

**EXTRACTION OF MARKET EXPECTATIONS FROM OPTION PRICES: EVIDENCE
FROM THE CURRENT FINANCIAL CRISIS**

presented by

Aimée R. Kanyankogote

B. Comm. Finance and Actuarial Mathematics, University of Manitoba 2003

Carlos A. Palomino,

B. Econ, ITESM, Monterrey, N.L. Mexico 2002

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Approval

Name: **Aimée Kanyankogote and Carlos Palomino**

Degree: **Master of Business Administration**

Title of Project: **Extraction of Market Expectations from Option prices:
Evidence from the Current Financial Crisis**

Supervisory Committee:

ChristinaAtanasova

Senior Supervisor

Assistant Professor, Finance

Evan Gatev

Second Reader

Assistant Professor, Finance

Date Approved: August 10th, 2009

Abstract

This project estimates risk neutral parameters of a jump diffusion model, as in Bates (1991), implicit in the option prices on the S&P500 futures over the period 2006-2008. Additionally, it investigates the extent to which market participants anticipated the financial market crash of 2008. We find that high levels of skewness premium are detectable in the short maturity out-of-the-money put options as early as July 2007. Nevertheless, market expectations of an extreme downturn subsided after the collapse of Bear Stearns in April 2008. Overall, our findings indicate that the estimated parameters show the presence of crash expectations prior to September 2008 but there is no evidence that the magnitude of the crash was predictable.

Keywords: Option pricing; Jump-diffusion; Probability density function, Skewness premium; non-central moment; Martingale; Bubble

Dedication

I dedicate this paper to my beloved parents, Patience BISAGA and Prosper KANYANKOGOTE for all their unconditional love and support. I am forever grateful to them both for instilling the values and faith that are my moral compass today.

Thanks to my dear aunt and uncle, Mely and Bazir and to my brothers, sisters, cousins and the rest of the family for their support, love and honesty.

Special Thanks to my beloved grandparents for their endless support and encouragements to follow my dreams in the early years. I carry your memories in my heart every day.

Aimée R.K.

I dedicate this paper to my beloved parents, my two lovely sisters and the rest of the family. I could not have accomplished all I have in life so far without your unbounded love, endless support and encouragements. For this, I am forever grateful.

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1. Introduction

After every major market crash since the Great Depression, pundits, academics, financial reporters and professionals alike, rush to offer possible explanations for causes of the latest crash, warning signs that might have been missed, and propose preventive measures to avoid the next big one. Others are of the opinion that extreme financial crashes, popularly referred to as Black Swan events¹, cannot be foreseen thereby prevented.

Various factors contribute to the occurrence of crashes. There is an on-going debate and despite the well accepted Theory of Rational Bubbles in Asset Prices, it still remains unclear why bubbles burst at the time they do². However, the key issue is to understand what a financial bubble consists of. A bubble, according to Cox and Hobson, is “in economic terms a deviation between the trading price of an asset and its underlying value.” Please see Exhibit I for definitions and a list of characteristics of a bubble.

Andersen and Sornette (2004) provide an alternative model with “exponential growth followed by a downward jump (or crash) at an unpredictable stopping time”. Many analysts and academics have characterized the growth in the subprime mortgage segment and subsequent crash in similar terms. The reason bubbles exist is due to irrational behavior and fear in the market. Misleading credit ratings issued by major ratings agencies superimposed with a lack of transparency in terms of companies disclosing exposures to mortgage related derivatives are believed to have contributed to the irrational behavior.

¹Nassim Nicholas Taleb, “The Black Swan, 2007

²Benninga, Loewenstein, Sarig, Oded, Simon “A note on market expectation of risk-free rates and volatilities before and after 1987.” Journal of Banking & Finance, 1993

There is a consensus that financial markets are driven by expectations. Central banks, policy makers, financial analysts and other market participants rely on forecasting tools to extract expectations containing invaluable information when setting short term as well as long term economic policies and strategies. They use not only options but also financial instruments such as forwards to extract expectations of spot prices, yield curves to forecast estimates of inflation, and interest rates to name a few.

In the wake of the current crisis, market regulators have been accused of being asleep at the wheel but the brunt of the blame has fallen for the most part on financial institutions. The need for risk management has now become essential for the latter as they try to rebuild credibility and reassure investors, who have suffered significant decline in wealth in the last year, that they have the tools at their disposal to identify, measure, mitigate and control common but more importantly extreme risks.

The use of risk management techniques such as Value at Risk (VAR), Expected Shortfall (ES) that assume log-normality of returns (low tail risk) and scenario analysis that has been criticized for ruling out unlikely extreme events, is believed to have limited the incentive to measure rare and extreme risk events. This has, in turn, led to an incomplete market for options as other risks such as jump risk are assumed to have zero risk premia³. Additionally, it has been shown that VAR measures are only good if the forecasts of the risk factors and correlations are realized. Historical data modeling or Monte Carlo simulations have been shown not to be reliable in times of changing

³ See Pan, J., "The Jump-risk Premia Implicit in Options: Evidence from an Integrated Time-series Study," *Journal of Financial Economics*, volume 63, pages 3--50, 2002 for an approach to finding a price kernel that prices diffusive price shocks, jump risks, and volatility shocks.

regimes. Hence, there is a need in the market to extract expectations information implied in options data under the appropriate distributional hypotheses.

Some believe that record high levels in the S&P500, the Dow and other major indices reached in recent years pointed to a bubble that was ready to burst. So was the magnitude of the decline predictable? We attempt to provide an answer to this question however our primary objective for this research project is to analyze Bates' (1991) methodology for extracting information implicit in option prices. A secondary objective is to find evidence of a bubble, if one existed, that possibly burst in or prior to September 2008.

We adopt an asymmetric jump diffusion model as in Bates (1991) to extract market expectation of a crash prior to September 2008 implicit in the distribution of option prices on the S&P500 futures. We find that high levels of skewness premium are detectable in short maturity out of the money put options (OTM) as early as July 2007. OTM puts are widely used as crash insurance to protect against sharp downturns in the market.⁴ Nevertheless, market expectations of an extreme downturn subsided after the collapse of Bear Sterns in April 2008. Overall, our findings indicate the presence of crash expectations prior to September 2008 although there is no evidence that the magnitude of the crash was predictable.

The remainder of this paper is organized as follows. The next section provides a review of the related literature. Section 3 presents a background to the study and addresses model and estimation issues. Section 4 provides a detailed description of the data used in our research and some summary statistics. In Section 5, the estimation results

⁴ Bates, D.S, "The Market for Crash risk," *Journal of Economic Dynamics and Control* 32:7, July 2008, 2291-23212001

are presented and discussed in the context of the major financial events of 2008. Finally, Section 6 concludes and recommends areas for future research.

2. Review of related literature

2.1. Implied volatility

Implied volatilities from option prices have long been used as a measure of future expectations of underlying asset volatilities. However, Canina and Figlewski (1993) find that for most actively stock traded options, implied volatility has no correlation to future realized volatility. They argue that the high transaction costs of executing an arbitrage trade prohibits prices from approaching their theoretical values resulting in the realized volatility diverging from the implied volatility. Options on futures and futures series, on the other hand, tend not to be subject to limits to arbitrage as they trade on the same exchange and involve lower transaction costs. They suggest using implied volatility as one of the tools, rather than the tool in determining future expectations.

2.2. Risk neutral probability density distributions

Another approach to forecasting expectations is to extract information from the dispersion in the risk neutral density distribution of economic variables⁵ such as asset prices based on investor preferences. There is vast literature on inference of risk neutral probability density function (PDFs) from the cross-sections of option prices. Bliss and Panigirtzoglou (2001)⁶ survey other studies⁷ with findings similar to theirs, showing that

⁵ Schmalensee, R and Trippi, R.R., “Common stock volatility expectations implied by option premia”, The Journal of Finance Vol XXXIII- March 1978

⁶ Robert R. Bliss and Nikolaos Panigirtzoglou, “Option-Implied Risk Aversion Estimates: Robustness and Patterns,” Available at SSRN: <http://ssrn.com/abstract=301334> or DOI: 10.2139/ssrn.301334

implied risk neutral PDFs do not provide unbiased forecasts of the distribution of future underlying asset values. However, some financial regulators and others still employ risk neutral PDFs to infer market expectations for the future value of assets. This may have unintended consequences as most findings have shown that risk neutral PDFs do not predict future market turbulence (see Anagnou et al., 2001). Nevertheless, they cannot reject the hypothesis that subjective PDFs⁸ derived from risk neutral ones using a risk aversion utility function are good forecasts of the distributions of future prices. Risk neutral PDFs tend to include expectations as well as risk premium whereas subjective PDFs only contain expectations. Given a level of risk aversion of a market agent, a corresponding subjective PDF is inferred from the risk neutral one.

Of course, market expectation and degree of risk aversion can be extracted directly from other sources such as a security or future price however the forecast estimate provides only a single value at some future point in time. For instance under security valuation methods, securities are priced by discounting streams of future cash flows. Given a current security price and known future cash flows, single expectation measures such as a (constant) dividend growth rate and present value of future growth opportunities can be extracted. In contrast, market expectations inferred from risk neutral PDFs of option prices provide multiple distributions of possible future values of the underlying asset at different points in time as a result of the fact that options are available for different maturities and different strikes prices.

⁷ Anagnou, Bedendo, Hodges and Tompkins, “The relation between implied and realized probability density functions”, presents an extensive literature review of research in this area as of 2001.

⁸ Bliss and Panigirtzoglou (2001) obtain measure of risk aversion implied in option prices by inferring subjective PDFs from risk neutral ones.

In summary, market participants must be careful not to rely solely on a single measure of market expectation especially when there has not been substantial empirical evidence warranting its use.

2.3. Other considerations

Another method of extracting market expectation consists of conducting surveys to estimate the actual expectations present in the market. This kind of tool is useful although unreliable in some instances as it may draw on biased samples of selected individuals' opinion when in fact the information implicit in prices of financial products represent the entire market's aggregate belief.

Shiller, Kon-Ya and Tsutsui (1996) present a paper on the causes of the Nikkei crisis by summarizing the statistics contained in market surveys. Their justification is based on the fact that implied volatilities or other market-derived expectation data disregard the fundamental sociological fact that expectations that are relevant for market behavior diffuse across different subpopulations of the investing public at different rates, and that attention of certain subpopulations shifts from one market to others. If crash theory is considered, they state that actors will not have been giving consideration to the probability of a crash or have nothing to do in the derivative markets at all⁹.

However, there is empirical research that provides evidence that extracting information from the derivative market is among the most technical and reliable approaches. Michael P. Leahy and Charles P. Thomas (1996) present a framework to extract expectations from options on exchange rates (i.e. FX options). They analyze the period prior to the Quebec's referendum, a structural change event, to measure the

⁹ J. Shiller, Fumiko Kon-Ya, Yoshiro Tsutsui Source, "Why Did the Nikkei Crash? Expanding the Scope of Expectations Data Collection"

impact on the Canadian Dollar foreign exchange rates. They relied on the assumption that the distribution of underlying asset is completely described by the second derivative of the option price with respect to the strike price. This makes it feasible to model the distribution of option prices as a weighted mixture of lognormal distributions.

Mellick and Thomas (1996) propose the measurement of expectations in the oil market with the parameters extracted from option prices on oil futures. They make the assessment that the parameters extracted under the Brownian motion assumption yield martingale parameters¹⁰ rather than the objective ones. In order to solve this issue, they propose a model similar to Barone-Adesi and Whaley (1987) with a similar weighted lognormal distribution that is estimated with MLE and SLN models. The weights are assigned to solve the options' moneyness problem. They conclude that their model yield results coherent with realized parameters for the period under study (i.e. Persian Gulf War) unlike those estimated under the Brownian motion assumption.

3. Background to the study: Model and Estimation issues

3.1. Background to Models

In his 1991 paper and subsequent work, Bates presents empirical results supporting the fact that some of the widely used stochastic processes do not explain moneyness biases observable in option prices on index futures. For instance, processes that assume log normal return distribution, one of the assumptions that made the derivation of the Black-Scholes formula possible; others in the family of constant elasticity of variance (CVE)

¹⁰ The expected value of the parameters at time t are equal to their current values

processes¹¹; and stochastic volatility processes¹² have been shown not to capture the skewness present in the implicit distribution of option prices (Bates 1996, page 1-2).

Figure 1 shows the monthly probability density distributions (PDF's) of the future price proportional to the current price, F_{t+T}/F_t , from the period 2006 to 2008. The PDF's are clearly skewed and far from normal. Bates' central thesis states that given the existence of biases (skewness) implying that theoretical prices are skewed, better distributional hypotheses ought to be used such as the jump diffusion model to be examined in this paper. Bates' model for option pricing under asymmetric jump diffusion differs in significant ways from models introduced by Merton (1976a, b) and extended by Ball and Torous (1983, 1985).

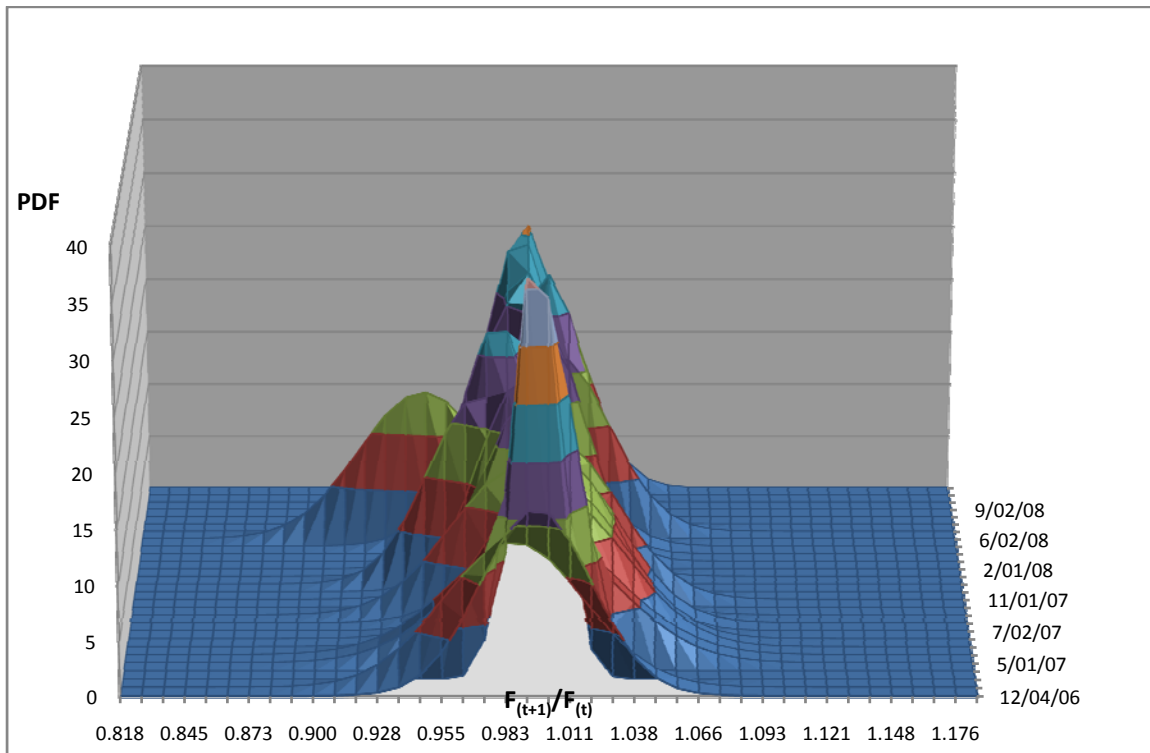


Figure 1: Probability density function of $F_{t+1\text{month}}/F_t$. Moments were calculating daily using the estimated parameters λ^* , γ^* , δ and σ . Refer to section four for model specifications.

¹¹ Special cases of CVE processes are arithmetic and geometric Brownian motion

¹² Similar to GARCH and ARCH processes

3.1.1. Merton (1976)

Merton presents a diffusion process for an asset or security which consists of both a continuous and a jump diffusion component. Merton recognizes that once the continuous time trading assumption of the Black-Scholes framework is relaxed, thereby allowing trading at discrete intervals, risk neutrality essential to derive the no-arbitrage price is no longer valid. The return on the non-arbitrage portfolio¹³ will contain a level of risk (Merton 1976, page 126). Although for a small interval that approaches zero, the difference between Black-Scholes and the option price evaluated at discrete point is quite small.

On the other hand, when pricing an option on an asset that follows a process that is not continuous, derivation of a risk neutral distribution from lognormal returns such as in the Black-Scholes model, will not yield tractable results. Merton presents an option pricing model under symmetric jump diffusion. The continuous part is modeled using a Wiener process, the jump counter using an independent Poisson process and the jump size a normal random variable with $\mu = 0$ and $\sigma = t$.

3.1.2. Ball and Torous (1983)

Ball and Torous conduct empirical tests on the returns of 47 stocks listed on the NYSE and find that more than 34 have experienced jumps. Similar to specifications of Merton's model, they define a diffusion component "conditional on no arrivals of abnormal information" and jump size variable "contingent upon the arrival of abnormal

¹³ The cash flow of the option can be replicated by a self-financing dynamic strategy in risk free bonds and stocks.

information.” However, the jump counter (for each interval) is defined as Bernoulli variable, which takes on either a value of zero or one, rather than Poisson.¹⁴

3.1.3. Numerical Technique for Estimation

Estimation of parameters λ^* , γ^* , δ , and σ from Bates’ jump diffusion model are extracted via non-linear regression. Trying to minimize unstable functions, as is the case for Bates option pricing equations, can be quite problematic. In Bates (1991), the Hillclimbing algorithm of Godfeld and Quandt(1966) is employed to extract parameters values.

This method consists of running numerous iterations to solve for unknown parameters values while minimizing a complex function. The method must be passed a vector of initial values R , which is updated after each iteration and fed back into the process. On one hand, this allows the process to advance faster when the function is approaching a local maxima or reaching convergence. On the other hand, the process slows down the further the function is from a maxima (i.e. diverging). The values in R are updated only if the function is locally maximized. Once the iterations for the first run are complete, the values in R are set as the starting values for the next run of Hillclimbing optimization. A global maxima is reached when the values in R are no longer significantly different from one run to the next. Picking good starting values is key as they determine the estimation accuracy. A drawback of the algorithm is that it requires a large amount of iterations for complex equations and large data sets.

¹⁴ The Bernoulli distribution is a special case of binomial distribution with $n=1$. As n goes to 1, the binomial distribution converges to Poisson.

3.2. Model Development

As mentioned above, Merton and Bates models differ in significant ways. Merton's model assumes that the percentage jump size follows a normal distribution with $\mu = 0$ and $\sigma = t$. Under Merton's and Ball and Torous' models, the assumption of zero mean implies that jump risk is non-systematic or diversifiable. On the other hand, Bates' model allows for the average jump size to have positive or negative skewness¹⁵ thereby allowing the distribution of the underlying asset to be more or less skewed than the lognormal distribution. Bates also points out that the degree of skewness "increases or decreases OTM call option prices relative to OTM put option prices." (Bates 1996, page 12).

The fact that Merton's jump diffusion model assumes non-systematic or diversifiable jump risk makes it inadequate to use for deriving a risk neutral distribution from option prices on the S&P500 futures. Using an equilibrium model, Bates derives an option pricing model that prices systematic (non-diversifiable) jump risk by making modifications to optimally invested wealth¹⁶. In brief, here are the three restrictions (Bates 1991, page 1024) that apply to Merton's model to derive an option pricing model under asymmetric jump diffusion:

- a. Markets are frictionless
- b. Optimally invested wealth¹⁷ follows a jump diffusion:

$$dW/W = (u_w - \lambda \bar{k}_w C/W) dt + \sigma_w dZ_w + k_w dq$$

- c. A consumer in this economy has time separable power utility:

¹⁵ A standard normal distribution has a skewness of 0

¹⁶ See Bates, D.S, "Pricing Options Under Jump-diffusion," 1988

¹⁷ A development of the modification is beyond the scope of this paper. Interested readers can refer to Cox, Ingersoll and Ross (1985b) and Bates (1988b).

$$E\tau \int_{\tau}^{\infty} e^{-\rho\tau} U(C\tau) d\tau, U(C) = \frac{(C^{1-R} - 1)}{(1 - R)}$$

3.2.1. Bates Jump diffusion Model

Rather than presenting a derivation of the asymmetric jump diffusion process here, we determine it more useful to present the model from an implementation standpoint¹⁸.

The jump stochastic process specification for the S&P500 index is as follows:

$$\frac{dS}{S} = (\mu - \lambda \bar{k} - d_t)dt + \sigma dZ + kdq \quad (1)$$

Where

μ – continuous expected return on the asset,

d_t – continuous dividend yield,

σ – variance conditional on no jumps (variance of diffusion component),

dZ – Wiener process (geometric Brownian motion),

k – random percentage jump size conditional on a jump event occurring;

$\ln(1+k) \sim N(\gamma - \frac{1}{2} \delta^2, \delta^2)$ and $E(k) \equiv \bar{k} = e^\gamma = 1$,

λ – annualized average number of Poisson events,

δ – standard deviation of jump sizes conditional on a jump

q – Poisson counter (number of Poisson events) with intensity λ - $\text{Prob}[dq=1] = \lambda dt$ and
($\text{Prob} [dq=0] = 1 - \lambda dt$)

Applying Ito's Lemma¹⁹:

$$S_t = S_0 e^{(b - \lambda k - \sigma^2 / 2) Y(n)} \quad (2)$$

where

$Y(n) = 1$ if $n = 0$ (i.e. no jump or Poisson event occur)

otherwise $Y(n) = \prod_{i=0}^{n(t)} (1 + k_i)$; $\ln(1 + k_i)$ is a normal deviate drawn from

¹⁸ Interest readers are referred to Bates, D.S., "The Crash of '87 -- Was It Expected? The Evidence from Options Markets," *Journal of Finance* 46:3, July 1991, pages 1023 to 1024

¹⁹ Refer to Hillard and Schwartz 2005, page 2-3

$N(\gamma - \frac{1}{2} \delta^2, \delta^2)$. It follows that the percentage size of the next jump which is lognormal is $e^{\ln(1+k_i)} \equiv 1+k_i$;
 $n(t)$ is Poisson distributed with parameter λ .

The steps to generate a sample path of the underlying asset from time t to $[t+T]$ are as follow:

Step 1: Draw a random normal deviate from $N(0,1)$ for the continuous diffusion process

Step 2: Select a time period T and length of interval dt (t to $t+1$) after which occurrence of a jump is determined

Step 3a: At the end of each interval of length dt , draw a random deviate q (Poisson event counter) from a Poisson distribution with λdt to evaluate whether jumps occurred or not. q - represents the number of jumps in the next time step.

Step 3b: Alternatively to step 3a, from Bates(1991) and Ball and Torous(1983, 1985), either one jump ($q=1$) occurs in the next interval or no jumps ($q=0$) occur. This is implemented by drawing a random uniform deviate $[0,1]$. If this deviate is less than λdt , a jump has occurred ($q=1$) then draw a normal random deviate from $N(\gamma - \frac{1}{2} \delta^2, \delta^2)$ to calculate the size of the jump in the underlying asset from the previous level. Else, $q=0$ and the asset drifts only due to the diffusion process i.e. there is no negative or positive asset price change caused by the jump diffusion. Note if $\lambda = 0$ then q is always = 0. In this case, the mixed diffusion process collapses to a Wiener process such Brownian motion used in the Black-Scholes option pricing model.

Here are two sample paths generated with different parameters under an asymmetric jump diffusion process:

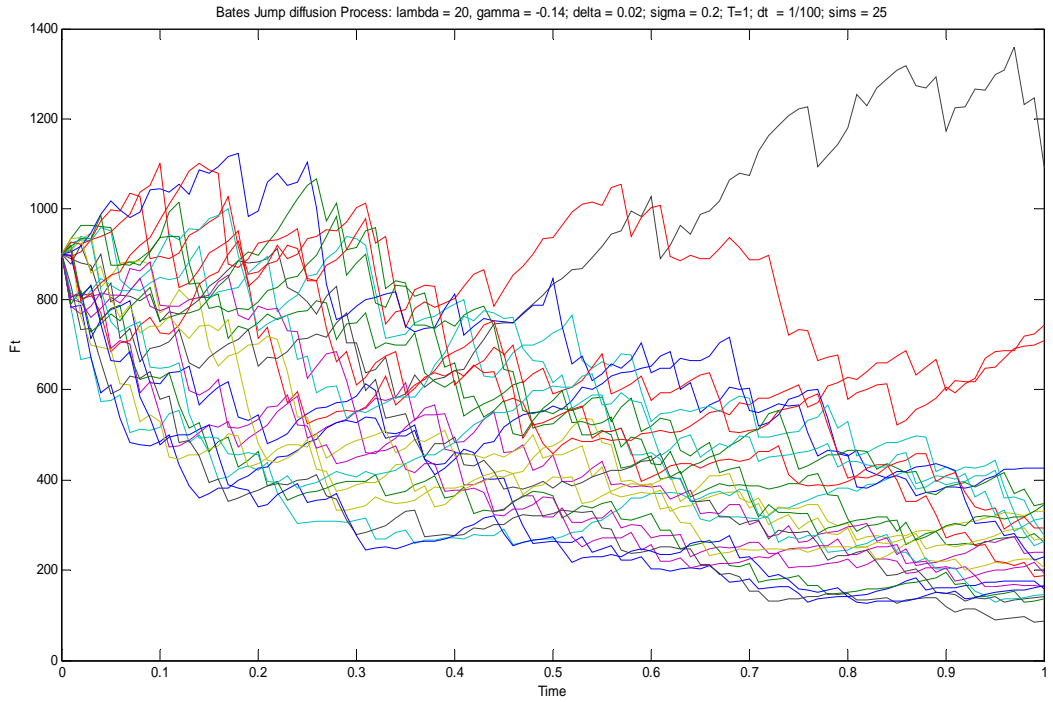


Figure 2: Asymmetric Jump Diffusion Process – Example 1: negative jump size

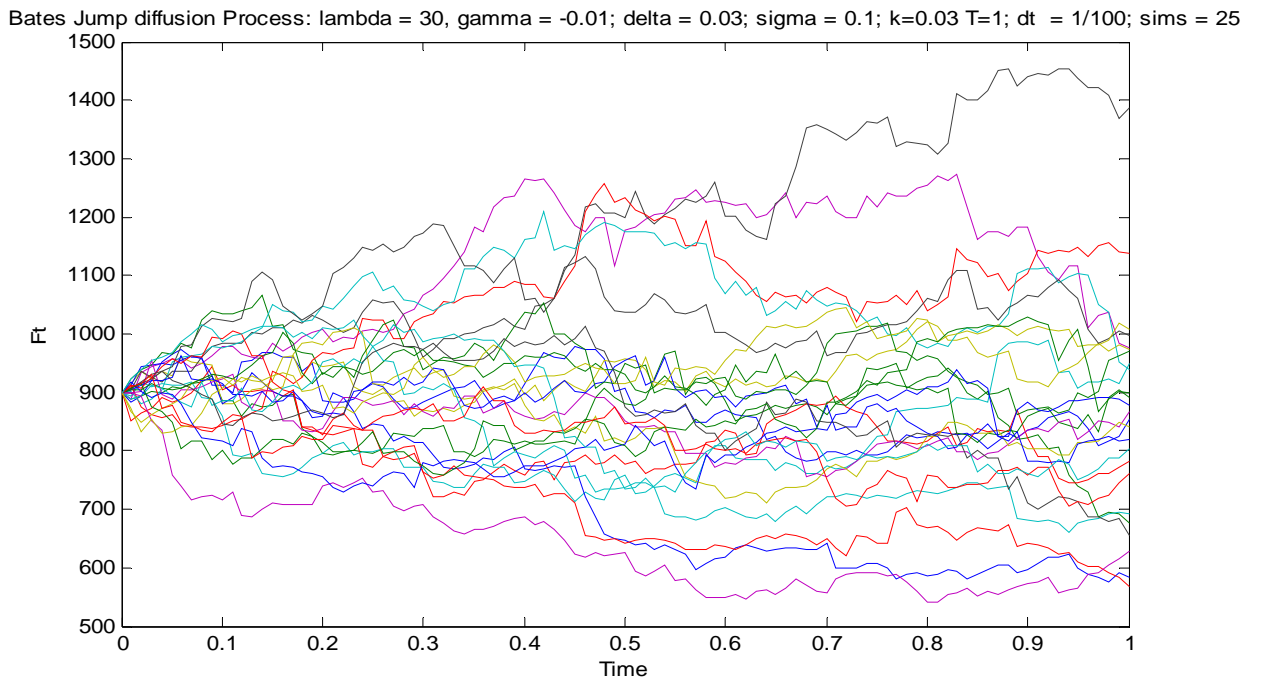


Figure 3: Asymmetric Jump Diffusion Process Example 2: positive jump size

3.2.2. Non-Linear Least Squared Regression: Hillclimbing method

The following regression equation is used to extract daily parameters λ^* , γ^* , δ , and σ :

$$V \left(1, T; \left(\frac{X}{F} \right)_j, \lambda^*, \gamma^*, \sigma, \delta \right) + \epsilon_j \quad (3a)$$

where V_j is the objective call or put price and $v(F, T, X)$ is the call or put price option formula under jump diffusion:

$$V(F, T; X) = e^{-rt} \sum_{n=0}^{\infty} \left[\left[\frac{e^{\lambda^* T} (\lambda^* T)^n}{n!} \right] [expected\ payoff\ give\ n\ jumps] \right] \quad (3b)$$

Note that we estimate γ^* in our implementation then \bar{k}^* is calculated as follows: $\bar{k}^* =$

$$e^{\gamma} = 1$$

To run the non-linear estimation equation #3a is rearranged in the form:

Daily pricing error term

$$= \sum_1^M (\text{market price of option} - \text{jump diffusion option price})^2$$

Where M is the number of observations and the jump diffusion option call and put price are calculated as follows:

(4)

$$c(F, T; X) = e^{-rt} \sum_{n=0}^{\infty} \left[\left[\frac{e^{\lambda^* T} (\lambda^* T)^n}{n!} \right] [F e^{b_n T} N(d_{1n}) - X N(d_{2n})] \right]$$

(5)

$$p(F, T; X) = e^{-rt} \sum_{n=0}^{\infty} \left[\left[\frac{e^{\lambda^* T} (\lambda^* T)^n}{n!} \right] [X N(-d_{2n}) - F e^{b_n T} N(-d_{1n})] \right]$$

$$b(n) = (b - \lambda^* \bar{k}^*) + \frac{n\gamma^*}{T} = -\gamma^* \bar{k}^* + \frac{n\gamma^*}{T}$$

$$d_{1n} = \frac{\ln \left(\frac{F}{X} \right) + b(n)T + \frac{1}{2}(\sigma^2 T + n \delta^2)}{(\sigma^2 T + n \delta^2)^{\frac{1}{2}}}$$

3.2.3. Option Pricing Model under Asymmetric Jump Diffusion

3.2.3.1. Martingale property

One important specification of the future price on an index is that at time t the expected value of the S&P500 at a future date T (denoted by S_{t+T}) is the value of the future contract maturity at T , or,

$$E_t(S_{t+T}) = F_{t,t+T}$$

Given that the cost of carry on future indices are typically zero, it follows that $F_{t,t+T} = S_t e^{bT} = S_t$. Therefore, the expected value of the S&P500 at time $[t+T]$ is its value at time $[t]$. In other words, $S_t = F_{t,t+T}$ is a martingale which is defined as the best forecast of tomorrow's price being today's price. At any point in time, the current price S_t or $F_{t,t+T}$ fully reflects all the information available in the market. The martingale property makes it possible to price the option on the futures as the discounted expectation of the payoff.

3.2.3.2. American options on the S&P500 futures

The daily estimated parameters λ^* , γ^* , δ , and σ from the non-linear least square regression are used to calculate the European call and put prices under the jump diffusion process. The option is evaluated as the discounted expectation of the expected payoff at time T given that n jumps occurred.

As shown in equation 3b, this consists of summing from $n=0$ to 1 the expected payoff at T given n jumps times the probability of n jumps. The implementation of the European option formula consists of truncating the summation at $\frac{(\lambda^*T e^{\gamma^*})^n}{n!}$ for calls and $\frac{(\lambda T)^n}{n!}$ for puts. Additional iterations are conducted until the difference between the

previous and next values in the summation is less than 104 or $n = 1000$. See equations 4 and 5. As there are no closed-form solution for American call and put prices²⁰, they are calculated using the quadratic approximation Bates derives from earlier work by MacMillian(1987) and Barone-Adesi and Whaley(1987). See equations 6 and 7.²¹

$$\begin{aligned} C(F, T;X) &= c(F, T;X) + A_2[(F/X)/y_c^*]^{q_2} && (F/X < y_c^*) \\ &= F - X && (F/X \geq y_c^*) \end{aligned} \quad (6)$$

$$\begin{aligned} P(F, T;X) &= p(F, T;X) + A_1[(F/X)/y_p^*]^{q_1} && (F/X > y_p^*) \\ &= X - F && (F/X \leq y_p^*) \end{aligned} \quad (7)$$

3.2.4. Skewness Premium Calculation

From Future series

To examine the skewness premium present in the underlying time series of S&P500 futures, Bates derives measures of non-central moments for S_{t+T}/S_t .²²

²⁰ Interested readers are referred to a book by Rama Cont and Peter Tankov, *Financial Modeling With Jump Processes* (Boca Raton, Fla. : Chapman & Hall/CRC, c2004) for an overview of theoretical, numerical and empirical research on the use of jump processes

²¹ C++ code to solve equations for A_2 and A_1 (Bates 1991, page 1027) and find root y_c^* and y_p^* using the Barone-Adesi and Whaley algorithm was borrowed from the “Financial Numerical Recipes in C++” by Bernt Arne. For our purpose, the source code was converted to MATLAB and modified to solve both equations using customized functions to find the European option prices. A function implementing the Newton algorithm written by Matt Fig was used to solve the roots q_1 and q_2 of the following equation:

$$\frac{1}{2}\sigma^2 q^2 + \left(-\lambda^* \bar{k}^* - \frac{1}{2}\sigma^2\right) q - \frac{r}{(1-e^{rT})} \lambda^* \left[e^{y^* q + \frac{1}{2} q(q-1)\delta^2} - 1 \right] = 0$$

²² Bates argues that examining central moments for $\ln(S_{t+T}, S_t)$ will not give a clear sense of the skewness premium as they approach those of a normal distribution (skewness = 0) as T increases from daily to weekly to monthly and so on time intervals.

$$M_n \equiv E \left[\left(\frac{S_{t+T}}{S_t} \right)^n \right]$$

$$= \exp\{n - d - \lambda k)T + 1/2 (n^2 - n)\sigma^2 T + \lambda T [e^{n\gamma + 1/2(n^2 - n)\delta^2} - 1]\} \quad (8)$$

$$\text{Var}(S_{t+T}/S_t) = M_2 - (M_1)^2$$

$$\text{SKEW} = [M_3 - 3M_1 M_2 + 2 (M_1)^3] / (\text{Var})^{3/2}$$

$$\text{KURT} = [M_4 - 4 M_1 M_3 + 6 (M_1)^2 M_2 - 3 (M_1)^4] / (\text{Var})^2$$

From Option Prices

Market expectation can be quantified by calculating the risk premium implicit in theoretical call and put prices using the four parameters estimated by performing a non-linear Hillclimbing regression. The risk premium (SK(x)) is defined as the ratio of the difference between call and put prices over the put prices.

$$SK(x) = \frac{c(F, T; X_c)}{p(F, T; X_p)} - 1 \quad (\text{European Options})$$

$$SK(x) = \frac{C(F, T; X_c)}{P(F, T; X_p)} - 1 \quad (\text{American Options})$$

Where

$$X_p = F(1+x) < F < X_c = F(1+x), \quad x > 0$$

3.3. Estimations

To deal with the infinite sum in the call and put price formula, we set the upper limit of the summation equal to the annualized expected number of jumps times the average size of jumps ($\lambda^* T e^{\gamma^*}$). Please see the next section for truncation values suggested by Bates).

The resulting function, although unstable, gives us a starting point to try out a number of

methods for minimizing the daily pricing error function. Bates enlists the help of Golfeld and Quandt's software, GQOPT method GRADX, with six starting values.

Our first approach is to use the MATLAB function FMINSEARCH to minimize the squared errors using the Nelder-Mead simplex method. However, given the non-stable nature of the error function, the results are not much better than the initial values. So we proceed to implement the Simplex Hillclimbing method algorithm designed by Kyriakos Tsourapas. The algorithm is modified to match specifications of the Bates Jump diffusion model as defined above. To ensure a solid and robustness estimation, additional important restrictions are made. First, a random number is generated for the first seeds of the starting values within the limits shown below.

	λ	ν	Δ	Σ
Upper Limit	10	1	1	1
Lower Limit	0	-1	0	0

Second, we run a number of iterations on the same day using six new starting values. To prevent the Hillclimbing function from getting trapped at a local minimum, we modify the algorithm by inserting the restriction that non-negative values for λ^* , δ , and σ are not permitted to be recognized as local minimums. The daily Hillclimbing estimation takes on average more than 1000 seconds of computing time for a total estimation time of approximately 40 hours. If sufficient computer power is available, we suggest increasing the number of starting points for the Hillclimbing function to 20 to improve accuracy.

4. Data and summary stats:

4.1.1. Options on S&P500 futures Data

Option prices on the S&P500 futures are obtained from the Chicago Mercantile Exchange (CME). The data series ranges from December 2006 to December 2008. There are approximately 108558 observations for each call and put series (Please see Appendix-Table 1 and Table 2 for descriptive statistics). We use end-of-day settlement prices which are closing prices adjusted by the Settlements Committee to take into account margin requirements. On any given trading day, options of different strikes and different maturities matching those of different futures contract series trade on the CME. Classes of options with different strikes but the same maturity mature one day prior to the expiration date of the futures contract they are written on. New options contracts are then issued once the futures contracts are rolled over.

Looking at objective option prices alone is difficult due to the sheer amount of options with different strikes and maturity trading daily. For this reason, the information available in option prices must be summarized by running a non-least linear estimation to extract daily parameters common across all classes of calls and puts. Before running the regression, a cubic spline extrapolation for each futures series is conducted on a representative sample of Option Price/Future Price ratios over the Strike Price/Future for puts and calls. The price ratio of 1 ought to be the point where the lower bounds of the calls and puts graph crosses. As seen in figure 4, the fitting of our representative sample agrees with option theory giving us an adequate level of confidence that other samples chosen similarly to run the regression will yield acceptable jump diffusion parameters.

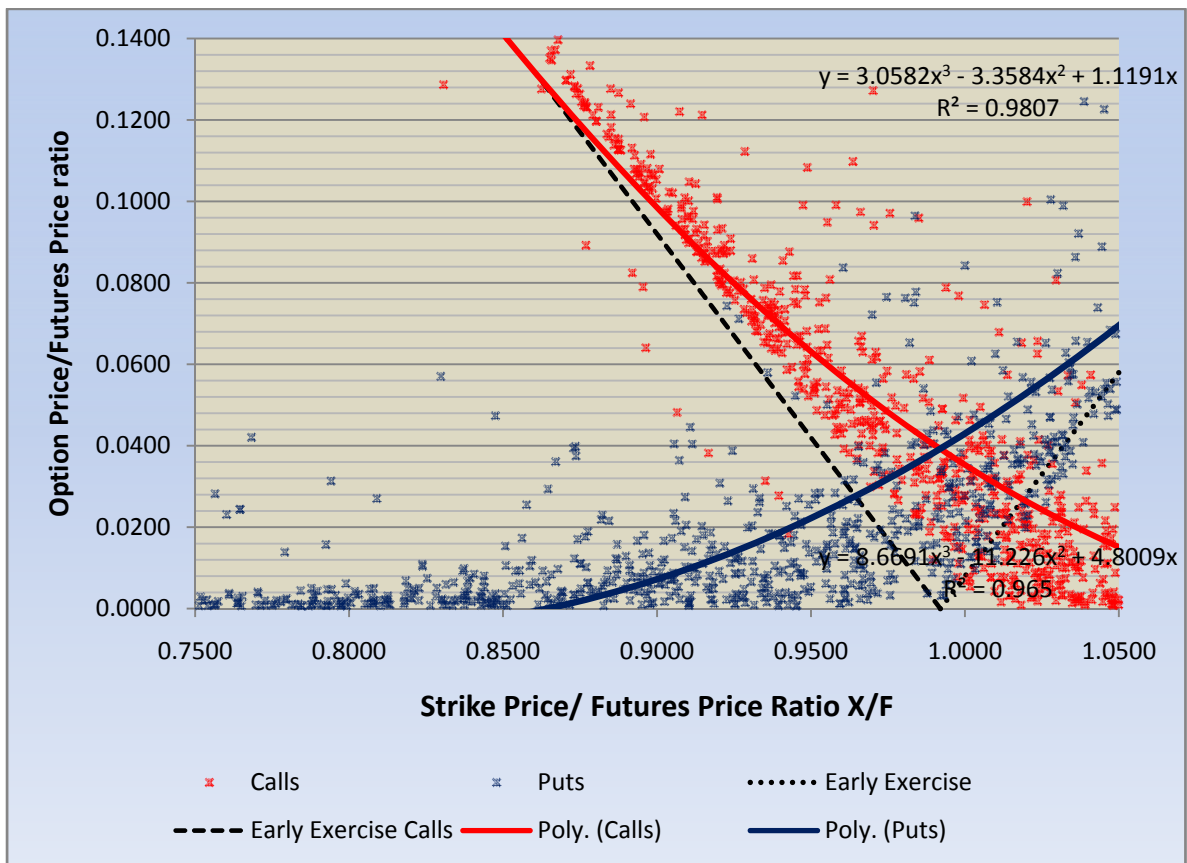


Figure 4: Transaction prices of calls and puts on the S&P500 futures (2006-2008): Cubic Spline of representative data set

4.1.2. S&P500 Index

When the dust settles and the world economies finally recover, the current global financial crisis that was triggered by the credit crunch of 2008 will be viewed by most as the worst since the great depression. As shown in figure 2, the S&P500 index started on a downward trend, after it peaked in April 2007, marking the first signs of turmoil. The first sharp decline occurred after the collapse of Bear Sterns in April 2008. However, it paled in comparison to systemic shock, which some argue was caused by the failure of

Lehman Brothers in late September 2008. In a span of two weeks, the S&P500 index lost more than 30% of its value dropping from around 1250 level to 900.

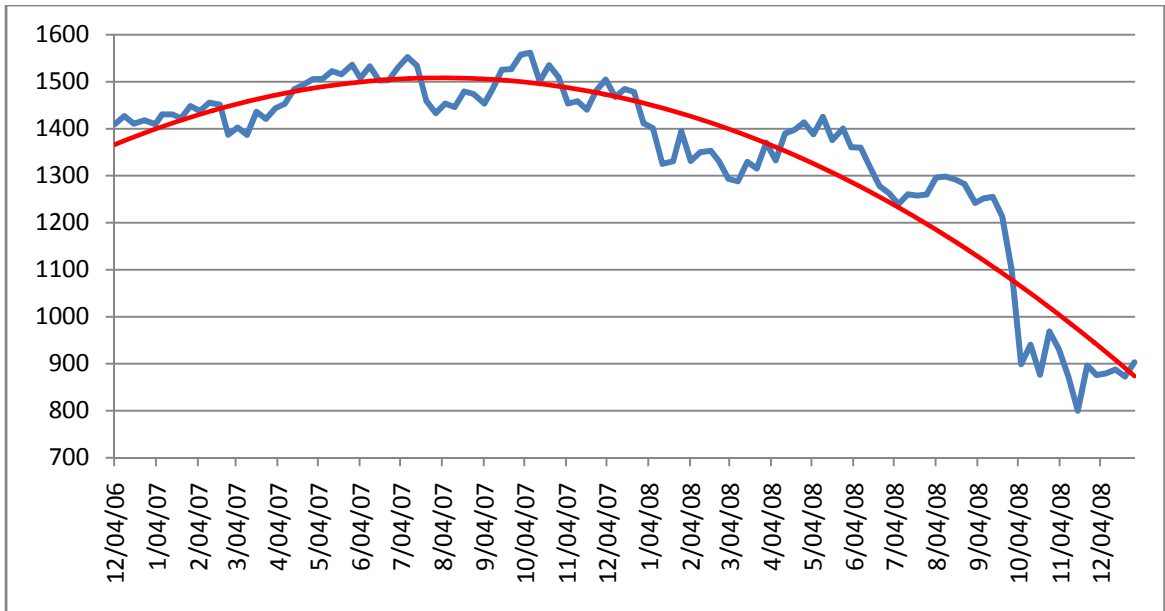


Figure 5: S&P500 index - December 2006 - December 2008. Source: yahoo/finance

4.1.3. S&P500 Futures series

The Futures series maturing in no more than 120 days are selected as follows: SPZ06, SPH07, SPM07, SPU07, SPZ07, SPH08, SPM08, SPU08, SPZ08, SPH09 (See Appendix – Exhibit II). Below is a histogram of the futures contract values from December 2006 to December 2008.

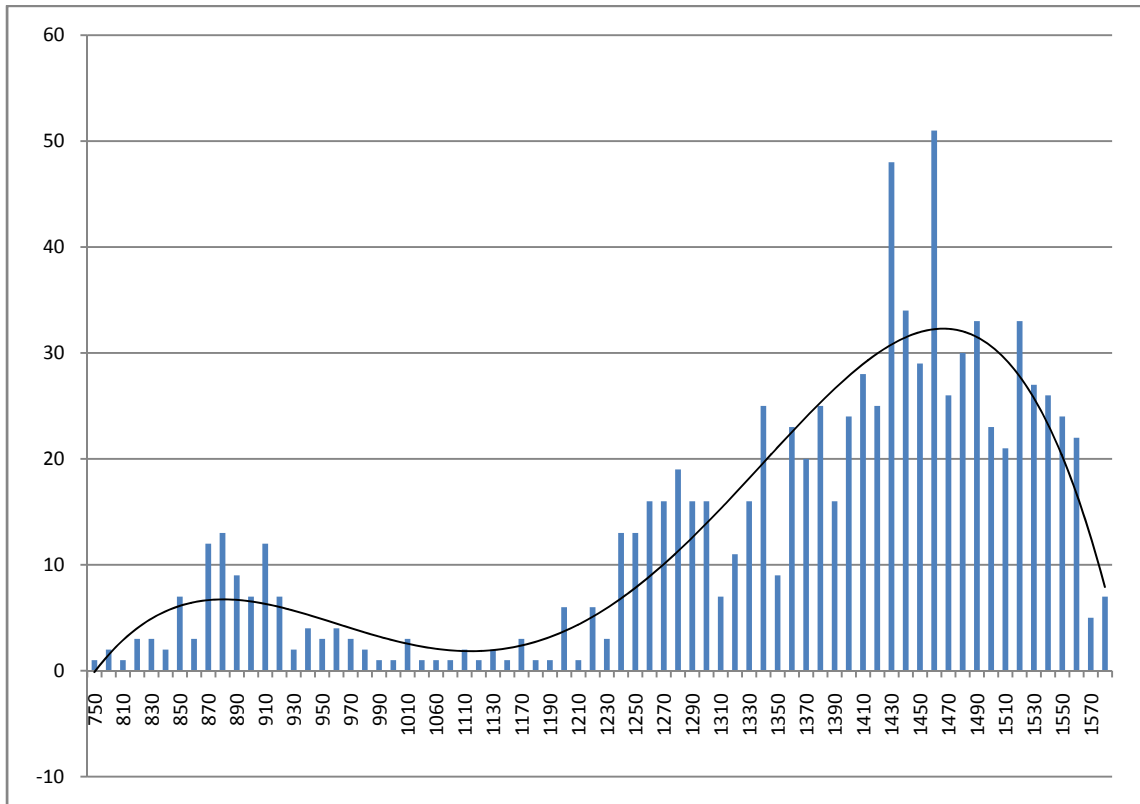


Figure 6: Histogram of Futures prices December 2006- December 2008. Data obtained from Datastream

We can see that the Futures series form a normal distribution with observations in the tail that appear to form a pseudo-normal distribution. This is evidence that conventional VAR measurements would result in underestimating the real risk. The switching regimen will be more detectable with the implicit volatility obtained from option prices. 912 observations are obtained from Datastream. This data is incomplete for some trading days. We use the cubic spline approximation to complete the series as mentioned above (See Appendix – Table 3 for descriptive statistics)

When we analyze each series individually, some interesting facts are observed. First of all, the downtrend starts with series SPH07 (March 07 maturity). Some have attributed the trend primarily to increasing signs of financial trouble in the mortgage

industry and a string of bankruptcy filings by a number of subprime mortgage lenders²³. However, this downtrend is not sustained in the next series SPM07 (June 07 maturity) as the futures prices go up. In the next two series, SPU07 and SPZ07, a lot of noise is detected but no steady trend is observed. Overall, the market remained stable in 2007. In the first quarter of 2008, the SPH08 series shows expectation of stability in the market. However, as can be seen in series SPM08 (March 08 maturity) things changed radically as volatility increased after Bear Stearns' collapse. Surprisingly, the next two futures series SPM08 and SPU08 do not give a preview of any drastic downward movement, much less an expectation of a crash. As expected, the effect of Lehman Brothers' demise can be observed in the SPZ08 (December 08 maturity) series.

After observing the S&P500 future series over the period of December 2006 to December 2008, our assessment is that a sharp market downturn was not expected until the second quarter of 2008. As illustrated in figure 7, the slope of the adjusted cubic spine tendency line (in green) suggests that given the information available in July 08, a decline was expected in the third quarter of 2008. However, crash fears were present but not as strong as to predict the magnitude of what actually transpired after the Lehman Brothers' failure. Of course, the magnitude of the drop in the S&P500 cannot be solely attributed to Lehman Brothers chapter 11 bankruptcy filings given the fact that a myriad of critical events and announcements were made in proximity albeit most agree that it accentuated crash fear already present.

²³ Several subprime lenders declared bankruptcy, announced significant losses, or liquidated their assets sale. These include Accredited Home Lenders Holding, New Century Financial, DR Horton and Countrywide Financial, J. Cox, "Credit Crisis Timeline" University of Iowa Center for International Finance and Development E-Book, 2008.

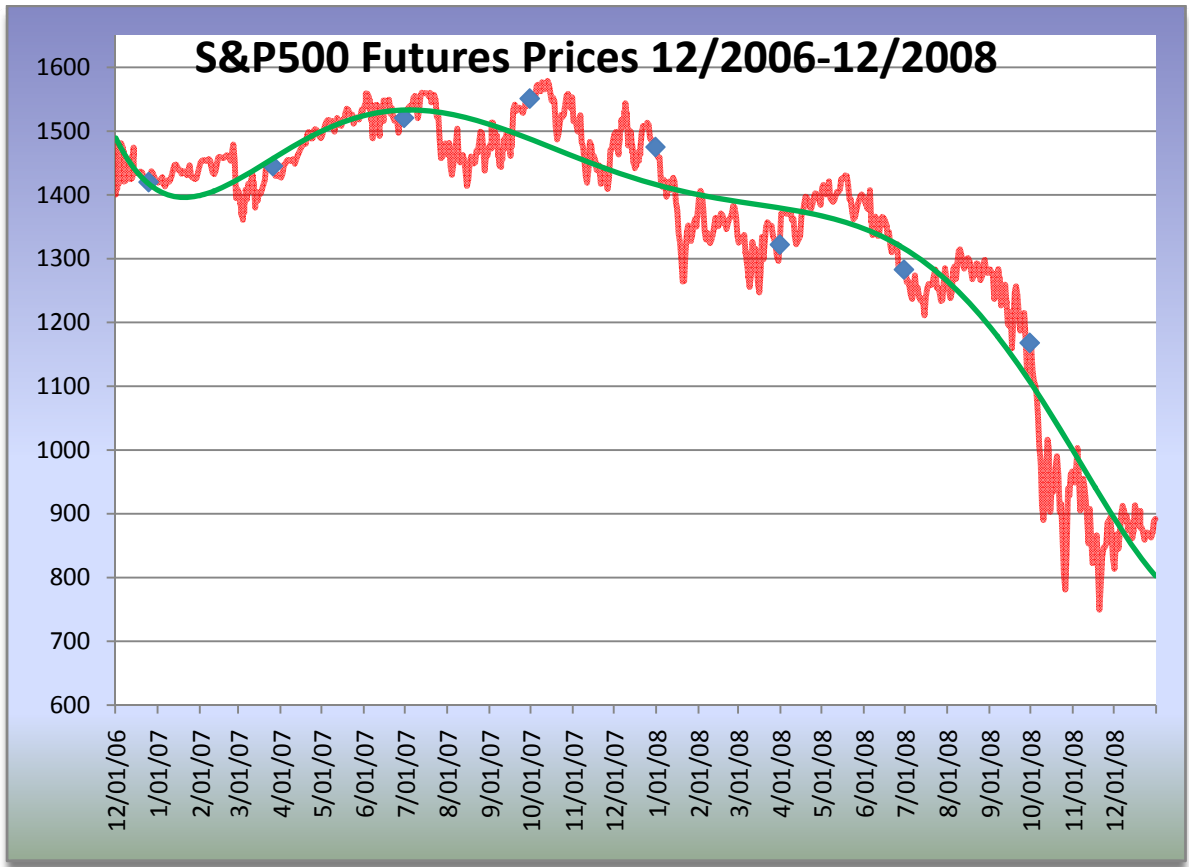


Figure 7: S&500 futures - Adjusted Cubic Spline Tendency Line

An analysis of the evolution of the PDFs for the Futures time series (see Figure 8) shows that the series starts with low kurtosis and is skewed ($\mu=1351$ See Appendix-Table 3) to the right. Progressively, the PDFs of later series increase in kurtosis and become more negatively skewed until Q4 2008 when the distribution shows an unusual clustering of observations in the left tail.

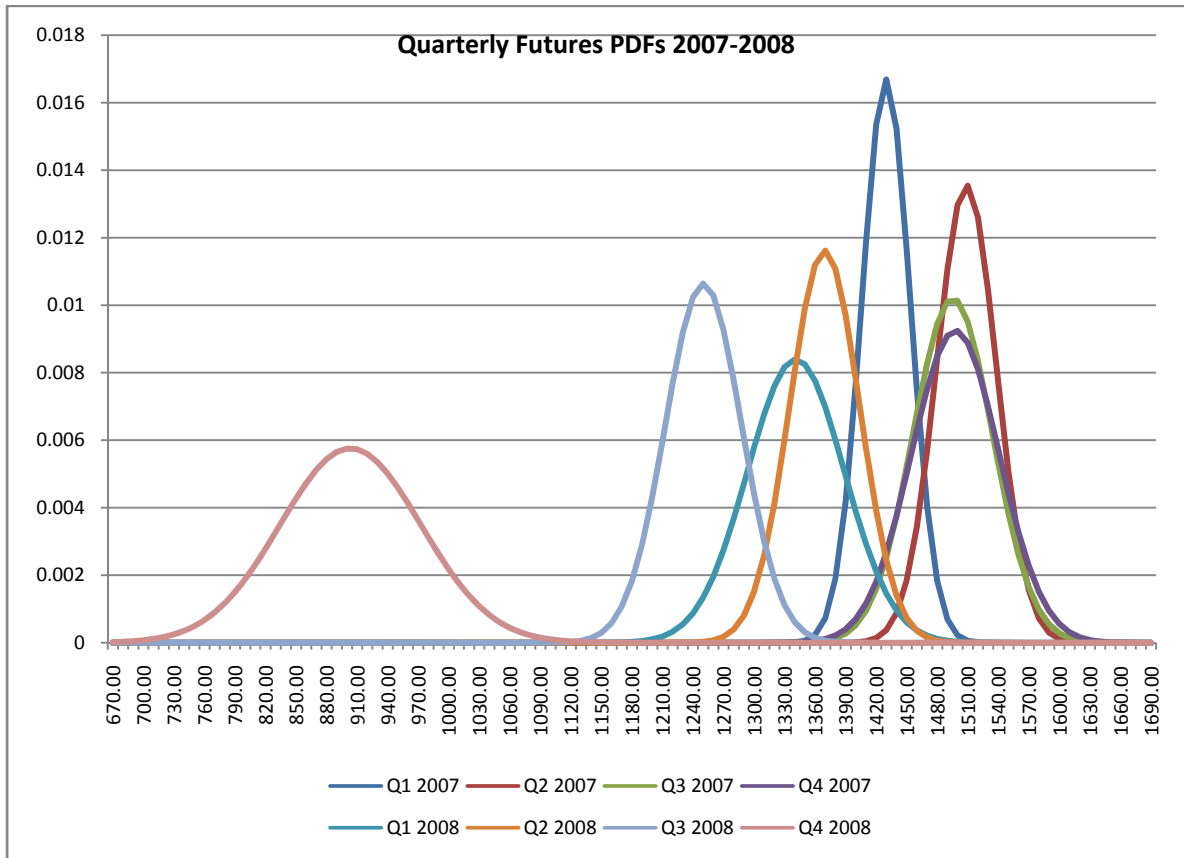


Figure 8: Quarterly PDFs of the Futures Series 2007-2008

4.1.4. Risk Premium

Although the analysis of the futures series gives a general idea of market expectation, it is not conclusive and is difficult to quantify. In order to measure the actual crash fears, we need to extract a value of the actual risk premia or crash insurance investors were paying.

Daily skewness premium can be quantified by calculating $SK(x)$, as detailed in section 3.2.4 Skewness Premium Calculation, from daily estimated risk neutral parameters which provide a way to estimate the size and probability of a correction $Prob[S_{t,t+T} < X]$. This is the left tail probability of the distribution of index prices being less than the strike price at maturity. This probability gets larger when expectation of a downturn intensifies which translates into OTM puts not only being priced higher than OTM calls for skewed distributions but also more likely to expire ITM. Note that if the

distribution of the underlying asset is assumed to be normal than the OTM calls and puts prices are identical stemming from the fact that the probability of an upward and downward trend are the same. Therefore, skewness premium defined as the percentage deviation of $x\%$ out of the money put prices from call prices gives a direct measure of downward expectations.

Regardless of the distribution hypotheses assumed (i.e. geometric Brownian motion, jump diffusion, etc.) calls are typically priced about 0% to 4% higher than puts (Bates 1996). So, ATM and OTM put options priced higher than calls points to the existence of skewness premium. Please note early exercise premium in options on future indices is negligible as the cost of carry which determine cash flow and the early exercise decision is zero. Therefore, it's reasonable to assume that the premium in option prices on index futures is attributed mostly to skewness in the underlying asset distribution.

5. Results

5.1.Data for Non Linear Estimation

There were approximately 600,000 data observations from December 2006 to December 2008. Running a regression on the entire data set of option prices would be computationally prohibitive. So, we proceed to generate separate random representative samples for call and put prices. Only options with maturity within 30 to 120 days matching the maturity on futures series March(SPH), June(SPM), September(SPU) and December(SPZ) are selected.

Additionally, only options 4% OTM and ITM options are used as they exhibit greater moneyness bias. The daily random representative sample consists of 50 call and 50 put options prices. For computing efficiency when running the daily regressions, the

sample for the entire two year period is divided in four groups. Option prices especially OTM can fluctuate widely depending on factors such as maturity term left and the strike price. Therefore, it is quite a challenge not only to select a set of options that contains mostly premium associated with moneyness biases, but also to interpret some of the results.

5.2. Test of validity of results from the regression and model implementation

5.2.1. Estimated Option prices

Given that American options on futures are bound between the European option price and the future value of the European option prices²⁴:

$$c(F, T : X) < C(F, T;X) < e^{rT} c(F, T : X)$$

$$p(F, T : X) < P(F, T;X) < e^{rT} p(F, T : X)$$

As you can see from Table 6 (see appendix), the estimation of European and American from both jump diffusion models we have implemented yield results within these bounds providing an adequate level of confidence from the implementation to continue our tests. Additionally, options prices replicated from Bates (1991) paper (Table 5A and 5B- Appendix) are for the most part accurate.

5.2.2. Estimated parameters

The put and call series were fitted with approximately 99% accuracy.²⁵ Figure 9 shows the robustness of implementation given that squared residuals from the parameters

²⁴ We credit Dr. Bates for personally reminding us of this fact

extracted from both call and put prices²⁶ are fairly congruent and for the most part move in this same direction. We are fairly confident that the parameters we estimate have the characteristics of those extracted from the risk neutral distribution implicit in option prices. In the period of March 2007 to November 2007, the put series lose some explanatory power. This could be attributed to the fact that implicit volatility increases with higher demand for puts. Also, the data sampling and picking of strike prices could affect implicit volatility.

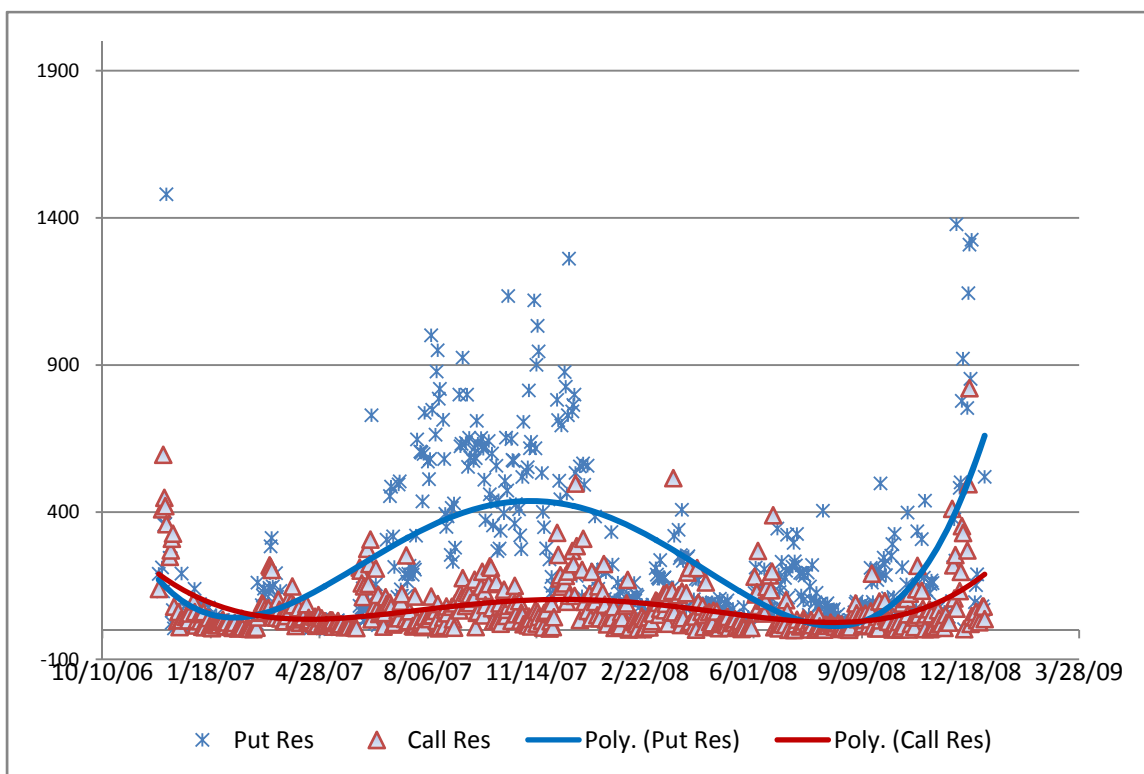


Figure 9: Squared Residuals from the Non-linear daily estimation using the Hillclimbing method.

For the most part, call and put parameters estimated separately via a non-linear Hillclimbing estimation exhibit characteristics of those implied from a risk neutral

²⁵ The R^2 was calculated assuming that the sample has the same distribution as the population and that the variance remains constant.

²⁶ Two different random samples were used for running the regression of puts and calls

distribution. We come to this conclusion after finding the same tendency in squared-residual errors of the calls and the puts. Additionally, most tables and graphs presented in the paper are estimated using both sets of parameters yielding for the most part similar results.

Nevertheless, we do find some discrepancies that may be due to various factors. Our assessment is that options are very sensitive to factors such as the strike price (moneyness bias) and time left to maturity, making it difficult to summarize information intrinsic in the options prices across all strikes and maturity terms at once. We suspect that estimating parameters on subsections (e.g. deep OTMs, OTM, ATM and ITM) of daily option prices would provide a better sense of the effect of various factors. The problem of the sensitivity of pricing errors to the moneyness of the options ought to be considered when using option based models to extract the implicit parameters of the distribution of underlying asset prices. The lack of a complete inventory of options could also pose problems to the usefulness of option-based tools in risk management given that some risks (e.g. tail risk) are not priced into the market.

To summarize, we reject Bates' hypothesis of identical jump-diffusion parameters for calls and puts as we find evidence that the fit is not as accurate depending on the factors detailed above. We recommend careful consideration when picking x% OTM strikes prices and term to maturity in the estimations.

5.3. Probability density functions

The daily μ and σ estimated using parameters from the regression are used to generate a daily probability density function (PDF) of the S&P500 future series. Note that the

reason for using estimated risk neutral parameters rather than the true parameters of the jump diffusion model to estimate moments of S_{t+T}/S_t is due to the fact that the true parameters of the underlying asset jump diffusion process are not observable. Here is another view of the monthly PDF graph shown in section two:

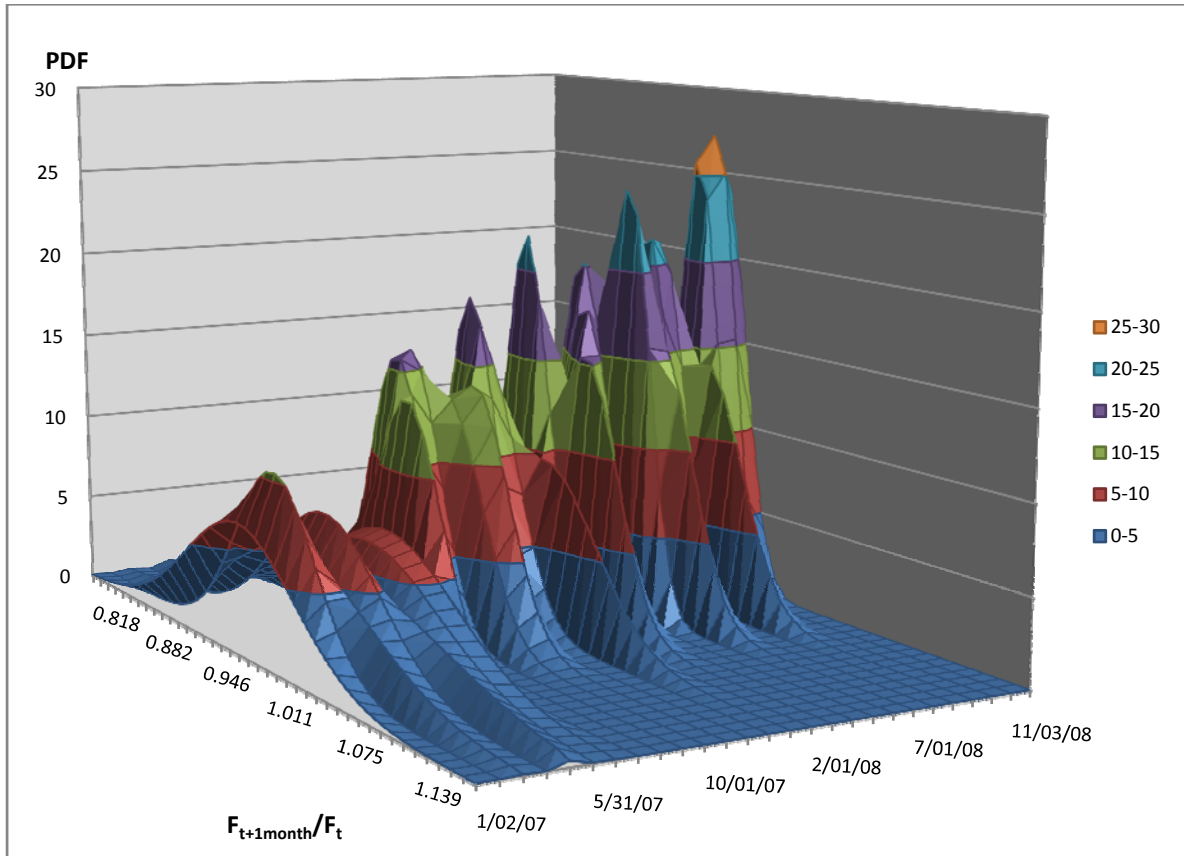


Figure 7: Probability density function of $F_{t+1\text{month}}/F_t$

5.3.1. Skewness and Kurtosis from PDFs

Figure 7 clearly shows that the PDFs of the future series are characterized mostly by negative skewness and lower kurtosis. Similarly to figure 6, the distribution of futures started to shift to the left starting as early as July 2007 suggesting that the market expected future downward movements in the S&P500 index.

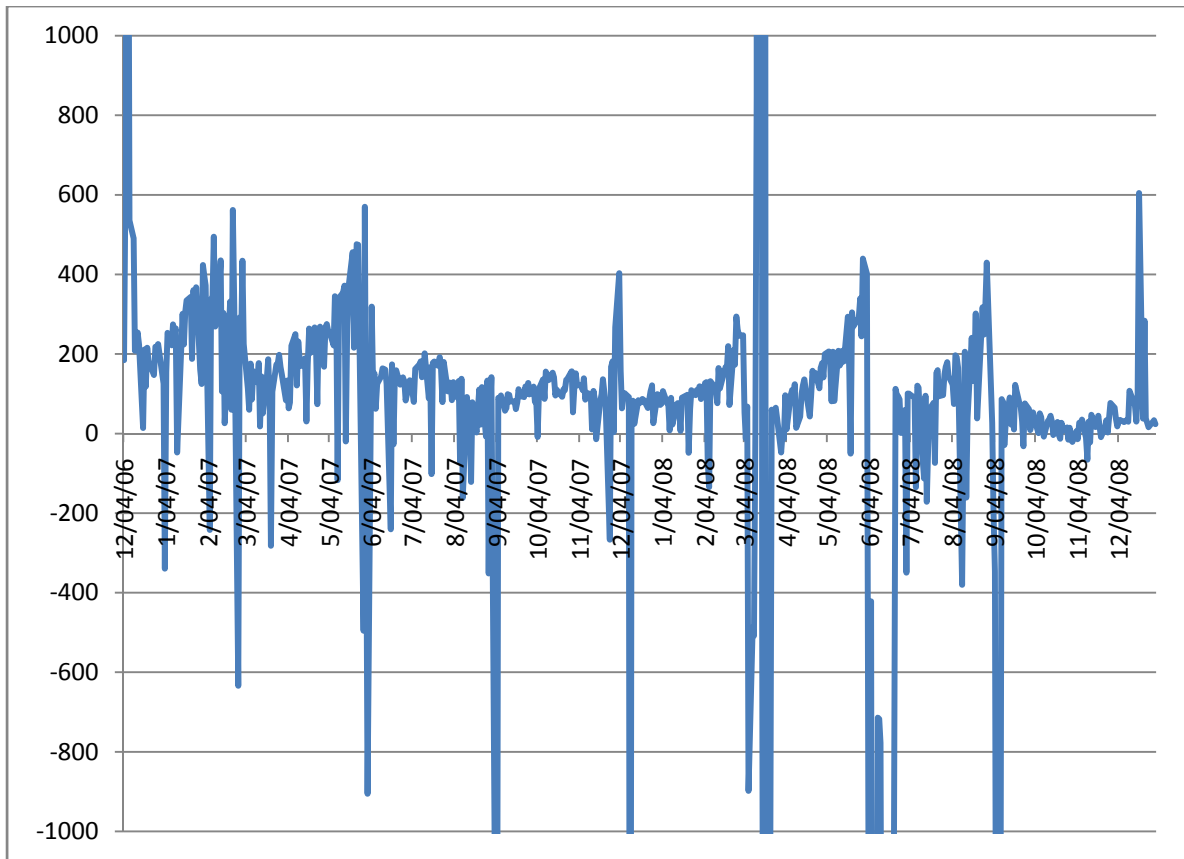


Figure 8: Skewness of $F_{t+1\text{month}}/F_t$

Figure 8 shows that monthly coefficient of skewness has been positive for the most part. However, we can observe infrequent but large negative jumps throughout the two year period which have resulted in the implicit distribution of F_{t+T}/F_t being negatively skewed. The negative jumps every quarter might be associated with the low trading volume prior to the futures contract rolling over. The coefficient of kurtosis hovered just above that of a normal distribution for December 2006 to June 2007. Then, as volatility picked up in the market in the summer of 2007 so did the coefficient of kurtosis. The combined effect of the sharp negative jumps experienced and higher kurtosis suggest an increasing left tail probability making OTM puts more valuable.

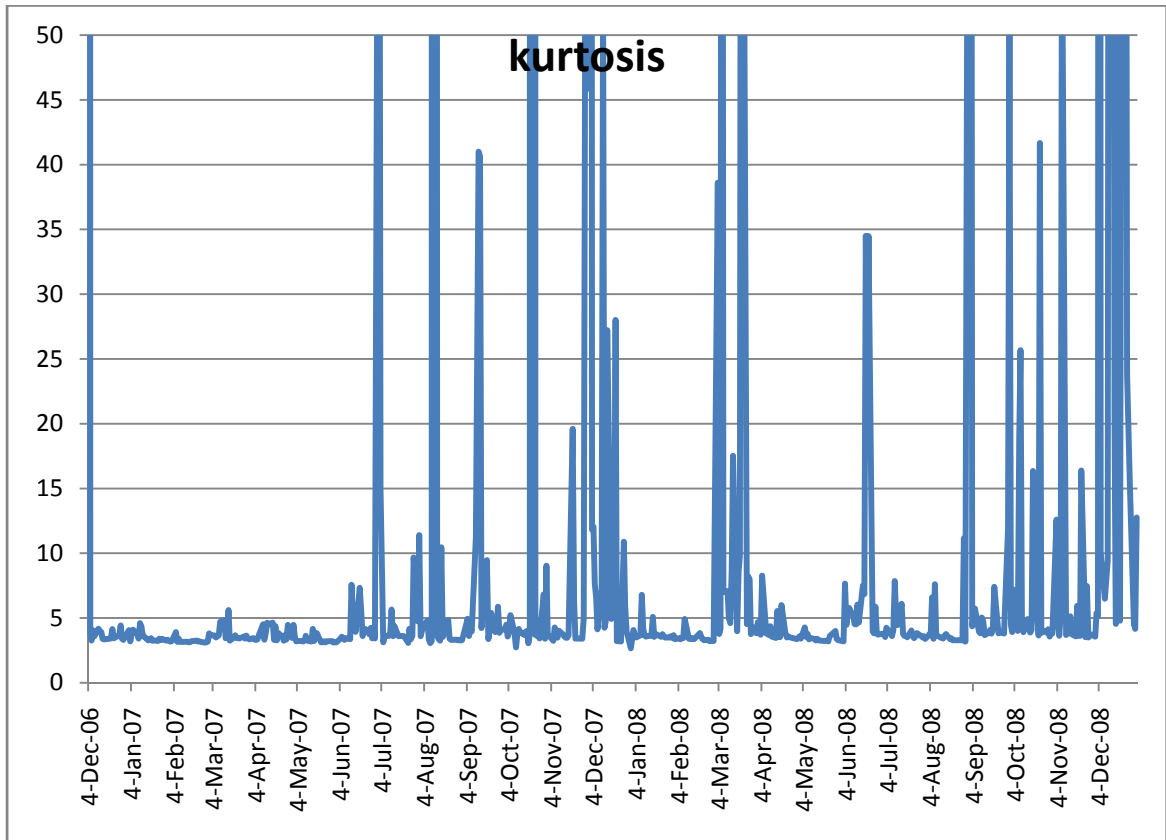


Figure 9: Kurtosis of $F_{t+1\text{month}}/F_t$

Note that estimated risk neutral parameters rather than the true parameters of the jump diffusion model are used to estimate moments of S_{t+T}/S_t as the true parameters of the underlying asset jump diffusion process are not observable.

5.4. Skewness premium estimated Options prices

We calculated daily call and put prices for 2 set of strikes then estimated the skewness premium:

$$SK(x) = \frac{C(F, T; X_c)}{P(F, T; X_p)} - 1 \quad (\text{American Options})$$

Where

$$X_p = F(1+x) < F < X_c = F(1+x), \quad x > 0$$

For ATM options, X_c and X_p were set to F . For OTM options, X_c and X_p were set based on the above restrictions ($x=4\%$). Estimated puts with prices lower than \$3 were excluded as most were closer to maturity and most likely did not contain any significant moneyness bias.

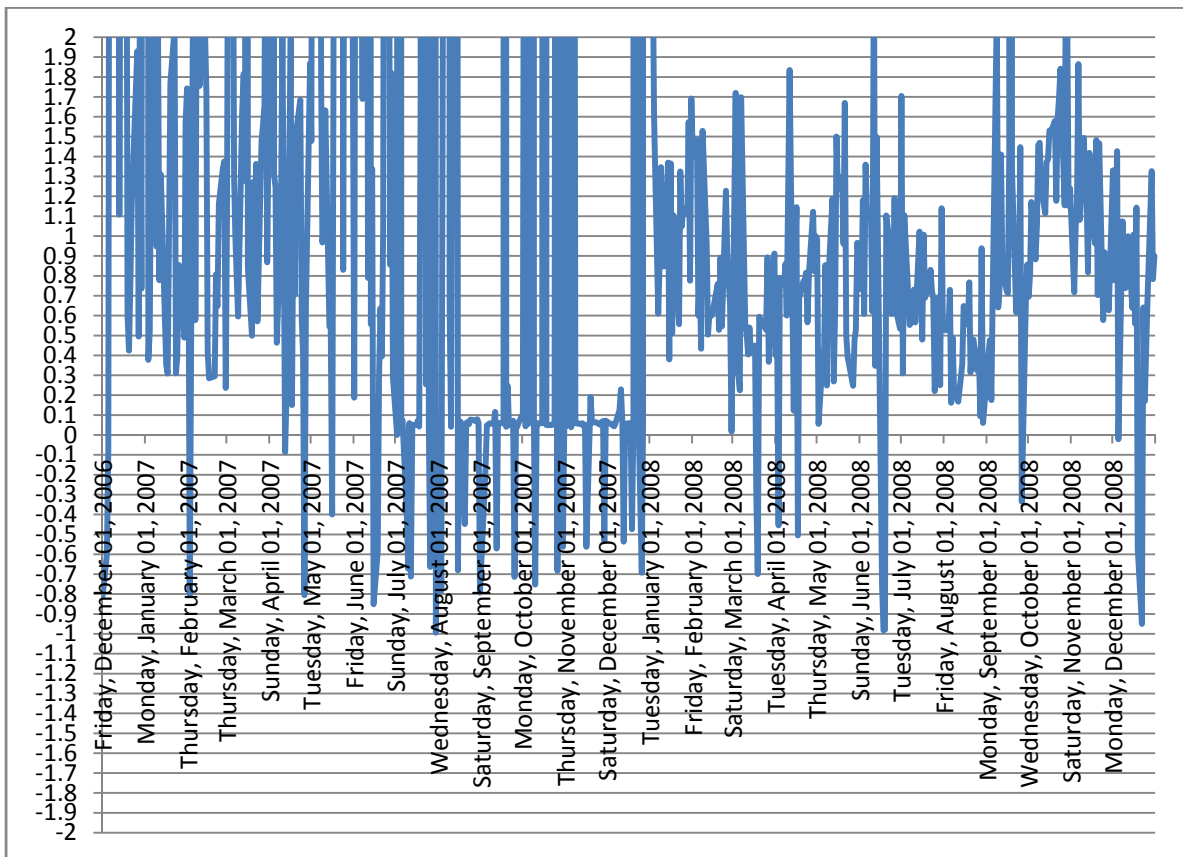


Figure 10: Skewness premium (C-P)/P against time

The OTM puts are priced much higher than calls after March 2007 yet crash fear eased after January 2008. We do see, however, high isolated jumps in skewness premium, most likely associated with major events, on March 2008, July and finally after the crash of September 2008.

5.4.1. Jumps

5.4.1.1. Annualized jump frequency

From figure 11 depicting the annualized jump frequency for puts and calls, we again observe similar trend lines except for the period July 2007 – January 2008 in which both set of parameters behave differently.

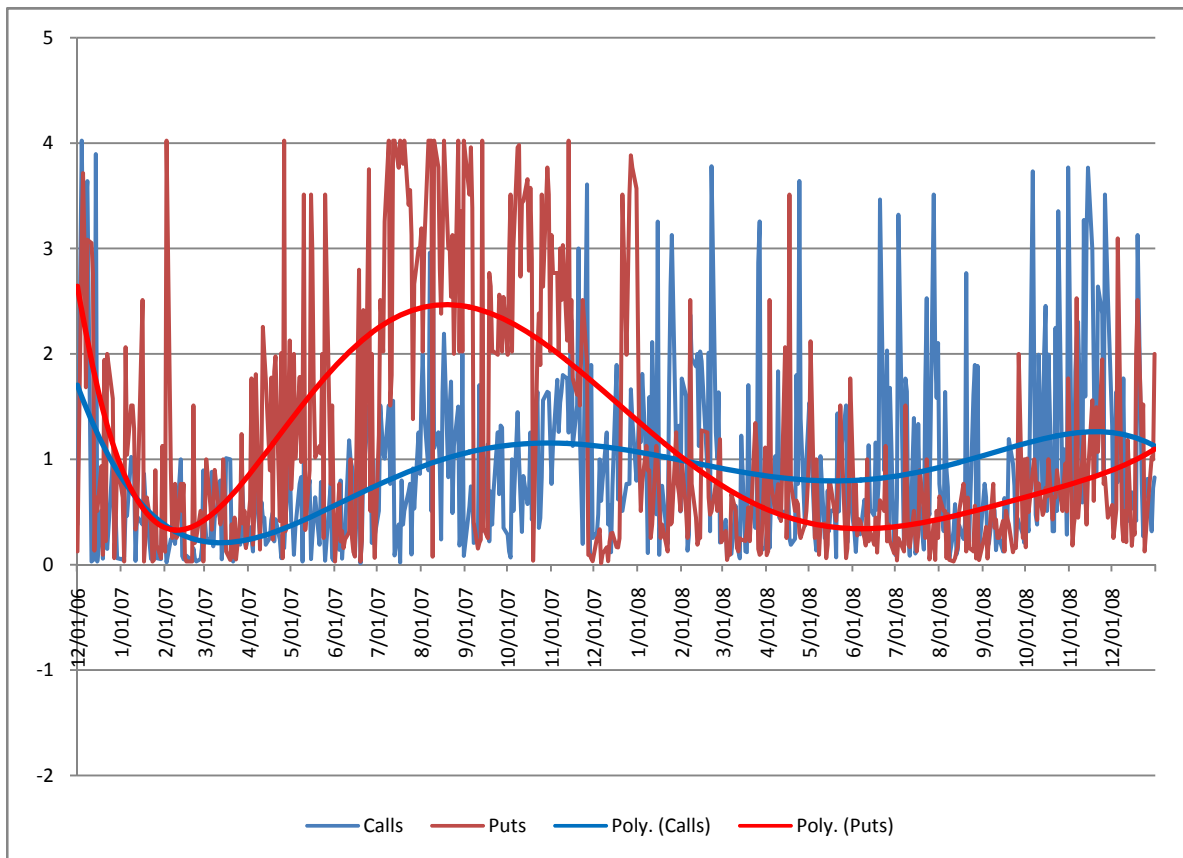


Figure 11: λ^* - Implicit annualized jump frequency estimated via Non-linear regression December 2006 December 2008

5.4.1.2. Average Jump Size and jump size volatility

Figure 12 illustrates estimated daily jump size parameters \bar{k}^* which measure the change in the distribution of the underlying asset, in this case the S&P500 index. Recall from the specifications of equation #2, the asset price changes by a factor if $(1+k^*)$. The implicit sizes of the corrections or jumps would indicate strong crash fears starting in December 2008 through the end of our testing window.

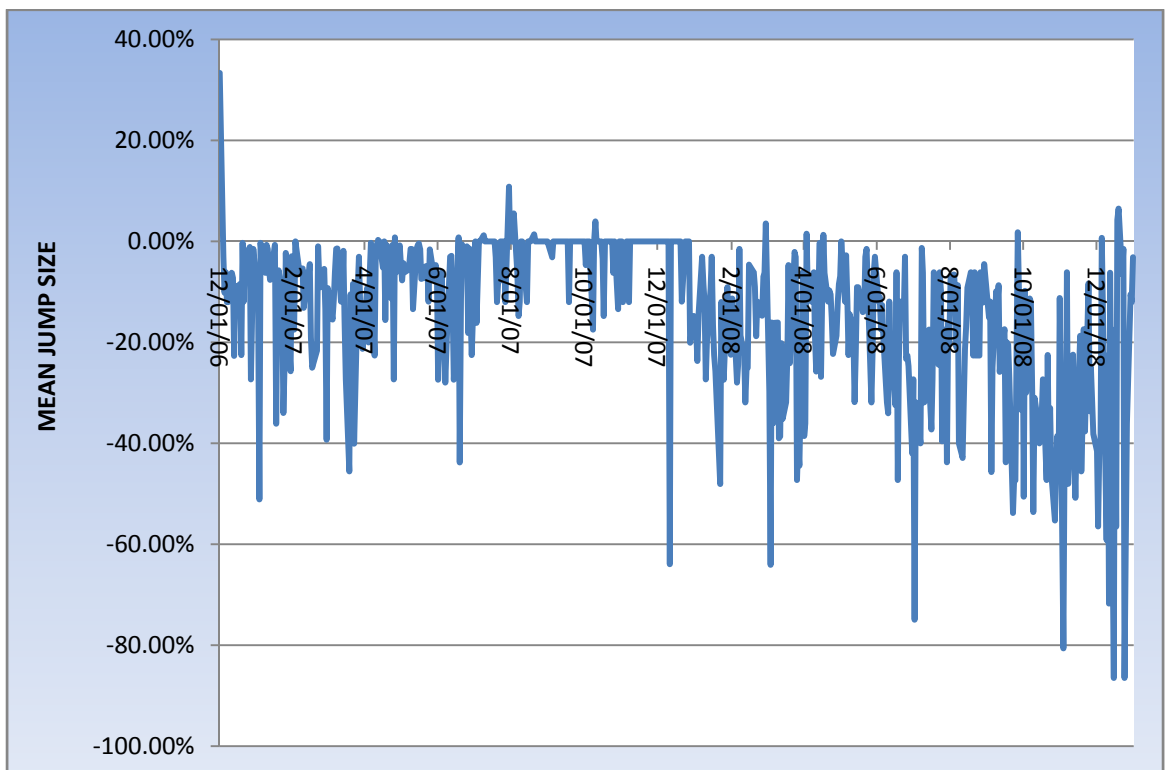


Figure 12: \bar{k}^* Daily average jump size from Non Linear Estimation December 2006 – December 2008

An interesting observation is that the sizes of the jump are quite muted between July 2007 and December 2007 compared to other months even though the implicit volatility and implicit annualized frequency of jumps were higher in this period.

Nevertheless, the daily delta parameter (Figure 13), volatility of jump sizes conditional on jump occurring, was below 0.2 for the most part from December 2006 to March 2008 although it spiked five times from July to the end of 2007.

We conclude that the volatility of the S&P500 index and the one associated with jumps or sharp movements in the index definitely increased starting in March 2007 which marks the beginnings of the mortgage crisis. Although we cannot definitely tell from our analysis what the expected size of the correction was a few months prior to the crash as other figures (see figure #14) depict a more muted correction.

We also note that expectations of further downward movements were present after the September 2008 crash actually reaching their peak in December. This hinted to a terrible first quarter in 2009 which in fact materialized.

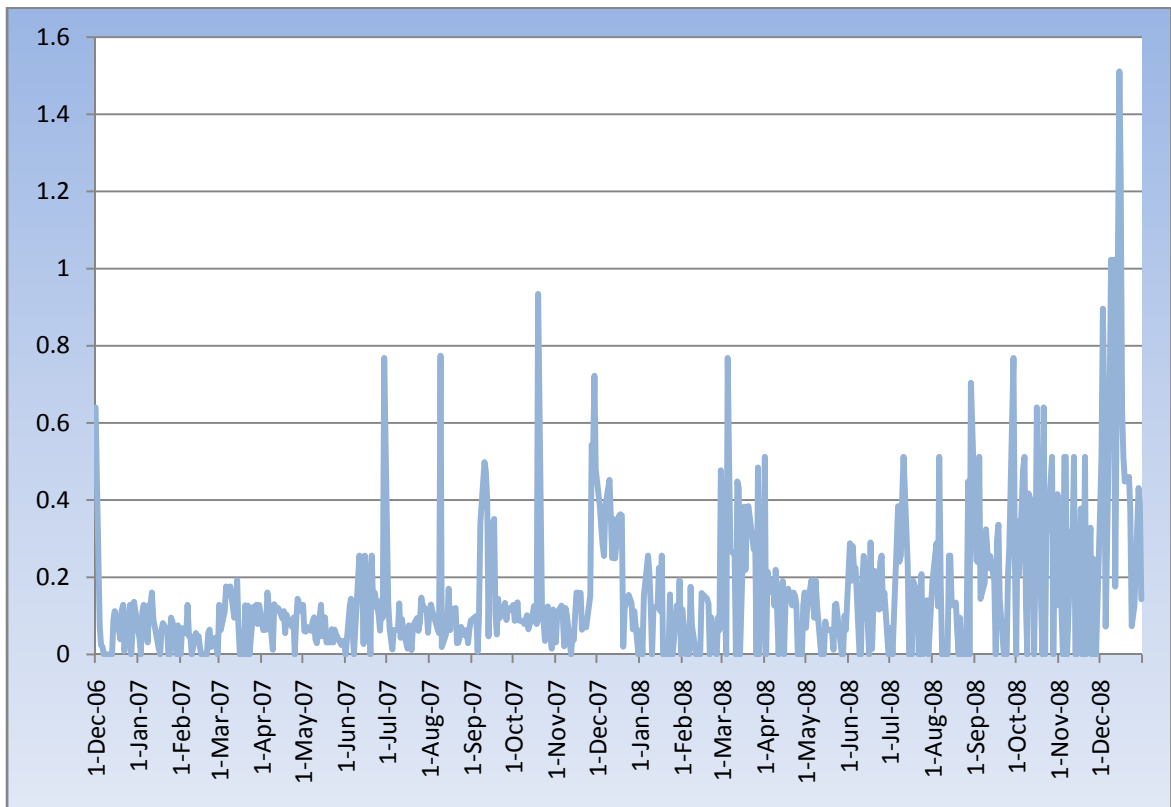


Figure 13: δ –Implicit jump standard deviation (percent) December 2006-December 2008

5.4.1.3. Jump frequency times Size

The jump frequency times jump size, $\lambda^* \bar{k}^*$, represents the expected jump loss or gain which summarizes the expectations of the market. In Figure 14, we observe a drastic change in expectations starting in July 2007. The period from July to November 2007 is characterized by the presence of higher expected jump losses in the put parameters compared to calls. Perhaps, the market was purchasing crash insurance to hedge against corrections expected in 2008 (refer back to Figure 12). Surprisingly, expected jump loss in the put series decreased after January 2008.

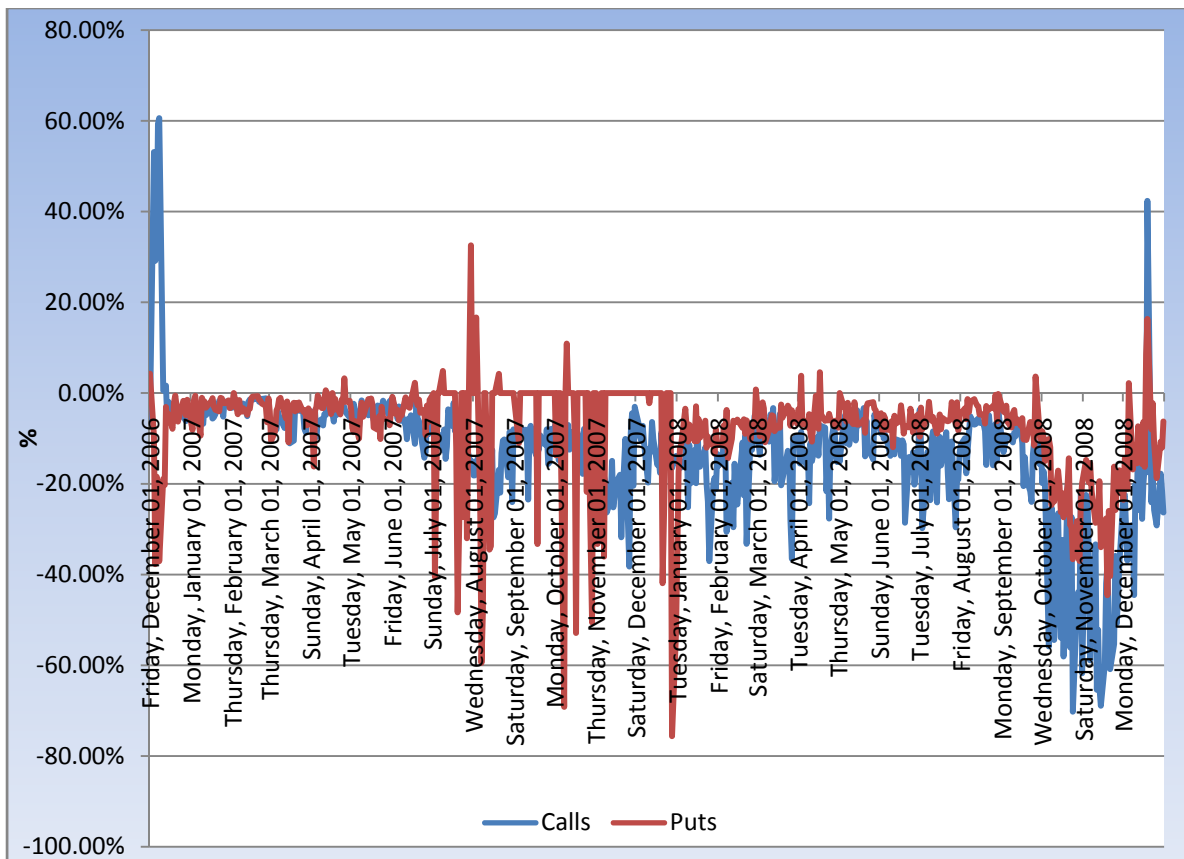


Figure 14: $\lambda^* \bar{k}^*$ of Non-linear Estimation for Options on Futures S&P500 December 2006-December 2008

In summary, our findings lead to the conclusion that there was a structural break in market behavior after Bear Stearns collapse in April 2008. As discussed above, our estimated parameters point to large negative jumps and high volatility associated with the jump size distribution after April 2008. However, no corrective actions seem to have been taken as calls were still priced at a premium for the most part. We offer the explanation that actions taken by the Fed (i.e. string of government bailouts) spread a false sense of confidence that the market had dodged a bullet. Signs of fear re-appeared after mid-July 2008 although it's unclear how and why the market did not grasp the extent of well document problems²⁷ in the financial system.

5.4.2. Implied volatility

The graph of the implied volatilities inferred from the call and put prices on the S&P500 future series show an increase in unconditional volatility (i.e. not dependent on jumps) from July 2007 to January 2008 pointing to increased uncertainty in the 3 month forward looking market outlook. Volatility stabilized then increased sharply after Lehman Brothers' collapse reaching its highest level at 0.5 then dropping down sharply below 0.1 for the March 2009 series.

²⁷ <http://www.creditwritedowns.com/credit-crisis-timeline>

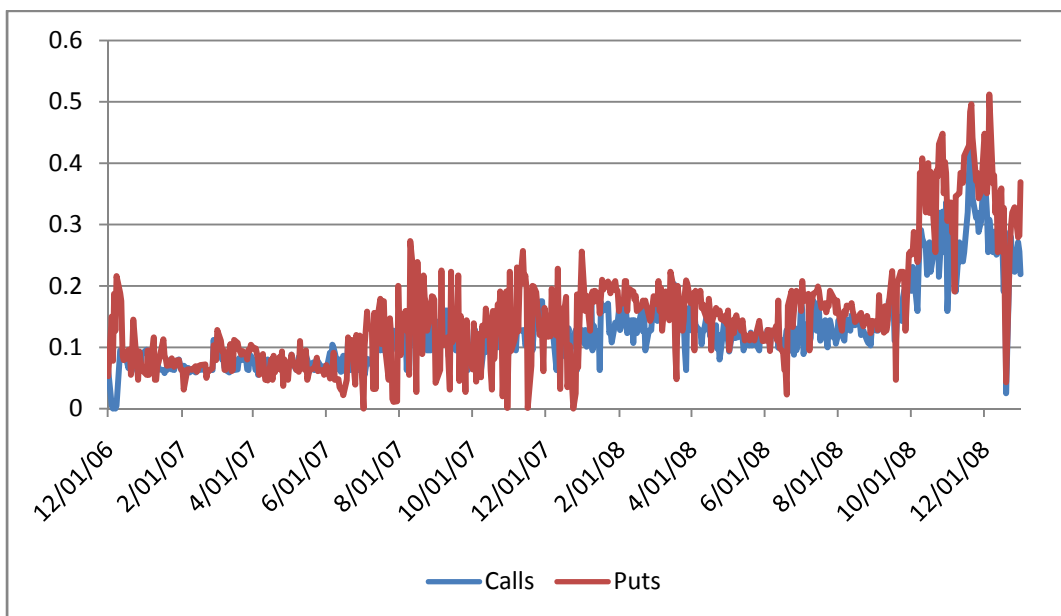


Figure 15: Implied Volatility from Non linear Estimation for Options on Futures of SP500
Dec2006-Dec2007

Figure 16 presents the differentials between ATM option prices estimated using the Black-Scholes and Bates jump diffusion. Differentials appear to increase in periods of high volatility (July 07 to Jan. 08 and Sept. 2008 to Dec. 2008). We suspect that these differentials could provide a measure of pure risk premium associated with jump risk. More analysis would be needed in this area before drawing conclusions.

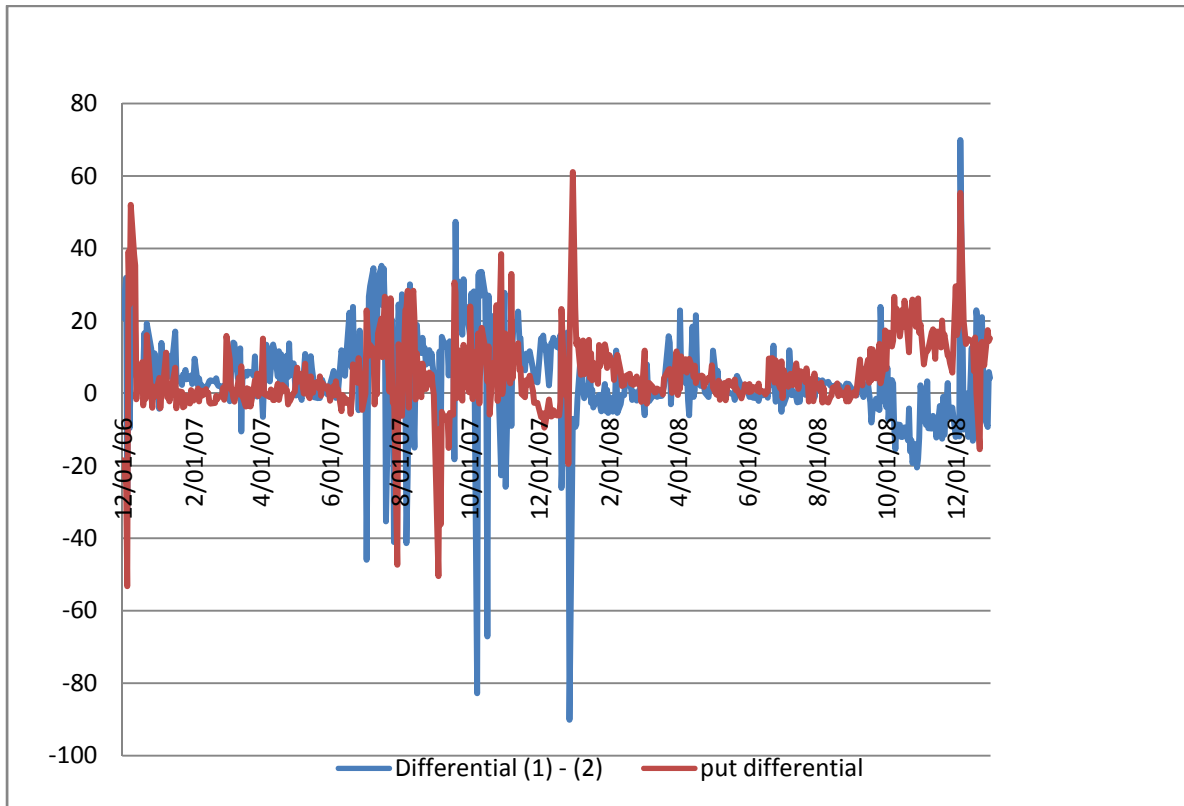


Figure 16: Differentials from Black and Scholes and Jump diffusion call and put option prices. Also refer to Table 6.

6. Conclusions and further research

Our research provides evidence that the information contained in option prices on the S&P500 futures between December 2006 and December 2008 are ample to forecast market expectation of a crash prior to it happening in “September 2008”. Our assessment is that Bates’ (1991) jump diffusion model is an adequate modeling tool to extract parameters of the risk-neutral distribution implicit in options prices. There is strong evidence of its robustness and the empirical data back up this affirmation. The results obtained in our research are consistent with those presented in Bates’ (1991) paper. The fitting is very good showing that the model is consistent and unbiased.

We find option-based forecast models to be superior to those using historical data. The former are also helpful in estimating tail risk when assuming an asymmetric jump diffusion process. Further model development is required for its use as an alternative measure, or at the minimum a supplementary one, to popular market risk management tools such as VAR. Further empirical studies into the effectiveness of option based forecasting tools and the use of implied volatility in option trading and risk management would be very valuable to gauge the accuracy of such models against realized volatility. Jorion (2007) states that “whenever possible, VAR should use implied parameters”.

After reviewing work by Bliss and Panigirtzoglou (2001) and others in the literature review section, who recommend using a subjective PDF inferred from the risk neutral one to extract expectation, we see the benefit of further research into the relative risk aversion parameter R . Based on the preliminary work Bates has laid out, R can provide a measure of the divergence between true parameters from the actual distribution of the underlying asset from risk neutral ones. It would be interesting to see how R evolves over time.

In the context of the current financial crisis, we conclude that expectation of market downturn started in the summer 2007 and remained through December 2007. That is, the information implied in the futures series and option prices on futures predicted a persistent downward trend until the end of March 2008. However, in early January 2008, the three month outlook seemed positive as apparent in the reduced oscillation of the implied volatility graph and the average jump times expected jump size figure (#16). There was a sizeable negative jump after April 2008, however sentiments of crash fears subsided until July 2008 when market expectation of a correction (larger

negative jump) intensified. Surprisingly, the market did not take corrective measures in the summer of 2008. Evidence of this is proven by the fact that crash insurance paid by put options remained somewhat at the same level from February 2008 to September 2008 as depicted in figure 16 showing the cost per unit time of jump insurance, λ^* , times the average size of the jumps, \bar{k} . Additionally, figure 18 shows minimal difference in the put prices estimated using Black-Scholes formula, which ignores jump risk, and option pricing under asymmetric jump diffusion, which factors in positive and negative jumps.

The parameters derived from our non-linear estimation show evidence of a rational bubble that was ready to burst in the period under observation. Similar to Bates (1991), we can assess that implicit jump risk assessments are strongly countercyclical after the crash. The bankruptcy of Lehman Brothers and liquidity constraints faced by other institutions holding mortgage backed securities magnified fears of widespread systemic risk. Similarly to events following the crash of October 1987, after mid September 2008, behavior of market participants reflected strong crash fear and irrationality as the market experienced large downward but also large upward movement in December 2008.

We conclude that the Theory of Rational Bubble cannot be rejected. We have gathered enough information to determine that a bubble did in fact burst not in September 2008 as reported but in April 2008. We are of the opinion that from information available in January 2008, a crash was expected in April 2008 but was delayed by actions of the Fed when Bear Sterns was liquidated and sold off to JP Morgan. We cannot say definitely what the magnitude of a crash would have been had the bubble burst in April rather than in September 2008. It also raises the question of whether or when the market would have crashed if Lehman Brothers was bailed out. It would be interesting to research the

question of the actions, whether delayed, inadequate or unnecessary, of the Fed introduced heightened fear of systemic risk in the market. So to answer the question: was the magnitude of the crash in September 2008 expected? The answer would be no. In July 2008, there was clear evidence in the future series maturing in September 2008 and information extracted from the option prices of an imminent sharp downward movement however the expected size of the negative jump in the S&P500 was moderate in comparison to events that transpired in September 2008.

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8. Appendix

---Exhibit I---

Bubble definition and characteristics

“The price process S has a bubble if S is a strict local martingale under the risk-neutral measure Q .

The key fact is that simply because S is a strict local martingale does not mean that there are arbitrage opportunities. In particular, the strategy of selling the asset short may not be admissible, since the liability is unbounded.”²⁸

To see this super martingale process we can check the following conditions:

S_T - asset price at time T

(ii) $E_t[S_T] < S_t$, so that the forward price is below the current price;

(iii) $C_t^E(K) - P_t^E(K) < S_t - K$ for some K , so that put-call parity fails;

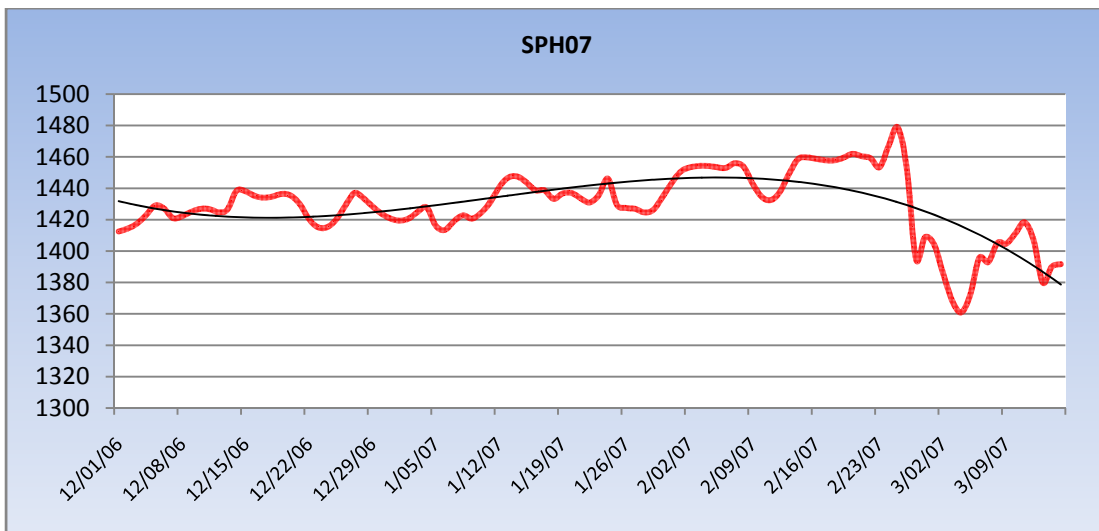
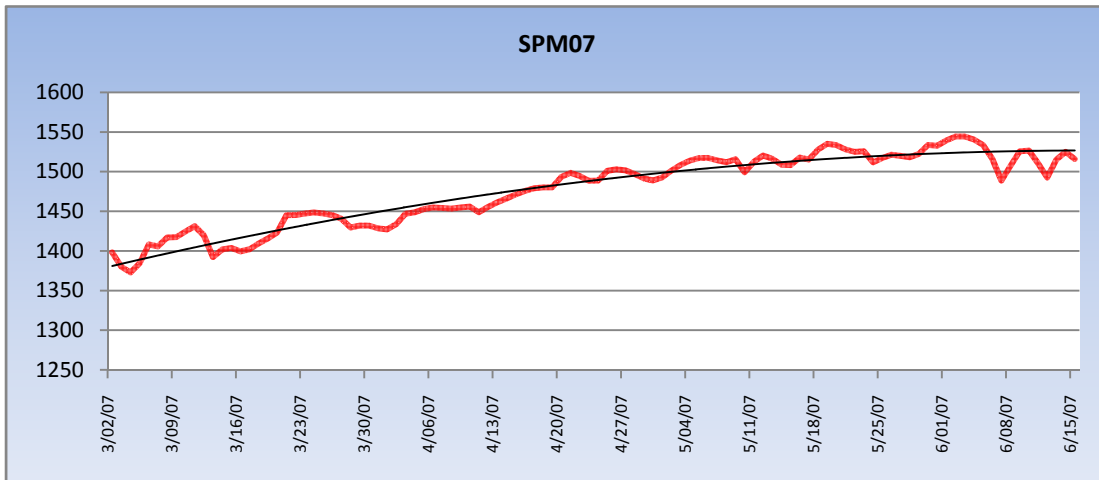
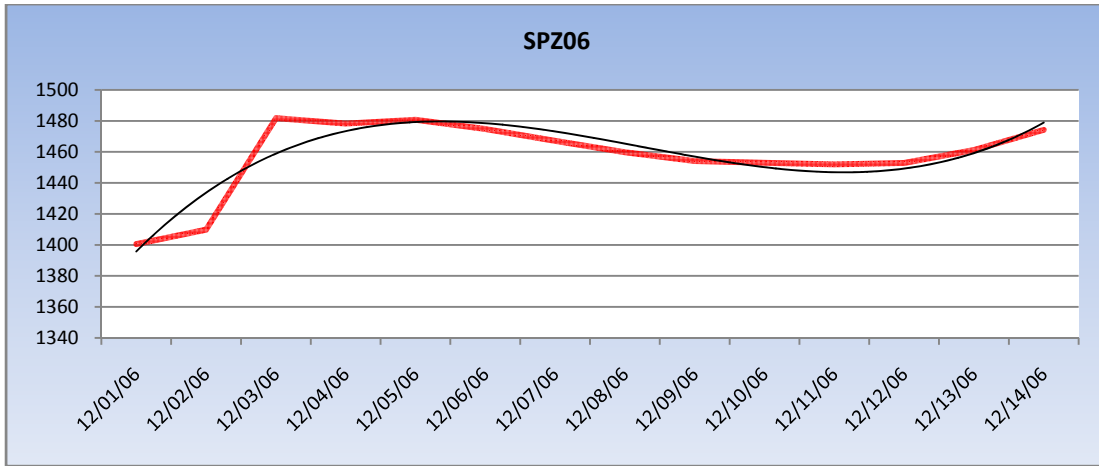
(iv) $\lim_{K \uparrow \infty} P_t^E(K) - K + S_t > 0$;

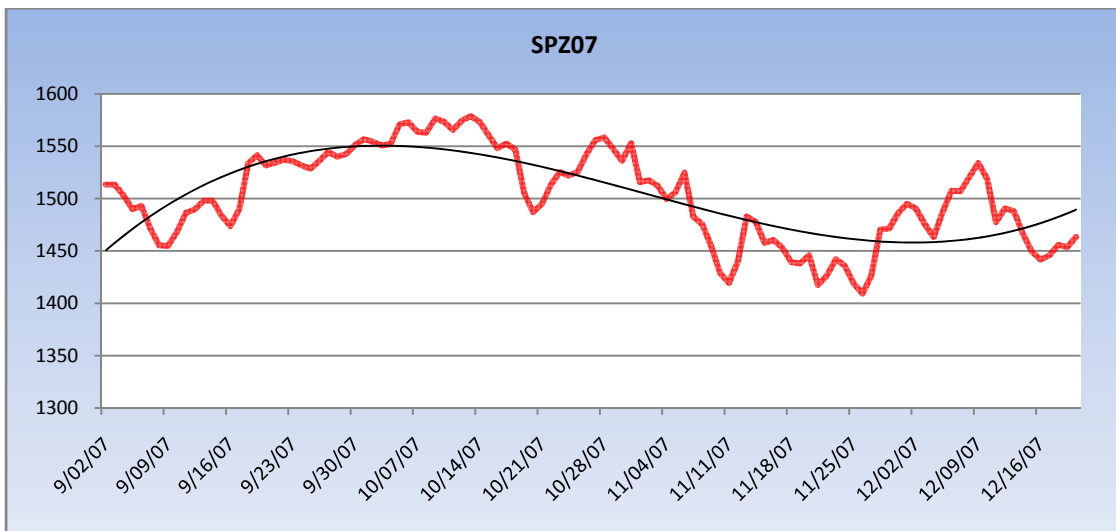
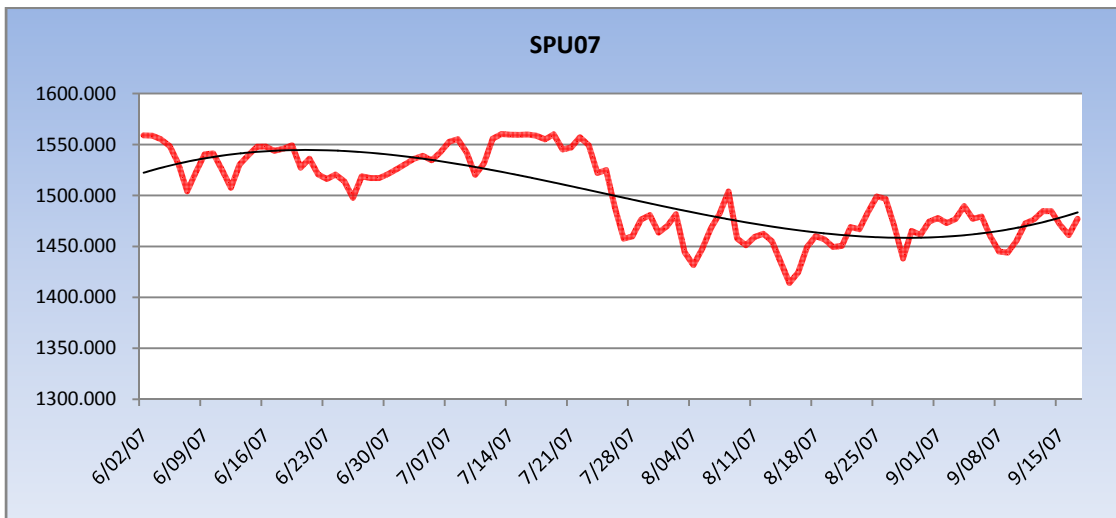
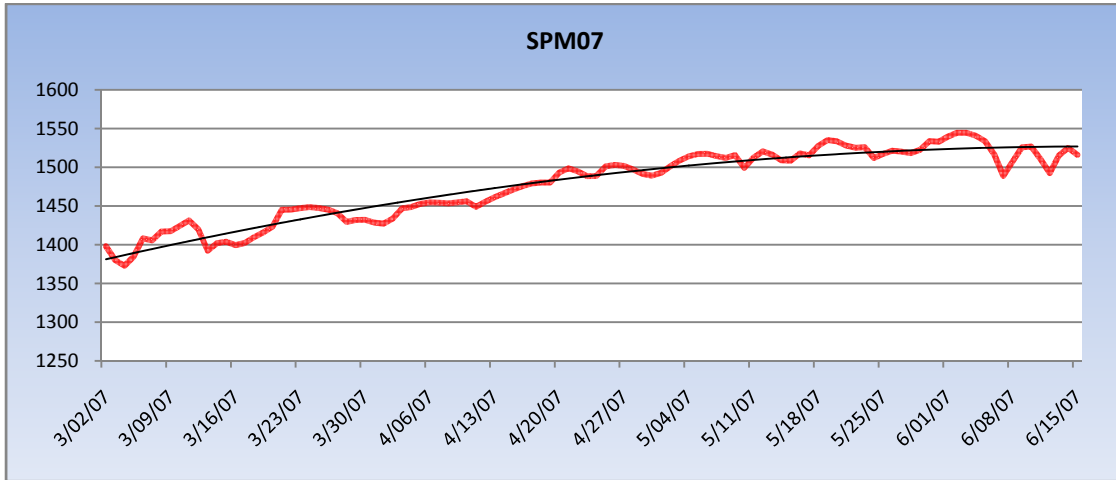
(v) $C_t^E(K) < C_t^A(K)$ for some K , so that American calls are more expensive than their European counterparts;

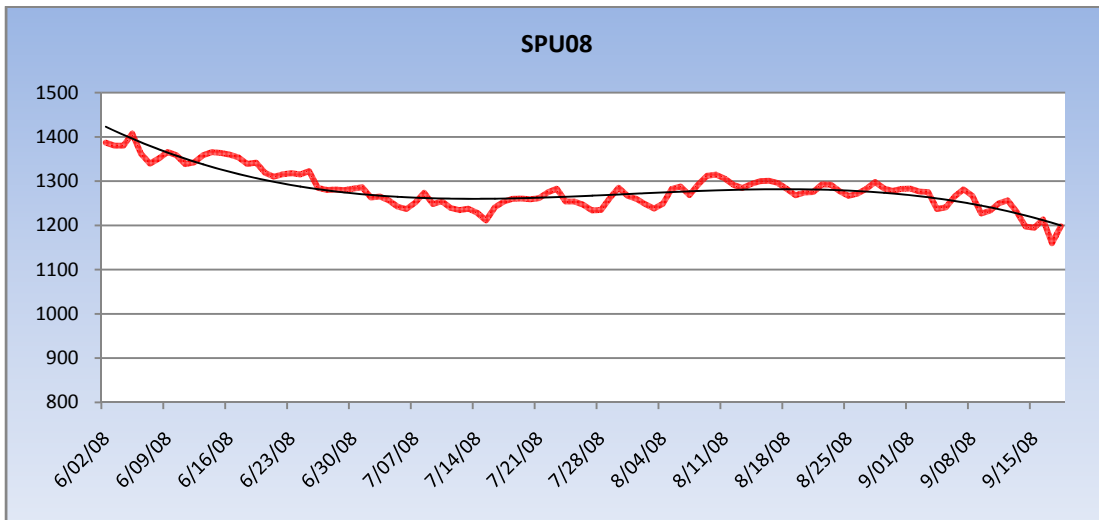
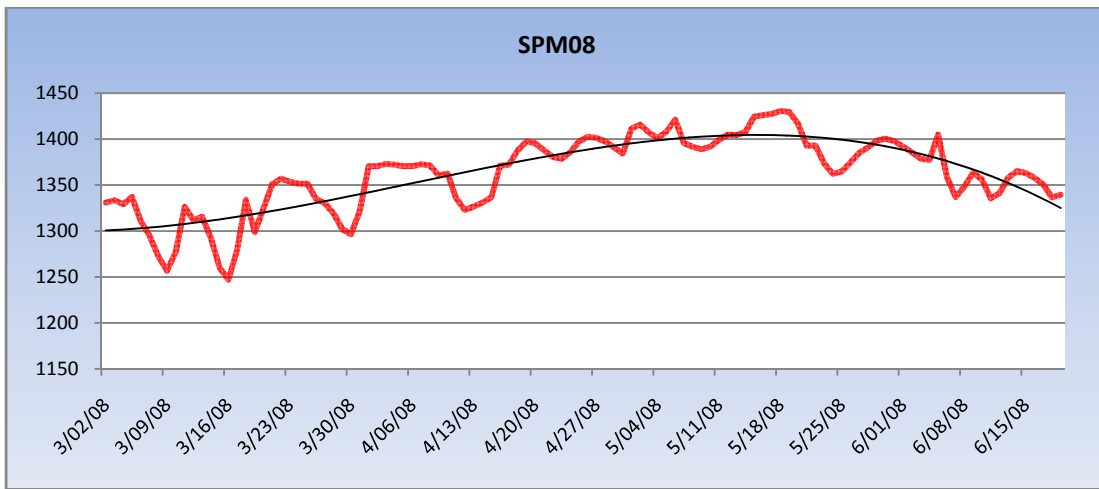
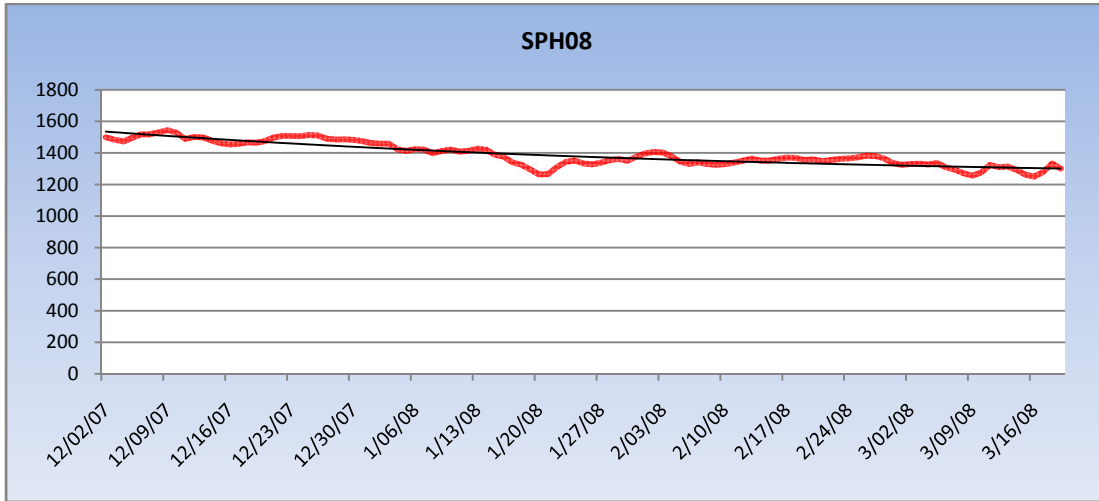
(vi) $\lim_{K \uparrow \infty} CA_t(K) > 0$.

²⁸ Alexander M.G. Cox¹, David G. Hobson, Local martingales, bubbles and option prices

---Exhibit II : Futures Series---







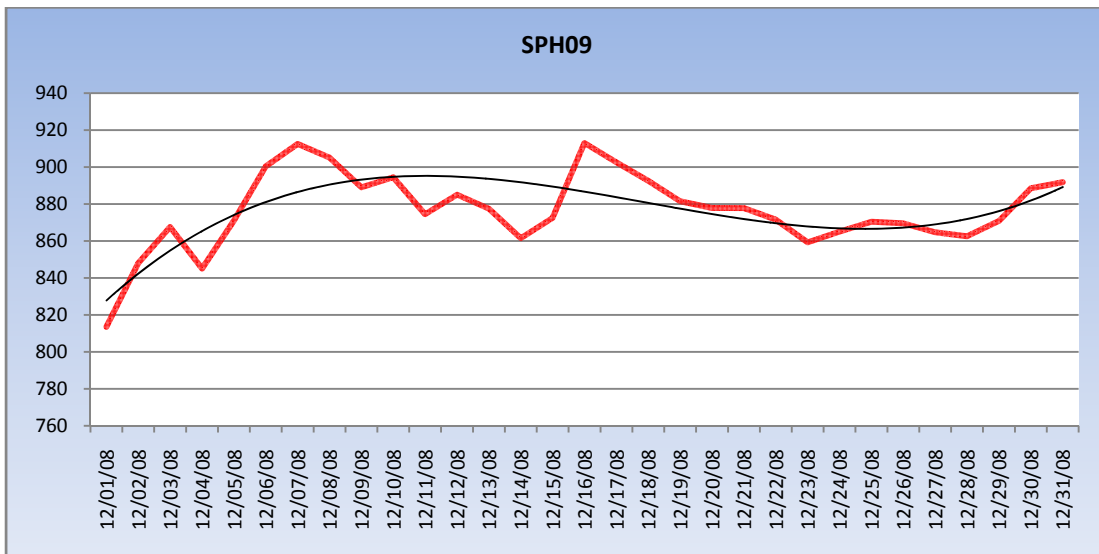
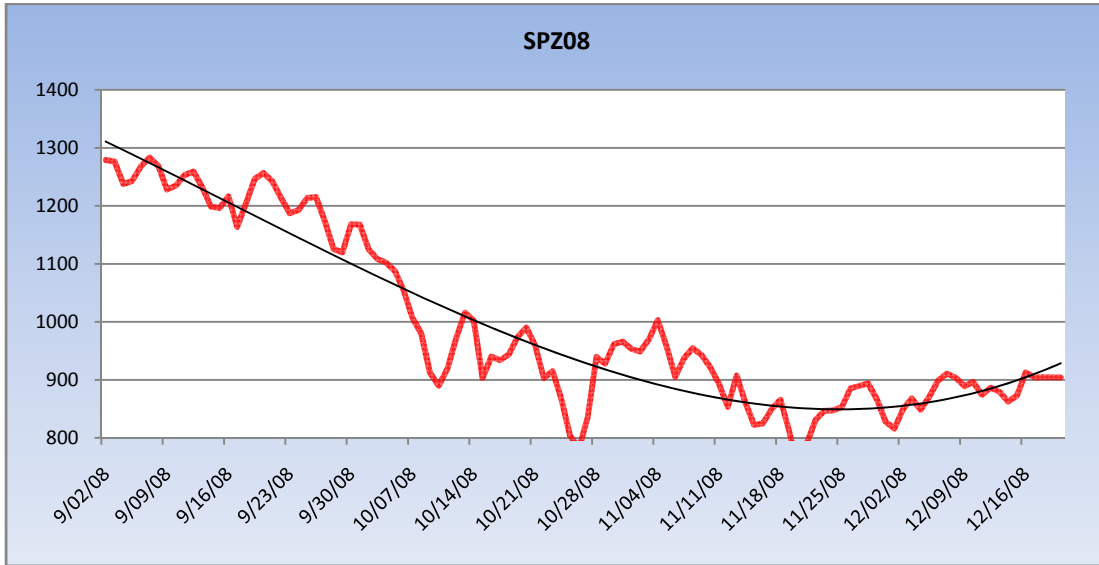


Table 1: Descriptive Statistics for Call Data from 12/ 2006 to 12/2008

	<i>X</i>	<i>V</i>	<i>V/F</i>	<i>X/F</i>
Mean	1442.79	45.45	0.03	1.12
Standard Error	0.71	0.29	0.00	0.00
Median	1465.00	4.40	0.00	1.07
Mode	1475.00	0.00	0.00	1.00
Standard Deviation	235.44	96.22	0.07	0.24
Sample Variance	55431.21	9258.66	0.00	0.06
Kurtosis	1.47	26.58	27.40	4.02
Skewness	-0.68	4.42	4.45	1.43
Range	2075.00	979.80	0.79	2.42
Minimum	200.00	0.00	0.00	0.22
Maximum	2275.00	979.80	0.79	2.64
Number of Observations	108558.00	108558.00	108558	108558.0
Confidence Level(95.0%)	1.40	0.57	.00	0

Table 2: Descriptive Statistics for Put Data from 12/ 2006 to 12/2008

	<i>X</i>	<i>V</i>	<i>V/F</i>	<i>X/F</i>
Mean	1250.9340	33.5262	0.1944	0.0036
Standard Error	0.7099	0.1816	0.0004	0.0000
Median	1285.0000	6.2000	0.2078	0.0028
Mode	1400.0000	0.0000	0.0000	0.0000
Standard Deviation	233.9096	59.8203	0.1266	0.0042
Sample Variance	54713.7186	3578.4691	0.0160	0.0000
Kurtosis	-0.0214	9.4684	-0.1503	59.9508
Skewness	-0.6096	2.8197	-0.0214	5.4016
Range	1250.0000	488.5000	0.9810	0.0818
Minimum	500.0000	0.0000	0.0000	0.0000
Maximum	1750.0000	488.5000	0.9810	0.0818
Number of Observations	108558	108558	108558	108558
Confidence Level(95.0%)	1.3915	0.3559	0.0008	0.0000

**Table 3: Descriptive Statistics for the Future Series from
12/ 2006 to 12/ 2008**

Mean	1351.03
Standard Error	6.34
Median	1414.2
Mode	1536
Standard Deviation	191.38
Sample Variance	36624.78
Kurtosis	1.37
Skewness	-1.49
Range	829.27
Minimum	749.5
Maximum	1578.77
Count	912

**Table 4: Mean and Volatility Historic for Future Series Q1 2007 -Q42008
(Daily Observations)**

	Q1-2007	Q2-2007	Q3-2007	Q4-2007	Q1-2008	Q2-2008	Q3-2008	Q4-2008
Variance	571.2121	865.9525	1528.508	1857.677	2258.227	1178.7578	1405.57	4793.667
STD	23.9	29.4271	39.0961	43.1008	47.5208	34.333	37.4909	69.2363
Mean	1429.79	1508.771	1495.37	1497.855	1341.128	1369.3831	1250.34	903.4645
Volatility	1.67%	1.95%	2.61%	2.88%	3.54%	2.51%	3.00%	7.66%

Table 5A: Theoretical Futures Options Values under Asymmetric Jump Diffusion Processes Futures Price (Bates 1991, page 1028-1029)

F= 250. Parameters: r=0.10, T=0.25

Jump Diffusion Parameters	Exercise Price X	Call Options			Put Options		
		European c(F,T;X)	American C(F,T;X)		European p(F,T;X)	American C(F,T;X)	
			Finite Difference Method ²⁹	Quadratic Approximation Method		Finite Difference Method	Quadratic Approximation Method
$\sigma = 0.11414$	220	29.49		30.03	0.23		0.23
$\lambda = 0$	235	16.39	N/A	16.53	1.76	N/A	1.77
$\gamma = 0$	250	6.88		6.92	6.88		6.92
$\delta = 0$	265	2.04		2.06	16.67		16.82
	280	0.42		0.43	29.68		30.12
$\sigma = 0.10$	220	29.45		30.01	0.19		0.19
$\lambda = 10$	235	16.25	N/A	16.42	1.62	N/A	1.63
$\gamma = 0.01$	250	6.81		6.86	6.81		6.85
$\delta = 0.03$	265	2.17		2.18	16.79		16.91
	280	0.56		0.56	29.82		30.19
$\sigma = 0.10$	220	29.58		30.04	0.33		0.33
$\lambda = 10$	235	16.49	N/A	16.61	1.86	N/A	1.88
$\gamma = -0.01$	250	6.79		6.83	6.79		6.85
$\delta = 0.03$	265	1.88		1.89	16.51		16.68
	280	0.35		0.35	29.61		30.09
$\sigma = 0.10$	220	29.3		30	0.04		0.04
$\lambda = 0.25$	235	15.62	N/A	15.92	0.99	N/A	0.99
$\gamma = 0.2$	250	6.28		6.42	6.28		6.28
$\delta = 0.03$	265	2.65		2.72	17.28		17.32
	280	1.42		1.45	30.68		30.81
$\sigma = 0.10$	220	30.14		30.32	0.88		0.90
$\lambda = 0.25$	235	16.71	N/A	16.75	2.08	N/A	2.13
$\gamma = -0.20$	250	6.02		6.02	6.02		6.14
$\delta = 0$	265	1.11		1.11	15.74		16.03
	280	0.09		0.09	29.35		30.01

²⁹ Finite Difference method for jump diffusion was not replicated

Table 5B: Replicated results (re: Table5A)

Jump Diffusion Parameters	Exercise Price X	Call Options			Put Options		
		European c(F,T;X)	American C(F,T;X)		European p(F,T;X)	American C(F,T;X)	
			Finite Difference Method ³⁰	Quadratic Approximation Method		Finite Difference Method	Quadratic Approximation Method
$\sigma=0.11414$ $\lambda = 0$ $\gamma = 0$ $\delta = 0$	220	29.48		30	0.23		0.23
	235	16.38		16.38	1.76		1.76
	250	6.88	N/A	6.91	6.88	N/A	6.88
	265	2.04		2.05	16.67		16.69
	280	0.42		0.42	29.68		29.74
$\sigma = 0.10$ $\lambda = 10$ $\gamma = 0.01$ $\delta = 0.03$	220	29.45		30	0.19		0.19
	235	16.25		16.3	1.62		1.62
	250	6.81	N/A	6.83	6.81	N/A	6.81
	265	2.17		2.17	16.79		16.8
	280	0.56		0.56	29.82		29.82
$\sigma = 0.10$ $\lambda = 10$ $\gamma = -0.01$ $\delta = 0.03$	220	29.58		30	0.33		0.33
	235	16.49		16.49	1.86		1.87
	250	6.79	N/A	6.82	6.79	N/A	6.8
	265	1.88		1.89	16.51		16.54
	280	0.35		0.35	29.61		29.68
$\sigma = 0.10$ $\lambda = 0.25$ $\gamma = 0.2$ $\delta = 0.03$	220	24.38		30	0.04		0.04
	235	11.58		15	0.99		0.99
	250	3.14	N/A	3.19	6.28	N/A	6.28
	265	0.39		0.41	17.28		17.28
	280	0.02		0.03	30.66		30.66
$\sigma = 0.10$ $\lambda = 0.25$ $\gamma = -0.20$ $\delta = 0$	220	30.11		30.11	0.01		0.02
	235	16.71		16.71	0.35		0.38
	250	6.02	N/A	6.03	3.41	N/A	3.46
	265	1.11		1.11	12.24		12.37
	280	0.09		0.09	24.97		30

Some replicated results (in red) differ from the Bates result. We re-estimated these prices using slightly lower values of lambda and observe the prices approaching those reported by Bates. Therefore, we conclude again as presented in our results section that prices estimated under a jump diffusion process is very sensitive to set strike price selection and maturity term.

³⁰ Finite Difference method for jump diffusion was not replicated

Table 6: Option prices with Black-Scholes and Jump Diffusion Model

Date	X=F	(1) JD European Call	(2) Black-Scholes Call	Call Differential (2) - (1)	(3) JD European Put	(4) Black-Scholes Put	Put Differential (4) - (3)
December-15-06	1438.10	30.60	35.03	4.43	13.87	16.36	2.49
January-16-07	1438.80	19.00	36.07	17.07	15.94	23.00	7.07
February-15-07	1459.70	15.05	18.41	3.37	12.96	10.52	-2.45
March-15-07	1403.80	40.06	29.58	-10.48	3.71	11.18	7.47
April-16-07	1475.50	24.08	28.72	4.64	12.48	15.14	2.67
May-15-07	1508.30	14.53	24.79	10.25	11.54	16.26	4.71
June-15-07	1547.80	11.68	21.52	9.84	3.88	1.24	-2.64
July-16-07	1559.70	22.97	58.18	35.20	22.97	43.63	20.66
August-15-07	1414.00	3.98	10.31	6.33	3.98	2.31	-1.67
September-14-07	1498.00	31.29	45.67	14.38	31.29	25.86	-5.43
October-15-07	1560.50	16.50	43.71	27.21	16.50	28.96	12.46
November-14-07	1478.00	35.17	50.17	15.00	35.17	41.63	6.46
December-14-07	1498.50	45.91	61.24	15.33	45.91	41.23	-4.67
January-15-08	1388.40	35.43	45.80	10.37	28.55	32.85	4.30
February-15-08	1351.00	30.03	33.89	3.86	21.72	26.41	4.69
March-14-08	1292.00	27.11	26.15	-0.95	22.11	23.45	1.34
April-15-08	1336.50	50.82	49.85	-0.96	32.82	37.39	4.57
May-15-08	1424.50	26.46	26.59	0.12	15.14	18.53	3.39
June-16-08	1357.80	11.37	10.97	-0.40	8.17	8.63	0.46
July-15-08	1211.10	52.34	49.90	-2.44	32.71	38.45	5.74
August-15-08	1299.90	28.06	29.72	1.66	20.82	22.37	1.56
September-15-08	1196.30	63.05	65.65	2.59	40.65	49.97	9.31
October-15-08	903.00	82.17	70.13	-12.04	38.50	61.60	23.10
November-17-08	850.00	55.65	52.19	-3.46	30.58	47.59	17.01
December-15-08	872.50	40.79	53.36	12.58	27.62	42.03	14.41

Table 7: Estimated Sample of Parameter with Non-Linear Estimation with the Hillclimbing method for Bates Jump Diffusion Process

Date	F	Lambda	Gamma	Delta	Sigma
12/01/06	459.286	0.3	-0.421	0.161	0.034
12/06/06	704.452	3.245	-0.036	0.063	0.119
12/07/06	444.299	3.373	-0.066	0.087	0.069
12/08/06	1703.414	3.447	-0.08	0.089	0.043
12/12/06	398.931	3.51	-0.061	0.083	0.039
12/14/06	94.580	3.023	-0.064	0.075	0.031
12/15/06	156.028	2.592	0.055	0.079	0.046
12/20/06	184.550	3.08	-0.128	0.027	0.118
12/21/06	105.499	3.192	-0.048	0.022	0.138
12/26/06	148.626	2.233	-0.064	0.023	0.121
12/28/06	141.934	3	-0.131	0.068	0.039
1/10/07	117.656	2.987	0.128	0.031	0.095
1/11/07	89.108	2.832	-0.048	0.033	0.101
1/22/07	96.946	3	-0.128	0.031	0.078
1/29/07	71.842	1.987	-0.032	0.055	0.063
2/02/07	20.674	4.023	-0.032	0.019	0.087
2/15/07	36.934	2.527	-0.064	0.014	0.077
3/05/07	354.231	3.512	0.031	0.058	0.121
3/07/07	380.221	3.793	-0.056	0.026	0.126
3/28/07	356.703	2.384	-0.064	0.086	0.052
3/30/07	314.072	2	-0.064	0.072	0.095
4/03/07	165.245	4.023	-0.111	0.031	0.127
4/05/07	292.625	1.831	-0.064	0.051	0.107
4/06/07	301.528	2	-0.112	0.006	0.125
4/23/07	196.863	3	-0.13	0.045	0.085
4/24/07	185.077	2.576	-0.108	0.067	0.039
4/25/07	120.810	3.965	-0.032	0.011	0.128
4/26/07	77.677	4.023	-0.064	0.055	0.054
4/30/07	130.685	3.896	-0.064	0.039	0.103
5/03/07	110.570	4.023	-0.128	0.05	0.045
5/07/07	82.025	4.023	-0.128	0.027	0.094
5/10/07	50.001	4.023	0.008	0.015	0.115
5/11/07	81.409	3	-0.051	0.057	0.039
5/15/07	53.994	4.023	-0.064	0.051	0.051
6/12/07	488.492	0.008	-1.023	0.944	0.095
6/14/07	570.823	0.026	-0.641	0.052	0.028
6/19/07	205.868	2.255	0.041	0.031	0.139
6/26/07	472.395	1.998	-0.064	0.071	0.15

---Exhibit III: Probability density functions for F_{t+1}/F_t ---

