

THE SIMULTANECUS OCCURRENCE OF HETEROSCEDASTICITY
AND AUTCCORRELATION: A MONTE CARLO STUDY

by

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THE SIMULTANEOUS OCCURRENCE OF HETEROSCEDASTICITY AND AUTOCORRELATION

: A MONTE CARLO STUDY

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ABSTRACT

Estimation when errors are simultaneously heteroscedastic and autocorrelated is simulated using the Monte Carlo technique. Five estimators under nine severity combinations of heteroscedasticity and autocorrelation are examined.

A major contribution is the development of a severity measure for heteroscedasticity, using the cosine concept.

The results indicate that the preferred estimator depends on the absolute and relative intensities of autocorrelation and heteroscedasticity.

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Any remaining errors are the responsibility of the author.

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INTRODUCTION

Jointly heteroscedastic-autocorrelated errors cause estimation problems. The ordinary least squares estimator (OLS) will be unbiased but inefficient. Moreover, if the incorrect estimated generalized least squares estimator (EGLS) is used the situation may be no better, and possibly worse. For instance, if both heteroscedasticity and autocorrelation exist, but correction is for one problem only, the estimators are still based on nonspherical errors and they will remain inefficient.

This paper, unlike the literature, studies the problem in detail. Five estimators are proposed and each is tested under nine severity combinations of heteroscedasticity and autocorrelation.

Determining the severity levels presented a major impasse. The absolute value of the autocorrelation coefficient is commonly used to indicate the degree of serial correlation, but no similar indicator exists for heteroscedasticity.

The first of two Monte Carlo studies is conducted. The postulated heteroscedasticity intensity measure is used to determine the severity level at which EGLS outperforms OLS. The results, for the data used, indicate estimated GLS has a smaller mean square error than OLS for even very weak levels of heteroscedasticity.

Finally, nine different severity combinations of heteroscedasticity and autocorrelation were chosen and used in a second Monte Carlo study to examine the performance of the five

estimators. The details of this study and the conclusions are in sections four and five.

I. LITERATURE SURVEY

Heteroscedasticity and autocorrelation can occur simultaneously in many different estimation contexts. Unfortunately, for the most part, the literature ignores this problem.

What their model in effect implies is an ordinary regression model with residuals showing both heteroscedasticity and autocorrelation. Regression models where both these problems are handled simultaneously have not been estimated. (Maddala, 1977, pp.398)

There have been only two articles, Harrison and McCabe (1975) and Epps and Epps (1977), directly concerned with the joint problem. However, they focus on the robustness of various tests for non-sphericalness, with no discussion on the best estimator under these circumstances.

In time series or cross-section regression analysis, the problems, of autocorrelation and heteroscedasticity may often stem from a common cause; they may, therefore, be reasonably expected to occur simultaneously. Yet most tests for autocorrelation are formulated using the assumption of homoscedasticity as a maintained hypothesis. Little is known about how these tests perform when this assumption is untrue. (Harrison and McCabe, 1975, p.215)

Harrison and McCabe conclude, on the basis of a Monte Carlo study, that tests for autocorrelation, in particular the Durbin-Watson test, are only very slightly affected by the presence of heteroscedasticity.

Their model has first-order autoregressive and multiplicative heteroscedastic disturbances:

$$e_t = \rho e_{t-1} + u_t \quad \text{and} \quad v(u_t) = x_t^\gamma.$$

Rho varies between 0 and .9. Gamma is set between 0 and 2 and is used to indicate the severity of heteroscedasticity.

Gamma is an inadequate measure since it ignores the magnitude of the x's. They do use two forms of x, one representing a random normal variable, the other a pure trend variable. The form of x is found to have an inappreciable effect on the powers of the tests, but the size of the x's should be used in conjunction with gamma to determine the severity of heteroscedasticity. Therefore, it is uncertain whether or not the autocorrelation tests have been rigorously examined.

Epps and Epps take a similar approach (1977, p.745):

Little study has been given to the properties of the four tests (Durbin-Watson, Geary, Goldfeld-Quandt, and Glejser) when BOTH autocorrelation and heteroskedasticity are present: yet in time-series models, especially, the effects of omitted variables may often give rise to both problems simultaneously. It is valuable, therefore, to know how robust are the standard tests for one problem in the presence of the other.

Epps and Epps reexamine the Durbin-Watson bounds test and the Geary tau test. They also investigate the robustness of the Goldfeld-Quandt and Glejser tests for heteroscedasticity. The four tests were examined both analytically and from Monte Carlo outcomes.

Both approaches concur with the previous work - the bounds test and the Geary test are robust in the presence of heteroscedasticity. The articles differ, however, in their

specification of the error term. Multiplicative

heteroscedasticity was replaced by additive heteroscedasticity.

$$e_t = \rho e_{t-1} + u_t \text{ so that } v(e_t) = \sigma_t^2 = \delta + \gamma x_t^2 .$$

The impetus behind this step was to facilitate the use of the ratio of the maximum to minimum disturbance variances as a heteroscedasticity measure that was independent of the degree of autocorrelation. For example,

$$\frac{v_{\max}(e_t)}{v_{\min}(e_s)} = \frac{\delta + \gamma x_t^2}{\delta + \gamma x_s^2}$$

If the earlier specification is used,

$$\frac{v_{\max}(e_t)}{v_{\min}(e_s)} = \frac{x_t \gamma + \rho^2 v(e_{t-1})}{x_s \gamma + \rho^2 v(e_{s-1})}$$

Obviously the ratio will vary with rho and thus depends on the severity of autocorrelation.

This ratio looks only at the range of variances, all other variances are neglected. Consequently, the degree of heteroscedasticity that the autocorrelation tests have been subjected to is ill-defined. Fortunately, the analytic results

are free of these measurement problems and as such are reliable.

The Goldfeld-Quandt and Glejser tests are shown to be invalid if the errors are serially correlated. Yet they do yield fairly good results if autocorrelation is corrected for first. As a result, Epps and Epps suggest a specific sequential testing procedure: if autocorrelation exists, apply the heteroscedasticity test to the Cochrane-Orcutt transformed residuals. They do not make the natural extension to estimation procedures. This paper fills the gap.

The only other area in the literature that is concerned with jointly heteroscedastic-autocorrelated disturbances is in the pooling of cross section and time series data. The fault here is that it is only indirectly discussed - in fact, only alluded to.

There are three basic models in which the pooled cross section time series data are slotted. They are: dummy variable, error components, and varying coefficient models. The most comprehensive model, the one that allows for dependence across all units and for all time periods, is discussed only theoretically by Judge et al. (1980, pp.327-328) and Kmenta (1971, pp.512-514). It does not seem to have been applied.

The general approach to estimation with panel data is to make assumptions about the intercept and slope coefficients: do they vary over time? Or over individuals? Or both? The next step is to decide whether or not the coefficients represent fixed parameters or random variables. These decisions are then

incorporated into the model's specification. The variance covariance matrix of the resulting disturbance term is merely a side-effect, and furthermore, is such that these disturbances do not have simultaneous heteroscedasticity and autocorrelation. Allowance is made for one or the other, but never both.

Judge's comments on these models are illuminating.

As pointed out by Swamy (1974), the assumptions of the error components model discussed in this section could be regarded as fairly restrictive. They imply that the contemporaneous covariance between observations on two individuals is the same for every pair of individuals and that the covariance between two observations on a given individual is constant over time and the same for every individual. It may be preferable to use ... (the comprehensive model). (1980, p.345)

The assumptions that have been made about the e could be regarded as fairly restrictive. There is no correlation between the disturbances corresponding to different individuals and, in addition, there is no serial correlation. (1980, p.351)

The random coefficients version of the dummy variable model can also be interpreted as an omitted variable model. "In this case we end up with a regression model in which the residuals are both heteroscedastic and autocorrelated." (Maddala, 1977, p.397). Unfortunately this is rarely, if ever, admitted in practice. For example, in the dummy variable context, homoscedasticity is assumed, and only a limited serial dependence for a given unit is allowed. Even in the broader omitted variables context, the common approach is, again, to allow for autocorrelation only. See for example Maddala (1977, p.326) and Griliches (1977).

II. MEASURING HETEROSCEDASTICITY

The Monte Carlo study reported here examines the performance of five procedures or estimators under nine different severity combinations of heteroscedasticity and autocorrelation.

Although the absolute value of the correlation coefficient (ρ) indicates the degree of autocorrelation, there does not exist in the literature a comparable severity measure, (h) for heteroscedasticity.

For multiplicative heteroscedasticity, $\sigma_t^2 = \sigma^2 x_t^\gamma$, Kmenta (1971, pp.263) Harrison and McCabe (1975) and Harvey (1976), to mention only a few, set $h = \gamma$. Epps and Epps use the ratio of the largest and smallest variance as an indicator of the degree of heteroscedasticity. The later measure considers only the range of variances, all intermediate information is neglected, and it depends on the form of heteroscedasticity used. Gamma is insufficient because the magnitude and variability of the underlying observations is ignored, and a specific form of heteroscedasticity is required. They are both unbounded, and therefore hard to interpret.

To proceed with the study, a severity measure of heteroscedasticity had to be developed. For the purposes of this study it is important to be able to specify, control, and compare the levels of autocorrelation and heteroscedasticity. To

illustrate this point: to examine the sampling distributions of the estimators when autocorrelation and heteroscedasticity are of different magnitudes it is necessary to: (a) specify the levels, say $p=.9$, $h=.5$, (b) to know that autocorrelation is more severe than heteroscedasticity, since p is greater than h and, (c) to make statements about their relative severity. For example, autocorrelation is 40 percent worse than heteroscedasticity.

To determine how heteroscedastic a given error structure is, its distance from the ideal homoscedastic case should be calculated. This is precisely what the cosine of the angle between two vectors captures - the relative direction of the two vectors, or the closeness of the angle between these vectors:

$$\text{Cos } \theta = \frac{\sum_{i=1}^N u_i v_i}{\sqrt{\sum_{i=1}^N u_i^2 \sum_{i=1}^N v_i^2}}$$

As a consequence, the cosine measuring the relative direction between a vector of constants, representing

homoscedasticity, and a vector of actual variances, representing heteroscedasticity, was chosen as the indicator of the degree of heteroscedasticity. It is easy to calculate, is bounded between zero and one, is influenced by all observations, and is scale invariant. Furthermore, as shown below, it is analogous to the severity measure of autocorrelation, ρ .

The v_i 's represent the vector of the error variance terms in the classical normal linear regression model, which without loss of generality, can be considered as a vector of 1's. The u_i 's denote the elements of a vector of the variances from a heteroscedastic model. (i.e., the diagonal elements of its covariance matrix) The cosine of the angle between U and V will indicate how far they are from each other. Thus it will measure the degree of heteroscedasticity of U.

For any two arbitrary vectors of two dimensions (i.e. two observations) the maximum cosine is 1, or $\theta = 0^\circ$. In other words, the vectors are zero distance apart. This situation, where the vectors lie atop each other, is homoscedastic (see Figure (1a)). The minimum $\text{Cos } \theta$ is 0 and occurs when the two vectors are at right angles ($\theta = 90^\circ$). (see Figure (1b)). Note that only the positive orthant is dealt with, since both of the vectors contain variances and thus contain only positive elements.

However, by constraining one of the vectors to consist of 1's, the minimum cosine is no longer zero. The maximum distance between the homoscedastic and heteroscedastic variances is no longer 90 degrees. For example, in Figure (1a), a comparative

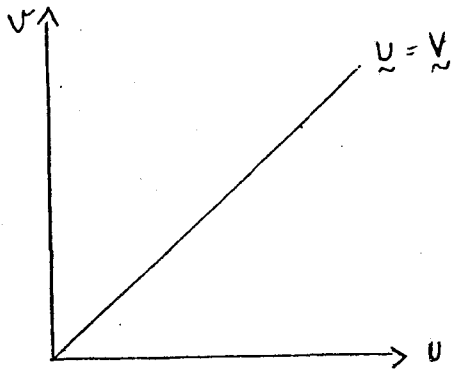


figure (1a) homoscedasticity

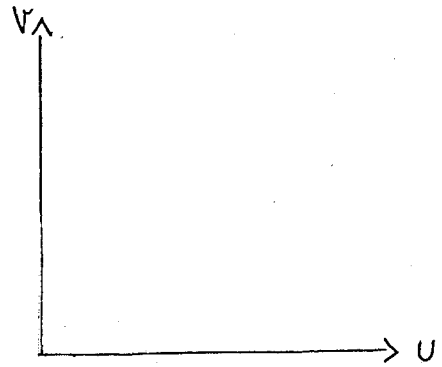


figure (1b) heteroscedasticity

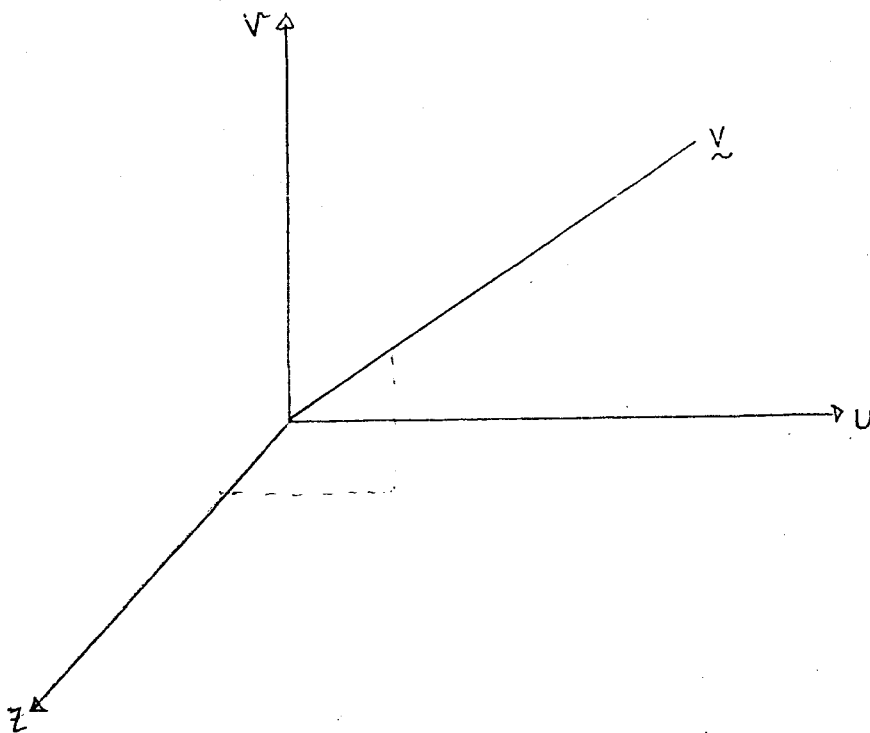


figure (1c) in 3 dimensions maximum $\text{Cos } \theta = 1$, minimum $\text{Cos } \theta = .57$

base has been specified, the homoscedastic vector, or a vector of 1's. It is obvious, from visual inspection, that the farthest any vector can be from this base is 45 degrees. (Cos 45 degrees=.71).

As the sample size increases a two-dimensional diagram can no longer be used to visualize the minimum cosine or the maximum angle between the two vectors. (see Figure(1c) for an illustration in 3 dimensions.) In all cases, Cos $\theta = 1$ still represents the maximum cosine, or the homoscedasticity case with the two vectors indistinguishable in space, but the minimum cosine clearly depends on the sample size. The proof in Appendix 1 shows that the minimum Cos $\theta = \frac{1}{\sqrt{N}}$ where N is the number of observations.

Incorporating all of this information leads to the following measure of the severity of heteroscedasticity:

$$h = 1 - \frac{\text{Cos } \theta - \frac{1}{\sqrt{N}}}{1 - \frac{1}{\sqrt{N}}}$$

This manipulation of the cosine compensates for the minimum cosine's dependence on N and thus ensures that $0 \leq h \leq 1$.

The cosine is subtracted from one such that when the two sets of variances are the same, or homoscedasticity prevails and $\text{Cos } \theta = 1$, the severity of heteroscedasticity, h , is 0. On the other hand, when the two vectors are as distant as possible, the severest case of heteroscedasticity, $h=1$.

In addition to the intuitiveness of this severity measure, as outlined above, it has many other advantages:

1. $\text{Cos } \theta$ is easy to calculate: with V equal to a vector of 1's it simplifies to:

$$\frac{\sum_{i=1}^N u_i}{\sqrt{N \sum_{i=1}^N u_i^2}}$$

2. it is scale invariant so that $\text{Cos}(3U, V) = \text{Cos}(U, V)$
3. It is between zero and one - the same as the severity measure of correlation, $|p|$. An h of .8 can be interpreted as being 80 percent as heteroscedastic as feasible.
4. h is analogous to rho thus the two severity measures can be

directly compared and contrasted.

$$\rho = \frac{\sum_{i=1}^N e_t e_{t-1}}{\sqrt{\sum_{i=1}^N e_{t-1}^2 \sum_{i=1}^N e_t^2}}$$

The formula for the correlation coefficient looks very similar to that given for the cosine. In fact, ρ is the cosine between the vector of e_t 's and lagged e_t 's. "In other words the geometric interpretation of correlation is the closeness of the angle θ ." (Wonnacott and Wonnacott 1970, p.303)

5. it does not depend on the form of heteroscedasticity, unlike other measures.

III. EVALUATING THE USE OF ESTIMATED GLS FOR HETEROSCEDASTICITY

A Monte Carlo study comparing OLS to estimated GLS (EGLS) for various severities of heteroscedasticity was run. The goal was to find the minimum h at which estimated GLS is preferred to OLS on the basis of their mean square errors (MSE).

OLS regression estimates are unbiased but inefficient if the error terms are heteroscedastic. Estimated GLS should increase efficiency, because it takes the size of the variances into account and weights the residuals accordingly.

The multiplicative heteroscedastic model, $w_t = x_t^{\frac{\gamma}{2}} v_t$, was chosen and estimation of gamma was undertaken following the method of Harvey (1971) and Park (1966).

The model:

$$y_t = \alpha + \beta x_t + w_t$$

$$w_t = x_t^{\frac{\gamma}{2}} v_t \quad v_t \sim N(0, 1), \quad w_t \sim (0, \Omega)$$

$$\sigma_{w_t}^2 = \sigma_v^2 x_t^\gamma$$

The x_t are the dollar returns on the New York Stock Exchange from 1955 to 1979 reported in tens of dollars.

The procedure:

$\sigma_{w_t}^2$ is estimated using the OLS residuals from a regression of y on x and a constant.

$$\hat{w}_t = y_t - \hat{\alpha} - \hat{\beta}x_t$$

$$\hat{\sigma}_{w_t}^2 = \hat{w}_t^2, \text{ an estimate of } x_t^\gamma \sigma_v^2$$

Thus $\log \hat{w}_t^2$ is an estimate of

$$\gamma \log x_t + \log \sigma_v^2$$

Gamma is estimated by regressing $\log \hat{w}_t^2$ on $\log x_t$ and a constant. Once gamma-hat is obtained, Ω can be estimated and the EGLS estimate of beta found. This estimated GLS procedure is implemented, as is customary, by transforming the raw data and using OLS on the transformed data.

$$\frac{y_t}{x_t^{\frac{\hat{\gamma}}{2}}} = \alpha \frac{1}{x_t^{\frac{\hat{\gamma}}{2}}} + \beta \frac{x_t}{x_t^{\frac{\hat{\gamma}}{2}}} + w_t^* \text{ where } w_t^* =$$

$$\frac{\frac{\gamma}{2} v_t}{x_t^{\frac{\hat{\gamma}}{2}}} \text{ .. Regress } \frac{y_t}{x_t^{\frac{\hat{\gamma}}{2}}} \text{ on } \frac{1}{x_t^{\frac{\hat{\gamma}}{2}}} \text{ and } \frac{x_t}{x_t^{\frac{\hat{\gamma}}{2}}} .$$

For the data used in this study the severity levels (h), and the corresponding gammas are given in Table 1.

TABLE 1

h	gamma	Preferred Estimator
0	0	OLS
.05	1.05	OLS
.132	2.00	GLS
.2	2.80	GLS
.3	4.16	GLS
.4	5.80	GLS
.6	10.20	GLS
.8	18.00	GLS
.9	29.00	GLS

Kmenta suggests gamma should never be greater than 6, and that a gamma of 2 is a more common level. Lancaster (1968) and Park (1966) concur with a maximum gamma of 2.

This is added evidence of the disadvantages of using the exponent as a severity measure. These authors represent severe heteroscedasticity with a gamma of 2. For the data used in this study, a gamma of 2 yields an h of .132, or only 13 percent as severe as possible. For some other data, it could represent an h of .99. The size and variability of the x's is vital, omitted information.

The Monte Carlo experiment consisted of 100 iterations for each of the severity levels. For a given severity level after 100 iterations were completed, the mean, variance, bias, and MSE of the 100 β^{OLS} and 100 $\hat{\beta}^{GLS}$ were calculated.

The results in Table 1 show that for $h > .132$ arising, in this example, from

$$v(w_t) = x_t^2 \sigma_v^2$$

EGLS outperforms OLS on the basis of the MSE criterion. OLS is the preferred estimator for cases of homoscedasticity, and when h is less than .132. It is a bit surprising that even at very weak levels of heteroscedasticity, ($h \geq .132$), EGLS will produce better estimates of the regression coefficients than OLS.

This severity measure for heteroscedasticity, combined with the existing severity measure of autocorrelation, allows development of the Monte Carlo study to investigate the properties of various estimators under different intensities of autocorrelation and heteroscedasticity.

IV. STRUCTURING THE MONTE CARLO STUDY

The model:

$$y_t = \alpha + \beta x_t + e_t$$

$$e_t = \rho e_{t-1} + x_t^{\frac{\gamma}{2}} v_t$$

The estimators examined were:

Procedure (1): OLS on the untransformed data

Heteroscedasticity and autocorrelation cause OLS estimates to be inefficient but unbiased and GLS is best linear unbiased (BLUE). In general, rho and gamma are unknown and must be estimated before EGLS can be used. Thus the performance of EGLS relative to OLS will also depend on the accuracy of rho-hat and gamma-hat. These estimates are unlikely to be efficient since they are usually calculated on the basis of non-spherical errors.

Under these conditions it may be best to ignore both problems and just run OLS. If OLS is preferred, when is this true, that is, for what p, h combinations?

Procedure (2): correct solely for heteroscedasticity

The researcher chooses to ignore, or for some reason, believes autocorrelation is absent. The form of heteroscedasticity is assumed known as multiplicative but the severity, and thus gamma, must be estimated.

$$w_t = x_t^{\frac{\gamma}{2}} v_t, \text{ where } v_t \sim n(0,1), v(w_t) = x_t^\gamma V(v_t).$$

Following the procedure suggested by Harvey (1971) and others, use the OLS residuals to estimate w_t .

$$\hat{w}_t = y_t - x_t \beta^{OLS} - \alpha^{OLS}$$

$$\hat{V}(\hat{w}_t) = \hat{w}_t^2 \cong x_t^\gamma V(v_t)$$

$$\log(\hat{w}_t^2) \cong \gamma \log x_t + \log \sigma_v^2$$

Regress $\log(\hat{w}_t^2)$ on $\log x_t$ to get an estimate of gamma. Gamma-hat indicates the relationship between the error variances and the independent variable. Consequently it is used to weight the raw data so that the resulting error term is approximately spherical, or the heteroscedastic influence is counter-balanced.

$$\text{Thus } \frac{y_t}{x_t^{\frac{\hat{\gamma}}{2}}} = \alpha \frac{1}{x_t^{\frac{\hat{\gamma}}{2}}} + \beta \frac{x_t}{x_t^{\frac{\hat{\gamma}}{2}}} + \frac{w_t}{x_t^{\frac{\hat{\gamma}}{2}}}.$$

OLS on the corrected data will be BLUE if the transformation is successful.

However, if the disturbances are also autocorrelated, $e_t = \rho e_{t-1} + w_t$, the new error term is

$$\frac{\rho e_{t-1}}{\frac{\hat{\gamma}}{x_t^2}} + \frac{\frac{\gamma}{2} v_t}{\frac{\hat{\gamma}}{x_t^2}} \approx \frac{\rho e_{t-1}}{\frac{\hat{\gamma}}{x_t^2}} + v_t$$

It is obviously both autocorrelated and heteroscedastic. One form of heteroscedasticity has been traded for another. Furthermore, because the autocorrelation has been dismissed, the estimate of gamma will be inefficient. $e_t = \rho e_{t-1} + w_t$ but ρe_{t-1} has been ignored. Thus gamma-hat was calculated from autocorrelated errors and hence will be inefficient. The right hand side of the above equation will be a poor estimate of the left hand side. In other words, the researcher has failed to transform w_t into a spherical error, that is, he has not even isolated v_t . As a result, this estimator will likely perform poorly, except, perhaps in cases where the autocorrelation is weak compared to the degree of heteroscedasticity.

Procedure (3): correct solely for autocorrelation

Autocorrelation is known to exist, but its severity, ρ , must be estimated. This is potentially the preferred procedure when the degree of autocorrelation is much greater than that of heteroscedasticity. Moreover, tests for autocorrelation are robust if heteroscedasticity is also present, whereas heteroscedasticity tests are not robust if errors are jointly serially dependent. (Epps and Epps (1977)) Thus, autocorrelation is more likely to be detected and corrected for. Also, autocorrelation is more frequently tested for than homoscedasticity, simply because it is a standard statistic in most regression packages.

The Cochrane-Orcutt iterative technique, with special transformation of the first observation, was used.

(a) $y_t = \alpha + \beta x_t + e_t$

lag (a) one period

(b) $y_{t-1} = \alpha + \beta x_{t-1} + e_{t-1}$

multiply (b) by ρ

(c) $\rho y_{t-1} = \rho\alpha + \rho\beta x_{t-1} + \rho e_{t-1}$

(d) Subtract (c) from (a)

$$y_t - \rho y_{t-1} = \alpha(1-\rho) + \beta(x_t - \rho x_{t-1}) + e_t - \rho e_{t-1}$$

where $e_t - \rho e_{t-1} = \frac{\gamma}{x_t} v_t$.

Steps (a) through (d) are an attempt to compensate for the autocorrelated error term.

Rho is unknown and is estimated by regressing the OLS residuals on \hat{e}_t lagged. The Cochrane-Orcutt iterative procedure is used to calculate rho. The final rho chosen is the rho-hat of the tenth iteration or when the absolute difference in two successive rho-hats is $\leq .001$.

This estimate of rho is used to transform the raw data according to (d). The first observations were transformed according to $(1 - \hat{\rho}^2)^{\frac{1}{2}}$. OLS is run on this corrected data.

The rationale behind this procedure is to create a spherical error term such that better estimates can be calculated. Unfortunately, the new disturbance term, $\hat{e}_t - \hat{\rho}\hat{e}_{t-1}$ is an estimate of $x_t^{\frac{y}{2}} v_t$ which is still heteroscedastic. This will cause the estimates to remain inefficient. On the other hand, if p is large relative to h , this procedure could be optimal.

Procedure (4): correction for both autocorrelation and heteroscedasticity

(a) use the modified Cochrane-Orcutt iterative technique of procedure (3) to find the estimates of the regression coefficients and rho.

(b) use the residuals from the autocorrelated-corrected data to determine the transformation required to remove heteroscedasticity.

$$v(w_t) = x_t^\gamma v(v_t)$$

$$\hat{w}_t = \hat{e}_t - \hat{\rho}\hat{e}_{t-1} = y_t - \hat{\rho}y_{t-1} - \hat{\alpha}(1-\hat{\rho}) - \hat{\beta}(x_t - \hat{\rho}x_{t-1})$$

$$\log \hat{v}(\hat{w}_t) \cong \log (\hat{w}_t^2) \cong \log x_t^\gamma v(v_t)$$

regress $\log (\hat{w}_t^2)$ on $\log x_t$ and a constant to find $\hat{\gamma}$.

(c) use $x_t^{\frac{\hat{\gamma}}{2}}$ to weight the non-autocorrelated data in an attempt to create a spherical disturbance term.

$$(i) \frac{y_t - \hat{\rho}y_{t-1}}{\frac{\hat{\gamma}}{2} x_t} = \frac{\hat{\alpha}(1-\hat{\rho})}{\frac{\hat{\gamma}}{2} x_t} + \beta \frac{(x_t - \hat{\rho}x_{t-1})}{\frac{\hat{\gamma}}{2} x_t}$$

The resulting error term, $\frac{e_t - \hat{\rho}e_{t-1}}{\frac{\hat{\gamma}}{2} x_t}$

is close to being spherical hence OLS is applied to (i).

This procedure is an outcome of Epps and Epps' (1977) result that if heteroscedasticity and autocorrelation occur simultaneously, only the autocorrelation tests are robust. In this case the data should first be transformed to correct for autocorrelation, and then tested for heteroscedasticity. Extending Epps and Epp's conclusion to an estimation procedure:

autocorrelation should be tested for first and corrected first, if it exists. Next, test the non-autocorrelated residuals for heteroscedasticity. Finally, if heteroscedasticity is present the appropriate correction is applied to the autocorrelation-free data. The necessary transformation is determined from the autocorrelated-corrected residuals. Kmenta suggests a somewhat similar method, (1971, p.513)

This estimator is estimated GLS like (2) and (3), however it takes both problems into account and consequently should perform well. In addition, $\hat{\gamma}$ is based on approximately non-autocorrelated errors, in contrast to its estimate in (2), and thus should be more efficient. It follows that the correction for heteroscedasticity should also be more successful than in (2).

Procedure (5): Two-stage correction for both autocorrelation and heteroscedasticity

The rationale behind this method is the same as that for procedure (4). First correct for autocorrelation and then for heteroscedasticity. But the $\hat{\rho}$ from (4) is calculated on the basis of heteroscedastic errors and thus is inefficient. This fifth procedure attempts to improve procedure (4) by allowing for heteroscedasticity when ρ is estimated. Hopefully this will improve $\hat{\rho}$.

In addition, the success of the estimated GLS depends partially on the estimates of ρ and γ . γ was estimated on the basis of autocorrelated-corrected residuals. ρ was

estimated on the basis of heteroscedastic residuals, thus it will be inefficient but unbiased. To improve this estimate of rho, and EGLS, heteroscedasticity should be taken into account.

Rho-star is calculated from the regression of

$$\frac{\hat{e}_t}{\frac{\hat{\gamma}}{x_t^2}} \text{ on } \frac{\hat{e}_{t-1}}{\frac{\hat{\gamma}}{x_t^2}} \quad \text{Or} \quad \frac{y_t - \hat{\alpha} - x_t \hat{\beta}}{\frac{\hat{\gamma}}{x_t^2}} \text{ on } \frac{y_{t-1} - \hat{\alpha} - x_{t-1} \hat{\beta}}{\frac{\hat{\gamma}}{x_t^2}} .$$

Gamma-hat is the same one found in procedure (4). Beta-hat and alpha-hat are the Cochrane-Orcutt iterative estimates from procedure (3), that is, they were based on autocorrelated-corrected data. The resulting estimate of rho, p*, and gamma-hat are used to transform the raw data.

$$(ii) \frac{y_t - \rho^* y_{t-1}}{\frac{\hat{\gamma}}{x_t^2}} = \alpha \frac{(1 - \rho^*)}{\frac{\hat{\gamma}}{x_t^2}} + \beta \frac{(x_t - \rho^* x_{t-1})}{\frac{\hat{\gamma}}{x_t^2}} + w_t^* .$$

w_t^* should be approximately spherical, so OLS is applied to (ii).

Whether this procedure is an improvement over procedure (4) will depend, in part, on the properties of p* relative to

$\hat{\rho}$. It should surpass estimators (1), (2), and (3) since it makes provisions for jointly heteroscedastic-autocorrelated errors.

Mechanics

The five estimators were examined under nine severity combinations of heteroscedasticity and autocorrelation. The sampling distribution of each estimator for each combination was based on one hundred iterations.

The true alpha and beta were set at 5 and 2 respectively, and remain unchanged throughout the study. The independent variable, x_t , is also predetermined, and consists of 25 observations. x_t is fixed in repeated samples and across experiments, as is the sample size. The x's are the same as used in the Monte Carlo study of section III.

The dependent variable Y, and e, vary from sample to sample and across experiments, depending on the v generated, rho, and gamma. Rho and gamma are adjusted for each experiment in accordance with the desired severity levels of autocorrelation and heteroscedasticity,

To generate the 25 standard normally distributed errors, v_t an IMSL subroutine was used. These errors were then used to generate

$$e_t = \rho e_{t-1} + \frac{\gamma}{x_t} v_t \quad \text{with } e_1 = \frac{\frac{\gamma}{x_1} v_1}{(1 - \rho^2)^{1/2}}$$

IMSL

Although this required $v(e_{t-1}) = v(e_t)$, which is false given the heteroscedastic disturbances, it was thought to be more reasonable than setting e_1 to zero, requiring an identical stationary error for all Monte Carlo iterations, or setting

$e_1 = \frac{Y}{x_1^2} v_1$, which would be inconsistent with the modified Cochrane-Orcutt estimator used to ensure no observations are lost.

Once the e_t were calculated, the y_t were constructed according to $y_t = 5 + 2x_t + e_t$.

Summarizing, in any given experiment, 25 v_t are generated and then 25 e_t , and finally 25 y_t . The first iteration of the first experiment is ready to proceed. The raw data consists of the constructed y_t and the predetermined x_t . Once this iteration is complete, that is the regression coefficients from each of the five procedures have been estimated, the second iteration begins. Twenty-five new v_t , e_t , and y_t are produced, while the x_t , rho, gamma, alpha, and beta remain constant. Again, the estimates of alpha and beta for the five procedures are calculated. This process continues until 100 iterations have been completed. The sampling distribution of each estimator is thus formulated on the basis of the 100 estimates calculated from its corresponding procedure. This requires the calculation of the mean and variance for the individual estimators. The MSE is used to determine the preferred estimator. This cycle is repeated for each of the eight remaining heteroscedasticity-autocorrelation severity combinations.

The detailed Fortran program for this Monte Carlo study is in Appendix 2.

V. RESULTS AND CONCLUSIONS

Tables 3 to 11 contain the outcomes of the nine combinations of $p=.2, .4,$ and $.6$ with $h=.132, .2,$ and $.3$. The overall results are summarized in Table 2.

The Monte Carlo study in Section III indicated that EGLS, with correction solely for heteroscedasticity, would outperform OLS for even very weak levels of heteroscedasticity ($h \geq .132$). When heteroscedasticity and autocorrelation occur jointly the best solution depends on the levels of $p, h,$ and on their relative magnitudes.

The problems in estimating the complicated variance covariance matrix (i.e. both ρ and γ) so that EGLS can be applied, outweigh the potential benefits when heteroscedasticity and autocorrelation are weak ($p=.2$ and $.4$ combined with $h=.132$). For these two experiments, OLS on the raw data was the optimal estimator, followed by EGLS with correction for heteroscedasticity. But OLS quickly loses ground when one or both problems are present to any degree. For $p=.6$ and/or $h>.132$, OLS frequently had the second largest MSE. Autocorrelation and heteroscedasticity can no longer be evaded, and choice of the best procedure depends on p and h .

In practice, procedure (3), correction solely for autocorrelation, is the most likely approach because: (i) tests for autocorrelation are robust if heteroscedasticity jointly

occurs, whereas the converse is not true, (ii) autocorrelation is, in general, more often tested and corrected for simply because it is a standard test statistic of most regression packages, (iii) the relevant literature tends to opt for autocorrelation if there is a possibility of both occurring, probably for reasons given in (ii), and because of uncertainty in dealing with the joint problem.

The Monte Carlo study to compare estimators under conditions of joint heteroscedasticity and autocorrelation, indicates procedure (3) is the worst estimator. It had the largest variance in seven of the nine experiments, the worst bias in all nine and hence, not surprisingly, the worst MSE in seven of the experiments. Its poor performance indicates that even weak heteroscedasticity ($h=.132$) cannot be ignored in the simultaneous situation. The transformations to correct for autocorrelation are ineffective because the parameters are estimated from non-trivially heteroscedastic residuals and hence are inefficient.

The results for procedure (2), correction solely for heteroscedasticity, are in sharp contrast to those for procedure (3). Procedure (2) had the smallest MSE more often than any other estimator (4 times) and it was first or second best in six of the nine combinations. This reinforces the previous conclusion that even weak heteroscedasticity causes non-negligible estimation problems. In other words, the strength and the severity of heteroscedasticity differ. h may indicate a

low intensity of heteroscedasticity yet it can have a powerful influence on the residuals and hence on estimation.

In two experiments correction solely for autocorrelation, procedure (3), was not the worst solution, but the third worst. This occurred when there was a substantial difference between the degrees of heteroscedasticity and autocorrelation ($p=.6$, $h=.132$) and ($p=.6$, $h=.2$). These are the two cases where the usually "good" estimator of procedure (2) had the largest MSE. These results lead to several interesting observations: although technically p may be greater than h this does not mean autocorrelation is the more serious problem. In fact, for the combinations studied, $(p-h)$ is approximately $.4$ before the problems caused by heteroscedasticity are overshadowed by those of autocorrelation. That is, procedure (2), correction solely for heteroscedasticity, has the highest MSE in these situations. Furthermore, procedure (3), transformation for autocorrelation while ignoring heteroscedasticity, is still not optimal. Instead, procedure (5) where compensations are made for both autocorrelation and heteroscedasticity, becomes the preferred estimator, followed closely by procedure (4) and then procedure (3).

This does not preclude the use of procedure (2). It could be chosen in cases where autocorrelation is causing a great deal more deviant behavior than heteroscedasticity. However, it appears that if this is ever to be true, p must be substantially greater than h .

Procedure (5) is optimal in three of the nine situations: when $p=.6$ $h=.132$, $p=.6$ $h=.2$, and $p=.6$ $h=.3$. In other words, it will be adopted when autocorrelation also starts to hinder estimation.

For combinations $p=.6$ $h=.132$ and $p=.6$ $h=.2$ autocorrelation becomes a greater obstacle to estimation than heteroscedasticity. The tables provide the evidence. Procedure (3) no longer has the largest MSE, ignoring autocorrelation and transforming solely for heteroscedasticity, procedure (2), suddenly becomes the worst estimator.

In the last experiment, $p=.6$ $h=.3$, autocorrelation is still a relatively important problem, however it is not overpowering heteroscedasticity ($p-h=.3$). For support of this see Table 11, where correcting solely for heteroscedasticity is the third best solution, while correcting solely for autocorrelation is, once again, the least desirable approach. Clearly autocorrelation cannot be swamping heteroscedasticity otherwise the ranking of these two estimators would be reversed.

Procedures (4) and (5) have the same rationale: both autocorrelation and heteroscedasticity are dealt with on a sequential basis. Further, the same $\hat{\gamma}$ is used in (4) and (5). This $\hat{\gamma}$ is estimated from non-autocorrelated residuals. On the other hand, procedure (4) estimates ρ from heteroscedastic residuals, while (5), when estimating ρ , attempts to create a spherical error by taking heteroscedasticity into account. The outcomes indicate success

for procedure (5): it generally has a smaller MSE than procedure (4). Furthermore, the $\hat{\rho}$ of procedure (5) has a consistently smaller, albeit for some combinations only slightly, MSE than the estimate of ρ from procedure (4). This reinforces the above observation that the effort to reestimate ρ on the basis of non-heteroscedastic residuals is worthwhile.

The importance of taking heteroscedasticity into account becomes more pronounced as ρ increases. The more severe the autocorrelation, the more problems it can cause and thus the greater the benefits of improving $\hat{\rho}$. As an example, see experiments 7, 8, and 9. For a given h , the positive difference between the MSE's of the $\hat{\rho}$'s from procedure (4) and procedure (5) becomes greater as autocorrelation becomes more intense. In addition, for a given severity of autocorrelation, as heteroscedasticity becomes more severe, the importance of removing the detracting heteroscedasticity becomes more pronounced. Combinations 3, 6, and 9 show that for a given ρ , as h increases, the difference between the MSE's of the $\hat{\rho}$'s from procedure (4) and procedure (5) increases.

No similar statements can be made about the estimates of γ . The $\hat{\gamma}$ of procedure (4), and (5), is calculated from approximately non-autocorrelated residuals and thus is expected to be an improvement over procedure (2)'s $\hat{\gamma}$. The estimate of γ found in procedure (2) is calculated from autocorrelated residuals, yet it invariably has the smaller MSE.

Unfortunately $\hat{\gamma}$ is conditional on the success of the transformation to correct for autocorrelation and thus on $\hat{\rho}$. As mentioned earlier, $\hat{\rho}$ can be improved by compensating for heteroscedasticity. Consequently the estimate of γ could also be improved if this reestimated ρ was used to remove the influence of autocorrelation before estimating γ .

Summarizing, for this study,

- i) There is a difference between the strength of heteroscedasticity and the severity of heteroscedasticity. In other words, for even small h 's, heteroscedasticity can be a powerful hinderance to estimation.
- ii) For equal severities of autocorrelation and heteroscedasticity, heteroscedasticity is the more troublesome problem. When h is at least $.132$, heteroscedasticity dominates autocorrelation unless $(p-h) > .4$.
- iii) When heteroscedasticity dominates autocorrelation the best approach is to correct solely for heteroscedasticity and ignore autocorrelation.
- iv) the exception to (iii) is when both autocorrelation and heteroscedasticity are weak ($p \leq .4$, $h \leq .132$). Then OLS has the

smallest MSE.

- v) When autocorrelation is as strong as, or stronger than heteroscedasticity, (p is much greater than h), correct for both autocorrelation and heteroscedasticity using procedure (5). However, if for some reason procedure (5) is unavailable, procedure (4) would, in most circumstances, be a good substitute, but not procedure (3). In fact,
- vi) the overall worst approach is to correct solely for autocorrelation. Even when autocorrelation dominates heteroscedasticity, correcting solely for autocorrelation is sub-optimal.

vii)

Procedure (5) is in general preferred to procedure (4): ρ should be reestimated on the basis of heteroscedastic corrected data. This is especially true if heteroscedasticity is strong (but not necessarily severe) or if p is at least of intermediate severity (i.e. $p \approx .6$)

In conclusion, the Monte Carlo results indicate that the simultaneous occurrence of heteroscedasticity and autocorrelation

should not be ignored or simplified into a one dimensional problem. The appropriate estimation procedure depends on the absolute and relative magnitudes of the severity of autocorrelation and heteroscedasticity. This emphasizes the need for a severity measure of heteroscedasticity. h , the cosine measure of the intensity of heteroscedasticity appears to be the best alternative.

TABLE 2

THE MSE RANKINGS FOR EACH EXPERIMENT AND EACH ESTIMATOR

EXPERIMENT:	SMALLEST MSE	SECOND	THIRD	FOURTH	LARGEST MSE
ONE (p=.2, h=.132)	(1)	(2)	(5)	(4)	(3)
TWO (p=.4, h=.132)	(1)	(2)	(5)	(4)	(3)
THREE (p=.6, h=.132)	(5)	(4)	(3)	(1)	(2)
FOUR (p=.2, h=.2)	(2)	(5)	(4)	(1)	(3)
FIVE (p=.4, h=.2)	(2)	(1)	(5)	(4)	(3)
SIX (p=.6, h=.2)	(5)	(4)	(3)	(1)	(2)
SEVEN (p=.2, h=.3)	(2)	(4)	(5)	(1)	(3)
EIGHT (p=.4, h=.3)	(2)	(4)	(5)	(1)	(3)
NINE (p=.6, h=.3)	(5)	(4)	(2)	(1)	(3)

TABLE OF THE SMALLEST MSE FOR EACH EXPERIMENT

	h=.132	h=.2	h=.3
p=.2	(1)	(2)	(2)
p=.4	(1)	(2)	(2)
p=.6	(5)	(5)	(5)

TABLE OF THE LARGEST MSE FOR EACH EXPERIMENT

	h=.132	h=.2	h=.3
p=.2	(3)	(3)	(3)
p=.4	(3)	(3)	(3)
p=.6	(2)	(2)	(3)

KEY:

THE NUMBER IN BRACKETS INDICATES THE SPECIFIC PROCEDURE

TABLE 3

COMBINATION ONE: $p=-.2$ $b=-.132$

	SUMMARY			
	MEAN	VARIANCE	BIAS	MSE
PROCEDURE 1	0.20605795E+01	0.68222462E+00	0.60579522E-01	0.68589450E+00
PROCEDURE 2	0.20012718E+01	0.71480728E+00	0.12717944E-02	0.71480889E+00
PROCEDURE 3	0.20668382E+01	0.83808049E+00	0.66838212E-01	0.84254784E+00
PROCEDURE 4	0.20364653E+01	0.74368004E+00	0.36465272E-01	0.74500976E+00
PROCEDURE 5	0.20374822E+01	0.73314739E+00	0.37482179E-01	0.73455231E+00
PHAT1	0.87920455E-01	0.52149265E-01	-0.11207954E+00	0.64711089E-01
PHAT2	0.10096784E+00	0.54498516E-01	-0.99032164E-01	0.64305885E-01
GAMMA1	0.16969801E+01	0.23059865E+01	-0.30201991E+00	0.23978076E+01
GAMMA2	0.15653124E+01	0.21172127E+01	-0.43468760E+00	0.23061660E+01

KEY:

Procedure 1=OLS
 Procedure 2=EGLS with correction for heteroscedasticity only
 Procedure 3=EGLS with correction solely for autocorrelation
 Procedure 4=FGLS correction for both problems
 Procedure 5=EGLS two-stage correction of both problems
 Phat1 =estimate of rho used in procedure (4)
 Phat2 =estimate of rho from procedure (5)
 gamma1 =estimate of gamma from procedure (2)
 gamma2 =estimate of gamma from procedure (4)

TABLE 4

COMBINATION TWO: $p=.4$ $h=.132$

	SUMMARY			
	MEAN	VARIANCE	BIAS	MSE
PROCEDURE 1	0.20510929E+01	0.99994059E+00	0.51092939E-01	0.10025511E+01
PROCEDURE 2	0.20093171E+01	0.10813815E+01	0.93170566E-02	0.10814683E+01
PROCEDURE 3	0.20728431E+01	0.12662702E+01	0.72843067E-01	0.12715763E+01
PROCEDURE 4	0.20510387E+01	0.11560665E+01	0.51038725E-01	0.11586715E+01
PROCEDURE 5	0.20381174E+01	0.11538277E+01	0.38117368E-01	0.11552806E+01
PHAT1	0.25466543E+00	0.54783393E-01	-0.14533457E+00	0.75905531E-01
PHAT2	0.26777222E+00	0.56513766E-01	-0.13222778E+00	0.73997953E-01
GAMMA1	0.16493879E+01	0.21576168E+01	-0.35061214E+00	0.22805457E+01
GAMMA2	0.16449333E+01	0.20184999E+01	-0.35506667E+00	0.21445722E+01

KEY:

Procedure 1=OLS
 Procedure 2=EGLS with correction for heteroscedasticity only
 Procedure 3=EGLS with correction solely for autocorrelation
 Procedure 4=EGLS correction for both problems
 Procedure 5=EGLS two-stage correction of both problems
 Phat1 =estimate of rho from procedure (4)
 Phat2 =estimate of rho from procedure (5)
 Gamma1 =estimate of gamma from procedure (2)
 Gamma2 =estimate of gamma from procedure (4)

TABLE 5

COMBINATION THREE: $p=.6$ $h=.132$

	SUMMARY			
	MEAN	VARIANCE	BIAS	RSE
PROCEDURE 1	0.20280056E+01	0.18400241E+01	0.28005597E-01	0.18408084E+01
PROCEDURE 2	0.20021466E+01	0.21526162E+01	0.21466155E-02	0.21526208E+01
PROCEDURE 3	0.20592505E+01	0.17706021E+01	0.59250527E-01	0.17741128E+01
PROCEDURE 4	0.20017129E+01	0.17105865E+01	0.17129103E-02	0.17105894E+01
PROCEDURE 5	0.19973902E+01	0.16842692E+01	-0.26097862E-02	0.16842761E+01
PHAT1	0.4170411E+00	0.54718324E-01	-0.18295889E+00	0.88192281E-01
PHAT2	0.43255908E+00	0.55070058E-01	-0.16744092E+00	0.83106519E-01
GAMMA1	0.17205745E+01	0.23600882E+01	-0.27942554E+00	0.24381668E+01
GAMMA2	0.16522403E+01	0.26065252E+01	-0.34775966E+00	0.27274620E+01

KEY:

Procedure(1) =OLS
 Procedure(2) =EGLS with correction for heteroscedasticity only
 Procedure(3) =EGLS correct solely for autocorrelation
 Procedure(4) =EGLS with correction for both problems
 Procedure(5) =EGLS with two-stage estimation for both problems
 Phat1 =the estimate of rho from procedure (4)
 Phat2 =the estimate of rho from procedure (5)
 Gamma1 =the estimate of gamma from procedure (2)
 Gamma2 =the estimate of gamma from procedure (4)

TABLE 6

COMBINATION FOUR: $p=.2$ $h=.2$

	SUMMARY			
	MEAN	VARIANCE	BIAS	MSE
PROCEDURE 1	0.20564622E+01	0.44167779E+00	0.56462214E-01	0.44486578E+00
PROCEDURE 2	0.20259192E+01	0.37441965E+00	0.25919214E-01	0.37509146E+00
PROCEDURE 3	0.20629992E+01	0.56377376E+00	0.62999192E-01	0.56774265E+00
PROCEDURE 4	0.20464976E+01	0.42338213E+00	0.46497632E-01	0.42554416E+00
PROCEDURE 5	0.20467358E+01	0.41795177E+00	0.46735797E-01	0.42013601E+00
PHAT1	0.87952202E-01	0.56130535E-01	-0.11204780E+00	0.68685244E-01
PUAT2	0.11469674E+00	0.54023494E-01	-0.85303261E-01	0.61300140E-01
GAMMA1	0.16145225E+01	0.10220976E+01	-0.11854768E+01	0.24274527E+01
GAMMA2	0.15795703E+01	0.10108191E+01	-0.12204289E+01	0.25002659E+01

KEY:

Procedure(1) =OLS
 Procedure(2) =EGLS with correction for heteroscedasticity only
 Procedure(3) =EGLS correct solely for autocorrelation
 Procedure(4) =EGLS with correction for both problems
 Procedure(5) =EGLS with two-stage estimation for both problems
 Phat1 =the estimate of rho from procedure (4)
 Phat2 =the estimate of rho from procedure (5)
 Gamma1 =the estimate of gamma from procedure (2)
 Gamma2 =the estimate of gamma from procedure (4)

TABLE 7

COMBINATION FIVE: $p=.4$ $h=.2$

	SUMMARY			
	MEAN	VARIANCE	BIAS	MSE
PROCEDURE 1	0.20506371E+01	0.62110490E+00	0.50637068E-01	0.62366901E+00
PROCEDURE 2	0.20219518E+01	0.60215478E+00	0.21951802E-01	0.60263666E+00
PROCEDURE 3	0.20710207E+01	0.80337312E+00	0.71020661E-01	0.80841705E+00
PROCEDURE 4	0.20490355E+01	0.67118298E+00	0.49035479E-01	0.67358746E+00
PROCEDURE 5	0.20438712E+01	0.67123106E+00	0.43871215E-01	0.67315575E+00
PHAT1	0.25427408E+00	0.57281424E-01	-0.14572592E+00	0.78517468E-01
PHAT2	0.28062576E+00	0.54428508E-01	-0.11937424E+00	0.68678716E-01
GAMMA1	0.16505785E+01	0.10117028E+01	-0.11494207E+01	0.23328709E+01
GAMMA2	0.16022202E+01	0.10899718E+01	-0.11977790E+01	0.25246463E+01

KEY:

Procedure (1) = OLS
 Procedure (2) = EGLS with correction for heteroscedasticity only
 Procedure (3) = EGLS correct solely for autocorrelation
 Procedure (4) = EGLS with correction for both problems
 Procedure (5) = EGLS with two-stage estimation for both problems
 Phat1 = the estimate of rho from procedure (4)
 Phat2 = the estimate of rho from procedure (5)
 Gamma1 = the estimate of gamma from procedure (2)
 Gamma2 = the estimate of gamma from procedure (4)

TABLE 8

COMBINATION SIX: $p=-6$ $h=-2$

SUMMARY				
	MEAN	VARIANCE	BIAS	MSE
PROCEDURE 1	0.20359370E+01	0.10882287E+01	0.35936999E-01	0.10895202E+01
PROCEDURE 2	0.20123494E+01	0.11831662E+01	0.12349359E-01	0.11833187E+01
PROCEDURE 3	0.20563942E+01	0.10805250E+01	0.56394214E-01	0.10837053E+01
PROCEDURE 4	0.20146555E+01	0.97216057E+00	0.14655495E-01	0.97237536E+00
PROCEDURE 5	0.20124112E+01	0.94462245E+00	0.12411184E-01	0.94477649E+00
PHAT1	0.41602719E+00	0.56954847E-01	-0.18397281E+00	0.90800842E-01
PHAT2	0.44833301E+00	0.52225896E-01	-0.15166699E+00	0.75228773E-01
GAMMA1	0.16436744E+01	0.11239589E+01	-0.11563249E+01	0.24610461E+01
GAMMA2	0.16474512E+01	0.12042081E+01	-0.11525480E+01	0.25325750E+01

KEY:

Procedure (1) =OLS
 Procedure (2) =EGLS with correction for heteroscedasticity only
 Procedure (3) =EGLS correct solely for autocorrelation
 Procedure (4) =EGLS with correction for both problems
 Procedure (5) =EGLS with two-stage estimation for both problems
 Phat1 =the estimate of rhc from procedure (4)
 Phat2 =the estimate of rhc from procedure (5)
 Gamma1 =the estimate of gamma from procedure (2)
 Gamma2 =the estimate of gamma from procedure (4)

TABLE 9

COMBINATION SEVEN: $p=.2$ $h=.3$

	SUMMARY			
	MEAN	VARIANCE	BIAS	MSE
PROCEDURE 1	0.20555299E+01	0.31588911E+00	0.55529857E-01	0.31897268E+00
PROCEDURE 2	0.20329262E+01	0.23007037E+00	0.32926152E-01	0.23115450E+00
PROCEDURE 3	0.20647966E+01	0.40694502E+00	0.64796631E-01	0.41114363E+00
PROCEDURE 4	0.20451434E+01	0.26654217E+00	0.45143432E-01	0.26858010E+00
PROCEDURE 5	0.20443513E+01	0.27541507E+00	0.44351261E-01	0.27738210E+00
PHAT1	0.88672070E-01	0.62375498E-01	-0.11132793E+00	0.74769406E-01
PHAT2	0.13958867E+00	0.54966367E-01	-0.60411326E-01	0.58615896E-01
GAMMA1	0.15991326E+01	0.55028278E+00	-0.25608672E+01	0.71083237E+01
GAMMA2	0.15361548E+01	0.46181972E+00	-0.26238451E+01	0.73463827E+01

KEY:

Procedure(1) =OLS
 Procedure(2) =EGLS with correction for heteroscedasticity only
 Procedure(3) =EGLS correct solely for autocorrelation
 Procedure(4) =EGLS with correction for both problems
 Procedure(5) =EGLS with two-stage estimation for both problems
 Phat1 =the estimate of rho from procedure (4)
 Phat2 =the estimate of rho from procedure (5)
 Gamma1 =the estimate of gamma from procedure (2)
 Gamma2 =the estimate of gamma from procedure (4)

TABLE 10

COMBINATION EIGHT: $p=-.4$ $h=-.3$

	SUMMARY		
	MEAN	VARIANCE	BIAS
PROCEDURE 1	0.20526928E+01	0.41776977E+00	0.52692797E-01
PROCEDURE 2	0.20239110E+01	0.36363189E+00	0.23910963E-01
PROCEDURE 3	0.20707017E+01	0.54359504E+00	0.70701698E-01
PROCEDURE 4	0.20412457E+01	0.39469203E+00	0.41245658E-01
PROCEDURE 5	0.20408191E+01	0.39861951E+00	0.40819147E-01
PHAT1	0.25470638E+00	0.61867379E-01	-0.14529362E+00
PHAT2	0.31476260E+00	0.54839418E-01	-0.85237400E-01
GAMMA1	0.15838648E+01	0.51897908E+00	-0.25761351E+01
GAMMA2	0.15375098E+01	0.51918463E+00	-0.26224901E+01
			MSE
			0.42054630E+00
			0.36420362E+00
			0.54859377E+00
			0.39639323E+00
			0.40028571E+00
			0.82977614E-01
			0.62104832E-01
			0.71554510E+01
			0.73966388E+01

KEY:

Procedure(1) =OLS
 Procedure(2) =EGLS with correction for heteroscedasticity only
 Procedure(3) =EGLS correct solely for autocorrelation
 Procedure(4) =EGLS with correction for both problems
 Procedure(5) =EGLS with two-stage estimation for both problems
 Phat1 =the estimate of rho from procedure (4)
 Phat2 =the estimate of rho from procedure (5)
 Gamma1 =the estimate of gamma from procedure (2)
 Gamma2 =the estimate of gamma from procedure (4)

TABLE 11

COMBINATION NINE: p=.6 h=.3

	SUMMARY			
	MEAN	VARIANCE	BIAS	RSE
PROCEDURE 1	0.20448375E+01	0.67540684E+00	0.44837530E-01	0.67741725E+00
PROCEDURE 2	0.20225133E+01	0.66663462E+00	0.22513251E-01	0.66714147E+00
PROCEDURE 3	0.20570729E+01	0.67958117E+00	0.57072920E-01	0.68283849E+00
PROCEDURE 4	0.20369682E+01	0.59415466E+00	0.36868181E-01	0.59552131E+00
PROCEDURE 5	0.20399647E+01	0.55892462E+00	0.39964727E-01	0.56052180E+00
PHAT1	0.41465963E+00	0.60542454E-01	-0.18534037E+00	0.94893508E-01
PHAT2	0.47831211E+00	0.51730279E-01	-0.12168789E+00	0.66538222E-01
GAMMA1	0.15323048E+01	0.57075624E+00	-0.26276951E+01	0.74755377E+01
GAMMA2	0.15696456E+01	0.55561246E+00	-0.25903543E+01	0.72655476E+01

KEY:
 Procedure (1) =OLS
 Procedure (2) =EGLS with correction for heteroscedasticity only
 Procedure (3) =EGLS correct solely for autocorrelation
 Procedure (4) =EGLS with correction for both problems
 Procedure (5) =EGLS with two-stage estimation for both problems
 Phat1 =the estimate of rho from procedure (4)
 Phat2 =the estimate of rho from procedure (5)
 Gamma1 =the estimate of gamma from procedure (2)
 Gamma2 =the estimate of gamma from procedure (4)

Appendix 1

Proof:

$$\cos \theta = \frac{\sum_{i=1}^N u_i v_i}{\sqrt{\sum_{i=1}^N u_i^2} \sqrt{\sum_{i=1}^N v_i^2}} = \frac{\sum_{i=1}^N u_i}{\sqrt{N \sum_{i=1}^N u_i^2}} \quad \text{for } v_i = 1 \forall i$$

minimize $\cos \theta$ with respect to u_i , subject to $u_i \geq 0$ (since u = a vector of variances)

$$\begin{aligned} \frac{\partial \cos \theta}{\partial u_j} &= \frac{\sqrt{N \sum u_i^2} - N u_j (N \sum u_i^2)^{-\frac{1}{2}} \sum u_i}{N \sum u_i^2} \\ &= \frac{N \sum u_i^2 - N u_j \sum u_i}{(N \sum u_i^2)^{3/2}} \end{aligned}$$

setting this equal to zero and solving for u_j :

$$u_j = \frac{\sum u_i^2}{\sum u_i}$$

since the right hand side is independent of j , this implies

$$u_j = u_k = u \quad \forall_{j,k}$$

then

$$\cos \theta = \frac{\sum u_i}{\sqrt{N \sum u_i^2}} = \frac{N u}{\sqrt{N^2 u^2}} = 1$$

But a cosine of 1 is a maximum not a minimum, thus

$u_j = u_k = u > 0$ is a local and an absolute maximum. This means that $\cos \theta$ cannot attain a minimum at an interior point in the positive orthant. The minimum must be at a boundary.

Since the minimum point lies on a boundary of the positive orthant it will lie either on a coordinate hyperplane or on a coordinate axis. $\cos \theta$ is completely symmetric with respect to u_1, \dots, u_N . Hence, without loss of generality, the starting point can be any coordinate hyperplane, in particular the u_N hyperplane - that is $u_N = 0$.

To find the minimum $\cos \theta$ on this plane, minimize $\cos \theta$ subject to $u_N = 0$. The result is $u_j = \frac{\sum_{i=1}^{N-1} u_i^2}{\sum_{i=1}^{N-1} u_i}$ for $j = 1, \dots, N-1$,

and $u_N = 0$ and $\cos \theta = \sqrt{\frac{N-1}{N}}$.

But this is a maximum $\cos \theta$ for this hyperplane ($u_N = 0$) since for any other combination of u_j 's $\cos \theta$ would be smaller.

For example, if $u_j = \dots = u_{N-2}$ and $u_{N-1} = u_N = 0$ $\cos \theta = \sqrt{\frac{N-2}{N}}$

which is $< \sqrt{\frac{N-1}{N}}$.

Again, the minimum must be a boundary solution on this hyperplane. This means the minimum must occur on some plane of lower dimension without loss of generality take $u_{N-1} = u_N = 0$ and minimize $\cos \theta$. Continue this process, until two dimensions

are left, each time taking one more component as zero, and searching the resulting hyperplane for a minimum cosine. The conclusion will be similar for each hyperplane: $u_j = u_k = u$ for $j = 1$ to $N-\tau$ with the remaining u_j 's constrained to zero. In each case, $\cos \theta$ will equal $\sqrt{\frac{N-\tau}{N}}$ and will be a maximum at $u_j = u_k = u$, not the desired minimum.

When two dimensions remain, the minimization of $\cos \theta$

subject to $u_3 = \dots = u_N = 0$ gives $u_j = \frac{\sum_{i=1}^2 u_i^2}{\sum_{i=1}^2 u_i}$. Or $u_1 = u_2 = u$,

$u_3 = \dots = u_N = 0$ and $\cos \theta = \sqrt{\frac{2}{N}}$. Once again, a maximum not a minimum is found. A minimum will occur if $u_1 = 0, u_2 = 1$ (the u_2 axis), or $u_2 = 0, u_1 = 1$ (u_1 axis), that is, on one of the boundaries of the positive u_1, u_2 quadrant and $\cos \theta$ will equal $\frac{1}{\sqrt{N}}$.

Consequently $\cos \theta$ attains a minimum when $u_j = 1$ for some j and $u_k = 0$ for $k \neq j, k = 1, \dots, N$. In other words, the minimum is when u lies along one of the N coordinate axis and $\cos \theta = \frac{1}{\sqrt{N}}$.

Appendix 2

PROGRAM FOR A MONTE CARLO STUDY TO COMPARE HETEROSCEDASTICITY AND AUTOCORRELATION

```

REAL*8 X(25),Y(25),E(25),V(25),EIND(25),TEX(25),NT(25),NSTAR(25),
&TEMPY(25),DOT(100),YTR(25),XTR(25),YTR2(25),XTR2(25),XTR2(25),
&BETA1(100),BETA2(100),BETA3(100),BETA4(100),BETA5(100),
&PHAT1(100),PHAT2(100),PTEMP(100),BTEMP(100),NTR(25),
&YSTAR(25),XSTAR(25),ESTAR(25),EHAT(25),ESLAG(25),DGLS(100)
REAL*8 LX(25),CEPT(100),POPHAT(100),TEA(25),DUMP(100),BOGS(100)
REAL*8 DSEED,ALPHA,BETA,RHO,M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,M11,
&V5,V6,V7,DF1,DF2,DF3,DF4,DF5,DF6,DF7,MS1,MS2,MS3,MS4,MS5,MS6,
&MS7,MX,MX,P1,B1,BOLS,P2,B2,BLOB,CEP,BOO,BO1,M8,M9,V8,V9,DF8
REAL*8 DF9,MS8,MS9,GAMMA(100),CON(25)
DIMENSION VS(25)

```

```

C READ(5,300,EHD=501) ( X(I), I=1,25)
300 FORMAT(D10.4)
501 CONTINUE
K=1
KMAX=100
ALPHA=5D0
BETA=2D0
EXP=4.16
RHO=-6D0
N=25
M=24
DSEED=299D0

```

GENERATION OF ERROR TERMS AND THE DEPENDENT VARIABLE.

```

C 5 CALL GGNML(DSEED,25,VS)
DO 101 I=1,25
V(I)=VS(I)
101 CONTINUE
10 E(I)=(X(I)/DSQRT(1D0 - RHO**2))*V(1)
Y(I)=ALPHA + BETA*X(I) + E(I)
DO 102 I=2,25
E(I)=RHO*E(I-1) + X(I)*V(I)
Y(I)=ALPHA + BETA*X(I) + E(I)
102 CONTINUE

```

PROCEDURE 1: ORDINARY LEAST SQUARES REGRESSION

```

CALL PEG(N,Y,X,BETA1(K),CEPT(K))

```

```

C
C
C
PROCEDURE 2: CORRECT RAW DATA FOR HETEROSEDASTICITY
DO 990 I= 1,25
CON(I)=1D0
EIND(I)=DLOG( (Y(I) - X(I)*BETA1(K) - CEPT(K))**2 )
LX(I)= DLOG(X(I))
990 CONTINUE
C
CALL REGNCH(N,EIND,LX,CON,GAMMA(K),DUMP(K))
C
DO 103 I=1,25
TEA(I)= 1D0/( X(I)**(GAMMA(K)/2) )
TEX(I)=X(I)/( X(I)**(GAMMA(K)/2) )
TEPY(I)=Y(I)/( X(I)**(GAMMA(K)/2) )
103 CONTINUE
CALL REGNCH(N,TEPY,TEX,TEA,BETA2(K),DUMP(K))
C
C
C
PROCEDURE 3: CORRECT RAW DATA FOR AUTOCORRELATION.
ITERATIVE CORCHRANE-ORCUTT METHOD. SPECIAL
TRANSFORMATION FOR FIRST OBSERVATION.
CEP=CEPT(K)
C
BLOB=BETA1(K)
CALL CORC(M,N,Y,X,BLOB,CEPT,PHAT1(K),BETA3(K),BOT(K))
C
C
C
C
PROCEDURE 4: CORRECTION FOR BOTH AUTOCORRELATION AND
HETEROSEDASTICITY.
NSTAR(1)=DSQRT(1D0 - PHAT1(K)**2)
XSTAR(1)=DSQRT(1D0 - PHAT1(K)**2)*Y(1)
YSTAR(1)=DSQRT(1D0 - PHAT1(K)**2)*Y(1)
EIND(1)=DLOG( (YSTAR(1)-BETA3(K)*XSTAR(1)-BOT(K)*NSTAR(1))**2)
DO 991 I=2,25
NSTAR(I)=(1D0 -PHAT1(K))
XSTAR(I)= X(I) - PHAT1(K)*X(I-1)
YSTAR(I)= Y(I) - PHAT1(K)*Y(I-1)
EIND(I)=DLOG( (YSTAR(I)-BETA3(K)*XSTAR(I)-BOT(K)*NSTAR(I))**2)
991 CONTINUE
CALL REGNCH(N,EIND,LX,CON,POWHAT(K),DUMP(K))
C
NT(1)=DSQRT(1D0 - PHAT1(K)**2)/( X(1)**(POWHAT(K)/2) )
YTR2(1)=DSQRT(1D0-PHAT1(K)**2)*Y(1)/( X(1)**(POWHAT(K)/2) )
XTR2(1)=DSQRT(1D0-PHAT1(K)**2)*X(1)/( X(1)**(POWHAT(K)/2) )
DO 108 I=2,25
YTR2(I)=(Y(I) - PHAT1(K)*Y(I-1))/( X(I)**(POWHAT(K)/2) )
XTR2(I)=( X(I) - PHAT1(K)*X(I-1))/( X(I)**(POWHAT(K)/2) )
NT(I)= (1D0 - PHAT1(K))/( X(I)**(POWHAT(K)/2) )
108 CONTINUE
CALL REGNCH(N,YTR2,XTR2,NT,BETA4(K),DUMP(K))
C

```

C C PROCEDURE 5: JOINT AUTOCORRELATION AND HETEROSCEDASTICITY CORRECTION
 C WITH RHO RE-ESTIMATED.

```

DO 109 I=1,25
  EIND(I) = (Y(J) - X(I)*BETA3(K) - BOT(K))
  IF (I.EQ. 1) GO TO 999
  ESTAR(I-1) = EIND(I)/( X(I)**(POWHAT(K)/2) )
  999 IF (I.EQ. 25) GO TO 109
  EIND(I) = EIND(I)/( X(I+1)**(POWHAT(K)/2) )
  109 CONTINUE

```

C CALL NCHREG(M, FSTAR, EIND, PHAT2(K))
 C THIS ESTIMATE OF RHO TAKES THE HETEROSEDASTICITY OF THE EPORR TERMS
 C INTO ACCOUNT.

```

NSTAR(I) = DSQRT(1D0 - PHAT2(K)**2)/( X(I)**(POWHAT(K)/2) )
YSTAR(I) = DSQRT(1D0 - PHAT2(K)**2)*Y(I)/( X(I)**(POWHAT(K)/2) )
XSTAR(I) = DSQRT(1D0 - PHAT2(K)**2)*X(I)/( X(I)**(POWHAT(K)/2) )
DO 110 I=2, 25
  NSTAR(I) = (1D0 - PHAT2(K))/( X(I)**(POWHAT(K)/2) )
  YSTAR(I) = (Y(I) - PHAT2(K)*Y(I-1))/( X(I)**(POWHAT(K)/2) )
  XSTAR(I) = (X(I) - PHAT2(K)*X(I-1))/( X(I)**(POWHAT(K)/2) )

```

110 CONTINUE
 CALL REGNCH(N, YSTAR, XSTAR, NSTAR, BETA5(K), DUMP(K))

C C THE NEXT TRIAL BEGINS.
 C IF (K-KMAX) 30,40,40
 30 DSEED=DSLEED+1D0
 K=K+1
 GO TO 5
 40 CONTINUE
 WRITE (6,302)
 302 FORMAT('1, //, 37X, ' EXPERIMENT ONE: WEAK HETEROSCEDASTICITY AND WEAK AUTOCORRELATION. ')
 WRITE (6,303)
 303 FORMAT('//, 40X, ' EXPECTED VALUE OR MEAN OF BETA FOR EACH ESTIMATOR.
 '(SAMPLE SIZE=100) ')

C
 CALL AVG(K, BETA1, M1)
 CALL AVG(K, BETA2, M2)
 CALL AVG(K, BETA3, M3)
 CALL AVG(K, BETA4, M4)
 CALL AVG(K, BETA5, M5)
 CALL AVG(K, PHAT1, M6)
 CALL AVG(K, PHAT2, M7)
 CALL AVG(K, GAMMA, M8)
 CALL AVG(K, POWHAT, M9)

```

WRITE(6,304) M1,M2,M3,M4,M5
304 FORMAT(//,1X,'PROCEDURE 1:',5X,D16.8,///,1X,'PROCEDURE 2:',5X,
1D16.8,///,1X,'PROCEDURE 3:',5X,D16.8,///,1X,'PROCEDURE 4:',5X,
2D16.8,///,1X,'PROCEDURE 5:',5X,D16.8)
WRITE(6,305)
305 FORMAT(///,42X,'MEANS OF THE ESTIMATORS OF RHO. (TRUE RHO=-.2)')
WRITE(6,306) M6,M7
306 FORMAT(//,1X,'CORCHRANE-CRUCUTT ESTIMATE OF RHO.',5X,D16.8,///,
6,'ESTIMATE OF RUC FROM PROCEDURE 5.',5X,D16.8)

```

C

```

CALL DEV(K,BETA1,M1,V1)
CALL DEV(K,BETA2,M2,V2)
CALL DEV(K,BETA3,M3,V3)
CALL DEV(K,BETA4,M4,V4)
CALL DEV(K,BETA5,M5,V5)
CALL DEV(K,PHAT1,M6,V6)
CALL DEV(K,PHAT2,M7,V7)
CALL DEV(K,GAMMA,M8,V8)
CALL DEV(K,POWHAT,M9,V9)

```

C

```

WRITE(6,307)
307 FORMAT(///,49X,'VARIANCE OF THE ESTIMATORS OF BETA.')
```

```

WRITE(6,308) V1,V2,V3,V4,V5
WRITE(6,308)
308 FORMAT(///,49X,'VARIANCE OF THE ESTIMATORS OF RHO.')
```

```

WRITE(6,309) V6,V7

```

C

```

CALL BIAS(M1,BETA,DP1)
CALL BIAS(M2,BETA,DP2)
CALL BIAS(M3,BETA,DP3)
CALL BIAS(M4,BETA,DP4)
CALL BIAS(M5,BETA,DP5)
CALL BIAS(M6,RHO,DP6)
CALL BIAS(M7,RHO,DP7)
CALL BIAS(M8,EXP,DP8)
CALL BIAS(M9,EXP,DP9)

```

C

```

WRITE(6,309)
309 FORMAT('1',///,47X,'BIAS OF THE ESTIMATORS OF BETA AND RHO.')
```

```

WRITE(6,304) DP1,DP2,DP3,DP4,DP5
WRITE(6,306) DP6,DP7
WRITE(6,310)

```

```

310 FORMAT(///,40X,'MEAN SQUARE ERROR OF THE ESTIMATORS FOR BETA AND RHO.')
```

1

1

C

```

CALL MSE(V1,DP1,MS1)
CALL MSE(V2,DP2,MS2)
CALL MSE(V3,DP3,MS3)
CALL MSE(V4,DP4,MS4)
CALL MSE(V5,DP5,MS5)
CALL MSE(V6,DP6,MS6)
CALL MSE(V7,DP7,MS7)
CALL MSE(V8,DP8,MS8)
CALL MSE(V9,DP9,MS9)

```

C VARIOUS SUBROUTINES FOLLOW THE MAIN PROGRAM.

```
C THIS SUBROUTINE CALCULATES BETA AND ALPHA WHEN THERE IS A CONSTANT TERM.  
C SUBROUTINE REG(N,Y,X,BETA,ALPHA)  
  REAL*8 X(25),Y(25)  
  REAL*8 XBAR,YBAR,NUM,DENOM,SUMY,SUMX,BETA,ALPHA  
  SUMY=0  
  SUMX=0  
  DO 200 I=1,N  
    SUMX = SUMX + X(I)  
    SUMY = SUMY + Y(I)  
  200 CONTINUE  
  XBAR = SUMX/N  
  YBAR = SUMY/N  
  NUM = 0  
  DENOM = 0  
  DO 201 I=1,N  
    NUM = (X(I)-XBAR)*(Y(I)-YBAR) + NUM  
    DENOM = (X(I) - XBAR)**2 + DENOM  
  201 CONTINUE  
  BETA = NUM/DENOM  
  ALPHA=YBAR - BETA*XBAR  
  RETURN  
  END
```

C THIS SUBROUTINE CALCULATES BETA WHEN THERE IS NO INTERCEPT.

```
C SUBROUTINE NCNREG(M,Y,X,BETA)  
  REAL*8 Y(25),X(25)  
  REAL*8 BETA,SUMX,NUME  
  SUMX=0  
  NUME=0  
  DO 204 I=1,M  
    SUMX=X(I)*X(I) + SUMX  
    NUME=Y(I)*X(I) + NUME  
  204 CONTINUE  
  BETA=NUME/SUMX  
  RETURN  
  END
```

```
C SUBROUTINE AVG(K,X,MEAN)  
  REAL*8 X(100)  
  REAL*8 MEAN,SUMX  
  SUMX=0  
  DO 202 I= 1,K  
    SUMX = X(I) + SUMX  
  202 CONTINUE  
  MEAN = SUMX/K
```

```
  RETURN  
  END
```

```

SUBROUTINE DEV (K, X, MEAN, VAR)
REAL*8 K(100)
RFAL*8 NUME,VAR,MEAN
NUME = 0
DO 203 I=1,K
NUME = (X(I) - MEAN)**2 + NUME
203 CONTINUE
VAR = NUME/(K-1)

```

```

RETURN
END

```

```

SUBROUTINE BIAS(BHAT, BETA, DF)
REAL*8 BETA, BHAT, DF
DF = BHAT - BETA
RETURN
END

```

```

SUBROUTINE MSE (VAR, BIAS, M)
REAL*8 VAR, BIAS, M
M = VAR + BIAS**2
RETURN
END

```

THIS SUBROUTINE CONTAINS THE ITERATIVE CORCHRAN-ORCUTT PROCEDURE.

```

SUBROUTINE CORC(M, N, X, Y, X, BLOB, CEP, PHAT1, BETA3, BOT)
REAL*8 Y(25), X(25), EIND(25), EHAT(25), YSTAR(25), XSTAR(25)
REAL*8 ESTAP(25), YTR(25), XTR(25), ESLAG(25), NSTAR(25), NTP(25)
REAL*8 P1, B1, P2, B2, DIE, PHAT1, BETA3, BLOB, CEP, BOO, B01, BOT

```

```

J=1
DO 4 I=1, N
EIND(I)=Y(I) - X(I)*BLOB-CEP
4 CONTINUE
DO 5 I=1, M
EHAT(I)=EIND(I+1)
5 CONTINUE

```

```

CALL NCNREG(M, EHAT, EIND, P1)

```

SPECIAL TRANSFORMATION OF FIRST OBSERVATION.

```

NSTAR(1)= DSORT(100 - P1**2)
YSTAR(1)= DSORT(100 - P1**2)*Y(1)
XSTAR(1)= DSORT(100 - P1**2)*X(1)
DO 10 I=2, N
NSTAR(I)= 100 - P1
YSTAR(I)= Y(I) - P1*Y(I-1)
XSTAR(I)= X(I) - P1*X(I-1)

```

```

10 CONTINUE
CALL PEGNCHM(M, YSTAR, XSTAR, NSTAR, B1, BOO)

```



```

C 11 DO 12 I=1,N
    ESLAG(I)= Y(I) - X(J) *B1-B00
12 CONTINUE
DO 13 I=1,M
    ESTAR(I)= ESLAG(I+1)
13 CONTINUE
    CALL NCNREG (M,ESTAR,ESLAG,P2)
C
    NTR(1)=DSQRT(1-P2**2)
    YTR(1)=DSQRT(100 - P2**2)*Y(1)
    XTR(1)=DSQRT(100 - P2**2)*X(1)
DO 14 I=2,N
    NTR(I)=(100 - P2)
    YTR(I) = Y(I) - Y(I-1)*P2
    XTR(I) = X(I) - X(I-1)*P2
14 CONTINUE
    CALL REGNCH(N,YTP,XTR,NTR,B2,B01)
C
    DIP=DABS(P1 - P2)
    IP (DIF-.001) 16,16,15
    IF (J-10) 17,16,16
C
17 B1=B2
    P1=P2
    B00=B01
    J=J+1
    GO TO 11
C
16 PHAT1=P2
    BETA3=B2
    BOT=B01
    RETURN
END
C
    THIS CALCULATES BETA2 AND BETA1 FOR A TWO VARIABLE REGRESSION.
SUBROUTINE REGNCH(N,YT,XT,NT,BGLS,BOGS)
    REAL*8 YT(25),XT(25),NT(25)
    REAL*8 SUMX1,SUMX2,SUMXX,SUMX1Y,SUMX2Y,DENOM,BGLS,BOGS
    SUMX1=0
    SUMX2=0
    SUMXX=0
    SUMX1Y=0
    SUMX2Y=0
DO 700 I=1,N
    SUMX1=SUMX1 + NT(I)**2
    SUMX2=SUMX2 + XT(I)**2
    SUMXX=SUMXX + XT(I)*NT(I)
    SUMX1Y=SUMX1Y + NT(I)*YT(I)
    SUMX2Y=SUMX2Y + XT(I)*YT(I)
    DENOM= SUMX1*SUMX2 - SUMXX**2
700 CONTINUE
    BGLS=(SUMX1*SUMX2Y - SUMXX*SUMX1Y)/DENOM
    BOGS=(SUMX1Y*SUMX2 - SUMX2Y*SUMXX)/DENOM
    RETURN
END

```

```

SUBROUTINE CLEF(EHAT,EIND,P1)
    REAL*8 EHAT(25),EIND(25)
    REAL*8 SUMX,SUMY,SUMXY,P1
    SUMX=0
    SUMY=0
    SUMXY=0
DO 778 I=1,24
    SUMX=SUMX + EIND(I)**2
    SUMY=SUMY + EHAT(I)**2
    SUMXY= EIND(I)*EHAT(I) + SUMXY
CONTINUE
    P1=SUMXY/DSQRT(SUMX*SUMY)
    RETURN
END

```

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