CANADIAN ZERO-COUPON YIELD CURVE SHOCKS AND STRESS TESTING

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Abstract

The yield curve movements have been the subject of many researches. As it is shown that the yield curve has the power to reflect major macroeconomic factors, the changes on it has been studied to predict future returns on portfolios and to identify some unusual events such as financial crisis. In this paper, a systematic procedure to identify yield curve shocks are presented mainly using the level, slope and curvature factors in the yield curves. The extreme changes happened in the level, slope, and curvatures are then provided for the stress testing purposes. This procedure should be simply applicable for any zero-coupon yield curve data, so the same tests are suggested for other curves to show this can be widely acceptable.

Keywords: Yield Curve Shocks, Zero-Coupon Bonds, Stress Testing, Interest Rate Risk

Dedication

To my lovely husband, Chongho, and my parents, of course!

- Annette

To my parents, for their endless love and support throughout the years. To Landers, for always being there for me.

- Tanya

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1: Introduction

1.1 Background

Macroeconomic impacts such as the large changes in expectation on the inflation, the monetary policy of the central bank, or high fiscal deficits are among the causes of significant yield curve changes. Financial crisis such as 1987 stock market crash or the recent credit crises are expected to be reflected in the shape of the yield curve. The normal shape of yield curve is generally recognized as increasing at decreasing rate as the time to maturity increases. The yield curve, formally called as term structure of interest rates, is often used as a tool to understand or predict the conditions in the financial market, since its movements are interpreted as a signal of changes in the market. The duration and convexity are commonly used for risk measures, but they are shown to have limitations in its efficiency of measuring risks. Consequently, the measure of changes indentified as shift, twist, and butterfly (STB) factor model is recognized to successfully overcome the limitations (Vannerem and Iyer, 2010).

Meanwhile, the recent financial crisis has highlighted the significance of stress testing based on the interest rate. Historically, there have been many factors that affected the yield curves. Some have affected the slope and others on the curvature. Thus, the stress testing for yield curves can be conducted by modelling the macroeconomic stress testing scenarios on different yield curves.

The purpose of our test is to identify shocks reflected in Canadian zero-coupon yield curves and to provide the STB factors of shocks for stress testing application. We attempt to create simply applicable tests for both identification of shocks and stress testing based on historical data.

1.2 Outline

In this paper, Section 2 starts with the literature reviews on selective yield curve models and the identification of shocks. In Section 3, the data and methodologies for the tests are discussed. Using annual average yields, the models are tested for its accuracy in fitting the Canadian zero-coupon Treasury curves in Section 4. The best fitted model is then used to identify the yield curve shocks reflected in the Canadian zero-coupon Treasury bill curves. The detailed tests conducted to identify yield curve shocks and the results are summarized in Section 5. Depending on the shocks identified, the changes in level, slope, and curvature factors will be provided for further stress testing in Section 6. Any further research that can be developed will be summarized in Section 7, and the paper will be finalized with the conclusion and discussion in Section 8.

2: Literature Review

Many models for fitting yield curves have been continuously developed since David Durand fit the yield curve by drawing monotonic envelop under the scatter points in 1942. A variety of parametric models was proposed to fit the yield curve by various other researchers afterwards. Some of them are based on polynomial regression while all include at least a linear term. A need for parsimonious modeling of yield curve has been recognized by Milton Friedman in 1977, and Nelson and Siegel built a widely used model that explains the term structure of yields using only a few parameters ten years later (Nelson and Sigel 1987). A number of authors have proposed extensions to the Nelson-Siegel model to enhance the flexibility; that is, to enhance the measure of the curvatures and humps (Diebond, Li, Perignon, 2008). For test purposes, three models are reviewed for the empirical yield curve fitting of Canadian zero-coupon Treasury bonds in this paper.

2.1 Yield Curve Model

2.1.1 Nelson- Siegel Model (1987)

Nelson-Siegel Model characterized the movement of three unobservable factors in the yield curve: level, slope and curvature. The instantaneous forward rate curve is:

$$f_t(\tau) = \beta_{1t} + \beta_{2t} e^{-\frac{\tau}{\lambda_t}} + \beta_{3t} e^{\frac{\tau}{\lambda_t}}$$

, which implies the continuously- compounded zero-coupon nominal yield at maturity τ is as follows:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right)$$

where β_{1t}, β_{2t}, β_{3t} and λ_t are time-varying parameters.

β_{1t} in the model indicates the level factor while β_{2t} and β_{3t} indicate the two shape factors, a slope factor and a curvature, respectively. Diebond and Li fixed the value of λ as 0.0609 (with maturities measured in months) and showed that this fixed λ not only helps simplify the test but also yields the most trustworthy estimates of the level, slope, and curvature factors (Diebond, Li, Perignon, 2008). This fixed value becomes 0.7308 for maturities measured in years: $\lambda = 0.0609 \times 12 = 0.7308$.

2.1.2 Svensson model (1994)

Svensson extended Nelson-Siegel model by allowing for two decay parameters λ_{1t} and λ_{2t}. His proposed forward rate curve equation is:

$$f_t(\tau) = \beta_{1t} + \beta_{2t} e^{-\lambda_{1t}\tau} + \beta_{3t}\left(\lambda_{1t}\tau e^{-\lambda_{1t}\tau}\right) + \beta_{4t}\left(\lambda_{2t}\tau e^{-\lambda_{2t}\tau}\right)$$

Then, the yield curve becomes as follows:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}\right) + \beta_{4t}\left(\frac{1-e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right)$$

The Svensson model allows two humps in the yield curve while Neslon-Siegel allows only one. Thus, the factors in this model can be interpreted as one level factor and three shape factors: a slope and two curvatures (humps). The first hump- the third term-is often placed in the relatively short horizons, so it often captures the effects of near term monetary policy. Meanwhile, the second hump-the fourth term-is located in the longer horizons (Diebond, Li, Perignon, 2008). In the Svensson model, it is plausible to set the λ_1 as 0.7308 and λ_2 as 0.08, respectively, according to the empirical tests done by Diebond and Li.

2.1.3 Bjork and Christensen model (1999)

Bjork and Christensen has argued that Nelson-Siegel model is not able to ensure that there is no arbitrage opportunities (Coroneo, Nyholm,Vidova-Koleva, 2008). Their five-factor model extended from Nelson-Siegel shows the following forward rate curve:

$$f_t(\tau) = \beta_{1t} + \beta_{2t}\tau + \beta_{3t}e^{-\lambda\tau} + \beta_{4t}\tau e^{-\lambda\tau} + \beta_{5t}e^{-2\lambda\tau}$$

, which implies the yield curve:

$$y_t(\tau)=\beta_{1t}+\beta_{2t}\left(\frac{\tau}{2}\right)+\beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)+\beta_{4t}\left(\frac{1-e^{-\lambda\tau}}{\lambda^2\tau}-\frac{e^{-\lambda\tau}}{\lambda}\right)+\beta_{5t}\left(\frac{1-e^{-2\lambda\tau}}{2\lambda\tau}\right)$$

In this model, the λ is fixed as 0.29 according to the empirical investigation done by Diebold et al (2008, p6).

2.2 Shift, Twist, and Butterfly (STB) Factors Model

The level and shape factors in the yield curves indicate how the yield curve behaves in accordance with its times to maturities; thus, how close or far they are from what we call “normal” yield curves. Whenever there are shocks on the yield curve, the level and shape factors show the abnormalities in the term structure.

The common practice to measure the shocks or abnormalities on the yield curve has been to measure its duration and convexity. Duration captures the interest rate risk- movements in the yield curve- that is associated with only with parallel shift. In reality, most movements happened in the yield curves are associated not only the parallel shift but also with the twist and butterfly in the yield curves (Vannerem and Iyer, p 3-4). Fabozzi has described in his textbook “Fixed Income Analysis”, the three factors movements- the shift, twist, and butterfly- are the driving factors that mostly describe yield curve dynamics. The Barra Risk Model Handbook confirmed this fact, as it proves the three factors “capture between 90-98% of interest rate variations in most developed markets” (2007). The Shift, Twist, and Butterfly (STB) factors are defined as follows:

Shift:	captures the changes in the level of yield curve
Twist:	captures the changes in the slope of yield curve
Butterfly:	captures the changes in the curvature of yield curve

This factor model together with the abnormalities found in the level, slope, and curvature will be the main methodology used to find the abnormalities in the Canadian zero-coupon yield curve in this paper.

3: Data and Methodology

3.1 Data

In the empirical analysis, the yields for Canadian zero-coupon bonds are obtained from the Bank of Canada. The daily data extend from January 1986 to February 2010, and they were generated using pricing data for Government of Canada bonds and treasury bills. Total 6,302 daily yield curve data with total 120 different maturities ranging from 0.25 to 30 years are gathered for the test purposes. Zero-coupon treasury rates were selected because corporate yield curves would include company-specific factors that possibly disguise the interest rate fluctuation.

3.2 Methodology

To identify shocks reflected in the Canadian zero-coupon Treasury yield curves, the annual average yields in each maturity is calculated and used for the regression in the three models selected: Nelson and Siegel, Svensson and Bjork and Christensen. The model that best fitted the Canadian zero-coupon Treasury yield is then selected to find out the abnormal slopes and curvatures. The abnormalities are evaluated based on 95% percentile of the distribution in the following maturity: 1, 2, 3, 5, 7, 15,20 and 30 years. The slopes and curvatures are also categorized to identify the abnormal movements in the yield curve. The same tests are conducted on the monthly data in order to specifically recognize when the shocks start to be reflected in the yield curves and how long it has taken to recover from the shocks. The categorized measures for shocks in its movements of shift, twist, and curvature are the key variables identified and provided for further stress testing.

4: Results on Yield Curve Models

To find the best-fitted yield curve model for the Canadian zero-coupon Treasury yields, three parsimonious models reviewed previously are tested in MATLAB. The factors (betas)- the level, slope, and curvature(s)- are estimated by the ordinary least squares and plugged into each model to evaluate how accurately the model fits the actual yield curve data.

From our tests, the model proposed by Bjork and Christensen is the best with highest R-square, average 0.9260 (compared to 0.7429 for Nelson-Siegel and 0.8685 for Svensson). At this point, only annual yield is tested since the purpose is to find out the best-fitted model, not to identify specific shocks in the very precise periods.

4.1 Tests on Yearly Average Yields

4.1.1 Nelson-Siegel Model

The yearly yield curves fitted by Nelson-Siegel model are presented in the Appendix A. The yearly time-varying factors estimated are summarized in the table A-1 by the level, slope, and curvature, while R squares of the model by year are in the table A-2. The fitted curves are illustrated by years in the graph A-1.

As table A-2 and graph A-1 indicate, Nelson-Siegel is generally good at fitting the curves from 1990 to 2007 with the exception of 2000 while it did a poor job before 1989.

4.1.2 Svensson Model

The yearly yield curve fitted by Svensson model are presented in the Appendix B. The yearly factors estimated are presented in the table B-1 by the level, slope, and two curvatures, while R squares of the model by year are in the table B-2. The fitted curves are illustrated by years in the graph B-2.

Svenssson model is fitting the curves with average R squared of 0.8685. Only in the years up to 1988 are quite bad in fitting the curves.

4.1.3 Bjork and Christensen Model

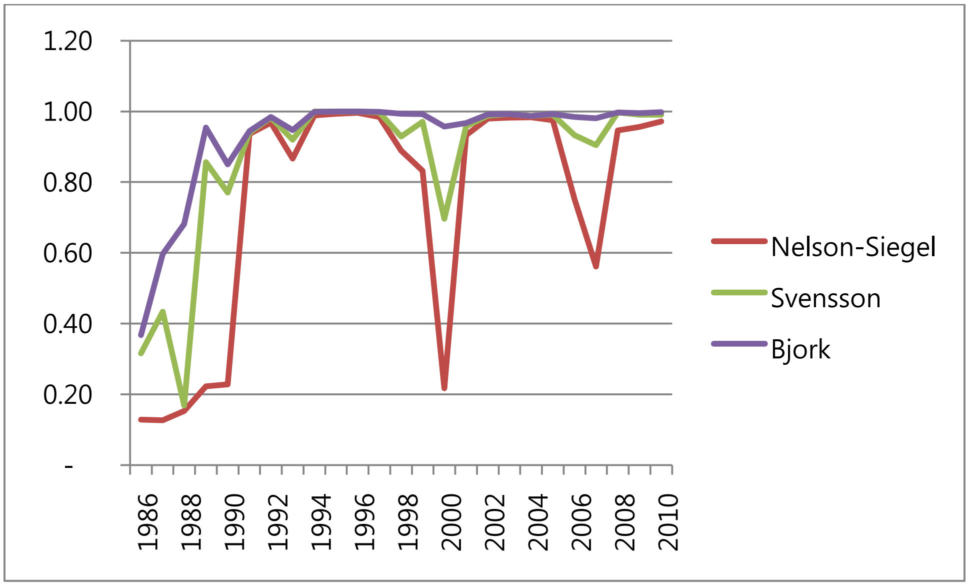
The yearly yield curve fitted by Bjork and Christensen model are presented in the Appendix C. The yearly factors estimated are summarized in the table C-1 by the level, slope, and three curvatures, while R squares of the model by year are illustrated in the table C-2. The fitted curves are shown by years in the graph C-2.

The R squares and yearly fitted curves in Figure 1 below illustrate that the Bjork and Christensen Model is the best fitted model among the three tested with highest R squares with average 0.9260. Bjork and Christensen Model have failed in precisely predicting the curves in 1986 and 1987, but it is generating mostly fitted yield curves in other years.

4.2 Comparison of R squares by Selected Models

The R squares by each model are summarized in the below graph for comparison purposes. As mentioned in the previous section, Bjork and Christensen Model is the best-fitted model to the Canadian zero-coupon Treasury yields; thus, it will be used for the test in identifying Canadian yield curve shocks in the following sections.

Figure 4.1 Comparison of R squares by Selected Models



5: Identification of Yield Curve Shocks

The level and shape of the yield curve has been altered when macroeconomic factors have changed. However, the shocks on the yield curve with severe changes in the level and shape have occurred when economic turmoil and natural or human-caused disasters take place. To identify the abnormality of the yield curve, the level, slope and curvature factors are closely examined in this paper. Monthly Canadian zero- coupon yields are used to precisely identify when the shocks happen and when the yield curve has recovered from the shocks using Bjork and Christensen Model.

5.1 Abnormality Test using Slope and Curvature

5.1.1 Slope and Curvature Calculation

The slope and curvature at each maturity is identified as follows:

Slope:

$$y_t{}'(\tau)=\frac{1}{2}\beta_{2t}+\beta_{3t}\left(\frac{1}{\tau}e^{-\lambda\tau}-\frac{1-e^{-\lambda\tau}}{\lambda\tau^2}\right)+\beta_{4t}\left(\frac{e^{-\lambda\tau}}{\lambda\tau}-\frac{1-e^{-\lambda\tau}}{\lambda^2\tau^2}+e^{-\lambda\tau}\right)+\beta_{5t}\left(\frac{e^{-2\lambda\tau}}{\tau}-\frac{1-e^{-2\lambda\tau}}{2\lambda\tau^2}\right)$$

Curvature:

$$y_t{}''(\tau)=\beta_{3t}\left(\frac{-\lambda e^{-\lambda\tau}}{\tau}-\frac{2}{\tau^2}e^{-\lambda\tau}+\frac{2(1-e^{-\lambda\tau})}{\lambda\tau^3}\right)+\beta_{4t}\left(\frac{-e^{-\lambda\tau}}{\tau}-\frac{2e^{-\lambda\tau}}{\lambda\tau^2}-\lambda e^{-\lambda\tau}+\frac{2(1-e^{-\lambda\tau})}{\lambda^2\tau^3}\right)+\beta_{5t}\left(\frac{-2\lambda e^{-2\lambda\tau}}{\tau}-\frac{2e^{-2\lambda\tau}}{\tau^2}+\frac{1-e^{-2\lambda\tau}}{\lambda\tau^3}\right)$$

Then, the monthly slopes and curvatures calculated from January 1986 to February 2010 are used to evaluate the abnormalities during the specified periods.

5.1.2 Test Methodologies

For test purposes, total nine time to maturities are selected in each month specifically: 1, 2, 3, 5, 7, 10, 15, 20, and 30 years. The abnormalities are evaluated based on each criterion below for the level, slope and curvature. Then, total 27 factors – nine maturities multiplied by three factors- are tested for abnormalities in each month. If more than 10, 11, or 12 factors fall out of the 27 selection parameters specified in Table 1, the month is selected as a ”shocked” month.

Table 5.1 Test Criteria for Abnormalities in the Yield Curves

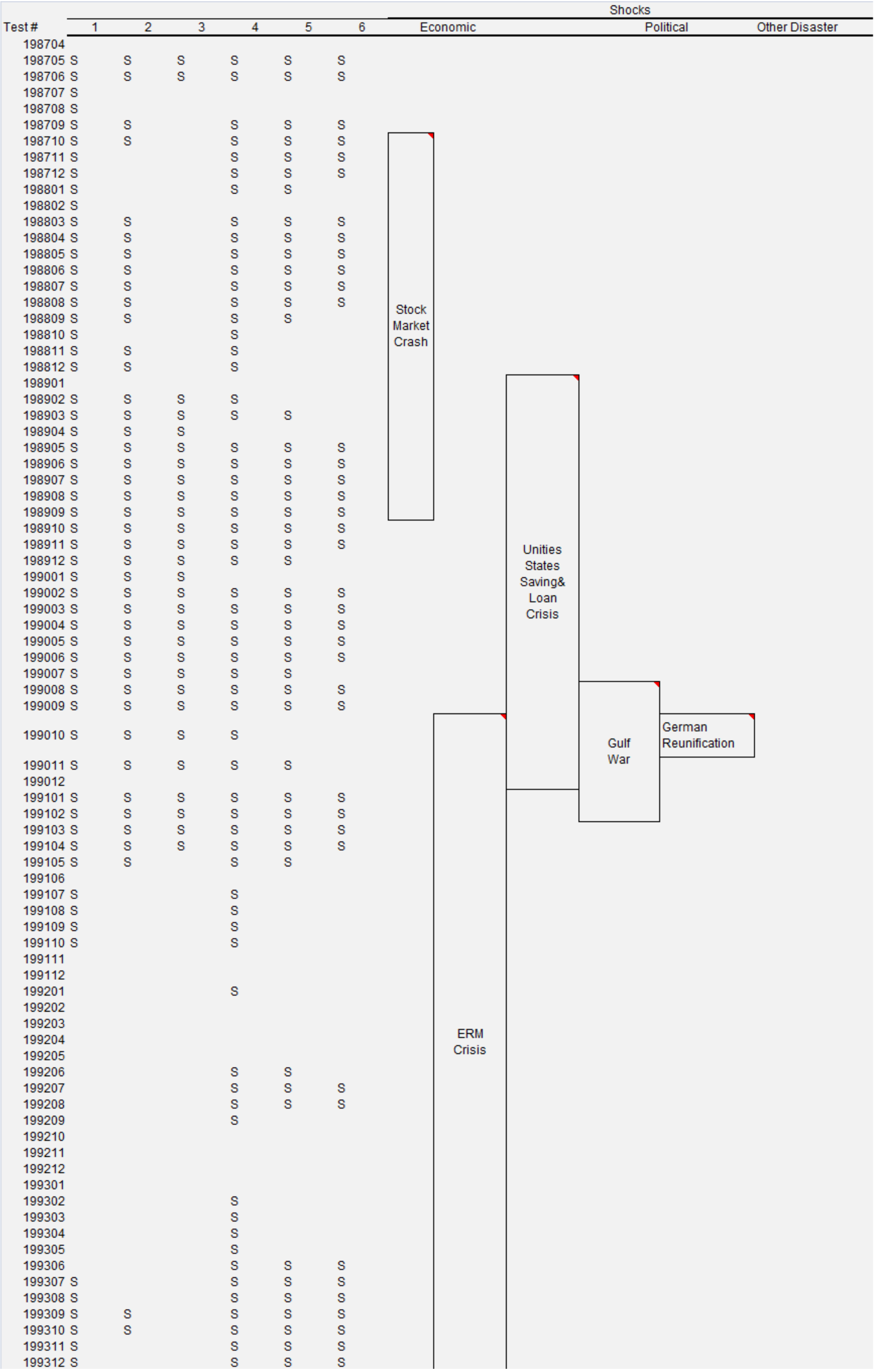
Test	Level	Slope	Curvature	Outlier #
1	2.5% Lower and Upper Bound (two-sided)	5% Lower bound	5% Upper bound	10
2	2.5% Lower and Upper Bound (two-sided)	5% Lower bound	5% Upper bound	11
3	2.5% Lower and Upper Bound (two-sided)	5% Lower bound	5% Upper bound	12
4		2.5% Lower and Upper Bound (two-sided)		10
5		2.5% Lower and Upper Bound (two-sided)		11
6		2.5% Lower and Upper Bound (two-sided)		12
7	5% Lower bound	5% Lower bound	5% Upper bound	3
8	5% Lower bound	5% Lower bound	5% Upper bound	2

The lower bound of first derivatives can capture the extreme negative slope, while the upper bound of second derivatives can capture the convex yield curves, which are classified as abnormalities in the yield curve. Thus, the first three tests were focused on finding out abnormalities in these two extremes. As the current steep yield curve is believed to signal the expected inflation, however, any big positive slope or big negative curvature are also classified as a sign for the recovery from shocks (Harvey). Accordingly, Test 4 to 6 include these possibilities by testing two-sided abnormalities. for the slope and curvature.

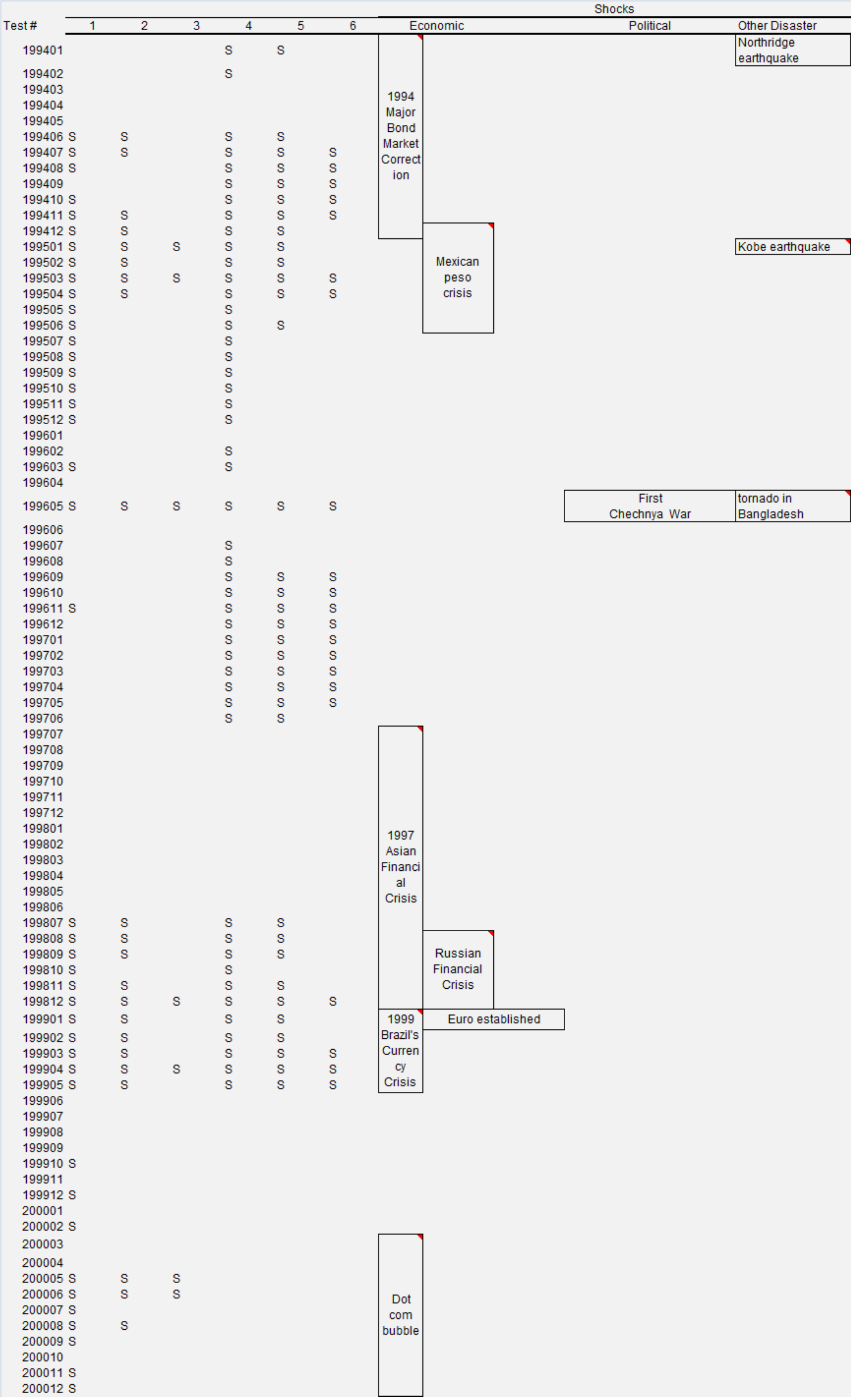
5.2 Results

According to the six criteria tested, the “shocked” months are identified in the following: Table 5.2 Months with Shocks Indentified by Six Different Criteria

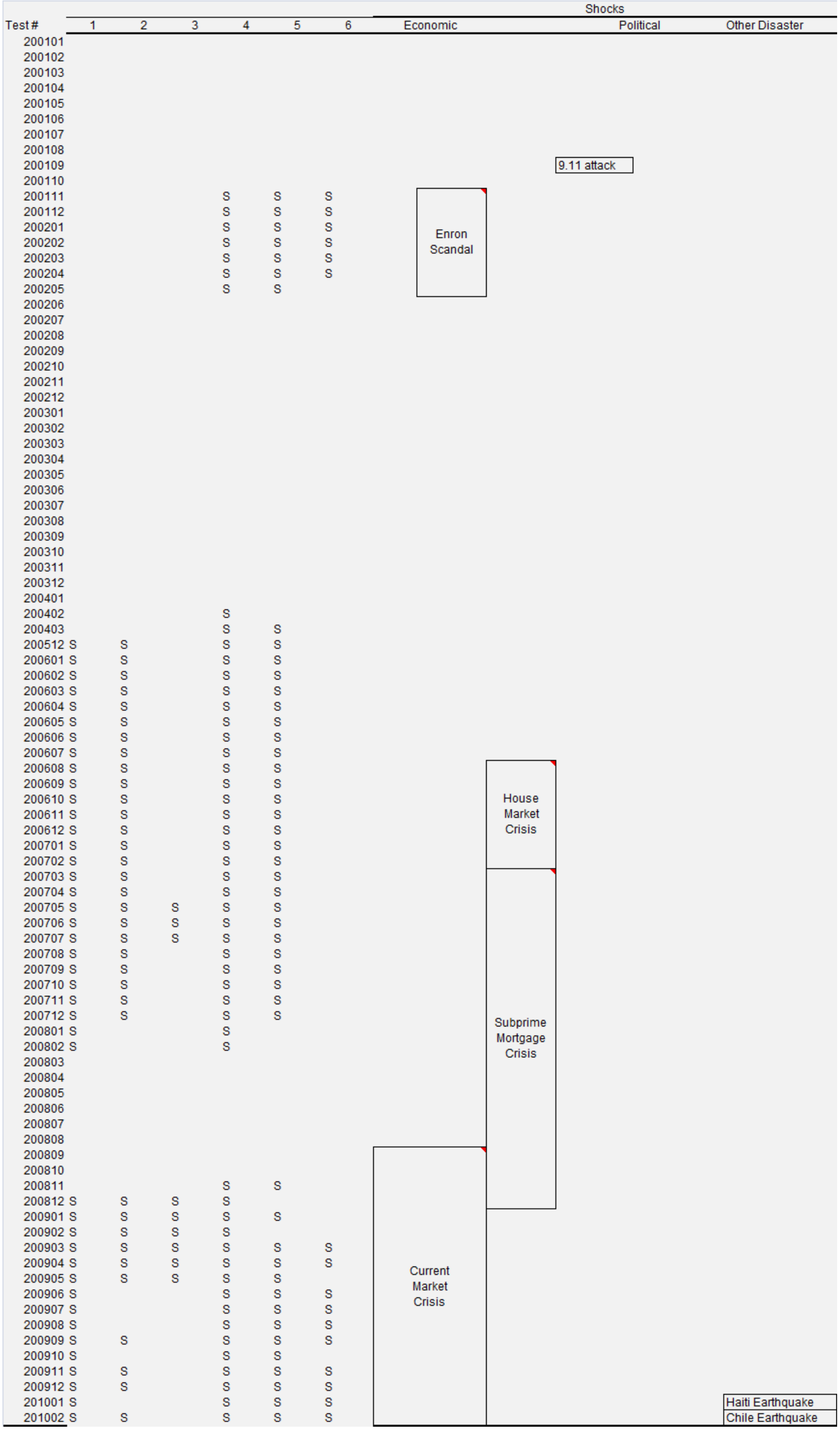
							Shocks		
Test #	1	2	3	4	5	6	Economic	Political	Other Disaster
198601									
198602									
198603									
198604	S			S	S				Chernobyl disaster
198605									
198606									
198607									
198608									
198609									
198610									
198611									
198612									
198701									
198702									
198703									



							Shocks		
Test #	1	2	3	4	5	6	Economic	Political	Other Disaster
198704									
198705	S	S	S	S	S	S			
198706	S	S	S	S	S	S			
198707	S								
198708	S								
198709	S	S		S	S	S			
198710	S	S		S	S	S	Stock Market Crash		
198711	S			S	S	S			
198712	S			S	S	S			
198801	S			S	S				
198802	S								
198803	S	S		S	S	S			
198804	S	S		S	S	S			
198805	S	S		S	S	S			
198806	S	S		S	S	S			
198807	S	S		S	S	S			
198808	S	S		S	S	S			
198809	S	S		S	S				
198810	S			S					
198811	S	S		S					
198812	S	S		S					
198901							Unities States Saving& Loan Crisis		
198902	S	S	S	S					
198903	S	S	S	S	S				
198904	S	S	S						
198905	S	S	S	S	S	S			
198906	S	S	S	S	S	S			
198907	S	S	S	S	S	S			
198908	S	S	S	S	S	S			
198909	S	S	S	S	S	S			
198910	S	S	S	S	S	S			
198911	S	S	S	S	S	S			
198912	S	S	S	S	S				
199001	S	S	S						
199002	S	S	S	S	S	S			
199003	S	S	S	S	S	S			
199004	S	S	S	S	S	S			
199005	S	S	S	S	S	S			
199006	S	S	S	S	S	S			
199007	S	S	S	S	S				
199008	S	S	S	S	S	S		Gulf War	
199009	S	S	S	S	S	S			
199010	S	S	S	S			ERM Crisis	German Reunification	
199011	S	S	S	S	S				
199012									
199101	S	S	S	S	S	S			
199102	S	S	S	S	S	S			
199103	S	S	S	S	S	S			
199104	S	S	S	S	S	S			
199105	S	S		S	S				
199106									
199107	S			S					
199108	S			S					
199109	S			S					
199110	S			S					
199111									
199112									
199201				S					
199202									
199203									
199204									
199205									
199206				S	S				
199207				S	S	S			
199208				S	S	S			
199209				S					
199210									
199211									
199212									
199301									
199302				S					
199303				S					
199304				S					
199305				S					
199306				S	S	S			
199307	S			S	S	S			
199308	S			S	S	S			
199309	S	S		S	S	S			
199310	S	S		S	S	S			
199311	S			S	S	S			
199312	S			S	S	S			



							Shocks		
Test #	1	2	3	4	5	6	Economic	Political	Other Disaster
199401				S	S		1994 Major Bond Market Correction		Northridge earthquake
199402				S					
199403									
199404									
199405									
199406	S	S		S	S				
199407	S	S		S	S	S			
199408	S			S	S	S			
199409				S	S	S			
199410	S			S	S	S			
199411	S	S		S	S	S			
199412	S	S		S	S		Mexican peso crisis		
199501	S	S	S	S	S				Kobe earthquake
199502	S	S		S	S				
199503	S	S	S	S	S	S			
199504	S	S		S	S	S			
199505	S			S					
199506	S			S	S				
199507	S			S					
199508	S			S					
199509	S			S					
199510	S			S					
199511	S			S					
199512	S			S					
199601									
199602				S					
199603	S			S					
199604									
199605	S	S	S	S	S	S		First Chechnya War	tornado in Bangladesh
199606									
199607				S					
199608				S					
199609				S	S	S			
199610				S	S	S			
199611	S			S	S	S			
199612				S	S	S			
199701				S	S	S			
199702				S	S	S			
199703				S	S	S			
199704				S	S	S			
199705				S	S	S			
199706				S	S				
199707							1997 Asian Financial Crisis		
199708									
199709									
199710									
199711									
199712									
199801									
199802									
199803									
199804									
199805									
199806									
199807	S	S		S	S				
199808	S	S		S	S		Russian Financial Crisis		
199809	S	S		S	S				
199810	S			S					
199811	S	S		S	S				
199812	S	S	S	S	S	S			
199901	S	S		S	S		1999 Brazil's Currency Crisis Euro established		
199902	S	S		S	S				
199903	S	S		S	S	S			
199904	S	S	S	S	S	S			
199905	S	S		S	S	S			
199906									
199907									
199908									
199909									
199910	S								
199911									
199912	S								
200001									
200002	S								
200003							Dot com bubble		
200004									
200005	S	S	S						
200006	S	S	S						
200007	S								
200008	S	S							
200009	S								
200010									
200011	S								
200012	S								



							Shocks		
Test #	1	2	3	4	5	6	Economic	Political	Other Disaster
200101									
200102									
200103									
200104									
200105									
200106									
200107									
200108									
200109								9.11 attack	
200110									
200111				S	S	S	Enron Scandal		
200112				S	S	S			
200201				S	S	S			
200202				S	S	S			
200203				S	S	S			
200204				S	S	S			
200205				S	S				
200206									
200207									
200208									
200209									
200210									
200211									
200212									
200301									
200302									
200303									
200304									
200305									
200306									
200307									
200308									
200309									
200310									
200311									
200312									
200401									
200402				S					
200403				S	S				
200512	S	S		S	S				
200601	S	S		S	S				
200602	S	S		S	S				
200603	S	S		S	S				
200604	S	S		S	S				
200605	S	S		S	S				
200606	S	S		S	S				
200607	S	S		S	S				
200608	S	S		S	S		House Market Crisis		
200609	S	S		S	S				
200610	S	S		S	S				
200611	S	S		S	S				
200612	S	S		S	S				
200701	S	S		S	S				
200702	S	S		S	S				
200703	S	S		S	S		Subprime Mortgage Crisis		
200704	S	S		S	S				
200705	S	S	S	S	S				
200706	S	S	S	S	S				
200707	S	S	S	S	S				
200708	S	S		S	S				
200709	S	S		S	S				
200710	S	S		S	S				
200711	S	S		S	S				
200712	S	S		S	S				
200801	S			S					
200802	S			S					
200803									
200804									
200805									
200806									
200807									
200808									
200809							Current Market Crisis		
200810									
200811				S	S				
200812	S	S	S	S					
200901	S	S	S	S	S				
200902	S	S	S	S					
200903	S	S	S	S	S	S			
200904	S	S	S	S	S	S			
200905	S	S	S	S	S				
200906	S			S	S	S			
200907	S			S	S	S			
200908	S			S	S	S			
200909	S	S		S	S	S			
200910	S			S	S				
200911	S	S		S	S	S			
200912	S	S		S	S	S			
201001	S			S	S	S			Haiti Earthquake
201002	S	S		S	S	S			Chile Earthquake

The table illustrates that Test 1, Test 2, and Test 5 successfully capture most economic or financial crisis while Test 3 and 6 do not. As Test 3 and 6 fail to identify some major crisis, it can be narrowed down that ten or eleven outliers are more appropriate numbers for testing purposes. However, Test 4 has picked up too many months as “shocked” months, so it is concluded that its results are containing “noises”. Interestingly, Test 1, 2, and 5 pick up the months right before the two major shocks with abnormalities: the 1987 stock market crash and the current crisis. These are not months with shocks, and may be considered as noises in the tests. However, it can also be interpreted that the Canadian yield curve, in fact, started to move abnormally right before the shocks and to provide “insights into the likely future paths of real economic activity” (Keen, 1989).

Apparently, the exact starting or ending months for certain shocks are hard to identify. In that case, the closest year is selected as the starting or ending period. The number of shock months and the percentage of total shock months that each test has identified are illustrated below:

Table 5.3 Number of Shock Months and Percentage Indentified Six Different Tests

Test	1	2	3	4	5	6
Number of Shock Months	142	99	44	163	129	76
% Shock Identified	0.91	0.63	0.28	1.04	0.83	0.49
Number of False Positive	24	11	2	33	22	12
% False Positive	0.08	0.04	0.01	0.11	0.08	0.04

According to Table 5.3, Test 1 with extremely small slope and high curvature delivers the best result as it successfully identifies 91% of “shocked” months. The false positive number of months identified by each test and the percentage out of total months tested are also included in the following rows in Table 5.3.

From this test, it has shown that the model can pick up abnormal movements in Canadian zero-coupon yield curve that is caused by major world-wide financial crisis. Meanwhile, it even reflected pre-shocks for major crisis such as 1987 stock market crash and recent one. Since Test 1 has successfully identified most of the crisis as “shocked” months, so it will be mainly used for stress test purposes in the following section.

6: Stress Testing

In practice, historical or hypothetical shocks can be used as the determination of the shocks (Diebond, Li, Perignon, 2008). In this paper, the historical shocks identified in the previous sections will be providing more realistic and plausible scenarios for future stress testing purposes.

6.1 Stress Testing Application for Yield Curve Shocks

As historical crises provide the plausible scenarios for the impact on the yield curve by the shocks, the changes in the level, slope, and curvatures in the identified “shocked” months in Section 5 are quantified. These changes are defined as shift, twist, and butterfly respectively for the changes for level, slope, and curvatures.

6.1.1 Shift, Twist, and Butterfly (STB) factors

The basic idea to apply the movements in “shocked” months is to measure the change in yield curve of this month from previous month. The shift, twist, and butterfly in each month are calculated as follows:

Shift = $y_t(\tau) - y_t(\tau - 1)$

Twist = $y_t'(\tau) - y_t'(\tau - 1)$

Butterfly = $y_t''(\tau) - y_t''(\tau - 1)$

6.1.2 Methodologies

In each maturity, the maximum, 95%, 5%, and minimum values for the shift, twist, and butterfly among 290 months from January 1986 to February 2010 are calculated. These figures are the extreme historical changes happened in each maturity that became a “shocked” month, which can be used as a more realistic stress testing scenario.

6.2 Results

The extreme values in the shift, twist, and butterfly identified from “shocked” months using Test 1 in Section 5 are summarized in the below tables.

Table 6.1 Shift, Twist, and Butterfly for Stress Testing of Yield Curve Shocks

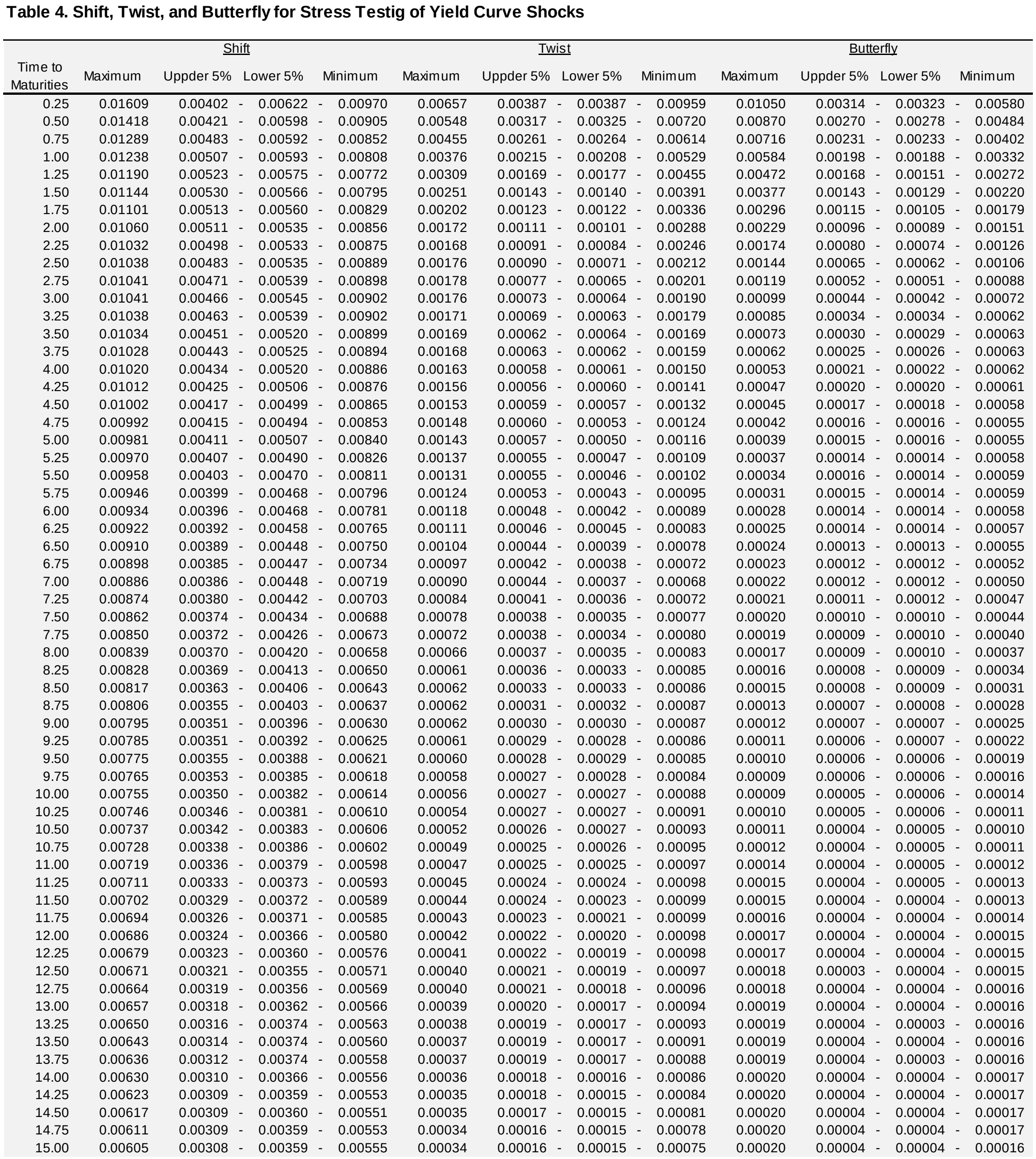


Table 4. Shift, Twist, and Butterfly for Stress Testig of Yield Curve Shocks

Time to Maturities	Shift				Twist				Butterfly			
	Maximum	Uppder 5%	Lower 5%	Minimum	Maximum	Uppder 5%	Lower 5%	Minimum	Maximum	Uppder 5%	Lower 5%	Minimum
0.25	0.01609	0.00402	- 0.00622	- 0.00970	0.00657	0.00387	- 0.00387	- 0.00959	0.01050	0.00314	- 0.00323	- 0.00580
0.50	0.01418	0.00421	- 0.00598	- 0.00905	0.00548	0.00317	- 0.00325	- 0.00720	0.00870	0.00270	- 0.00278	- 0.00484
0.75	0.01289	0.00483	- 0.00592	- 0.00852	0.00455	0.00261	- 0.00264	- 0.00614	0.00716	0.00231	- 0.00233	- 0.00402
1.00	0.01238	0.00507	- 0.00593	- 0.00808	0.00376	0.00215	- 0.00208	- 0.00529	0.00584	0.00198	- 0.00188	- 0.00332
1.25	0.01190	0.00523	- 0.00575	- 0.00772	0.00309	0.00169	- 0.00177	- 0.00455	0.00472	0.00168	- 0.00151	- 0.00272
1.50	0.01144	0.00530	- 0.00566	- 0.00795	0.00251	0.00143	- 0.00140	- 0.00391	0.00377	0.00143	- 0.00129	- 0.00220
1.75	0.01101	0.00513	- 0.00560	- 0.00829	0.00202	0.00123	- 0.00122	- 0.00336	0.00296	0.00115	- 0.00105	- 0.00179
2.00	0.01060	0.00511	- 0.00535	- 0.00856	0.00172	0.00111	- 0.00101	- 0.00288	0.00229	0.00096	- 0.00089	- 0.00151
2.25	0.01032	0.00498	- 0.00533	- 0.00875	0.00168	0.00091	- 0.00084	- 0.00246	0.00174	0.00080	- 0.00074	- 0.00126
2.50	0.01038	0.00483	- 0.00535	- 0.00889	0.00176	0.00090	- 0.00071	- 0.00212	0.00144	0.00065	- 0.00062	- 0.00106
2.75	0.01041	0.00471	- 0.00539	- 0.00898	0.00178	0.00077	- 0.00065	- 0.00201	0.00119	0.00052	- 0.00051	- 0.00088
3.00	0.01041	0.00466	- 0.00545	- 0.00902	0.00176	0.00073	- 0.00064	- 0.00190	0.00099	0.00044	- 0.00042	- 0.00072
3.25	0.01038	0.00463	- 0.00539	- 0.00902	0.00171	0.00069	- 0.00063	- 0.00179	0.00085	0.00034	- 0.00034	- 0.00062
3.50	0.01034	0.00451	- 0.00520	- 0.00899	0.00169	0.00062	- 0.00064	- 0.00169	0.00073	0.00030	- 0.00029	- 0.00063
3.75	0.01028	0.00443	- 0.00525	- 0.00894	0.00168	0.00063	- 0.00062	- 0.00159	0.00062	0.00025	- 0.00026	- 0.00063
4.00	0.01020	0.00434	- 0.00520	- 0.00886	0.00163	0.00058	- 0.00061	- 0.00150	0.00053	0.00021	- 0.00022	- 0.00062
4.25	0.01012	0.00425	- 0.00506	- 0.00876	0.00156	0.00056	- 0.00060	- 0.00141	0.00047	0.00020	- 0.00020	- 0.00061
4.50	0.01002	0.00417	- 0.00499	- 0.00865	0.00153	0.00059	- 0.00057	- 0.00132	0.00045	0.00017	- 0.00018	- 0.00058
4.75	0.00992	0.00415	- 0.00494	- 0.00853	0.00148	0.00060	- 0.00053	- 0.00124	0.00042	0.00016	- 0.00016	- 0.00055
5.00	0.00981	0.00411	- 0.00507	- 0.00840	0.00143	0.00057	- 0.00050	- 0.00116	0.00039	0.00015	- 0.00016	- 0.00055
5.25	0.00970	0.00407	- 0.00490	- 0.00826	0.00137	0.00055	- 0.00047	- 0.00109	0.00037	0.00014	- 0.00014	- 0.00058
5.50	0.00958	0.00403	- 0.00470	- 0.00811	0.00131	0.00055	- 0.00046	- 0.00102	0.00034	0.00016	- 0.00014	- 0.00059
5.75	0.00946	0.00399	- 0.00468	- 0.00796	0.00124	0.00053	- 0.00043	- 0.00095	0.00031	0.00015	- 0.00014	- 0.00059
6.00	0.00934	0.00396	- 0.00468	- 0.00781	0.00118	0.00048	- 0.00042	- 0.00089	0.00028	0.00014	- 0.00014	- 0.00058
6.25	0.00922	0.00392	- 0.00458	- 0.00765	0.00111	0.00046	- 0.00045	- 0.00083	0.00025	0.00014	- 0.00014	- 0.00057
6.50	0.00910	0.00389	- 0.00448	- 0.00750	0.00104	0.00044	- 0.00039	- 0.00078	0.00024	0.00013	- 0.00013	- 0.00055
6.75	0.00898	0.00385	- 0.00447	- 0.00734	0.00097	0.00042	- 0.00038	- 0.00072	0.00023	0.00012	- 0.00012	- 0.00052
7.00	0.00886	0.00386	- 0.00448	- 0.00719	0.00090	0.00044	- 0.00037	- 0.00068	0.00022	0.00012	- 0.00012	- 0.00050
7.25	0.00874	0.00380	- 0.00442	- 0.00703	0.00084	0.00041	- 0.00036	- 0.00072	0.00021	0.00011	- 0.00012	- 0.00047
7.50	0.00862	0.00374	- 0.00434	- 0.00688	0.00078	0.00038	- 0.00035	- 0.00077	0.00020	0.00010	- 0.00010	- 0.00044
7.75	0.00850	0.00372	- 0.00426	- 0.00673	0.00072	0.00038	- 0.00034	- 0.00080	0.00019	0.00009	- 0.00010	- 0.00040
8.00	0.00839	0.00370	- 0.00420	- 0.00658	0.00066	0.00037	- 0.00035	- 0.00083	0.00017	0.00009	- 0.00010	- 0.00037
8.25	0.00828	0.00369	- 0.00413	- 0.00650	0.00061	0.00036	- 0.00033	- 0.00085	0.00016	0.00008	- 0.00009	- 0.00034
8.50	0.00817	0.00363	- 0.00406	- 0.00643	0.00062	0.00033	- 0.00033	- 0.00086	0.00015	0.00008	- 0.00009	- 0.00031
8.75	0.00806	0.00355	- 0.00403	- 0.00637	0.00062	0.00031	- 0.00032	- 0.00087	0.00013	0.00007	- 0.00008	- 0.00028
9.00	0.00795	0.00351	- 0.00396	- 0.00630	0.00062	0.00030	- 0.00030	- 0.00087	0.00012	0.00007	- 0.00007	- 0.00025
9.25	0.00785	0.00351	- 0.00392	- 0.00625	0.00061	0.00029	- 0.00028	- 0.00086	0.00011	0.00006	- 0.00007	- 0.00022
9.50	0.00775	0.00355	- 0.00388	- 0.00621	0.00060	0.00028	- 0.00029	- 0.00085	0.00010	0.00006	- 0.00006	- 0.00019
9.75	0.00765	0.00353	- 0.00385	- 0.00618	0.00058	0.00027	- 0.00028	- 0.00084	0.00009	0.00006	- 0.00006	- 0.00016
10.00	0.00755	0.00350	- 0.00382	- 0.00614	0.00056	0.00027	- 0.00027	- 0.00088	0.00009	0.00005	- 0.00006	- 0.00014
10.25	0.00746	0.00346	- 0.00381	- 0.00610	0.00054	0.00027	- 0.00027	- 0.00091	0.00010	0.00005	- 0.00006	- 0.00011
10.50	0.00737	0.00342	- 0.00383	- 0.00606	0.00052	0.00026	- 0.00027	- 0.00093	0.00011	0.00004	- 0.00005	- 0.00010
10.75	0.00728	0.00338	- 0.00386	- 0.00602	0.00049	0.00025	- 0.00026	- 0.00095	0.00012	0.00004	- 0.00005	- 0.00011
11.00	0.00719	0.00336	- 0.00379	- 0.00598	0.00047	0.00025	- 0.00025	- 0.00097	0.00014	0.00004	- 0.00005	- 0.00012
11.25	0.00711	0.00333	- 0.00373	- 0.00593	0.00045	0.00024	- 0.00024	- 0.00098	0.00015	0.00004	- 0.00005	- 0.00013
11.50	0.00702	0.00329	- 0.00372	- 0.00589	0.00044	0.00024	- 0.00023	- 0.00099	0.00015	0.00004	- 0.00004	- 0.00013
11.75	0.00694	0.00326	- 0.00371	- 0.00585	0.00043	0.00023	- 0.00021	- 0.00099	0.00016	0.00004	- 0.00004	- 0.00014
12.00	0.00686	0.00324	- 0.00366	- 0.00580	0.00042	0.00022	- 0.00020	- 0.00098	0.00017	0.00004	- 0.00004	- 0.00015
12.25	0.00679	0.00323	- 0.00360	- 0.00576	0.00041	0.00022	- 0.00019	- 0.00098	0.00017	0.00004	- 0.00004	- 0.00015
12.50	0.00671	0.00321	- 0.00355	- 0.00571	0.00040	0.00021	- 0.00019	- 0.00097	0.00018	0.00003	- 0.00004	- 0.00015
12.75	0.00664	0.00319	- 0.00356	- 0.00569	0.00040	0.00021	- 0.00018	- 0.00096	0.00018	0.00004	- 0.00004	- 0.00016
13.00	0.00657	0.00318	- 0.00362	- 0.00566	0.00039	0.00020	- 0.00017	- 0.00094	0.00019	0.00004	- 0.00004	- 0.00016
13.25	0.00650	0.00316	- 0.00374	- 0.00563	0.00038	0.00019	- 0.00017	- 0.00093	0.00019	0.00004	- 0.00003	- 0.00016
13.50	0.00643	0.00314	- 0.00374	- 0.00560	0.00037	0.00019	- 0.00017	- 0.00091	0.00019	0.00004	- 0.00004	- 0.00016
13.75	0.00636	0.00312	- 0.00374	- 0.00558	0.00037	0.00019	- 0.00017	- 0.00088	0.00019	0.00004	- 0.00003	- 0.00016
14.00	0.00630	0.00310	- 0.00366	- 0.00556	0.00036	0.00018	- 0.00016	- 0.00086	0.00020	0.00004	- 0.00004	- 0.00017
14.25	0.00623	0.00309	- 0.00359	- 0.00553	0.00035	0.00018	- 0.00015	- 0.00084	0.00020	0.00004	- 0.00004	- 0.00017
14.50	0.00617	0.00309	- 0.00360	- 0.00551	0.00035	0.00017	- 0.00015	- 0.00081	0.00020	0.00004	- 0.00004	- 0.00017
14.75	0.00611	0.00309	- 0.00359	- 0.00553	0.00034	0.00016	- 0.00015	- 0.00078	0.00020	0.00004	- 0.00004	- 0.00017
15.00	0.00605	0.00308	- 0.00359	- 0.00555	0.00034	0.00016	- 0.00015	- 0.00075	0.00020	0.00004	- 0.00004	- 0.00016

Table 4. Shift, Twist, and Butterfly for Stress Testig of Yield Curve Shocks

	Shift				Twist				Butterfly			
Time to Maturities	Maximum	Uppder 5%	Lower 5%	Minimum	Maximum	Uppder 5%	Lower 5%	Minimum	Maximum	Uppder 5%	Lower 5%	Minimum
15.25	0.00599	0.00307	- 0.00360	- 0.00557	0.00033	0.00016	- 0.00015	- 0.00072	0.00020	0.00004	- 0.00004	- 0.00016
15.50	0.00593	0.00307	- 0.00360	- 0.00569	0.00033	0.00016	- 0.00015	- 0.00069	0.00020	0.00004	- 0.00004	- 0.00016
15.75	0.00588	0.00306	- 0.00360	- 0.00586	0.00032	0.00017	- 0.00014	- 0.00066	0.00019	0.00004	- 0.00004	- 0.00016
16.00	0.00582	0.00305	- 0.00360	- 0.00602	0.00033	0.00017	- 0.00015	- 0.00063	0.00019	0.00004	- 0.00004	- 0.00016
16.25	0.00577	0.00304	- 0.00360	- 0.00617	0.00035	0.00018	- 0.00014	- 0.00059	0.00019	0.00004	- 0.00004	- 0.00016
16.50	0.00572	0.00304	- 0.00358	- 0.00631	0.00036	0.00018	- 0.00014	- 0.00056	0.00019	0.00004	- 0.00004	- 0.00016
16.75	0.00567	0.00304	- 0.00356	- 0.00645	0.00038	0.00018	- 0.00014	- 0.00053	0.00019	0.00004	- 0.00004	- 0.00016
17.00	0.00561	0.00305	- 0.00354	- 0.00658	0.00040	0.00018	- 0.00014	- 0.00050	0.00018	0.00004	- 0.00004	- 0.00015
17.25	0.00556	0.00305	- 0.00352	- 0.00669	0.00044	0.00018	- 0.00015	- 0.00052	0.00018	0.00004	- 0.00004	- 0.00015
17.50	0.00552	0.00305	- 0.00350	- 0.00681	0.00049	0.00017	- 0.00016	- 0.00055	0.00018	0.00004	- 0.00004	- 0.00015
17.75	0.00547	0.00305	- 0.00349	- 0.00691	0.00053	0.00017	- 0.00016	- 0.00059	0.00018	0.00004	- 0.00004	- 0.00015
18.00	0.00542	0.00306	- 0.00347	- 0.00700	0.00058	0.00017	- 0.00016	- 0.00062	0.00017	0.00004	- 0.00004	- 0.00014
18.25	0.00537	0.00306	- 0.00346	- 0.00708	0.00062	0.00017	- 0.00016	- 0.00066	0.00017	0.00004	- 0.00004	- 0.00014
18.50	0.00533	0.00306	- 0.00357	- 0.00716	0.00066	0.00017	- 0.00016	- 0.00069	0.00017	0.00004	- 0.00004	- 0.00014
18.75	0.00528	0.00306	- 0.00353	- 0.00723	0.00071	0.00017	- 0.00016	- 0.00073	0.00017	0.00004	- 0.00003	- 0.00014
19.00	0.00535	0.00306	- 0.00349	- 0.00729	0.00075	0.00018	- 0.00016	- 0.00076	0.00016	0.00004	- 0.00003	- 0.00013
19.25	0.00554	0.00306	- 0.00345	- 0.00734	0.00079	0.00019	- 0.00017	- 0.00080	0.00016	0.00004	- 0.00003	- 0.00013
19.50	0.00574	0.00311	- 0.00343	- 0.00738	0.00083	0.00018	- 0.00017	- 0.00083	0.00016	0.00004	- 0.00003	- 0.00013
19.75	0.00595	0.00315	- 0.00349	- 0.00742	0.00087	0.00019	- 0.00017	- 0.00086	0.00015	0.00004	- 0.00003	- 0.00013
20.00	0.00617	0.00309	- 0.00356	- 0.00744	0.00090	0.00019	- 0.00018	- 0.00089	0.00015	0.00003	- 0.00003	- 0.00012
20.25	0.00641	0.00301	- 0.00358	- 0.00746	0.00094	0.00020	- 0.00019	- 0.00092	0.00015	0.00003	- 0.00003	- 0.00012
20.50	0.00665	0.00293	- 0.00358	- 0.00747	0.00098	0.00021	- 0.00020	- 0.00095	0.00015	0.00003	- 0.00003	- 0.00012
20.75	0.00689	0.00286	- 0.00358	- 0.00748	0.00101	0.00021	- 0.00021	- 0.00098	0.00014	0.00003	- 0.00003	- 0.00012
21.00	0.00715	0.00280	- 0.00357	- 0.00747	0.00105	0.00021	- 0.00021	- 0.00101	0.00014	0.00003	- 0.00003	- 0.00012
21.25	0.00742	0.00280	- 0.00358	- 0.00746	0.00108	0.00021	- 0.00022	- 0.00104	0.00014	0.00003	- 0.00003	- 0.00011
21.50	0.00769	0.00275	- 0.00359	- 0.00744	0.00112	0.00022	- 0.00023	- 0.00107	0.00013	0.00003	- 0.00003	- 0.00011
21.75	0.00798	0.00272	- 0.00356	- 0.00741	0.00115	0.00022	- 0.00024	- 0.00110	0.00013	0.00003	- 0.00003	- 0.00011
22.00	0.00827	0.00267	- 0.00355	- 0.00738	0.00118	0.00023	- 0.00024	- 0.00112	0.00013	0.00003	- 0.00003	- 0.00011
22.25	0.00857	0.00266	- 0.00353	- 0.00734	0.00121	0.00024	- 0.00025	- 0.00115	0.00013	0.00003	- 0.00003	- 0.00010
22.50	0.00888	0.00266	- 0.00350	- 0.00739	0.00125	0.00025	- 0.00026	- 0.00117	0.00012	0.00003	- 0.00003	- 0.00010
22.75	0.00919	0.00277	- 0.00351	- 0.00768	0.00128	0.00025	- 0.00027	- 0.00120	0.00012	0.00003	- 0.00003	- 0.00010
23.00	0.00951	0.00275	- 0.00353	- 0.00798	0.00131	0.00026	- 0.00027	- 0.00122	0.00012	0.00003	- 0.00003	- 0.00010
23.25	0.00984	0.00271	- 0.00355	- 0.00829	0.00133	0.00027	- 0.00027	- 0.00125	0.00011	0.00003	- 0.00003	- 0.00009
23.50	0.01018	0.00271	- 0.00358	- 0.00861	0.00136	0.00028	- 0.00027	- 0.00127	0.00011	0.00003	- 0.00003	- 0.00009
23.75	0.01053	0.00273	- 0.00359	- 0.00893	0.00139	0.00029	- 0.00027	- 0.00129	0.00011	0.00003	- 0.00003	- 0.00009
24.00	0.01088	0.00272	- 0.00358	- 0.00926	0.00142	0.00030	- 0.00027	- 0.00132	0.00011	0.00002	- 0.00002	- 0.00009
24.25	0.01123	0.00271	- 0.00351	- 0.00959	0.00144	0.00030	- 0.00028	- 0.00134	0.00011	0.00002	- 0.00002	- 0.00009
24.50	0.01160	0.00272	- 0.00349	- 0.00992	0.00147	0.00031	- 0.00028	- 0.00136	0.00010	0.00002	- 0.00002	- 0.00008
24.75	0.01197	0.00272	- 0.00350	- 0.01027	0.00150	0.00032	- 0.00028	- 0.00138	0.00010	0.00002	- 0.00002	- 0.00008
25.00	0.01235	0.00270	- 0.00354	- 0.01061	0.00152	0.00032	- 0.00029	- 0.00140	0.00010	0.00002	- 0.00002	- 0.00008
25.25	0.00431	0.00236	- 0.00288	- 0.00634	0.00048	0.00015	- 0.00016	- 0.00063	0.00009	0.00001	- 0.00002	- 0.00004
25.50	0.00428	0.00237	- 0.00292	- 0.00621	0.00050	0.00015	- 0.00016	- 0.00064	0.00009	0.00001	- 0.00002	- 0.00004
25.75	0.00430	0.00238	- 0.00297	- 0.00609	0.00052	0.00015	- 0.00016	- 0.00065	0.00009	0.00001	- 0.00002	- 0.00004
26.00	0.00433	0.00239	- 0.00301	- 0.00595	0.00054	0.00016	- 0.00017	- 0.00066	0.00008	0.00001	- 0.00002	- 0.00004
26.25	0.00436	0.00238	- 0.00304	- 0.00582	0.00056	0.00016	- 0.00017	- 0.00067	0.00008	0.00001	- 0.00002	- 0.00003
26.50	0.00438	0.00238	- 0.00303	- 0.00567	0.00058	0.00016	- 0.00017	- 0.00068	0.00008	0.00001	- 0.00001	- 0.00003
26.75	0.00441	0.00237	- 0.00303	- 0.00552	0.00060	0.00016	- 0.00017	- 0.00068	0.00008	0.00001	- 0.00001	- 0.00003
27.00	0.00445	0.00236	- 0.00303	- 0.00544	0.00062	0.00017	- 0.00018	- 0.00069	0.00008	0.00001	- 0.00001	- 0.00003
27.25	0.00448	0.00235	- 0.00303	- 0.00551	0.00064	0.00017	- 0.00018	- 0.00070	0.00008	0.00001	- 0.00001	- 0.00003
27.50	0.00451	0.00235	- 0.00303	- 0.00558	0.00066	0.00017	- 0.00018	- 0.00071	0.00007	0.00001	- 0.00001	- 0.00003
27.75	0.00454	0.00236	- 0.00314	- 0.00565	0.00068	0.00017	- 0.00018	- 0.00072	0.00007	0.00001	- 0.00001	- 0.00003
28.00	0.00458	0.00237	- 0.00329	- 0.00571	0.00070	0.00017	- 0.00019	- 0.00072	0.00007	0.00001	- 0.00001	- 0.00003
28.25	0.00461	0.00238	- 0.00331	- 0.00578	0.00071	0.00017	- 0.00019	- 0.00073	0.00007	0.00001	- 0.00001	- 0.00003
28.50	0.00465	0.00239	- 0.00334	- 0.00585	0.00073	0.00017	- 0.00019	- 0.00074	0.00007	0.00001	- 0.00001	- 0.00003
28.75	0.00469	0.00235	- 0.00333	- 0.00592	0.00075	0.00017	- 0.00020	- 0.00074	0.00007	0.00001	- 0.00001	- 0.00003
29.00	0.00472	0.00231	- 0.00332	- 0.00598	0.00076	0.00018	- 0.00020	- 0.00075	0.00006	0.00001	- 0.00001	- 0.00003
29.25	0.00476	0.00231	- 0.00330	- 0.00605	0.00078	0.00018	- 0.00020	- 0.00076	0.00006	0.00001	- 0.00001	- 0.00003
29.50	0.00480	0.00232	- 0.00329	- 0.00612	0.00080	0.00018	- 0.00020	- 0.00076	0.00006	0.00001	- 0.00001	- 0.00003
29.75	0.00484	0.00234	- 0.00328	- 0.00619	0.00081	0.00018	- 0.00021	- 0.00077	0.00006	0.00001	- 0.00001	- 0.00003
30.00	0.00488	0.00237	- 0.00318	- 0.00625	0.00083	0.00018	- 0.00021	- 0.00078	0.00006	0.00001	- 0.00001	- 0.00002

The values can be applied for stress testing as they were historically the extreme changes happened to the Canadian yeild curves when crisis took place. Applying the extreme shift, twist, and butterfly to the current yield curve, it can be expected how the yield curve will shape in the next period assuming there are big shocks. Same extreme change for each maturity is not suggested, because it would provide a seemingly parallel shift of the yield curve, which neglected the power of this methodology for yield curve shape changes.

7: Further Research

The magnitude of impact on its zero curves caused by same financial crisis or other disasters may vary by each country. However, the methodologies, presented in this paper, to single out the abnormalities will be applicable to each country’s yield curve. Major financial crisis are expected to be reflected in other countries’ yield curve, but with different magnitude. To show this, the same tests could be repeated with zero-coupon yield data from U.S., Europe, Japan, or other countries.

Then, the changes in the level, slope, and curvature factors in one country can be utilized as inputs in the stress testing in another country. The impact on Japanese yield curve by Asian crisis may be larger than that on Canadian yield curve. By applying the factor changes in Japanese yield curve by Asian crisis to Canadian yield curve, one plausible impact based on historical data could be quantified and applied in the stress testing based on the assumption that Canada may face large financial crisis if it is happened in North America.

8: Conclusion

The objective of this paper is to find a simple and easily applicable method to identify historical abnormalities reflected in the Canadian zero-coupon yield curve, which can be further applied for stress testing on other yield curves. To achieve the objective this paper presents the test methodologies as follows:

First, this paper has shown how to select the best fitting yield curves to specific country data- here, Canadian zero-coupon yields data- among various yield curve models developed historically. Second, the best model selected is used for calculating the level and shape factors of the specific yield curves. Then, the level and shape factors play the major roles to identify the abnormalities happened during the selected periods. The matching process of the abnormalities identified with the actual crisis happened acts to determine how each test accurately identify the months with “shocks” in the yield curves. Finally, the shift, twist, and butterfly calculated from the “shocked” months are provided for the further stress testing.

The significance of stress testing has been brought more attention after the recent financial crisis. Banks, insurance companies, and other financial institutions are building up their own stress testing models. The changes in interest rates are the most commonly used for stress testing. We aim that any institutions or people interested in zero curves movements can utilize this procedure to apply for their stress testing. To show this methodology is world-widely acceptable, the same tests could be repeated for different counties, but it is left as a further research. Additionally, the various methods to apply the suggested historical shocks for stress testing could be more developed, which is left to people who use this data for their own stress testing.

Regardless of these broad ranges of possible development in this paper, it has been clearly stated that the Canadian zero- coupon yield curves have reflected most major crisis happened in the last 25 years – from 1986 to 2010; thus, the methodologies presented in this paper could be further used for any identification of yield curve shocks and stress testing.

Appendix A. Yield Curve Model- Nelson and Siegel Model

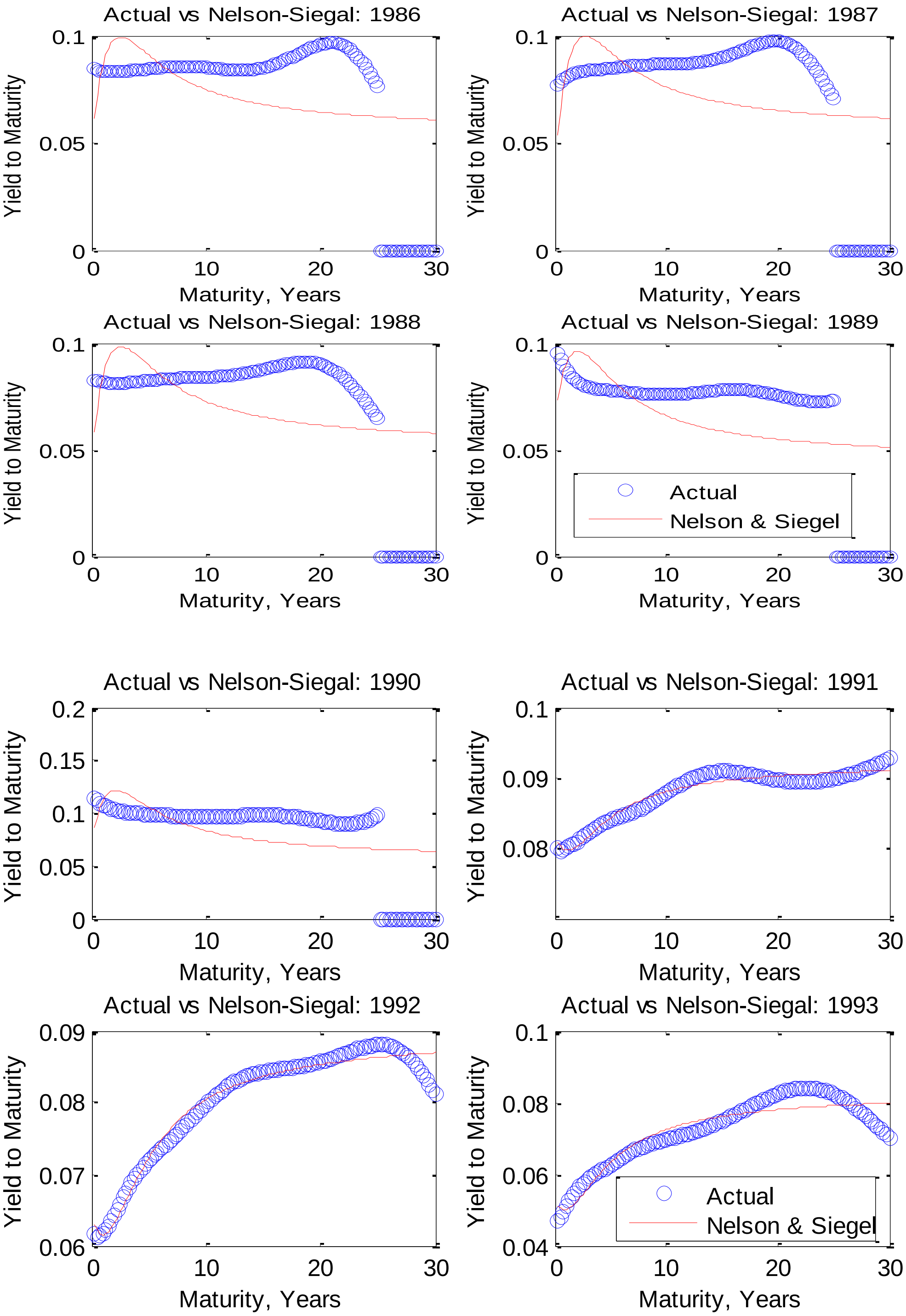
Table A-1 Factors from Nelson-Siegel Model

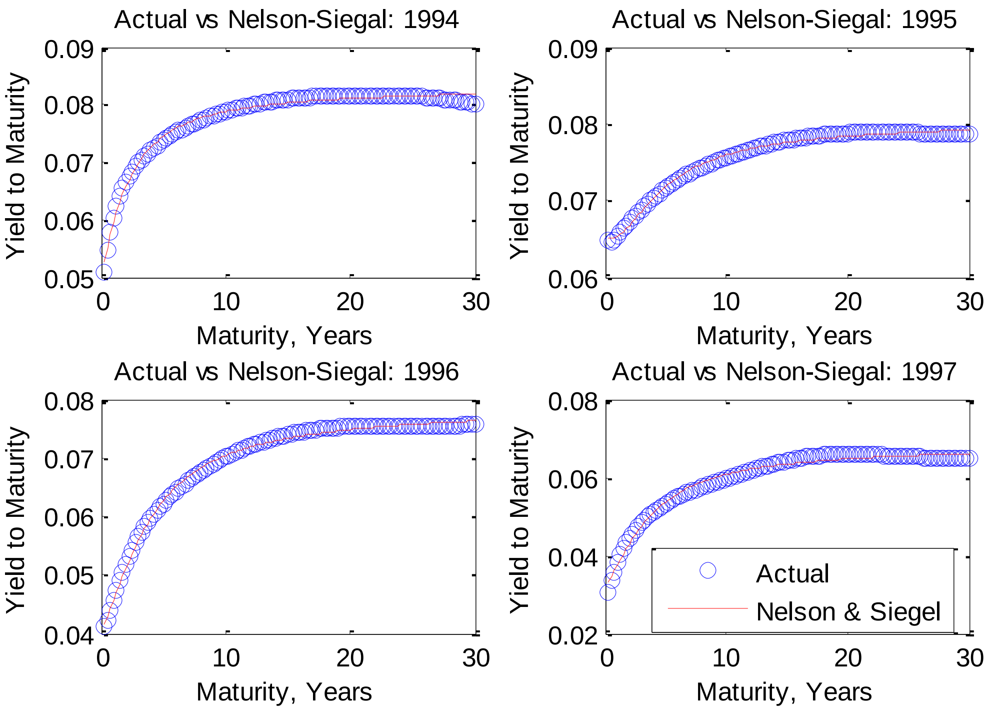
Factor Year	Level Beta1	Slope Beta2	Curvature Beta3
1986	0.0538	-0.0057	0.1609
1987	0.0541	-0.0162	0.1773
1988	0.0503	-0.0068	0.1714
1989	0.0438	0.0199	0.1426
1990	0.0542	0.0181	0.1945
1991	0.0925	-0.0109	-0.022
1992	0.0902	-0.0256	-0.0463
1993	0.0835	-0.0308	-0.0503
1994	0.0833	-0.0337	0.0006
1995	0.0808	-0.0153	-0.0207
1996	0.0793	-0.039	-0.0259
1997	0.0687	-0.0374	-0.0209
1998	0.0533	-0.0061	-0.0154
1999	0.0537	-0.0095	-0.0003
2000	0.053	-0.0049	0.0171
2001	0.058	-0.0233	-0.0157
2002	0.059	-0.0376	-0.0147
2003	0.0546	-0.0266	-0.036
2004	0.0532	-0.0316	-0.032
2005	0.0458	-0.0177	-0.0231
2006	0.0415	-0.0021	-0.0069
2007	0.0416	-0.0004	-0.0051
2008	0.0432	-0.0166	-0.0326
2009	0.0442	-0.0413	-0.0385
2010	0.0452	-0.0457	-0.0291

Table A-2. R squares by Year

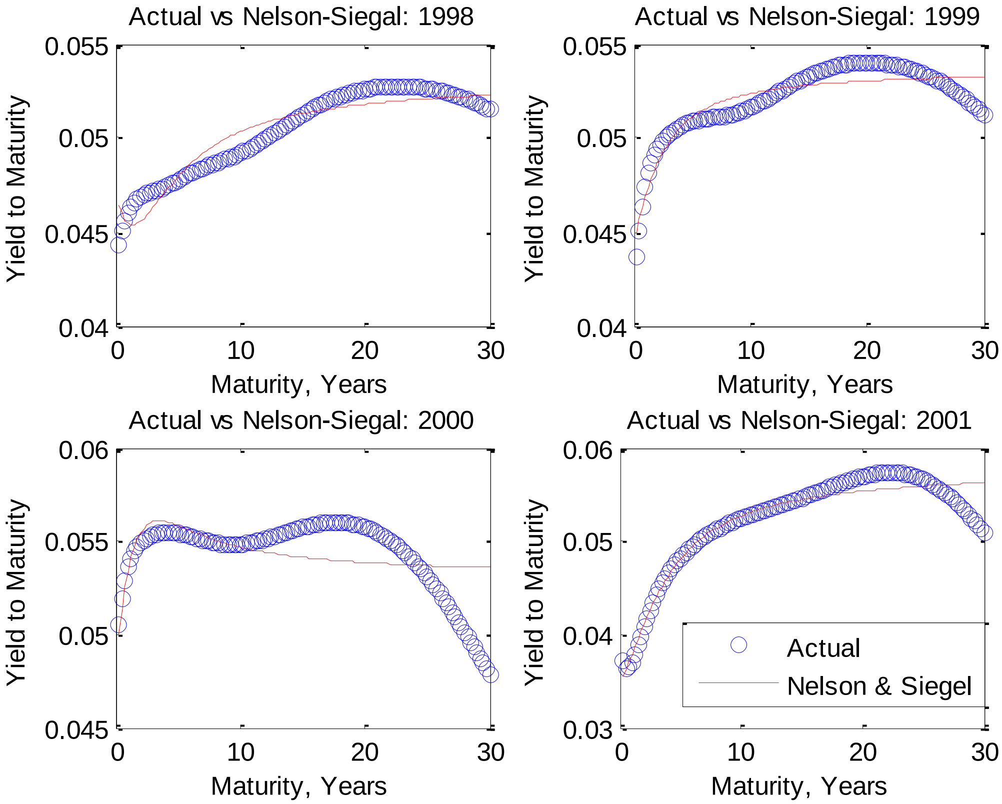
Year	R Squares
1986	0.128565
1987	0.127098
1988	0.15281
1989	0.222635
1990	0.228521
1991	0.936611
1992	0.967675
1993	0.866248
1994	0.988455
1995	0.993057
1996	0.996265
1997	0.984589
1998	0.888766
1999	0.832145
2000	0.217252
2001	0.932493
2002	0.979538
2003	0.982484
2004	0.983733
2005	0.975249
2006	0.751904
2007	0.561298
2008	0.946577
2009	0.956433
2010	0.971967

Graph A-1. Canadian Zero-Coupon Treasury Yields Compared with the Fitted Yield Curve by Nelson-Siegel Model

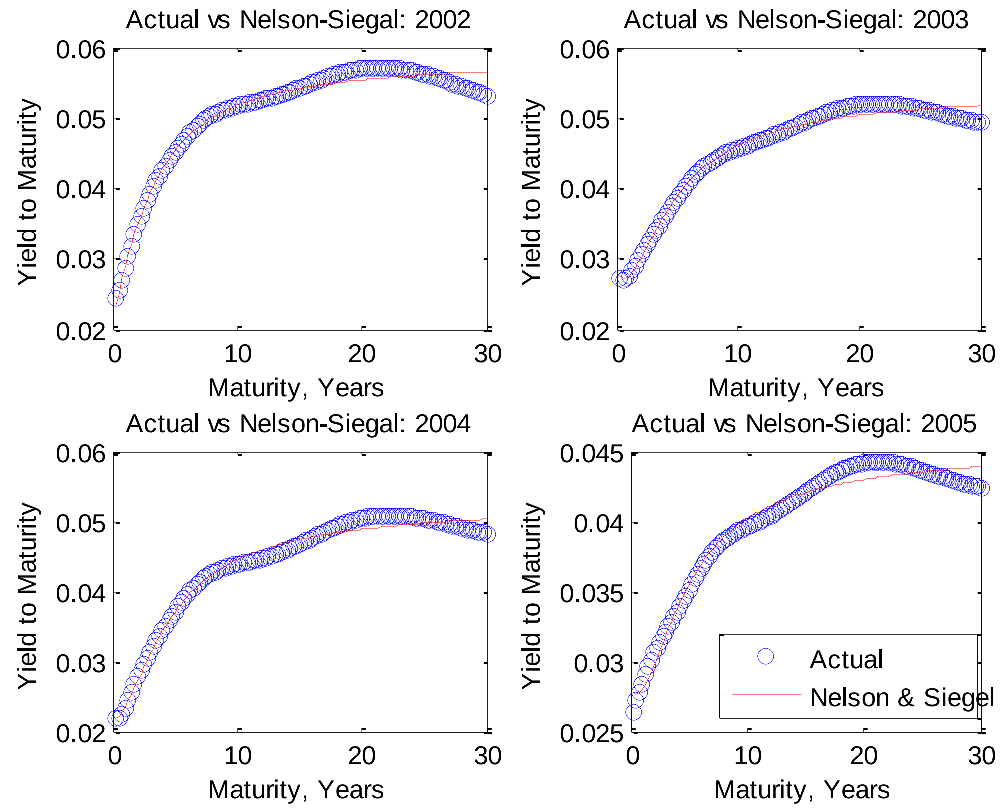


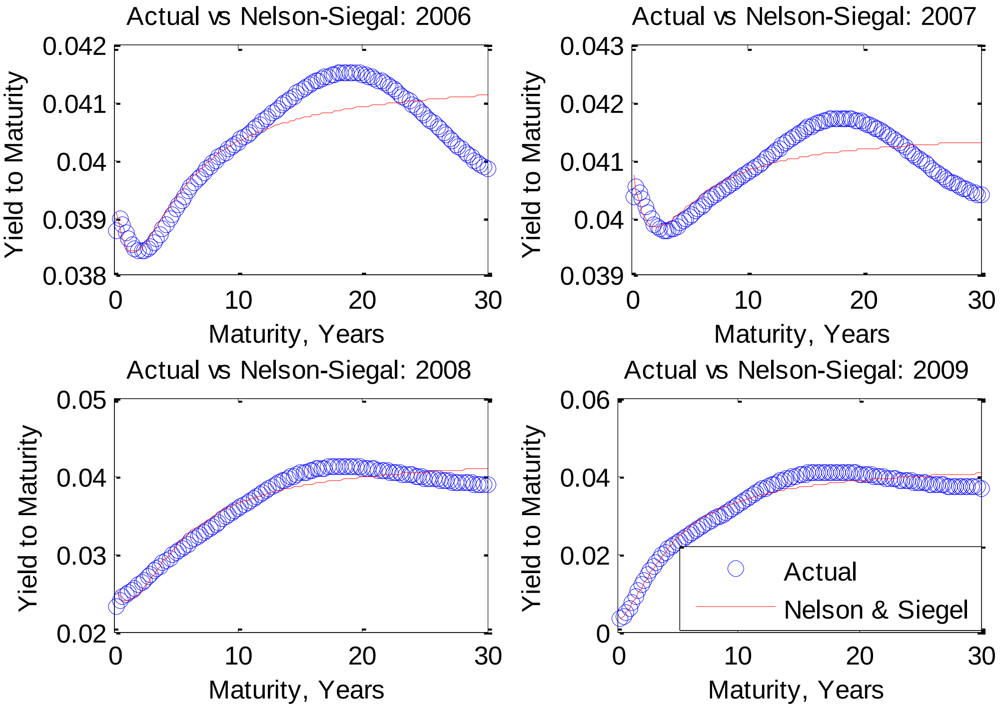


Actual vs Nelson-Siegal: 1994
Actual vs Nelson-Siegal: 1995
Actual vs Nelson-Siegal: 1996
Actual vs Nelson-Siegal: 1997
Yield to Maturity
Maturity, Years
0.09
0.08
0.07
0.06
0.05
0.04
0.02
0
10
20
30
Actual
Nelson & Siegel



Actual vs Nelson-Siegal: 1998
Actual vs Nelson-Siegal: 1999
Actual vs Nelson-Siegal: 2000
Actual vs Nelson-Siegal: 2001
Yield to Maturity
Maturity, Years
0.055
0.05
0.045
0.04
0.06
0.03
0
10
20
30
Actual
Nelson & Siegel


Actual vs Nelson-Siegal: 2002
Actual vs Nelson-Siegal: 2003
Actual vs Nelson-Siegal: 2004
Actual vs Nelson-Siegal: 2005
Yield to Maturity
Maturity, Years
0.06
0.05
0.04
0.03
0.02
0.045
0.04
0.035
0.03
0.025
0
10
20
30
Actual
Nelson & Siegel


Actual vs Nelson-Siegal: 2006
Actual vs Nelson-Siegal: 2007
Actual vs Nelson-Siegal: 2008
Actual vs Nelson-Siegal: 2009
Yield to Maturity
Maturity, Years
0.042
0.041
0.04
0.039
0.038
0.043
0.042
0.041
0.04
0.039
0.05
0.04
0.03
0.02
0.06
0.04
0.02
0
0
10
20
30
Actual
Nelson & Siegel

Appendix B. Yield Curve Model-Svensson Model

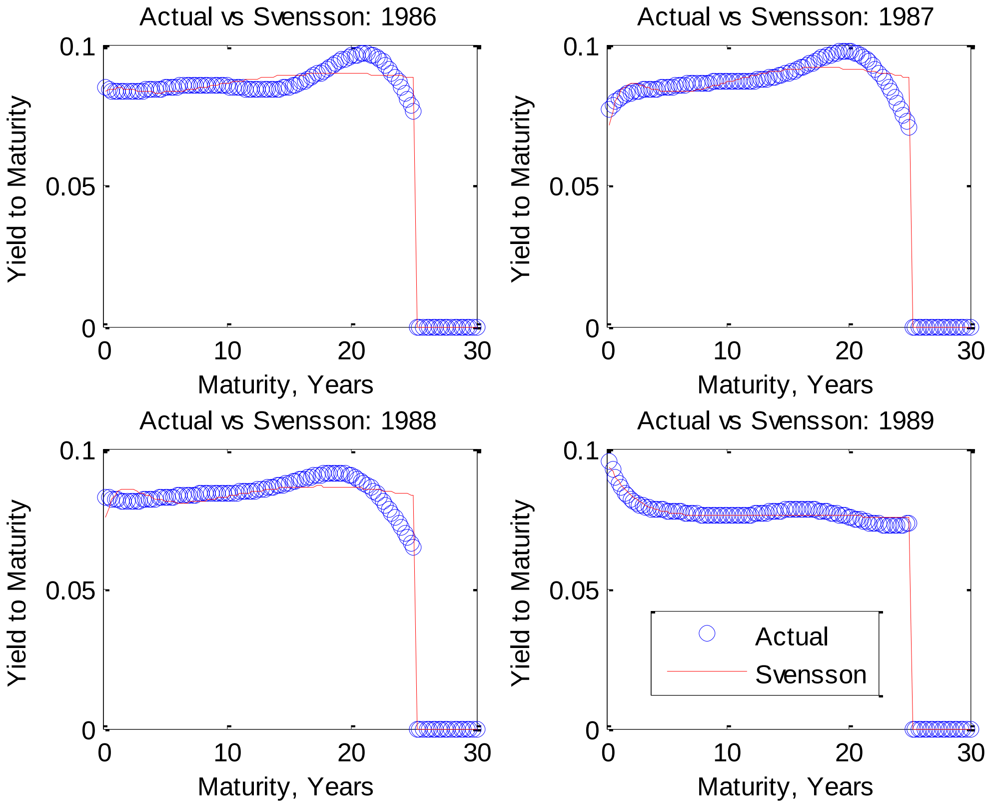
Table B-1. Factors from Svensson Model

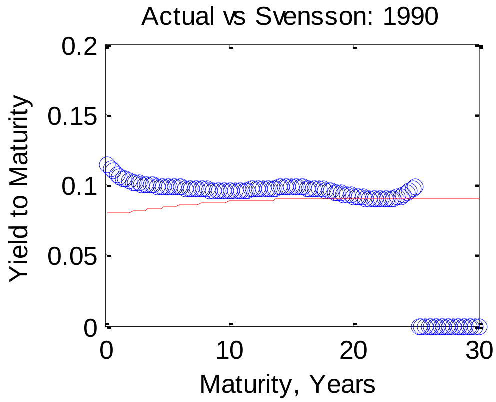
Factor	Level	Slope	Curvature	Curvature
Year	Beta1	Beta2	Beta3	Beta4
1986	0.034672	0.047294	0.043666	0.163836
1987	-0.03048	0.095166	0.13908	0.356207
1988	-0.01691	0.086992	0.120241	0.298872
1989	0.055706	0.039785	0.009295	0.05649
1990	0.096728	0.018414	-0.00598	-0.0103
1991	0.083474	-0.00275	-0.01147	0.02686
1992	0.048647	0.01195	0.002059	0.123411
1993	-0.01205	0.055471	0.060946	0.283709
1994	0.055867	-0.00893	0.032515	0.081509
1995	0.066695	-0.00254	-0.00425	0.041959
1996	0.057945	-0.01972	-0.00107	0.063263
1997	0.029372	-0.00181	0.02486	0.11684
1998	0.033426	0.011849	0.007664	0.058929
1999	0.023166	0.018074	0.035282	0.090686
2000	-0.00837	0.050553	0.088583	0.18226
2001	0.021061	0.010092	0.027333	0.109769
2002	0.028218	-0.00984	0.021075	0.091323
2003	0.024001	0.001115	-0.00032	0.09096
2004	0.03473	-0.01484	-0.01046	0.054932
2005	0.020311	0.005306	0.006527	0.075612
2006	0.023496	0.014238	0.014163	0.053597
2007	0.025754	0.013853	0.013301	0.046999
2008	-0.00839	0.02992	0.027421	0.153037
2009	-0.03658	0.031677	0.055552	0.239879
2010	-0.01547	0.009054	0.041517	0.180081

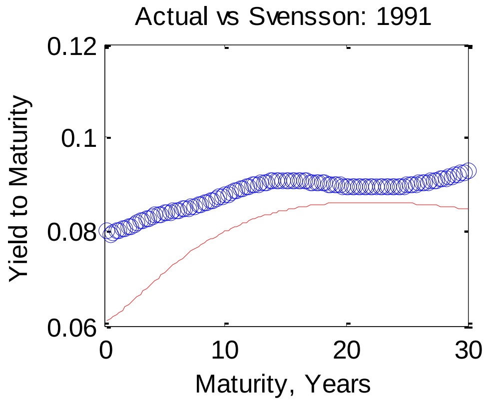
Table B-2. R squares by Year

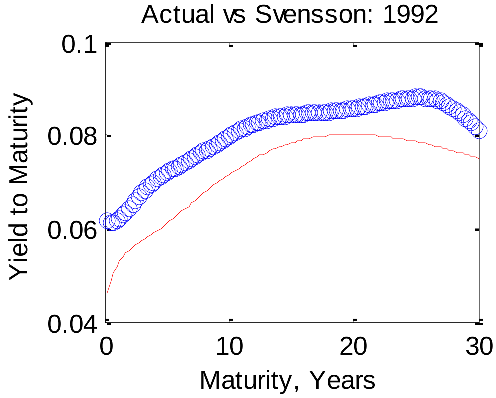
Year	R Squares
1986	0.3161
1987	0.4335
1988	0.1681
1989	0.8569
1990	0.7708
1991	0.9401
1992	0.9828
1993	0.92
1994	0.9993
1995	0.9994
1996	0.9995
1997	0.9973
1998	0.9292
1999	0.9708
2000	0.6966
2001	0.958
2002	0.988
2003	0.9922
2004	0.9868
2005	0.9907
2006	0.9324
2007	0.9045
2008	0.9967
2009	0.9907
2010	0.991

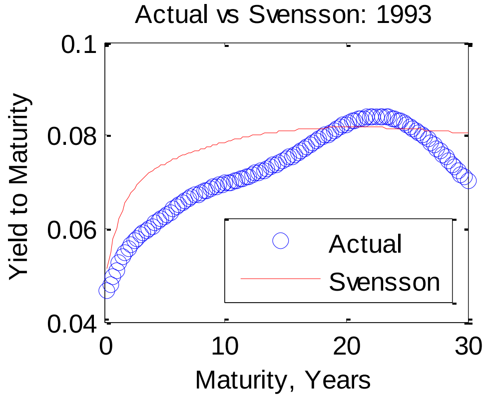
Graph B-1. Actual vs. Fitted Yield Curve by Svensson Model

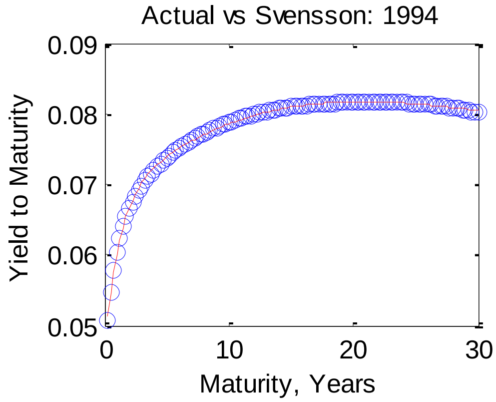


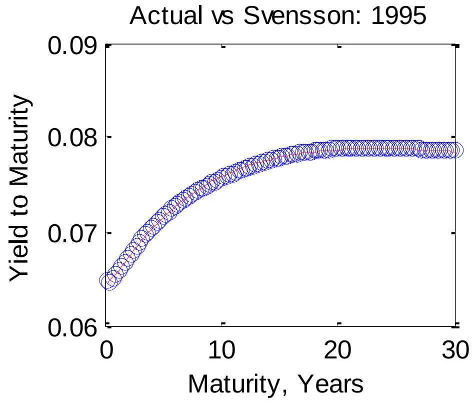

Actual vs Svensson: 1990

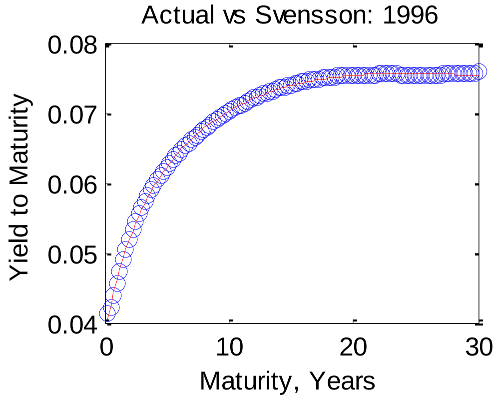

Actual vs Svensson: 1991

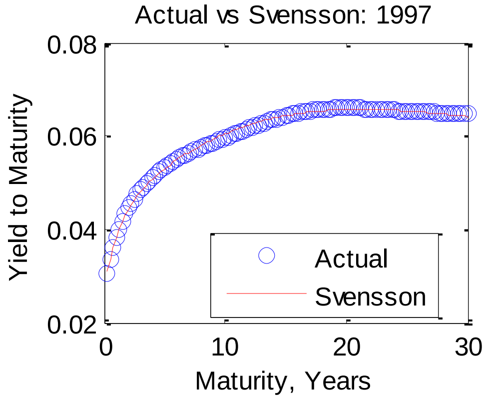

Actual vs Svensson: 1992

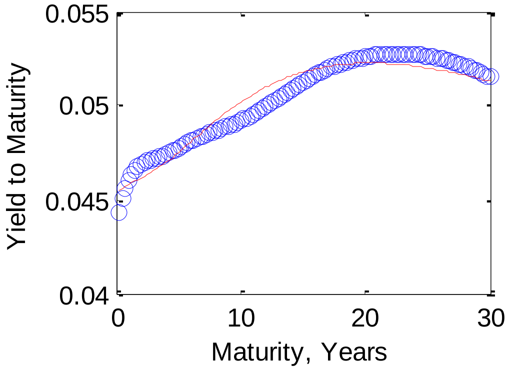

Actual vs Svensson: 1993


Actual vs Svensson: 1994

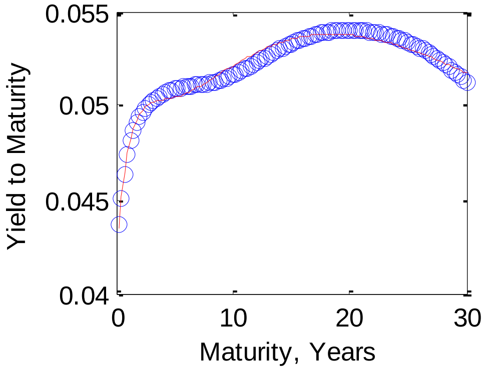

Actual vs Svensson: 1995


Actual vs Svensson: 1996

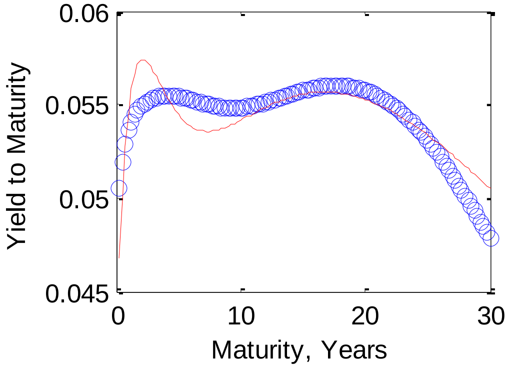

Actual vs Svensson: 1997



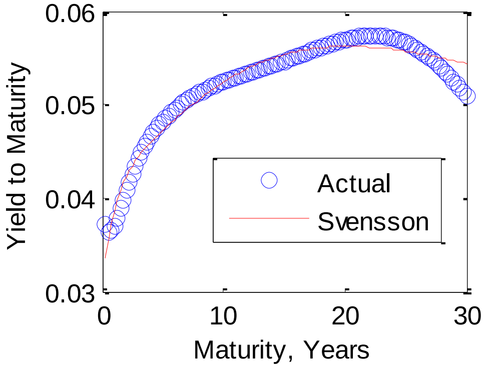
Actual vs Svensson: 1998



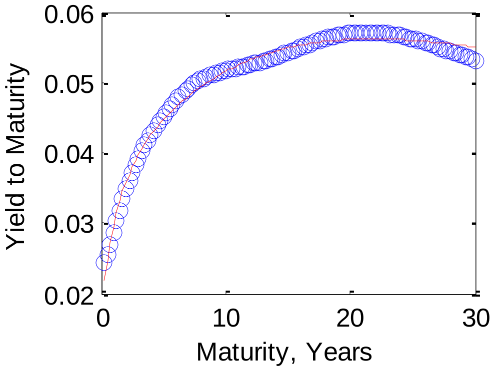
Actual vs Svensson: 1999



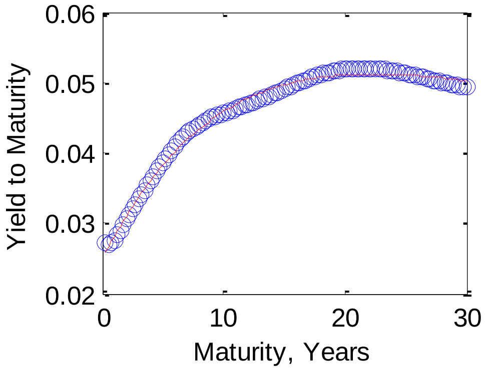
Actual vs Svensson: 2000



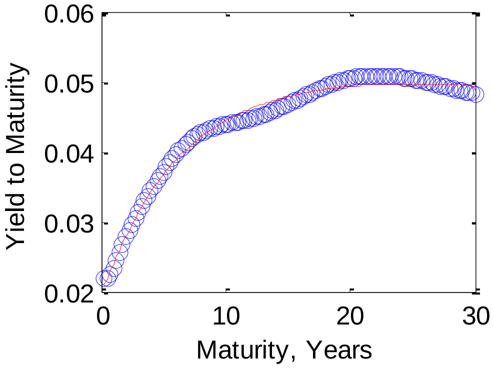
Actual vs Svensson: 2001



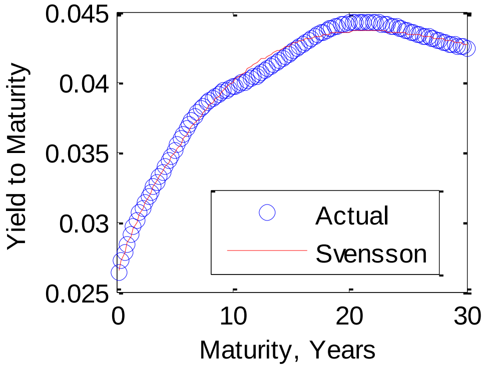
Actual vs Svensson: 2002



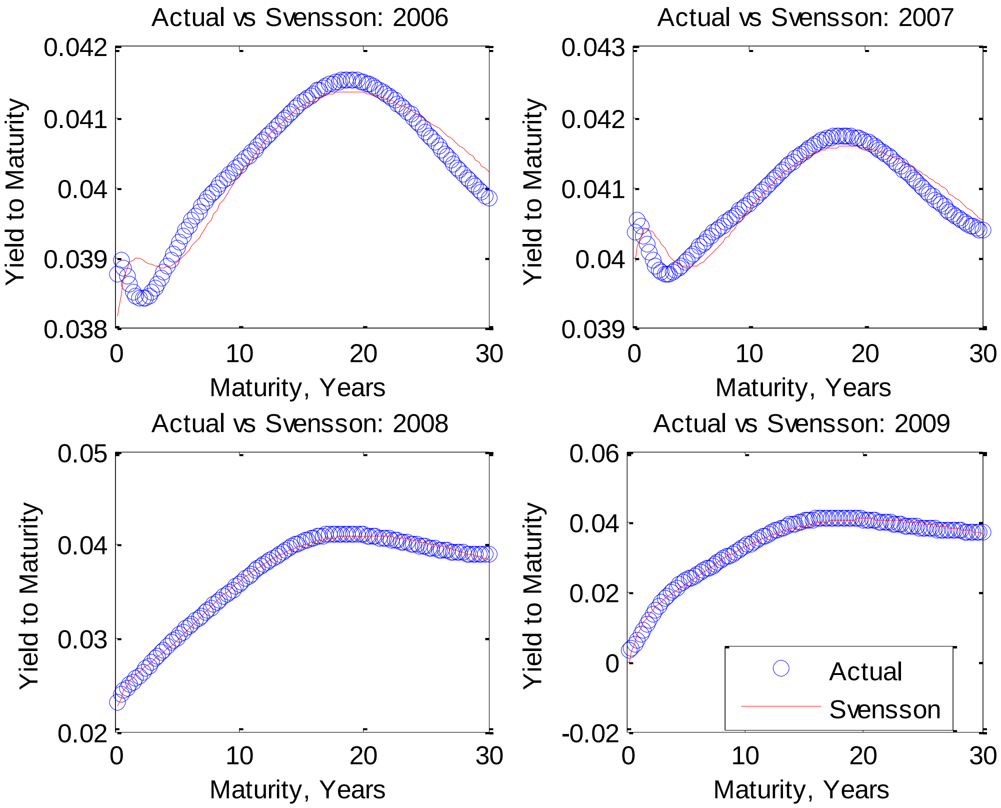
Actual vs Svensson: 2003



Actual vs Svensson: 2004



Actual vs Svensson: 2005


Actual vs Svensson: 2006
Actual vs Svensson: 2007
Actual vs Svensson: 2008
Actual vs Svensson: 2009
Yield to Maturity
Maturity, Years
0.042
0.041
0.04
0.039
0.038
0.043
0.042
0.041
0.04
0.039
0.05
0.04
0.03
0.02
0.06
0.04
0.02
0
-0.02
0
10
20
30
Actual
Svensson

Appendix C. Yield Curve Model- Bjork and Christensen model

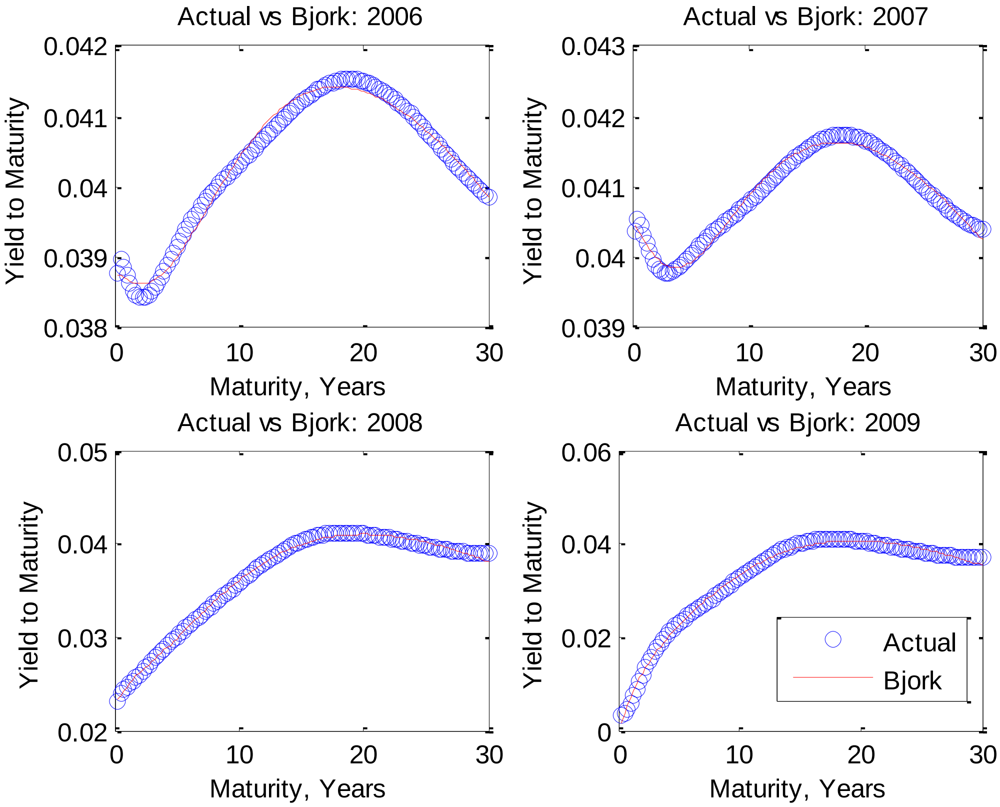
Table C-1. Factors

Factor Year	Level Beta1	Slope Beta2	Curvature Beta3	Curvature Beta4	Curvature Beta5
1986	0.1448	-0.0025	0.142	-0.0619	-0.2069
1987	0.29	-0.0106	0.3579	-0.1781	-0.5802
1988	0.3077	-0.0125	0.276	-0.1741	-0.5091
1989	0.1253	-0.003	-0.0228	-0.0267	-0.0063
1990	0.1189	-0.0017	-0.0601	-0.0035	0.0575
1991	0.0919	0	-0.0625	0.0083	0.0514
1992	0.123	-0.0016	-0.0131	-0.0239	-0.0508
1993	0.1843	-0.0047	0.2364	-0.1101	-0.3819
1994	0.1083	-0.0012	0.0777	-0.0271	-0.1373
1995	0.0903	-0.0005	-0.0209	-0.0047	-0.0057
1996	0.0942	-0.0008	-0.0107	-0.0096	-0.0454
1997	0.1048	-0.0017	0.0782	-0.0376	-0.1558
1998	0.0744	-0.0009	0.0573	-0.0256	-0.0884
1999	0.0852	-0.0015	0.0971	-0.0358	-0.1406
2000	0.1098	-0.0029	0.1281	-0.0526	-0.1899
2001	0.0933	-0.0018	0.0426	-0.0297	-0.1038
2002	0.0872	-0.0014	0.0342	-0.0233	-0.1019
2003	0.0779	-0.0012	-0.0299	-0.0132	-0.0235
2004	0.0708	-0.0008	-0.0043	-0.0137	-0.0486
2005	0.0675	-0.0011	0.0128	-0.0183	-0.0554
2006	0.0553	-0.0007	0	-0.009	-0.0165
2007	0.0537	-0.0006	0.003	-0.0084	-0.0162
2008	0.0819	-0.002	0.0041	-0.0283	-0.0634
2009	0.1054	-0.0032	0.0168	-0.0433	-0.1232
2010	0.0879	-0.0023	-0.0145	-0.0248	-0.0755

Table C-2. R Squares by Year

Year	R Squares
1986	0.3673
1987	0.5965
1988	0.6822
1989	0.9544
1990	0.8499
1991	0.9447
1992	0.9841
1993	0.9474
1994	0.9984
1995	0.9995
1996	0.9995
1997	0.9991
1998	0.9934
1999	0.9925
2000	0.9567
2001	0.9672
2002	0.9911
2003	0.9921
2004	0.9873
2005	0.9924
2006	0.9842
2007	0.9806
2008	0.9972
2009	0.9953
2010	0.9978

Graph C-1. Actual vs. Fitted Yield Curve by Bjork and Christensen Model



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