

THE EFFECTS OF ESTIMATION ERROR IN PORTFOLIO CREDIT RISK MODELING

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APPROVAL

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ABSTRACT

In this paper, we assess the effects of estimation error due to the impact of noisy input parameters in portfolio credit risk modelling by using Monte-Carlo simulations. We employ the methodology used in Löffler (2003) but apply different dataset to form two new portfolios: obligors with investment-grade credit rating and obligors with speculative-grade credit rating. The four sources of estimation risk are considered for each portfolio: default rate uncertainty only, recovery rate uncertainty only, correlation uncertainty only, and the three sources of uncertainty together. The resulting estimation error in the distribution of portfolio losses is considerable. The paper also shows that different credit datasets could result in different biases in value at risk (VaR) estimations in each portfolio.

Keywords: Credit Risk; Estimation error; Value at risk; Quantiles; Simulated distributions

DEDICATION

I wish to dedicate this paper to my dearest parents for supporting me during all the years of my study. Without their love and support, the completion of my study would not have been possible. Also, I wish to dedicate this paper to my boyfriend and all my friends, who care and support me during the years I study in Canada.

Ying

I wish to dedicate this paper to my dearest parents for fully supporting me the years to study in Canada. I also wish to dedicate this paper to my husband Dafang for consistently understanding and taking care of me throughout my study in this program.

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1. Introduction

Credit risk is one of the major issues in financial world. How to manage this risk has become an important concern for banks, regulators and academics in the past few years. In order to consistent with this trend, several models for the measurement and estimation of portfolio default and credit risk haven been proposed. (Crouhy et al., 2000) In the recent years, there is a considerable and growing literature about credit risk modeling. These credit risk models focus on characterizing, quantifying, forecasting and evaluating of the consequence of default of various contractual portfolios. (Jones, 2000) However, few of those literatures talked about the reliability of credit risk models (Löffler, 2003). Although some of the models have been already widely used in the market, analysts should notice that there are still uncertainties and estimation error exist. Sometimes, simply neglects estimation error would lead to significant flawed estimation results. Indeed, since analysts like to use those models to estimate portfolio losses due to default and credit risks, actual distribution of portfolio losses may vary from the estimated value because of estimation error.

In 2003, Löffler published a paper about analyzing estimation error in credit risk models in *Journal of Banking & Finance*, called *The Effects of Estimation Error on Measures of Portfolio Credit Risk*. Löffler (2003) doubted the reliability of existing credit risk model estimations and argued that there are uncertainties for the model input parameters, which should be taken into consideration. Also, even though the estimations of input parameters are correct, there is still considerable estimation error lead to biases in portfolios' value at risk (VaR) estimates, as well as the distribution of portfolio losses. (Löffler, 2003) The results showed in Löffler (2003) contributed in credit risk modeling, and provided a platform for further research. Therefore, in

our paper, we employ the idea and methodology used in Löffler (2003) to quantify the estimation error in portfolios' VaR calculations. By using different datasets, we would like to see if our results are consistent with those in Löffler (2003).

The necessary input parameters for modeling credit risk in the events of default are default rates, recovery rates, and default correlations. We obtain updated historical data for these three parameters, and provide estimations of portfolios' VaR based on the historical average of these data by using a credit risk model. These are estimations without considering any estimation error, called the base case estimation, and estimated portfolios' VaR in the base case is called conventional VaR. In order to see the effects of estimation error, we analyze the uncertainties of the three input parameters by providing Monte Carlo simulation for them (if applicable). We also provide simulations for portfolios' VaR due to those uncertainties. The estimated portfolios' VaR through simulation is called predictive VaR. The reason of using Monte Carlo simulation is to simulate the actual paths of model input parameters and potential losses for portfolios. Comparing the simulation results with those in the base case, we can see the differences between portfolios conventional VaR and predictive VaR. We also can analyze how significant are the effects of input parameter uncertainties and estimation error when estimating portfolio's credit risk. Besides, we compare our results with those concluded in Löffler (2003), and see whether those results are still applicable with new datasets used in our paper.

Our paper is presented as follows. In section 2, we provide a brief literature review on previous papers that discussed about the underlying idea of the VaR calculation, as well as estimation errors in estimating the portfolios' VaR. In section 3, we describe the methodologies for

estimating portfolios' conventional and predictive VaR due to credit risk. We also provide the methodologies for determining the uncertainties of all input parameters of the credit risk model by simulating the actual possible values for those parameters. In section 4, we provide the estimation results in the base case, and the simulating results on the accuracy of input parameters as well as portfolios' predictive VaR. In section 5, we conclude the paper and indicate possible areas for further research and analysis.

2. Literature Review

The modern financial institutions are very complex as they increasingly offer fee-based financial services and relatively new financial instruments and this has led to the creation of a number of new risks. Thus, risk management is playing on a more important role in modern finance. Essentially the riskier the bank's business, the more capital it should hold to be able to cover future fiscal losses. Although various banks face different risks (with regards to their category) some risks are common to most banks like Market risks, Liquidity risks, Credit risks and Operational risks (Jorion P.). However more serious risks pertain to losses which arise due to the failure of the obligator to perform (Credit Risk) and such losses are reported to be responsible for a significant amount of yearly bank losses. It is not enough for risk management practitioners to focus solely on a transactional approach to manage firm's credit risk. A more reliable way would be to pay more attention on the use of quantitative methods to manage such risk (The Committee on Regulation and Supervision, 1999).

Credit Risk has been defined as the “degree of value fluctuations in debt instruments and derivatives due to changes in the underlined credit quality of borrowers and counterparties” (Lopez & Saidenberg, 1999). While calculating the credit risk, when variables for the calculation of PD and LD have to be predicted, the prediction of those variables give rise to the estimation errors due to the fact that they are usually not present or available at the time of making the prediction (Hamerle, et al.).

The literature on this topic is relatively new and emerged since the 1970s. The emergence of BASEL II has given this area of research a new dimension as it has now been recognized as one area which has enormous practical as well as academic significance. Not only literature review in this area grew in numbers but also most of the commercial banks strived to develop their own internal models to map and measure credit risk (Lopez & Saidenberg, 1999). By doing so, banks could better assess their portfolio credit risks and assign economic capital properly (Lopez & Saidenberg, 1999). The constrained time horizon is a critical condition in term of evaluating a model’s prediction of credit losses in quantitative methods, especially with the small number of observations when their planning horizons are long (Lopez & Saidenberg, 1999). To be more specific, the daily time horizon in the models usually can generate a steady stream of observations for forecasting. However, it will not give the same steady results when running the yearly based data which is commonly used for credit risk models (Lopez & Saidenberg, 1999).

Value-at-Risk (VaR) is one of the risk management techniques widely used in financial risk management areas nowadays to measure credit risk in the context of a portfolio. It provides users with a summary measure of market risk and other types of financial risks. The VaR methodology

has been widely used not only to derivatives but also to all financial instruments and is definitely changing the way institutions approach their financial risk. The definition of VaR is “the worst loss over a target horizon that will not be exceeded with a given level of confidence (Jorion, 2003).” More specifically, VaR represents the percentile of the projected distribution of gains and losses over a fixed target horizon. If c is the chosen confidence interval, VaR corresponds to $1 - c$ of the lower tail level. A longer horizon will increase VaR as risk increases with respect to time (Jorion, 2003).

In contrast with traditional risk measures, VaR is a forward-looking risk measure. It describes an aggregate view of a portfolio’s market risk that accounts for leverage, correlations, and current positions (Jorion, 2003). Today VaR is being adopted by institutions all over the world, which includes: financial institutions, regulators, nonfinancial corporations and asset managers (Jorion, 2003). VaR is an integrated way to deal with various types of markets and exposure to risks and to combine all of them together into a single number which is a good indicator of the overall risk of different portfolios (Wiener, 1997). The major advantage of VaR is that it provides more consistent, accurate and timely measure of risk and it has been widely accepted by the industry and the regulators as the primary risk management tool (Wiener, 1997). Once there is a unified standard to look at, it will be easier to compare of risk between portfolios, institutions and financial intermediaries. Moreover, VaR methodology is relative simply to use and at the same time it unfolds stable results. (Wiener, 1997)

However, as VaR has moved far beyond use in financial institutions, it is used by an ever-increasing number of individual companies. The dangers of the widespread use of VaR are an

overreliance on the results it presents, misinterpretation, and even misuse of it (Krause, 2003). For example, VaR does not provide an accurate risk measure under particular circumstances when its estimation is subject to large estimation errors. A decreasing trend bias in the estimation could be easily manipulated by employees or the entire company to their own benefit (Krause, 2003). Practitioners should appropriate use VaR estimation with full awareness of its limitations, so that the decision making within the whole company will be improved. In our paper, we will mainly focus on the effects of estimation error in the portfolio of credit risk modeling.

As discussed above that the new regulatory announcements have made credit risk a challenge for the financial institutions thus the models prepared by the financial institutions are still under scrutiny due to the fact that model implementation is still a challenge for the financial institutions (Lopez & Saidenberg, 1999). The available credit models which have been developed in the recent past include many innovative and sophisticated models however most of these models are subject to model misspecification (violation of key modeling assumptions) and flawed calibration (wrong estimates of key parameters) errors (Tarashev & Zhu, 2007). The errors which emerge due to the portfolio risk measurements of credit portfolios of the financial institutions can emerge from many sources as during mapping of the credit risk, the essential portfolio risk measurements assumptions are violated thus creating a multiple of the sources from which those errors can emerge. Research has showed that a misspecification of model has little impact on the assessments of portfolio credit risk, especially for large and well diversified portfolios. By contrast, calibration error could have a substantial impact on measures of portfolio credit risk (Tarashev & Zhu, 2007).

Several aspects of the existing evaluation methodologies still require further research. For example, the impact of specific (noisy) input parameters, such as the number of credit observations to be included in a simulated portfolio and the essential of the simulated portfolio's weights, must be better understood (Lopez & Saidenberg, 1999). Research has suggested that the available models provide different results for the same borrower with same characteristics due to the fact that model differences arise due to the treatment of joint default behavior- one of the main assumptions of almost all the credit risk models. Thus effectively what is most important to understand is the fact that these models and the resulting estimation errors emerge at the macrolevel as at the microlevel (i.e. individual borrower's level), most of the models tend to converge together thus neglecting the impacts of the estimation errors on the individual borrower's basis. (Koyluoglu et al, 1999)

The existing literature therefore has largely focused on the estimation of errors which were based on the single sources of errors in credit risk models and failed to provide a unified framework for effectively measuring the errors during estimation of credit risk by the credit risk models. The assumptions of granularity which is considered as the backbone of any credit risk model are different in each model therefore the impact of noise in model parameters creates significant impact on the assessment of credit risk estimation of the models.

This gap in the literature was filled by the work of Löffler who attempted to include the noise into the parameters of the credit risk model in order to assess the impact of estimation errors. Löffler (2003) analyzed the effects of estimation error by comparing the estimated portfolios' VaR in two scenarios. Portfolios in the analysis contain either 50 or an infinite number of loans

to different obligors, whose credit quality is rated BBB or B by Standard & Poor's. (Löffler, 2003, p. 1429) The paper introduced a credit risk model for calculating portfolios' VaR, which contains three input parameters: default rates, recovery rates, and default correlations. The author firstly provided estimation of portfolios' VaR based on the model introduced by using historical average of those input parameters. Estimated VaR in this scenario did not consider any estimation error. Secondly, the author predicted portfolios' VaR through Monte Carlo simulations. Estimated VaR in the second scenario contained estimation errors and uncertainties of input parameters. After comparison estimated portfolios' VaR, the author concluded that the effects of estimation error in estimating portfolios' VaR are considerable (Jorion, 2003).

We also imitate the methodology of how to calculate portfolio loss from Giesecke (2004). This paper used a Bernoulli mixture model which explains the cyclical default dependence to construct the portfolio loss distribution. When the issuers are independent and have equally likely default probabilities, the sequence then follows a classical Bernoulli distribution. By having the individual default probability p and the asset correlation ρ , we can easily calculate the portfolio loss when default occurs. (Giesecke, 2004)

The approach in Löffler (2003) therefore proved one of the few academic attempts to help pave the way for the implementation of unified credit risk estimation parameters into credit risk models. The evaluation methodology could be implemented through various statistical tools. In our paper, we simply adopt the Monte-Carlo simulation methodology from Löffler (2003) and other statistical tools to assess the estimation error on measures of portfolio credit risk. We also use the simplified formula to calculate portfolio loss from Giesecke (2004).

3. Methodologies

3.1 Portfolio Losses and VaR Modeling

Löffler (2003) provided the analysis for four portfolios, which contain either 50 or an infinite number of obligors with BBB or B credit ratings. (Löffler, 2003, p. 1429) In our paper, we provide our analysis for just two infinite portfolios, obligors with investment-grade credit rating and obligors with speculative-grade credit rating. Portfolio of obligors with investment-grade credit rating contains bonds with high credit quality and medium credit quality, such as bonds with the rating AAA, AA, A, or BBB. Portfolio of obligors with speculative-grade credit rating contains other bonds with low credit quality, also called “junk bonds”, such as bonds with the rating BB, B, CCC, etc. (Investment Dictionary: Investment Grade) Thus, comparing with Löffler (2003), we provide the analysis based on portfolios with a larger variety of credit ratings. In order to assess the expect losses and VaR for each portfolio, we employ the methodology from both Löffler (2003) and Giesecke (2004). These papers talked about the one-factor model about asset returns for generating asset correlations, and provide formulas for calculating portfolio losses as well as different quantile VaR by using three input parameters: default rates, recovery rates, and correlations. In our paper, for portfolio of obligors with investment-grade credit rating, default rates for investment-grade rating bonds, recovery rates for senior secured bonds, and consistent correlation are taken into calculation, while for portfolio of obligors with speculative-grade credit rating, default rates for speculative-grade rating bonds, recovery rates for senior unsecured bonds, and consistent correlation are taken into calculation.

Let us first look at asset correlations. Same with Löffler (2003) the asset correlation in our paper is assumed to be constant, the expression is ρ . For any two firms i and j , the asset correlation matrix is of the form $\rho_{ji} = 1$ for $i = j$, and $\rho_{ij} = \rho$ for $i \neq j$. (Giesecke, 2004, p. 23) Both Löffler (2003) and Giesecke (2004) illustrate the one-factor linear model for asset return. Here is the expression (Löffler, 2003, p. 1430) , (Giesecke, 2004, p. 23):

$$X_i = \sqrt{\rho} Z_c + \sqrt{1 - \rho} Z_i \text{ --- (1)}$$

In this equation, X_i is the asset return, ρ is the asset correlation. Z_c is a systematic factor (common factor), and Z_i is an independent idiosyncratic factor. In Löffler (2003), the author assumed Z_i is normally distributed. However, the distribution of Z_c is a little bit more complicated. Z_c was assumed to be drawn from two other distributions with both mean zero but can differ in their variance (Löffler, 2003, p. 1430):

$$Z_c = \lambda Z_1 + (1 - \lambda) Z_2, \quad Z_1 \sim N(0, \sigma^2(Z_1)), \quad Z_2 \sim N(0, \sigma^2(Z_2)), \text{--- (2)}$$

where λ equal to 1 with probability γ , and 0 with probability $(1 - \gamma)$ $\gamma \sigma^2(Z_1) + (1 - \gamma) \sigma^2(Z_2)$ is the variance of Z_c . (Löffler, 2003, p. 1430) In our paper, in order to simplify the calculation process, we employ the assumption used in Giesecke (2004), which is assuming both the two factors are normally distributed with a mean of 0 and a variance of 1 (Giesecke, 2004, p. 24):

$$Z_c \sim N(0, \sigma^2(Z_c)), \quad Z_i \sim N(0, \sigma^2(Z_i)), \quad \sigma^2(Z_c) = \sigma^2(Z_i) = 1 \text{ --- (3)}$$

Also, $\sqrt{\rho} Z_c$ represents the systematic risk in asset returns, and $\sqrt{1 - \rho} Z_i$ represents the idiosyncratic risk in asset returns. (Giesecke, 2004, p. 24)

According to Löffler (2003), the probability of portfolio losses can be calculated through the following formula with known default correlation and default threshold (Löffler, 2003, p. 1430):

$$q(Z_c, p, \rho, \sigma^2(Z_i)) = \text{Prob} \left[Z_i \leq \frac{d - \sqrt{\rho} Z_c}{\sqrt{1 - \rho} \sigma(Z_i)} \right] = \Phi \left[\frac{d - \sqrt{\rho} Z_c}{\sqrt{1 - \rho} \sigma(Z_i)} \right] \text{ --- (4)}$$

The default threshold d can be expressed as (Löffler, 2003, p. 1430):

$$\gamma \text{Prob}(\sqrt{\rho} Z_1 + \sqrt{1-\rho} Z_i \leq d) + (1-\gamma) \text{Prob}(\sqrt{\rho} Z_2 + \sqrt{1-\rho} Z_i \leq d) = p, \text{ --- (5)}$$

$$\gamma \Phi\left(\frac{d}{\sqrt{\rho\sigma^2(Z1)+(1-\rho)\sigma^2(Zi)}}\right) + (1-\gamma) \Phi\left(\frac{d}{\sqrt{\rho\sigma^2(Z2)+(1-\rho)\sigma^2(Zi)}}\right) = p, \text{ --- (6)}$$

where Φ is the cumulative normal distribution function and p is the probability of default.

The above equations together with equation (1) demonstrate that the probability of portfolio losses rely on probability of default, asset correlation, the distribution of common factor, as well as the idiosyncratic factor. The probability of default is equal to the probability of occurrence of $X_i \leq d$ (Löffler, 2003). That means, if the value of portfolio's asset return is smaller than or equal to the value of default threshold, default will occur and then may cause losses on portfolios' value.

In our paper, since we assume Z_c and Z_i are normally distributed with mean 0 and variance 1, equation (5) and (6) from Löffler (2003) can be simplified as:

$$\text{Prob}(\sqrt{\rho} Z_c + \sqrt{1-\rho} Z_i \leq d) = p \text{ --- (7)}$$

$$\Phi(d) = p \text{ --- (8)}$$

Therefore, $d = \Phi^{-1}(p)$, and the probability of portfolio losses can be calculated though those simplified formulas, and can be expressed as:

$$q = \Phi\left[\frac{\Phi^{-1}(p) - \sqrt{\rho} Z_c}{\sqrt{1-\rho}}\right] \text{ --- (9)}$$

In the above equations, the common factor Z_c is a random number related to economy status. Positive values of Z_c correspond to a good state of the economy, which negative values of Z_c correspond to a distressed economy. (Giesecke, 2004, p. 24) Moreover, in our paper, we ignore the influence of the idiosyncratic factor Z_i . As the size of the portfolio goes to infinity, \bar{Z}_i would

be close to 0. Therefore, we do not take Z_i into account since the sample size of our portfolios is an infinite number.

This paper also considers the estimation error in portfolios' VaR. "VaR is the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger." (Jorion, 2003, p. 106) In our paper, portfolios' VaR are determined by those three input parameters, default rates, recovery rates, and default correlations. α VaR is the cutoff portfolio losses (in percentage of portfolio value) at $1 - \alpha$ confidence level. That means, the probability of experiencing a loss at α VaR percent of portfolio value is equal to α , or the probability of experiencing a greater loss than α VaR percent of portfolio value is less than α . (Jorion, 2003, p. 106) Different probabilities (α) analyzed in the paper are based on the distribution of common factors (Z_c) only, since we ignore the influence of Z_i because the sample size of our portfolios.

Employ the method using in Löffler (2003), portfolios' VaR at the percentile level α can be calculated using the following formula (Löffler, 2003, p. 1431):

$$\alpha\text{VaR} = q(1 - r) = \Phi\left[\frac{\Phi^{-1}(p) - \sqrt{\rho}Q_\alpha^z}{\sqrt{1-\rho}}\right] (1 - r), \text{ --- (10)}$$

where r is the expression for recovery rates, p is the probability of default, ρ is the correlation, and Q_α^z denotes the α quantile of risk factor Z_c . (Löffler, 2003, p. 1431) Since Z_c is a normally distributed random number, the values of Q_α^z are fixed by different α .

3.2 Input Parameter Uncertainties

According to equation (10), the formula has three input parameters: default rates, recovery rates, and correlations. When estimate portfolios' VaR, there are uncertainties exist in these parameters that could lead to estimation risk. In the following section, we analyze the uncertainty for each input parameter separately.

3.2.1 Default Rates Uncertainty

Bond default rates are varied based on different bond ratings, so that simply use the average default rates for all bonds is not an accurate analyze. For estimating the uncertainty of true mean default rate, Löffler (2003) used 18 annual default rates for BBB rating and B rating bonds from 1981 to 1998. (Löffler, 2003, p. 1432). In our paper, we use the updated historical annual default rates for bonds as the original data in the recent 20 years (1988 – 2007). Also, different with Löffler (2003), we divide all trading bonds into two categories, investment-grade rating bonds and speculative-grade rating bonds, in order to consistent with our portfolio classification.

Default rates of the former bond category are used for portfolio of obligors with investment-grade credit rating, and default rates of the latter category are used for portfolio of obligors with speculative-grade credit rating. The data source is *2007 Annual Global Corporate Default Study and Rating Transitions* report provided by Standard & Poor's (S&P) in 2008. Table 1 listed 20 historical annual default rates for two bond categories from S&P report. Since 20 default rates covers almost all possible value of default rates, and the numbers follow a reasonable distribution, it is easy for us to do the analysis based on this data source.

Usually, analysts average historical default rates and use the historical mean for estimation. This method neglect serial correlation between those historical default rates. By employing the method used in Löffler (2003), in order to estimate serial correlation, we need to run auto regressions based on the historical data. (Löffler, 2003, p. 1432) Here are the regression functions of default rates for both investment-grade rating bonds and speculative-grade rating bonds (t-statistics in parentheses):

Investment-grade:

$$\text{Rate}_t = 0.0406072524604043 + 0.702321203723159 * \text{Rate}_{t-1} - 0.144417926711624 * \text{Rate}_{t-2} \text{---} (11)$$

(1.195715) (2.83119) (-0.58218)

Speculative-grade:

$$\text{Rate}_t = 2.38991147193004 + 1.07722829673446 * \text{Rate}_{t-1} - 0.605964474888393 * \text{Rate}_{t-2} \text{---} (12)$$

(2.373332) (5.07544) (-2.74017)

We use the lag length of two which is same with the one used in Löffler (2003). Please refer to Table 2 and Table 3 for detail regression information obtained by using Excel. From t-stat values, we can see that there is a strong serial correlation existing in the default rates of speculative-grade rating bonds, because all of t-stat values for coefficients in the auto regression equation for speculative bonds are significant. For investment-grade rating bonds, not all of t-stat values for coefficients in the equation are significant. However, since the coefficients in equation (11) have the same sign as those in equation (12), we can still conclude that there is a serial correlation in the default rates of investment-grade rating bonds. These results are consistent with what it showed in Löffler (2003). In Löffler (2003), there were serial correlations existed in issuers with both BBB and B ratings, although the evidence for the BBB rated issuers is weaker. (Löffler, 2003, p. 1432)

Since there are serial correlations for the two portfolios, we can use a bootstrap procedure, which is used in Löffler (2003), based on the above auto regression functions to derive a distribution for the true mean default rates for each portfolio. (Löffler, 2003, p. 1432) We follow the instructions in Löffler (2003) to run the bootstrap step by step by using Java programming. Java codes are included in Appendix. We randomly draw two consecutive default rates from the 20 historical data for both investment-grade portfolio and speculative-grade portfolio, and substitute them into the regression equations provided above. This means, by knowing $RATE_{t-2}$ and $RATE_{t-1}$, we can get $RATE_t$. Then, we use $RATE_{t-1}$ and $RATE_t$ to get $RATE_{t+1}$. This process is repeated until all the remaining 18 default rates are obtained. Averaging the total new 20 default rates of the bootstrap sample gives a mean default rate. Given the history that was observed, this mean default rate might be a true mean. (Löffler, 2003, p. 1432) We simulate the whole process 20,000 times to obtain a distribution of the true mean default rates for each portfolio. (Löffler, 2003, p. 1433) The bootstrapped distribution would be used when estimating portfolios' predictive VaR.

3.2.2 Recovery Rates Uncertainty

When estimating the true mean recovery rate, we assume the investment-grade rating bonds analyzed in our paper share the properties of senior secured bonds, and the speculative-grade rating bonds share the properties of senior unsecured bonds. This assumption is different with that in Löffler (2003), since it assumed all bonds analyzed share the properties of senior unsecured bonds. (Löffler, 2003, p. 1434) The reason for the difference is that portfolios analyzed in Löffler (2003) did not contain bond with high level credit rating. In our paper, we intend to use historical annual recovery rate of senior secured bonds and senior unsecured bond for investment-grade portfolio and speculative-grade portfolio, respectively. The time period is

the same with the one we select in default rate estimation (1988 – 2007). *Altman High Yield Bond Default and Return* report in 2007 is our data source for historical recovery rate. However, the most recent recovery rate for both senior secured bonds and senior unsecured bonds in that report is in the year 2006. Therefore, we select 19 annual average recovery rates in each bond category from 1988 to 2006 as historical data source for analyzing each portfolio. Please refer to Table 4.

According to Löffler (2003), the true mean recovery rate in our paper is assumed to be normally distributed around the average of historical recovery rates, and the normal distribution would be used when estimating portfolios' predictive VaR. Based on our datasets, the historical mean of recovery rate is 39.68% for portfolio of obligors with investment-grade credit rating, and 17.63% for portfolio of obligors with speculative-grade credit rating. The standard deviations of historical data for the two portfolios are 17.50% and 12.45%, respectively. Therefore, according to the historical data, the lower and upper bounds for actual recovery rate at 95% confidence interval can be calculated as follows:

$$\bar{X} - \frac{1.645 \sigma(x)}{\sqrt{N}} \leq \mu_x \leq \bar{X} + \frac{1.645 \sigma(x)}{\sqrt{N}} \quad \text{--- (13)}$$

Thus, the 95% confidence interval of actual recovery is [33.08%, 46.29%] for investment-grade portfolio, and [12.93%, 22.33%] for speculative-grade portfolio. However, we still need to consider the standard error in the two cases, which is about 4.01% and 2.86%, respectively. In order to estimate portfolios' predictive VaR due to recovery rate uncertainty, we can run simulations by drawing different recovery rates as one input parameter from assumed normal

distribution with the mean and standard deviation by using Excel. We also add a constraint that the recovery rates should be larger than 0 and smaller than 1.

3.2.3 Correlations Uncertainty

According to Löffler (2003), a firm is assumed to default if its value falls below a critical level, which is defined by its value of liabilities. Thus, correlations of asset values can be translated into default correlations. (Löffler, 2003, p. 1430) In our paper, the meanings of default correlations and asset correlation are the same. Based on Löffler (2003), asset correlations are assumed to be constant. Since the historical data for asset correlation is difficult to obtain, we use the same estimate value 0.2 used in Löffler (2003) to be the mean of average asset correlations.

In order to determine the uncertainty of asset correlations, Löffler (2003) ran simulations to generate actual asset correlation values based on the following formula (Löffler, 2003, p. 1435):

$$\text{Corr}(X_i, X_j) = \frac{\text{Cov}(X_i, X_j)}{\sigma(X_i)\sigma(X_j)} = \frac{\rho \sigma^2(Z_c)}{\rho \sigma^2(Z_c) + (1-\rho)\sigma^2(Z_i)}$$

In this formula, analysts have to find the value for $\sigma^2(Z_c)$ and $\sigma^2(Z_i)$. Recall equation (2), Z_c was assumed to be drawn from two other distributions with both mean zero but can differ in their variance (Löffler, 2003, p. 1430). Therefore, the variance of Z_c is difficult to estimate. To simplify the simulation process, we assume the correlations used in our paper are normally distributed. This normal distribution would be used when estimating portfolios' predictive VaR. According to Table 2 in Löffler (2003), average asset correlations are in a range of [15.26%, 27.04%] at 95% confidence interval. Based on this range and the mean of 0.2, we estimate the standard deviation of correlation is probably around 27.73%. In order to estimate portfolios'

predictive VaR due to correlation uncertainty, we can run simulations by drawing different correlations as one input parameter from the assumed normal distribution with the mean and standard deviation using Excel.

3.3 Portfolios' VaR Calculation

3.3.1 Base Case VaR (Conventional VaR)

Estimating portfolios' VaR by simply using average historical data is called the base case estimation. The estimated VaR in this case is called base case VaR, or conventional VaR. In this section, VaR calculations are provided from both investment-grade portfolio and speculative-grade portfolio. For portfolio of obligors with investment-grade credit rating, one of the input parameters, default rate, is obtained by averaging the 20 annual default rates for investment-grade rating bonds from 1988 to 2007 based on S&P report, which is 0.10%. Another input parameter, recovery rate, is also obtained by averaging the 19 annual recovery rates from 1988 to 2006 based on Altman 2007 report, and the mean recovery rate to be used is 39.68%. For correlations, since we assume the asset correlations are constant between every two firms, the assumed number 0.2 in Löffler (2003) is a reasonable number to directly use in the calculation. For portfolio of obligors with speculative-grade credit rating, we use the same correlation 0.2 for VaR calculation. However, the other inputs using are different from those for investment-grade portfolio. We use the average of historical annual default rates and recovery rates for speculative-grade bonds from the same data sources as those for investment-grade bonds, which are 4.4945% and 17.63%, respectively.

Same with Löffler (2003), we calculate 1% VaR, 5% VaR and 10% VaR for both two portfolios. Here, the values of Q_{α}^z in equation (10) are -2.326 , -1.645 and -1.282 when α equals to 1%, 5% and 10% respectively. To be noticed that, this base case estimation does not consider any estimation error. If we consider uncertainties for the input parameters, the results would be different with this estimation.

3.3.2 Simulated VaR (Predictive VaR)

Estimated portfolios' VaR through simulations is called predictive VaR. In order to consistent with the analysis in the base case, based on equation (10), we simulate 1% VaR, 5% VaR and 10% VaR for both two portfolios. In order to estimate the effects of estimation errors and input uncertainties in portfolio VaR calculation, we divide our estimation into four situations. Firstly, we fix correlation and default rate with the value used in the base case, and then simulate portfolio's VaR 20,000 times by using different recovery rates. Recovery rates using here are drew from assumed normal distribution mentioned in Section 3.2.2. Then we can get the distribution of portfolio's VaR due to uncertainty of recovery rates. Secondly, we fix recovery rate and default rate with the value used in the base case, and then simulate portfolio's VaR 20,000 times by using different correlations. Correlations using here are drew from assumed normal distribution mentioned in Section 3.2.3. Then we can get the distribution of portfolio VaR due to uncertainty of correlations. Thirdly, we fix recovery rate and correlation with the value used in the base case, and then simulate portfolio's VaR 20,000 times by using different default rates. Same with the situations above, default rates using here are drew from the bootstrap distribution mentioned in Section 3.2.1. Then we can get the distribution of portfolio VaR due to uncertainty of correlations. Finally, in order to obtain portfolio's VaR due to

uncertainty of all the three input parameters, as well as estimation errors, we calculate VaR by drawing the values of those three parameters from their distributions at the same time. This process is repeated 20,000 times, and then we can get the distribution of portfolio's predictive VaR including all the uncertainties and estimation errors.

4. Result

4.1 Base Case VaR (Conventional VaR)

Table 5 presents the risk characteristics of portfolio of obligors of investment-grade rating and portfolio of obligors of speculative-grade rating as calculated above in the base case, where the asset correlation is 0.2 (the same assumption as in Löffler (2003)), mean recovery rates are 39.68% for Investment-grade portfolio and 17.63% for speculative-grade portfolio respectively, and default rates are 0.10% for investment-grade portfolio and 4.4945% for speculative-grade portfolio respectively. Here we ignore the estimation error as the same purpose in Löffler (2003), as there will be a comparison in the following section with the situation that the error term has been taken into account.

Let us review the definition of VaR. "VaR is the worst loss over a target horizon such that there is a low, pre-specified probability that the actual loss will be larger." (Jorion, 2003) For example, a 1% VaR of 0.66% (from Table 5) for the investment-grade bond is the cutoff probability of loss such that the probability of experiencing a greater loss is less than 1%.

As we can see from Table 5, the VaR number increases much from the portfolio of obligors with investment-grade credit rating to the portfolio of obligors with speculative-grade credit rating. When the credit rating goes down from investment to speculative, the bonds are affected to a higher degree by the possibility of default. Thus, it causes the VaR figures to increase. Another thing to be noticed from Table 5 is that the VaR figures decrease as the percentile gets bigger. It is because that the greater possibility of portfolio loss is more likely to be in the extreme quantiles.

The base case VaR figures of investment-grade portfolio in our paper are less than those figures of portfolio of BBB-rated bonds in Löffler's (2003), and the base case VaR figures of speculative-grade portfolio in our paper are greater than those figures of portfolio of B-rated bonds. This could be explained by the classification of the bond's credit rating. We divide the bond's credit rating into two grades in our paper: the investment-grade and the speculative-grade, instead of single rating bond such as BBB or B in Löffler (2003). The investment-grade portfolio contains bonds with credit rating equal to or higher than BBB, and the bonds with rest credit ratings (include junk bonds) go into the speculative-grade portfolio (Wikipedia).

4.2 Simulated VaR (Predictive VaR)

Table 6 represents the simulated distribution of the percentage portfolio VaR in the presence of estimation risk. Estimation error in the following input parameters used in the table is modeled: default rates (estimates based on S&P historical data of investment-grade rating bonds and speculative-grade rating bonds), recovery rates (estimates based on historical data of senior secured and unsecured bonds), and default correlations (estimates based on joint distribution of

asset values). In Table 6, we include the simulated confidence intervals for the 1% VaR, 5% VaR and 10% VaR of the two portfolios due to estimation error from different sources: uncertainty of recovery rates only, uncertainty of default correlations only, uncertainty of default rates only, and uncertainty of three input parameters together. We also include the simulated standard errors of those VaR figures. Figure 1 and Figure 2 show the confidence interval for each portfolio VaR in graphic forms, so that the width of the intervals can be presented more clearly.

For most of the cases of investment-grade portfolios, uncertainty of default rates is the most important source of estimation risk, as measured by the width of the confidence intervals or the standard error. However, the role of correlation uncertainty in more extreme percentile levels is larger. For example, uncertainty of default correlations is the most important source of estimation error for the precision of the 1% VaR of the investment-grade portfolio. It has the widest confidence interval (0.112%, 0.9917%) and the largest standard error (0.00197%) among all of three input parameters. When default rates rise, the elasticity of default correlations with respect to changes in asset correlations increases as well. (Morgan, 1997) especially in riskier portfolios or portfolios with more extreme percentiles. On the other hand, for speculative-grade portfolio, uncertainty of default correlations becomes the most important source of uncertainty in all three quantiles. The reasons for this could be the parameter estimation methodology used for default correlation (mentioned in section 3.2.3) or the range of the original datasets (speculative-grade portfolios).

Comparing Table 6 in our paper and Table 4 in Löffler (2003), the results we get are very similar to what Löffler got in his paper except for the correlation uncertainty in the portfolio of obligors

with speculative-grade credit rating. In this portfolio, default correlation becomes the most significant source of estimation risk in all three different quantiles (i.e. 1%, 5%, 10% VaR) in our paper. While in Löffler (2003), correlation uncertainty only matters in some more extreme quantiles as we mentioned in last paragraph, for example 1% VaR. The inconsistency may be due to the fact that we used different methodology to generate random numbers from the distribution of correlation, which was discussed previously in section 3.2.3, other than the methodology used in Löffler (2003). The small-sample estimation errors in the correlation parameters could still possibly lead to large flaws in quantifying of portfolio credit risk (Tarashev & Zhu, 2007).

4.3 Base Case VaR and Simulated VaR Comparison

Due to different market scenarios, the estimated VaR will sometimes overstate risk and sometimes understate risk. We need to take estimation error into account to the above two sides and assess its overall effects on the distribution of portfolio value (Löffler, 2003).

In Table 7, we put the conventional VaR (from base case parameters) and the predictive VaR (from 20,000 times simulated distributions) together. The results show that for the investment-grade portfolio, the conventional VaR overestimates the predictive VaR by considering the existence of estimation error. The magnitude of the bias ranges from a 4.5 to an 8.8 basis points. The documented biases thus appear to be very modest. From the numbers in Table 5 and the analysis here, we can conclude that the conventional VaR figures can be regarded as reasonable approximations to the true risk factors of a portfolio (Löffler, 2003). However, the estimation error adjustments would still be important in an economical use, especially for more extreme

events less than 1% quantile (Löffler, 2003). On the other hand, the conventional VaR underestimates the predictive VaR in speculative-grade portfolio. The magnitude of the bias ranges dramatically from 2.2 to 360 basis points. The differences are significant between the conventional ones and the predictive ones. Estimation error really plays a role in this case. Again, it returns to the question of the large data range of the speculative-grade rating bonds. In most of the cases, recovery rates are ranked as the third important uncertainties among the three. On the other hand, the conventional VaR underestimates the predictive VaR in speculative-grade portfolio. The magnitude of the bias ranges dramatically from 2.2 to 360 basis points. The differences are significant between the conventional ones and the predictive ones. Estimation error really plays a role in this case. Again, it returns to the question of the large data range of the speculative-grade portfolio. In most of the cases, recovery rates are ranked as the third most important uncertainties among the three.

Compared Table 7 in our paper with Table 7 in Löffler (2003), the conventional VaR estimates of the two different portfolios has different bias numbers in the presence of estimation risk. The bias for the portfolio of obligors with speculative-grade credit rating bonds is much higher than the bias for the portfolio of obligors with investment-grade credit rating bonds (i.e. a range of 0.045 to 0.088 for investment-grade portfolio and a range of 0.022 to 3.60 for speculative-grade portfolio). This is due to the large volatility of speculative-grade rating bonds (from BB to CCC). Therefore, the estimation error will matter much in the VaR calculation for the portfolio of speculative-grade portfolios.

5. Conclusion

This paper implements the idea and methodology mainly in Löffler (2003) to analyze the effects of estimation error in portfolios' VaR in the events of default. We attempt to replicate the main procedures showed in Löffler (2003) by using different and updated datasets. Our analysis is based on two portfolios, portfolio of obligors with investment-grade credit rating, and portfolio of obligors with speculative-grade credit rating. We simulate portfolios' predictive VaR values due to the uncertainty of each input, default rates, recovery rates, and correlations, as well as the VaR values due to the uncertainty of those input together. We compare the predictive VaR values estimated from simulations with those estimated by using historical means (base case estimation).

The results we get are basically consistent with those revealed in Löffler (2003). The estimates of portfolio credit risk are sensitive to uncertainty about input parameters. (Löffler, 2003, p. 1452) For portfolio of obligors with investment-grade credit rating, in the most of the cases, predicted portfolio VaR due to the uncertainty of default rates has higher value. That means, default rates are the most significant source of estimation risk. However, for portfolios of obligors with speculative-grade credit rating, correlations become the significant uncertainty. Besides, we find out biases in conventional (base case) VaR estimates, which are compared with predictive VaR through simulations, are small for investment-grade portfolio. However, the biases in conventional VaR estimates are larger for speculative-grade portfolios. This is slightly different with the result concluded in Löffler (2003).

Further research and analysis may focus on how to deal with those estimation errors when estimating portfolios' VaR. Since the estimations are sensitive to uncertainty about input parameters of VaR calculation, how to increase parameter accuracy may be the first step for the following research.

Table 1

Annual historical default rate from 1988 to 2007 for bonds with investment-grade credit rating and speculative-grade credit rating

Year	Investment-grade default rate(%)	Speculative-grade default rate(%)
1988	0.00	3.96
1989	0.14	4.53
1990	0.14	8.09
1991	0.14	11.04
1992	0.00	6.08
1993	0.00	2.50
1994	0.05	2.10
1995	0.05	3.51
1996	0.00	1.79
1997	0.08	1.98
1998	0.14	3.68
1999	0.17	5.45
2000	0.23	6.05
2001	0.26	9.64
2002	0.41	9.19
2003	0.10	4.88
2004	0.03	2.01
2005	0.03	1.41
2006	0.00	1.14
2007	0.00	0.86
Mean	0.10	4.4945

Source: Standard & Poor's 2008 Report: *Default, Transition, and Recovery: 2007 Annual Global Corporate Default Study and Rating Transitions*

Table 2

Summary output of regression for portfolio of obligors with investment-grade credit rating

<i>Regression Statistics</i>	
Multiple R	0.619816393
R Square	0.384172361
Adjusted R Square	0.302062009
Standard Error	0.093612111
Observations	18

<i>ANOVA</i>					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	2	0.082002	0.041001	4.678732	0.02636
Residual	15	0.131448	0.008763		
Total	17	0.21345			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.040607252	0.033961	1.195715	0.250369	-0.03178	0.112993	-0.03178	0.112993
X Variable 1	0.702321204	0.248066	2.83119	0.012638	0.173582	1.231061	0.173582	1.231061
X Variable 2	-0.144417927	0.248066	-0.58218	0.569096	-0.67316	0.384322	-0.67316	0.384322

Notes: This table shows the regression results for portfolio of obligors with investment-grade credit rating. Beside the coefficient values, the main information that our paper refers to is the t-state value in this table.

Table 3

Summary output of regression for portfolio of obligors with speculative-grade credit rating

<i>Regression Statistics</i>	
Multiple R	0.797636747
R Square	0.63622438
Adjusted R Square	0.587720964
Standard Error	2.062501476
Observations	18

<i>ANOVA</i>					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	2	111.598	55.79901	13.1171	0.000508
Residual	15	63.80869	4.253912		
Total	17	175.4067			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	2.3899114	1.00698	2.3733	0.0314	0.2435	4.5362	0.2435	4.53625
X Variable 1	1.0772282	0.21224	5.0754	0.0001	0.6248	1.5296	0.6248	1.529614
X Variable 2	0.6059644	0.22114	2.7401	0.0151	1.0773	0.1346	-1.07732	-0.13461

Notes: This table shows the regression results for portfolio of obligors with speculative-grade credit rating. Beside the coefficient values, the main information that our paper refers to is the t-state value in this table.

Table 4

Annual historical Recovery Rate from 1988 to 2006 for senior secured bond and senior unsecured bond

Year	Senior Secured Bond Recovery Rate (%)	Senior Unsecured Bond Recovery Rate (%)
1988	21	31
1989	12	21
1990	10	27
1991	3	44
1992	22	12
1993	6	22
1994	23	36
1995	15	27
1996	17	17
1997	16	48
1998	18	62
1999	11	47
2000	8	29
2001	3	67
2002	11	75
2003	28	53
2004	39	48
2005	54	36
2006	18	52
Mean	17.63	39.68

Source: Altman Edward I, and Suresh Ramayanam. *Default and Returns in the High Yield Bond Market 2006 in Review and Outlook*.

Table 5

Distribution of portfolios' VaR in the base case (in % of portfolio value)

Portfolio	1% VaR	5% VaR	10% VaR
Investment-grade	0.66070	0.25564	0.14758
Speculative-grade	19.0876	11.6543	8.62497

Notes: VaR in the base case also called conventional VaR, which is the estimation without considering estimation error. This table contains VaR values with different probability of α (1%, 5%, 10%), where the probability of loss is based on the distribution of common factor.

Table 6

Simulated distribution of portfolios' VaR (in % of portfolio value)

Source of estimation error	VaR of the portfolio of obligors with Investment-grade credit ratings					VaR of the portfolio of obligors with Speculative-grade credit ratings				
	Std. error	Quantiles				Std. error	Quantiles			
		2.5%	25%	75%	97.5%		2.5%	25%	75%	97.5%
Panel A: 1% VaR										
Recovery rate	0.00119	0.2515	0.4706	0.7012	0.9160	0.0179	12.7026	16.1697	19.8532	21.8237
Correlation	0.00197	0.1120	0.3805	0.8755	0.9917	0.09273	5.8189	13.0791	29.6916	54.8548
Default rate	0.00153	0.0939	0.3988	0.6916	0.9547	0.01678	13.1002	16.3912	19.5644	22.4183
All	0.00303	0.0291	0.2464	0.7996	1.6371	0.09787	5.3277	12.5178	29.5267	57.8024
Panel B: 5% VaR										
Recovery rate	5.19e ⁻⁴	0.1097	0.2052	0.3057	0.3994	0.0116	8.2353	10.4831	12.8712	14.1487
Correlation	4.59e ⁻⁴	0.0333	0.1677	0.2586	0.2682	0.03511	5.1058	9.2704	16.4055	23.6841
Default rate	7.22e ⁻⁴	0.0346	0.1674	0.3066	0.4378	0.01273	8.0598	10.4605	12.8739	15.1222
All	8.92e ⁻⁴	0.0022	0.0971	0.2661	0.4850	0.04175	4.4773	8.7457	16.3958	26.6273
Panel C: 10% VaR										
Recovery rate	3.00e ⁻⁴	0.0633	0.1185	0.1765	0.2306	0.00859	6.0947	7.7582	9.5256	10.4710
Correlation	2.96e ⁻⁴	0.0033	0.0887	0.1432	0.1485	0.01335	4.6598	7.3264	10.3068	10.9810
Default rate	4.36e ⁻⁴	0.0180	0.0944	0.1789	0.2608	0.01029	5.7701	7.6643	9.6180	11.4777
All	5.10e ⁻⁴	0.0001	0.0474	0.1458	0.2690	0.01992	3.8261	6.6436	10.4380	14.7284

Notes: VaR obtained through simulation also called predictive VaR, which is the estimation that take input parameter uncertainties and estimation error into account. This table contains VaR values with different probability of α (1%, 5%, 10%), where the probability of loss is based on the distribution of common factor. For each α VaR, the quantile value of the distribution is also showed in the table.

Table 7

Biases of conventional VaR estimates in the presence of estimation risk (in % of portfolio value)

Portfolio	Percentile level (%)	Conventional VaR	Predictive VaR	Difference
Investment-grade	1	0.66070	0.57301	0.08769
	5	0.25564	0.19131	0.06433
	10	0.14758	0.10258	0.045
Speculative-grade	1	19.0876	22.691	3.6034
	5	11.6543	13.044	1.3897
	10	8.62497	8.6472	0.02223

Notes: This table shows the value of conventional VaR and predictive VaR in two portfolios, as well as the difference between them.

Figure 1

Simulated Distribution of the VaR due to Default of Portfolio of Obligor with Investment-grade Credit Rating

Distribution of the 1% VaR



Distribution of the 5% VaR

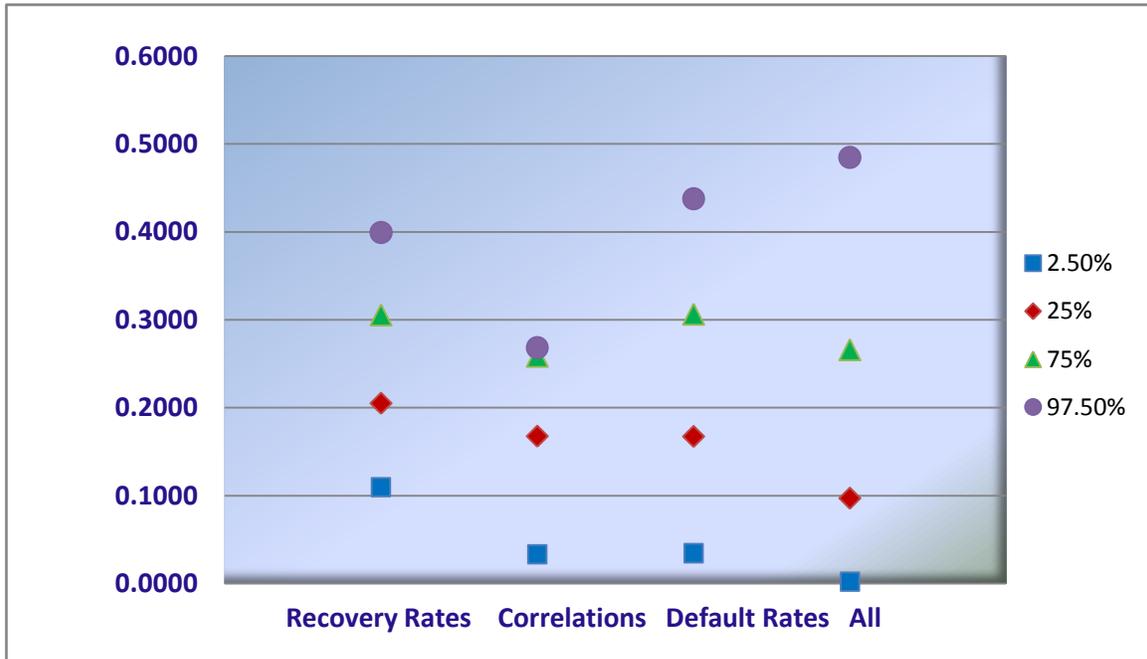
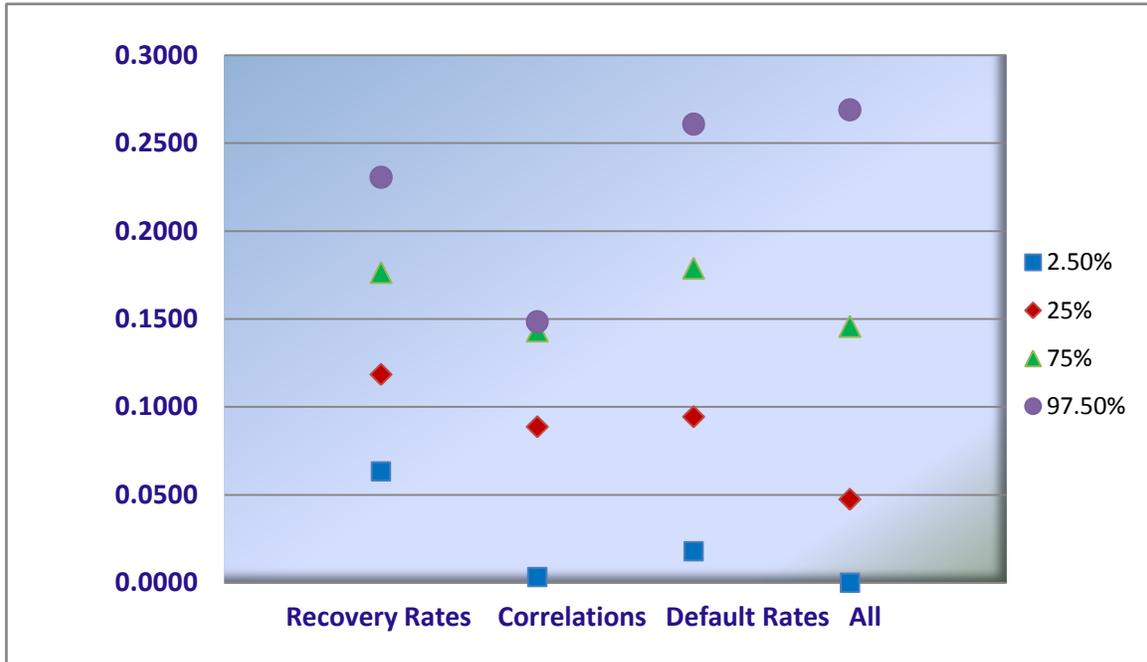


Figure 1: Con't

Distribution of the 10% VaR

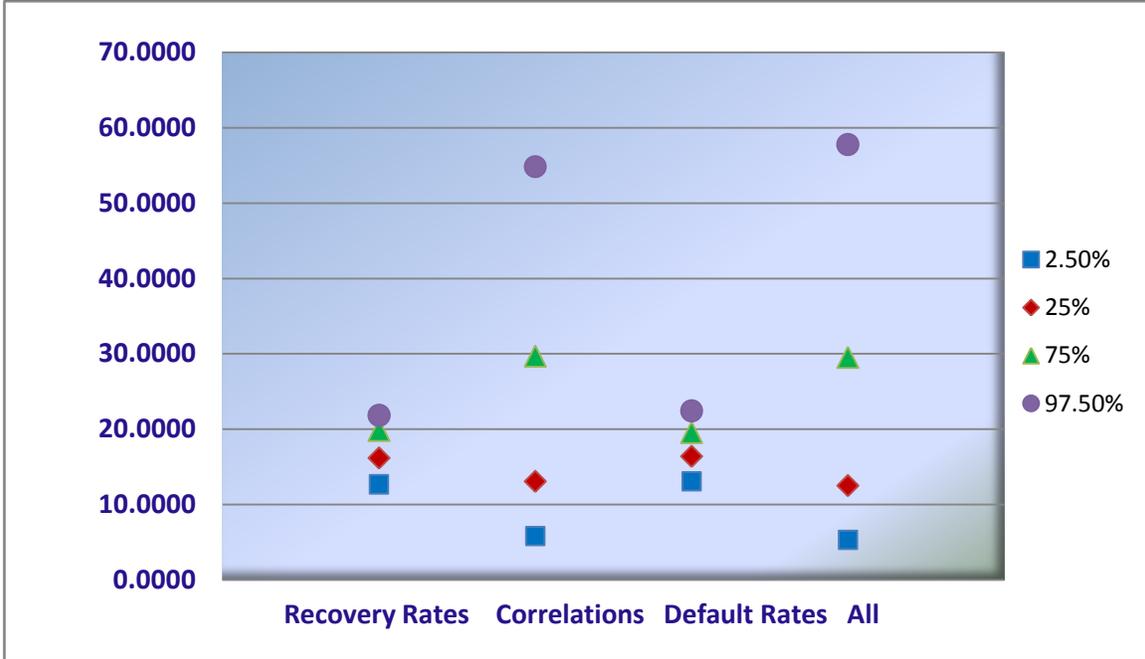


Notes: This figure shows simulated distribution of the 1% VaR, 5% VaR and 10% VaR due to default of portfolio of obligors with investment-grade credit rating in a graphic form. For each graph, the horizontal axis represents the source of estimation error: uncertainty of recovery rates only, uncertainty of default correlations only, uncertainty of default rates only, and uncertainty of three input parameters together. The vertical axis represents the confidence intervals for the portfolio VaR due to estimation error from different sources. Dots in the graphs represent different quantile of the confidence intervals for the portfolio VaR.

Figure 2

Simulated Distribution of the VaR due to Default of Portfolio of Obligators with Speculative-grade Credit Rating

Distribution of the 1% VaR



Distribution of the 5% VaR

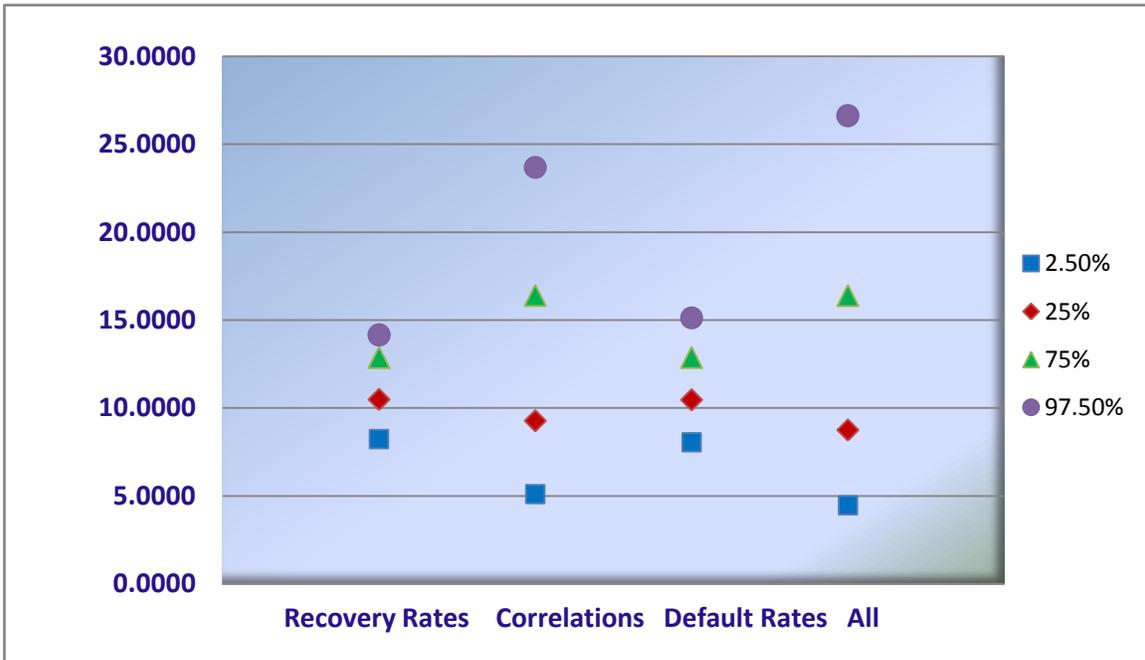
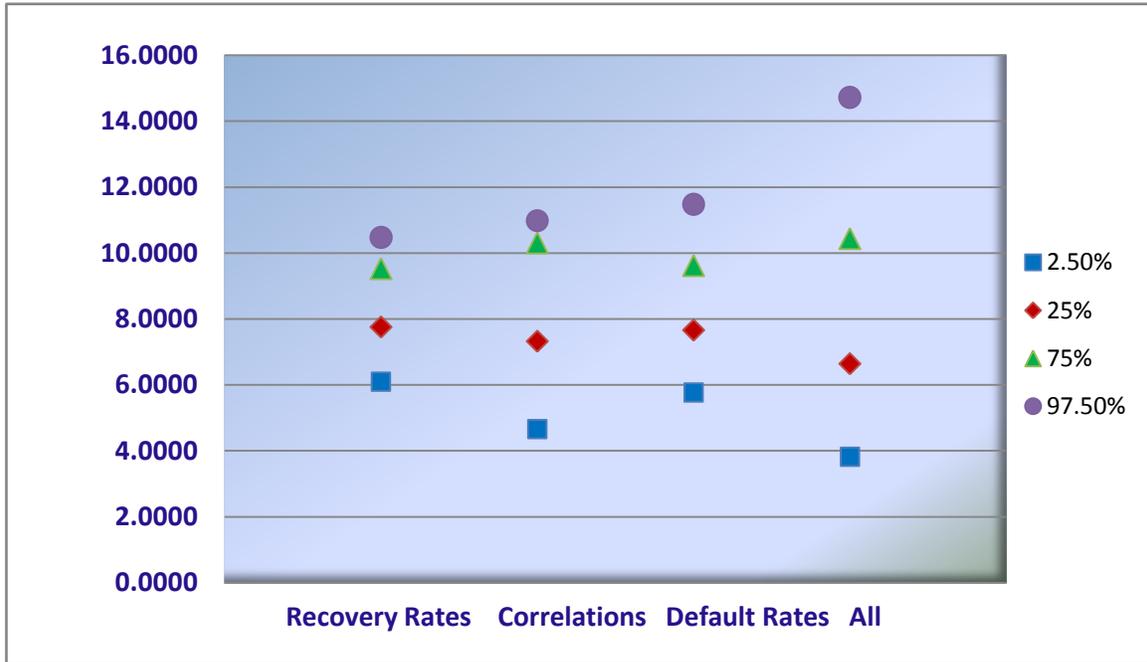


Figure 2 Con't

Distribution of the 10% VaR



Notes: This figure shows simulated distribution of the 1% VaR, 5% VaR and 10% VaR due to default of portfolio of obligors with Speculative-grade credit rating in a graphic form. For each graph, the horizontal axis represents the source of estimation error: uncertainty of recovery rates only, uncertainty of default correlations only, uncertainty of default rates only, and uncertainty of three input parameters together. The vertical axis represents the confidence intervals for the portfolio VaR due to estimation error from different sources. Dots in the graphs represent different quantile of the confidence intervals for the portfolio VaR.

Appendix

Java Code for bootstrap procedure of investment-grade rating bonds and speculative-grade rating bonds

```
public class MainLoop {
```

```
    static ArrayList<Double> rate = new ArrayList<Double>();  
    static ArrayList<Double> residuals = new ArrayList<Double>();  
    static ArrayList<Double> means = new ArrayList<Double>();
```

regression coefficients for bonds with two credit ratings: (investment-grade rating bonds/speculative-grade rating bonds)

```
    static double para1 = 0.0406072524604043;/2.389911472  
    static double para2 = 0.702321203723159;/1.077228297  
    static double para3 = 0.144417926711624;/0.605964475
```

historical data for bonds with two credit ratings: (investment-grade rating bonds/speculative-grade rating bonds)

```
    public void inital(){  
        rate.add(0.00);/(3.96)  
        rate.add(0.14);/(4.53)  
        rate.add(0.14);/(8.09)  
        rate.add(0.14);/(11.04)  
        rate.add(0.00);/(6.08)  
        rate.add(0.00);/(2.5)  
        rate.add(0.05);/(2.1)  
        rate.add(0.05);/(3.51)  
        rate.add(0.00);/(1.79)  
        rate.add(0.08);/(1.98)  
        rate.add(0.14);/(3.68)  
        rate.add(0.17);/(5.45)  
        rate.add(0.23);/(6.05)  
        rate.add(0.26);/(9.64)  
        rate.add(0.41);/(9.19)  
        rate.add(0.10);/(4.88)  
        rate.add(0.03);/(2.01)  
        rate.add(0.03);/(1.41)  
        rate.add(0.00);/(1.14)  
        rate.add(0.00);/(0.86)
```

Residual terms for bonds with two credit ratings: (investment-grade rating bonds/speculative-grade rating bonds)

```
    residuals.add(0.001068);/(3.219864)  
    residuals.add(0.021286);/(2.680331)  
    residuals.add(-0.11871);/(-3.30026)  
    residuals.add(-0.02039);/(0.250388)
```

```

residuals.add(0.009393);/(0.701282)
residuals.add(-0.02572);/(0.37282)
residuals.add(-0.0685);/(-3.10846)
residuals.add(0.046614);/(-0.21121)
residuals.add(0.043207);/(0.241853)
residuals.add(0.042621);/(0.295698)
residuals.add(0.090217);/(0.019144)
residuals.add(0.08241);/(4.035364)
residuals.add(0.220005);/(0.081693)
residuals.add(-0.19101);/(-1.56814)
residuals.add(-0.02163);/(-0.06797)
residuals.add(-0.01724);/(-0.18803)
residuals.add(-0.05734);/(-1.55081)
residuals.add(-0.03627);/(-1.90354)
}

public double calculate(double previous, double beforePrevious){
double result = para1 + para2 * previous - para3 * beforePrevious +
residuals.get(getRandomNumber(18)).doubleValue();
return result;
}

public int getRandomNumber(int range){
Random r = new Random();
int randint = r.nextInt(range);
return randint;
}

public void outputResult(int round, double result){
try{
// Create file
FileWriter fstream = new FileWriter("c:\\\\Means_Result2.xls", true);
BufferedWriter out = new BufferedWriter(fstream);
//out.write("mean "+round+" = "+Double.toString(result));
out.write(Double.toString(result));
out.newLine();
out.close();
}catch (Exception e){//Catch exception if any
System.err.println("Error: " + e.getMessage());
}
}

public static void main(String[] args) {
double buffer[] = new double[20];
double totalmeans = 0;
MainLoop mainLoop = new MainLoop();

```

```

//calculate for formula1
mainLoop.inital();

//calculation starts*****
System.out.println("Calculation starts.....");
for(int j = 0; j<20000; j++){
System.out.println("round " +j);
int random1 = mainLoop.getRandomNumber(20);
buffer[0] = rate.get(random1).doubleValue();
buffer[1] = random1 == 19?rate.get(random1 -1).doubleValue(): rate.get(random1 +
1).doubleValue();
double total = buffer[0]+buffer[1];
for(int i=2; i<20; i++){
buffer[i] = mainLoop.calculate(buffer[i-1], buffer[i-2]);
total = total +buffer[i];
}
means.add(j, new Double(total/20));
totalmeans = totalmeans + total/20;
mainLoop.outputResult(j, total/20);
} //end of outside for loop
System.out.println("Calculation done.....");

double minMean = (Double)Collections.min(means).doubleValue();
double maxMean = (Double)Collections.max(means).doubleValue();
System.out.println("min Mean is: "+minMean);
System.out.println("max Mean is: "+maxMean);
System.out.println("average of all means is: "+totalmeans/20000);
}
}

```

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