

**TEACHERS' MATHEMATICAL PRACTICES
INSIDE AND OUTSIDE THEIR CLASSROOMS**

by

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ABSTRACT

In this thesis, I analyzed the mathematical practices of two elementary school teachers across various contexts of their lives, in and out of their classrooms. This work was developed as an effort to reconcile the apparent contradictions between (a) recent research findings that suggest all mathematical activity is fundamentally situated and distributed across physical and social contexts such that there are distinct discontinuities between mathematics as taught in classrooms and as used outside classrooms (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a) and (b) recent proposals for teaching and learning mathematics in schools that have encouraged educators to connect mathematics instruction with students' experiences in out-of-school settings (NCTM, 1989). This contradiction prompted the effort to describe relationships between mathematical practices in and out of classrooms for two teachers.

I documented the teachers' mathematical practices as they taught in their elementary classrooms, completed assignments for a mathematics teaching methods course at the university, and engaged in mathematical practices outside their classrooms. Following the framework of interpretive research design (Erickson, 1986; Lincoln & Guba, 1985), interviews, observations, documents (research notebooks, university assignments, classroom lesson plans and resources), and researcher field notes served as primary data sources. The data were categorized and organized according to recurring themes and presented as two separate case studies, one for each participating teacher.

Consistent with earlier research findings, the two case studies document that the teachers displayed mathematical activity in everyday settings that differed from approaches taught in schools, including their own classrooms. However, the teachers did attempt, with at least partial success, to link their own and their students' non-school uses of mathematics with classroom instruction. Implications are drawn for how these connections could be strengthened in an effort to offer mathematics education as praxis.

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CHAPTER ONE

MATHEMATICS INSIDE AND OUTSIDE CLASSROOMS

I don't know about these things; I didn't go to school. I know about the oysters because we fish; the price of oysters we have to know. If they're selling 5 oysters at 750, then they're selling each one at 150.

—a woman who fishes for oysters
(Nunes, Schliemann, & Carraher, 1993, p. 11)

Introduction

Many people use mathematics in highly efficient and productive ways at work and in other areas of their daily lives, yet this mathematical activity may seldom be recognized. Some mathematical practices are so embedded in the form and process of other activities that the mathematical properties of the performance go unnoticed (Nunes, Schliemann, & Carraher, 1993). Consequently, people often do not recognize those structural equivalences in their mathematical activities across different settings that more formal mathematics would recognize. Mathematics educators have suggested that people understand mathematics when they can recognize structural equivalences across contexts, and when they can translate between representational systems and fluently integrate understandings across situations and contexts (Hiebert & Carpenter, 1992; Kaput, 1992). Such integration is also emphasized by recent curriculum improvement documents (National Council of Teachers of Mathematics [NCTM], 1989). An unanswered question is whether teachers themselves can make connections between the formal mathematics of their classrooms and the mathematics they use outside schools. This thesis is an analysis of the mathematical practices of two elementary school teachers across various contexts of their lives, in and out of their classrooms.

This study was based on recent research findings that mathematical practices are fundamentally situated and distributed across physical and social contexts (e.g., Lave, 1985, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a). Prior research indicates that people often spontaneously invent and use their own mathematical strategies that differ from school-taught algorithms, but that nonetheless lead to accurate

and efficient solutions in daily settings. For example, complicated ratio and proportion problems are embedded in calculating best buys at the grocery store (Murtaugh, 1985b), adapting recipe sizes (de la Rocha, 1985), buying and selling candy to maximize profit (Saxe, 1991), and reading blueprints (T. N. Carraher, 1986). However, these non-school uses of mathematics are seldom considered “real” mathematics. As Resnick (1987) concluded in a review of this research, school arithmetic and non-school uses of number knowledge do not map well to each other: There are distinct discontinuities of performance between mathematics as taught in classrooms and as used outside classrooms. Students who do well on textbook problems are often unable to use this school knowledge to interpret actual physical events (Masingila, 1992; Resnick, 1983). Similarly, dairy workers, grocery shoppers, child street vendors, carpenters, street corner bookies, and tailors who are highly accurate (~98%) in solving problems which emerge out of their everyday practices, drop significantly in accuracy on school-like, paper-and-pencil problems (Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a).

Problem Statement

Recent proposals for teaching and learning mathematics in schools have encouraged educators to integrate mathematics with other subjects and with children’s experiences in out-of-school settings (NCTM, 1989). Forging connections between school and non-school mathematical activity should provide a two-way bridge allowing students to draw on their out-of-school knowledge in the classroom, and their school knowledge outside the classroom, strengthening practice in both arenas. However, given the above cited studies, a yet to be resolved question is,

“If people do not make connections between their highly competent mathematical activity in everyday settings and structurally equivalent school problems, what degree of success can schools and teachers achieve in helping students to integrate mathematical experience across diverse settings?”

This question provided the focus for the present research study. I explored ways in which two elementary school teachers understood and used mathematics inside and outside their classrooms. I investigated the connections they made between school and non-school mathematics, in the ways they taught and learned mathematics in classrooms, and in the ways they used mathematics outside classrooms. Consistent with previous research on non-school mathematics (e.g., Lave, 1988; Scribner, 1984a), I tried to describe mathematical concepts and processes engaged in context, using an interpretive framework (Lincoln & Guba, 1985; Strauss & Corbin, 1990).

Overview

In this chapter, I provide a brief historical overview of research to lay the foundation for a description of three research programs concerned with discontinuities between school and non-school mathematics. This review culminates with a summary of the main findings from this (and other) research on out-of-school mathematical practices. I then juxtapose these research findings with recent proposals for teaching and learning mathematics that have encouraged educators to connect mathematics instruction with students' experiences in out-of-school settings. This juxtaposition highlights the contradiction between prior research and current proposals, and provides the frame for the research study described in the remainder of this thesis.

Historical Overview

D'Ambrosio has been credited with introducing the term ethnomathematics to describe "forms of mathematics that vary as a consequence of being embedded in cultural activities whose purpose is other than 'doing mathematics'" (Nunes, 1992, p. 557). Here "doing mathematics" refers to Western-style school mathematics with its algorithms and procedures. As Ascher (1991) suggested, ethnomathematics investigates the mathematical ideas of those who have traditionally been excluded from discussions of mathematics. Masingila (1992) suggests that ethnomathematical research examines relations of

sociocultural factors to teaching and learning mathematics. Scott (1985) defines ethnomathematics in the following way:

Ethnomathematics lies at the confluence of mathematics and cultural anthropology. At one level, it is what might be called “math in the environment” or “math in the community.” At another, related level, ethnomathematics is the particular (and perhaps peculiar) way of classifying, ordering, counting and measuring [and weighing].
(p. 2 as cited in Masingila, 1992, p. 16)

Thus, the term ethnomathematics applies to the kinds of mathematics used in everyday activities, that is, the mathematics involved in activities such as building houses, buying and selling, weighing products, calculating proportions for recipes, tallying sports statistics, and many other nonacademic uses of mathematics. As Lave (1988) explained, “‘Everyday’ is not a time of day, a social role, nor a set of activities, particular to social occasions, or settings for activity. Instead, the everyday world is just that: what people do in daily, weekly, monthly, ordinary cycles of activity” (p. 15). Of course, it can be argued that school mathematics is an everyday activity for students and teachers. To clarify this distinction, I contrast school mathematics with non-school or out-of-school mathematics in this thesis.

To further clarify this distinction, consider the following examples: dairy workers shift back and forth between different base number systems as they calculate cost per case using fixed-dimension cases that hold different numbers of unit containers of varying sizes (e.g., each case holds 4 gallon containers, 16 quart containers, or 32 pint containers) while pricing delivery tickets (Scribner, 1984b); a child in a Brazilian market counts by tens while separating out two lemons at a time (one lemon costs five cruzeiros) to complete a customer’s order for twelve lemons (Carraher, Carraher, & Schliemann, 1985); a dieter fills a measuring cup two-thirds full of cottage cheese, dumps it on the cutting board, pats it into a circle, marks a cross on it, scoops away one quadrant, and serves the rest to fulfill his Weight Watchers daily allotment of $\frac{3}{4}$ of $\frac{2}{3}$ of a cup (Lave, Murtaugh, & de la Rocha, 1984); and school children use paper, pencil and standard algorithms to fill in the teacher’s worksheet. The first three examples illustrate non-

school mathematical practices, whereas the final example describes school mathematical practices. As these examples illustrate, mathematics in non-school settings often looks very different from school mathematics because different numeration systems and different tools or devices are used. Ethnomathematics considers these distinctions, as well as a whole series of sociocultural factors in understanding mathematics teaching and learning.

Research in the past decade has examined mathematical understandings in such out-of-school contexts as candy selling (Carraher, Carraher & Schliemann, 1985, 1987; Saxe, 1988a, 1988b, 1990, 1991); grocery shopping (Lave, Murtaugh & de la Rocha, 1984; Murtaugh, 1985a, 1985b); working in a dairy (Fahrmeier, 1984; Scribner, 1984b, 1984c, 1984d, 1985); sewing and tailoring (Harris, 1988; Reed & Lave, 1979); dieting (de la Rocha, 1985, 1986/1987); reading blueprints (T. N. Carraher, 1986); installing carpet (Masingila, 1992, 1993a, 1993b); managing a restaurant (Prus-Wisniowska, 1993); and doing carpentry work (Millroy, 1992). To illustrate the methods and findings from this research, I briefly review three instances of this literature: candy sellers in Brazil (research conducted by D. W. Carraher, T. Nunes [Carraher], and Schliemann, as well as Saxe), the Industrial Literacy Project (Scribner, Fahrmeier, and colleagues), and the Adult Math Project (Lave, Murtaugh, and de la Rocha).

Candy Sellers in Brazil

Two separate research programs have focused on candy sellers in Brazil: Carraher and colleagues' research from the Federal University of Pernambuco, and Saxe's research. Much of this research, especially that from the Federal University of Pernambuco, is reviewed in D. W. Carraher (1991) and Nunes, Schliemann and Carraher (1993). Here, I review the original research focusing on candy sellers.

Carraher, Carraher and Schliemann (1985) analyzed strategies used by child vendors in real-life commercial transactions on the streets and in the markets of Recife, Brazil. Children's mathematical functioning in their vending roles was compared to their

performance on a formal test comprised of mathematically isomorphic word problems and computational exercises. For example, the following is an excerpt from the interview with a 12 year old, third grade coconut vendor:

Customer: How much is one coconut?
 Child: 35.
 Customer: I'd like ten. How much is that?
 Child: (Pause) Three will be 105; with three more, that will be 210.
 (Pause) I need four more. That is...(pause) 315...I think it is 350.
 (Carraher, Carraher & Schliemann, 1985, p. 23)

The researcher posed successive questions about potential or actual purchases in the market and then asked how the child arrived at an answer. The responses were mathematically represented by the researchers to produce a complete list of all problems and sub-problems entailed in each child's solutions. In the above example, the mathematical representation included the following problems:

- (a) 35×10 [the original posed question];
 - (b) 35×3 (which may have already been known);
 - (c) $105 + 105$;
 - (d) $210 + 105$;
 - (e) $315 + 35$;
 - (f) $3 + 3 + 3 + 1$ [monitoring the number of chained additions]
- (Carraher, Carraher & Schliemann, 1985, p. 23)

Overall, 63 questions were posed by the researcher in the naturalistic setting (ranging from 7 to 19 questions per child). An individualized list of word problems and computational exercises was composed for each child from a sample of researcher-posed questions and the child's spontaneously generated sub-problems. For this formal test, children were encouraged to use paper and pencil to compute solutions.

The researchers compared the number of questions children answered correctly in the three different situations (the naturalistic setting, word problems, and computational exercises). The average frequencies of correct answers in each of the settings were as follows: natural setting 98.2%, word problems 73.7%, and computational exercises 36.8%. Statistical analyses suggested that the naturalistic problems were more easily solved than computational exercises; there were no statistically detectable differences between accuracy on word problems and the other two contexts.

To investigate differences in mathematical functioning, the researchers analyzed solution strategies children used in the different settings. They concluded that the naturalistic setting induced more informal strategies such as convenient grouping, whereas the written tests induced the children to use school-taught algorithms. In the naturalistic setting, children tended to rely on mental calculations that were closely linked to the specific quantities concerned. The preferred strategy for multiplication problems tended to be a form of chaining additions, as in the example given. On the formal test, children's inaccurate uses of school-taught algorithms (typically confusion of addition and multiplication routines) and failures to monitor often led to absurd answers to given problems. These kinds of errors were virtually nonexistent when informal solution strategies were used, that is, in the naturalistic setting.

Carraher, Carraher, and Schliemann (1987) raised the concern that there was a potential confound in these studies because of differences in the relationship between researcher and participants for informal and formal tests. In the market/street setting, children perceived the researcher as a customer and, thus, had no reason to be anxious or nervous as was possible in the school setting (although there were no overt signs of anxiety during formal testing). To account for this, T. N. Carraher et al. (1987) adapted the earlier design to gain more experimental control and to streamline the interview process. For this study they selected grade 3 students ranging in age from 8 to 13 years, asking them to solve ten problems in each of three forms: (a) in a simulated store condition, in which the child played the role of storekeeper and the researcher played the role of customer; (b) embedded in word problems; and (c) as computation exercises. Arithmetic computations were matched across students for the different conditions. Although this design was not faithful to the children's actual performance in the markets and on the streets where they worked, it provided support for the earlier naturalistic findings because similar results were obtained in both cases (this bridging of laboratory and field research is essential according to Scribner, 1984a).

During the simulated store setting, concrete objects (cars, dolls, pencils, marbles, etc.) were spread out on the table, but there was no money available. Paper and pencil were available in all three contexts. The average frequencies of correct answers in each of the settings were as follows: simulated store 57%, word problems 56%, and computational exercises 38%. Statistical tests revealed that performance on simulated store questions and word problems was higher than on computational exercises (c.f., T. N. Carraher et al., 1985). The word problems and simulated store questions were more often solved with oral procedures, whereas written solutions were preferred for computational exercises. Most written solutions used school-taught algorithms, although often inappropriately (e.g., confusing addition and multiplication procedures).

As with the first study (T. N. Carraher et al., 1985), it was clear that computational problems elicited school-type algorithms, and these algorithms were far less likely to be correct than oral procedures. Based on these results, T. N. Carraher and colleagues (1987) postulated five generalizations about the kinds of heuristic procedures children use in practical settings (in this case, the simulated store and word problems):

- (1) Children altered problems to work with quantities that were easy to manipulate, an approach Reed and Lave (1979) called “manipulation of quantities” (in contrast to the “manipulation of symbols” entailed in algorithmic procedures).
- (2) Children considered the specific quantities to be worked with and considered solution options as they proceeded, rather than adopting a single uniform strategy for all problems.
- (3) Children preferred dealing with hundreds, tens, and then units—the opposite direction from that used in standard algorithms (except division).
- (4) Children generated solutions that, even when incorrect, were sensible, owing to their continuous monitoring of quantities during computation.

- (5) Children took advantage of known number facts by working mentally with quantities that would end in zeros if written, thus easing the processing load. (Zeros often provide complexity in written computations.)

These generalizations highlight the sensible and flexible nature of non-school mathematics. The children had a number of available strategies for solving practice-based problems, and they flexibly selected among these strategies to deal with constraints and affordances of particular situations. They capitalized on known number facts (landmarks) that helped them to navigate the conceptual domain (environment) of mathematics (see Greeno, 1991).

Like the research team at Federal University of Pernambuco, Saxe (1988a, 1988b, 1990, 1991) also studied child candy sellers in northeastern Brazil. The candy sellers in these studies, in contrast to those previously discussed, were totally responsible for purchasing candy at wholesale prices, determining prices for customers, and selling candy in the streets. Saxe was interested in children's mathematical problem solving in relation to the "web of such socio-cultural processes as an inflating monetary system, practice-linked conventions, and patterns of social interaction" (1988a, p. 15).

Saxe attempted to document four areas of candy sellers' mathematical understandings: "(a) representation of large numerical values, (b) arithmetical manipulation of large values, (c) comparison of ratios, and (d) adjustment for inflation in wholesale to retail markups" (1988a, p. 15). He interviewed urban sellers, urban non-sellers, and rural non-sellers of various ages and levels of education to identify influences on mathematical performance of schooling, age, candy-selling practice, and exposure to a monetary economy.

His results suggested that candy sellers' knowledge was influenced by an interplay between social and developmental processes:

[T]hese children construct and operate on mathematical problems that are influenced by social conventions (like pricing ratios), artifacts of culture (like the inflated currency), and social interactions (like assistance with computations provided by clerks). Moreover, the way these social processes

influence sellers' construction of mathematical problems shifts with their ages. (Saxe, 1988a, p. 19)

Younger sellers (age 5-7) used single ratios that entailed simple computations with customers, and received considerable assistance from retailers, older sellers and family members. Older sellers (age 8-13), on the other hand, used multiple ratios and calculated complex problems totally on their own. Schooling influenced strategy selection for many of the tasks, but had no effect on accuracy of solutions. Similarly, practical experience influenced strategy selection—sellers tended to use regrouping strategies, whereas non-sellers used school-taught algorithms—although accuracy was comparable.

All of the research on Brazilian candy sellers suggests that children do invent and use their own mathematical procedures. Schooling and practical experience influence strategy selection, but not necessarily accuracy. Setting influences both strategy selection and accuracy. In all cases, solutions generated in non-school settings were at least as accurate as those generated in school-like contexts. As a further example of this research, let us now look to American adults' practice-linked mathematical performance.

The Industrial Literacy Project

Scribner and her colleagues (Scribner, 1984a, 1984d, 1985) examined arithmetic practices of workers in a dairy factory. Their overall research aim “was to analyze the relationship between cognitive operations and behaviors in some of the principal work tasks in the dairy” (Scribner, 1985, p. 200). They conducted several studies to examine influences of job-related actions on the acquisition and organization of knowledge. Their research focused on four central tasks in the dairy: inventory counting, product assembly, pricing delivery tickets, and using computer forms to represent numbers. The research followed a three-phase pattern progressing from observations to simulations to laboratory experiments, with some movement back and forth between phases.

The first study (Fahrmeier, 1984) provided a focused analysis of inventory counting procedures observed in the warehouse. Taking inventory involved carefully counting the number of products and accurately recording these amounts on the proper forms.

Performing this task in a densely packed warehouse where some cases were hidden from view and boundaries between different products were obscured demanded ingenuity on the part of the inventory workers. As Fahrmeier (1984) explained, rather than counting individual cases, workers invented a number of highly efficient counting strategies:

The goal of inventory is to count the product in case amounts. Yet we never observed an inventory man using the single case as an operational unit of count. In all instances, inventory men reached the goal of a case count indirectly by working with units of other sizes. Their countables were, variously, rows of stacks, single stacks or case-multiples in partial stacks.
(p. 10)

Furthermore, these counting strategies were “employed flexibly, but not haphazardly” (Fahrmeier, 1984, p. 10) depending on the particular situation. Strategies were adjusted to the size, regularity, and location of the array of cases in an effort to respond to management-imposed goals of counting accurately and worker-imposed goals of simplifying the task.

A second study (Scribner, 1984c) examined product assembly in the dairy warehouse to see how practice-related knowledge influenced action. Specifically, Scribner investigated assemblers’ strategies for filling product orders, a task which entailed gathering information from order forms, locating exact amounts of required products in the warehouse, and sending these products to the loading platform. Orders were given in case equivalences, such that each order was expressed as a specific number of cases plus or minus some number of units. For example, there were 16 quarts in a case, so an order for 18 quarts would be expressed as $1 + 2$ (i.e., 1 case plus 2 units) and an order for 10 quarts would be $1 - 6$ (i.e., 1 case minus 6 units). Thus, to fill an order, assemblers could quite literally take the number of cases and add or subtract units as written on the order form. However, like the inventory workers in the previous study, assemblers used a number of “nonliteral” solutions to accomplish this task. For example:

The order “ $1 - 6$ quarts” (i.e., 10 quarts) recurred six times during the observation. On two occasions, assemblers filled the order by removing six quarts from a full case. Their behavioral moves were isomorphic to the symbolic moves in the problem presentation (subtract six quarts from one case). We refer to these as literal solutions. On two occasions, assemblers took

advantage of partially full cases in the area to modify the numbers in the subtraction problem: they removed four quarts from a partial case of 14, one quart from a case of 11. On the two remaining occasions, they behaviorally transformed the subtraction problem to an addition problem: they made up the required case of 10 by adding two quarts to a partial case of eight and four quarts to a partial case of six. We refer to these as nonliteral solutions.
(Scribner, 1984c, p. 12)

For each of these nonliteral solutions, workers assessed the situation to determine how they could fill the order and minimize the physical effort required. By using nonliteral solutions, assemblers substantially reduced the amount of lifting involved in their work. In this way, the extra effort required to calculate nonliteral solutions and to keep that quantity in mind while walking through the warehouse was justified in terms of physical effort saved. A simulation study designed to compare task performance by people with varying degrees of experience on the task (experienced product assemblers, inventory workers, wholesale drivers, dairy office workers, and ninth-grade students) supported this interpretation. Practice-linked knowledge resulted in a greater propensity to use nonliteral solutions, and this led to savings in physical effort.

Scribner, Gauvain, and Fahrmeier (1984) investigated a second important effort-saving strategy used in product assembly, minimizing distance traveled. During product assembly, workers did not necessarily fetch individual items in successive order from product lists; instead, they purposefully selected two, three, or four items conveniently located together or on some direct path through the warehouse. Assemblers flexibly used their knowledge about product location to organize the items on hand, thus saving themselves almost half the walking distance on a typical shift (10 922 feet versus 20 016 feet if they had collected each item individually in sequential order). The coordination of geometric knowledge and dairy knowledge resulted in optimized task performance.

A fourth study (Scribner, 1984b) investigated methods used by delivery drivers to price delivery tickets. This provided an example of standard “school arithmetic” in the workplace: Drivers entered the quantity of each product delivered, calculated the cost of the product, and totaled the delivery price. To complete this task, drivers frequently used

a “case-price technique,” converting unit number into cases and computing cost per case rather than per unit, even though orders were given in units. With the exception of one new employee, all drivers used the case-price technique on at least one problem. This technique necessitated continual shifts between multiple base number systems to accommodate the varying number of units of different sizes in fixed dimension cases (one case holds 4 gallons, 9 half-gallons, 16 quarts, 32 pints, or 48 half-pints¹).

A number of contingencies influenced drivers’ decisions to use case-price techniques. First, drivers only used case prices when they knew them from memory; even if the case price was readily available on a self-prepared “crib sheet,” drivers did not use this information. Second, case-price techniques were more common for single case orders than for multiple case orders. Third, drivers who typically used calculators tended to continue with unit price calculations even when they knew the case price. As Scribner (1984b), explained, case-price techniques combined number knowledge and dairy knowledge to save effort:

One of the factors moving drivers to take the indirect, case-mediated route to problem-solving was the desire to find the “easiest way.” We pushed the analysis far enough to demonstrate that the “easiest way” was not an absolute, but was contingent on an individual driver’s case price knowledge and calculating device. (p. 25)

These studies support the contention that people do invent and use mathematics in highly effective ways on the job. In all these examples, factory workers relied on their practice-based experience and their “number sense” (Greeno, 1991) to flexibly select from their strategy repertoires. Over time, they developed a strong sense for questions such as how many cases were in an array, how many units were in a case, what products were located close together. They knew their way around the dairy, and around the conceptual domains of taking inventory, filling product orders, and pricing delivery tickets. This number sense allowed them to flexibly select from various strategies to complete their work tasks in efficient, energy saving ways. But mathematics is not

¹Notice that the total volume in one case changes depending on the size of the unit containers.

relegated to school and work only. Mathematics is also used in everyday tasks such as shopping and cooking, the focus of the Adult Math Project.

The Adult Math Project

Dissertations by Murtaugh (1985a) and de la Rocha (1986/1987) form the basis of the Adult Math Project (other reports can be found in de la Rocha, 1985; Lave, 1988; Lave, Murtaugh, & de la Rocha, 1984; Murtaugh, 1985b). These studies investigated arithmetic practices by grocery shoppers and dieting cooks in California. In both investigations, the focus was on relationships between problem formation and problem solution, which the researchers considered as inextricably bound in real world behaviour: "It may be that the way people generate problems largely determines how they go on to solve them" (Murtaugh, 1985b, p. 186).

In the study of grocery shoppers, researchers accompanied each participant on a shopping excursion and asked the shoppers to think out loud as they shopped for groceries (Murtaugh, 1985a, 1985b). The unit of analysis for this study was defined as an instance of arithmetic use, that is, "any occasion on which a shopper associated two or more numbers with one or more arithmetic operation [sic]: addition, subtraction, multiplication, or division" (Murtaugh, 1985b, p. 187). According to this definition, arithmetic was observed on 213 occasions across all 24 shopping trips, that is, for approximately 25% of the products purchased. Supermarket arithmetic was virtually error-free with 98% accuracy, compared to 59% on a paper and pencil mathematics test. Furthermore, supermarket arithmetic performance was unrelated to number of years of schooling or years spent away from school. As Lave, Murtaugh, and de la Rocha (1984) concluded: "To the extent that correlational evidence provides clues, arithmetic problem-solving in test and grocery shopping situations appears quite different, or at least bears different relations with shoppers' demographic characteristics" (p. 83).

In the supermarket, mathematics was predominantly associated with best-buy calculations. In these situations, shoppers used a combination of unit price and ratio

calculations, but only after qualitative considerations about product quality, package size, and available storage space:

This kind of calculation occurs at the end of largely qualitative decision-making processes which smoothly reduce numerous possibilities on the shelf to single items in the cart. A snag occurs when the elimination of alternatives comes to a halt before a choice has been made. Arithmetic problem-solving is both an expression of and a medium for dealing with these stalled decision processes. It is, among other things, a move outside the qualitative characteristics of a product to its characterization in terms of a standard of value, money. (Lave, Murtaugh, & de la Rocha, 1984, pp. 80-81)

As this quote illustrates, the interaction between problem formation and problem solution was not merely a quantitative, straightforward calculation. Arithmetic problems tended to arise and be solved through reformulation rather than through application of school-taught algorithms. Relevant inputs, as well as solutions were negotiable:

Problems begin when the shopper is in some doubt about what item, or how many, to buy. The shopper then calls upon any information that is available and relevant for making the decision, including numerical information. The decision implications of the numerical information may not be immediately apparent; e.g., different prices may be associated with different quantities, thus obscuring which item is the best buy. In that case, shoppers perform some arithmetic operation that will transform the raw inputs into a clearer statement of the problem. Thus, clarifying the problem and solving it proceed hand in hand. (Murtaugh, 1985b, p. 192)

The link between problem formation and problem solving was further investigated in the Weight Watchers (dieting cooks) study (de la Rocha, 1985, 1986/1987). Arithmetic in this study involved mostly measurement situations: "Arithmetic problems arise for dieters in the course of handling individual food items in the relation of package sizes to serving sizes, in the conversion of one measuring scheme to another, and so forth" (de la Rocha, 1985, p. 193). Sometimes dieters used fairly precise measurements, but often they devised tactics and depended on contingencies in the environment to simplify solution procedures. For example, dieters often measured precisely the first time they used a product and then noted how much that amount filled some container or what package size had been used. They would then use that same container or package size as a tool to avoid measuring in the future.

The distinction between precise measurement and simplified tactics arose because of conflicts between dieting objectives and cooking objectives: Precise measurements require time and effort that may interfere with the careful timing and coordination of meal preparation. As de la Rocha (1986/1987) explained, this “is not difficult arithmetic to be sure, but while it is in progress, the salad, the family, and the evening meal await” (p. 134). Resolution of this conflict was influenced most strongly by dieters’ personal preferences along a strategy continuum concerning measurement use. Dieters fell into two distinct groups. One group adopted a “just eat less” strategy. They rarely measured, instead relying on their past dieting experience and using the Weight Watchers’ program as a guide to set upper limits on food portions. Simplified tactics (heuristics, approximations, anything that did not involve precise measurement) comprised 74% of their solutions. The second group was very methodical. They measured most items, using tactics for only 39% of their solutions. For this group, precise measurement constituted a source of discipline which provided the key to their dieting success. As one dieter explained when questioned why she continued to measure her lettuce even though she knew it was not necessary: “I’m getting now where I don’t measure my lettuce, but when I was new to the program still and I wanted to make sure I could keep track of my amounts, it really was just for me. Discipline is keeping track of amounts” (de la Rocha, 1985, p. 196, emphasis in original). Both groups also considered physical characteristics of food and packaging size in their efforts to resolve the cooking-dieting conflict. Solid foods were more often measured, whereas tactics were preferred with liquids. Matches between package size and serving size resulted in increased use of tactics. Thus, dieters’ use of precise measurement was dependent upon the particular context, and arithmetic was used for calculation reasons, as well as discipline in dieting.

The dieters’ arithmetic in the kitchen was very different from what would be expected in a classroom or from their performance on written mathematics tests:

While it would not be accurate to depict the dieters’ problem solving efforts [in the kitchen] as devoid of numerical operations, they used none of the

classroom trappings of problem representation beyond numbers and measuring schemes embedded in rules and devices. Although they were always available, neither pencil and paper nor calculators were employed by the dieters. In fact when they were given identical problems in written form and were then asked to solve them again in the course of preparing food, no one recognized that they were solving the same problem!

(de la Rocha, 1986/1987, p. 148)

These studies reveal that, not only do participants use mathematics in daily life, but they use mathematics in pursuit of multiple goals. In Murtaugh's work, shoppers engaged in calculations at the end of a decision process once they had considerably narrowed their choices. In de la Rocha's study, some dieters used mathematics as a form of dieting discipline. Mathematics, in these cases, was a means to an end rather than an end in itself, as it is in the classroom. The differences in these goals may partially explain why mathematical performance in grocery stores and kitchens is unrelated to mathematics in classrooms for it is well known that goals and task understandings strongly influence problem solving (see Doyle [1983] for a review).

Summary of Research Findings

From this brief sampling of research a few clear themes emerge. First, people can and do "invent" and use their own mathematics and, perhaps more importantly, this mathematics is highly accurate, efficient, and often superior to school mathematics. In almost all of these studies, participants demonstrated extensive use of solution strategies that differed markedly from school-taught algorithms. Out-of-school mathematical problem solving in these studies tended to be accomplished by different routines than school problem solving; there was an emphasis on manipulation of quantities rather than manipulation of symbols (see Reed & Lave, 1979). The production and use of informal, or invented, strategies is pervasive in research on mathematical functioning (see Resnick [1989] for a review). For example, Groen and Resnick (1977) used reaction time to document preschool children's invention of strategies when they were first taught to solve single-digit addition problems. After a period of extensive practice, many of the children developed their own, more efficient procedures rather than relying on the

instructed algorithm. Examples of such personally and socially constructed mathematical rules and systems are common in research, and in classrooms. As articulated by one of the student teachers working with BUGGY, a diagnostic modeling system, "I never realized how many different ways a child could devise to create his [or her] own system to do a problem" (Brown & Burton, 1978, p. 190). In daily tasks, such as dieting and grocery shopping (see, for example, de la Rocha, 1985; Murtaugh, 1985b), adults often construct solutions based on elaborate short cuts which circumvent many of the requisite calculations. Similarly, cross-cultural studies have documented people's abilities to construct their own mathematical systems by integrating traditional mathematical systems with school-taught Western strategies (Brenner, 1985; Reed & Lave, 1979; Saxe, 1985).

These studies suggest that there may be multiple systems of mathematics in the same culture, one related to school and another that flourishes outside school (Nunes, Schliemann & Carraher, 1993). Children develop informal or non-school mathematics when they are confronted with problems for which they do not have school mathematics, or if they have the school mathematics, they do not realize the connection between it and the problems they face (e.g., Ginsburg, Posner, & Russell, 1981; Petitto & Ginsburg, 1982; Saxe, 1985). Informal mathematics develops when there is a discrepancy between people's needs in problem solving and the amount of mathematics they have learned in school. For example, Saxe (1985) found that Oksapmin children with limited schooling invented their own counting systems that combined their traditional body part counting with Western base ten numbers, whereas children with more extensive school experience fully adopted the Western system.

Finally, it can be seen that these two kinds of mathematics do not map well to each other (Resnick, 1987). Students who do well on textbook problems are often unable to use this school knowledge to interpret actual physical events (Resnick, 1983). Similarly, students who perform complex computations at work often perform poorly on corresponding school mathematics tasks (e.g., Saxe, 1991). Research has shown some

limited transfer of out-of-school strategies to the classroom, but transfer of school algorithms to non-school problems seems infrequent (Carraher, Carraher & Schliemann, 1987; Saxe, 1988a; Saxe, 1990). It seems there is little correspondence between functioning in non-school contexts and performance in mathematics classrooms.

Connecting School and Non-School Mathematics

Ethnomathematical research often concludes with suggestions and implications for mathematics education focusing on the usefulness and importance of making connections between in-school and out-of-school mathematics. For example, Masingila (1992, 1993b) concludes with the suggestion that school mathematics curricula should include problems that build on mathematical understandings students bring from their everyday (non-school) experiences and engage students in doing mathematics as it is done in out-of-school situations. Similarly, Carraher, Carraher, and Schliemann (1987) conclude with the following plea:

We suggest that oral mathematics can no longer be treated merely as idiosyncratic procedures nor inconsequential curiosities ... mathematics teachers may profit from becoming acquainted with these procedures. Instead of dismissing them as lesser mathematics, irrelevant to formal learning, teachers can take advantage of this knowledge when promoting the development of their pupils' mathematical skills. (p. 96)

Such suggestions are consistent with the focus of the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). Throughout the Standards, there is an emphasis on making connections with students' daily lives outside schools. Consider as an example, Standard 4 for primary students:

In grades K-4, the study of mathematics should include opportunities to make connections so that all students can—

- link conceptual and procedural knowledge;
- relate various representations of concepts or procedures to one another;
- recognize relationships among different topics in mathematics;
- use mathematics in other curriculum areas;
- use mathematics in their daily lives. (NCTM, 1989, p. 32)

As this standard (and many others) suggest, problem solving in school and out of school provides important challenges for students. NCTM states that teachers must help students connect mathematics inside and outside classrooms. Forging connections between

mathematical practices inside and outside the classroom provides a two-way bridge allowing students to draw on their out-of-school knowledge in the classroom, and their school knowledge outside the classroom, strengthening practices in both arenas.

Students have experience solving mathematical problems outside the classroom, but this knowledge is typically unrecognized and devalued in classrooms:

The informal, sensible methods children have learned outside of school are ignored or discouraged, and little oral arithmetic or substantive discussion is related to the meaning of formal procedures that are taught. Thus, it is not surprising that students spend very little time using their intuition or making sense of what they do in school mathematics; they rarely are expected or given the opportunity to do so. (Lester, 1989, p. 34)

Drawing on students' pre-existing out-of-school knowledge can help develop conceptual understandings when new information is presented in the classroom. Furthermore, making links between school and non-school mathematics may help students to see the relevance of school mathematics to their daily lives. These connections can, therefore, improve mathematical functioning both inside and outside classrooms.

In addition to the cognitive ramifications of drawing connections between in-school and out-of-school mathematics, there may also be positive affective consequences. Validating the kinds of informal strategies that students already know and use may help to alleviate anxieties that often inhibit students in formal settings. For some, there may be a sense of empowerment in learning that they can and do use mathematics. Recognizing their own mathematical prowess outside the classroom may increase their comfort level with more formalized, school mathematics.

It is important to keep in mind that, as Cobb, Yackel, and Wood (1992) stress, the "goal here is not to advocate 'everyday' mathematics in school as an end in itself" (p. 13). Instead, the idea is to help students to understand and use mathematics better in and out of school. To do this, teachers could identify non-school situations that use mathematical concepts to be taught, and use those situations to promote awareness and understanding of mathematics. Nunes (1992) cautions, however, that connections between school and

non-school mathematics must be based on underlying mathematical invariants rather than surface level similarities between situations.

Considering the potential benefits of forging connections between in-school and out-of-school mathematics, it is not surprising that this has become a major focus for the NCTM Standards. However, given the consistency of research showing discontinuities between school and non-school mathematics, it is appropriate to question whether teachers can help students make these connections. If teachers do not make these connections in their own lives (as research with other adults suggests), how can they be expected to help students make these connections? This question prompted the current research effort to document and analyze the mathematical practices of two elementary school teachers across various contexts of their lives, in and out of their classrooms.

The remaining chapters of this thesis describe the steps I took and the understandings I gained in my efforts to reconcile the apparent contradictions between (a) recent research findings that suggest all mathematical activity is fundamentally situated and distributed across physical and social contexts such that there are distinct discontinuities between mathematics as taught in classrooms and as used outside classrooms (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a) and (b) recent proposals for teaching and learning mathematics in schools that have encouraged educators to connect mathematics instruction with students' experiences in out-of-school settings (NCTM, 1989). CHAPTER TWO: CONDUCTING THE RESEARCH outlines the theoretical perspectives from which this study emerged and the methods I adopted to conduct the study. In CHAPTER THREE: GEENA'S MATHEMATICS STORIES and CHAPTER FOUR: NORA'S MATHEMATICS STORIES I present and analyze the mathematical practices of the two participants. Finally, CHAPTER FIVE: CONCLUSIONS AND IMPLICATIONS summarizes the main findings of the study and provides implications for future research and teaching.

CHAPTER TWO

CONDUCTING THE RESEARCH

This case study investigated the mathematical practices of two elementary school teachers in different contexts of their everyday lives, inside and outside their classrooms. Following an interpretive research design (Erickson, 1986; Lincoln & Guba, 1985; Strauss & Corbin, 1990), I collected participant artifacts, conducted interviews, and made observations of the teachers' mathematical practices. In this chapter, I outline the theoretical perspectives that informed the research and describe the data collection and analysis procedures.

Theoretical Perspectives

I consider learning from a sociocultural, constructivist foundation. Like T. N. Carraher (1989), I recognize that "mathematical knowledge is not the result of unfolding of cognitive development but a cultural practice in which people become more proficient as they learn and understand particular ways of representing numbers and quantity and operating upon them" (p. 320). In this sense, knowledge is defined in terms of available resources and practices (see Roth, in press-b). This theoretical perspective guided my research interests and the methods I chose.

Mathematics, like all human activity, is socially situated (Lave, 1988). Therefore, I needed to adopt methodological approaches that recognized the situatedness of teachers' mathematical activities. I found this in naturalistic inquiry (Lincoln & Guba, 1985), also known as interpretivism (Eisenhart, 1988). This approach is founded on sociocultural and constructivist notions. As Eisenhart (1988) has described:

Central to interpretivism is the idea that all human activity is fundamentally a social and meaning making experience, that significant research about human life is an attempt to reconstruct that experience, and that methods to investigate the experience must be modeled after or approximate it.

(p. 102)

Consistent with previous research in everyday mathematics (e.g., Lave, 1988; Scribner, 1984a), I tried to describe mathematical concepts and processes engaged in context, using

thematic analysis following an interpretivist frame (Lincoln & Guba, 1985; Strauss & Corbin, 1990). My intention was to follow the precedent set by researchers who:

have consistently tried to understand mathematical problem solving in the same way as their subjects. They have lived with their subjects, interviewed them, and manipulated their mathematical environments over long periods of time in an attempt to learn how to respond to mathematical problems as their subjects do. (Eisenhart, 1988, p. 110, emphasis in original)

Of course it is never possible to understand “in the same way” as participants. Instead, interpretive research is founded on the idea of trying to see the world from the insider’s perspective (an emic perspective, see Fielding & Fielding, 1986). However, such understandings will always be coloured by the researcher’s perspective, for it is not possible to separate that which is observed from the researcher’s perception of what is observed. Phenomenology calls the inseparable connection between experience and perception of that experience “intentionality” (van Manen, 1990). Researchers are always a part of the social world they study, so all analyses are dependent on the researcher’s perspective (Hammersley & Atkinson, 1983). For this particular study, this means that my perspective colours the interpretations. For this reason I describe briefly my mathematical background and my interest in this research topic, before proceeding to a discussion of the methods I adopted and the data sources I collected.

The Researcher

I have always had a fascination with mathematics and could never understand why more people did not share this fascination. I have been particularly perplexed about the myths surrounding mathematics and mathematical competencies. Mathematics has been treated as an incomprehensible language only available to a select few (see Brookhart-Costa, 1993), yet I see mathematics as integral to all human action. Everyday activities such as baking, shopping, and doing home repairs all build from geometric, arithmetic, and other mathematical concepts. As Zaslavsky (1994) has written, “We all discover mathematical ideas, have mathematical experiences, and formulate opinions about mathematics” (p. 168). Yet, as argued in the opening chapter of this thesis, the

mathematical expertise practiced outside classrooms is seldom recognized or valued. I began this research with an interest in understanding the mathematics practiced in out-of-school settings to set the stage for valuing such mathematical practices. I was particularly interested in mathematical practices of a group of people who have been (by themselves and others) considered non-mathematical—elementary school teachers, in particular, female elementary school teachers.

My own experiences with formal school mathematics have been, for the most part, very positive. I experienced great success throughout my elementary and secondary school education, attaining top marks in most mathematics courses and high standings on province-wide mathematics contests. From a young age, I was always interested in “playing” with numbers and ideas. I received extensive encouragement to pursue a career in mathematics and sciences from my teachers and my oldest brother, and from career surveys administered by my secondary school guidance counselor. My original plan for my undergraduate education was to enroll in a mathematics major program with a teaching option. Instead, I chose to enroll in a psychology major program with the intention of taking enough mathematics and statistics courses to qualify for a mathematics minor, thereby fulfilling requirements for a secondary school teaching option in mathematics. Having made this decision, I approached an academic advisor from the mathematics department during registration week for my first year of university. After explaining my decision and my background, the senior male faculty member advised me to enroll in a mathematics course specifically designed for elementary school teachers, a course that recommended some grade 12 mathematics background. With top marks and awards from all three grade 13 mathematics courses, I argued that this course seemed inappropriate. After some further discussion and input from a second senior male faculty member, I accepted their second recommendation and enrolled in an introductory calculus course for non-mathematics majors (a course that duplicated much of my grade 13 coursework and ill-prepared me for advanced calculus courses). This advising

experience was the first time I felt my strengths in mathematics had been undermined because I was a woman and/or because I was interested in teaching. Many colleagues have experienced far worse discrimination which may be partially accountable for the high anxiety levels and professed ignorance surrounding mathematics.

My interests in mathematics and teaching continued, and informed the direction of my graduate work. In the early stages of my graduate career, I stumbled across the work by Carraher, Carraher and Schliemann (1985) while searching for information on assessing numeracy levels (see McGinn, 1992). This article led me to the ideas of situated cognition and the field of ethnomathematics (as outlined in CHAPTER ONE). From there it seemed like a small leap to the argument I proposed in the first chapter of this thesis: "If teachers do not make connections between their own mathematical practices (as research with other adults suggests), how can they be expected to help students make these connections?" I then sought to find teachers who would be willing to invite me into their lives and their classrooms to see if I could understand this topic a little further. This thesis is the result of that work.

At first glance, the stories I describe in this work may not look like mathematics yet, on closer reading, mathematical concepts and tools are inherent in all of these activities. However, mathematics involves more than the presence or use of mathematical tools; mathematical practices are processes wherein people learn to use mathematical tools and resources by engaging with those tools and resources in socially, situated contexts.² Mathematics need not be formalized or intentional, but can be realized through "intuitive" and unformulated actions. In my work, I have avoided the common practice of privileging formalized and formulated processes over unformulated actions, thus allowing me to develop understandings of all mathematical activity, unconstrained by the traditional preconception that only intentional acts can be considered mathematics. This

²This conception of mathematics fits well with the final internal conception described in Dossey's (1992) review of historical and modern conceptions of mathematics.

notion of mathematical practices fits well with Schoenfeld's (in press) description of thinking mathematically:

Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns—systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically (“pure mathematics”) or models of systems abstracted from real world objects (“applied mathematics”). The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. However, being trained in the use of these tools no more means that one thinks mathematically than knowing how to use shop tools makes one a crafts[person]. Learning to think mathematically means (a) developing a mathematical point of view—valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure—mathematical sense-making.
(as cited in Schoenfeld, 1992, p. 335)

I highlight in this thesis the mathematical concepts, processes, and actions indicated by the participants, as well as providing my own interpretations based on my understandings of mathematics. I draw on my own past successes with the established body of concepts, facts, principles, and skills associated with mathematics, as well as practice-based notions of personally constructed mathematical knowledge. It is not the objects themselves, but the practices involving those objects that qualify as mathematics. I document in this thesis those (formalized and non-formalized) practices foregrounded by participants, as well as those I observed in our time together.

Procedures

Gaining Entry and Securing Participation

I had realized early on that it would be necessary to limit participation to a small number of participants, so I could establish a detailed account of participants' relationships to mathematics within and across varied contexts of their lives. With this thought in mind, I approached one of the faculty members in my department who teaches mathematics teaching methods courses to see if she thought students from her class might be interested in participating in the study. She seemed quite enthusiastic and invited me to attend her class later that week to introduce my study and to solicit volunteers. As added

incentive for participating in the research study, the course instructor suggested that her students could write something about their research participation in place of one of the course assignments.³ Although the students seemed interested in the project, no one agreed to participate at that point. However, the following week two women, Geena and Nora⁴ contacted me about their willingness to participate.

After establishing initial contacts through Geena and Nora's mathematics methods course, we set up individual meetings where I could explain the study more fully and ensure that our interests were mutually compatible. During the first session with each participant, I carefully explained my research interests and my plans for the research project. I felt that providing a full description was important because I wanted the research to be conducted openly, with no hidden meanings or agendas. At this time, both women signed a letter of consent agreeing to participate in the study according to the following description:

The purpose of this research is to investigate how teachers understand and use mathematics inside and outside classrooms. The National Council of Teachers of Mathematics (NCTM) strongly advocates that teachers build connections with "real-world problems" in their teaching, drawing on mathematical understandings students bring from outside the classroom, and facilitating students' ability to solve problems from within and outside school mathematics. This research explores how you make these connections in your life, and how this translates into your classroom teaching and learning.

Your voluntary participation will involve spending time together over the next several weeks as you engage in mathematical activity in three contexts: your elementary school classroom, your university classes (specifically EDUC 475: Designs for Learning—Mathematics [Elementary]⁵), and your daily life outside the classroom (shopping, cooking, building, redecorating, gambling, wherever you use mathematics on a regular basis). I will observe while you engage in actual mathematical activity in these contexts. For example, I might observe while you prepare and teach a mathematics lesson, do an assignment for EDUC 475, or buy groceries. Occasionally, I will talk to you about what I observe and ask you for clarification or further input.

³Other assignments for the course were as follows: select and read a journal article related to the teaching of mathematics and write a short reflection about the article; complete a 1-hour mid-term examination; give a group presentation to the class about some mathematical topic; and write a final paper related to personal interests in mathematics (including lesson plans and/or theoretical analyses).

⁴All proper names (people, cities, and stores) used in this study are pseudonyms.

⁵This was the mathematics teaching methods course.

Initial Interview

After securing informed consent from Geena and Nora, I collected background information and contact numbers from both women. This provided a written record of key points regarding each woman's teaching and mathematics background. Then, I presented the following list of questions that I had generated after a series of brainstorming sessions (alone and with colleagues):

- Why are you interested in this research project?
- What role does mathematics play in your life?
- When and where do you use mathematics? What mathematical concepts and processes do you use?
- Are there any instances where you use mathematics in everyday life outside your classroom? When and how?
- Do you consider these applications when you are teaching or learning mathematics? When and how?
- What is your current position on the importance of drawing connections between school mathematics and non-school mathematics?
- How, if at all, have you attempted to make these connections for your students in your classroom?
- Has teaching mathematics influenced your answers to these questions? Have your courses at Simon Fraser influenced your answers to these questions? What other experiences, readings, discussions, etc. have influenced your answers?
- In general, how do you feel about mathematics?
- What other questions should I ask you?

I described these questions to Geena and Nora as “the kind of questions that I would like to talk about as we spend our time together” (G-1.1, 931021⁶). The questions

⁶Source codes are used throughout to index excerpts in terms of participant (G for Geena and N for Nora), source (interviews are numbered chronologically; other data sources are indicated by the following codes: fn for researcher field notes, nb for participant entries in the research notebook, jn for participants' journal

provided starting points for our discussions, but the interviews continued in an unstructured format.

This first interview, and all subsequent interviews, were conducted in an “interactive, dialogic manner” (Lather, 1991). This format is illustrative of “in-depth interviewing,” as defined by Marshall and Rossman (1989):

The researcher explores a few general topics to help uncover the participant’s meaning perspective, but otherwise respects how the participant frames and structures the responses. This, in fact, is an assumption fundamental to qualitative research—the participant’s perspective on the social phenomenon of interest should unfold as the participant views it, not as the researcher views it. (p. 82)

My intention was that we would discuss when and where participants used mathematics outside their classrooms in the first interview, and that we could select one of these contexts for me to observe during the second meeting. This is, in fact, what transpired during the first interview with Geena. In contrast, Nora and I spent a lot of time during our first interview talking about her family, her teaching, and her mathematics methods course, and waited until our second interview to select an example of non-school mathematics for me to observe. This flexibility was especially important at the beginning of the research as Nora and Geena and I got to know each other (establishing rapport).

During these interviews, our conversation flowed around topics of mutual interest, predominantly those topics related to mathematics, teaching, and learning. The underlying rationale for this approach was an effort to understand and to validate the knowledge participants were willing to share. I felt that it was important for me to share my thoughts and feelings, and that together, we could see what we could learn. People give a lot of themselves when they participate in research studies, and I felt it was imperative that I reciprocate this openness and willingness to share (Marshall & Rossman, 1989). For example, the topic of metric and imperial measures of temperature came up in my conversations with both Geena and Nora. As they explained their

assignments for the methods course, pr for the final project for the methods course, and fm for Nora’s functional mathematics program description), page numbers, and date. For example, the source code (G-1.1, 931021) refers to Geena interview 1, page 1 on 93/10/21.

understandings, I reciprocated with a description of my understandings, as depicted in the following conversation with Nora.

- M: The one thing that I think is most bizarre is, um, with Celsius and Fahrenheit for the temperature. For some reason I always think Celsius when it's cold like around the freezing point, I think of freezing as being zero. But when I think of heat and in the summer, I know what 80 degrees Fahrenheit is like.
- N: (laughs).
- M: So, I'm using the two different systems and I don't really know what a hot temperature is in Celsius. Like I know that something in the thirties is really hot, but I automatically think eighty degrees or ninety degrees.
- N: Yeah, I do the same actually. I find that of any of the metric stuff, the weather is what I can understand quicker. You know when they say, um, the Celsius degrees I can understand it in metric and imperial. Whereas in measurement I can understand it in imperial, but the metric I have to stop and think about it.
- M: Well, I just. I do it. It's partially depending on whether it's hot or cold.
(N-5.4, 940121)

Lather (1991) argued that interactive conversations and sequential interviewing facilitate collaboration and deep probing of issues, and this moves research toward reciprocity. Drawing on readings from Alcoff (1991) and Mishler (1986), I was concerned with allowing participants to take control of the process which gave their words and actions meaning in the context of this study. Throughout the research project I often brought excerpts from transcripts or documents back to the participants to ask them for elaboration or further explanations. In the following example, I had made copies of an excerpt from an interview transcript where Nora had described something she had been reading about using terminology in mathematics and another example where she had written about a similar topic in her final paper for the methods course. I asked Nora to elaborate on what she had said in these excerpts from the data collection.

- M: This is. Here are two excerpts. The first one is from something that you said in our first interview, and then the second thing is something you had written in your final assignment.
- N: Mmhmm.
- M: They're both related to issues of terminology and whether kids know what the words are, which I think it relates a lot to this issue of language and math and making those connections, but it's a different aspect of, than what we've talked about a lot. And I wondered if you had any other thoughts after you read these. If there's anything else
- N: To add to it.
- M: That it makes you think of.

- N: OK....(reading the interview excerpt) All right. I remember reading that, yeah. It's just. I can't really add to what you already said.
- M: OK.
- N: Putting a specific name or label to a particular activity without understanding the reason for it, this particular math activity. I mean, I can't.
- M: OK.
- N: I'm kind of, I'm just sounding like I talk in circles here but I don't have a particular activity to refer to.
- M: I know these little excerpts also are taken out of context so it's kind of hard to figure out.
- N: Think of.
- M: I just saw when I was going through the stuff again, I said, "Oh, here's an issue that we haven't really talked about."
- N: Yeah.
- M: I see it here and I see it here, but we haven't expanded on that. Because that is another important part of connecting language and math.
- N: Right.
- M: But, it's not the same as, you know, learning math through literature.
- N: You know, I can give you an example. For instance, it just happened recently... (N-6B.4-5, 940308)

Nora continued with an example of a child's struggles with terminology that clarified for me what she had meant in the two excerpts. Her clarifications and elaborations then provided additional data for the analysis (see *The Language of Mathematics*, CHAPTER FOUR).

My interviewing combined both formal and informal interviews. I set regular meetings throughout the research with the participants to discuss research questions, emerging analyses, and any issues that arose as the research progressed. Formal interviews loosely followed an interview schedule consisting of a general list of topics or questions to discuss, but new topics and materials were freely introduced by any of the participants (myself, Geena, or Nora) as they arose in the interview setting (Ellen, 1984). As an example, Nora brought a paper she had written in 1986 about her functional mathematics program because she thought it might be of interest to me. This program then became a focus for part of our conversations and in the analysis (see *Using Out-of-School Mathematics in the Classroom*, CHAPTER FOUR). Informal interviews involved questions that arose from the situation at hand during field observations rather than following a prespecified interview schedule. The terms that seem most appropriately to fit

the distinction between these two forms of interviewing are “research as conversation” and “research as chat” (Haig-Brown, 1992). I audiotaped conversations whenever feasible, supplementing this with written notes to help focus interviews.

Research Notebook

I gave each teacher a copy of the interview schedule and a small notebook to record any further thoughts they had about the interview questions. In addition, I asked them to record incidents, processes, concepts, perceptions, and understandings of: (a) school math learning, (b) school math teaching, and (c) non-school math functioning. They were asked to record what mathematical functioning was required, how the problem was solved, how they felt about the problem and its solution, and why they elected to record that problem. I explained that these instances of mathematics in use could come from their own actions or from their observations of others. Sample tasks I suggested included homework or in-class exercises from their university course, activities they assigned in their own elementary school class, tip calculations they made following a restaurant meal, or change confirmations from a purchase. In my introduction to the notebook, I explained to the participants that this was intended as a space for them to record what they wanted to record, they were the ones in control of the notebook, as shown in the following excerpt from my first interview with Nora.

M: I have a little notebook that you might want to record some stuff in and what I'll give you is a description. I just thought this might be one place, I don't know how helpful this is, but I will give you a copy of these kinds of questions that we sort of talked about [the interview schedule from the first interview, described above]. These are sort of some of the issues that I am most interested in. And this is sort of, um, these are some ideas that I threw together [the suggestions of what to record provided above]. Things that you might want to include in this notebook if you want to use it. It's not like this is an assignment and you must start writing in this notebook or something. But if you find that this is something helpful, maybe helpful as you're thinking about your project then you have some place to write things down or however you want to use that.

N: That's a good idea.

M: And then we can share that and maybe I can make photocopies or something like that.

N: Okay. Oh great.

M: So I can see where your thinking is, and use it for your purposes. And to let me see the kinds of things your thinking and if the notebook doesn't seem to be a good place for you to do it then maybe there are other ways for us to do that.

N: Yeah, okay.

M: So that was sort of my suggestion.

N: Okay, great. Oh no, that's great. That's a wonderful suggestion. Oh super, how special (admiring the notebook) Very nice.

(N-1.10-11, 931029)

I retained copies of all entries Nora and Geena made in their research notebooks for document analysis. During subsequent interviews, we discussed and elaborated these entries, providing further data. Both women made entries in the research notebook during the first few weeks of our interactions. After this point they discontinued using the notebook. Although some interesting data was garnered in this way, I did not pressure the women to continue using the notebooks because I felt that this was a research task that might not provide much direct benefit to the women. They were both extremely busy teaching and completing assignments for the mathematics methods course, so I did not want to burden them with more work. I had left the task as optional, and they had elected not to pursue it further; this seemed to be most consistent with my interest, developed from my readings of Alcoff (1991) and Mishler (1986), in letting them take some control of the research process.

Non-School Mathematics

In initial sessions, we discussed how and when each woman used mathematics inside and outside her classrooms (at the university and in the elementary school), and decided together where I could observe this activity. I explained my interest in selecting some mathematical task (or tasks) in which they engaged on a fairly regular basis, and that I would then arrange to go to the site of these mathematical tasks to observe them solving the task "in context."

Nora chose comparison shopping and, on two subsequent occasions, I accompanied her as she (a) compared the prices of pet products in pet stores in her neighbourhood; and (b) compared prices of food, clothing, and household items at an American factory outlet

to prices at local Canadian stores (necessitating a currency exchange). Geena chose baking, so I spent an evening in her kitchen as she prepared two items. Although I had not planned it this way, the chosen activities (shopping and baking) are distinctly gendered activities, providing excellent opportunities to discuss mathematical practices that have been consistently devalued. I return to this idea in the final chapter when I discuss my work in relation to Fasheh's (1982, 1990, 1991) reflections on the mathematics practiced by his illiterate, seamstress mother.

As the women engaged in their selected activities, we, simultaneously, carried on discussions to elaborate the mathematical concepts and processes involved (cf. de la Rocha, 1986/87). In this way, observations of non-school mathematics were "slightly artificial" because the women were not just engaging in the activity, but engaging in my presence as part of this research project. In this way, the descriptions I provide are not of mathematics in some abstract sense, but of situationally appropriate mathematics. For example, Geena was not just baking cookies, but baking cookies to show me her non-school mathematics. At the same time, the ongoing talk during the baking activity was directed to me in an effort to make her actions rationally accountable and to provide information she felt was necessary in the context of the research project. As a specific example, when Geena was measuring the shortening for her biscuits she explained how she would use volume displacement to measure the shortening if she was doing a lot of baking, but "I'm not going to do that because I've got this [box of shortening with a gauge], but I thought that was an important thing to say (laughs)" (G-2.1, 931026). In this way, the situated nature of the women's mathematical activity must be considered.

I documented my observations of the women's non-school mathematics in fieldnotes and with audio and videotapes, as appropriate. I was able to videotape Geena's entire baking session as we worked undisturbed in her kitchen. On the other hand, Nora's shopping expeditions required going to several different stores and it did not seem feasible to audio or videotape in these places of business. Instead, I took fieldnotes in the

stores, and then audiotaped our debriefing sessions back at Nora's home. Even taking fieldnotes in the stores was disruptive to the businesses during our cross-border factory outlet shopping trip. Nora explained the research project to her neighbourhood vendors when we were comparing prices for pet supplies, so I was able to freely take fieldnotes. For our cross-border shopping trip, Nora did not know the vendors, so she provided no explanations. In this way, the vendors were suspicious of my furtive note taking, and in one store we were treated as "corporate spies," that is, the salesclerks thought we were from their Head Office and had come to check their store, so they hovered around us the entire time we were in the store.

In addition to my observations of Nora's shopping and Geena's baking, other descriptions of non-school mathematics came up during interviews, in the research notebooks, and in methods course assignments. I did not observe activity in all these contexts, but I did document the women's verbal and written descriptions.

School Mathematics

I also observed both teachers in their classrooms for one day while they were teaching mathematics and language arts to see how mathematics was integrated into the teaching portion of their everyday lives. I collected lesson plans, materials, assignments, and class planning notes, and wrote field notes during and after my observation periods.

I also collected assignments from their methods course and any other materials they felt were important to their school mathematics. For example, Nora brought me a copy of her Functional Math Program for special needs students that she had designed and taught for several years (the paper she wrote in 1986, mentioned earlier).

Researcher Notes

Throughout the research process from initial planning to final stages of the analyses, I kept a record of my reflections, feelings, reactions, insights, and emerging interpretations. This reflective process is essential to research: researchers must constantly reflect on the self in relation to research because researchers are part of the social world they study

(Hammersley & Atkinson, 1983). These emerging interpretations and reflections provided an audit trail (Lincoln & Guba, 1985) and formed an integral part of the final analysis.

Trustworthiness

The research design included a number of steps to ensure trustworthiness of any data garnered from the study. According to Lincoln and Guba (1985), the following three activities increase the probability of producing credible findings. First, prolonged engagement allowed time to get to know participants and to build their trust as I attempted, with input from participants and others, to construct understandings about the participants' mathematical activities. Through repeated interviews, Nora, Geena, and I explored issues surrounding their uses of mathematics. Together, we tried to make sense of this information. The data collection process lasted six months, with analyses proceeding throughout that time and over the following year. Second, persistent observation through sequential interviewing and multiple research sessions brought depth to the research. The open-ended, unstructured interview format allowed deeper probing of questions to elicit further internal evidence for validation (Ellen, 1984). Third, triangulation of data within contexts by using multiple measures, data sources, methods, and schemes, and by seeking counter examples built the trustworthiness of data (Lather, 1991). However, because participants engaged in different mathematical activity depending on the setting, traditional criteria for establishing the credibility of an interpretation had to be modified. For example, it made little sense to triangulate data if a common structure of the participants' activity could not be assumed. In order to construct an understanding of participants' mathematical activity within and across contexts, the data gathered in each setting were searched for common themes. Emerging understandings were compared across different settings for each of the participants.

Later in this chapter (see also Tables 2.1 and 2.2), I document the steps I followed during data analysis. This was an essential step in establishing the trustworthiness of my

interpretations, for, “Categories do not simply ‘emerge’ from the data. In actuality, categories are created, and meanings are attributed by researchers, who wittingly or unwittingly, embrace a particular configuration of analytical preferences” (Constas, 1992, p. 254). Tables 2.1 and 2.2 along with the accompanying textual descriptions in this chapter illustrate the “emergence” of the final categories I used and how my thinking changed throughout the data analysis stages, thus providing an audit trail for my categorizations (Lincoln & Guba, 1985). When my initial descriptions and explanations were no longer able to adequately describe my understandings, I developed new categories and labels to better fit my understandings, that is, I developed a new “language game” (Gooding, 1992; Rorty, 1989).⁷ Constas (1992), Erickson (1986), Lincoln and Guba (1985) and others have argued that showing category development provides the necessary documentation for readers to assess the integrity and respectability of research, thus, building trustworthiness.

One potential threat to trustworthiness could be the observer-interaction effects afforded by the research design (Lincoln & Guba, 1985). I was directly involved with the participants throughout the research, and my presence no doubt had an influence on the way participants engaged and described their mathematics. As Hammersley and Atkinson (1983) accurately state, researchers cannot avoid having an effect on the social phenomena they study. Rather than engaging in futile attempts to eliminate these effects, I sought to understand them. I recognize the situated nature of all mathematical activity, and, in particular, I recognize that the mathematical activity I observed in this research was fundamentally situated in the context of my presence and the women’s interpretations of the goals of this research project.

In addition to these activities to increase the probability for arriving at credible findings, it is important to have participants themselves verify the credibility of findings

⁷This idea of appropriating a language game was evident in an e-mail message I received from Dr. W.-M. Roth (16 May, 1995) about a near-final version of my chapter about Geena. He wrote, “This is great. This shows that you appropriated a language game—you populate it with your own intentions.”

(Lincoln & Guba, 1985). This is primarily ensured through member checking where participants respond to and question data, analytic categories, interpretations, and conclusions. Geena and Nora were asked (and encouraged) to be involved at all stages of the data analysis, from the first day in the field until final reports were written. During data collection, I asked many clarifying questions and showed Geena and Nora outlines for what I proposed to write. Through our discussions, I revised and updated my initial understandings to reflect their comments. However, as the research progressed it became increasingly apparent that Nora and Geena's assistance would end when data collection terminated. Both women were leading busy lives, teaching and taking courses at the university, and had little time to read drafts of my analyses.

I was, however, able to use peer debriefing procedures to help ascertain the viability of my emerging understandings. Throughout the many months I worked on this thesis, I was able to share transcript excerpts, personal anecdotes, and many drafts of various bits of analyses with my colleagues. My senior supervisor, Dr. Wolff-Michael Roth spent much of his time pouring over transcripts and drafts of papers I had written. The teachers/graduate students in my thesis research group⁸ were infinitely patient and asked many clarifying questions, and relayed experiences and beliefs that helped to shape the analyses. Conversations with colleagues in hallways and offices, over the photocopier, via e-mail, and at educational research conferences forced me to clarify my understandings and elaborate the procedures I followed.

As will be seen in the following results chapters, the research design produced rich, thick description (a term popularized by Geertz, 1973). As Lincoln and Guba (1985) explain, thick description provides the empirical evidence necessary to form a base for judgments of transferability. According to the tenets of naturalistic inquiry, one cannot

⁸A group organized by Dr. Roth, comprised of Dr. Roth, myself, and four mathematics teachers working on their Masters theses in mathematics education. We met on a regular basis to work together on our research projects, acting as critics, cheerleaders, or research assistants depending on the circumstances. See Roth and McGinn (in press) for more information about this research group.

ensure the applicability of generalizations from research findings. Instead, in the following chapters, I attempt to provide sufficient detail for readers to make their own similarity judgments between contexts to see if these findings are transferable.

Data Sources

From the procedures section it is clear that this thesis draws from the primary methods of data collection used by ethnographers: observation, interviewing, document examination, and researcher introspection (Eisenhart, 1988). These methods provided various sources of information related to the research questions and allowed multiple perspectives and triangulation of data.

Meetings were conducted on an ongoing basis from October 1993 through February 1994. Overall, I met with Nora eight times for a total of 16 hours, and with Geena six times for a total of 10 hours. These meetings were held at Geena's home, Nora's home, my home, the university, in the women's elementary school classrooms, at stores (for Nora's comparison shopping), whatever location made the most sense for the planned activities. Time and location were decided based on mutual convenience.

Audiotaping, as well as researcher field notes, were used extensively to document observations and interviews. Audiotaping provided a permanent record of interactions which could then be reviewed and analyzed many times (Erickson, 1986; Marshall & Rossman, 1989; Merriam, 1988). To capture Geena's actions in the kitchen, I also used videotape. My intention had been to videotape mathematical practices across contexts, but this was not feasible. Videotaping in classrooms would have necessitated consent for all students in the class (from students, parents, principals, and school districts) even though my focus was on the teachers. Videotaping Nora's comparison shopping would have been too disruptive to the stores where we went. The audio and video records were extremely helpful when new hypotheses and theories emerged later in the research process, as I was then able to return to earlier segments to check the consistency of my interpretations.

I transcribed all audio and video recordings, and had these available to share with participants and others (with the participants' consent). As the data analysis proceeded I shared transcript excerpts and my emerging understandings with Dr. Roth and our thesis research group. This provided multiple interpretations and layers of interpretations throughout the research process, thereby strengthening the analysis. As Eisenhart (1988) explains "in ethnographic research, the more perspectives represented, the stronger the research design, because each additional perspective contributes to a more complete picture of the scene of interest" (p. 106). It is also possible that these alternative perspectives could have too much influence over the interpretations, so I have tried to document throughout the results section such major sources of influence. As an example of this documentation, I point out that I was alerted to this potential threat by Dr. Celia Haig-Brown. However, documenting these influences was an exceedingly difficult task. Many of the ideas were generated as a group, and no particular originator could be determined. Together, we were developing a community of practice and within that environment there were many "ideas in the air" (see Schoenfeld, 1989).

Data Analysis

Data from observations, conversations, documents, and researcher introspection were analyzed by following examples from previous research (e.g., Lave, 1988; Scribner, 1984a) and drawing on principles of interpretive research (Erickson, 1986; Lincoln & Guba, 1985; Strauss & Corbin, 1990). Data collection and analysis proceeded in an ongoing fashion (Merriam, 1988). I began analyses with the first notes, continuing in a reflective, dynamic manner—categories emerged and changed as data collection and analysis proceeded (such changes are documented in the next sections). I continuously compared and revised categories and began early trying to understand Nora and Geena's mathematical activity across varied settings and situations. I used my field notes, audio and video tapes, transcripts, teachers' notebooks, and input from others (participants, as

well as the thesis research group) to try to piece together a holistic picture of the participants' mathematical activity within and across contexts.

Consistent with my interest in keeping the research open, I encouraged both women to be involved in all aspects of data collection and analyses, but other contingencies (busy lives, research deadlines, and missed connections) prevented Nora and Geena from engaging in much of the analysis process for the research project. Their participation terminated before final drafts of these chapters were available for their input. However, both women felt that they had gained something from the data collection stage (and accompanying initial stages of analyses). For example, Geena explained during our initial phone conversation, "I have a lot of ideas. I think we can both get something out of this." Then, in our final meeting, she explained how the methods course and the meetings with me had reinforced what she was doing in her classroom, and everything had tied together very well for her. She left her course and the research project rejuvenated and excited about her teaching.

Developing Themes

Once data collection ended, I turned my full attention to data analysis. I read and re-read all transcripts, documents, and field notes accumulated over the course of the data collection stage. After several readings, I started marking the transcripts with themes that seemed relevant. These themes focused on my predetermined twin interests in (a) where mathematics is used and (b) how mathematics is used. This process resulted in a long list of themes that provided an overview of the information I had collected, but did little to help me make sense of this information or be able to communicate it. In particular, I was most concerned that I had not adequately discussed the processes, but had focused mostly on contexts for mathematics use.

Narrowing the Focus to Geena's Baking

I knew from the outset that, given the nature of the research project, it was probable that I would collect very different information from the two participants, such that any

comparisons between them would be minimally informative. As the research progressed, I realized that given their current goals, interests, and teaching assignments, comparisons between the two women could result in inappropriate and inaccurate impressions. In particular, Nora and Geena enrolled in the mathematics methods course for very different reasons and subsequently had very different experiences in the course. For these reasons, it made more sense to concentrate on the two participants individually, and to only discuss any common themes or differences briefly. Similar arguments could be made for separately discussing school and non-school mathematics. Thus, I decided to narrow my initial focus to just one participant (Geena) and to confine my attention to a description of her baking, since this seemed to provide the richest information about non-school mathematics.

I tried writing detailed descriptions of the baking evening, drawing on past research and showing what Geena had done. After various iterations of these descriptions, I ended up with several “stories” about Geena’s mathematics. These stories included descriptions of Geena’s method for doubling ingredients for a biscuit recipe, her flexible adaptations of a written recipe, her solution to cookie dough that was too dry to roll-out, and her use of non-standard measurements. In addition, although I did not finish writing the story for some time, it was apparent that an additional story was needed about drawing connections, for this was a theme to which Geena kept returning throughout our interactions. As Erickson (1986) has argued, writing stories (or narrative vignettes) helps to stimulate analysis by forcing analytic choices. Writing the stories of Geena’s mathematics forced me to select episodes that I found compelling and to explain why I found them compelling. Van Manen (1990) describes writing as the method of phenomenology, and possibly of all research in the social and human sciences.

After writing these stories, I had to prematurely write a three-page abstract of my research findings as a proposal for a conference presentation. This required me to explain the connections between the stories I had written, and resulted in the following list of

dimensions relevant to Geena's mathematical activity: calculating new quantities, measuring, establishing the precision and appropriateness of measurements, following plans/algorithms, and finding and establishing the appropriateness of solutions. To economically highlight these dimensions for the conference proposal, I selected one excerpt from the story about Geena doubling ingredients to bake her biscuits. This excerpt clearly related to all five dimensions. Counting emerged as an issue of relevance as I considered what I had written, but I had not yet created a category for this.

Although this process worked well for the abstract, it did not allow full description of Geena's mathematics, so I was left with the problem of clarifying the links between the dimensions and deciding how to best present results from the full research project. The chronological stories of Geena's baking provided a disorganized format for presenting research findings, so I began to think about how I could break the descriptions up into assertions, allowing me to present data, analyses, and discussion for each theme separately (Erickson, 1986). As I considered this task, it became clear that following plans/algorithms was an overriding concern, within which measuring, counting, calculating new values, etc. were situated actions. This understanding developed from my considerations of how my findings related to Roth's (1994) model of situated cognition and Suchman's (1987) work on following plans. As a test of the fit, I tried mapping my findings onto Roth's model, but I could see that this did not present all I wanted to say. Instead, it seemed more appropriate to consider how the following of plans was a self-regulated activity with multiple iterations of goals, actions, and monitoring. With this in mind, I turned to Butler and Winne's (in press) model of self-regulated learning and explored the fit between my findings and their model. This seemed to illustrate the evolving nature of Geena's mathematical activity as she baked the biscuits and cookies/squares, but again it did not capture all I wanted to say. In the end, I turned away from both models and focused exclusively on my data. I decided that the most important

thing to consider was that Geena's activity revolved around her flexibly modifying plans, a notion that was also prevalent in Suchman's (1987) work.

Expanding to Consider School Mathematics

While writing about Geena's non-school mathematics I had been keeping notes about how those dimensions were relevant to Geena's school mathematics, but had not pursued this area of investigation fully. I began writing stories about Geena's school mathematics, and found that important themes were making sense⁹, using physical representations, getting an answer/stating a solution, and recognizing multiple solutions. Each of these themes are described and elaborated in the chapter on Geena's Mathematics Stories (see CHAPTER THREE).

Geena's School and Non-School Mathematics

Next, I tried drawing links between these categories and across the school and non-school contexts. This is when it became very clear that similar categories were appropriate to describe both school and non-school mathematics. The final set of categories to describe all Geena's mathematical activity were: Flexibly Modifying Plans, Making Sense, Using Physical Objects, Stating Solutions, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Your Work, and Drawing Connections. Table 2.1 presents the chronological development from my initial stories through the dimensions of Geena's non-school mathematics and themes prevalent in her school mathematics to these final categories.

Next, it was important to organize these categories and define links among them (see Tables 2.1 and 3.1). As described above, *Flexibly Modifying Plans* seemed to frame the self-regulated nature apparent in all of Geena's mathematical activity. Further, *Making Sense* was an over-riding concern in all of Geena's mathematical activity. Her efforts to regulate her behaviour (while flexibly modifying plans) were all centered around the

⁹By "making sense" I refer to Schoenfeld's (1992) notion of mathematical sense-making as developing competence with mathematical tools and using those tools to understand the structure of mathematical ideas.

Table 2.1 Documenting the “Emergence” of Categories to Describe Geena’s Mathematics

Stories of Geena’s Non-School Mathematics	Dimensions of Geena’s Non-School Mathematics for a Conference Abstract	Stories of Geena’s School Mathematics	Final Categories of Geena’s School and Non-School Mathematics
Flexible adaptations of a written recipe	Following plans/algorithms	Making sense	Flexibly modifying plans
		Making sense	Making sense
		Using physical objects	Using physical objects
Use of non-standard measurements	Measuring		Measuring and calculating new measures
	Establishing the precision and appropriateness of measures		
Doubling ingredients for biscuit recipe	Calculating new quantities		
	Counting		
		Multiple Solutions	Recognizing multiple solutions
			Checking work
Solution to too-dry cookie dough	Finding and establishing the appropriateness of solutions	Stating solutions/Getting an answer	Stating solutions
Drawing connections			Drawing connections

importance of Making Sense. Using Physical Objects, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Her Work, and Stating Solutions are more specific mathematical activities in which Geena engaged. These five categories are presented according to the typical chronological order for solving one problem. For example, when Geena was preparing the squares, she used her grater (*Using Physical Objects*) to add “this many scrapes” (*Measuring and Creating New Measures*) of fresh nutmeg. She then explained how she had in other cases used a measuring spoon to measure pre-grated (by her or from the store) nutmeg (*Recognizing Multiple Solutions*). She also explained that she used a second measurement based on her sense of smell to confirm the appropriateness of the nutmeg measure (*Checking Work*), but that the real test was in the tasting (*Stating Solutions*). Finally, *Drawing Connections* is the category that seemed to summarize all that Geena and I did together, so I use this category to summarize my understandings about Geena’s mathematics.

Having established the eight categories to describe Geena’s mathematical practices in and out of her classrooms, I turned to the task of pulling my stories together into a draft of a chapter about Geena for this thesis, as well as a paper for the conference for which I had written the proposal described earlier (see McGinn, 1995).

Categories of Nora’s Mathematical Activity

At this point, I turned my attention to Nora’s mathematical activities. Again, I had read through all data sources repeatedly trying to make sense of the data I had collected. During the final stages of the data collection, I had also compiled a proposed outline for the thesis which I had discussed with Nora. This outline focused on the connections that Nora drew among different contexts for mathematics. In particular, I had indicated that three sections would be needed to describe her mathematical connections: language and mathematics, incidental mathematics, and everyday mathematics. I set this outline aside to start writing stories about Nora’s non-school mathematics (consistent with my approach to Geena’s stories). In particular, I wrote stories about Nora’s pet shopping and

her cross-border shopping. These, I considered in relation to the three sections I had proposed, and realized they fit under the heading of everyday mathematics. Next, I turned my attention to Nora's school mathematics. I wrote stories about her functional mathematics program, the mathematical games she played in her English as a Second Language classroom, and her final project in the methods course describing her calendar mathematics. As I considered what I had written, I realized that none of these stories focused on what could actually be considered school mathematics. Because Nora was not teaching mathematics at the time of this research study, there was little data to reflect specific school mathematics practices. It was evident that a direct comparison between school and non-school mathematics would not be possible or helpful in the analysis of Nora's mathematical practices. Instead, I returned to the three kinds of connections proposed in my earlier outline and matched the newly written stories to that framework. These three categories seemed to provide a good organizational format, but they needed to be considered in relation to Nora's goals and her understandings of mathematics that informed the connections she made. As I continued writing, I refined my understandings and determined that goals were prevalent throughout the descriptions, but that a separate category was needed for mathematical sense, a category I considered to include descriptions of Nora's "number sense" (Greeno, 1991) and her understandings of what counts as mathematics. A further refinement came when I realized that recognizing incidental mathematics was a different kind of connection than the other two. Incidental mathematics was at the heart of all the connection Nora drew because she needed to recognize incidental mathematics to present an activity as an example of her mathematics. Table 2.2 presents the chronological development from the proposed outline through the various stories to the final categories used.

From these categories, I created a four-faceted model to reflect Nora's mathematics across contexts (see Figure 4.1). This model highlights the centrality of Nora's *Mathematical Sense*. Her (a) number sense and (b) understandings of what counts as

Table 2.2 Documenting the “Emergence” of Categories to Describe Nora’s Mathematics

Sections to Describe Nora’s Mathematical Connections	Stories of Nora’s Non-School Mathematics	Stories of Nora’s “School” Mathematics	Final Categories of Nora’s Mathematics Across Contexts
			Mathematical Sense
Language and Mathematics		Playing Games in English as a Second Language Classroom	Connecting Language and Mathematics
Incidental Mathematics		Calendar Math Activities	Recognizing Incidental Mathematics
Everyday Mathematics	Pet Food Shopping Cross-Border Shopping		Using Mathematics Outside School
		Functional Mathematics Program	

mathematics affected what behaviours she presented in the context of our interactions and her descriptions of those behaviours. When asked to describe her non-school mathematics, Nora only described activities which she classified as mathematical. This mathematical sense was central to her *Recognizing Incidental Mathematics*, the second facet of the model. Throughout our time together, Nora emphasized “incidental” mathematics, that is, mathematics that was inherent in an activity, but that was not the focus of that activity. By recognizing incidental mathematics in her activities (teaching, shopping), Nora was able to draw connections between school mathematics and language, and from school mathematics to uses of mathematics outside school. These two types of connections form the third and fourth facets of the model: *Connecting Language and Mathematics* and *Using Mathematics Outside School*.

With these four facets to describe Nora’s mathematical practices, I continued writing stories and shaped these into a draft of a chapter for this thesis. I submitted my draft chapters for both Geena and Nora to Dr. Roth and Dr. Haig-Brown. After several drafts of these chapters, I came to the understandings that I present in the next two chapters of this thesis.

CHAPTER THREE

GEENA'S MATHEMATICS STORIES

In this chapter, I first present a brief description of Geena's mathematical biography and our interactions during the research project. This information sets a framework, for the subsequent descriptions of Geena's mathematics inside and outside her classrooms.

Geena's Mathematical Biography and the Context of our Interactions

Geena was a full-time grade two teacher with seven years of teaching experience (six of which were in grade 2 or grade 2/3 classrooms) and nine years experience as a staff assistant at the local school board office. She had decided to return to university on a part-time basis to complete her degree with minors in English and in Learning Disabilities. As one of her elective courses, she decided to enroll in the elementary mathematics methods course because she "thought [her] math program is kind of boring it needs something to, I want to rewrite it. I want to do something, I'm just not sure, I don't know what, but it's time to do this course" (G-1.3, 931026). As she started getting involved in the course, she found that everything was really coming together for her. From the first assignment, she realized the importance of connections in mathematics, in particular, connecting mathematics to the outside world. This realization was the impetus for her participating in this research project. As she explained:

I had done it [the initial journal assignment] all before I had even thought. That's what triggered me into doing this [agreeing to meet me to find out more about this research project]. And I'm just thinking, "Oh my god, this is just too good to be true. It's all, it's all tying in. Math connecting to the outside world. Oh my god, I guess I should do my major project on this because it just seems to be falling into my lap." You know?

(G-1.1, 931021)

Throughout our conversations, Geena emphasized the connections among her mathematics teaching, her methods course, her non-school mathematics, and her participation in this research project. In the end, she rated the course as excellent and found that both the course and our time together really helped reinforce what she was doing in her own classroom.

I found that any of the work that I had done, all of a sudden it started to tie together. Like when I first went in I thought, "I want to revise my math program. I want to work at it. This is my goal, here." And what I found is that, I didn't get done what I wanted to do, what I thought I wanted to do. I thought I wanted to revamp everything that I was doing. I was getting bored, blah, blah, blah. And what I found I was doing in that course was, I was totally reinforcing that everything that I was doing really was what I really believed in, and perhaps I felt it was boring, um, because I was unsure of myself. I wasn't really bored, I was just unsure.... And then just all of a sudden noticing, and meeting with you and talking about things really helped me.
(G-4.8-4.9, 940202)

Although Geena had enrolled in the methods course with the intention of revamping her entire mathematics program, she realized over the course of the term that this was not what she really needed; instead, she gained confidence in her own teaching and a greater understanding of the theoretical base that supported what she was already doing. She learned to build from her existing program rather than starting from scratch with an entirely new program. In particular, she learned to integrate themes and ideas across contexts, such that she drew many connections among her mathematics teaching, her methods course, her non-school mathematics, and the research project. This integration was accomplished only after a long history of struggling to make sense of mathematics. As a child, she found mathematics unpleasant and incomprehensible. Her background consisted primarily of memorization and rote operations, with few opportunities to build conceptual understandings. She articulated her early frustrations with mathematics in the introduction to her final project for the mathematics methods course:

Math was often a puzzle for me. It was a special secret that the teacher had and only a select few could get in on the secret if they were "smart enough." Sure, I could memorize and do the rote equations, but often not without endless fights and tears with my mom. I was told why it was important to know my addition and multiplication tables, etc., and I could even relate to some "real life situations," but I sure didn't understand what it was, I was memorizing. I just didn't get it!
(G-pr.1, 931206)

Geena finally started to make sense of school mathematics as a 25 year old, trying to teach young children. Throughout the research study it was clear that Geena's struggles to make sense and draw connections with mathematics continued to inform her actions and

her teaching practice, as will be described throughout this chapter. She had always worked hard to make sense of mathematics, but had only achieved success as an adult.

Geena and I met initially to discuss the research project and talk about the contexts where she used mathematics in her life outside the classroom. I gathered evidence of her mathematics inside and outside her classrooms from our conversations, my observations of her activities (including her teaching), her mathematics teaching methods course assignments, and her teaching resources and plans (see Appendix A for a list of all sources of information concerning Geena's mathematics). From our time together, we compiled a list of contexts where Geena used mathematics on a regular basis outside her classroom (see Appendix B). The first observation that can be made from this extensive list is that Geena obviously did use mathematics outside her classrooms, contrary to the existing myth that mathematics is not useful outside the classroom (see Civil, 1990).¹⁰ Second, it is essential to note that Geena recognized that this list was inherently incomplete. Throughout our conversations she adamantly stated that "Mathematics is everywhere," and for that reason a list of all contexts for her mathematics would be infinitely long. This list is, therefore, only a sampling of Geena's contexts for using mathematics outside school. The remainder of this chapter is devoted to an analysis of these samples of Geena's non-school mathematics, plus documentation of her school mathematics.

Geena's Mathematics Across Contexts

To understand Geena's mathematics across the various contexts of this study, I examined the entire data corpus (taped interviews, transcripts of said interviews, observations, research notebook, methods course assignments, and all other documents), looking for themes. From this analysis, the following eight categories emerged: Flexibly

¹⁰Since beginning this research project, I have lost track of the number of people who, when told the topic of my research, have asked, "Well, where do you use mathematics?" This myth often goes hand in hand with a second, equally damaging myth that mathematics is for a special "breed" of people, not the general populace (Brookhart-Costa, 1993; Roth, McGinn, & Bowen, in press).

Modifying Plans, Making Sense, Using Physical Objects, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Work, Stating Solutions, and Drawing Connections (see Table 3.1). These categories highlight the major themes that emerged as I considered how Geena's used mathematics in her life, and they relate to Geena's mathematics both inside and outside her classrooms (For further description of the category development please see relevant sections in CHAPTER TWO). *Flexibly Modifying Plans* seemed to frame the self-regulated nature apparent in all of Geena's mathematical activity; she started with general plans, but these evolved over the course of her activity such that artifacts emerged from interactions with multiple goals, other artifacts, tools, and resources in her environment. Further, *Making Sense* was an overriding concern in all of Geena's mathematical activity. Her efforts to regulate her behaviour (while flexibly modifying plans) were all centered around the importance of Making Sense. Using Physical Objects, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Work, and Stating Solutions are more specific mathematical (problem-solving) activities in which Geena engaged. These five categories are presented according to the typical chronological order for solving one problem. For example, when Geena was preparing the squares, she used her grater (*Using Physical Objects*) to add "this many scrapes" (*Measuring and Creating New Measures*) of fresh nutmeg. She then explained how she had in other cases used a measuring spoon to measure pre-grated (by her or from the store) nutmeg (*Recognizing Multiple Solutions*). She also explained that she used a second measurement based on her sense of smell to confirm the appropriateness of the nutmeg measure (*Checking Work*), but that the real test was in the tasting (*Stating Solutions*). Finally, *Drawing Connections* is the category that seemed to summarize all that Geena and I did together, so I use this category to summarize my understandings about Geena's mathematics.

Table 3.1. Map of Episodes Highlighting the Eight Categories of Geena's School and Non-School Mathematics.

Category	Non-School Mathematics	School Mathematics
Flexibly Modifying Plans	She had multiple goals that provided a general plan for her baking, but the plan evolved over the course of her actions.	She designed the unit plan, but ended up using only a few exercises.
	She used the recipes as guides, but they underspecified the necessary actions (She substituted ingredients, doubled measures, used approximate measures, abandoned cookies to create apple squares).	She took time to discuss mathematics across the curriculum (not tied to specific time frame). Students did not need to use manipulatives to complete their worksheets.
Making Sense	She defined mathematics as a "way of thinking," not calculations.	School mathematics was a puzzle for her as a child.
	She could operate in multiple measurement systems, but did not convert.	She and her student used the disk abacus to make sense of the subtraction with borrowing algorithm.
		She sent students to consult the commercial calendar to find out why the month did not start on Sunday.
		She tried to pick up on mathematical themes whenever they surfaced in the classroom. Rather than telling students what the length meant, the class laid metre sticks end to end to visualize length of the snake.

(table continues...)

Using Physical Objects	Measuring cups, utensils, shortening gauge, etc. used in the kitchen.	The worksheet exercise in her class used manipulatives.
	She used the gauge to measure the shortening, but described a measuring-cup alternative.	Students cut out and manipulated greater than and less than signs for the bulletin board.
	The 1-cup measure with 1/4-cup increments prompted her strategy for doubling the flour.	She used the disk abacus to understand subtraction with borrowing.
	She used the inside perimeter and thickness of the glass to select a biscuit cutter.	The class laid metre sticks end to end to visualize length of the snake.
		There was a strong emphasis on manipulatives in the methods course.

Measuring and Calculating New Measures	She developed estimation skills in Weight Watchers.	Measurement was the intended focus in mathematics for the month of May.
	She used situationally appropriate approximate measures.	The class laid metre sticks end to end to visualize length of the snake.
	She used repeated addition with commutation to double the flour.	
	She created new measures for the apple squares.	
	She kept a running tally to track her measures.	

(table continues...)

Recognizing Multiple Solutions	She explained conservation of volume as an alternate way to measure shortening.	The math games had multiple solution paths, but only one “right” answer.
	She combined “this many scrapes” with smell to measure the nutmeg.	She used a goal-free worksheet.
	She used different doubling methods for different ingredients.	

Checking Work	When cookies did not work as planned, she checked the recipe step-by-step, three times.	She checked students’ worksheets and called them back if there were any difficulties
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Stating Solutions	She cut the chunk of cheese she needed, but only assigned a number to the volume as an afterthought.	Worksheet blanks had to be completed. All math games had a “right” answer.
	She doubled the flour, but did not label the measure used.	Quiet Math required accurate hand signals.

Drawing Connections	She defined mathematics as a “way of thinking.”	She wove themes across the curriculum.
	She recognized that “mathematics is everywhere.”	She consciously picked up on mathematical themes whenever they arose.
		She designed a home mathematics unit plan.
		Geena and her students graphed information about their lives and their environment.

She read the special theme issue on Connections for her journal assignment in the methods course.

“Connections” were a recurring theme in all of our discussions.

The eight categories match across school and non-school mathematics.

In the following sections, I highlight findings relevant to each of the eight final categories to provide a flavour for Geena's mathematics within and across contexts. Table 3.1 presents a map of the episodes that support each of these categories as an advanced organizer to help the reader navigate through the text. I begin with a description of Geena's non-school mathematics, then her school mathematics, followed by some summary descriptions.

Non-School Mathematics

Geena chose baking as a prominent example of her non-school mathematics, so I spent an evening in her kitchen as she prepared two items. First, she baked biscuits by doubling one of her favourite recipes, substituting for some of the ingredients to match her preferences. Next, she attempted to make roll-out cookies from a standard recipe that she had only used once or twice. After mixing all the ingredients together, she realized that the dough was too dry and could not be rolled and cut into cookies. After checking the recipe and hypothesizing the reason for this "failure," she decided to combine the dough with some leftover apples from her classroom to make apple squares. In addition to the evidence gathered during our baking session, Geena also provided verbal and written descriptions of her other uses of non-school mathematics in her research notebook, her methods course assignments, and in our interviews. The remainder of this section, describes the mathematical activity inherent in Geena's non-school mathematics, focusing on the eight categories represented in Table 3.1.

Flexibly Modifying Plans

The concept of following plans provides an overall framework for Geena's mathematical activity across the various settings observed. In her kitchen, Geena's behaviour was directed by her goals for the activity, which included baking biscuits for herself, baking cookies for teachers in her school, not being wasteful, displaying her non-school mathematics, and talking with me about mathematics and baking. These multiple goals reflect "interpenetration" of different contexts (Lave, 1988, p. 40); Geena's

activities were multiply situated by her different objectives. Although, we came together for me to observe her baking as an example of her non-school mathematics and to talk about this process, the task of baking also introduced considerations of what to do with the baked goods resulting from our evening together. When we were planning our evening, I asked Geena, "What if you're doing baking? Are you going to end up with all these baking soda biscuits?" Geena interrupted me before I could even complete my question, exclaiming, "Oh heavens no. What I'll do with the baking is I will, um, keep some for me, throw them in my freezer, and then I often take some to school. Not for the kids, well sometimes for the kids, sometimes for the staff" (G-1.11, 931021). On our evening of baking, she explained how she would put the baking powder biscuits in the freezer for herself, and take the cookies as a treat for the staff room. When, the cookies turned into apple squares, Geena explained how this would still be suitable because she still had a treat for the staff room, plus she had a "fun" story to share with her colleagues about the intended cookies. Her concerns for not being wasteful were evident in her careful collection of food wastes to take to the school composter, and in her use of the leftover apples for the apple squares.

Together, these general goals provided a plan for Geena's activity, but this plan evolved over the course of her actions. In the context of pursuing her multiple goals (or enacting the plan), other aspects of the environment contributed to the emerging product. Geena used the written recipes as guides to direct her behaviour, but substituted different ingredients, doubled the written measures, approximated measures, and even abandoned the cookies to make squares instead. Consider, for example, the contrast between the written recipe Geena used to prepare her baking powder biscuits (see Figure 3.1) and her actions as she prepared the biscuits.

The recipe (in Figure 3.1) provided a list of ingredients and step-by-step instructions for making the biscuits. However, as with all written instructions, this recipe underspecified the actions necessary to make biscuits (see Amerine & Bilmes, 1988;

Gooding, 1990; Suchman, 1987). The sparse directions could be interpreted in multiple ways, and did not provide adequate information to allow exact replication. It was necessary for Geena to fill in details as she proceeded to prepare the biscuits. As Amerine and Bilmes (1988) explain:

Dealing competently with the instructions requires not just the apprehension of bare imperatives, but an understanding of general relationships and possible connections between a projected outcome and a corresponding course of action, of which the instructions are indexical. (p. 333)

Figure 3.1 Geena's Recipe for Baking Powder Biscuits.

<u>Basic Tea Biscuits</u>
1 3/4 cup all-purpose flour
3 tsp. baking powder
1 tsp. salt
1/4 cup shortening
3/4 cup milk
Stir dry ingredients together. Cut in shortening. Add milk. Knead 20 times.
Roll out with a rolling pin. Cut with a 2" biscuit cutter.
Bake at 450° F for 8-10 minutes.
Yield one dozen.

In the course of her actions, Geena followed some of the written instructions, but modified others, resulting in a uniquely situated enactment. First, she cut and grated some cheese, which she placed on the stove top. Nowhere in the recipe did it mention cheese, yet Geena added this extra ingredient. Then, she measured twice the listed amount of flour into her mixing bowl, explaining that she had decided to double the recipe. Contrary to the recipe instructions, Geena chose to double all the measures. She also substituted whole wheat flour for the all-purpose (white) flour indicated in the recipe. At that time, she also measured an additional "about half a cup" of flour, which she dumped at the

edge of her work space. The recipe did not mention the need for flour to roll her biscuits, but Geena knew it would be required, and decided to get it while the flour bag was open. Next, she counted six teaspoons of baking powder into her mixing bowl, followed by two teaspoons of salt. She then mixed these dry ingredients together, as indicated in the recipe. Next, she lined up the gauge on the shortening package and cut the shortening at the half cup mark. Using a pastry cutter, she cut the shortening into the dry ingredients. She then measured one and a half cups of buttermilk, substituting for the regular milk specified in the recipe. She used a fork to form a well in the mixed ingredients and poured in the milk, without any indication from the written instructions. She then stirred all the ingredients with a fork. When thoroughly mixed, she dumped in the grated cheese, dipped her hands in the flour she had set aside, and kneaded the dough. Although the recipe had not mentioned kneading with floured hands, she recognized this as important. She continued kneading the dough for a few minutes, with no reference to the listed "20 times." She then divided the dough into two approximately equal sections, returning one to the mixing bowl and flattening the second on the floured counter top with her hands. The recipe had specified rolling the dough with a rolling pin, but Geena used her floured hands on a floured counter top. Geena then cut the biscuits with a floured glass instead of a biscuit cutter as indicated in the recipe. By substituting ingredients, doubling measures, and approximating measurements, Geena flexibly modified the plan represented by the written recipe.

The modification of plans was even more apparent when Geena attempted to make the cookies. Again, she started with a written recipe (see Figure 3.2), but after mixing all the ingredients together, she realized that the dough was too dry and abandoned the cookies to make squares instead. Geena began by reading the first ingredient on the recipe, butter, but then proceeded to substitute shortening. She used the gauge to measure one cup of shortening, as she had for the biscuits. She then consulted the recipe before proceeding to mash the shortening with a wooden spoon. For the biscuits she had cut the

shortening with a pastry cutter, but this time she opted for a wooden spoon to cream the shortening, which is more faithful to the directions on this recipe (“Cream butter and sugar with an electric mixer.”). As she creamed the shortening, she explained why she was not using an electric mixer as indicated in the recipe, “It says electric mixer but my electric mixer isn’t working very well so I always do it by hand. Not always, but since it hasn’t been working very well, I do it by hand” (G-2.4, 931026). This explanation underscored Geena’s close attention to the details of the recipe, but also highlighted her modifications to the recipe. She not only substituted ingredients (shortening for butter), but she substituted tools (wooden spoon for electric mixer). Similarly, in measuring the sugar, she commented that she did not have dry ingredient measuring cups, so she had to use wet ingredient cups:

Now, if I had them I would use the dry ingredient measuring cups. I just don’t have them, so. They make a difference sometimes. Not enough of a difference for me to go out, it’s not a priority to go out and buy them right now, but if I had them, I would use them for sure. (G-2.4, 931026)

She mixed the shortening and sugar together, cracked an egg into the mixture and then measured in a teaspoon of vanilla. After stirring the mixture, she carefully leveled one teaspoon of baking powder and mixed it in. Then, she carefully scooped out each cup of flour and added it to the mixture. Again, she set aside a bit of flour for rolling, and then mixed all the ingredients together, first with the spoon and then with her hands.

After mixing for a few minutes, Geena looked down at the cookie dough with an expression of concern and announced “This seems awfully dry” (G-2.6, 931026). This assessment of the mismatch between the dough and her expectations for its consistency triggered a “snag” (Lave, Murtaugh, & de la Rocha, 1984) in her performance that led to several attempts to explain this discrepancy. As will be described in the *Checking Work* section, Geena referred back to the recipe three times checking her performance, but could not explain what went wrong with the dough. At this point, she abandoned the cookies and decided to make apple squares instead, as described in the following quote:

Well, it's dry, it's cookie. OK, this is what I'm going to do. I'm going to get my pan and I'm going to put. I think I'll cut up those apples 'cause I wanted to make some applesauce or something out of them because they were going bad at school. So, I think I'll put some apples in. Now, should I put this on the bottom, or maybe I'll do double layer? I'll put some on the bottom and then I'll put some apples and then I'll put some of this on the top.

(G-2.6, 931026)

The too-dry cookie dough provided the bottom layer and a crumbled top layer for apple squares with a filling of sliced apples, egg, brown sugar, cinnamon, nutmeg, and allspice.

Figure 3.2 Geena's Recipe for Roll-Out Cookies

<u>Roll-Out Cookies</u>
1 cup butter
1 cup sugar
1 large egg
1 teaspoon vanilla
2 teaspoons baking powder
3 cups flour
Cream butter and sugar with an electric mixer. Beat in egg and vanilla.
Add baking powder and flour, one cup at a time.
The dough will be very stiff. Do not chill. Divide dough into two balls.
Roll out into a 12" by 1/8" circle on a floured surface. Dip cutters in flour.
Bake on an ungreased cookie sheet at 350° for 6-7 minutes.

The "failure"¹¹ of the cookies highlighted the underspecification of the written recipe. Geena followed the recipe quite carefully, but she did not achieve the predicted outcome.

¹¹The term failure is used with extreme caution, here. Geena failed to achieve the predicted outcome of cookies, but she did achieve (triple) success with her apple squares: they tasted good, the story provided a humorous anecdote for the staff room, and she used up the leftover apples (see Amerine & Bilmes, 1988, for further discussion of success versus failure in instruction-following).

Instead, she flexibly modified the plan provided by the recipe, in relation to her multiple goals, to create the squares instead.

The too-dry cookie dough prompted Geena to abandon the cookies to make apple squares, and her baking knowledge and the available ingredients in her kitchen dictated what she included in the squares. First, she knew that she wanted to use up the leftover apples, so she sliced all of the apples into her mixing bowl. Second, because the cookie dough was quite dry, she thought it would be a good idea to add an egg to the filling to moisten the squares. Because she only had two eggs left, she decided to put them both in to use them up, as well. To moisten the squares even more and also to complement the apples, she thought lemon would be a good idea. After a quick search of her refrigerator, she found that she did not have any lemon, so she decided to skip that ingredient. Next, brown sugar and cinnamon complement apples very nicely, so she decided to include those. While looking for the cinnamon on her shelf, she came across nutmeg and allspice, so she added those complementary spices, too. Geena had started with a general plan, but the product emerged over the course of her baking activity.

The two episodes illustrate the ways in which Geena's artifacts (the biscuits and the squares) emerged from the interaction of available tools (measuring cups and spoons, the gauge on the shortening box), materials (the written recipes, baking ingredients specified in the recipe, substitutable ingredients not specified in the recipe, apples left behind by students in her class), the setting (her kitchen), community standards (taste, cookies should not be made with whole wheat flour), and the artifacts themselves (cookie dough that was too dry). In this sense, Geena started with a general plan, but the products emerged over the course of her baking activity. This evolution of a plan is analogous to the way in which elementary students' bridges and towers evolved from "vague" ideas through interactions with the learning environment (Roth, 1994). A similar process of evolving flexible plans and interpretations of artifacts has been observed in the work of scientists and engineers (Pinch & Bijker, 1984; Starling, 1992; Suchman, 1987). This

flexible evolution of plans was central to Geena's baking: She started with a vague plan which set up an event horizon (Roth, 1994); as she went through her cupboards, specific items became significant with respect to her plan. At the same time, her plan became more concrete (less vague) through her actions. For example, the moment Geena pressed the cookie dough into the bottom of the baking dish, the squares took on a form that could not be undone. In this sense, her actions projected into the future, constraining possible future events and reducing the flexibility of the initially vague plan. However, the fact that actions constrain future actions does not mean that individuals know or even ought to know all the implications of a vague plan beforehand. What makes human action so powerful is that it can be flexibly constituted in situations. Humans do not have to follow specific plans, but we can have fairly nonspecific plans which we realize ("concretize") as a matter of course. Realizing plans in this evolutionary sense represents an accomplishment of sense-making efforts. Rather than following some plan (or algorithm) indiscriminately, people make sense through their efforts to navigate the possibilities available on the event horizon. The next section of this chapter focuses on Geena's sense-making efforts in the context of her non-school mathematics.

Making Sense

In addition to flexibly modifying her plans, Geena had an over-riding concern with making sense of her own actions. Through her descriptions of her activities and from what I observed it was clear that Geena was concerned with making sense of her mathematics, rather than indiscriminately applying algorithms.

In her non-school mathematics, Geena's concern with making sense was evidenced through her focus on mathematics as a "way of thinking" rather than an emphasis on calculations. Geena explained her feelings about mathematics in her initial journal assignment for the methods course, in which she reflected on the article by Corwin (1993):

Corwin's article has helped to reinforce my beliefs that math is a way of thinking. It's around us all the time in our daily lives. We don't just "learn"

Math. We “live” it....Math is more than numbers, equations, and algorithms....Math is a part of everyday life...Mathematics is not just a lesson given at a specific time of day. As we identify that Reading, Spelling and Writing are used constantly in everyday life, so do we experience Mathematics in the same ways. It is because of our mindsets that we may not realize this. We no longer need to set Math aside for manipulation. We need to use Math for understanding. “Mathematics helps (students) make sense of the world (p.339).” (G-jn.1-3, 931018)

Evidence of one of the ways that Geena used mathematics for understanding rather than manipulation came from her description of how she could operate within different systems of measurement, but did not convert between them. When she was measuring the brown sugar for her cookies, which had by that time been modified to become apple squares, Geena used a metric measurement for the first time (150 mL). When I asked her about this in a later meeting, she explained:

Oh, I know why. Because I probably measured it just by, I just grabbed a cup that I measure with and it's easier to measure brown sugar with the metal. I think I must have used the metal cup....And that, I don't have one in cups, it's millilitres. That's why I switched. (laughs) (G-4.2-4.3, 940202)

Geena was quite comfortable using either metric or imperial measurements. For the brown sugar, she used metric because of the tool that was available to her. She liked measuring brown sugar with the metal cup, and that particular cup happened to be metric. She had a strong enough sense of both metric and imperial measures and did not need to convert between the two systems. She elaborated this idea by describing her understanding of Fahrenheit and Celsius, which did not require converting between the two systems of measurement:

See, I don't convert when it comes to. Same as the millilitres and cups, I don't convert. I use, this is what I have, so this is what I use. This is, I know what eighty degrees is, but I also know what 25 and 30 degrees is in Celsius. I know. I don't convert. Well 80 is minus this and do this. I just go, okay if it's a really hot day, 80 degrees is quite a hot day here in Vancouver. If it's 25 or 30 degrees that's really hot here in Vancouver too. If it's minus, um, um, if it's 32 degrees that's cold, if it's zero that's cold, if it's minus five it's cold....And if I had, if I was following a recipe that was in the metric then I would be reading the metric stuff. I wouldn't convert. I would just pull out my stuff and I would measure on the millilitre side. (G-4.3, 940202)

In each of these cases, Geena relied on her understandings of mathematics rather than manipulating numbers or using algorithms. She used available tools and materials (“This

is what I have, so this is what I use.”) and operated in whichever system was convenient. Her descriptions and the activities I observed, illustrated a personal, experiential dimension to her knowing akin to Greeno’s (1991) notion of number sense. She was able to move around in both imperial and metric environments, and could use the resources of both environments. As she explained, “I know what eighty degrees is, but I also know what 25 and 30 degrees is in Celsius.” She had a strong personal sense of both systems of measurement, and found no need to convert between the two. It was not important for her to engage in algorithmic calculations to convert between the two systems (“Well 80 is minus this and do this.”) when she understood both systems individually. This number sense was at the heart of all of Geena’s mathematical activity.

Within her attempts to make sense and to flexibly modify her plans, Geena’s use of physical objects was very important. Her mathematical sense was often framed in terms of available tools, and these tools were critical interactants in the emergence of her flexibly constituted plans and actions. The use of physical objects was also central to Geena’s problem solving activities. For these reasons, I turn now to a description of Geena’s use of physical objects as the starting point in my discussion of her mathematical problem solving activities.

Using Physical Objects

Geena’s problem solving activities in the kitchen relied upon and were constituted by the tools available in her environment. For example, to measure the shortening, Geena used the gauge on the package, rather than a measuring cup as she did on other occasions. She explained this tool selection in the following excerpt:

OK. Now this [shortening] is really easy to measure because it’s got a measure, a tape measure right here. When I’m doing something where, if I’m doing a lot of baking. Quite often as a Christmas present, I’ll do a lot of baking. I buy the big huge tub of Crisco. So when I’m measuring Crisco or shortening whatever, what I do. If I need, I would use a bigger measuring cup, but just say I just need a quarter of a cup, what I would do is just fill, is fill this half mark with cold water. Then I would scoop out of the tub the shortening until it actually reached the three quarters, and then I would have a quarter cup measured. I find that, my Dad taught me that. When you take

shortening and put it in a container and then you take a spatula at it, it gets real mucky and messy and sometimes you're not getting as much.

(G-2.1, 931026)

The gauge was available and it was easy to use. However, if she was doing a "marathon baking session" she would buy a large tub of shortening which would not have a gauge. In that case, she would use a measuring cup and the principle of conservation of volume to measure the shortening. In proposing this alternative strategy, she explicitly had considered tool availability (the large tub would not have a gauge), convenience ("When you take shortening and put it in a container and then you take a spatula at it, it gets real mucky and messy."), and measurement precision ("Sometimes you're not getting as much."). Relative cost and amount of packaging for the large tub versus multiple cartons may have also been considerations in her alternative strategy.

The different tools available for measuring the shortening helped to determine Geena's mathematical activities. Geena measured her shortening according to the units marked on the gauge. If she had used the alternative strategy that she described, she would have used the markings on a measuring cup to measure (a) the amount of water and then (b) the volume displacement as the shortening was added. Calculation of the volume displacement would require mathematical comparison between the measure before and after the shortening was added. In the example she described, if she wanted a quarter cup of shortening, she would start with half a cup of water and then add shortening until the combined water and shortening reached the three quarter cup mark. In this case, one way of considering the mathematical comparison is that three quarters of a cup of water and shortening less a half cup of water equals the desired quarter cup of shortening.

Similarly, Geena's access to a 1-cup measuring cup marked with $\frac{1}{4}$ cup increments may have prompted her repeated addition strategy (described in the *Calculating New Measures* section) for doubling the $1\frac{3}{4}$ cups of flour for the baking powder biscuits. Also, when measuring the brown sugar for the cookies, which had by that time been

modified to become apple squares, Geena used a metric measurement for the first time (150 mL). “Because I probably measured it just by, I just grabbed a cup that I measure with and it’s easier to measure brown sugar with the metal....And that, I don’t have one in cups, it’s millilitres” (G-4.2-4.3, 940202). To cut her biscuits, Geena selected a glass with an inside perimeter adequate to create biscuits of the size she preferred.¹²

M: Now is this the specific biscuit glass?

G: (She dips the glass in the flour and begins cutting out the biscuits)
That’s the one I usually use.

M: Why that one?

G: Because I like them big. (She gestures in a circular motion around the perimeter of the glass)...It’s the right size. Sometimes when I want a little one, that’s okay too and I’ll use a smaller glass, but this is usually the size I use. (G-2.3, 931026)

In all these examples, the available tools shaped Geena’s activity by prompting certain actions and preventing other possible actions. In this way, the tools contributed to Geena’s evolving plans (see *Flexibly Modifying Plans* described above; cf. Roth, 1994), particularly with reference to her measurement procedures.

Measuring and Calculating New Measures

Measuring is one of the six fundamental activities that Bishop (1991) argued are “necessary and sufficient for the development of mathematical knowledge” (p. 32). He defined measuring as “quantifying qualities like length and weight for the purposes of comparing and ordering objects” (Bishop, 1991, p. 32). Measuring occurred frequently in Geena’s mathematics across the contexts I observed. As elaborated above, measuring in Geena’s everyday mathematics was deeply tied to the presence and use of tools in her environment—the assorted measuring cups and spoons, the gauge on the shortening box, the cheese grater, and other supplies she used in her kitchen.

Four interrelated practices were involved in Geena’s measuring: estimating, determining appropriate levels of precision for measurements, calculating new measures,

¹²Further, although Geena did not point out this characteristic, the glass she selected was quite thin, thus better for cutting the biscuits. I owe this interpretation to Dr. Celia Haig-Brown.

and counting as a running tally. These four practices are elaborated in the following sections.

Estimating

For Geena's activity in the kitchen, measuring typically involved estimating and using non-standard measurements, as exemplified when she measured the cheese for her biscuits.

Geena eyed the package of cheese and cut off a chunk. Without prompting, she explained, "I have no idea how much this is, but this is the size of chunk I want." Then she queried whether she still had her scale (a remainder from her Weight Watchers dieting days), but made no attempt to look for it. She quickly changed approaches and told me, "I can figure it out." She held the chunk of cheese she had cut off and marked a smaller chunk with her knife, explaining, "This is one ounce, so I can figure it out." She further explained that her ability to estimate the size of an ounce of cheese was one of the estimation skills refined during her Weight Watchers' experience. She proceeded to mark off further portions of the cheese, counting, "One, two, three,...eight ounces." After this estimation, Geena then grated the large chunk of cheese and set it aside. (G-fn.1, 931026)

In this example, Geena made two related estimates. First, she estimated how much cheese to add to the biscuits. This estimate was enacted when she chopped the large chunk from the cheese block. Second, through her fractionating strategy, she estimated the volume of this chunk of cheese, stating "it's about eight ounces." Her estimations were based on a combination of her cooking, tasting, and dieting experiences. She recalled that a chunk of cheese the size she originally cut was appropriate for her cooking goal to make cheese biscuits. The estimation was influenced by her taste preferences because she knew that she liked biscuits when they had that amount of cheese. Further, her experience in the Weight Watchers dieting program had raised her awareness and increased her ability to estimate food amounts such that she "knew" that her small markings were equal to one ounce chunks. Again, this was an experiential sense of knowing (Greeno, 1991). Her tacit understandings developed out of her Weight Watchers dieting experience and her cooking practice, not through formalized instruction. A similar estimation sense was evident in de la Rocha's (1985) study. There, many of the dieting cooks gained estimation sense as they moved toward core participation in the Weight

Watchers community. The Weight Watchers program emphasizes measuring all food stuffs to keep track of how much of each type of food you are ingesting and the caloric and nutritional content of that food. Initially, the dieters in de la Rocha's study used their scales to measure most items, but progressively they developed a stronger estimation sense and no longer had to rely on their scales.¹³ Similarly, Geena and I had discussed during our first interview how the focus on measurement in the Weight Watchers program had stayed with her over the intervening years:

G: I found that um very much, it [attending Weight Watchers] definitely got me into the measuring and that kind of stuff. And then I just found that all I ever did was think about food. Since I stopped dieting, I lost a lot of weight....And it is basically because I stopped dieting. I thought, "No, I know when I'm full and I know when I'm not hungry," etc., etc. But, I have to admit that it very much got me thinking, even dieting got me thinking in different portions and into measuring portions and thinking about it and maybe not necessarily. But, now I don't. If I want a teaspoon of butter in something, I don't take out my teaspoon and level it, but I have a very good eye for a teaspoon of butter and I know when I'm putting butter on whatever I'm putting it on, I know that that's probably two teaspoons and not one, and I'm very conscious of that.

M: Which that was then influenced by taking it out first and measuring and seeing how much is a teaspoon?

G: Very much. Yes, that's very much. Practicing. (G-1.8, 931021)

As this quote illustrates, the estimation sense that Geena developed during her time in the Weight Watchers program had stayed with her over the intervening years, and continued to affect her performance. Even without her scale, she had a strong sense of how much cheese she had cut.

Central to Geena's estimating was her understanding of the level of precision required for a particular ingredient for a particular recipe. Measurement precision is the topic of the next section of this case.

¹³Some of the dieting cooks continued to use the scale long after they had developed a strong enough estimation sense to render the scale unnecessary. In these cases, the dieters seemed to rely on precise measurements to provide discipline in their dieting. In the words of one respondent, "Discipline is keeping track of amounts" (de la Rocha, 1985, p. 196, emphasis in the original).

Determining Appropriate Levels of Precision for Measures

To measure the cheese she added to her biscuits, Geena relied completely on her estimation skills; she cut the piece of cheese without engaging in any explicit measuring. To measure the flour for her biscuits, she filled the measuring cup, shook the cup slightly and checked the level, but showed little concern for the precision of her measures. Approximate measures are close enough to recipe directions to yield successful performance, therefore validating the situational appropriateness of the level of precision she adopted for her measurements (cf. Lynch, 1991).

Geena used a similar level of precision to measure the baking powder for her biscuits, as the following excerpt indicates:

With that I'm going, it says 3 teaspoons of baking powder, so I will add 6 teaspoons of baking powder. Three, four, five, six (counting teaspoons of baking powder. Not too concerned with leveling spoonfuls, just quickly fills it.) (G-2.1, 931026)

Here, for the baking powder biscuits it was very evident that Geena quickly scooped in the baking powder, paying little attention to the precision of her measure. Contrast this with the level of precision she adopted for the cookies/squares. She scooped out the baking powder with the teaspoon and then carefully leveled it off with a knife. Experience had taught her, as she explained, that baking powder was an important ingredient that could have a dramatic impact on the success of cookies.

Okay, two teaspoons of baking powder. And usually with cookies I usually really level the teaspoon because I find that too much can really make or break cookies, I don't know why. (G-2.1, 931026)

Of course, even this greater level of precision would not be considered exact enough for say a physics experiment where precision is defined by a certain number of significant digits and all measuring implements are carefully calibrated. However, as Geena indicated, more precise measures were not essential. The point is that the level of precision needed to be appropriate to the situation, and, in this case, this match was achieved through an approximate filling of the measuring spoon. As Lynch (1991) has argued, "Folk' measures are performed for all practical purposes and without need for

comparison with a more 'exact' or academic paradigm of measuring" (pp. 91-92). Situational appropriateness is central to human everyday activity and contrasts the indiscriminate accuracy of school mathematics.

Using the precise measures and controlled experimentation that is expected in physics, Kurti and This-Benckhard (1994) have demonstrated that many modifications can be made to a recipe with very little effect. Geena, however, did not need to rely on physicists' forays into the kitchen. She knew from her experience that some recipes and some ingredients demanded more precision than others. Her baking powder biscuits were flexible and did not require much precision. When it came to the cookies, with which she was less familiar, she took greater care with her measurement, especially with the baking powder which "can really make or break cookies." However, she never resorted to precise calibration of her measurements as would be expected (required?) in physics experimentation.

The "academic paradigm of measuring" (Lynch, 1991, p. 92) as required in physics and other scientific pursuits is not only unnecessary for the success of baked goods, but would be difficult, if not impossible, to achieve given typical kitchen tools. Measuring cups and spoons are seldom precisely calibrated, and actions such as shaking a powdery ingredient influence the settling of contents, thus impacting the precision of a measurement. In fact, I recall being admonished for shaking the measuring cup in my elementary school cooking classes because it would lead to an incorrect measure. Geena, on the other hand, consistently shook the cup as she was measuring. It is not clear whether Geena or my cooking teacher was more accurate, but it is certain that such differences would be problematic in physics, but seem not to be so in the kitchen.

In addition to measurement issues related to estimation and precision, Geena's actions also raised algebraic issues as she calculated new quantities. These issues are described in the next section.

Calculating New Measures

An important example of Geena's measuring is evidenced in the new measures she created by doubling the biscuit recipe. The first quantity listed in the written recipe was $1 \frac{3}{4}$ cups of flour, which when doubled would yield $3 \frac{1}{2}$ cups of flour. Transcripts of the videotaped observation and field notes recorded that Geena engaged in the following actions:

One and three quarter cups. (Reading the recipe. She fills the 1 cup measure) I'm terrible with math. If I know the, when I go to double a recipe if I can double it really easily then I do. If I'm not sure then I do, (Adds a second full cup) I put two in because I need two full cups and I need two three quarter cups. So, one three quarters of a cup. (She fills the measuring cup to the $\frac{3}{4}$ line, shakes it slightly, and checks the level, but shows little concern for the precision of her measure.) And then I need one [$\frac{3}{4}$ cup] more (Fills the measuring cup to the $\frac{3}{4}$ line and adds the flour to the mixing bowl).
(G-2.1, 931026)

Here, Geena translated the multiplicative problem, doubling $1 \frac{3}{4}$ cups, into a repeated addition problem. However, her actions showed more than simply repeated addition of the form $1 \frac{3}{4}$ cup + $1 \frac{3}{4}$ cup. She performed a commutation such that the addition was performed as $1 + 1 + \frac{3}{4} + \frac{3}{4}$. This representation led to the solution of adding $3 \frac{1}{2}$ cups of flour, although Geena's method made no explicit reference to this total amount—her calculation of the new quantity was situationally embedded in the context.

Geena's creation of this new measure is reminiscent of the story of the dieter from the Adult Math Project who attempted to measure three-quarters of two-thirds of a cup of cottage cheese. "He filled a measuring-cup two-thirds full of cottage cheese, dumped it out on the cutting board, patted it into a circle, marked a cross on it, scooped away one quadrant, and served the rest" (Lave, 1988, p. 165). This dieter produced "measurements' without requiring a separation between measuring device (or model) and what is measured" (Lynch, 1991, p. 106), and in this way, he actively distributed his calculations in the environment. Likewise, Geena actively distributed her calculations; she used the measuring cup to perform the calculation rather than performing the calculation in her head. It was quite clear that Geena could have performed this

calculation in her head. As she later admitted, she performed this very calculation in other instances, but had simply elected not to do so at this point. As our conversation continued, she indicated, “Half is fine, but when I double three quarters I think that’s one and a half, but also what’s the big deal” (G-2.1, 931026). She treated the problem of doubling one half as a multiplicative problem, but switched to repeated addition to double three quarters. This statement indicates that she did actually know that the mathematical answer to the problem $2 \times \frac{3}{4}$ was $1 \frac{1}{2}$, yet as she exclaimed “what’s the big deal.” It was not important for her purposes (baking the biscuits) to know this amount. Lynch’s (1991) statement about folk measures is relevant to Geena’s creation of new measures, “‘Folk’ measures are performed for all practical purposes and without need for comparison with a more ‘exact’ or academic paradigm of measuring” (pp. 91-92). Geena’s actions embodied the newly created measures, allowing her to bake the biscuits, but labeling the actual measure was irrelevant to her purposes.

After Geena decided that the cookie dough would be used for apple squares, she had to create new measures for all the ingredients for the filling. She wanted to use up all the leftover apples and the two eggs, so those measures were determined by availability of ingredients.¹⁴ Conversely, her decision not to include lemon was based on the lack of available ingredients (“I think I’ll put in a little lemon. I usually put it in apple pie. Do I have any? No I don’t. Never mind, I won’t...Forget the lemon.”). She relied on her memory of other recipes and her sense of smell to dictate how much of each of the spices to include. For the brown sugar, she used a small measuring cup to carefully measure the amount she added, as the following transcript excerpt indicates:

G: Fifty millilitres. (She packs the brown sugar into the 50 mL measuring scoop.) 100 millilitres. (She adds another full scoop and mixes it in.) I think 150 millilitres. That should be plenty. (She adds a third 50 mL scoop). So that’s one hundred and fifty millilitres of packed brown sugar.

¹⁴Certainly she also used her understandings of what would be reasonable amounts for these ingredients. If there had been a bushel of apples or a dozen eggs, she certainly would not have included the entire amount.

- M: How do you know that amount?
 G: Just by looking I guess. I won't know. When we go to taste it if it's too sweet, I'll know to cut back next time.
 M: You're just going by how much is on the apples?
 G: Yep. Basically coating, yep. (G-2.10, 931026)

This excerpt also suggests that, despite her careful measuring, Geena's measure was based on the "whole-person enactment" (Lave, 1992, p. 80) of seeing and feeling the brown sugar coating the apples, rather than the amount indicated by the measuring tool. She relied on her tacit understandings (measurement sense, see Greeno, 1991). In the act of stirring the filling Geena felt the proportions of apples and brown sugar.¹⁵

I turn now to the final category of Geena's measuring, that is, her use of counting to keep track of her measures.

Counting as a Running Tally of Measures

Although, Bishop (1991) listed counting as a separate "fundamental activity" from measuring, I combine the two in this description because Geena used counting as a strategy to keep track of her measures. Measuring individual ingredients for the biscuits and the cookies/squares often required multiple filling actions. In order to keep track of how many filling actions she had completed, Geena often counted aloud. For example, Geena's fractionating strategy to estimate the volume of the cheese for her biscuits included a running tally of the number of one ounce pieces of cheese. Similarly, Geena counted aloud as she added, first, two full cups and then, two three-quarter cups of flour to her biscuits. For the baking powder in her biscuits, she counted out all six teaspoons.

Counting as a strategy to keep track of her measuring actions is the last of Geena's four interrelated measuring practices (the other three practices being estimating, determining appropriate levels of precision for measures, and calculating new measures). However, as a further example of her flexibility, Geena was careful to explain alternate

¹⁵This statement is a modification of Lave's (1992) description, "In the act of stirring the pancake batter I feel the proportions of milk and flour" (p. 80).

ways of measuring. These alternatives are representative of the multiple solutions she recognized for her non-school mathematics.

Recognizing Multiple Solutions

In her baking, Geena's actions served as one embodied solution to the problem of making the biscuits and making the cookies/squares. However, as she worked, she frequently explained alternative solutions she could have adopted, or that she had adopted in the past. To measure the shortening, she used the supplied gauge, but described how she could have used volume displacement instead. To measure the nutmeg for her squares, she used a combined strategy of measuring "this many scrapes" on her spice grater and confirmed the appropriateness of that measure with a second measure based on her sense of smell. To double the flour, she enacted a repeated addition strategy (with commutation), but explained that she could have multiplied instead.

Throughout our conversations, Geena emphasized the existence of multiple solutions and described some of the constraints affecting her selections among those alternatives. As an example of these constraints, consider her response to my question about whether the repeated addition strategy was easier than multiplying to double the flour:

I guess it is easier, but it's not that I can't do it...if I sat down and I calculated it out. There are some recipes where I have actually calculated it out and written it right on the recipe. But I don't always double this, and sometimes I triple it, so I haven't written on this recipe. But, for me, sometimes it's faster, especially if I'm tired, and I'm tired. (G-2.1, 931026)

Here, it became very evident that Geena's goals and actions were influenced by multiple constraints. The particular recipe she worked with was one that she sometimes doubled, sometimes tripled, and sometimes made a single batch. Regardless of which multiple she used, her general approach of using repeated addition would work. For other recipes where she consistently doubled the ingredients, she calculated the values and wrote those down. Her admission of tiredness revealed another potential impact of my presence and the research needs. After a long day in her classroom, normally she would not bother with baking, let alone explaining each of her actions to an onlooker.

As Geena negotiated between multiple solutions, she continually monitored her actions, checking her work on an ongoing basis.

Checking Work

Geena's flexible modification of her plans required continual monitoring of her work. She confirmed the appropriateness of her actions by monitoring the resulting products against the projected outcomes, based on her baking experience and the recipe directions (cf. Amerine & Bilmes, 1988).

This monitoring was critical to problem recognition (see Starling, 1992), as when Geena recognized that her cookie dough was too dry. As she mixed the cookie ingredients together she observed, "This seems awfully dry." Her initial reaction to this observation was to confirm the accuracy of her procedure and hypothesize what might have gone wrong:

Butter, no, egg, vanilla, sugar, baking powder, flour. (Reading the recipe) It's very dry. (Her brow is furrowed) That's interesting. It molds, but I don't know. No, that's going to crumble....Okay, so now what am I going to use it for? Well, isn't that interesting. I wonder why? Flour's old? I wonder if that makes a difference? It shouldn't. I mean, flour sits in flour mills. That's really, it's not going to do anything. Well, let me see. (Molding dough in her hands as she speaks)...I don't understand why. *The dough will be very stiff* (Reading). Well, yes I can see it is very stiff. *Do not chill.* Oh yeah. *Divide it into two balls.* I can't divide it into two balls, I can't get even one. (laughs) *On a floured surface.* Like more flour's going to make this work. Right?! (laughs) Nope....I measured a *cup* [of butter], a *cup* [of sugar], *one large egg, teaspoon of vanilla, two teaspoons of baking powder, 3 cups of flour.* (Reading again) But I've done it before. I mean. Well, the flavour will be there. We'll adapt.
(G-2.6, 931026)

Throughout the interaction, Geena referred to the authority of the recipe, reading the list of ingredients, the directions, and then the amounts, as well as relying on her own knowledge (what cookie dough should look like, where flour comes from, her past success with these cookies). She seemed to be searching her knowledge base for an explanation for the too-dry cookie dough. Her reference to the written recipe is consistent with Amerine and Bilmes' (1988) students who relied on written instructions to classify their results as successes or failures when engaging in science experiments. It was, as Suchman (1987) would say, a retroactive determination of the plan as a fitting description

of action. The actual cookies reached back and made (did not make) the recipe a sufficient description for what Geena had done; the recipe and the product were mutually constituted.

After a few minutes, she turned her focus from “I wonder why?” to the question of “what am I going to use this for?” This problem reformulation provided a solution to her dilemma, and she proceeded to make the apple squares. The problem of too-dry cookie dough was resolved by reinterpreting the dough as a base for the apple squares. However, this reformulation beget new problems as Geena then had to decide what ingredients and what steps were required to make the squares.¹⁶ As documented in the *Calculating New Measures* section, Geena’s solution was again based on “whole-person enactment” (Lave, 1988). Discussion of Geena’s solution process would be incomplete without considering the form of her solutions, as addressed in the next section.

Stating Solutions

In the kitchen, getting an answer typically does not require stating a solution, rather the answer is realized through embodied actions. As described in the *Measuring and Calculating New Measures* section, Geena cut a chunk of cheese for the biscuits and then, as an afterthought, attempted to assign a value to this amount of cheese. Initially, she said, “I have no idea how much this is, but this is the size of chunk I want,” but then quickly went on to “figure it out” by marking off smaller chunks and tallying the number of smaller chunks. This effort to assign a value to the amount of cheese illustrates the impact of the contrived context for her baking. Geena was not just baking, but she was baking as part of my thesis research where the intent was to investigate the mathematics in her everyday situations. My presence and the task were part of the context to which Geena’s actions and her talk were directed. Assigning a value to the amount of cheese

¹⁶The idea of solutions to one problem begetting new problems (or “problem-children,” as Popper argued) was focal to Starling’s (1992) analysis of the space shuttle *Challenger* research and development project. This also highlights the inextricable relation between problem formation and problem solution (Lave, Murtaugh, & de la Rocha, 1984).

served to rationalize her behaviour for me, an observer in her kitchen, rather than serving an integral function in the act of baking.

Similarly, in doubling the $1 \frac{3}{4}$ cups of flour indicated in the biscuit recipe, Geena performed repeated addition with commutation, but made no explicit reference to the total amount of flour she added. In the kitchen, an appropriate solution was to add the required amount of flour rather than name the amount added. This paralleled the episode with the cheese, where Geena began by deciding how much cheese should be added, and only later tried to assign a value to that amount of cheese. In the classroom a numerical answer is required; in the kitchen the embodied action of adding the right amount of flour (or cheese) is required. This distinction reveals an underlying difference in goals for the two settings. As Lave (1988) described, mathematically isomorphic problems presented in different situations may entail different goals requiring different solution processes.

It is essential to consider that although Geena's solution did not include a school math answer, it certainly provided a situationally appropriate solution. Furthermore, Geena's ability to find the appropriate solution to the problem of doubling $1 \frac{3}{4}$ cups of flour indicated that her later statement, "I'm terrible with math" was inaccurate. Within the constraints of the situation, Geena appropriately solved the problem. During our conversations and in her assignments for her mathematics teaching methods course at the university, Geena made several references to her limited mathematical abilities, yet in the day-to-day transactions I observed and in those she described, she accurately solved many mathematical problems as this example illustrates.

Drawing Connections

In our conversations, Geena recognized the prevalence of mathematics in all of her activities. She continually referred to the fact that mathematics was everywhere, that mathematics was a "way of thinking." She elaborated this view in her initial journal assignment for the methods course where she reflected on the article by Corwin (1993), as described in the *Making Sense* section. This view was especially evident in our final

meeting, when I showed Geena the list of contexts for non-school mathematics that I had created from our conversations and the documents she wrote. I asked her if I should add anything else to the list and she responded that the list could be expanded indefinitely because “Math is literally everywhere”:

There is so much of math that is in everything that just happens (snaps her fingers twice) instantaneously and then you move on. You know, that kind of thing. So, it would be like, um, relating your language arts. You could literally videotape somebody all day long and everything that they do, you would pull out, and that’s. I think that’s why I am, I am, so adamant in that, and I’m sure that Nora is the same way. That math literally is everywhere. So you could take all day long and you could pull out math, all day long, as opposed to a specific situation. Yes, you can do specific situations, but I think the specific situations like baking, like banking, those kinds of things, um, the average person probably comes up with, “Oh yeah, that is math, for sure.” It’s the things that happen on a regular basis, all the time. The little things that are minutes long, seconds long, whatever. That they’re not making those connections with.

(G-4.11, 940202)

Clearly, Geena recognized that she was doing mathematics all the time, even if she was not always consciously aware of this fact. She connected mathematics and her everyday activities.

Summary

It is evident from these descriptions that Geena’s non-school mathematics was sensible, flexible, and appropriate. She relied on her tacit understandings and number sense (including her estimation sense and her sense of both metric and imperial measures) to enact solutions to the problems she encountered in the kitchen and in other non-school contexts. She started with vague plans of what she wanted to accomplish, but these plans evolved over the course of her actions based on multiple goals, available ingredients and tools, the setting, community standards, and the emerging artifacts. She used the available tools to negotiate between multiple possible solutions, with little concern for verbally stating the solutions she eventually adopted. Her measurements were enacted with situationally appropriate levels of precision leading to highly successful performance. She engaged in mathematics outside her classroom, and explicitly recognized many of the instances where she did so. However, she was also aware that it would be impossible to

describe all the mathematical ideas that were inherent in her activities, because “math is literally everywhere” and in everything. With this description in mind, I turn now to a discussion and analysis of Geena’s school mathematics.

School Mathematics

To provide a contrast to Geena’s non-school mathematics, I also collected data on Geena’s school mathematics. I attended her elementary school class one day, and her mathematics methods class one evening. I made copies of her lesson plans and her assignments. We discussed these activities together and I recorded these conversations. In the following sections, I use the eight categories to present my analysis of Geena’s school mathematics.

Flexibly Modifying Plans

In her classroom, Geena adopted a flexible scheduling approach, but allowed less flexibility to her students. At the beginning of the year, she wrote a Yearly Preview for her school principal outlining her planned activities, but she freely deviated from this schedule as the situation demanded. For example, she switched the order of classroom activities for January and February. The complete home mathematics unit she designed in her methods course to be implemented in January turned into a few exercises that she sent home in February and March. Geena described this flexibility as a propensity to “go with the flow” when it came to class scheduling. She was well-organized and planned her activities carefully, but followed her tacit understandings to modify those plans as dictated by the situation. Keep in mind that to “go with the flow” requires solid (often tacit) understandings of the situation, the kinds of understandings that develop over years of practice.

I’m very much. My year is planned. My month is planned. My days are even planned. But if something hits me on the way to work, like that math thing the other day I woke up in the morning and boom (snaps her fingers) “I’m going to do this.” I went and that’s what I did that day. And everything else well, “Okay, if this is important I will do it,” but “No, this can be put on hold,” or “No, I’ll wait to do that tomorrow” or whenever. Um, I’m very much with the, go with the flow. (G-4.7, 940202)

Similarly, individual classroom activities were flexible enough to allow modifications and redirections. In particular, Geena and I discussed how she picked up on mathematical themes whenever they surfaced in the classroom, rather than tying herself to a set time slot to pursue mathematics. For example, during language arts the day I was in her classroom, Geena read a chapter from the current class novel, Mrs. Piggle-Wiggle (MacDonald, 1947). At one point in the story, the author wrote that the mother spent three hours cleaning the boy's room and only one hour to do all the rest of her housecleaning. Geena paused in her reading to ask how much longer it took the mother to clean the boy's room than to do the rest of her housework. Several students raised their hands to respond, and Geena called on one child who accurately responded, "Two hours." I asked Geena about drawing mathematics into literature during our next interview, and she explained that she often took time to ask mathematical questions when they came up in literature:

Any time it comes up in the book, I pull it out. If I, If I, um, not if I think of it, but if I decide yes, this is worth pursuing right now or if it's not too far above them. If they're ready for it. That kind of thing. Sometimes we can get into real, um, multiplying, fractions, those kinds of things, depending on the story, what's happening. Those kinds of things. Um, it's way too high for them, especially if it's only in September or October. So, I might not worry too much about it, but I pull it out when I can. (G-4.6, 940202)

In this way, classroom activities evolved from interactions in the learning environment, just as Geena's non-school mathematics evolved (cf. Roth, 1994). Geena explained that this flexibility helped her students attach more meaning to the ideas presented in her classroom.

And I guess that's why when math issues, or whatever issues, but if I'm reading a story and a math issue comes up I go with it right now. OK, let's stop and let's have a look, let's deal with it now. I just feel that there is more meaning, um, and that, there's more understanding and more retention of something if there's more meaning to it. (G-4.7, 940202)

However, students in Geena's classroom were seldom able to make modifications to classroom plans on their own. For example, during mathematics the day I observed her classroom, Geena had designed a worksheet activity focusing on the concepts of "greater

than” and “less than.” Students were to draw two handfuls of differently coloured blocks and then create number sentences to specify the relationship between the two handfuls of blocks. The only flexibility students were afforded was in the decision to use manipulatives (some students completed the worksheet by brainstorming number sentences rather than counting handfuls of blocks, see *Using Physical Objects*); all students were required to fill each box on the worksheet with mathematically correct number sentences using greater than and less than signs. Even mathematically correct number sentences using equals signs were not permissible; students who drew equal numbers of two different coloured blocks were instructed to draw again. Geena’s instructions to the students underspecified what they were expected to do (Amerine & Bilmes, 1988; Suchman, 1987), but she was the one who decided if what they did fit her instructions (as outlined in *Checking Work*).

Although the students had few opportunities to modify classroom plans, Geena was very flexible with her own plans. Central to her flexible scheduling was her belief that this flexibility provided opportunities to help her students draw meaningful connections and make sense of mathematics, a topic to be elaborated in the next section.

Making Sense

Geena was interested in making sense of mathematics for herself and for her students. Her mathematics background consisted primarily of memorization and rote operations. In contrast, as we have seen, algorithmic approaches to mathematics did not play a great role in her kitchen mathematics. In the introduction to her final project for the mathematics methods course, Geena articulated that her difficulties with mathematics as a child were in making sense of mathematics rather than in manipulating the numbers:

Math was often a puzzle for me. It was a special secret that the teacher had and only a select few could get in on the secret if they were “smart enough.” Sure I could memorize and do the rote equations, but often not without endless fights and tears with my mom. I was told why it was important to know my addition and multiplication tables etc. and I could even relate to some “real life situations,” but I sure didn’t understand what it was, I was memorizing. I just didn’t get it! My mom was as upset as I was. She didn’t understand why I always had to know “why,” and was exasperated with the

amount of questions I asked. This often left us both in a state of frustration.
(G-pr.1, 931206)

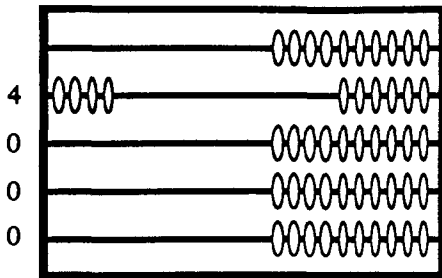
Similarly, when we discussed her understanding of school mathematics, Geena emphasized her efforts to make sense of school mathematics. As she explained during our first meeting, "I was 25 when I really understood, the real understanding of the concept of trading your tens for ones and when you had 4000 take away 2, I crossed off the zeroes and put a bunch of nines and put a ten at the end because I was told to" (G-1.2, 931021). Here, Geena is referring to the standard algorithm for performing subtraction with borrowing:

$$\begin{array}{r} 34909010 \\ - \quad \quad 2 \\ \hline 3998 \end{array}$$

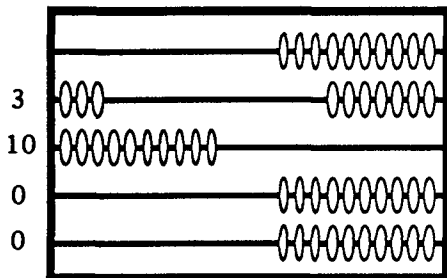
Geena's realization of the meaninglessness of this algorithm occurred while she was working with a student who was struggling to implement this algorithm. "And I said, 'Why do you do this?' And the kid said, 'My teacher tells me to.' And I'm thinking, 'Right on kid. Me, too'" (G-1.2, 931021). Rather than leaving it at this, she made a concerted effort to make sense of this algorithm, for herself and for the student. Using a disk abacus, the two experimented with a "really big question" to see if they could make sense.

Geena did not describe the step-by-step procedure they used nor did she explain what was the "really big question." Figure 3.3 illustrates the idea behind the use of a disk abacus to solve the equation $4000 - 2$, the sample problem Geena mentioned. Through this procedure both Geena and the student with whom she was working came to understand the concept of exchanging in place value. As Geena stated, "All of the sudden it was like this light went on for both of us. It was like wow, I get this. I really get this" (G-1.2, 931021).

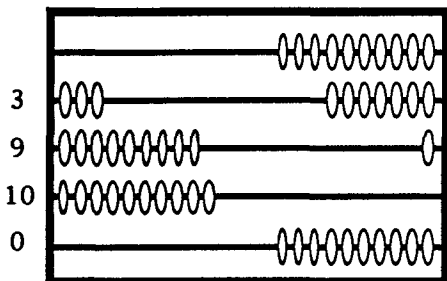
Figure 3.3 Using the Disk Abacus to Solve the Equation $4000 - 2$.



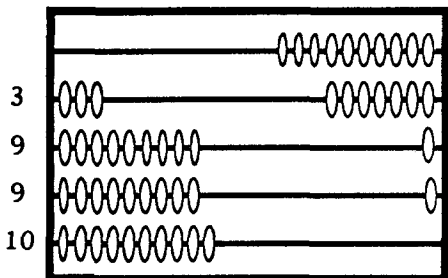
Begin with four disks in the 1000 row.



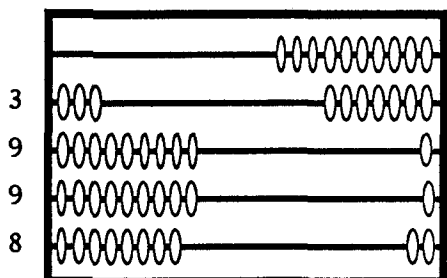
Exchange one disk from the 1000 row for ten disks in the 100 row. This leaves three disks in the 1000 row and ten disks in the 100 row.



Exchange one disk from the 100 row for ten disks in the 10 row. This leaves three disks in the 1000 row, nine disks in the 100 row, and ten disks in the 10 row.



Exchange one disk from the 10 row for ten disks in the 1 row. This leaves three disks in the 1000 row, nine disks in the 100 row, nine disks in the 10 row, and ten disks in the 1 row.



Subtract two from the 1 row. This gives the correct answer of 3998.

By “playing” around with the disk abacus, Geena and her student may have constructed an isomorphism between the movement of the disks on the abacus and the standard algorithm for subtraction with borrowing. Seeing the transition from zero to nine disks on a given row of the abacus may have allowed Geena and her student to construct more sophisticated concepts of place value and a deeper understanding of the standard algorithm. The disk abacus provided a “tool to think with” that allowed the reintegration of “thinking” and “acting” as did the design artifacts in Roth (in press-a). For Geena and her student, manipulating disks on the abacus allowed them to think and were the physical instantiation of that thinking process. As Cobb (1993) explained, with reference to grade two students’ using a hundreds board, “We see the abstract mathematical reality we create symbolically as we look thought [sic] the cultural tools we use” (p. 32). By looking at the disk abacus, Geena and her student came to see the abstract mathematical concept of exchanging in place value.

Through her efforts to teach the student, Geena herself came to understand the concept. This is not surprising given the consistency of research suggesting that teaching a concept to someone else is an excellent way to understand, and an excellent way to demonstrate understanding (see Hatano & Inagaki, 1987 as cited in Brown, 1988).

Geena’s efforts to make sense of mathematics extended to her classroom where she provided many opportunities for her students to make sense. For example, during calendar activities on the morning I attended Geena’s class, the students were looking for patterns on the posted classroom calendar and one student asked why the month did not start in the first square, that is, why it did not start on Sunday. Rather than explaining this (or saying “That’s just the way it is,” as many people might have), Geena sent this student and another student to the back of the classroom to check the two “real” calendars. These students came back to report that both commercial calendars started on Mondays, just like the class calendar. As a class, they talked about this finding and why it would be like that. When Geena and I talked about this during our debriefing interview

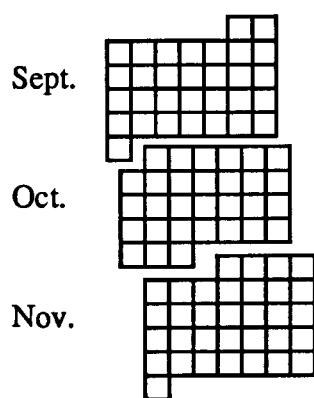
three months later, she explained that this was a common insight for children at that age.

She even went on to explain how it had happened again in her classroom:

That comes up all the time too. And it came up actually the other day (laughs). Because the kids who weren't ready for it then are ready to understand it now. So now they'll ask those questions they don't remember from before because it didn't mean anything to them. (G-4.6, 9402020)

In fact, in Nora's final project she included an activity building on the commercial calendar designed to highlight this very issue (see Figure 3.4).

Figure 3.4 A Suggested Activity for Helping Students Understand Calendar Layouts.



Children are often puzzled when they change the monthly calendar. They want to begin every month on Sunday. Here is an activity that might help them understand how the first day of the month is determined. At the end of the month, cut around the calendar's outline....Place that month's calendar so it fits with the previous month like a puzzle. (quoted in N-pr.12, 931206)

Geena did not engage in the activity described in Nora's final project, but she was certainly aware that this was a common question for grade two students, and took the time to discuss it with them when it came up in class. Her teaching, with its focus on making sense and flexible scheduling, gave students the freedom and perhaps the incentive to question.

Another example of this freedom to question relates to Geena's description of how she tried to pick up on mathematics whenever a mathematical issue was raised in her classroom, in mathematics, in language arts, in science, or at any time of the day. In our first interview, she described a newspaper clipping about a huge snake that one of her students had brought for Show and Tell. The article described the length of the snake in

metres, and students asked what that length meant. Rather than providing an answer (“Oh, it’s about...”), the whole class got out metre sticks and laid them end to end across the classroom to visually demonstrate the length of the snake. By looking through the metre sticks, the students saw the length (cf. Cobb, 1993).

Geena’s efforts to provide opportunities for her students to make sense are reminiscent of stories of the Nobel physicist (and inspirational teacher) Richard Feynman. In his memoirs, Feynman accredited his father as having had a strong influence on his development as a scientist because his father provided the same kinds of sense making opportunities as Geena provided in her classroom. Feynman’s father would read to him from an encyclopedia and upon reading that, say, the height of a tyrannosaurus rex was twenty five feet:

My father would stop reading and say, “Now, let’s see what that means. That would mean that if he stood in our yard, he would be tall enough to put his head through our window up here. But his head would be too wide to fit into the window.” Everything he read to me he would translate as best he could into some reality. (Feynman quoted in Ohanian, 1992, p. 189)

Perhaps, Geena’s teaching will provide similar inspiration for future scientists in her grade two classrooms.

Geena attributed many of the successful sense-making efforts by her and her students to the use of the physical representations provided by objects such as the disk abacus, the commercial calendars, and the metre sticks. The use of these physical objects provides the focus for the next section.

Using Physical Objects

Geena frequently mentioned the importance of using physical objects to do mathematics. In her classroom, so-called manipulatives were common, and students were encouraged to use these to make sense of mathematics. During our first meeting she described how the use of these physical objects improved students’ mathematical understandings. As a student herself, she never had access to manipulatives, and she felt that this resulted in limited understandings for her and her peers: “But people my age and

older, um, if they didn't understand it, it's because they just never had all this manipulative stuff to work with before." (G-1.2, 931021). Arguably, it is not the presence of manipulatives and other "cultural tools" that improve understandings, but the affordances these tools provide within the context of practice by providing "tools to think with" (Cobb, 1993; Roth, in press-a).

Consistent with Geena's assertions, manipulatives were an integral part of the lesson she gave during the math period I observed. At that time, the students had been working with inequalities and writing number sentences (e.g., $8 > 2$ or $2 < 8$) for the past few weeks. After distributing buckets of blocks to each grouping of desks, Geena began the lesson by grabbing a handful of orange blocks and a handful of red blocks. With the help of the students she counted the number of blocks of each colour (10 orange and 7 red). From this, she asked for the number sentence to represent the relationship between the orange and red blocks. One student responded that "ten is greater than seven." Geena wrote this number sentence on the board as " $10 > 7$." Then she asked if anyone could make a less-than sentence. A boy responded 7 less than 10. When asked to write this on the board, he wrote " $10 > 7$ " and then read it backwards as "7 is less than 10." When queried "Is that how you do a story?" he changed his written response to " $7 > 10$." Geena re-read her number sentence as "ten is greater than 7" and he recognized his error. He exclaimed, "Oh, I remember" and changed his written response to " $7 < 10$." Geena drew two new handfuls of blocks (2 white and 9 pink) and the class practiced the procedure. The two student volunteers for this second question wrote and then read the two number sentences $9 > 2$ and $2 < 9$ on their first attempts. Geena then handed out a worksheet, explaining that students should draw two handfuls of different coloured blocks, count them, and write the two number sentences to represent the relationship. The worksheet had space for eight pairs of number sentences.

As I circulated around the room, it was clear that most students were following the procedure and completing the worksheet as requested. There were a few exceptions

however. I noticed one boy who appeared to be struggling with the task. He kept mixing up the direction of his greater than and less than signs by writing inaccurate sentences such as “ $5 < 3$ ” or “ $2 > 9$.” When I talked with him, he was able to verbally state the correct sentences (5 is greater than 3, and 2 is less than 9), but did not always write them correctly. As it turned out, several of his digits were written backwards as well and his difficulty was the result of a visual motor learning disability. He could talk through the answers in words, but was unable to use the mathematical symbolism. Some of the other students, were writing mathematically correct number sentences without the use of blocks. For example, one child seemed to be experimenting with large numbers and wrote sentences such as “ $1000 > 2$.” Obviously, the child had not counted out 1000 blocks, rather she used her “number sense” (Greeno, 1991) and created her own number sentences without the use of physical manipulatives. This shortcut was accepted by Geena as fulfilling the task requirements and indicating conceptual understanding.¹⁷ Geena allowed students the flexibility to complete the worksheet with or without the use of manipulatives, as they so desired or needed.

Prior to my classroom visit, Geena had described in her research notebook an idea she had about another way she could use manipulatives to illustrate the concepts of greater than and less than in her classroom (G-nb.5, 931023). This notebook entry is re-presented in Figure 3.5.

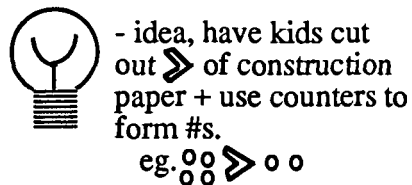
I later asked Geena whether she ever tried this idea and she responded in the affirmative:

I did. We actually did a great big huge bulletin board. You know when you walk into the room there's that huge wall over by the window. That's exactly what we did. We actually cut up, we cut out the physical things and then, um, they cut out little circles and squares, whatever they wanted to use, or drew

¹⁷ It is also possible that the child used this strategy because it was very clear that 1000 was greater than 2, whereas she may not have been able to write complete number sentences had she drawn two handfuls with similar numbers of blocks, e.g., 2 and 3 blocks. The important point in this analysis is that Geena allowed the children the flexibility to modify the worksheet plan as they wished, as long as she felt they demonstrated an ability to use the greater than and less than signs appropriately.

whatever it was. But, they made all kinds of number stories using the greater than, less than, put that symbol in the middle and then I just made this huge bulletin board.
(G-4.3-4.4, 940202)

Figure 3.5 Geena's Notebook Entry Describing a Bulletin Board Idea to Involve Students in Manipulating Greater Than and Less Than Symbols.



Examples of using manipulatives were common in Geena's mathematics classroom, as were examples of using physical objects to make meaning of school mathematics for herself (recall the earlier description of her use of the disk abacus to understand subtraction with borrowing and the metre sticks to understand snake length, see *Making Sense*) and as resources in her kitchen mathematics. Similarly, using manipulatives was common practice in Geena's mathematics methods course. The day I attended her methods course to solicit research participants, class began with the instruction for students to help themselves to the available manipulatives and experiment with them until others arrived.

Measuring and Calculating New Measures

Measuring was not an issue that came up frequently in the database regarding Geena's school mathematics, but her Yearly Preview indicated that measurement would be the focus for all mathematics instruction during the month of May (after data collection for this research project had already ended). Despite this limitation, measuring was still one of the practices Geena worked on in her classroom during the time of this research project. For example, she described laying metre sticks end to end across the classroom to demonstrate the length of the snake in one students' Show and Tell newspaper clipping. Measuring was one of the ways Geena helped students to make sense

of mathematics and, as with her non-school mathematics, measuring was dependent upon using physical objects as “tools to think with” (Roth, in press-a).

Recognizing Multiple Solutions

Geena allowed and encouraged students to consider multiple solutions in their mathematical work. As described earlier, they were allowed to complete their worksheet with or without manipulatives. Also, after students completed their worksheets, they were given free choice between numerous “math games.” Most of these activities focused on a “right” answer (cf. *Stating Solutions* section), but there were numerous ways to arrive at that answer. Two girls worked together with two sets of flash cards and two pictures of a flower with numbered parts. One girl, whose basic number facts seemed more automatic, used subtraction flash cards, while the other girl used addition flash cards. They took turns drawing a flash card and, if they got the correct answer, they covered the appropriate numbered space on the flower picture with a coloured marker. The object of the game was to cover all the numbers on the flower. The two flower pictures each had the same numbers, but one girl used subtraction while the other used addition to complete the task.

In a second group, three children asked me to join them for a board game. In this game, players took turns rolling the dice and moving forward the appropriate number of spaces. Some of the squares on the board had additional directions such as, “Go forward 2 spaces,” or “Go back 4 spaces.” The idea was to move through the maze of the board from the starting point to the end, as quickly as possible. This could be accomplished by a combination of high dice rolls, landing on “Go forward” squares, avoiding “Go back” squares, and landing on the square at the foot of a bridge that provided a short cut.

For part of the games period, Geena worked with two boys on a “math race.” Each of them (Geena and the two students) had a mini-chalkboard and chalk, and between them they had a pair of dice. The object of the game was to add to 100. Each person took turns rolling the dice, adding together the total, and keeping a running tally on their chalkboard.

To get started, Geena demonstrated how to play the game (see Figure 3.6). On her first turn, she rolled a 6 and a 3. She added these together, writing the problem and solution on her chalkboard. She then rolled a second time (2, 5) and explicitly described to the boys two ways of doing the next calculation. In the first scenario (middle column in Figure 3.6), she first added 2 to the previous sum of 9 to get a sub-total of 11, to which she added the 5 for a total of 16. She then described an alternate solution. First, add the two dice from the second roll (2, 5) to get 7, then add this to the previous total (9) to get 16 (see right column in Figure 3.6).

Figure 3.6 Geena's Demonstration of Multiple Solution Paths in the Math Race.

First role:	6		
Geena rolls 6 and 3	$\begin{array}{r} +3 \\ 9 \end{array}$		
Second role:	$\begin{array}{r} +2 \\ 11 \end{array}$	OR	2
Geena rolls 2, 5	$\begin{array}{r} +5 \\ 16 \end{array}$		$\begin{array}{r} +5 \\ 7 \end{array}$
			$\begin{array}{r} +9 \\ 16 \end{array}$

In each of these math games, multiple solution paths were available, but only one “right” answer was possible. The worksheet exercise was a goal-free problem whereby students could have picked any number of manipulatives to write any combination of number sentences, and had some flexibility in how they approached the task (i.e., they were not required to use handfuls of blocks). All of these exercises allow multiple solution paths, as was common in Geena’s mathematics outside her classroom, but did not allow alternative final solutions.

Checking Work

As the students worked on their mathematics, Geena circulated in the classroom checking their performance. When she observed difficulties, she questioned the students and provided hints to help them get back on track. During the worksheet exercise, students were encouraged to monitor their own performance, and when they finished they were allowed to move on to a math game of their choice. Based on Geena’s monitoring,

some students were called back from their games to answer questions or revise their worksheets. Just as Geena had retroactively fit the cookies/squares with the recipe, she fit the students' responses with her intentions (Amerine & Bilmes, 1988; Suchman, 1987). If the students filled the squares with correct number sentences, they were free to move on to the math games (even if the number sentences clearly did not use manipulatives, as for the student who wrote $1000 > 2$). If the squares did not match, students had to redo the exercise until they achieved a fit. This external comparison required (written) public statement of the solution which leads me to the next category, *Stating Solutions*.

Stating Solutions

Geena modified her inclination to state solutions when she moved from non-school to school mathematics. For non-school mathematics, Geena placed very little emphasis on stating a solution, whereas her classroom mathematics consistently involved stating a solution. Students had to fill all the blanks on the worksheet she presented during the mathematics class I observed. They worked toward single "right" answers for each of the available math games (although there were multiple ways to reach the single answer). For Quiet Math, students had to use accurate hand signals to indicate solutions to arithmetic problems. School mathematics involved rationally accountable practice and products; students were required to do something public and to produce appropriate documents (cf. Lynch, 1991). In contrast, in Geena's non-school mathematics her embodied actions represented solutions, but she rarely stated the solutions formally.

Drawing Connections

In her teaching, Geena flexibly wove together different subjects across the time slots of her day. The Yearly Preview she prepared emphasized cross-curricular themes that integrated subject areas. She planned to discuss pumpkins in math, science, and socials during the month of October to correspond with Halloween. She planned her geometry unit in mathematics to correspond with the space theme for centre-play time, art, and socials. During her unit on fairy tales in Language Arts, she drew in the stories of Robert

Munsch which have a fairy tale structure. At the same time, her students noticed the prevalence of numbers in these stories, and they discussed this further connection between the literary forms and the mathematical ideas:

We're actually studying fairy tales and doing a, um, Robert Munsch theme. I don't know if you're familiar with his writing or not, but the things that they look at, the things that are in common, between them, things that are common in fairy tales, the beginnings, the endings, that sort of thing. But they find that with Robert Munsch stories there is always numbers in them, and in fairy tales there are often numbers. And, um, just issues of, um, other kinds of questions that they're ready for.
(G-4.7, 940202)

Similarly, Geena explained how she picked up on mathematics issues when they came up in current events (e.g., the snake story) and other times during the school day. As Geena explained, "Whatever's happening, yeah. In the newspaper or in lit, always, any time it [mathematics] comes up in language arts or literature" (G-4.7, 940202). Even in her bulletin board displays, Geena focused on integrating across subjects, "I put everything from math stuff to a mixture of math and writing, to, to writing to art, a mixture of all three of them" (G-4.7, 940202).

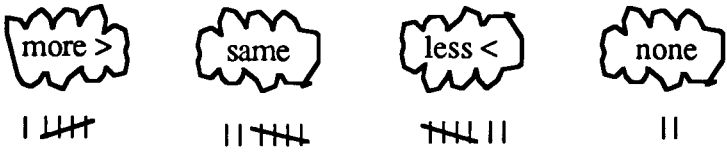
Not only did Geena emphasize cross-curricular connections in her teaching, but she also tried to connect to students' home lives. For her final project in the mathematics methods course she designed a geometry unit plan for her classroom that was comprised of activities for students to take home and work on with their families. Her unit grew from an interest in building "home-school relationships" by involving parents. At the same time, Geena explained in her paper that she wanted her students and their parents to recognize the importance of mathematics in their everyday lives:

Math is everywhere just as reading and writing are. The average person will say that most of the Math they learned in school was a waste of time and the only thing they use on a regular basis is the basic addition, subtraction, and multiplication tables which they use when they go shopping and when they do their banking. They are not aware that Math is so much more and that they use it on a day to day basis. Math doesn't stop at the classroom walls. We, as educators who believe this, need to continue to re-educate the public. Using a Home Math Program is one way to help parents and children realize that Math is everywhere for everyone.
(G-pr.3, 931206)

The activities Geena planned for this unit included games that students could play with their whole families and a series of worksheets where students identified geometric shapes in pictures and then looked for more shapes in their own environments (garden, yard, bathroom, kitchen, etc.). This worksheet series emphasized the second aspect of Geena’s two-pronged approach to environmental awareness: becoming more environmentally conscious (composting, recycling, not wasting food) and being more aware of the environment (drawing a map to get from school to home, graphing the difference in the amount of grass in their front and back yards, recognizing geometric shapes in their environment). This focus was evident on the day I observed in Geena’s classroom. When students first entered the classroom, they marked a tally on the board to create a class graph as shown in Figure 3.7.

Figure 3.7 Classroom Tally Graph Designed to Increase Students’ Awareness of Their Environment.

Do you have > or < grass in the front of your house than the back?



Geena explained that she had used this kind of tally graph as a daily activity in her classrooms for a number of years. She had started the first day of school creating these graphs with two categories and was slowly increasing the number of categories (four in this example). Before leaving in the afternoon, she would create a question and students would mark their responses when they arrived in the morning. Part way through the year (sometime prior to my November 16 visit) the students in this class had taken over the task of creating the graphs (this had never happened in any of her other classes). The day I was in the class, the student who was supposed to make the graph had forgotten, so Geena made the one shown in Figure 3.7. The students’ desire to take over the task of

creating the tally graphs demonstrates their high level of motivation and excitement. By allowing the students to take charge of their own learning in this way, Geena set up an environment that afforded motivation and learning (e.g., Brown, 1988).

This kind of graphing of students' own life experiences was also prevalent in the weather and lost tooth graphs posted near the class calendar that became a focus during early morning calendar activities each day. In all of these graphs, students were encouraged to reflect on and graphically represent aspects of their own lives (weather conditions, grass in their yard, infant teeth lost).

Summary

As these descriptions demonstrate, mathematical sense, flexibility, and connections informed Geena's school mathematics. She relied on her own tacit understandings to flexibly modify her teaching to meet the needs of different situations. She relied on manipulatives and other physical objects to make sense of school mathematics for herself and for her students. In her sense-making efforts, she attempted to render school mathematics more meaningful by connecting it to other curricular topics and to students' out-of-school experiences. In her teaching, she provided many opportunities for students to make sense, but gave them little freedom to modify the tasks placed before them. She had control of the modifications made, and determined whether students' adaptations suitably met her goals for instructional activities. Although, she emphasized the availability of multiple solution paths in mathematical problem solving, these paths tended to lead to a single, "correct" answer. This answer, in turn, needed to be appropriately documented through public performance. Thus, although mathematical sense, flexibility, and connections were prevalent themes in Geena's school mathematics, there were still distinct discontinuities between her school and non-school mathematical practices as will be highlighted in the next section.

Connecting School and Non-School Mathematics

Throughout all of our interactions, Geena continually came back to the idea of “connections.” During our first phone call to set up an interview, she indicated that she was interested in the idea of drawing connections between school and non-school mathematics. This focus was prevalent in her initial journal assignment for the mathematics methods course, the assignment that had convinced her to participate in this research project. As she explained during our first interview:

The journal assignment that I just handed in. I had done it all before I had even thought. That’s what triggered me into doing this [finding out about this research project]. And I’m just thinking, “Oh my god. This is just too good to be true. It’s all, it’s all tying in. Math connecting to the outside world, Oh my god, I guess I should do my major project on this because it just seems to be falling into my lap.” You know? (G-1.1, 931021)

For this assignment, Geena was supposed to flip through an issue of the Arithmetic Teacher and write a reflection about one of the articles, “What did it do for us? Did it talk about our experiences? Did it trigger anything?” Geena selected the February 1993 issue of the Arithmetic Teacher which was a special issue on “Empowering Students Through Connections” (Tunis, 1993). The articles in this issue discussed various connections among mathematical topics, between mathematics and other curriculum areas, between mathematics and students’ prior experiences, and between mathematics in the school and in the home. From these articles, Geena selected Corwin’s article on creating a mathematical culture. In her response, Geena drew connections to her own prior experiences and to events in her classroom. She also described the importance of making connections with colleagues, with students’ home lives, and to everyday uses of mathematics.

This initial assignment was the impetus for all of our interactions. During our interactions, Geena constantly drew connections between school and non-school uses of mathematics, school mathematics and other subjects, school and home learning, mathematics and art, etc. She reiterated this theme of connections in our final meeting when she evaluated her mathematics methods course:

All, every single thing that I did, tied together very nicely. Whereas some courses, um, depending on what they are, and if, if you're not able to tie them together, you do a paper on this and you do a paper on that and a paper on this. And there is no connection, they are just completely different. And, yeah maybe you've learned a lot, and maybe they've been great, but you can't connect them. But every paper, every thing I did in this class was able to be connected.
(G-4.9, 940202)

The overarching theme of drawing connections can also be seen by analyzing Table 3.1 and the eight categories that have framed discussion throughout this chapter. Similar themes were prevalent in Geena's school and non-school mathematics, highlighting the connectedness of her mathematics across contexts. Geena was able to draw many connections between school and non-school mathematics (see especially the sections titled *Drawing Connections*). In all of her activities, she relied heavily on her numerical sense and strove to make sense of the mathematics before her. She set up her classroom to facilitate students' sense-making activities, especially through the use of manipulatives and other physical objects. Manipulating these physical objects provided opportunities for Geena and her students to think and act in an integrated fashion (cf. Roth, in press-a). Many of their mathematical practices relied on and were constituted by these physical objects. By adopting different tools or adapting a particular tool for a new use, they were able to explore multiple solution strategies. However, in the classroom, these multiple solution strategies typically led to one "correct" answer and, therefore, suggested that there were some discontinuities in Geena's mathematical practices in and out of her classrooms (Resnick, 1987). Her practices were extremely flexible both in and out of school, but she did not allow her students a similar degree of flexibility. Geena's biscuits and squares, as well as her classroom lessons, evolved over the course of her actions. She started with vague plans, but flexibly modified these based on multiple goals, available tools and materials, community standards and reactions, and the products themselves. On the other hand, her students had few opportunities to shape the classroom activities in a similar fashion. They were free to use or not use manipulatives to complete the worksheet exercise, but Geena determined whether the end result of students' actions was

acceptable. To make this judgment of fit, she required that her students publicly state their solutions whereas, in out-of-school contexts, her own solutions seldom included public statements. In the kitchen, her embodied actions represented more appropriate solutions than a numerical statement.

Thus, consistent with earlier findings (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a), the present case study suggests that Geena displayed mathematical activity in non-school settings that differed from approaches taught in schools, including her own classrooms. Analysis of Geena's non-school mathematical activity revealed some marked contrasts from the mathematical activity that is legitimated in classrooms (including hers). However, several continuities were also realized. The over-arching themes of flexibility and sense-making, plus the emphasis on connections were important in both her school and non-school mathematics. These important themes will be raised again in the final chapter where suggestions are provided for how we may encourage other teachers to follow the model provided by Geena's stories while pushing the boundaries to wrestle with the remaining discontinuities. To elaborate these themes and provide further examples, I turn now to a discussion of Nora's mathematics stories.

CHAPTER FOUR

NORA'S MATHEMATICS STORIES

In this chapter, I first present a brief description of Nora's mathematical biography and our interactions during the research project. This information sets a framework, for the subsequent descriptions of Nora's mathematics across contexts, as drawn from the data corpus.

Nora's Mathematical Biography and the Context of our Interactions

Nora had 19 years of teaching experience, predominantly in special education across "multi-age and functioning levels" (i.e., students from age 3 to 25 with various intellectual disabilities), but had recently started focusing on primary age students. During the time we worked together she was employed on a part-time basis as an English as a Second Language and Learning Assistance support teacher for grades three through five. Her English as a Second Language classroom was held in an open common room that led to four different classes, including the children's homerooms. Her class consisted of only six children, all of whom were recent immigrants to Canada and spoke English as a second language (five spoke Cantonese and one spoke Spanish as first languages). The class was held for one hour, three afternoons per week.

The British Columbia College of Teachers had stipulated that Nora needed to complete 12 credit hours in teaching methods courses to qualify for a provincial teaching certificate after she relocated from out-of-province. Within these 12 hours, she needed to take three 4-credit teaching methods courses—one in mathematics, one in Language Arts or Reading, plus one other option. As the first of these courses, she had enrolled in the Designs for Learning—Mathematics (Elementary) course. Although she was not teaching mathematics at the time, Nora thought the mathematics methods course would help her to better understand mathematics learning, her own and that of her students. During our conversations and in her final project, she described how her limited mathematics background had inspired her to try to make mathematics better for children, and how her

former students' enthusiasm for mathematics had aroused her own interests in the subject and her attempts to improve her own mathematics:

I had a math phobia, however, I started teaching, I taught a grade 1/2 split class last year and the children are very, very into math, especially because now the trend is on manipulatives and a lot of involvement. Um, and it just piqued my interest and why couldn't I have been taught that way and maybe I wouldn't be so bad now, and it's almost like I want to go back and recoup what I didn't have a chance to get in the first place, um, and also because...I want to enhance my education in that area because I feel that it's just so, it's worse than limited, it's almost nonexistent. (N-1.1, 931029)

As this quote illustrates, Nora was dissatisfied with her own mathematics background and wished to improve. During our time together, Nora related stories of her experiences with school mathematics that had left her feeling unconfident, afraid, frustrated, and test anxious. Here, she referred to herself as having a "math phobia" and described her mathematics education as "almost nonexistent." In her final project for the methods course, she described herself as a teacher who "both as a young student and an adult, feared, failed and [had] been humiliated many a time because of [her] inability to easily grasp mathematical concepts" (N-pr.1, 931206). During our first interview, she described her difficulties understanding the metric system as a factor of the time it took to "penetrate this thick skull" (N-1.6, 931029). She also expressed great concern about having to write a test in the mathematics methods course:

And another thing that is concerning me is that we have to write a test. I mean I don't mind reading things because I learn that way, but to sit down and write a test really aggravates me because I am not a test person. I will get in front of it and I look at the questions and I just, I have always been like that. I'm not a test taker. I'm not looking forward to it, I know how I've been in the past so I'm expecting to be the same way again and I'm not looking forward to it. (N-1.10, 931029)

Her difficulties with mathematics had compelled her to try to make mathematics "fun, meaningful and exciting" for the children in her classroom and to provide a "stimulating, positive and secure environment."

I am a teacher, who has, both as a young student and an adult, feared, failed and have been humiliated many a time because of my inability to easily grasp mathematical concepts. Over the years, these feelings have compiled, especially as the mathematics activities became more complex. Working for many years in special education and at the early primary level, I wanted to

make math activities fun, meaningful and exciting for my students, as well as provide them with a stimulating, positive and secure environment, in which their natural curiosity and enthusiasm for solving problems could grow. I didn't want them to grow up feeling the way I did about math. Through reading, discussing with "math experts" and seeking out various elementary mathematics programmes and workshops, I realized that mathematics is an exciting and exploratory experience. (N-pr.1, 931206)

Nora wanted to provide very different experiences for her students, than she herself had experienced. For her, school mathematics had meant:

Open the math book and do that page or two pages whatever it was. And all it was sums and memorizing. Okay that's it. Close your book. Go on to something else... There wasn't any allowance, anything allowed for questioning. Well, why is it that way? It's this way because it's the rule, and I say so and no further explanation. Or manipulation or understanding in a concrete way rather than an intellectual sort of way. Because I found I had to teach and there are so many types of learning. People are visual learners, hands-on concrete learners or cerebral learners or a combination of all of those. And the special needs kids I worked with, they are so literal and so hands-on visual, the cerebral is the last that might come into play. It's the actual doing, the hands-on that has more influence. (N-1.4, 931019)

Nora's own mathematics education had been textbook-driven drill-and-practice, with few opportunities for manipulatives and active learning. In contrast, Nora felt manipulatives and active learning were critical to mathematics learning. The "cerebral" learning emphasized when she was a student was inadequate for the needs of many students. This was especially true for her special needs students, who had difficulties with abstract concepts, but could be quite successful with concrete materials. While teaching young adults with special needs, Nora had "concentrated mainly on math within daily living skills" (N-bg.2, 931029). She described that teaching assignment in positive terms, "It was a really, um, beneficial time and I miss it, you know, often. This friend I taught with, we're really good friends and we're still in contact since I've been here. And we still 'Remember when.' That was our ideal situation" (N-1.8, 931029). Teaching in that environment had been very rewarding for Nora, and liberating for her students.

N: Our program mainly relied on daily living and functioning skills because it wasn't any. Because it didn't make any sense talking about things that wasn't relevant. It made more sense dealing with things they were doing on a daily basis. For instance we taught them, um, they were used to coming to school and being inside the classroom doing whatever they did there and that was it. We got outside the

classroom and it involved mapping street location. Buses and schedules, where you go what bus you needed. What bus number you needed. One bus number takes you and it might be a different bus number to return and this was in Quebec so it also included the train, the metro, the subway. Um, shopping, banking skills, post office. Oh we did a lot! The most interesting got involved with the shopping we went out and bought lunch and it was so nice to see students who were used to literally being told what to do and what to eat, to them choosing and giving the money, things like that. If they bought what they wanted and they ate and were still hungry if they had the money what more could they buy and going from what do I do to being self-dependent, doing it themselves.

M: Which must have been really liberating for the students.

N: Oh it was! And very exciting for us, you know....I mean that was just so exhilarating for everybody. (N-1.4, 931029)

This teaching experience was also central to Nora's motivation to work with me on this research project. She had missed the class where I gave my presentation about the research study, but she immediately contacted me to introduce herself and express her interest when she found out about the project the following week. During our first session, she explained how the research project matched her own interests meaning that working together could enhance our mutual interests ("I could be giving you information you're looking for and I could be learning from you.").

When [the course instructor] first told me about your project I was quite enthusiastic about it. I was encouraged by it...The information I was given on your project hits on all the things I seem to have thought prior, previously on, "Gee, I'd like to know more about." That's why I decided to contact you. It was almost an added bonus because you were doing a project on something that I was interested in so maybe it could be a two-way street. I could be giving you information you're looking for and I could be learning from you. (N-1.1, 931029)

Consistent with this focus, in the end, Nora found that the time we had spent together was valuable. In particular, she found that it had increased her awareness of the mathematics that surrounded her and that she used on a regular basis. She explained, "Everything I do now, I think, 'Oh yeah, this is math.' (She laughed). I'm still thinking in

that mode right now” (N-5.7, 940121).¹⁸ This increased awareness of mathematics was the main thing that Nora felt she had learned during our time together.

It’s just the awareness, because the awareness is just again very wide. I mean it’s everywhere. And it makes you look at things, consider things differently than you, I might have otherwise done. Um, just goes to show how much you take for granted, you know. (N-6B.9, 940308)

In contrast, she was disappointed with her mathematics methods course. Initially, she was quite enthusiastic about the course, but this enthusiasm diminished over the term as she realized that the course was not meeting her needs. She experienced a great deal of anxiety over an exam she wrote mid-way through the course, and found that the assignments did not match her goals for the course. Her main complaint was that the course lacked “things to take away” which she felt should be the main focus of methods courses. Overall, she found that she learned very little from the instructor, only from her classmates and the textbook. She ended up with a B- in the course, which she interpreted as “at least I passed.” Her frustration with the course was evident in her response to her instructor’s feedback on her final project:

I don’t really know what she was thinking. She was looking more at something at a high school level or college level....She couldn’t relate to what I was talking to and that gets into what we had said in the car earlier that other people had said when we were talking is that she’s never been in a class, especially early, young primary kids. How they would use a calendar in a totally different manner. Where they’re doing patterns. I mean, her idea of patterns and my idea of patterns or somebody else’s idea of patterns are totally different, you know. (N-6B.6, 940308)

Nora’s disappointment with the course stemmed from her desire to accumulate a series of activities that she could use in teaching mathematics. She was little interested in theory, but wanted practical activities—activities that would get students actively involved in mathematics, using manipulatives, and having fun (in agreement with her goals for her mathematics teaching as outlined earlier). At the same time, she seemed to

¹⁸Nora’s increased awareness reveals an observer-interaction effect (Lincoln & Guba, 1985, see also the discussion of intentionality in van Manen, 1990). By focusing on her uses and understandings of mathematics, the research influenced her uses and understandings of mathematics; the act of observing changed what was observed. This issue will be addressed in the *Mathematical Sense* section of this chapter.

be looking for activities that were not too advanced.¹⁹ She had no interest in teaching students beyond the primary grades, so was not interested in mathematics that would be relevant for older elementary children. With her limited mathematics background, her preference to teach only at the primary level was probably well-placed.

Nora's limited mathematics background had great implications for her mathematical sense, as will be described later in this chapter. With a poor foundation in mathematics, she seemed to have a limited conception of what counts as mathematics and how much mathematics she actually engaged. The remainder of this chapter is devoted to a discussion of Nora's mathematical sense and its impact on her mathematics across contexts.

Before turning to a discussion of Nora's mathematics across contexts, it is important to remind the reader that, although Nora was not teaching mathematics at the time of this research study, she had taught mathematics in her primary classrooms and had included mathematics in her special needs classroom, so that it was possible for her to draw on recollections of her earlier teaching experiences in mathematics for the purposes of this study. Further, she was originally very enthusiastic about both the research project and her mathematics methods course, revealing a strong commitment to improving her own mathematics understanding, as well as her mathematics teaching. Her enthusiasm for the methods course diminished over the term because the course did not meet her expectations or her goals, but she remained enthusiastic about discussing her non-school mathematics and her mathematics teaching with me. For these reasons, I felt it was important to include her contributions as part of this research study even though she was not actually teaching mathematics at the time of the study. The reader is advised to

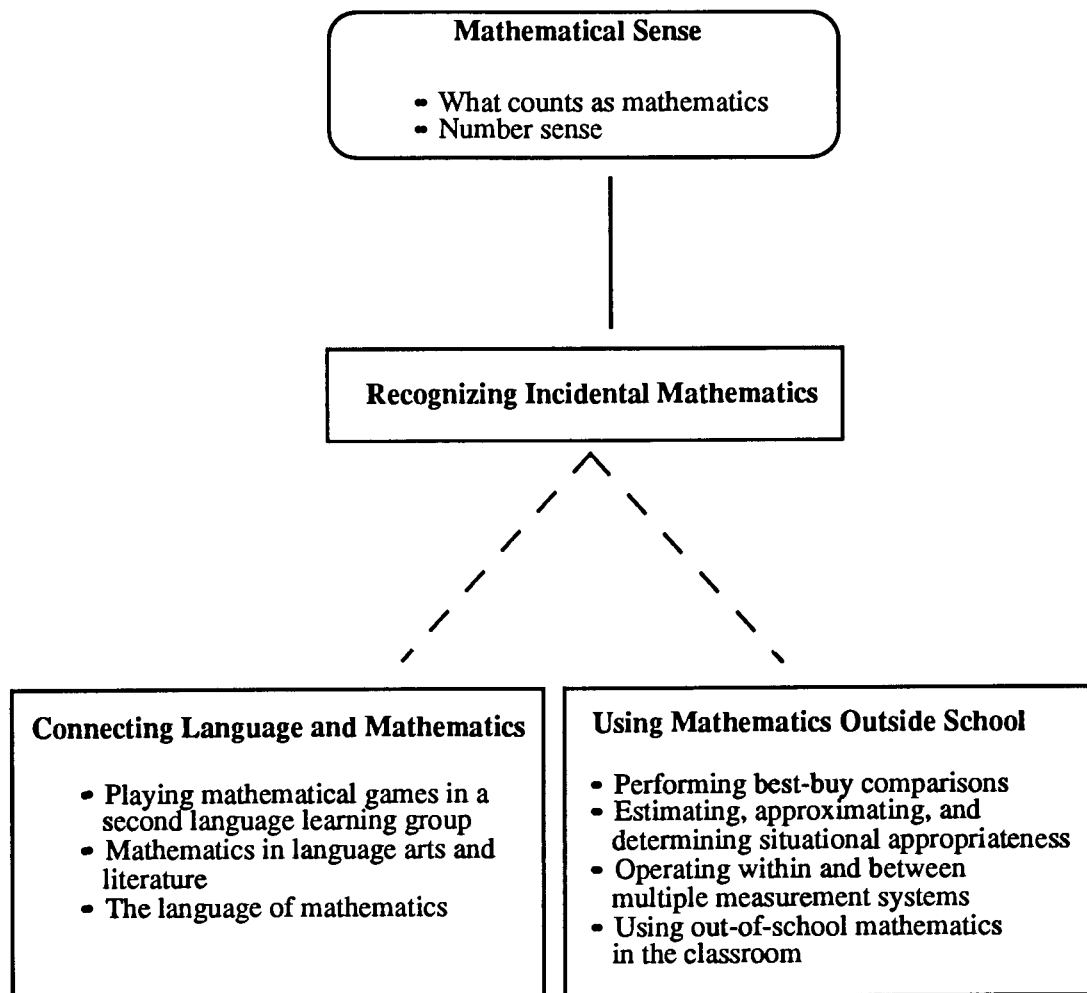
¹⁹I observed a similar interest in finding fun activities matched to an appropriate level of difficulty when we visited a teacher supply store together. Nora was discouraged that she could not find any "quick activities to do with her [English as a Second Language] class." She wanted some prepared activities that she could just use without much work, but she left discouraged because all the materials were "either too babyish, or too advanced. There was nothing mature, but easy to read" (ND fn.4, 931112).

consider these issues in interpreting the descriptions and analyses provided in this chapter.

Nora's Mathematics Across Contexts

To understand Nora's mathematics across the various contexts of this study, I looked for themes across the entire data corpus (interviews, observations, research notebook, mathematics methods assignments, and all other documents), as I had done for Geena. From this analysis, I created a four-faceted model—Mathematical Sense, Recognizing Incidental Mathematics, Connecting Language and Mathematics, and Using Mathematics Outside School—to reflect Nora's mathematics across contexts (see Figure 4.1). First, this model highlights the centrality of Nora's *Mathematical Sense*. Her (a) understandings of what counts as mathematics and (b) number sense affected what behaviours she presented in the context of our interactions and her descriptions of those behaviours. When asked to describe her non-school mathematics, Nora only described activities which she classified as mathematical. At the same time, however, her "number sense" (Greeno, 1991) informed her actions such that, for example, she automatically knew that \$5.49 American was more than \$6.00 Canadian. Nora's mathematical sense was central to her *Recognizing Incidental Mathematics*, the second facet of the model. Throughout our time together, Nora emphasized "incidental" mathematics, that is, mathematics that was inherent in an activity, but that was not the focus of that activity. This section will highlight the incidental mathematics that Nora recognized. At the same time, Nora's recognition of incidental mathematics in her own (and her students') activities increased over the course of the research project as she reflected on her activities. In this way, the act of researching her recognition of incidental mathematics increased her recognition of that mathematics, creating an observer-interaction effect (see CHAPTER TWO and Lincoln & Guba, 1985). By recognizing incidental mathematics in her activities (teaching, shopping), Nora was able to draw connections between school mathematics and language, and from school mathematics to uses of mathematics outside school. These two

Figure 4.1 Four Facets of Nora's Mathematics Inside and Outside Her Classrooms



types of connections form the third and fourth facets of the model, respectively.

Connecting Language and Mathematics describes (a) the mathematical games I observed in Nora's English as a Second Language classroom, (b) the mathematical ideas found in language arts and literature, and (c) the language Nora used to talk about mathematics.

Using Mathematics Outside School documents Nora's uses of mathematics outside the school in three examples: (a) performing best-buy comparisons, (b) estimating, approximating, and determining situational appropriateness, and (c) operating within and between multiple measurement systems. As well, this section documents how Nora (d) used out-of-school mathematics in a functional mathematics program she had designed as a special education teacher.

The reader will note that school mathematics itself is not described separately in these sections. Because Nora was not teaching mathematics at the time we worked together, the mathematics we discussed tended to come from contexts other than school mathematics. For example, in her English as a Second Language classroom, mathematics came in relation to language. In the class I observed, Nora showed the students how to play two mathematical games, but the focus was on the students communicating with their partners. Mathematics was also prevalent in the calendar activities Nora described in her final paper for the methods course. School mathematics is, therefore, woven across facets of the model. The remainder of this chapter is devoted to a discussion and description of Nora's mathematics as informed by the four facets: mathematical sense, recognizing incidental mathematics, connecting language and mathematics, and using mathematics outside school.

Mathematical Sense

The model in Figure 4.1 highlights the centrality of Nora's *Mathematical Sense*. I interpret her mathematical sense as encompassing two related considerations. First, Nora's understandings of what counted as mathematics affected what behaviours she presented in the context of our interactions and her descriptions of those behaviours.

Second, her “number sense” (Greeno, 1991) or “quantitative intuition” (Sowder, 1992) informed her actions such that, for example, she knew imperial measures, but had to convert to understand metric measures. These two aspects of Nora’s mathematical sense are elaborated in the following sections.

What Counts as Mathematics

Nora’s understanding of what counts as mathematics was central to our work. Because I asked Nora to select situations in which I could observe her school and non-school mathematics, I (only) found out about the mathematics as she found it in her own activities, not as an observer might have. If she did not recognize any mathematics in a given activity, she did not present that activity as an example of her mathematics. Alternatively, it may have been possible to shadow Nora through her daily activities and describe all the mathematics observed in that fashion. However, this would not have been faithful to Nora’s interpretation of her social reality. I was reluctant to force Nora’s activities to fit some pre-existing framework as has been done in other research. For example, Masingila (1993b, 1995) categorized the mathematics of carpet layers and middle school students according to Bishop’s (1991) six fundamental mathematical activities. Although there is merit to such an approach, I was more interested in finding out about mathematics from Nora’s perspective, not from an outsider’s perspective (i.e., an emic not an etic perspective, see Fielding & Fielding, 1986). Although Nora made the initial selections of the contexts to observe, my analysis of the mathematics within those contexts was informed by my own understandings of mathematics, for it is not possible to separate that which was observed from my perception of what was observed. This idea of the inseparability of my analyses of the mathematics from the mathematics itself is related to the phenomenologists’ conception of “intentionality” (van Manen, 1990). At the same time, Nora’s selections of what was mathematics reflected her intentionality; for her, only those things which she perceived as mathematical were presented as examples of her mathematics. However, just as Geena had stated, Nora did indicate on several

occasions that “mathematics is everywhere.” She recognized that the examples we discussed (see Appendix D) were only a sample of the actual mathematical activities she engaged.

As we worked together, it was apparent that Nora’s understanding of mathematics focused primarily on numbers, with some mention of arithmetical operations. Throughout our discussions, Nora often referred to numbers rather than the broader category of mathematics. In talking about the mathematics inherent in her calendar activities, she stated, “You know it’s all numbers and counting” (N-1.8, 931029). In describing how doctors and other medical professionals needed mathematics on the job, she stated, “Dentists and doctors must have to do a lot with numerical stuff because of medication” (N-5.9, 940121) and, later, “They have to be aware of numbers and their vital role in those kinds of situations” (N-6B.3, 940308). When describing the mathematics needed to track volunteer hours for a large community project, she summarized, “It’s again numbers. Numbers are involved” (N-6A.2, 940308). Each of these examples reflects a focus on mathematics as being related to numbers and arithmetical operations. This focus is prevalent in the list of all the contexts for non-school mathematics that Nora described (see Appendix D). Although a wide variety of mathematical ideas were represented in the activities Nora described, her focus was consistently on numbers and arithmetic. Taking the bus, in her estimation, required counting the correct change, knowing the proper bus number(s), and counting the number of stops to know when to get off. She made no mention of the geometry involved in spatially navigating the world or the complex calculations required to select one route over another alternative while minimizing factors such as distance, time or the probability of missing a bus. This unrecognized complexity will be described in the section, *Using Mathematics Outside School*. For example, in her comparison pet food shopping, Nora talked only about which store had higher or lower prices, but underestimated the complex optimization calculations required to balance all the related factors (driving distance, crowding of the store, availability of items, etc.).

Number Sense

Much of Nora's mathematical activity depended on her number sense, that is, her tacit understandings of mathematics which revealed personal, experiential dimensions to her knowing (Greeno, 1991). These are the kinds of understandings that develop out of practice and do not need to (and often cannot) be formalized. For example, Nora emphasized her number sense in reference to her understanding of imperial measures, "I'm still on imperial" (N-1.6, 931029) and "I think in feet and inches" (N-5.3, 940121). In contrast, she had a weaker understanding of metric measures, "Whereas in measurement I can understand it in imperial, but the metric I have to stop and think about it" (N-5.4, 940121). Nora had a strong sense for imperial measures, but not so for metric measures. In terms of Greeno's (1991) environmental metaphor, Nora knew her way around the domain of imperial measures, she had a strong sense of the imperial environment. On the other hand, she was less familiar with the domain of metric measures and could only find her way around the metric environment by referring to the guide (map) provided by her sense of the imperial environment. I return to this in the section, *Operating Within and Between Multiple Measurement Systems*. As a second example, Nora knew by looking that \$5.49 American was more than \$6.00 Canadian. She had a sense for exchange rates and knew that similar dollar figures would mean that the American would be more expensive than the Canadian. Again, this example is discussed in more detail in *Estimating, Approximating, and Determining Situational Appropriateness*. Other aspects of Nora's number sense helped to inform her practice inside and outside her classrooms, as will be described in the remainder of this chapter.

Recognizing Incidental Mathematics

From the beginning, Nora expressed an interest in what she called "incidental" mathematics, that is, mathematical activities that were "done on an incidental basis not necessarily preplanned verbatim, but how they sort of evolve[d] through something else"

(N-1.2, 931029). In our first meeting, Nora explained what she meant by the term incidental mathematics:

It's more the incidental math that I seem to be quite drawn to at the moment. Not a lesson, it just kind of comes through. Um, it is involved in a specific activity but it was set out to be maybe a language-oriented thing but it involved both [language and mathematics]. Because it sort of crosses over, I mean that's, you know, a big thing with math. (N-1.3, 931029)

Incidental mathematics, therefore, referred to mathematical ideas that came up in the context of other activities. We saw examples of incidental mathematics in Geena's classroom, such as when the newspaper clipping about the snake prompted the measuring activity or when the discussions about the similarities between fairy tales and Robert Munsch stories led to students' recognition of the prevalence of numbers in both (see CHAPTER THREE). Similarly, Nora provided an example of incidental mathematics from one of her colleague's classrooms:

For instance, one of the teachers I was talking to yesterday with the [grade] 1/2 split. And she said indirectly they started off keeping. They have a tally list. And I've done this, too. Where each day the child writes, okay yesterday we were in school since the beginning of the school year 40 days, that means today is 41 days, tomorrow might be 42 days. And then she started with straws, and it started out to be okay let's have a certain amount of straws for each day so that we can see how big our pile gets and she said it just kind of went from being this great big pile of straws to going into units, tens and hundreds. And she had the children, um, okay once we get 10 straws we have to bundle them and then she introduced units, tens and hundreds and that was not her original thought it just became that way because one child was having so much trouble. He said, "Oh, this is just too many straws." (N-1.3, 931029)

In this example, the task started out as an activity to increase students' numerical sense and understanding of time relative to the school year, but shifted to a discussion of place value. One boy was struggling with the large number of straws afforded by the original task. To accommodate this difficulty, the teacher introduced the idea of bundling the straws into groups of ten to make them more manageable. This grouping then provided a natural progression to a discussion of place value. Based on the boy's response, the teacher's original plan evolved to incorporate bundling practices, and then the bundles shaped the activity to allow the place value discussion. The teacher's plans

did not determine the actual course of her situated activities (Suchman, 1987), but provided a base from which the activity emerged. The place value activity emerged just as Geena's biscuits and squares emerged over the course of her baking activities (see CHAPTER THREE) and as the grade 4/5 students' bridges and towers emerged through interactions with their learning environment (Roth, 1994, in press-a).

Just as mathematics arose from the activity with the straws in Nora's colleague's classroom, mathematics often arose in Nora's English as a Second Language classroom. In her original notebook entry about her class, she created a list of language activities that raised mathematical issues (see Figure 4.2). Nora described the entries on this list as "things I could do with the kids in school that you might be interested in seeing" (N-2.1, 931105).

Figure 4.2 Nora's First Notebook Entry Showing Connections Between Mathematics and Language Activities in Her English as a Second Language Classroom.

E. S. L. Math/Language Activities

- cooking
- arts and crafts
- table games
- cards
- active games (whole body)
- time
- money
- amount of items

As we discussed this list over our next few meetings, Nora decided that it would be a good idea for me to come to her classroom and observe as the students' played some card games. She explained that she liked to teach the students games that they could play as rewards when they finished class assignments early, or in their free time at home with their families or with other students. These games provided two important kinds of

practice for the immigrant second language students in her classroom. First, they provided English language practice because they required students to communicate their intentions and, second, they introduced specific cultural activities with which these immigrant students tended not be familiar. Because these activities were all language-related, they will be discussed further in the section on *Connecting Language and Mathematics*.

All of the activities on Nora's list involved mathematics, but the mathematics were "frozen" or "hidden" beneath the language focus (Gerdes, 1986). Mathematical ideas are inherent in many human actions, but they may seldom be recognized. Gerdes (1986) has argued that "defrosting" this hidden mathematics gives status to the cultural activities in which the mathematics is hidden, leading to a cultural confidence. Although Gerdes was writing about instilling cultural pride in Mozambican weavers, a similar argument could be made for recognizing the hidden mathematics in any cultural activity. For Nora, this meant that she focused on cultural activities in the school and in her students' other daily activities. As she explained in her final project:

Often, incidental experiences and daily routines are used to reinforce mathematical ideas and concepts. Daily repetition and routine highlight and label these experiences for the students and this in turn, increases their awareness of the relevance and enjoyment of mathematics.

(N-pr.3, 931206)

As an example, Nora's final project focused on the important role of calendar activities in primary classrooms.²⁰ Nora was teaching part-time in the afternoons, so she was involved in few calendar activities at the time of this research project. However, to help her write her final paper, she made arrangements to attend three different classes during calendar activities. She also drew on what she had done in her grade 1/2 classroom the previous year.

²⁰Recall that calendar activities were a focal part of Geena's grade two classroom (see CHAPTER THREE).

In her paper, Nora outlined a series of activities that could be used to introduce and practice mathematical skills during calendar activities. The six main skills she referenced were: measuring, recognizing numerals, counting, patterning, sorting, and observing. The kinds of activities she presented revolved mainly around commercial and class-produced calendars, but she also introduced activities related to graphing and literature. What was common amongst these activities was that they served as opening activities for the school day. Fully one half (12 of 23 pages) of Nora's paper was devoted to reproducing activities from instructional booklets that mirrored the kinds of activities I had observed in Geena's classroom. Some of the ideas were as follows: creating a class calendar with date cards that were meant to be arranged in patterns representing some seasonal event (e.g., ghosts and pumpkins for the month of October in Nora's paper, poppies and doves for the month of November in Geena's classroom), creating bar graphs illustrating daily temperatures and weather, and graphing numbers of students who lost a tooth each month (Geena wrote students' names on a tooth-shaped poster labeled with the appropriate month, whereas Nora's paper suggested photocopying children's pictures and posting them on the tooth poster).

Nora believed that highlighting incidental mathematics in these ways helped increase her students' "awareness of the relevance and enjoyment of mathematics" (N-pr.3, 931206). In a similar fashion, our discussions about the incidental mathematics in Nora's activities inside and outside her classrooms, increased her awareness of the mathematics inherent in her activities. As she explained, "Everything I do now, I think, 'Oh yeah, this is math.'" (N-5.7, 940121). She even argued about the prevalence of mathematics with one of the other teachers in her social studies methods course the following semester:

Somebody said to me, "Well, where else do you use math, except figuring out?" You know, it was the typical question that we have discussed many, many times. I remember getting quite excited about it. And I said, "No, you use math all the time." And the person got almost defensive, and I went on to tell her, "I'm not, not trying to offend you, but I just want to bring up to you what's been brought to my attention because of this math course." Because I never looked at it that way, but the more you think about it, you're right. You

just don't think about it. When you think about it numbers are involved in everything. (N-6A.1, 940308)

Nora's increasing awareness of the incidental mathematics in her own (and her students') activities created an observer-interaction effect (see CHAPTER TWO and Lincoln & Guba, 1985), whereby questioning Nora about her incidental mathematics increased her recognition of that mathematics. As Nora reflected on and talked about her own incidental mathematics, her understanding of the role of that incidental mathematics in her life evolved. She increasingly became aware of her own mathematics, the mathematics that school typically "renders marginal, deems inferior and makes invisible" (Fasheh, 1991, p. 59). Through her own recognition of this incidental mathematics and her efforts to help students to do likewise, theory would suggest that Nora was poised to offer mathematics education as praxis (Fasheh, 1982, 1991; Millroy, 1992), an issue to which I return in the final chapter.

In the following section, I turn my attention to one of the prominent contexts for Nora's incidental mathematics, in connecting mathematics and language.

Connecting Mathematics and Language

Throughout our many conversations, Nora kept returning to the idea of connecting mathematics and language. As she described it, "Math and language are always intertwined. One complements the other" (N-nb.1, 931103). There were three main topics relevant to her connections of mathematics and language. First, discussions of Nora's classroom uses of mathematics often revolved around her second language learning group (her current teaching assignment at the time of the research project). In particular, I attended the class when Nora taught the students how to play two "mathematical games." Second, Nora discussed other connections between mathematics and language arts and literature, which were not specific to her second language learning group. Third, Nora emphasized the importance of language in mathematics, that is, the use of "proper" terminology to communicate mathematics. Each of these three topics are described in the following sections.

Playing Mathematical Games in a Second Language Learning Group

Because Nora was not teaching mathematics at the time of our interactions, our discussions of her classroom experiences often revolved around her second language learning group. Incidental mathematics often surfaced within the context of this group. Most prominently, on the day I observed in her classroom, she taught the students two “mathematical games”²¹ that each had a strong emphasis on communication. The focus in her language group was always on encouraging and providing opportunities for students to practice their language skills, and the “mathematical games” proved no exception. Students worked in pairs and responded verbally (in English, not their first languages). Nora purposefully selected dice and card games that involved mathematics, but were at the same time language-oriented and very social. Opportunities for the students to talk together in English were central to her language group, and she often used games with her students to achieve this goal. The day I observed, she also wanted to pick activities that were somehow mathematical as this would more closely match the goals of the research project. Of course, her selections of what was mathematical depended on her understandings of what counts as mathematics, as was described in the *Mathematical Sense* section.

Nora planned six games, knowing that this would be too many for the time allotted, but wanting to allow herself some flexibility. In the end, students completed two games. The first activity was a modification of Hundreds Chart Tic-Tac-Toe, drawn from a popular cards and dice book (Currah, Felling, & MacDonald, 1992, p. 23). Students played with a partner. Each pair of students had a hundreds chart (see Figure 4.3) and each student selected a coloured marker (a different colour from their partner). Nora explained that partners should take turns drawing two cards from a partial deck of cards (ace through nine only, no face cards). The first card represented the digit in the tens

²¹For an extended discussion and analysis of the relevance of games to mathematics learning see Bright, Harvey, and Wheeler (1985).

column and the second card represented the digit in the ones column. Students were then supposed to verbalize the number they had drawn and colour that number on their hundreds chart. Nora demonstrated the game by drawing two cards, a 6 and then a 5. She verbalized this as “6 tens and 5 ones equals 65” and then coloured the 65 square (see Figure 4.3). Rather than establishing a specific goal, Nora left the activity open-ended saying that the idea was to colour as many squares as possible. Together the class negotiated what to do if someone drew a number that had already been drawn. One child suggested, and others agreed that the individual should be allowed to redraw. As further language practice and to check students’ understandings, Nora asked one of the students to repeat the instructions back to her before play began.

Figure 4.3 Playing Board for the First Game in Nora’s Class.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The students got very involved in the game, and often had to be reminded to verbalize their responses rather than just racing to colour more squares. Because language and communication were the intended foci of the activity, Nora wanted students to verbalize their processes not just “jump to the answers”²²:

Um, as I did expect happened. Um, I knew they would jump to the answers instead of doing the verbalization. And constantly I had to remind them, which, it's not just pertaining to today. It happens at other times, too. And the other thing was speaking in their native language, but they are getting really good about correcting each other and reminding. (N-4.1, 931124)

Despite the focus on communication, mathematical ideas did arise during game play. After only a few minutes of play, one boy observed that it was not possible to fill the top row or the right column of the chart. This was a very astute observation based on the fact that there were no zeroes or tens involved in play. Rather than taking the opportunity to question why this might be or to highlight the mathematics behind this observation, Nora brushed off his comment as, “Oh, that's okay. Just see what you can get.” From a mathematics educator's perspective, this was an ideal opportunity to pick up on “incidental mathematics,” but Nora chose not to do so. We talked about this in our follow-up interview and Nora explained how she could “finagle it” by adding extra cards in future games:

N: Well I can always finagle it somehow. Put in other cards or have a jack, queen, king equal something else.
 M: Yeah, like there is some way you could add a zero or whatever.
 N: Zeroes, yeah. (N-4.2, 931124)

After my prompting, Nora realized that the missing zero was the problem, and agreed that she could add zeroes for future games, but she expressed no interest in following-up on the idea in class.

Mid-way through the game, in an effort to diminish the competition and to re-orient students toward communicating rather than filling squares, Nora prompted the class that

²²This emphasis matches the Standards focus on communicating mathematics (NCTM, 1989).

it was not important how many squares they filled, but the designs the colours made. She re-emphasized these patterns when the game finally ended (dictated by time, not some absolute goal) and then highlighted coloured patterns. However, in highlighting these patterns, she focused mainly on counting the number of like-coloured squares in a row, column, or diagonal (“Oh, look 4 in a row.”).

In our follow-up interview, Nora talked about why she modified the written objective of getting three squares in a row to filling the card. She explained that it was quite easy to get three squares in a row, such that the game would end very quickly. Instead she wanted the students to have more language practice. She felt that an objective of three in a row would make the students very goal-focused rather than thinking about the language. In her opinion, this more open-ended version really brought out the language and provided students with opportunities to discover patterns:

I purposely didn't do that [establish a definite ending] because I wanted to reinforce the rules of the game and the idea and the language. Whereas if I only did three in a row they would have been over really quickly and you know, then what? Whereas it really brought out the language and then they started discovering patterns. (N-4.2, 931124)

Despite her stated interest in “discovering patterns,” the only patterns she talked about were the patterns involving identically coloured squares in adjacent squares. Many other patterns based on the number and colour combinations could have been discussed, but Nora confined her description to how many adjacent squares were the same colour.

In the follow-up interview, Nora also explained that the game was not actually open-ended, that it did in fact have an ending, but it could be a long-term game. Given the fact that it was not possible to fill the top row or the right column, the objective as she specified it was not possible. However, it would be possible to fill all other squares, but it could take considerable time. As Nora explained, “It's a long term game, but it sometimes it can be quick as well. It's a chance thing” (N-4.2, 931124). With this comment, Nora highlighted the issue of probability in the game. This mathematical

concept came up again in the dice game, but Nora did not pursue a discussion of the idea of probability in the context of either game.

The second game was one that Nora had designed on her own, and had used in the past. Again, students worked in partners on one sheet of paper with a pair of dice. Nora asked me to help demonstrate the game. She went first and rolled a 3 and a 2. She verbalized “3 plus 2 equals 5,” and then wrote this on our sheet of paper. Next, I rolled the dice and got a 4 and a 6. She verbalized “4 plus 6 equals 10, and 10 plus 5 equals 15,” then she wrote this on the sheet of paper. The class negotiated what to do if someone rolled doubles (i.e., the same number on both dice) and decided that they could roll again, that is, they got an extra turn. The objective of this game was to work collaboratively and try to add to one hundred.²³ As with the first game, Nora asked one student to repeat the instructions back to her to confirm understandings and as language practice. The girl who volunteered stumbled over the instruction regarding shaking the dice, as she did not seem to know the word “shake.” With some prompting and help from her classmates, she was able to explain the task.

As Nora and I circulated around the table, listening in on students’ conversations, we again had to remind students to verbalize their steps in English. The students themselves also reminded their partners to verbalize, as was common practice in their classroom. After play had continued for a few minutes, Nora came to me and pointed out that one boy was having difficulty “rote counting.” I asked her to elaborate what she meant during our debriefing, and she explained how his addition difficulties were a failure to use a counting-on strategy:

That’s when I kind of turned to you and said he can’t rote count. I think he can do. It appeared because I did an, I was watching him in the forties and he was having difficulty, but he would come up with the right answer. Then we got past the fifties and even counting by one to one correspondence: 52 in your

²³Compare this game to Geena’s math race (described in CHAPTER THREE). Here, the students work collaboratively rather than competitively, and Nora demonstrates only one of the two solutions Geena demonstrated.

head, 53, 54, 55. He didn't have the sequence. I had to help him with it. Which I was surprised... I talked to his teacher about that actually, um, and it's in all areas. He just needs an opportunity to, um, he's having E. S. L. with me and he's having learning assistance with somebody else, and, um, she's trying to see if she can get some extra help for him. But, he is also a second language student and he's been one full, a year and a half in an English school from Spanish oriented originally. (N-4.3, 931124)

Notice, how Nora talked about the addition problem as one of "rote counting." She did not advocate the use of the standard arithmetic algorithm, but encouraged this student to consider the problem as counting. Rather than saying 52 plus 3, she talked about counting "52 in your head, 53, 54, 55." In contrast to the mathematical ideas raised in the first game (zeroes in place value, number patterns, and probability), Nora did follow up on the mathematics in this example. Although the focus of the activity was language, Nora was cognizant of the mathematics and was careful to talk with the child's homeroom teacher about the difficulty she had observed.

In this second game, the mathematics that arose was more blatant and simple, and Nora chose to follow up on the difficulty she observed. In contrast, during the first game, three fairly complex mathematical concepts arose—zeroes in place value, number patterns, and probability—and Nora downplayed their significance. This is a theme that I will pick up again in my descriptions of Nora *Using Mathematics Outside School*.

Throughout both of these games, the focus was on language, predominantly the need for students to communicate to their partners what they were doing. This language focus was entirely appropriate (and expected) for a second language learning group. If these mathematical games had been used in mathematics class, Nora may have adapted her teaching approach. With only one hour for students to engage in the games and practice their oral English language skills, there was little time for a mathematics lesson. Although games can be very effective instructional tools in mathematics, games need to be combined with other instruction to develop higher level skills (see the extensive research project investigating learning and mathematical games by Bright, Harvey, & Wheeler, 1985). An emphasis on language was essential for Nora's classroom, but she was able to

connect this focus with mathematics as well, something which NCTM (1989) guidelines have encouraged. This theme of connecting language and mathematics recurred throughout many of our discussions and in Nora's assignments for her mathematics methods course. Further examples of her concern for connecting mathematics with language arts and literature are addressed in the next section.

Mathematics in Language Arts and Literature

During our time together, Nora frequently wrote and talked about the connections between mathematics and language. She started her first notebook entry about her final project for the methods course with the statement, "Math and language are always intertwined. One complements the other" (N-nb.1, 931103). This highlights the recurring theme of connecting language and mathematics. She returned to this idea when she wrote her final paper for the mathematics methods course:

Mathematics and language go hand in hand and what better way than through calendar activities. Children learn mathematics through the use of language, therefore, opportunities for discussion during all stages of mathematical learning are important. For instance, the combination of mathematics and literature, used in conjunction with opportunities for talk and discussion, allow children to experience mathematical concepts in meaningful context.
(N-pr.1-2, 931206)

In this paragraph, Nora highlighted the interconnectedness of mathematics and language, making three important observations. First, calendar activities, as described throughout her paper, are very language-centered, but also allow ample opportunities to focus on mathematics. These calendar activities were discussed in the section, *Recognizing Incidental Mathematics*. Second, children learn mathematics by discussing mathematics. This is an idea that has received prominence in the Standards (NCTM, 1989) and will be further elaborated in the next section, *Language in Mathematics*. Finally, literature provides an excellent medium to connect mathematics and language, as has been recognized by NCTM (1989; see, for example, Welchman-Tischler, 1992). In the context of calendar activities, Nora had freedom to let her students explore both

language and mathematics, unlike in her second language group where she felt obliged to focus on language.

As Nora described in her final paper for the mathematics methods course, “Both math and literature serve the purpose or function in the following ways: ordering the world around us, classification, problem solving, relationships, patterns and structure, just to name a few” (N-pr.2, 931206). Her final project included several activities focusing on specific story books that could be used to introduce mathematical concepts in an engaging manner. Books provide excellent opportunities to weave mathematical concepts into language arts activities (see discussions of Mrs. Piggle Wiggle [MacDonald, 1947] and the Robert Munsch books in CHAPTER THREE).

Another issue regarding connecting language and mathematics came up in relation to Nora’s teaching in the special needs classroom. In that setting, students were connecting not only mathematics and language, but mathematics and two languages (French and English).

It used to just floor me, it was a so-called bilingual school and the number of children who were special needs kids [and] were bilingual, it was just unbelievable. I mean, and we also got involved in a rotated workshop situation...And so the kids would rotate, we would have the French and the English. Well, I spoke French and [my co-teacher] didn’t, which was fine for the two of us together. But in these rotating situations you would have French and English kids together, and inevitably I would speak English to the French and French to the English and then they would translate or the French would answer back in English and it just got, it was really neat and there wasn’t a language barrier...and it was nice to see the words, um, coming out in both languages, you know, especially while cooking. You know, how many cups of flour do you need or how many teaspoons and they were using them and counting and the words and the language and the labeling in both languages. It was just so neat. (N-1.7, 931029)

In this excerpt, Nora focused on the accomplishments of her special needs students and how exciting it was that they were able to communicate in both official languages (English and French). At the same time, these students were able to communicate the mathematics of cooking, “How many cups of flour do you need or how many teaspoons.” The students were able to communicate the mathematics in both languages, “They were using them and counting and the words and the language and the labeling in both

languages.” This idea of communicating mathematics is the focus of the next section of this chapter.

The Language of Mathematics

Discussing and communicating have gained popularity as important skills in the mathematics classroom. Consistent with this focus, Nora was interested in communication and the use of formal terminology for mathematics. In her final project for the mathematics methods course, she wrote about the importance of introducing students to formal terminology in mathematics. She cautioned, however, that one must be aware that knowing the words does not equate to understanding the concepts. You can use the terms without understanding, and you can understand without using the terminology:

At this early age [kindergarten], children may be informally introduced to words such as explore, justify, represent, solve, construct, discuss, use, investigate, describe, and predict...However, it is being cautioned to use these words “very informally”, as some children quickly absorb the formal language of mathematics, but its use does not always guarantee that it is understood. Too early an insistence on “correct” usage, may also make children unwilling to use their own language to express their ideas...Children develop mathematical concepts through the use of informal language and move gradually towards a formal terminology and a symbolic method of recording.
(N-pr.6, 931206)

In this quote, Nora demonstrated her preference for questions that ask students to do more than compute, that is, to “explore, justify, represent, solve, construct, discuss, use, investigate, describe, and predict.” These kinds of question words demand higher order thinking than mere calculation questions. At the same time, Nora recognized the potential language difficulties introduced by such question words. Students need to know what the words mean to understand what the question requires, and need to use language to present a response to the question. She was concerned that understanding not be circumvented by insistence on “correct” language; students need to be introduced to the formal terminology (or canonical language) slowly.

Nora's concern with introducing language slowly was also evident in our first interview where she explained that it was important to allow students opportunities to understand concepts before introducing the formal terminology:

Well one of things I was reading, too, was that communication, the language, understanding at that level before actually giving names to symbols. You know understanding at a manipulative level before. I read it in the text that [the methods instructor] has given us. They really seem to stress that. Don't be so quick to rush on to have the children into labeling you know quick, quick. Have them try to discuss and understand at a visual and a manipulative level before. And when it is really understood then bring in as they are ready to the symbols.
(N-1.12-13, 931029)

When I asked her to elaborate on the role of formal terminology in mathematics learning, she answered by providing an example of how limited access to the vocabulary of mathematics interfered with one student's ability to solve a long division problem. Notice, however, how Nora, herself, stumbled over the terminology in her description.

Um, they were doing multiplication, but it was, you know, there's a few steps involved. There's estimating first, and then multiplying, er, not multiplication, division. Dividing a number into another number. Estimating if the smaller number goes, or the divisor goes in, divider goes into the divisor. And then multiplying out to see if the answer is more or less than the number in the original....OK, three, 2 into 5 goes twice, remainder 4, um, goes in twice, 2 times 2 is 4, but there's a remainder of one, so you know you need to subtract 4 from 5, so that's another step. So, there's an estimation, there's an, um, dividing into which is the estimation, and multiplying and subtracting. And those are all steps. And I was doing that with one child and she got down to the subtraction part and I said, "Okay, subtract," oh no, I said, "6 minus 2," or whatever it was. And she says, "Minus?" And I said, "Take away. If you've got 6 and you take 2 away." So, then that's labeling. She knew what to do, but she didn't understand the word minus. As soon as I said it in a different way that she could relate to, that's the label she could relate to, she knew the actual activity requested of her. It was the label I assigned to it that she didn't understand.
(N-6B.5, 940308)

The girl did not understand the label, "minus," and, therefore, was unable to complete the task. When Nora changed her description to "take away," the girl knew exactly what to do. This child demonstrated an ability to perform the subtraction task, but did not understand the original terminology used.

Like the student she described, Nora also had difficulty understanding and appropriately using the terminology. In her description of the steps involved in doing long division, she mixed the labels for division and multiplication ("Multiplying, er not

multiplication, division”), and for divisor and dividend, to which she referred as divider (“The divisor goes in, divider goes into the divisor”), plus she stumbled over her description of testing divisors (“So, there’s an estimation, there’s an, um, dividing into which is the estimation”) and finding a remainder “2 into 5 goes twice, remainder 4, um, goes in twice, 2 times 2 is 4, but there’s a remainder of one”).

In other cases, it was evident that Nora invented her own language to talk about mathematical ideas. For example, after the pet shopping expedition, she explained that she had “looked for the same yeah size, the same brand, the same poundage, I guess, for lack of a better word” (N-3.1, 931112). She invented the language “poundage,” rather than using canonical language. Similarly, in describing the currency exchange rates for the cross-border shopping trip, she consistently referred to the rates as cents. She indicated that the rate of 27.8 was “up two cents, almost three cents” from last summer’s “25 cent rate.” Cents can only be considered rates when given as cents per dollar. Although, this was implicit in her descriptions, it was not explicit in her language. These examples illustrate Nora’s informal use of language, just as she had described for her students, “Children develop mathematical concepts through the use of informal language and move gradually towards a formal terminology and a symbolic method of recording” (N-pr.6, 931206). Both Nora and her students began with their own informal language, which should then evolve as new understandings emerge (Gooding, 1992; Latour & Woolgar, 1979; Rorty, 1989). As newcomers to a field work with new ideas, trying to understand and communicate, their language evolves to take on new meaning and increasing precision to allow enhanced communication. See McGinn, Roth, Boutonné, and Woszczyzna (1995) as an example of this evolution of language in a new field. In that study, grade 6/7 students adapted their language and adopted new language games as they constructed and presented simple machines in science. They invented labels such as “slippage” to talk about friction; they referred to “Lana’s law” as a resource to solve equal arm balance problems using the product moment rule; and they increasingly

adopted the label “mechanical advantage” to replace their earlier uses of advantage and disadvantage. The students in this study started from their informal language and gradually moved toward more formal (canonical) terminology. The snap shots of Nora and her students provided in this study do not allow a discussion of the evolutionary changes in their language games, but Nora certainly seemed accurate in her estimation that such changes must begin from the students’ current language “and move gradually towards a formal terminology.” At the time of this study, Nora had few opportunities to experiment with mathematical language because she was not teaching mathematics and the mathematics methods course was not meeting her needs. She had few opportunities to populate the language with her own intentions (Bakhtin, 1981) and, therefore, struggled to use the language in canonical forms.

This section has highlighted Nora’s many considerations about connecting language and mathematics, through games in her second language learning group, through other language arts and literature tasks, and through the language of mathematics, itself. Although it may appear on quick reading that Nora had failed to adequately connect language and mathematics, I would like to stress that such a reading is inappropriate. Nora did weave together mathematics and language, but only “simple” mathematics. However, given the constraints of provincially-mandated curriculum and available time, in conjunction with Nora’s instructional goals and limited mathematics background, limiting the scope of connections may be expected. Further, these students needed English language practice, with one exception (the child described above), they had few problems with mathematics. In addition to Nora’s concerns about connecting language and mathematics, she expressed a great deal of interest in connecting school mathematics to uses of mathematics outside school, as I describe in the next section.

Using Mathematics Outside School

In the following sections, I describe Nora’s own uses of mathematics outside school and how she used out-of-school mathematics in her teaching, in particular in the

functional mathematics program she designed for her special needs students. This discussion is segmented in four parts. First, I describe Nora's pet food comparison shopping under the heading, *Performing Best-Buy Comparisons*. Second, I describe her cross-border factory outlet shopping in *Estimating, Approximating, and Determining Situational Appropriateness*. Next, *Operating Within and Between Multiple Measurement Systems* outlines her uses of different measurement systems and how she converted between them. Finally, I document how she used out-of-school mathematics in the functional mathematics program she designed for her special needs students (see *Using Out-of-School Mathematics in the Classroom*).

Performing Best-Buy Comparisons

During our second interview, we continued our discussion of where and when Nora used mathematics outside her classroom, trying to pick an activity that I could observe. Nora's initial suggestion was shopping, but she admitted that she didn't really do a lot of grocery shopping because she was living on her own and would just go out and grab what she needed. Instead, she suggested a better example would be shopping for pet supplies. She had three pets—a cat, a puppy, and a dog (who was expected to return with her son in two months time, but did not return until June). She lived in a small suburban/rural community where she did all of her shopping, and a third pet store had just opened there. She thought it would be “really worthwhile” for her to compare prices at the three stores because she had recently noticed a huge discrepancy in prices:

I was even thinking pet supplies. I do have to go and get those, we could maybe go into the pet store because I've been thinking that there are three pet shops now in Kirktown and I would like to price their, I only get certain foods and I thought I would price them in all three stores and see which is the better deal. Because I've noticed, you know, I went to get the pup some toys and I walked into one store and I got exactly what I wanted, so I picked it up and it was like a little string, um, pull toy sort of thing and it was three dollars. So I bought it and took it home. She loves it. So I thought I should get another one, maybe a little bit bigger. So, I went into a different store and saw the same thing and priced the next size up and it was something like ten dollars and I thought that's crazy. The first one was just the next size down and it was only three dollars. So I thought, “How much is the small one, the identical one here?” And it was like seven dollars, and right there I thought, “Well, I know

where I'm not going to buy this toy." And that's what made me think, um, maybe I should shop around. (N-2.2-3, 931105)

So, the following week we met at the bus stop in Nora's neighbourhood and went to the three pet stores. First, we went to Pet Essentials, which was a big, new store that had just opened three weeks prior. Huge signs all over the store advertised the Grand Opening Sale that would start the next day. Nora explained that she would come back to check the sale prices the next day, but that we would do our comparisons based on the regular prices, since that was more appropriate for her long term budgeting. The second store was Cosby's, another fairly new store. This was the store where Nora had seen the expensive puppy toy which had prompted our outing. She suspected that the store would be more expensive in general. (I will describe how this expectation affected her comparisons later in this section.) Finally, we went to the oldest, original store. This was a family-run business which had been in the area for many years. The store was small and crowded, and had no name posted outside. Nora explained that she had no idea what the store was called, so made up the name Kirktown Pet Store to facilitate our discussions. No prices were listed on the products, so Nora had to check each price with the clerk (the son of the owner, as Nora pointed out). Nora suspected that this store would be cheaper than the other two because it was much older.

Nora had a list of four items that she wanted to purchase—puppy food, dog food (for the soon-to-be-returning adult dog, and, in time, for the growing puppy), cat food, and the puppy tug toy she had attempted to purchase and then realized the price discrepancy between Cosby's and Pet Essentials. While in the first store (Pet Essentials), she also noticed some treats for the puppy which she had received as a gift and which the puppy really liked. She then added the puppy treats to her list, so she was searching for five items in the three stores. For each of the items she knew exactly what she wanted:

I looked for the same yeah size, the same brand, the same poundage, I guess, for lack of a better word, and the ones that I use on a regular basis. Right now there are three pets and I have three different kinds of pet food only because one of the dogs is still a pup, so she's got to be on puppy food, but when she's

qualified as an adult she will be the same as the other dog's.
(N-3.1, 931112)

As we went to each store, Nora recorded the prices of all five products in a small notebook (one page per store). Then we went back to her house to discuss what she had found. Explicit recording of prices and making comparisons at home runs contrary to what one would normally expect for comparison shopping. Typically, shoppers make comparisons on-the-fly, that is, while they are in the store, they decide which product to purchase and where. Because Nora's comparisons were made on five products across three stores it made some sense to record prices in this fashion; remembering all fifteen prices would be difficult. Her recording may suggest another observer-interaction effect—it is quite possible that Nora would not have explicitly recorded the prices if it were not for the research (In fact, she may not have made the comparisons at all, if it were not for the research, although she did say she had wanted to do so).

Table 4.1 indicates the prices and Nora's commentary as we went over the figures she had recorded back at her home. From our discussion, it was clear that Nora selected the items to purchase based on brand loyalties (what her pets liked, what her veterinarian recommended) and size (larger packages were cheaper than multiple small packages,²⁴ larger packages lasted longer so fewer shopping trips were required, opened food could go stale over time, the four pound bag of cat food fit perfectly into the container she had for it, storage space was not an issue, and packages with enough food to last about a month were preferable), rather than cheapest product. After creating her list of products, she then compared the prices for the five specific items at the three different stores. As with Murtaugh's (1985b) grocery shoppers, aspects other than arithmetical calculations

²⁴This statement is not always true, but Nora reiterated twice in our conversation that this was a known fact, "When I had two cats I did [opt for the larger bag] because they both were on the same food and it's cheaper, obviously" and "When I have both dogs on the same [food] then I will go for the larger bag because it will definitely be cheaper." One of Nora's considerations was that larger packages were cheaper than multiple smaller packages, whether this statement was accurate is not at issue.

took precedence in Nora's best-buy comparisons. She engaged in arithmetical comparisons only after she had selected the particular items she wished to purchase.

Table 4.1 Nora's Price Comparisons and Comments Regarding Pet Supplies

Item	Kirktown Pet Store	Cosby's	Pet Essentials	Nora's Comments
Puppy Food	\$13.99	\$16.29		Wow, that's a big difference....That doesn't seem quite right, between the Kirktown Pet Store being fourteen. No, two dollars, a little over two dollars. I obviously won't go there for that. I know that.
			\$15.59	There's a difference of a dollar and some change. [Probably this comparison was to Kirktown Pet Store, but she did not indicate.]
Dog Food	\$10.99	\$11.49		So, again the Kirktown Pet Store is better.
			\$10.99	So it's the same as the Kirktown Pet Store, so Kirktown wins out again.
Cat Food	\$9.99	\$9.99	\$10.35	It's more expensive [at Pet Essentials]. So, for all of those three, the actual food, the Kirktown Pet Store is the cheaper place to go.
Puppy Treats	n/a	\$3.19	\$2.99	So, it's cheaper at Pet Essentials. And, that's not counting their opening special tomorrow. So Cosby's lost out.
Puppy Toy		not recorded	\$5.39	Cosby's I didn't even look because it was such a big difference. I just wasn't even going to consider it.
	\$6.99			So, that would have been cheaper at Pet Essentials.

(table continues...)

So the doggie cookies and the toy was cheaper at Pet Essentials, and at, um, but as far as daily food goes, the Kirktown Pet Store is better. [M: And you will have to compare if you go to the sale tomorrow to see if] Yeah. But looking at it as a long term thing, as a monthly purchase I guess the Kirktown Pet Store. And Cosby's loses out all together. I knew, I figured they would be a little more, but they seem to be quite a bit more than a little more....So it looks like it's Kirktown Pet Store where I will do my animal shopping, pet food shopping....So, there's our comparisons and, um, my answer is I think the Kirktown Pet Store is the store I will go to.

Because Nora already knew the sizes and brands of products she wished to purchase, the mathematical comparisons could be considered very simple. She could quickly assess which store was cheaper by looking at the prices of the five specific items. For example, it was very clear that \$13.99 was cheaper than \$16.29. Nora not only determined which prices were cheaper, but also by how much. Her comparisons of the magnitude of differences were all approximations. She described the difference between \$13.99 and \$16.29 as “between the Kirktown Pet Store being fourteen. No, two dollars, a little over two dollars.” Similarly, she described the difference between \$13.99 and \$15.59 as “a dollar and some change.” These estimates were situationally appropriate for Nora’s purposes. Her primary goal was to find out where she could purchase her specific pet supplies at the cheapest prices.

A closer analysis of Nora’s comparisons reveals that she was doing far more than simple numerical comparisons among prices at the three stores. In order to determine the best price, she had to engage a very sophisticated optimization problem. She considered a number of factors in deciding what to purchase and where. Brand loyalties based on pet taste preferences and veterinarian health recommendations seemed paramount in her

decisions. Size considerations incorporated several additional factors. First, she was quite convinced that larger packages were cheaper than multiple smaller packages containing the same volume of food. However, she needed to balance this against considerations of how long such a package would last. She was unconcerned about storage space, aside from the fact that she had a container for cat food that perfectly accommodated the four pound bag of food. She was, however, quite concerned about potential staleness. Larger packages last longer, so they have a greater potential of going stale before they can be used. At the same time, larger packages last longer, so fewer shopping trips would be required, thus saving time and money for additional trips to the store. As well, she preferred to consider her pet supplies as a regular monthly expense that she could budget for and purchase on a fixed schedule. This meant it was desirable to coordinate the pets so that all of their products would last about a month (the desired time to prevent staleness) and could be purchased together on one shopping outing. As for the shopping outing, Nora framed her solution in terms of one store, even though some of the products were the same price at more than one store. This meant she would only have to travel to one store to do all her shopping, thus saving time and money (fuel for her car) that would be required for additional stops. Product availability was critical to store selection, as were aspects of the store quality such as crowding and clearly posted prices that limited the need to ask for assistance. Finally, she wanted to select a store where she could do her shopping and feel confident that she was getting good service for her money, because she did not have time to continually compare prices.

Based on this analysis, Nora's best-buy comparisons involved the complex process of optimizing (at least) fifteen variables (see Table 4.2), rather than a simplistic numerical comparison of three numbers. Her solution to purchase the pet food at Kirktown Pet Store, and to get the "extras" at Pet Essentials was derived from her considerations of these various factors. Her solution that "as far as daily food goes, the Kirktown Pet Store is better," was framed in terms of one store, even though some of the products were the

same price at more than one store. Her assertion that Kirktown Pet Store was better was probably partially informed by her earlier sense that it would be cheaper and that Cosby's would be more expensive. Cat food was the same price at both stores, yet she starred Kirktown Pet Store as the better deal in her notebook. Dog food was more expensive at Cosby's, but the same price at Kirktown Pet Store and at Pet Essentials. Despite these price equivalencies, the large discrepancy in puppy food prices meant that Kirktown Pet Store had the best prices for all three products. By selecting one store at which she could purchase all her pet food, she simplified the task of pet food shopping since she would then only have to travel to one store to do her shopping. This would result in a further savings because, although all three stores were approximately equal distance from her home and all quite close together, having to travel to multiple stores would be more costly in time and money (fuel for her car) than having to travel to only one store.

Kirktown Pet Store did not sell the puppy treats Nora wanted, so it would be necessary to go to one of the other stores (probably Pet Essentials since it was cheaper). She would, however, request that Kirktown Pet Store order in the chosen puppy treats for future purchases. Also, the puppy toy was cheaper at Pet Essentials than either of the other stores, but this did not introduce a problem because this was a "one-time" purchase that she would pick up at Pet Essentials' grand opening sale the following day. Similarly, the puppy treats were an occasional purchase, rather than a regular monthly purchase, so she could plan to stop at Pet Essentials occasionally, but go to Kirktown Pet Store as her regular pet stop.

Table 4.2 Fifteen Factors Considered in Nora's Pet Food Comparison Shopping

Optimization Problem.

Factor

Pet taste preferences

Veterinarian recommendations

Large packages were cheaper

Storage space

Container for cat food

Potential staleness

Fewer shopping trips

Regular monthly expense

Purchased on a fixed schedule

Coordinate the pets

One store

Product availability

Crowding

Clearly posted prices

Time to continually compare

It is also important to keep in mind that the data Nora collected provided a snap shot of conditions at the three stores, but prices and product availability fluctuate over time in stores. Particularly, when a new store opens that is in direct competition with neighbouring stores, prices and product availability fluctuate as the stores vie for a limited customer base. Neither Nora nor I gathered data to substantiate such fluctuations,

but they certainly represent important considerations in everyday mathematics.²⁵ In fact, two weeks after our pet shopping expedition, Nora commented on the value of the comparisons we made and the need to repeat them on an ongoing basis, “I think I’ll do it on an irregular basis, check periodically and see if prices fluctuate because I am sure they must compare to each other to see, to keep track” (N-4.6, 931124).

Recognition of such price fluctuations may be important for school mathematics that attempts to be realistic. For example, Brenner (1995) described some comparative mathematics about pizzas conducted by an elementary school class. I would be interested to find out whether the price differences between the three pizza companies in the school neighbourhood changed as a result of this class’ activities. Certainly, if the entire class and their families decided to frequent one pizza shop over the others, they could agitate the competition between the pizza companies and lead to changes in the price differentials. Textbook mathematics problems, on the other hand, are always snap shots.²⁶

Whenever fluctuations occur in any of the variables of a best-buy optimization problem, re-optimization of the entire problem is required to find out the best buy. Although, I have no data to substantiate fluctuations in any of the fifteen factors outlined in Table 4.2, Nora did rethink her solution. When I checked back with her four months after our price comparisons, she explained that she was now shopping at Pet Essentials, although this was contrary to the data she collected during our first outing. She had found that Kirktown Pet Store was overcrowded and that it did not always have what she wanted. She still went there occasionally, but did most of her shopping at the new Pet Essentials. This finding provides further support for the conditional nature of Nora’s out-

²⁵Consider that the most astounding mathematical feats of the young Brazilian candy sellers were related to their abilities to maintain profits despite phenomenal inflation rates (Saxe, 1991).

²⁶Again, I am indebted to my colleagues in the thesis research group for pointing out this distinction between dynamic versus snap shot variations in “real” mathematics problems.

of-school mathematics. The stock at Kirktown Pet Store seemed unreliable, so she stopped shopping there. She adapted her solution, even after she had explicitly stated, “My answer is I think the Kirktown Pet Store is the store I will go to.”²⁷ As for Cosby’s, which she referred to as the “expensive place,” she had not been back since our shopping trip and had no plans to do so.

As this example shows, Nora was doing very sophisticated mathematics (a fifteen variable optimization problem), but it could be mistaken for very simple mathematics (a comparison of three numbers). Nora, herself, focused on the numerical comparisons between the three stores, and downplayed the more complex optimization that was involved. This is consistent with prior research that indicates that people structure their environments to avoid calculations (e.g., Lave, 1988). Given her tendency to pick up on the more blatant mathematics as opposed to the underlying mathematics, as revealed in the discussion of *Playing Mathematical Games in a Second Language Learning Group*, one would expect Nora to focus on the numerical comparisons at the expense of the more complex optimization. I now turn to a discussion of Nora’s cross-border shopping and the optimization problem relevant in that context.

Estimating, Approximating, and Determining Situational Appropriateness

After the pet shopping observation period, I suggested to Nora that she pick a second example of non-school mathematics that I could observe. One of the ideas that we had already discussed a few times was to cross the American border and go to a nearby factory outlet mall. This provided a second example of comparison shopping, but this time involving a currency exchange and different sales tax rates.²⁸ Nora explained, “The

²⁷Notice how she used the phrase “My answer is...” As pointed out by one of my colleagues from the thesis research group, this phrase suggests a school answer, thereby highlighting the situatedness of the activity. As with Geena, this was not just an exercise in shopping, but an effort to display her non-school mathematics to me.

²⁸Duty must be paid on some items when they are brought across the US-Canada border, but Nora did not mention this in her calculations. Further, when we reached the border crossing and she claimed her purchases, the customs official did not require her to pay the duty.

one thing that is kind of neat about the [factory outlet mall] I suppose is the fact that the difference in money comes into play. So price will look really great and then you start figuring the difference in money and you find out if it is a really good deal or not” (N-5.2, 940121).

Based on this decision, we planned a trip to the factory outlet mall for the following month. When we arrived at the mall, we decided to go to the cafe for a drink and a short discussion to focus our efforts. As soon as we walked into the cafe, Nora asked the clerk for the currency exchange rate, which was quoted as 27.8. Nora explained that this represented a significant increase from last summer’s rate of 25 (“Up two cents, almost three cents.”). Notice, how Nora converted the exchange rates to cents; implicit in her description was the fact that the cents were actually expressed as cents per dollar. She also explained that the rate of 25 was a lot easier to figure out because she could just add a quarter for every dollar. At the same time, she explained that different stores had different exchange rates, so you really had to be careful. In particular, the currency exchange market was worse than all the stores. As we went to the different stores, it was apparent that adding 25¢ per dollar was no longer adequate. Nora explained that, the increased exchange rate meant that it was more appropriate to “add about 33¢ per dollar” in most stores. In fact, the stores used their exchange rates in what seemed to be a counterintuitive (and misleading) fashion. For example, at the cafe the exchange rate was quoted as 27.8. However, this did not mean that American dollars were worth \$1.278 Canadian as one might expect. Instead, one Canadian dollar was worth $1.00 - .278 = .722$ American dollars (i.e., what would normally be quoted as an exchange rate of 38.5). Therefore, Nora’s 85¢ drink, which came to 91¢ American with tax, was converted to \$1.26 Canadian. That is, using Nora’s language of adding so many extra cents, she had to add 38.5¢ per dollar (or in this case 35¢ to the 91¢).

Two points are important about Nora’s implicit conversion of rates into sums and differences. First, she relied on her own informal language of adding cents to dollars

rather than adopting the canonical discussion of rates. This is consistent with her preference for informal language, as outlined in *The Language of Mathematics*. Second, adding cents is a more blatant and simple mathematical notion than the underlying notion of calculating rates. Again, this is consistent with prior research and with Nora's focus on superficial mathematics rather than underlying ideas (see *Playing Mathematical Games in a Second Language Learning Group* and *Performing Best-Buy Comparisons*).

After this experience with the exchange rates, we proceeded on our shopping spree. Nora had planned to check the prices on three items: dish cloths, pillow protectors, and women's trouser socks. Our first stop was the linen outlet where she would be able to find dish cloths and pillow protectors. She went directly to the back of the store to the display of towels, face cloths, dish clothes, and tea towels. The dish cloths were priced at 39¢. Nora compared this to what she considered a typical Canadian price of 99¢. With such a large difference, Nora felt there was no need to perform any calculations, she "knew" that the dish cloths were cheaper than in Canada, even with the exchange. Thus, she decided to purchase some dish cloths. Consider, however, the hour we spent driving to the factory outlet mall. The sixty cent savings on the dish cloths would not be worth the costs (in fuel for the car, time, and wear on the vehicle) of driving to the factory outlet, but Nora had already driven to the outlet, so it was worth purchasing the cloths at the lower price. That is, prior to our trip, the optimization problem included time and fuel costs, but once we had already arrived at the mall the re-optimization did not include these factors. Given that we were already there, it was economically viable to purchase the dish cloths; it would not, however, have been economically viable to drive there just to buy the dish cloths.

The next item on Nora's list was pillow protectors, which were priced at \$5.49. Again, no calculations were needed. Relying on her number sense, she could tell by looking that \$5.49 American was more expensive than the \$6.00 Canadian prices she had

seen. Back at her home afterwards, she explained how she had checked the prices at a local store before going to the factory outlet²⁹:

I specifically went into Mountain Linen because I knew we were coming down here....And um, I was deliberating over buying new ones [pillow protectors], and I thought well, I just want to see what the prices are. So I went into the one in Canada and saw they were between 6 and 10 dollars for a package of two, so I knew that if the American one was a similar price plus add on....it would be better to buy it here [in Canada]. If the prices are fairly close being American and Canadian, I know American is going to be more because of the difference. I mean if they're really close, and I'm looking at within a dollar range, then it's not worth it. But if there's a bigger difference then it's worth considering.
(N-6B.11, 940308)

Her number sense told her that if the American and Canadian prices were “within a dollar range” then the “American is going to be more.”³⁰ She did not need to calculate because \$5.49 and \$6.00 were so close, that the exchange would definitely make the American more expensive. Based on this description, she decided not to purchase any pillow protectors. She would wait until she returned home to buy these. Notice also that she talked about the difference between the Canadian and American prices rather than a ratio that would have permitted comparison with the exchange rate.

As we made our way to the check out in the linen store, Nora noticed some bed garters (elastics which clip to the bed sheets to hold them under the mattress). These were priced at \$3.99. She had not planned to buy these, but had seen them before and had been intrigued by them for sometime. She didn't know the exact comparison price in Canada, but thought that they were “something like ten dollars.” Assuming her memory was

²⁹Contrast her strategy of checking the range of prices for pillow protectors at the local store versus the explicit recording of all prices for the five desired items at the three pet stores, see *Performing Best-Buy Comparisons*. This is a sensible change given that the pillow protectors were an occasional purchase, whereas pet food was a regular monthly purchase.

³⁰It is not clear how Nora would transfer this strategy to more expensive items (e.g., televisions, computers, automobiles) where hundreds or even thousands of dollars difference on the ticket price could not compensate for the currency exchange rate. This question was posed by one of my colleagues in the thesis group, and may have provided an interesting avenue for Nora and I to pursue, but unfortunately the suggestion was made too late for me to follow-up. Further, Nora was not interested in buying such “big ticket” items, so there would have been few opportunities to see what she would have done in such cases.

correct, she believed that the American price was a bargain and decided to try them, after confirming with the sales clerk that they did in fact work.

We then went to a women's clothing store to look for her trouser socks. She found them at three pair for \$7.50. She explained that this was the only place she had found that sold these socks, so she always went there to buy them. She really liked the wear and durability of these particular socks, so she wanted to purchase some more. Plus, she explained that three pair of socks for \$7.50 was very cheap. She suggested that it often cost that much for one pair of socks. As she explained, these were "really good socks and a good price, even with the exchange." Again, she arrived at her solution without many formal calculations. As with the pet food, she knew the particular type of socks she wanted and it was only a matter of finding a reasonable price for them. This store was the only place she knew that sold the socks, so she bought them there. At the same time, she also recognized the huge discrepancy in price between these socks and other comparable quality socks available in Canada. With such a large difference in price, the socks were cheaper at the American outlet, even with exchange and duty.

We then continued looking in some of the other factory outlet stores. In the office supply store, she found some pencils that would be good for her students. These were priced at five for \$1.00. She counted the cost of a class set (30) on her fingers "5, 10, 15, ...\$6.00 plus tax plus exchange" and concluded that they were much cheaper at home where she could get a class set from one store for only a few dollars. Here, she calculated the price of the pencils in units equivalent to a class set because she knew that there was a teacher's supply store that sold class sets of pencils very cheaply. The class set unit, the price of a class set at the teacher's supply store, and the price for five pencils at the new store served as landmarks in her calculation space that allowed her to compare the prices across the two stores by relying on her number sense (Greeno, 1991). This demonstrates (a) counting by fives which is one of the skills that she included in her calendar activities for her final project in the methods course and (b) converting between different base units

(from base 5 for the pencils sold at five for a dollar to base 30 for the pencils sold in class sets). Nora's performance was analogous to the shifts between multiple base number systems Scribner (1984b) observed; delivery drivers converted unit numbers into cases and computed costs per case when they knew the case price from memory, that is, when they could use case price as a landmark. This strategy relies on the same kind of regrouping strategy that was prevalent in the street mathematics of the Brazilian candy sellers (Carraher, Carraher, & Schliemann, 1985).

Next, we went to a kitchen store. She picked up a grill basket for the barbecue which was priced at \$9.99. She described this as "10 dollars plus 33 per cent."³¹ She was interested in this, but wanted to postpone it until later in the season, and after she had a chance to research the prices in Canada. She thought it was a neat idea, but that she could probably find a better price.

Later, when we returned to Nora's home to tie up our shopping trip, I asked her about her calculations. She explained that she used a lot of estimating and rounding up, rather than calculations:

Well, basically I rounded off something to the nearest dollar, looking at the price. This cost 3.45, I said okay 4 dollars. You know put it at the higher level. And then I would look at, um, well, before it used to be easy, just add on another 25 cents for each dollar, but that's not what it is any more. So I would kind of do it 50 cents knowing I was really over. But it was a quick calculation. I mean this amount but less, you know, less than 5.50 or whatever, so that's how I did it. It was quick. I knew I was nowhere near it, but I was in the ballpark. (N-6B.10, 940308)

This level of precision was appropriate to determine whether prices were better at the American factory outlets or back home in Canada. For her calculations, she did not have to be "near" the price, just "in the ballpark" of the price (cf. Geena *Determining Appropriate Levels of Precision for Measures* and Lynch, 1991).

The situational appropriateness of Nora's approximate calculations with rounding up was further explained by her preference toward purchasing from Canadian stores. She

³¹The rate was actually 38.5 per cent.

believed that it was better to buy Canadian whenever possible, but would buy American when there were substantial price differences (e.g., the dish cloths) or when the products were unavailable locally (e.g., the socks). As with her pet food (and the groceries in Murtaugh's [1985b] study), conditions other than actual price differences were relevant in her best-buy comparisons.

What all these examples have in common is that they could be considered optimization problems, just like the pet food comparison shopping. This time the relevant factors were: different tax rates, currency exchange rates, product availability, product quality, preference to support local merchants, and travel time and expenses. In addition, she may have considered duty on items taken across the border and currency exchange rates from different sources.³²

These stories also demonstrate that Nora once again engaged in very sophisticated mathematics, but did not recognize it as such. She solved complex optimization problems and she operated between different base unit systems, but she interpreted her work as comparing prices, that is, as simple numerical comparisons. The next section, describes a third example of Nora's out-of-school mathematics, her attempts to convert between multiple measurement systems.

Operating Within and Between Multiple Measurement Systems

During our time together, the issue of operating within a given representational system, and converting between different systems came up on several occasions. Nora was quite adamant about the need for students to understand both analog and digital clocks. She recognized that "times are changing," but still felt it was important for students to be able to understand analog clocks. She discussed how this often came up in

³²Currency exchange rates from different sources could introduce additional optimization factors. For example, she could have obtained a more favourable exchange rate at a Canadian bank, but then she would have to determine in advance how much money to convert to American funds—a difficult task when she did not know what she would find to purchase on our trip. If she converted more money to American funds than needed, she would end up with extra American money that she did not want, and it would cost her money to convert it back to Canadian funds.

her classes. Students tended to have digital watches, and were unable to read her (analog) watch. She explained that she had made a deal with her son when he was younger that he could only get a digital watch if he first demonstrated that he knew how to tell time on an analog clock, a challenge he quickly mastered.

Conversely, when we talked about metric and imperial measurements, Nora revealed her own preference for imperial measurements and her limited knowledge of metric measurements:

And the metric you know, I'm still on imperial. I mean, certain metric things I know because it takes a while to penetrate this thick skull, but I mean I never learned the metric stuff and, um, I'm still kind of okay, I know such and such, like 1 pound is like 500 grams or something like that, thereabouts, so 250 is approximately half, but I'm still in pounds. (N-1.6, 931029)

Although she was adamant that her students learned to read both digital and analog clocks, she herself had not taken the time to learn to use both metric and imperial systems of measurement. She knew the imperial system ("I'm still on imperial"), but only limited metric ("Okay, I know such and such, like 1 pound is like 500 grams or something like that, thereabouts, so 250 is approximately half"). Through experience she had picked up some conversions, for example, the equivalent to a pound or half a pound, but still did not really know the metric system. She had a strong "number sense" (Greeno, 1991) for imperial measures, but not for metric measures. Greeno expressed this sense in terms of an environmental metaphor; we learn to navigate (gain a sense for) an environment by inhabiting that environment. Nora had been living with imperial measures for such a long time that she was highly familiar with them and had a strong "imperial sense."

To deal with her limited number sense for metric measures, she continually had to convert between metric and imperial measures. For example, when she and her son drove across the country, she was constantly trying to calculate how many miles they had travelled and how many more miles were left:

I know when my son and I drove across Canada, when we were moving here we, we had the map....And, um, we, ah, you know the signs such and such a city was, you know, 400 km. Okay so, you know, for me what does that mean in miles, so multiply by .6 or .06 or .6 or something like that. So, with my son

that kind of brought math into it for him as we were travelling for that short time he was interested. (N-1.6, 931029)

Nora preferred to operate in the imperial system, and had some difficulty understanding how to convert metric measures into the imperial measures that she understood more clearly. Unlike Geena, she found it was important to perform calculations to get a stronger sense. However, she wasn't sure whether the appropriate conversion factor was 0.6 or by 0.06, a definite problem for her calculations.

She had a strong number sense for imperial measures, and was trying to develop that sense for metric measures, but still very much relied on imperial measures for understanding:

I mean certain things that you hear and use on a regular basis I'm accustomed to, now but. Like when we were driving out here I was doing it constantly. Converting just to get a round idea about how far I had to go or I did go, even though I know a kilometre is less than a mile. Just to know, I understood miles, so....And even the same in school for measurement. You know you use feet and inches, I mean now I'm getting accustomed to the other because we are using it. But, even still, I think in feet and inches. Or if I go to sew. I don't sew very much, but I used to. But the odd time, I have to go buy material, and I think how many yards or how many inches, not metres. I mean I find that very confusing. (N-5.3, 940121)

As she continued, she elaborated on this idea of experience-based differences in her understandings. In Canada, temperature is always reported in metric, or both metric and imperial, so she was highly familiar with metric measurements related to temperature. Similarly, speed limits and gas purchases are given in metric, so she understands those things a bit better in metric (but, she still needed to convert distances into imperial to understand them, as described above):

I find that of any of the metric stuff, the weather is what I can understand quicker. You know when they say, um, the Celsius degrees I can understand it in metric and imperial. Whereas in measurement I can understand it in imperial, but the metric I have to stop and think about it...My, um, I notice the speedometer on the car is in metric but it has the imperial underneath. So, I refer to metric when I am driving, so I kind of know what it is. Actually, I don't. It's really interesting I understand it in metric, and when I start to think in imperial, I have to check to see where it falls. Because 100 kilometres is approximately 60 metric, I mean imperial. And, but it's really interesting because I'll go, "Oh yeah, but what is that?" and then I look down. Because I'm so used to it...and that's the larger indicator on the speedometer on the

car too. So, it's, I think what you use on a regular basis more than something that's occasional. (N-5.4, 940121)

These excerpts illustrate that Nora had a real sense for imperial measurements in some cases, and for metric in others, but did not have a strong sense about conversions between the two systems. Compare this with Geena's strong sense for both imperial and metric measures, but not for conversions between the two systems. Part of the reason behind Nora's preference for imperial measurement probably relates to her age as indicated in the following quote:

A couple teachers and I were putting up paper for the kids to do murals for Christmas and winter. And it was, OK we need so many feet or so many yards of the art paper to put up on the wall. And it was really interesting. The older teachers used feet and yards, and the younger teachers used the metric measurements. Which you know, we all kind of stand and look and laugh at each other, because we're dating ourselves right here. (N-5.3-5.4, 940121)

She learned the imperial system as a child, so she will probably always be more comfortable operating in that system than the new system (this is akin to processing in a first versus a second language). Geena, on the other hand, was younger and may have learned the metric system in school (although not so young that she would not have learned the imperial system as well).

Although Nora struggled to make sense of the metric system, preferring to rely on her understanding of imperial measures, she expected her students to operate freely in both analog and digital time. Although this distinction appears hypocritical, it must be noted that Nora learned both analog and digital time, whereas her students learned only metric measures, not imperial. Both analog and digital time are prevalent, so Nora and her students would probably be well-suited to learn both systems. On the other hand, metric is the official measuring system in this country, so it is more important that Nora learn metric than that her students learn imperial (although, her students would probably benefit from understanding both systems of measurement, just as Nora would and Geena did).

The last three sections have catalogued Nora's out-of-school uses of mathematics in (a) performing best-buy comparisons, (b) estimating, approximating, and determining situational appropriateness, and (c) operating within and between multiple measurement systems. In the next section, I focus on how Nora used out-of-school mathematics in the classroom.

Using Out-of-School Mathematics in the Classroom

For an earlier university course, Nora had prepared a description of the Functional Math program she had designed for her special needs students. During one of our meetings, she brought a copy of this 102 page document, comprised mainly of activities she had used in the classroom. The paper opened with a copy of a journal article describing a similar program. In this article, the authors defined functional mathematics as "the study of uses of mathematics needed for vocational, consumer, social, recreational, and homemaking activities" (Schwartz & Budd, 1981, p. 2). Nora's program focused on mathematics as daily living skills for her intellectually disabled students who would never be able to function totally on their own and would graduate to sheltered workshops or, for more highly functioning individuals, to low-skilled employment. Her intent was that most of the skills would be taught at a practical level. As a class, they traveled to stores in the community to make purchases, they ate in restaurants, they used the transit system to navigate the city, they made phone calls from pay telephones, and they cooked together.

Budgetary constraints prevented Nora from doing all the practical activities that she would have liked. For example, the students were unable to make bank deposits and withdrawals because of limited funding. Instead, the students practiced filling out deposit and withdrawal forms, but never actually deposited or withdrew money from a bank.³³ Similarly, Nora also provided worksheets to reinforce all the shopping and bus usage

³³This is akin to playing poker with play money, as indicated by one of my colleagues in the thesis research group.

skills that they practiced in real situations. She was dissatisfied with this approach, because she felt that many of her students were too literal to be able to transfer the contrived exercises to real-life situations. She felt that these constraints hindered her students' learning; to emphasize the limitation she relayed the story of how one student was misinformed as a direct result of the schools' attempts to save money:

We were on a budget and trying to teach them telephone skills. If they're outside in public and they needed to call home or if they needed to call an emergency number. They knew 9-1-1, we went over that, it wasn't a problem. We were with the group and one of the kids had to call home regarding something and, "Okay, fine. No problem. Here's a quarter, there's a pay phone, we'll wait for you here." She went and called, and she came back, "I can't get through." And in a couple minutes, she went to try again. And the third time, I said, "Okay, come on Sally, I'll come with you and you show me. Something is not quite right here." And the other teacher because we were on such a tight budget and we had 17 students, so she would put the quarter in and quickly press the release button, so we could get our quarter back and then have the child punch out their number and then pretend someone was on the other end of the phone. And that's what she did, literally. And I thought, and I got so angry, not at [her co-teacher] but angry at the whole situation. And here they wanted us to teach, but wouldn't give us a budget, of course. These kids are literal. I mean, she honestly thought she was doing the right thing and couldn't figure out why she wasn't. She had her right amount of money and she had her number, her phone number, you know, down pat. She knew the number, um, but she wasn't getting through. (N-1.4-5, 931029)

As this example illustrates, Nora's concerns about her students' inability to transfer from contrived tasks to real-life situations were certainly well-founded. This lack of transfer is not unexpected, however. Research has found that performance on contrived (paper) tasks has very little relationship with performance on actual tasks (see Lave, 1988). This may be even more apparent for students with intellectual disabilities who are believed to "learn best at a repetitive, concrete level and not so much in an abstract manner" (N-fm.1, 860723).

Despite her misgivings about the limitations of paper and pencil tasks to teach real-life skills, Nora provided templates of worksheets for several money skills that she felt were important for her students to learn. She clearly warned that these worksheets were useful to reinforce the actual skills as practiced, but were not sufficient to teach the skills. Referring once again to Greeno's (1991) metaphor, these worksheets could provide maps

to help students learn to navigate the environment, but could not replace actual navigation of the environment. As he explained:

We should view symbolic representations [including Nora's worksheet exercises] as analogous to maps, guidebooks, and written or spoken instructions that can be helpful—even essential—in the activities of learning to inhabit environments and as important resources for use in reasoning and communication, but they should not replace experiences in conceptual environments as the main learning activity that we provide for students.
(Greeno, 1991, p. 177)

The worksheets and other activities in Nora's program description included the following tasks:

- Recognizing coins.
- Adding coins.
- Adding and subtracting written monetary values.
- Rounding to the next highest dollar.
- Locating specific items and their prices in advertisements.
- Calculating change for purchases from a given dollar amount.
- Making shopping lists in a grocery store or a record store (including adding up total cost).
- Selecting and purchasing a meal at a restaurant or a counter in a food court (including calculating total price, rounding up, and confirming change due).
- Ordering take-out food (including calculating total price, rounding up, and confirming change due).
- Filling out bank deposit and withdrawal slips.
- Taking city buses from the school to specific destinations (recognizing bus numbers, knowing bus routes, knowing when/if to transfer buses, counting out proper fare).

It is clear from this list that this functional mathematics unit brought several examples of non-school mathematics into the classroom and, more importantly, Nora took the classroom out of the school. This approach is in keeping with the spirit of NCTM (1989)

guidelines, but is limited in that the paper versions of non-school tasks did not match the tasks as required out of school. For instance, in selecting and purchasing a meal at a restaurant or a counter in a food court, the students first decided on one restaurant or counter, and then selected a meal from that one place. In contrast, people often combine meals from different counters in a food court (typically, only if the line-ups are not too long). As another example, students made shopping lists for one single grocery store or one single record store. They did not make a shopping list for all their purchases at grocery stores, record stores, drug stores, and whatever other stores they needed to visit. For ordering take-out food, no consideration was made for driver's tips, yet these are expected in real-life. These tasks do not match the complexity of Nora's own functioning out of school where she enacted solutions to multi-variable optimization problems. Nora's functional mathematics program gave no consideration to such real-life concerns as product availability, crowding of stores, product freshness, and existing storage containers—the very factors that had influenced her own shopping habits. For these reasons, Nora's misgivings about these contrived tasks were well-founded. However, it is important to remember that Nora's program emphasized practical activities where students actually did go out into the community, and these contrived tasks were provided merely for practice.

Summary

From these descriptions of Nora's mathematics across contexts, the following general conclusions can be drawn. Nora was interested in recognizing incidental mathematics in her own and her students' actions, yet her ability to do so was constrained by her own understandings of mathematics. On the surface, the mathematical games I observed in Nora's classroom built on simplistic mathematical ideas, yet they also involved some more sophisticated mathematical concepts (zeroes in place value, number patterns, and probability). With her limited mathematics background and her focus on language rather than mathematics, Nora did not address these underlying mathematical concepts in the

class. She focused on the language (as was required for a second language learning group) and drew connections to simpler mathematical concepts—numerical recognition, place value, and addition. Similarly, Nora only recognized the simplistic, surface level numerical comparisons involved in her own shopping, but not the complex underlying optimization problems she actually solved. Without recognizing the sophistication underlying her own and her students' mathematical actions, the possibilities for mathematics as praxis were limited for Nora and her students (see Fasheh, 1982, 1991). However, it is essential to consider Nora's behaviour in the context of her goals. In the classroom, she wanted to provide language practice to her students. Out of the classroom, she wanted to find out where was the best place to shop. She did achieve the goals she had laid out for herself, but perhaps not all that she could have attained. Loftier goals of seamlessly weaving together school and non-school mathematics and offering mathematics education as praxis were still beyond her reach. In the final chapter, I consider what Nora (and Geena) achieved, and provide suggestions for taking things one step further.

CHAPTER FIVE

CONCLUSIONS AND IMPLICATIONS

Overview of the Study

In this thesis, I analyzed the mathematical activity of two elementary school teachers across various contexts of their lives, inside and outside their classrooms. This work emerged as an effort to reconcile the apparent contradictions between (a) recent research findings that suggest mathematical activity is fundamentally situated and distributed across physical and social contexts such that there are distinct discontinuities between mathematics as taught in classrooms and as used outside classrooms (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a) and (b) recent proposals for teaching and learning mathematics in schools that have encouraged educators to connect mathematics instruction with students' experiences in out-of-school settings (NCTM, 1989). This contradiction is captured in the focus question that prompted this study, "If people do not make connections between their highly competent mathematical activity in everyday settings and structurally equivalent school problems, what degree of success can schools and teachers achieve in helping students to integrate mathematical experience across diverse settings?"

In an effort to come to some understandings about this question, I documented the experiences of two teachers over a six month period as they taught in their elementary classrooms, completed assignments for a mathematics teaching methods course at the university, and engaged in mathematical practices outside their classrooms. Following the framework of interpretive research design, interviews, observations, documents (research notebooks, university assignments, classroom lesson plans and resources), and researcher field notes served as primary data sources. The data were categorized and organized according to recurring themes and presented as two separate case studies, one for each of the participating teachers.

Consistent with earlier research findings, the two case studies document that the teachers displayed mathematical activity in everyday settings that differed from approaches taught in schools, including their own classrooms. However, the teachers did attempt, with at least partial success, to link non-school uses of mathematics with their classroom instruction.

Eight common themes—Flexibly Modifying Plans, Making Sense, Using Physical Objects, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Work, Stating Solutions, and Drawing Connections (see Table 3.1)—were relevant to the analyses of both school and non-school mathematics for Geena. The existence of common themes across contexts suggested that Geena drew many connections between school and non-school mathematics. Her number sense and her sense-making efforts informed her mathematical practices both in and out of the classroom. Similarly, she provided many opportunities for her students to make sense of mathematics by relying on their number sense and connecting school mathematics to non-school mathematics and other familiar topics. She was very flexible in her approach to teaching mathematics, and using mathematics outside the classroom. However, she provided limited flexibility for her students to adopt and adapt different classroom tasks or goals. Students could select alternative solution strategies, but these strategies all led to single correct answers, unlike in Geena’s non-school mathematics where decisions about different solution paths led to completely different solutions (e.g., apple squares instead of cookies). Despite her emphasis on connecting school mathematics and non-school mathematics (and other topics), Geena’s own practices suggested discontinuities between school and non-school mathematics as have been discovered in previous research (see CHAPTER ONE).

The descriptions of Nora’s mathematics also highlighted the importance of mathematical sense and revealed a strong emphasis on connections (between mathematics and language, and between school and non-school mathematics). The four-faceted

model—Mathematical Sense, Recognizing Incidental Mathematics, Connecting Language and Mathematics, and Using Mathematics Outside School (see Figure 4.1)—indicated that Nora’s mathematical sense informed her mathematical practices and what she presented in the context of our interactions as mathematics. Although Nora was very interested in drawing on “incidental” mathematics and students’ own experiences with mathematics outside the classroom, her efforts were constrained by her own limited understandings of mathematics. She was able to make connections on the basis of simplistic, surface similarities, but appeared to be unaware of underlying, sophisticated mathematics in her own non-school practices (pet food shopping, cross-border shopping) and in the mathematical games played in her second language learning group. Without recognizing the sophistication underlying her own and her students’ mathematical actions, the possibilities for mathematics as praxis were limited for Nora and her students (see Fasheh, 1982, 1991), as I describe in the section *Mathematics Education as Praxis*.

Two major themes of Number Sense and Drawing Connections were prevalent across the two case studies, and I turn now to a brief discussion of these two main findings.

Number Sense

An important consideration underlying all mathematical practices is the form of cognitive expertise known as number sense. Greeno (1991) captured the essence of this notion through the use of an environmental metaphor:

Knowing the domain is knowing your way around in the environment and knowing how to use its resources....Knowing includes interaction with the environment in its own terms—exploring the territory, appreciating its scenery, and understanding how its various components interact. Knowing the domain also includes knowing what resources are in the environment that can be used to support your individual and social activities and the ability to recognize, find, and use those resources productively. Learning the domain, in this view, is analogous to learning to live in an environment: learning your way around, learning what resources are available, and learning how to use those resources in conducting your activities productively and enjoyably.
(p. 175)

Thus, number sense is characterized as the (often tacit) understandings that help one to function in a conceptual domain. Examples of number sense include such capabilities as

recognizing numerical equivalences that allow regrouping in mental multiplication or judging and making inferences about numerical quantities. These capabilities, although very important to mathematical performance, are difficult to define. Number sense seems to imply a personal, experiential dimension of knowing that is often difficult to put into words.

In the stories of Geena and Nora's non-school mathematical practices (see CHAPTER THREE and CHAPTER FOUR), examples of number sense were common. Geena had a strong sense for both imperial and metric measures, and was comfortable operating in either system, but found no need to convert between the two systems. She was able to move around in both imperial and metric environments, and could use the resources of both environments. Nora also understood imperial measures in an experiential sense, but only some metric measures (e.g., related to temperature and driving speed). She was accustomed to operating in imperial and understood that system well, plus she knew some metric measures based on landmark comparisons with the imperial (e.g., "Like 1 pound is like 500 grams"). Similarly, without reference to any formal measures, Geena was confident that the small markings she made on her chunk of cheese represented one-ounce chunks and that the amount of brown sugar coating the apples would make her squares suitably sweet. She relied on her number sense to flexibly modify her plans as the biscuits and cookies/squares evolved in her kitchen. At the American factory outlet, Nora used her number sense to circumvent calculations, or simplify necessary calculations. For example, she could tell by looking whether the American prices would be cheaper than the Canadian prices for many of the products she considered purchasing; if the numbers were comparable, then the Canadian price would be cheaper, if there was a large discrepancy in prices, then the American would be cheaper. With smaller discrepancies in prices, she could not distinguish just by looking and in those cases she resorted to (approximate) conversion calculations. She knew the Canadian prices and had a sense for how much the posted American prices would actually be when the exchange was added.

As another example, she relied on the Canadian price for a class set of pencils to serve as a landmark in her calculation of whether the individually-priced American pencils were a bargain.

Similarly, classroom examples of school mathematics showed some opportunities for students to use their number sense. For example, one of Geena's students relied on her number sense to construct number sentences for the worksheet exercise without using manipulatives. Given the prevalence and importance of number sense in Geena and Nora's mathematical practices, it would seem essential that a similar emphasis predominate in the classroom. According to Howden (1989), students' number sense "develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (as cited by Greeno, 1991, p. 173). Thus, it would seem that Geena's emphasis on making sense of mathematics for herself and her students set the stage for students' developing number sense. All of Geena's classroom instruction seemed to focus on making sense. She and her student used the disk abacus to understand why you "crossed off the zeroes and put a bunch of nines" in the subtraction with borrowing algorithm. Students were encouraged to consult commercial calendars in their efforts to find out why the class calendar did not begin on Sunday. The entire class laid metre sticks end-to-end across the classroom floor to visualize the length of the snake in their efforts to understand what the reported length meant.

This emphasis on sense-making was less prevalent in Nora's school mathematics, but this may be a function of the fact that she was not actually teaching mathematics at the time of this research project, so there were fewer opportunities for me to collect data on her school mathematics. Sense-making seemed to be at the heart of Nora's focus on practical life skills such as shopping and taking the bus for her special needs class. Nora took her students out into the community to work on "functional" mathematics. In this way, the students engaged in the actual skills, the "doing of (non-school) mathematics,"

which Howden (1989 as cited by Greeno, 1991) argues is essential to developing number sense. Nora also supplemented these practical skills with worksheet exercises that were intended to provide extra reinforcement to help her students develop number sense to a level where they could manage daily tasks on their own. According to Greeno's (1991) metaphor, the worksheets provided maps to help students learn to navigate the environment, but could not replace actual navigation of the environment (going out into the community to shop, to take the bus, etc.).

Developing number sense and providing sense-making opportunities seem to be critical in the mathematics classroom, as well as in life outside the classroom. Nora and Geena both relied on their number sense outside the classroom, and brought this into the classroom to varying degrees. This emphasis provides one important connection between school and non-school mathematics. Others are addressed in the next section.

Connections

I began this research project with an interest in investigating the connections that teachers made between school and non-school mathematics. Based on prior research comparing mathematics practice inside and outside classrooms (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984a), it seemed questionable whether teachers would be capable of integrating mathematics instruction with students' experiences in out-of-school settings as proposed in recent curriculum improvement documents (e.g., NCTM, 1989). As highlighted throughout this study, Geena and Nora were, at least partially, successful at this task. There was considerable overlap in Geena's mathematics practice inside and outside her classroom (most readily apparent in the match between categories used to describe her mathematical practices in the two arenas, see Table 3.1). In particular, number sense and sense-making efforts were central to all of Geena's mathematical practices, and to her students' school mathematics (see *Number Sense* above). Similarly, number sense was important in Nora's out-of-school mathematics practices, as well as in her classroom.

However, some discontinuities remained between Geena and Nora's mathematical practices and the practices in which they required their students to engage. In particular, Geena's practices were extremely flexible both in and out of school, but she did not allow her students a similar degree of flexibility. Her biscuits and squares, as well as her classroom lessons, evolved over the course of her actions. She started with vague plans, but flexibly modified these based on multiple goals, available tools and materials, community standards and reactions, and the products themselves. On the other hand, her students had few opportunities to shape the classroom activities in a similar fashion. They were free to use or not use manipulatives to complete the worksheet exercise, but Geena determined whether their actions were acceptable. In the kitchen, Geena shaped her solutions through her embodied actions, whereas her classroom activities included multiple solution strategies which all led to one "correct" answer. Further, she required her students to publicly state this final answer whereas, in out-of-school contexts, her own solutions seldom included public statements. The distinctions in flexibility and acceptable answers for school and non-school mathematics reveal two interrelated considerations about power and goals. Geena was the one in charge, not the students; she modified her plans (teaching and baking) and decided when solutions (hers and students') were appropriate. Students were able to negotiate with the tasks, but Geena had final say over what was or was not acceptable. Her judgments about what was acceptable were inherently tied to her goals for the situation. In the kitchen, she negotiated a plan of action based on multiple goals—baking biscuits for herself, baking cookies for teachers in her school, not being wasteful, displaying her non-school mathematics, talking with me about mathematics and baking, etc. In the classroom, her plan of action was also based on multiple goals—addressing curricular content, providing sense-making opportunities for her students, instilling a sense of self-worth in her students, etc. Based on her goals, she decided what activities to pursue, how much time to allot for a given task, whether students' responses were acceptable, and other concerns. The students, too, had multiple

goals that influenced their actions, such as completing the worksheet, being perceived as “good,” making friends, and a multitude of other options. The differences in goals are what determine task interpretation and solution processes selected. As Lave (1988) argued, such differences in goals are the reason that mathematically isomorphic problems presented in different situations are often approached very differently.

Much of the data describing Nora’s classroom activities focused on connections she drew between school mathematics and real-world uses of mathematics, between mathematics and language, and between other activities and incidental mathematics. However, she tended to base these connections on simplistic, surface level similarities, rather than underlying, sophisticated similarities. It appeared that Nora’s connections were constrained by her limited understandings of mathematics and were probably exacerbated by her fears and lack of self-confidence in mathematics. She may not have known the more sophisticated mathematics that was “hidden” in her own and her students’ activities, and without understanding this sophistication, she was unable to recognize the potential connections therein. Limitations in Nora’s mathematical understandings were not the only reason that she did not pick up on the sophistication underlying her own and her students’ practices. In her second language learning group the focus of instruction was on language not mathematics, thus it is not surprising that Nora chose not to follow-up on all the underlying mathematical ideas inherent in the activities. She may have been able to see the connections, but felt that they were irrelevant to her language goals.

If teachers are to draw connections between school mathematics and other contexts for mathematics, they must first understand the mathematical ideas they wish to integrate, and then see the value in such integration. When the goals of instruction do not support connecting school mathematics to other contexts than neither teachers nor students will make these connections. It has been suggested that education which emphasizes such connections may help to bridge the gap between classrooms and the “real world.” This

may increase engagement and learning in at least four ways: it may (1) help students to build conceptual understandings by connecting new information with prior knowledge; (2) help students to realize the relevance of curriculum materials to their lives; (3) validate the kinds of informal strategies that students already know and use; and (4) help reduce students' negative reactions to mathematics by increasing their comfort levels with their own intuitive mathematics, as well as more formal, school mathematics. Similar positive benefits may also accrue for teachers. Failure to draw these connections would circumvent these positive benefits.

The two major themes of Number Sense and Connections highlight the two main conclusions of this study: Geena and Nora displayed mathematical activity out of their classrooms that differed from approaches they used in their classrooms, but they attempted, with at least partial success, to link these different uses of mathematics (school and non-school). In this way, they approached the task of offering mathematics education as praxis, but their efforts seem to have fallen short.

Mathematics Education as Praxis

Fasheh (1990, 1991) contrasted his own mathematics, developed over years of formal institutionalized education, with the mathematics of his illiterate, seamstress mother who “unable to read or write...routinely took rectangles of fabric and, with few measurements and no patterns, cut them and turned them into beautiful, perfectly fitted clothing for people” (p. 21). Fasheh’s mother engaged in sophisticated mathematical practices, but she did not recognize her practices as such. This meant that her knowledge could not be praxis, that is, it lacked the constant interplay between (social, cultural, and material) conditions, reflection, and action necessary for it to be empowering. He writes:

She knew in practice much more than she was able to tell. In contrast, I was able to articulate words and manipulate symbols much more than I was able to put them in practice.

My mother’s math was biased toward life, action, production, and personal experience, and it was linked to immediate and concrete needs in the community. My math, on the other hand, was biased toward the manipulation of symbols and theories linked mainly to technological advancements and techniques that usually lead to military, political, and economic power and

control. What was lacking in my mother's knowledge was articulated structure and theory, while what was lacking in my knowledge was practice, relevance, and a context. In this sense, neither her knowledge nor mine was a praxis; each form of knowledge lacked one part of the dialectical relationship between life and mental construction, between practice and theory, between the world and our consciousness of it, between reality and our perception of it. (Fasheh, 1990, p. 22)

Like Fasheh's mother, Nora knew in practice much more than she was able to tell. For example, her pet shopping entailed solving a 15-variable optimization problem, but she viewed her practice as a simple comparison of prices. Without recognizing the complexity of her own mathematics, the possibilities for mathematics as praxis were limited for Nora and her students; like Fasheh's mother, Nora's knowledge lacked the essential theoretical half of the dialectic between theory and practice that is at the heart of praxis. In contrast, anyone who has been a student in our school system typically knows only the institutionalized mathematics that Fasheh knew; they lack the essential practical knowledge that comprises the other half of the dialectic. To develop mathematics education as praxis, teachers need to combine practical knowledge with theoretical knowledge for themselves and for their students. By coming to recognize and value their own "intuitive" and non-formalized mathematical practices, teachers (and their respective students) can bring their own practices into the domain of the represented (formulated) and, therefore, manipulatable. This will provide greater theoretical understandings of those mathematical practices. At the same time, thinking about formalized mathematics in relation to actual practices will provide "practice, relevance, and a context" for that mathematics. In this way, practical and theoretical knowledge can be combined to develop mathematics education as praxis.

In this work, I wanted to focus on non-school mathematical practices that have typically been devalued. Mathematicians and others have privileged intentional, formalized mathematics over non-formalized, out-of-school mathematical practices. This inequitable valuation has been most prevalent in distinctly gendered (and probably class-based) activities such as shopping and baking—the two primary examples of non-school

mathematics selected by Nora and Geena. These differences set up a contrast between the rational (and cold) mathematics of (mostly male) mathematicians against the embodied, non-formal, mathematical practices of non-academics, and especially women. In Fasheh's work, these contrasts were played out along gender lines: the author's institutionalized mathematics against his mother's practice-based mathematics. In my study, such a contrast was not at issue. Nora, Geena, and I each had a range of relations to both formal, institutionalized mathematics and informal, practice-based mathematics. By providing opportunities for the women to focus on their non-school mathematical practices as mathematical, I hoped to share my conception of mathematics as including such practices and to encourage the women to value these practices in a similar fashion. While a full analysis of the gender-based issues in mathematical practices is beyond the scope of this work, I felt it was important for the women to recognize their own gendered, practice-based understandings as mathematical. Recognizing and valuing one's own non-school practices is an essential first step toward praxis. I explore this issue further in the implications I draw from this study.

Implications

One of the first steps necessary to draw connections with school mathematics is that *teachers needs to recognize their own mathematics*. This idea was most prevalent in the chapter on Nora's mathematical practices where increased awareness and recognition of incidental mathematics became a topic of discussion and analysis (see CHAPTER FOUR). Geena also commented on how the methods course and the research project encouraged her to do a lot of self-reflection which increased her awareness of the mathematics around her and reinforced what she had been doing in her classroom.

Second, to recognize the often more sophisticated mathematical ideas that underlie their actions, *teachers need to have a strong understanding of mathematical ideas*. Teachers need to understand mathematics at an experiential, personal level (see the discussion on *Number Sense* and Greeno, 1991); they need to know the mathematics

curriculum they teach and how those ideas will be elaborated in future mathematics courses. Although Nora's awareness of numbers and arithmetical operations increased over our time together, her awareness of the underlying sophistication of her own and her students' actions changed very little. Without recognizing this sophistication, the possibilities for mathematics as praxis were limited. Geena treated mathematics as a puzzle on which she worked very hard to make sense. Nora was less confident and less successful with mathematics, so she seemed more hesitant to struggle to make sense of sophisticated mathematics, preferring instead to stick with topics with which she was more familiar (i.e., the more simplistic, surface ideas).

As well as reflecting on their own mathematical practices, teachers need to transfer these reflections to their classrooms. *They need to encourage students to reflect on their mathematical practices.* Although, there was evidence to suggest that Geena's students engaged in the kinds of sense-making activities that would provide opportunities for reflection, there was little direct evidence gathered on students' reflective practices.

Finally, *teachers need to provide opportunities for students to engage in diverse mathematical practices*, not just textbook exercises and worksheets. To have opportunities to reflect on various mathematical practices, students need opportunities to engage in those practices in the supportive environment of their classrooms. Students need to gain confidence and success at school and non-school mathematics practices. Non-school practices can include everyday activities such as cooking and shopping, as well as the everyday activities of scientists and mathematicians. School practices should be expanded to include diverse practices with opportunities for students to think mathematically across situations. By incorporating non-school mathematical practices into classrooms, students will have more opportunities to be successful in school mathematics and fewer reasons to be anxious about mathematics.

In order to achieve these aims, pre-service and in-service *teachers need forums for reflecting on their current mathematical practices in everyday situations outside schools,*

and to take these reflections as starting points for reflecting, with the help of a coach, on other mathematical practices in their own and other people's lives. By participating in this research project and the mathematics methods course, Nora and Geena had opportunities to reflect on their practices, and thus increased their awareness of their own mathematics practices. But, to achieve mathematics education as praxis, teachers need to do more than reflect on their out-of-school practices, they need to transfer these reflections to their classrooms. Classrooms provide new situations in new contexts, with new constraints that lead to different kinds of situated mathematical actions and new kinds of mathematical practices, so that teachers' abilities to reflect and recognize mathematical practices in their own lives, may not transfer to their mathematics classroom. Teachers need ongoing support and opportunities to discuss their reflections and how to translate these reflections to their classrooms. To facilitate this transfer, classrooms need to be transformed such that teachers are not subject to curricular constraints to "finish" the textbook at all costs or to segment their classroom schedule such that mathematics instruction is (only) provided from 9:15 to 9:55 each day. Further, teachers need to provide similar supports and encouragement for students to engage in and reflect on varied mathematical practices.

Future Research

The present research suggests three main avenues for future research, stemming from the interplay of reflection and action necessary for praxis. First, it would be beneficial to establish forums like the one suggested above and then to document their effectiveness in terms of providing opportunities for teachers to reflect on their practices. What are teachers' reflections? How helpful are forums that provide teachers with opportunities to reflect on their mathematical practices? What kinds of supports do teachers need to be self-reflective in this manner?

Second, how can teachers put this reflection in action? In this study, I was able to document over time teachers' increasing awareness of their own mathematical practices,

but there were few opportunities to observe changes over time in their classrooms.

Through our conversations I was able to pick up on the teachers' perceptions of what life was like in their classrooms, but I only entered their classrooms once. Future research should look at the ways teachers' increasing awareness of mathematics outside the classroom translates to their mathematics teaching. We need to document changes over time in teachers' classroom practices. What actions follow teachers' reflections? How do such reflections prompt transformations of classroom practices? How can forums support transforming classroom practices, particularly in school mathematics?

The third avenue for research stemming from the present study follows from students' reflection and action. Once teachers make changes in their classrooms, we need to document how this impacts on students' practices. In particular, we need to find out if the benefits of drawing connections that have been theorized in the existing research really do accrue. Can students be encouraged to reflect on their practices? Do opportunities to engage in various school and non-school mathematics practices provide opportunities for students' reflection? Do experiences with various mathematical practices increase students' understanding of and appreciation for school mathematics?

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APPENDIX A
GEENA: SOURCES OF INFORMATION

Interview 1 (introduction)

- introduce research project
- interest based on journal assignment (things coming together)
- background information
- goals for 475
- list of contexts for everyday use of mathematics
- select baking for next meeting

Journal Assignment

- teachers' struggles with math
- building math community in classroom

Interview 2 (baking)

- math while baking
- mixture of cooking knowledge and math knowledge
- baking in classroom
- research to justify classroom practice (for self and for parents)

Notebook

- lots more ideas about contexts for math in everyday
- idea for classroom using construction paper $>$, $<$ signs and concrete objects
- posting students' work on classroom walls

Interview 3 (classroom)

- what's posted on walls
- incidental math coming out of what's happening in the class
- talk about kids' lives and math where it fits
- begins to talk about final project
- embodies her philosophy
- lots of math connecting with literature and life

Final Project

- uses lots of research
- tries to build cohesive philosophy
- unit plan designed for her classroom

Interview 4 (tying it up)

- final impressions of 475 (very cohesive and enjoyable)
- plans to implement project unit, but only a few exercises sent home

Workshop

- topic was area
- brainstorming about all the places you use area (e.g., wrapping presents)

Yearly Preview

- yearly plan for her class
- highlights her focus on drawing connections between subjects (themes) and the flexibility of her plans

APPENDIX B
GEENA: CONTEXTS FOR MATHEMATICS IN DAILY LIFE

Grocery Shopping

keeping a running total on the calculator
 weighing fruits and vegetables on scale
 perimeter shopping

Other Shopping

comparison shopping (with and without GST)
 purchases from vending machines depend on how many coins in your pocket

Finances

Banking and Budgeting
 fees to change services when moving

Artistic Endeavours**Piano**

playing the piano
 transposing music (keep numerical fingering in mind)

Flower arranging**Drawing**

size of paper
 objects must look “proportionate”

Doing Home Repairs

painting
 hammering nails
 putting up shower curtain
 hanging pictures

Rearranging Furniture

measuring to determine if object will physically fit in a given spot
 aesthetic arrangement

Exercising**Aerobics**

pacing self
 taking pulse

Walking

timing
 distance
 patterns in trees, clouds, water ripples, etc.

Sewing and Crafts

repairing stuff (mending and alterations)
 making Christmas decoration for god child

Dining at a Restaurant

tax
 tip
 even sharing of bill among guests
 making change

(list continues...)

Dieting

- conscious of measurements
- fat intake (TV show mentioned in notebook)
- count cookies or chocolate bars, not just a handful or piece

Timing

- bus schedules
- making appointments
- business hours to call about moving
- coordinating multiple tasks
- estimating time based on lightness in room
- estimating time based on neighbours' noise
- how long to talk on long distance phone calls
- wait at the ferry terminal

Drawing Maps

- directions to your house

Counting

- to fill time while you wait

Doing Housework

- timing washer and dryer
- vacuuming (length of cord before changing sockets)
- ironing (cord must reach)

Studying

- certain number of pages or amount of time before break
- tuition fees

Playing Bingo

- numbers
- geometrically visualize patterns required

Baking

- Standard vs. non-standard measurements
 - depends on familiarity of recipe
 - depends whether exact measurement is critical to success of item
 - use smell for spices
 - recognize how much one ounce of cheese is from Weight Watchers

Converting recipe sizes

- Wet vs. dry measuring cups

Measuring shortening

- use tape on box
- using water displacement to measure tub (if doing a lot of baking)

Selecting size of glass to cut biscuits

- Geometric arrangement of biscuits on pan

Thickness

- all biscuits must be approx. same thickness to cook evenly
- thickness of cookie base determines how long to cook

Procedure to slice apple

- Temperature to set oven

- Knowing how long to cook something

APPENDIX C
NORA: SOURCES OF INFORMATION

Interview 1 (introduction)

introduce research project
 interest based on poor background (recover what she lost and not let it happen to her students)
 background information
 incidental math and selection of calendar activities as focus for final project
 some contexts for everyday math
 working with colleagues
 functional math program
 initial concerns about 475 course and how different from goals
 considerations about classroom visit (incidental math)

Functional math program

math in daily living skills from Special Ed. background
 making the abstract more concrete

Notebook

thoughts about classroom visit
 ideas for calendar math project
 few examples of her own everyday math (added during Interview # 2)

Interview 2 (picking contexts)

list of contexts for everyday math
 plan comparison pet food shopping
 some discussion of final project and classroom visit

Interview 3 (pet shopping)

math while shopping
 simplifying the problem based on consumer and pet-owner knowledge
 connecting with colleagues
 problems with 475
 playing games with students
 set up classroom visit

Interview 4 (classroom visit)

math games
 computation
 language connection (learning language through math)
 patterns
 plans for meeting with Gini (cancelled due to illness) and cross-border shopping

Final Project

calendar math/incidental math
 discussion of own math phobia and poor background as rationale
 connections with language and literature
 importance of using meaningful, familiar materials

(list continues...)

Interview 5 (catching up)

- more everyday contexts
- schedule cross border shopping
- spreading the word about “math is everywhere”
- clarifying project information

Workshop

- estimation
- using games to instruct

Interview 6 (cross-border shop and tying it up)

- math in shopping (exchange rates, tax, duty, and estimating)
- becoming aware of mathematics
- job-related uses of mathematics
- clarifying and elaborating on other sources of information
- reflections on course and project

APPENDIX D
NORA: CONTEXTS FOR MATHEMATICS IN DAILY LIFE

Budgeting

- setting up personal budget
- balancing personal budget
- conducting class within budget

Shopping

- grocery comparisons between products
- pet food comparisons between stores (also toys and litter)
- cross-border shopping—American exchange, duty, and free trade exemptions
- change
- tally items on a shopping list
- jewelry prices in Little India

Dining at a Restaurant

- tip
- change
- taxes—GST, PST, and luxury tax
- splitting bill among multiple guests

Purchasing Insurance

- cost
- deductible
- coverage options

Using a Post Office**Administering Medication and IVs (consider other drugs and health problems)****Making Phone Calls**

- remembering digits
- cost for pay phone

Playing Games

- card games
- table games
- secret message (adaptation of hang man)
- active games (whole body activities, gym activities)

Watching Television

- converting channel in TV guide to that on TV
- programming VCR

Doing Handiwork

- sewing
- arts and crafts

Baking, Cooking, and Candy Making

- measuring

(list continues...)

Using Language

- poetry and books (pattern of story structure, plus content)
- bring a number of books
- say a number of sentences

Conserving Energy

- thermostat setting vs. perception of temperature

Telling Time and Scheduling

- learning to tell time
- scheduling events
- bus schedules
- time to curl hair
- school-wide scheduling of shared spaces

Converting from Metric to Imperial

- travel distances
- speed
- temperature (weather and cooking)
- weight (products at store)
- lumber
- fabric
- art paper

Counting, Categorizing and Sorting

- words in language arts class
- students (for dismissal or lining up)
- days of year
- grids or lines in sheltered workshop rather than counting

Tallying and graphing

- birthdays
- lost teeth
- students' characteristics (height, hair colour, eye colour, skin colour, number of siblings, country of origin)

Sharing equally