TRANSACTIONS EXTERNALITIES AND HYSTERESIS IN THE LABOR MARKET

by

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Abstract

Transactions Externalities and Hysteresis in the Labor Market

This thesis addresses the issue of unemployment in the context of the microfoundations to the "natural" rate of unemployment. On a broader front, it questions the existence of a unique equilibrium in a market economy characterized by transactions externalities in the coordination of buying and selling activity. It is proposed that transactions externalities generate hysteresis in the equilibrium unemployment rate. The proposition has important macroeconomic policy implications.

The analysis centers on the labor market matching process. Workers and firms face a probability (less than one) of locating a vacancy or potential hiree in a market period. The expected return to seeking a match is a function of the expected wage and the probability of finding a match, which are jointly determined. This creates transactions externalities in the matching process, which lead to non-unique steady-state equilibria that can be permanently altered by exogenous shocks. Hence, transactions externalities generate hysteresis in the equilibrium unemployment rate. This is the first proposed hysteresis mechanism that does not rely on an assumption of market imperfection that is ad hoc.

Dedication

To Ken H. Van Wart, my father. The fondest memories enlighten.

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Introduction

1.1 Search Theory and Matching Theory

Search theory was proposed 25 years ago by George Stigler (1962) as a way to incorporate certain transactions costs into exchange behavior, specifically the cost of acquiring or learning market information. As a part of the broader literature on the economics of information and uncertainty, it was hoped that search theory would provide an explanation for such phenomena as the persistent dispersion of the price for identical goods, advertising, queuing and persistent unemployment of resources.

The standard search model has been developed in a labor market setting. As formalized by Phelps (1970, 1972), Mortensen (1970), Alchian (1970) and others, the analysis centres on the optimal decision of an individual facing a random distribution of wage offers, with a known, fixed cost of search. The key result is the reservation wage property: the optimal decision criterion to search is characterized by a reservation wage that divides the wage distribution into an acceptable and unacceptable class. This reservation wage is such that the marginal cost of obtaining one more offer is equal to the expected marginal return of that offer.

There is another type of transactions cost incurred in labor markets which does not imply the existence of search decision behavior, but might also provide an explanation for the existence of steady-state unemployment consistent with rational expectations and market equilibrium. In a labor market characterized by a known, single wage offer for homogeneous labor, frictional unemployment may arise if a transaction cost is incurred to acquire exchange opportunities and there is continuous natural turnover of market participants. It is useful to distinguish between frictional unemployment and search unemployment on this basis.

Analysis of frictional unemployment centers on, what I shall term, the labor market matching process, which is defined by the underlying matching technology. Market exchange, in general, requires the coordination of individual buyer-seeking and seller-seeking activities, constrained by the spatial and temporal dimensions of the market, the cost of information acquisition and the social organization of the market. The basic matching problem facing the individual is locating a potential partner for an exchange transaction. For example, in the labor market firms that are currently hiring are not uniformly or continuously distributed over space or time and, similarly, unemployed job seekers are not so distributed. As a result, resources must be expended to locate potential transaction partners.

Matching theory should be distinguished from what we call search theory because of the primary identification of the latter term with the optimal acceptance decision in the face of a random distribution of real wage offers. Matching theory is concerned with the process by which an individual obtains market offers, dispersed or not.[1] Clearly, all search theories must assume an underlying matching process and a concommitant theory of matching behavior.

A further, related reason why we wish to distinguish between matching and search theory is to clarify the necessary theoretical foundations of the models of transactions externalities and hysteresis which this thesis examines. Specifically, these two phenomena do not, in general, require the existence of a process of market search behavior, as we have defined it, but are rooted in the underlying market matching process. Hence, they can be fruitfully analyzed with a simple labor market matching model.

1.2 Literature Background

Briefly, the notion of transactions externalities in labor markets was introduced by Peter Diamond in a series of articles (1979 (with Maskin), 1981, 1982a, 1982b), followed up

[1] Some terminological confusion is inevitably engendered by the prevailing narrow useage of the term "search theory."

by Dale Mortensen, Christopher Pissarides, Peter Howitt and others. Howitt, in his Innis Lecture to the Canadian Economics Association (1986), makes transactions externalities the centrepiece of, what he calls, the "Keynesian recovery" in macroeconomics. This identification with Keynesianism might be justified in terms of the general theoretical and policy implications of this line of research. However, to be accurate, the microfoundations of the unemployment theory that underlies this concept (and that of hysteresis) was largely initiated by Arthur Pigou, in his extensive writings on unemployment during the 1930's and 1940's.[2]

Hysteresis in labor markets comes from a quite different literature, but is also identifiable as "neo-Keynesian". The term was first introduced in 1972 by Edmund Phelps (1972) and has been further developed by Robert Hall, Allan Drazen, S. P. Hargraves-Heap and Asser Lindbeck. A brief survey (and contribution to) the theoretical and empirical work on macroeconomic hysteresis is Blanchard and Summers (1986), with specific reference to an explanation of the persistent high unemployment rates in Europe over the past 15 years.

Let us now briefly define these terms.

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[2] In any event, the "Keynesian" label no longer tells us much about the content of any analysis, given the proliferation of models and stories reputedly belonging to this school.

1.3 Transactions externalities

In the labor market, transactions externalities result from stock-flow adjustments such as would occur with an exogeneous change in recruiting or job search intensity or with stochastic shifts in labor market entry and exit. The important transaction cost in these models is the cost of contacting potential trading partners. For example, a rise in the recruiting intensity of firms will make it easier for all unemployed job seekers to locate a partner. Hence, the firms' increased effort to transact confers an external benefit on unemployed laborers, whose marginal cost of transacting is directly reduced by the greater probability that a given contact effort will result in a successful match. This externality is entirely consistent with complete real wage flexibility.

This "thin market externality", as Howitt calls it, does not exhaust the potential transactions externalities arising from labor market stock-flow adjustments. A second is the external benefits (disbenefits) placed on future transactors on both sides of the market as a result of a persistent change in the ratio of unemployed labor to vacancies. A third external effect arises from congestion in the market matching process that occurs, for example, when more than one qualified unemployed job seeker contacts a single vacancy.

Taken singularly, or together, these transactions externalities are non-price market interactions which do not appear to be easily internalized into the individual wage bargain. If the wage bargain is struck after contact has been made between interested parties, the matching costs are essentially sunk costs. The private marginal cost (benefit) of changes in the volume of market transactions is less than the marginal social cost (benefit) to market participants as a whole, while there does not appear to exist an effective social mechanism to efficiently shift the social cost (benefit) to private transactors.

The existence of transactions externalities implies three important conclusions:

1. An increase in aggregate demand that raises the demand for labor will make jobs easier to locate by job seekers, as ratio of unemployed to vacancies falls, even if wages are perfectly flexible. This suggests that there is a multiplier process, as the increased matching activity by one group of agents induces the other to increase contact effort, due to the fall in its transaction cost.

2. Expectations can be self-fulfilling. If most agents believe that markets will be very active over the coming period they will anticipate a low cost of transacting,

which then induces them to transact. Hence, models incorporating transactions externalities tend to produce multiple- equilibria. In Diamond's stripped-down G.E. model, the high employment equilibria Pareto-dominate the low-level equilibria. Thus, the equilibria in these models are typically Pareto-inefficient.

3. The steady-state, or equilibrium, rate of unemployment, which is consistent with constant, stable expected inflation, is a function of the cost of transacting. Since the argument here indicates that this cost is not fixed, the steady-state rate of unemployment is endogeneous and is counter-cyclical with respect to changes in aggregate demand.[3]

1.4 Hysteresis

The term hysteresis has been applied to a number of features of the adjustment path of complex, multi-sectoral labor markets that alter the steady-state rate of unemployment. Hysteresis of the steady-state unemployment rate means that the rate fails to return fully to its initial

[3] This result was already clearly shown by Lucas and Prescott's (1974) general equilibrium search model, where changes in aggregate demand alter the optimization problem facing individuals and, hence, the simultaneously-determined equilibrium values of wages and unemployment. However, they did not recognize the dichotomy between the private and social costs of changes in recruitment or job-seeking activity.

long-run steady-state value following the impact of an exogeneous, surprise disturbance. In general, hysteresis is the property in dynamic systems where the adjustment path between steady-state positions determines the new steady-state position. The steady-state value, therefore, depends upon the history of shocks affecting the system. Strictly, we should say unemployment (or any endogeneous variable) exhibits hysteresis when current unemployment depends upon past values with coefficients summing to one. However, the term is used more loosely in the economics literature to refer to situations where the dependence upon the past history is high, but fundamental demographic and institutional factors affecting labor supply are still important. Thus, the steady-state unemployment rate is only partially affected by the history of temporary and permanent shocks through the hysteresis effects that have been cited by Phelps and others.

The movement over time of any variable subject to hysteresis can be approximated by a random walk. Blanchard and Summers (1986) present significant statistical evidence that this is a good characterization of the time-series behavior of unemployment in Great Britain, France and West Germany over the past twenty-five years, but not of unemployment in the United States, which shows some tendency to return to trend in the post-war period. They also present historical times series for the past century, from 1880 to present, which show a very high degree of persistence in unemployment, subject to periodic shifts, (characteristic of a random walk) in both the U.S. and Great Britain. Of course, time series evidence gives no indication of the cause of the infrequent changes in the mean level of unemployment that account for much of the observed persistence. It could be exogeneous or it could be due to hysteresis, that is triggered by changes in unemployment itself: a few years of high unemployment triggering an increase in the mean level, a few years of low unemployment triggering a fall. We require a clear specification of this "triggering mechanism" to be able to distinguish between these two possibilities.

Of the potential hysteresis effects that have been mentioned in the literature, two seem to be particularly interesting. The first arises from the effect of unemployment on human capital. The second arises from the notion of "insiders" versus "outsiders" in the labor force. This thesis proposes a new and perhaps more fundamental hysteresis mechanism.

1.4.1 Hysteresis via human capital depreciation

The human capital argument requires us to consider a world of heterogeneous capital and labor with technological change. In general, technological change is accompanied by the continual adaptation of employees' skills, particularly through on-the-job training. The loss of this workplace training, in addition to the deterioration of existing skills through lack of exercise, leads to depreciation in the value of the human capital of the unemployed. Hence, with prolonged spells of unemployment workers become increasingly unfit for their previous occupations and, perhaps, for any type of currently available employment. Some become virtually "unemployable".

This process is accentuated by, what Thurow (1983, p.83) calls, the "filtering effect" that occurs during periods of high unemployment. During an extended "buyers' market" the least preferred types of workers will be replaced with more preferred types, so that unemployment becomes increasingly concentrated among the least preferred. The unemployment rate of preferred workers (prime-aged white males) might actually drop over time with a constant or rising aggregate unemployment rate.

It needs to be stressed that these are not simply effects on the supply of labor, but also on the steady-state, or "natural rate", of unemployment in Friedman's (1968, p.8) well-known sense:

> The 'natural rate of unemployment' is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded within them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demand and supplies, the cost of gathering information about job vacancies and availabilities, the cost of mobility, and so on.

Therefore, given (1) the structural heterogeneity of labor and capital, (2) stochastic variability in demand and supply shocks across sectors (including those arising from non-neutral technological change), and (3) positive mobility costs, it follows that there is continuous, stochastic friction in the labor market due to the temporary, but slow to adjust, mismatch between desired labor skills and the available skills of unemployed job seekers at any point in space and time. This is, of course, the basis for what is commonly called structural unemployment. Clearly, structural unemployment is a component of the steady-state level of unemployment in a stochastic macro model, following Friedman's conception. It can also be usefully distinguished from purely frictional unemployment, which occurs even with perfectly homogeneous labor and capital, as I have indicated earlier.

A greater rate of depreciation of human capital during prolonged spells of high unemployment leads to a higher level of structural unemployment for a given skills composition of labor demand (or a given rate of change in this level of skills composition) because of the increased mismatch between available skills and jobs. The steady-state rate of unemployment is, therefore, increased by a protracted period of above average unemployment. On the other hand, a period of below average unemployment will work in the opposite direction, improving the match between the skills composition of labor demand and supply. Moreover, periods of tight labor demand will induce employers to hire even the least-desireable, "unemployable" types, therefore reducing the hard-core, long-duration component of the steady-state unemployment rate and providing on-the-job experience that will make them more desireable types in future.

1.4.2 Hysteresis via "insiders" versus "outsiders"

The second potentially important source of hysteresis in steady-state unemployment has led to a more extensive empirical literature. The distinction between employed "insiders" and unemployed "outsiders" in labor markets has been developed in a series of articles by Lindbeck and Snower (1985, 1987, 1988a, 1988b).

The simple story supposes that all wages are set by collective bargaining between employed workers, the insiders, and firms, with outsiders playing no role in the negotiations. Assume that insiders are concerned with maintaining their own jobs, but not the employment of outsiders. Insiders, therefore, set the wage, in implicit or explicit contract, so as to remain employed while extracting maximum monopoly rents. An adverse aggregate demand shock, which reduces employment, will reduce the proportion of insiders. Given the behavior specified, the remaining insiders will seek to raise the real wage sufficiently to maintain the new lower level of employment once the shock dissipates. This lower employment level will then persist as the new steady-state level following dissipation of the shock. On the other hand, a positive aggregate demand shock will serve to expand employment. If the workers, or the union, seek to maintain these new jobs then the average steady-state wage level must fall. Employment, therefore, does not tend to revert to its pre-shock value, but is determined by the history of shocks.

This story is, no doubt, over simplified, but indicates the nature of the problem. If union wage bargaining is prevalent in the labor market, the interaction between the size of the insider group and employment might generate substantial persistence in employment, with little tendency to revert to a mean level. This is closely related to the general issues of union membership and size. To the extent that insider membership is closely linked to being employed there will be an hysteresis effect on employment.

Clearly, in order to explain persistence of unemployment by this mechanism there has to an explanation for why the outsiders do not get jobs in an outside, or non-union, sector at a lower competitive wage. There are at least three oft-cited arguments why, granting the existence of a competitive sector, it is unlikely to absorb all of the displaced labor from the monopolistic sector:

1. Competitive firms may be reluctant to lower wages sufficiently because of the fear of unionization by the current workforce.

2. The union/non-union wage differential might be so high that unionized workers' reservation wages are above the prevailing wage in the competitive sector. In addition, the reservation wage depends upon the mobility cost of shifting sectors, the relative cost of search while employed to search while unemployed, any expected penalty incurred by quitting a job in the competitive sector and the value of leisure, including unemployment benefits. In one sense this unemployment is voluntary, since jobs are available. In another sense it is involuntary, since the workers wish to be employed at existing jobs requiring their skills at, or even below, the wage prevailing in the insider sector. In any case, this queue of unemployed outsiders is a component of the

steady-state level of unemployment.

3. Being unemployed could be useful for getting an insider job if queueing is required or if accepting a low-quality, outside job sends a negative signal to employers.

These features of labor markets which are segmented into insider and outsider groups generate a queue of outsiders attracted to the monopoly wages of the insiders. This queue will shrink permanently in response to higher employment, resulting from a surprise positive aggregate shock, since this reduces the relative wages of insiders, and expands permanently in response to a surprise fall in employment of insiders.

1.5 The relationship between transactions externalities and hysteresis

Although it has gone entirely unnoticed in the literature, there is a close relationship between the seemingly disparate concepts of transaction externalities and hysteresis. First, I will show that transaction externalities in labor markets are also an hysteresis mechanism. Surprise aggregate demand or supply shocks alter relative job seeking and recruitment transaction costs, measured by the ratio of unemployed to vacancies, if these costs are directly proportional to the time required to locate a partner. This permanently alters matching behavior and, hence, the steady-state rate of unemployment.

Second, the two hysteresis mechanisms discussed in the previous section also entail external social costs. For example, higher and longer duration unemployment may lead to increased crime, family breakups, alcohol and other substance abuse, dissaffection from labor force participation, racial and social strife or reduced growth in labor force productivity. When we add to this litany of potential hazards the apparently pervasive transactions externality, it is clear that the steady-state unemployment rate might not be Pareto optimal.

Third, the general macroeconomic and policy conclusions are substantially similar. The steady-state rate of unemployment is not exogeneous with respect to surprise aggregate disturbances. Hence, it is endogenous and cannot be correctly called a "natural" rate of unemployment. Furthermore, the long-run Phillips Curve (LRPC), which macroeconomic orthodoxy argues is vertical, is not stable, in general. While it may be unaffected by expected inflation, movement along the short-run Phillips Curve will tend to shift the LRPC. Therefore, maintaining unemployment above the steady-state rate for a prolonged period will cause that rate to rise, with the attendant slower growth of employment and real output. Vice versa, inducing unemployment to stay below the steady-state rate for a period of time will reverse the process. Clearly, this implies that the effectiveness of monetary policy in reducing currently unacceptable inflation must be assessed with respect to its cost in future higher steady-state unemployment.

Finally, the two phenomena, which have been discussed intuitively, can be rigorously modelled from the basic characteristics of individual interaction in a simple matching model of the labor market. To date the models of transactions externalities in the literature have tended to be highly abstract (particularly Diamond) or very mathematically complex (as Howitt), largely as a result of the unnecessarily sophisticated search framework used. Hysteresis models have tended, on the other hand, to be overly simple and to lack secure microfoundations.[4]

The following body of the thesis is divided into two parts. In part 1 a simple matching model of the labor market is formally developed within a single period. This model

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[4] It appears that only the inside-outside hysteresis mechanism implies the reservation wage property and, hence, requires a search decision model. In this case the simplest standard search model appears to be sufficient.

clearly develops the matching process and the closely related bargaining process, taking into account the pre-match decision of agents with rational expectations. It demonstrates the possibility of transactions externalities leading to multiple equilibria. Part 2 provides a richer model of the labor market and the probability of a match within a multi-period economy. This model analyzes the dynamic stock-flow adjustment of unemployed labor and vacant positions in a discrete time process subject to stochastic disturbances. It shows the relationship between the thin market transaction externality and hysteresis.

PART ONE

Transactions Externality and Multiple Equilibria

in a Single Period Economy

Chapter 2

Transactions Externality and Multiple Equilibria in a Single Period Economy

Before developing a fuller intertemporal model of matching behaviour, which will demonstrate how transaction externalities can generate unemployment hysteresis, I will provide a proof that in a one-period simple matching process, where the probability of a match is less than one, there is no unique equilibrium output and wage. It is shown that the pre-market participation decisions of agents impose external benefits or costs on other agents by affecting whether or not the economy moves to a high employment equilibrium or a low employment equilibrium. Thus, the reciprocal transaction externality is closely related to the problem of multiple equilibria in the matching model.

2.1 The matching process

In this story assume there are just two types of agents, type A and type B, who by matching can produce a good, x. The single period can be thought of as a finite period of time equal to 1. The simple team technology requires a fixed ratio of inputs, one agent of type A and one of type B, to produce a single unit of x within the period. Production also requires the expenditure by each agent of a fixed portion, h, of the total period, where $0 < h < 1$. Assume that agents match only once in the period and are homogeneous as inputs.

In addition to the good x, agents also value leisure, z, which is also measured as a portion of the period ($z \leq 1$). A convenient characterization of agents' preferences is the additive separable specification:

$$\phi(x, z) = \tau x + z \tag{1}$$

where τ is the marginal rate of substitution in consumption between x and z and $\tau > 0$.

The specification of the matching process is a crucial component of matching models that broadly affects wage determination (see Wolinsky (1987)). Let us assume that all agents who decide to participate in the matching process must proceed to a single location, where they are randomly matched such that the number of matches equals the lesser of A and B participants (where A and B are the numbers of type A and B participants). It follows that the probability of being matched for type A and B agents is:

$$\begin{aligned} P_A &= lesser\ of\left(\frac{B}{A}, 1\right).. \\ P_B &= lesser\ of\left(\frac{A}{B}, 1\right). \end{aligned} \tag{2}$$

Assuming, for simplicity, that the value of the product of employment exceeds the value of the leisure given up to produce it, then, if the matching process were costless, all agents would decide to seek a match independently of the match probabilities, since they would be better off if successful and no worse off if unsuccessful. Therefore, let α be a constant cost in time required to match, where $\alpha < 1-h$. Now, each agent faces a choice of whether to match or not.[1]

2.2 Wage bargaining

As Wolinsky (1987) has pointed out, wage determination in matching models requires careful specification of the relationship between the matching process and wage bargaining. While a number of wage bargaining processes could be considered, let us adopt the simplest useful specification. First, assume that agents do not bargain while engaged in the matching process. That is, the matching process is a purely stochastc pairing of type A with type B agents, who proceed to negotiate in a bilateral bargaining process. Since agents only match once, they cannot break off bargaining and seek a new match with an alternative partner. Matched agents may either consummate their match, and divide the surplus, or reject the match. In the latter case, they consume the

[1] The entry decision is called a "pre-market decision" in Marshall's discussion of the origins of demand and supply curves in Principles (book 5, chapter 3).

greater leisure, z, associated with being unmatched.

A solution to this bargaining game that does not incorporate the pre-market participation decision has been derived by Wolinsky (1987) using the concept of a perfect equilibrium. A perfect equilibrum is defined by a pair of strategies, one for each party, such that each strategy is the best strategy for the party after any possible history of the game. Since, in the model developed here, the quality of all matches is the same (including the value placed on not being matched) and search intensity is not a decision variable, the perfect equilibrium is the Nash bargaining solution relative to the disagreement points given by the values attributed by the parties to the prospect of being unmatched.[2]

The value of not consummating the match in the bargaining process we can call the agent's bargaining outside option, denoted by D_i. For the ith agent of type A or B, it is the utility derived from consumption of an additional h amount of leisure when unmatched:[3]

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[2] This is also the solution assumed in the market-matching models of Mortensen (1978, 1982), Diamond and Maskin (1979) and Diamond (1982).

[3] In the bargaining process the utility value of the agent's choices are:

$$Accept: \ \phi = w\tau + (1-\alpha-h)$$

$$Reject: \ \phi = 1-\alpha$$

where $w\tau$ is the wage offer (share of output). By not agreeing to

$$D_i = h \tag{3}$$

Following Wolinsky's general proof, in which D_A for the type A agent may differ from D_B for the type B agent, a perfect equilibrium wage pair for the assumed bargaining process is:

$$W_A = \begin{cases} 0 & \text{if } \tau < D_A+D_B \\ \frac{1}{2} & \text{if } D_A < \frac{1}{2}\tau,\ D_B < \frac{1}{2}\tau \\ \frac{D_A}{\tau} & \text{if } D_A \geq \frac{1}{2}\tau \\ 1-\frac{D_B}{\tau} & \text{if } D_B \geq \frac{1}{2}\tau \end{cases} \tag{4}$$

$$W_B = 1-W_A$$

where τ is the value of the total product realized by matching, which in this case is a single unit of x, from equation (1).

Since, under our special assumptions, $D_A = D_B = h$, equation (4) reduces to the same wage for each party:

$$W = \begin{cases} 0 & \text{if } \tau < 2h \\ \frac{1}{2} & \text{if } \tau \geq 2h \end{cases} \tag{5}$$

---

the match, the agent gains h hours of leisure at the cost of $w\tau$ income. So, the outside option is h. The agent accepts if the value of the outside option is less than the wage offer.

Note that matches of quality $\tau < 2h$ are not consummated, since agents would be better off by not matching. It is well known that the wage of 1/2 is the perfect equilibrium outcome for the bargaining game between two identical agents who have to divide the sum 1 and who possess no outside options. However, if h is smaller than $1/2\tau$ for both parties, then the value of the outside option does not affect the equilibrium outcome because the threat by either party to withdraw and realize her outside option is not credible. In the more general formulation of equation (4), the outside option affects the result when only one party prefers it to the Nash equilibrium that would obtain in its absence. The perfect equilibrium is then a corner solution which gives this party the sum D_i/τ.

2.3 Wage bargaining with a pre-market participation decision

We can extend this analysis to take into account the agents' pre-market participation decisions, which serves to further restrict the perfect equilibrium wage determined in bargaining. The value of the outside option for the participation decision is different than that of the bargaining outside option, but is derived similarly. It is the the value of leisure that an agent expects to give up by deciding to participate:[4]

$$D_i^* = \alpha + P_i h \tag{6}$$

No agent will choose to participate in matching if $D_i^* > P_i w\tau$, since the expected value of the wage is less than the value of the outside option.

Note that D_i^* may be greater or less than D_i depending upon the relative values of P_i and α. The critical difference between D_i and D_i^* is that the probability of a match affects the latter, but not the former. This leads to the following important result: changes in P_i, by changing the expected cost of seeking employment, may alter the expected equilibrium employment level without altering the equilibrium wage.

Assuming that agents know the values of all of the parameters in the model, no agent will choose to participate in matching if $h > 1/2\tau$, since they can already see that no match will be consummated. As a result, the perfect equilibrium wage for this simple case is unique:

$$W = \frac{1}{2}. \tag{7}$$

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[4] The utility of the two choices is:

Don't participate: $\phi = 1$

Participate: $\phi = 1-\alpha+P_i(\frac{1}{2}\tau-h)$

From the foregoing, it can be seen that the following two conditions must be satisfied for an agent to decide to participate in matching:

$$\begin{aligned} &(1) \quad h < \frac{1}{2}\tau \\ &(2) \quad \alpha + P_i h < \frac{1}{2}P_i\tau \end{aligned} \tag{8}$$

However, condition (8.2) alone is sufficient to ensure participation, since if $\alpha > 0$ then $P_i h < 1/2 P_i \tau$ must hold, the P_i cancels and (8.1) is satisfied. Condition (8.2) can be rewritten for the parameters α and P_i:

$$\alpha < P_i\left(\frac{1}{2}\tau - h\right)$$

$$P_i > \frac{2\alpha}{\tau - 2h}$$

It can be seen by inspection that these inequalities will hold for certain acceptable values of the parameters.

2.4 Multiple equilibria

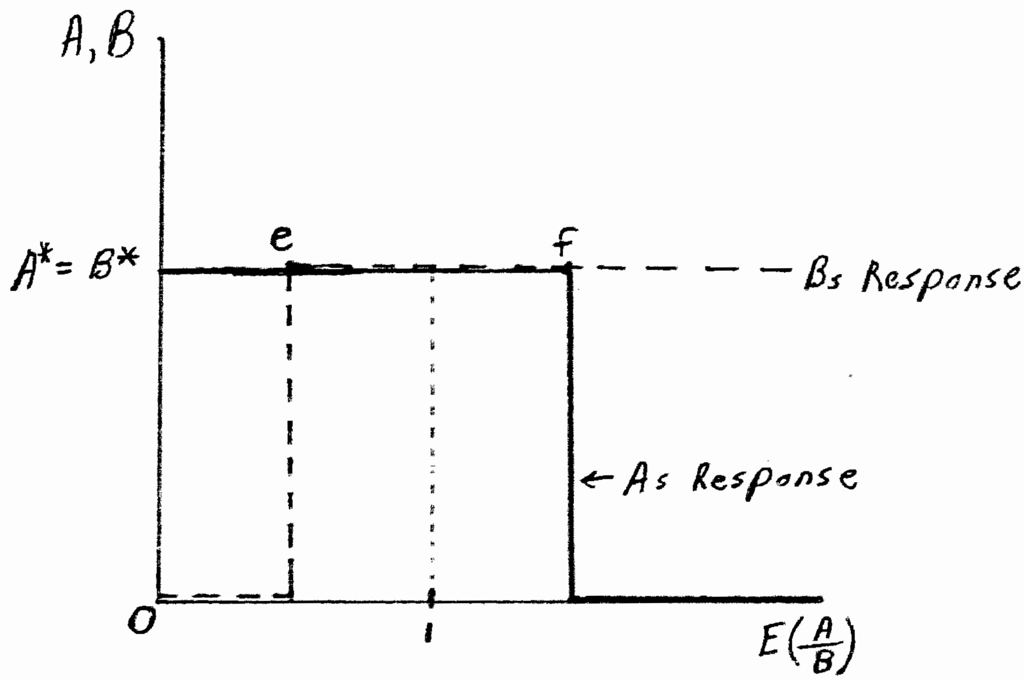
Now, it is reasonable to assume that some positive values of α and P_i are given such that (8.2) holds. But P_i is determined by the relative number of A and B agents who decide to participate in matching. The familiar conundrum here is that an agent's participation decision affects the decisions of all other agents. Consider the case where A > B. If all agents decided to participate, then $P_A < 1$ and $P_B = 1$. It is clear that all type B agents would decide to participate given that type A agents participate. However, type A agents may or may not decide to participate, depending upon whether the value of P_A is sufficiently large for condition (8.2) to hold.

If type A agents decide not to participate, then it would be irrational for type B agents to participate, given full information. In general, the value of P for agents on the long side of the market determines whether all agents seek and, in this special case, consummate a match or all agents refuse to seek a match. In addition, whether or not a type A agent (long side) decides to participate in matching also depends upon how many other type A agents decide to match. We, therefore, have a bootstraps equilibrium, such that everyone searches because everyone else is or no one searches because no one else is. There is at least one high employment equilibrium and one low employment equilibrium in the simple matching model.

Another way to show this multiple equilibria result is to consider the crucial role of expectations. Assume that there are an equal number of potential A and B agents, equal in value to $A^* = B^*$. Figure 1 shows the participation outcome with respect to the **expected** ratio of A to B. On the vertical axis is the actual number of type A and B agents who decide to participate in matching. The horizontal axis measures the expected probability of a match, held by each type of agent.

The decision of type A agents is shown by the solid line. At low values of $\epsilon(A/B)$, all type A agents will participate. As the ratio of expected A to B participants increases, P_A eventually declines. As shown above, there is likely to be a value of P_A less than 1 below which type A agents will decide not to seek to match. This is indicated by point f, where expected $A/B > 1$. The broken line shows the participation decision of type B agents. Since P_B increases as A/B increases, there is, similarly, a value less than 1 above which type B agents decide to participate. This is shown by point e, where expected $A/B < 1$. It follows, therefore, that there is a range of expectations of the ratio A/B (or equivalently of the Ps) between points e and f which yields a high employment equilibrium, where actual participation results in $A^* = B^*$. It can also be seen that expectations of A/B below and above this range will result in a low employment equilibrium.

Diagram 1



PART TWO

Transactions Externalities and Hysteresis in an Infinite Horizon Economy with Workers and Firms

Chapter 3

The Structure of Matching and Bargaining

3.1. Introduction

This chapter expands upon the one-period matching model of the previous chapter in a number of important ways. First, a much richer treatment of the probability of a match is developed, which captures both the thin-market effect of the quantity constraint of the supply of buyers (sellers) on the match probability of sellers (buyers) and the congestion effect of the supply of buyers (sellers) on the match probability of other buyers (sellers). Also, the match probability is made more general by introducing a parameter representing the degree of informational uncertainty.

Second, and more importantly, the model is developed intertemporally to capture the dynamics of transactions externalities over multiple market periods with an infinite horizon. This allows the incorporation of risk-averting savings behaviour and unanticipated wealth effects in the optimal plans of workers, thereby capturing the dynamics of employment and unemployment in the multi-period labor market. This allows a plausible argument for the hypothesis that transactions externalities generate unemployment hysteresis.

3.2. The Matching Process

Assumption 1: The labor market consists of a finite population of two discrete types of individual agents: F firms and L workers. Let j denote an individual firm and i denote an individual worker.

Ass. 2: All production requires coordination of agents - at least one firm plus one worker.

Ass. 3: Individual workers and firms are unaware of the exact location of firms with current job openings or workers currently seeking jobs. Hence, they incur costs in time and resources used to locate transaction partners.

Ass. 4: (i) Each worker may choose to seek or not to seek employment in a market period. Let S = # of job seekers in one market period.

(ii) Each firm may choose to recruit employees or not to recruit employees in a market period. Let J = # of firms engaged in recruitment search over one market period. Let each firm be limited to one job offer in a period, so that J = # of jobs made available in a market period.

Ass. 5: Matching Technology

Agents attempt to match at a given number, C, of locations called contact points. Within a discrete period of time, of given length δ hours, an agent can sample only one contact point, chosen at random. Thus, there is some probability, p, of a successful match in a period, i.e.,

$$0 < p < 1, \text{ as long as } S,J > 0 \text{ and } C > 1.$$

The matching process requires a fixed period of α hours, where $0 < \alpha < \delta$, at the start of a period.

This matching technology may be characterized as a spatially dispersed labor market with imperfect information. The number of contact points (C), relative to the number of agents (L and J), is arbitrarily set to represent the degree of information uncertainty of agents with respect to the location of job offers and searching workers.

Ass. 6: All agents are rational and possess complete, costless information about past and current parameters of the model, except for the location of potential transaction partners.

Ass. 7: For simplicity, all firms and all workers are homogeneous in technical productive characteristics.

Ass. 8: Turnover: All employment contracts vanish at the end of one period. I adopt the following definitions:

(i) U = # of job seekers who fail to obtain a job in a market period.

(ii) Each vacant job is associated with a unique job offer by a unique firm. Thus, V = # of job offers which are left unfilled in a market period.

Therefore, N = S - U is the number of employed workers in a period and N = J - V is the number of producing firms.[5]

3.3. Probability of a Match

The probability of a match for an individual agent is important because it will determine the cost of search perceived by the individual. A match is just a "success" in a binomial distribution. For simplicity I refer only to job seeking workers in deriving the probability of a match for an agent; the probabilities for recruiting firms are exactly symmetrical (replace S with J and vice versa).

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[5] An alternative turnover assumption is to have all employment contracts permanent for the life of the shorter-lived party. During each period a fixed percentage of firms and workers retire at random and an equal number of new potential participants enter as workers and firms. In this context the matching problem affects only a portion of the agents.

In order to arrive at a transactor's unconditional probability of matching in one contact period, note that if only one job seeking worker contacts a particular point, k (where k = 1,...,C), the probability of a match for that worker is:

$$p_s = p(\text{at least 1 of J on k}) = p(J_k \geq 1).$$

where J_k is the number of recruiting firms that contact k.[6] Now, the probability of at least one success in a series of binomial random draws is:

$$p(\text{successes} \geq 1) = 1 - p(\text{successes} = 0)$$

which can be written in general as:

$$p(\text{successes} \geq f) = 1 - p(\text{successes} \leq f-1) \quad (1)$$

for any $f \geq 1$.

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[6] It follows that the probability of at least one match, denoted M, on k is the joint probability of at least one job seeker and at least one recruiting firm contacting that point:

$$p(M_k \geq 1) = p(S_k \geq 1) \times p(J_k \geq 1)$$

which can be written in general as

$$p(M_k \geq f) = p(S_k \geq f) \times p(J_k \geq f)$$

for any $f = 0, 1, 2, \ldots, \infty$.

In general, the probability of exactly m job seeking workers contacting a particular point, k, is:

$$p(s_k = m) = \binom{S}{m} \left(\frac{1}{C}\right)^m \left(\frac{C-1}{C}\right)^{S-m} \tag{2}$$

Thus, the probability of at least one job seeking worker contacting a particular point, k, from (1) and (2) is:

$$p(s_k \geq 1) = 1 - \binom{S}{0} \left(\frac{1}{C}\right)^0 \left(\frac{C-1}{C}\right)^S = 1 - \left(\frac{C-1}{C}\right)^S \tag{3}$$

Note that a given job seeking worker will be matched if there are at least as many recruiting firms on the randomly chosen contact point, k, as there are job seeking workers on k. This probability is the sum of the joint probabilities:

$$\begin{aligned} p(J_k \geq S_k) = {} & (S_k-1 = 0) \times p(J_k \geq 1) \\ & + (S_k-1 = 1) \times p(J_k \geq 2) \\ & + \dots + (S_k-1 = S-1) \times p(J_k \geq S). \end{aligned}$$

This can be written as (following from (2) and (3)):

$$p(J_k \geq S_k) = \sum_{i=0}^{S-1,J} \left[\binom{S-1}{i}\left(\frac{1}{C}\right)^i\left(\frac{C-1}{C}\right)^{S-1-i}\right] \left[1 - \sum_{m=0}^{i} \binom{J}{m}\left(\frac{1}{C}\right)^m\left(\frac{C-1}{C}\right)^{J-m}\right] \tag{4}$$

where the first term in square brackets is the probability that i other job seeking workers also land on k and the second square bracketed term is the probability that at least i+1 recruiting firms also land on k.[7]

A congestion problem occurs, however, if more unemployed workers than recruiting firms land on k. When, for example, two job seeking workers contact a point which is contacted by only one recruiting firm we can suppose that one of the job seeking workers gets the match by the toss of a fair coin. Hence, the probability of a match for each job seeking worker in this case is 1/2. For three job seeking workers and two job openings on a single point the probability of a match for one of the job seeking workers is 2/3, and so on. Thus, the probability that an agent encounters a match under conditions of labor market congestion is the sum of the joint probabilities:

$$1/2\ p(S_k-1 = 1) \times p(J_k = 1) +$$
$$1/3\ p(S_k-1 = 2) \times p(J_k = 1) + \ldots +$$
$$1/S\ p(S_k-1 = S-1) \times p(J_k = 1) +$$
$$2/3\ p(S_k-1 = 3) \times p(J_k = 2) + \ldots +$$
$$J/S\ p(S_k-1 = S-1) \times p(J_k = J)$$

which can be written generally as:

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[7] Note also that the first Σ is to the lesser of S-1, the total remaining number of job seeking workers, and J, the total number of job openings.

$$\sum_{m=1}^{J} \sum_{i=m}^{S-1} \left(\frac{m}{i+1}\right) \left[\binom{S-1}{i} \left(\frac{1}{C}\right)^{i} \left(\frac{C-1}{C}\right)^{S-1-i}\right] \left[\binom{J}{m} \left(\frac{1}{C}\right)^{m} \left(\frac{C-1}{C}\right)^{J-m}\right] \tag{5}$$

which is similar in form to (4).

Adding this probability of a match on a congested point to the probability of a match on an uncongested point, (4), yields the general unconditional probability of a match for an agent:

$$\begin{aligned} p_s = & \sum_{i=0}^{S-1,J} \left[\binom{S-1}{i} \left(\frac{1}{C}\right)^{i} \left(\frac{C-1}{C}\right)^{S-1-i}\right] \left[1 - \sum_{m=0}^{i} \binom{J}{m} \left(\frac{1}{C}\right)^{m} \left(\frac{C-1}{C}\right)^{J-m}\right] + \\ & \sum_{m=1}^{J} \sum_{i=m}^{S-1} \left(\frac{m}{i+1}\right) \left[\binom{S-1}{i} \left(\frac{1}{C}\right)^{i} \left(\frac{C-1}{C}\right)^{S-1-i}\right] \left[\binom{J}{m} \left(\frac{1}{C}\right)^{m} \left(\frac{C-1}{C}\right)^{J-m}\right] \end{aligned} \tag{6}$$

Examining (6), note that if $C > 1$ then $p_s < 1$. An increase in J, holding S constant, raises p_s by raising the probability that at least as many job recruiters as job seekers contact k. Similarly, a rise in the number of job seeking workers, S, lowers the probability that an individual job seeking worker contacts a match by raising the probability that other job seeking workers also land on the same point. Raising C, the analog for the degree of market information imperfection, unambiguously lowers the individual probability of a match, while also reducing the degree of market congestion.[8] Note that the first term in (6), equation (4), while free of congestion by workers, includes congestion experienced by recruiting firms when $J_k > S_k$. Also, p_J is precisely symmetrical to p_s.

3.4. Aggregate Outcome of the Matching Process

Having determined the individual probability of being matched, the aggregate expected outcome of the matching process is all but explicitly stated. By definition, the total expected number of matches for a period, M^e, is the individual probability of a match times the number of individuals seeking a contact:

$$M^e = p_s S = p_J J, \tag{7}$$

where p_s and p_J are the probability of a match for a job seeking worker and a recruiting firm, respectively.

From (7) it follows that:

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[8] Better information will tend to reduce congestion in the sense that agents will match more efficiently. With perfect information, C = 1, everyone will go to one point and the lesser of S or J will be matched. This maximizes p for any given S and J. Increased information uncertainty reduces the expected number of congestion matches, since agents have a lower probability of contact on a point.

$$\frac{p_J}{p_S} = \frac{S}{J} \quad (8)$$

This relationship indicates that a change in the S/J ratio is directly equivalent to a change in the probability of firms finding a match relative to workers. Equation (6), by clearly distinguishing between congestion and non-congestion matches, shows that a rise in J, for example, not only causes p_s to rise in (8), but also p_J to fall (by a lesser amount).

Proposition 1: Assuming C is fixed, given values of S and J in (6) determines unique values of p_s and p_J, thereby fully specifying (8).

Proof: S and J are the sole endogenous variables in (6).

3.5. Transaction Costs

If we assume that some agent has a positive opportunity cost for time expended on contact effort (e.g., foregone leisure), then the stochastic matching process (where $0 < p < 1$), in itself, implies the existence of a transaction cost. We can prove this proposition by contradiction, as follows.

Consider a model combining the assumption that the matching process is costless with the assumption that the probability of a match is less than one (i.e., $0 < p < 1$). If agents have a positive opportunity cost of time, costless transacting implies instantaneous transacting. If, however, matching is instantaneous, each individual can sample repeatedly, within an instant, until she or he obtains the desired contact. Since there is no opportunity cost for attempting to match, all agents interested in employment will successfully match. Hence, no agent will ever be unemployed because of the matching process. But if this is the case, then the probability of contact in any period must be equal to one. This contradicts the original asumption that $0 < p < 1$. Hence, this assumption necessarily implies that matching is costly to some agent.

To formalize this transaction cost, assume that in the discrete time period of length δ hours[9] it takes a fixed number of hours, α, where $0 < \alpha < \delta$, to attempt to match. For a job seeking worker there is, then, an opportunity cost equal to the value of the loss of α hours of leisure in each period. Since $p_s < 1$, there is an expected transaction cost of locating a transaction partner equal to the present value of the expected number of periods required.[10]

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[9] This short discrete time period is some primitive unit, such as a working day.

[10] If $p_s = 1$, the fixed time cost per period would be identical to a unit tax on employment. It will be shown that there is no transaction externality in this case.

Faced with an opportunity cost to transact, firms adjust their contact effort such that, for a given S, profit is zero at the margin. For any lower level of contact effort there are positive expected profits from expending the time required to make a contact. However, the most direct way of modeling the firms transaction cost is to simply assume that resources are expended in the matching process. Firms spend money on advertising, recruitment fees, etc.. In this model I assume that there is some fixed cost incurred by both firms and workers per period of matching activity.[11] Thus, for simplicity I add to assumption 5:

Ass. 5a: Firms engaged in recruitment search incur a fixed cost of α^F units of output (x) in a period.

It follows that the lower is p_s or p_J, the higher is the expected cost of locating a match to the worker or firm, respectively.

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[11] This seems more realistic than assuming that the cost is proportional to the level of employment or output of the individual agents (e.g. Howitt (1985)). Alternatively, and perhaps capturing another important feature of some markets, Okun's (1981) "toll model" regards only matches by new market entrants as costly.

Chapter 4

The Worker's and Firm's Decisions

4.1. The matching entry decision

Let us analyze the behavior of the individual worker in the context of the matching process and resulting transaction costs in a general manner, using the following basic assumptions about workers and firms:

Ass. 9: To simplify aggregation, each agent has an infinite lifetime composed of discrete periods, each of length δ. The agent, therefore, has an infinite planning horizon.[12]

Ass. 10: For simplicity, there is a fixed number of work hours, h, which is a fraction of the market period ($0 < h < \delta$).

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[12] A finite lifetime horizon yields a set of time-specific demand and supply functions, since the individual's planning horizon shortens each period, except in the special case where the subjective rate of time preference is zero (i.e., the marginal utility of income is constant between the present and future periods). Excepting this case, the period t aggregate labor supply function for the finite lifetime model depends upon the age distribution of the labor force.

Note that overlapping finite generations implies an infinite planning horizon if the individual is concerned about the welfare of descendents. The infinite horizon model assures a time independent process, which under certain, quite reasonable, conditions is stable.

Ass. 11: All individual workers are identical in tastes and productivity.

Ass. 12: All firms have identical, given technology.

Ass. 13: Firms produce a homogeneous composite good x.

Ass. 14: The utilities of goods consumed in different periods are independent of each other. Hence, the utility index can be expressed as a sum of independent period utility functions.

Ass. 15: There are two goods: leisure (z) and a non-depreciating produced commodity (x). The individual's consumption plan in each period is defined by total consumption expenditures on x, total hours of z and the total stock of savings (y), which is x not consumed in a period.

Ass. 16: Let the produced good x be the numeraire, with price equal to one.

The individual's total utility function for the infinite, discrete time horizon can be written:

$$\Phi = \sum_{t=1}^{\infty} \beta^t \phi(x_t, z_t) \qquad (9)$$

where x_t: total consumption expenditure on goods in period t

z_t: total leisure consumed in period t

β: time discount factor.[13]

Ass. 17: $\beta = 1/(1+\rho)$, where ρ is the marginal rate of time preference, which is constant, equal for all individuals and greater than zero. This means that for each unit of real income (utility) given up today, we require $(1+\rho)$ units next period to maintain the same level of well-being.[14]

Ass. 18: There is no borrowing or lending between agents.

In general, individuals borrow or lend to smooth out their expected income stream across periods. This process would be stable if there is a binding intertemporal budget

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[13] Assume that ϕ is defined on $[0,\infty)$, is twice differentiable and that $\phi'(x_t) > 0$, $\phi''(x_t) < 0$ for all $x_t \geq 0$ and $\phi'(z_t) > 0$, $\phi''(z_t) < 0$ for all $z_t \geq 0$. Hence, $\phi(x_t, z_t)$ is a strictly monotone and strictly concave function.

[14] Note that this is different from the marginal rate of substitution between consumption today and consumption next period, ϕ_{xt}/ϕ_{xt+1}, which can be called the consumption rate of time preference. The latter is not constant, but depends upon the current consumption level relative to future levels. When $x_t = x_{t+1}$, then $MRS_{xt,xt+1} = \rho$.

constraint. However, the inclusion of a bond market would not necessarily alter the argument of this thesis. It is likely that borrowing and lending will mitigate, to some extent, the size of the intertemporal real wealth changes which generate the employment-unemployment dynamics underlying hysteresis in the unemployment rate, discussed in later sections. However, what is central to the dynamics of intertemporal matching is the saving process. The incentive to save can be introduced through a specific form of the utility function (9).

Given the stochastic nature of the matching process, the individual's future stream of income over the planning horizon is not known with certainty. Depending upon whether a worker actually works during the period, at the end of period t the present value of the individual's assets will be either:

$$y_{t-1} - x_t = y_t \tag{10a}$$

or, $$y_{t-1} + w_t - x_t = y_t, \tag{10b}$$

where y_{t-1}: initial stock of x (from the end of period t-1)

y_t: x left at the end of period t.

w_t: the share of the product of work in period t.

If the individual chooses to be unemployed, i.e. does not seek a contact, then constraint (10a) must hold. However, if the individual seeks employment, constraint (10b) will hold with a probability of p_s and constraint (10a) with a probability of $1 - p_s$. In the absence of borrowing, y_{t-1} and y_t cannot be negative.

Notice that employment in the previous period affects the level of savings carried over to the current period, y_{t-1}. This, in turn, will affect the choices of x_t, z_t and y_t and, hence, the discrete choice between participation or non-participation in the active labor force, S, (i.e., in the matching process). The individual maximizes consumption with respect to the objective function (9) over the infinite future horizon, but has a choice, whether or not to seek employment, that partially determines the effective budget constraint. To make this matching decision the individual will compare the expected utility of the two actions.

The two possible participation decisions that can be made at the beginning of period t can be denoted:

$\lambda_t = 0$: do not participate

$\lambda_t = 1$: participate.

Corresponding to these two decisions are three possible transformations of the stock of savings at the end of t:

y^1_t: when $\lambda_t = 0$

y^2_t: when $\lambda_t = 1$ and $\epsilon_t = 1$

y^3_t: when $\lambda_t = 1$ and $\epsilon_t = 0$

where ϵ is a discrete random variable that takes the value 1 for a successful match and 0 when there is no match. If the individual decides not to participate, $\lambda_t = 0$, the transformation is deterministic and $\epsilon_t = 0$. If the individual decides to participate, the transformation function is probabilistic and $\epsilon_t = 0$ or 1.

In a more general formulation, the individual attempts to maximize (9) subject to the budget constraints:

$$y_{t-1} + w_t - x_t = y_t \tag{11a}$$

$$y_t \geq 0 \tag{11b}$$

and to the additional constraints imposed by the matching technology:

$$\text{if } \lambda_t = 0, \text{ then } w_t = 0 \text{ and } z_t = 1 \tag{12a}$$

$$\begin{aligned} \text{if } \lambda_t = 1, \text{ then } w_t &= 0 \text{ when } \epsilon_t = 0 \\ & w_t \text{ when } \epsilon_t = 1 \\ \text{and } z_t &= \delta-\alpha \text{ when } \epsilon_t = 0 \\ & \delta-\alpha-h \text{ when } \epsilon_t = 1 \end{aligned} \tag{12b}$$

where $\epsilon_t = 0$ with probability $1-p_{s,t}$
1 with probability $p_{s,t}$.

4.2. Dynamic programing formulation of the worker's decision problem

The individual faces a multi-stage decision process in which the choices of λ_t, x_t and y_t must be made simultaneously with the contingent participation decisions λ_{t+1}, λ_{t+2}, ..., since λ_{t+i} (where i = 1, 2, ...) in turn depends partly upon the choice of y_t. The decision λ_{t+i} is contingent on λ_t and, if $\lambda_t = 1$, on the state outcome, ϵ_t. In addition, λ_{t+i} is also contingent on the market state variables, $p_{s,t+i}$ and w_{t+i}, which are exogeneous to the individual. Since ϵ_t and $p_{s,t+i}$ are random (hence w_{t+i} will be random, in general), it can only be expected lifetime utility that is maximized. This provides a deterministic measure of the utility of contingent future decisions in the presence of uncertainty. The optimal decision path for all future periods cannot be determined with certainty in any given period. What we seek, then, is an optimal decision rule which, when applied in every period, yields an optimal outcome in every period.

The state at the beginning of period t is that at the end of t-1. The state variables facing the individual at the beginning of period t are y_{t-1}, $P^e_{s,t-1}$, and W^e_{t-1}, where the latter two variables are vectors of the expectations of p and w for periods t, t+1,..., t+∞ held at the end of t-1. The individual, as an inframarginal actor, is assumed to believe that her current decisions have no effect on the future values of aggregate variables and, hence, forms her expectations independently of her current decisions.

The only decision variable (one over which the individual has control) affecting the current period choices that is state (time period) dependent is y_{t-1}, the stock of wealth carried over from the previous period. Therefore, the maximization problem has the Markovian property: $p\{S_{t+1} = y_t, p^e_{s,t}, W^e_t | S_0 = k_0, S_1 = k_1, \ldots, S_{t-1} = k_{t-1}, S_t = k\} = p\{S_{t+1} = y_t, p^e_{s,t}, W^e_t | S_t = k\}$ for t = 0, 1,... and for every sequence $k_0, k_1, \ldots, k_{t-1}, k$, where p denotes the probability of a state, S, and k is the set of state variables, $p^e_{s,t-1}$, W^e_{t-1} and y_{t-1}. In words, this means that knowledge of the current state conveys all information necessary to determine the expected utility maximizing policy for the current and future periods. An optimal policy for the path of labor market participation, consumption and savings over the horizon of an infinite, discrete period stochastic process possessing the Markovian property can, in principle, be derived using dynamic programming techniques.[15]

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[15] Assume that the set of attainable paths is non-empty. Then, as long as

$$\Phi(x_0, x_1, \ldots; z_0, z_1, \ldots) = \sum_{t=0}^{\infty} \phi(x_t, z_t)(1+\rho)^{-t}$$

is continuous and bounded, there exists an optimal attainable plan for the infinite horizon problem. See R. Beale and T.C. Koopmans, "Maximizing Stationary Utility in a Constant

Note that the matching process is not a Markov chain, since the transformation (or transition) probabilities change over time. The transformation probability, p_s, is stochastic, depending upon the aggregate outcome of the stochastic matching process. In addition, the multistage decision process is not strictly stationary, since the value of p_s affects both the transformation between states and the optimal return (utility) from decisions. For a non-stationary process, the optimal return over N stages starting at time $t = 1$ from state $S_1 = k$ is not necessarily the same as the optimal return over N stages starting at some other time, say $t = 100$, from the same initial state, $S_{100} = k$. However, the expected optimal return is always identical where the given k includes p_s, because the matching technology is constant. In this broader sense, the process determining the values of the state variables is stationary.

A general formulation for the decision problem where uncertainty affects both the transformation of state and the return function in dynamic programming is of little practical value. There is no general solution to such complex stochastic processes (see Jacobs, pp. 89-92). Nevertheless, the general dynamic programming formulation of the problem provides a useful heuristic depiction of the interaction

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Technology," SIAM Journal of Applied Mathematics, 17, Sept., 1969.

between discrete choices and the matching process, which is intuitive and particularly appropriate for a discrete period model. In addition, by adopting a particular specification of the return function (or utility function) that yields analytic tractability for certain restricted forms of the nested contingent choices, an approximate numerical solution may be found.

The contingent choices at the beginning of any period are fully defined by the state variables, y_{t-1}, $P^e_{s,t-1}$ and W^e_{t-1}. Thus, the expected value of total utility depends only upon the given values of these state variables and the series of savings decisions, y_t, y_{t+1}, ..., $y_{t+\infty}$.[16] Since y_t is chosen jointly with λ_t, the optimal choice of y_t also involves the optimal choice of λ_t. The transformation of state function that describes the contingent choice of y_t can be written:

$$y_t = F(y_{t-1},\ P^e_{s,t-1},\ W^e_{t-1},\ \epsilon_t) \tag{13}$$

where the first three state variables determine the participation decision, λ, and ϵ_t determines y_t when $\lambda_t = 1$.

This formulation of (13) assumes that the beliefs held by agents at the start of t are not updated immediately following matching (when ϵ_t is revealed) and before the consumption decision is finalized. This assumption simplifies the dynamic

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[16] In general, the chosen values of y_i are contingent on the values of λ_i and ϵ_i.

programming specification of the matching model considerably. It is also a reasonable restriction, implying that work and consumption are simultaneous within the market period and/or that there is a brief lag in learning the aggregate matching outcome and in updating beliefs about future values of p_s and w. Diagram 2 is a time line indicating exactly what is known, when, and when within the period each decision is made and each state variable changes.

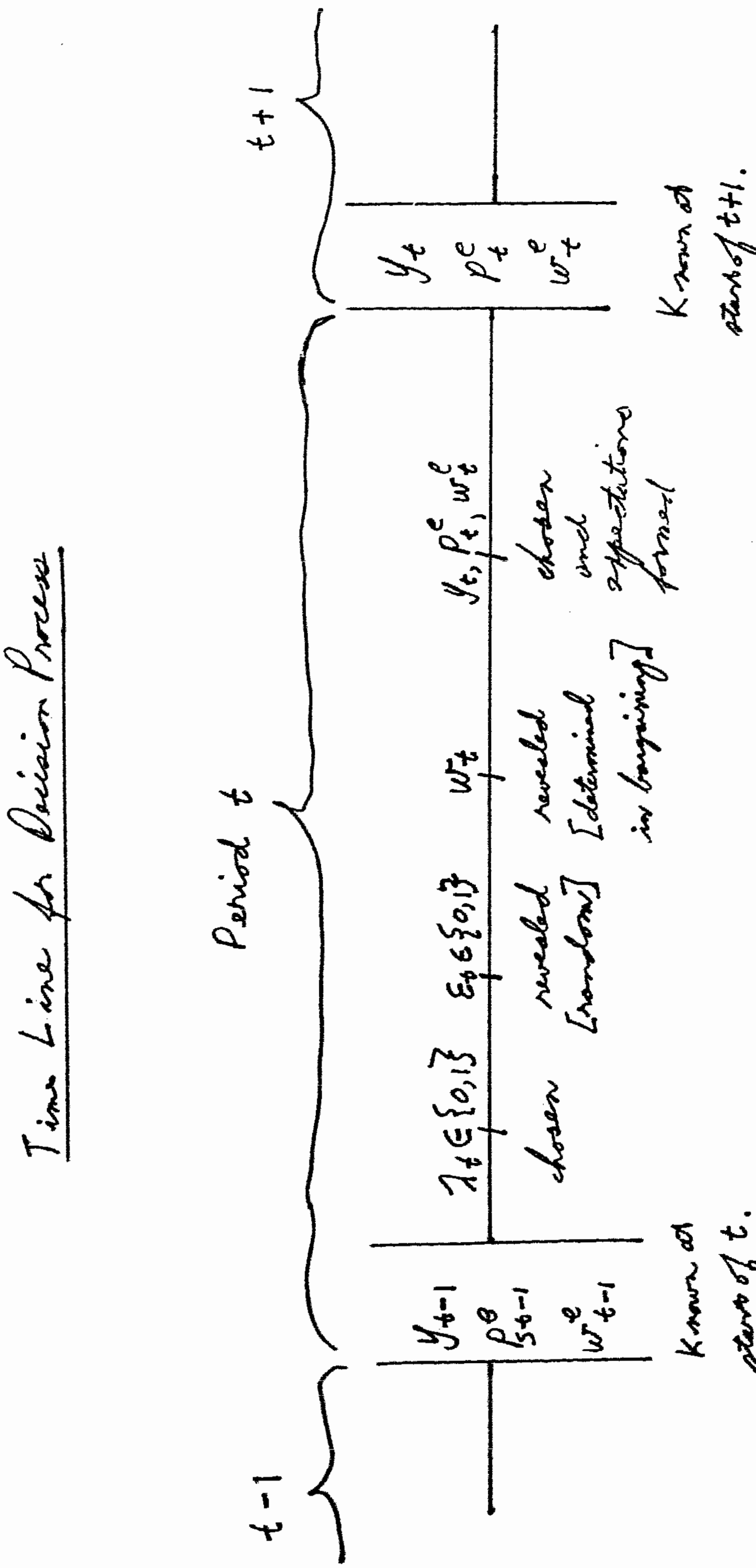
A key to the dynamic programming formulation of the problem is that the transformation function (13) assumes that the optimal decision is made each period. Given the nested character of the participation and consumption-saving choices, it is useful to rewrite (13) as a transformation of contingent states function, which holds at the beginning of each period before ϵ is revealed:

$$y_t = H(y_{t-1},\ P^e_{s,t-1},\ W^e_{t-1}) \tag{14}$$

where y_t takes one of three possible values, depending upon the choice between $\lambda_t=0$ and $\lambda_t=1$, and, in the latter case, whether $\epsilon_t=0$ or $\epsilon_t=1$.

An optimal contingent policy has the property that, whatever the initial state (y_{t-1}, $P^e_{s,t-1}$, W^e_{t-1}) and the optimal first decisions (λ_t, y_t), all future decisions constitute an optimal policy with regard to the state resulting from the first decision. Hence, let us define G as the maximum

Diagram 2


Time Line for Decision Process
Period t
t-1
t+1
y_{t-1} p^{e}_{s+t-1} w^{e}_{t-1}
Known at start of t.
$\lambda_t \in \{0,1\}$ chosen
$\varepsilon_t \in \{0,1\}$ revealed [random]
w_t revealed [determined in bargaining]
y_t, p^{e}_t, w^{e}_t chosen and expectations formed
y_t p^{e}_t w^{e}_t
Known at start of t+1.

expected discounted utility from the beginning of t onward:

$$G(y,\ P^e,\ W^e) = \max E[\sum_{i=t}^{\infty} \beta^i \phi(x_i,\ z_i) | y,\ P^e,\ W^e] \qquad (15)$$

where the t-1 subscripts have been dropped for convenience.

Substituting the constraints (11) and (12) into (15), the value function, G, satisfies the following recursive relationship, which is a functional equation of the optimal current and future values:[17]

$$G(y,P^e,W^e) = E_{P^e,W^e}\Big[\underset{\lambda}{MAX}\Big\{\underset{x}{MAX}\{\phi(x,\delta) + \beta G(y-x, P^e_{st}, W^e_t)\},$$
$$\underset{x}{MAX}\, E_\epsilon\{\phi(x,\delta-\alpha-\epsilon h) + \beta G(y+\epsilon w-x, P^e_{st}, W^e_t)\}\Big\}\Big] \qquad (16)$$

The first expression inside the curly brackets refers to the decision λ_t=0 and the second refers to λ_t=1. This equation tells us that the optimal policy is one that maximizes the expected value of the current and all future contingent labor and consumption choices, given the values of the expectations vectors, P_s^e and W^e. Note that, while these expectations are expected to change as a result of the stochastic outcome of aggregate market matching, it is assumed that the individual's decisions alone have no effect on their values. Therefore, P_s^e and W^e are not themseves contingent on the individual agent's decisions or luck at matching.

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[17] This is Bellman's "functional recurrence equation".

Note the P_{st}^e and W_t^e state variables on the right-hand side. These represent the updated beliefs the agent will hold <u>after</u> the uncertainties of period t are resolved (held at the beginning of t+1). Their values depend upon how the agent updates these beliefs, which we assume can be described by a learning rule. Some possible learning rules that may be considered are:

a). Fixed: $p_{t,t+i}^e = \bar{p}^e$, for all t and $_i$ = 1, 2, ..., where the first subscript represents the period in which the expectation is held and the second is the period for which it is held.

b). Static: $p_{t,t+i}^e = \bar{p}^e$, but p^e adjusts to p_t before t+1, unanticipated by the agent.

c). Myopic: $p_{t,t+i}^e = p_t$

d). Adaptive: $p_{t,t+i}^e = \sigma p_{t-1,t+i}^e + (1-\sigma)p_t$

e). Rational Expectations: the correct unbiased expectation, given an understanding of the equilibrium structure of the model, including inferences about the decisions of others (which, with identical agents, means inferences about the y's of others).

The fixed learning rule is, of course, really the absence of any learning process. Under this rule, P_{st}^e and W_t^e are invariant and, therefore, are not state dependent variables. This leaves y_{t-1} as the only state variable. The difference between the static and myopic learning rules does not lie in the value to which expectations adjust.[18] Rather, under static learning agents do not anticipate their expectations ever changing, while under myopic learning they do.

The general problem in (16) is conceptually solved with respect to the present expected values of future optimal decisions. These future optimal decisions are contingent, first, on the labor supply and consumption decisions made this period; second, on the individual's success in the stochastic matching process in this and intervening future periods; and, third, on the future values of p_s and w. Of course, expected future values of p_s will determine the expectation of ϵ.

4.3. Some general properties of the worker's decision

The general workers' choice problem is well defined in (16), given one of the above learning rules. The optimal policy would define λ^* as a function of y_{t-1}, P_s^e and W^e, and x^* as a function of y_{t-1}, P_s^e and W^e. Hence, G is a function of y_{t-1}, P_s^e and W^e. Because of the Markovian property, λ^*, x^* and G are independent of t. The λ^* can be characterized by a region in y_{t-1}, P_s^e and W^e space. The subset of this space where $\lambda^* = 1$

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[18] Since the p_t in the myopic and adaptive expectations formulations are not observed in the model, the individual's inference of this value based on the observed outcome of the matching process may be substituted.

we can call A. Hence, A complement is $\lambda^* = 0$.

What does A look like? Different perspectives are obtained by considering the boundary between A and its complement, where an agent decides to search or not. This boundary is delineated by the critical value of each of the state variables that determines the agent's choice between $\lambda = 0$ and $\lambda = 1$. These critical values, y_{t-1}^*, p_{st}^*, p_{st+i}^{e*}, w_t^* and w_{t+i}^{e*}, where $i = 1, 2, \ldots, \infty$, are alternate ways of describing the boundary of subset A. As pointed out previously, a general analytic solution to an optimization problem of this complexity has never been derived. It is likely that only numerical methods could be used to solve for these critical values. This is primarily due to what is called the free boundary problem, which results from the interdependence of y_{t-1}^*, p_{st}^*, p_{st+i}^{e*}, w_t^* and w_{t+i}^{e*}.

The critical values, w_t^* and w_{t+i}^{e*}, are the agent's current and future reservation wages. A general argument for the existence of a reservation wage runs as follows: when $p_{st} = 1$, there is a positive value of w_t below which the individual will never choose to seek employment because of the greater positive value of leisure when $\lambda_t = 0$; as p_{st} approaches 0, this value approaches ∞. The values of w_t^* and w_{t+i}^{e*} are single-valued for a given distribution of w. However, if the distribution of w is endogenously determined by the matching outcome and, therefore, uncertain, then these terms represent reservation distributions of w, as defined by their probability density functions, f(w).

From the characteristics of (16), discussed above, it can also be seen that the individual's reservation wage, w_t^*, will vary inversely with the expected value of p_{st}, but positively with the expected <u>future</u> values of p_s. This latter result is an intertemporal income effect; a higher future p_s increases the expected value of the future participation option and, hence, will increase expected lifetime income.

Since all workers are identical and the market parameters and variables are common to all,[19] the only factor leading some workers to decide $\lambda_t = 0$ and others $\lambda_t = 1$ is the state variable, y_{it-1}, which shifts the individual's labor-leisure trade-off. Since an increase in p_{st} or in w_t raises the value of seeking a match relative to not seeking a match in the current period, it can be conjectured that an increase in either of these variables causes the value of y_{t-1}^* to increase. Similarly, p_{st}^* is reduced by a higher value of w_t or y_{t-1}. These results will be shown explicitly in the next section for a simpler, more tractable specification of the matching model.

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[19] This would not be true if the agent's expectations are based on her own past matching experience.

From the above considerations we can define the individual's "labor search supply function" as the relationship between the expected wage rate (or expected wage distribution) and the hours of labor that the individual is prepared to supply at that wage, taking into account the transaction costs of matching that must be incurred. Since the work period is a constant, h, the individual's labor search supply function, S, is a simple step function corresponding to the two possible match participation decisions in (16). For the worker i:

$$S_{it} = \begin{array}{l} 0, \ if \ w_t^e < w_{sit}^* \\ 1, \ if \ w_t^e \geq w_{sit}^* \end{array} \tag{17}$$

where w^*_{sit} is the reservation wage (or family of wage distributions) below which the individual will choose $\lambda_t = 0$. This reservation wage will vary between individuals with identical tastes because y_{it-1} varies.

4.4. Analytically tractable specification

A general solution to (16) is unlikely to be very useful, since it can be seen that the qualitative effects of the variables affecting the labor-consumption decision depend crucially on the individual's labor-leisure preferences. In order to evaluate the problem more meaningfully, it is useful to specify the utility function, ϕ, so as to make it analytically tractable.

One useful form of ϕ is the simplest function of x and z that has the property of decreasing marginal utility of consumption within a period. This standard property generates an incentive to smooth out the intertemporal consumption path through saving, subject to the value of β and, therefore, maintains the general recursion relationship established in (16). Hence, I will specify the utility function (9) further, such that all individuals have the same additive exponential function.

For the analysis of this section, I greatly simplify the problem by placing the individual in a single period lifetime, with sequential generations, rather than an infinite lifetime. Each individual lives for one period and leaves a bequest for a single offspring. The wealth endowment y_{t-1} is, then, the value of the bequest from one's parent. This formulation of the problem has two advantages which render the solution to the choice problem more tractable: 1) it eliminates the complexity of the optimal intertemporal pattern of work and leisure, and 2) it allows a full analysis of the contingent nature of the decision, where realized values of p_s and w may differ from the expected values, while avoiding the additional complexity of the contingency on future expectations.

The sequential, single period generations model preserves the representative agent property of the infinite horizon model, since all individuals always belong to the same cohort. This is in contrast to the overlapping generations model, where the representative agent property is destroyed by the separation of agents into two distinct types. According to Farmer (1993), the representative agent property is the key characteristic distinguishing the infinite horizon model from the overlapping generations model. For this reason, it seems likely that the major qualitative results of the simplified approach used in this section should not differ from the more general infinite horizon model.

The modified utility function for the one period lifetime with inheritance and bequest is:

$$\phi = x^{\gamma} + z^{\gamma} + B^{\gamma} \tag{19}$$

where $0 < \gamma < 1$ and the individual's utility is also an additive increasing function of the bequest, B.

The agent faces the actual budget constraint:

$$x + B = y + \lambda \epsilon w \tag{20}$$

where the y refers to the inheritance from the previous period. However, this constraint is a function of the agent's decision for the λ value, while the values of ϵ and w are uncertain. The key point to notice is that the consequences for y of the agent's λ decision is contingent on the actual outcomes of ϵ and w. Therefore, the expected values of ϵ, which is p_s, and w cannot simply be plugged into (20) to determine λ.

For the given value of the state variable y, the agent must evaluate each possible outcome. The optimal consumption path for the match decision λ=0 is deterministic. The leisure consumed is a constant, δ. Maximizing (19) subject to the budget constraint

$$y - x - B = 0 \tag{21}$$

yields the result that the individual will split her inheritance equally between consumption, x, and bequest, B:

$$x = B = \frac{1}{2}y$$

It follows that

$$\frac{\partial x}{\partial y} = \frac{\partial B}{\partial y} > 0$$

and the cardinal utility of consumption associated with the λ=0 decision is:

$$\Phi_{\lambda=0} = 2\left(\frac{y}{2}\right)^{\gamma} + \delta^{\gamma} = 2^{1-\gamma}y^{\gamma} + \delta^{\gamma} \tag{22}$$

Note the following important properties of (22):

$$\frac{\partial\phi}{\partial y} > 0, \quad \frac{\partial\phi}{\partial\delta} > 0, \quad \frac{\partial\phi}{\partial\gamma} > 0 \ if\ \delta \geq 3$$

Now, if the agent were to decide $\lambda=1$, she maximizes utility subject to the budget constraint:

$$x + B = y + \epsilon w \tag{23}$$

The optimal consumption decision is, again, to split $y+\epsilon w$ equally between x and B. There are two possible values of ϵ and an infinite number of possible states of w that will determine the actual final consumption outcome.

It is not possible to simply use the expected value of w in (23) because the agent will decide to accept or reject a match depending upon the actual w that is realized. That is, there is a bargaining reservation wage, w', such that only for $w \geq w'$ will the individual accept a match when $\epsilon=1$. Maximizing utility for each of these two possible outcomes, it follows that

$$\begin{aligned} &\text{if } w \geq w', \text{ then } \phi = 2^{1-\gamma}(y+w)^{\gamma} + (\delta-\alpha-h)^{\gamma} \\ &\text{if } w < w', \text{ then } \phi = 2^{1-\gamma}y^{\gamma} + (\delta-\alpha)^{\gamma} \end{aligned}$$

Setting these two expressions equal yields the bargaining reservation wage:

$$w' = [y^{\gamma} + 2^{\gamma-1}((\delta-\alpha)^{\gamma} - (\delta-\alpha-h)^{\gamma})]^{\frac{1}{\gamma}} - y \tag{24}$$

which is always positive.[20]

Note the following properties of (24):

$$\frac{\partial w'}{\partial y} > 0, \quad \frac{\partial w'}{\partial \alpha} > 0.$$

The effect of the transaction cost, α, on the bargaining reservation wage is surprising. It follows from the fact that a higher value of α increases the relative value of the bargaining outside option, leisure when $\lambda=1$, given diminishing marginal utility.

It is clear that the agent's actual employment and consumption outcome is contingent on the value of w (similarly on the value of ϵ). Hence, in general, the matching entry decision must take into account the possibility of w being greater or less than w' and ascribe optimal utility values to each possibility. The expected value of the decision can be found by solving the dynamic programming problem:

$$\underset{\epsilon, w}{E}[\Phi_{\lambda=1}] = (1-p_s)[2^{1-\gamma}y^\gamma + (\delta-\alpha)^\gamma] + p_s \underset{w}{E}\, MAX\{2^{1-\gamma}(y+w)^\gamma + (\delta-\alpha-h)^\gamma, \\ 2^{1-\gamma}y^\gamma + (\delta-\alpha)^\gamma\} \quad (25)$$

[20] Proof: Since $(\delta-\alpha)^\gamma > (\delta-\alpha-h)^\gamma$, let $2^{\gamma-1}((\delta-\alpha)^\gamma-(\delta-\alpha-h)^\gamma)$ be represented by the positive constant a. By inspection,

$$[y^\gamma+a]^{\frac{1}{\gamma}} > y$$

for any positive value of a.

The expected utility on the left-hand side of (25) is with respect to the expected values of ϵ and w. The posssible values of ϵ are 0 with a probability of $1-p_s$ and 1 with a probability of p_s. To solve for the expected value of the choice to accept or reject the match, based on w, when $\epsilon=1$, it is necessary to know the complete distribution of w. Since a fully rational agent knows her w', she can determine the probability that $w \geq w'$ occurs and the converse.

Since (25) is not linear in w and the conditional distribution of $w \geq w'$ is not symmetrical (hence, the mean stochastic disturbance is not 0), certainty equivalence does not hold. The individual must evaluate the area under the distribution above w' to obtain the best value for w that falls in this region. The expected utility of the $\epsilon=1$ outcome in (25) can, therefore, be expressed as the sum of the utility times the probability that $w < w'$ and the utility times the probability that $w > w'$:

$$\underset{w}{E}\, MAX\ \{2^{1-\gamma}(y+w)^{\gamma}+(\delta-\alpha-h)^{\gamma},\ 2^{1-\gamma}y^{\gamma}+(\delta-\alpha)^{\gamma}\}$$

$$= \int_0^{\infty} MAX\ \{\ldots,\ldots\}\ f(w)\,dw \qquad (26)$$

$$= \int_0^{w'} [2^{1-\gamma}y^{\gamma}+(\delta-\alpha)^{\gamma}]\ f(w)\,dw + \int_{w'}^{\infty} [2^{1-\gamma}(y+w)^{\gamma}+(\delta-\alpha-h)^{\gamma}]\ f(w)\,dw$$

where f(w) is the probability density function of the distribution of w.

Taking the distribution of w as given, the key qualitative properties of (26) are:

$$\frac{\partial \phi}{\partial y} > 0, \quad \frac{\partial \phi}{\partial \alpha} < 0, \quad \frac{\partial \phi}{\partial p_s} \gtrless 0$$

Note that $\partial\phi/\partial p_s > 0$ when $\lambda=1$ dominates (is preferred to) $\lambda=0$ with $p_s=1$, for the given values of y and w. Otherwise, the individual is better off if she fails to match, in which case she would never choose $\lambda=1$. In addition, it can also be seen that an upward shift in the entire distribution of w will increase $\phi_{\lambda=1}$.

Clearly, the agent chooses $\lambda=1$ over $\lambda=0$ if (25) $\geq$ (22). Some important properties determining the agent's matching choice are revealed by comparing (25) with (22). An increase in y unambiguously increases the value of (22) by more than (25), which indicates the positive wealth effect on the value of leisure. Second, an increase in the expected distribution of w raises (25) relative to (22), as expected. Third, a higher value of α reduces (25) relative to (22). Finally, an increase in p_s increases (25) over (22), if (25) is preferred to (22) when $p_s=1$ for the given values of y and w.

These key properties of the decision process allow us to determine the change in an agent's state resulting from stochastic or exogeneous shifts in the parameters. They will be used later to derive the dynamic properties of the aggregate matching model.

An important characteristic of the agent's decision problem is the existence of a matching entry reservation wage, w^*, below which (22) is preferred to (25). This reservation wage delineates between two specific plans, one of which entails the decision to seek a match and the other not. In general, the w^* above which an agent chooses $\lambda=1$ over $\lambda=0$ can be derived by comparing the optimal plan for the decision $\lambda=1$ with the decision $\lambda=0$. We can solve for w^* by setting (25) equal to (22):

$$(1-p_s)\,[2^{1-\gamma}y^{\gamma}+(\delta-\alpha)^{\gamma}] + p_s\left[\int_0^{w'}[2^{1-\gamma}y^{\gamma}+(\delta-\alpha)^{\gamma}]\,f(w)\,dw + \int_{w'}^{\infty}[2^{1-\gamma}(y+w^*)^{\gamma}+(\delta-\alpha-h)^{\gamma}]\,f(w)\,dw\right] = 2^{1-\gamma}y^{\gamma}+\delta^{\gamma} \qquad (27)$$

where the value of w' is given by (24).

Solving (27) for w^* would be tedious and messy. The important qualitative properties of w^* can be found by inspection and comparison with (24), which determines w'. These are:

$$w^* > w',\ \frac{\partial w^*}{\partial y} > 0,\ \frac{\partial w^*}{\partial \alpha} > 0,\ \frac{\partial w^*}{\partial p_s} < 0.$$

First, it can be seen that $w^* > w'$ must always hold because of: 1) the higher value of the outside leisure option in the case of the market entry decision ($2^{1-\gamma}y^{\gamma}+\delta^{\gamma}$ versus $2^{1-\gamma}y^{\gamma}+(\delta-\alpha)^{\gamma}$), and

2) the non-zero probability of failing to match. A higher y raises w^* because y^γ increases relative to $(y+w)^\gamma$ and w' increases. A higher fixed transaction cost, α, raises the relative value of the outside option and reduces the value of $(\delta-\alpha)^\gamma$ by more than $(\delta-\alpha-h)^\gamma$. A higher value of p_s unambigously raises the value of seeking to match and has no effect on the outside option value.

As previously pointed out, there are also critical values of y^* and p_s^* which serve to separate the $\lambda=0$ from the $\lambda=1$ decision. One may solve for both of these values in the same way as for w* above, using (27). Since this approach is complicated by the distribution of the expected wage in (27), it is useful to derive the critical values in the much simpler case where w^e is known with perfect foresight. This strong assumption will be shown to be reasonable when we consider the wage bargaining process with rational expectations and an endogeneous match entry decision.

In the case where agents have perfect foresight of w, (27) can be written as

$$(1-p_s)\ [2^{1-\gamma}y^\gamma+(\delta-\alpha)^\gamma] + p_s\ [2^{1-\gamma}(y+w^*)^\gamma+(\delta-\alpha-h)^\gamma] = 2^{1-\gamma}y^\gamma+\delta^\gamma \quad (28)$$

Here, w' disappears because no agent would consider matching if she were not prepared to accept a match at the known wage. This expression can be solved for w^*, y^* and p_s^*.

The value of w^* from (28) is

$$w^* = \left[y^\gamma + 2^{\gamma-1}[(\delta-\alpha)^\gamma - (\delta-\alpha-h)^\gamma + \frac{1}{p_s}(\delta^\gamma - (\delta-\alpha-h)^\gamma)]\right]^{\frac{1}{\gamma}} - y \qquad (29)$$

It can be seen that $w^* > 0$, since $(y^\gamma+a)^{-\gamma} > y$ for $a > 0$. In addition, it can be confirmed that $\partial w^*/\partial y > 0$ and $\partial w^*/\partial p_s < 0$.

The value of y^* from (28) is

$$y^* = \left[\frac{1}{\gamma}w^{1-\gamma}\left[\left|\frac{1}{p_s}2^{\gamma-1}[\delta^\gamma - p_s(\delta-\alpha-h)^\gamma - (1-p_s)(\delta-\alpha)^\gamma] - w^\gamma\right|\right]\right]^{\frac{1}{\gamma-1}} \qquad (30)$$

Since this is a quadratic expression with $1/(\gamma-1) > 0$, the absolute value of the inside square bracketed expression is used. It can be ascertained that when

$$w > \left[\frac{1}{p_s}2^{\gamma-1}[\delta^\gamma - p_s(\delta-\alpha-h)^\gamma - (1-p_s)(\delta-\alpha)^\gamma]\right]^{\frac{1}{\gamma}}$$

then λ_1 is strictly preferred to λ_0, $y^* > 0$, and $\partial y^*/\partial w > 0$. In addition, it can be seen that $\partial y^*/\partial p_s > 0$.

Similarly, the value of p_s^* from (28) is

$$p_s^* = \frac{\delta^\gamma - (\delta-\alpha)^\gamma}{2^{\gamma-1}(\gamma y^{\gamma-1}w^{\gamma-1} + w^\gamma) + (\delta-\alpha-h)^\gamma - (\delta-\alpha)^\gamma} \qquad (31)$$

which is positive for values of y and w that ensure that λ_1 is strictly preferred to λ_0. When $p_s^* > 0$, then $\partial p_s^*/\partial y > 0$. In addition, $\partial p_s^*/\partial w < 0$ for all $w > 0$.

The dynamic behavior of the sequential generations process is generated by the bequest, B, which transmits the effect of events in generation t to generation t+1. The values of B that results from each possible outcome are:

$$\begin{aligned} &For\ \lambda = 0: \quad B = \tfrac{1}{2}y \\ &For\ \lambda = 1: \quad if\ \epsilon = 0: B = \tfrac{1}{2}y \\ &\qquad\qquad\qquad if\ \epsilon = 1: B = \tfrac{1}{2}(y+w) \end{aligned} \tag{32}$$

Hence, when an agent chooses not to match or fails to match, $B < y$. When an agent matches successfully, then $B > y$ if $w > y$ and $B < y$ if $w < y$.

Now, consider how the decision of an agent in t affects the decisions of her offspring in t+1, t+2, ..., t+∞. Note that $B = y_t$, where $y = y_{t-1}$ under the notation of the last section. In general, $\lambda=0$ for all successive generations over the infinite future cannot be optimal, since the lineage's stock of wealth asymptotically declines to zero. This causes the w^* of an agent's offspring to fall below w in some future period, the expected number of which can be computed.

Similarly, for $\lambda=1$, the wealth of the succeeding generation is kept constant only in the coincidental event that $y = w$. When $y > w$, then $B < y$, so the offspring is poorer. However, it follows that $y \leq w$ will occur in some future period, for any positive value of w. In the likely event that $y < w$ occurs, then $B > y$ results and the stock of savings begins to increase.

In the event that $y < w$, so $B > y$, the stock of savings increases in successive generations until either: 1) w^* rises above w (since $\partial w^*/\partial y > 0$) and the agent chooses $\lambda=0$, or 2) $y > w$ occurs and, therefore, $B < y$. When $w^* > w$ is a possibility, then the labor force participation path of successive generations is a recurring pattern of a number of periods of seeking to match followed by a number of periods out of the labor force.

To conclude the discussion of this analytically tractable specification of the matching model, it seems likely that the qualitative results will hold for the infinite horizon model, although I have no proof. The only substantive difference between the model of sequential one period generations and the more general model of infinitely lived agents is the incentive to more fully smooth out the consumption path over time in the latter model, taking into account the future expected values of p_s and w. How this smoothing works can be seen by briefly considering a two-period lifetime.

The modified utility function for the simple two period lifetime with inheritance and bequest is:

$$\Phi = x_t^\gamma + z_t^\gamma + \beta x_{t+1}^\gamma + \beta z_{t+1}^\gamma + \beta B_{t+1}^\gamma \tag{33}$$

where the individual's utility is an additive increasing function of the bequest, B_{t+1}.

The agent faces the actual budget constraint:

$$x_t + x_{t+1} + B_{t+1} = y_{t-1} + \lambda_t \epsilon_t w_t + \lambda_{t+1} \epsilon_{t+1} w_{t+1} \tag{34}$$

However, this constraint is a function of the agent's decision for the λ values, while the values of ϵ and w are uncertain. The key point to notice is that the agent's λ_{t+1} decision is contingent on the actual outcome of ϵ_t and w_t. Therefore, the expected value of ϵ_t, which is p_{st}, and w_t, which is $w_{t,t}^e$, cannot simply be plugged into (34) to determine λ_{t+1}.

Now, adopting the strong assumption that w in each period is known, the optimal solution to maximizing (33) subject to (34) can be found numerically by solving for all of the possible sets of decisions and outcomes, and then finding the λ_t decision that yields the greatest expected utility over the lifetime. Over the two periods there are four possible sets of participation decisions yielding nine possible outcomes:

The individual at time t doesn't need to decide λ_{t+1} until the next period. Hence, she compares the expected utility from $\lambda_t=0$ with $\lambda_t=1$. The expected utility from $\lambda_t=0$ is the greater of the expected lifetime utilities associated with each of the possible period t+1 participation choices. In this case, when $\lambda_{t+1}=0$, the optimal consumption path and associated utility is deterministic and easily solved. When $\lambda_{t+1}=1$, the expected utility is the weighted average of the utilities associated with the optimal consumption path for

$$\text{if } \lambda_t=\lambda_{t+1}=0, \quad x_t+x_{t+1} = \tfrac{3}{4}y_{t-1}$$

$$\text{if } \lambda_t=0,\ \lambda_{t+1}=1, \quad x_t+x_{t+1} = \tfrac{3}{4}y_{t-1}+w_{t+1} \text{ with probability } p^e_{t,t+1}$$

$$x_t+x_{t+1} = \tfrac{3}{4}y_{t-1} \text{ with probability } 1-p^e_{t,t+1}$$

$$\text{if } \lambda_t=1,\ \lambda_{t+1}=0, \quad x_t+x_{t+1} = \tfrac{3}{4}y_{t-1}+w_t \text{ with probability } p^e_{t,t}$$

$$x_t+x_{t+1} = \tfrac{3}{4}y_{t-1} \text{ with probability } 1-p^e_{t,t}$$

$$\text{if } \lambda_t=\lambda_{t+1}=1, \quad x_t+x_{t+1} = \tfrac{3}{4}y_{t-1}+w_t+w_{t+1} \text{ with probability } p^e_{t,t}p^e_{t,t+1}$$

$$x_t+x_{t+1} = \tfrac{3}{4}y_{t-1}+w_t \text{ with probability } p^e_{t,t}(1-p^e_{t,t+1})$$

$$x_t+x_{t+1} = \tfrac{3}{4}y_{t-1}+w_{t+1} \text{ with probability } (1-p^e_{t,t})\,p^e_{t,t+1}$$

$$x_t+x_{t+1} = \tfrac{3}{4}y_{t-1} \text{ with probability } (1-p^e_{t,t})(1-p^e_{t,t+1})$$

each possible outcome of ϵ_{t+1}, weighted by the probability of each outcome shown above. The expected utility for $\lambda_t=1$ is found, similarly, as the greater of the expected utilities associated with the two possible t+1 decisions. The agent then chooses the participation decision, λ_t, that yields the greater expected utility.

4.5. The Firms' Decision Problem

The objective of the firm is to maximize profit. Firms are initially assumed to be price-takers in labor and product markets. Assumptions 9 to 18 provide a general framework for the nature of the firm. The firm's expected profit function for the infinite, discrete time horizon can be written generally as:

$$\pi^e = \sum_{t=1}^{\infty} \lambda_t [b_t^e q_t^e - c_t^e - \alpha^F] \qquad (35)$$

where π: firm's expected lifetime profit

b_t^e: expected price of x in period t

q_t^e: expected quantity of x produced by firm in period t

c_t^e: expected cost of producing 1 unit of x in period t.

In the absence of a bond market the firm's expected lifetime profit is unbounded (approaching ∞). However, this is not a problem, since the lifetime expected profit in (35) is the sum of expected profit in each period. Given this separability, the firm's hiring decision each period is independent of all other periods. While the hiring transaction cost, α^F, is a constant, the firm has to decide each period to search (λ_t=1) or not (λ_t=0) and, therefore, to incur this cost or not.

We add one additional assumption to those above:

Ass. 19). All firms are risk neutral.

The individual firm assumes that its current decisions have no effect on future values of w and p_j. Therefore, its own recruitment in each period is independent of that in previous or future periods and the firm will seek to hire a worker when

$$\pi_t^e \geq 0 \tag{36}$$

The firm's production function can be specified in the simplest useful manner by adding to assumption 13:

Ass. 13b: The firm's given technology utilizes fixed factor proportions and labor is the only input. Let output, q_t, equal the quantity of labor input.

Since ϵ_t indicates the stochastic outcome of a matching attempt:

$$q_t = \epsilon_t, \tag{37}$$

where $\epsilon_t = 1$ if the firm hires successfully

(i.e., if $\lambda = 1$ and $\epsilon = 1$)

$\epsilon_t = 0$ if the firm does not hire

(i.e., if $\lambda = 0$ or if $\epsilon = 0$).

If a firm chooses to match, one of two outcomes occurs:

if $\epsilon_t = 0$, then $\pi_t = -\alpha^F$

if $\epsilon_t = 1$, then $\pi_t = 1 - w_t - \alpha^F$.

The firm's expected profit from seeking to recruit in any period is then:

$$\pi_t^e = p_{Jt}^e(1-w_t^e) - \alpha^F \tag{38}$$

It follows from (38) that a firm will attempt to match if

$$1-w_t^e \geq \frac{1}{p_{Jt}^e}\alpha^F \tag{39}$$

Hence, we can write the recruitment decision criterion with respect to the given expected wage rate as:

$$w_t^e \leq 1 - \frac{\alpha^F}{p_{Jt}^e} \tag{40}$$

This yields the firm's recruitment labor demand function:

$$J_{mt} = \begin{cases} 0, & if\ w_t^e > 1 - \dfrac{\alpha^F}{p_{Jt}^e} \\ 1, & if\ w_t^e \leq 1 - \dfrac{\alpha^F}{p_{Jt}^e} \end{cases} \tag{41}$$

Note that (41) establishes a direct relationship between the acceptable wage and the probability that the firm will locate a match. The firm will not seek to match if its share of the product in the event of a match is expected to be less than 1 minus the ratio of the firm's transaction cost to the expected probability of finding a match. If $p^e_{Jt} = 1$, then the wage rate need only be greater than or equal to the transaction cost to induce the firm to attempt to recruit. It can also be seen that $\alpha^F < p^e_{Jt}$ is a necessary condition for the firm to attempt to match. In more general terms, if the ratio of α^F to total revenue is equal to or greater than p^e_{Jt}, then $\pi^e \geq 0$ cannot hold for any $w^e_t > 0$.

Chapter 5

Market Equilibrium and Wage Determination

5.1. Equilibrium in the market period

The stick-figure depiction of the firm presented in the last chapter is intended to capture the essence of the market transaction for labor required for production in a modern market economy. The firm is an agent that facilitates the transformation of labor into a consumption good. Since all firms are assumed to be identical, they all have identical recruitment labor demand functions. This means that all firms will make the same decision, either to attempt to match in a period or not, with respect to a common market wage. The market recruitment demand for labor function is simply the summation of the F individual demand functions (41), which yields the step function:

$$J_t = \begin{cases} 0, \; if \; w_t^e > 1 - \dfrac{\alpha^F}{p_{Jt}^e} \\ F, \; if \; w_t^e \leq 1 - \dfrac{\alpha^F}{p_{Jt}^e} \end{cases} \tag{42}$$

For a given value of p^e_{Jt}, recruitment labor demand is a negative function of the wage rate. However, it is not possible to determine a unique recruitment labor demand with respect to price alone, holding other prices and income constant, as in the case of the neoclassical labor demand function. As (42) shows, recruitment labor demand is also directly affected by the quantities transacted in the market, which are not independent of the wage; p_{Jt} is endogenously determined, according to equation (6), by the ratios S_t/C and J_t/C, which are, in turn, endogenous with respect to w.

Consider a single firm. Rewriting (8) in terms of p_J yields:

$$p_J = \frac{S}{J}\, p_S .$$

Since J takes the discrete values of 0 and F, let J = F, which is a constant. Then, substituting for p^e_{Jt} in (41), we can express W^*_{Jt} as:

$$w^*_{Jt} = 1 - \frac{\alpha^F}{\left(\frac{S^e}{F}\right) p^e_{st}} \qquad (43)$$

But, p_s is itself a function of the level of match activity, S, for a fixed level of J. From (6) it is known that a rise in S will reduce p_s somewhat through congestion, but by less than the increase in p_J through the thin-market effect. Therefore, it follows:

<u>Proposition 2</u>: A rise in the expected level of S, for a given value of J, causes the firm's reservation wage, w^*_J, to increase, i.e, $\partial w^*_{Jt}/\partial S^e > 0$.

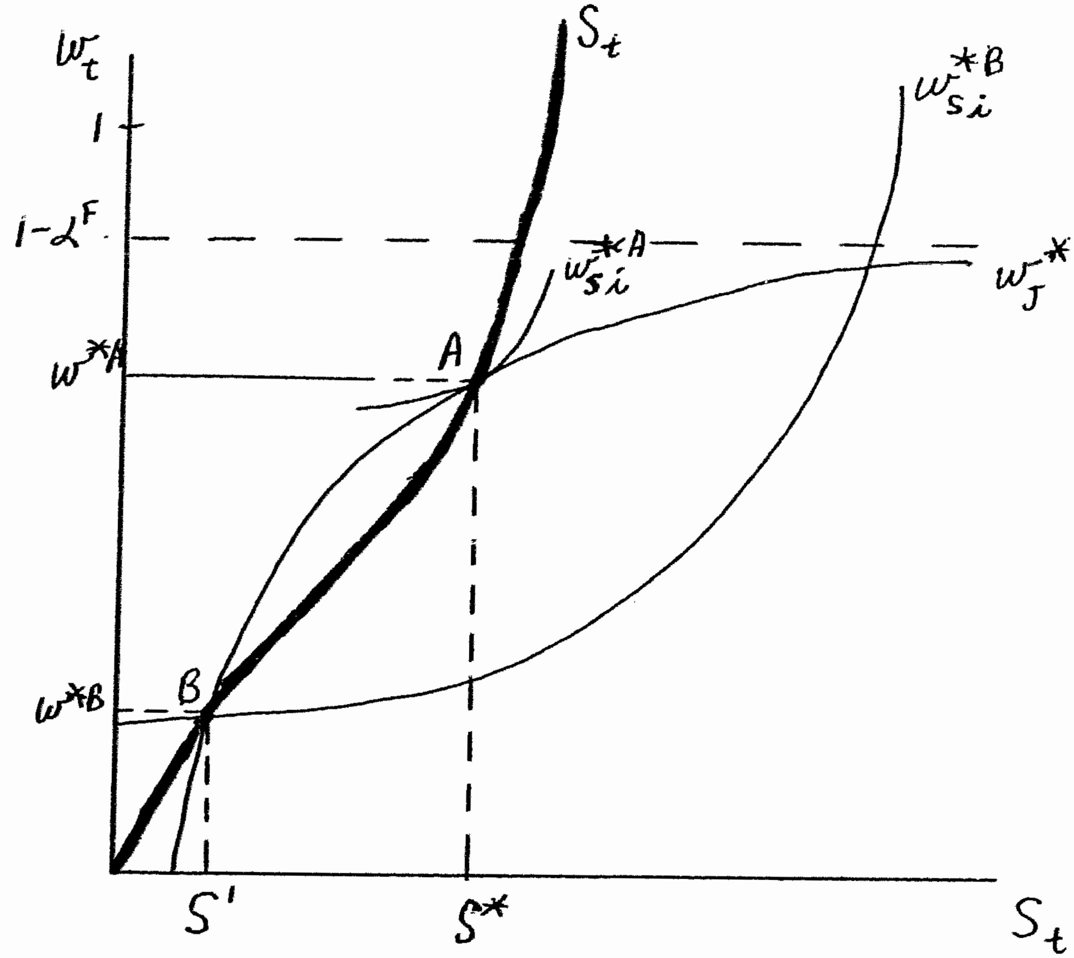
Argument: Sustitute for p^e_{Jt} in (41) from (6). Extensive numerical simulations of (6) show that an increase in S, ceteris paribus, unambiguously raises p^e_J, which reduces w^*_J in (41).

This positive feedback is a result of the external benefit to firms of a larger number of potential hirees. It also follows that, for a given level of S, a rise in J will cause w^*_J to fall.

The J_{mt} function, (41), defines a locus of the firm's reservation wage rates, w^*_{Jt}, in wage rate - expected number of match seeking workers (S^e_t) space, as shown in diagram 3. It can be seen by inspection that α^F/p_J can be greater or less than 1 for a positive value of S^e and that $1-(\alpha^F/p_J)$ asymptotically approaches $1-\alpha^F$ as S^e increases. Numerical simulations of (43) for various values of F and C indicate that w^*_J is monotonically positive and decreasing with respect to S^e, as shown in diagram 3. For any value of S^e on the x-axis, the locus of w^*_J defines the wage rate above which the firm will not seek to match and at or below which the firm will decide to seek a match. As S^e increases along the x-axis, p^e_s decreases and p^e_J increases at a more rapid rate.

Of course, since all firms are identical in all respects, the w^*_J locus in diagram 3 is common to all firms. The w^*_J

Diagram 3



curve delineates the wage rate above which no firm will seek to match and at or below which all firms will seek to match.

The aggregate labor search supply function is, similarly, the summation of the L individual labor search supply functions, shown in (17). If all potential workers had the same y_{t-1}, the aggregate labor search supply function would be:

$$S_t = \begin{cases} 0, & if\ w_t^e < w_{st}^* \\ 1, & if\ w_t^e \geq w_{st}^* \end{cases} \quad (44)$$

While (44) is also a simple step function, w^*_{st} is similarly a function of p^e_{st}, as shown in equations (27) or (29), which is, in turn, endogenous with respect to S and J by (6).

Consider an individual potential worker. To compare with the firm's w^*_{Jit} in (41), w^*_{sit} in (17) can be expressed in general functional terms as:

$$w_{sit}^* = 1 - f(\tfrac{a}{\delta}, \tfrac{h}{\delta}, p_s^e) \quad (45)$$

where the period is normalized to one. This means that the worker will not seek to match if the expected share of the product is less than 1 minus the value of the leisure given up to transact and the expected value of the leisure given up to work, both adjusted by the probability of matching.

Since w^*_{sit} from (27) or (29) is a negative function of p_{st} and p_{st} is a negative function of S, the locus of w^*_{sit} below which a worker will refuse to seek a match is monotonically positive and increasing with respect to S^e, as shown in diagram 3. There is a positive intercept on the y-axis if the individual has positive y_{t-1}, and the curve asymptotically approaches vertical as the degree of congestion causes p_s to approach 0.

Examining diagram 3, it can be seen that for a particular worker, the locus of w^*_{sit} may or may not intersect the locus of w^*_{Jt}. In general, if the two curves intersect, they will intersect twice, defining an elliptical area, a core, within which any combination of wage and number of match seekers will induce the particular worker to seek a match.

In general, y_{t-1} will vary among otherwise identical workers because of their stochastic work histories. As shown by (27) and (29), a lower (higher) value of y_{t-1} for an individual monotonically shifts the w^*_{sit} schedule down (up) and to the right (left), as the agent will seek a match at a lower (higher) wage for a given match probability. There is, therefore, a family of w^*_{sit} curves for a given distribution of y_{t-1} values.

From diagram 3, we can derive the aggregate labor search supply function, S_t, using the following important assumption:

Ass. 20: All workers share the same expectations of S^e_t and J^e_t.

The next assumption merely simplifies the diagrammatic exposition:

Ass.21: y_{it-1} is continuously distributed.

From (17) it follows that the aggregate S_t is the number of workers for whom $w_t \geq w^*_{it}$. Now, if the distribution of y_{t-1} is given by the distribution function, $g(y_{t-1})$, then the aggregate S_t can be written as the integral of $g(y_{t-1})$ from 0 to y_{t-1}^* for all L potential workers:

$$S_t = L\int_0^{y^*_{t-1}} g(y_{t-1})\, dy_{t-1} \equiv L\, G(y^*_{t-1}) \qquad (46)$$

which is equivalent to L times the cumulative density function of y_{t-1}^*, $G(y_{t-1}^*)$.

As shown in the last section, y_{t-1}^* is a function of w_t and p_{st}^e. However, with the number of recruiting firms fixed at F, p_{st}^e is fully determined by the expected number of match seeking workers, S_t^e, as shown by (6). Therefore, in functional form,

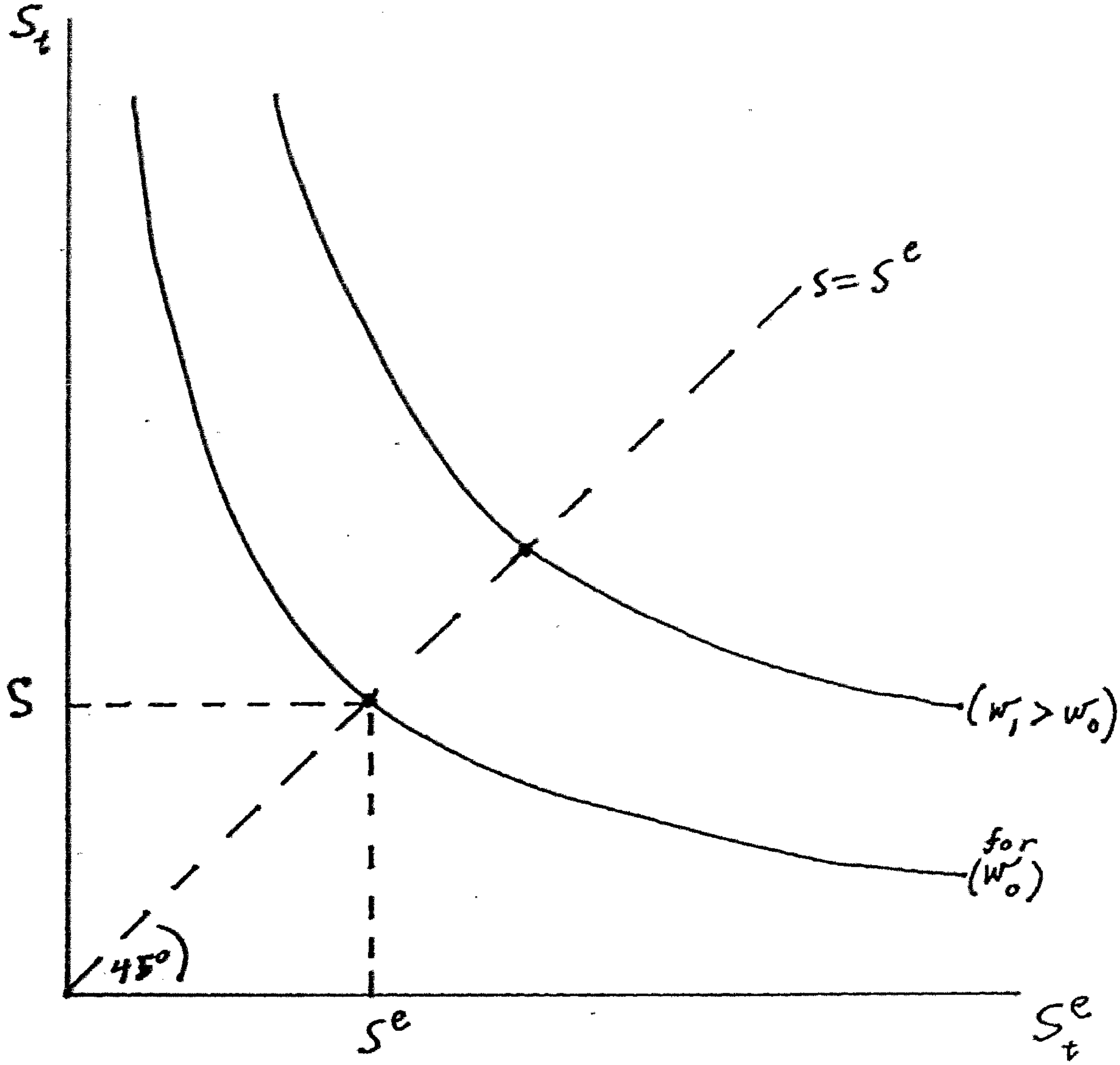
$$S_t = L\, G(y^*_{t-1}) = L\, G\big(y^*_{t-1}(w_t, S^e_t)\big) \qquad (47)$$

where $\partial y^*_{t-1}/\partial w_t > 0$ and $\partial y^*_{t-1}/\partial S^e < 0$, as previously shown.

Diagram 4 plots the number of workers who choose to enter the matching process in a period, S_t, against the number of workers who are expected to enter, S_t^e. The downward sloping curve for a given value of w_t, w_0, is equation (47). For a higher value of w_t, $w_1 > w_0$, this curve shifts to the right. Since all agents share the same S_t^e, the only level of S for which expectations are rational is where $S_t = S_t^e$. With this outcome all agents who would choose to seek a match for the given values of w_t and S_t^e and the given distribution $G(y_{t-1}^*)$, actually seek to match.

In diagram 3, it follows that the aggregate S_t curve is derived from the condition that the number of individual w^*_{sit} curves added vertically must equal the value of S^e_t on the x-axis. Therefore, the aggregate labor search supply function in diagram 3 will be positively sloped, like the individual w^*_{sit} curves, but will intersect the y-axis at 0, if $y_{t-1} = 0$ for some worker, and have everywhere a steeper slope. Also, because of the concavity of the w^*_{sit} curves, S_t approaches vertical at some value of S_t^e. S_t can also be interpreted as the aggregate reservation wage offer curve for various values of S^e_t, and, consequently, of p^e_{st} and p^e_{Jt}. It is the locus of the w^*_{sit} of the marginal worker who is indifferent to seeking a match at the given value of S^e_t. It also indicates that a higher expected value of S increases the reservation wage below which each individual worker will not seek to match.

Diagram 4



Now we adopt an even stronger assumption about expectations than that of assumption 20, in order to derive some important equilibrium results:

Ass. 22: Workers and firms share the same expectations of the level of S_t and J_t.

Given this assumption, the field of w^*_{sit} curves is tangent to the w^*_{Jt} curve at point A in diagram 3, so $w^*_{sit} = w^*_{Jt}$. At this point there are S^*_t expected match seeking workers, according to the w^*_{Jt} curve. Since the individual w^*_{sit} curve tangent to this point represents the agent with the highest value of y_{t-1} who would consider seeking a match for the given w^*_{Jt} schedule, then this agent is the S^*_tth worker. Therefore, the aggregate S_t curve passes through this point. S^* is the maximum number of workers who might seek to match in the period with the given w^*_{Jt} function. S^* workers will seek to match at the unique wage, w^*, if the anticipated probabilty of a match is less than or equal to the unique value, p_s^* given by S^* and F match seekers. However, at w^* no firm will seek to match if S_t is less than S_t^*, which implies a lower value of p_J.

The aggregate labor search supply curve also passes through point B, where S'_t offer curves added vertically also yield $w^*_{sit} = w^*_{Jt}$. At this point only S'_t workers seek to match. This is the minimum positive number of match seekers that is feasible, since for any lower value of S^e_t, and associated p^e_{st} and p^e_{Jt}, no firm will be willing to seek a match at the reservation wage demanded by workers.

Now, considering the S_t and w^*_{Jt} curves jointly, we note the following important properties:

a. below and to the left of point B, $w^*_{st} > w^*_{Jt}$,
b. in the elliptical area defined by the S_t and w^*_{Jt} curves, between point B and point A, $w^*_{st} < w^*_{Jt}$, and
c. above and to the right of point A, $w^*_{st} > w^*_{Jt}$.

These properties follow from the differing impact of the quantities offered for transaction on the matching costs of the two types of agents. The increase in S, holding J constant, implies a thin-market external benefit to the firms and a congestion external cost to workers.

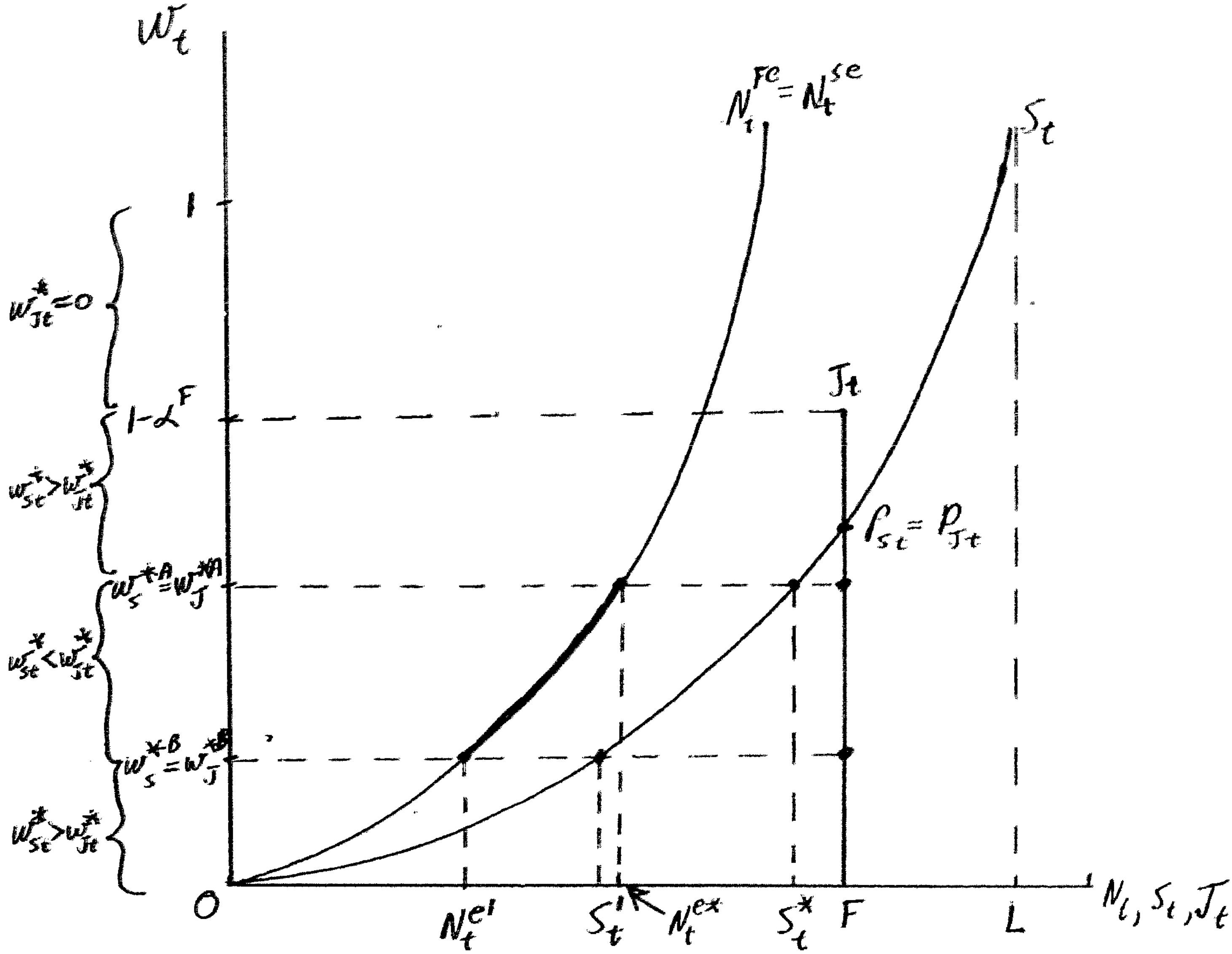
Diagram 3 clearly confirms the potential for multiple equilibria in the matching model, as shown in chapter 2 in a simpler context. There are three points at which $w^*_{st} = w^*_{Jt}$, including the degenerate equilibrium at the origin with 0 match seekers. Whether agents within the feasible core are driven to transact at A can only be determined through an analysis of the wage bargaining processes that are consistent with the matching technology. This issue will be considered in the next section.

Assuming, for a moment, that all successful matches are consummated at a single wage, note that there is no unique, Pareto optimal level of match seeking activity within a period. At S^*_t, the only feasible wage is w^{*A}_t. Those workers who would have decided to seek a match at a lower wage will be better off if they successfully match above their reservation wage. However, a greater number of those who would have sought to match at some lower wage are expected to be unemployed at S^*_t because of the lower match probability. Therefore, moving from any point on the aggregate S_t curve to any other point is expected to generate both winners and losers.

Now, we can illustrate J_t, the aggregate labor recruitment demand function, (42), and S_t, the aggregate labor search supply function, (47), in wage-quantity space, as in diagram 5. S_t, J_t and N_t, which denotes employment, are shown on the x-axis. This diagram helps to show an important proposition.

From (42), J_t will be vertical at F below the particular reservation wage, w^*_j, that corresponds with the expected value of S_t. Therefore, the height of the J_t line in diagram 5 is dependent upon the particular value of S^e_t. An increase

Diagram 5


w_t
1
$w^*_{Jt}=0$
$1-\alpha^F$
$w^*_{St}>w^*_{Jt}$
$w_S^{*A}=w_J^{*A}$
$w^*_{St}<w^*_{Jt}$
$w_S^{*B}=w_J^{*B}$
$w^*_{St}>w^*_{Jt}$
$N_t^{Fe}=N_t^{se}$
S_t
J_t
$P_{St}=P_{Jt}$
O
$N_t^{e'}$
S_t'
N_t^{ex}
S_t^*
F
L
N_t, S_t, J_t

(decrease) in the expected labor search supply, S^e_t, raises (reduces) the value of w^*_{Jt} at or below which all firms will seek to hire. Once again, this is due to the positive feedback of quantity decisions by potential workers on the expected hiring costs of firms.

The S curve in diagram 5 is the same as that derived in diagram 3. The higher is the value of S^e anticipated by a worker, the higher is the reservation wage below which the worker will not seek to match. Again, assuming that workers and firms share the same expectations of S_t and J_t, then at S'_t and at S^*_t, $w^*_{st} = w^*_{Jt}$, so the height of the J_t line is equal to the height of the S_t curve at these points. As explained with respect to diagram 3, for values of S_t below S'_t $w^*_{st} > w^*_{Jt}$, for S_t between S'_t and S^*_t $w^*_{st} < w^*_{Jt}$, and for S_t greater than S^*_t, $w^*_{st} > w^*_{Jt}$.

Diagram 5 helps to illustrate the following proposition:

<u>Proposition 3</u>: Within a period, $S^* = F$ is not necessary for $w^*_s = w^*_J$ to hold, and, in general, is not true. That is, a feasible equilibrium does not require labor search supply to equal labor recruitment demand.

<u>Proof</u>: Consider the condition $w^*_s = w^*_J$. From (43) and (45), this condition can be written as:

$$1 - \frac{\alpha^F}{\left(\frac{S_t^e}{F}\right)p_{st}^e} = 1 - f\left(\frac{a}{\delta}, \frac{h}{\delta}, p_{st}^e\right)$$

Assuming, for convenience, that the expectation of S_t is correct and dropping the period subscript, this can be simplified to read:

$$F\frac{\alpha^F}{p_s} = Sf\left(\frac{a}{\delta}, \frac{h}{\delta}, p_s\right) \tag{48}$$

Clearly, when F = S, then $w^*_s \neq w^*_J$, since, in general,

$$\frac{\alpha^F}{p_s} \neq f\left(\frac{a}{\delta}, \frac{h}{\delta}, p_s\right).$$

The intuitive explanation of this result is that the transactions costs to the firm and to the worker are not the same, in general.

5.2 Effective supply and demand

At the beginning of a period the actual number of matches is unknown, even if S_t and J_t are known. Labor matching decisions, indicated by the S_t and J_t offers to transact at a particular expected wage, generate only expected values of the numbers of agents that will be able to transact, N_{st}^e and N_{Jt}^e. These will be referred to as the expected effective labor supply and expected effective labor demand. These may be defined as

$$N_t^{se} = p_s S_t$$
$$N_t^{Je} = p_J J_t \qquad (49)$$

The expected level of employment, N_t^e, is also stochastic.

Note that $p_{st}S_t$ is a random subset of S_t, the aggregate labor search supply function (47). Hence, in diagram 5 the N_t^{se} function is the S_t function shifted to the left by the proportion p_{st}. Similarly, the subset $p_{Jt}J_t$ reduces the number of firms being aggregated from F to $p_{Jt}F$, when $w_t \le w^*_{Jt}$. The actual N_t will lie between, and could include, 0 and the lesser of S_t and F.

The following proposition may now be stated:

<u>Proposition 4</u>: $N_t^e \equiv N_{st}^e \equiv N_{Jt}^e$ for any S_t and J_t functions.

<u>Proof</u>: The values of N_{st}^e, N_{Jt}^e and N_t^e are associated with unique values of S_t, given constant J. From (7) and (8),

$$N_t^e = M_t^e = p_{st}S_t = p_{Jt}J_t$$

Since $N_{st}^e = p_{st}S_t$ and $N_{Jt}^e = p_{Jt}J_t$, by definitions (49), then it is always true that $N_{st}^e \equiv N_{Jt}^e$.

This property of the model follows from the assumption that only one worker matches with one firm in a successful match.[21] In diagram 5 a unique N_t^e curve is associated with each combination of S_t and J_t functions.

5.3 A wage bargaining equilibrium

A major objection to the assumption of a unique equilibrium wage in the matching model is that, once potential transaction partners have been matched, there is no apparent mechanism that drives them to settle the wage on the basis of the postulated S_t and J_t curves.[22] The agents who locate a match are effectively in a one-on-one bargaining game; no other agents can affect this negotiation within the current market period. In fact, this is at the very heart of the notion of transactions costs; in order to bargain with other agents or to sample a distribution of wage offers, an agent must incur an opportunity cost. Having matched within the

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[21] In a more general sense, it can be seen from section 3.3 that there may be more or less workers than firms landing on a single contact point in a period. Therefore, in aggregate, there can be more or less workers than firms making contact. The probability of a firm contacting a point with one or more workers is given by (3); the probability of a worker landing on a point with one or more firms is symmetrical. These probabilities are independent of the number of agents of one's own type, but there remains a congestion externality if the number of matches on a point is restricted to the lesser of the firms and workers who land there. The worker's decision problem and the wage bargaining process become more complex in this context.

[22] This objection also applies to the effective supply, N_t^s, and effective demand, N_t^d, curves derived in the previous section.

market period, it is to be expected that agents will negotiate for the best deal possible subject only to the particular constraints that affect the one-on-one bargaining.

This problem complicates the matching model considerably in the general setting. However, in the model developed here, I will argue that the wage rate negotiated by every matched pair is w^*_{Jt} if we adopt the following strong, but reasonable, assumption about the information available to agents:

Ass. 23: The recruitment reservation wage of the firm for all values of S_t is known to all agents. The search reservation wage of each individual worker is known only to that individual. However, all agents know the distribution of the w_{ts}^*, i.e., the labor search supply function, S_t.

This assumption is consistent with the simple structure of the matching model. The asymmetry between the knowledge of an individual characteristic and the knowledge of an objective constraint common to all firms gives a strategic advantage to the worker in the bargaining game.

Let me recapitulate briefly the structure of the matching and bargaining process. Agents do not bargain while engaged in the matching process. The matching process is a stochastic pairing of participants, who then proceed to negotiate in a bilateral bargaining process. Agents only match once in the period, so they cannot break off bargaining and seek an alternative partner. Matched agents may either reach a bargain, and divide the surplus, or reject the match. In the latter case, workers incur the loss of α and consume a greater amount of leisure associated with being unmatched, while firms incur the loss of α^f.

There are a number of potential bargaining outcomes that may be considered. First, it can be argued that the equilibrium wage settlement will be $w = 1$ because the transaction cost to the firm is sunk after the firm has matched. In this case, the argument goes, the firm has no credible threat point below $w = 1$ because rejecting any such offer would not minimize its period costs.

The problem with the $w = 1$ settlement is that it implies that firms always incur a loss when they successfully match. If workers know that firms will not refuse a wage approaching 1, none will settle for less than $w = 1$. This implies, assuming that firms' budget constraints are time consistent over the infinite horizon, that there is no positive-valued equilbrium matching outcome. In the matching entry decision, no rational firm will decide to seek to match given $w = 1$ as the expected bargaining outcome.

As briefly mentioned in chapter 2, the notion of a "pre-market decision" appears in Marshall's discussion of the origins of demand and supply curves in Principles (book 5, chapter 3). Gale (1985) argues for the general proposition that the strategic bargaining approach yields the competitive equilibrium wage and employment outcome when there is a market entry decision and the supply-demand curves are interpreted as information about the sellers and buyers who consider entering the market. Without endorsing this far-reaching conclusion, the idea that wage determination should be endogeneous within the context of the market, or matching, entry decision by rational agents is powerful and useful. In chapter 2, the perfect equilibrium solution for the one-on-one bargaining game incorporating the pre-market participation decision was derived for the case of perfect, symmetrical information with agents who had identical threat points.

Incorporating the pre-market entry decision into the wage bargaining process for the more general intertemporal model of workers and firms leads to the following proposition:

Proposition 5: A perfect equilibrium wage in the intertemporal labor market matching model with workers and firms is $w^*_{Jt} = 1-(\alpha^F/p_{Jt})$, which is consistent with a positive employment outcome. This requires each firm to pre-commit to reject any wage demand greater than w^*_{Jt}.

<u>Argument</u>: Firms will decide to seek to match iff

$$w_t^e \le w_{Jt}^* \equiv 1 - \frac{\alpha^F}{p_{Jt}}$$

It would be irrational for a firm to seek to match if it were prepared to turn around and accept a wage greater than w^*_{Jt}. In this case, $w^e_{Jt} > w^*_{Jt}$. Given the assumption of asymmetric information, each worker will refuse to accept a wage below w^*_{Jt} (a bluff for all but one) because they know that no firm will ultimately refuse to employ a worker at w^*_{Jt}. Hence, a positive equilibrium wage must be w^*_{Jt}. With a positive equilibrium outcome, all workers have the option to seek employment; hence, given the choice repeatedly over an infinite horizon all workers will have a higher utility consumption path, as shown in chapter 3. Firms are indifferent between the wage outcomes $w = 0$ and $w = w^*_{Jt}$. Therefore, the perfect equilibrium wage, consistent with optimal bargaining strategies for any period, is w^*_{Jt}.

An important result following from this special bargaining solution is that all agents who are successful in matching will agree to transact on the w^*_{Jt} curve in diagram 3. Moreover, it suggests a unique wage-expected employment outcome:

Proposition 6: All agents attempting to transact in the feasible core of diagram 3, except at point B, would be driven by a hypothetical tatonnement process to transact at the wage w_t^{*A}.

Argument: Consider a given expected value of S_t, such that $S'_t < S_t^e < S^{*t}$. This is consistent with $w^*_{st} < w^*_{Jt}$ associated with S_t^e. With rational expectations, knowledge of the wage bargaining process and knowledge of the firms' w^*_{Jt} curve, $S_t > S_t^e$ would decide to match, expecting w^*_{Jt}. But, this reduces p_{st} and, hence, increases w^*_{Jt} further, inducing even more workers to seek to match. This process iterates until $w^*_{st} = w^*_{Jt}$ at A.

To show that point A is a locally stable equilibrium, meaning that actual matching entry equals expected matching entry, we also need to consider what happens when S_t^e is greater than S^{*t}. In this case, $w^*_{sit} > w^*_{Jt}$. Those workers who would not be prepared to work for w^*_{Jt} will not attempt to match, reducing S_t and, therefore, reducing w^*_{Jt} and increasing p_{st}; this process iterates until $w^*_{st} = w^*_{Jt}$ at A.

Point B is a locally unstable, or knife-edge, equilibrium. For $S_t > S'_t$, we have argued that a tatonnement process would converge to point A. For $S_t < S'_t$, we have a similar situation as for $S_t > S^{*t}$. In this case, the process converges to the degenerate equilibrium of $S_t = 0$.

The degenerate outcome, $S_t = 0$ and $w_t = 0$, is also locally stable from the perspective of the hypothetical tatonnement wage-adjustment mechanism. However, as argued above the positive wage-expected employment outcomes at A and B are both strictly preferred to the no activity outcome.

We conclude that there are three possible equilibrium positions for the economy under the assumed matching and bargaining processes, two of which are locally stable. Note that, according to an interpretation of $w^*_{st} = w^*_{Jt}$ as a "market-clearing" condition and the long-run profit constraint

$$\sum_{t=1}^{\infty} \pi_t^e = 0,$$

each of these is consistent with a "competitive equilibrium" outcome.

The result that the wage bargaining process generates a "competitive" equilibrium wage is sensitive to the particular matching and bargaining technology assumed in the model and to the information assumed to be known by agents. The determination of the matching/bargaining wage has been based on the same method used in chapter 2. The outcome in that simpler model, a Nash equilibrium, differs from this model because of the different assumptions with respect to information and the threat points of the agents. Fisher (pp. 49-50, 1983) emphasizes that when sellers face a declining demand curve they ought to behave as monopolists and that search models have typically avoided this problem by assuming a price adjustment mechanism in which sellers act as if they perceive the demand curves they face to be flat. The matching model developed here avoids this objection by appealing to a special, plausible bargaining theory.

The basic issues of price determination in the context of matching or search models are difficult and largely unresolved in the literature. There are at least two schools of thought: those who believe that sensible price setting processes must entail the exercise of monopoly power, which must lead to a "noncompetitive" outcome, in general, and those who believe that sensible price setting processes should converge to an equilibrium with "competitive" properties in the presence of a market entry decision. A full exploration of these issues is beyond the scope of this study. However, the simple matching model constructed here illustrates three points: 1) under somewhat restrictive assumptions,[23] a matching model can be consistent with a "competitive" equilibrium wage, 2) there need be no unique equilibrium, and 3) the assumptions about

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[23] I leave the discussion here vague because, without a full analysis of these issues, it is not clear which assumptions are necessary for a competitive outcome and which are merely sufficient.

the information known to agents and the expectations formed on the basis of that information are critical to the determination of equilibria.

5.4 Multiple equilibria more generally

The equilibria determined above are based on the critical assumption of an exogeneous level of J (=F), the positive level of matching activity by firms. It has been shown for a restrictive type of bargaining process that, for a given positive expected value of J_t, there are just two S_t, w_t combinations at which rational agents would contract. It was shown in proposition 1 of chapter 3 that exogeneous values for either the set of p_{st} and p_{Jt} or of S_t and J_t are sufficient to determine unique values of the other set. The specific wage bargaining process developed above shows, further, that the discrete equilibria can be uniquely derived given just one of these quantity variables.

For the equilibria to be uniquely determined in this market, either the match quantities or the wage must be exogeneous. It can be seen from the solution to (42) and (47), shown in diagrams 3 and 5, that, for a given wage, there is a unique level of S_t consistent with the given value of J_t. Consider, now, the more general case in which J_t is, symmetrically to S_t, a function of the given value of S_t. It appears likely that there would then be a unique S_t, J_t combination consistent with a given wage, as determined through the bargaining process. However, since neither transactions quantities nor the wage is fully determined by exogeneous factors, such an equilibrium is a "boot-strap" equilibrium; it must be based on the acceptance by agents of given values of either quantities or prices.

The assumption of a given wage rate is not appealing for the general analysis of labor market behaviour. In a market economy prices are decided in an interactive bargaining process between buyers and sellers. This is an essential aspect of maximizing behaviour by individual agents.

As Marshall contended in his Principles, the assumption of given quantities of potential transactors for a pre-market decision is more defensible. If we regard the market period as a moment in a continuous process of buying and selling, and assume that the market is composed of a large number of agents, it is reasonable for the individual agent considering entry into the market to view the quantities of potential transactors and, hence, the probability of transacting, as given. The individual's decision is infra-marginal.

This argument does not mean that individual agents will not base their matching participation decisions on the basis of rational expectations of the interactions of all variables in the model. Agents may be assumed to realize and to take fully into account the influence of their decisions on expected equilibria quantity and wage outcomes. However, given the interdependence of the quantity, probability and wage variables, the model is underdetermined. The two unique equilibria are defined only for a given initial state of the economy. In the example developed above, a change in the given value of F will change the equilibrium levels of all aggregate variables. The implications of this for the non-uniqueness of the dynamic steady-state will be more fully explored in chapter 6.

5.5 The equilibrium unemployment rate

Having shown how an equilibrium wage and level of matching activity may be determined in the matching model, the equilibrium unemployment rate can be derived.

First, note that actual unemployment in period t is, by definition,

$$U_t = S_t - N_t \tag{50}$$

where N_t is the actual number of job-seeking workers who become employed. In (50) N_t is a stochastic variable, which depends upon the random matching outcomes, ϵ_{it}, for each job-seeking worker. From this, the unemployment rate for period t can be defined as

$$UR_t = \frac{U_t}{S_t} \tag{51}$$

which is the proportion of active job (match) seekers who fail to become employed.[24]

If all agents correctly anticipate S_t, then the expected level of unemployment is defined from (50) as:

$$U_t^e = S_t - N_t^e \tag{52}$$

Since, under the bargaining assumptions, all agents who succeed in contact will agree to transact, then the expected level of employment equals the expected number of matches, i.e. $N_{te} = M_{te}$. The expected number of matches is $p_{st}S_t$. Therefore,

$$U_t^e = (1-p_{st})\ S_t. \tag{53}$$

and

$$UR_t^e = 1 - p_{st}. \tag{54}$$

This result is intuitively straightforward, since the unemployment rate is defined with respect to those workers who seek to match.

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[24] Similarly, the number of vacancies is:

$$V_t = F - N_t$$

and, hence, the vacancy rate is:

$$VR_t = \frac{V_t}{F}.$$

Proposition 7: UR_t^e is the expected equilibrium rate of unemployment.

Argument: The period t equilibrium unemployment rate is that determined, from (54), by the equilibrium value of p_{st}, which is uniquely associated with S^*_t or S'_t. If rational individuals know this value of p_{st}, then UR_t^e must equal the equilibrium rate of unemployment in order for those expectations to be consistent. Otherwise, rational individuals will base their decisions on their best estimate of the equilibrium rate of unemployment, which is the expected rate of unemployment.

Chapter 6

Dynamic Adjustment in the Steady-State

6.1 The steady-state

Having derived the general equilibria for the matching model within the market period, the question arises whether or not the feasible single period equilibria are consistent with a multi-period steady-state. A steady-state can be defined as any set of market outcomes in period t which does not alter G, the optimal contingent policy of workers for periods t+1, t+2, ..., t+∞ formed in period t, as determined by (16).

For example, the steady-state rate of unemployment for period t can be defined as that rate which is consistent with the expectations vectors, P^e and W^e, which help to define G. If we let UR_t^* be the steady-state unemployment rate, then

Proposition 8:

$$UR_t^* = UR_t^e \tag{55}$$

Argument: From (16), the only value of p_{st} which does not, in general, alter the period t plans of any worker formed in t-1 is that used in the period t participation decision, i.e., $p_{st} = E_{t-1}(p_{st})$, where the E refers to the expection formed on the basis of information available at the end of the subscripted period. From (53) it follows that UR_t^e, the expected equilibrium rate of unemployment, is the steady-state rate of unemployment.

This argument can be generalized to derive steady-state values of all the aggregate variables. It rests on the strong assumptions of rational expectations, perfect understanding of the matching model and identical expectations among all workers. However, it does not assume that agents have perfect information, which would be contrary to the nature of transactions externalities. If agents correctly understand the matching model, then rational contingent plans, according to (16), must be based on expectations that are consistent with the equilibrium and dynamic properties of the model. Therefore, I state without further proof that the steady-state values of the macro variables are $N^*_t = N_t^e$, $UR^*_t = UR_t^e$, $w^*_t = w_t^e$, $S^*_t = S_t^e$ and $V^*_t = V_t^e$.

6.2 Stability of the steady-state

The stability of the steady-state over time is an important issue. A steady-state must be somewhat stable to be of practical interest. The following analysis considers the behavior of the steady-state of the matching model across time periods.

The aggregate matching outcome can yield any value of N_t between 0 and the lesser of S_t and F. When the actual matching outcome is such that $N_t = N_t^e$, then $UR_t = UR_t^e$ and, hence, aggregate period t income equals the expected period income, $wN_t = w^eN_t^e$. This I define as an aggregate steady-state. Rational individuals can use these aggregate variables as an indicator of whether their contingent expectations for future periods need to be revised.

However, even in the steady-state, when expectations of the aggregate variables are realized, the stochastic matching process makes it likely that $UR_{t+1}^* \neq E_tUR_{t+1}$ and, hence, some agents will need to revise their steady-state contingent plans after the matching outcome is revealed in t+1. To see this, we need to reconsider the values of y_{it} decided on by each of the L workers. The relationship between y_{it} and y_{it-1} for the three possible situations is shown in (32), p. 66, with respect to the sequential generations specification, where $y = y_{it-1}$ and $B = y_{it}$.

First, it is clear that for all L - S_t workers who decided not to seek a job, the expectation of y_t held at the beginning of period t is exactly realized, i.e., $y_{it} \equiv y_{it}^e$. Hence, the value of y_t affecting their decisions at the start of period t+1 is the value anticipated by the plan at the start of period t. It follows from the optimal policy

$G(y, P^e, W^e)$, defined in (16), that, ex post, if $p_{st} = p_{st}^e$ and $w_t = w_t^e$, then there would be no change in this groups' future expectations, P_{st}^e and W_{st}^e, or contingent t+1 decisions. For the individual, this could entail either seeking to match or not in t+1, since $y_{t+1} < y_t$ in general for this group.[25]

For the S_t agents who seek employment, y_{it} is contingent on the particular outcome of the matching process for each individual. Of the $p_{st}S_t$ workers who find employment in the case of an aggregate steady-state outcome, those with little wealth may be expected to be net savers in the period, so $y_{it} > y_{it-1}$, and those with greater wealth may be expected to be net consumers. In the sequential generations model of chapter 3, an agent is a net saver, according to (32), if $y_{it-1} < w_t$.

Suppose that the exact distribution of wealth among the L potential workers is known by all agents. It follows that the S_t curve, shown in diagrams 3 and 5, can be precisely determined. Then, the proportion of workers with $y_{it-1} < w_t$, who choose $y_{it} > y_{it-1}$ is

$$p_s \int_0^{y_{t-1}^*} \int_0^{w_t} f(y_{t-1})\, dy_{t-1} L$$

where y_{t-1}^* may be greater or less than w_t. From this, the aggregate flows into and out of employment resulting from the

[25] This means that $w^*_{sit+1} < w^*_{sit}$ for each worker not in the labor force. In diagram 3, the individual's reservation wage schedule shifts down and to the right.

effect of the expected changes in each individual's wealth in t on their t+1 participation decisions can be estimated with a known probability.

Even with the realization of UR_t^e and $w_t^e N_t^e$, some agents will alter their matching decision in t+1, consistent with their contingent plans held at the start of t. Some proportion of the N_t successfully matched workers who choose $y_{it} > y_{it-1}$ will be expected to not seek jobs in t+1 in the steady-state, while some proportion of the $L - S_t$ non-participants in the period t market will be expected to seek jobs in t+1. In a steady-state, if the flow into the labor market between t and t+1 equals the flow out, then $S_{t+1}^{e*} = S_t^{e*}$.

In general, however, the distribution of y_{t-1} of the N_t^e successfully matched workers will not be identical to the distribution of y_{t-1} among the S_t job seekers because of stochastic matching. This means that the expectation of S_{t+1}^*, formed by agents on the basis of the <u>aggregate</u> period t steady-state outcome, as above, will generally be false. The actual S_{t+1} curve that is consistent with the optimal plans of all individuals will shift by a random factor related to the change in the distribution of total wealth. Even with an aggregate steady-state outcome, the steady-state values of future periods will randomly fluctuate to some extent, requiring continual revisions of optimal contingent plans.

This dynamic random behaviour suggests that only equilibria of type A in diagram 3, where $\partial w_s^*/\partial S = \partial w_J^*/\partial S$, are consistent with a stable steady-state. As argued in section 5.3, pp.92-3, the type B equilibrium is a knife-edge. Random deviations of the S_t function, resulting from stochastic matching, will cause the economy to either collapse to zero activity or to jump to the higher level of activity at the type A equilibrium in period t+1.

6.3 Persistence of shocks to the equilibrium unemployment rate

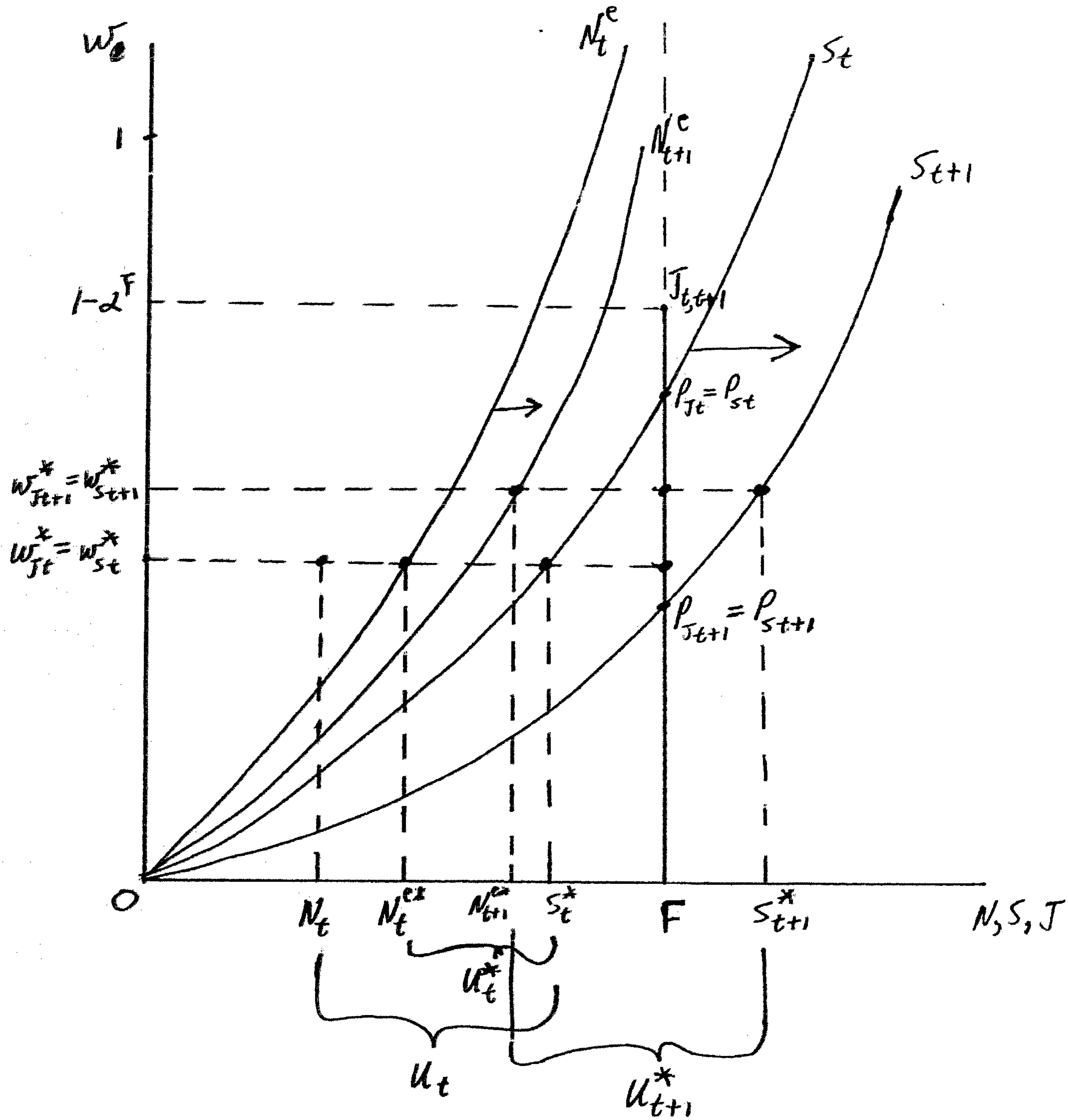
Consider the consequences of the most likely outcome of the matching process: that the rational expected number of matches is not exactly realized. That is, given a steady-state in t-1, the aggregate outcome in t is different than the expected outcome based on the actual matching probabilities. For example, assume that $N_t < N_t^e$. Since fewer agents have matched than expected, it follows that the contingent labor-leisure plans of workers, formed at the beginning of t, need to be revised.

To see how these plans are revised, note first that the wage in t is unaffected by the actual outcome of the matching process in t because agents revise their plans after the aggregate outcome of the matching-bargaining process becomes known. It follows that $w_t N_t < w_t^e N_t^e$ because of the lower value of N_t. This negative wealth shock reduces the y_{it} of each adversely affected individual, which then reduces the reservation wage schedule of those agents in t+1, used to determine their t+1 matching activity. As a result, fewer of the S_t participating workers will be expected to drop out from seeking employment in t+1. Therefore, $E_t S_{t+1}^* > E_{t-1} S_{t+1}^*$.

Agents know that an increase in the expected number of job seeking workers imposes external costs and benefits on other agents which affects their t+1 and future plans. First, the higher expected value of S_{t+1} raises p_{jt+1}, such that $E_t p_{jt+1}^* > E_{t-1} p_{jt+1}^*$. In the wage bargaining process, firms will be prepared to offer a higher w, so that $E_t w_{t+1}^* > E_{t-1} w_{t+1}^*$. This not only induces even more of the S_t participants to seek employment in t+1, but also some of the $L - S_t$ workers who had not planned to participate in t. This shows the multiplier effect of the thin-market externality, which has been pointed out by Diamond (1984) and Howitt (1983). It also indicates that the multiplier effect can occur even when the externality is internalized in the wage response.

On the other hand, the higher value of S_{t+1} also increases congestion among job seeking workers, as shown in section 3.3. That is $E_t P_{st+1}^* < E_{t-1} P_{st+1}^*$, but by a lesser amount than the change in expected p_{jt+1}. This serves to partially offset the

Diagram 6


w_e
1
$1-\alpha^F$
$w^*_{Jt+1} = w^*_{St+1}$
$w^*_{Jt} = w^*_{St}$
N^e_t
N^e_{t+1}
S_t
S_{t+1}
$J_{t,t+1}$
$\rho_{Jt} = \rho_{st}$
$\rho_{Jt+1} = \rho_{st+1}$
O
N_t
N^{e*}_t
N^{e*}_{t+1}
S^*_t
F
S^*_{t+1}
N, S, J
U^*_t
U_t
U^*_{t+1}

positive response to the change in the thin-market externality. The net result of these interactions is shown in diagram 6, where the S_{t+1} curve is below and to the right of the S_t curve and J_{t+1} rises vertically to w^*_{Jt+1}. The thin-market externality is indicated by the rise in the J function, while the congestion externality is indicated by the curvature of the S function. The fact that the J function is fixed forces the internalization of the net impact of the transactions externalities to firms through the firms' wage offer. The net result is unambiguous, $E_tS_{t+1}^* > E_{t-1}S_{t+1}^*$ by a greater amount than in the absence of the quantity effect. Therefore,

<u>Proposition 9</u>: The thin-market externality dominates the congestion externality.

There are also some secondary inter-temporal effects of the below-expected number of matches in t. Since both w_{t+1}^* and N_{t+1}^* have increased, $E_tw_{t+1}^*N_{t+1}^* > E_{t-1}w_{t+1}^*N_{t+1}^*$, which partially offsets the negative wealth impact in t. The net wealth effect is captured in the illustrated shift of the S curve from t to t+1.

An important dynamic property of the model is:

<u>Proposition 10</u>: When J is constant, a quantity disturbance arising from the matching process in t-1, such that

$S_t > S_t^*$, or $S_t < S_t^*$, implies that $UR_{t+1}^* < UR_t^*$, or $UR_{t+1}^* > UR_t^*$, respectively.

Argument: $S_t > S_t^*$ generates $w_tN_t > w_tN_t^*$. Agents respond through (16) such that $S_{t+1}^* < S_t^*$. From (5), this implies that $p_{st+1}^* > p_{st}^*$. By (54), $UR_{t+1}^* < UR_t^*$. $S_t < S_t^*$ is symmetric.

This proposition indicates that the steady-state rate of unemployment is not invariant to temporary market disturbances affecting the quantity of agents who match. A temporary market disturbance is here defined as a market outcome where $N_t \neq N_t^e$. Since the stochastic matching shock is a temporary one-time disturbance, the optimal reaction to the change in wealth will be to spread the adjusted consumption of x and leisure over future periods at a declining rate, as determined by the time rate of discount. Hence, as the unanticipated windfall loss is absorbed and in the absence of further shocks, the aggregate S curve in periods t+2, t+3, ..., t+∞ will tend to shift back toward S_t. This suggests that temporary shocks will lead to persistent deviations in UR^*.[26]

[26] Some authors define persistent deviations in UR^* that tend to return to their pre-shock value gradually as "weak hysteresis" (e.g., Blanchard and Summers (1986)). Such persistence may be important for economic policy, but is not consistent with the technical definition of the term and, in principle, is not as serious a concern as the possible existence of strict hysteresis in the steady-state unemployment rate.

The dislocation of UR^* is temporary in the above example because we have defined stable steady-state equilibria to which rational agents will tend to converge in the absence of shocks. A critical assumption underlying these path-independent equilibria is that the value of F, the positive matching quantity of firms, is a constant. In the next section this assumption is relaxed, as in chapter 2. By taking into account the reciprocal nature of the thin-market externality, the likely existence of strict hysteresis in steady-state employment and in the unemployment rate will be postulated.

6.4 Reciprocal externality and unemployment hysteresis

A strict definition of hysteresis requires that the current value of a variable is completely defined by the history of stochastic shocks affecting that variable. In this case, there is no tendency for the variable to return to an initial steady-state value and its value is not uniquely defined by the state variables.

In this section, I propose that the steady-state unemployment rate in the matching model exhibits strict hysteresis as a consequence of the reciprocal thin-market externality. This result is closely related to the existence of multiple equilibria when the search activity of firms, as well as workers, is endogenous. The existence of multiple equilibria in this case was proven in chapter 2 in a simpler, static model. While the thin-market externality has received some attention in the literature, its potential importance as a hysteresis mechanism, inherent in the nature of markets with significant costs of transacting, has not been previously suggested.

Assuming that J_t is, symmetrically to S_t, a function of the given value of S_t, it will be shown that there are non-unique S_t, J_t and w combinations consistent with the matching and bargaining process. Since transactions quantities and wage are determined only for given values of search probabilities, the equilibria are based on self-fulfilling expectations. The range of positive-valued equilibria in the model are defined only for a given initial state of the economy. A change in the value of either S or J may change the equilibrium levels of all variables.

The exercise of the previous sections was restricted from examining the reciprocal nature of the thin-market externality by the assumption of a fixed quantity of firms seeking to match. In general, of course, the search activity of firms is not constant. To allow the recruitment activity of firms to vary requires that firms differ in some characteristic that affects recruitment decisions. Therefore, I adopt the following assumption:

Ass. 24: Firms differ in recruitment search productivity, such that there is a distribution of fixed transaction costs, α_j^F, by firm, where j = 1, 2, ..., F, such that $\alpha_F^F > \alpha_{F-1}^F > \ldots > \alpha_1^F$.

This assumption is weak and ad hoc. It serves the purpose of easily differentiating otherwise identical firms in order to derive a continuous labor recruitment demand function. The variance in α^F implies that the aggregate J function in w, N space (as in diagrams 5 and 6) is downward sloping and increasing with S. A more useful graphical device, however, is to plot the matching reservation wage schedule of the firms for each value of α_F in w, S space, as was done for the representative firm in diagram 3. Diagram 7 shows the field of such functions for a sample of four firms. Note that for a given value of S, an increase in w reduces the number of firms willing to recruit, while for a given w, an increase in S increases the number of firms willing to recruit. Hence, there is a unique number of firms that will seek to match for every expected value of S and w in a period.

Diagram 8 illustrates the same field of w_J^* curves together with the corresponding field of w_s^* curves for each value of y_{t-1}, as in diagram 3. The w_s^* curves are here plotted

Diagram 7

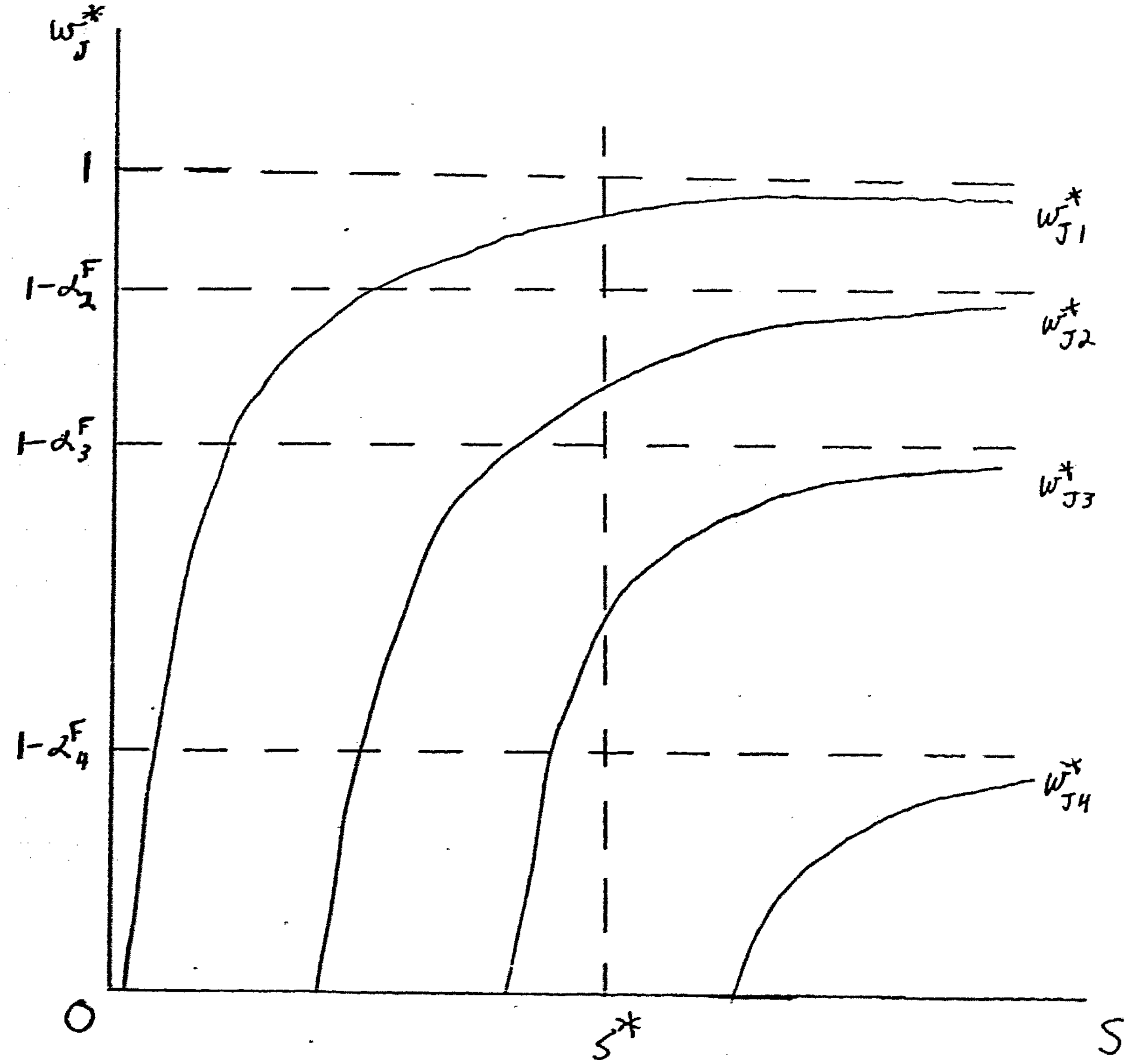
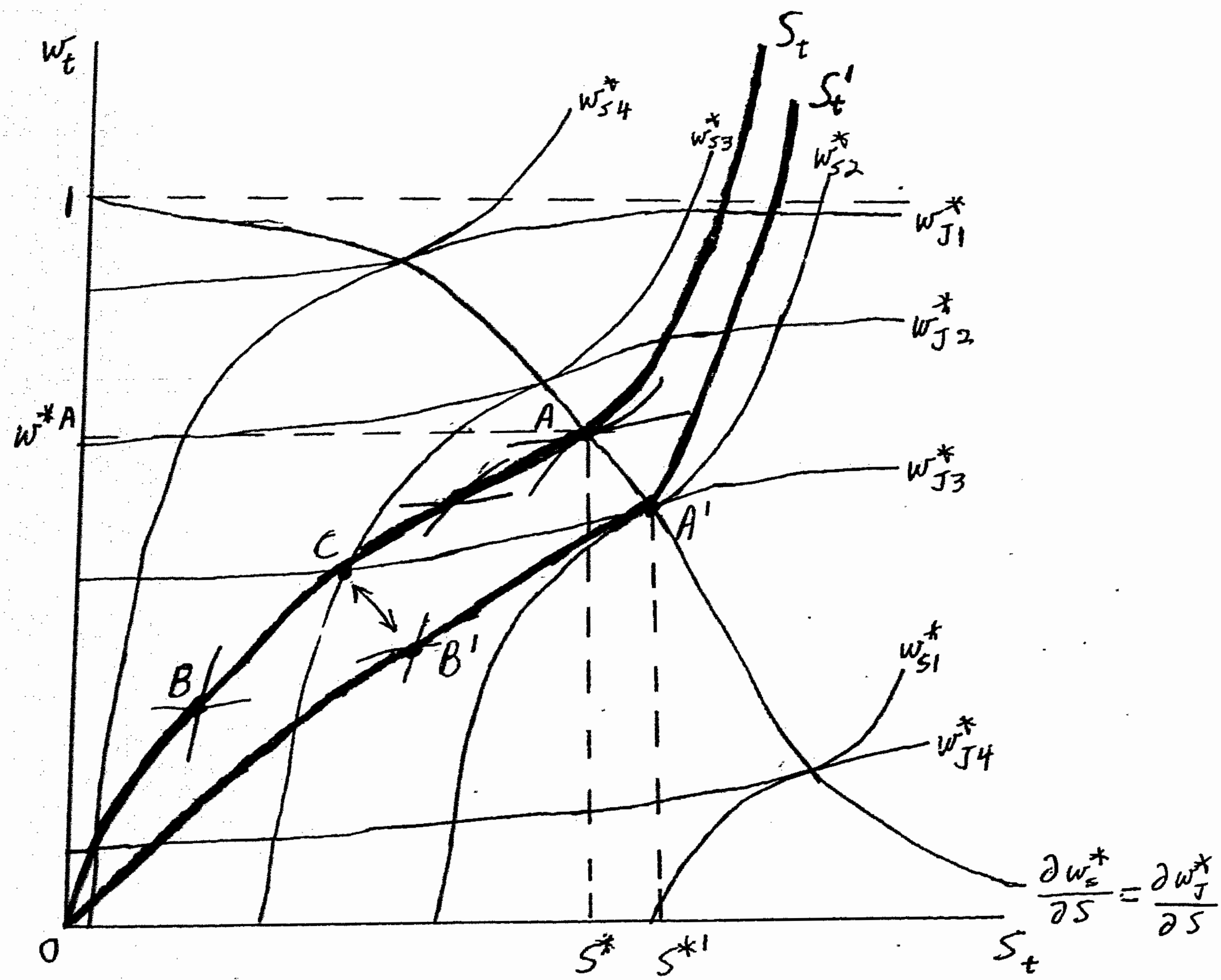


Diagram 8



w_t
S_t
S_t'
w^*_{S4}
w^*_{S3}
w^*_{S2}
1
w^*_{J1}
w^*_{J2}
w^{*A}
A
w^*_{J3}
C
A'
B
B'
w^*_{S1}
w^*_{J4}
$\frac{\partial w^*_S}{\partial S} = \frac{\partial w^*_J}{\partial S}$
0
S^*
$S^{*\prime}$
S_t

with respect to both the expected level of S, as in diagram 4, **and** the expected level of J that is associated with each w, S point. As S rises, the level of J rises endogenously due to the lower p_J, which in turn reduces the rate of increase in congestion, p_s. The shape of the w_J^* curves shows that J rises almost as rapidly as S for S close to 0 and that, as congestion increases, the rate of increase in J relative to S declines as S increases. Therefore, the w_s* curves with endogeneous firm entry are flatter than in the case of a fixed J in diagram 3.

Now, as in diagram 3, for a given distribution of y_{t-1}, there is a unique locus of points along which the marginal worker, who is indifferent to matching at the expected wage and the expected level of aggregate S, is the Sth worker, so that $S = S^e$. This locus, labelled S_t in diagram 8, is the aggregate labor search supply function. It is the supply of workers seeking to match at each value of w and S, with endogenous J, consistent with rational expectations by correctly informed workers.

The first important difference to notice between diagrams 3 and 8 is that, while the former has two distinct equilibrium points along S_t at which $w_J^* = w_s^*$, every point on S_t from 0 to A in diagram 8 is a potential equilibrium with $w_J^* = w_s^*$. To the right of A, where $\partial w_s^*/\partial S > \partial w_J^*/\partial S$, it is impossible to have both increasing S and J simultaneously and, as argued in section 5.3, p. 95, a hypothetical tatonnement type process would tend to move agents back to A. This result leads to a revision of proposition 6, p. 100. In the case of a fixed J, proposition 6 indicated that equilibrium point A was the only stable equilibrium and, hence, the only positive-valued steady-state position according to section 6.2.

Proposition 11: The reciprocal thin-market externality, where the matching effort of both workers and firms is endogeneous to the level of aggregate matching activity, leads to multiple steady-state outcomes. There exist unique combinations of S, J and w values that are consistent with a given value of S or J over a finite range.

Having established this key property of the model, we can repeat the exercise of section 6.3 for the case of endogeneous J. That exercise considers a random shock arising from the stochastic nature of the matching process. It was assumed that fewer agents were matched than expected in period t, such that $N_t < N_t^e$.

The lower value of N_t creates an aggregate negative wealth shock, since $w_tN_t < w_tN_t^e$, which is concentrated among those individuals who failed to match. As pointed out in section 6.3, the number of workers seeking to match in period t+1 is increased. To show this movement on diagram 8, note that some of the w_s^* curves shift down and to the right. This means that each of the illustrated w_s^* curves in diagram 8 is now consistent with a higher level of S than that indicated on the x axis. The S_t curve, along which $S = S^e$, must shift downward to a path such as OA' along which $w_J^* = w_s^*$.[27]

We can now consider the dynamic adjustment to the negative wealth shock in period t on the period t+1 steady-state, starting from different potential equilibrium positions. If expectations are such that the economy in t was at point A, then the path of adjustment in equilibrum values may lead back to the original position at point A on OA with the dissipation of the wealth shock. This result seems unlikely, however, in the case where the economy starts at any other point on OA, such as point B.

If the economy in t starts at point A, the higher expected value of S_{t+1} raises p_{Jt+1}, causing firms, moving along their wage offer curves, to offer a higher w_J^*. At the same time, p_{st+1} is reduced, causing workers, moving along their now lower wage demand curves, to demand a higher w_s^*. However, when the economy is at a point on the locus of tangencies

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[27] Notice that the w_s^* curve at point A is now associated with the $S^{*\prime}$ + kth wealthiest worker.

between the w_J^* and w_S^* curves, it is not possible to increase both S and J and w and still satisfy the equilibrium conditions. The positive externalities of the higher level of matching activity are thwarted at this starting position. The shift in the S_{t+1} function then dominates, forcing the rational expectation equilibrium in t+1 to the lower wage, higher level of matching activity point, A'. Therefore, as the negative wealth impact on the matching supply is dissipated, in the absence of any further disturbances, the steady-state equilibrium can be expected to return to point A, with the same equilibrium levels of N^* and U^*, and the same equilibrium rate of UR^*.

Now consider the adjustment process starting at point B in t. The equilibria locus 0A shifts to 0A', as before. In this case, however, the positive multiplier effect of the increase in period t+1 matching activity by workers is unchecked. The increased p_{Jt+1} induces a greater level of J and a higher w_J^*, while the reduced p_{St+1} partially offsets the multiplier impact of the higher J on S and generates a higher w_S^* offer. While the adjustment path is unclear without a full derivation of the quantitative magnitudes of these parameters for the given initial conditions, it appears likely that the economy would move to a point such as B' in period t+1. At B' the equilibrium wage rate might even be higher than at B in period t.

Note that B' is fully consistent with the new lower level of aggregate wealth and with rational expectations about the levels of matching activity. With the dissipation of the wealth shock, the S_{t+1} curve will be expected to move gradually back to the position of S_t, and the state of the market to a point such as C, which is associated with the higher matching activity at the initial level of aggregate wealth and the absence of any unexpected shocks.

Movements along the OA multiple steady-state locus, that may be due to unexpected exogeneous shocks or the stochastic outcomes of labor market matching, have predictable impacts on UR^*, the steady-state rate of unemployment. The movement from point B to C, in the example, is associated with a higher ratio of S/J. This means that p_s facing each worker is unambiguously lower at point C, after the new steady-state is established. By (54), $UR^* = 1 - p_s$. Therefore, the negative wealth shock leads to a permanently higher steady-state rate of unemployment. I expect that a positive wealth shock would operate in the opposite direction. Similarly, I expect that an exogenous increase, say, in the level of recruitment activity, J, would lead to a permanently lower UR^*, as it would be associated with a lower equilibrium ratio of S/J.

This argument supports the conjecture that transactions externalities generate strict hysteresis in the steady-state rate of unemployment in markets characterized by a congestible matching technology.

6.5 Conclusion

In the model developed in this thesis, workers and firms in a labour market face some probability (less than one) of locating a vacancy or potential hiree in a market period. The market entry decision of agents is based both on the expected wage and the probability of finding a match. However, the market wage and match probabilities are determined jointly. Therefore, when matching probabilities matter, the equilibrium output and wage are not uniquely determined, in general. The equilibria are "bootstraps" equilibria, in that the rational expectations of agents must be based on the acceptance of some intial value of quantities or price.

In an intertemporal framework, it was argued that the transaction externalities arising from the matching process lead to non-unique steady-state equilibria which can be permanently altered by exogenous shocks affecting the match probabilities and by shocks arising from the stochastic matching process itself. Hence, transaction externalities generate hysteresis in the equilibrium unemployment rate. This is the first hysteresis mechanism proposed in the literature that does not rely on an assumption about labor market imperfection that is largely ad hoc. Transaction costs and the related externalities are inherent in the matching process of the labor market.

The aggregate quantities of agents seeking employment affect the return to employment of other agents by affecting the probability of finding employment. Thus, the participation decisions of agents impose external costs or benefits on other agents, including the positive feedback (multiplier effect) that the wage adjustment has on the quantity of hopeful transactors. Whether these costs or benefits are internalized in the wage depends on the the bargaining process. It was shown that internalization (at least on one side of the market) is possible with strong assumptions about information and expectations.

Two types of transactions externalities have been identified. The thin-market externality results from the impact of the search, or matching, decisions of agents on one side of a market on agents on the other side of the market. There is also a counteracting transaction externality, the congestion externality, which serves to stabilize market adjustment by providing an offsetting change in the probability of a match to agents on the same side of the market.

It has been shown that the existence of transaction externalities affecting the quantity decisions of agents on just one side of the labor market is sufficient to generate persistence in the impact on UR^* of exogeneous shocks or of shocks arising from the stochastic nature of the matching process. Such persistence is often confused with actual hysteresis in the empirical literature and has been called "weak hysteresis". This result alone provides a potential explanation for such much-discussed phenomena as "Eurosclerosis." True hysteresis provides a foundation for the reconsideration of public policy designed to deal with the problem of persistent high unemployment.

A direction for future research suggested by this thesis is to determine how well the matching model can explain the actual institutional nature of labor markets. Are the observed institutional arrangements of labor markets designed to minimize these costs? It might be argued, for example, that infinitely lived agents would transact once and maintain the contract forever. Finite lived agents with an infinite horizon might pass on jobs to their offspring, thus maintaining contracts permanently. However, labor turnover inevitably leads to the transactions costs of matching. When agents vary in their characteristics, the matching process also involves search for an appropriate match. Modern industrial capitalist economies are characterized by a very high rate of labor turnover in comparison with earlier economies. Technological change and the business cycle contribute to this high rate of labor turnover and, consequently, to the problem of transactions externalities.

The effects of transaction externalities on real economic activity in the labor market are transmitted inter-temporally in three distinct ways. First, expected inter-temporal differences in the wage and in match probabilities create incentives for inter-temporal substitution of work (and matching effort) for leisure. Second, the changes brought about in workers' wealth will alter their consumption-work plan for all future periods. Third, and most important, changes in match probabilities alter the steady-state values of variables. While the first two traditional transmission mechanisms are likely to dampen the effect of shocks, the latter mechanism implies that such shocks may never be fully offset by market adjustments.

The quantity rationing effect of transaction externalities in a stochastic congestible matching technology makes the stability of the market uncertain. Even in the case where the expected number of transactions are exactly realized, an aggregate steady-state outcome, the optimal values of real variables will generally not remain constant in the next period. With multiple equilibria, such as a high employment - low unemployment outcome and a low employment - high unemployment outcome, stochastic or other disturbances could lead to a socially less desireable steady-state rate of unemployment.

As a final comment, the matching model of the labour market has been developed most fully in the literature, although with significant differences from the model of this thesis, in the book, Equilibrium Unemployment Theory, by Christopher Pissarides (1990). The most important departure of the model developed in this thesis from Pissarides is the emphasis on multiple equilibria arising from reciprocal transaction externalities. This departure from the literature (although already suggested by Diamond (1984)) shares the spirit of Roger Farmer's (1993) book, The Macroeconomics of Self-fulfilling Prophesies, wherein he argues that multiple equilibria arise in our economic models because we have failed to specify the beliefs of agents as a basic economic parameter.

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