UNPACKING STUDENT DISCOURSE IN MIDDLE SCHOOL MATHEMATICS

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ABSTRACT

This research accepts the communicational framework that relates thinking as a special case of the activity of communicating. My study observes the communication of 24 students working in pairs, involved in solving various mathematical problems. It stays tuned to factors that have been identified to be necessary for learning to occur in group situations. Based on these recordings, and the communicational framework for learning, the interactions were then rigidly analyzed to determine what parts of the learning were successful, and what parts were unsuccessful. The results indicate whether the factors accepted by several researchers to be necessary for learning to occur happen naturally when students work together. The analysis looks at each encounter to see if these factors affected the ultimate success of learning.

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FOREWORD

Do the factors that affect successful learning occur happen naturally when students work together? How do these factors affect the ultimate success of learning? After realizing that many students do not naturally exhibit all the criteria, it becomes evident that more instructional time should be spent with students and parents in order to reinforce factors that enhance mathematical learning regardless of a student's natural ability in the subject.

One place to start would be to spend time demonstrating to students and parents factors that affect successful learning in mathematics. But from an educator's point of view the obstacle is how should we run a classroom to enhance the ability of students to internalize characteristics of successful students as defined by Anna Sfard and Noreen Webb. This is not a question that I try to answer in this thesis, yet as a teacher, the purpose for conducting this research never loses sight of the need for research to improve the ability of all student to learn.

CHAPTER 1: INTRODUCTION

Researchers have long been trying to solve the problem of what prevents mathematical understanding from being accessible to all students. Should we be focusing our efforts on the best ways to explain mathematical concepts? Should we focus on the ideal set-up of the classroom and the environment in which students learn? Should we focus on creating the best resources from which students and teachers work? Or should we focus on the best ways to motivate students to work harder? Answers to all of these questions will need to be found in order to satisfy parents and students with the math programs at their schools.

At the root of all of these questions is human thought. Teaching students how to think must come after we understand human thought: What is thinking? How do we think? My interest in making mathematics more accessible to all students starts with creating a model that describes human thought. Once we have a model that describes how we think, it might be easier to create a model for how we can influence the way students think.

This research accepts the communicational framework that relates thinking as a special case of the activity of communicating (Sfard, 2001). My study observes the communication of 24 students working in pairs, involved in solving various mathematical problems. It stays tuned to factors that have been identified to affect learning in group situations (Webb, 2001). Some learning situations involve help from the teacher, but most are students working together to solve a problem. The encounters were set up with the goal of having students interact in their natural way as much as possible. The students knew that a camera was recording them, and that it would be watched by the teacher, but it was hoped that this would not affect the students' interaction enough to discredit the results as being unnatural.

Based on these recordings, and the communicational framework for learning, the interactions were then rigidly analyzed to determine what characteristics of students made learning successful and what characteristics limited the learning that occured. Successful completion of a task does not necessarily translate into a successful encounter. I want to look beyond the ultimate success or failure on each task to unpack what are the characteristics of a good learning encounter. The analysis attempts to unravel the complexities of the intangible factors that control the success of each encounter. Three instruments will be used both independently and together to unpack the tacit elements of effective group work. The video recordings were first transcribed to document both what the students were saying and what they were doing during the encounter. The transcriptions were then mapped to an interactivity flow chart similar to those used by Sfard. (Sfard, 2001b) The flowchart converts each utterance into an arrow that reflects whether or not the student was asking a question or answering a question of their partner. The flowcharts are helpful in determining the degree to which students worked together and listened to one another or whether they were more interested in a conversation with themselves.

Group interactions of students solving math problems are very complex to analyze. I am going to relate two distinct theories proposed by Anna Sfard and Noreen Webb that both relate to the challenges of making group work successful. (Sfard 2001a, Sfard 2001b, Webb 2001) I have combined these two frameworks into one tool for analysis called the *group work analysis checklist*, which covers all the factors that work together to create successful group work. One by one, each factor is analyzed to determine whether or not it contributed to a successful encounter or whether its absence inhibited success.

For each encounter I used the transcriptions, flowchart and checklist to support Webb and Sfard's ideas on what makes the interaction successful or not, but often one tool became more illuminating than another did when analyzing the results. As such, I was able to look at each encounter from a different angle, and then chose the tool that provided the most insight on the important factors of that particular encounter.

The thesis is broken up into six chapters. Chapter 2 looks at past research in the area of mathematical understanding and group work. It looks at the theories of Anna Sfard and Noreen Webb and how each recognizes that simply having students work in pairs does not guarantee successful learning. (Sfard 2001a, Sfard 2001b, Webb 2001) Only when numerous conditions are met does mathematical communication actually lead to successful learning. Each researcher outlines exactly what these conditions are. Noreen Webb's conditions are on a very practical level and are usually more easily observable than the conditions outlined by Anna Sfard. Anna Sfard articulates the underpinnings of what Webb says on a more theoretical level. Sfard also provides practical conditions necessary for successful group work and provides a framework for measuring those often-tacit conditions, which make or break any given interaction.

Chapter 3 is the methodology, which specifically describes how the data was collected, encoded and analyzed. In Chapter 4, the results of the encounter between Alexis and Siobhan are shown in full with all three tools used to analyze what lead to the successful learning encounter between the girls. Chapter 5 presents partial results from 11 encounters between students that highlight the most prominent and relevant themes emerging from the data. Chapter 6 looks more closely at the results in Chapter 5 in general, and at the themes that emerge from these results more particularly. Chapter 7 is the conclusion, which ends with implications for practice and questions that remain unanswered.

CHAPTER 2: LITERATURE REVIEW

In this section I look at the development of mathematical understanding and how it has led to the way we teach mathematics. I look at the models that have been created to help describe how we build mathematical understanding, and how our brain stores information that it has learned. I then look at two contrasting views on how we learn mathematics. The first is the acquisitionist metaphor that stems from theories that conceptualize learning as storing information in the form of mental representations. The second is called the communicational approach to cognition which views learning as becoming a participant in certain distinct activities.

The learning of mathematics in school classrooms has seen radical shifts over the past 100 years. Curricula have constantly changed in an effort to adopt the current educational philosophies of each period. The NCTM published its Principles and Standards document that outlines the current reform movement. The standards comment on the need for a well-connected curriculum where ideas build on and connect with other ideas. It also comments on the need for a balance between factual knowledge, procedural proficiency, and conceptual understanding. The standards state that a students understanding will be deeper and last longer if it is well connected.

> "Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and interconnect their knowledge" (NCTM 1993)

2.1 Hiebert and Carpenter

These statements are based on models of how we build understanding, and what it means to understand something. Hiebert and Carpenter state that a mathematical idea or procedure or fact is understood if it is part of an internal network.

> More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of the connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections. (Hiebert and Carpenter 1993, p 67)

> These networks of knowledge are built gradually as new information is connected to existing networks, or as new relationships are constructed between previously disconnected information. (Hiebert and Carpenter 1993, p 69)

Changes in these networks may be thought of as reorganizations where representations are rearranged making new connections, and old connections may be modified or deleted. A student's understanding is increased as they reorganize their knowledge to yield richly connected networks. (Hiebert and Carpenter, 1993)

It is also widely accepted that students construct their own mathematical knowledge rather than receiving it in finished form from the teacher or textbook.

This idea contends that students create their own internal representations of their interactions with the world and build their own networks of representations. Piaget and others (Piaget, 1973; Resnick, 1980; Wittrock, 1974) looked at the inventiveness of children and how this can be used to build understanding. In this philosophy, mental representations are enriched if they are connected within a network. Students' new inventions are then stimulated, and monitored by related knowledge in the network. If the inventions are not connected with related knowledge, then the inventions are more likely to contain errors and to be unproductive. It is believed that inventions can push students' current understanding to generate new understanding in a snowball effect. (Hiebert and Capenter, 1993)

Hiebert and Carpenter believe that communication and reflection work together to produce new relationships and connections. Students who reflect on what they do and communicate their thinking with others are in the best position to build useful connections in mathematics. If it is true that reflection and communication promote the development of connections, then classrooms that focus on building understanding will emphasize reflection, and mathematical communication. Hiebert and Carpenter highlight several ideas on how classrooms can best facilitate understanding. (Hiebert and Carpenter, 1993)

The Principles and Standards also comment on the importance of communication in developing students' understanding. Student's understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills. Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge (Principles and Standards, 1994)

2.2 Anna Sfard

Anna Sfard calls the theories proposed by Hiebert and Carpenter learning acquisitionist theories, where learning is thought of as a personal acquisition. The word personal emphases that it is an individual behaviour that takes place through passive reception, or by active construction, and that it results in a personalized account of the concepts and procedures. This reflects the classic Piagetian idea of concept schemas – organized mental structures everyone supposedly constructs for oneself from elementary building blocks called conceptions. Initially mental representations of mathematical concepts may not accurately reflect the official "correct" version of the concept. Cognitive psychology relates learning as the process where new knowledge is related to knowledge already possessed. The mind is like a toolbox where once tools are placed in the toolbox they can be carried to different situations and taken out to be used whenever necessary.

These theories search for cognitive invariants and consider thinking to be a personal endeavour. Although such theories offer excellent ideas, experience working with young children suggests that there is more to learning. Sfard steps away from the cognitivist approach to thinking but admits that her theory is meant to complement the traditional ideas.

Sfards calls her theory "learning by participation," where one is a participant in a mathematical discourse. It grows from the socio-cultural tradition of education that is sceptical about various cognitive invariants, which are expected to remain unchanged from person to person. She views learning as, "becoming a participant in certain distinct activities rather than as becoming a possessor of generalized, context-independent conceptual schemas." (Sfard, 2001b, p7) Sfard emphasizes, "That learning is first and foremost about the development of ways in which an individual participates in well-established communal activities." (Sfard, 2001b, p7)

Communication becomes paramount in the study of students learning according to Sfard's ideas and much of her research involves studying the discursive ways of children.

Sfard defines communication as, "a person's attempt to make an interlocutor act, think, or feel according to her intensions" (Sfard, 2001b, p9). When looking at human cognition as a communicational activity we look at how students regulate the communication between them. When one considers thinking as a form of communication it becomes clear that the social norms of the participants will have a large impact on learning. Sfard believes that this is true whether an individual is communicating with others or privately with themselves. (Sfard, 2001b)

It follows then that we must understand the social forces that guide human actions if we are to better understand how humans think. Followers of the traditional cognitivist approach would argue that communication is a result of thinking, and studying communication could act only as a window to the mind. In the participationist framework, "thinking is subordinate to, and informed by, the demands of making communication effective." (Sfard, 2001b, p10) Although Sfard believes that there is still room to talk about thought and speech as two different things, she understands them to be inseparable aspects of one and the same phenomenon, with neither being prior to the other.

Researchers using the participationist framework need to combine traditional evidence of learning with an awareness of the ongoing interactions that guide communication. They must be more focused on the growth of mutual understanding and coordination between the learner and the rest of the community than with unobservables such as mental schemas.

Research must look for all the elements that make mathematical communication ineffective. These elements are no longer in the individual alone but take into account that the learner is a part of a larger whole.

> Describing all that happens between the interlocutors exclusively in terms of stand alone cognition, that is of the actor's abilities and the contents of their minds, means overlooking a great many aspects and factors of change. (Sfard, 2001b, p8)

When we observe students engaged in a mathematical discourse we must appreciate how anything that goes into communication and influences its effectiveness – body movements, situational clues, interlocutor's histories, etc, must be included in our analysis.

Sfard's beliefs have a tremendous practical application to the classroom setting. She admits that individuals who can construct their own understandings through abstract thought do have a theoretical advantage, but in reality, it would prove less effective than apprentice–like participation. The behaviourist model of learning sees learning as "legitimate peripheral participation."(Sfard, 2001b, p8) When analyzing students' participation in a mathematical discourse we must have a high sensitivity of our ways of acting to social, cultural, historical, and situational contexts.

Sfard has taken the communicational approach that is rooted in Vygotskian ideas and turned it into a research framework. The basic foundation of this framework is "that thinking may be conceptualized as a case of communication, that is, as one's communication with oneself." (Sfard , 2001b, p9) Even when we are thinking to ourselves, we are still communicating. While we inform ourselves, we argue, we ask questions, and wait for our own response. Furthermore, the basic mechanisms of thinking are likely to be similar whether the communication is with oneself or with others.

Learning mathematics is defined as, "an initiation, that is initiation to a special form of communication known as mathematical." (Sfard, 2001b, p10) A person who learns about mathematics alters or extends their discursive skills as to become able to communicate with expert interlocutors. The new discourse makes it possible to solve problems that could not be solved previously.

Mathematical communication is distinct because it does not describe physical objects that can be seen. One of the reasons that math is so difficult to learn is because it uses virtual objects. Students must discuss things that are usually not physical objects but virtual creations. The vocabulary that describes these virtual objects is unique to mathematics and it is one of the first obstacles for many students trying to enter into a mathematical discourse. Students must learn terms they have never used before and that are unique to the virtual objects in mathematics. Often mathematical words have different uses outside the domain of mathematics and students must learn to use them in different ways.

Sfard highlights three other dimensions that distinguish mathematical discourse from other types of discourse: Mediating tools, their distinct routines, and their endorsed narratives. Mediating tools are visual devices that people use to help themselves while communicating. Unlike everyday discourses that can be supported by images of material objects that exist independent of the discourse itself, mathematical communication is mediated by symbolic artifacts specifically designed for the sake of communication. Mathematic symbols are not used:

> As a mere auxiliary means of "conveying" or "giving expression" to pre-existing thought. Rather, one thinks about them as part and

> parcel of the act of communicating, and thus the cognitive processes themselves. (Sfard, 2001a, p10)

Sfard believes that introduction of new names and symbols should be the starting point when introducing students into a new discourse. Sfard admits that this is contrary to the prevailing view that students must have some prior idea of a mathematical object before naming it. Sfard uses the example of teaching negative numbers for the first time. She believes that if we want to initiate students into a discourse on new mathematical objects, then we must start by using the new discourse. The objects of this new discourse must be identified with words or symbols by the teacher. Sfard believes that it is unlikely that students will appreciate the value of a discourse before they get experience applying the new discourse with an experienced interlocutor. Her research looks at students learning the rules for operations with negative numbers and highlights the impossible hope that students can somehow construct the rules themselves.

Sfard uses the term routine "to refer to patterns that can be spotted in discursive activities." (Sfard, 2001a, p10) It is easy for a teacher to think of a repetitive pattern in the discourse of mathematics. The same routine is crucial for the interlocutor's ability to apply mathematical discourse whenever appropriate.

> Human tendency to reiterate previously learned discursive behaviour is spontaneous and interlocutors are mostly unaware of the fact that their actions disclose structural regularities. (Sfard 2001a, p10)

It is up to the observer to detect the discursive patterns that govern the discourse. These observed rules are not anything the speaker would follow in a conscious way, but they do regulate the discursive flow. Sfard uses the term meta discursive rules, they can be wide ranging but can be used to describe the when and how actions happen. Sfard investigates the routines that children use to decide if a shape is a triangle. At first the children decide if a shape is a triangle based only by how it compares to other shapes they have seen that have been called triangles. For students to move beyond this stage they must make deep modifications at mostly invisible meta-discursive levels. As students adopt new discursive routines it allows them to analyze problems in more complex ways.

The goal in education is to turn these mathematical routines into endorsed narratives, or narratives that are accepted by the mathematical community as truth. These narratives are more commonly referred to as mathematical theories, and are included in discursive constructs like definitions, proofs, and theorems, some of which are in the form of formulae and identities. In the previous example the teacher is trying to alter the student's routine into identifying a triangle based on the number of edges and vertices it has. Previously the student's were relating the figure in front of them to other figures they have seen in the past which were classified as triangles. If the new figure did not look exactly like ones they had seen before, they dismissed them as not being triangles.

Under this framework Sfard proposes that asking what children have yet to learn is "equivalent to inquiring how students' ways of communicating should change if they are to become skilful participants of mathematical discourse." (Sfard 2001a, p 11) This change is often not an easy undertaking as it involves "a true upheaval in deeply rooted discursive routines." (Sfard 2001a, p11) As teachers we must be aware that although the narrative may be easy for us, it is quite unlike anything children have experienced so far. Not only do students need to learn new vocabulary, they also must learn specially designed symbols and models to mediate the communication. On top of this they will need to change certain routines that are discourse-guided principles. An example would be changing a child's routine of identifying a triangle based on other triangles he has seen before, to counting the number of vertices and edges. As this discursive routine is developed, a skinny three sided figure which did not look like previous shapes referred to as triangles, may now be correctly identified by a student.

If we accept the participationist model of learning the next question becomes what is the best means for becoming a participant in a new mathematical discourse. Sfard believes that the starting point should be the introduction of new vocabulary and symbols. As mentioned, this view is contrary to other beliefs that students should understand a mathematical object before introducing the symbolic representation of the object. It is this view that reflects Hiebert and Carpenters model, which suggests the need for building a conceptual understanding before practicing procedural skills. However, Sfard claims that it is inevitable that if we want to initiate children to a discourse on new objects, then we have to use the discourse itself. We must have a way of identifying the object in words or symbols if we are to use it.

It cannot be forgotten that children bring with them "prior knowledge" to each new learning situation. This can now be restated by saying that students possess discursive routines from previous situations, which they will try to incorporate into the new discourse. This relates to the previous example where students learning about triangles may have already seen shapes called triangles in their past. Although there may be some similarities between an old routine and the routine they will have to adopt for the new learning situation, there are also differences. The change is deep enough for Sfard to call it an upheaval, and further development in the new discourse, "will be stymied unless the students are ready to adopt a whole new set of meta-rules for the endorsement of the new mathematical narrative." (Sfard, 2001a, p16)

As teachers it is helpful to be aware of the routines children bring with them to new learning situations and the changes that that they will need to undergo. If we can understand how children identify triangles naturally it can aid us in guiding them into adopting the new routine.

Sfard introduces the phrase "communicational conflict" to describe the "discrepancy between interlocutor's discursive ways" (Sfard, 2001a, p24). Teachers can witness communicational conflict on a regular basis when students with different answers justify their answers with conflicting narratives. Sfard separates communicational conflict from the more common term 'cognitive conflict' that is associated with the acquisitionist framework. Cognitive conflict is defined "as resulting from a contradiction between one's concepts about the world and the real state of affairs." (Sfard 2001a, p24) The person will use the world as an ultimate arbitrator in their attempt to resolve the conflict. The idea of communicational conflict arises from the assumption that learning is a change of discourse resulting from interaction with others. As a result, most opportunities for learning come from differences in interlocutors' ways of communicating (communicational conflict), not from discrepancies between one's endorsed narratives or routines and certain external evidence (cognitive conflict). Sfard challenges the assumption that what we say (think) about the world is determined by what we find in the world. She prefers to think that our, "discourse remains fully consistent with the realty we experience until a discursive change opens our eyes to new possibilities and to new visions. We thus often need a change in how we talk (think) before we can experience a change in what we see." (Sfard 2001a, p24)

Sfard uses this theory to defend her idea that students should first be introduced to new vocabulary and symbols and through practicing the new discourse students then build an understanding of the objects. Further complicating the teaching of mathematics is the tacit nature of the contradiction. "Trying to modify something that, cannot be described in words, and thus cannot be regulated by explicit instruction is a formidable task." (Sfard, 2001a, 25)

Changing a student's discourse often requires a change at the meta level and this kind of change is, "not a matter of logical necessity for the student." (Sfard 2001a, p33) For example, when students are learning about rules for operations with negative numbers, there is no logical reason that they can comprehend why the product of two negative numbers is positive. However, a change in the routine students use when multiplying negative numbers is necessary. This highlights the fact that teaching should focus at least in part to changes that are supposed to occur in the meta-rules of mathematical discourse. Sfard uses these ideas to build her philosophy of how students should learn mathematics. She believes that students should learn by observing more experienced interlocutors and by joining them in their actions. When students try to make sense of the actions of more experienced interlocutors, they can learn the new meta-rules that guide the discourse.

Sfard claims that communicational conflict is a necessary condition for learning. As such, without other people's example, the children would have no incentive for the change in one's discursive ways.

> Young learners have no means of envisioning or of appreciating the value of a discourse before actually gaining some experience in applying this discourse in problem solving. (Sfard 2001a, 2001, p39)

Sfard highlights the circular nature of changing one's meta-discursive rules

> One is unlikely to change her discursive ways without a proper motivation, but such motivation, so it seems, can only result from the actual use of the discourse. (Sfard, 2001a).

Teachers must understand that we cannot expect the change to occur rapidly, and time is an extremely important factor. This makes it difficult to say whether learning has yet occurred in any particular instance, as it may be that the period of observation was too short to show the complex process involved in changing one's meta-discursive ways. However, passage of time is not the only condition for learning. Students must also become aware of the conflict. Even this though, is not a guarantee, as students may not overcome the conflict.

One of the central factors that makes the difference in instructionally effective communicational conflicts is a realistic learning teaching agreement – "a set of unwritten understandings between the participants." (Sfard, 2001a, p40) They must be unanimous on three basic aspects of the learning teaching process: the leading discourse, their own respective roles, and the nature of the expected change.

2.2.1. Agreement on the leading discourse.

This is traditionally the teacher or textbook, but under the reform movement this may not always be the case. However there must be a "well-defined, explicitly present model discourse for the learner to follow." (Sfard 2001a, p41) Overcoming a communicational conflict will not be possible if interlocutors insist on acting according to their own discursive rules. They must agree on which participant should be given the lead.

2.2.2 Agreement on interlocutor's roles.

Those who are given the lead must be willing to play the role of teachers. They must feel responsible for the change in the student's discourse. Likewise those who agree to be the learner must, "show confidence in the leader's guidance and must be willing to follow in the expert participant's discursive footsteps." (Sfard 2001a, p41) Students and teachers must realize that this does not equate to mindless imitation, but a "genuine interest in the new discourse and a strong will to explore its inner logic."

2.2.3. Agreement on the necessary course of the discursive change.

Learners must participate in the leading discourse, "even before its inner logic and its advantages are clear to them." (Sfard 2001a, p41) During these early stages the inner logic of the concepts will only be acquired through participation.

> The children's participation is possible only if initiated and heavily scaffolded by the expert participants. For some time to come the child cannot be expected to be a proactive user of the new discourse: At this point in time, the discourse may only be practiced by the learner as a *discourse for others.* (Sfard 2001a, p41)

Sfard believes that the goal of further learning will be to turn this "discourse into a *discourse for oneself."* (Sfard, 2001a, p41)

For learning to be successful, the teacher and the students must live with the fact that participating students will initially see the new discourse as somehow foreign. This is a difficult step for many children who cannot get over the fact that practicing the ‘discourse for others’ without being able to see its inner logic, and consequently its relevance, has little appeal to them. However, through practice of this initial discourse for others, it will gradually be transformed into a discourse for themselves. Sfard claims that because:

> children’s need to communicate is so strong in children, and in particular with interlocutors whose authority is recognized and acknowledged, that this powerful motivational force is strong enough to overcome all other difficulties. (Sfard, 2001a, p42)

Sfard highlights the fact that for many students who struggle with math it is because they never get past the discourse for other stage. She understands that communicational conflict and a strong learning teaching agreement does not guarantee by itself substantial learning will occur. By substantial she means that students fully complete the discourse for others/discourse for oneself transition. The question of how to encourage the transition in students is an intricate one, but motivation plays a strong role. Motivation is not always a matter of what happens in school, but is greatly affected by cultural factors that come from outside of the mathematics classroom.

However Sfard also believes that there are things that teachers can do to help the students in turning the discourse for other into a discourse for themselves. She believes that a proper message about the sources of mathematical discourse would put the, “human agent back into talk about “mathematical objects,” by making it clear that mathematics is a matter of human decision rather than an externally imposed necessity.” (Sfard, 2001a, p45) One could share the historical evolution of how things were created to show that they are often matter of human decision, which, although made freely, does usually have well-defined reasons. In doing so we should be trying to lure students into the mathematical discourse that has been going on for centuries as opposed to understanding an externally mind-independent truth about the world.

Sfard suggests that having classroom conversation with students about meta-discursive rules and the changes they are supposed to undergo might also be beneficial. She believes that discussing the history of mathematical developments and how humans decided the agreed endorsed narratives. As well, the more teachers appreciate the demanding nature of the required change in their meta-rules, the more sensitive they will be in leading students through the change. "By getting more knowledgeable about the hidden mechanisms of learning they would become more realistic in their expectations, and their help to the learners will be more effective." (Sfard, 2001a, p45)

Sfard has created two separate but interrelated tools that probe into the effectiveness of communication. She believes that "effectiveness of communication may be presented as dependent on the degree of clarity of the discursive focus." (Sfard, 2001b, p14) By clarity, she means that, "at any given moment, all the participants must seem to know what they are talking about and feel confident that all parties involved refer to the same thing when using the same words." (Sfard, 2001b, p14)

Sfard splits the word focus up into three different but related components: pronounced, attended, and intended focus. The pronounced focus is the words used in the sentence and the attended focus is how one attends, (looking at, listening to etc.) while speaking or thinking. The following quote would be a typical conversation where a student shows their partner an algebra question. "We divide both sides of the equation by two to cancel out this two on the x." The pronounced focus is cancelling while the attended focus is the scanning procedure or the movements he makes while showing that the two's cancel.

Sfard realized that there is more to communication than just what words a person chooses, or what they are looking at and pointing to, "Whatever is pronounced or seen evokes a whole cluster of experiences, and relates the person to an assortment of statements he or she is now able to make on the entity identified by the pronounced focus." (Sfard 2001b, p34)

Sfard uses the term intended focus to describe all the statements an interlocutor is likely to make, and all the attended foci he is likely to enact, while using the cancelling as a pronounced focus. The intended focus, which Sfard realizes to be the crux of the matter, is primarily a private entity of the speaker that changes from one utterance to another. By splitting focus into three entities, Sfard emphasizes that students do not always say what they mean and that words can be interpreted in different ways depending on how they are said. An important example is when students call each other stupid. It is difficult for a researcher to always know whether the student is joking or is intending to be hurtful. The solution for researchers and teachers is that the attended focus is a public representative of the intended focus.

Sfard uses focal analysis of a mathematical dialogue to determine whether students have a stable intended focus. Students who are describing a mathematical object with a consistent intended focus can do so in the same way they would describe a physical object. This object can "preserve its identity while its image and its attended aspects are changing from one utterance to another." (Sfard 2001b, p15) For example, a student can describe a function either by using its equation or the graph of the function. Students who speak about mathematical objects as if they were a material object can zoom in and zoom out of particular aspects of the object. Sfard refers to this kind of discourse as objectified. This description of understanding can be compared to the model of a well-connected web of understanding.

Sfard further highlights the things students must do to become participants in a fully-fledged objectified mathematical discourse. "Pointing to the attended focus, *per se*, is not in itself enough to create an adequate intended focus." (Sfard, 2001b, p16) The student must attempt to coordinate intended and attended foci by specifying the attending procedure. Second, the leading student must probe the thinking of the other student. Students who are only interested in their own understanding often do not care about why the other student is confused. Sfard believes that this is often the hidden reason for many communication breeches.

> Students working in groups may have disparate expectations and wishes with respect to the interaction, as well as some interpersonal, mathematics unrelated, goals and desires that may be preoccupying them as they are talking to the other. (Sfard, 2001b, p16)

Sfard uses a second tool called preoccupational analysis to investigate these hidden reasons behind the failure of students to communicate. She stresses that there are two types of intentions that may be conveyed through communicative actions. First, there are object level (cognitive) intentions, related to the declared goal of the activity, in most case solving a problem and learning the math. This is often looked at through focal analysis of the interaction. The second type is usually less visible but not less influential. It is "related to various aspects of the interaction, and thus have the discourse itself as their object."(Sfard, 2001b, p17) Sfard uses the term meta-discursive rules that determine on one hand the students concerns about, "the way the interaction is being managed, and on the other hand, the weighty and sometimes quite charged issue of relationship between interlocutors." (Sfard, 2001b, p17) Sfard states that, "every instance of communication is an occasion for re-negotiating interlocutors mutual positioning and their respective identities." (Sfard 2001b, p17) Clues that provide insight into meta discursive rules are found in students' speech and in the mechanism of the interaction rather than direct messages. The meta-level intentions often remain invisible to those they affect.

Sfard uses the interactivity flow chart as the principal tool of preoccupational analysis. The flow chart uses different arrows to evaluate the student's interest in activating different channels and in creating a real dialogue with their partners. "These arrows express the participant's meta-discursive wishes: the wish to react to a previous contribution of a partner or the wish to evoke a response in another interlocutor." (Sfard, 2001b, p17) The arrows are separated into reactive and proactive categories and often reveal whether participants are, "addressing and interpreting their partners or, in fact, are concentrating on a "conversation with themselves"." (Sfard, 2001b, p17)

Sfard sees a constant tension between object and meta-level intentions where they compete for being the focus. Interpersonal communication is a complex phenomenon because each participant is:

> simultaneously involved in a number of object-level and meta-level activities: in trying to understand explicit contents of previous utterances and to produce new ones, in monitoring the interactions, in presenting oneself to others the way the person would like to be seen, in engineering one's position within the given group, and so on. (Sfard 2001b, p17)

Because students attend to all of these intentions at the same time, it is no wonder that mathematical communication is so challenging.

2.3 Noreen Webb

The past twenty years has seen a large increase in group work in North American classrooms. When students work in pairs or small groups, it gives them a chance to learn from each other and act as help givers and help receivers. It also provides an opportunity for, "recognizing and resolving contradictions between their own and other students' perspectives, and by internalizing problem solving processes and strategies that emerge during group work." (Webb, 2001, p2)

Webb has done research that looks at the mechanism of the helping behaviour, specifically the exchange of explanations about the content being learned. She stresses that just because students are working in pairs does not in itself guarantee improved learning, but that several conditions must be satisfied for given help to be effective. In fact, receiving non-elaborated help can be negatively related to achievement. "When and how exchanging help promotes learning is not fully understood." (Webb, 2001, p3)

There are several theoretical benefits to having students work in groups as both the help giver and the help receiver stand to benefit from *elaborated help*. Giving elaborated help, "encourages explainers to clarify and reorganize the material in their own minds to make it understandable to others." (Webb, 2001, p3) In doing so, it helps them "develop new perspectives and recognize and fill in gaps in their own understanding." (Webb, 2001, p3) The receiver of help can, "fill in gaps in their understanding, correct misconceptions, and strengthen connections between new information and previous learning." (Webb, 2001, p3) Webb also comments that peers may be more effective explainers than adults because they share a similar language, and they can simplify difficult vocabulary into a language that their peers can understand. Giving non-elaborated help may not have the same benefits as giving elaborated help.

Webb begins with four conditions that must be satisfied for received help to be effective. Explanations "must be relevant to the target student's need for help, timely, correct, and sufficiently elaborated to enable the target student to correct his or her misconception or lack of understanding." (Webb, 2001, p4)

Webb also identified three conditions for how the student receiving help needs to respond after receiving help. "The target student must understand the explanation, the target student must have an opportunity to use the explanation to solve problems or carry out the task for herself or himself, and the target student must use the opportunity for practice by attempting to apply the explanation received to the problem at hand." (Webb, 2001, p5)

Receiving elaborated help was found to predict how actively students responded to the help. Students who received high-level help were more likely to rework the problem on their own. How actively students used the help received was a strong predictor of post-test scores. Webb believes that failure of students to apply the help they received may be one explanation why some students who received elaborated explanations did not show an improvement in their achievement. "To benefit from receiving help, the learner must be an active participant in the learning process." (Webb, 2001, p6)

Students who are seeking out help must clearly convey the area of difficulty or misunderstanding. To do this Webb identifies five necessary steps. The student must be "aware that he or she needs help, be willing to seek help, identify someone who can provide help, use effective strategies to elicit help (ask explicit, precise, questions, and direct questions) and be willing to reassess his or her strategies for obtaining help." (Webb, 2001, p7) Webb found a large variance in students' willingness to seek help. As well, she found students who only wanted to know the answer instead of asking someone to teach them the how to do the question. Finally, only the few students who persisted in asking for help until they received an answer that helped them solve the problem truly benefited.

As well as persistence, the kind of questions students asked affected the level of help they received. Specific questions like, "Why did you multiply both sides by 5?" were much more likely to receive elaborated help than general questions like, "How did you do that?" Webb suggests that specific errors or questions make it easier for the group to identify a student's problem and formulate appropriate explanations. Specific questions imply that students are motivated to learn, have some level of understanding, and consequently, deserve receiving explanations. Finally, Webb found that students who were so "disruptive, aggressive, or unpleasant to other group members (e.g. interrupting, distracting, insulting, or ridiculing other) that the group would not help them even when they asked for help." (Webb, 2001, p8)

Webb discovered that student's "willingness to give elaborate help depends partly on group norms that supporting working together and helping others, and a focus on understanding and learning." (Webb 2001, p8) Web found large differences in students' perceptions about their responsibilities in the classroom. Some students emphasized the importance of working together while others valued individual work. Some groups were willing to provide help where others were not. Some groups recognized the value of explaining and understanding where others justified copying as instance of sharing ideas. "Webb found that groups that emphasized the importance of working together, helping each other, explaining, and understanding were more likely to give high level help than were other groups." (Webb, 2001, p9)

Not only do students need to be willing to give elaborated help, Webb emphasizes the importance of students giving clear explanations that are meaningful to the help seeker. Giving elaborated help includes comprehension of the material, understanding the difference between explaining and just giving answers, and an ability to communicate clearly. Webb found that students' help often consisted only of numerical procedures without verbal labelling of the numbers. That is students did not appreciate the importance of labelling numbers (e.g. Speed = 50 km/hr instead of just writing the number 50). Webb suggested that making students aware of the importance of verbally labelling numbers might help them formulate more effective explanations.

Webb also found that many groups that started off giving elaborated help showed a sharp decrease in the level of elaboration over time. Even groups who gave a high level of help initially showed a decrease in the level of elaboration on the same problem in response to repeated requests for help. This often arose out of frustration of the target student's inability to understand the help and often led to encouraging them to just copy the answer. Webb also observed that students missed mistakes that others had made because they were not continually monitoring each other's work.

Webb found that of students who received elaborated help, only about half applied the help they received to try to solve problems for themselves. A critical predictor of applying help they received was their focus on understanding instead of getting the answer correct. Students who emphasized understanding as indicated by the kinds of questions they asked ("why did you divide by three?") were more likely to apply the help they received than students who did not emphasize understanding. Webb highlights the need to apply the help one receives as a key process for learning math. Therefore, the help giver must provide the help seeker with an opportunity to solve problems themselves.

Obviously the actions of the teacher play an important role in effectiveness of group work. Webb highlighted five areas where the teacher could promote productive group interactions. She found that the teacher must establish positive norms for group work that support seeking and giving elaborated help instead of just answers, understanding concepts instead of memorizing procedures, monitoring one's understanding, collaborating rather than working independently, and creating a positive group atmosphere that encourages all students to participate.

As mentioned previously, making understanding the goal of group work is key. Teachers can encourage this by asking students to explain the concepts under mathematical procedures as well as encouraging them to label numbers. Giving students a reasonable number of questions with no time pressure reduces their need to rush to finish. The teacher must model the desired behaviour of students with the class to show them what is expected in their interactions. Finally the teacher must monitor the group interactions to assess their effectiveness and redirect those who are not interacting effectively.

2.4 Summary

While Hiebert and Carpenter focus on changes that takes place within an individual when learning occurs, Sfard and Webb both look at the common belief that mathematics is best learned in an interactive way through communication with others. Webb analyzes numerous student interactions and highlights key features that best predict successful learning. She analyzes the responsibilities of the student seeking help, the students giving help, and the responsibilities of the teacher towards making group work successful. Sfard also looks at key features that determine the success of mathematical interactions but her research focuses more on the microscopic level of interactions where as Webb looks more at macroscopic levels of interactions. Sfards addresses the need for a realistic teacher learner agreement that highlights different factors that the help giver and the receiver must possess for learning to be effective. She stresses the need for the learner to have a model discourse provided by a more experienced interlocutor that the learner can follow. Although Sfard's research concludes with very practical ideas, it also contains the theoretical grounding to complement many of Webb's conclusions. The two theories combine very well when looking at a particular interaction between students to determine whether or not it led to successful learning.

CHAPTER 3: METHODOLOGY

The data for this research was collected over a two-month period. The following chapter introduces the participants and the setting under which they worked.

3.1 The Students

Participants of this study were 24 students in grades 8 and 9. Sixteen of the students were girls and eight students were boys. Four of the students were from an honours class while the rest were from regular stream mathematics classes. The students worked out of the New Elementary mathematics textbooks from Singapore, as well as Math Power, which is an approved curricular resource for mathematics 8 and 9 in British Columbia. The students chose their own partner and volunteered to work through the problems while being recorded.

3.2 The School

The research took place at a private day school in a higher socio-economic area of Vancouver, British Columbia. The school is an open entry school, which does not require any minimum academic achievement for acceptance. The school promotes a well-rounded approach to education where strengths in the arts, athletics, and service are highly valued. Consequently there is a wide range of students' mathematical ability. The parent body is generally well educated and successful. Expectations that students attend post secondary education after graduation are very high, both by parents and by students.

3.3 The Courses

The courses were taught by the author. The students were frequently asked to work in pairs up on whiteboards. Three of the four walls of the classrooms were completely covered in white boards so there was generally enough space to have all the students in the class up at the boards at one time. The teacher did take some time at the beginning of the year to emphasize some of the qualities of successful group learning and did review them from time to time. Students were encouraged to work together and help each other understand the questions.

3.4 Source of the Data

The data consists of videotapes of the students working on problems on the white board that vary in both topic and difficulty. Each question worked on by a pair of students is referred to as an encounter. There were 12 encounters in all. The students in the study were taken to a neighbouring classroom because the noise of other students made recording difficult.

The students in the study were working on the same questions as the rest of the class. There was no thought as to what questions to give particular students chosen for taping. Usually the teacher would teach the lesson with students sitting in their seats, then allow the students to work in pairs at the board for the second part of class. At other times students would be given the entire period to work together on questions. This was especially true when preparing for a test. All questions had answers either on the back of the sheet or in the back of the book and students were expected to check their answers with these sources.

3.5 List of Tasks:

Christine and Mary

An isosceles trapezoid with base lengths of 15cm and 25cm has an area of $240cm^2$.

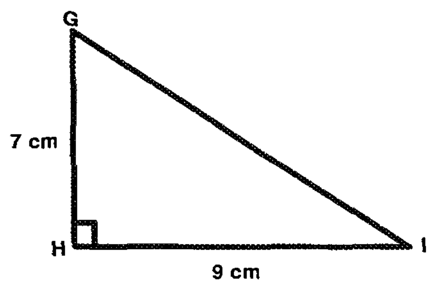
a) Find the height b) Find the perimeter

Jerry and John

A path, 2m wide, surrounds a garden, 30m long and 16 m wide. Find the area of the path.

Hillary and Sarah

Solve the right triangle HGI

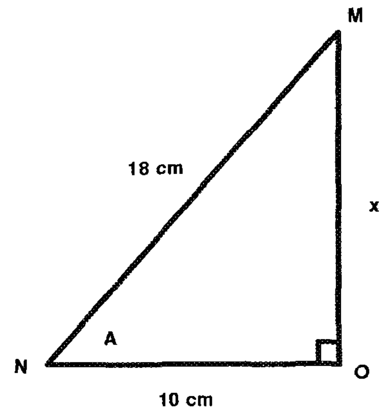


Kristy and Alison

Find the shaded area between two concentric circles with radii 4cm and 6cm. Compare this area with the smaller circle (take pi = 22/7)

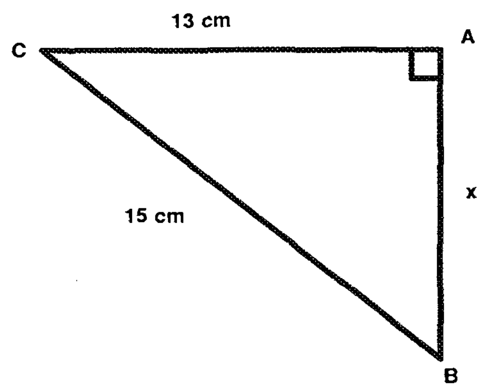
Anna and Kara

Use the cosine ratio to find angle A.



Alexis and Siobhan

Use the **cosine ratio only** to find the length of x



Quinton and Khris

#6 Solve the right triangle WXY with angle x = 90^0, XY = 25cm, and WY = 54cm

#8 Solve the right triangle GFB with angle G = 65^0, angle F = 90^0, and FB = 29cm (Mathpower p247)

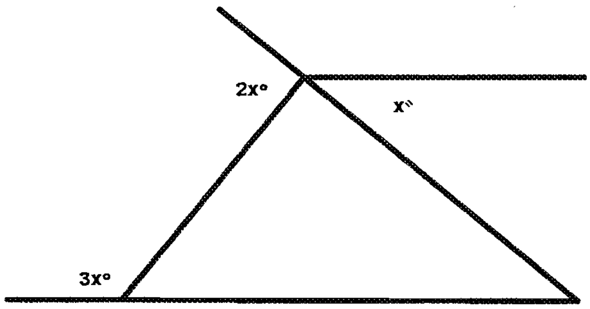
Evan and Mick

Truncated cone with top radius of 3 units and a bottom radius of 9 units. The entire height is 12 units and the height of the top missing cone is 4 units. Find the surface area.

Kerry and Jill

A cylindrical beaker is filled with water. It has a volume of 500 cm^3 and a height of 10 cm. Find the height of a similar beaker that has a radius of 7.98 cm.

Erica and Justine



Form an equation in x in each case and solve the equation.

Nicholas and Jim

A path, 2m wide, surrounds a garden, 30m long and 16 m wide. Find the area of the path.

3.6 Method of Collecting Data

A video camera was set up on a tripod with a microphone placed near the white board. The camera was turned on and the teacher left the room. To prevent background noise from distorting the audio recordings, the students were the only ones in the room. The students were asked to try not to block the camera whenever possible and to verbalize their thinking as much as possible. They were also encouraged to act normally and forget that they were being recorded.

3.7 Transcriptions

For each pair of students one encounter was chosen for transcription. The transcriptions do not only document what the students are saying but also what the students do. This is because what students do is an important part of the tripart focus, as described by Sfard (Sfard 2001b). Table 3.1 shows an example of such a transcription taken from an encounter between Anna and Kara.

Use the cosine ratio to find angle A.

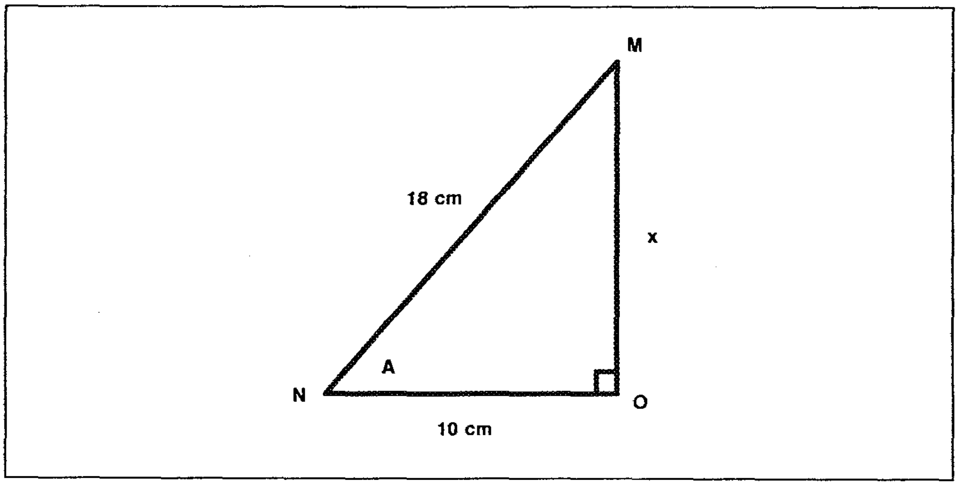


Figure 3.1: Task for Anna and Kara

Table 3.1: Transcription of Anna and Kara

What is done	What is said
K writes question on board	1) K: Use the cosine ratio to find angle A ...Okay so we have the hypotenuse
Labels hyp and adj	2) K: and we have the adjacent, this is the adjacent?
	3) A: Yes and this is the opposite

Says in a funny voice	4) A: Okay cosine
Writes HA	5) A: So we have the hypotenuse, and we have the adjacent
Crosses out HA and writes CAH	6) A: Its cosine okay so that wasn't right
Both girls begin writing out the cosine ratio	7) A: So it is Cos
not cos A!	8) K: cos A
A:repeats the cos A	9) A: Cos A, cos A
Writes the complete ratio	10) K: equals adjacent 10 over
Both girls working individually	11) A: 10 over 18
	12) A: So then we do it from there!, no calculations... mumbles
Puts brackets around each side. She wants to multiply both sides to remove the fraction	13) K: times both sides
Points to K. equation then her own	14) A: No no no because then, because then all it is, ya you just, cause then it will be like 18 times A and you still don't know A and its like, what the heck?
Erases brackets and watches Anna work	15) K: Oh oh oh ya
	16) A okay 10 divided by 18 equals
	17) A: We're trying to find an angle, yes, 2^{nd} Cos equals

3.2 Interactivity Flowchart

Each encounter was transcribed and sections of the transcriptions that best captured the essence of the encounter were chosen for analysis. From the transcriptions and the video clips, each session was then mapped to an interactivity flow chart. This tool is used in Sfard's framework for assessing certain regularities in the conversation. It consists of mapping the conversation using arrows. Each student has their own column and the numbers beside each arrow refer to the corresponding line in the transcriptions (See Table 3.1). There are two types of arrows that originate from each interlocutor.

Reactive arrows

Point vertically or diagonally backwards or upwards. This type of arrow expresses the fact that the source of the utterance (the one in which the arrow originates) is a reaction to the target utterance (to the one it is pointing).

Proactive arrows

Point vertically or diagonally, forward or downward. This type of arrow symbolizes the fact that the source utterance invites a response, so the following utterance is expecting to be a reaction. The proactive arrow will be drawn even if, in fact, the target utterance turns out to not relate to the source utterance and thus cannot count as a reaction. In other words, what should be considered in deciding about the nature of utterance are the intentions of the speaker, and not of their partner.

The arrows on the diagonal that point towards the partner, indicate dialogue which acts along interpersonal channels. Vertical arrows act in an intrapersonal channel, which reflect the fact that the interlocutor is communicating with himself or herself. Solid lines symbolize object level utterances, whereas meta-level interactions are marked with dotted lines. [1]

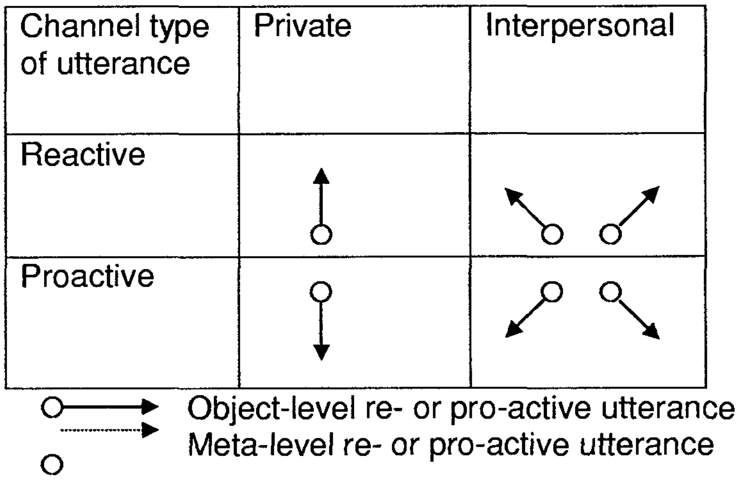
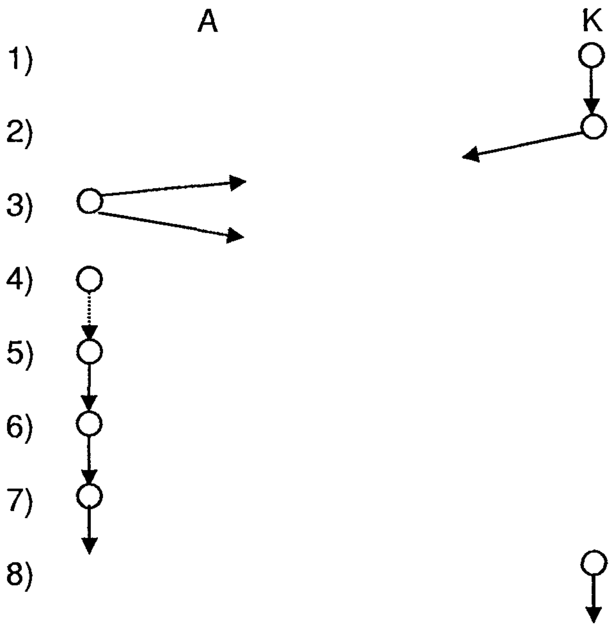


Figure 3.2: Interactivity Flow Chart Descriptors

The following interactivity flow chart maps the encounter of Anna and Kara taken from the previous transcription.



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[1] recall that meta-level discourse is when the focus of an utterance is on discursive elements rather than on the objects of mathematics (Sfard 2001 p59)

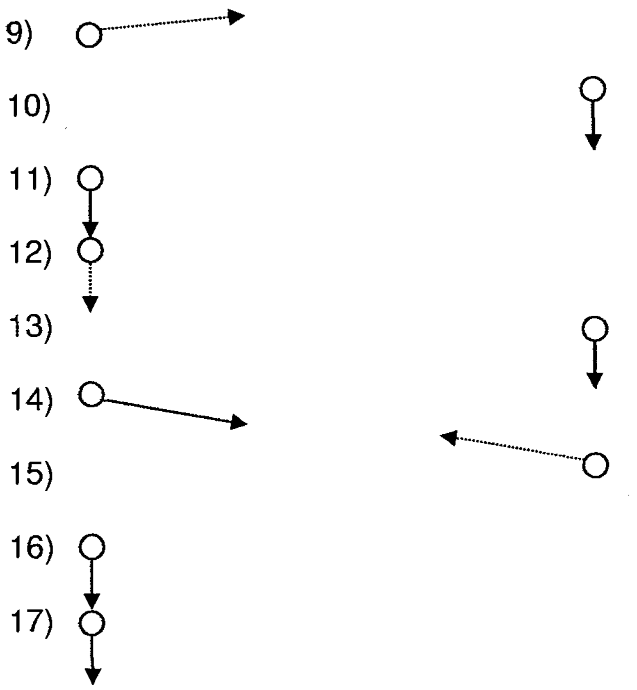


Figure 3.3: Interactivity Flow Chart for Anna and Kara

3.8 Analysis Checklist

Sfard stresses that students learn through participation in mathematical discourse. As such, studying the communication between students is paramount to the study of students learning. When learning is seen as participation in mathematical discourse, it becomes more obvious that the social norms of the participants will affect the success or failure of the encounter. Webb also looked at the characteristics of successful group interactions and concludes that there are several key factors that must be present for an encounter to lead to successful learning.

I combined these key factors that Web highlights for predicting learning in groups with Sfard's factors for a successful teacher-learner agreement into what I call the *group work analysis checklist*. The checklist strips down the complicated interactions into individual elements that can be studied to see if the dialogue contributed to a positive learning encounter. In theory, the ideal group interaction between two students would meet each of the criteria in the *analysis checklist*. In reality, few interactions could meet every criterion, but the degree to which these criteria are met is a very good indicator of how successful the learning was for both students. The transcriptions and the interactivity flow chart both provide insight on the elements for successful group work and aided in the creation of the checklist for each encounter. Figure 3.4 gives an example of a checklist for the encounter between Anna and Kara.

1) Agreement on the leading discourse

q **Was there an agreement on who should lead the discourse?**

At times A. tried to lead the discourse when she saw that Kara did not understand. (Question #4 line 14 - 26, Question #94 line 82, 94.) Although A.'s explanations were not always clear, K. quickly indicated that she understood thus not allowing A. to really help K. clear up her misunderstandings.

q **Did this person provide a well-defined, explicitly present model discourse for the learner to follow?**

A. had a difficult time precisely describing why K. should use the inverse cosine. Often A. gave good explanations that K. could follow.

q **Did the learner accept this new discourse or insist on acting according to their own discursive rules?**

K. was not overly responsive when A. tried to correct her misunderstandings. It was as though she did not want A. to think that she did not know what she was doing.

q **2) Agreement on interlocutor's roles.**

q **Did the leading student accept playing the role of the teacher?**

A. did accept the role of changing K. discursive habits but found it difficult to verbalize her own understanding. Although she tried to explain things to K., K. quickly brushed off A. trying to save face and not look like she did not understand what they were doing.

q **Did they feel responsible for changing the student's discourse?**

A showed some sense of responsibility, by stopping K. and trying to correct her mistakes

q **Did the receiver show confidence in the leader and were they willing to follow in the expert participants discursive footsteps?**

K. did not want to look like she did not know what was going on and therefore quickly changed the subject.

3) Agreement on the necessary course of the discursive change.

- **Did the learners engage in the leading discourse even before its inner logic and its advantages were clear to them?**

  K. did do as A. suggested (inverse cosine) to get the right answer but did not want to dwell on the fact that she did not know it herself. Lso when A tried to explain why A. could have found the third angle of the triangle from 90 – 30 instead of 180 – (90 + 30) K. refused saying that her way was more straightforward.

- **Did the expert participant heavily scaffold them at first?**

  A. tried corrected K. when she made mistakes with the inverse cosine. By the end of the interaction, K. did successfully calculate an angle using the inverse cosine. A.'s lack of ability to explain the reasoning of the inverse cosine did limit her ability to scaffold K.

- **Was the goal of learning to turn the *discourse of others* into a *discourse for oneself?***

  K. seemed more worried about saving face than she did adopting A.'s discursive routine.

- **Did the help receiver embrace the difficult step before they could see its inner logic**

  Both students were willing to begin a calculation to see if it would advance them closer to the solution even before they were confident that it would work. They hoped that once they saw the result, the process would become clear.

- **Were they motivated to make it through this transition?**

  Both students were motivated to make it through the initial challenges of the new discourse they were practicing. Their was a definite sense of uneasiness but they both were focused and concentrating on making it through.

4) <u>Was received help effective?</u>

- **Did they answer the questions?**

  Despite a few wrong turns, they were able to get back on course and navigate through to the correct answer.

- **Did they get answers when they needed them?**

  Both girls were good a responding to questions. A. was definitely willing to interrupt her own private conversation to bring S. back into the dialogue.

- **Was the answer correct?**

  The answer was correct and they confirmed this with the answer key

- **Was it sufficiently elaborated?**

  Although the girls were not able to perfectly explain their thinking, the help each received at different times was sufficiently elaborated to narrow down the discourse to one that would lead to the correct answer.

<u>5) How actively did students use the help received?</u>

- **Was the receiver an active part in the learning process?**

  Yes, both students were active in providing a new leading discourse when the previous one seemed destined to fail.

- **Did the students use the help to do the questions themselves or just watch the helper do it?**

  Each partner took turns writing different parts of the calculation and closely monitored the other persons work.

<u>6) Did the student clearly convey his need for help?</u>

- **Where they aware they needed help?**

  Each student was initially aware that they did not have the solution but both wanted to try to figure it out without just letting the partner do it for them.

- **Where they willing to seek out help?**

  Both student asked question, sometimes they waited for a response, other times they tried to answer their own questions.

- **Did they identify someone who could help? N/A**
- **Use effective strategies to get help (ask precise questions)**

  Generally, their questions were precise

- **Were they just looking for an answer or did they want to understand?**

Each student was interested in understanding

- q Were they persistent until they got the help they needed?

  Each student was persistent in asking questions until they got the answer they needed whether they answered their own questions or got answers from the partner.

7) Was the help giver effective?

- q Was the helper able to clearly communicate?

  The students seemed able to understand the explanations given

- q Did they write neatly and clearly label numbers and diagrams?

  Their diagram and calculations were neat

- q Did they understand the question?

  Yes

- q Did they value having the partner understand or just give an answer?

  A. seemed especially interested that S. understood and took the time to ask her if she was okay.

8) Miscellaneous

- q Did the helping process sustain itself or did frustration begin to hinder group work over time?

  As the were able to get to the solution without too much trouble frustration did not impede their work.

- q Did students support group work and helping others over individual work?

  The students seem to listen to each other, they answer each other's questions, and they check for understanding. They both contributed to solving the problem.

- q Were students rushing to get through the questions at the expense of understanding?

  The students were definitely interested in getting the answer as efficiently as possible, but A. took the time to check to make sure S. understood. Also, the girls spent the time at the end of the question to write down their answer.

Were the participants interested in:

Object level activities

- q Did the students understand explicit contents of previous utterances and produce new ones of their own.

  The students' answers to each other's questions indicated that they did understand each other, and although A. spoke more than S., S. did contribute to the conversation that led to a successful answer.

- q Were they interested in activating different channels and in creating a real dialogue with their partners

  Each partner although involved in a interpersonal conversation while they tried to solve the problem in their head, also benefited from the interpersonal conversation that helped keep them on track.

- q Was there a clear discursive focus? (Sfard defines clarity by saying that at any given moment, all the participants must seem to know what they are talking about and feel confident that all parties involved refer to the same thing when using the same words.)

  There was a clear discursive focus

- q Did students combining their attended and pronounced focus into a clear intended focus

  The students had a clearly labelled diagram and often pointed to specific parts of the diagram or calculation during the conversation.

- q Did they probe the other students thinking?

  Each student asked questions and waited for responses.

- q Did they care when their partner was confused?

  A. Specifically asked S. if she understood and seemed to genuinely care about S.'s understanding. When S. thought A. was going the wrong way she interrupted A. to ask a question that brought them back on track.

- q Did they look for mistakes that the partner made?

  A. watched S. work and did jump in to indicate inverse cosine. While calculating the third angle of the triangle.

q **Were their description objectified?**

Both students spoke about the cosine ratio in an objectified way although they stumbled at times applying it correctly in a new situation.

Meta-level activities

q **Monitoring the interaction.**

Both students seemed interested in figuring out the answer and showed concern for their partners progress as well

q **How they were presenting themselves in a way they would like to be seen.**

Neither student seemed particularly interested in this.

q **Engineering ones social position within the group.**

Neither student seemed particularly interested in this

q ***Are participants addressing and interpreting there partners or, in fact, are concentrating on a conversation with themselves?***

Although there are times when each student is involved in an interpersonal conversation, they do not give up on the interpersonal conversation.

q **Were they distracted by interpersonal, mathematics unrelated goals and desires that preoccupied the conversation.**

No

q **Did they sacrifice private communication to ensure interpersonal communication was not uncomfortable or did not make them look bad?**

There seemed to be a good balance of private and interpersonal conversation.

Figure 3.4: Analysis Checklist for Anna and Kara

For each pair of students the entire encounter was first transcribed. Sections of the transcription were then used as the basis to create the interactivity flowchart. The Analysis Check List was completed by attending to the video, the transcriptions and the flowchart. Critical themes were identified by a variety of means. Sometimes a theme would be evident first from the video. The corresponding line in the flowchart was then studied to see if it would provide insight on the theme. Other times a theme would be evident first from the flowchart, and then the video was watched to relate that event to what actually happened. Some encounters were analyzed more from the video and transcription, while others were analyzed more from the flowchart. The tools combined well to give me the opportunity to look at each encounter from a variety of angles and to more completely (and accurately) fill out the *analysis checklist*.

CHAPTER 4: RESULTS: ONE CASE STUDY

The encounter between Alexis and Siobhan demonstrates an example of successful group learning while working on the following task:

Use the cosine ratio only to find the length of x

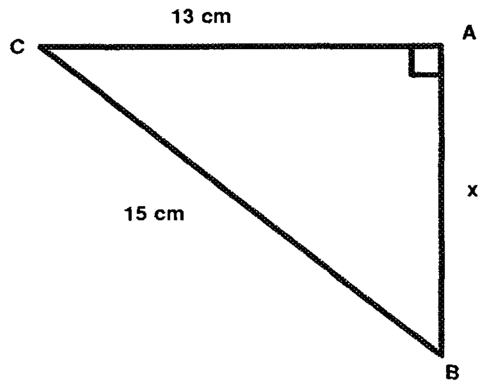


Figure 4.1: Task for Alexis and Siobhan

The transcription in Table 4.1 documents what each girl says and does during the encounter.

Table 4.1: Transcription for Alexis and Siobhan

What is done	What is said
A. draws question on board	1) A: Cosine equals adjacent over hypotenuse
Points to the side labelled x	2) A: This being......(mumbles to herself

Labels 15cm the hypotenuse	3) S: So
	4) A: But we can't use the Pythagoras theorem it says only use the cosine ratio to find the length of x
Both students stare at their sheets	5) S: okay...but there is no angle...or there is
	6) A: there is an angle?
	7) S: There is
	8) A: Where?
	9) S: While I mean except for the 90 degrees but...
A. reads the question again	10) A: Use the cosine ratio only to find the length of x
	11) S: You can just like say like...
	12) A: While we know that no matter what
S. points to angle C and writes x	13) S: Can't you just put it in and be like that's x, angle x, and then just
A. changes S.'s x to a y and then points to the length x	14) A: Or Y right? It has to be y because this is x right?
S. starts writing opp, then changes to adj	15: S: Right, and this is opposite
	16) S and A: No adjacent (both say at the same time)
	17) A: adjacent is next
	18) S: I think?
	19) A: Ya adjacent is next, to the thing
	20) S: Ya, its in-between
A. writes opp next to side x	21) A: And this is opposite, ya, I think
	22) S: Ya
	23) A. Do you think this will work? (Girls giggle)
S. starts write the cosine ratio for angle y	24) A. okay so 13 being adjacent of angle y
Both girls point to y	25) S: so... then you don't know that
A. points to the triangle and the formula quickly. Mean while S. puts a –1 exponent on the cosine	26) A: so the adjacent being 13 and then the hypotenuse being 15, so we can write inverted one

ratio.	
	27) S: you have to invert it, okay
	28) A: okay, this will just give us y, but we're supposed to….wait a minute…oh ya
	29) S: what are we trying to find here?
Circle angle x	30)A: x
Points to ratio	31)S: but this will just give us what y is
Looks at question on sheet	32) A: right
	33) S: Oh, wait so once we find out what y is then we can find out what x is?
Points to angle B	34)A: Well…ya…except that...wait why didn't we make this y
Points to side x	35) A: because then this would become the adjacent, because this is the opposite then the o6posite isn't the adjacent or the hypotenuse so its not included in the cosine
	37) A: Sometimes it is better not to pick any angle, sometimes you actually have to …. Okay
S. changes x to be the adjacent	38) S: so if we make this y
A. changes 13 to be the opp	39) A: right, and this is opposite
Writes the new ratio	40) S: So cosine is
	41) A: Adjacent over hypotenuse is x over
	42) S: 15
	43) A: right
	44) S: and then you have to invert, so
	45) A: you could times both side by 15
Points to the x and y	46) S: But how can you do it with two variable
	47) A: But know we have two variables (both say this at the same time)
Looks back at her sheet	48) A: But we can only use the cosine ratio?
Points to angle C	49) S: I don't get while that wouldn't have worked
Points to 13 and 15 respectively	50) A: Well because wouldn't this become hm, the adjacent and the hypotenuse, and x would become the opposite but if we're only allowed to use cosine then cosine is only adjacent and hypotenuse. Unless…No

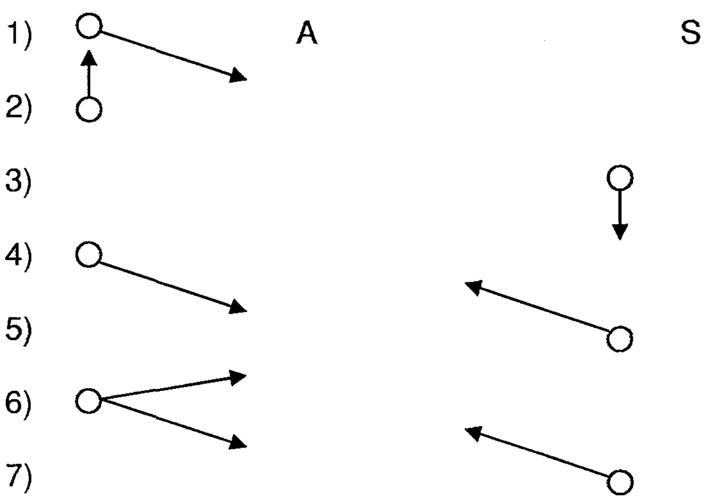
	51) S: Because this obviously can't work with two variables
	52) A: we could move .oh, why don't we do what we did at the beginning
	53) S: and see were it goes
	54) A: We could find out as many angles as we can using cosine and then see if it helps
Points to angle C	55) A: Oh right you're right and then we would do 180 – 90 plus this
	56) S: Oh Ya
	57) A: But then that would be...
	58) S: But we're trying to find what x is
	59) A: Oh right
S. labels angle C as Y and writes ratio	60) S: Not the other angle, lets just try it anyways. Okay
A: Changes 13 to the adj	61) A; Wait now this is the adjacent again
	59) S: Yes
Changes side x to opp, and then adds a –1 exponent to S. ratio	60) A: and this is the opposite, inverted cosine of y
Works it out on calculator and then writes it on the board cos y = 0.9	61) S: So what is 13 divided by 15...so 0.9
Adds the –1 again	62) A: inverted
	63) S: Ya
S: follows along on the calculator	64) A: So do you have 0.9 on your calculator? Okay then to get cos inverted we have to do 2nd then cosine
	65) S: okay twenty ...30
	66) A: 30 so
A. changes the y to 30	67) S: so that is 30..ya
	68) A: is it rounded
	69) S: Ya
	70) A: To what?
	71) S: I rounded it to 30 but its actually 29.9
	72) A: He likes decimals because then its more exact
A. changes it to 29.9	73) S: okay 29.9

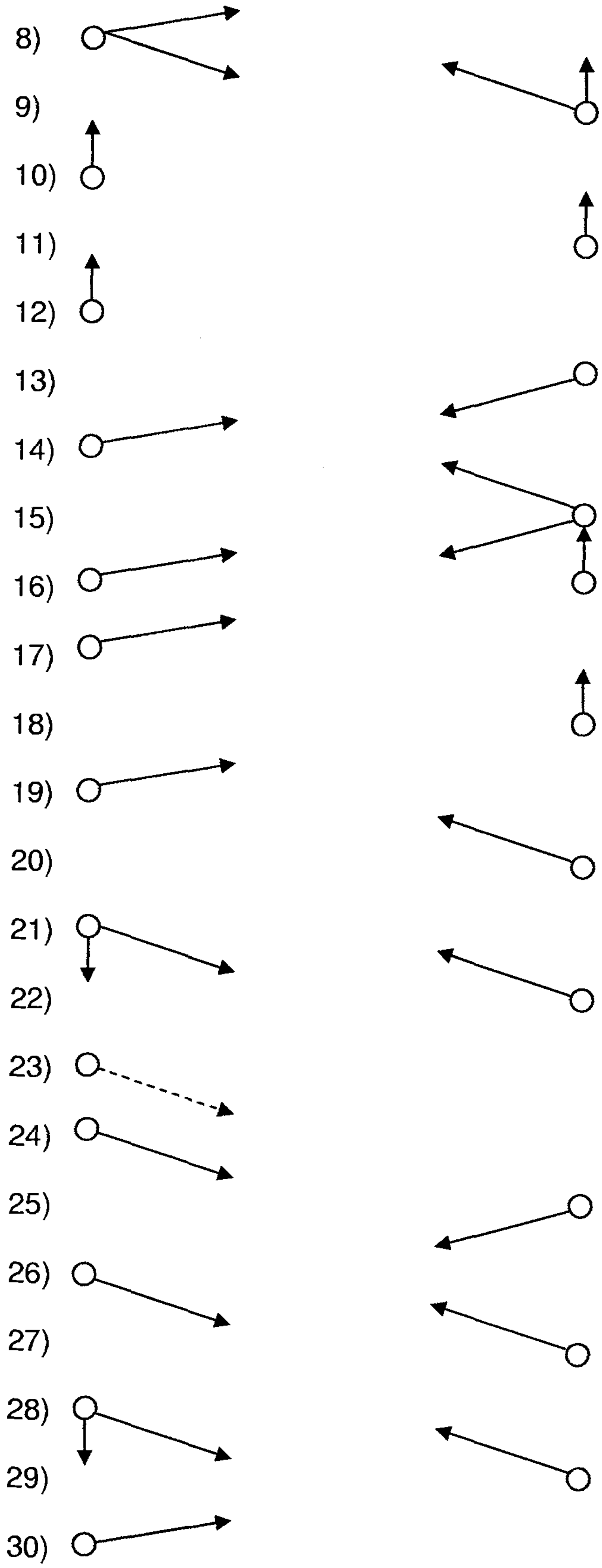
degrees	
Points to x	74) S: So once we have that angle know we can find what that is
Writes 90 beside the right angle	75) A: This is 90 degrees.. well supposedly
Points to B	76) A: Well ya because can't we do 180, we get this angle
Points to B	77) S: Why do you want to find what that is? Oh because that would be oh ya right
Points to 29.9 then to length x	78) A: if we keep that angle then this is still the opposite
	79) S: Okay
	80) A: Okay
S erases ratio while A. writes	81) A: So 180 degrees because that's how many are in a triangle minus 90 plus, why don't we just round it to 30 that will make it so much easier, plus 30..Is that 60
	82) S: Yup
S. write 60 for angle B	83) A: Because this is 120
	84) S: it is
A. points to 60	85) A: so then if we use this angle
S. points to x	86) S: so then can we change that
S. Labels x adj, and A. 13 the opp	87) A: Ya, then this becomes the adjacent and this is the opposite
S. write the ratio	88) A: The cosine of 60 equals x over 15. right, so now to get x by itself
	89) S: this time you know what the angle is
Siobhan has already started to do this	90) A: right so no inverted, regular cosine. Times both sides by 15 to get x by itself
Both girls use their calculators for 15(Cos 60)	91) A: 60 cos
	92) S: times 15
	93) A: what did you get
A. writes this down	94) S: 7.5
	95) A: 7.5 is equal to x
	96) S: okay there we go
A. checks her answer	97) A: 7.5 cm.. We got it right

with the back	
	98) S: Yes!
	99) A: I wasn't expecting it
	100) S: It was right the first time
	101) A: Wait should we copy this down?

When the transcription is mapped into the flow chart, there are important features that provide insight into the positive nature of the encounter. The first is that there is a balance of interpersonal and intrapersonal arrows. This reflects that the girls are communicating with themselves and also with each other as they try to solve the problem. Secondly, the arrows are mainly object level arrows and very few are meta-level in nature as both girls were focused on solving the problem and were sensitive to each other's needs. Finally there are several double arrows, which reflects that the girls are asking each other questions, listening to the questions, and formulating responses based on those questions

<u>Interactivity Flowchart for Alexis and Siobhan</u>

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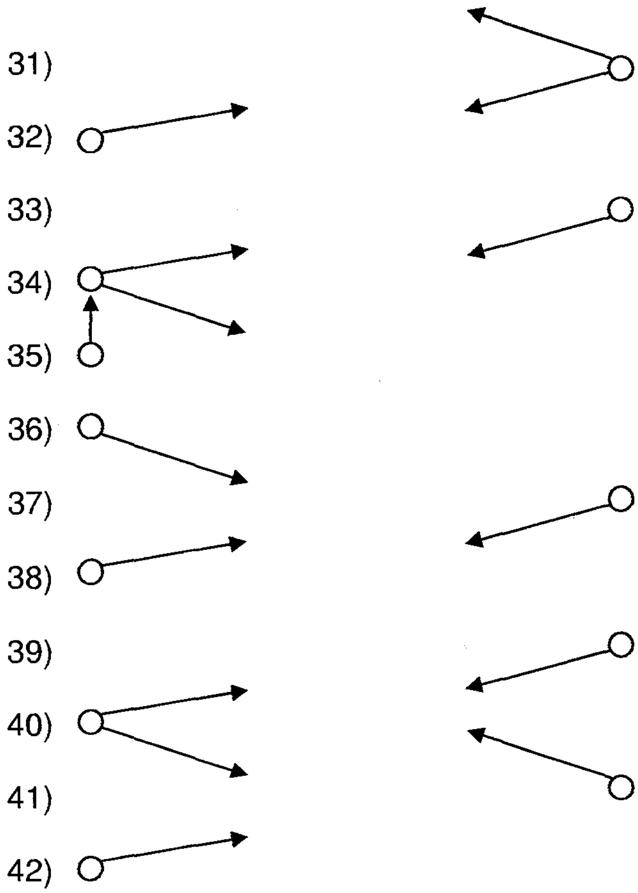


Figure 4.2: Interactivity Flowchart for Alexis and Siobhan

The *analysis checklist* then goes through the encounter and analyzes each of the factors deemed critical for successful group work, to see if it was successfully met.

<u>1) Agreement on the leading discourse</u>

q **Was there an agreement on who should lead the discourse?**

The students worked well together. The video shows that both students shared the lead when they had ideas. This is reflected in the interactivity flowchart with a variety of reactive and proactive arrows from both girls.

q **Did this person provide a well-defined, explicitly present model discourse for the learner to follow?**

The video shows that both students were good at explaining their arguments. At times the conversation that was occurring in their minds did not express itself clearly when they verbalized their thinking.

q **Did the learner accept this new discourse or insist on acting according to their own discursive rules?**

The video combined with the flow chart show that each student listened to each other and considered the ideas suggested by their partner.

q **<u>2) Agreement on interlocutor's roles.</u>**

q **Did the leading student accept playing the role of the teacher?**

The video shows that each student took turns being the leaner at times, A. seemed especially happy to take the lead role. S. was a little more hesitant, but did make attempts at leading the discourse; she did show less self-confidence in following through with her ideas.

q **Did they feel responsible for changing the student's discourse?**

Each student seemed generally concerned about helping his or her partner understand. There is a healthy mix of both proactive and reactive arrows from both students as they include each other in the conversation.

q **Did the receiver show confidence in the leader and were they willing to follow in the expert participants discursive footsteps?**

The partner taking the lead role was not always right. Each partner tried to understand the leader's ideas and explanations, but never gave up on their own thinking. Stronger students do not blindly follow a partner without checking to see if the new information fits with their previous knowledge. At the same time the video shows that they do listen to each other and consider the suggestions made by their partner to see if it will fit their way of thinking or contradict it.

<u>3) Agreement on the necessary course of the discursive change.</u>

q **Did the learners engage in the leading discourse even before its inner logic and its advantages were clear to them?**

S. was wiling to follow A's lead even though she did not completely see her reasons. She was able to detect when a current idea was not going to work

q **Did the expert participant heavily scaffold them at first?**

A. closely watched S's calculations and did jump in at times to correct S.'s work (adding the inverse cosine)

q **Was the goal of learning to turn the *discourse for others* into a *discourse for oneself?***

Both student saw the question as a challenge and wanted to be able to understand the inner workings of the new discourse.

q **Did the help receiver embrace the difficult step before they could see its inner logic**

Both students were willing to begin a calculation to see if it would advance them closer to the solution even before they were confident that it would work. They hoped that once they saw the result, the process would become clear.

q **Were they motivated to make it through this transition?**

Both students were motivated to make it through the initial challenges of the new discourse they were practicing. There was a definite sense of uneasiness but they both were focused and concentrating on making it through.

4) <u>Was received help effective?</u>

q **Did they answer the questions?**

Despite a few wrong turns they were able to get back on course and navigate through to the correct answer.

- ### Did they get answers when they needed them?

The video and the flow chart show that both girls were good a responding to questions. A. was definitely willing to interrupt her own private conversation to bring S. back into the dialogue. This can be seen in lines 12 – 14 where A.'s interpersonal arrows are interrupted by a proactive statement by S., which is then followed by a reactive arrow from A.

- ### Was the answer correct?

The answer was correct and they confirmed this with the answer key

- ### Was it sufficiently elaborated?

Although the girls were not able to perfectly explain their thinking, the help each received at different times was sufficiently elaborated to narrow down the discourse to one that would lead to the correct answer.

5) How actively did students use the help received?

- ### Was the receiver an active part in the learning process?

Yes both students were active in providing a new leading discourse when the previous one seemed destined to fail. The mix of reactive and proactive arrows from each student shows that they were both playing an active role in the conversation.

- ### Did the students use the help to do the questions themselves or just watch the helper do it?

Each partner took turns writing different parts of the calculation and closely monitored the other persons work.

6) Did the student clearly convey his need for help?

- ### Were they aware they needed help?

Each student was initially aware that they did not have the solution but both wanted to try to figure it out without just letting the partner do it for them. Both girls were motivated to figure out the problem but not so overconfident that they did not need to listen to each other's thoughts and ideas.

- ### Were they willing to seek out help?

Both students asked question, sometimes they waited for a response, other times they tried to answer their own questions

- ### Did they identify someone who could help? N/A

- ### Use effective strategies to get help (ask precise questions)

Generally their questions were precise

- ### Were they just looking for an answer or did they want to understand?

Each student was interested in understanding

- ### Were they persistent until they got the help they needed?

Each student was persistent in asking questions until they got the answer they needed whether they answered their own questions or got answers from the partner. The girls were good at responding quickly to each other so they did not need to be overly persistent.

7) Was the help giver effective?

- ### Was the helper able to clearly communicate?

The students seemed able to understand the explanations given

- ### Did they write neatly and clearly label numbers and diagrams?

Their diagram and calculations were neat.

- Did they understand the question?

  Yes

- Did they value having the partner understand or just give an answer?

  A. seemed especially interested that S. understood and took the time to ask her if she was okay.

8) Miscellaneous

- Did the helping process sustain itself or did frustration begin to hinder group work over time?

  As they were able to get to the solution without too much trouble frustration did not impede their work.

- Did students support group work and helping others over individual work?

  The video and the flowchart show that the students seem to listen to each other, they answer each other's questions, and they check for each other's understanding. The rich mix of proactive arrows followed by reactive arrows from both girls reflects that they both contributed to solving the problem. Line 4 shows a proactive arrow from A. followed by a reactive arrow by S. in line 5. The next two lines from A. show her responding to S.'s comment and then finishing with a new proactive arrow. These areas of the dialogue where students respond to their partners thoughts and then present their own thoughts show the most support for the cooperative nature of learning.

- Were students rushing to get through the questions at the expense of understanding?

  The students were definitely interested in getting the answer as efficiently as possible, but A. took the time to check to make sure S. understood. Also the girls spent the time at the end of the question to write down their answer. (line 101)

Were the participants interested in:

Object level activities

- Did the students understand explicit contents of previous utterances and produce new ones of their own.

  The students' answers to each other's questions indicated that they did understand each other, and although A. spoke more than S., S. did contribute to the conversation that led to a successful answer.

- Were they interested in activating different channels and in creating a real dialogue with their partners

  Each partner, although involved in an interpersonal conversation while they tried to solve the problem in their head, also benefited from the interpersonal conversation that helped keep them on track.

- Was there a clear discursive focus? (Sfard defines clarity by saying that at any given moment, all the participants must seem to know what they are talking about and feel confident that all parties involved refer to the same thing when using the same words.)

  There was a clear discursive focus

- Did students combining their attended and pronounced focus into a clear intended focus

  The students had a clearly labelled diagram and often pointed to specific parts of the diagram or calculation during the conversation.

- **Did they probe the other students thinking?**

  Each student asked questions and waited for responses. There are several area in the flowchart where proactive arrows represent questions they asked each other.

- **Did they care when their partner was confused?**

  A. Specifically asked S. if she understood and seemed to genuinely care about S.'s understanding. When S thought A. was going the wrong way she interrupted A. to ask a question that brought them back on track.

- **Did they look for mistakes that the partner made?**

  A. watched S. work and did jump in to indicate inverse cosine. While calculating the third angle of the triangle

- **Were their description objectified?**

  Both students spoke about the cosine ratio in an objectified way although they stumbled at times applying it correctly in a new situation.

Meta-level activities

- **Monitoring the interaction.**

  Both students seemed interested in figuring out the answer and showed concern for their partners progress as well

- **How they were presenting themselves in a way they would like to be seen.**

  Neither student seemed particularly interested in this

- **Engineering ones social position within the group.**

  Neither student seemed particularly interested in this

- **Are participants addressing and interpreting there partners or, in fact, are concentrating on a conversation with themselves?**

  Although there are times when each student is involved in an interpersonal conversation, they do not give up on the interpersonal conversation.

- ***Were they distracted by interpersonal, mathematics unrelated goals and desires that preoccupied the conversation.***

  No

- **Did they sacrifice private communication to ensure interpersonal communication was not uncomfortable or did not make them look bad?**

  There seemed to be a good balance of private and interpersonal conversation.

Figure 4.3: Analysis Checklist for Alexis and Siobhan

The checklist shows that most of the features of successful group work were met. The girls were highly motivated to get the answer correct but also had the confidence to persevere, even when they hit roadblocks. The students listened to each other and valued each other's thoughts. At the same time, they never gave up on their own thinking to blindly follow what their partner was saying. The students seemed concerned about each other's understanding and seemed to value doing the question together instead of individually. They wrote their calculations clearly on the board and pointed to the diagram and the calculations when they spoke. Meta-level dialogue did not interfere with the students' learning. They were not interested in sounding funny or worried about how they appeared to the other partner. Most importantly, both students used the help received to do the question themselves. After the solution was verified with the answer key, they both redid the question on their own question sheets.

CHAPTER 5: RESULTS AND ANALYSIS ORGANIZED BY ENCOUNTERS: SELECTION OF ENCOUNTERS

As already mentioned data from each encounter was recorded in the transcriptions and then converted to the interactivity flow chart. This data was then analyzed using the *group work analysis checklist*. In this Chapter, I summarize the dominant themes that emerged from each checklist in the form of analyzing text for each encounter. The analysis pays special attention to characteristics that Sfard and Webb believe affect successful learning. It stays attuned to which characteristics are present and which ones are absent and how this ultimately led to success or failure of each encounter.

Christine and Mary

An isosceles trapezoid with base lengths of 15cm and 25cm has an area of $240cm^2$.

a) Find the height b) Find the perimeter

Figure 5.1: Task for Christine and Mary

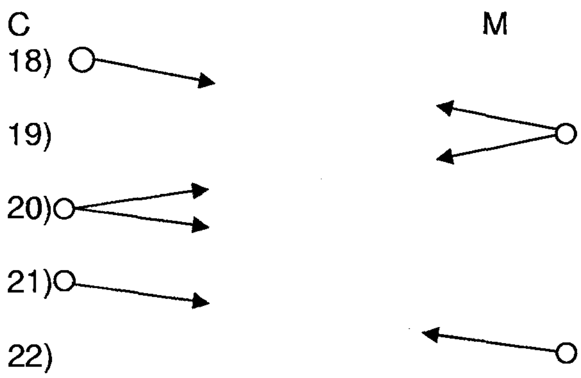
One of the challenges of teaching mathematics is working with students of different abilities as well as different personalities. This difference can be utilized to provide effective group learning as can be seen in the encounter with Christine and Mary working on a question to find the height of a trapezoid (Figure 5.1). Mary is a quieter girl who often struggles to learn concepts. Christine has a much more extroverted personality and is much more confident in her abilities. The key factor in this encounter is that although Christine could have done the calculation by herself and left Mary to just watch her, Christine stepped back and allowed Mary to write while she talked her through the calculation.

Table 5.1: Transcription for Mary and Christine

What is done	What is said
C. writes A= ½h(a + b)	18) And the area is equal to one half h a plus b
	19) M: but it has what the area is... the area is 246
	20) C: No I'm just writing down the formula, right, do you want to write down the 240
M. substitutes 240 for A	21) C: So a plus b then, that would be 15 plus 25....one half 8..15 plus 25...25
	22) M: Oh
Points to cm^2	23) C: and then you can remove that
	24) C: So 15 plus 25 would be ...(mumbles)
	25) M: It would be 40 right?
Teacher enters	26) T: you okay?
M. has 240 cm^2 = ½ h(40)	27) C: Yup
	28) C: Not 50 but 40
	29) M: Yup (giggles)
M. writes $240cm^2$=h20 and then divides the right by 20	30) C. Okay which then we... wait what do we do? Oh ya we divide 40 by 2, so 240... would be equal to, and then we divide by 20
	31) C: which therefore h is equal to 240 divided by 20...Lets see if our answer is correct.

M. works out on the calculator	32 M: 12
	33) C: Yup, good job

In line 19, Christine first realized that Mary might not know exactly what she needed to solve the problem. In line 20, Christine immediately took over the lead discourse and accepted playing the role of the teacher. Christine stepped back from the board and let Mary do all the written calculations. Before Christine let Mary do the writing, it appeared as though Mary was content to just watch Christine do the problem. When Mary had the pen, it forced her to engage in the problem. Mary is a very quiet girl and did not practice the discourse verbally but did write down the solution as Christine talked her through it. Although it would have been better if Mary spoke as she wrote down the symbolic representation of the conversation, Christine's voice provided the scaffolding for Mary to solve a problem that she may not have been able to solve on her own. Although she did not verbally turn the discourse into her own, by having the chance to take over the writing and do it herself she meet a necessary condition for effective group work.



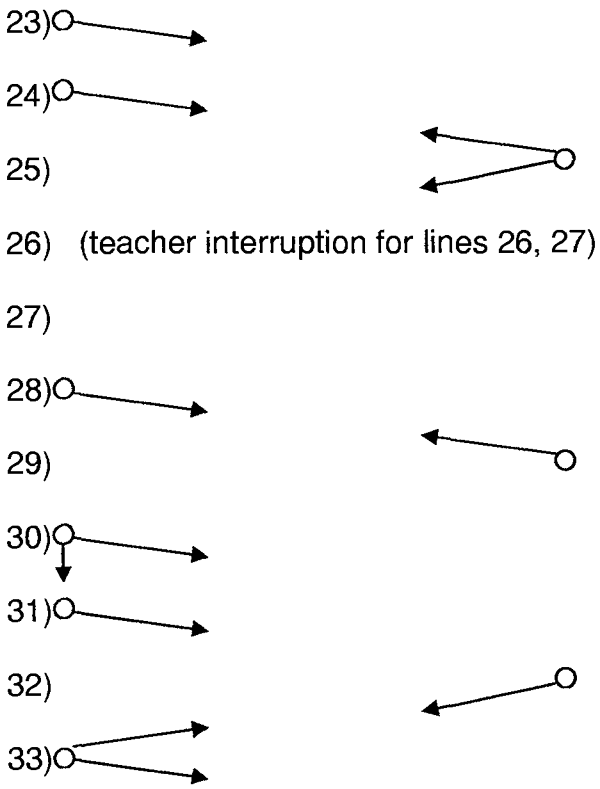


Figure 5.2: Flowchart for Mary and Christine

The interactivity flow chart shows Mary and Christine both starting the discourse with reactive and proactive statements (line 19 and 20). These combinations show that the students are listening and responding to their partner while at the same time making proactive statements that invite a response. In this case, Christine observes the communicational conflict that has arisen and then takes on the role of the teacher to lead the discourse. Her explanation of the mathematical procedure used to rearrange the formula was not perfect, but Mary was able to follow as Christine spoke and write down the proper symbolic representation. Although the flow chart shows a series of one-way proactive statements by Christine, Mary's response came in the form of what she wrote down instead of how she verbally replied. This is consistent with Mary's personality, which is on the quieter side. Often her writing was one step ahead of Christine's verbal answer.

The *analysis checklist* for this encounter highlights that Christine's discourse involves almost exclusively object level comments. There was a clear discursive focus displayed by Mary's ability to turn what Christine was saying verbally into the correct symbolic representation. The correlation of Christine's pronounced focus and Mary's attended focus created a clear intended focus for the successful communication. Meta-level activities were almost non-existent in the interaction and both girls seemed most interested in learning and working together productively. The fact that Mary got to do the question while using the help she received gives a greater chance that successful learning occurred.

> Jerry and John
>
> A path, 2m wide, surrounds a garden, 30m long and 16 m wide. Find the area of the path.

Figure 5.3: Task for Jerry and John

Many teachers assume that placing a strong student with a weaker student will automatically lead to effective learning. The encounter with Christine and Mary shows that it certainly can. However, students with different personalities do not always work in a mutually beneficial manner. This can be seen in the encounter between Jerry (J) and John (S) working on their question to find the area of a path (Figure 5.3). Jerry begins the question but cannot do it.

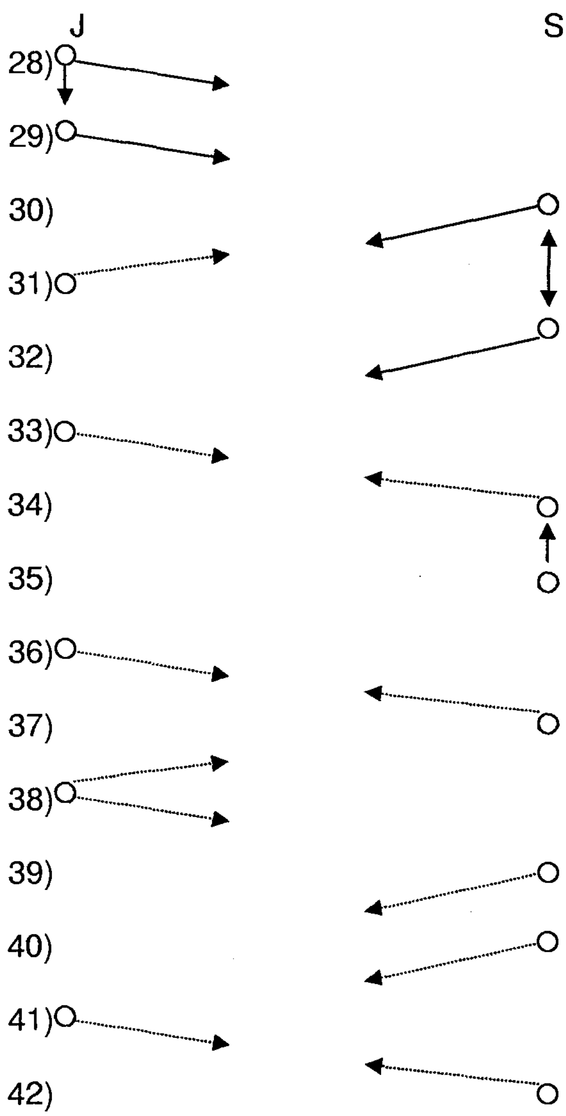
John takes over and does the question himself but does not demonstrate qualities of a good teacher.

Table 5.2: Transcription for Jerry (J) and John (S)

What is done	What is said
J. points to the outer length	28) J: While we need to know what that is
	29) J: I don't understand the question
S. comes to the diagram and points to the picture as he talks	30) S: If its 2 meters wide then its 2 meters wide this way, so this all the way from here its 2 meters, so you add two to that and two to that. So it makes it 34.
S. Begins labelling the diagram	31) J: Good accusation Mr. Steiner
	32) S: Ya 34 from there and 16 plus 4 is 20 across and 34 up, so you go 34 times 20.
	33) J: I'll take it after this
S; does the calculations on the board	34) S: No you won't
	35) S: 8 and 6, so it's 680 for all of it.
J. tries to grab the pen but John pulls it a way and hit J.'s shirt accidentally	36) J: No
	37) S: No
	38) J: I get to do half of it
	39) S: Look at your shirt (S. laughs)
J. pats S. on the shoulder	40) S: Then you go 16 times by 30, ahh, I hate this, okay its just taking up way to much space, you made it so big Jerry
J. continues to try to	41) J: Just hang in there big boy

clean his shirt	
	42) S: Shut up
	43) J: You don't see a stain on my shirt
S. continues to calculate	44) S: Okay good
J. slams the board pretending he's mad	Giggling
	45) J: S.

Interactivity Flow Chart for John (S) and Jerry (J)



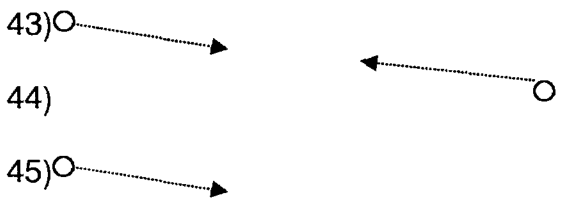


Figure 5.4: Interactivity flowchart for John (S) and Jerry (J)

John and Jerry's interaction created a different flowchart than the previous encounter. Perhaps more typical of some boys it is immediately noticeable that there are far more dotted lines in this interaction. When one analyzes the interaction closely, there are clues that predict this type of behaviour. John begins the question by trying to draw a picture while he listens to Jerry reading the question out loud. His drawing is wrong so Jerry fixes the drawing but then cannot seem to find a way to start the problem. Once appreciating the proper drawing, John knows what must be done to solve the problem. In line 30, John's proactive arrows are accompanied by intrapersonal arrows. The double headed interpersonal arrows representing John's dialogue in lines 30 and 32 show that although he is speaking out loud, he is really just talking to himself. John's solution is well articulated, but it is really just a conversation with himself.

John takes the lead and demonstrates that he does have an objective understanding of the concepts but shows little interest in playing the role of the teacher. He seems more interested in showing off his abilities than he does helping Jerry overcome his difficulties in solving the problem. John never stopped to let Jerry work on the problem. In fact, in line 33, Jerry asked if he could take over but John refused to let him. After that Jerry seemed to lose interest in the problem and began acting silly as he waited for John to finish his calculations. As a result, almost all qualities that Webb highlights for successful group work, and those that Sfard stresses as important for a successful teacher learner relationship, are not met. Because Jerry does not get a chance to practice John's discourse, he does not get a chance to make it his own, and loses motivation. The lack of motivation is reflected in the change almost entirely to mathematics unrelated comments as shown in the flow chart.

Hillary and Sarah

Solve the right triangle HGI

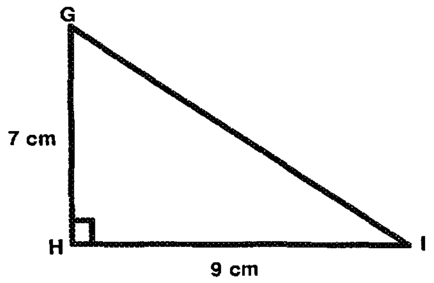


Figure 5.5: Task for Hillary and Sarah

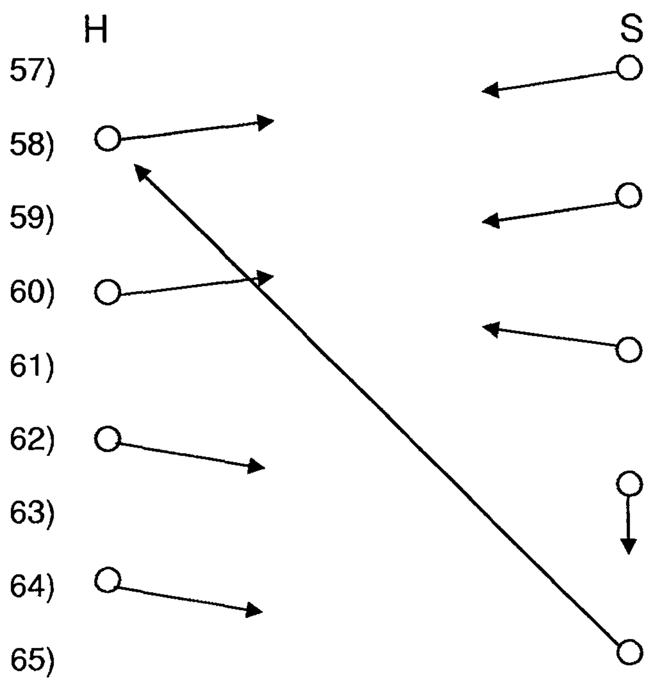
Hillary and Sarah are both in the process of learning to solve right triangles using trigonometric ratios as they work on the task shown in Figure 5.5. Both students are hard working and motivated to learn but their confidence levels differ greatly. Hillary is usually successful at mastering concepts where Sarah often takes a little longer and thus struggles to keep up. In stark contrast to John, when Sarah thought she has the question figured out she did not race ahead to demonstrate her ability but stopped to let Sarah write on the board despite Sarah's initial hesitation (line 72). This was a critical step that led to a successful learning encounter.

Table 5.3: Transcription for Sarah and Hillary

What is done	What is said
	57) S: Why are you doing Tan
H: labels the sides as she speaks	58) H: Because TOA, opposite, this is the opposite, and adjacent, and Mr. Rickard said to use
	59) S: Mmm, are you sure this is the opposite
H: points on the diagram	60) H: Ya because here's the angle, and that's opposite
	61) H: Oh ya
H: points to angle G	62) H: But if we use this one, then we'd have to... like, well
	63) S: So
	64) H: You can use
	65) S: cause, cause tangent is opposite and hypotenuse
H: thinks for a second	66) H: No no no opposite and adjacent and you want to use, you don't want to use hypotenuse because it has a decimal in it
	67) S: Ya
	68) H: and so it is easier to do it without a decimal
	69) S: Okay
	70) H: You get that?
	71) S: Ya
	72) H: Okay do you want to do this part then
	73) S: Not really (smiles)
	74) H: So Tan

	75) S: So Tan
H. Writes out tan theta and stops for S. to carry on	76) H: Tan theta equals…
	77) S: um, equals, okay so then, wait, so then, so then you go
	78) H: we're doing TOA
H. writes what S. says	79) S: Tan so... Tan theta equals opposite over adjacent so equals 7 over 9
S. puts nine on each side	80) S: and then, and then times each side by nine
	81) S: So 9 tan theta equals 7
	82) H: seven, so now we have to
	83) S: No because we don't have the angle so you have to go shift don't you?
	84) H: Ya

<u>Interactivity flow chart for Hillary and Sarah</u>



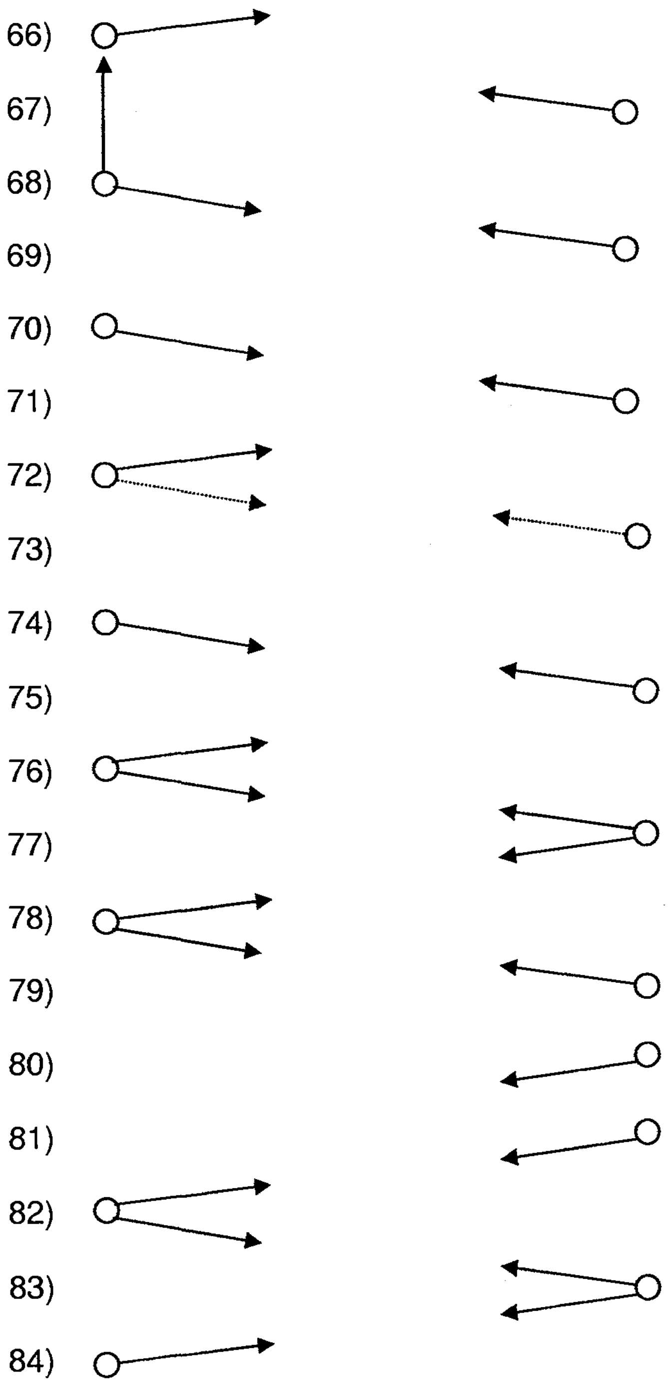


Figure 5.6: Interactivity flow chart for Hillary and Sarah

The interaction between Hillary and Sarah shows many of the qualities necessary for successful mathematical communication. The interactivity flowchart shows that the conversation begins with Sarah making proactive statements that Hillary responds to. In lines 61 – 75 Hillary makes mostly proactive statements to which Sarah responds. At this stage, Hillary is leading the discourse and Sarah tries her best to follow. In line 70, Hillary makes an important statement intended to check Sarah's understanding. Hillary is wise enough to tell from Sarah's yes reaction that she does not really understand.

Hillary stops working ahead and asks Sarah if she wants to try herself. The flowchart shows an immediate change in the configuration of the arrows (line 72). Hillary's arrows are both proactive and reactive as she scaffolds Sarah's conversation. By stopping and letting Sarah try it on her own she forced her to actively use the help she received and showed that Hillary was interested in making sure her partner understood and got the most out of the interaction. She forced Sarah to practice the discourse, which at that point was still a discourse for others. Similar to Christine and Mary's interaction almost all of the statements are object level activities. Metal-level activities do not control the situation in any way. Although Sarah answers yes when Hillary asks her whether she understands, Sarah is not trying to hide her difficulties. She does not seem embarrassed to be less able than Sarah and once she gets over her initial hesitation, does seem interested in practicing the discourse to make it her own. Sarah's agreement to embrace the difficult step before she could see its inner logic combined with Hillary stopping and letting Sarah take over, helped her begin to adopt Hillary's discourse and make it her own. These factors contributed to a successful learning situation.

Kristy and Alison
Find the shaded area between two concentric circles with radii 4cm and 6cm. Compare this area with the smaller circle (take pi = 22/7)

Figure 5.7: Task for Kristy and Alison

Perhaps the best result of having students work in-groups is the ability of students to get help when they are stuck. Having a long line of student's waiting to ask questions to one teacher is not efficient when many of the students' questions could be answered by their peers. The encounter between Kristy and Alison shows the two girls working on a challenging area question (Figure 5.7) where both students contributed to the correct solution. If the students were working on their own, they probably would not have solved the question. The encounter does end up leading to a successful outcome, but Kristy's unwillingness to listen to her partner's suggestion almost prevented their success.

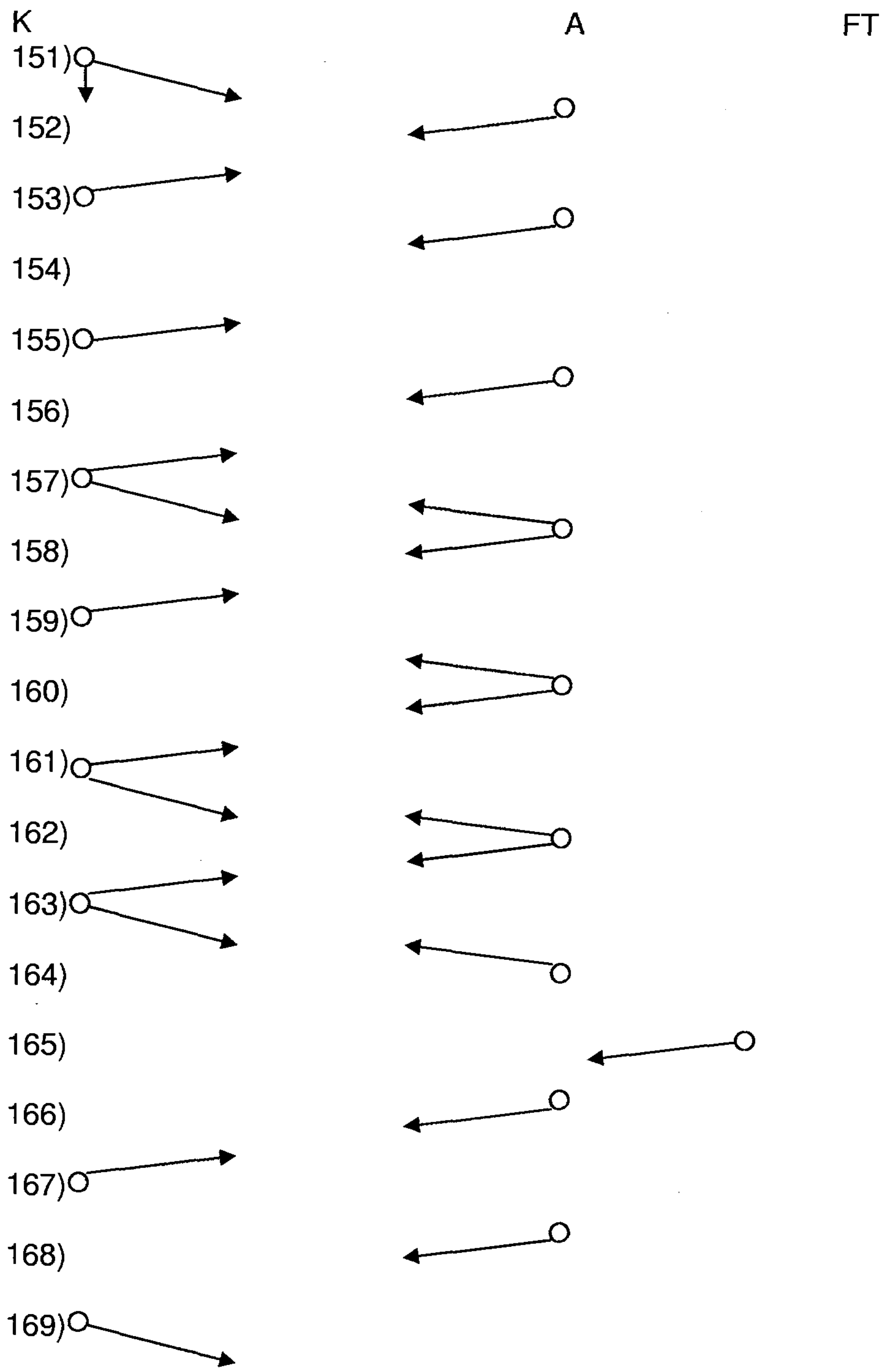
Table 5.4: Transcription for Anna and Kara

What is done	What is said
	151) K: Wait I need to times this by 11 right because the... okay what do I do
	152) A: you mean 22 over 7
Makes a mistake in what she says as she points to her calculation	153) K: No 11 over 7 because I divided them both by 2
A points to the 22	154) A: Why is that down there?

	155) K: Because they cross cancel
	156) A: Pi is 22 over 7 not 7 over 22
	157) K: I know, I know, but you have to divide 22 over 7 and then you have to flip em
	158) A: Why divide?
	159) K: No you have to flip them (mumbles something)
	160) A: I know but aren't you supposed to multiply
	161) K: Ya I know but I flipped them
	162) A: Why
	163) K: Isn't that what you supposed to do?
	164) A: Not when its multiple
French teacher shouts from the background	165) FT: Only invert when you divide, not when you multiply
	166) A: You multiply by the reciprocal
A. erases 7/22 and replaces it with 22/7	167) K: Oh
K. then cuts off A.	168) A: There, there's nothing you can do because 7 doesn't
K. is off camera	169) K: And then I times that by 7 right?
	170) A: not with multiplication,
	171) K: 16 so then times that by 7 okay
K: erases A's work without listening to her	172) A: that's only with adding and subtracting
Does it on a calculator	173) K: Um okay so that will equal 16 x 7
	174) A: No, why 16, no not with multiplication, that's only with addition and subtraction
K. erases her attempt	175) K: Alrighty, that makes way more sense
A. puts in equals	176) A: 16 times 22, that's why I have the equals sign

sign	there
K reaches for calculator, and then writes the answer	177) K: um, 352 over 7

Interactivity flowchart of Alison and Kristy



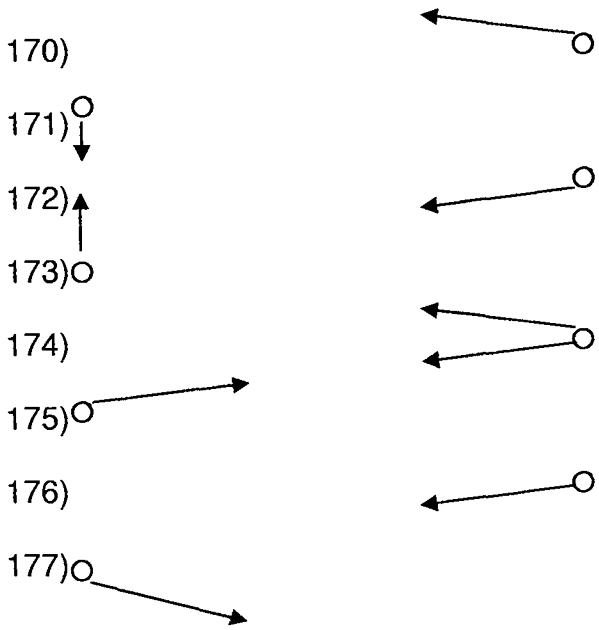


Figure 5.8: Interactivity flowchart of Alison and Kristy

Like most other pairs of students, their mathematical ability is different. Kristy is quite confident but does make mistakes. Alison is less confident and likes to be able to ask questions. At the beginning of the interaction, Kristy had to help Alison changing improper fractions into mixed numbers. Kristy seemed very confident and although she was willing to come over and help Alison, she seemed more interested in working in her private channel. Alison did understand Kristy's explanation and was able to do the work on her own after Kristy had left. Although a little unwilling at first, Kristy did show the patience and qualities of a successful help giver, and Alison showed the qualities of a successful help receiver.

The powerful element about group work is that one second a student can be the help receiver, and the next second they become the help giver. This is exemplified in this encounter when Kristy has to show Alison how to change an improper fraction into a mixed number, and then later Alison has to show Kristy how to calculate the area of a circle using 22/7 as π. In this encounter, having the students work in pairs allowed Alison to see that Kristy was making a mistake when she was multiplying by pi. Having a partner close at hand made it easy for Alison to see Kristy's mistake and help her.

A communicational conflict arose when both students had different explanations of how to multiply by π, and the girls knew that one of them had to be wrong. When Kristy's calculation seemed to confuse her, she was unwilling at first to accept Alison's discourse and wanted to stick to her own discursive ways. Thankfully, a French teacher happened to be setting up for another class and confirmed what Alison was saying. This can be seen in line 165 of the transcription. Once it was two against one Kristy was willing to adopt Alison's discourse and overcome the communicational conflict caused by the two contrasting ways of explaining the procedure. If the French teacher was not present, it is difficult to know whether Kristy would have accepted what Alison was saying. Kristy did not seem to have confidence in Alison's ability and may have considered her a less able participant. This encounter reinforces the need for students to be good listeners even when their partner may not be as strong a student as they are.

Kara and Anna

Use the cosine ratio to find angle A.

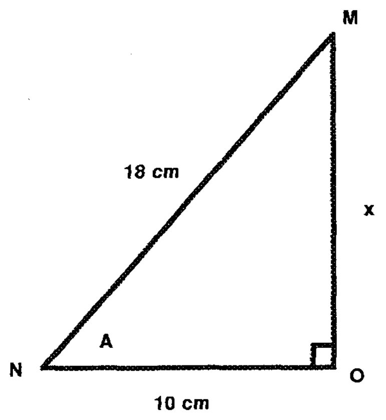


Figure 5.9: Task for Kara and Anna

The encounter between Anna and Kara shows another quality of students that can lead to unsuccessful group work. The girls work on the same problems that Alex and Sabrina worked on (Figure 5.9). The research of Webb and Sfard both emphasize the role the help receiver plays in learning. Often it is assumed that if a student has not learned it is the fault of the help giver. In this encounter, Anna tries her best to help Kara. Although her discourse it not totally objectified Kara does not want to accept the lead that Anna tries to take.

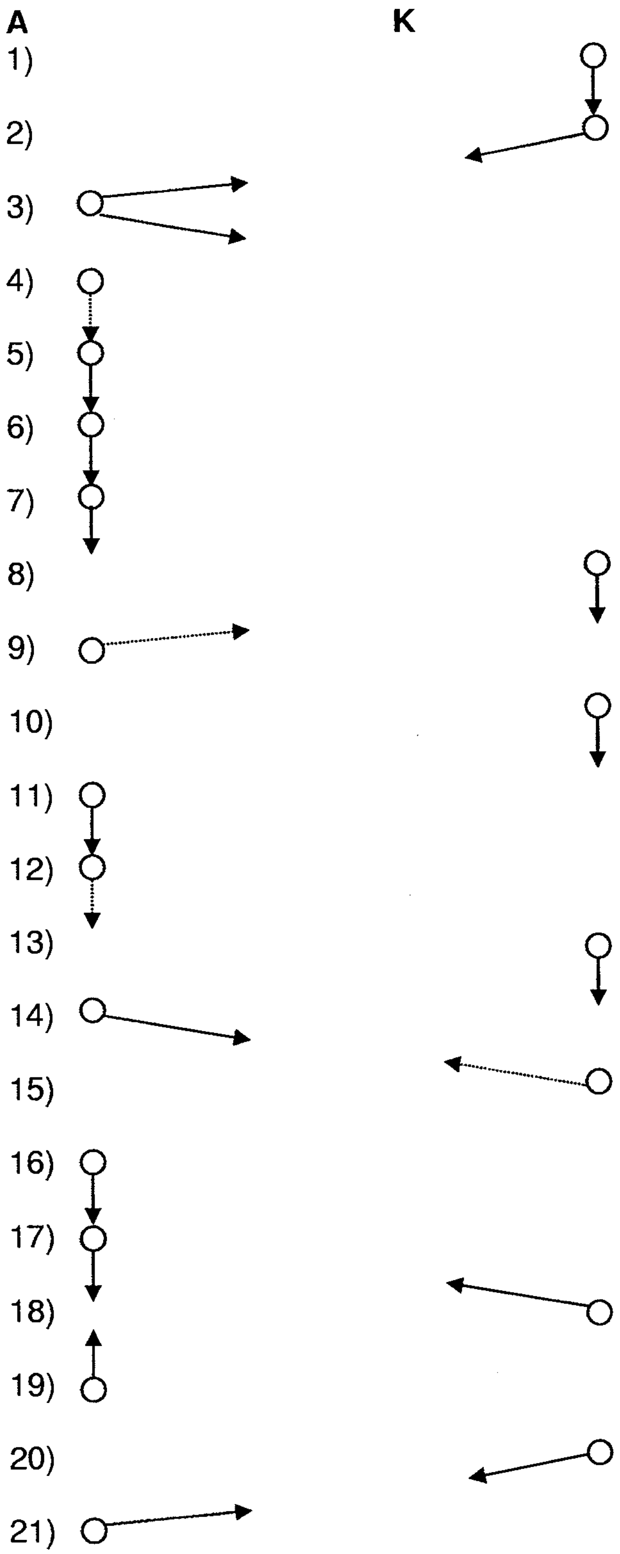
Table 5.5: Transcription for Anna and Kara

What is done	What is said
K writes question on board	1) K: Use the cosine ratio to find angle A ...Okay so we have the hypotenuse

Labels hyp and adj	2) K: and we have the adjacent, this is the adjacent?
	3) A: Yes and this is the opposite
Says in a funny voice	4) A: Okay cosine
Writes HA	5) A: So we have the hypotenuse, and we have the adjacent
Crosses out HA and writes CAH	6) A: Its cosine okay so that wasn't right
Both girls begin writing out the cosine ratio	7) A: So it is Cos
not cos A!	8) K: cos A
A: repeats the cos A	9) A: Cos A, cos A
Writes the complete ratio	10) K: equals adjacent 10 over
Both girls working individually	11) A: 10 over 18
	12) A: So then we do it from there!, no calculations… mumbles
Puts brackets around each side. She wants to multiply both sides to remove the fraction	13) K: times both sides
Points to K. equation then her own	14) A:No no no because then, because then all it is, ya you just, cause then it will be like 18 times A and you still don't know A and its like, what the heck?
Erases brackets and watches Anna work	15) K: Oh oh oh ya
	16) A okay 10 divided by 18 equals
	17) A: We're trying to find an angle, yes, 2^{nd} Cos equals
K. writes 2^{nd} cos above her ratio	18) K: So its 2^{nd}
	19) A: 56.25

K. writes cos A = 56.25	20) K: so cos, cos A, A equals 56.25
	21) A: well ya but it will just be like plain old A
Crosses out the cosine	22) K: and then you ... oh ya and then you... and then you did that so its
A. Points to K. equation	23) A: No....no no no no, its cos, it's A divided by, no no no no, it Cos, its cos 10 over 18 it has nothing to do with A yet
	24) K: oh oh ya ya ya
Looking at K. as she speaks Points to Cos A = 56.25	25) A: it doesn't really have any...its hard to explain...its not Cos A, its not that
	26) A: Its cos equals, cos minus one equals A, its really confusing, okay
Looks at answer	27) A: Question four, 56.25 yes yes!
In a funny accent!	28) A: We are correct, correctomundo

Interactivity flowchart for Kara and Anna



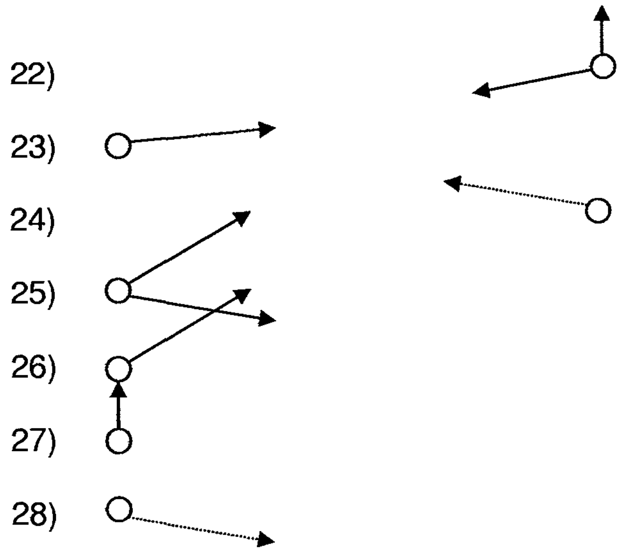


Figure 5.10: Interactivity flowchart for Kara and Anna

The interaction between Kara and Anna demonstrates characteristics that are different from those outlined previously. At the beginning of this interaction, the girls read the question and Kara asks a question to Anna regarding the labelling of the sides. After that, the girls work separately trying to solve the problem. At this point, the flow chart shows arrows mainly in their intrapersonal channels reflecting the fact that initially the girls are practicing the discourse with themselves. It is not until line 14 that Anna's focus changes as she realizes the communicational conflict that has arisen. The conflict occurs as Anna reaches for the calculator to use the inverse cosine button while Kara is trying to rearrange the equation using algebra. The change to a proactive arrow signifies Anna trying to take over the role of teacher as she attempts to change Kara's discourse. Anna has a difficult time finding the words to explain her understanding and this takes away from her ability to be an effective help giver. It appears that she can see what must be done in an objective way, but is not yet able to explain her understanding.

In line 15 of the transcription, Kara's reaction is not an object level reaction which might lead to further clarification of what Anna is trying to say. Instead, she brushes off Anna's help stating that she understands. However, on several occasions throughout this interaction she shows that she does not understand, and that very concept was an obstacle that prevented her from successfully solving the problem. Kara tries to do as much of the problem as she can on her own, which demonstrates how motivated she is to be successful. Although it is positive that Kara is trying to do the questions on her own it appears that either her pride or competitive nature prevents her from allowing Anna to help her become successful.

Later in the question she was unable to use the inverse cosine button to find the unknown angle which was the first key to unlocking the question. With the teacher's prompting she was able to see the need to use the inverse cosine.

Table 5.6: Transcription for Anna and Kara (a)

What is done	What is said
	52) T: so Kara do that
	53) K: well I just did that and I got the wrong answer
K. re-writes her ratio from before then looks at teacher. A takes calculator	54) K: so its like cos theta equals 13 over 15, and then I did that and cos theta equals 0.86 repeated
A. continues her calculations separately	55) T: And what did theta equal
Points to 0.86 R	56) K: Sighs, oh and then do I .... 2^{nd} Cos that?

Teacher nods and K. reaches for calculator excitedly	57) K: oh!!!

At the end of the question Anna had another chance to show her objective understanding of the cosine concept. Again Kara does not engage in a conversation about the difference between 2^{nd} cosine verses cosine.

Table 5.7: Transcription for Anna and Kara (b)

What is done	What is said
K. erases board and picks up calc	92) A: So then its times 15, okay so its 15 bracket cos 60.11 equals x, so you want to do 15 times 60.11 cos
	93) K: Wait 2nd cos?
K. begins calculation and does not listen to A's explanation	94) A: No because we are finding a side, not an angle.
	95) K: 7.475

A similar situation occurred when Anna tried to show Kara that to find the third angle of a triangle all she needed to do was go 90 minus the given angle. Kara didn't even react to Anna's attempt to teach her something new.

Table 5.8: Transcription for Anna and Kara (c)

What is said	What is done
K. Writes equation 180 – (90 + 29.89)	80) K: 90 plus 29.89
K. giggles as well	81) A: 90 minus 29, is (giggles) ...something I don't know
k. continues her way and works it out on	82) A: Just do 90 minus 29.89 because 180 minus 90 is always equals 90, so you can just go from 90, okay..

the calc (A. makes a further attempt later to show this to K. but K. prefers her way	
	83) K: 60.11

Later in question 7, Kara does properly use the inverse cosine to find the unknown angle. Anna tries again to show Kara a quicker method for getting the third angle of the triangle.

Table 5.9: Transcription for Anna and Kara (d)

What is done	What is said
	K) Okay so then 180 minus
	A) Or you could just do 30
	K) 30.1 No no no do that um
	A) For angles it will screw it up so bad, oh what ever
	K) It doesn't matter, um
	A) Its not, just, do you understand what I say when I say 90 minus 30.1 like instead of doing 180 minus 90 minus
	K) I know, I know, but that's just more straight forward this way
	A) Okay

Although Kara says she knows Anna's method it is doubtful that she does. One might argue that she is keen to focus on practicing one discourse before she tries to learn a new one. At the same time it is another example of Anna trying to take over the role of teacher and Kara quickly brushing off her attempts to help. Although strongly motivated to be successful, it seems that she wants her partner to see her as an equally able participant. By not engaging in the conversations Anna tries to start about the 2nd cosine button, and the third angle of the triangle, she prevents herself from practicing the discourse that underlies the concepts they are learning. By not engaging in a discourse about concepts Kara learns the procedures, but does not necessarily learn the concepts behind the procedures.

Quinton and Khris #6: Solve the right triangle WXY with angle x = 90^0, XY = 25cm, and WY = 54cm #8: Solve the right triangle GFB with angle G = 65^0, angle F = 90^0, and FB = 29cm. (Mathpower p.247)

Figure 5.11: Task for Quinton and Chris

The encounter between Khris and Quinton is also a typical situation of two grade nine boys who want to work together, and want to help each other, but perhaps do not fully understand the needs of the learner. Patience is a quality that all good teachers possess. Balancing the need for providing a model discourse for the learner, with backing off and giving the learner the chance to practice the discourse on their own, is a challenge. In this encounter, Quinton has a difficult time backing off to let Khris practice the discourse himself. The boys were working on the same task that Sarah and Hillary worked on (figure 4).

Table 5.10: Transcription for Khris and Quinton

What is Done	What is said

	1: Q: Okay I'm drawing this one
	2: Q: Where's the right angle
	3) K: uh it's the top right corner
Q. finally sees it and starts to draw	4) Q: Oh (mumbles numbers as he draw the diagram)
Q. labels 25 adj. and the opp while K. labels 54 Hyp	5) Q: Okay so lets find y first Khris, so then this is adjacent, and opposite
K. erases the opp and writes it clearer.	6) Q: So this one's the SOH CAH, so it's the cos of y....
Q. Bumps Khris aside while he writes out the ratio	7: Q: Move, y equals so 25 over 54, so it would be negative cos 25 divided by 54 equals
K. makes a bigger Y in Q.'s ratio. Q. looks at him, scribbles it out and re-writes his own y	8) Q: No what are you doing?
K. goes back into darken Q.'s new Y	9) Q: no no
Q: reads from his calculator, (K writes pi = 3.14)	10) Q: Okay so then that equals, y equals 62.423
Again Q. does in his head then writes it on the board	11) Q: y... 90 minus 62.423 equals 27.58
K: writes it down himself	12) K: What is that
	13) Q: That's W, x equals 90

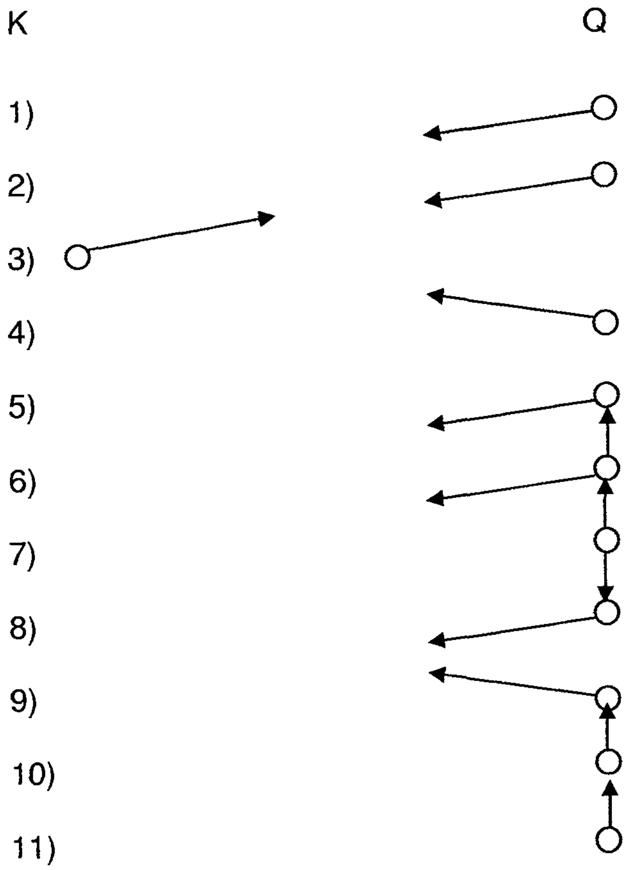
K; really likes the dark black marker he has
Boys don't check their answers with the back and don't realize that they were supposed to find the sides as well as the angles.
Question #8 (there are interruptions from another class coming in)

What is done	What is said
	1) T: Once Khris' got it let him do it and you can just watch
K. draws diagram and labels Hyp, and adj	2) Q: Okay
Labels 29 opp and	3) K: um this is opp
points to the adjacent	4) K: Okay....equals....what's this?
	5) Q: What?
	6) K: Wait
	7) Q: That's x
	8) K: Okay
	9) Q: Do it x
K. has Tan 65 = 29/x	10) Q: While which one do you want to do?
Q. crosses out the x in the denominator and puts an x next to Tan	11) Q: Okay so then you times both sides, okay what do you want to do?
K. pushes away Q.'s hand, K. fixes it so it is more neat	12) K: No
Q. divides both sides by tan 65	13) Q: So then, divide by tan 65
K. Writes 29 over tan 65, Q. calculates	14) K: 29 over tan 65
Q. writes x = 13.55	

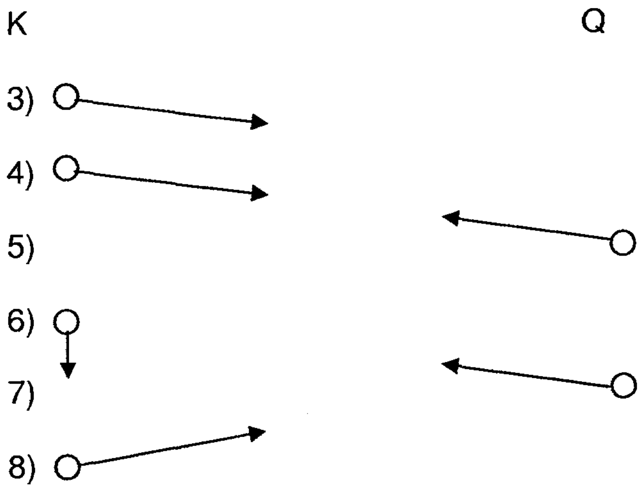
K erases question and they run out of time. They do not check the answer.

Interactivity flowchart for Khris and Quinton

Question #6



Question #8



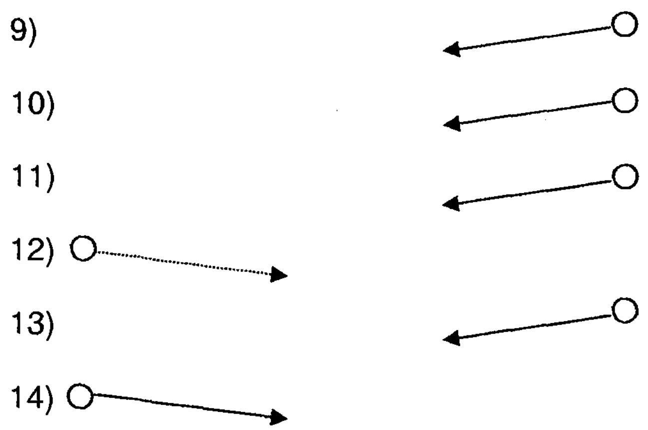


Figure 5.12: Interactivity flowchart for Khris and Quinton

The flow chart for this interaction looked quite similar to that of Christine and Mary's where there was a clear leader and one partner was trying to teach the other partner. Quinton controls the discussion and there are only a few comments made by Khris. The difference in this interaction is not so much in the flow chart alone, but in the combination of the flow chart with what is done. Christine let Mary do all the work on the board. At key moments, Quinton jumped into Khris' work and prevented him from doing the question himself. At the beginning of the question, Quinton tells Khris that he should do the next one by himself. This demonstrates that Quinton was interested in Khris' learning. In Question #8 line 12 Khris tries to push Quinton out of the way indicating that he does really want to do the question himself and learn. Both boys have the best intentions but their interaction falls short of counting as successful learning as described by Sfard and Webb. Quinton doesn't quite appreciate the skills necessary to be a good teacher despite his best intentions. By not letting Khris set up the equation himself, he does not give Khris a chance to change his discourse, but only listens to Quinton's. If asked Quinton probably would say that he did value having Khris understand, but by jumping in, he was giving answers before Khris had a chance to practice the discourse and develop his own thinking.

Evan and Mick Truncated cone with top radius of 3 units and a bottom radius of 9 units. The entire height is 12 units and the height of the top missing cone is 4 units. Find the Surface area.

Figure 5.13: Task for Evan and Mick

It is very easy for teachers to assume that if their students are working and are on task, then they must be learning. The encounter between Evan and Mick reminds us that learning math is far more complex then just having to put time in. Evan and Mick are on task, they are trying to solve the volume and surface area (Figure 5.13) problems, and they are both very capable learners of mathematics. Further analysis of the flowchart and *analysis checklist* reveals some of the problems that prevent successful communication.

Table 5.11: Transcription for Evan and Mick

What is done	What is said
	56) M: Okay this is the whole thing okay?
M. writes 594 on board	57) E: Ya, that's the whole thing are you sure?
	58) M: Here
	59) E: just do it, okay that's the whole thing
M.erases previous	60) M: Here, lets do it differently okay, lets not add the pi

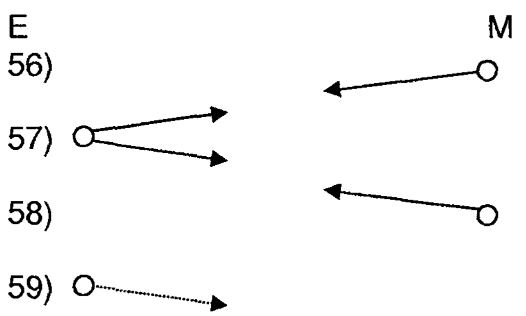
calculations	okay
E. points to top cone	61) E: That's the whole thing, and now you have to find this one, right?
	62) M: Here we've 81pi, we'll just dot it like this okay, like go like
	63) E: Is that how its...its 81pi are you sure?
	64) M: Ya
	65) E: Kay cool, so this whole thing is 81pi
E. knocks head on board, boys smile	66) M: No, no shsh, wait wait
E. walks around while M. calculates	67) E: Oh I know, its okay
M. looks at E.	68) M: While they give us the length so it just....
	69: M; plus 108 pi
	70) E: Ya, what time is this over at?
M. writes 81pi + 108pi	71) M: I don't know
	72) E: twenty right, ten minutes
Writes this number at the top of the board under 594	73) M: So the whole thing is 182pi
Points to 594, then interrupted by M. dropping the eraser	74) E: Which is that, a few times, okay sweat, okay we get it we get it, so the whole thing....
	75) E; got that on camera M., good one
	76) M: Okay so now we find the small one
	77) E: Ya, so 3 times pi, no radius squared, what the hell, pi times radius squared
M. Writes 9pi	78) M: So 9 pi
Labels the height of the top cone 4	79) E: Yes its so simple, 9 pi, oh now you have to find the length again, what, we don't know the height though, oh ya we do, 8, 12, 8, 4, 4

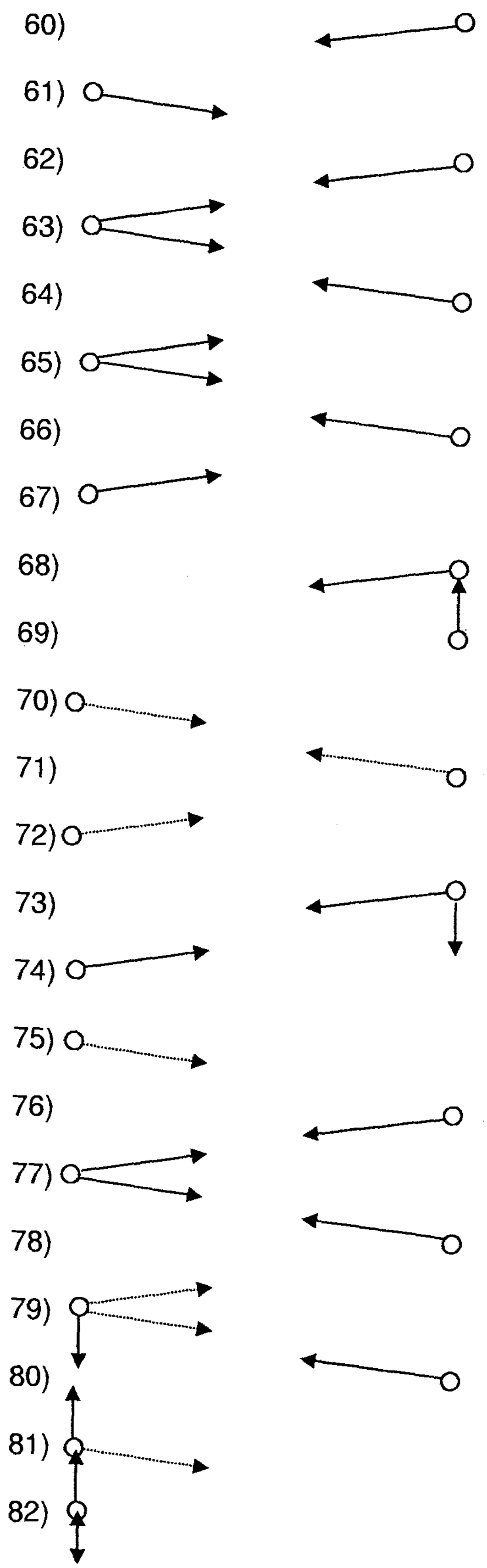
	80) M: Are you sure?
	81) E: 4 time 3 ...mumbles 9...mumbles 16 plus nine, what's 16 plus nine?
E. calculates on board, labels length as 25	82) E: I'm on a role come on come on, 16 Plus 9 equals 25
Points to lengths of the small cone, then erases 2 from the 25	83) E: that doesn't look right mumbles, you see squared, oh ya, that's that, what's, that's 5, that makes so much sense
	84) M: nine and 16
E. very animated	85) E: Come on I'm way ahead of you, look I'm way ahead of you, nine plus 16, 25 divided by five...five
	86) M: Ya
	87) E: Okay see, way ahead of you
	88) M: that's not a squared
E. smiles	89) E: shut up
	90) M: Okay so...
Points to 9pi which was left from earlier	91) E: Now we have to find the surface area of that so the bottom is so the bottom is 3 uh , pi times 9, oh 9 pi, we already got it right here
	92) M: Ya
	93) E: 9 pi
	94) M: plus
	95 the/Hmm (both together)
E. holds his hands to his head trying to remember	96) E: Oh wait, radius, no, pi radius L
	97) M: 15 pi
	98) E: What the, why did you get...
M. 9pi + 15pi	99) M: 24
M. writes 24	100) E: Oh ya, no, no, uh hm

	101) M: Why not?
	102) E: Uh hm
	103) M: Why not
	104) E: No
	105) M: Can you explain?
They laugh	106) E: Okay, here go ahead
M. writes calculation	107) E: 24 pi, that will give us the, are you sure that's right?, check, oh ya we had to minus that from 182pi and then times…mumbles come on answer standing by
	108) M: I know its wrong
M. continues calculating	109) E: Contestant..contestent (giggles) go
	110) M: Ohh
	111) E: it's .. mumbles... it's not that hard
	112) M: Yes it is
Boys laugh	113) E: we learned this in second grade
M. writes 182pi-24pi = 158pi	114) M: No why ya okay
	115) E: not the second grade, more like the third grade
Looks at M's answer	116) E: 158Pi what number is this

Boys check their answer with the back and realize that it is not correct. They think about it for a second and then decide to try to find the shapes volume.

<u>Flowchart for Evan and Mick</u>

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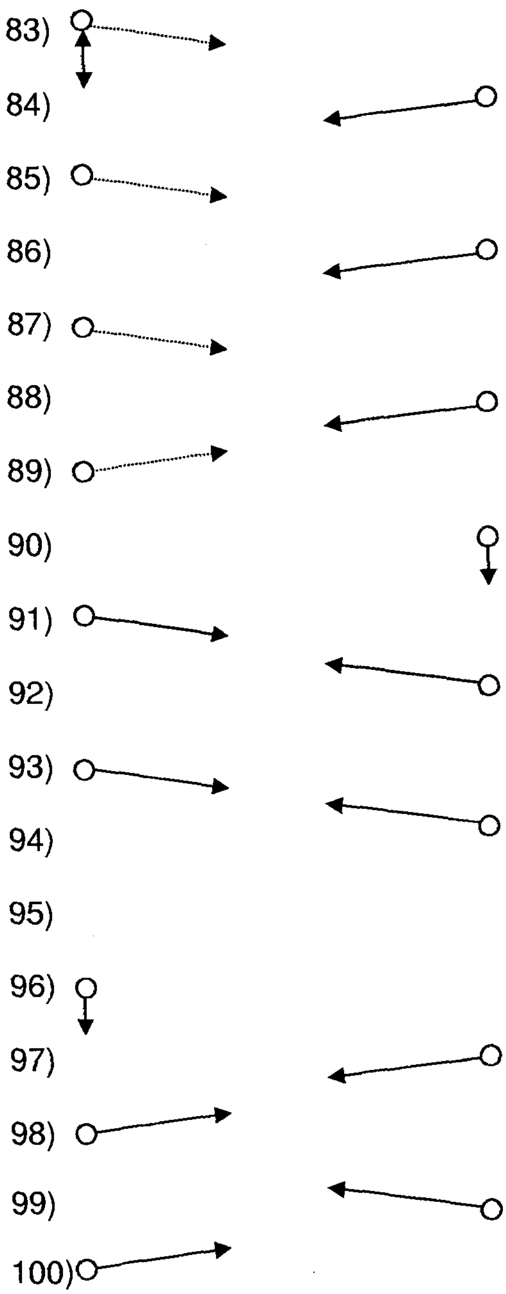


Figure 5.14: Flowchart for Evan and Mick

The two boys both contributed to working on the problems but neither provided a clear discourse for their partner to follow. The students fumbled through the questions and rarely got correct answers. The boys did not show any interest in changing each other's discourse. The flow chart from lines 56 –

67 shows that the two boys are working together initially. Lines 79 – 90 show a less positive picture but perhaps more indicative of why the interaction was unsuccessful. The boys did draw a diagram of the question and labelled the lengths but they did not write the numbers involved in the calculations or the formulas themselves. They tried to work out many of the calculations in their head. This severely limited the conversation, as they could not match their attended focus with their pronounced focus. Without a clear discursive focus, the boys often did not really get a chance to understand what the other boy was thinking. Consequently, the boys did not build on each other's ideas and had less chance of successfully solving the difficult problems.

The boys tried numerous problems and did not get correct answers. Instead of finding help from the teacher, they continued moving on to new questions. Line 83 shows a good example of the disjointed discourse Evan uses during the interaction. At the end, he claims that it makes so much sense. In line 79, Evan also shows his overconfidence stating that the calculations are so easy even though they still don't have a clear understanding of how to solve the problem. The problem is far from easy but Evan's overconfidence prevents them from seriously realizing their need for help.

It is unfortunate that although these boys did work together it is obvious to conclude that the interaction did not produce meaningful learning. Without a more able participant, the students have little chance of moving forward.

Kerry and Jill A cylindrical beaker is filled with water. It has a volume of 500 cm^3 and a height of 10 cm. Find the height of a similar beaker that has a radius of 7.98 cm.

Figure 5.15: Task for Kerry and Jill

The encounter between Kerry and Jill would probably be considered successful group learning because although they do mistakes as they go, they are able to see each other's mistakes and work towards the correct solution. Unfortunately being able to get the correct answer does not always indicate that successful learning occurred as defined by Sfard and Webb. In this encounter, two bright girls work out the solution to a volume and surface area problem (Figure 5.15), but do not get a chance to really understand why their answer works.

Table 5.12: Transcription for Jill and Kerry

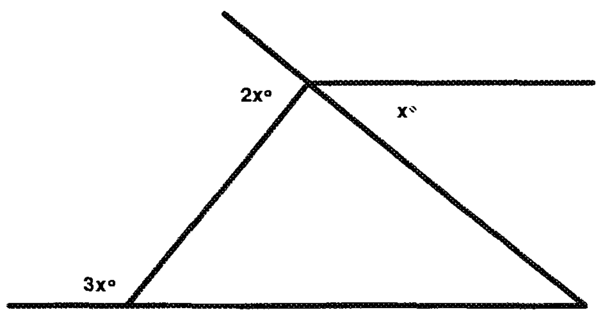
What is Done	What is said
J. watches K but does not read the question	42) K: Now so we're trying to find um, okay so now we want to find the height of a similar beacon that has a radius of 7.98
	43) K: So here's the similar beacon, beaker
K. draws a new cylinder	44) J: Doesn't matter we're in a science lab, wait it does matter
K. labels x on her drawing	45) K: 7.98, okay and we need to find the height so the height is going to be x

	46) J: Okay so do we do cross multiplication or something
K. labels both the old and new diagram	47) K: and the radius for this is 3.98, 7.98, so we want to find the height, so we can do similar triangles, or similar for this so
	48) J: Why don't we just do cross multiplication, lets just see if it works then we can like
	49) K: It's the same thing as doing similar triangles because you put 3.98 over 7, wait, 98 equals 10 over x
	50) J: why wouldn't you do radius over height over radius over height
	51)K: Why don't we just do similar triangles, or I don't know, but wait I...
	52) J: Because how can we do similar triangles if its like not a triangle
	53) K: Oh right ya
	54) J: Laughs, the one little thing there
J. starts writing and then uses calc	55) J: 7.98 times 10 divided by 3.989
	56) K: So the height should be then 20
	57) J: Ya
	58) So we can check on our answer

Both girls are quite bright students and are able to get the correct answer to this question. They have learned about similar triangles but never have been asked if the principle would apply to other shapes. Kerry suggests using similar triangles and it appears that she wants to use the same reasoning for this problem (line 47). Jill goes straight to setting up the ratio and suggesting that they use cross multiplication (line 48). This does get the right answer but we are left feeling unsatisfied with the learning that has occurred. Jill tells Kerry that they can't use similar triangles because they aren't triangles (line 52).

The girls need a more experienced interlocutor to tie similar triangles to similar figures, to link the ratio of corresponding sides of a triangle, which is a discourse that the girls are familiar with to a similar discourse on similar figures. The girls are able to apply the correct procedure but not explain the concept behind it. The girls seemed satisfied that their answer matches the correct one and see no reason in further developing the discourse but are keen to move on to the next question. The constructivist might claim that the girls were able to solve a problem that they have never seen before using skills that they already know. But the participationist would not be as satisfied because they have not yet learned the proper discourse, which would explain the missing link between what they know previously, and what they need to know to solve this new problem.

Erica and Justine



Form an equation in x in each case and solve the equation.

Figure 5.16: Task for Erica and Justine

The Encounter between Erica and Justine demonstrates how meta-level factors can work behind the scenes to prevent learning from being successful. As with Kerry and Jill, both girls are very capable and very motivated learners. As they work through questions using the exterior angle theorem (figure 4) a communicational conflict arises that never gets resolved. Although the interactivity flow chart did not show anything abnormal, watching the encounter and having the luxury of rewinding and analyzing what each student is trying to do make it easy to see why the communicational conflict never got resolved. Justine tries her best to resolve the conflict but does not understand that Erica is using a different set of meta-discursive rules to solve the problem. This shows that she has not developed the objective understanding that Justine has. Justine does not quite figure this out and ends up thinking that Justine is not even using the exterior angle theorem. This dilemma is combined with another meta-level issue that prevents successful communication. Erica's desire to be seen as an equal participant prevents her from slowing down to discuss why Justine's answer is different than her own.

Table 5.13: Transcription for Erica and Justine

What is Done	What is Said
J draws the picture	1) E: Okay that one is 3x, 3x, that's 3x
	2) J: Here
	3) E: uh hmm, 2x and x
	4) J: Okay
	5) E: Wait

J. labels the third angle 3x	6) J: So now we know that this here is 3x
	7) E: uh hmm
	8) J: So then
	9) E: Oh 180
	10) J: We have to do that like exterior
	11) E: I don't get that, oh so...
J starts writing	12) J: So that would be 180
E also writes it	13) E: equals
Labels the supplement to 3x and its opposite angle 180 – 3x	14) J: Minus 3x is this
	15) E: Ya no wait, no way, because then 180 equals x plus 2x
J adds 3x to E.'s equation	16) J: Plus 3x
J. adds 3x again	17) E 180 equals 3x, and then divide by three
	18) J: Plus...
	19) E: Oh wait no
	20) J: No
	21) E: Plus
	22) J: Its 6x
	23) E: No Justine
	24) J: ya, you forgot
E. write 30 = x	25) E: Oh that's 6x and then 30, 30 equals x
	26) J: Ya, but how do you know like the exterior
	27) E: That's it, because we added these two
	28) J: Unless
	29) E: Oh

E. erases some of here work	30) J:180 minus 3 equals, maybe its like 180 minus 3x
J. erases the rest	31) E: No J., its 60, I think its 60
	32) J: 60
J. watches E. write	33) E: Yes because we add these two, so wouldn't it be 180 equals x plus 2x which is 180 equals 3x which equals, divide by three, divide by three, 60 equals, is it?
	34) J: Yes
E. looks in the back of the book	35) E: can I just check I want to see the answer
	36) J: and if it is then I'll, what is it?
	37) E: Wait what question are we on?
	38) J: 4
	39) E: 4a, 30
	40) J: Okay so, so let me, okay and so equals x plus 2x,
	41) E: Oh I know
	42) J.and so then it would be 180…equals 3x
	43) E: And so then plus 3x plus 3x, 180 equals 6x
J works faster than J can keep up with	44) J: No that doesn't make sense
J starts writing plus 3x on both sides	45): J: Ya so plus 3x
	46) E: 30 equals x
	47) J: Yes
J sees where the 6x comes from	48) J: Plus x, cross that out, plus, okay I got it
	49) E: Do you get it
	50) J: Yes
	51) E: Are you sure
	52) J: Yup

	53) E: Okay

This conversation is the second question that the girls worked on together, which asks the students to use the exterior angle theorem to solve for x. On the first question Justine successfully led Erica through the solution using the exterior angle theorem. The second question is a little more challenging but Justine is already in the process of developing her understanding of the exterior theorem and how it relates to a similar theorem, which states that the sum of angles in a triangle is 180 degrees. Erica has an objective understanding of the latter theorem but tells Justine that she doesn't get the exterior angle theorem (line 11). When Erica begins trying her answer, Justine confuses herself thinking that Erica is using the sum of the angle's procedure, when in fact she is trying to use the exterior angle theorem.

The miscommunication never did totally resolve itself. Meta-level activities seemed to be working behind the scenes as they often do. Erica seems highly motivated to get the correct answer and to appear as an equal participant in the discourse. Similar to Kara, Erica does not want to look less able than Justine and does not stop to ask Justine why she keeps telling her to add 3x to her equation. She seems to be in a rush to get an answer and quickly takes over from Justine after Justine correctly sets up the equation.

Unfortunately the understanding does not come from solving the equation, but from setting up the equation. Instead of taking the time to converse about how the equation was set up, she wants to show how able she is by racing ahead of Justine to get the answer first. At the end of the question Erica turns to Justine to ask if she understands (line 49), but in fact it is Erica who did not develop an objective understanding of the equation. This again allows Erica to hide her difficulties and appear as an equally able participant.

The combination of Erica's difficulty seeing how the sum of the angles of a triangle is related to the exterior angle theorem makes it difficult for Justine to help Erica because Justine was never clear which theorem Erica was trying to use. Erica's pride and desire to appear as an equal participant added to the challenge and made it very difficult for successful communication and thus learning to occur.

Nicholas and Jim A path, 2m wide, surrounds a garden, 30m long and 16 m wide. Find the area of the path.

Figure 5.17: Task for Nicholas and Jim

Anna Sfard's tripartite focus stresses the need for students to match their pronounced focus with their attended focus while communicating. Using diagrams is often equally important to symbols for grounding verbal mathematical communication. The encounter between Nicholas and Jim also shows two intelligent and motivated students who fail to get a correct answer. Their encounter shares similarities to Mick and Evan's encounter where they often try to do calculations in their head without writing them down as they go, and they do not spend the necessary time labelling their diagram carefully. The boys work on a task where they are meant to find the area of a path surrounding a garden (figure 4).

Table 5.14: Transcription for Nicholas and Jim

What is Done	What is Said
J. draw a picture	1) J: A path with a two meters wide surrounds a rectangle garden, so lets draw the rectangular garden
	2) N: Okay so draw a rectangle 30m long and 16m wide
	3) J: 30
J. continues to write	4) N: 30 and 16 meters
	5) J: And there's a path around it?
	6) N: uh hmm
	7) N: 2 meters wide, so technically, so 16
	8) J: like that 2m, that's 2 m
N. erases J.'s boarder and makes it smaller	9) N: I want you to erase this, no but technically it wouldn't be like that it would be like this see its right around so it would be like this.
	10) J: okay okay
	11) N: And you extend the pass and then this span right here equals 2m, so what we do
	12) J: That's 2 meters?
	13) N: um hmm, so technically
J points to the path	14) J: So this is 2 m
N. points to the diagram as he talks	15) N: Ya cause your adding 2m to each thing, so technically if we want to find the whole thing including the path we do this
J. points to the diagram	16) J: Hey boy I don't think that's two meters, this is two meters

	17) N: Ya that is two meters not this
J. cont. to point	18) J: Not this
	19) N: No no no that is 30 meters still, no this is 32, its 32, cause your adding two meters here, so it would be 32
	20) J What's the question asking
	21) N: They want to find the area of the path
	22) J: Okay so
N. starts to write	23) N: So 2 times 18
	24) J: No its not 32..sorry
	25) N: It is 32 because when the path
J. pushes N. and takes the pen	26) J: No, give me the pen
	27) N: No I want you to listen to this (giggle)
	28) J: Your doing it wrong (giggle)
	29) N: I'm not doing it wrong, the path is 2m wide so
	30) J: Ya
N points	31) N: Technically your adding 2m to each length
	32) J: Oh your trying to find the area, oh
N repeats what J; says	33) N: Area
	34) N: Exactly so if you find the outer area and then the inner one and subtract them you get the area
J. motions that N. should calm down	35) J: or, or, or, Shhh
J. points as he talks. J does not include all of the area	36) J: Or you can do this, this is 30 and 2, multiply that out, that's 60 the area is 60, plus 60, plus 16, 16 times 2, and 16 times 2 so its 4 times 4, wait no
J. writes the number down	37) N: Okay to dude you make....that makes no logical sense.
	38) J: Okay wait, wait, wait well you do 16 times 2 which equals 12 and 3, 32 and 32 equals 64

	39) N: Here move ahead
J: continues to write down the numbers	40) J: And then you add up and you get 4, and your get 6 times 3 is 20 or no is 1, 184 is the area of the path
	41) N: Now let do it my way
Writes what he says	42) J: Area of path
	43) N: Now lets do it my way
	44) J: Fine fine
	45) N: My way, 18 times 32
	46) J: Where are you getting your 8, oh right, I see
N. works the calculation out on his calculator	47) N: Of course you see it it right in front of you
J. drops the pen, N. works on his calculator	48) N: 576 centimetres squared, go pick it up, equals this area so then we want to do 30 times 16, 30 times 16 equals
	49) J: Are you sure you know what your doing bud?
	50) N: I do know what I'm doing (mumbles as he calculates), equals 480
	51) J: Okay so lets see whose right
J. is not watching Nicholas work, N. writes down his answer under his calculations	52) N: 576 minus 480 equals 96 and 96 equals the area of the
	53) J: Lets see whose right, let's see whose right
	54) N: Okay
	55) J: Check it in, I, I want to see it
	56) N: Check it, check it
	57) T: You guys okay
	58) N: Oh Ya
	59) T: Do you have a calculator, yes, okay good carry

	on
	60) N: Teachers these days (giggle)
	61) N: # okay to its 12.3 b.
J; plays with the microphone while N. looks in the answer key	62) J: (mumbles) what's the answer?
	63)N: 200 centimetres squared
	64) J: So we're both wrong
	65) N: But that makes no sense how is that 200
J still playing with microphone	66) J: (giggles) let's fix this.....Okay we're good
	67) J: Okay forget it lets start from phase one again
	68) N: I'm not stopping till I get this question right (laughs) (mumbles)
J. says in a funny voice, J. pushes N.	69) J: Okay, okay, okay...so...so how do you want to go about this hey
	70) N: Oh...no
	71) J: So we know, okay (mumbles)
	72) N: 32 times 18
	73) J: What does it want can I read the question
N. paces as J. looks at the question	74) N: They want to find, they want to find the area of the path, my way, my way works really well
J: erases the questions	75) J: okay, okay, okay so we have a path, lets draw this out this looks really messy
	76) N: Okay lets do it again
J. begins drawing	77) J: Okay so we have our path, or no this isn't our path this is a rectangle. So then there's our rectangle then we have (mumbles). Okay we're like the biggest retards because we can't get the easiest question right.
N. points to the picture that J. just drew	78) N: Ya but still, this question we did it logically and correctly, we, if you subtract, if this is 2m right here so

	technically if you....ohhhhh, ohhhhh, we're supposed to add 4! Because this is 2 and this is 2 so you have to add 4. Because technically your adding to 2 because 2 plus 2 is 4, so you're adding 4 to the total thing
	79) J: Okay but no we're trying to find the area of the path.
N. points at the picture	80) N: No that's right though, of course but you have to find the area of the total then the area of the box and subtract the two
	81) J: Okay so let's do that, so its 8, or no it's....
	82) N: 18 no it would be 20
	83) J: No its 20, 20 and uhhhhh 34
	84) N: yup
	85) J: So 34
	86) N: I'll get the calculator
J. does calc in head, N. uses the calculator	87) J: You could just do 2, its uhhhhh 68 uhhh 6(mumbles) 680
	88) N: 680
	89) N:So 680 subtract 30 times 18
J. writes on board, then tries to calc in his head again	90) J: 30 times 18 (mumbles) that's really easy
J. writes this down	91) N: equals 540
J does subtraction on board	92) J: 540 and then that equals zero, 140
	93) N: 140 centimetres
	94) J: squared
	95) N: So this should give us
	96) J: This should give us the right answer

Boys check answer with the back and realize that it is not correct. They decide to move on to the next question instead of finding their mistake.

The boys are working on an area problem that is difficult but both boys have ideas on how they could solve it. The interactivity flow chart again showed no abnormal signs as each boy did contribute to the interaction, although they were silly at times. Both boys are insistent that their own method is right but neither can get the correct answer at first. Nicholas eventually realizes the error of his ways and explains his reasoning to Jim. Jim agrees and the boys work well together solving the problem. There were several important factors that led to successful communication. The first was that Jim was highly motivated to learn and wanted to do the work himself. At first he didn't give up his own thinking to blindly follow Jim and wanted to figure out the problem using his own method.

His method would have worked if done properly, but eventually he agreed with Nicholas that his way would be more efficient. By having the pen and talking with Nicholas through the question he had a chance to develop the discourse that Nicholas initiated. Unfortunately the boys made a careless error that prevented them from getting the question correct. Although the understanding was there, the boys could have found their mistake if they tried a little harder. Although they drew their diagram again to make it neater, it still did not show both the inner and outer dimensions. This, coupled with the fact that the boys were doing calculations in their head without writing the numbers down, prevented them from realizing the correct inner dimensions that were already given to them. At the end the boys quit and went on to the next question without checking over their work to look for careless mistakes. The lack of a neatly labelled diagram did not give them the attended focus that they needed to match with their pronounced focus. It is clear what they were intending to say, but the careless mistake prevented them from matching their pronounced and intended focus.

CHAPTER 6: FURTHER ANALYSIS ACROSS THE ENCOUNTERS

When unpacking the complex nature of group interactions using the transcriptions, the interactivity flowchart, and the *analysis checklist*, many themes appeared. Most of these themes directly relate to the conditions for successful group work as outlined by Sfard and Webb. Because different personalities of children can have such a large affect on the ultimate success of group work, it is unlikely that this small sample of group encounters displayed all of the possible themes. It did highlight some very important themes that all teachers should be aware of when prepping students to work cooperatively.

Grouping students has always been a challenge with group work. In this study students were free to choose their own partners and often chose people they were friends with. As such most partners had very different mathematical abilities. The results show differences in how students react when one partner knows more than the other. This difference is one of the most important factors that contribute to the successful nature of the communication between the students, and thus their learning.

The encounter between Christine and Mary showed that having the less able partner work with the pen while the help giver provides verbal scaffolding can lead to successful communication. Christine had a clear understanding of how this problem should be solved. The critical step that made this encounter successful was that as soon as Christine realized that Mary's dialogue did not fit with her own, she let her write on the board and worked with her to get the answer. The flow chart for the interaction may look like Christine was having a conversation with herself because Mary did not say much. In fact, Mary's dialogue is not verbal but symbolic as can be seen from her calculations done on the video.

Mary did get a chance to use the help provided by Christine and did not just listen to Christine solving the problem for her. The communicational conflict that arose showed that Mary did need Christine to take over the lead role and provide a model discourse for her to follow. The fact that Mary had the pen and was doing the calculations as Christine talked her through them gave her the chance to practice the discourse. Being a quieter girl prevented her verbal conversation from showing in the video, but when watching her do the calculations on the board it is possible to envision her having the conversation in her head.

Mutual understanding was emphasized often in the class. Students were constantly being reminded to make sure that their partner understood the answer before they moved on. Some of the students showed qualities of good teaching. After years of being taught by teachers it is inevitable that students have observed some of the qualities that make teachers effective. Scaffolding students to figure out questions on their own, checking for understanding, and being patient are all qualities of effective teachers. Not all humans possess these talents naturally and this is especially true of children who have had few opportunities to play the role of teacher in the past. However, when student help givers do understand the problem themselves and do care about helping their partner understand they can be very effective teachers.

The encounter between Hillary and Sarah was another good example of effective group learning. Similar to Christine and Mary's interaction almost all of the statements are object level activities. It took Hillary a few minutes to understand what the correct steps would be. It is doubtful that her partner Sarah had yet developed the objective understanding needed to solve this problem on her own. When Hillary figured out the question she stopped to let Sarah solve it. Even though Sarah was hesitant at first she encouraged her to do it anyway. This forced Sarah to practice the discourse while being scaffolded by a more able partner. Although still a discourse for others, Sarah began to learn it, and started the process of changing it into a discourse for herself. Having the help giver letting the help receiver do the writing can be the best way to push them into the discourse for others.

Not all students who participated in the study showed the ability to be good teachers, or perhaps they just cared more about their own understanding than their partners. The encounter between John and Jerry contains an episode that epitomizes the possible down sides of having students working in pairs. Both boys are quite motivated students but seem to be quite competitive and perhaps more motivated by looking smart and getting the answer first. At the beginning the students struggle to understand what the question is asking. As there is no diagram the boys must convert the written words into their own diagram. It takes them a few minutes to understand what the question is asking.

The interactivity flow chart shows that the boys are quite silly at times and often the remarks are derogatory to each other. Although they are not serious comments they do hint to the competition between the boys. Once John finally realizes what he needs to do he immediately takes over and works out the problem. Jerry watches John but John does not allow Jerry a chance to practice himself. He seems much more interested in getting the answer himself and shows no interest in checking Jerry to see if he understands. The competition that exists between the boys to one up the other does not lead to successful communication as the actual mathematical conversation takes a second seat to meta-level intentions. Perhaps this behaviour is more typical of boys; it demonstrates that the desire to show of your ability must take a second seat to making sure your partner can do it themselves. It is important that students appreciate the benefits for both the help giver and the help receiver in mathematical communication.

The preceding two encounters show the opposite ends of the spectrum for group learning. In any given class a teacher should expect to see example of both types of interactions. The encounter between John and Jerry did show some areas where the boys worked cooperatively, but those times were usually when both students knew what they were doing and were not learning from each other. During any group work situation the most revealing aspects of the encounter are displayed when one student understands the question, and one student does not. It is at these crucial moments when the data reveal whether or not the help giver and help receiver will work together to align their discourses and over come a communicational conflict. The next encounter shows how even though the help receiver may not want to take over, a good teacher will stop and give them the opportunity.

Alex and Siobhan are both bright students who are successful at arriving at the solution to a challenging problem. The interactivity flowchart quantifies what makes this a successful group work encounter. The arrows show that there is a mix of interpersonal and intrapersonal arrows. This reflects that both students are thinking by communicating with themselves and with their partner. The mix of both private and public communication is an important part of learning. The arrows are all objective and most proactive arrows are followed by a reactive arrow showing that the girls are listening to each other and responding to each other's questions. Perhaps the most illuminating feature of the interaction is when dialogue results in both a proactive and a reactive arrow. In this type of dialogue the interlocutor comments on what their partner said, while at the same time making a remark that invites a response back from their partner. It is these arrows that result from the richest conversation and several can be seen in this encounter. The girls bounce ideas off each other but are not worried that they might be wrong. Compared to other students who tried this same problem, they seem to have the creativity and confidence to keep going and keep working at it even though they struggle at first. Other students had good ideas but did not stick with the idea long enough to see if it would work.

The encounter between Kristy and Alison showed several examples of critical factors that can make or break successful group learning. The question that the girls worked on was one where they both could separate to work on their own part of the calculation and then come together to work out the final solution. Kristy was hesitant to interrupt her interpersonal conversation to go and help Alison when asked. Alison was very good at asking for help when she knew she wasn't sure how to continue. She did not show any embarrassment in needing help as some other students showed. Alison asked for help and patiently watched Kristy show her how to convert fractions. Afterwards Alison was able to do the rest of her calculation properly herself and was thankful for the help she received.

The encounter also showed that although one student may appear to be the stronger and more confident mathematician at first, you never know when having another person can fill in something that they missed. This was shown when Kristy was trying to multiply by pi (22/7) but was reciprocating the fraction. Alison saw her mistake and stopped her but Kristy insisted that she was correct. Alison did not let it go and thankfully there was another teacher nearby that confirmed what Alison was saying. At this point Kristy finally decided to agree with Alison and the girls were able to work together to get the final answer. The encounter is a good example that students do not always have to work together for all calculations, but they must be willing to be interrupted to assist their partner. Also it shows that students must listen to the ideas of other students and appreciate that every student has the ability to contribute. Giving up on one's intrapersonal conversation is not always easy for students but they must appreciate that helping others understand is an essential part of the group work process.

One quality that made Alison a successful learner was her ability to accept Kristy as a knowledgeable helper. She was not embarrassed to ask Kristy for help when she was confused. She did not try to hide the fact that she couldn't do the question herself. In the encounter between Anna and Kara, pride seemed to play a much larger role. Kara did not want to let Anna see that she was a less able partner. This meta level concern was a hindrance to the communication between the girls. Although Anna struggled to find the correct vocabulary, Kara did not seem to be interested in engaging in the conversation. By not opening herself up to watching and learning from Anna it took her much longer to correctly use the inverse sin to get the unknown angles. She was also reluctant to learn how the third angle of a right triangle could be found by subtracting the known angle from 90 degrees.

Like Anna and Kara, Justine and Erica were also highly motivated to learn the questions and get them right. But the two encounters show that motivation alone does not ensure that students working in pairs will be successful learners. The encounter shows a clear example of a communication conflict that did not get cleared up between the girls. Because of this both girls were playing by different meta-discursive rules. Justine thought that Erica was using the sum of angles in a triangle is 360^0, when in fact she was trying to use the exterior angle theorem, but just didn't understand it properly. Erica was trying to hide her lack of objective understanding to not seem less able than her partner does. Like Kara it appeared as though Erica's pride was interfering with her ability to learn from her partner.

Hillary and Mary both showed great patience when helping their partners. Patience is definitely a quality of a good teacher. If students are going to be good teachers to their peers then they must also appreciate the need for patience. Patience is a quality that not all student possess, especially some boys. In the encounter between Khris and Quinton, Quinton was motivated to help Khris who had missed a number of classes on trigonometry. After showing Khris the first few questions, Quinton did let Khris take over the question for a time but couldn't resist jumping back in to take over. Instead of talking him through the question as Christine did for Mary, he simply began answering the question for himself. Although Quinton had the best intentions, Khris never got a chance to practice himself, a critical requirement of successful group learning.

In many of the encounters there were times when the students couldn't work out the answer for themselves and had to come for extra help. Sfard clearly stresses the need for a lead discourse. When neither of the partners is capable of taking the lead discourse then they must seek help of the teacher or someone else who can help them. Having student's work on questions, which they are getting wrong, is not useful. When students finish a question they must have a way of verifying their answer and have the ability to get help when they just can't figure it out. This is evident in the encounter between Mick and Evan. The boys do several questions but are not getting any of them right. The teacher does pop in a number of times but they tell him that they are doing fine. The boys are over confident and don't seem to think it is worrying that they can't get the answers right. After missing the easier questions they attempt a difficult one and just haven't built up the skills to solve it.

Mathematical communication involves both oral language and symbolic representation. Writing down the calculations aids in the ability of each partner to focus on what is being done. Evan and Mick try to do the calculations in their head or just on the calculator. Their work consists mainly of numbers written on the board. This did not allow the boys to really understand each other's thinking, and they could not find their mistakes. The encounter clearly is an example of unsuccessful group work for several reasons, but not linking one's pronounced focus to a clear symbolic representation makes successful communication very difficult. Neither of the boys could take the lead with a model discourse and consequently neither of the boys learned much from the interaction.

The encounter between Jim and Nicholas shares similarities to Mick and Evan's encounter where they often try to do calculations in their head without writing them down as they go, and they do not spend the necessary time labelling their diagram carefully. The boys came very close to finding the solution of the two problems they worked on but small errors prevented them from ultimate success. Although some learning did occur in this encounter not getting the correct answer on either of the questions did not contribute to their confidence. Students working in pairs need to carefully watch each other to pick up on mistakes. They must show their thinking by writing down their wok on the board for the other student to follow. Verbal communication in itself makes mathematical communication difficult without symbols to ground their thinking.

There were many factors that prevented successful learning between Mick and Evan. Not having a model discourse that could show them where their mistakes were was a large contributing factor. This theme was also evident with Jill and Kerry. These two girls did write down their calculations and did work towards correct answers, which they confirmed, with the answer key. Although their answer did match the back of the book, neither girl was able to come up with the proper explanation. This is another example where students working cooperatively need to be able to get help from another expert who can properly steer their discourse.

CHAPTER 7: CONCLUSIONS

When Anna Sfard started her research she felt, as many reform teachers do, felt that emphasizing students' collaboration and mathematical conversation was the best way to learn mathematics. Sfard quickly realized that the benefits of learning by talking could not be taken for granted. She found that interactions between students could turn out to be unhelpful to either of the students. The common idea that students speak a similar language and therefore are better suited to help each other is not necessarily true. Consequently Sfard came up with her criteria for successful communication. The criteria that Sfard uses for successful mathematical communication runs very parallel to the characteristics that Webb found were critical for successful group work.

By combining the two theories I came up with a checklist of necessary components to measure whether student interactions were successful or what components were missing. The study focused on the mathematical communication of 24 boys and girls in grade eight and nine. Their conversations were transcribed and then analyzed to see if they met the requirements for successful learning as described by Sfard and Webb. As would be expected the results varied from group to group. Some students were very good communicators and had a natural ability to communicate and even possessed skills that made them good teachers. Other students were lacking in several areas and this resulted in less successful learning or no learning at all. Most students fell in between the two extremes and showed some qualities of good communicators but had areas that could be improved

Their strengths and weaknesses became noticeable when the transcriptions of their interaction were converted into the activity flow chart and then analyzed using the *analysis checklist*. The two methods of analysis combine to provide insight on the students' objective understanding as well as meta-level issues that strongly affect successful communication, especially between children. The meta-level issues often get overlooked in favour of cognitive deficiencies when looking at students who struggle. The communicational approach to mathematics education looks less at cognitive invariants that prevent students from understanding, such as learning disabilities, and more at social factors that prevent successful learning.

The interaction between private intrapersonal communication with oneself, and interpersonal conversation with another can be complicated. Often at the beginning of questions students are more interested in private thought until they have a better idea of what is being asked and what they must do. At this initial stage, it is perhaps unrealistic to expect students to have a focused conversation with their partner, while at the same time being creative as they try to figure out the problem themselves. This leads many people to believe that mathematics is better learned in isolation. Anna Sfard concludes that despite the difficult circumstances needed for successful mathematical communication between students, it is still a potentially useful strategy. For communication to be successful we must teach students the art of communicating. Although the idea that understanding is more important than just getting the answer is frequently emphasized in class, some students seemed driven more by the answer and less by being able to explain their thinking. But perhaps the most important observations that came from these students were personality traits that worked behind the scenes to hinder students' learning, despite the fact that they seemed highly motivated to learn. As mentioned in the foreword, teaching the characteristics of successful students was not a goal in this research, but it would follow as a logical next step.

Kara and Erica both appeared to be motivated to learn and worked hard at the questions. But they also were concerned about being equally able partners who did not need to ask questions or listen to what they could learn from each other. Mick and Evan seemed to be over confident that they knew how to find volume and did not need to come forward and admit that their answers weren't matching the answer key and that they really needed help. Kerry and Jill were getting answers that matched the back of the book but couldn't always explain the reasoning behind their answer. Jim and Nicholas showed that perceptual mediation is critical for successful communication. By not labelling their diagram they were not able to develop a useful attended focus. This may also have been a factor working against Christine and Mary when trying to find the perimeter of a trapezoid. Quinton and John showed that sometimes the temptation to take over and show the partner solutions is much easier and satisfying than waiting for their partner to figure out how to do it themselves. Kirsten and Morgan showed that the desire to appear funny could be a powerful distracter for teen-age kids. Unsupervised work with no expectations on behaviour can prevent successful communication from having a chance.

On the other hand Hillary showed how effective student communication can be. By letting Sarah take the pen and waiting patiently for her to transfer the verbal contents of the conversation into symbolic form she forced Sarah to practice the discourse. It is the practice of the discourse even when it is still a foreign one that is the first step to her making it her own. Sarah would not have gotten this same chance if she were sitting alone in a quiet classroom. Alex and Siobhan also showed that two students could combine their creative energy into developing a successful discourse about a problem that at first seems insurmountable. Other students came very close to solving this problem or similar problems but did need the advice of a more experienced interlocutor (in this case the teacher) to find the missing link.

This research does support Sfard's hesitation to have students work in pairs just because they "share a similar language." This research gives educators a tool for assessing how effective communication is between students. According to Sfard's learning by participation theory, those students who are strong communicators should be effective learners. Communication skills often correlate with a student's social skills and personality traits. Accordingly a student's personality plays a large role in determining their ability to internalize new mathematical discourses. The *analysis checklist* makes it easy to highlight personality traits that interfere with successful communication and learning.

Highlighting these factors can give teachers a new perspective on understanding why some student's learning is delayed.

Simply telling these students not to act in a certain way would be difficult if not impossible. So can the art of communicating be taught? Although this study does not attempt to answer this question the results of the data do make one think. Perhaps we cannot make all students perfect teachers and learners, but focusing on making students aware of what successful communication requires might end up being equally important to teaching the curricular content that we want them to know. Future research might focus on classroom strategies for developing the qualities that successful communicators possess. Training students to be better communicators might make them more aware of their own personalities, and what characteristics they need to develop to be more successful in general.

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