TOWARD AN INTERNAL VIEW OF MATHEMATICS:
A STORY OF DYNAMIC GROWTH IN A SECONDARY SCHOOL

by

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Abstract

This thesis explores views of the nature of mathematics, the nature of learning mathematics, and the nature of teaching mathematics held by a selection of teachers and students at New Venture Secondary School (NVSS), a new school in suburban Vancouver. A review of the literature provides a general perspective on these three areas. This view is related to the specific context of the mathematics program at NVSS based on interviews with students and teachers, questionnaire data, and the author's reflective journal entries.

The literature provides a description of an internal view of the nature of mathematics wherein mathematics is seen as being created by humans, and its truth being socially negotiated. This view implies that mathematics is subjective, influenced by culture and time, and fallible. Learning is seen as a process of active meaning construction whereby students require time to sort out ideas for themselves. Communication and interaction are vital because learning is viewed as a social phenomenon. Some implications for teaching mathematics are that teachers should work collaboratively with students, take their lead and attend to the affective domain.

The structure of the mathematics program at NVSS allows these implications to be addressed. It is based on self-paced mastery learning where students work both individually and together through a series of "learning guides". They do not work in isolation; interactions among students and between teachers and students are vital to the learning process.

The perspectives of teachers and students in this setting were gathered using qualitative methods. Transcripts from interviews and student responses on both questionnaires and course-end summary sheets were used to depict teacher and student views. The author's journal writing was critical both as the original impetus for research as well as a tool for an ongoing opportunity for reflection.

The study concludes that the data reveal a common vision shared amongst teachers and students, one that parallels many aspects of the internal view discussed in the literature.

Further conclusions include possible explanations for this shared vision, as well as an update of current practices and directions in mathematics at NVSS.

Dedication

This thesis is dedicated to our nini–I know it kept you waiting.

You have been a wonderful source of motivation for my writing.

Now we're looking forward to your arrival...

only you know how close together your birth and that of this work will be!

Acknowledgments

There are many people who helped in the development of my thesis. Firstly, without the staff and students whom I interviewed, my writing could not have existed. I thank them for their time and thoughtful responses. Other staff and students were a great source of support and ideas; I value all of their input. In fact, I would like to formally acknowledge the huge contribution that the on-going dialogue with people at NVSS has been to the development of this study.

Secondly, my supervising professors were an invaluable source of help; I could not have asked for better assistance and encouragement. I thank Dr. A. J. (Sandy) Dawson for all of the time he spent working with me. I always felt comfortable asking him for advice and his suggestions helped me greatly. I never knew editing could be such fun–thanks again for all of your hard work! Dr. Allan MacKinnon always seemed to help out right when I needed him; he encouraged me with my journal writing at just the right time. His positive approach with students has been an inspiration to me.

Thirdly, the encouragement and support from all of my family were very important to me; however, two people deserve special mention. My husband, Ramin, was–as always–my motivator and the person to whom I first turned for help. My sister-in-law, Mahtab, helped me more times than I can list. Everyone should be so lucky to have such a fantastic "personal assistant." I would not even have the pleasure of writing these acknowledgments without these two amazing people; thank-you so much!

Table of Contents

Chapter 1
Introduction: The Beginnings Of Research

As a means of introducing the reader to the research, I will describe how I arrived at my research topic. I am a mathematics teacher of six years and have been working at a non-traditional high school, referred to here as New Venture Secondary School (NVSS),[1] since its opening in 1992. I chose to work at this new school because I agreed with its approach to learning and its philosophy aligned with my world view.

The focus of my study is to explore views of the nature of mathematics, the nature of learning mathematics, and the nature of teaching mathematics in general, and at NVSS in particular. My involvement in this study resulted from my desire to learn more about these three areas and to investigate issues relevant to my teaching situation. I also wanted an opportunity to relate my reflective journal entries (which I had started writing at the same time as I had started teaching at NVSS) with the perceptions of my colleagues and students. As a relatively new teacher, I felt this study would help me to improve my teaching methods and aspects of the mathematics program at New Venture by enriching my understandings of mathematics, its learning and teaching. It is my hope that my findings will contribute to discussion of these issues in the community of mathematics educators, and provide a description of the unique teaching and learning environment at NVSS. Moreover, this thesis provides an account of one teacher's change in thinking through teacher and student discourse at NVSS as well as through private discourse by means of a journal.

The process of my research has been extremely evolutionary in nature—so much so that, at times, it has been quite disquieting. At some points, I wasn't certain where I was heading but through reading, talking with colleagues and supervisors, and recording and reviewing my own thoughts in journal form, themes and questions emerged, leading to the

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[1]New Venture is the pseudonym used to assure the anonymity of the school.

next steps. This chapter reveals that journal writing has been an integral part of this evolutionary process, particularly in the initial development of the research topics.

Journal Writing: A tool for concept identification

Journal writing has intermittently been a part of my teaching practice since I entered the pre-service Professional Development Program (PDP) at Simon Fraser University in 1988. One PDP assignment was to keep a reflective journal throughout the practica. This assignment was one among many, an extra thing to do which could be dropped when I began "teaching for real." However, it became the most useful assignment, forcing me to slow down and look at situations in new ways, and to focus on my own learning and development. In my first years of teaching, with a seemingly endless list of things to do, I discontinued my journal writing, but began again when I undertook the Master's program at SFU.

"What is it about journal writing that is so useful?" the reader may ask. I re-read journal entries to analyze my teaching and to reflect upon the ideas recorded. This reflection has improved my ability to deal with similar teaching situations as they arise. As Taylor, a teacher in Kilbourn's book, *Constructive Feedback*, states, "What this is doing for us is it is getting down on paper all the things that we think about; and we experience them again the next year with all this to look at" (1990, p. 45). By recording in journal form, I am forced to reflect on my experiences. I found this to be a powerful experience. For me, it added interest and relieved pressure when I treated teaching as a learning experience. "Every teacher can be a researcher, and every lesson can provide opportunities to find out more about how we can help pupils to learn" (Ernest, 1989, p. 174).

Teacher as researcher, that is what I have become, and journal writing has been a significant tool in that process. Support for that role was found in Joe Belanger's article in the *BC Teacher*:

> Teachers need to be at the center of educational research, not only because daily immersion in the practical problems of education provides a stream of research questions, but also because research is professionally liberating: it leads

> professionals to deeper understandings of why they do what they do, and it prompts them to question their assumptions. (1992, p. 6)

Merely knowing that I am engaged in journal writing puts me in a different mode of thought while working. I become more attentive to the actions of my students and myself. I am slower to jump in to give students answers, and more willing to allow them time to work ideas out for themselves. My conception of what mathematics is expands; thus, I feel less restricted in what I can discuss with the students and less concerned with getting the curriculum objectives taught in a quick and efficient manner. Rather, I place importance in discussing many "off-topic" items with students, allowing them to initiate discussion on subjects they see as important in their lives.

My teaching situation changed significantly early in my Master's program when an opportunity arose to teach at NVSS. The concept for this school included a building and curriculum design which was quite different from those of traditional high schools, and a philosophy with which I felt aligned. The reader is referred to Sanders' thesis "Introducing [New Venture Secondary School]: 'To Seek Challenge And To Experience Success'" for information regarding how the school concept came into being.

The opening of NVSS coincided with the resurrection of my journal writing as part of a Master's course. The experience of writing and reading the journal entries enriched my teaching in those opening months of the new school. However, despite the numerous benefits of keeping a journal, I again stopped using this tool once the particular course for which it was an assignment ended. Looking back, I realize that during the semester in which I discontinued journal writing, I lost some of the excitement and enjoyment of teaching. I "survived" each day, carrying out my tasks and responding to situations as they arose without taking the time to reflect on them. In this way, I was reacting to events around me. Writing allows me to step back and see myself as an integral part of the events in which I work. The nature of my teaching at New Venture was (and still is) constantly evolving and writing about it leads to a feeling of some control over that evolution.

I resumed journal writing once again as part of a subsequent course. In addition, I reviewed the document in search of common themes and discovered two: helping students sort out meaning for themselves and the importance of off-topic ideas in my teaching of mathematics. These themes are evidenced in the following two journal entries:

> I found it very hard to step back. I had to bite my tongue several times and, on some occasions, I did not succeed in shutting myself up. I wanted to help the students, guiding them toward a solution. I found it nearly impossible to leave them alone and let them think, especially in the beginning if they had trouble getting started. I wanted to tell them exactly how to begin so that they would get going. Sometimes it seemed that they were doing nothing and, at those times, I longed to jump in!...They resolved it without my help and I think that perhaps backing off actually helped them, by allowing them to think it out....I have to allow them to do it themselves more often. It's difficult for me to think that the assistance I offer may not actually help my students but this experience has certainly made me think carefully about that very topic. As Mary points out on page 125 of Borasi's book[2], "I learned a lot more, because I tend to learn a lot faster when I do it wrong first...if I was just handed on a silver platter exactly how to do it I probably wouldn't have learned a great lot." (Journal, 92/11/19)

> [Before working at NVSS, I was often] frustrated with students getting "off topic" during "my" lesson. I can hear myself saying, "We have to concentrate on factoring today", without giving the students a chance to pursue their thoughts. By seeing so many connections in mathematics recently...I know that a seemingly off-topic question in mathematics can quite possibly lead to some very interesting connections with the topic at hand. However...I had often opted for the "safe" pre-planned route. Now, [at NVSS] I have most often pursued the students' questions. It doesn't seem like such a risk as I am usually only working with only one or two students at a time and nothing is really off-topic for it is the students themselves who provide the topic. I find that I really enjoy discussing practically anything with the students and I myself am seeing many more connections through these discussions. (Journal, 92/11/05)

Concepts from the Journal

I did not begin with these two concepts, helping students sort our meaning for themselves (constructivism) and the importance of off-topic ideas in teaching mathematics (human nature of mathematics), in mind and then search out situations which illustrated them.

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[2]Raffaella Borasi's *Learning Mathematics Through Inquiry* was a book used in my first Master's course in which journal writing was assigned. In it, Borasi describes a "mini" mathematics course on definitions which she teaches using a constructivist approach. Mary is one of the two students in this course.

Nor did I discover the concepts after writing about simply any experience that popped into my mind. Kilbourn's views on concepts have value here; that is, "Some concepts may emerge from the data" but sometimes, "concepts are brought to the situation and constitute the lenses through which the situation is seen" (1990, p. 105). Both constructivism and the human nature of mathematics were discussed extensively in the Master's course work so the concepts were certainly part of my thinking from the start. This may have increased the significance of the experiences in my mind and thus pushed me to write about them. Nevertheless, they did occur. I experienced the situations about which I wrote and, through my writings, the concepts became more clear to me. In fact, it was not until I read my journal as a whole in search of common themes that I became fully aware of their existence in my writing. It actually came as a surprise to me that I could connect each of my entries to either constructivism or the human nature of mathematics.

Constructivism, as characterized by Ernest, is "the view that children construct their own knowledge of mathematics over a period of time in their own, unique ways, building on their pre-existing knowledge" (1989, p. 151). John Mason's simple observation was extremely helpful: "I can't *do* the learning for my students" (1989, p. 155). This truth is a clear example of constructivist thinking applied to teaching. My introduction to this theory of knowledge coincided with my introduction to teaching at New Venture Secondary. It was the simultaneous occurrence of these two events that illustrated the significance of constructivism for me as a mathematics teacher.

Jaworski echoes Mason's writing. At SFU in 1992, Jaworski presented dichotomous views of mathematical knowledge: sophisticated realism, where knowledge is viewed as absolute and "out there"; and the constructivist view that (mathematical) knowledge is individually constructed—not passively received, but actively built up by the individual. I began to see the intimate link between one's views of knowledge, and learning and teaching (these three concepts are explored in more detail in later chapters as they developed into the topics around which my research centred.) This link highlighted the importance of having a philosophical base for my research of mathematics teaching and learning. I began to look at how my teaching was encouraging and challenging New Venture students while I listened to and interacted with them. Instead of feeling that I am the conveyor of knowledge, I see the students as the constructors of their knowledge.

The question of the nature of mathematics was one with which I initially struggled while in the Professional Development Program. My impending job as a mathematics teacher necessitated thoughtful reflection upon what is important to teach in mathematics and why. An article written by David Burghes would have helped me at that time. Burghes' position is that

> it really does not matter what mathematical content we teach since it can all be done by computers now—it is the process skills that are important. These are skills such as: understanding problems, implementing strategies for solution, interpreting data, transferring skills to new problems, tackling unseen problems, critical judgment, and working positively in a group. It is this type of criterion which is far more important to later life than the usual stress on yet more and more abstract content. (1989, p. 87)

I would add some affective goals to Burghes' list of process skills, such as a positive attitude toward and confidence in doing mathematics. I see this affective side of mathematics as being extremely important to the students' ability to do mathematics. As Novak states, "We are moving towards research that shows the important interplay between thinking and feeling" (1986, p. 183).

I initially clarified my own position on this issue when I wrote my philosophy statement during the teacher education program. In a vein similar to Burghes, I stated, "The facts that students are often required to memorize will, in all likelihood, be forgotten very quickly; however, interpersonal, communication, thinking, reasoning, and problem-solving skills will be used throughout their life." I did not, however, go as far as Burghes does by saying that it does not matter what content we teach. In fact, I continued by stating, "That is not to say that the subjects themselves are not worthwhile; knowledge is extremely important and thinking skills would be useless without having content to think about."

By reflecting upon the question of "What does it mean to say something is mathematical?", my position has become closer to that of Burghes. Initially, I did not really question the content of the intended learning outcomes in our provincial curriculum as I felt that it had obviously been determined by knowledgeable people with far more experience than I. Thus, I thought, it must be what our students "need to know". As I found myself rushing the students along in an attempt to "cover" everything, I began questioning what students were actually getting out of the course: did they really need to know it all? If "all" means all of the curriculum objectives, students will be missing much of what mathematics is by "knowing it all."

This was an exciting and confusing time. I came to realize how far-reaching mathematics is and what a limited view of mathematics students may have if we continue to proceed step-by-step through the curriculum. I still do not go so far as to say that the curriculum is totally unnecessary or that the content is valueless. However, I recognize that there are many amazing mathematical experiences that my students will miss if I adhere strictly to the limiting examples provided in the curriculum and not allow room for student-initiated discussion.

The two themes of "humanizing" mathematics and teaching mathematics by helping students construct their own meanings are present in my journal entries. Paul Ernest merges these two ideas when he writes, "If mathematics…is to empower learners to become active and confident problem-solvers, they need to experience a *human* mathematics *which they can make their own*" (italics added, 1989, p. 228). As such, the two themes of my journal (constructivism and the nature of mathematics) are themselves connected. Students will be more likely to forge their own connections if they are given the opportunity to explore their ideas. These ideas often lead into interesting discussions on the human aspect of mathematics and its nature; that is, why is this concept important to learn? Who came up with these ideas? When I began to see mathematics as a richer subject—more as a way to view the world rather than a series of theories and rules—I also began to realize that students need to come up with their own views. As they construct their own meanings, they begin to see the human nature of mathematics for themselves.

Writing, reading, and reviewing journal entries began merely as course work yet eventually formed the basis of my research interest. This tool continued to be helpful throughout the study by clarifying my thinking and providing data as to the development of ideas and teaching practices at NVSS. Also, the themes identified in my journal led to the questions I pursued at NVSS. That setting, which became my research site, is described in the next chapter.

Chapter 2

The Defining Characteristics of New Venture High

This chapter describes the setting in which the research was conducted. As background for the reader, a description is provided of the educational model upon which the mathematics program at NVSS is based. As well, the evolution of the first years of the mathematics program is described.

The School's Mission

I have been teaching at New Venture Secondary School since it opened in September of 1992. I was also involved in pre-planning sessions during the year preceding the school's opening. Though the description of the setting is from my unique perspective, there are other sources which corroborate my depiction: papers and, in one case, a Master's thesis written by a colleague of mine. Further sources include a pamphlet produced to explain the school program to students and parents, an external team formative evaluation report written about the school, and a School Design Statement written by the school principal and school district officials. These items are listed in the bibliography.

The school mission statement, "All students will seek challenge and experience success" reflects the belief that all students can be successful if allowed to work at a rate that suits them. In fact, stated in a Guide For Parents pamphlet is the following key operating principle: "People learn in a variety of ways and at different rates" (School District No. XX, 1992, p. 4).[1] The remaining six principles of learning for New Venture are listed in the School Design Statement with the following introduction:

In order to respond to both individual needs and societal expectations, the school embraces these principles to guide educational practice:

[1] The school district number is replaced by XX to assure anonymity.

- Whether one learns is more important than when.
- Learning requires the active participation of the learner.
- Learning is both an individual and a social process.
- Learning takes place in safe, healthy, orderly environments.
- All learners can be successful learners.
- Learning is a lifelong process. (School District No. XX, 1993, p. 5)

Although teaching staff were not involved in the development of these principles, many of us worked cooperatively to develop the school's mission statement, as well as the Student Exit Outcomes which are also listed in the School Design Statement as follows:

Students will be:

- High Quality Producers,
- Collaborative Workers,
- Global Citizens,
- Socially Responsible Contributors,
- Self-Actualized Individuals,
- Communicative Persons, and
- Creative Contributors. (School District No. XX, 1993, p. 6)

Each of these outcomes is followed by descriptors which were also written by the staff during the early planning days.

Although at this stage in the school's history—three years after the opening—not all staff are satisfied with the Student Exit Outcomes, the staff as a whole seem to have a clear, common philosophical base and vision. My basis for this assumption arises not only from daily contact with colleagues, but also from professional development day discussions where we frequently revisit our philosophical roots and rediscover that we still have a common vision. These views were corroborated by the Formative Evaluation Team which visited our school during the week of January 30th through to February 4th, 1994. In their summary statement, they wrote:

> The development of this far reaching model has been made possible only by the strong vision, support, courage, and leadership of teachers, school based administrators, District staff and the Board of School Trustees....There is a cohesiveness among the teaching staff with a sense of vision of how learning is meant to be. (Hanos, Harper, Morrison, Taylor, and Walker, 1994, pp. 35-36)

The School's Learning Model: Teacher and Student Roles

The school's philosophy is put into practice through its model based on self-paced and mastery learning. Students establish their own timetable by setting long- and short-term goals in consultation with a teacher advisor (TA). Each teacher acts as a TA, monitoring the progress of and advocating for approximately twenty students in grades 8 through 12. The teacher also acts as a subject teacher, responsible for working with and evaluating a number of students with a load roughly equivalent to a typical "full course load" in any other school. The subject teacher role is very different from that of a subject teacher in a traditional school. Students do not come to the teacher as a group at and for a fixed time, but, rather, as individuals learning concepts at their own rate. To progress, they require a minimum 70% grade. Teachers' interactions with students are not limited to just those students for whom the teacher submits marks. A teacher may interact with any student who happens to be working in the physical area the teacher is overseeing. Students and staff identify the teacher responsible for evaluating a particular student by calling him/her the student's *marker*.

The daily life of a student at NVSS is significantly different from that of fellow students across the province. A colleague describes the students' daily activities:

> At NVSS, students are responsible for their own programs in that they move through individualized modules at their own pace (as fast or as slow as they can handle). What this means for the students is that they spend some time in the morning planning their daily activities (with a Teacher Advisor) and then they are on their own to move throughout the school according to this prearranged plan. There are no classes per se, but teachers are available in open working areas (divided by curricula) to interact with students on topics related to that particular subject area. (Banks, 1995a, p. 1)

It is towards developing this interaction with students that the teachers in the Mathematics/Science Department have been working as a group for the past three years. At a recent year-end department retreat (June 22-26, 1995), these teachers agreed that they have developed a much more common approach towards this teacher-student interaction than they held in the first two years of the school's operation. In fact, at the previous year's retreat, teachers' behaviours and the way in which they interact with students on a daily basis was a main topic of discussion, resulting in some significant changes in the following year. As one of the teachers reflects in a recent write-up about that retreat:

> A key element of the department's discussion last spring centered around how staff can best serve the students under their supervision in the science lab or math work areas. It was identified at that time, that some of us were still heavily "marker oriented" and tended to function as an "over the counter sales person", addressing mainly those students (usually ones on your own marking lists) that came to you for help. The suggestion was that instead of being passive and waiting for students to approach us, we should all be employing "door to door salesmanship", actively initiating contact and interaction with students, regardless of whether they were on our marking lists or not.
>
> When all staff bought into this principle last September, it had a marked, positive impact on the daily operations in our general work areas. With staff taking the initiative to open dialogue with each of the students using the facility, students in our areas became much more focused and less reluctant to ask for help when it was required. As a result of these changes, extremely exciting and valuable staff/student interactions became common place and often they involved teachers who did not have "marking" responsibility for the student. (Walsh, 1995, p. 1)

The Curriculum

In the NVSS model, teachers have had to redefine their role. For both teachers and students, there have been significant adjustments to their daily lives, as well as to perceptions of their respective roles. This adjustment, both to our mind-sets and to the practical elements of our daily lives, has not come without a struggle. One of the main sources of the struggle for both the teachers and the students was the way in which the curriculum is delivered at NVSS.

The School Design Statement explains, "the curriculum is organized to permit individual-pacing and independent learning. Each course is divided into units and 'Learning Guides.' Five learning guides are written for each unit" (School District No. XX, 1993, p. 7). In practice, this format is more or less followed. Courses are, for the most part, broken down into twenty learning guides which are usually grouped into units. However, teachers revising the learning guides have sometimes strayed from this strict organizational scheme to break the course into content topics which make sense to them and may not necessarily contain five learning guides per unit.

> It is estimated that each course will require 100 hours to complete. Each unit requires approximately twenty-five hours and each learning guide about five hours. These time allotments are simply estimates to correspond with the time requirements established by the Ministry of Education. In reality, students work at their own pace and thus may move more quickly or slowly depending on individual needs and characteristics. (School District No. XX, 1993, p. 7)

Despite this disclaimer, the time allotments proved unrealistic in many cases and as the current school principal wrote recently, "it became apparent early in our first year that there was too much work in the modules for students to experience success easily" (Bell, 1995, p. 1). He explained this phenomenon as follows:

> It is important to remember that the original guides were written by teachers who were trying to envision how New Venture would work, yet at the same time writing it from the context of a classroom teacher's experience of working within a traditional classroom...The other concern associated with the original guides was that the contracted writers did not want to be accused of writing "watered-down" learning guides. Each writer took great pride in ensuring that all of their best activities were included within the guides. (Bell, 1995, p. 1)

Teacher-student interactions

The problem of lengthy learning guides was apparent in the mathematics program from the beginning, and attempts were made to make changes prior to the school opening. Many of the learning guides called for mandatory topic-specific presentations by teachers as well as optional remedial tutorials. Often more than one of these presentation "lessons" were listed per guide. During preliminary discussions, the mathematics teachers (the author included) could not envision how they could possibly manage their time since, between them, they were expected to provide presentations for a multitude of topics in each course. In addition, their presence was being continuously required to assist students with individual or group difficulties. The mathematics teachers thus made the decision to cancel all presentations, but they did not have time to edit the guides; hence, the presentation lessons were still listed.

Throughout the three years, the mathematics teachers maintained their controversial decision and gradually established it in print as well. In fact, they have become more strongly opposed to the existence of these presentation lessons for reasons other than simply time-management impossibilities. Despite complaints from some students and parents, they have not instituted these lecture style teacher-directed presentations on philosophical grounds. They do not want to "stand and deliver." Instead, they opted to use their teacher time to assist students on an individual or small group basis, often working as peers alongside the students.

The mathematics teachers also encouraged students in the same course to meet and work together in a variety of ways, rather than simply working in the general multi-graded open mathematics area. Initially, the teachers attempted a mandatory "discussion session" per guide at the grade ten level, since this notion was written into those learning guides. The mathematics teachers did not want students to be delayed for any reason, and so did not want students to have to wait for their particular learning guide's session. The solution to this dilemma was to offer open sessions to any student at any part of the course. The discussion session group could potentially be working on a variety of topics. An early journal entry reveals my difficulty in adjusting to a new role in this new setting:

> In the first week, when all students were on the same learning guide, I fell comfortably into my routine of "giving" a lesson. Although I told the students that I would follow their lead and make their questions my top concern, I ended up doing a mini-lecture and then posing questions to check for understanding. I fell back into the role that I was used to and they remained in their "student as receiver" roles. It seemed that we both stayed with what was familiar.
>
> About one week later, however, this approach was no longer possible. The nature of the system forced us to make changes as students began to spread out amongst several learning guides. If we as a department were going to stick with our original goal of offering discussion sessions at set times, but not on set topics, then I would just have to change. I was not very comfortable with this change and was quite nervous as discussion session time approached every two days. Fortunately, I and the students improved; I now feel quite comfortable with the notion that I have no idea what will be discussed ahead of time at our sessions.
>
> This does not mean however, that I have applied Raffaella Borasi's style of "Well, maybe, let's try it." I still find myself saying, "No you're wrong. Yes, you're right," quite often. Just because I am now directing this to smaller groups of students does not make me any less guilty of it. The students are helping me to change this style though. Because I do not work with all of the

> students in a session as one group, and ask the ones with whom I am not working to try to sort out some of their questions on their own, the students do get a fair chance to "just try it." Recently I have found that by the time I get to another group, they are intent upon working out a question and I almost feel that I am intruding by asking them if I can help.
>
> Initially I felt a bit hurt when they declined my offer of assistance. I realized that I could not hope my students would become independent learners and yet, still want them to depend on me for help. Nonetheless, I felt a bit rejected. My colleagues helped me by pointing out that what had happened should be seen as a very positive change. It was certainly true that they had changed and I was often amazed at their ability to help themselves and each other to learn. They were applying Mary's prescription for learning as given on page 125 of Raffaella Borasi's *Learning Mathematics Through Inquiry*; namely, "we would try to figure it out and we would make errors first. Which is a regular way of learning things: you make an error and you try to correct it and you just work it through." (Journal, 92/10/10)

Despite strong belief in the beneficial nature of the discussion sessions, they were abandoned early in the first year due to administrative difficulties. It was difficult to meet the need to have students work and discuss in larger group settings; nonetheless, it remains a pressing need in my mind and in the minds of the interviewees. As one teacher looks to the future, he says,

> we're going to tap into that kind of cooperative thing a little bit...And I think that will be a benefit for everybody...I think that we'll be able to use some of the cooperative learning styles that are out there today. So I think we're going to have to do some more research on that and make sure that we've got a good tutorial system set up so that it will be meaningful for the kids. (Scott)

This need for students to work in groups together is echoed by another interviewed teacher,

> We also have to work on developing more group situations where there is a *bit more* sense of class...we'll find out...whether or not it'll have any effect on building groups of learners that feel comfortable with each other and, can somehow become a reliable group, meaning they'll rely on each other a little bit more. (Perveen)

Other ways that staff attempted to get students together in larger group settings include opening a room for senior math 11 and 12 students only, where a teacher would be present at set times during the day (this room is an ongoing project and will be expanded in the fall of 1995), offering drop-in sessions for teacher-perceived or student-requested problematic aspects of the course, and introducing "Fun Fridays" where games were offered to interested students. In addition to these activities, plans to introduce videos, guest speakers, and hands-on sessions in constructing polyhedrons, for example, are under development. There is nothing mandatory about these activities; students decide whether to attend or not. Teachers felt that if students found them beneficial and worthy of their attendance, then they would come and the word would spread. As one of the teacher interviewees explained:

> We'll know after the first few...whether we're doing it right or not because they'll want to come back and bring their friends if we're doing it right. If we're not doing it right, then it will die a natural death and so it should and we'll have to look at something else. I think that that's one of the neat things that we've been able to do is to let things die. That they aren't so sacred that you can't let them go. And again, that's been one of the *neat* experiences and I've learned a lot about letting things die or allowing my view to be challenged and changed. (Scott)

Learning Guides (LG's)

Students have influenced the development of the LG's. Their complaints regarding the length of time necessary to complete every assignment in the guide (and the fact that the actual time required seemed to consistently exceed the suggested time limits), indicated that the length of the learning guides was still a problem. Although teachers had omitted the presentation lessons prior to the opening of the school, further adjustments to the LG's were required.

Students' time was further drained in constantly having their work checked by a teacher. For example, the Math 9 learning guides stated:

> This assignment must be done to the satisfaction of your instructor before you go on to the next activity. Once it is completed, have your instructor check your work and sign the checklist at the end of this Learning Guide. (Secondary School Consortium, 1992, p. 3)

Teachers' immediate reaction was to eliminate all need for the students to ask them to check their work by having students mark all of their own work, either from the back of the text, or from hastily assembled answer binders for worksheets and quizzes. All quizzes were, within a few weeks of marking and teacher monitoring, left for the students' self-assessment, with the learning guide mark being determined solely by one end-of-guide test.

It took teachers somewhat longer to allow students the freedom to choose how many questions they felt they needed to complete in order to master a concept. However, within the first month, it became apparent that the reality of a student working through a learning guide was completely different from the instructions provided in the guide. A mathematics learning guide thus quickly evolved into something different for each student. Students had the option of challenging a guide by writing the end-of-guide test upon completion of a review assignment marked by the student. Other options were endless in that, depending upon the topic and the student, a variety of numbers and levels of questions could be attempted, ranging from the entire guide to a question or two from each assignment in addition to the review assignment. The decision was left to the student with guidance from a teacher, if requested, or upon unsuccessful completion of a test.

After the first year, *test slips* were instituted which had to be signed by a math teacher, preferably by the student's marker. The teacher would confer with the student as to what had been done in the guide, areas of difficulty, etc., before signing the slip. In this way, teachers were better able to monitor and offer assistance before the test was written. However, the majority of decisions were still very much in the students' control and the learning guide remained just a guide to be altered by each individual student according to his/her own needs. Finally, at the end of the third year, these changes are now written into many of the guides. As an example, at the front of the newly revised Math 9 guides it reads: "Select the activities in this learning guide which match the expectations you need to practice. Please contact a teacher if you aren't sure which activities to select" (NVSS, 1995, p. 1).

The activities are thus selected based on the expectations or outcomes of the guide. They are listed at the front of each guide, prefaced with "Can you do the following?" (NVSS, 1995, p. 1), or "on the completion of this learning guide you will be able to..." (NVSS, 1994, p. 1). In mathematics, as in other courses:

> the learning outcomes for each course are derived from the existing provincial curriculum which has been determined by subject specialists. However, this fact does not ensure that the outcomes successfully define mastery of a topic, but it does ensure that learning outcomes for individual courses are at least as complete as those used in local traditional schools. (Sanders, 1993, p. 5)

Reflective Learning

The changes made to streamline the learning guides in mathematics addressed how much work the students did, but not the manner in which they learned. As Banks, a colleague, wrote in a 1995 paper:

> I realize that I need to change the structure of these learning guides if I am going to implement some dynamic ideas. A typical math learning guide shows students the appropriate sections in the text to read and then assigns a series of questions followed by a test. If students are having problems, they are encouraged to seek help from the teacher. This is probably not that much different than most regular classrooms with a teacher's lecture replacing some of the student's reading of the text. Neither of these methods, in themselves, force students to deal with their preconceptions and the conflict that may arise when presented with these new concepts. I must determine a way of incorporating a reflective process into my learning guides. (1995b, p. 3)

I also felt a strong need to include a formal reflective process for students as evidenced by the following journal entry:

> I enjoyed reading the final chapter of Borasi's book, especially the section on student reflection. I agree with her position on the positive effects of providing opportunities for students to reflect on their learning. From my own experience in this class, I realize that reflecting on mathematical activities is difficult at first, but very worthwhile. It has provided much insight for me; why not pass that opportunity on to my students? I thus want to incorporate this element into the math programs at New Venture, but I feel it must be done carefully as I further agree with Borasi's comment regarding the importance of providing structure to help students with their reflections. (Journal, 92/12/06)

I continued with this line of thought while working at summer school:

> I have yet to act upon engaging students in a reflective process. I feel it is an important goal, mainly because I myself have positive experiences with writing my reflective journal, but also because I have often seen it recommended in print. For example, in an article by Cobb and Steffe, who quote von Glasersfeld on this topic, "for the constructivist teacher, the key is to help children hold their own mathematical activity at a distance and take it as its own object. This is the crucial aspect of reflection" (1983, p. 86). The only way I see for reflection to become a formal part of my students' learning at New Venture is for me to write that expectation into the learning guides. I need to provide a structure for them to engage in reflection about what they are learning and how they are learning on an ongoing basis. If it is a part of the guide for which students get credit then it will at least promote discussion on this topic. I also think reading student reflections can provide a tool for evaluation, not only of student progress, but also of the learning process at New Venture. (Journal, 93/07/09)

Banks offers some practical suggestions as to how to incorporate student reflection into the learning guides. He writes,

> students could be asked to brainstorm with a partner (keeping a written account) about different conceptions before they proceed with the material....The learning guide could key on a question that is habitually problematic for the students and have them do a detailed analysis of their own thinking....I often do this now in an informal way by having the student describe their thought processes as they worked through a problem. I really like, however, the intense reflection that actually writing out their thoughts requires....By reflecting on how we conceptualize something, we are forced to deal with the conflicts that many of our ideas hold. (1995b, pp. 3-4)

Further Elements to be Incorporated

Peer interaction is another element that Banks feels is informally present at NVSS, but not written into the guides.

> I have been working at having students work with other students...and the feedback has been extremely positive. By articulating how they solved a problem, students claim they are able to come to a better understanding of the material....The benefits are two-fold as the person they helped receives an explanation in a math language that is probably more similar to their own than it is to mine. When the students are unable to come to a consensus, they have grappled with the question and are in a much better position to understand the nuances of their own misconceptions. I can then enter the discussion and ask questions that have them reflect back on some of their own inconsistencies. I really believe that this type of student discourse is critical if we are going to enable students to reach a higher level of understanding. (1995b, p. 4)

Teachers also indicated the need to incorporate directly into the guides positive practices that they would like to encourage for all students, but which currently take place only on an informal basis. A teacher interviewee, in discussing the idea of encouraging students to get outside of the textbook world of math and see where people use it in their lives says, "we've got to put it into the learning guides. If it isn't in there, then the only way it happens is haphazardly" (Scott, p. 11).

A final important aspect about learning guides which has become apparent is that they are never finished. As Sanders stated in his thesis, "[NVSS] teachers view curriculum writing as one of the dimensions of the program which will have to be worked on continuously, in terms of staff training and re-writing of learning guides to improve effectiveness" (1993, p. 6).

Assessment Methods

One aspect of the NVSS mathematics program upon which attention has recently been focused is assessment; it was the major topic of the 1994 Mathematics/Science Department retreat. Authentic assessment is a school-wide concern. One professional development day was devoted to the topic during the 1994/95 school year and such days are in the planning stages for the future. Bell writes from a school-wide perspective:

> It became apparent, therefore, early in our first year that assessment was going to be the key problem we would face in trying to make our students successful. To this end we spent considerable time in the first and second years of operation in revising and editing the guides....These steps went a long way to reducing the work load of the students and teacher-markers but they still were not addressing the need to move to a criterion based or authentic assessment model....We are now making inroads in this direction in this, our third year of operation....Departments are looking at how activities can take on more of a demonstration aspect, and, in fact, how certain activities can be cross-referenced for credit in more than one course....Students are now finding ways to approach their teacher-markers and propose alternative activities that might be done for course credit....The more students are seeing these types of activities, the more they are creatively attacking their course work. (1995, p. 2)

As for the future, Bell states:

> We have committed the first two days of our Professional Days to a two day retreat environment in late September, 1995...to examine authentic assessment....This will set the stage for the rest of our Professional Development throughout the 95-96 school year. (1995, pp. 3-4)

In terms of mathematics assessment, once again it has been an evolutionary process, the beginnings of which are documented in Sanders' thesis:

> For the most part, the evaluation procedures used at [NVSS] are the same as those used in local traditional schools. However there is a recognition by [NVSS] teachers that they need to identify other ways for students to demonstrate their knowledge and understanding. (1993, p. 5)

The author's journal documents some of the evolution of the development of assessment tools during the first year of operation:

> [There were no assessment tools included with the original learning guides so we implemented a "quick fix" solution to our dilemma.]...I and others simply altered tests we had written in previous years to match the learning objectives for each guide. We all agree that we do not want to use only end-of-guide tests to assess student progress and we have come up with some ideas about which I feel good....[These ideas have quite often arisen as a reaction to the students' needs. For example,] one student raced through the learning guides in his

> course and the marker did not have enough time to prepare each test for him....[The solution, reached through discussions between the marker and student, was a] "final exam" as a set of "tasks," one for each of the main topics included in the course....The tasks may be done over an extended period of time...[and] students are allowed to get help from various sources as long as they are all referenced. [The final stage of this assessment is a discussion of the student's product with his marker which, in itself, I feel is a good assessment procedure.] (Journal, 92/11/11)

Later in this first school year, the teachers became more comfortable with the notion of alternatives to end-of-guide tests for student assessment. For example, the librarian and I collaboratively designed a geometry poster project as a replacement to the cumulative exam called for in the learning guide. As a partial solution to frustration with burgeoning marking loads, the mathematics/science department head suggested using oral tests, with which I later experimented.

Assessment in a Constructivist Environment

The above assessment tools have many advantages over the end-of-guide test. The discussions after the "final task exam" and during the oral test provide both the student and teacher with an insight into the student's mathematical abilities. They are a useful learning experience for both teacher and student. During the poster project and the task exam, students are encouraged to collaborate on their work and thus gain insight from each other. They are also encouraged to use the library where they may discover that there are many sources of mathematics writing accessible to them other than their texts. Students are not constrained by the parameters of the test, but, rather, can use their creativity to pursue their own ideas. Even those students who were opposed to any assessment other than a test engaged in some very worthwhile discussions with teachers about the purposes of evaluation and were then encouraged to suggest their own alternative to an end-of-guide test. These ideas are consistent with Jonassen's comments about assessment in a constructivist environment:

> Evaluating how learners go about constructing knowledge is more important, from a constructivist perspective, than the resulting product. This suggests that effective assessment should be integrated into instruction, that is, become a part of the instructional process. So, as learners are acquiring knowledge, evaluation guidelines should be available, so that both the student and the

> teachers may know how the student is progressing. The metacognitive awareness of learning that would result from this process should improve learning and, of course, the product as well. (1991, p. 30)

The second year saw further changes in our assessment ideas. Several journal entries during that second year attest to the changing nature of the assessment tools as well as to the importance of discussing ideas with colleagues.

> My teaching partner and I are car-pooling and we often use the opportunity to discuss new possibilities at NVSS, as well as to debrief about some new idea we've tried. Lately we've been discussing alternative assessment tools. A recent staff committee meeting in which we discussed ways to improve student progress was one of the things that prompted us to make changes in our assessment. We were thinking about it quite a lot prior to this, but now we've made a commitment to do something concrete. We even plan to apply for a grant to get release time to work on our ideas. (93/11/15)

Forms of constructivist assessment

During the second year, we developed other assessment tools. One was student-generated step-by-step workings of assigned questions accompanied by thorough descriptions of their way of thinking so that other students could learn from their work. An extension of this assessment is for students to make "how-to" videos showing their solutions. Beyond using questions provided by the text, students can also generate their own and provide solutions to be used by other students as quizzes or study notes.

Another way that students could simultaneously be assessed and help others was through peer tutoring. The idea of using peer tutoring as an assessment tool went through various stages of development as evidenced by the author's journal entries:

> Peer tutoring has been occurring naturally since NVSS opened and students are becoming more and more proficient at it and accepting of it [but it originally did not "count for marks"]....This year when thinking about ways to improve the rate of student progress, one staff member suggested that students get credit for peer tutoring....We began with two grade nine students who had finished the course and had prepared for the final exam but had not yet written it....[They were both hesitant initially, but] by the end of their helping time, both girls' confidence levels were much higher. One said, "Hey I even helped a grade 10 student!"...It was wonderful for us to see these girls recognized that the concepts they had been working on stretched across other courses [and to learn further mathematics concepts themselves]....We were so excited to see this learning aspect as merely writing the final exam does not provide this benefit. In fact, this assessment tool touched on many affective goals that a final exam

> never can reach such as improved communication skills, learning to help others, bolstered confidence, feeling good about themselves and is, as one staff member said, "what this school is all about." (Journal, 92/11/16)

After the initial excitement of using peer tutoring as a type of assessment, steps were taken to formalize it. A peer tutoring course summary sheet was developed which students must complete to get credit.

> The students use the form to keep a record of their time spent tutoring as well as to journalize their activities. In writing the journal, students are expected to reflect on their experience and to comment on how their peer tutoring related to the school exit outcomes. They also have a place to tell us how they would improve the math course and if they enjoyed the peer tutoring portion. (Journal, 94/01/22)

After reviewing several summary sheets teachers felt confident of their importance, but also felt that improvements were required. "We want to restructure the form so that it is more clear and provides more help since students find it difficult at first to relate what they are doing in math to these overall general goals [school exit outcomes]" (Journal, 94/02/18).

Another assessment tool arose from a change in the previously mentioned student-generated step-by-step workings of assigned questions.

> We now tell students that any assessment is simply a way to ensure that they have mastered the learning guide expectations. These expectations are printed at the beginning of the LG so that instead of us assigning questions to the students, we have them write out each expectation as well as some sample questions to illustrate their understanding. (Journal, 94/01/06)

There were some unexpected benefits to this assessment method. Students became aware that teachers develop assessment tools directly from the learning guide expectations. Further, the expectations (taken from the BC Curriculum Guide) exposed students to, and allowed them to practice using, formal mathematical language.

A grant for release time to collaboratively develop a common approach to assessment provided a boost to the development of alternative assessment tools. A journal entry reveals the author's goals for student assessment:

> I feel that we are aiming for a well-rounded student who communicates effectively, both in written and verbal format, helps others, approaches new problem situations in a positive and effective manner, seeks assistance from a variety of sources when necessary, has a good sense of what he/she knows,

> and performs successfully both under traditional test situations and in interview settings. (Journal, 94/01/16)

The grant was helpful in providing release time for the two full-time mathematics teachers (the author included) to work together to further develop and implement a variety of assessment tools. In the end, the release time proved to be invaluable. In fact, more was accomplished than initially planned, as evidenced in the following excerpt of the final report to the committee that had granted the release time:

> Although our main objective was to develop alternative assessment tools, we found that these new assessment techniques impacted on our entire mathematics program. We began to look at our program as a whole by examining carefully the secondary mathematics curriculum (as this is what we in fact assess)....We used what we had learned from our readings and our own experience to develop alternative assessment tools to be incorporated at any grade level. (Mehrassa and Gardiner, 1994, p. 1)

This work led to a stronger vision of the program—of individual mathematics courses as being part of a coherent whole—and a desire to promote that vision amongst students. The fact that the assessment practices can be used at any level and provide options for all students is one important component in viewing the program as a whole. Another vital step in this vision is the component of peer tutoring and students working with each other at all grade levels, both formally and informally.

<u>Problems implementing alternate assessment techniques</u>

Work and discussion of assessment continued during the third school year. Once the different methods were put into use, the challenge then was to determine how marks would be assigned using these alternative assessment techniques. It became clear that students were being marked differently by different markers within the same course, and were not being given the same options.

Reflection yielded reasons why this discrepancy existed. Personal apprehensions and uncomfortable feelings about going into unknown territory had to be overcome. It was difficult for teachers to give themselves permission to try new ideas and ways of assessing. Furthermore, with only brief discussions amongst *all* mathematics teachers, a final copy of the assessment options was created, yet not all teachers had been a part of the development process. As a result, some did not buy into the options in practice, although they verbally agreed to them. As stated in an article on the challenge of implementing the NCTM Standards, "Experience has revealed that the process of change requires time and acceptance and that the steps of change in people and programs are incremental" (Frye, 1989, p. 5).

Teacher Collegiality

So much has been gained (and still is, on a daily basis) from working closely with a variety of colleagues. This collegiality is a vital part of the evolution of the NVSS program:

> Today I spoke with someone who is working on his doctoral thesis and I found his questions quite insightful. He came to our school to discover more about the alternative type of education we offer....I realized, through his questions, that I kept repeating one important reason why I enjoy working at this school so much; that is, the collegiality.
>
> I feel that the structure of New Venture necessitates...a wide variety of communication....As a TA I need to know the expectations and procedures of each department in order to act effectively in my role as an advisor....The building design encourages collegiality since, with one main teacher preparation area, it is relatively easy to see every teacher every day.
>
> I am sure that increased communication, no matter what the means, can only translate into more creative ideas and positive action in the school. This year I am discovering what a truly powerful force teachers working together can be. (Journal, 92/12/08)

I continued to discuss the impact of working with others in the second year of the school's operation:

> I realize that working closely with others from different departments in the school (through discussion of our TA students and the need to know about the various programs offered) is shaping my view of mathematics and its teaching. At our first Pro-D, a teacher commented on how one of her TA students is frustrated at the lack of applicability of the mathematics she learns in school to her everyday life. She suggested that we bring in speakers who could explain where and how they use mathematics in their jobs. Another teacher joined in by suggesting people who might be willing to act in such a capacity. I feel that the structure of NVSS fosters this type of communication in that TA's act as advocates for their students and in this capacity, suggest improvements in programs that they would not normally do when each teacher is shut inside his/her own classroom. They openly criticize the daily operations of other areas in the school if they feel a complaint by one of their TA's is justified. They try to accommodate changes that will help their TA's succeed in a course or improve their outlook on a particular subject. Also, they tell us which parts of

our program their TA's see as being positive and ask about how they can incorporate similar ideas in their area of the school if they have heard it works well for us (hearing this often via students).

Ideas and feedback from other staff members are important measures of our effectiveness, as well as being springboards of ways to improve our program....Ideas can be discussed amongst a group of teachers with the common concern of improving the teaching at our school together. There is not the ownership problem of, "I will do this in my classroom, I don't care what you do in yours." We share courses and students so any idea that we develop is important to our program as a whole. Also, once we decide to put a plan in motion, a division of labour makes a once onerous task much more do-able.

My vision of mathematics is also shaped by comments from other staff members on what mathematics means to them. When a TA tells me that she is afraid of mathematics and doesn't want that to rub off on her TA students, I am once again reminded that discussion of feelings when people are learning mathematics is vital. When a colleague lends me a magazine with an article on geometries found in nature, I am again amazed at the pervasiveness of mathematics in our world and hope that we are imparting that in how we teach. When a teacher reads an article on mathematicians that I have posted near my desk and we discuss with humour how human these people really are, I am struck with the need for a human view of mathematics in society. (Journal, 93/10/03)

My view of mathematics teaching and learning has been challenged and shaped through on-going discussions with colleagues and students, as well as throughout the trials of the ideas which result from these discussions. This important realization resulted in the following journal entry:

I find I am often pushed by the students to attempt something new and when I am open to listening to them and try the change, I am usually amazed by the great results. I feel that students...who want to try it their way, my colleagues who encourage me to try new ideas or offer some ideas, the professional freedom offered at the school, and my continued work in the Master's program all contributed to changes in my thinking. This is an amazing atmosphere in which to work and learn. (Journal, 94/10/29)

Chapter Summary

This chapter "set the scene" for the research by introducing NVSS where the author has taught since its opening in 1992. The school's mission statement, "All students will seek challenge and experience success," was developed by teachers prior to the school opening, as were the school exit outcomes. The current staff has a strong common philosophical base and vision. This vision was translated into practice through the self-paced model of learning in which all students are expected to achieve a minimum 70% level of mastery at a rate which corresponds with their ability.

The roles of the teachers and students are quite different from traditional roles. Students are self-directed, and teachers play a dual role of advisor and subject teacher. Personal, one-to-one teacher-student interactions are key to the subject teacher role.

The organization of the curriculum is around learning guides. These guides have undergone several revisions: teachers eliminated teacher-directed presentations from the outset and work towards establishing forums for increased student group interactions on a more formal basis.

Mathematics learning guide expectations, listed at the beginning of the guide, focus the students' selection of activities. The number of activities selected by each student varies, but the goal is for students to understand the concepts listed in the expectations. The teacher's role in this selection process includes guiding student choices as requested and intervening as warranted by monitoring student progress.

Several elements of learning present in the learning environment required formalization in the learning guides, and in teacher practice. One main area was the process of students reflecting on their thinking and learning. Though teachers have formalized some student interactions in the form of peer tutoring, further work on making peer interactions a more explicit part of the mathematics program remains.

Assessment of students was a large issue; in particular, authentic assessment is a school-wide concern. As the mathematics teachers worked through an evolutionary process in the development of assessment tools, attempts to meet both student and teacher needs occurred. The process is on-going as teachers strive toward a more constructivist approach.

Teacher collegiality was key to the development of all aspects of the program. Including all colleagues in the development of ideas was important to enhance the participants' ownership of ideas and their implementation. I have personally experienced many benefits of working together in this learning environment, not the least of which is that my own views of mathematics teaching and learning have been challenged and shaped through on-going discussions with colleagues and students.

Chapter 3
Review of the Literature

This literature review is structured according to the three "key belief components of the mathematics teacher" as listed by Ernest. They are the teacher's: "view or conception of the nature of mathematics, model or view of the nature of mathematics teaching, model or view of the process of learning mathematics" (1989, p. 250). Borasi also uses these three categories when she articulates her "fundamental beliefs" as follows:

> that mathematics is a humanistic discipline, that learning mathematics results from personal inquiry and the construction of meaning within a community of learners, that teaching mathematics involves creating a stimulating and supportive environment for the students' own mathematical inquiries. (1992, p. 189)

These three components are used as headings into which the literature reviewed is organized. These headings further correspond to questions asked during the interview phase of the research and will help to inform the reader of the author's beliefs in each of these areas.

While a number of frameworks could have been used for this organization, this one seemed to be a natural fit for me. I was exposed to Borasi and Ernest simultaneously in the course in which journal writing was first assigned. I incorporated their ideas (which aligned with the way I viewed the world) into my writing and later discovered that this framework also fit well with the data. Ernest and Borasi's three concepts, then, provided both an organizational structure on which to "hang my ideas" as well as a link between the data and the literature. They are thus incorporated as headings for the remainder of this chapter.

Nature of Mathematics

> *Mathematics is similar to an ideology, a religion, or an art form; it deals with human meanings, and is intelligible only within the context of culture. In other words, mathematics is a humanistic study. It is one of the humanities. (Davis & Hersh, 1981, p. 410)*

Researchers feel that there is a connection between teachers' conceptions of the nature of mathematics and of its teaching and learning (Ernest, 1989). As Feldt states, "Researchers (Cooney 1988; Feldt, 1991; Fennema, Carpenter, and Peterson 1986; Thompson 1982, 1984) have found that teachers' conceptions about mathematics and mathematics instruction profoundly affect their teaching and the learning process" (1993, p. 400).[1] In fact, some would argue that an exploration of mathematics philosophy is necessary to mathematics teaching. As Hersh states, "The issue, then, is not, What is the best way to teach? but, What is mathematics really all about?...Controversies about...teaching cannot be resolved without confronting problems about the nature of mathematics" (Cited in Ernest, 1991, p. xiii).[2]

Dossey concurs that teachers' beliefs will influence their teaching practices and explains how these beliefs and practices may affect their students. He writes in his 1992 article about the role and influence of the nature of mathematics,

> The conception of mathematics held by the teacher may have a great deal to do with the way in which mathematics is characterized in classroom teaching. The subtle messages communicated to children about mathematics and its nature may, in turn, affect the way they grow to view mathematics and its role in their world. (p. 22)[3]

Various authors (Quaal, 1993; Borasi, 1992) link teachers' conceptions of the nature of mathematics with how they themselves were taught. Feldt summarizes,

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[1]In my original application to enter the Master's program for mathematics teachers, I stated, "I look to this Master's program then, as a means to explore not only math methodology but also the nature of mathematics itself." At that stage, I did not realize how closely linked my views on the nature of mathematics would become with my views on its teaching and learning.

[2]Interestingly, during my pre-service education in the Professional Development Program at Simon Fraser University, I viewed attempts to have me articulate my views on the nature of mathematics as an intrusion into my valuable time of learning how to teach. There were a few short discussions and assignments in a mathematics methods course I took which were related to the student-teachers' views of mathematics, but after class, my classmates and I expressed our frustration at how these philosophical side-tracks were wasting our time when we were soon to be out in the "real world" of teaching. We were looking for quick practical "teaching tips", but now I realize that we would have done well to read Hersh's comments.

[3]This notion of students' classroom experiences affecting their views of mathematics has certainly been true for me. The teacher interviewees also expressed how their experiences as students in mathematics classes, both prior to university and throughout their undergraduate courses, affected their views of mathematics and its teaching.

> As mathematics students, many of us experienced mathematics as a static body of rules, definitions, and algorithms. We learned by watching the teacher do model problems on the chalkboard, copying the steps into our notes, and trying similar problems by ourselves for homework. The pursuit involved no invention, no discovery, and no group work. Although we may not have constructed mathematics, we did construct a conception of mathematics and of what it meant to learn and teach mathematics. Mathematics was static, students were passive, and teachers supplied the information. (1993, p. 400)[4]

Dossey provides an early historical perspective on two contrasting conceptions of mathematics:

> Discussion of the nature of mathematics dates back to the fourth century BC. Among the first major contributors to the dialogue were Plato and his student, Aristotle. Plato took the position that the objects of mathematics had an existence of their own, beyond the mind, in the external world. In doing so, Plato drew clear distinctions between the ideas of the mind and their representations perceived in the world by the senses....Aristotle's view of mathematics was not based on a theory of an external, independent, unobservable body of knowledge. Rather it was based on experienced reality, where knowledge is obtained from experimentation, observation, and abstraction. This view supports the conception that one constructs the relations inherent in a given mathematical situation. In Aristotle's view, the construction of a mathematical idea comes through idealizations performed by the mathematician as a result of experience with objects....Thus, the works and ideas of Plato and Aristotle molded two of the major contrasting themes concerning the nature of mathematics. (1992, p. 40)

These two major categories of conceptions of mathematics have continued through to this day (Dossey, 1992). Fisher also describes the two contrasting views as external (Platonic) and internal (Aristotelian):

> On the one hand, mathematical knowledge can be assumed to be an external entity existing independently of human thought and action, and hence, something about which one can be objective. From this perspective, mathematical knowledge is viewed to be independent of human values and unrelated to the contexts within which humans find themselves when they inquire into or apply it. When viewed as external and objective, it is as though all mathematical knowledge were always in existence and occasionally mathematicians unconceal or discover another element, making it generally accessible. An alternative view posits that mathematical knowledge is internal and therefore subjective. From this point of view, mathematical knowledge depends on the values of the persons working with it and the context within

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[4]Because of my eventual belief in comments such as the following by Higginson: "We will not begin to make significant progress in dealing with this question [Why should it be that so many children have so much difficulty in learning mathematics?] until we more fully acknowledge the foundations of our discipline" (1980, p. 3), I included an interview question about the nature of mathematics for both students and teachers. In chapters four and five, the reader will find further discussions on this topic.

> which that work is conducted. From this perspective, mathematical knowledge is not so much discovered as created by social groups. (1990, p. 82)

This notion of mathematics being created—not discovered—by social groups is one main tenant of the internal view of mathematics. Several authors (Hitchcock, 1992; Ernest, 1989; Davis & Hersh, 1981; Gattegno, 1970) support this view of mathematics as being created by humans, and developed and negotiated within a community. As Schoenfeld writes,

> Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns....Truth in mathematics is that for which the majority of the community believes it has compelling arguments. In mathematics truth is socially negotiated, as it is in science. (Cited in Borasi, 1992, p. 163)

Because mathematics is a human creation, it is subject to all of the qualities of any social construct. That is, as Borasi writes,

> Both mathematical results and their truth are socially constructed—they are sanctioned by a community of practice (the mathematical community of the time) on the basis of agreed on criteria, which may change over time and in different contexts (Schoenfeld, in press; Lerman, 1989). Thus, mathematical results and procedures are not totally objective. They can be influenced by cultural values, political agendas, or even just the desire to solve specific problems deemed important by the contemporary mathematics community. (1992, p. 162)

The notion that mathematics changes under the influence of culture and time is an important one as it leads to the rejection of mathematics as an absolute fixed body of truth (Feldt, 1993; Borasi, 1992; Dossey, 1992; Ernest, 1991; Fisher, 1990; Davis & Hersh, 1981; Gattegno, 1980). For example, Lerman writes, "The history of mathematics is not one of the gradual revelation of absolute truths, but, as with all knowledge, the consequence of people's ideas, interests, conflicts and patronage, and is culturally and temporally relative" (Cited in Borasi, 1992, pp. 162-163). Dhombres echoes this sentiment, "...math has its schools and its fashions. Thus in many respects mathematics evolves like a culture" (1993, p. 402). Because mathematics itself changes with time, so too does its definition. "The definition of mathematics changes. Each generation and each thoughtful mathematician within a generation formulates a definition according to his lights" (Davis & Hersh, 1981, p. 8).

Because mathematics is changeable, it is fallible. This notion of fallibility, rejecting the idea of mathematics as a fixed and absolute body of knowledge, is an important element of the internal view of mathematics (Ernest, 1991; Lampert, 1990; Davis & Hersh, 1981). Borasi, too, supports this notion of mathematics as fallible:

> Mathematicians concluded that mathematical knowledge is neither absolutely true nor fully verifiable but, just as in any other science, only falsifiable and open to continuous revision....Once we realize that mathematical results are neither predetermined nor absolute, we also have to accept the fact that mathematics as we know it now is as fallible as any other product of human activity. (1992, p. 162)

This internal view of mathematics—namely its subjective social construction, changing nature, and fallibility attest to its human element.

> We can only reach the human element in mathematics when we remember that behind each scientific paper there is a *person* who has written it after days, weeks or even years of fumblings, doubts, certainties, minute examinations of special cases (never before mentioned), often with a feeling that the task as it stands is not yet completed and needs re-examination. First come mathematicians and only then mathematics. (Gattegno, 1970, p. 137)[5]

A mathematician himself speaks to this theme:

> In 1891 I have been able to solve a few problems in mathematics and physics....But any pride I might have felt in my conclusions was perceptibly lessened by that fact that I knew the solution of these problems has always come to me as the gradual generalization of favorable examples, by a series of fortunate conjectures, after many errors. I am fain to compare myself with a wanderer on the mountains, who, not knowing the path, climbs slowly and painfully upwards, and often has to retrace his steps because he can go no farther—then, whether by thought or from luck, discovers a new track that leads him on a little, till at length when he reaches the summit he finds to his shame that there is a royal way, by which he might have ascended, he had only the wits to find the right approach to it. In my works I naturally said nothing about my mistakes to the reader, but only described the made track by which he may now reach the same heights without difficulty. (Cited in Bishop, 1985, p. 28)

Davis and Hersh argue that any view of the nature of mathematics should fit with humankind's experience. "The actual daily experience of mathematicians—shows that

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[5]The human element in mathematics then, to me, characterizes its internal view. My own formulation of a definition of mathematics recognizes that it is socially constructed, fallible, subjective, contextualized, and influenced by factors such as culture and time.

mathematical truth, like other kinds of truth, is fallible and corrigible" (1981, p. 406). They thus conclude that any philosophy of mathematics must accept its human element. Borasi also emphasizes the human element in mathematics:

> I have used the term *humanistic* to try to convey the complexity of this view of mathematics—that is, mathematics as a fallible, socially constructed, contextualized, and culture—dependent discipline driven by the human desire to reduce uncertainty but without the expectation of ever totally eliminating it....My choice of the term *humanistic* has been motivated by the current usage of this term within the mathematics and mathematics education community and by my belief that emphasizing its human and humane elements can help us realize that mathematics is closer to other fields, and thus more approachable, than is usually perceived. (1992, p. 163)

Nature of Mathematics Learning

> *Those of us in education have a special reason for wanting this more human view of mathematics. Anything else alienates and disempowers learners. (Ernest, 1991, p. xii)*

Both the internal, human view and the external view of mathematics have implications for the nature of mathematics learning. As Fisher explains,

> If mathematical knowledge is external and objective...then learning tends to be seen as acquiring computational procedures or algorithms. In this view, learning is passive, time-limited, and usually carried out by individuals in isolation. Learning occurs when individuals receive knowledge. That is, objective knowledge, existing independently of the learner, is passively received by, or transmitted to, the learner. On the other hand, if mathematical knowledge is internal and subjective...then learning is conceptualized as an active construction, not time-limited but lifelong, and collaborative in the sense that learning is a social phenomenon. (1990, p. 83)

This section explores the literature on mathematics learning from an internal perspective of the nature of mathematics because, as Borasi articulates,

> The careful analysis of what people do when they engage in challenging mathematical tasks and of the explanations they provide for their actions (see, for example, Schoenfeld, 1985, 1987; Solver, 1985) has caused many mathematics education researchers to challenge the assumption, implicit in behaviorist theories, that students can be treated as "empty vessels" to be filled with information and "knowledge." These studies suggest that in order to understand and use mathematical concepts and techniques, students need to elaborate and reconstruct the information proposed to them until it makes sense and fits in with their preexisting knowledge. (1992, p. 174)

Borasi goes on to demonstrate the current support expressed for the constructivist notion of learning by those in the mathematics education community. This view holds that learning is an active process involving interpretation, justification and meaning construction on the part of the learner (1992).

The student's own construction of meaning is an important part of this perspective of learning. Support for this view is exemplified by Jaworski:

> My thinking about teaching is based on the belief that knowledge is constructed by the individual, not passively received from the environment, and that learning, or *coming to know*, is an adaptive process which tries to make sense of experience. It is not a process of discovering an independent pre-existing world outside the mind of the knower. [von Glasersfeld, 1988; Klipatrick, 1987]. Two immediate consequences of this are (1) that it denies the transmission metaphor of teaching and learning...and (2) that *if* there exists some absolute body of knowledge, based on some external ontological reality, then this can never be known. Knowledge, for any learner, can be no more than their own construction....What it recognizes is that learning is a constant process of meaning making and common knowledge is a negotiated synthesis of such things. (1992, p. 13)

This definition of learning as "coming to know" and "meaning making" requires the learner to play an active role in his/her construction of knowledge. Others (Borasi, 1992; Fielker, 1981; Easley, 1980) support the view that we learn by "discovering things for ourselves" (Davis & Hersh, 1981, p. 398) and further, that students need time to grapple with ideas and sort them out for themselves.

This sorting out does not come without a struggle and it is beneficial for students to witness others engaging in this struggle (Borasi, 1992; Hitchcock, 1992). This notion corresponds to the view of the nature of mathematics as constructed by humans through a difficult process, as described earlier. Lampert describes the process as "cross-country mathematics." That is, "in contrast to walking on a well marked path, the cross-country terrain is jagged and uncertain; watching someone traverse it is a key to learning how to traverse it yourself" (1990, p. 41).

This struggling process of coming to know does not take place in a vacuum. Firstly, "a new idea is meaningful to the extent that it makes connections with the individual's present knowledge" (Bishop, 1985, p. 26). As it is a personal process,

> it is obvious therefore that no two people will have the same sets of connections and meanings, and in particular teacher and learner will have very different meanings associated with mathematics. The teacher will "know" the ideas she is teaching in terms of the knowledge. The learner however is the "meaning maker" [Postman and Weingartner, 1971] in the educational enterprise and must establish the connections between the new idea and her existing knowledge, if the idea is to be learnt meaningfully. (Bishop, 1985, p. 26)[6]

Borasi articulates the idea of teacher and student working together as learners, stressing the importance of communication. "Within a community of learners engaged in the creation of knowledge, mathematical communication becomes an essential way of sharing guesses and ideas, providing and using feedback constructively, and ultimately building the consensus that sanctions new knowledge" (1992, p. 170). Bishop lists several forms of communication:

> Communication in a mathematics classroom is therefore concerned with sharing mathematical meanings and connections. We can only share ideas by exposing them, and "talk" is clearly a most important vehicle for exposing connections. Also important are symbolism, uses of diagrams for conveying images, examples from different contexts, analogies and metaphors, and written accounts and descriptions....Moreover if we add into the construct of communication the dimension of sharing, then the three-way process, from the pupil to teacher as well as teacher to pupil and pupil to pupil, shows us how ignorant we are about pupils' analogies, metaphors, contexts, examples, etc. and about ways of enabling these to be exposed and shared. (1985, p. 27)

This idea that communication is important in the meaning-making process takes into account that the interactions students have are an integral part of their learning. Again, this view rejects a "receiving" mode of learning. "The underlying assumption is that learning does not proceed as a one-way transmission of knowledge from teacher to student but results from the interaction of the learner with the social and natural environment" (D'Ambrosio, 1990, p.

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[6] I would argue that I, as a teacher, often feel like the learner as I make meanings and ideas connect for me when working with students. In fact, it is often not until I work through a question with a student that I feel more confident in my own understandings of the concepts involved. At New Venture then, I feel that I often take on the role of the learner with my students. However, I do believe that there is an important consequence of Bishop's view. That is, because the teacher and learner each have their own meanings, the idea of communication becomes extremely important.

22). Borasi relates the importance of communication to the idea of learning in a social environment:

> Vygotsky has pointed out the importance of also looking at the *social* aspects of learning. In this framework, language and communication take on a key role, since they are seen not merely as vehicles for expressing and receiving already formed ideas but as "tools for thinking"—for helping individuals clarify and refine their tentative interpretations and explanations even as they try to articulate them in an effort to share them with others. (1992, p. 176)

A view of mathematics learning cannot be adequately articulated without taking a student's interactions with others into account (Lester, Garofalo, & Kroll, 1986). "Sociologists of science have alerted us to the limitations of this [not including students' interactions in examining their construction of meaning] approach [e.g. Brannigan, 1981; Barnes, 1977, 1982; Bloor, 1976; Knorr, 1980; Knorr-Cetina, 1981]" (Cobb, 1986, p. 6). In fact, Sigel states that while the entire social experience of individuals is important, the element of interaction is crucial to understanding the process of cognitive growth (Cited in Cobb, 1986).

Student interactions are important when considering not only cognitive development but also emotional growth, as both occur together (McDonald, 1989). This idea of the simultaneous occurrence of emotional and cognitive development is important because "emotion is not only anecdotally and phenomenally part of human thought and action; there is now a burgeoning body of evidence that emotional states interact in important ways with traditional cognitive functions" (Mandler, 1989a, p. 4). McLeod acknowledges that emotions are an important consideration in a student's learning when he argues that, "the cognitive processes involved in problem solving are particularly susceptible to the influence of the affective domain" (1989, p. 24).

The term "affect" requires clarification as it is used in many forms in the literature. Here it is used in a general sense, one that Hart suggests many educators share, "as a blanket term to describe attitudes, appreciations, tastes and preferences, emotions, feelings, and values" (1989, p. 41). These aspects of the affective domain are important to students' learning in mathematics.

There is increasing recognition of the emotional element in the classroom environment (Adams, 1989). Students' emotions will include both positive and negative ones; both must be taken into consideration in the learning environment. In fact,

> The absence of negative emotions, such as frustration and anxiety, among students need not to be taken as an indication of a desirable affective state, because expert problem solvers also experience such emotions. The important issue is not whether such negative emotions occur, but how students cope with them when they occur and what drives students to persevere at a task even when it entails some degree of struggle. (Thompson and Thompson, 1989, p. 174)

Perseverance is an important aspect in student learning. Grouws and Cramer studied six urban junior high school mathematics classrooms and noted that:

> Three dimensions of the affective domain were prevalent in the classrooms we observed. Basically, there was evidence that students were willing to attempt problems, persevered in seeking a solution, and enjoyed problem-solving lessons. Lester (1980), in his review of the literature on problem solving, indicated that these three affective factors (willingness, perseverance, and self-confidence) are some of the most important influences on problem-solving performance. Reyes (1984) has also suggested that the students' willingness to work on a variety of mathematics tasks and their persistence in dealing with these tasks might make a difference in the degree to which a class is task oriented and easy to motivate. (1989, p. 160)

To attend to the role of emotions, students' beliefs and expectations must also be considered for, "it is the values of mathematics that the learner brings to the situation that will color the emotions that are experienced" (Mandler, 1989b, p. 239). Further, Cobb, Yackel and Wood (1989), relate how students' beliefs about themselves, their roles and others' roles, how mathematics is learned, and the nature of mathematics itself may affect their learning. For example, "if students have expectations that they cannot perform well in mathematics learning, this may influence the actual course of learning" (McDonald, 1989, p. 225). So, the development of positive beliefs about mathematics and the student's role within a mathematical community is an important part of learning. Borasi advocates that students should come to think of themselves as mathematicians and that process includes,

> interiorizing the set of beliefs and values that belong to mathematics as a "culture" (Schoenfeld, 1988, in press; Bishop, 1988) and feeling part of a community of practice (Lave and Wenger, 1989; Greeno, 1988)....Feeling like an "insider" in the mathematics community is also crucial if students are to stop being intimidated and marginalized by school mathematics, "victims" of the decrees of "experts" in the field and of the myths that surround it (Atwell, 1987). (1992, pp. 170-171)

Other authors (Cobb, Yackel, & Wood, 1989; Fennema & Peterson, Cited in Fennema, 1989) argue that successful students are independent problem solvers and take responsibility for their own learning.[7] This view of success was outlined by Cobb, Yackel and Wood in their teaching experiment in a second-grade classroom. As they observed students in this positive learning environment,

> Engaging in mathematical activity was an end in itself, and as long as they did their best they considered themselves to be succeeding. Indeed, they were succeeding in a classroom in which the social norms obliged them to think their problems through for themselves and take responsibility for their own learning. (1989, p. 144)

Nature of Mathematics Teaching

> *From Pythagoras in antiquity to Bourbaki in our own day, there has been maintained a tradition of instruction—religion which sacrifices free understanding to the recitation of formal and ritual catechisms, which creates docility and which only simulates sense. All this has gone on while the High Priests of the subject stand in their corners and laugh. But some of the High Priests of mathematics must weep, because it is in the very nature of mathematics that its abstract symbols suffer a loss of meaning even as they gain in generality. Over the ages, mathematicians have struggled to restore thought and meaning to mathematics instruction, to provide alternatives to the formal and ritualistic mode of learning in most mathematics classrooms, but in spite of new theories, new applications, new courses, new instruments, the battle is never won. The fight against formalized, unthinking action is perpetual. (Baruk cited in Davis & Hersh, 1986, p. 287)*

The internal and external views of mathematics have consequences for the nature of mathematics teaching. As Fisher explains,

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[7]This thinking is in-line with the philosophy of learning at New Venture. In fact, it is one of the measures of a successful student at the school as outlined in the school exit outcomes.

> [In the external perspective]...teaching is characterized by telling. That is, teachers who possess relevant mathematical knowledge, transmit it to learners simply by telling them about it. Teachers are seen as content area experts who specify all aspects of all learning tasks for students. Students are primarily engaged with the teacher and the teacher's representations of mathematics as opposed to being engaged with mathematics itself. Several aspects of this conception of teaching have been captured by describing a teacher as a "sage on a stage."...In the beta paradigm [the internal view], on the other hand, teaching is more like coaching, where the student (as opposed to the teacher) is the primary performer. From this perspective, teaching frequently takes on characteristics of co-inquiry; students and teachers share the function of task specification; and students engage the subject matter directly. (1990, p. 83)

This section of the chapter will explore the literature on mathematics teaching from an internal perspective because, as Borasi articulates,

> The view that mathematical knowledge is socially constructed and contextualized on one hand, and that learning mathematics is a personal construction of meaning, shaped by context and purpose as well as social interaction, on the other, challenges the very foundations of a transmission model of teaching. Thus, they also require us to rethink our notion of what teaching mathematics is all about. (1992, pp. 179-180)

This view of teaching mathematics involves several elements which relate to those articulated in the previous section on learning. For example, the constructivist theory of learning which views learners as meaning-makers corresponds to "the educational goal we are concerned with here,...that of sharing, and developing, mathematical meaning" (Bishop, 1985, p. 26). Borasi provides an idea as to how the above goal may be approached:

> Good mathematics teaching should be conceived not as the "clear and efficient" transmission of established mathematical results but as the creation of a community of learners engaged collaboratively in the construction of mathematical knowledge in order to increase their understanding of the world, to solve specific problems, and to come to appreciate and expand their mathematical ability. This, in turn, will involve the development of a "rich" classroom environment, which can stimulate students to engage in humanistic inquiries about mathematics and provide the necessary support for pursuing such inquiries. (1992, p. 181)

An implication arising from Bishop's notion that "a new idea is meaningful to the extent that it makes connections with the individual's present knowledge" (1985, pp. 25-26), is that a teacher must take the student's lead, rather than worrying about his/her own agenda. "We learn from both ethnology and developmental psychology that it is useless to work on a stage prior to the student's ability to be in that stage" (Agassi, 1980, p. 30). However, this is not to say that teachers must break subject matter into compartmental pieces and have students progress through the pieces in a fixed order (Borasi, 1992).[8]

Students may not follow a "logical" order in their learning and, perhaps as a consequence, they may experience more of a struggle. This struggle, however, is crucial to the development of the students' understandings. Lampert's reference to cross-country mathematics (1990) which demonstrates the difficult construction of mathematical ideas by humans leads to the notion that teachers should share their own struggles with students. She states,

> If the teacher only demonstrates that she knows how to explain the rules, and whether or not students' answers are correct, the student will get an unfortunately limited picture of mathematical expertise, and it is unlikely that he or she would learn how to walk on any but the most well-marked paths. (p. 42)

Borasi echoes this idea:

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[8]It was not until I had worked individually with students at New Venture for some time that I began to question my previous assumption that students must progress through mathematics concepts in a step-by-step, ordered manner. A journal entry documents my change in thinking:

> When I first saw the learning guides..., I was pleased at their arrangement into what I felt was the logical flow of topics throughout the course....The order was very similar to what I had used to structure my teaching in previous years, an ordered structure that I had always felt helped students to develop concepts in a logical (so it seemed to me) fashion.
>
> All of this reasoning was mine before the students at NVSS entered the picture. In fact, my reasoning did not change initially as all students were given LG 1 as a starting point and most then went on to do LG 2, etc. However, several students found the second guide on rational numbers (in math 9) to be quite difficult [and they became quite frustrated]....[They] asked if they could move on to a completely different section such as geometry. I agreed because I too was getting frustrated and wanted these students to experience success not only in mathematics but also in working through some learning guides.
> So they began to work on geometry. This approach worked very well and boosted several students' confidence levels so it soon became common practice for me to ask students which topics they would like to proceed to next, allowing them to work through the course in an order with which they felt comfortable even though I still did not see it as a logical order for learning. After some time I became quite comfortable with this practice and reading an article by Jacqueline Grennon Brooks helped me to feel not merely comfortable but actually good that students were proceeding with their order not someone else's. She writes, "Constructivism reminds us that order exists only in the minds of people, so when we as teachers impose our order on students, we rob them of the opportunity to create knowledge and understanding themselves" (1990, p. 70). (Journal, 93/06/13)

I still feel that the learning guide structure imposes some of this hierarchical approach, but allowing students to choose their own order is a start in breaking away from it.

> Another common assumption of the transmission model of teaching is the expectation that good mathematics teachers should always provide very *clear* explanations. A humanistic inquiry approach instead values uncertainty, even a certain degree of confusion (see Brown, in preparation), as a positive element in a mathematics lesson, since it can generate doubt and thus stimulate students' thinking....Absolute clarity on the part of the teacher may in fact turn out to be counterproductive in the long run for students' mathematical learning. (1992, p. 182)

Interactions and communication are two important aspects in student learning as outlined in the previous section. Both of these notions have implications for the nature of mathematics teaching as well. The types of teacher-student interactions required are of a negotiation nature rather than an imposition of the teachers' ideas over those of the students (Bishop, 1985).

> It is the case that the teacher is given authority and power by the society for the specific education of her pupils. This authority means that the teacher has certain goals and intentions for the pupils and these will be different from the pupils' goals and intentions in the classroom. Negotiation is a goal-directed interaction, in which the participants seek to attain their respective goals....[Teachers should] use their power *not* to impose their knowledge on the pupils...[instead]...teachers can encourage the negotiation process...[and] can encourage pupils to play a greater part in the development of their own mathematical meanings. (Bishop, 1985, p. 27)

A negotiation type of interaction involves students and teachers each adjusting their goals and expectations (Cobb, 1986). "In the absence of dialogue, there is a gross mismatch between the goals that the teacher thinks he or she is getting for students and the goals that students actually seek to achieve" (Cobb, 1986, p. 8). Thus, communication is an essential element to teaching and learning.

As discussed earlier, communication is essential for cognitive development and emotional growth. Therefore, an essential task for teachers is to encourage meaningful communication in their classrooms. Communication in mathematics classrooms is too often in the form of the teacher doing most of the talking to the students (Fielker, 1981). Some authors have advocated alternative roles for teachers in their communications such as "close listening" (Easley, 1980) and "three-way communication" (Bishop, 1985). In these forms, communication takes time and effort to incorporate into classroom activities. "Communication takes time—to play, relax, discuss, absorb, assimilate and understand" (Maxwell, 1989, p. 225). The time teachers take to foster meaningful communication is worthwhile since communication is an essential element in their students' cognitive and affective development. Because these two forms of development are linked,

> mathematics teachers should consider the effects of affective components of learning mathematics in planning effective instruction. In fact, attention to both affect and cognition may be needed in order to accomplish the goal of developing positive attitudes toward mathematics. (Adams, 1989, p. 192)

Fennema echoes this sentiment. "Both outcomes of mathematical education (positive affect concerning mathematics and the learning of mathematics) must be taken into consideration as scholarly activity is planned" (1989, p. 210).

One way in which teachers can address the affective domain is to provide a supportive learning environment by developing a positive atmosphere. Many authors encourage teachers to invest time in developing this supportive environment for students (Borasi, 1992; Jaworski, 1992; Lampert, 1990; Thompson & Thompson, 1989; McDonald, 1989). Some authors provide concrete suggestions as to how this positive learning atmosphere may be created. The way a teacher treats students appears to be important (Fennema, 1989; Hoyles, 1982). For example, Thompson and Thompson observed an elementary school mathematics classroom and noticed, "Mr. E's friendly and supportive manner and his tendency to accept students' contributions unquestioningly appeared to contribute to students' openness and to the low-stress atmosphere that prevailed in the class" (1989, p. 174).

Another way to build a supportive learning environment is to change the traditional role played by teachers. Borasi encourages teachers to work as partners with students instead of setting themselves up as the authority figure (1992). D'Ambrosio concurs as he advises the following:

> teachers should see themselves as partners in a common search, in the common and shared process of building up knowledge. The clear edge that the teacher most often has over the students may be adapted to become a congenial partnership which builds up positive self-esteem in the students and never adopts an arrogant, imposing, authoritative attitude. (1990, p. 23)

Agassi combines both of these pieces of advice when, referring to teachers he says, "let him treat his students as respectfully as possible...[I am] not for the method of coaxing and cajoling and manipulating children, but of talking to them as to equals" (1980, p. 31).

At the Conference on Affective Issues in Mathematical Problem Solving, participants discussed,

> the definition of the "good teacher."...Carmen Overson suggested that the discussion sounded at times much like the equally groping searches for the "good therapist." If as I believe, the parallel is real, then we are looking for teachers who communicate acceptance to their students (no matter how disappointing their performance) and who elicit trust by demonstrating their involvement in the students' problems. Teachers who will understand the relative salience of different attitudes and values, who will know when to intervene and when to abstain from doing so, should provide the emotional and intellectual support that the affectively handicapped student of mathematics needs. (Mandler, 1989b, p. 242)

Because students need to learn how to deal with negative emotions, teachers will need to be particularly supportive at times and address students' feelings in a straightforward manner. As McLeod states,

> Teachers need to know how to deal with these emotions, both the joys and the frustrations of problems solving. Traditionally, teachers have tried to control or eliminate emotional responses in mathematics classrooms, and traditional content, with its emphasis on learning low-level skills, tended to minimize the opportunities for more emotional involvement in learning. Now, however, the emphasis on higher-order thinking and problem solving makes the classroom a more exciting and more emotional place to be (1989, pp. 251-252).

Cobb, Yackel and Wood provide a better way to deal with students' emotions, rather than attempting to eliminate them or the mathematics that elicit them:

> Surely, the solution to the problem of students' negative emotions during mathematical problems solving is not to quash mathematical problem solving. We suggest instead that...the teacher should renegotiate the social context within which children attempt to solve mathematical problems and thus influence their beliefs about their own and the teacher's roles and the nature of mathematical activity. (1989, pp. 143-145)

Bishop also places importance on the creation of a social context and on attempting to address students' beliefs and expectations. He states,

> I assumed that they [students] would not learn a different way of thinking about what it means to know mathematics simply by being told what to do...I assumed that changing students' ideas about what it means to know and do mathematics was in part a matter of creating a social situation that worked

> according to rules different from those that ordinarily pertain in classrooms, and in part respectfully challenging their assumptions about what knowing mathematics entails. (1985, p. 58)

Beliefs about mathematics are important in students' learning, but teachers must also address their students' expectations about how they learn and thus their own and the teacher's roles within the learning environment. As Lampert states:

> When classroom culture is taken into consideration, it becomes clear that teaching is not only about teaching what is conventionally called *content*. It is also teaching students what a lesson is and how to participate in it (Florio, 1978; Jackson, 1968; Mehan, 1979)....The teacher has more power over how acts and utterances get interpreted, being in a position of social and intellectual authority, but these interpretations are finally the result of negotiation with students about how activity is to be regarded. (1990, pp. 34-35)

Borasi also emphasizes that both students' and teachers' roles and their expectations about learning must be redefined. She further states that this redefinition may be a difficult process for many students and teachers.[9]

Teachers may find comfort in the comment found in the NCTM's Professional Standards for Teaching Mathematics, "Teachers are in a constant state of 'becoming.' Being a teacher implies a dynamic and continuous process of growth that spans a career" (1991, p. 125). Feldt summarizes some suggestions to aid in this "becoming" as he states that the

> key elements of activity, reflection, and social interaction are woven throughout the Professional Teaching Standards (NCTM, 1991) in the suggestion that teachers work with colleagues and supervisors to modify and improve their

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[9]I have personally noted this difficulty at New Venture, to which the following journal entry attests:

> Working with these [summer school] students [new to NVSS] helped me to re-live September 1992 and reminded me just how strange the New Venture model is to students. I've heard several times, "Why can't you just teach me exactly what I need to know then I'll do it." I have seen anew students' frustration as they come to realize that I, their teacher, am not going to live up to their expectations of a teacher (telling them step by step what to do and how to do it). Their role as students undergoes a big shift...
>
> An article by D. N. Perkins states exactly what I have been thinking this past week. That is, "one basically is asking for the students to learn two things at once —*X* (by a route that looks roundabout to them) and a new theory of learning (that says that the route isn't so roundabout after all). This is a lot to ask" (1991, p. 20). (Mehrassa journal, 93/07/09)

It seems to me that this new way of viewing mathematics, its learning and its teaching is a lot to ask not only of the student, but also of the teacher.

> teaching....The coaching relationship gave teachers the opportunity to reflect mutually and check their perceptions, share their successes and frustrations, and think through mutual problems. (1993, pp. 401-402)[10]

Thus, as teachers themselves develop, they must focus on developing a supportive learning environment and creating a renegotiated social setting to support students' emotional and belief-laden learning. This mathematics community within a classroom is described by Bishop:

> each classroom group is still a unique combination of people—it has its own identity, its own atmosphere, its own significant events, its own pleasures and its own crises. As a result, it has its own history created by, shared between, and remembered by the people in the group. (1985, p. 26)

Aspects of the mathematics community created by the people at New Venture Secondary School are described in chapters five and six of this thesis.

Chapter Summary

This chapter presented two contrasting views (internal and external) of the nature of mathematics. The literature reviewed supported elements of the internal view; namely, that mathematics is created by social groups and thus is subjective, fallible, and influenced by such factors as culture and time. The authors suggest that the teacher's conception of the nature of mathematics affects how the subject is taught, how the student learns, and how the student comes to view mathematics. The student's conceptions thus formed, in turn affect their expectations, their view of classroom roles, and therefore their learning.

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[10] I readily concur with these suggestions as I have personally found that my own understandings of the nature of mathematics, its learning and its teaching have developed not only through reading, but also by reflecting on my teaching both with colleagues and through writing journal entries. I can further relate to the secondary school teacher whom Feldt describes preparing to teach a new course, "Even as an experienced teacher, she felt the need to 'examine and revise [her] assumptions about the nature of mathematics, how it should be taught, and how students learn mathematics' (NCTM 1991, p. 160). Finding herself in uncharted territory, she felt, once again, that teaching is a complex, problem-solving task." (1993, p. 402)

I, too, found myself in uncharted territory as I began to teach at New Venture and so it was an exciting setting in which to conduct my research.

Learning, from an internal perspective, is depicted as an active construction of meaning involving a lifelong struggle of connecting new ideas to preexisting knowledge and a social phenomenon where collaboration, communication, and interaction are important. The internal view also values affective development as a vital aspect of cognitive development. Consequently, teaching becomes a process of coaching and co-inquiry in which teachers follow the students' lead in their personal development of mathematical meaning, while also attending to the affective domain. The teacher becomes a facilitator of a supportive learning environment.

Chapter 4
Methodology

To explore the ideas emerging from the journal review, discussed in Chapter one, and to compare my perceptions with those of my colleagues and students at NVSS, data were collected from a variety of sources. Interviews with both teachers and students constituted the major source and further data was collected from students through questionnaires and peer tutoring summary sheets.

Interview questions for both students and teachers were generated based on my own insights gained through journal writing and review (see Appendix for the original interview questions). As the semi-structured interviews progressed, the original questions were expanded to pursue issues that were raised by the interviewees. Very early in the student interview process, I discovered that the students had a rich insight into questions that were originally reserved for the teachers. From that point on, both teachers and students were asked similar questions during the interviews.

The students interviewed were chosen through purposeful sampling. I attempted to have an equal number of male and female respondents, as well as representatives from every mathematics course offered at the school. The mathematics/science department comprised two full-time mathematics teachers (myself being one), four part-time mathematics teachers and a number of science teachers. The full-time mathematics teacher, Scott, and two of the part-time mathematics teachers, Perveen and Bruce, were interviewed. These teachers were selected based on the number of mathematics students for whom they were responsible and the level of their involvement in the department.

The interviews were conducted, for the most part, at NVSS and at a time that was mutually convenient for each interviewee and myself. Two teacher interviews took place off-site for the convenience of the interviewees. They began in June of 1994 and concluded in December of that same year. While most interviews were conducted one-to-one, one was grade eight students' time restrictions and to help them to feel more comfortable with the interview. Throughout all interviews conducted–with both teachers and students–member checking was used to clarify the responses. The interviews were tape-recorded, with the permission of each interviewee, and later transcribed. The recorded interviews and transcripts were then analyzed for general impressions and common themes. Respondent validation was carried out with the teachers in order to determine whether they found the themes identified to be valid. In one case, this procedure expanded into a second, longer interview.

As the student interviewees were not randomly selected and a relatively small sample were interviewed, a validity check was necessary to ensure that the themes derived from the student interviews were representative of the student body at large. To this end, a questionnaire was designed based on summaries of the student responses from the interview transcripts. Each question was reworded to fit a "Check every response with which you agree" format as follows: "These are ways that students said they learn math at [New Venture]. Check as many that apply to you." The questionnaires were distributed in early spring of 1995 to five randomly selected teacher advisor groups, each comprising approximately twenty students ranging from grades eight to twelve. The teacher advisors administered the questionnaires, following the instructions supplied on the cover sheet (see Appendix for sample). Eighty-five completed questionnaires were received out of a total of ninety-seven distributed.

Another source of data on student perceptions was the course-end peer tutoring summary sheet written by each student as a part of their final learning guide (see Appendix for sample). This sheet included three questions, of which I used two. These two questions were, "How did you benefit from this [peer tutoring] experience?" and "Please give your likes, dislikes, and ways to improve this math course."

Limitations

Throughout the course of this study, several limitations emerged with respect to inconsistencies in the questions put to the respondents, both in the interviews and on the questionnaire. Questions regarding the nature of mathematics were asked differently of the teachers than of the students. The method used for students did not produce very informative responses and the method used for teachers introduced an outside author's bias. Even within their respective groups, the interview questions used were not identical from one person to the next. Although follow-up interviews were conducted with the teachers as a means of respondent validation, none were carried out with the students. Instead, a questionnaire was used for the students, which in itself constituted a source of limitations. Not all of the questions asked in the interviews were reflected in this questionnaire. Of those that were included, some were worded differently than in the interviews and others had only been put to some of the interviewees.

Given the semi-structured nature of the interviews, it was not possible to maintain total consistency in the questions. This difficulty was compounded in the case when multiple students were interviewed at once. Further, the presence of their peers may have affected the responses provided in that case. In all interviews, at times, questions emerged from the participants' answers, revealing other topics to explore. If an "off-topic" comment illustrated the participants' view of the nature of learning or teaching at NVSS, it was followed up even though the probes used were not on the original list of questions. In subsequent interviews, these "side-line" questions were often incorporated to get the perspectives of other interviewees. Often these questions provided insight that would have been missed had the interviewees' off-topic ideas not been pursued. However, this technique introduced inconsistencies in the interview questions as, in the case of the students, everyone interviewed prior to the newly-developed question did not have the opportunity to respond.

Follow-up interviews using the new questions did not take place for students for several reasons. It would have required too much time to conduct follow-up interviews with each of the students. I also did not want to impose on the students' own time (which was given on a voluntary basis) once again to arrange and conduct another interview. In addition, the students were difficult to locate; many of them were no longer working on mathematics and thus, were not in my teaching area. Access to the teachers was much easier to obtain. As the teachers often have informal discussions in the common teacher preparation area, it was possible to ask them to clarify a part of the transcript or to discuss further an aspect of their interview. Being a burden on the teachers was not a concern.

The limitations apparent in the student interviews, in conjunction with the fact that a purposeful (non-random) sampling method had been used to select student interviewees, necessitated the use of a questionnaire as a means of carrying out respondent validation for the students. The questionnaire contained questions which had not been asked of all student interviewees. A prime example is question number eight: "It is good to be able to talk about things other than math with my teachers because...." This question was generated by an interviewee who brought up the importance of being able to stray away from the original mathematics question with teachers. It was followed up in subsequent interviewees by asking students if it was important to have the opportunity to explore off-topic ideas with teachers and, if so, why?

The format of the questionnaire posed an additional problem in that it dictated the manner in which information was obtained. In contrast to the direct questions asked in the interviews, the questionnaire made a statement and asked the students to check as many responses as they thought were applicable. As such, the wording of some statements on the questionnaire differed from the wording in the interviews. While these differences were not great, at times they were enough to slightly alter the substance of the question.

A question regarding the teacher's role was worded differently when posed to students than to teachers. On the questionnaire, this question was worded as, "Math teachers should..." whereas in the interviews, teachers were asked, "What is your role?" The teachers' responses were about what they see themselves as doing whereas the students provided answers as to what they felt the teachers should be doing.

For some questions, the problem was not a matter of wording, but of inclusion. Originally, some questions were designed to be asked only of the teachers. As the student interviews progressed, however, it became apparent that they had meaningful insight to offer on the teachers' questions. Thus, these questions were asked in all subsequent interviews. The questions with regard to the nature of mathematics proved to be difficult to incorporate consistently.

Not surprisingly, most people had a very difficult time answering the question, "What is mathematics according to you?" Further, the question itself was too limited in scope; it failed to address the many aspects of the nature of mathematics. Following a review of transcripts of these interviews, the question was redesigned to stimulate discussion at a broader level. At that point in time, all of the student interviews had already been completed and thus, there was no opportunity to put the reworked question to them. As the questionnaire was developed solely from interview responses and this new question had not been used in the student interviews, it was therefore also excluded from the questionnaire. Teachers, on the other hand, were exposed to this question in their interviews. This redesigned question involved providing the respondents with two passages to read describing both the internal and external views of mathematics. They were then asked to indicate with which of the two views they felt more aligned. This procedure was, in itself, a limitation of the study as, when reading the passages, the teachers were provided with that author's bias. However, this method did produce some fruitful discussion (far beyond the responses to the original question) and would have, in hindsight been a good way to foster discussion with the students as well.

Despite these limitations (most of which I was aware and willing to accept from the outset), the interviews provided rich insight into the philosophy and thoughts of the major stakeholders in the NVSS mathematics program: the teachers and the students. The data also provided a "snapshot" picture of the actual teaching and learning processes involved in this program during its second and third years of operation.

Chapter 5
Results

Data were obtained from both teachers and students in interviews and, in the case of the students, from a questionnaire and course end peer tutoring summary sheets. General results from these sources are presented here. The raw data from the questionnaire are presented in the Appendix. The framework used in this chapter mirrors that of the literature review. That is, there are three sections organized around (1) Nature of Mathematics, (2) Learning Mathematics, and (3) Teaching Mathematics. The first section is an exception, containing only teacher results due to the limitations on the nature of mathematics question as discussed in the previous chapter.

Teacher Views on the Nature of Mathematics

When the question, "What is mathematics according to you?" was posed to the interviewees, not surprisingly, most had a difficult time responding. When the teachers were presented with a written summary of two opposing views—internal and external (see Appendix for summary)—each professed an internal view of mathematics in which humans are viewed as math makers (not discoverers), and see mathematics in much of life. Perveen explained,

> I lean more towards the view that mathematical knowledge is internal...because I believe that as human beings, we have a unique ability to be able to construct knowledge; we don't necessarily just see it for face value, we don't just accept everything that's there in front of us, and the external view of math says its out there, it's just a matter of discovering it. I don't think it's that simple anymore. I think our view of the world is changing consequently *because* we're able to construct it from a very sort of subjective and value-based perspective.

Bruce similarly professes an internal view. The following excerpt discusses one element, the fallibility of mathematics:

> [So often] people have said "This *is* the way it is and you all believe it" and we all do. We take sides—we say yeah those guys know what they're talking about....and twenty years down the road, some guy comes through with this great discovery and says, "Oh, everybody before this was wrong. This is the new belief."

Each teacher also spontaneously described how their view had changed and developed over the years. Perveen stated,

> my view of math has changed through my teaching....When we look at the internal/external views of math, the external view of math is how we were taught...through university, through college, through high school, whatever– that it was something out there that we had to try and learn. And our teachers were telling us how to do that...And since I grew up that way, learning math that way and being taught math that way, that's how I viewed it when I initially started teaching it. But as I developed as a teacher — not just a math teacher, but as a teacher of all other subjects — and developed in my view of teaching and in my role of teaching, and especially now I've had a chance to explore *how* students learn it in this setting and in various settings, because the element of time is gone, for that reason, I think my view of math is changed to the internal where it's something that students *have* some knowledge, they come in with something and we work from there.

Scott felt that his work at NVSS, in particular, has changed the way he views mathematics. He explained,

> the one thing that has been of greatest impact to me [is] the idea that there's more than one way to approach the area of mathematics. I think our work on authentic assessment has opened my eyes, has broadened my vision of what mathematics is and we're only scratching the surface.

Having gained this new appreciation of mathematics, all teachers further stated that they would like to help students develop a similar wide view of mathematics, certainly beyond what is merely in the textbook.

Student and Teacher Views of Learning

When teachers were asked how students learn mathematics at NVSS, they responded that students learn by doing questions, on their own and with other students, and through interactions with teachers. The practice of doing questions was identified as being important to the development of student understanding because, as Perveen stated, "more is learned by actually doing in this setting than just by observing examples and trying to copy what's been done before. And therefore there's an element of construction of the understanding based solely on the part of the student themselves."

Students similarly indicated, both in interviews and questionnaires, that they learn mathematics at NVSS by doing questions, reading the LG and the text, and asking a teacher if they "get stuck." When asked in interviews what other ways are good to learn mathematics, responses included making posters, doing statistics projects, peer tutoring, and using the computer. When asked on the questionnaire what benefits they derive from this type of work, students most frequently chose the following responses: it puts my skills to use, it is more fun to learn, I develop a better understanding, and it gives me a bigger picture of what math is.

Learning through individual interaction with teachers was cited by students as being a positive and desirable way of learning. Students interviewed frequently mentioned that they prefer one-to-one help, feeling that they learn more because they have a greater opportunity to talk with the teacher (Nicole). However, the teachers are not always able to respond to every student's questions immediately, leading most students to indicate on the questionnaire that they don't like it when math teachers aren't available or are busy.

Both students and teachers viewed the teacher-student interactions as being crucial in developing student understanding. Perveen stated,

> I think the students are learning from the interactions they have with teachers. They're learning from the constructions of knowledge that they have about their math based on the work that they've worked through and they're building their knowledge base from the example that instructors and markers provide.

However, Perveen also acknowledged that more effort is required to ensure that all students benefit from such interaction:

> I'm still seeing not enough students feeling as comfortable yet in approaching teachers generally. So, to initiate that interaction is a difficult one. And it's very difficult because our time is *completely dominated* in this school with interaction of students. And the ones that *have* found that way of communicating with adults in the building, are the ones that are benefiting the best, the most. So we're not reaching our whole population yet.

One way in which teacher-student interactions are encouraged is by providing the opportunity for off-topic or "non-mathematical" talk. The two most common questionnaire responses with respect to this issue were: "it is good to be able to talk about things other than math with my teachers, because it helps me to feel comfortable with my teachers" and "it helps me relate to my teacher more." One student wrote on the questionnaire, "it is easier to learn from teachers when you get along well, when there is a friendship almost, as compared to at a traditional school where you sit and listen to some stranger talk to you for an hour." Other benefits of being able to talk with teachers mentioned by students were that it allows students to get their frustrations out, gives them a chance to voice their opinion and express their ideas, and adds interest. Teachers also felt that this emphasis on teacher-student interactions and the opportunity for off-topic discussions result in greater learning for the students.

Self-pacing is an aspect of learning at NVSS that the majority of students in both the interviews and questionnaires cited as a benefit. One student said, "I like that you can go fast or slow so that you learn it as you go not at the teacher's pace" (David) and another commented, "You're not pressured to go on, you can still make sure you understand it fully before moving on" (Jeremy). This idea of "fully understanding" concepts before moving on (a mark of 70% is required to progress) was important to several interviewees. One stated,

> I feel I understand it better even though it takes more time. Before [in my previous school], the course just moved on and you could go to the next grade with 50%. You could coast by before with a C-, but here you have to do the work. I like that you have to get a good understanding before moving on. (Kevin)

Self-pacing also gives students the ability to move more quickly through the course if they already understand the concepts. The teachers interviewed also discussed the merits of self-pacing, "[Students] are able to work at their own pace through something and *really* understand at a more basic, fundamental level in their own way, with the intervention and the instruction and the assistance of a teacher or another student" (Perveen).

According to some of the students interviewed, the fuller understanding developed through self-pacing leads to higher self-confidence in mathematics. One interviewee said, "your confidence can get built up by working on one subject for longer periods of time without interruption and, as you experience success, you begin to want to do it" (David).

Again, the teachers agree:

> Self-pacing allows the student to work at a pace that they are comfortable with...they are not put down in front of the class for asking a question that is

> three chapters behind. Or conceptually a little bit lower on the level. So, right there, you're not putting the kid in a situation like they typically would be in. Where they're afraid to ask a stupid question. ...*that's* the critical thing that it is self-paced and that students *can* work individually and *can* work in small groups and can find some sense of community with other kids that are in the same boat as they are. And not feel like they are just the only ones in the class in that boat. (Perveen)

Other positive aspects of learning at NVSS identified by the students surveyed are the ability to redo their work if they are unhappy with their mark, the ability to challenge work if they know the concepts, and the ability to talk and work with other students. These students also discussed some difficulties they face when learning mathematics at NVSS; namely, difficulty in focusing at times and the length of time they sometimes require to complete an LG.

The teacher interviewees discussed the importance of students working with each other and learning through helping each other. As Bruce said,

> I think the working together is really helping. I see more and more students helping one another and working together and discussing things and saying "This is how it's done." And whenever I see it, I encourage it, push for it and say, "Yeah, go ahead. Please do."

One of the main benefits of students working together mentioned by both teachers and students was the resulting positive feelings. The students indicated that they feel good from gaining a greater understanding of the material themselves, as well as from the mere act of helping someone. "This experience showed me that I had a decent understanding of math. I felt more confident using some of the concepts I had learned" (peer tutoring summary sheet). They also feel good as they become aware of their increased understanding. "I really enjoyed helping everyone, it made me feel smart" (peer tutoring summary sheet).

Scott discussed the effect of the peer-tutoring program on the tutors:

> Having students help students...In some respects, it has nothing to do with math so much as learning self-confidence and allowing them the opportunity to actually help somebody. There's nothing stronger in this world than to be able to make a bond with somebody and to help them out of something....You probably get more charge out of helping somebody than the student that was actually helped. I mean, that's our feedback; that's what keeps us going to work every day. And when you start to see students receive some of that, then you just go "Yes!" That's what we're after, those kinds of things.

Many students replied on the questionnaire that helping and/or getting help from peers, "teaches me social skills (to work with people, to get along)." An interviewee commented, "You have to know how to work with people because that's what you have to do when you're in a job. You end up working with people because they help you" (Jane). Scott expressed a similar thought,

> We're teaching *life*. Life skills, how to handle people, and *we're* learning. I mean, we're in a mini lab. It's like you're in a small, little world there and you all have to get along because you all have a common goal of getting through.

Another such benefit identified by students was improved communication skills. "When you ask [for] help...you have to be able to communicate well so that they understand it and so that when they tell you something, that you understand it" (Jeremy). The act of explaining to others was also mentioned by Scott as being important to the development of a higher level of communication in mathematics. This act of communicating in turn leads to an improved understanding. He said,

> communication has taken a notch up in the whole math area simply because we're having student talk to student. And get away from the repetition of the teacher....The whole idea of communication and language thing is probably [at] a higher level than I have seen in my previous years teaching....And they say, by communicating, you have to internalize it, you have to know it probably better than just listening to somebody talk about it. So in a way, we hope that they're learning it better this way than before.

The students' responses support this view. Most indicated that helping and/or getting help from other students developed their understanding of math concepts in a variety of ways because "I think if you can explain it, you understand it better yourself" (Heather). A similar sentiment expressed on a peer-tutoring summary sheet was, "It taught me in-depth these concepts better than when I did them. I learned more teaching, than when I did it myself." Another student explained that by "helping you may have to go through the book yourself to review a bit...[and] the other person may help you even if you're trying to help them...then the helper gets benefit too because they get something pointed out to them" (David).

This idea that helping other students benefits the helpers by providing them with a review was the second most frequent response on the questionnaire. One interviewee elaborated, "if the person helping isn't in the same grade then it's harder because they have to remember stuff from before, but that's good" (Jeremy). A comment from the summary sheets supports this view, "Helping people with their math has helped refresh my memory of all the math I have done this year." Similarly, helping students who are at a higher level gives the helpers a preview of future concepts. As another interviewee stated, "it gives me a view of what is to come in later courses if I help someone in the next course up from me" (Jeff).

Another benefit the helpers themselves derive from the peer-tutoring process is a greater awareness; they are not just learning about mathematics but they are also learning about their own level of understanding. That is, as an interviewee stated, "When you go to another student [for help] and they don't understand it either...that person finds out that they need some help too" (Jeremy). Thus, both the tutors and the tutees benefit from the peer-tutoring experience. In fact, it can be viewed as a collaborative effort. Comments from the summary sheets reveal that the tutors found it beneficial to work collaboratively with their tutees. "I liked to interact with the students and both work together on the problems." An interviewee further explained, "I do better work when I work with other people because I don't have only my ideas, but I have their ideas too on how something works or how something is" (Jeremy).

When asked on the questionnaire what impact the mathematics program at NVSS had on other aspects of their learning, students most frequently indicated that they learn how to ask for help when they need it and they learn how to work and learn on their own. One interviewed student said, "you have to figure things out yourself so it helps you in life–in life you're not always going to have someone there to tell you how to do stuff" (Amanda). Another student said that you learn how to, "find a person (not necessarily the teacher) to help you with the work...[and] you learn how to talk to your marker and have confidence to get things straightened around with him or her" (David).

The next three most frequently chosen responses on the questionnaire were related to the idea of learning on one's own. That is, learning how to, "use answers to help me do the question...push myself along...be more responsible for what I do and what I learn."

Interviewed students elaborated by saying that they learn to be more responsible in that "you're not in a class being spoon-fed by a teacher" (Amanda). They learn to push themselves along by "finding a place where you work best and not worrying about your outside environment so that you don't get distracted" (Kevin) and learning "how to say I'm sorry I'm trying to work can you please get lost" (David). Some students learned techniques in how to push themselves and stay focused. An interviewee responded, "[you] leave your work sometimes and go on to something else even in another subject until help is available. Sometimes when you come back to it you understand it" (Heather).

The teachers interviewed added that through the mathematics program, the students learn to access a variety of resources including readings and other students, in addition to the teacher. "*Here*, there's a totally and completely different perception because the information base is so much greater. The information base includes other students, other teachers, other resource material, *and* examples given by the instructor in a small group setting" (Perveen).

Every student in the mathematics program at NVSS may not develop each of the skills discussed in this chapter. However, both students and teachers agree that they consider successful mathematics students to be those who are persistent in their attempt to learn. On the questionnaire, the three most popular responses to the question regarding successful math students were that they don't give up, they want to try hard, and they ask for help then keep going back to the teacher until they're sure.

The quality of persistence was cited often in the student and teacher interviews as well. For example, "I think a successful math student is someone who will continue to try. That's important" (Bruce). Willingness to learn was a further important aspect. In the interviews, this willingness was mentioned in different ways. One student said, "you have to want to do it ... be willing to try new things, think new ways" (Jeff). The literature reviewed also cited persistence and willingness as important qualities of success.

The next most frequently chosen group of learning qualities on the questionnaire supported the view that persistence and willingness were important. These qualities included the ability to push themselves, to overcome challenges, and to accept responsibility. Most responses were related to learning in general and did not even mention mathematics.

The number and variety of affective qualities chosen were intriguing. During the interviews, students described feelings that they thought were important to success. Feeling confident, being happy and feeling good with what they're getting were cited in the interviews and chosen in the second highest group on the questionnaire. Again, the same theme was present in the literature that was reviewed. Scott spoke of successful students enjoying themselves and feeling confident in what they do. He said,

> [A successful student is one who] not necessarily gets the highest marks, but one who finally sees the joy of getting, of digging through some questions, of struggling a bit and then that "Aha" and "Yes, I can do it"; that confidence they get that they can do the questions....They were successful because they learned that they could do it. The mathematics they learned was immaterial. The fact that they had the confidence within themselves that they could tackle a math problem and get it, *that* was success for me.

Bruce commented that not being afraid of getting something wrong was also linked to high self-confidence:

> I think somehow, some students have built up the idea that errors are bad so I think they should realize that an error is not an error until you...don't correct it. So as long as you do a correction, learn from your error, this is how we learn.

Student interviewees mentioned that while an understanding of mathematics concepts was important, the time taken to develop this understanding was unimportant in determining success. Teachers' responses also supported this view. One student related the unimportance of time taken for understanding to the importance of feeling confident when he said,

> You have to have a confident feeling of yourself that you know what to do and that you're capable of doing that question. Even if you don't know how to do it right away — but have enough confidence to say if I work at it a bit I could do it. (Jeremy)

Even negative feelings were acknowledged by students describing successful math students as long as these feelings were dealt with quickly. That is, "deal with feelings–get over feeling stupid to ask for help–don't avoid stuff you don't like" (Jane).

Other indicators of a confident, and thus successful, student include an ability to access resources and the ability to seek help when required: "a successful math student is someone who has the ability to recognize their areas of weakness in future, that they know when and how to ask help" (Perveen). The "ability to recognize their areas of weakness" was also noted as a sign of success. Bruce felt,

> that's a skill that's going to take a little longer to get in...to develop their own learning style. Not [to] have to take a learning style test, but [they] can actually say, "Oh, I learned this best when I did 20 problems, or I learned this best when I went home and told my folks how to do this because they were trying to teach me and I said no, no, this is how it's done, but can you check to make sure I'm doing it right."

Student and Teacher Views of Teaching

All teachers interviewed felt that they need to start from where the student is and help the student to build from his/her own understanding.

> [The teacher should] draw from what they've got because they have a framework set up in their mind already of what they've got. And you've got to build from that framework...I firmly believe that there is no way that you can come in and bulldoze what they've got and start a new building. (Perveen)

Fortunately, the structure of the mathematics program facilitates this process.

> *Here* , because there's fewer students to deal with at any one given time, you have the time and the opportunity and the kind of interaction to ask the student "Where are you at first of all? What have you understood to this point? What framework have you got coming in with this question?" (Perveen)

The students also stated that teachers should build from the student's own understanding. Some mentioned that a teacher should be a good listener and should give the student a chance to talk. They want the teachers to start by asking them how they have attempted the questions and follow up on their lead.

Both student and teacher interviewees agreed that students need to be actively learning concepts themselves rather than having the teacher demonstrate to and direct them. Scott explained,

> it's up to us as teachers to try and *wean* the students off of that, where we give information right from the start of something new and take them right through

> it. I think we should be careful not to do that, but rather help them struggle a bit and learn on their own.

Bruce mentioned how effective he has found backing off to be:

> And I'm so amazed because...I'm finding that when I step out of the picture and then stop being so much Mr. Teacher-in-charge who knows it all, that things are happening probably better than if I were trying to control this all and push them to do all this.

The students surveyed seem to agree because this aspect of lecturing or giving a presentation was completely missing from their answers to the question of what a teacher's role should be. On the questionnaire, the least popular responses were assigning questions and telling students to be quiet, two common activities in teacher-directed instruction. One student interviewee said that teachers should, "Work with the students, helping the student to understand it themselves. Students should do most of the work because you won't learn it if someone else does it for you" (Jane).

The most common response on the questionnaire to the question about the role of the mathematics teacher was that math teachers should help students, with another common response being that teachers should explain things to students one-to-one. The teachers interviewed agree that they should help students one-to-one by providing encouragement, guidance, and suggestions for alternative possibilities. The word guide was mentioned often with respect to how teachers feel they behave as was the idea of teachers working alongside students in the learning process. "I think it's extremely vital to the way that we interact with students in this setting to be...an advisor. Somebody who can relate to the learning process or the struggle of the learning process" (Perveen). Scott asserted that teachers need to stay active, "but not in the sense of *teaching* actively, but in the sense of driving, of nudging, pulling, offering different ways of going about things."

The teachers felt that they should be available to answer questions for students, through techniques such as redirecting the questions to the students themselves. Scott explained his technique, "I try to put in their mind: what questions do you ask yourself in order to solve this problem?...I try to make it so that I don't go in and *solve* the problem. I try to ask them a question to get their mind thinking."

Scott's method of asking the students questions is not always how he responds to their queries. He also answers them directly, but, in doing so, he tries to go beyond that one question, emphasizing general techniques. As he described it, "I don't ever try to solve a question *just* to get the answer to that question. I try to make it bigger so that they can take away a bigger picture and not just the picture of that particular question." A technique that Bruce uses to help students is to suggest possibilities to help them solve their problems. "Have you tried this? Have you tried looking in the index to see if the formula is there? Have you tried asking your friend over there who's a learning guide ahead of you?"

The teacher interviewees felt that another component of a teacher's job at NVSS is to manage student learning. This includes helping students to establish positive working groups:

> Maybe you've just worked with one student who's solved a certain problem, along comes a different student with a similar problem, so you put those two together. Many instances like that have happened. If two students are doing the same test, I will put them together and have them work through it together because they will probably dovetail their information...So I think [part of] our role then is one of putting people together. (Scott)

Teachers also manage student learning by helping students to set goals and timelines. As Bruce explains, "you set simple goals and [say] 'Here's the goal kids, you need to be able to do this type of question.'...'Get to it however you can'."

The teachers identified the ability to address a variety of affective issues as being a vital aspect of their jobs. "I see that there is definitely a whole attitudinal and affective component to mathematics at NVSS....we're helping students to develop more positive attitudes" (Perveen). For students to develop a positive attitude, they must be immersed in a positive and comfortable learning environment, the creation of which is viewed by teachers as part of their role.

> [We have] the ability to create a community for those students to *belong in.* And the community that's been created is one of confidence, one of respect for what students have to say, and one of mutual respect amongst the teachers. And that, in itself, is a place where you can ensure good interaction...The ease

> in which that interaction happens depends solely on the type of community or learning environment that's created. (Perveen)

Perveen later contrasted the environment created at NVSS with that in a closed-in classroom and its implications for how the students view their teachers.

> I don't think they [students] view us the same way they would view us...sitting in the front of [the] classroom. They just can't—as a student myself, I don't view my professors the same way in my Master's program because we sit with them.

This one simple act, sitting with the students, encourages teacher–student interaction.

Teachers suggested further reasons why interaction with students is facilitated at NVSS. "We've got smaller numbers [at any one time] so we can *have* more meaningful interactions" (Perveen). Scott added that the removal of a time constraint and his expanded vision of mathematics have also contributed to his ability to have positive personal interactions with students:

> I've never had so many *meaningful* conversations with kids around mathematics as I've had this past year. And good discussions [that start] even by asking the question "Where do you use this?" I think my mind-set now being much more world-oriented, I'm starting to answer those questions better about where students can use and will use mathematics in the real world. Whereas before, in a class situation where you have thirty other kids looking at you for an answer, you tend to be under more pressure, you have to get other people through the curriculum, so you shortchange the question and you carry on by saying, "Oh, it's part of the curriculum, we have to learn it. Let's get going."...Whereas sitting around the table, one-on-one, you develop a relationship with the student and you kind of know where they're coming from and so you know how far to take the conversation with respect to mathematics and the world around us....Many times, the conversation isn't necessarily on mathematics, but it *is* about things that are of concern to students and I treat it as a time of building bridges that I use to cross when mathematics comes along. They're not afraid to ask questions. They're also not afraid to ask questions outside of the math area because they realize I am more than a math teacher; I am a person number one and that I have areas of expertise in many different subjects and I am willing to share that expertise with them...a teacher first, a math teacher second.

The notion of developing confidence and a positive attitude was viewed as being important by both teachers and students. Bruce strongly stated that it should be,

> part of math teaching, part of all teaching. "You can do this, You *can* do this! You're capable."...I think I have to keep encouraging and saying, "Yeah, right on!" and we cheer together when someone passes a test and they get that grin on their face.

In the interviews and on the questionnaire, students indicated again and again that mathematics teachers should be friendly and patient. In interviews, students explained that friendliness and patience on the part of their teachers help to develop their self-esteem and their understanding of the mathematics. They appreciate the teachers' efforts "to make sure students are feeling good about themselves by talking to them" (Jane). The way teachers treat them is important to their developing understanding because, as one interviewee stated, "I pay attention if they're friendly. If I don't like them and they get mad all the time then I don't pay attention so I'm not learning anything" (Nick). Some students apply this principle themselves when tutoring other students. "I found that friendliness made the students much more receptive to my tutoring" (peer tutoring summary sheet).

Students are clear about the negative effect an unfriendly or impatient teacher can have on their learning as "it's important how a teacher deals with you because if they get real snarky then you just want to come back at another time because they fully destroy your thought for the question" (Jeremy). Similarly, a negative attitude on the part of the teacher can damage the students' self-confidence for "if they're kind of mean then you get shy and you feel stupid" (Jane).

Trying to deal with the students' fears is another important step in the process of developing their self-confidence:

> There seems to be this fear of math....The classic scenario you get is, "Oh, I never got math when I was in school so he's never gonna get it." And you hear that from parents...Now we're saying take math and embrace it. It's not there to fear. (Perveen)

Scott echoed this point:

> we've worked with kids that the first thing they say is, I hate this stuff. I can't do it, I don't want to do it and then we go to work and try to break down those fears and I think, working one-on-one, we're probably doing a better job at getting those fears broken down. Because we can actually dig out that particular person's hang-ups or fears. "What are you really afraid of? Let's read the question. What don't *you* understand about the question? What is there, a word there or something that's bugging you there?" Whereas in a normal classroom, you might generically go through that for thirty people, but you don't get any feedback from the individual. Whereas when you're working one-on-one, you can actually watch the body language of the

> person...Sitting around the table, one-on-one, you develop a relationship with the student and you kind of know where they're coming from...It's a two-way conversation; it's not a one-way. Whereas you don't have conversations in classrooms.

Chapter Summary

The results reveal that the teachers' views of the nature of mathematics have changed over the years to the point where now, working at NVSS, they profess an internal conception of mathematics. Characteristics of this internal view articulated by the teachers include the subjectivity and fallibility of mathematics. They characterize mathematics as being is created–not discovered–by social groups and thus value-based. They felt that this view is in tune with the individualized, self-paced structure of the mathematics program at New Venture.

Both students and teachers agreed that students are responsible for their learning at New Venture. Learning is accomplished by working on one's own and with others. Both groups felt that communication and interaction with teachers and their peers are important to this learning process. They further indicated that the value placed on collaboration amongst students and between students and teachers, as well as the attention paid to the affective domain, result in greater understanding and personal development.

Both students and teachers emphasized the benefits of the "student directed" approach taken at NVSS in which the teachers offer one-to-one coaching, taking the students' lead rather than lecturing to them. Teachers further felt that they should manage student learning by helping students with goal setting and forming positive working groups. Both groups agreed that attending to affective issues by creating a positive atmosphere and supportive learning environment is a vital role of teachers.

The data reveal common responses among students and teachers to the questions asked. The nature of the environment created for learning and teaching at NVSS may foster this common vision. Several elements of this environment with accompanying possible explanations for the shared vision are presented in the final chapter.

Chapter 6
Conclusions

This chapter summarizes the results of the study, linking them back to the literature reviewed. The results provided strong support for the ideas presented in the literature. This connection was evident in each of the three areas investigated: the nature of mathematics, the nature of learning, and the nature of teaching mathematics. The common vision shared amongst teachers and students revealed in the data may be the result of several factors unique to New Venture. These factors as well as an update of current practices and directions in the mathematics program at NVSS are also presented in this final chapter.

Nature of Mathematics

Teachers stated that their views of the nature of mathematics have changed over the years, having been affected both by how they were taught and their own teaching. The literature supports this relationship and also makes a connection between teachers' conceptions of the nature of mathematics and of its teaching and learning. A "reciprocal relationship" develops in which conceptions of the nature of mathematics drive conceptions of learning and teaching, which affect teaching practices. Reflection on those practices may, in turn, instigate further reflection and change of personal conceptions of mathematics and its learning and teaching.

The elements of the internal view of mathematics discussed in the literature review are present in the teachers' articulation of their views of the nature of mathematics. They see mathematics as being created, not discovered, by humans. They view it as being subjective, value-based, and fallible. The interviewed teachers see mathematics in much of life and stated a desire to help students gain a wider view themselves.

Nature of Mathematics Learning

The literature reveals that learning is a process of active meaning construction wherein students elaborate and reconstruct ideas until they connect with their preexisting knowledge. At NVSS, teachers and students agreed that one learns through doing questions, citing sources of help such as working with others (peers and teachers) and reading. As such, learning is characterized as an active process—students are not viewed as passive receptors of knowledge.

The importance of interactions with peers and teachers in students' learning was stressed by the interviewees. This notion is also present in the literature where learning is viewed as a social phenomenon and a process of co-inquiry between students and teachers. Learning at NVSS takes place both on one's own and with others. The process of learning collaboratively with others and by helping peers was seen by both students and teachers as being vital. Communication improves through interactions and leads to increased understanding. The notion that communication as a "tool for thinking" is important in the learning process was articulated by both students and teachers and also stated in the literature.

The literature reveals that students need time to sort out and struggle with their ideas. NVSS's self-paced approach seems to address this learning need. Teachers and students agreed that having the student (not the teacher) decide the pace of learning allows for fuller understanding which in turn leads to improved self-confidence.

Self-confidence is also increased by working with others at NVSS. When students work with their peers, for example as tutors, they feel good about themselves from the mere act of helping as well as from an increased understanding and an increased awareness of that improved understanding. One-on-one interactions with teachers help students to feel more comfortable and to relate better to their teacher. They allow students to get their frustrations out and to deal with negative emotions. The fact that addressing the affective domain is important to the learning process was evident in the literature. This link between cognitive and emotional growth, articulated by both teachers and students, was also identified in the literature. As well, the results and the literature agreed that three important influences on learning were perseverance, willingness, and self-confidence.

The literature revealed that students' beliefs and expectations with regard to mathematics and how it is learned, the roles of teachers, and their own roles affect their learning. At NVSS, students are expected to be responsible for their own learning which is a difficult role adjustment. Students felt that, through learning mathematics at New Venture, they learn how to learn on their own. This learning involves how to figure things out for oneself, including accessing a variety of resources; how to ask for help, including how to talk with teachers; how to get along with others while working with them, including how to be a good listener; how to push oneself; and how to be more responsible.

An interesting parallel emerged in the data between the roles of teachers and of students when acting as peer tutors. Before looking to a teacher, students are encouraged to ask the tutors. In their peer tutoring role, students are asked a variety of questions from different courses just as teachers are. In this way, students learn about the teacher's role. On the peer tutoring course summary sheets, some students commented that they felt it best to work collaboratively with tutees and to have a friendly manner when helping which mirrors students' and teachers' expectations of teachers.

Nature of Mathematics Teaching

The interviewees agreed that teachers should follow the students' lead when helping them, starting from the students' ideas and giving them a chance to talk. The literature suggests that teachers should take the students' lead because the ideas will only be meaningful for the students if they make connections with their present knowledge.

In the literature, teaching is viewed as coaching, a co-inquiry process of working alongside the student as partners. Interviewees felt that teachers should work collaboratively with students, helping them one-on-one by guiding, encouraging and suggesting alternative possibilities. Teachers should not demonstrate, direct, or lecture to students. This idea, espoused by both teachers and students, is also advocated in the literature in that negotiation, not imposition, is the ideal in teacher-student interactions. Further, communication should be encouraged and, at NVSS, the peer tutoring and interactions (both amongst students and between teachers and students) facilitates this important process. Both the data and the literature recommend that teachers communicate and demonstrate their own struggle in the learning process rather than provide clear, step-by-step explanations for students.

Addressing affective issues with students was viewed by teachers as an important part of their job. Paying attention to aspects of the affective domain and thus providing emotional and intellectual support was similarly discussed in the literature. Ways of dealing with affective issues offered in the literature had exact parallels in the data. Both the literature and the data support providing a comfortable, supportive learning environment and positive atmosphere for students. They both discuss treating students in a friendly, respectful manner. Students felt that teachers should be patient and friendly because these qualities help to develop their self-esteem as well as their understanding. The literature addresses self-esteem when advising teachers not to adopt an authoritative attitude, but rather view oneself as a partner with the student. Interviewees agreed that it is important to have meaningful personal interactions between teachers and students through two-way conversations. These "equal" conversations demonstrate teachers' respect for students and develop confidence and a positive attitude in students at NVSS. Another similarity between the literature and the data is in dealing with students' feelings. Students felt that teachers should help them to address negative emotions and teachers agreed that they should deal with students' fears. The literature also advises that teachers address negative feelings rather than negating or attempting to eliminate or control them.

Growth Toward a Shared Vision

The data reveal that a similar sense of the nature of mathematics and its learning and teaching exists amongst the interviewees. This common vision may be the result of several factors. Firstly, the teachers originally chose to work at New Venture after learning about its philosophy and non-traditional program. Secondly, although an initial model of the New Venture program existed, teachers have been the force in shaping current practices. From the beginning, teachers were involved in developing the school's mission statement and its exit outcomes. With this common starting point, teachers worked to put ideas into action through the day-to-day operations of the school. These factors created the necessary foundation for implementing change as described by Ernest:

> Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change....Thus the practice of teaching mathematics depends on a number of key elements, most notably: the teacher's mental contents or schemes, particularly the system of beliefs concerning mathematics and its teaching and learning; the social context of the teaching situation, particularly the constraints and opportunities it provides; and the teacher's level of thought processes and reflection....These factors determine the autonomy of the mathematics teacher, and hence also the outcome of teaching innovations...which depend on teacher autonomy for their successful implementation. (1989, p. 249)[1]

Teachers currently have a great deal of autonomy in the development of the program at New Venture and regularly work together to implement further change. The nature of the working environment—very open, sharing, team-oriented—facilitates collegial planning and program implementation. This collaboration naturally encourages dialogue and reflection on the nature of teaching and learning. Further, interviewees mentioned how teaching at NVSS, in particular, has changed their view of mathematics, learning, and teaching. This change is part of the "reciprocal relationship" because the nature of the program at NVSS forces teachers to think about how it affects their own teaching practices and views. This contemplation may result in changes to their views or teaching practices, thus resulting in further change to the program.

[1]These notions have a strong personal connection. The teaching situation at NVSS has provided many opportunities for me to become more aware of and develop my system of beliefs. Teacher autonomy is ensured as I have no set model to follow but, along with my colleagues and students, create a unique learning culture. As mentioned in chapter two, my own conceptions of the nature of mathematics and its teaching and learning have been challenged and shaped through dialogue with colleagues and students, as well as through the reflective process of my journal writing. I would now add that the journey of writing this thesis has also contributed to my emerging conceptions.

This dynamic nature of the NVSS mathematics program is also facilitated by the teachers' ability to "let things die." Given that, for the most part, a single teacher is not solely responsible for a particular course because teachers share that responsibility, there is not too much difficulty in letting go of practices that are not working well. Further, the practice of teachers working together as a team to improve not just single courses but instead the mathematics program as a whole, helps to eliminate a lone teacher taking ownership of a course. Thus, when courses are not viewed as personal property, making changes to improve them can be viewed as a positive group task rather than as an attack on someone's own work. Students' input is also valued and their views are incorporated into program changes. This collaboration, along with other factors noted above, may be responsible for the common, connected vision present in the data. Further study is required to reveal whether these possibilities can be substantiated.

And the Vision Continues to Expand

While this thesis presents a snapshot of the students' and teachers' views of the mathematics teaching and learning at NVSS at a particular point in time, many characteristics present in this snapshot continue through to this day. One such characteristic is mutability—the program at New Venture is far from being static. Its dynamic nature is due to the work of the staff and students who ensure that the program evolves through continual positive change. The reader may be interested to learn about some areas of improvement on which teachers are currently focussed.

Much time is devoted to working with students in developing and implementing alternative assessment tools. Initial work was discussed in chapter two and teachers are now focussing on encouraging students to explore the options, then take advantage of those which suit their personal style. Assigning marks to assessments (other than tests) has been a challenge and currently teachers are developing and implementing rubrics to assist them in this task as well as to show students how their work will be evaluated. Part of teachers' motivation for developing these alternatives is to promote a wider vision of mathematics for students. Further work in developing this vision remains. This year, at the grade eight level, teachers are planning large group sessions to take mathematics outside of the classroom. Activities include groups solving complex problems such as "What size of heater would be needed to keep 100 parrots alive in the rotunda?" and community speakers discussing their use of and need for mathematics. Although teachers advocate active meaning construction and reflection in the students' learning process, they acknowledge that activities designed to foster these processes must be formally incorporated into the program. This idea, mentioned in chapter two, continues to be a concern.

The ideas for change currently under consideration attest to the dynamic nature of the New Venture mathematics program. As such, this thesis represents the mathematics culture given the players involved. As the players change, so too will the program.

Bibliography

Adams, V. M. (1989). Affective issues in teaching problem solving: A teacher's perspective. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 192-201). New York: Springer-Verlag.

Agassi, J. (1980). On mathematics education: The Lakatosian revolution. *For the Learning of Mathematics, 1*(1), 27-31.

Banks, M. (1995a). *Autobiography.* Unpublished document.

Banks, M. (1995b). *Book analysis.* Unpublished document.

Belanger, J. (1992). Teachers doing research: It's a natural fit. *Teacher, 4*(8), 6-7.

Bell, S. (1995). *[New Venture Secondary School]: Evolution of the New Venture assessment model.* Unpublished document.

Bishop, A. (1985). The social construction of meaning–a significant development for mathematics education? *For the Learning of Mathematics, 5*(1), 24-27.

Borasi, R. (1992). *Learning mathematics through inquiry.* Portsmouth, England: Heinemann Educational Books, Inc.

Brooks, J. G. (1990). Teachers and students: Constructivists forging new connections. *Educational Leadership, 47*(5), 68-71.

Burghes, D. (1989). Mathematics education for the twenty-first century: It's time for a revolution! In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 83-93). London: The Falmer Press.

Cobb, P. (1986). Contexts, goals, beliefs, and learning mathematics. *For the Learning of Mathematics, 6*(2), 2-9.

Cobb, P. & Steffe, L. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(2), 83-94.

Cobb, P., Yackel, E., & Wood, T. (1989). Young children's emotional acts while engaged in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 117-148). New York: Springer-Verlag.

D'Ambrosio, U. (1990). The role of mathematics education in building a democratic and just society. *For the Learning of Mathematics, 10*(3), 20-23.

Davis, P. J. & Hersh, R. (1981). *The mathematical experience.* Boston: Birkhauser.

Davis, P. J. & Hersh, R. (1986). *Descartes' dream.* San Diego, CA: Harcourt Brace Jovanovich.

Dhombres, J. (1993). Is one proof enough: Travels with a mathematician of the Baroque period. *Educational Studies in Mathematics, 24*(2), 401-419.

Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 39-48). New York: MacMillan.

Easley, J. (1980). Alternative research metaphors and the social context of mathematics teaching and learning. *For the Learning of Mathematics, 1*(1), 32-40.

Ernest, P. (Ed.). (1989). *Mathematics teaching: The state of the art.* London: The Falmer Press.

Ernest, P. (1991). *The philosophy of mathematics education.* London: The Falmer Press.

Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 205-219). New York: Springer-Verlag.

Fielker, D. (1981). Communicating mathematics is also a human activity. *For the Learning of Mathematics, 2*(1), 3-7.

Fisher, C. W. (1990). Review of the research agenda for mathematics education. *Journal for Research in Mathematics Education, 21*(1), 81-89.

Frye, S. (1989). NCTM Standards for school mathematics: Visions for implementation. The challenge of implementation. *Mathematics Teacher, 82*(8), 664-671.

Gattegno, C. (1970). The human element in mathematics. In Association of Teachers of Mathematics (Eds.), *Mathematical Reflections.* (pp. 131-137). London: Cambridge University Press.

Gattegno, C. (1980). The foundations of geometry. *For the Learning of Mathematics, 1*(1), 10-16.

Grouws, D. A., & Cramer, K. (1989). Teaching practices and student affect in problem-solving lessons of select junior-high mathematics teachers. In D. B. McLeod & V. M Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 149-161). New York: Springer-Verlag.

Hanos, A., Harper, G., Morrison, T., Taylor, R., & Walker, K. (1994). *[New Venture Secondary School]: Formative evaluation project.* Unpublished document.

Hart, L. E. (1989). Describing the affective domain: Saying what we mean. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 37-45). New York: Springer-Verlag.

Higginson, W. (1980). On the foundations of mathematics education. *For the Learning of Mathematics, 1*(2), 3-7.

Hitchcock, G. (1992). The "Grand Entertainment": Dramatising the birth and development of mathematical concepts. *For the Learning of Mathematics, 12*(1), 21-27.

Hoyles, C. (1982). The pupil's view of mathematics learning. *Educational Studies in Mathematics, 13*, 349-372.

Jaworski, B. (1992). Mathematics teaching: What is it? *For the Learning of Mathematics, 12*(1), 8-14.

Jonassen, D. (1991). Evaluating constructivistic learning. *Educational Technology, 31*(9), 28-33.

Kilbourn, B. (1992). *Constructive feedback: Learning the art.* Toronto: The Ontario Institute for Studies in Education.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behavior. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 75-88). New York: Springer-Verlag.

Mandler, G. (1989a). Affect and learning: Causes and consequences of emotional interactions. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 3-19). New York: Springer-Verlag.

Mandler, G. (1989b). Affect and learning: Reflections and prospects. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 237-244). New York: Springer-Verlag.

Mason, J. (1989). Teaching (pupils to make sense) and assessing (the sense they make). In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 153-166). London: The Falmer Press.

Maxwell, J. (1989). Mathephobia. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 221-226). London: The Falmer Press.

McDonald, B. A. (1989). Psychological conceptions of mathematics and emotion. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 220-234). New York: Springer-Verlag.

McLeod, D. B. (1989). Beliefs, attitudes, and emotions: New views of affect in mathematics education. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 245-258). New York: Springer-Verlag.

Mehrassa, E. & Gardiner, S. (1994). *Educational Change–Implementation Project: Final report.* Unpublished document.

National Council of Teachers of Mathematics, Commission on Teaching Standards for School Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: The Council.

Novak, J. D. (1986). The importance of emerging constructivist epistemology for mathematics education. *Journal of Mathematical Behavior, 5*, 181-184.

NVSS. (1994). *Ma10 LG#1.* Unpublished document.

NVSS. (1995). *Ma9 LG1.* Unpublished document.

Perkins, D. N. (1991). What constructivism demands of the learner. *Educational Technology, 31*(9), 19-21.

Quaal, D. (1993). Mathematics teaching–a student's view. *Mathematics Teacher, 86*(6), 441.

Sanders, C. (1993). *Introducing [New Venture Secondary School]: "To seek challenge and to experience success."* Unpublished master's thesis, Simon Fraser University, Burnaby, British Columbia, Canada.

School District No. XX. (1992). *[New Venture Secondary School]: Guide for parents.* Unpublished document.

School District No. XX. (1993). *School Design Statement: [New Venture Secondary School].* Unpublished document.

Secondary School Consortium. (1992). *Math 9: Learning Guide #1.* Unpublished document.

Thompson, A. G., & Thompson, P. W. (1989). Affect and problem solving in an elementary school mathematics classroom. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 162-176). New York: Springer-Verlag.

Walsh, C. (1995). *Proposal for student progress journals.* Unpublished document.

Appendices

Consent Form Used for Students

SIMON FRASER UNIVERSITY

INFORMED CONSENT FOR MINORS BY PARENT, GUARDIAN AND/OR OTHER APPROPRIATE AUTHORITY TO PARTICIPATE IN A RESEARCH PROJECT

NOTE: The University and those conducting this project subscribe to the ethical conduct of research and to the protection at all times of the interests, comfort, and well-being of subjects. This form and the information sheet with it are given to you to ensure your full understanding of the procedures involved. Your signature on this form will signify that you have received the information sheet regarding this project, that you have received an adequate opportunity to consider the information, and that you voluntarily agree to allow the student for whom you are responsible to participate in the study.

As (parent/guardian) ________ of (name of student) ________________, I consent to the above named engaging in the interviews specified in the information sheet titled NVSS/SFU Mathematics Project to be carried out at New Venture Secondary School from May to November of 1994 in a research project supervised by Mrs. Beth Mehrassa of Simon Fraser University and School District No XX.

I certify that I understand the procedures to be used in this project and have fully explained them to (name of student) ________________________. In particular, (name of student) ______________________ knows that he/she has the right to withdraw his/her participation in this project at any time. He/she also knows that his/her identity will be protected because of the use of a pseudonym in the thesis, in any publication or conference presentation or in any discussion about the study. I understand that I may register any complaint I might have about the project with the chief researcher named above, or with Dr. Michael Manley-Casimir, Director of Graduate Programs, Faculty of Education, Simon Fraser University.

I may obtain a copy of the results of this study, upon its completion, by contacting Dr. A. Dawson, Faculty of Education, Simon Fraser University.

NAME (please print): ______________________________________
ADDRESS: ____

SIGNATURE: ______________________ DATE: _____________

NOTE: Once signed, a copy of this consent form and a subject feedback form should be provided to you.

Information Sheet Given to Parents/Guardians

NVSS/SFU MATHEMATICS PROJECT
INFORMATION SHEET FOR PARENTS/GUARDIANS

From: Mrs. Beth Mehrassa April 18, 1994
Teacher

As part of my Masters thesis at Simon Fraser University, I am planning to conduct research on the mathematics program at New Venture Secondary School. Since __________________ has expressed interest in volunteering for the study, and since your approval is necessary for him/her to participate, I would like to describe my plans.

I wish to interview students and mathematics teachers at New Venture and then use the transcribed interviews along with literature on New Venture as some of my data sources. My interest in this study is to develop and communicate my own understanding of the NVSS mathematics program as well as of mathematics teaching and of mathematics itself. I wish to enrich that understanding by seeking the perspectives of my colleagues as well as of students at New Venture.

You and your child can be assured of the following:

1) Students' participation in the study is completely voluntary. There will be no prejudice whatsoever towards those students who initially elect not to participate.

2) Parental consent is required for participation and may be withdrawn at any time.

3) Students can withdraw from the study at any time, with no negative consequences.

4) Students will be encouraged to be forthright and spontaneous in their responses. For instance, negative as well as positive comments about the mathematics program will be sought.

5) There will be complete anonymity and confidentiality of research findings. Students will be protected because of the use of a pseudonym in the thesis, in any publication or conference presentation or in any discussion about the study.

6) Any concerns about the study may be addressed at any time to me, to Dr. Michael Manley-Casimir, Director of Graduate Programs, Faculty of Education, Simon Fraser University, or to Mr. Estergaard, our principal.

7) A copy of the results of this study, once completed, may be obtained by contacting Dr. A. Dawson, Faculty of Education, Simon Fraser University.

If you have any concerns or questions about this study, please feel free to contact me at the school.

Questionnaire Consent

If you have never taken a math course at New Venture, return this questionnaire to your TA - do not fill it out.

I, (print name)____________________, voluntarily agree to fill out this questionnaire. I am aware that the results will be kept confidential and my identity will not be revealed in the thesis, in any publication or conference presentation or in any discussion about the study.

(signature)____________________(date)____________________

Directions Given to TA's for Administering the Questionniare

Students should not fill out questionnaire if:

•they have never taken a math course at NVSS

•they are an LD or ESL student who finds the reading too much (in this case, please return their blank questionnaire to me and tell me that student's name)

•they feel strongly against filling it out (should not be pressured into doing it)

General directions to announce:

•All questions are about the math program, not other programs at NVSS

•Students can check off as many items as they agree with for each question (they could even check off every single one for a particular question if they agree with all of the responses)

•For questions #2-9, the capital letter heading is important. It should precede every option since the student may agree with the option but not in the context of the question's heading. This is especially true with #2&3

Please return all completed questionnaires and cover sheets as well as any blank leftover ones to me

Thank you very much (please pass my thanks along to your TA as well) and sorry for the inconvenience.

Summary of Questionnaire Responses

Listed beside each response is the number of students who selected that response. There were a total of 85 completed questionnaires.

1) These are ways that students said they learn math at New Venture
Check as many that apply to you.
__63__ I read the LG.
__72__ I do questions.
__66__ I read the text / follow examples in the text.
__14__ I write rules out in my own words.
__33__ I copy examples from the text and go through them trying the steps on my own.
__54__ I use the answers to help figure out how to do the question (do question reverse style from answer to question).
__38__ I work with other students to get their input on how they do questions.
__74__ I ask a teacher if I get stuck.
__55__ I ask another student if I get stuck.
__54__ I do a test after working.

Check here if none of the above apply to you _____
List any other ways that you learn math here at New Venture:

2) These are things students said they liked about learning math this way
Check as many that apply to you.
I LIKE THAT:

__67__ I can go at my own pace
__41__ I prefer one-to-one help
__49__ I understand better even though it sometimes takes more time.
__42__ I like that I have to learn it in order to move on
__30__ My confidence can get built up by working on one subject for longer periods of time and experiencing success.
__61__ I can ask other students for help (we are allowed to talk)
__45__ I have easy access to teachers located all in one spot
__44__ I can ask many different teachers for help
__28__ I am exposed to different methods by different teachers and/or students
__44__ There are other options besides tests to show my understanding
__65__ If I know something, I can challenge it
__45__ I know what I'm supposed to do (so if I'm away I don't have to ask a teacher what to do)
__44__ Marking happens quickly
__77__ I can re-do it if I don't like my mark

Check here if none of the above apply to you _____
List any other things that you like about how you learn math at New Venture:

3) These are things students said they did not like about learning math this way.
Check as many that apply to you.
I DON'T LIKE THAT:

__61__ I can't get focussed sometimes
__60__ I sometimes find it difficult and it takes a long time to get through a LG
__49__ Some LG's are longer than others
__62__ When math teachers aren't available/are busy
__45__ When math marking doesn't get done right away
__28__ There are no classroom discussions
__11__ I can "cheat the system" by getting a test slip signed when I'm not really ready for the test, then bomb the test, then study for the rewrite

Check here if none of the above apply to you _____
List any other things that you do not like about how you learn math at New Venture:

4) Students felt that mathematics teachers should do the following:
Check as many as you agree with.
MATH TEACHERS SHOULD:

__76__ help students
__64__ explain things to students one-to-one
__52__ give students a chance to talk/be a good listener
__44__ start by asking students what they know
__51__ steer students, don't do the questions for them
__68__ be friendly and nice
__59__ build up student's self-esteem
__69__ be patient
__35__ assign questions
__59__ mark work
__33__ tell students to be quiet / make sure nobody is goofing off

Check here if you don't agree with any of the above_____
List any other things you think a math teacher should do here at New Venture.

5) When asked what qualities a successful math student has, students gave the following answers. Check as many as you agree with.
SUCCESSFUL MATH STUDENTS ARE THOSE WHO:

__55__ understand the basics of math course / but the time it takes to get an overall broad knowledge does not matter
__71__ want to try hard
__65__ have confidence in what they're doing
__64__ feel good / are happy with what they're getting
__64__ have the ability to push themselves
__64__ overcome challenges
__64__ accept responsibility
__74__ don't give up
__39__ can figure it out for themselves
__53__ are willing to try new things, think new ways
__43__ know themselves, know their ability
__58__ read the directions and understand them
__46__ are good listeners
__69__ ask for help and keep going back to the teacher until they're sure
__61__ don't avoid stuff they dislike or think is hard
__56__ don't let the teacher do all the work

Check here if you don't agree with any of the above _____
List any other qualities of a successful math student.

6) Students felt the math course at NVSS had an impact on other aspects of their learning. Here are those other things they said they learn. Check as many that apply to you.

I LEARN HOW TO:

__55__ be more responsible for what I do and what I learn
__49__ be focussed
__59__ push myself along
__66__ work/learn on my own
__44__ leave my work sometimes and go on to something else
__52__ be organized
__44__ be flexible
__66__ ask for help
__44__ help others/work with others
__51__ avoid giving teachers a hard time and to respect them so they will give that respect back to me.
__34__ develop the confidence to say to another person "this is how I do it" and to question what they're doing if I don't agree
__48__ communicate
__41__ be a good listener
__51__ get my mind working, I have to really think
__43__ be a good reader to understand how to do the questions
__51__ use a text (e.g., index)
__50__ mark work and check my answers
__58__ use answers to help me do the question

Check here if none of the above apply to you _____

List any other things you think you learn by doing your math at NVSS

7) These are some benefits students said they noticed from helping and getting help from other students. Check as many that apply to you.

HELPING AND/OR GETTING HELP FROM OTHER STUDENTS:

__63__ develops my understanding of math concepts
__53__ reviews parts of previous math for me
__41__ gives me a view of what is to come in later courses if I help someone in the next course up from me
__44__ improves communication skills
__52__ teaches me social skills (to work with people, to get along)
__48__ makes for a friendly atmosphere
__40__ makes me feel good
__44__ makes me feel more confident
__36__ is less intimidating than asking a teacher for help
__40__ helps me to see different ways of doing questions
__46__ shows me sometimes that I need some help myself when I can't help someone else
__44__ is faster than waiting for a teacher to help me

Check here if none of the above apply to you _____

List any other benefits you get from helping and/or getting help from students:

8) This is why students said it was important to be able to talk about off-topic things (not necessarily about your math questions) with teachers

Check as many as you agree with

IT IS GOOD TO BE ABLE TO TALK ABOUT THINGS OTHER THAN MATH WITH MY TEACHERS BECAUSE IT:

__60__ helps me relate to my teacher more, at the same level
__52__ gives me a chance to voice my opinion
__62__ helps me to feel comfortable with my teachers
__50__ adds interest
__33__ makes me think about other subjects
__41__ helps me to concentrate more on what the teacher is saying about the math when we first say something friendly
__44__ gets my frustrations out

Check here if you don't agree with any of the above_____

List any <u>other</u> reasons why you think it is good to be able to talk about off-topic things (not necessarily about your math questions) with teachers:

9) These are the benefits that students said they got from doing work other than questions from the textbook (e.g., making a poster, tutoring, doing a project)

Check as many as you agree with.

IT IS GOOD TO TRY DIFFERENT WAYS OF LEARNING MATH (OTHER THAN DOING TEXT QUESTIONS) BECAUSE:

__63__ it puts my skills to use
__44__ I think back to other units
__55__ I develop a better understanding
__42__ it ties in other subjects too
__55__ it gives me a bigger picture of what math is
__58__ it is more fun to learn

Check here if you don't agree with any of the above_____

List any <u>other</u> benefits you get from doing project-type work (work other than text questions)

Original Interview Questions

Students will be asked the following:

•How do you learn math at NVSS?
[this one question may lead to many others, depending on the student's answer; for example, "How do you and/or your teacher know that you've learned the math concepts?"]

•Tell me some things you like about or see as beneficial to your learning math this way.

•Tell me some things you dislike about or see as detrimental to your learning math this way.

•By doing your math course, are there other things (not necessarily math concepts) you learn or get out of it?
Do you consider these mathematical?

•What is a successful math student according to you?

In addition to the above questions (with slight alterations to make them applicable), teachers will be asked the following:

•What is mathematics to you?

•Do you express your vision of mathematics to your students?

•What is mathematics teaching, in your opinion?

Peer Tutoring / Course Summary

Name: ____________________ TA #______ Course _______ Date: ___________

Please journalize your activities as you help at the kiosk and engage in peer tutoring. (Please refer to the **Outcomes** on the other side of this page and comment on them with respect to your time in this LG.)

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How did you benefit from this experience and would you choose this option **again?**

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Also, please give your likes, dislikes and ways to improve the math course you just completed.

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Please return form to your marker for course completion! *PTO*

Summaries of External and Internal Views of Mathematics

The following are the two written summaries presented to the teachers (taken from Fisher, 1990, p. 82):

External View of Mathematics

On the one hand, mathematical knowledge can be assumed to be an external entity existing independently of human thought and action, and hence, something about which one can be objective. From this perspective, mathematical knowledge is viewed to be independent of human values and unrelated to the contexts within which humans find themselves when they inquire into or apply it. When viewed as external and objective, it is as though all mathematical knowledge were always in existence and occasionally mathematicians unconceal or discover another element, making it generally accessible.

Internal View of Mathematics

An alternative view posits that mathematical knowledge is internal and therefore subjective. From this point of view, mathematical knowledge depends on the values of the persons working with it and the context within which that work is conducted. From this perspective, mathematical knowledge is not so much discovered as created by social groups.