

AFTER-TAX ASSET ALLOCATION

by

Jerry (Jian Qing) Chen
B.Sc., Simon Fraser University, 2005

and

Genica (Xin) Gao
B.Sc., Southwestern University of Finance & Economics, 2009
B.Mgt Southwestern University of Finance & Economics, 2009

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APPROVAL

Name: Jerry (Jian Qing) Chen
Degree: Master of Financial Risk Management

Name: Genica (Xin) Gao
Degree: Master of Financial Risk Management

Title of Project: After-Tax Asset Allocation

Supervisory Committee:

Dr. Peter Klein
Senior Supervisor

Dr. Jijun Niu
Second Reader

Date Approved: _____

ABSTRACT

This paper discusses after-tax asset allocation for individual investors, investigates mean-variance optimization models, and applies asset location under the after-tax framework. We demonstrate how the traditional allocation approaches fail to take tax properly into consideration. Based on Reichenstein's early after-tax asset allocation researches, we improve the adjustment for risks of portfolio, especially for fixed income, by choosing appropriate tax rate. Also we test Reichenstein's and the adjusted models by changing parameters and inputs to evaluate the new model. We illustrate how taxes and saving vehicles affect mean variance optimization and conclude the individual investors should locate bonds in tax-deferred accounts and stocks in taxable accounts.

Keywords: after-tax; asset allocation; asset location; risk adjustment; saving vehicles; private investment; individual investor; mean variance optimization

DEDICATION

This paper is dedicated to our respective families, who have been always supportive and devoted in our pursuit of further education.

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1. Introduction

In recent decades, wealth managers for both individual and institutional investors have focused on asset allocation, since it is crucial in determining returns. Asset allocation helps investors diversify their investments among asset categories, reduce risk and smoothen overall investment returns.

There are a number of factors (e.g. age, current assets, savings per year, income required, marginal tax rate, risk tolerance, economic outlook, etc.) that need to be considered for constructing an optimal asset allocation, and the process can be complex. Traditionally, time horizon and risk tolerance are two most important factors while tax is almost neglected.

In our study we researched two important after-tax asset allocation approaches, one is the after-tax approach in Reichenstein (2006) and the other is the tax-equivalent approach in Horan (2007). The disagreement between these two models is how to value \$1 in a tax-deferred account or tax-exempt account when calculating asset allocation for individual investors. Between these two models, after careful demonstration we prefer Reichenstein's after-tax approach and derive our model based on it. For a detailed explanation please see Chapter 4.

Nonetheless, the past studies seldom discussed how to adjust portfolio's risk in the after-tax calculation so based on the after-tax approach in Reichenstein (2006), we made the appropriate adjustment of portfolio's returns as well as risks.

In Reichenstein (2006), the author pointed out that we should convert all asset values to after-tax values before calculating asset allocation, because the measurement errors caused by traditional models can be substantial. When the author calculated the after-tax values of stocks and bonds, he adjusted the returns by multiplying them by $(1 - \text{tax rate})$ and the variance by multiplying it by $(1 - \text{tax rate})^2$, where for stocks the tax rate is capital gain tax rate and for bonds the tax rate is normal/ordinary income tax rate. The author then calculated the asset allocation based on these after-tax values.

One of the contributions of our paper is to show that Reichenstein's approach can lead to perverse results. When we followed Reichenstein's after-tax approach to calculate the after-tax asset allocation, we observed the perverse results that taxable investors locate more in bonds after tax adjustment for returns and variances of stocks and bonds with corresponding tax rates. What's more the perverse result is robust to different parameters input, as different return, variance and tax rate etc, so we reconsidered the adjustment for returns and variance of stocks and bonds. In Reichenstein (2006), the author adjusted

stocks by capital gain tax rate and adjusted bonds by normal income tax rate, for both returns and variances respectively.

Our intuition arises from considering how the variance of bonds arises. We consider since bonds are fixed income, the variance of bonds should only be provided by capital gains or losses, and not the income. Based on this idea we readjusted the variance of bonds by multiplying by $(1 - \text{capital gain tax rate})^2$ instead of $(1 - \text{normal income tax rate})^2$, and the perverse results disappeared. So it is concluded that the after-tax approach in Reichenstein (2006) is not appropriate for the adjustment of fixed-income investments.

In this study we illustrate the after-tax approach to calculate asset allocation in Reichenstein (2006) would lead the perverse results that taxable investor would allocate more in bonds. We then adjust the after-tax approach and compare asset allocation results of our adjusted approach with results of Reichenstein's after-tax approach under different parameters and inputs. Finally, we refer to some implications of this adjusted after-tax asset allocation framework.

2. Literature review

For asset allocation of individual investors, several studies have concluded that the traditional approach to calculate an individual's asset allocation is incorrect. because it fails to distinguish between the pretax funds and after-tax funds. In Apelfeld, Fowler, and Gordon (1996), it was concluded that on an after-tax basis portfolios that are constructed using a tax-aware optimizer outperform those that are constructed using traditional tax-unaware mean-variance optimization.

Also in Reichenstein (2006) the author compared traditional approach and after-tax approach when calculating asset allocation for individual investors it pointed out that traditional approach fails to distinguish the pre-tax funds and after-tax funds, which the measure errors can be substantial.

As well In Horan (2007), it demonstrates the importance of converting both taxable and tax-advantaged accounts values to after-tax values when calculating asset allocation, based on the implication that the after-tax present value can be substantially less than its pretax value.

In our thesis we will also illustrate why tax matters for individual investors in Chapter 3. After taking tax into consideration when calculating asset allocation,

previous documented studies also imply that individual investors should place equity securities in taxable account while bonds in tax-deferred account. In Poterba and Samwick (1999), it suggests that with the marginal tax rate increasing, holdings of interest bearing assets decline, such as corporate bonds.

In Reichenstein (2006), Reichenstein (2007), Reichenstein (2008), the author concluded that in general, assets whose returns are taxed at ordinary income tax such as bonds rates should be held in retirement accounts, while stocks, especially passively managed equity securities, should be held in taxable accounts.

Also in Dammon, Spatt, and Zhang (2004), the author investigated optimal intertemporal asset allocation and location decisions for investors making taxable and tax-deferred investments. This paper shows a strong preference for holding taxable bonds in the tax-deferred account and equity in the taxable account, reflecting the higher tax burden on taxable bonds relative to equity.

Shoven and Sialm (2003) derived optimal asset allocations and asset locations for a risk-averse investor saving for retirement. It was concluded that taxable bonds have a preferred location in the tax-deferred account and tax-exempt bonds have a preferred location in the taxable account for investors in sufficiently high tax brackets. Also tax-efficient stock portfolios should be held in the taxable

account and tax-inefficient stock portfolios should be held in the tax-deferred account.

In the Chapter 6 we will also illustrate the asset location for bonds and stocks in taxable accounts and tax-deferred accounts.

3. Tax Matters!

Why tax matters to asset allocation, particularly to private investors? Our hypothetical example below answers this question.

Mr. Daniel Ocean, a local contractor, has \$1.2 million in bonds held in tax-deferred accounts and \$800,000 in stocks held in taxable accounts, respectively. He will be in the 35% tax bracket during retirement.

Table 3.1 Asset allocation comparisons between traditional approach and after-tax approach

Savings Vehicle	Asset	Market Value	After-Tax Value
Tax-Deferred Account	Bonds	\$1,200,000	\$780,000
Taxable Account	Stocks	\$800,000	\$800,000

According to the traditional approach to calculating asset allocation, the weight for bonds is calculated as:

$$\frac{\$1,200,000}{\$1,200,000 + \$800,000} = 60\%$$

and for stocks

$$\frac{\$800,000}{\$1,200,000 + \$800,000} = 40\%$$

So Danny has a 60% bonds-40% stocks asset allocation; however, according to Reichenstein's after-tax approach the value for bonds will be converted to after-tax value:

$$\$1,200,000 * (1 - 35\%) = \$780,000$$

Then the weight for bonds is

$$\frac{\$780,000}{\$780,000 + \$800,000} = 49\%$$

and for stocks

$$\frac{\$800,000}{\$780,000 + \$800,000} = 51\%$$

So Danny has a 49% bonds-51% stocks allocation; therefore, by choosing traditional method, the profession actually mismanages individual investors' asset allocation in following ways:

- 1) Confuse pretax funds and after-tax funds. As the traditional models, developed within an institutional setting where investors are normally not subject to tax, inappropriately state that Mr. Ocean's asset mix is 60% bonds - 40% stocks, while the after-tax or spendable values have a totally different ratio.
- 2) Ignoring taxes in portfolio management may cause latent cost. Taxes represent one of the foremost barriers to long-term wealth accumulation for individual investors. For some asset classes, taxes can take away nearly 50% percent of an investor's pre-tax return. In this case, the tax arm of the

government can share 35% or \$21,000 of Mr. Ocean's interest income every year, if we make the assumption that his bond portfolio has a 5% of annual return.

- 3) Unlike institutional investors, pension funds, are tax-exempt, miscalculation on asset allocations due to failing to incorporate investment tax implication tends to result substantial measurement errors for individual investors, who are often taxable. In our simple case, Mr. Ocean's asset mix was misstated by nearly 10%.

So for asset allocation, taking tax into consideration is necessary and crucial. By researching for different after-tax asset allocation approaches, we mainly focused on Reichenstein's after-tax approach and Horan's tax equivalent approach. In the following chapter we compare these two approaches and conclude that we prefer Reichenstein's approach.

4. Why Reichenstein's After-Tax Asset Allocation Model

Reichenstein's and Horan's approaches both agree that the traditional approach when calculating individuals' asset allocation is wrong because it doesn't take tax into consideration. However, these approaches disagree how to value \$1 in tax exempt account (like Roth IRA) and tax-deferred account (TDA).

Horan's approach calculates the number of after-tax dollars in a taxable account that will provide the same expected after-tax future value as \$1 in a Roth IRA and TDA respectively. It then advocates using these tax equivalent values to calculate the current asset allocation.

Reichenstein's approach advocates converting assets' market values to after-tax values and then calculating asset allocation based on these after-tax values. It is concluded that \$1 in a Roth IRA has an after-tax value of \$1, while \$1 in TDAs has an after-tax value of $(1-t_n)$.

So we can see the difference between these approaches is that after-tax asset allocation should be based on after-tax values or tax equivalent values.

Here we first discuss Horan's tax equivalent approach. Consider three types in savings vehicles: Roth IRA, tax-deferred accounts (TDAs) and taxable accounts. Also assume the pretax market value is \$1. The different after-tax discount rates and the resulting after-tax present value calculations for the three savings vehicles are as following:

Table 4.1 Horan's after-tax discount rates and present values for different savings vehicles

Account type	Ending Wealth	Discount Rate	After-Tax Present Value
Roth IRA	$(1+r)^n$	r	$\frac{(1+r)^n}{(1+r)^n} = 1$
TDA	$(1+r)^n(1-t_n)$	r	$\frac{(1+r)^n(1-t_n)}{(1+r)^n} = (1-t_n)$
Taxable accounts	$(1+r^*)^n(1-T^*) + T^*$	$r_f + (1 - p_{oi}t_{oi} - p_{cg}t_{cg})\beta_i[E(r_m) - r_f]$	$\frac{(1+r^*)(1-T^*) + T^*}{(1+r_f + (1 - p_{oi}t_{oi} - p_{cg}t_{cg})\beta_i[E(r_m) - r_f])^n} < 1$

(r is the pretax return, n is the investment horizon, t_{oi} is the ordinary income tax rate, t_{cg} is the capital gains tax rate p_{oi} is the ordinary income, p_{cg} is the capital gains

r^ represents the effective annual after-tax return, T^* represents the effective capital gains tax rate*

$$r^* = r(1 - p_{oi}t_{oi} - p_{cg}t_{cg}) \text{ and } T^* = t_{cg}(1 - p_{oi} - p_{cg}) / (1 - p_{oi}t_{oi} - p_{cg}t_{cg})$$

Because the annual returns of assets in Roth IRA and tax-deferred account are not taxed, investors receive all returns and bear all risks. The discount rate is $(1+r)^n$ where n is the relevant number of periods for discounting. Then the after-tax present value of \$1 pretax market value in Roth IRA and tax-deferred account are \$1 and $(1-t_n)$ respectively.

In Horan's model, the current value of \$1 in a tax deferred account or Roth IRA depends on the projected rate of return, investment horizon, investment style (i.e., p_{oi}) and management style (i.e., p_{cg}), and tax rates.

However, in Reichenstein (2007), the author argues that the discount rate reflects the risk of an asset held in a taxable account. It is inappropriate to discount the future value in tax-deferred account or tax-exempt account by the risk borne when holding the asset in a taxable account.

For example, according to Horan, \$1 in a tax-deferred account has equivalent value of $(1+r)^n(1-t_n) / [(1+r^*)^n(1-T^*) + T^*]$. The projected future after-tax value of the tax deferred account, $(1+r)^n(1-t_n)$, is discounted at the after-tax rate of return on assets held in taxable accounts. However, the individual bears all risk of assets held in a tax-deferred account. So the appropriate discount rate should be r , and the present value is $(1+r)^n(1-t_n) / (1+r)^n$ or $\$1(1-t_n)$. It means the current after-tax value of \$1 of pretax funds in a TDA is $\$1(1-t_n)$. The investor effectively owns $(1-t_n)$ of the principal.

The same applies to assets in taxable accounting. Consider \$1 current value of after-tax funds held in taxable accounts, based on Horan's model, the present value of the projected future value is

$$\$1 \frac{[(1+r^*)^n(1-T^*)+T^*]}{[(1+r^*)^n(1-T^*)+T^*]} \text{ or } \$1.$$

Table 4.2 Present asset value of the projected future value in different accounts

	Projected future value	Discount rate	Present value
Tax-exempt account	$(1+r)^n$	$(1+r)^n$	\$1
Tax-deferred account	$(1+r)^n(1-t_n)$	$(1+r)^n$	$\$1(1-t)_n$
Taxable account	$(1+r(1-t))^n$	$(1+r(1-t))^n$	\$1

In conclusion, we consider Reichenstein's after-tax approach has two advantages:

1) When calculate an individual's asset allocation, only t_n need to be considered, the tax rate when funds are withdrawn. Unlike Horan's model, we need to estimate rates of return, length of investment horizon, and tax rates. So the after-tax approach is easier to apply.

2) Under the after-tax approach there is no need to discount the asset values, which means no need to distinguish an individual's current asset allocation from his future asset allocation.

5. Logic of After-Tax Asset Allocation

5.1 Revisit Reichenstein's After-Tax Ending Wealth Model

To calculate the asset allocation we need to convert all assets values to after-tax value. We are in agreement with Reichenstein's after-tax ending wealth model. Compared with Stephen M. Horan's approach in "Applying After-tax Asset Allocation" (2007) and other scholars' models, we are also convinced by how Reichenstein deal with after-tax returns of different asset classes across saving vehicles. We, however, question his after-tax adjustment for the fixed income component of the portfolio, and we suggest that it is perhaps more reasonable to apply the capital gains tax rate versus the ordinary income tax rate for bonds. Here we assume our whole bond portfolio will not constantly mark to the market within the investment horizon, but instead it will adopt a buy-and-hold strategy for the length of the single period optimization only, and the maturities for all bonds are longer than one period. We also assume a liquidation of bonds by the end of the single holding period. Therefore, the bond portfolio will only result capital gains or losses at the end, and the only portion of the bonds will be taxed by the ordinary tax rate is coupon payment, which should have no impact on the bond's volatility.

Table 5.1 After-tax ending wealth model for bonds and stocks in tax deferred account and taxable account

Savings Vehicle	Bonds	Stocks
Tax Deferred Account	$(1+r)^n (1-taxN)$	$(1+r)^n (1-taxN)$
Taxable Account	$(1+ r(1-taxN))^n$	$(1+r(1-taxC))^n$

(r is the assumed rate of return for any asset, taxN is the normal or ordinary income tax rate, taxC is the capital gains tax rate, and n is number of the investment periods)

We demonstrate his approach in our model, where the portfolio consists of three types of risky assets: S&P/TSX Composite Index (TSX), DEX Universe Bond Index (DEX), and S&P500 index (SNP). The choice of asset classes should reflect a typical Canadian private investor’s preference. The return and standard deviation of these three asset classes are as below:

Table 5.2 Returns and standard deviation of three asset classes in out model

Asset Class	Return	Standard Deviation
TSX	0.10	0.20
DEX	0.06	0.10
SNP	0.10	0.22

We followed Black-Litterman’s historical simulation method (1992), using a set of historical data to estimate a variance-and-covariance matrix of returns TSX, DEX, and SNP. We did not assume that expect returns will equal their historical averages. The problem with the historical average approach is that historical means provide poor forecasts of future returns. For instance, TSX’s last ten years’ (Jan 2000 to Jan 2010) average return is negative. Instead, we reverse

engineer expected return values to these assets from the weights of a stereotypical investor's portfolio.

We employ Matlab's financial toolbox to realize mean-variance optimization for this study. Optimize on the three risky assets in *frontcon* and then pass those frontier weights into *portalloc*; and compare the results from their tangency portfolios within *portalloc*.

Portalloc computes the optimal risky portfolio on the efficient frontier, based on the risk-free rate, the borrowing rate and the investor's degree of risk aversion. Also it generates the capital allocation line, which provides the optimal allocation of funds between the risky portfolio and the risk-free asset.

Frontcon computes portfolios along the efficient frontier for a given group of assets. The computation is based on constraints representing the maximum and minimum weights for each asset, and the maximum and minimum total weight for specified groups of assets.

To calculate after-tax asset allocation both TSX and SNP will be taxed by a capital gains tax rate while DEX will be taxed a normal tax rate (according to Reichenstein's approach). We then calculate the asset allocation for these three asset classes.

Table 5.3 Asset allocation of three asset classes by Reichenstein’s approach

Asset Class	Pretax	After-Tax
TSX	0.3365	0.3194
DEX	0.4638	0.4881
SNP	0.1996	0.1924
Risky Fraction	0.9714	1.5491

(The normal tax rate is 50% and the capital gains tax rate is 12.5%. When the investment is held for more than “the long-term holding period”, capital gains will be taxed at a discounted rate, while the normal tax rate can be very high in some countries like Canada. The tax rate used to adjust bonds is the normal tax rate here. Risky Fraction means the fraction of the complete portfolio allocated to the risky portfolio, following the same.)

As the table shows, we were surprised by the results found by a simplified demonstration of our model when we explored Reichenstein’s approach a bit further. The perverse results indicate that taxable investors would actually hold MORE bonds than non-taxable investors, which conflicts the conventional investment knowledge. We therefore tested the perverse results by changing inputs of different parameters.

5.1.1 Results by different tax rates

By altering tax rates, the perverse results are caused by the spread of the normal and capital gains tax rates. From our results, when both tax rates are relatively low, a 15% spread at least is necessary to show perverse outcomes. As tax rates increase, the required spread of normal tax rate and capital gain tax rates that cause the perverse results is diminishing.

Table 5.4 Asset allocation of three asset classes with different tax rates

Asset Class	Pretax	After-tax
TSX	0.3365	0.3270
DEX	0.4638	0.4765
SNP	0.1996	0.1965
Risky Fraction	0.9714	1.4907
<i>The normal tax rate is 45% and the capital gains tax rate is 15%. The normal tax rate is used to adjust bonds' standard deviation</i>		
TSX	0.3365	0.3285
DEX	0.4638	0.4741
SNP	0.1996	0.1973
Risky Fraction	0.9714	1.6619
<i>The normal tax rate is 50% and the capital gains tax rate is 25%. The normal tax rate is used to adjust bonds' standard deviation</i>		

5.1.2 Results by different bond returns

If we only permit bond return vary, but still adjust the bond variance/covariances by $(1 - \text{normal tax rate})^2$, then it shows that when the bond return is high (hold others variables constant), it is more probable result perverse outcomes. Also from the results, when the bond return is higher, the smaller spread of the normal tax rate and capital gain tax rate is needed to result such outcomes.

Table 5.5 Asset Allocation of four asset classes with higher bond return

Asset Class	Pre-Tax	After-Tax	Bond Return
TSX	0.3365	0.3341	0.06
DEX	0.4638	0.4658	0.06
SNP	0.1996	0.2001	0.06
Risky Fraction	0.9714	1.3563	0.06
<i>The normal tax rate is 35% and the capital gain tax rate is 15%. The normal tax rate is used to adjust bonds' standard deviation.</i>			
TSX	0.2806	0.2720	0.065
DEX	0.5561	0.5674	0.065
SNP	0.1633	0.1607	0.065
Risky Fraction	1.1261	1.5715	0.065
<i>The normal tax rate is 35% and the capital gains tax rate is 15%. The normal tax rate is used to adjust bonds' standard deviation.</i>			

So it can be concluded that the perverse results that tangency portfolio for taxable investor has more bonds is robust by different inputs, which conflicts with common investment knowledge.

For after-tax allocation, the observed research studies indicated that for taxable investors, equities are more tax-favorable compared to fixed income. We therefore doubt whether Reichenstein's after-tax adjustment for variance and covariances of bonds is accurate since bonds are fixed income, and none of the variance from bonds is provided by the income (or interest) but only from capital gains. We should therefore adjust variance of bonds by $(1 - \text{capital gains tax rate})^2$ instead.

5.2 The Adjusted After-Tax Asset Allocation Model

We believe Reichenstein's model set a good foundation for after-tax asset allocation but his approach appears to overlook the complication of the tax treatment for bonds. Indeed, it is a function of the character of the income, the netting, and the applicable tax rate for each component [of the fixed-income investments] (Horan, 2009). As we follow our personal insight, we should tax the income on bonds at the usual rate but adjust the covariance by $(1 - \text{capital gains tax rate})^2$ - the results almost change immediately.

5.2.1 Results by different tax rates under adjusted approach

If we only change the tax rates:

Table 5.6 Asset allocation after only changing the capital gain tax rate to adjust bonds

Asset Class	Pre-Tax	After-Tax
TSX	0.3365	0.5197
DEX	0.4638	0.1619
SNP	0.1996	0.3183
Risky Fraction	0.9714	0.9754
<i>The normal tax rate is 45% and the capital gains tax rate is 15%. The capital gains tax rate is used to adjust bonds' standard deviation.</i>		
TSX	0.3365	0.5111
DEX	0.4638	0.1762
SNP	0.1996	0.3127
Risky Fraction	0.9714	1.1103
<i>The normal tax rate is 50% and the capital gains tax rate is 25%. The capital gains tax rate is used to adjust bonds' standard deviation.</i>		

The results are affected more by the difference between tax rates than the level of the rates themselves. When the tax rates are higher, the same spread of the two tax rates will result more obvious outcomes.

5.2.2 Results by different bond returns under adjusted approach

If we only permit bond return to vary, all results are in line with conventional understanding. When the bond return level is high, the bond's allocation weight change is more obvious. For the same level of bond return, the larger spread between tax rates can cause a greater weight change.

Table 5.7 Asset allocation comparisons

Asset Class	Pre-Tax	After-Tax	Bond Return
TSX	0.5279	0.5488	0.05
DEX	0.1485	0.0943	0.05
SNP	0.3236	0.3569	0.05
Risky Fraction	0.6617	0.8215	0.05
<i>The normal tax rate is 30% and the capital gains tax rate is 15%. The capital gains tax rate is used to adjust bonds' standard deviation.</i>			
TSX	0.2806	0.3583	0.065
DEX	0.5561	0.4280	0.065
SNP	0.1633	0.2137	0.065
Risky Fraction	1.1261	1.2128	0.065
<i>The normal tax rate is 30% and the capital gains tax rate is 15%. The capital gains tax rate is used to adjust bonds' standard deviation.</i>			

TSX	0.2806	0.3307	0.065
DEX	0.5561	0.4734	0.065
SNP	0.1633	0.1958	0.065
Risky Fraction	1.1261	1.2504	0.065

The normal tax rate is 25% and the capital gains tax rate is 15%. The capital gains tax rate is used to adjust bonds' standard deviation.

6 Asset Location in an After-Tax Framework

6.1 Asset Location by Replicating Reichenstein FSR paper

To further explore the after-tax variance adjustment for fixed-income asset class, we further stretched Reichenstein's "after-tax mean variance optimization" model with asset location considered. Therefore, within our mean-variance optimization process, we constructed the following VCV matrix as below.

$$\begin{aligned}
 \text{VCV} = & \begin{bmatrix}
 \text{SrSD}^2 & 0.1 * \text{SrSD} * \text{BrSD} & \text{SrSD} * \text{StSD} & 0.1 * \text{SrSD} * \text{BtSD} \\
 0.1 * \text{BrSD} * \text{SrSD} & \text{BrSD}^2 & 0.1 * \text{BrSD} * \text{StSD} & \text{BrSD} * \text{BtSD} \\
 \text{StSD} * \text{SrSD} & 0.1 * \text{StSD} * \text{BrSD} & \text{StSD}^2 & 0.1 * \text{StSD} * \text{BtSD} \\
 0.1 * \text{BtSD} * \text{SrSD} & \text{BtSD} * \text{BrSD} & 0.1 * \text{BtSD} * \text{StSD} & \text{BtSD}^2
 \end{bmatrix}
 \end{aligned}$$

When replicating Reichenstein FSR paper, we obtained the same allocation weights; therefore, it allows us to compare methods on a same basis.

In this model there are two asset classes: bonds and stocks - investors can position bonds and stocks in both retirement accounts and taxable accounts. Here we denote stocks in retirement accounts, bonds in retirement accounts; stocks in taxable accounts and bonds in taxable accounts as Sr, Br, St, and Bt, respectively. To be consistent with Reichenstein's model, we assume bonds have a 4% return and 5% standard deviation while stocks have an 8% return and

15% standard deviation. In addition, the cost bases and market values are the same for assets held in taxable accounts.

Table 6.1 After-tax mean variance optimization by Reichenstein's approach

	After-Tax Values	Optimal Weights	Expected Returns	Standard Deviations
Stocks in retirement accounts	\$45,000	4.45%	8.0%	15%
Bonds in retirement accounts	\$405,000	40.55%	4.0%	6%
Stocks in taxable accounts	\$550,000	55.00%	6.8%	12.75%
Bonds in taxable accounts	\$0	0.00%	3.0%	4.5%
	\$1,000,000			

(Maximize Utility= $ER - SD^2 / RT$, where ER is portfolio expected returns, SD is portfolio standard deviation and RT, the investor's risk tolerance, is set at 49.9.

Constraint: $S_r, B_r, S_t, B_t \geq 0$; $S_t + B_t = 0.45$; and $S_t + B_t + S_r + B_r = 1.0$.

The correlation coefficient between stock and bond returns is 0.1.

The values reflect the investor in the 25% normal tax rate and 15% capital gains tax rate.

Optimizations were performed in MatLab.)

6.2 Asset Location Under Adjusted After-Tax Asset Allocation Model

In our enhanced model, we adjust the return on bonds by the normal tax rate but the variance and covariances of bonds by the capital gains tax rate $taxC$, instead of the same normal tax rate $taxN$ applied for the bond return in Reichenstein's approach.

Table 6.2 The adjusted after-tax model for bonds and stocks in retirement account and taxable account

Asset Location	Sr	Br	St	Bt
Return	0.0800	0.0400	$0.08*(1-taxRateC)$	$0.04*(1-taxRateN)$
Standard Deviation	0.15	0.06	$0.15*(1-taxRateC)$	$0.06*(1-taxRateC)$

When the tax rates are low, the results are obscure, i.e., weight changes are insignificant when we adjust from $(1-taxN)$ to $(1-taxC)$ for "BtSD", which denotes the standard deviation of Bond-in-taxable-account.

But as we hypothetically change all tax rates to usual high values: $taxN = 0.75$, $taxC = 0.45$; then the results start to vary significantly.

We first calculate the asset location by Reichenstein's after-tax approach, and we got the below results:

Table 6.3 The asset location under Reichenstein's approach

Sr	Br	St	Bt
0	0.4500	0.4525	0.0975

However, we doubt whether the taxable investor should allocate close to 10% of his/her assets to bond-in-taxable-accounts. So we recalculate the asset location by our improved after-tax approach. The results from the recalculations are:

Table 6.4 The asset location under adjusted after-tax approach

Sr	Br	St	Bt
0	0.4500	0.5453	0.0047

The result are more reasonable.

Also we can change the tax rates for another case of comparison:

Table 6.5 Comparison of asset location under Reichenstein's and Adjusted after-tax model

Asset Class	Optimal Weight	
	Reichenstein's	Adjusted approach
Sr	0	0
Br	0.4500	0.4500
St	0.5145	0.5500
Bt	0.0355	0

The normal income tax rate is 50% and the capital gains tax rate is 12.5%.

As many tests we have run, we probably can say that the impact of applying “(1-taxN)” or “(1-taxC)” is larger when tax rates are higher. Here we assume that the highest tax rate is 50%.

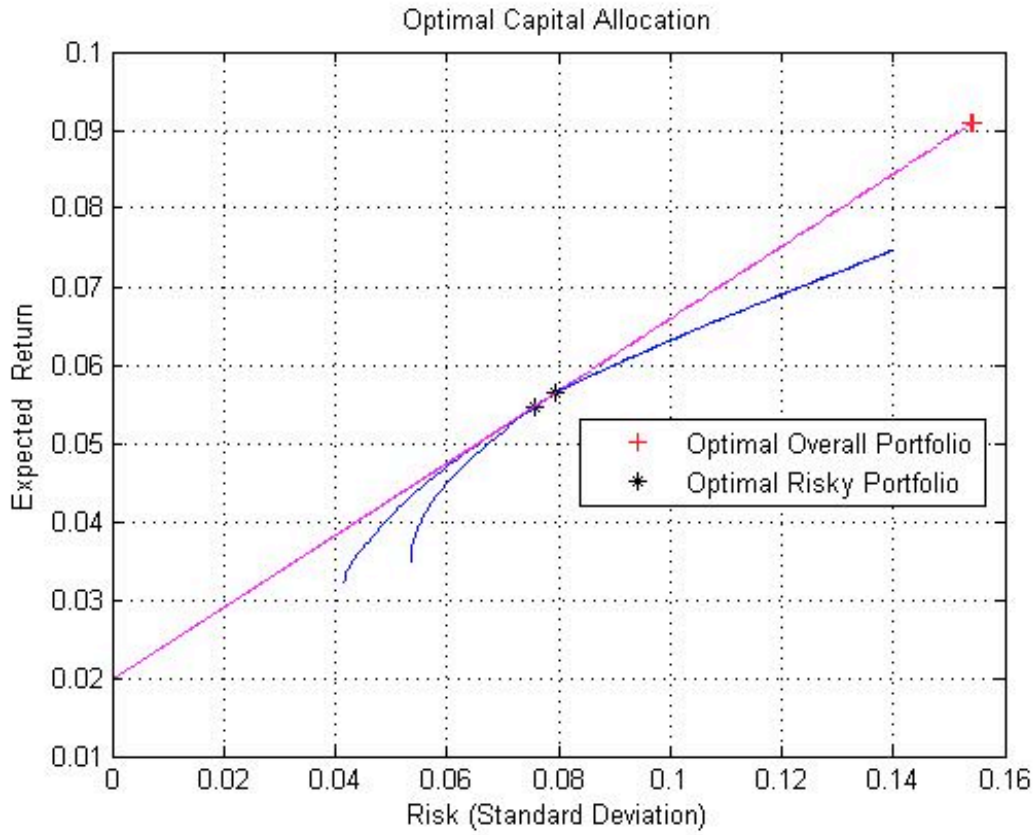


Figure 6.1 The optimal capital allocation under adjusted after-tax model

6.3 After-tax Variance adjustment for floating rate notes

Another alternative asset class for fixed-income bonds will be floating rate notes (FRNs), which are bonds that have a variable coupon, equal to a money market reference rate, like LIBOR or central bank rate, plus a spread. It is an asset class with historically lower volatility and less interest rate risk. Thus, FRNs would have a quite small variance, and the after-tax adjustment on variance and covariances should use $(1 - \text{Ordinary Income Tax Rate})^2$ because their price gain or loss are mainly caused by interest rate change. Therefore, perverse results should appear for asset allocation with FRNs included. Our research based on USD 3 Month LIBOR data since 1990 actually confirms our speculation.

7. Summary

This study further explores earlier works of Reichenstein, Horan, and other researchers, and compares and contrasts two after-tax mean-variance optimizations. Here we conclude that the adjustment for variance and covariances of bond assets by different tax rates has significant implication on asset allocation results. While the after-tax asset allocation becomes more important to the profession, more precise and sophisticated models can be seen as a new challenge for academics.

APPENDICES

Appendix A Standard Deviation of After-Tax Returns in Horan (2007)

Suppose the after-tax return equal $r(1-t_{oi})$ and σ equal the standard deviation of pretax returns; the variance of after-tax returns in a taxable account can then be written as

$$\sigma_{AfterTax, Taxable}^2 = \frac{1}{m} \sum_{j=1}^m [r_j(1-t_{oi}) - \bar{r}(1-t_{oi})]^2$$

where r_j is the return in period j . Factoring our $(1-t_{oi})$ and taking the square root yields the standard deviation of after-tax returns.

$$\sigma_{AfterTax, Taxable}^2 = (1-t_{oi})^2 \frac{1}{m} \sum_{j=1}^m [r_j - \bar{r}]^2 = (1-t_{oi})^2 \sigma^2$$

$$\sigma_{AfterTax, Taxable} = (1-t_{oi})\sigma$$

Source: Horan (2007) "Applying After-Tax Asset Allocation"

Reference:

- Apelfeld, Roberto, Fowler, Gordon B., Jr, & Gordon, James P., Jr. 1996. *Journal of Portfolio Management* Winter (1996): 18-28
- Barber, Brad M. & Odean, Terrance. 2003. Are individual investors tax savvy? Evidence from retail and discount brokerage accounts. *Journal of Public Economics* 88 (2003): 419-442
- Dammon, Robert M., Spatt, Chester S. & Zhang, Harold H. 2004. Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing. *Journal of Finance* June:999-1037
- Horan, Stephen M. 2009. *Private Wealth: Wealth Manegment in Practice*. NJ: John Wiley & Sons 2009.
- Horan, Stephen M.. 2007. Applying After-Tax Asset Allocation. *The Journal of Wealth Management* Fall 2007: 84-93
- Poterba & Samwick, 1999. Taxation and Household Portfolio Composition: U.S Evidence From the 1980s and 1990s. *National Bureau of Economic Research* Ocotober No. H24, G11(1999)
- Reichenstein, William. 2006. After-Tax Asset Allocation. *Financial Analysts Journal*, vol. 62, no.4: 14-19
- Reichenstein, William. 2007. Calculating After-Tax Asset Allocation is Key to Determining Risk, Returns, and Asset Location. *Journal of Financial Planning* 20,7 (July):66-77
- Reichenstein, William. 2008. How to Calculate an After-Tax Asset Allocation. *Journal of Financial Planning* August 2008:62-69
- Reichenstein, William. 2007. Note on "Applying After-Tax Asset Allocation". *The Journal of Wealth Management* Fall 2007: 94-97
- Reichenstein, William. 2007. Implications of Principal, Risk, and Returns Sharing Across Savings Vehicles. *Financial Service Review* Spring 2007:1-17
- Shoven, John B. & Sialm, Clemens 2003. Asset Location in Tax-Deferred and Conventional Savings Accounts. *Journal of Public Economics* 88(2003): 23-38
- Stein, David M., Siegel, Andrew F., Narasimhan, Premkumar, & Appeadu, Charles E. Diversification in the presence of taxes. *Journal of Portfolio Management* Fall (2000): 61-71