurring foreclosure costs $l(s,t)$. If the market value net of foreclosure costs exceeds the outstanding balance

We assume that $s(t)$ is costlessly and continuously observed by both parties to the contract.

2.3 The market

We assume that the borrower and lender have access to a market in which they can construct a transaction cost-free hedge against s-risk. Such a market is said to be dynamically complete with respect to s-risk. At each instant there exist securities or portfolios of securities that are locally perfectly correlated with s, allowing either party to hedge against the random variations in s. For example, if s is the value of the assets of a borrowing firm, risk of fluctuation in their value might be hedged by selling short shares of publicly traded firms in the same industry (Jones, 1995, p.4).

In addition, both parties can trade in default free bonds that provide a constant continuously compounded yield of r per year.[9]

Working in a dynamically complete market setting with symmetric information allows

[8] For the stochastic differential equation (4) to describe a unique stochastic (Ito) process, $\alpha(s,t)$ and $\sigma(s,t)$ must be Borel measurable and satisfy Lipschitz and growth conditions (see Duffie, 1988, p.225).

[9] For loans of moderate duration the loan contract described here is roughly equivalent to a floating rate contract with a constant 'credit spread', $c - r$. Contractual payments would be adjusted as r changes to maintain the same balance schedule $b(t)$ as in the fixed rate case. It seems contradictory to assume that the borrower can borrow elsewhere at default free rates. For the arbitrage valuation argument which follows, we require that the party in control of the default option is in this situation. This party could be the borrowing firm itself provided that the collateral supporting the loan is only some part of the firm's assets, and the lender does not have recourse to the remaining assets. Alternatively, the shareholders of the firm, protected by limited liability, may be in control of the default decision (Jones, 1995, p.4f).

one to obtain equilibrium option exercise strategies and contract values that are independent of the risk attitudes, personal circumstances and expectations about future collateral value of the contracting parties. It enforces consistency between collateral characteristics such as cash flows and capital appreciation. It also facilitates a tractable analysis of the welfare implications of the various contractual indentures referred to here (Jones, 1995, p.3).

Chapter 3

The games borrowers and lenders play

Once the debt contract is established, the borrower and lender engage in a noncooperative game in which they choose strategies to maximize the values of their claims. Given the stochastic process for $s(t)$, we describe a continuous time stochastic game of perfect information.[1] The players have complete information with respect to the environment (i.e. the stochastic process governing collateral values and the 'history' of collateral values to the present time), their payoffs and the game itself.

The game is essentially one of ongoing contract renegotiation, in which the agents attempt to deviate from the terms of the agreement whenever it is advantageous to do so. We develop a number of variations on the following basic subgame. At every point in time, the borrower exercises choice over the instantaneous debt service flow which he offers the lender, p^*. The borrower makes this offer with full knowledge of the rational response which it will induce from the lender. The lender's rational response maximizes the value of her claim, given the borrower's offer and the indentures of the loan contract. For example, if the offer falls short of the contractual flow, p, the borrower is in default. Default entitles the lender to foreclose or to invoke other remedies afforded her by the contract. The contract may also

[1]The basic property of stochastic games is that the history of the game at each point in time can be summarized by a 'state'. Current payoffs depend on this state and on current actions. (Fudenberg and Tirole, 1991, p.503). Continuous time stochastic games are known as 'differential games' since the evolution of the state variables are described by differential equations. Perfect information implies that all information sets in the extensive form of the game are singletons. In other words, players 'move' sequentially and their actions are observed before the next move occurs (Fudenberg and Tirole, 1991, pp.72-73).

entitle the lender to foreclose, under certain circumstances, in the absence of default on the part of the borrower, or in cases where the borrower is in default due to some prior breach of the contract which has not yet been remedied (see section 2.1). Thus the lender's response to the borrower's offer determines whether or not the game continues and the payoffs to the agents.

We restrict our analysis to the Markov perfect equilibria of the game. We find these equilibria by restricting the strategy space of the players to the set of 'Markov' or 'state-space' strategies in which the past influences current play only through its effect on a finite number of state variables that summarize the direct effect of the past on the current environment. In other words, the past matters only to the extent that it directly affects the current payoffs of the players.[2] A Markov perfect equilibrium is a profile of Markov strategies for the players that yields a Nash equilibrium in every proper subgame (Fudenberg and Tirole, 1991, p.501). Each player's choice of an optimal strategy is a control problem in which the player takes into account the influence of his actions on the state, both directly and indirectly through the influence of the state on the strategies of the player's opponent.

Modeling noncooperative games in continuous time can present subtle difficulties (see Fudenberg and Tirole, 1991, pp.118-119). We heed the advice of Fudenberg and Tirole (1985) by describing the equilibrium of a discretized version of the game and then take limits as the time interval goes to zero. Discretization allows the specification of the sequence of moves by the agents in a coherent fashion.

We divide the time to maturity of the loan contract into a number of small intervals, each of length dt. At the start of every interval, the borrower offers to service the loan at a rate of p^* for the duration of the interval which implies a payment of $p^* dt$.[3] Similarly, the contractual payment for the interval is pdt. No further action is taken by either agent until the start of the next time interval when the borrower makes a new offer.

We develop two classes of Markov games. First we describe one state variable games in which the market value of the collateral $s(t)$, is the only state variable. Then we describe games of two state variables in which we add a second state variable which captures some

[2]In games of repeated play, past play may influence current and future strategies, not because it has a direct effect on the environment, but rather because players believe that the past matters in some way. By restricting the strategy space to Markov strategies we ignore such beliefs. In other words, different histories of the state of the game which have a common current state are assumed to imply the same payoffs for the players for any given set of current actions.

[3]For the purposes of the description of the game, the payment, appropriately discounted, can be made at any point in time during the interval, or it may be paid continuously over the interval.

aspects of the history of the game in a 'payoff relevant' fashion. In each case we impose the indentures of the stylized loan contract, described in section 2, on the borrower and lender.

3.1 One state variable games

We assume that the current values of s and t embody all relevant information upon which the current actions of the lender and borrower are based. In particular the borrower's choice of the debt service flow can be expressed as $p^*[s(t), t]$. There is no scope here for past play to influence current payoffs. At any point in time the state of the game is determined by the current realization of $s(t)$ and the current actions of the players.

For a finite term loan, the state space $\mathsf{S} \times \mathsf{T}$, where $\mathsf{S} \equiv [0, \infty)$ denotes the range of values for s, and $\mathsf{T} \equiv [0, T]$, denotes the range of values for t, contains all possible states for the players strategies.[4] A strategy constitutes the specification of a number of regions or closed subsets in $\mathsf{S} \times \mathsf{T}$ in which specific actions are taken by the player.[5] For example, the borrower defaults whenever $(s, t) \in \mathsf{D}$, where D is a closed subset of $\mathsf{S} \times \mathsf{T}$. His prepayment policy, P, is another closed subset of $\mathsf{S} \times \mathsf{T}$. The lender forecloses whenever $(s, t) \in \mathsf{F}$, where $\mathsf{F} \subset \mathsf{S} \times \mathsf{T}$. Similarly, any other actions which the contract may afford the players may be represented by closed subsets of $\mathsf{S} \times \mathsf{T}$.

The loan contract is terminated whenever foreclosure or prepayment occurs, or when the maturity date is reached. The boundaries of F and P are referred to as the termination boundaries of the game, and the regions themselves are the termination regions. The open subset of $\mathsf{S} \times \mathsf{T}$ in which the loan contract is not terminated (the complement of $\mathsf{F} \cup \mathsf{P}$) is referred to as the continuation region, C.

Let $\Omega_L(s, t)$ and $\Omega_B(s, t)$ represent the termination values of the lender's claim and borrower's claim, respectively. The continuation value of the lender's claim, $L(s, t)$, is simply the value to the lender of the remaining cash flows from the loan if the collateral value at time t is s and the loan has not been terminated at an earlier date. $L(s, 0)$, the value of the lender's claim at the loan origination date, represents the maximum amount of credit that the lender would extend to the borrower in exchange for the promised sequence of contractual payments. Similarly, $B(s, t)$ represents the continuation value of the borrower's

[4]For infinite horizon cases, $T = \infty$, players' strategies depend only on s. In this case the game is said to be 'stationary' (Fudenberg and Tirole, 1991, p.521).

[5]For the valuation problems to be solved by the borrower and lender over the course of the game it is necessary that these sets be closed subsets of $\mathsf{S} \times \mathsf{T}$.

position, taking into account his options under the contract, assuming the contract has not yet been terminated.

In discretized form the continuation values of the claims may be expressed as

$$\begin{aligned} L(s,t) &= p^*[s(t),t]\mathrm{d}t + \mathrm{E}_t^Q\Big[L(s+\mathrm{d}s, t+\mathrm{d}t)\Big]e^{-r\mathrm{d}t} \\ &= p^*[s(t),t]\mathrm{d}t + L^-(s,t) \end{aligned} \tag{3.1}$$

$$\begin{aligned} B(s,t) &= \Big[d(s,t) - p^*[s(t),t]\Big]\mathrm{d}t + \mathrm{E}_t^Q\Big[B(s+\mathrm{d}s, t+\mathrm{d}t)\Big]e^{-r\mathrm{d}t} \\ &= \Big[d(s,t) - p^*[s(t),t]\Big]\mathrm{d}t + B^-(s,t) \end{aligned} \tag{3.2}$$

In the above E^Q is the expectation operator under the equivalent martingale - or risk adjusted probability measure, Q. Since the values of the claims depend on future realizations of s, they are uncertain. The assumption that markets are complete with respect to s-risk allows us to assume that the borrower and lender evaluate future payoffs or cash flows using the same martingale equivalent probability measure (see Harrison and Kreps, 1978). $L^-(s,t)$ and $B^-(s,t)$ are respectively the 'ex debt service' and 'ex dividend' values of the claims. The default free instantaneous interest rate , r, is the discount rate.

Given the assumptions specified in sections 2.2 and 2.3, the standard arbitrage or replication arguments of contingent claims pricing imply that $L(s,t)$ and $B(s,t)$ satisfy the following stochastic partial differential equations in C when $\mathrm{d}t \to 0$[6]

$$\frac{1}{2}\sigma^2 s^2 L_{ss} + [rs - d(s,t)]\, L_s + L_t + p^*[s(t),t] = rL \tag{3.3}$$

$$\frac{1}{2}\sigma^2 s^2 B_{ss} + [rs - d(s,t)]\, B_s + B_t + d(s,t) - p^*[s(t),t] = rB \tag{3.4}$$

With the exception of the $p^*[s(t),t]$ term on the left-hand side of both equations, these equations are the standard partial differential equations which emerge repeatedly in the valuation of claims contingent on a state variable which follows the Markov process described in equation (2.8). A heuristic derivation of these equations is provided in appendix 1.[7]

[6]The arguments of $L(s,t)$ and $B(s,t)$ are suppressed in the equations.

[7]Duffie (1988, sections 15 and 21) is one of many sources for a rigorous derivation of the partial differential equations used in the valuation of contingent claims.

Many solutions exist for these equations. Invoking the appropriate boundary conditions at maturity and the so-called 'free-boundary' conditions which must hold on the termination boundaries of $S \times T$, allows us to select the appropriate solutions for the players' optimal control problems. These boundary conditions will be determined by the specific indentures of the contract, and the restrictions imposed on the strategy space of the borrower and lender.

The free boundary conditions which characterize optimal policies, and determine the sets D, F and P are termed 'value matching' and 'high contact' or 'smooth pasting' conditions (Dixit, 1993).[8] The value matching condition requires that the continuation value and the termination value of a particular claim be equal on the boundaries of the termination regions. The smooth pasting condition requires that the first derivative in the s direction of the value function of the option exerciser be continuous on the boundary of these sets. For example, suppose that the borrower is in control of termination of the game along a particular boundary, $\underline{s}(t)$. The value matching condition implies $B(\underline{s}, t) = \Omega_B(\underline{s}, t)$, and the smooth pasting implies $B_s(\underline{s}, t) = \partial\Omega_B(\underline{s}, t)/\partial s$. This calculation assumes that the strategies followed by the players are fixed. Consequently, it determines a subgame perfect Nash equilibrium in the Markov strategies which is characteristic of a Markov perfect equilibrium.

Equations (3.3) and (3.4) constitute the continuous time representation of the solutions to the claim values, $L(s, t)$ and $B(s, t)$, for a general class of one state variable games. This formulation gives us much of the facility of contingent claims analysis while at the same time allowing us to build on game theoretic modeling of financial distress and contract renegotiation. The same general solution techniques are applicable to a variety of problems. We may consider a number of variations on the game. In each case we solve the same partial differential equations. All that changes from one case to another will be the specification of $p^*[s(t), t]$ and the boundary conditions.

We consider two versions of the one state variable game. First, we describe a benchmark case in which the scope for the borrower and lender to behave strategically to effectively renegotiate the terms of the loan contract, is limited. Our approach here is very similar to that of Jones (1995). Then we expand the scope for strategic behaviour, particularly for the borrower, proceeding along the lines of Anderson and Sundaresan (1996) and Anderson, Sundaresan and Tychon (1996).

[8] Samuelson (1965) seems to be the first person to have coined the phrase 'high contact' in his pioneering efforts in this area.

3.1.1 Terminating default

We proceed by initially ignoring the technical default provision. We assume that the lender does not entertain any attempt by the borrower to alter the terms of the contract. In particular, the lender *always* forecloses whenever the borrower offers $p^* < p$ for any $t < T$, or if he offers $P^* < P$ at T. Thus default is 'terminating' in that it forces foreclosure which terminates the game. There is no scope here for the borrower to explore the possibility of offering the lender payments which, while they fall short of the contractual amounts, do not induce the lender to foreclose.[9] Thus default is always characterized by the borrower offering the lender a debt service flow of zero, while the debt service flow is always equal to the contractual payment flow in the continuation region.

$$p^*[s(t), t] = \begin{cases} 0 & \text{for}(s,t) \in \mathsf{D} \\ p & \text{for}(s,t) \notin \mathsf{D} \end{cases}$$

Since his control variable, $p^*[s(t), t]$, is binary, the borrower's control problem is reduced to an 'optimal stopping' problem. At every point in time he can either terminate the game (default or prepay) or continue (make the contractual debt service payment). The borrower has a clear 'first-mover advantage'. The lender cannot foreclose until the borrower defaults, in the absence of a technical default provision. Under these circumstances, $\mathsf{D} \equiv \mathsf{F}$. The lender 'chooses' an optimal strategy in name only. The foreclosure restriction together with the absence of a technical default provision means that her actions are completely determined for every $(s,t) \in \mathsf{S} \times \mathsf{T}$.

Consequently we can describe the Markov perfect equilibrium of the game in the case of a (finite) term loan, by restricting our attention to the borrower's optimal stopping problem. We consider the optimal actions for the borrower to pursue at maturity which determine the boundary conditions for his problem. Then we 'step back' through time considering his optimal actions until the origination date ($t = 0$) is reached. This allows us to describe the free boundary conditions in the state space which characterise his rational behaviour.

At maturity, the borrower offers a lump sum payment, P^* to offset the outstanding balance, $P = b(T)$. Since an offer of $P^* < P$ forces foreclosure, his rational offer is

$$P^* = \begin{cases} 0 & \text{for} \quad s(T) \leq P \\ P & \text{for} \quad s(T) > P \end{cases}$$

[9]Strategic default or strategic debt service is considered in the next version of the game.

So, at maturity there is a single, (lower) termination region, $(s,T) \in \mathsf{F}$ if $s(T) \leq P$. The values of the claims at T are

$$L(s,T) = \max\{0, s(T) - l(s,T)\} \qquad B(s,T) = 0 \qquad \text{for} \quad s(T) \leq P \tag{3.5}$$

$$L(s,T) = P \qquad B(s,T) = s(T) - P \qquad \text{for} \quad s(T) > P \tag{3.6}$$

The lower termination region extends back from T to the loan origination date, $t = 0$. Default occurs in this region when the continuation value of the borrower's claim is driven to zero. Since default forces foreclosure, the termination values of the claims in this region are

$$\begin{aligned} \Omega_L(s,t) &= \max\{0, s(t) - l(s,t)\} \\ \Omega_B(s,t) &= 0 \end{aligned} \tag{3.7}$$

Note that the value of the lender's claim is never less than zero. This follows from the assumption that the lender can abandon the collateral if the foreclosure costs exceed its market value. The value of the borrower's claim is also never less than zero since his liability under the loan contract is limited to the market value of the collateral.

The boundary of this region, $\underline{s}(t)$, is also the lower termination boundary for the game. On this boundary the value matching condition for the borrower's problem, $B(\underline{s},t) = \Omega_B(\underline{s},t)$, implies

$$[d(\underline{s},t) - p]\,dt + B^-(\underline{s},t) = 0 \tag{3.8}$$

$B(s,t)$ is strictly positive whenever the dividend flow from the collateral exceeds the contractual debt service flow, $d(s,t) > p$, since $B^-(s,t) \geq 0$.[10] Thus any (s,t), such that $d(s,t) > p$, is not an element of the lower default region since it is not rational for the borrower to default under these circumstances.

It is rational for the borrower to continue servicing the debt when the dividend flow from the collateral falls short of the contractual payment flow, $d(s,t) < p$, if the ex dividend

[10] This is a consequence of the borrower's limited liability under the loan contract.

value of his claim is sufficiently large, $B^-(s,t) > [p - d(s,t)]\mathrm{d}t$.[11]

Prior to maturity, $\underline{s}(t) \leq b(t) + l(\underline{s},t)$, since there is some finite probability that the value of the collateral will recover sufficiently such that $B(s,T) > 0$.[12] Thus, $B(s,t) > 0$ for $\underline{s}(t) < s(t) < b(t) + l(s,t)$, even though the borrower would receive nothing in the event of default and foreclosure. The borrower's default decision problem is similar to the stopping problem faced by the holder of an American option. In the interval, $\underline{s}(t) < s(t) < b(t)+l(s,t)$, the 'intrinsic value' of the borrower's claim is zero, but the 'time value' is positive.

As we move back in time, sufficiently far away from T, a second termination region emerges for sufficiently 'high' values of the collateral if $c - r$ is sufficiently large. This upper region has a lower bound, $\overline{s}(t)$ which is also the upper termination boundary for the game.[13] As the collateral value rises, the probability of default diminishes. The credit spread, $c - r$, originally set when the collateral value was lower, now seems unwarranted. Faced with the prospect of making the high contractual payments for the remaining term to maturity of the loan contract, the borrower will choose to default or prepay the loan if his proceeds from doing so exceed the continuation value of his claim.[14] If the cost incurred by the borrower in negotiating a new loan to refinance the outstanding balance, $f[b(t)]$, is less than the foreclosure costs, $l(s,t)$, the borrower will prepay the loan instead of defaulting.[15] The termination values of the claims in this region are

$$
\begin{aligned}
\Omega_L(s,t) &= b(t) \\
\Omega_B(s,t) &= s(t) - \min\left\{l(s,t), f[b(t)]\right\} - b(t)
\end{aligned}
\tag{3.9}
$$

[11] The borrower may have other resources to draw on to finance the contractual payments in these circumstances, or, in the absence of appropriate contractual indentures, the borrower may be able to issue additional claims against the collateral.

[12] In fact, for 'reasonable' parameter values the boundary of the lower default region, $\underline{s}(t)$ can be significantly lower than the outstanding balance, $b(t)$ if there is sufficient time remaining to maturity.

[13] For valuation purposes we only consider foreclosure costs of the linear form, $l(s,t) = l_0 + l_1 s(t)$. In this case, the upper default region is a compact set for finite term loans, i.e.; there is an upper bound to the region. Since foreclosure costs are monotonically increasing in s, at sufficiently high levels of s, the foreclosure costs will exceed the benefits associated with termination in order to avoid the seemingly unwarranted credit spread.

[14] Since the foreclosure cost are a 'deadweight loss' form the point of view of the borrower and lender, a clear incentive exists for the parties to renegotiate the terms of the contract (the credit spread, in particular) as $s(t)$ approaches $\overline{s}(t)$. Such renegotiation is ruled out here. Prepayment or default are (costly) substitutes for renegotiation. In the strategic default version of the game, section 3.1.2, we allow for 'de facto' contract renegotiation via the strategic behaviour of the contracting parties.

[15] We assume that refinancing costs are of the form $f[b(t)] = f_0 + f_1 b(t)$, for valuation purposes. Since these costs are not increasing in s, the prepayment region will not have an upper bound.

On the boundary of the upper termination region, the value matching condition, $B(\bar{s},t) = \Omega_B(\bar{s},t)$, implies that the foreclosure- or refinancing costs incurred by the borrower in default are exactly equal to the present value of the extra cost associated with servicing the loan over the remaining term to maturity at the contractual rate c which is now greater than the fair market rate for the lower default risk.[16] The value matching condition may be expressed as

$$[d(\bar{s},t) - p]\,\mathrm{d}t + B^{-}(\bar{s},t) = \bar{s}(t) - \min\left\{l(\bar{s},t), f[b(t)]\right\} - b(t) \tag{3.10}$$

The borrower's optimal strategy is characterized by a termination set which consists of two disjoint regions or subsets in $\mathsf{S} \times \mathsf{T}$. The continuation region C is then defined by $\underline{s}(t) < s(t) < \bar{s}(t)$.

We summarize the borrower's optimal stopping problem in the following Bellman equation

$$B(s,t) = \max\left\{\Omega_B(s,t)\,,\, [d(s,t) - p]\mathrm{d}t + B^{-}(s,t)\right\} \tag{3.11}$$

where $\Omega_B(s,t)$ combines the termination values of the borrower's claim in the two termination regions

$$\Omega_B(s,t) = \max\left\{0, s(t) - \min\{l(s,t), f[b(t)]\} - b(t)\right\} \tag{3.12}$$

From the lender's perspective, default does not occur 'soon enough' along the lower termination boundary. The lender would always prefer the borrower to follow a strategy of defaulting at the last moment the loan could be fully repaid by the liquidated collateral, $\underline{s}(t) = b(t) + l(\underline{s},t)$. The borrower's rational behaviour of timing default so as to maximise the value of his claim, is detrimental to the value of the lender's claim.

Technical default

Under appropriate restrictions on the loan parameters, a technical default provision will remedy this situation to some extent by allowing the lender to pursue an 'active' foreclosure strategy for 'low' values of s. Recall that the technical default provision allows the lender to

[16] With sufficiently little time remaining to maturity the cost incurred in servicing the loan at a rate greater than the fair market rate for the reduced default risk will be less than the foreclosure costs incurred by defaulting. Thus, the upper stopping region and the upper stopping boundary do not extend to T for any s.

foreclose whenever $s(t) < b(t)$. Provided that the credit spread and the contractual payment flow are confined to 'reasonable' ranges, the boundary of the technical default region, $b(t)$, will lie above the boundary of the lower default region.[17] This allows the lender to foreclose at levels of $s(t)$ above those at which the borrower would choose to default, enhancing the value of her claim.[18]

For $\underline{s}(t) < s(t) < b(t)$, the lender solves the following optimal stopping problem[19]

$$L(s,t) = \max\{\Omega_L(s,t)\,,\, p\mathrm{d}t + L^-(s,t)\} \tag{3.13}$$

where the termination value of her claim is

$$\Omega_L(s,t) = \max\{0, s(t) - l(s,t)\} \tag{3.14}$$

Now the value matching condition for the lender on the lower termination boundary of the game, $\underline{s}(t)$, for some interval $[0,t]$, satisfies[20]

$$p\mathrm{d}t + L^-(\underline{s},(t)) = \max\{0, \underline{s}(t) - l(\underline{s},t)\} \tag{3.15}$$

Note that if we change the technical default provision such that the lender may foreclose whenever $s(t) < b(t) + l(s,t)$, the loan is effectively riskless. The lender always recovers the contractual balance in the event of foreclosure or prepayment. Since the riskless interest rate is assumed to be constant, there is no 'reinvestment risk' if the contract is terminated prior to maturity. In this case the lender would be willing to lend \$1 at the riskless rate, i.e., $c = r$.[21]

Figure 3.1 depicts the strategy space for the terminating default game. We assume that prepayment is preferred to terminating default for 'high' values of the collateral.

[17] By reasonable we mean values which are not too large. For example, for a 5 year loan, $c - r = 0.03$, $p = 0.1$, and $l(s,t) = .15s(t)$ will suffice.

[18] Under these conditions, $\mathsf{F} \not\equiv \mathsf{D}$.

[19] We assume here that the borrower continues to offer a debt service flow of p in this region since it lies 'outside' his lower default region. There is however, an incentive for the borrower to consider offering debt service flows which exceed p in an attempt to stave off foreclosure if this enhances the value of his claim. We allow for this in the strategic default game.

[20] The borrower is no longer in control of the termination of the game along the lower termination boundary so long as the technical default boundary lies above his lower default boundary. Thus, the lower boundary is no longer a 'free boundary' for the borrower.

[21] We are able to generate numerical results, for the case of a term loan, based on the methods described in chapter 5 which are consistent with this observation.

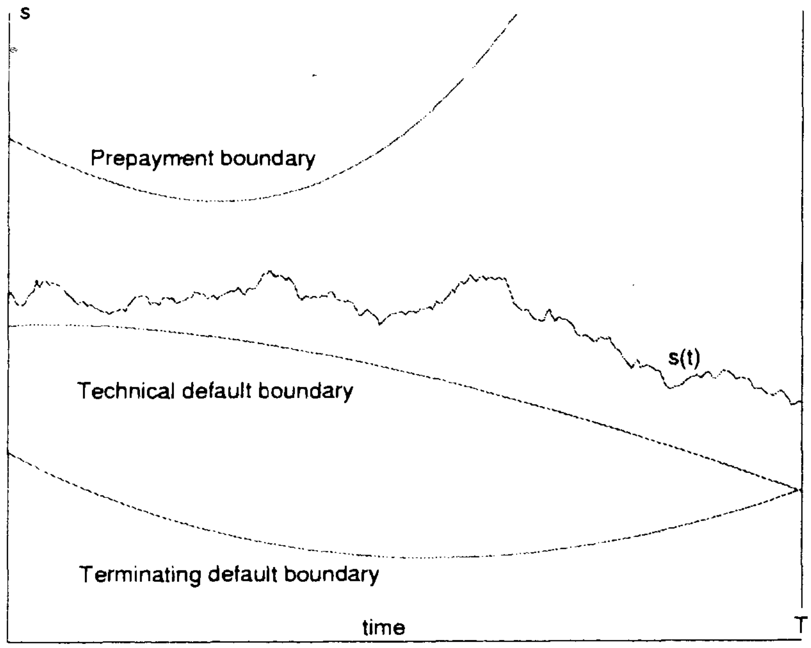

Figure 3.1: Strategy space - Terminating Default

This completes the description of the Markov perfect equilibrium for the terminating default game in the case of a term loan. For perpetual loans similar reasoning applies except that we simply need to specify the boundaries of D and P in terms of s for any t, as these boundaries are invariant with respect to t.[22]

3.1.2 Strategic default

We relax the assumption that the lender always forecloses in the event of default. Default merely 'activates' the lender's foreclosure option. The lender will not foreclose if doing so does not increase the value of her claim. This allows the borrower to explore the possibility of offering the lender a debt service flow which falls short of the contractual payment flow and hence implies default, but does not induce foreclosure. Thus, we allow for deviations from the terms of the original contract, or ongoing contract renegotiation.

We assume that if the lender accepts a debt service offer which is less than the contractual payment she surrenders any claim on the unpaid amount. In other words the outstanding

[22] In the case of a perpetual loan, the control problems for the borrower and lender are stationary.

balance, $b(t)$, is adjusted as if the full contractual payment had been made. Hence the contract is effectively renegotiated, in favour of the borrower, whenever such an offer is accepted. We refer to this strategic behaviour on the part of the borrower with respect to the debt service flow as strategic default.

As before we proceed by initially ignoring technical default. In the absence of a technical default provision the lender can never foreclose if the borrower offers the contractual instantaneous debt service flow, p. Since the continuation value of the borrower's claim is strictly decreasing in p^*, his debt service offer will never exceed p for $t < T$. Similarly, his offer at maturity, P^*, will not exceed P, the contractual balance at maturity.

We describe the Markov perfect equilibrium for a term loan. At maturity the borrower offers a lump-sum payment in lieu of the outstanding balance, $P = b(T)$. The borrower offers the smallest payment, P^*, which does not provoke foreclosure

$$P^* = \min\left\{P, \max\{0, s(T) - l(s,T)\}\right\} \tag{3.16}$$

This implies a single default region at maturity, $(s,T) \in \mathrm{D}$ if $s(T) \leq P + l(s,T)$.[23] Consequently the values of the claims at maturity are

$$\begin{aligned} L(s,T) &= P^* \\ B(s,T) &= s(T) - L(s,t) \\ &= \max\{l(s,T), s(T) - P\} \end{aligned} \tag{3.17}$$

For $P > s(T) - l(s,T) > 0$, the borrower avoids foreclosure by offering the lender an amount equal to what she would receive if she liquidated the collateral, $s(T) - l(s,T)$. This allows the borrower to retain $l(s,T)$, the amount which would be dissipated if foreclosure occurred. If $s(T) - l(s,T) \leq 0$, the borrower retains $s(T)$, while the lender receives nothing.[24] Clearly it is never rational for the borrower to provoke foreclosure at maturity.

For $t < T$, the rational strategies of the players are based on similar reasoning. Since foreclosure imposes a 'dead-weight' loss on the borrower, he never induces foreclosure along the equilibrium path of the game. Similarly, for high collateral values, where prepayment was

[23]This default region is *not* a termination region for the game.

[24]We assume here that since the lender has nothing to gain by foreclosing, she does not foreclose. In a setting in which borrowers and lenders have occasion to enter into contracts repeatedly over time, this assumption may not be reasonable. Lenders may foreclose with nothing to gain to temper borrowers' incentive to behave strategically in future contracts.

rational in the terminating default game, the borrower now prefers to avoid the refinancing costs associated with prepayment by engaging in strategic default instead. In other words, ongoing debt renegotiation in favour of the borrower is preferred to prepayment. For every time interval in the discretized form of the game, there exists a critical instantaneous debt service flow, $\hat{p}$ which leaves the lender indifferent between foreclosing and allowing the loan to continue until the start of the next time interval

$$\hat{p}[s(t), t]\mathrm{d}t = \max\{0\, , \, \Omega_L(s, t) - L^-(s, t)\} \tag{3.18}$$

where $\Omega_L(s, t)$ represents the value of the lender's claim if she forecloses

$$\Omega_L = \min\Big\{b(t), \max\{0, s(t) - l(s, t)\}\Big\} \tag{3.19}$$

To determine this critical level of the instantaneous debt service flow, the borrower must take into account the value to the lender of the subgames along which the contract is not terminated. The borrower evaluates future payoffs to the lender using the martingale equivalent probability measure, Q.

Suppose at t, $s(t)$ is realized. The borrower's optimal debt service offer for the next interval, $\mathrm{d}t$, is:

$$p^*[s(t), t]\mathrm{d}t = \min\{p\mathrm{d}t\, , \, \hat{p}\mathrm{d}t\} \tag{3.20}$$

The continuation values of the claims are again expressed in equations (3.1) and (3.2).

As we move back in time from T, we observe the emergence of two disjoint default regions.[25] Within these regions, $p^*[s(t), t] < p$. Strategic default, which does not induce foreclosure, is preferred to terminating default which terminates the contract. The lender is willing to accept debt service flows below the contractual flow since the probability of (terminating) default diminishes as $s(t)$ increases. Unlike the upper termination region in the terminating default game, this region extends to T. As the term to maturity declines, the lower boundary of the upper default region, $\bar{s}(t)$, declines as well. With less time remaining to maturity, the risk of the collateral value deteriorating before maturity, becomes smaller and the lender is willing to accept progressively smaller debt service flows without foreclosing.

[25]The upper default region only exists if the credit spread, $c - r$, is positive.

Prepayment, if permitted, may still occur just below the lower bound of the upper default region, but at significantly higher levels of $c - r$ than in the terminating default game. Again, it may be rational for the borrower to incur the refinancing costs associated with prepayment rather than to pay the high credit spread over the remaining term of the loan.

In the lower default region the borrower is able to offer a debt service flow less than the contractual flow due to the low collateral value, and consequently, the low value of the lender's claim in foreclosure. The presence of foreclosure costs would further reduce the value of the lender's claim. This adds to the borrower's ability to 'extract' value from the lender.[26] As the term to maturity diminishes, the upper boundary of the lower default set increases. With less time remaining to maturity the probability that the collateral value will recover diminishes, lowering the ex debt service value of the lender's claim. Progressively larger debt service flows are required to keep the lender from foreclosing.

So, the borrower's choice of the instantaneous debt service flow solves the following control problem[27]

$$B(s,t) = \max_{p^*} \left\{ [d(s,t) - p^*]\mathrm{d}t + B^-(s,t) \right\} \tag{3.21}$$

Technical default

The presence of a technical default provision alters the borrower's strategy. Assume again that the lender can foreclose if $s(t) < b(t)$, *irrespective* of the debt service flow offered by the borrower. To avoid foreclosure in the technical default region the borrower's offer must now satisfy

$$p^*[s(t),t]\mathrm{d}t = \hat{p}\mathrm{d}t \tag{3.22}$$

Again, if the technical default boundary lies above the boundary of the lower default region, $\underline{s}(t) < b(t)$, over some range of $[0,T]$, the borrower may be able to offer the lender

[26]While positive foreclosure costs enhance the scope for strategic default or strategic debt service when the collateral value is low, they are not necessary for strategic default. In the absence of foreclosure costs, the borrower can avoid foreclosure at low collateral values by offering the lender the entire dividend flow from the collateral if the dividend flow is less than the contractual debt service flow. From the lender's point of view, receiving the dividend flow is just as good as owning the collateral. Of course, the borrower prefers this strategy to inducing foreclosure since there is some positive probability that the collateral value will recover.

[27]The borrower now has a continuous control variable, $p^*[s(t),t] \in [0,p]$ which he chooses at every point in time to maximize the value of his claim.

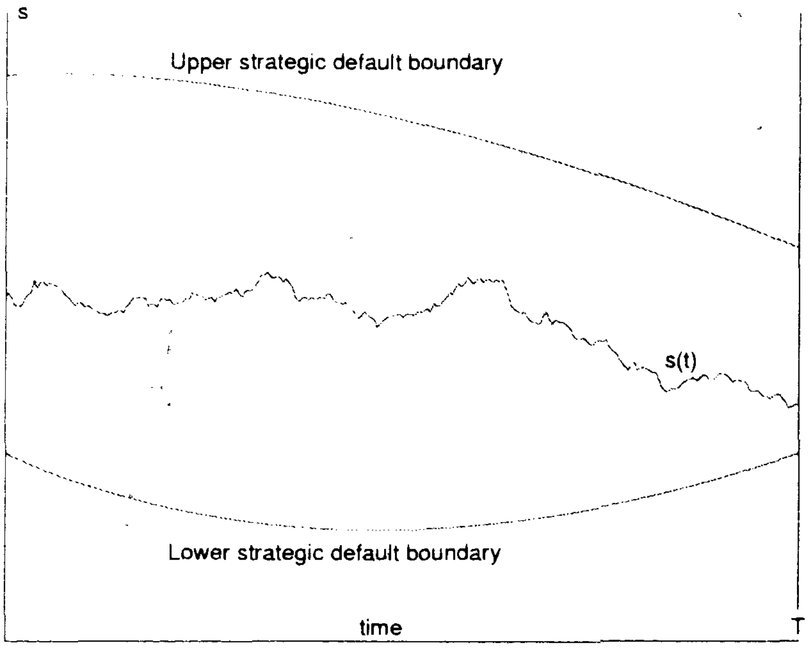

Figure 3.2: Strategy space - Strategic Default

debt service flows which exceed the contractual flow to prevent foreclosure whenever $s(t)$ satisfies $\underline{s}(t) < s(t) < b(t)$. We assume that any offered debt service flow in excess of the contractual flow is not reflected in the contractual balance $b(t)$. The balance continues to be adjusted as if the contractual debt service payments are being made. Thus,the technical default provision allows for renegotiation of the contract in favour of the lender.

The case of a perpetual loan is similar except that the boundaries of the default regions remain unchanged through time as the borrower and lender solve stationary control problems.

Figure 3.2 depicts what the strategy space for the terminating default game.

This completes the construction of a Markov perfect equilibrium for the strategic default game. The most important feature is that strategic default does not provoke foreclosure and the borrower never exercises his prepayment option. This can be interpreted as the outcome of the ongoing negotiation process between borrower and lender over the term of the loan.

The absence of foreclosure along the equilibrium path of the strategic default game is troublesome in that it is plainly unrealistic. In reality foreclosure does occur. Anderson and Sundaresan (1996) obtain foreclosure in equilibrium in some states by assuming that debt service flows must be financed by the dividend flow generated by the collateral. This implies the following restriction on the borrower's debt service offers

$$p^*[s(t), t] \in [0, d(s,t)] \quad \text{for all} \quad (s,t) \in \mathsf{S} \times \mathsf{T}$$

This can be viewed as a simple indenture designed to protect the lender's claim on the underlying collateral. In the case where the collateral is the assets of a firm, the indenture effectively prohibits the firm from issuing additional claims on the collateral (debt or equity) or selling assets to finance the debt service flow.

The dividend flow constraint is binding whenever $\hat{p} > d(s,t)$. In these states it is rational for the lender to foreclose. The precise effect of this indenture on the equilibrium strategies of the borrower and lender, and hence the location of the foreclosure set in the state space, $\mathsf{S} \times \mathsf{T}$, will depend on assumptions about the dividend flow, $d(s,t)$.[28] In cases where the constraint is binding in some subset of $\mathsf{S} \times \mathsf{T}$, the effect is to enhance the value of the lender's claim at the expense of the borrower.

3.2 Two state variable games

In the terminating default game, any concessions made by either party in the process of contract renegotiation are assumed to be irreversible. For example, suppose that, for some time, the collateral value deteriorates significantly and the borrower successfully negotiates a debt service flow smaller than the contractual flow over this period. However, after this period the collateral value recovers such that the terms of the original contract become binding once more. The contract does not provide for the lender to recover any of the concessions which she made to the borrower over this interval.

We relax the assumption of irreversible concessions for the lender.[29] Now the lender may agree to renegotiated terms with the understanding that in the event that circumstances reverse themselves, she will have some recourse to recover any concessions which she made. In particular, we assume that the loan contract includes an indenture which allows the

[28] Of course, if the loan contract were a pure discount note ($p = 0$), the dividend flow constraint would have no effect on the strategies of the borrower and lender. Also, if the dividend flow were sufficiently large the constraint would never binding.

[29] We continue to assume that any concessions made by the borrower, such as offering $p^* > p$, in the technical default region are irreversible.

lender to add outstanding debt service payments, which originate from concessions made, to the contractual loan balance. Furthermore, the contract may provide for a 'penalty' rate of interest to be applied to these outstanding amounts. The purpose of this feature of the contract is to temper the borrower's incentive to service the debt strategically, increasing the value of the lender's position. The lender has a claim to these outstanding payments at maturity or at the time that default or foreclosure occurs.

3.2.1 Penalized default

We model the behaviour of the borrower and lender within this revised negotiating environment by developing a game in which past play has a bearing on the current actions chosen by the borrower and the lender. Let $k(t)$ represent the balance of outstanding debt service payments at t. Over the time interval dt, the change in this balance is given by equation 2.6 on page 9.

We retain the Markov property of the games to be described by assuming that the current values of s, k and t embody all relevant information upon which the current actions of the borrower and lender are based. So, at any point in time the state of the game is determined by the current values of the state variables, $s(t)$ and $k(t)$, and the current actions of the borrower and lender. In particular the borrower's choice of the instantaneous debt service flow can be expressed as $p^*[s(t), k(t), t]$.

For a term loan, the state space is now $\mathsf{S} \times \mathsf{K} \times \mathsf{T}$, where $\mathsf{K} \equiv [0, K]$. K represents the maximum value of outstanding debt service payments which can accumulate over the term of the loan.[30] Strategies are described by the location of the closed sets D, F and P in $\mathsf{S} \times \mathsf{K} \times \mathsf{T}$.

In discretized form, the value of the claims in C are

$$\begin{aligned} L(s,k,t) &= p^*[s(t),k(t),t]\mathrm{d}t + \mathrm{E}_t^Q\Big[L(s+\mathrm{d}s, k+\mathrm{d}k, t+\mathrm{d}t)\Big]e^{-r\mathrm{d}t} \\ &= p^*[s(t),k(t),t]\mathrm{d}t + L^-(s,k,t) \end{aligned} \tag{3.23}$$

$$B(s,k,t) = \Big[d(s,t) - p^*[s(t),k(t),t]\Big]\mathrm{d}t + \mathrm{E}_t^Q\Big[B(s+\mathrm{d}s, k+\mathrm{d}k, t+\mathrm{d}t)\Big]e^{-r\mathrm{d}t}$$

[30] If the borrower does not make any payments over the term of the loan, and the loan survives to maturity, the value of the outstanding debt service flow at maturity is $K = \max k(T) = p/w(e^{wT} - 1)$.

$$= \Big[d(s,t) - p^*[s(t),k(t),t]\Big]\,\mathrm{d}t + B^-(s,k,t) \tag{3.24}$$

By the arbitrage arguments which apply in our complete markets setting, $L(s,k,t)$ and $B(s,k,t)$ must satisfy the following stochastic partial differential equations in C when $\mathrm{d}t \to 0$[31]

$$\frac{1}{2}\sigma^2 s^2 L_{ss} + [rs - d]L_s + [wk + p - p^*]L_k + L_t + p^* = rL \tag{3.25}$$

$$\frac{1}{2}\sigma^2 s^2 B_{ss} + [rs - d]B_s + [wk + p - p^*]B_k + B_t + d - p^* = rB \tag{3.26}$$

We consider two versions of the foreclosure 'rule'. First, we assume the loan contract stipulates that the lender can only foreclose in the event of *current* default. Any past action by the borrower which constituted default at that time cannot be invoked for the purpose of foreclosure at the present time. In other words, the lender has the opportunity to foreclose at the time default occurs, but not thereafter.[32] Hence the lender accepts the renegotiated terms of the contract, for the next time interval if she chooses to accept a debt service flow which falls short of the contractual flow. This is identical to the foreclosure rule in the strategic default game. However, unlike the strategic default game, the lender does not 'forget' the default in the sense that the contract allows her to add the outstanding debt service payments, $k(t)$, to her claim on the collateral. We refer to this version of the foreclosure rule as the 'current default' foreclosure rule.

The second version of the default rule considered allows the lender to foreclose at any time, t, if $k(t) > 0$. The borrower must restore the balance of outstanding debt service flows, $k(t) = 0$, in order to 'deactivate' the lender's foreclosure option. We refer to this version of the foreclosure rule as the 'outstanding payment' foreclosure rule.

We proceed, again, by ignoring technical default for the moment. At maturity, the borrower offers a lump sum payment

$$P^* = \min\Big\{P + k(T), \max\{0, s(T) - l(s,T)\}\Big\} \tag{3.27}$$

This implies a single default region at maturity, $(s,T) \in \mathrm{D}$ if $s(T) \leq P + k(T) + l(s,T)$. The values of the claims at maturity are

[31] The arguments of the functions are suppressed in these equations.

[32] Unless, of course, default occurs at a later time again.

$$
\begin{aligned}
L(s,k,T) &= P^* \\
B(s,k,T) &= s(T) - L(s,k,t) \\
&= \max\{l(s,T), s(T) - P - k(T)\}
\end{aligned}
\tag{3.28}
$$

For $t < T$, the critical debt service flow which leaves the lender indifferent between foreclosing and allowing the loan to continue is

$$\hat{p}[s(t), k(t), t]\mathrm{d}t = \max\left\{0, \Omega_L(s,k,t) - L^-(s,k,t)\right\} \tag{3.29}$$

where, the foreclosure value of the lender's claim is

$$\Omega_L(s,k,t) = \min\left\{b(t) + k(t), \max\{0, s(t) - l(s,k,t)\}\right\} \tag{3.30}$$

Under the 'current default' rule, the lender cannot foreclose if $p^* \geq p$, whereas under the outstanding balance rule the lender cannot foreclose if $k(t) = 0$.

We state the borrower's optimal control problem as

$$B(s,t) = \max\left\{\Omega_B(s,k,t)\,,\, \max_{p^*}\left\{[d(s,t) - p^*]\mathrm{d}t + B^-(s,k,t)\right\}\right\} \tag{3.31}$$

The presence of a second state variable complicates the analysis of the rational behaviour of the borrower, for $t < T$, considerably. The continuation value of the borrower's claim (equation 3.24) is no longer monotonically decreasing in the instantaneous debt service flow offered, p^*. A 'low' debt service flow offer allows the borrower to retain a greater share of the dividend flow, increasing, ceteris paribus, the value of his claim. At the same time, however, the instantaneous rate of growth in $k(t)$ increases by the difference between the contractual flow and the offered flow, which lowers the ex-dividend value of his claim.[33] Thus, we can no longer assert that the borrower will always make the smallest offer which prevents foreclosure. Unlike the strategic default game it may now be rational for the borrower to engage in terminating default or prepayment or to induce foreclosure, even in the absence of a dividend flow constraint or other constraints on his debt service strategy. Consequently, in the absence of assumptions about the values of the parameters which define the precise

[33] It is possible for there to be more than one level of the debt service flow over the next interval which maximizes the value of the borrower's claim. In such cases we assume that the borrower makes the lowest offer.

nature of the loan contract, general propositions about the strategies of the borrower and lender are not forthcoming.

However, some observations can be made. For collateral values significantly higher than $b(t) + k(t)$ there is a high probability that the collateral value will be greater than $P + k(T)$ at maturity. At these high collateral values the borrower will not choose to offer $p^* < p$ as this would increase the amount owing at maturity. Thus a debt service offer lower than the contractual flow, if it is accepted by the lender, does not constitute renegotiation of the contract in favour of the borrower. In fact, if interest accrues at a 'penal' rate, $w > c$, on the outstanding debt service flows, then renegotiation of this kind favours the lender!

We defer further remarks pertaining to the properties of this version of the renegotiation game to the next chapter where we employ numerical methods to glean further insights.

Chapter 4

Analytical results

We consider the effects of the behaviour described in chapter 3 on the values of the claims of the borrower and lender, and the debt servicing strategies which emerge.

To facilitate valuation we make a number of 'time independence' assumptions which improve the tractability of the analysis. First, we assume that the instantaneous drift in the collateral value is time independent, $\alpha(s,t) = \alpha s(t)$

$$ds(t) = \alpha s(t)dt + \sigma s(t)dz(t) \tag{4.1}$$

Furthermore we assume that the dividend flow generated by the collateral, loan refinancing costs and bankruptcy costs are independent of time and are homogeneous of degree one in their remaining arguments; $d(s,t) = d_1 s$, $f(b) = f_1 b$ and $l(s,t) = l_1 s$. This allows the solutions obtained for $L(s,t)$ and $B(s,t)$ to be interpreted as the values of the agents' claims *per* dollar of credit extended at the contract origination date. These values are independent of loan scale and $s(t)$ may be interpreted as the collateral value per dollar of credit initially extended.

Analytical solutions to the linked partial differential equations (3.3), (3.4), (3.25) and (3.26) are generally not available. However, imposing either one of two additional assumptions does yield solutions if we ignore indentures such as prepayment, technical default and dividend constraints.

First, if the contracting agents are restricted to exercising their options at the contract maturity date only, then optimal exercise policies are described by single critical values of the collateral at maturity. The values of agents' positions are then easily determined. Second, if the contract does not have a maturity date (the loan is perpetual), then the strategies of the borrower and lender are invariant with respect to time, and can be described by constant critical values of the collateral. Again, some solutions are forthcoming.

In chapter 5 we relax these assumptions. We employ a finite difference procedure which allows us to consider cases in which the options available to the contracting parties may be rationally exercised prior to maturity. This numerical framework also allows us to consider the effects of prepayment and technical default options and dividend constraints which are often contained within the set of indentures in actual loan contracts.

We consider a number of stylized loan contracts.

4.1 Pure discount loans

Since a discount loan specifies a single contractual payment at T, the borrower will not rationally default at 'low' values of s prior to T. In the absence of regular debt service payments, the value of the borrower's claim can never fall below zero, even as $s(t) \to 0$. However, if the credit spread $c - r$ is sufficiently large, it may be rational for the borrower to default at 'high' collateral values for the reasons described in section 3.1.1.[1] In order to obtain closed from expressions for $L(s,0)$ and $B(s,0)$, we allow the borrower to exercise his 'high default' option at maturity only.

4.1.1 Terminating default

Given the assumptions with respect to the contract parameters and the constraints on the behaviour of the contracting parties, we now restate (3.6), the value of the lender's claim and the borrower's claim at maturity

$$L(s,T) = \begin{cases} \max\{0, (1-l_1)s(T)\} & \text{for} \quad s(T) \leq P \\ P & \text{for} \quad s(T) > P \end{cases}$$

$$B(s,T) = \max\{0, s(T) - P\}$$

The values of these claims at $t = 0$ are stated in the following proposition

[1] If the foreclosure costs are sufficiently large, it would not be rational for the borrower to exercise his 'upper default' option for a given credit spread. For example, if $l_1 = 1$, the borrower will never default prior to maturity for any $s(t)$ and any $c - r$, in the case of a pure discount bond.

Proposition 1 *If the borrower and lender play the terminating default game, foreclosure costs and dividend flows are proportional to the collateral value, and there are no debt service payments over the term of the loan, then the loan value is*

$$L(s,0) = (1-l_1)\left[s(0)e^{-dT} - c\,(s,T;P)\right] + l_1 P\tilde{c}(s,T;P)$$

while the value of the borrower's claim is

$$B(s,0) = s(0)(1-e^{-dT}) + c\,(s,T;P)$$

where $c(s,T;P)$ is the value of a European call option on the collateral with expiry date T and exercise price equal to P, and $\tilde{c}(s,T;P)$ is the value of a European 'digital' call option with the same terms.[2]

Proof of the proposition is in appendix A.2. The value of the lender's claim is simply the 'after foreclosure cost' value of the collateral 'stripped' of its dividend flow, net of the value of the borrower's call option on the collateral, plus the value of a digital call option on the collateral. This option appears in the value function due to the discontinuity in the lender's payoff at maturity if $l_1 > 0$. The value of the borrower's claim is simply the sum of a European call option on the collateral and the present value of the dividend stream generated by the collateral over the term of the loan.

The expected present value of the foreclosure costs, $F(s,0) = s(0) - B(s,0) - L(s,0)$, is

$$F(s,0) = l_1\left[s(0)e^{-dT} - c(s,T;P) - P\tilde{c}(s,T;P)\right] \tag{4.2}$$

In the absence of foreclosure costs, $F(s,0) = 0$, which implies that the sum of the claims equals the market value of the collateral.

4.1.2 Strategic default

The values of the lender's claim and the borrower's claim at maturity (3.17) are now

$$\begin{aligned} L(s,T) &= \min\{P, (1-l_1)s(T)\} \\ B(s,T) &= \max\{l_1 s(T), s(T) - P\} \end{aligned} \tag{4.3}$$

[2] A European digital option pays at maturity, one unit of currency if it is in-the-money, and pays zero otherwise. This option appears in the value function due to the discontinuity in the lender's payoff at maturity if $l_1 > 0$.

The rational borrower will engage in strategic default for $s(T) \leq P/(1-l_1)$, offering the lender a payment of $(1-l_1)s(T)$.

Proposition 2 *If the borrower and lender play the strategic default game, bankruptcy costs and dividend flows are proportional to the collateral value, and there are no debt service payments over the term of the loan, then the loan value is*

$$L(s,0) = (1-l_1)\left[s(0)e^{-dT} - c\left(s,T;\frac{P}{1-l_1}\right)\right]$$

while the value of the borrower's claim is

$$B(s,0) = s(0)\left[1-(1-l_1)e^{-dT}\right] + (1-l_1)c\left(s,T;\frac{P}{1-l_1}\right)$$

where $c(s,T;P/(1-l_1))$ is the value of a European call option on the collateral with expiry date T and exercise price equal to $P/(1-l_1)$

Proof of the proposition is in appendix A.2. The value of the loan or the lender's claim is simply the 'after foreclosure cost' value of the collateral 'stripped' of its dividend flow, less the value of the borrower's call option on the collateral. Note that the value of claims sum to the value of the collateral

$$L(s,0) + B(s,0) = s(0)$$

The borrower's strategic behaviour ensures that foreclosure never occurs. Hence, $F(s,0) = 0$.

4.1.3 Penalized default

Since the contract calls for a single payment at maturity, the penalized default game collapses to the strategic default game. Any shortfall between the borrower's offered payment, P^* and the contractual balance, P, is immediately due in the form of outstanding debt service payments, $k(T) = P - P^*$. Thus the borrower offers

$$P^* = \min\left\{P, \max\{0,(1-l_1)s(T)\}\right\} \tag{4.4}$$

This is the same offer made in the strategic default game. Consequently the values of the claims at maturity are identical to the corresponding values in the strategic default game. The same applies to the values of the claims at $t = 0$.

4.2 Term loans with debt service payments

We continue to assume that the borrower can only exercise his default options at maturity. Similarly, the lender can only foreclose at maturity. Hence, the loan continues to maturity with certainty.[3]

4.2.1 Terminating default

The terminal values of the claims, $L(s,T)$ and $B(s,T)$, are identical to those in section 4.1.1. The values of the claims at $t = 0$ are

Proposition 3 *If the borrower and lender play the terminating default game, foreclosure costs and dividend flows are proportional to the collateral value, contractual payment flows are constant at p, and default cannot occur prior to maturity, then the loan value is*

$$L(s,0) = (1 - l_1)\left[s(0)e^{-dT} - c\left(s,T;P\right)\right] + \frac{p}{r}\left(1 - e^{-rT}\right) + l_1 P\tilde{c}(s,T;P)$$

while the value of the borrower's claim is

$$B(s,0) = s(0)(1 - e^{-dT}) - \frac{p}{r}\left(1 - e^{-rT}\right) + c\left(s,T;P\right)$$

where $c(s,T;P)$ is the value of a European call option on the collateral with expiry date T and exercise price equal to P, and $\tilde{c}(s,T;P)$ is the value of a European 'digital' call option with the same terms.

The values of the claims are identical to those in section 4.1.1 except for the presence of the present value of the debt service payments to be made over the term of the loan. There is no change in the expected foreclosure costs at the loan origination date $F(s,0)$.

4.2.2 Strategic default

$L(s,T)$ and $B(s,T)$ are identical to the expressions in section 4.1.2. $L(s,0)$ and $B(s,0)$ differ from the expressions in propostion 2 only due to the presence of the debt service payments

[3]Restricting the default options of the borrower in this fashion implies that the borrower always offers the lender the full contractual debt service payments, even if this means that the value of his claim is negative!

Proposition 4 *If the borrower and lender play the strategic default game, foreclosure costs and dividend flows are proportional to the collateral value, contractual payment flows are constant at p, and default cannot occur prior to maturity, then the loan value is*

$$L(s,0) = (1-l_1)\left[s(0)e^{-dT} - c\left(s,T;\frac{P}{1-l_1}\right)\right] + \frac{p}{r}\left(1-e^{-rT}\right)$$

while the value of the borrower's claim is

$$B(s,0) = s(0)\left[1-(1-l_1)e^{-dT}\right] + (1-l_1)c\left(s,T;\frac{P}{1-l_1}\right) - \frac{p}{r}\left(1-e^{-rT}\right)$$

where $c\left(s,T;P/(1-l_1)\right)$ is the value of a European call option on the collateral with expiry date T and exercise price equal to $P/(1-l_1)$

As in section 4.1.2, $F(s,0)=0$.

4.2.3 Penalized default

Since the behaviour of the contracting parties is constrained such that default and foreclosure may only occur at maturity, the penalized default game is indistinguishable from the strategic default game.

4.3 Perpetual loans

Consider a perpetual loan with contractual coupon rate c applied to a notional principle P. This implies a continuous stream of contractual payments $p = cP$. Since the dividend flow and the Markov process followed by s are assumed to be time independent, the strategies employed by the borrower and lender are stationary. The (current) value functions $L(s,t)$ and $B(s,t)$ are independent of time, and the exercise policies can be characterized as constant critical- or 'trigger' values of s at which the default options of the borrower and the foreclosure option of the lender are exercised. This time independence also implies that $L_t(s,t)$ and $B_t(s,t)$ are zero in the partial differential equations (3.3), (3.4), (3.25) and (3.26). Hence we are left with ordinary differential equations for which analytical solutions may be determined under appropriate assumptions. In particular, if we restrict the behaviour of the borrower such that he only defaults in either the lower or the upper default region, then closed form solutions for $B(s,0)$ and $L(s,0)$ are forthcoming. Allowing default in both regions yields boundary conditions for the control problems which result in a pair of simultaneous quadratic equations in the optimal default levels of the collateral, $\underline{s}$ and $\overline{s}$ that would require numerical solution for particular parameter values.

4.3.1 Terminating default

Jones (1995) presents analytical results under similar assumptions to those specified in the terminating default game. Constraining the borrower's behaviour such that default only occurs in the lower default region, he demonstrates that the value of the collateral which triggers default by the borrower, is strictly less than value of the remaining contractual payments capitalized at the risk-free rate of interest, $\underline{s} < p/r$ (Jones, 1995, p.12). This is in accordance with our assertion that from the lender's perspective rational default by the borrower occurs at values of $s(t)$ which are 'too low'. Jones (1995) also observes that the value of the loan or the lender's claim is decreasing in p in the vicinity of the default region. "There is thus a positive incentive for the lender to offer *permanently reduced* payments if default is imminent, ..." (Jones, 1995, p. 13). It is precisely this willingness to accept reduced payments in certain states which makes strategic default by the borrower possible.

4.3.2 Strategic default

Anderson, Sundaresan and Tychon (1996) provide analytical results for a perpetual loan contract within the context of the strategic default game with constant bankruptcy costs. By assuming a negative credit spread, $c - r < 0$, the borrower's optimal default strategy is characterized by a single lower default region and hence closed form solutions for the claim values, the critical level of the collateral value, $\underline{s}$, below which strategic default occurs, and the strategic debt service payment flow p^*, are forthcoming. They find that the critical value of s is smaller than the notional principal, $\underline{s} < P$, and that the strategic debt service payments offered are a small fraction of the contractual payments.

While closed form solutions are not forthcoming when $c - r > 0$, it is possible to find closed form expressions for the strategic debt service flows in the default regions without solving for the boundaries of these regions explicitly.

Proposition 5 *Strategic default will occur in the case of a perpetual, interest-only loan with a constant, instantaneous payment flow $p = cP$, and collateral paying constant proportional dividends d_1 at sufficiently low levels of the collateral value, $\underline{s}$ and at sufficiently high levels of the collateral value, $\overline{s}$ if $c - r > 0$, with strategic debt service payments of*

$$p^*(s) = \begin{cases} (1-l)sd_1 & \textit{for} \quad s \leq \underline{s} \\ rP & \textit{for} \quad s \geq \overline{s} \end{cases}$$

If $c - r < 0$ there will be a single lower strategic default boundary with strategic debt ser ·ce payments of

$$p^*(s) = (1-l)sd_1 \quad \textit{for} \quad s \leq \underline{s}$$

Proof of the proposition is in appendix A.3.

4.3.3 Penalized default

In the context of a perpetual loan, the indenture which allows the lender to apply a (penalty) rate of interest to outstanding debt service payments has no effect on the behaviour of the contracting parties in the absence of a technical default provision.[4] There exists no way for the lender to recoup oustanding payments. Hence the penalized default game will produce the same behaviour as in the strategic default game.

4.4 The limits to lending

In the case of a pure discount loan, when foreclosure costs and dividend flows are absent, the supply of credit is limited to

$$L(s,0) = s(0) - c\,(s,T;P) \tag{4.5}$$

for a given credit spread, $c - r$, when the borrower and lender play the terminating default game. This follows from proposition 1.

Consider what happens as the contractual rate c, rises. The principal due at maturity, P, increases. Consequently the probability of default at maturity increases. In other words, the probability of the call option on the collateral being 'in the money' at maturity declines,

[4] The presence of a technical default provision effectively tempers the borrower's incentive to engage in strategic default. Whenever the borrower engages in strategic default, the balance of outstanding debt service payments, k, increases. This increases the upper boundary of the technical default region, increasing the probability of technical default at some time in the future. When the lender exercises her technical default option at t, she has a claim to $b(t) + k(t)$. Thus, the greater the extent to which the borrower indulges in strategic default, the greater the probability of technical default at ever increasing levels of s.

and hence the value of this option at the loan origination date declines. It is clear that as $c - r \to \infty$, $c(s, T; P) \to 0$, and, $L(s, 0) \to s$. The same reasoning produces the same result in the strategic default game (proposition 2). In fact, this result applies to all forms of loan contracts under any assumptions about the rational strategic behaviour of the contracting parties.

In the absence of dividend flows and foreclosure costs, the rational lender will lend the full market value of the collateral, if offered a sufficiently high contractual interest rate. With a sufficiently high rate, default by the borrower occurs with certainty. The lender is effectively purchasing the collateral. With zero bankruptcy costs, the full value of the collateral is preserved in the foreclosure process. Zero dividend flows imply that the borrower cannot 'extract' value from the collateral. This insight is due to Jones (1995) which we summarize in the following proposition

Proposition 6 *(Jones, 1995, p.10) If there are no bankruptcy costs and no dividend flows from the collateral, then the supply of credit approaches the collateral value as the contractual loan rate approaches ∞. That is*

$$\lim_{c-r\to\infty} L(s,0) = s$$

Consider, again, the supply of credit in the case of the pure discount loan under the assumptions of the terminating default game. With a positive dividend flow, d_1, and no foreclosure costs, the supply of credit is limited to

$$L(s,0) = s(0)e^{-dT} - c(s,T;P) \tag{4.6}$$

for a given credit spread, $c - r$. Now, as $c - r \to \infty$, $L(s, 0) \to s(0)e^{-dT}$. Allowing for positive foreclosure cost as well, implies that $L(s, 0) \to (1 - l_1)s(0)e^{-dT}$ as $c - r \to \infty$. The same result is forthcoming in the strategic default game. The following proposition summarizes

Proposition 7 *If dividend flows and foreclosure costs are proportional to the collateral value, the supply of credit under pure discount loans is limited to*

$$L(s,0) \le (1 - l_1)s(0)e^{-dT}$$

In general we assert that for any type of loan contract, in this world of symmetric information, credit will only be 'rationed' to less than the full market value of the collateral if foreclosure costs are positive and/or the collateral generates a dividend flow. The upper bound on the amount of credit a rational lender would extend is strictly less than the collateral value, no matter how high the contractual interest rate specified in the loan contract. Note as well that the amount of credit extended is independent of the *objective* expected rate of capital appreciation in the collateral.

4.5 Contract design

A contract which allows the borrower and lender to engage in the strategic default game is efficient in the sense that there is no deadweight loss due to foreclosure. However, the terminating default game yields a higher value for the lender's claim at $t = 0$, if $l_1 > 0$, and hence implies that a greater amount of credit will be extended for any given credit spread. Of course, the increase in value to the lender moving from the strategic default game to the terminating default game comes at the expense of the borrower. However, the borrower may be willing to enter into a contract which tempers his incentive to engage in strategic default and lowers the value of his claim, if it means that his project is funded.

In section 3.2.1 we suggested that the penalized default game might be effective in mitigating credit rationing. However, given the constraints imposed on the behaviour of the contracting parties here for the sake of generating analytical solutions, the penalized default game 'collapses' into the strategic default game for all the loan contracts considered. To assess the effectiveness of penalized default in mitigating credit rationing we must relax these constraints. To this end, we employ a numerical approach to finding solutions in the next chapter.

While our analytical results are based on restrictive assumptions about the behaviour of the contracting parties, a number of important insights are forthcoming. Two are of particular interest. (1) Positive dividend flows from the collateral or positive foreclosure costs are necessary and sufficient for the existence of credit rationing as we have defined it. (2) Strategic debt service, while it reduces the value of the lender's claim, ceteris paribus, is efficient in that it removes the possibility of foreclosure and hence avoids the deadweight costs associated with foreclosure.

Chapter 5

Numerical results

For loans of finite maturity with regular debt service payments, the critical levels of s at which terminating or strategic default occurs vary with the remaining time to maturity of the loan contract. Analytical solutions for these levels of s are not available. Instead, we employ a finite difference procedure to approximate the functions which satisfy the partial differential equations for representative cases or boundary conditions. The state space is represented by a discrete grid of s and t (and k in the two state variable case) values. A solution is a set of L and B values for these gridpoints, together with an indication whether each point is in one or more of the termination regions (e.g. F). Working 'backwards' from maturity, T, the pde's are 'solved' for each time step using a Crank-Nicholson discrete approximation for the partial derivatives. At each time step the values of the agents' positions are checked to determine whether these values could be increased by exercising options available to the agents at that time.[1] Listings of the FORTRAN code used to implement the Crank-Nicholson algorithm are in appendix E.[2] This approach allows us to consider the full range of contractual indentures simultaneously. We have two objectives. First, we attempt to establish whether the propositions in chapter 4 apply in the case of term loans with debt service payments. Secondly, we explore the impact of the various indentures on credit rationing and the expected foreclosure costs at the origination date of the contract.

[1] See Hull(1997) and Wilmott et al (1993) for accessible treatments of the Crank-Nicholson finite difference method.

[2] A routine written by Prof. R. A. Jones which implements the Crank-Nicholson algorithm in Jones (1995) is included.

5.1 Effects of the loan parameters

Appendix B illustrates how $L(s,0)$, $B(s,0)$, $F(s,0)$ and the loan to value ratio, '*%Loan*', change as we alter various loan parameters. *%Loan* is the contractual value of the loan, \$1, as a proportion of the minimum collateral value for which $L(s,0) = 1$ at the given contractual rate. The details of the benchmark contract, a pure discount loan, are specified on page 64.[3] Note that we assume that foreclosure costs are 10% of the value of the collateral at the time that foreclosure occurs.[4]

In the tables, panels labeled (a) and (c) report results for the terminating default game, while panels labeled (b) and (d) report results for the strategic default game. We report results based on two assumptions regarding the borrower's strategic behaviour. In panels labeled (a) and (c) the borrower is denied the opportunity to exercise his *terminating* default option in the upper default region. In panels labeled (b) and (d) the borrower is free to exercise his terminating default option in both upper- and lower regions.

Table (i) demonstrates how higher dividend rates generated by the collateral (without commensurate increases in the contractual payment flows) reduce the willingness to lend against given collateral. Since the results are identical for the two versions of the terminating default game, we report a single panel for (a) and (c). Similarly, we report a single panel for (b) and (d).[5] Note that the willingness to lend is greater in the terminating default games than in the strategic default games. This is consistent with propositions 1 and 2 in chapter 4. Also, since foreclosure never occurs in the strategic default games, expected foreclosure costs at the loan origination date are zero.

Table (ii) reveals the negative effect of increasing foreclosure costs on the willingness to lend. In both games the presence of an upper terminating default region only matters when foreclosure costs are zero. In this case the willingness to lend is greater when the borrower is denied the option of terminating default in the upper region. There is an initial increase in the willingness to lend as l_1 rises above zero. This is due to the decline in terminating default in this upper region. As l_1 continues to increase the willingness to lend declines. The effect of increasing foreclosure costs is more pronounced in the case of the strategic default games

[3] We consider a pure discount loan so that our results may be compared to the propositions in section 4.1.

[4] We include foreclosure costs in our benchmark contract so that we can contrast the effects of variations in the loan parameters in both the terminating default game and the strategic default game. In the absence of foreclosure costs, the equilibria of these games 'converge'.

[5] For the term of the loan, $T = 5$, and the foreclosure costs $l_1 = 0.1$, a credit spread below 0.06 does not induce the borrower to rationally exercise his terminating default option in the upper default region.

than in the terminating default games. Table (iii) shows the negative impact of the loan term, while table (iv) reveals the negative impact of increased uncertainty about the future value of the collateral. Table (v) indicates that larger payment flows, which imply faster amortization, increase willingness to lend. Table (vi) makes the important observation that the amount of credit extended, expressed as a *proportion* of the initial collateral value, is *not* affected by the level of the risk free interest rate, r. Under the risk-adjusted probability measure, Q, the expected rate of return on all assets is equal to r.[6]

Tables (vii)-(ix) reveal properties of the 'supply curve' for credit.[7] In table (vii) with $l_1 = 0$, the credit supply curve slopes 'upward' for all cases, (a)-(d). In the absence of foreclosure costs there is no difference between the results generated by the terminating default games and the strategic default games. There is, however, a significant difference between the games which permit terminating default in the upper default region, (c) and (d), and those that do not, (a) and (b). The willingness to lend is significantly greater in the case of the latter games. In cases (c) and (d) the borrower exercises his terminating default option costlessly in the upper region, restricting the value of the of the lender's claim to the contractual balance, $b(t)$, at every point in time. In cases (a) and (b) the value of the lender's claim exceeds $b(t)$ for 'high' values of s since the borrower cannot exercise his terminating default option.

In table (viii), with $l_1 = 0.1$, we continue to observe that, when terminating default in the upper region is not allowed, the willingness to lend is greater, albeit much less so, in the terminating default game (a) than in the strategic default game (b). The same result does *not* hold in the cases where upper terminating default is permitted. For $c - r < 0.06$, the willingness to lend is greater in the terminating default game (c), but for $c - r \geq 0.06$ it is greater for the strategic default game. For credit spreads greater than or equal to 0.06 upper terminating default is rational for sufficiently high values of s. This default occurs more frequently in the terminating default game than in the strategic default game where the borrower has the additional option of strategic default.[8] In table (ix) we observe a 'backward bending' supply curve for credit in the terminating default games.[9] Foreclosure costs are sufficiently onerous such that terminating default never occurs in the strategic

[6] An increase in r may reduce the market value of the collateral which would lead to a decline in the absolute amount of credit that would be extended.

[7] By supply curve we mean the required credit spread, $c - r$ as a function of the loan amount.

[8] Compare the values for F(s,0) in (c) and (d).

[9] A backward bending supply curve was never observed in the strategic default game.

default games.

5.2 Rational default and foreclosure strategies

5.2.1 Interest-only loans

Appendix C.1 (page 71) reports results for an experiment designed to illustrate proposition 5 in chapter 4. We use a long term loan ($T = 50$ years) to approximate the perpetual loan.[10] We ignore technical default and prepayment here. The 50 year term is divided into 500 intervals of length, $dt = 0.1$. At the contractual rate, $c = 0.08$, the contractual payments are $pdt = 0.008$ per time interval. The contractual payments only cover the interest on the principal. The principal remains $P = 1$ over the term of the loan. The contracting parties play the strategic default game.

The first panel of table 1 displays the value of the lender's claim, $L(s,t)$. The second panel displays the value of the borrower's claim, $B(s,t)$, while the third panel displays the debt service payments. At $t = 50$, the boundary of the upper strategic default region lies in the interval $1.85 < \bar{s} < 1.90$. In the debt service payments table, we see that the payments offered by the borrower are less than the contractual amount for $s > 1.85$. The boundary of the upper default region, $\bar{s}$, remains in this interval for the first 35 years of the loan term. Thereafter $\bar{s}$ declines with the declining term to maturity.[11] Within the upper default region the debt service payments offered by the borrower reach a minimum of 0.005 which amounts to a return on the principal equal to the risk free interest rate of 0.05% per annum. The value of the lender's claim is always equal to the principal ($1) in the upper default region.

The boundary of the lower strategic default region, $\underline{s}$, lies in the interval $0.70 < \underline{s} < 0.75$ over the entire term of the loan. For $s \leq 0.6$ the payments offered by the borrower are $p^*dt = (1 - l_1)d_1 sdt$. Thus, in both default regions, the strategic payments offered by the borrower converge to the levels for a perpetual loan determined in proposition 5.

[10]The parameter values used to generate the tables in appendix C.1 are presented on page 71.

[11]In the case of a perpetual loan, the boundaries of the upper and lower default regions remain unchanged. See section 4.3.2.

5.2.2 Partially amortizing loans

Appendix C.2 reports results of experiments conducted under the assumptions specified in chapter 3 for 'representative' parameter values.[12] Tables 1 - 10 display numerically obtained values for $L(s,t)$ and $B(s,t)$, identifying the regions of $\mathsf{S} \times \mathsf{T}$ or $\mathsf{S} \times \mathsf{K} \times \mathsf{T}$, where the various options, available to the contracting parties are exercised. When these regions overlap, terminating default takes precedence over foreclosure due to a binding dividend flow constraint, over foreclosure due to technical default. Tables 1-3 report results for the terminating default game described in section 3.1.1. In table 1 we observe the lower terminating default region indicated by '$*$'. There is no prepayment option here, and given the credit spread $c - r = 0.03$, the upper terminating default region is not visible in the subset of $\mathsf{S} \times \mathsf{T}$ displayed.[13]

Table 2 allows for prepayment and technical default. We now observe an upper termination region (the prepayment region) identified by '+'. This prepayment region disappears well before maturity, T. With little time remaining to maturity, the cost of paying a now unwarranted high interest rate over the remaining term, falls short of the cost of refinancing the loan. The lower terminating default region lies well below the region of technical default when there remains a significant period of time until maturity, T. The borrower must have substantial negative equity before rationally defaulting if time remains for the collateral value to recover. This region is also larger in table 2 than in table 1, i.e. terminating default occurs at higher values of s in table 2 than in table 1, save for a curious 'dip' in the default region over the interval $t = 1$ to $t = .5$. This serves as an example of the complex effects of contractual indentures, such as technical default, on the rational behaviour of the contracting parties.

Of course, all of this is moot. The borrower never gets to exercise this terminating default option since the lender preempts him by foreclosing along the boundary of the technical default region. This foreclosure is indicated by ':'. States in which the lender does not exercise her technical default option, even though the technical default condition is satisfied, are identified by '-'.

The presence of the prepayment provision reduces the value of L for 'high' values of s

[12] The parameter values used to generate the tables in appendix C.2 are presented on page 71. The results are based on a time interval of $dt \approx .0208$ years. The contractual debt service payment over a single interval is $pdt \approx .0021$.

[13] In fact, for this credit spread, term of the loan, $T = 5$ years and magnitude of the foreclosure costs, $l_1 = 0.1$, there is no upper default region for any (s,t).

compared to the corresponding values in table 1, while L is higher for 'low' values of s in table 2 than the corresponding values in table 1 due to the presence of the technical default provision.

For tables 1 and 2 the combined value of B and L approaches $(1 - l_1)s(t)$ as $s(t) \to 0$, approaches $s(t) - f_1 b(t)$ as $s(t) \to \infty$ when prepayment is a viable option (the prepayment region of table 2) and approaches $s(t)$ outside of the prepayment region. The value of the positions of all parties to the contract, including bankruptcy trustees in the event of default or foreclosure, and new lenders in the event of prepayment, is conserved and sums to $s(t)$.

Table 3 allows for a dividend flow constraint in addition to prepayment and technical default. Foreclosure due to technical default in table 2 is now replaced by foreclosure due to the binding cash flow constraint, indicated by '#'. Furthermore, the dividend constraint yields foreclosure in some states in which it was not possible for the lender to foreclose in table 2. For example, at $(s, t) = (0.95, 2.0)$ the lender forecloses in table 3 since the dividend flow off the collateral is insufficient to cover the contractual payment flow.[14] In table 2, however, the borrower is not constrained to service the debt out of the dividend flows, and consequently does not default.

The dividend constraint has a significant effect on the borrower's lower terminating default region, and on the borrower's rational prepayment strategy. The borrower now prepays at lower levels of s over the interval 4 years to maturity ($t = 4$) to .5 years to maturity ($t = 0.5$).[15] Over the interval, $t = 1$ to $t = 0.5$ the prepayment region has an upper bound.[16] Here the borrower is not prepaying due to a suddenly unreasonable credit spread, he is prepaying to avoid the ever increasing likelihood that the dividend constraint will become binding and that the lender will foreclose. The refinancing costs are significantly less than the foreclosure costs that would be imposed on him at these levels of s. The lower terminating default region has expanded to extend to the boundary of the foreclosure region. The dividend constraint lowers the value of the borrower's claim for 'low' values of s and hence increases the region in which terminating default is rational. Again, however, terminating default does not occur in this game as the borrower is always preempted by the lender's foreclosure.

[14] The contractual debt service payment over a single interval, $pdt \approx .0021$, is greater than the dividend flow off the collateral, $d_1 s \approx .0020$ when its market value is $s = 0.95$.

[15] In the tables t refers to 'time to maturity' whereas in the preceding analysis t referred to calendar time.

[16] Recall that we asserted in section 3.1.1 that for the functional form of the refinancing costs we employ there will be no upper bound to the prepayment region, in the *absence* of a dividend constraint.

It is interesting to note that this loan contract does not appear to 'survive' to maturity. There does not seem to be a 'path' for s to maturity ($t = 0$) which does not traverse a boundary of a termination region.[17]

Table 4 reports the results for the strategic default game without a dividend constraint on the debt service payments offered by the borrower. As expected the value of the borrower's position for any $s(t)$ is higher than the values reported in tables 1-3, while the converse is true for the lender. The third panel of table 4 displays the strategic debt service payments. It is clear that strategic default occurs at both low and high collateral values, which is consistent with the analytical results.[18] In the strategic default regions, L is always equal to minimum of the contractual balance and the value of the collateral net of foreclosure costs ('*sliq*'). Also, the strategic behaviour of the borrower, unfettered by cash flow constraints, successfully avoids foreclosure or default in *all* states of the contract. In this game the values of the claims of the borrower and lender always sum to s.

Table 5 includes the prepayment and technical default provisions. The value of the lender's claim is now greater for all (s,t) where the borrower makes the full contractual payment, while the value of the borrower's claim is diminished in this region. The third panel of table 5 reveals that the strategic default region is now larger. The presence of the prepayment provision allows the borrower to extract more value from the lender at 'high' levels of s. Note however that on the boundary of the technical default region the borrower offers the lender debt service payments which are considerably greater than the contractual payment. This is to avoid foreclosure due to technical default. The borrower offers the lender debt service payments such that $L = (1 - l_1)s$, since this is what the lender would receive if she chose to exercise her foreclosure option. This accounts for the lower values of the borrower's claim in table 5 compared to the values in table 4 at corresponding (s,t).

Thus it appears that L increases while B falls with the introduction of the technical default indenture and the prepayment option. Tables 6 and 7 provide an interesting insight into the effect of the prepayment option on the values of the contracting parties' claims. Close study of the tables reveals that in the presence of strategic debt service with the cash flow constraint in place, the prepayment option *enhances* the value of the lender's position in the continuation region of the state space. For example, in table 6 $L(1.45, 5.0) \approx 0.997$

[17]We should be careful in asserting that is unambiguously true, given the 'coarseness' of the grid in table 3. A finer grid may yield a path to maturity for s.

[18]Strategic default occurs whenever the debt service payments are less than the contractual payments of .0021.

and $L(1.30, 3.5) \approx 0.960$, while in table 7, $L(1.45, 5.0) \approx 1$ and $L(1.30, 3.5) \approx 0.965$.[19] This increase in the value of the lender's claim is not due to higher debt service payments. In fact, the debt service payments in the third panel of table 7 are all less than or equal to the debt service payments in the third panel of table 6.

This occurs despite the fact that the prepayment option is never exercised! Note as well that the cash flow constraint induces foreclosure and hence bankruptcy at 'intermediate' levels of $s(t)$. Quite surprisingly, bankruptcy does not occur at low levels of $s(t)$ as one would intuitively expect, the technical default option notwithstanding.

Tables 8-10 report results for the penalized default game. The strategy space for this game is three dimensional ($\mathsf{S} \times \mathsf{K} \times \mathsf{T}$). The tables represent cross sections of this space at $s = 1.1$, the assumed fair market value of the collateral at the origination date. Each table includes an extra panel in which the critical debt service payments $\hat{p}$ are revealed.[20] Along the vertical axis of the $\mathsf{K} \times \mathsf{T}$ space in each panel of the tables we measure the actual outstanding debt service payments $k(t)$, and along the horizontal axis we measure time to maturity, t. Also, the maximum outstanding debt service payment amount $K(t)$, and the contractual balance $b(t)$, is indicated for every t along the horizontal axis.[21] For all combinations of $k(t)$ and $K(t)$ which are infeasible (i.e. $k(t) > K(t)$), the values of the claims and the debt service payments are set to zero.[22] This has no bearing on the values of the claims in the feasible region of the strategy space.

Table 8 reports results for the current default rule. For 'low' values of $k(t)$ there seems to be no clear relationship between the value of the claims and $k(t)$. For 'higher' values of $k(t)$, the value of the borrower's claim is decreasing in $k(t)$ while the value of the lender's claim is increasing in $k(t)$. Similarly, the strategic debt service payments offered by the borrower exhibit no clear relationship to $k(t)$ when these values are 'low', but are increasing in $k(t)$ when theses values are 'high'. Also, the debt service payments offered, p^* are significantly smaller than the critical debt service payments, $\hat{p}$ for all (k, t), but equal to or greater than the contractual payment, p. Hence the borrower avoids foreclosure in this region of the

[19] This result seems to be pervasive. In more than 100 cases studied, the inclusion of the prepayment provision in the strategic default game with a dividend constraint was never associated with a decline in the value of the lender's claim.

[20] The critical debt service payments are payments which render the lender indifferent between foreclosing and allowing the loan to continue, see section 3.1.2.

[21] The maximum outstanding debt service payments at any t is the amount which would be owing to the lender if the borrower had made no payments since the origination date of the loan to the present time.

[22] For example, at $t = 5$, $K(5) = 0$. So, for $k(5) > 0$, the values of the claims are set to zero.

strategy space.[23]

The observed behaviour of the values of the claims and the strategic debt service payments appears to be consistent with our conjecture on page 32.

Table 9 includes the prepayment, technical default and dividend constraint indentures in the penalized default game with the current default rule. The inclusion of these indentures affects the values of the claims and the strategic debt service payments significantly. L is greater for all (s,t) in table 9 than in table 8, while the converse is true for B. Also, the 'non-monotonicity' of B and L with respect to $k(t)$ is absent. It seems as if the presence of the contractual indentures dominates the opposing effects of lower debt service payments, p^* and higher $k(t)$ on L and B.

Again, $p^* < \hat{p}$ in table 9, but this does not induce foreclosure since $p^* = p$ for all (s,t) in this region of the strategy space. More importantly, since the borrower always makes the contractual debt service payments, we can conclude that, at least for $s = 1.1$, $k(t) = 0$ over the entire term of the loan contract. The borrower never engages in strategic default in this region of the strategy space. This explains the changes in L and B when moving from table 8 to table 9.

Table 10 reports results for the same set of contractual indentures, save one. Foreclosure is now governed by the outstanding payment rule. For $k(t) > 0$, the values of L, B and p^* are significantly different from those in table 9. This is due to the fact that for any $k(t) > 0$ it is rational for the lender to default. As in table 9, however, $k(t) = 0$ for all t over the term of the loan as the borrower always offers $p^* = p$.

The numerical results reported in appendix C appear to be broadly consistent with the analysis developed in chapter 3.

5.3 Credit rationing

Appendix D reports the results for a number of experiments conducted to determine the effect of changes in various parameters on the 'loan to value' ratio and the expected foreclosure costs at the loan origination date, $F(s,0)$.

Table 1 (i) reports the effect of changes in the credit spread on the loan to value ratio for a number of games and combinations of contractual indentures. Consider columns (a), (b),

[23]Under the current default rule the lender cannot foreclose if the borrower offers $p^* \geq p$, for any $k(t)$, see section 3.2.1.

(c), (d), (f) and (g). These columns report results for the three classes of games described in chapter 3 without any additional contractual indentures such as prepayment or technical default. Refer to the key on page 93 for a description of these games. As we would expect, the loan to value ratios in column (c) are lower than those in column (a) for a large range of credit spreads ($c - r < 0.16$). This is a consequence of the negative effect which strategic debt service has on L as the borrower effectively renegotiates the contract in his favour in the strategic default regions. It is interesting to note that for 'high' credit spreads ($c - r > 0.16$) the loan to value ratios are greater in column (c) than in column (a). We conjecture that this is due to the fact that at these excessive levels of the credit spread, default in the terminating default game is very likely to occur for high values of s (i.e. there is a 'large' upper terminating default region). This imposes an upper bound on L which is absent in the strategic default game (column (c)).

Note also that at as we move from $c - r = 0.06$ to $c - r = 0.07$ there is a decline in the loan to value ratio in column (a). This is due to the emergence of the upper terminal default region which reduces L at 'high' levels of s. For the strategic default game (column (c)) this upper terminal default region emerges at much higher credit spreads (c - r ¿ 0.12) and there is no decline in L near the boundary of this region due to the strategic debt service payments offered by the borrower.

Columns (b) and (d) add the dividend constraint to the terminating default game and the strategic default game respectively. Again, we observe a decline in the loan to value ratio in column (b) as we move from $c - r = 0.05$ to $c - r = 0.06$, as the upper terminating default region emerges. There is no such 'dip' in $\%Loan$ for the strategic default game (column (d)).

Adding the dividend constraint increases the loan to value ratios in the terminating default game for $c - r \leq 0.05$ (compare columns (a) and (b)). For $c - r \geq 0.06$, the loan to value ratios fall with the addition of the dividend constraint. This, we infer, is a consequence of the interaction between the dividend flow constraint and the rational default behaviour of the borrower. In the case of the strategic default game, the addition of the dividend flow constraint increases $\%Loan$ for all but two levels of $c - r$ where the ratio remains the same. In general we conclude that the dividend flow constraint mitigates credit rationing in the strategic default game while its effect in the terminating default game is ambiguous.

Column (f) reports the results for the penalized default game with the current default rule for foreclosure and a dividend constraint, while (g) reports the results for the same game with the outstanding payment rule for foreclosure. It is clear that the form of the foreclosure rule has no effect on the extent of credit rationing. The penalized default game yields less credit rationing than the strategic default game with the same indentures (d). Notice also that in terms of mitigating credit rationing, this game fares almost as well as the terminating default game with a dividend constraint (b).[24]

Consider columns (e), (h) and (i) which report results for the strategic default game (e), the penalized default game with the current default rule for foreclosure (h) and the outstanding payment rule for foreclosure (i) when the prepayment and technical default indentures are included. The effect on credit rationing is clear. In every case credit rationing is reduced by adding these indentures to the respective games. Furthermore the penalized default games yield modest reductions in credit rationing compared to the strategic default game for some credit spreads. Again we observe that the form which the foreclosure rule takes is of no consequence.

In chapter 1 we reported that the strategic debt service models of Anderson and Sundaresan (1996), Anderson, Sundaresan and Tychon (1996) and Mella-Barral and Perraudin (1996) for valuing default risky bonds, generate higher credit spreads than models based on approach of Merton (1974). The implication of this result within the framework developed here is that we should observe a greater degree of credit rationing in the strategic default game than in the terminating default game, which we do.[25] However, when we include common indentures such as prepayment and technical default, or we relax the assumption that concessions extracted from the lender are irreversible (the penalized default game), we observe levels of credit rationing which are comparable to those generated by the terminating default game. Consequently we should be weary in assuming that strategic behaviour on the part of the borrower (and the lender) will have a significant impact on credit rationing or, alternatively, on the credit spreads associated with default risky loan contracts.

Table 1 (ii) repeats the exercise discussed above for foreclosure costs of, $l_l = 0.35$. Increasing the foreclosure costs increases the extent of credit rationing across all the cases considered. The same general result prevails with respect to the extent to which particular

[24] In fact, for $c - r > 0.12$, the penalized default game is characterized by *less* credit rationing than the terminating default game.

[25] In the strategic debt service models, the loan to value ratio is exogenous. The credit spread is determined in the Markov perfect equilibria of the games considered. In the framework developed here, the credit spread is exogenous and the loan to value ratio is determined in the Markov perfect equilibria of the games considered. Hence, factors which increase the credit spread in the strategic debt service models should, in principle, increase the extent of credit rationing in this context.

combinations of contractual indentures are most effective at mitigating credit rationing.[26] Again, the penalized default game with either foreclosure rule, and the prepayment and technical default indentures yields the smallest scope for credit rationing.

Table 1 (iii) repeats the exercise for a dividend rate of, $d_l = 0.2$. Again, the overall extent of credit rationing increases, as we would expect. There is little to choose between the penalized default game (with either foreclosure rule) and the strategic default game with the prepayment and technical default indentures included for the purposes of minimizing credit rationing. With large dividend flows, the dividend constraint is only binding for very small values of s and has no effect on the extent of credit rationing (compare (a) and (b), and (c) and (d)).

5.4 Foreclosure costs

Table 2 (i) reports the expected foreclosure costs at the origination date, $F(s, 0)$, for the same parameter values employed to generate table 1 (i). As expected, the terminating default game with the dividend constraint (b) yields the highest F, while the strategic default game without any additional contractual indentures yields $F = 0$ for all credit spreads. The strategic default game with only the dividend constraint (d) yields roughly the same F as the penalized default games with only a dividend flow constraint, (f) and (g). These games with the full complement of contractual indentures, yield significantly lower values for F for every level of the credit spread.

Table 2 (ii) repeats the exercise for $l_l = 0.35$ and corresponds to table 1 (ii). Now, we observe that the strategic default games,(c) and (d), and the penalized default games, (f) and (g), which do not include the prepayment and technical default indentures yield $F = 0$ for all levels of the credit spread. The onerous foreclosure costs dissuade the lender from foreclosing at any (s, t) over the term of the loan contract. For the games which include the prepayment and technical default indentures, (e), (h) and (i), $F > 0$ for at least some levels of $c - r$.

Table 2 (iii) reports the expected foreclosure costs for $d_l = 0.2$ and corresponds to table 1 (iii). Again for games (c), (d), (f) and (g), $F = 0$ for all levels of $c - r$. For games (e),

[26] One exception is that for a credit spread of zero, the terminating default game (a) yields the highest loan to value ratio. Loan to value ratios of zero imply that the value of s at which $L(s, 0) = 1$ exceed 5 which is the upper limit for s on the grid of s and t values employed to generate the numerical solutions for L and B. In other words, zeros imply loan to value ratios smaller than 0.2.

(h) and (i), F is positive for all levels of $c - r$. The high dividend rate allows the borrower to extract value from the collateral at a greater rate. This increases the willingness of the lender to invoke the appropriate indentures (such as technical default) to terminate the loan prior to maturity.

Tables 3 and 4 provide another perspective on the effects of foreclosure costs and dividend flows. Consider table 3. There is a general tendency for L and B to decrease when l_1 increases, notwithstanding a small number of exceptions. The loan to value ratio, $\%Loan$, is decreasing in l_1 for games (d) and (f), but the results in games (a) and (h) are ambiguous. In table 4, L is decreasing in d_1 and B is increasing in d_1 for all games considered. Similarly $\%Loan$ is decreasing in d_1 for all the games.

Based on the small set of results reported in this chapter, a number of general conclusions can be drawn. First, contracts which tend to be effective in mitigating credit rationing tend to be associated with significant levels of expected foreclosure costs at the time of origination. Second, contractual indentures such as prepayment and technical default, when included in the loan contract, tend to interact in a complex manner, rendering the relationship between variables such as the value of the contracting parties claims, or the extent of credit rationing and the various loan parameters, ambiguous. However we have demonstrated that these indentures are important in reducing the extent to which credit rationing occurs.

Chapter 6

Conclusion

The primary objective of this study was to develop a general framework to study the rich possibilities and subtle interactions that occur in ostensibly 'simple' (standard) loan contracts. In doing so, the framework developed Anderson and Sundaresan (1996) has been extended such that the de facto contract renegotiation which occurs is not necessarily irreversible and not entirely one-sided. We have developed games in which the lender is able to extract concessions from the borrower in certain states of nature, in the presence of the appropriate contractual indentures.

It is worth reiterating that none of the results obtained in this study rely on any elements of asymmetric information, adverse selection or costly state verification. Instead, the key ingredients in this analysis are costly foreclosure and 'risky' collateral.

To summarize, we restate the major qualitative results reported here. (1) The upper limit on what a rational lender would lend may be a modest fraction of the current market value of the collateral, regardless of the interest rate the borrower offers. (2) The loan supply curve to a particular borrower may be backward bending, with the lender preferring a lower loan rate over a higher one. (3) The amount lendable is sensitive to the scope for opportunistic behaviour on the part of the borrower. This scope for opportunism is increasing in the costs associated with seizure of the collateral in the event of foreclosure.

A number of interesting implications emerge for loan contract design. Loan contracts, which penalize the borrower for strategic default, by applying penalty rates of interest to outstanding interest balances, temper the incentive for the borrower to engage in strategic debt servicing in many instances. This reduces the severity of credit rationing. Common contractual indentures such as a prepayment option for the borrower and a technical default provision are also very effective in mitigating credit rationing.

Furthermore, the inclusion of a prepayment option for the borrower in the presence of considerable scope for strategic debt service, may *enhance* the value of the lender's position. Conventional wisdom suggests that prepayment options on debt contracts diminish the value of the lender's position.

Loan contracts which tend to be effective in reducing credit rationing may be inefficient in the sense that there are significant levels of expected 'deadweight' foreclosure costs associated with them. On the other hand, contracts which are efficient in the sense that they minimize expected foreclosure costs are associated with significant levels of credit rationing.

Some implications for policy with respect to loan contracts and bankruptcy proceedings follow from the analysis. Most importantly, if credit rationing-like phenomena naturally occur without information asymmetry or moral hazard, then there is little reason to suspect market failure requiring government action. If action is called for, it suggests policies of *removing* regulatory restrictions on the enforceable forms loan contracts can take.

Appendix A

Analytical results

A.1 Risk neutral valuation

We assume that the Markov process describing the evolution of the collateral value is time independent.

$$ds = \alpha s\mathrm{d}t + \sigma s\mathrm{d}z$$

If markets are complete with respect to 's-risk', there exists a unique probability measure, Q equivalent to the true measure, P such that

$$ds = (r - d)s\mathrm{d}t + \sigma s\mathrm{d}z'$$

where z' is a Wiener process under Q.[1] The expression $(r - d)s$ is the 'risk-adjusted drift' in the collateral value, i.e. the expected rate of capital appreciation on the collateral in an equilibrium where agents are risk neutral. Under this measure, the value of agents' claims is the expected value of all future income flows, discounted at the risk-free interest rate.

By equivalent risk neutral valuation, the value of the lender's claim becomes

$$L(s,t) = E_t^Q \left[\int_t^T p(s,t)e^{-r\tau}\mathrm{d}\tau + L(s,0)e^{-r(T-t)} \right]$$

[1] A rigorous treatment of the 'equivalent martingale measure' is provided by Harrison and Kreps (1979) and Harrison and Pliska (1981). The existence of the measure Q implies the absence of arbitrage opportunities, while its uniqueness is a consequence of market completeness. A market is complete with respect to 's-risk' if all s states of nature can be spanned by existing securities (see Huang and Litzenberger, 1988, pp. 126 -129).

Allowing the passage of a small interval of time, dt, and rewriting the lender's claim in a form reminiscent of a Bellman equation

$$L(s,t) = p(s,t)\mathrm{d}t + E_0^Q\left[L(s+\mathrm{d}s, t+\mathrm{d}t)\right]e^{-r\mathrm{d}t}$$

Taking a Taylor series expansion of the right side of this expression, applying Ito's lemma and ignoring terms which approach zero 'faster' than dt as d$t \to 0$

$$L(s,t) = p(s,t)\mathrm{d}t + (1 - r\mathrm{d}t)\left[L(s,t) + L_t(s,t)\mathrm{d}t + (r-d)sL_s(s,t)\mathrm{d}t + \frac{1}{2}\sigma^s s^2 L_{ss}(s,t)\mathrm{d}t\right]$$

Suppressing the arguments of the functions and rearranging

$$\frac{1}{2}\sigma^2 s^2 L_{ss} + (r-d)sL_s + L_t + p = rL$$

Similarly, the value of the borrower's claim is

$$B(s,t) = E_t^Q\left[\int_t^T (ds(t) - p(s,t))e^{-r(\tau-t)}\mathrm{d}\tau + B(s,T)e^{-r(T-t)}\right]$$

This yields

$$\frac{1}{2}\sigma^2 s^2 B_{ss} + (r-d)sB_s + B_t + ds - p = rB$$

A.2 Pure discount loans

Proof of Proposition 1

The value of the lender's claim at maturity, T is

$$L(s,T) = \begin{cases} \max\{0,(1-l_1)s(T)\} & \text{for} \quad s(T) \leq P \\ P & \text{for} \quad s(T) > P \end{cases}$$

This 'payoff' can be replicated by a portfolio containing $1-l_1$ units of the collateral and a short position in $1-l_1$ units of a European call contract on the collateral with expiry date T and exercise price , P, and a long position in $l_1 P$ units of a European 'digital' call.[2]

Consequently the value of the lender's position at the origination of the loan contract is

$$\begin{aligned} L(s,0) &= (1-l_1)\left[s(0) - \mathrm{E}_0^Q\left(\int_0^T s(\tau)de^{-r\tau}\mathrm{d}\tau\right) - c\,(s,T;P)\right] + l_1 P\tilde{c}(s,T;P) \\ &= (1-l_1)\left[s(0) - \int_0^T s(0)de^{(r-d)\tau}e^{-r\tau}\mathrm{d}\tau - c\,(s,T;P)\right] + l_1 P\tilde{c}(s,T;P) \\ &= (1-l_1)\left[s(0)e^{-dT} - c\,(s,T;P)\right] + l_1 P\tilde{c}(s,T;P) \end{aligned}$$

At T, the value of the borrower's claim is

$$B(s,T) = \max\{0, s(T) - P\}$$

This is simply the terminal payoff on a European call option on the collateral with exercise price P and expiry date, T. In addition the borrower retains the dividend flow generated by the collateral

Thus, at the origination of the loan, the value of the borrower's claim is

$$B(s,0) = s(0)\left[1 - e^{-dT}\right] + c\,(s,T;P)$$

[2] A European digital option pays at maturity, one unit of currency if it is in-the-money, and pays zero otherwise. This option appears in the value function due to the discontinuity in the lender's payoff at maturity if $l_1 > 0$.

Proof of Proposition 2

The value of the lender's claim at maturity, T is

$$L(s,T) = \min\{(1-l_1)s, P\}$$

This 'payoff' can be replicated by a portfolio containing $1-l_1$ units of the collateral and a short position in a European call contract on $(1-l_1)$ units of the collateral at an exercise price of $P/(1-l_1)$.[3] Consequently the value of the lender's position at the origination of the loan contract is

$$\begin{aligned}
L(s,0) &= (1-l_1)\left[s(0) - E_0^Q\left(\int_0^T ds(\tau)e^{-r\tau}\mathrm{d}\tau\right) - c\left(s,T;\frac{P}{1-l_1}\right)\right] \\
&= (1-l_1)\left[s(0) - \int_0^T ds(0)e^{(r-d)\tau}e^{-r\tau}\mathrm{d}\tau - c\left(s,T;\frac{P}{1-l_1}\right)\right] \\
&= (1-l_1)\left[s(0)e^{-dT} - c\left(s,T;\frac{P}{1-l_1}\right)\right]
\end{aligned}$$

Since the strategic behaviour of the lender prevents foreclosure at T, the value of the borrower's claim is simply

$$B(s,T) = s(T) - L(s,T)$$

Thus, at the origination of the loan, the value of the borrower's claim is

$$B(s,0) = s(0)\left[1-(1-l_1)e^{-dT}\right] + (1-l_1)c\left(s,T;\frac{P}{1-l_1}\right)$$

[3]Of course, this payoff profile can also be replicated by a long position in a risk-free bond and a short position in a European put contract on the collateral, see Merton(1974).

A.3 Perpetual loans

Proof of Proposition 5

In the case of perpetual loans the value functions L and B are independent of time. Consequently, the pde's become

$$\frac{1}{2}\sigma^2 s^2 L_{ss} + (r-d)sL_s + p^*(s) = rL \tag{A.1}$$

$$\frac{1}{2}\sigma^2 s^2 B_{ss} + (r-d)sB_s + ds - p^*(s) = rB \tag{A.2}$$

where $p^*(s)$ is the instantaneous debt service flow offered by the borrower. If $c - r > 0$, there will be two strategic default regions where the borrower chooses $p^*(s)$ so that

$$L(s) = \begin{cases} (1-l_1)s & \text{for} \quad s \leq \underline{s} \\ P & \text{for} \quad s \geq \overline{s} \end{cases}$$

Substituting into equation A.1 yields

$$\begin{aligned} (r-d_1)s(1-l_1) + p^* &= r(1-l_1)s \quad && \text{for} \quad s \leq \underline{s} \\ p^* &= rP && \text{for} \quad s \geq \overline{s} \end{aligned}$$

which implies

$$p^*(s) = \begin{cases} (1-l_1)sd_1 & \text{for} \quad s \leq \underline{s} \\ rP & \text{for} \quad s \geq \overline{s} \end{cases}$$

If $c - r < 0$, there exists only a lower strategic default region where the borrower offers

$$p^*(s) = (1-l_1)sd_1 \quad \text{for} \quad s \leq \underline{s}$$

Appendix B

Effects of the contract parameters

Results presented here are based on the following parameter values, unless otherwise indicated:

Collateral:

σ	d_0	d_1	l_0	l_1	$s(0)$
0.2	0	0	0	0.1	1.1

Contract:

T	r	p
5	0.05	0

The following combinations of games and contractual indentures are studied:

(a) Terminating default - lower terminating default only

(b) Strategic default - lower terminating default only

(c) Terminating default - lower- and upper terminating default

(d) Strategic default - lower- and upper terminating default

(i) d_1

		(a) (c)				(b) (d)			
d_1	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.492	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.02	1.492	0.822	0.223	0.055	0.645	0.818	0.282	0.000	0.636
0.04	1.492	0.764	0.279	0.057	0.583	0.760	0.340	0.000	0.576
0.06	1.492	0.705	0.337	0.058	0.528	0.701	0.399	0.000	0.521
0.08	1.492	0.647	0.396	0.057	0.478	0.644	0.456	0.000	0.471
0.10	1.492	0.591	0.453	0.056	0.432	0.589	0.511	0.000	0.426
0.12	1.492	0.538	0.509	0.053	0.391	0.537	0.564	0.000	0.386
0.14	1.492	0.489	0.561	0.050	0.354	0.488	0.613	0.000	0.349
0.16	1.492	0.444	0.610	0.046	0.320	0.443	0.658	0.000	0.316
0.18	1.492	0.402	0.656	0.042	0.290	0.402	0.700	0.000	0.286
0.20	1.492	0.364	0.698	0.038	0.262	0.364	0.738	0.000	0.258

(ii) l_1

		(a) (c)				(b) (d)			
l_1	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.492	(a) 0.929	0.171	0.000	0.781	(b) 0.929	0.171	0.000	0.781
		(c) 0.912	0.188	0.000	0.635	(d) 0.912	0.188	0.000	0.635
0.10	1.492	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.20	1.492	0.829	0.171	0.100	0.660	0.807	0.293	0.000	0.625
0.30	1.492	0.778	0.171	0.151	0.619	0.730	0.370	0.000	0.547
0.40	1.492	0.728	0.171	0.201	0.587	0.641	0.459	0.000	0.469
0.50	1.492	0.678	0.171	0.251	0.561	0.543	0.557	0.000	0.388
0.60	1.492	0.628	0.171	0.301	0.539	0.438	0.662	0.000	0.000
0.70	1.492	0.577	0.171	0.352	0.521	0.326	0.774	0.000	0.000
0.80	1.492	0.527	0.171	0.402	0.506	0.220	0.880	0.000	0.000
0.90	1.492	0.477	0.171	0.452	0.493	0.110	0.990	0.000	0.000
1.00	1.492	0.426	0.171	0.503	0.481	0.000	1.100	0.000	0.000

(iii) T

		(a) (c)				(b) (d)			
T	P	L	B	F	%Loan	L	B	F	%Loan
1	1.083	0.938	0.124	0.038	0.777	0.928	0.172	0.000	0.756
2	1.174	0.916	0.142	0.042	0.750	0.907	0.193	0.000	0.732
3	1.271	0.900	0.154	0.046	0.733	0.893	0.207	0.000	0.719
4	1.377	0.887	0.163	0.050	0.719	0.882	0.218	0.000	0.710
5	1.492	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
6	1.616	0.871	0.177	0.052	0.706	0.867	0.233	0.000	0.698
7	1.751	0.864	0.182	0.054	0.701	0.861	0.239	0.000	0.694
8	1.896	0.860	0.187	0.053	0.698	0.855	0.245	0.000	0.691
9	2.054	0.854	0.191	0.055	0.694	0.851	0.249	0.000	0.688
10	2.226	0.850	0.194	0.056	0.690	0.847	0.253	0.000	0.684
11	2.411	0.846	0.198	0.056	0.685	0.842	0.258	0.000	0.678

(iv) σ

		(a) (c)				(b) (d)			
σ	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.492	0.995	0.001	0.104	0.905	0.990	0.110	0.000	0.900
0.05	1.492	1.002	0.026	0.072	0.910	0.986	0.114	0.000	0.895
0.10	1.492	0.967	0.073	0.060	0.868	0.957	0.143	0.000	0.854
0.15	1.492	0.924	0.122	0.054	0.797	0.917	0.183	0.000	0.785
0.20	1.492	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.25	1.492	0.834	0.219	0.047	0.623	0.830	0.270	0.000	0.615
0.30	1.492	0.789	0.267	0.044	0.533	0.786	0.314	0.000	0.527
0.35	1.492	0.745	0.314	0.041	0.438	0.742	0.358	0.000	0.431
0.40	1.492	0.700	0.361	0.039	0.000	0.697	0.403	0.000	0.000
0.45	1.492	0.654	0.409	0.037	0.000	0.651	0.449	0.000	0.000
0.50	1.492	0.603	0.460	0.037	0.000	0.601	0.499	0.000	0.000

(v) p

	(a) (c)					(b) (d)			
p	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.492	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.02	1.369	0.921	0.132	0.047	0.772	0.904	0.196	0.000	0.685
0.04	1.246	0.950	0.105	0.045	0.822	0.924	0.176	0.000	0.656
0.06	1.123	0.971	0.084	0.045	0.861	0.939	0.161	0.000	0.690
0.08	1.000	0.986	0.070	0.044	0.888	0.949	0.151	0.000	0.714
0.10	0.877	0.997	0.059	0.044	0.905	0.956	0.144	0.000	0.741
0.12	0.754	1.005	0.052	0.043	0.915	0.961	0.139	0.000	0.755
0.14	0.631	1.010	0.048	0.042	0.922	0.965	0.135	0.000	0.755
0.16	0.508	1.014	0.045	0.041	0.926	0.968	0.132	0.000	0.769
0.18	0.385	1.018	0.045	0.037	0.930	0.970	0.130	0.000	0.784
0.20	0.262	1.020	0.045	0.035	0.933	0.972	0.128	0.000	0.784

(vi) r

	(a) (c)					(b) (d)			
r	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.162	0.879	0.171	0.050	0.712	0.874	0.226	0.000	0.703
0.01	1.221	0.880	0.171	0.049	0.713	0.874	0.226	0.000	0.703
0.02	1.284	0.879	0.171	0.050	0.712	0.874	0.226	0.000	0.703
0.03	1.350	0.880	0.171	0.049	0.714	0.874	0.226	0.000	0.703
0.04	1.419	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.05	1.492	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.06	1.568	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.07	1.649	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.08	1.733	0.879	0.171	0.050	0.712	0.874	0.226	0.000	0.703
0.09	1.822	0.879	0.171	0.050	0.713	0.874	0.226	0.000	0.703
0.10	1.916	0.879	0.171	0.050	0.712	0.874	0.226	0.000	0.703

(vii) $c - r$

$l_1 = 0$

		(a) (b)				(c) (d)			
c - r	P	L	B	F	%Loan	L	B	F	%Loan
0.000	1.284	0.860	0.240	0.000	0.000	0.860	0.240	0.000	0.000
0.020	1.419	0.908	0.192	0.000	0.717	0.897	0.203	0.000	0.571
0.040	1.568	0.950	0.150	0.000	0.827	0.925	0.175	0.000	0.678
0.060	1.733	0.986	0.114	0.000	0.890	0.947	0.153	0.000	0.741
0.080	1.916	1.017	0.083	0.000	0.930	0.963	0.137	0.000	0.784
0.100	2.117	1.041	0.059	0.000	0.955	0.976	0.124	0.000	0.816
0.120	2.340	1.060	0.040	0.000	0.972	0.985	0.115	0.000	0.851
0.140	2.586	1.073	0.027	0.000	0.983	0.991	0.109	0.000	0.870
0.160	2.858	1.083	0.017	0.000	0.990	0.996	0.104	0.000	0.889
0.180	3.158	1.090	0.010	0.000	0.994	0.999	0.101	0.000	0.889
0.200	3.490	1.094	0.006	0.000	0.997	1.000	0.100	0.000	0.909

(viii) $c - r$

$l_1 = 0.1$

		(a)				(b)			
c - r	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.284	0.823	0.240	0.037	0.000	0.819	0.281	0.000	0.000
0.02	1.419	0.862	0.192	0.046	0.656	0.857	0.243	0.000	0.646
0.04	1.568	0.895	0.150	0.055	0.754	0.889	0.211	0.000	0.745
0.06	1.733	0.921	0.114	0.065	0.808	0.916	0.184	0.000	0.801
0.08	1.916	0.942	0.083	0.075	0.843	0.938	0.162	0.000	0.837
0.10	2.117	0.958	0.059	0.083	0.865	0.954	0.146	0.000	0.860
0.12	2.340	0.970	0.040	0.090	0.878	0.967	0.133	0.000	0.875
0.14	2.586	0.978	0.027	0.095	0.887	0.975	0.125	0.000	0.885
0.16	2.858	0.983	0.017	0.100	0.893	0.981	0.119	0.000	0.891
0.18	3.158	0.986	0.010	0.104	0.896	0.985	0.115	0.000	0.895
0.20	3.490	0.988	0.006	0.106	0.898	0.987	0.113	0.000	0.897

$l_1 = 0.1$

		(c)				(d)			
c - r	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.284	0.823	0.240	0.037	0.000	0.819	0.281	0.000	0.000
0.02	1.419	0.862	0.192	0.046	0.656	0.857	0.243	0.000	0.646
0.04	1.568	0.895	0.150	0.055	0.754	0.889	0.211	0.000	0.745
0.06	1.733	0.914	0.114	0.072	0.761	0.916	0.184	0.000	0.795
0.08	1.916	0.919	0.090	0.091	0.747	0.928	0.164	0.008	0.804
0.10	2.117	0.927	0.073	0.100	0.757	0.936	0.150	0.014	0.816
0.12	2.340	0.934	0.061	0.105	0.773	0.944	0.140	0.016	0.829
0.14	2.586	0.941	0.052	0.107	0.789	0.951	0.133	0.016	0.841
0.16	2.858	0.947	0.045	0.108	0.800	0.957	0.128	0.015	0.852
0.18	3.158	0.952	0.039	0.109	0.800	0.962	0.124	0.014	0.860
0.20	3.490	0.956	0.035	0.109	0.816	0.967	0.121	0.012	0.867

(ix) $c - r$

$l_1 = 0.35$

		(a)				(b)			
c - r	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.284	0.732	0.240	0.128	0.000	0.666	0.434	0.000	0.000
0.02	1.419	0.749	0.192	0.159	0.561	0.681	0.419	0.000	0.466
0.04	1.568	0.757	0.150	0.193	0.631	0.692	0.408	0.000	0.538
0.06	1.733	0.757	0.114	0.229	0.662	0.700	0.400	0.000	0.579
0.08	1.916	0.757	0.083	0.260	0.681	0.706	0.394	0.000	0.604
0.10	2.117	0.752	0.059	0.289	0.689	0.709	0.391	0.000	0.621
0.12	2.340	0.745	0.040	0.315	0.689	0.712	0.388	0.000	0.631
0.14	2.586	0.738	0.027	0.335	0.687	0.713	0.387	0.000	0.638
0.16	2.858	0.732	0.017	0.351	0.682	0.714	0.386	0.000	0.640
0.18	3.158	0.727	0.010	0.363	0.676	0.714	0.386	0.000	0.641
0.20	3.490	0.723	0.006	0.371	0.671	0.715	0.385	0.000	0.650

$l_1 = 0.35$

		(c)				(d)			
c - r	P	L	B	F	%Loan	L	B	F	%Loan
0.00	1.284	0.732	0.240	0.128	0.000	0.666	0.434	0.000	0.000
0.02	1.419	0.749	0.192	0.159	0.561	0.681	0.419	0.000	0.466
0.04	1.568	0.757	0.150	0.193	0.631	0.692	0.408	0.000	0.538
0.06	1.733	0.757	0.114	0.229	0.662	0.700	0.400	0.000	0.579
0.08	1.916	0.757	0.083	0.260	0.681	0.706	0.394	0.000	0.604
0.10	2.117	0.752	0.059	0.289	0.689	0.709	0.391	0.000	0.621
0.12	2.340	0.745	0.040	0.315	0.689	0.712	0.388	0.000	0.631
0.14	2.586	0.738	0.027	0.335	0.687	0.713	0.387	0.000	0.638
0.16	2.858	0.732	0.017	0.351	0.681	0.714	0.386	0.000	0.640
0.18	3.158	0.727	0.010	0.363	0.663	0.714	0.386	0.000	0.641
0.20	3.490	0.722	0.006	0.372	0.642	0.715	0.385	0.000	0.649

Appendix C

Rational default and foreclosure strategies

C.1 Interest-only loans

Results presented here are based on the following parameter values:

Collateral:

σ	d_0	d_1	l_0	l_1	f_0	f_1	$s(0)$
0.2	0	0.1	0	0.1	0	0.04	1.1

Contract:

T	r	$c-r$	p	P
50	0.05	0.03	0.08	1

Key to tables:

$L(s,t)$	=	value of lender's claim
$B(s,t)$	=	value of borrower's claim
$sliq$	=	$(1-l_1)s$, value of collateral net of foreclosure costs
div	=	$d_1 s dt$, dividend per time interval
$b(t)$	=	contractual balance at t
t	=	time remaining to maturity

Table 1: Strategic default

s	sliq	L(s,t):										
2.20	1.980	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.15	1.935	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.10	1.890	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.05	1.845	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2.00	1.800	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.95	1.755	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.90	1.710	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.85	1.665	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.80	1.620	0.997	0.997	0.997	0.997	0.997	0.998	0.998	0.998	0.999	1.000	1.000
1.75	1.575	0.994	0.994	0.994	0.994	0.994	0.994	0.995	0.995	0.996	1.000	1.000
1.70	1.530	0.990	0.990	0.990	0.990	0.990	0.990	0.991	0.991	0.993	0.999	1.000
1.65	1.485	0.985	0.985	0.985	0.985	0.985	0.985	0.986	0.986	0.988	0.996	1.000
1.60	1.440	0.979	0.979	0.979	0.979	0.979	0.979	0.979	0.980	0.983	0.992	1.000
1.55	1.395	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.973	0.976	0.987	1.000
1.50	1.350	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.965	0.968	0.980	1.000
1.45	1.305	0.955	0.955	0.955	0.955	0.955	0.955	0.955	0.956	0.959	0.972	1.000
1.40	1.260	0.945	0.945	0.945	0.945	0.945	0.945	0.945	0.946	0.949	0.963	1.000
1.35	1.215	0.933	0.933	0.933	0.933	0.933	0.933	0.934	0.934	0.938	0.952	1.000
1.30	1.170	0.921	0.921	0.921	0.921	0.921	0.921	0.921	0.922	0.925	0.939	1.000
1.25	1.125	0.907	0.907	0.907	0.907	0.907	0.907	0.907	0.908	0.911	0.925	1.000
1.20	1.080	0.892	0.892	0.892	0.892	0.892	0.892	0.892	0.893	0.896	0.910	1.000
1.15	1.035	0.875	0.875	0.875	0.875	0.875	0.875	0.875	0.876	0.879	0.892	1.000
1.10	0.990	0.857	0.857	0.857	0.857	0.857	0.857	0.857	0.858	0.860	0.873	0.990
1.05	0.945	0.837	0.837	0.837	0.837	0.837	0.837	0.837	0.838	0.840	0.852	0.945
1.00	0.900	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.816	0.818	0.828	0.900
0.95	0.855	0.792	0.792	0.792	0.792	0.792	0.792	0.792	0.792	0.794	0.802	0.855
0.90	0.810	0.766	0.766	0.766	0.766	0.766	0.766	0.766	0.766	0.767	0.774	0.810
0.85	0.765	0.737	0.737	0.737	0.737	0.737	0.737	0.737	0.737	0.738	0.743	0.765
0.80	0.720	0.705	0.705	0.705	0.705	0.705	0.705	0.705	0.705	0.706	0.709	0.720
0.75	0.675	0.670	0.670	0.670	0.670	0.670	0.670	0.670	0.670	0.670	0.672	0.675
0.70	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630
0.65	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585
0.60	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540
0.55	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495	0.495
0.50	0.450	0.450	0.450	0.450	0.450	0.450	0.450	0.450	0.450	0.450	0.450	0.450
0.45	0.405	0.405	0.405	0.405	0.405	0.405	0.405	0.405	0.405	0.405	0.405	0.405
0.40	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360	0.360
0.35	0.315	0.315	0.315	0.315	0.315	0.315	0.315	0.315	0.315	0.315	0.315	0.315
0.30	0.270	0.270	0.270	0.270	0.270	0.270	0.270	0.270	0.270	0.270	0.270	0.270
0.25	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225	0.225
0.20	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180
0.15	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135	0.135
0.10	0.090	0.090	0.090	0.090	0.090	0.090	0.090	0.090	0.090	0.090	0.090	0.090
0.05	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045	0.045
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	b(t):	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	t:	50.00	45.00	40.00	35.00	30.00	25.00	20.00	15.00	10.00	5.000	0.000

B(s,t)

2.20	1.980	1.211	1.211	1.211	1.211	1.210	1.210	1.210	1.209	1.207	1.204	1.200
2.15	1.935	1.161	1.161	1.161	1.160	1.160	1.160	1.159	1.158	1.157	1.154	1.150
2.10	1.890	1.110	1.110	1.110	1.110	1.110	1.110	1.109	1.108	1.107	1.104	1.100
2.05	1.845	1.060	1.060	1.060	1.060	1.060	1.059	1.059	1.058	1.056	1.054	1.050
2.00	1.800	1.010	1.010	1.010	1.010	1.010	1.009	1.009	1.008	1.006	1.004	1.000
1.95	1.755	0.960	0.960	0.960	0.959	0.959	0.959	0.958	0.958	0.956	0.954	0.950
1.90	1.710	0.909	0.909	0.909	0.909	0.909	0.909	0.908	0.907	0.906	0.904	0.900
1.85	1.665	0.860	0.860	0.860	0.859	0.859	0.859	0.858	0.857	0.856	0.854	0.850
1.80	1.620	0.811	0.811	0.811	0.811	0.811	0.811	0.810	0.809	0.807	0.804	0.800
1.75	1.575	0.764	0.764	0.764	0.764	0.764	0.764	0.763	0.762	0.759	0.753	0.750
1.70	1.530	0.718	0.718	0.718	0.718	0.718	0.717	0.717	0.716	0.713	0.705	0.700
1.65	1.485	0.673	0.673	0.673	0.673	0.672	0.672	0.672	0.670	0.667	0.657	0.650
1.60	1.440	0.629	0.629	0.629	0.628	0.628	0.628	0.627	0.626	0.622	0.611	0.600
1.55	1.395	0.585	0.585	0.585	0.585	0.585	0.585	0.584	0.583	0.579	0.566	0.550
1.50	1.350	0.543	0.543	0.543	0.543	0.543	0.543	0.542	0.541	0.537	0.523	0.500
1.45	1.305	0.502	0.502	0.502	0.502	0.502	0.502	0.501	0.500	0.496	0.481	0.450
1.40	1.260	0.462	0.462	0.462	0.462	0.462	0.462	0.461	0.460	0.455	0.440	0.400
1.35	1.215	0.423	0.423	0.423	0.423	0.423	0.423	0.422	0.421	0.417	0.401	0.350
1.30	1.170	0.386	0.386	0.386	0.385	0.385	0.385	0.385	0.383	0.379	0.363	0.300
1.25	1.125	0.349	0.349	0.349	0.349	0.349	0.349	0.348	0.347	0.343	0.327	0.250
1.20	1.080	0.314	0.314	0.314	0.314	0.314	0.314	0.313	0.312	0.308	0.293	0.200
1.15	1.035	0.280	0.280	0.280	0.280	0.280	0.280	0.280	0.278	0.275	0.260	0.150
1.10	0.990	0.248	0.248	0.248	0.248	0.248	0.248	0.247	0.246	0.243	0.229	0.110
1.05	0.945	0.218	0.218	0.218	0.218	0.218	0.218	0.217	0.216	0.213	0.200	0.105
1.00	0.900	0.189	0.189	0.189	0.189	0.189	0.189	0.189	0.188	0.185	0.174	0.100
0.95	0.855	0.163	0.163	0.163	0.163	0.163	0.163	0.162	0.161	0.159	0.149	0.095
0.90	0.810	0.139	0.139	0.139	0.139	0.139	0.139	0.138	0.138	0.136	0.128	0.090
0.85	0.765	0.117	0.117	0.117	0.117	0.117	0.117	0.117	0.116	0.115	0.108	0.085
0.80	0.720	0.099	0.099	0.099	0.099	0.099	0.099	0.098	0.098	0.097	0.092	0.080
0.75	0.675	0.084	0.084	0.084	0.084	0.084	0.084	0.084	0.083	0.082	0.080	0.075
0.70	0.630	0.073	0.073	0.073	0.073	0.073	0.073	0.073	0.073	0.072	0.071	0.070
0.65	0.585	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.067	0.066	0.065
0.60	0.540	0.063	0.063	0.063	0.063	0.063	0.063	0.063	0.062	0.062	0.061	0.060
0.55	0.495	0.058	0.058	0.058	0.058	0.058	0.058	0.057	0.057	0.057	0.056	0.055
0.50	0.450	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.052	0.051	0.050
0.45	0.405	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.047	0.046	0.046	0.045
0.40	0.360	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.041	0.041	0.040
0.35	0.315	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.036	0.036	0.036	0.035
0.30	0.270	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.031	0.030
0.25	0.225	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.026	0.025	0.025
0.20	0.180	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.020	0.020
0.15	0.135	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.015	0.015	0.015
0.10	0.090	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
0.05	0.045	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
s	sliq											
	b(t):	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	t:	50.00	45.00	40.00	35.00	30.00	25.00	20.00	15.00	10.00	5.000	0.000

Debt service payments:

Contractual payment (p*dt) = .0080

2.20	.0220	.0050	.0050	.0050	.0050	.0050	.0050	.0050	.0050	.0050	.0050	1.000
2.15	.0215	.0050	.0050	.0050	.0050	.0050	.0050	.0050	.0050	.0050	.0050	1.000
2.10	.0210	.0051	.0051	.0051	.0051	.0051	.0051	.0051	.0051	.0050	.0050	1.000
2.05	.0205	.0052	.0052	.0052	.0052	.0052	.0052	.0052	.0052	.0051	.0050	1.000
2.00	.0200	.0054	.0054	.0054	.0054	.0054	.0054	.0054	.0053	.0052	.0050	1.000
1.95	.0195	.0058	.0058	.0058	.0058	.0058	.0058	.0058	.0057	.0055	.0051	1.000
1.90	.0190	.0068	.0068	.0068	.0068	.0068	.0068	.0067	.0066	.0061	.0052	1.000
1.85	.0185	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0074	.0054	1.000
1.80	.0180	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0060	1.000
1.75	.0175	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0073	1.000
1.70	.0170	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.65	.0165	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.60	.0160	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.55	.0155	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.50	.0150	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.45	.0145	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.40	.0140	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.35	.0135	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.30	.0130	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.25	.0125	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.20	.0120	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.15	.0115	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	1.000
1.10	.0110	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.9900
1.05	.0105	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.9450
1.00	.0100	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.9000
0.95	.0095	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.8550
0.90	.0090	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.8100
0.85	.0085	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.7650
0.80	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.7200
0.75	.0075	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.0080	.6750
0.70	.0070	.0078	.0078	.0078	.0078	.0078	.0078	.0078	.0078	.0077	.0073	.6300
0.65	.0065	.0060	.0060	.0060	.0060	.0060	.0060	.0060	.0060	.0060	.0059	.5850
0.60	.0060	.0054	.0054	.0054	.0054	.0054	.0054	.0054	.0054	.0054	.0054	.5400
0.55	.0055	.0049	.0049	.0049	.0049	.0049	.0049	.0049	.0049	.0049	.0049	.4950
0.50	.0050	.0045	.0045	.0045	.0045	.0045	.0045	.0045	.0045	.0045	.0045	.4500
0.45	.0045	.0040	.0040	.0040	.0040	.0040	.0040	.0040	.0040	.0040	.0040	.4050
0.40	.0040	.0036	.0036	.0036	.0036	.0036	.0036	.0036	.0036	.0036	.0036	.3600
0.35	.0035	.0031	.0031	.0031	.0031	.0031	.0031	.0031	.0031	.0031	.0031	.3150
0.30	.0030	.0027	.0027	.0027	.0027	.0027	.0027	.0027	.0027	.0027	.0027	.2700
0.25	.0025	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.2250
0.20	.0020	.0018	.0018	.0018	.0018	.0018	.0018	.0018	.0018	.0018	.0018	.1800
0.15	.0015	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.1350
0.10	.0010	.0009	.0009	.0009	.0009	.0009	.0009	.0009	.0009	.0009	.0009	.0900
0.05	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0450
0.00	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
s	div											
	b(t):	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	t:	50.00	45.00	40.00	35.00	30.00	25.00	20.00	15.00	10.00	5.000	0.000

C.2 Partially amortizing loans

Results presented here are based on the following parameter values:

Collateral:

σ	d_0	d_1	l_0	l_1	f_0	f_1	$s(0)$
0.2	0	0.1	0	0.1	0	0.04	1.1

Contract:

T	r	$c-r$	$w-c$	p	P
5	0.05	0.03	0.05	0.1	0.877

Key to tables:

$L(s,t)$ = value of lender's claim
$B(s,t)$ = value of borrower's claim
$sliq$ = $(1-l_1)s$, value of collateral net of foreclosure costs
$b(t)$ = contractual balance at t
t = time remaining to maturity
$K(t)$ = maximum outstanding debt service payments at t
$k(t)$ = actual outstanding debt service payments at t

Loan status:

'*' = terminating default
'+' = prepayment
'-' = technical default
':' = foreclosure due to technical default
'#' = foreclosusre due to cash flow constraint

Table 1: Terminating default

L(s,t):

1.60	1.440	1.056	1.046	1.035	1.023	1.009	0.993	0.976	0.955	0.931	0.905	0.877
1.55	1.395	1.048	1.039	1.028	1.017	1.004	0.990	0.973	0.954	0.931	0.905	0.877
1.50	1.350	1.039	1.031	1.021	1.011	0.999	0.986	0.970	0.952	0.930	0.905	0.877
1.45	1.305	1.029	1.022	1.013	1.003	0.993	0.981	0.967	0.950	0.930	0.905	0.877
1.40	1.260	1.018	1.011	1.004	0.995	0.986	0.975	0.962	0.947	0.929	0.905	0.877
1.35	1.215	1.006	1.000	0.993	0.986	0.977	0.967	0.956	0.943	0.927	0.904	0.877
1.30	1.170	0.993	0.987	0.981	0.974	0.967	0.959	0.949	0.938	0.924	0.904	0.877
1.25	1.125	0.978	0.973	0.968	0.962	0.956	0.949	0.941	0.932	0.920	0.903	0.877
1.20	1.080	0.961	0.957	0.952	0.947	0.942	0.936	0.930	0.923	0.915	0.902	0.877
1.15	1.035	0.942	0.939	0.935	0.931	0.926	0.922	0.917	0.912	0.907	0.898	0.877
1.10	0.990	0.921	0.918	0.915	0.912	0.908	0.905	0.902	0.899	0.896	0.893	0.877
1.05	0.945	0.898	0.895	0.893	0.890	0.888	0.885	0.883	0.881	0.881	0.884	0.877
1.00	0.900	0.871	0.870	0.868	0.866	0.864	0.862	0.861	0.861	0.862	0.869	0.877
0.95	0.855	0.842	0.841	0.839	0.838	0.837	0.836	0.835	0.835	0.838	0.846	0.877
0.90	0.810	0.809	0.808	0.807	0.806	0.805	0.805	0.805	0.805	0.808	0.815	0.877
0.85	0.765	0.772	0.772	0.771	0.770	0.770	0.770	0.770	0.770	0.772	0.776	0.765*
0.80	0.720	0.731	0.730	0.730	0.729	0.729	0.729	0.729	0.730	0.731	0.729	0.720*
0.75	0.675	0.684	0.684	0.683	0.683	0.683	0.683	0.683	0.684	0.684	0.677	0.675*
0.70	0.630	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*
0.65	0.585	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*
0.60	0.540	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

1.60	1.440	0.527	0.539	0.552	0.567	0.583	0.601	0.621	0.644	0.669	0.695	0.723
1.55	1.395	0.483	0.494	0.507	0.521	0.536	0.553	0.572	0.594	0.619	0.645	0.673
1.50	1.350	0.440	0.451	0.462	0.475	0.490	0.506	0.524	0.545	0.569	0.595	0.623
1.45	1.305	0.398	0.408	0.418	0.430	0.444	0.459	0.476	0.496	0.519	0.545	0.573
1.40	1.260	0.357	0.366	0.376	0.387	0.399	0.413	0.429	0.448	0.470	0.495	0.523
1.35	1.215	0.316	0.325	0.334	0.344	0.355	0.368	0.383	0.400	0.421	0.445	0.473
1.30	1.170	0.278	0.285	0.293	0.303	0.313	0.324	0.338	0.353	0.372	0.396	0.423
1.25	1.125	0.240	0.247	0.254	0.262	0.272	0.282	0.293	0.307	0.324	0.346	0.373
1.20	1.080	0.204	0.210	0.217	0.224	0.232	0.241	0.251	0.263	0.277	0.297	0.323
1.15	1.035	0.170	0.176	0.181	0.187	0.194	0.202	0.210	0.220	0.232	0.248	0.273
1.10	0.990	0.138	0.143	0.148	0.153	0.158	0.165	0.171	0.179	0.188	0.200	0.223
1.05	0.945	0.109	0.112	0.116	0.121	0.125	0.130	0.135	0.141	0.147	0.155	0.173
1.00	0.900	0.082	0.085	0.088	0.091	0.095	0.099	0.102	0.106	0.109	0.113	0.123
0.95	0.855	0.058	0.060	0.062	0.065	0.068	0.071	0.073	0.075	0.076	0.075	0.073
0.90	0.810	0.037	0.039	0.041	0.042	0.044	0.046	0.048	0.049	0.048	0.044	0.023
0.85	0.765	0.020	0.021	0.023	0.024	0.025	0.026	0.027	0.027	0.026	0.021	0.000*
0.80	0.720	0.008	0.009	0.009	0.010	0.011	0.012	0.012	0.012	0.010	0.007	0.000*
0.75	0.675	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.002	0.002	0.000	0.000*
0.70	0.630	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.65	0.585	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.60	0.540	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 2: Terminating default: prepayment, technical default

L(s,t):

s	sliq											
1.60	1.440	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.931	0.905	0.877
1.55	1.395	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.945	0.931	0.905	0.877
1.50	1.350	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.951	0.930	0.905	0.877
1.45	1.305	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.950	0.930	0.905	0.877
1.40	1.260	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.947	0.929	0.905	0.877
1.35	1.215	0.992	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.943	0.927	0.904	0.877
1.30	1.170	0.984	0.979	0.970	0.961	0.951	0.942	0.936	0.938	0.924	0.904	0.877
1.25	1.125	0.973	0.969	0.961	0.953	0.943	0.937	0.934	0.932	0.920	0.903	0.877
1.20	1.080	0.960	0.956	0.950	0.942	0.934	0.929	0.927	0.923	0.915	0.902	0.877
1.15	1.035	0.945	0.941	0.936	0.930	0.922	0.918	0.917	0.913	0.907	0.898	0.877
1.10	0.990	0.927	0.924	0.920	0.916	0.909	0.904	0.903	0.899	0.896	0.893	0.877
1.05	0.945	0.906	0.904	0.902	0.898	0.893	0.886	0.885	0.883	0.881	0.884	0.877
1.00	0.900	0.882	0.881	0.880	0.878	0.875	0.865	0.864	0.862	0.862	0.869	0.877
0.95	0.855	0.855:	0.855:	0.855:	0.855:	0.855:	0.839	0.839	0.838	0.839	0.846	0.877
0.90	0.810	0.810*	0.811-	0.811-	0.811-	0.811-	0.810:	0.810:	0.810:	0.810:	0.815	0.877
0.85	0.765	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.773-	0.776-	0.765*
0.80	0.720	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.731-	0.729-	0.720*
0.75	0.675	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.684-	0.677-	0.675*
0.70	0.630	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*
0.65	0.585	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*
0.60	0.540	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

s	sliq											
1.60	1.440	0.560+	0.571+	0.582+	0.593+	0.605+	0.618+	0.631+	0.644+	0.669	0.695	0.723
1.55	1.395	0.510+	0.521+	0.532+	0.543+	0.555+	0.568+	0.581+	0.594	0.619	0.645	0.673
1.50	1.350	0.460+	0.471+	0.482+	0.493+	0.505+	0.518+	0.531+	0.545	0.569	0.595	0.623
1.45	1.305	0.410+	0.421+	0.432+	0.443+	0.455+	0.468+	0.481+	0.496	0.519	0.545	0.573
1.40	1.260	0.360+	0.371+	0.382+	0.393+	0.405+	0.418+	0.431+	0.447	0.470	0.495	0.523
1.35	1.215	0.311	0.321+	0.332+	0.343+	0.355+	0.368+	0.381+	0.399	0.421	0.445	0.473
1.30	1.170	0.264	0.272	0.282	0.293	0.306	0.318	0.332	0.351	0.372	0.396	0.423
1.25	1.125	0.218	0.226	0.235	0.246	0.259	0.270	0.284	0.303	0.324	0.346	0.373
1.20	1.080	0.175	0.182	0.189	0.199	0.213	0.224	0.238	0.257	0.277	0.297	0.323
1.15	1.035	0.135	0.140	0.146	0.155	0.169	0.181	0.193	0.210	0.231	0.248	0.273
1.10	0.990	0.097	0.101	0.106	0.113	0.126	0.139	0.149	0.165	0.186	0.200	0.223
1.05	0.945	0.062	0.064	0.068	0.073	0.083	0.099	0.108	0.121	0.142	0.155	0.173
1.00	0.900	0.030	0.031	0.033	0.036	0.042	0.063	0.069	0.078	0.097	0.113	0.123
0.95	0.855	0.000:	0.000:	0.000:	0.000:	0.000:	0.030	0.033	0.038	0.051	0.075	0.073
0.90	0.810	0.000*	0.000-	0.000-	0.000-	0.000-	0.000:	0.000:	0.000:	0.000:	0.044	0.023
0.85	0.765	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.006-	0.021-	0.000*
0.80	0.720	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.004-	0.007-	0.000*
0.75	0.675	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000-	0.000-	0.000*
0.70	0.630	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.65	0.585	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.60	0.540	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 3: Terminating default: prepayment, technical default, cash flow constraint

L(s,t):

1.60	1.440	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.931	0.905	0.877
1.55	1.395	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.931	0.905	0.877
1.50	1.350	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.931	0.905	0.877
1.45	1.305	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.931	0.905	0.877
1.40	1.260	1.000+	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.930	0.905	0.877
1.35	1.215	0.993	0.990+	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.929	0.904	0.877
1.30	1.170	0.985	0.981	0.979+	0.968+	0.957+	0.945+	0.932+	0.919+	0.928	0.904	0.877
1.25	1.125	0.974	0.971	0.968	0.962	0.957+	0.945+	0.932+	0.919+	0.926	0.903	0.877
1.20	1.080	0.962	0.958	0.956	0.952	0.948	0.945+	0.932+	0.919+	0.922	0.902	0.877
1.15	1.035	0.946	0.944	0.941	0.939	0.936	0.934	0.932+	0.919+	0.906+	0.901	0.877
1.10	0.990	0.928	0.926	0.924	0.923	0.921	0.921	0.921	0.919+	0.906+	0.899	0.877
1.05	0.945	0.907	0.906	0.904	0.903	0.903	0.903	0.905	0.919+	0.906+	0.895	0.877
1.00	0.900	0.882	0.882	0.881	0.881	0.880	0.880	0.882	0.888	0.906+	0.892+	0.877
0.95	0.855	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.906+	0.892+	0.877
0.90	0.810	0.810*	0.810#	0.810#	0.810#	0.810#	0.810#	0.810#	0.810#	0.810#	0.810#	0.877
0.85	0.765	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*	0.765*
0.80	0.720	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*	0.720*
0.75	0.675	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*	0.675*
0.70	0.630	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*	0.630*
0.65	0.585	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*	0.585*
0.60	0.540	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*	0.540*
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

1.60	1.440	0.560+	0.571+	0.582+	0.593+	0.605+	0.618+	0.631+	0.644+	0.667	0.695	0.723
1.55	1.395	0.510+	0.521+	0.532+	0.543+	0.555+	0.568+	0.581+	0.594+	0.617	0.645	0.673
1.50	1.350	0.460+	0.471+	0.482+	0.493+	0.505+	0.518+	0.531+	0.544+	0.567	0.595	0.623
1.45	1.305	0.410+	0.421+	0.432+	0.443+	0.455+	0.468+	0.481+	0.494+	0.516	0.545	0.573
1.40	1.260	0.360+	0.371+	0.382+	0.393+	0.405+	0.418+	0.431+	0.444+	0.465	0.495	0.523
1.35	1.215	0.311	0.321+	0.332+	0.343+	0.355+	0.368+	0.381+	0.394+	0.414	0.444	0.473
1.30	1.170	0.263	0.272	0.282+	0.293+	0.305+	0.318+	0.331+	0.344+	0.362	0.393	0.423
1.25	1.125	0.218	0.225	0.234	0.243	0.255+	0.268+	0.281+	0.294+	0.311	0.342	0.373
1.20	1.080	0.175	0.181	0.188	0.196	0.206	0.218+	0.231+	0.244+	0.259	0.289	0.323
1.15	1.035	0.135	0.139	0.145	0.151	0.159	0.168	0.181+	0.194+	0.208+	0.236	0.273
1.10	0.990	0.097	0.100	0.104	0.109	0.114	0.122	0.131	0.144+	0.158+	0.181	0.223
1.05	0.945	0.062	0.064	0.066	0.069	0.073	0.078	0.084	0.094+	0.108+	0.126	0.173
1.00	0.900	0.030	0.031	0.032	0.034	0.036	0.038	0.041	0.046	0.058+	0.073+	0.123
0.95	0.855	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.008+	0.023+	0.073
0.90	0.810	0.000*	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.023
0.85	0.765	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.80	0.720	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.75	0.675	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.70	0.630	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.65	0.585	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
0.60	0.540	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 4: Strategic default

L(s,t):

s	sliq											
1.60	1.440	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.55	1.395	0.999	0.989	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.50	1.350	0.996	0.987	0.978	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.45	1.305	0.991	0.983	0.975	0.966	0.956	0.945	0.932	0.919	0.906	0.892	0.877
1.40	1.260	0.985	0.978	0.971	0.962	0.954	0.944	0.932	0.919	0.906	0.892	0.877
1.35	1.215	0.977	0.971	0.964	0.957	0.949	0.940	0.931	0.919	0.906	0.892	0.877
1.30	1.170	0.967	0.962	0.956	0.950	0.943	0.935	0.927	0.918	0.906	0.892	0.877
1.25	1.125	0.955	0.950	0.945	0.940	0.934	0.928	0.921	0.914	0.904	0.892	0.877
1.20	1.080	0.941	0.937	0.933	0.928	0.924	0.918	0.913	0.907	0.901	0.892	0.877
1.15	1.035	0.924	0.921	0.918	0.914	0.910	0.906	0.902	0.898	0.894	0.889	0.877
1.10	0.990	0.905	0.903	0.900	0.897	0.894	0.891	0.888	0.885	0.884	0.883	0.877
1.05	0.945	0.883	0.881	0.879	0.877	0.875	0.872	0.870	0.869	0.869	0.872	0.877
1.00	0.900	0.859	0.857	0.855	0.854	0.852	0.850	0.849	0.849	0.850	0.855	0.877
0.95	0.855	0.830	0.829	0.828	0.827	0.826	0.825	0.824	0.824	0.826	0.831	0.855
0.90	0.810	0.798	0.797	0.797	0.796	0.795	0.795	0.794	0.795	0.796	0.801	0.810
0.85	0.765	0.762	0.761	0.761	0.761	0.760	0.760	0.760	0.760	0.761	0.764	0.765
0.80	0.720	0.720	0.720	0.720	0.720	0.720	0.720	0.720	0.720	0.720	0.720	0.720
0.75	0.675	0.675	0.675	0.675	0.675	0.675	0.675	0.675	0.675	0.675	0.675	0.675
0.70	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630	0.630
0.65	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585	0.585
0.60	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540	0.540
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

s	sliq											
1.60	1.440	0.601	0.611	0.621	0.632	0.644	0.656	0.668	0.681	0.694	0.708	0.723
1.55	1.395	0.552	0.561	0.571	0.582	0.594	0.606	0.618	0.631	0.644	0.658	0.673
1.50	1.350	0.505	0.513	0.523	0.532	0.544	0.556	0.568	0.581	0.594	0.608	0.623
1.45	1.305	0.460	0.467	0.475	0.484	0.494	0.506	0.518	0.531	0.544	0.558	0.573
1.40	1.260	0.416	0.423	0.430	0.438	0.447	0.457	0.468	0.481	0.494	0.508	0.523
1.35	1.215	0.374	0.380	0.386	0.393	0.401	0.410	0.420	0.431	0.444	0.458	0.473
1.30	1.170	0.334	0.339	0.345	0.351	0.358	0.365	0.373	0.383	0.394	0.408	0.423
1.25	1.125	0.296	0.300	0.305	0.310	0.316	0.322	0.329	0.337	0.346	0.358	0.373
1.20	1.080	0.260	0.264	0.268	0.272	0.277	0.282	0.287	0.293	0.299	0.308	0.323
1.15	1.035	0.226	0.229	0.233	0.236	0.240	0.244	0.248	0.252	0.256	0.261	0.273
1.10	0.990	0.195	0.198	0.200	0.203	0.206	0.209	0.212	0.215	0.217	0.217	0.223
1.05	0.945	0.167	0.169	0.171	0.173	0.176	0.178	0.180	0.181	0.181	0.178	0.173
1.00	0.900	0.142	0.143	0.145	0.147	0.148	0.150	0.151	0.151	0.150	0.145	0.123
0.95	0.855	0.120	0.121	0.122	0.124	0.125	0.126	0.126	0.126	0.124	0.119	0.095
0.90	0.810	0.102	0.103	0.104	0.104	0.105	0.106	0.106	0.105	0.104	0.099	0.090
0.85	0.765	0.089	0.089	0.089	0.090	0.090	0.090	0.090	0.090	0.089	0.087	0.085
0.80	0.720	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080
0.75	0.675	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075
0.70	0.630	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070	0.070
0.65	0.585	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065
0.60	0.540	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Debt service payments:

Contractual payment (p*dt) = .0021

1.60	.0033	.0020	.0017	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.55	.0032	.0021	.0021	.0019	.0016	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.50	.0031	.0021	.0021	.0021	.0021	.0017	.0015	.0015	.0015	.0015	.0015	.8770
1.45	.0030	.0021	.0021	.0021	.0021	.0021	.0019	.0016	.0015	.0015	.0015	.8770
1.40	.0029	.0021	.0021	.0021	.0021	.0021	.0021	.0020	.0016	.0015	.0015	.8770
1.35	.0028	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0020	.0016	.0015	.8770
1.30	.0027	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0019	.0015	.8770
1.25	.0026	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0016	.8770
1.20	.0025	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.15	.0024	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.10	.0023	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.05	.0022	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.00	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
0.95	.0020	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8550
0.90	.0019	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8100
0.85	.0018	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.7650
0.80	.0017	.0018	.0019	.0019	.0019	.0019	.0020	.0020	.0020	.0019	.0016	.7200
0.75	.0016	.0014	.0014	.0014	.0014	.0014	.0014	.0014	.0014	.0014	.0014	.6750
0.70	.0015	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.0013	.6300
0.65	.0014	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.0012	.5850
0.60	.0013	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.0011	.5400
s	div											
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 5: Strategic default: prepayment, technical default

L(s,t):

1.60	1.440	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.55	1.395	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.50	1.350	0.999	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.45	1.305	0.997	0.988	0.978	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.40	1.260	0.993	0.985	0.976	0.966	0.955	0.945	0.932	0.919	0.906	0.892	0.877
1.35	1.215	0.987	0.979	0.971	0.962	0.952	0.942	0.932	0.919	0.906	0.892	0.877
1.30	1.170	0.979	0.972	0.965	0.956	0.947	0.938	0.929	0.918	0.906	0.892	0.877
1.25	1.125	0.969	0.963	0.957	0.949	0.940	0.932	0.924	0.915	0.905	0.892	0.877
1.20	1.080	0.956	0.952	0.946	0.939	0.931	0.924	0.917	0.909	0.901	0.892	0.877
1.15	1.035	0.942	0.938	0.934	0.928	0.919	0.913	0.907	0.901	0.894	0.889	0.877
1.10	0.990	0.925	0.922	0.919	0.914	0.906	0.899	0.895	0.890	0.884	0.883	0.877
1.05	0.945	0.904	0.903	0.900	0.897	0.891	0.882	0.879	0.875	0.871	0.872	0.877
1.00	0.900	0.881	0.880	0.879	0.877	0.874	0.862	0.860	0.857	0.854	0.855	0.877
0.95	0.855	0.855-	0.855-	0.855-	0.855-	0.855-	0.838	0.837	0.835	0.833	0.832	0.855
0.90	0.810	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.801	0.810
0.85	0.765	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765
0.80	0.720	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720
0.75	0.675	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675
0.70	0.630	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630
0.65	0.585	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585
0.60	0.540	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

1.60	1.440	0.601	0.611	0.621	0.632	0.644	0.656	0.668	0.681	0.694	0.708	0.723
1.55	1.395	0.551	0.561	0.571	0.582	0.594	0.606	0.618	0.631	0.644	0.658	0.673
1.50	1.350	0.501	0.511	0.521	0.532	0.544	0.556	0.568	0.581	0.594	0.608	0.623
1.45	1.305	0.454	0.463	0.472	0.483	0.494	0.506	0.518	0.531	0.544	0.558	0.573
1.40	1.260	0.408	0.416	0.425	0.435	0.445	0.456	0.468	0.481	0.494	0.508	0.523
1.35	1.215	0.364	0.371	0.379	0.388	0.398	0.408	0.419	0.431	0.444	0.458	0.473
1.30	1.170	0.322	0.328	0.336	0.344	0.353	0.362	0.371	0.382	0.394	0.408	0.423
1.25	1.125	0.282	0.287	0.294	0.302	0.311	0.318	0.326	0.335	0.346	0.358	0.373
1.20	1.080	0.244	0.249	0.254	0.261	0.270	0.276	0.283	0.291	0.299	0.308	0.323
1.15	1.035	0.209	0.212	0.217	0.223	0.231	0.237	0.243	0.249	0.256	0.261	0.273
1.10	0.990	0.176	0.178	0.182	0.187	0.194	0.201	0.206	0.211	0.216	0.217	0.223
1.05	0.945	0.146	0.148	0.150	0.153	0.159	0.168	0.171	0.175	0.179	0.178	0.173
1.00	0.900	0.119	0.120	0.121	0.123	0.127	0.138	0.140	0.143	0.146	0.145	0.123
0.95	0.855	0.095-	0.095-	0.095-	0.095-	0.095-	0.112	0.113	0.115	0.117	0.118	0.095
0.90	0.810	0.090-	0.090-	0.090-	0.090-	0.090-	0.090-	0.090-	0.090-	0.090-	0.099	0.090
0.85	0.765	0.085-	0.085-	0.085-	0.085-	0.085-	0.085-	0.085-	0.085-	0.085-	0.085-	0.085
0.80	0.720	0.080-	0.080-	0.080-	0.080-	0.080-	0.080-	0.080-	0.080-	0.080-	0.080-	0.080
0.75	0.675	0.075-	0.075-	0.075-	0.075-	0.075-	0.075-	0.075-	0.075-	0.075-	0.075-	0.075
0.70	0.630	0.070-	0.070-	0.070-	0.070-	0.070-	0.070-	0.070-	0.070-	0.070-	0.070-	0.070
0.65	0.585	0.065-	0.065-	0.065-	0.065-	0.065-	0.065-	0.065-	0.065-	0.065-	0.065-	0.065
0.60	0.540	0.060-	0.060-	0.060-	0.060-	0.060-	0.060-	0.060-	0.060-	0.060-	0.060-	0.060
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Debt service payments:

Contractual payment (p*dt) = .0021

1.60	.0033	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.55	.0032	.0018	.0016	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.50	.0031	.0021	.0021	.0018	.0016	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.45	.0030	.0021	.0021	.0021	.0021	.0019	.0016	.0015	.0015	.0015	.0015	.8770
1.40	.0029	.0021	.0021	.0021	.0021	.0021	.0021	.0017	.0016	.0015	.0015	.8770
1.35	.0028	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0019	.0016	.0015	.8770
1.30	.0027	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0018	.0015	.8770
1.25	.0026	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0016	.8770
1.20	.0025	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.15	.0024	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.10	.0023	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.05	.0022	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.00	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
0.95	.0020	.0041-	.0042-	.0044-	.0046-	.0050-	.0021	.0021	.0021	.0021	.0021	.8550
0.90	.0019	.0018-	.0018-	.0018-	.0018-	.0019-	.0036-	.0037-	.0039-	.0041-	.0021	.8100
0.85	.0018	.0016-	.0016-	.0016-	.0016-	.0016-	.0017-	.0017-	.0017-	.0017-	.0025-	.7650
0.80	.0017	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.7200
0.75	.0016	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.6750
0.70	.0015	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.6300
0.65	.0014	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.5850
0.60	.0013	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.5400
s	div											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 6: Strategic default: technical default, cash flow constraint

L(s,t):

s	sliq											
1.60	1.440	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.55	1.395	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.50	1.350	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.45	1.305	0.997	0.989	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.40	1.260	0.993	0.986	0.977	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.35	1.215	0.987	0.981	0.973	0.965	0.955	0.945	0.932	0.919	0.906	0.892	0.877
1.30	1.170	0.980	0.974	0.967	0.960	0.952	0.943	0.932	0.919	0.906	0.892	0.877
1.25	1.125	0.970	0.965	0.959	0.953	0.946	0.938	0.929	0.919	0.906	0.892	0.877
1.20	1.080	0.958	0.954	0.949	0.944	0.939	0.932	0.924	0.916	0.905	0.892	0.877
1.15	1.035	0.943	0.940	0.936	0.932	0.928	0.923	0.917	0.910	0.901	0.891	0.877
1.10	0.990	0.925	0.923	0.921	0.918	0.915	0.911	0.906	0.901	0.895	0.888	0.877
1.05	0.945	0.905	0.904	0.902	0.900	0.898	0.896	0.893	0.889	0.885	0.881	0.877
1.00	0.900	0.881	0.881	0.880	0.879	0.878	0.877	0.875	0.874	0.872	0.869	0.877
0.95	0.855	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855
0.90	0.810	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810#	0.810#	0.810
0.85	0.765	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765
0.80	0.720	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720
0.75	0.675	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675
0.70	0.630	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	[illegible]	[illegible]	0.630-	0.630-	0.630
0.65	0.585	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585
0.60	0.540	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

s	sliq											
1.60	1.440	0.559	0.573	0.587	0.603	0.619	0.637	0.655	0.674	0.693	0.708	0.723
1.55	1.395	0.506	0.520	0.534	0.550	0.566	0.584	0.603	0.622	0.642	0.658	0.673
1.50	1.350	0.453	0.466	0.481	0.496	0.512	0.530	0.549	0.570	0.590	0.608	0.623
1.45	1.305	0.402	0.414	0.427	0.442	0.458	0.476	0.496	0.517	0.538	0.558	0.573
1.40	1.260	0.353	0.363	0.375	0.388	0.404	0.422	0.441	0.463	0.486	0.507	0.523
1.35	1.215	0.305	0.314	0.324	0.336	0.350	0.367	0.386	0.408	0.432	0.456	0.473
1.30	1.170	0.258	0.267	0.276	0.286	0.299	0.313	0.331	0.352	0.377	0.404	0.423
1.25	1.125	0.214	0.221	0.229	0.238	0.249	0.261	0.277	0.296	0.321	0.351	0.373
1.20	1.080	0.172	0.178	0.184	0.192	0.201	0.211	0.224	0.241	0.264	0.296	0.323
1.15	1.035	0.133	0.137	0.142	0.148	0.155	0.164	0.174	0.188	0.208	0.238	0.273
1.10	0.990	0.096	0.099	0.102	0.107	0.112	0.118	0.126	0.137	0.153	0.179	0.223
1.05	0.945	0.061	0.063	0.066	0.069	0.072	0.076	0.082	0.089	0.099	0.119	0.173
1.00	0.900	0.030	0.031	0.033	0.034	0.036	0.038	0.040	0.044	0.049	0.060	0.123
0.95	0.855	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.095
0.90	0.810	0.017-	0.017-	0.017-	0.018-	0.018-	0.018-	0.018-	0.016-	0.000#	0.000#	0.090
0.85	0.765	0.029-	0.029-	0.029-	0.030-	0.030-	0.031-	0.031-	0.029-	0.025-	0.033-	0.085
0.80	0.720	0.037-	0.038-	0.038-	0.039-	0.040-	0.040-	0.041-	0.040-	0.043-	0.054-	0.080
0.75	0.675	0.044-	0.044-	0.045-	0.045-	0.046-	0.047-	0.048-	0.049-	0.054-	0.065-	0.075
0.70	0.630	0.048-	0.048-	0.049-	0.050-	0.050-	0.051-	0.053-	0.055-	0.060-	0.067-	0.070
0.65	0.585	0.050-	0.050-	0.051-	0.052-	0.052-	0.054-	0.055-	0.057-	0.061-	0.064-	0.065
0.60	0.540	0.050-	0.051-	0.051-	0.052-	0.053-	0.054-	0.055-	0.057-	0.059-	0.060-	0.060
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

```
Debt service payments:

Contractual payment (p*dt) = .0021
```

1.60	.0033	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.55	.0032	.0017	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.50	.0031	.0021	.0019	.0016	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.45	.0030	.0021	.0021	.0021	.0017	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.40	.0029	.0021	.0021	.0021	.0021	.0019	.0016	.0015	.0015	.0015	.0015	.8770
1.35	.0028	.0021	.0021	.0021	.0021	.0021	.0021	.0016	.0015	.0015	.0015	.8770
1.30	.0027	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0017	.0015	.0015	.8770
1.25	.0026	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0017	.0015	.8770
1.20	.0025	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0017	.8770
1.15	.0024	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.10	.0023	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.05	.0022	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.00	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
0.95	.0020	.0041#	.0042#	.0043#	.0044#	.0045#	.0021#	.0021#	.0021#	.0021#	.0021#	.8550
0.90	.0019	.0018-	.0018-	.0018-	.0018-	.0018-	.0018-	.0019-	.0019-	.0019#	.0019#	.8100
0.85	.0018	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.7650
0.80	.0017	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.7200
0.75	.0016	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.6750
0.70	.0015	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.6300
0.65	.0014	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.5850
0.60	.0013	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.5400
s	div											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 7: Strategic default: prepayment, technical default, cash flow constraint

L(s,t):

1.60	1.440	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.55	1.395	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.50	1.350	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.45	1.305	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.40	1.260	0.997	0.989	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.35	1.215	0.991	0.985	0.976	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
1.30	1.170	0.983	0.978	0.971	0.965	0.956	0.945	0.932	0.919	0.906	0.892	0.877
1.25	1.125	0.972	0.968	0.963	0.958	0.953	0.943	0.930	0.919	0.906	0.892	0.877
1.20	1.080	0.960	0.956	0.952	0.947	0.944	0.938	0.925	0.917	0.906	0.892	0.877
1.15	1.035	0.944	0.942	0.939	0.935	0.932	0.929	0.918	0.912	0.904	0.892	0.877
1.10	0.990	0.927	0.925	0.922	0.920	0.917	0.915	0.908	0.904	0.900	0.890	0.877
1.05	0.945	0.906	0.904	0.903	0.901	0.900	0.898	0.894	0.892	0.897	0.886	0.877
1.00	0.900	0.882	0.881	0.880	0.880	0.879	0.878	0.876	0.875	0.891	0.880	0.877
0.95	0.855	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.855#	0.867	0.855
0.90	0.810	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.810-	0.812	0.810
0.85	0.765	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765-	0.765
0.80	0.720	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720-	0.720
0.75	0.675	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675-	0.675
0.70	0.630	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630-	0.630
0.65	0.585	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585-	0.585
0.60	0.540	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540-	0.540
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

B(s,t):

1.60	1.440	0.568	0.581	0.595	0.610	0.626	0.642	0.660	0.677	0.693	0.708	0.723
1.55	1.395	0.516	0.529	0.543	0.557	0.573	0.590	0.608	0.626	0.643	0.658	0.673
1.50	1.350	0.463	0.476	0.490	0.504	0.520	0.537	0.556	0.575	0.592	0.608	0.623
1.45	1.305	0.411	0.423	0.436	0.451	0.467	0.484	0.503	0.523	0.541	0.558	0.573
1.40	1.260	0.360	0.371	0.383	0.398	0.413	0.430	0.449	0.470	0.490	0.508	0.523
1.35	1.215	0.311	0.321	0.332	0.345	0.360	0.376	0.394	0.417	0.438	0.457	0.473
1.30	1.170	0.264	0.272	0.282	0.293	0.307	0.322	0.339	0.362	0.386	0.406	0.423
1.25	1.125	0.219	0.226	0.234	0.244	0.255	0.269	0.285	0.307	0.332	0.355	0.373
1.20	1.080	0.176	0.182	0.189	0.197	0.206	0.218	0.232	0.252	0.277	0.302	0.323
1.15	1.035	0.135	0.140	0.145	0.152	0.159	0.169	0.181	0.198	0.222	0.248	0.273
1.10	0.990	0.098	0.101	0.105	0.110	0.115	0.122	0.132	0.145	0.167	0.193	0.223
1.05	0.945	0.063	0.065	0.067	0.071	0.074	0.079	0.085	0.095	0.112	0.139	0.173
1.00	0.900	0.031	0.032	0.033	0.035	0.037	0.039	0.043	0.048	0.062	0.086	0.123
0.95	0.855	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.000#	0.047	0.095
0.90	0.810	0.017-	0.018-	0.018-	0.019-	0.020-	0.021-	0.022-	0.025-	0.036-	0.055	0.090
0.85	0.765	0.030-	0.030-	0.031-	0.032-	0.033-	0.035-	0.037-	0.042-	0.052-	0.063-	0.085
0.80	0.720	0.039-	0.039-	0.040-	0.041-	0.043-	0.045-	0.048-	0.053-	0.062-	0.068-	0.080
0.75	0.675	0.045-	0.046-	0.047-	0.048-	0.050-	0.052-	0.055-	0.060-	0.066-	0.070-	0.075
0.70	0.630	0.049-	0.050-	0.051-	0.052-	0.054-	0.056-	0.059-	0.062-	0.066-	0.069-	0.070
0.65	0.585	0.051-	0.052-	0.053-	0.054-	0.055-	0.057-	0.059-	0.061-	0.063-	0.065-	0.065
0.60	0.540	0.051-	0.052-	0.053-	0.054-	0.055-	0.056-	0.057-	0.058-	0.059-	0.060-	0.060
s	sliq											
	b(t):	1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
	t:	5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Debt service payments:

Contractual payment (p*dt) = .0021

1.60	.0033	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.55	.0032	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.50	.0031	.0016	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.45	.0030	.0021	.0017	.0017	.0015	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.40	.0029	.0021	.0021	.0021	.0016	.0015	.0015	.0015	.0015	.0015	.0015	.8770
1.35	.0028	.0021	.0021	.0021	.0021	.0016	.0015	.0016	.0015	.0015	.0015	.8770
1.30	.0027	.0021	.0021	.0021	.0021	.0021	.0019	.0021	.0016	.0015	.0015	.8770
1.25	.0026	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0020	.0016	.0015	.8770
1.20	.0025	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0020	.0016	.8770
1.15	.0024	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0018	.8770
1.10	.0023	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.05	.0022	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
1.00	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.0021	.8770
0.95	.0020	.0040#	.0041#	.0042#	.0043#	.0044#	.0021#	.0021#	.0021#	.0021#	.0000	.8550
0.90	.0019	.0018-	.0018-	.0018-	.0018-	.0018-	.0018-	.0019-	.0019-	.0017-	.0000	.8100
0.85	.0018	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0016-	.0014-	.7650
0.80	.0017	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.0015-	.7200
0.75	.0016	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.0014-	.6750
0.70	.0015	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.0013-	.6300
0.65	.0014	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.0012-	.5850
0.60	.0013	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.0011-	.5400
s	div											
b(t):		1.000	0.990	0.979	0.968	0.957	0.945	0.932	0.919	0.906	0.892	0.877
t:		5.000	4.500	4.000	3.500	3.000	2.500	2.000	1.500	1.000	0.500	0.000

Table 8: Penalized default, current default rule

L(s,k,t): s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8998	0.8972	0.8954	0.8945	0.9900
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8992	0.8966	0.8948	0.8939	0.9900
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8986	0.8960	0.8942	0.8934	0.9900
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8980	0.8954	0.8936	0.8928	0.9900
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.9004	0.8974	0.8949	0.8930	0.8922	0.9900
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.8998	0.8968	0.8943	0.8924	0.8916	0.9900
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.8992	0.8962	0.8937	0.8918	0.8910	0.9900
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.8986	0.8957	0.8931	0.8912	0.8904	0.9900
0.2113	0.0000	0.0000	0.0000	0.0000	0.9011	0.8980	0.8951	0.8925	0.8907	0.8898	0.9900
0.1937	0.0000	0.0000	0.0000	0.0000	0.9005	0.8974	0.8945	0.8919	0.8901	0.8893	0.9900
0.1761	0.0000	0.0000	0.0000	0.0000	0.8999	0.8968	0.8939	0.8913	0.8895	0.8887	0.9900
0.1585	0.0000	0.0000	0.0000	0.9023	0.8993	0.8962	0.8933	0.8907	0.8889	0.8881	0.9900
0.1409	0.0000	0.0000	0.0000	0.9018	0.8987	0.8957	0.8927	0.8902	0.8883	0.8875	0.9900
0.1232	0.0000	0.0000	0.0000	0.9012	0.8981	0.8951	0.8921	0.8896	0.8877	0.8869	0.9900
0.1056	0.0000	0.0000	0.9035	0.9006	0.8975	0.8945	0.8915	0.8890	0.8871	0.8863	0.9827
0.0880	0.0000	0.0000	0.9029	0.9000	0.8970	0.8939	0.8910	0.8884	0.8866	0.8857	0.9651
0.0704	0.0000	0.0000	0.9023	0.8994	0.8964	0.8933	0.8904	0.8878	0.8860	0.8851	0.9475
0.0528	0.0000	0.0000	0.9036	0.9006	0.8976	0.8946	0.8916	0.8891	0.8872	0.8864	0.9299
0.0352	0.0000	0.9052	0.9024	0.8995	0.8964	0.8934	0.8904	0.8879	0.8860	0.8852	0.9123
0.0176	0.0000	0.9054	0.9026	0.8996	0.8966	0.8935	0.8906	0.8880	0.8862	0.8854	0.8947
0.0000	0.9054	0.9027	0.8999	0.8970	0.8939	0.8909	0.8880	0.8854	0.8835	0.8827	0.8770
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

B(s,k,t): s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2027	0.2053	0.2070	0.2078	0.1100
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2033	0.2058	0.2076	0.2084	0.1100
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2039	0.2064	0.2082	0.2090	0.1100
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2045	0.2070	0.2088	0.2096	0.1100
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.2022	0.2051	0.2076	0.2094	0.2102	0.1100
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.2028	0.2057	0.2082	0.2100	0.2107	0.1100
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.2034	0.2063	0.2088	0.2106	0.2113	0.1100
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.2040	0.2068	0.2094	0.2112	0.2119	0.1100
0.2113	0.0000	0.0000	0.0000	0.0000	0.2015	0.2045	0.2074	0.2099	0.2117	0.2125	0.1100
0.1937	0.0000	0.0000	0.0000	0.0000	0.2021	0.2051	0.2080	0.2105	0.2123	0.2131	0.1100
0.1761	0.0000	0.0000	0.0000	0.0000	0.2027	0.2057	0.2086	0.2111	0.2129	0.2137	0.1100
0.1585	0.0000	0.0000	0.0000	0.2003	0.2033	0.2063	0.2092	0.2117	0.2135	0.2143	0.1100
0.1409	0.0000	0.0000	0.0000	0.2009	0.2039	0.2069	0.2098	0.2123	0.2141	0.2149	0.1100
0.1232	0.0000	0.0000	0.0000	0.2015	0.2045	0.2075	0.2104	0.2129	0.2147	0.2154	0.1100
0.1056	0.0000	0.0000	0.1992	0.2021	0.2050	0.2081	0.2109	0.2135	0.2153	0.2160	0.1173
0.0880	0.0000	0.0000	0.1997	0.2026	0.2056	0.2087	0.2115	0.2141	0.2159	0.2166	0.1349
0.0704	0.0000	0.0000	0.2003	0.2032	0.2062	0.2092	0.2121	0.2146	0.2164	0.2172	0.1525
0.0528	0.0000	0.0000	0.1991	0.2020	0.2050	0.2080	0.2109	0.2134	0.2152	0.2160	0.1701
0.0352	0.0000	0.1975	0.2003	0.2032	0.2062	0.2092	0.2121	0.2146	0.2164	0.2171	0.1877
0.0176	0.0000	0.1974	0.2001	0.2030	0.2060	0.2090	0.2119	0.2144	0.2162	0.2170	0.2053
0.0000	0.1951	0.1977	0.2004	0.2034	0.2063	0.2094	0.2122	0.2148	0.2166	0.2173	0.2230
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Debt service payments: s = 1.10
Dividend (d_1*dt) = .0023, Contractual payment (p*dt) = .0021

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0118	0.0118	0.0118	0.0118	0.9900
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0112	0.0112	0.0112	0.0112	0.9900
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0106	0.0106	0.0106	0.0106	0.9900
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0100	0.0100	0.0100	0.0100	0.9900
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0095	0.0095	0.0095	0.0095	0.0095	0.9900
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0089	0.0089	0.0089	0.0089	0.0089	0.9900
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0083	0.0083	0.0083	0.0083	0.0083	0.9900
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0077	0.0077	0.0077	0.0077	0.0077	0.9900
0.2113	0.0000	0.0000	0.0000	0.0000	0.0071	0.0071	0.0071	0.0071	0.0071	0.0071	0.9900
0.1937	0.0000	0.0000	0.0000	0.0000	0.0065	0.0065	0.0065	0.0065	0.0065	0.0065	0.9900
0.1761	0.0000	0.0000	0.0000	0.0000	0.0059	0.0059	0.0059	0.0059	0.0059	0.0059	0.9900
0.1585	0.0000	0.0000	0.0000	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.0054	0.9900
0.1409	0.0000	0.0000	0.0000	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.9900
0.1232	0.0000	0.0000	0.0000	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.0042	0.9900
0.1056	0.0000	0.0000	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.0036	0.9827
0.0880	0.0000	0.0000	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.9651
0.0704	0.0000	0.0000	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.9475
0.0528	0.0000	0.0000	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.0037	0.9299
0.0352	0.0000	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.9123
0.0176	0.0000	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.8947
0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.8770
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Critical payments: s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1020	0.1046	0.1065	0.1073	0.0000
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1020	0.1046	0.1065	0.1073	0.0000
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1020	0.1046	0.1065	0.1073	0.0000
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1020	0.1046	0.1065	0.1073	0.0000
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.2113	0.0000	0.0000	0.0000	0.0000	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.1937	0.0000	0.0000	0.0000	0.0000	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.1761	0.0000	0.0000	0.0000	0.0000	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.1585	0.0000	0.0000	0.0000	0.0930	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.1409	0.0000	0.0000	0.0000	0.0930	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.1232	0.0000	0.0000	0.0000	0.0930	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.1056	0.0000	0.0000	0.0901	0.0930	0.0961	0.0991	0.1020	0.1046	0.1065	0.1073	0.0000
0.0880	0.0000	0.0000	0.0901	0.0930	0.0961	0.0991	0.1020	0.1046	0.1065	0.0970	0.0000
0.0704	0.0000	0.0000	0.0901	0.0930	0.0961	0.0991	0.1020	0.1043	0.0926	0.0794	0.0000
0.0528	0.0000	0.0000	0.0901	0.0930	0.0961	0.0991	0.0970	0.0866	0.0750	0.0618	0.0000
0.0352	0.0000	0.0873	0.0901	0.0930	0.0961	0.0890	0.0794	0.0690	0.0574	0.0442	0.0000
0.0176	0.0000	0.0873	0.0901	0.0887	0.0803	0.0714	0.0618	0.0514	0.0398	0.0265	0.0000
0.0000	0.0867	0.0892	0.0813	0.0732	0.0648	0.0558	0.0463	0.0359	0.0243	0.0110	0.0000
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Table 9: Penalized default: current default rule, prepayment, technical default, cash flow constraint

L(s,t): s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.9178-	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.9178-	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.9178-	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.9178-	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.2113	0.0000	0.0000	0.0000	0.0000	0.9216-	0.9178-	0.9156-	0.9075-	0.9005-	0.8927-	0.9900
0.1937	0.0000	0.0000	0.0000	0.0000	0.9216-	0.9178-	0.9156-	0.9075-	0.9005	0.8927	0.9900
0.1761	0.0000	0.0000	0.0000	0.0000	0.9216-	0.9178-	0.9156-	0.9075	0.9005	0.8927	0.9900
0.1585	0.0000	0.0000	0.0000	0.9242-	0.9216-	0.9178-	0.9156	0.9075	0.9005	0.8927	0.9900
0.1409	0.0000	0.0000	0.0000	0.9242-	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9900
0.1232	0.0000	0.0000	0.0000	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9900
0.1056	0.0000	0.0000	0.9268	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9827
0.0880	0.0000	0.0000	0.9268	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9651
0.0704	0.0000	0.0000	0.9268	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9475
0.0528	0.0000	0.0000	0.9268	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9299
0.0352	0.0000	0.9288	0.9268	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.9123
0.0176	0.0000	0.9288	0.9268	0.9242	0.9216	0.9178	0.9156	0.9075	0.9005	0.8927	0.8947
0.0000	0.9287	0.9266	0.9247	0.9220	0.9194	0.9157	0.9135	0.9054	0.8984	0.8906	0.8770
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

B(s,t): s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.1220-	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.1220-	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.1220-	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.1220-	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.2113	0.0000	0.0000	0.0000	0.0000	0.1147-	0.1220-	0.1313-	0.1453-	0.1674-	0.1944-	0.1100
0.1937	0.0000	0.0000	0.0000	0.0000	0.1147-	0.1220-	0.1313-	0.1453-	0.1674	0.1944	0.1100
0.1761	0.0000	0.0000	0.0000	0.0000	0.1147-	0.1220-	0.1313-	0.1453	0.1674	0.1944	0.1100
0.1585	0.0000	0.0000	0.0000	0.1088-	0.1147-	0.1220-	0.1313	0.1453	0.1674	0.1944	0.1100
0.1409	0.0000	0.0000	0.0000	0.1088-	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1100
0.1232	0.0000	0.0000	0.0000	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1100
0.1056	0.0000	0.0000	0.1040	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1173
0.0880	0.0000	0.0000	0.1040	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1349
0.0704	0.0000	0.0000	0.1040	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1525
0.0528	0.0000	0.0000	0.1040	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1701
0.0352	0.0000	0.0998	0.1040	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.1877
0.0176	0.0000	0.0998	0.1040	0.1088	0.1147	0.1220	0.1313	0.1453	0.1674	0.1944	0.2053
0.0000	0.0961	0.0997	0.1038	0.1087	0.1146	0.1219	0.1311	0.1451	0.1673	0.1942	0.2230
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Debt service payments: s = 1.10

Dividend (d_1*dt) = .0023, Contractual payment (p*dt) = .0021

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.2113	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.0021-	0.0021-	0.9900
0.1937	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021-	0.0021	0.0021	0.9900
0.1761	0.0000	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021	0.0021	0.0021	0.9900
0.1585	0.0000	0.0000	0.0000	0.0021-	0.0021-	0.0021-	0.0021	0.0021	0.0021	0.0021	0.9900
0.1409	0.0000	0.0000	0.0000	0.0021-	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9900
0.1232	0.0000	0.0000	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9900
0.1056	0.0000	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9827
0.0880	0.0000	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9651
0.0704	0.0000	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9475
0.0528	0.0000	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9299
0.0352	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.9123
0.0176	0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.8947
0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.8770
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Critical payments: s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743-	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743-	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743-	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743-	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.2113	0.0000	0.0000	0.0000	0.0000	0.0706-	0.0743-	0.0765-	0.0846-	0.0916-	0.0994-	0.0000
0.1937	0.0000	0.0000	0.0000	0.0000	0.0706-	0.0743-	0.0765-	0.0846-	0.0916	0.0994	0.0000
0.1761	0.0000	0.0000	0.0000	0.0000	0.0706-	0.0743-	0.0765-	0.0846	0.0916	0.0994	0.0000
0.1585	0.0000	0.0000	0.0000	0.0680-	0.0706-	0.0743-	0.0765	0.0846	0.0916	0.0994	0.0000
0.1409	0.0000	0.0000	0.0000	0.0680-	0.0706	0.0743	0.0765	0.0846	0.0916	0.0994	0.0000
0.1232	0.0000	0.0000	0.0000	0.0680	0.0706	0.0743	0.0765	0.0846	0.0916	0.0994	0.0000
0.1056	0.0000	0.0000	0.0653	0.0680	0.0706	0.0743	0.0765	0.0846	0.0916	0.0994	0.0000
0.0880	0.0000	0.0000	0.0653	0.0680	0.0706	0.0743	0.0765	0.0846	0.0916	0.0891	0.0000
0.0704	0.0000	0.0000	0.0653	0.0680	0.0706	0.0743	0.0765	0.0843	0.0777	0.0715	0.0000
0.0528	0.0000	0.0000	0.0653	0.0680	0.0706	0.0743	0.0715	0.0667	0.0601	0.0539	0.0000
0.0352	0.0000	0.0634	0.0653	0.0680	0.0706	0.0642	0.0539	0.0491	0.0425	0.0363	0.0000
0.0176	0.0000	0.0634	0.0653	0.0637	0.0548	0.0466	0.0363	0.0315	0.0249	0.0187	0.0000
0.0000	0.0634	0.0653	0.0566	0.0482	0.0393	0.0311	0.0209	0.0160	0.0095	0.0032	0.0000
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Table 10: Penalized default: outstanding payment rule, prepayment, technical default, cash flow constraint

L(s,t): s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.2113	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.1937	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.1761	0.0000	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.1585	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.1409	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.1232	0.0000	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900
0.1056	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9827
0.0880	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9797#	0.9651
0.0704	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9900#	0.9896#	0.9761#	0.9621#	0.9475
0.0528	0.0000	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9850#	0.9720#	0.9585#	0.9445#	0.9299
0.0352	0.0000	0.9900#	0.9900#	0.9900#	0.9900#	0.9799#	0.9674#	0.9544#	0.9409#	0.9269#	0.9123
0.0176	0.0000	0.9900#	0.9900#	0.9857#	0.9742#	0.9623#	0.9498#	0.9368#	0.9233#	0.9093#	0.8947
0.0000	0.9287	0.9266	0.9247	0.9220	0.9194	0.9157	0.9135	0.9054	0.8984	0.8906	0.8770
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

B(s,t): s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.2113	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.1937	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.1761	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.1585	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.1409	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.1232	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1100
0.1056	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.1173
0.0880	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0103#	0.1349
0.0704	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0004#	0.0139#	0.0279#	0.1525
0.0528	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0050#	0.0180#	0.0315#	0.0455#	0.1701
0.0352	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0101#	0.0226#	0.0356#	0.0491#	0.0631#	0.1877
0.0176	0.0000	0.0000#	0.0000#	0.0043#	0.0158#	0.0277#	0.0402#	0.0532#	0.0667#	0.0807#	0.2053
0.0000	0.0961	0.0997	0.1038	0.1087	0.1146	0.1219	0.1311	0.1451	0.1673	0.1942	0.2230
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Debt service payments: s = 1.10
Dividend (d_1*dt) = .0023, Contractual payment (p*dt) = .0021

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.2113	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.1937	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.1761	0.0000	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.1585	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.1409	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.1232	0.0000	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9900
0.1056	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9827
0.0880	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9651
0.0704	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9475
0.0528	0.0000	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9299
0.0352	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.9123
0.0176	0.0000	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.0000#	0.8947
0.0000	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.0021	0.8770
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Critical payments: s = 1.10

0.3521	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.3345	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.3169	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.2993	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.2817	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.2641	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.2465	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.2289	0.0000	0.0000	0.0000	0.0000	0.0000	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.2113	0.0000	0.0000	0.0000	0.0000	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.1937	0.0000	0.0000	0.0000	0.0000	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.1761	0.0000	0.0000	0.0000	0.0000	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.1585	0.0000	0.0000	0.0000	0.0680#	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.1409	0.0000	0.0000	0.0000	0.0680#	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.1232	0.0000	0.0000	0.0000	0.0680#	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.1056	0.0000	0.0000	0.0653#	0.0680#	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0994#	0.0000
0.0880	0.0000	0.0000	0.0653#	0.0680#	0.0706#	0.0743#	0.0765#	0.0846#	0.0916#	0.0891#	0.0000
0.0704	0.0000	0.0000	0.0653#	0.0680#	0.0706#	0.0743#	0.0765#	0.0843#	0.0777#	0.0715#	0.0000
0.0528	0.0000	0.0000	0.0653#	0.0680#	0.0706#	0.0743#	0.0715#	0.0667#	0.0601#	0.0539#	0.0000
0.0352	0.0000	0.0634#	0.0653#	0.0680#	0.0706#	0.0642#	0.0539#	0.0491#	0.0425#	0.0363#	0.0000
0.0176	0.0000	0.0634#	0.0653#	0.0637#	0.0548#	0.0466#	0.0363#	0.0315#	0.0249#	0.0187#	0.0000
0.0000	0.0634	0.0653	0.0566	0.0482	0.0393	0.0311	0.0209	0.0160	0.0095	0.0032	0.0000
k(t)											
K(t):	0.0000	0.0517	0.1068	0.1656	0.2284	0.2954	0.3669	0.4432	0.5246	0.6115	0.7043
b(t):	1.0000	0.9898	0.9792	0.9681	0.9566	0.9446	0.9322	0.9192	0.9057	0.8917	0.8770
t:	5.0000	4.5000	4.0000	3.5000	3.0000	2.5000	2.0000	1.5000	1.0000	0.5000	0.0000

Appendix D

Credit rationing

Results presented here are based on the following parameter values, unless otherwise indicated:

Collateral:

σ	d_0	d_1	l_0	l_1	f_0	f_1	$s(0)$
0.2	0	0.1	0	0.1	0	0.04	1.1

Contract:

T	r	$w-c$	p	P
5	0.05	0.05	0.1	0.877

The following combinations of games and contractual indentures are studied:

(a) Terminating default
(b) Terminating default, dividend flow constraint
(c) Strategic default
(d) Strategic default, dividend flow constraint
(e) Strategic default, dividend flow constraint, technical default, prepayment
(f) Penalised default, current default rule, dividend flow constraint
(g) Penalised default, oustanding payment rule, dividend flow constraint
(h) Penalised default, current default rule, dividend flow constraint, technical default, prepayment
(i) Penalised default, outstanding default rule, dividend flow constraint, technical default, prepayment

Table 1: Loan to value ratios

1 (i)

$c-r$	P	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
		% Loan								
0.00	0.716	0.268	0.399	0.207	0.320	0.348	0.518	0.518	0.523	0.523
0.01	0.767	0.620	0.706	0.500	0.580	0.580	0.617	0.617	0.617	0.617
0.02	0.820	0.704	0.759	0.571	0.635	0.635	0.658	0.658	0.660	0.660
0.03	0.877	0.752	0.788	0.625	0.667	0.741	0.678	0.678	0.715	0.715
0.04	0.937	0.787	0.807	0.656	0.690	0.755	0.703	0.703	0.769	0.769
0.05	1.000	0.811	0.822	0.678	0.702	0.784	0.715	0.715	0.784	0.784
0.06	1.067	0.825	0.816	0.702	0.727	0.816	0.729	0.729	0.833	0.833
0.07	1.137	0.812	0.799	0.714	0.741	0.833	0.742	0.742	0.833	0.833
0.08	1.211	0.805	0.793	0.741	0.741	0.833	0.755	0.755	0.851	0.851
0.09	1.290	0.804	0.793	0.741	0.755	0.851	0.770	0.770	0.851	0.851
0.10	1.372	0.806	0.797	0.755	0.769	0.851	0.771	0.771	0.851	0.851
0.12	1.552	0.813	0.804	0.789	0.800	0.870	0.800	0.800	0.870	0.870
0.14	1.751	0.821	0.813	0.816	0.816	0.870	0.820	0.820	0.870	0.870
0.16	1.973	0.829	0.822	0.829	0.832	0.889	0.834	0.834	0.889	0.889
0.18	2.220	0.837	0.828	0.836	0.838	0.889	0.844	0.844	0.889	0.889
0.20	2.494	0.842	0.835	0.845	0.847	0.889	0.851	0.851	0.889	0.889

1 (ii)

$l_1 = 0.35$

$c-r$	P	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
		% Loan								
0.00	0.716	0.243	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.01	0.767	0.541	0.448	0.360	0.360	0.360	0.000	0.000	0.000	0.000
0.02	0.820	0.606	0.516	0.412	0.412	0.412	0.442	0.442	0.442	0.442
0.03	0.877	0.637	0.561	0.449	0.449	0.449	0.468	0.468	0.468	0.468
0.04	0.937	0.662	0.596	0.476	0.476	0.476	0.489	0.489	0.489	0.489
0.05	1.000	0.679	0.624	0.494	0.494	0.494	0.503	0.503	0.503	0.503
0.06	1.067	0.687	0.643	0.506	0.506	0.506	0.517	0.517	0.556	0.556
0.07	1.137	0.691	0.656	0.519	0.519	0.519	0.527	0.527	0.625	0.625
0.08	1.211	0.694	0.665	0.533	0.533	0.533	0.541	0.541	0.645	0.645
0.09	1.290	0.696	0.672	0.541	0.541	0.541	0.544	0.544	0.645	0.645
0.10	1.372	0.697	0.676	0.548	0.548	0.548	0.556	0.556	0.645	0.645
0.12	1.552	0.691	0.675	0.571	0.571	0.563	0.573	0.573	0.667	0.667
0.14	1.751	0.687	0.673	0.588	0.588	0.580	0.590	0.590	0.714	0.714
0.16	1.973	0.683	0.671	0.598	0.598	0.597	0.606	0.606	0.741	0.741
0.18	2.220	0.677	0.666	0.606	0.606	0.606	0.609	0.609	0.769	0.769
0.20	2.494	0.672	0.661	0.612	0.612	0.606	0.615	0.615	0.800	0.800

1 (iii)

$d_1 = 0.2$

% Loan

$c-r$	P	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
0.00	0.716	0.000	0.000	0.000	0.000	0.000	0.349	0.349	0.383	0.383
0.01	0.767	0.399	0.399	0.323	0.323	0.377	0.409	0.409	0.463	0.463
0.02	0.820	0.460	0.460	0.377	0.377	0.471	0.438	0.438	0.606	0.606
0.03	0.877	0.495	0.494	0.417	0.417	0.656	0.459	0.459	0.645	0.645
0.04	0.937	0.521	0.521	0.444	0.444	0.702	0.484	0.484	0.690	0.690
0.05	1.000	0.540	0.540	0.465	0.465	0.714	0.500	0.500	0.690	0.690
0.06	1.067	0.552	0.552	0.482	0.482	0.784	0.516	0.516	0.800	0.800
0.07	1.137	0.562	0.562	0.494	0.494	0.784	0.527	0.527	0.800	0.800
0.08	1.211	0.569	0.569	0.506	0.506	0.800	0.535	0.535	0.800	0.800
0.09	1.290	0.575	0.575	0.513	0.513	0.816	0.541	0.541	0.800	0.800
0.10	1.372	0.576	0.576	0.519	0.519	0.816	0.547	0.547	0.833	0.833
0.12	1.552	0.577	0.577	0.541	0.541	0.833	0.560	0.560	0.833	0.833
0.14	1.751	0.580	0.579	0.550	0.550	0.851	0.573	0.573	0.833	0.833
0.16	1.973	0.582	0.582	0.561	0.561	0.870	0.588	0.588	0.870	0.870
0.18	2.220	0.584	0.584	0.566	0.566	0.870	0.589	0.589	0.870	0.870
0.20	2.494	0.586	0.586	0.571	0.571	0.870	0.595	0.595	0.870	0.870

Table 2: Expected foreclosure costs at origination

2 (i)

$F(s,0)$

$c-r$	P	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
0.00	0.716	0.028	0.084	0.000	0.084	0.075	0.084	0.084	0.071	0.071
0.01	0.767	0.032	0.084	0.000	0.083	0.078	0.084	0.084	0.077	0.077
0.02	0.820	0.037	0.084	0.000	0.084	0.080	0.084	0.084	0.079	0.079
0.03	0.877	0.042	0.085	0.000	0.084	0.078	0.084	0.084	0.078	0.078
0.04	0.937	0.046	0.085	0.000	0.084	0.077	0.084	0.084	0.077	0.077
0.05	1.000	0.051	0.085	0.000	0.084	0.076	0.084	0.084	0.076	0.076
0.06	1.067	0.054	0.087	0.000	0.084	0.076	0.084	0.084	0.074	0.074
0.07	1.137	0.065	0.094	0.000	0.083	0.074	0.083	0.083	0.073	0.073
0.08	1.211	0.072	0.096	0.000	0.083	0.073	0.082	0.082	0.072	0.072
0.09	1.290	0.078	0.098	0.000	0.082	0.071	0.082	0.082	0.070	0.070
0.10	1.372	0.082	0.099	0.000	0.081	0.070	0.080	0.080	0.069	0.069
0.12	1.552	0.085	0.099	0.000	0.080	0.067	0.078	0.078	0.066	0.066
0.14	1.751	0.088	0.100	0.000	0.079	0.065	0.076	0.076	0.065	0.065
0.16	1.973	0.088	0.100	0.000	0.079	0.064	0.075	0.075	0.059	0.059
0.18	2.220	0.088	0.100	0.000	0.079	0.059	0.074	0.074	0.058	0.058
0.20	2.494	0.088	0.100	0.000	0.079	0.057	0.073	0.073	0.059	0.059

2 (ii)

$l_1 = 0.35$

$F(s,0)$

$c-r$	P	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
0.00	0.716	0.100	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.01	0.767	0.114	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	0.820	0.130	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.03	0.877	0.149	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.04	0.937	0.161	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.05	1.000	0.177	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.06	1.067	0.191	0.297	0.000	0.000	0.000	0.000	0.000	0.067	0.067
0.07	1.137	0.208	0.299	0.000	0.000	0.000	0.000	0.000	0.136	0.136
0.08	1.211	0.218	0.302	0.000	0.000	0.000	0.000	0.000	0.172	0.172
0.09	1.290	0.231	0.305	0.000	0.000	0.000	0.000	0.000	0.195	0.195
0.10	1.372	0.240	0.307	0.000	0.000	0.000	0.000	0.000	0.214	0.214
0.12	1.552	0.258	0.315	0.000	0.000	0.000	0.000	0.000	0.235	0.235
0.14	1.751	0.271	0.320	0.000	0.000	0.000	0.000	0.000	0.244	0.244
0.16	1.973	0.279	0.324	0.000	0.000	0.026	0.000	0.000	0.243	0.243
0.18	2.220	0.284	0.328	0.000	0.000	0.025	0.000	0.000	0.241	0.241
0.20	2.494	0.287	0.330	0.000	0.000	0.019	0.000	0.000	0.232	0.232

2 (iii)

$d_1 = 0.2$

$F(s,0)$

$c-r$	P	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
0.00	0.716	0.032	0.033	0.000	0.000	0.087	0.000	0.000	0.082	0.082
0.01	0.767	0.034	0.035	0.000	0.000	0.088	0.000	0.000	0.084	0.084
0.02	0.820	0.036	0.037	0.000	0.000	0.089	0.000	0.000	0.085	0.085
0.03	0.877	0.038	0.039	0.000	0.000	0.089	0.000	0.000	0.085	0.085
0.04	0.937	0.039	0.041	0.000	0.000	0.088	0.000	0.000	0.084	0.084
0.05	1.000	0.040	0.041	0.000	0.000	0.088	0.000	0.000	0.085	0.085
0.06	1.067	0.041	0.042	0.000	0.000	0.088	0.000	0.000	0.086	0.086
0.07	1.137	0.043	0.043	0.000	0.000	0.087	0.000	0.000	0.085	0.085
0.08	1.211	0.043	0.044	0.000	0.000	0.086	0.000	0.000	0.084	0.084
0.09	1.290	0.044	0.045	0.000	0.000	0.084	0.000	0.000	0.084	0.084
0.10	1.372	0.043	0.045	0.000	0.000	0.083	0.000	0.000	0.082	0.082
0.12	1.552	0.044	0.045	0.000	0.000	0.080	0.000	0.000	0.080	0.080
0.14	1.751	0.044	0.046	0.000	0.000	0.077	0.000	0.000	0.078	0.078
0.16	1.973	0.045	0.046	0.000	0.000	0.077	0.000	0.000	0.078	0.078
0.18	2.220	0.045	0.046	0.000	0.000	0.073	0.000	0.000	0.074	0.074
0.20	2.494	0.045	0.046	0.000	0.000	0.071	0.000	0.000	0.072	0.072

Table 3: Foreclosure costs

3 (a)

l_1	P	L	B	F	%Loan
0.000	0.877	0.947	0.153	0.000	0.690
0.020	0.877	0.941	0.144	0.015	0.697
0.040	0.877	0.941	0.138	0.021	0.750
0.060	0.877	0.937	0.138	0.025	0.779
0.080	0.877	0.928	0.138	0.034	0.765
0.100	0.877	0.920	0.138	0.042	0.752
0.120	0.877	0.911	0.138	0.051	0.740
0.140	0.877	0.903	0.138	0.059	0.729
0.160	0.877	0.894	0.138	0.068	0.718
0.180	0.877	0.886	0.138	0.076	0.707
0.200	0.877	0.877	0.138	0.085	0.698

3 (d)

l_1	P	L	B	F	%Loan
0.000	0.877	0.996	0.104	0.000	0.851
0.020	0.877	0.986	0.097	0.017	0.800
0.040	0.877	0.974	0.092	0.034	0.755
0.060	0.877	0.961	0.089	0.050	0.727
0.080	0.877	0.946	0.086	0.068	0.690
0.100	0.877	0.931	0.085	0.084	0.667
0.120	0.877	0.916	0.083	0.101	0.635
0.140	0.877	0.900	0.082	0.118	0.615
0.160	0.877	0.884	0.082	0.134	0.597
0.180	0.877	0.868	0.081	0.151	0.571
0.200	0.877	0.851	0.084	0.165	0.556

3 (f)

l_1	P	L	B	F	%Loan
0.000	0.877	0.990	0.110	0.000	0.800
0.020	0.877	0.980	0.104	0.016	0.770
0.040	0.877	0.969	0.099	0.032	0.744
0.060	0.877	0.956	0.096	0.048	0.717
0.080	0.877	0.942	0.094	0.064	0.692
0.100	0.877	0.928	0.093	0.079	0.669
0.120	0.877	0.914	0.092	0.094	0.646
0.140	0.877	0.899	0.091	0.110	0.627
0.160	0.877	0.883	0.090	0.127	0.607
0.180	0.877	0.868	0.090	0.142	0.591
0.200	0.877	0.852	0.091	0.157	0.573

3 (h)

l_1	P	L	B	F	%Loan
0.000	0.877	0.990	0.110	0.000	0.800
0.020	0.877	0.980	0.104	0.016	0.770
0.040	0.877	0.969	0.099	0.032	0.744
0.060	0.877	0.956	0.096	0.048	0.717
0.080	0.877	0.942	0.095	0.063	0.690
0.100	0.877	0.930	0.095	0.075	0.714
0.120	0.877	0.917	0.095	0.088	0.714
0.140	0.877	0.904	0.096	0.100	0.714
0.160	0.877	0.891	0.096	0.113	0.714
0.180	0.877	0.876	0.096	0.128	0.714
0.200	0.877	0.857	0.099	0.144	0.714

Table 4: Dividend Flow

4 (a)

d_1	P	L	B	F	%Loan
0.000	0.877	0.997	0.059	0.044	0.905
0.020	0.877	0.987	0.070	0.043	0.888
0.040	0.877	0.974	0.083	0.043	0.865
0.060	0.877	0.959	0.099	0.042	0.834
0.080	0.877	0.941	0.117	0.042	0.796
0.100	0.877	0.920	0.138	0.042	0.752
0.120	0.877	0.896	0.162	0.042	0.703
0.140	0.877	0.869	0.190	0.041	0.650
0.160	0.877	0.839	0.220	0.041	0.597
0.180	0.877	0.809	0.251	0.040	0.544
0.200	0.877	0.778	0.284	0.038	0.495

4 (d)

d_1	P	L	B	F	%Loan
0.000	0.877	0.990	0.000	0.110	0.889
0.020	0.877	0.990	0.000	0.110	0.889
0.040	0.877	0.990	0.000	0.110	0.889
0.060	0.877	0.990	0.000	0.110	0.889
0.080	0.877	0.990	0.000	0.110	0.889
0.100	0.877	0.931	0.085	0.084	0.667
0.120	0.877	0.891	0.149	0.060	0.588
0.140	0.877	0.862	0.201	0.037	0.541
0.160	0.877	0.834	0.267	0.000	0.500
0.180	0.877	0.805	0.296	0.000	0.455
0.200	0.877	0.775	0.326	0.000	0.417

4 (f)

d_1	P	L	B	F	%Loan
0.000	0.877	0.990	0.000	0.110	0.870
0.020	0.877	0.990	0.000	0.110	0.870
0.040	0.877	0.990	0.000	0.110	0.870
0.060	0.877	0.990	0.000	0.110	0.870
0.080	0.877	0.990	0.000	0.110	0.870
0.100	0.877	0.928	0.093	0.079	0.669
0.120	0.877	0.894	0.152	0.054	0.609
0.140	0.877	0.869	0.203	0.028	0.573
0.160	0.877	0.843	0.258	0.000	0.532
0.180	0.877	0.819	0.282	0.000	0.503
0.200	0.877	0.791	0.311	0.000	0.459

4 (h)

d_1	P	L	B	F	%Loan
0.000	0.877	1.000	0.060	0.040	0.909
0.020	0.877	1.000	0.060	0.040	0.909
0.040	0.877	1.000	0.060	0.040	0.909
0.060	0.877	1.000	0.060	0.040	0.909
0.080	0.877	0.990	0.062	0.048	0.870
0.100	0.877	0.930	0.095	0.075	0.714
0.120	0.877	0.922	0.100	0.078	0.714
0.140	0.877	0.916	0.104	0.080	0.690
0.160	0.877	0.910	0.107	0.083	0.690
0.180	0.877	0.907	0.109	0.084	0.667
0.200	0.877	0.902	0.113	0.085	0.645

Appendix E

Computer Code

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***********************************************************************
*                                CREDTAB                              *
***********************************************************************
      This program employs the Crank Nicholson finite difference
      algorithm to determine the values of L(s,0) and B(s,0) in the
      terminating default game and the strategic default game.
***********************************************************************
      implicit  double precision (a-h,k-l,o-z)
      integer sstep, tstep, scount, smaxcount, smincount, timesteps,
     &        tabstep, dcount
      parameter (sstep=100, tstep=240)
      dimension vl(0:sstep), vb(0:sstep), s(0:sstep), balt(0:tstep),
     &          div(0:sstep), liq(0:sstep), sliq(0:sstep),
     &          arr(0:sstep,1:4), stratpay(0:sstep), parm(15),
     &          vlex(0:sstep,0:tstep), vbex(0:sstep,0:tstep),
     &          vlcum(0:sstep,0:tstep), vbcum(0:sstep,0:tstep),
     &          stratpaytab(0:sstep,0:tstep), timetab(0:tstep),
     &          critpay(0:sstep), pay(0:sstep)
      character*8 A, B, change, strat, amort, coup, prep,
     &            tdef, cash, hdef
      dimension SYM(0:sstep,0:tstep)
      character*1 SYM
      character*12 infile

***** input ***********************************************************

c     set input and output files

      write(*,*) 'Enter input file'
      read(*,5) infile
 5    format (a12)
      open(3, file=infile, status='old', form='formatted')
      open(8, file='credtab.out', form='formatted')

c     crank-nicholson algorithm parameters:
```

```
      data imin, imax, smin, smax, ifut / 1, 0, 0.0, 5.0, 0 /

c     contract parameters:

      read(3,*) sigma, div0, div1, tmat, cspread, r,
     &          p, liq0, liq1, s0, ref1

      write(*,700) 'Collateral:'
      write(*,720) 'r', 'sigma', 'div0', 'div1', 'liq0', 'liq1',
     &             'ref1', 's(0)'
      write(*,730) r, sigma, div0, div1, liq0, liq1,
     &             ref1, s0
      write(*,740) 'Contract:'
      write(*,750) 'T', 'c-r', 'p'
      write(*,760) tmat, cspread, p

      write(*,*) 'Change parameter value (y/n):'
      read(*,20) A
      if (A .eq. 'y' .or. A .eq. 'Y') then
 10      continue
         write(*,*) 'Enter parameter:'
         read(*,20) B
         if (B .eq. 'c-r' .or. B .eq. 'C-R') then
            write(*,*) 'Enter c-r:'
            read*, cspread
         else if ( B .eq. 'p' .or. B .eq. 'P') then
            write(*,*) 'Enter p:'
            read*, p
         else if ( B .eq. 'tmat' .or. B .eq. 'TMAT') then
            write(*,*) 'Enter tmat:'
            read*, tmat
         else if ( B .eq. 'liq0' .or. B .eq. 'LIQ0') then
            write(*,*) 'Enter liq0:'
            read*, liq0
         else if ( B .eq. 'liq1' .or. B .eq. 'LIq1') then
            write(*,*) 'Enter liq1:'
            read*, liq1
         else if ( B .eq. 'sigma' .or. B .eq. 'SIGMA') then
            write(*,*) 'Enter sigma:'
            read*, sigma
         else if ( B .eq. 'div0' .or. B .eq. 'DIV0') then
            write(*,*) 'Enter div0:'
            read*, div0
         else if ( B .eq. 'div1' .or. B .eq. 'DiV1') then
            write(*,*) 'Enter div1:'
            read*, div1
         else if ( B .eq. 's0' .or. B .eq. 'S0') then
            write(*,*) 'Enter s0:'
            read*, s0
         else if ( B .eq. 'r' .or. B .eq. 'R') then
            write(*,*) 'Enter r:'
```

```
         read*, r
      else if ( B .eq. 'ref1' .or. B .eq. 'REF1') then
         write(*,*) 'Enter ref1:'
         read*, ref1
      endif
      write(*,*) 'Change another parameter (y/n):'
      read(*,20) change
      if (change .eq. 'y' .or. change .eq. 'Y') goto 10
   endif

c  set table dimensions for output

   write(*,*) 'Enter s : stabmax, stabmin, sint'
   read*, stabmax, stabmin, sint
   write(*,*) 'Enter t: start, stop, step'
   read*, tabstart, tabstop, tabstep

   dt = tmat / dble(tstep)
   ds = (smax - smin) / dble(sstep)
   scount = idnint((s0 - smin) / ds)
   smaxcount = idnint((stabmax - smin) / ds)
   smincount = idnint((stabmin - smin) / ds)
   dcount = idnint(sint / ds)
   startc = idnint(tabstart / dt)
   stopc = idnint(tabstop / dt)

c  set pde coefficient parameters

   c = r + cspread
   parm(1) = sigma
   parm(2) = r
   parm(3) = div0
   parm(4) = div1

c  set behavioural assumptions and contract details

   write(*,*) 'Strategic debt service (y/n):'
   read(*,20) strat

   if (strat .eq. 'y' .or. strat .eq. 'Y') then
      write(8,*) ' Strategic debt service'
   endif

   write(*,*) 'Cash flow constraint (y/n):'
   read(*,20) cash
   if (cash .eq. 'y' .or. cash .eq. 'Y') then
      write(8,*) ' Cashflow constraint on'
   endif

   write(*,*) 'Full amortisation (y/n):'
   read(*,20) amort
   if (amort .eq. 'y' .or. amort .eq. 'Y') then
```

```
         write(8,*) ' Full amortisation'
         p = c*exp(c*tmat)/(exp(c*tmat)-1d0)
      else
         write(*,*) 'Coupon loan (y/n):'
         read(*,20) coup
         if (coup .eq. 'y' .or. coup .eq. 'Y') then
            write(8,*) ' Coupon loan: p = c'
            p = c
         endif
      endif

      conpay = p*dt

      write(*,*) 'Prepay (y/n):'
      read(*,20) prep
      if (prep .eq. 'y' .or. prep .eq. 'Y') then
         write(8,*) ' Prepayment'
      endif

      write(*,*) 'Technical default (y/n):'
      read(*,20) tdef
      if (tdef .eq. 'y' .or. tdef .eq. 'Y') then
         write(8,*) ' Technical default'
      endif

      print*, 'High default (y/n):'
      read(*,20) hdef
      if (hdef .eq. 'y' .or. hdef .eq. 'Y') then
         write(8,*) ' High default'
      endif

 20   format(a8)

c     outstanding loan balance at maturity

      balt(0) = exp(c*tmat)-(exp(c*tmat)-1)*(p/c)

***** solve L(s,0) and B(s,0) *****************************************

      call cnset (sstep,smin,smax,dt,ifn,ifut,imin,imax,parm,arr)

      do 100 i = 0, sstep

         s(i) = smin + dble(i) ** ds
         div(i) = (div0 + s(i)*div1) ** dt
         liq(i) = s(i)*liq1 + liq0
         sliq(i) = max(0d0, s(i) - liq(i))

         if (strat .eq. 'y' .or. strat.eq. 'Y') then
            vl(i) = min(sliq(i), balt(0))
            vb(i) = s(i) - vl(i)
            stratpaytab(i,0) = vl(i)
```

```
            SYM(i,0) = ' '
         else
            if (s(i) - balt(0) .lt. -1d-10) then
               vb(i) = 0.0d0
               vl(i) = sliq(i)
               SYM(i,0) = '*'
            else
               vb(i) = s(i) - balt(0)
               vl(i) = balt(0)
               SYM(i,0) = ' '
            endif
         endif

         vlex(i,0) = vl(i)
         vlcum(i,0) = vl(i)
         vbex(i,0) = vb(i)
         vbcum(i,0) = vb(i)

 100  continue

c     time loop

      t = 0d0
      timetab(0) = 0d0
      j = 0

 110  continue

      call cnstep (t, vb, arr)
      call cnstep (t, vl, arr)

      t = t + dt
      j = j + 1
      timetab(j) = t
      balt(j)=exp(c*(tmat-t))-(exp(c*(tmat-t))-1d0)*(p/c)
      ref = balt(j) * (1d0 + ref1) + 1d-10

      do 120 i = 0, sstep
         vlex(i,j) = vl(i)
         vbex(i,j) = vb(i)

 120  continue

      do 140 i = 0, sstep

         vlliq = min(sliq(i), balt(j))
         vbliq = sliq(i) - vlliq
         vldlow = sliq(i)
         vbdlow = 0d0
         vldhi = balt(j)
         vbdhi = sliq(i) - balt(j)
         vlprep = balt(j)
```

```
vbprep = s(i) - ref
critpay(i) = max(0d0, vlliq - vl(i))
stratpay(i) = min(conpay, critpay(i))

if (tdef .eq. 'y' .and. strat .eq. 'y') then
   if (s(i) .lt. balt(j)) then
      stratpay(i) = critpay(i)
   endif
endif

if (strat .eq. 'y' .or. strat .eq. 'Y') then
   pay(i) = stratpay(i)
else
   pay(i) = conpay
endif

vlcont = vl(i) + pay(i)
vbcont = vb(i) + div(i) - pay(i)
vl(i) = vlcont
vb(i) = vbcont
SYM(i,j) = ' '

if (vbcont .lt. vbdlow) then
   vb(i) = vbdlow
   vl(i) = vldlow
   SYM(i,j) = '*'
goto 140
endif

if (hdef .eq. 'y' .and. prep .eq. 'y') then
   if (vbcont .lt. max(vbdhi,vbprep)) then
      if (vbprep .gt. vbdhi) then
         vb(i) = vbprep
         vl(i) = vlprep
         SYM(i,j) = '+'
      else
         vb(i) = vbdhi
         vl(i) = vldhi
         SYM(i,j) = '*'
      endif
   goto 140
   endif
else if (hdef .eq. 'y' .and .prep .eq. 'n') then
   if (vbcont .lt. vbdhi) then
      vb(i) = vbdhi
      vl(i) = vldhi
      SYM(i,j) = '*'
   goto 140
   endif
else if (hdef .eq. 'n' .and .prep .eq. 'y') then
   if (vbcont .lt. vbprep) then
      vb(i) = vbprep
```

```
               vl(i) = vlprep
               SYM(i,j) = '+'
            goto 140
            endif
         endif

         if (cash .eq. 'y' .or. cash .eq. 'Y') then
            if (pay(i) .gt. div(i)) then
               vb(i) = vbliq
               vl(i) = vlliq
               SYM(i,j) = '#'
               goto 140
            endif
         endif

         if (tdef .eq. 'y' .or. tdef .eq. 'Y') then
            if (s(i) .lt. balt(j)) then
               SYM(i,j) = '-'
               if (vlcont .lt. vlliq) then
                  vl(i) = vlliq
                  vb(i) = vbliq
                  SYM(i,j) = ':'
               goto 140
               endif
            endif
         endif

 140  continue

      do 160 i = 0, sstep
         vlcum(i,j) = vl(i)
         vbcum(i,j) = vb(i)
         stratpaytab(i,j) = pay(i)
 160  continue

      if (t .lt. tmat - 1d-10) goto 110

***** output ************************************************************

      write(8,700) 'Collateral:'
      write(8,720) 'r', 'sigma', 'd_0', 'd_1', 'l_0', 'l_1',
     &              'f_1', 's(0)'
      write(8,730) r, sigma, div0, div1, liq0, liq1,
     &        ref1, s(scount)
      write(8,740) 'Contract:'
      write(8,755) 'T', 'c-r', 'p', 'P'
      write(8,760) tmat, cspread, p, balt(0)

c     print L(s,t) and B(s,t) tables

      write(8,620) 'VLEX:'
      do 500 i = smaxcount, smincount, -dcount
```

```
         write(8,630) s(i), sliq(i), (vlex(i,j),
     &                SYM(i,j), j = startc,stopc,-tabstep)
 500  continue
      write(8,660) 's |', 'sliq|'
      write(8,665) 'bal:', (balt(j),j=startc,stopc,-tabstep)
      write(8,670) 't:', (timetab(j),j=startc,stopc,-tabstep)

      write(8,620) 'VLCUM:'
      do 505 i = smaxcount, smincount, -dcount
         write(8,630) s(i), sliq(i), (vlcum(i,j),
     &                SYM(i,j), j = startc,stopc,-tabstep)
 505  continue
      write(8,660) 's |', 'sliq|'
      write(8,665) 'bal:', (balt(j),j=startc,stopc,-tabstep)
      write(8,670) 't:', (timetab(j),j=startc,stopc,-tabstep)

      write(8,620) 'VBEX:'
      do 510 i = smaxcount, smincount, -dcount
         write(8,630) s(i), sliq(i), (vbex(i,j),
     &                SYM(i,j), j = startc,stopc,-tabstep)
 510  continue
      write(8,660) 's |', 'sliq|'
      write(8,665) 'bal:', (balt(j),j=startc,stopc,-tabstep)
      write(8,670) 't:', (timetab(j), j=startc,stopc,-tabstep)

      write(8,620) 'VBCUM:'
      do 520 i = smaxcount, smincount, -dcount
         write(8,630) s(i), sliq(i), (vbcum(i,j),
     &                SYM(i,j), j = startc,stopc,-tabstep)
 520  continue
      write(8,660) 's |', 'sliq|'
      write(8,665) 'bal:', (balt(j),j=startc,stopc,-tabstep)
      write(8,670) 't:', (timetab(j), j=startc,stopc,-tabstep)

c     print stratpay table

      write(8,620) 'Stratpay table:'
      write(8,625) 'Contractual payment (p*dt) = ', conpay
      do 550 i = smaxcount, smincount, -dcount
         write(8,640) s(i), div(i),(stratpaytab(i,j),
     &                SYM(i,j), j=startc,stopc,-tabstep)
 550  continue
      write(8,660) 's |', 'div |'
      write(8,680) 'bal:', (balt(j),j=startc,stopc,-tabstep)
      write(8,680) 't:', (timetab(j),j=startc,stopc,-tabstep)

 620  format (/ a19 /)
 625  format (a35, f5.4 /)
 630  format (1x, f4.2, '|', 1x, f5.3, '|', 2x, 80(f5.3, a1, 1x))
 640  format (1x, f4.2, '|', 1x, f5.4, '|', 2x, 80(f5.4, a1, 1x))
 660  format (2x, a4, a7)
 665  format (a13, 2x, 80(f5.3, 2x) /)
```

```
 670  format (a13, 2x, 80(f5.3, 2x) /)
 680  format (9x, a4, 2x, 80(f5.3, 2x))

 700  format(/ 2x, a11 /)
 720  format(2x, 8(a6, 2x))
 730  format(2x, 8(f6.2, 2x) /)
 740  format(/ 2x, a9 /)
 750  format(2x, 3(a6, 2x))
 755  format(2x, 4(a6, 2x))
 760  format(2x, 3(f6.2, 2x), f6.4 / /)

      stop
      end

***** function definitions **********************************************

      double precision function coeff()

      implicit  double precision (a-h,k-l,o-z)
      dimension parm(15)

      entry fna(s,ifn,parm)
         sigma = parm(1)
         fna   = sigma * sigma * s * s * 0.5d0
      return

      entry fnb(s,ifn,parm)
         r    = parm(2)
         div0 = parm(3)
         div1 = parm(4)
         fnb  = (r - div1) * s - div0
      return

      entry fnc(s,ifn,parm)
         fnc = -r
      return

      entry fmin(t,ifn,parm)
         fmin = 0.0
      return

      entry fmax(t,ifn,parm)
         fmax = 0.0
      return
      end
```

```
************************************************************************
*                                LOANTAB                               *
************************************************************************
      This program employs the Crank Nicholson finite difference
      algorithm to determine the values of L(s,0) and B(s,0) in the
      penalized default game.
************************************************************************

      implicit double precision (a-h, o-z)
      integer sstep, tstep, outstep, stratstep, scount, osmax
      parameter (sstep =100, tstep =240, outstep =40, stratstep =30)
      double precision liq(0:sstep), loanrat, maxoutpay(0:tstep),
     &                 newoutpay, maxstratpay, liq0, liq1
      dimension vltab(0:sstep,0:outstep,0:tstep),
     &          vbtab(0:sstep,0:outstep,0:tstep),
     &          vlgrid(0:sstep,0:outstep,0:tstep),
     &          vbgrid(0:sstep,0:outstep,0:tstep),
     &          stratab(0:sstep,0:outstep,0:tstep),
     &          crittab(0:sstep,0:outstep,0:tstep)
      dimension vl(0:sstep,0:outstep), vb(0:sstep,0:outstep),
     &          s(0:sstep), arr(0:sstep,1:4), parm(15),
     &          vltemp(0:sstep), balt(0:tstep), timetab(0:tstep),
     &          outpay(0:outstep), vbintrp(0:stratstep),
     &          vlintrp(0:stratstep), stratpay(0:stratstep),
     &          sliq(0:sstep), div(0:sstep), critpay(0:stratstep)
      dimension SYM(0:sstep,0:outstep,0:tstep,0:stratstep),
     &          SYMOUT(0:sstep,0:outstep,0:tstep)
      character*1 SYM, SYMOUT
      character*8 A, B, change, amort, coup, prep, tdef, cash, out,
     &            hdef
      character*12 infile

****** input ***********************************************************

      print*, 'Enter input file'
      read(*,5) infile
5     format (a12)
      open(3, file=infile, form='formatted')
      open(8, file='loantab.out', form='formatted')

c     crank-nicholson algorithm parameters:
      data imin, imax, smin, smax, ifut / 1, 0, 0.0, 5.0, 0 /

c     financial model parameters:

      read(3,*) sigma, div0, div1, tmat, cspread, r,
     &          p, liq0, liq1, s0, ref1, pspread

      print 900, 'Collateral:'
      print 920, 'r', 'sigma', 'div0', 'div1', 'liq0', 'liq1',
     &           'ref1', 's(0)'
```

```
      print 930, r, sigma, div0, div1, liq0, liq1, ref1, s0
      print 940, 'Contract:'
      print 945, 'T', 'c-r', 'p-c', 'p'
      print 960, tmat, cspread, pspread, p

      print*, 'Change parameter value (y/n):'
      read(*,20) A
      if (A .eq. 'y' .or. A .eq. 'Y') then
10     continue
      print*, 'Enter parameter:'
      read(*,20) B

    if (B .eq. 'c-r' .or. B .eq. 'C-R') then
       print*, 'Enter c-r:'
       read*, cspread
    else if (B .eq. 'p-c' .or. B .eq. 'P-C') then
       print*, 'Enter p-c:'
       read*, pspread
    else if ( B .eq. 'p' .or. B .eq. 'P') then
       print*, 'Enter p:'
       read*, p
    else if ( B .eq. 'tmat' .or. B .eq. 'TMAT') then
       print*, 'Enter tmat:'
       read*, tmat
    else if ( B .eq. 'liq0' .or. B .eq. 'LIQ0') then
       print*, 'Enter liq0:'
       read*, liq0
    else if ( B .eq. 'liq1' .or. B .eq. 'LIq1') then
       print*, 'Enter liq1:'
       read*, liq1
    else if ( B .eq. 'sigma' .or. B .eq. 'SIGMA') then
       print*, 'Enter sigma:'
       read*, sigma
    else if ( B .eq. 'div0' .or. B .eq. 'DIV0') then
       print*, 'Enter div0:'
       read*, div0
    else if ( B .eq. 'div1' .or. B .eq. 'DIV1') then
       print*, 'Enter div1:'
       read*, div1
    else if ( B .eq. 's0' .or. B .eq. 'S0') then
       print*, 'Enter s0:'
       read*, s0
    endif
      print*, 'Change another parameter (y/n):'
      read(*,20) change
      if (change .eq. 'y' .or. change .eq. 'Y') goto 10
      endif

      print*, 'Enter t: start, stop, step'
      read*, tabstart, tabstop, tabstep
      print*, 'Enter osmax: 0 - ', outstep
      read*, osmax
```

```
c = r + cspread
prate = c + pspread
dt = tmat / dble(tstep)
ds = (smax - smin) / dble(sstep)
scount = idnint((s0 - smin) / ds)
cstrt = idnint(tabstart / dt)
cstp = idnint(tabstop / dt)
parm(1) = sigma
parm(2) = r
parm(3) = div0
parm(4) = div1

print*, 'Cash flow constraint (y/n):'
read(*,20) cash
if (cash .eq. 'y' .or. cash .eq. 'Y') then
   write(8,*) ' Cashflow constraint'
endif

print*, 'Full amortisation (y/n):'
read(*,20) amort
if (amort .eq. 'y' .or. amort .eq. 'Y') then
   write(8,*) ' Full amortisation'
   p = c*exp(c*tmat)/(exp(c*tmat)-1d0)
else
   print*, 'Coupon loan (y/n):'
   read(*,20) coup
   if (coup .eq. 'y' .or. coup .eq. 'Y') then
      write(8,*) ' Coupon loan: p = c'
      p = c
   endif
endif

print*, 'Prepay (y/n):'
read(*,20) prep
if (prep .eq. 'y' .or. prep .eq. 'Y') then
   write(8,*) ' Prepayment'
endif

print*, 'Technical default (y/n):'
read(*,20) tdef
if (tdef .eq. 'y' .or. tdef .eq. 'Y') then
   write(8,*) ' Technical default'
endif

print*, 'High default (y/n):'
read(*,20) hdef
if (hdef .eq. 'y' .or. hdef .eq. 'Y') then
   write(8,*) ' High default'
endif

print*, 'Foreclose on outpay (y/n):'
```

```
      read(*,20) out
      if (out .eq. 'y' .or. out .eq. 'Y') then
         write(8,*) ' Foreclosure on outpay'
      endif

 20   format(a8)

      pay = p*dt

      do 40 i = 0, sstep
         s(i) = smin + dble(i) * ds
         div(i) = (div0 + s(i)*div1) * dt
         liq(i) = s(i)*liq1 + liq0
         sliq(i) = max(0d0, s(i) - liq(i))
40    continue

      do 80 i = 0, sstep
         do 70 j = 0, outstep
            do 60 k = 0, tstep
               SYMOUT(i,j,k) = ' '
               do 50 l = 0, stratstep
                  SYM(i,j,k,l) = ' '
 50            continue
 60         continue
 70      continue
 80   continue


c     outstanding loan balance at maturity

      balt(0) = exp(c*tmat)-(exp(c*tmat)-1d0)*(p/c)
      maxoutpay(0) = (exp(prate*tmat)-1d0)*(p/prate)
      doutpay = maxoutpay(0)/dble(outstep)


***** solve L(s,0) and B(s,0) *******************************************

      call cnset (sstep,smin,smax,dt,ifn,ifut,imin,imax,parm,arr)

      do 110 j = 0, outstep
         outpay(j) = 0d0 + dble(j)*doutpay
         do 100 i = 0, sstep
            vl(i,j) = min(sliq(i), balt(0) + outpay(j))
            vb(i,j) = s(i) - vl(i,j)
            vbtab(i,j,0) = vb(i,j)
            vltab(i,j,0) = vl(i,j)
            vbgrid(i,j,0) = vb(i,j)
            vlgrid(i,j,0) = vl(i,j)
            stratab(i,j,0) = vl(i,j)
            SYMOUT(i,j,0) = ' '
100      continue
110   continue
```

```
c     time loop

      t = 0d0
      k = 0
      timetab(0) = 0d0
130   continue

      do 145 j = 0, outstep
         call cnstep (t, vb(0,j), arr)
         call cnstep (t, vl(0,j), arr)
         do 135 i = 0, sstep
            vbgrid(i,j,k+1) = vb(i,j)
            vlgrid(i,j,k+1) = vl(i,j)
 135     continue
 145  continue

      write(*,*) 'Time loop: k = ', k

      t = t + dt
      k = k + 1
      timetab(k) = t
      maxoutpay(k) = (exp(prate*(tmat-t))-1d0)*(p/prate)
      balt(k) = exp(c*(tmat-t))-(exp(c*(tmat-t))-1d0)*(p/c)

c     loan balance loop

      j = 0

150   continue

      ref = (balt(k) + outpay(j)) * (1d0 + ref1) + 1d-10

      do 250 i = 0, sstep
         if (cash .eq. 'y' .or. cash .eq. 'Y') then
            maxstratpay = div(i)
         else
            maxstratpay = outpay(j) + pay
         endif
         dstrat = maxstratpay / dble(stratstep)
         vlliq = min(sliq(i), balt(k) + outpay(j))
         vbliq = sliq(i) - vlliq
         vbdlow = 0d0
         vldlow = sliq(i)
         vbdhi = sliq(i) - balt(k) - outpay(j)
         vldhi = balt(k) + outpay(j)
         vbprep = s(i) - ref
         vlprep = balt(k) + outpay(j)

c     stratpay loop

         do 170 l = 0, stratstep
```

```
         stratpay(l) = 0d0 + dble(l) * dstrat
         netout = outpay(j) + pay - stratpay(l)
         if (netout .gt. 0d0) then
            newoutpay = netout*exp(prate*dt)
         else
            newoutpay = 0d0
         endif

         n = 1
160      if (outpay(n) .lt. newoutpay) then
            n = n + 1
            goto 160
         endif

         alpha = (newoutpay-outpay(n))/(outpay(n-1)-outpay(n))
         vlintrp(l) = alpha*vl(i,n-1) + (1-alpha)*vl(i,n)
         vbintrp(l) = alpha*vb(i,n-1) + (1-alpha)*vb(i,n)
         critpay(l) = max(0d0, vlliq - vlintrp(l))
         vlcont = vlintrp(l) + stratpay(l)
         vbcont = vbintrp(l) + div(i) - stratpay(l)
         vbintrp(l) = vbcont
         vlintrp(l) = vlcont

         if (vbcont .lt. vbdlow) then
            vbintrp(l) = vbdlow
            vlintrp(l) = vldlow
            SYM(i,j,k,l) = '*'
         goto 170
         endif

         if (hdef .eq. 'y' .and. prep .eq. 'y') then
            if (vbcont .lt. max(vbdhi,vbprep)) then
               if (vbprep .gt. vbdhi) then
                  vbintrp(l) = vbprep
                  vlintrp(l) = vlprep
                  SYM(i,j,k,l) = '+'
               else
                  vbintrp(l) = vbdhi
                  vlintrp(l) = vldhi
                  SYM(i,j,k,l) = '*'
               endif
            goto 170
            endif
         else if (hdef .eq. 'y' .and. prep .eq. 'n') then
            if (vbcont .lt. vbdhi) then
               vbintrp(l) = vbdhi
               vlintrp(l) = vldhi
               SYM(i,j,k,l) = '*'
            goto 170
            endif
         else if (hdef .eq. 'n' .and. prep .eq. 'y') then
            if (vbcont .lt. vbprep) then
```

```
                vbintrp(l) = vbprep
                vlintrp(l) = vlprep
                SYM(i,j,k,l) = '+'
              goto 170
              endif
            endif

            if (out .eq. 'y' .or. out .eq. 'Y') then
              if (stratpay(l) .lt. min(outpay(j) + pay,
     &          critpay(l))) then
                vlintrp(l) = vlliq
                vbintrp(l) = vbliq
                SYM(i,j,k,l) = '#'
              goto 170
              endif
            else if (stratpay(l) .lt. min(pay, critpay(l))) then
              vlintrp(l) = vlliq
              vbintrp(l) = vbliq
              SYM(i,j,k,l) = '#'
            goto 170
            endif

            if (tdef .eq. 'y' .or. tdef .eq. 'Y') then
              if (s(i) .lt. balt(k) + outpay(j)) then
                SYM(i,j,k,l) = '-'
                if (vlintrp(i) .lt. vlliq) then
                  vlintrp(l) = vlliq
                  vbintrp(l) = vbliq
                  SYM(i,j,k,l) = ':'
                goto 170
                endif
              endif
            endif

 170        continue

            vbmax = 0d0
            do 190 l = 0, stratstep
              vbmax = max(vbintrp(l), vbmax)
 190        continue

            l = 0
 200        if (vbintrp(l) .lt. vbmax - 1d-10) then
              l = l + 1
            goto 200
            endif

            vb(i,j) = vbintrp(l)
            vl(i,j) = vlintrp(l)
            vbtab(i,j,k) = vbintrp(l)
            vltab(i,j,k) = vlintrp(l)
            stratab(i,j,k) = stratpay(l)
```

```
            SYMOUT(i,j,k) = SYM(i,j,k,l)
            crittab(i,j,k) = critpay(l)

250   continue

      j = j + 1
      if (outpay(j) .lt. maxoutpay(k)) goto 150

      if (t .lt. tmat -1d-10) goto 130

***** sfair, %Loan ****************************************************

         i = 1
260      if (vl(i,0) .lt. 1d0 - 1d-10) then
            i = i + 1
            if (i .lt. sstep) goto 260
         endif

         if (i .eq. sstep .and. vl(i,0) .lt. 1d0 - 1d-10) then
            write(*,*) 'vl(',i,'0) = ', vl(i,0)
            sfair = 0d0
            loanrat(j) = 0d0
         else
            alpha = (1d0-vl(i,0))/(vl(i-1,0)-vl(i,0))
            sfair = alpha*s(i-1) + (1-alpha)*s(i)
            loanrat(m) = 1d0/sfair
         endif

         sfairout(m) = sfair
         vlout(m) = vl(scount,0)
         vbout(m) = vb(scount,0)

 350  continue

***** output **********************************************************

      write(8,900) 'Collateral:'
      write(8,920) 'r', 'sigma', 'd_0', 'd_1', 'l_0', 'l_1',
     &               'f_1', 's(0)'
      write(8,930) r, sigma, div0, div1, liq0, liq1,
     &         ref1, s(scount)
      write(8,940) 'Contract:'
      write(8,950) 'T', 'c-r', 'p-c', 'p', 'P'
      write(8,960) tmat, cspread, pspread, p, balt(0)
      write(8,970) 'Claim values:'
      write(8,980) 'L', 'B', 'Sfair', '%Loan'
      write(8,990) vlout, vbout, sfairout, loanrat

      write(8,800) 'VBGRID: s = ', s(scount)
      do 650 j = osmax, 0, -1
         write(8,810) outpay(j),(vbgrid(scount,j,k),
     &                SYMOUT(scount,j,k),  k = cstrt, cstp, -tabstep)
```

```
650   continue
      write(8,820) 'outpay '
      write(8,820) 'maxout: ', (maxoutpay(k), k=cstrt,cstp,-tabstep)
      write(8,820) 'balt: ', (balt(k), k=cstrt,cstp,-tabstep)
      write(8,825) 't: ', (timetab(k), k = cstrt, cstp, -tabstep)

      write(8,800) 'VLGRID: s = ', s(scount)
      do 670 j = osmax, 0, -1
         write(8,810) outpay(j),(vlgrid(scount,j,k),
     &                SYMOUT(scount,j,k), k = cstrt, cstp, -tabstep)
670   continue
      write(8,820) 'outpay '
      write(8,820) 'maxout:', (maxoutpay(k), k=cstrt,cstp,-tabstep)
      write(8,820) 'balt: ', (balt(k), k=cstrt,cstp,-tabstep)
      write(8,825) 't: ', (timetab(k), k = cstrt, cstp, -tabstep)

      write(8,800) 'VB: s = ', s(scount)
      do 700 j = osmax, 0, -1
         write(8,810) outpay(j), (vbtab(scount,j,k),
     &                SYMOUT(scount,j,k), k = cstrt, cstp, -tabstep)
700   continue
      write(8,820) 'outpay '
      write(8,820) 'maxout:', (maxoutpay(k), k=cstrt,cstp,-tabstep)
      write(8,820) 'balt: ', (balt(k), k=cstrt,cstp,-tabstep)
      write(8,825) 't: ', (timetab(k), k = cstrt, cstp, -tabstep)

      write(8,800) 'VL: s = ', s(scount)
      do 720 j = osmax, 0, -1
         write(8,810) outpay(j),(vltab(scount,j,k),
     &                SYMOUT(scount,j,k), k = cstrt, cstp, -tabstep)
720   continue
      write(8,820) 'outpay '
      write(8,820) 'maxout:', (maxoutpay(k), k=cstrt,cstp,-tabstep)
      write(8,820) 'balt: ', (balt(k), k=cstrt,cstp,-tabstep)
      write(8,825) 't: ', (timetab(k), k = cstrt, cstp, -tabstep)

      write(8,800) 'Stratpay: s = ', s(scount)
      write(8,805) 'Div = ', div(scount)
      write(8,805) 'Pay = ', pay
      do 730 j = osmax, 0, -1
         write(8,810) outpay(j),(stratab(scount,j,k),
     &                SYMOUT(scount,j,k), k = cstrt,cstp,-tabstep)
730   continue
      write(8,820) 'outpay '
      write(8,820) 'maxout:', (maxoutpay(k), k=cstrt,cstp,-tabstep)
      write(8,820) 'balt: ', (balt(k), k=cstrt,cstp,-tabstep)
      write(8,825) 't: ', (timetab(k), k = cstrt, cstp, -tabstep)

      write(8,800) 'Critpay: s = ', s(scount)
      write(8,805) 'Div = ', div(scount)
      write(8,805) 'Pay = ', pay
      do 740 j = osmax, 0, -1
```

```
        write(8,810) outpay(j),(crittab(scount,j,k),
     &                SYMOUT(scount,j,k), k = cstrt,cstp,-tabstep)
740   continue
      write(8,820) 'outpay '
      write(8,820) 'maxout:', (maxoutpay(k), k=cstrt,cstp,-tabstep)
      write(8,820) 'balt: ', (balt(k), k=cstrt,cstp,-tabstep)
      write(8,825) 't: ', (timetab(k), k = cstrt, cstp, -tabstep)

 800  format ( / a20, f4.2 /)
 805  format (a6, f5.4 /)
 810  format (2x, f6.4, '|', 2x, 80(f6.4, a1, 1x))
 820  format (a8, '|', 2x, 80(f6.4, 2x))
 825  format (a8, '|', 2x, 80(f6.4, 2x))

 900  format(/ 2x, a11 /)
 920  format(2x, 8(a6, 2x))
 930  format(2x, 8(f6.2, 2x) /)
 940  format(/ 2x, a9 /)
 945  format(2x, 4(a6, 2x))
 950  format(2x, 5(a6, 2x))
 960  format(2x, 4(f6.2, 2x), f6.4 /)
 970  format(/ 2x, a12 /)
 980  format(2x, 4(a8, 2x) /)
 990  format(2x, 4(f8.6, 2x))

      stop
      end

***** function definitions *********************************************

      double precision function coeff()

      implicit  double precision (a-h,k-l,o-z)
      dimension parm(15)

      entry fna(s,ifn,parm)
        sigma = parm(1)
        fna   = sigma **sigma * s **s **0.5d0
      return

      entry fnb(s,ifn,parm)
         r      = parm(2)
         div0 = parm(3)
         div1 = parm(4)
         fnb   = (r - div1) * s - div0
      return

      entry fnc(s,ifn,parm)
         r = parm(2)
         fnc   = -r
      return
```

```
entry fmin(t,ifn,parm)
   fmin    = 0.0d0
return

entry fmax(t,ifn,parm)
   fmax    = 0.0d0
return

end
```

```
      SUBROUTINE CNSET (IN,SMIN,SMAX,K,IFN,IFUT,ISMN,ISMX,PARM,ARR)

C***********************************************************************
C
C  SUBROUTINE CNSET (...)
C
C  Subroutine sets up coefficient array needed for Crank-Nicholson
C  algorithm to solve 1 state variable plus time partial differential
C  equations.  Use in conjunction with routine CNSTEP.  PDE has form
C
C                 FNA * Uss + FNB * Us + FNC * U - Ut = 0
C
C  Arguments: IN       number of grid intervals in state space S
C             SMIN     minimum value of state variable S
C             SMAX     maximum value of state variable S
C             K        step size in time direction
C             IFN      flag available for passing to coeff. fcns.
C             IFUT     flag setting FNC = 0 for futures contract pricing
C             ISMN     flag for SMIN boundary (0 for quadratic extrapol.)
C             ISMX     flag for SMAX boundary (1 for given values       )
C             PARM     vector of model parameters for coeff. fcns.
C             ARR      output array of coefficients for CNSTEP
C                      dimension (4,IN+1) by calling program
C
C  Other routines called:  functions FNA, FNB, FNC(S,IFN,PARM) must be
C                      externally defined and available to subroutine.
C
C  Author:     R. A. Jones     15 December 1988
C
C***********************************************************************

      IMPLICIT  DOUBLE PRECISION ( A-H, K-L, O-Z )
      COMMON   /CNCOM/ N,ISMIN,ISMAX,IIFN,XPARM(15)
      DIMENSION PARM( 15 ), ARR( 0:IN, 4 )

      N      = IN
      IIFN   = IFN
      ISMIN  = ISMN
      ISMAX  = ISMX
      DO 50 I = 1, 15
   50    XPARM(I) = PARM(I)
      H      = ( SMAX - SMIN ) / DBLE( N )
      FUTURE = 1D0
      IF ( IFUT .EQ. 1)    FUTURE = 0D0

C**** FIRST DO 'INTERIOR' COEFFICIENTS *********************************

      DO 100  I = 1, N-1

         S         =  SMIN +  DBLE(I) * H
         AX        =  FNA(S,IFN,PARM) * 2D0 * K
```

```
      BX       =  FNB(S,IFN,PARM) * H * K
      CX       =  FNC(S,IFN,PARM) * FUTURE * 2D0 * H * H * K
      DENOM    =  CX - 2D0 * AX - 4D0 * H * H

      ARR(I,1) =  ( AX - BX ) / DENOM
      ARR(I,2) =  1D0
      ARR(I,3) =  ( AX + BX ) / DENOM
      ARR(I,4) =  1D0 + 8D0 * H * H / DENOM
C***TEST ONLY
C     IF (I.EQ.21) PRINT*,'ARR: ',(ARR(I,III),III=1,4)

 100  CONTINUE

C**** THEN HANDLE BOUNDARIES ACCORDING TO FLAGS ************************

      IF ( ISMIN .EQ. 1 )  THEN

C        CASE OF KNOWN VALUE AT SMIN: ISMIN = 1
         ARR(0,1) =  0D0
         ARR(0,2) =  1D0
         ARR(0,3) =  0D0

      ELSE

C        CASE OF QUADRATIC EXTRAPOLATION AT SMIN: ISMIN = 0
         G        =  ARR(1,3) / ( ARR(2,2) + 3D0 * ARR(2,3) )
         ARR(0,1) =  0D0
         ARR(0,2) =  G * ARR(2,3) - ARR(1,1)
         ARR(0,3) =  G * ( ARR(2,1) - 3D0 * ARR(2,3) ) - ARR(1,2)
         ARR(0,4) =  G

      ENDIF

      IF ( ISMAX .EQ. 1 )  THEN

C        CASE OF KNOWN VALUE AT SMAX: ISMAX = 1
         ARR(N,1) =  0D0
         ARR(N,2) =  1D0
         ARR(N,3) =  0D0

      ELSE

C        CASE OF QUADRATIC EXTRAPOLATION AT SMAX: ISMAX = 0
         G        =  ARR(N-1,1) / ( ARR(N-2,2) + 3D0 * ARR(N-2,1) )
         ARR(N,1) =  G * ( ARR(N-2,3) - 3D0 * ARR(N-2,1) ) - ARR(N-1,2)
         ARR(N,2) =  G * ARR(N-2,1) - ARR(N-1,3)
         ARR(N,3) =  0D0
         ARR(N,4) =  G

      ENDIF

      RETURN
```

```
      END

      SUBROUTINE CNSTEP ( T, U, ARR )

C************************************************************************
C
C  SUBROUTINE CNSTEP (...)
C
C  Subroutine takes 1 step in time direction in solving 1 state variable
C  PDE using Crank-Nicholson algorithm.  T is current time used only for
C  passing to boundary value functions FMIN(T) and FMAX(T) if ISMIN or
C  ISMAX are set to 1.  U(0:N) is N+1 dimensional vector of solution so
C  far.  ARR() is coefficient array set up by CNSET().
C
C************************************************************************

      IMPLICIT   DOUBLE PRECISION ( A-H, K-L, O-Z )
      COMMON    /CNCOM/ N, ISMIN, ISMAX, IFN, PARM(15)
      DIMENSION  ARR( 0:N, 4 ), U( 0:N )

C     NOTE: PARAMETER NMAX MUST BE .GE. N FOR TRIDAG ALGORITHM
      PARAMETER ( NMAX = 200 )
      COMMON    /TRICOM/ D( 0:NMAX ), GAM( 0:NMAX )

C     SET UP RIGHT HAND SIDE OF SYSTEM TRIDIAGONAL SYSTEM (ABC)U = D

      DO 100  I = 1, N-1
         D(I) = - ARR(I,1)*U(I-1) - ARR(I,4)*U(I) - ARR(I,3)*U(I+1)
 100  CONTINUE

      IF ( ISMIN .EQ. 1 )  THEN
C        GET SOLUTION VALUE AT RMIN
         D(0) = FMIN(T,IFN,PARM)
      ELSE
         D(0) = D(2) * ARR(0,4) - D(1)
      ENDIF

      IF ( ISMAX .EQ. 1 )  THEN
C        GET SOLUTION VALUE AT RMAX
         D(N) = FMAX(T,IFN,PARM)
      ELSE
         D(N) = D(N-2) * ARR(N,4) - D(N-1)
      ENDIF

      CALL TRIDAG ( ARR(0,1), ARR(0,2), ARR(0,3), D, GAM, U, N )

      RETURN
      END

C**** TRIDIAGONAL SOLN. ALGORITHM FROM "NUMERICAL RECIPES", P. 40 ******

C     SOLVES: (ABC)X = D FOR X.  N=DIMENSION.  A,B,C,D, NOT ALTERED
```

```
C     NOTE: SUBSCRIPTS RUN FROM 0 AND SCRATCH VECTOR GAM VARIABLE DIMEN.

      SUBROUTINE TRIDAG ( A, B, C, D, GAM, X, N )

      IMPLICIT  DOUBLE PRECISION ( A-H, K-L, O-Z )
      DIMENSION A(0:*), B(0:*), C(0:*), D(0:*), GAM(0:*), X(0:*)

      BET  = B(0)
      X(0) = D(0) / BET
      DO 10 J = 1, N
         GAM(J) = C(J-1) / BET
         BET    =  B(J) - A(J) * GAM(J)
         X(J)   = (D(J) - A(J) * X(J-1)) / BET
   10 CONTINUE

      DO 20 J = N-1, 0, -1
   20    X(J)   =  X(J) - GAM(J+1) * X(J+1)
      RETURN
      END
```

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