

ESTIMATION AND FAULT DIAGNOSTICS IN
NONLINEAR AND TIME DELAY SYSTEMS BASED ON
UNKNOWN INPUT OBSERVER METHODOLOGY

by

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Abstract

In recent years, fault detection and isolation (FDI) problem has been receiving a great deal of attention in a wide variety of industries. State estimation through design of proper observer is at the heart of many proposed FDI approaches and poses many theoretical research challenges that exists in model based FDI. Estimation and observer design problems are dual of controller design problems, and as such they have been a subject of long study by the control community over the years. However, just as controller design problem, there are variety of unresolved and research problems dealing with state observation problems in a number of dynamical systems. On the other hand, observer design for FDI purposes poses additional constraints and challenges to this problem. As an example consider the problem of state observation in an uncertain linear or nonlinear dynamical system. Generally for control purposes, it is desired to design an observer that is robust to all uncertainties in the system—already a difficult problem depending on the system and the nature of uncertainties. Now imagine that it is also desired to accomplish FDI using this observer. A robust observer that is simply insensitive to all uncertainties will no longer be adequate to accomplish the task. The reason is that such an observer could be robust to faults as well, and as a result, FDI would not be possible. Therefore, ideally one would like to design an specialized observer for FDI—one that is robust to system uncertainties and certain external disturbances but at the same time would be highly sensitive to failures. This makes the state estimation problem particularly difficult and challenging. The overall area of nonlinear control/estimation is still in its infancy. As a result, the FDI problem is even more challenging in such systems. This thesis is an attempt to accomplish model based FDI in certain classes of linear and especially

nonlinear systems using the unknown input observer framework.

Unknown Input Observer (UIO) is an estimator which is decoupled from the unknown inputs (certain disturbances, or faults) that may be acting on the system and the measurements. This particular class of observer has been the subject of study by researchers in the FDI field, since it is particularly attractive for accomplishing certain FDI tasks. However, most of the existing literature on this class of observers deals with linear systems. In this thesis, we show how reliable FDI can be accomplished in a number of systems through the use of UIO methodology. One of the main contribution of this thesis is generalization of the UIO from linear systems to other classes of systems, namely the state retarded (time delay) systems, bilinear systems (a special class of nonlinear systems), and affine nonlinear systems. For bilinear as well as time-delay systems, we propose a reduced order observer, and illustrate how FDI of actuators as well as sensors can be achieved in these systems. Conditions for existence of the proposed observer, plus the stability and convergence proof of the observer based on the Lyapunov Approach and Razumikhin Theorem are given. For affine nonlinear systems, we give conditions under which the system can be diffeomorphically transformed into a particular form which observers with linear error dynamics can be designed, and we apply the typical UIO as well as the fault diagnostic algorithm to this type of nonlinear systems. Finally, the thesis investigates certain connections between Unknown Input Observer (UIO) methodology and Sliding Mode Observer (SMO) which through the discontinuous switching term in their structure can compensate for certain unknown inputs or disturbances so long that they are bounded, and the upper bound is known.

The applicability and effectiveness of our methods in estimation and FDI are illustrated by numerical examples through out the thesis.

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Dedication

To Hedy, Joy and my loving parents.

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Chapter 1

Introduction

System monitoring and timely fault detection capabilities are critical requirements of many modern control systems. Traditionally these features have been of utmost importance in safety critical systems such as civil and military aviation, or nuclear power plants, etc. However, in recent years, other factors have been playing a major role in recognizing the need for these capabilities in other technical systems. Broadly speaking, by the term *fault* we mean failures, errors, malfunctions or disturbances in the functional units that can lead to undesirable or intolerable behavior of the system. Some of the contributing factors that have made the automatic fault detection, isolation and accommodation (FDIA) problem to become an active area for research in a wide variety of industries and systems are:

- i. The increased level of sophistication of many industrial and consumer goods due to the advances in electronics and computer technology, and at the same time decrease in processor's costs. Today's automobiles are a good example of this trend. The auto manufacturers have introduced a tremendous amount

of electronics in recent models. Many functions such as powertrain control, anti-lock brake, chassis control, climate control, traction control, etc. are now performed electrically and are available on many vehicles.

- ii. Many manufacturing and process industries are highly interested in FDIA capabilities due to the fact that timely detection of early faults can result in unexpected and total failure that can lead to plant shutdown and loss of revenues. Therefore, economics is now an important factor in incorporation of FDIA techniques in many industries.
- iii. The environmental concern is now a new driving force for FDIA requirement. An example of this is the new California Air Resource Board (CARB), and Environmental Protection Agency (EPA) legislations which require that by 1998, On Board Diagnostics II (OBD-II) to be rolled into all light duty vehicles sold in North American fleet. Essentially, OBD-II requires fault detection capability for all vehicle components whose failure can result in emission levels beyond a certain level. It would not be surprising if similar tight restriction were to be placed on control and fault diagnosis of other internal combustion engines such as those of boats or lawn mowers.

In any of the systems that were discussed above, in order to have the efficient operation of the process and to increase the reliability and safety, prompt detection of anomalous situations (fault detection) and the fast identification (isolation) of the most probable causes (faults) need to be addressed. As was discussed above, there have been new incentives for requiring FDIA capabilities, but the fact remains that

still the prime reason for it is safety. The 1979 accident in Three Mile Island-2 (TMI-2) nuclear site which resulted in nearly total destruction of the reactor core, and the 1985 explosion of the space shuttle Challenger could have indeed been preventable by proper system monitoring, and timely alarm, isolation, and accommodation.

In the past few years a great deal of effort has gone into providing the operators of critical systems or processes with better graphics, user interface and displays, where an operator (e.g. pilot of an aircraft) can get information regarding a particular part of the plant by typing a few keys on the computer. However, with the increase in complexity of systems, it is humanly impossible to carry the entire monitoring and diagnosis. This is particularly true for low probability events, since detection and diagnosis by human operators require cognitive skills, and efficient retrieval of knowledge from long term memory depends on the frequency of use. In those systems where a human operator is involved, such as flying an aircraft, automatic monitoring could be more reliable than a human operator who is vulnerable to boredom, and stress which can cause errors and impaired judgment. It is expected that more and more systems will require higher degree of autonomous operation that allow for health monitoring and fault tolerance over long periods of time without human intervention.

The Fault Detection and Isolation (FDI) can be achieved by using a replication of hardware (e.g., computers, sensors, actuators, and other components) in what is known as a hardware redundant system in which outputs from identical components are compared for consistency. This approach may be costly, bulky, and the added instruments and periphery hardware adds to the size of the system as well. One example of hardware redundancy is multiple sensors measuring the same quantity. Faults can simply be detected through a majority vote logic rule. Alternatively, FDI

can be carried out using analytical or functional information about the system being monitored, i.e., based on a mathematical model of the system. The latter approach is known as analytical redundancy, which is also known invariably as model-based or quantitative FDI. Model-based FDI is currently the subject of extensive research and is being used in highly reliable control systems due to the fact that analytical redundancy based techniques are more economical and at times more powerful. These methods are also capable of detecting soft incipient faults even during the system's transient operation.

Research attention in recent years has been focused on robust methods for FDI, which are able to detect incipient (soft or small) faults in a system before they are manifested as problems requiring either operator or automatic system intervention (accommodation or control reconfiguration). Since model-based FDI use the mathematical knowledge of the system for their diagnostics purposes, the FDI system would register numerous false alarms if the process model is not accurately represented, or if some parameters in the system change due to aging, corrosion, etc. It is clear that under such conditions robust control and FDI should be used. However, although there is a wealth of research in the area of robust control, the problem of robust FDI is different, and a lot more difficult. Basically, the problem is that we demand precise answers in an imprecise system, that is, high sensitivity to instrument faults, but robustness to the process uncertainties. In other words, robustness is the ability to isolate the fault in the presence of modeling errors. This clearly is not an easy task to accomplish, since there may be only very limited degrees of freedom in the design. Another very difficult bottleneck in FDI is that although practically almost all physical systems are nonlinear, there is very little work available in the literature

on the subject of FDI for nonlinear systems. It should also be pointed out that the general area of nonlinear control is still in its infancy, and thus it is not surprising that work on FDI for nonlinear system is even more scarce.

The problem treated in this thesis is the use of analytical redundancy for fault detection and isolation, i.e., the model-based FDI approach. The work here is mainly on the problem of FDI in certain classes of nonlinear and time delay systems. In this thesis a fault is considered as a defect in actuator, or sensor, or system structure, which may cause problems or unwanted changes in system dynamics.

Application of analytical redundancy based FDI generally require a mathematical model of the process under consideration without any faulty signal as well as certain amount of information (measurement) from the actual (be it faulty or fault free) plant. As long as the deviation (residual) between the system information (measurement) and the information supplied from the fault free model is close to zero, then one can *probably* claim that there is no fault in the system, i.e., the system operation is normal. The reason that a conclusion with certainty can not be reached in the previous scenario is that in theory the faulty signal may be decoupled from the output, i.e., the faulty signal may not have an effect on the system output. Based on this discussion it is clear that the distinction between the uncertainties and the soft failure effects is an important consideration in the process of designing an FDI system. The main task here is to design an FDI approach which is robust to model uncertainties and sensitive to faults that may occur within the system.

Model based FDI approach is generally composed of two main tasks:

- i. *Residual Generation*

The residual generation amounts to generating signals or symptoms which reflect the faults. The signal comes from the reconstruction of measurements of faulty plant based on the mathematical model (which is robust to model uncertainties). It should be noted that at times transformation of the measurements can emphasize particular faults. This is a useful means for detection and isolation tasks.

Generally speaking, there are three basic schemes that accomplish the task of residual generation. These are:

(a) *Parity Space Approach*

This approach is based on checking (i.e., the parity check) the consistency of the mathematical relations (parity equations—properly modified system equation) between the outputs (or a subset of outputs) and inputs. These relations may lead a direct redundancy, which gives the static algebraic relations between sensor outputs, or temporal redundancy, which gives the dynamic relations between inputs and outputs, in other words, from the inconsistency of the parity equations we can detect the faults (see [7, 36]). This approach has also been formulated in frequency domain [11, 15]. Parity based approaches are commonly considered to be open loop approaches since they essentially use the input and output of the system. Parity equations do not involve any comparison or feedback of information from the plant. It has been shown by Massouminia [33, 34] that for linear systems it is possible in theory to filter the residuals generated through the parity equations so as to get the same residual dynamics as in the case of observer based schemes discussed in the next class of approaches. However, this is

in general not practical and the design is not systematic. Furthermore, parity equations may represent totally meaningless quantities, and can be of use only for FDI purposes.

(b) *Observer Based Approach*

The idea is to reconstruct the outputs of the system from the measurements or a subset of measurements by using either (Luenberger, sliding-mode, unknown input, etc.) observers in a deterministic setting or Kalman Filters, etc. in a stochastic setting. Then the output estimation error or innovations in the stochastic case are used as a residual. In comparison to parity based techniques, this class of residual generators use error feedback and thus can be more useful for robust FDI. In addition, estimators provide useful and meaningful quantities, i.e. the state of the dynamical systems which are often needed for control purposes as well. Finally, observer/estimator design is a well established area in the systems and control and there exists vast number of studies on the subject. The design is systematic and the flexibility in selecting observer gains has been fully exploited in the literature, yielding a rich variety of fault detection schemes [5, 13, 14, 38, 40, 42, 55, 57].

(c) *Parameter Estimation and Identification Approach*

In this approach, system parameters are estimated and identified on-line to monitor the changes for fault detection and diagnostics purpose.

Component faults can be considered as deviation of physical parameters, this method is simple and direct. Generally, in the approach fault decision logic can employ the estimates of some physical parameters such

as efficiency, fuel consumption, etc., which can effectively be used (see [24, 25, 26]).

In this class of techniques, the system's parameters are continuously monitored as they are generated through some recursive identification scheme. The FDI would conclude that there is a fault in the plant or a subsystem within the plant when the corresponding identified parameter takes a sudden jump from its nominal value. The detection logic of course should be intelligent enough to distinguish between a change in the dynamic model of the system due to faults and other normal effects that may lead to changes in the dynamics.

In this thesis, we will confine ourselves to observer based approaches. The advantage of using observer over parity space was discussed above and it is the author's opinion that the observer based techniques are the most powerful of techniques discussed above.

ii. *Residual Evaluation*

The alarm strategy (logical decision-making) is chosen based on the generated residual and experiences on the fault occurrences. Generally the residuals are further processed in either a deterministic or stochastic decision process for the purpose of detection and more commonly isolation of failures.

From a historical perspective, the first comprehensive work in the area of analytical redundancy based FDI appeared in the early 70's. Notably, was the work of Beard (1971) [2] and Jones (1973) [27] who reported an observer-based fault diagnosis in linear systems. A survey and summary of various works in the field including those

of the Beard and Jones was given by Willsky (1976) [54]. Many of the early works were performed based on the basic assumption that the dynamics of the system under consideration was precisely known. That is, no consideration was given to the effect of always present uncertainties, disturbances, as well as the nonlinearities which were ignored in the development of linear models. Later on, Clark (1978) [8] showed the possibility of using inherent analytical redundancy of multiple observers to diagnose instrument faults. The main question that was being addressed in that paper and many subsequent studies was how to isolate faulty instrument(s) once a fault was detected via the observer based approach. This requirement of the isolability of faults led Clark and a number of other researchers to look at FDI using a bank of observers driven by different and at times somewhat overlapping set of measurements. Again the basic assumption of these works was that a sufficiently accurate model of the system as well as independent number of measurements was available [13, 14, 49, 55].

Later results on perfect decoupling between the residuals and the unknown inputs were given by many authors (Watanabe and Himmelblau [52], Ge and Fang [16], Guan and Saif [18], White and Speyer [53]) under different conditions. For linear systems, Massounmia [33, 34] gave a clear treatment of the FDI via geometric approach. His results covered most of the existing works in linear systems. In addition, the connection between the parity based and observer based techniques were brought forward in his work for the first time.

Other researchers such as Ding [11] and Kinnaert et al [30] gave some insight and relationship between the observer based schemes and certain optimization based techniques in frequency domain.

The contents of this thesis is organized as follows:

In Chapter 2, we will give a problem formulation and a brief outline of the Fault Detection and Isolation in linear systems by using Unknown Input Observer (UIO).

In Chapter 3, the FDI results in linear systems are generalized to bilinear systems [57]. From practical point of view, for the bilinear systems considered, we assume that the input is bounded. With some additional conditions, we present sufficient conditions for the design of proper UIO suitable for FDI purpose. The main idea there is to treat the faults as unknown inputs and design observer for state estimation which is decoupled from the fault. Next an inverse transformation is performed to calculate the estimates of the faults. The advantage of this method is that not only it enables us to detect the fault, but it also provides an immediate means for isolation of the fault. Furthermore, another by product of this approach is the exact knowledge of the shape of the fault. This feature is an important piece of information that many of the existing techniques do not provide, and it can be effectively used in the accommodation phase, if necessary. Of course accommodation of the fault may not be a requirement in certain applications, but where such requirement is present either due to safety or other practical reasons, accommodation task can insure that the system can function suboptimally until such times that repairs can be made.

Time delay is also a common phenomenon in many complicated industrial systems. As a result, in Chapter 4 we apply the UIO theory to time-delay systems, which, to our knowledge, has not been considered by other researchers to date. The results here are generalization of the ones in bilinear systems. In the process of the proof of the observer's stability and convergence, we use the result of Razumikhin's theorem – a generalization of Lyapunov result in retarded differential equations. The remainder of the FDI design procedure for this class of systems is similar to that in the case of

the bilinear systems [59, 61].

In Chapter 5, we tackle the fault diagnosis in the systems with perhaps more general forms of nonlinearities. We give an analysis on observer design and FDI for the class of nonlinear systems discussed in that Chapter which uses state transformation that can transform the nonlinear system into a special form. Once the system is transformed into the desired canonical form, we apply some of the results for observer design and FDI in linear systems. Also, certain types of uncertainties are dealt with by using an adaptive observer that can not only estimate the relevant states but also the unknown system's parameters [58, 60].

In Chapter 6, an approach that perhaps could be applicable to a wider class of nonlinear systems is presented. This approach uses ideas that come from the area of Sliding Mode Observer (SMO) plus the UIO domain and combines them to arrive at a more powerful means for FDI. Fault Diagnosis in both linear and nonlinear systems through the use of this hybrid observer is discussed in this chapter.

Finally, the contribution of this thesis are summarized, conclusions are drawn and future developments are discussed in Chapter 7.

Chapter 2

Fault Detection and Isolation in Linear Systems

In this chapter, we will deal with the Fault Detection and Isolation problem in linear systems. From historical point of view, fault diagnosis was first tried in linear systems. This is logical due to the fact that the theory of control system design and estimation are more mature for the class of linear systems than those of nonlinear systems.

Observer based approach was one of the several methods which have been popularly used since seventies. Saif & Guan [41] applied Unknown Input Observer (UIO) in fault diagnosis process. The advantage of doing so is that UIO is insensitive to (or decoupled from) unknown inputs, so that more accurate, robust (to disturbances, or unknown inputs) estimation on state, or even on faulty signals, can be achieved [41]. As a matter of fact, the subject of designing Unknown Input Observer has attracted a lot of attention from many different authors for many years [5, 4, 18, 22, 23, 39, 45, 49]. Due to the fact that the FDI requirements are becoming more and more stringent in variety of systems, there is perhaps a stronger argument for proposing to use observer

based fault diagnosis in complex systems.

In this chapter, we will give the Unknown Input Observer design for linear time invariant systems and also discuss the fault diagnosis approach based on UIO. As far as the design and analysis of the UIO is concerned, we believe that all the existing design methods in the literature are basically equivalent. They are only expressed in different ways, or emphasized under different conditions. However, in general it can be shown that mathematically one can be induced from one the other. Based on the unifying idea, we explain UIO in a simple and clear way, and also compare some different results in the next sections.

2.1 Problem Formulation

Consider a linear time invariant system of the following form,

$$\begin{cases} \dot{x} = Ax + Bu + E_1d_1 + F_1f_a \\ y = Cx + Du + E_2d_2 + F_2f_s \end{cases} \quad (2.1)$$

where state $x \in \mathbb{R}^n$, output $y \in \mathbb{R}^p$, input $u \in \mathbb{R}^m$, actuator fault $f_a \in \mathbb{R}^{m_a}$, sensor fault $f_s \in \mathbb{R}^{m_s}$, d_1, d_2 are disturbances or uncertainties with dimensions r_1 and r_2 respectively, and $A, B, C, D, E_1, E_2, F_1, F_2$ are the corresponding matrices with proper dimensions.

In order to detect faults in linear system (2.1), we propose to design a residual generator of following form

$$\begin{cases} \dot{w} = \xi(w, u, y) \\ r = \eta(w, u, y) \end{cases} \quad (2.2)$$

where r is the residual signal such that

$$\begin{aligned} r &= 0, & \text{if } f_a &= 0 \text{ and } f_s = 0; \\ r &\neq 0, & \text{if } f_a &\neq 0 \text{ or } f_s \neq 0. \end{aligned}$$

Through exploitation of certain redundancies in linear systems more information could be extracted in the above system. This can then assist us in isolating the faulty component(s). This is generally done by making certain elements of the residual vector r sensitive to a certain group of faults and others decoupled from them.

Given the fact that in this work we propose to adopt the observer based approach for FDI, the first equation in (2.2) can be designed as Luenberger Observer for the linear system (2.1). The second equation in (2.2) can be set as the difference between the output and its estimation, or just the estimate of the unknown input, by using the estimate of the observer. Generally in observer based FDI approach, the diagnosis procedure consists of two steps: (1) observer design; (2) residual design.

2.2 Unknown Input Observer

In this section, we discuss Unknown Input Observer. By UIO, we mean that the observer is designed in such a way that its estimate of the state is completely decoupled from the unknown exogenous inputs. The results in Subsection 2.2.1 directly come from [18]. Results in Subsections 2.2.2 and 2.2.3 are also based on the similar idea in [18], but they are derived for general cases.

2.2.1 UIO for Linear Systems—Case 1

In this subsection, we consider a simplified form of system (2.1) with the following two conditions:

1. No fault;
2. No disturbance acting on measurement.

In this situation, the original system (2.1) becomes,

$$\begin{cases} \dot{x} = Ax + Bu + E_1 d_1 \\ y = Cx + Du \end{cases} \quad (2.3)$$

Without any loss of generality, we assume $C = [I_p \ 0]$. In this case, we can partition the system (2.3) into the following form,

$$\begin{cases} \dot{x} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} u + \begin{bmatrix} E_{11} \\ E_{12} \\ E_{13} \end{bmatrix} d_1 \\ y = \begin{bmatrix} I_{r_1} & 0 & 0 \\ 0 & I_{p-r_1} & 0 \end{bmatrix} x, \end{cases} \quad (2.4)$$

where the matrices $A_1 \in \mathbb{R}^{r_1 \times n}$, $A_2 \in \mathbb{R}^{(p-r_1) \times n}$, $A_3 \in \mathbb{R}^{(n-p) \times n}$, $B_1 \in \mathbb{R}^{r_1 \times q}$, $B_2 \in \mathbb{R}^{(p-r_1) \times q}$, $B_3 \in \mathbb{R}^{(n-p) \times q}$, $E_{11} \in \mathbb{R}^{r_1 \times r_1}$, $E_{12} \in \mathbb{R}^{(p-r_1) \times r_1}$, $E_{13} \in \mathbb{R}^{(n-p) \times r_1}$, I_p is the identity matrix with dimension $p \times p$, and the state vector x is partitioned as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ x_3 \end{bmatrix},$$

so that the estimation of state vector is reduced to be the estimation of certain part of the state, i.e., x_3 , that means we need to design a reduced order observer for x_3 .

Assumption 2.2.1 Rank $CE_1 = \text{Rank } E_1 = r_1$ and $r_1 \leq p$. □

Lemma 2.2.1 The rank condition,

$$\text{Rank } CE_1 = \text{Rank } E_1$$

is equivalent to

$$\text{Ker } C \cap \text{Im } E_1 = \{0\}.$$

□

Proof. For any vector $v \in \text{Ker } C \cap \text{Im } E_1$, then $v \in \text{Ker } C$ and $v \in \text{Im } E_1$. So that we know there exists r_1 dimensional vector α , such that

$$v = E_1\alpha, \quad Cv = 0.$$

We have

$$CE_1\alpha = 0.$$

We know α has unique solution 0 is equivalent to CE_1 has full rank, i.e., Rank $CE_1 = \text{Rank } E_1$. This completes the proof. ■

Equivalently, we have

$$\text{Rank} \begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} = r_1. \tag{2.5}$$

Since $E_{11} \in \mathbb{R}^{r_1 \times r_1}$, without loss of generality, can be assumed nonsingular, therefore the following transformation matrix can be defined

$$T = \begin{bmatrix} I & 0 & 0 \\ -E_{12}E_{11}^{-1} & I & 0 \\ -E_{13}E_{11}^{-1} & 0 & I \end{bmatrix}. \quad (2.6)$$

Premultiplying (2.4) with (2.6) results in

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 - E_{12}E_{11}^{-1}\dot{y}_1 \\ \dot{x}_3 - E_{13}E_{11}^{-1}\dot{y}_1 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 - E_{12}E_{11}^{-1}A_1 \\ A_3 - E_{13}E_{11}^{-1}A_1 \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 - E_{12}E_{11}^{-1}B_1 \\ B_3 - E_{13}E_{11}^{-1}B_1 \end{bmatrix} u + \begin{bmatrix} E_{11} \\ 0 \\ 0 \end{bmatrix} d \quad (2.7)$$

Based on the above, it is clear that only the estimate of the x_3 is required. The reason being that x_1 and x_2 are the same as measurements y_1 and y_2 respectively. Before designing an observer for x_3 , we shall introduce the following simpler notation

$$\begin{cases} \bar{A}_i \triangleq A_i - E_{1i}E_{11}^{-1}A_1, & i = 1, 2. \\ \bar{B}_i \triangleq B_i - E_{1i}E_{11}^{-1}B_1, & i = 1, 2. \end{cases} \quad (2.8)$$

As a result, the dynamics of x_3 and the dynamics of y_2 which are not directly affected by uncertainty d_1 , are described by the following equations,

$$\begin{cases} \dot{x}_3 - E_{13}E_{11}^{-1}\dot{y}_1 = \bar{A}_3x + \bar{B}_3u, \\ \dot{y}_2 - E_{12}E_{11}^{-1}\dot{y}_1 = \bar{A}_2x + \bar{B}_2u. \end{cases} \quad (2.9)$$

Partitioning \bar{A}_i as

$$\bar{A}_i \triangleq [\bar{A}_{i1} \quad \bar{A}_{i2} \quad \bar{A}_{i3}], \quad (2.10)$$

and substituting it into (2.9) yields,

$$\dot{x}_3 = \bar{A}_{33}x_3 + \gamma, \quad (2.11)$$

and

$$z = \overline{A}_{23}x_3, \quad (2.12)$$

where

$$\gamma \triangleq \overline{A}_{31}y_1 + \overline{A}_{32}y_2 + E_{13}E_{11}^{-1}y_1 + \overline{B}_3u, \quad (2.13)$$

$$z \triangleq \dot{y}_2 - E_{12}E_{11}^{-1}\dot{y}_1 - \overline{A}_{21}y_1 - \overline{A}_{22}y_2 - \overline{B}_2u. \quad (2.14)$$

Note that γ and z are known because they are expressed by input and output measurements (though the derivative of output is needed in (2.14), certain transformation (2.17) can eliminate the explicit expression of output's derivatives). To design an estimator for x_3 , consider taking (2.11) as the state equation, and (2.12) as the output equation. Based on this system, then we propose a Luenberger observer of the form

$$\dot{\hat{x}}_3 = \overline{A}_{33}\hat{x}_3 + \gamma + K(z - \overline{A}_{23}\hat{x}_3) \quad (2.15)$$

where K is the observer's gain matrix. Substituting (2.13) and (2.14) into (2.15), we get

$$\begin{aligned} \dot{\hat{x}}_3 &= (\overline{A}_{33} - K\overline{A}_{23})\hat{x}_3 + (\overline{A}_{31} - K\overline{A}_{21})y_1 + (\overline{A}_{32} - K\overline{A}_{22})y_2 \\ &\quad + (\overline{B}_3 - K\overline{B}_2)u + [(E_{13} - KE_{12})E_{11}^{-1}\dot{y}_1 + K\dot{y}_2]. \end{aligned} \quad (2.16)$$

As was pointed out above, there are certain terms in the above involving the derivative of the system's output. Since these derivatives are not available and differentiation can lead to noise problems, we shall introduce a coordinate transformation which would lead to cancellation of the terms involving the output's derivatives.

Define

$$w \triangleq \hat{x}_3 - [(E_{13} - KE_{12})E_{11}^{-1}y_1 - Ky_2], \quad (2.17)$$

then we have

$$\begin{aligned} \dot{w} = & (\overline{A}_{33} - K\overline{A}_{23})w + [(\overline{A}_{31} - K\overline{A}_{21}) + (\overline{A}_{33} - K\overline{A}_{23})(E_{13} - KE_{12})E_{11}^{-1}]y_1 \\ & + [(\overline{A}_{32} - K\overline{A}_{22}) - (\overline{A}_{33} - K\overline{A}_{23})K]y_2 + (\overline{B}_3 - K\overline{B}_2)u. \end{aligned} \quad (2.18)$$

Note now that besides the variable w , every terms in (2.18) are known or measurable. Therefore equation (2.18) can be solved.

The following theorem will summarize the design of the Unknown Input Observer (UIO) proposed in the above.

Theorem 2.1 [18] *If the pair $\{\overline{A}_{33}, \overline{A}_{23}\}$ is observable, then the state of the dynamical system given in (2.4) can be estimated by using the UIO proposed in (2.18). The estimation of the state is given by*

$$\hat{x} = \begin{bmatrix} y \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} w + \begin{bmatrix} I \\ [-(E_{13} - KE_{12})E_{11}^{-1} \quad -K] \end{bmatrix} y \quad (2.19)$$

also the eigenvalues of $\overline{A}_{33} - K\overline{A}_{23}$ (i.e., the convergent rate of error dynamics) can be arbitrarily designed by choosing proper gain matrix K .

The proof is clear from the above discussion.

Proposition 2.2.2 For system (2.3), we have

$$\text{Rank} \begin{bmatrix} \lambda I - A & E_1 \\ C & 0 \end{bmatrix} = \text{Rank} \begin{bmatrix} \overline{A}_{23} \\ \lambda I - \overline{A}_{33} \end{bmatrix} + r_1 + p \quad (2.20)$$

□

Proof. According to the transformation (2.6), we know

$$\begin{aligned}
\text{Rank} \begin{bmatrix} \lambda I - A & E_1 \\ C & 0 \end{bmatrix} &= \text{Rank} \left[\begin{array}{ccc|c} & & & E_{11} \\ & \lambda - \bar{A} & & 0 \\ & & & 0 \\ \hline I_{r_1} & 0 & 0 & \\ 0 & I_{p-r_1} & 0 & 0 \end{array} \right] \\
&= \text{Rank} \left[\begin{array}{cc|cc} \lambda I - \bar{A}_{11} & -\bar{A}_{12} & -\bar{A}_{13} & E_{11} \\ -\bar{A}_{21} & \lambda I - \bar{A}_{22} & -\bar{A}_{23} & 0 \\ -\bar{A}_{31} & -\bar{A}_{32} & \lambda I - \bar{A}_{33} & 0 \\ \hline I_{r_1} & 0 & 0 & 0 \\ 0 & I_{p-r_1} & 0 & 0 \end{array} \right] \\
&= \text{Rank} \begin{bmatrix} -\bar{A}_{23} \\ \lambda I - \bar{A}_{33} \end{bmatrix} + p + r_1
\end{aligned}$$

And this completes the proof. ■

Remark 2.2.1 The observability of the pair $(\bar{A}_{33}, \bar{A}_{23})$ is equivalent to

$$\text{Rank} \begin{bmatrix} \lambda I - A & E_1 \\ C & 0 \end{bmatrix} = n + r_1. \quad (2.21)$$

□

Remark 2.2.2 Based on the rank condition, we know that the derivative of y contains all the information regarding E . Thus, we only need to estimate the part of x whose derivative is not directly affected by the unknown input. From the proof of Proposition 2.2.2, we notice that the number of the transmission zeros are same as

the dimensions of the unobservable space of $(\overline{A_{33}}, \overline{A_{23}})$. Based on the above discussion, Theorem 2.1 and Proposition 2.2.2, we know that the number of non-assignable eigenvalues consist of r_1 , i.e. the dimension of E and the number of transmission zeros. By considering the number of transmission zeros, we have an alternative way to verify the condition described in Theorem 2.1. This is basically the same result as that stated in Theorem 1 of [45]. \square

2.2.2 UIO for Linear Systems—Case 2

In Subsection 2.2.1, we considered the case that there was no disturbance or uncertainty in the output. In this subsection, we will consider system (2.1) with the presence of d_2 under the following two conditions:

1. No fault.
2. Disturbances (uncertainties) d_1 and d_2 are independent.

Besides Condition 2, there is no special restriction on d_2 , that means d_2 may appear in the measurable output. In this situation, system (2.1) becomes,

$$\begin{cases} \dot{x} = Ax + Bu + E_1 d_1, \\ y = Cx + Du + E_2 d_2. \end{cases} \quad (2.22)$$

The only difference between systems (2.22) and (2.3) is the presence of certain disturbance on measurement. So in system (2.22), part of the output has been corrupted by the disturbance (unknown input) signals, this may prevent us from using certain part of the output to design a correct unknown input observer as long as the dynamic disturbance (uncertainties) d_1 and measurement disturbance d_2 are independent. If d_1

and d_2 are not completely independent, i.e., there exist none zero matrices $\mathcal{D}_1, \mathcal{D}_2$ with proper dimensions, such that $\mathcal{D}_1 d_1 = \mathcal{D}_2 d_2$, then we can still use certain “corrupted” part of the output to design unknown input observer as long as the corrupted term in measurement is cancelled out by the corresponding part of the dynamic disturbance d_1 in the observer dynamics, and this will be discussed in the next subsection.

Theorem 2.2 *The unknown input observer can be designed for system (2.22) if the following conditions are satisfied,*

1.

$$\text{Ker}((\text{Ker}(E_2^T))^T C) \cap \text{Im}(E_1) = \{0\}; \quad (2.23)$$

2.

$$(A, [E_1 \ 0], C, [0 \ E_2]) \quad (2.24)$$

has no transmission zeros.

Proof. From dimension and rank of E_2 , we know there exists a full rank matrix $T_0 \in \mathbb{R}^{(p-r_2) \times p}$, such that $T_0 E_2 = 0$ and $T_0^T \in \text{Ker}(E_2^T)$. Based on Condition 1, it is easy to see that no vector in the span-space of E_1 that can be selected from kernal space of $T_0 C$, so that

$$\text{Rank } T_0 C E_1 = \text{Rank } E_1,$$

and also Condition 1 implies that $r_1 \leq p - r_2$, i.e.,

$$p \geq r_1 + r_2.$$

Taking

$$\bar{T} = \begin{bmatrix} (E_2^T E_2)^{-1} E_2 \\ T_0 \end{bmatrix},$$

left-multiplying y by \bar{T} , we get,

$$\bar{T}y = \bar{T}Cx + \bar{T}E_2d_2 + \bar{T}Du,$$

correspondingly we have

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} I \\ 0 \end{bmatrix} d_2 + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} u.$$

Now the output y_2 can be used for the design of UIO. However, in order to apply Theorem 2.1, we need $\text{Rank } C_2 E_1 = \text{Rank } E_1$, but this turns out to be $\text{Rank } T_0 C E_1 = \text{Rank } E_1$ which is guaranteed from Condition 1. Also,

$$\begin{aligned} \text{Rank} \begin{bmatrix} \lambda I - A & E_1 & 0 \\ C & 0 & E_2 \end{bmatrix} &= \text{Rank} \begin{bmatrix} \lambda I - A & E_1 & 0 \\ C_1 & 0 & I \\ C_2 & 0 & 0 \end{bmatrix} \\ &= \text{Rank} \begin{bmatrix} \lambda I - A & E_1 & 0 \\ 0 & 0 & I \\ C_2 & 0 & 0 \end{bmatrix} \\ &= \text{Rank} \begin{bmatrix} \lambda I - A & E_1 & 0 \\ C_2 & 0 & 0 \\ \hline 0 & 0 & I \end{bmatrix} = \text{Rank} \begin{bmatrix} \lambda I - A & E_1 \\ C_2 & 0 \end{bmatrix} + r_2 \end{aligned}$$

From Condition 2, we know

$$\text{Rank} \begin{bmatrix} \lambda I - A & E_1 \\ C_2 & 0 \end{bmatrix} = n + r_1.$$

Then by combination of Theorem 2.1 and Proposition 2.1, the UIO can be designed. This ends the proof. ■

Remark 2.2.3 If $E_2 = 0$, that means there is no uncertainty in the output, i.e., system (2.22) reduces to Case 1 described in Subsection 2.2.1. In such situation, Condition 1 (equation (2.23)) becomes $\text{Ker}(C) \cap \text{Im}(E_1) = \{0\}$ which is equivalent to $\text{Rank } CE_1 = \text{Rank } E_1$. Theorem 2.2 is generalization of Theorem 2.1. □

2.2.3 UIO for Linear Systems—Case 3

In this subsection, we consider the same kind of system as before, but without any restriction on the independence of d_1 and d_2 . We have the following conditions,

1. no fault;
2. disturbance d_1 and d_2 are dependent in such a way that there exists $d_0 \in \mathbb{R}^{r_0}$, $\underline{d}_1 \in \mathbb{R}^{r_1-r_0}$, $\underline{d}_2 \in \mathbb{R}^{r_2-r_0}$, such that

$$d_1 = \begin{bmatrix} d_0 \\ \underline{d}_1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} d_0 \\ \underline{d}_2 \end{bmatrix}, \quad \underline{d}_1, \underline{d}_2 \text{ are independent.}$$

System (2.1) becomes of the following form,

$$\begin{cases} \dot{x} = Ax + Bu + E_{10}d_0 + \underline{E}_1\underline{d}_1 \\ y = Cx + Du + E_{20}d_0 + \underline{E}_2\underline{d}_2 \end{cases} \quad (2.25)$$

where E_{i0}, \underline{E}_i , are the partition of E_i according to the dimension of $\begin{bmatrix} d_0 \\ \underline{d}_i \end{bmatrix}$, $i = 1, 2$.

By premultiplying by a nonsingular matrix, y can be transformed into

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} u + \begin{bmatrix} E_{20}^1 \\ E_{20}^2 \end{bmatrix} d_0 + \begin{bmatrix} I_{r_2-r_0} \\ 0 \end{bmatrix} \underline{d}_2 \quad (2.26)$$

where y_1 is corrupted by independent signal \underline{d}_2 , so for observer design we can only use y_2 .

Theorem 2.3 *Unknown Input Observer can be designed for system (2.25), if the following conditions hold,*

1.

$$\text{Rank} [C_2 \underline{E}_1 \quad E_{20}^2] = r_1; \quad (2.27)$$

2.

$$\text{Rank} \left[\begin{array}{c|cc} \lambda I - A & E_{10} & \underline{E}_1 \\ \hline C_2 & E_{20}^2 & 0 \end{array} \right] = n + r_1. \quad (2.28)$$

Proof. Set $X = \begin{bmatrix} x \\ d_0 \end{bmatrix}$, $\underline{d}_0 = \dot{d}_0$, then (2.25) combining with (2.26) becomes

$$\begin{cases} \dot{X} = \begin{bmatrix} A & E_{10} \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} \underline{E}_1 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \underline{d}_1 \\ \underline{d}_0 \end{bmatrix} \\ y = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \begin{bmatrix} E_{20}^1 \\ E_{20}^2 \end{bmatrix} X + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} u + \begin{bmatrix} I_{r_2-r_0} \\ 0 \end{bmatrix} \underline{d}_2. \end{cases} \quad (2.29)$$

Applying Theorem 2.2 to the above system, UIO for (2.29) exists if the following two equations hold,

$$\text{Rank} \left[[C_2 \ E_{20}^2] \begin{bmatrix} \underline{E_1} & 0 \\ 0 & I \end{bmatrix} \right] = \text{Rank} \begin{bmatrix} \underline{E_1} & 0 \\ 0 & I_{r_0} \end{bmatrix} = r_1; \quad (2.30)$$

$$\text{Rank} \left[\begin{array}{c|c|c} \lambda I - \begin{bmatrix} A & E_{10} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \underline{E_1} & 0 \\ 0 & I_{r_0} \end{bmatrix} & \\ \hline \begin{bmatrix} C_2 & E_{20}^2 \end{bmatrix} & & 0 \end{array} \right] = n + r_0 + r_1. \quad (2.31)$$

Clearly

$$\begin{aligned} & \text{Rank} \left[\begin{array}{c|c|c} \lambda I - \begin{bmatrix} A & E_{10} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} \underline{E_1} & 0 \\ 0 & I_{r_0} \end{bmatrix} & \\ \hline \begin{bmatrix} C_2 & E_{20}^2 \end{bmatrix} & & 0 \end{array} \right] \\ &= \text{Rank} \left[\begin{array}{c|c|c} \lambda I - A & -E_{10} & \underline{E_1} & 0 \\ 0 & \lambda I & 0 & I_{r_0} \\ \hline C_2 & E_{20}^2 & 0 & 0 \end{array} \right] \\ &= r_0 + \text{Rank} \begin{bmatrix} \lambda I - A & E_{10} & \underline{E_1} \\ C_2 & E_{20}^2 & 0 \end{bmatrix}. \end{aligned}$$

so that we know equation (2.31) is same as Condition 2 described by equation (2.28).

Also

$$\text{Rank} \left([C_2 \ E_{20}^2] \begin{bmatrix} \underline{E_1} & 0 \\ 0 & I_{r_0} \end{bmatrix} \right) = r_1$$

is equivalent to

$$\text{Rank} [C_2 \underline{E_1} \ E_{20}^2] = r_1$$

which is the Condition 1, i.e., the equation (2.27). This completes the proof. ■

2.3 Fault Diagnosis

We assume that the actuator fault term $F_1 f_a$ in system (2.1) can be modeled in such a way that it can be taken as part of the unknown input. So for system (2.1), as long as the existence conditions of unknown input observer (UIO) are guaranteed, then by using the Theorems 2.1–2.3 the actuator fault as well as the disturbance can be estimated by using other information such as outputs and estimated states. This is described in more details in the following.

From system (2.1), we have

$$[E_1 \quad F_1] \begin{bmatrix} d_1 \\ f_a \end{bmatrix} = \dot{x} - Ax - Bu, \quad (2.32)$$

so estimate of the unknown inputs/faults can be outlined through

$$\begin{bmatrix} \hat{d}_1 \\ \hat{f}_a \end{bmatrix} = \left(\begin{bmatrix} E_1^T \\ F_1^T \end{bmatrix} [E_1 \quad F_1] \right)^{-1} \begin{bmatrix} E_1^T \\ F_1^T \end{bmatrix} (\dot{x} - A\hat{x} - Bu), \quad (2.33)$$

the estimate of \hat{x} comes from UIO, and the derivative of \hat{x} can be eliminated by introducing a state transformation.

Similarly, under the presence of sensor fault term $F_2 f_s$ in system (2.1), we can also estimate the sensor fault as long as UIO exists taking f_a, f_s as unknown input. The sensor fault can be approximated by the following information,

$$\begin{bmatrix} \hat{d}_2 \\ \hat{f}_s \end{bmatrix} = \left(\begin{bmatrix} E_2^T \\ F_2^T \end{bmatrix} [E_2 \quad F_2] \right)^{-1} \begin{bmatrix} E_2^T \\ F_2^T \end{bmatrix} (y - C\hat{x} - Du) \quad (2.34)$$

In our case, the residual signal r (defined in equation (2.2)) is \hat{f}_a or \hat{f}_s itself. Due to the unstructured disturbances, approximation during the calculations, the estimate of sensor or actuator faults may not be very accurate. Therefore certain threshold value δ should be set in order to give reliable alarms. We claim that there exists actuator fault if $\|\hat{f}_a\| \geq \delta$; sensor fault presents if $\|\hat{f}_s\| \geq \delta$.

The advantages of the above fault diagnosis are,

- Simple and easy to implement;
- Accurate estimation of disturbance;
- Accurate estimation of actuator faults, so that the isolation is also achieved.

The disadvantage of the above discussed fault diagnosis is, that the unknown input observer has to exist. If a single UIO that can account for all the sensor, and actuator faults plus the disturbances does not exist, the situation may be remedied by considering to use a bank of observers in order to give the diagnosis as well as certain degree of isolation for the faults in the systems. This idea is illustrated in Figure 2.1.

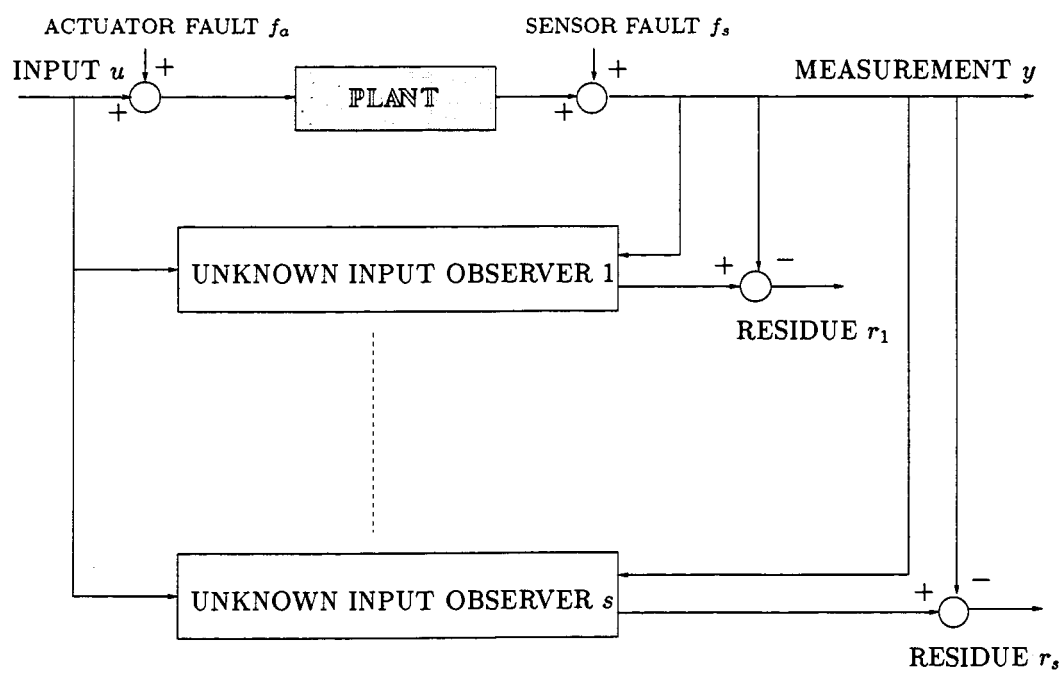


Figure 2.1: FDI Using a Bank of Observers

Chapter 3

UIO Design and FDI in Bilinear Systems

This chapter explores the design of a reduced order observer with unknown inputs for the purpose of fault detection and isolation (FDI) in a class of bilinear systems. An approach for sensor and actuator failure detection and isolation (FDI), based on the proposed observer is presented. Finally, the applicability and effectiveness of the proposed FDI scheme is illustrated on an electrohydraulic servovalve system. The main results are also reported in [57].

3.1 Introduction

Model based approach to failure detection, isolation, and accommodation (FDIA) is now recognized as an important area of research in system and controls. However, the current state of the art mainly deals with FDIA problem in linear systems, although a trend in extending these methodologies to nonlinear systems is currently under way

[14].

Here we deal with the problem of FDI in a class of nonlinear systems, namely bilinear systems. Bilinear systems are a special class of nonlinear systems in which the control appears in both additive and multiplicative terms. The bilinearity is an important phenomenon which arises in a variety of physical systems and even offer advantages to linear systems [3]. As estimation plays a crucial role in many FDI studies, it is not surprising that it is an important part of the development presented in this chapter as well.

The minimal order observer designs for bilinear systems has been discussed by Hara & Furuta [21], and Derese, Stevens & Noldus [10] and others.

In this chapter, we develop a minimal order observer with unknown inputs for the bilinear systems. Later, the unknown input formulation is used to model the effects of failures into the system. This work is a further extension of the work by Guan and Saif [18] to bilinear case, and an alternative approach to the design of Saif [40]. The observer design proposed here is more powerful than that of Hara & Furuta [21] since it can cope with the presence of additional unknown disturbances. In addition, as opposed to the observers proposed in [21], [40], [18], and [62], the estimator proposed here is applicable to a wider class of bilinear systems. As a result, unlike those works, the estimation error of the observer discussed here depends on the control input to the system, i.e., we don't need the complete cancellation of the input in error dynamics. Thus we claim that the proposed observer is suitable for a wider class of bilinear systems. The proposed approach provides a transparent means for verification of the existence conditions for the estimator as well.

In the second part of this chapter, the proposed observer is used for the purpose

of sensor and actuator failure detection and isolation in bilinear systems, also the relations between the faulty input and faulty output channels is discussed. Fault detection in bilinear systems was also considered in [62], however, as was mentioned before the proposed approach in this chapter is applicable to a wider class of problems, and also fault detection as well as isolation of both sensor and actuators are discussed in this chapter. Finally, the proposed bilinear observer and the FDI strategy, are tested in simulation on a bilinear model of an electrohydraulic drive which is of use in applications where large forces with high force to weight ratio are required.

3.2 Reduced Order Observer for Bilinear Systems

In this section, we will study the observer design problem for bilinear systems. Hara and Furuta [21], were the first to design a minimal order state observer for the bilinear systems with observation error dynamics independent of the control input. As an extension, we outline an observer design for bilinear systems driven by completely unknown disturbances or faults. In addition, unlike the observer of [21] the proposed observer error convergence is dependent upon the control input. This will allow observer design for a wider class of bilinear systems than considered in [21].

The idea of this observer design is similar as the one mentioned in Guan and Saif [18], and Saif [40]. The underlying idea to the design of such an estimator is to separate the state manifold as the output manifold and another decoupled manifold on which the tangent vector fields will not be affected by the faulty signals, i.e. the dynamics of the variable on that manifold will not be directly influenced by the faults. With certain matching conditions, the states on that manifold can be observed by using the information of output y and the input u .

3.2.1 Single Input Case

Consider the single input bilinear system described as

$$\begin{cases} \dot{x} = Ax + Dxu + Bu + Ef_a \\ y = Cx \end{cases} \quad (3.1)$$

where C, E have full rank p, m respectively, and $p \geq m$, also $\text{Rank } E = \text{Rank } CE = m$.

Input $u \in \mathbb{R}$, states $x \in \mathbb{R}^n$, actuator faults $f_a \in \mathbb{R}^m$, output $y \in \mathbb{R}^p$.

Since the rank of E is preserved under left multiplication by C , one can always use row permutation for C in the form $C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$, such that $CE = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} E = \begin{bmatrix} C_1 E \\ C_2 E \end{bmatrix}$, where $C_1 \in \mathbb{R}^{m \times n}$, $C_2 \in \mathbb{R}^{(p-m) \times n}$ and $C_1 E$ is an $m \times m$ nonsingular matrix.

Now let us consider the derivative of y_1 and y_2 ,

$$\dot{y}_1 = C_1 \dot{x} = C_1 Ax + C_1 Dxu + C_1 Bu + C_1 E f_a,$$

$$\dot{y}_2 = C_2 \dot{x} = C_2 Ax + C_2 Dxu + C_2 Bu + C_2 E f_a.$$

If we assume that $y_2^* = y_2 - C_2 E (C_1 E)^{-1} y_1$, then the derivative of y_2^* is not affected by the faults f_a , i.e.,

$$\dot{y}_2^* = (C_2 - C_2 E (C_1 E)^{-1} C_1) Ax + (C_2 - C_2 E (C_1 E)^{-1} C_1) Dxu + (C_2 - C_2 E (C_1 E)^{-1} C_1) Bu.$$

Based on the above analysis, one can easily construct a coordinate transformation T ,

$$T = \begin{bmatrix} C_1 \\ C_2 - C_2 E (C_1 E)^{-1} C_1 \\ N \end{bmatrix} \quad (3.2)$$

where N is selected such that $NN^T = I$, N^T belongs to the null space of $\begin{bmatrix} C_1 \\ C_2 - C_2E(C_1E)^{-1}C_1 \end{bmatrix}$ and also make the matrix T nonsingular, i.e., choose $N^T \in \text{Ker } C$ and $\dim N = n - p$ (This is guaranteed by state transformation).

Through the use of the transformation

$$\begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} = Tx,$$

the original system can be written as following,

$$\begin{aligned} \begin{bmatrix} \dot{y}_1 \\ y_2^* \\ z \end{bmatrix} &= \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} \\ \hat{A}_{21} & \hat{A}_{22} & \hat{A}_{23} \\ \hat{A}_{31} & \hat{A}_{32} & \hat{A}_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} + \begin{bmatrix} \hat{D}_{11} & \hat{D}_{12} & \hat{D}_{13} \\ \hat{D}_{21} & \hat{D}_{22} & \hat{D}_{23} \\ \hat{D}_{31} & \hat{D}_{32} & \hat{D}_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} u \\ &+ \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \\ \hat{B}_3 \end{bmatrix} u + \begin{bmatrix} C_1E \\ 0 \\ 0 \end{bmatrix} f_a \end{aligned} \quad (3.3)$$

where $\hat{A} = TAT^{-1}$, $\hat{B} = TB$, $\hat{D} = TDT^{-1}$.

Remark 3.2.1 Note from the above that if $\hat{D}_{23} = 0$, then $\hat{A}_{23}z$ can be expressed by y , u , and \dot{y} . □

Express T^{-1} as $[T_1 \ T_2 \ T_3]$ such that $T[T_1 \ T_2 \ T_3] = \text{diag}(I, I, I)$ (as a matter of fact $T_3 = N^T$), so we know that

$$\hat{D}_{23} = [C_2 - C_2E(C_1E)^{-1}C_1]DT_3,$$

in order to have $\hat{D}_{23} = 0$, we need $[C_2 - C_2E(C_1E)^{-1}C_1]DT_3 = 0$.

From $TT^{-1} = I$, we know that $NT_3 = I$, $CT_3 = 0$, and also $\text{Rank } C + \text{Rank } T_3 = n$. So $\hat{D}_{23} = [C_2 - C_2E(C_1E)^{-1}C_1]DT_3 = 0$ is equivalent to $[C_2 - C_2E(C_1E)^{-1}C_1]D \in \text{span}C$, where $\text{span}C$ means the span space of rows of C , i.e.

$$\hat{D}_{23} = 0 \leftrightarrow [C_2 - C_2E(C_1E)^{-1}C_1]D \in \text{span}C.$$

Assumption 3.2.1 $[C_2 - C_2E(C_1E)^{-1}C_1]D \in \text{span}C$. □

Remark 3.2.2 $[C_2 - C_2E(C_1E)^{-1}C_1]D \in \text{span}C$ is less restrictive than $D \in \text{span}C$. If $D \in \text{span}C$, then there would exist matrix \bar{D} , such that Dxu can be expressed as $\bar{D}Cxu = \bar{D}yu$ which gives not only $\hat{D}_{23} = 0$, but also $\hat{D}_{13} = 0, \hat{D}_{33} = 0$. □

Assumption 3.2.2 $(\hat{A}_{33}, \hat{A}_{23})$ is detectable.

Given the above, we can design the reduced order observer for z as,

$$\dot{\hat{z}} = \hat{A}_{33}\hat{z} + \hat{A}_{31}y_1 + \hat{A}_{32}y_2^* + \hat{D}_{31}y_1u + \hat{D}_{32}y_2^*u + \hat{D}_{33}\hat{z}u + \hat{B}_3u + K(\hat{A}_{23}z - \hat{A}_{23}\hat{z}). \quad (3.4)$$

Then the dynamics of the error is governed by

$$\dot{\tilde{z}} = (\tilde{A}_{33} + \hat{D}_{33}u)\tilde{z}, \quad (3.5)$$

where $\tilde{z} = z - \hat{z}$ and $\tilde{A}_{33} = (\hat{A}_{33} - K\hat{A}_{23})$.

Consider now the following Lemma which would prove useful in establishing the stability of the error dynamics and hence guaranteed convergence of the observer's estimates.

Lemma 3.2.3 [28] Assume A is Hurwitz, then for any positive definite matrix Q , there exists a unique solution $P > 0$ for

$$PA + A^TP = -Q.$$

□

Lemma 3.2.4 [29] Assume that the matrix A is Hurwitz, there exists $Q > 0$ such that H (which may be time variant) satisfies

$$\|H\| < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)},$$

where P is the solution of $PA + A^T P = -Q$, then the system

$$\dot{x} = (A + H)x$$

is asymptotically stable. □

Proof. Consider a Lyapunov candidate $V = x^T P x$, $\dot{V} = x^T (PA + A^T P)x + 2x^T P H x = -x^T Q x + 2x^T P H x \leq (-\lambda_{\min}(Q) + 2\|P\| \cdot \|H\|)\|x\|^2$. In order to make $\dot{V} \leq 0$, we need

$$\|H\| < \frac{\lambda_{\min}(Q)}{2\|P\|} = \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}.$$

That completes the proof. ■

Remark 3.2.5 If Q is set as I (or even a matrix with same eigenvalues), then $\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}$ becomes maximum. This will give a more relaxed (better) bound for perturbation matrix H . □

Definition 3.2.6 For any vector $x \in \mathbb{R}^n$, $\|x\| \triangleq (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$, i.e., Euclidean norm; and for any vector $A \in \mathbb{R}^{n \times m}$, $\|A\| \triangleq \lambda_{\max}^{\frac{1}{2}}(A^T A)$. □

Theorem 3.1 *If the Assumptions 3.2.1 and 3.2.2 are satisfied, and there is a stabilizing matrix K such that*

$$\sup_{t \geq 0} \|D\| \cdot |u(t)| < \frac{1}{2\lambda_{\max}(P_0)},$$

where P_0 is a solution of

$$P_0 \tilde{A}_{33} + \tilde{A}_{33}^T P_0 = -I,$$

then the system (3.4) is a reduced order asymptotical observer.

Proof. From Assumption 3.2.2, we know that we can select matrix K to place the eigenvalues of $(\hat{A}_{33} - K\hat{A}_{23})$ in the left-hand side of the complex plane. So that the equation $P_0 \tilde{A}_{33} + \tilde{A}_{33}^T P_0 = -I$ has a unique solution P_0 .

By using Lemma 3.2.2, we know that as long as $\forall t$,

$$\|\hat{D}_{33}\| \cdot |u(t)| < \frac{1}{2\lambda_{\max}(P_0)},$$

then (3.5) is asymptotically stable. But as a matter of fact,

$$\begin{aligned} \|\hat{D}_{33}\| \cdot |u(t)| &= \|NDT_3\| \cdot |u(t)| \leq \|N\| \cdot \|T_3\| \cdot \|D\| \cdot |u(t)| \\ &= \|D\| \cdot |u(t)| \quad (\text{since } NT_3 = NN^T = I, \|N\| = \|T_3\| = 1). \end{aligned}$$

So we conclude that the system $\dot{\tilde{z}} = (\tilde{A}_{33} + \hat{D}_{33}u(t))\tilde{z}$ is asymptotically stable. ■

In the reduced order observer (3.4), we need information $\hat{A}_{23}z$ while z is not measurable. But note that from (3.3) we get,

$$\hat{A}_{23}z = \dot{y}_2^* - \hat{A}_{21}y_1 - \hat{A}_{22}y_2^* - \hat{D}_{21}y_1u - \hat{D}_{22}y_2^*u - \hat{B}_2u$$

this requires differentiation of y_2^* since \dot{y}_2^* is not directly measurable. To alleviate this problem a transformation should be introduced,

$$w = \hat{z} - Ky_2^* = \hat{z} - K(y_2 - C_2E(C_1E)^{-1}y_1)$$

in which case the reduced order observer (3.4) is transformed to

$$\begin{aligned} \dot{w} &= (\hat{A}_{33} - K\hat{A}_{23} + \hat{D}_{33}u)w + \\ &\quad \underline{-K\hat{B}_2u + \hat{B}_3u + (\hat{A}_{31} + \hat{D}_{31}u - K\hat{A}_{21} - K\hat{D}_{21}u)y_1 +} \\ &\quad \underline{[(\hat{A}_{33} - K\hat{A}_{23} + \hat{D}_{33}u)K + (\hat{A}_{32} + \hat{D}_{32}u - K\hat{A}_{22} - K\hat{D}_{22}u)]y_2^*}. \end{aligned}$$

The above equation can be solved on line (the underlined terms are contributed from the output y_1, y_2 and input u). From the solution of w and output y_1, y_2 , we get

$$\hat{z} = w + K(y_2 - C_2 E (C_1 E)^{-1} y_1), \quad (3.6)$$

thus the estimate \hat{z} for states z is constructed, and the estimates are asymptotically convergent even under the presence of failure or disturbance f_a .

3.2.2 Multiple Input Case

Assume the dimension of input u is m_u , then the original system (3.1) is modified as

$$\begin{cases} \dot{x} = Ax + \sum_{i=1}^{m_u} D^{(i)} x u_i + Bu + E f_a \\ y = Cx \end{cases} \quad (3.7)$$

After using the same transformation T mentioned previously, we get the following system

$$\begin{aligned} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2^* \\ \dot{z} \end{bmatrix} &= \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} \\ \hat{A}_{21} & \hat{A}_{22} & \hat{A}_{23} \\ \hat{A}_{31} & \hat{A}_{32} & \hat{A}_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} + \sum_{i=1}^{m_u} \begin{bmatrix} \hat{D}_{11}^{(i)} & \hat{D}_{12}^{(i)} & \hat{D}_{13}^{(i)} \\ \hat{D}_{21}^{(i)} & \hat{D}_{22}^{(i)} & \hat{D}_{23}^{(i)} \\ \hat{D}_{31}^{(i)} & \hat{D}_{32}^{(i)} & \hat{D}_{33}^{(i)} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} u_i \\ &+ \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \\ \hat{B}_3 \end{bmatrix} u + \begin{bmatrix} C_1 E \\ 0 \\ 0 \end{bmatrix} f_a \end{aligned} \quad (3.8)$$

where $\hat{A} = TAT^{-1}$, $\hat{B} = TB$, $\hat{D}^{(i)} = TD^{(i)}T^{-1}$

Assumption 3.2.3 $[C_2 - C_2 E (C_1 E)^{-1} C_1] D^{(i)} \in \text{span} C$, $i = 1, \dots, m_u$. \square

From Assumption 3.2.3, we know that $\hat{D}_{23}^{(i)} = 0$ and also $\hat{A}_{23}z$ can be expressed by using the output y , \dot{y} as well as u . Based on this, we build the observer,

$$\dot{\hat{z}} = \hat{A}_{33}\hat{z} + \hat{A}_{31}y_1 + \hat{A}_{32}y_2^* + \sum_{i=1}^{m_u} (\hat{D}_{31}^{(i)}y_1 + \hat{D}_{32}^{(i)}y_2^* + \hat{D}_{33}^{(i)}\hat{z})u_i + \hat{B}_3u + K(\hat{A}_{23}z - \hat{A}_{23}\hat{z}) \quad (3.9)$$

Then the error dynamics becomes

$$\dot{\tilde{z}} = (\hat{A}_{33} - K\hat{A}_{23} + \sum_{i=1}^{m_u} \hat{D}_{33}^{(i)}u_i)\tilde{z}. \quad (3.10)$$

Theorem 3.2 *If Assumptions 3.2.2 and 3.2.3 hold, and there exists a stabilizing matrix K such that*

$$\sup_{t \geq 0} \max_{i=1, \dots, m_u} |u_i| \cdot \left\| \sum_{i=1}^{m_u} |u_i| D^{(i)T} D^{(i)} \right\| \leq \frac{1}{4\lambda_{\max}(P_0)^2}, \quad (3.11)$$

where P_0 is the solution of

$$P_0 \tilde{A}_{33} + \tilde{A}_{33}^T P_0 = -I,$$

then the observer (3.9) is an asymptotically stable one.

Proof. Similar to the proof of Theorem 3.1, as long as

$$\left\| \sum_{i=1}^{m_u} \hat{D}_{33}^{(i)} u_i \right\| \leq \frac{1}{2\lambda_{\max}(P_0)},$$

then the error dynamics (3.10) is asymptotically stable.

On the other hand,

$$\begin{aligned} \left\| \sum_{i=1}^{m_u} \hat{D}_{33}^{(i)} u_i \right\| &= \left\| \sum_{i=1}^{m_u} \hat{D}_{33}^{(i)} \operatorname{sgn}(u_i) |u_i| \right\| \\ &\leq \left\| [\sqrt{|u_1|} I \cdots \sqrt{|u_{m_u}|} I] \right\| \cdot \left\| \begin{bmatrix} \hat{D}_{33}^{(1)} \operatorname{sgn}(u_1) \sqrt{|u_1|} \\ \vdots \\ \hat{D}_{33}^{(m_u)} \operatorname{sgn}(u_{m_u}) \sqrt{|u_{m_u}|} \end{bmatrix} \right\| \end{aligned}$$

$$\begin{aligned}
&= \max_{i=1, \dots, m_u} (\sqrt{|u_i|}) \cdot \left\| \sum_{i=1}^{m_u} |u_i| \hat{D}_{33}^{(i)T} \hat{D}_{33}^{(i)} \right\|^{\frac{1}{2}} \\
&= \max_{i=1, \dots, m_u} (\sqrt{|u_i|}) \cdot \|N[D^{(1)T} \sqrt{|u_1|} \cdots D^{(m_u)T} \sqrt{|u_{m_u}|}]\| \\
&\quad \begin{bmatrix} N^T N & & \\ & \ddots & \\ & & N^T N \end{bmatrix} \begin{bmatrix} D^{(1)} \sqrt{|u_1|} \\ \vdots \\ D^{(m_u)} \sqrt{|u_{m_u}|} \end{bmatrix} N^T \|^{\frac{1}{2}} \\
&\leq \max_{i=1, \dots, m_u} (\sqrt{|u_i|}) \cdot \left\| \sum_{i=1}^{m_u} |u_i| D^{(i)T} D^{(i)} \right\|^{\frac{1}{2}} \leq \frac{1}{2\lambda_{\max}(P_0)} \quad (\text{by (3.11)})
\end{aligned}$$

therefore (3.9) is an asymptotically stable observer under the given condition. \blacksquare

While $\hat{A}_{23}z$ can be expressed as,

$$\hat{A}_{23}z = \hat{y}_2^* - \hat{A}_{21}y_1 - \hat{A}_{22}y_2^* - \sum_{i=1}^{m_u} (\hat{D}_{21}^{(i)}y_1 + \hat{D}_{22}^{(i)}y_2^*)u_i - \hat{B}_2u$$

with transformation (3.6), the observer of z can be reformulated as

$$\begin{aligned}
\dot{w} &= (\hat{A}_{33} - K\hat{A}_{23} + \sum_{i=1}^{m_u} \hat{D}_{33}^{(i)}u_i)w + \\
&\quad -K\hat{B}_2u + \hat{B}_3u + (\hat{A}_{31} + \sum_{i=1}^{m_u} \hat{D}_{31}^{(i)}u_i - K\hat{A}_{21} - K\sum_{i=1}^{m_u} \hat{D}_{21}^{(i)}u_i)y_1 + \\
&\quad \underline{[(\hat{A}_{33} - K\hat{A}_{23} + \sum_{i=1}^{m_u} \hat{D}_{33}^{(i)}u_i)K + (\hat{A}_{32} + \sum_{i=1}^{m_u} \hat{D}_{32}^{(i)}u_i - K\hat{A}_{22} - \sum_{i=1}^{m_u} K\hat{D}_{22}^{(i)}u_i)]y_2^*}
\end{aligned}$$

where the underlined terms are contributed from the output y_1, y_2 and input u . The solution w can be calculated from the above equation, so \hat{z} can be estimated again from (3.6).

To summarize the steps involved in designing the unknown input bilinear observer, we shall state the following algorithm.

The Algorithm for Reduced Order Observer's Construction:

1. Find the null space of C , and construct N with full rank $n - p$.
2. Construct the matrix T as described in (3.2).
3. Calculate $\hat{A} = TAT^{-1}$, $\hat{B} = TB$, $\hat{D}^{(i)} = TD^{(i)}T^{-1}$.
4. Verify that Assumptions 3.2.1–3.2.3 and the conditions stated in Theorems 3.1–3.2 hold.
5. If the assumptions and conditions are satisfied, then construct the reduced order observer as (3.9) via introducing the variable w and using the transformation (3.6). Then the state z can be asymptotically estimated as \hat{z} even with the presence of faulty signal f_a . If the assumptions and conditions are not satisfied, such an observer can not be constructed.

3.3 Fault Diagnosis via UIO for Bilinear Systems

3.3.1 Actuator Faults Diagnosis

In order to consider the actuator fault detection problem, we assume that the actuator failures can be modeled by the term Ef_a in (3.1). It can be shown that variety of actuator failure models can be realized with this type of formulation, see Saif and Guan [41]. Therefore, given actuator failure as described in (3.1), a simple approach to detect and isolate the actuator faults would be to try to estimate the magnitude of the actuator failure vector f_a . If an actuator failure is present, then this vector would have a nonzero norm, otherwise, its estimate should have a zero (or very small) norm. Additionally, the failure can be isolated by checking the orientation of the actuator

fault vector estimate. An estimate of the actuator fault can easily be obtained after discretization of system (3.8),

$$\begin{aligned} \delta C_1 E f_a(k) &= y_1(k+1) - y_1(k) - \delta[\hat{A}_{11}y_1(k) + \hat{A}_{12}y_2^*(k) + \hat{A}_{13}z(k) \\ &\quad + \sum_{i=1}^{m_u} (\hat{D}_{11}^{(i)}y_1(k) + \hat{D}_{12}^{(i)}y_2^*(k) + \hat{D}_{13}^{(i)}z(k))u_i(k) + \hat{B}_1u(k)] \end{aligned} \quad (3.12)$$

where k represents the k th time step, and δ is the sampling period.

Assuming that no failure takes place during the initial short transient of the observer, and using the estimate $\hat{z}(k)$ for $z(k)$, actuator fault can be approximately estimated as

$$\begin{aligned} \hat{f}_a(k) &= (C_1 E)^{-1} \frac{y_1(k+1) - y_1(k)}{\delta} - (C_1 E)^{-1} [\hat{A}_{11}y_1(k) + \hat{A}_{12}y_2^*(k) + \hat{A}_{13}\hat{z}(k) \\ &\quad + \sum_{i=1}^{m_u} (\hat{D}_{11}^{(i)}y_1(k) + \hat{D}_{12}^{(i)}y_2^*(k) + \hat{D}_{13}^{(i)}\hat{z}(k))u_i(k) + \hat{B}_1u(k)]. \end{aligned} \quad (3.13)$$

As a result, actuator fault detection and isolation is easily accomplished.

3.3.2 Actuator and Sensor Fault Diagnosis

Consider the bilinear system with the effect of both sensor and actuator failures modeled as

$$\begin{cases} \dot{x} = Ax + \sum_{i=1}^{m_u} D^{(i)}xu_i + Bu + Ef_a \\ y = \tilde{C}x + \tilde{E}_s f_s \end{cases} \quad (3.14)$$

where the sensor fault $f_s \in \mathbb{R}^{m_s}$, and $\text{Rank } \tilde{C}E = \text{Rank } E = m$, \tilde{E}_s is a matrix with only m_s nonzero rows and also $\text{Rank } \tilde{E}_s = m_s$.

Without loss of generality, we may renumber the outputs such that $y = \tilde{C}x + \tilde{E}_s f_s$, and

$$\tilde{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}, \quad \tilde{E}_s = \begin{bmatrix} 0 \\ 0 \\ E_s^3 \end{bmatrix}$$

and E_s^3 is an $m_s \times m_s$ nonsingular matrix. $C_1 : m \times n$, $C_2 : (p-m-m_s) \times n$, $C_3 : m_s \times n$.

Remark 3.3.1 Note that through appropriate selection of \tilde{E}_s , faults in different sensors can be modeled. If the sensor faults are independent then generally E_s^3 can be set as an identity matrix. This allows the isolation of the sensor faults in the last m_s outputs. \square

As long as the following relation holds,

$$p - m_s - m \geq 1, \quad (3.15)$$

then we can design a diagnostic strategy so that m actuator, and m_s sensor faults can be detected and isolated.

Remark 3.3.2 Notice that the total number of instruments (sensor and actuators) that can be detected with the proposed approach is closely related. Under the proposed scheme with a single observer, the lower the number of actuators m , the larger the number of sensor faults m_s that could be detected and visa versa. \square

Theorem 3.3 If $\text{Rank} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} E = \text{Rank } E = m$, then the system in (3.14) can be

transformed to the following form,

$$\begin{aligned}
 \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} &= \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} \\ \hat{A}_{21} & \hat{A}_{22} & \hat{A}_{23} \\ \hat{A}_{31} & \hat{A}_{32} & \hat{A}_{33} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} + \sum_{i=1}^{m_u} \begin{bmatrix} \hat{D}_{11}^{(i)} & \hat{D}_{12}^{(i)} & \hat{D}_{13}^{(i)} \\ \hat{D}_{21}^{(i)} & \hat{D}_{22}^{(i)} & \hat{D}_{23}^{(i)} \\ \hat{D}_{31}^{(i)} & \hat{D}_{32}^{(i)} & \hat{D}_{33}^{(i)} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} u_i \\
 &+ \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \\ \hat{B}_3 \end{bmatrix} u + \begin{bmatrix} C_1 E \\ 0 \\ 0 \end{bmatrix} f_a \\
 y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ E_s^3 \end{bmatrix} f_s
 \end{aligned} \tag{3.16}$$

where E_s^3 is $m_s \times m_s$ nonsingular matrix, $\begin{bmatrix} y_1 \\ y_2^* \\ z \end{bmatrix} = Tx$, with T given in (3.2).

Proof. Since $\text{Rank} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} E = \text{Rank } E = m_s$, without loss of generality, we assume $\text{Rank } C_1 E = \text{Rank } E$. The proof is obvious by using the transformation T defined in (3.2). ■

Assuming $C \triangleq \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$, and as long as Assumptions 3.2.1–3.2.3 and conditions in Theorem 3.1–3.2 are still valid, then the actuator faults can be diagnosed by (3.13).

Note that as in actuator FDI, sensor FDI would easily be possible if we could obtain an estimate of the sensor fault vector f_s . In the above case this is simply

achieved by using (3.16) to get

$$\hat{f}_s(t) = (E_s^3)^{-1} \left(y_3(t) - C_3 T^{-1} \begin{bmatrix} y_1(t) \\ y_2^*(t) \\ \hat{z}(t) \end{bmatrix} \right) \quad (3.17)$$

which as in the actuator FDI case, provides a simple means for sensor FDI. When the norm of $\hat{f}_s(t)$ is greater than certain threshold value, the alarm will be switched on, and isolation is accomplished by using the orientation of the fault vector.

Assume now that the condition in (3.15) does not hold. In such a case, it would not be possible to detect and isolate sensor failures using a single unknown input bilinear observer as described previously. However, when $p - m - m_s = 0$, sensor faults can be detected if we assume that simultaneous failure of sensors and actuators is an unlikely event.

Different from the output described in (3.16), we may have the output equation in the following form

$$y = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} x + \begin{bmatrix} 0 \\ \overline{E}_s^2 \\ \overline{E}_s^3 \end{bmatrix} f_s, \quad (3.18)$$

where C_1, C_2 are assumed to satisfy the Assumptions 3.2.1–3.2.3, and $\begin{bmatrix} \overline{E}_s^2 \\ \overline{E}_s^3 \end{bmatrix}$ is an $m_s \times m_s$ matrix. The implication of the above case is that there would be a steady state error in the estimate of the observer due to the presence of the sensor faults. It is possible to distinguish the sensor faults from actuator faults by using the \hat{z} instead of z in (3.16) to get an estimate of the y_2^* , as follows

$$\hat{y}_2^* = \hat{A}_{21} y_1 + \hat{A}_{22} \hat{y}_2^* + \hat{A}_{23} \hat{z} + \sum_{i=1}^{m_u} (\hat{D}_{21}^{(i)} y_1 + \hat{D}_{22}^{(i)} \hat{y}_2^* + \hat{D}_{23}^{(i)} \hat{z}) u_i + \hat{B}_2 u. \quad (3.19)$$

After discretization of (3.19), we get

$$\begin{aligned} \hat{y}_2^*(k+1) = & (I + \delta \hat{A}_{22} + \delta \sum_{i=1}^{m_u} \hat{D}_{22}^{(i)} u_i(k)) \hat{y}_2^*(k) + \\ & \delta [\hat{A}_{21} y_1(k) + \hat{A}_{23} \hat{z}(k) + \sum_{i=1}^{m_u} (\hat{D}_{21}^{(i)} y_1(k) + \hat{D}_{23}^{(i)} \hat{z}(k)) u_i(k)]. \end{aligned} \quad (3.20)$$

Next, a residual can be generated for the purpose of sensor fault detection as

$$r_s(k) = \|\hat{y}_2^*(k+1) - y_2^*(k+1)\|. \quad (3.21)$$

If the residue $r_s(k)$ is greater than a certain threshold value d , then it would be indication of a sensor fault and an alarm can be issued.

The following will summarize the FDI approach.

The Actuator and Sensor FDI Algorithm:

1. Check if (3.14) can be written as the form of (3.16),

If yes, go to 2; If not, go to 6.

2. Use y_1, y_2 to construct observer (3.9),

If output equation is in form of (3.16), then calculate \hat{f}_s via (3.17), and go to 5.

If output equation is in form of (3.18), then go to 3.

3. Construct (3.20) to estimate $\hat{y}_2^*(k)$. Go to 4.

4. Use (3.21) to generate residue $r_s(k)$.

If $r_s(k) \geq d$, then there is a sensor failure. Go to 6.

If $r_s(k) < d$, then no sensor fault in y_2^* . Go to 5.

5. Verify (3.13) to find $\hat{f}_a(k)$.

6. Stop.

3.4 Illustrative Examples

To illustrate the bilinear observer and FDI capability of our proposed approach, we consider the following two examples.

Example 3.4.1 Consider the bilinear model of an electrohydraulic drive given in [43]. For this bilinear system, the matrices in (3.1) are given by

$$A = \begin{bmatrix} 0 & 0 & 0 & -1.8 \times 10^{-3} \\ 1 & 0 & 0 & -1.33 \times 10^{-2} \\ 0 & 1 & 0 & -5.95 \times 10^{-1} \\ 0 & 0 & 1 & -3.88 \end{bmatrix}, \quad D = \begin{bmatrix} -46.5 & 37.7 & -130.3 & 483.8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times 10^{-3},$$

$$B = \begin{bmatrix} 1.6 \\ 0 \\ 0 \\ 0 \end{bmatrix} \times 10^{-3}, \quad E = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad E_s = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\text{or } \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right)$$

where it is assumed that two of the sensors are highly reliable, whereas one of them is subject to failures. Also in this system, the actuator is subject to failures. Using

the transformation

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

the above system can be transformed to the following form,

$$\hat{A} = \begin{bmatrix} 0 & -1.8 \times 10^{-3} & 0 & 0 \\ 0 & -3.88 & 0 & 1 \\ 1 & -1.33 \times 10^{-2} & 0 & 0 \\ 0 & -5.95 \times 10^{-1} & 1 & 0 \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} -46.5 & 483.8 & 37.7 & -130.3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times 10^{-3},$$

$$\hat{B} = \begin{bmatrix} 1.6 \\ 0 \\ 0 \\ 0 \end{bmatrix} \times 10^{-3}, \quad \hat{E} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \hat{E}_s = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\text{or } \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right).$$

In this system, we need to diagnosis the actuator fault f_a and sensor fault f_s .

Note that all the assumptions in the proposed observer design are satisfied. Thus, we can construct the following observer

$$\dot{w} = \begin{bmatrix} 0 & -K_1 \\ 1 & -K_2 \end{bmatrix} w + \begin{bmatrix} 1 \\ 0 \end{bmatrix} y_1 + \begin{bmatrix} -K_1 K_2 - 0.0133 + 3.88 K_1 \\ K_1 - K_2^2 - 0.595 + 3.88 K_2 \end{bmatrix} y_2 \quad (3.22)$$

where the estimate of $z = [x_3, x_4]^T$ is $\hat{z} = w + [K_1 \ K_2]^T x_2$, and the estimation error is $\tilde{z} = \hat{z} - z$. Thus, the error dynamics is given by

$$\dot{\tilde{z}} = \begin{bmatrix} 0 & -K_1 \\ 1 & -K_2 \end{bmatrix} \tilde{z} \quad (3.23)$$

where \tilde{z} is independent of the actuator fault f_a , i.e., the estimate of z will be always accurate, even in presence of actuator faults f_a . The parameters K_1, K_2 have to be chosen such that the error dynamics is asymptotically stable. For simulation, we assume that the output measurement is corrupted by white noise.

Figure 3.1 shows the response of the system along with that of the observer when there are no faults in the system. In this case, the control input was selected to be 0.5, and the estimator's gain was set at $K_1 = 1, K_2 = 0.5$. The first two sub-plots in Figure 3.1 show the outputs of the valve, and the noise is noticeable in both outputs. The next two subplots show the estimates (dotted) and actual (solid) values of the third and fourth states which are assumed to be not available for measurements. Clearly, the observer estimates these two states with good accuracy and the estimates converge to their actual values in a very short time. Finally, the last two sub-plots illustrate the estimation's errors which again are shown to converge to zero in a very short amount of time.

Next, we investigated the capability of the algorithm to detect simultaneous sensor and actuator failures.

Figure 3.2, illustrates the simulation result where soft actuator and sensor failures were introduced into the system described by the following (where $E_s = [0, 0, 1]^T$),

$$f_a = \begin{cases} 0 & \text{for } 0 \leq t \leq 20; \\ 0.3 + 0.1rand & \text{for } 20 < t \leq 30; \\ -0.4 + 0.1rand & \text{for } 30 < t \leq 40; \\ 0 & \text{for } 40 < t \leq 45; \\ 0.45 + 0.1rand & \text{for } 45 < t \leq 60. \end{cases}$$

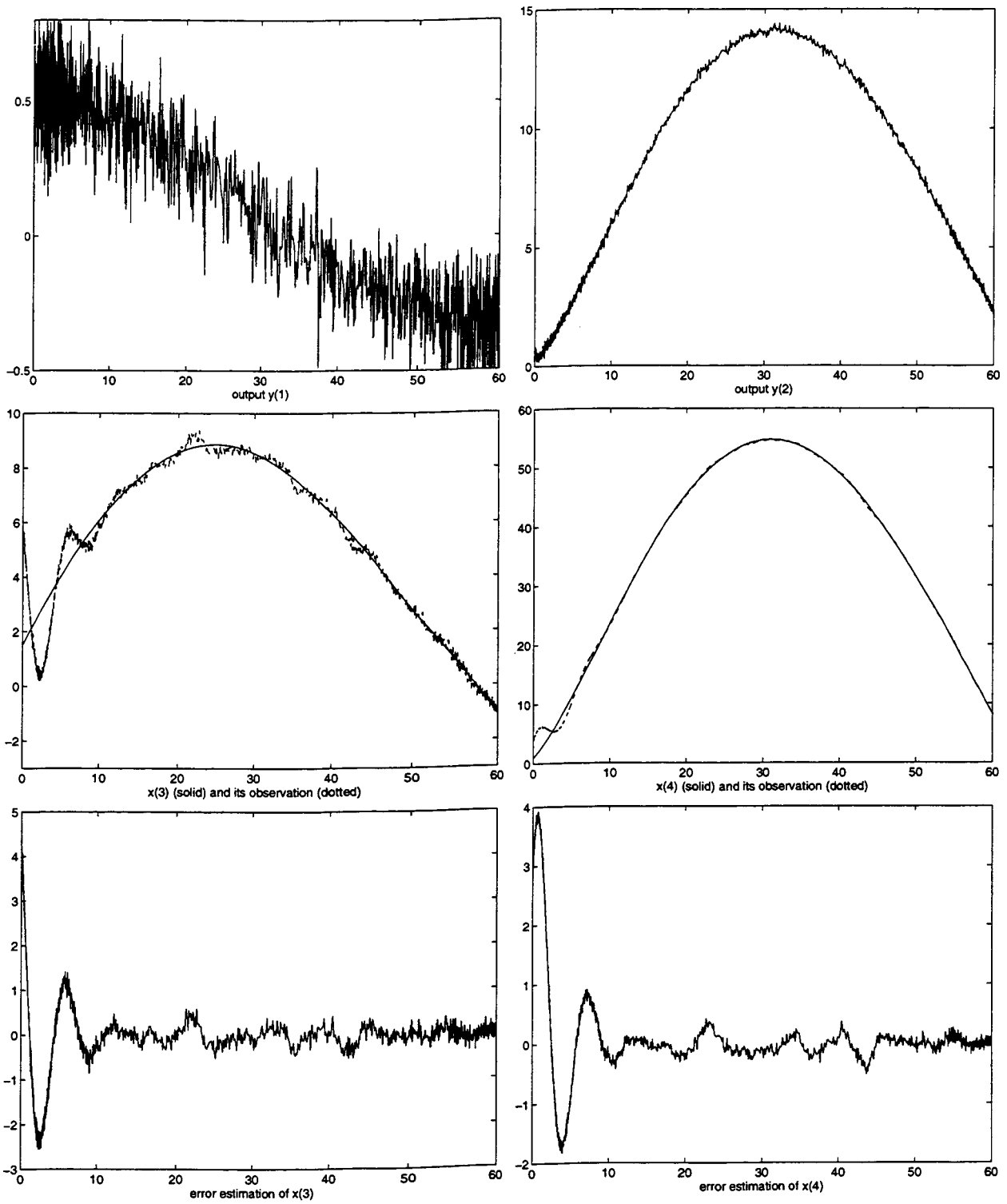


Figure 3.1: Hydraulic Drive Observer Design without any Faults

$$f_s = \begin{cases} 0 & \text{for } 0 \leq t \leq 40; \\ 1 + 0.2rand + 0.1 \sin(t) & \text{for } 40 < t \leq 60. \end{cases}$$

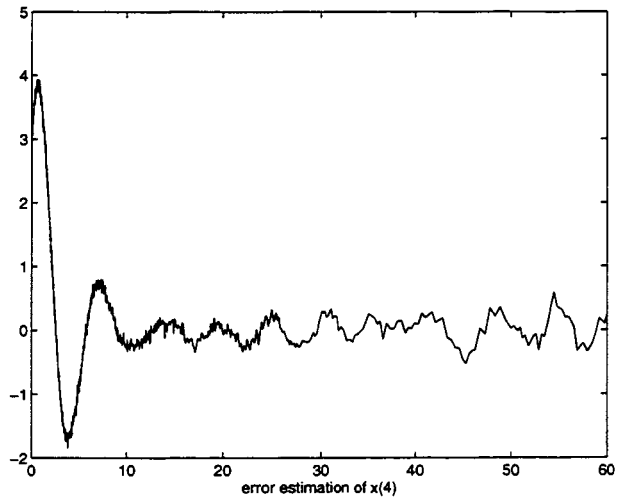
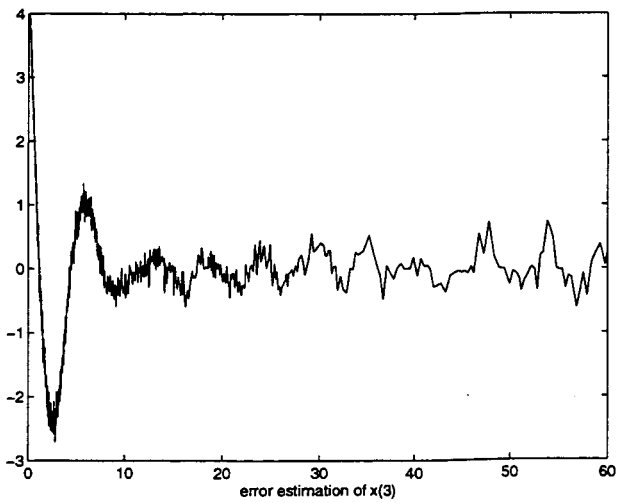
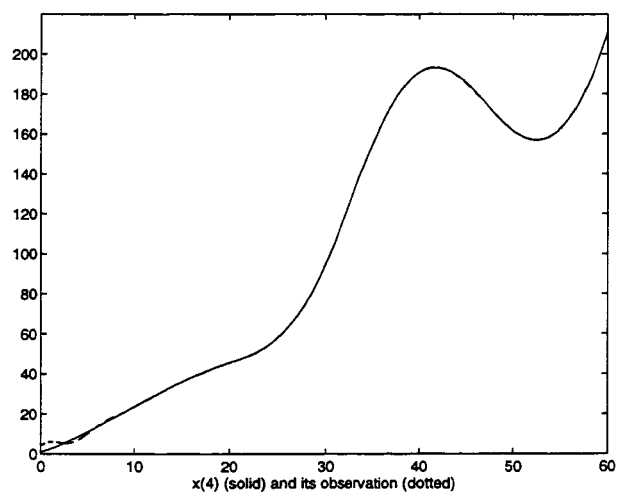
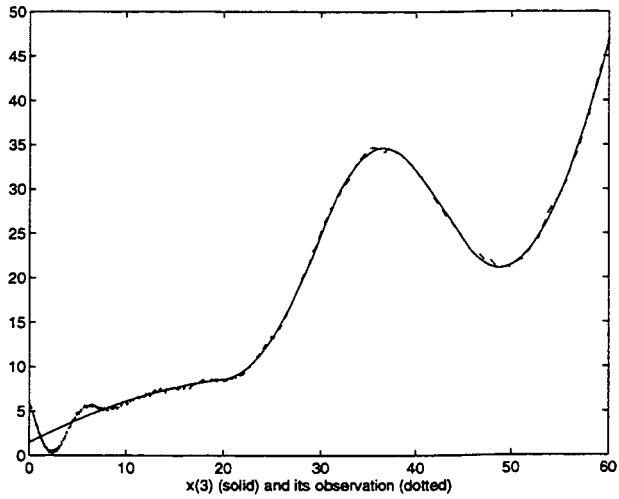
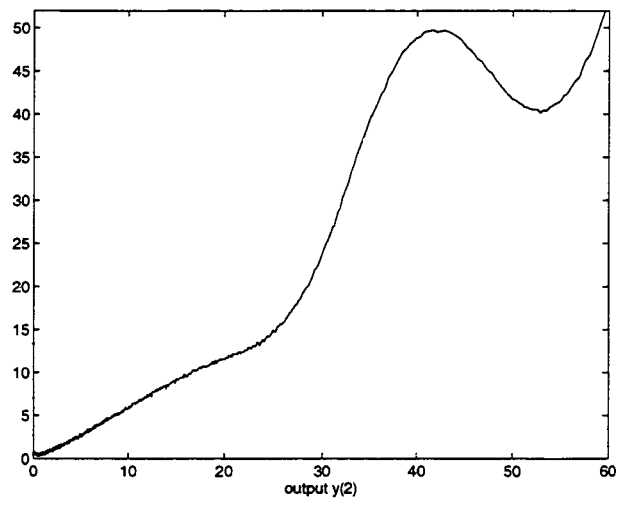
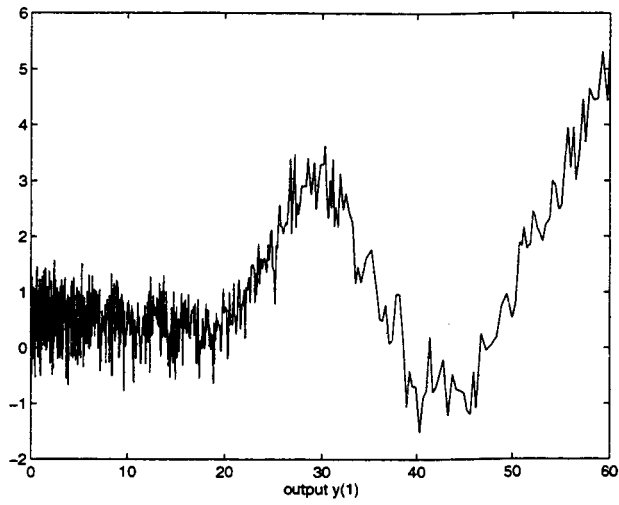
The first six sub-plots in Figure 3.2 correspond to the same plots in Figure 3.1. Again it is clear that the bilinear observer successfully estimates the correct value of the states that are not measurable, even when there are faults in the system. The last two additional sub-plots in Figure 3.2 show the estimates of the actuator and sensor faults. Examining the plot that is labeled as “actuator fault” clearly illustrates how and when the actuator fault is detected. Ordinarily, the value of this estimate would be monitored against a threshold and once it passes the threshold, actuator fault is declared. Based on this, it is clear that the actuator fault first occurs at $t = 20$ seconds. Similarly, the figure labeled “sensor fault” reveals that the sensor failure occurs at $t = 40$ seconds.

Figure 3.3 shows the result of simulation when the faults occur at different times and are described as (where $E_s = [0, 1, 1]^T$),

$$f_a = \begin{cases} 0 & \text{for } 0 \leq t \leq 20; \\ 0.3 + 0.1rand & \text{for } 20 < t \leq 30; \\ -0.4 + 0.1rand & \text{for } 30 < t \leq 40; \\ 0 & \text{for } 40 < t \leq 80. \end{cases}$$

$$f_s = \begin{cases} 0 & \text{for } 0 \leq t \leq 50; \\ 1 + 0.5rand + 0.1 \sin(t) & \text{for } 50 < t \leq 80. \end{cases}$$

Again, Figure 3.3 indicates that the observer is successfully estimating the states and the faults can be detected based on monitoring their estimates. In summary then, it can be seen from Figures 3.2 and 3.3, that the algorithm successfully detects and identifies the failures and their shapes.



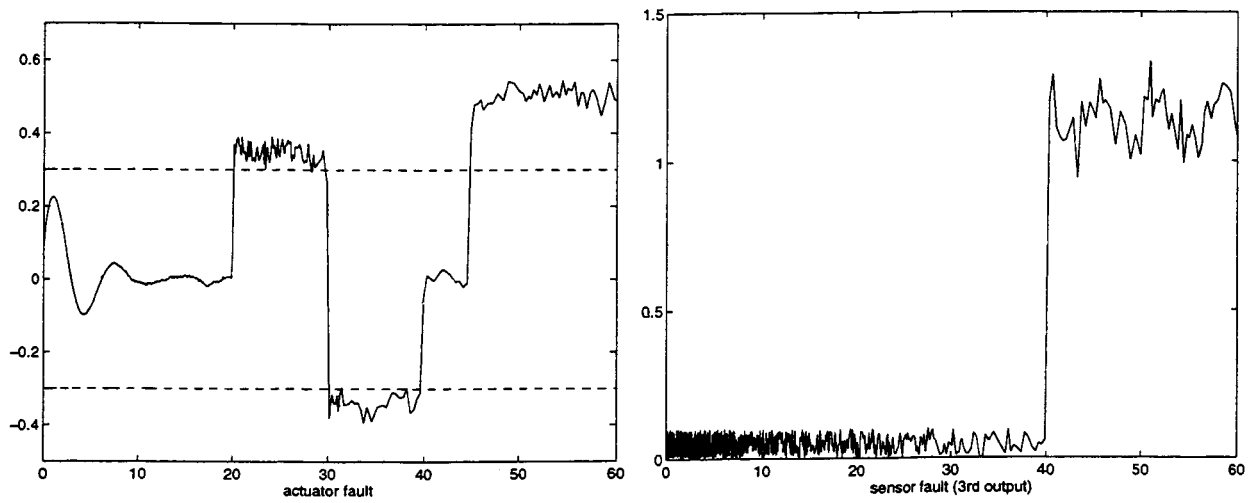
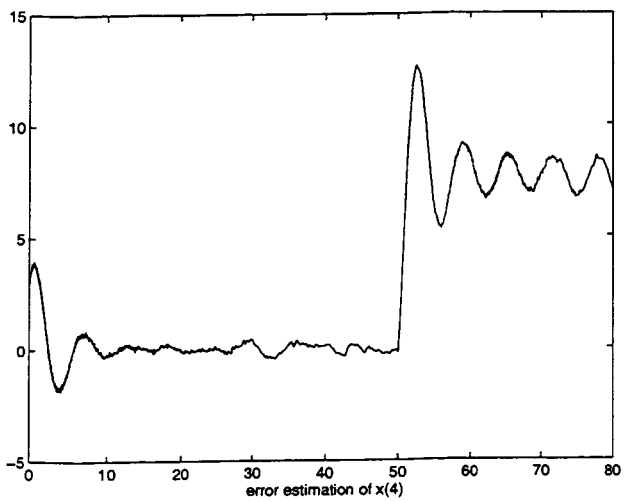
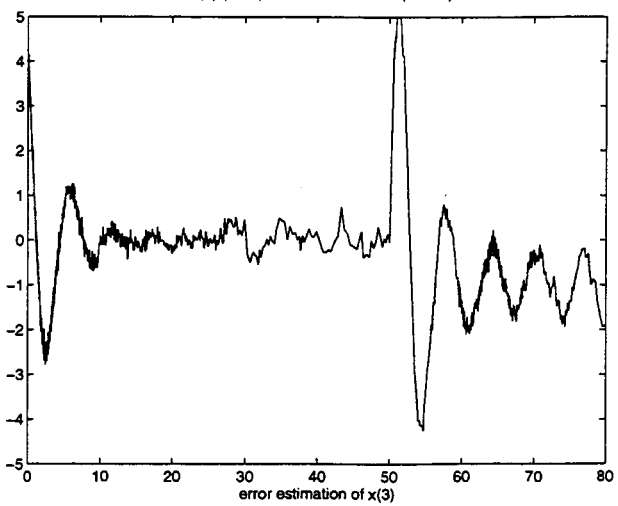
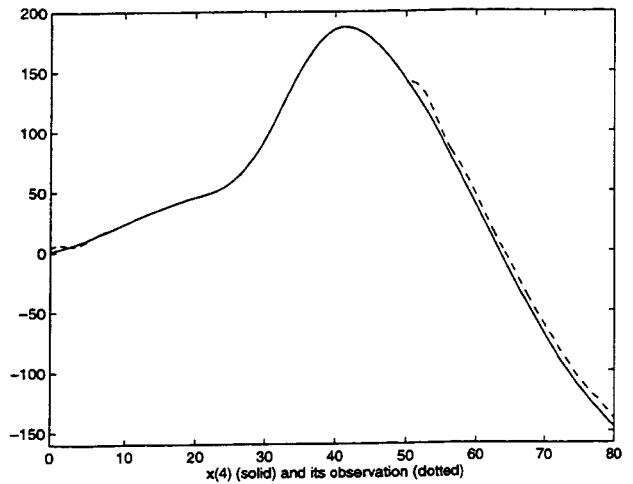
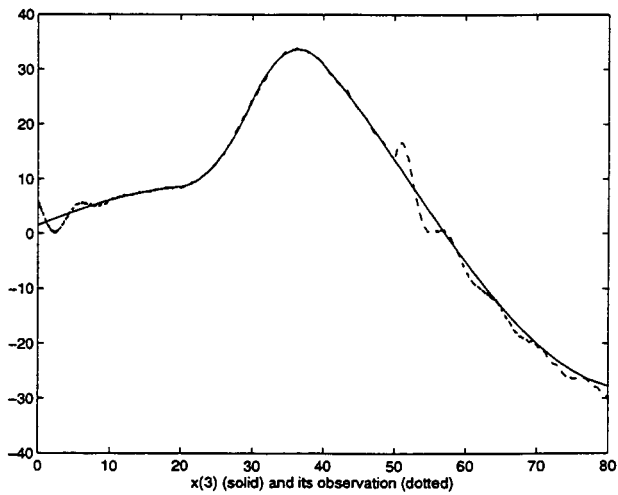
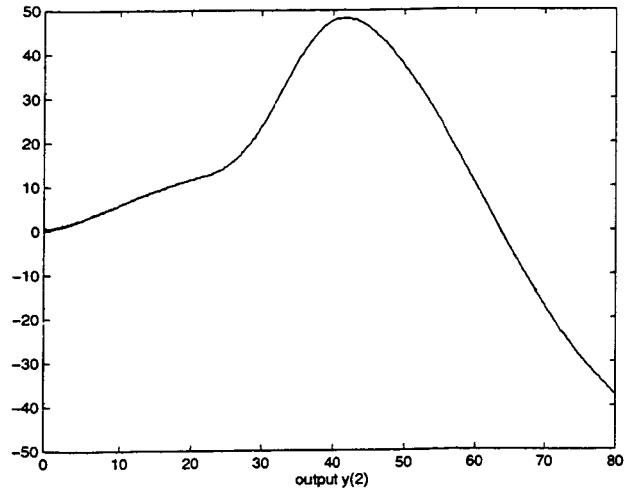
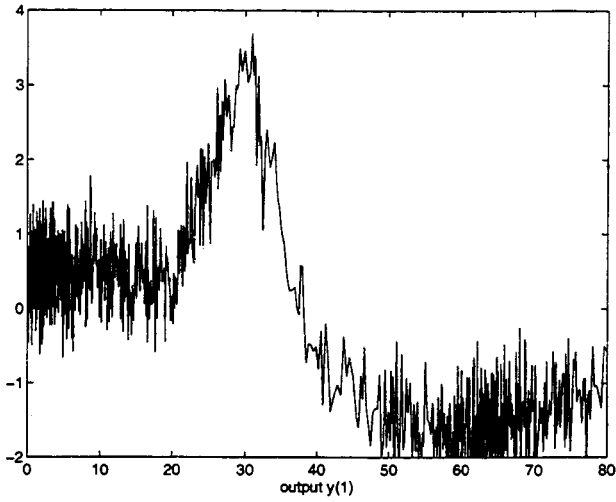


Figure 3.2: Hydraulic Drive Observer Design with Actuator and Sensor Faults (3rd output)

□

Example 3.4.2 Consider a bilinear system as described in (3.1) with the numerical value of the matrices given in the following,

$$A = \begin{bmatrix} -4 & -2 & 0.5 & 0 & 0 & 1.2 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 3 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1.2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.02 & 0 \\ 0 & 0 & 0 & -0.4 & 0.14 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0.6 \end{bmatrix},$$



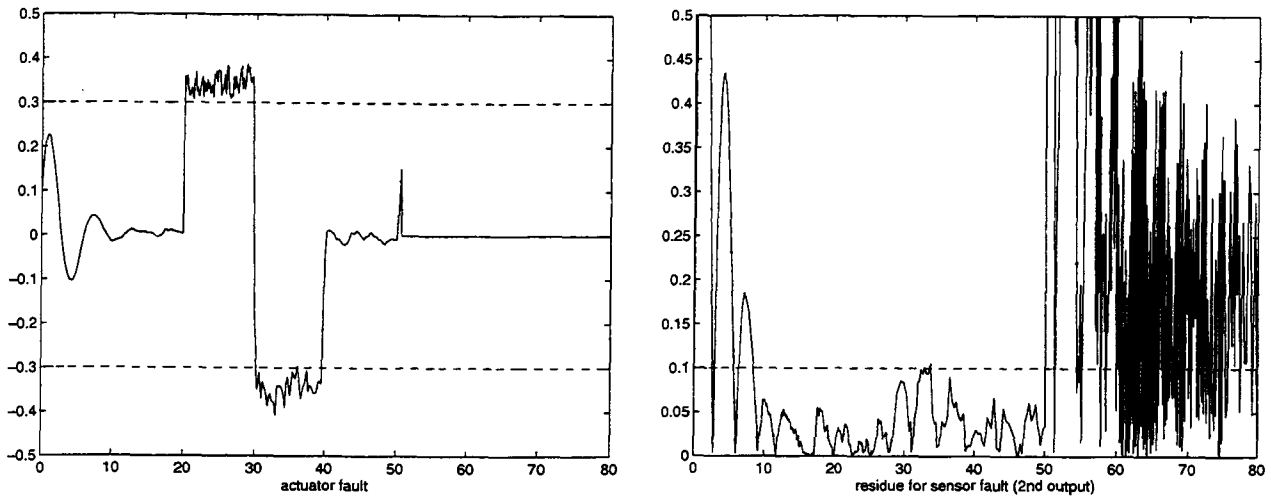


Figure 3.3: Hydraulic Drive Observer Design with Actuator and Sensor Faults (2nd output)

$$D^{(2)} = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1.2 & 0 \\ 0 & 0 \\ 3 & 1 \\ 0 & -2 \\ 0.5 & -0.5 \\ 1 & -1.5 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad E_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \left(\text{or } \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right).$$

It is easy to verify that the Assumptions 3.2.2–3.2.3 are satisfied. Also, it can be verified that with the observer gain given as $K = [6, 13, 10]^T$, the solution P_0 of Lyapunov equation $\tilde{A}_{33}^T P_0 + P_0 \tilde{A}_{33} = -I$ has maximum eigenvalue 2.9123, i.e.,

$\lambda_{\max}(P_0) = 2.9123$. Furthermore, given the inputs

$$u_1 = 0.5 \sin(1.5t), \quad u_2 = 0.5 \cos(1.5t),$$

it can be verified that condition (3.11) is also satisfied. It is therefore possible to detect and isolate both actuators as well as some sensor faults in this system.

Following the procedures introduced before, we can estimate states x_4, x_5, x_6 , and the faults as well.

Figure 3.4 illustrates the state variables along with their estimates as well as the estimation errors. It can also be seen that the estimation errors converge to zero in a fairly short time.

Figure 3.5 illustrates the results for the case where the actuators as well as the fourth sensor fail. Although uncommon in practice, for the purpose of illustration in this case, the actuator and sensor failures happen simultaneously. The faults that were simulated for the purpose of this test are given as

$$f_{a1} = \begin{cases} 0 & \text{for } 0 \leq t \leq 3; \\ 14 + 0.1rand & \text{for } 3 < t \leq 4; \\ 0 & \text{for } 4 < t \leq 5; \\ 13 + 0.1rand & \text{for } 5 < t \leq 6. \end{cases} \quad f_{a2} = \begin{cases} 0 & \text{for } 0 \leq t \leq 3.5; \\ -13 + 0.1rand & \text{for } 3.5 < t \leq 4.5; \\ 0 & \text{for } 4.5 < t \leq 5; \\ 14 + 0.1rand & \text{for } 5 < t \leq 6. \end{cases}$$

$$f_s = \begin{cases} 0 & \text{for } 0 \leq t \leq 4.2; \\ 8 + 0.2rand + 2 \sin(100t) \times rand & \text{for } 4.2 < t \leq 6. \end{cases}$$

Next we considered the possibility of both actuator as well as the third sensor failure. As opposed to the previous case, here fault detection and isolation is possible as long as the actuator and sensor faults do not happen simultaneously. In Figure 3.6,

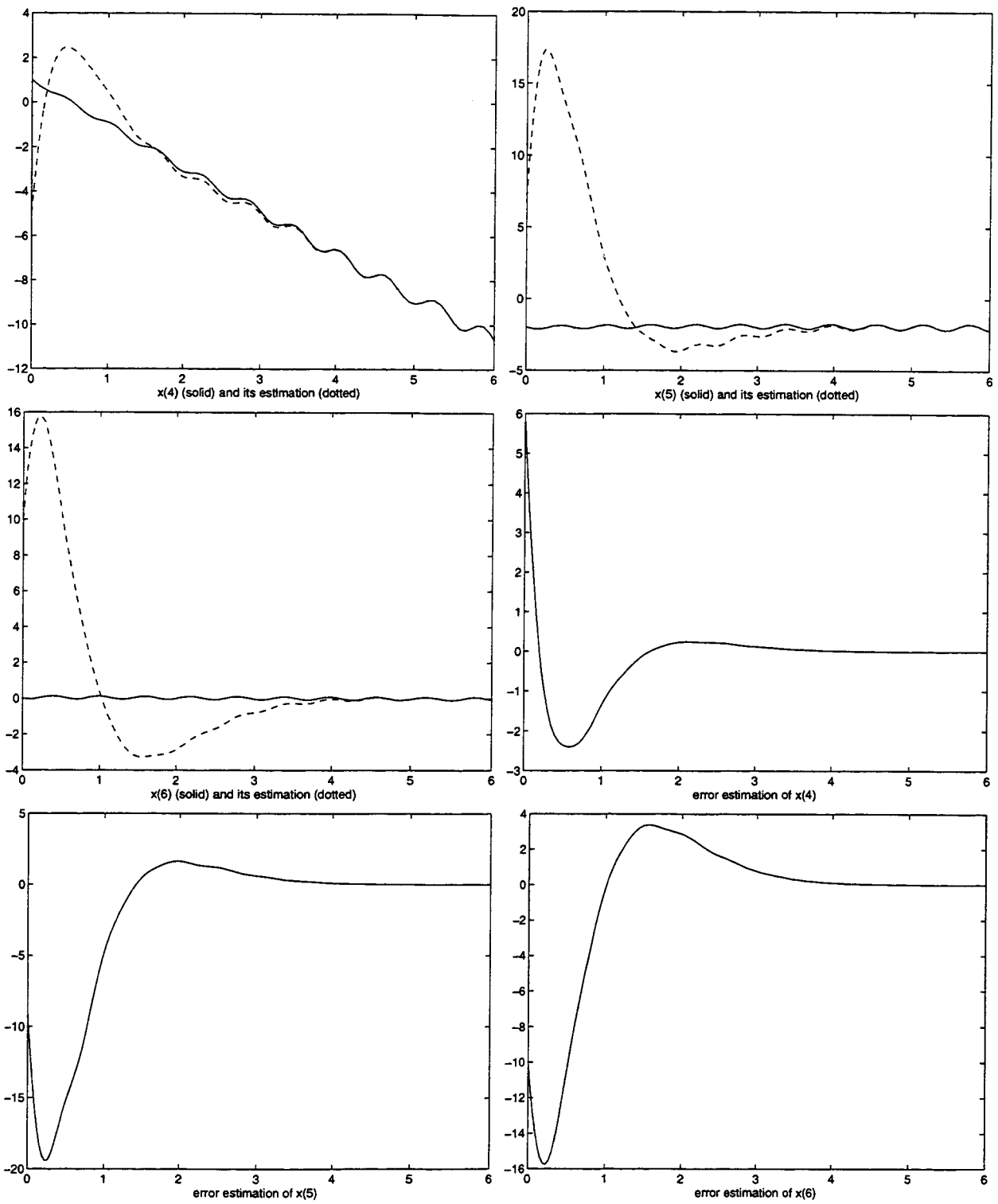
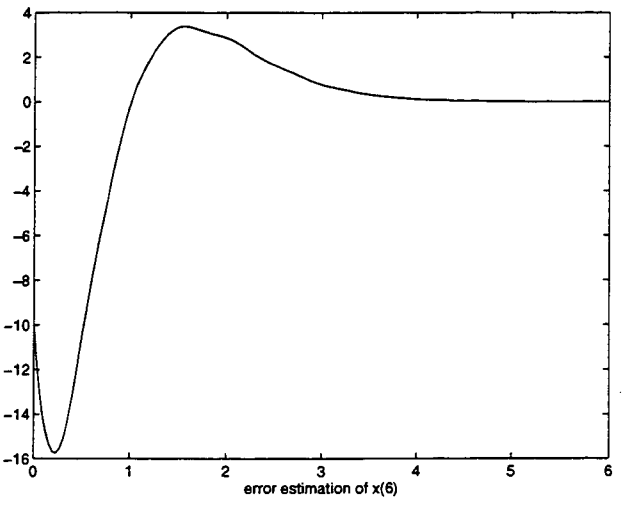
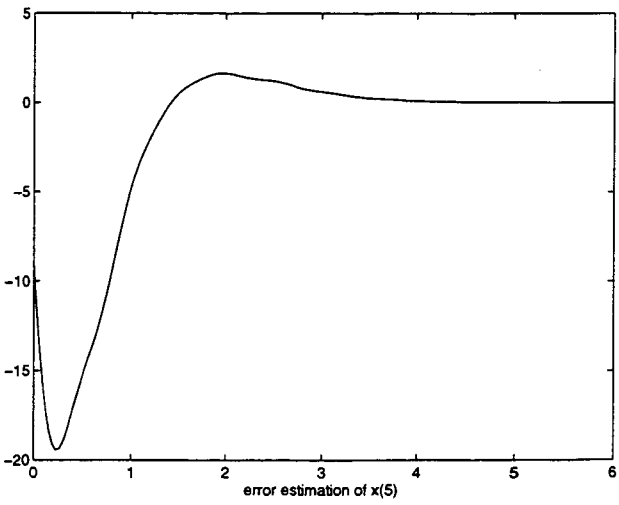
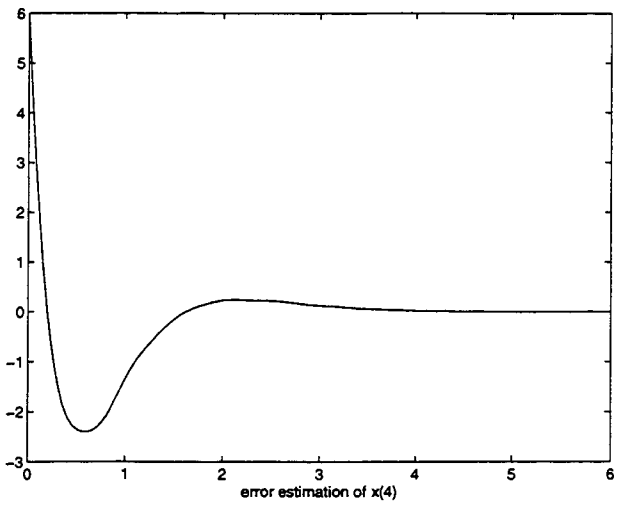
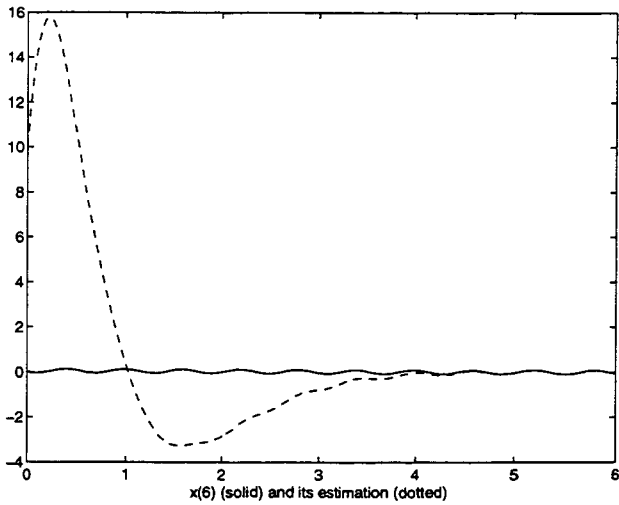
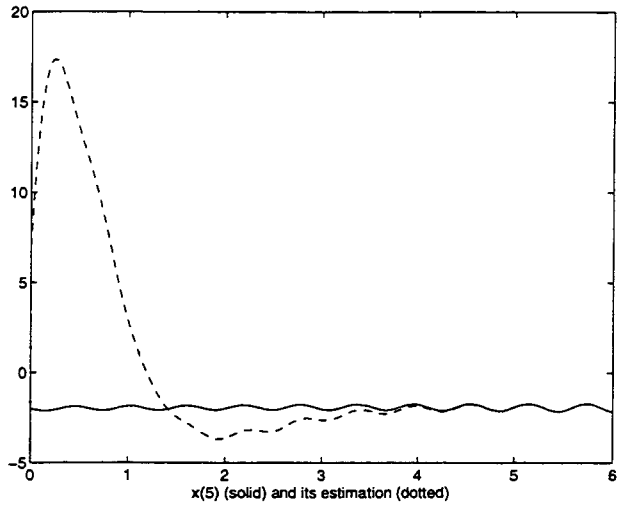
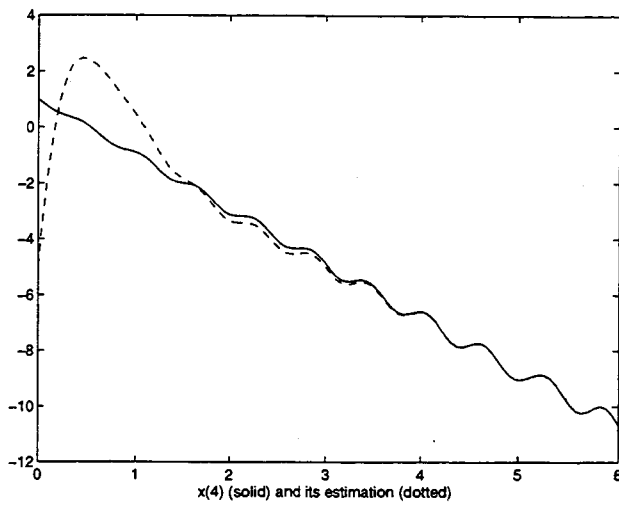


Figure 3.4: Bilinear System Observer Design without any Faults



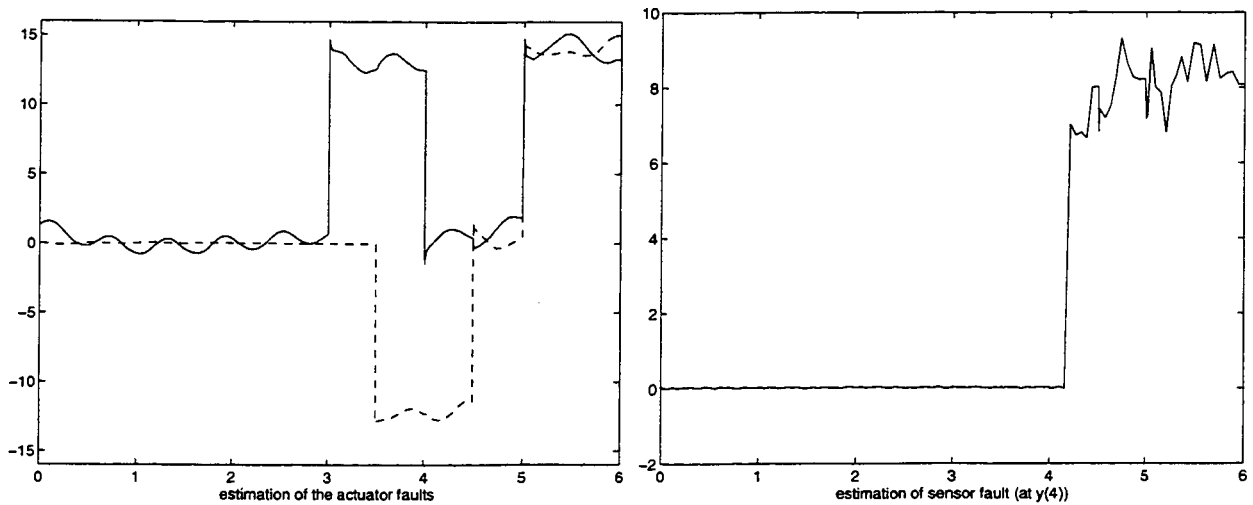


Figure 3.5: Bilinear System Observer Design with Actuator and Sensor (4th output) Faults

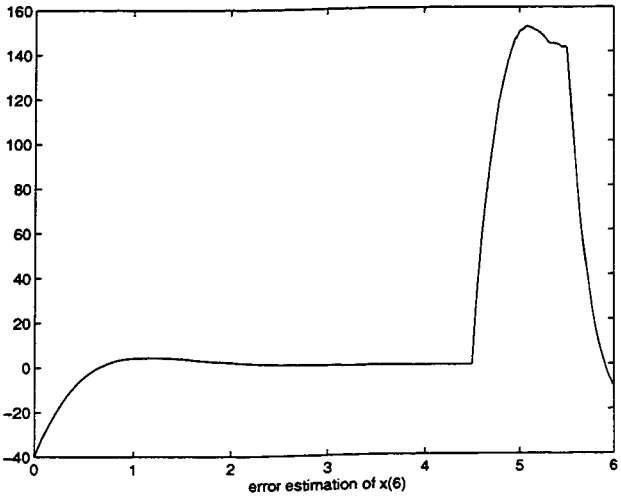
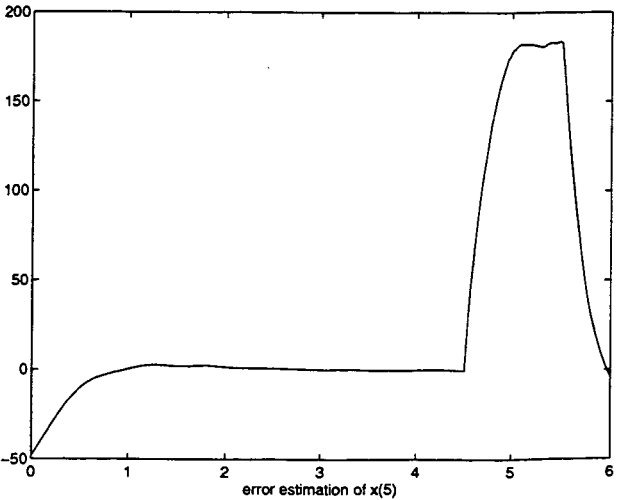
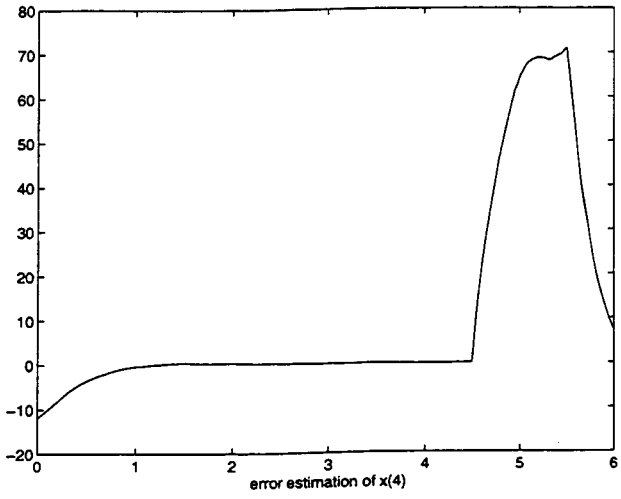
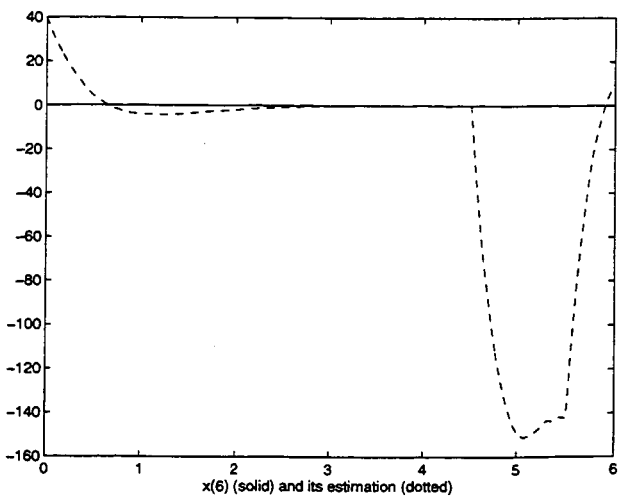
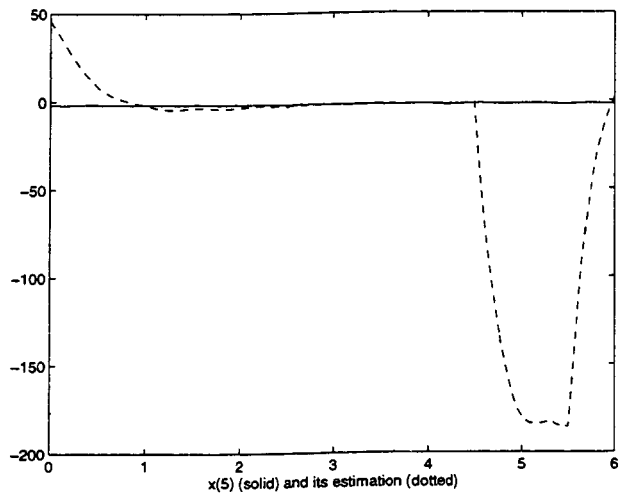
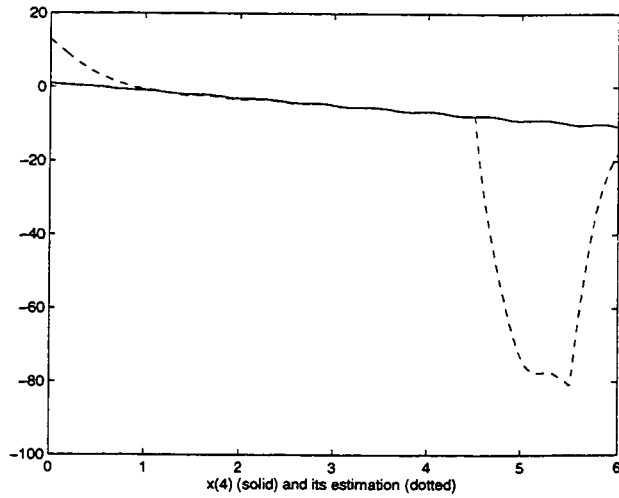
fault detection and isolation result is illustrated. It can be seen that we can detect this sensor fault as well as other actuator faults.

Once again the faults that were used in the simulation are given by,

$$f_{a1} = \begin{cases} 0 & \text{for } 0 \leq t \leq 3; \\ 14 + 0.1rand & \text{for } 3 < t \leq 4; \\ 0 & \text{for } 4 < t \leq 6. \end{cases} \quad f_{a2} = \begin{cases} 0 & \text{for } 0 \leq t \leq 3.5; \\ -13 + 0.1rand & \text{for } 3.5 < t \leq 4.5; \\ 0 & \text{for } 4.5 < t \leq 6. \end{cases}$$

$$f_s = \begin{cases} 0 & \text{for } 0 \leq t \leq 4.5; \\ 12 + 0.5rand + \sin(10t \times rand) & \text{for } 4.5 < t \leq 5.5; \\ 0 & \text{for } 5.5 < t \leq 6. \end{cases}$$

□



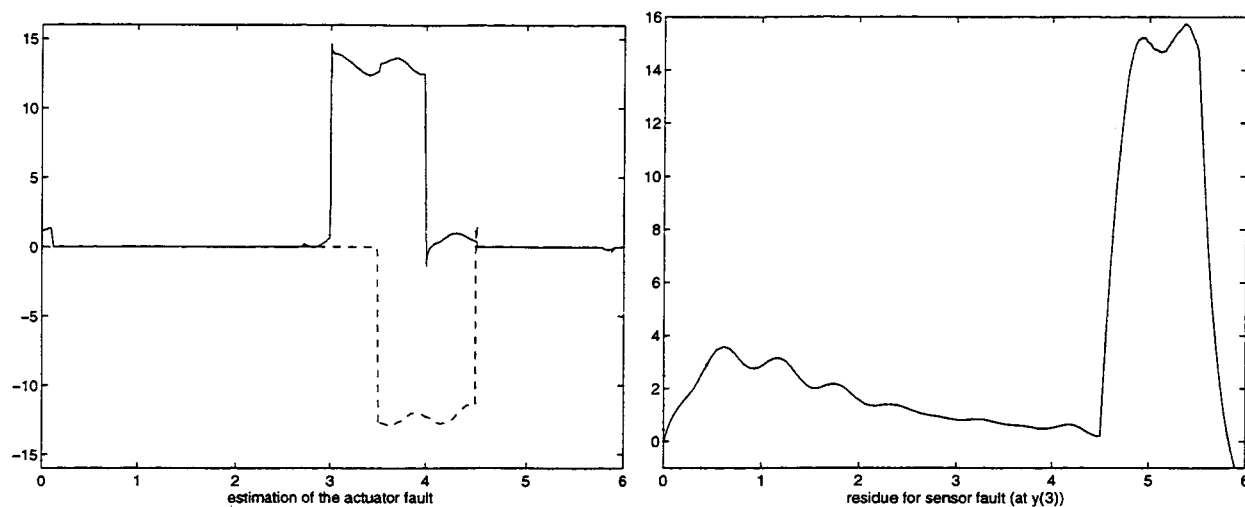


Figure 3.6: Bilinear System Observer Design with Actuator and Sensor (3rd output) Faults

3.5 Summary

A simple approach for designing unknown input bilinear observers was presented in this chapter. The observer design was then extended for fault diagnostic purposes in bilinear systems. The fault diagnostic approach that was proposed in this chapter uses a single observer to detect and identify sensor and actuator faults. As a result, some rather stringent conditions need to be satisfied for the possible use of single observer in FDI. However, generally detection and isolation of all instrument faults with a single estimator is not possible. In such cases, the requirements could perhaps be relaxed if multiple observers were to be used rather than a single observer that is used in the current scheme.

Chapter 4

Robust Estimation and Fault

Diagnostics in Time Delay Systems

In this chapter, we propose a reduced order observer, for state estimation in a class of state delayed dynamical systems driven by known as well as unknown inputs. Conditions for the existence of the proposed observer, along with the stability and convergence proof for the observer based on the Razumikhin Theorem are given. Additionally, the proposed observer is utilized in an analytical redundancy based approach for sensor and actuator failure detection problem. Finally, the applicability and effectiveness of the proposed FDI scheme is illustrated by numerical examples.

4.1 Introduction

Time delay is an inherent property of many physical systems—rolling mills, chemical processes, water resources, biological, economic and traffic control systems to name a few. Time delays whether inherently present or as a result of feedback are troublesome

in that they could cause oscillations or instability in the system. As a result, a great deal of studies have been performed on the subject of stability, control, and state estimation for time delay systems over the years.

In recent years, due to the increased complexity of the industrial systems, as well as the need for reliability, safety, and efficient operation of industrial systems, a great deal of attentions have been focused on the subject of fault detection and isolation (FDI) in dynamical systems. In general, most of these studies have concentrated on linear systems [14], [41], some considering bilinear and nonlinear systems [57, 62]. One of the objectives of this chapter is to address the FDI problem in time delay systems which to our knowledge has not been considered before.

The proposed FDI approach belongs to the class of analytical redundancy based schemes, where the mathematical model of the system along with the input and output information are used to generate redundant information about the system. This redundant information is then used for FDI purposes. Under the general umbrella of analytical redundancy based schemes there are two main approaches to FDI: 1) parity based schemes, and 2) observer based schemes [14]. The proposed approach of this chapter belongs to the second class. As a result, in this chapter we design an observer for time delay system driven by known as well as unknown inputs. Next, we will present an approach that utilizes the input, output, and the state estimate to detect and isolate sensor and actuator faults.

There exist limited results on designing observers for time delay systems, and almost no results for time delay systems with unknown inputs [35], [46], and [20]. Some of the existing results require certain conditions which may not be easily satisfied [35]. In this chapter, we have taken a similar approach as in [18], [40], [57] for designing an

unknown input observer (UIO) for the retarded system. The observer will provide an asymptotically converging estimate of the state of the time delay system under the presence of completely unknown inputs, which can include actuator faults as well as other effects such as disturbances and higher order nonlinearities.

This chapter is arranged as follows: Section 4.2 gives the preliminary assumptions and unknown input observer design. Section 4.3 presents the observer-based fault detection and isolation approach. In section 4.4, illustrative examples are given and finally, the conclusion is included in section 4.5.

4.2 Mathematical Preliminaries

In this section we shall present a brief overview of the stability analysis of retarded functional differential equations (RFDE). The treatment is based upon the Razumikhin-type theorems [19].

Consider the RFDE described by

$$\dot{x}(t) = f(t, x_t) \tag{4.1}$$

with the initial condition

$$x(t) = \phi(t); \quad t \in [-r, 0], r > 0$$

and

$$x_t \triangleq x(t + \theta); \quad \theta \in [-r, 0]$$

where $f : \mathbb{R} \times \mathbb{C} \rightarrow \mathbb{R}^n$ is continuous, and $\mathbb{C} = \mathbb{C}([-r, 0], \mathbb{R}^n)$ is the Banach space of all the continuous functions mapping from $[-r, 0] \rightarrow \mathbb{R}^n$. For $\phi \in \mathbb{C}$, $\|\phi\| \triangleq$

$\sup_{-r \leq \theta \leq 0} \|\phi(\theta)\|$, $\phi(\theta) \in \mathbb{R}^n$, where $\|\cdot\|$ on the right hand side of the definition equation is the norm $\|\cdot\|_2$ in \mathbb{R}^n .

If $V : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ is a continuous function, then $\dot{V}(t, \phi(0))$, the derivative of V along the solution of a RFDE is defined to be

$$\dot{V}(t, \phi(0)) = \overline{\lim}_{h \rightarrow 0^+} \frac{1}{h} [V(t+h, x(t, \phi)(t+h)) - V(t, \phi(0))] \quad (4.2)$$

where $x(t, \phi)(\cdot)$ is the solution of the RFDE(f) through (t, ϕ) .

Razumikhin Theorem [19]: *Suppose that f takes $\mathbb{R} \times$ (bounded sets of C) into bounded sets of \mathbb{R}^n and consider the RFDE(f). Suppose $u, v, w, p : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ are continuous, nondecreasing functions with $u(s), v(s), w(s) > 0$, and $p(s) > s$ for $s > 0$, additionally, $u(0) = v(0) = 0$. If there is a continuous function $V : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}$ such that*

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|), \quad t \in \mathbb{R}, x \in \mathbb{R}^n \quad (4.3)$$

and

$$\dot{V}(t, \phi(0)) \leq -w(\|\phi(0)\|) \quad \text{if} \quad V(t+\theta, \phi(\theta)) < p(V(t, \phi(0))), \theta \in [-r, 0], \quad (4.4)$$

then the solution $x = 0$ of the RFDE(f) is uniformly asymptotically stable. If $u(s) \rightarrow \infty$ as $s \rightarrow \infty$, then the solution $x = 0$ is also a global attractor for the RFDE(f). \square

Remark 4.2.1 If u, v are monotonous increasing, then the condition in (4.4) can be changed. Assume there is a constant $\bar{p} > 1$, and $\|x(t+\theta)\| < \bar{p}\|x(t)\|$. Using (4.3), we obtain $v^{-1}(V(t+\theta, x(t+\theta))) \leq \|x(t+\theta)\| < \bar{p}\|x(t)\| \leq \bar{p}u^{-1}(V(t, x(t)))$, and $V(t+\theta, x(t+\theta)) < (v\bar{p}u^{-1}(V(t, x(t))))$. Taking $p = v(\bar{p}u^{-1})(\cdot)$ (from the definitions of $u, v, \bar{p}, p(s) > s$ for $s > 0$). So we have $V(t+\theta, \phi(\theta)) < p(V(t, \phi(0)))$.

Similarly, from $V(t+\theta, \phi(\theta)) < p(V(t, \phi(0)))$ and (3), we get $\|x(t+\theta)\| < u^{-1}(p(v(\|x(t)\|)))$, since u, v, p are nondecreasing, $p(s) > s$, so $u^{-1}(p(v(s))) >$

$u^{-1}(v(s)) \geq s$. Then $\exists \bar{p} > 1$, such that $\|x(t + \theta)\| < \bar{p}\|x(t)\|$. The condition in (3.4), can be changed to $\exists \bar{p} > 1, \|x(t + \theta)\| < \bar{p}\|x(t)\|$, and the Razumikhin Theorem will still hold. \square

In the remainder of this section, we will present necessary state transformation required for the design of the estimator. Additionally, we shall present existence conditions that need to be satisfied for the observer design.

Consider a linear time delay system described

$$\begin{aligned}\dot{\tilde{x}} &= \tilde{A}\tilde{x} + \sum_{i=1}^d \tilde{A}_i\tilde{x}(t + \theta_i) + \tilde{B}u + \tilde{E}f \\ y &= \tilde{C}\tilde{x}\end{aligned}\tag{4.5}$$

where input $u \in \mathbb{R}^q$, state $x \in \mathbb{R}^n$, unknown input $f \in \mathbb{R}^m$, output $y \in \mathbb{R}^p, p > m, \theta_i \in [-r, 0], r$ is a certain positive number, \tilde{E} and \tilde{C} are of full rank (i.e., $\text{Rank}(\tilde{C}) = p$ and $\text{Rank}(\tilde{E}) = m$). Also, it is assumed that $\text{Rank}(\tilde{C}\tilde{E}) = \text{Rank}(\tilde{E})$.

Lemma 4.2.2 There exist an equivalence transformation $x = T\tilde{x}$, where T is a nonsingular matrix that transforms the system (4.5) into

$$\begin{aligned}\dot{x} &= Ax + \sum_{i=1}^d A_i x(t + \theta_i) + Bu + Ef \\ y &= Cx\end{aligned}\tag{4.6}$$

given that $\text{Rank}(CE) = \text{Rank}(E)$. In this case, the transformed system matrices can be written in the following forms.

- A is in the form of

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & [A_{23}^1 \ 0] \\ A_{31} & A_{32} & \begin{bmatrix} A_{33}^{11} & 0 \\ A_{33}^{21} & A_{33}^{22} \end{bmatrix} \end{bmatrix}$$

where $A_{11} \in \mathbb{R}^{m \times m}$, $A_{21} \in \mathbb{R}^{(p-m) \times m}$, $A_{31} \in \mathbb{R}^{(n-p) \times m}$, $A_{12} \in \mathbb{R}^{m \times (p-m)}$, $A_{13} \in \mathbb{R}^{m \times (n-p)}$, $A_{22} \in \mathbb{R}^{(p-m) \times (p-m)}$, $A_{32} \in \mathbb{R}^{(n-p) \times (p-m)}$, $A_{23}^1 \in \mathbb{R}^{(p-m) \times r}$, $A_{33}^{11} \in \mathbb{R}^{r \times r}$, $A_{33}^{21} \in \mathbb{R}^{(n-p-r) \times r}$, $A_{33}^{22} \in \mathbb{R}^{(n-p-r) \times (n-p-r)}$ and also (A_{33}^{11}, A_{23}^1) is complete observable, and r is dimension of the observable space.

- E is of the form

$$\begin{bmatrix} E_1 \\ 0_{(n-m) \times m} \end{bmatrix}$$

where $E_1 \in \mathbb{R}^{m \times m}$ is nonsingular.

- C is of the form $C = [C_1 \ 0]$, C_1 is a $p \times p$ orthogonal matrix.

□

Proof. Without loss of generality, we assume $C = [I_{p \times p} \ 0]$, and partition $\tilde{E} = \begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_2 \end{bmatrix}$, where $\tilde{E}_1 \in \mathbb{R}^{p \times m}$, $\tilde{E}_2 \in \mathbb{R}^{(n-p) \times m}$. From $\text{Rank}(CE) = \text{Rank}(E)$, we know $\text{Rank}(\tilde{E}_1) = m$. So the generalized pseudo-inverse of \tilde{E}_1 given by \tilde{E}_1^+ exist, and $\tilde{E}_1^+ \triangleq (\tilde{E}_1^T \tilde{E}_1)^{-1} \tilde{E}_1^T$. Furthermore, there exists an orthogonal $p \times p$ matrix C_1 , such that $C_1^T \tilde{E}_1 = \begin{bmatrix} E_1 \\ 0_{(p-m) \times m} \end{bmatrix}$, where E_1 is $m \times m$ nonsingular matrix.

Then through the use of the following transformation,

$$T_1 = \begin{bmatrix} C_1^T & 0 \\ -\tilde{E}_2 \tilde{E}_1^+ & I \end{bmatrix},$$

the last two conditions are satisfied, i.e., E is of the form $\begin{bmatrix} E_1 \\ 0_{(n-m) \times m} \end{bmatrix}$, C is of the

form $[C_1 \ 0]$. Additionally, $A = T_1 \tilde{A} T_1^{-1}$ can be partitioned as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix},$$

$$A_{23} \in \mathbb{R}^{(p-m) \times (n-p)}, A_{33} \in \mathbb{R}^{(n-p) \times (n-p)}.$$

If (A_{33}, A_{23}) is partially observable and the observable space is of dimension r , then it can be partitioned by $(n-p) \times (n-p)$ coordinate transformation matrix T_2 to the following Kalman Decomposition form,

$$\left(\begin{bmatrix} A_{33}^{11} & 0 \\ A_{33}^{21} & A_{33}^{22} \end{bmatrix}, [A_{23}^1, 0] \right),$$

i.e., $T_2 A_{33} T_2^{-1} = \begin{bmatrix} A_{33}^{11} & 0 \\ A_{33}^{21} & A_{33}^{22} \end{bmatrix}$, $A_{23} T_2^{-1} = [A_{23}^1, 0]$, where (A_{33}^{11}, A_{23}^1) is completely observable, with dimension r .

By taking transformation $T = \begin{bmatrix} I_{p \times p} & 0 \\ 0 & T_2 \end{bmatrix} T_1$, we change the matrices $\tilde{A}, \tilde{E}, \tilde{C}$ in system (4.6) to the form described in the Lemma. This completes the proof. ■

Based on the Lemma 4.2.2, we may assume that the time-delay system (4.7) has the following structure,

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \\ x_3 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \sum_{i=1}^d A_i \begin{bmatrix} x_1(t + \theta_i) \\ x_2(t + \theta_i) \\ x_3(t + \theta_i) \end{bmatrix} + Bu + Ef_a \quad (4.7)$$

$$y = Cx = C_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & \begin{bmatrix} A_{23}^1 & 0 \end{bmatrix} \\ A_{31} & A_{32} & \begin{bmatrix} A_{33}^{11} & 0 \\ A_{33}^{21} & A_{33}^{22} \end{bmatrix} \end{bmatrix}, \quad A_i = \begin{bmatrix} A_{11}^{(i)} & A_{12}^{(i)} & A_{13}^{(i)} \\ A_{21}^{(i)} & A_{22}^{(i)} & A_{23}^{(i)} \\ A_{31}^{(i)} & A_{32}^{(i)} & A_{33}^{(i)} \end{bmatrix},$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}, \quad E = \begin{bmatrix} E_1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [C_1 \ 0].$$

where the dimensions of the sub-blocks of A_i are same as the corresponding ones of A , $B_1 \in \mathbb{R}^{m \times q}$, $B_2 \in \mathbb{R}^{(p-m) \times q}$, $B_3 \in \mathbb{R}^{(n-p) \times q}$, $E_1 \in \mathbb{R}^{m \times m}$ is nonsingular, C_1 is a $p \times p$ orthogonal matrix, x_1, x_2 and x_3 are the corresponding dimensional vectors.

Since $y = Cx = C_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and C_1 is a $p \times p$ nonsingular orthogonal matrix,

so that the states $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ can be calculated as $C_1^{-1}y$. In order to observe the states of the system (4.7), we only need to design observer for estimating x_3 . Generally

speaking x_3 may not be completely observable, assume $x_3 = \begin{bmatrix} x_3^o \\ x_3^u \end{bmatrix}$, $x_3^o \in \mathbb{R}^r$, $x_3^u \in$

\mathbb{R}^{n-p-r} , x_3^o is completely observable, x_3^u is unobservable. Correspondingly, we de-

fine $A_{31} = \begin{bmatrix} A_{31}^o \\ A_{31}^u \end{bmatrix}$, $A_{32} = \begin{bmatrix} A_{32}^o \\ A_{32}^u \end{bmatrix}$, $A_{31}^{(i)} = \begin{bmatrix} A_{31}^{(i)o} \\ A_{31}^{(i)u} \end{bmatrix}$, $A_{32}^{(i)} = \begin{bmatrix} A_{32}^{(i)o} \\ A_{32}^{(i)u} \end{bmatrix}$, $A_{33}^{(i)} =$

$$\begin{bmatrix} A_{33}^{(i)11} & A_{33}^{(i)12} \\ A_{33}^{(i)21} & A_{33}^{(i)22} \end{bmatrix}.$$

Before we propose an approach for designing an estimator for the time delay

the rank we have the following (details are skipped for brevity),

$$\text{Rank } (\mathcal{O}(A, C)) = \text{Rank} \begin{bmatrix} C_1 & 0 \\ 0 & \begin{bmatrix} A_{13} \\ A_{23} \end{bmatrix} \\ 0 & \begin{bmatrix} A_{13} \\ A_{23} \end{bmatrix} A_{33} \\ * & * \end{bmatrix}$$

so the first result is proved.

2) Refer to the system matrices in (8), we have,

$$\text{Rank} \begin{bmatrix} \lambda I - A & E \\ C & 0 \end{bmatrix} = \text{Rank} \begin{bmatrix} \lambda I - A_{11} & -A_{12} & -A_{13} & E_1 \\ -A_{21} & \lambda I - A_{22} & -A_{23}^1 & 0 & 0 \\ * & & \lambda I - A_{33}^{11} & 0 & 0 \\ * & & -A_{33}^{21} & \lambda I - A_{33}^{22} & 0 \\ C_1 & & 0 & 0 & 0 \end{bmatrix}$$

by using fundamental transformation, we calculate that the above rank is

$$\text{Rank} \begin{bmatrix} 0 & 0 & E_1 \\ 0 & -A_{23}^1 & 0 & 0 \\ 0 & \lambda I - A_{33}^{11} & 0 & 0 \\ 0 & 0 & \lambda I - A_{33}^{22} & 0 \\ C_1 & 0 & 0 & 0 \end{bmatrix} = p + m + r + \text{Rank} [\lambda I - A_{33}^{22}],$$

so that $\text{Rank} \begin{bmatrix} \lambda I - A & E \\ C & 0 \end{bmatrix}$ decreases at $\lambda \in \text{eig}(A_{33}^{22})$. The proof is complete. ■

As a matter of fact, this is a generalization of Proposition 2.2.1.

Remark 4.2.3 As long as $\text{eig}(A_{33}^{22}) \in \mathbb{C}^-$, then the pair (A, C) is detectable. \square

Corollary 4.2.4 For time-delay system (4.6), the following conditions are equivalent,

1. $\text{Rank} \begin{bmatrix} \lambda I - \tilde{A} & \tilde{E} \\ \tilde{C} & 0 \end{bmatrix} = n + m, \quad \forall \lambda \in \mathbb{C}.$
2. For system (4.7), $\dim \mathcal{O}(A_{33}^{11}, A_{23}^1) = r = n - p$, i.e., (A_{33}, A_{23}) is completely observable.

Proof. From claim 1, $\text{Rank} \begin{bmatrix} \lambda I - \tilde{A} & \tilde{E} \\ \tilde{C} & 0 \end{bmatrix} = n + m$ for $\tilde{A}, \tilde{C}, \tilde{E}$ in system (4.5). Since

$$\begin{bmatrix} T & \\ & I \end{bmatrix} \begin{bmatrix} \lambda I - \tilde{A} & \tilde{E} \\ \tilde{C} & 0 \end{bmatrix} \begin{bmatrix} T^{-1} & \\ & I \end{bmatrix} = \begin{bmatrix} \lambda I - T\tilde{A}T^{-1} & T\tilde{E} \\ \tilde{C}T^{-1} & 0 \end{bmatrix},$$

so for system (4.7)

as well as system (4.5), $\begin{bmatrix} \lambda I - A & E \\ C & 0 \end{bmatrix}$ has constant rank $n + m$, i.e., the dimension of A_{33}^{22} shrinks to zero, so that (A_{33}, A_{23}) is completely observable, and as a result claim 2 is valid.

Similarly, from claim 2 we can prove that claim 1 valid. \blacksquare

4.3 Reduced Order Unknown Input Observer Design

In this section, we will study the design of an unknown input observer for a class of time delay systems. The fact that the proposed observer can provide correct estimate of the state of the time delay system in spite of the presence of totally unknown

inputs, makes it of general interests in variety of applications. However as mentioned before, the observer will be used here for fault detection and diagnosis purposes.

In order to construct an observer for the above system, we shall require the following assumptions to be valid.

Assumption 4.3.1 $A_{33}^{(i)12} = 0, i = 1, \dots, d.$ □

Assumption 4.3.2 $A_{23}^{(i)} = 0, i = 1, \dots, d.$ □

Remark 4.3.1 It should be noted that in practice there may be only few state variables which may be delayed. Therefore in general, matrices \tilde{A}_i may be very sparse. As a result, the above assumptions are not too restrictive. □

In this case, the dynamics of x_3 is described by the following equations,

$$\begin{aligned} \dot{x}_3^o &= A_{31}^o x_1 + A_{32}^o x_2 + A_{33}^{11} x_3^o + B_3^o u \\ &\quad + \sum_{i=1}^d [A_{31}^{(i)o} x_1(t + \theta_i) + A_{32}^{(i)o} x_2(t + \theta_i) + A_{33}^{(i)11} x_3^o(t + \theta_i)] \quad (4.8) \\ \dot{x}_3^u &= A_{31}^u x_1 + A_{32}^u x_2 + A_{33}^{21} x_3^o + A_{33}^{22} x_3^u + B_3^u u \\ &\quad + \sum_{i=1}^d [A_{31}^{(i)u} x_1(t + \theta_i) + A_{32}^{(i)u} x_2(t + \theta_i) + A_{33}^{(i)21} x_3^o(t + \theta_i) + A_{33}^{(i)22} x_3^u(t + \theta_i)] \quad (4.9) \end{aligned}$$

For the system (4.8), we can design observer for x_3^o as follows,

$$\begin{aligned} \dot{\hat{x}}_3^o &= A_{31}^o x_1 + A_{32}^o x_2 + A_{33}^{11} \hat{x}_3^o + B_3^o u \\ &\quad + \sum_{i=1}^d [A_{31}^{(i)o} x_1(t + \theta_i) + A_{32}^{(i)o} x_2(t + \theta_i) + A_{33}^{(i)11} \hat{x}_3^o(t + \theta_i)] \\ &\quad + K(A_{23}^1 x_3^o(t) - A_{23}^1 \hat{x}_3^o(t)). \quad (4.10) \end{aligned}$$

Notice that the above observer uses the information of $A_{23}^1 x_3^o$ which can not be measured directly. As a result, by introducing

$$w(t) = x_3^o(t) - Kx_2(t) = x_3^o(t) - K[0_{(p-m) \times m} I_{(p-m) \times (p-m)}] C_1^T y,$$

we may have (4.10) expressed in the following form,

$$\begin{aligned} \dot{w}(t) = & (A_{33}^{11} - KA_{23}^1)w(t) + \sum_{i=1}^d A_{33}^{(i)o} w(t + \theta_i) + (B_3 - KB_2)u \\ & + [A_{31}^o - KA_{21}^1, (A_{33}^{11} - KA_{23}^1)K + A_{32}^o - KA_{22}]C_1^T y \\ & + \sum_{i=1}^d [A_{31}^{(i)o} - KA_{21}^{(i)}, A_{33}^{(i)11}K + A_{32}^{(i)o} - KA_{22}^{(i)}]C_1^T y(t + \theta_i) \end{aligned} \quad (4.11)$$

In which case, the error dynamics of $e_3^o \triangleq \hat{x}_3^o - x_3^o$ is

$$\dot{e}_3^o(t) = (A_{33}^{11} - KA_{23}^1)e_3^o(t) + \sum_{i=1}^d A_{33}^{(i)11} e_3^o(t + \theta_i). \quad (4.12)$$

The following theorem will establish conditions under which the observer's error dynamics would converge to zero asymptotically.

Theorem 4.2 *The estimator in (4.11) is an asymptotical observer for the partial state x_3^o of system (4.7) given that Assumptions 4.3.1 and 4.3.2, and the following condition hold*

$$\sum_{i=1}^d \|A_{33}^{(i)11}\|_2 \leq \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} \quad (4.13)$$

where $\lambda_{\max}(P)$ is the maximum eigenvalue of P , $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q , and $Q > 0$, P is the positive definite matrix solution of

$$P(A_{33}^{11} - KA_{23}^1) + (A_{33}^{11} - KA_{23}^1)^T P = -2Q \quad (4.14)$$

Proof. Given that the pair (A_{33}^{11}, A_{23}^1) is observable, there exists matrix K , such that $A_{33}^{11} - KA_{23}^1$ is Hurwitz. Assuming that one such K is used, consider the estimator's error dynamics given by (4.12), and choose a Lyapunov function given by

$$V(e_3^o) = e_3^{oT} P e_3^o$$

By using (4.14), we have

$$\begin{aligned}\dot{V}(e_3^o) &= \dot{e}_3^{oT} P e_3^o + e_3^{oT} P \dot{e}_3^o \\ &= e_3^{oT} [P(A_{33}^{11} - K A_{23}^1) + (A_{33}^{11} - K A_{23}^1)^T P] e_3^o + 2e_3^{oT} P \left(\sum_{i=1}^d A_{33}^{(i)11} e_3^o(t + \theta_i) \right) \\ &\leq -2\lambda_{\min}(Q) \|e_3^o(t)\|^2 + 2\|e_3^o(t)\| \cdot \lambda_{\max}(P) \cdot \sum_{i=1}^d (\|e_3^o(t + \theta_i)\| \cdot \|A_{33}^{(i)11}\|).\end{aligned}$$

If $\|e_3^o(t + \theta_i)\| \leq \bar{p}\|e_3^o(t)\|$, then

$$\dot{V}(e_3^o) \leq -2(\lambda_{\min}(Q) - \lambda_{\max}(P) \cdot \left(\sum_{i=1}^d \|A_{33}^{(i)11}\| \right) \cdot \bar{p}) \cdot \|e_3^o(t)\|^2.$$

As long as

$$\sum_{i=1}^d \|A_{33}^{(i)11}\| < \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)},$$

we can choose $1 < \bar{p} < \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)(\sum_{i=1}^d \|A_{33}^{(i)11}\|)}$, so that the condition $\dot{V} \leq 0$ is satisfied.

Furthermore, equation (4.13) will also guarantee nonincreasing behavior of V . Finally, based upon the Razumikhin Theorem it is concluded that the system (4.12) is asymptotically stable.

Now we know that $w + K[0_{(p-m) \times m} \ I_{(p-m) \times (p-m)}] C_1^T y$ can asymptotically approximate the partial states x_3^o , so that state $\begin{bmatrix} x_1 \\ x_2 \\ x_3^o \end{bmatrix}$ can be estimated and is given by

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3^o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix} + \begin{bmatrix} I_{p \times p} \\ \left[\begin{array}{cc} 0 & K \end{array} \right] \end{bmatrix} C^T y.$$

This completes the proof. ■

The following result will relax the condition (4.13) in the above theorem.

Proposition 4.3.2 [29] The ratio $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$ reaches its best upper bound, if the matrix Q is an identity. \square

To show this, assume $Q > 0$, A is Hurwitz, P_1 and P_2 are the positive definite matrix solutions for the following two Lyapunov equations,

$$P_1 A + A^T P_1 = -Q, \quad (4.15)$$

$$P_2 A + A^T P_2 = -(\lambda_{\min}(Q) - \epsilon)I. \quad (4.16)$$

where ϵ is a small enough positive number which allows the right term of (4.15) remain positive definite. Subtracting (4.16) from (4.15), we get

$$(P_1 - P_2)A + A^T(P_1 - P_2) = -(Q - \lambda_{\min}(Q)I + \epsilon I).$$

The solution to the above is given by $P_1 - P_2 = \int_0^\infty e^{A^T t} (Q - \lambda_{\min}(Q)I + \epsilon I) e^{At} dt > 0$ which would imply that $\lambda_{\max}(P_1) > \lambda_{\max}(P_2)$. So as $\epsilon \rightarrow 0$, we conclude that

$$\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P_1)} \leq \sup_{\epsilon \rightarrow 0} \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P_2)} = \frac{1}{\lambda_{\max}(P)}$$

where P is the unique solution of the Lyapunov equation $PA + A^T P = -2I$.

4.4 Robust Unknown Input Observer for Time Delay Systems

Consider the system of the following form,

$$\begin{aligned} \dot{x} &= Ax + \sum_{i=1}^d A_i x(t + \theta_i(t)) + Bu + H(t, x, x_t) \\ y &= Cx \end{aligned} \quad (4.17)$$

where input $u \in \mathbb{R}^q$, state $x \in \mathbb{R}^n$, output $y \in \mathbb{R}^p$, $\theta_i \in [-r, 0]$, $i = 1, \dots, d$ is time-delay, H is unknown term composed of model uncertainties, high order nonlinearities, and disturbances.

Before we design robust observer, we need the following assumptions.

Assumption 4.4.1 (A, C) is observable.

Assumption 4.4.2 $\|H(t, x, x_i)\| \leq \alpha$, where α is a positive constant.

Theorem 4.3 Let the Assumptions 4.4.1-4.4.2 be true, and

$$\beta \triangleq \lambda_{\min}(Q) - 2 \sum_{i=1}^d \|PA_i\| > 0, \quad (4.18)$$

then we can design the following practical stable observer,

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \sum_{i=1}^d A_i\hat{x}(t + \theta_i) + Bu + K(\hat{y} - y) \\ \hat{y} &= C\hat{x} \end{aligned} \quad (4.19)$$

such that

$$\|\tilde{x}\| \leq 2 \frac{\lambda_{\max}(P)^{\frac{3}{2}} \alpha}{\lambda_{\min}(P)^{\frac{1}{2}} \beta} \quad (4.20)$$

where $\tilde{x} \triangleq x - \hat{x}$ and $\lambda_{\max(\min)}(\cdot)$ is maximum (minimum) eigenvalue of the matrix.

$Q > 0$, P is the positive definite matrix solution of

$$P(A - KC) + (A - KC)^T P = -Q \quad (4.21)$$

□

Proof. We only give a brief outline of the proof.

From equation (4.17) and observer (4.19), we have error dynamics described by,

$$\dot{\tilde{x}} = (A - KC)\tilde{x} + \sum_{i=1}^d A_i\tilde{x}(t + \theta_i) + H. \quad (4.22)$$

Consider Lyapunov candidate function, $V \triangleq \tilde{x}^T P \tilde{x}$, where P is positive definite, a solution of (4.21).

$$\begin{aligned} \dot{V} &= -\tilde{x}^T Q \tilde{x} + 2\tilde{x}^T P \left(\sum_{i=1}^d A_i \tilde{x}(t + \theta_i) \right) + 2\tilde{x}^T P H \\ &\leq -\lambda_{\min}(Q) \|\tilde{x}\|^2 + 2\tilde{x}^T P \left(\sum_{i=1}^d A_i \tilde{x}(t + \theta_i) \right) \\ &\quad + 2\alpha \|P\| \|\tilde{x}\| \end{aligned}$$

By using combination of Razumikhin Theorem, and some algebra, we can get the result (4.20) and complete the proof. \blacksquare

Similar to (4.6), we consider a time delay system described by

$$\begin{aligned} \dot{x} &= Ax + \sum_{i=1}^d A_i x(t + \theta_i) + Bu + Ef + H \\ y &= Cx \end{aligned} \quad (4.23)$$

all the conditions are same as those in (4.6). And Lemma 4.2.2 as well as Assumptions 4.3.1–4.3.2 hold. Also we assume the H satisfies the following Assumption.

Assumption 4.4.3 $\|H_1\| \leq \alpha_1$, $H_2 = 0$, $\|H_3\| \leq \alpha_2$, α_1, α_2 are constants. \square

Assumption 4.4.4 $\text{Rank} \begin{bmatrix} \lambda I - A & E \\ C & 0 \end{bmatrix} = n + m, \forall \lambda \in \mathbb{C}$. \square

From Theorem 4.1 and Corollary 4.2.4, we know Assumptions 4.4.1 and 4.4.4 will give us the result that dimension of A_{33}^{22} diminishes and (A_{33}, A_{23}) is completely observable, i.e., $r = n - p$.

In the case subject to Assumptions 4.4.3 and 4.3.2, the dynamics of x_3 is described by,

$$\begin{aligned} \dot{x}_3 &= A_{31}x_1 + A_{32}x_2 + A_{33}x_3 + B_3u \\ &\quad + \sum_{i=1}^d [A_{31}^{(i)}x_1(t + \theta_i) + A_{32}^{(i)}x_2(t + \theta_i) + A_{33}^{(i)}x_3(t + \theta_i)] + H_3 \end{aligned} \quad (4.24)$$

For the system (4.24), we can design observer for x_3 similar to (4.11),

$$\begin{aligned}\dot{\hat{x}}_3 &= A_{31}x_1 + A_{32}x_2 + A_{33}\hat{x}_3 + B_3u \\ &+ \sum_{i=1}^d [A_{31}^{(i)}x_1(t + \theta_i) + A_{32}^{(i)}x_2(t + \theta_i) \\ &+ A_{33}^{(i)}\hat{x}_3(t + \theta_i)] + K(A_{23}x_3(t) - A_{23}\hat{x}_3(t)).\end{aligned}\quad (4.25)$$

Notice that the above observer uses the information of $A_{23}x_3$ which is not directly available from output. As a result, by introducing

$$\begin{aligned}w(t) &= \hat{x}_3(t) - Kx_2(t) \\ &= \hat{x}_3(t) - K[0_{(p-m) \times m} I_{(p-m) \times (p-m)}]C_1^T y,\end{aligned}$$

we may have (4.25) expressed in the following form,

$$\begin{aligned}\dot{w}(t) &= (A_{33} - KA_{23})w(t) + \\ &\sum_{i=1}^d A_{33}^{(i)}w(t + \theta_i) + (B_3 - KB_2)u + [A_{31} - KA_{21}, \\ &(A_{33} - KA_{23})K + A_{32} - KA_{22}]C_1^T y \\ &+ \sum_{i=1}^d [A_{31}^{(i)} - KA_{21}^{(i)}, A_{33}^{(i)}K + A_{32}^{(i)} - KA_{22}^{(i)}]C_1^T y(t + \theta_i)\end{aligned}\quad (4.26)$$

In which case, the error dynamics of $\tilde{x}_3 \triangleq \hat{x}_3 - x_3$ is

$$\dot{\tilde{x}}_3(t) = (A_{33} - KA_{23})\tilde{x}_3(t) + \sum_{i=1}^d A_{33}^{(i)}\tilde{x}_3(t + \theta_i) + H_3. \quad (4.27)$$

The following theorem will establish conditions under which the observer's error dynamics would converge to a bounded set asymptotically.

Theorem 4.4 *If the Assumptions 4.4.1, 4.3.2, 4.4.3-4.4.4 and the following condition hold*

$$\beta_1 \triangleq \lambda_{\min}(Q) - 2 \sum_{i=1}^d \|PA_{33}^{(i)}\| > 0 \quad (4.28)$$

where $\lambda_{\max}(P)$ is the maximum eigenvalue of P , $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q , and $Q > 0$, P is the positive definite matrix solution of

$$P(A_{33} - KA_{23}) + (A_{33} - KA_{23})^T P = -Q, \quad (4.29)$$

then the system (4.23) can be estimated under the presence of unknown input f , and also the error of estimation for x_3 is bounded,

$$\|\tilde{x}_3\| \leq 2 \frac{\lambda_{\max}(P)^{\frac{3}{2}} \alpha_2}{\lambda_{\min}(P)^{\frac{1}{2}} \beta_1}. \quad (4.30)$$

□

The proof of this theorem is same as that of Theorem 4.1 with only additional consideration of system matrices transformation and assumptions.

We know that the difference of $w + K[0_{(p-m) \times m} \quad I_{(p-m) \times (p-m)}] C_1^T y$ and x_3 will converge into a bounded set (see (4.30)), so that state $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ can be estimated and is given by

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix} + \begin{bmatrix} I_{p \times p} \\ \begin{bmatrix} 0 & K \end{bmatrix} \end{bmatrix} C^T y.$$

4.5 Fault Detection and Isolation Approach

We assume that the actuator faults can be modeled by the term $\tilde{E}f$ in system (4.5) (from here on, we use f_a instead of f to refer to the actuator fault vector). It can be shown that variety of actuator faults can be realized with this type of formulation [41]. Therefore, given the actuator faults as described in (4.5), a simple approach

of detecting and isolating the actuator faults would be to try to estimate the magnitude of the actuator fault f_a . This is a kind of an inverse problem, f_a is directly reconstructed by other information. It is felt that this approach is more accurate and reliable than other fault diagnosis methods used in linear, bilinear and nonlinear systems which are more indirect.

Proposition 4.5.1 Actuator faults can be detected and isolated as long as Theorem 4.2 and Corollary 4.2.4 are satisfied. \square

The proof of the above result is obvious from the following discussion.

Based on the conditions in the above Proposition, we know that the system (4.7) is observable by combination of Theorem 4.2 and Corollary 4.2.4. If an actuator fault is present, then the actuator fault estimate \hat{f}_a would have a nonzero norm, otherwise, it should have a zero (or near zero) norm. Additionally, the fault can be isolated clearly by checking the nonzero entry, or the orientation of \hat{f}_a . An estimate of the actuator fault can easily be obtained after discretization of the system (4.7),

$$\delta E_1 f_a(k) = x_1(k+1) - x_1(k) - \delta(A_{11}x_1(k) + A_{12}x_2(k) + A_{13}x_3(k) + \sum_{i=1}^d (A_{11}^{(i)}x_1(k - \Theta_i) + A_{12}^{(i)}x_2(k - \Theta_i) + A_{13}^{(i)}x_3(k - \Theta_i))) \quad (4.31)$$

where k represents the k -th time step, and δ is the sampling period satisfying $\theta_i = -\Theta_i\delta$, Θ_i is positive integer, $i = 1, \dots, d$.

Based on the observer design approach described in the last section, we know that the observer (4.11) considers the fault f_a as an unknown input. So that even under the presence of fault, \hat{x}_3 is still going to approach x_3 asymptotically. Then the actuator fault can be isolated from the dynamics of x_1 . Assuming that no fault takes place

during the initial short transient of the observer, and using the estimation $\hat{x}_3(k)$ for $x_3(k)$, the actuator fault can be approximately estimated as

$$\begin{aligned} \hat{f}_a = & E_1^{-1} \frac{x_1(k+1) - x_1(k)}{\delta} - E_1^{-1} [(A_{11}x_1(k) + A_{12}x_2(k) + A_{13}\hat{x}_3(k) + \\ & \sum_{i=1}^d (A_{11}^{(i)}x_1(k - \Theta_i) + A_{12}^{(i)}x_2(k - \Theta_i) + A_{13}^{(i)}\hat{x}_3(k - \Theta_i))]. \end{aligned} \quad (4.32)$$

where x_1, x_2 are linear combination of y , so it is known. As a result, actuator fault detection and isolation is easily accomplished.

A similar approach as above can be employed for detection and isolation of sensor faults. To account for the effect of sensor failures, consider writing the output equation in system (4.5) as

$$y = \tilde{C}\tilde{x} + \tilde{E}_s f_s. \quad (4.33)$$

where matrix $\tilde{E}_s \in \mathbb{R}^{m_s \times m_s}$ is nonsingular, and $f_s \in \mathbb{R}^{m_s}$ represents the vector of sensor failures which is completely unknown.

Consider now representing the sensor fault vector f_s as the output of the following dynamical system [41]

$$\dot{f}_s = A_s f_s + v, \quad (4.34)$$

where v is unknown and A_s is Hurwitz.

Augmenting (4.5) with (4.34), we get

$$\begin{aligned} \begin{bmatrix} \dot{\tilde{x}} \\ \dot{f}_s \end{bmatrix} &= \underline{A} \begin{bmatrix} \tilde{x} \\ f_s \end{bmatrix} + \sum_{i=1}^d \underline{A}_i \begin{bmatrix} \tilde{x}(t + \theta_i) \\ f_s(t + \theta_i) \end{bmatrix} + \underline{B}u + \underline{E} \begin{bmatrix} f_a \\ v \end{bmatrix} \\ y &= \underline{C} \begin{bmatrix} \tilde{x} \\ f_s \end{bmatrix}, \end{aligned} \quad (4.35)$$

$$\underline{A} = \begin{bmatrix} \tilde{A} & 0 \\ 0 & A_s \end{bmatrix}, \quad \underline{A}_i = \begin{bmatrix} \tilde{A}_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix}, \quad \underline{E} = \begin{bmatrix} \tilde{E} & 0 \\ 0 & I \end{bmatrix}, \quad \underline{C} = \begin{bmatrix} \tilde{C} & E_s \end{bmatrix}.$$

Note now that the above system is again in the form of (4.5). For the above composite system, from system (4.5) $\text{Rank}(\tilde{C}\tilde{E}) = \text{Rank}(\tilde{E})$, we know that $\text{Rank}(\underline{C}\underline{E}) = \text{Rank}(\underline{E})$ is equivalent to $\text{Rank}[\tilde{C}\tilde{E} \quad \tilde{E}_s] = m_s + m$. If $\text{Rank}[\tilde{C}\tilde{E} \quad \tilde{E}_s] \neq m_s + m$, then we can not design UIO.

Proposition 4.5.2 Consider system (4.35) with full rank \tilde{E}_s , as long as,

1. $\text{Rank}[\tilde{C}\tilde{E} \quad \tilde{E}_s] = m_s + m$,

2. $\text{Rank} \begin{bmatrix} \lambda I - \underline{A} & \underline{E} \\ \underline{C} & 0 \end{bmatrix} = n + m + m_s, \quad \forall \lambda.$

Then system (4.35) can be transformed to the form of (4.7). In addition, for the newly transformed system, if Assumptions 4.3.1–4.3.2 and condition (4.13) are satisfied, then we can design UIO observer for system (35) to detect and isolate both the actuator and sensor faults. □

Proof. Consider system (4.35), condition 1 is to guarantee that Lemma 4.2.2 can be applied to system (4.35), so that it can be transformed to the form of (4.7). From Corollary 4.2.4, we know that condition 2 is used to guarantee the observability of $(\underline{A}, \underline{C})$. Assumptions 4.3.1–4.3.2 and (4.13) will allow UIO design for the time delay system (4.35), from the approach described in (4.33), so the actuator and sensor faults detection and isolation can be done simultaneously. ■

4.6 Illustrative Examples and Simulation Results

Example 4.6.1 Consider a time delay system described by,

$$\begin{aligned}
 \dot{x}(t) &= \begin{bmatrix} -2 & 1.5 & 0.5 & 0 \\ -0.5 & -1 & 1 & 0 \\ 3 & 0.5 & -3 & 1 \\ 0 & -0.2 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 2 & 1.5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0.5 & -0.6 & 0 \\ -0.5 & 0 & 0 & -0.7 \end{bmatrix} x(t + \theta) \\
 &+ \begin{bmatrix} 1 & 0 \\ 0 & -2 \\ 0 & 0.5 \\ 1 & 0 \end{bmatrix} (u(t) + f_a) \\
 y(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \end{bmatrix} x(t)
 \end{aligned} \tag{4.36}$$

We can use the approach in Lemma 4.2.2 to transform the above system into the form of (4.7) and we find that Assumptions 4.3.1 and 4.3.2 are hold. Therefore the observer can be designed, and also both actuator faults can be detected and isolated.

Figures 4.1–4.3 give the simulation results without (Figure 4.1) and with actuator faults (Figures 4.2 and 4.3). In the simulation study, we choose discretization time step of 0.01, $\theta = -0.2$, initial conditions $x_1(t) = -\sin t, x_2(t) = \cos t, x_3(t) = 0; x_4(t) = -2.5, \hat{x}_4(t) = 0, -0.2 \leq t \leq 0$; And $u = [2 \sin t, 2 \cos t]^T$. Note from the second and the third sub-plots in Figure 4.1 that the fault estimates are essentially zero during the system's operation and this indicates that there are no failures. On the other hand, the fault estimates of the first and the second actuator faults in Figure 4.2,

clearly indicate the onset of the faults at $t = 3$ seconds. The same is true in Figure 4.3. In practice, at the beginning of the FDI algorithm operation, the estimation of the faults may not be correct due to the transients of the observer. That is, in the first few moments of the operation, the algorithm may indicate a fault where in fact there are no failures in the system. This of course is due to the fact that the estimation errors are generally nonzero during this period. As a result, the FDI system alarms have to be disabled during this period, after which the alarm can be enabled. This in reality is not a restriction or a major problem since it is highly unlikely that failures can take place during such a short interval, and even if they did they certainly need not be detected immediately.

□

Example 4.6.2 We consider the time-delay system with actuator as well as sensor faults of the following form,

$$\begin{aligned} \dot{x}(t) = & \begin{bmatrix} -2 & 1.5 & 0.5 & 0 \\ -0.5 & -1 & 2 & 0 \\ 3 & 0.5 & -3 & 1 \\ 0 & -0.2 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & -0.7 \end{bmatrix} x(t + \theta) \\ & + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} (u(t) + f_a) \quad y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f_s \end{aligned} \quad (4.37)$$

where x is the state variable, y is output, u is input, $\theta = -0.2$ is time delay and f_a is the actuator fault, f_s sensor fault. For the form in (4.34), we take $A_s = 0$ for simulation. And the single input u is taken as $2\sin(t)$ for simulation.

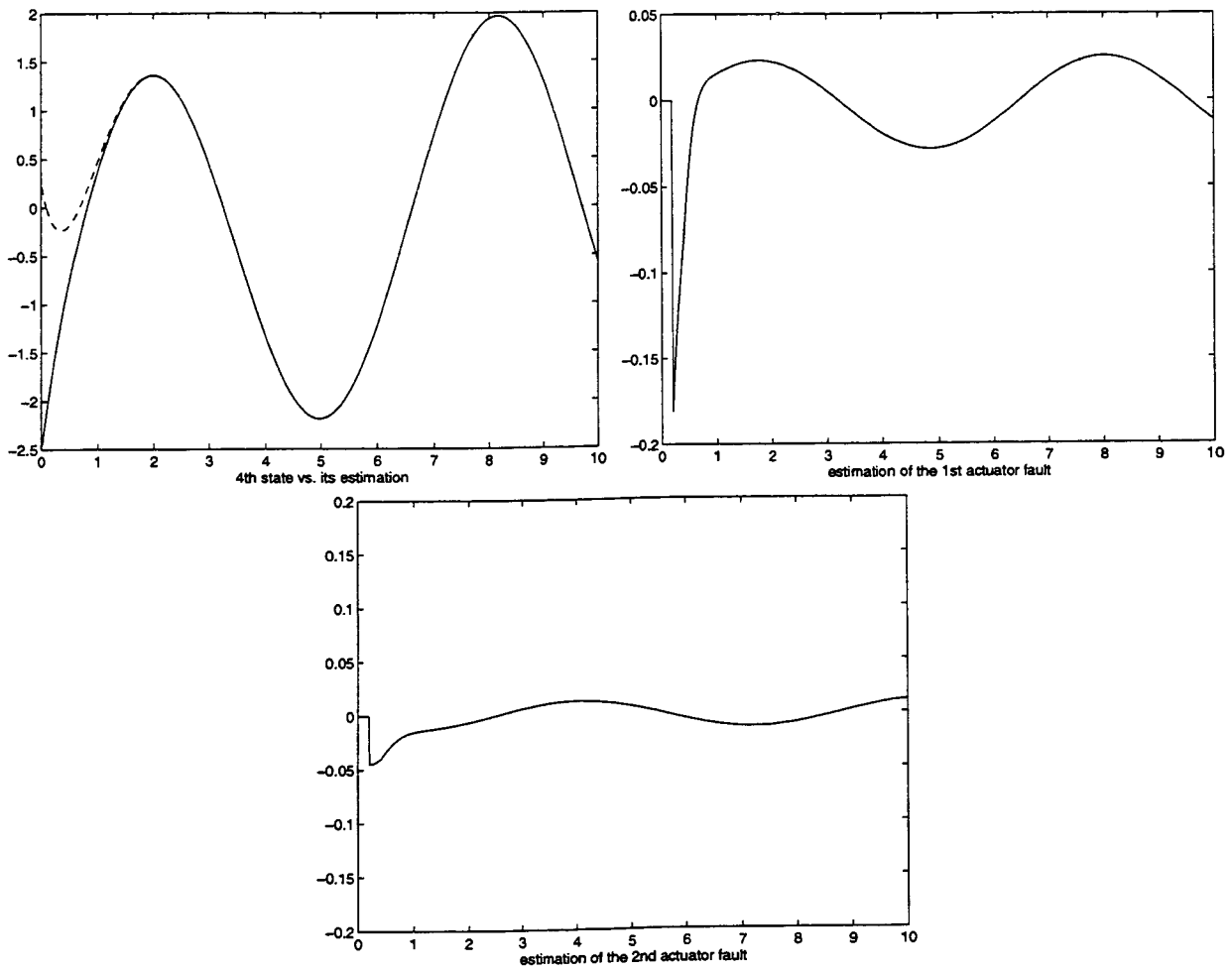


Figure 4.1: No Fault Signals and the Estimation

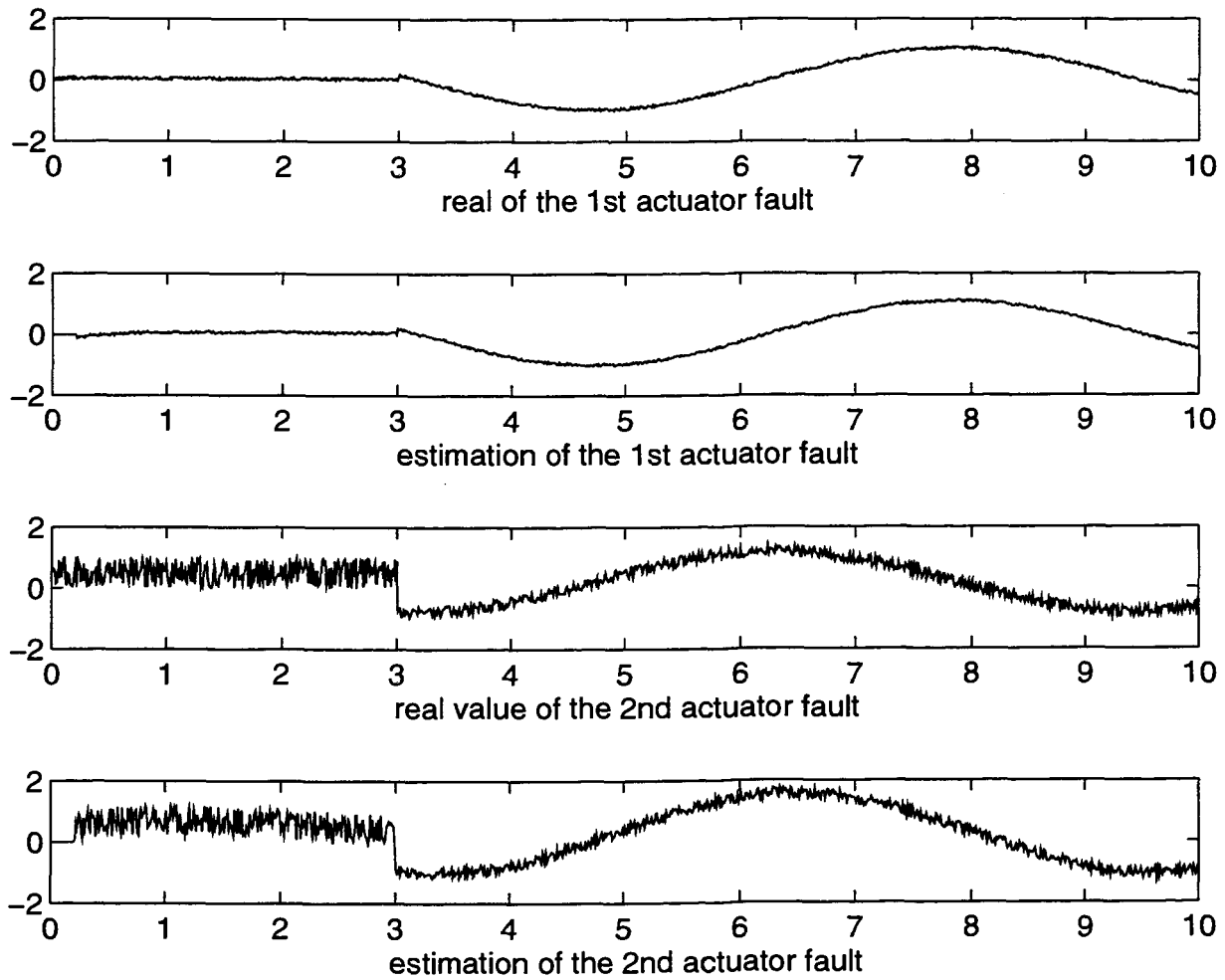


Figure 4.2: Fault Isolation and the Estimation

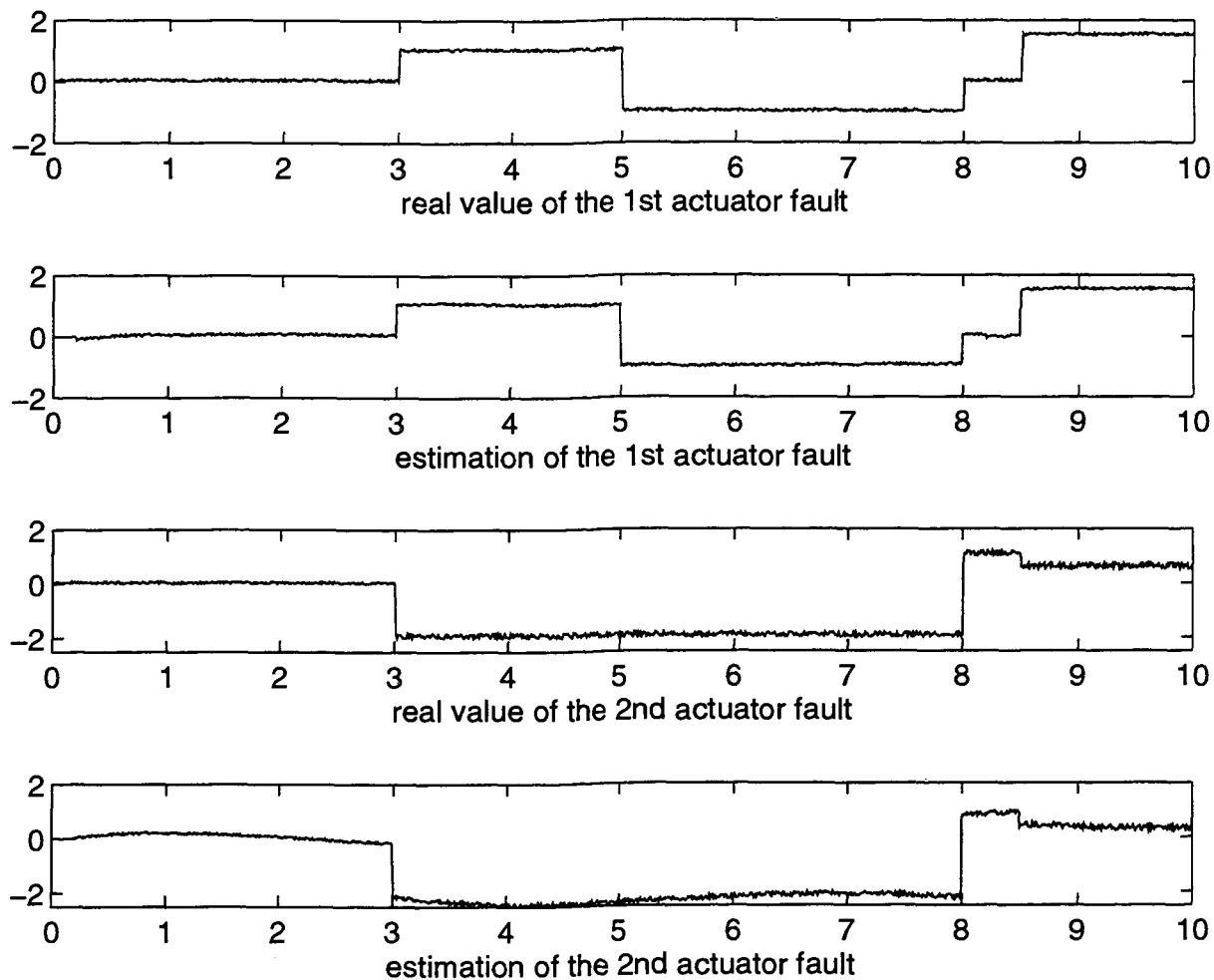


Figure 4.3: Fault Isolation and the Estimation

Using the approach described through (4.33)–(4.35), we have the following overall dynamics,

$$\begin{aligned}
 \begin{bmatrix} \dot{x} \\ \dot{f}_s \end{bmatrix} &= \begin{bmatrix} -2 & 1.5 & 0.5 & 0 & 0 \\ -0.5 & -1 & 2 & 0 & 0 \\ 3 & 0.5 & -3 & 1 & 0 \\ 0 & -0.2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & -0.7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t+\theta) \\ f_s(t+\theta) \end{bmatrix} \\
 + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_a \\ v \end{bmatrix} & \quad \dot{y}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix}
 \end{aligned} \tag{4.38}$$

by using the following transformation,

$$z = T \begin{bmatrix} x \\ f_s \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

the above equation (4.38) can be written as the form of (4.7),

$$\begin{aligned}
 \dot{z} = & \begin{bmatrix} -2 & 0 & 1.5 & 0 & 0.5 \\ 4 & 0 & 0.5 & 1 & -0.3 \\ -0.5 & 0 & -1 & 0 & 2 \\ 1 & 0 & -1.7 & -1 & -0.5 \\ 4 & 0 & 0.5 & 1 & -3 \end{bmatrix} z + \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ -0.2 & 0 & 0 & -0.7 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} z(t + \theta) \\
 & + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_a \\ v \end{bmatrix} \\
 y(t) = & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} z
 \end{aligned} \tag{4.39}$$

By using the FDI approach (4.32) for the composed system, sensor as well as actuator faults are detected and estimated. Simulation results for two cases are given in Figures 4.4 and 4.5, which show the close and accurate detection and isolation (estimation) of the sensor and actuator faults.

□

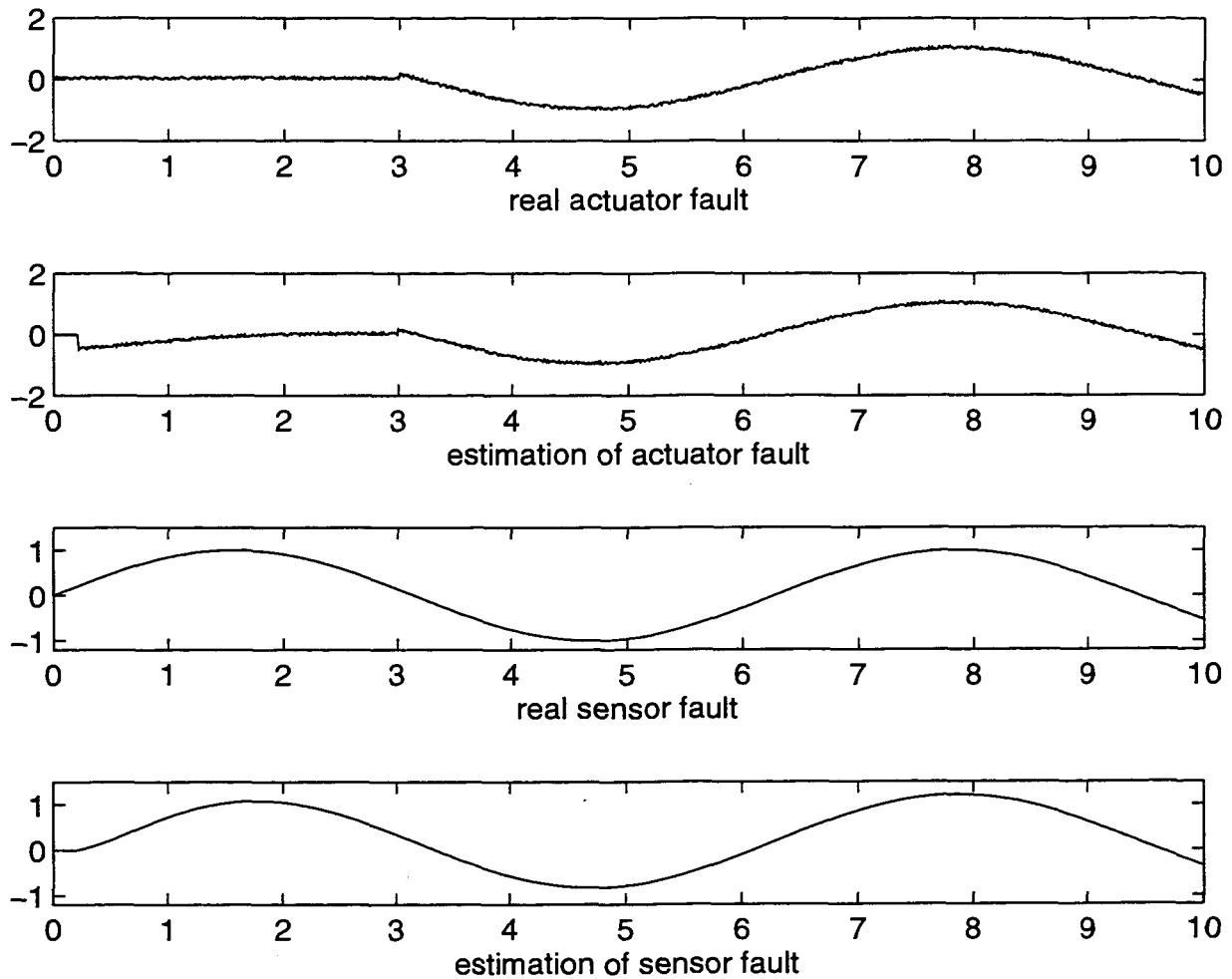


Figure 4.4: Actuator & Sensor Faults Isolation, Case 1

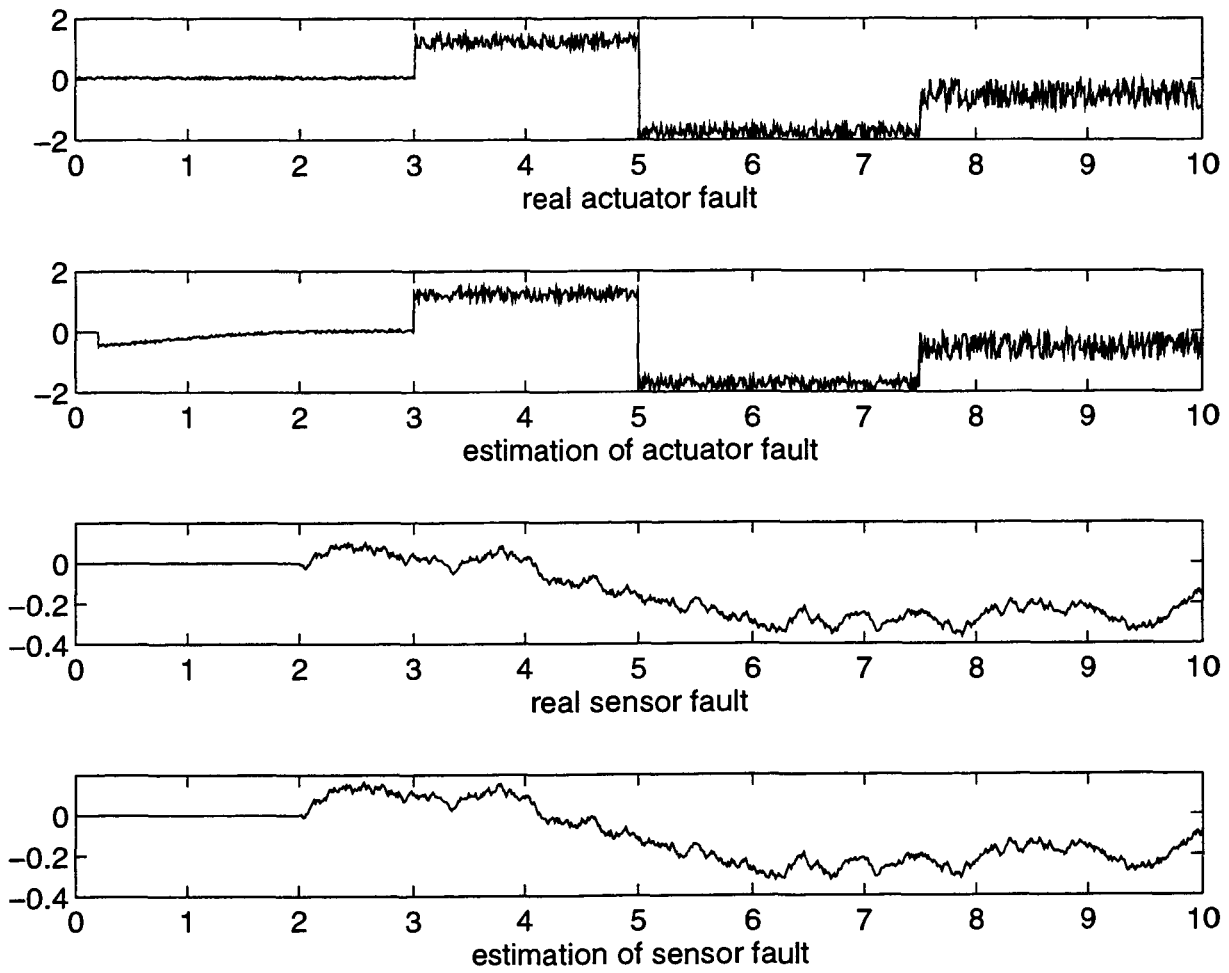


Figure 4.5: Actuator & Sensor Faults Isolation, Case 2

Example 4.6.3 Consider a time delay system of the form (4.17) with $E = B$,

$$\begin{aligned}
 A &= \begin{bmatrix} -2 & 1.5 & 0.5 & 1 \\ -0.5 & -1 & 1 & 0 \\ 3 & 0.5 & -3 & 1 \\ 0 & -0.2 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -2 \\ 0 & 0.5 \\ 1 & 0 \end{bmatrix}, \\
 A_1 &= \begin{bmatrix} -1 & 2 & 1.5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0.5 & -0.6 & 0 \\ -0.5 & 0 & 0 & -0.7 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ h \end{bmatrix}, \\
 C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \end{bmatrix}, E_s = 0, \tau = 0.2, \|h\| \leq 0.9.
 \end{aligned} \tag{4.40}$$

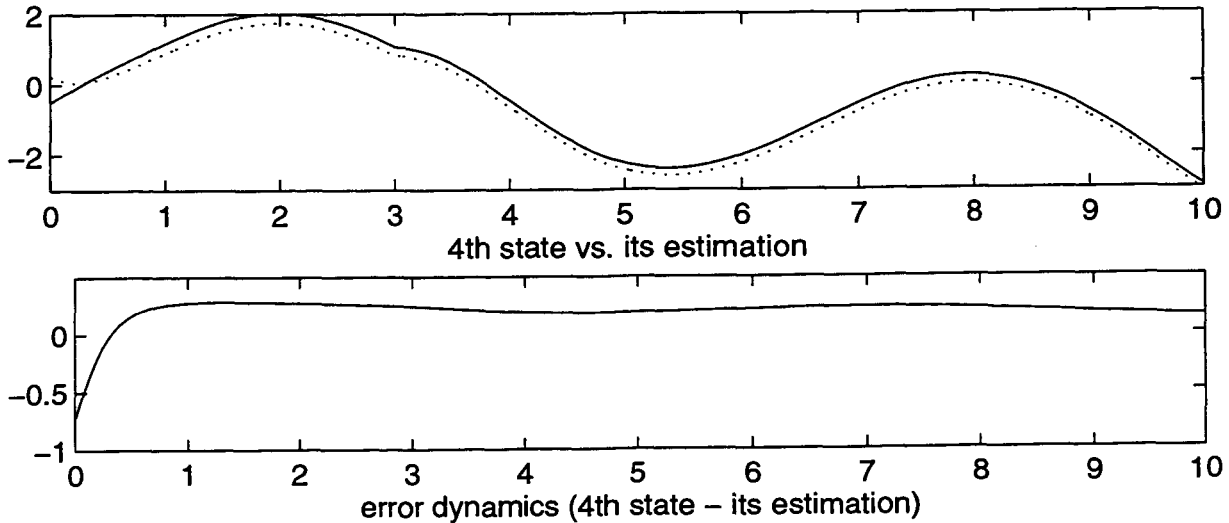


Figure 4.6: State Estimation under Fault Occurrence

Similar to Examples 4.6.1 and 4.6.1, this example is simulated based on the results in section 4.4.

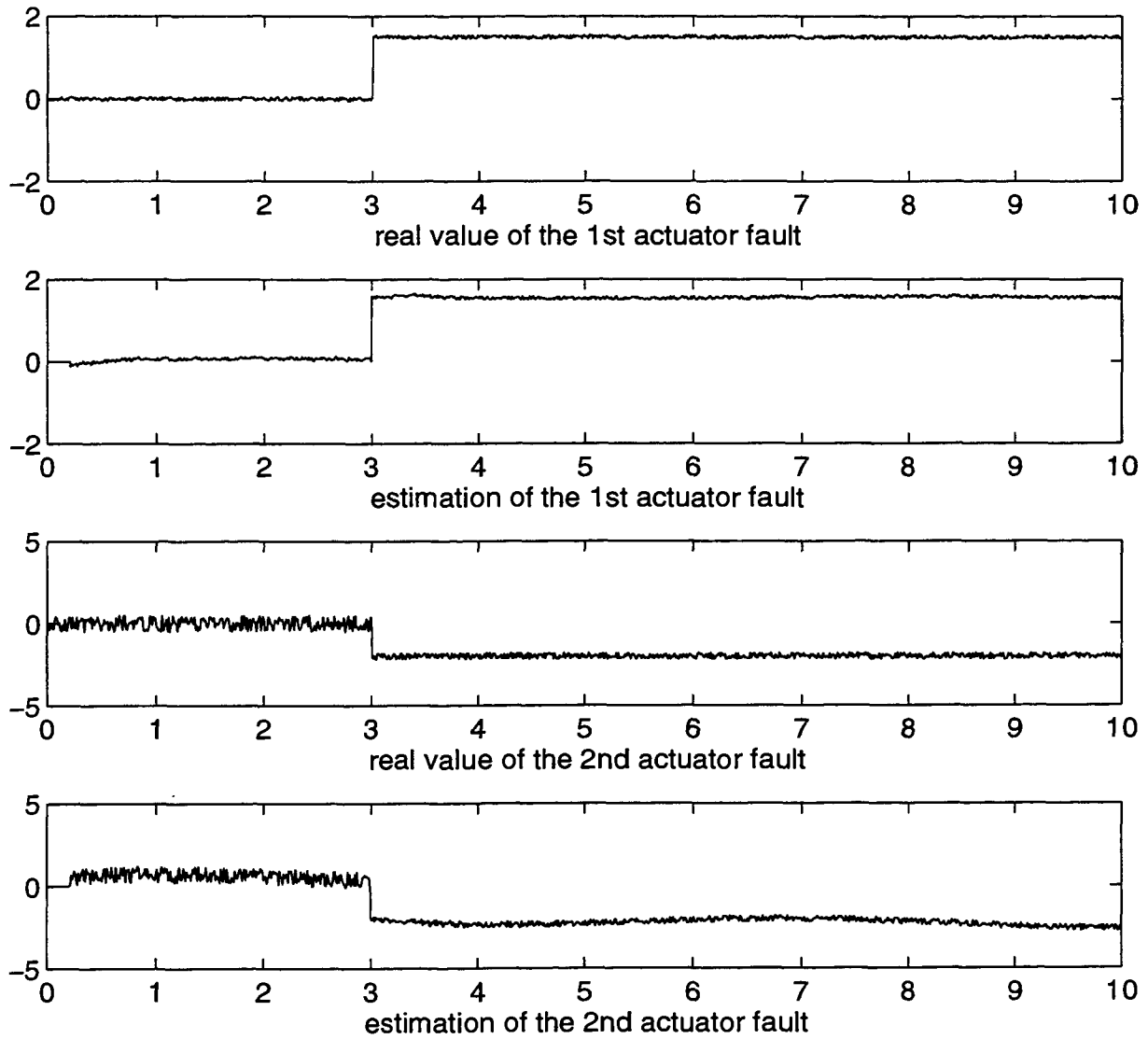


Figure 4.7: Fault Isolation and Estimation

In this example, as can be seen from Figure 4.6, the estimate of the state is not approaching the true value of the state asymptotically. This is caused by the disturbance term H . However, increasing the variable β as described in Section 4.4, can reduce the error, and the FDI may then be performed satisfactorily. This is illustrated in Figure 4.7, where although the disturbance is present, and the state estimate is not exact, still the estimate of the actuator faults are accurate enough that the actuator fault detection and identification can be accomplished with no difficulties.

□

4.7 Conclusions

The unknown input observer design philosophy, developed in previous chapters was extended to cover time delay systems where the state of the system is retarded. The observer design approach was extended for fault diagnostic purposes. Again, the fault diagnostic approach that was proposed in this chapter uses a single observer to detect and identify actuator and sensor faults. As described in the case of linear and bilinear systems, for more complex time delay systems, multiple observers (observer bank) may be required for FDI.

Chapter 5

FDI in a Class of Nonlinear Systems Using Observer Approach

In this chapter, we study a class of nonlinear systems with unknown inputs for which nonlinear observers with linearizable error dynamics in appropriate coordinates can be designed. Various conditions for accomplishing this task is stated. Furthermore, our findings evidently encompass some results on the Unknown Input Observers (UIO) for linear and bilinear systems.

The observation scheme is next utilized as a mean for model based monitoring and failure diagnosis within the system. More specifically, a simple approach for fault detection and isolation (FDI) of actuator faults is presented. Selection of threshold value with reliability is discussed. Finally, the chapter concludes with examples, illustrating applicability of the reported results in linear and nonlinear systems.

5.1 Introduction

Apart from the traditional areas such as aerospace and nuclear industries [37], the FDI research has been gaining momentum in other technical fields such as automotive, manufacturing, autonomous vehicles and robots, etc [6], [25], [42]. Among the approaches to the FDI problem, model based FDI has been the main subject of research due to the fact that it requires no redundant hardware. In addition, this approach has the potential for systematically detecting variety of failures. Although by no means a mature subject, FDI in linear systems has received a great deal more attention in the past than its nonlinear counterpart. As a result, there are still a great deal more work available in the linear FDI field [13].

In this chapter, we consider a special class of nonlinear systems for FDI purposes. The approach taken toward the monitoring and diagnostics is similar to that of [41] in which an unknown input observer was utilized for FDI. Essentially, for the class of nonlinear system considered, we shall decompose the state and outputs in two parts. One part is affected by the actuator faults, whereas the other is decoupled from them. Next, the subsystem that is decoupled from faults is used to design the nonlinear unknown input observer (NUIO) for the nonlinear system. The estimates are then used for FDI purposes. Generally speaking, the observer design approach for FDI is a continuation of the UIO design for linear, bilinear and time-delay systems, and its utilization for FDI as we discussed in the previous chapters [40], [41]. As a matter of fact, it will be shown that this similarity is not coincidental, and the present results will encompass the UIO design in linear and bilinear systems as well.

In the next section, we will give conditions for the existence of the transformation which will transform the original nonlinear system into the desired form. Also an

approach for the design of the unknown input observer and some discussions on the comparison with linear systems will be made. In section 5.3, an adaptive system's observer is designed. Afterwards we will present an approach for actuator FDI. Finally, the last section will provide numerical examples illustrating the practical application of the proposed FDI approach.

5.2 Transformation and Observer Design

Consider the following nonlinear time invariant system,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + \sum_{i=1}^p q_i(x)f_i^a \triangleq f(x) + g(x)u + Q(x)f_a \\ y &= h(x) = [h_1(x), \dots, h_d(x), h_{d+1}(x) \dots, h_l(x)]^T \end{aligned} \quad (5.1)$$

where states $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^l$, actuator faults $f_a = [f_1^a, \dots, f_p^a]^T \in \mathbb{R}^p$, $Q(x) = [q_1(x), \dots, q_p(x)]$. f, g, h are smooth vector fields with $f(0) = 0, g(0) \neq 0, h(0) = 0$. $l \geq p$ may be required.

Given the system (5.1), assume that there exists a transformation $\xi = F(x), (x = F^{-1}(\xi) = W(\xi))$, such that (5.1) can be transformed to

$$\begin{aligned} \dot{\xi} &= \begin{bmatrix} 0_{d \times d} & 0_{d \times (n-d)} \\ 0_{(n-d) \times d} & A_0 \end{bmatrix} \xi + \beta(y)u + \phi_0(y) + \sum_{i=1}^p \phi_i(\xi)f_i^a \\ y &= [\xi_1^{(1)}, \dots, \xi_d^{(1)}, \xi_{d+1}^{(1)}, \dots, \xi_l^{(1)}]^T = C_0 \xi \end{aligned} \quad (5.2)$$

where l -tuple of integers (r_1, \dots, r_l) satisfying $r_i = 1, i = 1, \dots, d; r_1 + \dots + r_d + r_{d+1} + \dots + r_l = n$. $\xi = [\xi_1^{(1)}, \dots, \xi_d^{(1)}, \xi_{d+1}^{(1)}, \dots, \xi_{d+1}^{(r_{d+1})}, \xi_{d+2}^{(1)}, \dots, \xi_{d+2}^{(r_{d+2})}, \dots, \xi_i^{(1)}, \dots, \xi_l^{(r_l)}]^T$ and ϕ_i is of the following form,

$$\phi_i^T = [* \dots * 0 \dots 0] \quad (\text{i.e., last } n - d \text{ elements are zero}) \quad (5.3)$$

$$\phi \triangleq [\phi_1, \phi_2, \dots, \phi_p].$$

$$A_0 = \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ 0 & & & 1 \\ 0 & 0 & \dots & 0 \\ & & & \ddots \\ & & & & 0 & 1 & & 0 \\ & & & & \vdots & & \ddots & \\ & & & & 0 & & & 1 \\ & & & & 0 & 0 & \dots & 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \\ & & & 1 & \dots & 0 \\ & & & & & \ddots \\ & & & & & & 1 & \dots & 0 \end{bmatrix}.$$

Consider the transformation $x = W(\xi)$, by differentiating we get

$$\begin{aligned} \dot{x} &= \frac{\partial W}{\partial \xi} \dot{\xi} = \frac{\partial W}{\partial \xi} \left(\begin{bmatrix} 0 & 0 \\ 0 & A_0 \end{bmatrix} \xi + \beta(y)u + \phi_0(y) + \sum_{i=1}^p \phi_i(\xi) f_i^a \right) \\ y &= h(x) = C_0 \xi \end{aligned} \quad (5.4)$$

Comparing the above with equation (5.1), we conclude that such transformation $x = W(\xi)$ exists as long as the following conditions are satisfied,

$$(i) \quad f(x) = \frac{\partial W}{\partial \xi} \left(\begin{bmatrix} 0 & 0 \\ 0 & A_0 \end{bmatrix} \xi + \phi_0(y) \right); \quad (5.5)$$

$$(ii) \quad g(x) = \frac{\partial W}{\partial \xi} \beta(y); \quad (5.6)$$

$$(iii) \quad Q(x) = \frac{\partial W}{\partial \xi} \phi(\xi); \quad (5.7)$$

$$(iv) \quad h(W(\xi)) = C_0 \xi. \quad (5.8)$$

By using the concepts of partial differentiation and Lie bracket with some derivation (for brevity, more details are deleted), it can be shown that condition (i) is

where $O(x)$ is defined as

$$O(x) \triangleq \begin{bmatrix} dh_1(x) \\ \vdots \\ dh_d(x) \\ dh_{d+1}(x) \\ \vdots \\ dL_f^{r_{d+1}-1}h_{d+1}(x) \\ \vdots \\ dh_l(x) \\ \vdots \\ dL_f^{r_l-1}h_l(x) \end{bmatrix}. \quad (5.13)$$

If equation (5.12) is valid, then we know condition (iii) is same as

$$O(x)Q(x) = D\left(\frac{\partial W}{\partial \xi}\right)^{-1}\left(\frac{\partial W}{\partial \xi}\right)\phi(\xi) = D\phi(\xi). \quad (5.14)$$

From (5.14), it can be shown that a necessary and sufficient condition for (5.3) to hold is that

$$\langle dL_f^j h_{d+i}(x), q_k(x) \rangle = 0 \quad i = 1, \dots, l-d; \quad j = 0, \dots, r_{d+i}-1; \quad k = 1, \dots, p \quad (5.15)$$

In other words, the faults will not directly effect the outputs $h_i, i = d+1, \dots, l$, if the following condition is satisfied,

$$q_k(x) \in \Delta^*(x) \triangleq \bigcap_{i=d+1}^l \bigcap_{j=1}^{r_i} \text{Ker}(dL_f^{j-1}h_i(x)) \quad k = 1, \dots, p. \quad (5.16)$$

Assume

$$s_i \triangleq \frac{\partial W}{\partial \xi_i^{(r_i)}}, \quad (5.17)$$

Theorem 5.2 *There exists a global state-space diffeomorphism $\xi = F(x)$ with $F(x_0) = 0$, transforming system (5.1) into (5.2) if, and only if the following conditions are satisfied,*

1. $[ad_f^i s_\eta, ad_f^j s_\gamma] = 0 \quad 0 \leq i \leq r_\eta - 1, \quad 0 \leq j \leq r_\gamma, \quad 1 \leq \eta, \quad \gamma \leq l.$
2. $[g_i(x), ad_f^j s_\eta] = 0 \quad i = 0, \dots, m; \quad j = 0, \dots, r_\eta - 2; \quad \eta = 1, \dots, l.$
3. *the vector fields $ad_f^i s_\eta, 0 \leq i \leq r_\eta - 1$, are complete, where s_η is the vector field satisfying,*

$$\left\langle \begin{bmatrix} dh_\eta \\ \vdots \\ d(L_f^{r_\eta-1} h_\eta) \end{bmatrix}, s_\eta \right\rangle = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \quad (5.21)$$

4. $q_k = \sum_{i=1}^d \phi_{ki}(\xi) s_i \quad k = 1, \dots, p.$ (where ϕ_{ki} is the i -th element of ϕ_k).

□

Proof. From the observability of (f, h) , we know that $(h_1, \dots, h_d, h_{d+1}, \dots, L_f^{r_l-1} h_l)$ is a diffeomorphism. Also from condition 3, we have

$$\left\langle \begin{bmatrix} dh_\eta \\ \vdots \\ d(L_f^{r_\eta-1} h_\eta) \end{bmatrix}, (s_\eta, \dots, (-1)^i ad_f^i s_\eta, \dots, (-1)^{r_\eta-1} ad_f^{r_\eta-1} s_\eta) \right\rangle = \begin{bmatrix} 0 & 1 \\ & \dots \\ 1 & * \end{bmatrix}$$

and therefore the vectors $s_\eta, \dots, ad_f^{r_\eta-1} s_\eta$ are linearly independent, also we have

$$ad_f^i s_\eta = (-1)^i \frac{\partial W}{\partial \xi_\eta^{(r_\eta-i)}} \quad 0 \leq i \leq r_\eta - 1.$$

The 1st condition is shown to be necessary and sufficient to make the transformed linear dynamics represented in the form of (5.2) without the input u and faults f_a .

The 2nd condition $[g_i, ad_f^j s_\eta] = 0, i = 0, \dots, p, j = 0, \dots, r_\eta - 2$ guarantees that $g_i(x)$ is independent from $(-1)^j \frac{\partial W}{\partial \xi_\eta^{(r_\eta-j)}}$, i.e., at most $g_i(x)$ is a function of $\frac{\partial W}{\partial \xi_{d+1}^{(1)}}, \dots, \frac{\partial W}{\partial \xi_i^{(1)}}, \frac{\partial W}{\partial \xi_1^{(1)}}, \dots, \frac{\partial W}{\partial \xi_d^{(1)}}$, i.e., $g_i(x)$ can be transformed to a function of output y .

Condition 4 is necessary and sufficient to transform q_i into ϕ_i which its last $n - d$ elements are zero in ξ -coordinates. Also equation (5.16) is equivalent to this condition.

So after the diffeomorphism transformation F , $g_i(x), q_i(x)$ become $\beta_i(y), \phi_i(\xi)$. Then system (5.1) can be transformed into (5.2). That completes the proof. \blacksquare

As a matter of fact, from the form of ϕ described in (5.3), we know that the output $h_{d+1}(x), \dots, h_l(x)$ and states $\xi_{d+1}^{(1)}, \dots, \xi_l^{(r_l)}$ are free from the faults, and those information will be used to construct the NUIO for the purpose of FDI.

Remark 5.2.1 Consider the unknown input observer (UIO) in linear and bilinear systems (see [4], [40]). The familiar condition $\text{Rank } CE = \text{Rank } E$ (i.e., $h(x) = Cx, Q(x) = E$) implies that all the information with regard to the unknown inputs (faults) are contained in the output y , and not necessarily in \dot{y} etc. Similarly, in the nonlinear case we have the faults only affecting the first d elements of the output. So that the faults information will still be remained in the dynamics stored in the output. This will help us to recover and isolate those faulty signals. As a matter of fact, just as in the case of linear and bilinear systems it is necessary that the number of the outputs for recovering and isolating the faulty signals must be greater than the number of unknown inputs. \square

Remark 5.2.2 $\text{Rank} \underbrace{\{dh_1(x), \dots, dh_d(x)\}}_{(I)}, \underbrace{\{dh_{d+1}(x), \dots, dL_f^{r_l-1} h_l(x)\}}_{(II)} = n$. (I) Observation does not need any dynamics of these outputs, instead a transformation will

generate the estimation of the partial states. This is the reason that no dynamic observer is needed for this part.

(II) This illustrates that dynamical equations of order $(n - d)$ are needed to recover the states which are affected by the faults. \square

Remark 5.2.3 If there is no faulty signals, this transformation is identical to the case described in [56] with $r_i = 1, i = 1, \dots, d$. \square

Design of the NUIO

Note that for those $r_i = 1$, the state variable is directly measured. Given that and the assumption that the nonlinear system can be transformed into the form described by (5.2), the NUIO can be designed by using the canonical normal observer approach.

Set

$$\xi_{d+1 \rightarrow n} = [\xi_{d+1}^{(1)} \cdots \xi_{d+1}^{(r_{d+1})} \cdots \xi_l^{(1)} \cdots \xi_l^{(r_l)}]^T, \quad \xi_{d+1 \rightarrow n}^{(1)} = [\xi_{d+1}^{(1)} \xi_{d+2}^{(1)} \cdots \xi_l^{(1)}]^T,$$

then we have the following dynamics,

$$\dot{\xi}_{d+1 \rightarrow n} = A_0 \xi_{d+1 \rightarrow n} + \beta_{(d+1 \rightarrow n)}(y)u + \phi_{0(d+1 \rightarrow n)}(y) \quad (5.22)$$

where $\beta_{(d+1 \rightarrow n)}$ represents the rows of β from $(d + 1)$ -th to the n -th, similarly is $\phi_{0(d+1 \rightarrow n)}$.

For the above system, we can design observer as following,

$$\dot{\hat{\xi}}_{d+1 \rightarrow n} = A_0 \hat{\xi}_{d+1 \rightarrow n} + \beta_{(d+1 \rightarrow n)}(y)u + \phi_{0(d+1 \rightarrow n)}(y) + K(\xi_{d+1 \rightarrow n}^{(1)} - \hat{\xi}_{d+1 \rightarrow n}^{(1)}) \quad (5.23)$$

It is clear that the error dynamics can be made asymptotically stable by choosing a proper K .

With the estimation $\hat{\xi}_{d+1 \rightarrow n}$ of $\xi_{d+1 \rightarrow n}$ as well as the output, we can find the estimation \hat{x} for states x by using the diffeomorphism transformation F . Subsequently, this information can be used for FDI purposes. However, before we do that a computational design procedure which is a modification to that of [56] is presented.

Algorithm for Observer Design:

- (i) Compute $O(x)$ in (5.13) (with possible reordering of h_i).
- (ii) Find solution s_i of (5.11) and (5.17), compute S as defined in (5.19).
- (iii) From (5.9) and (5.10), find

$$\frac{\partial[\phi_0(y) \quad \beta_k(y)]}{\partial \xi_i^{(1)}} = -\left(\frac{\partial W}{\partial \xi}\right)^{-1} \left[ad_f \frac{\partial W}{\partial \xi_i^{(1)}} \quad ad_{g_k} \frac{\partial W}{\partial \xi_i^{(1)}} \right]$$

and as long as the left hand side is a function of output, then solve for $\phi_0(y)$ and $\beta_k(y)$.

- (iv) Compute

$$\begin{aligned} \xi_i^{(1)} &= h_i(x), \quad \text{for } i = 1, \dots, l. \\ \xi_i^{(j+1)} &= L_f \xi_i^{(j)} - \phi_{0i}^{(j)} - \beta_i^{(j)} u, \quad \text{for } j = 1, \dots, r_i - 1. \end{aligned}$$

- (v) Verify condition (5.16); if it is satisfied, then we can transform system (5.1) into (5.2), and observer can be designed according to (5.23). Otherwise stop.

5.3 Further Discussion: Adaptive Observer Design

Consider the following nonlinear time invariant system,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + \sum_{i=1}^r e_i(x)\theta_i + \sum_{i=1}^p q_i(x)f_i^\alpha \triangleq f(x) + g(x)u + E(x)\theta + Q(x)f_a \\ y &= h(x) = [h_1(x), \dots, h_d(x), h_{d+1}(x) \dots, h_l(x)]^T \end{aligned} \quad (5.24)$$

where states $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^l$, uncertain parameters $\theta = [\theta_1, \dots, \theta_r]^T \in \mathbb{R}^r$, $E(x) = [e_1(x), \dots, e_r(x)]$, actuator faults $f_a = [f_1^a, \dots, f_p^a]^T \in \mathbb{R}^p$, $Q(x) = [q_1(x), \dots, q_p(x)]$. f, g_i, h are smooth vector fields with $f(0) = 0, g(0) \neq 0, h(0) = 0$. $l \geq p$ may be required. Also we assume the local observability of (f, h) , i.e.,

$$\text{Rank} \{dh_1, dL_f h_1, \dots, dL_f^{r_1-1} h_1, \dots, dh_l, \dots, dL_f^{r_l-1} h_l\} = n.$$

Given the system (5.24), assume that there exists a parameter independent, global state-space change of coordinates in \mathbb{R}^n , i.e., transformation $\xi = F(x), F(0) = 0, (x = F^{-1}(\xi))$, such that (5.24) can be transformed to

$$\begin{aligned} \dot{\xi} &= \begin{bmatrix} 0_{d \times d} & 0_{d \times (n-d)} \\ 0_{(n-d) \times d} & A_0 \end{bmatrix} \xi + \beta(y)u + \phi_0(y) + \sum_{i=1}^r \psi_i(y)\theta_i + \sum_{i=1}^p \phi_i(\xi)f_i^a \\ y &= [\xi_1^{(1)}, \dots, \xi_d^{(1)}, \xi_{d+1}^{(1)}, \dots, \xi_l^{(1)}]^T = C_0 \xi \end{aligned} \quad (5.25)$$

where l -tuple of integers (r_1, \dots, r_l) satisfying $r_i = 1, i = 1, \dots, d; r_1 + \dots + r_d + r_{d+1} + \dots + r_l = n$. $\xi = [\xi_1^{(1)}, \dots, \xi_d^{(1)}, \xi_{d+1}^{(1)}, \dots, \xi_{d+1}^{(r_{d+1})}, \xi_{d+2}^{(1)}, \dots, \xi_{d+2}^{(r_{d+2})}, \dots, \xi_l^{(1)}, \dots, \xi_l^{(r_l)}]^T$ and ϕ_i is of the following forms,

$$\phi_i^T = [* \dots * 0 \dots 0], \quad (\text{i.e., last } n-d \text{ elements are zero}) \quad i = 1, \dots, p. \quad (5.26)$$

$\phi \triangleq [\phi_1, \phi_2, \dots, \phi_p]$, $\psi \triangleq [\psi_1, \psi_2, \dots, \psi_r]$, and

$$A_0 = \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ 0 & & & 1 \\ 0 & 0 & \dots & 0 \\ & & & \ddots \\ & & & & 0 & 1 & & 0 \\ & & & & \vdots & & \ddots & \\ & & & & 0 & & & 1 \\ & & & & 0 & 0 & \dots & 0 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \\ & & & 1 & \dots & 0 \\ & & & & & \ddots \\ & & & & & & 1 & \dots & 0 \end{bmatrix}.$$

When the output is single, $d = 0$ and there is no faults, then there is asymptotic adaptive observer design available for system (5.24) [32]. Following [32], the global version of the transformation for (5.24) can be stated as follows.

Theorem 5.3 *There exists a global state-space diffeomorphism $\xi = F(x)$ with $F(x_0) = 0$, transforming system (5.24) into (5.24) if, and only if the following conditions are satisfied,*

1. $[ad_f^i s_\eta, ad_f^j s_\gamma] = 0 \quad 0 \leq i \leq r_\eta - 1, \quad 0 \leq j \leq r_\gamma, \quad 1 \leq \eta, \quad \gamma \leq l.$
2. $[g_i(x), ad_f^j s_\eta] = 0 \quad i = 1, \dots, m; \quad j = 0, \dots, r_\eta - 2; \quad \eta = 1, \dots, l.$
3. $[e_i(x), ad_f^j s_\eta] = 0 \quad i = 1, \dots, r; \quad j = 0, \dots, r_\eta - 2; \quad \eta = 1, \dots, l.$
4. *the vector fields $ad_f^i s_\eta, 0 \leq i \leq r_\eta - 1$, are complete, where s_η is the vector field*

satisfying,

$$\left\langle \begin{bmatrix} dh_\eta \\ \vdots \\ d(L_f^{r_\eta-1} h_\eta) \end{bmatrix}, s_\eta \right\rangle = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \quad (5.27)$$

$$5. \quad q_k = \sum_{i=1}^d \phi_{ki}(\xi) s_i \quad k = 1, \dots, p. \quad (\text{where } \phi_{ki} \text{ is the } i\text{-th element of } \phi_k).$$

□

Proof. Condition 1 is shown in [31] to be necessary and sufficient for (1) with $u = 0, \theta = 0, f_a = 0$ to be transformable via a local diffeomorphism in the neighborhood of $x = 0$ into system

$$\begin{aligned} \dot{\xi} &= \begin{bmatrix} 0_{d \times d} & 0_{d \times (n-d)} \\ 0_{(n-d) \times d} & A_0 \end{bmatrix} \xi + \beta(y)u + \phi_0(y) \\ y &= [\xi_1^{(1)}, \dots, \xi_d^{(1)}, \xi_{d+1}^{(1)}, \dots, \xi_l^{(1)}]^T = C_0 \xi \end{aligned}$$

The needed local coordinates $\xi = F(x)$ are defined, by virtue of condition 1, as those in which

$$ad_f^i s_\eta = (-1)^i \frac{\partial}{\partial \xi_\eta^{(r_\eta-1)}} \quad 0 \leq i \leq r_\eta - 1.$$

that is, the vector fields $ad_f^i s_\eta, 0 \leq i \leq r_\eta - 1, 1 \leq \eta \leq l$, are simultaneously rectified. Consequently, condition 2 guarantees that the vector fields g_i depend, in the ξ -coordinates, on the output y only. Similarly condition 3 is considered as a guarantee that $e_i(x)$ only depends on y in the new coordinates. Condition 4 is necessary and sufficient, according to [9], for the above change of coordinates to be a global one. When condition 4 fails, we only have a local change of coordinates. Condition 5 is necessary and sufficient to transform q_i into ϕ_i which its last $n - d$ elements are zero in ξ -coordinates.

Then system (5.24) can be transformed into (5.25) under the conditions 1–5 in Theorem 5.8. That completes the proof. \blacksquare

Remark 5.3.1 As long as the following condition is satisfied,

$$e_k = \sum_{\eta=d+1}^l \sum_{i=0}^{r_{\eta-1}} \psi_{ki}^{\eta}(y) a d_f^i s_{\eta} \quad k = 1, \dots, r. \quad (\text{where } \psi_{ki}^{\eta} \text{ is an element of } \psi_k),$$

then $\psi_i(y)$ will be of the following form

$$\psi_i^T = [0 \cdots 0 * \cdots *] \quad (\text{i.e., first } d \text{ elements are zero}), \quad i = 1, \dots, r. \quad (5.28)$$

i.e., the uncertain parameter will not directly affect the output dynamics. \square

As a matter of fact, from the form of ϕ described in (5.26), we know that the actuator fault f_a will not directly affect the derivatives of the output $h_{d+1}(x), \dots, h_l(x)$ and states $\xi_{d+1}^{(1)}, \dots, \xi_l^{(r_l)}$, and those information will be used to construct the adaptive observer for the purpose of FDI.

Design of the adaptive observer

Note that for those $r_i = 1$, the state variable is directly measured. Given that and the assumption that the nonlinear system can be transformed into the form described by (5.25), the nonlinear adaptive observer can be designed by using the canonical normal observer approach.

Set

$$\xi_{d+1 \rightarrow n} = [\xi_{d+1}^{(1)} \cdots \xi_{d+1}^{(r_{d+1})} \cdots \xi_l^{(1)} \cdots \xi_l^{(r_l)}]^T, \quad \xi_{d+1 \rightarrow n}^{(1)} = [\xi_{d+1}^{(1)} \xi_{d+2}^{(1)} \cdots \xi_l^{(1)}]^T,$$

then we have the following dynamics,

$$\dot{\xi}_{d+1 \rightarrow n} = A_0 \xi_{d+1 \rightarrow n} + \beta_{(d+1 \rightarrow n)}(y)u + \phi_{0(d+1 \rightarrow n)}(y) + \sum_{i=1}^r \psi_{i(d+1 \rightarrow n)}(y)\theta_i \quad (5.29)$$

where $\beta_{(d+1 \rightarrow n)}$ represents the rows of β from $(d+1)$ -th to the n -th, similarly are $\phi_{0(d+1 \rightarrow n)}$, $\psi_{i(d+1 \rightarrow n)}$.

For brevity, we define, $\Psi_{d+1 \rightarrow n}$ as the last $n-d$ rows of ψ , and \overline{C}_0 as a matrix composed by last $l-d$ rows and $n-d$ columns of C_0 .

Assumption 5.3.1 $\Psi_{d+1 \rightarrow n}(y)$ can be expressed as $B\overline{\Psi}_{d+1 \rightarrow n}(y)$ where B is a constant matrix. \square

Assumption 5.3.2 There exists $Q \in \mathbb{R}^{(n-d) \times (n-d)}$, symmetric and positive definite, such that for some matrix F ,

$$\overline{C}_0^T F^T = PB \quad (5.30)$$

where P is the unique positive-definite solution to the Lyapunov equation

$$(A_0 - K\overline{C}_0)^T P + P(A_0 - K\overline{C}_0) = -Q.$$

\square

Remark 5.3.2 Another interpretation of the Assumption 5.3.2 can be obtained from [44, 50]. If the triple $(A_0 - K\overline{C}_0, B, F\overline{C}_0)$ is controllable and observable, and a matrix F exists such that $FC[sI - (A_0 - K\overline{C}_0)]^{-1}B$ is strictly positive real, then Q exists such that Assumption 5.3.2 is valid. \square

Theorem 5.4 *If all the conditions in Theorem 5.3 and Assumptions 5.3.1–5.3.2 are satisfied, we can transform system (5.24) into (5.25) and design the following adaptive observer for (5.29),*

$$\dot{\hat{\xi}}_{d+1 \rightarrow n} = A_0 \hat{\xi}_{d+1 \rightarrow n} + \beta_{(d+1 \rightarrow n)}(y)u + \phi_{0(d+1 \rightarrow n)}(y) + \Psi_{d+1 \rightarrow n}(y)\hat{\theta} + K(\hat{\xi}_{d+1 \rightarrow n}^{(1)} - \hat{\xi}_{d+1 \rightarrow n}^{(1)}) \quad (5.31)$$

with adaptive law as follows,

$$\dot{\tilde{\theta}} = -G\bar{\Psi}_{d+1 \rightarrow n}^T(y)F\bar{C}_0\tilde{\xi}_{d+1 \rightarrow n} \quad (5.32)$$

then the above adaptive observer is asymptotically stable.

Proof. Take Lyapunov candidate function $V = \tilde{\xi}_{d+1 \rightarrow n}^T P \tilde{\xi}_{d+1 \rightarrow n} + \tilde{\theta}^T G^{-1} \tilde{\theta}$, where G is a positive definite matrix.

Using (5.29), (5.31) and Assumption 5.3.1, we get the error dynamics of $\xi_{d+1 \rightarrow n}$ as follows,

$$\dot{\tilde{\xi}}_{d+1 \rightarrow n} = (A_0 - K\bar{C}_0)\tilde{\xi}_{d+1 \rightarrow n} + B\bar{\Psi}_{d+1 \rightarrow n}\tilde{\theta} \quad (5.33)$$

By calculating the time derivative of V with Assumption 5.3.2 and adaptive law, we can obtain that

$$\begin{aligned} \dot{V} &= 2\tilde{\xi}_{d+1 \rightarrow n}^T P \dot{\tilde{\xi}}_{d+1 \rightarrow n} + 2\tilde{\theta}^T G^{-1} \dot{\tilde{\theta}} \\ &= -\tilde{\xi}_{d+1 \rightarrow n}^T Q \tilde{\xi}_{d+1 \rightarrow n} + 2\tilde{\xi}_{d+1 \rightarrow n}^T P B \bar{\Psi}_{d+1 \rightarrow n} \tilde{\theta} - 2\tilde{\theta}^T G^{-1} G \bar{\Psi}_{d+1 \rightarrow n}^T F \bar{C}_0 \tilde{\xi}_{d+1 \rightarrow n} \\ &= -\tilde{\xi}_{d+1 \rightarrow n}^T Q \tilde{\xi}_{d+1 \rightarrow n}. \end{aligned} \quad (5.34)$$

So that we claim that the observer (5.31) is an asymptotically stable one. \blacksquare

With the estimation $\hat{\xi}_{d+1 \rightarrow n}$ of $\xi_{d+1 \rightarrow n}$ as well as the output, we can find the estimation \hat{x} for states x by using the diffeomorphism transformation F .

Remark 5.3.3 Assume that $d = 0, l = 1, f_a = 0$ for system (5.24), i.e., the single output system without actuator fault, then adaptive observer design here is same as the approach in [32]. If $f_a = 0, \theta = 0$, then theorem 1 is equivalent to the result in [56]. \square

Remark 5.3.4 Basically speaking, for system (5.24), we first decouple (5.24) into two parts, one part the states are same as outputs, in the other part states are not outputs, and then we estimate the states as well as uncertain parameters. \square

5.4 Fault Detection and Isolation for Nonlinear Systems

For the sake of convenience, we only consider Fault Detection and Isolation for system (5.1), not (5.24). The FDI procedure for (5.24) should be same as the one of (5.1).

Assumption: $p \leq d$,

$$\text{Rank } \Phi_{1 \rightarrow d}(\xi) = p \quad (5.35)$$

where $\Phi_{1 \rightarrow d}(\xi) \triangleq [\phi_{1 \rightarrow d}(\xi), \dots, \phi_{p \rightarrow d}(\xi)] = \begin{bmatrix} L_{q_1} h_1(x) & L_{q_2} h_1(x) & \cdots & L_{q_d} h_1(x) \\ \vdots & \vdots & \ddots & \vdots \\ L_{q_1} h_d(x) & L_{q_2} h_d(x) & \cdots & L_{q_d} h_d(x) \end{bmatrix}$
and $\phi_{i \rightarrow d}$ represents the 1st d elements of ϕ_i . \square

For linear and bilinear cases, this assumption is equivalent to $\text{Rank } CE = \text{Rank } E$, where $Q(x) = E$, $h(x) = Cx$.

Theorem 5.5 *With all the conditions satisfied in Theorem 5.1 remaining valid, and the above Assumption, then the actuator faults in system (5.1) can be detected and isolated as well.*

Proof. Consider the dynamics of the fault-affected output described in system (2),

$$\dot{\xi}_{1 \rightarrow d} = \phi_{0(1 \rightarrow d)}(y) + \beta_{(1 \rightarrow d)}(y)u + \Phi_{1 \rightarrow d} f_a. \quad (5.36)$$

Discretize the above dynamic equation, we have

$$\frac{\xi_{1 \rightarrow d}(k+1) - \xi_{1 \rightarrow d}(k)}{\delta} = \phi_{0(1 \rightarrow d)}(y(k)) + \beta_{(1 \rightarrow d)}(y(k))u(k) + \Phi_{1 \rightarrow d}(\xi(k))f_a(k). \quad (5.37)$$

When Assumption hold, then we can estimate the faulty signal f_a as \hat{f}_a ,

$$\hat{f}_a = (\hat{\Phi}_{1 \rightarrow d}^T \hat{\Phi}_{1 \rightarrow d})^{-1} \hat{\Phi}_{1 \rightarrow d}^T \left\{ \frac{\xi_{1 \rightarrow d}(k+1) - \xi_{1 \rightarrow d}(k)}{\delta} - \phi_{0(1 \rightarrow d)}(y(k)) + \beta_{(1 \rightarrow d)}(y(k))u(k) \right\} \quad (5.38)$$

And

$$\hat{\Phi}_{1 \rightarrow d}(k) \triangleq \Phi_{1 \rightarrow d}(\xi_{1 \rightarrow d}(k), \hat{\xi}_{d+1 \rightarrow n}(k)).$$

So that the fault detection as well as isolation is done. As soon as any of the components of \hat{f}_a is greater than that of threshold value vector, then the alarm for the corresponding fault component will be on. ■

Convergence of the fault estimate:

Now we consider the efficiency of this fault detection and isolation scheme, i.e., the upper bound and lower bound of the estimation for faults. With this information, we know how accurate the fault estimation will be and how reliable the alarm will be in some sense.

Assume that $\dot{\xi}_{1 \rightarrow d}(k)$ represent the real derivative of $\xi_{1 \rightarrow d}$ at time step k , so from (5.36), we have

$$\dot{\xi}_{1 \rightarrow d}(k) = \phi_{0(1 \rightarrow d)}(y(k)) + \beta_{(1 \rightarrow d)}(y(k))u(k) + \Phi_{1 \rightarrow d}(\xi(k))f_a(k) \quad (5.39)$$

and for estimation of faults, we use the following equation,

$$\frac{\xi_{1 \rightarrow d}(k+1) - \xi_{1 \rightarrow d}(k)}{\delta} = \phi_{0(1 \rightarrow d)}(y(k)) + \beta_{(1 \rightarrow d)}(y(k))u(k) + \hat{\Phi}_{1 \rightarrow d}(k)\hat{f}_a(k). \quad (5.40)$$

Subtract (5.39) from (5.40), we have

$$\begin{aligned}\ddot{\xi}_{1 \rightarrow d}(k + \theta) \frac{\delta}{2} &= \hat{\Phi}_{1 \rightarrow d}(k) \hat{f}_a(k) - \Phi_{1 \rightarrow d}(k) f_a(k) \\ &= \hat{\Phi}_{1 \rightarrow d}(k) (\hat{f}_a(k) - f_a(k)) + (\hat{\Phi}_{1 \rightarrow d}(k) - \Phi_{1 \rightarrow d}(k)) f_a(k).\end{aligned}\quad (5.41)$$

Since disturbance and faults are not involved in the process of estimating $\xi_{d+1 \rightarrow n}$, so it can be made asymptotically, and $\hat{\Phi}_{1 \rightarrow d}$, $\Phi_{1 \rightarrow d}$ are continuous functions, $\hat{\Phi}_{1 \rightarrow d} - \Phi_{1 \rightarrow d}$ converge to zero asymptotically. As long as faulty signals are bounded, $(\hat{\Phi}_{1 \rightarrow d}(k) - \Phi_{1 \rightarrow d}(k)) f_a(k) \rightarrow 0$, i.e., $\forall \epsilon > 0, \exists K_0$, when $k > K_0$,

$$\|(\hat{\Phi}_{1 \rightarrow d}(k) - \Phi_{1 \rightarrow d}(k)) f_a(k)\| < \epsilon.$$

If $\|\ddot{\xi}_{1 \rightarrow d}\| < M_1$, then

$$\|\hat{\Phi}_{1 \rightarrow d}(k) (\hat{f}_a(k) - f_a(k))\| - \|(\hat{\Phi}_{1 \rightarrow d}(k) - \Phi_{1 \rightarrow d}(k)) f_a(k)\| \leq \frac{M_1 \delta}{2}$$

and

$$\|\hat{\Phi}_{1 \rightarrow d}(k) (\hat{f}_a(k) - f_a(k))\| \leq \frac{M_1 \delta}{2} + \epsilon$$

so

$$\|\hat{f}_a(k) - f_a(k)\| \leq \frac{M_1 \delta + 2\epsilon}{2\sqrt{\lambda_{\min}(\hat{\Phi}_{1 \rightarrow d}^T(k) \hat{\Phi}_{1 \rightarrow d}(k))}}.\quad (5.42)$$

If $\|\ddot{\xi}_{1 \rightarrow d}\| > M_0$, then

$$\|\hat{\Phi}_{1 \rightarrow d}(k) (\hat{f}_a(k) - f_a(k))\| + \|(\hat{\Phi}_{1 \rightarrow d}(k) - \Phi_{1 \rightarrow d}(k)) f_a(k)\| \geq \frac{M_0 \delta}{2}$$

and

$$\|\hat{\Phi}_{1 \rightarrow d}(k) (\hat{f}_a(k) - f_a(k))\| \geq \frac{M_0 \delta}{2} - \epsilon$$

so

$$\|\hat{f}_a(k) - f_a(k)\| \geq \frac{M_0 \delta - 2\epsilon}{2\sqrt{\lambda_{\max}(\hat{\Phi}_{1 \rightarrow d}^T(k) \hat{\Phi}_{1 \rightarrow d}(k))}}.\quad (5.43)$$

where $\lambda_{\min}(\lambda_{\max})$ represents the minimum (maximum) eigenvalue of the corresponding matrix.

From the above analysis, we know that the (adaptive) threshold value $T_r(k)$ can not be chosen smaller than $\frac{M_0\delta-2\epsilon}{2\sqrt{\lambda_{\max}(\hat{\Phi}_{1-d}^T\hat{\Phi}_{1-d})}}$, otherwise the alarm would be too sensitive to give the correct signal; the alarm for fault detection would be reliable if $T_r(k)$ chosen greater than $\frac{M_1\delta+2\epsilon}{2\sqrt{\lambda_{\min}(\hat{\Phi}_{1-d}^T\hat{\Phi}_{1-d})}}$. When $\|\hat{f}_a(k)\| > T_r(k)$, then the alarm for faults is on.

5.5 Illustrative Examples and Simulation Results

Example 5.5.1 Consider the following linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + E(x)f_a \\ y &= Cx \end{aligned} \quad (5.44)$$

where $x \in \mathbb{R}^5$, $u \in \mathbb{R}^2$, $f_a \in \mathbb{R}^2$ and

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 1 & 0 \\ -4 & 2 & 1 & 3 & 1 \\ 11 & -21 & -15 & -14 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ -1 & -5 \\ 5 & 11 \end{bmatrix}, \quad E(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ -1 & -4 \\ 5 & 5 \end{bmatrix} e(x),$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix}, \quad e(x) = 1 + |x_1 + 4x_2 + x_4|.$$

Following the procedures introduced before, we can find suitable transformation which transforms the system (5.44) into the form of (5.2). After designing the observer

for the fault-unaffected states, we can estimate the actuator fault.

For brevity, the details of calculation are deleted. The results on the estimation of states and actuator fault are shown in Figures 5.1–5.2. In Figure 5.1, it is shown that the estimate of the fault is essentially zero during the entire length of the simulation study, and thus the conclusion is that the system is healthy. However, the estimate of the actuator fault shown in Figure 5.2 indicates that the actuator is faulty.

Note again that the actual shape of the failure is also detected in our algorithm. This is not necessarily the case in many other studies. The fact that we can identify the shape of the failure is the by product of our algorithm which can prove useful perhaps in failure accommodation.

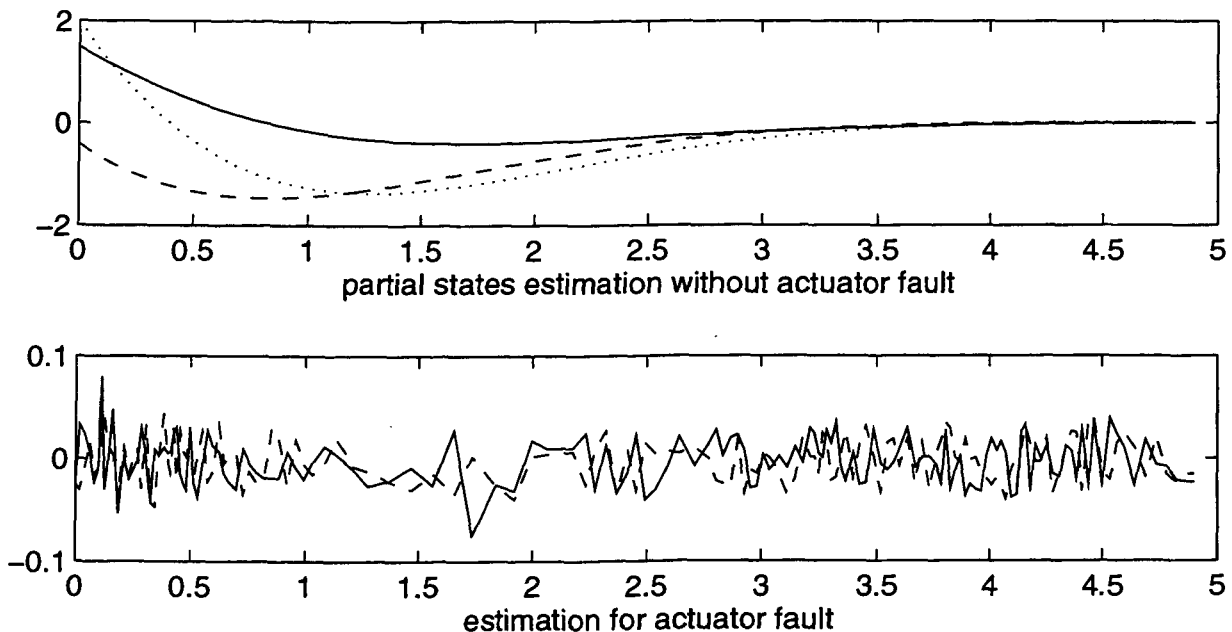


Figure 5.1: No Fault Signals and the Estimation

□

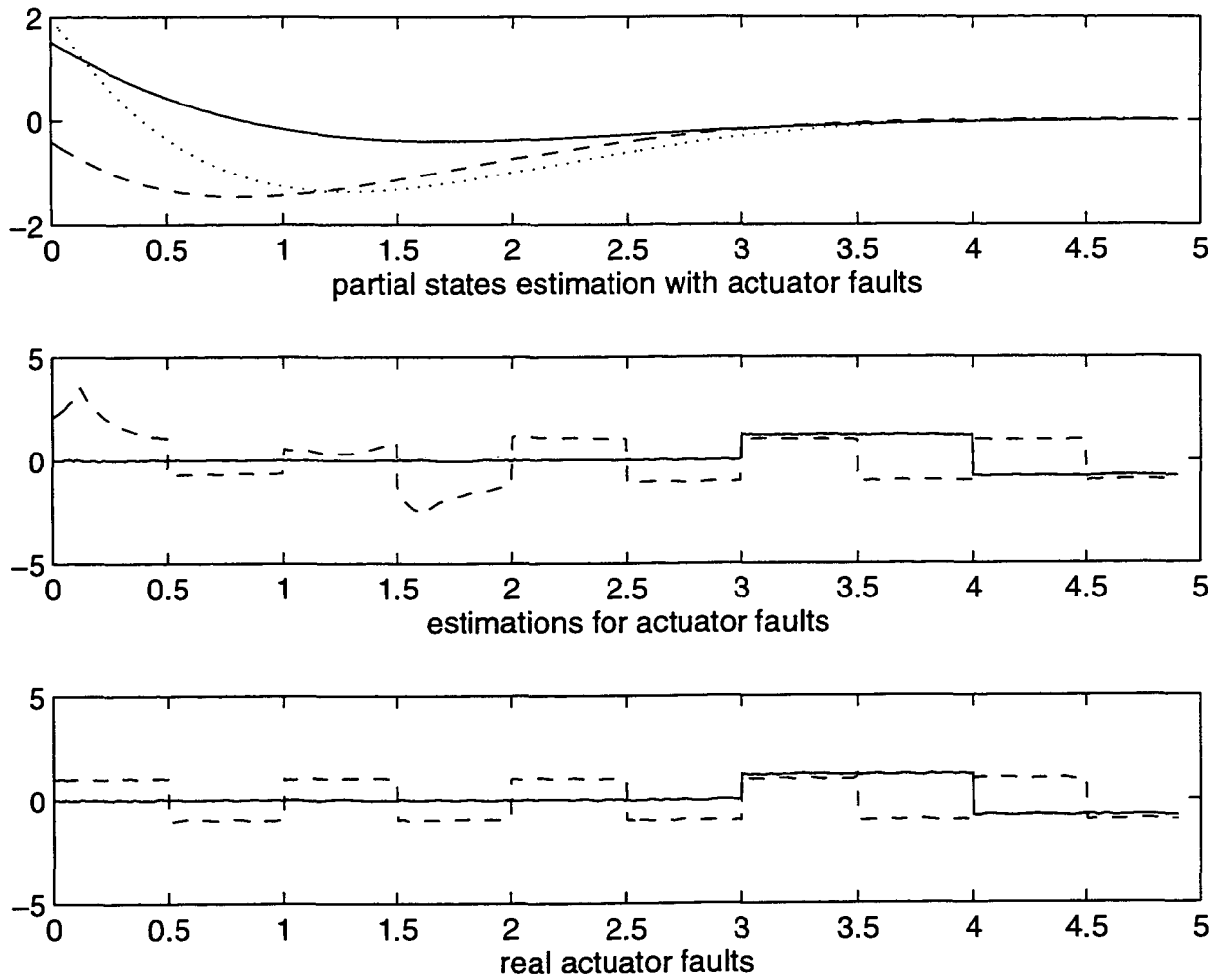


Figure 5.2: Fault Identification and the Estimation

Example 5.5.2 : We consider a nonlinear system of the following form,

$$\begin{aligned}
 \dot{x}_1 &= x_2 + (e^{x_1+2x_2+x_3} + 1)f_a - 0.1152\theta \\
 \dot{x}_2 &= x_3 - 0.3078\theta \\
 \dot{x}_3 &= u + 1.2308\theta \\
 y_1 &= x_1 + 2x_2 + x_3 \\
 y_2 &= x_2 + 2x_3
 \end{aligned} \tag{5.45}$$

where θ is an uncertain parameter and f_a is actuator faulty signal.

By using the coordinate transformation $\xi_1 = x_1 + 2x_2 + x_3$, $\xi_2 = x_2 + 2x_3$, $\xi_3 = x_3$, we can transform (5.45) to

$$\begin{aligned}
 \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} e^{y_1} - 1 \\ 0 \\ 0 \end{bmatrix} f_a + \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0.5 \\ 2.1538 \\ 1.2308 \end{bmatrix} \theta \\
 y &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}
 \end{aligned} \tag{5.46}$$

As a matter of fact, $q = [e^{x_1+2x_2+x_3} + 1, 0, 0]^T$.

For this form (5.46), since the dynamics of ξ_2, ξ_3 are not affected by faulty signal f_a , we can design the reduced-order adaptive observer as follows, by using Theorem 5.4,

$$\begin{bmatrix} \dot{\hat{\xi}}_2 \\ \dot{\hat{\xi}}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\xi}_2 \\ \hat{\xi}_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u + K(\xi_2 - \hat{\xi}_2) + \begin{bmatrix} 2.1538 \\ 1.2308 \end{bmatrix} \hat{\theta} \tag{5.47}$$

$$\dot{\hat{\theta}} = -(\hat{\xi}_2 - \xi_2) \tag{5.48}$$

where $\hat{\xi}_2, \hat{\xi}_3$ are the estimation of ξ_2 , and ξ_3 correspondingly. Assume the error dynamics are $\tilde{\xi}_2 = \hat{\xi}_2 - \xi_2, \tilde{\xi}_3 = \hat{\xi}_3 - \xi_3$, then we have the following equation,

$$\begin{bmatrix} \dot{\tilde{\xi}}_2 \\ \dot{\tilde{\xi}}_3 \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - K \begin{bmatrix} 1 & 0 \end{bmatrix} \right) \begin{bmatrix} \tilde{\xi}_2 \\ \tilde{\xi}_3 \end{bmatrix} + \begin{bmatrix} 2.1538 \\ 1.2308 \end{bmatrix} \tilde{\theta} \quad (5.49)$$

by properly choosing $K = [k_1, k_2]^T$ (here we choose $k_1 = k_2 = 2$), the error dynamics can be stabilized together with the adaptive law, so that the observer is asymptotical.

From equation (5.38), the fault f_a can be isolated from the discretization of the dynamics of ξ_1 . Also for simulation, we assume that $\theta = 0.5$, we can see that estimation of the parameter is approaching this value. Figure 5.3 shows the performance of the FDI algorithm under a no failure situation. The unknown parameter is adaptively estimated and converges to its true value of 0.5 and the fault estimate is zero after a short observer transient. Figure 5.4 and 5.5 illustrate the same things for two different shapes of failures. Clearly, in both cases, the uncertain system parameter is correctly identified and the estimate of the fault indicates that indeed a fault exists. Furthermore the actual shape and magnitude of the fault is detected.

□

Example 5.5.3 We consider a dc motor system with load (see Figure 5.6). The model is described by the following equation,

$$\begin{aligned} L_a \frac{di_a}{dt} &= -R_a i_a - K_v \omega i_e + v \\ J \frac{d\omega}{dt} &= K_v i_e i_a - D\omega \end{aligned} \quad (5.50)$$

Set $x_1 = i_a, x_2 = \theta, x_3 = \omega, u = i_e, v = v_a + f, i_e$ and v_a are ideal inputs and f is

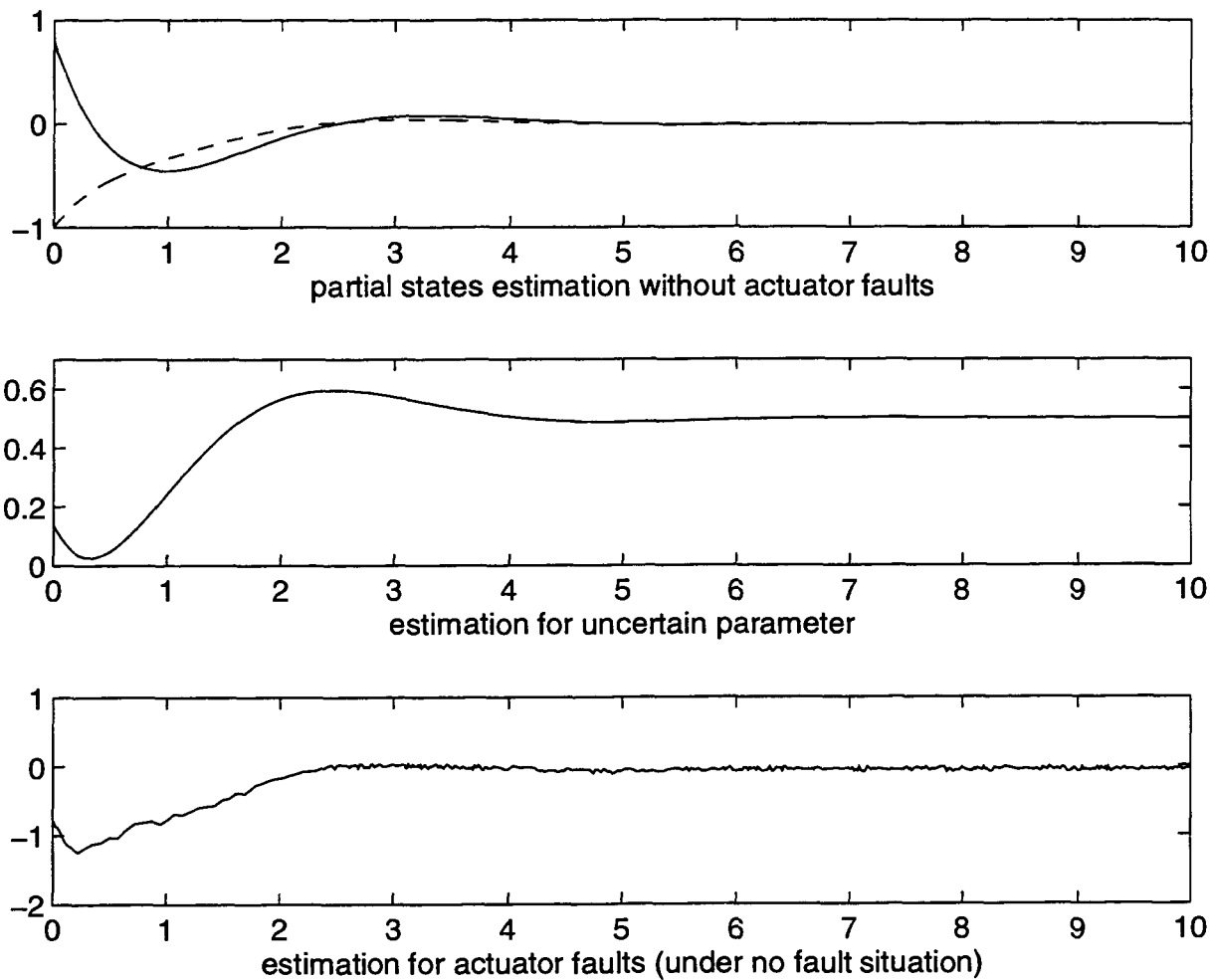


Figure 5.3: No Fault Signals and the Estimation

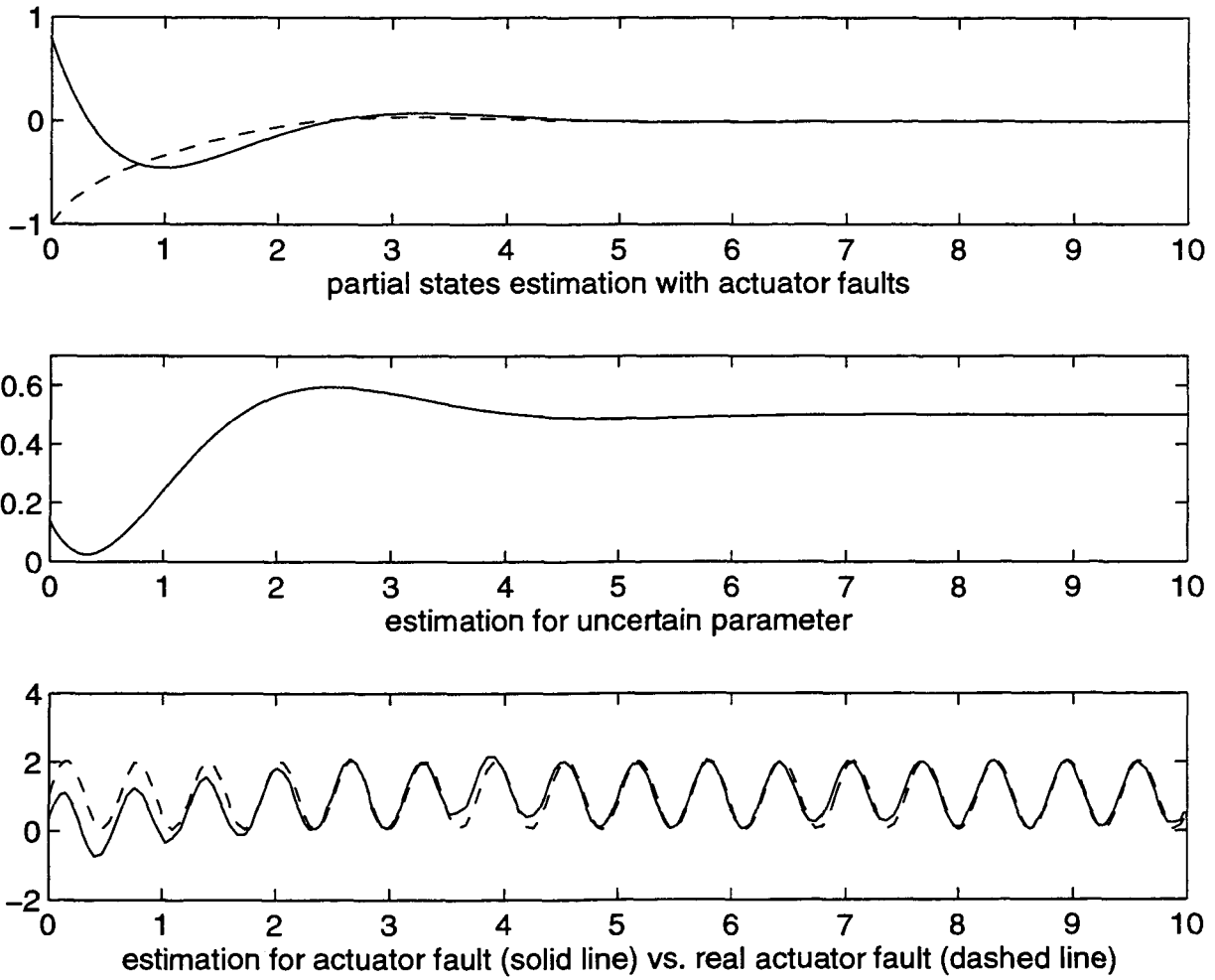


Figure 5.4: Fault Identification and the Estimation

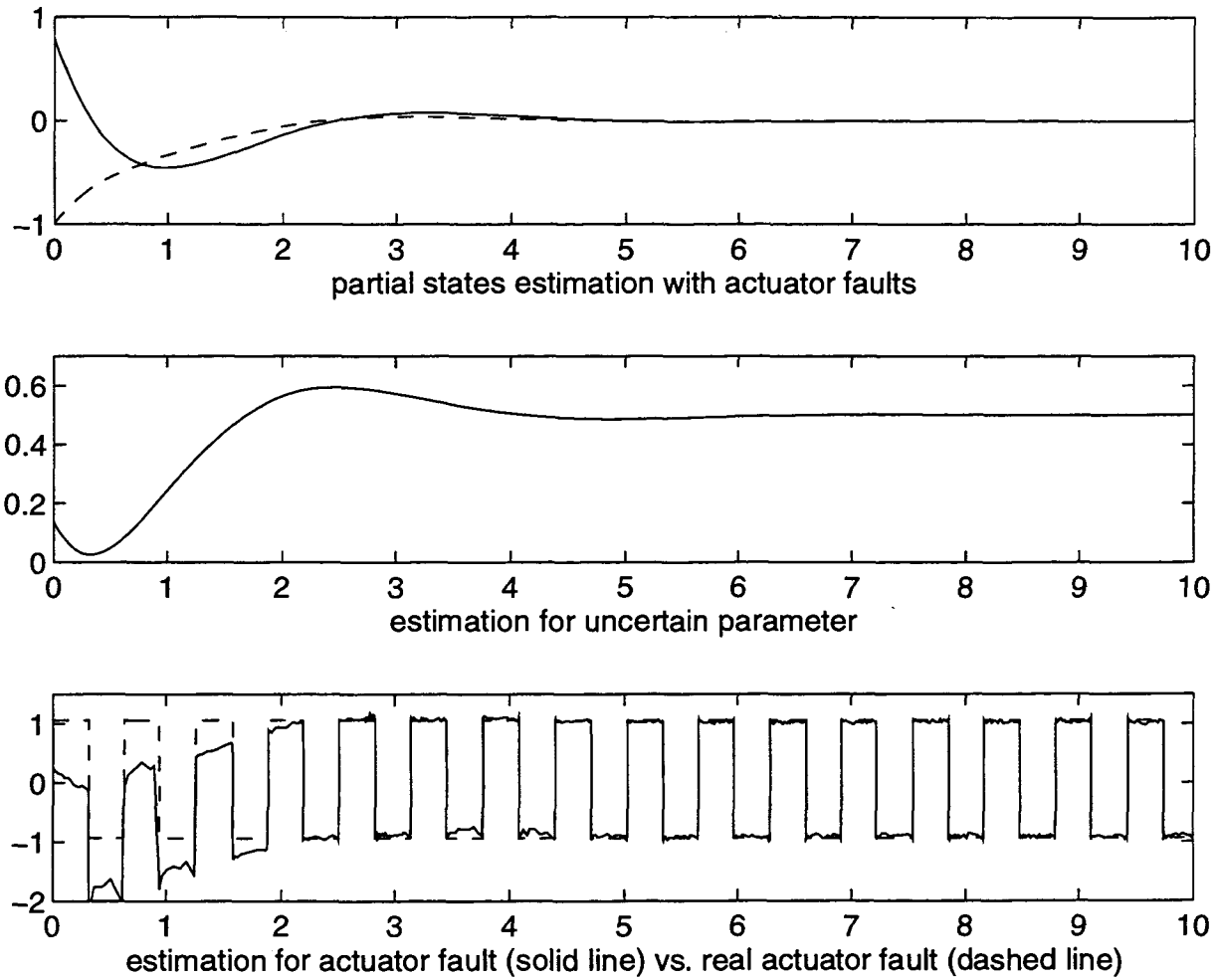


Figure 5.5: Fault Identification and the Estimation

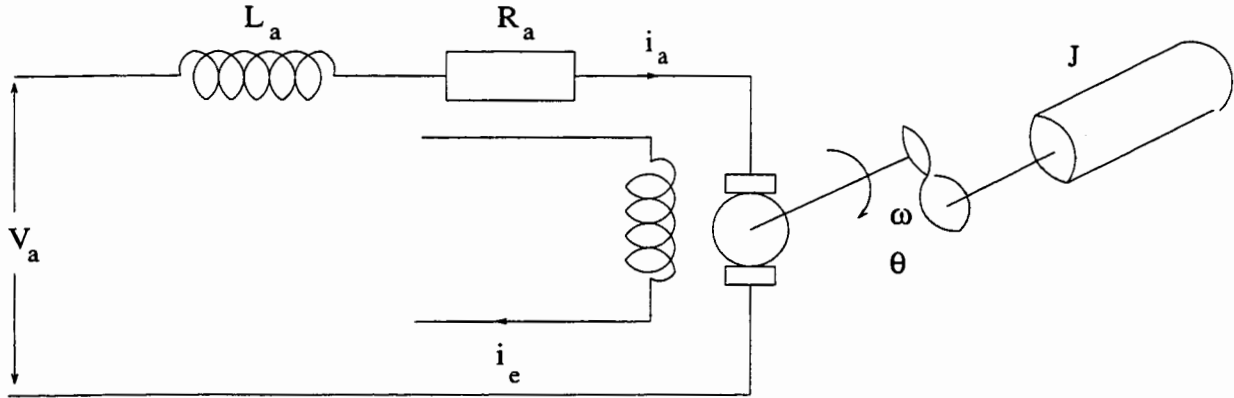


Figure 5.6: DC Motor System with Load

actuator fault, then (5.50) becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{D}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\frac{K_v}{L_a} \\ 0 & 0 & 0 \\ \frac{K_v}{J} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} u + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} \quad (5.51)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x \quad (5.52)$$

For simulation, we assume that $R_a = 1, L_a = 0.05, K_v = 10, D = 0.1, J = 0.2$.

And we transform coordinate by $\xi_1 = x_1, \xi_2 = x_2, \xi_3 = x_3 + 0.5x_2$, so the system becomes

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\frac{K_v}{L_a} \\ 0 & 0 & 0 \\ \frac{K_v}{J} & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} u + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} v \quad (5.53)$$

and output equation (5.52) remains the same, i.e., $y_i = \xi_i, i = 1, 2$.

To observe ξ_2, ξ_3 , we construct the following observer

$$\begin{bmatrix} \dot{\hat{\xi}}_2 \\ \dot{\hat{\xi}}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\xi}_2 \\ \hat{\xi}_3 \end{bmatrix} + \begin{bmatrix} 20 \\ 0 \end{bmatrix} v_a + \begin{bmatrix} 0 \\ 50 \end{bmatrix} y_1 u + K(\xi_2 - \hat{\xi}_2) \quad (5.54)$$

The faulty signal f can be detected and isolated based on the following equation,

$$\hat{f} = \frac{1}{20} \left(\frac{y_1(k+1) - y_1(k)}{\delta} + 20y_1(k) + 200\hat{\xi}_3(k)u(k) \right) - 20v_a \quad (5.55)$$

Figures 5.7–5.8 represent the simulation results. Again, these figures illustrate that the observer is successful in estimating the unavailable state of the system as well as the fault. When the estimate of the fault is anything other than zero, the fault is easily detected and declared.

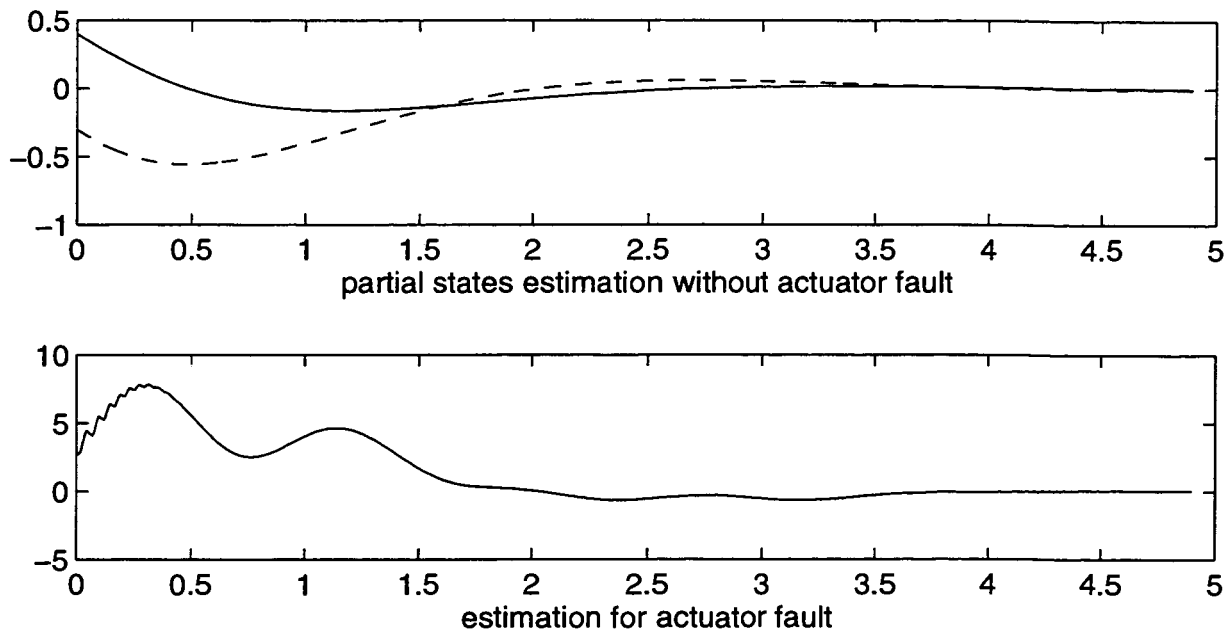


Figure 5.7: No Fault Signals and the Estimation

□

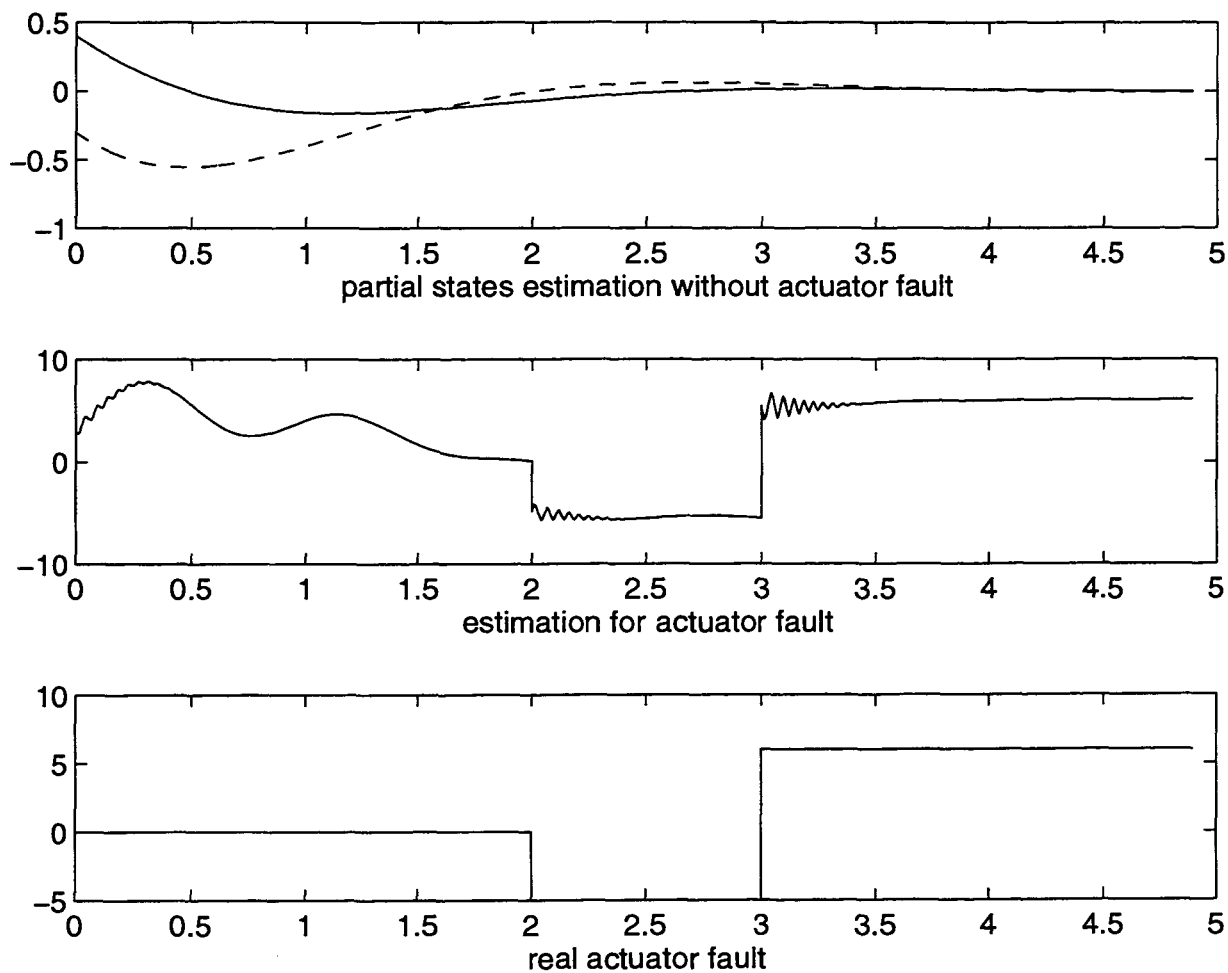


Figure 5.8: Fault Identification and the Estimation

Chapter 6

Practical Estimation for Fault Diagnosis

The last five chapters of this thesis concentrated on the application of observer based methodology for FDI. The main thread in all of those chapters was the notion of decoupling the faults and certain disturbances from parts of the system's dynamics, so that an unknown input decoupled observer could be designed. The resulting UIO was then used for both estimation as well as FDI. Clearly, at the heart of the FDI approach discussed in each of the last chapters then is the UIO. Recall to design UIO for linear systems, the condition $\text{Rank } CE = \text{Rank } E$ is needed. For nonlinear systems, strong conditions on linearization are also needed in our previous analysis. It is then fair to say that the estimation and diagnosis approaches, especially for nonlinear systems, are applicable to a limited class of control systems. The issue of designing FDI scheme, applicable to a wider classes of systems has been and will be the subject of many research studies in the past and in the years to come. The chapter is an attempt in this direction. It is the goal of this chapter to examine whether the

diagnostic schemes that were discussed in the previous chapters can be broadened so that they encompass perhaps some larger class of dynamical systems.

In this chapter, we will first introduce the notion of Sliding Mode Observer (SMO). We will then attempt to show that for the linear and nonlinear systems which do not satisfy the condition $\text{Rank } CE = \text{Rank } E$, it may be possible to combine SMO and UIO approaches and apply them to those systems. In this way, the UIO is generalized to a wider class of control systems. This is the subject of discussion in Section 6.2. In Section 6.3, a brief diagnostics approach is outlined.

6.1 Introduction to Sliding Mode Observer

Observers based on sliding mode concept [47] were first developed for linear systems [12, 50, 51]. The variable structure observer design approach or the SMO, has proved to be an effective estimator for certain nonlinear systems. Many applications of the SMO have been reported with success in the literature. The SMO approach has some advantages such as: cancelling certain nonlinearities, uncertainties or disturbances; estimating states without using exact linearization for nonlinear systems, etc. The basics of the SMO approach is outlined in the following.

Consider the following linear system in state space formulation,

$$\begin{cases} \dot{x} = Ax + Bu + Ed \\ y = Cx \end{cases} \quad (6.1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $d \in \mathbb{R}^r$ may be time-varying uncertainty.

The idea of classical SMO is: under certain assumptions (matching conditions), we can design an observer with one more nonlinear term in comparison to Luenberger

observer, such that the uncertainty d is cancelled by the extra term. As a result, the error dynamics are forced to an attracting region which is often referred to as the “sliding manifold”. Once the error trajectory is confined to the manifold, it will then asymptotically slides to the origin, and thus the estimation goal is achieved (see Figure 6.1).

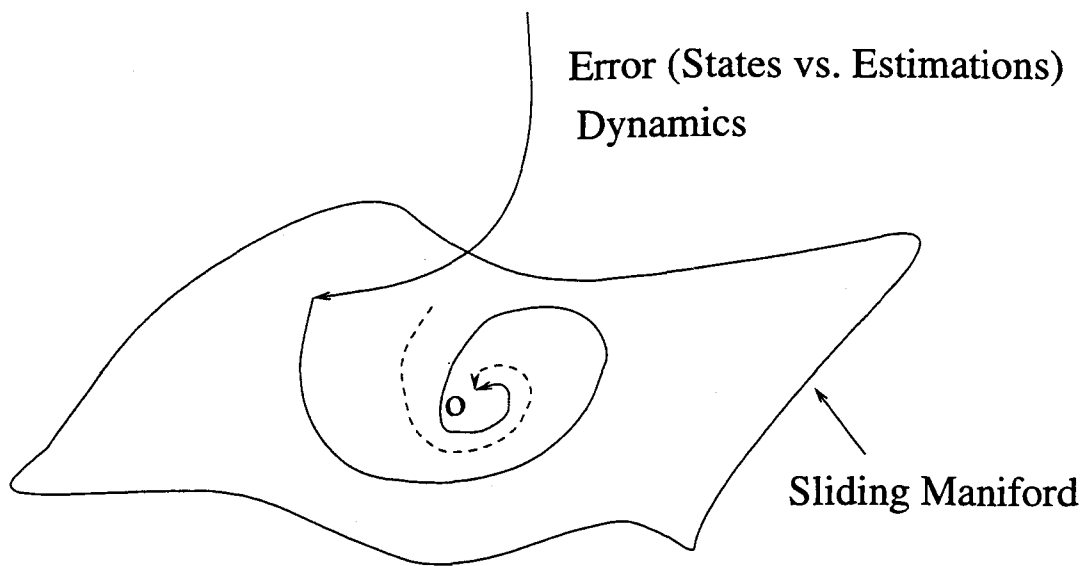


Figure 6.1: Idea of Sliding Mode Observer

In order to design such a SMO, the following certain requirements need to be satisfied. These are: **Assumption 6.1.1** (A, C) is observable. \square

From the Assumption 6.1.1, we know that for certain matrix K and $Q > 0$, there exists positive definite matrix P as a solution,

$$P(A - KC) + (A - KC)^T P = -Q \quad (6.2)$$

holds.

Assumption 6.1.2 (“Matching Condition”) Assume there exists L , such that

$$LC = E^T P \quad (6.3)$$

where positive definite matrix P is the solution of (6.2). \square

Under the Assumption 6.1.2, we can design an asymptotic SMO,

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + K(y - \hat{y}) + \gamma E \text{sign}(L(y - \hat{y})) \\ \hat{y} = C\hat{x} \end{cases} \quad (6.4)$$

where γ is just a scaling gain. This results in the error dynamics,

$$\dot{\tilde{x}} = (A - KC)\tilde{x} - \gamma E \text{sign}(L C \tilde{x}) + Ed \quad (6.5)$$

where $\tilde{x} \triangleq x - \hat{x}$.

Based on the result in [51], we know that when the Lyapunov function is set as $V = \tilde{x}^T P \tilde{x}$, then

$$\begin{aligned} \dot{V} &= \tilde{x}^T P \dot{\tilde{x}} + \dot{\tilde{x}}^T P \tilde{x} \\ &= \tilde{x}^T (P(A - KC) + (A - KC)^T P) \tilde{x} + 2\tilde{x}^T P E d - 2\tilde{x}^T P E \gamma \text{sign}(L \tilde{y}) \\ &= -\tilde{x}^T Q \tilde{x} + 2\tilde{x}^T P E d - 2\tilde{x}^T P E \gamma \text{sign}(L \tilde{y}) \\ &= -\tilde{x}^T Q \tilde{x} + 2(L \tilde{y})^T d - 2\|L \tilde{y}\|_1 \gamma \\ &\leq -\lambda_{\min}(Q) \|\tilde{x}\|^2 + 2\|d\| \|L \tilde{y}\| - 2\gamma \|L \tilde{y}\|_1. \end{aligned}$$

where $\|x\|_1 \triangleq \sum_{i=1}^n |x_i|$. We conclude from the above discussion that since $\|\cdot\| \leq \|\cdot\|_1$, and as long as γ is chosen such that $\gamma \geq \|d\|$, then the error dynamics approaches to zero asymptotically.

The above essentially sketched the SMO design approach for linear systems.

6.2 Practical Observations—Combined Sliding Mode, Unknown Input Observers (SM-UIO)

In this section, we shall treat linear and nonlinear systems separately.

6.2.1 Case 1: Linear System

Consider the following linear system,

$$\begin{cases} \dot{x} = Ax + Bu + E_1 d_1 + D\xi \\ y = Cx \end{cases} \quad (6.6)$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p, d_1 \in \mathbb{R}^{r_1}, \xi \in \mathbb{R}^s$, d_1 and ξ are uncertainties or unknown disturbances.

Similar to our discussions in Chapter 2, we can state the following result:

Lemma 6.2.1 If the following conditions,

1.

$$\text{Rank } CE_1 = \text{Rank } E_1 \quad (\text{Equivalently } \text{Ker}(C) \cap \text{Im}(E_1) = \{0\}) \quad (6.7)$$

2.

$$\text{Im}(D) \subset \text{Ker}(C) \quad (6.8)$$

then the system (6.6) can be transformed to

$$\begin{bmatrix} y_1 \\ y_2 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} u + \begin{bmatrix} E_1^1 \\ 0 \\ 0 \\ 0 \end{bmatrix} d_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ D_4 \end{bmatrix} \xi \quad (6.9)$$

where $y_1 \in \mathbb{R}^{r_1}$, $y_2 \in \mathbb{R}^{p-r_1}$, $z_1 \in \mathbb{R}^{n-p-s}$, $z_2 \in \mathbb{R}^s$, $E_1^1 \in \mathbb{R}^{r_1 \times r_1}$ is nonsingular matrix and so is $D_4 \in \mathbb{R}^{s \times s}$. The remaining matrices are of corresponding appropriate dimensions.

□

Proof. From condition ii, $\text{Im}(D) \subset \text{Ker}(C)$, we know

$$CD = 0.$$

Differentiate the output equation in system (6.6) to get

$$\dot{y} = C\dot{x} = CAx + CBu + CE_1d_1 + CD\xi,$$

i.e.,

$$\dot{y} = CAx + CBu + CE_1d_1.$$

Under condition i, we can apply the same partition process described in section 2.2.1 or Lemma 4.2.2 together with $\text{Rank } D = s$ and condition ii to system (6.6). Condition i guarantees the structure of parameter matrix in front of d_1 in (6.9), similarly condition ii will guarantee the form of parameter matrix in front of ξ . So that the system (6.6) can be transformed into (6.9) under conditions i and ii.

■

Basically, all the published literature of the design of the UIO, implicitly, or explicitly require that $\text{Rank } CE = \text{Rank } E$. The question is then what happen if this condition doesn't hold? This is addressed in the following:

Consider the following system,

$$\begin{cases} \dot{x} = Ax + Bu + Ed \\ y = Cx \end{cases} \quad (6.10)$$

where $d \in \mathbb{R}^r$, $p \geq r$, and the remaining matrices are the same as those in (6.6).

Lemma 6.2.2 If the rank condition discussed above is not satisfied, that is if,

$$\text{Rank } CE \neq \text{Rank } E, \quad (\text{Equivalently } \dim(\text{Ker}(C) \cap \text{Im}(E)) = s, s \neq 0),$$

where $s \triangleq \text{Rank } E - \text{Rank } CE$, then the system (6.10) can be transformed into (6.9).

□

Proof. Find linear independent and orthonormal vectors t_i , such that $t_i \in \text{Ker}(C)$, $i = 1, \dots, n - p$; and $T \triangleq [t_1, \dots, t_{n-p}]$; $\begin{bmatrix} C \\ T^T \end{bmatrix}$ is an $n \times n$ nonsingular matrix. It is easily verified that

$$\begin{bmatrix} C \\ T^T \end{bmatrix}^{-1} = [C^T(CC^T)^{-1} \quad T].$$

We know

$$\begin{aligned} E &= I \cdot E = [C^T(CC^T)^{-1} \quad T] \begin{bmatrix} C \\ T^T \end{bmatrix} E \\ &= C^T(CC^T)^{-1}CE + TT^TE = E_C + E_T \end{aligned}$$

where $E_C \triangleq C^T(CC^T)^{-1}CE$, $E_T \triangleq TT^TE$. Furthermore, we have $E = E_C + E_T$, also $CE = CE_C$, $CE_T = 0$, $E_C^TE_T = 0$, i.e., E_C and E_T are orthogonal. By Sylvester Inequality,

$$\text{Rank } CE \geq \text{Rank } C + \text{Rank } E - n = p + r - n$$

so that

$$s = \text{Rank } E - \text{Rank } CE \leq r - (p + r - n) = n - p.$$

Therefore, we can find a column transformation ($r \times r$ matrix) E_0 , such that $E_C E_0 = [\underline{E}_C \quad 0_{n \times s}]$, $E_T E_0 = [0_{n \times (r-s)} \quad \underline{E}_T]$ where \underline{E}_C is $n \times (r - s)$ with rank $r - s$, \underline{E}_T is $n \times s$ with rank s .

$$\underline{d} = E_0^{-1}d = \begin{bmatrix} \underline{d}_1 \\ \underline{d}_2 \end{bmatrix}, \quad \underline{d}_1 \in \mathbb{R}^{r-s}, \underline{d}_2 \in \mathbb{R}^s.$$

If we replace \underline{d}_1 with d_1 in (6.6) and \underline{d}_2 with ξ in (6.6), then the conditions in Lemma 6.2.1 hold for

$$\begin{cases} \dot{x} = Ax + Bu + \underline{E_C}d_1 + \underline{E_T}\xi \\ y = Cx \end{cases} \quad (6.11)$$

So, according to Lemma 6.2.1, the system (6.11) as well as (6.10) can be transformed to the form in (6.9). This completes the proof. ■

Theorem 6.1 For system (6.10), if

$$\text{Rank} \begin{bmatrix} \lambda I - A & E_C \\ C & 0 \end{bmatrix} = \text{constant}, \quad \forall \lambda \quad (6.12)$$

then UIO can be designed as long as $\text{Rank } CE = \text{Rank } E$.

Based on this theorem the following can be stated.

Corollary 6.2.1 When $s \neq 0$ for transformed equation (6.9), besides condition (6.12), if there exists matrix L , such that the matching condition

$$L[A_{23} \quad A_{24}] = [0 \quad D_4^T]P \quad (6.13)$$

holds, where the positive definite matrix P is the solution of

$$P \left(\begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} - K[A_{23} \quad A_{24}] \right) + \left(\begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} - K[A_{23} \quad A_{24}] \right)^T P = -Q \quad (6.14)$$

then the sliding mode observer for the transformed system (6.9) can be designed as following,

$$\begin{aligned} \dot{w} = & \left(\begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} - K[A_{23} & A_{24}] \right) w + \left(\begin{bmatrix} A_{31} \\ A_{41} \end{bmatrix} - K A_{21} \right) y_1 \\ & + \left\{ \left(\begin{bmatrix} A_{32} \\ A_{42} \end{bmatrix} - K A_{22} \right) - \left(\begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} - K[A_{23} & A_{24}] \right) K \right\} y_2 \\ & + \left(\begin{bmatrix} B_3 \\ B_4 \end{bmatrix} - K B_2 \right) u \end{aligned} \quad (6.15)$$

$$\begin{aligned} & + L \text{sign}(\dot{y}_2 - A_{21}y_1 - A_{22}y_2 - B_2u - [A_{23} \quad A_{24}](w + Ky_2)) \\ \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} = & w + Ky_2. \end{aligned} \quad (6.16)$$

Proof. The proof of the theorem and the corollary is provided here at once. If equation (6.12) holds, and $\text{Rank } CE = \text{Rank } E$, then from Lemma 6.2.2, we know that in the transformed system (6.9), $D_4 = 0$. Under such situation, we can use Proposition 2.2.2 and Theorem 2.1 to design a proper UIO.

The UIO is similar as the one described in subsection 2.2.1, the design process is same as in subsection 2.2.1,

$$\begin{aligned} \dot{w} = & \left(\begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} - K[A_{23} & A_{24}] \right) w + \left(\begin{bmatrix} A_{31} \\ A_{41} \end{bmatrix} - K A_{21} \right) y_1 \\ & + \left\{ \left(\begin{bmatrix} A_{32} \\ A_{42} \end{bmatrix} - K A_{22} \right) - \left(\begin{bmatrix} A_{33} & A_{34} \\ A_{43} & A_{44} \end{bmatrix} - K[A_{23} & A_{24}] \right) K \right\} y_2 \\ & + \left(\begin{bmatrix} B_3 \\ B_4 \end{bmatrix} - K B_2 \right) u \end{aligned} \quad (6.17)$$

$$\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} = w + Ky_2. \quad (6.18)$$

If $\text{Rank } CE \neq \text{Rank } E$, that means we can have system (6.10) transformed to (6.9) with $D_4 \neq 0$. Combine the UIO design together with sliding mode observer design in section 6.1. In this case, we know under conditions (6.13)–(6.14), we have sliding observer in form of (6.15)–(6.16), and the error dynamics approach to zero asymptotically. ■

Note that in the dynamics of the SMO described by (6.15) there appear \dot{y}_2 in the discontinuous part of the observer's dynamics. If the output y_2 is slow varying, then the approximation for y_2 can be derived and utilized in the implementation of the SMO. Also in discrete systems, \dot{y}_2 becomes $y_2(k+1)$ which is easy to implement.

Lemma 6.2.3 If (6.13) holds, the following

$$\text{Rank} \begin{bmatrix} A_{23} & A_{24} \end{bmatrix} \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = \text{Rank} \begin{bmatrix} 0 \\ D_4 \end{bmatrix}$$

is valid. □

Proof. Since $L[A_{23} \ A_{24}] = [0 \ D_4^T]P$, then

$$L[A_{23} \ A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = [0 \ D_4^T]P \begin{bmatrix} 0 \\ D_4 \end{bmatrix}$$

is valid.

On the other hand,

$$[0 \ D_4^T]P \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = [0 \ D_4^T] \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = D_4^T P_{22} D_4,$$

and from the property of positive definite matrix P we know $P_{22} > 0$, thus $\text{Rank } D_4^T P_{22} D_4 = s$.

Hence,

$$\text{Rank } L[A_{23} \quad A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = s.$$

Also,

$$\text{Rank } L[A_{23} \quad A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} \leq \text{Rank } [A_{23} \quad A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} \leq \text{Rank} \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = s.$$

Therefore we have,

$$\text{Rank } [A_{23} \quad A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = s,$$

i.e.,

$$\text{Rank } [A_{23} \quad A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = \text{Rank} \begin{bmatrix} 0 \\ D_4 \end{bmatrix}.$$

This completes the proof. ■

As in the above lemma and its proof, we know that condition (6.3) infers the rank condition, $\text{Rank } CE = \text{Rank } E$.

To summarize the SM-UIO design consider the following step by step procedure.

SM-UIO Design Algorithm: (Assuming $s \neq 0$ and (6.12) holds)

1. For system (6.10), using Lemmas 6.2.1–6.2.2, we can transform (6.10) into the form of (6.9).
2. If equation (6.13) holds for (6.9), then sliding mode observer for unknown input can be designed by (6.15) and (6.16); **STOP**.

Otherwise, **CONTINUE**.

3. The dynamics of z is taken as the first equation in system (6.10), output (this output is generated from the original output and its derivative, $\dot{y}_2 - A_{21}y_1 - A_{22}y_2 - B_2u$) equation is substituted with $y = [A_{23} \ A_{24}]z$.

4. If

$$\text{Rank} [A_{23} \ A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = \text{Rank} \begin{bmatrix} 0 \\ D_4 \end{bmatrix},$$

then UIO of the form (6.17)–(6.18) can be designed when $\text{Rank} [A_{23} \ A_{24}] > \text{Rank} D_4$ and (6.12) holds for the new system; only bounded estimation can be achieved when $\text{Rank} [A_{23} \ A_{24}] = \text{Rank} D_4$; **STOP**.

If

$$0 \neq \text{Rank} [A_{23} \ A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} < \text{Rank} \begin{bmatrix} 0 \\ D_4 \end{bmatrix},$$

then **REPEAT 1**;

If

$$\text{Rank} [A_{23} \ A_{24}] \begin{bmatrix} 0 \\ D_4 \end{bmatrix} = 0,$$

then only bounded estimation can be achieved according to the bounds of certain elements of d corresponding with D_4 . **STOP**.

Remark 6.2.4 If the above algorithm is terminated after σ -flops, then the discontinuous term in SMO involves σ -order differentiate of output as described in (6.15). If y and its derivatives of y introduce excessive noise during the implementation of the last algorithm, we have to resort to noise filtering to get better estimates. In fact, the higher the degree of the derivatives of output, the worse the estimation might be.

□

Remark 6.2.5 Since the matching condition for SMO can lead to the rank condition for UIO, one may ask: does that mean the existence of SMO will lead to the existence of UIO? As a matter of fact, in the UIO design the necessary condition for existence of observer with arbitrarily assignable rate of convergence is that the number of outputs has to be greater than the unknown inputs. This is necessary since the extra outputs are used for the design of decoupled reduced order observer. In SMO, there is no such restriction, instead there is requirements in terms of the boundedness of the unknown inputs. So if the number of outputs are greater than the number of unknown inputs, as long as we can design SMO, it is likely that we can also design UIO. In this case, UIO is even better, since we don't need to know the bound information for the unknown inputs. The only point is that for UIO, we need to do the transformation in order to design the observer. For SMO, the design is easier and no transformation is needed. However, if the number of outputs is same as the number of the unknown inputs, then UIO may or may not exist [41]. In such case, even if the UIO exist, its rate of convergence may not be controlled by the designer. SMO is more suitable in such case.

□

Remark 6.2.6 From Lemma 6.2.3, we know if the Assumptions 6.1.1–6.1.2 hold, then we get $\text{Rank } CE = \text{Rank } E$. For UIO, we need $\text{Rank } C > \text{Rank } CE = \text{Rank } E$, but for SMO, we may have $\text{Rank } C = \text{Rank } E$. The reason is that for SMO the uncertainty term is considered as bounded; and for UIO the uncertainty term (unknown input) is assumed completely unknown, so that extra information on output ($\text{Rank } C - \text{Rank } E$ elements) may be needed for observer design.

□

6.2.2 Case 2: Nonlinear System

In practice, many of nonlinear systems involve various discontinuous terms which are not as smooth as the systems we discussed in Chapter 5. Different observer design is therefore needed [1]. The SMO is one alternative for nonlinear dynamical systems. In the following, we shall briefly discuss the SMO design for nonlinear systems.

Consider the following state space representation of a nonlinear system

$$\begin{cases} \dot{x} = Ax + Bu + f(x, u, t) \\ y = Cx \end{cases} \quad (6.19)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, where $p \geq m$. Function $f(x, u, t)$ represents nonlinearity, uncertainty, disturbance, etc..

Assumption 6.2.1 There exists matrix E , such that f can be written as the form $f(x, u, t) = E\xi$. □

Under Assumption 6.2.1, the system (6.19) becomes

$$\begin{cases} \dot{x} = Ax + Bu + E\xi \\ y = Cx \end{cases} \quad (6.20)$$

which is same as the system described by (6.10).

If we know the bound of ξ , we can follow the procedures introduced in last section. If condition (6.12) is satisfied for equation (6.20), then at least bounded estimation can be achieved by using the SM-UIO Design Algorithm in the above section.

The approach used here may not give a very accurate state (asymptotic) estimation, but it gives us a simple and practical design for a large class of nonlinear systems as compared to the diffeomorphism linearization UIO discussed in the last chapter.

6.3 Fault Diagnosis

From the above discussion, we know that SMO is more suitable for those systems with bounded unknown inputs, and UIO is suitable for the systems with completely unknown information. So in such case, if the faulty signals are bounded, then we may try to use SMO, with regard to the detection approach we may use similar methods we used in previous chapters. If the faulty signals are unbounded, then we may try to use UIO.

For the systems whose outputs are slow varying, then the approximation of output's derivative is acceptable, so that the SM-UIO Design Algorithm can be utilized. For discrete systems, SM-UIO Design Algorithm can be easily implemented. Otherwise, different approach should be introduced for the fault diagnosis.

6.4 Summary

In conclusion, the previous chapters of this dissertation used the common thread of the unknown input observer methodology for designing observers for linear, bilinear, time delay, and nonlinear systems. In each case appropriate observers for the class of system under consideration was proposed. In each chapter, once the appropriate observer was designed, it was then used for the purpose of sensor and actuator FDI, and proper fault detection and isolation schemes were discussed. However, as we progressively moved from a simpler class of systems (i.e. linear systems) to a more difficult classes of systems (i.e. nonlinear and time delay) systems, it was realized that additional conditions needed to be satisfied for the UIO based FDI to be feasible. At this point It should be emphasized that at the moment, for the type of FDI problems

that we studied in this work, all the available existing approaches in the literature have similar restrictions and requirements. This is being pointed out so that the reader is not led to believe that it is the approach undertaken in this work that poses such limitations and restrictions. Nevertheless, these restrictions are there, and the classes of systems for which FDI using the proposed approaches discussed in this thesis are limited. Currently, much of the work in the research community deals with enlarging the class of systems for which model based FDI can be feasible. This chapter dealt with an attempt in this direction, based on the work presented in the previous chapters. In essence, it was shown that by combining the sliding mode and UIO methodologies, perhaps some of the restrictions that could arise in the previous chapters could be relaxed and the FDI may still be possible.

Chapter 7

Conclusions

The unknown input observer design and its application for fault detection and isolation of dynamical systems was considered in this thesis work. The main contribution of this thesis was the extension of the UIO theory for FDI in purposes in linear systems to time delay, bilinear, and a more general class of nonlinear systems.

The unknown input observer is designed in such a way that the estimation is decoupled from the unknown input. For UIO, we normally require that the number of outputs p , to be greater than the number of unknown inputs m , i.e. $p > m$, in order to guarantee the existence of UIO. One important conclusion that was drawn from the work that was developed here is that even with the use of an sliding mode observer (SMO) this condition can not be relaxed much further. In Chapter 6, we showed that with an SMO, still the requirement for the design of the fault detection observer would be $p \geq m$. This is a very important result in that it underscores the fundamental importance of this requirement. That the condition $p \geq m$ needs to be satisfied, and is not only a limitation in the UIO approach to FDI. Another interpretation of this requirement may be that in fact, the observer design is a kind of

inverse problem, so the largest number of independent unknown recoverable signals must be less than (or equal) the number of independent inputs to the observer.

From a practical point of view, it is very likely that in a complex dynamical system with many sensor and actuators, a single UIO may not possibly diagnose all the faults in the system. In such systems, it is likely that a bank of observers using different sets of information need to be used for a more complete FDI. By re-grouping different elements of output, and using them to design inherently different (unknown input) observers, we can detect and isolate actuator and sensor faults via some logic operation on residuals coming from different observers and the measurements.

At present, SMO is very popularly applied in nonlinear systems, and at a first glance may seem to be more powerful than UIO. This is the reason that in this thesis the possibility of using the SMO for FDI was investigated in the first place. However, from the results presented in Chapter 6 a conclusion can be drawn that UIO theory is just as powerful as that of the SMO. If the bounds on the unknown inputs are not known, then UIO is even more powerful than the SMO. For further development on observer-based fault diagnosis, more thorough investigation on the relationship between UIO and SMO will be needed. Additional results on the unifying design approaches may be achieved in the near future. For nonlinear systems, as long as the theory of observer design can be developed, then the observer-based fault detection and isolation techniques may also be improved. However, the fact remains that a major stumbling block in the FDI for general nonlinear system is the design of stable observers. This of course points to a more fundamental point that the control and system theory for nonlinear systems in general requires a great deal of maturing in the years to come.

In order to apply the FDI concepts in grand scale applications, such as at a factory wide level, it is necessary to broaden the approach by a great deal.

It is obvious to us that the analytical redundancy (AR) based techniques described in this thesis, although powerful will not be adequate to deal with diagnosing a large variety of faults (specifically those that can not be mathematically modeled). In addition, since the effect of certain faults may propagate through the system, this kind of diagnosis, and post fault analysis for determining possible compensation would require greater knowledge of the system and intelligent reasoning capabilities. The goal here is to expand the space of the possible faults from only controller faults i.e., sensors and actuators (as would be the case in our initial phase of the study), to a larger one consisting of plant components as well as controller failures. We feel that it may be still possible to detect such failures by using AR based schemes. However, the identification of the faulty device(s), effect of the failure on the system, and accommodation by issuing command for restructuring the control system, would most likely be not possible by AR means. Therefore, if a fault has been detected in the lowest level of the hierarchy, and it can not be identified, (e.g. if we arrive at the conclusion that all the sensor readings are abnormal and that no actuator has failed), the measurements and the estimates are to be supplied through the interface to the higher level where the inference engine would initiate a search (either depth first or breadth first) through the knowledge base for determining the cause of the failure. The knowledge about the system will be represented through the use of fault trees or cause and consequence diagrams [48]. These trees not only are of use in the search process but also provide valuable information as to the weak areas of the system that can cause frequent failures, and therefore are of use in the design stages as well. Once

the faulty instrument is detected a command may trigger the system reconfiguration subsystem to assess the impact of the fault and a command from this level would be issued to the lowest level for restructuring the control system, if necessary, to bring the plant into a safe state.

There are variety of other tasks that need to be considered at this level. These include issues such as interfacing with an operator, scheduling, overall optimization, global planning, and conflict resolution. These issues are all topics for future research.

In summary, it is felt that for complicated industrial systems, many tools and theories from model-based, knowledge or data-based methods, such as expert systems, fuzzy logic, neural networks, statistical techniques and their combinations need to be employed in order to arrive at a truly powerful and intelligent FDI architecture. Figure 7.1 illustrates one such possible architecture.

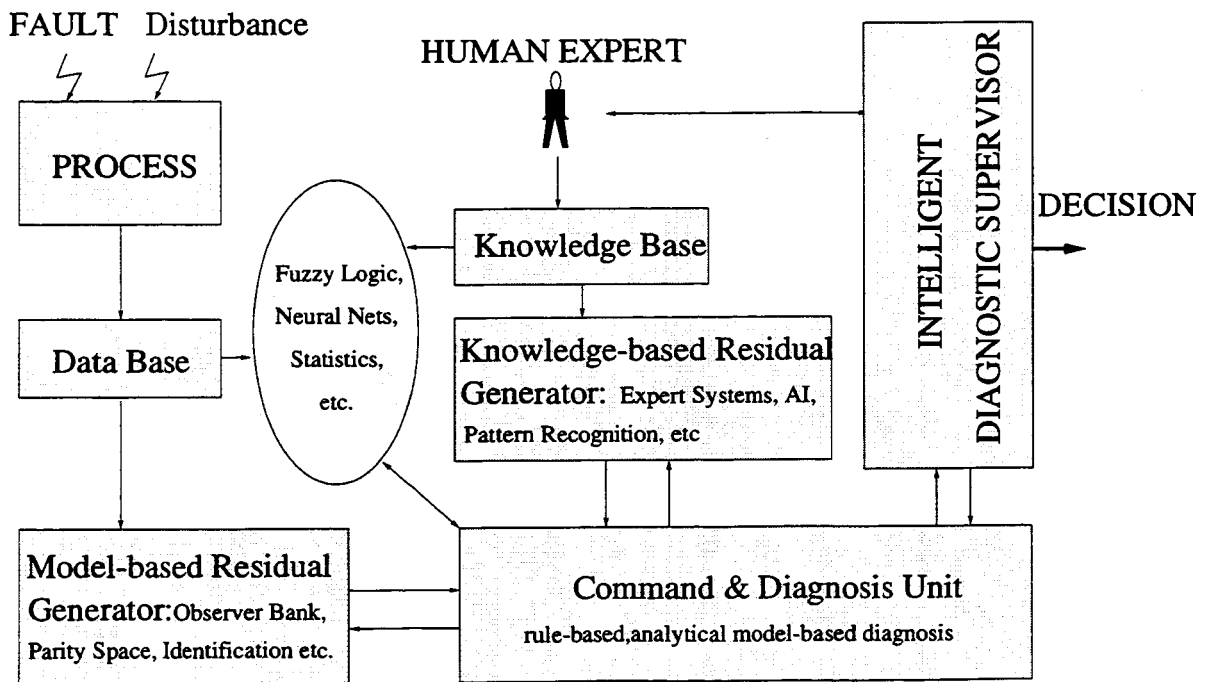


Figure 7.1: An Intelligent Monitoring and Diagnostic Architecture

At present, further investigation and theoretical foundations are needed for those knowledge-based fault diagnosis schemes. The inclusions of learning algorithms, and connections between model-based and knowledge-based techniques are also important topics in both research and applications.

Bibliography

- [1] K. Adjallah, D. Maquin, & J. Ragot, "Non-Linear Observer-Based Fault Detection," *Proceedings of IEEE Conference on Control and Applications*, pp. 1115–1120, 1994.
- [2] R.V. Beard, "Failure Accommodation in Linear Systems through Self-Reorganization," Ph.D. dissertation, Dept. Aeronautics and Astronautics, Mass. Inst. Technology, Cambridge, MA, February 1971.
- [3] C. Bruni, G. DiPillo & G. Koch, "Bilinear Systems: An Appealing Class of "Nearly Linear" Systems in Theory and Applications," *IEEE Trans. Aut. Control*, Vol. AC-19, No. 4, pp. 334–348, 1974.
- [4] S.K. Chang, W.T. You & P.L. Hsu, "General-Structured Unknown Input Observers," *Proceedings of the American Control Conference*, Baltimore, Maryland, USA, pp. 666–670, 1994.
- [5] J. Chen, R.J. Patton & H.-Y. Zhang, "Design of Unknown Input Observers and Robust Fault Detection Filters," *International Journal of Control*, Vol. 63, No. 1, pp. 85-105, 1996.

- [6] D. Cho & P. Paoella, "Model-Based Failure Detection and Isolation of Automotive Powertrain Systems," *Proceedings of the American Control Conference*, pp. 2898–2905, 1990.
- [7] E.Y. Chow & A.S. Wilsky, "Analytical Redundancy and the Design of Robust Detection Systems," *IEEE Transactions on Automatic Control*, Vol. AC-29, No. 7, pp.603–614, 1984.
- [8] R.N. Clark, "Instrument Fault Detection," *IEEE Transactions on Aero. & Electron. Syst.*, Vol. AES-14, pp. 456–465, 1978.
- [9] W.P. Dayawansa, W.M. Boothby & D.L. Elliott, "Global State and Feedback Equivalence of Nonlinear Systems," *Systems & Control Letters*, Vol. 6, pp. 229–234, 1985.
- [10] I. Derese, P. Stevens & E. Noldus, "Observers for Bilinear Systems with Bounded Input," *International Journal of Systems Science*, Vol. 10, No. 6, pp. 649–668, 1979.
- [11] X. Ding, "Frequenzbereichsverfahren zur beobachtergestützten Fehlerentdeckung," Ph.D. Thesis, University of Duisburg, Germany, 1992.
- [12] S.V. Drakunov, "Sliding-Mode Observers Based on Equivalent Control Method," *Proceedings of IEEE 31st Conference on Decision and Control*, pp. 2368–2369, Tucson, Arizona, 1992.
- [13] P.M. Frank, "Advances in Observer-Based Fault Diagnosis," *TOOLDIAG'93 International Conference on Fault Diagnosis*, Toulouse, France, 1993.

- [14] P.M. Frank, "Enhancement of Robustness in Observer-Based Fault Detection," *International Journal of Control*, Vol. 59, No. 4, pp. 955–981, 1994.
- [15] P.M. Frank, "Frequency Domain Approach to Optimally Robust Residual Generation and Evaluation for Model-based Fault Diagnosis," *Automatica*, Vol. 30, No. 5, pp. 789–804, 1994.
- [16] W. Ge & C.Z. Fang, "Detection of Faulty Components via Robust Observation," *International Journal of Control*, Vol. 49, pp. 1537–1553.
- [17] J. Gertler, "Analytical Redundancy Methods in Failure Detection and Isolation," *IFAC-Symposium SAFEPROCESS'91*, Baden-Baden, Vol. 1, pp. 9–21, 1991.
- [18] Y. Guan & M. Saif, "A Novel Approach to the Design of Unknown Input Observers," *IEEE Transaction on Automatic Control*, Vol. AC-36, No. 5, pp. 632–635, 1991.
- [19] J. Hale, *Theory of Functional Differential Equations*, Springer-Verlag, 1977.
- [20] H. Hamidi-Hashemi & C.T. Leondes, "Observer Theory for Systems with Time Delay," *International Journal of Systems Science*, Vol. 10, No. 7, pp. 797–806, 1979.
- [21] S. Hara & K. Furuta, "Minimal Order State Observers for Bilinear Systems," *International Journal of Control*, Vol. 25, No. 5, pp. 705–718, 1976.
- [22] M. Hou & P.C. Müller, "Design of Observers for Linear Systems with Unknown Inputs," *IEEE Transactions on Automatic Control*, Vol. AC-37, No. 6, pp. 871–874, June 1992.

- [23] S. Hui & S.H. Zak, "Low-Order State Estimators and Compensators for Dynamical Systems with Unknown Inputs," *Systems & Control Letters*, Vol. 21, pp. 493-502, 1993.
- [24] R. Isermann, "Process Fault Detection Based on Modelling and Estimation Methods: A Survey," *Automatica*, Vol. 20, pp. 387-404, 1984.
- [25] R. Isermann, "Model Based Fault Detection and Diagnosis Methods and Their Applications," *Proceedings of the American Control Conference*, Seattle, WA, USA, pp. 1605-1609, 1995.
- [26] R. Isermann & B. Freyermuth, "Process Fault Diagnosis Based on Process Model Knowledge—Part I: Principles for Fault Diagnosis with Parameter Estimation," *Transactions of the ASME*, Vol. 113, pp. 620-626, 1991.
- [27] H.L. Jones, "Failure Detection in Linear Systems," Ph.D. dissertation, Dept. Aeronautics and Astronautics, Mass. Inst. Technology, Cambridge, MA, August 1973.
- [28] T. Kailath, *Linear Systems*, PRENTICE-HALL, INC. 1980.
- [29] H. Khalil, *Nonlinear Systems*, Maxwell Macmillan, 1992.
- [30] M. Kinnaert & Y. Peng, "Residual Generator for Sensor and Actuator Fault Detection and Isoaltion: a Frequency Domain Approach," *International Journal of Control*, Vol. 61, No. 6, pp. 1423-1435, 1995.
- [31] A.J. Krener & A. Isidori, "Linearization by Output Injection and Nonlinear Observers," *System & Control Letters*, Vol. 3, No. 1, pp. 47-52, 1983.

- [32] R. Marino, "Adaptive Observers for Single Output Nonlinear Systems," *IEEE Transaction on Automatic Control*, Vol. AC-35, No. 9, pp. 1054-1058, 1990.
- [33] M.-A. Massoumnia, "A Geometric Approach to the Synthesis of Failure Detection Filters," *IEEE Transactions on Automatic Control*, Vol. AC-31, No. 9, pp. 839-846, September 1986.
- [34] M.-A. Massoumnia, G.C. Verghese & A.S. Wilsky, "Failure Detection and Identification," *IEEE Transactions on Automatic Control*, Vol. AC-34, No. 3, pp. 316-321, March 1989.
- [35] B.A. Ogunnaike, "A New Approach to Observer Design for Time-Delay Systems," *Internation Journal of Control*, Vol. 33, No. 3, pp. 519-542, 1981.
- [36] R.J. Patton & J. Chen, "A Review of Parity Space Approaches to Fault Diagnosis," *IFAC-Symposium SAFEPROCESS'91*, Baden-Baden, Vol. 1, pp. 239-255, 1991.
- [37] R.J. Patton & J. Chen, "Review of Parity Space Approaches to Fault Diagnosis for Aerospace Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 2, pp. 278-284, 1994.
- [38] R.J. Patton & S.M. Kangethe, "Robust Fault Diagnosis Using Eigenstructure Assignment of Observers," *Fault Diagnosis in Dynamic Systems, Theory and Application*, R.J. Patton, P.M. Frank, R.N. Clark (eds), Prentice Hall, 1989.
- [39] J. Ragot, D. Maguin & F. Kratz, "Analytical Redundancy for Systems with Unknown Inputs—Application to Faults Detection," "Unknown-Input Observers," *Control-Theory and Advanced Technology*, Vol. 9, No. 3, pp. 775-788, 1993.

- [40] M. Saif, "A Disturbance Accommodating Estimator for Bilinear Systems," *Control-Theory and Advanced Technology*, Vol. 10, No. 3, pp. 431–446, 1994.
- [41] M. Saif & Y. Guan, "A New Approach to Robust Fault Detection & Identification," *IEEE Transaction on Aerospace and Electronic Systems*, Vol. 29, No. 3, pp. 685–695, 1993.
- [42] H. Schneider & P.M. Frank, "Observer-based Supervision and Fault Detection for Robots," *TOOLDIAG'93 International Conference on Fault Diagnosis*, Toulouse, France, 1993.
- [43] H. Schwartz & R. Ingenbleek, "Observing the State of Hydraulic Drives via Bilinear Approximated Models," *Control Engineering Practice*, Vol. 2, No. 1, pp. 61–68, 1994.
- [44] A. Steinberg & M. Corless, "Output Feedback Stabilization of Uncertain Dynamical Systems," *IEEE Transaction on Automatic Control*, Vol. 30, No. 10, pp. 1025–1027, 1985.
- [45] R.H.C. Takahashi & P.L.D. Peres, "Unknown Input Observers for Uncertain Systems: A Unifying Approach and Enhancements," *Proceedings of the 35th Conference on Decision and Control*, pp. 1483–1488, Kobe, Japan, 1996.
- [46] A. Tornambe, "Simple Observer-Based Control Law for Time Lag Systems," *International Journal of Systems Science*, Vol. 23, No. 9, pp. 1463–1473, 1992.
- [47] V.I. Utkin, *Sliding Modes and Their Application Variable Structure Systems*, Moscow: MIR, 1978.

- [48] F.E. Villaseca, M. Saif & M. Fathizadeh, *Alternative Architectures for a Space Station Power System*, **Technical Report NAG 3-518**, submitted to NASA Lewis Research Center, Cleveland, Ohio, January 1985.
- [49] N. Viswanadham & R. Srichander, "Fault Detection Using Unknown-Input Observers," *Control-Theory and Advanced Technology*, Vol. 3, No. 2, pp. 91-101, 1987.
- [50] B.L. Walcott, M.J. Corless & S.H. Zak, "Comparative Study of Nonlinear State Observation Techniques," *International Journal of Control*, Vol. 45, No. 6, pp. 2109-2132, 1987.
- [51] B.L. Walcott & S.H. Zak, "Observation of Dynamical Systems in the Presence of Bounded Nonlinearities/Uncertainties," *Proceedings of IEEE 25th Conference on Decision and Control*, pp. 961-966, Athens, Greece, 1986.
- [52] K. Watanabe & D.M. Himmelblau, "Instrument Fault Detection in Systems with Uncertainties," *International Journal of System Science*, Vol. 13, pp. 137-158, 1982.
- [53] J.E. White & J.L. Speyer, "Detection Filter Design: Spectral Theory & Algorithm," *IEEE Transactions on Automatic Control*, Vol. AC-32, No. 7, pp. 593-603, 1987.
- [54] A.S. Wilsky, "A Survey of Design Methods for Failure Detection in Dynamic Systems," *Automatica*, Vol. 12, No. 6, pp. 601-611.
- [55] J. Wünnenberg, "Observer-Based Fault Detection in Dynamic Systems," Ph.D. Thesis, University of Duisburg, Germany, 1990.

- [56] X. Xia & W.-B. Gao, "Nonlinear Observer Design by Observer Error Linearization," *SIAM J. Control Optim.*, Vol. 27, pp. 199–216, 1989.
- [57] H. Yang & M. Saif, "State Observation, Failure Detection and Isolation (FDI) in Bilinear Systems," *Proceedings of IEEE 34th Conference on Decision and Control*, pp. 2391-2396, New Orleans, LA, 1995. (Revised version was accepted for publication by *International Journal of Control*).
- [58] H. Yang & M. Saif, "Monitoring and Diagnostics of a Class of Nonlinear Systems Using a Nonlinear Unknown Input Observer," *Proceedings of IEEE Conference on Control and Applications*, Dearborn, MI, 1996.
- [59] H. Yang & M. Saif, "Observer Design and Fault Diagnosis for Retarded Dynamical Systems," *Proceedings of IEEE 35th Conference on Decision and Control*, pp. 1149–1154, Kobe, Japan, 1996. (Revised version was submitted to *Automatica*).
- [60] H. Yang & M. Saif, "Fault Detection and Isolation for a Class of Nonlinear Systems Using an Adaptive Observer," to appear in *IEEE American Control Conference*, Albuquerque, NM, June 1997.
- [61] H. Yang & M. Saif, "Robust Observation and Fault Diagnosis in a Class of Time-Delay Control Systems," to appear in *IEEE American Control Conference*, Albuquerque, NM, June 1997.
- [62] D. Yu, D.N. Shields & J.L. Mahtani, "Fault Detection for Bilinear Systems with Application to a Hydraulic System," *Proceedings of IEEE CCA 94*, pp. 1379–1384, 1994.