ALGEBRA TILES AND LEARNING STYLES

by

Geesje Joke Thornton

Diploma Secondary Mathematics Education, University of Rhodesia, 1971

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APPROVAL

Name: Geesje Joke Thornton

Degree: Master of Science

Title of Thesis: Algebra Tiles and Learning Styles

Examining Committee:

Chair: Philip H. Winne

Thomas J. O'Shea
Senior Supervisor

Carolyn Mamchur
Associate Professor

Wolff-Michael Roth
Assistant Professor
Faculty of Education
Simon Fraser University
External Examiner

Date Approved March 31, 1995

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Author:

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Abstract

The purpose of this study was to investigate the effectiveness of a program of instruction in polynomials and factoring which made extensive use of the manipulative materials "Algebra Tiles".

A total of 132 students participated in the study. Each student followed the same program of instruction, extensively using the manipulative materials, over a period of approximately six weeks. Activities and strategies that appeal to students with different learning types were included in the program of instruction.

The instruments used in the data collection consisted of two researcher developed attitudinal questionnaires, a polynomial unit test, a factoring unit test and the Murphy-Meisgeier Type Indicator for Children, which was used to establish learning style preferences. Effectiveness was determined by gauging students' attitudes to the program by analyzing their responses to attitudinal questionnaires.

Analysis of variance tests were conducted to determine if differences were evident between students' learning style, their attitude toward and their success with the program of instruction. Success was measured using results from both the teacher constructed Polynomial and Factoring unit tests, and responses from attitudinal questionnaires. A successful transition from using the concrete materials to abstract manipulations of symbols was encouraged, although it was difficult to measure the success of this transition.

Positive responses to the Algebra Tiles Attitudinal Questionnaire indicated students enjoyed and understood the concepts involved in the program. Analysis of variance showed no significant difference between four learning type categories and the students' attitudes to, and success in, various aspects of the program of instruction. A structured regression indicated that ability to perform abstract operations with polynomials can be predicted by student success using the concrete materials regardless of students' self reported mathematical ability.

The researcher concluded that even though more extensive studies should be conducted in the areas of learning styles and the use of manipulatives, the program of instruction using the manipulative materials was effective and students given instruction using these concrete materials appeared to have a greater understanding of the concepts covered.

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CHAPTER I
INTRODUCTION

Educational and Instructional Change

During the 1980s a number of serious concerns were expressed about the education system in the United States. The state of mathematics education was high on the list of these concerns. Publications such as *An Agenda for Action* (National Council Teachers of Mathematics, 1980), *A Nation at Risk: The Imperative for Educational Reform* (National Commission on Excellence in Education, 1983), *Educating Americans for the 21st Century* (National Science Board Commission, 1983) and *The Mathematical Sciences Curriculum K-12: What is Fundamental and What is Not* (Pollak, 1983) caused educators, governments and the public to demand educational reform. The situation was similar in British Columbia where a Royal Commission on the state of the province's education system was set up in 1986.

The calls for educational reform in the 1980s took on such a sharpened focus that they could hardly be ignored. The findings of the Royal Commission in British Columbia spurred the introduction of what is commonly referred to as "The Year 2000", an educational reform policy portions of which are presently being implemented in this province. The National Council of Teachers of Mathematics (NCTM), as a national organization of mathematics educators in the United States, was aware that a national problem required a national solution. So in 1986 the NCTM set out on its ambitious *Standards* project for curricular reform in mathematics.

Three features of mathematics are embedded in the *Standards* (NCTM, 1989, p.7-8). First, "knowing" mathematics is "doing" mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. This active process is different from mastering concepts and procedures. They do not assert that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity. It is clear that the fundamental concepts and procedures from some branches of mathematics should be known by all students; established concepts and procedures can be relied on as fixed variables in a setting in which other variables may be known. But instruction should persistently emphasize "doing" rather than "knowing that."

Second, some aspects of doing mathematics have changed in the last decade. Because mathematics is a foundation discipline for other disciplines and grows in direct proportion to its utility, it is believed that the curriculum for all students must provide opportunities to develop an understanding of mathematical models, structures, and simulations applicable to many disciplines.

Third, changes in technology and the broadening of the areas in which mathematics is applied have resulted in the growth and changes in the discipline of mathematics itself. The new technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them. In developing the *Standards* the NCTM considered the curricular content appropriate for all students. It was recognized that students exhibit different talents, abilities, achievements, needs and interests.

Furthermore there is some consensus emerging in the psychology and education community regarding learning. Psychologists and educators are engaged in the study of cognition which provides evidence of how a student comes to acquire knowledge.

> Extensive data verify the existence of individual differences among youngsters - differences so extreme that identical methods, resources, or grouping procedures can prevent or block learning for the majority of our students. (Dunn & Dunn, 1978, p. xiii)

This emphasis on cognition and how students learn, coupled with the belief that learning mathematics is doing mathematics, has forced educators to re-evaluate the lecture/ algorithm/homework style of teaching mathematics.

Too many schools still rely on a rather mechanistic approach to learning. A 1988 National Assessment for Educational Progress (NAEP) survey gathered the following results on how secondary mathematics students were being instructed.

> Typical mathematics instruction apparently consists of listening to teacher explanations, watching teacher work problems at the chalkboard, using a mathematics textbook, and working alone to solve problems on worksheets. Over one half of the students reported never working in small groups to solve mathematical problems. Over eighty percent claimed that they had never worked on independent projects or investigations in mathematics class. The picture painted by the data is in stark contrast to the vision of mathematics instruction portrayed in the Standards . (Silver et al., p. 725)

Learning Styles and Teaching Strategies

> Learning is an activity of the mind, a fascinating interactive process, the product of student and teacher activity within a specific learning environment, that involves the application of specific and controlled operations to new information. The result is that this information becomes a part of long term memory. (Keefe, 1988, p. 22)

Every person's experience of learning is not the same, in fact learning can be categorized in several different ways and the learning process is subject to a wide variation in pattern, style and quality.

Distinctive learning strategies do exist. There are also certain distinct styles, or dispositions to accept certain types of strategy. In other words different people have different learning styles or preferences and, in order to accommodate these preferences, different teaching strategies need to be adopted. The existence of widely different learning styles prevents any possibility of any single 'correct' way to teach or to learn. Differences in personality and cognitive style lead to contrasting strategies in trying to assimilate information. Entwistle (1981) comments:

> These different styles of learning affect the levels of understanding across like a horizontal plane. They do not necessarily imply better or worse, they do imply qualitative differences in the flavor of understanding and recall. (pp. 270 - 271)

Educators are beginning to ask questions about student learning and how instruction can be improved to facilitate learning in all students. How does information enter the learning system? How is it processed? How is it stored in the brain? How is it retrieved for problem solving or new learning? How should information be sequenced, organized and presented for appropriate instruction? Given the realities of the education system today, what strategies or teaching/learning aids can be used to reach students with as many different learning styles as possible?

These questions obviously cut across every function and task of schooling. They challenge the organization of the learning environment, the choice of instructional methodology and strategy, and the way that teachers interact with individual students. Cognitive theory has implications for everything that teachers and students do in school, but its major impact is on the diagnosis and enhancement of student learning style.

Benjamin Bloom proposed a significant model of school learning. His theory deals with three important elements: student characteristics, instruction, and learning outcomes. His claim is that there are three interdependent variables that account for the greatest degree of variance in student learning. These are:

> 1) Cognitive entry behaviors - the extent to which the student has already learned the basic prerequisites.
> 2) Affective entry characteristics - the extent to which the student is or can be motivated to engage in the learning process.
> 3) Quality of instruction - the extent to which the instruction to be given is appropriate to the learner. (Keefe, 1989, p. 2)

The task of educators today is to try to address these three variables. The last two are the most attainable for the classroom teacher, motivating all students to learn and the quality and type of instruction required in order to motivate the student and allow learning to take place. Dunn and Dunn (1978) maintain that, "despite a wealth of well-conducted research, schools continue to function in the 'dark-ages' in terms of teaching-learning process" (p. xiii).

The reality in secondary mathematics classrooms is that every hour a teacher faces up to thirty students who can have a variety of different learning preferences or learning styles. In recent years, the focus of much educational research and programming has been toward accommodating individual differences. Materials, methods and management systems have proliferated rapidly. While strategies and technology have been developed that provide alternative learning opportunities for children the means to identify psychological type and learning preference in children have been practically non-existent. The Murphy - Meisgeier Type Indicator for Children (MMTIC) was developed out of this need. It was designed to provide an interpretation of type as it relates to how an individual child perceives and processes information and how that child prefers to interact socially and behaviorally with others. To provide a learning environment that can accommodate all student preferences, strengths and weaknesses in just the one hour of a classroom session is a difficult if not impossible task.

Psychological type theorists (Meisgeier & Murphy, 1987; Myers, 1980) believe that children have a high tendency toward a sensory learning style. This sensory learning style is similar to Piaget's (1977) concrete operational style. Sensing types learn best through the five senses and any learning involving concrete experiences would be well suited to their learning preference. Intuitive learners on the other hand "enjoy imagining, creating, and conceiving possibilities" (Murphy-Meisgeier, 1987, p. 3) and enjoy tasks that challenge the imagination. It is anticipated that intuitive learners would not respond favorably to a manipulative approach. Other instructional considerations should involve learning in various group settings. Here again children with different preferences would prefer different approaches. Those learners with a preference toward extraversion would function well with a partner or group, whereas an introvert learner needs time for individual thinking in order to function at his or her best. The challenge for educators is to try and come up with methods of instruction that will not only be workable for the teacher, but will include a variety of activities suited to the various learning preferences of students.

According to the NCTM *Curriculum and Evaluation Standards* (1989) and *Professional Standards for Teaching Mathematics* (1991) educators have learned a great deal in the last few decades about how students learn mathematics. New approaches to instruction make it possible to increase the scope and depth of the study of mathematical topics for wider and more diverse student populations. Learning materials are rapidly evolving to reflect the newly acquired knowledge of how students acquire mathematical ideas. Manipulative materials such as algebra tiles have been developed that claim to make algebraic ideas accessible to all students. The *Standards* actively encourage the use of manipulatives in all areas of mathematics instruction. The presumption here is that mathematics students in general like "hands-on" experiences. They like the opportunity to make, do, draw, fix, or manipulate something.

Purpose and Importance of the Study

Often in the course of teaching mathematics one hears some comments from students. "This is dumb". "It's too hard". "Why are we doing this". "This is boring". "I hate math". "I can't do this it is stupid anyway and I am never going to use it". All these comments show a high degree of frustration and result in a lack of success in mathematics. Positive comments about learning mathematics are equally as plentiful. "Easy stuff, anyone can do it". "I like math it makes my brain work". "I really love it when I puzzle over a problem and suddenly find the correct answer". "It is really frustrating when you work at it for a long time and then discover that the answer was so simple". These comments are from students who are successful at learning and applying mathematics (Thornton, 1992). Some students enjoy the activities and in fact like mathematics as a subject, some do not. Some students like some topics, others do not. These different attitudes and the varying success rates of students prompted me to consider and reflect on just how successful I was as a teacher of mathematics and what I could do in my instruction that would cause more students to experience success at, and enjoy learning, mathematics. These concerns prompted this investigation of how different students learn, what type of instruction is more suited to students and how different strategies are received and accepted by students.

The following question forms the basis of this study:

"Are algebra tiles an acceptable and effective educational tool suited to students with a variety of different learning styles?"

The study focuses on one particular teaching aid: the use of the manipulative "algebra tiles" to facilitate the learning of two particular algebra units, by a group of approximately 130 Grade 9 (14-15 year old) students with varied learning preferences. The manipulative will be used to aid in the instruction, and student discovery, of basic algebraic concepts in the polynomials and factoring units of the Grade 9 curriculum.

In order to fully analyze the research question on the success of algebra tiles as an educational tool suited to a variety of different learners, the basic research question is broken down into the following three sub questions.

1) **Do students with different learning styles differ in their attitudes towards mathematics, group activities and the use of the manipulative materials "Algebra Tiles"?**

2) **Do students with different learning styles differ in mathematical achievement using this manipulative based instructional strategy?**

3) **Does ability to do mathematics using the concrete materials "Algebra Tiles" help to predict achievement after self-report mathematical ability is taken into account?**

To answer these questions a series of questionnaires, lessons, and tests were designed for the polynomials and factoring units. Instruction of the two units using algebra tiles took place with five classes over a five to six week period. Throughout the two units students used the tiles to aid in their learning and the two teachers involved used a special set of overhead projector tiles during demonstrations and instruction. Different activities using the tiles were given to students; some were individual activities and some needed to be done in groups of various size. Reaction to the use of the tiles and to different student groupings were closely monitored. Initially students completed both a general attitudinal questionnaire about their attitudes and feelings toward mathematics, and the Murphy Meisgeier Type Indicator for Children (MMTIC) diagnostic test to ascertain individual learning preferences. Once the instruction on the units was completed students were asked to answer a second attitudinal questionnaire, specifically on their attitudes and feelings to the use of the algebra tiles and to whether or not they felt the use of these manipulatives enhanced their understanding of the units.

Questionnaire responses, various unit assignments, as well as results from the following four achievement tests were included in the data collected for each student.

1) Polynomials section A: a test on proficiency with the tiles, including drawing picture representations of algebraic expressions and finding algebraic expressions given picture representation of the tiles.
2) Polynomials section B: performing the operations of addition, subtraction and multiplication on algebraic expressions by abstract methods without the aid of the manipulatives.
3) Factoring section A: a test on proficiency in factoring with the tiles, including drawing picture representations of algebraic expressions and factored expressions.

4) Factoring section B: factoring algebraic expressions by abstract methods without the aid of the manipulatives.

Results from the four tests and unit assignments were used to assess the effectiveness of the tiles as a teaching tool. Learning style preferences as indicated by the Murphy Meisgeier Type Indicator for Children (MMTIC), together with the results from the attitudinal questionnaires and the unit tests are combined in order to determine if attitudes and achievements varied depending on learning style. Data gathering also included a journal of the researcher's observations, students comments, general impressions and recommendations for improvements.

Teachers often find that students experience difficulties making the transition from the explicit nature of ideas and symbols in arithmetic to their multiple meaning in algebra. These difficulties tend to lead to loss of confidence and lack of motivation in a number of students. Prigge (1978) suggests that there is considerable evidence that manipulative use has been successful in addressing transition problems in geometry, and that student understanding and motivation have been enhanced when manipulatives were used in this area of mathematics. There is every reason to expect that positive outcomes will be achieved when manipulatives are used in algebra. The best way to learn algebra is to understand the meaning of the symbols, techniques and properties. Actually understanding algebra must surely be more enjoyable and more efficient than memorizing a list of rules. Post (1980) notes that "researchers in mathematics education are in the process of accumulating a persuasive body of evidence that supports the use of manipulative materials in the mathematics classroom" (p. 109).

Delimitation of the Study

All subjects selected for the study were enrolled in Grade 9 mathematics classes. All the students at this grade level except those working on an alternate program participated in the study. The "polynomials" and "factoring" units are topics from the British Columbia Grade 9 curriculum guide (1993, pp. 119-120), see Appendix A for full details. The course of instruction followed and the content covered in these two units, was as recommended in the curriculum guide, except for the introduction of the algebra tiles as a teaching and learning tool. The MMTIC instrument selected for use in this study was deemed by the researcher to be the most effective tool for assessing learning preferences in teenagers.

Limitations of the Study

The limitations of this study were imposed by the students, by the curriculum time restrictions, by the learning style indicator itself, and by the potential for social demand characteristics to affect the responses given by students who were required to provided their names on all questionnaires and instruments used. The subjects were from one school in one district. The district itself is not representative of all districts because of its high percentage of English as a Second Language (ESL) students. In the classes used for the study approximately one - third of the students were ESL of Chinese origin. Although the MMTIC was selected because of its appropriateness for students of different backgrounds and experiences, some instructions and questions could have posed difficulties for students with limited experience of the English language. Although every effort was made when administering the test to overcome the language problem certain, inaccuracies in the outcome may result.

Attitudinal data was collected from self report questionnaires where students were required to provide their names. Responses may have been affected by influences such as the desire to conform to the teachers' attitudes or to the attitudes of peers, and could therefore have some influence on the reliability of results.

Organization of the Thesis

Chapter One discusses the need for educational and instructional change, specifically the need for instruction to move away from the lecture / algorithm / homework style of teaching, and for instruction to accommodate a variety of different learning preferences within the classroom. The purpose and importance of the research are outlined and its limitations are identified. The literature is reviewed in Chapter Two. This chapter is divided into five separate sections. The use of manipulative materials in mathematics, algebra tiles, learning styles, Murphy Meisgeier Type Indicator for Children, and different instructional techniques for students with different learning preferences as indicated by the MMTIC test, are examined.

The research methodology is outlined in Chapter Three. The subjects, the instruments, the treatment and the research design are described. The sequence of instruction, evaluating process and the data collecting procedure are also described. Chapter Four deals with the results of the study. Findings are discussed in Chapter Five and improvements to the instructional methods and topics for further research are suggested. A complete sequence of instruction including worksheets and tests is included in the Appendix section of the thesis.

CHAPTER II
REVIEW OF THE LITERATURE

The British Columbia curriculum guide for mathematics Grade 7-12 states that:

> Prior learning and cognitive development also influence individual achievement. Although students enter the school system at varying stages of development they all require extensive experience in concrete manipulations in order to form sound, transferable mathematical concepts. The curriculum should therefore assist students to develop an understanding of mathematics based on a sound foundation of concrete experiences in both pure and applied mathematics. It is through appropriate experiences presented in logical sequences that positive attitudes develop and effective learning occurs. (p. viii)

This supports the NCTM *Standards* notion that "knowing mathematics" is "doing mathematics". Bearing in mind that students have different talents, abilities, achievements, needs, interests and that they acquire knowledge in different ways, both the curriculum guide and the *Standards* also stress content appropriateness in mathematics education.

This chapter outlines historical developments, theories and researchers' experiences in both the use of manipulatives in mathematics education and in learning style, cognitive style and type preferences of individuals. A more in depth discussion on the manipulative "algebra tiles" and the current theories and instruments for assessing learning style will be included in this discussion chapter. The final section in this chapter is specifically related to this project in considering how the learning environment can be adapted to the style preferences of students as determined by the Murphy Meisgeier Type Indicator for Children, the learning style instrument used in this study.

Use of Manipulatives in Mathematics Education

Over the years the role of the teacher has changed from being the transmitter of knowledge to being the facilitator of the learner's discovery of knowledge. This means that the learner's role has changed from being a spectator in the game of learning to being an active participant. Scully, Scully and LeSage (1991) maintain

> To become an active participant, learners should experiment with hands-on materials to discover patterns, make conjectures, and test out these conjectures before they move on to the abstract stage of learning. (p. iii)

Teaching mathematics using manipulatives has a long history. In the nineteenth century Pestalozzi advocated their use. In the 1930s manipulative materials were included as activities in mathematics curricula. The mid-1960s began another period of emphasis on using concrete objects and pictorial representations in mathematics instruction. During the 1960s and 1970s researchers compared, in a number of educational settings, outcomes of mathematics instruction with concrete or pictorial materials to outcomes of instruction without such materials. The results were often mixed. Findings in some comparisons favored the group using the materials, whereas in other comparisons the control group achieved comparable or better results.

Some early reviewers of research on manipulatives simply summarized findings and let readers draw their own conclusions about the effectiveness of the materials. Other reviewers concluded that manipulative materials were beneficial for young children but were unnecessary for older children (Fennema, 1972a; Friedman, 1978; Johnson, 1971;Keiren, 1969; Scott & Neufeld, 1976; Wilkenson, 1974). Kieren (1971) claimed that students learn mathematics well in laboratory settings where manipulative materials are common, but that other methods of instruction work equally well.

Suydam and Higgins (1977) released a comprehensive review of studies of activity-based learning in mathematics instruction from Kindergarten through Grade 8. This review offered support for the assumption that concrete or manipulative approaches to instruction in mathematics are generally superior to a more abstract non-manipulative approach. Forty studies were reported, twenty-four showed significant positive effect of manipulatives on student achievement. Twelve showed no difference in achievement between the manipulative and non-manipulative approaches, four showed a more positive achievement using a non-manipulative approach.

Friedman (1978), however, reported that in the six years prior to 1978 four studies concerned with the effectiveness of a manipulative activity approach to teaching mathematics in the elementary schools were published in American journals. Three focused on the topic of multiplication and the fourth involved several mathematical concepts. The findings reported in all four were not favorable in their assessment of manipulative materials strategies.

Fennema (1972b) compared an instructional approach to learning basic multiplication facts that used repeated addition with a strategy based on the manipulation of Cuisenaire rods. Second Grade students were randomly assigned to either the manipulative instructional approach or the symbolic approach based on addition. When students were evaluated by means of a recall test and transfer tests the mean scores achieved favored the symbolic method.

Fennema (1972a) cited fifteen studies conducted prior to 1970 that were concerned with elementary school mathematics instruction. Three studies with First Grade students reported significant differences in favor of a manipulative approach. Of the remaining twelve studies, seven reported no significant differences between manipulative and symbolic approaches, one showed significant advantage to a manipulative approach, one showed significant difference in favor of non-manipulative approach, and three achieved mixed results.

Threadgill-Sowder and Juilfs (1980) claimed that the number of studies favoring the manipulative approach decrease, with results so mixed as to be inconclusive, at the junior high school level. Friedman (1978) suggested "it would appear that after the first grade, where the manipulative strategy has been effective in several situations, an instructional strategy that gives preeminence to the use of manipulative materials is unwarranted" (p. 79).

On the other side of the discussion, Post (1980) claimed that researchers in mathematics education are accumulating significant evidence in support of the use of manipulative materials in the mathematics classroom. Cheatham (1969), Dienes and Golding (1967), and Prigge (1978) concur that there is considerable support for the use of manipulative aid when teaching geometric concepts. In fact Prigge (1978) states that "children are better able to learn geometric concepts when manipulatives (geometric solids) are included in the presentation" (p. 367).

Cronbach and Snow (1977) suggest that perhaps difference between manipulative and non-manipulative treatments can be found when students' characteristics are more closely considered. For example, students who are low achievers in mathematics might benefit more from concrete materials and would find a manipulative approach more conducive to learning than a more abstract symbolic approach. High-achieving students on the other hand would likely be less affected by instructional methods and be able to process information from either approach. Good, Grouws, and Elmeir (1983) suggested strong evidence exists to indicate that achievement in mathematics is strongly related to topic coverage by teachers and not necessarily to the use or non use of manipulative materials. Raphael and Wahlstrom (1989) concur with these findings and argue that

> differences in students learning may be due not to the use of aids per se but rather the covering of course content by teachers. Finally, the effective use of instructional aids in the classroom may not be simply related to a greater use of the aids as excessive reliance on instructional aids may lead to poor presentation of content. (p. 173)

Sowell (1989) combined the results of sixty separate studies to investigate the question of how the outcomes of using concrete materials compare with those of abstract instruction and states:

> ...length of treatment was found to be related to achievement. When treatments lasted a school year or longer, the result was significant in favor of the concrete instructional conditions. Treatments of shorter duration did not produce statistically significant results. (p. 502)

According to Sowell's study, the effectiveness of manipulative materials is shown most clearly in comparisons of long term use of concrete materials with symbolic instruction. Furthermore the Sowell (1989) studies also indicated:

> Attitudes toward mathematics were related to instructional conditions depending on key design elements. In comparisons where students were assigned to groups randomly or treatments were assigned to groups randomly, attitude measures were significant in favor of the concrete instructional condition. When assignments were not made randomly, attitudes were negative and favored the abstract instructional condition. (p. 502)

Scully, Scully, and LeSage (1991) contend that research supports the notion that concepts that are introduced with the proper use of hands-on materials have a greater chance of producing improved understanding than concepts that are initially developed from the abstract stage. They claim that research reported by the British Audio Visual Association in their 1988 report found that we remember 10% of what we read, 20% of what we hear and 90% of what we say and do. This finding is confirmed by research undertaken by the National Council of Teachers of Mathematics and is incorporated into the *Curriculum and Evaluation Standards for School Mathematics.*

Studies have shown that when students are given extensive experience with concrete materials in ratio, integers, algebra and equation solving, they become more confident in their mathematical abilities. As a result their attitude to mathematics improves with this newly-found success (Scully, Scully, & LeSage, 1991).

It is generally accepted that other components must be in place for the use of hands-on materials to be effective. The classroom environment should allow students to work together, to learn by investigating, and to engage in communicating their ideas in their own words.

In designing their instructional program using manipulatives Scully, Scully and LeSage found from experience that once the students have discovered a pattern and tested out their conjecture they were ready to move on to the symbolic stage of learning. Students who are having trouble with the symbolic stage may need to refer back to the manipulatives by making sketches. In other words, students move from the actual materials to a model (on paper or in their minds) of the materials, to the strategy, algorithm, etc. This is not a one way trip for all students as some will need to temporarily return to the previous stage. In summary, the students will DO something with the materials, REFLECT on what they have done, formulate some theories (THINK), DO something with a model (on paper or in their minds) of the materials, REFLECT on what they have done, formulate a definite strategy or algorithm (THINK) and DO (using the strategy, algorithm etc.). Figure 1 shows this continuous learning process.

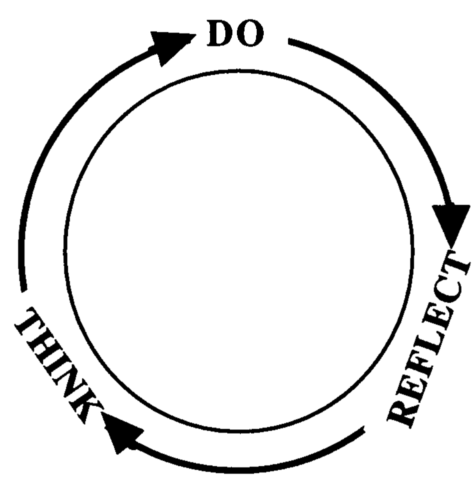


Figure 1 - Continuous Learning Process Using Manipulatives

This process is similar to Jerome Bruner's levels of concept development: enactive (manipulative); iconic (mental images); symbolic (manipulation of symbols). He of course was influenced by Piaget's levels:

> Any domain of knowledge (or any problem within that domain of knowledge) can be represented in three ways: by a set of actions appropriate for achieving a certain result (enactive representation); by a set of summary images or graphics that stand for a concept without defining it fully (iconic representation); and by a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming propositions (symbolic representation). (Bruner, 1966, p. 44 - 45)

The research then is inconclusive and mixed. Some studies reported success with the manipulative approach and others reported no difference in results obtained from a symbolic or abstract non-manipulative approach. There does not appear to be agreement between all educators regarding what the nature of instruction should be to most effectively achieve desired outcomes. For example there is no conclusive research as to whether instruction should be verbal, non-verbal, visual, auditory, or various combinations of them all. Gustad (1964) looking for an ideal learning design comments:

> At one time or another, radio, television, motion pictures, learning labs, and teaching machines have been hailed as the savior of education. As have large classes, small classes, seminars, tutorials, independent study, years abroad, work-study programs. None of these is either as bad as detractors assert or as good as zealots claim. Lacking an adequate theoretical framework in which to place these innovations, the pendulum continues to swing wildly from euphoria to cynicism. (p. 37)

Friedman (1978) after finding only a few positive results about manipulative use, concluded by saying, "we should urge the inclusion of the manipulative materials strategy in an instructional repertoire. And we should increase our efforts to determine those situations in which the strategy is most promising" (p. 80).

Sowell (1989) argued that the presentation and design of a program of instruction and using manipulative aids over an extended period of time, rather than in isolation, will have a positive effect on student learning. Howden (1985) maintained:

> It is generally recognized that understanding the meaning of a mathematics concept, as opposed to merely performing the associated computation, is an essential element of true learning and achievement.... research shows that the modeling and visualization promotes such understanding. (p. 3)

The Manipulative "Algebra Tiles"

Howden (1985) maintains that even though it is generally recognized that modeling and visualization promote understanding and give meaning to mathematical concepts, algebra is still traditionally taught at the symbolic level. Not all algebraic concepts can be modeled with manipulatives, but the concrete-pictorial-symbolic sequence suggested by Scully, Scully and LeSage (1991), Bruner (1966), and Howden (1985) applies readily to the basic operations of polynomials and factoring, which encompass a large part of the Grade 9 mathematics course.

The present study is concerned with one set of materials used to teach two specific units in the algebra component of the Grade 9 curriculum. The units in this research project are designed so that a concept is introduced on the concrete level and carried through the draw-a-picture and see-a-mental-image stages, before going onto the symbolic stage.

Algebra can be difficult to learn because it is often taught with no recognizable meaning. Edge and Kant (1992) provide an interesting analogy. They claim that learning a language is easy because it means something as words generally represent something touched or experienced. "If you look at a word like "banana" or "computer" you cannot help visualizing an object" (p. 1). Mathematics on the other hand can be difficult because it is often taught with no recognizable meaning:

> Do you visualize anything at all when you see 2x, or x^2? Do you know what these symbols mean? Because you do not know what the symbols stand for, learning mathematics can be like attempting to learn how to read without knowing what the words mean. (p. 1)

Algebra is a language where groups of symbols have specific meanings. Gatley (1991) claimed that a pilot project in algebra undertaken by some Vancouver schools, showed that students learned concepts more quickly and remembered them better when the manipulative "Algebra Tiles" was used. The Vancouver study also indicated that students are required to follow a certain process to ensure that the learning's are abstracted so that dependence on the tiles is eliminated. A three step process was recommended: (Gatley, 1991, p. 7)

Step 1) Use tiles to model questions and get answers.

Step 2) Draw pictures representing the tiles.

Step 3) Record the work using algebraic notation.

Considerable research with manipulatives confirms that the pictorial step is important in bridging from concrete to abstract as indicated by the results of the Vancouver project. The experience of the Vancouver teachers in this regard was further reinforced by Ross (1993). When designing the program for this study the three step process was incorporated into the sequence of instruction, worksheets and unit tests, so as to insure transfer of knowledge from concrete to abstract could take place.

Several commercial publishers have introduced a variety of packages to teach algebraic concepts and procedures. At least five commercial resources are available for teachers and students that claim to be a complete program of algebra textbooks and manipulatives.

1) Algebra Lab (1990) by Henri Picciotto available from Creative Publications.

2) Algeblocks (1994) by Anita Johnston available from South-Western Publishing.

3) Alge-Tiles (1991) by Jack LeSage, Barry Scully and Janet Scully available from Exclusive Educational Products.

4) Algebra Tiles for the Overhead Projector (1985) by Hilde Howden available from Cuisenaire Company of America.

5) Flip Chip Algebra by Frank Edge and Steven Kant available from Flip Chip Enterprises.

All of these products are very similar and consist of manipulatives, some teacher's notes and examples on how to use or teach with the manipulatives, and sets of exercises to be completed by students using the manipulative materials. All five resources were consulted when designing the program for this study and the manipulative materials used were those supplied by Flip Chip Enterprises. The reasons for the choice was the fact that the Flip Chips were made of cardboard and not plastics so they were cheaper to buy and to replace if lost. The Flip Chips were also a lot larger than most of the other products and therefore not as easy to lose.

Each student set of tiles or flip chips used in this study consists of 6 "x^2" tiles, 15 "x" tiles and 20 "1's" or "unit" tiles. There are also "y^2" tiles and "xy" tiles but these were removed and not included in the student sets as they were not required for the purposes of this research project. The tiles look similar to the representation shown in Figure 2 and Figure 3, but the actual size is approximately five times that of the representation shown here.

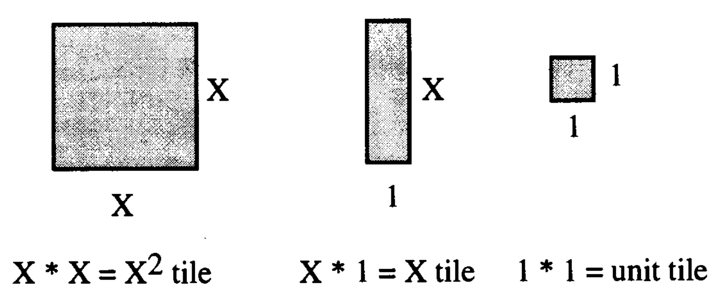


Figure 2 - Representation of Positive Algebra Tiles

The length of the rectangular "x" tile is not an integral number of "1's" tiles. This is not a construction defect but is done deliberately to guarantee uniqueness when students use the tiles for factoring.

The faces of the tiles are multi-colored and are considered to be positive (+) tiles. If the tiles are flipped they are all one color either white (as with Flip Chips) or black (as with home-made tiles). These white or black faces are referred to as negative tiles. Note that in printed worksheets and tests for this study white tiles are positive and black tiles are negative.

Figure 3 - Representation of Negative Algebra Tiles

> Bruner (1966) first devised the tiles when teaching quadratic equations.
>
> Each child was provided with building materials. These were large flat squares made of wood whose dimensions were unspecified and described simply as unknown, or x long and x wide. There were also a large number of strips of wood that were as long as the sides of the square and were described arbitrarily as having a width of "1" or simply as 1 by x. And there was a supply of little squares with sides equal to the width "1" of the strips, thus 1 by 1. (Bruner, 1966, p. 62)

It is interesting to note that the manipulative materials now referred to as algebra tiles have been around in one form or another for over 30 years, and have been used for helping students to learn a number of algebraic concepts

Learning Styles, Cognitive Styles and Type Preferences

Definitions

Learners seem to approach the learning process in many different ways. Much research has been done to determine what is referred to as the "cognitive style", "learning style" and "psychological type preference" of individuals. The term learning style is used very differently by various writers. Webster's Seventh New Collegiate Dictionary (1976) defines style as "a distinctive or characteristic manner". Random House Unabridged Dictionary Second Edition (1993) defines style as "a particular kind, sort or type" or "a particular, distinctive or characteristic mode of action." The manner in which students learn, their style of perceiving and organizing what they experience, has been referred to in a variety of ways in recent literature.

Some confusion exists between the terms "cognitive style" and "learning style." Random House defines cognitive as "mental processes involved in perception, learning, memory, and reasoning." Learning on the other hand is defined as "the act or process of acquiring knowledge or skill." Guild (1980) in her historical review of cognitive and learning style literature found that:

> The term cognitive style was used almost exclusively throughout the 1960s by psychologists to define various ways people perceived, thought, processed and thus learned. In the 1970s educators became more actively interested in the various personal and individualistic ways people learned. ... Recent literature most often uses the two terms cognitive style and learning style interchangeably, with psychologists more often discussing cognition and educators talking about learning. (p. 19)

As Gregorc (1979) suggested, "Style appears to be both nature/nurture in its roots." He presents the following definition of learning style: "Learning style consists of distinctive behaviors which serve as indicators of how a person learns from and adapts to his environment. It also gives clues as to how a person's mind operates" (p. 234). Claxton and

Ralston (1978) describe learning style as a "student's consistent way of responding and using stimuli in the context of learning" (p. ii).

Keefe (1979) maintains that learning style and cognitive style have often been used synonymously in the literature although they decidedly are not the same. Learning style, in fact, is the broader term and includes cognitive along with affective and physiological styles. Each learner has preferred ways of perception, organization and retention that are distinctive and consistent. These characteristic differences are called cognitive styles. According to Keefe the second dimension of learning style, affective style, encompasses those aspects of personality that have to do with attention, emotion, and values. Affective style is the result of motivational processes that are subject to a wide variety of influences. The third dimension physiological styles are characteristic learning related behaviors of the human body. The National Association of Secondary School Principals (NASSP) task force on learning styles define learning style as "the composite of characteristic cognitive, affective, and physiological factors that serve as relatively stable indicators of how a learner perceives, interacts with, and responds to the learning environment" (Keefe & Monk, 1990, p. 1). Cognitive styles then are information-processing habits; affective styles, motivationally-based processes; physiological styles, biologically-based responses. Keefe (1979) however suggests "most (learning style) research focused upon the perceptual mode that would best increase learning or retention and not on specific learning or cognitive styles" (p. 4). Although researchers vary on style definitions, there appear to be striking similarities among many diverse theories and studies.

History

The term "cognitive style" was coined by Allport in 1937 to refer to a quality of living and adapting influenced by distinctive personality types. Robinson and Gray (1974) however claims that Gardner was the first to use that specific term in 1953. Keefe contends that the term "learning style" was probably first used in 1954 by Thelen in discussing the dynamics of groups at work.

Early in the twentieth century, the Swiss psychiatrist Carl Gustav Jung did much research on cognitive style and he developed the concept of psychological type (how individuals absorb information about their environment and how they then order and make decisions about that information) to explain natural differences in human behavior. Psychological type theory seeks to describe how individuals perceive and make judgments about their perceptions. Katherine Briggs and her daughter Isabel Myers have devoted themselves to furthering Jung's work on the theory of psychological type preferences ("type preferences").

Educators since the early transformation of "school" from tutorials to group instruction, have been looking for ways to teach individual students in a common setting. After 2,500 years, research continues to try to attain success in this quest. Since the work of John Dewey educators have made Herculean efforts to make learning meaningful, to accommodate the learner and to acknowledge individual differences in students. Keefe (1979) surmises:

> We have only faintly understood the learner even as we accommodated school to the social disposition of children and youth. Often we adjusted the school setting with new formats ranging from non graded instruction to team teaching to open classrooms. Often we tinkered with new technical tools, including aptitude tests and teaching machines and visual devices. They all helped, a little, but none dramatically. (p. i)

Much of the educational change though well-intended and reasonably successful failed to focus on the unique learning proclivities of individual students. For example, changes can be made to instructional methods that will accommodate some students but are contradictory to the needs of others. One thing does appear obvious, however, and that is that no single approach to instruction is adequate. For some time, educators have focused not only on the abilities and achievements of students, and on methods of instruction, but also on the ways in which students acquire knowledge. Knowles (1973), has written that understanding how a person learns, and helping people understand how to learn is a major requisite for a successful educational program.

Elements of learning style appeared in the research literature as early as 1892. Most of that early research (before 1940) concerned the relationship between memory and oral or visual teaching methods. The findings were conflicting, no doubt due in large part to the differences in the populations, learning materials, and test instrumentation that were used. Early researchers were too preoccupied with finding the one perceptual mode that would best increase learning or retention. Even before 1900, Cattell and Jostrow attempted to relate differences in perceptual mode to general intelligence and learning performance without success. Vernon, Eysenck, and others described perceptual typologies such as analyzers vs. synthesizers and color vs. form reactors. In the 1940s, Thurstone and later Guilford identified factors of perceptual speed and flexibility which they believed were related to personality. Tyler (1933) wrote:

> No one series of learning experiences has proven equally effective with all students... learning activities should be supplemented by means of discovering for students where their difficulties are and of suggesting what kind of activities will be most helpful to them in overcoming these difficulties in learning. (p. 288)

Specific research on cognitive styles was greatly expanded in the 1950s. Asch and Witkin at Brooklyn College worked with the bi-polar trait of "field dependence-independence", the ability of a person to identify a figure against a background field. In time, Witkin and his associates broadened this notion to include "analytic-global" functions and the concept of "psychological differentiation". Witkin (1976) and Witkin et al. (1977) are most prominently associated with "cognitive style." They speaks of "field-dependence /interdependence", which is an attempt to understand, measure, and predict behavior based on the characteristic modes of perceptual and intellectual functioning of individuals. Cognitive style describes the manner and form that perception and cognition take in individuals.

At the Fels Institute, Kagan and his colleagues focused on analytic styles of thinking and problem solving. Research on analytic and non-analytic modes led to the identification of a "reflection-impulsivity" dimension. The reflective person tends to analyze and thoroughly differentiate a complete concept; an impulsive person is inclined to make quick and often erroneous responses. Gardner (1953), Gardner, Jackson and Messick (1960) and others at the Menninger group concentrated on cognitive style research with emphasis on differentiation and undifferentiation modes. Dewey, whose theory of experience has become the philosophical base of the experiential learning movement, formulated a two-phase cycle of trying and undergoing. Experience, according to Dewey (1966) includes an active and a passive element and states "when we experience something we act upon it, we do something to it; we suffer or undergo the consequences" (p.139). Keefe maintains that there is a similar active-passive dimension in the work of the Brooklyn, Menninger, and Fels groups (Witkin's Field independence, Gardner's Differentiation and Kagan's Reflection, are all active dimensions and Field dependence, Undifferentiation and Impulsivity are all passive dimensions).

The consideration of cognitive style widened after 1960 to include selection strategies (scanning and focusing), open/closed mindedness, memory or retention styles, risk taking vs. cautiousness and sensory modality preferences (kinesthetic, visual and auditory). Siegel and Siegel (1977) recognize two types of learning orientation, namely conceptual and factual. Conceptually oriented students prefer to learn principles and concepts, whereas factually oriented students prefer to learn facts.

Rosenberg (1956) uses the phrase "learning style" when writing about :

> ...an individuals' characteristic pattern of behavior when confronted with a problem. If a person is observed in a number of different problem solving situations, a modal pattern of behavior can usually be ascertained. (p.18)

Rosenberg postulated the existence of four learning styles (rigid-inhibited, undisciplined-impulsive, acceptant-anxious, independent-motivated). Hill and Nunnery(1973) offer a "cognitive mapping" scheme. This mapping instrument yields scores in twenty-eight different learning style categories. A number of these seem to overlap with categories of other theorists. Hunt and his associates define learning style in terms of the amount of structure individuals need. He suggests looking at conceptual levels (CL), and maintains that low CL students need high structure and high CL students need less structure.

The model of Dunn and Dunn (1978, 1993) and Dunn and Griggs (1988) deals with environmental, emotional, sociological, and physical factors affecting the learner and identifies eighteen learning styles in these four categories. Other researchers such as Bergquist, Bishop, Cross and Salmon refer to the sensitivity of students to specific sensory modes (visual, auditory, kinesthetic, olfactory, savory, tactile). Salmon deals with the tendency of some students to learn more efficiently when instruction is visual (films, slides, etc.) whereas others are more successful when the learning experience is predominantly auditory.

Reichmann and Grasha (1974) propose that students differ in the way they relate to their peers and to their instructors along three interpersonal dimensions (independence vs. dependence, competition vs. cooperation, participation vs. avoidance). Similarly, Mann (1970) isolates eight different styles: compliant students, anxious-dependent students, discouraged students, independent students, heroes, snipers, attention-seekers and silent students.

The learning theory of Kolb and his associates evolve out of Kolb's experiential learning theory based on that of Dewey. He conceives of learning as a four stage cycle in which experience is translated into concepts which in turn are used as guides in the choice of new experiences. In this context the learner needs four kinds of abilities - Concrete Experience abilities, Reflective Observation abilities, Abstract Conceptualization abilities, and Active Experimentation abilities. Using these learning modes, Kolb (1984) proposes the existence of four learning styles, which are determined by combining the scores measuring the four basic learning ability modes. In 1975, Kolb and Fry wrote that "the experiential learning theory has provided the basis for a framework to link whom we teach, how we teach, or to what purpose we teach. Individual learning styles can be related to preferred learning situations" (p. 90).

In 1974, Bergquist and Philips proposed three teaching and learning configurations; content-centered, instructor-centered and student-centered. Benjamin Bloom (1976, 1987) advocated a similar model of school learning dealing with three important elements: student characteristics, instruction, and learning outcomes.

Current efforts to explain the underlying processes of learning and teaching reflect two lines of research. Researchers such as, Hill, Dunn and Dunn, Letteri and Keefe, are working with applied models of learning style. The emphasis here is classifying individuals by the type of environmental, emotional, sociological, and physical conditions preferred.

The other line of research retains a strong preference for the cognitive style dimension. An early example is the model developed by McKenney and his associates at the Harvard Business School. This model is bi-dimensional rather than simply bi-polar. For McKenney, human information processing has two dimensions: information gathering (perceptive vs. receptive) and information evaluating (systematic vs. intuitive). A bi-polar model based on Carl Jung's theory of psychological "types" is developed by Myers, Briggs, McCaulley, Murphy and Meisgeier. Briefly the theory is that much seemingly chance variation in human behavior is not due to chance; it is in fact the logical result of a few basic, observable differences in mental functioning. These basic differences concern the way people prefer to use their minds, specifically the way they perceive and the way they make judgments.

Perceiving is understood to include the processes of becoming aware of things, people, occurrences, and ideas. Judging includes the processes of coming to conclusions about what has been perceived. Together, perception and judgment, which make up a large portion of people's total mental activity, govern much of their outer behavior, because perception by definition determines what people see in a situation and their judgment determines what they decide to do about it.

Keefe and Monk (1990) feel that learning style research has not been a high priority in the past decade or more:

> Educational psychologists maintained interest through the 1960s in the work on cognitive controls that had begun after World War II at Brooklyn College, the Menninger Foundation, and the Fels Institute. But this work floundered when many psychologists concluded that cognitive style research was unproductive or not a defensible independent field of inquiry. Educational practitioners discovered learning style technology at about the same time most psychologists were losing interest. The result until recently was slow progress in the field. (p. 1)

In late 1982 the National Association of Secondary School Principals (NASSP) assembled a distinguished task force to examine the concept of learning style. The task force spent some time documenting the fragmented nature of research up to that point, and then set about creating a single learning style instrument that would assess a broad spectrum of research-based style elements. Learning style elements were classified into cognitive, affective, and physiological/environmental domains. Keefe (1988) maintains that the "General Operations Model" advocated by Charles Letteri (see also Letteri, 1991) was used as the prototype for relating learning style to information processing. This model views learning as the storage and retrieval of information.

It appears that the dilemma is not in agreeing upon the existence of individual differences but rather in the exact definition of these differences. Reviewing this brief sampling of definitions and style descriptions, it is feasible to arrange theoretical approaches to learning styles into three orientations (as proposed by Bergquist et al., 1982, in Renner, 1984): 1) Media orientation: how the learner uses various media (visual, auditory, tactile, etc.), 2) Interpersonal orientation: how the learner relates to others in the learning environment (peers, individuals and groups, instructor), 3) Cognitive orientation: how the learner receives and works with the information from his environment.

Focus on Current Research and Diagnostic Instruments

For more than a century, educators have been searching for solutions to what may be education's most basic dilemma: What should schools do about individual differences among learners? It is crucial to emphasize here that, practically speaking, not all the elements of learning style are of equal importance. Some of the styles have no generally acceptable testing techniques and others are still vague enough that much more investigation is needed. What is important is a general understanding of learning style.

The school learning process reflects the interaction of student cognitive and affective behaviors and the organization of the instructional environment. School reform efforts of the 1970s and 1980s have moved the purposes and importance of effective instruction to the forefront of educational research. Learning style analysis emerges as a key element in this effort to make learning and instruction more responsive to the needs of individual students.

The goal of personalizing learning and instruction is an historical one. It is a quest that may now be within our grasp with the development of varied diagnostic and instructional techniques. The concept of learning style revives the hope for authentic personalized education since it starts with the learner and then proceeds logically to a consideration of the teaching and learning environment. An understanding of the ways students learn is the door to educational improvement. Learning style diagnosis is therefore the key to an understanding of student learning.

No education program can be successful without attention to the personal learning needs of individual students. Readiness and incentive, style and rate of learning, preferred methodology and content, all vary widely from person to person. A single approach to instruction, whether traditional or innovative, simply does not do the job.

A number of individuals or groups of individuals have made major contributions to research on how people learn in the last twenty years. There are some very distinct differences in the cognitive, affective and physiological styles that are emphasized by different research groups, but all have developed valid and reliable instruments for determining individual learning styles or learning preferences.

Experiential learning developed by Kolb and his associates has been presented as one model of how one learns. This theoretical model deals with how experiences are used to develop concepts, which then serve as guides in choosing new experiences. Kolb and Fry (1975) surmise that "learning and change result from the integration of concrete emotional experiences with cognitive processes: conceptual analysis and understanding" (p. 34). Kolb feels that new knowledge skills and attitudes are achieved through confrontation among the four perspectives in the experiential learning model. The learner needs to apply four different kinds of abilities, Concrete Experience abilities, Reflective Observation abilities, Abstract Conceptual abilities and Active Experimentation abilities. Kolb further maintains that:

> ...there are two primary dimensions to the learning process. The first represents the concrete experiencing of events at one end and abstract conceptualization at the other. The other dimension has active experimentation at one extreme and reflective observation at the other. (p. 36)

The Kolb Learning Styles Inventory (LSI) measures strengths and weaknesses in the four stages of experiential learning. It is a self-report instrument which measures relative importance placed on the four learning modes by asking the student to rank order a series of four words or statements that describe different approaches to dealing with learning tasks. It assesses the learner's perceived preference for concrete verses abstract learning and for active verses reflective learning.

Jung (1971) believes that mankind is equipped with two distinct and sharply contrasting ways of perceiving. One means of perception is the familiar process of sensing, by which a person become aware of things directly through the five senses. The other is the process of intuition, which is indirect perception by way of the unconscious. The existence of distinct ways of perceiving would seem self-evident. People perceive through their senses, and they also perceive things that are not and never have been present to their senses. The theory adds the suggestion that the two kinds of perception compete for a person's attention and that most people, from infancy up, enjoy one more than the other. When people prefer sensing, they are so interested in the actuality around them that they have little attention to spare for ideas coming faintly out of nowhere. Those people who prefer intuition are so engrossed in pursuing the possibilities it presents that they seldom look very intently at the actualities.

There are also two distinct and sharply contrasting ways of coming to conclusions. One way is by the use of thinking, that is, by a logical process, aimed at an impersonal finding. The other is by feeling, that is bestowing on things a personal, subjective value. These two ways of judging would also seem self-evident. People make some decisions with thinking and some with feeling, and the two methods do not always reach the same result from a given set of facts. The theory suggests that a person is almost certain to enjoy and trust one way of judging more than the other.

How people use perception and judgment depends on how they react to their outer and inner worlds. Individuals are either introverted or extraverted. The introvert's main interests are in the inner world of concepts and ideas, while the extravert is more involved with people and things.

The Myers-Briggs Type Indicator (MBTI) is a measure of personality dispositions and preferences based on Carl Jung's theory of psychological "types". Jung postulated two basic bi-polar mental processes (sensing-intuition and thinking-feeling) and two fundamental orientations to life (extroversion and introversion). The MBTI added the fourth dimension (judgment-perception) to identify the dominant mental process. Judgment and perception indicate a preference for an organized, judgment - oriented way of dealing with the world, or an adaptive perceiving - oriented way of dealing with the world. Although people use both perception and judgment most people find one attitude more comfortable than another. The resulting matrix formed by the eight preferences categorizes individuals into 16 types. The MBTI is a self-report instrument where the individual is asked to choose his or her preferred response from two choices, neither of which is right or wrong.

Mamchur (1984) reaffirms Jung's theory that individuals are constantly choosing between the open act of perceiving (finding out, discovering) through their senses or their intuition, and the closed act of judging (taking action, deciding, evaluating) through their thinking or feeling process. The four functions (sensing, intuition, thinking, feeling) all co-exist but one is most preferred and one is least preferred. Psychological type theory might be figuratively represented by Table 1: (adapted from Mamchur, 1994, p. 5)

The Murphy-Meisgeier Type Indicator for Children (MMTIC) is designed to provide type information about children. The MMTIC is built on the same foundation and developed within the same conceptual framework as the MBTI. The test contains 70 items designed to measure the same four preference scales as the MBTI, resulting in sixteen different type categories.

Table 1 - Defining Type Categories
Four preferences are scored to arrive at a person's type.

Does the person's interest flow mainly to the:

outer world of actions, objects and persons? **EXTRAVERSION**	inner world of concepts and ideas? **INTROVERSION**

Does the person prefer to perceive:

the immediate real, solid facts of experience? **SENSING**	the possibilities, meanings, and relationships of experience? **INTUITION**

Does the person prefer to make judgments or decisions:

objectively and impersonally, analyzing facts and ordering them, in terms of cause and effect? **THINKING**	subjectively and personally, weighing values for the importance of choices for one-self and other people? **FEELING**

Does the person prefer to live:

in a planned, orderly way, aiming to regulate and control events? **JUDGMENT**	in a flexible, spontaneous way, aiming to understand and to adapt to events? **PERCEPTION**

The Learning Style Inventory (LSI) developed by Dunn, Dunn, and Price is a widely used assessment instrument in elementary and secondary schools. The LSI incorporates many useful affective and physiological elements of learning style but only touches on the cognitive (in the area of perceptual modalities). The Dunns and Price define learning style in terms of four pervasive learning conditions and 18 elements. Students complete a 104-item self-report questionnaire that identifies learning preferences about immediate environmental conditions and emotional, sociological, and physical needs. The inventory is designed to support alternative approaches to instruction by profiling the elements of each individual's learning style.

The National Association of Secondary School Principals Learning Style Profile (LSP) created by a task force on learning style incorporates much work done by Dunn, Dunn and Price and the works of Letteri. The LSP assesses a broad spectrum of style elements and contains 24 independent scales representing four higher order factors: cognitive skills, perceptual responses, study preferences, and instructional preferences. Learning style elements are classified into cognitive, affective, and physiological / environmental domains. Cognitive controls are processes and skills that are prerequisite to learning itself. These cognitive skills in turn are influenced by various affective and environmental preferences that the individual brings to learning.

Experiential learning theory, psychological type theory and the learning style inventory or profile model have been included in the previous summaries because of the current significance of their research, their conceptual importance, or their practical utility. There are many other theories and styles that have been omitted from the current research summaries either because their validity is uncertain, their application is questionable, or their meaning is similar to another style that has been included.

Choice of Instrument

The researcher's interest in learning styles stems from a desire to develop instructional events which will be suitable for students with varied learning styles. For the purpose of this research project, Jung and Myers' definition of type preference and the subsequent effect of type preference on learning style, were influential in the choice of the Murphy-Meisgeier instrument (MMTIC). It was essential that the instrument chosen for this project was suitable for, and applicable to, children. The National Association of Secondary School Principals Learning Style Profile (LSP) was rejected because it was felt that only certain specific scales would be of use for this project. Although physical preferences such as lighting and sound, and study preference times such as morning or evening are important considerations, it was felt that in this case they were not relevant to the project.

Mamchur in her paper "Learning Style Comes to the Workplace", suggests the following criteria when examining learning style and choosing an instrument. These criteria were considered in choosing the instrument most suited to this project.

1) Personal Response - are the learning style attributes and personality characteristics described as a result of taking the instrument meaningful?
2) Theoretical Background - did the instrument grow out of a sound theoretical base?
3) Expense - was the cost of purchasing instrument for one hundred and thirty students excessive?
4) Implementability - is the information received practical, relevant and can changes based on the findings be implemented in a meaningful way for the students.

The last and perhaps one of the most important considerations for this project was readability. Could the students involved, some with limited English language skills, understand what was being asked of them?

The MMTIC instrument was one that fitted all the above criteria and expectations. Based on the MBTI, it has been specifically designed to identify psychological type in children:

> Designed to elicit information about individual differences in children through the identification of psychological type, it seeks to provide an interpretation of type as it relates to how an individual child best perceives and processes information and how that child prefers to interact socially and behaviorally with others. (Meisgeier and Murphy, 1987, p. 1)

Adaptation of Learning Environment to Style Preferences

Meisgeier and Murphy (1987), Myers (1962), Myers and McCaulley (1985), Lawrence (1980), and Mamchur (1984) advocate adapting classroom instruction to a student's type preference. The goal is not to teach solely to each student's type preference and dominant function, but to develop teaching methods that do not ignore any student's type preference or dominant function. Activities, methods and materials should be developed to address all preferred learning functions. The following section (adapted from Mamchur (1994) with her permission) explains the theory of how type preferences manifests itself in the classroom and what activities would best suit different learning types.

Extraversion is an outward focusing of energy, it causes the person to seek outside influences as a source of energy and pleasure and satisfaction. An extraverted learner clarifies thoughts by thinking out loud, sharing ideas, stories and personal experiences. An extravert craves variety and diversity in how and what is learned, and will rarely choose to learn alone. An extravert is a person of action who learns by doing. Learning must be practical and relevant. The extravert is very concerned about what others think and needs feedback, preferably constructive and non-critical. The key words in the extravert's learning pattern are: vocal, interactive, variety, movement, action, practical, approval.

In order to accommodate an extraverted learner a program that is practical and active, in which there is some opportunity to talk, to move, to discuss, to present ideas is best suited. The program design must also include some positive feedback.

Introversion is an inward focusing of energy, it causes the person to look to internal resources as a source of energy and satisfaction and safety. An introverted learner is quiet and thoughtful and needs to think everything through before responding in public. An introvert is a private person who needs to concentrate while learning and resents interruptions. Self-motivated and very focused, an introvert wants clear instructions to follow and the opportunity to explore ideas without supervision. The key words in the introvert's learning pattern are: private, territorial, independent, reflective, intense, uninterrupted, respectful, free from demanded verbal interactions.

When designing a program to accommodate those with an introvert preference, time and space to think and learn need to be incorporated in the course of instruction. Respect is a key word when dealing with the introvert. Respect for their space, their need for time to reflect, and for the introvert's need to keep to themselves.

Sensation is the perceiving function which seeks that which is immediately relevant and accessible through the senses, it causes the person to pay careful attention to each detail in their immediate environment in a very practical, focused way. As most learning happens in the perceiving mode, learning style is greatly influenced by one's preference for sensation and intuition. It is on these aspects of personality that educators need to focus most carefully so as to develop programs in which everyone can succeed. The sensing type needs to see a practical reason for learning and moves cautiously into new learning, preferring a set procedure, learning one step at a time. The sensing type likes to build on existing knowledge and abhors abstract theory. Learning very quickly when told step by step procedure, the sensing type is an open sponge, absorbing through all the senses. Distrustful of most forms of evaluation, the sensing type does not perform well in testing situations The key words in the sensing type learning patterns are: precise, familiar, procedural, practical, concrete, sequenced, hands-on.

For a program of instruction to be suitable for a sensing type it must be broken down into component parts, moving ahead slowly, with plenty of time for observation and practice. The program should avoid the theoretical, concentrating on practical, relevant knowledge which the sensing type already possess to some degree, but wants to increase.

Intuition is the perceiving function which makes sense of the world by creating patterns and inventing hypotheses. It causes the person to scan situations and data in order to see relationships between things in a way which is self-inspiring and inventive. The intuitive learner gets bored very easily and wants variety in how and what is learned. This type of learner thrives on the abstract and symbolic and wants to know the theoretical framework behind all practical knowledge. This is a learner who hates repetition and deeply resents being forced into a review situation. The intuitive tends to skip over details and works unevenly, in spurts and starts. Key words in the intuitive learner's pattern are: multi-level, inventive, variety, impulsive, complicated, challenging, inspirational, theoretical.

In designing a program the intuitive learner needs to be provided with plenty of opportunity to invent, guess, touch, work independently beyond the scope of the program. Theory must be presented at a sophisticated level.

Thinking is the judgment function which values objective, analytical ways to make decisions and evaluate situations. Not nearly as important as the perceiving functions of sensation and intuition, the judging functions of thinking and feeling affect tone more than actual method of learning. The thinking function causes the person to stand back, remaining cool and a bit aloof so that he or she can think logically and rationally, honestly and fairly. The thinker wants to communicate on an intellectual, not personal level and values honesty and fair play. The thinker values constructive criticism, needs a well organized, logically developed course of study. The thinker is not big on consensus, avoids group work and is very uncomfortable with emotional interactions. A very driven, independent learner who values and respects expert knowledge and wants at all times to appear confident especially in front of peers. Key words in the thinking type's learning pattern are: analytical, autonomous, objective, logical, scientific, impersonal, expert, critical, competitive, competent.

In the classroom a thinking type needs a well-organized program with cause and effect sequences clearly evident. A situation where this type of student would feel incompetent must be avoided.

Feeling is the judging function which values subjective analysis and empathetic understanding as a means of decision-making and evaluation . A person with a preference for this function seeks a personal and harmonious relationship with the environment, relying on a deep sense of personal values to guide behavior and judge the behavior of others. The feeler takes everything personally and needs a harmonious environment in which to learn. The feeler values cooperation, consideration and consensus, and dislikes competition. The feeler likes to please and wants a sense of decorum and respectful manners to prevail in the classroom. Key words in the feelers learning style are: harmonious, sensitive, consistent, personal, cooperative, aesthetic, values-driven, considerate.

For the feeling person to function well the teacher is the key factor. Above all, the teacher needs to be genuine and empathetic. A teacher who is prone to sarcasm or inconsistent behavior will rarely be successful with feeling students. Close attention must be paid to the learning environment, ensuring that it is pleasant and harmonious.

A judging person is inclined to use more energies in controlling than in understanding events. The judging type needs a structure, a framework or agenda, and wants that structure to be followed throughout the course of study. This type of person likes to plan and schedule and needs exact dates regarding course progress, exams, assignment deadlines. The judging type has a strong work ethic, is very responsible, expects a lot of feedback from the teacher, and takes learning very seriously with specific expectations about how everyone should behave. This type has a tendency to make hasty decisions and then stick to them even if some of the decisions weren't good ones. The judging type wants to complete every task started and craves a sense of closure. Keywords in the judging learning style are: systematic, decisive, responsible, closure, controlled, organized, monitored.

The judging type functions best with a carefully designed course, given agendas and time-tables and due dates. Adequate lead time needs to be given for any changes of routine thereby avoiding surprises and allowing the judging type to relax, feel safe, and go along with the potential changes. The teacher must endeavor to give consistent feedback.

The perceiving type is inclined to put off decision making until a chance to explore and investigate all the avenues of information has been given. Because the pleasure of process feels much more satisfying than having a final product, the perceiver may start more projects than they finish. Driven by a natural curiosity, the perceiving type enjoys the process of discovering new ideas, but without a lot of pressure. This type is good at explorative learning but avoids schedules and resents the notion of testing or proving capability. The perceiving type is quite relaxed and open to a variety of styles and ideas and appreciates any form of flexibility and spontaneity. Key words in the perceiving learning style are: understanding, flexible, receptive, process-oriented, pending, open full of surprise, curious, spontaneous.

To suit the perceiving learner a course which is flexible, with plenty of opportunity for exploration and discovery is most effective. The teacher and the course must give as much freedom, as many options, as few deadlines and "proving" tasks as possible.

It is important when designing a program such as the one used for this study that the preferences of all the learners are considered and accommodated. Every effort was made in designing the program of instruction for this study to incorporate activities relevant to each preference.

CHAPTER III
METHODOLOGY

This study explored the effectiveness of a particular program of instruction designed for use in the polynomial and factoring units of the Grade 9 curriculum. The instructional strategies employed included extensive use of manipulative materials to help enhance student understanding of the concepts involved. The subjects were all taught in the same way and followed the same program; no comparison study was done. Questionnaires and tests assigned to students were designed to measure both attitudes toward the instructional strategies used and the success experienced by students participating in this program of instruction using manipulative materials. Student learning style preferences were ascertained and data from tests was collected to establish if the method of instruction in the polynomial and factoring units appealed to students with different learning preferences. This chapter explains all aspects of the study. and describes the complete program of instruction.

Subjects

This study was conducted in a Junior Secondary School within the Richmond, BC School District. The school contains 650 students ranging from Grade 7 to Grade 10. The students from all but one of the Grade 9 mathematics classes were used as subjects in the study. The one class that did not participate was an alternate class of 24 students who were following a modified curriculum. There were 141 students involved, four of whom did not return consent forms, so their data were not included in the study. A further five students were recent immigrants with little or no English language skill, so they were not included in the study. Although one of the five classes was an honors class, the students in this class were not treated any differently and followed the same sequence and program of instruction. All five classes were taught by one of two teachers, both of whom made every effort to be consistent in their delivery of the program of instruction.

Instruments

The data collected from the five different instruments described in this section was analyzed to answer the research questions. The questions are restated here:

1) Do students with different learning styles differ in their attitudes towards mathematics, group activities and the use of the manipulative materials "Algebra Tiles"?
2) Do students with different learning styles differ in mathematical achievement using this manipulative based instructional strategy?
3) Does ability to do mathematics using the concrete materials "Algebra Tiles" help to predict achievement after self-report mathematical ability is taken into account?

Mathematics Attitudinal Questionnaire (1)

This questionnaire (Appendix B), designed by the researcher, was split into three sections A, B and C. The sixteen questions in Section A were designed to determine the student's attitude to mathematics in general. Section B (eight questions), focused on attitudes to group work in mathematics and Section C (three questions) tried to determine how successful students felt they were in mathematics.

To establish reliability, selected questions were administered to a random group of twenty one students on two occasions approximately two and one-half weeks apart. The similar responses received from students on both occasions gave a test-retest reliability. The validity of the measure was obtained by soliciting the opinions of colleagues and the university supervisor. A subjective opinion of the intent of each measure, and relative agreement on the type and validity of questions was sought.

Section A

This section asked students to respond to general questions about their attitude and feelings toward mathematics. Questions such as the following were asked:

I enjoy the challenge of a math problem.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

Math is dull.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

Which of the following best describes your feeling toward mathematics?

A) Fascinating and easy B) Interesting C) Indifferent (no feelings at all) D) Boring E) Frustrating and difficult

If students had a strong positive attitude to mathematics in a question in Section A they received five points. Four points were awarded for a positive attitude, three points for indifferent or undecided attitude, two points for a negative attitude and one point for a strong negative attitude. This section contained sixteen questions with a maximum possible score of 80 and a minimum possible score of 16.

Section B

In this section students responded to questions such as the following about their attitude and feelings toward group work and group activities in mathematics:

I prefer working on math assignments on my own rather than in groups during math class.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

I understand math better when we work in groups.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

As in section A if students expressed a strong positive attitude to a group work question in Section B they received five points. Four points were awarded for a positive attitude, three points for indifferent or undecided attitude, two points for a negative attitude and one point for a strong negative attitude. This section contained eight questions with a maximum possible score of 40 and a minimum possible score of 8.

<u>Section C</u>

This section consisted of the three questions listed below regarding how successful students felt they were in mathematics:

How would you rate your mathematical ability?

A) Excellent B) Good C) Average D) Weak E) Very weak

How do your math marks or comments on your report card compare to those in other subjects?

A) Usually better than most other subjects
B) Usually about the same as most other subjects
C) Usually worse than most other subjects

What letter grade do you usually get in math on your report card?

A) A B) B C) C+, C or C- D) D E) E

If students felt they were very successful at mathematics in a Section C question they received five points. Successful four points, moderately successful three points, unsuccessful two points and very unsuccessful one point. This section contained three questions with a maximum possible score of 15 and a minimum possible score of 3.

Murphy-Meisgeier Type Indicator for Children

Early in the twentieth century, the Swiss psychiatrist Carl Gustav Jung developed the concept of psychological type to explain natural differences in human behavior. Isabel Myers states that "the essence of the theory is that much seemingly random variation in human behavior is actually quite orderly and consistent, being due to certain basic differences in the way people prefer to use their perception and judgment" (Myers & McCaulley, 1985, p. 1). The patterns identified by Jung describe how people perceive information and how they reach decisions about it. The Myers-Briggs Type Indicator, on which the Murphy-Meisgeier Type Indicator for Children (MMTIC) is modeled, is based on Jung's theory which measures preferences on the following four bipolar dimensions: (a) Extraversion / Introversion (EI), (b) Sensing / Intuition (SN), (c) Thinking / Feeling (TF), and (d) Judging / Perceiving (PJ). The MMTIC employs the same bipolar dimensions. A description of the four dimensions and eight preferences follows. (Adapted from Murphy-Meisgeier Type Indicator for Children, p.3)

Table 2 - Definitions of Preferences

Where We Focus Our Attention: This dimension assesses whether individuals are oriented to the outer or inner world. The preferences are:

Extraversion (E)	Introversion (I)
Extraverted individuals respond to the environment and are stimulated by people and actions in the environment. Those with a preference for Extraversion tend to be sociable and enjoy active participation in tasks.	Introverted individuals are interested in the inner world of ideas, concepts, or impressions. Those preferring Introversion need privacy and do their best work when alone or with a few people.

How We Perceive or Take In Information: People receive information through two possible functions:

Sensing (S)	Intuition (N)
Sensing individuals receive information through the five senses. Those with a preference for Sensing tend to be practical and realistic, appreciating facts and important details. Their focus is usually on the present.	Intuitive individuals receive information through a "sixth sense". Individuals with a preference for Intuition enjoy imagining, creating, and conceiving possibilities. They attend to meanings, relationships, and symbols, and their focus is usually on the future.

How We Make Judgments or Decisions About Information: Once information is perceived, some kind of decision must be made about it. People can make decisions using one of two functions:

Thinking (T)	Feeling (F)
Thinking individuals make decisions based on logical, objective analysis. Those who adopt Thinking as a decision-making style are analytical and concerned with objective truth and justice.	Feeling individuals make decisions based on a person-centered value system. They consider the impact of decisions on others and are sensitive to the values of others.

Two ways of dealing with the outer world: When dealing with the outer world, an individual may rely upon a judging process (T or F) or upon a perceiving process (S or N). The process primarily used in dealing with the outer world is one of two attitudes:

Judging (J)	Perceiving (P)
Judging individuals prefer an ordered, planned, and structured lifestyle. Individuals with a preference for Judging tend to be organized and like to bring closure to projects, liking things decided and settled.	Perceiving individuals prefer a spontaneous, flexible lifestyle. Individuals with a Perceiving preference are adaptable and curious and like to keep options open. They aim to miss nothing.

A note on terminology: Following Jung's nomenclature, Sensing and Intuition are referred to as the perceiving functions and Thinking and Feeling as the judging functions. Extraversion and Introversion are called attitudes as are Judging and Perceiving.

Like the Myers-Briggs Type Indicator (MBTI) which assesses psychological type in adults, the MMTIC is a self report instrument in which the child is asked to choose a preferred response from two choices, neither of which is right or wrong. There are 70 short items, 18 items each for Sensing/Intuition (SN), Thinking/Feeling (TF), and Judging/Perceiving (JP) scales and 16 items for the Extraverted/Introverted (EI) scale. The student is asked to make a simple "A" or "B" choice as shown in the following sample questions taken from the MMTIC instrument. (Meisgeier and Murphy, 1993)

In a new school, making friends is:

A. Exciting
B. Hard

You like a:

A. Straight line
B. Zigzag/curly line

People do better when they:

A. Know the rules
B. Know someone cares about them

In assessing the effectiveness of the program of instruction on students with different learning preferences, the concentration will be on the functions, S, N, T, and F and only small consideration will be given to an individual's attitudes, E, I, J, and P.

The TF preference (thinking or feeling) is entirely independent of the SN preference (sensing or intuition). Either kind of judgment can team up with either kind of perception. Thus, four combinations occur (Myers, 1985, p. 4):

ST	sensing plus thinking
SF	sensing plus feeling
NF	intuition plus feeling
NT	intuition plus thinking

Each of these combinations produces a different kind of personality, characterized by the interests, values, needs, habits of mind, and surface traits that naturally result from the combination. The following paragraphs sketch the contrasting personalities found from each of the four possible combinations of perception and judgment (adapted from Myers, 1985, p. 5).

SENSING plus THINKING

The ST (sensing plus thinking) people rely primarily on sensing for purposes of perception and on thinking for purposes of judgment. Thus, their main interest focuses upon facts, because facts can be collected and verified directly by the senses - by seeing, hearing, touching, counting, weighing and measuring. ST people approach their decisions regarding these facts by impersonal analysis, because of their trust in thinking, with its step-by-step logical process of reasoning from cause to effect, from premise to conclusion. In consequence, their personalities tend to be practical and matter-of-fact, and their best chances of success and satisfaction lie in fields that demand impersonal analysis of concrete facts.

SENSING plus FEELING

The SF (sensing plus feeling) people also rely primarily on sensing for purposes of perception, but they prefer feeling for purposes of judgment. They approach their decisions with personal warmth, always conscious of their own feelings and the feelings of others. They are more interested in facts about people than in facts about things and, therefore, they tend to be sociable and friendly.

INTUITION plus FEELING

The NF (intuition plus feeling) people possess the same personal warmth as SF people because of their shared use of feeling for purposes of judgment, but because the NF person prefers intuition to sensing, they do not concentrate their attention on concrete situation. Instead they focus on possibilities, such as new projects (things that haven't ever happened but might be made to happen) or new truths (things that are not yet known but might be found out). The new project or the new truth is imagined by the unconscious processes and then intuitively perceived as an idea that feels like an inspiration. The personal warmth and commitment with which the NF people seek and follow up a possibility are impressive. They are both enthusiastic and insightful. Often they have a marked gift of language and can communicate both the possibility they see and the value they attach to it.

INTUITION plus THINKING

The NT (intuition plus thinking) people also use intuition but team it with thinking. Although they focus on a possibility, they approach it with impersonal analysis. Often they choose a theoretical or executive possibility and subordinate the human element. NT people tend to be logical and ingenious and are most successful in solving problems in a field of special interest.

Everyone has probably met all four kinds of people: ST people, who are practical and matter-of-fact; the sympathetic and friendly SF people; NF people, who are characterized by their enthusiasm and insight; and NT people, who are logical and ingenious.

Myers (1980) claimed that the most important learning style differences are those related to the perceiving and judging functions. Learning style can thus be split into four quadrants as shown in figure 5. It must be noted that learning style has many issues to consider and recent opinion expressed by McCaulley and Mamchur suggest that extraversion and introversion together with sensing and intuition are most important when considering learning style. However, for the purposes of this research project the Myers model based on Jung's theories, of considering the perceiving and judging functions as most important in learning style differences, was adopted.

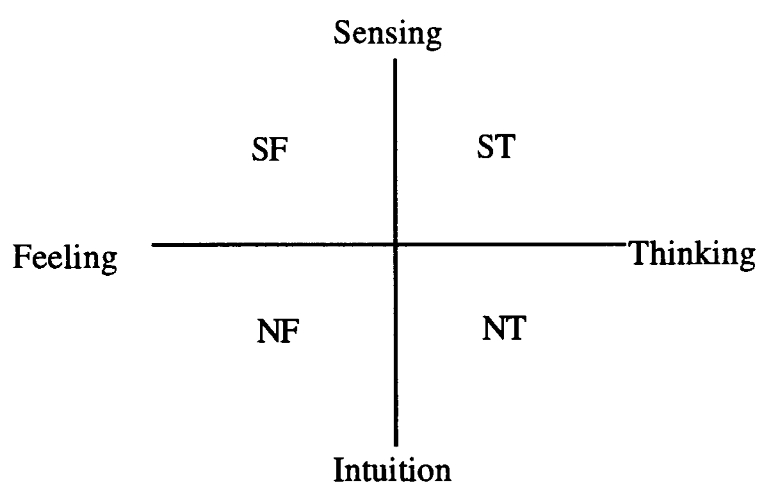


Figure 4 - Learning styles

Of the four functions S, N, T, and F, one leads and is called the dominant. This function is the most preferred and the first to develop. The function that balances the dominant is called the auxiliary. If the dominant is a perceiving function (S or N), the auxiliary will be a judging function (T or F), and vice-versa. For example, if an individual's dominant function is Sensing, then the auxiliary function will be either Thinking or Feeling. This provides a balance between taking in information and making decisions about that information. One of the four functions becomes dominant "because of an inborn predisposition that, in the course of normal development, makes the activities of that function more interesting and rewarding" (McCaulley, 1981, p.300).

As a child's dominant function is still developing throughout most of the school years, preferences in some instances may not be well differentiated. Allowances have been made on the MMTIC for the developmental nature of type by providing an "undetermined" category. Each of the bipolar dimensions, EI, SN, TF, and JP is scored on a continuous scale. Low scores in each scale would indicate a preference for E, S, T, or J, while high scores indicate clear preference for I, N, F, or P. In the middle of each of these continuous scales is the "U-band." Any score falling within the U-band indicates an undetermined preference.

The MMTIC was assessed by university supervisors to be both reliable and valid for purposes of this study. The developmental nature of type preference in children must not be forgotten and it must be realized that results obtained from this test are not necessarily the same type preferences students will express at another time.

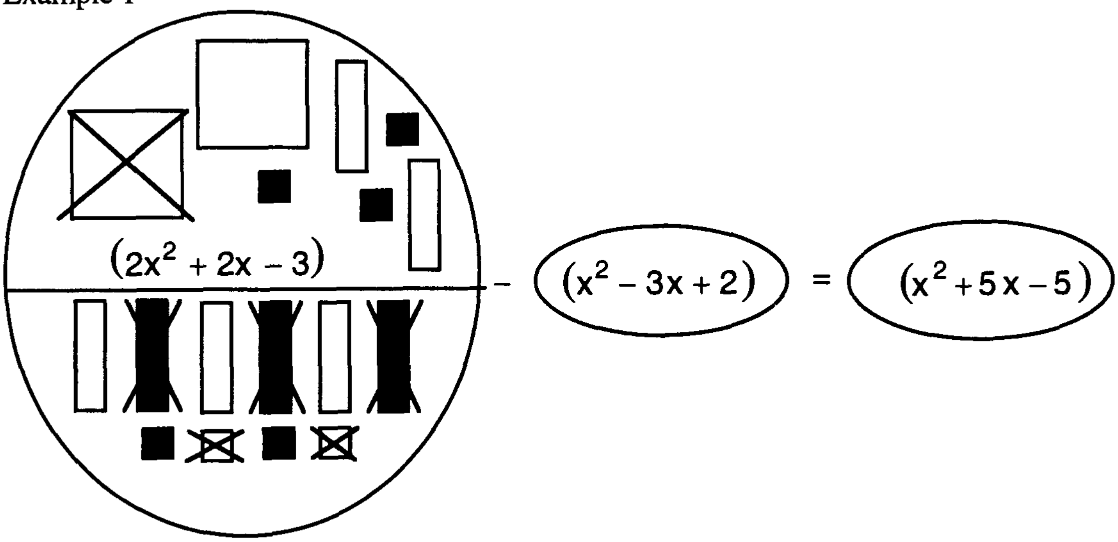
Polynomial and Factoring in Class Tests

Both the polynomial and factoring unit tests were split into two parts, A and B. Part A consisted of a number of questions where it was necessary to use tiles, or draw representations of tiles in order to answer the questions. Students were expected to perform the required operations using concrete materials and to draw a diagrammatic representation of the solutions. The following are questions taken from Section A of the tests.

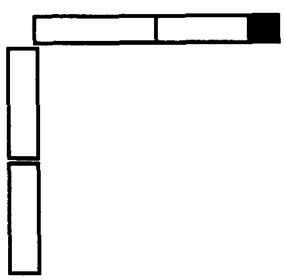
Polynomials Test (Appendix C)

The following is an example of what students were expected to do when performing subtraction of polynomials using tiles and diagrammatic representation of tiles. Required "zeros" need to be added to the bottom of the first circle as shown. Without these "zeros" the subtraction cannot be completed. Tiles that are being subtracted need to be crossed out as shown. The final answer to the subtraction is written in abstract form in the last oval.

Example 1



Example question 2 - Multiply (2x)•(2x - 1). The frame is provided for you as shown.

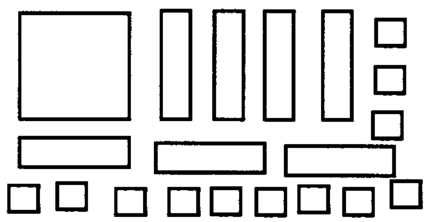


Students were expected to draw diagrammatic representations of tiles to fill in the rectangle. Tiles could be used as aids if required.

Factoring Test (Appendix D)

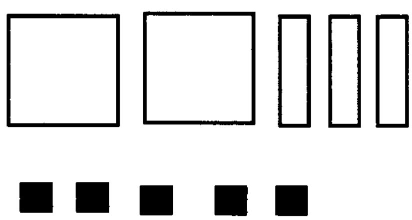
In the following example which appears on the factoring test the students were given the required area and shown the tiles that represent that area. They were then required to set up the tiles and maneuver them in such a way as to form a perfect rectangle. The dimensions of the rectangle then form the two factors whose product make up the area.

Example 3



Area = $x^2 + 7x + 12$

Example 4



Area = $2x^2 + 3x - 5$

In section B students were asked to use their knowledge to answer a number of questions involving addition, subtraction, multiplication and factoring of expressions. Students were required to use abstract methods to answer questions in part B. The tests were set up in two sections to encourage the transition from using the manipulative to using algorithmic solutions. Some examples from Section B in both tests are shown.

Expand $-x(-3 + 2x)$

= ____________________

Expand $5(2p - 7)(3p - 4)$

= ____________________

Factor $3a^2 - 18a + 24$

= ____________________

The two tests were designed by the teachers involved in the study and were thought to be a good test of student understanding of concepts involved in these two units. It was also felt that the test results would give a good indication as to the level of understanding and to whether students are comfortable using both concrete and abstract methods to solve problems.

Algebra Tiles Attitudinal Questionnaire (2)

As with the Mathematics Attitudinal Questionnaire (1) this questionnaire (Appendix E), designed by the researcher, was split into three sections A, B and C. The thirteen questions in Section A were designed to determine the student's attitude to algebra tiles as a fun manipulative, and whether or not students felt that the use of the tiles enhanced their learning of the two units. Section B (eight questions) focused on attitudes to group work with algebra tiles, and Section C (three questions), was to determine how successful students felt they were in the polynomial and factoring units.

<u>Section A</u>

This section asked students to respond to general questions about their attitude and feelings toward algebra tiles. Questions such as the following were asked:

I liked the algebra tiles units.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

Hands on materials such as algebra tiles make understanding math a lot easier.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

Which of the following best describes your feelings toward the algebra tiles units?

A) Fascinating and easy B) Interesting C) Indifferent (no feelings at all) D) Boring E) Frustrating and difficult

If you were asked by your teacher whether or not you would encourage the use of algebra tiles when learning polynomials and factoring, would you -

A) Strongly encourage B) Encourage C) Be indifferent D) Discourage E) Strongly discourage

As with attitudinal questionnaire (1) if students had a strong positive attitude to algebra tiles in a question in Section A they received five points. Four points were awarded for a positive attitude, three points for indifferent or undecided attitude, two points for a negative attitude and one point for a strong negative attitude. This section contained thirteen questions with a maximum possible score of 65 and a minimum possible score of 13.

<u>Section B</u>

Attitude and feelings of students toward group work and activities specifically in the algebra tiles units. Questions asked were similar to the examples shown:

I prefer working with the algebra tiles on my own rather than in groups during math class.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

I enjoy working in groups to study the algebra tiles units.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

I enjoyed helping others when we were working with the tiles.

A) Strongly agree B) Agree C) Undecided D) Disagree E) Strongly disagree

If students had a strong positive attitude to a group work question in Section B they received five points. Four points were awarded for a positive attitude, three points for indifferent or undecided attitude, two points for a negative attitude and one point for a strong negative attitude. This section contained eight questions with a maximum possible score of 40 and a minimum possible score of 8.

Section C

This section consisted of only three questions, listed below, regarding how successful students thought they were in the algebra tiles units:

How would you rate your ability to do polynomials and factoring using algebra tiles?

A) Excellent B) Good C) Average D) Weak E) Very weak

What letter grade did you get for the polynomials and factoring units?

A) A B) B C) C+, C or C- D) D E) E

How do your marks for the algebra tiles units compare to those you would expect to get in mathematics?

A) Better than I would expect to get in other math units.
B) About the same as other math units.
C) Worse than I would expect to get in other math units.

If students felt that they were very successful using algebra tiles in a Section C question they received five points. Successful four points, moderately successful three points, unsuccessful two points and very unsuccessful one point. This section contained three questions with a maximum possible score of 15 and a minimum possible score of 3.

Treatment

A full program of instruction, including sequence, worksheets and tests, was prepared by the researcher. The instruction program for both polynomials and factoring was based on the requirements in the Grade 9 curriculum guide for mathematics. Before beginning the instruction of the two units the students who had returned consent forms (Appendix K) completed both Mathematics Attitudinal Questionnaire (1) (Appendix B) and the Murphy-Meisgeier Type Indicator for Children. Once these initial tests were complete instruction in the two units began.

Murphy and Meisgeier claim that the goal of using information about learning style preferences in the classroom is not to teach solely to each student's preferred type, but to develop teaching methods that do not ignore any student's type. The teacher needs to develop activities, methods or materials that address all students' type preferences. Furthermore, Meisgeier and Murphy (1987) state:

> It would be just as wrong to expose a student to opportunities to learn in only one function, even if it were his or her dominant function, as it would be to ignore the dominant function completely. The student needs a chance to practice skills in all functions and attitudes. The problem develops for the student who is given few or no opportunities to learn through his or her dominant function. (p.16)

Jung also suggests that the goal of education should not be to help the child develop all preferences equally, but rather to provide opportunities for the child to discover and express his or her own unique gifts. Every effort was made when designing the program of instruction to include activities that would appeal to students with a variety of different learning preferences. Suggestions made by Mamchur (1994), on adapting the learning environment to style preferences as discussed in chapter two, plus some of the kinds of activities suggested by Meisgeier and Murphy (1987, p. 12) are shown in Table 3. These suggestions were considered when designing the program of instruction for this study.

Table 3 - Description of Learning Styles

TYPE PREFERENCE	PREFERED LEARNING SITUATIONS
EXTRAVERSION	Let students problem solve with a partner. Student needs time for social conversation. Demonstrate work through modeling. Permit an active manipulation of materials. Let the student experiment before explaining the concept. Allow time for trial and error learning.
INTROVERSION	Needs to consider possibilities or all facts before answering. Must have time for individual thinking. Needs to understand the concept before experiencing it.
SENSING	"Seeing is believing" for this student. Use films, TV, and other audiovisual aids. Enjoys activities that require, observation, memory and hands on experiences. Always give practical examples, apply to real life situations. Introduce new materials one step at a time.
INTUITIVE	Explain anticipated results before breaking lesson into parts. Student focuses on the global picture. Offer tasks that challenge the imagination. Give open ended problems that allow for many different solutions. This student needs variety, enjoys self paced learning and can quickly become bored with routine.
THINKING	This student is logical and objective. Give opportunity to solve problems by collecting, organizing, and evaluating data. Explain cause and effect, needs to know why something is being done.
FEELING	Needs harmony, frequent praise and positive feedback. Likes team work, group projects and working with a partner. Enjoys pleasing others and dislikes competition.
JUDGING	Likes strict routine. Likes to finish one thing before beginning next assignment. Explain exactly what is expected. May enjoy organized group activities.
PERCEIVING	Loves to explore, needs freedom to move around and learn. Unorganized, acts spontaneously. This student is curious and loves to discover new things. Dislikes routine and may leave assignments to the last minute. Loves to explore ideas and concepts.

Instruction as indicated by the sequence that follows, was conducted over a period of six weeks in January, February and March 1994. The researcher was responsible for three of the five classes while the other two classes received instruction from the only other mathematics teacher in the school. Both teachers involved adhered to the following detailed sequence of instruction very closely and had daily discussions so as to be consistent with all five classes involved in the study.

Sequence of Instruction

A) POLYNOMIALS

Period 1) Distribute Algebra Tiles, one set for each pair of students. Students sit in clusters of four (two pairs).

- Activity 1 Construct a shape either two dimensional or three dimensional using as many of the set of tiles as possible. This activity is designed to help the students to become familiar with the different tiles that make up a set. In playing they will hopefully also discover that a whole number of "unit" tiles does not make up an "x" tile. It is important that the students understand from the beginning that no number of "unit" tiles make an "x" tile.
- Activity 2 Discuss the different shapes within the foursome and with the teacher. It is important to have dialogue within pairs and groups.

- Activity 3 Give the different tiles in the set an identity. "x^2" tile, "x" tile and "unit" tile. Discuss with students the negative tiles, and that a negative tile and positive tile cancel each other out (zero concept). Overhead set to be used for this. On overhead place certain tiles, ask class as a whole to tell you what algebraic expression those tiles represents. Use as many examples as necessary including negatives. In pairs each student to test their partner by using certain tiles and giving the algebraic expression which represents those tiles.

Distribute Algebra Tiles Project (1) (Appendix F) to each pair of students. Students to complete page 2, 3, and top of 4 plus page 7 and top of 8.

Period 2) Adding Polynomials.

- Activity 1 Each pair to place a number of tiles on the desk. Write down an algebraic expression for the tiles. Now combine the tiles with those of the other pair of students in the foursome and write down an algebraic expression for the combined tiles. The students have now added two algebraic expressions together with the aid of tiles. This activity can be extended to include more foursomes, hence adding a string of algebraic expressions together.
- Activity 2 Teacher using overhead to go through example of addition on bottom of page 4 of project 1. Do a number of similar examples and allow individual students to come and solve examples for the class.

Assign each pair to complete page 5, 6, bottom 8 and top 9 on addition of polynomials project 1 (Appendix F). Students must complete the circle, drawing in the tiles, as well as completing the algebraic expressions for their answers.

Period 3) Subtracting Polynomials.

- Activity 1 Set up examples, such as those on page 9 and 10 of project 1, on overhead. Ask students to think of ways in which to do the subtraction questions and to share their ideas with the rest of the class. It is important to spend some time on this activity so that all students understand how to do the subtraction, and are comfortable with the **"ZERO RULE"**.

Students to go through examples on bottom of page 11 and top of page 12 in project 1 (Appendix F) then complete subtraction exercise pages 12, 13, and 14 exactly as shown in examples, drawing in the tiles and writing down the algebraic expressions.

Period 4) Transition between using the tiles to assist in adding and subtracting algebraic expressions and using only formal algebraic methods to complete addition and subtraction problems.

- Activity 1 Write some addition and subtraction problems on overhead. Use examples such as those in Section B and C on pages 15, 16 and 17 of project 1 (Appendix F). Ask students to discuss the problems within their pairs and foursomes, without the aid of algebra tiles. One volunteer from each foursome to come up and explain a solution to the rest of the class. Students will often come up with some excellent and different ideas on how to correctly complete the examples and these ideas should be encouraged.

Assign Section B and C pages 15, 16, 17 of project 1 (Appendix F). Project 1 should be completed and handed in to the teacher to be marked.

Period 5) Multiplying Polynomials.

- Activity 1 Teacher starts by doing some very simple and basic multiplication such as 3 x 4. Show that by creating a rectangle 3 units wide by 4 units long and filling in the rectangle the answer is 12. Using the same method multiply algebraic expressions together and fill in the area of the rectangle to find the product. Continue to do a number of examples using algebra tiles as shown on page 2 of Algebra Tiles Project (2) - Multiplication (Appendix G). Show students how to create a frame and then fill in the rectangle. Use only a monomial multiplied by a binomial initially.

Distribute Project (2) (Appendix G) to each individual student. Students may discuss questions and work in pairs but each individual must complete the project. Assign section A pages 2/3 numbers 1 to 4.

- Activity 2 Continue on from activity 1, this time multiplying a binomial by a binomial as shown in example on page 3 project 2. Get students to demonstrate to the class using the overhead.

Assign section B pages 3/4 numbers 1 to 4.

Period 6) Multiplying Binomials involving the "DOUBLE COVER-UP RULE"

- Activity 1 Continue as for activity 2 in previous lesson. Overlapping negatives are difficult to represent. Review with students the "DOUBLE COVER-UP RULE" where overlapping negative tiles result in positive units. Do a number of examples similar to the one shown on page 4 of project 2. Allow the students to explain to the class using overhead.

Assign section C pages 4/5 numbers 1 to 6 to be completed using the algebra tiles.

Period 7) Transition between using the tiles to assist in multiplying algebraic expressions and using only formal algebraic methods to complete multiplication problems.

- Activity 1 Write some multiplication problems on overhead. Problems such as those in Section D and E on pages 6 to 8 of project 2. (Appendix G) Ask students to discuss the problems within their pairs and foursomes, without the aid of algebra tiles. One volunteer from each foursome to come up and explain a solution to the rest of the class. Students often come up with some excellent and different ideas on how to correctly complete the multiplication questions.
- Assign section D and E of project 2 (Appendix G)

Period 8) Review of entire polynomials unit in preparation for test including use of tiles.

Period 9) Test on polynomials unit. Test comprises two sections A and B where A involve the use and manipulation of algebra tiles and section B requires solutions without the aid of tiles. (Appendix C)

B) FACTORING (Change pairs and foursomes)

Period 10) Factoring. The dimensions of a given area are the factors of the expression representing that area.

- Activity 1 Get students to lay out 6 or 8 or 12 of the "unit" tiles. Arrange the 6 or 8 or 12 square units in such a way that a rectangle is formed. Students will discover that the length and width are factors of the expression for the area. Allow students to explain their findings to the entire class.
- Activity 2 Extension of activity 1 but instead of area being represented as square units an algebraic expression is used, as in example 2 on page 3 of project 3. (Appendix H) Do a number of similar examples for the entire class using overhead. Let the students set up some examples of their own for their partners and foursomes.

Distribute Algebra Tiles Project (3) - Factoring and Division. (Appendix H) One assignment for each group of two students. Assign pages 2 to 5 to be completed by the pair.

Period 11) Division of polynomials where the area and one factor are given and the second factor or dimension is needed.

- Activity 1 Go over some numerical and algebraic examples such as those shown on page 6 of project 3. (Appendix H) Let students do these examples within their own groups and then get one person from each foursome to explain an example to the class. Remind students about overlapping double negatives making a positive. It may be necessary to review the "double cover-up" concept.

Assign pages 7 and 8 of project 3. (Appendix H)

Period 12) Division of more difficult polynomials where there are too few tiles from a given trinomial to establish the dimensions specified by the divisor.

- Activity 1 Go through example as shown on page 9 of project 3. (Appendix H). Review the "zero" principle with the students. Let the students do a number of examples for themselves which involve adding "zeros" in order to complete the division.

Assign exercise pages 9 and 10 of project 3. (Appendix H)

Period 13) Factoring involving the use of the "zero rule".

- Activity 1 Do examples such as those shown on page 11 of project 3 (Appendix H). As always rely on students to provide solutions.

Assign exercise pages 12 to 14 of project 3. (Appendix H)

Period 14) Transition between using the tiles to assist in factoring algebraic expressions and using only formal algebraic methods to complete factoring problems.

- Activity 1 Write some factoring problems on overhead. Problems such as those in Section D on pages 15 and 16 of project 3. Ask students to discuss the problems within their pairs and foursomes, without the aid of algebra tiles. One volunteer from each foursome to come up and explain a solution to the rest of the class.

Assign section D on pages 15 and 16 of project 3. (Appendix H)

Period 15) Further examples and exercises using formal algebraic methods to complete factoring problems. Common factors, difference of two squares and trinomials to be covered. Use examples from text book.

Period 16) As for period 15 above using examples from text book and puzzle sheets.

Period 17) Review of entire factoring unit in preparation for test including use of algebra tiles.

Period 18) Test on factoring unit. Test comprises two sections A and B where A involve the use and manipulation of algebra tiles and section B requires solutions without the aid of tiles. (Appendix D)

Once the instruction and testing of the units was completed the students were asked to complete the final questionnaire of the study; Algebra Tiles Attitudinal Questionnaire (2). (Appendix E)

Data Analysis

The student data collected were entered on a Macintosh Excel spreadsheet file. All raw scores can be found in appendix I. Each student was given an identification number as shown in column 1 of the raw scores, this was followed by a class identification number. The third column showed the students type preference including those undecided in one or more areas. Undecided preference is indicated by a U. According to Jungian theory, as explained earlier in this chapter, learning style was then broken into one of five categories ST, SF, NF, NT, undecided Each category was given a code number of 1, 2, 3, 4 or 5 respectively.

Scores for the first attitudinal questionnaire were recorded by results for all three sections, A, B and C. The next three columns consist of the total scores for sections A, B and C of attitudinal questionnaire (1) divided by the number of questions per section, to get an average mark out of five for each section. There were a different number of questions for sections in the two attitudinal questionnaires, so to keep results consistent for comparison purposes the results were calculated as averages out of five.

Following attitudinal questionnaire (1) scores the next three entries were polynomial test results, split into section A, section B and total score. Three columns of factoring test results were then recorded. The next six columns were for attitudinal questionnaire (2) results treated in the same way as questionnaire (1). The last four columns of raw scores consist of responses to certain specific single questions from attitudinal questionnaires (1) and (2).

Statistical analysis was conducted using various options within Minitab. Minitab is a powerful tool originally developed at Pennsylvania State University. It is one of the most widely accepted statistical analysis packages and has proven itself as an established research tool. To obtain results for this study, means and standard deviations for various data were calculated using analysis of variance (Anova) procedure, which not only shows significant differences but also indicates means and standard deviations. An analysis of variance was conducted on both learning preferences and success with the two algebra tiles units, as well as learning preferences and attitudes expressed in both attitudinal questionnaire. If significant differences were found at the P= 0.05 level the Tukey Multiple Comparison Test was conducted to find where those differences existed.

Structured regression tests were performed on section B of both polynomial and factoring unit tests, using students self perceived success in mathematics as a first predictor, followed by the variable representing students' achievement on sections A of both tests as a second predictor. The regression analysis was performed to answer research question three as to the predictability of achievements in abstract test questions, based on results achieved on questions where concrete materials were used to determine the solutions.

Reliability analysis using SPSS statistical package was conducted on the responses to questions in all three sections of both attitudinal questionnaires to establish the reliability of those responses.

CHAPTER IV
RESULTS

In this chapter the results of the procedures listed in Chapter 3 are reported and an attempt is made to answer the three research questions. The data from the entire sample of 132 students was analyzed as a whole or by learning style preferences and not by separate classes. As the responses to both attitudinal questionnaires are used extensively in determining a number of answers to questions related to student attitudes, reliability testing on these responses was conducted. SPSS statistical package for the Macintosh was used to perform reliability tests on the responses of all three sections of both attitudinal questionnaires. The Cronbach's alpha reliability coefficients generated by the test are as follows.

	QUESTIONNAIRE (1)	CRONBACH'S ALPHA
Section A:-	Attitude and feelings to Mathematics	.92
Section B:-	Attitude to group activities in Mathematics	.75
Section C:-	Success in Mathematics	.78
	QUESTIONNAIRE (2)	
Section A:-	Attitude and feelings to Algebra Tiles	.95
Section B:-	Attitude to group activities in Algebra Tiles units	.84
Section C:-	Success in Algebra Tiles units	.79

The outcomes for each section of .75 or higher indicate a reasonably strong reliability of the responses for all sections of both questionnaires.

First Research Question

Do students with different learning styles differ in their attitudes towards mathematics, group activities and the use of the manipulative materials "Algebra Tiles"?

Learning style research indicates that there are individual differences among children "so extreme that identical methods, resources, or grouping procedures can prevent or block learning for the majority of our students" (Dunn & Dunn, 1978, p.xiii). Table 3 page 58, outlines a number of different instructional strategies suited to students with different learning styles. For example intuitive learners prefer tasks that challenge the imagination, whereas to sensing learners "seeing is believing", these students prefer hands on experiences. Some learning types have a preference to the security of working with partners or in groups, whereas others work better individually. Based on research we would expect there to be significant differences in the attitudes of students with different learning preferences toward mathematics, group activities and the "hands on" approach used in the algebra tiles units.

As discussed in Chapter 3 on methodology, an individual's psychological type is the combination of one of the two attitudes [Extraversion, Introversion (EI)], one of the two interfaces [Judging, Perception (JP)] and two of the four functions [Sensing, Intuition (SN); Thinking, Feeling (TF)] preferred by the individual. When all eight preferences are combined in all possible ways, sixteen types result. Jung and researchers such as Myers feel that the most important type differences are those related to the perceiving and judging functions and that one way to define learning styles is to split the preferences into the four main quadrants of: Sensing, Thinking (SN); Sensing, Feeling (SF); Intuition, Feeling (NF); and Intuition, Thinking (NT).

An analysis of variance was performed on the responses to sections A and B of both attitudinal questionnaires to establish if significant differences in attitude do exist based on learning styles. Table 4a and 4b show test results comparing attitude to mathematics and the four learning style groups. Table 5a, 5b, 6a and 6b compare attitudes of different learning style groups towards group activities in mathematics and in the algebra tiles units respectively. Table 7a and 7b compare attitudes and feelings to the algebra tiles units.

Table 4a - Means and Standard Deviations of Mathematics Attitudinal Questionnaire(1) Section A - (Attitude and feelings toward Mathematics) for Four Major Type Preferences

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	58.19	9.36
SF	51	55.94	7.61
NF	24	55.13	9.94
NT	14	54.93	10.21

Table 4b - Analysis of Variance of Mathematics Attitudinal Questionnaire(1) Section A - (Attitude and feelings toward Mathematics) for Four Major Type Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	134.9	45.0	0.58	**0.632**
Error	106	8275.6	78.1		
TOTAL	109	8410.6			

Table 5a - Means and Standard Deviations of Mathematics Attitudinal Questionnaire(1) Section B - (Attitude and feelings toward group activities in Mathematics) for Four Major Type Preferences

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	24.05	3.83
SF	51	24.23	3.89
NF	24	26.83	3.81
NT	14	25.50	4.93

Table 5b - Analysis of Variance of Mathematics Attitudinal Questionnaire(1) Section B - (Attitude and feelings toward group activities in Mathematics) for Four Major Type Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	131.1	43.7	2.73	**0.048**
Error	106	1699.5	16.0		
TOTAL	109	1830.6			

Table 6a - Means and Standard Deviations of Algebra Tiles Attitudinal Questionnaire(2) Section B - (Attitude and feelings toward group activities in Algebra Tiles units) for Four Major Type Preferences

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	25.05	4.51
SF	51	25.37	4.06
NF	24	25.79	3.87
NT	14	26.64	4.16

Table 6b - Analysis of Variance of Algebra Tiles Attitudinal Questionnaire(2) Section B - (Attitude and feelings toward group activities in Algebra Tiles units) for Four Major Type Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	25.0	8.3	0.49	**0.689**
Error	106	1800.0	17.0		
TOTAL	109	1825.1			

Table 7a - Means and Standard Deviations of Algebra Tiles Attitudinal Questionnaire(2) Section A - (Attitude and feelings toward Algebra Tiles) for Four Major Type Preferences

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	43.86	10.81
SF	51	43.63	10.15
NF	24	47.42	8.30
NT	14	45.00	5.51

Table 7b - Analysis of Variance of Algebra Tiles Attitudinal Questionnaire(2) Section A - (Attitude and feelings toward Algebra Tiles) for Four Major Type Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	251.9	84.0	0.94	**0.424**
Error	106	9468.3	89.3		
TOTAL	109	9720.2			

Although differences in attitudes were expected between different learning style groups the results show that this was not the case in this study. No significant differences in attitude were found at the 0.05 level except for the attitudes to group work in mathematics (Table 5a and 5b). The Anova test showed a p-value of 0.048, indicating that there were differences at the 0.05 level. The Tukey test was used to determine which means differed from one another, but no pairwise differences were found. Therefore, although there appear to be some slight differences in attitude to group activities in mathematics, these differences not considered to be significant. Surprisingly then the results in this study indicate that no significant difference in attitudes towards mathematics, group activities and the use of the manipulative materials “Algebra Tiles”, was expressed by students with different learning styles.

To determine whether or not the concrete approach using the algebra tiles was effective as a teaching strategy, a number of other questions need to be answered. Did the students enjoy using the manipulative materials? Did students feel that the concrete “hands on” approach was beneficial to their understand of some of the concepts involved in the polynomial and factoring units.?

No significant difference in attitudes to the tiles by students with different type preferences was found and mean scores shown in Table 7a indicate that means range between 43 and 48 out of a possible 65 . These mean scores show a fairly positive attitude indicating that algebra tiles were favorably and positively received. Table 8 shows the responses to Algebra Tiles Attitudinal Questionnaire (2) Section A, regarding general attitude and feeling of students toward use of algebra tiles in the teaching of the polynomial and factoring units.

Table 8 - Algebra Tiles Attitudinal Questionnaire (2) Responses Section A (General attitude and feeling toward use of algebra tiles in the teaching of Grade 9 polynomial and factoring units)

ATTITUDE	SCORE RANGE	NUMBER OF STUDENTS
STRONG POSITIVE	58 or more	10
POSITIVE	46 - 57	58
INDIFFERENT / UNDECIDED	33 - 45	50
NEGATIVE	20 - 32	13
STRONG NEGATIVE	less than 20	0

According to the responses 52% of students had a positive or strongly positive attitude to algebra tiles, while only 10% had a negative or strongly negative attitude. An attitude of indifference was expressed by 38% of the students. This data as well as the mean scores show an encouraging positive attitude to algebra tiles and we can safely surmise that the majority of students in all four learning style groups enjoyed using the manipulative.

One further question remains. Did students perceive that their knowledge of polynomials and factoring was enhanced by the use of concrete materials? Altizer-Tuning (1984) argued that, "the conceptualization and understanding of problems should be valued more highly than just correct solutions to routine exercises" (p. 2). Ross and Kurtz (1993) maintained that the effective use of manipulatives contributes to conceptualizing and understanding. The data received from students in this study appears to support Ross and Kurtz in this position.

A one-way analysis of variance of each of the learning style groups with the responses given to question 8 and question 9 combined of the Algebra Tiles Attitudinal Questionnaire (Appendix E), indicated no significant difference between groups, as shown in Table 9a and 9b.

Question 8

The algebra tiles helped me to understand the polynomials and factoring units a lot better than I would have done without the tiles.

Question 9

Hands on materials such as algebra tiles make understanding math a lot easier.

Table 9a - Means and Standard Deviations of combined responses to Question 8 and Question 9 of Attitude to Algebra Tiles Questionnaire (2)

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	7.23	2.10
SF	51	7.28	1.96
NF	24	8.25	1.60
NT	14	7.93	1.69

Table 9b - Analysis of Variance of combined responses to Question 8 and Question 9 of Attitude to Algebra Tiles Questionnaire (2)

SOURCE	DF	SS	MS	F	p
Type pref	3	19.66	6.55	1.85	**0.143**
Error	106	375.39	3.54		
TOTAL	109	395.05			

Of special interest here is that the mean response from all the students was close to the 7.5 level out of a possible 10 for the combined questions 8 and 9, indicating that students generally felt positive toward the use of the manipulative and they felt that the manipulative enhanced their understanding of the two units.

Second Research Question

Do students with different learning styles differ in mathematical achievement using this manipulative based instructional strategy?

An Anova test was conducted to determine whether each learning style group was equally successful at both the polynomial and factoring units. Test results for both the units were compared with learning style preference as shown in Tables 10a, 10b, 11a, 11b. Results once again indicate that at the $p < 0.05$ level there is no significant difference between the achievements of the students and their preferred learning style.

Table 10a - Means and Standard Deviations of Polynomial Unit Test for Four Major Learning Style Preferences.

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	56.14	16.40
SF	51	54.98	14.27
NF	24	56.00	11.55
NT	14	47.14	16.91

Table 10b - Analysis of Variance of Polynomial Unit Test for Four Major Learning Style Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	880	293	1.39	**0.250**
Error	106	22345	211		
TOTAL	109	23225			

Table 11a - Means and Standard Deviations of Factoring Unit Test for Four Major Learning Style Preferences.

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	40.90	11.14
SF	51	37.08	12.92
NF	24	34.63	13.50
NT	14	34.07	15.08

Table 11b - Analysis of Variance of Factoring Unit Test for Four Major Learning Style Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	575	192	1.13	**0.341**
Error	106	17980	170		
TOTAL	109	18555			

Further variance analysis was performed on the separate sections of both unit tests to determine if differences existed between learning style and achievement on these sections. It should be stressed that section A questions required the use of the manipulative materials and section B questions were solved by abstract methods without the use of concrete materials. It was expected that those students with a learning style that favored a "hands on" approach would show significant differences in achievement in section B portions of the tests. Likewise students whose style favors the abstract approach were expected to show significant differences of achievement in section A portions of the tests. However, results as shown in Tables 12a , 12b, 13a, 13b, 14a, 14b, 15a and 15b indicate that there was no significant differences in achievement by learning style groups on either section A or section B results.

Table 12a - Means and Standard Deviations for Section A of Polynomial Unit Test for Four Major Learning Style Preferences.

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	27.48	5.59
SF	51	26.75	4.85
NF	24	27.42	4.76
NT	14	24.00	7.91

Table 12b - Analysis of Variance for Section A of Polynomial Unit Test for Four Major Learning Style Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	127.1	42.4	1.43	**0.237**
Error	106	3134.8	29.6		
TOTAL	109	3261.9			

Table 13a - Means and Standard Deviations for Section B of Polynomial Unit Test for Four Major Learning Style Preferences.

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	28.67	11.83
SF	51	28.24	10.93
NF	24	28.58	8.63
NT	14	23.14	11.92

Table 13b - Analysis of Variance for Section B of Polynomial Unit Test for Four Major Learning Style Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	343	114	0.98	**0.403**
Error	106	12333	116		
TOTAL	109	12677			

Table 14a - Means and Standard Deviations for Section A of Factoring Unit Test for Four Major Learning Style Preferences.

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	18.81	5.55
SF	51	17.61	6.58
NF	24	17.50	6.05
NT	14	16.43	6.79

Table 14b - Analysis of Variance for Section A of Factoring Unit Test for Four Major Learning Style Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	49.7	16.6	0.42	**0.742**
Error	106	4222.8	39.8		
TOTAL	109	4272.6			

Table 15a - Means and Standard Deviations for Section B of Factoring Unit Test for Four Major Learning Style Preferences.

TYPE	NUMBER	MEAN	STANDARD DEVIATION
ST	21	22.10	6.64
SF	51	19.47	7.56
NF	24	17.13	8.17
NT	14	17.64	9.04

Table 15b - Analysis of Variance for Section B of Factoring Unit Test for Four Major Learning Style Preferences.

SOURCE	DF	SS	MS	F	p
Type pref	3	317.0	105.7	1.77	**0.158**
Error	106	6338.4	59.8		
TOTAL	109	6655.3			

The overall mean results of the unit tests are very encouraging for students of all learning style preferences. In fact a high success rate was achieved by students in both sections of both unit tests as shown by the average percent scores

.

POLYNOMIAL TEST	AVERAGE (%) (132 students)
Section A (out of 32)	84%
Section B (out of 40)	69%
Entire test (out of 72)	76%
FACTORING TEST	
Section A (out of 24)	76%
Section B (out of 28)	71%
Entire test (out of 52)	73%

It appears from the results that different learning styles were accommodated by using this manipulative based instructional strategy. Students were successful at learning using the manipulative approach irrespective of learning style preferences and achievement levels were high.

Third Research Question

Does ability to do mathematics using the concrete materials "Algebra Tiles" help to predict achievement after self-report mathematical ability is taken into account?

How can teachers help students make the transition from the use of manipulative materials to abstract mathematical symbols? Heddens (1986) divides the stage between the concrete and abstract level into two levels -- semi-concrete and semi-abstract. The semi-concrete level is a representation of a real situation; pictures of the real items are used rather than the items themselves. The semi-abstract level involves a symbolic representation of concrete items by symbols or pictures that do not look like the items they represent.

As stated in the methodology chapter both the polynomials and factoring unit tests were set up in two parts. Part A required the use of the manipulatives themselves and diagrammatic representations of the manipulative (semi-concrete and semiabstract) to answer the questions. Part B involved abstract methods to answer the questions. Section B was set up in such a way that students could not fall back on the concrete materials to solve the problems. A regression test using Minitab was performed on both polynomials and factoring section B results. A structured model was used with fixed entry of the results from the self-report on mathematical ability, followed by achievement based on the use of the algebra tiles as the second predictor. The outcomes of this regression test were used to assess whether students could apply the knowledge they had acquired from concrete experiences in abstract situations.

Table 16 - Regression-Polynomial and Factoring Tests Section B with Self-report Mathematical Ability and Polynomial and Factoring Tests Section A as predictors.

	R^2_{full}	R^2_{red}	R^2_{ch}	p
Polynomials	41.16	31.78	9.38	< 0.05
Factoring	57.07	41.53	15.54	< 0.05

The self-report on mathematical ability variable was entered first and explained **31.78%** of the variance in performance on the polynomial unit test section B, and **41.53%** of the variance in performance on the factoring unit test section B as shown in Table16. In the second step, the results from the polynomials unit test section A, and factoring unit test section A were entered into the equations. This model explained **41.16%** of the variance in polynomials section B, and **57.07%** of the variance in factoring section B. The change in R^2 of 9.4% and 15.5% respectively was statistically significant at the 0.05 level.

Results in both cases indicate that there is a large influence of concrete on abstract even once general mathematical ability has been taken into account. A significant relationship was found between the section B abstract scores and the section A concrete scores. Ability to do mathematics using the concrete materials "Algebra Tiles" does in fact help to predict achievement in abstract situations.

Summary of Results

1) Analysis of variance tests showed that students with different learning styles did not differ significantly in their attitudes toward mathematics, group activities or the use of algebra tiles.

2) Results from mean tests indicate that students generally enjoyed using the manipulative and this enjoyment was consistent among all learning style groups

3) Students with different learning styles did not differ in achievement using this manipulative based instructional strategy

4) Results indicate that students felt that their level of understanding was enhanced by use of concrete materials. This appeared to be true for all learning style groups.

5) Students appeared to be able to apply the knowledge they had acquired from concrete experiences in abstract situations. Stepwise regression indicates that results in abstract procedures are predictable by outcomes using concrete materials.

CHAPTER V
DISCUSSION

Teaching mathematics using manipulative materials has a long history. Instruction using concrete materials was included in the mathematics curricula as far back as the 1930s and before. Even as far back as the nineteenth century Pestalozzi advocated manipulative use. Parker and Dalida (1993) claim that the goals and priorities for schools have changed from "increased attendance" at the turn of the century, to "equality of educational opportunity" in the 1950s and 1960s, to "academic achievement for all students" in the 1990s. Furthermore, "as one aspect of this emphasis on outcomes and success for all students in the 1990s, some people are rethinking the traditional algebra course which many consider to be the gatekeeper to success beyond school" (p. 1).

Many studies have been conducted to establish effectiveness of a manipulative activity approach to teaching mathematics. Kieren (1971) cited widespread support for the manipulative activity approach to mathematics instruction. Others however disagree and claim that research findings are not always encouraging to proponents of manipulative materials strategies. Friedman (1978) claims that eighteen doctoral dissertations between 1970 and 1978 have compared the achievements of groups given a manipulative treatment with the achievement of groups given a non-manipulative treatment. In ten of these studies, there were no significant differences between the groups. This confirms the findings of Fennema (1972b), who cited fifteen studies conducted prior to 1970 that were concerned with elementary school mathematics instruction. Only three of the fifteen reported significant differences in favor of a manipulative approach.

Sowell (1989) claims that instruction using manipulative materials can be effective under certain conditions, and that this effectiveness is shown most clearly when the materials are used over an extended period of time. Parker and Dalida (1992) conclude, "While the idea of learning algebra via a concrete approach is not new, the idea that students will use concrete materials throughout their algebra experience is new." (p. 1)

Throughout the instruction of the program in this study, both teachers participating agreed with Sowell, and Parker and Dalida, in feeling that the concrete materials would have had more impact if students had been introduced to them at an earlier stage. As mentioned previously a journal was kept by the researcher during the six week course of instruction. The following extracts related to starting the use of concrete materials at an earlier stage, and providing more time for manipulative instruction were taken from the journal

> *Perhaps a little more time on this, students confused. More time discussing sides x and 1 and stressing by doing that no integral number of 1's will fit into the x side. In order to get the most benefit from using the manipulative extra time needs to be allocated to these units.*
>
> *This transition lesson went well with addition and subtraction easy to understand. Some students asked for tiles - "Can we have tiles?" ; "We want to use tiles." Others drew tiles on their paper to help solve the problems. Should start using tiles with all kids at a much earlier age or whenever they work on integers.*
>
> *Tile problems assigned 1) $(x + 1)\bullet(x + 2)$; 2) $(x - 2)\bullet(x + 1)$; 3) $(x - 3)\bullet(x + 2)$; 4) $(x - 2)\bullet(x - 3)$. Students worked enthusiastically on these problems, with a lot of discussion and excellent ideas. Much more satisfying day for me as students actually seemed to understand something from using the tiles, whereas in previous classes with addition and subtraction some students were wondering why I was using tiles for what seemed to them basic obvious concepts. In future I think it is necessary to get the most out of this manipulative we will have to start to use it in a much earlier grade before any "rules" are drilled into the students.*

The algebra tiles manipulative could be introduced in Grade 8 or earlier when work begins on integers and equations. This prior use of the materials would speed up the understanding of some of the concepts introduced in the Grade 9 course. The use of manipulatives when working with integers has been found to be successful, yet use of these materials in secondary classrooms is very limited largely due to time constraints. It would be interesting for further study to introduce a program of instruction which made use of concrete materials throughout the entire algebra course as suggested by Parker and Dalida.

Results indicate that the manipulative based program of instruction was suited to students with a variety of different learning styles. Most students demonstrated a positive attitude to the manipulative materials and it can be claimed with reasonable certainty that students felt algebra tiles were an acceptable and effective educational tool when used to enhance student understanding of basic algebraic concepts. It would be beneficial to this discussion to take a more critical look at the results obtained and problems experienced during the course of this study.

The majority of students felt that learning with algebra tiles as tools was enjoyable. The best indication of this was the positive responses received for Section A on Algebra Tiles Attitudinal Questionnaire (2):- Attitudes and feelings toward algebra tiles. Only 13 (9.8%) of the 132 students indicated that they did not enjoy using the tiles, nor did they feel that the tiles were beneficial to their learning outcomes. Some students indicated that they were indifferent in their attitudes to the use of the concrete materials. The majority of students however, appear to have enjoyed the units and felt the manipulatives helped enhance and broaden their understanding. With positive student reactions, confidence levels increase and meaningful learning takes place. VanEngen (1953), when discussing a concrete materials approach to learning concludes:

> Reactions to the world of concrete objects are the foundation stones from which structure of abstract ideas arise. these reactions are refined, reorganized, and integrated so that they become even more useful and even more powerful than the original responses. (p. 86)

Of interest and concern is the negative attitude expressed by the 13 students who obviously did not feel that the program of instruction was suited to their needs. A few comments such as the one included in the following extracted from the researcher's journal where made by some students during the course of instruction.

> *The reason for discovery learning seems to have eluded some students with comments like - "Why did we have to do the tile stuff if it is so easy to do in a formal*

> *way?" and We hope we never use tiles again as they were very hard to know what to do.*

The data collected also indicated that some of the students who felt very positive toward mathematics and were very successful at mathematics did not enjoy the manipulative approach used in the algebra tiles units. No analysis was done as to the reasons why some students reacted negatively to the use of the tiles, however a comment from the researcher's journal is interesting and may partially explain some of the negative responses:

> *A few of the ESL students are reluctant to make use of the tiles. Some of these students are very new to Canada and claim to have done polynomials and factoring before. These students (8) in my three classes wanted to give the answer symbolically and not use the tiles at all, especially in the factoring unit.*

Richmond School District has an ESL population of approximately 40%. The majority of these students are new Canadians with varying degrees of English proficiency. When one of the students who recently arrived in Canada was questioned about how mathematics was taught in her previous school she responded:

> *"We would sit in our class and wait for the teacher. Every day we would start with a test on our homework. The teacher would teach us our math and do some examples on the chalk board. Every example we would write down in our books and ask the teacher questions if we did not understand. Every day we would get lots of homework so we can exercise our math. If we do not write things down correctly than we must do it again." (Research journal)*

Although this is only one student's point of view, other students have experienced similar situations. It appears that the emphasis in some countries with regard to mathematics education, is weighted toward the delivery of content in the more traditional, lecture / algorithm / homework format. It is not surprising then, that some students are reluctant to participate in, or feel uncomfortable with, a "hands on" approach, something which is foreign to their previous experience of learning mathematics.

Further study could be conducted to establish why some students dislike the concrete approach to learning mathematics. In teaching a program such as this one, are teachers doing an injustice to a small group of students who dislike a concrete approach and if so, how can the needs of these students be accommodated within a manipulative based program of instruction? Certainly in this study every effort was made to get a balance between the concrete and the abstract, but responses show that the needs of some students were definitely not met. With increasing numbers of ESL students enrolling in a lot of school districts, more consideration needs to be given to developing programs and teaching strategies that accommodate the needs of these students, some of whose previous educational experiences may have been vastly different to Canadian educational practices.

Results show that there was no significant difference in attitude to algebra tiles by pupils from the four type preference categories, nor was there any significant difference in the achievements of students in each of the four categories. It must be stressed, however, that 22 out of the 132 students who participated indicated an undetermined learning preference for either their dominant or auxiliary function. Even with the encouraging attitudinal and achievement results indicated, the sample (110) used, was too small to make any categorical statements about different styles being accommodated by this instructional strategy.

Another question of interest arises. Even though it appears from results in this small sample that different learning styles were accommodated, no comparative research was done on whether or not any of the four type groups would have been more successful with a different method of instruction. For example, were intuitive thinking (NT) type students well served by this manipulative approach? According to Myers (1980), the NT type student focuses on the global picture and enjoys self-paced learning. The averaged mean scores for the polynomial and factoring tests were not significantly different for students in the NT group, however, the mean scores for this group were the lowest of all four learning style groups. Myers maintains the NT type of student enjoys to learn and is successful at learning new skills and therefore they would not be expected to achieve the lowest mean score of the four groups. The teachers and some of the students felt that for some learners the materials based approach may not have been as effective as a more symbolic traditional approach would have been. These students were in the minority, and even though they were successful they perhaps could have done even better had they been instructed using a different strategy. Mamchur (1994) suggests that it is easier for the intuitive learner to understand the sensing approach than it is for the sensing learner to understand the intuitive approach. Therefore it might be possible that even though intuitive learners would not prefer the hands on manipulative approach, they could learn that way, whereas the sensing learner might be lost if instruction involved only the abstract approach. To shed light on this hypothesis and to determine whether or not a manipulative approach does in fact bring out the best in all groups of students, a comparison study could be conducted using two groups with both sensing and intuitive learners in both. One group would receive instruction through a manipulative approach and the other group would be instructed using only an abstract approach.

The most gratifying moment for a teacher is that moment when student understanding occurs; when suddenly the student "sees the light", and from that moment on becomes excited and stimulated by the activities presented to them.

> *Good feedback today with students proud of themselves and sharing what they had learned with other members of the class. In general a satisfying day or me and the students.*
>
> *Students found zero rule easy to understand using tiles. In fact most developed the rule for themselves. Addition and subtraction easy to follow and students made comments such as "This is cool I'm going to pass this . It was interesting to see some students drawing tiles to help them with transition. (Researcher's journal)*

The time and effort that went into the designing of the program of instruction became worthwhile when seventy percent of the students involved responded very favorably to the following two questions:

1) The algebra tiles helped me to understand the polynomials and factoring units a lot better than I would have done without the tiles.(Algebra Tiles Attitudinal Questionnaire (2), Appendix E, question 8)

2) Hands on materials such as algebra tiles make understanding math a lot easier. (Algebra Tiles Attitudinal Questionnaire (2), Appendix E, question 9)

The favorable responses agree with the comments made by Ross and Kurtz (1993), that effective use of manipulatives contributes to conceptualizing and understanding. It is extremely difficult for the researcher to truly assess whether or not understanding was enhanced by the use of the manipulative, however, the majority of students themselves seem certain that their understanding was increased. It would have been interesting to repeat the unit tests a few months after the completion of instruction to determine how much retention took place, after all if students fully understood the concepts one would assume there to be greater retention of knowledge.

Moody et al. (1971), in a comparison study between manipulative and symbolic instruction found there were "no significant differences between the manipulative and symbolic groups in tests of transfer and retention" (p. 210). Heddens (1986), however, claims that many students have difficulty understanding mathematics because they are unable to make connections between the physical world and the abstract world and states:

> The gap between concrete and abstract functioning should be considered as a continuum. Helping children bridge this gap is crucial because many children cannot cross it without the teacher's assistance. Learners must internalize new knowledge at the concrete level and systematically progress along the continuum to arrive at the abstract representation of that knowledge. Two processes of interaction between reality and the mind are accommodation and assimilation. Piaget refers to these processes as equilibration. New knowledge can be assimilated very rapidly by children. Other children who must accommodate, or reorganize, their mental structures to incorporate new knowledge, need considerably more time. A provision must be made for students to bridge the gap at different rates. (p. 14)

This program of instruction was designed so that the learner could consciously build on many concrete experiences and emerge with mathematical concepts at the abstract, symbolic level. Even the British Columbia curriculum guide makes mention of the fact that, "it is through concrete manipulations that concepts and relations are understood at an intuitive level and are later given conventional mathematical notations" (p.viii). The ever present problem of time constraints and curriculum pressures was very noticeable during the transition phase, where students were encouraged and expected to rely less on the concrete materials and start to apply a more intuitive abstract approach. There was a tendency on the part of the teachers to try and hurry the transition phase along, when in fact more time should have been allocated to allow students to become comfortable with an abstract approach. This is probably one of the most important aspects of the entire course of instruction, yet both teachers worried about some students taking too much time in their progress away from reliance on the concrete materials. Results obtained from the unit test indicate that the "gap" between the concrete and the abstract was bridged, but more research and time needs to be spent in trying to determine just how successful this transition was with all students regardless of mathematical ability.

Suggestions for Further Research

1) The results of this study support some of the existing research which supports the inclusion of manipulative materials in mathematics programs. The study was however, done with a small number of students and needs to be replicated with a much larger sample from more than one school. Future studies should perhaps also include the use of concrete materials throughout the entire algebra course and not just certain aspects of the course.
2) Does the concrete approach to learning mathematics appeal to students of all learning types? Future study could examine whether teaching a program

such as this one puts some students who dislike a concrete approach at a disadvantage. If so, how can the needs of these students be addressed?

3) Even though it appears from results achieved in this project that different learning styles are accommodated, no comparative research was done on whether or not some learning style groups would have achieved more from a non-manipulative approach. A comparative study with a larger sample could be of value to determine whether or not a manipulative approach does in fact bring out the best in all groups of students.

4) Can intuitive learners learn from a sensory method of instruction better than sensory learners can learn from an intuitive method of instruction? An answer to this question could be useful in determining appropriate teaching strategies and methods of instruction in mathematics classrooms.

5) More extensive studies need to be carried out allowing more time for treatment, as well as follow-up studies to determine whether students retain the skills they have learned from a manipulative based program of instruction.

Conclusion

Based on the results of this investigation the manipulative algebra tiles has potential as a tool to aid instruction in algebra courses. Indications are that students of all learning style preferences are well served by this method of instruction using concrete materials. A word of caution however, when dealing with children there is some doubt as to the reliability of any instrument to determine individual learning style, as self report instruments may not yield objective data. Judging by the high number of students (30%) who reported to have undetermined type preferences in one or more of the eight preference area, and bearing in mind the developmental nature of personality in children, it would be inaccurate to assume that the same type preferences would be reported by a student at a different time.

> Personality is a germ in the child that can develop only by slow stages in and through life. No personality is manifest without definitiveness, fullness, and maturity. These three characteristics do not, and should not, fit the child, for they would rob it of its childhood. (Jung 1940, p. 285)

Educational practitioners should give consideration to the assertion that learners prefer certain learning styles and that learning events can be designed to accommodate different preferences. Instructional planning must accommodate the learning characteristics of individual students to be effective with these students.

The teachers in this study would claim that the program of instruction was effective, because students enjoyed it and expressed a positive attitude to the algebra tiles units, results for unit tests were good, and students generally felt that they had a better understanding of the concepts. There does, however, appear to be a need for some changes and adjustments to the program to make it a more effective learning tool. More detailed consideration should be given to the groupings used for activities. During the instruction of the units some students commented that they wanted to do more individual assignments.

> *Some students protested about having to work in pairs or fours and wanted to work on their own. (Researcher's journal)*

As these units were designed to incorporate considerable group activities some analysis was done on the attitude to group work by students with different learning preferences. Results show that there is no measurable difference between learning preference and attitude to group work activities, both in mathematics in general and in algebra tiles in particular. The mean responses however indicate that students are only moderately happy with certain groupings. When preparing for future instruction of these units some choices of activities, both co-operative groups and individual should be included, to facilitate all student preferences in this area.

The most important criticism of the project from a teacher's viewpoint is time constraints. The feelings of both teachers involved was that the students were pushed through the program leaving the concrete materials for the abstract solutions too quickly, with little time given to consolidate concepts. It takes time to think! The six weeks allocated to the teaching of these two algebra units did not provide students with enough time to consolidate in their own minds the concepts and skills learned using the algebra tiles. If "learning mathematics is doing mathematics" is considered to be an essential strategy for teaching mathematics, then curriculum developers and classroom teachers should re-examine their priorities and the content of courses. Present curriculum and time constraints in secondary classrooms are definitely not conducive to "doing mathematics".

In order for concrete materials, such as algebra tiles, to become more prevalent in secondary classrooms, educators, educational advisors and administrators need to be committed to the advantages of manipulatives. An economic commitment is also required, teachers need to be trained in the effective use of manipulative materials, text books and curriculum need to be restructured away from more traditional approaches. Projects such as the NCTM *Standards* go a long way to helping mathematics educators embrace new ideas, but without considerable funding and teacher support it is unlikely that many new initiatives will ever reach full potential.

The positive attitude, and high success rate, of the large majority of the students, regardless of learning style, to this instructional process was very gratifying. If this study has even partially achieved what it set out to do, that is to develop a course of instruction using manipulative materials which would be beneficial to the learning of all students, then it was successful and worth doing.

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APPENDIX A

BRITISH COLUMBIA CURRICULUM GUIDE EXTRACTS

Prerequisite Skills	Intended Learning Outcomes	Limiting Examples	References
The student should be able to			
• collect like terms having integral coefficients. (8.68)	• 9.38 add polynomials.	Simplify: $(3x^2 - 2xy + 6y^2) + (xy - 7y^2)$	
	• 9.39 subtract polynomials.	Simplify: $(3a^2 - 2a + 4) - (a^2 - 5)$	
	• 9.40 multiply: a) monomial x monomial b) monomial x polynomial c) binomial x binomial.	Simplify the following: a) $-3x^3 \times 6xy^2$ b) $\frac{3}{2}x^5(2x^2 - 3x + 2)$ c) $(3xy + y)(2x^2 - 5y)$	

Prerequisite Skills	Intended Learning Outcomes	Limiting Examples	References
The student should be able to • determine the greatest common factor of two or more whole numbers. (7.066, 8.08)	• 9.41 find the greatest common factor of monomial terms.	Find the greatest common factor of: $6x^3y^2$, $-15x^4y^2$	
	• 9.42 factor expressions containing a common monomial factor.	Factor completely: $6x^3y^2 + 10x^2y$	
	• 9.43 factor the difference of two squares.	Factor: $49x^2 - 9y^2$	
	• 9.44 factor trinomials with leading coefficient of 1.	Factor: $x^2 - 5x - 24$	
	• 9.45 factor expressions containing combinations of the above.	Factor completely: $3x^3 + 9x^2 - 30x$	

APPENDIX B

MATHEMATICS ATTITUDINAL QUESTIONNAIRE (1)

MATHEMATICS ATTITUDINAL QUESTIONNAIRE (1)

Answer the following questions as accurately as possible. Only circle the response which is closest to the way you feel. Respond to all questions.

NAME:____________________

(A)

1. If I don't get a math question right away, I usually give up.

   A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

2. I like to study math in school.

   A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

3. Math is dull.

   A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

4. I enjoy math.

   A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

5. Math is fun.

   A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

6. I enjoy the challenge of a math problem.

   A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

7. I get nervous whenever I have to write a math test.

   A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

8. The math taught in schools is useless.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

9. I enjoy going to math class.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

10. Math is a very boring subject.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

11. I plan to study math after high school.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

12. When I get a question wrong, I go over it until I find the mistake.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

13. I enjoy math when I can do the questions.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

14. Math is one of my favourite subjects at school

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

15. Which of the following best describes your feelings toward mathematics.

A. Fascinating and easy
B. Interesting
C. Indifferent (no feelings at all)
D. Boring
E. Frustrating and difficult

16. Do you believe math skills will help you in later life?

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

(B)

17. I prefer working on math assignments on my own rather than in groups during math class.

A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

18. I understand math better when we work in groups.

A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

19. I enjoy doing math questions on the board in front of the whole class.

A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

20. I enjoy working in groups to study and work on mathematics.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

21. I enjoy helping others with mathematics.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

22. I find it easier to learn math by -

A. Listening to the teacher explain
B. Watching the examples the teacher does
C. Listening to one of my friends explain
D. Watching the examples my friends are doing
E. By working on a problem on my own

23. When given a math problem, I like to start immediately rather than having to discuss it with others in my group.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

24. When given a math problem, I like to wait until others have started to make sure that I am doing it right.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

(C)

25. How would you rate your mathematical ability?

A. excellent B. good C. average D. weak E. very weak

26. How do your math marks or comments on your report card compare to those in other subjects?

A. Usually better than most other subjects
B. Usually about the same as most other subjects
C. Usually worse than most other subjects

27. What letter grade do you usually get in math on your report card?

A. A B. B C. C+, C or C–
D. D E. E

APPENDIX C

POLYNOMIAL UNIT TEST

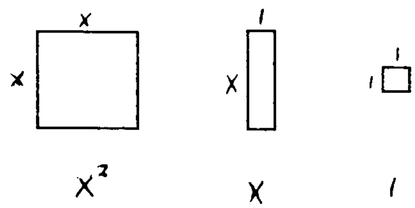
POLYNOMIALS TEST

NAME: ____________________

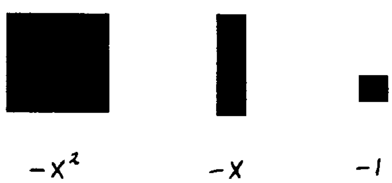
SECTION A

You may use your Algebra Tiles to help you answer the questions in this section. In most cases examples of how to answer the questions are given.

Use the following values for the tiles.

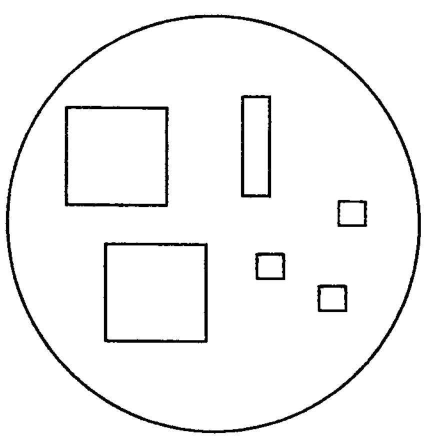


Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)



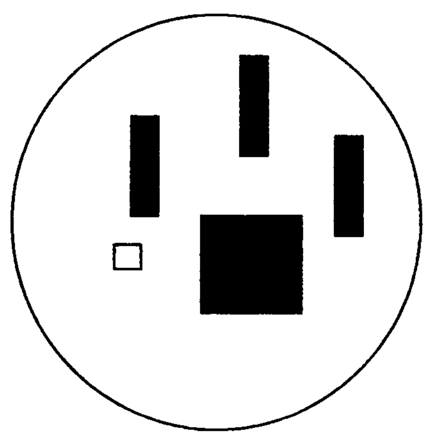
1) What Polynomials are represented in each of the following circles? Write your answers on the line underneath the question.

a)



____

b)



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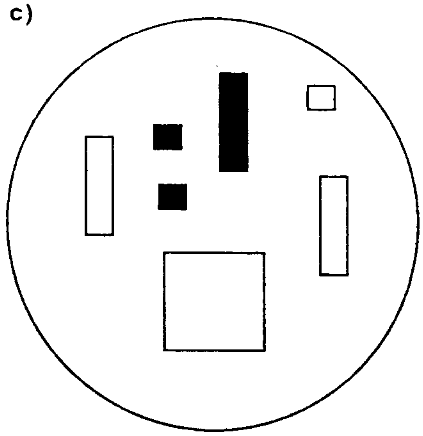
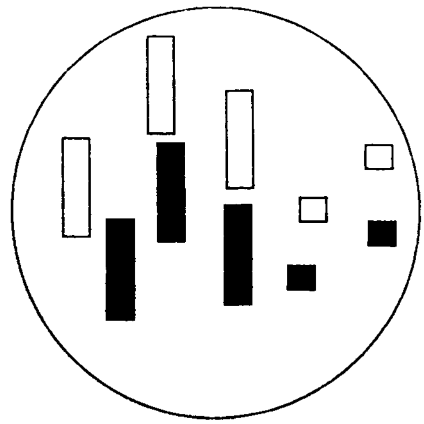
c)

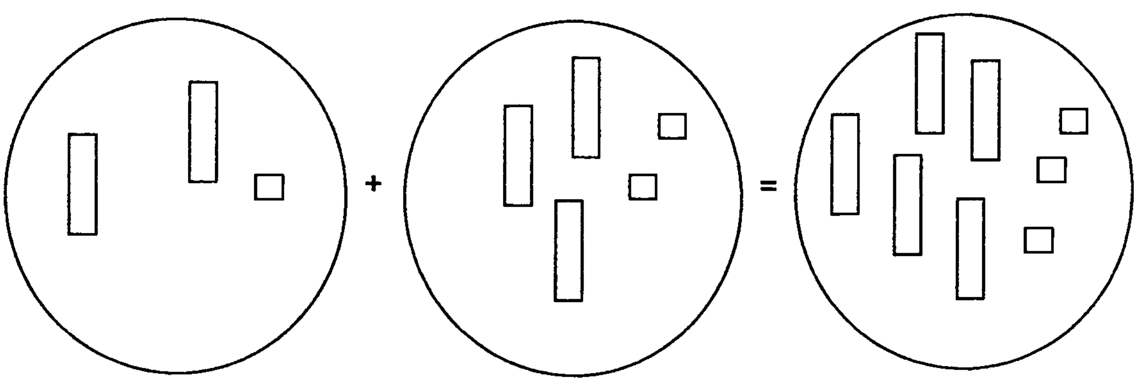


d)



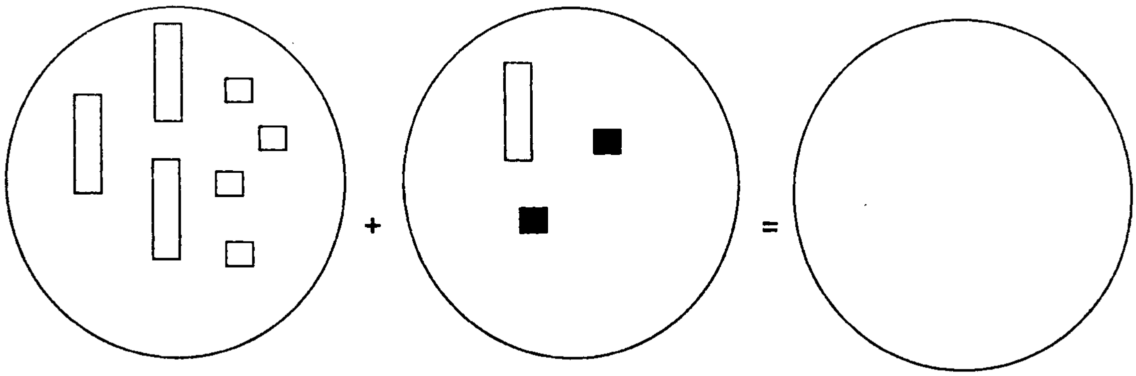
Look at the following example of addition. Complete questions 2, 3, 4, and 5 by adding the tiles together, filling in the third circle and writing the whole operation down symbolically in the space provided underneath the question.

Example



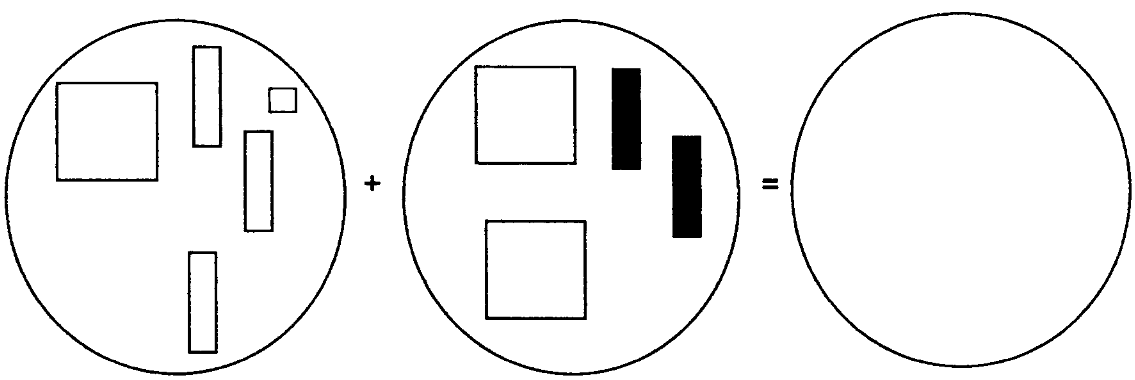
(2x + 1) + **(3x + 2)** = **(5x + 3)**

2)



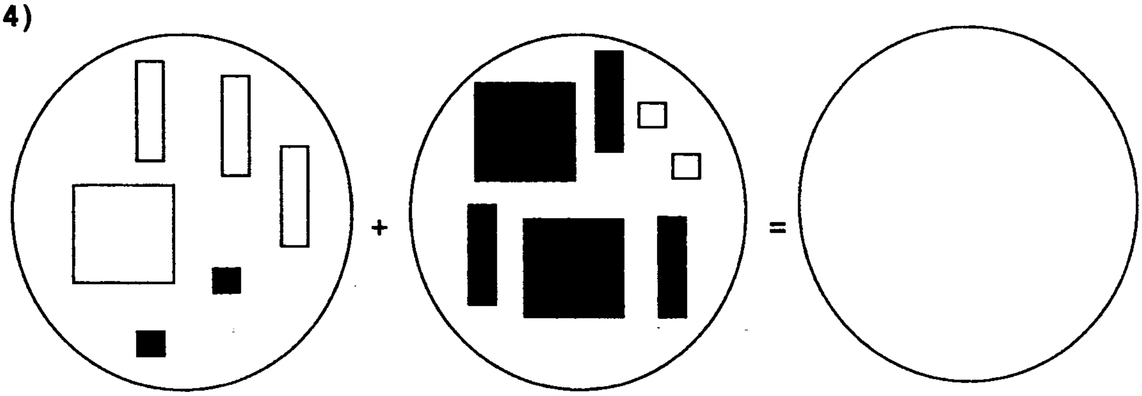
________ + ________ = ________

3)



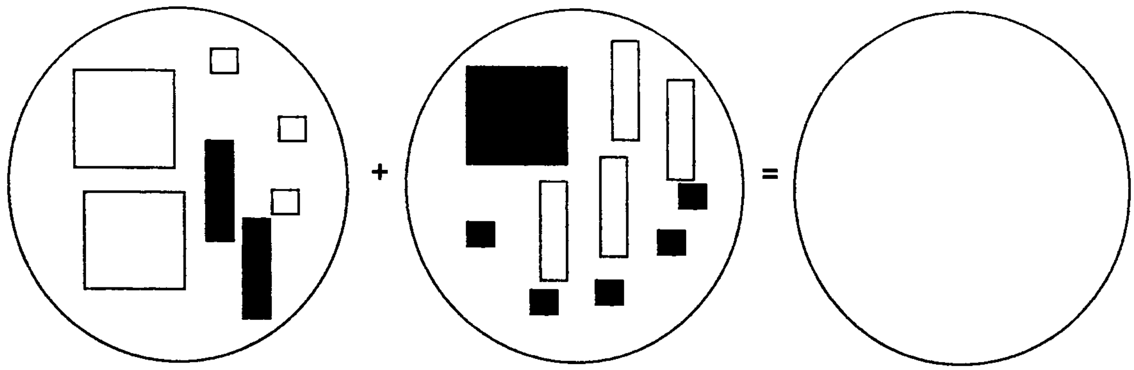
________ + ________ = ________

4)



________ + ________ = ________

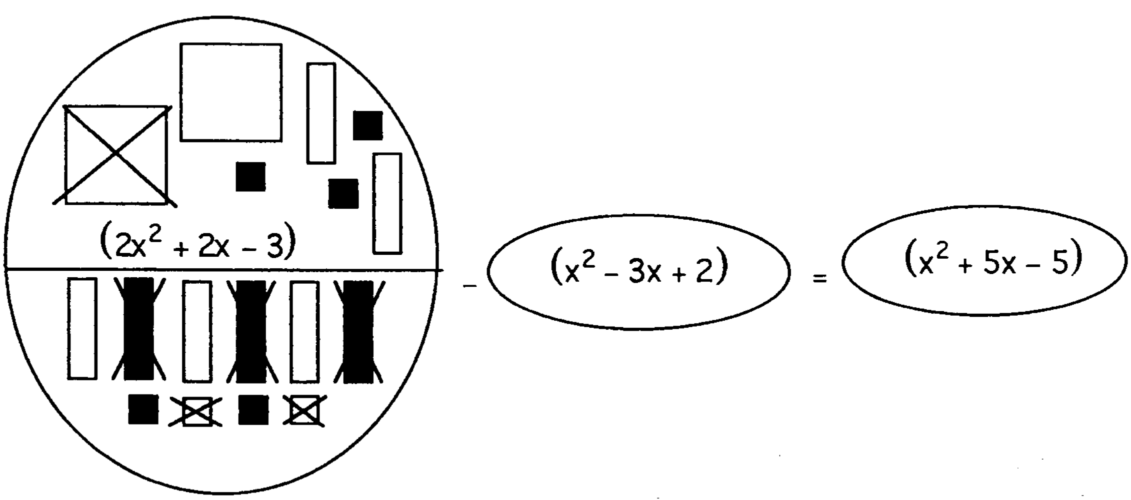
5)



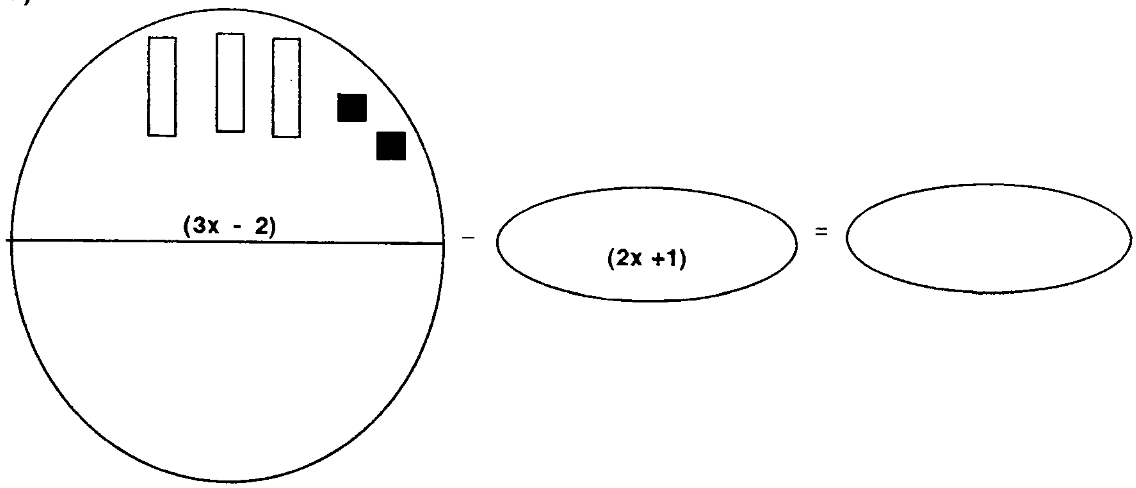
________ + ________ = ________

Look at the following example of subtraction. Complete questions 6,7,8, and9. Remember you may need to add "zero's" in order to complete the subtraction. Draw the zero's you are adding in the bottom half of the first circle as shown in the example. Also remember to cross out the tiles which are being subtracted as shown in the example.

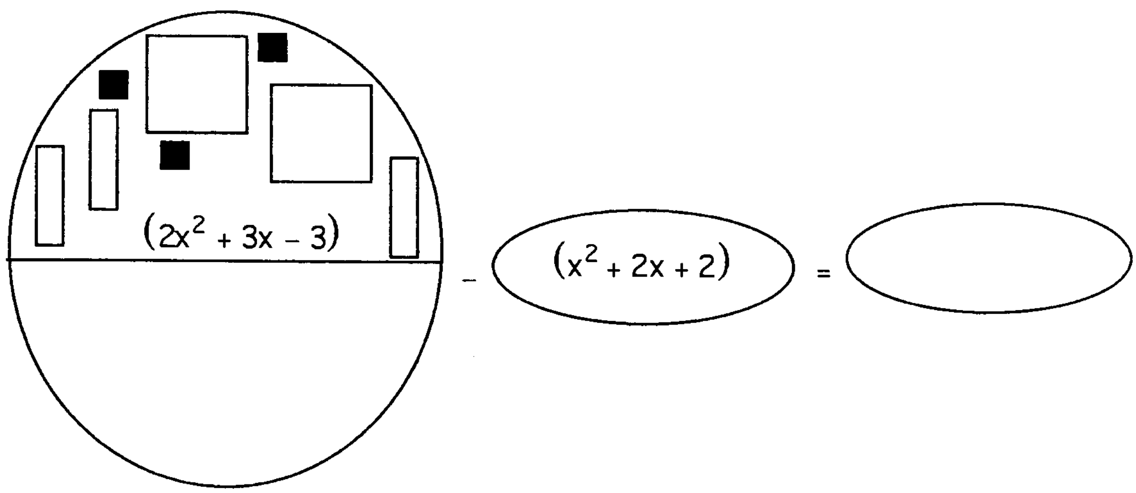
Example



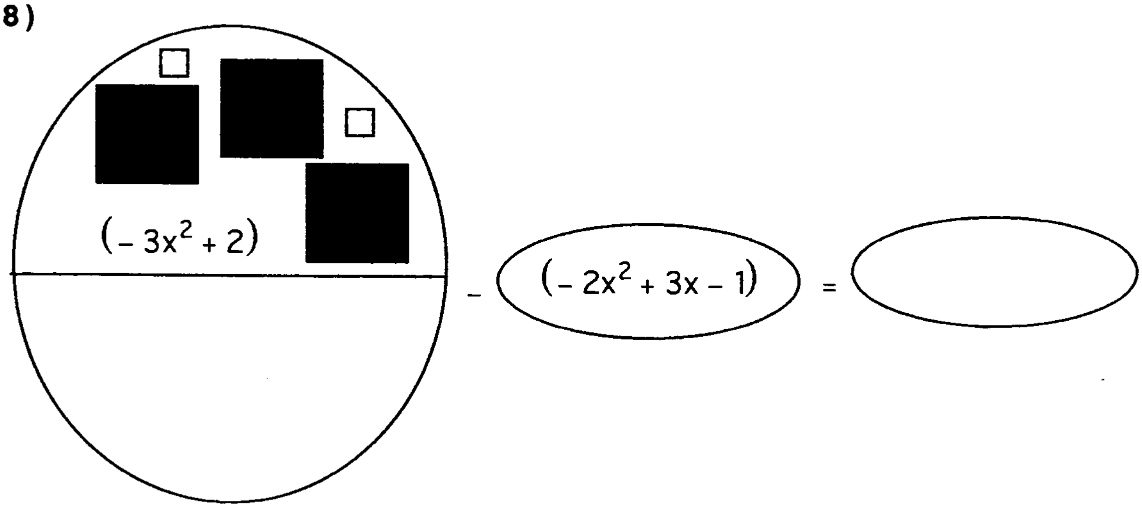
6)



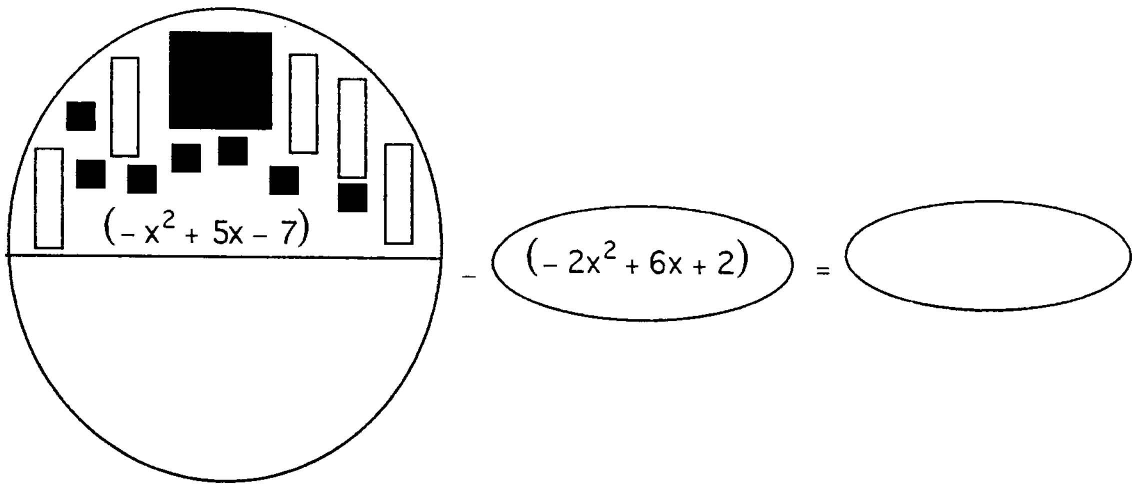
7)



8)



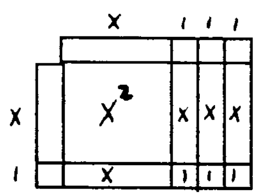
9)



Look at the following example of multiplication. Complete questions 10,11,12,13,14 and15. Remember overlapping negatives are called "double coverup" and positive units will result.

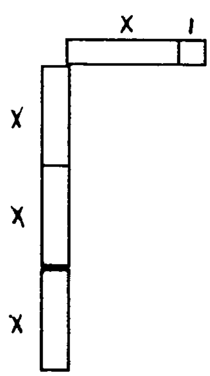
Example

$(x + 1)(x + 3)$



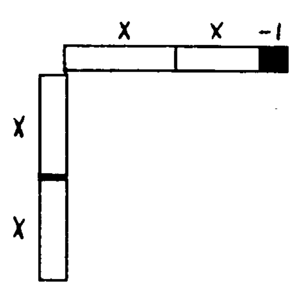
= $x^2 + 4x + 3$

10) $3x(x + 1)$



= ______________

11) $2x(2x - 1)$



= ______________

The next 4 questions are the same as 10 and 11, except the frame is not provided for you. You will need to draw your own frame.

12) $(x + 3)(2x + 1)$

= ____________________

13) $(2x + 2)(x + 1)$

= ____________________

14) $(x - 1)(x + 3)$

= ____________________

15) $(2x + 1)(x - 2)$

= ____________________

SECTION B

Complete the following questions without using Algebra Tiles. Show all your working in the space provided.

1) $(5x - 2) + (3x - 4)$

= ____________________

2) $(5x - 2) - (3x - 4)$

= ____________________

3) $(x^2 - 3x + 1) + (x^2 - 7)$

= ____________________

4) $(x^2 + 3x + 3) - (2x^2 - x)$

= ____________________

5) $(8x - 2) - (7 - 2x)$

= ____________________

6) $(3x^2 - 5x + 1) - (x^2 - 3x - 2)$

= ____________________

7) $(2x^2+x)+(-3x^2-2x)$

= ____________________

8) $(2x^2+x)-(-3x^2-2x)$

= ____________________

9) $(4x^2-7x+3)-(x^2-5x+9)-(8x^2+6x-11)$

= ____________________

10) $(2ab-2ac-2bc)+(2ca+2cb-2ba+3)$

= ____________________

11) $(3x^2-2y^2)+(y^2-2x^2)-(4x^2+2)$

= ____________________

12) $5(2x + 1)$

= ______________________

13) $(x + 5)(x + 1)$

= ______________________

14) $(x + 1)(2x - 4)$

= ______________________

15) $(4x - 4)(3x - 2)$

= ______________________

16) $-x(-3 + 2x)$

= ______________________

17) $(6s - 2t)^2$

= ______________________

18) $(2x + 4)(3x - 2) + (5x - 2)(3x - 4)$

= ____________________

19) $5(2p - 7)(3p - 4)$

= ____________________

20) $(3x - 1)(2x^2 + 3x - 4)$

= ____________________

APPENDIX D

FACTORING UNIT TEST

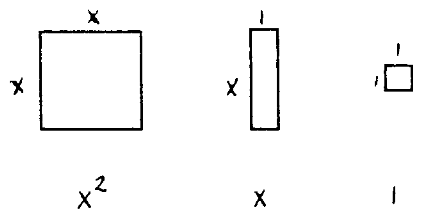
FACTORING TEST

NAME: ____________________

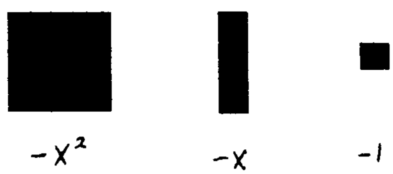
SECTION A

You may use your Algebra Tiles to help you answer the questions in this section. In most cases examples of how to answer the questions are given.

Use the following values for the tiles.

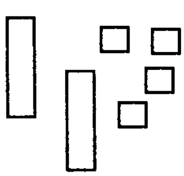


Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)

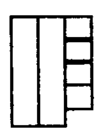


In the following question arrange the given tiles into a rectangle to find a product of two factor which make up the given area. Draw in the tiles that make the rectangle and write the area as a product of the length and width as shown in the example.

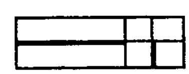
Example 1



<u>2x + 4</u>

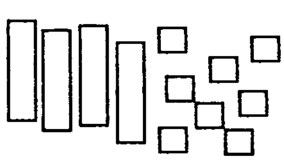


<u>Not a rectangle not a solution</u>



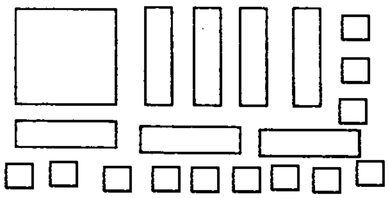
<u>Solution 2 X (x + 2)</u>
<u>2(x + 2)</u>

1)



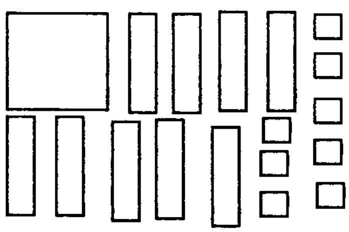
<u>Area = 4x + 8</u> ____________________

2)



Area = $x^2+7x+12$ ______________

3)



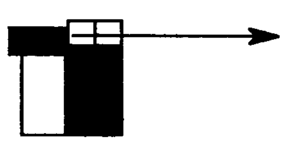
Area = x^2+9x+8 ______________

Sometimes it is impossible to complete a rectangle without adding extra tiles. When this occurs either the trinomial cannot be factored at all, or else tiles are added using the "zero" rule. For the next few questions also keep in mind that sometimes when the tiles are arranged to form a rectangle a "double coverup" occures. Go through the following example then complete questions 4 to 9.

Example 2

Given the following trinomial x^2-3x+2

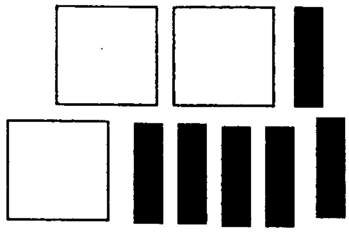
Notice when the tiles are arranged to form a rectangle a "double coverup" occures. The effect of the overlapping of the negative tiles creates the need for the positive unit tiles.



Note double coverup resulting in two positive unit tiles.

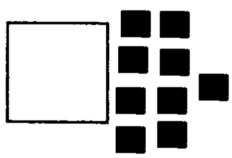
THE FACTORS OF x^2-3x+2 ARE $(x-1),(x-2)$

4)



Area = $3x^2 - 6x$

5)

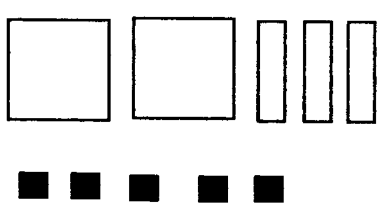


Area = $x^2 - 9$

6)

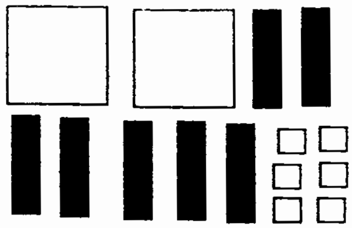
Area = $x^2 - x - 6$

7)



Area = $2x^2 + 3x - 5$

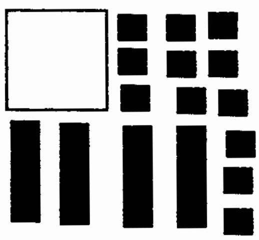
8)



Area = $2x^2 - 7x + 6$

____

9)

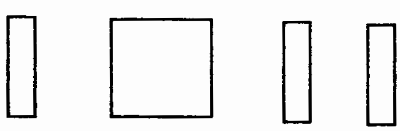


Area = $x^2 - 4x - 12$

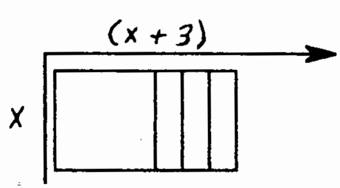
The following questions involve division of polynomials, where the area and one dimension (factor) are given and the other dimension (factor) needs to be found.

Example 3

$$(x^2 + 3x) \div x$$



One dimension has to be x because that is what you are dividing by. Rearrange the tiles so that one dimension is x.



Notice that thehorizontal dimension is (x + 3). The new dimension is therefore the answer to your division problem.

Using the above example as a guide complete questions 10, 11 and 12.

10) $(2x^2 - 4x) \div 2x$

2 x

$(2x^2 - 4x) \div 2x$ = ______________________________

11) $(x^2 - 5x + 6) \div (x - 3)$

(x − 3)

$(x^2 - 5x + 6) \div (x - 3)$ = ______________________________

12) $(x^2 + 2x - 8) \div (x + 4)$

$(x^2 + 2x - 8) \div (x + 4)$ = ______________________________

SECTION B

Factor the questions in this section without using tiles. You will be required to factor using common factors, difference of two squares and trinomials.

1) $12x^2+6x$

____

2) $4x^2-25$

____

3) x^2+4x-5

____

4) $x^2-7x-60$

____

5) $x^2-10x+9$

____

6) $-12p^2q^3 - 20p^3q^3 + 8p^2q^4$

____

7) $15x^4y^3 - 25x^3y^4 + 10x^2y^3 - 5x^3y^2 + 30x^4y^4 - 35x^2y^2$

____

8) $3a^2 - 18a + 24$

____

9) $15 - 8y + y^2$

____

10) $x^2 + 7xy - 18y^2$

____

11) $x^2 - 17x + 72$

____

12) $3a^2 - 48$

____

13) $49b^2 - 1$

____

14) $9m^2 - 16n^2$

____

APPENDIX E

ALGEBRA TILES ATTITUDINAL QUESTIONNAIRE (2)

ALGEBRA TILES ATTITUDINAL QUESTIONNAIRE

Answer the following questions as accurately as possible. Only circle the response which is closest to the way you feel. Respond to all questions.

NAME:____________________

(A)

1. I liked the algebra tiles units.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

2. I found the units using algebra tiles very dull.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

3. I enjoyed the polynomial and factoring units.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

4. Algebra tiles were fun.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

5. The algebra tiles units were very boring.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

6. The algebra tiles units are one of my favourite topics in math.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

7. Which of the following best describes your feelings toward the algebra tiles units.

A. Fascinating and easy
B. Interesting
C. Indifferent (no feelings at all)
D. Boring
E. Frustrating and difficult

8. The algebra tiles helped me to understand the polynomials and factoring units a lot better than I would have done without the tiles.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

9. Hands on materials such as algebra tiles make understanding math a lot easier.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

10. Which of the following methods do you think would best help you to learn?

A. Listening to the teacher explain what to do and then doing an exercise.
B. Going through examples in the text book and doing lots of practice questions from the exercises
C. Using manipulatives such as algebra tiles to discover how to do certain examples.

11. The polynomials and factoring units would have been difficult to understand without the help of the tiles.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

12. If you were asked by your teacher whether or not you would encourage the use of algebra tiles when learning polynomials and factoring, would you -

A. Strongly encourage B. Encourage C. Be indifferent D. Discourage E. Strongly discourage.

13. Did you find using algebra tiles very time consuming and not very helpful to your understanding of the polynomial and factoring units?

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

(B)

14. I prefered working with the algebra tiles on my own rather than in groups during math class.

A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

15. I understood the algebra tile topics better when we worked in groups.

A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

16. I enjoyed doing algebra tiles questions on the board in front of the whole class.

A. strongly agree B. agree C. undecided D. disagree E. strongly disagree

17. I enjoy working in groups to study and work on the algebra tiles units.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

18. I enjoy helping others when we were working with the tiles.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

19. I found it easier to learn how to do the algebra tiles problems by -

A. Listening to the teacher explain
B. Watching the examples the teacher does
C. Listening to one of my friends explain
D. Watching the examples my friends are doing
E. By working on a problem on my own

20. When given a problem, I liked to start immediately rather than having to discuss it with others in my group.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

21. When given a problem, I waited until others had started to make sure that was doing it right.

A. Strongly agree B. Agree C. Undecided D. Disagree E. Strongly disagree

(C)

22. How would you rate your ability to do polynomials and factoring using algebra tiles?

A. excellent B. good C. average D. weak E. very weak

23. What letter grade did you get for the polynomials and factoring units?

A. A B. B C. C+, C or C–
D. D E. E

24. How do your marks for the algebra tiles units compare to those you would expect to get in mathematics?

A. Better than I would expect to get in other math units

B. About the same as other math units.

C. Worse than I would expect to get in other math units.

APPENDIX F

ALGEBRA TILES PROJECT (1)

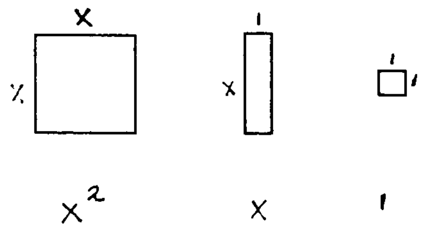
ALGEBRA TILES PROJECT (1) - ADDITION/SUBTRACTION

NAME: ________________

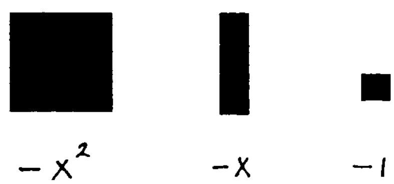
NAME: ________________

ALGEBRA TILES

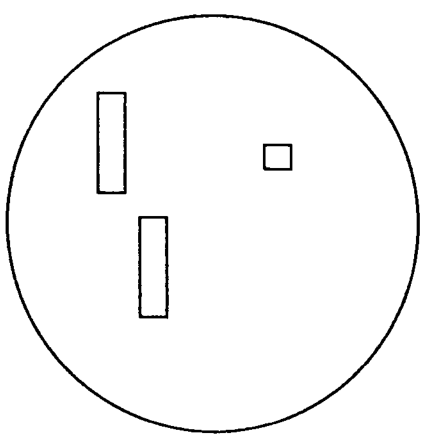
Tiles can be given any values we will use the following values for the tiles until you are told to change the values.

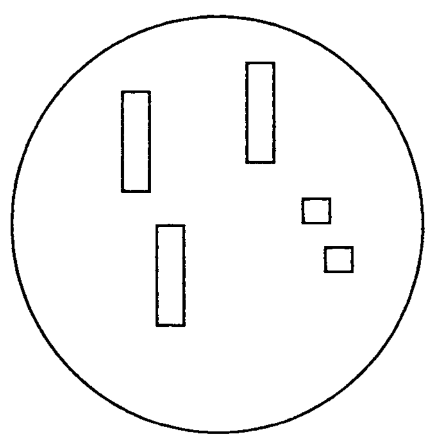


Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)

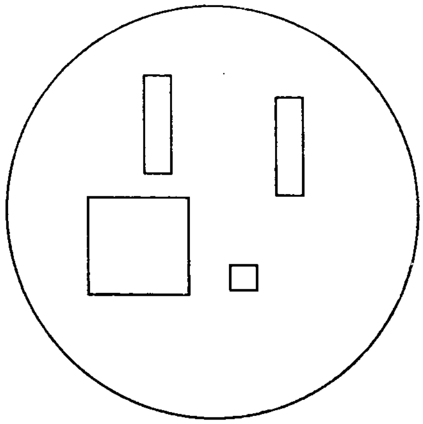


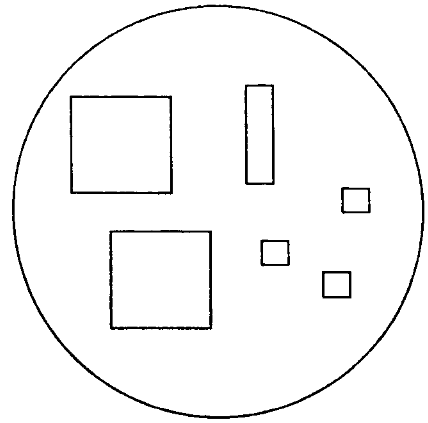
What Polynomials are represented in each of the following circles? Write your answers on the line underneath the question.

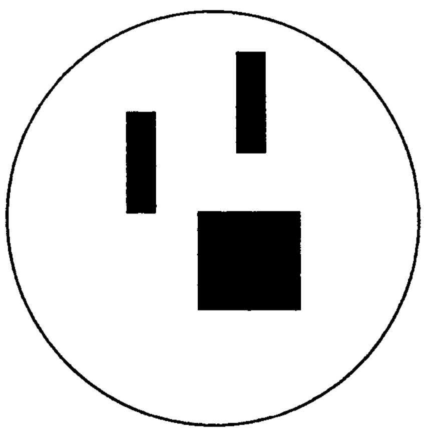


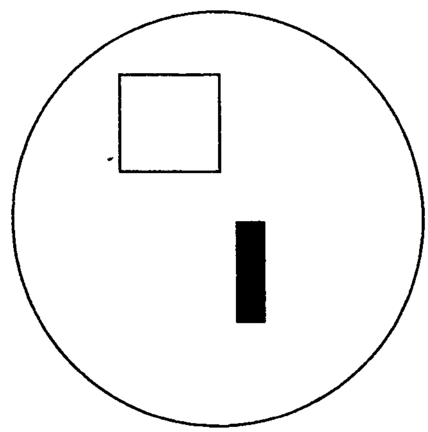


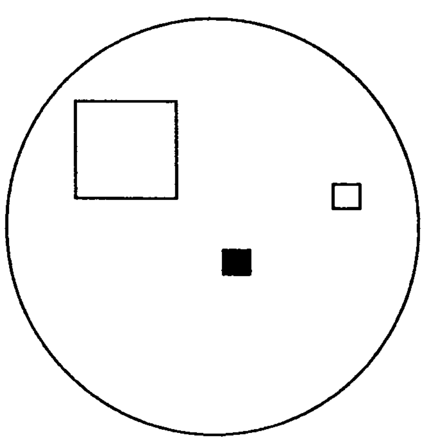
____________________ --------------------

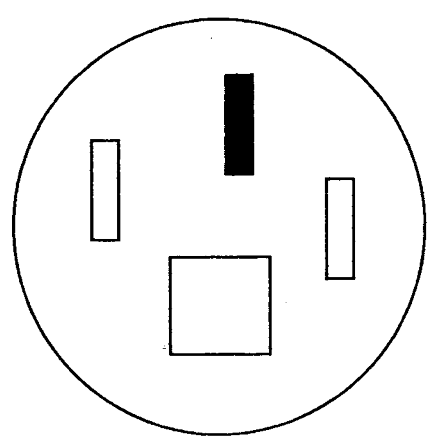


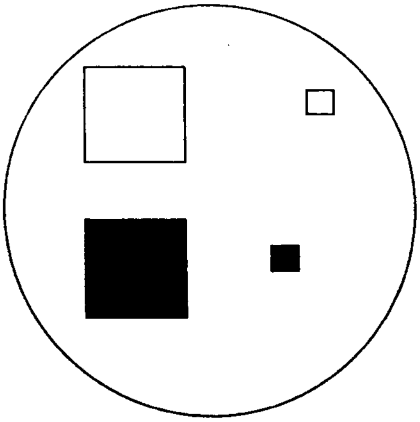


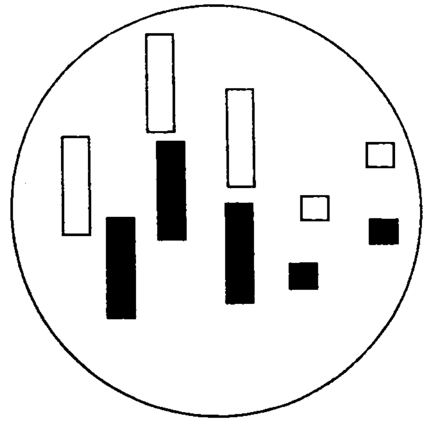






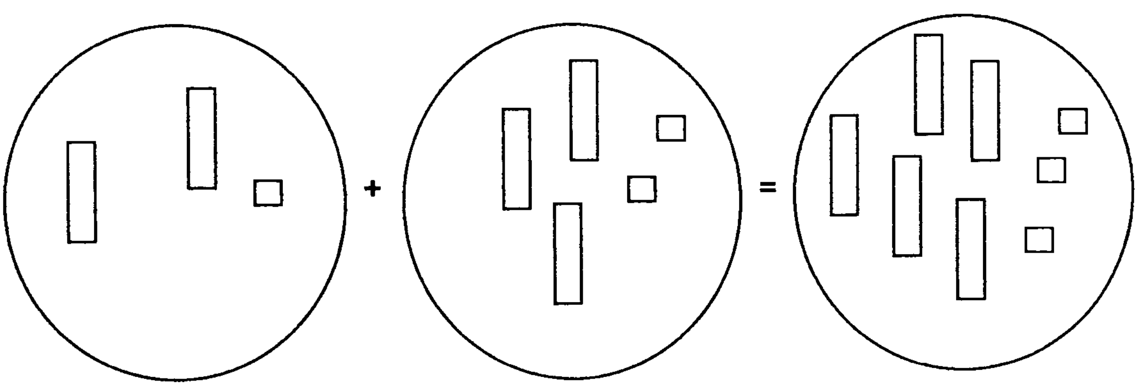




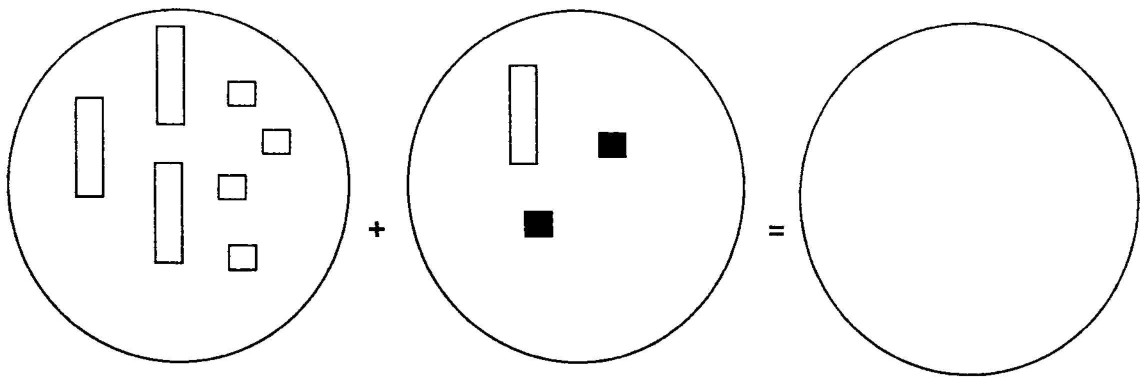


ADDITION OF POLYNOMIALS

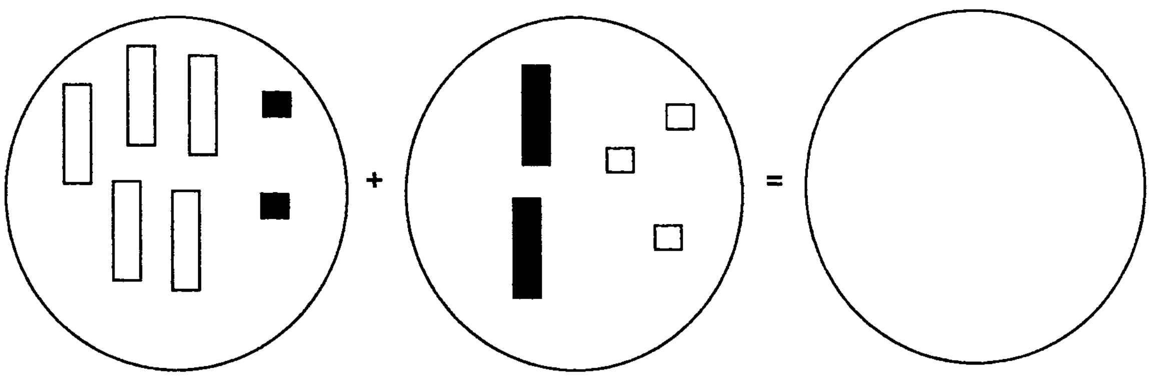
LOOK AT THE FOLLOWING EXAMPLE THEN DO THE NEXT 6 QUESTIONS BY ADDING THE TILES TOGETHER, FILLING IN THE THIRD CIRCLE AND WRITING THE WHOLE OPERATION DOWN SYMBOLICALLY IN THE SPACE PROVIDED UNDERNEATH.



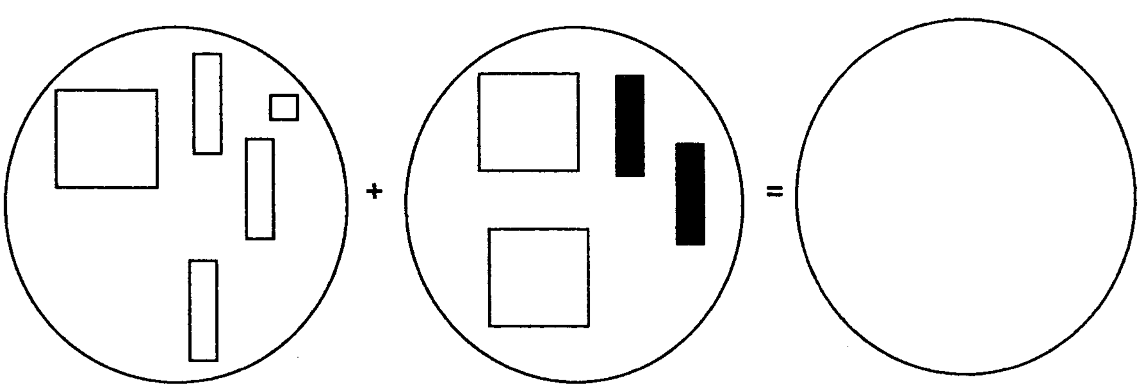
(2x + 1) + **(3x + 2)** = **(5x + 3)**



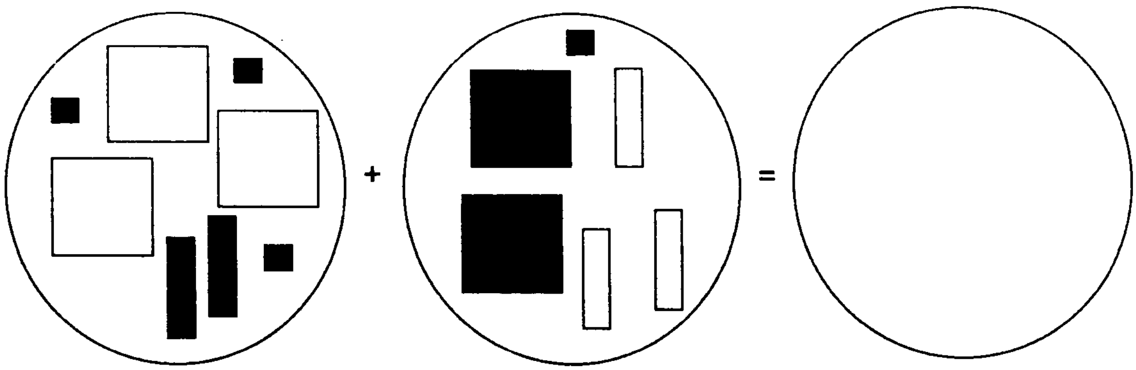
_____ + _____ = _____



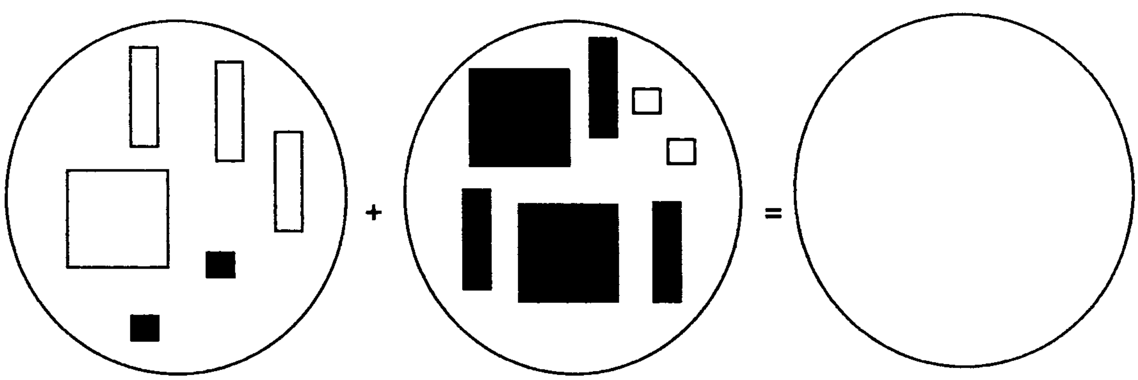
_____ + _____ = _____



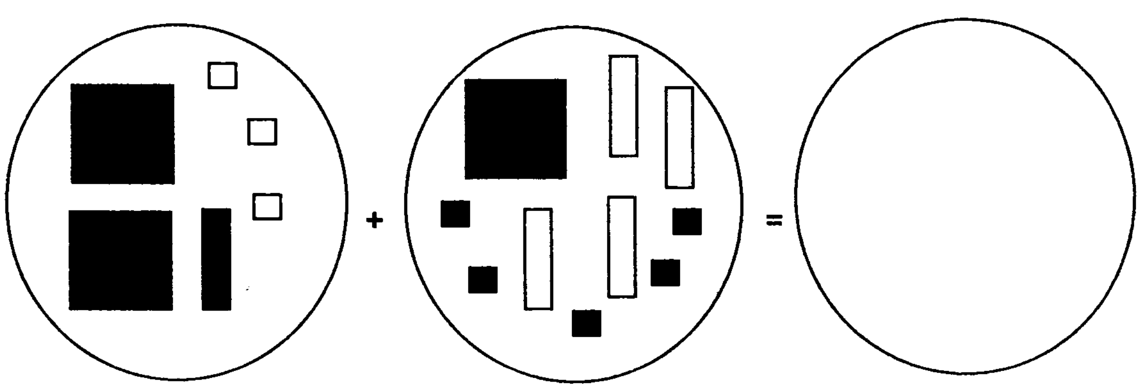
_____ + _____ = _____



______ + ______ = ______

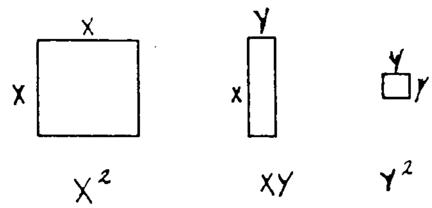


______ + ______ = ______

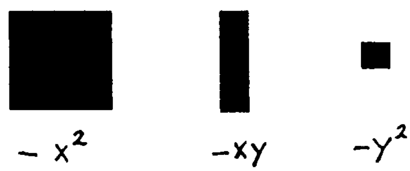


______ + ______ = ______

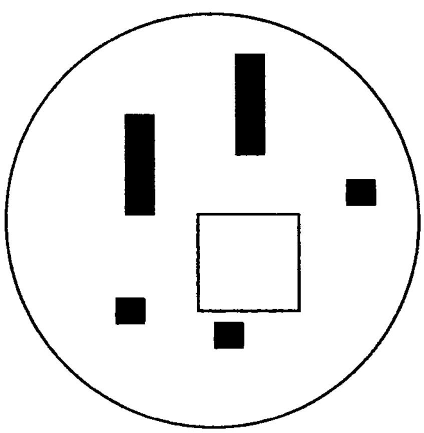
NOW LET US CHANGE THE VALUES OF THE TILES

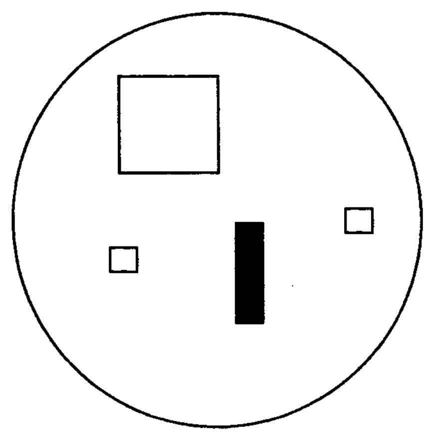


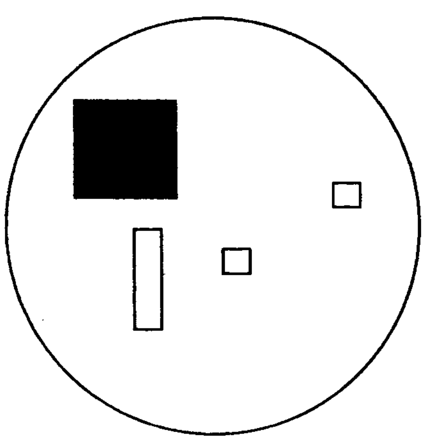
Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)

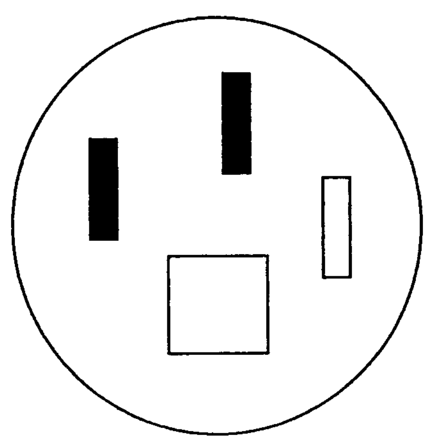


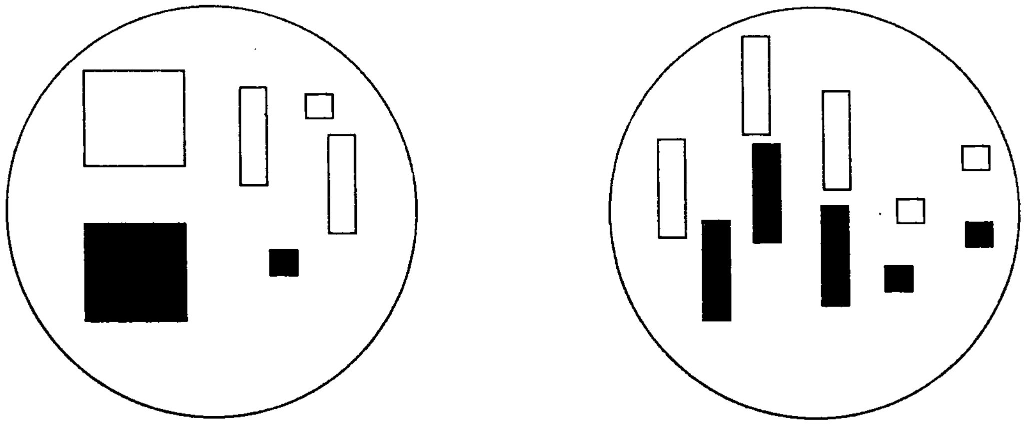
<u>What Polynomials are represented in each of the following circles? Write your answers underneath the question. Remember the tiles have different values.</u>



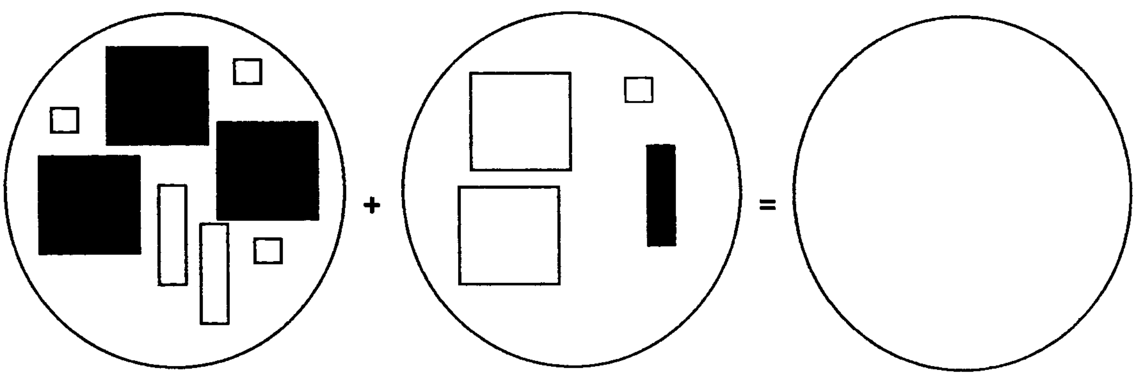




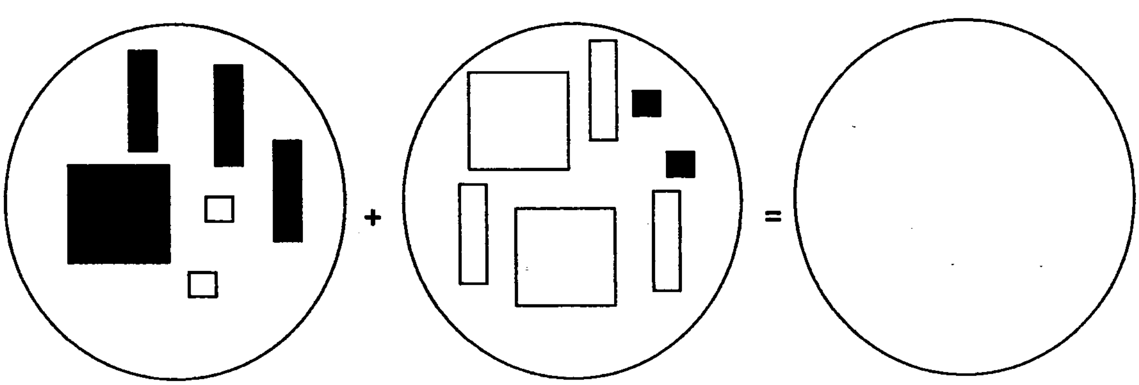




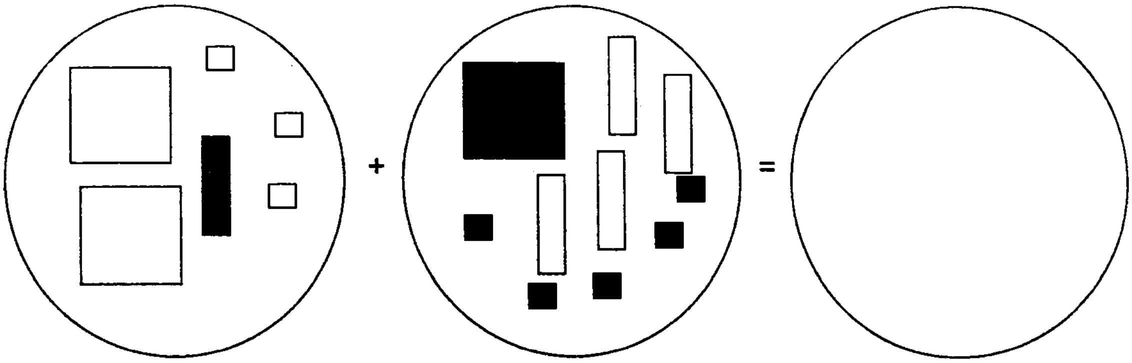
DO THE NEXT 3 QUESTIONS BY ADDING THE TILES TOGETHER, FILLING IN THE THIRD CIRCLE AND WRITING THE WHOLE OPERATION DOWN SYMBOLICALLY IN THE SPACE PROVIDED UNDERNEATH.



__________ + __________ = __________



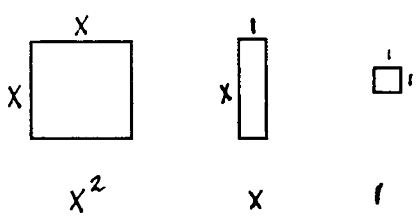
__________ + __________ = __________



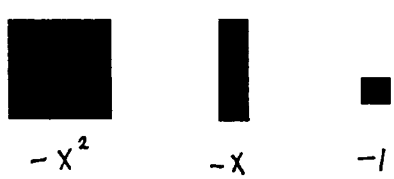
________ + ________ = ________

SUBTRACTION OF POLYNOMIALS

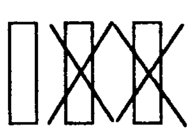
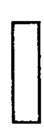
LET US CHANGE THE VALUES OF THE TILES BACK TO WHAT THEY WERE AT THE BEGINNING



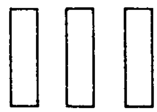
Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)



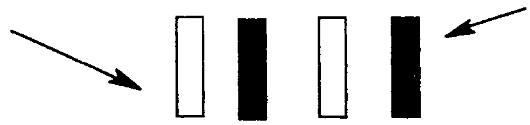
LOOK THROUGH THE FOLLOWING EXAMPLES

1) 3x - (2x) =  =  = **x**

2) $3x - (-2x)$ =

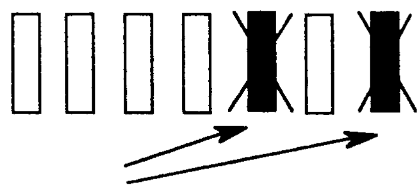


?? We do not have 2 negative tiles to take away, so we add zero. Remember this technique, we will call it the "ZERO RULE"



Two -x's and two +x's cancel each other out so in effect we are adding zero.

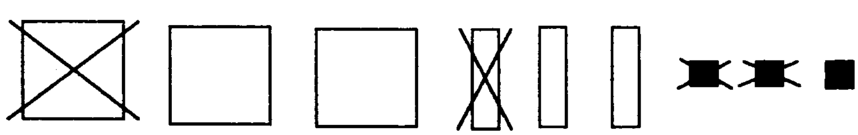
We now have



Take away the 2 "(-x)'s"

We are left with 5x ---- Therefore $3x - (-2x) = 5x$

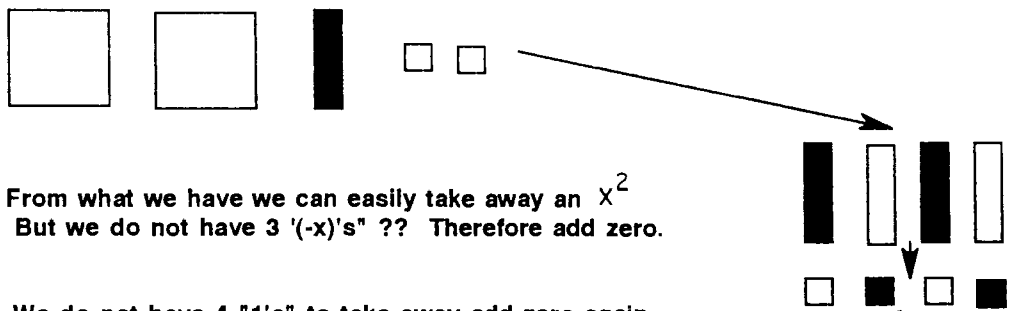
3) $(3x^2 + 3x - 3) - (x^2 + x - 2)$



with this problem we can simply remove an x^2 , x and 2 (-1)'s

Therefore $(3x^2 + 3x - 3) - (x^2 + x - 2) = (2x^2 + 2x - 1)$

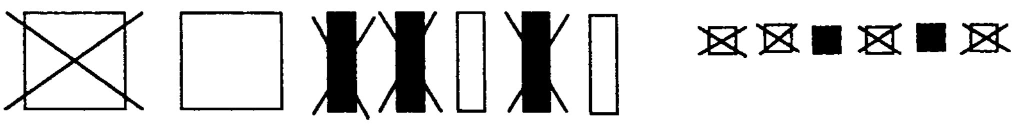
4) $(2x^2 - x + 2) - (x^2 - 3x + 4)$



From what we have we can easily take away an x^2
But we do not have 3 '(-x)'s" ?? Therefore add zero.

We do not have 4 "1's" to take away add zero again

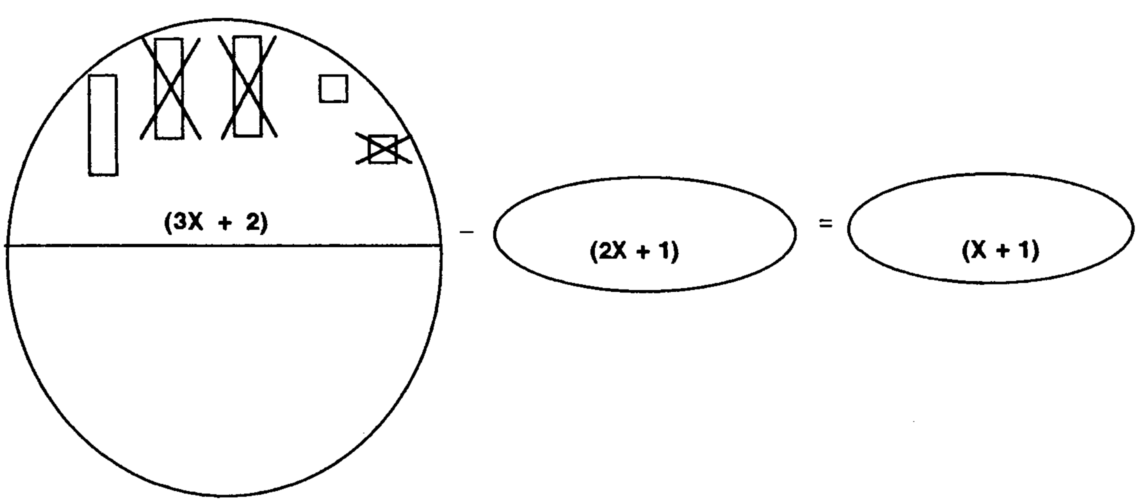
Altogether we now have

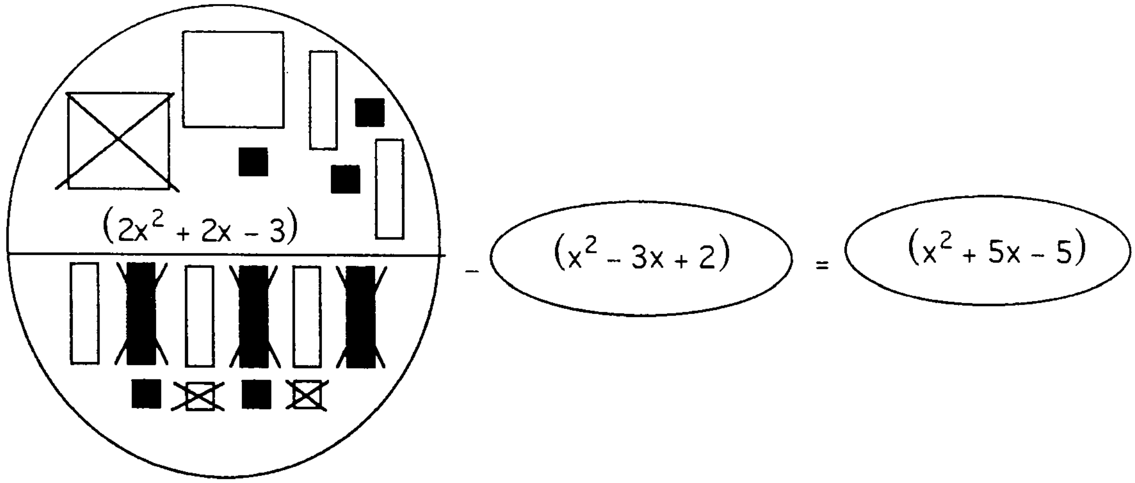


Now we can take away $(x^2 - 3x + 4)$ by crossing out the boxes

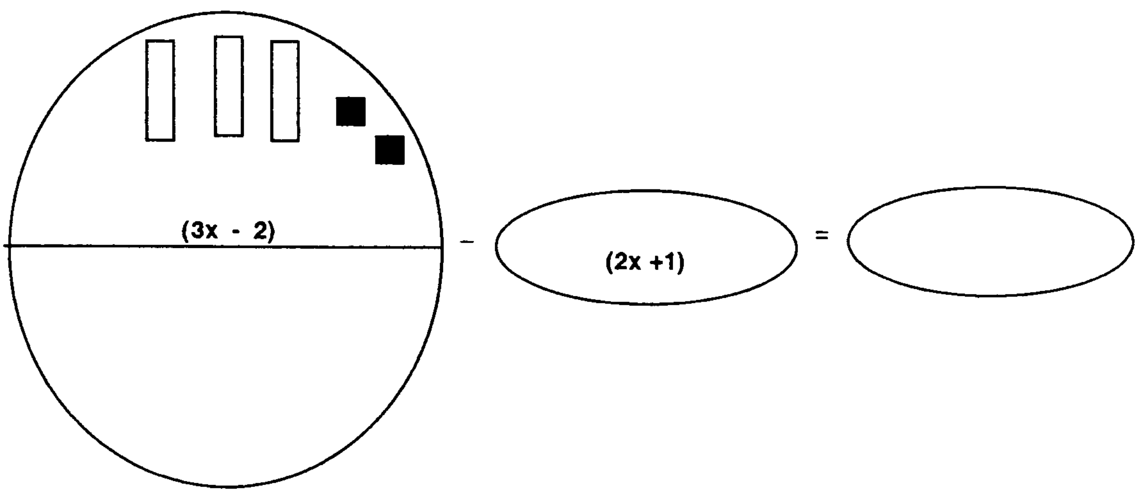
ANSWER $(x^2 + 2x - 2)$

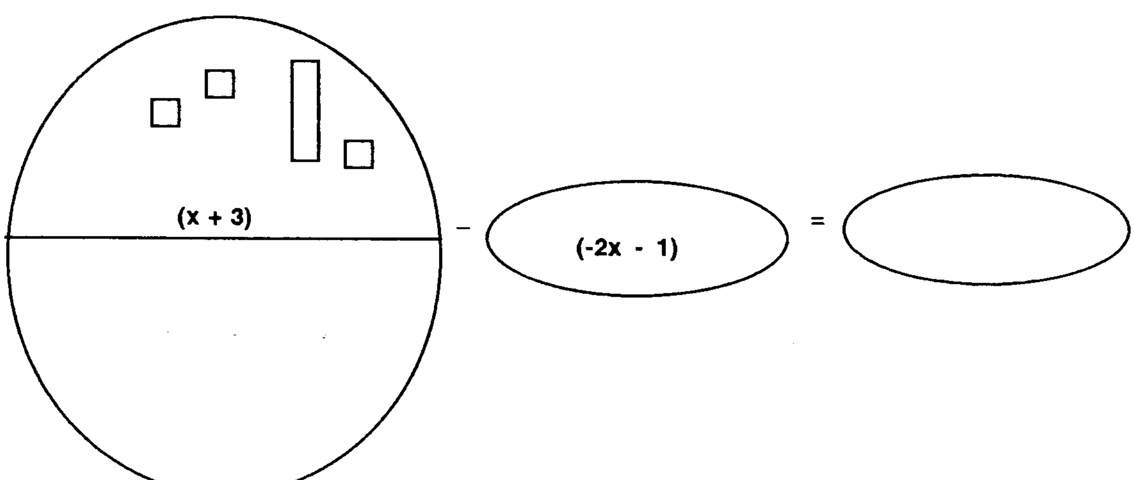
USING YOUR ALGEBRA TILES AND THE KNOWLEDGE YOU HAVE BEEN GIVEN IN THE EXAMPLES COMPLETE THE FOLLOWING POLYNOMIAL SUBTRACTIONS. SET YOUR WORK OUT AS YOU ARE SHOWN IN THE NEXT TWO EXAMPLES, THE BOTTOM SECTION OF THE FIRST OBLONG IS FOR YOU TO DRAW IN ANY TILES (ZERO'S) YOU NEED TO COMPLETE YOUR SUBTRACTION.

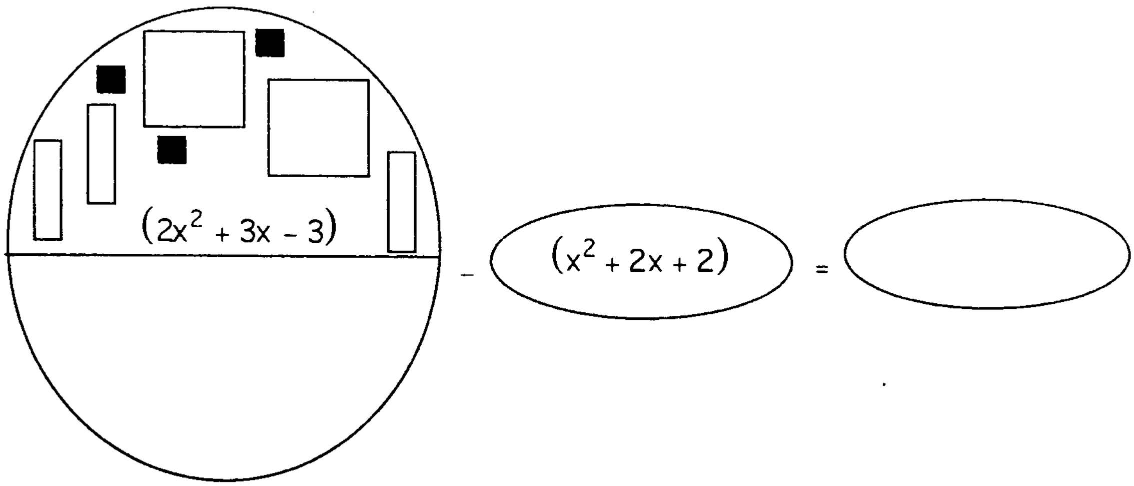


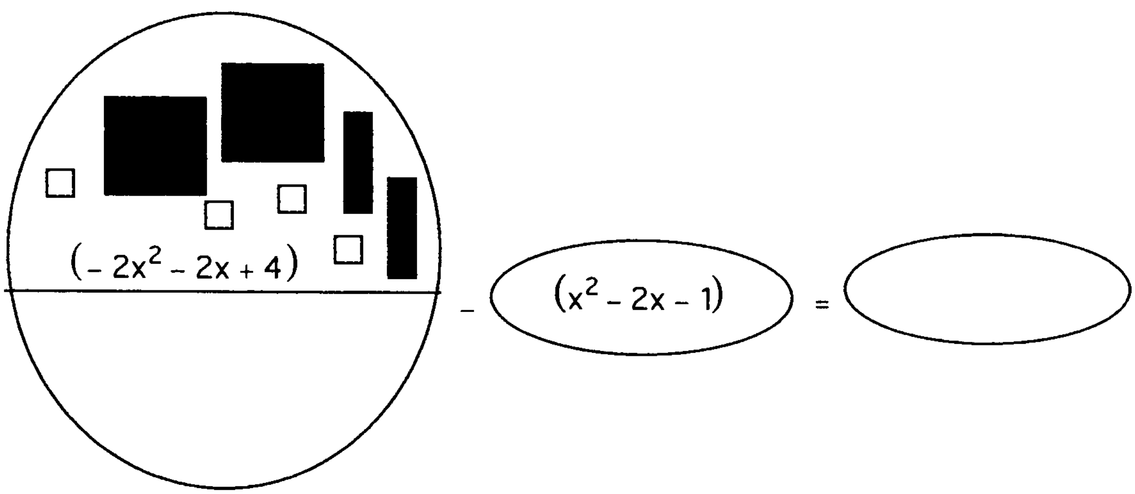


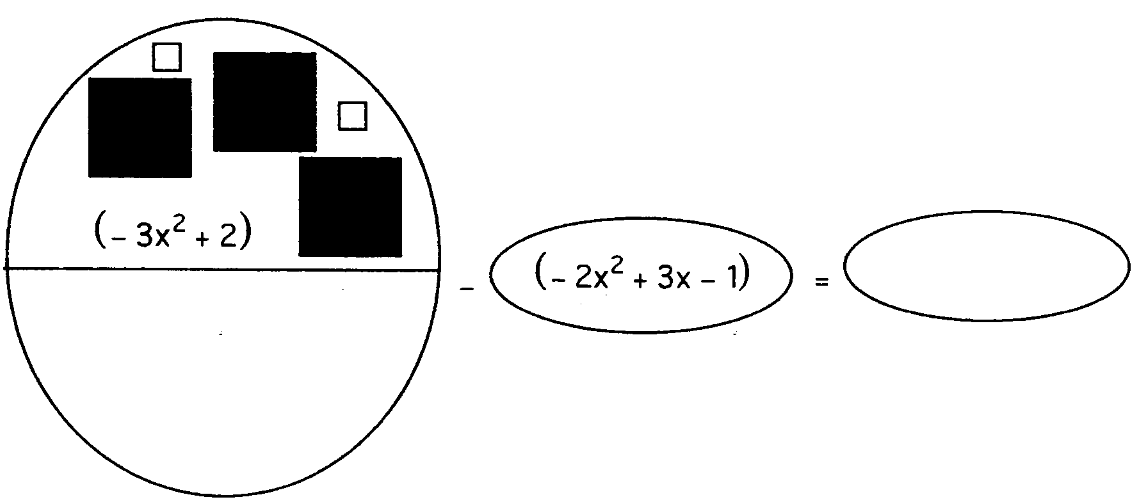
NOW DO THE FOLLOWING PROBLEMS IN THE SAME WAY AS THE ABOVE TWO EXAMPLES

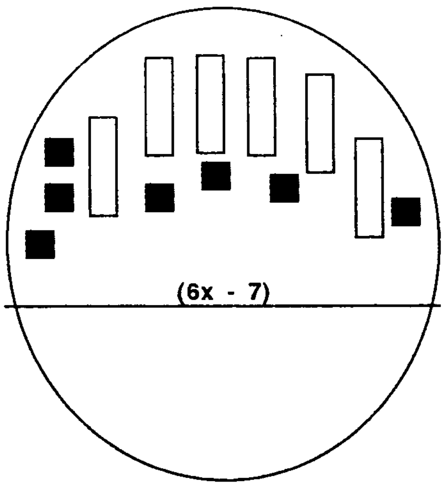




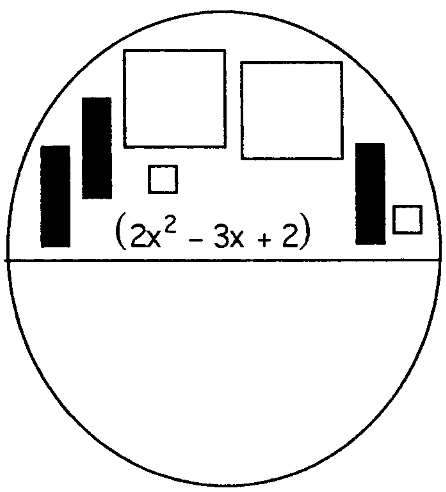

$(2x^2 + 3x - 3)$
–
$(x^2 + 2x + 2)$
=


$(-2x^2 - 2x + 4)$
–
$(x^2 - 2x - 1)$
=

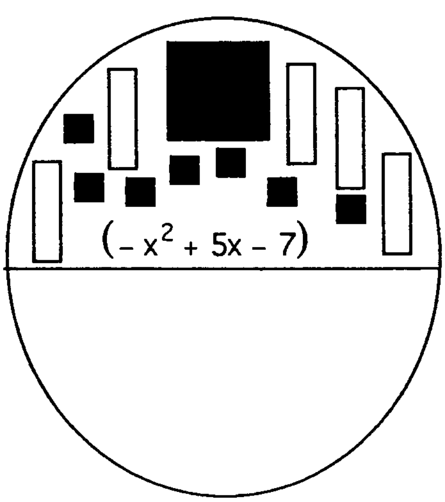

$(-3x^2 + 2)$
–
$(-2x^2 + 3x - 1)$
=



$- \left(x^2 - 2x - 4\right) =$



$- \left(-x^2 + 2x + 2\right) =$



$- \left(-2x^2 + 6x + 2\right) =$

SECTION B COMPLETE THE FOLLOWING EXAMPLES <u>WITHOUT</u> USING ALGEBRA TILES

1) $(3x - 2) + (5x - 6)$

= ____________________

2) $(3x - 2) - (5x - 6)$

= ____________________

3) $(x^2 + 3x + 3) + (2x^2 - x)$

= ____________________

4) $(-2x^2 - x - 1) + (2x^2 + x - 1)$

= ____________________

5) $(x^2 - 3x + 1) + (x^2 - 7)$

= ____________________

6) $(x^2 + 3x + 3) - (2x^2 - x)$

= ____________________

7) $(-2x^2 - x - 1) - (2x^2 + x - 1)$

= ____________________

8) $(6x - 2) - (3 - 2x)$

= ____________________

9) $(x^2+3x-1)-(x^2-2x+5)$

= ____________________

10) $(2x+3)-(x^2-5x)$

= ____________________

11) $(2x^2-5)-(x^2+5x-6)$

= ____________________

12) $(-3x+5)+(4x^2-5)$

= ____________________

13) $(x^2+3x-2)+(3x^2-x-5)$

= ____________________

14) $(3x^2-5x+1)-(x^2-3x-2)$

= ____________________

15) $(2x-5)+(x^2+3x+2)$

= ____________________

16) $(2x^2+x)+(-3x^2-2x)$

= ____________________

17) $(2x^2+x)-(-3x^2-2x)$

= ____________________

SECTION C (Extensions)

1) $(3m^2 - 5m + 9) + (8m^2 + 2m - 7)$

= ____________________

2) $(x^2 - 5x + 6y) - (4x^2 + 15x - 11y)$

= ____________________

3) $(5t^2 - 13t + 17) + (9t^3 - 7t^2 + 3t - 26) - (16t^2 + 5t - 8)$

= ____________________

4) $(4x^2 - 7x + 3) - (x^2 - 5x + 9) - (8x^2 + 6x - 11)$

= ____________________

5) $(p^3q^2 + 7q^2p^2 - 3p) + (2q - 6p^2q^2 - q^2p^3)$

= ____________________

6) $(2ab - 2ac - 2bc) + (2ca + 2cb - 2ba + 3)$

= ____________________

7) $(3x^2 - 2y^2) + (y^2 - 2x^2) - (4x^2 + 2)$

= ____________________

8) $(4x^2y - 2yx) + (3yx^2 - 6xy^2) - (3x^2y^2 + 2y^2x^2 - xy)$

= ____________________

APPENDIX G

ALGEBRA TILES PROJECT (2)

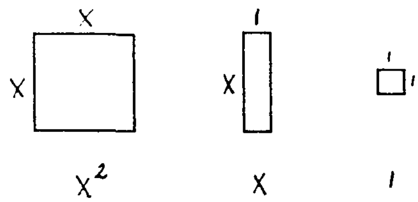
ALGEBRA TILES PROJECT (2) - MULTIPLICATION

NAME: ____________________

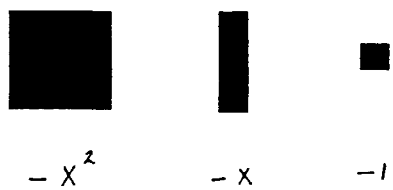
NAME: ____________________

ALGEBRA TILES MULTIPLICATION

For this assignment the tiles will be given these values:



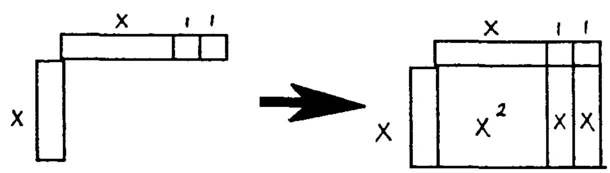
Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)



A) FOR EACH EXERCISE, COMPLETE THE RECTANGLE TO OBTAIN THE PRODUCT. WRITE YOUR ANSWER ON THE LINE PROVIDED.

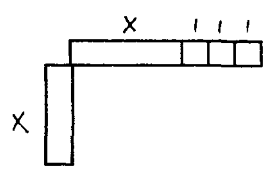
eg.

x(x + 2)



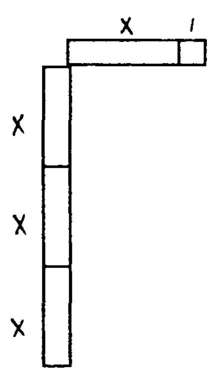
$= x^2 + 2x$

1) **x(x + 3)**



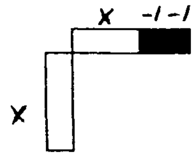
= ____________

2) **3x(x + 1)**



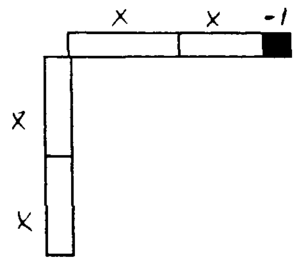
= ____________

3) x(x - 2)



= ______________

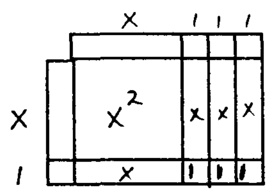
4) 2x(2x - 1)



= ______________

B) FOR EACH OF THE FOLLOWING, DRAW THE TILE DIAGRAM NEEDED TO FIND THE PRODUCT. WRITE YOUR ANSWER ON THE LINE PROVIDED.

eg: (x + 1)(x + 3)



= $x^2 + 4x + 3$

1) (x + 2)(x + 2)

= ______________

2) (x + 2)(2x + 1)

= ______________

3) (3x + 1)(x + 3)

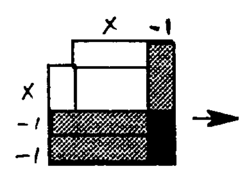
= _______________

4) (2x + 2)(x + 1)

= _______________

C) FOR EACH OF THE FOLLOWING DRAW THE TILE DIAGRAM NEEDED TO FIND THE PRODUCT. OVERLAPPING NEGATIVE NUMBERS WILL BE DIFFICULT TO REPRESENT. ONE METHOD WOULD BE TO LIGHTLY SHADE IN NEGATIVE AND TO BLACKEN OVERLAPPING NEGATIVES. WHEN WE GET OVERLAPPING NEGATIVES WE WILL CALL THIS "DOUBLE COVERUP" AND POSITIVE UNITS WILL RESULT.

eg: (x - 2)(x - 1)



$= x^2 - 3x + 2$ ← This would be overlapping negative tiles

1) (x - 1)(x - 3)

= _________________

2) (2x - 1)(x - 1)

= _________________

3) (x - 3)(x - 2)

= ________________

4) (3x - 1)(x - 1)

= ________________

5) (x - 1)(x + 3)

= ________________

6) (2x + 1)(x - 1)

= ________________

D) IN THE FOLLOWING SECTION FIND THE EXPANDED ANSWER WITHOUT THE USE OF ALGEBRA TILES. CAN YOU FIND AN EASY METHOD OR RULE TO DO THESE EXPANSIONS?

1) $2(x + 2)$

= ________________

2) $3(2x + 1)$

= ________________

3) $3(-x + 1)$

= ________________

4) $-2(x - 3)$

= ________________

5) $-3(-x + 3)$

= ________________

6) $x(x - 6)$

= ________________

7) $-x(3x - 2)$

= ________________

8) $(x + 4)(x + 1)$

= ________________

9) $(x - 3)(x + 4)$

= ________________

10) (x - 1)(x - 5)

= __________________

11) (2x + 1)(x - 4)

= __________________

12) (2x - 3)(3x + 2)

= __________________

13) (3x - 4)(4x - 2)

= __________________

14) (5x - 2)(2x - 1)

= __________________

15) -x(3 - 2x)

= __________________

16) (7x + 2)(-3x -5)

= __________________

E) EXTENSIONS

1) $(5m - 2n)(7m - n)$

= ____________________

2) $(6s - 2t)^2$

= ____________________

3) $(-5f + 4g)^2$

= ____________________

4) $(2x + 4)(3x - 2) + (5x - 2)(3x - 4)$

= ____________________

5) $5(2p - 7)(3p - 4)$

= ____________________

6) $5(2m - 4)(3m + 2) - 2(4m - 1)(3m - 4)$

= ____________________

7) $(3x - 1)(2x^2 + 3x - 4)$

= ____________________

8) $(5a - 3)^2(2a - 7)$

= ____________________

APPENDIX H

ALGEBRA TILES PROJECT (3)

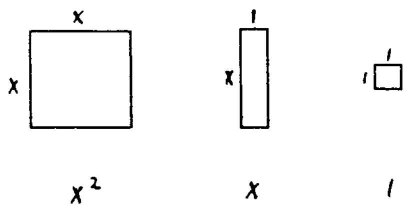
ALGEBRA TILES PROJECT (3) - FACTORING/DIVISION

NAME: ____________________

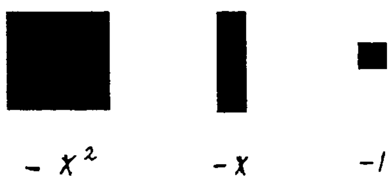
NAME: ____________________

ALGEBRA TILES FACTORING/DIVISION

For this assignment the tiles will be given these values:



Remember dark tiles are the opposite of light tiles. (Dark tiles are negative tiles)



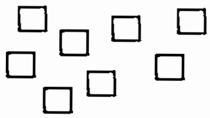
A) POSITIVE FACTORING

IN THE FOLLOWING TABLE YOU ARE GIVEN THE AREA OF A RECTANGLE FILL IN THE MISSING DIMENSIONS.

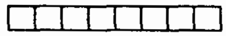
Area	Length	Width
$4m^2$	4m	
	2m	
$10m^2$		
		2m
$24m^2$		1m
	12m	
		3m
	6m	

NOTICE THAT THE DIMENSIONS OF A GIVEN AREA ARE THE FACTORS OF THE NUMBER REPRESENTING THE AREA. NORMALLY YOU ARE GIVEN THE DIMENSIONS AND ASKED TO FIND THE AREA, BUT FOR THE NEXT EXERCISE YOU WILL BE GIVEN THE AREA AND WITH THE HELP OF YOUR TILES YOU WILL BE ASKED TO FIND THE DIMENSIONS. GO THROUGH THE FOLLOWING TWO EXAMPLES THEN COMPLETE SECTION A.

Example 1



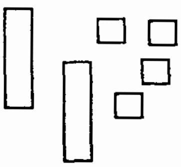
8 UNITS



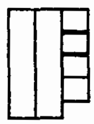
Possible solution is 8 x 1

Possible solution 4 x 2

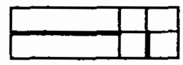
Example 2



2x + 4



Not a rectangle not a solution

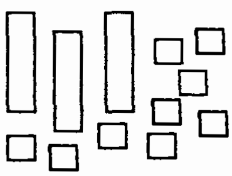


Solution 2 X (x + 2)
2(x + 2)

SECTION A

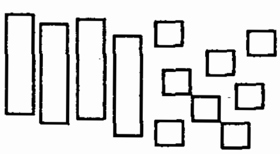
Arrange the given tiles into a rectangle to find a product of two factor which make up the given area. Draw in the tiles that make the rectangle and write the area as a product of the length and width as shown in the examples.

1)



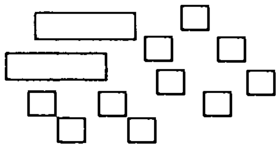
Area = 3x + 9 ____________________

2)



Area = 4x + 8 ____________________

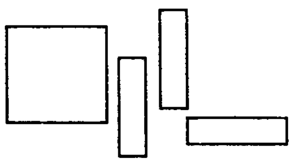
3)



Area = $2x + 10$ ____________________

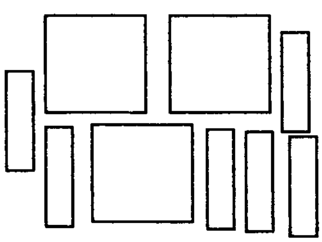
Now try the following examples

4)



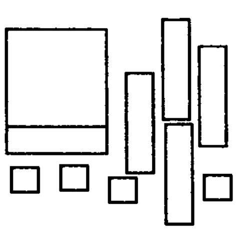
Area = x^2+3x ____________________

5)



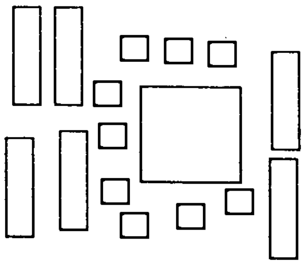
Area = $3x^2+6x$ ____________________

6)



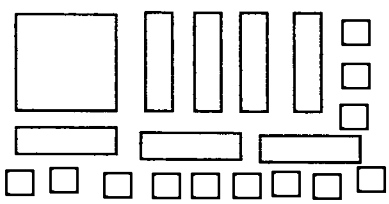
Area = x^2+5x+4 ____________________

7)



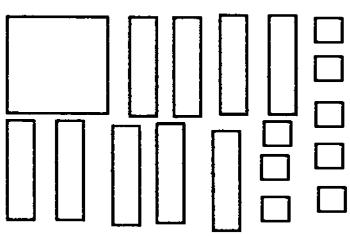
Area = x^2+6x+9 ____________________

8)



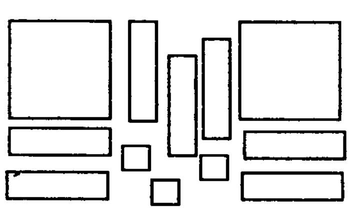
Area = $x^2+7x+12$ ____________________

9)



Area = x^2+9x+8 ____________________

10) Challenge



Area = $2x^2+7x+3$ ____________________

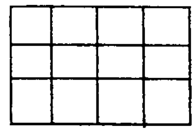
NOTE THAT THE LENGTH AND WIDTH ARE FACTORS OF THE EXPRESSION FOR THE AREA.

B) DIVISION OF POLYNOMIALS

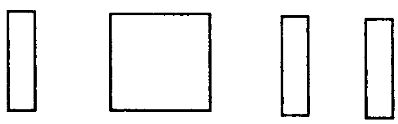
In division the area of the product rectangle and one of its dimensions are given and the other dimension is to be determined.

eg:

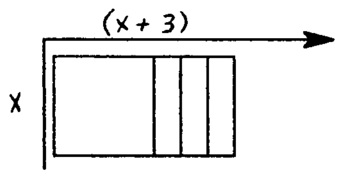
$12 \div 3 = 4$



Using Algebra Tiles $(x^2+3x) \div x$



One dimension has to be x because that is what you are dividing by. Rearrange the tiles so that one dimension is x.

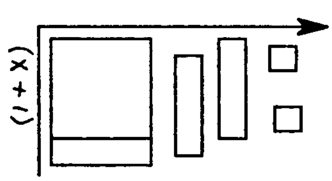


Notice that the horizontal dimension is (x + 3). The new dimension is therefore the answer to your division problem.

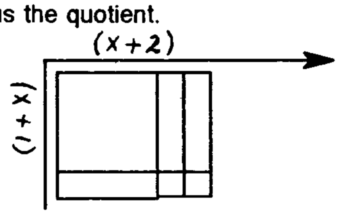
WHAT ABOUT

$$(x^2+3x+2) \div (x+1)$$

a) Arrange the x^2 , three "x's" and the two unit tiles so that one dimension is (x + 1).



b) Now complete the rectangular array with the remaining tiles to determin the other dimension and thus the quotient.



c) Therefore $(x^2+3x+2) \div (x+1) = (x+2)$

Now complete the following division problems as done in the previous example.

1) $(2x^2+6x) \div x$

x

$(2x^2+6x) \div x$ = ______________________________

2) $(2x^2-4x) \div 2x$

$2x$

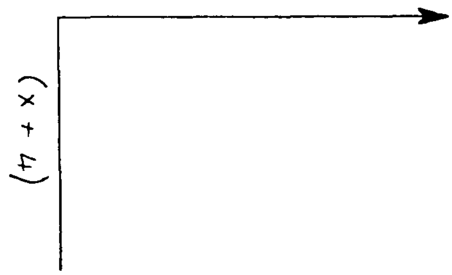
$(2x^2-4x) \div 2x$ = ______________________________

3) $(3x^2+6x) \div 3x$

$3x$

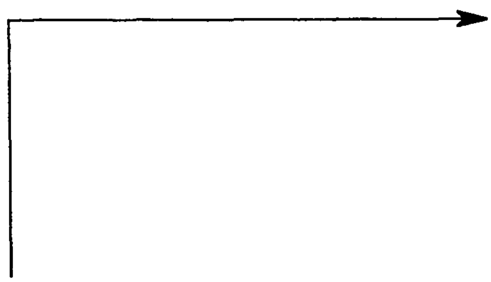
$(3x^2+6x) \div 3x$ = ______________________________

4) $(x^2+6x+8) \div (x+4)$



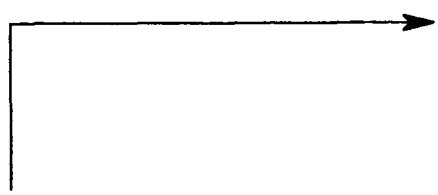
$(x^2+6x+8) \div (x+4)$ = ____________________

5) $(x^2+7x+12) \div (x+3)$



$(x^2+7x+12) \div (x+3)$ = ____________________

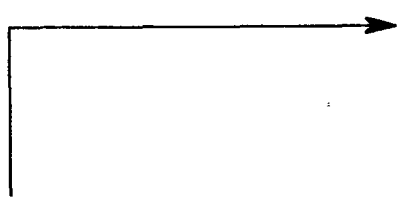
6) $(x^2-3x+2) \div (x-1)$



$(x^2-3x+2) \div (x-1)$ = ____________________

Remember overlapping double negatives makes positive

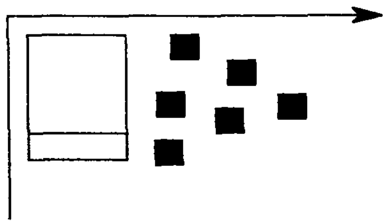
7) $(x^2-5x+6) \div (x-3)$



$(x^2-5x+6) \div (x-3)$ = ____________________

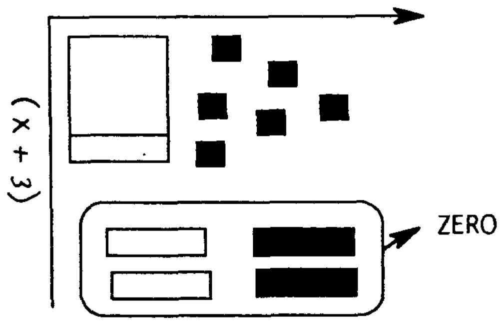
Sometimes there are too few tiles from the given trinomial to establish the dimensions specified by the divisor. In such cases the zero principal must be applied as shown in the example.

eg: $(x^2+x-6)\div(x+3)$

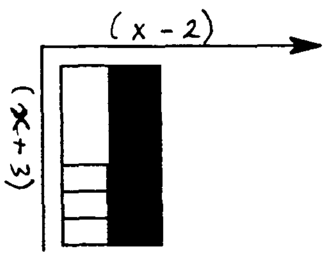


Notice there are two more "x" tiles needed to establish the (x + 3) dimension.

They can be added if the two -x tiles are also added.



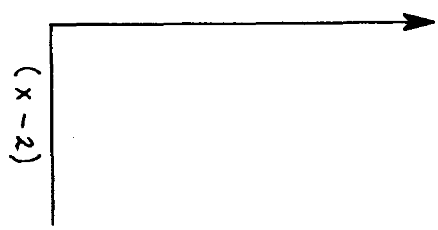
Now the tiles can be arranged to complete the rectangle whose dimensions are (x + 3) and (x - 2)



Thus $(x^2+x-6)\div(x+3)=(x-2)$

Now complete the following problems remembering to add "ZERO'S" when required.

1) $(x^2-x-2)\div(x-2)$



$(x^2-x-2)\div(x-2)$ = ______________________________

2) $(x^2 - x - 12) \div (x - 4)$

$(x - 4)$

$(x^2 - x - 12) \div (x - 4)$ = ______________________________

3) $(x^2 + 2x - 8) \div (x + 4)$

$(x + 4)$

$(x^2 + 2x - 8) \div (x + 4)$ = ______________________________

4) $(2x^2 + 5x - 3) \div (x + 3)$

$(2x^2 + 5x - 3) \div (x + 3)$ = ______________________________

5) $(3x^2 - x - 2) \div (x - 1)$

$(3x^2 - x - 2) \div (x - 1)$ = ______________________________

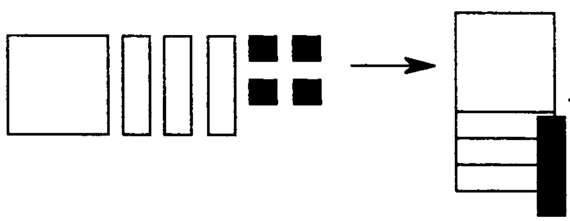
C) FACTORING Part 2 (includes negatives)

Consider the following trinomial. Sometimes it is impossible to complete a rectangle without adding extra tiles. When this occurs either the trinomial cannot be factored at all or else tiles are added using the" zero" rule.

Example 1

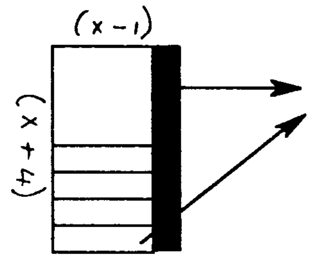
Factor the following by first creating a rectangle and then finding the dimensions.

$$x^2+3x-4$$



This trinomial requires the use of extra tiles.

To create a rectangle there is a need for an extra positive x tile -x tile. The two tiles +x and -x, make no difference to the original trinomial.



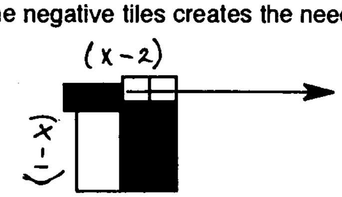
Extra +x and -x tiles

THE FACTORS OF x^2+3x-4 are (x - 1), (x + 4)

Example 2

Given the following trinomial x^2-3x+2

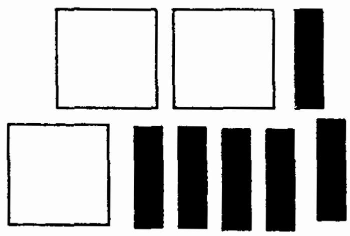
Notice when the tiles are arranged to form a rectangle a "double coverup" occures. The effect of the overlapping of the negative tiles creates the need for the positive unit tiles.



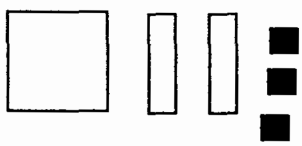
Note double coverup resulting in two positive unit tiles.

THE FACTORS OF x^2-3x+2 ARE (x - 1), (x - 2)

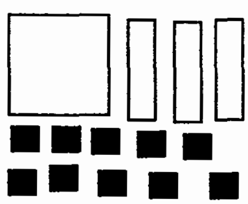
In the following exercise you will be given the area. Use your tiles to form a rectangular array and hence find the product of the factors which result in the given area. Complete the examples as you did for section A in this project. Remember in some examples the" zero rule" will need to be applied to give you extra tiles and in some examples" double coverup" will occur, so make sure you understand these two principles.

1) 

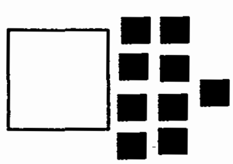
Area = $3x^2 - 6x$ ____________________

2) 

Area = $x^2 + 2x - 3$ ____________________

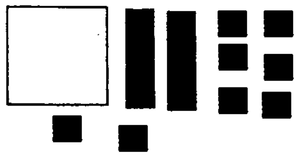
3) 

Area = $x^2 + 3x - 10$ ____________________

4) 

Area = $x^2 - 9$ ____________________

5)

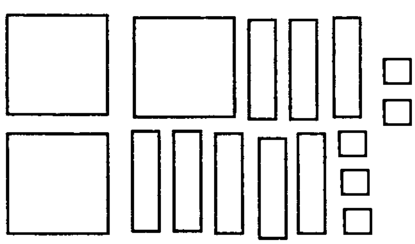


Area = $x^2 - 2x - 8$ ____________________

6)

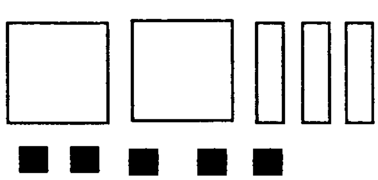
Area = $x^2 - x - 6$ ____________________

7)



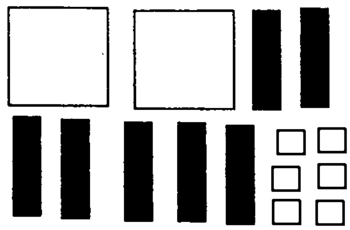
Area = $3x^2 + 8x + 5$ ____________________

8)



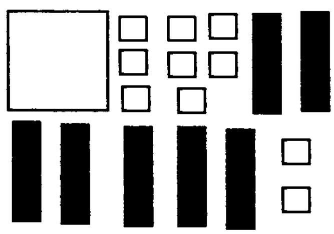
Area = $2x^2 + 3x - 5$ ____________________

9)



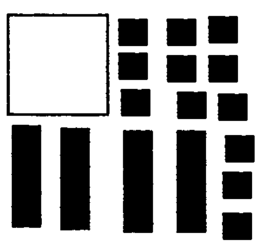
Area = $2x^2 - 7x + 6$ ____________________

10)



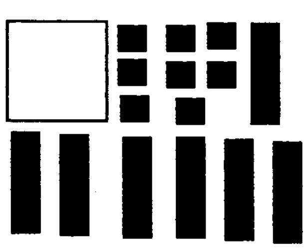
Area = $x^2 - 7x + 10$ ____________________

11)



Area = $x^2 - 4x - 12$ ____________________

12)



Area = $x^2 - 7x - 8$ ____________________

D) FACTORING without the use of tiles

In the following examples you may use tiles to help you factor, but try to develop a method of factoring which you can use without the aid of the tiles.

1) $12x^2+6x$

____

2) x^2-4

____

3) $4x^2-25$

____

4) x^2+5x+6

____

5) x^2+4x-5

____

6) x^2-x-20

____

7) $x^2 - 7x - 60$

____

8) $x^2 + 40x - 41$

____

9) $x^2 - 21x - 72$

____

10) $x^2 - 10x + 9$

____

Extension

11) $3x^2 - 10x + 8$

____

12) $2x^2 - x - 21$

____

APPENDIX I

RAW SCORES

Stu ID#	Cls ID#	MMTIC Type	Type Cat	Att(1) A-80	Att(1) B-40	Att(1) C-15	A	B	C	P test A-32	P test B-40	P test Tot-72	F test A-24	F test B-28	F test Tot-52	Att(2) A-65	Att(2) B-40	Att(2) C-15	A	B	C	#27	#5	#8	#9
1	1	ESTP	1	44	28	9	2.8	3.5	3	29	17	46	14	17	31	44	34	12	3.4	4.3	4	3	3	4	4
2	1	UNFP	3	56	30	9	3.5	3.8	3	32	9	41	13	16	29	56	27	9	4.3	3.4	3	3	5	5	5
3	1	USTP	1	51	25	8	3.2	3.1	2.7	29	29	58	16	15	31	51	27	9	3.9	3.4	3	3	5	5	5
4	1	ENUP	5	64	29	12	4	3.6	4	30	39	69	29	25	54	38	25	13	2.9	3.1	4.3	4	3	4	4
5	1	ISTP	1	42	21	4	2.6	2.6	1.3	31	15	46	20	11	31	62	21	12	4.8	2.6	4	1	5	5	5
6	1	ESTJ	1	63	26	14	3.9	3.3	4.7	32	39	71	24	27	51	43	29	14	3.3	3.6	4.7	5	3	2	3
7	1	ENFP	3	62	29	12	3.9	3.6	4	24	21	45	9	6	15	44	27	8	3.4	3.4	2.7	4	4	4	4
8	1	UNTJ	4	54	33	11	3.4	4.1	3.7	22	21	43	20	24	44	41	27	14	3.2	3.4	4.7	4	3	4	4
9	1	ESUJ	5	54	33	11	3.4	4.1	3.7	30	15	45	19	15	34	42	25	11	3.2	3.1	3.7	4	3	5	4
10	1	ENFP	3	57	29	9	3.6	3.6	3	19	15	34	15	13	28	54	23	8	4.2	2.9	2.7	2	4	5	5
11	1	UNTP	4	68	24	14	4.3	3	4.7	26	17	43	16	20	36	49	28	9	3.8	3.5	3	4	4	4	4
12	1	ISFJ	2	71	20	11	4.4	2.5	3.7	32	24	56	21	20	41	59	27	11	4.5	3.4	3.7	3	5	5	5
13	1	ENFP	3	63	26	14	3.9	3.3	4.7	32	37	69	22	23	45	40	24	11	3.1	3	3.7	4	3	2	3
14	1	ISFU	2	52	23	10	3.3	2.9	3.3	32	36	68	24	27	51	50	25	11	3.8	3.1	3.7	3	4	5	3
15	1	UUFP	5	62	21	9	3.9	2.6	3	32	33	65	20	27	47	49	27	11	3.8	3.4	3.7	3	3	4	4
16	1	ESFP	2	52	25	10	3.3	3.1	3.3	31	11	42	8	16	24	30	25	7	2.3	3.1	2.3	3	2	2	2
17	1	ESFP	2	62	23	12	3.9	2.9	4	32	36	68	23	28	51	36	23	12	2.8	2.9	4	4	2	2	2
18	1	ENFP	3	34	29	10	2.1	3.6	3.3	32	35	67	22	23	45	50	32	13	3.8	4	4.3	3	4	5	5
19	1	USTJ	1	61	26	9	3.8	3.3	3	29	23	52	9	20	29	52	31	12	4	3.9	4	3	4	4	4
20	1	ENTJ	4	49	21	9	3.1	2.6	3	23	34	57	15	23	38	44	24	15	3.4	3	5	3	3	4	5
21	1	USFU	2	49	25	8	3.1	3.1	2.7	32	9	41	18	7	25	49	25	10	3.8	3.1	3.3	2	4	4	4
22	1	ESFJ	2	54	27	6	3.4	3.4	2	20	23	43	14	9	23	49	27	10	3.8	3.4	3.3	1	4	4	4
23	1	ENFP	3	57	30	9	3.6	3.8	3	30	18	48	24	22	46	56	27	9	4.3	3.4	3	3	5	5	5
24	1	ESFJ	2	64	22	13	4	2.8	4.3	24	23	47	24	28	52	45	28	14	3.5	3.5	4.7	4	4	3	3
25	1	EUUU	5	58	15	13	3.6	1.9	4.3	19	30	49	12	25	37	40	25	7	3.1	3.1	2.3	4	3	1	4
26	3	ENFP	3	60	27	12	3.8	3.4	4	26	30	56	21	24	45	49	27	11	3.8	3.4	3.7	3	4	4	4
27	3	ESFP	2	51	25	8	3.2	3.1	2.7	20	18	38	19	17	36	49	26	10	3.8	3.3	3.3	2	4	4	4
28	3	ESFP	2	63	26	8	3.9	3.3	2.7	30	36	66	24	24	48	47	28	13	3.6	3.5	4.3	4	4	3	3
29	3	INFP	3	53	22	11	3.3	2.8	3.7	26	22	48	16	7	23	41	24	7	3.2	3	2.3	3	3	4	4
30	3	ENFP	3	43	32	7	2.7	4	2.3	23	26	49	9	1	10	30	30	3	2.3	3.8	1	3	2	3	4
31	3	USFP	2	53	26	7	3.3	3.3	2.3	23	25	48	23	8	31	61	26	11	4.7	3.3	3.7	3	5	5	5
32	3	INTP	4	42	23	6	2.6	2.9	2	11	27	38	9	3	12	41	18	10	3.2	2.3	3.3	2	3	4	4
33	3	ESFP	2	52	25	12	3.3	3.1	4	25	37	62	20	8	28	39	24	8	3	3	2.7	4	3	3	3
34	3	ENFP	3	50	31	7	3.1	3.9	2.3	20	34	54	9	0	9	28	25	5	2.2	3.1	1.7	1	1	2	2

Stu	Cla	MMTIC	Type	Att(1)	Att(1)	Att(1)	A	B	C	P test	P test	P test	F test	F test	F test	Att(2)	Att(2)	Att(2)	A	B	C	#27	#5	#8	#9
ID#	ID#	Type	Cat	A-80	B-40	C-15				A-32	B-40	Tot-72	A-24	B-28	Tot-52	A-65	B-40	C-15							
35	3	ESFP	2	43	24	8	2.7	3	2.7	17	7	24	10	13	23	20	23	4	1.5	2.9	1.3	2	1	1	1
36	3	ENTP	4	62	24	13	3.9	3	4.3	31	36	67	24	27	51	47	22	12	3.6	2.8	4	5	4	4	3
37	3	ESFP	2	62	25	11	3.9	3.1	3.7	31	37	68	21	22	43	42	25	7	3.2	3.1	2.3	4	3	3	4
38	3	ENFP	3	64	27	13	4	3.4	4.3	31	30	61	8	19	27	49	24	13	3.8	3	4.3	4	4	5	5
39	3	ENTP	4	48	31	7	3	3.9	2.3	8	6	14	5	10	15	34	27	7	2.6	3.4	2.3	1	2	4	4
40	3	EUFP	5	63	25	10	3.9	3.1	3.3	28	36	64	22	24	46	46	25	12	3.5	3.1	4	4	4	5	3
41	3	ESFJ	2	55	25	11	3.4	3.1	3.7	23	29	52	1	1	2	43	22	8	3.3	2.8	2.7	3	3	4	4
42	3	INFP	3	38	21	9	2.4	2.6	3	22	31	53	16	21	37	47	28	10	3.6	3.5	3.3	3	4	4	4
43	3	ESTP	1	42	26	11	2.6	3.3	3.7	25	21	46	12	24	36	30	29	7	2.3	3.6	2.3	3	2	4	3
44	3	ISTP	1	64	25	15	4	3.1	5	27	38	65	22	24	46	53	28	13	4.1	3.5	4.3	4	4	4	5
45	3	EUFP	5	41	33	6	2.6	4.1	2	18	14	32	21	16	37	40	35	5	3.1	4.4	1.7	3	3	4	5
46	3	ISFP	2	65	20	9	4.1	2.5	3	26	33	59	21	22	43	47	24	10	3.6	3	3.3	4	4	4	4
47	3	ESFP	2	47	29	6	2.9	3.6	2	27	31	58	8	19	27	52	35	9	4	4.4	3	2	5	4	4
48	3	ENFU	3	59	32	9	3.7	4	3	20	24	44	5	5	10	44	29	9	3.4	3.6	3	2	4	4	3
49	3	INFP	3	49	21	8	3.1	2.6	2.7	32	35	67	22	21	43	36	21	10	2.8	2.6	3.3	3	2	4	3
50	3	ISFJ	2	63	25	9	3.9	3.1	3	29	38	67	20	25	45	44	27	14	3.4	3.4	4.7	3	3	3	4
51	3	ESFP	2	52	21	9	3.3	2.6	3	23	7	30	9	0	9	42	27	4	3.2	3.4	1.3	3	3	5	4
52	5	ESTP	1	47	30	7	2.9	3.8	2.3	14	8	22	7	6	13	44	27	5	3.4	3.4	1.7	1	3	3	4
53	5	INFP	3	55	21	6	3.4	2.6	2	29	32	61	22	16	38	48	16	9	3.7	2	3	3	4	5	5
54	5	ISFP	2	74	24	15	4.6	3	5	24	38	62	19	26	45	34	32	12	2.6	4	4	5	2	4	3
55	5	INFJ	3	61	26	13	3.8	3.3	4.3	32	40	72	24	23	47	47	24	11	3.6	3	3.7	4	4	4	4
56	5	ISTP	1	71	14	13	4.4	1.8	4.3	22	30	52	18	21	39	35	21	9	2.7	2.6	3	4	3	3	3
57	5	ESFP	2	39	32	5	2.4	4	1.7	32	20	52	13	20	33	60	33	12	4.6	4.1	4	2	5	5	5
58	5	ESFP	2	57	24	7	3.6	3	2.3	19	14	33	4	12	16	49	21	8	3.8	2.6	2.7	2	4	4	5
59	5	ESFP	2	51	25	4	3.2	3.1	1.3	18	11	29	15	15	30	52	29	8	4	3.6	2.7	2	4	4	4
60	5	ESFU	2	50	22	11	3.1	2.8	3.7	30	33	63	21	17	38	43	21	10	3.3	2.6	3.3	4	4	4	4
61	5	ESFP	2	43	24	6	2.7	3	2	17	11	28	8	14	22	29	26	9	2.2	3.3	3	2	2	1	3
62	5	ESFJ	2	60	28	10	3.8	3.5	3.3	25	34	59	16	22	38	36	25	9	2.8	3.1	3	3	3	2	2
63	5	ESFP	2	49	20	14	3.1	2.5	4.7	27	39	66	21	26	47	28	18	11	2.2	2.3	3.7	4	2	3	3
64	5	ESFU	2	59	28	11	3.7	3.5	3.7	15	22	37	6	16	22	26	24	6	2	3	2	3	2	3	2
65	5	ESUP	5	67	24	14	4.2	3	4.7	27	38	65	21	27	48	43	25	13	3.3	3.1	4.3	5	3	4	4
66	5	ISTJ	1	73	26	13	4.6	3.3	4.3	24	25	49	22	26	48	33	22	11	2.5	2.8	3.7	4	3	3	2
67	5	ENFP	3	39	23	9	2.4	2.9	3	27	32	59	14	10	24	50	20	10	3.8	2.5	3.3	3	4	4	4
68	5	ESFP	2	54	22	10	3.4	2.8	3.3	23	18	41	13	26	39	39	24	10	3	3	3.3	3	3	2	3

Stu	Cla	MMTIC	Type	Att(1)	Att(1)	Att(1)	A	B	C	P test	P test	P test	F test	F test	F test	Att(2)	Att(2)	Att(2)	A	B	C	#27	#5	#8	#9
ID#	ID#	Type	Cat	A-80	B-40	C-15				A-32	B-40	Tot-72	A-24	B-28	Tot-52	A-65	B-40	C-15							
69	5	USFU	2	62	19	13	3.9	2.4	4.3	28	37	65	23	23	46	38	18	11	2.9	2.3	3.7	4	3	3	3
70	5	ENFP	3	46	27	6	2.9	3.4	2	24	14	38	16	18	34	55	22	12	4.2	2.8	4	3	5	5	4
71	5	UUUV	5	73	18	13	4.6	2.3	4.3	23	10	33	14	21	35	53	18	12	4.1	2.3	4	4	4	4	4
72	5	ESFP	2	49	17	11	3.1	2.1	3.7	30	29	59	23	24	47	25	21	7	1.9	2.6	2.3	3	1	3	2
73	5	ESFP	2	50	25	12	3.1	3.1	4	30	34	64	17	25	42	51	26	11	3.9	3.3	3.7	4	4	5	4
74	5	ISTJ	1	73	20	13	4.6	2.5	4.3	24	16	40	17	26	43	28	28	8	2.2	3.5	2.7	4	2	2	2
75	5	ENTP	4	59	30	13	3.7	3.8	4.3	24	27	51	22	20	42	44	30	11	3.4	3.8	3.7	3	3	4	4
76	5	USFU	2	62	30	13	3.9	3.8	4.3	30	22	52	18	17	35	53	30	10	4.1	3.8	3.3	3	4	5	5
77	5	ESUP	5	56	25	9	3.5	3.1	3	28	32	60	12	26	38	47	26	9	3.6	3.3	3	3	4	4	4
78	5	ESFP	2	60	15	14	3.8	1.9	4.7	31	36	67	12	22	34	30	15	9	2.3	1.9	3	4	2	4	2
79	5	ESFP	2	55	27	12	3.4	3.4	4	31	39	70	22	27	49	46	26	11	3.5	3.3	3.7	5	4	4	4
80	6	ESFP	2	64	29	10	4	3.6	3.3	32	35	67	21	24	45	56	28	14	4.3	3.5	4.7	4	5	5	4
81	6	ISTU	1	62	20	12	3.9	2.5	4	32	30	62	14	27	41	25	22	9	1.9	2.8	3	4	2	3	4
82	6	EUUP	5	55	27	12	3.4	3.4	4	26	15	41	22	24	46	55	25	14	4.2	3.1	4.7	4	5	5	4
83	6	ESFP	2	52	32	11	3.3	4	3.7	24	39	63	23	19	42	58	26	11	4.5	3.3	3.7	3	5	5	5
84	6	ENTJ	4	68	18	14	4.3	2.3	4.7	30	40	70	24	28	52	40	23	12	3.1	2.9	4	5	4	4	4
85	6	ESUJ	5	74	23	13	4.6	2.9	4.3	32	39	71	24	26	50	44	30	11	3.4	3.8	3.7	4	4	4	4
86	6	ISTJ	1	64	24	14	4	3	4.7	31	38	69	23	27	50	34	21	12	2.6	2.6	4	5	3	2	3
87	6	EUUU	5	61	24	12	3.8	3	4	30	34	64	24	24	48	57	22	14	4.4	2.8	4.7	4	5	5	5
88	6	INTP	4	41	25	9	2.6	3.1	3	30	14	44	23	20	43	48	26	9	3.7	3.3	3	3	4	5	4
89	6	ESFP	2	50	28	12	3.1	3.5	4	31	38	69	24	27	51	51	23	13	3.9	2.9	4.3	4	4	5	4
90	6	UNFP	3	73	24	13	4.6	3	4.3	32	38	70	24	28	52	43	25	12	3.3	3.1	4	4	3	5	4
91	6	ESFP	2	62	29	15	3.9	3.6	5	32	39	71	21	25	46	42	32	13	3.2	4	4.3	5	3	4	5
92	6	ESTJ	1	66	28	12	4.1	3.5	4	32	38	70	22	26	48	60	29	14	4.6	3.6	4.7	4	5	5	5
93	6	USTP	1	57	23	12	3.6	2.9	4	32	39	71	24	26	50	48	27	14	3.7	3.4	4.7	5	4	5	4
94	6	INTU	4	75	24	15	4.7	3	5	27	15	42	16	27	43	43	26	8	3.3	3.3	2.7	5	4	2	2
95	6	UNTP	4	48	31	8	3	3.9	2.7	31	8	39	24	22	46	55	30	15	4.2	3.8	5	4	5	4	5
96	6	ISTU	1	57	20	15	3.6	2.5	5	26	40	66	24	27	51	32	20	10	2.5	2.5	3.3	5	4	2	2
97	6	ESFP	2	44	18	8	2.8	2.3	2.7	31	38	69	24	27	51	39	19	14	3	2.4	4.7	3	3	4	4
98	6	ISTJ	1	56	24	14	3.5	3	4.7	31	39	70	24	25	49	42	23	11	3.2	2.9	3.7	5	3	3	3
99	6	INFP	3	54	25	14	3.4	3.1	4.7	32	38	70	23	20	43	51	28	11	3.9	3.5	3.7	5	4	4	4
100	6	ESFJ	2	49	27	10	3.1	3.4	3.3	32	38	70	21	22	43	51	29	10	3.9	3.6	3.3	3	4	4	4
101	6	ENFJ	3	73	24	13	4.6	3	4.3	32	38	70	22	28	50	48	25	12	3.7	3.1	4	4	4	3	4
102	6	EUFP	5	59	30	10	3.7	3.8	3.3	32	34	66	22	26	48	49	28	13	3.8	3.5	4.3	4	4	4	4

Stu	Cla	MMTIC	Type	Att(1)	Att(1)	Att(1)	A	B	C	P test	P test	P test	F test	F test	F test	Att(2)	Att(2)	Att(2)	A	B	C	#27	#5	#8	#9
ID#	ID#	Type	Cat	A-80	B-40	C-15				A-32	B-40	Tot-72	A-24	B-28	Tot-52	A-65	B-40	C-15							
103	6	ISTJ	1	61	23	13	3.8	2.9	4.3	31	40	71	24	28	52	54	15	14	4.2	1.9	4.7	4	5	5	5
104	6	ESFP	2	62	23	15	3.9	2.9	5	30	40	70	24	27	51	24	26	13	1.8	3.3	4.3	5	1	2	3
105	6	ESUP	5	63	29	11	3.9	3.6	3.7	31	15	46	21	26	47	45	27	12	3.5	3.4	4	3	4	5	5
106	6	ENTP	4	49	31	7	3.1	3.9	2.3	32	36	68	15	16	31	45	34	9	3.5	4.3	3	3	4	5	5
107	6	ESTP	1	61	28	12	3.8	3.5	4	31	39	70	24	24	48	44	23	12	3.4	2.9	4	4	4	2	4
108	6	IUTP	5	52	24	7	3.3	3	2.3	32	37	69	24	26	50	44	24	14	3.4	3	4.7	3	3	3	4
109	7	ISTJ	1	52	20	10	3.3	2.5	3.3	32	37	69	23	28	51	55	22	12	4.2	2.8	4	3	5	3	3
110	7	ESFJ	2	67	27	15	4.2	3.4	5	30	35	65	24	26	50	48	30	13	3.7	3.8	4.3	5	4	4	5
111	7	ISFP	2	59	23	11	3.7	2.9	3.7	28	40	68	23	28	51	51	27	12	3.9	3.4	4	3	4	5	4
112	7	UUFU	5	52	23	12	3.3	2.9	4	32	40	72	24	28	52	36	27	13	2.8	3.4	4.3	4	3	3	4
113	7	ISFJ	2	60	18	14	3.8	2.3	4.7	30	40	70	23	28	51	51	26	15	3.9	3.3	5	5	4	4	4
114	7	ESFP	2	56	27	9	3.5	3.4	3	22	23	45	12	7	19	50	25	8	3.8	3.1	2.7	3	3	4	4
115	7	USUJ	5	70	26	13	4.4	3.3	4.3	23	15	38	23	19	42	43	25	11	3.3	3.1	3.7	4	3	4	3
116	7	EUFU	5	48	25	8	3	3.1	2.7	23	18	41	20	21	41	47	18	13	3.6	2.3	4.3	3	4	4	4
117	7	USUU	5	48	21	7	3	2.6	2.3	24	35	59	21	25	46	53	30	13	4.1	3.8	4.3	3	5	5	5
118	7	ENFP	3	62	26	11	3.9	3.3	3.7	20	24	44	18	23	41	51	29	11	3.9	3.6	3.7	3	4	4	4
119	7	EUFP	5	61	29	14	3.8	3.6	4.7	30	31	61	23	24	47	48	29	11	3.7	3.6	3.7	5	4	3	3
120	7	ESTU	1	55	28	9	3.4	3.5	3	13	1	14	12	9	21	52	27	8	4	3.4	2.7	2	4	5	5
121	7	ISFP	2	50	18	7	3.1	2.3	2.3	21	6	27	3	13	16	37	21	5	2.8	2.6	1.7	2	3	4	4
122	7	UNTP	4	55	18	8	3.4	2.3	2.7	28	35	63	12	4	16	54	26	9	4.2	3.3	3	3	5	5	5
123	7	UNFJ	3	59	27	10	3.7	3.4	3.3	30	27	57	24	21	45	61	29	12	4.7	3.6	4	3	5	5	5
124	7	ESUP	5	59	28	7	3.7	3.5	2.3	30	34	64	19	13	32	59	26	12	4.5	3.3	4	2	5	4	4
125	7	ENFP	3	56	35	9	3.5	4.4	3	31	36	67	22	23	45	60	33	15	4.6	4.1	5	3	5	4	5
126	7	EUFP	5	59	29	10	3.7	3.6	3.3	29	18	47	13	21	34	60	31	9	4.6	3.9	3	3	5	5	5
127	7	ESFP	2	51	29	7	3.2	3.6	2.3	22	11	33	8	6	14	46	32	5	3.5	4	1.7	2	3	4	4
128	7	USFP	2	68	28	9	4.3	3.5	3	28	16	44	17	16	33	48	18	8	3.7	2.3	2.7	3	4	2	4
129	7	ENTP	4	51	24	5	3.2	3	1.7	13	8	21	5	3	8	45	32	9	3.5	4	3	1	4	2	3
130	7	ESFU	2	66	23	11	4.1	2.9	3.7	25	32	57	23	20	43	60	28	11	4.6	3.5	3.7	3	5	5	5
131	7	ESFP	2	58	21	11	3.6	2.6	3.7	27	34	61	24	26	50	37	24	13	2.8	3	4.3	3	3	2	2
132	7	ESFJ	2	51	19	12	3.2	2.4	4	32	39	71	24	26	50	33	24	14	2.5	3	4.7	4	2	3	3
		ST - 1	SF - 2																						
		NF - 3	NT - 4																						
			U - 5																						

APPENDIX J

LETTER OF CONSENT

CAMBIE JUNIOR SECONDARY SCHOOL

3751 Sexsmith Road, Richmond, B.C. V6X 2H6
Tel. (604) 668-6430 Fax. (604) 668-6132

Principal: Mr. P. S. Healy Vice-Principal: Mrs. J. McKnight

January 10th, 1994.

Dear Parent/Guardian and Student,

As part of the requirements for a Master of Science (Secondary Mathematics Education) degree, I would like to complete a study involving all the Grade 9 mathematics students at Cambie Junior Secondary. The project involves determining the learning style of each student and then assessing whether classroom instruction using the manipulative, Algebra tiles, is effective for all students even though they have varied learning styles and preferences. The outcome of the study will be useful in the future development of our algebra units in Grade 9.

In addition to the regular classroom instruction and tests, students will be asked to write the Murphy-Meisgeier Type Indicator for Children (MMTIC). The results of the MMTIC will not be used for anything other than this project. The identity of the students will not be revealed and all answer sheets will be destroyed on completion of the study. The test will be administered during a regular mathematics class for half an hour. Participation is voluntary and your child can choose not to write this test. However, I hope most parents and students will agree to participate and help with this project. For anyone interested a copy or research results will be available in my thesis.

Please complete the consent form below and return it to Ms Sadler or Mrs. Thornton. If you have any questions or concerns please call me at school (668-6430) or contact my research supervisor Dr Tom O'Shea at Simon Fraser University (291-4453).

Yours truly

Mrs. Geesje Thornton
(Mathematics Department Head)

Name of Student:

____

I consent to my son/daughter participating in this study.

I agree to take part in this study

____________________________ ____________________________

Signature of Parent or Guardian Signature of Student

"A Community of Learners"