STOCHASTIC OPTIMAL CONTROL WITH A CANADIAN ECONOMETRIC MODEL

by

Malcolm E. Turner

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APPROVAL

Name: Malcolm E. Turner

Degree: Doctor of Philosophy

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Examining Committee:

Chairman: Clyde G. Reed

Charles E. Love
Senior Supervisor

Richard Holmes

John Chant

Martin Puterman
External Examiner
Associate Professor
Faculty of Commerce
University of British Columbia

Date Approved: April 3, 1984

ABSTRACT

Stochastic optimal control methods were applied to an eight-equation, log-linear econometric model of the Canadian economy. Certain variables were selected as control variables which were used as instruments to direct a group of target variables towards preset target values over a planning time horizon.

Several stochastic control models were tested on this econometric model and an expected penalty cost, an additive quadratic function of the deviations from the preset targets, was used as a standard of comparison. The control models ranged from simple models which ignored uncertainty to more sophisticated adaptive control models, designed to compensate for the continual revision of the model parameters over time. Testing took the form of computer simulation, generating many typical time spans while recording the model's overall performance against the preset targets.

An important finding was that the optimal control Hamiltonian could be optimized recursively, enabling the stochastic control problems to be solved by conventional stochastic dynamic programming methods.

A simple model, although one that allowed for uncertainty in the regression coefficients, performed consistently well, even when tested in an adaptive environment. Such a model could be used to perform control optimizations on a 50-60 equation, econometric model without excessive computational cost.

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I. INTRODUCTION

A large portion of economic textbook theory is static and deterministic. A static theory is a timeless concept. Very often a system is assumed to be in static equilibrium without knowledge of how it actually got there or where it might move next. The position is timeless in the sense that there are no links with the past or with the future. Some textbooks, when covering dynamics, concentrate only on equilibrium dynamics. This is the situation where a system is moving (or growing) uniformly over time. Disequilibrium dynamics, a more difficult concept, assumes non uniform movement through time and normally involves lagged variables along with error-adjustment or control mechanisms. Frequently, uncertainty in the system values or parameters is ignored and the economic system is treated as if it were deterministic. In the work that follows, disequilibrium dynamics of a particular stochastic system will be carefully examined and, amongst other things, the effects of ignoring uncertainty will be evaluated.

The system chosen for study is a model of the Canadian economy but it could just as easily have been a model of a business firm or some other institution. It is assumed that the system must meet certain objectives for its target variables, e.g., specific levels of inflation or unemployment over the planning time period. Some of the variables in the model are termed instrument or control variables, e.g., money supply or level of government expenditure, and these variables are seen as being available for adjustment to steer the system as close as possible to its targets. This in a nut shell is the optimal control problem. In a deterministic environment, where the number of instruments is equal to the number of target variables, the targets can be achieved precisely. For a stochastic model where the number of instruments might be less than the number of target variables, the optimal control solution will be the one that is closest to achieving all the targets.

To determine the optimal values of the control variables under uncertainty, stochastic optimal control theory will be used. This theory was originally developed by engineers and scientists for the control of hardware systems. In the early seventies much interest was shown in applying the theory to economic systems.

The research objectives for this study are:

1. to develop, test and calibrate a set of computer algorithms for analysing the stochastic optimal control problem under varying degrees of uncertainty
2. to apply the algorithms to a relatively-small, econometric model of the Canadian economy
3. to assess the practical applications of stochastic optimal control theory.

In Chapter II some of the fundamental ideas of general control theory are developed. The intent here is to clear up some of the confusing definitions and to set the stage for some later theoretical analysis. Chapter III presents the basic theory behind the linear-quadratic-tracking controller for simple feedback control. Again, the goal is to define and clarify but also it is to provide a framework for comparing the various approaches to adaptive control developed in Chapter IV. The methodology for the study is outlined in Chapter V where special mention is made of Monte Carlo testing to be carried out on each proposed theoretical solution. Unfortunately, there has been a lot of theorizing and very little in the way of comprehensive testing in many economic control studies. The results and conclusions follow in Chapters VI and VII.

II. GENERAL CONTROL THEORY

General control theory was originally developed for industrial or scientific applications. In this chapter, the general theory will be very briefly summarized not merely to show the similarities with the newer economic control theories but to define some of the different types of control. In the same way that there are confusions in terms between the two schools of forecasting: econometric forecasting and time series forecasting, so there is confusion between the older scientific control theories and the newer economic control theories. In a 1979 article by Zellner,[1] a plea is made for less conflict and more cooperation between the forecasting schools with the aim of improved forecasts. Even though there is some terminology confusion, there appears to be great potential for cooperation between the disciplines of control theory. An excellent example is a recent economic control study[2] by two economists: Kendrick and Norman, and two control engineers: Tse and Bar-Shalom.

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1 Arnold Zellner, "Statistical Analysis of Econometric Models," Journal of the American Statistical Association, Vol. 74, No. 367 (September, 1979), pp. 628-643.

2 David A. Kendrick, Stochastic Control for Economic Models (New York: McGraw-Hill Book Company, 1981)

Process Control

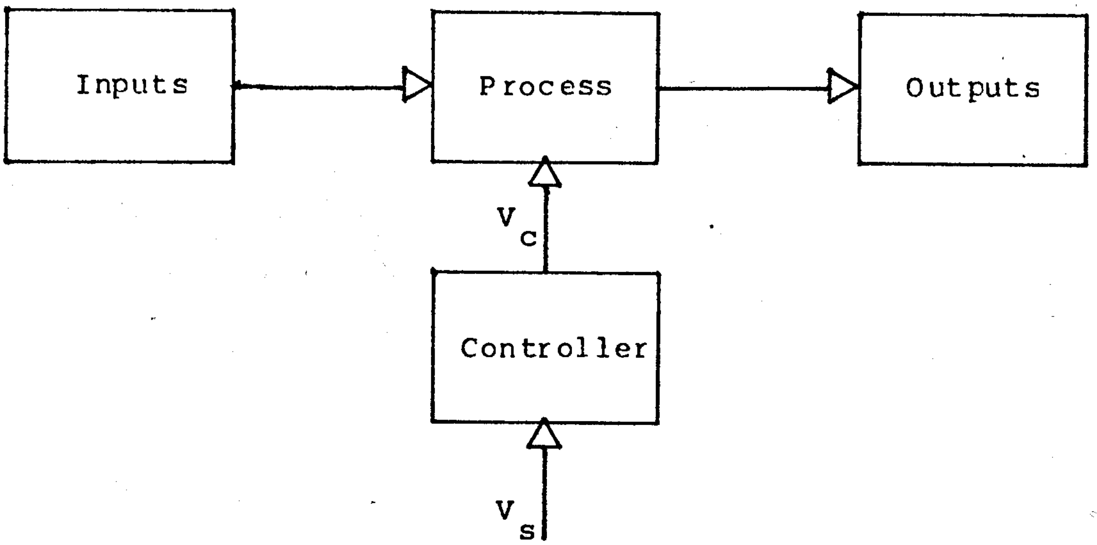

FIG. 2.1 OPEN-LOOP CONTROL.

Lets begin by looking at some of the basic ideas behind industrial process control. Figure 2.1 illustrates one of the simplest types of control: open-loop control. Given some requirements for the system which are translated into electronic or mechanical signals termed set points, V_s, the black box called a controller generates a controlling signal, V_c. This signal may trigger a switch or a lever in a correction unit to control the process to its desired levels. If the set points are fixed, then the control unit is termed a regulator. A controller has the capability to handle varying set points, which are normally changed by some external master unit, e.g., during the warm-up phase of some complex process.

Notice that the control signal V_c is independent of the outputs of the process. Should an abnormal event take place, the control signal may not respond correctly and the system could move towards explosive instability. To correct this weakness, closed-loop feedback control was developed and is shown in Figure 2.2. Here a detector is used to sense the output stream and feedback an output signal to the controller, V_o, in effect closing the control loop. Any sudden shocks to the system will now be sensed by the controller and the necessary control strategy implemented.

There are further refinements in process control to improve the quality of control. For example, another detector may be placed in the input stream or at some other leading position to detect shocks before they reach the process. This new loop, a feedforward loop, gives the system warning of things to come and allows extra time to react smoothly to the new situation. Feedforward control is illustrated in Figure 2.3.

Yet another variation in control strategy is cascade control which has the effect of distributing control intelligence more widely over the system. Additional controllers are placed in the input stream, one for each input, with a

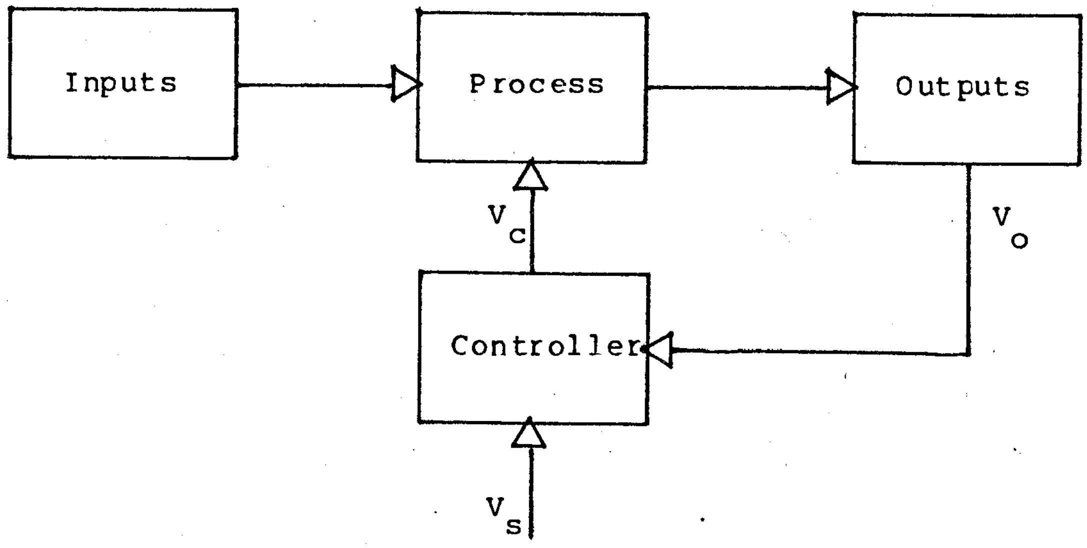

FIG. 2.2 CLOSED-LOOP FEEDBACK CONTROL.

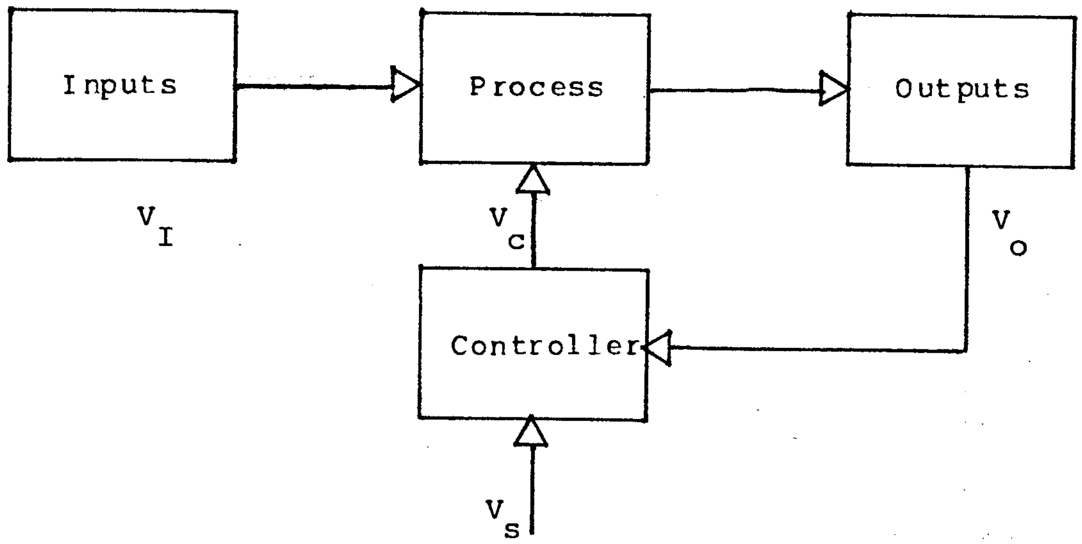

FIG. 2.3 FEEDFORWARD AND FEEDBACK CONTROL.

master controller in charge of the input controllers. The master controller controls the input controllers' set points which it changes according to the overall state of the process. With the advent of microprocessors and improved computer network communication, cascade control with feedforward and feedback control loops can be readily implemented in modern process control applications.

Proportional, Integral and Derivative Control

As its name implies, under proportional control, the controlling signal is in direct proportion to the difference between the set-point signal and the feedback signal.

$$V_c = K (V_s - V_o) \qquad (2.1)$$

Unfortunately, the lag between the detected output signal, V_o, and the applied control signal, V_c, causes bias problems or offset, as it is known in process control terminology. Figure 2.4 clearly shows this problem for the case where the set-point demand is in the form of a linear ramp function. In practice, the set point movements will rarely follow such a simple function but the figure is intended to show that, during a period of adjustment, the process may not be at the desired level when only under proportional control. This bias problem

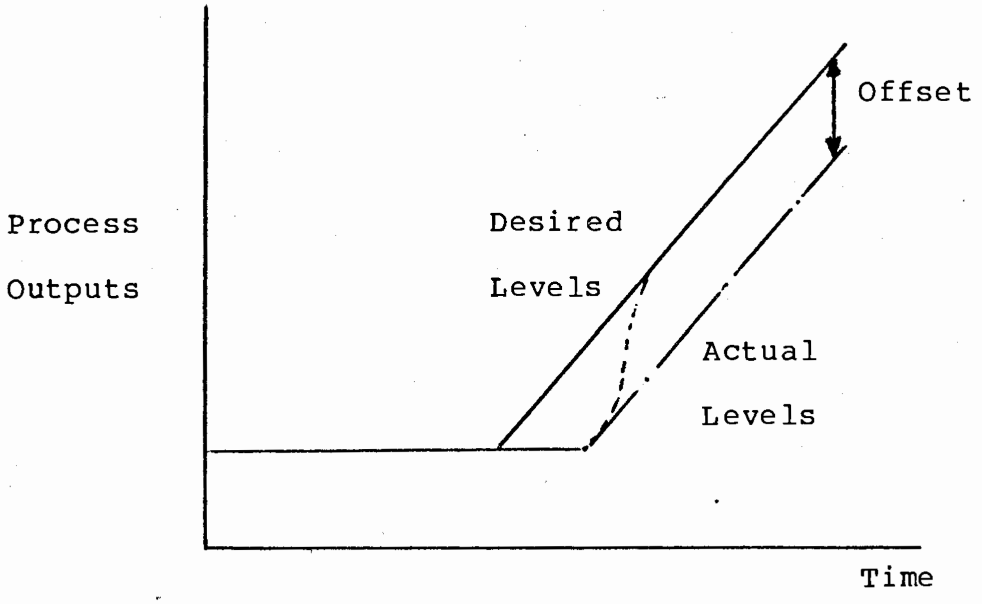

FIG. 2.4 PROPORTIONAL CONTROL OFFSET EFFECTS.

can be removed by adding another component to the control rule, an integral component. Here differences between set-point signals and feedback signals are constantly integrated and an addition is made to the control signal in proportion to the integral. This has the effect of removing bias and, if applied to the beginning of the ramp in Figure 2.4, would gradually bring the process to its desired levels as indicated by the dotted curve. Another component is often added to dampen irregulaties and stabilize the process: the derivative component. The other components could, if necessary, be used by themselves but the derivative component, though a stabilizing influence in combination with the others, could be unstable when used by itself.[3] The full proportional, integral and derivative controller, sometimes referred to as the PID controller, can now be formulated:

$$V_c = K + K_p V + K_I \int V\,dt + K_D \frac{dV}{dt} \qquad (2.3)$$

where K, K_p, K_I, and K_D are constants,

and $V = (V_s - V_o)$.

Control of Economic Systems

In process control, after considerable testing, one of the critical functions of the control engineer is to tune the process by judicious selection of the PID controller constants. In economic systems, the control rules can only be developed from a mathematical model of the system. The model is typically derived from a limited number of past observations. The truth and the accuracy of the model, given the many specification assumptions and limited information, become vital considerations later, at the time of implementing the control rules. The control of an economic system is schematically illustrated in Figure 2.5.

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[3] E. I. Lowe and A. E. Hidden, Computer Control in Process Industries (London: Peter Peregrinus Ltd., 1971), p. 31.

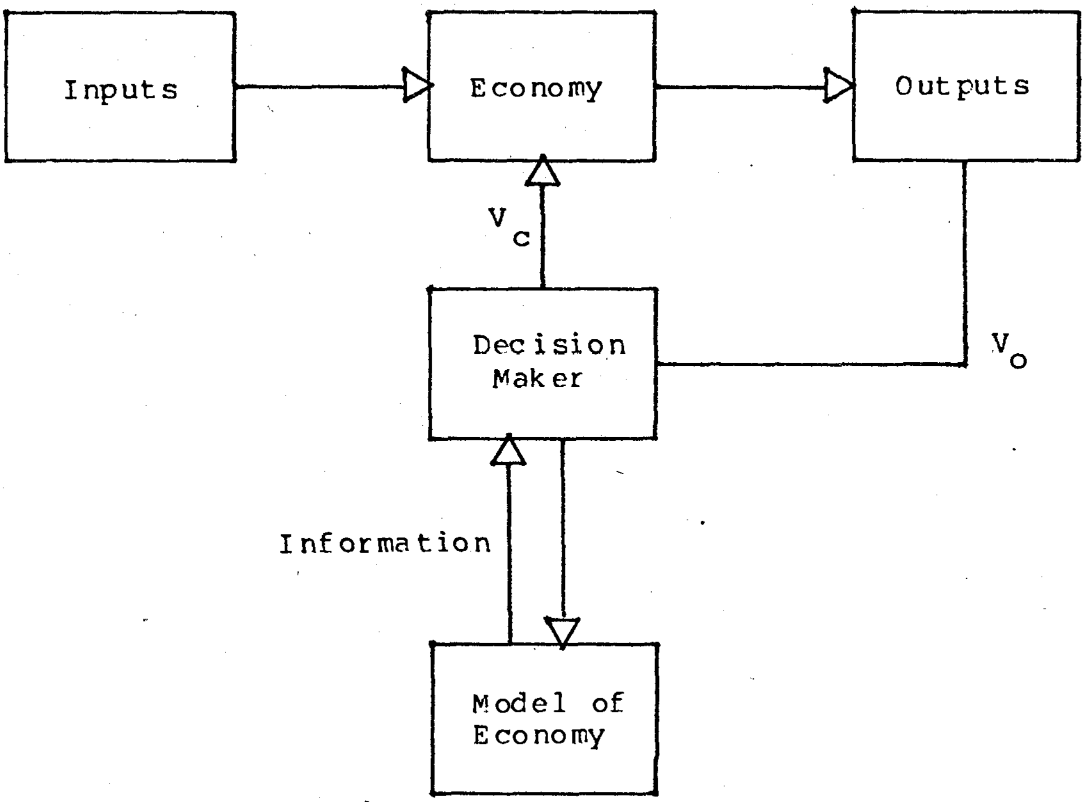

FIG. 2.5 CONTROL OF AN ECONOMY

There are some major differences between process control and the control of an economic system. The planning time horizon is often short, perhaps only six to ten time periods into the future. This would probably mean that the economic system is not in a steady state; the economic control problem is one of disequilibrium dynamics and the goal is to produce a stable, short-term, controller able to keep the economic system on track even in times of unforeseen events or shocks. Perhaps the biggest difference between process and economic control is the inclusion of a decision-maker in the loop, acting as an information broker and a correction unit.

To prepare for future theoretical analysis, the following definitions are pertinent. Open-loop control will indicate the absence of feedback control, i.e., a sequence of control strategies, set beforehand, which are independent of the effects that they produce. Under normal conditions, one would expect the model of the economic system to be updated with the latest observations before developing new control strategies. Assuming the model to be well structured, just like wine, it will improve with age; the precision of the model will increase as more observations are included. It could be said that the model passively learns from these new observations. Some control rules for economic systems take explicit account of this updating procedure i.e., that the uncertainty in the model will decrease over the planning time span. Methods that actively take this learning into account will be said to produce adaptive control rules. In the literature, such names as dual control, closed-loop control and active-learning stochastic control are all used interchangeably to mean adaptive control.

The stage is now set for the simplest of controllers, the linear-quadratic-tracking controller. It looks completely different to the PID controller at first sight but it turns out to be remarkably similar in many respects.

III. LINEAR QUADRATIC TRACKING CONTROLLER

The following derivation[1] of the linear-quadratic-tracking controller (LQT controller) serves the dual purpose of defining terminology to be used later in this work and to raise a framework on which to build the more complex adaptive control models in the next chapter.

Much of scientific control theory focuses on continuous systems and dynamic relations are normally represented by a set of simultaneous differential equations. Economic systems, with longer time periods, can be assumed discrete in time and the dynamics can therefore be introduced through a set of finite difference equations. What better source for the dynamics than an econometric model of the system. These models, normally used for testing 'what if' situations or forecasting, contain through their lag structures all the necessary information to perform dynamic analysis. Further, an analyst needing an econometric model for a control study, is likely to pay much more attention to lag structures and the residuals in the model development phase. Indirectly, this is just the goal of Zellner,[2] stated earlier, for generally improving the specification of structural econometric models.

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[1] Gregory Chow, Analysis and Control of Dynamic Economic Systems (New York: John Wiley and Sons, 1975), pp. 226 - 232.

[2] Zellner, Statistical Analysis of Econometric Models.

State-Space Transformations

First, lets define how time is going to be treated in this derivation. Assume that N time periods of data exist already and they are available for the development of the econometric model. The object of the control study, then, is to set control variable values T time periods into the future given N time periods of historical data. A time period can be three months or it could be a year; it depends only on the frequency with which the historical data were gathered. Time period 0, in effect, represents the last time period at which a set of observations were included in the model data. Time period 1, will, of course, be the first time period into our planning time span of T time periods; this is normally the most important time period for the decision-maker in terms of immediate implementation of the control study results. Now, lets consider the reduced form of a general econometric model:

$$y_t = A_{1t}\, y_{t-1} + \ldots + A_{mt}\, y_{t-m} + C_{ot}\, x_t + \ldots + C_{nt}\, x_{t-n}$$

$$+ E_t w_t + \varepsilon_t \qquad (3.1)$$

where y_t is a p-vector of endogenous variables, x_t is a q-vector of instrument or control variables, A_t , C_t and E_t are matrices of coefficients and w_t is an r-vector of exogenous variables which are not subject to control. The matrices are labelled with time subscripts to indicate that they or their probability distributions may vary over time. The error vectors are assumed to be serially uncorrelated with a mean of zero and a covariance matrix V_t.

The analysis can be simplified by converting the set of equations in (3.1) to a first order system. This is achieved by rearranging them to reduce the lags in the redefined states to be at most one time period as follows in equations (3.2):

$$\begin{bmatrix} y_t \\ y_{t-1} \\ \cdot \\ \cdot \\ y_{t-m+1} \\ x_t \\ x_{t-1} \\ \cdot \\ \cdot \\ x_{t-n+1} \end{bmatrix} = \begin{bmatrix} A_{1t} & A_{2t} & \cdots & A_{mt} & C_{1t} & \cdots & C_{nt} \\ I & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ 0 & 0 & \cdots I & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & I & \cdots & 0 \\ \cdot & \cdot & & \cdot & 0 & & 0 \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ 0 & 0 & \cdots & 0 & 0 & \cdots I & 0 \end{bmatrix} \cdot \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \cdot \\ \cdot \\ y_{t-m} \\ x_{t-1} \\ x_{t-2} \\ \cdot \\ \cdot \\ x_{t-n} \end{bmatrix} + \begin{bmatrix} C_{ot} \\ 0 \\ \cdot \\ \cdot \\ 0 \\ I \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} x_t + \begin{bmatrix} b_t \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

The system of equations in (3.2) can be written in a more compact form after suitable redefinition of the vectors and matrices:

$$y_t = A_t y_{t-1} + C_t x_t + b_t + \varepsilon_t \qquad (3.3)$$

where $b_t = E_t w_t$

$$\text{or} \quad y_t = D_t z_t + \varepsilon_t \tag{3.4}$$

$$\text{where} \quad D_t = (A_t \mid C_t \mid E_t)$$

$$\text{and} \quad z'_t = (y'_{t-1} \mid x'_t \mid w'_t)$$

and | represents a matrix or vector partition

Clearly, the systems (3.3) and (3.4) are much simpler and consequently should be easier to analyse. This system is known as a state-space representation of (3.1) and, in fact, it is an art to select the best representation. Here, convenience has guided the transformation along with meaningful economic representation. Wall[3] suggests methods for obtaining optimum state-space forms which have minimum realizations. The best realization, according to Wall, is the one that produces the smallest dimension of the state vector. Often sacrificed in this realization is economic meaning for some of the state variables. An efficient realization can, however, considerably reduce the computational cost; Norman[4] states that the computational cost rises exponentially (as a cubic function of the state vector dimension) for his methods of optimal control computation.

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[3] Kent D. Wall and J. H. Westcott, "Macroeconomic Modelling for Control," IEEE Transactions on Automatic Control, AC-19, Vol. 6. (December, 1974) pp. 862-873.

[4] Alfred L. Norman, "Linear Quadratic Control for Models with Long Lags", Econometrica, (forthcoming).

System Objective Criteria

The performance of the system can be measured as some function of the deviations of the state variables, y_t, from their predetermined targets, a_t. One particularly convenient representation (and the one most often used in economic control studies) is the additive quadratic penalty cost function:

$$W = \sum_{t=1}^{T} \beta_t^{-t} (y_t - a_t)' K_t (y_t - a_t) \tag{3.5}$$

The objective is clearly to minimize the weighted sum of the squares of the deviations of the state variables from their targets. The relative importance of achieving certain targets can be introduced by the appropriate selection of the weights in the matrices K_t. A larger weight for a particular variable will give preference to its target achievement over other state variables. It is interesting to note that the control variables also appear in the state vector. They too can be given targets even though they are instrumental in helping others to achieve theirs. Indirectly, the weights given to them represent the price of control. Clearly, control can be made expensive or cheap. In many control studies, as is the case in this work, the weighting matrices are positive definite diagonal matrices which are held constant throughout the planning time span. To express the importance of achieving the last period targets over the others, a terminal factor, α, can be used to factor the last weighting matrix, K_T. The discount factor,[5] β_t, is included so that future penalty costs can be appropriately discounted before addition to the total penalty cost. Values for a_t, K_t, α, and, β_t would be assigned by the decision-maker. It is quite possible that some parameters will be controversial and the decision-maker will need to perform sensitivity analysis for different value combinations. An obvious example of this in economic applications is the relative weights to assign the targets of inflation and unemployment. The decision-maker probably represents an executive committee or cabinet with each member having different targets or weights for the stochastic control analysis. There might also be several scenarios to explore corresponding to different forecasts for the exogenous variables. The simple task of assigning values to the parameters, mentioned above, may, in fact, be the most difficult and time consuming part of a stochastic control study.

Questions often asked are: why is the penalty cost a quadratic function? - why not a cubic or some other function? The answer is difficult and certainly it would be more elegant to leave the function open for a later choice. Unfortunately, the controller derivation becomes unwieldy for anything other than a quadratic function. Similarly, if cross-terms are included in the penalty function to take account of the

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5 For simplicity, both the discount factor, β_t, and the terminal factor, α, are excluded from the derivation of the linear-quadratic tracking controller. It is a simple matter to include them later.

covariance between the state variables (or even worse, covariance between state variables in different time periods), the derivation becomes even more difficult. The choice of a quadratic function, much like the choice of simple least squares for regression analysis, represents a compromise between an elegant yet insoluble problem and a much simpler yet tractable approach.

Another undesirable feature of the penalty function is the equal treatment it hands out to undershooting versus overshooting the targets. This problem can be relieved by careful selection of the targets so that the solution path consistently overshoots or undershoots the targets. Where gradient methods are used in the stochastic control analysis, another way to relieve this problem is to impose inequality constraints on both the state and control variables, restraining them from entering forbidden zones.

Derivation of the LQT Controller

It is shown in Appendix A that classical optimal control theory for economic systems with discrete time periods is directly equivalent to conventional dynamic programming. Starting at the end of the planning time span and working backwards, the control variable values would be sought to minimize the total penalty cost-to-go at each time period. Lets begin by looking at the penalty cost for the last time period:

$$w_T = E_\tau (y_T - a_T)' K_T (y_T - a_T)$$

$$= E_\tau (y'_T K_T y_T - 2 y'_T K_T a_T + a'_T K_T a_T) \quad (3.6)$$

$$= E_\tau (y'_T H_T y_T - 2y'_T h_T + c_T) \quad (3.7)$$

In the above equations, E_τ, is the conventional expectation operator, using all the probabilistic information available at time period τ. For the present, it can be assumed that τ has a value of zero, i.e., that the expectation operator utilizes only the N historical observations. Later, the range for the expectation operator will be expanded. The substitution in (3.7) was purposely made in anticipation of a later recurrence relation and, of course:

$$H_T = K_T \qquad h_T = K_T a_T \qquad c_T = a'_T K_T a_T \quad (3.8)$$

The next step is to substitute the state-space relations (3.3) into the penalty cost (3.7) to get:

$$
\begin{aligned}
w_T = {} & E_\tau (A_T y_{T-1} + C_T x_T + b_T)' H_T (A_T y_{T-1} + C_T x_T + b_T) \\
& + E_\tau \varepsilon_T' H_T \varepsilon_T - 2 E_\tau (A_T y_{T-1} + C_T x_T + b_T)' h_T + E_\tau c_T \\
= {} & E_\tau (A_T y_{T-1} + b_T)' H_T (A_T y_{T-1} + b_T) \\
& + x_T' E_\tau (C_T' H_T C_T) x_T + 2 x_T' E_\tau C_T' H_T (A_T y_{T-1} + b_T) \\
& + E_\tau \varepsilon_T' H_T \varepsilon_T - 2 E_\tau (A_T y_{T-1} + b_T)' h_T \\
& - 2 x_T' E_\tau C_T' h_T + E_\tau c_T
\end{aligned}
\tag{3.9}
$$

It is now possible to find an expression for the optimal x_T by differentiating the above expression and applying first order conditions:

$$
\begin{aligned}
0 = {} & 2 E_\tau C_T' H_T (A_T y_{T-1} + b_T) + 2 E_\tau (C_T' H_T C_T) x_T^* \\
& - 2 (E_\tau C_T') h_T
\end{aligned}
\tag{3.10}
$$

The expression for the feedback control function can now be derived from (3.10).

$$
\begin{aligned}
x_T^* &= G_T y_{T-1} + g_T \\
G_T &= -(E_\tau C_T' H_T C_T)^{-1} (E_\tau C_T' H_T A_T) \\
g_T &= -(E_\tau C_T' H_T C_T)^{-1} (E_\tau C_T' H_T b_T - (E_\tau C_T') h_T)
\end{aligned}
\tag{3.11}
$$

By substitution of the expression (3.11) for x^*_T into the terminal penalty cost (3.9), we get:

$$w^*_T = E [(A_T + C_T G_T) y_{T-1} + b_T + C_T g_T]' H_T$$
$$\times [(A_T + C_T G_T) y_{T-1} + b_T + C_T g_T]$$
$$+ E_\tau \varepsilon'_T H_T \varepsilon_T - 2 E_\tau [(A_T + C_T G_T) y_{T-1} + b_T + C_T g_T]' h_T$$
$$+ E_\tau c_T$$

$$= y'_{T-1} [E_\tau (A_T + C_T G_T)' H_T (A_T + C_T G_T)] y_{T-1}$$
$$+ 2 y'_{T-1} E_\tau [(A_T + C_T G_T)' (H_T b_T - h_T)] \quad (3.12)$$
$$+ E_\tau (b_T + C_T g_T)' H_T (b_T + C_T g_T) + E_\tau \varepsilon'_T H_T \varepsilon_T$$
$$- 2 E_\tau (b_T + C_T g_T)' h_T + E_\tau c_T$$

Now lets move to the next-to-last time period and formulate its penalty cost:

$$w_{T-1} = E_\tau (y'_{T-1} K_{T-1} y_{T-1} - 2 y'_{T-1} K_{T-1} a_{T-1}$$
$$+ a'_{T-1} K_{T-1} a_{T-1} + w^*_T) \quad (3.13)$$

It is possible to substitute for the optimal last period cost (3.12) into (3.13) to get:

$$w_{T-1} = E_\tau (y'_{T-1} H_{T-1} y_{T-1} - 2 y'_{T-1} h_{T-1} + c_{T-1}) \quad (3.14)$$

The quadratic constants in (3.14) can be obtained from the following so-called Riccati recurrence relations:

$$H_{T-1} = K_{T-1} + E_\tau (A_T + C_T G_T)' H_T (A_T + C_T G_T)$$
$$= K_{T-1} + E_\tau (A'_T H_T A_T) + G'_T (E_\tau C'_T H_T A_T) \quad (3.15)$$

$$h_{T-1} = K_{T-1} a_{T-1} + E_\tau (A_T + C_T G_T)' (h_T - H_T b_T)$$
$$= K_{T-1} a_{T-1} + E_\tau (A_T + C_T G_T)' h_T$$
$$- E_\tau A'_T H_T b_T - G'_T (E_\tau C'_T H_T b_T) \quad (3.16)$$

$$c_{T-1} = E_\tau (b_T + C_T g_T)' H_T (b_T + C_T g_T)$$
$$- 2 E_\tau (b_T + C_T g_T)' h_T + a'_{T-1} K_{T-1} a_{T-1}$$
$$+ E_\tau \varepsilon_T' H_T \varepsilon_T + E_\tau c_T \quad (3.17)$$

The similarity between (3.14) and (3.7) should now be evident and fortunately the derivation is now complete. In fact the above Riccati relations hold for every time period and can be so written:

$$H_{t-1} = K_{t-1} + E_\tau (A_t + C_t G_t)' H_t (A_t + C_t G_t)$$
$$= K_{t-1} + E_\tau (A'_t H_t A_t) + G'_t (E_\tau C'_t H_t A_t) \quad (3.18)$$

$$h_{t-1} = K_{t-1} a_{t-1} + E_\tau (A_t + C_t G_t)' (h_t - H_t b_t)$$
$$= K_{t-1} a_{t-1} + E_\tau (A_t + C_t G_t)' h_t$$
$$- E_\tau A'_t H_t b_t - G'_t (E_\tau C'_t H_t b_t) \quad (3.19)$$

$$c_{t-1} = E_\tau \,(b_t + C_t g_t)' \, H_t \, (b_t + C_t \, g_t)$$
$$- 2 \, E_\tau \, (b_t + C_t \, g_t)' \, h_t + a'_{t-1} \, K_{t-1} a_{t-1}$$
$$+ E_\tau \, \varepsilon'_t \, H_t \, \varepsilon_t + E_\tau \, c_t \qquad (3.20)$$

Starting at the last time period, we simply backtrack, using the Riccati recurrence relations, storing the values H_t, h_t, c_t, G_t, and g_t all the way back to the first time period. But it is now possible to substitute actual values into the first feedback equation:

$$x_1 = G_1 \, y_o + g_1 \qquad (3.21)$$

The first period control values, x_1, can be used in (3.3) to find the first set of state values. Clearly, all the state values can be determined in a similar manner using a forward sweep to the last time period in the planning time span. A full stochastic solution for the LQT controller has now been obtained. This solution will minimize the total expected penalty cost as long as E(C'HC) is positive definite or, equivalently, as long as H_t is positive definite. The symmetrical definition of penalty cost in (3.5) ensures that H_t will be positive definite.

It is also interesting to compare the LQT controller with the PID controller from Chapter II. First, because the control variables in (3.3) have an immediate effect (they are not lagged compared to y_t), there is no need for the integral component; there are no bias or offset problems. At first sight, the feedback function (3.11) appears to be a simple proportional or linear controller. However, there may already be first differences (equivalent to derivatives in a continuous system) in the state vector. If not they could be added using a relationship such as:

$$y_t - y_{t-1} = (A_t - I) y_{t-1} + C_t x_t + b_t \qquad (3.22)$$

Both proportional and derivative components would now be present in the LQT controller. The decision-maker will have to decide the weight to place on the first differences compared to the other state variables. But this is just like the control engineer in process control tuning the process by experimentation with the PID constants.

So far no mention has been made regarding the calculation of the expected values in the feedback and Riccati equations. One approach is to use only the mean values of the model coefficients or parameters, ignoring any covariance relations. This approach, very appealing in its simplicity, yields the certainty equivalence solution. A more difficult but perhaps more realistic approach is the uncertain parameters method which incorporates the covariance relations. The probability theory required for these methods is to be found in Appendix B and an example of their application to a relatively simple econometric model is shown in Appendix D. Both methods will be explored later, along with adaptive control methods which are the subject of the next chapter.

IV. ADAPTIVE CONTROL

The LQT controller developed in the last chapter does not take into account how the model will actually be applied; that the model will probably be revised with new information each time that a new set of control policies are required. Adaptive controllers compensate for this updating procedure by allowing for a decrease in parameter uncertainty over the planning time span. Predicted state variable values are treated as if they were new observations and the model is appropriately updated with them at each time period under consideration. In effect, the expectation parameter,[1] τ , will take on a value of (t-1) when the time period t is under investigation.

For small models with large quantities of historical data and relatively short planning time spans, the simple LQT controller from the last chapter should be quite sufficient. Typical practical situations, however, tend to produce large models with a limited number of historical observations. Here adaptive control policies could be quite different to the simple LQT control solutions.

In the last chapter, with τ equal to zero, it was obvious that an earlier state value in the planning time span could not affect the values of the feedback matrices, (3.11), for a later time period. With adaptive controllers, this is no longer the

[1] Unless otherwise stated the value of τ will be (t-1) throughout this chapter.

case. Earlier state values do in fact affect later feedback matrices by working indirectly through their covariance relations. It would seem that under these conditions, the basic foundation of dynamic programming: Bellman's Principle of Optimality[2] would be violated. Amazingly, with care, dynamic programming can still be used under these circumstances.

There have been several attempts to produce efficient adaptive controllers in the past. Because the covariance relations are so complex and not easily represented, they are all approximations and differ markedly in their initial assumptions and computational approaches. A few of the more popular ones will be described in the following sections.

The MacRae Adaptive Control Model

Elizabeth Chase MacRae[3] developed one of the simplest yet most elegant adaptive control models. Before describing her

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[2] The Principle of Optimality states:

> An optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

Taken from: Richard E. Bellman, Adaptive Control Processes: A Guided Tour, (Princetown, New Jersey: Princetown University Press, 1961), p. 57.

[3] Elizabeth C. MacRae, "An Adaptive Learning Rule for Multiperiod Decision Problems, " Econometrica, Vol. 43 No. 5-6, (September - November, 1975), pp. 893-906.

model in detail, a small digression on the mechanics of updating an econometric model would be useful. Assume that a set of exogenous observations are stored in a matrix Z and that we are about to revise the model with a single set of observations in a column vector z_t. Clearly, as indicated in Appendix B, the old parameters of the model, π_{t-1} were derived from simple regression analysis:

$$\pi_{t-1}' = (Z' Z)^{-1} Z' Y \tag{4.1}$$

and after revision the new parameters will be:

$$\pi_t' = (Z' Z + z_t z_t')^{-1} (Z' \mid z_t) Y \tag{4.2}$$

The coefficient or parameter covariance matrix,[4] Γ_{t-1}, at time (t-1) is:

$$\Gamma_{t-1} = V_{t-1} \otimes (Z' Z)^{-1} \tag{4.3}$$

and at time period t is :

$$\Gamma_t = V_t \otimes (Z' Z + z_t z_t')^{-1} \tag{4.4}$$

A simple covariance recurrence relation is thus revealed:

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[4] For a fuller description of the updating variance covariance relations, see Appendix B and in particular note the derivation of equation (B.17).

$$\Gamma_t^{-1} = \Gamma_{t-1}^{-1} + V_t^{-1} \otimes (z_t \; z_t') \tag{4.5}$$

MacRae adds this recurrent covariance relationship to the previous penalty cost function (3.5) with appropriate multiplication by a matrix of Lagrange multipliers, M_t:

$$W = E \sum_{t=1}^{T} \beta_t^{-t} [(y_t - a_t)' K_t (y_t - a_t)$$

$$+ M_t \circledast \{\Gamma_t^{-1} - \Gamma_{t-1}^{-1} V_t^{-1} \otimes (z_t z_t')\}] \tag{4.6}$$

which can be rearranged[5] into the following form:

$$\begin{aligned} W = E \sum_{t=1}^{T} \beta_t^{-t} [(y_t - a_t)' K_t (y_t - a_t) \\ - z_t' (V_t^{-1} \otimes M_t) z_t \\ + M_t \circledast (\Gamma_t^{-1} - \Gamma_{t-1}^{-1})] \end{aligned} \tag{4.7}$$

The middle term in (4.7) can be expanded by substituting for z_t from (3.4) and splitting $M = V_t^{-1} \circledast M_t$ into partitions corresponding to A, C, and E in the system equations (3.3):

$$(y_{t-1}' \mid x_t' \mid w_t') \begin{bmatrix} M^{AA} & M^{AC} & M^{AE} \\ M^{CA} & M^{CC} & M^{CE} \\ M^{EA} & M^{EC} & M^{EE} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_t \\ w_t \end{bmatrix} \tag{4.8}$$

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[5] The Kronecker product operator, $\otimes$, and the star product, $\circledast$, used in (4.3) through (4.7) are defined in Appendix B.

Similarly the first term in (4.7), $(y_t' K_t y_t)$ can be partitioned after substituting for y_t from (3.3):

$$(y_{t-1}' \mid x_t' \mid w_t') \begin{bmatrix} A H A & A H C & A H E \\ C H A & C H C & C H E \\ E H A & E H C & E H E \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_t \\ w_t \end{bmatrix} \qquad (4.9)$$

From (4.8) and (4.9) it is obvious that wherever the expectation of a triple product occurs in the MacRae model, it will be reduced by the appropriate partition matrix of (4.8). For example, E (A H A) in the simple LQT controller will become E (A H A − M^{AA}) in the MacRae model.

By simply augmenting the triple product expectations we have, in effect, completed the differentiation of (4.7) with respect to x_t and the Riccati recurrence relations (3.18), (3.19) and (3.20) suitably modified[6] can be used. But we can also differentiate (4.7) with respect to the coefficient variance, Γ_{t-1}, and obtain yet another recurrence relationship:

$$\beta_t M_{t-1} = \{M_t + \Gamma_{t-1} [H_t \times E_\tau (z_t z_t')] \Gamma_{t-1} \} \qquad (4.10)$$

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[6] In actual fact the triple product expectations should be reduced by only a proportion of M , for example, E (C H C) would become E (C H C − γ M^{CC}), where the stability factor, γ , is in the range $0 \leq \gamma \leq 1$. This is to guard against the augmented triple product of E (C H C) becoming negative definite, changing the optimization from one of minimization to maximization. The value of the stability factor would be ascertained by experiment and for most models it is expected to be unity.

With this recurrent relation we are able to tailor the coefficient covariance matrices for minimum penalty cost.

Unfortunately, this control problem cannot be solved as easily as in the last chapter because of the complexity of the new covariance relations. Numerical techniques must therefore be used. One approach is to start the solution off with the simple LQT controller solution assuming uncertain parameters. These values can be used to update the covariance relations in the planning time span. Assuming that M_T is zero, the MacRae recurrence relations (4.10) can be applied to yield a sequence of Lagrange multiplier matrices. When the first time period is reached, a new solution can be obtained by substitution in the system equations (3.3). This new solution can then be used to update the covariance relations in the planning time span and another iteration of the MacRae model undertaken. With each iteration the penalty cost should get smaller and when the decrease is below a predetermined level, the iterations are terminated, yielding a full adaptive solution. It is a simple matter to repeat the above steps for the case where the stability factor, γ , is zero. This too will yield a learning solution but without the MacRae recurrence relations (4.10). In fact this model embodies accidental learning. It will be referred to as the heuristic uncertain parameters model and it too will be tested later in this work.

The Chow Adaptive Control Model

Chow[7] also developed an adaptive control model. He incorporated the state covariance relations in the planning time span by using a non-additive penalty cost function.

$$W = \sum_{t=1}^{T} y_t' K_{t,t} y_t + \sum_{t=1}^{T} \sum_{s<t} y_t' K_{t,s} y_s + \sum_{t=1}^{T} y_t' K_t \qquad (4.11)$$

Notice the large number of quadratic forms that will be generated from (4.11) which not only include interstate impacts within a particular time period but also interstate impacts between time periods. Chow applies the normal dynamic programming approach except that he splits the penalty function into two parts at each time period: a part that is dependent on x_t , $E_\tau w_t$, and a part w_{Nt} that is independent of x_t . For example, at the last time period, the dependent part is :

$$w_T = E_\tau [y_T' H_{T,T} y_T + y_T'(\sum_{s=1}^{T} H_{T,s} y_s + h_T)] \qquad (4.12)$$

His approach directly parallels that of the simple LQT controller in the last chapter except that the expressions are longer and more unwieldy. Chow's model, like any other adaptive model which includes the complex covariance relations, does not necessarily have a quadratic cost-to-go penalty function. Chow

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[7] Gregory Chow, "A Solution to Optimal Control of Linear Systems with Unknown Parameters, " The Review of Economics and Statistics, (August, 1975), pp. 338-345.

in effect approximates it as a quadratic by numerically fitting a second order Taylor series. In this way he is able to produce simple recurrence relations. Unfortunately, the computational cost for these simple recurrence relations can be enormous. Just to give an idea, consider the case of a model with 10 states, 10 exogenous variables for regression and 10 time periods in the planning time span. At each iteration, 220 Hessians or matrices of numerical second order derivatives of order 10 by 10, would be required. Each entry in a Hessian matrix would require 2 regression calculations, each one requiring the inversion of a 10 by 10 matrix; in total, just for part of the overall analysis per iteration, this is 44,000 inversions. A marathon of computation fraught with many sources of error and loss of accuracy.

The Norman First Order Dual Control Model

Norman[8] developed a simple yet efficient open-loop adaptive control model. His approach depends on the separation of the control problem into a deterministic part and a stochastic part. The system equations from (3.3) can be rewritten to reflect this separation. The deterministic component of y_t would be:

$$\bar{y}_t = \bar{A}_t \bar{y}_{t-1} + \bar{C}_t \bar{x}_t + \bar{b}_t \qquad (4.13)$$

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[8] Alfred L. Norman, "First Order Dual Control," Annals Of Economic and Social Measurement, Vol. 5 No. 3. (1976) pp. 311-321

and the stochastic component of y_t would be:

$$\Delta y_t = \bar{A}_t \Delta y_{t-1} + \bar{C}_t \Delta x_t + \Delta A_t \bar{y}_{t-1} + \Delta C_t \bar{x}_t + \Delta b_t + \varepsilon_t \tag{4.14}$$

where $y_t = \bar{y}_t + \Delta y_t$

and $x_t = \bar{x}_t + \Delta x_t$

Again general dynamic programming can be applied using an objective function of the following form:

$$w = E \sum_{t=1}^{T} \beta_t^{-t} [(\bar{y}_t - a_t)' K_t (\bar{y}_t - a_t) + \phi_t] \tag{4.15}$$

where ϕ_t, in effect, includes the effect of the coefficient covariance relations over the planning time horizon and can be formulated as follows:

$$\phi_t = E_\tau (\Delta y_t' H_t \Delta y_t)$$

$$= (\bar{y}_{t-1}' \mid \bar{x}_t \mid \bar{w}_t) \; E_\tau \begin{pmatrix} A'HA & A'HC & A'HE \\ C'HA & C'HC & C'HE \\ E'HA & E'HC & E'HE \end{pmatrix} \begin{pmatrix} \bar{y}_{t-1} \\ \bar{x}_t \\ \bar{w}_t \end{pmatrix} \tag{4.16}$$

Norman uses a gradient method to obtain a stochastic control solution and, at each stage, he obtains a simple certainty equivalence solution using a modified version of the Riccati equations (3.20). The only change is to the last equation for c_{t-1} and simply requires the addition of a new term, ϕ_t, on the right hand side.

The method can be explained more easily with reference to the flowchart shown in Figure 4.1. To begin, a simple certainty equivalence solution is derived. This is used to generate the covariance relations, (4.18), and enables the penalty cost for all time periods to be evaluated. Gradients are then derived for each control variable to see if the optimum has been reached. Gradients close to zero signal that an optimal solution has been found. If the search is not over, a new set of control values are computed[9] and the preceding steps repeated. When the search is over, the optimal control is applied to determine the state values at time period t; this new observation set is recorded and the next time period is then considered.

The method is fast and has the advantage that constraints can be applied to the state values if so required. The disadvantage is that it is open-loop control and may not perform as well as feedback control in the later time periods.

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[9] Normally some type of gradient search algorithm would be used here such as ZXCGR or ZXMIN from the computer package IMSL by IMSL Inc., NBC Building, Houston, Texas, U.S.A. described in:

John R. Rice, Numerical Methods, Software, and Analysis: IMSL Reference Edition, (New York, New York: McGraw-Hill Book Company, 1983), p. 632.

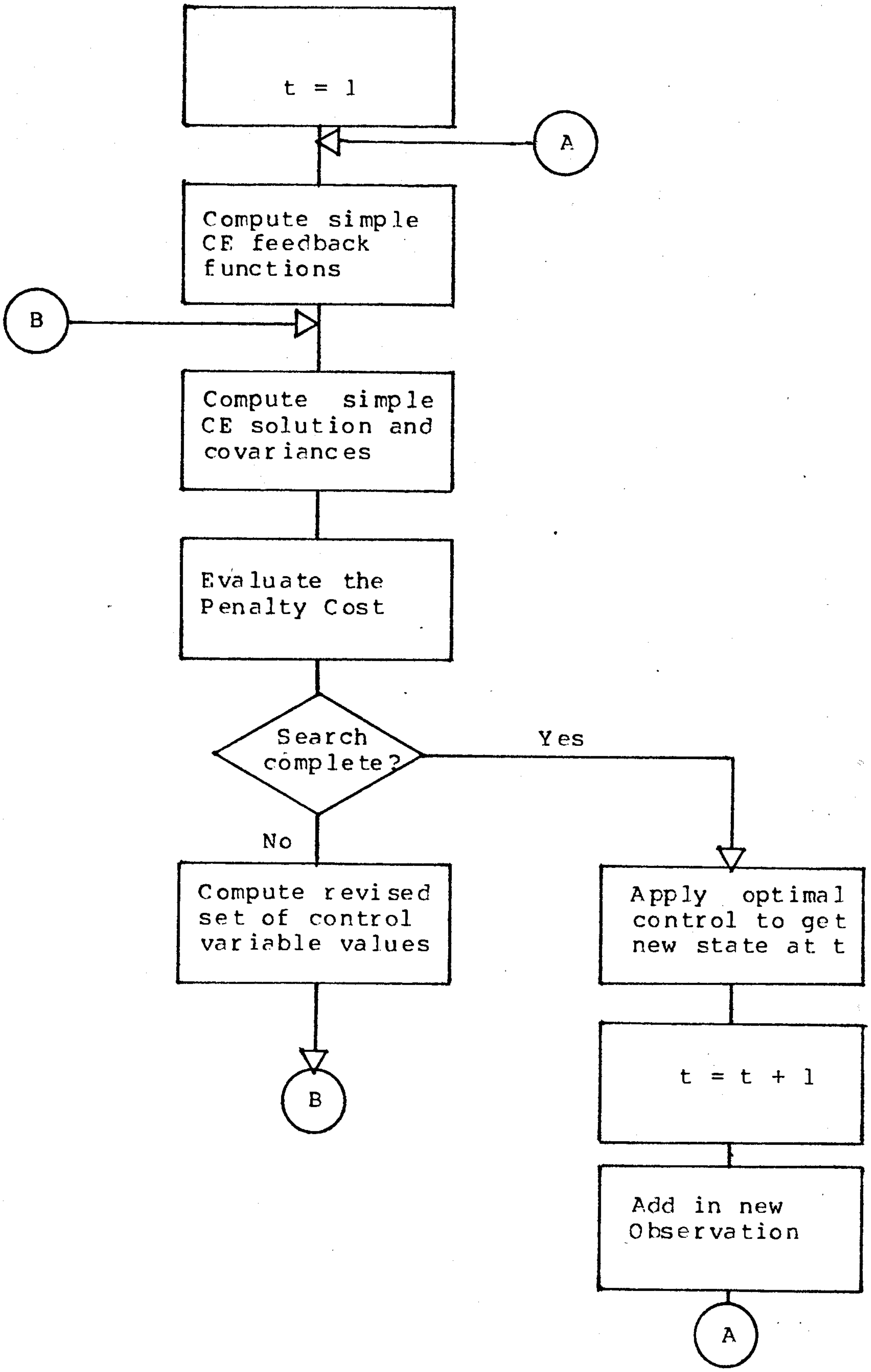

FIG 4.1 NORMAN FIRST ORDER DUAL CONTROL FLOWCHART

Other Adaptive Control Models

The literature abounds with other adaptive control methods. One of the most famous is that of Bar-Shalom and Tse.[10] Their model is a more refined version of the Norman model and can handle more difficult situations. For example, the systems equations can be non-linear or the state values can contain measurement error. Given some assumptions about the parameter relationships, the model can also cope with unknown parameters. The disadvantage of the method is the high computational cost, like Chow's model, attributable to a second order Taylor series approximation[11] compared to the first order approximation of Norman. This same criticism can be levelled at some other fine adaptive control methods: Kendrick[12] and Upadhyay[13] using the adaptive model of Deshpande.[14] Prestcott[15] also produced some very interesting adaptive control results. Unfortunately, the

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[10] Y. Bar-Shalom and Edson Tse, "Caution Probing and the Value of information in the Control of Uncertain Systems," Annals of Economic and Social Measurement, Vol. 5 (1976), 323-338.

[11] Bar-Shalom and Tse apply the second order Taylor series approximation before taking expectations whilst Chow does it afterwards. They claim their method is more efficient.

[12] Kendrick, Stochastic Control for Economic Models.

[13] Treveni Upadhyay, "Application of Adaptive Control to Economic Stabilization Policy," ,International Journal of Systems Science, Vol. 7 No. 10. (1976) pp. 641-650.

[14] J. G. Deshpande, T. N. Upadhyay and D. G. Lainiotis, "Adaptive Control of Linear Stochastic Systems," Automatica, Vol. 9. (1973), pp. 107-115.

[15] Edward C. Prestcott, "The Multi-Period Control Problem," Econometrica, Vol. 40, No. 6 (November,1972) pp. 1043-1058.

results were found by complete enumeration for a very simple model and the approach would not be practical for most situations. Rausser and Freebairn[16] have also contributed to the wealth of adaptive control information. They looked at several adaptive control methods and tried varying the number of time periods over which learning took place. They referred to the method as M-Control, M being the number of periods over which learning was considered and applied the simple LQT controller for the the remaining (T-M) periods in the planning time span.

An important research objective in this work is to test the application of control methods to practical situations. Some of the adaptive controllers described above will be tested to see how they measure up to this objective. The rationale for choosing controllers and the testing procedure itself will be the subject of the next chapter.

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[16] G. C. Rausser and J. W. Freebairn, "Approximate Adaptive Control Solutions to US Beef Trade Policy," Annals of Economic and Social Measurement, Vol. 3, (January, 1974), pp. 177-203.

V. METHODOLOGY

In this chapter, the first item to be considered will be checking and testing of the control algorithms. As mentioned in the last chapter, there are many control studies in the literature, each differing slightly in data used, assumptions made or numerical procedures followed. The second part of this chapter will concentrate on the selection of some of these control models for testing. Finally, we will be describing the testing procedure itself.

A set of computer programs (the control algorithms) will be used to test the various models on several econometric models. The author has tried to resurrect the control models as accurately as possible from articles in the literature. With ambiguity or vague descriptions, there might be slight differences to the original models. Similarly, the econometric models were also taken from articles in the control literature. Differences in regression methods or errors in data communication may cause small variations from the original models. The author has tried to be as accurate as possible.

Algorithm Development and Calibration

A search was conducted to find a simple econometric model for checking the computer algorithms. Preference was given to one that had been used previously in other control studies; especially one that had been subjected to a wide range of control models. The one eventually selected was developed by Abel[1] and was originally used for comparing monetary and fiscal policies. Chow[2] also has used it for some control investigations as has Kendrick,[3] employing an updated version of the Bar-Shalom and Tse model.[4] The Abel model along with its data is fully described in Appendix C.

At the heart of the computer algorithms is the simple LQT controller which, as mentioned in Chapter III, was based upon the work of Chow.[5] Abel's computer algorithms were also based upon the work of Chow and therefore should provide an interesting comparison.

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1 Andrew B. Abel, "A Comparison of Three Control Algorithms as Applied to the Monetarist-Fiscalist Debate," Annals of Economic and Social Measurement, Vol. 4, No. 2 (1975) pp. 239-253

2 Chow, Control of Economic Systems, p. 271.

3 David Kendrick, "Caution and Probing in a Macroeconomic Model," Journal of Economic Dynamics and Control, Vol. 4, No. 2, (May 1982) pp. 149-170.

4 Bar-Shalom, Control of Uncertain Systems.

5 Chow, Control of Economic Systems.

An Econometric Model of the Canadian Economy

A key research objective from Chapter I is to conduct a control study on a relatively-small, econometric model of the Canadian economy. The Laidler model[6] was selected for the purpose and it is described more fully in Appendix E.

The Laidler model contains seven equations and, although developed in a monetarist environment, is neutral with respect to the monetarist-fiscalist debate. It contains both monetarist and fiscalist control variables. The suspicious absence of the interest rate as an endogenous variable is explained by the use of a much wider definition of money, M3, rather than the more common definition, M1. Testing revealed that M3 is not sensitive to interest rate. The model is log linear and quite robust. With minor differences, the model has been applied to the economies of Britain,[7] USA,[8] and Italy.[9]

It should be noted that Laidler used constrained, full information, maximum likelihood regression techniques in

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[6] David Laidler et al., "A Small Macroeconomic Model of an Open Economy: The Case of Canada," Paper Presented at the Fifth Paris-Dauphine Conference on International Monetary Economics, Paris, June 1981.

[7] David Laidler and P O'Shea, "An Empirical Macromodel of an Open Economy Under Fixed Exchange Rates: The United Kingdom 1954-1970," Economica, Vol. 47, (1980), pp. 141-158.

[8] David Laidler and B. Bentley, "A Small Macromodel of the Post-War United States," University of Western Ontario Research Report 8101, (1981), Mimeo.

[9] F. Spinelli, "Fixed Exchange Rates and Monetarism: The Italian Case," University of Western Ontario Research Report 7915, (1979), Mimeo.

developing his model. The frequency with which regressions are required, especially in adaptive control models, make this approach computationally impractical for this control study. However, one method that can be used, again a compromise approach, is simple regression utilizing extraneous information (see Appendix B). The extraneous information would take the form of a set of zero linear constraints on the model's coefficients. At least some of the major constraints can be incorporated in this way.

Control Models

The simple LQT control model from Chapter III is an obvious candidate for testing, both certainty equivalence (no parameter covariance relationships assumed) and with uncertain parameters. It will also be interesting to test them in a learning environment comparing their performance with some of the more complex adaptive models from Chapter IV.

In choosing adaptive control models for testing, a compromise has to be struck between accuracy of representation and computational cost. The models of Chow, Bar-Shalom, Kendrick, Prestcott, and Upadhyay were felt to be too computationally demanding, especially when including extraneous information in the analysis. However, the MacRae model, including the heuristic uncertain parameters model as a special case, looks computationally possible. Arguing along similar lines, the Norman model also should be chosen; it contains the elements of some of the more complex models but without the concomitant computational cost.

Monte Carlo Testing

The testing of each control model is considered an extremely important part of this work and something often neglected in previous control studies. Consider the simple certainty equivalence solution for a moment. Having assumed away the covariance relations, the expected penalty cost must look very attractive to a decision-maker; it could be much lower than for the uncertain parameters solution. However, if the certainty equivalence solution were actually implemented and the estimated covariance relations were reasonably realistic, the tables could be turned; it might perform badly against the uncertain parameters solution. It is just this sort of effect that the Monte Carlo testing is designed to capture.

It should be possible to simulate typical planning time spans, using covariance relations for the error terms and the parameters, starting at the first time period. A method for generating a vector of typical values for a multivariate normal distribution with a given covariance matrix is described in Appendix F. At each time period in the planning time horizon, a set of typical coefficients and error terms for the econometric model could be generated. Using the appropriate feedback control matrices (or control variable values in the case of open-loop control) and the previous state values, a new set of state values could be derived. With a full set of state variable values over the planning time span and a given set of weighting parameters, a typical value for the penalty cost could be obtained. Over many time spans, it would be a simple matter to obtain an average penalty cost and standard deviation. At least 50 simulations would be required for this average penalty cost, although 100 or more simulations would be preferred if computer CPU time permits.

As long as the weighting matrices and control factors are kept constant, simulations of the various LQT or adaptive controllers should be directly comparable. Obviously, a full regime of model variations could be tested. Model size, parameter uncertainty and model dynamics could be tested by comparing the Abel model to the Laidler model and the simple Laidler model to the restricted Laidler model. The importance of current versus future penalty costs could be tested by varying the terminal factor or discount factor. Finally, to test the anticipatory characteristics of the adaptive models, a sudden step change to large terminal targets could be incorporated in the model.

The simulation of adaptive controllers raises a computational challenge; the regression analysis and covariance determination must be repeated T times per planning time span simulation. If 50 or more time spans are simulated and extraneous information is incorporated, the required computation cost could be excessively large.

VI. RESULTS AND DISCUSSION

Checking the control algorithms against the previous work of Abel, Chow and Kendrick occupies the first two sections of this chapter. In the next section the Laidler model is summarized followed by some preliminary trials using the control algorithms. The chapter concludes with a discussion of the simulation results for both simple and adaptive control models.

Comparison with the Abel Control Study

The data for the Abel model were taken from a study by Kendrick.[1] Given that only 39 of the original 40 observations were available from this study, slight differences in the regression equations would be expected. In fact they are remarkably similar and the derived model with coefficient standard deviations is:

$$\underset{}{C_t} = \underset{(0.0522)}{0.9144\, C_{t-1}} - \underset{(0.0927)}{0.0173\, I_{t-1}} + \underset{(0.1470)}{0.3037\, G_t} + \underset{(0.1890)}{0.4270\, M_t} - \underset{(24.50)}{59.66}$$

$$\underset{}{I_t} = \underset{(0.0780)}{0.0973\, C_{t-1}} + \underset{(0.1385)}{0.4244\, I_{t-1}} - \underset{(0.2196)}{0.1036\, G_t} + \underset{(0.2824)}{1.4589\, M_t} - \underset{(36.17)}{184.62}$$

As to be expected for time series regression, the R^2 values are

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[1] Kendrick, "Caution and Probing in a Macroeconomic Model".

high at 0.996 and 0.875 and autocorrelation of the residuals is present, but not excessive, as indicated by the Durbin-Watson statistics of 1.69 and 1.72.

A comparison of stochastic control results from this work and the Abel study are shown in Table 6.1. Here are shown for the first six time periods some of the key matrices for the simple LQT solution derivation. A detailed derivation of these results is to be found in Appendix D. The quadratic matrix H_1, derived from (3.18), the feedback matrices G_1 and g_1, derived from (3.11), and the solution for the control variables government expenditure G_1, and money supply M_1, derived from (3.21), are all shown in Table 6.1. Notice the similarity in the results.

A full stochastic control solution for the uncertain parameters case using this regression model is shown in Table 6.2. A schedule of state and control variable values and their targets over the planning time span is illustrated. For convenience all values and targets have been divided by one thousand; the penalty cost, as a result, is very small and therefore is not shown in Table 6.2.

The results from this section indicate that the basic regression, covariance and control algorithms appear to be working satisfactorily.

TABLE 6.1
COMPARISON OF STOCHASTIC CONTROL SOLUTIONS
WITH THE ABEL MODEL

		This Control Study				Abel Control Study			
Certainty Equivalence	H_1	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
		0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	G_1	-2.65	0.42	0.00	0.00	-2.61	0.37	0.00	0.00
		-0.26	-0.26	0.00	0.00	-0.22	-0.23	0.00	0.00
	g_1	1023.57		260.10		1013.01		243.13	
	x_1	114.11		147.59		111.72		142.90	
Uncertain Parameters	H_1	1.44	-0.14	0.00	0.00	1.41	-0.13	0.00	0.00
		-0.14	1.09	0.00	0.00	-0.13	1.08	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	G_1	-1.67	0.08	0.00	0.00	-1.69	0.06	0.00	0.00
		-0.27	-0.23	0.00	0.00	-0.25	-0.20	0.00	0.00
	g_1	701.50		261.60		709.04		249.70	
	x_1	114.00		147.44		111.78		142.85	

TABLE 6.2

STOCHASTIC CONTROL SOLUTIONS USING THE ABEL MODEL

UNCERTAIN PARAMETERS SOLUTION

	Consumption		Investment		Government Spending		Money Supply	
	Control	Target	Control	Target	Control	Target	Control	Target
1	0.362	0.36	0.089	0.09	0.114	0.14	0.147	0.15
2	0.367	0.37	0.090	0.09	0.113	0.14	0.146	0.15
3	0.371	0.37	0.091	0.09	0.114	0.14	0.146	0.15
4	0.376	0.38	0.092	0.09	0.115	0.14	0.147	0.15
5	0.380	0.38	0.093	0.09	0.116	0.15	0.147	0.16
6	0.384	0.38	0.094	0.09	0.116	0.15	0.147	0.16

Comparison with the Kendrick Control Study

Kendrick[2] used the same Abel data for testing his adaptive control algorithms except that he applied a high terminal factor of 10,000. This high terminal factor, in effect, placed greater importance on achieving the terminal targets. Figures 6.1 and 6.2 attempt to replicate part of his study with the Norman model used in place of the Kendrick model. As mentioned in Chapter IV, the Norman model is a simpler version of the Bar-Shalom/Tse model which in turn is a simpler version of the Kendrick model. Although we would not expect to get the same results,[3] we might expect to see similar patterns. In fact the patterns shown in Figures 6.1 and 6.2 were taken from the last simulation[4] of 50 for this study. The last simulation was chosen quite arbitrarily and, like Kendrick, the intent is to show patterns rather than actual values.

As in the Kendrick study, the simple LQT controllers for certainty equivalence (labelled CERTAINTY) and for uncertain parameters (labelled UNCERTAIN), used here in a learning environment, are very close in pattern and in value. The Norman model (labelled NORMAN) is more aggressive than the other models, displaying some violent fluctuations in the early time

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[2] *Ibid.*

[3] The error that Kendrick made in the initial states was faithfully reproduced in this control study to bring the results as close as possible to his.

[4] Kendrick was only able to undertake 20 simulations per case because each simulation consumed roughly eight minutes of computer CPU time.

FIG. 6.1 ADAPTIVE CONTROL SIMULATION — CONSUMPTION

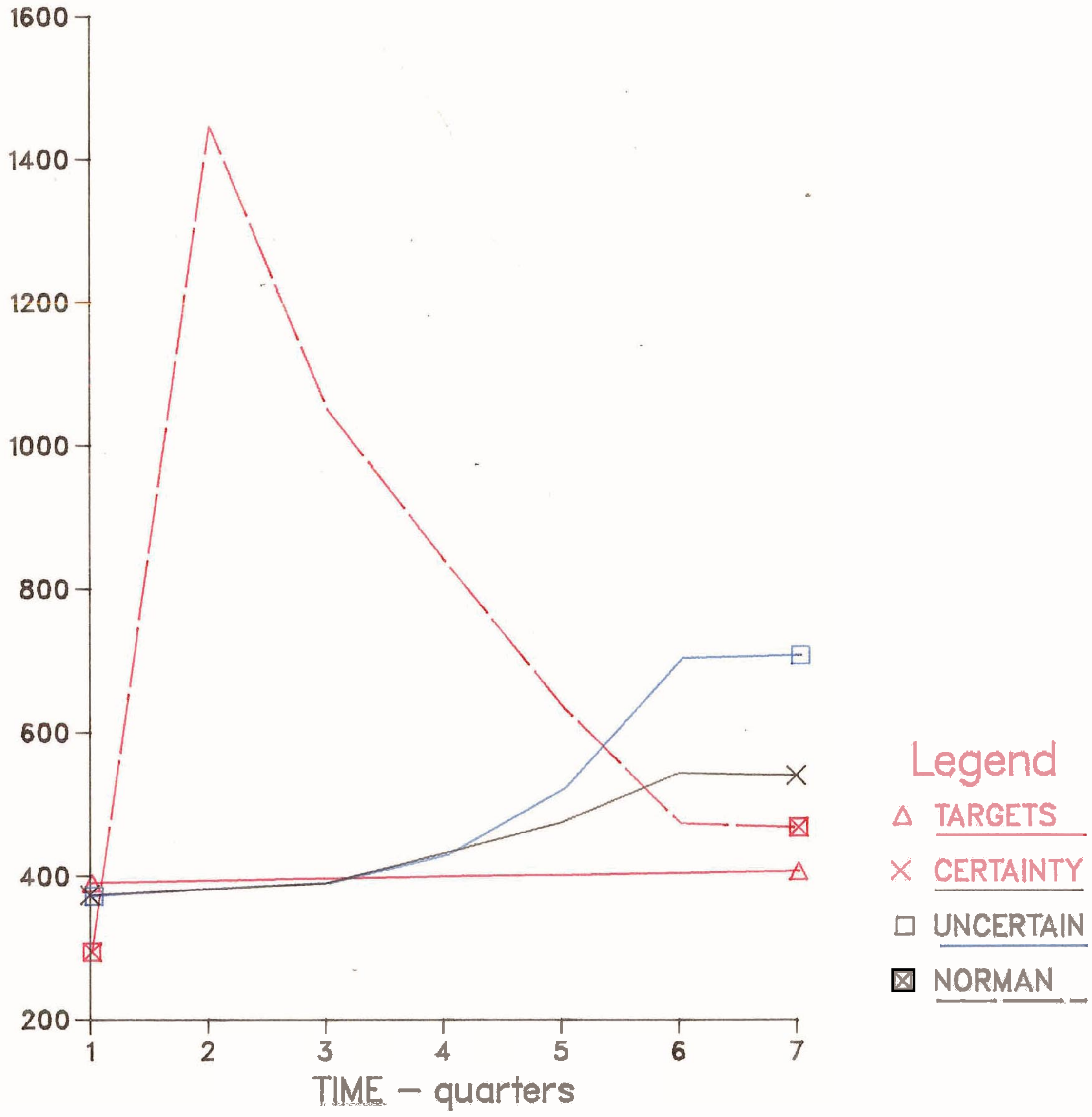
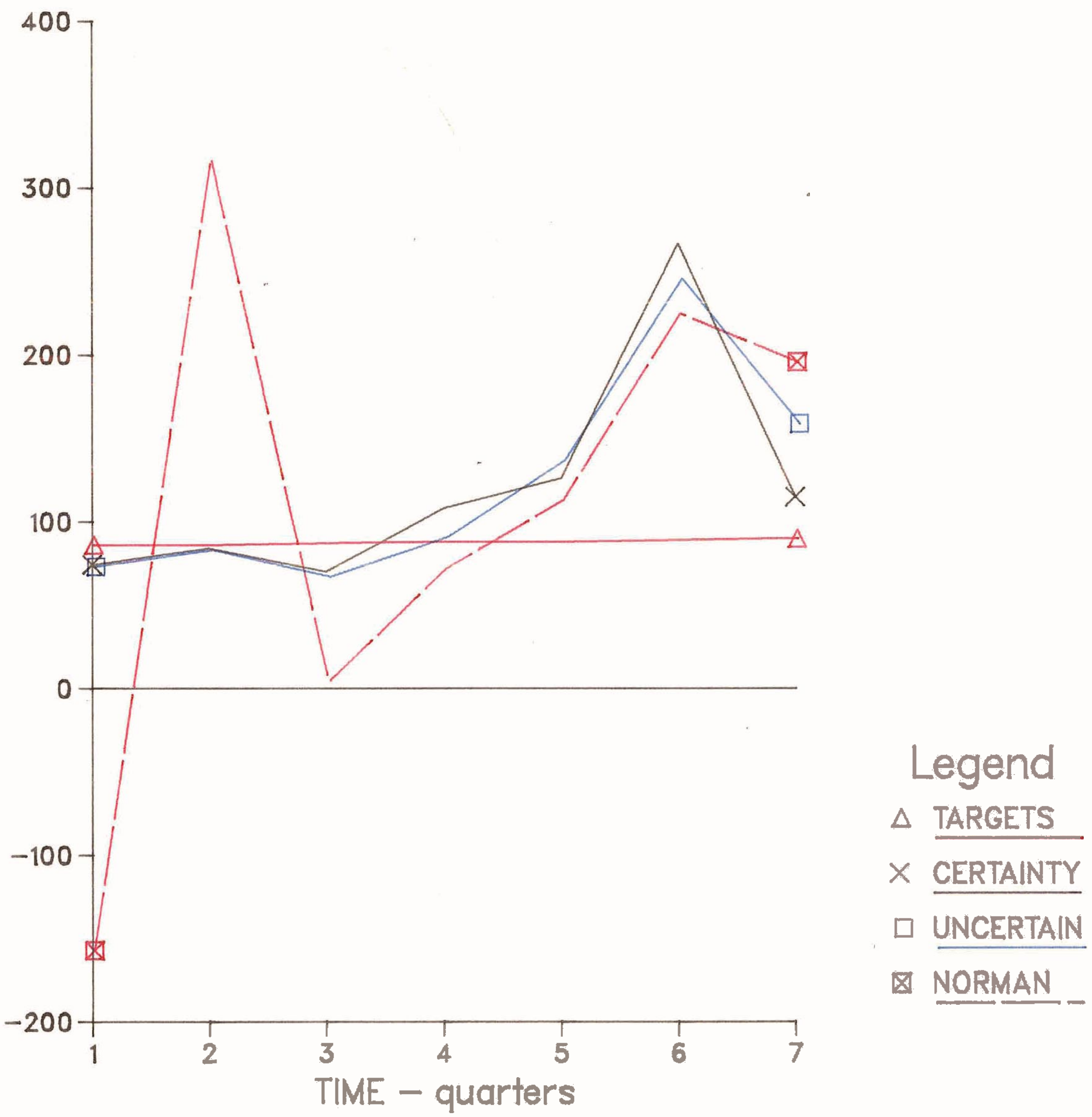


FIG. 6.2 ADAPTIVE CONTROL SIMULATION – INVESTMENT



periods, and achieves the smallest overall penalty cost. Like the Kendrick model, the Norman model seems to probe and experiment in the earlier less costly time periods in order to improve control for the more important last period.

The expected penalty costs and their standard deviations, as derived from these 50 simulations, are shown in Table 6.3. A summary of the computer runs from which these entries were taken is to be found in Appendix G.

TABLE 6.3

EXPECTED PENALTY COSTS FOR THE KENDRICK STUDY SIMULATIONS.

Control Model	Expected Penalty Cost	Standard Deviation
Certainty Equivalence	1980	11070
Uncertain Parameters	4604	19910
Norman Model	157	288

Clearly, the Norman model dominates the others. More comprehensive testing is required to see if this dominance is lasting.

The Laidler Model

In most control studies, as in the Abel and Kendrick studies, simple regression analysis is used to estimate the parameters of the model. For the Laidler model, more fully described in Appendix E, the simple regression coefficients are shown in Table 6.4. The model in this form is not very satisfactory; the standard errors for the second equation are large and some of the coefficients do not agree in magnitude or sign with those derived by Laidler. As mentioned previously, he used full information, maximum likelihood techniques to obtain his estimates. Using zero constraints as extraneous information for the Laidler model, regression yields the results shown in Table 6.5. These coefficients look much better and the standard deviations are smaller and more manageable. The simultaneous nature of the regression, as outlined in Appendix B, has still been retained even under extraneous information. The model is now much more in the spirit of the original Laidler model. Most of the coefficients agree in sign, approximately in magnitude and variance. Except for the domestic price equation, there is little evidence of autocorrelation as indicated by the Durbin-Watson statistics in Table 6.5. The model is not perfect but it is much closer to the original model compared to that obtained by simple regression. As a test bed for comparing controllers, it should be more than adequate.

TABLE 6.4
LAIDLER MODEL COEFFICIENTS

		Lagged Variables						Control Variables			Durbin Watson Statistic
		Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Domestic Credit	Government Spending	Tax Rate	
Transitory Income		0.3564	0.1081	0.3140	-1.6221	-0.6733	0.8380	0.1057	0.2300	-0.1520	1.977
	Stand. Error	0.1576	0.0690	0.1360	0.5859	0.6384	0.5233	0.0840	0.1479	0.0915	
Reserves		0.3985	-0.2258	0.5683	-3.3952	3.4059	-3.1258	-1.5758	2.2825	-0.1099	2.198
	Stand. Error	1.4810	0.6481	1.2767	5.5038	5.9979	4.9170	0.7896	1.3893	0.8601	
Exchange Rate		-0.5079	-0.0852	0.2489	-1.9201	-0.1184	0.3563	0.1917	-0.7674	0.4160	2.937
	Stand. Error	0.1730	0.0757	0.1491	0.6213	0.7005	0.5742	0.0922	0.1623	0.1005	
Domestic Price		0.3383	-0.0057	0.0539	1.1157	0.1328	-0.0234	0.0536	-0.1640	-0.0022	2.088
	Stand. Error	0.0407	0.0178	0.0351	0.1512	0.1648	0.1351	0.0217	0.0382	0.0237	
Money Supply		0.0614	0.0195	0.0545	-0.0931	-0.1274	0.0147	0.6313	0.2331	-0.0122	2.252
	Stand. Error	0.0263	0.0680	0.1339	0.5771	0.6289	0.5156	0.0828	0.1457	0.0902	

TABLE 6.5
RESTRICTED LAIDLER MODEL COEFFICIENTS

		Lagged Variables						Control Variables			Durbin Watson Statistic
		Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Domestic Credit	Government Spending	Tax Rate	
Transitory Income		-	-	0.3999	-1.4610	0.4834	-	-	0.2257	-0.1079	1.709
	Stand. Error	-	-	0.0427	0.4471	0.0342	-	-	0.0580	0.0182	
Reserves		-	0.4150	-0.0426	-	-0.0517	-	-	-	-	2.437
	Stand. Error	-	0.0820	1.6792	-	1.1892	-	-	-	-	
Exchange Rate		-	-	0.6156	-1.0540	-0.1150	-	-	-	-	1.769
	Stand. Error	-	-	0.0448	0.7932	0.0324	-	-	-	-	
Domestic Price		0.3187	-	0.0574	0.9890	-	-	-	-	-	1.082
	Stand. Error	0.0047	-	0.0036	0.0792	-	-	-	-	-	
Money Supply		-	-	0.0109	0.1545	0.4626	-0.3483	0.7349	-	-	2.689
	Stand. Error	-	-	0.02448	0.6737	0.1332	0.0930	0.0143	-	-	

Simple LQT Runs on the Laidler Model

Using the Laidler model, a series of simple LQT control runs were made for a variety of initial and terminal conditions. Table 6.6 contains the targets used for all of these variations. The targets were chosen to represent choices that should be quite acceptable to a Canadian policy-maker. They require modest growth for the economy with little inflation and growth in the money supply. The base run with unit control factors and unit diagonal weighting matrices, K_t , is shown in Table 6.7 for the case of certainty equivalence and in Table 6.8 for for the uncertain parameters case. At first sight, the certainty equivalence case, with a penalty cost of 4.3, looks much better than the uncertain parameters case with a penalty cost of 13.0. All the variations are applied to the uncertain parameters base run, one at a time, and the results for these cases, are shown in Tables 6.9 to 6.11.

Table 6.9 illustrates the situation of a high terminal factor; in this case the penalty costs for the last time period were multiplied by 10. As expected, the targets for the last time period are achieved more closely at the expense of the others. The reverse effect is produced in Table 6.10 where a discount factor of 1.2 is operating on the penalty costs. Here the future is less important and the near-term targets are more closely followed. Finally, in Table 6.11, the effects of a step change in the terminal targets are explored. The reserves and

TABLE 6.6

TARGETS FOR THE LAIDLER MODEL

	State Variables					Control Variables		
Time Period	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1	0.01	1.60	4.68	5.00	4.30	4.20	0.00	0.00
2	0.02	1.66	4.68	5.06	4.38	4.27	0.00	0.00
3	0.03	1.72	4.69	5.12	4.46	4.34	0.00	0.00
4	0.04	1.78	4.69	5.18	4.54	4.41	0.00	0.00
5	0.05	1.84	4.70	5.24	4.62	4.48	0.00	0.00
6	0.06	1.90	4.70	5.30	4.70	4.55	0.00	0.00
7	0.07	1.96	4.70	5.36	4.78	4.62	0.00	0.00
8	0.08	2.02	4.71	5.42	4.86	4.69	0.00	0.00
9	0.09	2.18	4.71	5.48	4.94	4.76	0.00	0.00
10	0.10	2.24	4.72	5.54	5.02	4.83	0.00	0.00

Note: All values, except time period, are given in natural logarithms.

TABLE 6.7

CERTAINTY EQUIVALENT STOCHASTIC CONTROL SOLUTION FOR THE LAIDLER MODEL - BASE RUN

Total Penalty Cost: 4.270

	State Variables					Control Variables		
Time Period	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1	-0.102	1.686	4.673	5.073	4.146	3.968	-0.339	0.162
2	-0.393	1.768	4.369	5.165	4.321	4.161	-0.178	0.085
3	-0.526	1.849	4.107	5.113	4.459	4.304	-0.031	0.015
4	-0.472	1.930	3.025	5.008	4.539	4.398	0.040	-0.019
5	-0.316	2.005	4.116	4.922	4.586	4.467	0.039	-0.019
6	-0.143	2.071	4.299	4.895	4.635	4.544	0.007	-0.003
7	-0.015	2.130	4.474	4.939	4.712	4.645	-0.011	-0.005
8	0.030	2.184	4.567	5.037	4.813	4.759	0.007	-0.003
9	-0.020	2.239	4.550	5.158	4.912	4.850	0.045	-0.021
10	-0.156	2.299	4.441	5.266	4.966	4.870	0.058	-0.028

Note: All values, except time period, are given in natural logarithms.

TABLE 6.8

UNCERTAIN PARAMETERS STOCHASTIC CONTROL SOLUTION FOR THE LAIDLER MODEL - BASE RUN

Total Penalty Cost: 13.006

	State Variables					Control Variables		
Time Period	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Govern- ment Spending	Tax Rate
1	-0.209	1.686	4.673	5.073	3.948	3.698	-0.735	0.327
2	-0.518	1.779	4.392	5.131	4.190	3.980	-0.286	0.131
3	-0.506	1.859	4.172	5.041	4.314	4.111	0.055	-0.028
4	-0.377	1.939	4.158	4.947	4.371	4.183	0.154	-0.082
5	-0.236	2.011	4.282	4.898	4.422	4.259	0.098	-0.060
6	-0.119	2.075	4.444	4.907	4.506	4.376	0.012	-0.017
7	-0.039	2.131	4.565	4.967	4.625	4.523	-0.021	0.004
8	-0.012	2.185	4.604	5.062	4.746	4.658	0.019	-0.011
9	-0.067	2.241	4.554	5.172	4.857	4.765	0.073	-0.036
10	-0.193	2.303	4.435	5.265	4.972	4.868	0.086	-0.042

Note: All values, except time period, are given in natural logarithms.

TABLE 6.9

UNCERTAIN PARAMETERS STOCHASTIC CONTROL SOLUTION FOR THE LAIDLER MODEL - HIGH TERMINAL FACTOR

Total Penalty Cost: 19.924

	State Variables					Control Variables		
Time Period	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1	-0.226	1.686	4.673	5.073	3.919	3.659	-0.802	0.346
2	-0.527	1.780	4.396	5.125	4.204	3.999	-0.270	0.120
3	-0.471	1.859	4.178	5.032	4.309	4.105	0.125	-0.059
4	-0.349	1.938	4.171	4.950	4.304	4.094	0.209	-0.115
5	-0.267	2.014	4.294	4.912	4.282	4.068	0.096	-0.077
6	-0.216	2.083	4.453	4.911	4.334	4.136	-0.051	-0.016
7	-0.150	2.143	4.587	4.940	4.509	4.360	-0.125	0.026
8	-0.041	2.195	4.659	5.002	4.707	4.605	-0.056	0.015
9	0.064	2.244	4.655	5.107	4.787	4.681	0.217	-0.105
10	-0.098	2.303	4.575	5.247	4.963	4.873	0.060	-0.029

Note: All values, except time period, are given in natural logarithms.

TABLE 6.10

UNCERTAIN PARAMETERS STOCHASTIC CONTROL SOLUTION FOR THE LAIDLER MODEL - DISCOUNTED PENALTIES

Total Penalty Cost: 6.433

	State Variables					Control Variables		
Time Period	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1	-0.140	1.686	4.673	5.073	4.066	3.860	-0.479	0.220
2	-0.440	1.773	4.379	5.153	4.236	4.045	-0.211	0.099
3	-0.525	1.855	4.135	5.086	4.358	4.167	0.040	-0.020
4	-0.439	1.936	4.081	5.984	4.430	4.254	0.153	-0.078
5	-0.282	2.011	4.189	4.911	4.484	4.333	0.140	-0.074
6	-0.129	2.076	4.366	4.900	4.551	4.431	0.070	-0.041
7	-0.026	2.133	4.520	4.952	4.644	4.551	0.021	-0.015
8	0.003	2.187	4.590	5.049	4.751	4.669	0.029	-0.016
9	-0.053	2.242	4.559	5.163	4.857	4.770	0.067	-0.033
10	-0.179	2.303	4.448	5.261	4.970	4.869	0.083	-0.040

Note: All values, except time period, are given in natural logarithms.

TABLE 6.11

UNCERTAIN PARAMETERS STOCHASTIC CONTROL SOLUTION FOR THE LAIDLER MODEL - HIGH TERMINAL TARGETS

Total Penalty Cost: 1752.157

	State Variables					Control Variables		
Time Period	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1	-0.221	1.686	4.673	5.073	3.907	3.643	-0.778	0.346
2	-0.556	1.781	4.397	5.127	4.141	3.913	-0.352	0.160
3	-0.541	1.863	4.184	5.025	4.279	4.063	-0.013	0.003
4	-0.376	1.941	4.186	4.921	4.379	4.196	0.121	-0.067
5	-0.169	2.010	4.325	4.875	4.493	4.362	0.152	-0.084
6	0.015	2.069	4.487	4.908	4.619	4.538	0.188	-0.097
7	0.107	2.121	4.578	5.013	4.695	4.624	0.251	-0.123
8	0.025	2.176	4.556	5.155	4.646	4.515	0.257	-0.123
9	-0.259	2.245	4.438	5.273	4.618	4.416	0.112	-0.054
10	-0.485	2.322	4.285	5.296	20.684	26.215	0.147	-0.071

Note: All values, except time period, are given in natural logarithms.

money supply targets for the tenth time period are factored by 10 compared to the base runs. As would be expected for a simple LQT controller, this variation should produce confusion in the latter time periods along with a high penalty cost. For adaptive controllers, to be tested in the next section, it will be interesting to see how well they anticipate this step change and prepare for it in the earlier time periods.

Adaptive Control Runs Using the Laidler Model

Figures 6.3 through 6.7 display the simulated movement of the endogenous variables over the planning time span under an adaptive or learning environment. As with Figures 1 and 2, they represent the results of just one simulation, quite arbitrarily chosen to be the last of 50. It is the general trend and patterns that are of interest rather than the actual values.

Again the Norman model shows a different tracking pattern to the others but this time it is not the closest to the targets. Actually, this looks a particularly bad simulation for the Norman model and illustrates the situation where everything goes wrong. One or two simulations like this in 50 can drastically increase the overall average penalty cost.

The other control models are very similar and, except for the Reserves simulation in Figure 6.4, the certainty equivalence model (labelled CERTAINTY) and the uncertain parameters model (labelled UNCERTAIN) are almost identical. The latter model, not designed for an adaptive environment, does almost as well as the best adaptive model, the MacRae model. Actually the MacRae model, achieved the lowest overall penalty cost for this simulation, clearly dominating its rival adaptive controller, the Norman model. The heuristic model, the model which learns by chance, and is in fact the MacRae model with a stability factor of zero, seems to fall between the certainty equivalence and uncertain parameters models.

FIG. 6.3 ADAPTIVE CONTROL SIMULATION — TRANSITORY INCOME

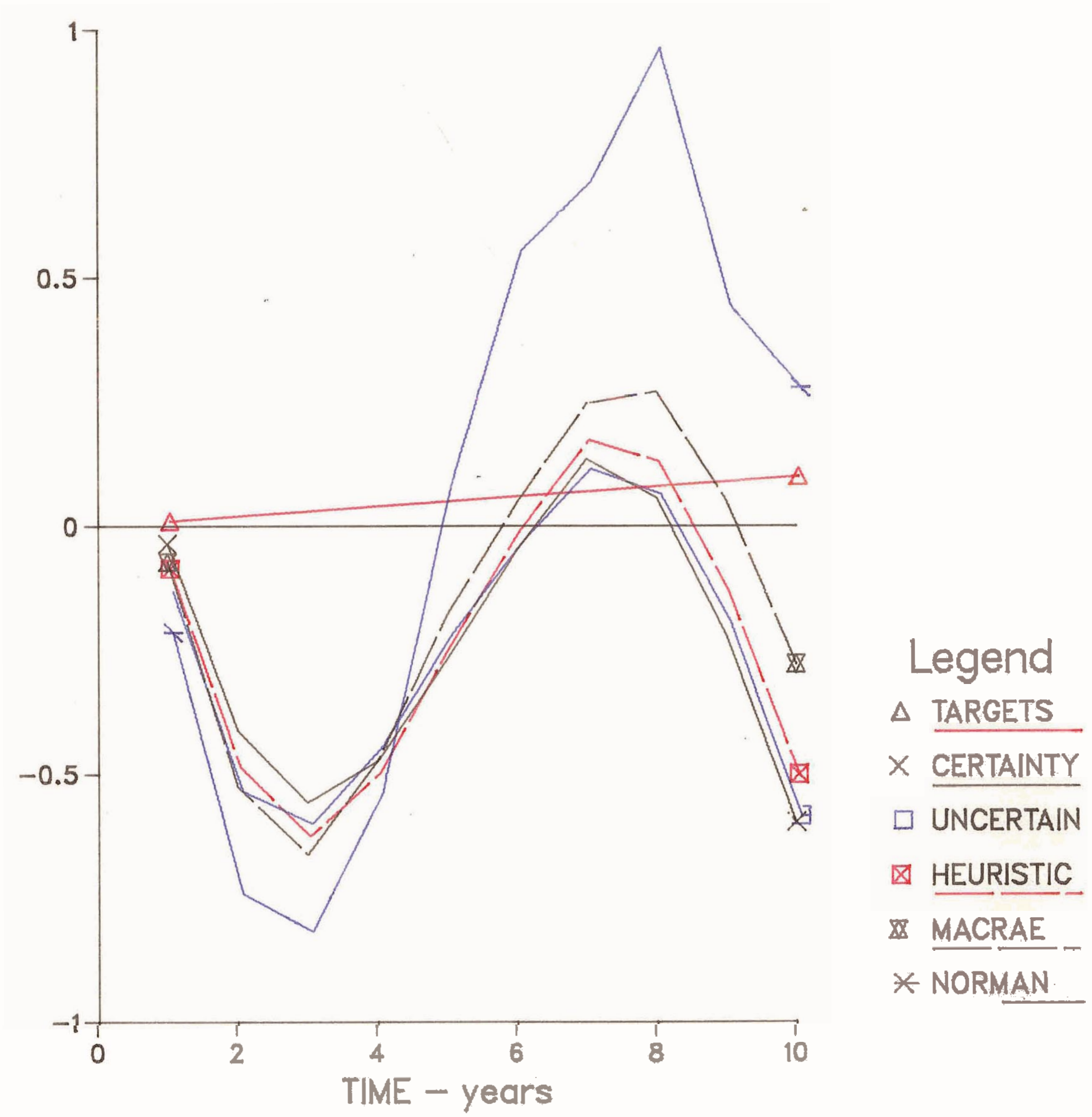


FIG. 6.4 ADAPTIVE CONTROL SIMULATION – RESERVES

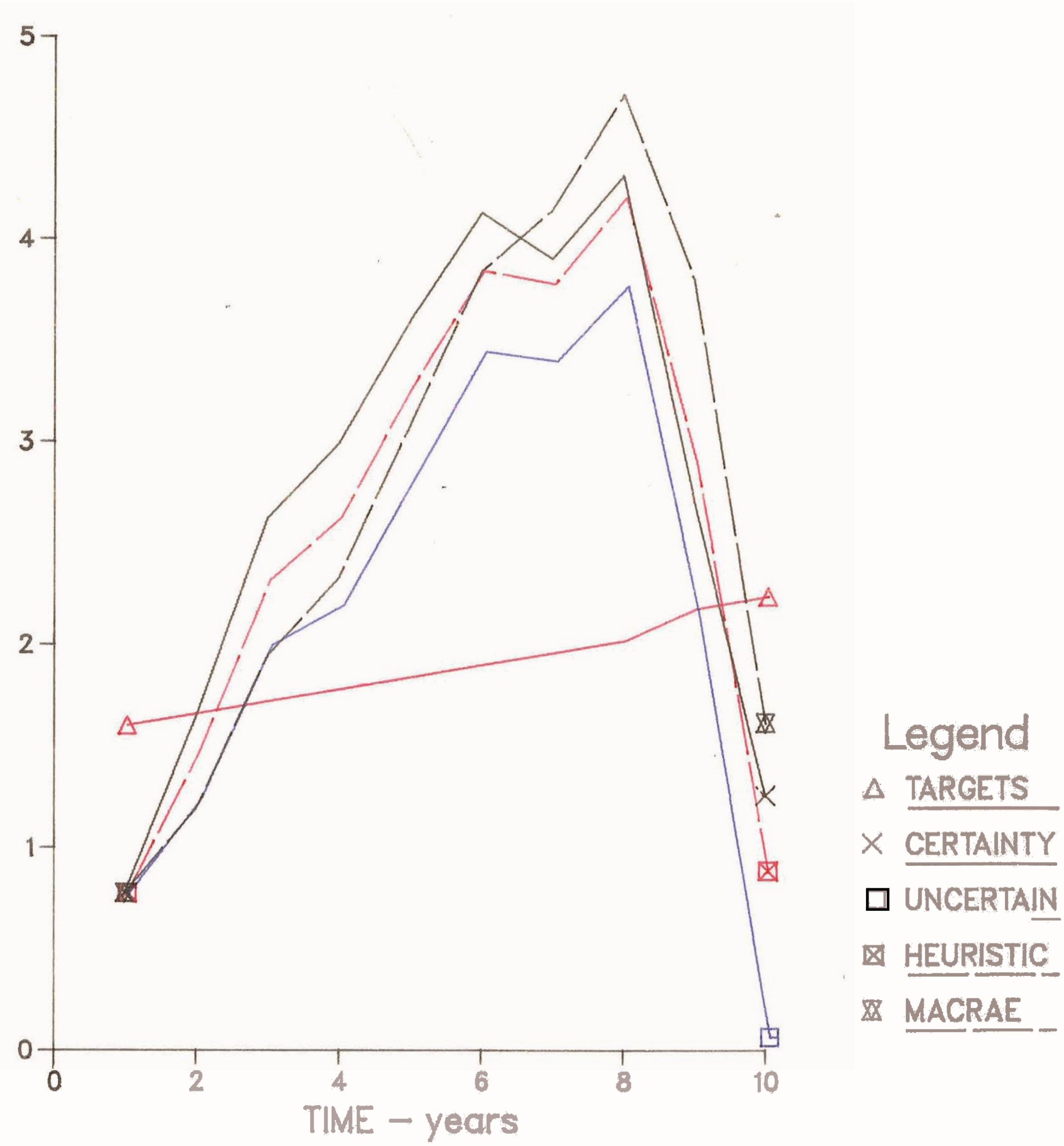


FIG. 6.5 ADAPTIVE CONTROL SIMULATION – EXCHANGE RATE

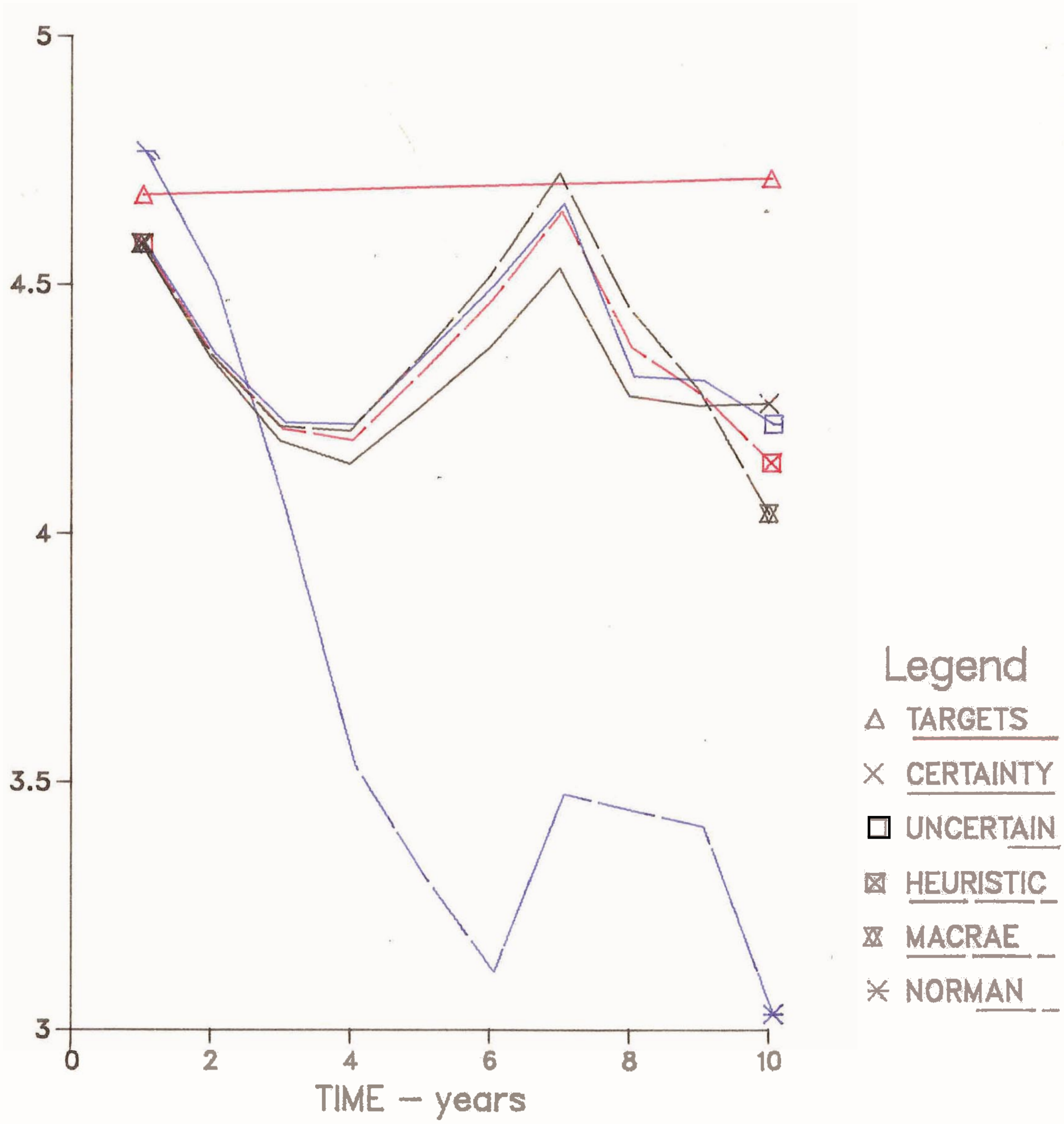


FIG. 6.6 ADAPTIVE CONTROL SIMULATION – PRICE INDEX

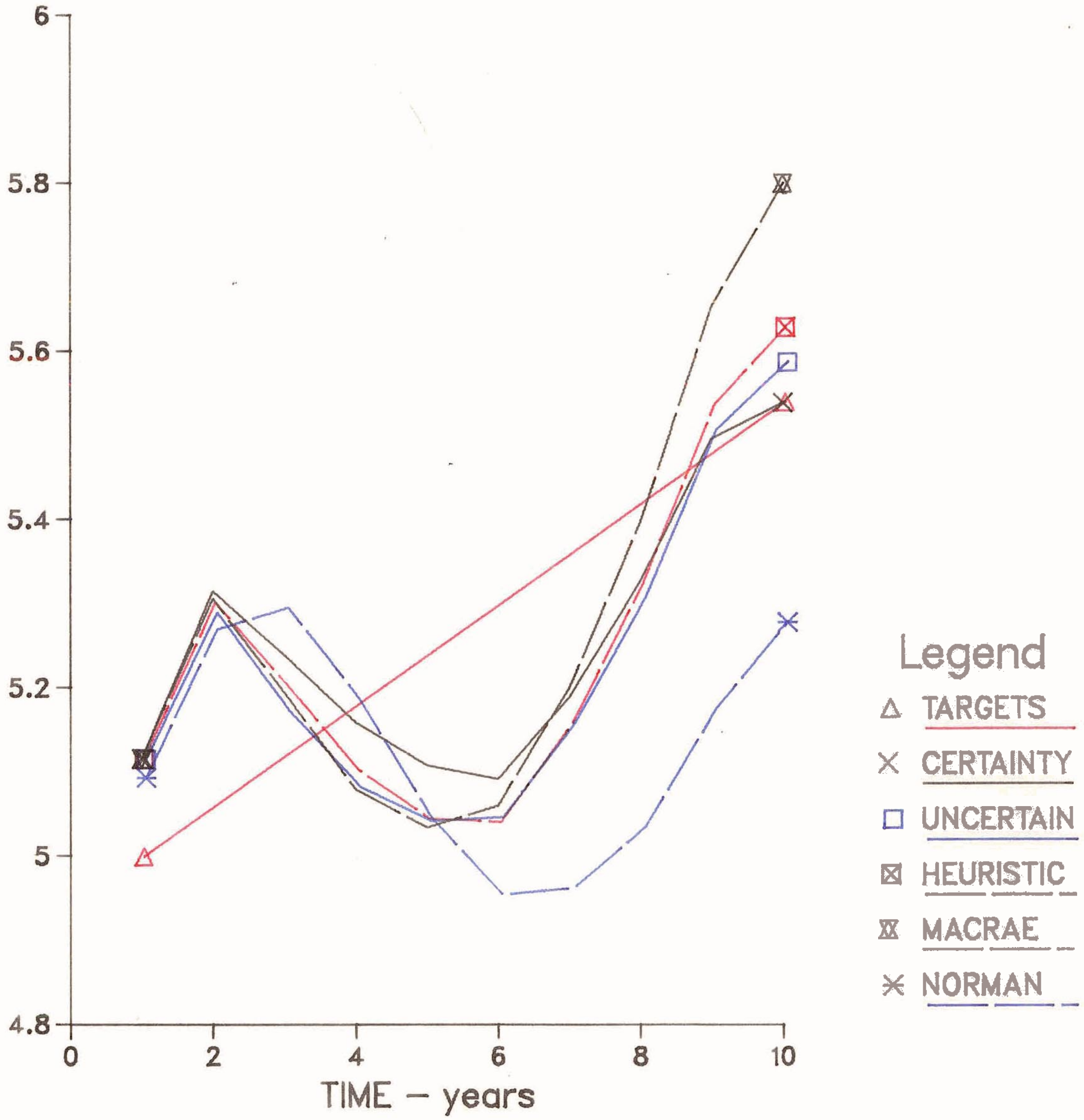
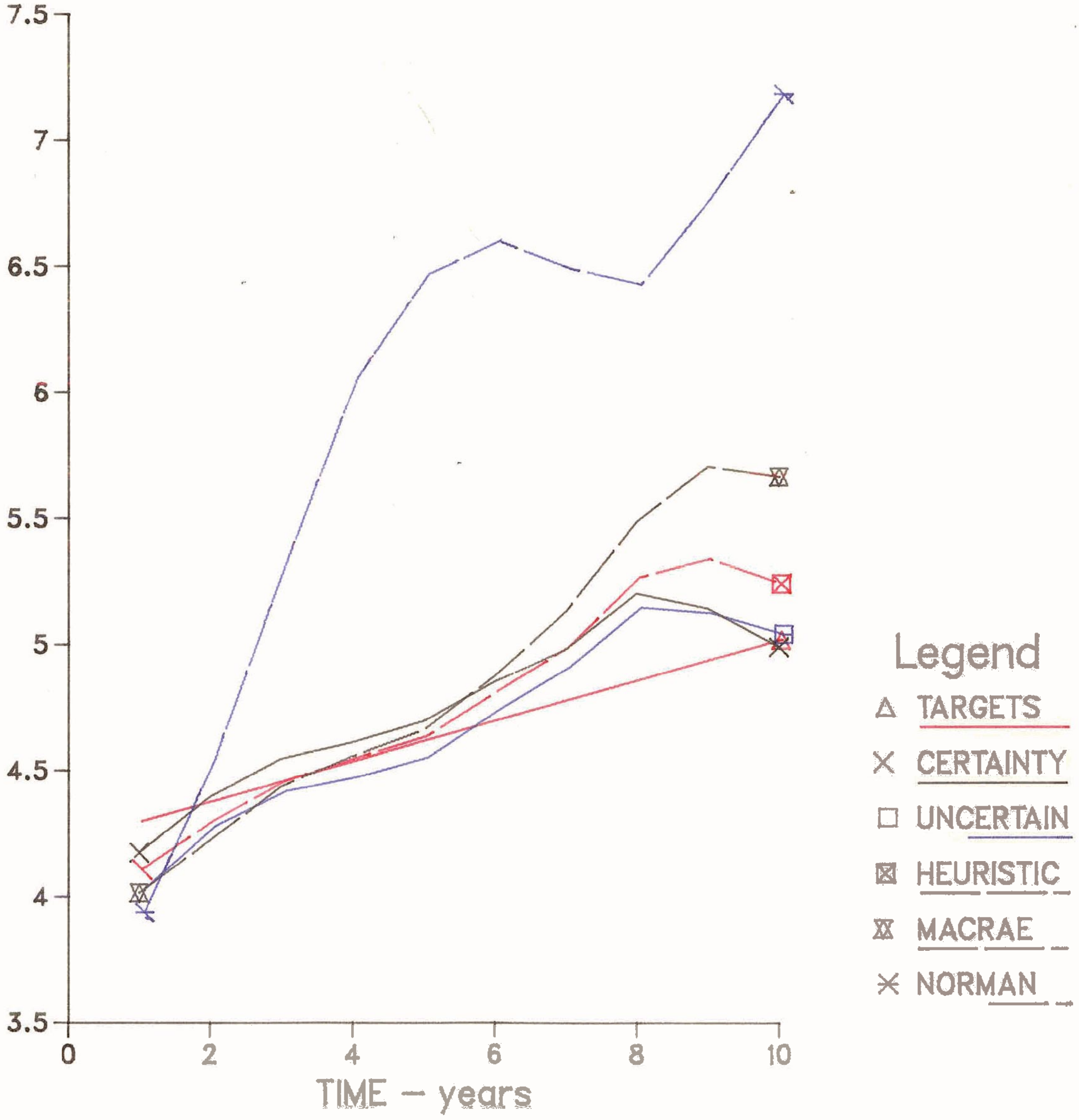


FIG. 6.7 ADAPTIVE CONTROL SIMULATION - MONEY SUPPLY



The Overall Adaptive Simulation Averages

The results from averaging 50 adaptive control simulations are shown in Table 6.12 for the Laidler model and in Table 6.13 for the Abel model.

The earlier dominance of the Norman model is clearly broken by the results in Table 6.12. There is no clear winner, except that the uncertain parameters model does consistently well. The Norman model encountered difficulty with the high terminal factor case, the gradients taking a long time to converge. On the other hand, the MacRae model had difficulties with the high terminal targets case and required the selection of a smaller stability factor to solve the convergence problem.

The results in Table 6.13 are similar to the Kendrick results in Table 6.3. Here the base run for the Abel model, shown in Table 6.2, has been changed to generate a set of variations similar to the Laidler model. For the high terminal factor case, the Kendrick figure of 10,000 has been used and a discount factor of 1.2 was used in the discounted penalties case. The high terminal targets case was generated by multiplying the terminal targets for consumption and investment by 100.

The Norman model repeats its excellent performance although unlike the Kendrick simulations, it does not totally dominate the others. The uncertain parameters model does extremely well for the high terminal targets case.

TABLE 6.12

STOCHASTIC ADAPTIVE CONTROL SIMULATION

PENALTY COSTS[a] FOR THE LAIDLER MODEL

Solution[c]	Base Run	High Terminal Factor $\alpha = 100$	Discounted Penalty Costs $\beta = 1.2$	High Terminal Targets[b]
Certainty Equivalence	2,681,717 (16,843,126)	40,312,279 (265,822,044)	725,557 (4,765,233)	1,000,555 (5,305,561)
Uncertain Parameters	597,126 (2,936,271)	4,881,208 (24,030,976)	227,126 (1,290,508)	485,157 (2,410,785)
Heuristic Uncertain Parameters	1,138,224 (6,264,002)	12,992,322 (75,927,452)	372,341 (2,283,329)	739,348 (3,714,662)
MacRae Model	522,681 (2,580,855)	5,495,886 (27,273,212)	169,720 (898,611)	739,330 (3,714,554)
Norman Dual Control	686,779 (4,760,975)	3,889,679 (26,683,289)	63,856 (431,579)	801,624 (5,602,102)

[a]The figures in parenthesis are estimates of population standard deviations.

[b]The last period base run targets for Reserves and Money Supply were multiplied by 10.

[c]The computer runs from which the table entries were recorded are to be found in Appendix G.

TABLE 6.13

STOCHASTIC ADAPTIVE CONTROL SIMULATION

PENALTY COSTS[a] FOR THE ABEL MODEL

Solution[c]	Base Run	High Terminal Factor $\alpha = 10000$	Discounted Penalty Costs $\beta = 1.2$	High Terminal Targets[b]
Certainty Equivalence	1.2 (4.6)	9,749 (39,760)	0.4 (1.6)	9,131 (9,743)
Uncertain Parameters	0.2 (0.7)	10,276 (43,369)	0.1 (0.2)	342 (257)
Heuristic Uncertain Parameters	0.2 (0.7)	3,178 (11,932)	0.1 (0.2)	8,602 (9,427)
MacRae Model	0.2 (0.7)	26 (42)	0.1 (0.2)	3,199 (5,157)
Norman Dual Control	0.0 (0.1)	63 (99)	0.0 (0.0)	742 (907)

[a]The figures in parenthesis are estimates of population standard deviations.

[b]The last period targets for Consumption and Investment were multiplied by 100.

[c]The computer runs from which the table entries were recorded are to be found in Appendix G.

Comparison of Stochastic Control Solutions with Actual Data

Actual results (measured in natural logarithms for ease of comparison) for the Canadian economy over the time span 1976 to 1981 are summarized in Table 6.14. These values were estimated as closely as possible, using the same data sources as those listed in Laidler (1981).

Stochastic control predictions for six time periods using the uncertain parameters control model are shown in Table 6.15. The targets used in this table are the same as those used previously over the first six time periods. As stated before, they were chosen quite arbitrarily and the results in Table 6.15 are heavily influenced by their choice. In fact, the actual values in Table 6.14 could have been used as targets for this run and, of course, the results would then have been close to the actual values. Unfortunately, we can only speculate what targets were behind the actual values in Table 6.14.

There are some obvious differences between the two sets of results. The actual figures reflect a larger money supply, higher inflation, and smaller reserve levels. Transitory income is also higher and even though a weighting factor of 100 (as opposed to unity for the base run) was applied to the transitory income penalties, the Laidler model seemed reluctant to move closer to its income targets.

TABLE 6.14

ACTUAL CANADIAN ECONOMIC AGGREGATES

1976 - 1981

Year	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1976	0.178	1.765	4.671	5.032	4.443	4.371	0.094	-0.405
1977	0.143	1.528	4.752	5.108	4.554	4.504	0.089	-0.439
1978	0.107	1.520	4.833	5.195	4.762	4.722	0.040	-0.480
1979	0.098	1.358	4.818	5.282	4.868	4.837	-0.013	-0.532
1980	0.059	1.394	4.840	5.376	5.010	4.983	-0.012	-0.514
1981	0.023	1.475	4.833	5.496	5.148	5.122	-0.040	-0.494

TABLE 6.15

STOCHASTIC CONTROL PREDICTIONS FOR

CANADIAN ECONOMIC AGGREGATES 1976 - 1981

Year	Transitory Income	Reserves	Exchange Rate	Domestic Price	Money Supply	Domestic Credit	Government Spending	Tax Rate
1976	-0.141	1.686	4.673	5.073	4.204	4.047	-0.554	0.080
1977	-0.259	1.765	4.363	5.152	4.627	4.579	0.249	1.006
1978	-0.181	1.833	4.081	5.142	4.877	4.881	0.689	-0.154
1979	-0.095	1.903	3.929	5.146	5.070	5.125	0.912	-0.308
1980	-0.034	1.971	3.851	5.172	5.228	5.324	1.033	-0.392
1981	-0.017	2.036	3.798	5.217	4.687	4.568	1.076	-0.425

NOTE: All values, except time period, are given in natural logarithms.

VII. CONCLUSIONS

Many of the control studies reviewed used small, linear regression models in which standard regression methods were employed. More practical situations, as in this study, often demand non linear models and special regression techniques. The results from simple regression were simply not good enough for the Laidler model. The need to impose extraneous information on the regression by way of zero restrictions resulted in a large computational overhead, especially noticeable in the simulation phase of the analysis. These restrictions did bring the model closer to the actual Laidler model in which maximum likelihood, full information methods were used in the parameter estimation.

Without simulation, it would be tempting to conclude that the simple certainty equivalence model is the best stochastic control model. Certainly, it is computationally efficient, has the smallest penalty cost and its always easy to ignore uncertainty. However, when tested with simulation, it is clearly inferior to the uncertain parameters model, a model specifically designed to handle uncertainty in the error terms and in the regression coefficients. One would expect this superiority to increase, the more the uncertainty in the regression model.

The testing of the various control models under simulation was an important part of this study. Fifty simulations, representing 500 years for the Laidler model, were performed for each control model. Without the computational burden mentioned above, a higher number of simulations would have been undertaken, especially in view of the high penalty cost distribution variances. A small number of outliers were the culprits for these large variances, producing high positive skewness in the penalty cost distributions, much like a chi-square probability distribution.

The tracking patterns for the various adaptive control models, under simulation, showed how similar most models were in their stochastic control solutions, the only dissident being the Norman model with its very aggressive control fluctuations in the early stages of the planning time span. Norman would argue that learning is taking place in the early stages, just like a captain with a newly commissioned vessel, rapidly manoeuvring the ship from the dock in order to quickly learn its response characteristics. One should also note that the Norman model is completely different in its approach to stochastic control analysis; one obvious difference is that it moves forward through the planning time span whilst all the others move backwards.

Simulation was particularly useful in examining the performance of the adaptive controllers. None of them came close to their theoretical penalty cost predictions which underscores the complexity and the difficulty in modelling the adaptive or learning environment. For the relatively simple, Abel econometric model, the Norman adaptive control model is by far the best choice. For the Laidler econometric model, however, the

MacRae adaptive control model did well, even though it suffered information overload for the high terminal targets case. A simple model, the uncertain parameters model, not even designed to cope with the adaptive environment, performed well for both econometric models. These confusing results are typical for stochastic control studies. Rausser, Norman and Kendrick also found that there was no clear winner; that different models excelled for different control problems.

Based upon a simple average rank index, the uncertain parameters and the MacRae model emerge just ahead of the Norman model as the best control models for the Laidler econometric model. A key advantage for the uncertain parameters model is its relative computational efficiency, requiring only 12 seconds of computer central processing unit (CPU) time for its Laidler control solution as opposed to 60 seconds for the MacRae model and 200 seconds for the Norman model (all of the models took about 400 seconds of CPU time for the 50 adaptive simulations). In fact, its required computer central processing time seems to grow exponentially with the dimension of the state variable vector at a rate of 2.6. For the Abel model, with a state variable vector dimension of 4, the CPU time was 2 seconds while, for the Laidler model, with a state vector dimension of 8, the CPU time was 12 seconds. It would seem to indicate that the uncertain parameters model could be applied to 50-60 equation regression models without placing too much strain on the computer facilities (the required CPU time would be about 30 minutes for this situation).

An important message for the policy-maker emerges from this study: economic policies should be investigated in a stochastic dynamic framework, not deterministic and static. Even a very simple econometric model, such as the one in this study, can yield surprising results which are sometimes quite counter-intuitive. A medium-sized model of the economy, 50-60 equations, say, developed with a keen eye on the error terms and system dynamics, could be an extremely valuable policy-making tool when incorporated in a stochastic control analysis, much like the one described in this study.

APPENDIX A

OPTIMAL CONTROL THEORY
AND DYNAMIC PROGRAMMING

Optimal control theory was originally developed for continuous, deterministic control problems. For the moment, therefore, stochastic variations will be ignored. We begin by augmenting the penalty cost function with a state equation constraint:

$$W = \sum_{t=1}^{T} \left[(y_t - a_t)' K_t (y_t - a_t) + 2\lambda_t' (y_t - D_t z_t) \right] \qquad (A.1)$$

Optimal control theory provides a solution to the above equation along with the Lagrange multipliers or costate variables, as they are called. A careful choice[1] for the Hamiltonian is next made:

$$H_t = \tfrac{1}{2} (y_t - a_t)' K_t (y_t - a_t) + 2\lambda_t' (y_t - D_t z_t) \qquad (A.2)$$

Pontryagin's maximum or minimum principle[2] is then applied in order to minimize H_t for each time period over all possible values for the control variables:

$$\frac{\partial H_t}{\partial x_t} = 0 \qquad \frac{\partial H_t}{\partial y_t} = 0 \qquad \frac{\partial H_t}{\partial \lambda_t} = 0 \qquad (A.3)$$

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[1] Arthur E. Bryson and Yu-Chi Ho, Applied Optimal Control, (Waltham, Mass: Blaisdell Publishing Company, 1969), p. 44.

[2] L. S. Pontryagin et al., The Mathematical Theory of Optimal Processes, (New York: Intersciences, 1962).

The first two conditions yield:

$$C'_t \lambda_t = 0$$

$$K_t (y_t - a_t) - \lambda_t + A'_{t+1} \lambda_{t+1} = 0 \qquad (A.4)$$

The logical place to start the solution is at the last time period, T, where $\lambda_{T+1} = 0$, which gives:

$$\lambda_T = K_T (y_T - a_T) \qquad (A.5)$$

We would then continue moving backwards in time, solving for y_t and λ_t , using the recurrence relations (A.4). But this is equivalent to applying Bellman's Principle[3] and therefore to using conventional dynamic programming methods. If dynamic programming methods are used, without regard to minimizing the Hamiltonian, the same results will be obtained except that the costate variables will not be evaluated. Obviously, it would be a simple matter to evaluate them, if required, by using a recurrence relationship developed from (A.4).

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[3] Bellman, Adaptive Control Processes, p. 57.

APPENDIX B

REGRESSION THEORY FOR STOCHASTIC CONTROL

Derivation of Regression Coefficients

Given a set of observations on the endogenous variables, Y, and a set of observations on the lagged endogenous, control and exogenous variables, Z, related by $Y = Z\,\Pi' + \varepsilon$, the ordinary least squares estimate of the coefficient matrix, Π, is:

$$\Pi' = (Z'Z)^{-1} Z'Y \tag{B.1}$$

If we impose zero restrictions on Π by way of extraneous information, e.g., $R_i\,\pi_i' = 0$ for the i th equation, then:

$$\hat{\pi}_i = \pi_i R_i' \tag{B.2}$$

$$R_i = I - (Z'Z)^{-1} R_i' \left[R_i (Z'Z)^{-1} R_i' \right]^{-1} R_i \tag{B.3}$$

$\hat{\pi}_i$ is a set of restricted coefficients to be found in the i th row of $\hat{\Pi}$, the restricted coefficient matrix. The sum of squares, residual cross-product matrix, S, can be written:

$$S = (Y' - \hat{\Pi} Z')(Y - Z\hat{\Pi}') \tag{B.4}$$

This cross-product matrix, S, can then be used along with N, the number of observations, s, the number of columns in $\hat{\Pi}$ and p, the number of endogenous variables to obtain an estimator for the residual covariance matrix, $\bar{V}_t$:

$$\overline{V} = \frac{1}{(N - s - p - 1)} \quad S \qquad (B.5)$$

It should be noted that the regression coefficients, $\hat{\pi}$, only represent part of the D_t matrix in (3.4) of the main text. In fact, they are the first row of matrices in (3.2), the remainder being either null or identity matrices. In the next section, the regression coefficient covariance relationships will be analysed. The covariance matrix derived will be for the first row of coefficient matrices in matrix D_t.

<u>Derivation of the Coefficient Covariance Relationships</u>

This section closely follows the work of Elizabeth MacRae.[1] We begin by making the assumption that the regression coefficient matrix, D_t, will have a multivariate normal probability distribution with covariance, Γ_t, of order equal to the number of rows in D_t multiplied by the number of columns in D_t. The probability distribution of D_t, under these assumptions, will be:

$$P(D_t | y_{t-1}, z_{t-1}, z_{t-2}, \ldots) \, \alpha$$

$$\text{Exp} - \tfrac{1}{2}[P(D'_t) - P(D'_{t-1})]' \, \Gamma^{-1}_{t-1} [P(D'_t) - P(D'_{t-1})] \qquad (B.6)$$

Where P is a pack operator which forms a vector from a matrix by stacking the matrix columns, one above the other. The

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[1] MacRae, <u>An Adaptive Learning Rule</u>, p. 905.

conditional distribution of D_t, given the next period observations, may then be written, using Bayes rule:

$$P(D_t | y_t, z_t, z_{t-1}, \dots) =$$

$$\frac{P(y_t | D_t, z_t, z_{t-1}, \dots) \; P(D_t | z_t, z_{t-1}, \dots)}{P(y_t | z_t, z_{t-1}, \dots)} \tag{B.7}$$

The first term of the numerator in (B.7) can be simplified by substituting for the system relations (3.4) to yield:

$$P (y_t | D_t, z_t, z_{t-1}, \dots) \propto$$

$$\text{Exp} - \tfrac{1}{2} [(y_t - D_t z_t)' V_t^{-1} (y_t - D_t z_t)] \tag{B.8}$$

Because x_t and w_t in z_t contribute nothing to the posterior distribution beyond that contributed by y_{t-1} (also in z_t), the other term in the numerator of (B.7) is proportional to the right-hand side of (B.6). The denominator in (B.7) is independent of D_t and can therefore be ignored by simply changing the equality sign in (B.7) to a proportional sign, $\propto$.

After substitution into the numerator of (B.7) and expansion of the quadratic forms:

$$P(D_t \mid y_t , z_t , z_{t-1} , \ldots) \alpha$$

$$\text{Exp } -\tfrac{1}{2} [y'_t V_t^{-1} y_t - 2 z'_t D'_t V_t^{-1} y_t + z'_t D'_t V_t^{-1} D_t z_t$$

$$+ P'(D'_t) \Gamma_{t-1}^{-1} P(D'_t) - 2 P'(D'_t) \Gamma_{t-1}^{-1} \bar{P}(D'_{t-1})$$

$$+ P'(D'_{t-1}) \Gamma_{t-1}^{-1} P(D'_{t-1})] \qquad (B.9)$$

Now the pack operator in (B.9) has the following property:

$$D_t z_t = P (D_t z_t) = P (z'_t D'_t) = (I \otimes z'_t) P (D'_t) \qquad (B.10)$$

In (B.10), $\otimes$ is termed the Kronecker product in which each element to the left of the operator is multiplied by the matrix on the right of the operator. This should not be confused with the MacRae star product operator[2] which is a special form of matrix multiplication defined by:

$$C = A \circledast B \equiv \sum_{ij} a_{ij} B_{ij} \qquad (B.11)$$

The matrices in (B.11) must be compatible such that, if A is an m by n matrix, and the submatrix B_{ij} is of order p by q, then the size of B must be mp by nq and the resulting C will be a p by q matrix.

---

[2] Ibid, p. 904.

Exploiting the Kronecker product relationship in (B.9) gives:

$$P(D_t \mid y_t, z_t, z_{t-1}, \ldots) \; \alpha$$

$$\text{Exp} - \tfrac{1}{2}\{ P'(D_t') [\Gamma_{t-1}^{-1} + (V_t^{-1} \otimes z_t z_t')] P(D_t')$$

$$- 2P'(D_t') [\Gamma_{t-1}^{-1} P(D_{t-1}') + (V_t^{-1} \otimes z_t) y_t]$$

$$- \text{(other terms independent of } D_t \text{)} \qquad \text{(B.12)}$$

After completing the square, neglecting terms that are independent of D_t, (B.12) can be rewritten to give (B.13):

$$P(D_t \mid y_t, z_t, z_{t-1}, \ldots) \; \alpha$$

$$\text{Exp} - \tfrac{1}{2} [P(D_t') - \mu]'[\Gamma_{t-1}^{-1} + (V_t^{-1} \otimes z_t z_t')] [P(D_t') - \mu]$$

where

$$\mu = [\Gamma_{t-1}^{-1} + (V_t^{-1} \otimes z_t z_t')]^{-1} [\Gamma_{t-1}^{-1} P(D_{t-1}') + (V_t^{-1} \otimes z_t) y_t]$$

In fact, by comparing (B.13) with (B.8), a multivariate normal probability distribution is revealed with a mean of μ and a precision given by:

$$\Gamma_t^{-1} = \Gamma_{t-1}^{-1} + (V_t^{-1} \otimes z_t z_t') \qquad \text{(B.14)}$$

The mean, μ, can then be further simplified after substituting for Γ_t and the state equations, (3.4):

$$P(D'_t) = \Gamma_t [\Gamma_{t-1}^{-1} P(D'_{t-1}) + (V_t^{-1} \otimes z_t z'_t) P(D'_{t-1})]$$

$$= \Gamma_t [\{ \Gamma_{t-1}^{-1} + (V_t^{-1} \otimes z_t z'_t) \} P(D'_{t-1})]$$

$$= \Gamma_t \Gamma_t^{-1} P(D'_{t-1}) = P(D'_{t-1}) \qquad (B.15)$$

The above equation, (B.15), implies that all the coefficient means are equal to the prior mean and are therefore constant over the planning time span. It is also possible to develop an expression for covariance after substituting:

$$Z'_t Z_t = (Z'_{t-1} Z_{t-1} + z_t z'_t) \qquad (B.16)$$

into equation (B.14) to get:

$$\Gamma_t = V_t \otimes (Z'_t Z_t)^{-1} \qquad (B.17)$$

The above equation, (B.17), can be used to obtain the coefficient covariance matrix, Γ_t, at any time period. For simple regression, the covariance between the i th and j th equations is $v_{ij} (Z'_t Z_t)^{-1}$ while, for restricted regression, it is $v_{ij} R_i (Z'_t Z_t)^{-1} R_j'$.

APPENDIX C

THE ABEL MODEL

The Abel model is a two endogenous, two control variable model of the U.S. economy. It is quarterly and covers the period from about the end of the Korean war (1954-I) to the beginning of heavy involvement in the Vietnam war (1963-IV). The theory for the model is based on the concept of a private consumption accelerator developed by Paul A. Samuelson[1] and takes the following form:

$$C_t = \alpha_1 C_{t-1} + \alpha_2 I_{t-1} + \alpha_3 G_t + \alpha_4 M_t + \alpha_5 \quad \text{(C.1)}$$

$$I_t = \beta_1 C_{t-1} + \beta_2 I_{t-1} + \beta_3 G_t + \beta_4 M_t + \beta_5 \quad \text{(C.2)}$$

where C_t is personal consumption expenditures in billions of 1958 dollars, I_t is gross private investment in billions of 1958 dollars, G_t is government purchases of goods and services in billions of 1958 dollars, M_t is money stock defined as currency plus demand deposits (M1), and GDC is a fixed weight price index for personal consumption expenditure. The data which was used to develop the model is shown in Table C.1 and was taken from a study by Kendrick.[2]

---

[1] Paul A. Samuelson, "Interactions Between the Multiplier Analysis and the Principle of Acceleration," Review of Economic Statistics, Vol. 21 (May 1939), pp. 75-78.

[2] Kendrick, "Caution and Probing in a Macroeconomic Model," p. 68.

TABLE C.1
DATA USED IN DEVELOPING ABEL MODEL

	Consumption	Investment	Government Expenditure	Money Stock	Price Index
1954-I	250.8	56.3	94.1	129.1	92.6
1954-II	253.3	57.0	88.8	129.4	92.6
1954-III	256.9	59.8	87.2	130.6	92.4
1954-IV	261.9	64.3	85.4	131.9	92.3
1955-I	267.6	70.8	85.5	133.5	92.6
1955-II	273.0	75.5	84.2	134.3	92.6
1955-III	276.3	76.9	85.8	134.9	92.9
1955-IV	279.9	78.5	85.1	135.1	93.0
1956-I	279.8	75.5	85.2	135.5	93.6
1956-II	280.3	74.5	85.8	135.9	94.3
1956-III	280.8	74.0	84.3	135.9	95.3
1956-IV	284.7	73.3	85.7	136.6	95.8
1957-I	286.6	70.5	89.0	136.8	96.7
1957-II	287.0	69.9	89.4	136.9	97.3
1957-III	289.3	70.9	89.1	136.9	98.1
1957-IV	289.7	64.0	89.9	136.2	98.5
1958-I	285.6	57.5	91.8	136.0	99.6
1958-II	287.5	56.0	93.6	137.6	100.0
1958-III	291.9	61.6	94.8	139.0	100.1
1958-IV	295.2	68.5	96.5	140.7	100.3
1959-I	302.3	70.9	95.5	142.6	100.6
1959-II	307.0	78.5	95.1	143.8	100.9
1959-III	309.9	70.2	94.3	144.5	101.6
1959-IV	310.0	75.0	94.2	143.6	102.0
1960-I	313.8	79.9	93.9	143.0	102.3
1960-II	317.7	73.5	94.7	142.7	102.7
1960-III	316.4	71.0	95.4	143.9	103.0
1960-IV	316.4	65.2	95.9	144.2	103.6
1961-I	316.2	62.4	97.6	144.8	103.8
1961-II	320.4	67.8	99.5	146.0	103.7
1961-III	323.9	71.2	102.0	146.8	104.0
1961-IV	329.5	74.7	102.9	148.3	104.2
1962-I	333.3	77.2	105.5	149.1	104.5
1962-II	335.7	79.0	107.8	149.8	104.7
1962-III	340.1	80.6	107.8	149.5	105.0
1962-IV	344.6	80.7	108.5	150.4	105.3
1963-I	348.5	78.7	110.2	151.8	105.6
1963-II	350.9	80.6	108.7	153.3	106.0
1963-III	356.1	83.1	110.0	154.8	106.2
1963-IV	357.7	87.7	109.5	156.4	106.7

APPENDIX D

SIMPLE LQT SOLUTION FOR THE ABEL MODEL.

The Abel econometric model can be derived from the 39 sets of observations[1] in Table C.1, Appendix C, using the following regression formula:

$$\pi' = (Z'Z)^{-1} Z'Y \qquad (D.1)$$

giving the following reduced form (D.2):

$$C_t = \underset{(0.0522)}{0.9144}\, C_{t-1} - \underset{(0.0927)}{0.0173}\, I_{t-1} + \underset{(0.1470)}{0.3037}\, G_t + \underset{(0.1890)}{0.4270}\, M_t - \underset{(24.50)}{59.66}$$

$$I_t = \underset{(0.0780)}{0.0973}\, C_{t-1} + \underset{(0.1385)}{0.4244}\, I_{t-1} - \underset{(0.2196)}{0.1036}\, G_t + \underset{(0.2824)}{1.4589}\, M_t - \underset{(36.17)}{184.62}$$

It will be convenient later to consider three submatrices of the coefficients from (D.2) in the form $\pi = [\pi_A \; \pi_C \; \pi_E]$ where:

$$\pi_A = \begin{bmatrix} 0.9144 & -0.0173 & 0.0 & 0.0 \\ 0.0973 & 0.4244 & 0.0 & 0.0 \end{bmatrix}$$

$$\pi_C = \begin{bmatrix} 0.3037 & 0.4270 \\ -0.1036 & 1.4589 \end{bmatrix} \qquad \pi_E = \begin{bmatrix} -59.66 \\ -184.2 \end{bmatrix}$$

---

[1]Note that there are two endogenous variables for the same set of four exogenous variables. Consequently, Z will be a 39 by 4 matrix and Y will be a 39 by 2 matrix.

The coefficient matrices in the state equation (3.3) can also be defined:

$$y_t = A\, y_{t-1} + C\, x_t + b$$

$$A = \begin{bmatrix} 0.9144 & -0.0173 & 0.0 & 0.0 \\ 0.0973 & 0.4244 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.3037 & 0.4270 \\ -0.1036 & 1.4589 \\ 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \qquad b = \begin{bmatrix} -59.66 \\ -184.2 \\ 0.0 \\ 0.0 \end{bmatrix}$$

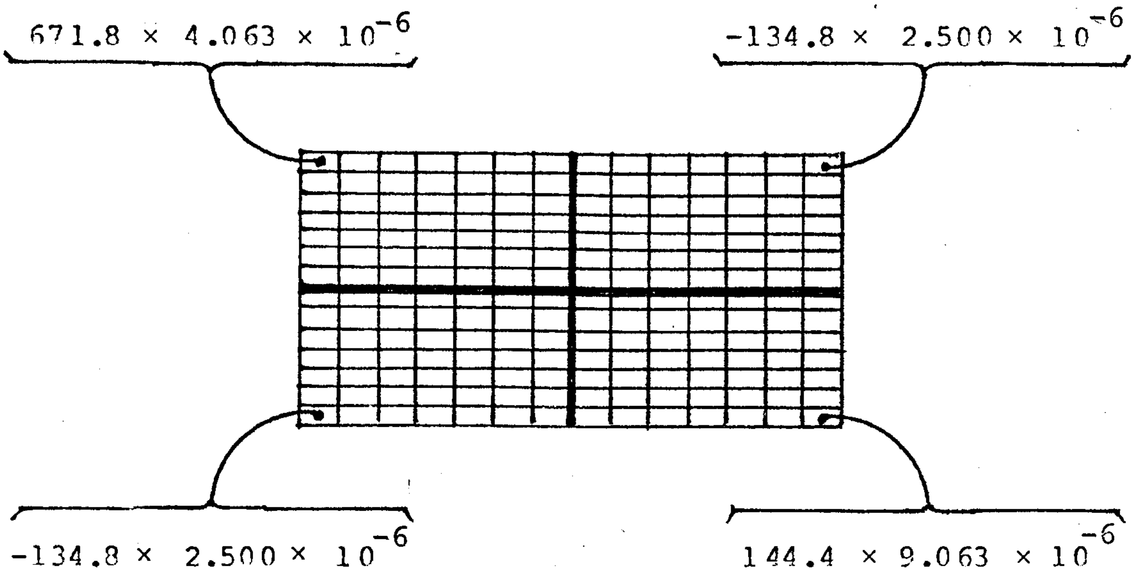
The inverse, $(Z'Z)^{-1}$, a by-product from the derivation of (D.2), is tabulated below:

$$\begin{bmatrix} 671.8 & -945.2 & 0.0 & 0.0 & -1811.1 & 1210.8 & -134.8 \\ -945.2 & 2115.2 & 0.0 & 0.0 & 2323.4 & -2963.0 & 333.7 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -1811.1 & 2323.4 & 0.0 & 0.0 & 5321.9 & -3090.3 & 312.9 \\ 1210.8 & -2963.0 & 0.0 & 0.0 & -3090.3 & 8800.0 & -1108.5 \\ -134.8 & 333.7 & 0.0 & 0.0 & 312.9 & -1108.5 & 144.4 \end{bmatrix}$$

An estimate of the error term covariance matrix, $\bar{V}$, using (B.4) and (B.5) from Appendix B, again a by-product of the regression analysis, is:

$$\bar{V} = \begin{bmatrix} 4.063 & 2.500 \\ 2.500 & 9.063 \end{bmatrix} \times 10^{-6}$$

It is now possible to derive the full 14 by 14 coefficient covariance matrix[2] by applying (B.17) from Appendix B, i.e., that $\Gamma = \bar{V} \otimes (Z'Z)^{-1}$ to get:



At each stage in the stochastic control analysis, the next set of values, H_{t-1}, h_{t-1} and c_{t-1}, are obtained from the current set

[2]For clarity, only the corner terms have actually been derived. The objective here is to show how (B.17) can be applied.

of values, H_t, h_t, c_t, G_t and g_t. The Riccati equations (3.20) are used for this purpose and they require, amongst other things, the evaluation of several triple matrix product expectations, e.g., E(A'HA). These matrix products can be easily derived from the following matrix product.[3]

$$P = \pi' H_t^{11} \pi + [\,\mathrm{Trace}\,(H_t^{11}\bar{V})\,]\,(Z_t' Z_t) \qquad \text{(D.3)}$$

where H_t^{11} is a submatrix of the current H_t matrix:

$$H_t = \begin{bmatrix} H^{11} & H^{12} \\ H^{21} & H^{22} \end{bmatrix}$$

For example, the submatrix H_2^{11} for the second time period, uncertain parameters quadratic matrix, H_2, is:

$$H_2^{11} = \begin{bmatrix} 1.4358 & -0.1415 \\ -0.1415 & 1.0893 \end{bmatrix}$$

---

[3]The equation (D.3), as it stands, would be used in the development of the uncertain parameters solution. By dropping the last term, it can also be used for the certainty equivalence solution.

The triple matrix product expections can now be readily derived. E(A'HA) is simply P^{AA} and, for example,

$$E\ (C'\ H\ A) = P^{CA} + H^{21}\ \pi_A \qquad (D.4)$$

A summary of the control analysis for the last two stages (or first two time periods) is shown in Table D.1. For simplicity, only the derivation of E(A'HA) is shown but the other terms E(C'HA), E(C'HC), E(A'Hb) or E(C'Hb) are obtained just as easily.

TABLE D.1

SIMPLE LQT SOLUTIONS
FOR THE ABEL MODEL

	Certainty Equivalence				Uncertain Parameters			
H_2	1	0	0	0	1.4358	-.1415	0	0
	0	1	0	0	-.1415	1.0893	0	0
	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
$E(A'H_2A)$	.8457	.0255	0	0	1.196	-.0469	0	0
	.0255	.1805	0	0	-.0469	.2311	0	0
	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
H_1	1	0	0	0	1.1362	-.1417	0	0
	0	1	0	0	-.1417	1.0893	0	0
	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0
G_1	2.6529	.4238	- 0	0	-1.6690	.0825	0	0
	.25508	-.2608	- 0	0	- .2664	-.2323	0	0
g_1				1.0236				.7015
				.2601				.2616
y_1	.3622	.0888	.1141	.1476	.3621	.0886	.1140	.1474

APPENDIX E

THE LAIDLER MODEL

The Basic Model

The Laidler model[1] is in the tradition of open-economy monetariasm and was designed to cope with data generated under fixed exchange rates. Thus the interaction of the balance of payments with the supply and demand for money, and the interaction of actual and expected inflation rates with fluctuations in real income, are its key ingredients. The model is in log-linear form and may be set out as follows:

$$y = \alpha_1 (m_s - m_d)_{-1} + \alpha_2 (\pi + e + a - p)_{-1} + \alpha_3 t_{-1} + \alpha_4 g \qquad (E.1)$$

$$\Delta r = \gamma_1 (m_s - m_d)_{-1} + \gamma_2 (\pi + e + a - p)_{-1} \qquad (E.2)$$

$$m_d = \delta_o + \delta_1 \tilde{y} + p \qquad (E.3)$$

$$\Delta_p = \beta y_{-1} + (p^e - p)_{-1} \qquad (E.4)$$

$$(p^e - p) = \varepsilon_1 \Delta\pi + \varepsilon_2 (\pi + e + a - p) \qquad (E.5)$$

$$\Delta m_s = (1 - \mu) \Delta c + \mu \Delta r \qquad (E.6)$$

$$\tilde{y} = \theta_o + \theta_1 \tau \qquad (E.7)$$

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[1] In part, this material was taken directly from: Laidler et al., "A Small Macroeconomic Model of an Open Economy".

The variables used above are all, with the exception of (time), natural logarithms and are defined as follows: y the transitory component of the log of real income; m_s the log of the nominal money supply; m_d the log of the long-run quantity of nominal money demanded; π the log of the price level ruling in the world economy; e the log of the exchange rate or price of foreign currency; p the log of domestic price level; p^e the value that the log of the domestic price level is expected to take next period; t the deviation of the log of the economy's average tax rate from trend; g the deviation of the log of government expenditure from trend; r the log of foreign exchange reserves; $\tilde{y}$ the permanent component of the log of income; c the log of domestic credit extended by the consolidated banking system; and 'a' is a constant to be described below.

Equation (E.1) may be regarded as a reduced form of an income expenditure system that tells us that deviations of output from its 'permanent' level (or natural level, the concepts are interchangeable in this model) depend upon the size of any difference between the supply of money on the one hand and the long-run demand for money on the other, the deviation of domestic prices from some equilibrium value relative to world prices and the exchange rates (with units of measurement chosen so that the equilibrium value of that relative price term is zero) and the deviation of the tax rate and government expenditure from trend. In fact, equation (E.1) is a log-linear approximation to the following, more conveniently expressed relationship, where capital letters are used to indicate the natural values of the relevant variables, with C being real consumption, I investment and X net exports. The approximation in question is obtained by linearising the logarithmic form of the relationship about the steady state values of the logarithms of the variables following a procedure set out in Wymer.[2]

$$Y = C + I + X + G = k \left[\left(\frac{M_s}{M_d}\right)^{\alpha_1'} \frac{\pi}{P} E^{\alpha_2'} T^{\alpha_3'} \right] \tilde{Y} + G \quad (E.8)$$

The parameters of equation (E.1) bear the following relationships to those of equation (E.8):

$$\alpha_1 = k\alpha_1' \; ; \; \alpha_2 = k\alpha_2' \; ; \; \alpha_3 = k\alpha_3' \; ; \; \alpha_4 = (1-k)$$

It is worth noting two points explicitly here. First, to enter fiscal period variables as deviations from trend while making permanent, or steady state, income solely a function of time imposes long-run crowding out of fiscal policy on the behaviour of the model, while leaving short-run crowding out open as an empirical matter. Second, the relative price level includes a constant 'a': in empirical work, prices and the exchange rate are measured as index numbers with a base of 1970 (i.e., the 1970 index is one) so their logs are zero in that year, and the constant 'a' is thus a measure of the proportion

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[2] C. R. Wymer, "Linearization of Non-Linear Systems", Supplement #15, Computer Programmer, London School of Economics, 1976, Mimeo.

by which the home currency, in this case the Canadian dollar, was undervalued in 1970.

Equation (E.2) is an equation that determines the balance of official settlements concept of the balance of payments and makes it depend upon the excess supply of money, as well as upon relative price levels and the exchange rate.

Equation (E.3) tells us that the long-run demand for real money balances depends upon permanent income, and omits an opportunity cost argument. This last omission might disturb some readers, but the following points should be noted. First, and most important, because this is a model in which the economy is allowed to be off its long-run demand for money function, to leave an interest rate out of this relationship does not, as it would in a conventional IS-LM framework, ensure that only money matters as far as the determination of real income and prices is concerned. Second, some preliminary work carried out using ordinary least squares on equation (E.1) suggests that to add an interest rate to the demand for money function makes no important difference to the results obtained for other parameters of the system, a conclusion that receives further support from the work of Laidler and Bentley[3] (1981) on United States data. Third, and of considerable importance, the omission of interest rates from the model enables us to keep the structure simple. Were such a variable included, the model would

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[3] Laidler and Bentley, "A Small Macromodel of the Post-War United States".

have to be extended in order to cope with its determination, and that would be no easy matter. Finally, omitting the interest rate from the model does not imply that the variable is unimportant in the transmission mechanism of monetary policy. The model may be interpreted as ignoring interest rate variations, not because they do not matter, but because they are an intermediate step in that mechanism.

Equation (E.4) is a conventional expectation augmented Philips curve while equation (E.5) tells us that inflation expectations depend both upon the rate of inflation ruling in the world economy, and on the deviation of the domestic price level from its long-run equilibrium level. Equation (E.6) is a standard log-linear approximation to the money supply identity while equation (E.7) defines the permanent or natural component of the logarithm of income as a simple time trend.

<u>The Data Used in Developing the Laidler Model</u>

The output variable upon which y and $\tilde{y}$ are based is real gross domestic product. The price variable is the consumer price index, chosen because it is the most readily comparable with the data upon which the world price level variable is based. The latter variable is a GNP weighted geometric average of the consumer or retail price indices of sixteen other countries, while the exchange rate, defined as the domestic currency price of foreign currency, is a similarly weighted index of the individual exchange rates on the same sixteen countries.

The government expenditure and tax variables appertain to federal, provincial and local government activities. As with the output series, they represent deviations of the logs of the variables in question from trends fitted by ordinary least squares to the original data.

The 'reserve' series is constructed by starting in a bench mark year and then adding, to the value of reserves observed in that year, the current Canadian dollar value of the balance of official settlements concept of the balançe of payments observed in the next and each successive year. This practice ensures that the proportional change in reserves variables on the left hand side of the equation (E.2) is appropriately measured. The money supply identity, to which equation (E.6) is a log-linear approximation, is then preserved by subtracting the reserves variable from the money supply (M3) to generate a series for domestic credit. The latter variable is treated as exogenous, and all changes in foreign exchange reserves arising from exchange rate changes that were at any time permitted to influence the money supply rather than the net worth of the banking system are hence included in it. They are thus treated as exogenous. The actual data used in developing the Laidler model is shown in Tables E.1 and E.2.

TABLE E.1

DATA USED IN DEVELOPING THE LAIDLER MODEL

	Real GDP $ Billion	Consumer Prices 1970 = 100	World Prices 1970 = 100	Exchange Rate $/FCU 1975 = 100
1952	31.84	69.5	64.5	96.8
1953	33.46	68.9	65.0	97.1
1954	33.57	69.3	65.5	96.2
1955	36.93	69.4	65.7	97.3
1956	40.65	70.5	67.0	97.1
1957	41.38	72.7	68.7	94.3
1958	41.63	74.7	71.0	94.7
1959	43.48	75.5	71.9	92.5
1960	44.75	76.4	73.2	93.4
1961	45.84	77.2	74.2	97.9
1962	49.14	78.1	76.0	103.5
1963	51.83	79.4	77.9	104.6
1964	55.25	80.9	79.6	104.6
1965	59.05	82.8	81.8	104.6
1966	63.75	85.9	84.6	104.6
1967	66.06	89.0	87.0	104.5
1968	69.30	92.6	90.4	103.3
1969	73.12	96.8	94.7	103.2
1970	75.43	100.0	100.0	100.0
1971	80.53	102.8	104.9	97.5
1972	86.01	107.8	109.5	99.7
1973	94.76	115.9	117.7	106.2
1974	102.85	128.6	132.5	102.3
1975	105.87	142.5	145.8	107.6

TABLE E.2

DATA USED IN DEVELOPING THE LAIDLER MODEL
(CONTINUED)

	Money Supply $ Cdn. Bill.	Reserves $ Cdn. Bill.	Domestic Credit $ Cdn. Bill.	Real Govt. Exp. $ Cdn. Bill.	Taxes as a Proportion of GDP
1952	9.26			5.21	.176
1953	9.32	1.75	7.57	5.55	.174
1954	10.14	1.88	8.26	5.52	.161
1955	10.88	1.38	9.50	5.82	.164
1956	11.19	1.88	9.31	6.28	.172
1957	11.50	1.77	9.73	6.29	.166
1958	12.93	1.89	11.04	6.50	.137
1959	12.79	1.87	10.92	6.59	.149
1960	13.40	1.47	11.57	6.91	.146
1961	14.58	2.13	12.45	7.24	.164
1962	15.12	2.28	12.84	7.59	.168
1963	16.15	2.42	13.73	7.81	.166
1964	17.35	2.78	14.57	8.29	.179
1965	19.34	2.94	16.40	8.88	.187
1966	20.58	2.59	17.99	9.96	.199
1967	23.68	2.60	21.08	10.96	.202
1968	27.19	2.96	24.23	11.92	.209
1969	27.91	3.02	24.89	12.75	.224
1970	31.16	4.56	26.60	14.46	.221
1971	36.35	5.33	31.02	15.49	.214
1972	41.42	5.55	35.87	16.31	.207
1973	49.03	5.08	43.95	17.22	.205
1974	57.44	5.10	52.34	18.68	.212
1975	67.38	4.70	62.68	20.16	.172

APPENDIX F

MULTIVARIATE NORMAL COMPUTER SIMULATION

In stochastic control simulation, it is often required to generate a vector of typical values for a multivariate normal distribution with a mean vector, μ, and a variance-covariance matrix, Ω, of order n.

The method is extremely simple and begins by factoring[1] the variance-covariance matrix into a lower triangular matrix, P, such that:

$$\Omega = P P' \tag{E.1}$$

Using conventional Monte Carlo sampling, a vector, z, can be created where each of its n elements is a random drawing from an independent standard normal distribution. Now define another vector, x, to be:

$$x = P z + \mu \tag{E.2}$$

Clearly the vector x will have a multivariate normal distribution with mean μ and variance $E(P z z' P')$ or $E(P P')$ which, by definition, is the required variance-covariance matrix, Ω.

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[1] The Choleski factorization method is ideal for this purpose and is described in:
Richard L. Burden, et al., Numerical Analysis, (Boston, Mass: Prindle, Weber and Schmidt, 1981), p. 309.

APPENDIX G

COMPUTER RUNS FOR ADAPTIVE CONTROL SIMULATION

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

USING THE KENDRICK MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 1979.638

WITH STANDARD DEVIATION 11070.592

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	0.063		1.000		1.000		0.444	
1	0.374	0.39	0.074	0.09	0.119	0.11	0.145	0.15
2	0.382	0.39	0.084	0.09	0.125	0.11	0.151	0.15
3	0.390	0.40	0.070	0.09	0.128	0.11	0.149	0.15
4	0.432	0.40	0.108	0.09	0.129	0.11	0.153	0.15
5	0.475	0.40	0.126	0.09	0.099	0.11	0.130	0.15
6	0.544	0.41	0.267	0.09	-0.027	0.12	0.090	0.15
7	0.541	0.41	0.115	0.09	0.116	0.12	0.094	0.16

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

USING THE KENDRICK MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 4604.299

WITH STANDARD DEVIATION 19910.291

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	0.063		1.000		1.000		0.444	
1	0.373	0.39	0.073	0.09	0.115	0.11	0.144	0.15
2	0.383	0.39	0.083	0.09	0.128	0.11	0.151	0.15
3	0.391	0.40	0.067	0.09	0.129	0.11	0.146	0.15
4	0.430	0.40	0.091	0.09	0.120	0.11	0.142	0.15
5	0.524	0.40	0.137	0.09	0.050	0.11	0.125	0.15
6	0.705	0.41	0.246	0.09	-0.063	0.12	0.093	0.15
7	0.709	0.41	0.159	0.09	0.114	0.12	0.090	0.16

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE KENDRICK MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 157.593

WITH STANDARD DEVIATION 253.536

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	0.063		1.000		1.000		0.444	
1	0.295	0.39	-0.157	0.09	0.319	0.11	-0.072	0.15
2	1.447	0.39	0.318	0.09	2.879	0.11	1.080	0.15
3	1.051	0.40	0.004	0.09	-0.327	0.11	-0.295	0.15
4	0.839	0.40	0.072	0.09	-0.373	0.11	-0.034	0.15
5	0.637	0.40	0.113	0.09	-0.423	0.11	-0.001	0.15
6	0.474	0.41	0.225	0.09	-0.565	0.12	0.121	0.15
7	0.469	0.41	0.196	0.09	0.117	0.12	0.148	0.16

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 2681717.436

WITH STANDARD DEVIATION 16843126.110

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.036	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.175	4.30	3.968	4.20	-0.339	0.0	0.162	0.0
2	-0.412	0.02	1.643	1.66	4.353	4.68	5.315	5.06	4.398	4.38	4.171	4.27	-0.181	0.0	0.087	0.0
3	-0.558	0.03	2.620	1.72	4.186	4.69	5.239	5.12	4.547	4.46	4.313	4.34	-0.053	0.0	0.026	0.0
4	-0.474	0.04	2.993	1.78	4.139	4.69	5.159	5.18	4.613	4.54	4.387	4.41	-0.001	0.0	0.000	0.0
5	-0.268	0.05	3.601	1.84	4.253	4.70	5.108	5.24	4.696	4.62	4.456	4.48	-0.008	0.0	0.004	0.0
6	-0.048	0.06	4.131	1.90	4.374	4.70	5.092	5.30	4.855	4.70	4.512	4.55	-0.068	0.0	0.032	0.0
7	0.136	0.07	3.904	1.96	4.534	4.70	5.189	5.36	4.981	4.78	4.584	4.62	-0.102	0.0	0.049	0.0
8	0.057	0.08	4.318	2.02	4.277	4.71	5.327	5.42	5.205	4.86	4.723	4.69	-0.045	0.0	0.021	0.0
9	-0.220	0.09	2.712	2.18	4.258	4.71	5.496	5.48	5.146	4.94	4.824	4.76	0.110	0.0	-0.052	0.0
10	-0.598	0.10	1.259	2.24	4.262	4.72	5.539	5.54	4.992	5.02	4.790	4.83	0.163	0.0	-0.078	0.0

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 40312278.615

WITH STANDARD DEVIATION 265822044.099

TERMINAL FACTOR - 10. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.016	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.214	4.30	4.040	4.20	-0.258	0.0	0.123	0.0
2	-0.370	0.02	1.754	1.66	4.351	4.68	5.320	5.06	4.423	4.38	4.222	4.27	-0.112	0.0	0.053	0.0
3	-0.539	0.03	2.775	1.72	4.174	4.69	5.256	5.12	4.521	4.46	4.295	4.34	-0.050	0.0	0.024	0.0
4	-0.528	0.04	3.040	1.78	4.131	4.69	5.180	5.18	4.544	4.54	4.306	4.41	-0.061	0.0	0.029	0.0
5	-0.362	0.05	3.458	1.84	4.241	4.70	5.098	5.24	4.582	4.62	4.330	4.48	-0.129	0.0	0.062	0.0
6	-0.134	0.06	3.775	1.90	4.363	4.70	5.034	5.30	4.681	4.70	4.342	4.55	-0.273	0.0	0.131	0.0
7	0.129	0.07	3.248	1.96	4.519	4.70	5.081	5.36	4.756	4.78	4.390	4.62	-0.436	0.0	0.209	0.0
8	0.073	0.08	3.546	2.02	4.347	4.71	5.215	5.42	5.049	4.86	4.672	4.69	-0.474	0.0	0.227	0.0
9	-0.191	0.09	1.777	2.18	4.372	4.71	5.407	5.48	5.323	4.94	5.317	4.76	0.198	0.0	-0.095	0.0
10	-0.422	0.10	-0.296	2.24	4.201	4.72	5.494	5.54	5.086	5.02	4.839	4.83	0.108	0.0	-0.052	0.0

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST 725556.554

WITH STANDARD DEVIATION 4765233.389

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.200

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.007	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.227	4.30	4.063	4.20	-0.219	0.0	0.105	0.0
2	-0.355	0.02	1.789	1.66	4.350	4.68	5.323	5.06	4.421	4.38	4.224	4.27	-0.099	0.0	0.047	0.0
3	-0.530	0.03	2.809	1.72	4.169	4.69	5.263	5.12	4.550	4.46	4.341	4.34	0.013	0.0	-0.006	0.0
4	-0.503	0.04	3.185	1.78	4.118	4.69	5.189	5.18	4.619	4.54	4.420	4.41	0.059	0.0	-0.028	0.0
5	-0.313	0.05	3.724	1.84	4.219	4.70	5.118	5.24	4.702	4.62	4.493	4.48	0.058	0.0	-0.027	0.0
6	-0.073	0.06	4.192	1.90	4.326	4.70	5.077	5.30	4.854	4.70	4.552	4.55	0.008	0.0	-0.004	0.0
7	0.135	0.07	3.859	1.96	4.485	4.70	5.156	5.36	4.936	4.78	4.613	4.62	-0.034	0.0	0.016	0.0
8	0.057	0.08	4.251	2.02	4.258	4.71	5.286	5.42	5.123	4.86	4.729	4.69	-0.006	0.0	0.003	0.0
9	-0.233	0.09	2.466	2.18	4.312	4.71	5.454	5.48	5.004	4.94	4.818	4.76	0.107	0.0	-0.051	0.0
10	-0.605	0.10	1.208	2.24	4.386	4.72	5.494	5.54	4.872	5.02	4.823	4.83	0.160	0.0	-0.077	0.0

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 1000555.354

WITH STANDARD DEVIATION 5305561.073

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.065	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.098	4.30	3.827	4.20	-0.453	0.0	0.217	0.0
2	-0.491	0.02	1.427	1.66	4.357	4.68	5.308	5.06	4.306	4.38	4.009	4.27	-0.342	0.0	0.163	0.0
3	-0.640	0.03	2.281	1.72	4.203	4.69	5.210	5.12	4.497	4.46	4.211	4.34	-0.195	0.0	0.093	0.0
4	-0.471	0.04	2.685	1.78	4.173	4.69	5.103	5.18	4.656	4.54	4.410	4.41	-0.047	0.0	0.023	0.0
5	-0.142	0.05	3.501	1.84	4.308	4.70	5.059	5.24	4.858	4.62	4.620	4.48	0.093	0.0	-0.045	0.0
6	0.191	0.06	4.417	1.90	4.477	4.70	5.102	5.30	5.112	4.70	4.742	4.55	0.178	0.0	-0.085	0.0
7	0.424	0.07	4.996	1.96	4.696	4.70	5.298	5.36	5.284	4.78	4.687	4.62	0.190	0.0	-0.091	0.0
8	0.313	0.08	6.011	2.02	4.363	4.71	5.552	5.42	5.387	4.86	4.458	4.69	0.144	0.0	-0.069	0.0
9	-0.319	0.09	4.287	2.18	4.135	4.71	5.811	5.48	5.282	4.94	4.193	4.76	0.138	0.0	-0.066	0.0
10	-0.971	0.10	3.036	22.40	3.952	4.72	5.797	5.54	21.142	50.20	26.191	4.83	0.257	0.0	-0.123	0.0

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 597126.061

WITH STANDARD DEVIATION 2936271.302

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.131	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.028	4.30	3.698	4.20	-0.735	0.0	0.327	0.0
2	-0.535	0.02	1.230	1.66	4.361	4.68	5.290	5.06	4.281	4.38	3.960	4.27	-0.312	0.0	0.141	0.0
3	-0.601	0.03	1.997	1.72	4.222	4.69	5.175	5.12	4.419	4.46	4.105	4.34	0.007	0.0	-0.001	0.0
4	-0.445	0.04	2.194	1.78	4.219	4.69	5.083	5.18	4.475	4.54	4.154	4.41	0.109	0.0	-0.055	0.0
5	-0.218	0.05	2.821	1.84	4.359	4.70	5.042	5.24	4.553	4.62	4.215	4.48	0.041	0.0	-0.028	0.0
6	-0.034	0.06	3.446	1.90	4.500	4.70	5.047	5.30	4.742	4.70	4.302	4.55	-0.092	0.0	0.036	0.0
7	0.116	0.07	3.399	1.96	4.665	4.70	5.157	5.36	4.913	4.78	4.409	4.62	-0.132	0.0	0.059	0.0
8	0.065	0.08	3.772	2.02	4.387	4.71	5.308	5.42	5.149	4.86	4.546	4.69	-0.043	0.0	0.020	0.0
9	-0.195	0.09	2.157	2.18	4.309	4.71	5.507	5.48	5.126	4.94	4.651	4.76	0.138	0.0	-0.066	0.0
10	-0.585	0.10	0.064	2.24	4.221	4.72	5.588	5.54	5.043	5.02	4.771	4.83	0.199	0.0	-0.095	0.0

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 4881208.163

WITH STANDARD DEVIATION 24030976.237

TERMINAL FACTOR - 10. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.146	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.006	4.30	3.659	4.20	-0.802	0.0	0.346	0.0
2	-0.546	0.02	1.169	1.66	4.361	4.68	5.286	5.06	4.297	4.38	3.985	4.27	-0.308	0.0	0.135	0.0
3	-0.574	0.03	1.902	1.72	4.222	4.69	5.167	5.12	4.424	4.46	4.137	4.34	0.055	0.0	-0.020	0.0
4	-0.421	0.04	2.060	1.78	4.224	4.69	5.084	5.18	4.412	4.54	4.098	4.41	0.147	0.0	-0.076	0.0
5	-0.262	0.05	2.593	1.84	4.354	4.70	5.050	5.24	4.419	4.62	4.055	4.48	0.005	0.0	-0.024	0.0
6	-0.165	0.06	3.063	1.90	4.474	4.70	5.038	5.30	4.545	4.70	4.047	4.55	-0.238	0.0	0.080	0.0
7	-0.064	0.07	2.634	1.96	4.577	4.70	5.095	5.36	4.699	4.78	4.165	4.62	-0.401	0.0	0.158	0.0
8	-0.104	0.08	2.969	2.02	4.350	4.71	5.163	5.42	4.955	4.86	4.395	4.69	-0.345	0.0	0.152	0.0
9	-0.184	0.09	1.032	2.18	4.397	4.71	5.322	5.48	4.933	4.94	4.541	4.76	0.283	0.0	-0.136	0.0
10	-0.468	0.10	-1.025	2.24	4.411	4.72	5.434	5.54	5.014	5.02	4.806	4.83	0.145	0.0	-0.070	0.0

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST 227126.132

WITH STANDARD DEVIATION 1290507.767

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.200

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.070	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.116	4.30	3.860	4.20	-0.479	0.0	0.220	0.0
2	-0.459	0.02	1.477	1.66	4.356	4.68	5.306	5.06	4.320	4.38	4.035	4.27	-0.210	0.0	0.099	0.0
3	-0.591	0.03	2.355	1.72	4.203	4.69	5.216	5.12	4.459	4.46	4.152	4.34	0.025	0.0	-0.010	0.0
4	-0.486	0.04	2.643	1.78	4.176	4.69	5.127	5.18	4.535	4.54	4.218	4.41	0.117	0.0	-0.057	0.0
5	-0.257	0.05	3.259	1.84	4.304	4.70	5.070	5.24	4.617	4.62	4.287	4.48	0.090	0.0	-0.047	0.0
6	-0.037	0.06	3.814	1.90	4.439	4.70	5.057	5.30	4.794	4.70	4.366	4.55	-0.007	0.0	-0.001	0.0
7	0.138	0.07	3.673	1.96	4.609	4.70	5.160	5.36	4.940	4.78	4.451	4.62	-0.056	0.0	0.024	0.0
8	0.077	0.08	4.072	2.02	4.344	4.71	5.310	5.42	5.164	4.86	4.567	4.69	-0.005	0.0	0.002	0.0
9	-0.196	0.09	2.510	2.18	4.296	4.71	5.499	5.48	5.148	4.94	4.658	4.76	0.140	0.0	-0.067	0.0
10	-0.570	0.10	0.712	2.24	4.239	4.72	5.563	5.54	5.109	5.02	4.768	4.83	0.196	0.0	-0.094	0.0

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 485156.709

WITH STANDARD DEVIATION 2410784.613

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.142	0.01	0.776	1.60	4.582	4.68	5.116	5.00	3.997	4.30	3.643	4.20	-0.778	0.0	0.346	0.0
2	-0.565	0.02	1.144	1.66	4.362	4.68	5.287	5.06	4.237	4.38	3.887	4.27	-0.381	0.0	0.173	0.0
3	-0.641	0.03	1.874	1.72	4.231	4.69	5.163	5.12	4.391	4.46	4.052	4.34	-0.065	0.0	0.031	0.0
4	-0.456	0.04	2.063	1.78	4.239	4.69	5.056	5.18	4.488	4.54	4.164	4.41	0.078	0.0	-0.041	0.0
5	-0.161	0.05	2.751	1.84	4.394	4.70	5.010	5.24	4.629	4.62	4.316	4.48	0.104	0.0	-0.056	0.0
6	0.106	0.06	3.531	1.90	4.563	4.70	5.040	5.30	4.860	4.70	4.462	4.55	0.093	0.0	-0.049	0.0
7	0.308	0.07	3.952	1.96	4.779	4.70	5.210	5.36	5.041	4.78	4.508	4.62	0.132	0.0	-0.063	0.0
8	0.234	0.08	4.806	2.02	4.465	4.71	5.439	5.42	5.196	4.86	4.389	4.69	0.164	0.0	-0.076	0.0
9	-0.274	0.09	3.106	2.18	4.283	4.71	5.701	5.48	5.199	4.94	4.270	4.76	0.159	0.0	-0.075	0.0
10	-0.770	0.10	1.654	22.40	4.086	4.72	5.734	5.54	20.693	50.20	26.041	4.83	0.258	0.0	-0.122	0.0

HEURISTIC STOCHASTIC CONTROL MODEL

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 1138923.538

WITH STANDARD DEVIATION 6264002.278

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	TRANCCME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.086	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.108	4.30	3.845	4.20	-0.544	0.0	0.248	0.0
2	-0.486	0.02	1.456	1.66	4.357	4.68	5.302	5.06	4.299	4.38	3.999	4.27	-0.312	0.0	0.153	0.0
3	-0.626	0.03	2.314	1.72	4.210	4.69	5.205	5.12	4.456	4.46	4.142	4.34	-0.089	0.0	0.048	0.0
4	-0.498	0.04	2.626	1.78	4.187	4.69	5.103	5.18	4.550	4.54	4.232	4.41	-0.007	0.0	0.006	0.0
5	-0.244	0.05	3.272	1.84	4.325	4.70	5.045	5.24	4.635	4.62	4.295	4.48	-0.032	0.0	0.015	0.0
6	-0.010	0.06	3.845	1.90	4.470	4.70	5.041	5.30	4.815	4.70	4.361	4.55	-0.123	0.0	0.057	0.0
7	0.174	0.07	3.780	1.96	4.649	4.70	5.157	5.36	4.991	4.78	4.433	4.62	-0.152	0.0	0.072	0.0
8	0.131	0.08	4.210	2.02	4.375	4.71	5.324	5.42	5.268	4.86	4.566	4.69	-0.078	0.0	0.038	0.0
9	-0.131	0.09	2.897	2.18	4.281	4.71	5.536	5.48	5.342	4.94	4.732	4.76	0.094	0.0	-0.044	0.0
10	-0.501	0.10	0.887	2.24	4.143	4.72	5.629	5.54	5.243	5.02	4.732	4.83	0.154	0.0	-0.073	0.0

HEURISTIC STOCHASTIC CONTROL MODEL

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 12992321.616

WITH STANDARD DEVIATION 75927452.091

TERMINAL FACTOR - 10. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.083	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.123	4.30	3.873	4.20	-0.534	0.0	0.241	0.0
2	-0.471	0.02	1.499	1.66	4.356	4.68	5.303	5.06	4.321	4.38	4.039	4.27	-0.276	0.0	0.137	0.0
3	-0.609	0.03	2.381	1.72	4.207	4.69	5.210	5.12	4.471	4.46	4.172	4.34	-0.072	0.0	0.042	0.0
4	-0.500	0.04	2.684	1.78	4.180	4.69	5.114	5.18	4.515	4.54	4.190	4.41	-0.040	0.0	0.022	0.0
5	-0.302	0.05	3.233	1.84	4.315	4.70	5.054	5.24	4.537	4.62	4.176	4.48	-0.173	0.0	0.079	0.0
6	-0.134	0.06	3.618	1.90	4.451	4.70	5.026	5.30	4.661	4.70	4.196	4.55	-0.425	0.0	0.194	0.0
7	0.052	0.07	3.143	1.96	4.595	4.70	5.088	5.36	4.783	4.78	4.257	4.62	-0.602	0.0	0.276	0.0
8	-0.003	0.08	3.398	2.02	4.371	4.71	5.205	5.42	5.032	4.86	4.424	4.69	-0.575	0.0	0.267	0.0
9	-0.201	0.09	1.752	2.18	4.346	4.71	5.384	5.48	5.106	4.94	4.635	4.76	0.212	0.0	-0.103	0.0
10	-0.504	0.10	-0.230	2.24	4.262	4.72	5.471	5.54	5.136	5.02	4.775	4.83	0.124	0.0	-0.060	0.0

HEURISTIC STOCHASTIC CONTROL MODEL

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST 372341.255

WITH STANDARD DEVIATION 2283328.993

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.200

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.037	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.166	4.30	3.952	4.20	-0.343	0.0	0.159	0.0
2	-0.415	0.02	1.618	1.66	4.354	4.68	5.315	5.06	4.335	4.38	4.072	4.27	-0.172	0.0	0.085	0.0
3	-0.594	0.03	2.531	1.72	4.189	4.69	5.239	5.12	4.474	4.46	4.185	4.34	-0.006	0.0	0.006	0.0
4	-0.531	0.04	2.858	1.78	4.154	4.69	5.145	5.18	4.576	4.54	4.293	4.41	0.048	0.0	-0.021	0.0
5	-0.294	0.05	3.476	1.84	4.275	4.70	5.069	5.24	4.668	4.62	4.370	4.48	0.041	0.0	-0.020	0.0
6	-0.030	0.06	4.019	1.90	4.405	4.70	5.039	5.30	4.841	4.70	4.435	4.55	-0.025	0.0	0.012	0.0
7	0.182	0.07	3.848	1.96	4.581	4.70	5.139	5.36	4.985	4.78	4.498	4.62	-0.069	0.0	0.033	0.0
8	0.134	0.08	4.297	2.02	4.326	4.71	5.298	5.42	5.221	4.86	4.609	4.69	-0.035	0.0	0.018	0.0
9	-0.156	0.09	2.839	2.18	4.288	4.71	5.503	5.48	5.230	4.94	4.745	4.76	0.095	0.0	-0.044	0.0
10	-0.541	0.10	1.110	2.24	4.218	4.72	5.582	5.54	5.127	5.02	4.762	4.83	0.153	0.0	-0.072	0.0

HEURISTIC STOCHASTIC CONTROL MODEL

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 739347.875

WITH STANDARD DEVIATION 3714661.903

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.103	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.067	4.30	3.770	4.20	-0.616	0.0	0.281	0.0
2	-0.537	0.02	1.340	1.66	4.359	4.68	5.297	5.06	4.246	4.38	3.909	4.27	-0.446	0.0	0.216	0.0
3	-0.680	0.03	2.147	1.72	4.221	4.69	5.185	5.12	4.423	4.46	4.076	4.34	-0.210	0.0	0.105	0.0
4	-0.500	0.04	2.466	1.78	4.213	4.69	5.065	5.18	4.567	4.54	4.235	4.41	-0.044	0.0	0.023	0.0
5	-0.159	0.05	3.189	1.84	4.368	4.70	5.008	5.24	4.723	4.62	4.392	4.48	0.069	0.0	-0.033	0.0
6	0.156	0.06	3.944	1.90	4.542	4.70	5.042	5.30	4.957	4.70	4.518	4.55	0.120	0.0	-0.057	0.0
7	0.373	0.07	4.324	1.96	4.755	4.70	5.229	5.36	5.168	4.78	4.556	4.62	0.145	0.0	-0.068	0.0
8	0.309	0.08	5.180	2.02	4.427	4.71	5.477	5.42	5.356	4.86	4.415	4.69	0.122	0.0	-0.056	0.0
9	-0.214	0.09	3.780	2.18	4.224	4.71	5.759	5.48	5.400	4.94	4.236	4.76	0.117	0.0	-0.054	0.0
10	-0.730	0.10	2.421	22.40	4.003	4.72	5.811	5.54	21.159	50.20	26.086	4.83	0.221	0.0	-0.104	0.0

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 522681.055

WITH STANDARD DEVIATION 2580854.599

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.020 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.072	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.014	4.30	3.673	4.20	-0.488	0.0	0.224	0.0
2	-0.526	0.02	1.190	1.66	4.360	4.68	5.306	5.06	4.225	4.38	3.868	4.27	-0.286	0.0	0.141	0.0
3	-0.663	0.03	1.943	1.72	4.215	4.69	5.195	5.12	4.438	4.46	4.112	4.34	-0.050	0.0	0.033	0.0
4	-0.473	0.04	2.331	1.78	4.206	4.69	5.079	5.18	4.558	4.54	4.229	4.41	0.010	0.0	-0.000	0.0
5	-0.178	0.05	3.086	1.84	4.355	4.70	5.034	5.24	4.662	4.62	4.284	4.48	-0.050	0.0	0.025	0.0
6	0.048	0.06	3.843	1.90	4.518	4.70	5.060	5.30	4.876	4.70	4.338	4.55	-0.160	0.0	0.076	0.0
7	0.247	0.07	4.149	1.96	4.726	4.70	5.200	5.36	5.133	4.78	4.396	4.62	-0.193	0.0	0.093	0.0
8	0.271	0.08	4.721	2.02	4.455	4.71	5.397	5.42	5.491	4.86	4.511	4.69	-0.120	0.0	0.059	0.0
9	0.052	0.09	3.798	2.18	4.289	4.71	5.653	5.48	5.708	4.94	4.681	4.76	0.078	0.0	-0.035	0.0
10	-0.278	0.10	1.618	2.24	4.042	4.72	5.800	5.54	5.668	5.02	4.637	4.83	0.153	0.0	-0.071	0.0

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 5495885.505

WITH STANDARD DEVIATION 27273212.419

TERMINAL FACTOR - 10. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.020 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.067	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.039	4.30	3.718	4.20	-0.467	0.0	0.216	0.0
2	-0.501	0.02	1.259	1.66	4.359	4.68	5.307	5.06	4.240	4.38	3.895	4.27	-0.225	0.0	0.112	0.0
3	-0.647	0.03	2.040	1.72	4.212	4.69	5.204	5.12	4.442	4.46	4.113	4.34	-0.012	0.0	0.017	0.0
4	-0.486	0.04	2.422	1.78	4.194	4.69	5.094	5.18	4.522	4.54	4.159	4.41	-0.004	0.0	0.010	0.0
5	-0.247	0.05	3.128	1.84	4.340	4.70	5.043	5.24	4.568	4.62	4.136	4.48	-0.190	0.0	0.093	0.0
6	-0.078	0.06	3.755	1.90	4.486	4.70	5.039	5.30	4.730	4.70	4.135	4.55	-0.494	0.0	0.232	0.0
7	0.126	0.07	3.624	1.96	4.646	4.70	5.127	5.36	4.919	4.78	4.161	4.62	-0.728	0.0	0.338	0.0
8	0.094	0.08	3.831	2.02	4.372	4.71	5.280	5.42	5.231	4.86	4.296	4.69	-0.777	0.0	0.366	0.0
9	-0.128	0.09	2.557	2.18	4.235	4.71	5.485	5.48	5.435	4.94	4.540	4.76	0.158	0.0	-0.074	0.0
10	-0.418	0.10	-0.063	2.24	4.009	4.72	5.587	5.54	5.497	5.02	4.681	4.83	0.133	0.0	-0.064	0.0

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL UNCERTAIN PARAMETERS SOLUTION
(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST 169719.598
WITH STANDARD DEVIATION 898611.346

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.200
STABILITY FACTOR - 0.020 NO. SIMULATIONS - 50

	TRABCCME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.023	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.096	4.30	3.823	4.20	-0.288	0.0	0.134	0.0
2	-0.444	0.02	1.419	1.66	4.356	4.68	5.318	5.06	4.280	4.38	3.967	4.27	-0.163	0.0	0.082	0.0
3	-0.618	0.03	2.259	1.72	4.192	4.69	5.234	5.12	4.475	4.46	4.161	4.34	0.013	0.0	-0.000	0.0
4	-0.500	0.04	2.702	1.78	4.155	4.69	5.133	5.18	4.608	4.54	4.297	4.41	0.051	0.0	-0.020	0.0
5	-0.233	0.05	3.462	1.84	4.279	4.70	5.076	5.24	4.720	4.62	4.365	4.48	0.024	0.0	-0.010	0.0
6	0.018	0.06	4.168	1.90	4.420	4.70	5.077	5.30	4.931	4.70	4.416	4.55	-0.051	0.0	0.025	0.0
7	0.236	0.07	4.349	1.96	4.612	4.70	5.198	5.36	5.171	4.78	4.463	4.62	-0.096	0.0	0.047	0.0
8	0.250	0.08	4.933	2.02	4.344	4.71	5.379	5.42	5.506	4.86	4.550	4.69	-0.064	0.0	0.033	0.0
9	0.024	0.09	4.021	2.18	4.206	4.71	5.614	5.48	5.703	4.94	4.679	4.76	0.084	0.0	-0.038	0.0
10	-0.294	0.10	2.147	2.24	3.996	4.72	5.739	5.54	5.729	5.02	4.643	4.83	0.148	0.0	-0.068	0.0

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL UNCERTAIN PARAMETERS SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 739329.504

WITH STANDARD DEVIATION 3714553.999

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.103	0.01	0.776	1.60	4.582	4.68	5.116	5.00	4.067	4.30	3.770	4.20	-0.616	0.0	0.281	0.0
2	-0.537	0.02	1.340	1.66	4.359	4.68	5.297	5.06	4.246	4.38	3.909	4.27	-0.446	0.0	0.216	0.0
3	-0.680	0.03	2.147	1.72	4.221	4.69	5.185	5.12	4.423	4.46	4.076	4.34	-0.210	0.0	0.105	0.0
4	-0.500	0.04	2.466	1.78	4.213	4.69	5.065	5.18	4.567	4.54	4.235	4.41	-0.044	0.0	0.023	0.0
5	-0.159	0.05	3.189	1.84	4.368	4.70	5.008	5.24	4.723	4.62	4.392	4.48	0.069	0.0	-0.033	0.0
6	0.156	0.06	3.944	1.90	4.542	4.70	5.042	5.30	4.957	4.70	4.518	4.55	0.120	0.0	-0.057	0.0
7	0.373	0.07	4.324	1.96	4.755	4.70	5.229	5.36	5.168	4.78	4.556	4.62	0.145	0.0	-0.068	0.0
8	0.309	0.08	5.180	2.02	4.427	4.71	5.477	5.42	5.356	4.86	4.415	4.69	0.122	0.0	-0.056	0.0
9	-0.214	0.09	3.780	2.18	4.224	4.71	5.759	5.48	5.400	4.94	4.236	4.76	0.117	0.0	-0.054	0.0
10	-0.730	0.10	2.421	22.40	4.003	4.72	5.811	5.54	21.159	50.20	26.086	4.83	0.221	0.0	-0.104	0.0

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 686778.955

WITH STANDARD DEVIATION 4760974.969

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.214	0.01	0.589	1.60	4.767	4.68	5.094	5.00	3.937	4.30	3.968	4.20	-0.339	0.0	0.162	0.0
2	-0.742	0.02	-0.662	1.66	4.507	4.68	5.270	5.06	4.549	4.38	4.161	4.27	-0.178	0.0	0.085	0.0
3	-0.818	0.03	-3.243	1.72	4.052	4.69	5.296	5.12	5.325	4.46	4.304	4.34	-0.031	0.0	0.015	0.0
4	-0.537	0.04	-7.180	1.78	3.534	4.69	5.186	5.18	6.059	4.54	4.398	4.41	0.040	0.0	-0.019	0.0
5	0.088	0.05	-14.413	1.84	3.310	4.70	5.046	5.24	6.469	4.62	4.467	4.48	0.039	0.0	-0.019	0.0
6	0.555	0.06	-26.923	1.90	3.117	4.70	4.955	5.30	6.602	4.70	4.544	4.55	0.007	0.0	-0.003	0.0
7	0.693	0.07	-46.173	1.96	3.476	4.70	4.963	5.36	6.495	4.78	4.645	4.62	-0.011	0.0	0.005	0.0
8	0.964	0.08	-78.789	2.02	3.442	4.71	5.035	5.42	6.431	4.86	4.759	4.69	0.007	0.0	-0.003	0.0
9	0.444	0.09	*******	2.18	3.410	4.71	5.176	5.48	6.782	4.94	4.850	4.76	0.045	0.0	-0.021	0.0
10	0.279	0.10	*******	2.24	3.032	4.72	5.279	5.54	7.185	5.02	4.870	4.83	0.058	0.0	-0.028	0.0

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 3889678.713

WITH STANDARD DEVIATION 26683289.419

TERMINAL FACTOR - 10. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.176	0.01	0.589	1.60	4.767	4.68	5.094	5.00	4.007	4.30	4.040	4.20	-0.258	0.0	0.123	0.0
2	-0.662	0.02	-0.514	1.66	4.480	4.68	5.284	5.06	4.547	4.38	4.205	4.27	-0.112	0.0	0.054	0.0
3	-0.804	0.03	-2.930	1.72	4.044	4.69	5.325	5.12	5.286	4.46	4.290	4.34	-0.006	0.0	0.003	0.0
4	-0.559	0.04	-6.712	1.78	3.577	4.69	5.219	5.18	6.082	4.54	4.312	4.41	0.000	0.0	-0.000	0.0
5	0.151	0.05	-13.932	1.84	3.417	4.70	5.086	5.24	6.680	4.62	4.325	4.48	-0.068	0.0	0.033	0.0
6	0.758	0.06	-26.089	1.90	3.276	4.70	5.054	5.30	7.126	4.70	4.399	4.55	-0.140	0.0	0.067	0.0
7	0.987	0.07	-44.203	1.96	3.476	4.70	5.184	5.36	7.416	4.78	4.570	4.62	-0.147	0.0	0.069	0.0
8	1.202	0.08	-73.410	2.02	3.538	4.71	5.467	5.42	7.915	4.86	4.822	4.69	-0.016	0.0	0.008	0.0
9	0.996	0.09	********	2.18	2.761	4.71	5.842	5.48	8.913	4.94	4.764	4.76	0.222	0.0	-0.087	0.0
10	0.282	0.10	********	2.24	1.902	4.72	6.244	5.54	10.159	5.02	4.876	4.83	0.039	0.0	-0.019	0.0

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST 63858.970

WITH STANDARD DEVIATION 431578.908

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.200

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.158	0.01	0.589	1.60	4.767	4.68	5.094	5.00	4.029	4.30	4.063	4.20	-0.219	0.0	0.105	0.0
2	-0.644	0.02	-0.464	1.66	4.472	4.68	5.292	5.06	4.538	4.38	4.213	4.27	-0.109	0.0	0.052	0.0
3	-0.803	0.03	-2.793	1.72	4.042	4.69	5.336	5.12	5.328	4.46	4.335	4.34	0.011	0.0	-0.005	0.0
4	-0.494	0.04	-6.539	1.78	3.580	4.69	5.232	5.18	6.278	4.54	4.427	4.41	0.076	0.0	-0.036	0.0
5	0.369	0.05	-13.988	1.84	3.416	4.70	5.125	5.24	7.117	4.62	4.500	4.48	0.080	0.0	-0.038	0.0
6	1.168	0.06	-26.671	1.90	3.268	4.70	5.185	5.30	7.835	4.70	4.571	4.55	0.047	0.0	-0.022	0.0
7	1.409	0.07	-45.680	1.96	3.324	4.70	5.499	5.36	8.469	4.78	4.657	4.62	0.016	0.0	-0.007	0.0
8	1.372	0.08	-75.782	2.02	3.123	4.71	6.022	5.42	9.480	4.86	4.755	4.69	0.014	0.0	-0.007	0.0
9	0.862	0.09	*******	2.18	1.964	4.71	6.610	5.48	11.413	4.94	4.842	4.76	0.040	0.0	-0.019	0.0
10	0.339	0.10	*******	2.24	0.389	4.72	7.138	5.54	14.157	5.02	4.871	4.83	0.054	0.0	-0.026	0.0

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION WITH INITIAL CERTAINTY EQUIVALENCE SOLUTION

(USING LAIDLER DATA FOR YEARS 1954 TO 1975) - HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 801623.571

WITH STANDARD DEVIATION 5602102.188

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	TRANCOME	TGT	RESERVES	TGT	EXCHRATE	TGT	DOMPRICE	TGT	$SUPPLY	TGT	CREDIT	TGT	GVTSPNDG	TGT	TAXRATE	TGT
WT	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
1	-0.268	0.01	0.589	1.60	4.767	4.68	5.094	5.00	3.803	4.30	3.827	4.20	-0.453	0.0	0.217	0.0
2	-0.908	0.02	-0.937	1.66	4.559	4.68	5.248	5.06	4.462	4.38	4.010	4.27	-0.330	0.0	0.158	0.0
3	-0.908	0.03	-3.417	1.72	4.134	4.69	5.245	5.12	5.281	4.46	4.210	4.34	-0.165	0.0	0.079	0.0
4	-0.510	0.04	-6.339	1.78	3.623	4.69	5.119	5.18	5.994	4.54	4.426	4.41	-0.008	0.0	0.004	0.0
5	0.175	0.05	-11.065	1.84	3.392	4.70	4.985	5.24	6.305	4.62	4.636	4.48	0.133	0.0	-0.064	0.0
6	0.662	0.06	-19.433	1.90	3.213	4.70	4.909	5.30	6.227	4.70	4.783	4.55	0.247	0.0	-0.118	0.0
7	0.695	0.07	-32.444	1.96	3.657	4.70	4.951	5.36	5.866	4.78	4.778	4.62	0.303	0.0	-0.145	0.0
8	0.684	0.08	-56.357	2.02	3.614	4.71	5.033	5.42	5.279	4.86	4.550	4.69	0.255	0.0	-0.122	0.0
9	-0.281	0.09	-91.300	2.18	4.455	4.71	5.129	5.48	5.145	4.94	4.272	4.76	0.110	0.0	-0.053	0.0
10	-0.176	0.10	*******	22.40	4.306	4.72	5.240	5.54	29.454	50.20	26.410	4.83	0.159	0.0	-0.076	0.0

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

USING THE ABEL MODEL FOR THE CASE: BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 1.154

WITH STANDARD DEVIATION 4.631

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'I	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.148	0.15
2	0.346	0.37	0.078	0.09	0.112	0.14	0.138	0.15
3	0.334	0.37	0.053	0.09	0.163	0.14	0.155	0.15
4	0.321	0.38	0.007	0.09	0.197	0.14	0.166	0.15
5	0.343	0.38	0.041	0.09	0.225	0.15	0.183	0.16
6	0.388	0.38	0.116	0.09	0.194	0.15	0.170	0.16

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 9749.013

WITH STANDARD DEVIATION 39760.226

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.148	0.15
2	0.346	0.37	0.078	0.09	0.112	0.14	0.138	0.15
3	0.334	0.37	0.053	0.09	0.163	0.14	0.155	0.15
4	0.321	0.38	0.007	0.09	0.197	0.14	0.166	0.15
5	0.343	0.38	0.041	0.09	0.225	0.15	0.183	0.16
6	0.388	0.38	0.116	0.09	0.194	0.15	0.170	0.16

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

USING THE ABEL MODEL FOR THE CASE: DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST	0.401
WITH STANDARD DEVIATION	1.604

TERMINAL FACTOR -	1.	DISCOUNT FACTOR -	1.200
STABILITY FACTOR -	1.000	NO. SIMULATIONS -	50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.148	0.15
2	0.346	0.37	0.078	0.09	0.112	0.14	0.138	0.15
3	0.334	0.37	0.053	0.09	0.163	0.14	0.155	0.15
4	0.321	0.38	0.007	0.09	0.197	0.14	0.166	0.15
5	0.343	0.38	0.041	0.09	0.225	0.15	0.183	0.16
6	0.388	0.38	0.116	0.09	0.194	0.15	0.170	0.16

CHOW STOCHASTIC CONTROL MODEL - CERTAINTY EQUIVALENCE SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 9130.651

WITH STANDARD DEVIATION 9743.196

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.148	0.15
2	0.346	0.37	0.078	0.09	0.112	0.14	0.138	0.15
3	0.334	0.37	0.053	0.09	0.163	0.14	0.155	0.15
4	0.321	0.38	0.007	0.09	0.197	0.14	0.166	0.15
5	0.343	0.38	0.041	0.09	0.225	0.15	0.183	0.16
6	-43.571	38.45	-99.948	9.43	105.992	0.15	14.082	0.16

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

USING THE ABEL MODEL FOR THE CASE: BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 0.196

WITH STANDARD DEVIATION 0.707

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GV'SPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.345	0.37	0.081	0.09	0.107	0.14	0.138	0.15
3	0.334	0.37	0.060	0.09	0.149	0.14	0.154	0.15
4	0.322	0.38	0.018	0.09	0.175	0.14	0.164	0.15
5	0.347	0.38	0.056	0.09	0.199	0.15	0.178	0.16
6	0.397	0.38	0.137	0.09	0.165	0.15	0.165	0.16

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 10276.136

WITH STANDARD DEVIATION 43368.539

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.369	0.36	0.116	0.09	0.119	0.14	0.147	0.15
2	0.350	0.37	0.089	0.09	0.112	0.14	0.139	0.15
3	0.335	0.37	0.075	0.09	0.155	0.14	0.152	0.15
4	0.314	0.38	0.041	0.09	0.192	0.14	0.160	0.15
5	0.313	0.38	0.060	0.09	0.238	0.15	0.174	0.16
6	0.349	0.38	0.130	0.09	0.216	0.15	0.173	0.16

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

USING THE ABEL MODEL FOR THE CASE: DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST	0.066
WITH STANDARD DEVIATION	0.236

TERMINAL FACTOR -	1.	DISCOUNT FACTOR -	1.200
STABILITY FACTOR -	1.000	NO. SIMULATIONS -	50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.345	0.37	0.081	0.09	0.107	0.14	0.138	0.15
3	0.334	0.37	0.059	0.09	0.148	0.14	0.154	0.15
4	0.322	0.38	0.017	0.09	0.174	0.14	0.164	0.15
5	0.348	0.38	0.056	0.09	0.197	0.15	0.178	0.16
6	0.398	0.38	0.138	0.09	0.164	0.15	0.164	0.16

CHOW STOCHASTIC CONTROL MODEL - UNCERTAIN PARAMETERS SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 342.091

WITH STANDARD DEVIATION 256.649

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.437	0.36	0.073	0.09	0.340	0.14	0.149	0.15
2	0.592	0.37	0.018	0.09	0.659	0.14	0.136	0.15
3	1.025	0.37	-0.152	0.09	1.735	0.14	0.120	0.15
4	2.342	0.38	-0.605	0.09	4.996	0.14	0.093	0.15
5	6.656	0.38	-1.419	0.09	15.129	0.15	0.354	0.16
6	31.923	38.45	14.990	9.43	56.491	0.15	12.847	0.16

HEURISTIC STOCHASTIC CONTROL MODEL

USING THE ABEL MODEL FOR THE CASE: BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 0.196

WITH STANDARD DEVIATION 0.707

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.345	0.37	0.081	0.09	0.107	0.14	0.138	0.15
3	0.334	0.37	0.058	0.09	0.150	0.14	0.154	0.15
4	0.322	0.38	0.016	0.09	0.176	0.14	0.164	0.15
5	0.348	0.38	0.054	0.09	0.200	0.15	0.179	0.16
6	0.399	0.38	0.138	0.09	0.165	0.15	0.165	0.16

HEURISTIC STOCHASTIC CONTROL MODEL

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 3178.011

WITH STANDARD DEVIATION 11932.101

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.346	0.37	0.080	0.09	0.110	0.14	0.139	0.15
3	0.332	0.37	0.053	0.09	0.158	0.14	0.154	0.15
4	0.318	0.38	0.005	0.09	0.192	0.14	0.164	0.15
5	0.341	0.38	0.036	0.09	0.220	0.15	0.180	0.16
6	0.401	0.38	0.135	0.09	0.180	0.15	0.171	0.16

HEURISTIC STOCHASTIC CONTROL MODEL

USING THE ABEL MODEL FOR THE CASE: DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST 0.067

WITH STANDARD DEVIATION 0.239

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.200

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.345	0.37	0.081	0.09	0.107	0.14	0.138	0.15
3	0.334	0.37	0.058	0.09	0.149	0.14	0.154	0.15
4	0.322	0.38	0.016	0.09	0.175	0.14	0.164	0.15
5	0.349	0.38	0.055	0.09	0.199	0.15	0.179	0.16
6	0.400	0.38	0.138	0.09	0.164	0.15	0.164	0.16

HEURISTIC STOCHASTIC CONTROL MODEL

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 8602.205

WITH STANDARD DEVIATION 9427.214

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 0.0 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.113	0.14	0.147	0.15
2	0.345	0.37	0.080	0.09	0.108	0.14	0.138	0.15
3	0.333	0.37	0.055	0.09	0.156	0.14	0.154	0.15
4	0.305	0.38	-0.027	0.09	0.223	0.14	0.160	0.15
5	-0.334	0.38	-1.523	0.09	1.468	0.15	0.176	0.16
6	-36.792	38.45	-90.745	9.43	105.878	0.15	14.729	0.16

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 0.201

WITH STANDARD DEVIATION 0.727

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'I	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.345	0.37	0.081	0.09	0.107	0.14	0.138	0.15
3	0.334	0.37	0.059	0.09	0.150	0.14	0.154	0.15
4	0.322	0.38	0.016	0.09	0.176	0.14	0.164	0.15
5	0.347	0.38	0.054	0.09	0.200	0.15	0.179	0.16
6	0.398	0.38	0.137	0.09	0.166	0.15	0.165	0.16

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 25.846

WITH STANDARD DEVIATION 42.170

TERMINAL FACTOR - 10000.   DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.270	0.36	-0.085	0.09	-0.006	0.14	0.023	0.15
2	0.102	0.37	-0.232	0.09	-0.154	0.14	-0.075	0.15
3	0.148	0.37	-0.053	0.09	0.199	0.14	0.167	0.15
4	0.305	0.38	-0.289	0.09	0.658	0.14	0.184	0.15
5	0.379	0.38	0.197	0.09	0.157	0.15	0.241	0.16
6	0.398	0.38	0.077	0.09	0.127	0.15	0.123	0.16

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST	0.068
WITH STANDARD DEVIATION	0.245

TERMINAL FACTOR -	1.	DISCOUNT FACTOR -	1.200
STABILITY FACTOR -	1.000	NO. SIMULATIONS -	50

	CONSUM'N	TGT	INVEST'T	TGT	GV'SPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.367	0.36	0.117	0.09	0.114	0.14	0.147	0.15
2	0.345	0.37	0.081	0.09	0.107	0.14	0.138	0.15
3	0.334	0.37	0.058	0.09	0.149	0.14	0.154	0.15
4	0.322	0.38	0.016	0.09	0.175	0.14	0.164	0.15
5	0.348	0.38	0.055	0.09	0.199	0.15	0.179	0.16
6	0.399	0.38	0.138	0.09	0.165	0.15	0.165	0.16

MACRAE STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 3199.260

WITH STANDARD DEVIATION 5157.128

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.373	0.36	0.099	0.09	0.142	0.14	0.141	0.15
2	0.359	0.37	0.094	0.09	0.111	0.14	0.146	0.15
3	0.393	0.37	0.119	0.09	0.271	0.14	0.139	0.15
4	0.379	0.38	0.102	0.09	0.155	0.14	0.149	0.15
5	0.396	0.38	0.121	0.09	0.203	0.15	0.143	0.16
6	40.254	38.45	38.904	9.43	105.426	0.15	13.625	0.16

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: BASE RUN

SOLUTION SUMMARY FOR TOTAL COST 0.041

WITH STANDARD DEVIATION 0.066

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.366	0.36	0.107	0.09	0.114	0.14	0.148	0.15
2	0.367	0.37	0.105	0.09	0.113	0.14	0.146	0.15
3	0.392	0.37	0.106	0.09	0.114	0.14	0.146	0.15
4	0.373	0.38	0.088	0.09	0.115	0.14	0.147	0.15
5	0.347	0.38	0.086	0.09	0.116	0.15	0.147	0.16
6	0.307	0.38	0.073	0.09	0.117	0.15	0.147	0.16

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL FACTOR

SOLUTION SUMMARY FOR TOTAL COST 63.032

WITH STANDARD DEVIATION 98.721

TERMINAL FACTOR - 10000. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVTSPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	1.445	0.36	0.411	0.09	3.178	0.14	0.578	0.15
2	0.381	0.37	0.087	0.09	-2.710	0.14	-0.222	0.15
3	0.410	0.37	0.109	0.09	0.115	0.14	0.154	0.15
4	0.379	0.38	0.081	0.09	0.112	0.14	0.144	0.15
5	0.321	0.38	0.069	0.09	0.004	0.15	0.131	0.16
6	0.310	0.38	0.063	0.09	0.208	0.15	0.159	0.16

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: DISCOUNTED PENALTIES

SOLUTION SUMMARY FOR TOTAL COST	0.017
WITH STANDARD DEVIATION	0.027

TERMINAL FACTOR -	1.	DISCOUNT FACTOR -	1.200
STABILITY FACTOR -	1.000	NO. SIMULATIONS -	50

	CONSUM'N	TGT	INVEST'T	TGT	GV'SPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.366	0.36	0.107	0.09	0.114	0.14	0.148	0.15
2	0.367	0.37	(.105	0.09	0.113	0.14	0.146	0.15
4	=/403	=/48	=/2=7	=/=0	=/114	0.14	0.146	0.15
4	0.373	0.38	0.088	0.09	0.115	0.14	0.147	0.15
5	0.347	0.38	0.086	0.09	0.116	0.15	0.147	0.16
6	0.307	0.38	0.073	0.09	0.117	0.15	0.147	0.16

NORMAN STOCHASTIC CONTROL MODEL - ADAPTIVE SOLUTION

USING THE ABEL MODEL FOR THE CASE: HIGH TERMINAL TARGETS

SOLUTION SUMMARY FOR TOTAL COST 741.687

WITH STANDARD DEVIATION 907.432

TERMINAL FACTOR - 1. DISCOUNT FACTOR - 1.000

STABILITY FACTOR - 1.000 NO. SIMULATIONS - 50

	CONSUM'N	TGT	INVEST'T	TGT	GVISPEND	TGT	$SUPPLY	TGT
WT	1.000		1.000		0.0		0.0	
1	0.510	0.36	0.143	0.09	0.528	0.14	0.202	0.15
2	0.368	0.37	0.088	0.09	-0.267	0.14	0.099	0.15
3	0.393	0.37	0.095	0.09	0.114	0.14	0.146	0.15
4	0.367	0.38	0.080	0.09	0.115	0.14	0.147	0.15
5	0.337	0.38	0.077	0.09	0.116	0.15	0.147	0.16
6	40.530	38.45	12.787	9.43	105.915	0.15	14.059	0.16

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