Polynomials With Plus or Minus One Coefficients: Growth properties on the unit circle

by

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Abstract

In the 1950's and 1960's Littlewood, Erdös and others made conjectures concerning the modulus of ± 1 polynomials on the unit circle in the complex plane. One such conjecture asks whether there are positive numbers A_1 and A_2 such that, for every degree n, there exists a polynomial $P_n(z)$ having only ± 1 coefficients, with the property that

$$A_1\sqrt{n+1} \leq |P_n(z)| \leq A_2\sqrt{n+1}$$

for $|z| = 1$. Another asks whether ± 1 polynomials exist, for every degree n, satisfying the one-sided inequality

$$A_1\sqrt{n+1} \leq |P_n(z)|$$

Both are unsolved. This thesis describes the background of these problems and presents numerical evidence on polynomials up to degree 64 supporting the conjectures. Numerical data on mean and extreme values of norms L_1, L_3, L_4 and Mahler measure is also presented. Graphical computer programs to view the image and modulus of ± 1 polynomials on the unit circle were developed, and their usage explained.

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Dedication

To my Father

James Sutherland

In Memory

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Chapter 1

Introduction

There are many interesting facts known regarding the modulus of an analytic function, particularly in regards to bounds on the modulus. The principal question addressed by this thesis is concerned with the modulus of a certain subset of analytic functions—polynomials having only ± 1 coefficients, and with a domain consisting of the unit circle in the complex plane.

About forty years ago conjectures concerning the bounds on the modulus of the image of the unit circle under such polynomials were posed by Erdös, and discussed in papers by, amongst others, Clunie, Hayman, Littlewood, Shapiro and, more recently by Newman and Byrnes, Spencer, and Jozsef Beck.

Some of the conjectures raised in these papers remain unanswered. In particular, the question of whether there are positive numbers A_1 and A_2 such that, for every degree n, there exists a polynomial $P_n(z)$ having only ± 1 coefficients, with the property that

$$A_1\sqrt{n+1} \le |P_n(z)| \le A_2\sqrt{n+1}$$

for $|z| = 1$. Furthermore, it is not known whether ± 1 polynomials exist, for every degree n, satisfying the one-sided inequality

$$A_1\sqrt{n+1} \le |P_n(z)|$$

This thesis describes the background of these problems and presents numerical evidence that suggests both conjectures may be true. Numerical data, theorems and conjectures regarding norms L_1, L_3, L_4 and Mahler measure are also presented.

As one might expect there is a close relationship between these problems and various problems in signal processing. One of these connections is with the so-called Barker polynomials (See Saffari [18]). However, this thesis does not elaborate on these applications.

1.1 Notation Used in This Thesis

Throughout this thesis a ± 1 polynomial refers a function of the form

$$P(z) = \sum_{k=0}^{n} a_k z^k$$

where $a_k \in \{-1, 1\}$ and z is a complex variable. Since we are considering the behaviour of the polynomials on the unit circle, sometimes it is more convenient to consider the polynomial as a function of θ

$$P(\theta) = \sum_{k=0}^{n} a_k e^{ik\theta}$$

or

$$P(\theta) = \sum_{k=0}^{n} a_k(\cos k\theta + i \sin k\theta),\ \theta \in [0, 2\pi],\ a_k \in \{-1, 1\}$$

The notation μ_n will represent the quantity $\sqrt{n+1}$, where n generally denotes the degree of a polynomial.

If the modulus or other quantity concerning a ± 1 polynomial of degree n is said to be *normalized*, this means it has been divided by by μ_n.

A polynomial is sometimes represented by a binary number. Each digit in the binary number represents a term of the polynomial, with a 0 denoting $-$ and a 1 denoting $+$. For example the polynomial $P(z) = -z^3 + z^2 + z + 1$ is represented by the binary number 0111.

The notation $E(\cdot)$ indicates an expected value, or mean.

Chapter 2

History of the Problem

2.1 The Modulus of ± 1 Polynomials

2.1.1 Rudin-Shapiro Polynomials

In 1951 Shapiro [20] discovered a recursive sequence of ± 1 polynomials having modulus on the unit circle bounded above by $\sqrt{2}\mu_n = \sqrt{2}(n+1)^{\frac{1}{2}}$, where n is the degree of the polynomial. These polynomials were rediscovered independently by Rudin in 1959, and are often referred to as Rudin-Shapiro Polynomials. Some authors refer to them as Shapiro-Rudin Polynomials, and, more recently, as Golay-Rudin-Shapiro Polynomials, since it has been revealed that they were discovered in a different form by Golay prior to 1951 [4].

Theorem 1 (Rudin-Shapiro) *Let $P_0(z) = Q_0(z) = 1$ and let*

$$P_{m+1}(z) = P_m(z) + z^{2^m} Q_m(z) \tag{2.1}$$

$$Q_{m+1}(z) = P_m(z) - z^{2^m} Q_m(z) \tag{2.2}$$

for each natural number m. Then for $|z| = 1$,

$$\frac{|P_m(z)|}{\mu_n} \leq \sqrt{2}$$

where $\mu_n = \sqrt{2^m}$, $n = 2^m - 1$ being the degree of $P_m(z)$.

Proof: Sum equations (2.1) and (2.2) to obtain

$$P_{m+1}(z) + Q_{m+1}(z) = 2P_m(z)$$

Now take their difference to obtain

$$P_{m+1}(z) - Q_{m+1}(z) = 2z^{2^m} Q_m(z)$$

Hence

$$|P_{m+1}(z) + Q_{m+1}(z)|^2 = |2P_m(z)|^2 = 4|P_m(z)|^2$$

and

$$|P_{m+1}(z) - Q_{m+1}(z)|^2 = |2z^{2^m} Q_m(z)|^2 = 4|Q_m(z)|^2$$

whenever $|z| = 1$. Summing the above expressions yields:

$$|P_{m+1}(z) + Q_{m+1}(z)|^2 + |P_{m+1}(z) - Q_{m+1}(z)|^2 = 4(|P_m(z)|^2 + |Q_m(z)|^2)$$

But

$$\begin{aligned} |P_{m+1}(z) + Q_{m+1}(z)|^2 + |P_{m+1}(z) - Q_{m+1}(z)|^2 = \\ \{\Re P_{m+1}(z) + \Re Q_{m+1}(z)\}^2 + \{\Im P_{m+1}(z) + \Im Q_{m+1}(z)\}^2 + \\ \{\Re P_{m+1}(z) - \Re Q_{m+1}(z)\}^2 + \{\Im P_{m+1}(z) - \Im Q_{m+1}(z)\}^2 = \\ 2\{\Re P_{m+1}(z)^2 + \Im P_{m+1}(z)^2\} + 2\{\Re Q_{m+1}(z)^2 + \Im Q_{m+1}(z)^2\} \\ = 2\{|P_{m+1}(z)|^2 + |Q_{m+1}(z)|^2\} \end{aligned}$$

Thus we have

$$|P_{m+1}(z)|^2 + |Q_{m+1}(z)|^2 = 2(|P_m(z)|^2 + |Q_m(z)|^2)$$

Considering the above expression for $m = 0$ as a basis, and proceeding inductively, we obtain

$$|P_m(z)|^2 + |Q_m(z)|^2 = 2^{m+1} = 2\mu_n^2$$

Hence

$$|P_m(z)|, |Q_m(z)| \leq \sqrt{2}\mu_n$$

completing the proof. □

The constructive nature of this proof allows us to create Rudin-Shapiro polynomials and thus to examine their properties, although the degrees of the polynomials increase exponentially with m, limiting the practical uses of this construction. The first few Rudin-Shapiro polynomials are given below:

$$P_0(z) = 1 \quad Q_0(z) = 1$$

$$P_1(z) = 1 + z \quad Q_1(z) = 1 - z$$

$$P_2(z) = -z^3 + z^2 + z + 1 \quad Q_2(z) = z^3 - z^2 + z + 1$$

$$P_3(z) = z^7 - z^6 + z^5 + z^4 - z^3 + z^2 + z + 1$$

$$Q_3(z) = -z^7 + z^6 - z^5 - z^4 - z^3 + z^2 + z + 1$$

$$P_4(z) = -z^{15} + z^{14} - z^{13} - z^{12} - z^{11} + z^{10} + z^9 + z^8 + z^7 - z^6 + z^5 + z^4 - z^3 + z^2 + z + 1$$

$$Q_4(z) = z^{15} - z^{14} + z^{13} + z^{12} + z^{11} - z^{10} - z^9 - z^8 + z^7 - z^6 + z^5 + z^4 - z^3 + z^2 + z + 1$$

$$\begin{aligned} P_5(z) = {} & z^{31} - z^{30} + z^{29} + z^{28} + z^{27} - z^{26} - z^{25} - z^{24} + z^{23} - z^{22} \\ & + z^{21} + z^{20} - z^{19} + z^{18} + z^{17} + z^{16} - z^{15} + z^{14} - z^{13} - z^{12} - z^{11} \\ & + z^{10} + z^9 + z^8 + z^7 - z^6 + z^5 + z^4 - z^3 + z^2 + z + 1 \end{aligned}$$

$$\begin{aligned} Q_5(z) = {} & -z^{31} + z^{30} - z^{29} - z^{28} - z^{27} + z^{26} + z^{25} + z^{24} - z^{23} + z^{22} \\ & - z^{21} - z^{20} + z^{19} - z^{18} - z^{17} - z^{16} - z^{15} + z^{14} - z^{13} - z^{12} - z^{11} \\ & + z^{10} + z^9 + z^8 + z^7 - z^6 + z^5 + z^4 - z^3 + z^2 + z + 1 \end{aligned}$$

Figures 2.1 to 2.4 represent the modulus of the first five Shapiro-Rudin polynomials. Their normalized modulus around the unit circle is plotted as a function of θ. The plot of modulus verses angle appears to have a characteristic shape for odd-indexed Rudin-Shapiro polynomials, with the modulus achieving its upper bound of $\sqrt{2}\mu_n$ at $\theta = 0$ and $\theta = 2\pi$ and then dipping down in a graceful spike to give $P_n(\pi) = 0$, thus ruling out the possibility of the odd-indexed sequence solving the two-sided inequality conjecture described in the next section.

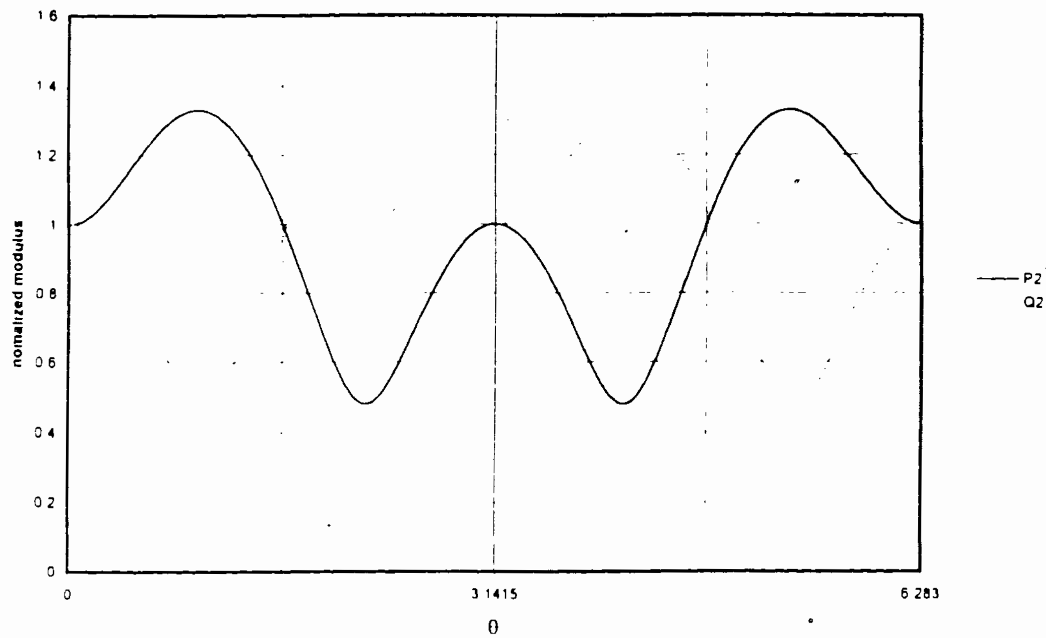

Figure 2.1: Modular plot of Rudin-Shapiro polynomials $P_2(z)$ and $Q_2(z)$

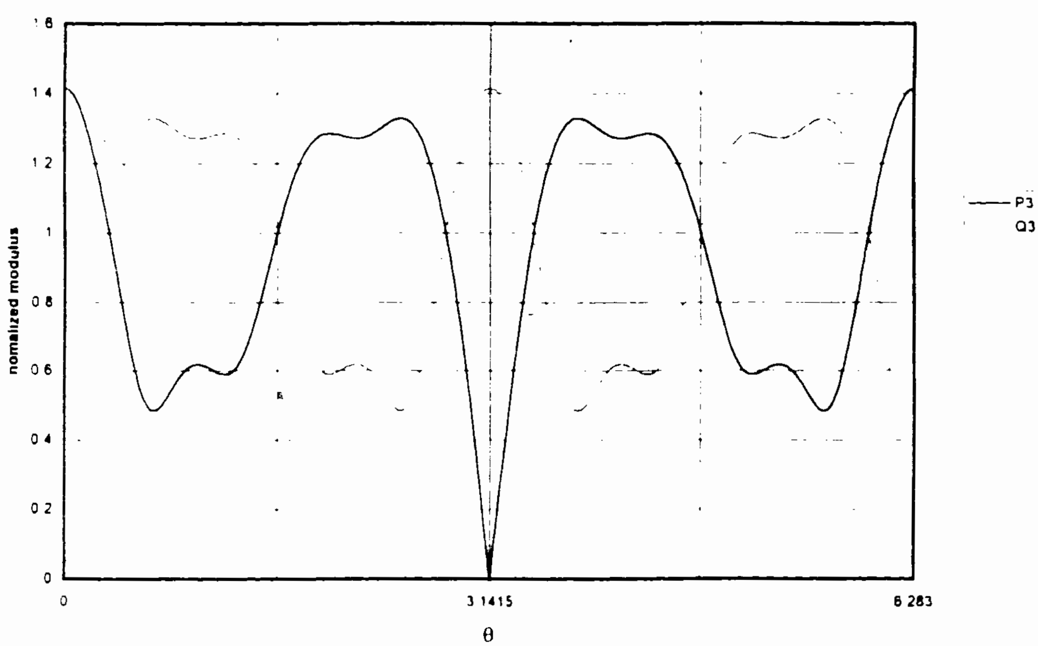

Figure 2.2: Modular plot of Rudin-Shapiro polynomials $P_3(z)$ and $Q_3(z)$

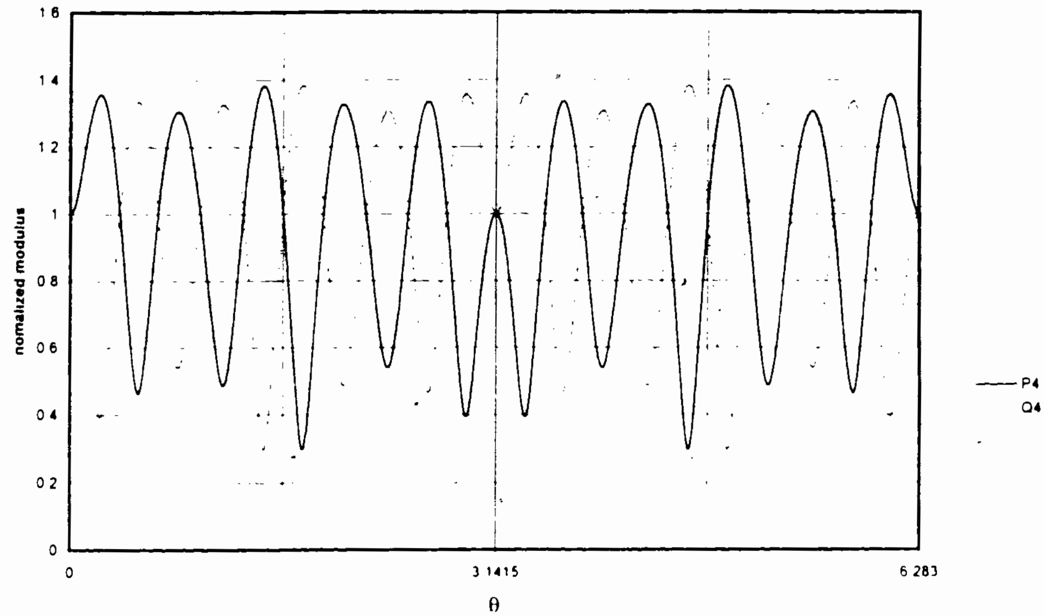

Figure 2.3: Modular plot of Rudin-Shapiro polynomials $P_4(z)$ and $Q_4(z)$

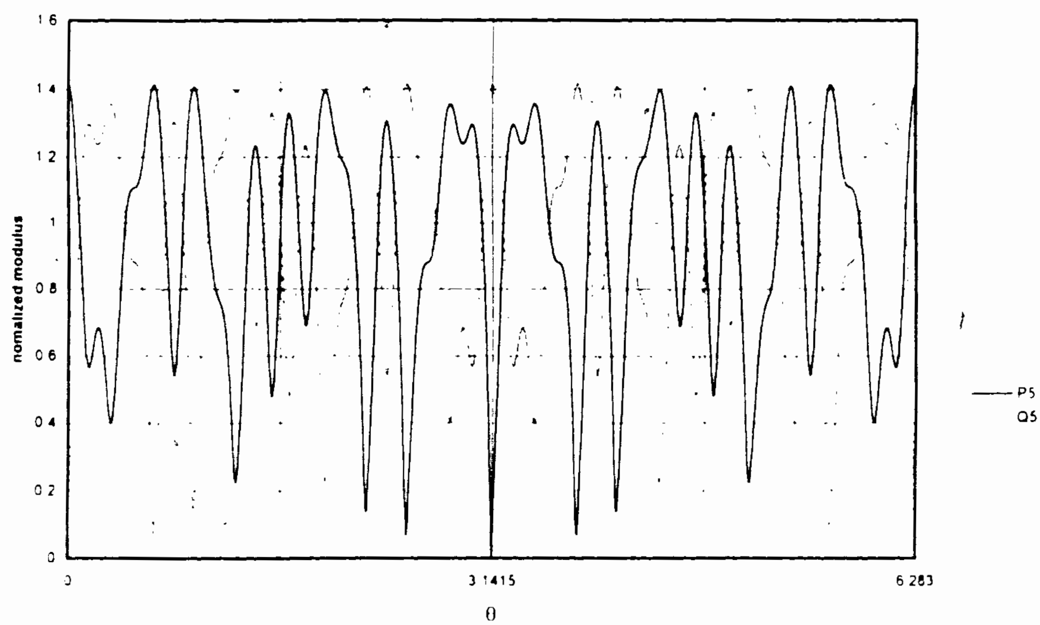

Figure 2.4: Modular plot of Rudin-Shapiro polynomials $P_5(z)$ and $Q_5(z)$

2.1.2 Some Conjectures

Shapiro-Rudin polynomials provide an example of an infinite sequence of ± 1 polynomials whose maximum modulus on the unit circle is bounded above by an absolute constant. After learning about this sequence, it is perhaps natural to ask: Is there a sequence having a minimum modulus that is bounded from below? Since Shapiro's result appeared in 1951 [20] several authors have made conjectures concerning the minimum modulus.

Conjecture 1 (Minimum Modulus) *There exists a positive constant A, independent of n, such that for arbitrarily large degree n there exists*

$$P_n(z) = \sum_{k=0}^{n} a_k z^k,\ a_k \in \{1, -1\}$$

for which

$$A < \frac{\min_{|z|=1} |P_n(z)|}{\mu_n}$$

The above problem was published by Clunie in 1959 [7]. In his paper Clunie tells us that the problem was posed by Erdös and conveyed to him by Hayman. Hayman [10] published the problem again in 1967. It remains unsolved as of 1997.

Littlewood [12] combined the idea of an upper bound and a lower bound into a two-sided inequality.

Conjecture 2 (The Two-Sided Conjecture) *There exist positive constants A_1 and A_2 such that, for arbitrarily large degree n, there is a polynomial*

$$P_n(z) = \sum_{k=0}^{n} a_k z^k, a_k \in \{1, -1\}$$

for which

$$A_1 \leq \frac{|P_n(z)|}{\mu_n} \leq A_2$$

for all $|z| = 1$.

Littlewood's conjecture remain open. Clunie and Littlewood also considered versions of these conjectures for polynomials with complex coefficients of modulus one, that is: $|a_k| = 1$, rather than $a_k \in \{1, -1\}$. These conjectures have been solved. In 1990 Jozsef Beck [1] proved the following theorem, regarding polynomials whose coefficients are 400^{th} roots of unity:

Theorem 2 (Beck) *For sufficiently large n, there exists a polynomial*

$$P(z) = \sum_{k=0}^{n} a_k z^k$$

with $a_k \in \{e^{2\pi i j/400} : j = 0, 1, \ldots, 399\}$ *and positive constants* A_1 *and* A_2, *independent of* n, *such that*

$$A_1 < \left|\frac{P(z)}{\mu_n}\right| < A_2$$

whenever $|z| = 1$.

The proof for this theorem appears in Beck's paper *Flat Polynomials on the Unit Circle Note on a Problem of Littlewood.*

In 1980 Kahane [14] proved the existence of "ultra-flat" polynomials having arbitrary complex coefficients of modulus one. These polynomials have the property that

$$\lim_{n\to\infty} \frac{\max_\theta |P_n(\theta)|}{\min_\theta |P_n(\theta)|} = 1$$

Kahane's result disproved the following conjecture of Erdös [9] from 1962:

Conjecture 3 *Let* $P(z)$ *be a degree* n *polynomial with complex coefficients of modulus 1. There exists a constant* $c > 0$, *independent of* n, *so that*

$$\max_{|z|=1} |P(z)| > (1+c)\mu_n$$

The conjecture of Erdös remains unsolved for the case of ± 1 coefficients.

Theorems of Beck and Kahane are mentioned here for completeness, but this thesis is concerned only with polynomials having ± 1 coefficients. The two-sided conjecture and the minimum modulus conjecture for polynomials having ± 1 coefficients have been open problems now for about 40 years.

2.1.3 Spencer's Theorem

J. Spencer generalized the idea of Rudin-Shapiro polynomials, calling any ± 1 polynomial of degree n with

$$\max_{|z|=1} |P(z)| \leq K\sqrt{n+1}$$

a *Rudin-Shapiro function for* K. In 1985 he provided a new proof for the existence of Rudin-Shapiro polynomials of arbitrary degree n. In addition, Spencer demonstrated that such polynomials are exponential in number for each degree n.

Spencer's proof relies of the following theorem, which is the main result of his paper *Six Standard Deviations Suffice.* [21]

Theorem 3 (Spencer) *Let*

$$\mathbf{v_1}, \ldots, \mathbf{v_n} \in \mathbf{R^n}, \ \textit{where} \ \|\mathbf{v_i}\| \leq 1 \ \textit{for} \ 1 \leq i \leq n$$

Then there exists $\epsilon_1 \ldots \epsilon_n \in \{-1, 1\}$ *such that*

$$\|\epsilon_1 \mathbf{v_1} + \cdots + \epsilon_n \mathbf{v_n}\| \leq K\sqrt{n}$$

where K *is a constant independent of* n.

In the above theorem the notation $\|\mathbf{x}\|$ is defined as $\|\mathbf{x}\| = \max|x_i|$, $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbf{R^n}$

To apply Spencer's theorem to ± 1 polynomials on the unit circle, we consider the terms of the polynomial as vectors in $\mathbf{R^2}$. But for each of the infinitely many θ's on the unit circle $P(\theta)$ represents a different set of vectors. Spencer reduces it to a discrete problem by considering $(4n)^{th}$ roots of unity and the classic Bernstein inequality $|P'(z)| \leq n\|P\|$ where $\|P\| = \max_{|z|=1} |P(z)|$

Spencer also proved the existence of "many" Rudin-Shapiro polynomials in the following theorem.

Theorem 4 (Spencer) *There are at least* $(2-\delta_k+o(1))^{n+1}$ *polynomials with* $\max_{|z|=1}|P(z)| \leq K\sqrt{n+1}$. δ_k *is defined for all* $K \geq K_0$, K_0 *an absolute constant.*

2.2 Mahler Measure and Norms of ±1 Polynomials

2.2.1 Introduction

The root mean square value of the modulus of a polynomial $|P(z)|$ on the unit circle, also known as the L_2 norm, is given by

$$\|P\|_{L_2} = \left(\frac{1}{2\pi}\int_0^{2\pi} |P(\theta)|^2 \, d\theta\right)^{\frac{1}{2}}$$

For ± 1 polynomials of degree n, the value of this quantity is constant:

$$||P||_{L_2} = (n+1)^{\frac{1}{2}}$$

since

$$\begin{aligned}\int_0^{2\pi} |P(\theta)|^2 &= \int_0^{2\pi} \{\Re P(\theta)\}^2 + \int_0^{2\pi} \{\Im P(\theta)\}^2 \\ &= \int_0^{2\pi} \left\{\sum_{k=0}^{n} a_k \cos k\theta\right\}^2 + \int_0^{2\pi} \left\{\sum_{k=0}^{n} a_k \sin k\theta\right\}^2\end{aligned}$$

The terms of the integrands are, up to sign, products of two elements from the orthogonal set $\{1, \cos\theta, \sin\theta, \cos 2\theta, \sin 2\theta, \ldots, \cos n\theta, \sin n\theta\}$ with

$$\int_0^{2\pi} \sin^2 k\theta \, d\theta = \int_0^{2\pi} \cos^2 k\theta \, d\theta = \pi, \quad k > 0$$

$$\int_0^{2\pi} \sin k\theta \sin j\theta \, d\theta = \int_0^{2\pi} \cos k\theta \cos j\theta \, d\theta = 0, \quad k \neq j$$

$$\int_0^{2\pi} \cos^2 0\theta \, d\theta = 2\pi$$

$$\int_0^{2\pi} \sin^2 0\theta \, d\theta = 0$$

So it follows that

$$\left(\frac{1}{2\pi}\int_0^{2\pi} |P(\theta)|^2 \, d\theta\right)^{\frac{1}{2}} = \left(\frac{1}{2\pi}\{(n\,\pi + 2\,\pi) + (n\,\pi)\}\right)^{\frac{1}{2}} = (n+1)^{\frac{1}{2}}$$

More generally, the above method may be used to show that for any polynomial having real coefficients $a_0 \ldots a_n$

$$\left(\frac{1}{2\pi}\int_0^{2\pi} |P(\theta)|^2 \, d\theta\right) = \sum_{k=0}^{n} a_k^2$$

As noted previously, the quantity $(n+1)^{\frac{1}{2}}$ will be denoted μ_n, and division by this quantity is used to normalize other measures and facilitate comparison between various degrees.

The L_p norm defined as

$$||P||_{L_p} = \left(\frac{1}{2\pi}\int_0^{2\pi} |P(\theta)|^p \, d\theta\right)^{\frac{1}{p}}$$

is computed in this thesis for $p = 1$, $p = 3$, and $p = 4$. Sometimes known as the L_0 norm, the quantity

$$\exp\left\{\int_0^{2\pi} \log|P(\theta)| \, d\theta\right\}$$

is also computed. It was studied by Kurt Mahler [15] and is often known as the *Mahler Measure*.

2.2.2 Theorems and Conjecture on the L_4 Norm

Littlewood [12] considered the quantity

$$J_n = \frac{1}{2\pi}\int_0^{2\pi} |P(\theta)|^4 \, d\theta - \mu_n^4$$

Since

$$J_n = \frac{1}{2\pi}\int_0^{2\pi} \left\{|P(\theta)|^2 - \mu_n^2\right\}^2 d\theta$$

He proved the following theorem regarding the L_4 norm of Rudin-Shapiro polynomials [14].

Theorem 5 (Littlewood) *If $n = 2^k - 1$ and $P(z)$ is the Rudin-Shapiro polynomial of degree n, then*

$$||P||_{L_4}^4 = \frac{4(n+1)^2 - (-1)^k(n+1)}{3}$$

Newman and Byrnes [16] are motivated to study the L_4 norm because the L_p norm (for any $p > 2$) is a lower bound for the maximum modulus of a ± 1 polynomial on $|z| = 1$. They rediscovered the theorem of Littlewoood (given above) and also prove:

Theorem 6 (Newman and Byrnes) *In the set of $\pm$ polynomials of degree n, $E(||P||_{L_4}^4) = 2(n+1)^2 - (n+1)$.*

Newman and Byrnes make the following conjecture. They note that, because the L_4 norm is a lower bound for the maximum modulus, that the truth of the conjecture would imply that the Erdös conjecture is also true with $c = 1 - (\frac{6}{5})^{\frac{1}{4}}$.

Conjecture 4 (Newman and Byrnes) *The minimum L_4 norm among all ± 1 polynomials of degree n is asymptotic to $(\frac{6}{5})^{\frac{1}{4}}\mu_n$*

2.2.3 Theorem and Conjecture of Borwein and Lockhart

Borwein and Lockhart prove the following theorem in [3]. It applies to a more general class of random polynomials and, in particular, applies to ± 1 polynomials.

Theorem 7 (Borwein and Lockhart) *Let*

$$q_n(\theta) = \sum_0^n X_k e^{ik\theta}$$

be a polynomial having coefficients X_k that are independent, identically distributed random variables with mean 0, variance equal to 1 and, if $p > 2$ a finite p^{th} moment $E(|X_k|^p)$. Then

$$\frac{E(\|q_n\|_{L_p}^p)}{(n+1)^{p/2}} \to \Gamma(1 + p/2)$$

as $n \to \infty$.

As we shall see, the numerical evidence gathered for this thesis is consistent with this theorem (in the case of ± 1 polynomials for $p = 3$ and $p = 4$) and also supports the following conjecture:

Conjecture 5 (Borwein and Lockhart) *Under the conditions of Theorem 7*

$$\frac{\|q_n\|_{L_p}^p}{(n+1)^{p/2}} \to \Gamma(1 + p/2)$$

almost surely as $n \to \infty$. If so then also

$$\frac{E(\|q_n\|_{L_p})}{(n+1)^{1/2}} \to \Gamma(1 + p/2)^{1/p}$$

as $n \to \infty$ by dominated convergence.

2.2.4 Mahler Measure

In 1962 Kurt Mahler [15] studied the measure of a polynomial which now carries his name.

$$M(P) = \exp\left\{\int_0^{2\pi} \log|P(\theta)|\, d\theta\right\}$$

The Mahler measure of a polynomial is multiplicative $M(PQ) = M(P)M(Q)$ and may be calculated from the roots of the polynomial according to the following theorem, which follows from Jensen's formula (see, for example [2]).

Theorem 8 (Mahler) *If $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0$ has roots $\{\alpha_1, \ldots, \alpha_n\}$ then*

$$\exp\left\{\int_0^{2\pi} \log|P(\theta)|\, d\theta\right\} = |a_n| \Pi_{j=1}^n \max(1, |\alpha_j|)$$

This theorem was used to calculate the Mahler measure numerically while gathering data for this thesis.

Chapter 3

Some General properties of ±1 Polynomials

While there are 2^{n+1} ±1 polynomials of a given degree, there are not 2^{n+1} distinct sets of modulii values. Indeed, ±1 polynomials of a given degree may be partitioned into families that share the same set of modulii values. As in Littlewood [12], two polynomials in the same such family will be called *conjugates*.

3.1 Conjugates

Definition 1 (Conjugate) *If $P(z)$ and $Q(z)$ are said to be* conjugate *provided one of the following holds:*

1. $Q(z) = -P(z)$
2. $Q(z) = \pm P(-z)$
3. $Q(z) = \pm z^n P(\frac{1}{z})$
4. $Q(z) = \pm z^n P(\frac{-1}{z})$

Conjugate polynomials always share the same set of modulii values. That is, whenever $P(z)$ and $Q(z)$ are conjugate, we have:

$$\{|P(z)| : |z| = 1\} = \{|Q(z)| : |z| = 1\}$$

A pair of conjugates may be classified according to whether $|P(z)| = |Q(z)|$ or $|P(z)| = |Q(-z)|$ for z on the unit circle in the complex plane.

- When $P(z)$ and $Q(z)$ are related according to expressions 1. or 3. in the definition, then $|P(z)| = |Q(z)|$, for all $|z| = 1$. In case 3., we have $Q(z) = z^n P(\frac{1}{z})$. Then $|z^n P(\frac{1}{z})| = |z^n||P(\frac{1}{z})| = |P(\overline{z})| = |P(z)|$ whenever $|z| = 1$.

  The plot of modulus verses angle for P and Q are, of course, identical, but the image of the unit circle under P and Q may be different. The image of the unit circle for two such conjugates are shown in figures 3.1 and 3.2.

- When $P(z)$ and $Q(z)$ are related according to expressions 2. or 4. in the definition, then $|P(z)| = |Q(-z)|$, for all $|z| = 1$. In case 2. it follows immediately that $|P(z)| = |Q(-z)|$ In case 4., we have $Q(z) = z^n P(\frac{-1}{z})$. Then $|z^n P(\frac{-1}{z})| = |z^n||P(\frac{-1}{z})| = |P(-\overline{z})| = |P(-z)|$ whenever $|z| = 1$. In cases 2. and 4. the modular plot of $P(\theta)$ on $[0, \pi]$ is a mirror image of the modular plot of $Q(\theta)$ on the same interval. The plots of modulus verses angle for a conjugate pair such as this is shown in figure 3.3.

Although conjugates must always share the same set of modular values, it is not true that all polynomials sharing the same modular values are necessarily conjugate. Consider the following pair:

$$P(z) = -z^{14} + z^{13} - z^{12} + z^{11} - z^{10} - z^9 + z^8 + z^7 - z^6 - z^5 + z^4 + z^3 + z^2 + z + 1$$

$$Q(z) = -z^{14} - z^{13} - z^{12} + z^{11} + z^{10} + z^9 - z^8 + z^7 + z^6 + z^5 - z^4 + z^3 + z^2 - z + 1\,.$$

Numerical evidence suggests that $|P(z)| = |Q(z)|$ for all $|z| = 1$, yet the two are not a conjugate pair.

One may ask, is it possible to have polynomials $P(z)$ and $Q(z)$ with

$$\{|P(z)| : |z| = 1\} = \{|Q(z)| : |z| = 1\}$$

yet neither $|P(z)| = |Q(z)|$ for all $|z| = 1$, nor $|P(z)| = |Q(-z)|$ for all $|z| = 1$? No such example was apparent in the data collected for this thesis.

Rudin-Shaprio polynomials, which were discussed in Chapter 2, are examples of conjugate pairs. This is proved in the following theorem.

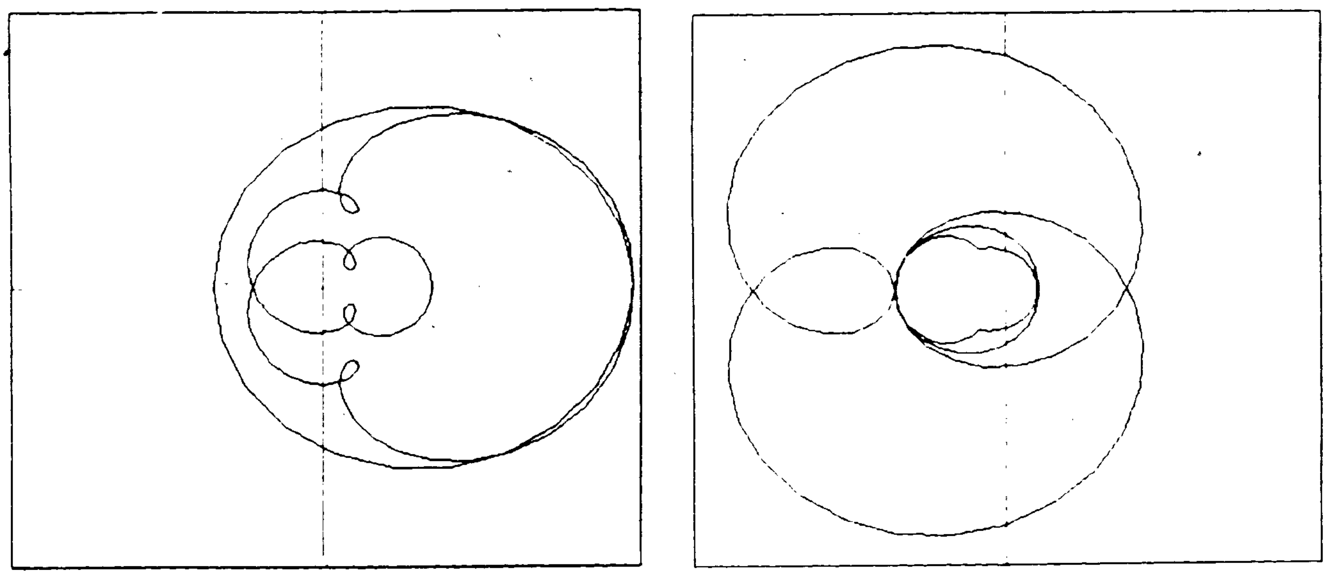

Figure 3.1: The Unit circle transformed under $P(z) = -z^7 - z^6 - z^5 - z^4 - z^3 + z^2 + z + 1$ (left) and $Q(z) = z^7 P(\frac{1}{z})$ (right)

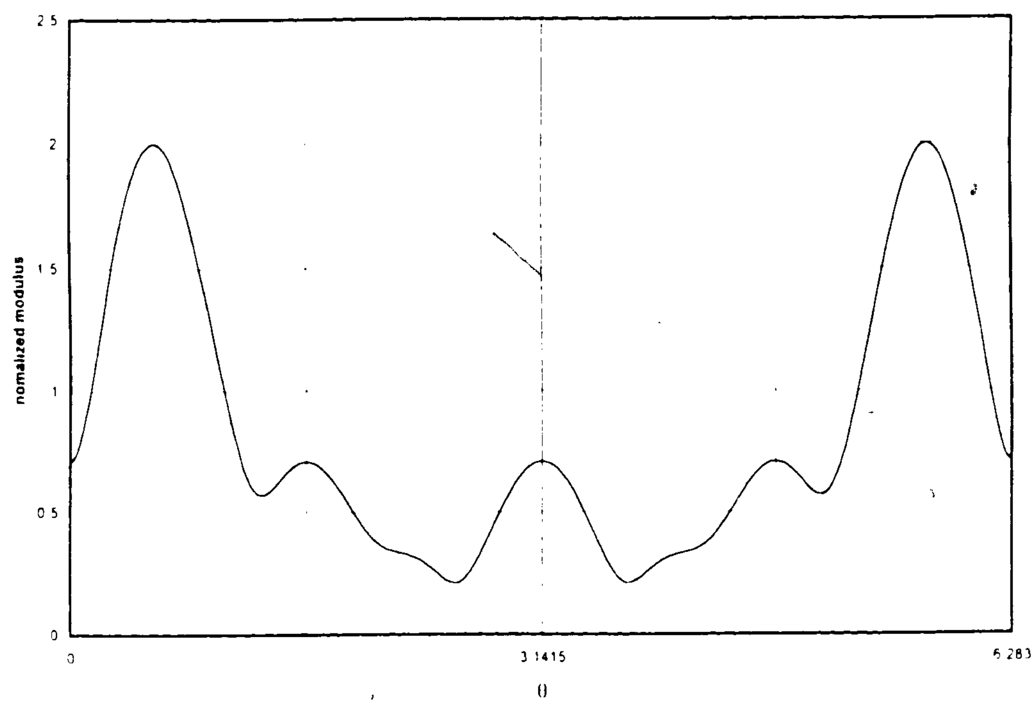

Figure 3.2: Joint modular plot of conjugates shown in Figure 3.1

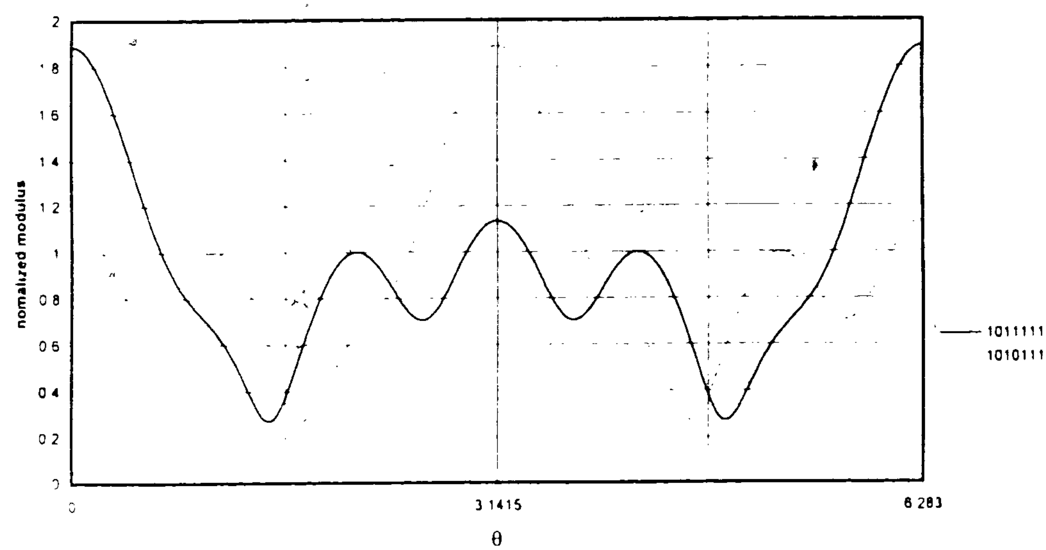

Figure 3.3: Example: Modular Plots of Conjugates $P(z) = z^6 - z^5 + z^4 + z^3 + z^2 + z + 1$ (solid) and $Q(z) = z^6 P(\frac{-1}{z})$ (dotted)

Theorem 9 *Rudin-Shapiro polynomials $P_m(z)$ and $Q_m(z)$ of degree $2^m - 1$ (as defined in Theorem 1) are a conjugate pair with*

$$Q_m(z) = z^{2^m-1} P_m(-\frac{1}{z}), \quad m \ even$$

$$Q_m(z) = -z^{2^m-1} P_m(-\frac{1}{z}), \quad m \ odd$$

Proof (by induction) Note that the result is true for $m = 0$ and $m = 1$. Suppose it is true for $m = k$ and k is odd. Then from the definition the Rudin-Shapiro polynomials

$$z^{2^{k+1}-1} P_{k+1}(-\frac{1}{z}) = z^{2^{k+1}-1} \left\{ P_k(-\frac{1}{z}) + (-\frac{1}{z})^{2^k} Q_k(\frac{-1}{z}) \right\} \tag{3.1}$$

Since k is odd, by the induction hypothesis

$$Q_k(z) = -z^{2^k-1} P_k(-\frac{1}{z}).$$

From this it follows that

$$Q_k(-\frac{1}{z}) = -(-\frac{1}{z})^{2^k-1} P_k(-\frac{1}{\frac{-1}{z}}) = \frac{1}{z^{2^k-1}} P_k(z)$$

and

$$P_k(-\frac{1}{z}) = \frac{1}{-z^{2^k-1}}Q_k(z)$$

Substituting these results into equation 3.1 yields:

$$z^{2^{k+1}-1}P_{k+1}(-\frac{1}{z}) = z^{2^{k+1}-1}\left\{\frac{1}{-z^{2^k-1}}Q_k(z) + (-\frac{1}{z})^{2^k}\frac{1}{z^{2^k-1}}P_k(z)\right\} = P_k(z) - z^{2^k}Q_k(z) = Q_{k+1}(z)$$

Thus $Q_{k+1}(z) = z^{2^{k+1}-1}P_{k+1}(-\frac{1}{z})$ and since similar results follow when k is even the stated result is true for all integers m. □

The conjugate relationship between $P_m(z)$ and $Q_m(z)$ is depicted in figures 2.1 to 2.4, which show, for $m = 2\ldots5$, that the modular plot of $P_m(z)$ on $[0,\pi]$ is a mirror image of the plot of $Q_m(z)$ on the same interval, just as for the more typical conjugate pair plotted in figure 3.3.

3.2 Self-Conjugates: Reciprocal and Symmetric

In most cases a ±1 polynomial $P(z)$ has seven distinct conjugates. However, sometimes not. In a case where $P(z)$ is equal to one of its conjugates, Littlewood calls $P(z)$ a *self-conjugate*.

There are two types of self-conjugates, here called *reciprocal* and *symmetric*. Both types of self-conjugates appear to have some extremal properties among all ±1 polynomials of the same degree.

Definition 2 (Reciprocal) *A ±1 polynomial $P(z)$ is said to be a* reciprocal *provided $P(z) = z^n P(\frac{1}{z})$ or $P(z) = -z^n P(\frac{1}{z})$.*

Reciprocal polynomials, like palindromic words, are the same (up to sign) when they are reversed. They occur in both odd and even degrees. A few examples are:

$$z^5 + z^4 - z^3 - z^2 + z + 1$$

$$z^3 - z^2 + z - 1$$

$$z^4 - z^3 + z^2 - z + 1$$

An odd degree reciprocal polynomial $P(z) = \sum_{k=0}^{n} a_k z^k$ has the property that for each $k \in \{0\ldots n\}$, $a_k = a_{n-k}$ or, for each $k \in \{0\ldots n\}$, $a_k = -a_{n-k}$. An even degree reciprocal polynomial must have $a_k = a_{n-k}$ for each k, since the middle coefficient $a_{\frac{n}{2}}$ remains unchanged. When n is odd, their are $2(2^{\frac{n+1}{2}})$ degree n reciprocal polynomials, and when n is even there are $2(2^{\frac{n}{2}})$ degree n reciprocal polynomials.

Definition 3 (Symmetric) *A ±1 polynomial $P(z)$ is said to be* symmetric *provided $P(z) = z^n P(\frac{-1}{z})$ or $P(z) = -z^n P(\frac{-1}{z})$.*

Symmetric polynomials occur only for even degrees. The name *symmetric* is not a standard definition, but has been used in [6] to describe polynomials of this sort. The name reflects the symmetric nature of the modular plots. While the modular plot of *every* ±1 polynomial is symmetric about $\theta = \pi$ due to the fact that $|P(z)| = |P(\bar{z})|$ (and hence $|P(\theta)| = |P(2\pi - \theta)|$), *symmetric* polynomials have the property that $|P(z)| = |P(-z)|$, hence $|P(\theta)| = |P(\pi - \theta)|$. An example of the modular plot of a symmetric polynomial is shown in figure 3.4.

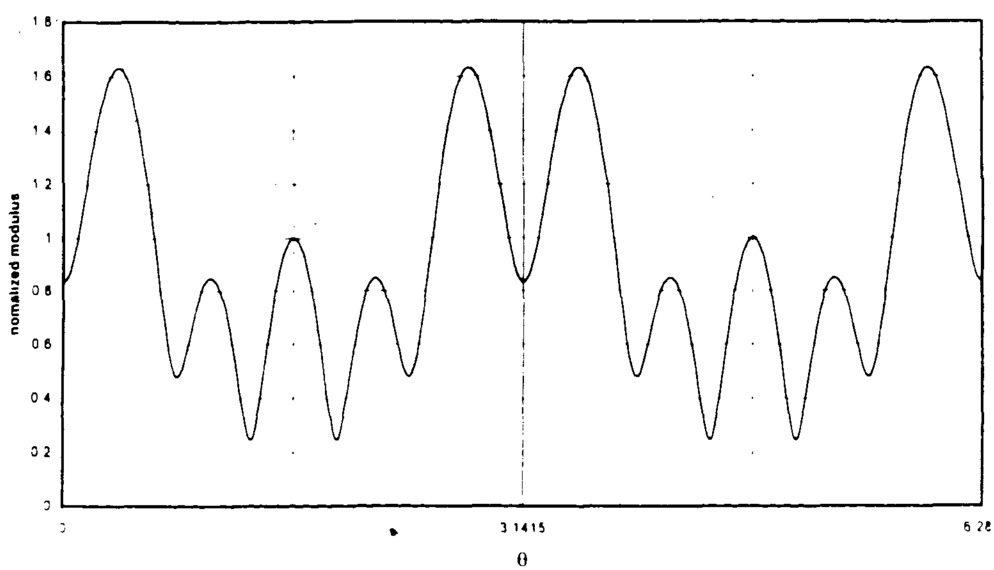

Figure 3.4: Example: Modular Plot of Symmetric self-conjugate $P(z) = -z^{12} - z^{11} + z^{10} - z^9 + z^8 - z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z - 1$. Note symmetry about $\theta = \frac{\pi}{2}$

When the degree is divisible by 4, symmetric polynomials satisfy $P(z) = z^n P(\frac{-1}{z})$. In this case the middle term is raised to an even power. An example of such a symmetric polynomial is

$$P(z) = z^8 + z^7 + z^6 - z^5 + z^4 + z^3 + z^2 - z + 1$$

Symmetric polynomials with degrees divisible by 4 have the property that $a_k = a_{n-k}$ when k is even and $a_k = -a_{n-k}$ when k is odd.

When the degree is even, but not divisible by 4, symmetric polynomials satisfy $P(z) = -z^n P(\frac{-1}{z})$. In this case the middle term is raised to an odd power, and $a_k = -a_{n-k}$ when k is even and $a_k = a_{n-k}$ when k is odd. An example of such a symmetric polynomial is

$$P(z) = z^6 - z^5 + z^4 + z^3 - z^2 - z - 1$$

For all even degrees there are $2(2^{\frac{n}{2}})$ symmetric polynomials. There are no odd degree symmetric polynomials, since if n is odd and $P(z) = \sum_{k=0}^{n} a_k z^k = z^n P(\frac{-1}{z}) = z^n\{\sum_{k=0}^{n} a_k (\frac{-1}{z})^k\}$ then, equating coefficients of the leading term gives $a_n = a_0$, while equating the constant term give $a_0 = -a_n$. However, there *are* odd degree polynomials having the property that $a_k = a_{n-k}$ when k is odd and $a_k = -a_{n-k}$ when k is even (or vice versa). Curiously, the Rudin-Shapiro polynomials appear to have this trait.

The total number of self-conjugates for even degrees is $2^{\frac{n+4}{2}}$ while the total number of self-conjugates for odd degrees is $2^{\frac{n+3}{2}}$. So as the degree increases, the total number of self-conjugates form a smaller and smaller portion of all ±1 polynomials. Yet as we shall see, they appear to have some interesting tendencies towards extremal properties.

Chapter 4

Experimental Methods and Results

For degrees up to 24 the complete sets of ± 1 polynomials were studied. For even degrees to 44 the set of symmetrical polynomials were studied and for degrees up to 42 reciprocal polynomials of the form $P(z) = z^n P(1/z)$ were studied.

Data for degrees higher than 24 was obtained among the self-conjugates since as the degree increases they form a diminishing fraction of total ± 1 polynomials. For example symmetric polynomials are $1/2^{\frac{n}{2}}{}^{th}$ of the entire set. This means, for instance, that degree 44 symmetric polynomials form only about 0.000023845 percent of the total set, which consists of $2^{45} = 35184372088832$ polynomials.

For each polynomial the following quantities were estimated:

1. Minimum modulus on the unit circle, $\min_{|z|=1} \frac{P(z)}{\mu_n}$
2. Maximum modulus on the unit circle, $\max_{|z|=1} \frac{P(z)}{\mu_n}$
3. Normalized Mahler Measure
4. Normalized L_1, L_2, L_3, and L_4 norms.

The expected value of each quantity was calculated for each degree, and a list of 100 polynomials having extremal properties for each quantity was maintained.

In addition more limited calculations involving the minimum modulus of symmetric polynomials up to degree 64 were undertaken.

4.1 How the Data was Calculated

Data for this thesis was gathered using programs written in C++. Graphs were created with spreadsheet software. The minimum and maximum modulus of a polynomial was estimated simply by evaluating the modulus of the polynomial on the unit circle at regular intervals on $[0, \pi]$. For a degree n polynomial, the number of intervals used was $20 \times n$, the same density of intervals used to create the modular plots which illustrate this thesis. Results on certain polynomials were verified using up to $1000 \times n$ intervals, and the initial estimates appear to be accurate to within at least 2 decimal places.

L_1, L_2, L_3, and L_4 norms were calculated by numerical quadrature (Composite Simpson's algorithm [5, page 176] using $20 \times degree$ intervals). Although $L_2/\mu_n = 1$ for all ± 1 polynomials, it was calculated along with the other norms to varify data integrity. The norms appear to be accurate to at least 5 decimal places.

Mahler measure was calculated according to the formula

$$M(P) = \Pi_{i=1}^{n} max\{1, |\alpha_i|\}$$

where α_i, $i = 1 \ldots n$ are the roots of the polynomial. The roots were extracting numerically.

4.2 Means Over Various Sets of ± 1 Polynomials

Mean values over all ± 1 polynomials degree 4 to 24 are summarized in Table 4.1. Data for complete sets was gathered only up to degree 24 because of the large amount of time taken for the programs to run.

Table 4.2 displays mean values for L_1, L_2^2, L_3^3, and L_4^4 over all ± 1 polynomials degrees 6 to 18. This data was collected as an afterthought, which is why processing continued only to degree 18. The data in Table 4.2 is displayed graphically in Figure 4.1 and Figure 4.2, along with least-squares estimation curves. Although the curves for L_2^2 and L_4^4 are already known (Theorem 6), were included in the calculations to verify the integrity of the data. The least-squares curve for L_3^3 is

$$y = 1.3357(n+1)^{\frac{3}{2}} - 0.07(n+1) + 0.0188(n+1)^{\frac{1}{2}} - 0.3255$$

and is the best approximating function of the form $a(n+1)^{\frac{3}{2}} + b(n+1) + c(n+1)^{\frac{1}{2}} + d$. This form was chosen because the value of mean L_3^3 appears to fall between $o(n)$ and $o(n^2)$.

This result is consistent with Theorem 7, which states that $E(L_p^p)/(n+1)^{\frac{p}{2}} \to \Gamma(1+p/2)$, since $\Gamma(1+3/2) \approx 1.33$.

Salem and Zygmund [19] show that for the set of ± 1 polynomials there are constants c and d with

$$d\sqrt{(n+1)\log(n+1)} \leq E(M) \leq c\sqrt{(n+1)\log(n+1)}$$

where M is the maximum modulus. Figure 4.14 shows the expected maximum modulus of symmetric, reciprocal and all ± 1 polynomials after division by $\sqrt{(n+1)\log(n+1)}$. The normalized curves appear to approach three different constant asymptotes, suggesting that the expected maximum modulus of reciprocal and symmetric polynomials also grow according to $c\sqrt{(n+1)log(n+1)}$, but with different constants c.

Mean values over all symmetrical ± 1 polynomials degree 6 to 44 are summarized in Table 4.3. Mean values over reciprocal polynomials of the form $P(z) = z^n P(1/z)$ of degrees 6 to 42 are given in Table 4.4. Data from Tables 4.1, 4.3 and 4.4 are displayed graphically in Figures 4.3 to 4.14

4.3 Extremal Data for Various sets of ± 1 Polynomials

This section contains tables and charts containing extremal data for modulus, Mahler measure and Norms L_1 through L_4. In the tables a polynomial is represented by a binary number as described in section 1.1.

Generally more than one polynomial attains the extremal value, but only one extremal polynomial for each category is given in the tables. The polynomial selected for inclusion is that having the smallest binary number representation among the set of polynomials attaining the extreme value.

In tables that contain extremal polynomials over the entire set of ± 1 polynomials, a * symbol indicates a symmetric polynomial. The * symbol is omitted in tables that contain extremal polynomials over the set of symmetric polynomials.

The caption for a table indicates which category of extreme values the table represents, but often a polynomial is extremal in more than one category. All extremal values in the tables (except the reciprocal tables) are shown in bold letters.

degree	Min mod	Max mod	Mahler	L_1 norm	L_3 norm	L_4 norm
4	0.35393	1.60169	0.78372	0.89805	1.08302	1.14744
5	0.14882	1.69173	0.72178	0.88765	1.08590	1.15356
6	0.24925	1.71462	0.77687	0.89444	1.08759	1.15764
7	0.10593	1.72591	0.73573	0.88852	1.08914	1.16092
8	0.17034	1.76916	0.75996	0.89101	1.09019	1.16360
9	0.09950	1.80244	0.74869	0.88951	1.09110	1.16581
10	0.15240	1.81554	0.76205	0.89046	1.09182	1.16766
11	0.10051	1.84023	0.74340	0.88834	1.09247	1.16921
12	0.13605	1.86152	0.76040	0.88976	1.09297	1.17055
13	0.08218	1.87751	0.75048	0.88872	1.09343	1.17171
14	0.11030	1.89472	0.75541	0.88897	1.09382	1.17273
15	0.08020	1.90949	0.74903	0.88828	1.09418	1.17364
16	0.10600	1.92400	0.75722	0.88880	1.09447	1.17445
17	0.07037	1.93960	0.74923	0.88810	1.09475	1.17518
18	0.09581	1.95162	0.75610	0.88849	1.09499	1.17584
19	0.069	1.964	0.75000	0.88796	1.09522	1.17644
20	0.083	1.976	0.75404	0.88816	1.09542	1.17698
21	0.063	1.987	0.75116	0.88790	1.09560	1.17748
22	0.081	1.997	0.75467	0.88805	1.09577	1.17794
23	0.058	2.007	0.74943	0.88765	1.09592	1.17837
24	0.076	2.017	0.75411	0.88789	1.09606	1.17876
Asymptotic values predicted by Borwein-Lockhart conjecture:					**1.09954**	**1.18921**

Table 4.1: Normalized mean values over all ± 1 polynomials of degrees 8 to 24. The asymptotic values predicted by Borwein and Lockhart in Conjecture 5 are shown for $p = 3$ and $p = 4$

Degree	$E(L_1)$	$E(L_2^2)$	$E(L_3^3)$	$E(L_4^4)$
6	2.36647	7	23.9685	91
7	2.51312	8	29.3963	120
8	2.67303	9	35.1622	153
9	2.81288	10	41.2715	190
10	2.95331	11	47.6933	231
11	3.0773	12	54.4246	276
12	3.20808	13	61.4364	325
13	3.32527	14	68.7324	378
14	3.44296	15	76.2928	435
15	3.55311	16	84.1145	496
16	3.66463	17	92.1828	561
17	3.76787	18	100.497	630
18	3.87283	19	109.045	703

Table 4.2: Expected Values for L_1,L_2^2, L_3^3, and L_4^4 over all ±1 Polynomials degrees 6 to 18

degree	Mahler	Max mod	Min mod	L_1 norm	L_3 norm	L_4 norm
8	0.808313	1.63063	0.295031	0.909184	1.07367	1.13195
10	0.78050	1.63765	0.23094	0.90077	1.07839	1.13983
12	0.78380	1.70459	0.21541	0.90109	1.08066	1.14573
14	0.78546	1.71903	0.23917	0.89970	1.08317	1.15068
16	0.77596	1.75885	0.19737	0.89680	1.08507	1.15444
18	0.77432	1.78021	0.17431	0.89613	1.08655	1.15773
20	0.77290	1.80298	0.16044	0.89552	1.08763	1.16023
22	0.765862	1.83057	0.139826	0.893574	1.08871	1.1625
24	0.76464	1.85437	0.12968	0.89307	1.08953	1.16444
26	0.767591	1.87227	0.1298	0.893279	1.09025	1.16606
28	0.76565	1.89304	0.12216	0.89272	1.09083	1.16746
30	0.762377	1.906	0.110252	0.891923	1.0914	1.16871
32	0.761756	1.92301	0.106202	0.89159	1.09187	1.16982
34	0.76150	1.93486	0.10084	0.89134	1.09230	1.17082
36	0.76052	1.95129	0.09595	0.89106	1.09267	1.17171
38	0.75966	1.96205	0.09122	0.89075	1.09302	1.17253
40	0.75937	1.97669	0.08808	0.89056	1.09332	1.17326
42	0.759474	1.98552	0.0854632	0.890433	1.09361	1.17393
44	0.75912	1.99771	0.08321	0.89027	1.09386	1.17455

Table 4.3: Normalized mean values over Symmetric ±1 polynomials of even degrees 8 to 44

degree	Max mod	Min mod	Mahler	L_1 norm	L_3 norm	L_4 norm
6	1.97803	0.0126426	0.614107	0.82437	1.13876	1.24334
7	1.97486	0.01133	0.51464	0.80239	1.14638	1.25515
8	2.04942	0.01135	0.57139	0.81125	1.14622	1.25858
9	2.08147	0.00801	0.55275	0.80727	1.15009	1.2653
10	2.1534	0.01006	0.57207	0.81124	1.1499	1.26735
11	2.15807	0.00273	0.49563	0.79627	1.15338	1.27224
12	2.21482	0.00595	0.56683	0.80942	1.15281	1.27443
13	2.18378	0.00589	0.55	0.80436	1.15516	1.27709
14	2.24031	0.00538	0.5511	0.80629	1.15476	1.2788
15	2.22538	0.0029	0.5218	0.79972	1.15685	1.28104
16	2.29231	0.00433	0.55393	0.80583	1.15626	1.28231
17	2.28119	0.00429	0.53372	0.80075	1.15805	1.28434
18	2.33272	0.00398	0.54836	0.80461	1.1575	1.28533
19	2.3133	0.00208	0.52514	0.79973	1.15907	1.28703
20	2.35879	0.00372	0.54556	0.80393	1.15848	1.28772
21	2.34587	0.0033	0.53873	0.80094	1.15986	1.28925
22	2.3885	0.00329	0.54733	0.80368	1.15931	1.28982
23	2.37944	0.00208	0.52241	0.79888	1.16059	1.29114
24	2.41544	0.00276	0.54506	0.80308	1.16002	1.29162
25	2.40277	0.00285	0.53741	0.80039	1.16115	1.29278
26	2.44307	0.00261	0.54187	0.80253	1.16062	1.29318
27	2.42862	0.00157	0.52801	0.79933	1.16167	1.2942
28	2.46625	0.00257	0.54298	0.80236	1.16114	1.29454
29	2.45177	0.00217	0.53141	0.79954	1.1621	1.29546
30	2.48827	0.00236	0.5415	0.802	1.1616	1.29575
31	2.47389	0.00151	0.52845	0.79917	1.1625	1.29658
32	2.50894	0.00214	0.53998	0.80171	1.16201	1.29683
33	2.49483	0.00215	0.53488	0.79964	1.16283	1.29757
34	2.52906	0.00189	0.54015	0.80152	1.16237	1.29779
35	2.51338	0.00129	0.52668	0.79884	1.16315	1.29847
36	2.54698	0.00194528	0.540032	0.801354	1.16269	1.29866
37	2.53207	0.00187	0.53408	0.79941	1.16342	1.29928
38	2.56343	0.00178	0.53821	0.8011	1.16298	1.29945
39	2.54931	0.00117	0.52879	0.79893	1.16367	1.30002
40	2.57918	0.00171	0.53863	0.80098	1.16325	1.30017
41	2.56485	0.00168	0.53148	0.79907	1.1639	1.3007
42	2.59556	0.00163	0.53816	0.80083	1.16349	1.30083

Table 4.4: Normalized mean values over reciprocal ±1 polynomials of degrees 6 to 42

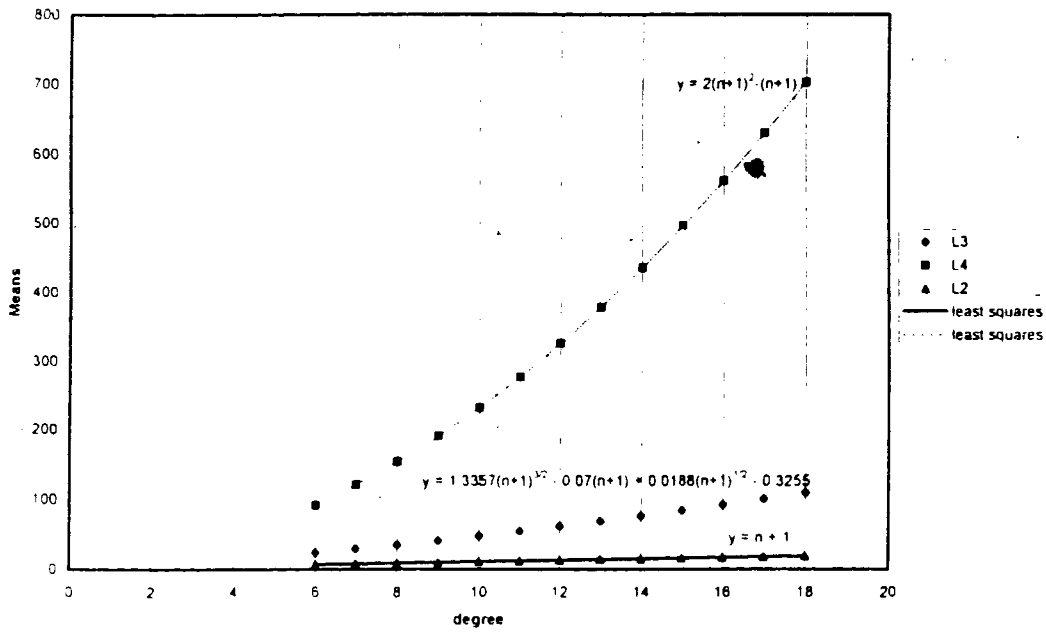

Figure 4.1: Mean L_2^2, L_3^3, and L_4^4 with least-squares approximation equations

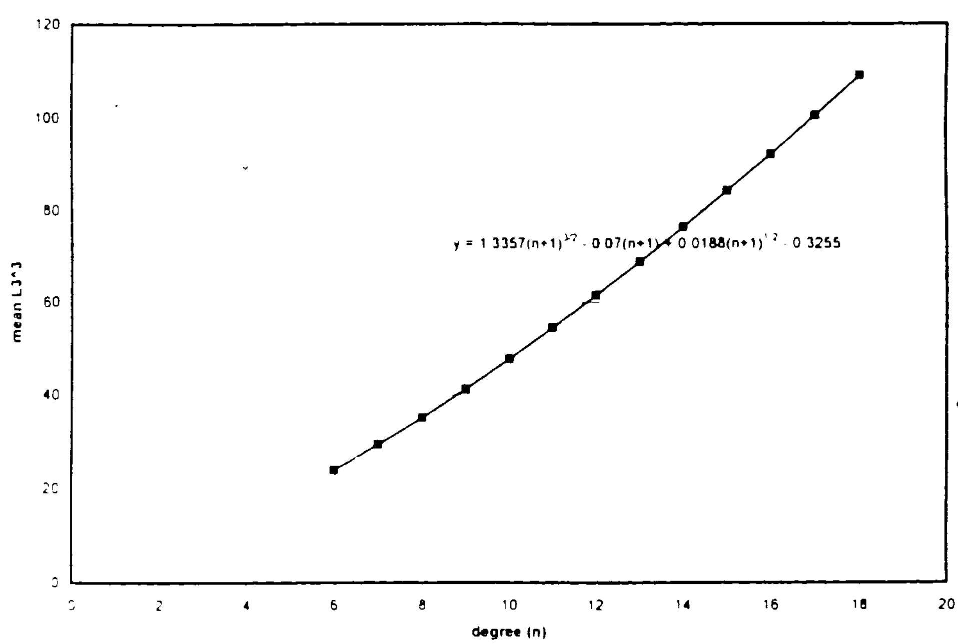

Figure 4.2: Mean L_3^3 with least-squares approximation equation

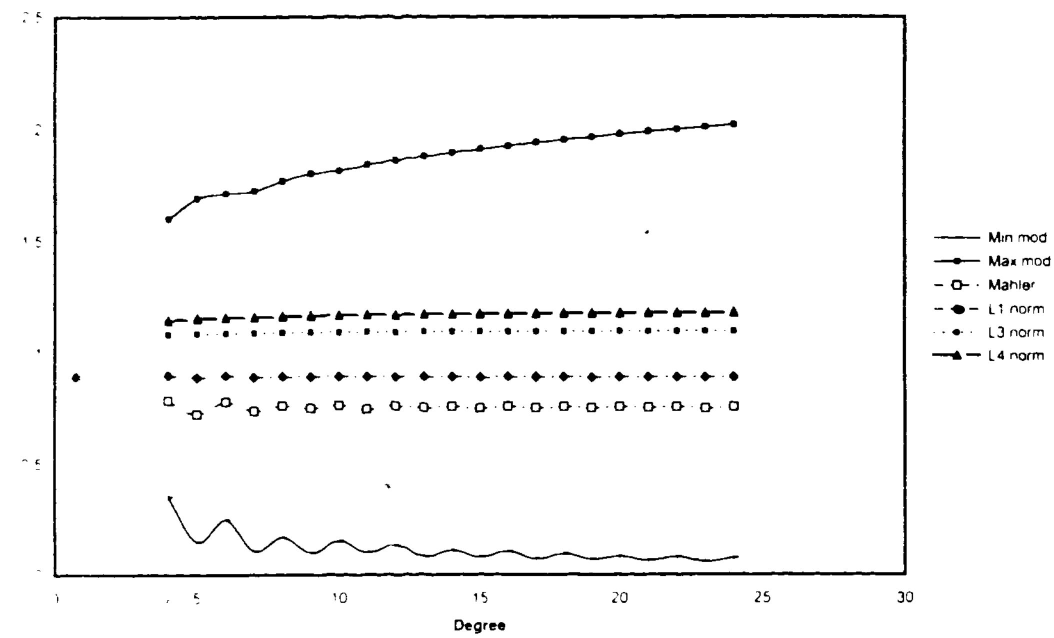

Figure 4.3: Normalized Means over all ±1 polynomials Degrees 4 to 24

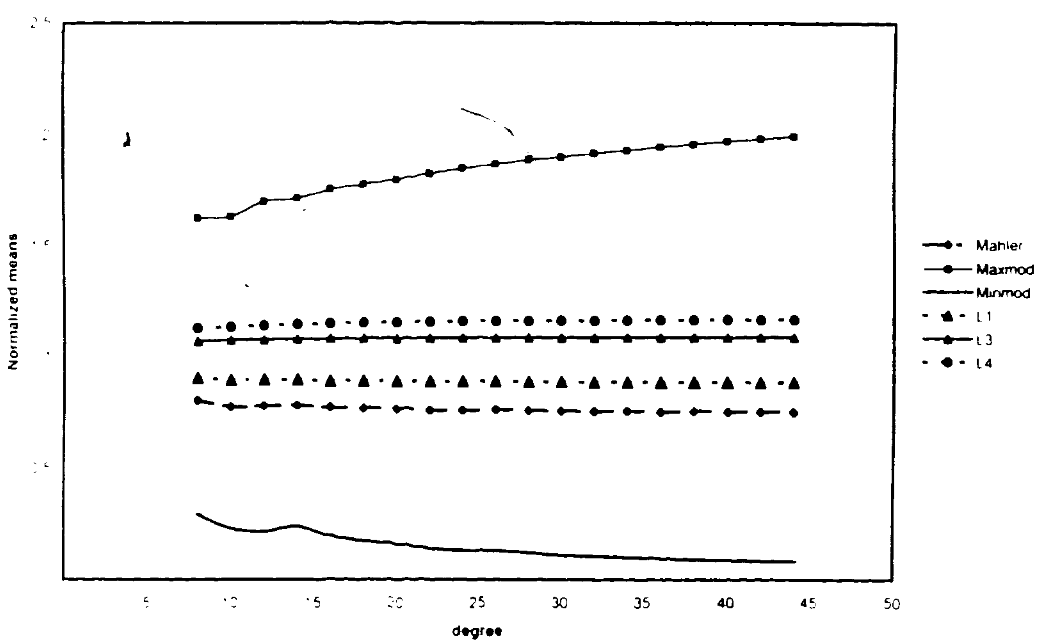

Figure 4.4: Normalized Means over symmetric ±1 polynomials Degrees 6 to 44

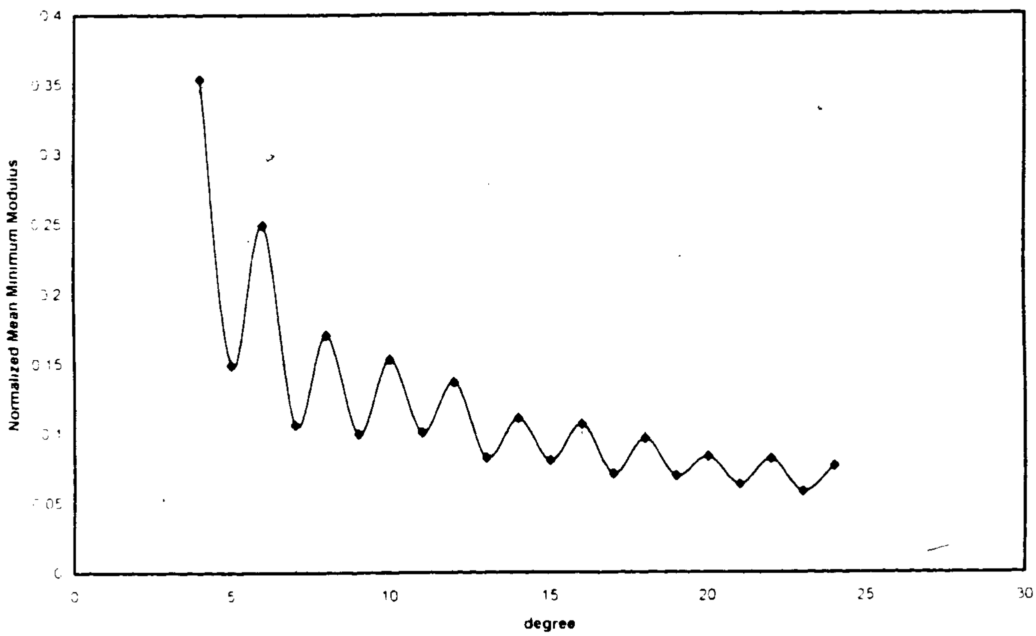

Figure 4.5: Normalized Mean Minimum Modulus Degrees 4 to 24

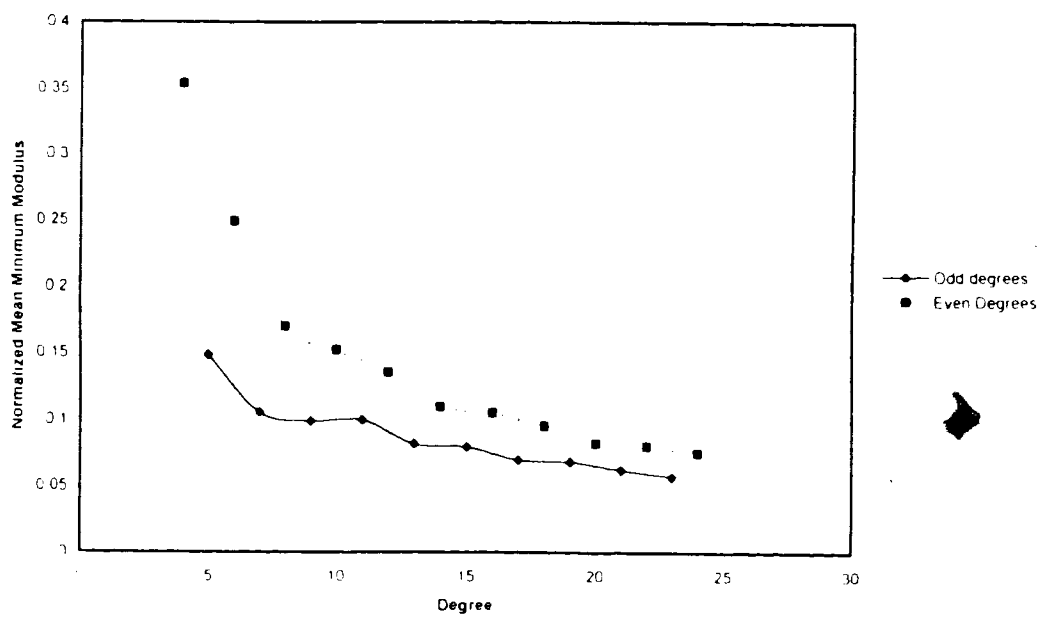

Figure 4.6: Normalized Mean Minimum Modulus Degrees 4 to 24—Even and Odd degrees

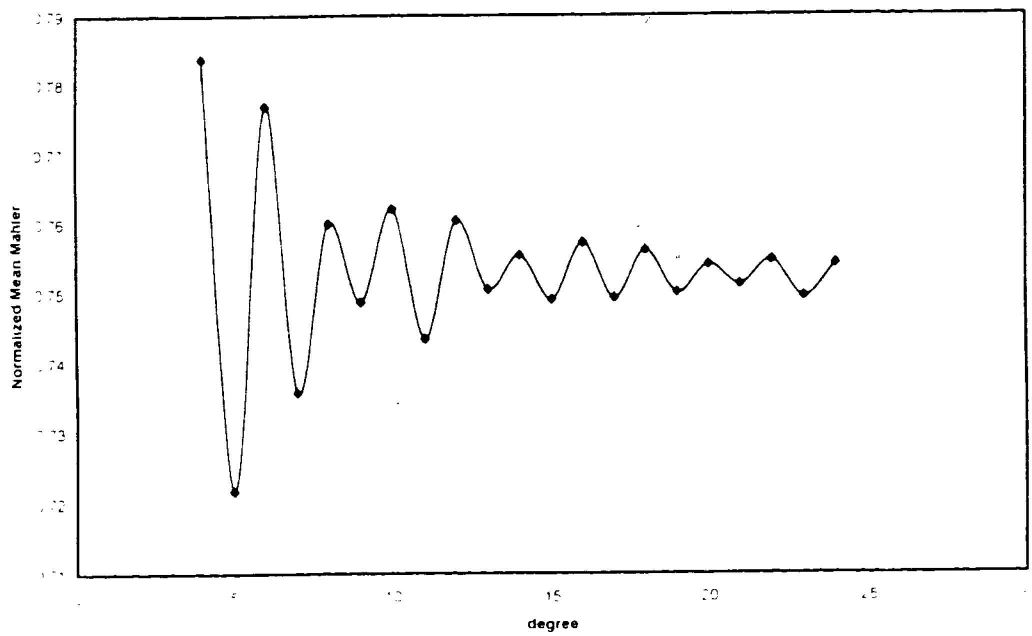

Figure 4.7: Normalized Mean Mahler Degrees 4 to 24

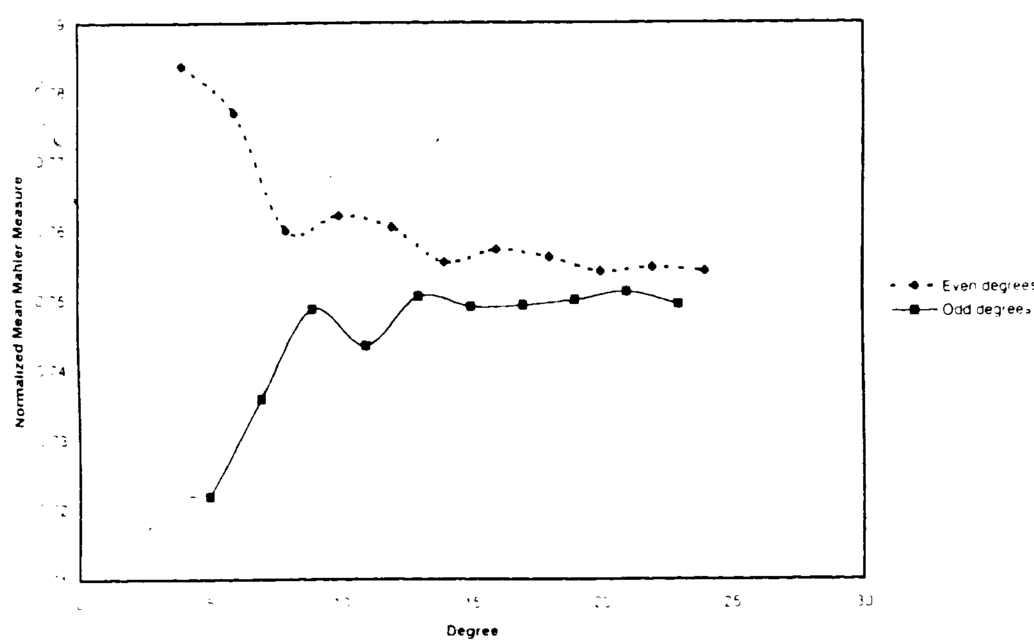

Figure 4.8: Normalized Mean Mahler Measure—Even and Odd degrees

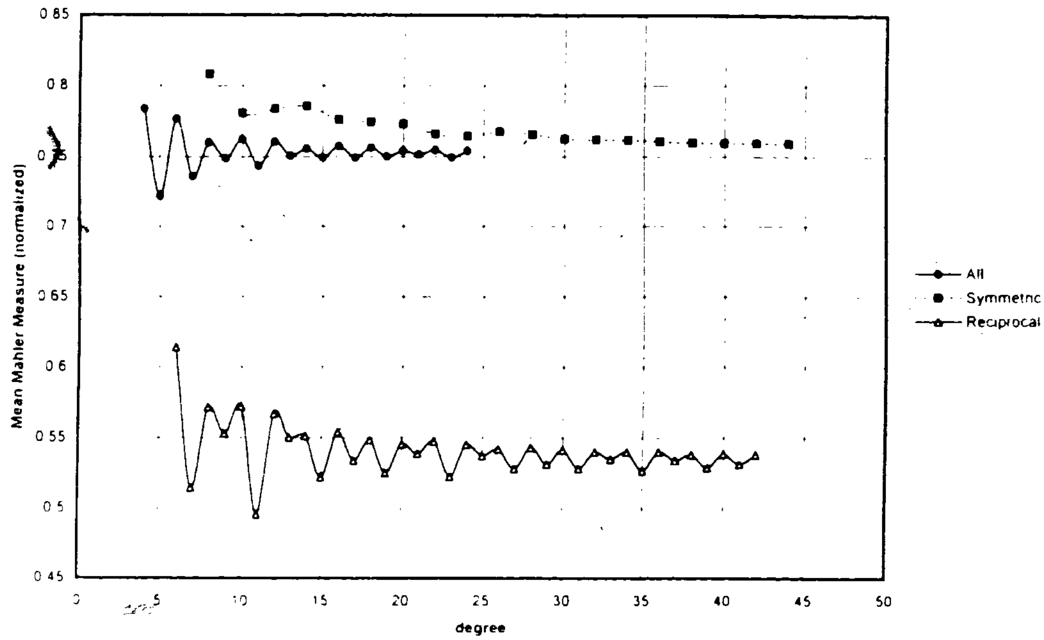

Figure 4.9: Mean Mahler Measure, All, Symmetric and Reciprocal

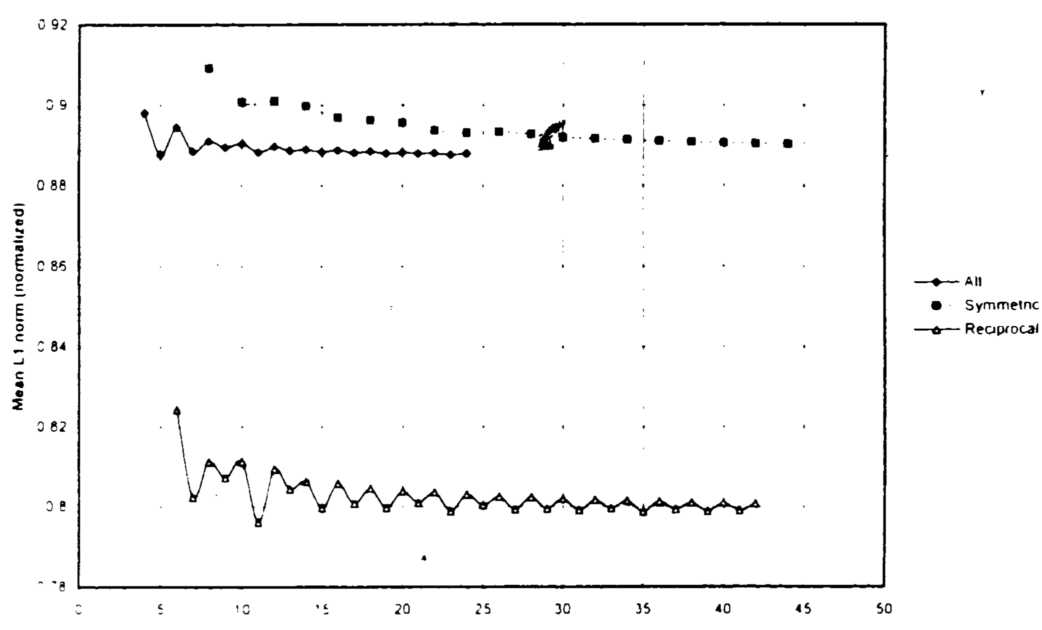

Figure 4.10: Mean Mean L_1 Norms, All, Symmetric and Reciprocal

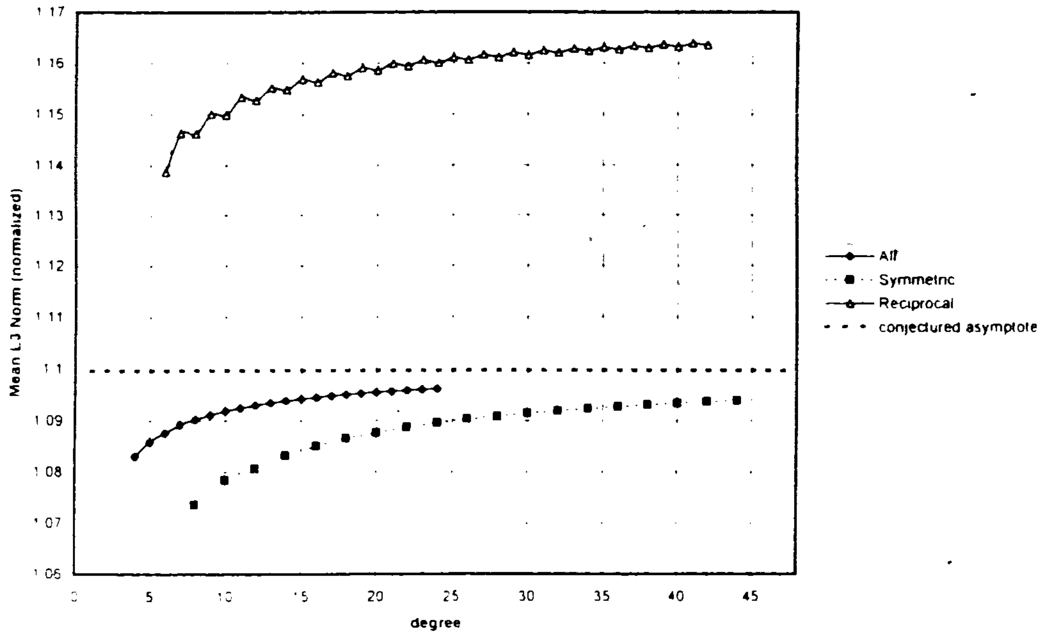

Figure 4.11: Mean L_3 norms with asymptote conjectured by Borwein and Lockhart

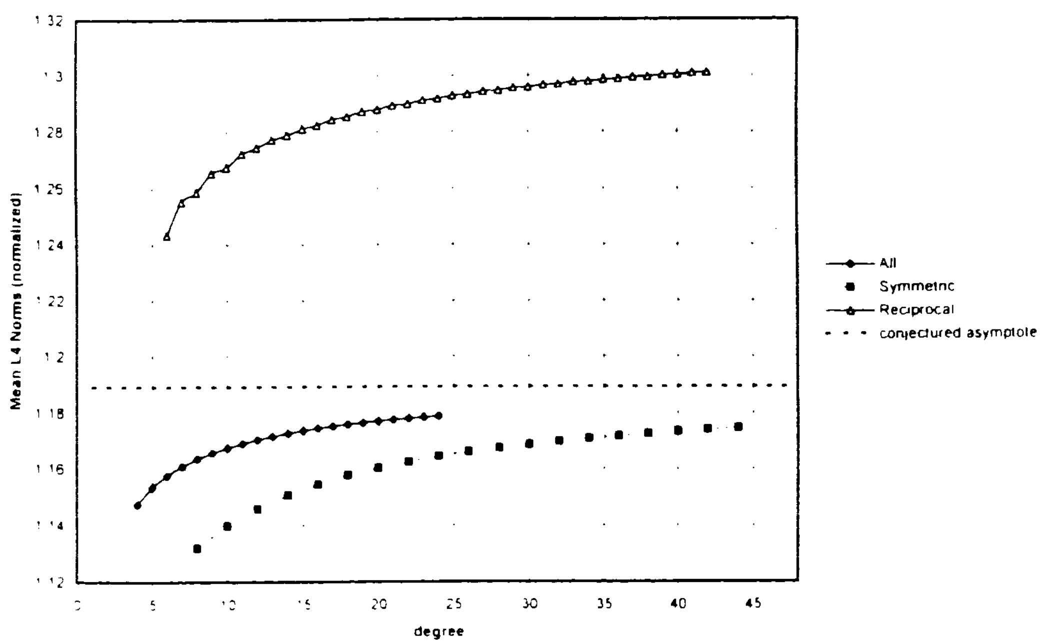

Figure 4.12: Mean L_4 norms with asymptote conjectured by Borwein and Lockhart

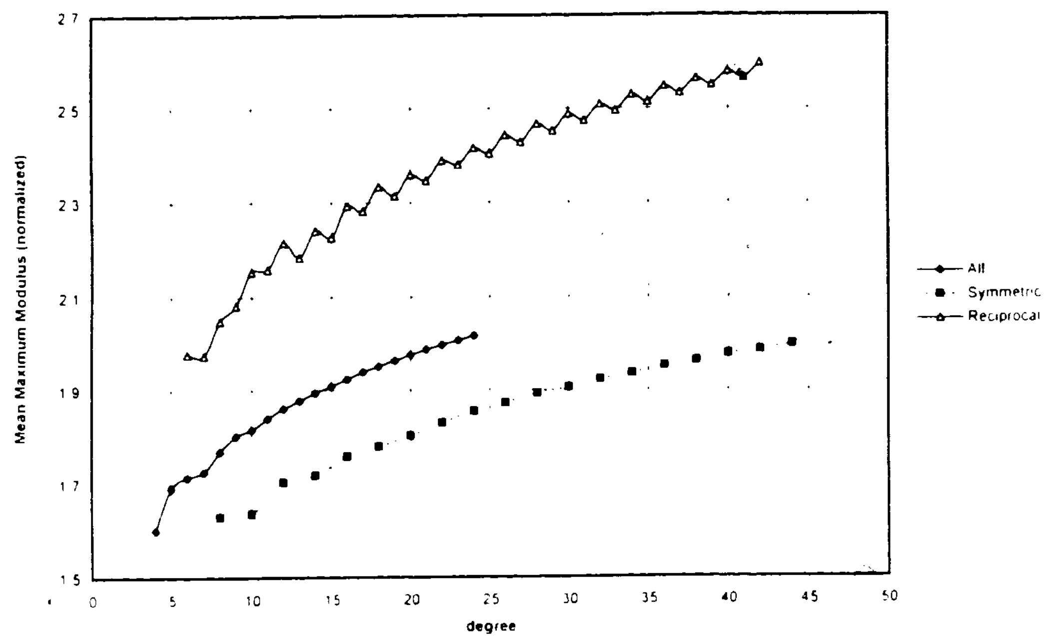

Figure 4.13: Mean Maximum Modulus normalized by $\mu_n = \sqrt{n+1}$

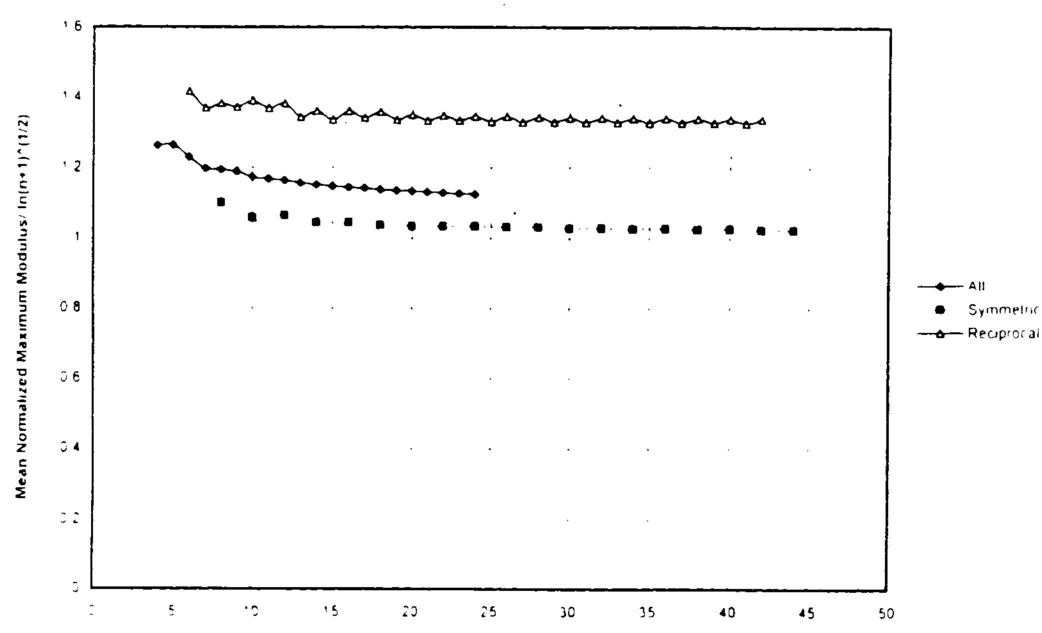

Figure 4.14: Mean Maximum Modulus normalized by $\sqrt{(n+1)\log(n+1)}$

degree Mahler	polynomial max mod	**min mod**	L_1	L_3	L_4
7	00001101				
0.93412	1.38137	**0.56021**	**0.96761**	**1.03014**	**1.05737**
8	000001000				
0.81116	2.33333	**0.37007**	0.8947	1.12053	1.24242
9	0001011001				
0.89248	1.57938	**0.41706**	0.94737	1.04837	1.09162
10	00001100101*				
0.94725	1.38336	**0.62268**	0.97367	1.02558	1.04986
11	000001100101				
0.96262	**1.28077**	**0.6306**	**0.98171**	**1.01718**	**1.03305**
12	0000011001010*				
0.98637	1.38675	**0.83793**	**0.99276**	**1.00816**	**1.0173**
13	00000011001010				
0.95556	1.60357	**0.67253**	**0.97757**	**1.02262**	**1.0453**
14	000001100110101*				
0.96589	1.29099	**0.72036**	**0.98305**	**1.01635**	**1.03179**
15	0111110101100110				
0.94148	1.521	**0.61211**	0.97051	1.02917	1.05737
16	00000001100101010*				
0.92346	1.69775	**0.60748**	0.95993	1.04236	1.08533
17	001010000011001100				
0.92505	1.68924	**0.56503**	0.96157	1.03928	1.07828
18	0010111010000100001*				
0.95499	1.40534	**0.60664**	**0.97755**	1.02189	1.04287
19	01001001000101110001				
0.90444	1.78646	**0.5748**	0.95127	1.04806	1.09354
20	000000111001101101010*				
0.95459	1.39564	**0.65465**	0.97707	1.0227	1.04455
21	0011101111010010111010				
0.933	1.64808	**0.56061**	0.96618	1.03363	1.06643
22	00000001110011011010101*				
0.95329	1.44352	**0.62554**	**0.9767**	1.02288	1.04507
23	000000001110011011010101				
0.94911	1.59639	**0.71358**	0.97301	1.02943	1.0603
24	0011100111111010100110110*				
0.95752	1.61245	**0.65609**	0.97764	1.02442	1.05045

Table 4.5: ±1 Polynomials having highest minimum modulus, degrees 7 to 24. * indicates a symmetric polynomial

degree Mahler	polynomial max mod	**min mod**	L_1	L_3	L_4
6	0100111				
0.94366	**1.17282**	**0.37796**	0.97665	**1.0168**	**1.0293**
8	000110010				
0.89	1.37437	**0.36256**	0.95148	1.03757	1.06703
10	00001100101				
0.94725	1.38336	**0.62268**	0.97367	1.02558	1.04986
12	0000011001010				
0.98637	1.38675	**0.83793**	**0.99276**	**1.00816**	**1.0173**
14	000001100110101				
0.96589	**1.29099**	**0.72036**	**0.98305**	**1.01635**	**1.03179**
16	00000001100101010				
0.92346	1.69775	**0.60748**	0.95993	1.04236	1.08533
18	0010111010000100001				
0.95499	1.40534	**0.60664**	**0.97755**	**1.02189**	**1.04287**
20	000000111001101101010				
0.95459	1.39564	**0.65465**	0.97707	1.0227	1.04455
22	00000001110011011010101				
0.95329	1.44352	**0.62554**	**0.9767**	**1.02288**	**1.04507**
24	0011100111111010100110110				
0.95752	1.61245	**0.65609**	**0.97764**	1.02442	1.05045
26	000000000110001001101010101				
0.87942	1.7907	**0.58046**	0.93455	1.07089	1.14087
28	00000011110001100100101101010				
0.92304	1.53739	**0.56278**	0.96092	1.03866	1.07545
30	0000000011100011001001001010101				
0.91448	1.61645	**0.59114**	0.95569	1.04471	1.08727
32	000000000111110011001010010101010				
0.91272	1.64753	**0.629**	0.95327	1.05042	1.10114
34	00000000011000001100101001101010101				
0.90042	1.87769	**0.6375**	0.94494	1.06382	1.13193
36	0010000101000011100110110100000101110				
0.94865	1.41214	**0.60876**	0.97462	1.02425	1.04699
38	000011100000000011001100101010100100101				
0.92556	1.76141	**0.64861**	0.96041	1.04273	1.08643
40	00001111100000100100010001110101101011010				
0.93056	1.90017	**0.64392**	0.96124	1.04724	1.10132
42	0000000011011000011011000110100111001010101				
0.96612	1.44163	**0.67861**	**0.98287**	1.0172	1.03417
44	000000001100011110001001110110100100110101010				
0.96986	1.41638	**0.68203**	**0.98493**	1.01497	1.02976

Table 4.6: Symmetric polynomials having highest minimum modulus, even degrees 6 to 44.

degree	polynomial				
	max mod	min mod	L_1	L_3	L_4
46	00111111100000100100001100101110001010010101001				
	1.50288	**0.60773**	0.96565	1.03473	1.06847
48	0000000000011011000011011000110100111001010101010				
	1.57143	**0.70049**	0.97704	1.02438	1.04951
48	0010101110101001001111001100101100011111101111110				
	1.32844	0.61841	0.9819	1.01717	1.03329
50	000001100000000110110110010001100011100101010011010 1				
	1.47219	**0.61382**	0.97818	1.0214	1.04206
50	001100000000001111000011001100110100101101010101 1001				
	1.40726	0.60517	0.97863	1.01931	1.03658

Table 4.7: Polynomials with a high minimum modulus. Those with minimum modulus in bold have the highest minimum modulus among symmetric polynomials of the same degree.

degree	polynomial				
Mahler	max mod	min mod	L_1	L_3	L_4
52	00000111111100011100011001000110010010010010101001010				
0.97112	1.43489	0.62171	0.98535	1.01501	1.03028
54	0010011011001001111110001010000010010101100011000110001				
0.93481	1.66095	0.64461	0.96637	1.03445	1.0685
56	001110000011100010001101010100000000110111011011010110110				
0.95874	1.53272	0.62236	0.97944	1.02023	1.04001
58	000111000001110001000110101010000000001101110110110101101101				
0.95418	1.52923	0.61162	0.97694	1.02304	1.04574
60	00101000011001110010110101000100010000000111100100110010111110				
0.9424	1.72256	0.60365	0.97056	1.02997	1.05974
62	0000011101111110001011001000110100001101110011110110101110110101				
0.93807	1.56389	0.6272	0.96856	1.0313	1.06149
64	01110100011100010111001010011101000010011111001000010010010000100				
0.92862	1.68024	0.60094	0.96274	1.03928	1.0791

Table 4.8: These ±1 polynomials have a minimum modulus > 0.6, but they are not known to be extremal. All are symmetric.

degree Mahler	polynomials **Max mod**	Min mod	L_1	L_3	L_4
7 0.85943	00010110 **1.28869**	0.03886	0.95143	1.03275	1.05737
8 0.88257	000101001 **1.37237**	0.23216	0.94998	1.03769	1.06703
9 0.89234	0000011010 **1.38608**	0.33024	0.95434	1.03371	1.05948
10 **0.95466**	00011101101* **1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
11 **0.96262**	000001100101 **1.28077**	**0.6306**	**0.98171**	**1.01718**	**1.03305**
12 0.82133	0001110010010* **1.27388**	0.01879	0.94234	1.03509	1.05955
13 0.93942	00001010011001 **1.28812**	0.49712	0.97157	1.02449	1.0453
14 0.92122	000111010100100 **1.29088**	0.23274	0.96616	1.0261	1.0476
15 **0.94411**	0001000111010010 **1.30838**	0.53678	**0.97347**	**1.02344**	**1.0439**
16 0.83514	00111111101010110* **1.32623**	0.10891	0.93549	1.04344	1.07434
17 0.91537	001100001101101010 **1.29013**	0.19008	0.96456	1.02664	1.04744
18 0.93368	0001111101110101101 **1.28279**	0.39903	0.97055	1.02363	1.04287
19 0.88775	00000110011001011010 **1.32575**	0.04802	0.95856	1.02975	1.05312
20 0.88599	000010101101101100111 **1.32577**	0.01811	0.95582	1.03187	1.05629
21 0.90541	0101101001111100010001 **1.30024**	0	0.96596	1.02513	1.04537
22 0.80941	00000001111001011010101* **1.28816**	0.11049	0.93205	1.04187	1.07061
23 0.87903	000000110001011011001010 **1.25837**	0.16413	0.95145	1.03338	1.05737
24 0.91694	0010010110111011100001110* **1.28914**	0.2	0.96604	1.02517	1.04489

Table 4.9: ±1 Polynomials having smallest maximum modulus, degrees 7 to 24. The symbol * indicates that a polynomial is a symmetric.

Degree Mahler	Polynomial **max mod**	min mod	L_1	L_3	L_4
6	0100111				
0.94366	**1.17282**	**0.37796**	0.97665	**1.0168**	**1.0293**
8	000110010				
0.89	**1.37437**	0.36256	0.95148	1.03757	1.06703
10	00011101101				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
12	0001110010010				
0.82133	**1.27388**	0.01879	0.94234	1.03509	1.05955
14	000001100110101				
0.96589	**1.29099**	**0.72036**	**0.98305**	**1.01635**	**1.03179**
16	00111111101010110				
0.83514	**1.32623**	0.10891	0.93549	1.04344	1.07434
18	0001111101110101101				
0.93368	**1.28279**	0.39903	0.97055	1.02363	1.04287
20	001111111001101010110				
0.95984	**1.32737**	0.43167	**0.98201**	**1.01511**	**1.02826**
22	00000001111001011010101				
0.80941	**1.28816**	0.11049	0.93205	1.04187	1.07061
24	0010010110111011100001110				
0.91694	**1.28914**	0.2	0.96604	1.02517	1.04489
26	000000110110110001110010101				
0.94284	**1.21369**	0.32957	0.97534	1.01918	1.03452
28	00100101101110111011100001110				
0.9295	**1.24568**	0.19468	0.97226	1.01978	1.03498
30	0000000110110100111100011010101				
0.90508	**1.25006**	0.14737	0.96402	1.02445	1.04248
32	001001000000001110110101010001110				
0.90674	**1.24793**	0.32686	0.95962	1.03062	1.0542
34	00001000011000110111001001101000101				
0.82575	**1.29676**	0.07622	0.93984	1.03586	1.06008
36	0011100010101110000101101111110110110				
0.89176	**1.3033**	0.0641	0.96171	1.02537	1.04443
38	000000011111000111001101101101011010101				
0.94944	**1.27585**	0.16013	0.97755	1.01913	1.03583
40	00000001111100011100110010010010100101010				
0.94217	**1.2873**	0.11876	0.97732	1.01767	1.03285
42	0011110000000000011001100110101010101101001				
0.91993	**1.26787**	0.14205	0.96768	1.02364	1.04191
44	000010100000111001110011001101100100101111 1010				
0.90602	**1.29459**	0.14907	0.96262	1.02681	1.04739

Table 4.10: Symmetric ±1 Polynomials having smallest maximum modulus amongst the set of symmetric polynomials, degrees 6 to 44.

degree Mahler	polynomial **max mod**	min mod	L_1	L_3	L_4
6	0010100				
0.73584	**1.466**	0.0173	0.91138	1.05649	1.09715
8	000101000				
0.73686	**1.66667**	0.02946	0.89772	1.07824	1.1404
10	00101110100				
0.58073	**1.61825**	0.00063	0.84594	1.09549	1.15988
12	0011010101100				
0.66623	**1.61303**	0.0046	0.88382	1.07274	1.12322
14	000101101101000				
0.63583	**1.63624**	0.0005	0.86504	1.08979	1.15398
16	00001101010110000				
0.75069	**1.69375**	0.01837	0.91021	1.06486	1.11644
18	0001110110110111000				
0.76992	**1.62142**	0.00135	0.91833	1.05695	1.10128
20	001101011111110101100				
0.66219	**1.57882**	0.01053	0.8805	1.07712	1.13233
22	00001010011011001010000				
0.72493	**1.59237**	0.0006	0.89964	1.06983	1.12222
24	0011110110101010110111100				
0.61291	**1.59933**	0	0.8747	1.07609	1.12943
26	001001010001111100010100100				
0.69812	**1.63945**	0.00398	0.88887	1.0772	1.13528
28	01100101011110001111010100110				
0.69524	**1.66867**	0.00945	0.88693	1.07859	1.13689
30	0010111000100001000010001110100				
0.67622	**1.62986**	0.01595	0.88329	1.07822	1.13575
32	001100111110101111101011111001100				
0.70155	**1.60456**	0.00004	0.89188	1.07533	1.13276
34	00110100111111101010111111100101100				
0.65391	**1.61136**	0.00022	0.882	1.07488	1.12854
36	0011101101000101000001010001011011100				
0.70865	**1.59322**	0.00066	0.89375	1.07306	1.12757
38	001110111010010110000011010010111011100				
0.68905	**1.61914**	0.00075	0.89187	1.0719	1.12531
40	00111011101011010011111001011010111011100				
0.66517	**1.58988**	0.00154	0.88426	1.0728	1.12378
42	0010101100011111011001001101111100011010100				
0.72619	**1.57135**	0.00196	0.90319	1.06588	1.11511

Table 4.11: Reciprocal ±1 Polynomials having smallest maximum modulus amongst the set of reciprocal polynomials, even degrees 6 to 42.

degree	polynomial				
Mahler	**max mod**	min mod	L_1	L_3	L_4
7	01111110				
0.6982	**1.55176**	0.012	0.89223	1.07371	1.12905
9	0110000110				
0.67454	**1.72754**	0.02704	0.86822	1.09891	1.17405
11	000101101000				
0.70283	**1.65301**	0	0.89493	1.07007	1.12174
13	00011011011000				
0.62003	**1.56088**	0.00159	0.86433	1.08327	1.13911
15	0010001111000100				
0.69863	**1.62509**	0.00001	0.89189	1.07382	1.12905
17	000010110011010000				
0.6947	**1.60944**	0.00375	0.89237	1.0678	1.11568
19	01011110011001111010				
0.74139	**1.58236**	0.01576	0.90657	1.06285	1.10852
21	0001110110110110111000				
0.73285	**1.54286**	0.00239	0.90546	1.06393	1.11123
23	010011100000000001110010				
0.67699	**1.63299**	0.00033	0.88535	1.07498	1.12905
25	01011111001100110011111010				
0.71254	**1.52097**	0.00274	0.89999	1.06559	1.11317
27	0111110011010110101100111110				
0.7427	**1.59197**	0.00165	0.90572	1.06635	1.11689
29	011001010000111111000010100110				
0.68194	**1.63211**	0.00287	0.88681	1.07494	1.12973
31	00001001110100111100101110010000				
0.66338	**1.61848**	0.00595	0.87867	1.07963	1.13711
33	0110100111011100000011101110010110				
0.64713	**1.57064**	0.00046	0.87549	1.07777	1.13137
35	0101110000110111111111110110000111010				
0.69402	**1.60029**	0.00315	0.88378	1.07816	1.13358
37	01010110000001100111100110000001101010				
0.74638	**1.58736**	0.00115	0.9059	1.06707	1.11825
39	0011011000010101111111111010100001101100				
0.72175	**1.59461**	0.00018	0.89995	1.0694	1.12204
41	000111010001001011011110110100100010111000				
0.69827	**1.58534**	0.0002	0.89416	1.06836	1.1174

Table 4.12: Reciprocal ±1 Polynomials having smallest maximum modulus amongst the set of reciprocal polynomials, odd degrees 7 to 41.

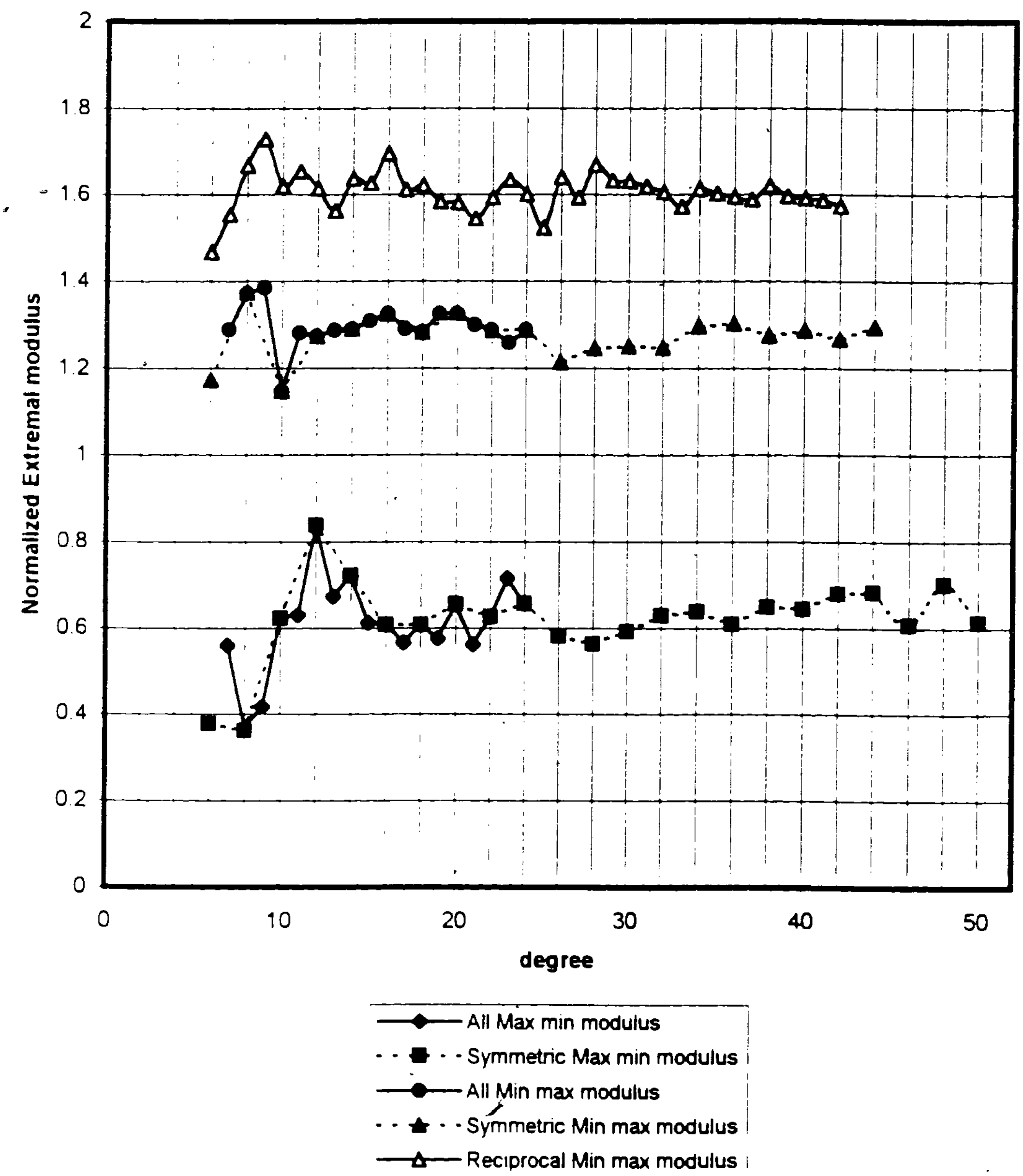

Figure 4.15: Smallest maximum modulus and largest minimum modulus over all ± 1 polynomials, and over symmetric and over reciprocal polynomials.

degree **Mahler**	Polynomial max mod	 min mod	 L_1	 L_3	 L_4
7	00001101				
0.93412	1.38137	**0.56021**	**0.96761**	**1.03014**	**1.05737**
8	001101000				
0.90834	1.46305	0.32976	0.95795	1.03593	1.06703
9	0011010000				
0.92363	1.498[illegible]9	0.35262	0.96446	1.03138	1.05948
10	00011101101*				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
11	000001100101				
0.96262	**1.28077**	**0.6306**	**0.98171**	**1.01718**	**1.03305**
12	0000011001010*				
0.98637	1.38675	0.83793	0.99276	1.00816	1.0173
13	000000011001010				
0.95556	1.60357	0.67253	0.97757	1.02262	1.0453
14	000001100110101*				
0.96589	1.29099	0.72036	0.98305	1.01635	1.03179
15	0001000111010010				
0.94411	**1.30838**	0.53678	**0.97347**	**1.02344**	**1.0439**
16	00100110000101011				
0.94067	1.39918	0.50775	**0.97106**	**1.02685**	**1.05128**
17	000100011101001011				
0.94612	1.41722	0.4714	**0.97368**	1.02466	1.04744
18	0010111010000100001*				
0.95499	1.40534	**0.60664**	**0.97755**	1.02189	1.04287
19	00000101110100111001				
0.96188	1.3405	0.44721	**0.98192**	**1.01631**	**1.03103**
20	001111111001101010110*				
0.95984	1.32737	0.43167	**0.98201**	**1.01511**	**1.02826**
21	0000010011010100111000				
0.95411	1.38287	0.43486	**0.97818**	**1.01987**	**1.03806**
22	00000001110011011010101*				
0.95329	1.44352	**0.62554**	**0.9767**	1.02288	1.04507
23	001100011111101010110110				
0.96237	1.36319	0.3978	**0.98244**	**1.01567**	**1.02988**
24	0001110000000101011011001				
0.96683	1.38019	0.48233	**0.98438**	**1.01427**	**1.02763**

Table 4.13: ±1 Polynomials having largest Mahler measure, degrees 7 to 24.

degree **Mahler**	polynomial max mod	min mod	L_1	L_3	L_4
6	0100111				
0.94366	**1.17282**	**0.37796**	0.97665	**1.0168**	**1.0293**
8	000011010				
0.8917	1.37994	0.33333	**0.95274**	**1.03704**	**1.06703**
10	00011101101				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
12	0000011001010				
0.98637	1.38675	**0.83793**	**0.99276**	**1.00816**	**1.0173**
14	000001100110101				
0.96589	**1.29099**	**0.72036**	**0.98305**	**1.01635**	**1.03179**
16	00111110011010110				
0.93885	1.35059	0.57835	**0.97025**	**1.02715**	**1.05128**
18	0010111010000100001				
0.95499	1.40534	**0.60664**	**0.97755**	**1.02189**	**1.04287**
20	001111111001101010110				
0.95984	**1.32737**	0.43167	**0.98201**	**1.01511**	**1.02826**
[illegible]2	00000001110011011010101				
[illegible]5329	1.44352	**0.62554**	**0.9767**	**1.02288**	**1.04507**
24	0011100111111010100110110				
0.95752	1.61245	**0.65609**	**0.97764**	1.02442	1.05045
26	000111100010001000100101101				
0.96894	1.31082	0.50748	**0.98548**	**1.01288**	**1.02446**
28	00011000111111101010110110010				
0.96268	1.40513	0.55709	**0.98163**	**1.01779**	**1.03498**
30	0001100011010101111111001001101				
0.95476	1.53455	0.53882	**0.97759**	1.02169	1.04248
32	001111000101110110001000010010110				
0.94978	1.45473	0.52223	**0.9756**	1.02301	1.04467
34	00000111110011011001110011010110101				
0.96105	1.35186	0.50709	**0.98109**	**1.0178**	**1.0345**
36	0001110111110001001100010010100010010				
0.95736	1.65219	0.56018	**0.97851**	1.02195	1.04443
38	000000001101101001001110000111001010101				
0.96383	1.40184	0.54716	**0.98265**	**1.01607**	**1.03107**
40	00011110001010101100110011111111011010010				
0.96104	1.36576	0.5019	**0.98169**	**1.01622**	**1.03068**
42	0000000011011000011011000110100111001010101				
0.96612	1.44163	**0.67861**	**0.98287**	1.0172	1.03417
44	000000001100011110001001110110100100110101010				
0.96986	1.41638	**0.68203**	**0.98493**	1.01497	1.02976

Table 4.14: Symmetric ±1 Polynomials having largest Mahler measure amongst all symmetric polynomials degrees 6 to 44.

degree	polynomial				
Mahler	max mod	min mod	L_1	L_3	L_4
6	0001000				
0.7545	1.88982	0.02348	0.89545	1.0843	1.15433
8	000101000				
0.73686	1.66667	0.02946	0.89772	1.07824	1.1404
10	00110101100				
0.69266	1.68657	0.01504	0.88923	1.0778	1.13809
12	0000100010000				
0.71912	2.49615	0.0043	0.88131	1.11766	1.23802
14	001000111000100				
0.70453	1.94678	0.00877	0.88706	1.08905	1.16538
16	00000110101100000				
0.75911	1.99709	0.01868	0.9031	1.08125	1.15426
18	0001110110110111000				
0.76992	1.62142	0.00135	0.91833	1.05695	1.10128
20	000001100101001100000				
0.77033	1.96396	0.00519	0.91208	1.06589	1.11963
22	00110011110101111001100				
0.74886	2.04894	0.00579	0.88996	1.10173	1.19506
24	0011001111101011111001100				
0.75896	1.81725	0.0198	0.89927	1.08062	1.1468
26	000000011001010100110000000				
0.76048	2.50185	0.0074	0.90193	1.0909	1.18117
28	00101011110011011001111010100				
0.77272	1.82595	0.00371	0.91502	1.06427	1.11696
30	0011111110011010101100111111100				
0.75987	1.86362	0.01708	0.90233	1.0774	1.14139
32	000111110111011010110111011111000				
0.75696	1.6848	0.00114	0.90913	1.06581	1.11728
34	00100110000101011111010100001100100				
0.75273	1.64131	0.0012	0.91591	1.05589	1.09777
36	0011111001000101001001010001001111100				
0.7613	1.9214	0.00156	0.90909	1.07191	1.13364
38	001111011111010001101100010111110111100				
0.75816	1.84928	0.002	0.90892	1.06808	1.12346
40	00011110110100100010001000100101101111000				
0.76474	1.88852	0.00256	0.91052	1.0704	1.13043
42	0001000111011110100111110010111101110001000				
0.77236	2.03127	0.00895	0.91132	1.07072	1.13189

Table 4.15: Reciprocal ±1 Polynomials having largest Mahler measure amongst all reciprocal polynomials even degrees 6 to 42.

degree **Mahler**	polynomial max mod	 min mod	 L_1	 L_3	 L_4
7 **0.7753**	01000010 1.68485	 0.02524	 0.91135	 1.07049	 1.12905
9 **0.75333**	0001001000 1.89737	 0.00399	 0.91236	 1.06518	 1.12115
11 **0.70283**	000101101000 1.65301	 0	 0.89493	 1.07007	 1.12174
13 **0.70937**	00111011011100 1.8939	 0.02753	 0.88286	 1.0911	 1.16577
15 **0.7514**	0101100000011010 1.81337	 0.00149	 0.90717	 1.07049	 1.12905
17 **0.75639**	010100111111001010 1.90022	 0.01029	 0.90344	 1.07981	 1.14966
19 **0.76631**	01011001111110011010 1.70993	 0.00457	 0.91554	 1.06063	 1.10852
21 **0.76351**	0101111100110011111010 1.63921	 0.00215	 0.91946	 1.05534	 1.09898
23 **0.76086**	010100000110011000001010 1.92647	 0.00342	 0.91439	 1.06424	 1.11927
25 **0.76251**	00100010000111100001000100 1.96116	 0.00149	 0.91327	 1.06254	 1.11317
27 **0.75619**	0011001111101001011111001100 1.81474	 0	 0.90748	 1.07309	 1.13477
29 **0.762**	011100010110111111011010001110 2.01517	 0.00188	 0.91675	 1.06227	 1.11719
31 **0.77461**	00101110100001000010000101110100 1.71819	 0.01474	 0.91615	 1.06214	 1.1124
33 **0.75853**	0010000100101110000111010010000100 1.92385	 0.00284	 0.90987	 1.06881	 1.12656
35 **0.76389**	000001011101001110011100101110100000 1.90062	 0.00153	 0.90956	 1.0712	 1.13146
37 **0.76097**	00110111000101101111110110100011101100 1.92378	 0.00407	 0.91007	 1.06852	 1.12609
39 **0.75826**	0001011001011110111001110111101001101000 1.66088	 0.00985	 0.91563	 1.05695	 1.10017
41 **0.76478**	001000111011111001011110100111110111000100 1.69827	 0.00561	 0.91332	 1.06215	 1.11084

Table 4.16: Reciprocal ± 1 Polynomials having largest Mahler measure amongst all reciprocal polynomials even degrees 6 to 42.

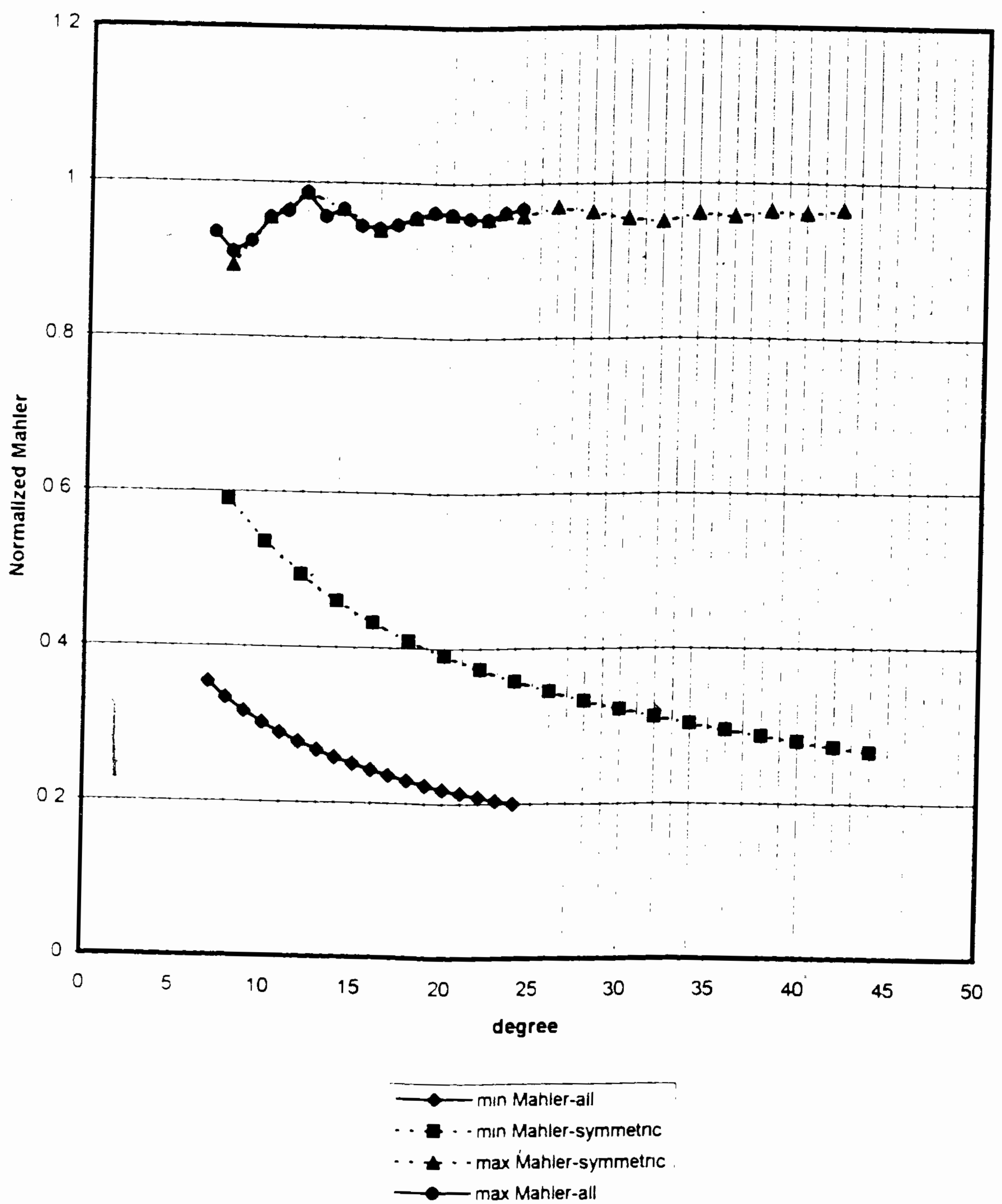

Figure 4.16: Extremal Mahler measure over all ±1 polynomials, over symmetric and over reciprocal polynomials.

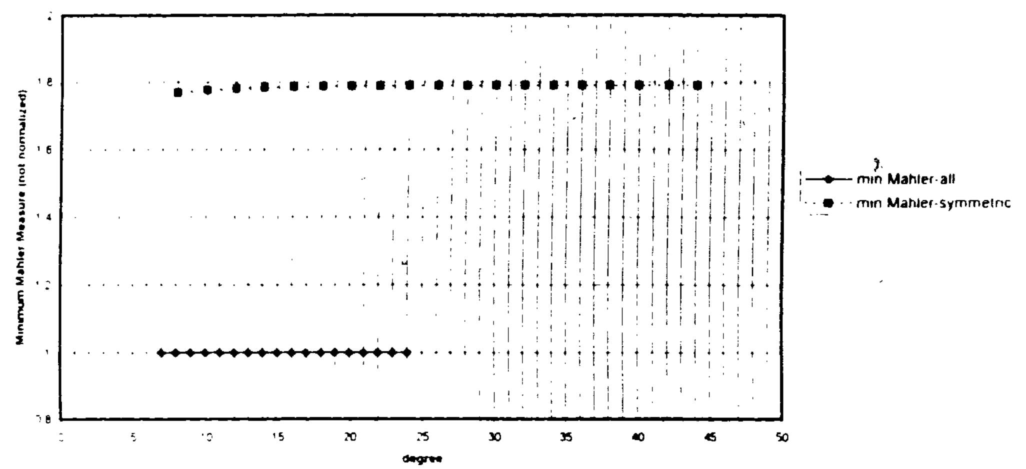

Figure 4.17: Smallest Mahler measure, not normalized. All and symmetric

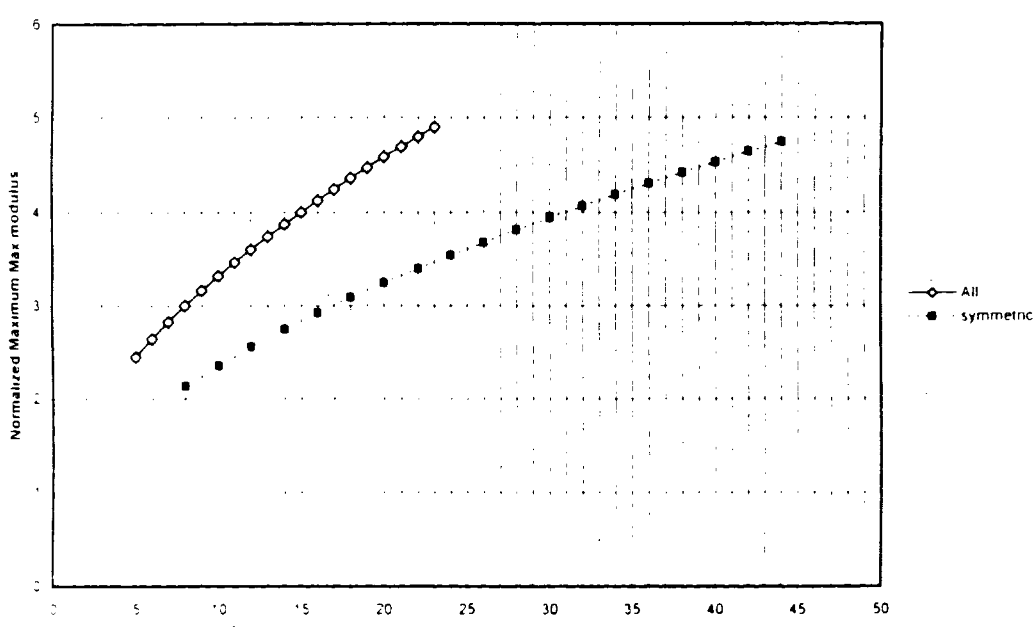

Figure 4.18: Largest maximum modulus, all and symmetric

Degree Mahler	Polynomial max mod	min mod	$\mathbf{L_1}$	L_3	L_4
7	00001101				
0.93412	1.38137	**0.56021**	**0.96761**	**1.03014**	**1.05737**
8	000101100				
0.90834	1.46305	0.32976	**0.95795**	1.03593	1.06703
9	0000101100				
0.92363	1.49869	0.35262	**0.96446**	1.03138	1.05948
10	00011101101*				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
11	000001100101				
0.96262	**1.28077**	**0.6306**	**0.98171**	**1.01718**	**1.03305**
12	0000011001010*				
0.98637	1.38675	0.83793	**0.99276**	1.00816	1.0173
13	00000011001010				
0.95556	1.60357	0.67253	**0.97757**	1.02262	1.0453
14	000001100110101*				
0.96589	1.29099	0.72036	**0.98305**	1.01635	1.03179
15	0001000111010010				
0.94411	**1.30838**	0.53678	**0.97347**	**1.02344**	**1.0439**
16	00100110000101011				
0.94067	1.39918	0.50775	**0.97106**	**1.02685**	**1.05128**
17	000100011101001011				
0.94612	1.41722	0.4714	**0.97368**	1.02466	1.04744
18	0010111010000100001*				
0.95499	1.40534	**0.60664**	**0.97755**	1.02189	1.04287
19	00000101110100111001				
0.96188	1.3405	0.44721	**0.98192**	**1.01631**	**1.03103**
20	001111111001101010110*				
0.95984	1.32737	0.43167	**0.98201**	**1.01511**	**1.02826**
21	0000010011010100111000				
0.95411	1.38287	0.43486	**0.97818**	**1.01987**	**1.03806**
22	00000001110011011010101*				
0.95329	1.44352	**0.62554**	**0.9767**	1.02288	1.04507
23	00110001111110101010110110				
0.96237	1.36319	0.3978	**0.98244**	**1.01567**	**1.02988**
24	0001110000000010101011011001				
0.96683	1.38019	0.48233	**0.98438**	**1.01427**	**1.02763**

Table 4.17: Polynomials having largest L_1 norm , degrees 7 to 24.

degree Mahler	Polynomial max mod	min mod	$\mathbf{L_1}$	L_3	L_4
6	0011101				
0.88532	1.39578	0.28258	**0.95115**	1.03652	1.06484
8	000011010				
0.8917	1.37994	0.33333	**0.95274**	**1.03704**	**1.06703**
10	00011101101				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
12	0000011001010				
0.98637	1.38675	**0.83793**	**0.99276**	**1.00816**	**1.0173**
14	000001100110101				
0.96589	**1.29099**	**0.72036**	**0.98305**	**1.01635**	**1.03179**
16	00111110011010110				
0.93885	1.35059	0.57835	**0.97025**	**1.02715**	**1.05128**
18	0010111010000100001				
0.95499	1.40534	**0.60664**	**0.97755**	**1.02189**	**1.04287**
20	001111111001101010110				
0.95984	**1.32737**	0.43167	**0.98201**	**1.01511**	**1.02826**
22	00000001110011011010101				
0.95329	1.44352	**0.62554**	**0.9767**	**1.02288**	**1.04507**
24	0011100111111010100110110				
0.95752	1.61245	**0.65609**	**0.97764**	1.02442	1.05045
26	000111100010001000100101101				
0.96894	1.31082	0.50748	**0.98548**	**1.01288**	**1.02446**
28	00011000111111101010110110010				
0.96268	1.40513	0.55709	**0.98163**	**1.01779**	**1.03498**
30	0001100011010101111111001001101				
0.95476	1.53455	0.53882	**0.97759**	1.02169	1.04248
32	001111000101110110001000010010110				
0.94978	1.45473	0.52223	**0.9756**	1.02301	1.04467
34	00000111110011011001110011010110101				
0.96105	1.35186	0.50709	**0.98109**	**1.0178**	**1.0345**
36	0001110111110001001100010010100010010				
0.95736	1.65219	0.56018	**0.97851**	1.02195	1.04443
38	000000001101101001001110000111001010101				
0.96383	1.40184	0.54716	**0.98265**	**1.01607**	**1.03107**
40	00011110001010101100110011111111011010010				
0.96104	1.36576	0.5019	**0.98169**	**1.01622**	**1.03068**
42	0000000011011000011011000110100111001010101				
0.96612	1.44163	**0.67861**	**0.98287**	1.0172	1.03417
44	000000001100011110001001110110100100110101010				
0.96986	1.41638	**0.68203**	**0.98493**	1.01497	1.02976

Table 4.18: Symmetric polynomials having largest L_1 norm amongst the set of all symmetric polynomials, degrees 6 to 44

degree	polynomial				
Mahler	Maxmod	Minmod	**L_1**	L_3	L_4
6	0010100				
0.73584	1.466	0.0173	**0.91138**	1.05649	1.09715
8	000101000				
0.73686	1.66667	0.02946	**0.89772**	1.07824	1.1404
10	00110101100				
0.69266	1.68657	0.01504	**0.88923**	1.0778	1.13809
12	0001101011000				
0.71561	1.74885	0.02687	**0.89161**	1.07862	1.13957
14	001000111000100				
0.70453	1.94678	0.00877	**0.88706**	1.08905	1.16538
16	00001101010110000				
0.75069	1.69375	0.01837	**0.91021**	1.06486	1.11644
18	0001110110110111000				
0.76992	1.62142	0.00135	**0.91833**	1.05695	1.10128
20	000011001010100110000				
0.76291	1.76614	0.00259	**0.9166**	1.05913	1.10647
22	00110010111111101001100				
0.74008	1.97182	0.00111	**0.90166**	1.07699	1.14303
24	0000111101110111011110000				
0.75059	1.88838	0.01826	**0.90175**	1.08082	1.15102
26	000111011101101101110111000				
0.75913	1.77908	0.01823	**0.90765**	1.07057	1.12771
28	00101011110011011001111010100				
0.77272	1.82595	0.00371	**0.91502**	1.06427	1.11696
30	0001010111100110110011110101000				
0.75631	1.90113	0.00495	**0.90604**	1.07254	1.13289
32	000111110111011010110111011111000				
0.75696	1.6848	0.00114	**0.90913**	1.06581	1.11728
34	00100110000101011111010100001100100				
0.75273	1.64131	0.0012	**0.91591**	1.05589	1.09777
36	0010111010000100001000010000101110100				
0.75264	1.80839	0.00074	**0.91087**	1.06409	1.11514
38	000101110110111100111001111011011101000				
0.75157	1.67396	0.00552	**0.91477**	1.05726	1.10051
40	00001011101101111001110011110110111010000				
0.75099	1.74136	0.00262	**0.91233**	1.06107	1.10838
42	0001001011110011011101011101100111101001000				
0.75838	1.69565	0.00246	**0.91252**	1.06325	1.11355

Table 4.19: Reciprocal polynomials having largest L_1 norm amongst the set of all reciprocal polynomials, even degrees 6 to 44

degree	polynomial				
Mahler	Maxmod	Minmod	$\mathbf{L_1}$	L_3	L_4
7	01000010				
0.7753	1.68485	0.02524	**0.91135**	1.07049	1.12905
9	0001001000				
0.75333	1.89737	0.00399	**0.91236**	1.06518	1.12115
11	000101101000				
0.70283	1.65301	0	**0.89493**	1.07007	1.12174
13	01110100101110				
0.69596	1.65864	0.0159	**0.88701**	1.07937	1.13911
15	0101100000011010				
0.7514	1.81337	0.00149	**0.90717**	1.07049	1.12905
17	010100111111001010				
0.75639	1.90022	0.01029	**0.90344**	1.07981	1.14966
19	01011001111110011010				
0.76631	1.70993	0.00457	**0.91554**	1.06063	1.10852
21	0101111100110011111010				
0.76351	1.63921	0.00215	**0.91946**	1.05534	1.09898
23	010001000011110000100010				
0.76083	1.63299	0.00243	**0.91578**	1.05663	1.09892
25	00000110010100101001100000				
0.75295	1.96116	0.01767	**0.9149**	1.05849	1.10449
27	0010011110101111010111100100				
0.73729	1.72636	0.0056	**0.90823**	1.06356	1.11321
29	011100010110111111011010001110				
0.762	2.01517	0.00188	**0.91675**	1.06227	1.11719
31	00101110100001000010000101110100				
0.77461	1.71819	0.01474	**0.91615**	1.06214	1.1124
33	0010000100101110000111010010000100				
0.75853	1.92385	0.00284	**0.90987**	1.06881	1.12656
35	010001011011100111111001110110100010				
0.7514	1.69548	0.00274	**0.91034**	1.06394	1.11401
37	00101100101111011100111011110100110100				
0.75851	1.99561	0.00583	**0.91042**	1.06825	1.12609
39	0001011001011110111001110111101001101000				
0.75826	1.66088	0.00985	**0.91563**	1.05695	1.10017
41	001000111011111001011110100111110111000100				
0.76478	1.69827	0.00561	**0.91332**	1.06215	1.11084

Table 4.20: Reciprocal polynomials having largest L_1 norm amongst the set of all reciprocal polynomials, odd degrees 7 to 41

degree Mahler	polynomial Maxmod	Minmod	$\mathbf{L_1}$	L_3	L_4
7	00000000				
0.35355	2.82843	0	**0.64757**	1.30023	1.52263
8	000000000				
0.33333	3	0.0051	**0.62671**	1.32518	1.56749
9	0000000000				
0.31623	3.16228	0	**0.60803**	1.34804	1.60886
10	00000000000				
0.30151	3.31662	0.00314	**0.5914**	1.36914	1.6473
11	000000000000				
0.28868	3.4641	0	**0.57637**	1.38876	1.68325
12	0000000000000				
0.27735	3.60555	0.00757	**0.56269**	1.4071	1.71705
13	00000000000000				
0.26726	3.74166	0.00331	**0.55027**	1.42433	1.74898
14	000000000000000				
0.2582	3.87298	0	**0.53886**	1.44059	1.77927
15	0000000000000000				
0.25	4	0	**0.52826**	1.456	1.80809
16	00000000000000000				
0.24254	4.12311	0.00226	**0.51849**	1.47063	1.8356
17	000000000000000000				
0.2357	4.24264	0.00284	**0.50936**	1.48458	1.86193
18	0000000000000000000				
0.22942	4.3589	0.001	**0.50081**	1.4979	1.88719
19	00000000000000000000				
0.22361	4.47214	0	**0.49274**	1.51067	1.91148
20	000000000000000000000				
0.21822	4.58258	0.00086	**0.48521**	1.52291	1.93488
21	0000000000000000000000				
0.2132	4.69042	0.00211	**0.47807**	1.53469	1.95747
22	00000000000000000000000				
0.20851	4.79583	0.00143	**0.47131**	1.54604	1.9793
23	000000000000000000000000				
0.20412	4.89898	0	**0.46489**	1.55698	2.00043
24	0000000000000000000000000				
0.2	5	0	**0.45882**	1.56756	2.02092

Table 4.21: Polynomials having smallest Mahler measure, smallest L_1 norm , largest L_3 and largest L_4 norms, degrees 7 to 24.

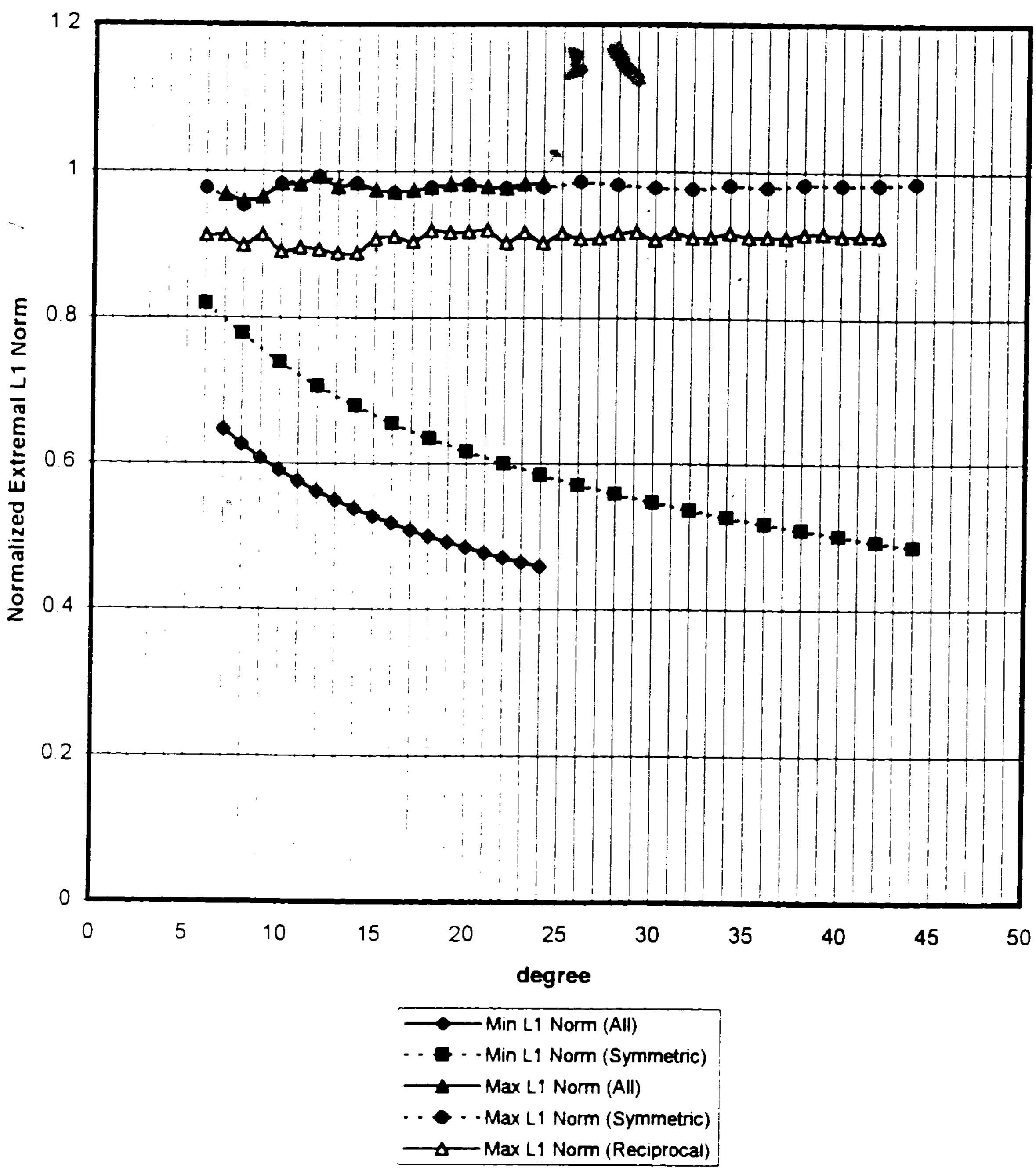

Figure 4.19: Extremal L_1 norms over all ± 1 polynomials of degree to 24, and over symmetric polynomials of degrees to 44.

degree Mahler	Polynomial max mod	 min mod	 L_1	 L_3	 L_4
8 0.58968	001100110 2.13437	 0.25238	 0.7781	 1.18634	 1.32412
10 0.5356	00110011001 2.35488	 0.2232	 0.73909	 1.22393	 1.38947
12 0.49387	0011001100110 2.55704	 0.20208	 0.70677	 1.25676	 1.44705
14 0.46046	001100110011001 2.74469	 0.18815	 0.67936	 1.28595	 1.49866
16 0.43297	00110011001100110 2.92052	 0.17656	 0.65569	 1.31224	 1.54554
18 0.40984	0011001100110011001 3.08647	 0.16469	 0.63494	 1.3362	 1.58858
20 0.39004	001100110011001100110 3.24404	 0.15625	 0.61654	 1.35822	 1.62842
22 0.37284	00110011001100110011001 3.39437	 0.1506	 0.60005	 1.37862	 1.66556
24 0.35773	0011001100110011001100110 3.53836	 0.14631	 0.58515	 1.39763	 1.7004
26 0.3443	001100110011001100110011001 3.67675	 0.13986	 0.57158	 1.41544	 1.73324
28 0.33228	00110011001100110011001100110 3.81015	 0.13494	 0.55915	 1.43221	 1.76432
30 0.32144	0011001100110011001100110011001 3.93905	 0.13096	 0.54769	 1.44807	 1.79385
32 0.31158	001100110011001100110011001100110 4.06388	 0.1256	 0.53709	 1.4631	 1.822
34 0.30258	00110011001100110011001100110011001 4.18501	 0.12098	 0.52723	 1.47741	 1.8489
36 0.29432	0011001100110011001100110011001100110 4.30273	 0.11701	 0.51803	 1.49106	 1.87469
38 0.28669	001100110011001100110011001100110011001 4.41733	 0.11363	 0.50942	 1.50411	 1.89945
40 0.27963	00110011001100110011001100110011001100110 4.52904	 0.11079	 0.50132	 1.51663	 1.92329
42 0.27307	0011001100110011001100110011001100110011001 4.63806	 0.10844	 0.49369	 1.52865	 1.94627
44 0.26694	001100110011001100110011001100110011001100110 4.74459	 0.10652	 0.48649	 1.54022	 1.96848

Table 4.22: Symmetric polynomials having largest L_3 and L_4 norms and smallest Mahler measure and L_1 norm among the set of all symmetric polynomials, degrees 8 to 44

degree	Polynomial				
Mahler	max mod	min mod	L_1	$\mathbf{L_3}$	L_4
7	00001101				
0.93412	1.38137	**0.56021**	**0.96761**	**1.03014**	**1.05737**
8	010000110				
0.90834	1.46305	0.32976	0.95795	**1.03593**	1.06703
9	0101111001				
0.92363	1.49869	0.35262	0.96446	**1.03138**	1.05948
10	00011101101*				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
11	000001100101				
0.96262	**1.28077**	**0.6306**	**0.98171**	**1.01718**	**1.03305**
12	0000011001010*				
0.98637	1.38675	0.83793	0.99276	**1.00816**	1.0173
13	00000110010101				
0.95556	1.60186	0.67253	0.97757	**1.02262**	1.0453
14	000111011101101*				
0.96589	1.29099	0.72036	0.98305	**1.01635**	1.03179
15	0001000111010010				
0.94411	**1.30838**	0.53678	**0.97347**	**1.02344**	**1.0439**
16	00100110000101011				
0.94067	1.39918	0.50775	**0.97106**	**1.02685**	**1.05128**
17	011001000011110101				
0.92364	1.41157	0	0.97298	**1.02003**	**1.03653**
18	0000101011110011011				
0.946	1.3648	0.22942	0.97611	**1.02016**	1.03795
19	00000101110100111001				
0.96188	1.3405	0.44721	**0.98192**	**1.01631**	**1.03103**
20	001111111001101010110*				
0.95984	1.32737	0.43167	**0.98201**	**1.01511**	**1.02826**
21	0000010011010100111000				
0.95411	1.38287	0.43486	**0.97818**	**1.01987**	**1.03806**
22	00011000000110110101010				
0.94858	1.45816	0.34554	0.97602	**1.02169**	**1.04174**
23	001100011111101010110110				
0.96237	1.36319	0.3978	**0.98244**	**1.01567**	**1.02988**
24	0001110000000101011011001				
0.96683	1.38019	0.48233	**0.98438**	**1.01427**	**1.02763**

Table 4.23: Polynomials having smallest L_3 norm , degrees 7 to 24.

degree Mahler	polynomial max mod	min mod	L_1	L_3	L_4
6	0100111				
0.94366	**1.17282**	**0.37796**	0.97665	**1.0168**	**1.0293**
8	000011010				
0.8917	1.37994	0.33333	**0.95274**	**1.03704**	**1.06703**
10	00011101101				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
12	0000011001010				
0.98637	1.38675	**0.83793**	**0.99276**	**1.00816**	**1.0173**
14	000001100110101				
0.96589	**1.29099**	**0.72036**	**0.98305**	**1.01635**	**1.03179**
16	00111100011010110				
0.93885	1.35059	0.57835	0.97025	**1.02715**	1.05128
18	0010111010000100001				
0.95499	1.40534	**0.60664**	**0.97755**	**1.02189**	**1.04287**
20	001111111001101010110				
0.95984	**1.32737**	0.43167	**0.98201**	**1.01511**	**1.02826**
22	00000001110011011010101				
0.95329	1.44352	**0.62554**	**0.9767**	**1.02288**	**1.04507**
24	0000001110001101101101010				
0.94154	1.32071	0.34021	0.974	**1.02133**	**1.03923**
26	000111100010001000100101101				
0.96894	1.31082	0.50748	**0.98548**	**1.01288**	**1.02446**
28	00011000111111101010110110010				
0.96268	1.40513	0.55709	**0.98163**	**1.01779**	**1.03498**
30	0000001101100101111001110010101				
0.93775	1.44802	0.17961	0.97367	**1.02099**	**1.03879**
32	001100000001111001101001010100110				
0.93704	1.41614	0.13298	0.9744	**1.02043**	**1.03816**
34	00000111110011011001110011010110101				
0.96105	1.35186	0.50709	**0.98109**	**1.0178**	**1.0345**
36	0011110000000001100110010101010010110				
0.93779	1.31325	0.29351	0.97381	**1.0202**	**1.03665**
38	000000001101101001001110000111001010101				
0.96383	1.40184	0.54716	**0.98265**	**1.01607**	**1.03107**
40	00011110001010101100110011111111011010010				
0.96104	1.36576	0.5019	**0.98169**	**1.01622**	**1.03068**
42	0001001101001001111111010101100011110011101				
0.94732	1.28797	0.09092	0.97985	**1.01535**	**1.02826**
44	000000001101101001001110110001111000110101010				
0.96756	1.32454	0.53456	0.98437	**1.0145**	**1.02794**

Table 4.24: Symmetric polynomials having smallest L_3 norm among the set of all symmetric polynomials, degrees 6 to 44

degree	polynomial				
Mahler	max mod	min mod	L_1	$\mathbf{L_3}$	L_4
6	0010100				
0.73584	1.466	0.0173	0.91138	**1.05649**	1.09715
8	000101000				
0.73686	1.66667	0.02946	0.89772	**1.07824**	1.1404
10	00110101100				
0.69266	1.68657	0.01504	0.88923	**1.0778**	1.13809
12	0011010101100				
0.66623	1.61303	0.0046	0.88382	**1.07274**	1.12322
14	000110101011000				
0.66565	1.65188	0.00838	0.88024	**1.08134**	1.14223
16	00001101010110000				
0.75069	1.69375	0.01837	0.91021	**1.06486**	1.11644
18	0001110110110111000				
0.76992	1.62142	0.00135	0.91833	**1.05695**	1.10128
20	000011001010100110000				
0.76291	1.76614	0.00259	0.9166	**1.05913**	1.10647
2[illegible]	00001100101010100110000				
0.72248	1.61865	0.00319	0.9001	**1.06955**	1.12222
24	0000101011001001101010000				
0.70928	1.79883	0.00568	0.89739	**1.0704**	1.12496
26	001101111010111010111101100				
0.7184	1.65635	0.00878	0.89877	**1.06881**	1.11998
28	00101011110011011001111010100				
0.77272	1.82595	0.00371	0.91502	**1.06427**	1.11696
30	0011101101000001000001011011100				
0.68806	1.68261	0.00036	0.89383	**1.07083**	1.12421
32	000111110111011010110111011111000				
0.75696	1.6848	0.00114	0.90913	**1.06581**	1.11728
34	00100110000101011111010100001100100				
0.75273	1.64131	0.0012	0.91591	**1.05589**	1.09777
36	0010111010000100001000010000101110100				
0.75264	1.80839	0.00074	0.91087	**1.06409**	1.11514
38	000101110110111100111001111011011101000				
0.75157	1.67396	0.00552	0.91477	**1.05726**	1.10051
40	00001011101101111001110011110110111010000				
0.75099	1.74136	0.00262	0.91233	**1.06107**	1.10838
42	0001001011110011011101011101100111101001000				
0.75838	1.69565	0.00246	0.91252	**1.06325**	1.11355

Table 4.25: Reciprocal polynomials having smallest L_3 norm among the set of all reciprocal polynomials, even degrees 6 to 42

degree	polynomial				
Mahler	max mod	min mod	L_1	$\mathbf{L_3}$	L_4
7	01000010				
0.7753	1.68485	0.02524	0.91135	**1.07049**	1.12905
9	0001001000				
0.75333	1.89737	0.00399	0.91236	**1.06518**	1.12115
11	000101101000				
0.70283	1.65301	0	0.89493	**1.07007**	1.12174
13	01110100101110				
0.69596	1.65864	0.0159	0.88701	**1.07937**	1.13911
15	0101100000011010				
0.7514	1.81337	0.00149	0.90717	**1.07049**	1.12905
17	010110000000011010				
0.7059	1.67591	0.00189	0.89727	**1.06[illegible]**	1.11568
19	01011001111110011010				
0.76631	1.70993	0.00457	0.91554	**1.06063**	1.10852
21	0101111100110011111010				
0.76351	1.63921	0.00215	0.91946	**1.05534**	1.09898
23	010001000011110000100010				
0.76083	1.63299	0.00243	0.91578	**1.05663**	1.09892
25	00000110010100101001100000				
0.75295	1.96116	0.01767	0.9149	**1.05849**	1.10449
27	0010011110101111010111100100				
0.73729	1.72636	0.0056	0.90823	**1.06356**	1.11321
29	011100010110111111011010001110				
0.762	2.01517	0.00188	0.91675	**1.06227**	1.11719
31	00101110100001000010000101110100				
0.77461	1.71819	0.01474	0.91615	**1.06214**	1.1124
33	0111100101000100110010001010011110				
0.73366	1.58453	0.01237	0.90527	**1.06411**	1.11175
35	011000110100010111111010001011000110				
0.71337	1.61929	0	0.90893	**1.05895**	1.10268
37	01000110000000101101101101000000001100010				
0.718	1.62221	0.00237	0.90348	**1.06335**	1.11024
39	0001011001011110111001110111101001101000				
0.75826	1.66088	0.00985	0.91563	**1.05695**	1.10017
41	001000111011111001011110100111110111000100				
0.76478	1.69827	0.00561	0.91332	**1.06215**	1.11084

Table 4.26: Reciprocal polynomials having smallest L_3 norm among the set of all reciprocal polynomials, odd degrees 7 to 41

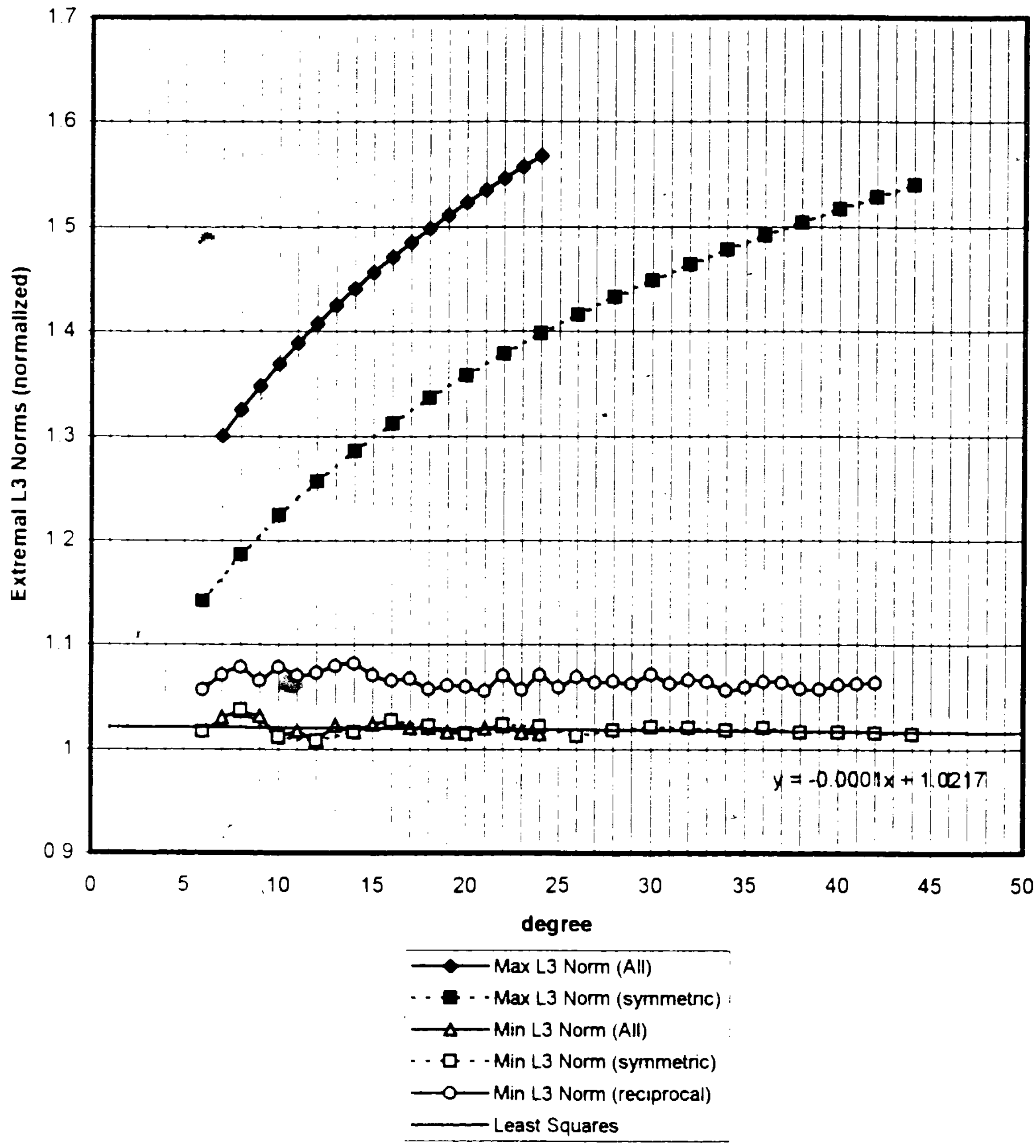

Figure 4.20: Extremal L_3 norms over all ± 1 polynomials, over symmetric and reciprocal polynomials. Linear least squares estimate for minimum L_3 among symmetric polynomials is shown.

degree Mahler	polynomial Max mod	Min mod	L_1	L_3	L_4
7	00001101				
0.93412	1.38137	**0.56021**	**0.96761**	**1.03014**	**1.05737**
8	010000110				
0.90834	1.46305	0.32976	0.95795	1.03593	**1.06703**
9	0110000101				
0.92363	1.49869	0.35262	0.96446	1.03138	**1.05948**
10	00011101101*				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
11	000001100101				
0.96262	**1.28077**	**0.6306**	**0.98171**	**1.01718**	**1.03305**
12	0000011001010*				
0.98637	1.38675	0.83793	0.99276	1.00816	**1.0173**
13	01010011000001				
0.95092	1.60059	0.57465	0.97587	1.02314	**1.0453**
14	010010001000111*				
0.96589	1.29099	0.72036	0.98305	1.01635	**1.03179**
15	0001000111010010				
0.94411	**1.30838**	0.53678	**0.97347**	**1.02344**	**1.0439**
16	00100110000101011				
0.94067	1.39918	0.50775	**0.97106**	**1.02685**	**1.05128**
17	011001000011110101				
0.92364	1.41157	0	0.97298	**1.02003**	**1.03653**
18	0000101011110011011				
0.946	1.3648	0.22942	0.97611	1.02016	**1.03795**
19	00000101110100111001				
0.96188	1.3405	0.44721	**0.98192**	**1.01631**	**1.03103**
20	001111111001101010110*				
0.95984	1.32737	0.43167	**0.98201**	**1.01511**	**1.02826**
21	0000010011010100111000				
0.95411	1.38287	0.43486	**0.97818**	**1.01987**	**1.03806**
22	00011000000110110101010				
0.94858	1.45816	0.34554	0.97602	**1.02169**	**1.04174**
23	001100011111101010110110				
0.96237	1.36319	0.3978	**0.98244**	**1.01567**	**1.02988**
24	0001110000000101011011001				
0.96683	1.38019	0.48233	**0.98438**	**1.01427**	**1.02763**
Asymptotic value predicted by Newman-Byrnes conjecture					**1.04664**

Table 4.27: Polynomials having smallest normalized L_4 norms , degrees 7 to 24. The asymptotic value of $\min(L_4/\mu_n)$ predicted by Newman and Byrnes in Conjecture 1 is also shown

degree Mahler	Polynomial Max mod	 Min mod	 L_1	 L_3	 L_4
6	0100111				
0.94366	**1.17282**	**0.37796**	0.97665	**1.0168**	**1.0293**
8	000011010				
0.8917	1.37994	0.33333	**0.95274**	**1.03704**	**1.06703**
10	00011101101				
0.95466	**1.14643**	0.30151	**0.98243**	**1.01172**	**1.02005**
12	0000011001010				
0.98637	1.38675	**0.83793**	**0.99276**	**1.00816**	**1.0173**
14	000001100110101				
0.96589	**1.29099**	**0.72036**	**0.98305**	**1.01635**	**1.03179**
16	00111110011010110				
0.93885	1.35059	0.57835	**0.97025**	**1.02715**	**1.05128**
18	0010111010000100001				
0.95499	1.40534	**0.60664**	**0.97755**	**1.02189**	**1.04287**
20	001111111001101010110				
0.95984	**1.32737**	0.43167	**0.98201**	**1.01511**	**1.02826**
22	00000001110011011010101				
0.95329	1.44352	**0.62554**	**0.9767**	**1.02288**	**1.04507**
24	0000001110001101101101010				
0.94154	1.32071	0.34021	0.974	**1.02133**	**1.03923**
26	000111100010001000100101101				
0.96894	1.31082	0.50748	**0.98548**	**1.01288**	**1.02446**
28	00011000111111101010110110010				
0.96268	1.40513	0.55709	**0.98163**	**1.01779**	**1.03498**
30	0000001101100101111001110010101				
0.93775	1.44802	0.17961	0.97367	**1.02099**	**1.03879**
32	001100000001111001101001010100110				
0.93704	1.41614	0.13298	0.9744	**1.02043**	**1.03816**
34	00000111110011011001110011010110101				
0.96105	1.35186	0.50709	**0.98109**	**1.0178**	**1.0345**
36	0011110000000001100110010101010010110				
0.93779	1.31325	0.29351	0.97381	**1.0202**	**1.03665**
38	000000001101101001001110000111001010101				
0.96383	1.40184	0.54716	**0.98265**	**1.01607**	**1.03107**
40	00011110001010101100110011111111011010010				
0.96104	1.36576	0.5019	**0.98169**	**1.01622**	**1.03068**
42	0001001101001001111111010101100011110011101				
0.94732	1.28797	0.09092	0.97985	**1.01535**	**1.02826**
44	000000001101101001001110110001111000110101010				
0.96756	1.32454	0.53456	0.98437	**1.0145**	**1.02794**
Asymptotic value predicted by Newman-Byrnes conjecture					**1.04664**

Table 4.28: Symmetric polynomials having smallest L_4 norm among the set of all symmetric polynomials, degrees 6 to 44. The asymptotic value of $\min(L_4/\mu_n)$ predicted by Newman and Byrnes in Conjecture 4 is also shown

degree Mahler	polynomial Max mod	Min mod	L_1	L_3	$\mathbf{L_4}$
6	0010100				
0.73584	1.466	0.0173	0.91138	1.05649	**1.09715**
8	000101000				
0.73686	1.66667	0.02946	0.89772	1.07824	**1.1404**
10	00110101100				
0.69266	1.68657	0.01504	0.88923	1.0778	**1.13809**
12	0011010101100				
0.66623	1.61303	0.0046	0.88382	1.07274	**1.12322**
14	000110101011000				
0.66565	1.65188	0.00838	0.88024	1.08134	**1.14223**
16	00001101010110000				
0.75069	1.69375	0.01837	0.91021	1.06486	**1.11644**
18	0001110110110111000				
0.76992	1.62142	0.00135	0.91833	1.05695	**1.10128**
20	000011001010100110000				
0.76291	1.76614	0.00259	0.9166	1.05913	**1.10647**
22	01011001111111110011010				
0.72248	1.61865	0.00319	0.9001	1.06955	**1.12222**
24	0000101011001001101010000				
0.70928	1.79883	0.00568	0.89739	1.0704	**1.12496**
26	001101111010111010111101100				
0.7184	1.65635	0.00878	0.89877	1.06881	**1.11998**
28	00011110110111011101101111000				
0.73414	1.67207	0.00437	0.90445	1.06501	**1.11353**
30	0011101101000001000001011011100				
0.68806	1.68261	0.00036	0.89383	1.07083	**1.12421**
32	000100110101111000111101011001000				
0.67335	1.65316	0.00024	0.89523	1.06627	**1.11464**
34	00100110000101011111010100001100100				
0.75273	1.64131	0.0012	0.91591	1.05589	**1.09777**
36	0000100111001011101011101001110010000				
0.74315	1.67661	0.01262	0.90764	1.0642	**1.11303**
38	000101110110111100111001111011011101000				
0.75157	1.67396	0.00552	0.91477	1.05726	**1.10051**
40	00001011101101111001110011110110111010000				
0.75099	1.74136	0.00262	0.91233	1.06107	**1.10838**
42	0001001011110011011101011101100111101001000				
0.75838	1.69565	0.00246	0.91252	1.06325	**1.11355**

Table 4.29: Reciprocal polynomials having smallest L_4 norm among the set of all reciprocal polynomials, even degrees 6 to 42.

degree Mahler	polynomial Max mod	 Min mod	 L_1	 L_3	 **L_4**
7 0.7753	01000010 1.68485	 0.02524	 0.91135	 1.07049	 **1.12905**
9 0.75333	0001001000 1.89737	 0.00399	 0.91236	 1.06518	 **1.12115**
11 0.70283	000101101000 1.65301	 0	 0.89493	 1.07007	 **1.12174**
13 0.65773	01000111100010 1.60689	 0.00692	 0.87517	 1.08164	 **1.13911**
15 0.69863	0010001111000100 1.62509	 0.00001	 0.89189	 1.07382	 **1.12905**
17 0.6947	000010110011010000 1.60944	 0.00375	 0.89237	 1.0678	 **1.11568**
19 0.76631	01011001111110011010 1.70993	 0.00457	 0.91554	 1.06063	 **1.10852**
21 0.76351	0101111100110011111010 1.63921	 0.00215	 0.91946	 1.05534	 **1.09898**
23 0.73926	000010100110011001010000 1.63299	 0.00324	 0.91185	 1.05712	 **1.09892**
25 0.75295	00000110010100101001100000 1.96116	 0.01767	 0.9149	 1.05849	 **1.10449**
27 0.73729	0010011110101111010111100100 1.72636	 0.0056	 0.90823	 1.06356	 **1.11321**
29 0.75481	001101111101011110101111101100 1.82574	 0.0036	 0.91207	 1.06436	 **1.11719**
31 0.77461	00101110100001000010000101110100 1.71819	 0.01474	 0.91615	 1.06214	 **1.1124**
33 0.73366	0111100101000100110010001010011110 1.58453	 0.01237	 0.90527	 1.06411	 **1.11175**
35 0.71337	011000110100010111111010001011000110 1.61929	 0	 0.90893	 1.05895	 **1.10268**
37 0.718	01000110000001011011011010000001100010 1.62221	 0.00237	 0.90348	 1.06335	 **1.11024**
39 0.75826	0001011001011110111001110111101001101000 1.66088	 0.00985	 0.91563	 1.05695	 **1.10017**
41 0.75002	010110111101110100011110001011101111011010 1.64656	 0.00067	 0.91002	 1.06285	 **1.11084**

Table 4.30: Reciprocal polynomials having smallest L_4 norm among the set of all reciprocal polynomials, odd degrees 7 to 41.

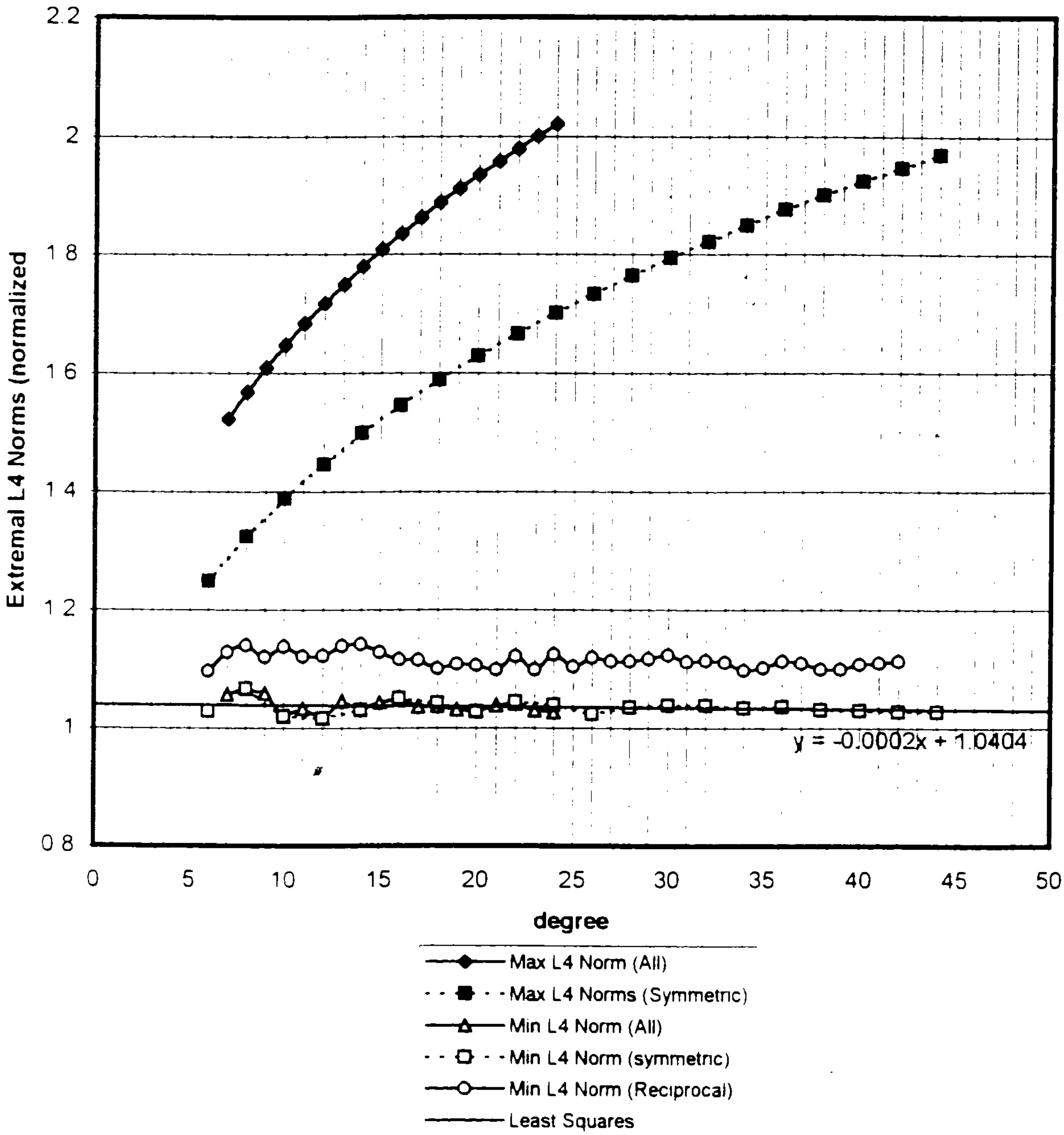

Figure 4.21: Extremal L_4 norms over all ± 1 polynomials, over symmetric and reciprocal polynomials. Linear least squares estimate for minimum L_4 among symmetric polynomials is shown.

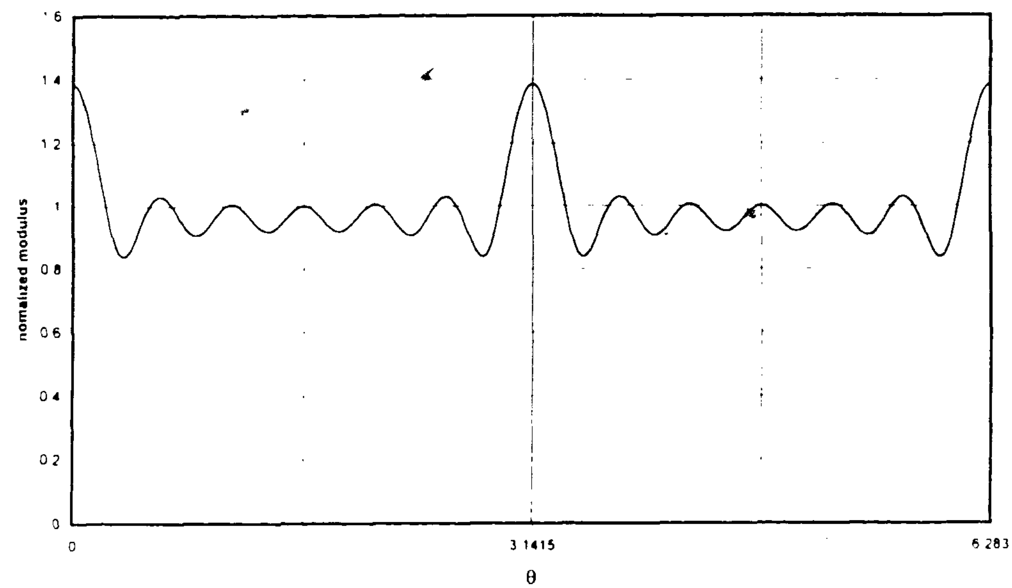

Figure 4.22: Highest minimum modulus for degree 12—0000011001010: a "particularly striking" polynomial, according to Littlewood.

4.4 Some Modular Plots of Extreme Polynomials

In [12], Littlewood provided the modular plot of polynomials having highest minimum modulus for degrees 12, 14, 18 and 20. His plots where on the interval $[0, \pi/2]$, the rest, he says, "coming by symmetries". Carrol, Eustice and Figiel [6] point out that a special subset of even degree ± 1 polynomials (namely the symmetric polynomials) display this symmetry.

Littlewood singled out the highest minimum modulus degree 12 polynomial as being a "particularly striking" example of a flat polynomial. It has an exceptionally high minimum modulus, ≈ 0.8379. The plot of this polynomial on the interval $[0, 2\pi]$ is given in Figure 4.22.

Figure 4.23 shows a high (but not known to be extremal) minimum modulus degree 64 polynomial. Figure 4.24 depict the highest minimum modulus (also the flattest) degree 44 symmetric polynomial. This figure also contains a plot of the symmetric polynomial having the highest *maximum* modulus, since the "flat" nature of the other polynomial is better appreciated when seen in comparison to a polynomial with a fairly high maximum modulus.

Figure 4.25 depicts the highest minimum modulus symmetric polynomial of degree 32.

Not all flat and high minimum modulus polynomials are symmetric. There are plenty of

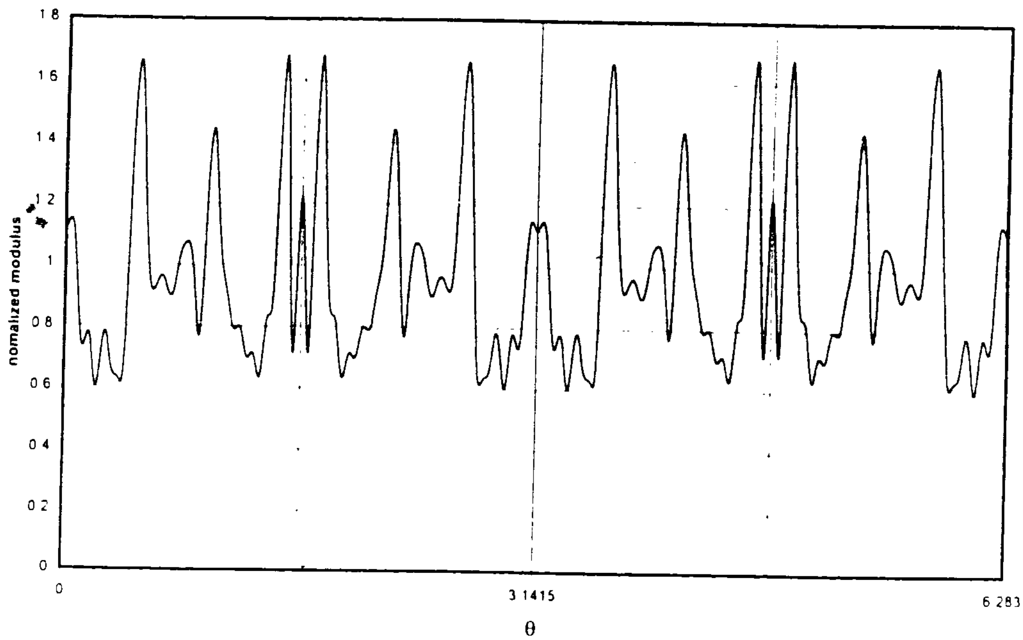

Figure 4.23: High minimum modulus symmetric, degree 64

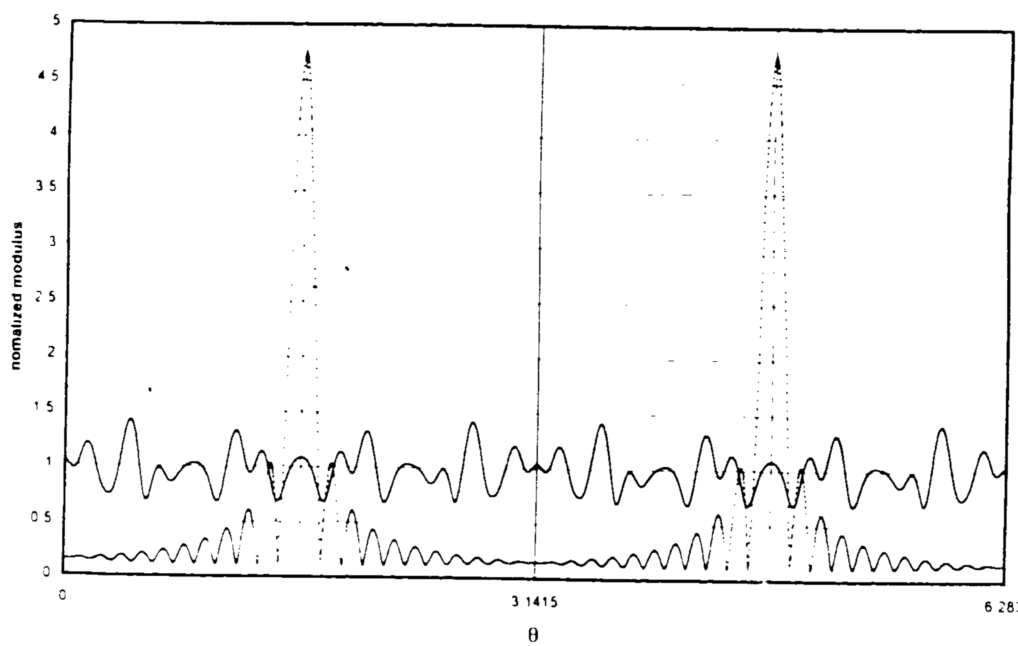

Figure 4.24: Highest minimum modulus symmetric (solid) plotted with highest maximum modulus symmetric (dotted), degree 44

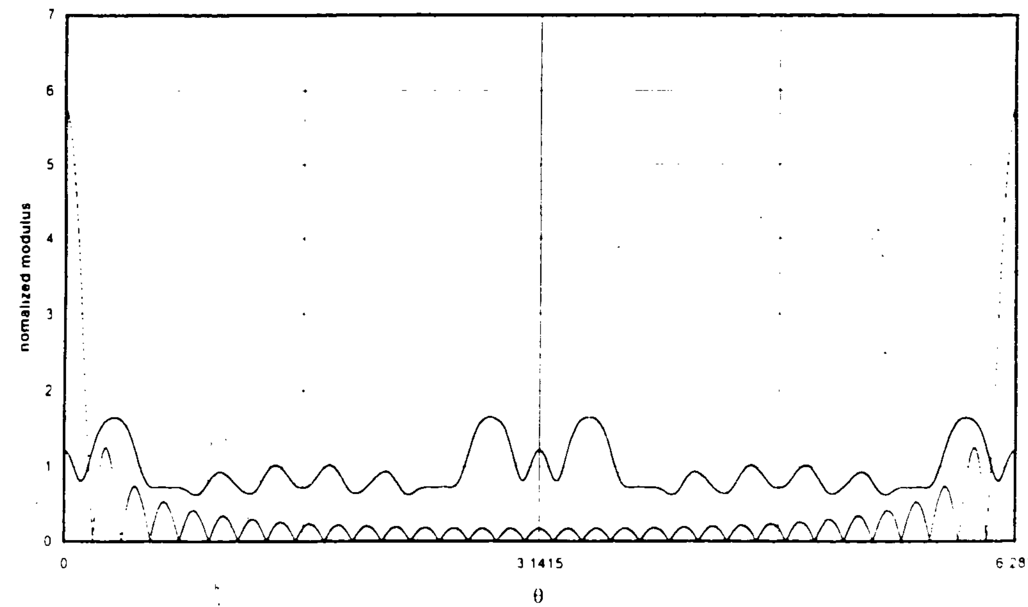

Figure 4.25: Highest minimum modulus symmetric degree 32 polynomial 00000000011111001100101001010101010, and the highest maximum modulus degree 32 polynomial 00000000000000000000000000000000

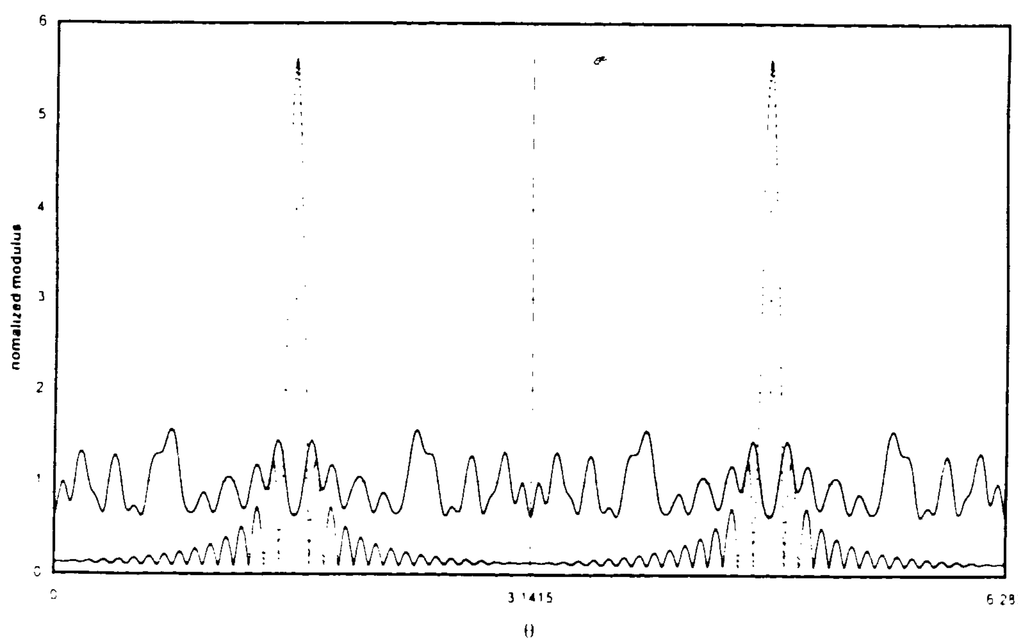

Figure 4.26: High minimum modulus symmetric degree 62 polynomial 00000111011111000101100100011010000110111001111011010110110101, together with high maximum modulus symmetric polynomial 0011001100110011001100110011001100110011001100110011001100110011001

odd degree and non-symmetric even degree polynomials that have these characteristic. A example of a non-symmetric "flattish" polynomial is given in Figure 4.27.

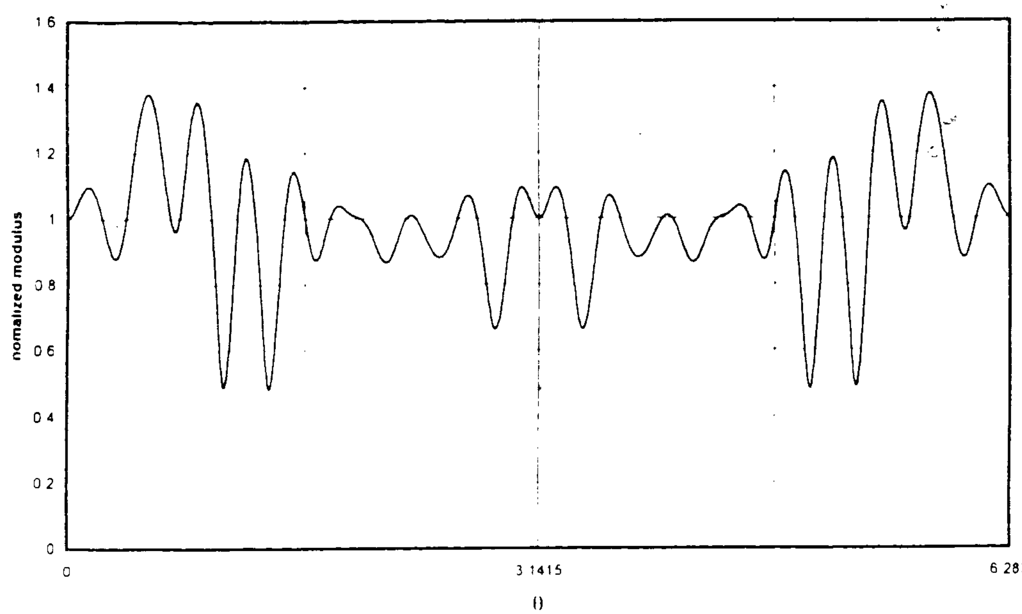

Figure 4.27: An example of a non-symmetric "flat" polynomial, degree 24 0001110000000101011011001

This section represents only a small selection of modular plots. The Java PolyApplets that were written for this thesis, and are described in the Appendix, allow the modular plots for your choice of ± 1 polynomials to be easily viewed.

Chapter 5

Observations and Conclusions

5.1 Evidence on the One and Two-sided Conjectures

For each degree up to 24, and for even degrees up to 64, we see there are ± 1 polynomials having a normalized minimum modulus that is bounded well away from zero. Furthermore, it appears polynomials with a high minimum modulus also tend to have a below average to average maximum modulus. Figure 5.1 plots the highest minimum modulus for each degree along with the associated maximum modulus of each polynomial.

Polynomial having highest minimum modulus are not usually the "flattest" for the degree. Figure 5.2 depicts a selection of symmetric polynomials that have a fairly flat modulus with

$$0.4 \leq \frac{|P_n(z)|}{\mu_n} \leq 1.5$$

for even degrees between 10 and 44. (Note that these polynomials were found on lists containing polynomials with high Mahler measure.) If we are concerned only with the one-sided inequality, we may improve on the lower bound, finding polynomials of degrees 10 to 23 and even degrees up to 64 that satisfy

$$0.55 \leq \frac{|P_n(z)|}{\mu_n}$$

As the degree increases, the lower bound appears to gradually drift upwards, giving

$$0.6 \leq \frac{|P_n(z)|}{\mu_n}$$

for even degrees between 32 and 64.

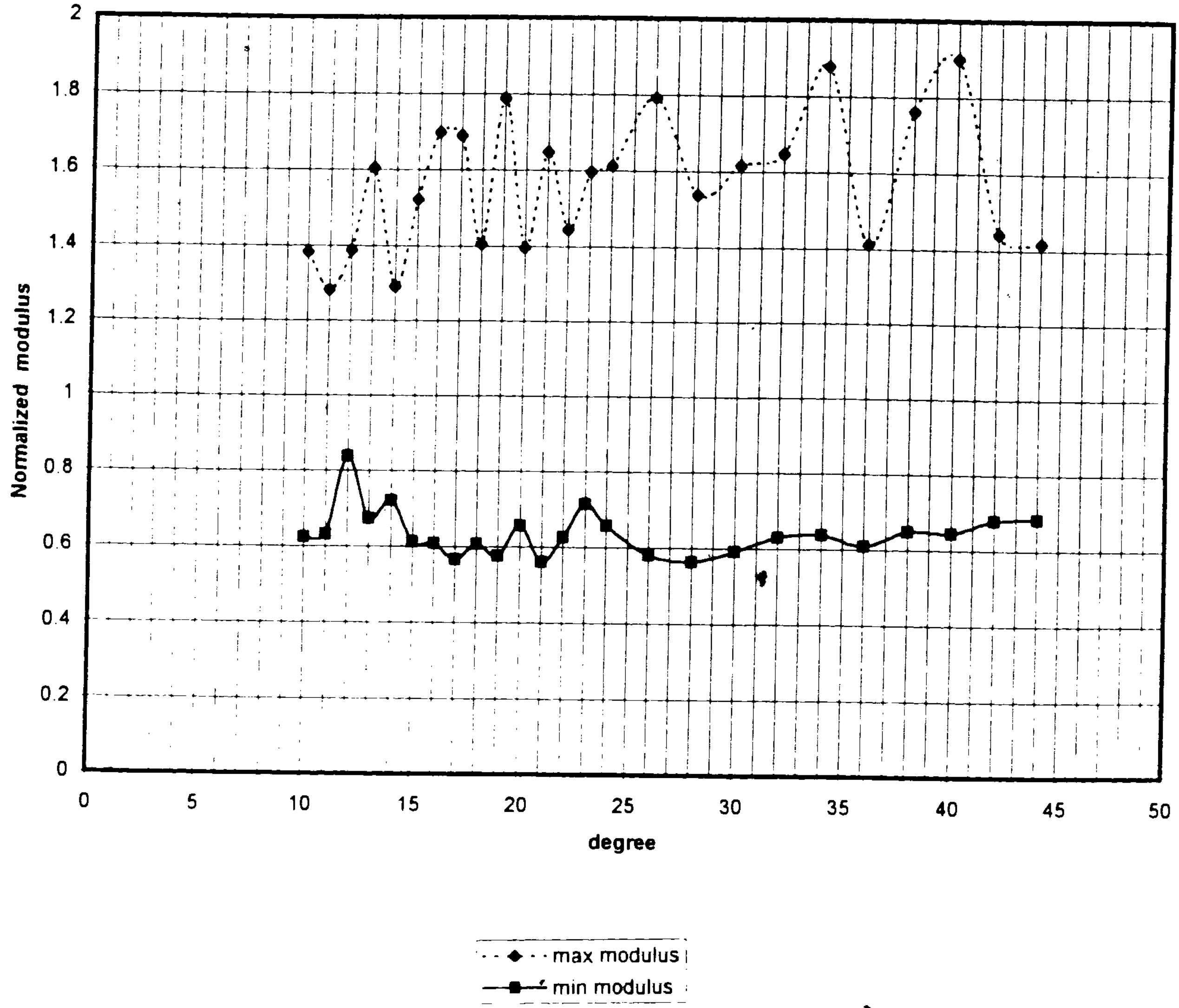

Figure 5.1: For each degree shown, the maximum and minimum modulus of the polynomial attaining the highest minimum modulus (highest among the symmetrics for degrees > 24) is plotted

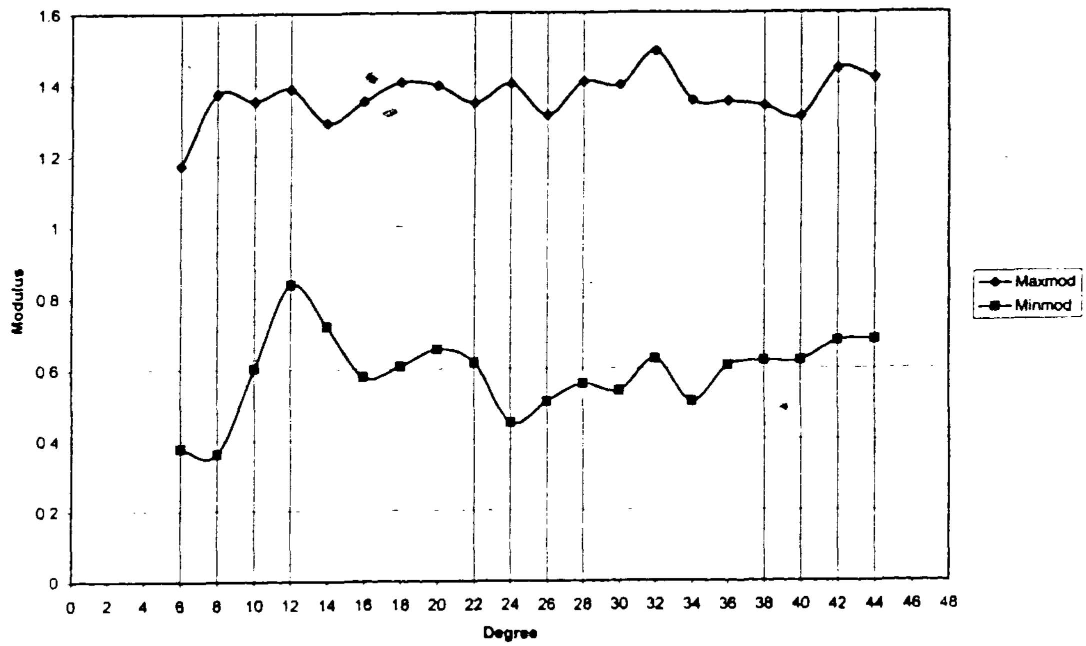

Figure 5.2: The maximum and associated minimum modulus of a selection of flat polynomials

5.2 Evidence on Conjectures about Norms

5.2.1 Evidence on the Newman-Byrnes Conjecture

In 1990 Newman and Byrnes [16] conjectured that the minimum normalized L_4 norm among all ± 1 polynomials of the same degree is asymptotically about 1.04664 (Conjecture 4). They say that their conjecture is based on "extensive numerical evidence" but do not indicate what degrees were studied. Data gathered for this thesis up to degree 44 do not provide evidence that would suggest the conjecture is true. The extreme value of the L_4 norm was determined to degree 24, and an upper bound for that value for even degrees 26 to 44. Figure 5.3 reveals that the smallest L_4 norms do not fall along a smooth curve and appear to drift downwards from the predicted asymptote.

5.2.2 Evidence on the Borwein-Lockhart Conjecture

Borwein and Lockhart have conjectured that $\frac{E(L_p)}{\mu_n} \to \Gamma(1+p/2)^{\frac{1}{p}}$ as $n \to \infty$. (Conjecture 5) Expected values of L_3 and L_4 norms were computed to degree 24. The normalized values are

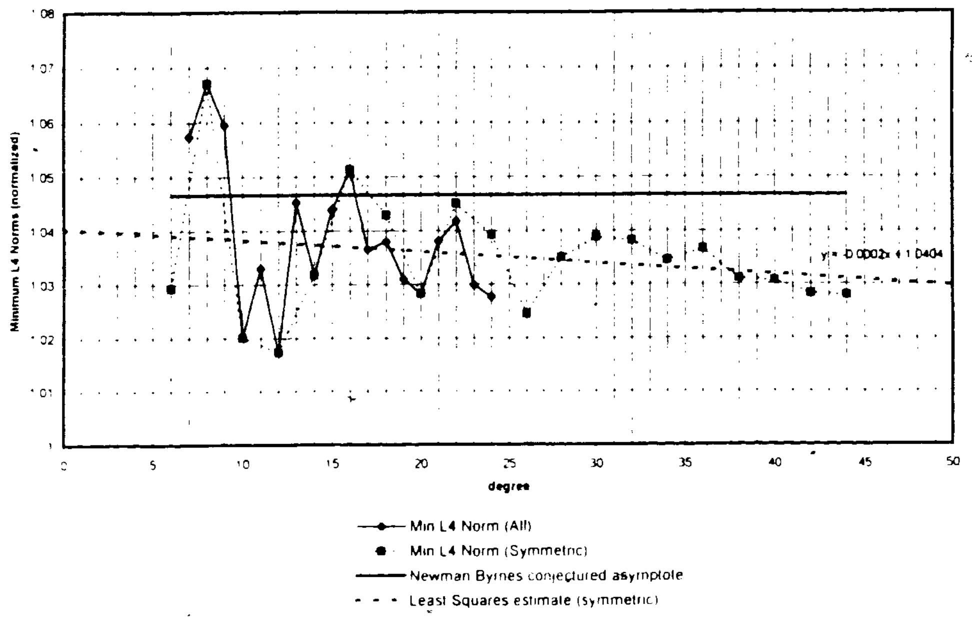

Figure 5.3: Minimum L_4 norms and asymptotic value conjectured by Newman and Byrnes

plotted against degree in Figure 5.4. The data falls along smooth curves that approach the predicted asymptotes. $E(L_3^3)/\mu_n^3$ and $E(L_4^4)/\mu_n^4$ are also shown, along with the asymptotic values proven by Borwein and Lockhart in Theorem 7.

5.3 More Observations about Maximum and Minimum the Modulus

Symmetric polynomials tend to have higher minimum modulus than non-symmetric polynomials. This can be seen for degree 14 in the histogram depicted in Figure 5.6, where the distributions of minimum modulus for symmetric and non-symmetric degree 14 polynomials are displayed.

Figure 5.7 depicts symmetric polynomials as a percent of all ± 1 polynomials and as a percent of a list of 100 ± 1 polynomial having highest minimum modulus. Symmetric degree 22 polynomials, for example, constitute only about 0.0005 percent of all degree 22 ± 1 polynomials, yet 20 of the top 100 minimum modulus degree 22 polynomials are symmetric. However, not all symmetric polynomials have a high minimum modulus—the next section

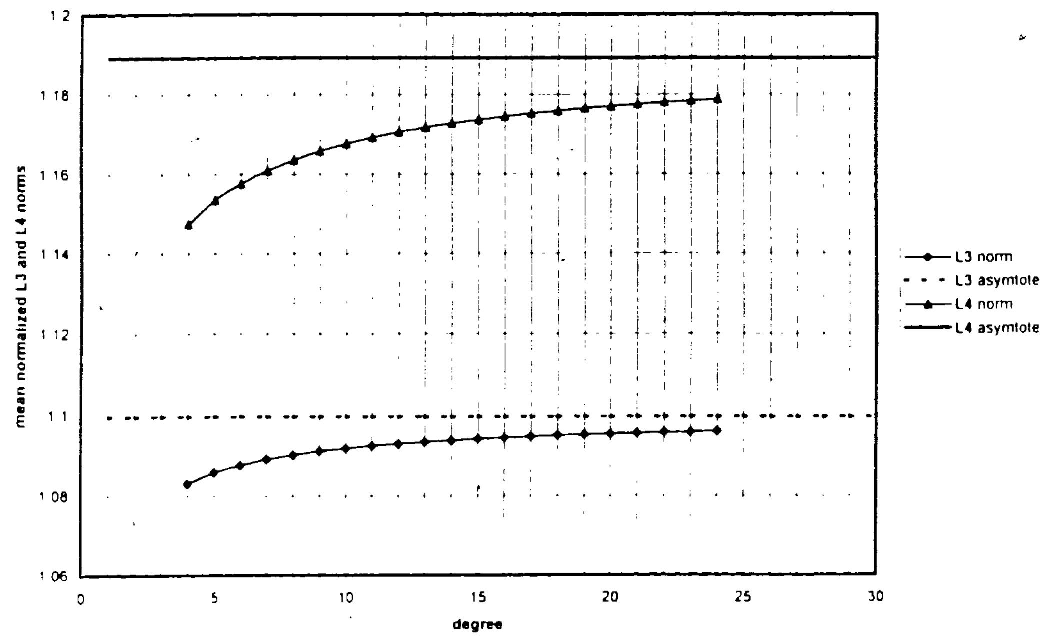

Figure 5.4: Mean normalized L_3 and L_4 norms and asymptotic values conjectured by Borwein and Lockhart

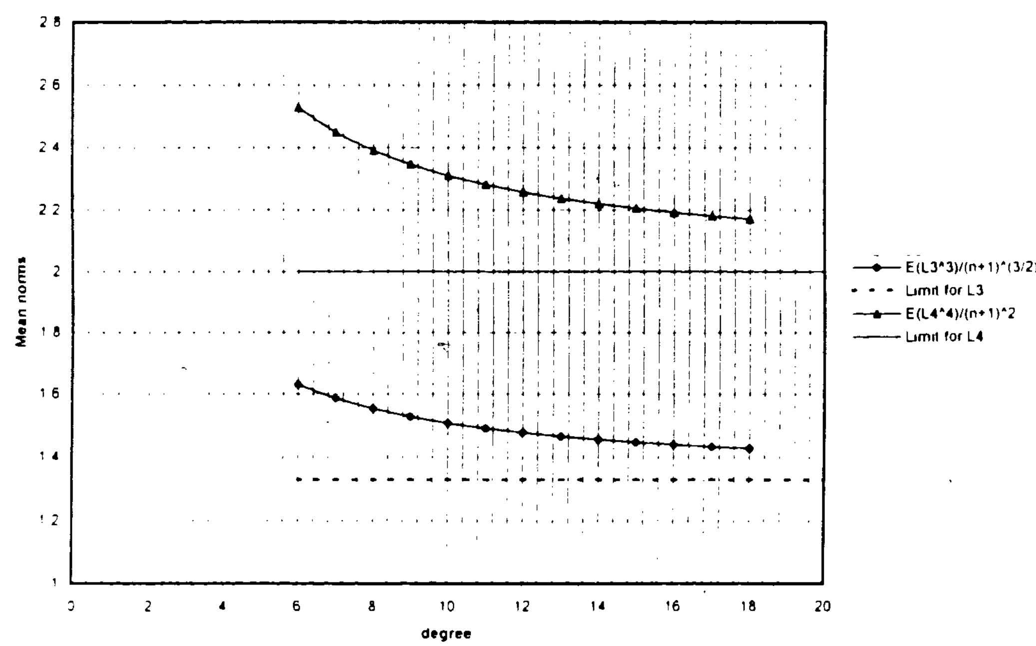

Figure 5.5: $E(L_3^3)/\mu_n^3$ and $E(L_4^4)/\mu_n^4$ and asymptotic values proven by Borwein and Lockhart

describes a series of symmetric polynomials having a maximum minimum modulus tending to zero.

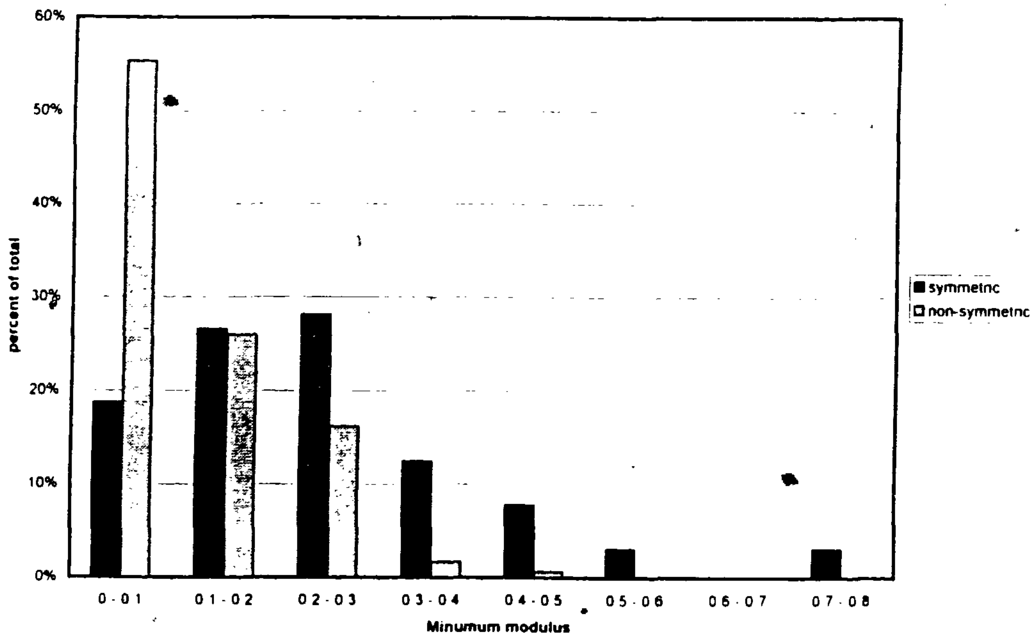

Figure 5.6: Distribution of minimum modulus for degree 14, symmetric and non-symmetric

Carroll, Eustice and Figiel [6] note that Littlewood's table containing high minimum modulus polynomials (in [12]) reveals that the extremes are attained by symmetrics for the even degrees from 10 to 20. They mention that it "would be interesting to know" whether or not the extremal polynomials must always be symmetric for even degrees greater than 10. Data gathered for this thesis indicate that the result is true for all even degrees from 10 up to and including 26, but it appears that it is not true for degree 28. The highest minimum modulus degree 28 polynomial is not symmetrical, although its maximum modulus is only slightly higher than that of the extreme symmetrical for the same degree, based on evaluating the modulii at 28000 evenly spaced points on $[0, \pi]$. The modulii of the highest symmetric and overall highest degree 28 polynomials are compared in Figures 5.8 and 5.9. There are two non-symmetric degree 28 polynomials having a higher minimum modulus than the extremal among the symmetrics. (see Table 5.1)

Properties of symmetric, reciprocal and non-symmetric degree 14 polynomials are shown in Figure 5.10, which is a scatter-plot of maximum vs. minimum modulus. Each dot in this

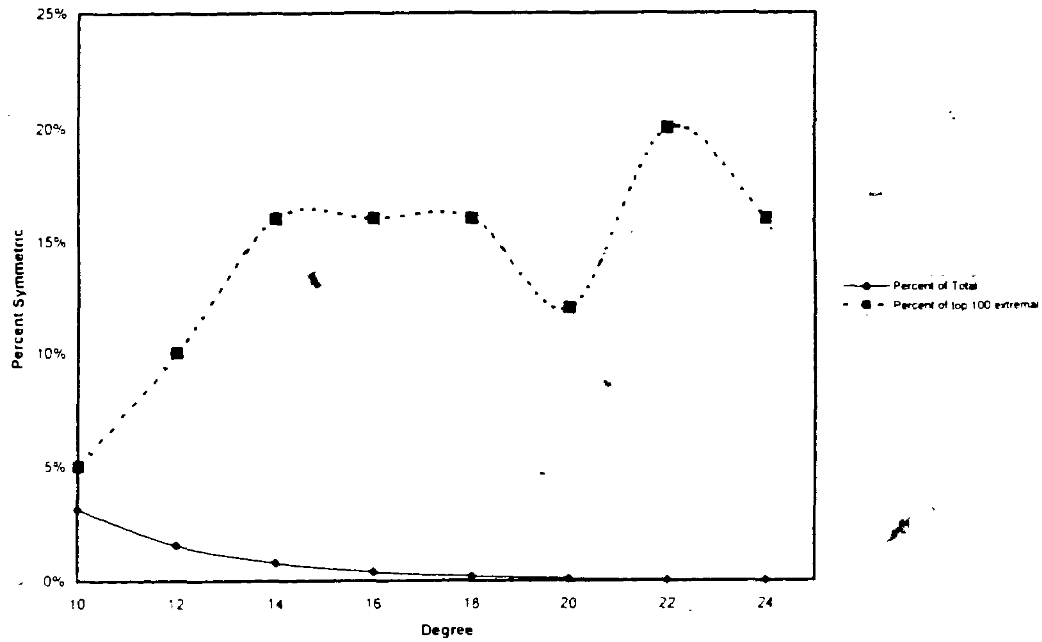

Figure 5.7: Symmetric polynomials as a percent of total and a percent of 100 high minimum modulus polynomials

degree / Mahler	polynomial / max mod	min mod	L_1	L_3	L_4
28	00000000001101110101100001111				
	2.08240	**0.57407**	0.93191	1.07515	1.15119
28	00010101000000100100111001110				
	1.52503	0.56610	0.96652	1.03279	1.06379
28	00000011110001100100101101010*				
	1.53744	0.56276	0.96092	1.03866	1.07545

Table 5.1: The three degree 28 polynomials having highest minimum modulus, based on evaluating the moduli at 28000 points in $[0, \pi]$. The * indicates a symmetric polynomial and bold type indicates the extremal value.

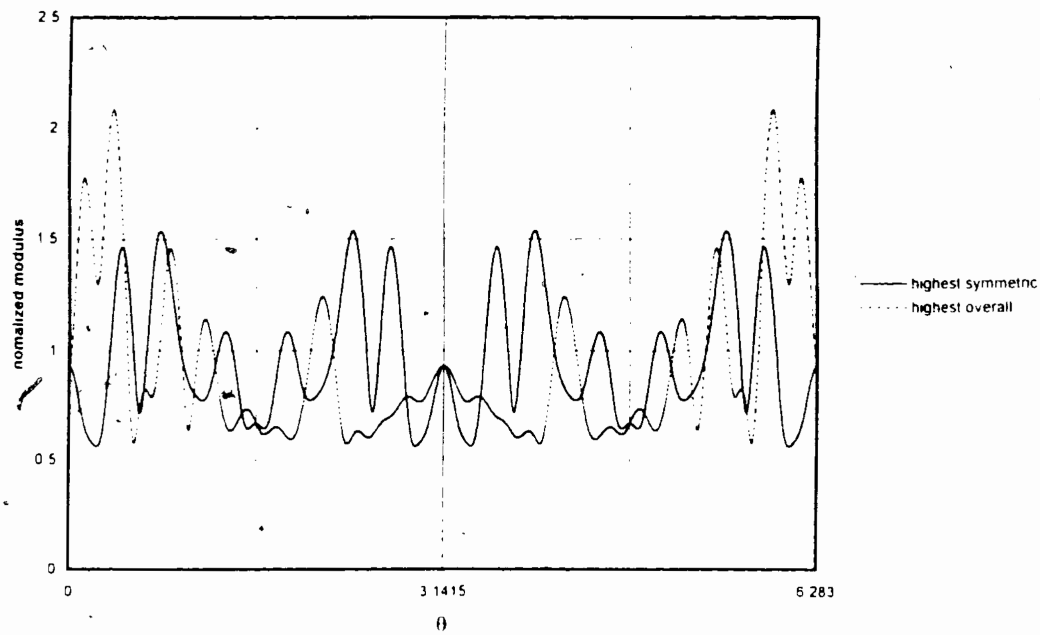

Figure 5.8: Highest minimum modulus degree 28 polynomial, compared to the highest minimum modulus symmetric.

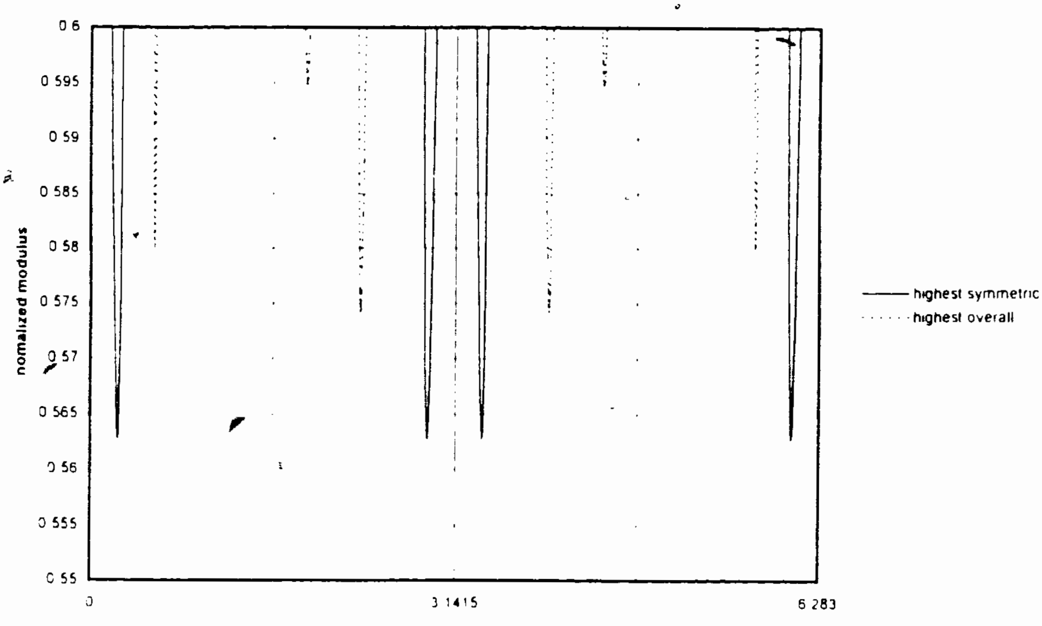

Figure 5.9: Close up of modulus of polynomials shown above.

plot represents a ± 1 polynomial of degree 14. A dot is located in such a way that the maximum modulus of the polynomial it represents may be read along the x-axis and the minimum modulus along the y-axis.

As seen in Figure 5.10 for degree 14, the reciprocal polynomials have a minimum modulus at or near zero. In fact, odd degree reciprocal polynomials of the form $P(z) = z^n P(1/z)$ must always have a zero at $z = -1$, since such polynomials are of the form

$$P(z) = a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \ldots + a_2 z^2 + a_1 z + a_0$$

Since n is odd, we have

$$P(-1) = -a_0 + a_1 - a_2 + \ldots + a_2 - a_1 + a_0 = 0$$

Numerical evidence strongly suggests that even degree reciprocals must also have zeros on the unit circle, although the zeros are not located in such regular places as the they are for the odd degree reciprocals.

Even degree polynomials having the property that $a_k = -a_{n-k}$ except when $k = n/2$ are a kind of "semi-reciprocal" because $P(z)$ is the same as $-z^n P(1/z)$ except for the middle coefficient. Curiously, these "semi-reciprocals" appear to always have a minimum modulus of 1 ($1/\mu_n$ when normalized), attained at $z = 1$ and $z = -1$, among other places. In Figure 5.10, the horizontal line of dots with minimum modulus of $1/\mu_4 \approx 0.258$ is composed mainly of these "semi-reciprocals".

Note that the upper right-hand quadrant of the chart in Figure 5.10, where we would expect to find polynomials having a high maximum and a high minimum modulus, is lacking in points. (This is also true for degrees less than 14. Degrees greater than 14 have too many points for the graphing software to handle.) Perhaps it is not possible for a polynomials to have both a high minimum modulus and a high maximum modulus. This observation leads to the following conjecture:

Conjecture 6 *If there is an infinite sequence of* ± 1 *polynomials that satisfies the Minimum Modulus Conjecture, then it also satisfies the Two-sided conjecture.*

This conjecture is only weakly supported by the data, since the maximum modulus of polynomials attaining the highest minimum modulus, while remaining below average, appears to gradually drift upwards. (see Figure 5.1)

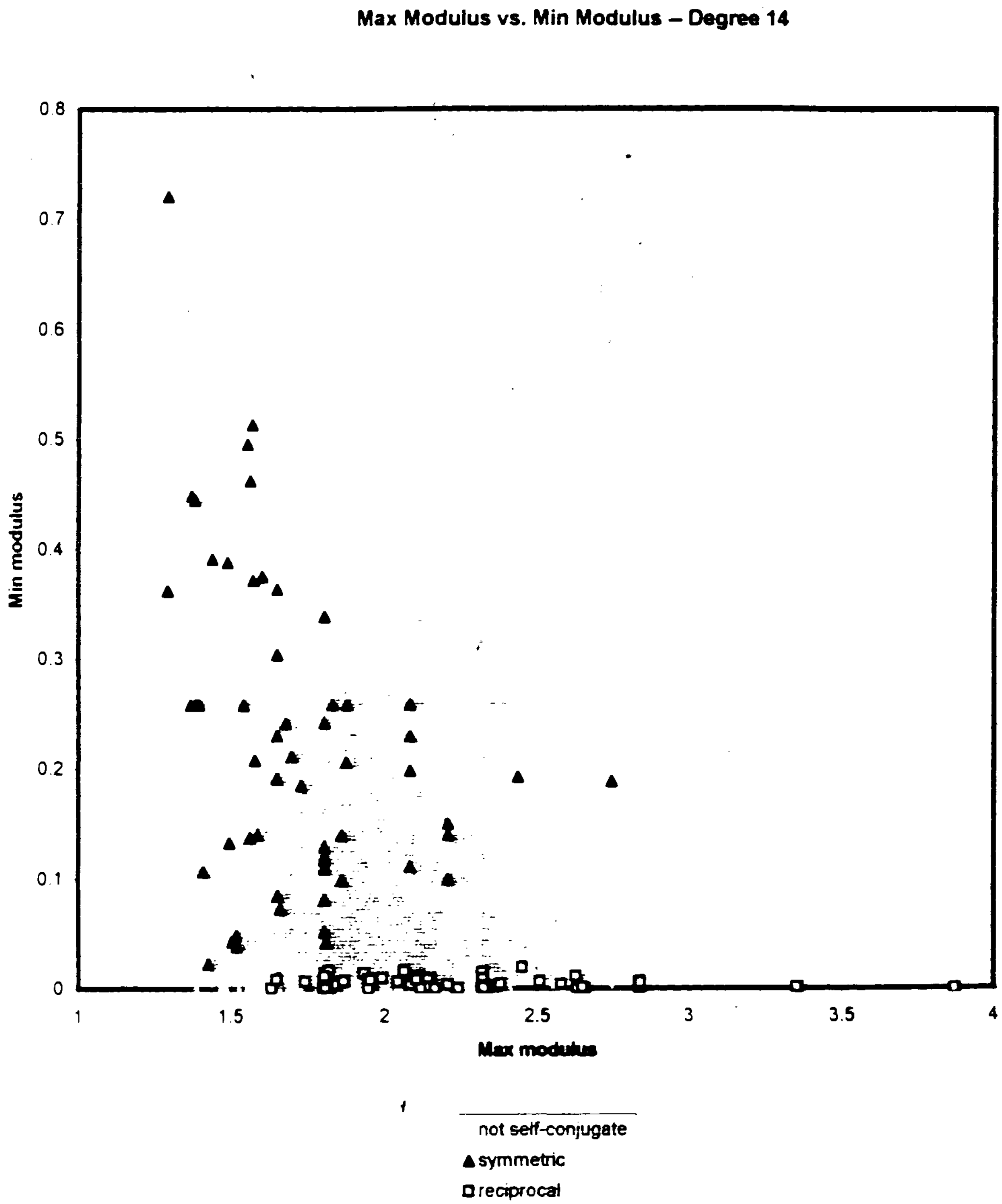

Figure 5.10: Minimum Modulus vs. Maximum Modulus, degree 14

5.4 Some Sets of Extremal Polynomials

While flat and high minimum modulus polynomials exist for every degree studied, unfortunately no pattern or general rule was determined to facilitate finding such polynomials for arbitrarily high degrees. However, for other extreme values, obvious patterns exist.

The largest possible maximum modulus occurs when, for some angle θ, the terms of the polynomial represent vectors all going in the same direction. This occurs, for example, at $\theta = 0$ when all terms of the polynomial have the same sign. So the value of the maximum modulus for a degree n polynomial is $n+1$.

Polynomials having coefficients all of the same sign have other extremal properties besides highest maximum modulus. The normalized Mahler measure of these polynomials is just $1/\mu_n$, since all their roots lies on the unit circle, so the smallest Mahler measure among the ± 1 polynomials is attained by such polynomials for every degree n. For degrees up to 24, we see that such polynomials also attain the highest L_3 norm and L_4 norm, and the lowest L_1 norm.

For even values of $p > 2$, the following theorem shows that the highest L_p norm is attained by this type of polynomial for each degree n.

Theorem 10 (Borwein) *For even values $p > 2$ and for every n, the maximum L_p norm among ± 1 polynomials of degree n is attained by a polynomial having coefficients all of the same sign*

Proof

Since p is an even number greater than 2, $p = 2k$ for some positive integer k. So for every ± 1 polynomial $P(\theta)$ of degree n,

$$\begin{aligned} ||P(\theta)||_{L_p} &= \left(\frac{1}{2\pi}\int_0^{2\pi} |P(\theta)|^p \, d\theta\right)^{\frac{1}{p}} \\ &= \left(\frac{1}{2\pi}\int_0^{2\pi} |P(\theta)^k|^2 \, d\theta\right)^{\frac{1}{p}} \\ &= \left(||P(\theta)^k||_{L_2}\right)^{\frac{2}{p}} \\ &= \left(\sum_{i=0}^{nk} a_i^2\right)^{\frac{1}{p}} \end{aligned}$$

if $P(z)^k = \sum_{i=0}^{nk} a_i z^i$. The squares of the coefficients of $P(z)^k$, and hence the L_2 norm of

$P(z)^k$ and the L_{2k} norm of $P(z)$, will be maximized when the coefficients of $P(z)$ all have the same sign □.

Based on the numerical evidence, we further conjecture:

Conjecture 7 *For each degree n, among the ±1 polynomials of degree n the lowest L_1 norm and the highest L_p norm for every $p > 2$ are attained by a polynomial having coefficients all of the same sign.*

Among the set of symmetric polynomials there are none having coefficients all equal. However, symmetric degree n polynomials of the form

$$00110011\ldots0011001$$

when $n \equiv 2 \pmod 4$ or

$$00110011\ldots00110$$

when $n \equiv 0 \pmod 4$ appear to have extremal properties among the symmetric polynomials similar to those of polynomials having coefficients all equal. (see Table 4.22) This observation leads to Conjecture 8.

Conjecture 8 *For every even degree n, among the symmetric ±1 polynomials of degree n the highest maximum modulus, the lowest L_1 norm and Mahler measure, and the highest L_p norm, $p > 2$, are attained by a polynomial having the form given above.*

Certainly a series of high minimum modulus or flat polynomials is not to be found among symmetrics having this form—their normalized minimum modulus tends to zero, since

$$\min_{|z|=1} \frac{|P(z)|}{\mu_n} \leq \frac{|P(1)|}{\mu_n} = \frac{1}{\mu_n}$$

The modular plots of these symmetric polynomials appear to have a characteristic shape (see Figure 4.24), with a maximum modulus of $\left(\frac{n^2+2n+2}{2}\right)^{\frac{1}{2}}$ attained at $\theta = \pi/2$ and $\theta = 3\pi/2$. (±1 polynomials having coefficients all of the same sign also have a characteristic modular plot -see Figure 4.25, with maximum modulus $n+1$ attained at $\theta = 0$ and $\theta = 2\pi$.)

5.5 Some Smooth Curves—a Summary

A glance at the charts of mean and extreme data shows that some sets of data fall along smooth, regular curves while other data does not. The following, when plotted against degree, appear to fall along distinctly smooth curves. Bold type indicates that an exact expression for the quantity in terms of the degree has already been established.

1. **E(L_2^2) over all ±1 polynomials and all symmetric polynomials** (Figure 4.1) For every ±1 polynomial $L_2^2 = n + 1$, hence $E(L_2^2) = n + 1$.

2. E(L_3^3) over all ±1 polynomials (Figures 4.1 and 4.2)

3. **E(L_4^4) over all ±1 polynomials** (Figure 4.1). Newman and Byrnes have proven that this is $2(n+1)^2 - (n+1)$. More generally, Borwein and Lockhart have proven that $E(L_p^p) \to \Gamma(1 + p/2)(n+1)^{p/2}$ as $n \to \infty$ (Figure 5.5 for $p = 3$ and $p = 4$).

4. E(L_3) over all ±1 polynomials and all symmetric polynomials (Figure 4.11)

5. E(L_4) over all ±1 polynomials and all symmetric polynomials (Figure 4.12). Borwein and Lockhart have conjectured that the expected value of normalized L_p approaches $\Gamma(1 + p/2)^{1/p}$ as $n \to \infty$.

6. E(maximum modulus) over all ±1 polynomials (Figures 4.13 and 4.14) Salem and Zygmund have shown that this quantity grows according to $c\sqrt{(n+1)\log(n+1)}$.

7. E(maximum modulus) over all symmetric polynomials (Figures 4.13 and 4.14)

8. E(maximum modulus) over all reciprocal polynomials (Figures 4.13 and 4.14)

9. **Smallest Mahler measure over all ±1 polynomials** (Figure 4.16) Equals 1 (not normalized) and occurs for polynomials having coefficients all of the same sign, since their roots lie on the unit circle.

10. Smallest Mahler measure over all symmetric polynomials (Figure 4.16 and 4.17) Appears to be fairly constant at about 1.79 when not normalized.

11. Smallest L_1 norm over all ±1 polynomials and all symmetric polynomials (Figure 4.19)

12. **Largest maximum modulus over all ±1 polynomials** (Figure 4.18) This is just $n + 1$.

13. Largest maximum modulus over all symmetric ±1 polynomials (Figure 4.18) Appears to be $\left(\frac{n^2+2n+2}{2}\right)^{\frac{1}{2}}$.

14. Largest L_3 norm over all ± 1 polynomials and all symmetric polynomials (Figure 4.20)

15. Largest L_4 norm over all ± 1 polynomials and all symmetric polynomials (Figure 4.21)

The following quantities do not follow quite such smooth curves, but they display marked differences between odd and even degrees:

- E(minimum modulus) over all ± 1 polynomials (Figure 4.6)
- E(Mahler measure) over all ± 1 polynomials (Figure 4.8)

5.6 Concluding Remarks

The one-sided and two-sided conjectures, which have been unsolved since they were first posed over forty years ago, are supported by the data presented here. No doubt future research[1] will study higher degrees and reveal whether or not the trends described in this thesis regarding the modulus of ± 1 polynomials on the unit circle continue beyond the degrees studied here.

[1] Andrew Odlyzko has done extensive calculations on some of these problems and his results will be reported later this year [17].

Appendix A

Guide to PolyApplets

Two Java applets that can be run on the World Wide Web and may be useful for studying ± 1 polynomials are described here. They have been tested on a PC running Windows 95 with Microsoft Internet Explorer. In theory, Java is a "platform independent" language: the applets should run in the same manner on any platform running a Java-enabled Web browser. But at the time of this writing this does not appear to always be true in practice, so if you are using a browser other than Microsoft Internet Explorer the behaviour of the applets may not be exactly as described. At the time of this writing the applets are available on the World Wide Web at the URL's given below. If the URL's given are not available, contact the *Centre For Experimental and Constructive Mathematics* (http://cecm.sfu.ca) at Simon Fraser University to obtain updated URL's.

PolyApplet 1—Select From a Set of ± 1 Polynomials

http://polymath.cecm.sfu.ca/java/gallery/lesley/thesis1.html

This applet is useful for studying a large number of ± 1 polynomials having degrees not greater than 15.

To run the applet, select from the lists shown:

1. A plot type

2. A degree

3. A polynomial

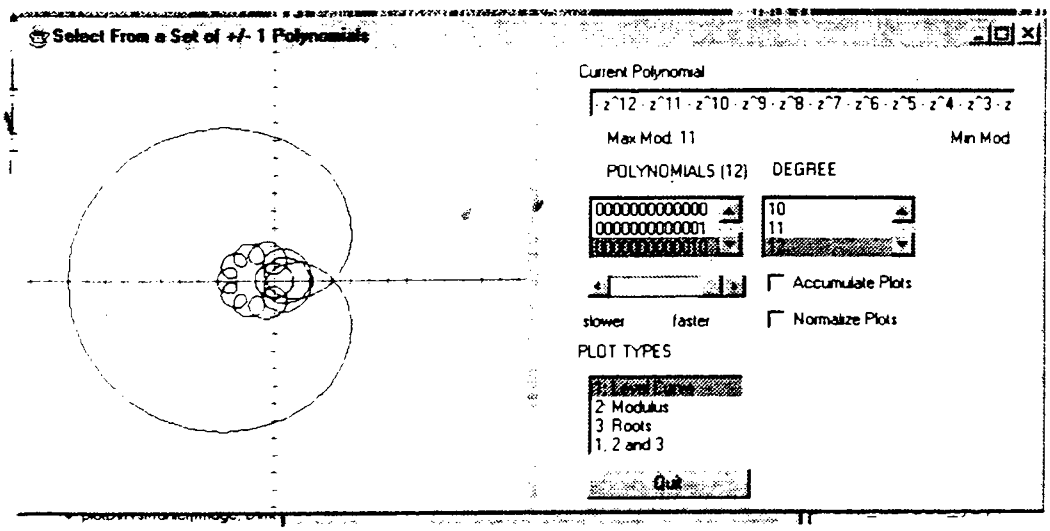

Figure A.1: Polyapplet 1, showing a plot of type 1

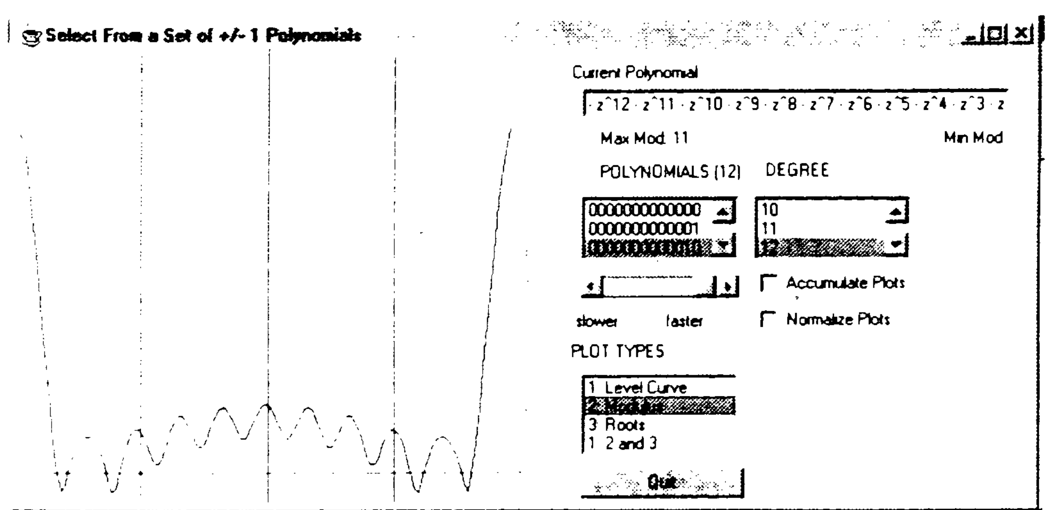

Figure A.2: Polyapplet 1, showing a plot of type 2

To select a degree, use the mouse to scroll up and down the list then click the mouse on the desired degree.

Once a degree is selected, the polynomial list will become populated with a binary representation of every ± 1 polynomial of the selected degree. For example, the binary number 00101 represents the polynomial $-z^4 - z^3 + z^2 - z + 1$. Note that it may take a few minutes for the list to be constructed, particularly for higher degrees.

To select a polynomial from the list click with the mouse on the polynomial list to activate it, then scroll up and down the list either with the mouse or with the up and down arrow keys to select a polynomial. Every time you select a new polynomial, a plot corresponding to that polynomial will appear in the plane to the left of the lists. The plot that appears depends on what plot type has been selected. There are three plot types available:

1. Level Curve - the image of the unit circle in the complex plane under the selected polynomial. The red circle is the unit circle, the blue curve is its image. The unit circle and its image are drawn simultaneously. You may control the speed at which the curves are drawn using the "faster...slower" slider.

2. Modulus - the modulus of the selected polynomial on the unit circle plotted as a function of angle from 0 to 2π.

3. Roots - Roots of the polynomial are plotted in the complex plane. The red circle is the unit circle.

To plot more than one polynomial at the same time, use the "Accumulate Plots" option. The "Normalize" option causes the modulus to be divided by $(degree + 1)^{\frac{1}{2}}$

PolyApplet 2—View a ±1 Polynomial

http://polymath.cecm.sfu.ca/java/gallery/lesley/thesis2.html

This applet is useful for studying polynomials up to about degree 100. It is similar to PolyApplet 1, except that no polynomial list is created. You must enter a polynomial as a binary string, either by typing it at the keyboard or copying from a file and pasting.

To begin, use the mouse to select a degree from a degree list. Next, enter a binary string having length one greater than the selected degree into the text-box labelled "Enter polynomial here:". After activating the text-box by clicking on it with the mouse, you may type in the text-box or paste from the clip-board. To paste from the clip-board, copy a binary string from a file onto the clip-board and paste it into the text-box with the "Ctrl - v" key combination.

After a new binary string is placed in the text-box (or after changing any other setting) activate the current polynomial by pressing the "Enter" key when the cursor is visible in the text-box. You may adjust the scale of the plots by typing an integer in the "Enter scale:" box. You must re-activate the current polynomial for the scale change to take effect.

Show Vectors

The plot types available in this applet are the same as those available in Polyapplet 1, but one additional option is available in Polyapplet 2: the "show vectors" option. Clicking the "show vectors" button causes an animated string of vectors, added together nose to tail, to appear. Each vector represents one term of the polynomial. The vectors change as θ changes in increments from 0 to 2π around the unit circle, creating the animation. The speed at which θ changes is controlled by the "slow-fast" slider. After theta reaches 2π, the animation stops and two fixed strings of vectors appear. These fixed strings of vectors represent the modulus at approximately its maximum and minimum values.

The "show vectors" option was added in the hope that studying the pattern of the vectors might provide some clues about the behaviour of the modulus. One half of the vector string appears in blue, the other half in red. For even degrees the two halves are separated by a single white vector that represents the middle term of the polynomial. Reciprocal polynomials have "symmetrical" vector string, with the first half of the vector string a mirror image of the second half, allowing, for example, both halves to be stretched out at the same time.

Symmetric polynomials, on the other hand, tend to have the two halves of the vector string distinctly different shapes: whenever one half is stretched out fairly flat, the other half tends to be crunched up. In [14] Littlewood describes symmetric polynomials as having

> *... a central term and two stretches of $n/2$ terms on either side, the end one having the coefficients of the front one written backwards, but affected with signs alternately $-$ and $+$.*

The idea of considering a polynomial as two vector strings joined together may be traced back to Rudin-Shapiro polynomials, because a Rudin-Shapiro polynomial having 2^n terms is created by joining together a pair of Rudin-Shapiro polynomials each having 2^{n-1} terms.

Figures A.3 to A.5 show Polyapplet 2 in typical applications.

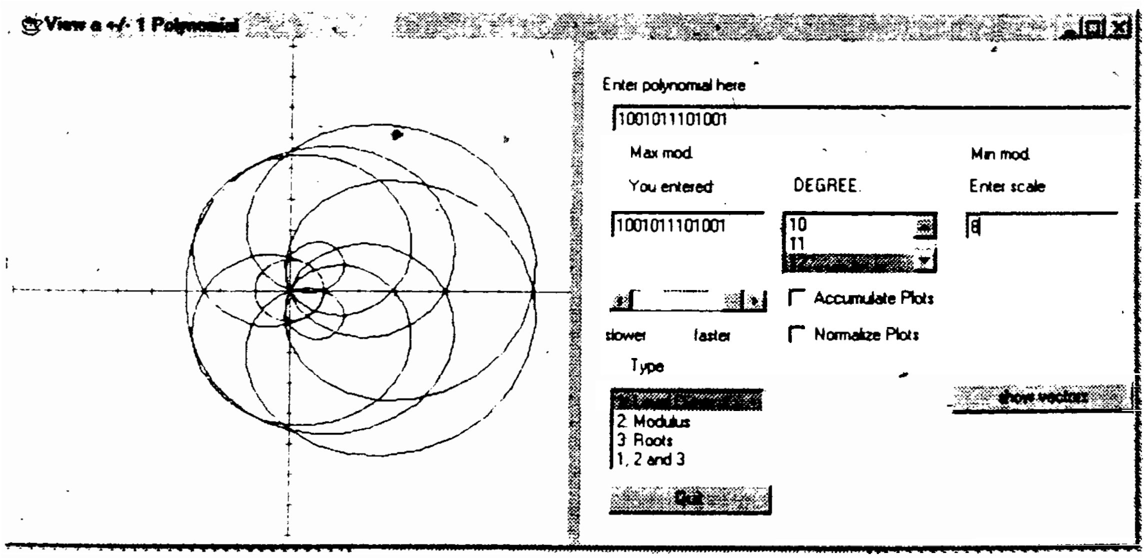

Figure A.3: Polyapplet 2, showing a plot of type 1

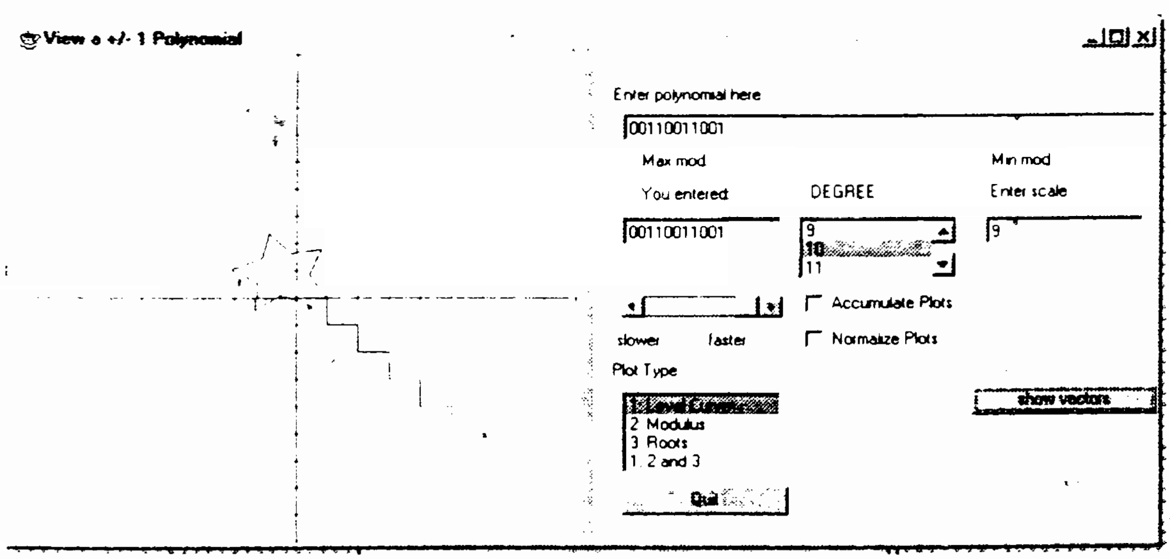

Figure A.4: Polyapplet 2 showing maximum and minimum vector strings of degree 10 symmetric polynomial 00110011001

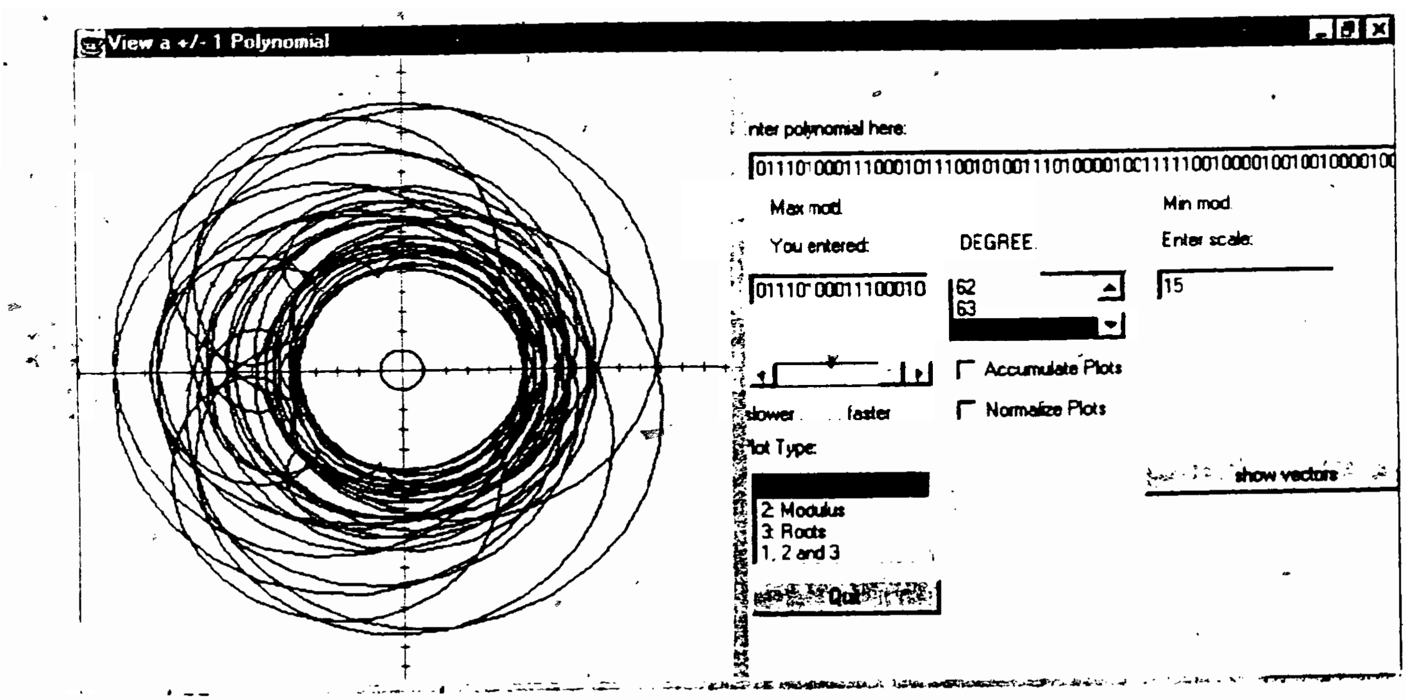

Figure A.5: Polyapplet 2 showing the image of the unit circle under a degree 64 polynomial with a high minimum modulus. The small circle in the middle of the image is the unit circle

Appendix B

Selected Data

Lists containing the 50 most extreme polynomials in various categories are given here for degrees 23, 24, 42 and 44. For degrees 23 and 24 the lists contain polynomials extremal among the complete set of ±1 polynomials of the same degree. For degrees 42 and 44 the lists contain polynomials extremal among the set of symmetric polynomials of the same degree.

The extremal quantity is shown in bold italic type if it is minimal and in bold type if it is maximal. For degrees 23 and 24, reciprocal polynomials are marked with a tilde and symmetric polynomials are marked with an asterisk.

Degree 23 ***symmetric** **~reciprocal**

50 smallest Mahler measure

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000000000000000000000000~	***0.20412***	4.89898	0	0.46489	1.55698	2.00043
010101010101010101010101~	***0.20412***	4.89351	0	0.46489	1.55698	2.00043
000000001111111100000000~	***0.20412***	3.33395	0	0.59598	1.34014	1.60769
010101011010101001010101~	***0.20412***	3.33395	0	0.59598	1.34014	1.60769
001100111100110000110011~	***0.20412***	2.77338	0	0.65883	1.23944	1.42031
011010011001011001101001~	***0.20412***	2.7567	0	0.68408	1.22914	1.40554
001100110011001100110011~	***0.20412***	3.47219	0	0.55439	1.3906	1.68543
010101101010010101101010~	***0.20412***	3.26524	0	0.60631	1.3261	1.58377
010110101010010101011010~	***0.20412***	3.2059	0	0.64544	1.30621	1.55503
011001100110011001100110~	***0.20412***	3.47219	0	0.55439	1.3906	1.68543
000011110000111100001111~	***0.20412***	3.24766	0	0.60108	1.32757	1.58377
010110100101101001011010~	***0.20412***	3.24766	0	0.60108	1.32757	1.58377
000011111111000000001111~	***0.20412***	3.2059	0	0.64544	1.30621	1.55503
010010010010010010010010~	***0.20412***	3.29102	0	0.58733	1.34358	1.60769
000011110000000011110000~	***0.20412***	2.92112	0	0.65875	1.25388	1.45309
000000000000111111111111~	***0.20412***	3.55431	0	0.57811	1.38515	1.68543
011001101001100101100110~	***0.20412***	2.77338	0	0.65883	1.23944	1.42031
011010010110011010010110~	***0.20412***	2.67757	0	0.69691	1.21453	1.37983
000000111111000000111111~	***0.20412***	3.26524	0	0.60631	1.3261	1.58377
001111000011001111000011~	***0.20412***	2.67757	0	0.69691	1.21453	1.37983
010101010101101010101010~	***0.20412***	3.55431	0	0.57811	1.38515	1.68543
010110100101010110100101~	***0.20412***	2.92112	0	0.65875	1.25388	1.45309
001100110011110011001100~	***0.20412***	2.76393	0	0.65603	1.2544	1.44395
001100001100001100001100~	***0.20412***	2.88669	0	0.64793	1.25616	1.45309
000111000111000111000111~	***0.20412***	3.29102	0	0.58733	1.34358	1.60769
001111000011110000111100~	***0.20412***	3.22051	0	0.62384	1.31998	1.57317
011010010110100101101001~	***0.20412***	3.22051	0	0.62384	1.31998	1.57317
011001011001011001011001~	***0.20412***	2.88669	0	0.64793	1.25616	1.45309
001111001100001100111100~	***0.20412***	2.7567	0	0.68408	1.22914	1.40554
011001100110100110011001~	***0.20412***	2.76393	0	0.65603	1.2544	1.44395
010010101101010010101101~	***0.20413***	3.19529	0	0.64307	1.30663	1.55503
001100001100110011110011~	***0.20413***	2.66322	0	0.69551	1.21516	1.37983
000000111111111111000000~	***0.20413***	3.26497	0	0.63828	1.31788	1.57317
000111000111111000111000~	***0.20413***	2.79424	0	0.67023	1.23807	1.42031
000111111000000111111000~	***0.20413***	3.19529	0	0.64307	1.30663	1.55503
010010010010101101101101~	***0.20413***	2.79424	0	0.67023	1.23807	1.42031
011001011001100110100110~	***0.20413***	2.66322	0	0.69551	1.21516	1.37983
010101101010101010010101~	***0.20413***	3.26497	0	0.63828	1.31788	1.57317
010010101101101101010010~	***0.20425***	2.78305	0	0.69387	1.22765	1.40554
000111111000111000000111~	***0.20426***	2.78305	0	0.69387	1.22765	1.40554
011001101010101010011001~	***0.30744***	2.07834	0	0.72516	1.1735	1.29261
010111111010010111111010~	***0.30744***	2.24049	0	0.72938	1.17189	1.29261
001100111111111111001100~	***0.30744***	2.07834	0	0.72516	1.1735	1.29261
000010101111000010101111~	***0.30744***	2.24049	0	0.72938	1.17189	1.29261
011010011010010110010110~	***0.30744***	2.97781	0	0.68968	1.25658	1.46649
001110011100001110011100~	***0.30744***	2.97345	0	0.67802	1.25834	1.46649
010101011001100110101010~	***0.30744***	2.69896	0	0.7201	1.19455	1.34696
011011001001011011001001~	***0.30744***	2.97345	0	0.67802	1.25834	1.46649
010110010110011010011010~	***0.30744***	2.54888	0	0.71905	1.19717	1.34696
000011000011001111001111~	***0.30744***	2.54888	0	0.71905	1.19717	1.34696

Degree 23 ***symmetric** **~reciprocal**

50 largest Mahler measure

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
00111000000010101011011001	**0.96237**	1.36319	0.3978	0.98244	1.01567	1.02988
01100100101011111111100011	**0.96237**	1.36319	0.3978	0.98244	1.01567	1.02988
01101101010111111110001100	**0.96237**	1.36319	0.3978	0.98244	1.01567	1.02988
00110001111110101011010110	**0.96237**	1.36319	0.3978	0.98244	1.01567	1.02988
01001011100001110111011 1	**0.95397**	1.50953	0.61993	0.97644	1.02418	1.04845
000100010001111000101101	**0.95397**	1.50953	0.61993	0.97644	1.02418	1.04845
000111101101001000100010	**0.95397**	1.50953	0.61993	0.97644	1.02418	1.04845
010001000100101101111000	**0.95397**	1.50953	0.61993	0.97644	1.02418	1.04845
010101101101100111000000	**0.95108**	1.43816	0.40825	0.97688	1.0207	1.03929
010101101100111000111111	**0.95108**	1.43816	0.41123	0.97688	1.0207	1.03929
000000111001101101101010	**0.95108**	1.43816	0.40825	0.97688	1.0207	1.03929
000000111000110010010101	**0.95108**	1.43816	0.41123	0.97688	1.0207	1.03929
001101100001111011110101	**0.95077**	1.39811	0.40676	0.97742	1.01935	1.03618
010100001000011110010011	**0.95077**	1.39811	0.40676	0.97742	1.01935	1.03618
000001011101001011000110	**0.95077**	1.39811	0.40676	0.97742	1.01935	1.03618
011000110100101110100000	**0.95077**	1.39811	0.40676	0.97742	1.01935	1.03618
001110011011010101111111	**0.9499**	1.63299	0.57092	0.97422	1.02688	1.05442
011011001110000000101010	**0.9499**	1.63283	0.57092	0.97422	1.02688	1.05442
000000010101001001100011	**0.9499**	1.63299	0.57092	0.97422	1.02688	1.05442
010101000000011100110110	**0.9499**	1.63283	0.57092	0.97422	1.02688	1.05442
000000011100110110101010	**0.94911**	1.59639	0.71358	0.97301	1.02943	1.0603
010101001001100011111111	**0.94911**	1.59639	0.71358	0.97301	1.02943	1.0603
010101011011001110000000	**0.94911**	1.59639	0.71358	0.97301	1.02943	1.0603
000000001110011011010101	**0.94911**	1.59639	0.71358	0.97301	1.02943	1.0603
010101100011000000110110	**0.94884**	1.63299	0.61023	0.97298	1.0294	1.0603
011011000000110001101010	**0.94884**	1.63299	0.61023	0.97298	1.0294	1.0603
001110010101100100111111	**0.94884**	1.63299	0.61023	0.97298	1.0294	1.0603
000000110110010101100011	**0.94884**	1.63299	0.61023	0.97298	1.0294	1.0603
010001000100101001111000	**0.94861**	1.43076	0.41584	0.97499	1.0235	1.04542
000100010001111100101101	**0.94861**	1.43076	0.40825	0.97499	1.0235	1.04542
010010110000011101110111	**0.94861**	1.43076	0.40825	0.97499	1.0235	1.04542
000111100101001000100010	**0.94861**	1.43076	0.41584	0.97499	1.0235	1.04542
000001001101010011100011	**0.94759**	1.58117	0.4625	0.97357	1.027	1.05442
010100011000000110110110	**0.94759**	1.58117	0.4625	0.97357	1.027	1.05442
011011011000000110001010	**0.94759**	1.58117	0.4625	0.97357	1.027	1.05442
001110001101010011011111	**0.94759**	1.58117	0.4625	0.97357	1.027	1.05442
001110000000101011001101	**0.94583**	1.56961	0.41534	0.9738	1.02478	1.04845
011011010101111110011000	**0.94583**	1.56961	0.41534	0.9738	1.02478	1.04845
010011001010111111100011	**0.94583**	1.56961	0.41534	0.9738	1.02478	1.04845
000110011111101010110110	**0.94583**	1.56961	0.41534	0.9738	1.02478	1.04845
010101000000111001101001	**0.94547**	1.72347	0.52012	0.97212	1.02938	1.0603
000000010101101100111100	**0.94547**	1.72347	0.52012	0.97212	1.02938	1.0603
011010011000111111010101	**0.94547**	1.72347	0.52012	0.97212	1.02938	1.0603
001111001101101010000000	**0.94547**	1.72347	0.52012	0.97212	1.02938	1.0603
011110100001100101110111	**0.94445**	1.52102	0.47218	0.97318	1.02503	1.04845
001011110100110000100010	**0.94445**	1.52102	0.47218	0.97318	1.02503	1.04845
010001000011001011110100	**0.94445**	1.52102	0.47218	0.97318	1.02503	1.04845
000100010110011110100001	**0.94445**	1.52102	0.47218	0.97318	1.02503	1.04845
011110000010010100011001	**0.94377**	1.51724	0.46124	0.97287	1.02516	1.04845
001100100000111010110100	**0.94377**	1.51724	0.46124	0.97287	1.02516	1.04845

Degree 23 ***symmetric** **~reciprocal**

50 smallest max modulus

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
010100110110100011000000	0.87903	***1.25837***	0.16413	0.95145	1.03338	1.05737
000000110001011011001010	0.87903	***1.25837***	0.16413	0.95145	1.03338	1.05737
000001100011110110010101	0.87903	***1.25837***	0.16413	0.95145	1.03338	1.05737
010101100100001110011111	0.87903	***1.25837***	0.16413	0.95145	1.03338	1.05737
001110011001001010100000	0.9318	***1.27336***	0.30074	0.96963	1.02481	1.04542
000001010100100110011100	0.9318	***1.27336***	0.30074	0.96963	1.02481	1.04542
010100000001110011001001	0.9318	***1.27336***	0.30074	0.96963	1.02481	1.04542
011011001100011111110101	0.9318	***1.27336***	0.30074	0.96963	1.02481	1.04542
000001100000101101101010	0.83109	***1.29028***	0.01223	0.93481	1.04372	1.07457
010101101101000001100000	0.83109	***1.29028***	0.01223	0.93481	1.04372	1.07457
010100110101111000111111	0.83109	***1.29028***	0.01223	0.93481	1.04372	1.07457
000000111000010100110101	0.83109	***1.29028***	0.01223	0.93481	1.04372	1.07457
010111111100011100110110	0.90513	***1.29617***	0.08255	0.96379	1.02567	1.04542
011011001110001111111010	0.90513	***1.29617***	0.08255	0.96379	1.02567	1.04542
001110011011011010101111	0.90513	***1.29617***	0.08255	0.96379	1.02567	1.04542
000010101001001001100011	0.90513	***1.29617***	0.08255	0.96379	1.02567	1.04542
010010101000000001111001 1	0.86587	***1.29966***	0	0.95596	1.02801	1.04845
001100001111111010101101	0.86587	***1.29966***	0	0.95596	1.02801	1.04845
000111111101010110100110	0.86587	***1.29966***	0	0.95596	1.02801	1.04845
011001011010101111111000	0.86587	***1.29966***	0	0.95596	1.02801	1.04845
010011101100011110111110	0.88707	***1.29994***	0.13213	0.95447	1.03268	1.05737
011111011110001101110010	0.88707	***1.29994***	0.13213	0.95447	1.03268	1.05737
001010001011011000100111	0.88707	***1.29994***	0.13213	0.95447	1.03268	1.05737
000110111001001011101011	0.88707	***1.29994***	0.13213	0.95447	1.03268	1.05737
010100110110001110000000	0.92501	***1.31336***	0.192	0.96847	1.02483	1.04542
000000011100011011001010	0.92501	***1.31336***	0.192	0.96847	1.02483	1.04542
000001100011011011010101	0.92501	***1.31336***	0.192	0.96847	1.02483	1.04542
010101001001001110011111	0.92501	***1.31336***	0.192	0.96847	1.02483	1.04542
000000101010011010011100	0.88589	***1.31786***	0.14862	0.95227	1.03566	1.0632
001110010110010101000000	0.88589	***1.31786***	0.14862	0.95227	1.03566	1.0632
011011000011000000010101	0.88589	***1.31786***	0.14862	0.95227	1.03566	1.0632
010101111111001111001001	0.88589	***1.31786***	0.14862	0.95227	1.03566	1.0632
010101011011000011000000	0.85052	***1.31843***	0.07701	0.94548	1.03641	1.0632
000000110000110110101010	0.85052	***1.31843***	0.07701	0.94548	1.03641	1.0632
010101100101100011111111	0.85052	***1.31843***	0.07701	0.94548	1.03641	1.0632
000000001110010110010101	0.85052	***1.31843***	0.07701	0.94548	1.03641	1.0632
010100100010110011100000	0.87625	***1.31885***	0.11216	0.9517	1.03312	1.05737
000001110011010001001010	0.87625	***1.31885***	0.11216	0.9517	1.03312	1.05737
010100100110000100011111	0.87625	***1.31885***	0.11216	0.9517	1.03312	1.05737
000001110111100110110101	0.87625	***1.31885***	0.11216	0.9517	1.03312	1.05737
011010001100010000010111	0.86418	***1.32224***	0	0.95621	1.02794	1.04845
000101111101110011101001	0.86418	***1.32224***	0	0.95621	1.02794	1.04845
010000101000100110111100	0.86418	***1.32224***	0	0.95621	1.02794	1.04845
001111011001000101000010	0.86418	***1.32224***	0	0.95621	1.02794	1.04845
011001101011000010100000	0.87816	***1.32557***	0.09256	0.95353	1.0326	1.05737
000001010000110101100110	0.87816	***1.32557***	0.09256	0.95353	1.0326	1.05737
010100000101100000110011	0.87816	***1.32557***	0.09256	0.95353	1.0326	1.05737
001100111110010111110101	0.87816	***1.32557***	0.09256	0.95353	1.0326	1.05737
011001101101000001010000	0.88257	***1.32711***	0.06341	0.95194	1.03571	1.0632
000010100000101101100110	0.88257	***1.32711***	0.06341	0.95194	1.03571	1.0632

Degree 23 ***symmetric** **~reciprocal**

50 largest min modulus

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000000001110011011010101	0.94911	1.59639	**0.71358**	0.97301	1.02943	1.0603
010101001001100011111111	0.94911	1.59639	**0.71358**	0.97301	1.02943	1.0603
010101011011001110000000	0.94911	1.59639	**0.71358**	0.97301	1.02943	1.0603
000000011100110110101010	0.94911	1.59639	**0.71358**	0.97301	1.02943	1.0603
000111101101001000100010	0.95397	1.50953	**0.61993**	0.97644	1.02418	1.04845
010001000100101101111000	0.95397	1.50953	**0.61993**	0.97644	1.02418	1.04845
000100010001111000101101	0.95397	1.50953	**0.61993**	0.97644	1.02418	1.04845
010010111000011101110111	0.95397	1.50953	**0.61993**	0.97644	1.02418	1.04845
000000110110010101100011	0.94884	1.63299	**0.61023**	0.97298	1.0294	1.0603
001110010101100100111111	0.94884	1.63299	**0.61023**	0.97298	1.0294	1.0603
011011000000110001101010	0.94884	1.63299	**0.61023**	0.97298	1.0294	1.0603
010101100011000000110110	0.94884	1.63299	**0.61023**	0.97298	1.0294	1.0603
011000110010010100000001	0.91068	1.80657	**0.60842**	0.95149	1.05386	1.10923
011111110101101100111001	0.91068	1.80657	**0.60842**	0.95149	1.05386	1.10923
001101100111000001010100	0.91068	1.80657	**0.60842**	0.95149	1.05386	1.10923
001010100000111001101100	0.91068	1.80657	**0.60842**	0.95149	1.05386	1.10923
010010111111001110101010	0.92293	1.72174	**0.57611**	0.9588	1.04562	1.09364
010101011100111111010010	0.92293	1.72174	**0.57611**	0.9588	1.04562	1.09364
000111101010011011111111	0.92293	1.72174	**0.57611**	0.9588	1.04562	1.09364
000000001001101010000111	0.92293	1.72174	**0.57611**	0.9588	1.04562	1.09364
011000110100101111110101	0.93266	1.71028	**0.57481**	0.96541	1.03572	1.07176
010100000010110100111001	0.93266	1.71028	**0.57481**	0.96541	1.03572	1.07176
001101100001111010100000	0.93266	1.71028	**0.57481**	0.96541	1.03572	1.07176
000001010111100001101100	0.93266	1.71028	**0.57481**	0.96541	1.03572	1.07176
010101000000011100110110	0.9499	1.63283	**0.57092**	0.97422	1.02688	1.05442
011011001110000000101010	0.9499	1.63283	**0.57092**	0.97422	1.02688	1.05442
000000010101001001100011	0.9499	1.63299	**0.57092**	0.97422	1.02688	1.05442
001110011011010101111111	0.9499	1.63299	**0.57092**	0.97422	1.02688	1.05442
011110001110111011010010	0.9388	1.55998	**0.5668**	0.96902	1.03072	1.0603
010010110111011100011110	0.9388	1.55998	**0.5668**	0.96902	1.03072	1.0603
001011011011101110000111	0.9388	1.55998	**0.5668**	0.96902	1.03072	1.0603
000111100010001001001011	0.9388	1.55998	**0.5668**	0.96902	1.03072	1.0603
001110110001010011110111	0.9141	1.92877	**0.56637**	0.9541	1.05068	1.10411
000100001101011100100011	0.9141	1.92877	**0.56637**	0.9541	1.05068	1.10411
011011100100000110100010	0.9141	1.92877	**0.56637**	0.9541	1.05068	1.10411
010001011000001001110110	0.9141	1.92877	**0.56637**	0.9541	1.05068	1.10411
001110001101101010111111	0.93809	1.48773	**0.55639**	0.96911	1.02968	1.05737
000000101010010011100011	0.93809	1.48773	**0.55639**	0.96911	1.02968	1.05737
010101111111000110110110	0.93809	1.48773	**0.55639**	0.96911	1.02968	1.05737
011011011000111111101010	0.93809	1.48773	**0.55639**	0.96911	1.02968	1.05737
011011010101111110011100	0.94244	1.53817	**0.55603**	0.97175	1.02664	1.05145
001110011111101010110110	0.94244	1.53817	**0.55603**	0.97175	1.02664	1.05145
001110000000101011001001	0.94244	1.53817	**0.55603**	0.97175	1.02664	1.05145
011011001010111111100011	0.94244	1.53817	**0.55603**	0.97175	1.02664	1.05145
011111101101000111011000	0.93078	1.74164	**0.55373**	0.96476	1.03587	1.07176
000110111000101101111110	0.93078	1.74164	**0.55373**	0.96476	1.03587	1.07176
001010111000010010001101	0.93078	1.74164	**0.55373**	0.96476	1.03587	1.07176
010011101101111000101011	0.93078	1.74164	**0.55373**	0.96476	1.03587	1.07176
010101001011000001000110	0.91041	1.99323	**0.55264**	0.95196	1.05385	1.11177
011000100000110100101010	0.91041	1.99323	**0.55264**	0.95196	1.05385	1.11177

Degree 23 ***symmetric** **~reciprocal**

50 smallest L1 norm

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
010101010101010101010101~	0.20412	4.89351	0	***0.46489***	1.55698	2.00043
000000000000000000000000~	0.20412	4.89898	0	***0.46489***	1.55698	2.00043
001100110011001100110011~	0.20412	3.47219	0	***0.55439***	1.3906	1.68543
011001100110011001100110~	0.20412	3.47219	0	***0.55439***	1.3906	1.68543
010101010101101010101010~	0.20412	3.55431	0	***0.57811***	1.38515	1.68543
000000000000111111111111~	0.20412	3.55431	0	***0.57811***	1.38515	1.68543
000111000111000111000111~	0.20412	3.29102	0	***0.58733***	1.34358	1.60769
010010010010010010010010~	0.20412	3.29102	0	***0.58733***	1.34358	1.60769
010101010101010101010100	0.40825	4.48663	0.06678	***0.59146***	1.48085	1.8803
001010101010101010101010	0.40825	4.48663	0.06678	***0.59146***	1.48085	1.8803
011111111111111111111111	0.40825	4.49073	0.06678	***0.59146***	1.48085	1.8803
000000000000000000000001	0.40825	4.49073	0.06678	***0.59146***	1.48085	1.8803
010101011010101001010101~	0.20412	3.33395	0	***0.59598***	1.34014	1.60769
000000001111111100000000~	0.20412	3.33395	0	***0.59598***	1.34014	1.60769
000011110000111100001111~	0.20412	3.24766	0	***0.60108***	1.32757	1.58377
010110100101101001011010~	0.20412	3.24766	0	***0.60108***	1.32757	1.58377
000000111111000000111111~	0.20412	3.26524	0	***0.60631***	1.3261	1.58377
010101101010010101101010~	0.20412	3.26524	0	***0.60631***	1.3261	1.58377
010101010101010101010111	0.43321	4.4864	0.04965	***0.61468***	1.47114	1.86977
000101010101010101010101	0.43321	4.4864	0.04965	***0.61468***	1.47114	1.86977
010000000000000000000000	0.43321	4.49073	0.04965	***0.61468***	1.47114	1.86977
000000000000000000000010	0.43321	4.49073	0.04965	***0.61468***	1.47114	1.86977
001111000011110000111100~	0.20412	3.22051	0	***0.62384***	1.31998	1.57317
011010010110100101101001~	0.20412	3.22051	0	***0.62384***	1.31998	1.57317
001110001110001110001110	0.33028	3.2719	0	***0.62581***	1.33808	1.60097
011100011100011100011100	0.33028	3.2719	0	***0.62581***	1.33808	1.60097
001001001001001001001001	0.33028	3.2719	0	***0.62581***	1.33808	1.60097
011011011011011011011011	0.33028	3.2719	0	***0.62581***	1.33808	1.60097
011010101010101010101010	0.40815	4.07978	0	***0.62685***	1.41925	1.7727
010101010101010101010110	0.40815	4.07978	0	***0.62685***	1.41925	1.7727
000000000000000000000011	0.40815	4.08248	0.01393	***0.62685***	1.41925	1.7727
001111111111111111111111	0.40815	4.08248	0.01393	***0.62685***	1.41925	1.7727
000000000001111111111111	0.40356	3.53874	0.04785	***0.62761***	1.37791	1.6774
000000000000011111111111	0.40356	3.53874	0.04785	***0.62761***	1.37791	1.6774
010101010101001010101010	0.40356	3.53874	0.04785	***0.62761***	1.37791	1.6774
010101010100101010101010	0.40356	3.53874	0.04785	***0.62761***	1.37791	1.6774
010101010101010110101010	0.33028	3.3263	0	***0.63486***	1.33555	1.60097
010101011010101010101010	0.33028	3.3263	0	***0.63486***	1.33555	1.60097
000000001111111111111111	0.33028	3.3263	0	***0.63486***	1.33555	1.60097
000000000000000011111111	0.33028	3.3263	0	***0.63486***	1.33555	1.60097
010101010101010101010001	0.47938	4.4862	0.09814	***0.63695***	1.46261	1.86013
011101010101010101010101	0.47938	4.4862	0.09814	***0.63695***	1.46261	1.86013
001000000000000000000000	0.47938	4.49073	0.09814	***0.63695***	1.46261	1.86013
000000000000000000000100	0.47938	4.49073	0.09814	***0.63695***	1.46261	1.86013
010101010101011010101010	0.39742	3.49222	0	***0.63762***	1.36475	1.65717
010101010110101010101010	0.39742	3.49222	0	***0.63762***	1.36475	1.65717
000000000011111111111111	0.39742	3.49222	0.00287	***0.63762***	1.36475	1.65717
000000000000001111111111	0.39742	3.49222	0.00287	***0.63762***	1.36475	1.65717
000000111111111111000000~	0.20413	3.26497	0	***0.63828***	1.31788	1.57317
010101101010101010010101~	0.20413	3.26497	0	***0.63828***	1.31788	1.57317

Degree 23 ***symmetric** **~reciprocal**

50 largest L1 norm

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
001110000000101011011001	0.96237	1.36319	0.3978	**0.98244**	1.01567	1.02988
011001001010111111100011	0.96237	1.36319	0.3978	**0.98244**	1.01567	1.02988
011011010101111110001100	0.96237	1.36319	0.3978	**0.98244**	1.01567	1.02988
001100011111101010110110	0.96237	1.36319	0.3978	**0.98244**	1.01567	1.02988
011000110100101110100000	0.95077	1.39811	0.40676	**0.97742**	1.01935	1.03618
000001011101001011000110	0.95077	1.39811	0.40676	**0.97742**	1.01935	1.03618
010100001000011110010011	0.95077	1.39811	0.40676	**0.97742**	1.01935	1.03618
001101100001111011110101	0.95077	1.39811	0.40676	**0.97742**	1.01935	1.03618
010101101100111000111111	0.95108	1.43816	0.41123	**0.97688**	1.0207	1.03929
000000111000110010010101	0.95108	1.43816	0.41123	**0.97688**	1.0207	1.03929
000000111001101101101010	0.95108	1.43816	0.40825	**0.97688**	1.0207	1.03929
010101101101100111000000	0.95108	1.43816	0.40825	**0.97688**	1.0207	1.03929
010001000100101101111000	0.95397	1.50953	0.61993	**0.97644**	1.02418	1.04845
000111101101001000100010	0.95397	1.50953	0.61993	**0.97644**	1.02418	1.04845
010010111000011101110111	0.95397	1.50953	0.61993	**0.97644**	1.02418	1.04845
000100010001111000101101	0.95397	1.50953	0.61993	**0.97644**	1.02418	1.04845
000000011100011001001010	0.94003	1.63299	0.06687	**0.97501**	1.02195	1.04237
010100100110001110000000	0.94003	1.63299	0.06687	**0.97501**	1.02195	1.04237
010101001001001100011111	0.94003	1.63077	0	**0.97501**	1.02195	1.04237
000001110011011011010101	0.94003	1.63077	0	**0.97501**	1.02195	1.04237
010001000100101001111000	0.94861	1.43076	0.41584	**0.97499**	1.0235	1.04542
000111100101001000100010	0.94861	1.43076	0.41584	**0.97499**	1.0235	1.04542
000100010001111100101101	0.94861	1.43076	0.40825	**0.97499**	1.0235	1.04542
010010110000011101110111	0.94861	1.43076	0.40825	**0.97499**	1.0235	1.04542
000000010101001001100011	0.9499	1.63299	0.57092	**0.97422**	1.02688	1.05442
001110011011010101111111	0.9499	1.63299	0.57092	**0.97422**	1.02688	1.05442
011011001110000000101010	0.9499	1.63283	0.57092	**0.97422**	1.02688	1.05442
010101000000011100110110	0.9499	1.63283	0.57092	**0.97422**	1.02688	1.05442
011000110000001011101001	0.94278	1.47102	0.37587	**0.974**	1.02241	1.04237
011010001011111100111001	0.94278	1.47102	0.37587	**0.974**	1.02241	1.04237
001111011110101001101100	0.94278	1.47102	0.37587	**0.974**	1.02241	1.04237
001101100101011110111100	0.94278	1.47102	0.37587	**0.974**	1.02241	1.04237
010011001010111111100011	0.94583	1.56961	0.41534	**0.9738**	1.02478	1.04845
001110000000101011001101	0.94583	1.56961	0.41534	**0.9738**	1.02478	1.04845
011011010101111110011000	0.94583	1.56961	0.41534	**0.9738**	1.02478	1.04845
000110011111101010110110	0.94583	1.56961	0.41534	**0.9738**	1.02478	1.04845
011011011000000110001010	0.94759	1.58117	0.4625	**0.97357**	1.027	1.05442
010100011000000110110110	0.94759	1.58117	0.4625	**0.97357**	1.027	1.05442
001110001101010011011111	0.94759	1.58117	0.4625	**0.97357**	1.027	1.05442
000001001101010011100011	0.94759	1.58117	0.4625	**0.97357**	1.027	1.05442
011010100010011111001111	0.94118	1.37344	0.28818	**0.97338**	1.02266	1.04237
000011000001101110101001	0.94118	1.37344	0.28818	**0.97338**	1.02266	1.04237
010110010100111011111100	0.94118	1.37344	0.28818	**0.97338**	1.02266	1.04237
001111110111001010011010	0.94118	1.37344	0.28818	**0.97338**	1.02266	1.04237
010001000011001011110100	0.94445	1.52102	0.47218	**0.97318**	1.02503	1.04845
001011110100110000100010	0.94445	1.52102	0.47218	**0.97318**	1.02503	1.04845
011110100001100101110111	0.94445	1.52102	0.47218	**0.97318**	1.02503	1.04845
000100010110011110100001	0.94445	1.52102	0.47218	**0.97318**	1.02503	1.04845
010101011011001110000000	0.94911	1.59639	0.71358	**0.97301**	1.02943	1.0603
000000011100110110101010	0.94911	1.59639	0.71358	**0.97301**	1.02943	1.0603

Degree 23 ***symmetric** **~reciprocal**

50 smallest L3 norm

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
001100011111101010110110	0.96237	1.36319	0.3978	0.98244	***1.01567***	1.02988
011011010101111110001100	0.96237	1.36319	0.3978	0.98244	***1.01567***	1.02988
011001001010111111100011	0.96237	1.36319	0.3978	0.98244	***1.01567***	1.02988
001110000000101011011001	0.96237	1.36319	0.3978	0.98244	***1.01567***	1.02988
010100001000011110010011	0.95077	1.39811	0.40676	0.97742	***1.01935***	1.03618
001101100001111011110101	0.95077	1.39811	0.40676	0.97742	***1.01935***	1.03618
011000110100101110100000	0.95077	1.39811	0.40676	0.97742	***1.01935***	1.03618
000001011101001011000110	0.95077	1.39811	0.40676	0.97742	***1.01935***	1.03618
010101101101100111000000	0.95108	1.43816	0.40825	0.97688	***1.0207***	1.03929
000000111001101101101010	0.95108	1.43816	0.40825	0.97688	***1.0207***	1.03929
000000111000110010010101	0.95108	1.43816	0.41123	0.97688	***1.0207***	1.03929
010101101100111000111111	0.95108	1.43816	0.41123	0.97688	***1.0207***	1.03929
010101001001001100011111	0.94003	1.63077	0	0.97501	***1.02195***	1.04237
000001110011011011010101	0.94003	1.63077	0	0.97501	***1.02195***	1.04237
010100100110001110000000	0.94003	1.63299	0.06687	0.97501	***1.02195***	1.04237
000000011100011001001010	0.94003	1.63299	0.06687	0.97501	***1.02195***	1.04237
001111011110101001101100	0.94278	1.47102	0.37587	0.974	***1.02241***	1.04237
001101100101011110111100	0.94278	1.47102	0.37587	0.974	***1.02241***	1.04237
011010001011111100111001	0.94278	1.47102	0.37587	0.974	***1.02241***	1.04237
011000110000001011101001	0.94278	1.47102	0.37587	0.974	***1.02241***	1.04237
011011001010111101111000	0.9292	1.37985	0.03092	0.97277	***1.02245***	1.04237
000111101111010100110110	0.9292	1.37985	0.03092	0.97277	***1.02245***	1.04237
010010111010000001100011	0.9292	1.37985	0.03092	0.97277	***1.02245***	1.04237
001110011111101000101101	0.9292	1.37985	0.03092	0.97277	***1.02245***	1.04237
001110010110100010111111	0.92182	1.48264	0.0229	0.9713	***1.02259***	1.04237
000000101110100101100011	0.92182	1.48264	0.0229	0.9713	***1.02259***	1.04237
011011000011110111101010	0.92182	1.48264	0.0229	0.9713	***1.02259***	1.04237
010101111011110000110110	0.92182	1.48264	0.0229	0.9713	***1.02259***	1.04237
011010100010011111001111	0.94118	1.37344	0.28818	0.97338	***1.02266***	1.04237
010110010100111011111100	0.94118	1.37344	0.28818	0.97338	***1.02266***	1.04237
001111110111001010011010	0.94118	1.37344	0.28818	0.97338	***1.02266***	1.04237
000011000001101110101001	0.94118	1.37344	0.28818	0.97338	***1.02266***	1.04237
001001110010000111010111	0.92938	1.38243	0	0.97152	***1.02286***	1.04237
000101000111101100011011	0.92938	1.38243	0	0.97152	***1.02286***	1.04237
011100100111010010000010	0.92938	1.38243	0.06964	0.97152	***1.02286***	1.04237
010000010010111001001110	0.92938	1.38243	0.06964	0.97152	***1.02286***	1.04237
000001110011101010110110	0.92875	1.35294	0	0.9712	***1.02302***	1.04237
011011010101110011100000	0.92875	1.35294	0	0.9712	***1.02302***	1.04237
010100100110111111100011	0.92875	1.35294	0.0641	0.9712	***1.02302***	1.04237
001110000000100110110101	0.92875	1.35294	0.0641	0.9712	***1.02302***	1.04237
001100110100010100101111	0.91575	1.37269	0	0.96831	***1.02341***	1.04237
000010110101110100110011	0.91575	1.37269	0	0.96831	***1.02341***	1.04237
011001100001000001111010	0.91575	1.37269	0.0585	0.96831	***1.02341***	1.04237
010111100000100001100110	0.91575	1.37269	0.0585	0.96831	***1.02341***	1.04237
011101110110000011010010	0.91927	1.36294	0	0.9686	***1.02349***	1.04237
010010110000011011101110	0.91927	1.36294	0	0.9686	***1.02349***	1.04237
000111100101001110111011	0.91927	1.36294	0.05852	0.9686	***1.02349***	1.04237
001000100011010110000111	0.91927	1.36294	0.05852	0.9686	***1.02349***	1.04237
010010110000011101110111	0.94861	1.43076	0.40825	0.97499	***1.0235***	1.04542
000100010001111100101101	0.94861	1.43076	0.40825	0.97499	***1.0235***	1.04542

Degree 23 ***symmetric** **~reciprocal**

50 largest L3 norm

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000000000000000000000000~	0.20412	4.89898	0	0.46489	**1.55698**	2.00043
010101010101010101010101~	0.20412	4.89351	0	0.46489	**1.55698**	2.00043
011111111111111111111111	0.40825	4.49073	0.06678	0.59146	**1.48085**	1.8803
000000000000000000000001	0.40825	4.49073	0.06678	0.59146	**1.48085**	1.8803
010101010101010101010100	0.40825	4.48663	0.06678	0.59146	**1.48085**	1.8803
001010101010101010101010	0.40825	4.48663	0.06678	0.59146	**1.48085**	1.8803
010000000000000000000000	0.43321	4.49073	0.04965	0.61468	**1.47114**	1.86977
000000000000000000000010	0.43321	4.49073	0.04965	0.61468	**1.47114**	1.86977
010101010101010101010111	0.43321	4.4864	0.04965	0.61468	**1.47114**	1.86977
000101010101010101010101	0.43321	4.4864	0.04965	0.61468	**1.47114**	1.86977
001000000000000000000000	0.47938	4.49073	0.09814	0.63695	**1.46261**	1.86013
000000000000000000000100	0.47938	4.49073	0.09814	0.63695	**1.46261**	1.86013
011101010101010101010101	0.47938	4.4862	0.09814	0.63695	**1.46261**	1.86013
010101010101010101010001	0.47938	4.4862	0.09814	0.63695	**1.46261**	1.86013
000100000000000000000000	0.49495	4.49073	0.05554	0.64893	**1.45536**	1.85144
000000000000000000001000	0.49495	4.49073	0.05554	0.64893	**1.45536**	1.85144
010101010101010101011101	0.49495	4.48601	0.05554	0.64893	**1.45536**	1.85144
010001010101010101010101	0.49495	4.48601	0.05554	0.64893	**1.45536**	1.85144
000010000000000000000000	0.50344	4.49073	0.14211	0.65581	**1.44928**	1.84373
000000000000000000010000	0.50344	4.49073	0.14211	0.65581	**1.44928**	1.84373
010111010101010101010101	0.50344	4.48584	0.14211	0.65581	**1.44928**	1.84373
010101010101010101000101	0.50344	4.48584	0.14211	0.65581	**1.44928**	1.84373
000001000000000000000000	0.49601	4.49073	0.02773	0.6583	**1.44422**	1.83705
000000000000000000100000	0.49601	4.49073	0.02773	0.6583	**1.44422**	1.83705
010101010101010101110101	0.49601	4.4857	0.02773	0.6583	**1.44422**	1.83705
010100010101010101010101	0.49601	4.4857	0.02773	0.6583	**1.44422**	1.83705
000000100000000000000000	0.49614	4.49073	0.05529	0.66084	**1.44007**	1.83142
000000000000000001000000	0.49614	4.49073	0.05529	0.66084	**1.44007**	1.83142
010101110101010101010101	0.49614	4.48557	0.05529	0.66084	**1.44007**	1.83142
010101010101010100010101	0.49614	4.48557	0.05529	0.66084	**1.44007**	1.83142
000000010000000000000000	0.51487	4.49073	0.20655	0.66672	**1.43677**	1.82688
000000000000000010000000	0.51487	4.49073	0.20655	0.66672	**1.43677**	1.82688
010101010101010111010101	0.51487	4.48547	0.20655	0.66672	**1.43677**	1.82688
010101000101010101010101	0.51487	4.48547	0.20655	0.66672	**1.43677**	1.82688
000000001000000000000000	0.4806	4.49073	0.03665	0.66022	**1.43468**	1.82346
000000000000000100000000	0.4806	4.49073	0.03665	0.66022	**1.43468**	1.82346
010101011101010101010101	0.4806	4.48539	0.03665	0.66022	**1.43468**	1.82346
010101010101010001010101	0.4806	4.48539	0.03665	0.66022	**1.43468**	1.82346
000000000001000000000000	0.49263	4.49073	0.07462	0.65828	**1.43377**	1.82001
000000000000100000000000	0.49263	4.49073	0.07462	0.65828	**1.43377**	1.82001
010101010100010101010101	0.49263	4.48526	0.07461	0.65828	**1.43377**	1.82001
010101010101110101010101	0.49263	4.48526	0.07461	0.65828	**1.43377**	1.82001
000000000100000000000000	0.5116	4.49073	0.17404	0.66722	**1.43312**	1.82116
000000000000001000000000	0.5116	4.49073	0.17404	0.66722	**1.43312**	1.82116
010101010101011101010101	0.5116	4.48532	0.17404	0.66722	**1.43312**	1.82116
010101010001010101010101	0.5116	4.48532	0.17404	0.66722	**1.43312**	1.82116
000000000010000000000000	0.51472	4.49073	0.16065	0.66806	**1.43264**	1.82001
000000000000010000000000	0.51472	4.49073	0.16065	0.66806	**1.43264**	1.82001
010101010111010101010101	0.51472	4.48528	0.16065	0.66806	**1.43264**	1.82001
010101010101000101010101	0.51472	4.48528	0.16065	0.66806	**1.43264**	1.82001

Degree 23 ***symmetric** **~reciprocal**

50 smallest L4 norm

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
011001001010111111100011	0.96237	1.36319	0.3978	0.98244	1.01567	***1.02988***
001100011111101010110110	0.96237	1.36319	0.3978	0.98244	1.01567	***1.02988***
001110000000101011011001	0.96237	1.36319	0.3978	0.98244	1.01567	***1.02988***
011011010101111110001100	0.96237	1.36319	0.3978	0.98244	1.01567	***1.02988***
010100001000011110010011	0.95077	1.39811	0.40676	0.97742	1.01935	***1.03618***
001101100001111011110101	0.95077	1.39811	0.40676	0.97742	1.01935	***1.03618***
011000110100101110100000	0.95077	1.39811	0.40676	0.97742	1.01935	***1.03618***
000001011101001011000110	0.95077	1.39811	0.40676	0.97742	1.01935	***1.03618***
010101101101100111000000	0.95108	1.43816	0.40825	0.97688	1.0207	***1.03929***
000000111001101101101010	0.95108	1.43816	0.40825	0.97688	1.0207	***1.03929***
010101101100111000111111	0.95108	1.43816	0.41123	0.97688	1.0207	***1.03929***
000000111000110010010101	0.95108	1.43816	0.41123	0.97688	1.0207	***1.03929***
010101001001001100011111	0.94003	1.63077	0	0.97501	1.02195	***1.04237***
000001110011011011010101	0.94003	1.63077	0	0.97501	1.02195	***1.04237***
011011000000111001110101	0.90816	1.35272	0	0.96665	1.02382	***1.04237***
010100011000111111001001	0.90816	1.35272	0	0.96665	1.02382	***1.04237***
011011010101110011100000	0.92875	1.35294	0	0.9712	1.02302	***1.04237***
000001110011101010110110	0.92875	1.35294	0	0.9712	1.02302	***1.04237***
000011111011011100110101	0.89169	1.33263	0.01875	0.9625	1.02445	***1.04237***
010100110001001000001111	0.89169	1.33263	0.01875	0.9625	1.02445	***1.04237***
010110101110001001100000	0.89169	1.33263	0.01875	0.9625	1.02445	***1.04237***
010010110000011011101110	0.91927	1.36294	0	0.9686	1.02349	***1.04237***
011101110110000011010010	0.91927	1.36294	0	0.9686	1.02349	***1.04237***
011011001010111101111000	0.9292	1.37985	0.03092	0.97277	1.02245	***1.04237***
011010100010011111001111	0.94118	1.37344	0.28818	0.97338	1.02266	***1.04237***
011010001011111100111001	0.94278	1.47102	0.37587	0.974	1.02241	***1.04237***
011000110000001011101001	0.94278	1.47102	0.37587	0.974	1.02241	***1.04237***
010110010100111011111100	0.94118	1.37344	0.28818	0.97338	1.02266	***1.04237***
001111110111001010011010	0.94118	1.37344	0.28818	0.97338	1.02266	***1.04237***
001111011110101001101100	0.94278	1.47102	0.37587	0.974	1.02241	***1.04237***
001110010110100010111111	0.92182	1.48264	0.0229	0.9713	1.02259	***1.04237***
001101100101011110111100	0.94278	1.47102	0.37587	0.974	1.02241	***1.04237***
001100110100010100101111	0.91575	1.37269	0	0.96831	1.02341	***1.04237***
001001110010000111010111	0.92938	1.38243	0	0.97152	1.02286	***1.04237***
000111101111010100110110	0.9292	1.37985	0.03092	0.97277	1.02245	***1.04237***
000101000111101100011011	0.92938	1.38243	0	0.97152	1.02286	***1.04237***
000011000001101110101001	0.94118	1.37344	0.28818	0.97338	1.02266	***1.04237***
000010110101110100110011	0.91575	1.37269	0	0.96831	1.02341	***1.04237***
000001100100011101011010	0.89169	1.33263	0.01875	0.9625	1.02445	***1.04237***
000000101110100101100011	0.92182	1.48264	0.0229	0.9713	1.02259	***1.04237***
001110011111101000101101	0.9292	1.37985	0.03092	0.97277	1.02245	***1.04237***
010010111010000001100011	0.9292	1.37985	0.03092	0.97277	1.02245	***1.04237***
011011000011110111101010	0.92182	1.48264	0.0229	0.9713	1.02259	***1.04237***
010101111011110000110110	0.92182	1.48264	0.0229	0.9713	1.02259	***1.04237***
011100100111010010000010	0.92938	1.38243	0.06964	0.97152	1.02286	***1.04237***
010000010010111001001110	0.92938	1.38243	0.06964	0.97152	1.02286	***1.04237***
010100100110001110000000	0.94003	1.63299	0.06687	0.97501	1.02195	***1.04237***
000000011100011001001010	0.94003	1.63299	0.06687	0.97501	1.02195	***1.04237***
001110000000100110110101	0.92875	1.35294	0.0641	0.9712	1.02302	***1.04237***
010100100110111111100011	0.92875	1.35294	0.0641	0.9712	1.02302	***1.04237***

Degree 23 ***symmetric** **~reciprocal**

50 largest L4 norm

Polynomials	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000000000000000000000000~	0.20412	4.89898	0	0.46489	1.55698	**2.00043**
010101010101010101010101~	0.20412	4.89351	0	0.46489	1.55698	**2.00043**
011111111111111111111111	0.40825	4.49073	0.06678	0.59146	1.48085	**1.8803**
000000000000000000000001	0.40825	4.49073	0.06678	0.59146	1.48085	**1.8803**
010101010101010101010100	0.40825	4.48663	0.06678	0.59146	1.48085	**1.8803**
001010101010101010101010	0.40825	4.48663	0.06678	0.59146	1.48085	**1.8803**
010000000000000000000000	0.43321	4.49073	0.04965	0.61468	1.47114	**1.86977**
000000000000000000000010	0.43321	4.49073	0.04965	0.61468	1.47114	**1.86977**
010101010101010101010111	0.43321	4.4864	0.04965	0.61468	1.47114	**1.86977**
000101010101010101010101	0.43321	4.4864	0.04965	0.61468	1.47114	**1.86977**
001000000000000000000000	0.47938	4.49073	0.09814	0.63695	1.46261	**1.86013**
000000000000000000000100	0.47938	4.49073	0.09814	0.63695	1.46261	**1.86013**
011101010101010101010101	0.47938	4.4862	0.09814	0.63695	1.46261	**1.86013**
010101010101010101010001	0.47938	4.4862	0.09814	0.63695	1.46261	**1.86013**
000100000000000000000000	0.49495	4.49073	0.05554	0.64893	1.45536	**1.85144**
000000000000000000001000	0.49495	4.49073	0.05554	0.64893	1.45536	**1.85144**
010101010101010101011101	0.49495	4.48601	0.05554	0.64893	1.45536	**1.85144**
010001010101010101010101	0.49495	4.48601	0.05554	0.64893	1.45536	**1.85144**
000010000000000000000000	0.50344	4.49073	0.14211	0.65581	1.44928	**1.84373**
000000000000000000010000	0.50344	4.49073	0.14211	0.65581	1.44928	**1.84373**
010111010101010101010101	0.50344	4.48584	0.14211	0.65581	1.44928	**1.84373**
010101010101010101000101	0.50344	4.48584	0.14211	0.65581	1.44928	**1.84373**
000001000000000000000000	0.49601	4.49073	0.02773	0.6583	1.44422	**1.83705**
000000000000000000100000	0.49601	4.49073	0.02773	0.6583	1.44422	**1.83705**
010101010101010101110101	0.49601	4.4857	0.02773	0.6583	1.44422	**1.83705**
010100010101010101010101	0.49601	4.4857	0.02773	0.6583	1.44422	**1.83705**
000000100000000000000000	0.49614	4.49073	0.05529	0.66084	1.44007	**1.83142**
000000000000000001000000	0.49614	4.49073	0.05529	0.66084	1.44007	**1.83142**
010101110101010101010101	0.49614	4.48557	0.05529	0.66084	1.44007	**1.83142**
010101010101010100010101	0.49614	4.48557	0.05529	0.66084	1.44007	**1.83142**
000000010000000000000000	0.51487	4.49073	0.20655	0.66672	1.43677	**1.82688**
000000000000000010000000	0.51487	4.49073	0.20655	0.66672	1.43677	**1.82688**
010101000101010101010101	0.51487	4.48547	0.20655	0.66672	1.43677	**1.82688**
010101010101010111010101	0.51487	4.48547	0.20655	0.66672	1.43677	**1.82688**
000000000000000100000000	0.4806	4.49073	0.03665	0.66022	1.43468	**1.82346**
000000001000000000000000	0.4806	4.49073	0.03665	0.66022	1.43468	**1.82346**
010101011101010101010101	0.4806	4.48539	0.03665	0.66022	1.43468	**1.82346**
010101010101010001010101	0.4806	4.48539	0.03665	0.66022	1.43468	**1.82346**
000000000000001000000000	0.5116	4.49073	0.17404	0.66722	1.43312	**1.82116**
000000000100000000000000	0.5116	4.49073	0.17404	0.66722	1.43312	**1.82116**
010101010101011101010101	0.5116	4.48532	0.17404	0.66722	1.43312	**1.82116**
010101010001010101010101	0.5116	4.48532	0.17404	0.66722	1.43312	**1.82116**
000000000010000000000000	0.51472	4.49073	0.16065	0.66806	1.43264	**1.82001**
000000000001000000000000	0.49263	4.49073	0.07462	0.65828	1.43377	**1.82001**
000000000000100000000000	0.49263	4.49073	0.07462	0.65828	1.43377	**1.82001**
000000000000010000000000	0.51472	4.49073	0.16065	0.66806	1.43264	**1.82001**
010101010111010101010101	0.51472	4.48528	0.16065	0.66806	1.43264	**1.82001**
010101010101000101010101	0.51472	4.48528	0.16065	0.66806	1.43264	**1.82001**
010101010101110101010101	0.49263	4.48526	0.07461	0.65828	1.43377	**1.82001**
010101010100010101010101	0.49263	4.48526	0.07461	0.65828	1.43377	**1.82001**

Degree 24 ***symmetric** **~reciprocal**

50 smallest Mahler measure

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0000000000000000000000000~	***0.2***	5	0	0.45882	1.56756	2.02092
0101001010010100101001010~	***0.2***	3.30761	0	0.59913	1.33112	1.59302
0000011111000001111100000~	***0.2***	3.30761	0	0.59913	1.33112	1.59302
0101010101010101010101010~	***0.2***	4.99443	0	0.45882	1.56756	2.02092
0110110110110110110110110~	***0.32361***	3.40512	0.00009	0.61614	1.35601	1.63561
0011100011100011100011100~	***0.32361***	3.40512	0.00009	0.61614	1.35601	1.63561
0110011001100011001100110~	***0.32916***	3.22764	0.00009	0.63277	1.32149	1.57203
0011001100110110011001100~	***0.32916***	3.22764	0.00009	0.63277	1.32149	1.57203
0000101000011100001010000~	***0.33793***	2.39942	0	0.74513	1.1864	1.32819
0101111101001001011111010~	***0.33793***	2.39942	0	0.74513	1.1864	1.32819
0000011111111111111100000~	***0.34442***	3.03605	0	0.67749	1.27752	1.49774
0000000000111111111100000	***0.34442***	3.27189	0	0.66478	1.31451	1.57285
0101001010101011010101010	***0.34442***	3.27189	0	0.66478	1.31451	1.57285
0101001010101010101001010~	***0.34442***	3.03605	0	0.67749	1.27752	1.49774
0111001110011100111001110~	***0.34442***	3.23607	0.00096	0.65914	1.31426	1.56872
0101010101101011010101010~	***0.34442***	2.99452	0	0.67313	1.27871	1.49774
0011011001001101100100110	***0.34442***	3.25122	0.00333	0.65285	1.31658	1.57285
0010000100001000010000100~	***0.34442***	3	0.00203	0.69631	1.23442	1.41492
0110010011011001001101100	***0.34442***	3.25122	0.00333	0.65285	1.31658	1.57285
0101010101101010101001010	***0.34442***	3.27189	0	0.66478	1.31451	1.57285
0010011011001001101100100~	***0.34442***	3.23607	0.00096	0.65914	1.31426	1.56872
0000011111111110000000000	***0.34442***	3.27189	0	0.66478	1.31451	1.57285
0000000000111110000000000~	***0.34442***	3	0	0.67313	1.27871	1.49774
0110001100011000110001100	***0.34442***	3.25122	0.00333	0.65285	1.31658	1.57285
0011000110001100011000110	***0.34442***	3.25122	0.00333	0.65285	1.31658	1.57285
0111010001011101000101110~	***0.34442***	2.99657	0.00203	0.69631	1.23442	1.41492
0000011111111110000011111	***0.35665***	3.00802	0	0.69982	1.25345	1.46323
0101010101010100101010101	***0.35665***	3.53622	0	0.62994	1.36779	1.66341
0101001010010101010110101	***0.35665***	3.00802	0	0.69982	1.25345	1.46323
0011111000001111100000111	***0.35665***	3.25691	0.00406	0.65726	1.31609	1.57285
0001111100000111110000011	***0.35665***	3.25691	0.00406	0.65726	1.31609	1.57285
0000011111000000000011111	***0.35665***	3.00802	0	0.69982	1.25345	1.46323
0110110010011011001001101	***0.35665***	3.25415	0.00354	0.65811	1.31589	1.57285
0110101101011010110101101	***0.35665***	3.25691	0.00406	0.65726	1.31609	1.57285
0101001010101011010110101	***0.35665***	3.00802	0	0.69982	1.25345	1.46323
0011100111001110011100111	***0.35665***	3.25415	0.00354	0.65811	1.31589	1.57285
0000000000000001111111111	***0.35665***	3.53622	0	0.62994	1.36779	1.66341
0100101001010010100101001	***0.35665***	3.25691	0.00406	0.65726	1.31609	1.57285
0100110110010011011001001	***0.35665***	3.25415	0.00354	0.65811	1.31589	1.57285
0001100011000110001100011	***0.35665***	3.25415	0.00354	0.65811	1.31589	1.57285
0000000000111111111111111	***0.35665***	3.53622	0	0.62994	1.36779	1.66341
0101010101101010101010101	***0.35665***	3.53622	0	0.62994	1.36779	1.66341
0011001100110011001100110*	***0.35773***	3.53836	0.14631	0.58515	1.39763	1.7004
0110011001100110011001100*	***0.35773***	3.53836	0.14631	0.58515	1.39763	1.7004
0011011000110110001101100~	***0.36098***	2.67605	0.0008	0.70372	1.21784	1.38217
0110001101100011011000110~	***0.36098***	2.67605	0.0008	0.70372	1.21784	1.38217
0101010101000001010101010~	***0.36153***	4.19445	0	0.67845	1.35681	1.68856
0000000000010100000000000~	***0.36153***	4.2	0	0.67845	1.35681	1.68856
0111000111100011110001110~	***0.36355***	3.08562	0	0.68794	1.28047	1.51275
0010010010110110100100100~	***0.36355***	3.08562	0	0.68794	1.28047	1.51275

Degree 24 ***symmetric** **~reciprocal**

50 largest Mahler measure

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000111000000010101011011001	**0.96683**	1.38019	0.48233	0.98438	1.01427	1.02763
010010010101000000011100 11	**0.96683**	1.38019	0.48233	0.98438	1.01427	1.02763
011001001010111111111000111	**0.96683**	1.38019	0.48233	0.98438	1.01427	1.02763
001100011111101010110110 1	**0.96683**	1.38019	0.48233	0.98438	1.01427	1.02763
0110110101011111100011000	**0.96261**	1.41909	0.55855	0.9818	1.01721	1.03348
0011100000001010110110010	**0.96261**	1.41909	0.55855	0.9818	1.01721	1.03348
0100110110101000000011100	**0.96261**	1.41909	0.55855	0.9818	1.01721	1.03348
0001100011111101010110110	**0.96261**	1.41909	0.55855	0.9818	1.01721	1.03348
0011100111111010100110110*	**0.95752**	1.61245	0.65609	0.97764	1.02442	1.05045
0110110010101111110011100*	**0.95752**	1.61245	0.65609	0.97764	1.02442	1.05045
0010001000100101101111000	**0.95339**	1.45852	0.5714	0.97676	1.0228	1.04489
0111011101110000111010010	**0.95339**	1.45852	0.5714	0.97676	1.0228	1.04489
0001111011010010001000100	**0.95339**	1.45852	0.5714	0.97676	1.0228	1.04489
0100101110000111011101110	**0.95339**	1.45852	0.5714	0.97676	1.0228	1.04489
0000001110011011011010101	**0.95313**	1.39941	0.2	0.97805	1.02026	1.03923
0101011011001110001111111	**0.95313**	1.4	0.21333	0.97805	1.02026	1.03923
0101010010010011000111111	**0.95313**	1.39941	0.2	0.97805	1.02026	1.03923
0000000111000110010010101	**0.95313**	1.4	0.21333	0.97805	1.02026	1.03923
0101011011011000000011000	**0.95145**	1.38079	0.50526	0.97704	1.02062	1.03923
0000001110001101010110010	**0.95145**	1.38079	0.50526	0.97704	1.02062	1.03923
0100110101011000111000000	**0.95145**	1.38079	0.50526	0.97704	1.02062	1.03923
0001100000001101101101010	**0.95145**	1.38079	0.50526	0.97704	1.02062	1.03923
0111100010010110111011101	**0.95137**	1.58717	0.56584	0.97524	1.02522	1.05045
0010110111000011101110111	**0.95137**	1.58717	0.56584	0.97524	1.02522	1.05045
0100010001001011011100001	**0.95137**	1.58717	0.56584	0.97524	1.02522	1.05045
0001000100011110001001011	**0.95137**	1.58717	0.56584	0.97524	1.02522	1.05045
0101011110111100001101100	**0.95078**	1.39366	0.21153	0.97881	1.01772	1.03348
0110001101001011101000000	**0.95078**	1.39366	0.21153	0.97881	1.01772	1.03348
0000001011101001011000110	**0.95078**	1.39366	0.21153	0.97881	1.01772	1.03348
0011011000011110111101010	**0.95078**	1.39366	0.21153	0.97881	1.01772	1.03348
0101111110100110100011001	**0.94996**	1.57466	0.6	0.97476	1.02534	1.05045
0000101011110011110110011	**0.94996**	1.57466	0.6	0.97476	1.02534	1.05045
0110011101001101000000101	**0.94996**	1.57466	0.6	0.97476	1.02534	1.05045
0011001000011000010101111	**0.94996**	1.57466	0.6	0.97476	1.02534	1.05045
0110010011110000100001010	**0.94831**	1.47053	0.33558	0.97653	1.02052	1.03923
0101000010000111100100110	**0.94831**	1.47053	0.33558	0.97653	1.02052	1.03923
0011000110100101110100000	**0.94831**	1.47053	0.33558	0.97653	1.02052	1.03923
0000010111010010110001100	**0.94831**	1.47053	0.33558	0.97653	1.02052	1.03923
0100100101010001100011111	**0.94777**	1.32474	0.2	0.97665	1.01957	1.03637
0001110000000100110110101	**0.94777**	1.32474	0.21474	0.97665	1.01957	1.03637
0101001001101111111000111	**0.94777**	1.32474	0.21474	0.97665	1.01957	1.03637
0000011100111010101101101	**0.94777**	1.32474	0.2	0.97665	1.01957	1.03637
0101110100111111011000110	**0.94709**	1.39848	0.32153	0.97538	1.02201	1.04207
0011011000101011000010000	**0.94709**	1.4	0.32153	0.97538	1.02201	1.04207
0000100001101010001101100	**0.94709**	1.4	0.32153	0.97538	1.02201	1.04207
0110001101111111001011101 0	**0.94709**	1.39848	0.32153	0.97538	1.02201	1.04207
0110110000000101010011100*	**0.94696**	1.61245	0.47685	0.97388	1.02549	1.05045
0011100101010000000110110*	**0.94696**	1.61245	0.47685	0.97388	1.02549	1.05045
0101011011001000000011100	**0.94669**	1.33497	0.21228	0.97652	1.01952	1.03637
0110110101011100111000000	**0.94669**	1.33497	0.2	0.97652	1.01952	1.03637

Degree 24 ***symmetric** **~reciprocal**

50 smallest max modulus

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0111000011101110110100100*	0.91694	***1.28914***	0.2	0.96604	1.02517	1.04489
0010010110111011100001110*	0.91694	***1.28914***	0.2	0.96604	1.02517	1.04489
0101001010001110110011111	0.88164	***1.31061***	0.19219	0.95115	1.03508	1.06132
0000011001000111010110101	0.88164	***1.31061***	0.19219	0.95115	1.03508	1.06132
0101001100010010000011111	0.88164	***1.31061***	0.19219	0.95115	1.03508	1.06132
0000011111011011100110101	0.88164	***1.31061***	0.19219	0.95115	1.03508	1.06132
0111111011011000111010100*	0.92633	***1.31078***	0.29062	0.96694	1.02741	1.05045
0010101110001101101111110*	0.92633	***1.31078***	0.29062	0.96694	1.02741	1.05045
0010010000101100010101110	0.88948	***1.31601***	0.03352	0.95939	1.02854	1.05045
0111010100011010000100100	0.88948	***1.31601***	0.03352	0.95939	1.02854	1.05045
0010000000100111101001110	0.88948	***1.31601***	0.03352	0.95939	1.02854	1.05045
0111000101111001000000100	0.88948	***1.31601***	0.03352	0.95939	1.02854	1.05045
0000001110001101101101010*	0.94154	***1.32071***	0.34021	0.974	1.02133	1.03923
0101011011011000111000000*	0.94154	***1.32071***	0.34021	0.974	1.02133	1.03923
0010010010101111110001110*	0.93435	***1.32089***	0.2	0.97114	1.02411	1.04489
0111000111111010100100100*	0.93435	***1.32089***	0.2	0.97114	1.02411	1.04489
0111000111011000000010010	0.90509	***1.32204***	0.0795	0.96217	1.02814	1.05045
0100100000001101110001110	0.90509	***1.32204***	0.0795	0.96217	1.02814	1.05045
0001110101011000100100100	0.90509	***1.32204***	0.0795	0.96217	1.02814	1.05045
0010010010001101010111000	0.90509	***1.32204***	0.0795	0.96217	1.02814	1.05045
0011100000010101011011011	0.91773	***1.32254***	0.14926	0.96588	1.0263	1.04768
0010010010101011111100011	0.91773	***1.32254***	0.14926	0.96588	1.0263	1.04768
0111000111111110101001001	0.91773	***1.32254***	0.14926	0.96588	1.0263	1.04768
0110110101000000001110001	0.91773	***1.32254***	0.14926	0.96588	1.0263	1.04768
0000101001111101001101110	0.88961	***1.32287***	0.07609	0.95816	1.02982	1.0532
0111011001011111001010000	0.88961	***1.32287***	0.07609	0.95816	1.02982	1.0532
0101111100101000011000100	0.88961	***1.32287***	0.07609	0.95816	1.02982	1.0532
0010001100001010011111010	0.88961	***1.32287***	0.07609	0.95816	1.02982	1.0532
0100100101010001100011111	0.94777	***1.32474***	0.2	0.97665	1.01957	1.03637
0000011100111010101101101	0.94777	***1.32474***	0.2	0.97665	1.01957	1.03637
0101001001101111111000111	0.94777	***1.32474***	0.21474	0.97665	1.01957	1.03637
0001110000000100110110101	0.94777	***1.32474***	0.21474	0.97665	1.01957	1.03637
0101011111100011100100100	0.89968	***1.3259***	0.04377	0.96042	1.02968	1.0532
0010010011100011111101010	0.89968	***1.3259***	0.04377	0.96042	1.02968	1.0532
0000001010110110110001110	0.89968	***1.3259***	0.04377	0.96042	1.02968	1.0532
0111000110110110101000000	0.89968	***1.3259***	0.04377	0.96042	1.02968	1.0532
0110110101010000000011100*	0.91411	***1.32602***	0.24287	0.96357	1.02799	1.05045
0011100000000101010110110*	0.91411	***1.32602***	0.24287	0.96357	1.02799	1.05045
0110110001110000000010101	0.92133	***1.32686***	0.15591	0.96762	1.02475	1.04489
0101011111111000111001001	0.92133	***1.32686***	0.15591	0.96762	1.02475	1.04489
0011100100100101010111111	0.92133	***1.32686***	0.15591	0.96762	1.02475	1.04489
0000001010101101101100011	0.92133	***1.32686***	0.15591	0.96762	1.02475	1.04489
0101001010011001111100000*	0.88206	***1.32694***	0.1283	0.95269	1.03458	1.06132
0000011111001100101001010*	0.88206	***1.32694***	0.1283	0.95269	1.03458	1.06132
0000011011011001110010101	0.91653	***1.33139***	0.09618	0.96776	1.02344	1.04207
0101011000110010010011111	0.91653	***1.33139***	0.09618	0.96776	1.02344	1.04207
0101001110001100100111111	0.91653	***1.33139***	0.09618	0.96776	1.02344	1.04207
0000001101100111000110101	0.91653	***1.33139***	0.09618	0.96776	1.02344	1.04207
0000000110101001110100110	0.91336	***1.33226***	0.19657	0.9646	1.02653	1.04768
0110010111001010110000000	0.91336	***1.33226***	0.19657	0.9646	1.02653	1.04768

Degree 24 ***symmetric** **~reciprocal**

50 largest minimum modulus

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0011100111111010100110110*	0.95752	1.61245	**0.65609**	0.97764	1.02442	1.05045
0110110010101111110011100*	0.95752	1.61245	**0.65609**	0.97764	1.02442	1.05045
0111001011100110111100100*	0.92027	1.96977	**0.6**	0.95721	1.04856	1.10178
0110011101001101000000101	0.94996	1.57466	**0.6**	0.97476	1.02534	1.05045
0101111110100110100011001	0.94996	1.57466	**0.6**	0.97476	1.02534	1.05045
0101010000110000100110111	0.93271	1.73862	**0.6**	0.96533	1.03582	1.07187
0100011000101100101111111	0.93271	1.73862	**0.6**	0.96533	1.03582	1.07187
0000000101100101110011101	0.93271	1.73862	**0.6**	0.96533	1.03582	1.07187
0011001000011000010101111	0.94996	1.57466	**0.6**	0.97476	1.02534	1.05045
0010011110110011101001110*	0.92027	1.96977	**0.6**	0.95721	1.04856	1.10178
0001001101111001111010101	0.93271	1.73862	**0.6**	0.96533	1.03582	1.07187
0000101011110011110110011	0.94996	1.57466	**0.6**	0.97476	1.02534	1.05045
0011111100110100110101010	0.9291	1.79958	**0.59159**	0.96313	1.03908	1.07959
0101010110010110011111100	0.9291	1.79958	**0.59159**	0.96313	1.03908	1.07959
0000000011000011001010110	0.9291	1.8	**0.59159**	0.96313	1.03908	1.07959
0110101001100001100000000	0.9291	1.8	**0.59159**	0.96313	1.03908	1.07959
0101010110110011100000000*	0.93774	1.62215	**0.58891**	0.96738	1.03517	1.07187
0000000011100110110101010*	0.93774	1.62215	**0.58891**	0.96738	1.03517	1.07187
0000111101101001000100010	0.93608	1.66662	**0.58229**	0.9667	1.03538	1.07187
0100010001001011011110000	0.93608	1.66662	**0.58229**	0.9667	1.03538	1.07187
0101101000111100010001000	0.93608	1.66662	**0.58229**	0.9667	1.03538	1.07187
0001000100011110001011010	0.93608	1.66662	**0.58229**	0.9667	1.03538	1.07187
0100001011111011001111100	0.91773	1.80333	**0.57914**	0.95691	1.04566	1.09208
0011111001101111101000010	0.91773	1.80333	**0.57914**	0.95691	1.04566	1.09208
0001011110101110011010110	0.91773	1.80333	**0.57914**	0.95691	1.04566	1.09208
0110101100111010111101000	0.91773	1.80333	**0.57914**	0.95691	1.04566	1.09208
0101010010010001110000000*	0.93549	1.8	**0.57274**	0.96604	1.03723	1.07703
0000000111000100100101010*	0.93549	1.8	**0.57274**	0.96604	1.03723	1.07703
0001111011010010001000100	0.95339	1.45852	**0.5714**	0.97676	1.0228	1.04489
0010001000100101101111000	0.95339	1.45852	**0.5714**	0.97676	1.0228	1.04489
0111011101110000111010010	0.95339	1.45852	**0.5714**	0.97676	1.0228	1.04489
0100101110000111011101110	0.95339	1.45852	**0.5714**	0.97676	1.0228	1.04489
0110000110011111100101011	0.94174	1.50055	**0.57119**	0.97083	1.02856	1.05593
0010101100000011001111001	0.94174	1.50055	**0.57119**	0.97083	1.02856	1.05593
0011010011001010110000001	0.94174	1.50055	**0.57119**	0.97083	1.02856	1.05593
0111111001010110011010011	0.94174	1.50055	**0.57119**	0.97083	1.02856	1.05593
0011100101011001001111111	0.94621	1.65843	**0.57097**	0.97278	1.02776	1.05593
0000000110110010101100011	0.94621	1.65843	**0.57097**	0.97278	1.02776	1.05593
0101010011100111111001001	0.94621	1.65843	**0.57097**	0.97278	1.02776	1.05593
0110110000001100011010101	0.94621	1.65843	**0.57097**	0.97278	1.02776	1.05593
0101011000110000001101101	0.94001	1.65921	**0.57042**	0.96955	1.03078	1.06132
0100100111111001110010101	0.94001	1.65921	**0.57042**	0.96955	1.03078	1.06132
0001110010101100100111111	0.94001	1.65921	**0.57042**	0.96955	1.03078	1.06132
0000001101100101011000111	0.94001	1.65921	**0.57042**	0.96955	1.03078	1.06132
0000000001110011011010101	0.93021	1.73627	**0.56773**	0.96386	1.03802	1.07703
0101010010011000111111111	0.93021	1.73627	**0.56773**	0.96386	1.03802	1.07703
0101010100100110001111111	0.93021	1.73627	**0.56773**	0.96386	1.03802	1.07703
0000000111001101101010101	0.93021	1.73627	**0.56773**	0.96386	1.03802	1.07703
0001000100011110001001011	0.95137	1.58717	**0.56584**	0.97524	1.02522	1.05045
0010110111000011101110111	0.95137	1.58717	**0.56584**	0.97524	1.02522	1.05045

Degree 24 ***symmetric** **~reciprocal**

50 smallest L1 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0101010101010101010101010~	0.2	4.99443	0	***0.45882***	1.56756	2.02092
0000000000000000000000000~	0.2	5	0	***0.45882***	1.56756	2.02092
0101010101010101010101011	0.4	4.59577	0.06883	***0.58311***	1.49361	1.90418
0010101010101010101010101	0.4	4.59577	0.06883	***0.58311***	1.49361	1.90418
0111111111111111111111111	0.4	4.6	0.06882	***0.58311***	1.49361	1.90418
0000000000000000000000001	0.4	4.6	0.06882	***0.58311***	1.49361	1.90418
0110011001100110011001100*	0.35773	3.53836	0.14631	***0.58515***	1.39763	1.7004
0011001100110011001100110*	0.35773	3.53836	0.14631	***0.58515***	1.39763	1.7004
0101001010010100101001010~	0.2	3.30761	0	***0.59913***	1.33112	1.59302
0000011111000001111100000~	0.2	3.30761	0	***0.59913***	1.33112	1.59302
0101010101010101010101000	0.42412	4.59556	0.03432	***0.60585***	1.48447	1.89437
0001010101010101010101010	0.42412	4.59556	0.03432	***0.60585***	1.48447	1.89437
0100000000000000000000000	0.42412	4.6	0.03432	***0.60585***	1.48447	1.89437
0000000000000000000000010	0.42412	4.6	0.03432	***0.60585***	1.48447	1.89437
0000000000000111111111111	0.3946	3.62328	0.2	***0.6098***	1.39196	1.7004
0000000000001111111111111	0.3946	3.62328	0.2	***0.6098***	1.39196	1.7004
0101010101011010101010101	0.3946	3.62328	0.2	***0.6098***	1.39196	1.7004
0101010101010010101010101	0.3946	3.62328	0.2	***0.6098***	1.39196	1.7004
0011100011100011100011100~	0.32361	3.40512	0.00009	***0.61614***	1.35601	1.63561
0110110110110110110110110~	0.32361	3.40512	0.00009	***0.61614***	1.35601	1.63561
0101010101010101010101001	0.4	4.19713	0.0098	***0.61812***	1.43317	1.7989
0110101010101010101010101	0.4	4.19713	0.0098	***0.61812***	1.43317	1.7989
0000000000000000000000011	0.4	4.2	0.0098	***0.61812***	1.43317	1.7989
0011111111111111111111111	0.4	4.2	0.0098	***0.61812***	1.43317	1.7989
0101010101011101010101010~	0.40419	4.59443	0.00215	***0.62566***	1.44917	1.84595
0000000000001000000000000~	0.40419	4.6	0.00215	***0.62566***	1.44917	1.84595
0101010101010110101010101	0.39653	3.59417	0.04685	***0.62612***	1.38161	1.68657
0101010101001010101010101	0.39653	3.59417	0.04685	***0.62612***	1.38161	1.68657
0000000000011111111111111	0.39653	3.59417	0.04685	***0.62612***	1.38161	1.68657
0000000000000011111111111	0.39653	3.59417	0.04685	***0.62612***	1.38161	1.68657
0101010101010101010101110	0.4687	4.59536	0.06221	***0.6276***	1.47642	1.88537
0111010101010101010101010	0.4687	4.59536	0.06221	***0.6276***	1.47642	1.88537
0010000000000000000000000	0.4687	4.6	0.06221	***0.6276***	1.47642	1.88537
0000000000000000000000100	0.4687	4.6	0.06221	***0.6276***	1.47642	1.88537
0101010101010100101010101	0.35665	3.53622	0	***0.62994***	1.36779	1.66341
0101010101101010101010101	0.35665	3.53622	0	***0.62994***	1.36779	1.66341
0000000000111111111111111	0.35665	3.53622	0	***0.62994***	1.36779	1.66341
0000000000000001111111111	0.35665	3.53622	0	***0.62994***	1.36779	1.66341
0110011001100011001100110~	0.32916	3.22764	0.00009	***0.63277***	1.32149	1.57203
0011001100110110011001100~	0.32916	3.22764	0.00009	***0.63277***	1.32149	1.57203
0111000111000111000111000	0.39851	3.30531	0.04771	***0.63418***	1.34249	1.60862
0001110001110001110001110	0.39851	3.30531	0.04771	***0.63418***	1.34249	1.60862
0100100100100100100100100	0.39851	3.30531	0.04771	***0.63418***	1.34249	1.60862
0010010010010010010010010	0.39851	3.30531	0.04771	***0.63418***	1.34249	1.60862
0101010101010101010101101	0.4	3.79854	0.01415	***0.63535***	1.38652	1.71008
0100101010101010101010101	0.4	3.79854	0.01415	***0.63535***	1.38652	1.71008
0001111111111111111111111	0.4	3.8	0.01415	***0.63535***	1.38652	1.71008
0000000000000000000000111	0.4	3.8	0.01415	***0.63535***	1.38652	1.71008
0101010101010101101010101	0.39886	3.45633	0.0316	***0.63761***	1.35327	1.63561
0101010100101010101010101	0.39886	3.45633	0.0316	***0.63761***	1.35327	1.63561

Degree 24 ***symmetric** **~reciprocal**

50 largest L1 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0110010010101111111000111	0.96683	1.38019	0.48233	**0.98438**	1.01427	1.02763
0001110000000101011011001	0.96683	1.38019	0.48233	**0.98438**	1.01427	1.02763
0100100101010000001110011	0.96683	1.38019	0.48233	**0.98438**	1.01427	1.02763
0011000111111010101101101	0.96683	1.38019	0.48233	**0.98438**	1.01427	1.02763
0011100000001010110110010	0.96261	1.41909	0.55855	**0.9818**	1.01721	1.03348
0100110110101000000011100	0.96261	1.41909	0.55855	**0.9818**	1.01721	1.03348
0001100011111101010110110	0.96261	1.41909	0.55855	**0.9818**	1.01721	1.03348
0110110101011111100011000	0.96261	1.41909	0.55855	**0.9818**	1.01721	1.03348
0000001011101001011000110	0.95078	1.39366	0.21153	**0.97881**	1.01772	1.03348
0110001101001011101000000	0.95078	1.39366	0.21153	**0.97881**	1.01772	1.03348
0101011110111100001101100	0.95078	1.39366	0.21153	**0.97881**	1.01772	1.03348
0011011000011110111101010	0.95078	1.39366	0.21153	**0.97881**	1.01772	1.03348
0101011011001110001111111	0.95313	1.4	0.21333	**0.97805**	1.02026	1.03923
0000000111000110010010101	0.95313	1.4	0.21333	**0.97805**	1.02026	1.03923
0101010010010011000111111	0.95313	1.39941	0.2	**0.97805**	1.02026	1.03923
0000001110011011011010101	0.95313	1.39941	0.2	**0.97805**	1.02026	1.03923
0110110010101111110011100*	0.95752	1.61245	0.65609	**0.97764**	1.02442	1.05045
0011100111111010100110110*	0.95752	1.61245	0.65609	**0.97764**	1.02442	1.05045
0100110101011000111000000	0.95145	1.38079	0.50526	**0.97704**	1.02062	1.03923
0000001110001101010110010	0.95145	1.38079	0.50526	**0.97704**	1.02062	1.03923
0101011011011000000011000	0.95145	1.38079	0.50526	**0.97704**	1.02062	1.03923
0001100000001101101101010	0.95145	1.38079	0.50526	**0.97704**	1.02062	1.03923
0010001000100101101111000	0.95339	1.45852	0.5714	**0.97676**	1.0228	1.04489
0001111011010010001000100	0.95339	1.45852	0.5714	**0.97676**	1.0228	1.04489
0111011101110000111010010	0.95339	1.45852	0.5714	**0.97676**	1.0228	1.04489
0100101110000111011101110	0.95339	1.45852	0.5714	**0.97676**	1.0228	1.04489
0001110000000100110110101	0.94777	1.32474	0.21474	**0.97665**	1.01957	1.03637
0101001001101111111000111	0.94777	1.32474	0.21474	**0.97665**	1.01957	1.03637
0100100101010001100011111	0.94777	1.32474	0.2	**0.97665**	1.01957	1.03637
0000011100111010101101101	0.94777	1.32474	0.2	**0.97665**	1.01957	1.03637
0000010111010010110001100	0.94831	1.47053	0.33558	**0.97653**	1.02052	1.03923
0011000110100101110100000	0.94831	1.47053	0.33558	**0.97653**	1.02052	1.03923
0110010011110000100001010	0.94831	1.47053	0.33558	**0.97653**	1.02052	1.03923
0101000010000111100100110	0.94831	1.47053	0.33558	**0.97653**	1.02052	1.03923
0101011011001000000011100	0.94669	1.33497	0.21228	**0.97652**	1.01952	1.03637
0011100000001001101101010	0.94669	1.33497	0.21228	**0.97652**	1.01952	1.03637
0000001110011101010110110	0.94669	1.33497	0.2	**0.97652**	1.01952	1.03637
0110110101011100111000000	0.94669	1.33497	0.2	**0.97652**	1.01952	1.03637
0011011000101011000010000	0.94709	1.4	0.32153	**0.97538**	1.02201	1.04207
0000100001101010001101100	0.94709	1.4	0.32153	**0.97538**	1.02201	1.04207
0101110100111111011000110	0.94709	1.39848	0.32153	**0.97538**	1.02201	1.04207
0110001101111111001011010	0.94709	1.39848	0.32153	**0.97538**	1.02201	1.04207
0111100010010110111011101	0.95137	1.58717	0.56584	**0.97524**	1.02522	1.05045
0100010001001011011100001	0.95137	1.58717	0.56584	**0.97524**	1.02522	1.05045
0010110111000011101110111	0.95137	1.58717	0.56584	**0.97524**	1.02522	1.05045
0001000100011110001001011	0.95137	1.58717	0.56584	**0.97524**	1.02522	1.05045
0101111110100110100011001	0.94996	1.57466	0.6	**0.97476**	1.02534	1.05045
0110011101001101000000101	0.94996	1.57466	0.6	**0.97476**	1.02534	1.05045
0011001000011000010101111	0.94996	1.57466	0.6	**0.97476**	1.02534	1.05045
0000101011110011110110011	0.94996	1.57466	0.6	**0.97476**	1.02534	1.05045

Degree 24 ***symmetric** **~reciprocal**

50 smallest L3 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
0011000111111010101101101	0.96683	1.38019	0.48233	0.98438	***1.01427***	1.02763
0100100101010000001110011	0.96683	1.38019	0.48233	0.98438	***1.01427***	1.02763
0001110000000101011011001	0.96683	1.38019	0.48233	0.98438	***1.01427***	1.02763
0110010010101111111000111	0.96683	1.38019	0.48233	0.98438	***1.01427***	1.02763
0110110101011111100011000	0.96261	1.41909	0.55855	0.9818	***1.01721***	1.03348
0001100011111101010110110	0.96261	1.41909	0.55855	0.9818	***1.01721***	1.03348
0100110110101000000011100	0.96261	1.41909	0.55855	0.9818	***1.01721***	1.03348
0011100000001010110110010	0.96261	1.41909	0.55855	0.9818	***1.01721***	1.03348
0101011110111100001101100	0.95078	1.39366	0.21153	0.97881	***1.01772***	1.03348
0011011000011110111101010	0.95078	1.39366	0.21153	0.97881	***1.01772***	1.03348
0110001101001011101000000	0.95078	1.39366	0.21153	0.97881	***1.01772***	1.03348
0000001011101001011000110	0.95078	1.39366	0.21153	0.97881	***1.01772***	1.03348
0110110101011100111000000	0.94669	1.33497	0.2	0.97652	***1.01952***	1.03637
0000001110011101010110110	0.94669	1.33497	0.2	0.97652	***1.01952***	1.03637
0101011011001000000011100	0.94669	1.33497	0.21228	0.97652	***1.01952***	1.03637
0011100000001001101101010	0.94669	1.33497	0.21228	0.97652	***1.01952***	1.03637
0100100101010001100011111	0.94777	1.32474	0.2	0.97665	***1.01957***	1.03637
0000011100111010101101101	0.94777	1.32474	0.2	0.97665	***1.01957***	1.03637
0101001001101111111000111	0.94777	1.32474	0.21474	0.97665	***1.01957***	1.03637
0001110000000100110110101	0.94777	1.32474	0.21474	0.97665	***1.01957***	1.03637
0101010010010011000111111	0.95313	1.39941	0.2	0.97805	***1.02026***	1.03923
0000001110011011011010101	0.95313	1.39941	0.2	0.97805	***1.02026***	1.03923
0000000111000110010010101	0.95313	1.4	0.21333	0.97805	***1.02026***	1.03923
0101011011001110001111111	0.95313	1.4	0.21333	0.97805	***1.02026***	1.03923
0110010011110000100001010	0.94831	1.47053	0.33558	0.97653	***1.02052***	1.03923
0101000010000111100100110	0.94831	1.47053	0.33558	0.97653	***1.02052***	1.03923
0011000110100101110100000	0.94831	1.47053	0.33558	0.97653	***1.02052***	1.03923
0000010111010010110001100	0.94831	1.47053	0.33558	0.97653	***1.02052***	1.03923
0001100000001101101101010	0.95145	1.38079	0.50526	0.97704	***1.02062***	1.03923
0101011011011000000011000	0.95145	1.38079	0.50526	0.97704	***1.02062***	1.03923
0100110101011000111000000	0.95145	1.38079	0.50526	0.97704	***1.02062***	1.03923
0000001110001101010110010	0.95145	1.38079	0.50526	0.97704	***1.02062***	1.03923
0101011011000000011000011	0.94049	1.35391	0.18264	0.97466	***1.02091***	1.03923
0011110011111110010010101	0.94049	1.35391	0.18264	0.97466	***1.02091***	1.03923
0000001110010101001101001	0.94049	1.35391	0.18264	0.97466	***1.02091***	1.03923
0110100110101011000111111	0.94049	1.35391	0.18264	0.97466	***1.02091***	1.03923
0110101100011101000100000	0.9429	1.35402	0.27944	0.97461	***1.02112***	1.03923
0000010001011100011010110	0.9429	1.35402	0.27944	0.97461	***1.02112***	1.03923
0101000100001001001111100	0.9429	1.35402	0.27944	0.97461	***1.02112***	1.03923
0011111001001000010001010	0.9429	1.35402	0.27944	0.97461	***1.02112***	1.03923
0101011011011000111000000*	0.94154	1.32071	0.34021	0.974	***1.02133***	1.03923
0000001110001101101101010*	0.94154	1.32071	0.34021	0.974	***1.02133***	1.03923
0110001101111110010111010	0.94709	1.39848	0.32153	0.97538	***1.02201***	1.04207
0101110100111111011000110	0.94709	1.39848	0.32153	0.97538	***1.02201***	1.04207
0011011000101011000010000	0.94709	1.4	0.32153	0.97538	***1.02201***	1.04207
0000100001101010001101100	0.94709	1.4	0.32153	0.97538	***1.02201***	1.04207
0101101011111000001001100	0.94174	1.36606	0.2	0.97386	***1.0224***	1.04207
0011001000001111101011010	0.94174	1.36606	0.2	0.97386	***1.0224***	1.04207
0110011101011010111110000	0.94174	1.36606	0.20525	0.97386	***1.0224***	1.04207
0000111110101101011100110	0.94174	1.36606	0.20525	0.97386	***1.0224***	1.04207

Degree 24 ***symmetric** **~reciprocal**

50 largest L3 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0000000000000000000000000~	0.2	5	0	0.45882	**1.56756**	2.02092
0101010101010101010101010~	0.2	4.99443	0	0.45882	**1.56756**	2.02092
0111111111111111111111111	0.4	4.6	0.06882	0.58311	**1.49361**	1.90418
0000000000000000000000001	0.4	4.6	0.06882	0.58311	**1.49361**	1.90418
0101010101010101010101011	0.4	4.59577	0.06883	0.58311	**1.49361**	1.90418
0010101010101010101010101	0.4	4.59577	0.06883	0.58311	**1.49361**	1.90418
0100000000000000000000000	0.42412	4.6	0.03432	0.60585	**1.48447**	1.89437
0000000000000000000000010	0.42412	4.6	0.03432	0.60585	**1.48447**	1.89437
0101010101010101010101000	0.42412	4.59556	0.03432	0.60585	**1.48447**	1.89437
0001010101010101010101010	0.42412	4.59556	0.03432	0.60585	**1.48447**	1.89437
0010000000000000000000000	0.4687	4.6	0.06221	0.6276	**1.47642**	1.88537
0000000000000000000000100	0.4687	4.6	0.06221	0.6276	**1.47642**	1.88537
0101010101010101010101110	0.4687	4.59536	0.06221	0.6276	**1.47642**	1.88537
0111010101010101010101010	0.4687	4.59536	0.06221	0.6276	**1.47642**	1.88537
0001000000000000000000000	0.48256	4.6	0.0159	0.63922	**1.46954**	1.8772
0000000000000000000001000	0.48256	4.6	0.0159	0.63922	**1.46954**	1.8772
0101010101010101010100010	0.48256	4.59519	0.01589	0.63922	**1.46954**	1.87719
0100010101010101010101010	0.48256	4.59519	0.01589	0.63922	**1.46954**	1.87719
0000100000000000000000000	0.49443	4.6	0.16356	0.64646	**1.46373**	1.8699
0000000000000000000010000	0.49443	4.6	0.16356	0.64646	**1.46373**	1.8699
0101110101010101010101010	0.49443	4.59503	0.16356	0.64646	**1.46373**	1.8699
0101010101010101010111010	0.49443	4.59503	0.16356	0.64646	**1.46373**	1.8699
0000010000000000000000000	0.48861	4.6	0.04442	0.64887	**1.45886**	1.8635
0000000000000000000100000	0.48861	4.6	0.04442	0.64887	**1.45886**	1.8635
0101010101010101010001010	0.48861	4.59489	0.04442	0.64887	**1.45886**	1.8635
0101000101010101010101010	0.48861	4.59489	0.04442	0.64887	**1.45886**	1.8635
0000001000000000000000000	0.4923	4.6	0.1112	0.65197	**1.45481**	1.85804
0000000000000000001000000	0.4923	4.6	0.1112	0.65197	**1.45481**	1.85804
0101011101010101010101010	0.4923	4.59477	0.1112	0.65197	**1.45481**	1.85804
0101010101010101011101010	0.4923	4.59477	0.1112	0.65197	**1.45481**	1.85804
0000000100000000000000000	0.49849	4.6	0.15279	0.6554	**1.45154**	1.85353
0000000000000000010000000	0.49849	4.6	0.15279	0.6554	**1.45154**	1.85353
0101010101010101000101010	0.49849	4.59467	0.15279	0.6554	**1.45154**	1.85353
0101010001010101010101010	0.49849	4.59467	0.15279	0.6554	**1.45154**	1.85353
0000000010000000000000000	0.48073	4.6	0.063	0.65173	**1.44928**	1.85
0000000000000000100000000	0.48073	4.6	0.063	0.65173	**1.44928**	1.85
0101010101010101110101010	0.48073	4.59458	0.063	0.65173	**1.44928**	1.85
0101010111010101010101010	0.48073	4.59458	0.063	0.65173	**1.44928**	1.85
0000000000001000000000000~	0.40419	4.6	0.00215	0.62566	**1.44917**	1.84595
0101010101011101010101010~	0.40419	4.59443	0.00215	0.62566	**1.44917**	1.84595
0000000001000000000000000	0.49667	4.6	0.13697	0.65628	**1.44757**	1.84747
0000000000000001000000000	0.49667	4.6	0.13697	0.65628	**1.44757**	1.84747
0101010101010100010101010	0.49667	4.59452	0.13697	0.65628	**1.44757**	1.84747
0101010100010101010101010	0.49667	4.59452	0.13697	0.65628	**1.44757**	1.84747
0000000000010000000000000	0.5028	4.6	0.10939	0.65692	**1.44691**	1.84544
0000000000000100000000000	0.5028	4.6	0.10939	0.65692	**1.44691**	1.84544
0101010101010001010101010	0.5028	4.59444	0.10939	0.65692	**1.44691**	1.84544
0101010101000101010101010	0.5028	4.59444	0.10939	0.65692	**1.44691**	1.84544
0000000000100000000000000	0.50105	4.6	0.15306	0.65812	**1.44668**	1.84595
0000000000000010000000000	0.50105	4.6	0.15306	0.65812	**1.44668**	1.84595

Degree 24 ***symmetric** **~reciprocal**

50 smallest L4 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0011000111111010101101101	0.96683	1.38019	0.48233	0.98438	1.01427	***1.02763***
0100100101010000001110011	0.96683	1.38019	0.48233	0.98438	1.01427	***1.02763***
0001110000000101011011001	0.96683	1.38019	0.48233	0.98438	1.01427	***1.02763***
0110010010101111111000111	0.96683	1.38019	0.48233	0.98438	1.01427	***1.02763***
0101011110111100001101100	0.95078	1.39366	0.21153	0.97881	1.01772	***1.03348***
0011011000011110111101010	0.95078	1.39366	0.21153	0.97881	1.01772	***1.03348***
0110110101011111100011000	0.96261	1.41909	0.55855	0.9818	1.01721	***1.03348***
0001100011111101010110110	0.96261	1.41909	0.55855	0.9818	1.01721	***1.03348***
0011100000001010110110010	0.96261	1.41909	0.55855	0.9818	1.01721	***1.03348***
0100110110101000000011100	0.96261	1.41909	0.55855	0.9818	1.01721	***1.03348***
0110001101001011101000000	0.95078	1.39366	0.21153	0.97881	1.01772	***1.03348***
0000001011101001011000110	0.95078	1.39366	0.21153	0.97881	1.01772	***1.03348***
0110110101011100111000000	0.94669	1.33497	0.2	0.97652	1.01952	***1.03637***
0000001110011101010110110	0.94669	1.33497	0.2	0.97652	1.01952	***1.03637***
0100100101010001100011111	0.94777	1.32474	0.2	0.97665	1.01957	***1.03637***
0000011100111010101101101	0.94777	1.32474	0.2	0.97665	1.01957	***1.03637***
0101001001101111111000111	0.94777	1.32474	0.21474	0.97665	1.01957	***1.03637***
0001110000000100110110101	0.94777	1.32474	0.21474	0.97665	1.01957	***1.03637***
0101011011001000000011100	0.94669	1.33497	0.21228	0.97652	1.01952	***1.03637***
0011100000001001101101010	0.94669	1.33497	0.21228	0.97652	1.01952	***1.03637***
0000001110011011011010101	0.95313	1.39941	0.2	0.97805	1.02026	***1.03923***
0101010010010011000111111	0.95313	1.39941	0.2	0.97805	1.02026	***1.03923***
0110010011110000100001010	0.94831	1.47053	0.33558	0.97653	1.02052	***1.03923***
0101000010000111100100110	0.94831	1.47053	0.33558	0.97653	1.02052	***1.03923***
0001100000001101101101010	0.95145	1.38079	0.50526	0.97704	1.02062	***1.03923***
0101011011011000000011000	0.95145	1.38079	0.50526	0.97704	1.02062	***1.03923***
0011000110100101110100000	0.94831	1.47053	0.33558	0.97653	1.02052	***1.03923***
0000010111010010110001100	0.94831	1.47053	0.33558	0.97653	1.02052	***1.03923***
0101011011000000011000011	0.94049	1.35391	0.18264	0.97466	1.02091	***1.03923***
0100110101011000111000000	0.95145	1.38079	0.50526	0.97704	1.02062	***1.03923***
0011110011111110010010101	0.94049	1.35391	0.18264	0.97466	1.02091	***1.03923***
0000001110001101010110010	0.95145	1.38079	0.50526	0.97704	1.02062	***1.03923***
0110101100011101000100000	0.9429	1.35402	0.27944	0.97461	1.02112	***1.03923***
0000010001011100011010110	0.9429	1.35402	0.27944	0.97461	1.02112	***1.03923***
0110100110101011000111111	0.94049	1.35391	0.18264	0.97466	1.02091	***1.03923***
0000001110010101001101001	0.94049	1.35391	0.18264	0.97466	1.02091	***1.03923***
0101000100001001001111100	0.9429	1.35402	0.27944	0.97461	1.02112	***1.03923***
0011111001001000010001010	0.9429	1.35402	0.27944	0.97461	1.02112	***1.03923***
0101011011011000111000000*	0.94154	1.32071	0.34021	0.974	1.02133	***1.03923***
0000001110001101101101010*	0.94154	1.32071	0.34021	0.974	1.02133	***1.03923***
0101011011001110001111111	0.95313	1.4	0.21333	0.97805	1.02026	***1.03923***
0000000111000110010010101	0.95313	1.4	0.21333	0.97805	1.02026	***1.03923***
0110001101111110010111010	0.94709	1.39848	0.32153	0.97538	1.02201	***1.04207***
0101110100111111011000110	0.94709	1.39848	0.32153	0.97538	1.02201	***1.04207***
0100110100111001010111111	0.93975	1.39863	0.21832	0.97332	1.02248	***1.04207***
0000001010110001101001101	0.93975	1.39863	0.21832	0.97332	1.02248	***1.04207***
0010101100100111100010000	0.92518	1.3988	0.08165	0.97005	1.02305	***1.04207***
0000100011110010011010100	0.92518	1.3988	0.08165	0.97005	1.02305	***1.04207***
0111111001110010110111010	0.92518	1.4	0.08165	0.97005	1.02305	***1.04207***
0101110110100111001111110	0.92518	1.4	0.08165	0.97005	1.02305	***1.04207***

Degree 24 ***symmetric** **~reciprocal**

50 largest L4 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0000000000000000000000000~	0.2	5	0	0.45882	1.56756	**2.02092**
0101010101010101010101010~	0.2	4.99443	0	0.45882	1.56756	**2.02092**
0111111111111111111111111	0.4	4.6	0.06882	0.58311	1.49361	**1.90418**
0000000000000000000000001	0.4	4.6	0.06882	0.58311	1.49361	**1.90418**
0101010101010101010101011	0.4	4.59577	0.06883	0.58311	1.49361	**1.90418**
0010101010101010101010101	0.4	4.59577	0.06883	0.58311	1.49361	**1.90418**
0100000000000000000000000	0.42412	4.6	0.03432	0.60585	1.48447	**1.89437**
0000000000000000000000010	0.42412	4.6	0.03432	0.60585	1.48447	**1.89437**
0101010101010101010101000	0.42412	4.59556	0.03432	0.60585	1.48447	**1.89437**
0001010101010101010101010	0.42412	4.59556	0.03432	0.60585	1.48447	**1.89437**
0010000000000000000000000	0.4687	4.6	0.06221	0.6276	1.47642	**1.88537**
0000000000000000000000100	0.4687	4.6	0.06221	0.6276	1.47642	**1.88537**
0111010101010101010101010	0.4687	4.59536	0.06221	0.6276	1.47642	**1.88537**
0101010101010101010101110	0.4687	4.59536	0.06221	0.6276	1.47642	**1.88537**
0001000000000000000000000	0.48256	4.6	0.0159	0.63922	1.46954	**1.8772**
0000000000000000000001000	0.48256	4.6	0.0159	0.63922	1.46954	**1.8772**
0101010101010101010100010	0.48256	4.59519	0.01589	0.63922	1.46954	**1.87719**
0100010101010101010101010	0.48256	4.59519	0.01589	0.63922	1.46954	**1.87719**
0000100000000000000000000	0.49443	4.6	0.16356	0.64646	1.46373	**1.8699**
0000000000000000000010000	0.49443	4.6	0.16356	0.64646	1.46373	**1.8699**
0101110101010101010101010	0.49443	4.59503	0.16356	0.64646	1.46373	**1.8699**
0101010101010101010111010	0.49443	4.59503	0.16356	0.64646	1.46373	**1.8699**
0000010000000000000000000	0.48861	4.6	0.04442	0.64887	1.45886	**1.8635**
0000000000000000000100000	0.48861	4.6	0.04442	0.64887	1.45886	**1.8635**
0101010101010101010001010	0.48861	4.59489	0.04442	0.64887	1.45886	**1.8635**
0101000101010101010101010	0.48861	4.59489	0.04442	0.64887	1.45886	**1.8635**
0000001000000000000000000	0.4923	4.6	0.1112	0.65197	1.45481	**1.85804**
0000000000000000001000000	0.4923	4.6	0.1112	0.65197	1.45481	**1.85804**
0101011101010101010101010	0.4923	4.59477	0.1112	0.65197	1.45481	**1.85804**
0101010101010101011101010	0.4923	4.59477	0.1112	0.65197	1.45481	**1.85804**
0000000100000000000000000	0.49849	4.6	0.15279	0.6554	1.45154	**1.85353**
0000000000000000010000000	0.49849	4.6	0.15279	0.6554	1.45154	**1.85353**
0101010101010101000101010	0.49849	4.59467	0.15279	0.6554	1.45154	**1.85353**
0101010001010101010101010	0.49849	4.59467	0.15279	0.6554	1.45154	**1.85353**
0000000010000000000000000	0.48073	4.6	0.063	0.65173	1.44928	**1.85**
0000000000000000100000000	0.48073	4.6	0.063	0.65173	1.44928	**1.85**
0101010111010101010101010	0.48073	4.59458	0.063	0.65173	1.44928	**1.85**
0101010101010101110101010	0.48073	4.59458	0.063	0.65173	1.44928	**1.85**
0000000001000000000000000	0.49667	4.6	0.13697	0.65628	1.44757	**1.84747**
0000000000000001000000000	0.49667	4.6	0.13697	0.65628	1.44757	**1.84747**
0101010101010100010101010	0.49667	4.59452	0.13697	0.65628	1.44757	**1.84747**
0101010100010101010101010	0.49667	4.59452	0.13697	0.65628	1.44757	**1.84747**
0000000000100000000000000	0.50105	4.6	0.15306	0.65812	1.44668	**1.84595**
0000000000000010000000000	0.50105	4.6	0.15306	0.65812	1.44668	**1.84595**
0000000000001000000000000~	0.40419	4.6	0.00215	0.62566	1.44917	**1.84595**
0101010101010111010101010	0.50105	4.59447	0.15306	0.65812	1.44668	**1.84595**
0101010101110101010101010	0.50105	4.59447	0.15306	0.65812	1.44668	**1.84595**
0101010101011101010101010~	0.40419	4.59443	0.00215	0.62566	1.44917	**1.84595**
0000000000010000000000000	0.5028	4.6	0.10939	0.65692	1.44691	**1.84544**
0000000000000100000000000	0.5028	4.6	0.10939	0.65692	1.44691	**1.84544**

Degree 42 Symmetric Polynomials

50 smallest Mahler measure

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0000100101111100110101111100110101111100010	***0.1525***	2.03877	0.06248	0.90237	1.0893	1.17086
0111110011100101101010000001111001001101 01	***0.1525***	1.73729	0.03541	0.91212	1.0669	1.12207
0111100011001010000010001010000011001001 01	***0.1525***	1.8907	0.0912	0.88037	1.0918	1.16406
0100000000000110101110111011111001010101011	***0.1525***	1.96254	0.0223	0.85496	1.1121	1.20319
0010100110011001110010001101100110011000 00	***0.1525***	2.53808	0.06211	0.8523	1.142	1.27479
0001000001101110010001101110010001101011 10	***0.1525***	1.78157	0.03453	0.88076	1.0956	1.17354
0110011001100110011001100110011001100110 01	***0.27307***	4.63806	0.10844	0.49369	1.5287	1.94627
0011001100110011001100110011001100110011 00	***0.27307***	4.63806	0.10844	0.49369	1.5287	1.94627
0000000000000000000000101010101010101010 10	***0.3391***	3.20247	0.02299	0.59823	1.3648	1.63863
0101010101010101010101111111111111111111 11	***0.3391***	3.20247	0.02299	0.59823	1.3648	1.63863
0011001100110010011001100110001100110011 00	***0.34329***	3.5207	0.01274	0.63664	1.3175	1.56835
0110011001100111001100110011011001100110 01	***0.34329***	3.5207	0.01274	0.63664	1.3175	1.56835
0011001001100011001001100011001001100011 00	***0.35511***	3.38602	0.01451	0.66066	1.2904	1.52449
0110011100110110011100110110011100110110 01	***0.35511***	3.38602	0.01451	0.66066	1.2904	1.52449
0011100100111001001110010011100100111001 00	***0.35657***	3.21696	0.00172	0.65168	1.2953	1.52081
0110110001101100011011000110110001101100 01	***0.35657***	3.21696	0.00172	0.65168	1.2953	1.52081
0111011101110111011101110111011101110111 01	***0.36033***	3.35843	0.02299	0.60525	1.364	1.63863
0010001000100010001000100010001000100010 00	***0.36033***	3.35843	0.02299	0.60525	1.364	1.63863
0000101111101000010111110100001011111010 00010	***0.38904***	3.12997	0.01325	0.64964	1.3196	1.56835
0101111010000101111010000101111010000101 11	***0.38904***	3.12997	0.01325	0.64964	1.3196	1.56835
0111011000100111011000100111011000100111 01	***0.39386***	3.5207	0.01529	0.65552	1.3148	1.56835
0010001101110010001101110010001101110010 00	***0.39386***	3.5207	0.01529	0.65552	1.3148	1.56835
0101010101010100000000010101011111111111 11	***0.40059***	2.70509	0.00172	0.68956	1.2682	1.4731
0000000000000001010101111111101010101010 10	***0.40059***	2.70509	0.00172	0.68956	1.2682	1.4731
0001101110010001101110010001101110010001 10	***0.40121***	3.33062	0.03843	0.65824	1.3034	1.54012
0100111011000100111011000100111011000100 11	***0.40121***	3.33062	0.03843	0.65824	1.3034	1.54012
0011111000011111000011110100101101001011 0100	***0.40185***	2.96313	0.03444	0.66483	1.2937	1.52081
0110100101101001011010000111100001111000 01	***0.40185***	2.96313	0.03444	0.66483	1.2937	1.52081
0111100001111000011110010110100101101001 01	***0.40365***	3.0354	0.01999	0.668	1.3002	1.53596
0010110100101101001011000011110000111100 00	***0.40365***	3.0354	0.01999	0.668	1.3002	1.53596
0011001001100110011100110110011001100011 00	***0.40563***	3.38602	0.01171	0.6888	1.2788	1.51338
0110011100110011001001100011001100110110 01	***0.40563***	3.38602	0.01171	0.6888	1.2788	1.51338
0110011001100011001100110011001001100110 01	***0.40907***	3.38602	0.00418	0.66622	1.3035	1.54249
0011001100110110011001100110011100110011 00	***0.40907***	3.38602	0.00418	0.66622	1.3035	1.54249
0110011011000110011011000110011011000110 01	***0.40928***	3.54703	0.01314	0.68909	1.2918	1.54484
0011001110010011001110010011001110010011 00	***0.40928***	3.54703	0.01314	0.68909	1.2918	1.54484
0011001101100110011001100110011001110011 00	***0.41105***	3.44052	0.00928	0.66584	1.3085	1.55873
0110011000110011001100110011001100100110 01	***0.41105***	3.44052	0.00928	0.66584	1.3085	1.55873
0000000010101011111111010101000000000101 010	***0.41162***	3.02343	0.02588	0.68453	1.2901	1.52449
0101010111111110101010000000001010101111 111	***0.41162***	3.02343	0.02588	0.68453	1.2901	1.52449
0110011001101100110011001100110001100110 01	***0.41195***	3.21696	0.00172	0.66635	1.295	1.52081
0011001100111001100110011001100100110011 00	***0.41195***	3.21696	0.00172	0.66635	1.295	1.52081
0010100000010100000010100000010100000010 100000	***0.41287***	3.20247	0.01221	0.66835	1.2928	1.51773
0111110101111101011111010111110101111101 01	***0.41287***	3.20247	0.01221	0.66835	1.2928	1.51773
0110011001100110011000100110011001100110 01	***0.41459***	4.43296	0.11505	0.60558	1.4621	1.85155
0011001100110011001101110011001100110011 00	***0.41459***	4.43296	0.11505	0.60558	1.4621	1.85155
0110110001101101001110010011110001101100 01	***0.41484***	2.38731	0.00917	0.72336	1.2113	1.3667
0011100100111000011011000110100100111001 00	***0.41484***	2.38731	0.00917	0.72336	1.2113	1.3667
0010001000100011011101110111001000100010 00	***0.41562***	3.38602	0.02246	0.68409	1.2881	1.52449
0111011101110110001000100010011101110111 01	***0.41562***	3.38602	0.02246	0.68409	1.2881	1.52449

Degree 42 Symmetric Polynomials

50 largest Mahler measure

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
0101010110001101001110010011110010011111111	**0.96612**	1.44163	0.67861	0.98287	1.0172	1.03417
0000000011011000011011000110100111001010l0	**0.96612**	1.44163	0.67861	0.98287	1.0172	1.03417
00001000010100101110011001000011110100010	**0.9606**	1.45289	0.43499	0.98122	1.0175	1.03417
010111010000011110110011000101101011110111	**0.9606**	1.45289	0.43499	0.98122	1.0175	1.03417
0001111110000010001110111011011101011010110	**0.95727**	1.40659	0.46605	0.97957	1.0188	1.03612
0100101001010001001000100011101111110000011	**0.95727**	1.40659	0.46605	0.97957	1.0188	1.03612
001101101111110101101000011111010100011100	**0.9552**	1.34599	0.38032	0.97927	1.0181	1.03417
0110001110101000001111010010100000001001001	**0.9552**	1.34599	0.38032	0.97927	1.0181	1.03417
010101110100100111101100010110001111011111	**0.95518**	1.37249	0.41212	0.97928	1.0181	1.03417
000000100001110010111001000011011010001010	**0.95518**	1.37249	0.41212	0.97928	1.0181	1.03417
000111111000111011111101010001001001010110	**0.95514**	1.43737	0.57805	0.97753	1.0222	1.04382
0100101011011011101010000000100011100000011	**0.95514**	1.43737	0.57805	0.97753	1.0222	1.04382
000010010000011110110011000101101011100010	**0.95504**	1.40597	0.37767	0.979	1.0189	1.03612
010111000101001011100110010000111110110111	**0.95504**	1.40597	0.37767	0.979	1.0189	1.03612
000111000100001011100010010000101110110110	**0.95453**	1.38762	0.50183	0.97854	1.0191	1.03612
010010010001011110110111000101111011100011	**0.95453**	1.38762	0.50183	0.97854	1.0191	1.03612
000000000111001011000110110000110110101010	**0.95377**	1.39015	0.53037	0.97725	1.0216	1.04191
010101010010011110010011100101100011111111	**0.95377**	1.39015	0.53037	0.97725	1.0216	1.04191
010101011000111100010011101101001001111111	**0.95354**	1.48115	0.59677	0.9762	1.0246	1.04948
000000001101101001000110111000011100101010	**0.95354**	1.48115	0.59677	0.9762	1.0246	1.04948
010101001010010010011001100011100000111111	**0.95353**	1.32106	0.1525	0.98002	1.0163	1.03024
00000001111100011100110011011010110101010	**0.95353**	1.32106	0.1525	0.98002	1.0163	1.03024
001100000000011110011101100101101010101100	**0.95278**	1.39215	0.4575	0.97722	1.0209	1.03999
011001010101001011001000110000111111111001	**0.95278**	1.39215	0.4575	0.97722	1.0209	1.03999
010001110000100011101000010010001011011011	**0.95269**	1.33665	0.48229	0.97788	1.0193	1.03612
000100100101110110111101000111011110001110	**0.95269**	1.33665	0.48229	0.97788	1.0193	1.03612
000000000111100001100110011010010110101010	**0.95258**	1.3957	0.51654	0.97687	1.0217	1.04191
010101010010110100110011001111000011111111	**0.95258**	1.3957	0.51654	0.97687	1.0217	1.04191
000000001111100011100110010010010100101010	**0.9522**	1.52783	0.34256	0.97739	1.0213	1.04191
010101011010110110110011000111000001111111	**0.9522**	1.52783	0.34256	0.97739	1.0213	1.04191
0101001000110001110101111110110110010001111	**0.95216**	1.40597	0.59777	0.97635	1.0226	1.04382
000001110110010010000010100011100111011010	**0.95216**	1.40597	0.59777	0.97635	1.0226	1.04382
000000100100110111101000010111001110001010	**0.95153**	1.53769	0.50569	0.97651	1.0223	1.04382
010101110001100010111101000010011011011111	**0.95153**	1.53769	0.50569	0.97651	1.0223	1.04382
001100000000011110011001100101101010101100	**0.95074**	1.47901	0.4418	0.97618	1.0225	1.04382
011001010101001011001100110000111111111001	**0.95074**	1.47901	0.4418	0.97618	1.0225	1.04382
010010110111011110111001000101110111000011	**0.95001**	1.50914	0.50336	0.97618	1.0218	1.04191
000111100010001011101100010000100010010110	**0.95001**	1.50914	0.50336	0.97618	1.0218	1.04191
0000100000101100011100110010010011110100010	**0.94979**	1.35584	0.4686	0.97594	1.022	1.04191
010111010000110110110011000111001011110111	**0.94979**	1.35584	0.4686	0.97594	1.022	1.04191
0100101001010001001001100011101111110000011	**0.94949**	1.53485	0.4575	0.97515	1.0241	1.0476
0001111110000010001110011011011101011010110	**0.94949**	1.53485	0.4575	0.97515	1.0241	1.0476
001001011011111101100110011101010001111000	**0.94925**	1.63486	0.6089	0.9735	1.0286	1.05871
0111000011101010001100110010000000100101101	**0.94925**	1.63486	0.6089	0.9735	1.0286	1.05871
010100101001001101011101111100111000001111	**0.9486**	1.3576	0.46402	0.97532	1.0228	1.04382
000001111100011000001000101001101101011010	**0.9486**	1.3576	0.46402	0.97532	1.0228	1.04382
000001011001001111110111010100111001111010	**0.94851**	1.45392	0.4636	0.97529	1.0228	1.04382
010100001100011010100010000001101100101111	**0.94851**	1.45392	0.4636	0.97529	1.0228	1.04382
011100001100010111111101010111101100101101	**0.94821**	1.37249	0.37283	0.97557	1.022	1.04191
0010010110010000101010000000010111001111000	**0.94821**	1.37249	0.37283	0.97557	1.022	1.04191

Degree 42 Symmetric Polynomials

50 smallest maximum modulus

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
0011111000000000000110011001101010101010110100	0.91993	***1.2679***	0.14205	0.96768	1.0236	1.04191
0110100101010101001100110011111111111100001	0.91993	***1.2679***	0.14205	0.96768	1.0236	1.04191
0100011000011100101010000000110110100110111	0.94732	***1.288***	0.09092	0.97985	1.0154	1.02826
0001001101001001111111010101100011110011110	0.94732	***1.288***	0.09092	0.97985	1.0154	1.02826
0011100010101111000101111011010000000100100	0.90464	***1.3084***	0.15095	0.96206	1.027	1.0476
0110110111111010010000101110000101011110001	0.90464	***1.3084***	0.15095	0.96206	1.027	1.0476
0010001111101100001011000010110001010010000	0.90817	***1.3098***	0.02883	0.96384	1.0275	1.04948
0111011010111001011110010111100100000111101	0.90817	***1.3098***	0.02883	0.96384	1.0275	1.04948
0110110110111111100000010101101010001110001	0.89854	***1.3108***	0.15005	0.95803	1.0311	1.05505
0011100011101010010101111111000000100100100	0.89854	***1.3108***	0.15005	0.95803	1.0311	1.05505
0101010010100101100111011001111000000111111	0.89023	***1.3125***	0.04753	0.95846	1.0292	1.05134
0000000111110000110010001100101101011010100	0.89023	***1.3125***	0.04753	0.95846	1.0292	1.05134
0100101001111011100010001001000101100000011	0.91194	***1.3126***	0.16786	0.96307	1.0284	1.05134
0001111100101110110111011100010000110101100	0.91194	***1.3126***	0.16786	0.96307	1.0284	1.05134
0000000010010110110011001100011110001010100	0.9104	***1.3149***	0.12455	0.96332	1.0276	1.04948
0101010111000011100110011001001011011111111	0.9104	***1.3149***	0.12455	0.96332	1.0276	1.04948
0000010000111101110011001101110100101110100	0.90093	***1.3152***	0.1525	0.95929	1.03	1.0532
0101000101101000100110011000100001111011111	0.90093	***1.3152***	0.1525	0.95929	1.03	1.0532
0001000010100101100011001001111000001011100	0.91822	***1.3152***	0.04232	0.96868	1.0233	1.04191
0100010111110000110110011100101101011110111	0.91822	***1.3152***	0.04232	0.96868	1.0233	1.04191
0011010101000101101011000001111011111111100	0.89353	***1.3209***	0.10754	0.95824	1.0293	1.05134
0110000000010000111110010100101110101010010	0.89353	***1.3209***	0.10754	0.95824	1.0293	1.05134
0101010010100100100110011000111000000111111	0.95353	***1.3211***	0.1525	0.98002	1.0163	1.03024
0000000111110001110011001101101101011010100	0.95353	***1.3211***	0.1525	0.98002	1.0163	1.03024
0010100101010001110011001101101111111000000	0.90727	***1.3306***	0.09901	0.96434	1.0256	1.04571
0111111000000010010011001100011101010110101	0.90727	***1.3306***	0.09901	0.96434	1.0256	1.04571
0010100111101110001001100010010001011000000	0.91813	***1.331***	0.31461	0.96445	1.0276	1.04948
0111110010111011011100110111000100001101010	0.91813	***1.331***	0.31461	0.96445	1.0276	1.04948
0010010010010101111011000101111110001110000	0.90994	***1.3319***	0.1525	0.96411	1.0258	1.04571
0111000111000000101110010000101011011011010	0.90994	***1.3319***	0.1525	0.96411	1.0258	1.04571
0001011111100011001001000111001101101011110	0.90131	***1.3334***	0.18601	0.95865	1.031	1.05505
0100001010010011000111011011001110000010110	0.90131	***1.3334***	0.18601	0.95865	1.031	1.05505
0100101101111111110011001100101010111000011	0.9205	***1.3339***	0.06717	0.96962	1.0229	1.04191
0001111000101010110011001100000000010010110	0.9205	***1.3339***	0.06717	0.96962	1.0229	1.04191
0001001001011101101111010001110111100011100	0.95269	***1.3367***	0.48229	0.97788	1.0193	1.03612
0100011100001000111010000100100010110110110	0.95269	***1.3367***	0.48229	0.97788	1.0193	1.03612
0100100100010110101011000000011110111000110	0.83024	***1.3378***	0.10759	0.93391	1.0445	1.07649
0001110001000011111110010101001011101101100	0.83024	***1.3378***	0.10759	0.93391	1.0445	1.07649
0011101011100001110110011101101001000001000	0.89245	***1.3387***	0.03551	0.9595	1.0297	1.0532
0110111110110100100011001000111100010100010	0.89245	***1.3387***	0.03551	0.9595	1.0297	1.0532
0010100111111010110010001100000101011000000	0.91185	***1.3388***	0.20252	0.96272	1.0285	1.05134
0111110010101111110011101100101000000110101	0.91185	***1.3388***	0.20252	0.96272	1.0285	1.05134
0101011011010010000110011010001111000111110	0.85053	***1.3392***	0.02627	0.94126	1.0404	1.06949
0000001110000011101001100111101101001001010	0.85053	***1.3392***	0.02627	0.94126	1.0404	1.06949
0000011111001011001001100011000011010110100	0.89087	***1.3407***	0.17483	0.95374	1.035	1.06234
0101001010011110011100110110010110000011110	0.89087	***1.3407***	0.17483	0.95374	1.035	1.06234
0000000010010110001101110010011110001010100	0.88388	***1.3415***	0.24408	0.95108	1.0363	1.06414
0101010111000011011000100111001011011111111	0.88388	***1.3415***	0.24408	0.95108	1.0363	1.06414
0000011111100011100011001001001001010110100	0.85245	***1.3437***	0.09027	0.94458	1.0364	1.06234
0101001010110110110110011100011100000011110	0.85245	***1.3437***	0.09027	0.94458	1.0364	1.06234

Degree 42 Symmetric Polynomials

50 largest minimum modulus

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000000001101100001101100011010011100101010	0.96612	1.44163	**0.67861**	0.98287	1.0172	1.03417
010101011000110100111001001111001001111111	0.96612	1.44163	**0.67861**	0.98287	1.0172	1.03417
010101001000100100101000001110001000111111	0.93245	1.65963	**0.65133**	0.96402	1.0393	1.07994
000000011101110001111101011011011101101010	0.93245	1.65963	**0.65133**	0.96402	1.0393	1.07994
010000000111110100110111001111010110101011	0.93741	1.53489	**0.63132**	0.96775	1.0331	1.06593
000101010010100001100010011010000011111110	0.93741	1.53489	**0.63132**	0.96775	1.0331	1.06593
000000001111000011100110010010110100101010	0.92161	1.71671	**0.60986**	0.9582	1.0454	1.09177
010101011010010110110011000111100001111111	0.92161	1.71671	**0.60986**	0.9582	1.0454	1.09177
001001011011111101100110011101010001111000	0.94925	1.63486	**0.6089**	0.9735	1.0286	1.05871
0111000011101010001100110010000000100101101	0.94925	1.63486	**0.6089**	0.9735	1.0286	1.05871
000000000111100000110011001010010110101010	0.93205	1.59213	**0.60258**	0.96492	1.0359	1.07125
010101010010110101100110011111000011111111	0.93205	1.59213	**0.60258**	0.96492	1.0359	1.07125
000001110110010010000010100011100111011010	0.95216	1.40597	**0.59777**	0.97635	1.0226	1.04382
010100100011000111010111110110110010001111	0.95216	1.40597	**0.59777**	0.97635	1.0226	1.04382
010101011000111100010011101101001001111111	0.95354	1.48115	**0.59677**	0.9762	1.0246	1.04948
000000001101101001000110111000011100101010	0.95354	1.48115	**0.59677**	0.9762	1.0246	1.04948
0101111110100000101100110011110101111010111	0.94287	1.50194	**0.59242**	0.97221	1.0258	1.04948
000010100001010000110011001011111010000010	0.94287	1.50194	**0.59242**	0.97221	1.0258	1.04948
000001111100000100100010001110101101011010	0.91737	1.78397	**0.58448**	0.9559	1.0487	1.09999
0101001010010100011101110110111111000001111	0.91737	1.78397	**0.58448**	0.9559	1.0487	1.09999
011110001101010100110011001111111100100101	0.91965	1.78342	**0.57988**	0.95773	1.0453	1.09177
001011011000000001100110011010101001110000	0.91965	1.78342	**0.57988**	0.95773	1.0453	1.09177
000111111100011101111111010100010010010110	0.95514	1.43737	**0.57805**	0.97753	1.0222	1.04382
0100101011011011101010000000100011100000011	0.95514	1.43737	**0.57805**	0.97753	1.0222	1.04382
000010100111000001000110111010110110000010	0.93565	1.7275	**0.57798**	0.96658	1.0352	1.07125
010111110010010100010011101111100011010111	0.93565	1.7275	**0.57798**	0.96658	1.0352	1.07125
0100100101001110111110111010001001111100011	0.94503	1.43497	**0.57768**	0.97264	1.0264	1.05134
000111000001101110100010000100011010110110	0.94503	1.43497	**0.57768**	0.97264	1.0264	1.05134
010101010110100101001100111110000111111111	0.92021	1.62739	**0.57473**	0.95951	1.04	1.07822
000000000011110000011001101011010010101010	0.92021	1.62739	**0.57473**	0.95951	1.04	1.07822
000111100010101011101100010000000010010110	0.92299	1.62879	**0.57339**	0.96066	1.0396	1.07822
010010110111111110111001000101010111000011	0.92299	1.62879	**0.57339**	0.96066	1.0396	1.07822
0110110110111111101100010011101010001110001	0.9419	1.69135	**0.57017**	0.97012	1.0315	1.06414
0011100011101010001101110010000000100100100	0.9419	1.69135	**0.57017**	0.97012	1.0315	1.06414
000000000001111000110011001001011010101010	0.94802	1.46202	**0.57007**	0.97416	1.0252	1.04948
010101010100101101100110011100001111111111	0.94802	1.46202	**0.57007**	0.97416	1.0252	1.04948
000000100111001011000110110000110110001010	0.94633	1.70499	**0.56957**	0.9726	1.0282	1.05688
010101110010011110010011100101100011011111	0.94633	1.70499	**0.56957**	0.9726	1.0282	1.05688
010010001000010010010011100011101000100011	0.93036	1.59213	**0.56795**	0.96522	1.0333	1.06414
000111011101000111000110110110111101110110	0.93036	1.59213	**0.56795**	0.96522	1.0333	1.06414
001101111111100001101100011010010101011100	0.94761	1.57753	**0.56778**	0.97377	1.026	1.05134
011000101010110100111001001111000000001001	0.94761	1.57753	**0.56778**	0.97377	1.026	1.05134
000011000001100010110111000010011010110010	0.9377	1.69339	**0.56318**	0.96851	1.0319	1.06414
010110010100110111100010010111001111100111	0.9377	1.69339	**0.56318**	0.96851	1.0319	1.06414
0001111110000000000110011001010101011010110	0.91341	1.64952	**0.55826**	0.95551	1.0445	1.08675
010010100101010101100110011111111110000011	0.91341	1.64952	**0.55826**	0.95551	1.0445	1.08675
010000000010100000110111001010000010101011	0.8992	1.82067	**0.55129**	0.94594	1.059	1.11899
0001010101111110101100010011111010111111110	0.8992	1.82067	**0.55129**	0.94594	1.059	1.11899
000000000101100011000110110010011110101010	0.93694	1.6669	**0.55035**	0.96807	1.0322	1.06414
010101010000110110010011100111001011111111	0.93694	1.6669	**0.55035**	0.96807	1.0322	1.06414

Degree 42 Symmetric Polynomials

50 smallest L1 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
0110011001100110011001100110011001100110011001	0.27307	4.63806	0.10844	***0.49369***	1.5287	1.94627
0011001100110011001100110011001100110011001100	0.27307	4.63806	0.10844	***0.49369***	1.5287	1.94627
0101010101010101010101111111111111111111	0.3391	3.20247	0.02299	***0.59823***	1.3648	1.63863
0000000000000000000001010101010101010101010	0.3391	3.20247	0.02299	***0.59823***	1.3648	1.63863
0111011101110111011101110111011101110111011101	0.36033	3.35843	0.02299	***0.60525***	1.364	1.63863
0010001000100010001000100010001000100010001000	0.36033	3.35843	0.02299	***0.60525***	1.364	1.63863
0011001100110011001101110011001100110011001100	0.41459	4.43296	0.11505	***0.60558***	1.4621	1.85155
0110011001100110011000100110011001100110011001	0.41459	4.43296	0.11505	***0.60558***	1.4621	1.85155
0111001100110011001100110011001100110011001101	0.42261	4.2399	0.03345	***0.61822***	1.4449	1.81685
0010011001100110011001100110011001100110011000	0.42261	4.2399	0.03345	***0.61822***	1.4449	1.81685
0001100110011001100110011001100110011001100110	0.44164	4.2179	0.1525	***0.6197***	1.4457	1.81649
0100110011001100110011001100110011001100110011	0.44164	4.2179	0.1525	***0.6197***	1.4457	1.81649
0100011001100110011001100110011001100110011011	0.43905	4.2179	0.04964	***0.63111***	1.436	1.80482
0001001100110011001100110011001100110011001110	0.43905	4.2179	0.04964	***0.63111***	1.436	1.80482
0010001100110011001100110011001100110011001000	0.44308	4.2399	0.02217	***0.63405***	1.4357	1.80593
0111011001100110011001100110011001100110011101	0.44308	4.2399	0.02217	***0.63405***	1.4357	1.80593
0110011001100111001100110011011001100110011001	0.34329	3.5207	0.01274	***0.63664***	1.3175	1.56835
0011001100110010011001100110001100110011001100	0.34329	3.5207	0.01274	***0.63664***	1.3175	1.56835
0011001100110011001110010011001100110011001100	0.4257	4.2179	0.06413	***0.64146***	1.4072	1.76212
0110011001100110011011000110011001100110011001	0.4257	4.2179	0.06413	***0.64146***	1.4072	1.76212
0110111001100110011001100110011001100110010001	0.45954	4.2179	0.03178	***0.64621***	1.4273	1.79443
0011101100110011001100110011001100110011000100	0.45954	4.2179	0.03178	***0.64621***	1.4273	1.79443
0011011100110011001100110011001100110011011100	0.45882	4.2399	0.03806	***0.64679***	1.4276	1.7963
0110001001100110011001100110011001100110001001	0.45882	4.2399	0.03806	***0.64679***	1.4276	1.7963
0101111010000101111010000101111010000010111	0.38904	3.12997	0.01325	***0.64964***	1.3196	1.56835
0000101111010000101111010000101111010000010	0.38904	3.12997	0.01325	***0.64964***	1.3196	1.56835
0011011001100110011001100110011001100110011100	0.42056	3.89693	0.02182	***0.64993***	1.3797	1.70395
0110001100110011001100110011001100110011001001	0.42056	3.89693	0.02182	***0.64993***	1.3797	1.70395
0011100110011001100110011001100110011001100100	0.42017	3.84889	0.01034	***0.65093***	1.3796	1.7022
0110110011001100110011001100110011001100110001	0.42017	3.84889	0.01034	***0.65093***	1.3796	1.7022
0110110001101100011011000110110001101100 01	0.35657	3.21696	0.00172	***0.65168***	1.2953	1.52081
0011100100111001001110010011100100111001 00	0.35657	3.21696	0.00172	***0.65168***	1.2953	1.52081
0011001100110011001001100011001100110011001100	0.42125	4.06059	0.04713	***0.65291***	1.3736	1.70088
0110011001100110011100110110011001100110011001	0.42125	4.06059	0.04713	***0.65291***	1.3736	1.70088
0110011001100110011101110110011001100110011001	0.45384	4.2399	0.04687	***0.65409***	1.4087	1.76802
0011001100110011001000100011001100110011001100	0.45384	4.2399	0.04687	***0.65409***	1.4087	1.76802
0101010101010101010111011111111111111111	0.44738	3.20247	0.02084	***0.65481***	1.3484	1.62064
0000000000000000000010001010101010101010101010	0.44738	3.20247	0.02084	***0.65481***	1.3484	1.62064
0010001101110010001101110010001101110010 00	0.39386	3.5207	0.01529	***0.65552***	1.3148	1.56835
0111011000100111011000100111011000100111 01	0.39386	3.5207	0.01529	***0.65552***	1.3148	1.56835
0110011101100110011001100110011001100111011001	0.4685	4.2399	0.02927	***0.65647***	1.4208	1.78803
0011001000110011001100110011001100110010001100	0.4685	4.2399	0.02927	***0.65647***	1.4208	1.78803
0011000100110011001100110011001100110011101100	0.47651	4.2179	0.08771	***0.65764***	1.4199	1.78538
0110010001100110011001100110011001100110111001	0.47651	4.2179	0.08771	***0.65764***	1.4199	1.78538
0100111011000100111011000100111011000100 11	0.40121	3.33062	0.03843	***0.65824***	1.3034	1.54012
0001101110010001101110010001101110010001 10	0.40121	3.33062	0.03843	***0.65824***	1.3034	1.54012
0011001100110001100110011001101100110011001100	0.41779	3.33062	0.01881	***0.65898***	1.3055	1.54012
0110011001100100110011001100111001100110011001	0.41779	3.33062	0.01881	***0.65898***	1.3055	1.54012
0011001001100011001001100011001001100011000 1100	0.35511	3.38602	0.01451	***0.66066***	1.2904	1.52449
0110011100110110011100110110011100110110 01	0.35511	3.38602	0.01451	***0.66066***	1.2904	1.52449

Degree 42 Symmetric Polynomials

50 largest L1 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
000000001101100001101100011010011100101010	0.96612	1.44163	0.67861	**0.98287**	1.0172	1.03417
010101011000110100111001001111001001111111	0.96612	1.44163	0.67861	**0.98287**	1.0172	1.03417
010111010000011110110011000101101011110111	0.9606	1.45289	0.43499	**0.98122**	1.0175	1.03417
000010000101001011100110010000111110100010	0.9606	1.45289	0.43499	**0.98122**	1.0175	1.03417
010101001010010010011001100011100000111111	0.95353	1.32106	0.1525	**0.98002**	1.0163	1.03024
000000011111000111001100110110110101101010	0.95353	1.32106	0.1525	**0.98002**	1.0163	1.03024
010001100001110010101000000011011010011011	0.94732	1.28797	0.09092	**0.97985**	1.0154	1.02826
00010011010010011111101010110001111001110	0.94732	1.28797	0.09092	**0.97985**	1.0154	1.02826
010010100101000100100010001110111110000011	0.95727	1.40659	0.46605	**0.97957**	1.0188	1.03612
000111110000010001110111011011101011010110	0.95727	1.40659	0.46605	**0.97957**	1.0188	1.03612
010101110100100111101100010110001111011111	0.95518	1.37249	0.41212	**0.97928**	1.0181	1.03417
000000100001110010111001000011011010001010	0.95518	1.37249	0.41212	**0.97928**	1.0181	1.03417
001101101111110101101000011111010100011100	0.9552	1.34599	0.38032	**0.97927**	1.0181	1.03417
011000111010100000111101001010000001001001	0.9552	1.34599	0.38032	**0.97927**	1.0181	1.03417
010111000101001011100110010000111110110111	0.95504	1.40597	0.37767	**0.979**	1.0189	1.03612
000010010000011110110011000101101011100010	0.95504	1.40597	0.37767	**0.979**	1.0189	1.03612
010010010001011110110111000101111011100011	0.95453	1.38762	0.50183	**0.97854**	1.0191	1.03612
000111000100001011100010010000101110110110	0.95453	1.38762	0.50183	**0.97854**	1.0191	1.03612
010001110000100011101000010010001011011011	0.95269	1.33665	0.48229	**0.97788**	1.0193	1.03612
000100100101110110111101000111011110001110	0.95269	1.33665	0.48229	**0.97788**	1.0193	1.03612
010010101101101110101000000100011100000011	0.95514	1.43737	0.57805	**0.97753**	1.0222	1.04382
000111111000111011111101010001001001010110	0.95514	1.43737	0.57805	**0.97753**	1.0222	1.04382
000000001111100011100110010010010100101010	0.9522	1.52783	0.34256	**0.97739**	1.0213	1.04191
010101011010110110110011000111000001111111	0.9522	1.52783	0.34256	**0.97739**	1.0213	1.04191
010101010010011110010011100101100011111111	0.95377	1.39015	0.53037	**0.97725**	1.0216	1.04191
000000000111001011000110110000110110101010	0.95377	1.39015	0.53037	**0.97725**	1.0216	1.04191
001100000000011110011101100101101010101100	0.95278	1.39215	0.4575	**0.97722**	1.0209	1.03999
011001010101001011001000110000111111111001	0.95278	1.39215	0.4575	**0.97722**	1.0209	1.03999
010101010010110100110011001111000011111111	0.95258	1.3957	0.51654	**0.97687**	1.0217	1.04191
000000000111100001100110011010010110101010	0.95258	1.3957	0.51654	**0.97687**	1.0217	1.04191
001001101111110000010100000101001010001100	0.94142	1.37249	0.11549	**0.97654**	1.0184	1.03417
011100111010110101111101011111000001001101	0.94142	1.37249	0.11549	**0.97654**	1.0184	1.03417
010101110001100010111101000010011011011111	0.95153	1.53769	0.50569	**0.97651**	1.0223	1.04382
000000100100110111101000010111001110001010	0.95153	1.53769	0.50569	**0.97651**	1.0223	1.04382
010100100011000111010111110110110010001111	0.95216	1.40597	0.59777	**0.97635**	1.0226	1.04382
000001110110010010000010100011100111011010	0.95216	1.40597	0.59777	**0.97635**	1.0226	1.04382
010101011000111100010011101101001001111111	0.95354	1.48115	0.59677	**0.9762**	1.0246	1.04948
000000001101101001000110111000011100101010	0.95354	1.48115	0.59677	**0.9762**	1.0246	1.04948
000111100010001011101100010000100010010110	0.95001	1.50914	0.50336	**0.97618**	1.0218	1.04191
010010110111011110111001000101110111000011	0.95001	1.50914	0.50336	**0.97618**	1.0218	1.04191
001100000000011110011001100101101010101100	0.95074	1.47901	0.4418	**0.97618**	1.0225	1.04382
011001010101001011001100110000111111111001	0.95074	1.47901	0.4418	**0.97618**	1.0225	1.04382
011100011101010011101000010011111101101101	0.93856	1.41256	0.09674	**0.97609**	1.0185	1.03417
001001001000000110111101000110101000111000	0.93856	1.41256	0.09674	**0.97609**	1.0185	1.03417
010111010000110110110011000111001011110111	0.94979	1.35584	0.4686	**0.97594**	1.022	1.04191
000010000101100011100110010010011110100010	0.94979	1.35584	0.4686	**0.97594**	1.022	1.04191
001001011001000010101000000001011100111100	0.94821	1.37249	0.37283	**0.97557**	1.022	1.04191
011100001100010111111101010111101100101101	0.94821	1.37249	0.37283	**0.97557**	1.022	1.04191
010100101010011010001100100001100000001111	0.94689	1.41482	0.37223	**0.97533**	1.022	1.04191
000001111111001111011001110100110101011010	0.94689	1.41482	0.37223	**0.97533**	1.022	1.04191

Degree 42 Symmetric Polynomials

50 smallest L3 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000100110100100111111101010110001111001110	0.94732	1.28797	0.09092	0.97985	***1.0154***	1.02826
010001100001110010101000000011011010011011	0.94732	1.28797	0.09092	0.97985	***1.0154***	1.02826
010101001010010010011001100011100000111111	0.95353	1.32106	0.1525	0.98002	***1.0163***	1.03024
000000011111000111001100110110110101101010	0.95353	1.32106	0.1525	0.98002	***1.0163***	1.03024
010101011000110100111001001111001001111111	0.96612	1.44163	0.67861	0.98287	***1.0172***	1.03417
000000001101100001101100011010011100101010	0.96612	1.44163	0.67861	0.98287	***1.0172***	1.03417
010111010000011110110011000101101011110111	0.9606	1.45289	0.43499	0.98122	***1.0175***	1.03417
000010000101001011100110010000111110100010	0.9606	1.45289	0.43499	0.98122	***1.0175***	1.03417
010101110100100111101100010110001111011111	0.95518	1.37249	0.41212	0.97928	***1.0181***	1.03417
000000100001110010111001000011011010001010	0.95518	1.37249	0.41212	0.97928	***1.0181***	1.03417
011000111010100000111101001010000001001001	0.9552	1.34599	0.38032	0.97927	***1.0181***	1.03417
001101101111110101101000011111010100011100	0.9552	1.34599	0.38032	0.97927	***1.0181***	1.03417
011100111010110101111101011111000001001101	0.94142	1.37249	0.11549	0.97654	***1.0184***	1.03417
001001101111100000101000001010010100011000	0.94142	1.37249	0.11549	0.97654	***1.0184***	1.03417
001001001000000110111101000110101000111000	0.93856	1.41256	0.09674	0.97609	***1.0185***	1.03417
011100011101010011101000010011111101101101	0.93856	1.41256	0.09674	0.97609	***1.0185***	1.03417
010010100101000100100010001110111110000011	0.95727	1.40659	0.46605	0.97957	***1.0188***	1.03612
000111110000010001110111011011101011010110	0.95727	1.40659	0.46605	0.97957	***1.0188***	1.03612
010111000101001011100110010000111110110111	0.95504	1.40597	0.37767	0.979	***1.0189***	1.03612
000010010000011110110011000101101011100010	0.95504	1.40597	0.37767	0.979	***1.0189***	1.03612
010010010001011110110111000101111011100011	0.95453	1.38762	0.50183	0.97854	***1.0191***	1.03612
000111000100001011100010010000101110110110	0.95453	1.38762	0.50183	0.97854	***1.0191***	1.03612
010001110000100011101000010010001011011011	0.95269	1.33665	0.48229	0.97788	***1.0193***	1.03612
000100100101110110111101000111011110001110	0.95269	1.33665	0.48229	0.97788	***1.0193***	1.03612
001100000000011110011101100101101010101100	0.95278	1.39215	0.4575	0.97722	***1.0209***	1.03999
011001010101001011001000110000111111111001	0.95278	1.39215	0.4575	0.97722	***1.0209***	1.03999
010101011010110110110011000111000001111111	0.9522	1.52783	0.34256	0.97739	***1.0213***	1.04191
000000001111100011100110010010010100101010	0.9522	1.52783	0.34256	0.97739	***1.0213***	1.04191
010001011111000011001000110010110101111011	0.94539	1.40597	0.38356	0.97491	***1.0214***	1.03999
000100001010010110011101100111100000101110	0.94539	1.40597	0.38356	0.97491	***1.0214***	1.03999
010101010010011110010011100101100011111111	0.95377	1.39015	0.53037	0.97725	***1.0216***	1.04191
000000000111001011000110110000110110101010	0.95377	1.39015	0.53037	0.97725	***1.0216***	1.04191
011000001100010111111001010111101100101001	0.94309	1.35738	0.41292	0.97407	***1.0217***	1.03999
001101011001000010101100000010111001111100	0.94309	1.35738	0.41292	0.97407	***1.0217***	1.03999
010101010010110100110011001111000011111111	0.95258	1.3957	0.51654	0.97687	***1.0217***	1.04191
000000000111100001100110011010010110101010	0.95258	1.3957	0.51654	0.97687	***1.0217***	1.04191
010010110111011110111001000101110111000011	0.95001	1.50914	0.50336	0.97618	***1.0218***	1.04191
000111100010001011101100010000100010010110	0.95001	1.50914	0.50336	0.97618	***1.0218***	1.04191
010111010000110110110011000111001011110111	0.94979	1.35584	0.4686	0.97594	***1.022***	1.04191
000010000101100011100110010010011110100010	0.94979	1.35584	0.4686	0.97594	***1.022***	1.04191
010100101010011010001100100001100000001111	0.94689	1.41482	0.37223	0.97533	***1.022***	1.04191
000001111111001111011001110100110101011010	0.94689	1.41482	0.37223	0.97533	***1.022***	1.04191
011100001100010111111101010111101100101101	0.94821	1.37249	0.37283	0.97557	***1.022***	1.04191
001001011001000010101000000010111001111000	0.94821	1.37249	0.37283	0.97557	***1.022***	1.04191
010010101101101110101000000100011100000011	0.95514	1.43737	0.57805	0.97753	***1.0222***	1.04382
000111111000111011111101010001001001010110	0.95514	1.43737	0.57805	0.97753	***1.0222***	1.04382
010100011100110101101000011111001101101111	0.93926	1.48094	0.23345	0.97345	***1.0223***	1.04191
000001001001100000111101001010011000111010	0.93926	1.48094	0.23345	0.97345	***1.0223***	1.04191
000000100100110111101000010111001110001010	0.95153	1.53769	0.50569	0.97651	***1.0223***	1.04382
010101110001100010111101000010011011011111	0.95153	1.53769	0.50569	0.97651	***1.0223***	1.04382

Degree 42 Symmetric Polynomials

50 largest L3 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
001100110011001100110011001100110011001100	0.27307	4.63806	0.10844	0.49369	**1.5287**	1.94627
011001100110011001100110011001100110011001	0.27307	4.63806	0.10844	0.49369	**1.5287**	1.94627
001100110011001100110111001100110011001100	0.41459	4.43296	0.11505	0.60558	**1.4621**	1.85155
011001100110011001100010011001100110011001	0.41459	4.43296	0.11505	0.60558	**1.4621**	1.85155
010011001100110011001100110011001100110011	0.44164	4.2179	0.1525	0.6197	**1.4457**	1.81649
000110011001100110011001100110011001100110	0.44164	4.2179	0.1525	0.6197	**1.4457**	1.81649
011100110011001100110011001100110011001101	0.42261	4.2399	0.03345	0.61822	**1.4449**	1.81685
001001100110011001100110011001100110011000	0.42261	4.2399	0.03345	0.61822	**1.4449**	1.81685
010001100110011001100110011001100110011011	0.43905	4.2179	0.04964	0.63111	**1.436**	1.80482
000100110011001100110011001100110011001110	0.43905	4.2179	0.04964	0.63111	**1.436**	1.80482
011101100110011001100110011001100110011101	0.44308	4.2399	0.02217	0.63405	**1.4357**	1.80593
001000110011001100110011001100110011001000	0.44308	4.2399	0.02217	0.63405	**1.4357**	1.80593
011000100110011001100110011001100110001001	0.45882	4.2399	0.03806	0.64679	**1.4276**	1.7963
001101110011001100110011001100110011011100	0.45882	4.2399	0.03806	0.64679	**1.4276**	1.7963
011011100110011001100110011001100110010001	0.45954	4.2179	0.03178	0.64621	**1.4273**	1.79443
001110110011001100110011001100110011000100	0.45954	4.2179	0.03178	0.64621	**1.4273**	1.79443
011001110110011001100110011001100111011001	0.4685	4.2399	0.02927	0.65647	**1.4208**	1.78803
001100100011001100110011001100110010001100	0.4685	4.2399	0.02927	0.65647	**1.4208**	1.78803
011001000110011001100110011001100110111001	0.47651	4.2179	0.08771	0.65764	**1.4199**	1.78538
001100010011001100110011001100110011101100	0.47651	4.2179	0.08771	0.65764	**1.4199**	1.78538
011001100010011001100110011001100010011001	0.47708	4.2399	0.0295	0.66289	**1.4154**	1.78118
001100110111001100110011001100110111001100	0.47708	4.2399	0.0295	0.66289	**1.4154**	1.78118
011001101110011001100110011001100100011001	0.47996	4.2179	0.04412	0.6638	**1.4139**	1.77773
001100111011001100110011001100110001001100	0.47996	4.2179	0.04412	0.6638	**1.4139**	1.77773
011001100111011001100110011001110110011001	0.48209	4.2399	0.02946	0.66669	**1.4115**	1.7758
001100110010001100110011001100100011001100	0.48209	4.2399	0.02946	0.66669	**1.4115**	1.7758
011001100100011001100110011001101110011001	0.46885	4.2179	0.06199	0.66317	**1.4095**	1.77153
001100110001001100110011001100111011001100	0.46885	4.2179	0.06199	0.66317	**1.4095**	1.77153
011001100110001001100110011000100110011001	0.47369	4.2399	0.06557	0.66316	**1.4093**	1.77192
001100110011011100110011001101110011001100	0.47369	4.2399	0.06557	0.66316	**1.4093**	1.77192
011001100110011001110111011001100110011001	0.45384	4.2399	0.04687	0.65409	**1.4087**	1.76802
001100110011001100100010001100110011001100	0.45384	4.2399	0.04687	0.65409	**1.4087**	1.76802
011001100110011001101100011001100110011001	0.4257	4.2179	0.06413	0.64146	**1.4072**	1.76212
001100110011001100111001001100110011001100	0.4257	4.2179	0.06413	0.64146	**1.4072**	1.76212
011001100110011000100110001001100110011001	0.456	4.2399	0.06319	0.6607	**1.4065**	1.76724
001100110011001101110011011100110011001100	0.456	4.2399	0.06319	0.6607	**1.4065**	1.76724
011001100110111001100110011001000110011001	0.49013	4.2179	0.1183	0.67058	**1.4061**	1.76684
001100110011101100110011001100010011001100	0.49013	4.2179	0.1183	0.67058	**1.4061**	1.76684
001100110011001000110011001000110011001100	0.48376	4.2399	0.04105	0.67218	**1.4055**	1.76802
011001100110011101100110011101100110011001	0.48376	4.2399	0.04105	0.67218	**1.4055**	1.76802
001100110011001100010011101100110011001100	0.49131	4.2179	0.1525	0.66814	**1.4033**	1.76054
011001100110011001000110111001100110011001	0.49131	4.2179	0.1525	0.66814	**1.4033**	1.76054
011001100110011011100110010001100110011001	0.47786	4.2179	0.05368	0.67002	**1.402**	1.76054
001100110011001110110011000100110011001100	0.47786	4.2179	0.05368	0.67002	**1.402**	1.76054
011001100110010001100110011011100110011001	0.53748	4.2179	0.1525	0.69005	**1.4013**	1.76212
001100110011000100110011001110110011001100	0.53748	4.2179	0.1525	0.69005	**1.4013**	1.76212
011100110011001100110111001100110011001101	0.44102	4.06059	0.02675	0.66095	**1.3887**	1.73085
001001100110011001100010011001100110011000	0.44102	4.06059	0.02675	0.66095	**1.3887**	1.73085
001000110011001100110111001100110011001000	0.46992	4.06059	0.03067	0.67697	**1.3809**	1.7216
011101100110011001100010011001100110011101	0.46992	4.06059	0.03067	0.67697	**1.3809**	1.7216

Degree 42 Symmetric Polynomials

50 smallest L4 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
0001001101001001111111010101100011110011100	0.94732	1.28797	0.09092	0.97985	1.0154	***1.02826***
0100011000011100101010000000110110100110111	0.94732	1.28797	0.09092	0.97985	1.0154	***1.02826***
0101010010100100100110011000111000001111111	0.95353	1.32106	0.1525	0.98002	1.0163	***1.03024***
0000000111110001110011001101101101011010100	0.95353	1.32106	0.1525	0.98002	1.0163	***1.03024***
0000001000011100101110010000110110100010100	0.95518	1.37249	0.41212	0.97928	1.0181	***1.03417***
0101011101001001111011000101100011110111111	0.95518	1.37249	0.41212	0.97928	1.0181	***1.03417***
0111001110101101011111010111110000010011011	0.94142	1.37249	0.11549	0.97654	1.0184	***1.03417***
0010011011111100000101000001010010100011000	0.94142	1.37249	0.11549	0.97654	1.0184	***1.03417***
0111000111010100111010000100111111011011011	0.93856	1.41256	0.09674	0.97609	1.0185	***1.03417***
0010010010000001101111010001101010001110000	0.93856	1.41256	0.09674	0.97609	1.0185	***1.03417***
0110001110101000001111010010100000010010011	0.9552	1.34599	0.38032	0.97927	1.0181	***1.03417***
0011011011111101011010000111110101000111000	0.9552	1.34599	0.38032	0.97927	1.0181	***1.03417***
0000100001010010111001100100001111101000100	0.9606	1.45289	0.43499	0.98122	1.0175	***1.03417***
0101110100000111101100110001011010111101111	0.9606	1.45289	0.43499	0.98122	1.0175	***1.03417***
0101010110001101001110010011110010011111111	0.96612	1.44163	0.67861	0.98287	1.0172	***1.03417***
0000000011011000011011000110100111001010100	0.96612	1.44163	0.67861	0.98287	1.0172	***1.03417***
0001001001011101101111010001110111100011100	0.95269	1.33665	0.48229	0.97788	1.0193	***1.03612***
0100011100001000111010000100100010110110111	0.95269	1.33665	0.48229	0.97788	1.0193	***1.03612***
0100100100010111101101110001011110111000111	0.95453	1.38762	0.50183	0.97854	1.0191	***1.03612***
0001110001000010111000100100001011101101100	0.95453	1.38762	0.50183	0.97854	1.0191	***1.03612***
0100101001010001001000100011101111110000011	0.95727	1.40659	0.46605	0.97957	1.0188	***1.03612***
0001111110000010001110111011011101011010110	0.95727	1.40659	0.46605	0.97957	1.0188	***1.03612***
0000100100000111101100110001011010111000100	0.95504	1.40597	0.37767	0.979	1.0189	***1.03612***
0101110001010010111001100100001111101101111	0.95504	1.40597	0.37767	0.979	1.0189	***1.03612***
0100010111111000011001000110010110101111011	0.94539	1.40597	0.38356	0.97491	1.0214	***1.03999***
0001000010100101100111011001111000001011100	0.94539	1.40597	0.38356	0.97491	1.0214	***1.03999***
0110010101010010110010001100001111111110011	0.95278	1.39215	0.4575	0.97722	1.0209	***1.03999***
0011000000000111100111011001011010101011000	0.95278	1.39215	0.4575	0.97722	1.0209	***1.03999***
0110000011000101111110010101111011001010011	0.94309	1.35738	0.41292	0.97407	1.0217	***1.03999***
0011010110010000101011000000101110011111000	0.94309	1.35738	0.41292	0.97407	1.0217	***1.03999***
0010100100000100011000100110111010111000000	0.93586	1.37249	0.22791	0.97224	1.0226	***1.04191***
0111110001010001001101110011101111101101011	0.93586	1.37249	0.22791	0.97224	1.0226	***1.04191***
0111010100010111100011001001011110111111011	0.93627	1.37806	0.28982	0.97201	1.0228	***1.04191***
0010000000100001011011001110000101110101000	0.93627	1.37806	0.28982	0.97201	1.0228	***1.04191***
0101010110110101100011001001111100011111111	0.88624	1.37249	0.04067	0.96212	1.0241	***1.04191***
0000000011100000110110011100101001001010100	0.88624	1.37249	0.04067	0.96212	1.0241	***1.04191***
0111000011100101010111011111111001001011011	0.93736	1.37249	0.23004	0.97266	1.0226	***1.04191***
0010010110110000000010001010101100011110000	0.93736	1.37249	0.23004	0.97266	1.0226	***1.04191***
0111000011000101111111010101111011001011011	0.94821	1.37249	0.37283	0.97557	1.022	***1.04191***
0010010110010000101010000000010111001111000	0.94821	1.37249	0.37283	0.97557	1.022	***1.04191***
0111010110110001010010001111101100011111011	0.94065	1.35774	0.32652	0.97307	1.0226	***1.04191***
0010000001110010000011101101011100100101000	0.94065	1.35774	0.32652	0.97307	1.0226	***1.04191***
0101011110001111001100110011010010010111111	0.92925	1.36054	0.20608	0.97044	1.023	***1.04191***
0000001011011010011001100110000111000010100	0.92925	1.36054	0.20608	0.97044	1.023	***1.04191***
0000000011111000111001100100100101001010100	0.9522	1.52783	0.34256	0.97739	1.0213	***1.04191***
0101010110101101101100110001110000011111110	0.9522	1.52783	0.34256	0.97739	1.0213	***1.04191***
0100101101111111110011001100101010111000011	0.9205	1.33388	0.06717	0.96962	1.0229	***1.04191***
0001111000101010110011001100000000010010110	0.9205	1.33388	0.06717	0.96962	1.0229	***1.04191***
0100101101110111101110010001011101110000110	0.95001	1.50914	0.50336	0.97618	1.0218	***1.04191***
0101110100001101101100110001110010111101110	0.94979	1.35584	0.4686	0.97594	1.022	***1.04191***

Degree 42 Symmetric Polynomials

50 smallest L4 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
0011001100110011001100110011001100110011001100	0.27307	4.63806	0.10844	0.49369	1.5287	**1.94627**
0110011001100110011001100110011001100110011001	0.27307	4.63806	0.10844	0.49369	1.5287	**1.94627**
0011001100110011001101110011001100110011001100	0.41459	4.43296	0.11505	0.60558	1.4621	**1.85155**
0110011001100110011000100110011001100110011001	0.41459	4.43296	0.11505	0.60558	1.4621	**1.85155**
0111001100110011001100110011001100110011001101	0.42261	4.2399	0.03345	0.61822	1.4449	**1.81685**
0010011001100110011001100110011001100110011000	0.42261	4.2399	0.03345	0.61822	1.4449	**1.81685**
0001100110011001100110011001100110011001100110	0.44164	4.2179	0.1525	0.6197	1.4457	**1.81649**
0100110011001100110011001100110011001100110011	0.44164	4.2179	0.1525	0.6197	1.4457	**1.81649**
0111011001100110011001100110011001100110011101	0.44308	4.2399	0.02217	0.63405	1.4357	**1.80593**
0010001100110011001100110011001100110011001000	0.44308	4.2399	0.02217	0.63405	1.4357	**1.80593**
0100011001100110011001100110011001100110011011	0.43905	4.2179	0.04964	0.63111	1.436	**1.80482**
0001001100110011001100110011001100110011001110	0.43905	4.2179	0.04964	0.63111	1.436	**1.80482**
0110001001100110011001100110011001100110001001	0.45882	4.2399	0.03806	0.64679	1.4276	**1.7963**
0011011100110011001100110011001100110011011100	0.45882	4.2399	0.03806	0.64679	1.4276	**1.7963**
0110111001100110011001100110011001100110010001	0.45954	4.2179	0.03178	0.64621	1.4273	**1.79443**
0011101100110011001100110011001100110011000100	0.45954	4.2179	0.03178	0.64621	1.4273	**1.79443**
0110011101100110011001100110011001100111011001	0.4685	4.2399	0.02927	0.65647	1.4208	**1.78803**
0011001000110011001100110011001100110010001100	0.4685	4.2399	0.02927	0.65647	1.4208	**1.78803**
0110010001100110011001100110011001100110111001	0.47651	4.2179	0.08771	0.65764	1.4199	**1.78538**
0011000100110011001100110011001100110011101100	0.47651	4.2179	0.08771	0.65764	1.4199	**1.78538**
0011001101110011001100110011001100110111001100	0.47708	4.2399	0.0295	0.66289	1.4154	**1.78118**
0110011000100110011001100110011001100010011001	0.47708	4.2399	0.0295	0.66289	1.4154	**1.78118**
0110011011100110011001100110011001100100011001	0.47996	4.2179	0.04412	0.6638	1.4139	**1.77773**
0011001110110011001100110011001100110001001100	0.47996	4.2179	0.04412	0.6638	1.4139	**1.77773**
0110011001110110011001100110011001110110011001	0.48209	4.2399	0.02946	0.66669	1.4115	**1.7758**
0011001100100011001100110011001100100011001100	0.48209	4.2399	0.02946	0.66669	1.4115	**1.7758**
0110011001100010011001100110011000100110011001	0.47369	4.2399	0.06557	0.66316	1.4093	**1.77192**
0011001100110111001100110011001101110011001100	0.47369	4.2399	0.06557	0.66316	1.4093	**1.77192**
0110011001000110011001100110011001101110011001	0.46885	4.2179	0.06199	0.66317	1.4095	**1.77153**
0011001100010011001100110011001100111011001100	0.46885	4.2179	0.06199	0.66317	1.4095	**1.77153**
0110011001100110011101110110011001100110011001	0.45384	4.2399	0.04687	0.65409	1.4087	**1.76802**
0011001100110011001000100011001100110011001100	0.45384	4.2399	0.04687	0.65409	1.4087	**1.76802**
0011001100110010001100110010001100110011001100	0.48376	4.2399	0.04105	0.67218	1.4055	**1.76802**
0110011001100111011001100111011001100110011001	0.48376	4.2399	0.04105	0.67218	1.4055	**1.76802**
0110011001100110001001100010011001100110011001	0.456	4.2399	0.06319	0.6607	1.4065	**1.76724**
0011001100110011011100110111001100110011001100	0.456	4.2399	0.06319	0.6607	1.4065	**1.76724**
0110011001101110011001100110011001000110011001	0.49013	4.2179	0.1183	0.67058	1.4061	**1.76684**
0011001100111011001100110011000100110011001100	0.49013	4.2179	0.1183	0.67058	1.4061	**1.76684**
0011001100110011001110010011001100110011001100	0.4257	4.2179	0.06413	0.64146	1.4072	**1.76212**
0110011001100110011011000110011001100110011001	0.4257	4.2179	0.06413	0.64146	1.4072	**1.76212**
0110011001100100011001100110111001100110011001	0.53748	4.2179	0.1525	0.69005	1.4013	**1.76212**
0011001100110001001100110011101100110011001100	0.53748	4.2179	0.1525	0.69005	1.4013	**1.76212**
0110011001100110010001101110011001100110011001	0.49131	4.2179	0.1525	0.66814	1.4033	**1.76054**
0011001100110011000100111011001100110011001100	0.49131	4.2179	0.1525	0.66814	1.4033	**1.76054**
0110011001100110111001100100011001100110011001	0.47786	4.2179	0.05368	0.67002	1.402	**1.76054**
0011001100110011101100110001001100110011001100	0.47786	4.2179	0.05368	0.67002	1.402	**1.76054**
0111001100110011001101110011001100110011001101	0.44102	4.06059	0.02675	0.66095	1.3887	**1.73085**
0010011001100110011000100110011001100110011000	0.44102	4.06059	0.02675	0.66095	1.3887	**1.73085**
0111011001100110011000100110011001100110011101	0.46992	4.06059	0.03067	0.67697	1.3809	**1.7216**
0010001100110011001101110011001100110011001000	0.46992	4.06059	0.03067	0.67697	1.3809	**1.7216**

Degree 44 Symmetric Polynomials

50 smallest Mahler measure

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
00001010111101101111000100101110001011111	***0.14907***	2.04698	0.02309	0.89087	1.09968	1.18943
01111011100100001011000100111101000110111	***0.14907***	2.09231	0.09614	0.87012	1.10715	1.19756
01111010111100010011111010110001001011111	***0.14907***	2.04646	0.03678	0.87852	1.09869	1.18112
01111001010011010100111011000000110000011	***0.14907***	1.84071	0.07636	0.92863	1.06208	1.11741
01110110101110000111010000100101101111100	***0.14907***	1.6866	0.02899	0.89652	1.07501	1.13267
01101100001111110111100110100010101101000	***0.14907***	1.7852	0.12497	0.93868	1.05295	1.10002
01101010011110001001011000011101101001111	***0.14907***	2.31297	0.00607	0.85813	1.12683	1.24009
01100101001010111111110101010111111000000	***0.14907***	1.70801	0.20038	0.92611	1.06359	1.11882
01100101001010000010100111110101111100000	***0.14907***	1.85732	0.04916	0.88262	1.09265	1.16771
01100101001001111110110011101010011100000	***0.14907***	2.00555	0.14907	0.94142	1.05129	1.09853
01100010000001011101010000001000010101110	***0.14907***	2.03222	0.0875	0.8662	1.10436	1.18943
01011010000101100101100110000110000101110	***0.14907***	2.13151	0.07427	0.86743	1.10616	1.19525
01010100101001111000010001011010011111000	***0.14907***	1.81341	0.11844	0.8816	1.09255	1.16647
01010000110101100111111010100110000011010	***0.14907***	1.96468	0.01799	0.90005	1.0902	1.17018
01001110111011001101110010001100111011100	***0.14907***	3.14466	0.05374	0.83176	1.18974	1.38471
01001100000100011111000100101001000101000	***0.14907***	1.6607	0.09602	0.92015	1.06128	1.1117
01001001000100100000010001010111000100010	***0.14907***	2.23607	0.0346	0.9071	1.09139	1.17872
01000111011110110101101110000011101000100	***0.14907***	1.5823	0.08331	0.9344	1.05344	1.09853
01000110000101011010001101111000000101100	***0.14907***	2.03156	0.09631	0.87077	1.10397	1.1906
01000100100011000010111011110100110111000	***0.14907***	2.2161	0.01747	0.88488	1.09428	1.17508
01000011100000101010111011111111010110110	***0.14907***	2.0846	0.03249	0.87355	1.10629	1.19641
01000001110100100111110010100111000010010	***0.14907***	2.1929	0.06178	0.82652	1.14173	1.25535
01000001000000101111111010101111010100010	***0.14907***	2.46915	0.15068	0.86076	1.13842	1.27006
00111100010100011000101111011001000001000	***0.14907***	1.78741	0.21661	0.9123	1.07314	1.13402
00111010101000110001000100010011011111110	***0.14907***	2.48104	0.00925	0.85823	1.13122	1.24931
00111001100110010111110010100001100110010	***0.14907***	2.33333	0.03844	0.86508	1.11336	1.21113
00110110100000100100110011000111010111100	***0.14907***	1.91799	0.06005	0.90448	1.07643	1.14207
00110101101011011111110010101000111110000	***0.14907***	1.7911	0.22921	0.92298	1.06624	1.12303
00101101100101101000110011011110000110000	***0.14907***	1.78989	0.04307	0.88302	1.08312	1.14864
00101100111111000110000101100100101011000	***0.14907***	1.88493	0.06098	0.8768	1.08963	1.15893
00101010101111001110110011101100101111110	***0.14907***	1.93793	0.05374	0.90358	1.08226	1.15253
00100110101010101101010000001111111111100	***0.14907***	1.86509	0.07183	0.91028	1.06929	1.12857
00100101010010101110100111101111110000000	***0.14907***	1.56072	0.06302	0.93441	1.0511	1.09252
00100011010000100000010001010111010000110	***0.14907***	1.96073	0.07155	0.9135	1.07214	1.13402
00100001100111010000111011010000100110010	***0.14907***	1.65716	0.08555	0.92945	1.05231	1.09403
00010101110101101100111011001110000010000	***0.14907***	1.89525	0.03771	0.89254	1.08339	1.14994
00010101010100100101000100000111000000000	***0.14907***	2.83235	0.04345	0.86944	1.13685	1.2796
00001100101001100000110011010110011111000	***0.14907***	2.15616	0.11241	0.88351	1.10855	1.20553
00001011010000101111010000101111010000110	***0.14907***	3.03604	0.07371	0.74311	1.27283	1.50616
01100110011001100110011001100110011001100	***0.26694***	4.74459	0.10652	0.48649	1.54022	1.96848
00110011001100110011001100110011001100110	***0.26694***	4.74459	0.10652	0.48649	1.54022	1.96848
01010101010101010101010000000000000000000	***0.33148***	3.42864	0.02637	0.59089	1.374	1.65709
00000000000000000000001010101010101010100	***0.33148***	3.42864	0.02637	0.59089	1.374	1.65709
01100011011000110110001101100011011000110	***0.34869***	3.44158	0.00226	0.64258	1.31137	1.55517
00110110001101100011011000110110001101100	***0.34869***	3.44158	0.00226	0.64258	1.31137	1.55517
00010001000100010001000100010001000100010	***0.35223***	3.42864	0.03334	0.59715	1.37412	1.65709
01000100010001000100010001000100010001000	***0.35223***	3.42864	0.03334	0.59715	1.37412	1.65709
01101001011010010110100111100001111000010	***0.39278***	3.08578	0.0397	0.65391	1.31196	1.55517
00111100001111000011110010110100101101000	***0.39278***	3.08578	0.0397	0.65391	1.31196	1.55517
01001011010010110100101111000011110000110	***0.39406***	2.99272	0.00907	0.66069	1.30248	1.53591

Degree 44 Symmetric Polynomials

50 largest Mahler measure

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
0000000011000111100010011101101001001101	**0.96986**	1.41638	0.68203	0.98493	1.01497	1.02976
0101010110010010110111001000111100011000	**0.96986**	1.41638	0.68203	0.98493	1.01497	1.02976
0101010110001111000110111001001011011000	**0.96756**	1.32454	0.53456	0.98437	1.0145	1.02794
0000000011011010010011101100011110001101	**0.96756**	1.32454	0.53456	0.98437	1.0145	1.02794
01010001011111100001001100111010010100001	**0.96007**	1.40652	0.57078	0.98036	1.01887	1.03692
0000010000101001011100110010000111110100	**0.96007**	1.40652	0.57078	0.98036	1.01887	1.03692
0101010111000011111001100110101101001000	**0.95939**	1.51057	0.62241	0.97936	1.02105	1.04219
0000000010010110101100110011111000011101	**0.95939**	1.51057	0.62241	0.97936	1.02105	1.04219
0101000111010110100011001101111000001001	**0.9584**	1.32988	0.37431	0.98121	1.01592	1.02976
00000100100000011110110011000101101011100	**0.9584**	1.32988	0.37431	0.98121	1.01592	1.02976
01101101001000000010011001100010101110000	**0.95823**	1.49815	0.59638	0.97873	1.02184	1.04393
001110000111010100011001100100000000100101	**0.95823**	1.49815	0.59638	0.97873	1.02184	1.04393
01100100100000001010010111100000101011100	**0.95681**	1.36125	0.47873	0.97921	1.01921	1.03692
0011000111010100000111101001010000001001	**0.95681**	1.36125	0.47873	0.97921	1.01921	1.03692
0010010101101101110101000000100011100000	**0.95624**	1.39139	0.53993	0.97905	1.01922	1.03692
0111000000111000100000001010111011010101	**0.95624**	1.39139	0.53993	0.97905	1.01922	1.03692
0101101010101100101001100111110011111111	**0.95554**	1.44836	0.55711	0.9777	1.02218	1.04393
0000111111111001111100110010100110101010	**0.95554**	1.44836	0.55711	0.9777	1.02218	1.04393
00011000000000011110011001100101101010101	**0.9555**	1.36723	0.44679	0.97883	1.01928	1.03692
01001101010101101001100110011110000000000	**0.9555**	1.36723	0.44679	0.97883	1.01928	1.03692
0011111111110001111001100110100100101010	**0.95497**	1.40943	0.501	0.97855	1.01939	1.03692
0110101010100100101100110011110001111111	**0.95497**	1.40943	0.501	0.97855	1.01939	1.03692
01011010111101111110011001100101000101111	**0.95331**	1.45593	0.53071	0.97723	1.02164	1.04219
0000111110100010100110011001111101111010	**0.95331**	1.45593	0.53071	0.97723	1.02164	1.04219
01110001101110000101000100000010110111001	**0.95329**	1.60343	0.60659	0.97603	1.02503	1.05081
00100100111011010000010001010000011101100	**0.95329**	1.60343	0.60659	0.97603	1.02503	1.05081
0101101111001010001110111011011111001011	**0.95316**	1.33815	0.42228	0.97811	1.01945	1.03692
0000111010011111011011101110001010011110	**0.95316**	1.33815	0.42228	0.97811	1.01945	1.03692
0111001111010110101011101111111000001011	**0.95286**	1.43479	0.49159	0.97788	1.01953	1.03692
0010011010000011111110111010101101011110	**0.95286**	1.43479	0.49159	0.97788	1.01953	1.03692
0101010110001111000100110001001011011000	**0.95283**	1.35526	0.45935	0.97796	1.01948	1.03692
0000000011011010010001100100011110001101	**0.95283**	1.35526	0.45935	0.97796	1.01948	1.03692
0000000011111000111001100110110110101101	**0.9528**	1.35965	0.14907	0.97943	1.01759	1.03336
0101010110101101101100110011100011111000	**0.9528**	1.35965	0.14907	0.97943	1.01759	1.03336
0010010100101000100100110001110111110000	**0.95241**	1.41433	0.52827	0.97685	1.02178	1.04219
0111000001111101110001100100100010100101	**0.95241**	1.41433	0.52827	0.97685	1.02178	1.04219
0111111110010011110010011100101100011010	**0.95206**	1.65094	0.5422	0.97607	1.02424	1.0491
0010101011000110100111001001111001001111	**0.95206**	1.65094	0.5422	0.97607	1.02424	1.0491
0111000001111101110001000100100010100101	**0.95189**	1.4463	0.4915	0.97635	1.02259	1.04393
0010010100101000100100010001110111110000	**0.95189**	1.4463	0.4915	0.97635	1.02259	1.04393
0111010110000101001110111011000001011000	**0.9503**	1.5462	0.57457	0.97502	1.02482	1.0491
00100000011010000011011101110010100001101	**0.9503**	1.5462	0.57457	0.97502	1.02482	1.0491
01011010111101111110011101100101000101111	**0.9501**	1.3556	0.48461	0.97567	1.02281	1.04393
0000111110100010100110111001111101111010	**0.9501**	1.3556	0.48461	0.97567	1.02281	1.04393
0101010101101011010011001100001111100000	**0.95002**	1.51992	0.45137	0.97595	1.02256	1.04393
0000000000111110000110011001011010110101	**0.95002**	1.51992	0.45137	0.97595	1.02256	1.04393
0001011011110101100100110001100000101110	**0.94979**	1.40916	0.44581	0.97592	1.02206	1.04219
0100001110100000110001100100110101111011	**0.94979**	1.40916	0.44581	0.97592	1.02206	1.04219
0010011010000011010100010000001101011110	**0.94973**	1.49156	0.48369	0.97576	1.02265	1.04393
0111001111010110000001000101011000001011	**0.94973**	1.49156	0.48369	0.97576	1.02265	1.04393

Degree 44 Symmetric Polynomials

50 smallest maximum modulus

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
000010100000111001110011001101100100101111	0.90602	***1.29459***	0.14907	0.96262	1.02681	1.04739
010111110100100110110011001110011100010	0.90602	***1.29459***	0.14907	0.96262	1.02681	1.04739
001001011011000000001100110101010011100O	0.88313	***1.29546***	0.19944	0.95282	1.0331	1.05755
011100001110010101011001100000000110110 1	0.88313	***1.29546***	0.19944	0.95282	1.0331	1.05755
000000111110110110111001101110001110101 1	0.92733	***1.30164***	0.22268	0.96955	1.02336	1.04219
010101101011100011101100111011011011111 0	0.92733	***1.30164***	0.22268	0.96955	1.02336	1.04219
001010110001100011101011111011011001001 1	0.92962	***1.30366***	0.24184	0.96933	1.02426	1.04393
011111100100110110111110101110001100011 0	0.92962	***1.30366***	0.24184	0.96933	1.02426	1.04393
001001111011101000001110110101111011101 0	0.90329	***1.30799***	0.00799	0.96232	1.02829	1.05081
011100101110111101011011100000101110111 1	0.90329	***1.30799***	0.00799	0.96232	1.02829	1.05081
000110001111010111101100111010000010110 1	0.88423	***1.31552***	0.12251	0.95758	1.02751	1.04739
010011011010000010111001101111010111100 0	0.88423	***1.31552***	0.12251	0.95758	1.02751	1.04739
011110111100001010011011100111110100101 1	0.91285	***1.31679***	0.22382	0.96281	1.02839	1.05081
001011101001011111001110110010100001111 0	0.91285	***1.31679***	0.22382	0.96281	1.02839	1.05081
000111110110001011101011111011110110001 0	0.92305	***1.31992***	0.0894	0.96902	1.02404	1.04393
010010100011011110111111010111010001101 1 1	0.92305	***1.31992***	0.0894	0.96902	1.02404	1.04393
000110001100001010000000101011111010011 0 1	0.90488	***1.32091***	0.12604	0.96144	1.0285	1.05081
010011011001011111010100000010100001100 0	0.90488	***1.32091***	0.12604	0.96144	1.0285	1.05081
010011110011101000000010101000010011010	0.90958	***1.32336***	0.04512	0.96568	1.02469	1.04393
000110101100100001010100000000101110011 11	0.90958	***1.32336***	0.04512	0.96568	1.02469	1.04393
011111010000110010010001000111001101000 0	0.92077	***1.32341***	0.18397	0.9655	1.02778	1.05081
001010000101100111000100010010011000010 1	0.92077	***1.32341***	0.18397	0.9655	1.02778	1.05081
000000100100110010011110100111001100011 1	0.91632	***1.32445***	0.16119	0.96527	1.02695	1.0491
010101110001100111001011110010011001001 0	0.91632	***1.32445***	0.16119	0.96527	1.02695	1.0491
000000001101101001001110110001111000110 1	0.96756	***1.32454***	0.53456	0.98437	1.0145	1.02794
010101011000111100011011100100101101100 0	0.96756	***1.32454***	0.53456	0.98437	1.0145	1.02794
000110101111001010001100110111110010111 1	0.92662	***1.32465***	0.32533	0.96699	1.02692	1.0491
010011111010011111011001100010100111101 0	0.92662	***1.32465***	0.32533	0.96699	1.02692	1.0491
011011101001001010001011110111111000111 10	0.87964	***1.32488***	0.0505	0.95696	1.02895	1.05081
001110111100011111011110100010100100101 1	0.87964	***1.32488***	0.0505	0.95696	1.02895	1.05081
010101011001101101001011110000111001100 0	0.9165	***1.3251***	0.02372	0.96926	1.02314	1.04219
000000001100111000011110100101101100110 1	0.9165	***1.3251***	0.02372	0.96926	1.02314	1.04219
001001001011111110100011011110101011110 0	0.93144	***1.3267***	0.472	0.9679	1.02751	1.05081
011100011110101011110110001011111110100 1	0.93144	***1.3267***	0.472	0.9679	1.02751	1.05081
011011001001000010101001111111010001110 0	0.88784	***1.32691***	0.0854	0.95661	1.03075	1.0542
001110011100010111111100101010000100100 1	0.88784	***1.32691***	0.0854	0.95661	1.03075	1.0542
011101010001111010010011000111101001000 0	0.93294	***1.32772***	0.15235	0.97173	1.0228	1.04219
001000000100101111000110010010111100010 1	0.93294	***1.32772***	0.15235	0.97173	1.0228	1.04219
000000100001100111001011110010011001011 1	0.87886	***1.32959***	0.03605	0.9517	1.03537	1.06253
010101110100110010011110100111001100001 0	0.87886	***1.32959***	0.03605	0.9517	1.03537	1.06253
000001001000000111101100110001011010111 00	0.9584	***1.32988***	0.37431	0.98121	1.01592	1.02976
010100011101011010001100110111110000010 01	0.9584	***1.32988***	0.37431	0.98121	1.01592	1.02976
011110111101111000100100011101101000101 1	0.90117	***1.33149***	0.06146	0.96047	1.03066	1.05588
001011101000101101110001001000111101111 0	0.90117	***1.33149***	0.06146	0.96047	1.03066	1.05588
010101001010010010011001100111000111110 0	0.92988	***1.33288***	0.06134	0.97306	1.02029	1.03692
000000011111000111001100110010010010100 1	0.92988	***1.33288***	0.06134	0.97306	1.02029	1.03692
010000101001111010110011001111101001111 1	0.8885	***1.33301***	0.04616	0.95466	1.03353	1.05922
000101111100101111110011001101011110010 10	0.8885	***1.33301***	0.04616	0.95466	1.03353	1.05922
010001011101100100101001111100011000100 0	0.9156	***1.33327***	0.06744	0.96999	1.02072	1.03692
000100001000110001111100101001001101110 1	0.9156	***1.33327***	0.06744	0.96999	1.02072	1.03692

Degree 44 Symmetric Polynomials

50 largest minimum modulus

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
010101011001001011011100100011110001100 0	0.96986	1.41638	**0.68203**	0.98493	1.01497	1.02976
000000001100011110001001110110100100110 1	0.96986	1.41638	**0.68203**	0.98493	1.01497	1.02976
010101011100001111100110011010110100100 0	0.95939	1.51057	**0.62241**	0.97936	1.02105	1.04219
000000001001011010110011001111100001110 1	0.95939	1.51057	**0.62241**	0.97936	1.02105	1.04219
011001010101001011001110110011110000000 0	0.94368	1.55635	**0.61923**	0.97146	1.0284	1.05588
001100000000011110011011100110100101010 1	0.94368	1.55635	**0.61923**	0.97146	1.0284	1.05588
000000001111000011100110011011010010110 1	0.93145	1.65593	**0.61423**	0.96424	1.03755	1.07546
010101011010010110110011001110000111100 0	0.93145	1.65593	**0.61423**	0.96424	1.03755	1.07546
001001001110110100000100010100001110110 0	0.95329	1.60343	**0.60659**	0.97603	1.02503	1.05081
011100011011100001010001000000101101110 01	0.95329	1.60343	**0.60659**	0.97603	1.02503	1.05081
011101010001111011000110010011101001000 0	0.94221	1.54575	**0.60377**	0.97097	1.02851	1.05588
001000000100101110010011000110111100010 1	0.94221	1.54575	**0.60377**	0.97097	1.02851	1.05588
010010001001001010000001010111110001110 1	0.94211	1.49085	**0.60333**	0.97061	1.02928	1.05755
000111011100011111010100000010100100100 0	0.94211	1.49085	**0.60333**	0.97061	1.02928	1.05755
000000000110111000011100100101101110010 1	0.93994	1.50197	**0.59701**	0.96978	1.02954	1.05755
010101010011101101001001110000111011000 0	0.93994	1.50197	**0.59701**	0.96978	1.02954	1.05755
011011010010000001001100110001010111000 0	0.95823	1.49815	**0.59638**	0.97873	1.02184	1.04393
001110000111010100011001100100000001001 01	0.95823	1.49815	**0.59638**	0.97873	1.02184	1.04393
010101101010110011111001101011001111111 0	0.94218	1.66357	**0.59581**	0.97032	1.03089	1.06253
000000111111100110101100111110011010101 1	0.94218	1.66357	**0.59581**	0.97032	1.03089	1.06253
010110100101010011011011100011000000011 1	0.93272	1.82795	**0.59511**	0.9643	1.03944	1.08176
000011110000000110001110110110010101001 0	0.93272	1.82795	**0.59511**	0.9643	1.03944	1.08176
000000001100011000010110000101100100110 1	0.92243	1.93793	**0.58137**	0.95913	1.04434	1.091
010101011001001101000011010000110001100 0	0.92243	1.93793	**0.58137**	0.95913	1.04434	1.091
010101010100011000110100001101100100000 0	0.88829	2.23607	**0.57709**	0.93925	1.06909	1.14339
000000000001001101100001011000110001010 1	0.88829	2.23607	**0.57709**	0.93925	1.06909	1.14339
011101011000010100111011101100000101100 0	0.9503	1.5462	**0.57457**	0.97502	1.02482	1.0491
001000000110100000110111011100101000011 01	0.9503	1.5462	**0.57457**	0.97502	1.02482	1.0491
010101010110101100011001100100111110000 0	0.93654	1.63904	**0.57393**	0.96786	1.0323	1.06417
000000000111110010011001100011010110101	0.93654	1.63904	**0.57393**	0.96786	1.0323	1.06417
010100010111111000010011001110100101000 01	0.96007	1.40652	**0.57078**	0.98036	1.01887	1.03692
000001000010100101110011001000011111010 0	0.96007	1.40652	**0.57078**	0.98036	1.01887	1.03692
010101010100111000011100100101101100000 0	0.92115	1.63978	**0.56861**	0.96115	1.0362	1.06906
000000000001101101001001110000111001010 1	0.92115	1.63978	**0.56861**	0.96115	1.0362	1.06906
011110111111010011100011011011000010101 1	0.94863	1.41564	**0.56682**	0.97508	1.02299	1.04393
001011101010000110110110001110010111111 0	0.94863	1.41564	**0.56682**	0.97508	1.02299	1.04393
001100011000001110000001010110110101100 1	0.90531	2.09231	**0.56618**	0.94875	1.05764	1.11882
011001001101011011010100000011100000110 0	0.90531	2.09231	**0.56618**	0.94875	1.05764	1.11882
010101010110100000110110001101011110000 0	0.89838	1.69118	**0.56473**	0.94771	1.05215	1.1015
000000000011110101100011011000001011010 1	0.89838	1.69118	**0.56473**	0.94771	1.05215	1.1015
001000000000100010100101111000001000101 01	0.88703	1.93793	**0.56467**	0.93849	1.0679	1.13672
011101010100010000011110100101000100000 0	0.88703	1.93793	**0.56467**	0.93849	1.0679	1.13672
010101011100001111100100011010110100100 0	0.94573	1.55847	**0.56354**	0.97317	1.02592	1.05081
000000001001011010110001001111110000111 01	0.94573	1.55847	**0.56354**	0.97317	1.02592	1.05081
000001111000000001001001100011101010110 10	0.92809	1.54431	**0.56342**	0.96342	1.03622	1.07067
010100101101010111000110010010000000111 1	0.92809	1.54431	**0.56342**	0.96342	1.03622	1.07067
010101011011001101100011011000110011100 0	0.94219	1.86786	**0.56339**	0.96914	1.03543	1.07546
000000001110011000110110001101100110110 1	0.94219	1.86786	**0.56339**	0.96914	1.03543	1.07546
000011100000110010000000101011100110101 10	0.93075	1.39439	**0.56121**	0.96635	1.03055	1.05755
010110110101100111010100000001001100000 11	0.93075	1.39439	**0.56121**	0.96635	1.03055	1.05755

Degree 44 Symmetric Polynomials

50 smallest L1 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
001100110011001100110011001100110011001100110011	0.26694	4.74459	0.10652	***0.48649***	1.54022	1.96848
011001100110011001100110011001100110011001100110	0.26694	4.74459	0.10652	***0.48649***	1.54022	1.96848
010101010101010101010101000000000000000000000	0.33148	3.42864	0.02637	***0.59089***	1.374	1.65709
000000000000000000000010101010101010101010101	0.33148	3.42864	0.02637	***0.59089***	1.374	1.65709
010001000100010001000100010001000100010001000100	0.35223	3.42864	0.03334	***0.59715***	1.37412	1.65709
000100010001000100010001000100010001000100010001	0.35223	3.42864	0.03334	***0.59715***	1.37412	1.65709
011001100110011001100100011001100110011001100110	0.40668	4.53382	0.10926	***0.59765***	1.47388	1.87351
001100110011001100110001001100110011001100110011	0.40668	4.53382	0.10926	***0.59765***	1.47388	1.87351
001001100110011001100110011001100110011001100110	0.41314	4.3538	0.02078	***0.6087***	1.45912	1.84304
011100110011001100110011001100110011001100110011	0.41314	4.3538	0.02078	***0.6087***	1.45912	1.84304
010011001100110011001100110011001100110011001100	0.43179	4.33333	0.14983	***0.61009***	1.4599	1.84273
000110011001100110011001100110011001100110011001	0.43179	4.33333	0.14983	***0.61009***	1.4599	1.84273
000100110011001100110011001100110011001100110011	0.4293	4.33333	0.03579	***0.62127***	1.45071	1.8319
010001100110011001100110011001100110011001100110	0.4293	4.33333	0.03579	***0.62127***	1.45071	1.8319
011101100110011001100110011001100110011001100110	0.43494	4.3538	0.01517	***0.62442***	1.45037	1.83286
001000110011001100110011001100110011001100110011	0.43494	4.3538	0.01517	***0.62442***	1.45037	1.83286
011001100110011001100011011001100110011001100110	0.41422	4.3538	0.05006	***0.6298***	1.42537	1.79582
001100110011001100110110001100110011001100110011	0.41422	4.3538	0.05006	***0.6298***	1.42537	1.79582
001110110011001100110011001100110011001100110011	0.45056	4.33333	0.03291	***0.63624***	1.4425	1.82218
011011100110011001100110011001100110011001100110	0.45056	4.33333	0.03291	***0.63624***	1.4425	1.82218
011000100110011001100110011001100110011001100111	0.44937	4.3538	0.01867	***0.63703***	1.44272	1.82381
001101110011001100110011001100110011001100110010	0.44937	4.3538	0.01867	***0.63703***	1.44272	1.82381
011000110011001100110011001100110011001100110011	0.40896	4.01386	0.00606	***0.64022***	1.39494	1.73253
001101100110011001100110011001100110011001100110	0.40896	4.01386	0.00606	***0.64022***	1.39494	1.73253
011011001100110011001100110011001100110011001100	0.41194	3.96933	0.02579	***0.64125***	1.39495	1.731
001110011001100110011001100110011001100110011001	0.41194	3.96933	0.02579	***0.64125***	1.39495	1.731
011000110110001101100011011000110110001101100011	0.34869	3.44158	0.00226	***0.64258***	1.31137	1.55517
001101100011011000110110001101100011011000110110	0.34869	3.44158	0.00226	***0.64258***	1.31137	1.55517
010101010101010101010001000000000000000000000	0.43838	3.42864	0.0404	***0.64616***	1.35893	1.64124
000000000000000000000100010101010101010101010101	0.43838	3.42864	0.0404	***0.64616***	1.35893	1.64124
001100110011000100110011001100110001001100110011	0.4356	4.33333	0.05157	***0.64631***	1.4204	1.79206
011001100110010001100110011001100100011001100110	0.4356	4.33333	0.05157	***0.64631***	1.4204	1.79206
001100100011001100110011001100110011001100110111	0.46131	4.3538	0.02476	***0.64647***	1.43623	1.81595
011001110110011001100110011001100110011001100010	0.46131	4.3538	0.02476	***0.64647***	1.43623	1.81595
001100110011001100111001101100110011001100110011	0.41402	4.14461	0.04861	***0.64651***	1.38284	1.71869
011001100110011001101100111001100110011001100110	0.41402	4.14461	0.04861	***0.64651***	1.38284	1.71869
001100010011001100110011001100110011001100110001	0.46185	4.33333	0.02436	***0.64685***	1.43546	1.81363
011001000110011001100110011001100110011001100100	0.46185	4.33333	0.02436	***0.64685***	1.43546	1.81363
001100110011001100111011101100110011001100110011	0.45203	4.33333	0.07118	***0.64729***	1.41976	1.78862
011001100110011001101110111001100110011001100110	0.45203	4.33333	0.07118	***0.64729***	1.41976	1.78862
001011111010000101111101000010111110100001011111010000010	0.40027	3.16926	0.03212	***0.648***	1.32393	1.57576
011111010000101111101000010111110100001011111010000010111	0.40027	3.16926	0.03212	***0.648***	1.32393	1.57576
010011001100110011001110110011001100110011001100	0.42992	4.14461	0.09794	***0.64866***	1.40268	1.75488
000110011001100110011011100110011001100110011001	0.42992	4.14461	0.09794	***0.64866***	1.40268	1.75488
011001100110011001110011001001100110011001100110	0.40774	4.01386	0.03721	***0.6494***	1.36164	1.67503
001100110011001100100110011100110011001100110011	0.40774	4.01386	0.03721	***0.6494***	1.36164	1.67503
001100110010001100110011001100110011001101110011	0.45487	4.3538	0.04777	***0.65103***	1.42712	1.80395
011001100111011001100110011001100110011000100110	0.45487	4.3538	0.04777	***0.65103***	1.42712	1.80395
011001100010011001100110011001100110011001110110	0.45961	4.3538	0.03834	***0.65107***	1.43097	1.80931
001100110111001100110011001100110011001100100011	0.45961	4.3538	0.03834	***0.65107***	1.43097	1.80931

Degree 44 Symmetric Polynomials

50 largest L1 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
000000001100011110001001110110100100110 1	0.96986	1.41638	0.68203	**0.98493**	1.01497	1.02976
01010101100100101101110010001111000110 00	0.96986	1.41638	0.68203	**0.98493**	1.01497	1.02976
000000001101101001001110110001111000110 1	0.96756	1.32454	0.53456	**0.98437**	1.0145	1.02794
01010101100011110001101110010010110110 00	0.96756	1.32454	0.53456	**0.98437**	1.0145	1.02794
00000100100000111101100110001011010111 00	0.9584	1.32988	0.37431	**0.98121**	1.01592	1.02976
01010001110101101000110011011110000010 01	0.9584	1.32988	0.37431	**0.98121**	1.01592	1.02976
01010001011111000010011001110100101000 01	0.96007	1.40652	0.57078	**0.98036**	1.01887	1.03692
00000100001010010111001100100001111101 00	0.96007	1.40652	0.57078	**0.98036**	1.01887	1.03692
01010101101011011011001100111000111110 00	0.9528	1.35965	0.14907	**0.97943**	1.01759	1.03336
00000000111110001110011001101101101011 01	0.9528	1.35965	0.14907	**0.97943**	1.01759	1.03336
00000000100101101011001100111110000111 01	0.95939	1.51057	0.62241	**0.97936**	1.02105	1.04219
01010101110000111110011001101011010010 00	0.95939	1.51057	0.62241	**0.97936**	1.02105	1.04219
01100100100000010100101111000001010111 00	0.95681	1.36125	0.47873	**0.97921**	1.01921	1.03692
00110001110101000001111010010100000010 01	0.95681	1.36125	0.47873	**0.97921**	1.01921	1.03692
01110000001110001000000101011101101101 01	0.95624	1.39139	0.53993	**0.97905**	1.01922	1.03692
00100101011011011101010000000100011100 000	0.95624	1.39139	0.53993	**0.97905**	1.01922	1.03692
01001101010101101001100110011110000000 00	0.9555	1.36723	0.44679	**0.97883**	1.01928	1.03692
00011000000000111100110011001011010101 01	0.9555	1.36723	0.44679	**0.97883**	1.01928	1.03692
00111000011101010001100110010000000100 101	0.95823	1.49815	0.59638	**0.97873**	1.02184	1.04393
01101101001000000100110011000101011100 00	0.95823	1.49815	0.59638	**0.97873**	1.02184	1.04393
00111111111100011110011001101001001010 10	0.95497	1.40943	0.501	**0.97855**	1.01939	1.03692
01101010101001001011001100111100011111 11	0.95497	1.40943	0.501	**0.97855**	1.01939	1.03692
01011011110010100011101110110111110010 11	0.95316	1.33815	0.42228	**0.97811**	1.01945	1.03692
00001110100111110110111011100010100111 10	0.95316	1.33815	0.42228	**0.97811**	1.01945	1.03692
01010101100011110001001100010010110110 00	0.95283	1.35526	0.45935	**0.97796**	1.01948	1.03692
00000000110110100100011001000111100011 01	0.95283	1.35526	0.45935	**0.97796**	1.01948	1.03692
01110011110101101010111011111110000010 11	0.95286	1.43479	0.49159	**0.97788**	1.01953	1.03692
00100110100000111111011101010110101111 0	0.95286	1.43479	0.49159	**0.97788**	1.01953	1.03692
01011010101011001010011001111100111111 11	0.95554	1.44836	0.55711	**0.9777**	1.02218	1.04393
00001111111110011111001100101001101010 10	0.95554	1.44836	0.55711	**0.9777**	1.02218	1.04393
00001111101000101001100110011111011110 10	0.95331	1.45593	0.53071	**0.97723**	1.02164	1.04219
01011010111101111100110011001010001011 11	0.95331	1.45593	0.53071	**0.97723**	1.02164	1.04219
01110000011111011100011001001000101001 01	0.95241	1.41433	0.52827	**0.97685**	1.02178	1.04219
00100101001010001001001100011101111100 00	0.95241	1.41433	0.52827	**0.97685**	1.02178	1.04219
00001001111110111100111011001011101010 01	0.94926	1.37437	0.44424	**0.97661**	1.01991	1.03692
01011100101011101001101110011110111111 00	0.94926	1.37437	0.44424	**0.97661**	1.01991	1.03692
01110000011111011100010001001000101001 01	0.95189	1.4463	0.4915	**0.97635**	1.02259	1.04393
00100101001010001001000100011101111100 00	0.95189	1.4463	0.4915	**0.97635**	1.02259	1.04393
01111111100100111100100111001011000110 10	0.95206	1.65094	0.5422	**0.97607**	1.02424	1.0491
00101010110001101001110010011110010011 11	0.95206	1.65094	0.5422	**0.97607**	1.02424	1.0491
01110001101110000101000100000101101110 01	0.95329	1.60343	0.60659	**0.97603**	1.02503	1.05081
00100100111011010000010001010000111011 00	0.95329	1.60343	0.60659	**0.97603**	1.02503	1.05081
00111000111010101111010000101111111011 01	0.94624	1.34164	0.33333	**0.976**	1.01993	1.03692
01101101101111111010000101111010101110 00	0.94624	1.34164	0.33333	**0.976**	1.01993	1.03692
01010101011010110100110011000011111000 00	0.95002	1.51992	0.45137	**0.97595**	1.02256	1.04393
00000000011111000011001100101101011010 1	0.95002	1.51992	0.45137	**0.97595**	1.02256	1.04393
01000011101000001100011001001101011110 11	0.94979	1.40916	0.44581	**0.97592**	1.02206	1.04219
00010110111101011001001100011000001011 10	0.94979	1.40916	0.44581	**0.97592**	1.02206	1.04219
01110011110101100000010001010110000010 11	0.94973	1.49156	0.48369	**0.97576**	1.02265	1.04393
00100110100000110101000100000011010111 10	0.94973	1.49156	0.48369	**0.97576**	1.02265	1.04393

Degree 44 Symmetric Polynomials

50 smallest L3 norm

Polynomial	**Mahler**	**Maxmod**	**Minmod**	**L1**	**L3**	**L4**
00000000110110100100111011000111110001101	0.96756	1.32454	0.53456	0.98437	***1.0145***	1.02794
01010101100011110001101110010010110110000	0.96756	1.32454	0.53456	0.98437	***1.0145***	1.02794
01010101100100101101110010001111000110000	0.96986	1.41638	0.68203	0.98493	***1.01497***	1.02976
00000000110001111000100111011010010011101	0.96986	1.41638	0.68203	0.98493	***1.01497***	1.02976
00000100100000011110110011000101101011100	0.9584	1.32988	0.37431	0.98121	***1.01592***	1.02976
01010001110101101000110011011110000010011	0.9584	1.32988	0.37431	0.98121	***1.01592***	1.02976
00000000111111000111001100110110110101101	0.9528	1.35965	0.14907	0.97943	***1.01759***	1.03336
01010101101011011011001100111000111110000	0.9528	1.35965	0.14907	0.97943	***1.01759***	1.03336
01010001011111000010011001110100101000011	0.96007	1.40652	0.57078	0.98036	***1.01887***	1.03692
00000100001010010111001100100001111101000	0.96007	1.40652	0.57078	0.98036	***1.01887***	1.03692
01100100100000001010010111100000101011100	0.95681	1.36125	0.47873	0.97921	***1.01921***	1.03692
00110001110101000000111101001010000000100	0.95681	1.36125	0.47873	0.97921	***1.01921***	1.03692
01110000000111000100000010101110110110101	0.95624	1.39139	0.53993	0.97905	***1.01922***	1.03692
00100101011011011101010000000100011100000	0.95624	1.39139	0.53993	0.97905	***1.01922***	1.03692
00011000000000111100110011001011010101010	0.9555	1.36723	0.44679	0.97883	***1.01928***	1.03692
01001101010101101001100110011110000000000	0.9555	1.36723	0.44679	0.97883	***1.01928***	1.03692
01101010101001001011001100111100011111111	0.95497	1.40943	0.501	0.97855	***1.01939***	1.03692
00111111111100011110011001101001001010100	0.95497	1.40943	0.501	0.97855	***1.01939***	1.03692
01011011110010100011101110110111110010110	0.95316	1.33815	0.42228	0.97811	***1.01945***	1.03692
00001110100111110110111011100010100111100	0.95316	1.33815	0.42228	0.97811	***1.01945***	1.03692
00000000110110100100011001000111100011010	0.95283	1.35526	0.45935	0.97796	***1.01948***	1.03692
01010101100011110001001100010010110110000	0.95283	1.35526	0.45935	0.97796	***1.01948***	1.03692
00100110100000111111101110101011010111100	0.95286	1.43479	0.49159	0.97788	***1.01953***	1.03692
01110011110101101010111011111110000010110	0.95286	1.43479	0.49159	0.97788	***1.01953***	1.03692
01011100101011101001101110011110111111000	0.94926	1.37437	0.44424	0.97661	***1.01991***	1.03692
00001001111110111100111011001011101010010	0.94926	1.37437	0.44424	0.97661	***1.01991***	1.03692
00111000111010101111010000101111111011010	0.94624	1.34164	0.33333	0.976	***1.01993***	1.03692
01101101101111111010000101111010101110000	0.94624	1.34164	0.33333	0.976	***1.01993***	1.03692
01010101101101001000110011011100001110000	0.9425	1.44935	0.20303	0.97534	***1.01994***	1.03692
00000000111000011101100110001001011011010	0.9425	1.44935	0.20303	0.97534	***1.01994***	1.03692
01000100100010010110100111100001110111000	0.92905	1.37437	0.01799	0.9739	***1.01997***	1.03692
00010001110111000011110010110100100010010	0.92905	1.37437	0.01799	0.9739	***1.01997***	1.03692
01110001010111000010110011110100100000010	0.94234	1.33366	0.32896	0.97499	***1.02007***	1.03692
00100100000010010111100110100001110101000	0.94234	1.33366	0.32896	0.97499	***1.02007***	1.03692
01010100101001001001100110011100011111000	0.92988	1.33288	0.06134	0.97306	***1.02029***	1.03692
00000001111100011100110011001001001010010	0.92988	1.33288	0.06134	0.97306	***1.02029***	1.03692
01000101110110010010100111110001100010000	0.9156	1.33327	0.06744	0.96999	***1.02072***	1.03692
00010000100011000111110010100100110111010	0.9156	1.33327	0.06744	0.96999	***1.02072***	1.03692
01011111010000110011100110110011010000100	0.94207	1.34883	0.2287	0.97456	***1.02092***	1.03868
00001010000101100110110011100110000101110	0.94207	1.34883	0.2287	0.97456	***1.02092***	1.03868
00000000100101101011001100111110000111010	0.95939	1.51057	0.62241	0.97936	***1.02105***	1.04219
01010101110000111110011001101011010010000	0.95939	1.51057	0.62241	0.97936	***1.02105***	1.04219
00111000100011101000010001011110110111010	0.94454	1.34239	0.14907	0.97519	***1.02144***	1.04044
01101101110110111110100010000101110001000	0.94454	1.34239	0.14907	0.97519	***1.02144***	1.04044
01011011010100110010111011110011000000110	0.94158	1.41753	0.14907	0.97456	***1.02146***	1.04044
00001110000000110011110111010011001010110	0.94158	1.41753	0.14907	0.97456	***1.02146***	1.04044
01011010111101111110011001100101000101111	0.95331	1.45593	0.53071	0.97723	***1.02164***	1.04219
00001111101000101001100110011111011110100	0.95331	1.45593	0.53071	0.97723	***1.02164***	1.04219
01100101001101111010100111111010001100000	0.93666	1.41669	0.14907	0.97311	***1.02177***	1.04044
00110000011000101111110010101111011001010	0.93666	1.41669	0.14907	0.97311	***1.02177***	1.04044

Degree 44 Symmetric Polynomials

50 largest L3 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
001100110011001100110011001100110011001100110011	0.26694	4.74459	0.10652	0.48649	**1.54022**	1.96848
011001100110011001100110011001100110011001100110	0.26694	4.74459	0.10652	0.48649	**1.54022**	1.96848
011001100110011001100100011001100110011001100110	0.40668	4.53382	0.10926	0.59765	**1.47388**	1.87351
001100110011001100110001001100110011001100110011	0.40668	4.53382	0.10926	0.59765	**1.47388**	1.87351
000110011001100110011001100110011001100110011001	0.43179	4.33333	0.14983	0.61009	**1.4599**	1.84273
010011001100110011001100110011001100110011001100	0.43179	4.33333	0.14983	0.61009	**1.4599**	1.84273
011100110011001100110011001100110011001100110011	0.41314	4.3538	0.02078	0.6087	**1.45912**	1.84304
001001100110011001100110011001100110011001100110	0.41314	4.3538	0.02078	0.6087	**1.45912**	1.84304
010001100110011001100110011001100110011001100110	0.4293	4.33333	0.03579	0.62127	**1.45071**	1.8319
000100110011001100110011001100110011001100110011	0.4293	4.33333	0.03579	0.62127	**1.45071**	1.8319
011101100110011001100110011001100110011001100110	0.43494	4.3538	0.01517	0.62442	**1.45037**	1.83286
001000110011001100110011001100110011001100110011	0.43494	4.3538	0.01517	0.62442	**1.45037**	1.83286
011000100110011001100110011001100110011001100111	0.44937	4.3538	0.01867	0.63703	**1.44272**	1.82381
001101110011001100110011001100110011001100110010	0.44937	4.3538	0.01867	0.63703	**1.44272**	1.82381
011011100110011001100110011001100110011001100110	0.45056	4.33333	0.03291	0.63624	**1.4425**	1.82218
001110110011001100110011001100110011001100110011	0.45056	4.33333	0.03291	0.63624	**1.4425**	1.82218
011001110110011001100110011001100110011001100010	0.46131	4.3538	0.02476	0.64647	**1.43623**	1.81595
001100100011001100110011001100110011001100110111	0.46131	4.3538	0.02476	0.64647	**1.43623**	1.81595
011001000110011001100110011001100110011001100100	0.46185	4.33333	0.02436	0.64685	**1.43546**	1.81363
001100010011001100110011001100110011001100110001	0.46185	4.33333	0.02436	0.64685	**1.43546**	1.81363
011001100010011001100110011001100110011001110110	0.45961	4.3538	0.03834	0.65107	**1.43097**	1.80931
001100110111001100110011001100110011001100100011	0.45961	4.3538	0.03834	0.65107	**1.43097**	1.80931
011001101110011001100110011001100110011001101110	0.47032	4.33333	0.03792	0.65401	**1.42963**	1.8063
001100111011001100110011001100110011001100111011	0.47032	4.33333	0.03792	0.65401	**1.42963**	1.8063
011001100111011001100110011001100110001001100110	0.45487	4.3538	0.04777	0.65103	**1.42712**	1.80395
001100110010001100110011001100110011011100110011	0.45487	4.3538	0.04777	0.65103	**1.42712**	1.80395
001100110011001100110110001100110011001100110011	0.41422	4.3538	0.05006	0.6298	**1.42537**	1.79582
011001100110011001100011011001100110011001100110	0.41422	4.3538	0.05006	0.6298	**1.42537**	1.79582
011001100100011001100110011001100110010001100110	0.47352	4.33333	0.0412	0.65769	**1.4251**	1.80024
001100110001001100110011001100110011000100110011	0.47352	4.33333	0.0412	0.65769	**1.4251**	1.80024
011001100110001001100110011001100111011001100110	0.47453	4.3538	0.03973	0.65811	**1.42423**	1.7999
001100110011011100110011001100110010001100110011	0.47453	4.3538	0.03973	0.65811	**1.42423**	1.7999
011001100110011001110110001001100110011001100110	0.46895	4.3538	0.04125	0.65438	**1.42188**	1.79446
001100110011001100100011011100110011001100110011	0.46895	4.3538	0.04125	0.65438	**1.42188**	1.79446
001100110011101100110011001100111011001100110011	0.47272	4.33333	0.05636	0.66005	**1.42171**	1.79548
011001100110111001100110011001101110011001100110	0.47272	4.33333	0.05636	0.66005	**1.42171**	1.79548
011001100110011000100110011101100110011001100110	0.46166	4.3538	0.04197	0.65748	**1.42059**	1.79446
001100110011001101110011001000110011001100110011	0.46166	4.3538	0.04197	0.65748	**1.42059**	1.79446
001100110011000100110011001100010011001100110011	0.4356	4.33333	0.05157	0.64631	**1.4204**	1.79206
011001100110010001100110011001000110011001100110	0.4356	4.33333	0.05157	0.64631	**1.4204**	1.79206
011001100110011101100110011000100110011001100110	0.50152	4.3538	0.0314	0.672	**1.42011**	1.79582
001100110011001000110011001101110011001100110011	0.50152	4.3538	0.0314	0.672	**1.42011**	1.79582
011001100110011001101110111001100110011001100110	0.45203	4.33333	0.07118	0.64729	**1.41976**	1.78862
001100110011001100111011101100110011001100110011	0.45203	4.33333	0.07118	0.64729	**1.41976**	1.78862
011001100110011001000110010001100110011001100110	0.47226	4.33333	0.05433	0.65856	**1.41755**	1.78793
001100110011001100010011000100110011001100110011	0.47226	4.33333	0.05433	0.65856	**1.41755**	1.78793
011001100110011011100110011011100110011001100110	0.47053	4.33333	0.05955	0.66128	**1.41692**	1.78862
001100110011001110110011001110110011001100110011	0.47053	4.33333	0.05955	0.66128	**1.41692**	1.78862
010011001100110011001110110011001100110011001100	0.42992	4.14461	0.09794	0.64866	**1.40268**	1.75488
000110011001100110011011100110011001100110011001	0.42992	4.14461	0.09794	0.64866	**1.40268**	1.75488

Degree 44 Symmetric Polynomials

50 smallest L4 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
000000001101101001001110110001111000110 1	0.96756	1.32454	0.53456	0.98437	1.0145	***1.02794***
01010101100011110001101110010010110110 00	0.96756	1.32454	0.53456	0.98437	1.0145	***1.02794***
0101000111010110100011001101111000001001	0.9584	1.32988	0.37431	0.98121	1.01592	***1.02976***
0000010010000011110110011000101101011100	0.9584	1.32988	0.37431	0.98121	1.01592	***1.02976***
0101010110010010110111001000111100011000	0.96986	1.41638	0.68203	0.98493	1.01497	***1.02976***
0000000011000111100010011101101001001101	0.96986	1.41638	0.68203	0.98493	1.01497	***1.02976***
0101010110101101101100110011100011111000	0.9528	1.35965	0.14907	0.97943	1.01759	***1.03336***
0000000011111000111001100110110110101101	0.9528	1.35965	0.14907	0.97943	1.01759	***1.03336***
0101000101111110000100110011101001010000 1	0.96007	1.40652	0.57078	0.98036	1.01887	***1.03692***
0000010000101001011100110010000111110100	0.96007	1.40652	0.57078	0.98036	1.01887	***1.03692***
0110010010000000101001011110000010101110 0	0.95681	1.36125	0.47873	0.97921	1.01921	***1.03692***
0011000111010100000111101001010000001001	0.95681	1.36125	0.47873	0.97921	1.01921	***1.03692***
0000000011011010010001100100011110001101	0.95283	1.35526	0.45935	0.97796	1.01948	***1.03692***
0101010110001111000100110001001011011000	0.95283	1.35526	0.45935	0.97796	1.01948	***1.03692***
0110110110111111101000010111101010111000	0.94624	1.34164	0.33333	0.976	1.01993	***1.03692***
0011100011101010111101000010111111101101	0.94624	1.34164	0.33333	0.976	1.01993	***1.03692***
0100010010001001011010011110000111011100	0.92905	1.37437	0.01799	0.9739	1.01997	***1.03692***
0001000111011100001111001011010010001001	0.92905	1.37437	0.01799	0.9739	1.01997	***1.03692***
0000000111110001110011001100100100101001	0.92988	1.33288	0.06134	0.97306	1.02029	***1.03692***
0101010010100100100110011001110001111100	0.92988	1.33288	0.06134	0.97306	1.02029	***1.03692***
0100010111011001001010011111000110001000	0.9156	1.33327	0.06744	0.96999	1.02072	***1.03692***
0001000010001100011111001010010011011101	0.9156	1.33327	0.06744	0.96999	1.02072	***1.03692***
0111000000111000100000010101110110110101	0.95624	1.39139	0.53993	0.97905	1.01922	***1.03692***
0010010101101101110101000000100011100000	0.95624	1.39139	0.53993	0.97905	1.01922	***1.03692***
0001100000000011110011001100101101010101	0.9555	1.36723	0.44679	0.97883	1.01928	***1.03692***
0100110101010110100110011001111000000000	0.9555	1.36723	0.44679	0.97883	1.01928	***1.03692***
0010011010000011111110111010101101011110	0.95286	1.43479	0.49159	0.97788	1.01953	***1.03692***
0111001111010110101011101111111000001011	0.95286	1.43479	0.49159	0.97788	1.01953	***1.03692***
0101010110110100100011001101110000111000	0.9425	1.44935	0.20303	0.97534	1.01994	***1.03692***
0000000011100001110110011000100101101101	0.9425	1.44935	0.20303	0.97534	1.01994	***1.03692***
0110101010100100101100110011110001111111	0.95497	1.40943	0.501	0.97855	1.01939	***1.03692***
0011111111110001111001100110100100101010	0.95497	1.40943	0.501	0.97855	1.01939	***1.03692***
0111000101011100001011001111010010000001	0.94234	1.33366	0.32896	0.97499	1.02007	***1.03692***
0010010000001001011110011010000111010100	0.94234	1.33366	0.32896	0.97499	1.02007	***1.03692***
0101110010101110100110111001111011111100	0.94926	1.37437	0.44424	0.97661	1.01991	***1.03692***
0000100111111011110011101100101110101001	0.94926	1.37437	0.44424	0.97661	1.01991	***1.03692***
0101101111001010001110111011011111001011	0.95316	1.33815	0.42228	0.97811	1.01945	***1.03692***
0000111010011111011011101110001010011110	0.95316	1.33815	0.42228	0.97811	1.01945	***1.03692***
0101111101000011001110011011001101000010	0.94207	1.34883	0.2287	0.97456	1.02092	***1.03868***
0000101000010110011011001110011000010111	0.94207	1.34883	0.2287	0.97456	1.02092	***1.03868***
0110110111011011110100010000101110001000	0.94454	1.34239	0.14907	0.97519	1.02144	***1.04044***
0011100010001110100001000101111011011101	0.94454	1.34239	0.14907	0.97519	1.02144	***1.04044***
0101101101010011001011101111001100000011	0.94158	1.41753	0.14907	0.97456	1.02146	***1.04044***
0000111000000110011110111010011001010110	0.94158	1.41753	0.14907	0.97456	1.02146	***1.04044***
0001110111110000111011001110110100101000	0.92822	1.37189	0.14907	0.97037	1.02243	***1.04044***
0100100010100101101110011011100001111101	0.92822	1.37189	0.14907	0.97037	1.02243	***1.04044***
0110010100110111101010011111101000110000	0.93666	1.41669	0.14907	0.97311	1.02177	***1.04044***
0011000001100010111111001010111101100101	0.93666	1.41669	0.14907	0.97311	1.02177	***1.04044***
0011100111010101111010011110100000001001	0.93067	1.37437	0.14907	0.97163	1.02202	***1.04044***
0110110010000000101111001011110101011100	0.93067	1.37437	0.14907	0.97163	1.02202	***1.04044***

Degree 44 Symmetric Polynomials

50 largest L4 norm

Polynomial	Mahler	Maxmod	Minmod	L1	L3	L4
00110011001100110011001100110011001100110011	0.26694	4.74459	0.10652	0.48649	1.54022	**1.96848**
01100110011001100110011001100110011001100110	0.26694	4.74459	0.10652	0.48649	1.54022	**1.96848**
01100110011001100110010001100110011001100110	0.40668	4.53382	0.10926	0.59765	1.47388	**1.87351**
00110011001100110011000100110011001100110011	0.40668	4.53382	0.10926	0.59765	1.47388	**1.87351**
01110011001100110011001100110011001100110011	0.41314	4.3538	0.02078	0.6087	1.45912	**1.84304**
00100110011001100110011001100110011001100110	0.41314	4.3538	0.02078	0.6087	1.45912	**1.84304**
00011001100110011001100110011001100110011001	0.43179	4.33333	0.14983	0.61009	1.4599	**1.84273**
01001100110011001100110011001100110011001100	0.43179	4.33333	0.14983	0.61009	1.4599	**1.84273**
00100011001100110011001100110011001100110011	0.43494	4.3538	0.01517	0.62442	1.45037	**1.83286**
01110110011001100110011001100110011001100110	0.43494	4.3538	0.01517	0.62442	1.45037	**1.83286**
01000110011001100110011001100110011001100110	0.4293	4.33333	0.03579	0.62127	1.45071	**1.8319**
00010011001100110011001100110011001100110011	0.4293	4.33333	0.03579	0.62127	1.45071	**1.8319**
01100010011001100110011001100110011001100111	0.44937	4.3538	0.01867	0.63703	1.44272	**1.82381**
00110111001100110011001100110011001100110010	0.44937	4.3538	0.01867	0.63703	1.44272	**1.82381**
01101110011001100110011001100110011001100110	0.45056	4.33333	0.03291	0.63624	1.4425	**1.82218**
00110110011001100110011001100110011001100111	0.45056	4.33333	0.03291	0.63624	1.4425	**1.82218**
01100111011001100110011001100110011001100010	0.46131	4.3538	0.02476	0.64647	1.43623	**1.81595**
00110010001100110011001100110011001100110111	0.46131	4.3538	0.02476	0.64647	1.43623	**1.81595**
01100100011001100110011001100110011001100100	0.46185	4.33333	0.02436	0.64685	1.43546	**1.81363**
00110001001100110011001100110011001100110001	0.46185	4.33333	0.02436	0.64685	1.43546	**1.81363**
00110011011100110011001100110011001100100011	0.45961	4.3538	0.03834	0.65107	1.43097	**1.80931**
01100110001001100110011001100110011001110110	0.45961	4.3538	0.03834	0.65107	1.43097	**1.80931**
01100110111001100110011001100110011001101110	0.47032	4.33333	0.03792	0.65401	1.42963	**1.8063**
00110011101100110011001100110011001100111011	0.47032	4.33333	0.03792	0.65401	1.42963	**1.8063**
01100110011101100110011001100110011000100110	0.45487	4.3538	0.04777	0.65103	1.42712	**1.80395**
00110011001000110011001100110011001101110011	0.45487	4.3538	0.04777	0.65103	1.42712	**1.80395**
01100110010001100110011001100110011001000110	0.47352	4.33333	0.0412	0.65769	1.4251	**1.80024**
00110011000100110011001100110011001100010011	0.47352	4.33333	0.0412	0.65769	1.4251	**1.80024**
01100110011000100110011001100111011001100110	0.47453	4.3538	0.03973	0.65811	1.42423	**1.7999**
00110011001101110011001100110010001100110011	0.47453	4.3538	0.03973	0.65811	1.42423	**1.7999**
01100110011001110110011001100010011001100110	0.50152	4.3538	0.0314	0.672	1.42011	**1.79582**
00110011001100100011001100110111001100110011	0.50152	4.3538	0.0314	0.672	1.42011	**1.79582**
00110011001100110011011000110011001100110011	0.41422	4.3538	0.05006	0.6298	1.42537	**1.79582**
01100110011001100110001101100110011001100110	0.41422	4.3538	0.05006	0.6298	1.42537	**1.79582**
01100110011011100110011001100110011011100110	0.47272	4.33333	0.05636	0.66005	1.42171	**1.79548**
00110011001110110011001100110011001110110011	0.47272	4.33333	0.05636	0.66005	1.42171	**1.79548**
01100110011001100111011000100110011001100110	0.46895	4.3538	0.04125	0.65438	1.42188	**1.79446**
01100110011001100010011001110110011001100110	0.46166	4.3538	0.04197	0.65748	1.42059	**1.79446**
00110011001100110111001100100011001100110011	0.46166	4.3538	0.04197	0.65748	1.42059	**1.79446**
00110011001100110010001101110011001100110011	0.46895	4.3538	0.04125	0.65438	1.42188	**1.79446**
01100110011001000110011001100100011001100110	0.4356	4.33333	0.05157	0.64631	1.4204	**1.79206**
00110011001100010011001100110001001100110011	0.4356	4.33333	0.05157	0.64631	1.4204	**1.79206**
01100110011001100110111011100110011001100110	0.45203	4.33333	0.07118	0.64729	1.41976	**1.78862**
00110011001100110011101110110011001100110011	0.45203	4.33333	0.07118	0.64729	1.41976	**1.78862**
01100110011001101110011001101110011001100110	0.47053	4.33333	0.05955	0.66128	1.41692	**1.78862**
00110011001100111011001100111011001100110011	0.47053	4.33333	0.05955	0.66128	1.41692	**1.78862**
00110011001100110001001100010011001100110011	0.47226	4.33333	0.05433	0.65856	1.41755	**1.78793**
01100110011001100100011001000110011001100110	0.47226	4.33333	0.05433	0.65856	1.41755	**1.78793**
00011001100110011001101110011001100110011001	0.42992	4.14461	0.09794	0.64866	1.40268	**1.75488**
01001100110011001100111011001100110011001100	0.42992	4.14461	0.09794	0.64866	1.40268	**1.75488**

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