EMPIRICAL RESULTS FROM VAR PREDICTION USING PEARSON'S TYPE IV DISTRIBUTION

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Abstract

Two most important characteristics of equity returns time series data are volatility clustering and non-normality. GARCH model has been widely used to forecast dynamic volatilities and hence has been used for value-at-risk (VaR) estimation. (Bhattacharyya et al 2008) has developed a new VaR estimation model for equity return time series using a combination of the Pearson's Type IV distribution and the GARCH(1,1) approach which showed superior predictive abilities. This new model was tested on indices of eighteen countries [3] on daily return up to March 1^{st}, 2005. In this project, we replicate the results in [3], and test the model for its predictive power over a more volatile period (i.e. 350 trading days prior to July 18^{th}, 2008). We backtest the validity of the VaR estimations and compare the predictive power of this model over both of the above time periods on indices of eight countries. We discover that the Pearson's type IV model still remains a good predictive ability during the more volatile period.

Keywords: Risk Management, GARCH, Pearson's Type IV Distribution,Value-at-Risk, Volatility Forecast, Backtesting

To my beloved Mengying Wang who introduced me to FRM.

Liang Chen

To my beloved family for their endless love, tremendous support and encouragement to me all through these years of my studies.

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Chapter 1

Introduction

In response to the financial disasters of the early 1990s, Value-at-Risk (VaR), a new method to measure financial-market risk has been developed. Since its adoption by the Basel Committee, VaR becomes the most common tool of measuring risk in financial sector. VaR was initially used to measure market risk. Nowadays, VaR is not only being adopted to control and manage risk activities, but also has been extended to quantify operational risk and credit risk. As increasing financial uncertainty and the VaR applications, intensive research has been done by financial institutions, regulators and academics for developing sophisticated models for VaR estimation. In recent years, especially under the sub-prime crisis, we've witnessed billions of dollars of financial losses and several financial institutions failure that should draw highly attention to risk management systems.

"VaR uses standard statistical techniques used routinely in other technical field, and it can be defined as the worst loss over a target horizon that will not be exceeded with a given level of confidence" (Jorion 2007). Thus, VaR is calculated based on the statistical distribution of the asset returns. Parametric approach is one reasonable way of estimating VaR, which involves estimation of parameters such as the standard deviation. If the distribution of asset returns can be assumed to belong to a particular parametric family, VaR computation can be reduced considerably. The important issue is whether the distribution assumption is realistic. Originally, many implementations of conditional VaR assume that asset returns are normally distributed such that it results in underestimating VaR because of lowering the likelihood of extreme returns. In fact, empirical evidence shows that the distribution of asset return is leptokurtic, and it shows volatility clustering and heteroscedasticity. In order to capture this empirical fact, GARCH models have been widely used with several other distributions which have been proposed in terms of VaR estimation. Examples of proposed distribution are the student t distribution (Bollerslev, 1987), stable distribution (McCulloch, 1996), and mixture of normal distribution (Alexander and Lazar, 2006). Whatever the assumed distribution, most of the models focus on the first two moments. That is, the mean and conditional variance. The second moment, variance, is a measure of risk that has been widely recognized as proxy market volatility. Recent studies tilted to the importance of higher than second moment effect on risk management of asset return and portfolio construction. The characteristic of Pearson's Type IV distribution has been applied in such respect. Pearson's type IV distribution captures great range of skewness and kurtosis that is a better fit in explaining excess kurtosis in the financial data. This distribution was first introduced by Permaratne and Bera (2001) in use of extended GARCH model for capturing asymmetry and fat-tail of asset returns. In addition to it, Yan (2005) again used Pearson's Type IV distribution in autoregressive conditional density models to accommodate time-varying parameters in modeling non-normal innovation. Due to the natural of financial return data, simultaneously combining asymmetry, excess kurtosis and volatility clustering of financial data by using Pearson's Type IV distribution in estimation of VaR theoretically would yield more accurate results.

In the literature, Bhattacharyya et al(2008) is known as the first one using Pearson's Type IV distribution for estimation of VaR. They found that Pearson-GARCH is a more robust method for estimating conditional VaR, and they also asserted Pearson-GARCH model performed very well for holdout sample. Stated in Bhattacharyya et al (2008) that "The high kurtosis values of Pearson's type IV curves ensure that even extreme events can be protected against". From this respect, we follow their approaches and then update the sample size to verify the performance of estimating VaR of market indices during the high volatility period i.e. 350 trading days prior to Jul 18^{th}, 2008. The rest of this paper is organized as follows. Chapter 1 provides an introduction of background knowledge of VaR estimation such as, GARCH model, Pearson IV distribution and backtesting methods. Chapter 2 describes the methodology and detailed settings of the models that are used. In the last two sections of Chapter 2, we assess the VaR predictive power of the models over eight indices at different confidence levels. In addition, we also compared the predictive abilities of the models over two periods of time with emphasis on the new innovations with Pearson's Type IV distribution. Chapter 3 gives us the concluding remarks.

1.1 GARCH

Modeling volatility in time series of financial data has been very popular since early 1980. Engel (1982) initially developed the Autoregressive Conditional Heteroscedasticity (ARCH). ARCH models have been proposed by researchers with various refinements to improve the model performance. GARCH (Bollerslev 1986) is one of the most common ways used in estimating conditional mean and volatility of the returns needed for implantation of conditional VaR. The GARCH model (Akgiray, 1989) assumes that the variance of returns follows a predictable process. The conditional variance not only depends on the all past innovations of order p but also on all pervious conditional variances of order q. A general GARCH(p, q) process can be described as follows:

$$R_t|\Omega_{t-1} \sim F(\mu_t, \sigma_t^2), \tag{1.1}$$

$$\mu_t = c + \rho R_{t-1}, \tag{1.2}$$

$$\sigma_t^2 = \omega + \Sigma_{i=1}^{p} \alpha_i e_{t-i}^2 + \Sigma_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{1.3}$$

and

$$e_t = R_t - \mu_t \tag{1.4}$$

Where $p > 0$, and $q \geq 0$ are the orders of the process, and the parameters satisfy the conditions $\omega > 0$, $\alpha_i, \beta_j \geq 0$, $i = 1, \ldots, p$, $j = 1, \ldots, q$. $F(\mu_t, \sigma_t^2)$ is the conditional distribution of the variable, with conditional mean μ_t and variance σ_t^2. Ω_{t-1} is the set of all information available at time t ($i.e. R_{t-1}, R_{t-2}, \ldots$).

It is well known that financial data are not normally distributed with fatter tails and peaked around mean that are caused by volatility clustering. GARCH model captures the dynamic volatility in returns, however, it assumes the conditional kurtosis is constant over the estimated period.

1.2 Pearson's Type IV Distribution

To better modeling conditional skewness and conditional kurtosis presented in the financial data, many distribution of return residuals are assumed to take these into account. Introduced by Pearson (1895), the Pearsons family of curves encompasses a wide range of distribution such as normal, beta, student t, gamma, and inverse Gaussian. To see this, lets present the probability density function of such distribution:

$$\frac{1}{f(x)}\frac{df(x)}{dx} = \frac{x-\alpha}{c_0 + c_1 x + c_2 x^2} \tag{1.5}$$

where α, c_0, c_1, c_2 are the parameters, some special cases are state as follow:

i If $c_1 = c_2 = 0$, it is normal distribution which gives first order estimation to the unknown distribution.

ii If $\alpha = 0$ and $c_1 = 0$, integrate the equation then it gives a symmetric leptokurtic density

iii If $c_1 \neq 0$ and $c_2 \neq 0$, the roots of quadratic equation $c_0 + c_1 x + c_2 x^2 = 0$ are imaginary. Equation 1.5 integrates to following density

$$f(x)dx = k[1 + (\frac{x-\lambda}{a})^2]^{-m} \exp[-v \tan^{-1}(\frac{x-\lambda}{a})]dx \tag{1.6}$$

where $m > 1/2$, v, a, and λ are real valued parameters, $-\infty < x < \infty$ and k is a normalization constant that depends on m , v, a.

Equation 1.6 is known as Pearson's type IV distribution. v can be interpreted as skewness parameters (e.g. when $v = 0$, it is symmetric). m can be interpreted as kurtosis parameter that controls the thickness of the tail. In another word, increases m decreases the kurtosis. When $m \to \infty$, it becomes normal distribution. The first four moments: mean μ, variance σ^2, kurtosis s and skewness k of Pearson's Type IV are given below.

$$\mu = \lambda - \frac{av}{2(m-1)}, (m > 1), \tag{1.7}$$

$$\sigma^2 = \frac{a^2}{r^2(r-1)}(r^2 + v^2), (m > 3/2), \tag{1.8}$$

$$s = -\frac{4a^3v(r^2 + v^2)}{r^3(r-1)(r-2)}, (m > 2), \tag{1.9}$$

$$k = \frac{3a^4(r^2 + v^2)[(r+6)(r^2 + v^2) - 8r^2]}{r^4(r-1)(r-2)(r-3)}, (m > 5/2), \tag{1.10}$$

where $r = 2(m - 1)$.

1.3 Backtesting Methods

VaR models should always be backtested because they are only useful when they produce acceptable results. "Backtesting is a formal statistical framework that consists of verifying that actual losses are in line with projected losses" (Jorion 2007). It is essential for VaR users to check that their VaR models are well calibrated. If not, the models should be examined for faulty assumptions, wrong parameters or inaccurate modeling. It is also important from a regulator's point of view that financial institutions do not understate their risks.

Since VaR is usually reported at a specific statistic level (95% or 99%), it still can be acceptable if the VaR estimation does not exceed a given confidence level. The issue is how to make the decision whether to accept or reject the result produced by VaR models. This decision is made based on some confidence levels: 95% confidence level (mostly used) and 99% confidence level (the Basel rule). One way to verify the accuracy of the model is to record the failure rate, which is the proportion of VaR exceeds in a given sample. Let N be the number of VaR exceedances over the sample period of T days, p be the quantile level, N/T is the failure rate. Ideally, the failure rate should be unbiased and converge to p as the sample size increases. To test whether N is too many or too few under the null hypothesis given a confidence level, it is important to point out that return distribution can be normal, skewed or fat-tailed. However, there is no assumption for return series distribution. The setup for such a test is the classic testing framework for a sequence of success and failures, also called Bernoulli trials. Under the null hypothesis that the model is correctly calibrated, the number of exceptions x follows a binomial probability distribution:

$$f(x) = \begin{pmatrix} T \\ x \end{pmatrix} p^x (1-p)^{T-x} \tag{1.11}$$

We also know that x has expected value of $E(x) = pT$ and variance $V(x) = p(1-p)T$. When T is large, we can use the central limit theorem and approximate the binomial distribution by the normal distribution (Equation 1.12).

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0,1) \tag{1.12}$$

The cutoff value of $|\ z\ |$ is 1.96 for 95% confidence level of decision rule. If $|\ z\ |$ is larger than the cutoff value of 1.96, then the VaR model is biased.

For any backtesting model, there is always a trade-off between type 1 and type 2 errors. Type 1 error occurs when we reject a correct model and type 2 error occurs when we do not reject an incorrect model. Our goal is to get the lowest type 2 error for a given type 1 error.

Kupiec (1995) developed approximate 95 percent confidence regions for such a test. The regions are defined by the tail points of the Log-likelihood Ratio (LLR):

$$LR_{uc} = -2ln[(1-p)^{T-N}p^{N}] + 2ln[(1-\frac{N}{T})^{T-N}(\frac{N}{T})^{N}]. \tag{1.13}$$

This LLR follows the Chi-square distribution with one degree of freedom. The null hypothesis is that p is the true probability, and we will reject the VaR model if $LR_{uc} > 3.841$. Statistical decision theory shows that the LLR test is the most powerful backtesting method among its class. This test is also easy to be done by plugging the values of p, N and T into the the LLR formula and compare the result with the critical value to see if we should reject the VaR model at the 95% confidence level.

Chapter 2

Testing the Pearson-GARCH Model

2.1 Description of Data

We obtain from *http://finance.yahoo.com* the daily adjusted closing index values of eight countries: Australia (All Ordinary), Hong Kong (Hang Seng), Japan (N225) Korea (KOSPI), France (CAC 40), Germany (DAX), UK (FTSE 100) and USA (NYSE Composite). We have shown in Table 2.1 the periods for which data have been analyzed and the number of obervations in each data set. As we can see in Table 2.1, the starting dates of our data series are the same as those in [3] for all eight indices that we choose among the eighteen indices analyzed.

For each index series we obtained, we denote the index observations made at time t and

Country	Index	Ticker	Data range		Data points
			From	To	
Australia	All Ordinary	AORD	1984-8-3	2008-7-18	6061
Hong Kong	Hang Seng	HSI	1986-12-31	2008-7-18	5342
Japan	N225	N225	1984-1-4	2008-7-18	6040
Korea	KOSPI	KS11	1997-7-1	2008-7-18	2715
France	CAC 40	FCHI	1995-3-1	2008-7-18	3391
Germany	DAX	GDAXI	1995-3-1	2008-7-18	3384
UK	FTSE 100	FTSE	1995-3-1	2008-7-18	3380
USA	S&P 500	GSPC	1995-3-1	2008-7-18	3371

Table 2.1: Countries analyzed and data ranges

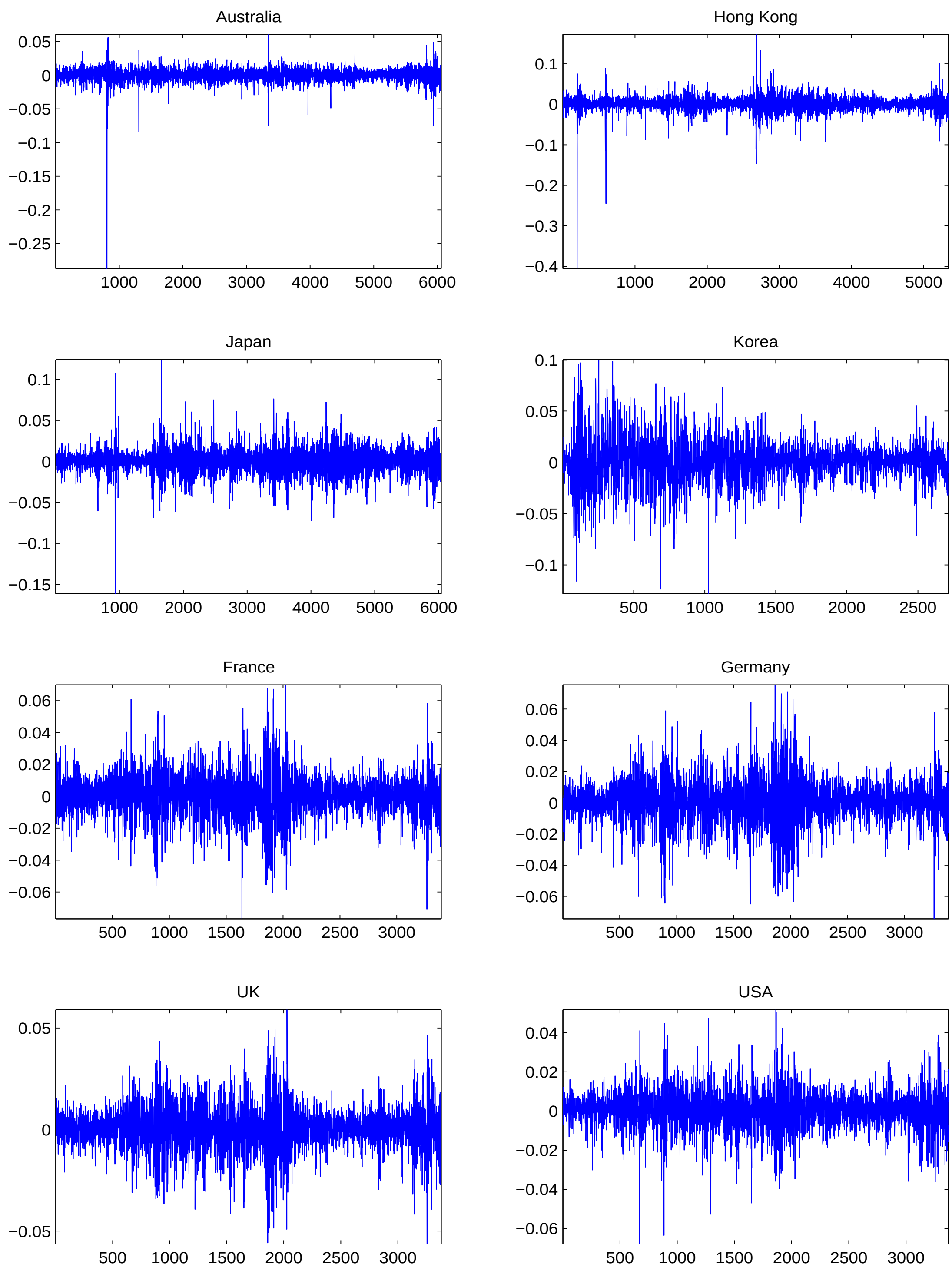
$t + 1$ as P_t and P_{t+1} respectively. We use the continuous compounding to transform the index series $\{P_t\}$ into a return series $\{R_t\}$ such that

$$R_t = \log \frac{P_{t+1}}{P_t} = \log P_{t+1} - \log P_t. \tag{2.1}$$

We calculate log return series for all eight indices and plot them in Figure 2.1.

By looking at the return series, it is clearly shown in each return series the volatility clustering property of the series and a high volatile period in the last 350 observations. In our experiments, we divide each of the sample return series into two parts: estimation sample and prediction sample. Estimation sample is used to estimate the initial values of AR(1)-GARCH(1,1) parameters and the Pearson's Type IV distribution parameters (Section 2.3). The parameters are then used to predict the VaR in the next time period (i.e. VaR in the next day). In later sections, we will refer these two parts of the samples as in-sample and holdout-sample respectively.

Figure 2.1: Log returns of the entire sample data



2.2 Test for Autocorrelation and Stationary

The return series are tested for autocorrelation using the MATLAB 'autocorr' function. A plot of the autocorrelations is shown in Figure 2.2. The return series are also tested for stationary using augmented Dickey-Fuller tests and none of the series are found to have unit roots. This is achieved using the MATLAB 'dfARDTest' function. The tests for autocorrelation and stationary are against the entire sample data.

2.3 VaR Estimation using GARCH and Pearson's Type IV Distribution

In this section, we will describe two approaches that we have adopted for VaR estimation. Both of the approaches combine the GARCH model with the Pearson's Type IV distribution. Before we get into these two approaches, let's first look at the GARCH model settings that we are using for both of the approaches. To be consistent with the settings used in [3], we choose AR(1) model without a constant term for the conditional mean and GARCH(1,1) model for the conditional variance. This AR(1)-GARCH(1,1) model is expressed as follows:

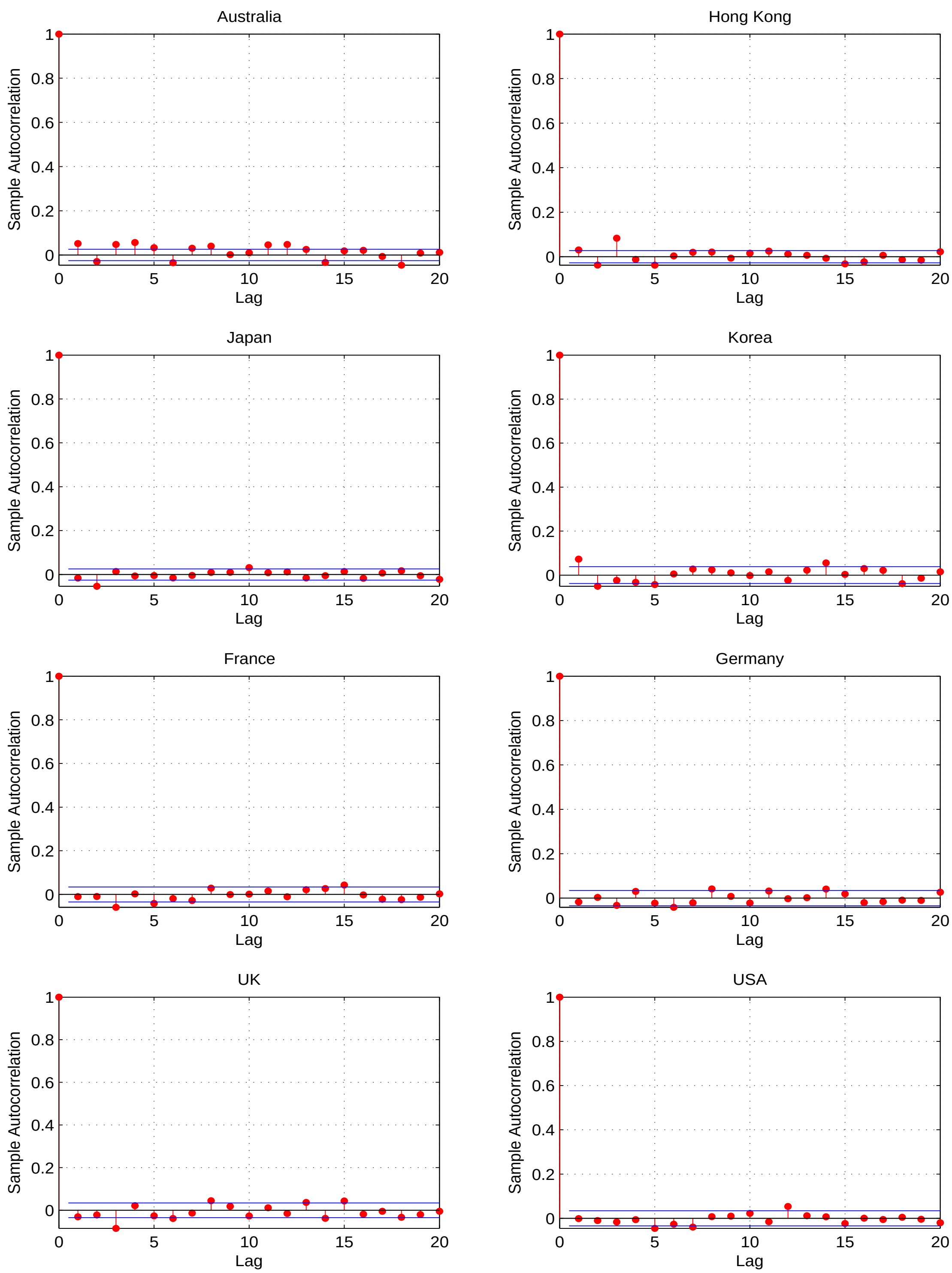
$$\mu_t = \rho R_{t-1}, \tag{2.2}$$

$$\sigma_t^2 = \omega + \alpha(R_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2, \tag{2.3}$$

and

$$e_t = R_t - \mu_t = R_t - \rho R_{t-1} \tag{2.4}$$

Figure 2.2: Autocorrelations of the entire sample data



where $|\rho| < 1, \omega, \alpha, \beta > 0$ and $\alpha + \beta < 1$. We denote h_t to be the variance (i.e. σ_t^2) and Z_t to be the *i.i.d.* standardized innovations. In GARCH models, Z_t is assumed to follow the standard normal distribution that is

$$Z_t \sim N(0,1) \Rightarrow (e_t \mid F_{t-1}) \sim N(0, h_t) \tag{2.5}$$

$$f(e_t \mid F_{t-1}) = \frac{1}{\sqrt{2\pi h_t}} \exp(-\frac{e_t^2}{2h_t}) \tag{2.6}$$

The log-likelihood (LL) of the model is given by $\sum l_t$ where l_t is the conditional LL defined as:

$$l_t = -\frac{1}{2}[\log(2\pi h_t) + \frac{e_t^2}{h_t}] \tag{2.7}$$

The maximum likelihood estimates (MLE) for ρ, ω, α and β are computed by minimizing the negative of the LL. We refer this model as normal innovation for AR-GARCH model. In the following two models, we will use the same setting for GARCH as shown in equations 2.2 and 2.3 except that the distribution of Z_t is assumed to be Pearson's Type IV. Assume the 1-day ahead mean and variance prediction to be μ_t and σ_t^2, the VaR prediction at day t for the return can be written as

$$VaR_q^t = \mu_{t+1} + \sigma_{t+1} VaR(Z)_q \tag{2.8}$$

where $VaR(Z)_q$ denotes the qth quantile of the residuals Z_t.

2.3.1 GARCH-PIV: Pseudo Maximum Likelihood - Method of Moments Estimation

In this approach, we estimate the model coefficients in two steps, then obtain VaR_q^t from both normal and Pearson's Type IV fitted standardized residuals. First of all, an AR(1)-GARCH(1,1) model is fitted to the return series using the Pseudo Maximum Likelihood (PML) method. The standardized residuals $\{Z_t\}$ are extracted and the model is used to make 1-day ahead predictions of μ_{t+1} and σ_{t+1}^2. Then, we fit a Pearson's Type IV curve to $\{Z_t\}$ using the Method of Moments (MM). At last, the VaR_q^t is calculated using Equation 2.8. We denote this VaR estimation model as **Approach 1**.

The MM estimates the Pearson's Type IV parameters r, ν, a and λ by matching the first four moments (Equations 2.9, 2.10, 2.11 and 2.12) using (Heinrich, 2004).

$$r = 2(m-1) = \frac{6(k - s^2 - 1)}{2k - 3s^2 - 6} \tag{2.9}$$

$$\nu = -\frac{r(r-1)s}{\sqrt{16(r-1) - s^2(r-2)^2}} \tag{2.10}$$

$$a = \frac{\sigma\sqrt{16(r-1) - s^2(r-2)^2}}{4} \tag{2.11}$$

$$\lambda = \mu - \frac{(r-1)s\sigma}{4} \tag{2.12}$$

where μ, σ^2, s and k are the mean, variance, skewness and kurtosis of Z_t.

The reason of using PML instead of MLE is that the PML method yields consistent and asymptotically normal estimators (Gourieroux, 1997). This approach is computationally faster than the next approach.

2.3.2 GARCH-PIV: Maximum Likelihood Estimation

In this approach, we use the maximum likelihood (ML) method to estimate all model coefficients (i.e. AR-GARCH coefficients and PIV coefficients) together. We assume that Z_t follows a Pearson's Type IV distribution with parameters m, ν, a and λ.

$$Z_t \sim PIV(k, m, \nu, a, \lambda) \tag{2.13}$$

This choice results in Z_t being a unit variance process, but the mean is not necessarily zero. The theoretical justification for such a choice is given by Newey and Steigerwald (1997) [13], who proved that if the conditional distribution of innovations in a GARCH model is asymmetric, then an additional location parameter is required to satisfy identification condition for the consistency of the parameter estimates. Due to the non-zero mean of the innovation series, the expected conditional return needs to be adjusted and is given by

$$E(R_t \mid F_{t-1}) = \mu_t + \sqrt{h_t}(\lambda - \frac{a\nu}{r}) \tag{2.14}$$

As the normalizing constant k is inversely proportional to the scale parameter a,

$$Z_t \sim PIV(k, m, \nu, a, \lambda) \Rightarrow (e_t \mid F_{t-1}) \sim PIV(\frac{k}{\sqrt{h_t}}, m, \nu, a\sqrt{h_t}, \lambda\sqrt{h_t}) \tag{2.15}$$

Therefore, we can compute MLEs of ρ, ω, α, β, r, ν and λ. At last, the VaR_q^t is calculated using Equation 2.8. We refer this VaR estimation model as **Approach 2** in later sections.

2.4 Data Analysis and Parameter Interpretation

Although the Bhattacharyya et al (2008) provided an excellent example to study VaR models in actual market with Pearson's type IV distribution, this method has not been widely studied and hence is not commonly used. We have repeated the same experiments that Bhattacharyya et al (2008) did with updated data. The data set that we used has been described in Section 2.1.

For each index series, we obtain its log-return series via continuous compounding. For each return series, we divide it into two parts: in-sample data and holdout-sample data. We fix the holdout-sample data size to be 350 for all indices. This means we use the data from in-sample to estimate model parameters and then predict the VaR in the next day for 350 days appear in the end of the return series.

2.4.1 AR-GARCH Model Parameters

From the log return series of the samples (Figure 2.1), we can observe volatility clustering on log returns for all eight indices. After testing for autocorrelation and stationary of the return series, we obtain the parameters of AR(1)-GARCH(1,1) model with both normal innovations and Pearson's Type IV innovations using the MATLAB code suggested by Bhattacharyya et al (2008). This code is freely available under the GNU General Public License at *http://nmisra.googlepages.com*. The estimated AR(1)-GARCH(1,1) parameters for both of the approaches are reported in Table 2.2. The coefficients of the mean and variance equations are found to be significant by looking at the t-statistic values.

2.4.2 Standardized Residuals

Characteristics of the standardized residual series in AR(1)-GARCH(1,1) model are summarized in Table 2.3. It lists the mean, standard deviation, skewness and kurtosis of the

Table 2.2: AR(1)-GARCH(1,1) estimated model parameters.

Country	Mean Equation Coefficients		Variance Equation Coefficients					
	ρ		ω		α		β	
	N	P	N	P	N	P	N	P
Australtia	0.10132	0.0638	6.2E-06	2.1E-06	0.2173	0.0867	0.7222	0.8834
Hong Kong	0.10819	0.05173	6.6E-06	3.8E-06	0.1358	0.0833	0.847	0.9019
Japan	0.02949	0.0002	2.6E-06	1.5E-06	0.1251	0.0946	0.8716	0.9037
Korea	0.07241	0.04778	2.1E-06	1.8E-06	0.0802	0.069	0.9192	0.9298
France	-0.01496	-0.0279	1.5E-06	1.2E-06	0.0724	0.0683	0.9206	0.9259
Germany	-0.015	-0.035	2.3E-06	1.5E-06	0.0922	0.0848	0.8977	0.9101
UK	-0.01908	-0.0315	9.5E-07	8.8E-07	0.0851	0.0821	0.9085	0.9118
USA	0.03031	0.00644	1.3E-06	9.4E-07	0.083	0.0785	0.9064	0.9146

N: Normal Innovation for AR-GARCH (Approach 1); P: Pearson's Type IV innovations for AR-GARCH (Approach 2).

Table 2.3: Standardized residual series characteristics.

Country	Mean		SD		Skewness		Kurtosis	
	N	P	N	P	N	P	N	P
Australtia	0.0378	0.0409	1	1.0378	-1.0219	-1.8337	13.215	30.766
Hong Kong	0.0304	0.0337	0.9996	1.0169	-0.8194	-1.0024	9.6338	11.629
Japan	0.0049	0.0063	0.9994	1.0102	-0.6026	-0.6955	10.369	11.683
Korea	0.0168	0.0171	0.9983	0.9966	-0.3605	-0.3786	4.9356	5.0143
France	0.0215	0.0219	1	1.0014	-0.3048	-0.3108	3.8474	3.8622
Germany	0.0324	0.0335	0.9991	1.0008	-0.3914	-0.4157	4.1024	4.2056
UK	0.0192	0.0196	0.9991	1	-0.3113	-0.3157	3.5326	3.5294
USA	0.0341	0.0356	0.998	1.0008	-0.5094	-0.523	5.031	5.0927

N: Normal Innovation for AR-GARCH (Approach 1); P: Pearson's Type IV innovations for AR-GARCH (Approach 2).

Table 2.4: Q-statistic for standardized residuals

Country	Lag 5		Lag 10		Lag 15	
	MM	MLE	MM	MLE	MM	MLE
Australtlia	9.5453	16.177	14.906	21.337	31.34	39.583
Hong Kong	21.901	37.285	28.74	44.704	35.719	52.53
Japan	1.1855	5.7245	15.193	20.728	19.889	25.851
Korea	3.4446	6.3598	8.2706	11.396	17.559	20.545
France	7.6594	8.5719	15.013	15.758	19.898	20.881
Germany	4.7431	7.2579	9.0422	11.515	16.936	19.258
UK	6.1435	7.5983	9.6466	11.182	17.063	18.566
USA	7.8781	9.7751	13.848	15.616	23.036	24.681

MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2).

standardized residuals. In all cases, the skewness statistic on the standardized residuals is negative. The excess kurtosis statistic is larger than that of normal distribution which show the characteristic of heavy tail in all markets.

The Q-statistics at lag 5, 10, and 15 for the standardized residuals are listed in Table 2.4. The null hypothesis is that autocorrelations up to a given number of lags are all simultaneously zero. Low values of the Q-statistic mean that we cannot reject the null hypothesis. The statistic follows a chi-quare distribution with degrees of freedom equal to the number of lags. The threshold values for lags 5, 10, and 15 are 15.09, 23.21 and 30.58 at 1% significance level. The statistical values shown in Table 2.4 tell that we cannot reject the null hypothesis for all indices except Australia and Hong Kong.

Table 2.5: Pearson's Type IV parameter estimates.

Country	μ		ν		α		λ	
	MM	MLE	MM	MLE	MM	MLE	MM	MLE
Australtia	2.9374	4.5841	1.1411	1.5112	1.6264	2.4302	0.5168	0.5569
Hong Kong	3.1227	3.2389	1.1211	0.3021	1.741	1.8606	0.4902	0.1655
Japan	2.9795	3.8063	0.6895	0.7397	1.6936	2.1293	0.2998	0.2872
Korea	4.2794	4.4278	1.1608	1.159	2.3178	2.386	0.427	0.42
France	6.8356	8.0222	2.7031	3.3673	3.1823	3.5122	0.7585	0.8642
Germany	6.0699	7.2287	2.7695	3.8303	2.9138	3.2354	0.8283	1.0283
UK	10.435	10.436	6.1651	6.196	4.0144	4.0166	1.3308	1.3383
USA	4.4458	4.7154	1.8299	1.6215	2.3412	2.4776	0.6558	0.577

MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2)

2.4.3 Pearson's Type IV Distribution Parameters

We obtain the estimated parameters of Pearson's Type IV distribution that fits the standardized residual (Approach 1) by PML estimation and the parameters of the Pearson's Type IV distribution (Approach 2) by ML estimation. Table 2.5 lists the Pearson's Type IV parameter estimations. We can see that values of m (i.e. μ shown in the table) are all greater than 5/2 in all cases so than they all satisfying the conditions for a type IV distribution.

2.4.4 VaR Exceedances over the Entire Sample

The model is fitted on the eight indices in our study. The goodness of fit of the model is measured in terms of the number of exceptions observed. An exception occurs when a realized daily loss (i.e. negative of daily return) is greater than the estimated VaR for a

Table 2.6: VaR exceedances for returns of entire sample.

Country	95 percentile				97.5 percentile				99 percentile			
	N	Pearson		E	N	Pearson		E	N	Pearson		E
		MM	MLE			MM	MLE			MM	MLE	
Australtia	269	279	287	303	158	117	140	151	85	32	57	60.6
Hong Kong	232	240	285	267	141	101	129	134	70	34	50	53.4
Japan	302	321	318	301.9	181	135	144	151	84	35	45	60.4
Korea	145	143	142	135.7	83	64	66	67.8	41	19	19	27.1
France	183	176	175	169.5	100	82	81	84.7	51	27	30	33.9
Germany	183	171	167	168.7	102	76	79	84.3	45	27	29	33.7
UK	183	169	171	168.9	107	93	93	84.5	58	34	35	33.8
USA	175	173	175	168.5	109	81	86	84.2	61	24	30	33.7

N: Normal innovation for AR-GARCH; Pearson: Pearson's Type IV innovations for AR-GARCH; E: Expected number of exceptions; MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2).

given percentile. If the model is correct, the expected exception is the tail area of each quantile times the total number of observation. A high exception number implies that the model excessively underestimated the realized VaR, and a low exception number implies it overestimated the realized VaR. The number of VaR exception for our entire sample for each index with normal and Pearson's Type IV innovations is given in the Table 2.6 along with the expected number of exceptions. As we can see from the table, in most cases, the Pearson's Type IV distribution with MLE method out performs the normal distribution at all three percentiles and the Pearson's Type IV distribution with PML estimation also out performs the normal distribution at 95 and 97.5 percentile.

2.4.5 VaR Predictions

To test the predictive power of the models, we need to identify an holdout-sample. It is usually the last N data points in the entire sample. In our case, we choose the last 350 points in each return series to justify model predictions and use the data points that are prior to it to estimate the model parameters. As we can see, the return series between Jan 1^{th}, 2007 and July 18^{th}, 2008 is very volatile compare to a prior to it. As we all know, the stock markets around the world have been extremely volatile since 2007. We choose such period to see the performance of the models under this particular situation. The expected numbers of exceptions for an holdout-sample of size 350 are 17.5, 8.75 and 3.5 at 5%, 2.5% and 1% significance, respectively. The actual number of exceptions are disclosed in Tables 2.7, 2.8 and 2.9 with different window sizes that we will explain in the following paragraphs.

Default Sliding Window

We use the same method described in Bhattacharyya et al (2008) that is after computing the VaR at the end of a day, we roll the window forward by one point such that we drop the least recent data point and repeat the same steps for the next VaR computation. The in-sample size, which is also the sliding window size, equal to the number of observations in the entire sample for each return series minus the holdout-sample size (i.e. 350). The VaR violations for the holdout-sample are reported in Table 2.7.

Table 2.7: VaR exceedances for returns for holdout-sample using moving window size equal the length of in-sample.

Country	Holdout sample VaR violations								
	95 percentile			97.5 percentile			99 percentile		
	N	Pearson		N	Pearson		N	Pearson	
		MM	MLE		MM	MLE		MM	MLE
Australia	<u>28</u>	<u>30</u>	24	<u>18</u>	14	<u>17</u>	<u>11</u>	3	<u>8</u>
Hong Kong	25	26	<u>29</u>	<u>16</u>	14	15	<u>8</u>	4	6
Japan	26	<u>29</u>	<u>27</u>	<u>17</u>	12	13	7	3	6
Korea	20	20	23	15	12	12	7	3	5
France	<u>29</u>	29	<u>30</u>	15	13	13	5	4	4
Germany	21	21	21	13	13	13	6	4	5
UK	23	23	23	<u>16</u>	15	14	<u>11</u>	<u>9</u>	<u>9</u>
USA	26	25	26	<u>22</u>	15	15	<u>13</u>	4	5

N: AR-GARCH model with normal innovation; MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2). The size of the holdout-sample is 350. Therefore, the expected number of violations are 17.5, 8.75 and 3.5 respectively. The underlined numbers of exceptions should be rejected at 95% level of test confidence. See Table 2.10 for model backtesting 95% non-rejection test confidence regions.

Table 2.8: VaR exceedances for returns for holdout-sample using incremental window.

Country	Holdout sample VaR violations								
	95 percentile			97.5 percentile			99 percentile		
	N	Pearson		N	Pearson		N	Pearson	
		MM	MLE		MM	MLE		MM	MLE
Australia	29	30	27	18	16	17	11	4	8
Hong Kong	25	26	31	18	12	15	8	4	6
Japan	26	27	27	17	12	14	7	3	6
Korea	20	19	19	13	11	11	7	3	3
France	30	28	29	17	13	14	6	3	4
Germany	20	20	20	13	13	13	6	4	4
UK	23	23	23	16	15	15	13	9	9
USA	26	26	26	21	15	16	13	4	6

N: AR-GARCH model with normal innovation; MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2). The size of the holdout-sample is 350. Therefore, the expected number of violations are 17.5, 8.75 and 3.5 respectively. The underlined numbers of exceptions should be rejected at 95% level of test confidence. See Table 2.10 for model backtesting 95% non-rejection test confidence regions.

Incremental Window and Customized Sliding Window

As pointed out in [3] that the right window size can be obtained by trial and error. We adopted two rolling window algorithms other than the default sliding window algorithm. The first one is called incremental window such that the window size keeps increasing as predictions go on. The difference between the incremental window and the default sliding window is that, after we compute the VaR at the end of a day, we roll the window forward to include one more data points without dropping off the least recent data in the in-sample. Therefore, the window size increases by one after each prediction. The VaR prediction of using the incremental window is described in Table 2.8.

Table 2.9: VaR exceedances for returns for holdout-sample using moving window size equal to 2000.

Country	Holdout sample VaR violations								
	95 percentile			97.5 percentile			99 percentile		
	N	Pearson		N	Pearson		N	Pearson	
		MM	MLE		MM	MLE		MM	MLE
Australia	25	24	26	<u>19</u>	14	15	<u>12</u>	7	7
Hong Kong	23	25	24	<u>18</u>	15	<u>17</u>	<u>9</u>	6	6
Japan	25	24	23	<u>16</u>	12	12	7	6	6
Korea	21	21	20	<u>17</u>	11	11	7	3	3
France	<u>31</u>	<u>27</u>	<u>29</u>	<u>16</u>	11	12	6	3	3
Germany	19	19	19	13	10	12	5	4	4
UK	23	22	22	<u>17</u>	14	14	<u>12</u>	<u>9</u>	<u>9</u>
USA	26	26	26	<u>20</u>	<u>16</u>	<u>17</u>	<u>13</u>	6	6

N: AR-GARCH model with normal innovation; MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2). The size of the holdout-sample is 350. Therefore, the expected number of violations are 17.5, 8.75 and 3.5 respectively. The underlined numbers of exceptions should be rejected at 95% level of test confidence. See Table 2.10 for model backtesting 95% non-rejection test confidence regions.

The second rolling window algorithm that we use is the customized sliding window in which we fix the sliding window size to a specific number and hence ignoring the data that is prior to the end of the (N + holdout-sample size) observations. The motivation of choosing a smaller window size is trying to put more weight on the recent data. We have done experiments with window size equal to 1000 and 2000. The results shown in Table 2.9 are those produced with window size equal 2000. This result is much better than the result produced with window size equal 1000.

Table 2.10: Model backtesting 95% non-rejection test confidence regions.

Nonrejection Region for Number of Exceptions N			
Probability level p	VaR Confidence level c	T = 350 Days	T = 500 days
0.01	99%	0 < N < 8	1 < N < 10
0.025	97.5%	3 < N < 16	6 < N < 20
0.05	95%	10 < N < 27	16 < N < 36

N is the number of failures that could be observed in a sample size T without the null hypothesis that p is the correct probability at the 95 percent level of test confidence.

2.5 Backtesting

We use the log-likelihood ratio (LLR) method mentioned in Section 1.3 to check the validity of the predictions made by the models. The null hypothesis of the test is that p is the correct probability at the 95 percent level of test confidence. We have generated the non-rejection regions for all three different quantiles. The model backtesting 95% non-rejection test confidence is shown in Table 2.10. The results of the backtesting are shown in Tables 2.7, 2.8 and 2.9 with the rejections underlined.

2.6 Comparing the Two Periods of Experiments

The tables shown in Appendix A give us the the empirical results that we have generated using Approach 1 and Approach 2 for the date range that have been analyzed in Table 1 of [3]. As described in Section 2.3, we used the same settings for the models and the parameters, and we have obtained similar parameter estimations as in [3]. We have reproduced the tables that are in the same format of those shown in [3] for the ease of comparison. Table A.1 corresponds to Table 2 of [3]; Table A.2 corresponds to Table 3 of [3]; Table A.3 corresponds to Table 4 of [3]; Table A.4 corresponds to Table 5 of [3]; Table A.5 corresponds to Table 7 of [3]; Table A.6 corresponds to Table 8 of [3] with additional information disclosed for approach 1 and normal innovation of GARCH.

Now, let's look at Tables A.6, and 2.9 for the comparison of the predictive power during the two periods. For period one (i.e. 500 trading days prior to March 1^{st}, 2005), we should reject the MLE model for 3, 1 and 2 countries for 95%, 97.5% and 99% VaR predictions respectively; for period two (i.e. 350 trading days prior to July 18^{th}, 2008), we should reject the MLE model for 1, 2 and 1 countries for 95%, 97.5% and 99% VaR predictions respectively. This is under the change that we used moving window size equal to 2000. Therefore, we see a similar predictive power of the Pearson's Type IV model among those two periods. Another observation is that, for all of those rejections during period 1 shown in Table A.6, the model is rejected because of overestimating the VaR during period 1; however, for all of those rejections during period 2 shown in Table 2.9, the model is rejected because of underestimating the VaR. Nevertheless, the number of rejections of the models during the two periods are similar which indicates a similar predictive power.

Chapter 3

Conclusion

Superior performance in measuring market risk is on demanding with increasing occurrence of extreme events in recent years. Models that can better explain the left tail behavior of returns are required, since otherwise the reported VaR with thinner tail leads to an underestimation of risk. This paper has analyzed the application of newly proposed GARCH model with Pearson's Type IV distribution to measure VaR in order to account for a non-normal return distribution. Most existing GARCH models of the estimation of VaR capture time-varying volatility in return and the leptokurtosis in the distribution of returns. Literally we can say they ignore the role of skewness and kurtosis in the return distribution. Thus, such models are likely to provide poor estimations of VaR when high confidence levels are set. Empirical evidence approves this while more study on skewness and excess kurtosis discovers the important role they play on asset returns. In this respect, the high kurtosis value of Pearson's Type IV distribution is adopted. Applying the Pearson-GARCH procedure to the in-sample period of eight indices, the results show that at high confidence levels, it generates VaR forecasts that are more accurate than those generated by normal distribution. For the out samples period, in the case of high market volatility, the performances are pretty good at high confidence level and are generally good at lower confidence level.Our study shows the Pearson- GARCH model provides good tail estimates, and therefore more reliable VaR predictions in turbulent times. In other words, this approach dose reliably captures the risk of the extreme events. Furthermore, the size of rolling window used in estimation is important. In our case, window size equals to 2000 gives us the best performance of the Pearson-GARCH model. In both of the two periods that we have tested the model on, it shows similar power of VaR predictions.

Appendix A

Empirical Results of Replicating Bhattacharyya 2008

The following tables show the empirical results that have been generated using Approach 1 and Approach 2. Both of the approaches are using the same AR(1)-GARCH(1,1) settings mentioned in section 2.3. The date ranges of the data that have been analyzed are the same as the date ranges stated in Table 1 of [3]. Table A.1 corresponds to Table 2 of [3]; Table A.2 corresponds to Table 3 of [3]; Table A.3 corresponds to Table 4 of [3]; Table A.4 corresponds to Table 5 of [3]; Table A.5 corresponds to Table 7 of [3]; Table A.6 corresponds to Table 8 of [3] with additional information.

Table A.1: AR(1)-GARCH(1,1) estimated model parameters on return series used in [3].

Country	Mean Equation Coefficients		Variance Equation Coefficients					
	ρ		ω		α		β	
	N	P	N	P	N	P	N	P
Australtia	0.13194	0.09572	7.8E-06	2.4E-06	0.237	0.087	0.6793	0.8751
Hong Kong	0.12693	0.06928	8.8E-06	5.4E-06	0.1431	0.0874	0.8329	0.891
Japan	0.03657	0.00926	2.7E-06	1.5E-06	0.1326	0.0987	0.8661	0.9003
Korea	0.07791	0.05876	4.8E-06	3.5E-06	0.0799	0.0699	0.9156	0.9272
France	0.00137	-0.0077	1.4E-06	1.2E-06	0.0661	0.0629	0.9278	0.9318
Germany	-0.00731	-0.0242	2E-06	1.3E-06	0.0909	0.0847	0.9024	0.9121
UK	0.00437	-0.0048	9E-07	7.8E-07	0.077	0.0747	0.9162	0.9195
USA	0.06642	0.0335	1.2E-06	9.4E-07	0.0958	0.0822	0.8966	0.9112

N: Normal Innovation for AR-GARCH (Approach 1); P: Pearson's Type IV innovations for AR-GARCH (Approach 2). The coefficients of the mean and the volatility equations are all found to be significant.

Table A.2: Standardized residual series characteristics.

Country	Mean		SD		Skewness		Kurtosis	
	N	P	N	P	N	P	N	P
Australtia	0.0388	0.0423	1.0001	1.0441	-1.0415	-1.9898	13.745	34.172
Hong Kong	0.0275	0.0305	0.9996	1.0202	-0.8567	-1.0406	9.9857	11.983
Japan	0.0048	0.0061	0.9993	1.0124	-0.6086	-0.711	11.109	12.647
Korea	0.004	0.0041	0.9973	0.9975	-0.3427	-0.3585	5.3012	5.4018
France	0.0246	0.025	0.9995	1.0005	-0.2227	-0.2264	3.6009	3.609
Germany	0.0276	0.0284	0.9989	0.999	-0.2708	-0.289	3.5943	3.6707
UK	0.0205	0.0209	0.999	0.9993	-0.2444	-0.2481	3.434	3.4347
USA	0.0361	0.0379	0.9979	1.0009	-0.4792	-0.4992	4.7411	4.8492

N: Normal Innovation for AR-GARCH (Approach 1); P: Pearson's Type IV innovations for AR-GARCH (Approach 2). The kurtosis values of the standardized residuals clearly show that the residuals have a much thicker tail than that of a normal distribution.

Country	Lag 5		Lag 10		Lag 15	
	MM	MLE	MM	MLE	MM	MLE
Australtia	7.0000	12.1156	10.4937	15.4084	21.5422	27.6593
Hong Kong	14.1600	28.3219	18.8729	33.9399	25.4967	41.0269
Japan	0.7698	3.3626	10.9956	14.2687	16.4206	19.9810
Korea	2.9929	4.4244	4.1841	5.6621	10.8778	12.3991
France	7.5548	8.0707	12.2331	12.6896	17.0246	17.6879
Germany	2.7751	3.7869	7.0958	8.0121	15.9481	16.8966
UK	7.9229	8.7156	13.0790	13.9174	17.3068	18.1242
USA	6.6033	8.8989	10.7058	12.6492	19.9185	22.0288

Table A.3: Q-statistic for standardized residuals.

Table A.4: Pearson's type IV parameter estimates

Country	μ		ν		α		λ	
	MM	MLE	MM	MLE	MM	MLE	MM	MLE
Australtia	2.9183	4.3862	1.1361	1.1091	1.6151	2.371	0.5171	0.4353
Hong Kong	3.1033	3.1976	1.1511	0.1606	1.7265	1.8414	0.4999	0.1057
Japan	2.9339	3.6649	0.6584	0.564	1.6683	2.0693	0.2888	0.2255
Korea	3.9532	4.255	0.9028	0.6572	2.1837	2.3355	0.3378	0.2404
France	8.2681	8.7855	2.8084	3.0659	3.6105	3.7453	0.7222	0.7625
Germany	8.8081	8.9167	3.8791	4.2335	3.7062	3.7207	0.9482	1.0229
UK	11.341	11.318	5.5079	5.585	4.2828	4.2775	1.1611	1.1786
USA	4.7714	5.1001	2.0278	1.7507	2.465	2.6242	0.6987	0.5989

MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2).

Table A.5: VaR exceedances for returns for entire sample.

Country	95 percentile				97.5 percentile				99 percentile			
	N	Pearson		E	N	Pearson		E	N	Pearson		E
		MM	MLE			MM	MLE			MM	MLE	
Australtia	220	233	245	260.05	129	94	118	130	68	24	51	52.01
Hong Kong	185	187	224	224.6	111	75	110	112	57	29	46	44.92
Japan	261	270	271	260.25	153	113	129	130	75	25	39	52.05
Korea	91	92	92	93.75	51	41	43	46.9	26	14	15	18.75
France	134	130	132	126.25	72	63	63	63.1	36	25	25	25.25
Germany	136	122	123	126.05	74	58	58	63	32	21	22	25.21
UK	136	130	129	126.2	79	70	69	63.1	39	24	24	25.24
USA	122	123	127	125.85	76	58	62	62.9	43	22	24	25.17

N: AR-GARCH model with normal innovation; MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2). The expected number of violations is based on the actual number of observations. For example, for 99% VaR for Australia index, the expected number of violations is 0.01*5201 = 52.01.

Table A.6: VaR exceedances for returns for holdout-sample.

Country	Holdout sample VaR violations								
	95 percentile			97.5 percentile			99 percentile		
	N	Pearson		N	Pearson		N	Pearson	
		MM	MLE		MM	MLE		MM	MLE
Australia	3	5	7	1	1	1	0	0	0
Hong Kong	17	18	21	5	4	7	3	3	3
Japan	22	22	25	17	11	14	6	3	3
Korea	18	17	19	12	9	12	5	3	4
France	14	14	13	12	12	12	7	6	6
Germany	19	17	16	13	10	9	6	4	4
UK	19	18	17	9	8	8	5	4	3
USA	17	15	19	9	6	6	3	0	0

N: AR-GARCH model with normal innovation; MM: Method of moments (using Approach 1); MLE: Method of maximum likelihood (using Approach 2). The size of the holdout-sample is 500. Therefore, the expected number of violations are 25, 12.5 and 5 respectively. The underlined numbers of exceptions should be rejected at 95% level of test confidence. See Table 2.10 for model backtesting 95% non-rejection test confidence regions.

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