THREE EMPIRICAL ESSAYS IN INTERNATIONAL FINANCIAL ECONOMICS

by

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Three Empirical Essays in International Financial Economics

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Abstract

This dissertation is comprised of three empirical essays in international finance where each essay deals with a different foreign exchange related market.

The first essay, "implied volatilities and informational efficiency of the Tokyo currency option market," focuses on assessing the informational efficiency of the Tokyo currency option market. Restrictions on the term structure of implied volatilities is examined and we find evidence that the option prices may not be informationally efficient given that the option pricing model and the model of the term structure is correct. The ability of implied volatilities to forecast realized volatility is evaluated relative to volatility forecasts based on historical spot rate data using both in-sample and out-of-sample tests. Implied volatilities are obtained via a stochastic volatility and stochastic interest rate currency option pricing model. Historical data based forecasts are calculated by using a "naive" lagged volatility and a GARCH(1,1) conditional variance model. In-sample tests where the implied volatility is included in the conditional variance of a GARCH(1,1) model and out-of-sample criterion based on RMSE provide evidence that the Tokyo currency option market is informationally efficient. When regression based tests, where the realized volatility is regressed on the four forecasts is applied, we find that the implied volatilities have substantial predictive ability but that the forecasts based on historical spot rate data also provide useful information.

In the second essay, "an empirical study of swap covered interest parity," we examine the largest financial market that exists to date. The covered swap interest rate parity condition is empirically investigated for the British pound, German mark, and Japanese yen using daily data. We find that covered swap parity holds for the British pound on average based on the MAE criterion. The German mark and Japanese yen swaps exhibit a significant amount of deviation from parity. To further

investigate the deviations for the yen swap we take into account transaction costs. Overall, the swap covered interest parity deviation is shown to hold. Substantial deviations are not rare but the magnitude of the deviations has decreased in recent years.

Finally, the third essay, "cointegration and long dated forward rates," documents the relationship between long dated forward exchange rates and the underlying spot exchange rate for the US/Canadian dollar using cointegration. A simple model is developed which motivates cointegration of forward rates and realized spot rates. The shorter dated forward rates share a common trend with the spot rates whereas the longer dated three and five year forward rates do not share such a common stochastic trend with the spot rate. Given our model, this implies that the long dated US and Canadian interest rates do not share a common stochastic trend. Preliminary investigation of cointegration of the "term structure" of forward rates is also conducted using the Johansen trace test. We find evidence of two common stochastic trends in the "term structure" of forward rates.

Dedication

To my parents

Acknowledgments

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Essay One:
Implied Volatilities and Informational Efficiency of the Tokyo Currency Option Market

1. Introduction

The purpose of the present essay is to assess the predictive ability of the implied volatility for the yen/dollar exchange rate given currency option prices traded on the Tokyo market. This yen/dollar option is a European styled option traded on the inter-bank market in Tokyo. Given the recent volatility in the yen/dollar exchange rate, currency options have become an increasingly important instrument for Japanese firms looking to hedge against foreign exchange (transaction) exposure. Trading on currency options in the Tokyo market was in excess of 132 billion US dollars during the first three months of 1994. This is a 12.5 % increase over sales in the same period in 1993 (*Nihon Keizai Shimbun*, April 16, 1994). The focus of this essay could be regarded as a test of the informational efficiency of the option market as we look at the ability of implied volatility to predict realized volatility given that a particular option pricing model is correct.

We obtain implied volatilities (IV) from two option pricing models and compare the forecasts with those based on historical data. A GARCH(1,1) model and naive lagged historical model are used as benchmark forecasts using historical spot rate data. Implied volatilities are obtained from a stochastic volatility model and stochastic interest rate model. If markets are efficient in the sense that the option price contains all relevant information and the option pricing model is correct, then the forecasts based on historical data should not predict realized volatility any better than the implied volatilities.

Thus the present paper is in the spirit of recent studies investigating the predictive ability of implied volatilities against a class of generalized autoregressive conditional heteroskedastic models (GARCH). Much of the methodology employed in the paper parallels the work in Day and Lewis (1992), Lamoureux and Lastrepes (1993), and Xu and Taylor (1993). The information content of call option prices is inferred from the implied volatility's (IV) ability to predict the future volatility of the underlying asset. An in-sample based test is conducted where the IV is included as an exogenous variable in the conditional variance equation of a GARCH(1,1) model. Out-of-sample tests involve a battery of regressions comparing the marginal forecasting ability of IV and forecasts based on the underlying assets historical data series. Day and Lewis (1992) investigate the S&P100 index option at weekly intervals and find inconclusive evidence on the predictive ability of implied volatilities. Lamoureux and Lastrepes (1993) investigate implied volatilities for 10 stocks and compare the forecasts with a GARCH(1,1) model. Applying several out-of-sample tests they reject the orthogonality condition that forecasts from historical spot rates should not provide additional information in predicting realized volatility.

In the context of currency options, Xu and Taylor (1993) also compare GARCH forecasts with two different measures of implied volatility and find the PHLX currency option market to be informationally efficient in the sense that the implied volatility measure outperforms GARCH based forecasts of realized volatility. These results stand in contrast to those found for the US stock market. The focus of this paper, however, is in the Tokyo currency option market which has received very little attention in the academic literature to date. We find that implied volatilities have explanatory power in predicting realized volatility but that forecasts based on GARCH models provide additional information.

In addition, we present some preliminary empirical findings on the term structure of implied volatilities. The tests of the term structure restrictions could also be viewed as a test of informational efficiency provided the model is correct. Stein (1989) provides the basic theoretical framework and we apply an extension of that approach as outlined in Heynen, Kemna, and Vorst (1994).

Section 2 provides background on currency options including a review of some of the empirical literature on currency option pricing. This is followed by a review, in Section 3, of the option pricing models used to obtain implied volatilities in this essay. The data used in the present study is explained in Section 4. Empirical analysis begins with Section 5 where we investigate the bias from assuming the modified Black-Scholes model as an approximation to the Hull-White stochastic volatility model. The time series properties of the implied volatilities is examined in Section 6 in the context of term structure restrictions. Both the in-sample and out-of-sample results are presented in Section 7. A discussion of the findings in this study is found in Section 8 and the essay concludes with Section 9.

2. Background on Currency Options

A call (put) currency option gives the individual the right but not the obligation to purchase (sell) a pre-specified amount of a currency at a specified exchange rate (the strike or exercise price) on or before a specified date (maturity) in the future. If the option can be exercised before the maturity date, then the option is referred to as an American option. On the other hand, if the option can only be exercised on the maturity date then, this option is referred to as a European option. The currency options investigated in this paper are European options on the spot exchange rate.[1]

Much of the empirical research on currency options employs data from the Philadelphia Stock Exchange (PHLX).[2] Both American and European styled currency options are traded on the Philadelphia Stock Exchange beginning in 1982. Currencies traded include the Austrian dollar, British pound, Canadian dollar, European currency unit, German mark, French franc, Japanese yen, and Swiss franc for maturities ranging from one month to one year. Recently, PHLX trades cross-currency options of which the German mark-Japanese yen is most active.

This paper, however, uses data from the inter-bank market in Tokyo. The Tokyo over-the-counter currency options market began in 1984. The market, however, did not achieve any depth until after the Plaza agreement in 1985. And it was not until most recently that the Tokyo currency option market has been actively accessed by the major industrial, commercial, and investment banks [Harrington (1988)].

This Section reviews the empirical literature on currency options. The put-call parity condition is introduced in Section 2.1 in the context of currency options. We

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1 Other variations include options on foreign currency futures contracts and futures styled options on the underlying spot rate. A call on FX futures options would give the buyer the right to go long on a foreign currency futures contract at the specified strike price. Such options are currently traded on Chicago Mercantile Exchange.

In a futures styled option an exchange of cash flows is made daily between the buyer and writer of the option depending on whether the market value of the option increased or decreased. The buyer also has the right to purchase currency at the rate specified by the strike price. An American future-styled option on spot were traded on the London International Financial Futures Exchange in the past [Grabbe (1991)].

Exotic options traded over the counter include average rate or Asian options and barrier options [Levy (1992)].

2 Hilliard and Tucker (1992) document day of the week and intra-day patterns for PHLX traded yen, pound, and mark put options.

provide a brief review of several option pricing models and related pricing tests in Sections 2.2 and 2.3.

2.1 Put-Call Parity

The relationship between European calls and puts is given by the following put-call parity condition.

$$P = C - S\exp\{-r_f(T-t)\} + X\exp\{-r_d(T-t)\}$$

where P is the price of a European put, C the price for a call, S the underlying spot exchange rate, and X denotes the strike price for both the call and put. r_d and r_f are the domestic and foreign interest rates respectively. Thus, the price of the put can be determined given a corresponding call price, spot exchange rate, and the discount bond prices for the respective currencies.

This relationship is tested in several papers. Shastri and Tandon (1985) find evidence of significant violations of put-call parity and thus the potential of profit opportunities. Violations of the parity condition ranged from 28.25% to 39.52% of the sample observations. When transaction costs and simultaneity of the price data are carefully incorporated into the study both Tucker (1985) and Bodurtha and Courtadon (1986) find the currency option market to be "efficient." More recently, Knoch (1992) provides corroborating evidence using a weekly data series over a longer time span 1987-

1990. Tucker (1985) also examines a trading strategy by calculating "hedged" returns and does not find evidence of "abnormal" returns.

2.2 The Modified Black-Scholes Model

Feiger and Jacquillant (1979) develop a solution for pricing European options for foreign exchange implicitly before such instruments were traded on the PHLX. In essence, this is accomplished by pricing a two-currency, currency option bond with a replicating portfolio of a single currency option bond and a currency option. The closed form version of the pricing formula for European currency options based on a "modified" version of the Black and Scholes (1973) model, however, is often attributed to Biger and Hull (1983), Grabbe (1983), and Garman and Kohlhagen (1983). Biger and Hull (1983) treat the currency option pricing problem as an option on a stock which pays a continuous constant dividend yield. Under the assumption that the underlying spot exchange rate, S, follows a log normal diffusion process of the form

$$dS = \mu S dt + \sigma S dz \tag{2.2.1}$$

where μ and σ are the mean and volatility parameters (constant) and z follows a standard Wiener process. The closed form solution for the call option is given as

$$Call(F) = FN(d1)e^{-r_f(T-t)} - XN(d2)e^{-r_d(T-t)} \tag{2.2.2}$$

where $d1 = \dfrac{\ln(\frac{S}{X}) + (r_d - r_f + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$ and $d2 = d1 - \sigma\sqrt{T-t}$.

Garman and Kohlhagen (1983) derive identical closed form solutions by deriving a partial differential equation of the form

$$\frac{1}{2}\sigma^2 S^2 C_{SS} - r_d C + (r_d - r_f)C_S = C_\tau \tag{2.2.3}$$

where the subscript S and τ on C denotes the partial derivative of the call function with respect to the spot exchange rate and time to maturity respectively. This equation is derived by equating risk adjusted excess returns of portfolios as these returns must be equal for no-arbitrage to hold.

Another related and important result is that the valuation equation of the call can be rearranged such that it is a function of the forward rate, F. If we assume covered interest parity to hold, $F = S\exp\{(r_d - r_f)(T-t)\}$, and substitute this parity relationship into equation (2.2.2), then we have

$$call(F) = FN(d1)e^{-r_d(T-t)} - XN(d2)e^{-r_d(T-t)} \tag{2.2.4}$$

where $d1 = \frac{\ln(\frac{F}{X}) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$ and $d2 = d1 - \sigma\sqrt{T-t}$

A call option pricing formula identical to equation (2.2.4) will be derived when we examine the stochastic interest rate model formulated by Hilliard, Madura, and Tucker (1991).

The early tests of pricing bias tests for the modified Black-Scholes model described above include Goodman, Ross, and Schmidt (1985), Shastri and Tandon (1986a,b, 1987), Bodurtha and Courtadon (1987). Goodman et. al. (1985) simply compared the performance of the modified Black-Scholes model (2) with the "original" version of the Black-Scholes model which only incorporates the domestic interest rate. Shastri and Tandon (1986, 1987), and Bodurtha and Courtadon (1987) attempt to take into account the fact that PHLX options must be tested using an "American" option pricing formula.

2.3 Other Pricing Models

As the modified Black-Scholes model did not perform very well empirically, several other option pricing models were tested. As a consequence, empirical research turned to alternate forms of diffusion processes to capture the movement of foreign exchange rates. The constant elasticity variance (CEV) option pricing model is examined in Tucker, Peterson, and Scott (1988), and Melino and Turnbull (1990, 1991). Tucker et. al. (1988) examine the diffusion process of the form

$$\frac{dS}{S} = \mu dt + \delta S^{[\theta-2]/2} dZ \tag{2.3.1}$$

proposed originally by Cox and Ross (1976). If we assume, $\theta = 2$ and $\delta = \sigma$ then equation (2.3.1) reduces to a log normal diffusion process as in equation (2.2.1). Model parameters are estimated given data up to time t and then used to obtain theoretical prices for options t+k days later. The CEV outperforms the Black-Scholes for k<5 days.

Melino and Turnbull (1990), on the other hand, look at the CEV diffusion process of the form used in Marsh and Rosenfield (1983)

$$dS = (\alpha_1 S^{-(1-\beta)} + \alpha_2 S)dt + \sigma S^{\beta/2} dZ \tag{2.3.2}$$

where α_1, α_2, and β are constants such that $0 < \beta < 2$. When $\beta = 2$ the CEV diffusion process reduces to the log normal diffusion process in equation (2.2.1). When the pricing bias was regressed on "moneyness", time to maturity, and interest rates, the coefficients were all significantly different from zero.

Another process of interest is the jump diffusion process

$$\frac{dS}{S} = \alpha dt + \sigma dZ + dJ \tag{2.3.3}$$

where J follows a Poisson process and dJ is independent of dZ. "Abnormal" information is assumed to arrive a mean number of times, λ, per unit time and with a jump "size" of Y. Y is also independent of dZ, and lnY is normally distributed with a mean of μ_Y and variance V_Y. The jump diffusion model has been relatively successful when compared to the constant volatility Black-Scholes model [Borensztein and Dooley (1987), Shastri and Wethyavivorn (1987), Yagi (1988), Tucker (1991)]. Tucker (1991), for example, finds that the average absolute percentage pricing error for the jump model is about half of that for the modified Black-Scholes model; these results were robust across currency, maturity, and boundary status. Shastri and Wethyavivorn (1987) obtained simulated option prices for four different diffusion processes including the jump diffusion process. These simulated prices were then used to obtain a set of implied volatilities (IV) given the modified Black-Scholes model is correct. The IV from the simulated prices are then compared to the IV obtained from observed market currency option prices. They find that the IV from the jump diffusion process is highly consistent with the pattern exhibited by the IV from observed market prices across maturity and "moneyness." The present paper obtains implied volatilities from two models referred to as the stochastic volatility and stochastic interest rate models.

3. Option Pricing Models Used in Obtaining Implied Volatility

The Black-Scholes (1973) model often serves as the pricing model used to obtain implied volatilities (IV). Biger and Hull (1983), Grabbe (1983), and Garman and Kohlhagen (1983) provide modifications of the basic Black-Scholes (BS) model to accommodate pricing of currency options. An underlying assumption of the model is that the variance of the underlying spot rates are constant and that interest rates are constant as well. By deriving IV from the class of option pricing models represented by Hull and White (1987) we are able to take into account the effects of stochastic volatility. Also, we obtain another set of IV from a model developed in Grabbe (1983) and Hilliard, Madura, and Tucker (1991) which incorporates stochastic interest rates.

3.1 The Hull - White Stochastic Volatility Model

Hull and White (1987) obtained a closed form solution for a stochastic volatility European option under the assumption that volatility risk is not priced. Johnson and Shannon (1987), Scott(1987), and Wiggins (1987) develop similar stochastic volatility models also under the same assumption.

The model formulated in Hull and White (1987) assumes a pair of diffusion processes (risk neutral world) of the form

$$\begin{aligned} dS &= (r_d - r_f)Sdt + S\sqrt{V}dw \\ dV &= \mu V dt + V\theta dz \end{aligned} \qquad (3.1.1)$$

where S is the spot exchange rate and V is the volatility (variance) term. *dw* and *dz* are the (change) Wiener processes driving the exchange rate process and volatility process respectively. θ denotes a parameter reflecting variance of the volatility of the underlying exchange rate. When the exchange rate and the volatility are not correlated, then Hull and White obtain a closed form solution where the option price depends on the average volatility (over its path).

$$\bar{V} = \frac{1}{T-t}\int_t^T V_\tau d\tau \qquad (3.1.2)$$

The distribution of $\log\{S_T/S_t\}$ conditional on $\bar{V}$ is normal with a mean of $(r_d - r_f)T - \frac{\bar{V}T}{2}$ and variance of $\bar{V}T$. There are an infinite number of paths that a stochastic V could follow which produce the same mean $\bar{V}$. Thus, it follows that all of the these paths will give rise to the terminal log normal distribution for the underlying exchange rate. In other words, a call option is priced using the following set of equations,

$$C(\bar{V}) = Se^{-r_f} N(d_1) - Xe^{-r_d} N(d_2)$$

where,

$$d_1 = \frac{\log(S/X) + (r_d - r_f + \bar{V}/2)}{\sqrt{\bar{V}}} \qquad (3.1.3)$$

$$d_2 = d_1 - \sqrt{\bar{V}}$$

A correlation of ρdt, between dw and dz, however, requires a monte carlo simulation to obtain the option price since the mean and variance of the distribution of the exchange rate conditional on the volatility, V, is a function of ρ. Specifically, the distribution of log (S_t/S_{t-1}) conditional on V_t is normal with a mean of

$$(\mathrm{r_d}-\mathrm{r_f})\frac{\mathrm{T}}{\mathrm{n}}-\mathrm{V_{t-1}}\frac{\mathrm{T}}{2\mathrm{n}}+(\rho\frac{\sqrt{\mathrm{V_{t-1}}}}{\theta})\log(\frac{\mathrm{V_t}}{\mathrm{V_{t-1}}})-\frac{\mu\mathrm{T}}{\mathrm{n}}+\frac{\theta^2\mathrm{T}}{2\mathrm{n}}) \tag{3.1.4}$$

and variance

$$\frac{\mathrm{V_{t-1}T}}{\mathrm{n}}(1-\rho^2) \tag{3.1.5}$$

for maturity time T, and, n, the interval over which the volatility (variance) term changes. Note that under the assumption that the correlation between *dw* and *dz* is zero, then the $\log(S_t/S_{t-1})$ is normally distributed with mean $\frac{(r_d-r_f)T}{n}-\frac{V_{t-1}T}{2n}$ and variance of $\frac{V_{t-1}T}{n}$. The $\log(V_t/V_{t-1})$ is normally distributed with a mean of $\frac{\mu T}{n}-\frac{\theta^2 T}{2n}$ and variance of $\frac{\theta^2 T}{n}$.

In obtaining the implied volatility from this model we make the assumption of $\rho=0$. To check the possible effects (bias) of making such an assumption we conduct a simple simulation described in Section 5 which is grounded in the data. We find the assumption of $\rho=0$ is warranted given the sample data used in this study.

Under the assumption that $\rho=0$ and that volatility risk is not priced, the call price can also be written as

$$C_t=\int BS(\overline{V}_t)h(\overline{V}_t|F_t)d\overline{V}_t=E[BS(\overline{V}_t|F_t)] \tag{3.3.6}$$

where $h(.|.)$ is the density function of the average volatility conditional on information F at time t (in other words conditional on V_t), BS(.) is the Black-Scholes pricing formula described above in equation (3.1.3). The "subjective" variance formulated by market participants will equal the actual variance if participants form their expectations rationally. More specifically, the subjective expectation of the variance of the market participants is simply $E[\overline{V}_t|F_t]$ and will the equal variance given the actual density $h(.|.)$ or $\int \overline{V}_t h(\overline{V}_t|F_t)d\overline{V}_t$ under rational expectations.

For at the money options, we know from Cox and Rubinstein (1985) that the relationship between the call value and volatility is approximately linear. Thus, the implied volatility from the call price $E[BS(\overline{V}_t|F_t)]$ will be approximately equal to $E[\overline{V}_t|F_t]$. In other words,

$$E[BS(\overline{V}_t|F_t)] \cong BS[E(\overline{V})_t|F_t] \qquad (3.1.7)$$

It follows then that the implied volatility will yield the subjective expected volatility of market participants. Empirically, this is carried out by employing at the money call option market prices under the assumption that the Black-Scholes model (stochastic volatility) is the correct pricing model. The implied volatility which is the subjective volatility is compared to the realized volatility both in-sample and out-of-sample.

3.2 A Stochastic Interest Rate Model

In addition to the Hull-White model, we consider the implied volatility from a model developed in Grabbe (1983) and Hilliard, Madura and Tucker (1991) which represents a class of option pricing models employing stochastic domestic and foreign interest rates. As the underlying spot rate as well as the forward rate are related to the interest rate agio via covered interest parity we would expect movements in interest rates to affect the option price Hence, unlike the modified BS models with a single volatility term, the stochastic interest rate model has three volatility terms. The basic model is restated below (in similar notation to avoid confusion). The derivation of the model closely follows the original work in Hilliard et. al. (1991).

We assume the forward rate at time t with maturity T, $F(t,T)$, is in equilibrium according to the interest parity condition $F(t,T) = S(t)\dfrac{B_f(t,T)}{B_d(t,T)}$. $S(t)$ is the underlying spot exchange rate at time t, $B_f(t,T)$ is the price of the foreign bond with maturity T at time t, and $B_d(t,T)$ is the price of the domestic bond with maturity T at time t. Next let us assume that the underlying exchange rate, domestic bond price and foreign bond price each follow log normal diffusion processes of the form

$$\begin{aligned} dS &= \mu_S S dt + \sigma_s S dZ_s \\ dB_d &= \mu_d B_d dt + \sigma_d B_d dZ_d \\ dB_f &= \mu_f B_f dt + \sigma_f B_f dZ_f \end{aligned} \tag{3.2.1}$$

where μ is the drift term, σ the volatility term, and Z follows the standard Wiener process. Also note that there are two short term interest rate processes for the domestic, r, and foreign, f, interest rates.

$$dr = \alpha_r dt + \delta_r dw_r$$
$$df = \alpha_f dt + \delta_f dw_f \qquad (3.2.2)$$

where w is the Wiener process, δ the volatility term, and α the mean (drift) parameter.

In deriving a solution, firstly form a portfolio V composed of the currency call option, C, the domestic bond price $B_d(t,T)$, and $SB_f(t,T)$ the price of the foreign bond price in terms of domestic units. The value of the portfolio, M, is such that $M = C + h_d B_d + h_f SB_f$ where the weights h_d and h_f in the portfolio are such that

$$M = C - \frac{\partial C}{\partial B_d} B_d - \frac{\partial C}{\partial (SB_f)} SB_f \ . \qquad (3.2.3)$$

Applying Ito's lemma to the above equation (3.2.3) gives

$$\frac{1}{2}\{\frac{\partial^2 C}{\partial B_d^2} B_d^2 \sigma_{B_d}^2 + \frac{\partial^2 C}{\partial G^2} G^2 \sigma_G^2 + 2\frac{\partial^2 C}{\partial B_d \partial G} \rho_{dG,dB_d}\} - \frac{\partial C}{\partial \tau} = 0 \qquad (3.2.4)$$

where $G = SB_f$ and $dG = (\mu_S + \mu_d)Gdt + G(\sigma_S dZ_S + \sigma_d dZ_d)$. The time subscript τ (= T-t) is the time left to maturity. Given the parity condition, it follows that the terminal boundary condition (at T) is $\max[S(T) - X, 0] = \max[F(T,T) - X, 0]$. The call price is then given as $C = B_d \hat{E}[\max[F(T) - X, 0]$ where $\hat{E}$ denotes that we assume risk neutrality in taking the expectations.

Now, given the interest parity equilibrium and Ito's lemma the diffusion process for the forward rate process is given as

$$\frac{dF}{F} = \sigma_S dZ_S + \sigma_d dZ_d + \sigma_f dZ_f \qquad (3.2.5)$$

where F is now risk adjusted. Forward rates are assumed to have a zero drift term as there is zero initial cost at least theoretically. Since, we assume a log normal diffusion processes, the variance for this process $\frac{dF}{F}$ is given by

$$v2 = Var[\log(\frac{F_T}{F_t})| F_t] = \int_t^T Var[\frac{dF}{F}] \qquad (3.2.6)$$

or

$$Var[\log(\frac{F_T}{F_t})| F_t = \int_t^T [\sigma_S \ \sigma_d \ -\sigma_f]^T Cov(dZ, dZ^T)[\sigma_S \ \sigma_d \ -\sigma_f]$$

where $dZ = [dZ_S \ dZ_d \ dZ_f]$ and $Cov(dZ, dZ^T) = \begin{bmatrix} 1 & \rho_{Sd} & \rho_{Sf} \\ & 1 & \rho_{df} \\ & & 1 \end{bmatrix} dt \qquad (3.2.7)$

ρ_{ij} denotes the correlation coefficient for pairs diffusion processes i and j (Wiener process) where $i,j = S,d,f$.

The call price is given by

$$C(t,T,F) = B_d(t,T)[F(t,T)N(d1) - XN(d2)] \tag{3.2.8}$$

where $d1 = [\log(\frac{F}{X}) + 0.5v^2]/v$ and $d2 = d1 - v$. $N(.)$ denotes the cumulative $N(0,1)$, X the exercise price, and T maturity date. Assuming a Vasicek (1977) bond pricing model for both the domestic and foreign bonds, Hilliard, Madura, and Tucker (1991) derive an approximation for the variance of the forward rate $F(.)$ as

$$v^2 = \sigma_s^2\tau + \tau^3/3\,(\sigma_r^2 + \sigma_f^2 - 2\sigma_{rf}) + \tau^2(\sigma_{sr} - \sigma_{sf}) \tag{3.2.9}$$

where τ is the time to maturity. σ_s^2 is the variance of the exchange rate, σ_r^2 is the variance of the domestic interest rate, σ_f^2 is the variance of the foreign interest rate, σ_{rf} is the covariance of the domestic and foreign interest rates, σ_{sr} is the covariance of exchange rate and domestic interest rate, σ_{sf} is the covariance of the exchange rate and foreign interest rate. Although this is the variance for the forward rate, we could think of this as the variance of the future spot rate when movement in interest rates is taken into consideration.

4. The Data

The data set used in the present study was collected from various issues of the *Nihon Keizai Shimbun*. Call option prices with one month and three month maturities for the Yen/Dollar option traded in the Tokyo market are collected on a daily basis. Near-the-money call options were collected from January 7, 1992 to November 29, 1993. This gives us 466 observations in the sample. Corresponding one and three month Euro rates for the yen and US dollar from the Tokyo off-shore market were used as risk free rates. Closing forward and spot rates of the underlying exchange rate were collected on a daily basis as well.

As in Day and Lewis (1992) and Xu and Taylor (1993) among others, we use options which are nearest the money. This is to avoid potential strike bias effects as documented by recent studies on currency options [Knoch (1992), Heynen (1993) among others]. Also, the implied volatility should closely approximate mean expected volatility (over the remaining life of the option) for at-the-money options.

Table 1 provides a summary of the descriptive statistics of the data set used in the present study. As documented in the literature, the foreign exchange rate return series exhibits a relatively large excess kurtosis statistic and thus motivates the use of GARCH models in this paper. The excess kurtosis statistic at 4.1664 is substantially larger than zero and is statistically significant at the one percent level (against the null of the normal distribution which has a kurtosis of three). Boothe and Glassman (1987) also find evidence of a "fat tailed" distribution and account for this by describing the underlying distribution for the yen as a mixture of two normal distributions. The two normal distributions have equal mean but differ in their variance. Their findings are robust across the subsamples they examined. Hsieh (1988) examines whether exchange rate "fat tails"

Table 1: Descriptive Statistics of Data Set: January 7, 1992 - November 29, 1993

Variable	Mean	Standard Deviation	Skewness	Kurtosis	Number Observ.
Call Price (one month)	1.3480	0.1945	0.5359	-0.1095	466
Call Price (three month)	2.3106	0.1965	0.3273	-0.2464	466
Euro-yen (one month)	3.9122	0.8755	0.2031	-0.8626	466
Euro-yen (three month)	3.7853	0.8202	0.2692	-0.8220	466
Euro-dollar (one month)	3.5625	0.4139	0.8402	-0.8021	466
Euro-dollar (three month)	3.6109	0.3844	0.9217	-0.3921	466
FX Return	-0.000263	0.0065	-0.3466	4.1664	466
FX Return Squared	0.000042	0.0001	7.2408	66.4737	466

	LB(6)	LB(12)	ρ_1	ρ_2	ρ_3	ρ_6	ρ_{12}	Standard Error
FX Return	8.5	13.7	0.012	0.053	0.051	0.057	0.027	0.05
FX Return Squared	15.7	18.2	-0.017	0.061	0.014	-0.001	0.003	0.05

FX return is defined as $\ln(S_t / S_{t-1})$ where S_t is the closing yen/US dollar exchange rate at t. FX return squared is defined as $[\ln(S_t / S_{t-1})]^2$. ρ_i is the autocorrelation coefficient for the i th lag, and the standard error is in the last column. LB denotes the Ljung-Box-portmanteau test statistic with the number of lags in the parenthesis.

distributions result from a "fat tailed" distribution which is fixed of over time or from a distribution which is time varying. Hsieh finds that the exchange rate changes are not independent and identically distributed for five currencies. Furthermore, an ARCH type model was estimated to examine the time varying properties of the exchange rate but it did not account for the rejection of iid for the yen.

The high degree of autocorrelation in the squared returns also provides some evidence of the autoregressive nature of volatility which the GARCH model is designed to capture. The Ljung-Box-portmanteau statistics for six and twelve lags are 15.7 and 18.2 respectively and indicate autocorrelation in the squared returns.

In sum, the descriptive statistics for the foreign exchange series motivates our application of the GARCH model as we find evidence of kurtosis and autocorrelation in the squared returns (clustering).

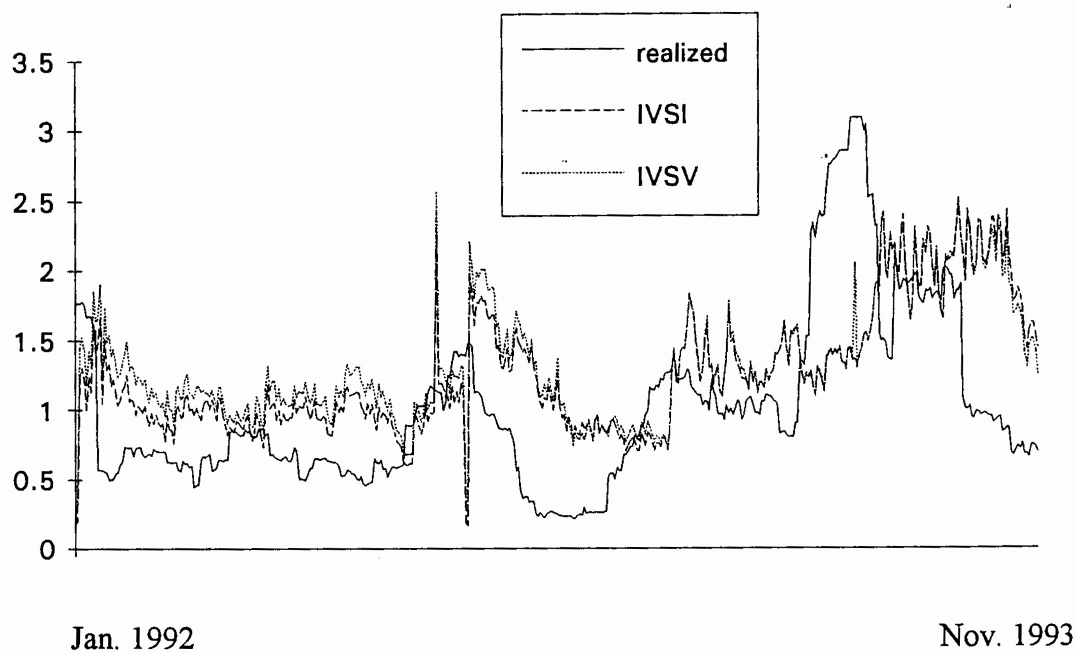
The implied volatilities (IV) are calculated by using an iterative procedure based on a Newton type routine [Benniga (1992)].

$$\sqrt{IV(k+1)} = \sqrt{IV(k)} - \frac{C[IV(k)] - C_{actual}}{C'[IV(k)]} \qquad (4.1)$$

where IV(k) is the implied volatility on the kth iteration and C[IV(k)] is the call price evaluated with the implied volatility from the kth iteration. The implied volatilities are defined as variances hence their square root would give us the standard deviation. C'[IV] is the first derivative of the call valuation function with respect to the volatility parameter and is equal to $S\sqrt{T-t}F'(d1)$ with T-t left to maturity. $F'(d1) = \frac{\exp(-d1^2/2)}{\sqrt{2\pi}}$ is the density of the normal distribution evaluated at d1. The iterations are repeated until the absolute error between C[IV(k)] and the observed call price is less than 0.00001.

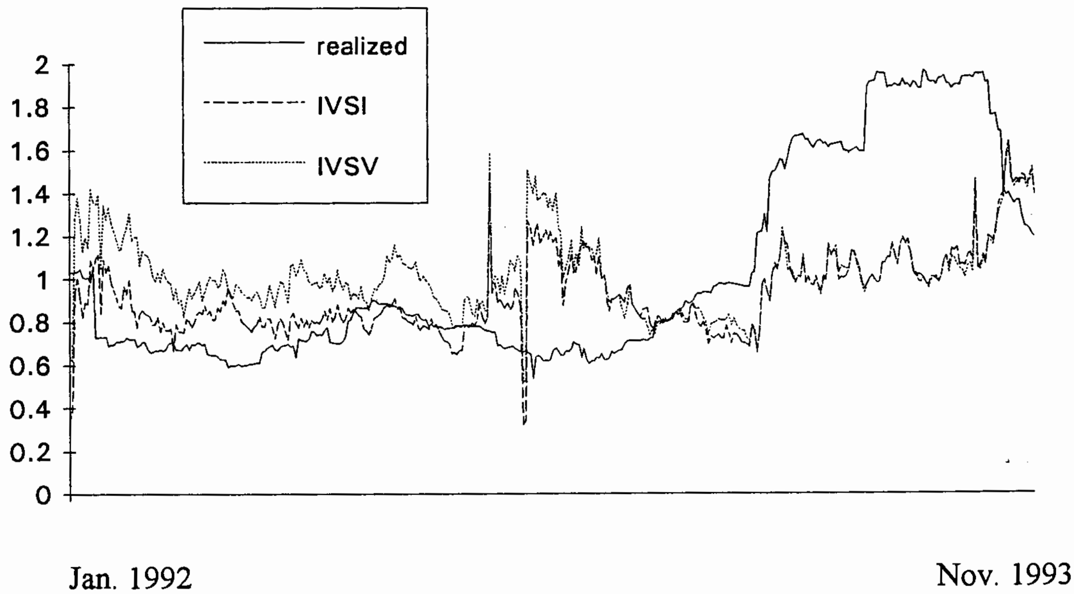
Figures 1 and 2 graph the implied volatilites for the one month and three month horizon respectively. The realized volatility over the sample period is plotted as a benchmark. It is clear that the movements of the implied volatility tend to be "smoother" relative to the realized volatility with the exception of a couple of "spikes."

Figure 1: Implied and Realized Volatility for One Month Horizon



IVSI is the implied volatility from the stochastic interest rate model, IVSV is the implied volatility from the stochastic volatility model, realized is the realized volatility obtained from the historical spot exchange rate series. All volatility measures are annualized standard deviations.

Figure2: Implied and Realized Volatility at Three Month Horizon



IVSI is the implied volatility from the stochastic interest rate model, IVSV is the implied volatility from the stochastic volatility model, realized is the realized volatility obtained from the historical spot exchange rate series. All volatility measures are annualized standard deviations.

5. Measuring Biases

We apply the model outlined in equation (3.1.3) to obtain the implied volatility for the stochastic volatility model. This is warranted, provided the correlation between the spot rate and volatility, ρ, is zero or has an insignificant effect in deriving the implied volatility. Firstly, we examine the bias directly by conducting a monte carlo simulation with GARCH model parameter estimates based on data from the underlying spot rate. We also calculate the effect on pricing as an indirect test.

5.1 Measuring Bias From Assuming ρ=0 in the Hull and White Model

The exchange rate and volatility path is simulated by the following pair of equations,

$$S_t = S_{t-1}\exp[(r_d - r_f - V_{t-1}/2)\Delta t + u_t\sqrt{V_{t-1}\Delta t}] \qquad (5.1.1)$$

$$V_t = V_{t-1}\exp[(\mu - \theta^2/2)\Delta t + \rho u_t \theta \Delta t + \psi_t \theta\sqrt{1-\rho^2}\sqrt{\Delta t} \qquad (5.1.2)$$

where u and ψ are generated from N(0,1). At each time interval, a new set of random numbers are generated and the lagged exchange rate (volatility) is replaced by the current exchange rate (volatility). An interval of one day is selected for simulation purposes. For each simulated path, the terminal exchange rate is obtained and the price of the option is calculated as $e^{-rd(T-t)}ax(S_T\text{-}X,0)$. This experiment is referred to as one simulation run, and is repeated N times yielding N currency option prices. The average of these currency prices is taken to be the models' option price.

In order to obtain initial parameter values, the GARCH(1,1) model of the form

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = C + \varepsilon_t$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} \qquad (5.1.3)$$

where, S is the spot rate and C a constant in the mean equation. The conditional variance is denoted by h_t [Bollerslev (1986)].[3]

More generally,

$$\varepsilon_t \mid I_{t-1} \sim N(0, h_t)$$

where,

$$\varepsilon_t = y_t - \underline{x}_t{}'\underline{b} \qquad \text{for } t = 1,\ldots,T$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}{}^2 + \beta h_{t-1} \qquad (5.1.4)$$

The error term, ε_t, from the time series regression of y_t on a vector of n explanatory variables, $\underline{x}_t$, follows a stochastic process (normally distributed) conditioned on the information set I_{t-1} [Bollerslev (1986)]. $\underline{b}$ [=(b_1, ..., b_n)] is a vector of n unknown coefficients to be estimated. The conditional variance of ε_t, h_t, follows a GARCH(1,1) process. The GARCH(1,1) process is a parsimonious representation of the variance since

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3 A literature review of ARCH (Engle, 1982) type models is found in Bollerslev, Chou, and Kroner (1992), and Bera and Higgins (1993).

it is equivalent in nature to an infinitely lagged ARCH process: To illustrate, in the GARCH(1,1) case, $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$ which can be rearranged to $h_t = \alpha_0(1-\beta L)^{-1} + \alpha_1 \varepsilon_{t-1}^2(1-\beta L)^{-1}$ where L is the lag operator. $(1-\beta L)^{-1}$ expands to an infinite series of lags and hence h_t in effect follows an ARCH(∞).

Also, note $\alpha_1+\beta< 1$ is required for stationarity. The closer this sum of the coefficients is to one then the greater the persistence of the shocks to the variance. Add and subtract $\alpha_1 h_{t-1}$ on the right-hand side of the GARCH(1,1) conditional variance equation to get,

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 - \alpha_1 h_{t-1} + \beta h_{t-1} + \alpha_1 h_{t-1}$$

$$= \alpha_0(1- \alpha_1 L- \beta L)^{-1} + \alpha_1 Z_{t-1}(1 -\alpha_1 L - \beta L)^{-1}$$

$$= \alpha_0(1 -(\alpha_1 + \beta)L)^{-1} + \alpha_1 Z_{t-1}(1 -(\alpha_1 + \beta)L)^{-1} \quad (5.1.5)$$

where L is the lag operator and $Z_{t-1} = \varepsilon_{t-1}^2 - h_{t-1}$. $(1- (\alpha_1 + \beta)L)^{-1}$ expands to an infinite series. Thus, $\alpha_1 + \beta< 1$ is required for stationarity.

The log likelihood function, $L_T(.)$, for the GARCH regression takes the following form (abstracting from constant)

$$L_T(\theta) = \Sigma_t l_t(\theta) \qquad \text{for } t= 1,...,T \quad (5.1.6)$$

where, $l_t(\theta) = -(1/2)\log(h_t) - (1/2)\,\varepsilon_t^2/h_t$, θ [= ($\underline{b}$, α_0, α_1, β)] is the vector of parameters to be estimated, and T represents the sample size.

The univariate GARCH(1,1) lends itself conveniently to this problem since it is consistent with the volatility diffusion processes (discrete to continuous)

$$d\sigma^2 = (\omega^* - \gamma^* \sigma^2)dt + \alpha^* \sigma^2 dZ \tag{5.1.7}$$

To illustrate, assume a univariate GARCH (1,1) conditional variance, h_t, model of the form

$$h_t = \omega + \alpha\, \varepsilon_{t-1}^2 + \beta\, h_{t-1} \tag{5.1.8}$$

Subtract h_{t-1} from both sides, and add $\alpha h_{t-1} - \alpha h_{t-1}$ (=0). Assume $h_t \zeta^2_t = \varepsilon_t^2$ where $\zeta \sim N(0,1)$. Rearranging yields,

$$\Delta h_t = \omega - (1-\alpha-\beta)\, h_{t-1} + \alpha\, h_{t-1}\, (\zeta^2_{t-1} - 1) \tag{5.1.9}$$

In the limit this approaches $d\sigma^2$ [Nelson (1990), Engle and Mustafa (1992), Kuwahara and Marsh (1992), Lamoureux and Lastrepes (1993)]. Specifically, $E\{dZ\}^2 = dt$, ω

approaches ω^*dt, $(1-\alpha-\beta)$ approaches γ^*dt, and α approaches $\alpha^*\sqrt{dt/2}$ in the limit. Note that when the stationarity condition, $\alpha+\beta < 1$, is not violated then $\gamma > 0$ which implies mean reversion.[4] The GARCH model described above is consistent with the diffusion process but not necessarily a unique approximation. In order to calculate the correlation coefficient between the exchange rate and volatility, ρ, we simply find the sample ρ using S_t and h_t.

Volatility forecasts based on historical data should not provide more information in predicting volatility than the implied volatility forecasts if markets are efficient (price reflects all available information) assuming the model used to obtain the implied volatility is the correct model. As shown in equation (5.1.9), the conditional variance equation for the GARCH(1,1) model is consistent with the stochastic volatility process. Thus the GARCH model is cast within the context of a stochastic volatility model developed in Hull and White (1987). Any information contained in the GARCH conditional variance equation should be reflected in the option price via the stochastic volatility model (which is assumed to be the correct model). If the option price contains all relevant information and the Hull-White model is correct, then we do not expect the GARCH model (or any other historical data series forecasts) to provide additional information in forecasting volatility. Thus, the GARCH model should not provide forecasts of realized volatility any better than those provided by the implied volatility from the stochastic volatility model. This is the empirical question addressed in Sections 6 and 7.

The GARCH parameter estimates used were $\alpha_1+\beta = 0.304$ and $\rho = -0.0231$. These parameter estimates were used as to obtain empirical approximations to the

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4 Nelson (1990) shows that a GARCH in mean model has a two state variable diffusion model as a limiting case (the two driving Wiener processes are independent). The fact that the "discrete" GARCH-M model has only a single source of randomness is thus not problematic in the limit.

parameters from the diffusion processes in equation (3.1.1). These diffusion process parameter estimates are then employed in the simulation process described in equations (5.1.1) and (5.1.2). Each simulated price is an average over 500 runs.[5] This was repeated 150 times so that the mean simulated variance and mean implied variance in Table 2 is averaged over a sample of 150. We find the bias which measures the difference between the simulated variance and the BS implied variance is relatively small. As expected, the percentage bias is larger for the longer maturity 90 day option but it is only 1.7%. Whereas for the 30 day option the bias is a mere -0.21%. These results are consistent with those found in Lamoureux and Lastrepes (1993) for options on stocks.

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5 Boyle (1977) provides an introductory discussion of applying monte carlo methods to the pricing of options. Antithetic values were used in the simulation as described in Hull and White (1987). u and ψ drawn from a standard normal distribution and a corresponding simulated price is obtained, C_1 . The negative of u and ψ (antithetic standard normal variables) is then taken a second simulated price is calculated as C_2 . The average of the two simulated prices is taken to be the simulated price for the Nth run used in the analysis or $C_N = (C_{1,N} + C_{2,N})/2$. Thus in actuality, each simulated price is an average over 1000 runs.

Table 2: Bias from Approximation in Obtaining Implied Volatility

The simulated variance is obtained via a monte carlo simulation using GARCH parameters and the Hull White model. A single model call price is obtained over 500 runs (1000 with anithetic variates) and a corresponding implied variance from the Black-Scholes formula is derived. This process is repeated 150 times. The mean and bias reported below is for the 150 variances. The bias is the percentage difference between the simulated and implied variance.

	Mean Simulated Variance	Mean Implied Variance	Bias
one month	0.01974	0.01969	-0.2110
option	0.000007	0.00214	0.1085
three month	0.01986	0.02007	1.7160
option	0.000007	0.00206	0.1046

5.2 Pricing Bias of At-the-Money Options

We investigate mispricing of at-the-money options using the Hull and White stochastic volatility model and the Hilliard, Madura, and Tucker (1991) stochastic interest rate model. The Hull White model prices are obtained in two different ways. Firstly, the Black-Scholes formula was employed in obtaining the implied variance from the call price and then used in calculating a price for the next period. Secondly, the GARCH model was estimated with spot data 466 times with a sample window of 200 days. The parameters were then used in a simulation as described in the previous section to derive the simulated option price. If the model is correct and market efficient, then both approaches should exhibit similar pricing bias, if effect of the correlation, ρ, is trivial. Finally, as a benchmark case, the Hilliard, Madura, Tucker(1991) model is used in obtaining a simulated price where the parameters are calculated simply from historical data.

This is not a test of pricing bias in general where we compare the models ability to predict market prices with other models across various strike prices and terms to maturity. This would require a more comprehensive option price data set not available to the author at present. These pricing bias tests are used as an indirect test of measuring the effect of assuming $\rho=0$ in the Hull-White model within a given sample.

Although the focus of the present paper is not to investigate the pricing bias of the stochastic volatility and interest rate models a brief review of related empirical findings using these models is useful. Chesney and Scott (1989) look at the stochastic volatility model for the Swiss franc currency option traded in Geneva, Switzerland. They find the modified Black-Scholes model with daily updated implied volatilities performs substantially better than the stochastic volatility option pricing model based on the mean absolute value criterion. Two forms of stochastic volatility models were employed in the

Chesney and Scott study. One model was simply a random walk model for stochastic volatility and the second was a mean reverting process. The random walk version performed extremely poorly but the MAE from using the mean reverting model was 0.204 compared to 0.104 for the Black-Scholes model. Chesney and Scott posit that the use of the modified Black-Scholes model in the market could possibly account for their findings as well as the belief that the Black-Scholes model serves as a close approximation to a broad class of models. As we have discussed earlier, the Black-Scholes model holds as an approximation to such stochastic volatility models when the correlation is close to zero.

Two more recent studies employing a stochastic volatility model are Melino and Turnbull (1991) and Knoch (1992). These studies, in contrast to Chesney and Scott, provided evidence that the predictive ability of (ex-post) stochastic volatility models is better than the modified Black-Scholes model. One reason for the difference in results could be that Melino and Turnbull (1991), and Knoch (1992) investigate currency options traded on the Philadelphia stock exchange. Melino and Turnbull focus on the US/Canadian dollar employing a mean reverting volatility process also. They find that pricing bias is substantially lower for both puts and calls when using the stochastic volatility model as opposed to a constant volatility model.

Finally, Knoch (1992) empirically tests a model developed by Heston (1993). Knoch examines options for the Canadian dollar, Japanese yen, and German mark traded on the PHLX. The stochastic volatility model accounts for much of the maturity bias as well as strike price bias documented in earlier studies. These findings are consistent with that of Melino and Turnbull (1991).

The stochastic interest rate model is empirically investigated in Hilliard, Madura and Tucker (1991), and Choi and Hauser (1990). Hilliard et. al. find the stochastic interest rate model to have better predictive ability (pricing) over the constant volatility model for options on six different currencies (PHLX). The estimates for the variance-covariance matrix of domestic interest, foreign interest, and spot rate was conducted using monthly data. As these parameter estimates are crucial in testing the model, a data set with a finer time interval would have provided convincing results. In a related study, Choi and Hauser recognize the importance of interest rate movements and examine the effect of non-flat yield curves on currency option prices. As the closed form solution formulated by Hilliard et. al. involves the forward rate as opposed to the spot rate, the option pricing formula is similar in form to pricing formulas for an option on a forward or futures. Adams and Wyatt (1987) take a slightly different but interesting approach to measuring the volatility parameter; the volatility of the appropriate forward rate is used in place of the spot rate volatility to take into account a "premium" effect.

The pricing bias for "near the money" options used in this study are displayed in Table 3. Three models were used in the analysis. The mean model or simulated price is in the first column followed by the standard deviation of the model prices in the second column. The mean absolute error (MAE) indicates the pricing bias between the model price and the observed market price.

The benchmark model is the modified Black-Scholes (BS) model with daily updated implied volatility. The stochastic volatility model (HW) with daily updated GARCH parameter estimates was used as the second model. As the GARCH parameters are updated each day using a sample estimation window of 200 days. A monte carlo simulation as described in section was conducted 466 times to obtain the corresponding theoretical (simulated) call option prices. The average of the GARCH parameters (sum) was 0.7103 indicating some persistence of shocks in the conditional variance. We used daily historical data to estimate the variance-covariance matrix of interest rates and the spot rate for evaluating the Hilliard, Madura, Tucker model (HMT). The pricing bias as measured by the mean absolute error for both the stochastic volatility methods BS and HW is similar for the one month option. The BS model performs much better than the HW model for three month options indicating the BS model is a good proxy for the Hull and White stochastic volatility model when the correlation is zero.

Table 3: Pricing Bias for at-the-money Call Options

Model	Simulated Mean	Standard Deviation	MAE(%)	Number Observations
BS(1)	1.8729	0.17184	27.95	466
HW(1)	1.8474	0.24944	27.50	466
HMT(1)	2.0897	0.3972	32.93	466
BS(3)	2.8607	0.2918	19.84	466
HW(3)	3.5633	0.4429	33.60	466
HMT(3)	3.2000	0.6841	55.41	466
α_1	0.0674	0.0566		466
β	0.6428	0.2633		466
$\alpha_1 + \beta$	0.7103	0.2585		466

The simulated mean prices are for 1)BS denotes Black Scholes model using implied volatilities, 2)HW denotes the Hull and White model using GARCH parameter estimates in a monte carlo simulation, and 3) HMT denotes the Hilliard Madura Tucker model using historical averages. The numbers in parenthesis indicate months to maturity of the option. The standard deviation of the simulated price is given in the following column. $MAE = \frac{1}{466}\sum_{i=1}^{466}\left|\frac{market_i - model_i}{model_i}\right|$. α_1 and β are GARCH parameters for deriving the Hull and White model prices.

6. Time Series Properties of Implied Volatilities

We present some preliminary findings on the time series properties of the implied volatilities obtained from options traded on the Tokyo market. As we are able to obtain implied volatilities over the next one and three months, this allows us to conduct empirical work on the relationship between IV for different maturities. In other words, we look at the term structure of IV. Stein (1989) provides us with the basic framework, and we replicate the approach developed in Stein (1989) to our data set. Then, we apply GARCH parameter estimates as developed in Heynen, Kemna, and Vorst (1994) to test the term structure restrictions. As the focus of this essay is to test the informationally efficiency of option prices relative to the underlying spot rate, testing the GARCH term structure restrictions is in the spirit of this paper.

Poterba and Summers (1986) looked at implied volatilities for stock indices and found that when a shock to current volatility expectations occur, the expected volatility in future periods also changes but by less than the change in the current volatility. Stein (1989) provides for a restriction on the term structure of volatility assuming that the volatility follows a mean reverting diffusion process. Weekly implied volatilities from the S&P 100 index options were obtained over the period December 1983 to September 1987. The nearby was zero to one month and the distant was between one to two months and provides evidence of overreaction in the longer term option market. Stein regresses the short term IV on the longer term IV to get an estimate of the elasticity and compares it with a "theoretically" implied rate. If the estimated elasticity is greater than the theoretical value, then this could be taken to mean overreaction exists in the market. As an additional test, the forecast error (derived from the term structure restriction) is regressed on the nearby IV and a negative coefficient on the nearby IV is estimated indicating overreaction as well.

Xu and Taylor (1995) have also conducted a similar study finding the PHLX currency option market informationally efficient. In particular they employ a volatility forecast based on a term structure model using a Kalman filtering procedure.

The present essay takes the approach originally developed in Stein (1989) and Heynen, Kemna, and Vorst (1994) to test the restrictions on the term structure of implied volatilities.

6.1 The Stein Model of the Term Structure of Implied Volatility

As in Stein (1989) we obtain an "elasticity" which measures the impact of a change in nearby IV on the longer dated (deferred) IV. Stein (1989)[6] provides the following framework, where it is assumed the volatility process is

$$d\sigma_t = -\alpha(\sigma_t - \overline{\sigma})dt + \beta\sigma_t dz \tag{6.1.1}$$

The expected value of the volatility can be derived by rearranging and manipulating equation (6.1.1). Let us divide by $\sigma_t - \overline{\sigma}$ and take expectations to yield

$$E_t[\frac{d\sigma_t}{\sigma_t - \overline{\sigma}}] = -\alpha dt$$

6 The description of the Stein method closely follows that of the original work, including notation.

Then integrate over the period t to t+j to get

$$E_t\{\ln\left(\frac{(\sigma_{t+j}-\overline{\sigma})}{(\sigma_t-\overline{\sigma})}\right)\} = -\alpha(t+j)+\alpha t$$

Then it follows that the expectation of the volatility at t for j periods ahead is given as $E_t(\sigma_{t+j}) = \overline{\sigma} + \phi^j(\sigma_t - \overline{\sigma})$, where $\phi = e^{-\alpha} < 1$.

In other words, volatility is expected to decay to a long run parameter $\overline{\sigma}$ (mean volatility level). Recall that in the Hull and White stochastic volatility model, a closed form solution was derived when the volatility term is defined as the average volatility over the remaining life of the option. The average volatility which we will refer to as the implied volatility (stochastic volatility model) can be defined as

$$IV_t(T) = \frac{1}{T}\int_{j=0}^{T}[\overline{\sigma}+\phi^j(\sigma_t-\overline{\sigma})]dj = \overline{\sigma}+\frac{\phi^T-1}{T\ln\phi}[\sigma_t-\overline{\sigma}] \qquad (6.1.2)$$

Stein then derives the restrictions given a nearby IV(T) with maturity T and a deferred IV(K) with a maturity of K [ie. T<K]. The deferred IV(K) is defined in a similar fashion to IV(T). Rearranging and substituting gives the restriction

$$(IV_t(K)-\overline{\sigma})=\frac{T(\phi^K-1)}{K(\phi^T-1)}(IV_t(T)-\overline{\sigma}) \tag{6.1.3}$$

The restrictions thus give us a relationship $(IV_t(K)-\overline{\sigma})=\varepsilon(\rho,T)(IV_t(T)-\overline{\sigma})$ where $\varepsilon(\phi,T)=\dfrac{T(\phi^K-1)}{K(\phi^T-1)}$ which we can use to test empirically. We conduct simple tests as outlined in Stein (1989).

The autocorrelation of the implied volatilities for the nearby (one month) options are calculated through lags 12 and displayed in Table 4. The autocorrelation displays a decaying pattern for ϕ. The estimates are raised to 1/k power where k is the lag length in days to obtain implied daily autocorrelations for comparative purposes. The daily autocorrelations are fairly stable ranging from a low of around 0.80 to a high of approximately 0.91 for IV from both the stochastic volatility and stochastic interest rate models. The elasticity is approximately 0.333 when the autocorrelations coefficient of 0.8 is used. The time to maturity applied in obtaining the elasticity is T=30 and K=90 days.

Table 4: Term Structure of Implied Volatilities

Autocorrelation of IVSV(1) and IVSI(1)

lag k	IVSV(1) autocorrelation	implied	IVSI(1) autocorrelation	implied
1	0.80	0.80	0.81	0.810
2	0.69	0.830	0.70	0.836
3	0.67	0.875	0.67	0.875
4	0.64	0.894	0.64	0.895
5	0.61	0.906	0.62	0.894
6	0.55	0.905	0.57	0.908
7	0.50	0.907	0.53	0.910
8	0.46	0.910	0.44	0.913
10	0.39	0.906	0.39	0.914
11	0.34	0.908	0.35	0.913
12	0.31	0.909	0.32	0.910

The autocorrelation is for the one month implied volatility. The implied autocorrelation is the autocorrelation raised to the 1/k power where k is the lag length in days.

IV(3)=constant + ε IV(1)

	constant	IV(1)	adj. R^2
Stochastic Volatility	-0.0005 (3.37)	0.9729 (81.81)	0.946
Stochastic Interest Rate	0.0002 (29.69)	0.8868 (26.69)	0.701

$$[E_t(IV_{t+30}) + E_t(IV_{t+60}) - 2(IV_t(T) - 3(IV_t(K) - IV_t(T))] = \text{constant} + \theta \text{ IV}(1)$$

	constant	θ	adj. R^2
Stochastic Volatility	0.0125 (16.69)	-0.8672 (14.19)	0.3675
Stochastic Interest Rate	0.0077 (9.38)	-0.5666 (6.93)	0.1202

t-statistics in parenthesis.

Given these estimates for rho and the specification for the "elasticity" from equation (6.1.3) we can calculate the theoretically implied ε. These theoretical values should be consistent with the empirically obtained estimate from regressing the three month IV on the one month IV. The results of the regressions are also presented in Table 4. For the stochastic volatility IV, the estimated elasticity is 0.9729 which is higher than the theoretical values derived above. Furthermore, the stochastic interest IV yields an elasticity of 0.8868 which is also far greater than 0.333 the theoretical value calculated above. These empirical estimates (from the regression) provided some evidence that there is some "overreaction" in the market given the model used in the analysis is correct.

An alternative test discussed in Stein is to regress a prediction error on the nearby IV. The prediction error is derived by assuming the deferred IV is a sum of the nearby IV and the expected nearby IV one and two months in the future.

$$(IV_t(K)-\overline{\sigma})=\frac{1}{3}[(IV_t(T)-\overline{\sigma})+E_t(IV_{t+30}-\overline{\sigma})+E_t(IV_{t+60}-\overline{\sigma})] \qquad (6.1.4)$$

Rearranging equation (6.1.4) yields the prediction error $[E_t(IV_{t+30})+E_t(IV_{t+60})-2(IV_t(T)-3(IV_t(K)-IV_t(T))]=0$. This forecast error is then simply regressed on IV(T). to look for overreaction in the market. If a high IV(T) is accompanied by an IV(K) which is higher then expected, then the prediction error would get smaller or become negative; thus, the prediction error and IV(T) should exhibit a negative correlation if there is overreaction in the market. Again the results are presented in Table 4. Realized values replace the expected values in equation (6.1.4), thus reducing the sample size from 466 to 406 overlapping observations. The coefficient on the IV(T) term is negative and statistically significant indicating what Stein refers to as "overreaction" in the market and is taken as evidence against informational efficiency in the market.

6.2 The GARCH Model and the Terms Structure of Implied Volatilities

We assume a discrete process, the GARCH(1,1) model developed in Bollerslev (1986).

$$\ln(\frac{S_t}{S_{t-1}}) = \lambda\sqrt{h_t} + \varepsilon_t$$
$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1}$$

The conditional variance in also included in the mean process giving us a GARCH(1,1) in mean model. Heynen et.al. (1994) then show that deviations from term structure restriction, δ_t, is

$$\delta_t = [IV_t(K) - \overline{\sigma}] - \frac{K}{T}\frac{(\alpha_1+\beta-1)^T}{(\alpha_1+\beta-1)^K}[IV_t(T) - \overline{\sigma}] \qquad (6.2.1)$$

where the mean volatility is given by $\overline{\sigma}^2 = \frac{\alpha_0}{1-\alpha_1-\beta}$. Thus, the elasticity is measured independent of the implied volatilities and the option prices. If the term structure restrictions hold, then we expect the deviations, δ_t, to be zero or at least zero on average.

In order to derive the above restriction let us assume that $\varepsilon_t^2 = h_t \zeta_t^2$ where $\zeta_t \sim N(0,1)$ and iid. We will denote h_t as σ_t^2 to be consistent with Heynen et. al. (1994). Rewrite the GARCH(1,1) conditional variance equation as

$$\begin{aligned}\sigma_{t+k}^2 &= \alpha_0 + (\alpha_1 \zeta_t^2 + \beta)\sigma_{t+k-1} \\ &= \alpha_0 + (\alpha_1 \zeta_t^2 + \beta)[\alpha_0 + (\alpha_1 \zeta_t^2 + \beta)\sigma_{t+k-2}] \\ &= \alpha_0 + \alpha_0 \sum_{m=1}^{k-1} \prod_{n=1}^{m} (\alpha_1 \zeta_{t+k-n}^2 + \beta) + \sigma_t \prod_{n=1}^{k} (\alpha_1 \zeta_{t+k-n}^2 + \beta)\end{aligned}$$

Next take expectations conditional on information at time t.

$$E_t \sigma_{t+k}^2 = \alpha_0 + \alpha_1 \sum_{m=1}^{k-1} (\alpha_1 + \beta)^m + (\alpha_1 + \beta)^{k-1} (\alpha_1 \zeta_t^2 + \beta)\sigma_t$$

where $E_t \sigma_{t+k}^2 = 1$ and ζ_t is iid. If we define $\gamma = \alpha_1 + \beta$ and use the definition of a sum of a geometric progression, then

$$E_t \sigma_{t+k}^2 = \alpha_0 + \alpha_0 \left(\frac{\gamma - \gamma^k}{1-\gamma}\right) + \gamma^{k-1} (\alpha_1 \zeta_t^2 + \beta)\sigma_t^2 \qquad (6.2.2)$$

Recall that the average volatility can be defined (for the discrete case) as

$$\sigma^2_{AVG}(t,T) = \frac{1}{T}\sum_{k=1}^{T} E_t \sigma^2_{t+k} \qquad (6.2.3)$$

Substituting equation (6.2.2) into equation (6.2.3) we get

$$\sigma^2_{AVG}(t,T) = \frac{1}{T}\sum_{k=1}^{T} E_t \sigma^2_{t+k}$$

$$= \frac{1}{T}\{\beta_0 + \beta_0 \frac{\gamma - \gamma}{1-\gamma} + \gamma^0(\alpha_1 \zeta_t^2 + \beta)\sigma_t^2$$

$$+ \beta_0 + \beta_0 \frac{\gamma - \gamma^2}{1-\gamma} + \gamma\ (\alpha_1 \zeta_t^2 + \beta)\sigma_t^2$$

$$+ \beta_0 + \beta_0 \frac{\gamma - \gamma^3}{1-\gamma} + \gamma^2(\alpha_1 \zeta_t^2 + \beta)\sigma_t^2$$

$$+ \dots\dots\dots\dots\dots\dots\}$$

$$= \frac{1}{T}[\beta_0 T + \frac{\beta_0 T\gamma}{1-\gamma} - \frac{\beta_0}{1-\gamma}(\gamma + \dots\dots + \gamma^T) + (\sigma^2_{t+1} - \beta_0)(1 + \dots\dots + \gamma^{T-1})]$$

$$= \frac{1}{T}[\frac{\beta_0 T}{1-\gamma} - \frac{\beta_0}{1-\gamma}(\gamma + \dots + \gamma^T) + \sigma^2_{t+k}(1 + \dots + \gamma^{T-1}) - \beta_0(\frac{1-\gamma}{1-\gamma})(1 + \dots + \gamma^{T-1})]$$

$$= \frac{1}{T}[\overline{\sigma}^2 T - \overline{\sigma}^2(\gamma + \dots\dots + \gamma^T) - \overline{\sigma}^2(1 + \gamma + \dots\dots + \gamma^{T-1}) + \overline{\sigma}^2(\gamma + \dots\dots + \gamma^T) + \sigma^2_{t+1}(1 + \gamma + \dots\dots + \gamma^{T-1})$$

$$= \frac{1}{T}[\overline{\sigma}^2 T + (\sigma^2_{t+1} - \overline{\sigma}^2)(1 + \dots\dots + \gamma^{T-1})]$$

$$= \overline{\sigma}^2 + \frac{1}{T}(\sigma^2_{t+k} - \overline{\sigma}^2)(\frac{1-\gamma^{T-1}}{1-\gamma}) \qquad (6.2.4)$$

where $\overline{\sigma}^2 = \dfrac{\alpha_0}{1-\alpha_1-\beta}$ and $\sigma^2_{t+k} = \alpha_0 + \alpha_1\varepsilon^2_t + \beta\sigma^2_t$. We can derive the same relationship for an average volatility over the period t to K instead of T. If we substitute the implied volatility for the average volatility then we obtain the restriction in equation (6.2.1).

As an alternative model we apply the mean reverting volatility process and test the following restriction

$$\delta_t = [IV_t(K) - \overline{\sigma}] - \frac{K}{T}\frac{(\phi-1)^T}{(\phi-1)^K}[IV_t(T) - \overline{\sigma}]$$

where ϕ is estimated from the process $IV_t(T) = \text{constant} + \phi IV_{t-1}(T) + \theta\varepsilon_t$. ε_t is white noise. The estimated implied volatility from the thirty day option is used to obtain estimates of θ and ϕ. The mean volatility is defined as $\overline{\sigma}^2 = \dfrac{\theta}{1-\phi}$ [Heynen, Kemna, and Vorst (1994)].

Parameter estimates of the GARCH and autoregressive models employed to obtain the elasticity measure (restriction) and the mean volatility are found in Table 5. The deviations from the term structure restrictions are documented in Table 6. The parameter estimates were obtained using an estimation window of 250 days. The implied volatilites for the remaining 216 days of our sample were used to calculate the deviations from the term structure restrictions. In order to assess whether the deviations were on average zero, we used three measures: the mean error (ME), mean absolute error (MAE), and root mean square error (RMSE). The error is simply the deviation the error is the difference between the deviation and zero. We find that the absolute magnitude of the mean error to be small for both the GARCH and autoregressive models. The GARCH mean error and

MAE is, however, slightly smaller than that of the autoregressive model indicating the GARCH model performed somewhat better. These findings are preliminary evidence.

Table 5: Parameter Estimates of the GARCH(1,1)-M and Autoregressive Model for the Term Structure Restriction

GARCH(1,1)-M

$$\ln(\frac{S_t}{S_{t-1}}) = \lambda\sqrt{h_t} + \varepsilon_t$$

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1}$$

λ	α_0(x1000)	α_1	β
0.0473	0.000047	0.0604	0.8607
(0.706)	(1.293)	(10.727)	(2.667)

LB(6)=4.84

LB is the Ljung-Box-portemanteau test of the normalized residuals. t-statistics in parenthesis.

Autoregressive Model

$$IV_t(T) = \text{constant} + \phi IV_{t-1}(T) + \theta\varepsilon_t$$

constant	ϕ	θ
0.0001729	0.9735	0.6928
(1.164)	(50.090)	(12.582)

LB(6)=2.91

LB is the Ljung-Box-portemanteau test. t-statistics in parenthesis.

Table 6: Deviations From the Term Structure Restrictions

$$\delta_t = [IV_t(K) - \overline{\sigma}] - \frac{K}{T}\frac{(\alpha_1 + \beta - 1)^T}{(\alpha_1 + \beta - 1)^K}[IV_t(T) - \overline{\sigma}]$$ deviation for the GARCH model.

$$\delta_t = [IV_t(K) - \overline{\sigma}] - \frac{K}{T}\frac{(\phi - 1)^T}{(\phi - 1)^K}[IV_t(T) - \overline{\sigma}]$$ deviation for the autoregressive model.

	ME	RMSE	MAE	LB(6)
Autoregressive	-0.0321	1.0197	0.7374	4.84
GARCH	-0.0119	1.0156	0.7230	5.66

LB is the Ljung-Box-portmanteau test for the deviations. ME is mean error, RMSE is root mean square error, and MAE is mean absolute error. The error is the difference between the deviation, δ_t, and zero.

7. Comparison of Forecasts

An in-sample and out-of-sample test is conducted to compare the forecasting ability of the implied volatility with the forecasts from the underlying spot rate. If the option market is informationally efficient and the option pricing models used to obtain the implied volatility are correct, then the forecasts based underlying spot rate data should not fare any better then the implied volatility (IV) forecasts from the stochastic volatility model (IVSV) and the stochastic interest rate model (IVSI). The two historical data based forecasts are the lagged historical forecast (HV) and the GARCH conditional variance (GV) forecast. In the case of the of the stochastic volatility model, the proposition is somewhat stronger as the GARCH model is imbedded into the stochastic volatility model. The conditional variance equation is consistent with the volatility diffusion process from the option pricing model hence information from the GARCH model should be reflected in the option price if the model is correct. In the case of the stochastic interest rate model, the GARCH model is simply an alternative competing forecasting model.

7.1 In-sample Results

Day and Lewis (1992), Lamoureux and Lastrepes (1993), and Xu and Taylor (1993) examine the informational content of option prices by including the implied volatility as an exogenous variable in the conditional variance equation of the GARCH model.[7]

7 If the exogenous variable accounts for the persistence in the conditional variance, then the magnitude of the conditional variance parameters will decrease when such a variable is included in the specification. Volume is often included when analyzing stock returns [Lamoureux and Lastrepes (1990)] and quote arrivals for foreign exchange [Takezawa (1994)] as proxies for information flow.

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = C + \varepsilon_t$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + \phi_1 IVSV_{t-1} + \phi_2 IVSI_{t-1} \qquad (7.1.1)$$

By including the implied volatility in the conditional variance equation, we test whether IV has additional predictive ability. If the currency option market is informationally efficient, then the implied volatilities should explain the persistence effects of the GARCH model. A priori, we expect that the estimates of α_1 and β to decrease in magnitude if all relevant information is imbedded in the option price and the option pricing model used to obtain IV is correct.

The GARCH model A in Table 7 is estimated over the entire sample period. The GARCH parameters are statistically significant but relatively small in magnitude. When we include the one month implied volatility from the stochastic volatility model (IVSV) or the stochastic interest rate model (IVSI), the GARCH parameters decrease in magnitude in either case. The β estimate declines to its limit of zero in both cases as well. Moreover, the ϕ_1 coefficient is both positive and significantly different from zero when the one month IV is included in the conditional variance equation. Finally, note that the coefficient on IVSV(1) is substantially larger than that for IVSI(1).

Table 7: GARCH (1,1) Models and In-sample Tests

$$\ln(S_t / S_{t-1}) = C + \varepsilon_t$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + \phi_1 IVSV_{t-1} + \phi_2 IVSI_{t-1}$$

where S is exchange rate, C the constant in the mean equation, IVSV the implied volatility from the stochastic volatility model (Hull and White) and IVSI the implied volatility from the stochastic interest rate model.

	A	IVSV(1)	IVSV(3)	IVSI(1)	IVSI(3)	IVSV(1) IVSV(3)	IVSI(1) IVSI(3)
C(x100)	0.048 (0.12)	0.059 (0.01)	0.190 (0.49)	0.080 (0.21)	0.080 (0.21)	0.003 (0.01)	0.089 (0.49)
α_0	0.055 (1.37)	0.145 (0.87)	0.024 (0.47)	0.062 (1.43)	0.087 (1.02)	0.221 (1.05)	0.459 (0.01)
α_1	0.012 (10.2)	0.145 (0.87)	0.024 (0.57)	0.062 (1.43)	0.087 (1.03)	0.0	0.0
β	0.292 (6.20)	0.0	0.0	0.0	0.0	0.0	0.095 (0.76)
ϕ_1		0.250 (16.34)		0.038 (8.37)		0.250 (16.37)	0.080 (7.24)
ϕ_2			0.070 (7.35)		0.040 (18.36)	0.0	0.0
Log Likelihood	929.49	946.62	945.47	937.59	937.59	946.62	945.47
Skewness	-0.4051	-0.3923	-0.3568	-0.4140	-0.3854	-0.3925	-0.4140
Kurtosis	4.3939	2.3258	3.0265	2.7189	3.5203	2.3259	2.7188
LB(6)	8.97	6.77	7.45	6.39	8.00	6.77	6.39
LB2(6)	16.4	17.6	17.5	17.3	17.0	17.7	16.2

t-statistics in parenthesis. The t-statistic are calculated by using "robust standard errors" as described in Weiss (1986). Skewness and kurtosis (excess) are for the normalized residuals. LB(6) is the Ljung-Box statistic for the normalized residuals. LB2(6) is the Ljung-Box statistic for the squared normalized residuals.

The implied volatilities for the three month maturity options also exhibit a positive and statistically significant effect in the conditional variance equation. Again the sum of the GARCH parameters declines in magnitude as $\beta = 0$. The log likelihood is for each of the models with IV in the conditional variance equation is larger than the log likelihood 929.49 for the restricted GARCH model A ($\phi_1 = \phi_2 = 0$) in Table 7. In all cases a likelihood ratio test statistic rejects the null of the restricted model A. Hence, we find evidence that implied volatilities capture the GARCH persistence effect satisfactorily.

These results are consistent with findings in Xu and Taylor (1993) for the PHLX currency option market. Xu and Taylor look at the British pound, German mark, Japanese yen, and Swiss franc and in all cases the GARCH parameters decline in magnitude when the IV is included in the conditional variance equation.

We next turn to a related question of whether the one month IV is sufficient to capture the GARCH effect. Both the one month and three month IV are included in the conditional variance to investigate this conjecture. The one month IV from both the stochastic volatility and stochastic interest rate models are positive and statistically significant. Furthermore, the coefficients on IVSV(3) and IVSI(3), ϕ_2, is zero indicating that the one month IV is sufficient to explain the persistence effect. Xu and Taylor (1995) take a similar approach but include a simulated short and deferred maturity IV from their term structure model. The longer dated IV has a zero coefficient for all currencies examined including the Japanese yen and the shorter dated IV is positive and significant. Thus, our in sample GARCH results are consistent with those for PHLX currency options examined in Xu and Taylor (1995).

7.2 Out-of-sample Results

The in-sample results decisively exhibit evidence that the IV measures are informationally efficient. As noted in past research, however, out of sample tests are required as well and this circumvents the problem of the time horizon mis-match inherent in the in-sample GARCH methods discussed in the previous Section. The maturity mis-match problem refers to the fact that the implied volatility forecasts volatility over the next month (or three months) whereas the GARCH model is estimated with daily data.

Two forecast methods rooted in the data are also employed as benchmark forecasts. The GARCH(1,1) forecast is based on the model in equation (5.1.4). Forecasts are obtained via a recursive formulation of the equation. Firstly, the GARCH model is fitted over an estimation period of 200 trading days. Based on these estimates the forecast for the next day is made. Then the estimation sample period is shifted up by one day while the window remains at 200 days. A forecast for the next day is conducted once again and this procedure repeated until 30 daily forecasts are obtained. The average of these 30 daily forecasts is taken to be the forecasts are over the next 30 days into the future and is denoted GV. Therefore, the GV forecasts are overlapping. This is referred to as the rolling GARCH forecast in Lamoureux and Lastrepes (1993). The 90 day forecast is made in the same manner except an average of 90 daily GARCH forecasts are used.

Table 8: Out-of-Sample Forecasts

$$ME = \frac{1}{N}\sum_{t=1}^{N}(RV_t - FV_t)$$

$$MAE = \frac{1}{N}\sum_{t=1}^{N}|RV_t - FV_t|$$

$$RMSE = \{\frac{1}{N}\sum_{t=1}^{N}(RV_t - FV_t)^2\}^{1/2}$$

where RV is the realized volatility and FV the forecasted volatility. HV is the lagged historical volatility forecast, GV is the GARCH volatility forecast, IVSV the Hull and White model implied volatility, IVSI the stochastic interest rate model implied volatility.

	ME	MAE	RMSE
HV(1)	0.0008227	0.0000057	0.0079261
GV(1)	0.0023410	0.0000106	0.0073630
IVSV(1)	0.0031209	0.0000127	0.0068704
IVSI(1)	0.0024585	0.0000172	0.0066181
HV(3)	0.0000028	0.0000126	0.0073456
GV(3)	0.0022536	0.0000100	0.0057704
IVSV(3)	-0.0000697	0.0000051	0.0045879
IVSI(3)	-0.0009805	0.0000052	0.0041233

A naive lagged (average) model of the residuals[8] is used as a second benchmark model.

$$HV_t = \frac{1}{t}\sum_{i=1}^{t} \varepsilon_i^2 \qquad (7.2.1)$$

HV is the lagged historical volatility measure. The historical volatility is simply the sum of the squared (demeaned) returns divided by the sample size. The 30 days prior to the forecast date is used.

The realized volatility is also simply an average of the squared residuals but summed over the time remaining before the option matures.

$$RV_t = \frac{1}{T}\sum_{i=1}^{T} \varepsilon_{t+i}^2 \qquad (7.2.2)$$

Thus, we note that although there are 466 observations for the options prices, we are limited to 436 and 376 out-of-sample observations for the 30 and 90 day forecasts respectively This occurs since we lose 30 observations from the calculation of the final realized volatility for the 30 day option and 90 observations for the 90 day option.

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8 The residuals that are obtained by regressing the FX return on a constant. This amounts to a return series adjusted by subtracting the mean.

The four forecasts are compared using three standard measures for of forecasting error: mean error (ME), mean absolute error (MAE), and root mean square error (RMSE).

$$ME = \frac{1}{N}\sum_{t=1}^{N}(RV_t - FV_t)$$
$$MAE = \frac{1}{N}\sum_{t=1}^{N}|RV_t - FV_t| \qquad (7.2.3)$$
$$RMSE = \{\frac{1}{N}\sum_{t=1}^{N}(RV_t - FV_t)^2\}^{1/2}$$

where RV is the realized volatility and FV is one of the four forecast volatilities. The equations in (7.2.3) are calculated for FV=HV, GV, IVSV, IVSI and results summarized in Table 8.

The mean error is relatively small for the lagged historical volatility as compared to the remaining forecasts. The errors are positive for the one month maturity indicating the forecasts underpredict the realized volatility on average. A similar pattern is exhibited by the three month forecasts except the implied volatilities have negative ME implying that the IV overpredict the realized volatility on average.

As the positive and negative errors could cancel producing a small ME, we then use the mean absolute error criteria. For the one month forecast, the lagged historical volatility (HV) produces the best forecast whereas both of the IV forecasts performed relatively poorly. The MAE for HV(1) is roughly half of that of the MAE for the GARCH forecast and stochastic volatility (IVSV) forecast. At the three month horizon the MAE results indicate the IV forecasts have predictive ability relative to the HV(3) and GV(3) forecasts. It appears that the implied volatilites perform better for longer horizons based on the MAE criteria.

We find the two implied volatility measures have the smallest RMSE and thus this provides evidence that IV is informationally efficient relative to the GARCH and HV forecast models for both the one and three month horizon. Hence our results based on MAE and RMSE are consistent for both the IVSV and IVSI at the three month horizon.

Due to the high degree of leptokurtosis documented in Table 1, the MAE criterion is probably the more appropriate measure by which to judge forecasting ability. Thus, we can conclude that implied volatility from the stochastic volatility model (IVSV) and stochastic interest rate model (IVSI) have superior predictive ability over the lagged historical forecast (HV) and GARCH based forecast (GV) at the three month horizon. Results, however, at the one month horizon are mixed.

Comparing forecasts with solely on the basis of MAE and RMSE has its limitations. Fair and Shiller (1990) suggest running a set of regressions of the realized volatility on the forecasts. In this way, we could discern the marginal effect of each forecast to predict realized volatility. As these regressions are in the spirit of the encompassing forecasts literature we shall refer to them as encompassing regression tests. The forecast horizon is relatively long given the number of observations (sample period). Thus in the present study we employ overlapping daily data to obtain a larger sample and avoid losing information. The use of overlapping data, however, induces a moving average error term in encompassing regressions. As a consequence, the parameter estimates and t-statistics are obtained by applying generalized method of moments [Hansen and Singleton (1982)]. A Bartlett kernel is employed as suggested in Newey and West (1987).

An estimate of the variance-covariance matrix is obtained by using

$$X'\Omega X = \sum_{j=0}^{L} \sum_{t=j+1}^{M} w_j \varepsilon_t \varepsilon_{t-j} (X_t X'_{t-j} + X_{t-j} X'_t)$$

where the window length is $w_j = 1 - \frac{j}{L+1}$, X is a MxK matrix of instrumental variables (M is the number of observations and K is the number of instruments), and ε is a vector of error terms.

Firstly, we regress realized volatility on all four forecast measures.

$$RV_{t+1} = \alpha_0 + \alpha_1 IVSV_t + \alpha_2 IVSI_t + \alpha_3 HV_t + \alpha_4 GV_t + \varepsilon_{t+1} \qquad (7.2.4)$$

For both the one and three month options, all the forecast measures provide some information in predicting the realized volatility as all the coefficients are statistically significant with the exception of IVSI for the one month forecast. This does not mean that the implied volatility is not a good forecast of future volatility but that the other forecasts possibly contain useful information (from historical spot rate data) not imbedded in the option price (Table 9). The GV forecast has a negative weight and the HV forecast has a positive weight in both regressions. The IV show different signs depending on the forecast horizon. IVSI has a negative weight and IVSV has a positive weight for the shorter horizon and vice versa for the longer three month horizon.

Table 9: Encompassing Regressions

$$RV_{t+1} = \alpha_0 + \alpha_1 IVSV_t + \alpha_2 IVSI_t + \alpha_3 HV_t + \alpha_4 GV_t + \varepsilon_{t+1}$$

	α_0	α_1	α_2	α_3	α_4
one month	0.0239	0.7806	-0.4991	0.1105	-1.5150
	(6.63)	(2.09)	(1.35)	(1.89)	(5.88)
three month	0.0162	-1.5248	2.0885	0.1210	-0.8799
	(4.31)	(6.09)	(8.00)	(2.01)	(3.09)

$$RV_{t+1} = \alpha_0 + \alpha_1 FV_t + \varepsilon_{t+1}$$

	α_0	α_1
IVBS(1)	0.0026 (2.35)	0.5687 (8.75)
IVSI(1)	0.0032 (3.54)	0.5535 (8.91)
IVBS(3)	0.0043 (5.31)	0.4342 (3.36)
IVSI(3)	-0.0002 (0.23)	1.1303 (8.50)

$$RV_{t+1} - IV_{it} = \alpha_0 + \alpha_1 GV_t + \alpha_2 HV_t + \varepsilon_{t+1}$$

	α_0	α_1	α_2
Error SV(1)	0.0076 (2.84)	-0.1811 (4.26)	-0.6982 (3.41)
Error SI(1)	0.0036 (1.33)	-0.1895 (4.40)	-0.3176 (1.53)
Error SV(3)	0.0233 (9.40)	0.0281 (0.47)	-1.8827 (8.23)
Error SI(3)	0.0166 (7.28)	0.0215 (0.39)	-1.2673 (6.02)

HV is the lagged historical volatility forecast, GV is the GARCH volatility forecast, IVSV is the Hull White implied volatility, IVSI is the stochastic interest rate model implied volatility, RV is the realized volatility and (RV-IV) is the forecast error of the implied volatility forecast. The number in the parenthesis indicates months to maturity of the option. Estimates of coefficients and t-statistics obtained using generalized method of moments (GMM) employing a Bartlett kernel.

The statistically insignificant coefficient for IVSI at the one month horizon is consistent with MAE results from Table 8 as the MAE was largest for IVSI(1) when compared to other one month forecasts. The absolute magnitude of the coefficients also provides us with information on the relative predictive ability of the forecasts. For the three month horizon regression, the absolute magnitude of the implied volatility coefficients are larger than those HV and GV which is consistent with MAE results. We do not necessarily discern such a pattern for the one month horizon regression. In a sense this is consistent with the MAE results in that we find mixed results.

A simple test of unbiasedness of forecasts of the implied volatilities is done by regressing the realized volatility on the implied volatility (forecast).

$$RV_{t+1} = \alpha_0 + \alpha_1 FV_t + \varepsilon_{t+1} \qquad (7.2.5)$$

These regressions are analogous to the empirical literature testing the whether the forward rate is an unbiased predictor of the future or realized spot rate. Instead of testing the unbiasedness of the mean or level of a spot exchange rate we are interested in the second moment. If the implied volatility forecast is an unbiased predictor of realized volatility, then $\alpha_1 = 1$ and $\alpha_0 = 0$. The results in Table 9 show that α_1 is positive and statistically significant. The α_1 coefficients are, however, not close to one with the exception of IVSI(3). The constant terms are small in magnitude but statistically significantly different from zero. Again the exception being the constant for the three month stochastic interest rate IV. IVSI(3) appears to be an unbiased predictor of the realized volatility and this is consistent with the encompassing regression test where the magnitude of the coefficient is relatively large and the MAE findings in Table 8.

To further test whether these implied volatilities are unbiased we regress the forecast error between IV and realized volatility on the historical forecasts. Again, if the market is efficient and the model used to obtain the implied volatility is correct, then the historical information should not add additional forecasting ability. In other words, the coefficients in equation (7.2.6), α_1 and α_2, should be zero.

$$RV_{t+1} - IV_{it} = \alpha_0 + \alpha_1 GV_t + \alpha_2 HV_t + \varepsilon_{t+1} \qquad (7.2.6)$$

In the case of one month options we find that the historical data provides information in explaining the forecast bias. The coefficients for GV and HV for the one month horizon are negative and significant. Thus, historical spot exchange rate data does provide additional information in forecasting realized volatility at the shorter one month horizon. For the three month option, however, only the historical lagged measure contains information to explain the bias; the HV coefficient is negative and statistically significant.

The coefficient on the HV(3) forecasts are statistically significant but negative. The GARCH forecast is not statistically significant in this regression and is consistent with the notion that GARCH forecasts for the longer term are less accurate. Although the in-sample GARCH results were consistent with the PHLX currency option market [Xu and Taylor (1995)], the out-of-sample results are completely reversed. We find that IV has predictive strength but that information from forecasts based on historical spot data such as the GARCH forecast and lagged historical forecast have incremental predictive ability as well for the Tokyo market.

8. Discussion and Interpretation of Results

The focus of the present paper was to test whether the Tokyo currency option market was informationally efficient. To test this conjecture, we obtain the implied volatility given the market currency option price and compare it to the realized volatility of the underlying spot rate. If the option price contains all relevant information in the market and the pricing model used to back-out the IV is correct, then the forecasts based on the underlying spot rate should not predict realized volatility any better than the forecast from implied volatilities. Within the context of the stochastic volatility models used in this paper this should be case. We indicate that a GARCH(1,1) model is consistent with the volatility process assumed in the stochastic volatility option pricing model. Hence forecast information based on the GARCH model should be incorporated into the stochastic volatility model and thus into the IV from such models. We also obtain two other forecasts as benchmark forecasts. Implied volatility from a stochastic interest rate model and lagged historical volatility forecasts are calculated.

Based on several out-of-sample regression tests we reject the joint hypothesis that the Tokyo currency option market is efficient and the option pricing models used to get the IV are correct. From encompassing regression tests we find that all forecasts have marginal yet significant predictive ability. Furthermore, the GARCH and lagged historical volatility forecasts were significant in the regression of the forecast error on forecasts based on spot rate data. The one possible exception was from the three month implied volatility from the stochastic interest rate model. From Table 9, IVSI(3) had the largest marginal predictive ability and is consistent with the unbiasedness hypothesis. Furthermore, IVSI(3) performed the best under the RMSE criteria and exhibited minimal bias under the MAE criteria.

Overall, however, we find the joint hypothesis is rejected. How do these results compare with other empirical findings? The results for the PHLX currency option market seem to indicate informationally efficiency based on criterion similar to those used in this paper. Xu and Taylor (1995), Scott and Tucker (1989), and Scott (1992) indicate that implied volatilities outperform forecasts based on historical spot rate data. Scott and Tucker (1989), however, find that the modified Black-Scholes IV performs equally as well as the IV from a CEV option pricing model discussed in Section 2. This can be taken to mean that the stochastic volatility model (approximation) performed relatively well. The realized volatility is regressed on the implied volatility and a lagged historical forecast. They find that the coefficients on the lagged historical forecasts are not statistically significant and thus is in contrast with the findings in this essay.

Xu and Taylor (1995) use an IV derived from the volatility term structure model [Xu and Taylor (1994)] and find evidence in favor of informational efficiency for the pound, mark and yen. Poor performance for the Swiss franc is attributed to data problems. The IV based on their volatility term structure model is the only statistically significant forecast in an encompassing regression test when stepwise regression techniques were applied to select statistically significant regressors. Furthermore, when the realized volatility is regressed against the term structure forecast without a coefficient, they report that the slope coefficient is close to one in all cases.

The IV from the Black-Scholes model is used again in Scott (1992) and employed in tests for unbiasedness using regressions of the form $\Delta RV_{t+1} = a + bRV_t + c[IV_t - RV_t] + e_t$. If IV is an unbiased predictor of realized volatility, then under the null a=b=0 and c=1. Interestingly, three of the four currencies (British pound, German mark, and Swiss franc) examined did not reject the null of unbiasedness. The exception was the Japanese yen.

Although we examine the Tokyo currency option market, our results are in a sense consistent with Scott (1992) results for the PHLX yen option. Scott also uses call option prices with three months to maturity to obtain IV but the time frame used in the present paper is more recent. Whether such rejections of the joint hypothesis are peculiar to the yen currency option or to the Tokyo market is subject to future research.

As the joint hypothesis suggests, a rejection of the null could be due to one of two reasons (or both). It could be argued that the currency market is not informationally efficient. According Stein (1989) the market exhibits an "overreaction." The market overreacts to any "recent" volatility shocks in the sense that shocks are perceived as having lasting or prolonged effects even though the shock could be transitory in nature. If such is the case, then the longer dated IV should be very sensitive to movements in shorter dated IV given the deferred IV incorporates expectations of volatility in the future. This is in fact what we document in Section 6 where the empirical elasticity measure is found to be larger than the proposed "theoretical" elasticity value. The negative coefficients on the HV and GV from equation (7.2.5) reinforce this interpretation since a negative sign would indicate that any shocks should be temporary.

An alternative view would be that the market is informationally efficient but the models used to impute the implied volatilities are not correct. In this case, we work under the assumption the models used in this study do not capture an important feature of the market which could lead to a rejection of the joint hypothesis. Lamoureux and Lastrepes (1993) suggest that a risk premium and in particular a time varying risk premium could be a major factor. Canina and Figlewski (1993)[9] in examining the implied volatility for S&P 100 index options suggest that factors such as liquidity considerations, interaction between the S&P 100 option and other derivative assets, and investor tastes are other factors which affect the demand and supply of options but are not incorporated into the model.

9. Conclusion

Based on in-sample GARCH and out-of-sample tests, we reject the conjecture that public information in the form of spot exchange rate data cannot be utilized to improve the volatility forecasts derived from observed currency option prices and currency option pricing model. These results stand in contrast to those presented in Xu and Taylor (1995) for the PHLX currency option market. The rejection of the joint hypothesis of informational efficiency and the correct pricing model could be attributed to "overreaction" in the market or simply by the fact that the pricing models used to obtain the implied volatilities do not account for a time varying risk premium. In any case, the implied volatilities do provide useful information in forecasting volatility which is not completely captured by either the GARCH or lagged historical forecasting models. In determining the expected value of the future volatility of the yen/US dollar exchange rate, the implied volatility should be included as part of the conditional information set along with the historical volatility measures when the implied volatility is derived from Tokyo currency option market prices.

9 Canina and Figlewski (1993) examine the implied volatilities for the S&P 100 index options. They regress realized volatility on implied volatility and historical volatility and find that implied volatilities are poor forecasts of realized volatility.

Essay Two:
An Empirical Study of Swap Covered Interest Parity

1. Introduction

The swap market, with an outstanding face amount of three trillion dollars, is among the largest and most active financial markets today.. In a recent article, Litzenberger (1992) highlights the need for greater academic research on interest rate and cross currency swaps. It is the smaller of the two swap markets, the currency swap market, which is the focus of the present paper. The currency swap market, broadly defined, however, has a relatively long history as compared to the interest rate swap market. Clinton (1988) notes that most "of the functions that economists somewhat loosely ascribe to the forward market are in fact performed in the swap market" (p.359). The swap in this context involves trades and cash flows in the short term of several days to several months. Over recent years, however, the currency swap market that has evolved refers to the long dated contract which is typically defined as having a maturity of over one year. These currency swaps (long dated) are done in various currencies. In 1989, for example, 40% of the outstanding currency swaps were in US dollars, 23% in yen, 7% in Swiss francs, and 4% in Canadian dollars [Beidleman (1992)].

The present paper empirically investigates whether covered interest parity conditions holds when the currency swap is employed as the hedging instrument. The well known covered interest parity condition has been empirically tested for numerous currencies and with a battery of statistical and econometric techniques. Baillie and McMahon (1989) and MacDonald and Taylor (1992) among others provide an extensive review of the empirical studies on short term covered interest parity. Deviations from short term covered interest parity are often explained, for example, by political risk factors

[Aliber (1973)] or transaction costs [Clinton (1988)]. An interesting example of an empirical study taking into account institutional features is in Poitras (1988a) who investigates CIP for the US/Canadian dollar. As traders cannot borrow but can lend in the t-bill market, it can be shown that the Canadian t-bill rate is bounded above by the covered Euro-US dollar rate. Employing methods for estimating stochastic frontier functions, Poitras (1988a) provides evidence that the Canadian t-bill rate is bounded above by the Euro-US dollar rate and bounded below by the US t-bill rate.[10] The quality of the data could also lead to deviations. Taylor (1988) uses high frequency data to covered interest parity on the London Euromarket. He finds the CIP holds well for 1985 but less so in the 1970's.

Investigation of long term covered interest parity is left relatively unexplored. Poitras (1992) takes the conventional short dated CIP condition and extends the framework to accommodate a test of whether CIP holds for long dated US/Canadian dollar forward rates. Swap covered versions of CIP are tested in Fletcher and Beidleman(1992), Popper (1993), and Fletcher and Taylor (1994). The present essay parallels these recent empirical studies but we examine a more recent time frame using a daily data series.

These findings shed some light on the Feldstein and Horioka (1980) conjecture that changes in national savings rates affect rates of investment substantially and as a consequence evidence of low capital mobility. The savings-investment differential could be linked to the lack of capital mobility in long dated markets. This, however, is largely an empirical question which very few studies address as data from markets of long dated instrumetns is not readily available. Frankel (1993), however, cites the evidence from

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10 Poitras(1988b) applies the covered interest parity conditions to creating to hedge Canadian T-bill positions using US money markets.

long term swap covered interest parity to argue that long term financial capital is mobile. The findings in the present essay confirm that long term financial capital is mobile overall and especially in recent years.

2. Background on Currency Swaps

In essence a currency swap is a contract which commits two counterparties to exchange streams of interest payments in different currencies over an agreed period of time (usually several years) and, at the end of the period, to exchange the corresponding principal amounts at an exchange rate agreed at the initiation of the contract.

One of the first currency swaps took place as early as August 1976 between Bos Kalis Westminister Group NV and ICI Finance Limited. Apparently, Goldman Sachs and Continental Illinois Limited arranged for the swap contract. It was not until the swap contract between the World Bank and IBM in 1981 arranged by Salomon Brothers that currency swaps received such high publicity. The World Bank would service IBM's Swiss franc and German mark denominated debt in exchange for IBM to service the World Bank's dollar debt [Price and Henderson (1988)].

The currency swap evolved as a successor to the back-to-back and parallel loans. Parallel loans originated in the U.K. as a way of avoiding domestic exchange controls. In particular, U.K. firms wishing to purchase US dollars within the U.K. for investment abroad were required to pay a tax. In a parallel loan, the U.K. company could obtain U.S. dollar funds for its subsidiary from a US firm, and in exchange the subsidiary (in U.K.) of the U.S. firm would receive British pounds from the U.K. firm. Legally, the parallel loan, as just illustrated, involves two contracts. Hence, if one party defaults, the other party is still obligated to pay.

Back-to-back loans were developed to avoid such default problems. In the back to back loan the transaction is made between the parent companies thus this loan involves a single contract. The currency swap and back-to-back loan essentially provide the same pay-off structure but in practice, however, legal and accounting treatment of the back-to-back loan remains problematic. Although, the back-to-back loans were originally designed to be off balance sheet items, the accounting practice is ambiguous and often back-to-back loans are treated as on balance sheet items.

The currency swap got around these problems. Instead of an exchange of cash flows in themselves, the process was simplified to the exchange of *net* cash flows. Default problems are less problematic. Finally, currency swaps are treated as off balance sheet items as a matter of accounting practice as swaps are forward contingent commitments.

At the early stage of the currency swap market, the contracts were essentially made on a matching basis where an offsetting counter party was usually required. Hence, these earlier swaps were more in the form of "custom-made" currency swaps. Currency swaps which first emerged involved an exchange of principal at the initiation of the contract and at maturity, however, an initial exchange of principal is not necessary. We note that when principal is exchanged initially and at maturity, the currency swap is similar to the foreign exchange swap.

The early intermediaries in the currency swap market tried to avoid taking risk in the swap market by acting as "arrangers" of swap deals between customers. Arrangers serve as agents introducing matching counterparties to each other and then stepping aside to avoid exposure to risk. They charge fees rather than dealing in spreads. Arrangers in the early swap market were typically merchant and investment banks.

As the market grew and the customers became more diverse in their needs, it became necessary for intermediaries to act as "principals" in swap deals. Some customer required anonymity thus disclosing the credit worthiness of the customer to a potential counter party was not possible. Moreover, many end-users lacked independent credit risk analysis capability and were thus reluctant to enter into a contract with a non-bank. This encouraged the entry of commercial banks into the swap market. Banks typically 1) are capable of credit risk analysis, 2) and are preferred to non-bank names. The off balance sheet nature is also attractive to banks as there balance sheets are often impaired by other debt problems. Initially, such intermediaries limited their activity to matched book swaps. That is swaps were initiated only if a counter party or reverse swap was found. Since, in practice, finding such counter parties could take several weeks, intermediaries charged dealing spreads. As the number of banks entering the market as intermediaries increased and the demand for swaps increased, banks formed "portfolios" of swaps. In other words, a bank would enter into an agreement on one leg of the swap without a counter party but hedge its positions within the futures or bond market, for example, until a counter party is found.

By the late 1980's the swap market became relatively standardized in terms of contracts and the market more liquid. This gave rise to market makers in the currency swap market quoting prices continuously. These market makers would engage in swaps without a readily available counterparty as long as the contract involves "reasonable" amounts. Warehousing swaps and recent portfolio management techniques for swaps are key factors which have allowed the market maker to accommodate exposure to risk from unmatched swap positions.

The motive for entering into a currency swap is not always clear. To date, risk sharing and arbitrage has often been cited as motivation for (currency) swaps. In a risk sharing scenario, one agent has an unmatched cash flow in a foreign currency and thus would be willing to swap with a party with a complementary mismatch. The arbitrage argument simply stated would be when it is "cheaper" to issue debt in a foreign currency and swap back to the home currency rather than directly issuing debt in the home currency. Melnik and Plaut (1992) provide an overview of the theoretical discussion.

Then there is the market completion argument such as posited by Smith, Smithson, and Wakeman (1986) that the growth in the swap market is due to the demand of creating a "synthetic" market where a market was non-existent. As an example, they use the synthetic Swiss Treasury bill market formed from a combination of interest rate and currency swaps.

The most popularized determinant of the growth of the currency swap market is the "comparative advantage" approach. To illustrate we use a stylized example [Hull (1993)]. Imagine two companies A and B and each of these companies are offered fixed interest rates in US dollars at and British pound as displayed in the Table 1. It is clear that company A has an absolute advantage over company B. Let us assume that A wants to borrow sterling and B wants to borrow dollars. But A and B each have a relative advantage in borrowing in dollars and sterling respectively. As a consequence, there is some motivation for exchanging or "swapping" payments.

Table 1: Borrowing Rates for Company A and B

	US dollar	British pound
Company A	8.0%	11.6%
Company B	10.%	12.0%

3. Literature Review

To date, several empirical papers examine swap covered interest parity using weekly data. Firstly, Popper (1993) uses weekly data from October 3, 1985 to February. 18, 1988 for the Canadian dollar, German mark, Japanese yen, Sterling pound, and Swiss franc five and seven year currency swaps. Deviations from swap covered parity are examined by looking at the mean absolute error (MAE) and root mean square error (RMSE). MAE and RMSE for long term swap covered parity is compared with those for short term covered interest parity. Popper shows that the difference in deviations is not large enough to support the notion that long term financial capital is less mobile than short term financial capital.

When onshore market bonds were used as benchmark bonds in the parity condition, Popper (1993) finds the MAE ranges from a low of 15.12 basis points for the five year Canadian dollar swap to high of 49.75 basis points for the five year British pound swap. A band of roughly 60 basis points captures 95% of the deviations for the seven year Canadian dollar swap whereas bands as large as 160 basis points is needed to account for deviations when the five year mark currency swap is used to cover. Overall, evidence is in favor of swap covered interest parity holding.

Fletcher and Beidleman (1992) and Fletcher and Taylor (1994) also examine long term covered parity but take into account transaction costs. They look at weekly data from October 3, 1985 to March 2, 1989 for the five year maturity swap, and October 3, 1985 to Sept. 24, 1987 for seven year and finally Oct. 1 1987 to March 2, 1989 for ten year. The Canadian dollar, German mark, Japanese yen, British pound, and Swiss franc were examined against the domestic currency the US dollar.

The absolute value of the deviations from parity is regressed on a constant and transaction costs using a Tobit model in Fletcher and Beidleman (1992).

$$|error| = r_{t,t+s} - r^{sw}_{t,t+s} - r^{*}_{t,t+s} + r^{sw*}_{t,t+s} = \beta_0 + \beta_1 TC_t + \varepsilon_t \qquad (3.1)$$

where TC denotes transaction costs and β_0 is a constant. The E (ε_t) is assumed to be equal to zero. If swap covered parity holds exactly then $\beta_0 = 0$ and $\beta_1 = 1$. In other words, all deviations from parity are captured by transaction costs with the exception of some random error. A modified specifications is also examined where the lagged deviations and lagged transaction costs are included to take into account possible autocorrelation. They find that lagged deviations are statistically significant. In most cases, a single lag was sufficient to capture autocorrelation.

Fletcher and Beidleman (1992) argue that recent developments in technology such as computer-information networks and telecommunication systems has enhanced greater integration in the international market. Furthermore, the increased competition among financial intermediaries and increased number of users of such instruments has also contributed to integration and a reduction in transaction costs.

Fletcher and Taylor (1994) provide a follow-up of the analysis in Fletcher and Beidleman (1992) by simply taking the difference between the absolute deviation and transaction costs. They find the yen swap deviations cannot be accounted for by transaction costs on average for the five, seven, and ten year maturity swaps.

Finally, in a related study, Poitras (1992) using long dated forward exchange rates finds evidence against covered interest parity holding in the long dated forward exchange market. In other words, long dated foreign exchange is not priced according to covered interest rate parity. Poitras looks at the Canadian-US dollar forward rate for three, five, seven, and ten years in maturity for the period July-December 1990 using daily data. He concludes, that there is a possibility that the long dated forward rates are not priced consistently with swaps.

4. Swap Covered Interest Parity

As in the case of forward contracts, the currency swap does not incur foreign exchange risk (transaction) of holding an asset denominated in a foreign currency. The mechanics of the currency swap, however, differ from the (short dated) forward contract or the foreign exchange swap.

To illustrate long term swap covered interest parity[11], let us assume that an investor has US$1.00 which can be invested in a domestic bond or a foreign bond both with maturity T.[12] Investment in the foreign bond requires conversion of the dollar to a foreign currency at the prevailing exchange rate S_t giving the investor S units of the foreign currency. Invest S units in a foreign bond with interest rate r_t^* and cover the return with a currency swap. A currency swap typically involves a stream of payments in one currency to be exchanged for a stream of payments in another currency (net flows are exchanged) over periods of several years. Thus by entering into a swap agreement, the investor pays the foreign currency swap fixed rate r_t^{sw*} and receives the dollar swap fixed rate r_t^{sw} . At maturity, the foreign currency return is converted to dollars at the initial

11 In this essay long term covered interest parity is achieved by using the currency swap as the covering instrument. Thus, we will refer to the long term covered interest parity condition as swap covered interest parity. This phrase is used in Popper (1993) and Fletcher and Taylor (1994).

12 We illustrate by assuming an investment scenario as opposed to a borrowing scenario to be consistent with the convention in discussing short term covered interest parity.

exchange rate, S_t, thus the covered return is $[S(r_t^* - r_t^{sw*})\frac{1}{S} + r_t^{sw}] = [r_t^* + r_t^{sw} - r_t^{sw*}]$. For the parity condition to hold equate the covered return with the return on the domestic investment, r_t. The swap covered interest parity condition is given as

$$r_t = r_t^* + r_t^{sw} - r_t^{sw*} \quad (4.1)$$

where * indicates foreign and no mark indicates domestic currency interest rate, and a sw superscript denotes that it is a swap rate. As equation (4.1) is not expected to hold perfectly, we can rewrite equation (4.1) as

$$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}] \quad (4.2)$$

The deviation term in equation (4.2) is calculated and statistically examined. If long term covered interest parity holds on average, then the deviation term should not be statistically different from zero. Statistical analysis of the deviations stated in equation (4.2) do not take into account transaction costs. If transaction costs explain the deviations, they should provide an upper bound for the absolute deviation. In other words, $|Deviation| < TC$ or,

$$|deviation| - TC < 0 \quad (4.3)$$

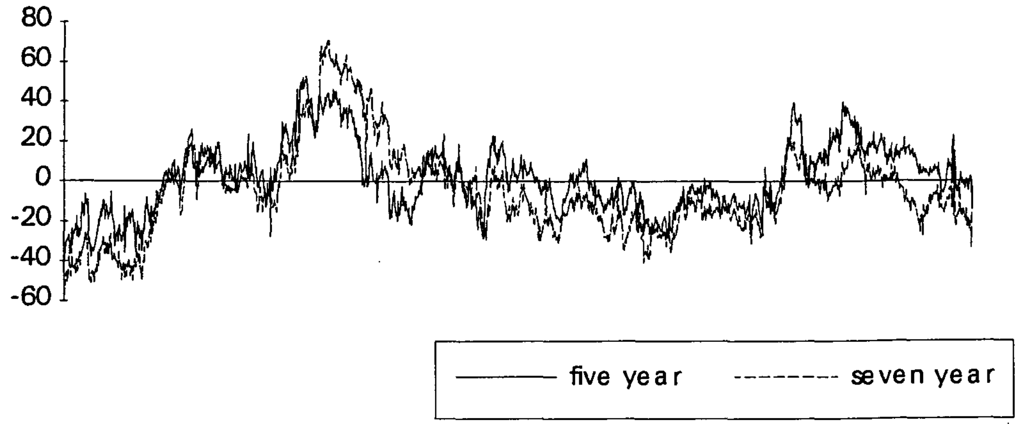
where TC denotes transaction costs. In this case, if long term covered interest parity holds, then we should not expect to get positive |deviation|-TC on average.

5. Empirical Results

The deviation as stated in equation (4.2) is measured using daily data from February 25, 1988 to February 22, 1994. The foreign currency swap rate and benchmark bond yield data[13] for the British pound, German mark, and Japanese yen as well as the US dollar swap rate at five and seven year maturities were kindly provided by J.P. Morgan Securities (Tokyo). The swap rates used in this study are over the counter rates for each trading day. The currency swap is conventionally quoted as a fixed non-dollar payments against floating dollar payments (LIBOR). Thus, to construct currency swap fixed payments, the floating dollar payments must be converted to fixed dollar payments by combining the currency swap with a US dollar interest rate swap. The currency swap is quoted as the non-US dollar fixed interest rate in exchange for floating LIBOR. The interest rate swap in contrast exchanges floating LIBOR for fixed rate over a US Treasury bond yield. Combining the

13 As the bond yields are benchmark rates, they do not necessarily reflect actual five year bond rate on a particular trading day. For example, a ten year bond with approximately five years left to maturity could be used as a five year benchmark rate.

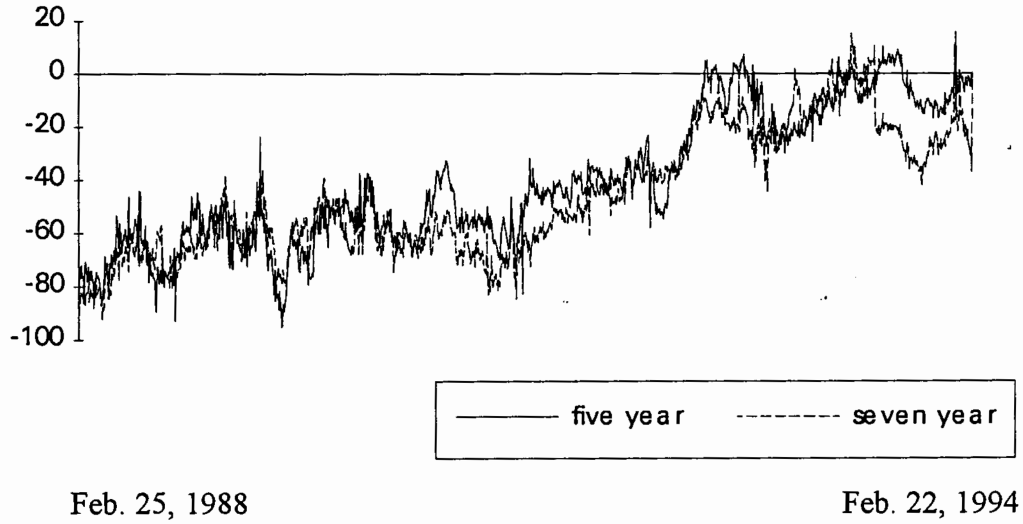
Figure 1: Deviations From Parity for the Sterling Pound Swap



Feb. 25, 1988 Feb. 22, 1994

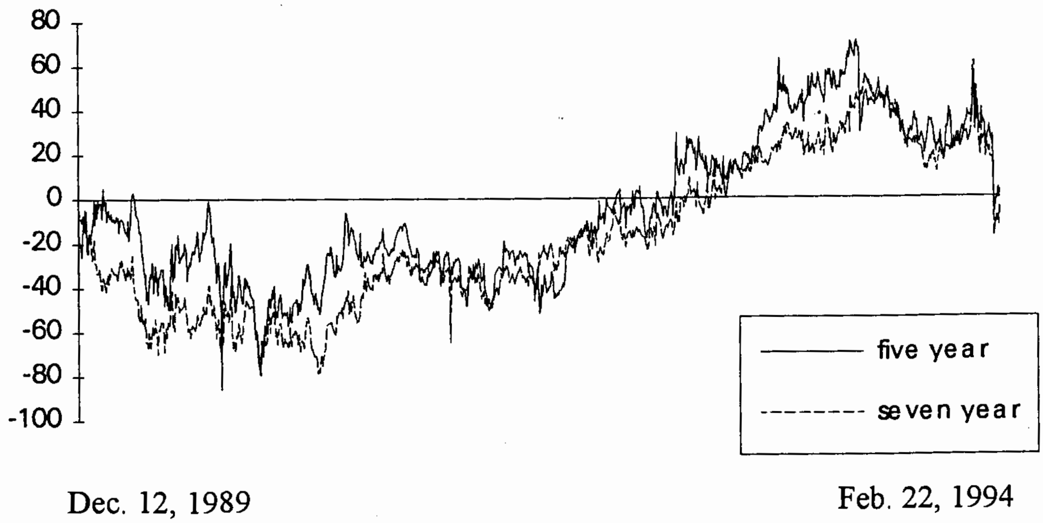
$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$. Deviation is in basis points.

Figure 2: Deviations From Parity for the Mark Swap



$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$. Deviation is in basis points.

Figure 3: Deviations From Parity For the Yen Swap



$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$. Deviation is in basis points.

Table 2: Deviations From Long Term Covered Interest Parity: February25, 1988 - February 22, 1994

$$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$$

	Sterling 5 year	Sterling 7 year	Mark 5 year	Mark 7 year	Yen 5 year	Yen 7 year
Mean (x100)	2.20	-4.9	-40.66	-45.69	-5.18	-17.39
Standard Deviation	0.1746	0.2272	0.2512	0.2295	0.3213	0.3327
Skewness	0.3640	0.8172	0.2803	0.3920	0.3890	0.3124
Kurtosis	-0.8470	1.1732	-1.0466	-0.8980	-0.8966	-1.0707
Maximum	53.00	71.10	10.5	15.30	70.80	55.20
Minimum	-40.10	-57.00	-95.00	-92.10	-85.60	-78.30
MAE (x100)	13.88 (0.1997))	17.79 (0.2346)	40.66 (0.0947)	45.91 (0.0414)	28.46 (0.0709)	33.26 (0.0559)
RMSE (x100)	3.09	5.40	22.44	26.14	10.58	14.08
Number Positive Deviations	777	539	98	34	391	337
Number Observation	1465	1465	1468	1468	1053	1053

For the yen, the sample period is from December 12, 1989- February 22, 1994. The p-value for t-statistic for the MAE is in parenthesis; the p-values are doubled to take into account absolute deviation.

currency swap and interest rate swap will yield a fixed-fixed swap since the LIBOR will cancel.

Summary statistics of the deviations from covered swap parity are displayed in Table D. The mean deviations are not statistically different from zero for the sterling and yen, but statistically different from zero for the mark. With the exception of the 5 year sterling, the deviations were negative on average indicating that the covered route was higher than the domestic US rate. As a consequence it is cheaper to borrow in the U.S. directly. The number of negative deviations was also large thus possibly giving rise to negative average values. Again the only exception was the 5 year sterling. For the remaining swaps, the number of positive deviations ranged from a low of 2.3% for 7 year mark to a high of 53.0% for the 7 year sterling. This indicates that deviations were predominately negative.

The large number of negative deviations as well as large positive deviations could serve to cancel extreme movements in the error to provide a mean error close to zero. If the deviations were large but averaged close to zero, then we cannot conclude that long term covered interest parity holds on average. Swings in the error term can be judged by the minimum and maximum deviations. The largest positive deviation was 70.80 basis points for the 5 year yen swap and the largest negative deviation was from the 5 year mark swap at -95.00 basis points. As these are obviously large deviations, we circumvent the potential canceling effect by using the mean absolute error

$$MAE_i = \frac{1}{n}\sum_{j=1}^{n} |error_i| \qquad (5.1)$$

Table 3: Deviations From Long Term Covered Interest Parity: February 25. 1988 - December 31, 1990

$$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$$

	Sterling 5 year	Sterling 7 year	Mark 5 year	Mark 7 year	Yen 5 year	Yen 7 year
Mean (x100)	3.01	23.80	-60.57	-63.37	-30.20	-49.66
Standard Deviation	0.2062	0.2912	0.1172	0.1001	0.1805	0.1490
Skewness	0.2837	0.1822	-0.2267	-0.1212	-0.3651	0.0237
Kurtosis	-0.4697	-0.3397	-0.0729	-0.0827	0.4853	-0.4546
Maximum	53.00	71.10	-23.60	-36.10	4.90	-5.20
Minimum	-40.10	-57.00	-95.00	-92.10	-85.60	-76.60
MAE (x100)	16.63 (0.1849)	22.15 (0.2449)	60.57 (0.0000)	63.37 (0.0000)	30.29 (0.0906)	49.66 (0.0009)
RMSE (x100)	4.03	8.52	38.06	41.16	12.37	26.87
Number Positive Deviation	384	379	0	0	4	0
Number Observation	676	676	677	677	262	262

For the yen, the sample period is from December 12, 1989- December 31, 1990. The p-value for t-statistic for the MAE is in parenthesis; the p-values are doubled to take into account absolute deviation.

Table 4: Deviations From Long Term Covered Interest Parity: January 3, 1991- February 22, 1994

$$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$$

	Sterling 5 year	Sterling 7 year	Mark 5 year	Mark 7 year	Yen 5 year	Yen 7 year
Mean (x100)	1.52	-11.20	-22.69	-30.55	3.11	-6.70
Standard Deviation	0.1419	0.1212	0.1980	0.1987	0.3147	0.3067
Skewness	0.3544	0.3730	-0.3675	-0.4108	-0.4736	-0.1414
Kurtosis	-0.3660	-0.2663	-0.9362	-0.4015	-0.4644	-1.1207
Maximum	40.00	21.30	10.5	15.30	70.80	55.20
Minimum	-27.30	-40.60	-75.80	-84.60	-51.60	-78.30
MAE (x100)	11.53 (0.1706)	14.07 (0.1032)	23.61 (0.2069)	30.97 (0.1071)	27.85 (0.0625)	27.83 (0.0553)
RMSE (x100)	2.03	2.72	9.06	13.28	9.99	9.84
Number Positive Deviation	393	160	98	34	387	337
Number Observation	789	789	791	791	791	791

The p-value for t-statistic for the MAE is in parenthesis; the p-values are doubled to take into account absolute deviation.

for the ith currency swap and observations j=1,....., n. The MAE is roughly consistent in magnitude with the Popper (1992) study except for the sterling. It appears the sterling swap follows long term covered interest parity more closely in recent years. The sterling 5 and 7 year swaps had an MAE of 49.5 and 46.58 basis points in the Popper study which looked at data from November 1985 to February 1988. As the data series in the present study begins in February 1988, the present study provides a follow-up of the Popper analysis. The mean absolute error for the deviations (MAE) is not significantly different from zero at the 1% significance level as well. [14] At the 10% significance level, however, the MAE for the mark and yen swaps is statistically different from zero. Thus, we conclude that long term covered interest parity holds on average for all the swaps examined but only marginally for the mark and yen swaps.

The skewness and kurtosis statistics provide some evidence that the deviations are non-normal. We reject the null of normality for both the skewness and excess kurtosis at the one percent significance level.

Since the data series covers roughly six years, we divided the sample into two subperiods to look for structural differences. The first subperiod begins February 1988 and ends December 1990. The yen data set is slightly shorter due to data limitations. A second subperiod was also analyzed ranging from January 1991 to February 1994. We find major differences in the deviations from the subperiods making any general conclusions from Table 1 difficult.

All of the deviations were negative for the mark and seven year yen swaps in the first period. Moreover, we find that the mean and mean absolute deviations are

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14 The p-value for the t-statistic is doubled to account for the fact we using the absolute value of the deviation [Popper (1993)].

statistically different from zero for the German mark swaps and seven year Japanese yen swap. These yen swap results are consistent with Fletcher and Taylor (1994) who find that for the five, seven, and ten year yen swaps, the deviations are in excess of transaction costs on average. Our findings for the mark swap although large could be partially accounted for by transaction costs. For the remaining currency swap markets, we find that swap covered interest parity holds under the MAE criterion (1% significance level).

The statistics for the five year sterling is consistent across subperiods and with the entire period. The mean deviation is positive but not significantly from zero in both periods. The number of positive and negative deviations is also evenly spread. Fluctuation in the deviations, however, has diminished in recent years as the standard deviation falls from 0.2062 to 0.1419. Also, the range between the minimum and maximum deviation decreases from roughly 90 basis points in the first subperiod to about 67 basis points in the second subperiod. This could be evidence that swap covered interest parity holds for the five year sterling and the margin of error is diminishing in recent years.

A pattern begins to emerge when we look at the seven year sterling,. The mean error moves from being positive to negative in the second subperiod. Although the mean deviation is not statistically different from zero, it appears there is a trend. 56% of the deviations were positive in the first period but this declined to 20.2% in the second period. This pattern in reverse form is marked for the mark and yen swaps.

The mean deviations are negative and statistically significant from zero for the yen and mark in the first subperiod. Moreover, none of the deviations were positive for the mark swaps and the 7 year yen swap, and only four deviations were positive for the 5 year yen. For the yen, the trend reverses in the second subperiod where we have an increased number of positive deviations and positive mean errors. The mean error remains negative for the mark but all positive deviations observed in this data set for the mark come from the second subperiod. As these deviation appear persistent over a period of at least one year, this could be evidence against swap covered interest parity. The deviations, however, must be greater than transaction costs to indicate the possibility of arbitrage opportunities.

Interestingly, in the second subperiod which covers the early 1990's, the mean and mean absolute deviations from parity are not significantly different from zero at the 1% significance level (Table 3). The p-values for the yen swap MAE are both under 0.1 and could be interpreted as being marginally significant, but these deviations could be accounted for by transaction costs.

Furthermore, the root mean square error (RMSE) declines from the first sub period to the second period providing evidence the error of margin from long term covered interest parity is declining. Thus we note swap covered parity holds on average especially in recent years due to deregulation and increased arbitrage activity.

The degree to which deviations of the same sign persist is also of import. The autocorrelation coefficients for lags of one, two, three, and six were calculated and displayed in Table 4. As the coefficients are positive and statistically different from zero, this provides evidence of persistence in the deviations. This, does not necessarily imply that arbitrage opportunities exist as Popper also notes that the autocorrelation could be due correlation of transaction costs or reflect political risk factors.

A graphical depiction of the deviations is presented in Figures 1, 2, and 3. A casual visual inspection reveals the patterns described above. The deviations are large and persistent especially for the mark and 7 year yen swaps. Transaction costs of 30 or 35 basis points would not be able to explain the deviations displayed in Figures 2 and 3 for the German mark and Japanese yen swaps, especially during the first subperiod.

Table 5: Autocorrelation of Deviations

	Sterling 5 year	Sterling 7 year	Mark 5 year	Mark 7 year	Yen 5 year	Yen 7 year
Feb. 1988 - Feb. 1994						
ρ_1	0.96	0.98	0.98	0.98	0.98	0.99
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
ρ_2	0.94	0.97	0.97	0.97	0.98	0.98
	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)
ρ_3	0.93	0.96	0.97	0.97	0.97	0.98
	(0.05)	(0.05)	(0.06)	(0.06)	(0.07)	(0.07)
ρ_6	0.88	0.93	095	0.95	0.95	0.97
	(0.08)	(0.08)	(0.08)	(0.08)	(0.10)	(0.10)
Subsample: Feb. 1988 - Dec 1990						
ρ_1	0.96	0.98	0.90	0.91	0.93	0.93
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
ρ_2	0.94	0.97	0.86	0.86	0.88	0.90
	(0.06)	(0.07)	(0.06)	(0.06)	(0.07)	(0.07)
ρ_3	0.93	0.96	0.84	0.82	0.84	0.87
	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)
ρ_6	0.88	0.93	0.78	0.72	0.75	0.80
	(0.12)	(0.12)	(0.11)	(0.11)	(0.13)	(0.13)
Subsample Jan. 1991 - Feb. 1994						
ρ_1	0.96	0.95	0.97	0.97	0.98	0.98
	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)
ρ_2	0.94	0.92	0.96	0.96	0.96	0.97
	(0.06)	(0.06)	(0.06)	(0.06)	(0.07)	(0.07)
ρ_3	0.92	0.90	0.96	0.95	0.95	0.96
	(0.07)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)
ρ_6	0.88	0.83	0.93	0.92	0.92	0.93
	(0.11)	(0.11)	(0.11)	(0.11)	(0.14)	(0.14)

For the yen, the sample begins from December 12, 1989. ρ_k denotes the sample autocorrelation for lag k. Standard error is in the parenthesis.

Table 6: Correlation Matrix of Deviations From Long Term Covered Interest Parity

	J5	J7	D5	D7	S5
J7	0.9328	1.0			
D5	0.7902	0.8432	1.0		
D7	0.7921	0.8456	0.8922	1.0	
S5	0.3971	0.3136	0.1409	0.1496	1.0
S7	0.0572	-0.0726	-0.2460	-0.1871	0.7110

J denote the Japanese yen swap, D denotes the German mark swap, and S denotes the British pound swap. The adjacent number 5 and 7 denote the maturity in years.

The relationship between the deviations is also of some interest. We calculate the correlation coefficient to discern any patterns. The swaps in the same currency but different maturity are highly correlated as expected. What is striking is the degree to which the yen and mark deviations are strongly correlated. In contrast, the sterling deviations have relatively little correlation with the mark or yen errors.

The yen error appeared to exhibit a market change over the sample period examined. The p-values for the MAE in the second period remain below 0.1 as well as for the entire sample period. We next seek to investigate the degree to which transaction costs can explain such deviations. Our analysis of transaction costs is limited to the yen currency swap due to data limitations. The transaction cost variable is constructed by adding the bid-ask spread for the interest rate swap and yen currency swap rate. As bid-ask spreads for the bond quotes were not available we are limited to the spreads for the swap quotes. This, however, should not necessarily be a major drawback as the "spread component in the securities market (benchmark bonds) therefore cancels and the only remaining transaction costs are those of the swap" [Fletcher and Taylor (1994), p.465].

The results are displayed in Table 7. The mean for the "disequilibrium" deviations is positive for the five and seven year swap. This indicates that the transaction costs as reflected in the bid-ask spreads does not fully explain the deviations from parity. Both the mean and absolute value of the mean is smaller than the mean and MAE deviations exhibited in Tables 2-4. Thus, TC partially accounts for the deviations. What is striking is that the number of observations which violate the transaction cost bounds. The percentage of violations was slightly larger for the five year swap. For the five year swap 92.1%, 89.7%, and 92.9% of the observations violated the transaction cost bound for the entire period, the first subperiod, and second period respectively. In the case of

Table 7: Deviations From Transaction Costs: The yen swap covered interest parity deviations.

$Deviation = r_t - [r_t^* + r_t^{sw} - r_t^{sw*}]$. $Mean = \frac{1}{T}\sum_{i=1}^{T} |deviation|_i - TC_i$

where T is the sample size. Excess mean takes average of all observations for which $|deviation| - TC > 0$ and observations in excess of TC is the number of observations which satisfy $|deviation| - TC > 0$.

	Dec. 1989- Feb. 1994	Dec. 1989 Dec. 1990	Jan. 1991 Feb. 1994
Mean 5 yr.	22.80	22.70	22.84
	16.93	21.35	15.21
|Mean| 5 yr	23.65	24.77	23.28
	15.72	18.89	14.52
Excess Mean 5 yr	25.21	26.46	24.81
	14.40	15.85	13.89
Observations in Excess of TC	970	235	735
Mean 7 yr	25.69	39.82	21.01
	16.59	15.03	14.29
|Mean| 7 yr	26.11	40.00	21.51
	15.93	14.55	13.52
Excess Mean 7yr	28.51	42.92	29.31
	15.23	15.85	16.99
Observations in Excess of TC	909	229	680

the seven year swap, the percentage of violations was 86.3%, 87.4%, and 85.9% for entire period, the first subperiod, and second period respectively.

6. Discussion and Conclusion

The present essay empirically investigated the long term swap covered interest parity condition. This is one of the few studies to examine whether parity conditions hold in long dated markets. Popper (1993), Fletcher and Beidleman (1992), and Fletcher and Taylor (1994) conduct such a study for currency swaps and provide the framework on which this study is largely based on. We, however, use daily as opposed to weekly data over a longer and more recent time period.

The findings in this study are summarized below:

1) The five and seven year sterling rate seems to be consistent with long term swap covered parity based on MAE and RMSE criteria.

2) The deviations for the yen and mark tend to be negative and large on average during the first subperiod suggesting violation of long term covered interest parity. In the second period, however, deviations become smaller on average. Thus, the margin of error and any potential arbitrage opportunities diminished in recent years.

3) Although the average deviations diminish in magnitude, the yen deviations are still statistically different from zero at least marginally. Transaction costs as defined as the sum of bid-ask spread quotes for the swap rates do not explain the deviations for the yen.

Thus, we do *not* find conclusive evidence that long term swap covered interest parity is maintained over the past six years especially for the mark and yen swaps. These results and their implications stand in contrast to the conclusion in Popper (1993) and Fletcher and Beidleman (1992) that long term financial capital is "mobile" and that capital markets are integrated for long dated assets. Fletcher and Taylor (1994), however, find that deviations from interest parity are not rare but have diminished over time. This consistent with our findings using daily data. Moreover, they find that transactions costs do not account for the deviations on average for the yen swap as well. They investigate yen swaps at five, seven, and ten year maturity.

The Tokyo offshore money market was established in January 1987 and the deregulation of the inter-bank market in November 1988 increased arbitrage activity between the inter-bank and open money market. Thus prior to such deregulation we could conjecture that Samurai bonds were viewed as more costly and thus required a premium. This could possibly account for the deviations observed in the data set examined by Fletcher and Taylor (1994) but the data set investigated in this essay begins in December 1989 which is a more than a year since the inter-bank went under deregulation.

The lack of liquidity could be the major reason for the deviations in the yen swaps. Currency swaps were slow to develop in Japan since two the fundamental prerequisite financial characteristics, low interest rates and high liquidity, were absent form the Japanese market [Malecka (1992)]. Japanese financial markets were highly regulated. In practice only the ten year Japanese Government Bond (JGB) market is liquid. Hence, initially the benchmark yield curve was hybrid of various long term rates including Eurobonds and fixed rate domestic lending rates. Swap rates were also very sensitive to demand and supply factors. This combined to yield relatively volatile swap rates for the yen. Managing portfolios was also difficult for yen related swaps. Hedging new swaps usually required an offsetting or reverse swap since other hedging instruments were virtually non-existent. This also contributed to the volatility of the swap rate. Meeting the capital adequacy requirements set by the Bank for International Settlements places pressure on banks to achieve higher returns. The dramatic decline in the Japanese stock market puts further pressure on the banks. The sample period examined in this essay coincides with drop in Japanese stock prices and could thus partially account for the observed deviations.

Essay Three:
Cointegration and Long Dated Forward Rates

1. Introduction

Financial time series are often documented as possessing a stochastic trend or equivalently a unit root in the levels. One well documented financial variable which is also the focus of this paper is the foreign exchange rate. Evidence of a unit root in the spot rates is documented by Meese and Singleton (1982) among others. A question of interest that arises is whether various rates ***share*** a common stochastic trend.[15] We examine whether there is a common stochastic trend driving a pair of spot and forward exchange rates in the context of the US/Canadian dollar rate. A common stochastic trend would imply that the spot and forward markets are linked. Within the context of the model presented in this paper this would also mean that the interest rate agio is stationary and as a consequence a common stochastic trend should drive the interest rates in the US and Canada. This would provide indirect evidence of international market integration.

The FX forward market has a relatively long history. Grabbe (1993) notes that FX forwards were traded as early as the 1880's on the Vienna stock exchange for the German mark at one, three, four, and six month maturities. Markets for the mark also existed in Berlin and St. Petersburg. Markets for the British pound and French franc shortly followed in Berlin and Vienna in the 1890's. Today FX forward contracts are

15 The common trends interpretation of cointegration is attributed to Stock and Watson (1988). Common stochastic trends are found for various time series in international finance. Stock indices from various countries are cointegrated as evidenced by Kasa (1992), Chan, Gup, and Pan (1992), Jeon and Chiang (1991), Taylor and Tonks (1989). Also find cointegration for various spot rates McDermott (1990), Copeland (1991), and Sephton and Larsen (1992). Within the US -Canadian context cointegration is found in long run relationships such as purchasing power parity [Johnson (1990)] and the currency substitution effect [Wong and Kennedy (1992)].

important in hedging transaction foreign exchange risk. As the earlier markets, the inter-bank market today usually deals with forwards with maturity of under a year. The present paper looks at long dated forward contracts which extend to maturity lengths of ten years for the US-Canadian dollar rate.[16]

The present paper examines what is often referred to as "outright" forward exchange rates, i.e., forward exchange rates which are quoted without reference to the underlying spot exchange rate. These trades do not involve a spot transaction such as that between banks. The contract in this case locks in a rate at which the client will purchase (or sell) a currency at a specified future date. Reference to forward rates in this paper should be taken to mean the "outright" forward exchange rate. Forward contracts are offered in the inter-bank traded often for standard quantity and maturity but these can be negotiated. We examine the "standard" maturity contracts ranging from one month to five years in maturity. As a secondary market for forward contracts does not exist, reversing a position would require entering another forward contract with the same bank to cancel the original position. However, an important feature of the forward contract is that they have a very high delivery rate. Forwards also do not have daily price limits. Settlement is made by depositing the currency in question in the account of the counter party. Note, however, if the domestic bank is not open then delivery cannot take place.

Unlike previous studies investigating spot and forward rates, we examine a spectrum of forward rates ranging from one month to five years in maturity. Furthermore, a modeling framework is developed which formalizes the notion that forward and spot rates could share a common stochastic trend. The spot exchange rate is

16 A related financial instrument is the dual currency bond. The dual currency bond involves purchasing a bond in one currency and receiving coupon payments in that same currency, but at maturity the principal is paid in a second currency. Thus, the dual currency bond can be viewed as combination of a bond and a forward contract.

assumed to follow a log normal diffusion process. Integrating and substituting the covered interest parity condition into the solution yields a spot and forward rate relationship which is consistent with the concept of cointegration. Given this framework, we document whether the realized spot rate and corresponding forward rate share a common stochastic trend; in other words, we test for stationarity of the forecast error. This is *not* a test of the unbiased expectations hypothesis which requires a stricter set of restrictions. We look for a "weaker" form of equilibrium which only requires that the spot and forward rates do not drift apart from each other. In fact we find the forecast error to be stationary and hence the forward and spot rates to share a common stochastic trend for shorter term maturities. This relationship, however, does not hold for the longer three and five year forward rates. As the long dated forward rates do not seem to share a common stochastic trend with the spot rate we make a preliminary investigation into the possibility that the forward rates themselves share a common stochastic trend. The long dated forward rates appear to share a common stochastic trend.

Firstly we motivate the possibility of cointegrated spot and forward rates in section 2. A review of cointegration and its implications for testing the unbiased expectations hypothesis is presented in section 3 and followed in section 4 by a discussion of unit roots and cointegration tests. A discussion of the multivariate testing procedure used in a preliminary investigation of the term structure of forward rates is given in section 5. The empirical results are covered in section 6 and the paper concludes with section 7.

2. Cointegration of Forward and Spot Exchange Rates

Assume the short term covered interest parity (CIP) relationship holds for all maturities of forward contracts examined.

$$F_t(t+1) = [(1+r_{t,t+1})/(1+r^*_{t,t+1})]S_t \qquad (2.1)$$

where $F_t(t+1)$ is the forward rate at t with maturity at t+1, $r_{t,t+1}$ is the domestic (zero coupon) annualized interest rate over the period t to t+1, $r^*_{t,t+1}$ the foreign (zero coupon) annualized interest rate, and S_t the spot exchange rate (domestic/foreign units) at t. This simply states that an investment of X units in the home currency will yield $X(1+r_{t,t+1})$ and this should be equivalent to investing the same X units abroad at rate $r^*_{t,t+1}$ and covering the foreign investment with a forward contract.

Let us assume that the spot exchange rate follows the log normal process of the form

$$dS = mSdt + \sigma Sdz \qquad (2.2)$$

where m is the mean, σ the volatility measure (standard deviation), and dz the standard (change) wiener process. The solution to the stochastic differential equation is given as

$$S_t = S_{t-1} \exp\{(m - ½\sigma) + \sigma(z_t - z_{t-1})\} \qquad (2.3)$$

Taking logs gives a linear form of

$$\ln S_t - \ln S_{t-1} = m - \tfrac{1}{2}\sigma + \sigma(z_t - z_{t-1}) \qquad (2.4)$$

where $z_t \sim N(0,\sigma^2)$, and m and σ are constant. We note that the above relationship indicates that the natural log of spot exchange rate time series (in levels) exhibits a unit root. By definition the exchange rates series contains a unit root if the difference of the series is represented by a deterministic mean, $m-\tfrac{1}{2}\sigma$ and stationary moving average process.[17] Equation (2.4) in conjunction with the equilibrium CIP relationship (2.1) yields the forward - spot relationship that is often tested using cointegration. To see this, substituting the CIP relationship (2.1) for $\ln S_{t-1}$ in equation (2.4) gives,

$$\ln S_t - \ln F_{t-1}(t) = m - \tfrac{1}{2}\sigma + \sigma(z_t - z_{t-1}) - \ln R_{t-1,t} \qquad (2.5)$$

If the interest rate ratio, $\ln R_{t-1,t}$, is stationary then the forward and spot rate should be cointegrated with a cointegrating vector of (1,-1). The same relationship should hold theoretically for forward contracts with different maturities. The above relationship is a "no arbitrage" condition or an exogoneously determined equilibrium relationship and is not necessarily a behavioral relationship. The only explicit behavioral assumption made is that

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17 By taking the natural log of the variables, the raw variables are assumed to be log-normally distributed. As a result, the spot and forward rates do not have any probability of becoming negative. The possible effects of currency bands or target zones [Krugman (1991)] on the spot or forward rates is not explicitly considered.

of the diffusion process for the spot exchange rate. Given the above analysis, it follows that the various forecast errors between the forward and future spot rate could be cointegrated. Brenner and Kroner (1995) motivate cointegration of financial variables in the same manner but provide discussion in greater depth.[18]

Crucial to this approach, however, is the assumption of stationary interest rate differentials; this, however, is largely an empirical question. It is plausible that a single common factor could drive the interest rates in two different countries. In the case of the US and Canada this is more likely at an intuitive level due to greater economic interaction between the two countries.

A term structure cointegration framework for interest rates is developed in Engle and Granger (1987), Campbell and Shiller (1987), Hall, Anderson and Granger (1992), Bradley and Lumpkin (1992), Zhang (1993), Wallace and Turner (1993), and Engsted and Tanggaard (1994). The evidence, to date, indicates that interest rates are cointegrated.[19]

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18 The framework in this paper and that in Brenner and Kroner (1995) were developed independently. Brenner and Kroner, however, provides further extensions and greater in-depth discussion of the implications.

19 The earlier studies focused on two rates: a long rate and a short rate. Engle and Granger, and Campbell and Shiller used a month T-bill rate as the short rate and a 20 year bond yield as the long rate. Both studies find evidence of cointegration. Instead of focusing on pairs of interest rates, some studies look at a range of rates to capture movements in the term structure. Hall et al employ the CRSP file of monthly rates ranging from one month to twelve months from 1970 to 1988. Both augmented Dickey Fuller and Johansen multivariate tests are applied to the spreads.

Although Hall et al look at several rates, they neglect to include long dated yields for bond/notes over one year in maturity. Bradley and Lumpkin (1992) look at T-bill and T-notes ranging from three months to thirty years using Engle-Granger cointegrating regressions with ADF test statistics. They find cointegration and then estimate an error correction model in order to forecast rates. Engsted and Tanggard (1994) also look at a spectrum of rates ranging from one month to ten years. Johansen test statistics are used. Finally, Zhang (1993) also examines probably the widest range of rates: one month T-bill to bonds with maturity of 30 years. Zhang employs the Johansen method and finds three

In particular, Boothe (1991) finds that Canadian and US interest rates are cointegrated for the period 1972 to 1989 using monthly data. Boothe looks at both a constant maturity data set from the Bank of Canada and US Federal Reserve publications which begins in 1972 and ends in 1989, and the Boothe-Glassman (1988) data set which spans from 1972 to 1988. The Boothe-Glassman set included three month T-bills, short term bonds (1-3 years), medium term bonds (10-13 years), and long term bonds (14 years and greater). The constant maturity set included three month T-bills, two, ten, and twenty year bond rates. They find strong evidence in favor of a common stochastic trend between US and Canadian rates with similar maturity using both data sets. The augmented Dickey-Fuller test was used. For the constant maturity data set four separate bivariate cointegrating regressions for the US and Canadian t-bill, short term bonds, medium term bonds, and long term bonds were estimated. In essence, they provide evidence that the interest rate differentials are stationary.

In a related study, Bosner-Neal and Roley (1994) examine covered interest parity for Japanese yen and U.S. dollar using cointegration. They find that the covered Eurodollar rate is cointegrated with the Japanese CD rates since June 1984 and that prior to 1984 the gensaki rate is cointegrated with the covered Eurodollar rate.

To summarize, if covered interest parity (2.1) holds and equation (2.2) is the correct underlying diffusion process of the spot exchange rate then the realized spot rate and corresponding forward rate should be cointegrated given a stationary interest rate agio. In the case the interest rate differential is stationary as documented by Boothe (1991), the spot and forward rates, $\ln S_t$ and $\ln F_{t-1}(t)$, should be cointegrated with cointegrating vector (1, -1). It follows that if the interest rate agio is non-stationary, then

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common stochastic trends for the entire spectrum of rates. Moreover, the three factors are interpreted as the shift, slope factor, and curvature factor.

the spot and forward rate will not be cointegrated, but would require the interest rates to form a trivariate cointegrated system.

3. Unbiased Expectations and Cointegration

Although this study focuses on documenting a common stochastic trend, evidence of cointegration does have implications for testing the unbiased expectations hypothesis. We should emphasize that testing for cointegration is not a test of the unbiased expectations hypothesis and necessarily the objective of this essay. Since, testing for a common stochastic trend is closely linked to such tests we discuss the implications of non-stationarity in this section.

The unbiased expectations hypothesis in its simplest form states that the forward rate is the best and an unbiased predictor of the future spot rate (the future being the maturity date of the forward contract). In testing the hypothesis, however, the realized spot rate is often regressed against the corresponding forward rate. As noted by Barnhart and Szakmary (1991) among others, the two most employed regression models are

$$\ln S_{t+1} = a + b\ln F_t(t+1) + e_{t+1} \quad (3.1)$$

$$\ln S_{t+1} - \ln S_t = a + b(\ln F_t(t+1) - \ln S_t) + e_{t+1} \quad (3.2)$$

where (3.1) is referred to as the level regression model and (3.2) the percentage change form. Longworth (1981) among others have used specifications (3.1) and (3.2) in empirically investigating the unbiased expectations in the context of the Canadian-US dollar exchange rate.

Baillie and McMahon (1989) and MacDonald and Taylor (1992) provide an excellent review of the testing literature including the various econometric specifications and methods used. We, however, are interested in the literature which employs cointegration techniques.

In either model (3.1) or (3.2) cointegration of the spot and forward rates would have implications for statistical inference. Corbae, Lim, and Ouliaris (1992), Baillie and Bollerslev (1989), Hakkio and Rush (1989), Barnhart and Szakmary (1991), Copeland (1991), Jung and Wieland (1990), Lai and Lai (1991), and Tronzano (1992) investigate the implications of unit roots and cointegration on the econometric specification of models used to test the unbiased expectations hypothesis.

In (3.1), cointegration would directly affect inference of b . If the spot and forward rate are not cointegrated, then (3.1) is referred to as a spurious regression (Granger and Newbold (1984)). Phillips (1986) shows that if the spot and forward rates are not cointegrated then the distribution for the conventional t-statistic for b (=0) diverges as the sample size approaches infinity. This implies that critical values for the statistic will not exist. As the change in the spot rate is most likely stationary the regression model (3.2) would require the forward premium to be stationary, i.e., the contemporaneous spot and forward rates need to be cointegrated.

The unbiased expectations hypothesis requires that a=0 and b=1 for both models which is a stricter set of restrictions than simply finding cointegration of the forecast error. To illustrate we rearrange equation (2.5) to yield,

$$\ln S_t = m - \tfrac{1}{2}\sigma + \ln R_T + \ln F_{t-1}(T) + \sigma (z_t - z_{t-1}) \qquad (3.3)$$

which resembles the specification of the level regression model (3.1). $m - \tfrac{1}{2}\sigma + \ln R_T$ is the constant term, a, given the interest rate agio is stationary. The unbiased expectations hypothesis requires that $a = m - \tfrac{1}{2}\sigma + \ln R_T = 0$ [Brenner and Kroner (1995)]. A summary of the cointegration studies to date is displayed in Table 1. Various exchange rates in differing time periods and data intervals were examined. The majority of these papers, however, focus on the one month forward rate with the exception of which investigates the three month contract. The present study examines forward rates for various maturities including long dated rates of up to five years.

Table 1: Summary of Studies which Cointegrate Realized Spot and Forward Rates

Study	Currency			Sample Period	Frequency of Data	Forward Maturity
Corbae, Lim, and Ouliaris (1992)	BP FF	CD JY	DM SF	Jan. 1976- Jan. 1985	Weekly	one month
Baillie and Bollerslev (1989)	BP FF SF	CD IL	DM JY	Mar. 1980- Jan. 1985	Daily	one month
Barnhart and Szakmary (1991)	BP	DM	FF	Jan. 1974 - Nov. 1988	Monthly	one month
Copeland (1991)	BP JY	DM SF	FF	1976-1990	Daily	one month
Hakkio and Rush (1989)	BP	DM		July 1975- Oct. 1986	Monthly	one month
Jung and Wieland* (1992)	BP IL	DG USD	FF SF	1979-1989	Monthly Quarterly	three month
Lai and Lai (1991)	CD SF	BP JY	DM	July 1973- Dec. 1989	Monthly	one month
Tronzano (1992)	DM	JY		Jan. 1973- Dec. 1989	Monthly	three month

CD denotes the Canadian dollar, BP the UK pound, DG the Dutch Guilder, DM the German mark, FF the French Franc, IL the Italian Lira, JY the Japanese yen, USD the US dollar.

* The currencies investigated in this study are against the DM. The remaining papers examine rates quoted against the US dollar.

We conclude that if the spot and forward rates are not cointegrated then the unbiased expectations hypothesis does not hold. This does not necessarily imply that markets are not informationally efficient. If the spot and forward rates are cointegrated then the constant term must be zero for unbiasedness. Thus it follows that cointegration of the realized spot rate and corresponding forward rate is a necessary but not sufficient condition for unbiased expectations to hold within the context of the model developed in this paper.

4. Cointegration and Unit Roots

The notion of stationarity employed in this paper is tied to the concept of difference stationarity. To illustrate the concept of difference stationarity, let us assume that the forward exchange rate series $\{f_t\}$ can be represented by the following ARMA(p,q) process

$$\alpha(L)\,(f_t - \mu) = \theta(L)\varepsilon_t \qquad (4.1)$$

where L is the lag operator for the autoregressive polynomial $\alpha(L) = [\,1 - \alpha_1 L - \ldots - \alpha_p L^p]$ and the moving average polynomial $\theta(L) = [\,1 - \theta_1 L - \ldots - \theta_q L^q\,]$ for $\varepsilon_t \sim N(0,\sigma^2)$. The roots of the AR polynomial are obtained by solving

$$\alpha(L) = [\,1 - \alpha_1 L - \ldots\ldots - \alpha_p L^p] = 0 \qquad (4.2)$$

The roots can take on values on the unit root circle such as +1 and -1, or imaginary numbers i and -i. The testing literature usually addresses tests for the root +1.[20] If $\alpha(L)$ has d unit roots then the AR polynomial can be rewritten as

$$\alpha(L) = \alpha'(L)\,(1-L)^d \qquad (4.3)$$

and thus, $\alpha'(L)(1-L)^d\,(f_t - \mu) = \theta(L)\,\varepsilon_t$ an ARIMA(p,d,q) process.[21] f_t is said to be difference stationary of order d or equivalently integrated of order d, denoted as I(d). Hence, if the series contains one unit root, denoted I(1), its first difference is stationary or I(0).

Cointegration maintains a specific relationship between non-stationary variables.[22] When, for example, the linear combination of I(1) variables is stationary or I(0), then the

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[20] These tests include the Dickey-Fuller (DF) tests, Augmented Dickey-Fuller (ADF) tests [Dickey and Fuller (1979), and Said and Dickey (1984)], and Phillips-Perron tests [Phillips (1987), and Phillips and Perron (1988)].

[21] More generally, $f_t = A(L)\varepsilon_t$ where $A(L) = \theta(L)/\alpha(L)$ and contains d unit roots, then
$(1-L)^d f_t = A'(L)\,\varepsilon_t$ where $A'(L)$ has no unit roots.

[22] Engle and Granger (1987) formally define cointegration as when "the components of the vector x_t are said to be cointegrated of order d, b, denoted $x_t \sim CI(d,b)$, if (i) all components of x_t are I(d); (ii) there exists a vector α (=0) so that $z_t = \alpha' x_t \sim I(d-b)$, b>0. The vector α is called the cointegrating vector."

respective variables are said to be cointegrated.[23] Denote a vector of spot and forward rates $\underline{F} = [\ln S_{t+T}, \ln F_t(T)]$ where each individual rate is I(1). For equilibrium to hold,

$$\underline{a}'\underline{F} = \text{constant} \qquad (4.4)$$

where $\underline{a} = [a_1, a_2]$ is a vector of coefficients. However, this above notion of equilibrium is exceptionally strong. We consider a weaker version which allows for an equilibrium error

$$\underline{a}'\underline{F} = w_t \quad \text{where } w_t \text{ is I(0).} \qquad (4.5)$$

In other words, the equilibrium error w_t is a stationary series wandering or oscillating about the mean which in this case is a constant [Engle and Granger (1987)]. If such a relationship exists, then the forward rates are said to be cointegrated with a cointegrating vector $\underline{a}$. The equilibrium notion entailed in cointegration relates variables which are individually non-stationary. A cointegrating relationship would indicate that these variables move together or do not drift far apart. In other words, imbedded in the definition of cointegration is an equilibrium relationship which allows for "equilibrium" errors. If the financial variables in question sustain such a stationary relationship then

23 For a survey of unit root tests refer to Diebold and Nerlove (1990) and Perron (1990). Surveys of cointegration include Dickey, Jansen, Thornton (1991), Dolado, Jenkins, Sosvilla-Rivero (1990) , Kennedy (1992), McDermott (1990), Muscatelli and Hurn (1992).

persistent profit opportunities may not exist. As a result, cointegration is often cited as evidence in favor of market efficiency [Dwyer and Wallace(1992)].

Testing for cointegration within the ordinary least squares (OLS) framework typically involves regressing an I(1) variable on other I(1) variables and testing the residuals from this cointegrating regression for stationarity using modified versions of the standard unit root tests often referred to as residual based tests for cointegration [Engle and Granger (1987), and Phillips and Ouliaris (1990)]. In the context of the present paper, however, the cointegrating vector is predetermined by the CIP relationship. We restrict the cointegrating vector to be (1,-1) and thus avoid the need to estimate the cointegrating regression, thereby allowing one to apply the standard unit root testing procedures mentioned above. Testing for cointegration would be a test of stationarity of the forecast error between the forward rate and future spot rate.

We begin by testing for a unit root in each forward rate series using the Phillips-Perron (PP) test [Phillips (1987), and Phillips and Perron (1988)].[24] The underlying null hypotheses for the various PP tests are based on the assumption that the underlying process is of the form

$$F_t = F_{t-1} + e_t \qquad (4.6)$$

24 The unit root (and residual based cointegration) tests as applied to financial data are criticized by Cochrane (1991), Hakkio and Rush (1991), and Sephton and Larsen (1991). By restricting the cointegration vector a priori, as opposed to estimating it by ordinary least squares techniques, the problem concerning the lack of power is partially addressed.

for t=1,2,3, ... Phillips and Perron (1989) assume the following set of conditions on the innovation term e_t (p. 336).

1) $E(e_t) = 0 \quad \forall t$

2) $\sup_t E|e_t|^{\eta+\zeta} < \infty$ for some $\eta > 2$ and $\zeta > 0$

3) $\sigma^2 = \lim E(\frac{1}{T} S_T^2)$ exists as $T \to \infty$. Variance is non - negative. Define $S_t = \sum_{\tau=1}^{\infty} e_\tau$

4) $\{e_t\}$ is strong mixing with mixing coefficient α_j such that $\sum_{j=1}^{\infty} \alpha_j^{1-2/\beta} < \infty$.

These are relatively general conditions and hence what is partially motivates our use of the PP tests. Possible heterogeneity in the process is accounted for by condition 2 and some forms of temporal dependence are allowed by condition 4. Conditions 1 and 2 simply impose basic restrictions on the first and second moments of the innovation series.

The test statistics are obtained by running the following pair of regressions.

$$F_t = \rho F_{t-1} + \mu + e_t \qquad (4.7)$$

$$F_t = \rho^* F_{t-1} + \mu^* + \theta^* T + e^*_t \qquad (4.8)$$

The t-statistics for ρ and ρ* are denoted as t and t* for equations (4.7) and (4.8) respectively.[25] The t and t* statistics are given a non-parametric adjustment in order to

25 The limiting distributions of the two τ statistics are non-standard (in distribution) in that they tend to be skewed. As a consequence larger critical values (absolute values) are necessary. The tables developed and compiled in MacKinnon (1990), and Phillips and Ouliaris (1990) are applied in the present paper. Also note that the OLS estimates of ρ are super-consistent for $F_t \sim I(1)$ and large sample size.

eliminate the dependence of nuisance parameters on the limiting distribution to yield corresponding Z(t) and Z(t*) statistics. The Phillips-Perron test allows for weak dependence and heterogeneity for e_t. This is important since the pairs of forward and spot rates are overlapping and could thus introduce serial correlation into the analysis. From model (4.7) the null hypothesis of a unit root is treated using the Z(t) statistic. For the same equation the null hypothesis, Ho': $\mu=0$ and $\rho=1$ is tested with the $Z(\Phi 1)$ statistic. In essence, the Z(t) and $Z(\Phi 1)$ statistics can be regarded as adjusted t and F test statistics respectively.

For equation (4.8) we consider three null hypotheses, Ho'': $\rho^*=1$, Ho''': $\rho^*=1$, $\mu^*=0$, and Ho'''': $\rho^*=1$, $\mu^*=0$, $\theta^*=0$ with the statistics Z(t*), $Z(\Phi 3)$, and $Z(\Phi 2)$ respectively.

If we accept Ho or Ho'', then we automatically reject the unbiased expectations hypothesis since the forecast error is non-stationary. Rejecting the null would imply stationarity of the forecast error and that the forward and spot rate share a common stochastic trend.

The statistics are obtained by using the following set of formulas.

$$Z(t) = (\frac{S_0}{S_{nl}})t - (\frac{1}{2}S_{nl})(S_{nl}^2 - S_0^2)[n^{-2}\sum_1^n(F_{t-1} - \bar{F}_{-1})^2]^{-1/2} \quad (4.9)$$

$$Z(\Phi 1) = (\frac{S_0}{S_{nl}})\Phi 1 - (\frac{1}{2}S_{nl}^2)(S_{nl}^2 - S_0^2)\{n(\rho-1) - \frac{1}{4}(S_{nl}^2 - S_0^2)[n^{-2}\sum_1^n(F_{t-1} - \bar{F}_{-1})^2]^{-1}\} \quad (4.10)$$

where $\Phi 1=(2S^{*2})^{-1}n(S_0^2-S^{*2})$, n denotes the sample size, and $\bar{F}_{-1}$ is the mean of [F1, ..., Fn-1]. S_0^2 and S^{*2} is the residual variance under the appropriate null and the residual variance employing OLS estimates for equation (4.7) respectively.

The set of statistics for equation (4.8) are obtained using the following set of formulas.

$$Z(t^*)=(\frac{S_0}{S_{nl}})t^*-(\frac{n^3}{4\sqrt{3}D_x^{1/2}S_{nl}^2})(S_{nl}^2-S_0^2) \qquad (4.11)$$

$$Z(\Phi 3)=(\frac{S_0^2}{S_{nl}^2})\Phi 3-(\frac{1}{2}S_{nl}^2)(S_{nl}^2-S_0^2)[n(\rho-1)-(\frac{n^6}{48D_x})(S_{nl}^2-S_0^2)] \qquad (4.12)$$

$$Z(\Phi 2)=(\frac{S_0^2}{S_{nl}^2})\Phi 2-(\frac{1}{3}S_{nl}^2)(S_{nl}^2-S_0^2)[n(\rho-1)-(\frac{n^6}{48D_x})(S_{nl}^2-S_0^2)] \qquad (4.13)$$

where $\Phi 2=(3\tilde{S}^2)^{-1}n(S_0^2-\tilde{S}^2)$ and $\Phi 3=(2\tilde{S}^2)^{-1}n(S_0^2-\tilde{S}^2-(\bar{F}-\bar{F}_{-1})^2)$.

The t-statistics denoted t and t* are the standard t-statistics under the null of $\rho=1$. S_0^2 denotes the variance under the null hypothesis, and S^{*2} the OLS residual variance. D is the determinant of X'X where X is the 3 x n explanatory variable matrix from the OLS regression (4.8).

Finally, S_{nl}^2 is the consistent estimator of the variance of the residuals.

$$S_{nl}^2=\frac{1}{n}\sum_{t=1}^{n}e_t^2+\frac{2}{n}\sum_{\tau=1}^{l}\sum_{t=\tau+1}^{n}e_te_{t-\tau}\omega_{\tau l} \qquad (4.14)$$

where $\omega_{\tau l} = 1 - \frac{\tau}{(l+1)}$. ω is employed to insure that the estimated variance is non-zero. The innovation terms in (4.14) can be replaced by the appropriate innovation from equation (4.7) or (4.8) depending on the null being tested.

The Z statistics involve replacing the standard errors in the regressions with more general standard errors which account for serially correlated innovations. The PP statistics also have a correction term between the variance estimates S_0^2 and S_{nl}^2 which in essence also captures any serial correlation.

5. Cointegration of Forward Rates

We also provide some preliminary evidence of cointegration in the forward rates across different maturities. In other words, we investigate for the possibility that a common stochastic trend drives the term structure of forward rates. Numerous papers have investigated the term structure of interest rates using cointegration. We employ a multivariate test, the Johansen (1988) trace test to examine the forward rates. When cointegration between more than two variables is investigated, the standard Phillips-Perron tests do not provide us with information on whether there exists a unique cointegrating vector. To circumvent this problem, we employ the Johansen trace test. We reject the possibility of a common stochastic trend for the entire spectrum of forward rates ranging from one month to five years.

The step involved in obtaining the Johansen trace test outlined in this section closely follows procedure described in Dickey, Jensen, and Thornton (1991). We begin by assuming that the variable in question follows the process of the form with lag k.

$$\Delta F_t = \delta + \Gamma_1 \Delta F_{t-1} + \ldots + \Gamma_{t-k} \Delta F_{t-k} + \Pi F_{t-k} + \varepsilon_t \qquad (5.1)$$

where $\Gamma_i = -(I - A_1 - \ldots - A_i)$ for i=1, ..., k

and $\Pi = -(I - A_1 - \ldots - A_k)$.

δ is the constant term, F is the n x 1 vector of forward rates which are assumed to be I(1), and ε_t is normal iid. I is an n x n identity matrix and A is an n x n coefficient matrix. The Johansen test, in essence, determines the rank of Π which in turn tells us the number of cointegrating vectors. A full rank would require the variables to be stationary given the stationary assumptions of the residuals. On the other hand, a null rank would reduce equation to a VAR in first differences. Finally, a rank of order z where $0<z<n$ would imply there cointegrating vectors such that ΠF_{t-1} is stationary. z cointegrating relationships would imply that there are n-r common stochastic trends.

In the case there are z cointegrating relationships, Π could be decomposed such that $\Pi = \alpha\beta'$ where α and β are n x z matrices. The columns (z) of β are the cointegrating vectors. We used the following procedure to obtain the Johansen statistic and determine the number of common stochastic trends.

Firstly we note that

$$\Delta F_t = \delta + \sum_{i=1}^{k-1} \Gamma_i \Delta F_{t-i} + \alpha\beta\, F_{t-k} + \varepsilon_t \qquad (5.2)$$

To obtain estimates of β , we first run a pair of regressions ΔF_t and F_{t-k} on [1, ΔF_{t-1}, , ΔF_{t-k}] to obtain vectors of residuals R_{0t} and R_{kt} respectively. Next regress R_{0t} on R_{kt}

$$R_{0t} = \alpha\beta R_{kt} + error \tag{5.3}$$

to yield

$$\hat{\alpha}(\beta) = S_{0k}\beta(\beta\, S_{kk}\beta)^{-1} \tag{5.4}$$

Equation (5.4) amounts to minimizing the squared residuals from (5.3)

$$\Sigma(\beta) = S_{00} - \hat{\alpha}(\beta)(\beta\, S_{kk}\beta)\hat{\alpha}(\beta)' \tag{5.5}$$

where $$S_{ij} = \frac{1}{T}\sum_{t=1}^{T} R_{it}R_{jt}' \tag{5.6}$$

Note that when i=j equation (5.6) reduces to the estimator for the relevant variance term and when i ≠ j, the it follows that it is simply an estimate for the covariance term.

An estimate of β is then obtained by solving

$$|\lambda S_{kk} - S_{0k} S_{00}^{-1} S_{0k}| = 0 \tag{5.7}$$

for eigenvectors associated with the eigenvalues λ such that the $\lambda_1 < \ldots < \lambda_n$. Johansen then proceeds to obtain a likelihood ratio test to determine the number of cointegrating vectors. The form of the statistic is

$$Trace = -T\sum_{i=q+1}^{n} \ln(1-\hat{\lambda}_i) \tag{5.8}$$

Under the null hypothesis there are at most q cointegrating vectors or $z \leq q$ cointegrating vectors. The alternative would be $z \leq n$ unrestricted cointegrating vectors exist.

6. Empirical Results

6.1 The Data

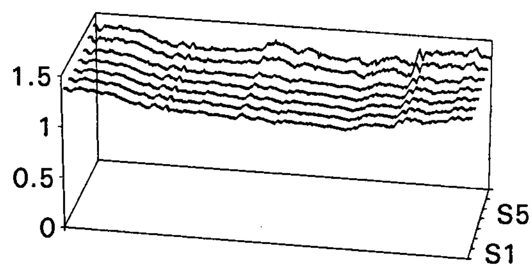
The data employed in the study was taken from various issues of the *Globe and Mail.* The quoted forward rates are for noon each trading day. Forward rates of one month, two months, three months, six months, one year, three years, and five years were taken. The rates are in Canadian dollars per U.S. dollar. The weekly series that was collected dates back to the first week the long dated forward rates were reported in the *Globe and Mail* (April 1987) and covers recent movements through to November 1993.

For the weekly series, Wednesday rates were taken; when the Wednesday rate was not available the Tuesday rate was used.

The descriptive statistics of the forward and spot rate data is presented in Table 1. The mean change in the various forward and spot rates is close to zero in all cases. The excess kurtosis statistics are relatively large and range from 1.733 to 2.728 for the change in forward rates and 104.178 for the change in the spot rate. We also note that the mean level of the forward rates increases with the maturity of the contract and thus indicates that on average that the forward rate structure was upward sloping. A visual inspection of Figure 1 also reveals an upward sloping forward rate term structure and that the rates seem to move together.

The forward and spot rate data are paired such that the spot rate is T days ahead of the forward rate observation where T is the time to maturity of the forward contract. Due to the limitation in time span of the data series overlapping pairs of data sets were created. Since the seven and ten year forward rates had maturity dates which exceeded the span of the spot rate series the analysis of spot and forward rate series was confined to forward rates ranging from one month to five years in maturity. More specifically, the one month forward quote for example on September 10 is paired with the realized spot rate on October 11 instead of October 10. The extra day takes into account the settlement procedure which usually requires one day in North America for the US-Canadian dollar exchange rate. In the case that October 11 is a holiday or weekend then the realized spot rate on the next nearest business day is used. An exception is when holidays or non-business days force the next nearest business day into the following month; in which case you move backwards to the nearest business day

Figure 1: US/CD $ Spot and Forward Rates from 1987-1993



April 1987 Nov. 1993

The spot rate is plotted as S1 (foremost on the graph). S2 to S5 indicate the forward rates with maturities of one month, two months, three months, six months, one year, three years, and five years.

Table 2: Descriptive Statistics of the Level and Change in Spot and Forward Rates

	mean (x1000)	standard deviation	skewness	kurtosis	number of observations
spot	0.00786	0.01472	0.58869	104.178	356
	1.2188	0.064313	0.44753	-0.92896	357
one month	0.00730	0.00721	0.45533	1.993	356
	1.22208	0.062747	0.52107	-1.07318	357
two month	0.00589	0.00720	0.52234	1.733	356
	1.22455	0.062164	0.51851	-1.06961	357
three month	0.00393	0.00734	0.41919	2.40098	356
	1.22706	0.061556	0.51615	-1.07034	357
six month	-0.001685	0.007869	0.54279	2.23037	356
	1.23419	0.060417	0.50861	-1.06714	357
one year	-0.016854	0.0089586	0.30350	2.37479	356
	1.24677	0.059165	0.49336	-1.07126	357
three year	-0.12219	0.011719	0.30089	2.19083	356
	1.29114	0.057982	0.3966	-1.18454	357
five year	-0.1382	0.013276	0.024214	2.72893	356
	1.32262	0.059887	0.31072	-1.22965	357

The change in rates is the log relative of the rates. The first row is for the change in rates and second row for the level series.

6.2 Unbiased Expectations: A Simple Test

Before testing for cointegration we look at the forecast error which is the difference the forward rate and the corresponding realized spot rate. As noted by Frankel (1993) this is simplest test of unbiased expectations. "The weakest possible test of rational expectations defines the information set to contain nothing other than a constant; the criteria is simply $E(S_{t+k} - F_t)=0$, or , in other words, the mean prediction error is zero. Given the length of our sample is very short relative to the time to maturity of the long dated forward rates, this simple test is in a sense the only feasible conventional test of unbiased expectations we can conduct.

A summary of the descriptive statistics of the forecast error, the difference between the forward and expected (realized) spot rate, is displayed in Table 2. The mean of the forecast error tends to be close to zero and negative in all cases. Negative means would indicate that the forward rate overpredicts the realized spot rate on average for all maturities. The mean is small in magnitude relative to the standard deviation hence we conclude that mean forecast error is not statistically different from zero. There is the possibility that positive and negative errors could cancel yielding a mean close to zero hence we examine the mean of the absolute value of the forecasts, |mean|. As with mean forecast error, the magnitude of the error increase with length of maturity of the forward contract. The |mean|, however, are still relatively small compared to the standard deviation. From these simple tests we find that the forward rate is an unbiased predictor of future spot rates for short dated as well as long dated forward rates. This, however, does not necessarily rule out the possibility of a risk premia.

Table 3: Descriptive Statistics of the Forecast Error

	mean	standard deviation	|mean|	standard deviation	number observ.
one month	-0.0032	0.0181	0.0108	0.0110	353
two month	-0.0057	0.0239	0.0146	0.0143	349
three month	-0.0084	0.0279	0.0181	0.0158	345
six month	-0.0171	0.0421	0.0306	0.0211	331
one year	-0.0333	0.0671	0.0526	0.0301	305
three year	-0.0894	0.0927	0.0842	0.0592	201
five year	-0.0947	0.1004	0.0866	0.0605	95

6.3 Spot and Forward Rates

All forward rates and the spot rate contain a unit root as indicated by the test statistics in Table 3. The null hypothesis which include testing for the mean and/or trend [Z(Φ1), Z(Φ2), Z(Φ3)] were not rejected in all cases. The first difference of the forward rates exhibit stationarity for all of the series examined. The evidence strongly suggests that the forward rates and spot rate are I(1).

The Phillips-Perron unit root test is applied again to test whether the forecast error is cointegrated. The lag length for estimate of the variance equation (4.14) was set at l=12. Test statistics with alternative lag lengths of 6 and 18 did not change the qualitative results. As shown in Table 4 the shorter dated forward rates are cointegrated with the spot rates at conventional significance levels and is thus consistent with the literature to date. As the forecast error is stationary we infer that the interest rate agio is I(0) as well and that a common stochastic trend drives short term interest rates in the US and Canada.

Table 4: Phillips Perron Unit Root Tests for Spot and Forward Rates

$$F(T)_t = \rho\, F(T)_{t-1} + \mu + e_t$$
$$F(T)_t = \rho*\, F(T)_{t-1} + \mu* + \theta*T + e*_t$$

Z(t) Ho: ρ=1
Z(Φ1) Ho: ρ=1 , μ=0

Z(t*) Ho: ρ*=1
Z(Φ3) Ho: ρ*=1, μ*= 0
Z(Φ2) Ho: ρ*=1, μ*=0, θ*=0

T	Z(t)	Z(Φ1)	Z(t*)	Z(Φ3)	Z(Φ2)
spot	-1.639	1.327	-1.257	1.803	2.704
	-30.389	461.000	-34.653	398.900	598.33
one month	-1.115	0.622	-0.684	2.553	3.829
	-17.179	147.510	-17.722	104.430	156.650
two month	-1.137	0.647	-0.669	2.599	3.899
	-17.472	144.630	-17.472	101.520	152.280
three month	-1.178	0.696	-0.737	2.454	3.681
	-16.740	140.05	-17.176	98.082	147.120
six month	-1.295	0.839	-0.948	2.071	3.107
	-179.904	150.890	-17.904	106.560	159.840
one year	-1.459	1.0643	-1.156	1.855	2.782
	-18.424	169.600	-18.995	119.960	179.940
three year	-2.163	2.357	-2.105	1.969	2.936
	-19.546	169.60	-18.995	119.96	179.94
five year	-2.283	2.357	-2.105	1.969	2.936
	-19.546	190.810	-19.858	131.140	196.710

The statistic in the first row is for the level and the second row the first difference.

Table 5: Cointegration of Spot and Forward Rates

$FE_t = \rho FE_{t-1} + \mu + e_t$
$FE_t = \rho FE_{t-1} + \mu* + \theta*T + e*_t$

where FE is the forecast error

Z(t) Ho: ρ=1
Z(Φ1) Ho: ρ=1 , μ=0

Z(t*) Ho: ρ*=1
Z(Φ3) Ho: ρ*=1, μ*= 0
Z(Φ2) Ho: ρ*=1, μ*=0, θ*=0

T	Z(t)	Z(Φ1)	Z(t*)	Z(Φ3)	Z(Φ2)
one month	-9.904	49.018	-9.895	32.560	48.838
two month	-7.431	27.617	-7.484	18.658	27.982
three month	-6.204	19.255	-6.964	16.179	24.268
six month	-3.396	5.766	-5.213	9.104	13.644
one year	-1.950	1.968	-3.966	5.275	7.844
three year	-0.3110	1.194	-3.618	5.163	6.787
five year	-0.149	3.423	-2.776	4.665	3.960

From equation (5), a stationary forecast error does not rule out the possibility of a time varying risk premium. The sum of $(m - \frac{1}{2}\sigma)$ and $\ln R_{t-1,t}$ from equation (5) could be interpreted as the risk premium. If the interest rate agio is assumed stochastic, then the risk premium is time varying in

nature. Furthermore, as the interest rate agio is implicitly stationary from our findings for the short dated market, the time varying risk premium is thus stationary as well. A time varying but stationary risk premium is consistent with a weak "equilibrium" relationship.

The long dated forward rates, namely the three and five year rates, are not cointegrated with the spot rate and thus stand in contrast to findings on the short dated market.[26] This would imply that the interest rate agio is non-stationary for long dated rates if the model based on equations (2.1) and (2.2) are correct. If the interest rate agio is not I(0), for long dated rates, then US and Canadian long dated interest rates are driven by different factors. This could be due to institutional differences and the relative liquidity of the bonds of longer maturity in the US as opposed to Canada. Boothe, however, as noted earlier finds that US and Canadian medium and long dated bond yields are cointegrated.

6.4 Johansen Trace Test Results

As the long dated forward rates are not cointegrated with the realized spot rate, we turn to the possibility that long dated forward rates are driven by common factor distinct from that for shorter dated forward rates. The Johansen trace test is applied to

26 The fact that the long dated forward rates are not cointegrated with the spot rate could be due to the power of the tests. Moreover, cointegration is expected to hold in the long run. As the maturity of the longer dated forward rates is extremely long relative to the sample period it would be difficult to discern a cointegrating relationship even if one existed.

the entire range of forward rates investigated in this paper. This amounted to looking for a common stochastic trend for the one month to five year forward rates.

If covered interest parity holds and the interest rate agio is stationary then we could expect six independent cointegrating relationships and hence a single common stochastic trend. To illustrate, let us assume covered interest parity holds so that $F(t,t+1)=\left(\frac{1+r^*}{1+r}\right)S(t)$ and that $F(t,t+n)=\left(\frac{1+r^*}{1+r}\right)^{n-t}S(t)$ then

$$F(t,t+n)=F(t,t+1)\left(\frac{1+r^*}{1+r}\right)^{n-t-1}S(t)\frac{1}{S(t)} \qquad (6.4.1)$$

for n>1. If we take natural logs and rearrange, then the difference between the nearby forward rate with maturity at t+1 is cointegrated with the deferred forward rate with maturity t+n so long as the interest rate agio is stationary. Thus, we could expect this relationship to hold between the one month forward rate and each of the other forward rates with two month, three month, six month, one year, three year, and five year maturities. Since yields a total of six bivariate cointegrating relationship. If each of these pair is stationary, then the linear combination of any of these pairs (spreads) would be stationary by definition. In other words, with a set of seven forward rates we could expect them to driven by a single common stochastic trend or equivalently have six independent cointegrating relationships.

The critical values used in this paper are obtained from Table 1 (p.239) in Johansen (1988). The approximation procedure as outlined in Johansen (1988) is used when testing the entire maturity spectrum of the forward rates. The approximation is calculated as $c\chi^2(f)$ where the degrees of freedom is $f=2m^2$ and $c = 0.85-0.58/f$. m denotes the number of common stochastic trends. The critical values calculated by Zhang (1993) are used in this essay.

The corresponding eigenvalues are given in the second column of the tables. The eigenvalues in all three cases are consistent with the structure outlined in section 5 where they are ordered in magnitude from largest to smallest. These statistics are obtained by assuming the autoregressive lag length to be k=1 for equation (20).

We find evidence of two common stochastic trends driving the forward foreign exchange rate "term structure." The trace test statistics are statistically significant at the conventional five percent significance level for three common stochastic trends and higher. In other words, we cannot reject that there are at most two common stochastic trends. Since we find two common stochastic trends, our original conjecture of a single common stochastic trend is rejected.

To verify this, we divided the forward rates into sub groups by their maturity. The shorter dated rates which range from one month to six months are in one group and the longer dated rates are bunched in a second group. The trace test is applied to each of these maturity groups. At the five percent significance level we find that each group has a single common stochastic trend. Thus, when the two maturity groups are combined there are two common stochastic trends which is consistent with the original test results for the entire spectrum of forward rates.

Table 6: Johansen Trace Test for Cointegration of Forward Rates

rank	m common trends	Trace Statistic	Eigenvalue
All Forward Rates			
0	7	510.01*	0.3897
1	6	333.73*	0.3537
2	5	177.92*	0.2326
3	4	83.39*	0.1249
4	3	35.77*	0.0725
5	2	8.90	0.0214
6	1	1.18	0.0033
Short Dated Forward Rates			
0	4	288.20*	0.3808
1	3	117.07*	0.2484
2	2	15.04*	0.0397
3	1	0.66	0.0018
Long Dated Forward Rates			
0	3	58.36*	0.1117
1	2	16.05*	0.0415
2	1	0.90	0.0025

All forward rates refers to one month, two month, three month, six month, one year, three year, and five year rates.
Short dated refers to one month, two month, three month, six month rates.
Long dated refers to one year, three year, five year rates.
* indicates significant at the five percent significance level.

As the long dated forward rates have a single common stochastic trend distinct from the shorter maturities, this provides some evidence that long dated forward contracts could be priced differently from the shorter dated contracts. The short dated forward rates are usually consistent with conventional covered interest parity. Poitras (1992) finds evidence that long dated forward rates are not consistent with a long dated version of conventional CIP. The long dated forward rates could be consistent with other long dated instruments as currency swaps and dual currency bonds. Implied forward rates are imbedded in currency swap agreements and thus "outright" forward contracts could be priced so that they are consistent with the implied forward rates in a currency swap. To illustrate, imagine a swap between yen and dollar payments. The present value of the swap is

$$Value = \sum_{t=1}^{T}(C_{yen}F_t - C_{dollar})e^{-r_t t} + (P_{yen}F_T - P_{dollar})e^{-r_T T} \qquad (6.4.2)$$

where C is the interest cash flow for periods t=1,....,T , P is the notional principal to be exchanged at maturity, and r is the dollar discount rate. F is the implied forward rate at which the yen cash flow is converted to dollars. Hence, an implied term structure of forward rates is imbedded in the swap. As swaps are long dated instruments, the implied term structure is for long dated forward rates. As a consequence, it is highly likely that long dated forward rate share a single common stochastic trend as we have documented in this paper.

7. Conclusion

The long dated forward exchange contract for the Canadian dollar is examined in this paper. The CIP framework employed in the present paper relies on the assumption that the interest rate differential is stationary. The empirical evidence cited from the literature imply that this assumption is not unreasonable. Given stationarity of the interest differential, the forward exchange rates and the (future) spot exchange rate should be stationary if the covered interest parity condition holds. The objective of this study was simply to document whether spot and forward rates share a common stochastic trend. Given the model developed in this paper, finding evidence of a common stochastic trend has implications for international market integration.

We find evidence that the forward and spot rates are cointegrated for shorter dated forward rates. This is consistent with the literature to date. As a stationary forecast error also implies a stationary interest rate agio, this also provides evidence that a common stochastic trend could be driving the interest rates in the US and Canada. This is consistent with the empirical evidence provided by Boothe (1991). For the longer dated forward contracts we do not find such a cointegrating relationship implying that the movement in the far end of the term structure of the US and Canada are driven by independent factors.

The Johansen trace test is applied to the entire spectrum of forward rates. We find evidence of two common stochastic trends driving the "term structure" of forward rates. When the forward rates were divided into two groups based on maturity, each maturity group has a single common stochastic trend. It is possible that the factor(s) driving forward rates at the short end of the spectrum are different from those driving the longer dated forward rates.

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