ENDOGENOUS GROWTH , HUMAN CAPITAL AND UNSKILLED LABOUR: A MODEL OF GROWTH IN A DEVELOPING COUNTRY WITH APPLICATION TO GROWTH AND OPENNESS IN DEVELOPING AMERICA

by

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Abstract

A model of growth in a developing economy is developed, extending previous models by Lucas (1988) and Dixit (1968). The model incorporates an elastic labour supply curve for unskilled labour that migrates from an informal sector to the formal sector over the course of development. It is shown that the transitional properties of the model with wage constraints are characterized by relatively physical capital intensive transitions and greater persistence of growth rates from balanced path growth rates. These properties improve the model's ability to fit with observed patterns of convergence of incomes and growth rates, across different developing and industrialized economies. A calibrated version of the model is used to assess the impact of trade liberalization on developing America.

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Table of Contents **Page**

List of Tables **Page**

List of Figures **Page**

I. Growth and openness in developing America: some issues

I.i Introduction

This dissertation is motivated by the ongoing trade policy reform in the developing American economies, (DAEs). In particular, the prospect of a Western Hemispheric Free Trade Area (WHFTA) has raised hopes that the developing American economies may be poised for a growth miracle, replicating the experience of East Asia.

In order to assess this possibility, an endogenous growth model is presented which describes accumulation in a developing country. The model follows Lucas (1988), so that savings decisions over physical and human capital are endogenous. Following Dixit (1968) however, the model is extended to allow for developing country characteristics. Over the course of development, unskilled labour migrates from an informal sector, where there is no accumulation, to the formal sector where accumulation of both human and physical capital occur.

The model is developed in chapter II and then is used to assess some of the dynamic effects of trade liberalisation in developing America in chapter III. These are evaluated on sectors where there is diminishing returns to accumulation of capital and, following the endogenous growth literature, on sectors where returns to human capital are non diminishing. In the latter case, policy changes which cause efficiency gains, such as trade liberalisation, result in changes in the long run equilibrium growth rate.

This chapter introduces some of the relevant concepts employed in the theoretical model of chapter II, and discusses some of the recent literature on growth and trade. It begins with a discussion of the post-war growth performance of the DAEs in *I.ii*. In section *I.iii* the Solow-Swan growth model is discussed as an introduction to the elements of growth theory, especially transition paths, balanced path growth rates and the role of technology. In section *I.iv*, a simple dual economy growth model is presented which attempts to integrate Lewis'(1954) model of development with growth theory. This provides an introduction to the model discussed in chapter II. The effects of productivity changes on growth are discussed in *I.v* and the recent empirical debate on the effects of openness are discussed in section *I.vi*

I.ii Recent growth patterns in South America

With the post-war success of Japan and the recent growth of other high performing Asian economies (HPAEs) it is difficult to avoid comparison with the performance of the DAEs. The comparison reveals that the DAEs grew relatively rapidly during the first half of the 20th century. Per capita GDP growth was around 1.5-2.0 percent, similar to that of the USA (Maddison 1994). In the post-war era, until 1980, growth remained respectably high, approximately 2.5-3.0 percent per year. Nevertheless, this was overshadowed by the performance of countries such as Japan, Korea, and Thailand, whose growth rates often exceeded 5-6 percent, Hofman (1993, p.244, Summers and Heston 1991). Moreover, in the 1980's growth in developing America came to a standstill due to the debt crisis.

The post-war performance of some of the larger developing American economies is compared with the USA in figures 1.1 and 1.2. The per capita GDP gap is shown in figure 1.1, which shows that although developing America kept pace with the USA, the gap was not closed very much. The growth rates over the post-war period are shown in figure 1.2, using a 10 year moving average. This highlights the convergence that occurred over the two decades after 1960. The average rate of convergence of income levels in this period - given by the difference between the two growth rates - was around 1.5-2 percent per year. The picture post 1980, however, is very different with negative annual growth rates in the region.

The growth rates of GDP per capita and GDP per worker are reported in tables 1.1-1.2 for the larger DAEs. Only Brazil achieved growth rates comparable with the HPAEs (see tables 1.5, 1.6). In the decade 1980-90, only Chile and Colombia achieved positive growth rates, with GDP per worker in Argentina and Venezuela falling almost 3 percent per year.

Thus, with the possible exception of Brazil, growth in developing America never reached levels like the HPAEs during the post-war era. Further, in all the

DAEs, growth collapsed in the 1980s. Thus, while the DAEs continued to catch up with the USA at a rate of 1.5-2 percent per year, this was much lower than the catch up in the HPAEs.

Hofman (1993) undertakes a growth accounting exercise comparing Latin America with other developing and developed economies. He shows that the Solow residual - which he interprets as reflecting disembodied technological change - in the HPAEs has been about 10% higher than the DAEs. By contrast, capital accumulation appears to be relatively more important in accounting for growth in the DAEs. The growth rates of net capital stocks in developing America are reported in tables 1.3, and 1.4. The data show that capital growth rates were higher than income growth rates, indicating considerable capital deepening over this period. Thus, while there has been considerable accumulation of capital, this did not result in as much income growth as might have been expected relative to the HPAEs.

Commentators on developing America's relative economic performance have attributed the difference in productivity growth to the different trade policy regimes of developing America and South East Asia. The latter were export orientated while the former followed import substitution industrialization (Edwards 1993a, Nogues and Gulati 1994, World Bank 1993, Hofman 1993, Maddison 1994).[1] To the extent that these policy differences may be partly responsible for the growth differences, there is some hope that recent changes in trade policy direction in the region may lead to a growth revival in developing America. Chile and Mexico in particular are currently involved in unilateral tariff reductions. Chile is seeking bilateral negotiations with the USA while Mexico has been committed to tariff reduction through the GATT and NAFTA. Other major South American countries have renewed regional trade blocs, the most important being the Andean pact, consisting of Bolivia, Colombia, Chile, Ecuador and Peru, and the MERCOSUR, signed by Argentina, Brazil, Paraguay and Uruguay. These developments may also be hastened by the Enterprise for the Americas' programme, for a WHFTA.

[1] These views are discussed further in section *I.vi*.

I.iii Accumulation in the Solow-Swan Model - explaining differences in growth rates

This section briefly reviews the Solow (1956) and Swan (1956) model of economic growth in a closed economy. This model reveals a number of insights into the nature of growth and thus also the potential effects of policy changes on growth. As such, the model has been used to analyse the growth performance of developing (and developed) economies.

Assume that aggregate income Y is equal to output which can be represented by an aggregate production function, with two inputs, capital, K, and labour, N.

$$Y = F(K, AN) \tag{1.1}$$

Capital is the accumulated factor while labour is given exogenously and grows at rate n. In addition, the parameter A captures any productivity increases, say due to technological change or knowledge. These are assumed to be Harrod neutral, and grow at rate g.

$$\frac{\dot{A}}{A} = g \tag{1.2}$$

The model assumes that a constant proportion of income, s, is saved. Ignoring depreciation the savings-investment condition is expressed ...

$$\dot{K} = sY \tag{1.3}$$

or

$$\frac{\dot{K}}{K} = s\frac{Y}{K} \tag{1.4}$$

This may be expressed in per worker terms as ...

(1.5) $$\frac{\dot{k}}{k}+n=s\frac{y}{k},$$

or in per effective worker terms, $\bar{k}=k/A$, as ...

(1.6) $$\frac{\dot{\bar{k}}}{\bar{k}}+n+g=s\frac{\bar{y}}{\bar{k}}.$$

From these expressions it can be seen that the growth rate of capital per worker depends on the average product of capital, which in turns depends on the level of A and the amount of capital per worker. For example, if the production function were Cobb-Douglas, then in per worker terms ...

(1.7) $$\frac{\dot{k}}{k}+n=s\left(\frac{A}{k}\right)^{1-\alpha}$$

Thus, the rate of accumulation is inversely related to the capital-labour ratio and positively related to the level of technology.

A balanced growth path can be defined as when the growth rate of capital per worker is constant. This requires that the left hand side of equations 1.5-1.7 be constant. Given that the production function is Cobb-Douglas, as in equation 1.7, the growth rate of A must equal the growth rate of k, so that the ratio $A/k=\bar{k}$ is also constant. Thus ...

(1.8) $$\frac{\dot{k}}{k}=g.$$

An increase in productivity will therefore have no effect on the long run growth rate. If $g > \dot{k}/k$, then $\dot{k}/k$ is increasing, whereas if $g < \dot{k}/k$, then $\dot{k}/k$ is decreasing. Thus the condition $g = \dot{k}/k$ is a stable equilibrium growth path - or balanced growth path.[2] The ratio of capital per effective labour on the balanced growth path is ...

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[2] Equivalently one could write $\dot{K}/K=n+g$ or $\dot{\bar{k}}/\bar{k}=0$.

$$(1.9) \qquad \left(\frac{k}{A}\right)^{*} = \left(\frac{s}{n+g}\right)^{\frac{1}{1-\alpha}}$$

This information is summarized in figure 1.3 which shows the relationship between $\dot{k}/k$ and k/A.

Growth in this model can be attributed to two factors. The vertical distance between the line g and the curve $s(A/k)^{1-\alpha}-n$, is the growth in capital per worker attributable to the convergence properties of the model, or the transitional dynamics. If the average product of capital is lower than the stable equilibrium level, for example at $(k/A)_0$, then growth will be faster than the balanced path rate. The transitional dynamics imply that an increase in the level of productivity, A, will raise the growth rate by reducing the ratio, k/A. The effect is temporary, however as the higher growth of capital eventually restores the equilibrium ratio of $(k/A)^*$. The second factor causing growth is simply the exogenously given increase in labour augmenting technology/knowledge, A. An increase in the rate of technology/knowledge growth, g, will also raise the rate of capital accumulation. In this case, however the increase in the growth rate is permanent.

These two sources of growth provide two potential explanations for why growth rates differ across countries. If two regions have the same long run balanced growth paths but one country has a lower per capita income, the transitional dynamics will cause that region to grow faster and "catch up" with the high income region in terms of per worker income levels. This is a potential explanation for the international convergence of incomes. The explanation depends on the equivalent levels of technology/knowledge across regions, which may be justified in that many forms of knowledge have public good characteristics and are thus available to all regions at zero cost (Fagerberg 1994). Alternatively, if technology levels are embodied in people or machines and thus do not have public good characteristics, then differences in tastes and technology will explain cross country differences in income levels and growth rates. In this case a low average product of capital relative to the equilibrium level may, nevertheless, still allow some partial, or conditional, convergence of incomes.

The second explanation for growth differences is, as suggested, simply that economies have different growth rates of knowledge or technology inputs - that g is different for different economies. These theories play down the role of accumulation of factor inputs and emphasizes the determinants of technical innovations, knowledge, human capital and related variables which affect the differences in technology and savings behavior between regions. An important reason why the growth rate, g, might be higher in developing economies is due to diffusion of technology. These explanations are outside the Solow-Swan framework described above, but a simple mechanism for endogenising technology/knowledge growth will be discussed below. First however, it remains to be seen how much growth the transitional dynamics can account for, without changes in the exogenous growth rate g.

Mankiw, Romer and Weil (1992) and Barro and Sala-i-Martin (1992, 1995) have used a Taylor series expansion around the steady state equilibrium to approximate the velocity of convergence of $\bar{k}$ or $\bar{y}$ to their steady state values.[3] This yields an equation which relates the log difference in capital to the savings rate, the steady state growth rate and population growth rate and initial income.

$$\ln(\bar{k}(t)) = (1 - e^{-\lambda t})\ln(\bar{k}^*) + e^{-\lambda t}\ln(k(0))$$

Using this model Barro and Sala-i-Martin show that regional districts in the USA, Europe and Japan approach their steady state growth rates at approximately 2% per year, that is $\lambda = 0.02$, implying a half-life of 35 years (Barro and Sala-i-Martin 1995, 1992, Sala-i-Martin 1994). The half-life measures the number of years the economy takes to move from an initial point a, in figure 1.3, to a point half way to the balanced path, b. It would then take another 35 years to reach point c and so on.

Similarly Mankiw, Romer and Weil (1992), estimate $\lambda = 0.02$ for the OECD between 1960 and 1985. For larger samples they estimate convergence rates of

[3] The convergence rate of capital and income per worker are equivalent, see Barro and Sala-i-Martin, (1995, p.36).

between 1.4 % per year and 1.8 % per year implying half lives of approximately 50 years, (Mankiw *et al* pp.426-429). Knight, Loayza and Villanueva (1993) employ the same model using panel data but obtain higher convergence rates of between 2.3 percent and 4.7 percent for developing economies.

The convergence rate, λ, has the interpretation of the speed at which the capital stock or income per effective worker approaches its steady state level. It is also the change in the growth rate of capital or income per effective worker for a given deviation of per worker income or capital from its steady state level. That is ...

$$\lambda = \frac{-d(\dot{\bar{y}} / \bar{y})}{d \ln(\bar{y} / \bar{y}^*)}.$$

The growth rate, $\dot{\bar{y}} / \bar{y}$, is zero on a balanced path, so $d(\dot{\bar{y}} / \bar{y})$ in the neighborhood of the balanced path, measures the change from zero, or the transitional growth rate. Thus, if $\bar{y}(t)=0.5\bar{y}^*$ and the convergence rate was 0.02, then the growth rate would be ln(0.5) 0.02 = 1.4 percent. Figure 1.4 shows the transitional growth rates (growth rates above the balanced path rate) predicted by a convergence coefficient of 0.02 and 0.005, for different levels of y/y^*. This is zero for $y(t)=y^*$, and tends to infinity as y/y^* approaches zero. Plotted against this curve are the observations for various countries. For each country the vertical axis shows the growth rate of GDP per worker between 1960 and 1990 minus the growth rate achieved for the USA over the same period. This can be interpreted as that country's transitional growth rate, assuming that the balanced path growth rate is equal to that of the USA. This is plotted against the country's 1960 GDP per worker relative to the USA, which has the interpretation of the distance from the balanced path assuming similar levels of technology exist in each country. It shows that a value of λ=0.02 is able to explain the East Asian miracle quite well as the curve comes close to the high growth countries in the sample, Hong Kong, Japan, Korea and Singapore and Taiwan. Despite this, there are many economies where growth rates are substantially below the predicted level, and even diverging, as indicated by negative transitional growth rates. Moreover, many DAEs fall in this category. The graph therefore, also highlights that differences in balanced path growth rates are an important part of the explanation of cross country growth differences.

A difficulty with the transition path explanation of cross-country differences in growth rates is that for these models to account for the empirically observed conditional convergence, after adjusting for difference in balanced paths, the capital share, α, must be large; around 0.75 which is twice the typically accepted value. Capital is therefore interpreted broadly as physical and human capital (Barro and Sala-i-Martin, 1995). King and Rebelo (1993) showed that if the capital share is large then models of this type with endogenous savings predict highly counterfactual investment/income ratios (King and Rebelo p.917). Barro and Sala-i-Martin, however, argue that savings rates in the order of 50%, (their finding for a capital share of 0.75) is reasonable given a broad view of capital (Barro and Sala-i-Martin, 1995, p.86). Similarly King and Rebelo (1993) and Pack and Page (1994) argue that the transition path explanation for cross country growth rates implies that the marginal product of capital must become unrealistically large when income levels are low relative to the steady state. This criticism also depends on more conventional values of the capital share·

A further difficulty is that the Solow-Swan model, as described above, predicts that the growth rates declines monotonically toward the balanced path level. However there is little evidence of this effect from economies which have made a successful transition. In the case of Japan, tables 1.5 and 1.6 show that growth was fairly constant between 1955 and 1970, when it dropped of sharply. Similarly, the post-war growth of Korea accelerated to high levels in the 1960's and has shown no evidence of a downward trend. In general, monotonically falling growth rates is not one of the stylized facts of development and growth experiences.[4]

Finally, Helliwell (1992a) and Helliwell and Chung (1992) found that the Mankiw *et al* model does not explain the cross sectional variation in growth rates in a sub-sample of Asian economies. Brander (1992) similarly notes that studies following Mankiw *et al*, have not found such strong correlations and

[4] For a recent list of "stylized facts" see Lau and Wan (1993).

that countries like South Korea are outliers in these regressions, after adjusting for balanced path characteristics.

The conclusion of these studies then, is that the transitional dynamics may account for growth and income differences across regions. Nevertheless, even if one accepts the findings of Mankiw *et al*, this still implies that changes in the balanced path growth rates are very important in explaining the cross country variation in growth rates. This is shown in particular by the fact that gross rates of convergence are often negative, and that in regressions, adjusting for different balanced path growth rates the HPAEs are outliers. Moreover there are further problems posed by the required size of the capital share, and the prediction of monotonically falling growth rates.

I.iv A dual economy growth model

In the preceding discussion, the supply of labour was assumed to be inelastic at every point in time. Dixit (1968) and Stern (1972) have developed growth models which incorporate Lewis' (1954) observation that the supply of labour to the industrial sector of a developing country may be highly elastic. Moreover, these labour market properties have been argued to be a crucial aspect of the development process (Minami, 1973, Kelly, Williamson and Chetham, 1972, Ohkawa and Rosovsky, 1973, Ranis and Fei 1961, Stiglitz 1992, Pack 1992). Similarly in recent cross country regression studies, there has been some evidence of the importance of this affect. For example Wolf (1995, p.756) finds strong convergence in agricultural sector and little convergence in labour productivity in manufacturing sectors, which he attributes to the vanishing surplus labour pool. Pack and Page (1994 p.211) argue that structural change between informal and formal sector in LDC's is correlated with initial levels, and may explain the significance of income levels in cross country regressions. Similarly, many studies by development economists such as Clark (1940) Kuznets (1966) and numerous studies by Chenery and Syrquin (see Syrquin 1988) have demonstrated the robustness of the structural transformation process across many different developing economies at different historical periods.

The standard Solow-Swan model can be modified to account for the dual structure in developing economies.[5] First the constraint is imposed that the real wage rate can not fall below the level of per capita income in the informal sector. Thus ...

$$(1.10) \qquad AF_2\,(K, AL) \geq \overline{w},$$

where L is the number of the total potential workforce N, who are employed in the formal sector, $L \leq N$. This reservation wage may be low in absolute terms but is nevertheless assumed to be initially above the minimum marginal product of labour required to fully employ the labour force. This in turn is low because the initial endowment of capital is assumed to be very small relative to labour. Thus the price at which firms are profitably able to employ labour is less than the price required to bring labour from the informal sector.[6]

Assuming that the production function is Cobb-Douglas, equation 1.10 can be rearranged to give a labour demand function.

$$(1.11) \qquad L = A^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha}{\overline{w}} \right)^{\frac{1}{\alpha}} K$$

Substituting back into the production function gives a reduced form equation for output.

$$(1.12) \qquad Y = K \left(\frac{(1-\alpha)A}{\overline{w}} \right)^{\frac{1-\alpha}{\alpha}}$$

This equation for output is linear in capital. Each additional unit of capital raises the amount of labour that can be employed, thus diminishing returns do

5 The treatment here follows Solow (1956), who used the same model to analyse "Keynesian" wage rigidities.

6 Following Dixit (1968) it is assumed that the wage is set by institutions so that the formal sector faces a constant wage for labour from informal sector. The assumption has been controversial. For overviews of the micro economic foundations and debates over the assumption of a fixed wage see Freeman (1993), Rosenzweig (1988) and Stiglitz (1988).

not set in as capital accumulates. To see the effect of this in the Solow-Swan growth rate equation 1.12 can be substituted into equation 1.4.

$$(1.13) \qquad \frac{\dot{K}}{K} = s\left(\frac{(1-\alpha)A}{\overline{w}}\right)^{\frac{1-\alpha}{\alpha}}.$$

In per worker terms,

$$(1.14) \qquad \frac{\dot{k}}{k} = s\left(\frac{(1-\alpha)A}{\overline{w}}\right)^{\frac{1-\alpha}{\alpha}} - n.$$

The average product of capital is now independent of the level of the capital stock. Accumulation occurs at a rate given by the expression on the right hand side of 1.14. If there is no technological change then the growth rate is constant. Alternatively if technological change is occurring, $g > 0$, then the growth rate of capital will be increasing as the level of technology rises.[7] With this modification the transitional dynamics are very similar to the linear '*AK*' endogenous growth model (see Barro and Sala-i-Martin, 1995). This contrasts with the conventional Solow-Swan model (or human capital augmented Solow-Swan model) where the growth rate falls monotonically toward the balanced path growth rate.

More generally, if the production function had more than two factors, the growth rate would not be constant but diminishing returns would set in less rapidly than in the conventional model. For example if the production function had three factors, physical capital, *K*, human capital, *H* and labour, *L*, ...

$$(1.15) \qquad Y = K^{\alpha} H^{\beta} N^{1-\alpha-\beta}.$$

Replicating the arguments above, it can be shown that growth of physical capital per worker is ...

[7] If wages grew exogenously at the same rate as technology, the rate of growth would also be constant.

(1.16) $$\frac{\dot{k}}{k}=s\left(\frac{h}{k}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{(1-\alpha-\beta)A}{\overline{w}}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}-n\,.$$

In equation 1.7 the elasticity of physical capital growth with respect to its level is $-(1-\alpha)$. With $\alpha = 1/3$, the elasticity would be -2/3. As discussed above, this appears to be too high to accord with the stylised facts of growth. In 1.14, the elasticity, is zero i.e. the level of the capital stock did not affect its rate of growth. Thus, aside from technological change, the growth rate is constant up until the turning point, thus showing strong persistence. In 1.16, the partial elasticity of the physical capital stock growth rate with respect to physical capital is $-\alpha/(\alpha+\beta)$, which, for example would be 1/2, if $\beta = 1/3$. This model would also be consistent with greater persistence of growth rates in transitions, though this depends on the accumulation process for human capital.

While the growth rate in the dual economy model has greater persistence over the transition, the growth rate will change at the point where the informal sector labour supply is exhausted. This occurs where the marginal product of labour evaluated at $L=N$ is equal to the wage, for example ...

(1.15) $$AF_2\,(K, AN) = \overline{w}$$

which for the Cobb-Douglas case implies ...

(1.16) $$(1-\alpha)y = \overline{w}$$

or by setting $L=N$ in 1.11,

(1.17) $$\tilde{k} = A^{\frac{-(1-\alpha)}{\alpha}}\left(\frac{1-\alpha}{\overline{w}}\right)^{\frac{-1}{\alpha}}$$

At this point the transitional dynamics will change from the dual economy case to the more conventional Solow-Swan case above.

Thus the dual economy assumptions applied to the Solow-Swan model results in a significant alteration to the transitional dynamics of that model. In particular it implies that diminishing returns will set in more slowly so that transitional growth rates will show more persistence than predicted by the standard formulation of the Solow-Swan model. This may therefore be a key issue in evaluating the pattern of growth in the DAEs and the effects of policy changes, such as trade liberalization, on the rate of growth.

I.v Growth and policy changes - the effects of openness.

This section attempts to assess how policy changes, such as trade openness, might be associated with changes in balanced path growth rates and with changes in the transition path.

The gains from trade have been divided into static gains and dynamic gains. The static gains are those welfare gains obtained by improving production and consumption efficiency to the internationally Pareto optimal allocation. Applied trade models, for example as described in Shoven and Whalley (1992), emphasise these static efficiency gains, but tend to ignore the dynamic gains from trade.[8] Dynamic gains from trade fall to two categories. First, there are the factor accumulation responses induced by the static impact on incomes (Corden, 1971, 1985). If some of the additional income is saved, the capital stocks may expand and further income effects are realised. If there are externalities associated with accumulation, as argued for example by Romer (1986) and Baldwin (1992), then this accumulation response will also realise additional welfare gains. Similarly, changing relative factor prices resulting from trade will also induce accumulation responses.[9]

Other dynamic gains are often less well defined and more difficult to quantify. They include changing savings and accumulation behavior, extending the size of the market, demonstration effects on learning and entrepreneurship and other behavioral impacts on productivity and technology transfer (Meier 1984, Myint 1958, 1977). The common feature of these ideas is that they indicate

[8] Recent applied general equilibrium models have however accounted for economies of scale effects of trade liberalisation, following Cox and Harris (1985).

[9] Baldwin (1992) argued, by the Stopler-Samuelson theorem, that trade policy may affect the rate of return to capital.

changes in production functions and taste parameters rather than simply increasing inputs for given tastes and technology.[10]

It is relatively straightforward to account for the accumulation effects as predicted by the models discussed above. Modeling the static gain as a productivity increase then, from equation 1.8, the increase in A requires an equivalent increase in k to restore the long run equilibrium. Given the production function in per capita terms is $y = k^{\alpha} A^{1-\alpha}$, the initial impact of the labour productivity increase is then simply $\frac{\partial \ln(y)}{\partial \ln(A)} = 1-\alpha$. From 1.9 however, as the economy returns to a balanced path, k must rise by the same fraction as A, so that the long run elasticity of per capita income with respect to A is unity. Thus there is a long run accumulation multiplier equal to the ratio of the long run effect to the short run effect, $1/(1-\alpha)$.

In addition to the effect on income, the immediate impact of the productivity change on the growth rate of capital can be seen, from equation 1.7, to be $\frac{\partial \ln(\dot{k}/k)}{\partial \ln(A)} = 1-\alpha$. This effect is temporary as the level of k also rises to reduce the growth rate of k. The growth rate thus rises by $1-\alpha$ %, then falls to zero. In the dual economy model the short run effect on the growth rate is different. From equation 1.14 it can be seen that the elasticity of capital accumulation is larger, $(1-\alpha)/\alpha$, in the two factor case.[11]. The effects on accumulation may, therefore, not only be more persistent than suggested by the conventional Solow-Swan model, but may also be larger.

These results apply to models where the aggregate production function exhibits diminishing returns to reproducible factors or where there are essential fixed factors in all the accumulation equations (Romer, 1991). If these conditions do not hold, the accumulation effects of the static gains may result in a permanent change in the growth rate, or a new balanced path growth rate. A simple example is provided by Lucas (1988, 1990, 1993) who adopts the assumption

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[10] Meier attributes these arguments to J.S. Mill *Principles of Political Economy* 2(3), 1848.

[11] The elasticity of the physical capital growth rate with respect to A, is $(1-\alpha-\beta)/(\alpha+\beta)$ in the three factor case.

that there are no fixed factors in the production of labour augmenting technology. He proposes a process for $A(t)$ ...

$$(1.18) \qquad \frac{dA}{dt} = \delta(1 - u(t))A(t)$$

where $u(t)$, represents a fraction of $A(t)$ being used on other activities such as physical-capital investment, or non economic activities. In this way, Lucas endogenises the supply of labour augmenting technology. Given a quantity of $A(t)$, the growth rate of $A(t)$ will depend on how much is being used for other activities.[12] A change in the productivity term δ generates a permanent increase in the growth rate of A.

Thus, if static welfare gains are realised in this sector of the economy, for example where R&D and learning activities occur, the economy will move to a higher balanced growth path. For example Lewis (1955), emphasizes that developing economies can only obtain the practical applications of knowledge if there is an appropriate institutional structure with which to market and profit from the ideas. Similarly classical economists such as Smith and Mill emphasised the educative effects of openness, (Meier, 1984 and Myint, 1977 p.247). According to Lewis, low income regions may face a lack of commercial incentives to exploit available technologies. These are enhanced through exposure to foreign cultures, foreign investment, and international trade, (Lewis 1955, pp.164-182, 280-282). Thus a policy change toward openness may result in a greater rate of technological progress though more efficient application of knowledge to economic activities.

Similarly Fagerberg (1994) reviews a number of empirical studies which emphasize the importance of domestic technological activities in developing regions in catching up with developed regions. Most prominent in this are Ohkawa and Rosovsky (1973) and Abramovitz (1994) who argue that 'social

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[12] A variation on this is provided by Rebelo (1992) and Azariadis and Drazen (1990). They argue that there may be a threshold level of human capital required for the accumulation process to occur. Below the threshold level there is no accumulation. Extensions to the basic idea that there is a sector producing "productivity improvements" is extended by Mulligan and Sala-i-Martin (1993) and Cabelle and Santos (1993).

capability' and/or 'technological congruence' largely determine a county's ability to realize its potential for catch up.

Lucas (1993) and Parente and Prescott (1994) modified 1.18 by allowing the growth of $A(t)$ to depend on the gap between $A(t)$ in the country and the global technology frontier. This then turns the model of domestic accumulation into a model of technology transfer or diffusion. The parameter δ can thus be seen as a barrier to technology adoption, representing the amount of resources it takes to get the technology frontier into the domestic country.[13] Other prominent studies emphasizing dynamic gains from technology diffusion are Nelson and Phelps (1966) World Bank (1991), various models by Grossman and Helpman (1990, 1992, 1994) and Barro and Sala-i-Martin (1995).

The analysis in chapter III attempts to quantify the accumulation effects of static efficiency gains from trade liberalisation. These will be considered both in the context of transitional effects and changes in balanced path growth. The former represent effects of higher savings on physical capital accumulation, as in Corden (1971, 1985). The latter represent the effect of efficiency gains in sectors which control the flow of technology or knowledge into economic inputs as suggested by classical economists, such as Mill and Smith, and development economists such as Lewis. While this by no means captures all of the dynamic trade gains discussed above, it is a useful first step in quantifying some of the dynamic effects of traditional, and relatively easily quantifiable efficiency gains.

I.vi Openness and Growth - empirical evidence.

The preceding discussion examined the potential effects of trade policy changes on models of accumulation. To complete the discussion this section briefly considers some recent empirical evidence on the relationship between growth and openness in developing economies, particularly in developing America.

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[13] Parente and Prescott present evidence showing that the rates of development have increased since the 1800's. It took economies in the post war era just 18-20 years to achieve the same change in per capita GDP as would have taken 45 years before 1913. They attribute this to the growth of world technology.

The World Bank (1993) has argued that the outward trade policy orientation of the HPAEs is largely responsible for their success.[14] Similarly recent surveys by Edwards (1993b), Havrylyshyn (1990), World Bank (1991) and Greenaway and Sapsford (1994) find general support for positive relationships between openness and growth. Edwards however notes the lack of explicit theories which predict higher growth rates resulting from an increased export orientation and some surveys note the lack of evidence on specific links between growth and trade policy. Havrylyshyn also notes that the evidence is weaker in the case of developing economies than industrialized economies.

Recent empirical cross country studies explicitly taking account of trade policy and recent developments in growth theory have been conducted by Benhabib and Speigal (1994) and Edwards (1992). They argue that the role of human capital in developing countries is to facilitate the diffusion of technology and that the human capital level also affects the endogenous rate of productivity growth. Whereas Benhabib and Speigal use estimates of average schooling to proxy for human capital levels, Edwards uses initial GDP per capita and the number of engineers engaged in R&D activities as measures of the technology gap. Edwards also directly incorporates indices of trade openness as explanatory variables in his regressions. Both studies obtain strong correlations and interpret these as suggesting that trade facilitates the rate of human capital diffusion from other countries.

Helliwell (1992a) and Pack and Page (1994) show that measures of trade openness have a positive partial correlation with growth rates in Asian economies. Helliwell uses frequency of non-tariff barriers, black market exchange premium and import duty collected as measures of openness (or closedness). He finds a significant relationship between openness and growth, the results are tenuous with a very small sample size. Pack and Page propose a slightly different thesis - that manufacturing exports growth, as opposed to openness, have led to productivity increases in manufacturing, and this has resulted in growth. According to Pack and Page, exports gave rise to unanticipated benefits in the form of increased ability to obtain knowledge

[14] This study and the lessons it draws for Latin America have been criticised by Felix (1994)

efficiently thus accounting for the link between productivity and exports, (Pack and Page 1994, p.228). Both Pack and Page and Helliwell conclude that their trade policy variables are unable to explain all the residual growth in the HPAEs, though they argue that trade policy has played a significant role.[15] Sachs and Warner (1995), however argue that a combination of three critera - relating to trade restrictions and the presence of a socialist economic structure - are sufficient to identify all of the slow growing economies in a sample of 117 countries. Thus they argue that economic policy, and in particular openness, is a crucial determinant of economic growth rates.

Two important voices of dissent for this explanation of the HPAEs growth performance come from Young (1994a, 1994b) and Levine and Renelt (1992). According to Young, a detailed analysis of factor accumulation eliminates the productivity residual obtained from growth accounting exercises on the HPAEs. The "Young hypothesis", thus contends that the growth miracles of the HPAEs are substantially explained by rapid factor accumulation and industrialization. The hypothesis is therefore consistent with the dual economy model above. Similarly, Levine and Renelt (1992) conduct sensitivity tests on many different regression equations on different data sets. They reject the emphasis on exports *per se* as a cause of growth. Nevertheless they find qualified support for a relationship between trade (exports or imports) and investment and growth.

Among these studies there are few which have attempted to explain growth in Latin America specifically. Nevertheless, among the many regression studies attempting to explain differences in cross country growth rates, many have

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15 The studies cited relate to developing country studies, or studies which include developing economies. There are a number of important studies showing links between trade and/or openness and convergence of incomes. Dowrick and Nguyen (1989) and Helliwell (1992b) have found that among trading groups such as the OECD and the G7 there is evidence of catch-up in the Solow residual, which they take as support for technological transfer. Likewise Ben-David (1991) uses the history of the EEC to test for increases in convergence effects resulting from trade liberalisation policies. He finds that during the nine year transition period when the EEC was undergoing tariff reduction, this difference fell to 0.6 of its initial level, which amounts to a half life for the disparity of 13 years. This is compared to a half life of 75 years for the same countries in the pre war, and pre EEC era. Similarly Coe and Helpman (1995) find positive correlations among industrial countries between total factor productivity and access to foreign trading partners R&D capital stock, which they interpret as evidence of R&D spillover affects from trade.

included dummy variables for Latin America as an "explanatory" variable. As shown in table 1.4, this variable always enters with a negative sign and is usually significant, with a low standard error.

The evidence from these studies suggests that Latin America has a worse than average growth rate after adjusting for a variety of potential explanatory variables, typically, the GDP level at the beginning of the growth period and estimates of factor inputs including schooling. Ades and Glaeser (1994), Levine and Renelt (1992) and Rebelo (1992) include measures of openness (the trade share of GDP and export share of GDP respectively) but obtain high standard errors on the Latin American dummy. According to Rebelo, this is due to multicollinearity between the explanatory variables, in particular between the dummy and initial income. Similarly, Barro and Lee (1994) use the black market premium on foreign exchange as a proxy for market distortions as an explanatory variable and find that this accounts for much of the low growth in Latin America, especially between fast and slow growing Latin American economies. Thus a possible interpretation of the lower growth rates in Latin America could be policy related variables and in particular differences in trade policy.

To the extent that models explaining the effects of trade policy on growth have been developed, many suggest that human capital, and in particular primary schooling may be important (Benhabib and Speigal, 1994, Wolff and Gittleman, 1993, Gemmell, 1995, Barro and Lee, 1994). Lau *et al* (1993) estimate production functions for Brazil and find that the output elasticity of schooling is very large and significant. They also find evidence of increasing returns to education in the form of a threshold effect at approximately 3-4 years of average minimum schooling. Moreover, recent estimates of human capital stocks by Nehru, Swanson and Dubey (1995) show that the recent growth rate of human capital has been much higher in East Asia than in Latin America.[16] Thus, human capital accumulation may also be an important part of the explanation for the DAEs relatively slow growth, either as a factor input,

[16] The level of average primary school enrollment rate in Latin America is, however, very similar to East Asia.

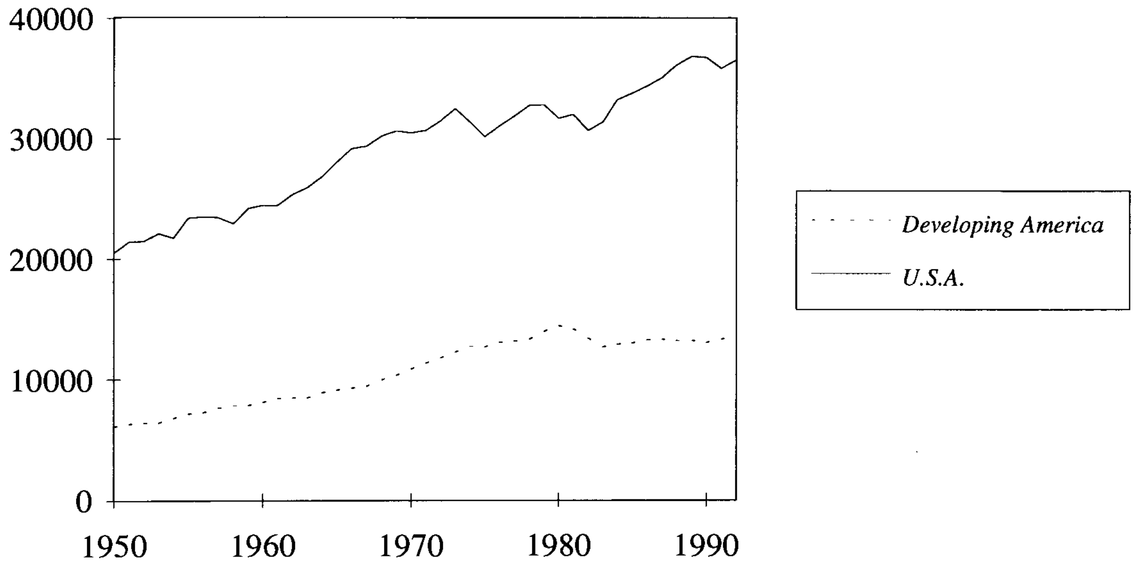
or as a means of facilitating technology transfer as suggested by Nelson and Phelps.

In sum, there is some qualified support for the hypothesis that trade openness raises growth rates over significant time periods, and some evidence that trade and human capital accumulation may be important explanatory factors in the relative growth performance of the DAEs. The weakness of the hypothesis, however is in identifying the specific links by which this process occurs.

I.vii Conclusion

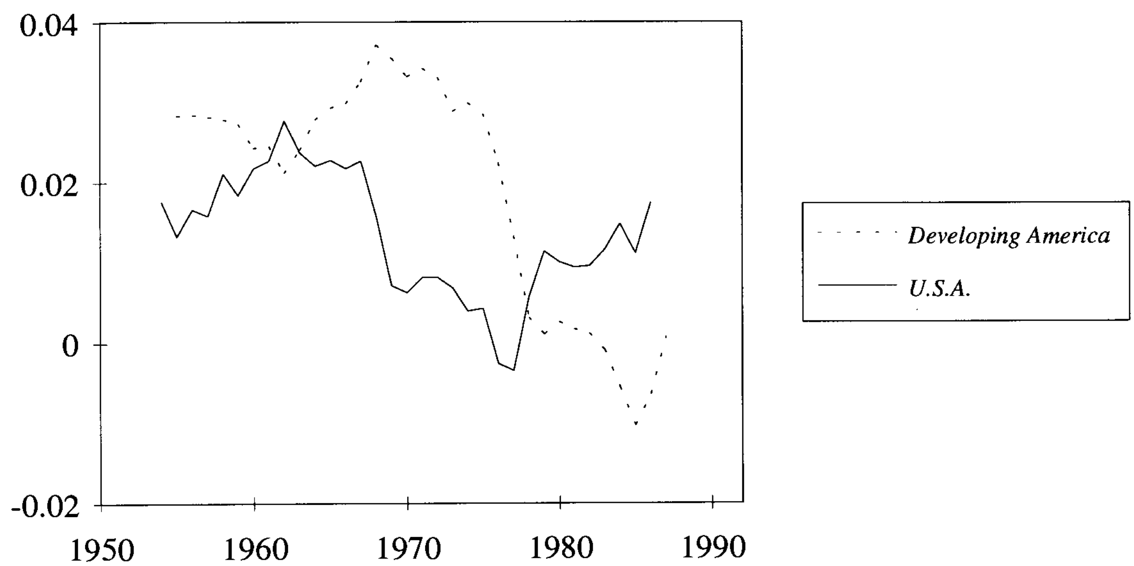
This chapter has introduced some of the main elements of growth theory and related them to the impact of trade policy changes and the implications for growth in developing America. The basis for this is the Solow-Swan growth model and the accumulation effects of static trade gains as discussed by Corden (1971, 1985). This theory is extended by considering the effects of different labour supply conditions on the transition path of the Solow-Swan model. It is also extended by considering the rationale for permanent changes in the rate of growth resulting from the traditional static trade gains. The discussion is supplemented with an overview of recent literature on the effects of trade policy on growth rates in developing economies which finds some evidence supporting the effects of trade policy and human capital accumulation on growth rates. This suggests that there is some qualified support for the hypothesis that domestic policies have been a significant negative factor in developing America's post-war growth.

Figure 1.1 - GDP per worker in South America and USA (constant 1985 PPP dollars)



Source: Penn World Tables Mk 5.6
Note: 1990-1992 are estimates based on World Bank (1994).

Figure 1.2 - Growth of GDP per worker in South America and USA ten year moving average, 1950-92



Source: Penn World Tables Mk 5.6
Note: 1990-1992 are estimates based on World Bank (1994).

Figure 1.3- Solow Swan Model

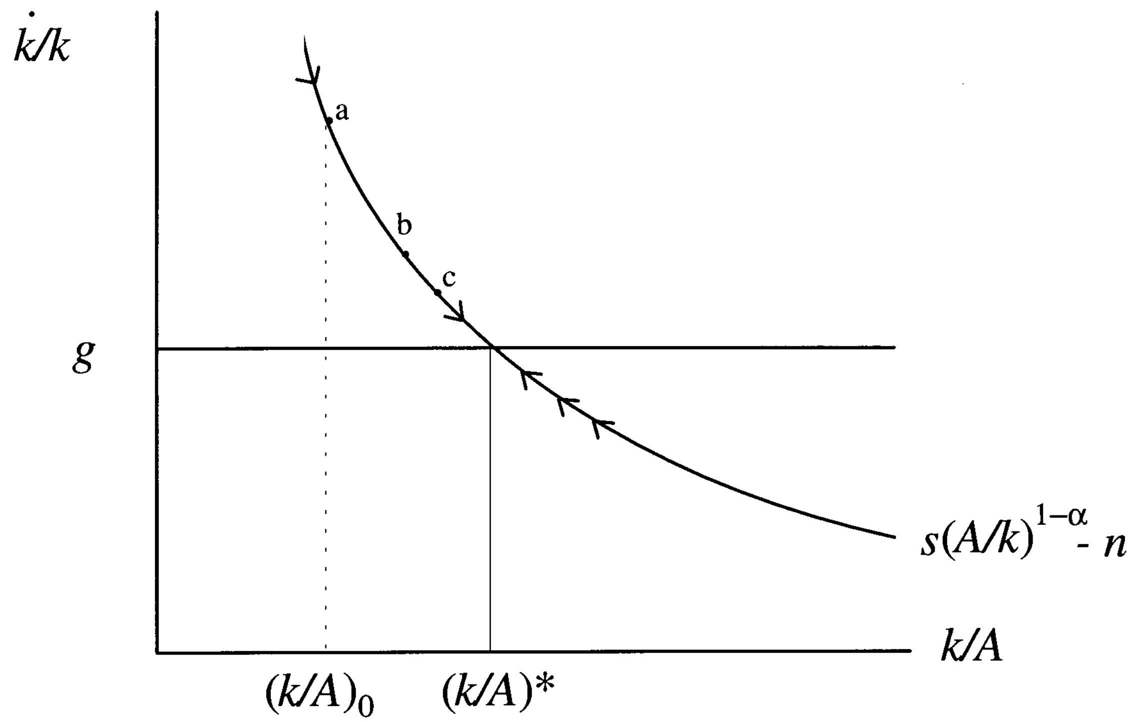
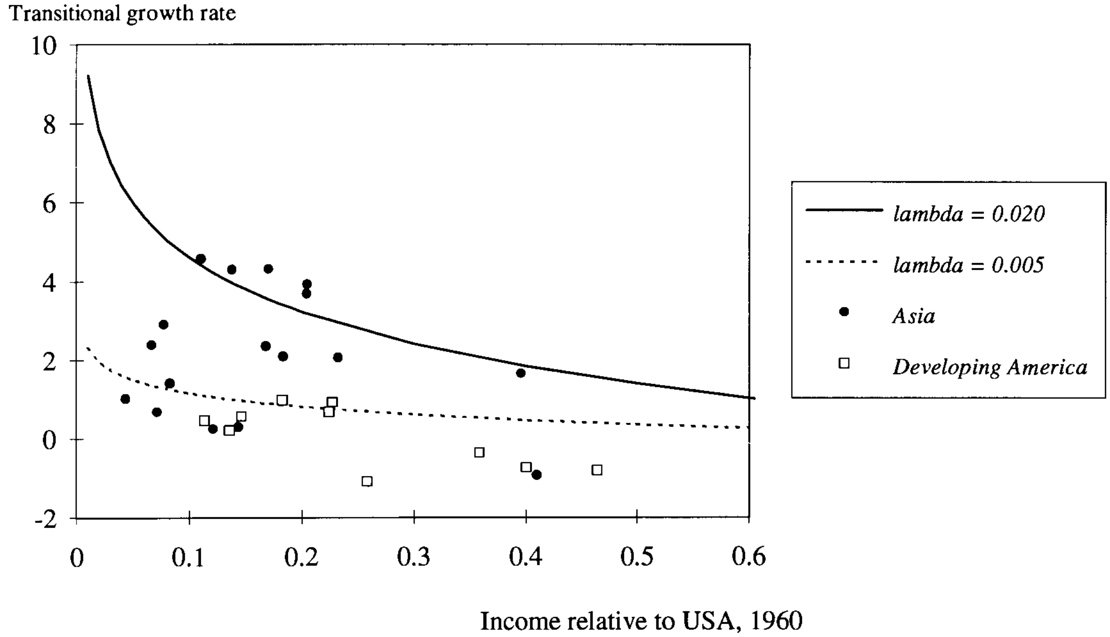


Figure 1.4 - Transitional Growth in Asia and Developing America and convergence properties of the Solow-Swan model
(GDP per worker)



Source: Penn World Tables Mk 5.6
Notes: Squares are Asian economies, circles are South American economies, see text for explanation.

Table 1.1 - Growth rates of real GDP per capita in developing American economies and the U.S.A.: 1950-1992

	Argentina	*Brazil*	*Chile*	*Colombia*	*Mexico*	*Venezuela*	*Average*	*U.S.A.*	*Difference*
1950-60	1.01	3.44	1.71	1.14	2.55	2.78	2.20	1.20	1.00
1960-70	2.34	3.11	2.23	2.40	3.41	2.02	2.79	2.70	0.09
1970-80	1.43	5.70	0.77	3.20	4.18	-0.46	3.66	1.65	2.00
1980-90	-3.24	-0.63	1.08	1.13	-0.38	-2.01	-0.79	1.66	-2.45
1990-92*	7.25	-2.02	5.99	1.20	3.53	7.83	2.12	-0.30	2.43

Source: Penn World Tables Mk 5.6
** Estimates based on World Bank (1994).*

Table 1.2 - Growth rates of real GDP per worker in developing American economies and the U.S.A.: 1950-1992

	Argentina	*Brazil*	*Chile*	*Colombia*	*Mexico*	*Venezuela*	*Average*	*U.S.A.*	*Difference*
1950-60	1.53	3.84	2.44	2.22	3.33	3.53	2.84	1.76	1.08
1960-70	2.44	2.88	2.76	2.64	3.92	2.68	2.93	2.21	0.73
1970-80	2.09	4.66	-0.04	2.86	2.93	-1.74	2.86	0.40	2.46
1980-90	-2.85	-0.65	0.30	0.62	-1.05	-2.54	-1.03	1.48	-2.52
1990-92*	7.52	-2.03	5.41	0.81	2.77	7.35	1.87	-0.31	2.19

Source: Penn World Tables Mk 5.6
** Estimates based on World Bank (1994).*

Table 1.3 - Growth rates of non residential net capital stocks in developing American economies: 1950-1992

	Argentina	*Brazil*	*Chile*	*Colombia*	*Mexico*	*Venezuela*	*Average*
1950-59	4.07	11.40	4.76	3.27	7.54	11.68	7.69
1960-70	4.93	7.13	4.70	3.90	6.00	2.66	5.54
1970-80	4.84	10.76	1.74	5.35	6.27	7.48	7.83
1980-89	-0.62	3.86	0.85	4.66	3.17	1.00	3.02

Source: Hofman (1992)

Table 1.4 - Growth rates of machinery and equipment net capital stocks in developing American economies: 1950-1992

	Argentina	*Brazil*	*Chile*	*Colombia*	*Mexico*	*Venezuela*	*Average*
1950-59	5.02	7.35	6.64	7.97	8.24	13.92	8.31
1960-70	7.31	5.01	3.84	2.59	14.89	0.46	6.58
1970-80	4.99	10.98	1.48	7.28	8.47	9.13	8.88
1980-89	-2.84	-0.26	2.54	4.39	1.63	0.28	0.51

Source: Hofman (1992)

Table 1.5 - Growth rates of real GDP per capita in selected Asian economies.: 1955-1992

	India	*China*	*Indonesia*	*Japan*	*Rep. Korea*	*Malaysia*	*Thailand*	*Philippines.*
1955-60	2.62	~	~	7.28	0.56	2.06	5.93	2.46
60-70	0.46	2.05	1.14	9.06	6.20	4.17	4.81	2.14
70-80	0.95	3.34	5.83	3.21	6.10	5.67	3.56	2.92
80-90	3.60	3.09	4.32	3.53	7.69	2.99	4.97	-0.64
90-92	0.71	6.01	3.14	2.63	~	5.73	4.82	-2.14

Source: Penn World Tables Mk 5.6
~ Not available

Table 1.6 - Growth rates of real GDP per worker in selected Asian economies.: 1955-1990

	India	*China*	*Indonesia*	*Japan*	*Rep. Korea*	*Malaysia*	*Thailand*	*Philippines.*
1955-60	3.26	~	~	6.58	1.50	3.18	6.61	3.73
60-70	1.21	2.28	1.39	8.36	5.53	4.33	5.00	2.55
70-80	1.43	2.69	6.03	3.46	5.33	4.35	3.24	3.00
80-90	3.45	2.12	3.79	3.29	6.94	2.46	4.52	-0.79

Source: Penn World Tables Mk 5.6
~ Not available.

Table 1.7-Coefficient on dummy variables for Latin America in recent cross-country regression studies.

Author-date	*Dependent variable GDP per capita*	*Coefficient on Latin American Dummy and standard error**	*Sample size*	R^2	*Trade related variables included*
Barro & Lee (1994)[1]	growth rate 1965-75/ 75-85	-0.0087 (0.0037)	85/95	0.57/0.60	black market premium
Barro (1991)	growth rate 1960-85	-0.0129 (0.003)	98	0.62	
Barro & Sala-i-Martin (1995)[1]	growth rate 1965-75/ 75-85	-0.0139 (0.004)	97	0.63/0.57	black market premium
Benhabib and Speigal (1994)	log difference in GDP 1965-85	-0.135 (0.065)	78	29.2[2]	
World Bank (1993)	growth rate 1960-85	-0.0131 (0.0039)	113	0.48	
Ades and Glaeser (1994)a	growth rate 1960-85	-0.0058 (0.0051)	65	0.66[3]	trade share in GDP (1960)
Ades and Glaeser (1994)b	growth rate 1960-85	-0.0061 (0.0068)	56	0.45[3]	trade share in GDP (1960)
Levine and Renelt (1992)a	growth rate 1960-85 (percent)	-1.18 (0.33)	103	0.68	
Levine and Renelt (1992) b	growth rate 1960-85 (percent)	-1.34 (0.38)	84	0.67	export share growth
Levine and Renelt (1992)c	growth rate 1960-85(percent)	-1.27 (0.36)	86	0.73	export share growth
Rebelo (1992)	savings rate	-0.001 (0.06)	98	0.24	export share of GDP
Rebelo (1992)	investment rate	-0.016 (0.013)	98	0.64	export share of GDP
Helliwell and Chung (1992)	log difference 1960-85[4]	-0.184 (0.102)	98	0.53	

* Standard error in brackets.
1 Restricted seemingly unrelated regression (SUR) over 2 periods.
2 F value with 71 degrees of freedom
3 Adjusted R^2
4 GDP per equivalent adult.

II. Human-capital accumulation and the supply of unskilled labour.

II.i Introduction

This chapter presents an aggregate endogenous growth model that attempts to explain the pattern of growth for a developing country or region. The analysis concentrates on three distinguishing features of less developed countries (LDCs). They are; the elasticity of unskilled labour supply with respect to price, the levels of human and physical capital stocks relative to unskilled labour, and savings decisions regarding human and physical capital investments.

As in models of dualistic development, the unskilled labour supply curve to the formal sector is assumed to be infinitely elastic up until the "turning point" where the whole of the informal sector labour force is integrated into the formal sector. The formal sector is defined to be where accumulation of physical and human capital occurs, while the informal sector is outside the model, being simply a supply of unskilled labour. Thus, with accumulation unskilled labour migrates to the formal sector where it may acquire human and physical capital. These conditions are similar to those employed in growth models by Dixit (1968, 1973), Stern (1972) and Solow(1956), which in turn are based on development models of Lewis (1954) and Sen (1966). Further, the initial conditions assumed are that the developing country faces a relatively large stock of unskilled labour and relatively small stocks of human and physical capital.

These features are included in an endogenous growth model, due to Lucas (1988), where human-capital is the "engine of growth". Lucas' model was in turn based on Uzawa (1965), and has been extended by Cabelle and Santos (1993) and Mulligan and Sala-i-Martin (1993). In these models the long run balanced path growth rate is determined by the utility and production parameters of the model, in particular the productivity of human-capital.[1] The model presented in this chapter also exhibits these features on the long run

[1] This is true of the models in Lucas (1988), Cabelle and Santos (1993) and Mulligan and Sala-i-Martin (1993), but not Uzawa (1965).

balanced growth path when all informal sector labour has been absorbed into the formal sector.

The point of departure of this model is, therefore, in the transitional dynamics rather than the balanced path properties. In particular, it is argued that the nature of labour supply in the early stages of development allows a persistent divergence of growth rates from their balanced path values. Second, contrary to Mulligan and Sala-i-Martin (1993), it is argued that the optimal strategy for a developing country is likely to involve relatively physical-capital intensive accumulation compared to a mature economy.

Section *II.ii* describes the model and derives the necessary conditions for a balanced growth path. Section *II.iii* discusses the balanced path properties of the model. The implications of the dual economic structure are explored further in *II.iv* and *II.v*. Numerical solutions are presented in sections *II.vi* and *II.vii*. The conclusions are summarised in section *II.viii*.

II.ii Lucas' endogenous growth model with a dual economy structure.

The relevant production functions are given by (2.1) and (2.2).

$$(2.1) \qquad AF(K(t), H(t), L(t), u(t), t) = AK(t)^{\alpha}(u(t)H(t))^{\beta}L(t)^{(1-\alpha-\beta)}$$

$$(2.2) \qquad E(H(t), u(t), t) = \delta(1-u(t))H(t)$$

Equation 2.1 is the production function of the physical-capital/consumption good assuming that physical-capital and consumption commodities are perfect substitutes. It is homogeneous of degree one in three inputs, physical-capital, $K(t)$, effective units of human-capital, $u(t)H(t)$, and employed labour $L(t) \leq N(t)$, where $N(t)$ is the total labour in the formal sector at time t. Equation 2.2 is the production function for human-capital. It is linear in the only input, effective units of human-capital, $(1-u(t))H(t)$. Agents must choose the fraction of human-capital to be allocated to the production of physical-capital/consumption products, $u(t)$, and the fraction to be allocated to human-capital investment $(1-u(t))$.

The accumulation equations are assumed to be ...

(2.3) $$\dot{K} = AF\,(K(t), H(t), L(t),\, u(t),\, t) - C(t)$$

(2.4) $$\dot{H} = E(H(t),\, u(t),\, t)$$

where $C(t)$ is consumption.

The production functions differ slightly from those employed by Lucas (1988). Lucas' production function for human-capital is written in per capita terms as ...

$$\dot{h} = h(t)\,\delta\,(1 - u(t))$$

which is equivalent to ...

$$\dot{H} = \delta(1 - u(t))H(t) + nH(t)$$

where $n \equiv \dot{N} / N$.[2] Thus the human-capital equation employed by Lucas (1988), embodies the implicit assumption that human-capital grows at the same rate as the labour force even if no time is allocated toward human-capital. That is, if $u(t)$=1, $\dot{H} / H = n$.

This requirement would be restrictive in analysing the growth problems of developing economies. A constant human-capital per worker ratio cannot always be maintained in the face of rapid population growth. In equation 2.2 however, a faster growth rate of population reduces human-capital per worker.[3] As in Lucas (1988), the human-capital production function implies that a worker's accumulation of human-capital depends only on the worker's

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2 Lucas' equation is written in per capita terms as $\dot{h} = h(t)\delta(1 - u(t))$. Letting H=Nh, then by the product rule gives, $\dot{H} = N(t)\dot{h} + h(t)\dot{N}$. Substituting $\dot{h}$ into this expression gives $\dot{H} = N(t)\delta(1 - u(t))h(t) + h(t)\dot{N}$ or, $\dot{H} = \delta(1 - u(t))H(t) + nH(t)$.

3 Cabelle and Santos (1993) and Mulligan and Sala-i-Martin (1993) also discuss a more general model where human capital is produced with inputs of several factors. In particular it would be desirable to impose diminishing marginal productivity of human capital with respect to labour inputs. These would be useful themes to explore further.

own inputs. The assumption is common in the literature on human-capital and earnings functions, for example, as surveyed by Weiss (1986). As a consequence of the different treatment of human-capital, equation 2.1 allows for a qualitative difference between unskilled and skilled labour (see for example Barro, Sala-i-Martin and Mankiw, 1995, and Mankiw, Romer and Weil, 1992).

Production is assumed to occur in firms that rent capital and labour services and maximise profits at every moment of time. Firms must adjust their labour hiring decision to allow for workers taking time out for training courses, education or lower labour inputs owing to on the job learning, such as apprenticeships. Profits are equal to revenue minus payments for factor services. Perfect competition is assumed among firms so that profits are equal to zero. Ignoring time subscripts, this gives equation 2.5.

(2.5) $$\Pi = A\,F(K,uH,L) - rK - q(uH) - wL = 0$$

where Π is firms' profits, r is the return to physical-capital, q is the return to human-capital and w the return to unskilled labour. The first order conditions for profit maximisation are ...

(2.5a) $$A\,F_1\,(K,\ uH,\ L) = r$$

(2.5b) $$A\,F_2\,(K,\ uH,\ L) = q$$

(2.5c) $$A\,F_3\,(K,\ uH,\ L) = w.$$

In addition, there is assumed to be a reservation wage, $\overline{w}$, which must be offered to attract labour from the informal sector, $w \geq \overline{w}$. All labour is identical so the wage $\overline{w}$ is sufficient to attract all available labour and the labour supply curve is a horizontal line at $\overline{w}$, up until $L(t)=N(t)$. At this point the labour supply curve at any time is vertical. The labour supply conditions mean that the labour employed in the formal sector at any time will be determined by the demand for labour. Thus, firms choose $L(t)$ given the constraint that ...

(2.5d) $$A\,F_3\,(K,\ uH,\ L) \geq \overline{w},\ \forall \quad L(t) < N(t).$$

The constraint will cease to bind when ...

$$(2.5e) \qquad A\,F_3\,(K,\, uH,\, N) = \overline{w}.$$

The first part of this constraint, 2.5d, combined with 2.5c, says that firms are not able to employ any unskilled labour for a wage less than $\overline{w}$. The second part limits the operation of the constraint to where $L(t)$ is strictly less than $N(t)$. When full employment is attained the constraint is assumed to disappear. This allows the wage to be less than $\overline{w}$, if $L(t)=N(t)$.[4] Combining 2.5c, 2.5d and 2.1 gives the labour demand equation when the constraint is binding.

$$(2.6) \qquad \tilde{L} = K^{\frac{\alpha}{(\alpha+\beta)}}(uH)^{\frac{\beta}{(\alpha+\beta)}}\left(\frac{(1-\alpha-\beta)A}{\overline{w}}\right)^{\frac{1}{(\alpha+\beta)}}$$

Equation 2.6 determines the equilibrium demand for labour when the full employment marginal product of labour is less than the reservation wage, $AF_3(K,\, uH,\, N) < \overline{w}$. At any time the equilibrium quantity of labour employed is given by ...

$$(2.7) \qquad L^* = \min\left[\tilde{L},\, N\right].$$

There are, therefore two phases to consider. They are the classical phase (phase I), where the reservation wage is binding, and the neoclassical phase where the wage is flexible and all agents are employed in the formal sector (phase II). When the minimum wage is in effect, the production function can be expressed in terms of K, H and $\overline{w}$ by using equation 2.6 to eliminate L. Thus in phase I ...

$$(2.8) \qquad AF(K,uH,L^*) = G(K,uH,A,\overline{w})$$
$$= K^{\frac{\alpha}{\alpha+\beta}}(uH)^{\frac{\beta}{\alpha+\beta}}\left(\frac{(1-\alpha-\beta)}{\overline{w}}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}A^{\frac{1}{\alpha+\beta}}$$

[4] This is discussed further in section *II.iv*.

The production function in phase I is homogeneous of degree one in physical-capital and human-capital. This follows from the fact that the demand for labour is also homogeneous of degree 1 in both capital inputs. If each unit of physical or human capital is increased by an amount, ϕ, from 2.6 this raises the demand for labour by ϕ, and so output in the formal sector also increases by ϕ.

It is assumed that a representative consumer maximises intertemporal utility given by...

$$(2.9) \qquad V = \int_{t=0}^{\infty} N(t)\, U(c(t))\, e^{-\rho t}\, dt$$

where $c(t)$ is consumption per capita, $c(t) = C(t)/N(t)$, ρ is the rate of time preference and $U(c(t))$ takes the form ...

$$(2.10) \qquad U(c(t)) = \begin{cases} \dfrac{c(t)^{\left(1-\frac{1}{\sigma}\right)}}{1-\frac{1}{\sigma}}, & \sigma \neq 1 \\ \ln(c(t)), & \sigma = 1 \end{cases}$$

where σ is the intertemporal consumption elasticity of substitution.[5]

The consumer's budget constraint equates income from physical-capital returns and human-capital returns and wages with expenditure on physical-capital accumulation and consumption. This is expressed ...

$$(2.11) \qquad N c + \dot{K} = wL + rK + q(uH),$$

where the right hand side is equal to $F(K, uH, L^*)$, by Euler's theorem of linearly homogeneous equations. The problem for the representative consumer is stated in Definition 1.

[5] Utility is assumed to depend on consumption per worker, rather than consumption per worker employed in the formal sector. This is mainly for simplicity. The implicit assumption is that workers in the formal sector share their consumption with those in the informal sector, through, for example remittances. Dixit (1968) has also justified the assumption by arguing that the lower utility implied by using consumption per worker could reflect a disutility associated with poverty in the informal sector.

Definition 1: Find an optimal set of paths $\{c(t), K(t), H(t), u(t)\}$ that maximise (2.9) subject to (2.1)-(2.4), (2.7), (2.10), (2.11) and the initial endowments $K(0)$, $H(0)$, over an infinite time horizon.

To solve this the current value Hamiltonian for the problem in Definition 1 is formed.

$$\begin{aligned} \boldsymbol{H}(c,u,\lambda_1,\lambda_2,K,H,L,t) = {} & NU(c) \\ & +\lambda_1[AF(K,\ (uH),L^*) - Nc] \\ & +\lambda_2[\delta(1-u)H] \end{aligned} \tag{2.12}$$

where the λ_i, are co-state variables. According to the Maximum Principle,[6] for an optimum solution to Definition 1 ($K(t)^*$, $H(t)^*$, $c(t)^*$ $u(t)^*$) there is a pair of costate variables $\lambda_1(t) > 0$, $\lambda_2(t) > 0$ that are continuous functions of time, t, such that $K(t)^*$, $H(t)^*$, $c(t)^*$ $u(t)^*$, $\lambda_1(t)$ and $\lambda_2(t)$ simultaneously satisfy equations 2.12a - 2.12f.

$$\boldsymbol{H}_c = c^{\frac{-1}{\sigma}} - \lambda_1 = 0 \tag{2.12a}$$

$$\boldsymbol{H}_u = \lambda_1\left[AHF_2(K,uH,L^*)\right] - \lambda_2\delta H = 0 \tag{2.12b}$$

$$\dot{K} = AF(K,\ uH,\ L^*) - Nc \tag{2.12c}$$

$$\dot{H} = \delta(1-u)H \tag{2.12d}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \lambda_1\left[AF_1(K,uH,L^*)\right] \tag{2.12e}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \lambda_1\left[AuF_2(K,uH,L^*)\right] - \lambda_2\delta(1-u) \tag{2.12f}$$

Equation 2.12f may be simplified using 2.12b. Simplifying 2.12b gives $\lambda_1 = \lambda_2\delta / AF_2(K,uH,L^*)$. Using this to eliminate $\lambda_2(t)$, 2.12f simplifies to ...

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6 See for example Leonard and van Long (1992), theorem 6.3.1, p.193.

$$\dot{\lambda}_2 = \rho\lambda_2 - \lambda_2 u\delta - \lambda_2\delta(1-u)$$

or

(2.13) $$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta \ .$$

II.iii Behaviour on the Balanced growth path

The problem 2.12a -2.12e and 2.13, is an autonomous infinite horizon problem and, as such, the solution must either be unstable, or saddle path stable. In such problems it is usual to employ steady state or balanced path conditions, rather than a transversality condition, as a boundary condition (Kaimen and Schwartz, 1981, p.159). In this section the necessary conditions for a balanced growth path are derived and the solution is described.[7]

The problem in Definition 1 requires four boundary conditions. Two boundary conditions are provided by the initial conditions $K(0)$ and $H(0)$. The remaining two are derived from the balanced path conditions. First the balanced path for phase II is defined in terms of the time derivative of the control variables.

Definition 2: A balanced growth path in phase II is obtained at any time $s \in t$, when $\dot{u} = 0$ and $\dot{c}/c = \kappa$, where κ is a positive constant.

The conditions in Definition 2 can be used to derive the behaviour of the state and costate variables on the balanced growth path. From 2.12d, when $\dot{u}$=0, then ...

(2.14a) $$\frac{\dot{H}(s)}{H(s)} = \delta(1-u^*) \equiv v + n,$$

or

[7] Following Lucas (1988) the existence of a balanced path is assumed, but not proven. This assumption, however has not been contradicted by numerical experiments described below. A proof of stability towards a balanced path in Lucas' (1988) endogenous growth model is given by Cabelle and Santos (1993).

(2.14b) $$u^* = 1 - \frac{v+n}{\delta}$$

where u^* is the balanced growth path value of u, v is the constant rate of growth of human-capital per worker and n is the exogenously given growth of labour. Thus on the balanced growth path the growth rate of human-capital is constant. From 2.12e...

(2.15) $$\frac{\dot{\lambda}_1}{\lambda_1} = \rho - AF_1(K, uH, L^*),$$

and from 2.12a,

(2.16) $$\frac{\dot{\lambda}_1}{\lambda_1} = -\frac{1}{\sigma}\frac{\dot{c}}{c}.$$

Combining 2.15 and 2.16 gives the "Keynes-Ramsey" equation, describing the relationship between the growth of consumption and the marginal product of capital.[8]

(2.17) $$\frac{1}{\sigma}\frac{\dot{c}}{c} = \left[AF_1(K, uH, L^*) - \rho\right].$$

The Keynes-Ramsey equation on the balanced path is therefore ...

(2.18) $$\frac{\kappa}{\sigma} + \rho \ = \ AF_1(K(s), u^* H(s), N(s)) .$$

Thus on the balanced growth path with $\dot{u} = 0$ and $\dot{c}/c = \kappa$, the marginal product of capital must also be constant and equal to $\kappa\sigma^{-1} + \rho$. Noting the specific form assumed for $F(.)$ in equation 2.1, along the balanced path it must be the case that ...

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[8] The name comes from Blanchard and Fisher (1989), who use it to describe the analogous condition in the standard growth model, which governs the evolution of optimal consumption in an infinite horizon. It is an acknowledgment to Ramsey (1928) who, in turn, attributes it to J.M. Keynes.

$$\text{(2.19a)} \qquad \frac{h^{\beta}}{k^{1-\alpha}} = \frac{\rho + \sigma^{-1}\kappa}{\alpha A u^{\beta}} \equiv z,$$

where $k=K/N$ and $h=H/N$. Equation 2.19a describes a curve, z, in $\{k,\ h\}$ space that intersects the origin. All the points on this curve that also satisfy $u(s)=u^*$, are points on the balanced path. Differentiating 2.19a with respect to time yields the required balance between the factor inputs.

$$\text{(2.19b)} \qquad \frac{\dot{k}(s)}{k(s)} = \frac{\beta}{1-\alpha}\, v$$

Equation 2.19b shows that the growth rate of the physical-capital stock must also be constant in the balanced path, and will be a constant fraction, $\beta/(1\text{-}\alpha)$, of the growth of human-capital. Thus for example, if $\alpha=\beta=1/3$ the growth of physical-capital per worker will be half the growth rate of human-capital per worker. Given that the factor shares must also be constant, 2.19b implies that the marginal product of human-capital must always be falling faster (or increasing slower), than the marginal product of physical-capital on a balanced path. From 2.18, however, the marginal product of physical-capital is constant along the balanced path, so the marginal product of human-capital must be falling. This can be seen by expressing the marginal product of human-capital in terms of the average product, that is $q = \beta \frac{y}{uh}$. Because, $v > \kappa$, the average product of human-capital, y/h, falls along the balanced path. Despite this, the quantity of human-capital per worker, h, is rising so that the average skilled wage per worker, $quh = \beta y$, is rising along a balanced path.[9]

Finally the relationship between the growth rate of consumption and the capital stock can be derived. This may be seen via the constant relationship between the marginal and average product of capital. The marginal product of capital is equal to $\alpha Y/K$. From this and combining 2.11 and 2.18 ...

$$\text{(2.20)} \qquad \alpha\left[\frac{N(s)c(s)}{K(s)} + \frac{\dot{K}(s)}{K(s)}\right] = \frac{\kappa}{\sigma} + \rho$$

9 This result follows from the assumption of constant factor shares.

The right hand side of 2.20 is constant as is $\dot{K}(s)/K(s)$, so it follows that $N(s)c(s)/K(s)$ is also constant on the balanced path. Differentiating 2.20 with respect to time therefore shows that the rate of growth of consumption per worker must be equal to the growth rate of the physical-capital stock per worker, $\dot{k}(s)/k(s) = \kappa$. Thus on the balanced growth path the growth rate of consumption and capital are constant and related to the growth rate of human-capital as in 2.21.

$$(2.21) \qquad \frac{\dot{k}(s)}{k(s)} = \frac{\dot{c}(s)}{c(s)} = \kappa = \frac{\beta}{1-\alpha} v$$

The value of k and v are determined by the parameters of the model. Following Lucas (1988) one may differentiate 2.12b and solve for v, the rate of growth of human-capital per capita.

$$(2.22) \qquad v(\beta - 1) = \rho - \delta - (\alpha - \sigma^{-1})\kappa.$$

Equation 2.21 can then be used to eliminate k. Solving for v then gives ...

$$(2.23) \qquad v = \frac{(1-\alpha)(\delta-\rho)}{\sigma^{-1}\beta + (1-\alpha-\beta)}.$$

Having determined the value of v, the value of k and the balanced growth path value of u are also determined by equations 2.14b and 2.19b.

These results are summarised in Result 1.

Result 1. If the economy reaches a balanced path, where $\dot{c}/c = \kappa$ and $\dot{u} = 0$, then $\dot{k}/k = \kappa = \beta/(1-\alpha)v$, where v is defined by 2.23, and all points lie on the curve in $\{k, h\}$ space defined by 2.19b. Along this curve the marginal product of capital is constant and equal to $\kappa/\sigma+\rho$, and the marginal product of human-capital is falling.

Having defined the necessary conditions for a balanced path, equation 2.18 can be used to impose an end boundary condition on either $K(s)$ or $H(s)$. On the balanced path these must be related by 2.18 so that the marginal product is constant. This condition imposes a restriction on the value of each state variable given the value of the other state variable. A further boundary condition may be obtained from 2.14. Rearranging gives ...

$$(2.24a) \qquad u = \left[\frac{\lambda_1 \beta A K^{\alpha} N^{(1-\alpha-\beta)}}{\lambda_2 \delta} \right]^{\frac{1}{(1-\beta)}} H^{-1}$$

so that in the balanced growth path the value of any one of the four state or costate variables (K, H, λ_1, λ_2) is determined by the other three contemporaneous endogenous variables. Thus, for example, the value of $H(s)$ must satisfy ...

$$(2.25) \qquad H(s) = \frac{1}{u*} \left[\frac{\lambda_1(s) \beta A K(s)^{\alpha} N(s)^{(1-\alpha-\beta)}}{\lambda_2(s) \delta} \right]^{\frac{1}{(1-\beta)}}$$

In addition to the four differential equations, 2.13c-2.13f, the solution to problem 1 also requires four boundary conditions. Two of these boundary conditions are the initial conditions on $K(0)$ and $H(0)$. The two end boundary conditions 2.18 and 2.25 ensures that the solution reaches a balanced path consistent with an infinite time horizon. Thus equations 2.12c-2.12f, 2.18 and 2.25, along with the two initial conditions, provide all the information required to obtain a solution in $K(t)^*$, $H(t)^*$, $\lambda_1(t)^*$ and $\lambda_2(t)^*$ from some initial point to some arbitrary point on the balanced path. Equations 2.12a and 2.13b can be used to find the implied optimal time path for $c(t)^*$ and $u(t)^*$ from this solution. The method of solving the model is described more fully in appendix A1.[10]

II.iv Behaviour in phase I.

To analyse the behaviour of the economy in phase I, a similar set of balanced path growth conditions are derived. Given the finite horizon, it is likely that the

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[10] It has not been proven that the solution reaches the balanced path described above. Numerical solutions have been used, however, to show that the balanced path is reached in the problems considered.

economy will never actually achieve a balanced path. It turns out, however, that the balanced path conditions in phase I are very useful in understanding the actual solution and for calibration purposes. The definition of a balanced path is the same as in Definition 2.

Definition 3: A balanced path in phase I is obtained when $\dot{u}=0$ and $\dot{c}/c=\gamma$, where γ is a positive constant.

If these conditions are met then the marginal product of capital will also be constant. In phase I, and equation 2.18 can be written ...

$$(2.18)' \qquad \frac{\gamma}{\sigma}+\rho = \left(\frac{\alpha}{\alpha+\beta}\right)K^{\frac{-\beta}{\alpha+\beta}}(uH)^{\frac{\beta}{\alpha+\beta}}\left(\frac{(1-\alpha-\beta)}{\overline{w}}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}A^{\frac{1}{\alpha+\beta}}$$

It was noted that the production function is homogeneous of degree 1 in $\{K, H\}$ in phase I. This implies that the marginal product of capital is homogeneous of degree 0 in $\{K, H\}$. This can be seen from 2.18′, where rearranging gives ...

$$(2.19a)' \qquad \left(\frac{H}{K}\right)^{\frac{\beta}{\alpha+\beta}} = \frac{\gamma\sigma^{-1}+\rho}{\left(\frac{\alpha}{\alpha+\beta}\right)\left(\frac{(1-\alpha-\beta)}{\overline{w}}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}A^{\frac{1}{\alpha+\beta}}u*^{\beta}}$$

The right hand side is constant and so the balanced path therefore lies on a ray from the origin in $\{K,H\}$, or $\{k,h\}$ space. Differentiating (2.18)′ gives ...

$$(2.19b)' \qquad \frac{\dot{K}}{K}=\frac{\dot{u}}{u}+\frac{\dot{H}}{H}.$$

If $u(t)$ is constant, as required on a balanced path, then the growth of the physical-capital stock will be equal to the growth rate of the human-capital stock. That is ...

$$\frac{\dot{k}}{k}=\frac{\dot{h}}{h}=\gamma.$$

A condition analogous to 2.22, describing the phase II balanced path growth rate of human-capital, can be derived for the economy in phase I. Differentiation of 2.12b gives ...

$$(2.26) \qquad \frac{\dot{\lambda}_1}{\lambda_1}+\frac{\alpha}{\alpha+\beta}\left(\frac{\dot{K}}{K}-\frac{\dot{u}}{u}\right)+\frac{\beta}{\alpha+\beta}\frac{\dot{H}}{H}=\frac{\dot{\lambda}_2}{\lambda_2}+\frac{\dot{H}}{H}$$

then substituting 2.12f and simplifying ...

$$(2.27) \qquad \frac{\dot{\lambda}_1}{\lambda_1}+\frac{\alpha}{\alpha+\beta}\left(\frac{\dot{K}}{K}-\frac{\dot{u}}{u}-\frac{\dot{H}}{H}\right)=\rho-\delta\,.$$

From 2.16, on a balanced path the growth rate of λ_1 is constant and equal to $\gamma\sigma^{-1}$. Further, from 2.19b′, the expression in brackets must be equal to zero. Thus on a balanced path 2.27 simplifies to ...

$$(2.28) \qquad \gamma\sigma^{-1}=\delta-\rho,$$

or

$$(2.23)' \qquad \gamma=\frac{\delta-\rho}{\sigma^{-1}}.$$

This condition holds whether or not $\dot{u}=0$ as long as $\dot{c}/c=\gamma$. Thus as long as the marginal product of physical-capital is close to its balanced path value in phase I, equation 2.23′ can be used to calculate the growth rate of consumption. This is useful because numerical experiments suggest that the growth rate of u does not tend toward zero. In particular $\dot{u}/u$ is often increasing as the boundary between phase I and phase II is approached.

If $\dot{u}/u$ approaches zero then the balanced path level of u can be obtained from 2.12d.

$$(2.29) \qquad u^*=1-\frac{\gamma+n}{\delta}.$$

Comparing 2.29 and 2.14b, if $\gamma < v$, then the balanced path value of u in phase I will be greater than that of phase II. Dividing 2.23′ by 2.23 it can be shown that $\gamma < v$ for all values of $\sigma < 1$. This ratio gives ...

$$\frac{\gamma}{v} = \frac{1-\alpha-\beta+\sigma^{-1}\beta}{(1-\alpha)\sigma^{-1}}$$

If $\sigma = 1$, then $\gamma = v$. In other cases, rearranging gives ...

$$\frac{\sigma^{-1}\frac{\gamma}{v}-1}{\sigma^{-1}-1} = \frac{\beta}{1-\alpha} < 1$$

so that if $\sigma < 1$, then $\gamma < v$ and the value of u^* is higher in phase I than phase II. Similarly taking the ratio of γ/κ gives ...

$$\frac{\gamma}{\kappa} = 1 + \frac{1-\alpha-\beta}{\beta\sigma^{-1}} > 1,$$

which is positive and greater than one for all values of σ. Thus, along the balanced path, growth rates of consumption and physical-capital are greater in phase I than in phase II. However it has not been proven that the transition path in phase I approaches a balanced path.

The major findings of this section are summarised in *Result 2*.

Result 2: In phase I, a balanced path requires that the rate of growth of consumption per worker is constant and equal to γ as given by 2.23′, and implies that the marginal product of capital is constant. This in turn requires that physical and human capital stocks per worker will also grow at rate γ, and that the locus of balanced path points is a ray intersecting the origin in $\{k, h\}$ space. The balanced path growth rate in phase I, γ, is greater than the phase II growth rate κ, and, if $\sigma < 1$, less than or equal to v.

II.v Transition between phase I and phase II.

In this section the behaviour of the model as it crosses the boundary between phase I and phase II is considered. The point of transition was defined above to be where;

(2.5e) $\quad A F_3 (K, uH, N) = \overline{w}$.

This equation defines a boundary in K, u, H, and N.[11] The boundary is shown in k, h space in figure 2.1. It forms a rectangular hyperbole between the two inputs. Thus reaching the boundary requires sufficiently high physical and human capital stocks, for a given level of u. Any combination of k and h above the boundary will be sufficient to employ all labour in the formal sector. An increase in u will decrease the required inputs of k, and h to reach phase II, while a decrease in u will increase the required inputs.

The boundary is well defined if the variables K, H, u and N are continuous across the boundary. The necessary conditions for the optimisation problem in Definition 1 do not permit any jumps in the state variables, K, H, but do permit discontinuities in $u(t)$.

From 2.8, in phase I,

(2.30) $\quad Y = AF(K, uH, L^*) = G(K, uH, A, \overline{w})$.

In the neighborhood of the phase I-phase II boundary, it is approximately true that, $L^*=N$ so that,

[11] In the numerical routines used for solving the system 2.12a - 2.12f, u is eliminated. The boundary condition is found by substituting 2.13b into 2.6 and setting $L=N$ gives ...

$$N = K\left(\frac{1-\alpha-\beta}{\overline{w}}\right)^{\frac{(1-\beta)}{\alpha}}\left(\frac{\lambda_1\,\beta}{\lambda_2\,\delta(\alpha+\beta)}\right)^{\frac{\beta}{\alpha}} A^{\frac{1}{\alpha}}$$

which defines the boundary in terms of the endogenous variables K, H, λ_1 and λ_2. See the appendix for details on the solution method.

(2.31) $Y = AF(K,uH,N) = G(K,uH,A,\overline{w})$

Thus for a given value of $u(t)$, Y will be constant across the phase boundary. Because the phase I production function is homogeneous of degree 1 in K and H, the partial derivatives with respect to K and H are homogeneous of degree 0. The partial derivatives of the production function in phase II, however, are homogeneous of degree 0 in three factors, K, H, and L. Thus the slope of the production function changes across the phase boundary. In particular ...

(2.32) $AF_1(K,uH,N) < G_1(K,uH,A,\overline{w})$

and

(2.33) $AF_2(K,uH,N) < G_2(K,uH,A,\overline{w})$.

For example the marginal product of physical-capital in phase I is, $r = \frac{\alpha}{\alpha+\beta}\frac{Y}{K}$, whereas in phase II, it is, $r = \alpha\frac{Y}{K}$. Thus as the economy crosses from phase I to phase II, the marginal product of physical-capital falls by a factor of $1/(\alpha+\beta)$. The change in the value of the partial derivative implies that $u(t)$ is discontinuous across the phase boundary. The solution for u in terms of K, H, λ_1, λ_2, is given in 2.13b. Rearranging this, in phase II and substituting for Y...

(2.24b) $u = \beta\frac{\lambda_1}{\lambda_2}\frac{A}{\delta}\frac{Y}{H}$.

In phase I,

(2.30b) $u = \frac{\beta}{\alpha+\beta}\frac{\lambda_1}{\lambda_2}\frac{A}{\delta}\frac{Y}{H}$.

From 2.31, in the neighborhood of the boundary, the value of Y is approximately the same K, H, λ_1, and λ_2 constant, so that from 2.24b and 2.30b the partial percentage change in $u(t)$, given Y, is equal to $\alpha+\beta-1$.[12] Thus

[12] This is calculated from equations 2.24b and 2.30b as (2.24b-2.30b)/2.30b.

the change in the slope of the production function causes a fall in $u(t)$ at the boundary.

From 2.17, the growth rate of consumption is determined by the difference between the marginal product of physical-capital and the rate of time preference, ρ. The fall in u also reduces the growth rate of consumption so that there is a fall in the growth rate of consumption across the boundary. Note however that the level of consumption is constant. These findings are summarised in result 3.

Result 3: The boundary between phase I and phase II is a rectangular hyperbole in k, h space. As the economy crosses from phase I to phase II the growth rate of c, K and Y, as well as the marginal products of human and physical capital and unskilled labour, will fall. In addition, the level of human-capital investment in human-capital production, $1-u(t)$, increases.

Thus given the continuity of K, H, λ_1, λ_2, the optimal value of $u(t)$ exhibits a discontinuity at the boundary between phases I and II. The potential indeterminacy is avoided as it has been assumed that the constraint only holds for values of $K(t)$, $H(t)$, $u(t)$ on or above the boundary. As the economy reaches the boundary, the wage is allowed to adjust in response to the jump in $u(t)$.

A possible trajectory is sketched in $\{k, h\}$ space in figure 2.2. Given some initial endowment $k(0)$, $h(0)$, the trajectory moves toward the boundary as the economy accumulates physical and human capital. Note that the boundary is a rectangular hyperbole, so that as long as either $\dot{k} > 0$ and $\dot{h} \geq 0$, or $\dot{k} \geq 0$ and $\dot{h} > 0$, then the trajectory must cross the boundary.

A possible solution path is plotted as the heavy line in figure 2.2. On a balanced growth path, k and h must lie on the curve, as drawn. This can be traced back to some point on the boundary, which does not have to be on the balanced path. Crossing the boundary the optimal path in $\{k, h\}$ must be continuous, so that the position that the phase II path cuts the boundary is also the terminal point for the phase I trajectory. It has been shown that the level of physical-capital investment falls and human-capital investment rises as the path crosses the boundary. Thus the slope of the solution path gets steeper as the economy crosses the boundary. Finally, the phase I path begins at the initial endowments $k(0)$ and $h(0)$ and moves toward the boundary. It does not necessarily approach the balanced path. The actual path followed in phase I and the point on the boundary at which the solution crosses are left as empirical matters.

II.vi Numerical Solutions for the model without a minimum wage constraint.

Mulligan and Sala-i-Martin (1993) use numerical methods to demonstrate the properties of their model, while Cabelle and Santos (1993) use more formal methods. This section uses numerical methods to examine the transition for the case where there is no reservation wage, so that the entire transition occurs in phase II. These indicate that in phase II, the transitional dynamics are the same as those found by Mulligan and Sala-i-Martin and Cabelle and Santos. The phase II solution provides a benchmark for comparing the phase I transition path, which is evaluated in section *II.viii.*

Experiments were conducted using numerical procedures based on FORTRAN routines described in Press *et al* (1990). The solution method is discussed in detail in Appendix A1. The initial values for labour and human-capital were set to 1, $H(0) = N(0) = 1$. The factor shares are assumed to be $\alpha = \beta = (1-\alpha-\beta) = 1/3$. With these parameters the balanced path growth rates, (equations 2.21, 2.23) simplify to ...

$$\kappa = \frac{\delta - \rho}{1 + \sigma^{-1}}$$

and

$$\nu = 2\kappa .$$

The experiments reported assume $\delta - \rho = 0.08 - 0.05 = 0.03$, which, with $\sigma = 1$, gives $r^* = 6.5\%$, which is consistent with the evidence of King and Rebelo (1993). The balanced path values are given in table 2.1 for different values of σ. The solutions for two different initial values $z(0)$ have been calculated. In the first, $z(0) = 2z^*$ and in the second, $z(0) = 1/2z^*$. Each experiment was conducted with a terminal horizon of 300 years as an approximation to an infinite horizon.

Figures 2.3a-2.3b show the transition of the per capita capital stocks from their initial position onto the balanced path, z*, where z* is equal to z (equation 2.19a) evaluated at $u(t)=u^*$. The capital stocks move towards the balanced path ratio with most of the adjustment occurring in the physical-capital rather than human-capital. The transition path then asymptotically approaches the growth path with increasing physical and human capital with constant $z = z^*$.

Figures 2.4a-2.4b show the transition in the space; $\bar{k}(t) = k(t)e^{-\kappa t}$ and $\bar{h}(t) = h(t)e^{-vt}$. In this space the balanced path becomes a steady state, and the set of steady state values is again given by the upward sloping curve z*. Again transitions are represented by movements onto the curve. The movement is south-east when $z>z^*$ as growth rates of k are greater than the balanced path levels but falling, and the growth rate of h is lower than the balanced path rate, v, but increasing. The results are reversed for $z < z^*$. The slope of these solution paths increases for lower values of σ. From equation 2.14, any deviation of h from its balanced path level relative to k is due to movements in u away from u^*. Thus the increasing slope of the transition paths, in $\{\bar{k}, \bar{h}\}$ space, for lower values of σ is due to relatively greater deviations in $u(t)$ from u^*.

Figures 2.5a-2.5b directly compare the evolution of $u(t)$ over the transition and the impact of the different initial conditions and values of σ on the path of u. The figures confirm that $u(t)>u^*$ for $z(t)>z^*$ and $u(t)<u^*$ for $z(t)<z^*$. Further they confirm that the variation in $u(t)$ from its balanced path value u^*, increases as σ gets smaller. When σ is low, there is relatively more substitution of human-capital between the two uses during the transition. When σ is high, there is relatively more substitution of consumption and investment.[13] Finally,

[13] Mulligan and Sala-i-Martin (1993) and Cabelle and Santos (1993) showed that for Lucas' model the relationship between the capital stocks and u depends on whether $\sigma < 1/\alpha$. If this condition holds the optimal choice of $u(t)$ reinforces the Solow-Swan and Keynes-Ramsey effects. The case where $\sigma > 1/\alpha$, has the opposite transitional behaviour but can be regarded as empirically less important. When $\sigma = 1/\alpha$, the value of $u(t)$ is constant in the transition, so that the transition reduces to that of the Ramsey model. Numerical experiments show that these results also hold for this model

figures 2.6a-2.6b show the evolution of the marginal product of capital, which is inversely correlated with $z(t)$. These findings confirm the behaviour of the transition path found by Mulligan and Sala-i-Martin (1993) and Cabelle and Santos (1993).

These figures show the pattern of the transition, but do not indicate how long the transition takes. King and Rebelo (1993) have argued that transition paths in neoclassical growth models do not provide a very good account of actual growth paths observed in economies where some transitional dynamics might be expected, for example in Japan's post war growth. Mulligan and Sala-i-Martin (1993), however, claim that the transition properties in their model can account for experiences of rapid industrialisation whereby countries such as Japan, Germany and Korea experienced 20-30 years of relatively high growth rates. They also argue that transitions involving relatively intensive accumulation of human-capital are slow and may be useful in understanding the accumulation behaviour of developing countries.[14]

The values, in tables 2.2a-2.2b, show the distance of the solution from the balanced path value attained after a given time. The distance is measured as the value of $z(t)$ as a percentage of the balanced path value. By definition, in the initial year, zero percent of the transition has been completed. The distance measures are also calculated for the growth rates of physical-capital, consumption and human-capital relative to their balanced growth path growth rates, κ and ν. The final row shows the percentage of the balanced path value that was attained after 100 years. Each table shows two transitions, one for $z(0) = 2z^*$ and one for $z(0) = 1/2z^*$.

Table 2.2a reports the results for $\sigma = 1$. When beginning with a high value of $z(0)$, indicating that physical-capital is relatively scarce, 99 percent of the

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[14] Mulligan and Sala-i-Martin (1993) argue further that the transition paths in Lucas' (1988) model have interesting empirical applications. For example the post war examples of Japan and Germany where the physical capital stock was destroyed probably gave a post war value of $z(t)$ greater than z^*. Lucas' model predicts that the transition will be characterised by rapid growth in capital and consumption, with high savings levels. On the other hand, a developing country, it could be argued, faces a high $z(t)$, relative to z^*, and this results in low savings and capital growth, but a relatively high growth of human capital. The optimal strategy for a post war economy is thus an industrial strategy, while the optimal strategy for a developing country is a renaissance strategy.

transition is completed within 40-50 years. However, 60-70 percent of the transition is completed in just eight years. Moreover this is true for much lower values of σ. It can be seen that the half-life of the transition, that is the time taken by each variable to move from 0% to 50%, or 50% to 75% etc, is approximately 4-5 years. This would appear to be too fast to accord with observed growth experiences. For example, Japan sustained a rate of growth around 5 percent above the USA for 15-20 years (table 1.6). According to these results, a growth rate of 5% above the balanced path growth rate would have fallen to just 2.5% in just 4-5 years.

In the case where $z(t)<z^*$, the growth rates of physical-capital and consumption are lower than the balanced path rates, and the growth rate of human-capital is higher than the balanced path rate. As found by Mulligan and Sala-i-Martin (1993), the transition is slower in this case. The results suggest that the half-life of the transition is around 15 years. This is still relatively fast compared to empirical estimates discussed in chapter I, where the half-life of converging regions is argued to be approximately 35 years.

The final rows of tables 2.2a-2.2b show the total variation in the particular variable, between the initial value and the balanced path value, as a percentage of the balanced path value.[15] Comparing these across the different values of σ one may confirm that the variation in physical-capital growth rates and consumption decline as σ gets smaller, and that the variation in human-capital growth rates gets larger as σ gets small.

Thus transition times are very fast, with half of the transition occurring within 5 years in the case where $z(0) > z^*$ and in 15 years when $z(0) < z^*$. Moreover the length of time for the total transition is similar for different values of σ. On the basis of these results Mulligan and Sala-i-Martin's use of transition paths is subject to the criticism made by King and Rebelo (1993), of neoclassical growth models. The results of this section are summaries in Result 4.

Result 4. The model developed in this chapter, without a constraint on the minimum wage, has a transition path similar to Lucas' (1988) model as

15 By construction this is always -50 percent or 100 percent for z.

described by Mulligan and Sala-i-Martin (1993) and Cabelle and Santos (1993). The numerical solutions, however, had transitions that displayed very short half lives of 5-15 years under a standard parameterisation.

II.vii Numerical Solutions for the constrained model.

In this section solutions are presented for the same parameters and initial values, but it is assumed that the wage constraint binds over part of the transition. The minimum wage was set at $\overline{w}=1$, which is above the initial marginal product of labour evaluated at $L=N$, so that economy is initially in phase I. Table 2.3 shows the phase I balanced path growth rates of c, k and y, equal to γ, and the phase I balanced path value u^*. These values are derived from equation 2.27 and 2.28 and show that, for different values of σ, the growth rates of c, k, y, are greater than the phase II balanced path rates. The growth rates of h, are also given by γ and, except for the case where $\sigma = 1$, are below the phase II balanced path values in table 2.1. Similarly, the phase I value of u^* is greater than in phase II, except for when $\sigma=1$.

The solution path and the phase boundary are plotted in $\{k, h\}$ space in figures 2.7a-2.7d. The solid line traces the capital stocks from their initial values to the boundary, and then from the boundary towards the balanced path curve. For $z>z^*$ (figures 2.7a, 2.7c) physical and human capital both accumulate steadily in phase I until intersecting the boundary. When $z(0) < z^*$ (2.7b, 2.7d) the per capita stock of human-capital falls initially, before increasing again towards the boundary. In both cases the phase I solution path intersects the boundary below z^*, so that the initial phase II solution path is a transition from beneath the z^* curve.[16]

Comparing the transition paths in phase I with the transition paths shown in figures 2.3a-2.3b, reveals that the former are more intensive in physical-capital. In figures 2.7a-2.7d, the transition paths begin at the same point and initially

[16] The phase I and phase II balanced paths only intersect at 0 and 1. For any pair of $\{k, h\}$ where both elements are greater than 1, the phase II balanced path curve lies above the phase I balanced path curve. Thus, assuming that the solution path is close to the phase I balanced path at the boundary, the economy will always traverse the boundary at a point below the phase II balanced path curve.

closely follow the phase II path. Rather than approaching the z* line, however, there is a relative increase in physical-capital, shown by the horizontal movements in the solution paths in figures 2.7a-2.7d. This suggests that the growth paths of developing economies may be relatively more capital intensive than suggested by the behaviour of the transition paths discussed by Mulligan and Sala-i-Martin (1993).

Figures 2.8a-2.8b show the same solutions in $\{\bar{k}, \bar{h}\}$ space. The solutions in this space can be seen to meet the z^* balanced path curve at lower points than the solutions shown in figures 2.4 a-b. This means that an economy that undergoes a phase I transition will always have a lower level of human and physical capital, and therefore output, than an economy that begins with all of its labour employed in the formal sector.

Figures 2.9a-2.9b and 2.10a-2.10b show the phase I transition of $u(t)$ and the marginal product of physical-capital (MPK). As discussed, the value of u falls as the economy crosses the phase boundary. The value of u after the economy crosses the boundary is below the phase II balanced path value, u^*, and u is always above the balanced path value as the economy reaches the boundary. This confirms that the phase I transition is relatively physical-capital intensive.

The figures also show that the value of $u(t)$ does not settle onto a balanced path in phase I but the MPK does tend to stabilise around its phase I balanced path value. Because the MPK depends positively on $u(t)$ it also falls as the economy intersects the phase boundary.

These findings are summarised in Result 5.

Result 5. When there is a binding wage constraint the economy will accumulate more physical-capital relative to human-capital and will have a permanently lower level of output, relative to an economy without a binding wage constraint. The economy does not approach the balanced path in phase I, but the marginal product of capital does approach the phase I balanced path value.

An important consideration in evaluating the descriptive merits of the transition path without wage constraints in the previous section, was the length of the transition and the persistence of the transitional growth rate. The length of phase I can be measured as the length of time it takes for the ratio *L/N* to increase from its initial value until it reaches unity. It follows from the assumption of constant factor shares, that the growth rate of *l=L/N* must be equal to the growth rate of income per worker, *y=Y/N,* in the formal sector. This can be seen from equation 2.5c, which can be expressed ...

$$(2.31) \qquad \overline{w} = (1-\alpha-\beta)\frac{y}{l}.$$

where *l=L/N*. Thus the constancy of $\overline{w}$ implies $\dot{y}/y = \dot{L}/L - n$. Letting x be the total growth in y required to bring the economy from an initial ratio of formal to informal sector labour $l(0)$, to $l = 1$, then 2.31 implies ...

$$(2.32) \quad l(0) = e^{-x}.$$

Chenery and Syrquin (see Syrquin 1988) have examined the typical patterns of employment for an economy undergoing approximately a 300 percent increase in income per worker - covering the development spectrum.[17] Equation 2.32 implies that for all labour to be employed in the formal sector after a 300 percent increase in income, then it must be that $l(0)$ = 5 percent. According to Syrquin (1988, p.238), however this growth involves the non-agricultural share of employment rising from 35 percent of total employment to 90 percent. Thus if the agricultural shares of employment were taken as a indicator of the size of the informal sector, then their study would suggest that this model does not fit the stylized facts.

While there is likely to be some overlap between the concept of an informal sector, as employed here, and the agriculture sector, the informal sector in a developing economy presumably initially extends beyond agriculture in the early stages of development. Similarly, the formal sector will include much of agriculture in the latter stages. Chenery and Syrquin's results also show that significant employment shifts have occurred by the time the economy's income

[17] That is, the natural logarithm of the income ratios is equal to 3.

increases by 200 percent, which would imply $l(0) = 15$ percent, by 2.32. This seems a more reasonable estimate of the size of the formal sector.

Figures 2.11a-2.11b show the evolution of L relative to N from the numerical solutions. Each figure also shows the labour force employed in the formal sector beginning below the total population and then converging. It can be seen that the length of the transition varies with σ and the value of $z(0)$. The ratios of L/N are reported in table 2.4a.[18] In the case where $\sigma = 1/2$ and $z(0)=2z^*$, the average growth rate is 1.55 percent over phase I. When $z(0)=z^*/2$ the average growth rate is 1.3 percent. Table 2.4b shows the growth rates of $l=L/N$ and $y=Y/N$ over the transition, and the final row of table 2.4c translates these growth rates into years, from alternative starting ratios of L/N. The low growth rates imply long transitions and, as discussed in chapter I, display much more persistence than the growth rates derived in the unconstrained model. Thus phase I transitions can be very long and the time that a developing economy's growth pattern can deviate from the phase II balanced path pattern is also relatively long.

In conclusion, this section has shown that the phase I transition is characterised by relatively physical-capital intensive accumulation and low levels of output relative to the model without a wage constraint. Further the growth rates of output per worker and of formal sector labour per worker display greater persistence than in the unconstrained model. Finally, the model with wage constraints can be considered to be in accordance with the stylized facts of development as presented by Syrquin (1988) although this is sensitive to one's assumptions regarding the size of the informal sector.

II.viii Conclusion.

This chapter has evaluated an endogenous growth model, after Uzawa (1965) and Lucas (1988), but where not all labour is available for accumulation activities due to the dualistic structure of economy. The introduction of a minimum wage level follows Dixit (1968, 1973), Stern (1971) and

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[18] The initial value of L/N is, however, not the same in each case owing to the different initial values of the state variables used, and the different value of σ.

Solow (1956), which in turn are based on development models of Lewis (1954) and Sen (1966).

The major results concerning this model have been summarised in results 1-5. A long run (phase II) balanced path solution was derived for the model in which the growth rates and the marginal product of capital were constant but the marginal product of unskilled labour and average skilled wage were rising (result 1). Numerical solutions were used to demonstrate that the model will converge to this balanced path, and to determine the behaviour of the transition path. The balanced path and transitional path in phase II, were shown to be similar to Lucas' (1988) model. Nevertheless the transition path in phase II displayed very short half lives, thus indicating a lack of persistence in transitional growth rates (result 4). This was a criticism raised by King and Rebelo (1993) regarding the use of transitional growth paths as descriptions of development processes.

Balanced path solutions for phase I were also derived and shown to result in high rates of physical-capital accumulation - relative to the model without a wage constraint (result 2). The behaviour of the economy across the phase boundary was examined and shown to exhibit shifts of human-capital effort toward the human-capital accumulation sector and falls in the marginal products of human and physical capital (result 3). Numerical solutions showed that while the growth path in phase I did not reach a balanced path, the solutions examined nevertheless implied higher levels of physical-capital accumulation relative to the case without a minimum wage constraint. Further, the numerical solutions demonstrated that the growth rates displayed much more persistence than in the unconstrained case. On this basis it was argued that the additions to Lucas' (1988) endogenous growth model may improve the ability of this model to describe the growth process in developing economies.

Table 2.1 – Balanced Path Values for Numerical experiments: no minimum wage constraint

σ	v	κ	r^*	u^*
1	0.03	0.015	0.065	0.375
1/2	0.02	0.010	0.070	0.050

Table 2.2a – Percentage of Transition Completed
(numbers report 100 minus the per cent change from the balanced path value)

	Z(0)>z^*				Z(0)<z^*			
Years	z	$\dot{k}/k$	$\dot{c}/c$	$\dot{h}/h$	z	$\dot{k}/k$	$\dot{c}/c$	$\dot{h}/h$
0	0	0	0	0	0	0	0	0
2	30	30	29	24	8	7	8	10
4	49	49	47	41	16	14	16	20
6	61	61	59	53	24	22	24	29
8	70	70	68	63	31	29	31	38
10	76	76	75	70	39	36	39	45
20	92	92	91	89	68	66	68	74
30	97	97	96	96	85	84	85	88
40	99	99	99	98	94	93	94	95
50	99	99	99	99	97	97	97	98
100	100	100	100	100	100	100	100	100
Total Change %	100	341	425	-16	-50	-154	-228	15

Table 2.2b – Percentage of Transition Completed σ=1/2
(numbers report 100 minus the per cent change from the balanced path value)

	Z(0)>z*				Z(0)<z*			
Years	z	$\dot{k}/k$	$\dot{c}/c$	$\dot{h}/h$	z	$\dot{k}/k$	$\dot{c}/c$	$\dot{h}/h$
0	0	0	0	0	0	0	0	0
2	30	29	28	24	6	6	7	8
4	47	48	45	41	13	12	14	17
6	59	60	58	53	20	18	20	24
8	68	68	66	62	27	24	27	32
10	74	75	73	69	33	31	34	39
20	91	90	90	88	61	59	62	67
30	96	96	96	95	79	79	80	83
40	99	98	98	98	89	90	91	92
50	100	99	99	99	94	95	96	96
100	100	100	100	100	99	100	100	100
Total % Change	100	356	380	-45	-50	-146	-192	34

Table 2.3 – Balanced path values numerical experiments in wage constrained model

σ	γ	u*
1	0.030	0.375
1/2	0.015	0.563
1/5	0.006	0.675

Table 2.4a – *L/N* Ratio in Phase I Transition

	$\sigma=1$		$\sigma=1/2$	
Years	z(*0*)>z*	z(*0*)<z*	z(*0*)>z*	z(*0*)<z*
0	0.22	0.50	0.28	0.54
10	0.33	0.57	0.35	0.61
20	0.47	0.70	0.41	0.70
30	0.63	0.87	0.48	0.81
40	0.83	1.00	0.56	0.92
50	1.00	1.00	0.65	1.00
60	1.00	1.00	0.75	1.00
70	1.00	1.00	0.87	1.00
80	1.00	1.00	0.98	1.00
90	1.00	1.00	1.00	1.00

Table 2.4b - Average growth rates by decade in phase I transition

	$\sigma=1$		$\sigma=1/2$	
Years	z(*0*)>z*	z(*0*)<z*	z(*0*)>z*	z(*0*)<z*
10	4.27	1.69	2.08	1.40
20	3.41	2.05	1.65	1.38
30	3.05	2.09	1.55	1.38
40	2.64	1.12	1.52	1.27
50	1.52	0.00	1.50	0.56
60	0.00	0.00	1.47	0.00
70	0.00	0.00	1.39	0.00
80	0.00	0.00	1.12	0.00
90	0.00	0.00	0.00	0.00

Table 2.4c - Average growth rates over entire phase I transition

	$\sigma=1$		$\sigma=1/2$	
Years	z(*0*)>z*	z(*0*)<z*	z(*0*)>z*	z(*0*)<z*
Average growth rate[a]	3.08	1.83	1.55	1.30
Time from L/N=0.35, in years[b]	34	57	68	81
Time from L/N=0.15, in years[b]	61	103	122	145

a: percent per year.
b: calculated as $t = (\ln(1)-\ln(L/N))/g$, where g is the average growth rate and t is the time in years.

Figure 2.1 - Balanced growth path and phase I-phase II boundaries in {*k*, *h*}

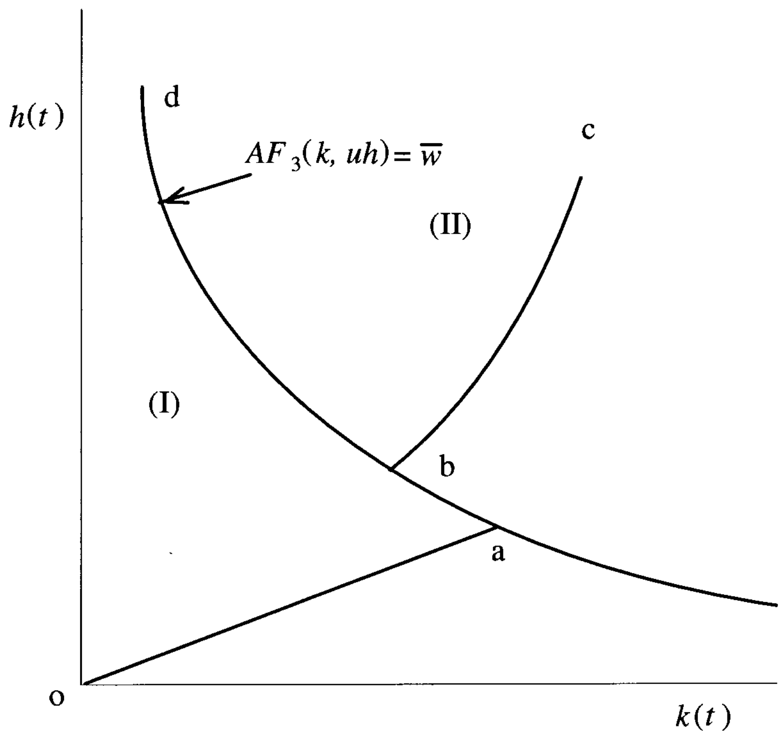


Figure 2.2 - An Optimal Solution Path in $\{k, h\}$ Space.

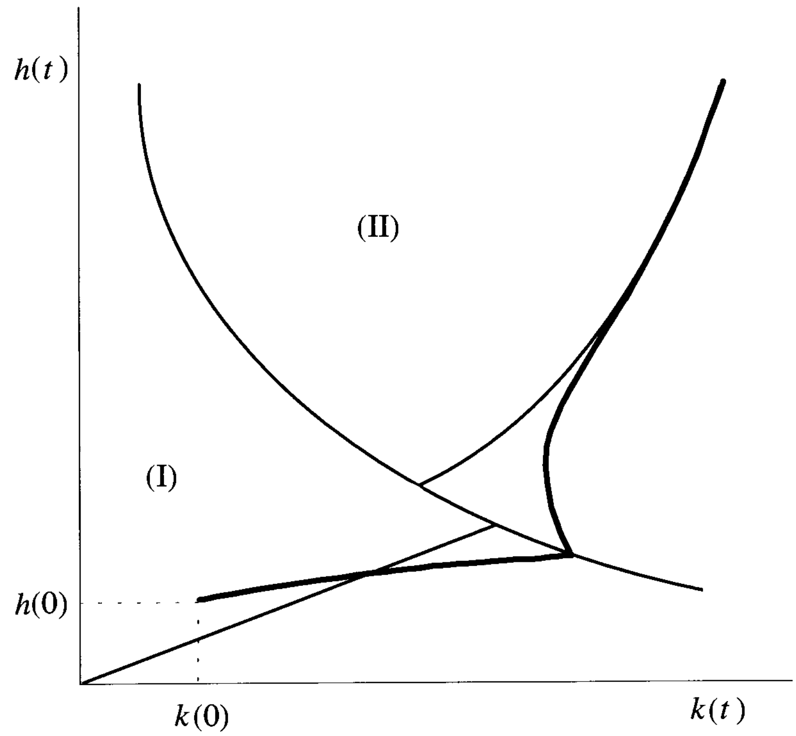


Figure 2.3a – Transition in {*k*, *h*}, σ = 1.

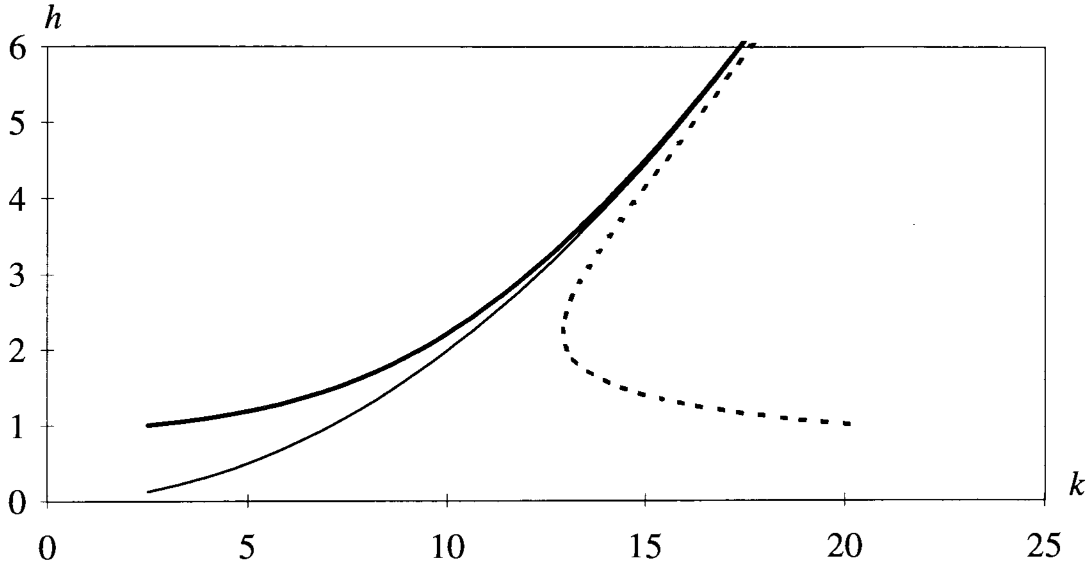
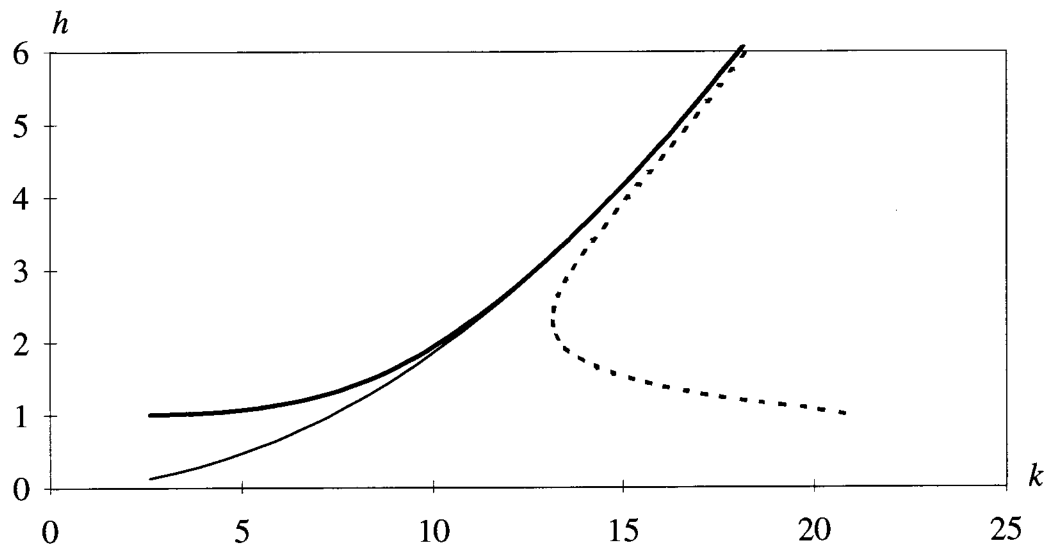


Figure 2.3b – Transition in {*k*, *h*}, σ = 1/2.



Key: solid bold line, transition when $z(0)=2z^*$, broken bold line, transition when $z(0)=1/2z^*$, solid line, *z^*.*

Figure 2.4a – Transition Paths in $\{\bar{k}, \bar{h}\}$ $\sigma = 1$

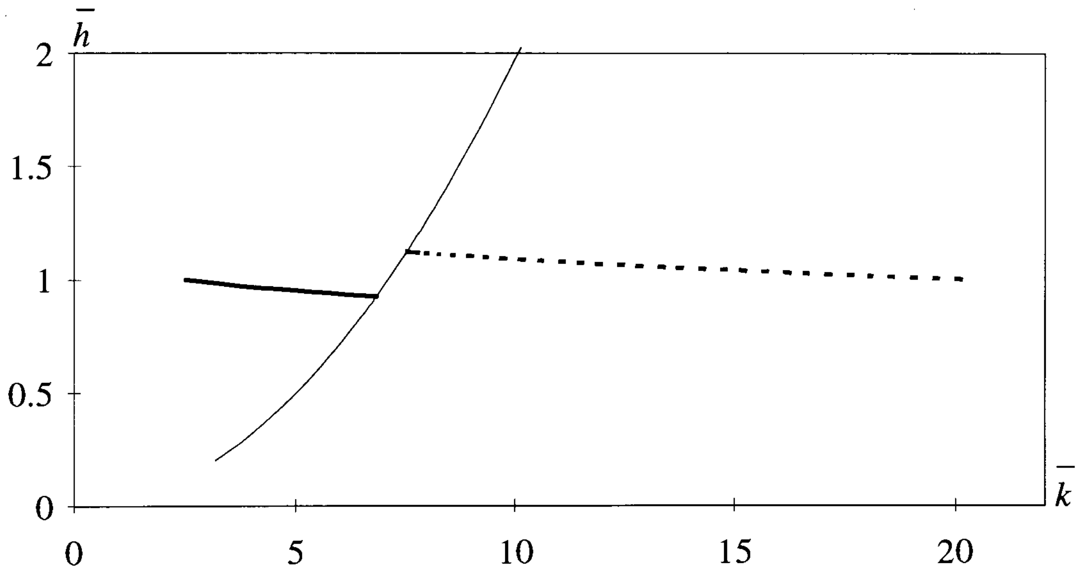
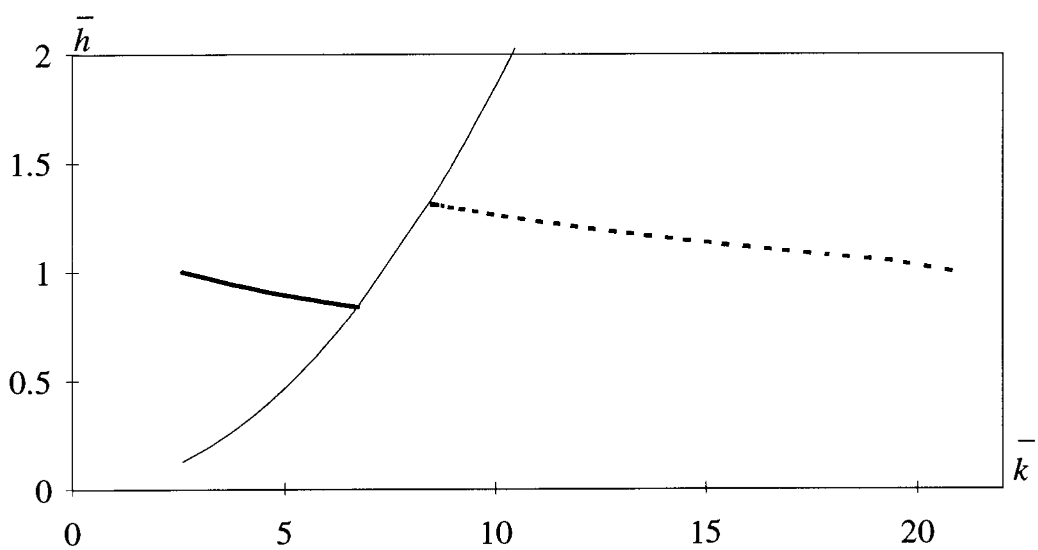
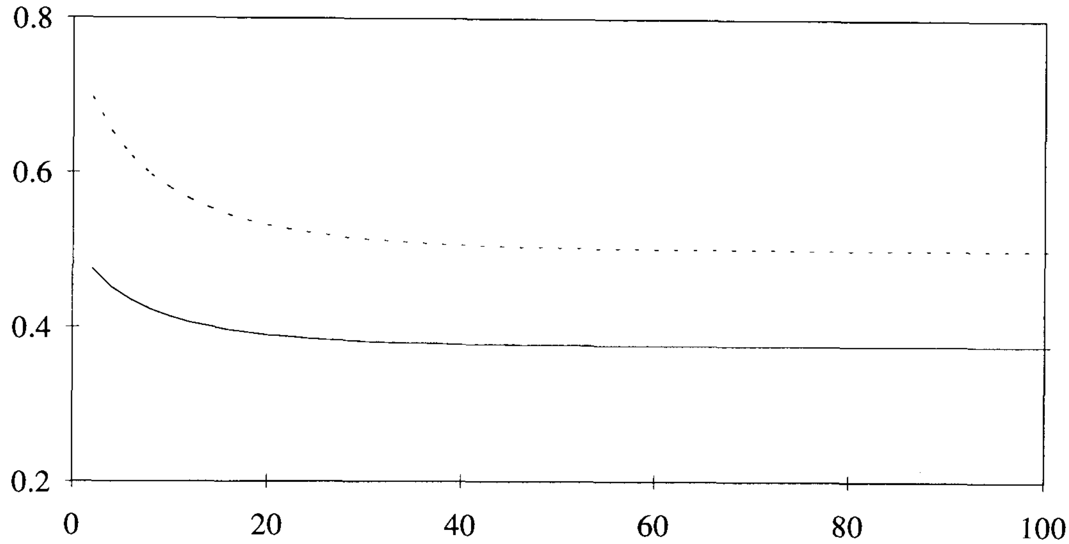


Figure 2.4b – Transition Paths in $\{\bar{k}, \bar{h}\}$ σ= 1/2



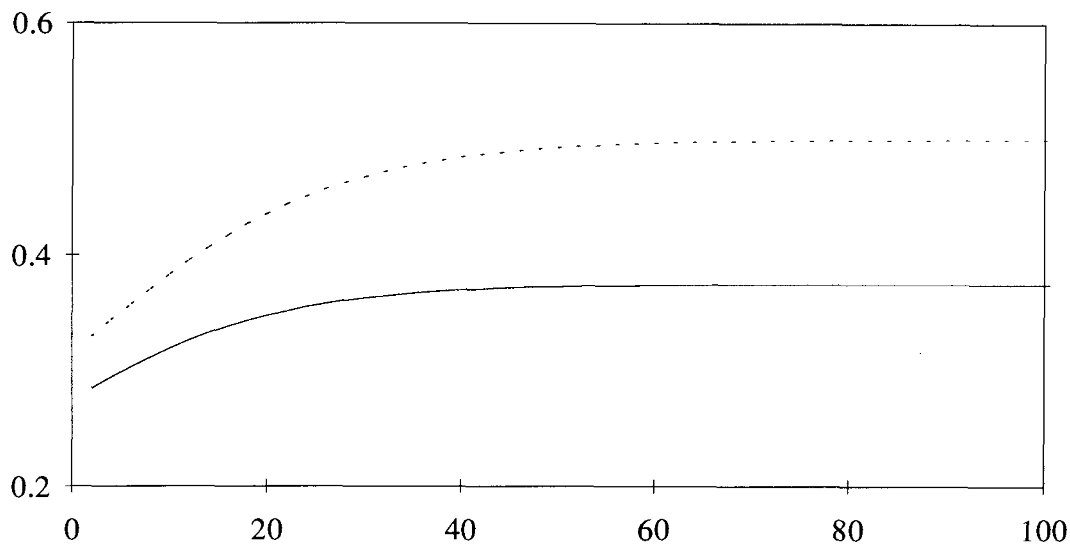
Key: solid bold line, transition when $z(0)=2z^*$, *broken bold line, transition when* $z(0)=1/2z^*$, solid line, z^*.

Figure 2.5a – Transition of *u*, z(0) = 2z*



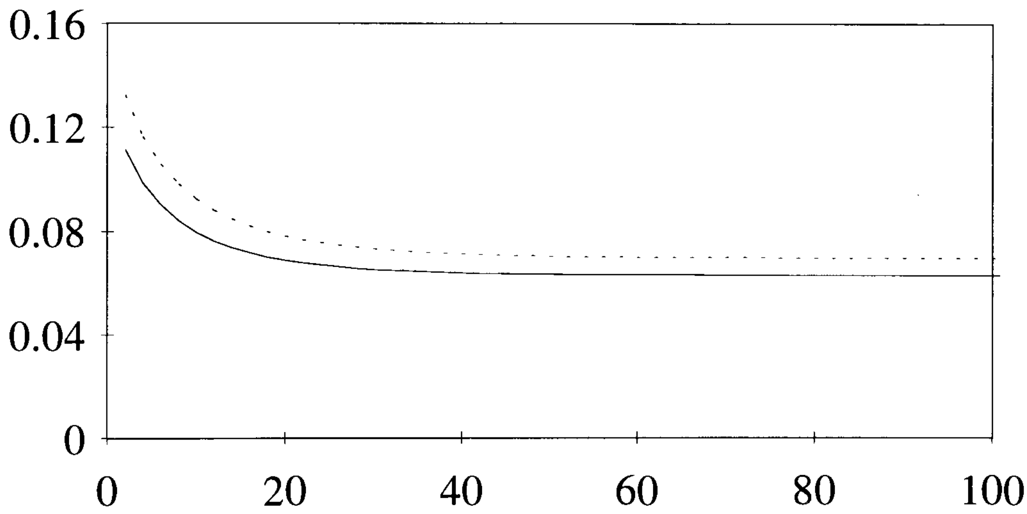
Key: filled line, $\sigma = 1$, dashed line, $\sigma = 1/2$.

Figure 2.5b – Transition of *u*, z(0) = 1/2z*



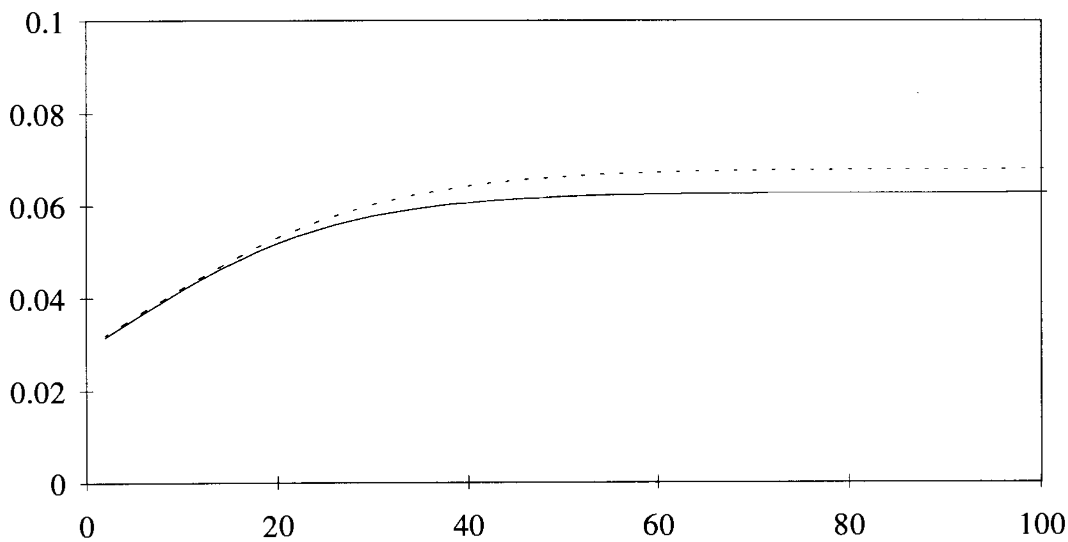
Key: filled line, $\sigma = 1$, dashed line, $\sigma = 1/2$.

Figure 2.6a – Transition of Marginal Product of Physical Capital (MPK), z(0) = 2z*



Key: filled line, $\sigma = 1$*, dashed line,* $\sigma = 1/2$*.*

Figure 2.6b – Transition of Marginal Product of Physical Capital (MPK), z(0) = 1/2z*



Key: filled line, $\sigma = 1$*, dashed line,* $\sigma = 1/2$*.*

Figure 2.7a – Transition in {*k*, *h*},
σ = 1, *z*(0)=2*z.**

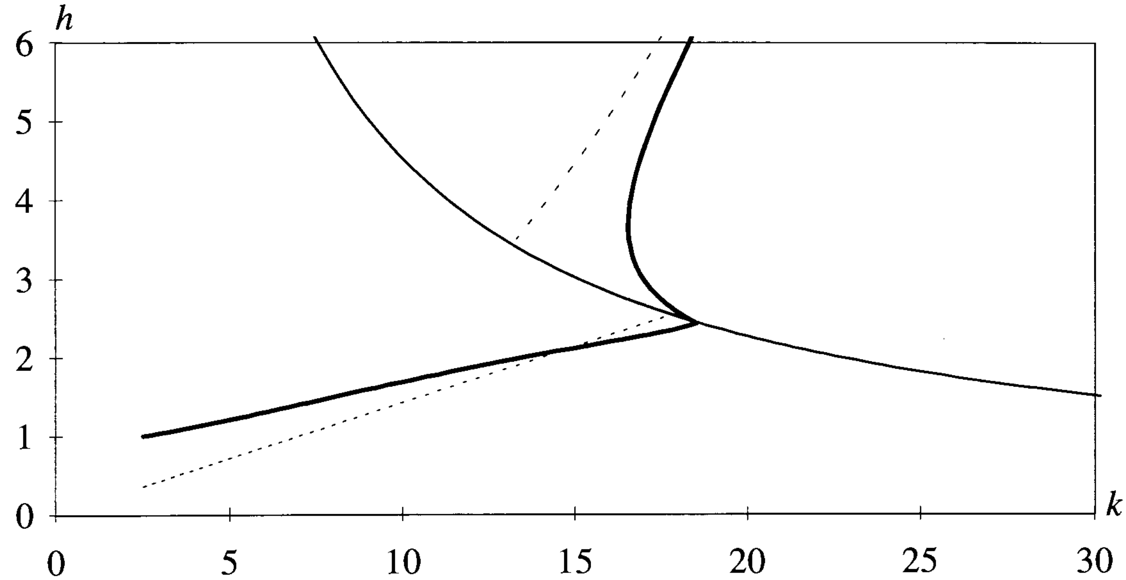
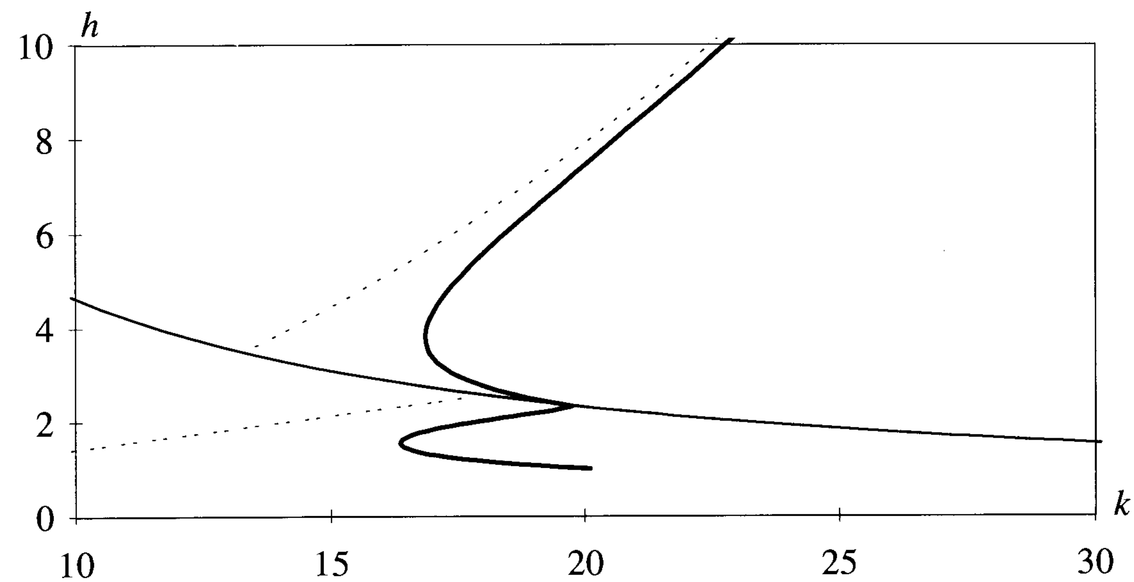


Figure 2.7b – Transition in {*k*, *h*},
σ = 1, *z*(0)=1/2*z.**



Key: solid bold line, solution path; solid line, phase I-phase II boundary; dotted line, balanced growth path.

Figure 2.7c – Transition in {*k*, *h*};
σ = 1/2, *z*(0)=2*z.**

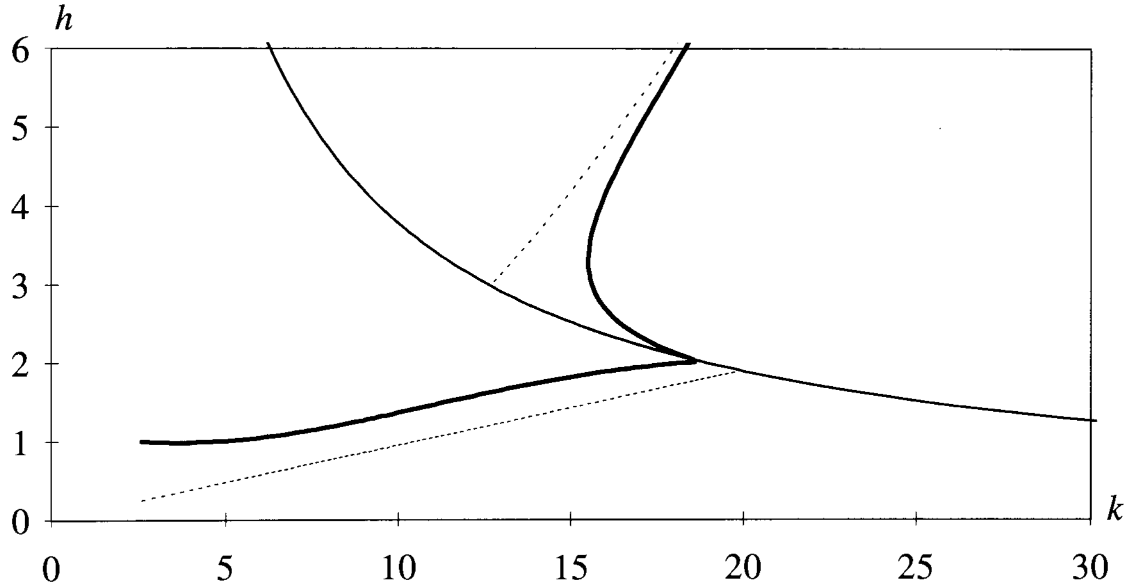
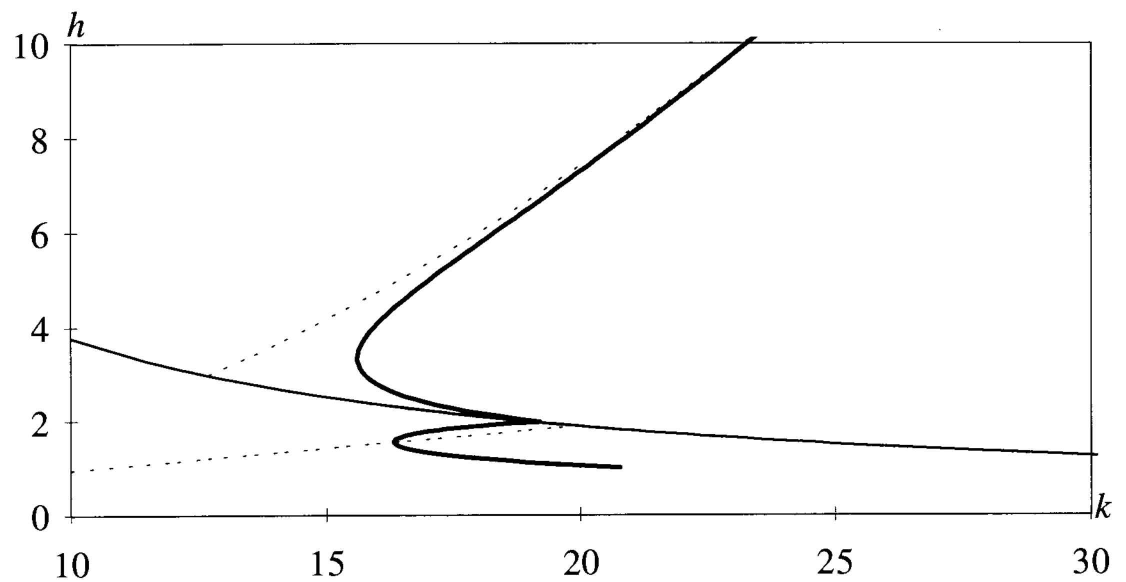


Figure 2.7d – Transition in {*k*, *h*};
σ = 1/2, *z*(0)=1/2*z.**



Key: solid bold line, solution path; solid line, phase I-phase II boundary; dotted line, balanced growth path.

Figure 2.8a – Transition Paths in $\{\bar{k}, \bar{h}\}$ σ= 1

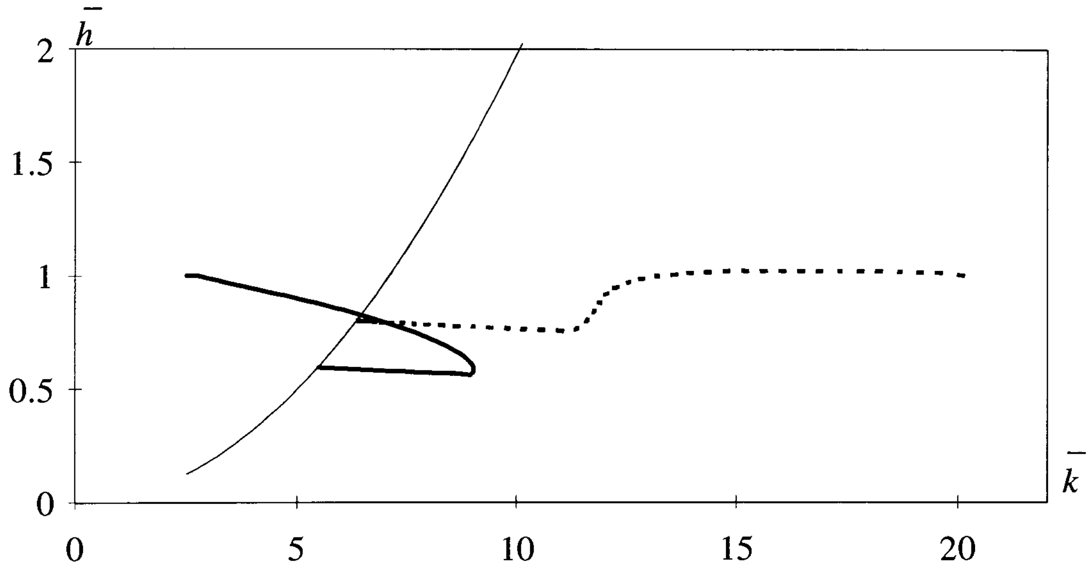
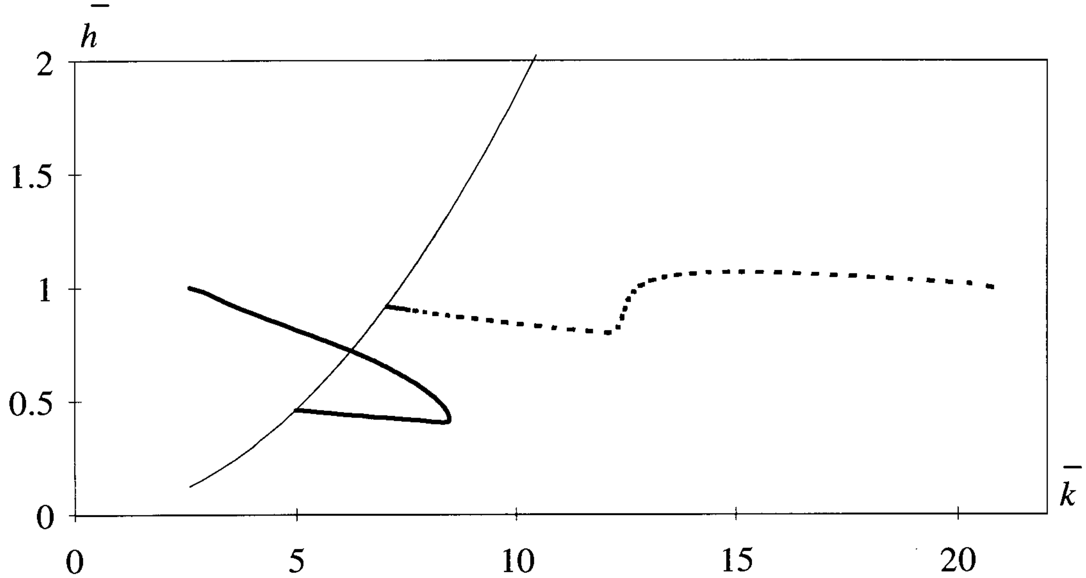


Figure 2.8b – Transition Paths in $\{\bar{k}, \bar{h}\}$ σ= 1/2



Key: solid bold line, transition when $z(0)>z^*$, broken bold line, transition when $z(0)<z^*$, solid line, z^*.

Figure 2.9a - Transition of u, z > z*

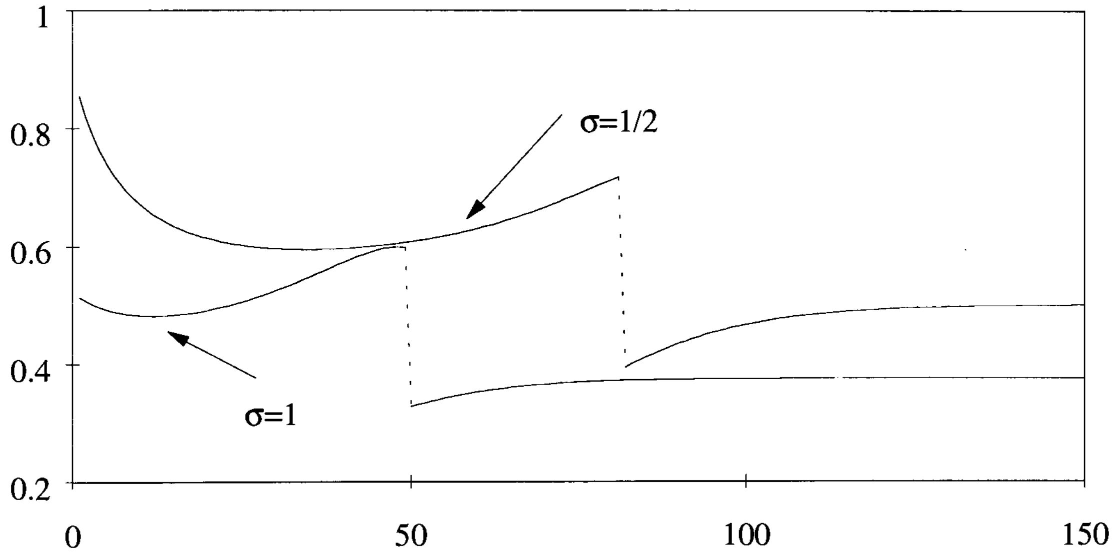


Figure 2.9b - Transition of u, z < z*

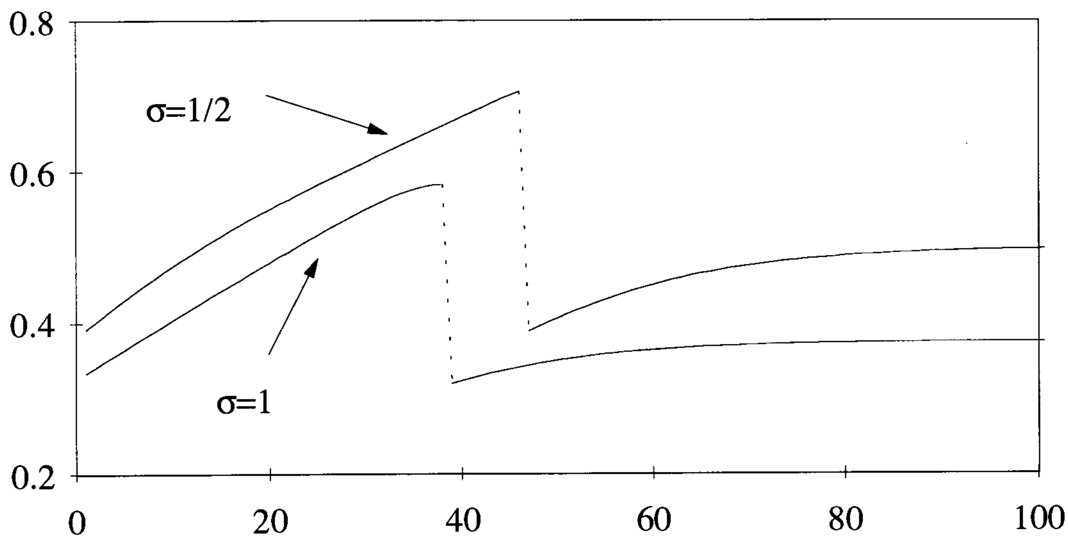


Figure 2.10a - Transition of Marginal Product of Physical Capital (MPK), σ=1

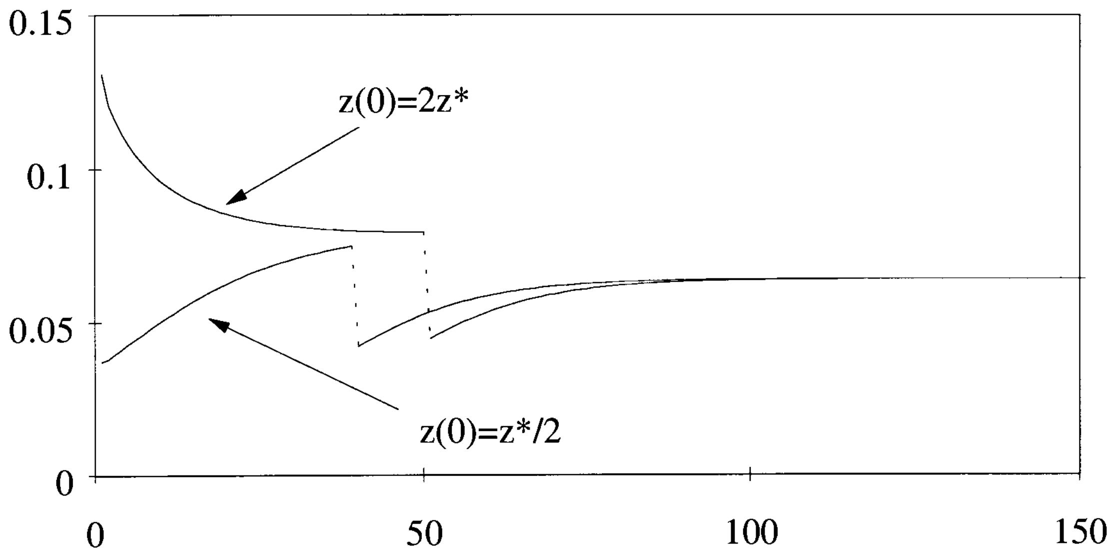


Figure 2.10b - Transition of Marginal Product of Physical Capital (MPK), σ=1/2

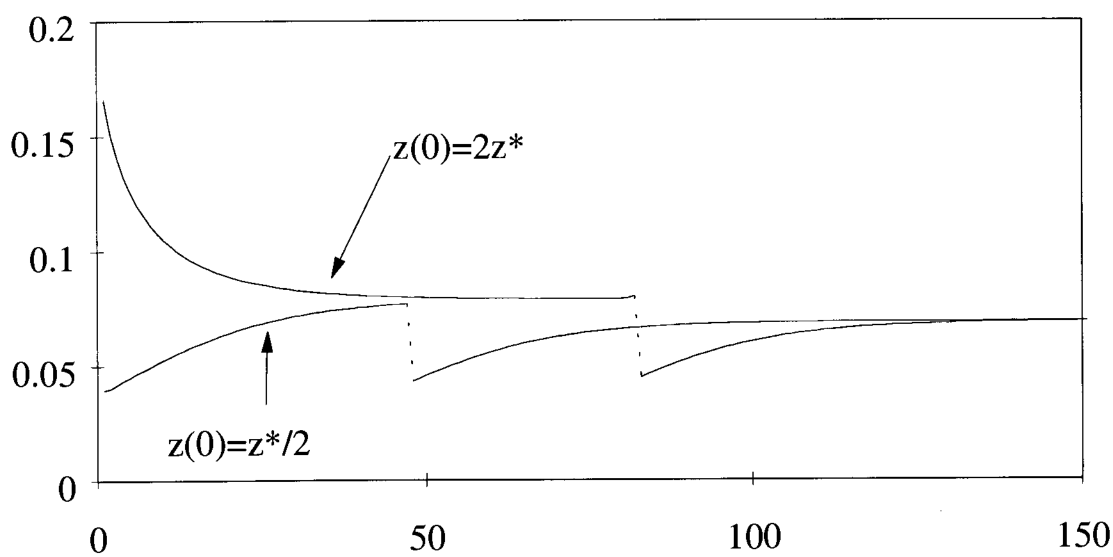


Figure 2.11a Path of *L*, *N* over Phase I
z(0)=1/2*z**

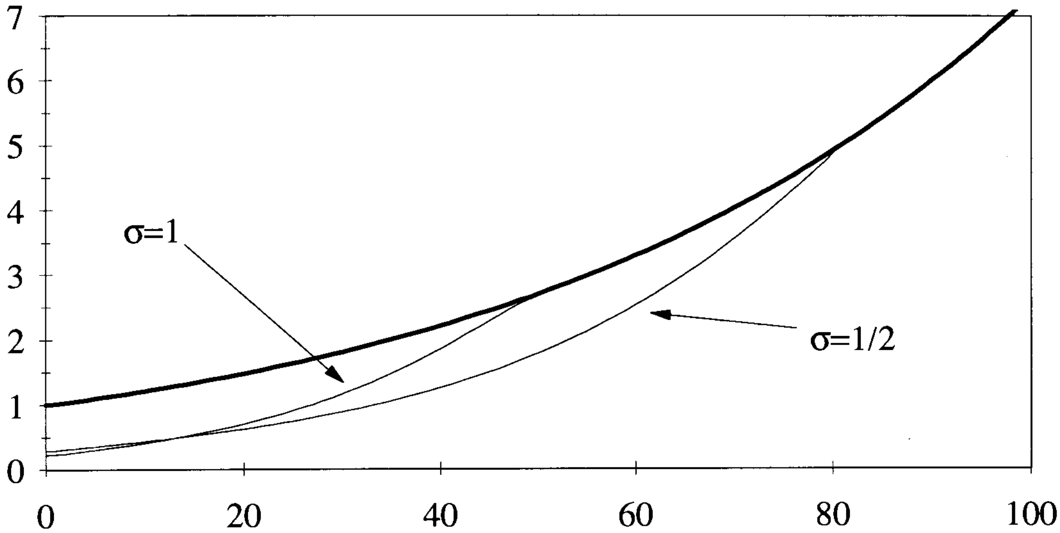
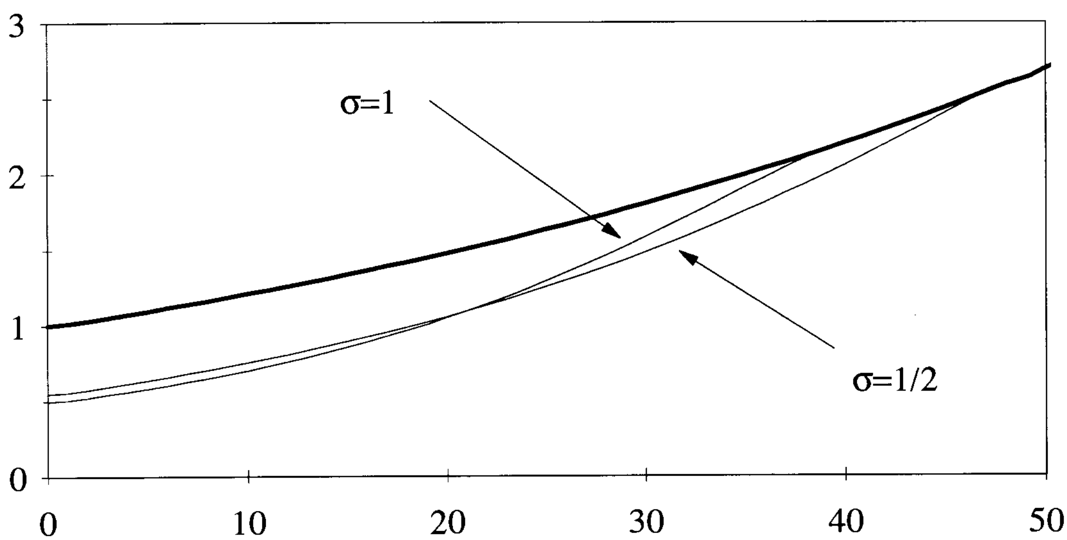


Figure 2.11b - Path of *L*, *N* over Phase I
z(0)=2*z**



Key: Solid bold line, N(t); solid line, L(t).

III. An application to changing trade policies in developing America

The model developed in chapter 2 is a closed economy model. As such, it makes no predictions about the effects of changes in the trade policy regime upon the economy. The model can nevertheless be employed to describe the dynamic behaviour of an economy, given some other theory about how trade policy changes will affect some of the exogenous variables. In this chapter the model is used to assess the dynamic impacts of a move to a more open trade regime in developing America.

It was argued, in chapter 1, that there is some empirical support for the hypothesis that open economies grow faster than closed economies. In this chapter, two theories about the effects of trade policy changes are considered. The first is the dynamic impact of the traditional static efficiency gains that have been argued to arise as a result of trade liberalisation. This is modelled by introducing an increase in total factor productivity, A, in the physical capital/consumption sector.[1]

Second, as noted in chapter 1, the static efficiency gains from trade may result in a higher long run growth rate if these efficiency gains occur in sectors producing human capital, (knowledge and R&D) where there is argued to be non diminishing returns to human capital accumulation. For this reason an unanticipated increase in δ, the productivity of human capital accumulation, is also considered. This diverts resources to the creation of human capital and sustains a higher long run growth path.

These experiments are shown conceptually in figures 3.1 and 3.2. In figure 3.1 there is a temporary increase in the growth rate of capital after time $t(0)$. Thus capital ceases to accumulate along its balanced path, (represented by the straight line against the logarithmic scale). The increase is temporary so that eventually the growth rate returns to the same balanced path rate, but the capital stock, and therefore income, is permanently higher as a result of the transitional dynamic response. In figure 3.2 however, there is a permanent increase in rate of growth. This is the result of an increase in the productivity of human capital accumulation in the model presented in chapter 2. The exact pattern of accumulation in each case depends on the parameters of the model. These in turn are determined by benchmarking the model to represent the patterns of growth in developing American countries.

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1 For example see Corden (1971, 1985)

Each experiment is conducted under two separate benchmarks. In the first benchmark calibration (BC1), it is assumed that there are no binding wage constraints, so that the dynamic growth path is restricted to phase II only. This exercise is mainly for the purposes of comparison with the second benchmark calibration. In the second case (BC2), it is assumed that there is a binding wage constraint, and that the economy is in phase I of a growth path. Aside from the different dynamic responses in each phase, this benchmarking procedure results in significantly different parameter values and thus has implications for the effects of the policy experiments.

III.i Benchmark calibration 1 (BC1) – all labour employed in formal sector, no binding wage constraint.

The parameters for the model were chosen under the assumption that the post-war developing American growth experience (1960-1980), approximates a balanced growth path. The assumption of a balanced growth path in this experiment is a simplification and will be relaxed in the second benchmark calibration. It was argued in chapter 1, however, that over this period the capital stock grew faster than income, which contradicts the assumption of a balanced growth path. Nevertheless, growth rates over this period were relatively stable, not showing any trend, (see figure 1.1 - 1.2).

Post-war data, 1960-1980, from Hofman (1992) were used to calibrate the benchmark solution. For purposes of this discussion "developing America" is defined to be Argentina, Brazil, Colombia, Chile, Mexico and Venezuela. The benchmark solution thus reproduces the historical growth path for these regions over this period.[2] Table 3.1 summarises some of the relevant aggregate data.

The calibration procedure for BC1, was to choose the parameters to replicate the steady growth from 1960 until 1980, assuming that the economy was approximately evolving along a balanced path during this period. The parameters of the production function were taken from Mankiw, Romer and Weil (1992) who estimated the share of capital, human capital and labour to be approximately 1/3, $\alpha = \beta = (1 - \alpha - \beta) = 1/3$.

2 No attempt was made to benchmark the model through the 1980s which saw the debt crisis and the Mexican earthquake.

Using these factor shares, the balanced path growth equations 2.19b and 2.23 simplify to ...

(3.1) $$\kappa = \frac{\delta - \rho}{1 + \sigma^{-1}},$$

(3.2) $$v = 2\kappa.$$

Further, rearranging 3.1 and letting r^* denote the steady state marginal product of capital we have ...

(3.3) $$r^* \equiv \rho + \sigma^{-1}\kappa = \delta - \kappa.$$

According to Hofman's (1992) data, the average growth rate of κ is 3% or 4%, depending on whether the estimate is based on capital growth or GDP growth (see Table 3.1). GDP figures are more reliable than capital stock figures (Ward, 1976, Scott 1993), and so κ is assumed to be 3%. It is assumed that the elasticity of substitution for consumption over time is unity, $\sigma = 1$, and $\rho = 0.05$. The assumption that $\sigma = 1$ is based on Blanchard and Fisher (1989, p.44). They argue that estimates of this value are variable but usually lie around or below unity. The value of ρ is similarly a convenient benchmark value and both are varied in sensitivity analyses.

The values of ρ and σ imply, by 3.3, that for κ to be 3%, δ must be 0.11. This implies also that r^*=0.08.[3] This is higher than values of the implied interest rates used in other studies. For example Trostel (1993) and King and Rebelo (1993) assume a long run interest rate of approximately 0.065 based on USA data. The average post-war growth rate of the USA is only 1.5%, half that of developing America. From equation 4.3 it can be seen that this low growth rate will imply a low value of r^*. Thus the higher

[3] The share of capital is assumed to be 1/3. The implicit value of r^* could then be obtained from the average capital-output ratio observed during the balanced path growth. The value according to Hofman's data is 1.33, implying a marginal product of capital of 0.25. This seems far too high. Further more it would imply extremely high values of r and delta. The problem has been noted by Mankiw *et al* (1992) and indirectly by King and Rebelo (1993). Other attempts to calibrate perfect foresight growth models (Trostel, King and Rebelo) have used conventional parameters, and discarded the implications for the size of the aggregate capital stock. With a long run MPK of 0.065, and a capital share of 1/3 the capital output ratio would have to be $(0.065\ 3)^{-1} = 5.13$. This is about 2-3 times the capital - output ratios observed using accounting definitions such as the perpetual inventory method. The marginal product of capital assumed here is 0.08. Assuming $\alpha = 1/3$, this implies capital output ratios of 4.16. This is considerably higher than Hofmans's estimate of 1.33.

marginal product of capital is a direct consequence of the higher growth rate, for a given value of ρ. There is also evidence that the marginal product of capital is higher in developing economies than developed economies. King and Levine (1994) use different methods of estimating capital stocks to show that capital-output ratios are lower in developing countries. They find that a lower income country typically has a capital-output ratio of 1.4, a country with half the GDP per capita of the USA has a capital-output ratio of 2.2, and a country with similar GDP per capita levels to the USA has a capital -output ratio of around 3.1, (King and Levine 1994, p.274-276).

An estimate of the parameter δ is obtained from the observations on the long run growth rate and the requirement that $\rho = 0.05$. By 3.2, $v = 6\%$. From equation 2.14, this implies that the balanced path value $u^* = 0.27$. The relevant parameters are summarised in table 3.2. They are thus based on the observed post-war growth experience in developing America and assumed values of $\sigma = 1$, $\rho = 0.05$ and factor shares, $\alpha = \beta = (1 - \alpha - \beta) = 1/3$

The initial values of capital $K(0)$, GDP, $Y(0)$ and labour, $N(0)$, are also taken from Hofman's data. This leaves two variables, A and $H(0)$ to complete the adding up requirement of the production function in the initial year, 1960.

Despite a large amount of recent evidence on human capital accumulation there have been few attempts to measure the stock of human capital. The major attempts to estimate stocks are for the USA, and these studies come up with very different values. Jorgenson and Fraumeni (1992), for example, report human capital stock estimates for the USA around 10 times larger than estimates by Kendrick (1976) and McMahon (1991).

More recently there have been some attempts to estimate human capital for large samples of countries (Nehru *et al* 1995, Gundlach 1994, Benhabib and Spiegal, 1994, Barro and Lee, 1993, Gemmell 1995). Typically these estimates are based on schooling data and thus have a quantity dimension, but not a value.[4] It is therefore difficult to make comparisons between the "size" of the human capital stock relative to physical capital stocks or income, required for calibration.

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[4] The exception is Gundlach (1994), who uses a method developed by Kreuger (1968) and Lucas (1990) which is based on relative Solow residuals between high and low income countries.

McMahon (1991) reports that the ratio of the human capital stock value to GDP was approximately 3 for the USA in 1980 and it was assumed that this ratio also held for developing America in 1980. Recent estimates of human capital stocks from Gundlach (1994 p.365), and Benhabib and Speigal (1994, p.171) are broadly consistent with the assumption of a common income/human capital ratio in these regions.[5] While this number is very uncertain, numerical estimates indicate that the results are highly insensitive to assumptions about the level of human capital versus the level of A. Due to this insensitivity, the results are reported for the base case only with $H/Y=3$.[6]

Rather than attempting to impose shocks to account for the debt crisis, a base solution was obtained with 1992 as the initial date.[7] To take into account the debt crises, however, the initial capital stock was assumed to be such that the capital-output ratio was off the balanced path predicted by the 1960-1980 benchmark solution. By 1992, capital-output ratios in developing America had risen to approximately 38% above the 1960-1980 average. To accommodate this, the initial capital stock for 1992 was assumed to be 38% above the balanced path level. The level of the 1992 human capital stock was then adjusted so that the predicted value of Y in the model was equal to the actual value, given the constant value of A obtained from the benchmark. This required an estimate at the initial value of u. Successive iterations between u and H eventually yield a solution path where the model correctly replicates the observed initial value of Y, with u endogenous. The resulting growth path is compared to the actual growth path in figure 3.3.

The experiments then consisted of applying an unanticipated shock to the accumulation equations in the year 1995. Harris and Robertson (1994) estimated the efficiency gains arising as a result of a reduction in tariff barriers between North and

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5 Gundlach and Benhabib and Speigal show that the estimated human capital stock per capita for Brazil, relative to the per capita estimate for the USA, can vary according to method from between 45.5%-25.1%. Estimates from Benhabib and Speigal for Argentina and Mexico are around 45% of the USA level. Summers and Heston's data, however, indicate that the GDP per worker in the developing America region was 46% of the USA in 1980 and GDP per capita was 33 % of the USA level.

6 Note that on a balanced growth path, the growth rate of human capital depends only on δ, and u^*, which are both constant. Thus, the behaviour of the model along a balanced path is independent of the choice between A and H.

7 Hofman's data series stops in 1989 and so was updated using comparable data from the World Tables, World Bank (1994).

developing America. Their estimates suggested that the efficiency gains in developing America represented an 8.5% increase in GDP. This included traditional net gains in consumer surplus, as well as economies of scale and terms of trade movements, Harris and Roberston (1994, p.18). This value was therefore used as an estimate of the increase in productivity from trade liberalisation, and was applied to both A and δ separately and jointly.

III.ii Results, Case BC1

Table R1 shows the results of a 8.5 productivity shock to Y. The results reflect the diminishing returns to human and physical capital accumulation in the production function. The shock causes a permanent rise in GDP, capital and human capital and consumption of 12.4%. Thus the system returns to its balanced path with the same capital/output and human capital/output ratio and MPK as before the shock. The permanent shock thus has a transitory effect on the growth rate.

The increase of 12.4% is only slightly different from that which could be obtained by a "back of the envelope" calculation, using the difference between two balanced paths and assuming u and h are constant.[8] There is, however, also a re-allocation of human capital between the two sectors. The rise in productivity increases u, the proportion of human capital allocated to consumption and physical capital investment, by almost 20%. The 8.5% increase in A thus causes output of the consumption/physical capital sector to increase by more than 8.5%, the sum $Y+(\lambda_1/\lambda_2)\dot{H}$, to increase by less than 8.5% and sets in place a transition involving faster physical capital and slower human capital accumulation.

Some of these results are also presented graphically in figure 3.5, 3.6 and 3.7, which plot the logarithm of consumption, GDP, physical capital and human capital respectively. They show the initial increases in consumption and income, which then increase further as the accumulation effects of the capital stocks are realised. The transition path of the physical capital (figure 3.6) follows the pattern shown in figure

8 On a balanced path we have $r^* = \alpha A k^{\alpha-1}(uh)^{\beta}$. Solving for k and taking logarithms, gives $\frac{\partial \ln(k)}{\partial \ln(A)} = \frac{1}{1-\alpha}$. Using this result it can be shown that $\frac{\partial \ln(y)}{\partial \ln(A)} = 1+\frac{\alpha}{1-\alpha}$, which in this case is 1.50. Thus the initial increase in A of 8.5% would have led to a 12.75% increase in y once accumulation effects have been accounted for, given constant values of u and h.

3.1, while human capital (figure 3.7) moves only slightly from the base path, as was shown in table R1.

Thus the final 12% increase is a result of general equilibrium re-allocation of resources and the transitional dynamics, and the effect of these changes on growth rates is temporary. In addition, the model predicts that the bias toward physical capital accumulation causes a temporary decline in human capital investment, and a lower stock of human capital. The result is the same as the transition paths discussed in chapter 2 and by Mulligan and Sala-i-Martin (1993) and Cabelle and Santos (1993).

In the second experiment, the effect of an 8.5% increase in the productivity of human capital, δ, was considered. As discussed above, and in chapter 1, a permanent increase in δ will result in a new higher balanced growth path. A similar effect could have been obtained in the Solow-Swan model, or other neoclassical models with exogenous growth, by assuming that trade liberalisation results in a higher value of the exogenous growth rate. In this model, however, the value of δ can be calibrated so that the change in the growth rate following a change in the productivity of the technology sector, is an endogenous response. Equation 3.1 shows that an 8.5% shock to δ, given σ and ρ, increases the long run growth rate of the economy, κ, from its initial benchmark growth rate of 3%, to approximately 3.5%. The effects of this, in terms of changes in levels, are quite dramatic and are reported in Table R2.

Compared to the modest 12% increase in the first experiment, Y, K, and c have all increased by approximately 20-30% by 2050. Because the economy is on a higher growth path, the increases over the base solution will always be increasing. In the short run, however the effects of this shock are very different. Y falls as resources are directed to human capital investment. This can be seen by the fall 10% - 11% fall in u. Similarly, figures 3.8 and 3.9 shows the initial downward movements in physical capital, and consumption. Figure 3.7, however shows the immediate increase in the growth rate of human capital. It takes approximately 15 years for consumption levels to re-gain ground as a result of the accelerated human capital investment. Because of the initial fall in consumption, the increase in utility in this experiment is much smaller than in the shock to A.

Table R3 shows the results of the 8.5% productivity shock applied to both sectors. A comparison of Tables R1 and R2 with R3 shows that the combined effects on GDP and consumption are greater than the sum of the two shocks separately. When the two shocks are considered together, there is a larger increase in u than is implied by the sum of the two previous experiments. This results in a greater capital stock, so that consumption and GDP are marginally greater in this case. In general however, the results of considering the two experiments separately or combining them have little effect on the path of accumulation. The estimates presented here suggest that in the best case scenario, GDP would increase by approximately 20-30% in 50 years. This is very large compared to the 12% gains estimated in table R1.

What do these numbers imply for the ability of the South to close the current gap between the North and the South? North America's GDP per worker is currently 2.8 times that of the South, which represents a 180% increase to eliminate the gap. The benchmark growth rate of 3% per year is approximately twice the post-war growth rate of the USA, as observed in Summers and Heston's 1991 data set. The time in the benchmark solution until the South catches up with the North is 77 years. The increase in δ reduces the time to 62 years.[9] With a change in δ and A the time is 59 years. Thus while the experiment suggests a seemingly large percentage increases in GDP, converting these to changes in the time required to catch up with the North, suggests that the results are quite modest.

It could be argued the growth path of developing America represents the transitional phase of the growth path rather than the balanced path. This constitutes a reasonable objection to the approach outlined above. As such, the transitional growth rate may be significantly higher than the balanced path rate. If this is the case then the assumption that the historical marginal product of capital and growth rate in South America are also the balanced path values, may be misleading. An alternative assumption is that the long run growth rate in developing America is closer to the USA's, approximately 1.5%, and the observed growth of 3% is partially due to transitional effects. As shown in chapter 2, however, the transition paths implied by the version of the model without wage constraints, are subject to the criticisms of King and Rebelo (1993), in that the transition path converges too rapidly to the balanced path. These issues are addressed by the alternative benchmarking procedure in section *II.iii* and *III.iv* based on the dual economy version of the model with a minimum wage constraint.

9 Assuming constant growth rates the South would eliminate the GDP gap in 69 years. The 0.5% increase in the growth rate of human capital predicted by the change in δ reduces this time to 52 years. The difference between these numbers and those in the text is due to the transitional dynamics.

III.iii Benchmark calibration 2 (BC2)– binding wage constraint, labour migration to formal sector.

In this case it is assumed that the post-war growth in South America occurred as labour was absorbed into the formal sector. The benchmark is therefore calibrated using the phase I growth path. This will affect the size of the calibrated parameters and therefore the potential response of various exogenous shocks or policy experiments on the model economy. In particular, benchmarking the model under the assumption that the model is in phase I, produces a long run growth rate that is lower than the observed benchmark growth rate. The assumed parameters and those derived from the benchmarking exercise are given in table 3.3.

To calibrate the benchmark parameters, it is convenient to assume that the observed growth path follows the phase I, balanced path conditions derived in Chapter 2. The parameters may then be adjusted iteratively so that the predicted growth path corresponds to the observed path.

On the phase I balanced path, the rates of growth of physical and human capital are given by γ where, from 2.23′ ...

$$(3.4) \qquad \gamma = \frac{\delta - \rho}{\sigma^{-1}}$$

It is assumed that $\sigma = 1$ and $\rho = 0.05$, as in case BC1. Given that the average growth rate of GDP per worker was 3 per cent per year, this implies that $\delta = 0.08$. The implied long run balanced path real interest rate is 0.065. This figure is the average return on capital for the United States cited by King and Rebelo (1993) and is close to that cited by Lucas (1988). It is evident that the value of δ is much lower than in BC1. The process of labour absorption requires a lower level of productivity of human capital in order to generate the observed growth of GDP per worker of three per cent per year. The observed growth in the post-war era will, therefore, not be sustained in the long run. The growth rate in the balanced path is predicted to be one half the post-war average growth rate.

To proceed further it is necessary to determine the extent of the division of the labour force between the formal and informal sectors. In this study, the formal sector has been defined as all economic activity that employs and potentially produces human and physical capital. It is difficult to find an empirical observation corresponding to this definition. Table 3.4 shows data on the urban rural differentials and the traditional labour force in 19 Latin American countries.

The data in table 3.4 show that there was considerable migration from the rural sector to the urban sector between 1970 and 1980. Despite the geographic migration, there was very little movement from the traditional sector into the modern sector.[10]

According to these data, in 1980 approximately 35 per cent of the total labour force still remained in the traditional sector, and 65 per cent were in the modern sector. The benchmark is calibrated so that the ratio L/N in 1980 is 0.65. There is a simple relationship between the marginal product of labour and GDP per worker, due to the constancy of factor shares. This is obtained by equating the marginal product of labour to the constant opportunity cost of labour. Recall equation 2.31 ...

$$(2.31) \qquad \overline{w} = (1 - \alpha - \beta)\frac{y}{l}.$$

where $y \equiv Y/N$ and $l \equiv L/N$. Thus given an estimate of y and l in 1980, 2.31 implies a value of $\overline{w}$. From table 3.4 the value of GDP per worker in South America in 1980 was $PPP (1985) 10 904. This then implies that $\overline{w}$ is $PPP (1985) 5 592, about half the GDP per worker.

Solving the model using these parameter values, using 1960 as the initial year then produces an approximate benchmark solution. As in case BC1, the value of A is calculated as a residual, under the assumption that H/Y in 1980 is 3. A is assumed constant so in the initial year, 1960, only the values of H and u are unknown. The actual growth of the capital stock was different from the growth rate of GDP, as would be implied by a balanced path solution. For this reason, the initial guess at the

[10] More recent estimates of urban/rural shares from the World Tables (World Bank, 1994) show that the share in South America has risen to levels similar to that of the USA. The data in table 3.4, however, suggest that the urban rural division is likely to be a poor indicator of the type of activity that labour is engaged in.

capital stock level in 1960 was the value implied by a phase I balanced path. That is the value implied by the condition,

$$(3.5) \qquad \frac{\alpha}{\alpha+\beta}\frac{Y}{K}=\delta\ .$$

The LHS of 4.6 is just the marginal product of capital which is equal to δ on the phase I balanced path.

The initial guess for H was obtained from 2.8 ...

$$(3.6) \qquad H=Y\left[Ku\frac{(1-\alpha-\beta)}{\overline{w}}\right]^{\frac{-\alpha}{\alpha+\beta}}A^{\frac{-1}{\alpha+\beta}},$$

where Y, K, $\overline{w}$ and A are determined as described above and u was at guess at the initial value determined by the solution. Because u is endogenous, the solution may not produce the correct values of Y in the initial year. The benchmark solution was thus obtained by iterations between H and u from the initial guess solution.

The initial trial values for ρ and δ were chosen assuming that the economy was in a phase I balanced path. If the solution does not grow along this path then the value of ρ and δ, chosen from equation 3.4, may not produce the desired growth rate. The initial guess of ρ and δ produced a good fit and it only remained to iterate, from an initial guess at the value of u, to a solution by adjusting H, as in 3.6. Figure 3.4, shows the level of Y in BC1 versus Hofman's (1992) data to which it was calibrated.

The long run growth balanced path growth rate of this benchmark solution is given by 3.1 rather than 3.4. Substituting the values of ρ and δ from table 4.3 into 3.1 shows that the long run growth rate is 1.5%. This is exactly half the observed growth of the 1960-1980 period and is approximately equal to the post-war growth of the USA.

Finally, as in case BC1, the first step in conducting the trade liberalisation experiment was to compute a base solution with 1992 as the initial year. As in BC1, the initial physical capital stock was set to 38% above the implied phase I balanced path level, so as to account for the changed economic structure since the debt crises. This means that the initial average and marginal products of capital are below the balanced path levels and as such the initial rate of physical capital accumulation is less than 3%. Iterations between u and H, were conducted until the model reproduced an initial level of Y consistent with the observed level.

In phase I of the model, however, Y does not represent all of the economic activity of the economy. It excludes all activity in the informal sector. If the informal sector is growing at a slower rate than the formal sector, or declining, then the observed path of GDP, is likely to understate the growth of the formal sector Y. Thus by calibrating the path of Y to GDP, the parameter δ may be also understated.

III.iv Results - Unanticipated productivity shocks δ, A for BC2.

The results, employing the same experiments explained above for the current benchmark procedure, are reported in tables R4-6. Before considering these results, it may be noted that the transition between phase I and II is associated with a fall in u, representing a re-allocation of resources in a response to changes in the marginal product of human capital in the physical capital/consumption sector. These features will also show up in the solutions considered here.

Table R3 shows the effect of the 8.5% increase in total factor productivity in the consumption/physical capital sector only. It presents percentage differences between the base case and the experiment, for each year. In the base case the turning point, or transition between phase I and phase II, was predicted to occur in 2014 - given the assumption that it was 65% complete in 1980. Thus in the base case the values of u and Y are predicted to fall after 19 years. The increase in A advances the date of the turning point 10 years to 2004. In table R3 it can be seen that the post productivity shock value of Y has fallen between 2004 and 2014, due to the overlap of the turning point dates. This is associated with a 40% fall in u. Throughout the two transitions however the value of $Y+(\lambda_1/\lambda_2)\dot{H}$ is relatively stable. The comparative dynamic response in this case in more clearly visible in figures 3.10 -3.12.

The immediate impacts on Y and $Y+(\lambda_1/\lambda_2)\dot{H}$ are 23.5% and 10.2% respectively. These are much larger than the changes observed in case BC1. The reason can be seen from considering the production function in phase I and II, equation 2.8. The elasticity of Y with respect to A in phase II is simply ...

$$\frac{\partial Y}{\partial A}\frac{A}{Y} = 1.$$

In phase I however, labour is endogenous, the elasticity becomes ...

$$\frac{\partial Y}{\partial A}\frac{A}{Y} = \left(\frac{\partial F}{\partial A} + \frac{\partial F}{\partial L}\frac{\partial L}{\partial A}\right)\frac{A}{Y} = \frac{1}{\alpha + \beta}.$$

Thus the impact of the shock is greater in BC2. Intuitively, this is because the change in A raises the marginal product of labour and therefore attracts more labour from the informal sector.[11] It can be seen from table R4, that there is a 24% increase in the quantity of labour in the formal sector after the shock to A.

According to the derivatives above, the change in Y in BC2, should be 1.5 times that of BC1, however there is also a change in u resulting from the shocks. The increase in u, of 19.5% is much larger than in case BC1, which was just 3.5%. This is because the increase in A raises the marginal product of human capital by more in phase I than in phase II. The change in the marginal product of human capital with respect to A is ...

$$\frac{\partial(AF_2)}{\partial A} = F_2 + \frac{\partial F_2}{\partial L}\frac{\partial L}{\partial A}.$$

The second term on the right hand side is positive in phase I but zero in phase II. This explains why the change is Y is very large in case BC2, and why the change in $Y + (\lambda_1 / \lambda_2)\dot{H}$ is large, but not as large as the change in Y. Thus the shift of labour from the informal sector to the formal sector raises the initial responses in the physical capital/consumption sector.

The major difference in the long run effects between the two cases is in the behaviour of human capital. In BC1, the increase in A diverted human capital to the physical capital/consumption sector and therefore reduced the level of human capital, by

[11] Implicitly therefore there has been a decline in the output of the informal sector, which is not accounted for in these results. This raises a problem in comparing the results of case BC2 to BC1, because the latter accounts for all economic activity. The problem does not occur in comparing the long run gains, however, where the base in both cases accounts for all economic activity, the informal sector having been absorbed.

reducing the rate of accumulation over a short period. In this case it can be seen that, although human capital falls initially, by 2010 there is a net increase in human capital. This is because u jumps downwards at the turning point and raises the rate of human capital accumulation. By getting to the turning point sooner, the higher level of human capital growth is reached sooner. This results in a permanent increase in human capital. Figure 3.12 shows the path of human capital bending upwards and intersecting the path for the base scenario.

Looking at the long run gains, for example in the new balanced path in year 2100, the variables, K, Y, c, $Y+(\lambda_1/\lambda_2)\dot{H}$ and MPL have all increased by 18.7% over the base (table R4). This is significantly larger than the 12.4% increase in BC1. The higher levels of output and consumption are associated with increases in both the human capital and physical capital stock. The intuition behind these results is that the shock to A has two effects. First, it raises output and leads to accumulation affects as discussed in BC1. Second, it reduces the total amount of time that the economy is operating under the constraint that prevents the full employment of labour resources in the sector where accumulation occurs.

Next, the effects of an unanticipated 8.5% increase in δ are recorded in table R5. In this experiment the value of δ increases from the calibrated value of 0.080 to 0.087. The long run growth rate κ, from equation 3.1 changes from 1.50% to 1.84%, which is a change of 0.34 per cent per year. This compares with a change of 0.47 per cent per year in case BC1. Thus the long run impact will not be as great as in case BC1, where the same percentage increase in δ caused a larger change in the growth rate.[12] It may be tempting to attribute this result simply to the different benchmarks. It should be recalled however that the lower value of δ in BC2 is a direct result of the differences in the model's dynamics between phase I and phase II. In order to calibrate the model to a given growth path, a lower value of δ is required if it assumed that the growth path represents phase I dynamics rather than phase II dynamics.

The increase in δ causes a large fall in u, -12%, as resources are attracted to human capital investment. It then asymptotically approaches the long run value of 7.8%,

[12] The percentage change in BC2 is greater than the percentage change in BC1, however it is the absolute change in the growth rate which is relevant for the comparison of percentage changes in levels over time

which represents the gap between the two balanced path values of u^*. The withdrawal of human capital reduces the marginal product of capital and the rate of accumulation, and therefore capital stock levels fall. Similarly, consumption falls so that utility, evaluated up until the current year, falls in the short run. The gain in utility and consumption are not as large in this experiment as in BC1. This is because of the larger change in the annual growth rate in the former case.

When δ is increased the time taken to reach the turning point is exactly the same as in the base case. This is despite the fact that the full GDP measure, $Y + (\lambda_1 / \lambda_2)\dot{H}$, increases by over 2% in the first year due to the higher levels of human capital investment. By assumption, human capital investment activities only employ human capital, not unskilled labour. Therefore the increased human capital investment activities do not attract more labour from the informal sector. To the contrary, it can be seen that there is a fall in the quantity of labour employed in the formal sector. The demand for unskilled labour contracts as the human capital resources are attracted away from the physical capital/consumption sector. Despite the lower level of labour employment as a result of the shock to δ, the faster growth rate of human capital offsets this so that there is no change in the date of the turning point.

The increase in human capital accumulation and decline in physical capital accumulation in the short term are shown in figures 3.12 and 3.14, while the effects on consumption and output are recorded in 3.13. Compared to BC1, there are two results. First the shock to δ in BC2 is more costly to the physical capital/consumption sector due to the withdrawal of labour. Second, the higher rate of accumulation in phase I, implies a lower value of δ in BC2 than BC1. Thus a given percentage change in delta has a smaller effect on the economy in BC2.

Under BC2 there is virtually no catch up between the North and the South. The long run growth rate implied by the calibration method was just 1.5% per year which is the same as the North, so that there is no catch up in the long run. The increase in δ raises the long run growth by 0.34 percentage points. This would close the gap in approximately 303 years. The implications for the long run gains from the increases in human capital productivity are, therefore, very different, once allowance has been made for the structural changes, from phase I to phase II.

Finally table R6 shows the effects of shocks to δ and to A combined. The results are not equal to the simple sum of the results in tables R4-R5. Nevertheless the differences are not great, especially in the short run. The long run increase in full GDP, after say 2100 is shown to be 65.7%. Under the alternative calibration method, BC1, the value was 81%. These gains average out to 0.63 and 0.77 per cent per year respectively.

III.v Conclusion

This chapter has attempted to quantify the effects of trade liberalisation in developing America. As discussed in chapter 1, there are many potential dynamic effects of trade liberalisation and these have been difficult to identify and quantify. The dynamic gains from trade considered were: 1. the transitional dynamic effects from changes in consumption behaviour and the allocation of human capital activity, resulting from static efficiency gains in the consumption/physical capital good sector; 2. permanent changes in the balanced growth path resulting from static efficiency gains in the sector producing human capital. These experiments were considered for the model described in chapter 2, under two scenarios. In the first case the model is close to Lucas' (1988) endogenous growth model but with three factors, physical capital, human capital and unskilled labour. The second case introduced a binding minimum on the wage rate of unskilled labour so that unskilled labour was elastically supplied from an informal sector.

The results from this analysis are summarised as follows;

1. An application of a standard endogenous growth model (following Lucas 1988) calibrated to South American data revealed that an 8.5% increase in the productivity of the consumption/physical capital sector, increased consumption by 5% initially. It also increased the rate of accumulation of physical capital, however, which resulted in a 12% increase in consumption after 35 years. This result follows traditional analysis of dynamic responses from trade, as for example, outlined by Corden (1971, 1985). In addition to this accumulation response, there was a 3.5% increase in the amount of human capital effort devoted to physical capital/consumption activities, which resulted in a permanent decline in the human capital stock.

2. Under the calibrated parameters, an 8.5% increase in the productivity of human capital creation was shown to raise the balanced path growth rate by 0.5 percentage

points. While this results in large percentage gains over the base solution, the implications for catch up with the North are still modest. The implied time until catch up between North and South America, under this calibration, was reduced from 77 years to 62 years. The increased investment in human capital required to reach the balanced growth path, occurred primarily at the expense of physical capital accumulation, with only a 0.2-0.8 % decline in consumption over 10 years. These results build on the traditional analysis of dynamic responses by incorporating endogenous growth theory, thus allowing for endogenous changes in the long run growth rate.

3. An application of the model presented in chapter 2, allowing for supply responses of unskilled labour had quite different outcomes. The increase in efficiency of the physical capital/consumption sector has a greater accumulation effect than the previous case, because each unit of capital also allows more labour to be employed for accumulation purposes. The initial 8.5% increase in consumption/physical capital efficiency therefore resulted in a long run increase in consumption and income of 18%. Additional welfare gains are obtained by reaching the turning point sooner. This is realised by a higher growth rate of human capital after reaching the turning point, so that there was a net increase in human capital over the base case. Although an increase in human capital productivity could also potentially reduce the transition time to the turning point, the assumption that human capital sector only employs human capital, meant that demand for unskilled labour falls in response to this experiment.

4. Allowing for unskilled labour supply responses resulted in different calibrated parameter values. In particular the calibrated value of δ was lower than in the standard model. This implied a much lower long run growth rate than that implied in the standard model. An implication of the different parameter values was that the 8.5% increase in human capital productivity only increased the long run growth rate by 0.34 percentage points. This combined with the result that the long run calibrated growth rate was just 0.15% per year, means that there would be very little catch up resulting from the changes, even within 100 years.

The results, especially with respect to the extent of catch up with the USA, indicate that the dynamic effects considered are not sufficient to produce a growth miracle of the scope seen in the HPAEs since WWII. If trade liberalisation is an important component of such a miracle, the dynamic benefits of trade must extend well beyond the accumulation effects considered here. This suggests some important avenues for further research such as quantifying the extent of technology diffusion and international factor movements and quantifying externalities associated with either human or physical capital accumulation in the formal sector.[13]

[13] An example of modelling dynamic gains with factor mobility and technology diffusion is Harris (1994).

Figure 3.1- An increase in productivity resulting in a temporary increase in the rate of capital accumulation

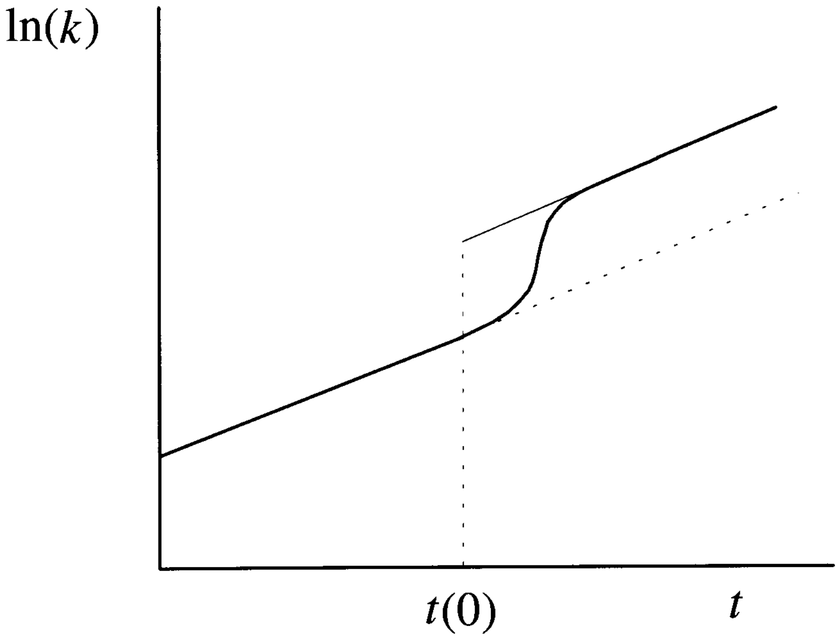

Figure 3.2- An increase in productivity resulting in a permanent increase in the rate of capital accumulation

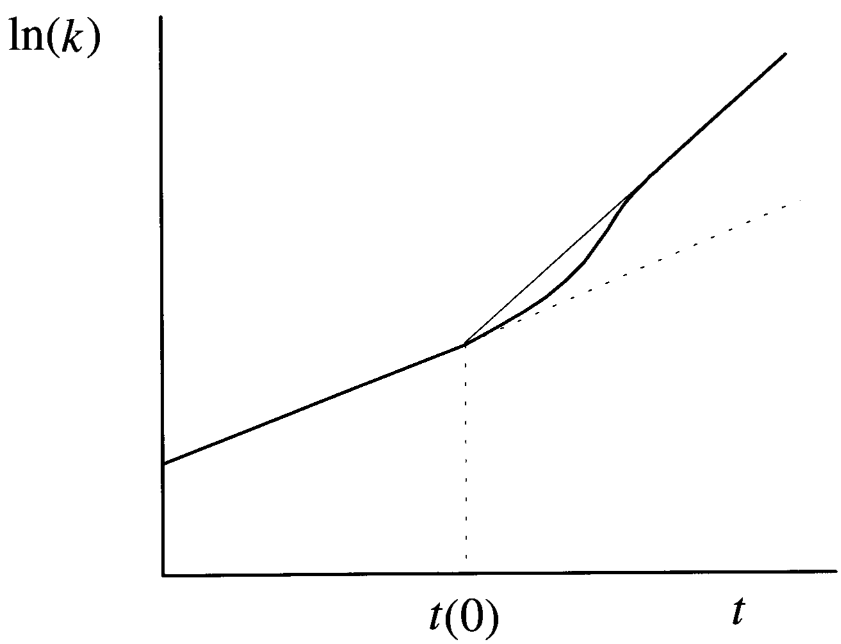

Figure 3.3 - GDP in Benchmark solution BC1 versus Hofman's (1992) data

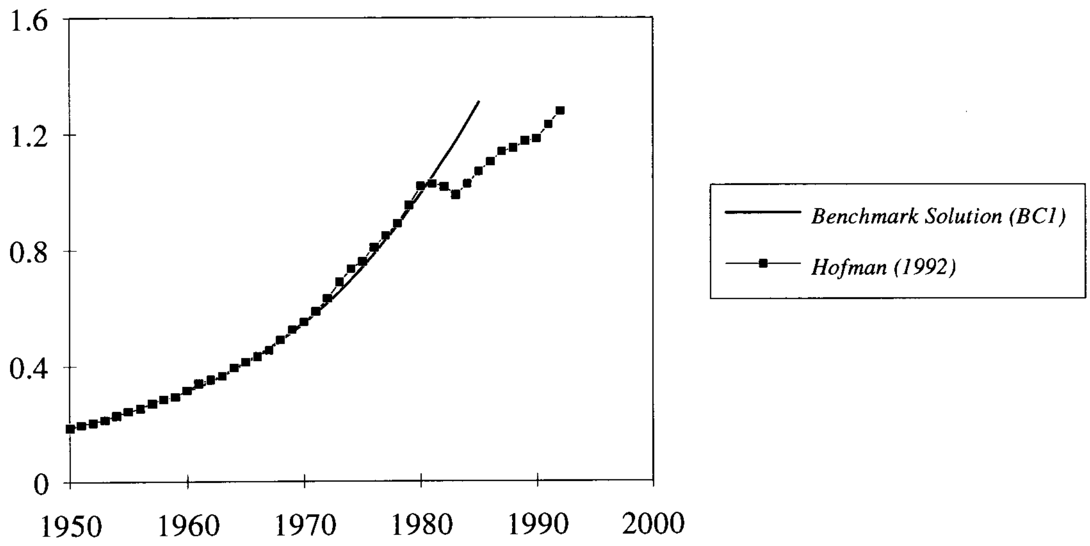

Figure 3.4 - GDP in Benchmark solution BC2 versus Hofman's (1992) data

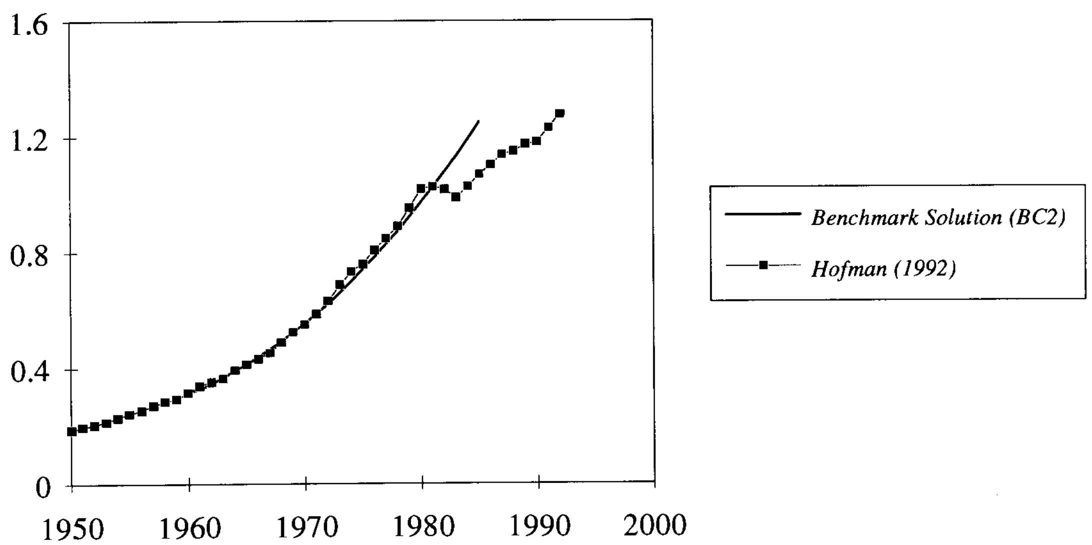

Figure 3.5 - Consumption and GDP after an 8.5% unanticipated increase in *A* (*BC1*)

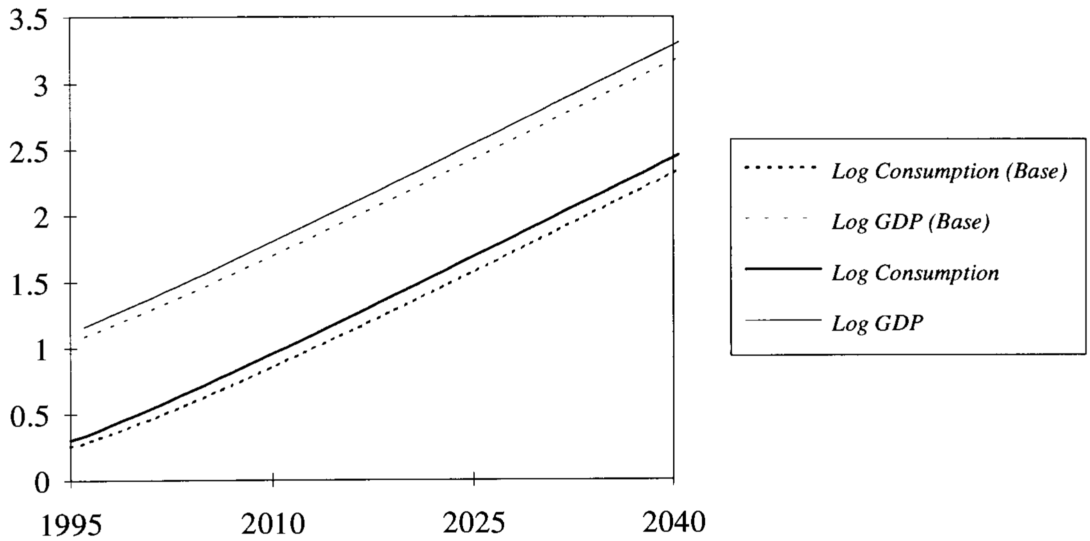

Figure 3.6 - Physical Capital after an 8.5% unanticipated increase in *A* (*BC1*)

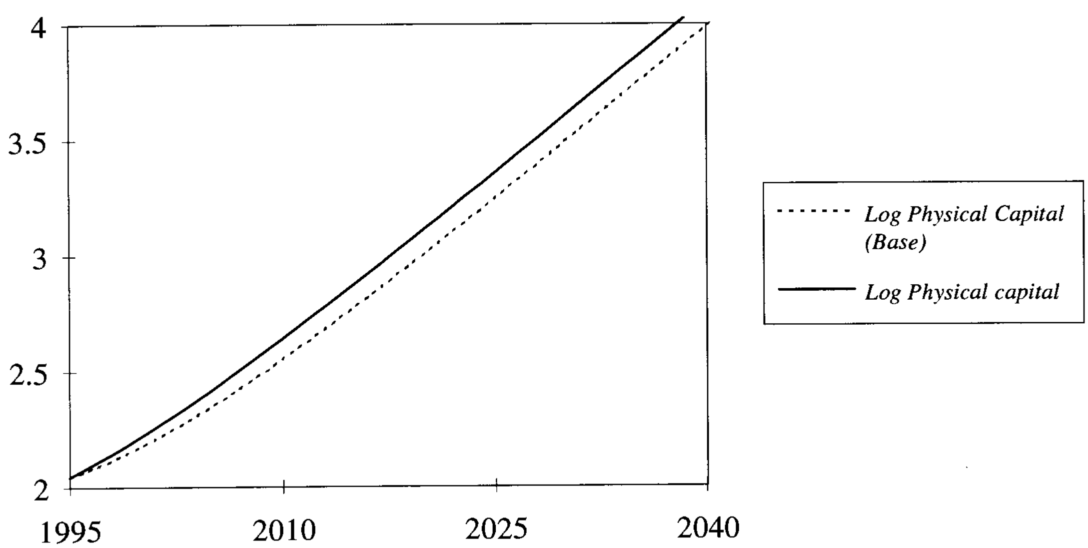

Figure 3.7 - Human Capital after an 8.5% unanticipated increase in *A* and δ (*BC1*)

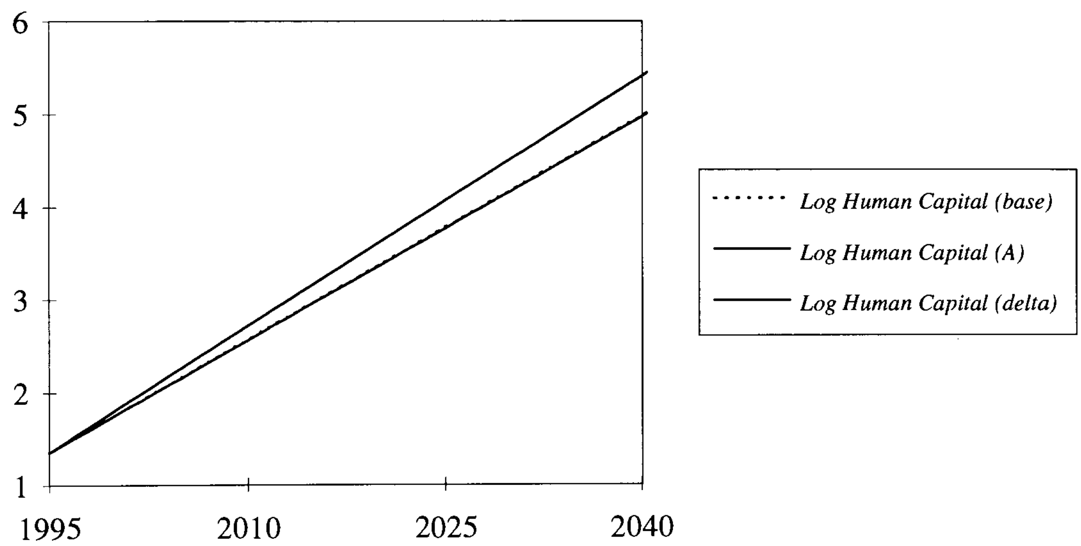

Figure 3.8 - Consumption and GDP after an 8.5% unanticipated increase in δ (*BC1*)

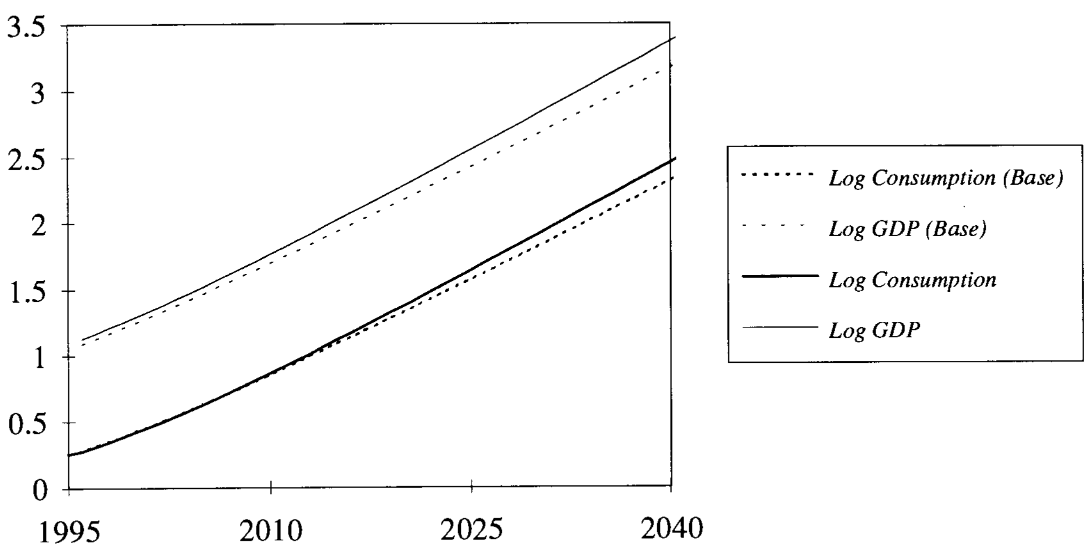

Figure 3.9 - Physical Capital after an 8.5% unanticipated increase in δ (*BC1*)

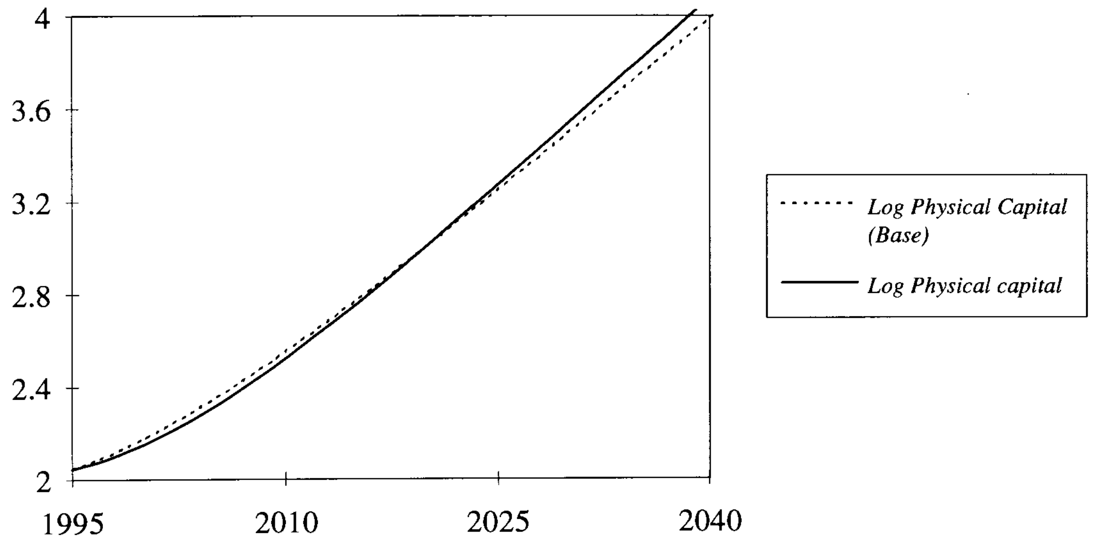

Figure 3.10 - Consumption and GDP after an 8.5% unanticipated increase in *A* (*BC2*)

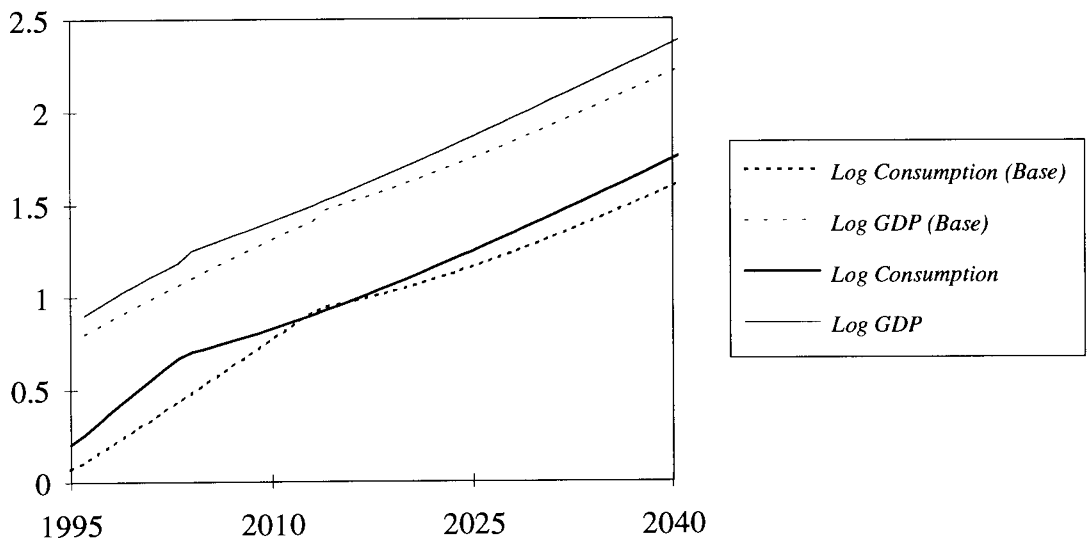

Figure 3.11 - Physical Capital after an 8.5% unanticipated increase in *A* (*BC2*)

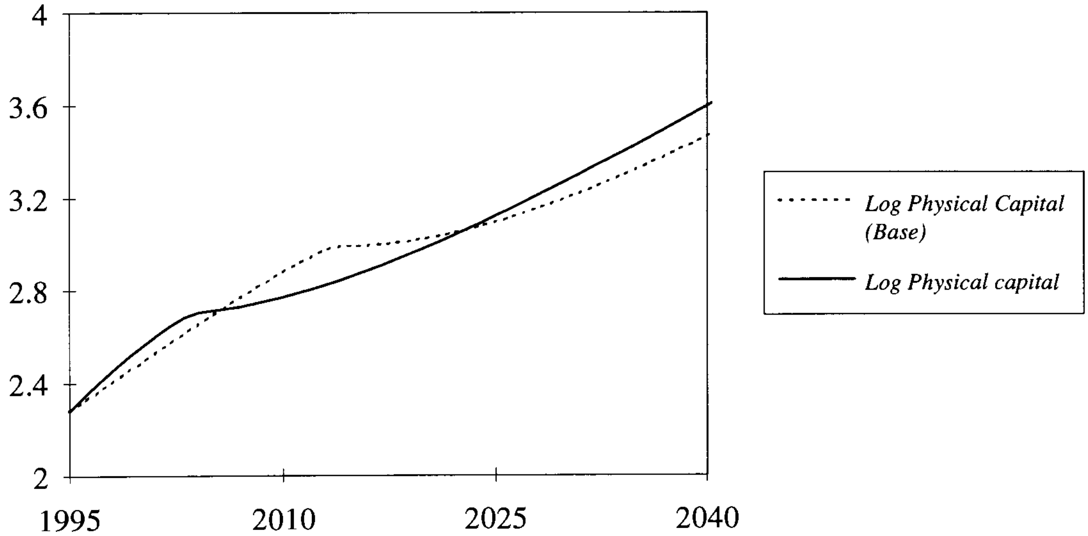

Figure 3.12 - Human Capital after an 8.5% unanticipated increase in δ and *A* (*BC2*)

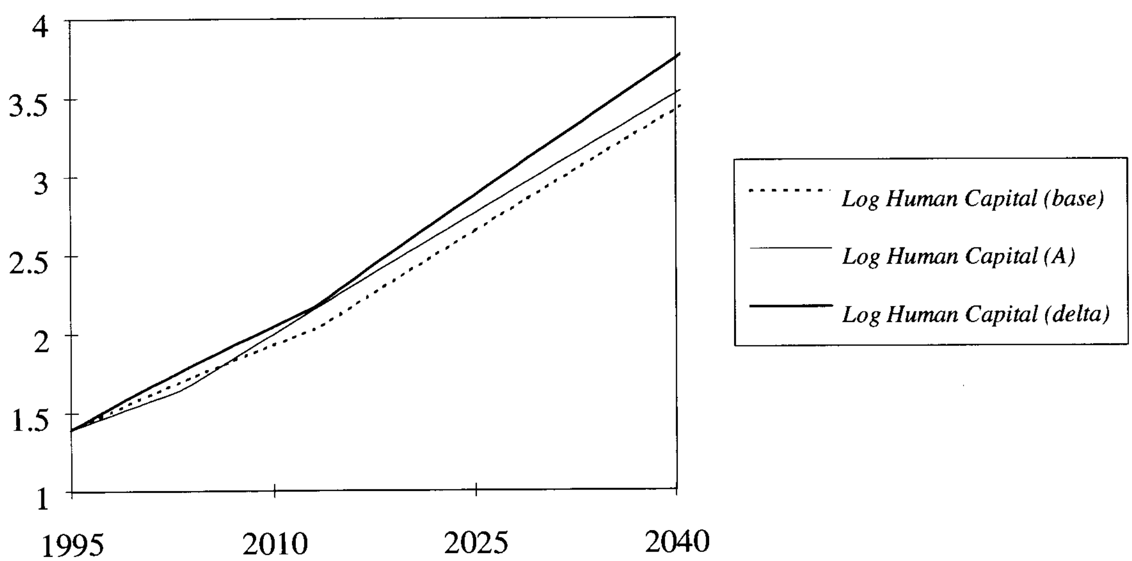

Figure 3.13 - Consumption and GDP after an 8.5% unanticipated increase in δ (*BC2*)

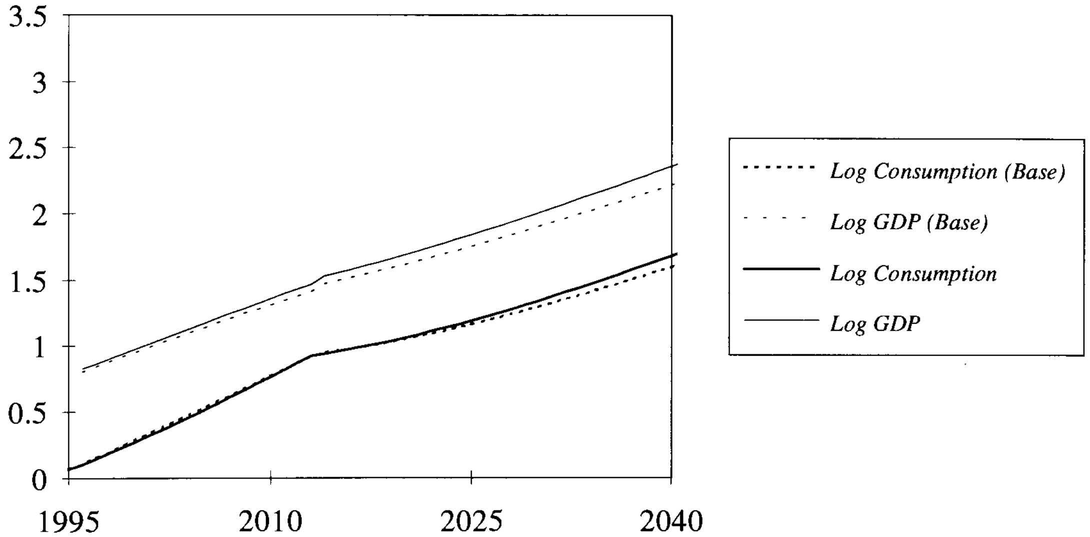

Figure 3.14 - Physical Capital after an 8.5% unanticipated increase in δ (*BC2*)

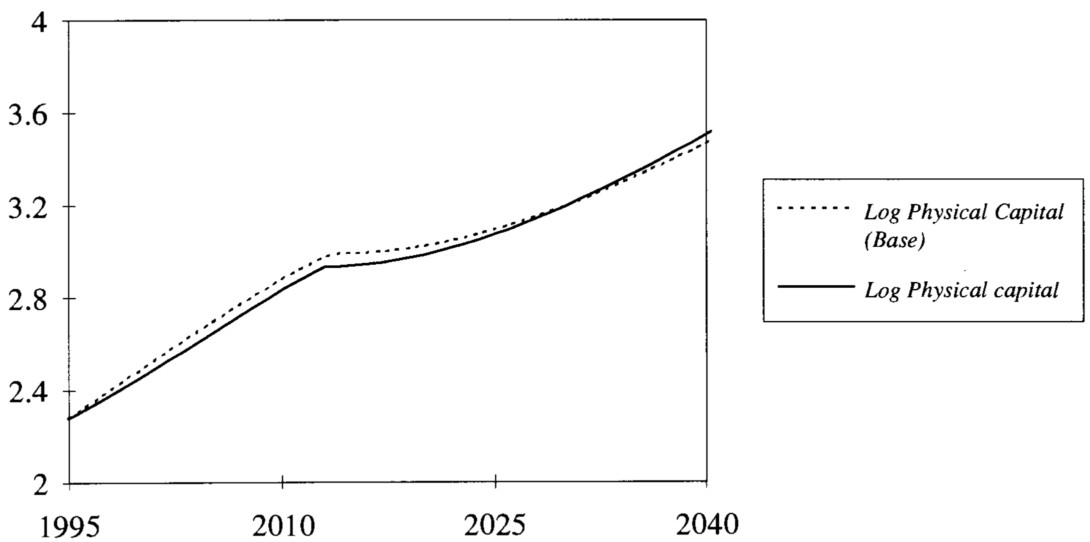

Table 3.1 – Summary data for developing American economies

Average rate of growth of GDP per worker, 1960-80, (per cent).	3.03
Average rate of growth of net capital stock per worker, 1960-80, (per cent).	3.81
Average growth rate of labour force, 1960-80, (per cent).	2.84
Average capital - GDP ratio, 1960-80.	1.38
Value of GDP in 1992 $1980 PPP (billions)	1277.15
Value of capital stock in 1992 $1980 PPP (billions)	2347.88
Labour force in 1992 (millions)	123.32

Source: Hofman (1992)

Table 3.2 – Parameter values used for calibration of Case BC1

Parameters	Symbol	Value
Balanced path growth rate of capital and GDP, per cent	κ	3.00
Constant growth rate of labour force, percent	n	2.00
Balanced path growth rate of human capital per capita, per cent	ν	6.00
Intertemporal elasticity of consumption	σ	1.00
Rate of time preference, per cent	ρ	5.00
Productivity of human capital investment	δ	0.11
Value share of physical capital	α	0.33
Value share of human capital	β	0.33
Value share of labour	$1-\alpha-\beta$	0.33
Total factor Productivity	A	1.47
Balanced path constants		
Balanced path marginal product of capital	r^*	8.00
Balanced path share of human capital in production	u^*	0.27

Table 3.3 – Benchmark parameter values used for calibration when wage constraint is binding, Case BC2

Parameters	Symbol	Value
Balanced path growth rate of capital in phase I, per cent	γ	3.00
Balanced path growth rate of capital and GDP in phase II, per cent	κ	1.50
Constant growth rate of labour force, percent	n	2.00
Balanced path growth rate of human capital per capita, per cent	v	3.00
Intertemporal elasticity of consumption	σ	1.00
Rate of time preference, per cent	ρ	5.00
Productivity of human capital investment	δ	0.08
Value share of physical capital	α	0.33
Value share of human capital	β	0.33
Value share of labour	$1-\alpha-\beta$	0.33
Total Factor Productivity	A	1.22
Balanced path constants		
Balanced path marginal product of capital, per cent	r^*	6.50
Balanced path share of human capital in production, per cent	u^*	37.5

Table 3.4 – Rural and Traditional Sector Shares in 19 Latin American Countries: 1970, 1980[a]

	Total labour share (per cent)	*Traditional sector share (per cent)*
1970		
Rural	50	64
Urban	50	17
Total	100	40
1980		
Rural	34	65
Urban	66	19
Total	100	35

a. Traditional sector labour is defined as own account workers and unpaid family members and paid domestic services in urban areas.
Source: International Labour Office (1987)

Table R1 – Impact of 8.5% productivity shock on physical capital accumulation, A: Case BC1
(numbers report the per cent change from base)

Years	K	H	Y	$Y+(\lambda_1/\lambda_2)\dot{H}$	c	U_0	u	MPK	MPL	MPH
1995	0.0	0.0	9.8	7.3	4.9	2.1	3.5	9.8	9.8	9.8
1996	1.0	-0.1	9.9	7.5	5.5	2.2	3.4	8.8	9.9	10.0
1997	1.9	-0.2	10.1	7.9	6.1	2.3	3.1	8.0	10.1	10.3
1998	2.8	-0.3	10.3	8.3	6.7	2.4	2.8	7.3	10.3	10.6
1999	3.6	-0.3	10.5	8.6	7.2	2.5	2.6	6.6	10.5	10.8
2000	4.4	-0.4	10.6	9.0	7.6	2.6	2.3	6.0	10.6	11.1
2005	7.4	-0.6	11.3	10.3	9.5	3.0	1.4	3.6	11.3	12.0
2010	9.4	-0.8	11.8	11.2	10.7	3.2	0.8	2.2	11.8	12.7
2015	10.6	-0.9	12.0	11.7	11.4	3.4	0.5	1.3	12.0	13.0
2020	11.4	-0.9	12.2	12.0	11.8	3.4	0.3	0.7	12.2	13.3
2025	11.8	-1.0	12.3	12.2	12.1	3.5	0.2	0.4	12.3	13.4
2050	12.4	-1.0	12.4	12.4	12.4	3.5	0.0	0.0	12.4	13.6
2100	12.4	-1.0	12.4	12.4	12.4	3.3	0.0	0.0	12.4	13.6

Key: K, physical capital stock; H, human capital stock; Y, consumption plus physical capital investment; $Y+(\lambda_1/\lambda_2)\dot{H}$, GDP and human capital investment weighted by the relative shadow price of human capital; c, consumption per capita; U_0, sum of discounted utility to relevant year; u, proportion of human capital allocated to physical capital investment and consumption; MPK, marginal product of physical capital; MPL, marginal product of labour; MPH, marginal product of human capital in producing capital and consumption output.

Table R2 – Impact of 8.5% productivity shock on human capital accumulation, δ: Case BC1
(numbers report the per cent change from base)

Years	K	H	Y	$Y+(\lambda_1/\lambda_2)\dot{H}$	c	U_0	u	MPK	MPL	MPH
1995	0.0	0.0	-3.9	4.2	-0.2	-0.1	-11.2	-3.9	-3.9	-3.9
1996	-0.7	1.0	-3.8	4.2	-0.4	-0.1	-11.0	-3.1	-3.8	-4.8
1997	-1.2	2.1	-3.5	4.2	-0.5	-0.2	-10.7	-2.3	-3.5	-5.5
1998	-1.7	3.1	-3.3	4.2	-0.7	-0.2	-10.4	-1.5	-3.3	-6.2
1999	-2.2	4.2	-3.0	4.3	-0.7	-0.2	-10.2	-0.8	-3.0	-6.9
2000	-2.5	5.2	-2.7	4.4	-0.8	-0.2	-9.9	-0.2	-2.7	-7.5
2005	-3.3	10.5	-1.1	5.3	-0.3	-0.2	-9.0	2.3	-1.1	-10.5
2010	-2.9	16.0	0.8	6.9	1.0	-0.1	-8.5	3.8	0.8	-13.0
2015	-1.6	21.6	3.0	8.9	2.7	0.0	-8.1	4.7	3.0	-15.3
2020	0.1	27.5	5.3	11.2	4.8	0.3	-8.0	5.1	5.3	-17.4
2025	2.2	33.6	7.7	13.7	7.1	0.5	-7.9	5.4	7.7	-19.4
2050	14.6	68.9	21.0	27.6	20.2	1.4	-7.8	5.6	21.0	-28.3
2100	44.8	169.6	52.9	61.3	51.9	2.5	-7.8	5.6	52.9	-43.3

Key: K, physical capital stock; H, human capital stock; Y, consumption plus physical capital investment; $Y+(\lambda_1/\lambda_2)\dot{H}$, GDP and human capital investment weighted by the relative shadow price of human capital; c, consumption per capita; U_0, sum of discounted utility to relevant year; u, proportion of human capital allocated to physical capital investment and consumption; MPK, marginal product of physical capital; MPL, marginal product of labour; MPH, marginal product of human capital in producing capital and consumption output.

Table R3 – Impact of 8.5% productivity shock on physical and human capital accumulation, A, δ: Case BC1
(numbers report the per cent change from base)

Years	K	H	Y	$Y+(\lambda_1/\lambda_2)\dot{H}$	c	U_0	u	MPK	MPL	MPH
1995	0.0	0.0	5.5	11.6	4.7	2.0	-7.9	5.5	5.5	5.5
1996	0.3	0.9	5.8	11.9	5.1	2.1	-7.9	5.5	5.8	4.8
1997	0.6	1.9	6.2	12.3	5.5	2.1	-7.9	5.6	6.2	4.2
1998	0.9	2.9	6.7	12.7	5.9	2.2	-7.8	5.7	6.7	3.7
1999	1.3	3.8	7.2	13.2	6.3	2.3	-7.8	5.8	7.2	3.2
2000	1.6	4.8	7.6	13.6	6.8	2.4	-7.8	5.9	7.6	2.7
2005	3.8	9.8	10.1	16.1	9.2	2.7	-7.7	6.1	10.1	0.3
2010	6.3	15.0	12.7	18.8	11.8	3.1	-7.7	6.1	12.7	-2.0
2015	8.9	20.5	15.4	21.7	14.5	3.4	-7.7	6.0	15.4	-4.3
2020	11.6	26.3	18.2	24.6	17.3	3.7	-7.7	5.9	18.2	-6.4
2025	14.4	32.3	21.0	27.6	20.1	3.9	-7.8	5.8	21.0	-8.6
2050	28.8	67.2	36.1	43.5	35.2	4.9	-7.8	5.6	36.1	-18.6
2100	62.8	166.9	71.9	81.3	70.8	5.8	-7.8	5.6	71.9	-35.6

Key: K, physical capital stock; H, human capital stock; Y, consumption plus physical capital investment; $Y+(\lambda_1/\lambda_2)\dot{H}$, GDP and human capital investment weighted by the relative shadow price of human capital; c, consumption per capita; U_0, sum of discounted utility to relevant year; u, proportion of human capital allocated to physical capital investment and consumption; MPK, marginal product of physical capital; MPL, marginal product of labour; MPH, marginal product of human capital in producing capital and consumption output.

Table R4 – Impact of 8.5% productivity shock on physical capital accumulation, A: Case BC2
(numbers report the per cent change from base)

Years	K	H	Y	$Y+(\lambda_1/\lambda_2)\dot{H}$	c	U_0	u	MPK	MPL	MPH	Labour
1995	0.0	0.0	23.5	10.2	14.2	6.3	19.5	23.5	0.0	23.5	23.5
1996	1.9	-0.8	23.7	10.6	16.1	6.6	19.1	21.4	0.0	24.6	23.7
1997	3.5	-1.5	23.8	11.3	18.0	7.0	18.1	19.6	0.0	25.7	23.8
1998	4.9	-2.2	23.7	11.8	19.7	7.3	17.2	18.0	0.0	26.5	23.7
1999	6.0	-2.9	23.4	12.2	21.2	7.5	16.2	16.5	0.0	27.1	23.4
2000	6.8	-3.5	23.0	12.5	22.7	7.8	15.1	15.2	0.0	27.5	23.0
2005	1.9	-2.5	-3.9	15.0	21.2	8.5	-40.5	-5.7	-17.6	-1.4	16.7
2010	-10.4	7.1	-8.4	10.1	5.8	7.3	-40.7	2.2	-13.3	-14.5	5.7
2015	-11.9	14.0	11.7	5.8	-0.8	5.9	9.5	26.8	11.7	-2.1	0.0
2020	-3.9	12.9	13.6	9.4	4.7	5.1	6.6	18.2	13.6	0.6	0.0
2025	2.6	12.0	15.1	12.3	8.9	4.8	4.5	12.2	15.1	2.8	0.0
2050	16.6	10.6	18.3	17.9	17.5	4.8	0.5	1.4	18.3	7.0	0.0
2100	18.7	10.4	18.7	18.7	18.7	4.9	0.0	0.0	18.7	7.6	0.0

Key: K, physical capital stock; H, human capital stock; Y, consumption plus physical capital investment; $Y+(\lambda_1/\lambda_2)\dot{H}$, GDP and human capital investment weighted by the relative shadow price of human capital; c, consumption per capita; U_0, sum of discounted utility to relevant year; u, proportion of human capital allocated to physical capital investment and consumption; MPK, marginal product of physical capital; MPL, marginal product of labour; MPH, marginal product of human capital in producing capital and consumption output; Labour, total volume of labour employed in the formal sector.

Table R5 – Impact of 8.5% productivity shock on human capital accumulation, δ: Case BC2

(numbers report the per cent change from base)

Years	K	H	Y	$Y+(\lambda_1/\lambda_2)\dot{H}$	c	U_0	u	MPK	MPL	MPH	Labour
1995	0.0	0.0	-6.4	2.4	-0.3	0.0	-12.4	-6.4	0.0	-6.4	-6.4
1996	-0.9	0.9	-6.3	2.3	-0.7	-0.2	-12.3	-5.5	0.0	-7.1	-6.3
1997	-1.7	1.8	-6.1	2.2	-1.1	-0.3	-11.8	-4.5	0.0	-7.7	-6.1
1998	-2.3	2.6	-5.8	2.2	-1.4	-0.4	-11.5	-3.6	0.0	-8.2	-5.8
1999	-2.9	3.5	-5.5	2.1	-1.6	-0.5	-11.1	-2.7	0.0	-8.7	-5.5
2000	-3.4	4.3	-5.2	2.2	-1.8	-0.5	-10.7	-1.9	0.0	-9.2	-5.2
2005	-4.7	8.5	-3.3	3.0	-1.7	-0.6	-9.3	1.4	0.0	-10.9	-3.3
2010	-4.6	12.4	-1.0	4.6	-0.6	-0.6	-8.4	3.7	0.0	-12.0	-1.0
2015	-5.2	17.5	0.9	5.8	-0.7	-0.5	-7.5	6.4	0.9	-14.2	0.0
2020	-3.8	21.5	2.5	7.4	0.9	-0.4	-7.4	6.6	2.5	-15.7	0.0
2025	-2.1	25.7	4.3	9.2	2.7	-0.3	-7.4	6.5	4.3	-17.0	0.0
2050	7.9	48.6	13.8	19.2	12.7	0.5	-7.7	5.5	13.8	-23.5	0.0
2100	28.5	108.7	35.0	41.5	34.0	1.5	-7.8	5.1	35.0	-35.3	0.0

Key: K, physical capital stock; H, human capital stock; Y, consumption plus physical capital investment; $Y+(\lambda_1/\lambda_2)\dot{H}$, GDP and human capital investment weighted by the relative shadow price of human capital; c, consumption per capita; U_0, sum of discounted utility to relevant year; u, proportion of human capital allocated to physical capital investment and consumption; MPK, marginal product of physical capital; MPL, marginal product of labour; MPH, marginal product of human capital in producing capital and consumption output; Labour, total volume of labour employed in the formal sector.

Table R6 – Impact of 8.5% productivity shock on physical and human capital accumulation, A, δ: Case BC2
(numbers report the per cent change from base)

Years	K	H	Y	$Y+(\lambda_1/\lambda_2)\dot{H}$	c	U_0	u	MPK	MPL	MPH	Labour
1995	0.0	0.0	14.5	12.5	11.7	5.2	2.7	14.5	0.0	14.5	14.5
1996	0.9	0.2	14.8	12.9	12.9	5.5	2.7	13.8	0.0	14.6	14.8
1997	1.7	0.4	15.3	13.5	14.1	5.7	2.5	13.4	0.0	14.8	15.3
1998	2.4	0.6	15.7	14.1	15.3	5.9	2.3	13.1	0.0	15.0	15.7
1999	2.9	0.9	16.1	14.6	16.4	6.1	2.0	12.8	0.0	15.1	16.1
2000	3.4	1.1	16.3	15.1	17.6	6.2	1.6	12.5	0.0	15.0	16.3
2005	3.5	2.7	15.4	16.8	23.1	7.1	-1.9	11.5	0.0	12.4	15.4
2010	-10.5	17.2	-8.9	16.4	7.1	6.5	-46.5	1.8	-13.8	-22.3	5.7
2015	-12.8	29.3	12.3	12.6	0.6	5.3	-0.4	28.8	12.3	-13.1	0.0
2020	-4.8	32.6	15.8	17.7	6.9	4.7	-2.6	21.7	15.8	-12.7	0.0
2025	2.3	36.3	19.1	22.2	12.5	4.6	-4.2	16.5	19.1	-12.6	0.0
2050	24.7	59.7	32.8	38.9	31.0	5.4	-7.4	6.5	32.8	-16.8	0.0
2100	50.4	124.0	58.0	65.7	56.8	6.4	-7.8	5.1	58.0	-29.5	0.0

Key: K, physical capital stock; H, human capital stock; Y, consumption plus physical capital investment; $Y+(\lambda_1/\lambda_2)\dot{H}$, GDP and human capital investment weighted by the relative shadow price of human capital; c, consumption per capita; U_0, sum of discounted utility to relevant year; u, proportion of human capital allocated to physical capital investment and consumption; MPK, marginal product of physical capital; MPL, marginal product of labour; MPH, marginal product of human capital in producing capital and consumption output; Labour, total volume of labour employed in the formal sector.

Appendix: Solving the model using numerical methods

The problem given by 2.12a-2.12g is a system of four ordinary differential equations for the state and costate variables $K(t)$, $H(t)$, $\lambda_1(t)$ and $\lambda_2(t)$ and two algebraic equations. While these represent the solution to the constrained maximization problem in definition 1 (or 2.12), the differential equations must still be integrated to give the solution path to this problem. The relevant differential equation system was shown to be ...

(2.12c) $$\dot{K} = AF(K, uH, L^*) - Nc$$

(2.12d) $$\dot{H} = \delta(1-u)H$$

(2.12e) $$\dot{\lambda}_1 = \rho\lambda_1 - \lambda_1\left[AF_1(K, uH, L^*)\right]$$

(2.12g) $$\dot{\lambda}_2 = (\rho - \delta)\,\lambda_2 \,.$$

The control variables $c(t)$ and $u(t)$ can be eliminated from the system using the algebraic first order conditions 2.12a and 2.12b, which relate the control variables to the state and costate variables. The substitution for $c(t)$ can be made directly while it is convenient to make the substitution for $u(t)$ numerically. Having made these substitutions one is left with a system of four ordinary differential equations in with four endogenous variables, $K(t)$, $H(t)$, $\lambda_1(t)$, $\lambda_2(t)$, two exogenous variables $N(t)$, $A(t)$ and the parameters α, β, δ, ρ, n, $\overline{w}$. In phase I, the exogenous variable $N(t)$ is superfluous, while in phase II the parameter $\overline{w}$ is superfluous.

The system cannot be solved without four values for the endogenous variables at some t. If the values $K(0)$, $H(0)$, $\lambda_1(0)$ and $\lambda_2(0)$ were known it would be straightforward to integrate the equations from the initial values along the time path using 12c-12g as updating rules. The nature of the problem however does not permit this as the prices $\lambda_1(0)$ and $\lambda_2(0)$ are determined by considering the value of the two capital stocks over the infinite horizon. Thus only two initial values are known $K(0)$, $H(0)$. In finite horizon problems the values of $K(T)$,

$H(T)$ may also be known. In that case the model can be solved by making an initial guess at $\lambda_1(0)$ and $\lambda_2(0)$, integrating along the path given by 12c-12g. If the initial guess is correct then the values of $K(T)$ and $H(T)$ will be equal to the known or desired values and a solution is found. If not then the error can be used to adjust the initial estimates of $\lambda_1(0)$ and $\lambda_2(0)$ and in this way one can iterate towards a solution This is known as the "shooting method", Press *et al* (1990).

In this case the model has an infinite horizon so that there are no terminal values for the capital stocks. What is known, however, are relations between the endogenous variables that must hold along a balanced growth path. Using the balanced path conditions two additional equations can be obtained which serve as end boundary conditions. Thus a solution can be approximated by choosing a sufficiently long time horizon and assuming that a balanced path is achieved in that time. The end boundary conditions were given by 2.18 and 2.25.

(2.18) $$\frac{\kappa}{\sigma}+\rho = AF_1\left(K(s), u^* H(s), N(s)\right)$$

(2.25) $$H(s)=\frac{1}{u^*}\left[\frac{\lambda_1(s)\beta A K(s)^{\alpha} N(s)^{(1-\alpha-\beta)}}{\lambda_2(s)\delta}\right]^{\frac{1}{(1-\beta)}}$$

Equation 2.18 relates the balanced path growth rate of consumption, κ, to the marginal product of capital.[1] In simpler models with one capital stock and in which the infinite horizon is a steady state, the equation describing this economic principle, i.e. the Keynes-Ramsey equation, could be used to obtain a value for the final capital stock per worker. In this case its gives a relation between the human and physical capital stocks. Similarly 2.25 gives a second relationship between the endogenous variables that must hold on a balanced path when marginal value product of human capital effort $\lambda_1 AHF_2(\cdot)$ must be

[1] In a steady state these variables would be constant and the equation simply says that the marginal product of capital must equal the intertemporal discount rate, ρ.

constant for a constant $u(t) = u^*$. These two equations along with $K(0)$, and $H(0)$ complete the required boundary conditions to obtain a solution to the model.

The iterative solution method outlined above is known as the shooting method, and is perhaps the simplest and most intuitive method of obtaining a solution to a two point boundary value problem. In many economic problems, however, the point which one wants to aim for, is a saddlepoint equilibrium and is thus unstable in some dimensions. This makes 'shooting' very difficult as extremely small changes in the guess of the initial conditions can lead to large errors at the steady state point or balanced path. For this reason a more reliable method, known as 'relaxation', has been employed[2]. Relaxation methods require transforming the system of differential equations into a system of discrete algebraic equations. Thus 2.12c-2.12g are replaced by A1-A4.

(A1) $$K_t - K_{t-1} = \frac{1}{2}\left((A_t + A_{t-1})(F_t(.) + F_{t-1}(.)) - (N_t + N_{t-1})(c_t + c_{t-1})\right)$$

(A2) $$H_t - H_{t-1} = \delta\left(1 - (u_t + u_{t-1})\frac{1}{2}\right)(H_t - H_{t-1})\frac{1}{2}$$

(A3) $$\lambda_{1t} - \lambda_{1t-1} = \left(\rho - (\lambda_{1t} + \lambda_{1t-1})(A_t + A_{t-1})(F_{1t}(.) + F_{1t-1}(.))\frac{1}{2}\right)$$

(A4) $$\lambda_{2t} - \lambda_{2t-1} = (\rho - \delta)\,(\lambda_{2t} + \lambda_{2t-1})\frac{1}{2}$$

The equations for the two end boundary conditions are similarly transformed. The solution method begins with a guess solution path for the system. This consists of an estimate of the values of each endogenous variable at M discrete points, t. In addition to the system of differential equations, the algorithm also requires equations for the partial derivatives of each difference equation for

[2] The method and techniques for obtaining numerical solutions are described in Press *et al* (1990). The following discussion draws heavily on the discussion in Press *et al* Chapter 16 on solving two point boundary value problems.

each endogenous variable. These are used to evaluate a Taylor series approximation to the discrete change at each discrete point. The solution algorithm then adjusts the $4 \times M$ endogenous variables by solving a $4 \times M$ linear simultaneous equation system.

The size of M will depend on the time period over which a solution is desired, i.e for how many years, and how many discrete intervals are required each year. The appropriate number will depend on the particular problem, however in most of the solutions obtained in this study M was set above 200 with each interval representing 1-2 years. This means that the time horizons considered were around 100-200 years. As discussed above, the long time horizon is necessary to ensure that the solution is sufficiently close to the balanced path in finite time.

The following text gives the FORTRAN subroutine used to solve the equation system described above. It draws on several other subroutines that are for controlling data inputs and outputs. The matrix Y contains the solution of the endogenous variables and S contains the partial derivatives. For further details on how the algorithm operates one is referred to Press *et al* (1990).

```
      SUBROUTINE DIFEQ(K,K1,K2,JSF,IS1,ISF,INDEXV,NE,S,NSI,
 &    NSJ,Y,NYJ,NYK,X,H,RBAR,M)
      IMPLICIT REAL*8(A-H,O-Z)
      DOUBLEPRECISION Y(NYJ,NYK),S(NSI,NSJ),INDEXV(NYJ),RBAR(4)
      DOUBLEPRECISION ATY(4),ATA,ATP,X(M),H,RHO,ALPHA,SIGMA,Z,F,FK,
 &    FKK,G,RLL,TEMPK,TEMPH,
 &    DELTA,ATU,FU,FKH,U1,UK,U2,UH,V,KAPPA,BETA,KTEMP,FH,FKU,
 &    ATLSTAR,WBAR,ATL,UM,FL,UBAR,ATLMAX,RSTAR

      COMMON /VALUES/ A(201),P(201),ATLABOUR(201),ATUUU(201),
      &      GDP1(301),GDP2(201),RMPK(201),RMPH(201),RMPL(201)
      COMMON /PARAMS/ RHO,Z,ALPHA,BETA,SIGMA,DELTA,G,WBAR,UBAR
      COMMON /SHOCKS/ BOUND

      *SET VALUES OF STATE VARIABLES AND EXOGENOUS ON GRID

      * AT INITIAL BOUNDARY
      IF (K.EQ.K1) THEN
            ATP=P(K1)
            ATA=A(K1)
            ATY(1)=Y(1,K1)
            ATY(2)=Y(2,K1)
            ATY(3)=Y(3,K1)
            ATY(4)=Y(4,K1)

      * AT END BOUNDARY
      ELSE IF(K.GT.K2) THEN
            ATP=(P(M))
            ATA=(A(M))
            ATY(1)=(Y(1,M))
            ATY(2)=(Y(2,M))
            ATY(3)=(Y(3,M))
            ATY(4)=(Y(4,M))
```

```
* AT INTERMEDIATE POINTS
ELSE
      DO 10 I=1,NE
            ATY(I)=(Y(I,K)+Y(I,K-1))/2
      CONTINUE
      ATP=(P(K)+P(K-1))/2
      ATA=(A(K)+A(K-1))/2
ENDIF

*PROCEEDURE FOR LABOUR AND DERIVS OF U
*CALCULATE ENDOGENOUS VALUE OF LABOUR ASSUMING L<N
*THIS IS EQUATION 2.6

ATL=((1-ALPHA-BETA)/WBAR)**((1-BETA)/ALPHA)*ATA**(1/ALPHA)
 & *ATY(3)*
 & ((ATY(1)*BETA)/(ATY(2)*DELTA*(ALPHA+BETA)))**(BETA/ALPHA)

*SEE IF IN PHASE I OR PHASE II. IF IN PHASE I THEN;
*ASSIGN PHASE II FORMULA FOR U
*SET L=N
*EVALUATE F(K,UH,L)
*EVALUATE VARIOUS USEFUL DERIVATIVES OF U AND F

IF(ATL.GT.ATP) THEN

      ATU=((ATY(1)*BETA*ATA*ATY(3)**ALPHA*ATP**
 &    (1-ALPHA-BETA))/(ATY(2)*DELTA))
 &    **(1/(1-BETA))*(1/ATY(4))
      ATLSTAR=ATP
      F=ATA*ATY(3)**ALPHA*(ATU*ATY(4))**
 &    BETA*ATLSTAR**(1-ALPHA-BETA)

      U1=0.5*(1/(1-BETA))*ATU/ATY(1)
      U2=-0.5*(1/(1-BETA))*ATU/ATY(2)
```

```
        UK=0.5*(ALPHA/(1-BETA))*ATU/ATY(3)
        UH=-0.5*ATU/ATY(4)

        FK=ALPHA*F/ATY(3)
        FKK=(ALPHA-1)*FK/ATY(3)
        FU=BETA*F/ATU
        FH=(BETA)*F/ATY(4)
        FKH=(BETA)*FK/ATY(4)
        FKU=BETA*FK/ATU
        FL=(1-ALPHA-BETA)*F/ATP


ENDIF

*RECORD PHASE SPECIFIC VALUES
IF(K.LE.M) THEN
RMPK(K)=FK
RMPH(K)=FH
RMPL(K)=FL
ENDIF

*IF IN PHASE I DO THE SAME, APPLYING PHASE I FORMULAS

ELSE
ATU=((ATY(1)*BETA)/(ATY(2)*DELTA*(ALPHA+BETA)))**
&       ((ALPHA+BETA)/ALPHA)*
&       ATA**(1/ALPHA)*
&       ((1-ALPHA-BETA)/WBAR)**((1-ALPHA-BETA)/ALPHA)*
&       (ATY(3)/ATY(4))

       ATLSTAR=ATL
       F=ATA*ATY(3)**ALPHA*(ATU*ATY(4))**BETA*ATLSTAR**
&       (1-ALPHA-BETA)
```

```
U1=0.5*((ALPHA+BETA)/ALPHA)*(ATU/ATY(1))
U2=-0.5*((ALPHA+BETA)/ALPHA)*(ATU/ATY(2))
UK=0.5*ATU/ATY(3)
UH=-0.5*(ATU/ATY(4))

FK=(ALPHA/(ALPHA+BETA))*F/ATY(3)
FKK=(-BETA/(ALPHA+BETA))*FK/ATY(3)
FH=(BETA/(ALPHA+BETA))*F/ATY(4)
FKH=(BETA/(ALPHA+BETA))*FK/ATY(4)
FKU=(BETA/(ALPHA+BETA))*FK/ATU
FU=BETA/(ALPHA+BETA)*F/ATU
RLL=ATLSTAR/ATP

*RECORD PHASE SPECIFIC VALUES
IF(K.LE.M) THEN
RMPK(K)=FK
RMPH(K)=FH
RMPL(K)=WBAR
ENDIF

ENDIF

*RECORD NON PHASE SPECIFIC VALUES

IF (K.LE.M) THEN
  ATLABOUR(K)=ATLSTAR
  ATUUU(K)=ATU
  GDP1(K)=F
  GDP2(K)=F+(ATY(2)/ATY(1))*DELTA*(1-ATU)*ATY(4)
ENDIF
(1-ALPHA)*(DELTA-RHO)/(SIGMA*BETA+(1-ALPHA-BETA))
KAPPA=((BETA)*V/(1-ALPHA))

KONST=2*NE+1
```

```
      DO 11 I=1,NE
            DO 12 J=1,KONST
                  S(I,J)=0
            CONTINUE
      CONTINUE

*****************************************************************
*******1ST BOUNDARY CONDITIONS, PARTIALS AND EQUATIONS***********
*****************************************************************

      * AT INITIAL BOUNDARY
      IF (K.EQ.K1) THEN
        S(3,JSF)=Y(3,1)-RBAR(3)
        S(3,INDEXV(1))=0
        S(3,4+INDEXV(1))=0
        S(3,4+INDEXV(3))=1
        S(3,4+INDEXV(2))=0
        S(3,4+INDEXV(4))=0

        S(4,JSF)=Y(4,1)-RBAR(4)
        S(4,4+INDEXV(1))=0
        S(4,4+INDEXV(2))=0
        S(4,4+INDEXV(3))=0
        S(4,4+INDEXV(4))=1


      ***************************************************************
      **********END BOUNDARY CONDITIONS, PARTIALS AND EQUATIONS******
      ***************************************************************

      ELSE IF(K.GT.K2) THEN
            UM=1-((V+Z)/DELTA)
            RSTAR=RHO+SIGMA*KAPPA
            TEMPKK=ALPHA*F/RSTAR
```

```
      TEMPK=((ALPHA*A(M)*(UM*Y(4,M))**BETA*P(M)**(1-ALPHA-BETA))
     &      /RSTAR)**(1/(1-ALPHA))
      S(1,JSF)=Y(3,M)-TEMPK
      S(1,4+INDEXV(1))=0
      S(1,4+INDEXV(2))=0
      S(1,4+INDEXV(3))=1
      S(1,4+INDEXV(4))=-(BETA/(1-ALPHA))*TEMPK/Y(4,M)

      TTEMPH=Y(1,M)*BETA*A(M)*Y(3,M)**ALPHA*P(M)**(1-ALPHA-BETA)
      TS(2,JSF)=Y(2,M)*DELTA*(Y(4,M)*UM)**(1-BETA)-TEMPH
      S(2,4+INDEXV(1))=-TEMPH/Y(1,M)
      S(2,4+INDEXV(2))=DELTA*(Y(4,M)*UM)**(1-BETA)
      S(2,4+INDEXV(3))=-TEMPH/Y(3,M)
      S(2,4+INDEXV(4))=(1-BETA)*DELTA*Y(4,M)**(-BETA)
     &      *UM**(1-BETA)

*****************************************************************
**************INTERMEDIATE VALUES********************************
*****************************************************************

   ELSE

      S(1,JSF)=Y(1,K)-Y(1,K-1)-((RHO-FK)*ATY(1))*H

      S(1,INDEXV(1))=-1-((0.5)*(RHO-FK)-ATY(1)*FKU*U1)*H
      S(1,INDEXV(2))=FKU*U2*ATY(1)*H
      S(1,INDEXV(3))=(FKK*(0.5)+FKU*UK)*ATY(1)*H
      S(1,INDEXV(4))=(FKH*(0.5)+FKU*UH)*ATY(1)*H

      S(1,4+INDEXV(1))=1-((0.5)*(RHO-FK)-ATY(1)*FKU*U1)*H
      S(1,4+INDEXV(2))=FKU*U2*ATY(1)*H
      S(1,4+INDEXV(3))=(FKK*(0.5)+FKU*UK)*ATY(1)*H
      S(1,4+INDEXV(4))=(FKH*(0.5)+FKU*UH)*ATY(1)*H
```

```
      S(2,JSF)=Y(2,K)-Y(2,K-1)-((RHO-DELTA)*ATY(2)+PSI*
&     UBAR/ATY(4))*H

      S(2,INDEXV(1))=0
      S(2,INDEXV(2))=-1-(0.5)*(RHO-DELTA)*H
      S(2,INDEXV(3))=0
      S(2,INDEXV(4))=0

      S(2,4+INDEXV(1))=0
      S(2,4+INDEXV(2))=1-(0.5)*(RHO-DELTA)*H
      S(2,4+INDEXV(3))=0
      S(2,4+INDEXV(4))=0

      S(3,JSF)=Y(3,K)-Y(3,K-1)-(F-ATP*ATY(1)**(-1/SIGMA))*H

      S(3,INDEXV(1))=-(FU*U1+(1/SIGMA)*ATP*(0.5)*ATY(1)**
&     ((-1/SIGMA)-1))*H
      S(3,INDEXV(2))=-FU*U2*H
      S(3,INDEXV(3))=-1-((FK*0.5)+FU*UK)*H
      S(3,INDEXV(4))=-(FH*(0.5)+FU*UH)*H

      S(3,4+INDEXV(1))=-(FU*U1+(1/SIGMA)*ATP*(0.5)*ATY(1)**
&     ((-1/SIGMA)-1))*H
      S(3,4+INDEXV(2))=-FU*U2*H
      S(3,4+INDEXV(3))=1-((FK*0.5)+FU*UK)*H
      S(3,4+INDEXV(4))=-(FH*(0.5)+FU*UH)*H

      S(4,JSF)=Y(4,K)-Y(4,K-1)-DELTA*(1-ATU)*ATY(4)*H
      S(4,INDEXV(1))=DELTA*ATY(4)*H*U1
      S(4,INDEXV(2))=DELTA*ATY(4)*H*U2
      S(4,INDEXV(3))=DELTA*ATY(4)*H*UK
      S(4,INDEXV(4))=-1-DELTA*H*((1-ATU)*0.5-UH*ATY(4))

      S(4,4+INDEXV(1))=DELTA*ATY(4)*H*U1
```

```
          S(4,4+INDEXV(2))=DELTA*ATY(4)*H*U2
          S(4,4+INDEXV(3))=DELTA*ATY(5)*H*UK
          S(4,4+INDEXV(4))=1-DELTA*H*((1-ATU)*0.5-UH*ATY(4))
      ENDIF
      RETURN
      END
```

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