TESTING FOR NORMALITY IN LINEAR REGRESSION AND AUTOREGRESSIVE TIME SERIES MODELS

by

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TESTING FOR NORMALITY IN LINEAR REGRESSION AND AUTOREGRESSIVE TIME SERIES MODELS

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ABSTRACT

Residuals in normal regression theory are used to test for normality of the unknown error term. This test examines the normal probability plot of the residuals, or suitable modifications of these residuals, for departure from linearity. Noticeable nonlinearity of this plot indicates that the residuals, and hence the unknown errors which they estimate, are not normal. Such a test is subjective at best. However, these plots are now a standard feature of most statistical packages, such as Minitab.

A large sample result of Pierce and Kopecky, combined with tables of Stephens, provides an easily applied goodness-of-fit test for normality of the error distribution in ordinary least squares regression.

This study uses simulation to examine the validity of applying the (large sample) test to samples of small and moderate size. Extensive Monte Carlo runs indicate that sample size, N=20, is large enough to justify the use of the test.

Pierce shows that the same test, using the residuals after fitting an autoregressive time series model, may be used to test for normality of the error term, in such a model. It is demonstrated empirically that sample size ,N=20, again is adequate for the application of the test.

Robustness of the Pierce-Kopecky goodness-of-fit test to mis-specification of the degree of the model in the linear regression case, and to the order of the model in the autoregressive case, is explored.

When a linear model is fitted to quadratic data with normal errors, the EDF tests reject normality, if the sample size exceeds n = 20. Similarly, when an AR(1) model is fitted to AR(2) data with normal errors, normality is rejected, even for n = 20. The EDF tests are robust to overfitting of the model in both the linear regression and autoregressive cases.

If the wrong model is fitted to data and the errors are non-normal, the EDF tests will reject normality for any sample size.

DEDICATION

To my mother, Mrs Alma, Agatha Croal

For your faith, dedication, and support.

To my late father, George Croal, in memoriam.

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CHAPTER 1

1.1 Introduction

Testing for normality is an old and very important area of statistical research, both practically and theoretically. In regression analysis, the statistical procedures, *eg.*, confidence intervals and significance tests for the regression estimates, are based on the assumed error distribution. The normal distribution is often the hypothesized error distribution, because of the desirable properties of the regression estimates, when the error distribution is indeed normal.

1.2 The Method of Least Squares

Consider the first order linear regression model in the form:

$$y_i = \beta_0 + \beta_1 x_i + \sigma\epsilon_i \qquad \ldots 0.1.1$$

where,

1. y_i is the i'th observation;
2. β_0 , β_1 are unknown parameters;
3. x_i is a known constant;
4. ϵ_i is an unknown random error term with mean $E[\epsilon_i] = 0$, and variance $var(\epsilon_i) = 1$; ϵ_i and ϵ_j are uncorrelated, *ie.*, $Cov(\epsilon_i,\epsilon_j) = 0$; for all i, j, $i \neq j$, $i = 1,\ldots, n$.

The Method of Least Squares gives estimates $\hat{\beta}_0$, $\hat{\beta}_1$ of β_0 , β_1 , which minimize the sum of squares: $Q = \sum_{i=1}^{n} [y_i-(\beta_0 + \beta_1 x_i)]^2$.

1.3 Properties of Estimators

We mention some properties of estimators which are desirable and are needed later. Let θ be an unknown parameter, and let $\hat{\theta}_n$, $\tilde{\theta}_n$ be estimators of θ based on samples of size, n.

1. The estimator, $\hat{\theta}_n$ is *unbiased* if:

   $E[\hat{\theta}_n] = \theta$ ...0.2.1

2. The estimator, $\hat{\theta}_n$ is a *consistent estimator* of θ if:

   $P(|\hat{\theta}_n - \theta| \geq \epsilon) \longrightarrow 0$ as $n \longrightarrow \infty$, for any $\epsilon > 0$.

3. The estimator, $\hat{\theta}_n$ is a *sufficient estimator* for θ, if the conditional joint probability density function, pdf, of the sample observations, given $\hat{\theta}_n$, does not depend on the parameter, θ.

4. The estimator, $\hat{\theta}_n$ is a *minimum variance estimator* of θ, if for any other estimator, $\tilde{\theta}_n$: $Var(\hat{\theta}_n) \leq Var(\tilde{\theta}_n)$.[1]

1.4 Properties of Least Squares Estimators

The *Gauss-Markov Theorem* states: under the conditions imposed on the model (0.1.1), the least squares estimators are best linear unbiased estimators, BLUES, *ie.*, they have *minimum variance* among all linear[2]

Inference procedures for the least squares estimates requires an assumption about the distribution of the error terms

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[1]see, *eg.*, Neter, Wasserman, and Kutner (1985), Chapters 1, and 2.

[2] the least squares estimators are linear combinations of the observations, y_i, i=1,...,n; *op.cit.*

ϵ_i, i = 1,...,n. If the ϵ_i are assumed to be N(0,1) random variables, then, the least estimates, in addition to being BLUES, have other useful properties. First, they are maximum likelihood estimators; and

1. They are consistent;
2. They are sufficient;
3. They are minimum variance unbiased, *ie.*, they have minimum variance among all unbiased estimators, whether linear or not.

Some inference procedures *eg.*, t-tests for the regression estimates are not sensitive to slight departures from normality of the error distribution. However, serious departures from normality will affect significance tests and confidence intervals for the regression estimates, especially when the sample size is small.

The residuals from regression are used check the assumptions of the model, mainly by means of the residual plots. Formal tests using the residuals face the problem that the residuals are correlated and are not even identically distributed. Most formal tests of significance require independent and identically distributed observations. If p parameters are estimated by a least squares regression, the residuals may be transformed to give n-p independent and identically distributed , i.i.d, random variables. These are linear combinations of the raw residuals. These i.i.d *best linear unbiased scaled, BLUS,* residuals[3] may be

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[3]see *Seber (1977) pp162-173*

used in any of the standard tests for normality. The practical tester would prefer to be able to use the residuals themselves rather than complicated linear combinations of them.

Pierce and Kopecky (1979) show that for large samples, the residuals may be used in formal EDF tests as if they were i.i.d observations.

Following Pierce and Kopecky(1979), we consider the linear regression model in the form:

$$y_i = \underline{x}_i' \underline{\beta} + \sigma\epsilon_i \tag{1.1}$$

where i=1,....n; the y_i are independent observations; the $\underline{x}_i$ are px1 vectors of known constants; $\underline{\beta}$ is a px1 vector of unknown parameters; σ (>0) is an unknown scale factor; and ϵ_i is an unknown error term of mean zero and variance one.

We wish to test whether the ϵ_i in (1.1) are independent random variables from some specified distribution such as the standard normal. Probability plots using the residuals, after fitting the regression, give rough assessments of the goodness-of-fit of the hypothesized error distribution. However, formal tests of significance are sometimes needed to supplement these pictures. This has become increasingly important in lifetime testing and survival analysis; see, for example, Lawless(1981).

1.5 Specification of the Model

The regression model given by equation (1.1) is specified by the following :

1. The (fixed) dimension, p, of the parameter vector , $\underline{\beta} = (\beta_0, \beta_1, \ldots, \beta_{p-1})'$. The main result of Pierce and Kopecky (1979) requires the model (1.1) to have a constant term, β_0. Hence the model equation can be written in such a way that the first element of each vector $\underline{x}_i$ equals one.
2. The null hypothesis distribution, F(.), of the error term, ϵ_i.

In this study we investigate cases with the dimension of $\underline{\beta}$ equal to 2, 3, and 4. For all cases the null hypothesis distribution is the standard normal distribution.

1.6 The standardized residuals

From equation (1.1), the true errors, ϵ_i, are given by the equation:

$$\epsilon_i = (y_i - \underline{x}_i' \underline{\beta})/\sigma \tag{1.2}$$

Let $\hat{\underline{\beta}}$, $\hat{\sigma}$ be the maximum likelihood estimators of $\underline{\beta}$, σ respectively. Define the standardized residuals by the equation,

$$\hat{\epsilon}_i = (y_i - \underline{x}_i' \hat{\underline{\beta}})/\hat{\sigma} \tag{1.3.1}$$

The standardized residuals have the useful property of being independent of the parameter vector, $\underline{\beta}$, and the scale factor, σ. This fact which is used to simplify the data generated for our simulation study, will be proved in chapter 2.

1.7 The Empirical (Sample) Distribution Function(EDF)

Let $x_1, \ldots, x_n$ be a sample from a population with the cumulative distribution function(cdf), $F(x)$. The empirical(sample)distribution function, $F_n(x)$, is defined by $F_n(x)$ = the proportion of sample values not exceeding x, i.e., $F_n(x) = \frac{1}{n} \text{card}\{i \le n : x_i \le x\}$. A form of $F_n(x)$ suitable for computation is, $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} H(x-x_i) = \frac{1}{n}\sum_{i=1}^{n} H(x-x_{(i)})$, where the Heaviside function, H(.), is given by

$$H(t) = \begin{cases} 1, & t \ge 0 \\ 0, & t < 0, \end{cases}$$

and $x_{(1)} \le x_{(2)} \le \ldots \le x_{(n)}$ are the ordered sample values.

We now state some well-known facts about $F_n(x)$.[4]

1. a. $F_n(x) = 0$, if $x < x_{(1)}$.
   b. $F_n(x) = 1$, if $x \ge x_{(n)}$.
   c. $F_n(x) = i/n$, if $x_{(i)} \le x < x_{(i+1)}$, $1 \le i \le n$.
2. $nF_n(x) \sim \text{Bin}(n, F(x))$, i.e., $nF_n(x)$ is a binomial random variable with parameters n and F(x).
3. The mean and variance of $F_n(x)$ are :

   $E[F_n(x)] = F(x)$,$\text{var}(F_n(x)) = F(x)[1-F(x)]/n$.

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[4] see, *eg.*, Darling (1957), or, Pratt and Gibbons (1981), Chapter 7, pp 318-344.

4. $F_n(x)$ is asymptotically normal with mean and variance given in (3).

5. Strong law of large numbers :

   $F_n(x) \longrightarrow F(x)$ with probability 1, for each x.

6. Glivenko-Cantelli lemma :

$$\sup_{-\infty<x<\infty} |F_n(x) - F(x)| \longrightarrow 0 \text{ with probability } 1.$$

7. $F_n(x)$ converges uniformly to $F(x)$ in probability, i.e.,

   $P\{|F_n(x) - F(x)| < \epsilon \text{ for all } x\} \longrightarrow 1$ as $n \longrightarrow \infty$, for all $\epsilon > 0$.

8. $Cov(F_n(x), F_n(y)) = E[F_n(x)F_n(y)] - F(x)F(y)$

$$= \frac{1}{n}\rho((F(x), F(y)), \text{ where}$$

$$\rho(s,t) = \min(s,t) - st = \begin{cases} s(1-t), & \text{if } s \le t \\ t(1-s), & \text{if } s \ge t, \end{cases}$$

   $0 \le s,t \le 1$.

9. Multivariate Central Limit Theorem :

   For any fixed $t_1, t_2, \ldots, t_k$, the random variables

$$\sqrt{n}[F_n(t_i) - F(t_i)], \quad i = 1, \ldots, k,$$

   have an asymptotic (k fixed, $n \longrightarrow \infty$) k-dimensional normal distribution, with mean vector, $\underline{0}$, and covariance matrix ,

$$\Sigma_{ij} = \rho(F(t_i), F(t_j)),$$

   with ρ given in (8).

1.8 The EDF Statistics

The EDF statistics are so called because they are derived from the empirical (sample) distribution function(EDF), $F_n(x)$, defined in (1.4). They are goodness-of-fit statistics which measure the discrepancy between $F_n(x)$ and the assumed cumulative distribution function(cdf), $F(x)$, from which the sample, x_i, $i = 1,\ldots,n$, comes.

This study investigates the distributions of 4 EDF statistics derived from the standardized residuals defined in (1.3). They are

1. The Kolmogorov 2-sided statistic, D_n:

$$D_n = \sup_{-\infty<x<\infty} |F_n(x) - F(x)|$$

2. The Cramér-von Mises statistic, W_n^2 :

$$W_n^2 = n \int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 \, dF(x).$$

3. The Watson statistic, U_n^2 :

$$U_n^2 = n \int_{-\infty}^{\infty}[F_n(x) - F(x) - \int_{-\infty}^{\infty}[F_n(x) - F(x)]dF(x)]^2 dF(x) .$$

4. The Anderson-Darling statistic, A_n^2 :

$$A_n^2 = n \int_{-\infty}^{\infty} \frac{[F_n(x) - F(x)]^2}{F(x)[1-F(x)]} \, dF(x) .$$

The definitions are not suitable for computational work with these statistics. Simple computational forms for them are given in Stephens(1974). These forms will be used in all computer routines which generate the EDF statistics.

1.9 The Single Sample Case

Suppose we wish to test whether the independent observations come from a normal population with both mean, μ, and variance, σ^2, unknown. We write $x_i = \mu + \sigma w_i$ where the w_i, $i = 1,\ldots,n$, are independent standard normal random variables.

$$w_i = (x_i - \mu)/ \sigma$$

The maximum likelihood estimators of μ and σ^2 are

$\hat{\mu} = \overline{x}$ and $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n} (x_i - \overline{x})^2$. By the Invariance Principle of Maximum Likelihood[5], $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$

Write

$$\hat{w}_i = (x_i - \hat{\mu}) / \hat{\sigma}$$
$$= (x_i - \overline{x}) / \hat{\sigma}$$

Let $\hat{w}_{(i)}$, $i=1,\ldots,n$, be the order statistics of the w_i, and let $z_i = \Phi(\hat{w}_{(i)})$ where Φ is the standard normal cumulative distribution function, cdf. Then $0 \leq z_i \leq 1$, and $z_1 \leq z_2 \leq \ldots \leq z_n$.

If $F_n(t)$ is the EDF of the z_i, the empirical process is :

$$y_n(t) = \sqrt{n}[F_n(t) - t]$$

Stephens(1976) shows that in the case of a single sample, where only a mean and scale are estimated,

$$y_n(t) \xrightarrow{D} y(t)$$

where $y(t)$ is a Gaussian process, with $y(0)=0$, $y(1)=0$ (the tied-down brownian bridge), with mean equal zero, and a covariance function, $\rho(s,t)$, which depends on Φ, the standard normal cdf, but is independent of the unknown parameters, μ and σ.[6]

This is case 3 of Stephens(1974,1976,*et seq.*).[7]

In chapter four, the EDF statistics are expressed as functionals of the empirical process, y_n. Arguments based on Durbin(1973a)

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[5] see Mood, Graybill, and Boes(1974)

[6] $\xrightarrow{D}$ means *converges in distribution to, or, converges weakly to.*

[7] Stephens uses s, instead of $\hat{\sigma}$ above; where s^2 is the usual unbiased estimator of σ^2

show that for an EDF statistic, of the form, $G(y_n(t))$;

$$G(y_n(t)) \xrightarrow{D} G(y(t)),$$

if G(.) is a continuous functional. This establishes the existence of the asymptotic distributions of the EDF statistics, in the single sample case.

1.10 Scope of this Work

Consider the linear regression model given by equation 1.1. For any (fixed) dimension of $\underline{\beta}$, the large sample distributions of the EDF statistics derived from the standardized residuals $\hat{\epsilon}_i$, i=1,...,n, are the same as the large sample distributions of these statistics for the single sample case, Pierce and Kopecky(1979).

Pierce(1985) showed that for a stationary autoregressive time series model of any fixed order, the EDF statistics from the standardized regression residuals have large sample distributions which , as in the linear regression case, are identical to the large sample distributions for the corresponding statistics in the single sample case.

The Pierce-Kopecky Theorem applies to tests for any error distribution. In the specific case of testing for normality, the case 3 tables of Stephens(1974,1976), give the upper tail percentage points for the asymptotic distributions of the EDF statistics for the single sample. Hence the case 3 tables may be used to provide a formal test of the residuals for normality.

This work uses simulation to investigate four areas determined by the specification of the problem, for both the linear regression and the autoregressive cases. We state the areas examined for the linear regression case:

1. The null hypothesis case:

   The correct model is fitted to the appropriate data, with normal errors, *eg* ,in the simple linear case the 'canonical', or, simplest, form of the data is generated and the model, $y_i = \beta_0 + \beta_1 x_i + \sigma\epsilon_i$ is fitted. We discuss the generated data in section 2.5 of Chapter 2.

2. The alternative hypothesis case:

   The correct model is fitted but with non-normal errors. We use two alternative error distributions. They are:

   a. The Laplace (double-exponential) distribution with scale factor 2 ;and,

   b. The U(-.5,.5)distribution, i.e., the uniform distribution over the interval (-.5,.5).

3. Mis-specified models with normal errors, *eg*, underfit a linear model to quadratic data; or, overfit a quadratic model to linear data.

4. Mis-specified models with non-normal errors; *eg*, underfit a linear model to quadratic data with non-normal errors.

Table 5 gives a list of all the cases examined when, either the error distribution is non-normal or the degree of the model is wrong. For the autoregressive case, we may substitute order for degree, and underfit an AR(1) model to AR(2) data, *etc*.

In chapter 2 we discuss the general linear model, and a special case, the simple trigonometrical model. Chapter 3 states the basic facts about the autoregressive time series model. In this work, we consider only the models of orders 1, and 2 : AR(1)and AR(2). Chapter 4 expresses the EDF statistics as functionals of the *Empirical Process*, and discusses the convergence of these Statistics for the regression case. Chapter 5 discusses the simulation design used for this study. Chapter 6 concludes with an assessment of our results.

The goal of the first area we study is to assess the validity of applying the Pierce-Kopecky test, valid for large samples, to samples of small and moderate size. A sample size, N=20, is found to be adequate to justify use of the test. Tables 7.1 - 7.4 validate this claim.

How robust is the test to mis-specification of the degree, in the ordinary regression case, or of the order in the autoregressive case? This is one objective of the second area of study. Results in this area are in tables 1.1a, 2.1a, 3.1a, and, 4.1a.

How good is the test at detecting non-normality whether the model is correct or not? Results in this area are in tables 1.1b - 1.1e, 2.1b - 2.1e, 3.1b - 3.1e, and 4.1b - 4.1e.

Finally we apply the EDF test to the standardized residuals from live data. The results of these applications of the test are collected in Tables 8.1 - 8.3.

CHAPTER 2

THE GENERAL(NORMAL)LINEAR MODEL

2.1 Introduction

We rewrite the linear regression model equation (1.1) in the matrix form :

$$\underline{Y} = \mathbf{X}\underline{\beta} + \sigma\,\underline{\epsilon} \qquad ...(2.1.1)$$

where

1. $\underline{Y}$ is an nx1 observable random vector;
2. $\mathbf{X}$, the design matrix, is an nxp matrix of known constants, assumed to be of full rank, $p(<n)$;
3. $\underline{\beta}$ is a px1 vector of unknown parameters;
4. $\underline{\epsilon}$ is an unknown random vector, with $\underline{\epsilon} \sim N(\underline{0}, I_n)$, i.e., $\underline{\epsilon}$ has a multivariate normal distribution with mean vector, $\underline{0}$, and covariance matrix equal to the nxn identity matrix;
5. $\sigma(>0)$ is an unknown scale factor.

Conditions (1)-(4) imply that $\underline{Y} \sim N(\mathbf{X}\underline{\beta}, \sigma^2 I_n)$, i.e., $\underline{Y}$ has a multivariate normal distribution with mean vector, $\mathbf{X}\underline{\beta}$, and covariance matrix, $\sigma^2 I_n$.

2.2 The Likelihood Function and the Regression Estimates

We need the maximum likelihood estimators $\hat{\underline{\beta}}$ and $\hat{\sigma}$ in order to compute the standardized residuals.

The likelihood function for the random vector, $\underline{Y}$, is

$$L(\underline{Y};\underline{\beta},\sigma^2) = \frac{1}{(2\pi)^{n/2}\sigma^n} \exp[-\frac{1}{2\sigma^2} (\underline{Y}-\mathbf{X}\underline{\beta})'(\underline{Y}-\mathbf{X}\underline{\beta})] \quad \ldots(2.2.1)$$

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}[(\underline{Y}-\mathbf{X}\underline{\beta})'(\underline{Y}-\mathbf{X}\underline{\beta})] \quad \ldots(2.2.2)$$

$$\frac{\partial}{\partial\underline{\beta}}\ln L = -\frac{1}{2\sigma^2}[-2\mathbf{X}'(\underline{Y}-\mathbf{X}\underline{\beta})] = \mathbf{X}'(\underline{Y}-\mathbf{X}\underline{\beta})/\sigma^2 \quad \ldots(2.2.3)$$

$$\frac{\partial^2}{\partial\underline{\beta}^2}\ln L = -\mathbf{X}'\mathbf{X}/\sigma^2 \quad \ldots(2.2.4)$$

$\mathbf{X}$ is an nxp matrix of full rank p, so $\mathbf{X}'\mathbf{X}$ is positive definite; hence $-\mathbf{X}'\mathbf{X}/\sigma^2$ is negative definite.

From equation (2.2.3) $\frac{\partial}{\partial\underline{\beta}}\ln L = 0$ gives $\mathbf{X}'\underline{Y} = \mathbf{X}'\mathbf{X}\hat{\underline{\beta}}$

...(2.2.5)

where $\hat{\underline{\beta}}$ is the maximum likelihood estimator of $\underline{\beta}$, *i.e.*, the value of $\underline{\beta}$ which makes $L(\underline{Y};\underline{\beta},\sigma^2)$ a maximum.

Equation (2.2.5) gives $\hat{\underline{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\underline{Y}$ ...(2.2.6)

$$\frac{\partial}{\partial\sigma^2}\ln L = -\frac{1}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}(\underline{Y}-\mathbf{X}\hat{\underline{\beta}})'(\underline{Y}-\mathbf{X}\hat{\underline{\beta}}) \quad \ldots 2.2.7$$

after substituting $\hat{\underline{\beta}}$ for $\underline{\beta}$ in equation (2.2.2)

$\frac{\partial}{\partial\sigma^2}\ln L = 0$ gives $\hat{\sigma}^2 = \frac{1}{n}(\underline{Y}-\mathbf{X}\hat{\underline{\beta}})'(\underline{Y}-\mathbf{X}\hat{\underline{\beta}})$

...(2.2.8)

$$\hat{\sigma}^2 = \frac{1}{n}\underline{Y}'[\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\underline{Y} = = \frac{1}{n}\underline{Y}'[\mathbf{I} - \mathbf{V}]\underline{Y} \quad \ldots(2.2.9)$$

where, $\mathbf{V} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$. $\mathbf{V}$ is called the 'hat matrix'. We note that $\mathbf{V}$ is symmetric and idempotent: *i.e.*,

1. $\mathbf{V}' = \mathbf{V}$ symmetry.
2. $\mathbf{V}^2 = \mathbf{V}\mathbf{V} = \mathbf{V}$ idempotence.

$\hat{\sigma}^2$ is the maximum likelihood estimator, mle, for σ^2. By the

Invariance Principle for mle's, $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$:

$$\hat{\sigma} = \sqrt{\left\{ \frac{1}{n} [\underline{Y}'(I-V)\underline{Y}] \right\}} \qquad \ldots(2.2.10)$$

2.3 The Standardized Residual Vector

The residual vector after fitting the regression is given by

$$\mathbf{e} = \underline{Y}-\mathbf{X}\hat{\underline{\beta}} = [I - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}']\underline{Y} = (I - \mathbf{V})\underline{Y} \qquad \ldots(2.3.1)$$

Since **V** is symmetric and idempotent, so is **P** = I-**V**. It is well known that **V** is the projection operator from $\mathbf{R}^n$ down to the space spanned by the column vectors of **X**. Similarly, **P** = I-**V** is the projection operator from $\mathbf{R}^n$ down to the space orthogonal to the column vectors.

$$\mathbf{VX} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X} = \mathbf{X} \qquad \ldots(2.3.2)$$

$$\mathbf{PX} = (I-\mathbf{V})\mathbf{X} = \mathbf{X}-\mathbf{VX} = \mathbf{X}-\mathbf{X} = \underline{0} \qquad \ldots(2.3.3)$$

where $\underline{0}$ is the zero matrix.

Equation(2.2.9) gives $\hat{\sigma} = \frac{1}{\sqrt{n}} |\underline{Y}-\mathbf{X}\hat{\underline{\beta}}|$ ...(2.3.4)

The standardized residual vector is

$$\hat{\underline{\epsilon}} = \mathbf{e} / \hat{\sigma} = \sqrt{n}(\underline{Y}-\mathbf{X}\hat{\underline{\beta}})/|\underline{Y}-\mathbf{X}\hat{\underline{\beta}}| = \sqrt{n}\mathbf{P}\underline{Y}/|\mathbf{P}\underline{Y}| \qquad \ldots(2.3.5)$$

Now $\mathbf{P}\underline{Y} = \mathbf{P}(\mathbf{X}\underline{\beta} + \sigma\underline{\epsilon}) = \mathbf{PX}\underline{\beta} + \sigma\mathbf{P}\underline{\epsilon} = \sigma\mathbf{P}\underline{\epsilon}$, since $\mathbf{PX} = \underline{0}$.

Hence, $\hat{\underline{\epsilon}} = \sqrt{n}\,\mathbf{P}\underline{\epsilon} / |\mathbf{P}\underline{\epsilon}|$ ...(2.3.6)

Equation (2.3.6) implies the following facts:

1. $|\hat{\underline{\epsilon}}| = \sqrt{n}$. Hence $\hat{\underline{\epsilon}}$ lies on a sphere of radius $\sqrt{n}$. This sphere lies in the space of dimension n-p, orthogonal to the column vectors of **X**.
2. $\hat{\underline{\epsilon}}$ depends only on $\mathbf{P} = I - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$, and the unknown error vector, $\underline{\epsilon}$; *i.e.,* $\hat{\underline{\epsilon}}$ depends only on **X** and $\underline{\epsilon}$. The

standardized residual vector is independent of the unknown parameters, $\underline{\beta}$ and σ. Then, any statistics computed from $\underline{\hat{\epsilon}}$ will depend only on **X** and $\underline{\epsilon}$. This is a good reason for using the standardized residuals rather than the raw residuals. In 2.4 we use this fact to simplify the data we generate for fitting the appropriate models in linear regression.

We state some facts about the connection between the hat matrix, **V** and the ordinary residual vector. We do not use these in the sequel, but they are relevant to the definition of the 'leverage', used in the study of the residuals in the autoregressive cases.

1. $\text{Cov}(\mathbf{e}) = \text{Cov}(\mathbf{P}\,\underline{\epsilon}) = \mathbf{P}\text{Cov}(\underline{\epsilon})\mathbf{P}' = \mathbf{P}\mathbf{I}_n\mathbf{P}' = \mathbf{P}\mathbf{P}' = \mathbf{P}\mathbf{P} = \mathbf{P}$, since **P** is symmetric and idempotent.
2. $\text{Cov}(\mathbf{e}) = \mathbf{I}_n - \mathbf{V}$
3. $\text{Var}(e_i) = 1 - v_{ii}$, where v_{ii} is the i'th diagonal element of **V**. For $1 \le i \le n$, $0 \le v_{ii} \le 1$, $k_i = \sqrt{(1-v_{ii})}$ is the i'th leverage. This quantity is used to modify the standardized residuals in the autoregressive cases.

2.4 Forms of the Model

We now discuss the forms of the model which we shall fit to data. The generation of the data will be discussed in section 2.5..

1. The **Simple Linear Model:**

   $y_i = \beta_0 + \beta_1 x_i + \sigma\epsilon_i$, for $i=1,\ldots,n$. For this model the mle's $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\sigma}$ are given by the equations:

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad \ldots(2.4.1)$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \qquad \ldots(2.4.2)$$

$$\hat{\sigma} = \sqrt{\left\{ \frac{1}{n} \left[\sum_{i=1}^{n} (y_i - \bar{y})^2 - \frac{\left[\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})\right]^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right] \right\}} \qquad \ldots(2.4.3)$$

2. The **Quadratic Model:**

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \sigma\epsilon_i, \; i=1,\ldots,n \qquad \ldots(2.4.4)$$

3. The **Cubic Model:**

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \sigma\epsilon_i, \qquad \ldots(2.4.5)$$

$i=1,\ldots,n$.

The mle's, $\underline{\hat{\beta}}$ and $\hat{\sigma}$, for forms 2, and 3 are obtained by using the IMSL subroutine LEQ2S.

4. The **Simple Trigonometric Model:**

$$y_i = \beta_0 + \beta_1 \cos\left(\frac{2\pi}{T}\right)x_i + \beta_2 \sin\left(\frac{2\pi}{T}\right)x_i + \sigma\epsilon_i, \qquad \ldots(2.4.6)$$

$i=1,\ldots,n$

This model provides an approximate description of some phenomena of a periodic type. **T** in equation (2.4.7) is a known positive integer, the *fundamental period* of the system. We make certain restrictions on the x_i, **T**, and n, in order to simplify the computations for the mle's of the parameters, $\underline{\beta}$ and σ. These are :

a. The observed x_i are integers, 1,2,...,n. In many applications, x_i is the i'th minute, hour, day, month, or year. The observations, y_i may be hourly temperature, daily commodity prices, or annual rainfall;

b. The *fundamental period*, **T**, is a known positive integer, and $T \geq 3$. Note that if x_i =integer and $T = 1$, or 2, the sine term in the above equation is absent. The requirement that $T \geq 3$ keeps the sine term in the equation.

c. The number of observations, n, is a known multiple of T, the *fundamental period*: $n = CT$ where C is a positive integer. Hence the observations are taken over C fundamental periods, and the x_i assume the integer values $1,2,\ldots,T$, $T+1,\ldots,2T$, $2T+1,\ldots,CT$. For this model specification we can derive simple formulae for the mle's $\hat{\underline{\beta}}$ and $\hat{\sigma}$:

$$\hat{\beta}_0 = \bar{y}$$

$$\hat{\beta}_1 = \frac{n}{2} \sum_{i=1}^{n} y_i \cos\left(\frac{2\pi}{T}\right) i$$

$$\hat{\beta}_2 = \frac{n}{2} \sum_{i=1}^{n} y_i \sin\left(\frac{2\pi}{T}\right) i$$

$$\hat{\sigma} = \surd\left\{ \frac{1}{n} \left[\sum_{i=1}^{n} (y_i - \bar{y})^2 - \frac{n}{2} (\hat{\beta}_1^2 + \hat{\beta}_2^2) \right] \right\}$$ [1]

2.5 Generated Data

For the null hypothesis distributions of the statistics derived from the residuals, we need to generate data which conform to the form of the model being studied. Since we showed that the statistics do not depend on the parameters, we need only consider generated data given by the *canonical* form of the data. Thus, for the simple linear model: $y_i = \beta_0 + \beta_1 x_i + \sigma\epsilon_i$, we

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[1] see Graybill(1976)

generate data with $\beta_0 = 0$, $\beta_1 = 1$, and $\sigma = 1$. For the quadratic case, $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \sigma\epsilon_i$, we generate data with $\beta_0 = 0$, $\beta_1 = 1 = \beta_2$, and $\sigma = 1$. We treat the cubic case in a similar manner, adding $\beta_3 = 1$. For the simple trigonometric model, we generate data with $\beta_0 = 0$, and $\beta_1 = 1 = \beta_1 = \beta_2 = \sigma$.

2.6 The Design Matrix

In 2.2 we showed that the standardized residuals depend only on **X** and $\underline{\epsilon}$. Hence the EDF statistics derived from these residuals may depend on **X** and $\underline{\epsilon}$. By keeping $\underline{\epsilon}$ fixed (by using the same seed to start the pseudo-random number generation) and varying **X** within any assumed form of the model, we may examine the effect of **X** on the limiting distributions of the statistics, or on the rates of convergence to the limits. We state the different designs used in this study. For the simple linear model, the designs used were:

1. case(a):

$$x_i = \begin{cases} -1, & \text{if } 2i \le n \\ 1, & \text{if } 2i > n \end{cases}$$

2. case(b): $x_i = i$, $i = 1,\ldots,n$
3. case(e): $x_i = (1.01)^i$, $i = 1,\ldots,n$
4. case(sq): $x_i = \sqrt{i}$, $i = 1,\ldots,n$
5. case(lg): $x_i = \ln i$, $i = 1,\ldots,n$

For the quadratic form of the model, 2 cases were examined :

1. case(q): $x_i = i/n$, $i = 1,\ldots,n$
2. case(qsq): $x_i = \sqrt{i}$, $i = 1,\ldots,n$

Only one case was examined for the cubic model :

1. case(cu): $x_i = i/n$, $i = 1,\ldots,n$

The simplifying restrictions imposed on the simple trigonometric form of the model limited the number of cases studied to one:

1. case(tr): $x_i = i$, $i = 1,\ldots,n$

For the quadratic and cubic forms of the model, the choice of $\mathbf{X}$, the design matrix, was limited by the computational difficulties involved in the inversion of the matrix, $\mathbf{X}'\mathbf{X}$.

CHAPTER 3

THE STATIONARY AUTOREGRESSIVE PROCESS

The general linear autoregressive process is of the form:

$$y_i = \mu+\phi_1(y_{i-1}-\mu)+\phi_2(y_{i-2}-\mu)+ \ldots+\phi_p(y_{i-p}-\mu)+\sigma\epsilon_i$$

and is called an autoregressive process of order p, or, an AR(p) process.In this study we consider only the AR(1) and AR(2) processes. The equations governing these processes are

$$AR(1): y_i = \mu + \rho(y_{i-1}-\mu)+ \sigma\epsilon_i$$

$$AR(2): y_i = \mu + \rho_1(y_{i-1}-\mu)+\rho_2(y_{i-2}-\mu)+\sigma\epsilon_i$$

where ϵ_i, $i = 1,2,\ldots,n$, are independent standard normal errors.

3.1 Stationarity

A stochastic process is said to be *strictly stationary* if its properties are unaffected by a change of time origin, *i.e.*, if the joint probability distribution of r observations, of the process made at any set of times $i_1,\ldots,i_r$, is the same as that associated with the r observations, made at times $i_1+k,\ldots,i_r+k$. For the AR(1) process, stationarity requires $|\rho|< 1$.

For the AR(2) process, the condition of being stationary requires the following inequalities be satisfied:[1]

$$|\rho_1| < 1$$

$$|\rho_2| < 1$$

$$\rho_1^2 < (\rho_2+1) / 2$$

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[1]see Box and Jenkins(1976)

3.2 The Standardized Residuals

For the AR(1) process, following Pierce(1985), we regress y_i on y_{i-1}. For the AR(2) process we regress y_i on y_{i-1} and y_{i-2}. We write the equation for the AR(1) model in the form : $y_i = \lambda+\rho y_{i-1}+\sigma\epsilon_i$, where $\lambda = \mu(1-\rho)$. After fitting the regression, we get the standardized residuals in the form:

$\hat{\epsilon}_i = (y_i-\hat{\lambda}-\hat{\rho}y_{i-1})/\ \hat{\sigma}$. For the AR(2) process, we write the model equation in the form: $y_i = \lambda + \rho_1 y_{i-1}+\rho_2 y_{i-2} + \sigma\epsilon_i$, where $\lambda = \mu(1-\rho_1-\rho_2)$. To find the maximum likelihood estimates of the coefficients we regress y_i on y_{i-1} and y_{i-2}. The standardized residual for the AR(2) process is

$$\hat{\epsilon}_i = (y_i - \hat{\lambda} - \hat{\rho}_1\ y_{i-1}-\hat{\rho}_2\ y_{i-2})/\ \hat{\sigma}.$$

The argument deriving the large sample distributions of the EDF statistics of the standardized residuals from autoregression is in terms of the *empirical process*: $y_n(t) = \sqrt{n}[F_n(t)-t]$. As in the case of linear regression, the limiting process, $y(t)$, is a Gaussian process, with mean zero. The covariance function, $\rho(s,t)$, depends on Φ, the standard normal cdf, but is independent of the parameters, λ, ρ_1, ρ_2. Moreover, the limiting process, $y(t)$, is the same as in the single sample case.

CHAPTER 4

THE EDF STATISTICS AND THE EMPIRICAL PROCESS

4.1 Computational Form of the EDF Statistics

Assume that the independent observations x_i, $i=1,\ldots,n$, come from a population with a cdf, $F(x)$, where $F(.)$ may contain estimated parameters. Let $z_i = F(x_{(i)})$. Then $0 \le z_i \le 1$, and $z_1 \le \ldots \le z_n$. The Kolmogorov-Smirnov Statistics are:

$$D_n^+ = \max_{1 \le i \le n} (i/n - z_i)$$

$$D_n^- = \max_{1 \le i \le n} (z_i - (i-1)/n)$$

The Kolmogorov statistic is :

$$D_n = \max(D_n^+, D_n^-).$$

The Cramér-von Mises statistic is:

$$W_n^2 = \sum_{i=1}^{n} [z_i - (2i-1)/2n]^2 + 1/(12n).$$

The Watson statistic is:

$$U_n^2 = W_n^2 - n(\bar{z} - .5)^2 \text{ , where , } \bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i.$$

The Anderson-Darling statistic is:

$$A_n^2 = -\frac{1}{n}\left[\sum_{i=1}^{n} (2i-1)\{\ln z_i + \ln(1 - z_{n+1-i})\}\right] - n.$$

The Kolmogorov statistic is usually used in the form $\sqrt{n}D_n$ rather than as D_n above.

Stephens(1974) modifies the EDF statistics so that a single table of asymptotic percentage points may be used in performing

EDF goodness of fit tests for any finite sample size, n. The modifications for the 4 EDF statistics given above, appropriate in the single sample case are:

1. $\sqrt{n}D_n(1 - 0.1/\sqrt{n} + 0.85/n)$
2. $W_n^2\ (1 + 0.5/n)$
3. $U_n^2\ (1 + 0.5/n)$
4. $A_n^2\ (1 + 0.75/n + 2.25/n^2)$ [1]

4.2 The Empirical Process for the Standardized Residuals

The standardized residuals are given by equation (1.3) as $\hat{\epsilon}_i = (y_i - \underline{x}_i' \hat{\underline{\beta}})/\hat{\sigma}$. Assume that we have ordered the $\hat{\epsilon}_i$. Let $z_i = \Phi(\hat{\epsilon}_i)$. The z_i are ordered in ascending order, and satisfy $0 \le z_i \le 1$, for $1 \le i \le n$. As in section 1.6 we can form the EDF of the z_i, $F_n(t)$, and then get the empirical process:

$$y_n(t) = \sqrt{n}\,[F_n(t) - t]$$

Pierce and Kopecky show that for the linear regression case regularity conditions are satisfied so that the empirical process converges weakly to a Gaussian process with mean zero and covariance function $\rho(s,t)$ which depends only on Φ and not on the parameters estimated. Moreover, they show that the limiting Gaussian process is identical to that obtained in the case 3 example.

We now express the EDF statistics in terms of the empirical process.

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[1] The modification for A_n^2 is different from that given in Stephens(1974), see Table1.3, p732.The new modification is from Stephens (personal communication).

$$\sqrt{n}\, D_n = \sup_{0 \le t \le 1} |y_n(t)|$$

$$W_n^2 = \int_0^1 y_n(t)^2 \, dt$$

$$U_n^2 = \int_0^1 (y_n(t) - \bar{y})^2 \, dt$$

$$A_n^2 = \int_0^1 \frac{y_n^2(t)}{t(1-t)} dt$$

W_n^2 , U_n^2 , A_n^2 are the quadratic EDF statistics. The EDF statistics are functionals of the empirical process. Pierce and Kopecky(1979), following Durbin(1973a), argue that continuity conditions are satisfied, so that the EDF statistics will converge weakly to the same functionals of the Gaussian limiting process, y(t), for the empirical process; *eg.*,

$$\sqrt{n} D_n \xrightarrow{D} \sup_{0 \le t \le 1} |y(t)|$$

$$W_n^2 \xrightarrow{D} \int_0^1 y(t)^2 \, dt$$

$$U_n^2 \xrightarrow{D} \int_0^1 (y(t) - \bar{y})^2 \, dt$$

$$A_n^2 \xrightarrow{D} \int_0^1 \frac{y^2(t)}{t(1-t)} \, dt$$

Since y(t) is the same for the regression case as for the single sample case, the large sample distributions of the EDF statistics, under the null hypothesis, *i.e.*, the model is the right one and the error distribution is normal, for the regression and single sample cases will be identical.

CHAPTER 5

THE SIMULATION DESIGN

5.1 The Null Hypothesis Case

For each case mentioned in chapter 2.5 we generated 10,000 samples of size, n =5, 8,10,12,15,20,30,50, 60, and 100, except for case(cu), where the lowest sample size is n = 8. The IMSL reference library routine GGNML was used to generate n pseudo-random standard normal errors. The normal errors were used to construct data appropriate to the model being fitted. For cases (q) and (cu), the IMSL subroutine LEQ2S was used to invert the matrix **X'X** in order to solve the normal equations.

For all the autoregression cases, when we needed to fit a model for a sample of size n, we generated a sample of size n+10, and used the last n pairs (y_i, y_{i-1}) for the AR(1) model, or, the last n triples (y_i, y_{i-1}, y_{i-2}) for the AR(2) model.

After fitting the model, the standardized residuals were computed. The IMSL routine MDNOR was used to compute the normal cdf of the standardized residuals.

For the autoregression cases we used the unbiased equivalent of $\hat{\sigma}$ in computing the standardized residuals $\hat{\epsilon}_i$, i = 1,..., n. Further, $\hat{\epsilon}_i/k_i$, the studentized residual, was used, rather than $\hat{\epsilon}_i$, in computing the EDF statistics.[1] This study uses the values

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[1]Recall 2.3, k_i is the i'th leverage for the design matrix, **X**. see Pierce(1985)

of the EDF statistics computed using the computational forms, rather than the *empirical process* , $y_n(t)$. The EDF statistics were computed using the case 3 modifications of Stephens(1974) given in chapter 4. The distribution of the 10,000 values of each of the 4 statistics was obtained, and the upper tail 15%,10%,5%,2.5%, and 1% were recorded, in Tables 1.1,2.1,3.1,4.1. The upper tails were recorded since the EDF tests are usually upper tail tests. Tables for the single sample case were also recorded for comparison.The tables show the same rate of convergence to the asymptotic percentage points for the single sample case and the null hypothesis case in both the ordinary regression and the autoregression cases.

5.2 The Simulation Design : non null case

For the alternative case 1,000 samples were used. Generally the sample sizes studied were n = 20,30,50. table 1.a gives a mnemonic key for the cases looked at. The IMSL routine GGUBS was used to generate uniform pseudo-random variates. Standard transformations were then applied to produce the U(-.5,.5) and Laplace(0,2) variates used in the power study. In general underfitting a linear model to quadratic data causes confounding of the effects of model mis-specification and non-normality. A similar situation occurs when an AR(1) model is fitted to AR(2) data. A separate power study was done for the cases when the correct model is fitted, and a mis-specified model is fitted, to data with non-normal errors. The alternative error distributions used are the Laplace(0,2) and the U(-.5,.5) distributions. In the case when the model was correct, the powers of the EDF tests were compared with those for the corresponding tests in the single sample case in Stephens(1974) Tables 5, and, 6,pp734,735.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

Tables 7.1 to 7.4 give a comparison of the estimated sizes of the tests compared with the nominal levels, for sample sizes 10 and 20. For n = 10, the estimated sizes of the 4 EDF statistics corresponding to the 10% nominal level, lie between 10.0% and 13.0%. For n = 20, the estimated sizes corresponding to the 10% level, lie between 10.0% and 11.4%. The estimated sizes corresponding to the 5% nominal level lie, for n = 10, between 4.2% and 6.7%; for n = 20, the corresponding range of the estimated size is 4.6% to 5.8%. Since, the estimated size is always close to the nominal level we conclude that sample size 20 is adequate to justify the use of the test.

The greatest difference between the estimated size and the nominal level occurs for A^2_n. This is seen in Tables 7.1 to 7.4.

Tables 1.1 - 4.1 show the Monte Carlo upper tail percentage points for the null hypothesis cases for each of the 4 EDF statistics. In each case the correct model is fitted to the data with N(0,1) errors. Case(1s) is Stephens case 3: the single sample case where only a mean and a scale are fitted. The last row of percentage points are the case 3 asymptotic percentage points.[1]

Examination of Tables 1.1 to 4.1 for the different designs, **X**, shows that from n = 10 the tables for the cases are very

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[1]see Stephens (1974) Table 1.3 p732.

similar to the case 3 table. This suggests that the rate of convergence to the case 3 asymptotic points, for the null hypothesis distributions of the EDF statistics is independent of the design.

Throughout this study we used the Stephens case 3 modifications for the EDF statistics. These modifications are appropriate for the single sample case. Their purpose is to enable the tester to use one table of asymptotic percentage points when performing a test for any sample size, n. For the null hypothesis case where the correct model is fitted to the appropriate data with nornmal errors, the similarity of the tables to the tables for the single sample case suggests that the case 3 modifications are appropriate for the linear regression case. In that case to carry out a test for normality of the error distribution, for any finite sample size, n, the tester would treat the residuals as though they formed a single independent sample of size = n, and perform the case 3 test as in 1.6.

The same tentative suggestion may be made for the autoregression case.

If a linear model is fitted to quadratic data (underfitting), the effects of non-normality are confounded with those of model mis-specification, for sample size N > 20. Large samples will nearly always reject normality. This is seen in Tables 1.1.a to 4.1.a, for cases

These tables show a maximum difference between the estimated size and the nominal level of 2.1 percentage points at the 10% nominal level, and 1.4% at the 5% level in any of the cases:(uflq1) and (uflq2).

1. overfitting a quadratic model to linear data.
2. overfitting a cubic model to quadratic data.
3. overfitting an AR(2)model to AR(1) data.

For the autoregressive case, underfitting an AR(1) model to AR(2) data with normal errors, leads to rejection of normality by the EDF statistics. This is seen in Tables 1.1a to 4.1a, case(ufar12). In both the linear and autoregressive cases underfitting can seriously undermine the test for normality. If in the linear regression case, the test rejects normality after fitting a linear model, the pattern of residuals should be checked. A set of residuals of one sign followed by a set of the other sign should alert thetester to the fact that he/she may have fitted a linear model to quadratic data. In the autoregressive case, case(ofar21) shows that, for the 4 EDF statistics, the maximum difference between the estimated size and the nominal level is 2.3% at the 10% level, and 1.6% at the 5% level. Overfitting an AR(2) model to suspected AR(1) data is recommended.

The U(-.5,.5) and Laplace distributions are natural symmetric alternatives to the normal distribution. The U(-.5,.5) is short-tailed, while the Laplace is long-tailed. We compared the power of the EDF test for normality against either alternative with Tables 5, 6 of Stephens(1974), where the power study is done for the EDF tests for case 3. This appears in Table 6. The powers are smaller in the regression case. This suggests that fitting a line to the data produces a better fit than if only a mean is fitted. The errors are then smaller and the test is then less likely to reject normality. This is shown in Tables 1.1b, 1.1d - 4.1b, 4.1d.

Mis-specification of the model alters the power of the test against both alternatives.

If the model is underfitted to the data and the error is non-normal, the power of the EDF test is greater than that for the single sample case. If the model is overfitted to the data, the power of the test is less than that for the single sample case. These power studies appear in Tables 1.1c, 1.1e to 4.1c, 4.1e.

6.1 Further Research

The Pierce-Kopecky result holds for any error distribution. Non-normal error distributions are now common in Survival Analysis.[2]. Further work in this direction should test the residuals from regression for the Logistic and Extreme-value distributions for which Stephens(1977,1979) has worked out the details and provided tables of the asymptotic percentage points, for the case of the single independent sample.

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[2]see Lawless(1981)

Comparison of the power of the EDF statistics against the U(-.5,.5) and the Laplace alternatives with the values obtained in Stephens(1974)[3], shows that, for all 4 statistics, the powers are only slightly smaller in both the ordinary regression and autoregression cases. This is the situation where the correct model is fitted, but the error is non-normal. The closeness of the powers in the regression case and case 3 suggests that the large sample distributions of the EDF statistics under the alternative distributions are the same as those for the case 3 situation.

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[3]see Table5, p734, and Table6, p735

Table 1.1 : Monte Carlo Upper tail significance points for $\sqrt{n}D_n$ based on 10,000 samples

Case(1s)

N	15.0	10.0	5.0	2.5	1.0
5	0.793	0.838	0.898	0.967	1.047
8	0.778	0.825	0.896	0.961	1.038
10	0.780	0.826	0.895	0.961	1.039
12	0.777	0.823	0.898	0.963	1.051
15	0.779	0.825	0.901	0.965	1.029
20	0.774	0.818	0.889	0.953	1.028
30	0.772	0.817	0.900	0.965	1.057
40	0.770	0.815	0.889	0.952	1.030
50	0.775	0.821	0.895	0.957	1.048
60	0.774	0.820	0.893	0.964	1.039
100	0.777	0.825	0.898	0.960	1.044
∞	0.775	0.819	0.895	0.955	1.035

Case(a)

N	15.0	10.0	5.0	2.5	1.0
5	0.778	0.826	0.883	0.920	0.951
8	0.792	0.836	0.904	0.964	1.037
10	0.786	0.831	0.902	0.966	1.041
12	0.785	0.832	0.900	0.966	1.033
15	0.784	0.835	0.906	0.972	1.051
20	0.774	0.820	0.896	0.966	1.056
30	0.773	0.818	0.894	0.964	1.040
40	0.774	0.818	0.883	0.952	1.029
50	0.777	0.825	0.899	0.957	1.034
60	0.776	0.822	0.894	0.960	1.049
100	0.776	0.826	0.902	0.960	1.037
∞	0.775	0.819	0.895	0.955	1.035

Case(b)

N	15.0	10.0	5.0	2.5	1.0
5	0.808	0.857	0.923	0.978	1.076
8	0.796	0.841	0.910	0.969	1.042
10	0.790	0.835	0.903	0.970	1.047
12	0.785	0.828	0.902	0.967	1.042
15	0.779	0.828	0.903	0.972	1.044
20	0.777	0.825	0.897	0.960	1.050
30	0.777	0.823	0.902	0.967	1.052
40	0.772	0.817	0.889	0.950	1.025
50	0.774	0.819	0.894	0.968	1.049
60	0.774	0.822	0.898	0.961	1.045
100	0.778	0.824	0.897	0.956	1.032
∞	0.775	0.819	0.895	0.955	1.035

Table 1.1 : Monte Carlo Upper tail significance points for $\sqrt{n}D_n$ based on 10,000 samples.

Case(e)

N	15.0	10.0	5.0	2.5	1.0
5	0.807	0.857	0.924	0.977	1.074
8	0.795	0.841	0.909	0.971	1.039
10	0.791	0.834	0.903	0.972	1.047
12	0.784	0.829	0.901	0.966	1.041
15	0.780	0.827	0.902	0.973	1.044
20	0.778	0.825	0.899	0.960	1.050
30	0.778	0.823	0.903	0.966	1.049
40	0.772	0.816	0.887	0.951	1.025
50	0.775	0.821	0.897	0.971	1.047
60	0.773	0.822	0.899	0.962	1.041
100	0.779	0.823	0.899	0.958	1.032
∞	0.775	0.819	0.895	0.955	1.035

Case(q)

N	15.0	10.0	5.0	2.5	1.0
5	0.823	0.880	0.931	0.954	0.965
8	0.786	0.831	0.895	0.955	1.019
10	0.785	0.832	0.896	0.958	1.025
12	0.775	0.821	0.895	0.961	1.028
15	0.781	0.824	0.894	0.954	1.033
20	0.775	0.819	0.890	0.955	1.026
30	0.779	0.827	0.899	0.965	1.047
40	0.775	0.819	0.889	0.956	1.039
50	0.777	0.823	0.893	0.960	1.045
60	0.777	0.823	0.895	0.961	1.044
100	0.779	0.824	0.898	0.964	1.056
∞	0.775	0.819	0.895	0.955	1.035

Case(sq)

N	15.0	10.0	5.0	2.5	1.0
5	0.819	0.863	0.917	0.979	1.060
8	0.795	0.843	0.909	0.975	1.056
10	0.790	0.839	0.910	0.963	1.042
12	0.789	0.839	0.911	0.979	1.064
15	0.784	0.828	0.905	0.978	1.048
20	0.778	0.822	0.892	0.962	1.044
30	0.778	0.823	0.901	0.974	1.060
40	0.773	0.816	0.887	0.949	1.028
50	0.775	0.823	0.897	0.963	1.040
60	0.779	0.824	0.890	0.956	1.044
100	0.776	0.824	0.891	0.951	1.026
∞	0.775	0.819	0.895	0.955	1.035

TABLE 1.1 : Monte Carlo Upper tail significance points for $\sqrt{n}D_n$ based on 10,000 samples.

Case(lg)

N	15.0	10.0	5.0	2.5	1.0
5	0.827	0.872	0.928	0.976	1.041
8	0.796	0.841	0.915	0.977	1.052
10	0.791	0.837	0.909	0.977	1.052
12	0.783	0.830	0.900	0.967	1.039
15	0.785	0.832	0.903	0.967	1.039
20	0.778	0.821	0.893	0.957	1.043
30	0.774	0.823	0.898	0.967	1.053
40	0.773	0.819	0.890	0.947	1.023
50	0.778	0.825	0.899	0.968	1.040
60	0.775	0.821	0.894	0.960	1.039
100	0.776	0.824	0.897	0.959	1.028
∞	0.775	0.819	0.895	0.955	1.035

Case(qsq)

N	15.0	10.0	5.0	2.5	1.0
5	0.776	0.780	0.784	0.804	0.817
8	0.785	0.831	0.898	0.958	1.018
10	0.779	0.829	0.901	0.959	1.019
12	0.778	0.827	0.894	0.959	1.036
15	0.782	0.825	0.891	0.960	1.043
20	0.776	0.823	0.898	0.958	1.041
30	0.776	0.821	0.896	0.968	1.044
40	0.772	0.817	0.891	0.958	1.035
50	0.775	0.822	0.895	0.959	1.049
60	0.776	0.820	0.892	0.959	1.043
100	0.781	0.826	0.897	0.962	1.045
∞	0.775	0.819	0.895	0.955	1.035

Case(cu)

N	15.0	10.0	5.0	2.5	1.0
8	0.786	0.834	0.906	0.957	1.013
10	0.794	0.835	0.907	0.970	1.041
12	0.780	0.827	0.897	0.951	1.021
15	0.783	0.830	0.903	0.971	1.055
20	0.772	0.818	0.889	0.954	1.028
30	0.778	0.825	0.896	0.964	1.039
40	0.775	0.818	0.888	0.953	1.027
50	0.779	0.827	0.901	0.969	1.050
60	0.776	0.822	0.896	0.963	1.055
100	0.778	0.825	0.901	0.966	1.046
∞	0.775	0.819	0.895	0.955	1.035

TABLE 1.1 : Monte Carlo Upper tail significance points for $\sqrt{n}D_n$ based on 10,000 samples.

N	Case(tr) 15.0	10.0	5.0	2.5	1.0
5	0.691	0.699	0.706	0.710	0.712
8	0.788	0.838	0.916	0.980	1.063
10	0.784	0.829	0.897	0.966	1.042
12	0.778	0.825	0.894	0.965	1.036
15	0.782	0.829	0.901	0.970	1.041
20	0.777	0.824	0.896	0.964	1.045
30	0.776	0.824	0.894	0.958	1.039
40	0.772	0.822	0.899	0.961	1.028
50	0.774	0.817	0.889	0.962	1.049
60	0.778	0.824	0.897	0.962	1.041
100	0.779	0.824	0.896	0.956	1.040
∞	0.775	0.819	0.895	0.955	1.035

N	AR(1) Case(μ=.5,ρ=.9,σ=1) 15.0	10.0	5.0	2.5	1.0
5	0.955	0.989	1.038	1.099	1.146
8	0.808	0.857	0.926	0.982	1.050
10	0.798	0.845	0.919	0.982	1.065
12	0.793	0.835	0.908	0.975	1.050
15	0.782	0.828	0.904	0.961	1.044
20	0.779	0.825	0.896	0.960	1.038
30	0.775	0.821	0.896	0.965	1.048
40	0.781	0.826	0.896	0.958	1.053
50	0.772	0.819	0.884	0.955	1.040
60	0.777	0.822	0.894	0.971	1.050
100	0.776	0.821	0.896	0.960	1.047
∞	0.775	0.819	0.895	0.955	1.035

N	AR(1) Case(μ=.5,ρ=.5,σ=1) 15.0	10.0	5.0	2.5	1.0
5	0.956	0.988	1.031	1.093	1.141
8	0.811	0.855	0.925	0.989	1.070
10	0.799	0.840	0.914	0.982	1.057
12	0.788	0.834	0.908	0.969	1.047
15	0.789	0.837	0.901	0.967	1.058
20	0.780	0.829	0.898	0.960	1.038
30	0.777	0.824	0.898	0.964	1.046
40	0.779	0.829	0.898	0.964	1.036
50	0.775	0.822	0.892	0.959	1.030
60	0.774	0.825	0.902	0.965	1.061
100	0.773	0.818	0.894	0.957	1.042
∞	0.775	0.819	0.895	0.955	1.035

TABLE 1.1 : Monte Carlo Upper tail significance points for $\sqrt{n}D_n$ based on 10,000 samples.

AR(1)Case(μ=3.5,ρ=-.9,σ=2)

N	15.0	10.0	5.0	2.5	1.0
5	0.840	0.882	0.938	0.988	1.063
8	0.802	0.843	0.916	0.977	1.059
10	0.791	0.835	0.909	0.974	1.056
12	0.790	0.838	0.915	0.978	1.060
15	0.786	0.834	0.907	0.974	1.058
20	0.783	0.830	0.902	0.967	1.047
30	0.775	0.822	0.896	0.953	1.032
40	0.778	0.825	0.901	0.962	1.051
50	0.771	0.816	0.887	0.951	1.032
60	0.776	0.824	0.898	0.969	1.059
100	0.775	0.819	0.894	0.967	1.037
∞	0.775	0.819	0.895	0.955	1.035

AR(2) Case(ρ_1=.5,ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.937	0.997	1.1 2	1.171	1.235
8	0.818	0.866	0.939	1.002	1.071
10	0.793	0.840	0.910	0.974	1.046
12	0.790	0.834	0.909	0.979	1.042
15	0.784	0.831	0.906	0.977	1.072
20	0.784	0.831	0.907	0.971	1.042
30	0.783	0.828	0.899	0.965	1.047
40	0.776	0.820	0.893	0.956	1.041
50	0.774	0.819	0.892	0.958	1.036
60	0.780	0.824	0.896	0.971	1.057
100	0.779	0.825	0.900	0.968	1.035
∞	0.775	0.819	0.895	0.955	1.035

AR(2) Case(ρ_1=.5,ρ_2=.4)

N	15.0	10.0	5.0	2.5	1.0
5	0.944	1.010	1.1 6	1.180	1.251
8	0.819	0.866	0.940	1.009	1.100
10	0.797	0.843	0.912	0.971	1.055
12	0.793	0.841	0.917	0.987	1.066
15	0.785	0.834	0.906	0.978	1.056
20	0.780	0.827	0.902	0.970	1.049
30	0.780	0.823	0.892	0.958	1.036
40	0.774	0.819	0.895	0.967	1.049
50	0.772	0.816	0.887	0.949	1.023
60	0.780	0.825	0.898	0.962	1.056
100	0.776	0.821	0.893	0.961	1.048
∞	0.775	0.819	0.895	0.955	1.035

TABLE 1.1 : Monte Carlo Upper tail significance points for $\sqrt{n}D_n$ based on 10,000 samples.

AR(2) Case(ρ_1=-.5, ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.938	1.000	1.095	1.171	1.242
8	0.816	0.863	0.936	0.995	1.080
10	0.803	0.850	0.918	0.989	1.064
12	0.794	0.842	0.905	0.978	1.054
15	0.790	0.834	0.909	0.984	1.068
20	0.782	0.827	0.904	0.976	1.062
30	0.778	0.826	0.903	0.972	1.056
40	0.782	0.828	0.902	0.976	1.052
50	0.775	0.820	0.885	0.951	1.035
60	0.772	0.819	0.894	0.972	1.057
100	0.777	0.827	0.897	0.965	1.044
∞	0.775	0.819	0.895	0.955	1.035

AR(2) Case(ρ_1=-.5, ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.891	0.948	1.031	1.094	1.174
8	0.805	0.852	0.923	0.988	1.059
10	0.796	0.842	0.914	0.975	1.053
12	0.785	0.836	0.910	0.970	1.052
15	0.786	0.833	0.907	0.967	1.052
20	0.780	0.827	0.898	0.967	1.041
30	0.777	0.823	0.900	0.967	1.049
40	0.777	0.823	0.899	0.965	1.045
50	0.769	0.815	0.884	0.947	1.032
60	0.776	0.823	0.893	0.957	1.042
100	0.776	0.824	0.901	0.971	1.060
∞	0.775	0.819	0.895	0.955	1.035

AR(2) Case(ρ_1=.1, ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.851	0.898	0.957	1.020	1.132
8	0.806	0.854	0.926	0.983	1.055
10	0.794	0.840	0.907	0.972	1.057
12	0.793	0.841	0.915	0.979	1.051
15	0.787	0.836	0.908	0.972	1.060
20	0.779	0.828	0.900	0.960	1.044
30	0.775	0.822	0.894	0.961	1.043
40	0.777	0.827	0.899	0.964	1.056
50	0.767	0.815	0.890	0.961	1.035
60	0.777	0.824	0.896	0.961	1.040
100	0.777	0.823	0.898	0.964	1.043
∞	0.775	0.819	0.895	0.955	1.035

Table 2.1 : Monte Carlo Upper tail significance points for W_n^2 based on 10,000 samples

case(1s)

N	15.0	10.0	5.0	2.5	1.0
5	0.091	0.102	0.121	0.144	0.173
8	0.090	0.102	0.123	0.146	0.173
10	0.091	0.104	0.126	0.149	0.177
12	0.091	0.104	0.127	0.150	0.179
15	0.093	0.106	0.130	0.151	0.178
20	0.091	0.105	0.126	0.148	0.177
30	0.090	0.103	0.127	0.152	0.182
40	0.090	0.103	0.124	0.148	0.178
50	0.091	0.105	0.127	0.148	0.181
60	0.091	0.104	0.126	0.148	0.179
100	0.090	0.104	0.128	0.151	0.182
∞	0.091	0.104	0.126	0.148	0.178

case(a)

N	15.0	10.0	5.0	2.5	1.0
5	0.090	0.099	0.111	0.118	0.125
8	0.095	0.105	0.124	0.145	0.172
10	0.094	0.106	0.127	0.148	0.176
12	0.094	0.107	0.130	0.150	0.171
15	0.095	0.108	0.129	0.152	0.183
20	0.092	0.105	0.127	0.146	0.178
30	0.091	0.105	0.126	0.151	0.181
40	0.091	0.103	0.125	0.146	0.178
50	0.091	0.104	0.127	0.150	0.184
60	0.092	0.105	0.126	0.147	0.178
100	0.092	0.105	0.127	0.150	0.181
∞	0.091	0.104	0.126	0.148	0.178

case(b)

N	15.0	10.0	5.0	2.5	1.0
5	0.092	0.104	0.128	0.150	0.182
8	0.094	0.106	0.127	0.147	0.174
10	0.094	0.107	0.128	0.151	0.177
12	0.092	0.106	0.128	0.150	0.176
15	0.093	0.107	0.130	0.151	0.180
20	0.092	0.106	0.128	0.148	0.179
30	0.091	0.105	0.127	0.152	0.181
40	0.090	0.103	0.124	0.149	0.177
50	0.091	0.104	0.128	0.149	0.180
60	0.091	0.104	0.125	0.148	0.178
100	0.091	0.104	0.126	0.149	0.183
∞	0.091	0.104	0.126	0.148	0.178

Table 2.1 : Monte Carlo Upper tail significance points for W_n^2 based on 10,000 samples

case(e)

N	15.0	10.0	5.0	2.5	1.0
5	0.091	0.104	0.127	0.150	0.181
8	0.094	0.106	0.127	0.147	0.174
10	0.094	0.107	0.128	0.151	0.177
12	0.092	0.106	0.128	0.150	0.177
15	0.093	0.107	0.130	0.151	0.179
20	0.092	0.106	0.128	0.148	0.179
30	0.091	0.104	0.127	0.152	0.181
40	0.090	0.103	0.125	0.149	0.176
50	0.091	0.104	0.128	0.149	0.181
60	0.091	0.104	0.125	0.149	0.179
100	0.091	0.104	0.127	0.150	0.181
∞	0.091	0.104	0.126	0.148	0.178

case(q)

N	15.0	10.0	5.0	2.5	1.0
5	0.086	0.100	0.115	0.122	0.125
8	0.091	0.103	0.123	0.139	0.162
10	0.093	0.105	0.123	0.144	0.169
12	0.092	0.103	0.124	0.144	0.171
15	0.094	0.106	0.127	0.148	0.172
20	0.091	0.104	0.126	0.147	0.174
30	0.091	0.105	0.128	0.149	0.182
40	0.090	0.103	0.125	0.147	0.177
50	0.091	0.104	0.128	0.149	0.179
60	0.091	0.103	0.124	0.148	0.178
100	0.090	0.104	0.126	0.149	0.182
∞	0.091	0.104	0.126	0.148	0.178

case(sq)

N	15.0	10.0	5.0	2.5	1.0
5	0.095	0.105	0.125	0.145	0.180
8	0.094	0.107	0.128	0.148	0.172
10	0.095	0.106	0.129	0.151	0.176
12	0.095	0.107	0.130	0.153	0.186
15	0.093	0.106	0.128	0.151	0.177
20	0.092	0.106	0.127	0.148	0.177
30	0.093	0.106	0.129	0.151	0.183
40	0.091	0.103	0.125	0.146	0.178
50	0.091	0.104	0.127	0.149	0.180
60	0.091	0.104	0.127	0.149	0.177
100	0.091	0.104	0.126	0.149	0.180
∞	0.091	0.104	0.126	0.148	0.178

Table 2.1 : Monte Carlo Upper tail significance points for W_n^2 based on 10,000 samples

case(lg)

N	15.0	10.0	5.0	2.5	1.0
5	0.099	0.112	0.125	0.137	0.162
8	0.094	0.107	0.127	0.148	0.180
10	0.094	0.107	0.128	0.152	0.180
12	0.093	0.106	0.127	0.149	0.178
15	0.095	0.108	0.129	0.151	0.181
20	0.092	0.105	0.126	0.148	0.178
30	0.091	0.105	0.127	0.150	0.182
40	0.091	0.103	0.124	0.147	0.176
50	0.092	0.105	0.128	0.149	0.182
60	0.091	0.104	0.126	0.147	0.177
100	0.091	0.104	0.127	0.149	0.183
∞	0.091	0.104	0.126	0.148	0.178

case(qsq)

N	15.0	10.0	5.0	2.5	1.0
5	0.077	0.081	0.087	0.090	0.090
8	0.092	0.103	0.121	0.140	0.161
10	0.092	0.104	0.125	0.144	0.168
12	0.092	0.104	0.125	0.146	0.175
15	0.094	0.106	0.128	0.149	0.177
20	0.091	0.104	0.125	0.148	0.174
30	0.091	0.104	0.126	0.150	0.180
40	0.090	0.103	0.123	0.145	0.174
50	0.092	0.104	0.128	0.149	0.178
60	0.092	0.104	0.125	0.148	0.177
100	0.091	0.105	0.128	0.149	0.184
∞	0.091	0.104	0.126	0.148	0.178

case(cu)

N	15.0	10.0	5.0	2.5	1.0
8	0.091	0.104	0.126	0.144	0.175
10	0.094	0.106	0.128	0.147	0.171
12	0.092	0.104	0.124	0.143	0.172
15	0.093	0.106	0.128	0.151	0.174
20	0.090	0.103	0.125	0.145	0.169
30	0.092	0.105	0.127	0.149	0.176
40	0.092	0.103	0.125	0.146	0.178
50	0.092	0.104	0.128	0.150	0.176
60	0.091	0.104	0.125	0.147	0.178
100	0.091	0.105	0.125	0.150	0.180
∞	0.091	0.104	0.126	0.148	0.178

Table 2.1 : Monte Carlo Upper tail significance points for W_n^2 based on 10,000 samples

N	case(tr) 15.0	10.0	5.0	2.5	1.0
5	0.072	0.077	0.081	0.083	0.084
8	0.095	0.110	0.136	0.160	0.190
10	0.092	0.105	0.125	0.147	0.174
12	0.092	0.104	0.126	0.147	0.177
15	0.092	0.105	0.128	0.150	0.178
20	0.092	0.105	0.128	0.150	0.179
30	0.092	0.104	0.126	0.147	0.172
40	0.091	0.104	0.127	0.151	0.178
50	0.091	0.103	0.125	0.149	0.183
60	0.091	0.104	0.126	0.148	0.180
100	0.091	0.104	0.128	0.150	0.178
∞	0.091	0.104	0.126	0.148	0.178

N	AR(1)case(μ=.5,ρ=.9,σ=1) 15.0	10.0	5.0	2.5	1.0
5	0.126	0.138	0.155	0.173	0.189
8	0.099	0.111	0.131	0.151	0.178
10	0.097	0.110	0.132	0.152	0.183
12	0.095	0.108	0.130	0.151	0.185
15	0.094	0.107	0.128	0.148	0.180
20	0.093	0.107	0.128	0.149	0.180
30	0.091	0.105	0.128	0.152	0.181
40	0.092	0.105	0.127	0.149	0.178
50	0.090	0.103	0.126	0.146	0.172
60	0.090	0.103	0.126	0.149	0.178
100	0.089	0.102	0.125	0.147	0.176
∞	0.091	0.104	0.126	0.148	0.178

N	AR(1)case(μ=.5,ρ=.5,σ=1) 15.0	10.0	5.0	2.5	1.0
5	0.126	0.137	0.155	0.172	0.187
8	0.099	0.111	0.130	0.151	0.179
10	0.097	0.110	0.130	0.153	0.182
12	0.095	0.108	0.129	0.152	0.180
15	0.094	0.107	0.130	0.150	0.179
20	0.094	0.107	0.129	0.152	0.179
30	0.092	0.104	0.127	0.150	0.180
40	0.092	0.106	0.127	0.149	0.175
50	0.091	0.104	0.127	0.148	0.176
60	0.091	0.103	0.126	0.148	0.175
100	0.090	0.103	0.125	0.147	0.175
∞	0.091	0.104	0.126	0.148	0.178

Table 2.1 : Monte Carlo Upper tail significance points for W_n^2 based on 10,000 samples

AR(1)case(μ=3.5, ρ=-.9, σ=2)

N	15.0	10.0	5.0	2.5	1.0
5	0.103	0.114	0.132	0.147	0.170
8	0.097	0.109	0.130	0.151	0.180
10	0.094	0.106	0.130	0.150	0.180
12	0.095	0.108	0.129	0.151	0.179
15	0.094	0.108	0.131	0.152	0.182
20	0.093	0.107	0.130	0.152	0.183
30	0.092	0.105	0.128	0.150	0.176
40	0.092	0.104	0.126	0.153	0.183
50	0.091	0.103	0.125	0.146	0.175
60	0.090	0.103	0.126	0.150	0.186
100	0.091	0.104	0.126	0.149	0.179
∞	0.091	0.104	0.126	0.146	0.178

AR(2) case(ρ_1=.5, ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.136	0.154	0.189	0.213	0.238
8	0.103	0.117	0.138	0.158	0.192
10	0.096	0.109	0.131	0.150	0.178
12	0.096	0.109	0.131	0.153	0.182
15	0.094	0.107	0.130	0.156	0.189
20	0.093	0.106	0.128	0.153	0.186
30	0.093	0.107	0.128	0.149	0.179
40	0.091	0.104	0.126	0.150	0.180
50	0.091	0.103	0.125	0.144	0.173
60	0.091	0.105	0.129	0.149	0.183
100	0.091	0.105	0.128	0.149	0.178
∞	0.091	0.104	0.126	0.148	0.178

AR(2) case(ρ_1=.5, ρ_2=.4)

N	15.0	10.0	5.0	2.5	1.0
5	0.137	0.156	0.189	0.217	0.240
8	0.103	0.116	0.138	0.163	0.195
10	0.097	0.110	0.130	0.151	0.176
12	0.096	0.110	0.132	0.155	0.187
15	0.094	0.107	0.129	0.153	0.181
20	0.093	0.106	0.128	0.149	0.184
30	0.093	0.106	0.128	0.150	0.181
40	0.092	0.104	0.127	0.150	0.179
50	0.091	0.102	0.125	0.145	0.171
60	0.091	0.105	0.127	0.149	0.175
100	0.091	0.104	0.127	0.151	0.183
∞	0.091	0.104	0.126	0.148	0.178

Table 2.1 : Monte Carlo Upper tail significance points for W_n^2 based on 10,000 samples

AR(2) case(ρ_1=-.5, ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.135	0.153	0.185	0.211	0.237
8	0.101	0.114	0.135	0.156	0.186
10	0.098	0.111	0.134	0.155	0.180
12	0.096	0.109	0.130	0.153	0.186
15	0.094	0.108	0.132	0.158	0.190
20	0.093	0.106	0.129	0.152	0.183
30	0.093	0.105	0.127	0.152	0.182
40	0.093	0.105	0.127	0.149	0.185
50	0.090	0.103	0.122	0.146	0.174
60	0.089	0.103	0.126	0.147	0.178
100	0.091	0.104	0.126	0.147	0.178
∞	0.091	0.104	0.126	0.146	0.178

AR(2) case(ρ_1=-.5, ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.122	0.137	0.157	0.180	0.212
8	0.098	0.111	0.131	0.150	0.176
10	0.095	0.108	0.130	0.152	0.180
12	0.095	0.107	0.131	0.153	0.178
15	0.094	0.107	0.128	0.152	0.187
20	0.093	0.106	0.127	0.149	0.175
30	0.092	0.104	0.126	0.150	0.178
40	0.092	0.105	0.126	0.150	0.181
50	0.090	0.102	0.124	0.144	0.173
60	0.091	0.104	0.125	0.147	0.178
100	0.091	0.104	0.127	0.150	0.182
∞	0.091	0.104	0.126	0.146	0.178

AR(2) case(ρ_1=.1, ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.113	0.123	0.141	0.161	0.198
8	0.097	0.109	0.130	0.153	0.180
10	0.095	0.108	0.130	0.153	0.180
12	0.096	0.110	0.132	0.155	0.182
15	0.095	0.109	0.131	0.154	0.186
20	0.092	0.105	0.128	0.148	0.179
30	0.092	0.104	0.126	0.149	0.177
40	0.092	0.105	0.126	0.152	0.183
50	0.090	0.103	0.125	0.146	0.172
60	0.091	0.104	0.127	0.146	0.177
100	0.091	0.104	0.126	0.149	0.182
∞	0.091	0.104	0.126	0.146	0.178

Table 3.1 : Monte Carlo Upper tail significance points for U_n^2 based on 10,000 samples

case(ls)

N	15.0	10.0	5.0	2.5	1.0
5	0.087	0.098	0.116	0.136	0.163
8	0.085	0.096	0.115	0.135	0.161
10	0.085	0.097	0.117	0.138	0.164
12	0.086	0.097	0.118	0.139	0.165
15	0.087	0.099	0.121	0.141	0.163
20	0.085	0.097	0.118	0.137	0.165
30	0.084	0.096	0.118	0.139	0.168
40	0.084	0.096	0.116	0.137	0.162
50	0.085	0.098	0.118	0.137	0.167
60	0.085	0.097	0.117	0.138	0.165
100	0.084	0.097	0.118	0.138	0.166
∞	0.085	0.096	0.116	0.136	0.163

case(a)

N	15.0	10.0	5.0	2.5	1.0
5	0.089	0.097	0.108	0.115	0.122
8	0.089	0.099	0.116	0.134	0.161
10	0.088	0.100	0.118	0.136	0.161
12	0.088	0.100	0.120	0.139	0.160
15	0.089	0.101	0.121	0.141	0.168
20	0.086	0.098	0.118	0.135	0.163
30	0.085	0.098	0.118	0.139	0.165
40	0.085	0.097	0.116	0.136	0.161
50	0.086	0.097	0.118	0.139	0.166
60	0.085	0.097	0.118	0.138	0.160
100	0.085	0.098	0.118	0.138	0.165
∞	0.085	0.096	0.116	0.136	0.163

case(b)

N	15.0	10.0	5.0	2.5	1.0
5	0.087	0.099	0.122	0.141	0.167
8	0.089	0.100	0.119	0.137	0.160
10	0.088	0.100	0.120	0.140	0.164
12	0.087	0.099	0.119	0.138	0.162
15	0.087	0.100	0.121	0.140	0.166
20	0.086	0.098	0.118	0.136	0.165
30	0.085	0.098	0.118	0.141	0.167
40	0.084	0.096	0.116	0.136	0.161
50	0.085	0.097	0.118	0.138	0.167
60	0.085	0.097	0.116	0.138	0.166
100	0.085	0.097	0.117	0.137	0.165
∞	0.085	0.096	0.116	0.136	0.163

Table 3.1 : Monte Carlo Upper tail significance points for U_n^2 based on 10,000 samples

case(e)

N	15.0	10.0	5.0	2.5	1.0
5	0.086	0.099	0.122	0.141	0.166
8	0.089	0.100	0.119	0.137	0.160
10	0.088	0.100	0.119	0.139	0.163
12	0.087	0.099	0.119	0.138	0.163
15	0.087	0.100	0.121	0.140	0.165
20	0.086	0.098	0.118	0.136	0.165
30	0.085	0.097	0.118	0.140	0.167
40	0.084	0.096	0.116	0.136	0.161
50	0.085	0.097	0.118	0.139	0.167
60	0.085	0.097	0.117	0.138	0.167
100	0.085	0.097	0.118	0.136	0.164
∞	0.085	0.096	0.116	0.136	0.163

case(q)

N	15.0	10.0	5.0	2.5	1.0
5	0.081	0.094	0.108	0.116	0.119
8	0.086	0.097	0.115	0.132	0.151
10	0.087	0.098	0.116	0.134	0.155
12	0.086	0.097	0.116	0.133	0.159
15	0.087	0.099	0.117	0.136	0.158
20	0.085	0.096	0.118	0.134	0.161
30	0.085	0.098	0.119	0.140	0.165
40	0.084	0.096	0.115	0.135	0.161
50	0.085	0.097	0.118	0.139	0.165
60	0.084	0.096	0.115	0.137	0.164
100	0.085	0.097	0.117	0.138	0.165
∞	0.085	0.096	0.116	0.136	0.163

case(sq)

N	15.0	10.0	5.0	2.5	1.0
5	0.090	0.099	0.118	0.136	0.164
8	0.088	0.100	0.120	0.138	0.160
10	0.088	0.100	0.120	0.140	0.162
12	0.089	0.101	0.120	0.141	0.172
15	0.087	0.099	0.119	0.140	0.163
20	0.086	0.098	0.119	0.138	0.165
30	0.087	0.099	0.119	0.140	0.170
40	0.085	0.096	0.116	0.134	0.159
50	0.085	0.097	0.119	0.138	0.165
60	0.084	0.096	0.117	0.136	0.166
100	0.085	0.097	0.117	0.137	0.164
∞	0.085	0.096	0.116	0.136	0.163

Table 3.1 : Monte Carlo Upper tail significance points for U_n^2 based on 10,000 samples

case(lg)

N	15.0	10.0	5.0	2.5	1.0
5	0.094	0.105	0.119	0.129	0.147
8	0.089	0.099	0.119	0.138	0.164
10	0.089	0.100	0.120	0.141	0.165
12	0.087	0.098	0.118	0.139	0.163
15	0.089	0.101	0.120	0.139	0.167
20	0.085	0.098	0.118	0.136	0.165
30	0.085	0.097	0.119	0.140	0.167
40	0.085	0.097	0.116	0.135	0.161
50	0.085	0.098	0.118	0.138	0.166
60	0.084	0.097	0.117	0.137	0.164
100	0.085	0.097	0.118	0.137	0.168
	0.085	0.096	0.116	0.136	0.163

case(qsq)

N	15.0	10.0	5.0	2.5	1.0
5	0.072	0.076	0.082	0.085	0.087
8	0.086	0.097	0.115	0.131	0.149
10	0.086	0.097	0.117	0.134	0.154
12	0.086	0.097	0.116	0.134	0.160
15	0.088	0.099	0.119	0.138	0.162
20	0.085	0.097	0.117	0.135	0.160
30	0.085	0.098	0.118	0.139	0.164
40	0.084	0.096	0.115	0.134	0.159
50	0.086	0.097	0.118	0.138	0.165
60	0.085	0.097	0.117	0.136	0.163
100	0.085	0.098	0.118	0.139	0.166
	0.085	0.096	0.116	0.136	0.163

case(cu)

N	15.0	10.0	5.0	2.5	1.0
8	0.086	0.098	0.118	0.135	0.161
10	0.088	0.099	0.119	0.137	0.161
12	0.086	0.097	0.115	0.132	0.157
15	0.087	0.099	0.120	0.138	0.162
20	0.085	0.097	0.116	0.134	0.156
30	0.086	0.097	0.118	0.138	0.162
40	0.085	0.096	0.115	0.134	0.161
50	0.086	0.098	0.118	0.138	0.162
60	0.085	0.096	0.116	0.137	0.161
100	0.085	0.097	0.116	0.138	0.164
∞	0.085	0.096	0.116	0.136	0.163

Table 3.1 : Monte Carlo Upper tail significance points for U_n^2 based on 10,000 samples

N	case(tr) 15.0	10.0	5.0	2.5	1.0
5	0.072	0.077	0.081	0.083	0.084
8	0.090	0.104	0.129	0.150	0.179
10	0.087	0.098	0.118	0.137	0.163
12	0.086	0.098	0.117	0.137	0.162
15	0.086	0.098	0.119	0.139	0.163
20	0.086	0.098	0.119	0.139	0.163
30	0.086	0.097	0.116	0.135	0.160
40	0.085	0.097	0.118	0.139	0.166
50	0.085	0.096	0.117	0.137	0.166
60	0.085	0.096	0.116	0.138	0.165
100	0.085	0.097	0.117	0.138	0.164
	0.085	0.096	0.116	0.136	0.163

N	AR(1) case(μ=.5,ρ=.9,s=1) 15.0	10.0	5.0	2.5	1.0
5	0.105	0.114	0.126	0.137	0.154
8	0.091	0.103	0.119	0.136	0.160
10	0.090	0.102	0.122	0.140	0.164
12	0.089	0.100	0.120	0.140	0.168
15	0.088	0.099	0.119	0.138	0.164
20	0.087	0.099	0.119	0.139	0.162
30	0.085	0.097	0.118	0.140	0.166
40	0.086	0.098	0.118	0.139	0.164
50	0.084	0.096	0.117	0.134	0.159
60	0.084	0.096	0.116	0.137	0.165
100	0.084	0.095	0.115	0.135	0.160
	0.085	0.096	0.116	0.136	0.163

N	AR(1)case(μ=.5,ρ=.5,σ=1) 15.0	10.0	5.0	2.5	1.0
5	0.104	0.113	0.125	0.136	0.151
8	0.092	0.102	0.119	0.137	0.161
10	0.090	0.101	0.120	0.140	0.169
12	0.089	0.100	0.120	0.140	0.163
15	0.087	0.099	0.120	0.139	0.164
20	0.088	0.100	0.119	0.140	0.162
30	0.085	0.097	0.117	0.138	0.165
40	0.086	0.098	0.118	0.137	0.162
50	0.085	0.097	0.117	0.137	0.162
60	0.085	0.096	0.118	0.137	0.162
100	0.084	0.095	0.116	0.137	0.159
∞	0.085	0.096	0.116	0.136	0.163

Table 3.1 : Monte Carlo Upper tail significance points for U_n^2 based on 10,000 samples

AR(1)case(μ=3.5, ρ=-.9, σ=2)

N	15.0	10.0	5.0	2.5	1.0
5	0.094	0.104	0.120	0.134	0.151
8	0.090	0.101	0.120	0.139	0.162
10	0.088	0.099	0.120	0.140	0.166
12	0.088	0.100	0.119	0.138	0.163
15	0.089	0.100	0.121	0.141	0.165
20	0.087	0.100	0.120	0.139	0.167
30	0.086	0.098	0.118	0.137	0.159
40	0.085	0.097	0.118	0.139	0.167
50	0.084	0.096	0.115	0.134	0.161
60	0.084	0.097	0.116	0.137	0.168
100	0.085	0.096	0.117	0.137	0.163
∞	0.085	0.096	0.116	0.136	0.163

AR(2) case(ρ_1=.5, ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.106	0.117	0.131	0.144	0.158
8	0.093	0.105	0.124	0.142	0.166
10	0.089	0.100	0.120	0.138	0.160
12	0.089	0.101	0.120	0.140	0.165
15	0.087	0.099	0.120	0.144	0.173
20	0.087	0.099	0.119	0.140	0.170
30	0.086	0.100	0.118	0.139	0.161
40	0.084	0.097	0.117	0.138	0.165
50	0.084	0.096	0.116	0.133	0.158
60	0.086	0.097	0.119	0.140	0.166
100	0.085	0.098	0.119	0.139	0.163
∞	0.085	0.096	0.116	0.136	0.163

AR(2) case(ρ_1=.5, ρ_2=.4)

N	15.0	10.0	5.0	2.5	1.0
5	0.107	0.117	0.133	0.145	0.161
8	0.093	0.105	0.124	0.144	0.173
10	0.089	0.101	0.119	0.137	0.161
12	0.089	0.102	0.122	0.142	0.168
15	0.088	0.100	0.118	0.140	0.166
20	0.087	0.098	0.118	0.139	0.170
30	0.087	0.099	0.119	0.138	0.165
40	0.086	0.097	0.118	0.138	0.165
50	0.085	0.095	0.115	0.135	0.156
60	0.085	0.098	0.117	0.138	0.160
100	0.084	0.097	0.118	0.140	0.168
∞	0.085	0.096	0.116	0.136	0.163

Table 3.1 : Monte Carlo Upper tail significance points for U_n^2 based on 10,000 samples

AR(2) case(ρ_1=-.5,ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.106	0.115	0.130	0.143	0.157
8	0.091	0.102	0.121	0.142	0.163
10	0.090	0.102	0.123	0.142	0.164
12	0.088	0.101	0.120	0.140	0.169
15	0.088	0.100	0.122	0.143	0.172
20	0.087	0.098	0.120	0.140	0.165
30	0.086	0.098	0.118	0.140	0.165
40	0.086	0.098	0.119	0.139	0.169
50	0.084	0.095	0.113	0.133	0.157
60	0.084	0.096	0.115	0.137	0.165
100	0.085	0.097	0.117	0.135	0.162
∞	0.085	0.096	0.116	0.136	0.163

AR(2) case(ρ_1=-.5,ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.101	0.112	0.127	0.138	0.150
8	0.090	0.102	0.120	0.138	0.160
10	0.088	0.100	0.121	0.138	0.167
12	0.087	0.099	0.120	0.139	0.160
15	0.087	0.100	0.120	0.139	0.167
20	0.086	0.098	0.118	0.137	0.162
30	0.086	0.097	0.117	0.137	0.165
40	0.085	0.097	0.117	0.139	0.164
50	0.083	0.095	0.115	0.133	0.158
60	0.085	0.096	0.116	0.135	0.167
100	0.084	0.096	0.117	0.137	0.170
∞	0.085	0.096	0.116	0.136	0.163

AR(2) case(ρ_1=.1,ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.103	0.112	0.124	0.136	0.149
8	0.090	0.101	0.120	0.138	0.161
10	0.088	0.101	0.121	0.141	0.164
12	0.088	0.101	0.122	0.141	0.166
15	0.089	0.101	0.121	0.141	0.169
20	0.086	0.098	0.117	0.138	0.161
30	0.086	0.097	0.116	0.137	0.164
40	0.086	0.098	0.118	0.140	0.168
50	0.084	0.095	0.116	0.136	0.158
60	0.085	0.097	0.117	0.136	0.162
100	0.085	0.097	0.116	0.137	0.168
∞	0.085	0.096	0.116	0.136	0.163

Table 4.1 : Monte Carlo Upper tail significance points for A_n^2 based on 10,000 samples

case(ls)

N	15.0	10.0	5.0	2.5	1.0
5	0.584	0.646	0.756	0.872	1.018
8	0.564	0.627	0.740	0.864	1.027
10	0.563	0.634	0.752	0.879	1.026
12	0.562	0.634	0.754	0.890	1.034
15	0.571	0.648	0.770	0.888	1.027
20	0.559	0.635	0.750	0.877	1.025
30	0.558	0.626	0.761	0.884	1.045
40	0.556	0.625	0.750	0.869	1.026
50	0.562	0.631	0.754	0.883	1.054
60	0.562	0.626	0.753	0.874	1.025
100	0.558	0.633	0.756	0.876	1.062
∞	0.561	0.631	0.752	0.873	1.035

case(a)

N	15.0	10.0	5.0	2.5	1.0
5	0.613	0.650	0.712	0.750	0.795
8	0.606	0.666	0.762	0.867	1.013
10	0.595	0.668	0.789	0.890	1.041
12	0.591	0.660	0.776	0.890	1.057
15	0.584	0.654	0.772	0.901	1.047
20	0.577	0.646	0.761	0.876	1.042
30	0.572	0.638	0.762	0.883	1.047
40	0.564	0.631	0.747	0.869	1.042
50	0.567	0.634	0.757	0.886	1.057
60	0.566	0.637	0.752	0.873	1.017
100	0.566	0.636	0.753	0.878	1.046
∞	0.561	0.631	0.752	0.873	1.035

case(b)

N	15.0	10.0	5.0	2.5	1.0
5	0.621	0.694	0.838	0.975	1.134
8	0.611	0.673	0.789	0.912	1.045
10	0.600	0.669	0.786	0.916	1.062
12	0.594	0.666	0.795	0.927	1.098
15	0.586	0.654	0.773	0.895	1.045
20	0.576	0.653	0.771	0.878	1.063
30	0.572	0.643	0.769	0.890	1.060
40	0.559	0.630	0.747	0.873	1.040
50	0.564	0.631	0.757	0.884	1.054
60	0.565	0.633	0.751	0.868	1.018
100	0.558	0.631	0.746	0.870	1.039
∞	0.561	0.631	0.752	0.873	1.035

Table 4.1 : Monte Carlo Upper tail significance points for A_n^2 based on 10,000 samples

case(e)

N	15.0	10.0	5.0	2.5	1.0
5	0.620	0.691	0.833	0.966	1.131
8	0.610	0.674	0.788	0.911	1.043
10	0.600	0.667	0.787	0.915	1.060
12	0.594	0.667	0.796	0.931	1.098
15	0.585	0.655	0.772	0.892	1.041
20	0.575	0.652	0.772	0.879	1.056
30	0.573	0.644	0.767	0.888	1.061
40	0.557	0.629	0.749	0.875	1.045
50	0.564	0.633	0.758	0.884	1.049
60	0.566	0.634	0.752	0.870	1.027
100	0.564	0.635	0.751	0.870	1.043
∞	0.561	0.631	0.752	0.873	1.035

case(q)

N	15.0	10.0	5.0	2.5	1.0
5	0.594	0.679	0.769	0.816	0.839
8	0.591	0.652	0.756	0.845	0.982
10	0.590	0.659	0.759	0.868	1.015
12	0.576	0.644	0.760	0.866	1.016
15	0.590	0.658	0.768	0.883	1.015
20	0.567	0.640	0.752	0.863	1.015
30	0.568	0.644	0.766	0.883	1.047
40	0.560	0.631	0.745	0.870	1.051
50	0.564	0.634	0.768	0.878	1.038
60	0.564	0.628	0.746	0.867	1.030
100	0.560	0.634	0.756	0.872	1.058
∞	0.561	0.631	0.752	0.873	1.035

case(sq)

N	15.0	10.0	5.0	2.5	1.0
5	0.637	0.692	0.808	0.932	1.102
8	0.608	0.674	0.788	0.911	1.047
10	0.601	0.671	0.793	0.910	1.064
12	0.588	0.656	0.783	0.892	1.045
15	0.594	0.665	0.791	0.906	1.057
20	0.577	0.647	0.768	0.878	1.063
30	0.565	0.640	0.767	0.886	1.061
40	0.564	0.633	0.744	0.868	1.040
50	0.565	0.635	0.758	0.886	1.043
60	0.567	0.633	0.748	0.871	1.025
100	0.562	0.632	0.752	0.872	1.048
∞	0.561	0.631	0.752	0.873	1.035

Table 4.1 : Monte Carlo Upper tail significance points for A_n^2 based on 10,000 samples

case(lg)

N	15.0	10.0	5.0	2.5	1.0
5	0.668	0.741	0.818	0.884	1.011
8	0.609	0.673	0.785	0.905	1.081
10	0.602	0.674	0.787	0.918	1.077
12	0.588	0.658	0.781	0.891	1.042
15	0.597	0.667	0.786	0.905	1.075
20	0.577	0.650	0.768	0.879	1.051
30	0.565	0.639	0.765	0.888	1.071
40	0.567	0.631	0.741	0.865	1.040
50	0.569	0.641	0.757	0.882	1.039
60	0.568	0.634	0.751	0.864	1.017
100	0.561	0.632	0.756	0.871	1.058
∞	0.561	0.631	0.752	0.873	1.035

case(qsq)

N	15.0	10.0	5.0	2.5	1.0
5	0.530	0.576	0.602	0.624	0.633
8	0.595	0.654	0.757	0.854	0.977
10	0.587	0.655	0.765	0.872	0.995
12	0.576	0.649	0.765	0.876	1.047
15	0.590	0.659	0.772	0.890	1.047
20	0.569	0.642	0.759	0.870	1.023
30	0.567	0.640	0.763	0.870	1.065
40	0.561	0.630	0.744	0.855	1.026
50	0.569	0.635	0.763	0.882	1.040
60	0.565	0.635	0.749	0.876	1.022
100	0.565	0.639	0.763	0.877	1.055
∞	0.561	0.631	0.752	0.873	1.035

case(cu)

N	15.0	10.0	5.0	2.5	1.0
8	0.583	0.649	0.770	0.869	1.072
10	0.599	0.663	0.783	0.885	1.010
12	0.582	0.649	0.754	0.864	1.013
15	0.588	0.661	0.780	0.889	1.019
20	0.567	0.637	0.754	0.867	1.013
30	0.572	0.645	0.768	0.872	1.039
40	0.565	0.633	0.748	0.865	1.048
50	0.563	0.638	0.764	0.881	1.034
60	0.563	0.633	0.748	0.863	1.043
100	0.564	0.636	0.752	0.868	1.052
∞	0.561	0.631	0.752	0.873	1.035

Table 4.1 : Monte Carlo Upper tail significance points for A_n^2 based on 10,000 samples

	case(tr)				
N	15.0	10.0	5.0	2.5	1.0
5	0.509	0.537	0.564	0.578	0.586
8	0.604	0.690	0.834	0.966	1.138
10	0.586	0.651	0.768	0.873	1.013
12	0.579	0.645	0.773	0.879	1.039
15	0.575	0.647	0.773	0.881	1.024
20	0.575	0.646	0.763	0.889	1.054
30	0.570	0.635	0.750	0.863	1.015
40	0.565	0.637	0.762	0.889	1.050
50	0.562	0.631	0.749	0.878	1.048
60	0.561	0.633	0.750	0.871	1.040
100	0.562	0.635	0.758	0.883	1.042
∞	0.561	0.631	0.752	0.873	1.035

	AR(1) case(μ=.5,ρ=.9,σ=1)				
N	15.0	10.0	5.0	2.5	1.0
5	0.886	0.960	1.074	1.159	1.236
8	0.632	0.698	0.803	0.909	1.052
10	0.613	0.681	0.804	0.915	1.063
12	0.598	0.670	0.789	0.909	1.097
15	0.592	0.660	0.770	0.885	1.044
20	0.581	0.654	0.770	0.891	1.036
30	0.570	0.645	0.770	0.887	1.047
40	0.570	0.641	0.758	0.877	1.034
50	0.565	0.631	0.757	0.863	1.006
60	0.561	0.633	0.751	0.878	1.040
100	0.556	0.621	0.741	0.856	1.026
	0.561	0.631	0.752	0.873	1.035

	AR(1) case(μ=.5,ρ=.5,σ=1)				
N	15.0	10.0	5.0	2.5	1.0
5	0.889	0.962	1.065	1.148	1.236
8	0.632	0.696	0.810	0.912	1.058
10	0.611	0.680	0.789	0.906	1.064
12	0.600	0.673	0.787	0.920	1.068
15	0.590	0.667	0.783	0.891	1.049
20	0.586	0.656	0.777	0.893	1.052
30	0.570	0.641	0.758	0.883	1.032
40	0.576	0.645	0.760	0.873	1.034
50	0.567	0.637	0.759	0.870	1.019
60	0.561	0.633	0.762	0.881	1.026
100	0.556	0.623	0.741	0.855	1.020
∞	0.561	0.631	0.752	0.873	1.035

Table 4.1 : Monte Carlo Upper tail significance points for A_n^2 based on 10,000 samples

AR(1)case($\rho=-.9, \mu=3.5, \sigma=2$)

N	15.0	10.0	5.0	2.5	1.0
5	0.688	0.758	0.862	0.957	1.089
8	0.617	0.683	0.804	0.915	1.071
10	0.596	0.666	0.790	0.904	1.076
12	0.597	0.665	0.781	0.899	1.065
15	0.592	0.663	0.785	0.906	1.065
20	0.583	0.654	0.783	0.900	1.065
30	0.569	0.640	0.768	0.885	1.037
40	0.568	0.634	0.758	0.885	1.062
50	0.562	0.630	0.748	0.853	1.034
60	0.563	0.631	0.747	0.877	1.075
100	0.563	0.630	0.757	0.882	1.038
∞	0.561	0.631	0.752	0.873	1.035

AR(2) case($\rho_1=.5, \rho_2=0.2$)

N	15.0	10.0	5.0	2.5	1.0
5	0.874	0.972	1.142	1.346	1.535
8	0.647	0.717	0.838	0.950	1.110
10	0.605	0.675	0.790	0.906	1.058
12	0.598	0.671	0.786	0.911	1.066
15	0.585	0.658	0.786	0.920	1.083
20	0.578	0.647	0.772	0.905	1.075
30	0.580	0.649	0.767	0.878	1.021
40	0.567	0.637	0.752	0.881	1.025
50	0.564	0.630	0.739	0.856	1.013
60	0.569	0.641	0.759	0.876	1.047
100	0.565	0.638	0.761	0.881	1.029
∞	0.561	0.631	0.752	0.873	1.035

AR(2) case($\rho_1=.5, \rho_2=0.4$)

N	15.0	10.0	5.0	2.5	1.0
5	0.879	0.978	1.156	1.346	1.562
8	0.649	0.719	0.837	0.962	1.141
10	0.607	0.678	0.787	0.892	1.035
12	0.603	0.674	0.793	0.913	1.103
15	0.586	0.657	0.787	0.904	1.059
20	0.579	0.648	0.770	0.891	1.054
30	0.578	0.651	0.772	0.887	1.048
40	0.569	0.642	0.756	0.866	1.044
50	0.563	0.625	0.740	0.857	0.997
60	0.567	0.638	0.757	0.872	1.042
100	0.562	0.639	0.763	0.889	1.044
∞	0.561	0.631	0.752	0.873	1.035

Table 4.1 : Monte Carlo Upper tail significance points for A_n^2 based on 10,000 samples

AR(2) case(ρ_1=-.5,ρ_2=.2)

N	15.0	10.0	5.0	2.5	1.0
5	0.872	0.969	1.138	1.331	1.576
8	0.633	0.706	0.816	0.938	1.088
10	0.613	0.687	0.808	0.926	1.058
12	0.595	0.669	0.788	0.908	1.073
15	0.588	0.665	0.787	0.928	1.111
20	0.580	0.654	0.774	0.901	1.064
30	0.572	0.643	0.769	0.897	1.049
40	0.576	0.643	0.755	0.882	1.064
50	0.558	0.627	0.737	0.854	1.007
60	0.558	0.630	0.747	0.869	1.045
100	0.563	0.635	0.749	0.873	1.031
∞	0.561	0.631	0.752	0.873	1.035

AR(2) case(ρ_1=-.5,ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.803	0.882	1.003	1.132	1.319
8	0.613	0.685	0.800	0.892	1.022
10	0.598	0.671	0.787	0.895	1.049
12	0.590	0.658	0.779	0.903	1.031
15	0.588	0.655	0.778	0.888	1.064
20	0.579	0.651	0.771	0.877	1.019
30	0.569	0.640	0.757	0.879	1.047
40	0.565	0.637	0.760	0.878	1.046
50	0.559	0.625	0.748	0.853	1.010
60	0.564	0.629	0.744	0.860	1.029
100	0.561	0.631	0.759	0.886	1.036
∞	0.561	0.631	0.752	0.873	1.035

AR(2) case(ρ_1=.1,ρ_2=-.9)

N	15.0	10.0	5.0	2.5	1.0
5	0.776	0.833	0.926	1.024	1.206
8	0.615	0.683	0.796	0.906	1.047
10	0.599	0.669	0.789	0.904	1.052
12	0.600	0.671	0.793	0.912	1.076
15	0.591	0.668	0.791	0.921	1.073
20	0.576	0.645	0.765	0.883	1.044
30	0.573	0.641	0.758	0.871	1.034
40	0.568	0.637	0.761	0.897	1.047
50	0.560	0.629	0.744	0.857	1.000
60	0.566	0.633	0.752	0.858	1.032
100	0.564	0.638	0.757	0.874	1.060
∞	0.561	0.631	0.752	0.873	1.035

TABLE 5 : Mnemonic Key to Alternative Models and Error Distributions.

Key	Explanation	EC[1]
uflq1	*fit a linear model to quadratic data*	1
ofql1	*fit a quadratic model to linear data*	1
uflq2	*fit a linear model to quadratic data*	1
ofqle	*fit a quadratic model to linear data*	1
ufqc	*fit a quadratic model to cubic data*	1
ofqc	*fit a cubic model to quadratic data*	1
ufar12	*fit an AR(1) model to AR(2) data*	1
ofar21	*fit an AR(2) model to AR(1) data*	1
ble1	*fit a linear model to linear data*	2
ble2	*fit a linear model to linear data*	2
ble2	*fit a quadratic model to quadratic data*	2
ar1le	*fit an AR(1)to AR(1) data*	2
ar2le	*fit an AR(2) model to AR(2) data*	2
uflqle	*fit a linear model to quadratic data*	2
ofqlle	*fit a quadratic model to linear data*	2
ar12le	*fit an AR(1) model to AR(2) data*	2
ar21l2	*fit an AR(2) model to AR(1) data*	2
blue1	*fit a linear model to linear data*	3
blue2	*fit a linear model to linear data*	3
que1	*fit a quadratic model to quadratic data*	3
que2	*fit a quadratic model to quadratic data*	3
ar1ue	*fit an AR(1) model to AR(1) data*	3
ar2ue	*fit an AR(2) model to AR(2) data*	3

Key	*Explanation*	EC
uflque	*fit a linear model to quadratic data*	3
ofqlue	*fit a quadratic model to linear data*	3
ar12ue	*fit an AR(1) model to AR(2) data*	3
ar21ue	*fit an AR(2) model to AR(1) data*	3

[1]EC gives the error distribution code. 1 = Normal, 2 = Laplace(double exponential), 3 = U(.-5,.5), i.e.,uniform over the interval (-.5,.5). Except for ufar12 and ofar21,keys ending in 1 refer to x_i even-spaced (= i, or, = i/5); keys ending in 2 refer to $x_i = \sqrt{i}$.

Table 1.1a : Estimated Sizes (%) of $\sqrt{n}D_n$ compared with nominal levels, based on 1,000 samples

MIS-SPECIFIED MODELS(Normal Errors)
Estimated Sizes(%) based on 1,000 samples
compared with the nominal levels(%)

case (uflq1)

N	15.0	10.0	5.0	2.5	1.0
20	15.5	10.1	5.0	2.7	1.2
30	56.7	46.5	29.9	19.5	9.6
50	56.0	46.2	31.4	20.7	10.5
100	99.7	99.7	99.7	99.7	99.7

case (ofql1)

N	15.0	10.0	5.0	2.5	1.0
20	16.0	10.6	5.7	2.8	1.2
30	14.5	9.6	4.3	2.9	1.1
50	16.6	11.7	6.4	3.3	1.4
100	14.4	9.9	4.9	2.4	0.9

case (uflq2)

N	15.0	10.0	5.0	2.5	1.0
20	14.9	11.2	6.4	3.4	1.7
30	28.0	21.3	13.0	7.8	3.8
50	80.3	70.6	54.6	39.6	26.0

case (ofql2)

N	15.0	10.0	5.0	2.5	1.0
20	16.8	12.1	6.2	3.7	1.8
30	15.2	9.7	4.4	2.3	1.3
50	17.2	12.7	5.8	4.1	1.9

case (ufqc)

N	15.0	10.0	5.0	2.5	1.0
20	16.3	11.7	5.7	3.2	1.6
30	14.8	10.0	4.7	2.6	0.8
50	15.5	10.1	4.9	3.1	1.1
100	14.4	9.5	4.3	2.1	0.7

case (ofcq)

N	15.0	10.0	5.0	2.5	1.0
20	17.1	12.2	5.5	3.3	1.4
30	16.2	10.8	5.5	2.6	1.0
50	16.5	10.6	5.1	2.8	1.2
100	15.9	10.8	5.2	3.1	1.2

Table 1.1a : Estimated Sizes (%) of $\sqrt{n}D_n$ compared with nominal levels, based on 1,000 samples

	case (ufar12)				
N	15.0	10.0	5.0	2.5	1.0
20	16.8	11.9	6.7	3.6	1.4
30	22.5	17.5	10.3	6.4	3.0
50	29.0	22.1	13.4	8.6	4.2

	case (ofar21)				
N	15.0	10.0	5.0	2.5	1.0
20	15.2	10.3	5.6	3.1	1.2
30	17.6	12.3	5.6	3.5	1.3
50	15.0	9.7	4.5	2.4	0.9

Table 1.1b : Power (%) of $\sqrt{n}D_n$ against the Laplace distribution, based on 1,000 samples

CORRECT MODELS with LAPLACE errors
Power(%) at various significance levels

case (ble1)

N	15.0	10.0	5.0	2.5	1.0
20	32.0	25.0	16.1	11.6	6.4
30	41.8	36.0	25.3	17.8	10.7
50	59.6	52.3	40.7	31.9	22.9

case (ble2)

N	15.0	10.0	5.0	2.5	1.0
20	32.8	26.2	16.6	11.5	6.9
30	41.7	35.0	25.6	18.6	11.0
50	57.1	49.1	36.5	27.2	17.9

case (qle1)

N	15.0	10.0	5.0	2.5	1.0
20	33.4	25.4	15.3	9.5	5.3
30	39.9	30.7	20.0	13.5	9.2
50	56.8	49.6	37.0	28.5	19.9

case (qle2)

N	15.0	10.0	5.0	2.5	1.0
20	32.7	25.3	16.4	11.1	6.7
30	39.4	32.0	20.6	14.8	8.7
50	55.7	47.0	33.5	24.8	16.1

case (ar1le)

N	15.0	10.0	5.0	2.5	1.0
20	32.6	26.7	18.2	12.1	7.3
30	42.0	35.4	23.9	17.2	10.2
50	56.9	48.7	35.7	27.2	17.4

case (ar2le)

N	15.0	10.0	5.0	2.5	1.0
20	31.4	25.2	15.9	10.9	6.2
30	36.2	29.1	19.3	13.5	8.9
50	56.1	47.9	34.5	26.0	16.3

Table 1.1c : Power (%) of $\sqrt{n}D_n$ against the Laplace distribution, based on 1,000 samples

MIS-SPECIFIED MODELS(LAPLACE Errors)
Power(%) at various significance levels

case (uflqle)

N	15.0	10.0	5.0	2.5	1.0
20	26.4	19.9	11.6	7.2	3.8
50	33.3	25.1	14.9	9.8	5.8

case (ofqlle)

N	15.0	10.0	5.0	2.5	1.0
20	27.8	20.8	13.3	8.8	4.6
50	57.7	49.5	37.8	28.6	21.0

case (ar12le)

N	15.0	10.0	5.0	2.5	1.0
20	28.1	21.7	13.9	9.1	3.8
50	31.8	26.7	17.6	12.9	7.4

case (ar21le)

N	15.0	10.0	5.0	2.5	1.0
20	32.1	25.5	15.1	10.5	6.4
50	54.8	47.0	33.6	26.3	16.9

Table 1.1d : Power(%) of $\sqrt{n}D_n$ against the U(-.5,.5) distribution, based on 1,000 samples

CORRECT MODELS with U(-.5,.5)errors
Power (%) at different significance levels

case (blue1)

N	15.0	10.0	5.0	2.5	1.0
20	28.3	22.1	11.1	6.1	2.1
30	35.3	26.2	13.4	7.8	3.4
50	51.5	40.8	23.5	13.6	6.1

case (blue2)

N	15.0	10.0	5.0	2.5	1.0
20	28.1	20.1	11.2	5.4	2.4
30	34.5	25.5	13.7	7.8	2.9
50	53.0	42.7	25.7	15.6	6.9

case (que1)

N	15.0	10.0	5.0	2.5	1.0
20	23.0	16.3	9.7	5.5	2.4
30	31.7	22.2	11.9	6.5	2.7
50	50.8	41.3	24.5	14.5	5.7

case (que2)

N	15.0	10.0	5.0	2.5	1.0
20	23.8	16.9	9.0	4.9	2.0
30	31.1	22.8	12.3	6.9	2.6
50	51.6	40.7	24.6	14.8	6.0

case (ar1ue)

N	15.0	10.0	5.0	2.5	1.0
20	27.6	19.5	10.1	4.5	1.4
30	35.5	27.0	15.0	8.3	3.5
50	52.4	42.3	26.3	16.7	8.7

case (ar2ue)

N	15.0	10.0	5.0	2.5	1.0
20	21.9	16.3	8.1	4.1	2.0
30	32.3	23.9	12.4	6.2	3.2
50	48.3	38.4	21.2	13.1	6.5

Table 1.1e : Power(%) of $\sqrt{n}D_n$ against the U(-.5,.5) distribution, based on 1,000 samples

MIS-SPECIFIED MODELS with U(-.5,.5) errors
Power(%) at various significance levels

			case (uflque)		
N	15.0	10.0	5.0	2.5	1.0
20	47.1	34.7	20.5	10.0	4.1
50	99.6	99.6	97.0	87.5	58.6
			case (ofqlue)		
N	15.0	10.0	5.0	2.5	1.0
20	24.5	17.3	8.7	4.6	2.4
50	50.4	38.4	22.6	13.1	5.7
			case (ar12ue)		
N	15.0	10.0	5.0	2.5	1.0
20	19.6	14.0	6.4	3.8	1.6
50	43.8	36.1	24.2	16.5	9.7
			case (ar21ue)		
N	15.0	10.0	5.0	2.5	1.0
20	23.4	16.8	8.4	4.6	1.7
30	35.4	26.6	14.0	8.4	3.9
50	49.0	37.9	24.3	15.4	6.6

Table 2.1a : Estimated Sizes (%) of W_n^2 compared with nominal levels, based on 1,000 samples

MIS-SPECIFIED MODELS(Normal Errors)
Estimated Sizes (%) based on 1,000 samples
compared with nominal levels(%)

case (uflq1)

N	15.0	10.0	5.0	2.5	1.0
20	14.9	10.6	5.0	2.7	1.1
30	67.3	55.8	40.0	26.4	11.6
50	64.6	55.7	40.5	28.0	15.0
100	100.0	100.0	100.0	100.0	100.0

case (ofql1)

N	15.0	10.0	5.0	2.5	1.0
20	16.0	10.9	5.8	2.7	1.0
30	14.6	10.5	5.7	2.9	1.4
50	16.8	11.5	5.7	3.3	1.5
100	14.7	9.6	4.5	2.3	1.4

case (uflq2)

N	15.0	10.0	5.0	2.5	1.0
20	17.1	10.9	5.8	3.5	1.8
30	33.2	25.1	16.2	8.5	4.1
50	90.6	83.9	69.6	55.0	35.6

case (ofql2)

N	15.0	10.0	5.0	2.5	1.0
20	15.8	11.1	5.7	2.8	0.9
30	14.0	9.2	3.7	2.5	1.0
50	16.6	11.1	5.7	3.0	1.4

case (ufqc)

N	15.0	10.0	5.0	2.5	1.0
20	16.5	10.8	5.3	2.8	1.0
30	15.1	9.6	4.5	2.2	1.1
50	15.8	11.2	5.7	2.7	1.0
100	14.3	9.1	4.0	2.0	0.9

case (ofcq)

N	15.0	10.0	5.0	2.5	1.0
20	17.1	11.6	5.8	2.3	0.8
30	16.2	10.4	4.6	2.2	1.1
50	16.4	11.7	5.7	2.8	0.9
100	15.7	9.4	4.8	2.4	1.3

Table 2.1a : Estimated Sizes (%) of W_n^2 compared with nominal levels, based on 1,000 samples

	case (ufar12)				
N	15.0	10.0	5.0	2.5	1.0
20	20.9	14.6	9.1	4.8	2.3
30	23.4	17.6	11.0	6.8	3.5
50	30.2	24.3	17.6	11.3	6.3

	case (ofar21)				
N	15.0	10.0	5.0	2.5	1.0
20	15.2	9.4	4.7	2.8	0.7
30	16.2	11.6	5.7	3.1	1.8
50	15.1	10.0	5.6	2.7	1.1

Table 2.1.b : Power (%) of W_n^2 against the Laplace distribution, based on 1,000 samples

CORRECT MODELS with LAPLACE Errors
Power (%) at various significance levels

case (ble1)

N	15.0	10.0	5.0	2.5	1.0
20	36.7	29.7	19.9	13.2	9.0
30	48.0	40.4	29.8	23.3	13.9
50	65.4	59.2	49.3	41.0	31.2

case (ble2)

N	15.0	10.0	5.0	2.5	1.0
20	37.1	29.3	19.9	13.0	9.3
30	47.3	40.0	29.7	22.9	15.0
50	63.1	56.0	45.1	37.5	26.0

case (qle1)

N	15.0	10.0	5.0	2.5	1.0
20	35.2	28.0	18.4	12.6	7.8
30	42.9	36.3	26.5	18.1	12.0
50	62.9	55.4	46.1	38.0	28.6

case (qle2)

N	15.0	10.0	5.0	2.5	1.0
20	35.5	28.6	19.2	12.7	7.8
30	44.4	36.9	25.8	19.3	12.3
50	61.3	53.9	42.9	33.9	23.6

case (ar1le)

N	15.0	10.0	5.0	2.5	1.0
20	35.2	28.3	20.7	15.8	9.5
30	46.3	38.3	28.8	20.9	14.8
50	65.9	58.0	46.4	37.6	27.3

case (ar2le)

N	15.0	10.0	5.0	2.5	1.0
20	33.6	27.0	18.5	13.0	8.0
30	41.9	34.5	24.4	17.6	11.5
50	65.1	56.8	44.6	34.8	26.1

Table 2.1c : Power (%) of W_n^2 against the Laplace distribution, based on 1,000 samples

MIS-SPECIFIED MODELS(LAPLACE Errors)
Power (%) at various significance levels

case (uflqle)

N	15.0	10.0	5.0	2.5	1.0
20	28.7	22.7	14.5	8.4	4.4
50	40.6	31.3	19.6	12.0	6.9

case (ofqlle)

N	15.0	10.0	5.0	2.5	1.0
20	32.1	24.0	15.6	10.4	6.3
50	63.7	57.7	47.0	38.9	28.4

case (ar12le)

N	15.0	10.0	5.0	2.5	1.0
20	29.8	23.3	14.7	9.0	4.9
50	35.9	29.6	21.4	15.0	10.0

case (ar21le)

N	15.0	10.0	5.0	2.5	1.0
20	33.1	27.2	19.5	13.8	8.1
50	61.1	54.3	43.0	34.8	25.5

Table 2.1d : Power (%) of W_n^2 against the U(-.5,.5) distribution, based on 1,000 samples

CORRECT MODELS with U(-.5,.5)Errors
Power (%) at various significance levels

case (blue1)

N	15.0	10.0	5.0	2.5	1.0
20	35.3	26.5	15.2	8.4	3.3
30	47.2	35.0	22.0	12.4	6.0
50	69.0	58.3	41.0	28.4	14.9

case (blue2)

N	15.0	10.0	5.0	2.5	1.0
20	34.7	25.7	14.3	8.5	3.4
30	47.2	36.2	22.3	12.3	5.9
50	71.8	60.9	44.4	30.0	17.0

case (que1)

N	15.0	10.0	5.0	2.5	1.0
20	30.2	21.4	11.8	6.4	2.5
30	41.8	30.1	18.5	9.9	4.8
50	67.4	56.8	39.6	27.6	15.2

case (que2)

N	15.0	10.0	5.0	2.5	1.0
20	29.4	20.7	10.6	5.9	2.7
30	40.5	30.3	18.3	10.2	4.7
50	67.4	55.9	39.6	28.4	15.0

case (ar1ue)

N	15.0	10.0	5.0	2.5	1.0
20	33.0	23.2	14.3	7.0	3.3
30	46.1	36.1	23.2	14.1	7.0
50	69.1	61.5	46.9	32.4	19.2

case (ar2ue)

N	15.0	10.0	5.0	2.5	1.0
20	26.7	18.0	11.8	5.3	1.7
30	39.7	30.4	18.6	11.1	5.4
50	63.1	53.7	38.8	26.2	13.9

Table 2.1e : Power (%) of W_n^2 against the U(-.5,.5) distribution, based on 1,000 samples

MIS-SPECIFIED MODELS with U(-.5,.5)Errors
Power (%) at various significant levels

case (uflque)

N	15.0	10.0	5.0	2.5	1.0
20	53.0	39.5	20.5	11.0	3.4
50	99.8	99.8	99.8	99.8	99.8

case (ofqlue)

N	15.0	10.0	5.0	2.5	1.0
20	30.3	21.2	11.1	6.1	3.1
50	64.4	54.0	38.5	27.2	13.9

case (ar12ue)

N	15.0	10.0	5.0	2.5	1.0
20	21.2	15.4	8.1	4.3	1.6
50	52.2	43.9	32.9	24.1	15.2

case (ar21ue)

N	15.0	10.0	5.0	2.5	1.0
20	29.2	20.4	10.9	5.3	1.9
30	42.2	33.5	21.9	12.5	4.8
50	64.2	54.2	38.4	25.3	15.5

Table 3.1a : Estimated Sizes (%) of U_n^2 compared with nominal levels, based on 1,000 samples

MIS-SPECIFIED MODELS(Normal Errors)
Estimated Sizes (%) based on 1,000 samples
compared with the nominal levels

case (uflq1)

N	15.0	10.0	5.0	2.5	1.0
20	14.5	10.4	5.4	2.9	1.0
30	65.1	54.9	38.8	24.6	11.6
50	62.1	54.1	37.5	25.7	13.8
100	100.0	100.0	100.0	100.0	100.0

case (ofql1)

N	15.0	10.0	5.0	2.5	1.0
20	16.0	11.2	5.7	2.8	1.2
30	14.5	10.1	5.6	2.9	1.2
50	16.7	11.9	6.0	3.8	1.5
100	14.1	9.9	4.5	2.2	1.3

case (uflq2)

N	15.0	10.0	5.0	2.5	1.0
20	16.0	10.4	5.8	3.5	1.7
30	30.4	22.8	14.0	7.7	3.1
50	83.9	76.3	60.8	45.8	27.5

case (ofql2)

N	15.0	10.0	5.0	2.5	1.0
20	16.1	11.2	5.3	2.6	1.1
30	14.0	9.1	4.1	2.2	1.1
50	16.4	11.7	5.8	3.1	1.3

case (ufqc)

N	15.0	10.0	5.0	2.5	1.0
20	16.1	11.3	5.2	2.9	1.2
30	14.3	10.3	4.7	2.2	0.9
50	15.2	10.0	5.5	2.9	1.0
100	14.0	9.7	4.9	2.1	1.2

case (ofcq)

N	15.0	10.0	5.0	2.5	1.0
20	17.6	11.8	5.5	3.0	0.7
30	15.5	11.3	4.2	2.1	1.1
50	16.4	11.9	6.0	2.8	1.1
100	15.6	9.9	5.1	2.4	1.4

Table 3.1a : Estimated Sizes (%) of U_n^2 compared with nominal levels, based on 1,000 samples

case (ufar12)

N	15.0	10.0	5.0	2.5	1.0
20	23.0	17.6	11.5	6.4	3.6
30	23.0	17.6	10.2	6.7	3.1
50	30.3	23.8	16.7	11.3	6.2

case (ofar21)

N	15.0	10.0	5.0	2.5	1.0
20	14.5	9.9	4.9	3.0	1.0
30	16.5	11.3	5.7	3.1	2.1
50	14.1	10.7	6.2	3.2	1.4

Table 3.1b : Power (%) of U_n^2 against the Laplace distribution, based on 1,000 samples

CORRECT MODELS with LAPLACE Errors
Power (%) at various significance levels

case (ble1)

N	15.0	10.0	5.0	2.5	1.0
20	35.9	29.0	20.4	13.1	8.8
30	46.7	39.6	30.4	23.0	13.8
50	66.0	59.7	49.7	42.6	32.5

case (ble2)

N	15.0	10.0	5.0	2.5	1.0
20	35.8	28.8	19.7	13.4	8.9
30	46.1	39.6	30.0	23.1	14.9
50	62.7	57.1	47.3	38.6	26.6

case (qle1)

N	15.0	10.0	5.0	2.5	1.0
20	34.7	27.0	18.4	12.2	7.3
30	42.1	35.3	25.7	18.7	11.8
50	63.6	56.4	46.5	39.0	29.9

case (qle2)

N	15.0	10.0	5.0	2.5	1.0
20	34.1	28.4	19.1	12.3	7.6
30	43.4	35.8	25.8	19.8	12.2
50	61.2	54.9	43.0	34.9	24.4

case (ar1le)

N	15.0	10.0	5.0	2.5	1.0
20	34.9	28.3	20.4	15.9	9.1
30	45.6	38.2	28.9	21.3	14.4
50	66.0	59.5	48.0	38.4	28.6

case (ar2le)

N	15.0	10.0	5.0	2.5	1.0
20	32.3	26.6	17.6	11.8	7.8
30	40.4	34.6	25.2	17.7	11.4
50	65.1	57.9	45.7	35.7	27.1

Table 3.1c : Power (%) of U_n^2 against the Laplace distribution, based on 1,000 samples

MIS-SPECIFIED MODELS(LAPLACE Errors)
Power (%) at various significance levels

case (uflqle)

N	15.0	10.0	5.0	2.5	1.0
20	27.5	21.8	13.8	7.9	3.8
50	37.4	30.1	18.2	10.9	5.8

case (ofqlle)

N	15.0	10.0	5.0	2.5	1.0
20	30.0	23.5	15.6	10.4	6.0
50	63.9	58.5	47.8	39.4	30.3

case (ar12le)

N	15.0	10.0	5.0	2.5	1.0
20	29.6	24.0	15.1	8.8	5.3
50	35.6	29.8	21.0	14.7	9.4

case (ar21le)

N	15.0	10.0	5.0	2.5	1.0
20	32.0	26.7	18.4	12.9	7.7
50	61.1	55.0	43.9	35.8	26.6

Table 3.1d : Power (%) of U_n^2 against the U(-.5,.5) distribution, based on 1,000 samples

CORRECT MODELS with U(-.5,.5) Errors
Power (%) at various significance levels

case (blue1)

N	15.0	10.0	5.0	2.5	1.0
20	36.9	29.4	17.8	9.8	4.4
30	49.7	39.1	25.0	15.5	7.0
50	71.2	62.2	46.3	32.8	19.1

case (blue2)

N	15.0	10.0	5.0	2.5	1.0
20	36.5	28.9	16.5	10.1	4.3
30	49.9	40.4	25.2	15.9	7.3
50	74.3	65.4	49.8	35.1	22.1

case (que1)

N	15.0	10.0	5.0	2.5	1.0
20	31.1	23.9	13.2	7.2	3.2
30	43.7	35.2	21.2	12.3	5.4
50	70.5	61.4	44.8	31.5	19.4

case (que2)

N	15.0	10.0	5.0	2.5	1.0
20	30.8	23.6	13.0	6.8	3.0
30	43.7	33.2	21.1	11.4	6.2
50	70.1	60.8	44.0	32.8	19.0

case (ar1ue)

N	15.0	10.0	5.0	2.5	1.0
20	35.3	26.5	16.4	9.3	4.3
30	47.7	40.0	26.6	17.6	9.6
50	71.6	64.6	50.5	37.1	23.9

case (ar2ue)

N	15.0	10.0	5.0	2.5	1.0
20	28.2	20.7	12.7	6.5	2.5
30	42.6	33.4	21.7	12.9	7.1
50	65.1	56.7	43.9	30.3	17.8

Table 3.1e : Power (%) of U_n^2 against the U(-.5,.5) distribution, based on 1,000 samples

MIS-SPECIFIED MODELS with U(-.5,.5) errors
Power (%) at various significance levels

case (uflque)

N	15.0	10.0	5.0	2.5	1.0
20	47.7	35.5	18.5	8.8	3.1
50	99.8	99.8	99.8	99.8	99.0

case (ofqlue)

N	15.0	10.0	5.0	2.5	1.0
20	32.9	23.7	12.9	7.2	3.6
50	68.1	58.1	42.5	31.4	17.7

case (ar12ue)

N	15.0	10.0	5.0	2.5	1.0
20	20.1	14.3	7.6	3.7	1.2
50	51.2	43.9	33.3	24.6	15.2

case (ar21ue)

N	15.0	10.0	5.0	2.5	1.0
20	31.3	22.8	12.1	6.8	2.6
30	44.7	36.4	23.9	15.1	7.0
50	67.2	58.9	43.5	29.3	18.8

Table 4.1a : Estimated Sizes (%) of A_n^2 compared with nominal levels, based on 1,000 samples

MIS-SPECIFIED MODELS(Normal Errors)
Estimated Sizes(%) based on 1,000 samples
compared with nominal levels(%)

case (uflq1)

N	15.0	10.0	5.0	2.5	1.0
20	15.8	10.8	5.7	2.9	1.3
30	72.8	61.9	43.0	27.9	13.7
50	67.0	58.3	41.6	29.2	17.0
100	100.0	100.0	100.0	100.0	100.0

case (ofql1)

N	15.0	10.0	5.0	2.5	1.0
20	16.4	10.9	5.5	3.1	1.1
30	16.0	10.4	5.7	3.0	1.4
50	17.6	11.1	5.6	2.9	1.4
100	14.9	10.1	4.2	2.4	1.4

case (uflq2)

N	15.0	10.0	5.0	2.5	1.0
20	17.4	12.3	6.4	3.6	2.1
30	34.9	27.0	16.4	9.7	3.9
50	93.1	88.6	75.1	60.4	39.1

case (ofql2)

N	15.0	10.0	5.0	2.5	1.0
20	17.8	12.0	5.5	2.5	1.0
30	15.0	9.3	4.6	2.5	1.0
50	17.6	12.0	5.4	2.8	1.3

case (ufqc)

N	15.0	10.0	5.0	2.5	1.0
20	16.4	10.9	5.5	2.9	1.3
30	15.5	9.9	4.8	2.3	1.0
50	16.5	10.8	4.9	2.7	0.9
100	14.0	9.3	4.1	2.1	1.0

case (ofcq)

N	15.0	10.0	5.0	2.5	1.0
20	17.6	11.7	6.1	2.6	0.8
30	16.5	10.8	5.1	2.3	1.1
50	17.3	11.7	5.5	2.7	1.0
100	14.8	10.2	4.5	2.6	1.4

Table 4.1a : Estimated Sizes (%) of A_n^2 compared with nominal levels, based on 1,000 samples

case (ufar12)

N	15.0	10.0	5.0	2.5	1.0
20	20.4	16.1	9.3	4.9	2.6
30	24.6	19.5	11.4	7.5	3.6
50	33.1	26.2	18.8	13.2	7.0

case (ofar21)

N	15.0	10.0	5.0	2.5	1.0
20	15.4	9.8	4.7	2.8	0.8
30	16.2	11.4	6.0	2.8	1.8
50	15.6	10.5	6.1	2.8	1.2

Table 4.1b : Power (%) of A_n^2 against the Laplace distribution, based on 1,000 samples

CORRECT MODELS with LAPLACE Errors
Power (%) at various significance levels

case (ble1)

N	15.0	10.0	5.0	2.5	1.0
20	40.6	31.9	21.7	14.7	9.5
30	49.6	42.0	32.9	24.4	15.7
50	67.9	61.0	52.5	42.2	32.3

case (ble2)

N	15.0	10.0	5.0	2.5	1.0
20	39.9	32.1	21.7	14.8	9.4
30	49.5	42.6	32.7	23.9	16.3
50	64.9	58.0	48.5	39.2	28.3

case (qle1)

N	15.0	10.0	5.0	2.5	1.0
20	38.0	29.4	20.4	13.8	8.8
30	45.8	38.4	28.5	20.6	13.6
50	65.7	57.6	48.3	39.4	31.0

case (qle2)

N	15.0	10.0	5.0	2.5	1.0
20	37.8	30.8	21.2	14.1	9.2
30	46.8	38.8	29.0	20.9	14.0
50	62.5	56.2	46.1	36.3	25.7

case (ar1le)

N	15.0	10.0	5.0	2.5	1.0
20	38.1	30.7	22.2	17.7	10.9
30	48.4	40.2	30.6	22.7	16.1
50	67.7	60.9	49.3	39.6	29.9

case (ar2le)

N	15.0	10.0	5.0	2.5	1.0
20	35.7	28.3	20.3	13.7	9.1
30	43.9	37.4	26.9	20.0	13.5
50	65.9	59.2	47.4	36.6	27.6

Table 4.1c : Power (%) of A_n^2 against the Laplace distribution, based on 1,000 samples

MIS-SPECIFIED MODELS(LAPLACE Errors)
Power (%) at various significance levels

	case (uflqle)				
N	15.0	10.0	5.0	2.5	1.0
20	31.2	25.5	16.3	9.4	4.9
50	41.7	33.8	23.0	14.3	7.4

	case (ofqlle)				
N	15.0	10.0	5.0	2.5	1.0
20	34.4	27.4	18.4	11.5	7.4
50	66.2	59.1	49.0	39.9	31.5

	case (ar12le)				
N	15.0	10.0	5.0	2.5	1.0
20	31.5	24.9	15.5	9.4	5.5
50	38.7	31.4	23.4	16.7	10.9

	case (ar21le)				
N	15.0	10.0	5.0	2.5	1.0
20	36.0	28.6	20.4	14.7	8.9
50	63.3	55.8	45.2	36.3	26.9

Table 4.1d : Power (%) of A_n^2 against the U(-.5,.5) distribution, based on 1,000 samples

CORRECT MODELS with U(-.5,.5) Errors
Power (%) at various significance levels

case (blue1)

N	15.0	10.0	5.0	2.5	1.0
20	37.8	29.1	17.6	10.2	3.8
30	52.4	42.7	26.2	16.1	6.9
50	77.8	68.8	52.7	37.8	23.2

case (blue2)

N	15.0	10.0	5.0	2.5	1.0
20	37.6	28.3	17.4	10.0	4.1
30	53.5	43.1	27.6	16.0	7.5
50	79.7	71.7	55.3	40.3	25.1

case (que1)

N	15.0	10.0	5.0	2.5	1.0
20	32.5	25.6	13.5	7.2	3.5
30	45.6	36.8	22.1	12.9	5.3
50	76.3	66.2	50.0	35.0	21.9

case (que2)

N	15.0	10.0	5.0	2.5	1.0
20	33.5	23.7	11.9	7.3	2.6
30	45.2	35.8	21.6	12.7	5.1
50	75.8	66.6	48.9	36.3	22.0

case (ar1ue)

N	15.0	10.0	5.0	2.5	1.0
20	36.2	26.6	16.4	8.6	3.9
30	51.4	42.6	28.1	17.5	9.6
50	78.5	69.8	56.9	43.5	26.6

case (ar2ue)

N	15.0	10.0	5.0	2.5	1.0
20	29.0	20.8	11.3	5.7	2.2
30	43.7	35.1	22.1	12.5	6.6
50	70.3	61.5	47.0	34.2	19.8

Table 4.1e : Power (%) of A_n^2 against the U(-.5,.5) distribution, based on 1,000 samples

MIS-SPECIFIED MODELS with U(-.5,.5) Errors
Power (%) at various significance levels

case (uflque)

N	15.0	10.0	5.0	2.5	1.0
20	55.6	41.5	21.2	10.5	3.1
50	99.8	99.8	99.8	99.8	99.8

case (ofqlue)

N	15.0	10.0	5.0	2.5	1.0
20	33.1	23.4	12.3	6.8	3.3
50	73.2	64.3	48.4	35.9	20.4

case (ar12ue)

N	15.0	10.0	5.0	2.5	1.0
20	23.2	16.5	8.6	4.6	1.9
50	56.7	49.6	38.5	29.1	19.3

case (ar21ue)

N	15.0	10.0	5.0	2.5	1.0
20	32.0	22.5	11.8	5.9	2.7
30	46.0	37.3	24.5	15.0	6.4
50	72.3	63.8	48.1	33.3	20.7

Table 6 : Comparison of Powers(%) against the Laplace and U(-.5,.5) alternatives with those for the single sample case, at the 5% and 10% significance levels.[1]

Dist.	n	D_n	W^2_n	U^2_n	A^2_n
Laplace 5%	20	16.1(22)	19.9(26)	20.4(25)	21.7(26)
	30	25.3(29)	29.8(35)	30.4(34)	32.9(-)
U(-.5,.5) 5%	20	11.1(12)	15.2(16)	17.8(18)	17.6(21)
	30	13.4(17)	22.0(26)	25.0(29)	26.2(-)
	50	23.5(28)	41.0(47)	46.3(52)	52.7(-)
Laplace 10%	50	52.3(-)	59.2(63)	59.7(-)	61.0(64)
U(-.5,.5) 10%	50	40.8(-)	58.3(61)	62.2(-)	68.8(75)

[1]The numbers in brackets are from Stephens(1974), Tables 5,6, pp734,735. These are the powers of the EDF tests for normality against the 2 alternatives, for the single sample case (case 3).

Table 7.1 : Comparison of Estimated Sizes(%) with Nominal Levels for sample sizes n = 10, 20, for $\sqrt{n}D_n$, based on 10,000 samples.[1]

Case	sample size	Nominal Levels (%)			
		10.0	5.0	2.5	1.0
(a)	10	11.3	5.5	2.9	1.1
	20	10.1	5.1	2.9	1.4
(b)	10	11.8	5.6	3.1	1.2
	20	10.6	5.1	2.7	1.3
(e)	10	11.7	5.6	3.1	1.2
	20	10.6	5.3	2.7	1.3
(q)	10	11.4	5.1	2.6	0.8
	20	10.0	4.6	2.5	0.8
(sq)	10	12.0	6.1	2.9	1.1
	20	10.3	4.8	2.8	1.1
(lg)	10	12.0	6.0	3.3	1.3
	20	10.2	4.9	2.6	1.1
(cu)	10	12.0	5.8	3.1	1.1
	20	9.9	4.6	2.5	0.9
(qsq)	10	11.0	5.4	2.7	0.6
	20	10.4	5.2	2.6	1.1
(tr)	10	11.1	5.1	2.9	1.1
	20	10.5	5.1	2.8	1.1

[1]The first row of numbers are the nominal levels. The rows below give the Monte Carlo estimated sizes.

Table 7.2 : Comparison of Estimated Sizes(%) with Nominal Levels for sample sizes n = 10, 20, for W_n^2 , based on 10,000 samples.

Case	sample size	Nominal Levels (%)			
		10.0	5.0	2.5	1.0
(a)	10	10.8	5.2	2.5	0.9
	20	10.4	5.2	2.2	1.0
(b)	10	11.2	5.5	2.8	0.9
	20	10.7	5.5	2.5	1.0
(e)	10	11.2	5.5	2.8	0.9
	20	10.7	5.5	2.5	1.0
(q)	10	10.4	4.2	2.0	0.5
	20	10.0	5.0	2.4	0.8
(sq)	10	10.9	5.7	2.8	0.9
	20	10.7	5.2	2.5	0.9
(lg)	10	11.2	5.5	2.9	1.1
	20	10.4	5.0	2.5	1.0
(cu)	10	10.8	5.5	2.4	0.6
	20	9.6	4.7	2.1	0.4
(qsq)	10	10.0	4.8	2.0	0.4
	20	10.0	4.8	2.5	0.8
(tr)	10	10.4	4.8	2.4	0.8
	20	10.4	5.4	2.7	1.1

Table 7.3 : Comparison of Estimated Sizes(%) with Nominal Levels for sample sizes n = 10, 20, for U_n^2 , based on 10,000 samples.

Case	sample size	Nominal Levels (%)			
		10.0	5.0	2.5	1.0
(a)	10	11.7	5.6	2.5	0.9
	20	10.8	5.5	2.4	1.0
(b)	10	11.7	6.0	3.0	1.1
	20	10.8	5.5	2.5	1.1
(e)	10	11.7	5.8	2.9	1.0
	20	10.8	5.5	2.5	1.1
(q)	10	10.9	5.0	2.2	0.4
	20	10.0	5.5	2.2	0.9
(sq)	10	11.7	6.0	3.0	0.9
	20	10.8	5.7	2.8	1.1
(lg)	10	11.8	6.0	3.1	1.1
	20	10.8	5.7	2.8	1.1
(cu)	10	11.4	5.8	2.6	0.8
	20	10.4	5.0	2.2	0.5
(qsq)	10	10.5	5.3	2.2	0.3
	20	10.4	5.3	2.4	0.8
(tr)	10	10.9	5.5	2.6	1.0
	20	10.8	5.7	2.9	1.0

Table 7.4 : Comparison of Estimated Sizes(%) with Nominal Levels for sample sizes n = 10, 20, for A_n^2 , based on 10,000 samples.

Case	sample size	Nominal Levels (%)			
		10.0	5.0	2.5	1.0
(a)	10	12.5	6.5	2.9	1.4
	20	11.1	5.4	2.6	1.1
(b)	10	12.8	6.5	3.3	1.3
	20	11.4	5.8	2.6	1.2
(e)	10	12.7	6.5	3.3	1.3
	20	11.4	5.8	2.6	1.2
(q)	10	12.0	5.4	2.4	0.8
	20	10.6	5.0	2.3	0.9
(sq)	10	12.7	6.7	3.3	1.3
	20	11.0	5.7	2.6	1.2
(lg)	10	13.0	6.5	3.4	1.4
	20	11.3	5.7	2.6	1.1
(cu)	10	12.5	6.3	2.8	0.7
	20	10.4	5.1	2.4	0.8
(qsq)	10	11.8	5.4	2.5	0.5
	20	10.8	5.3	2.4	0.9
(tr)	10	11.5	5.7	2.5	0.8
	20	11.0	5.5	2.9	1.2

TABLE 8.1 : Air quality Data

x_i	y_i	$\hat{\epsilon}_i$
164.2	181	2.52
156.9	156	0.82
109.8	115	0.27
111.4	132	1.64
87.0	96	0.08
82.9	90	-0.18
78.9	86	-0.27
161.8	170	1.72
230.9	193	-0.73
106.5	110	0.05
97.6	94	-0.77
79.7	77	-1.11
100.8	88	-1.49
387.8	310	-0.89
118.7	106	-1.07
248.8	204	-0.95
102.4	98	-0.73
64.2	76	-0.20
89.4	89	-0.68
117.9	130	1.05
135.0	141	0.91
108.1	102	-0.75
89.4	91	-0.51
76.4	97	0.85
131.7	128	0.00

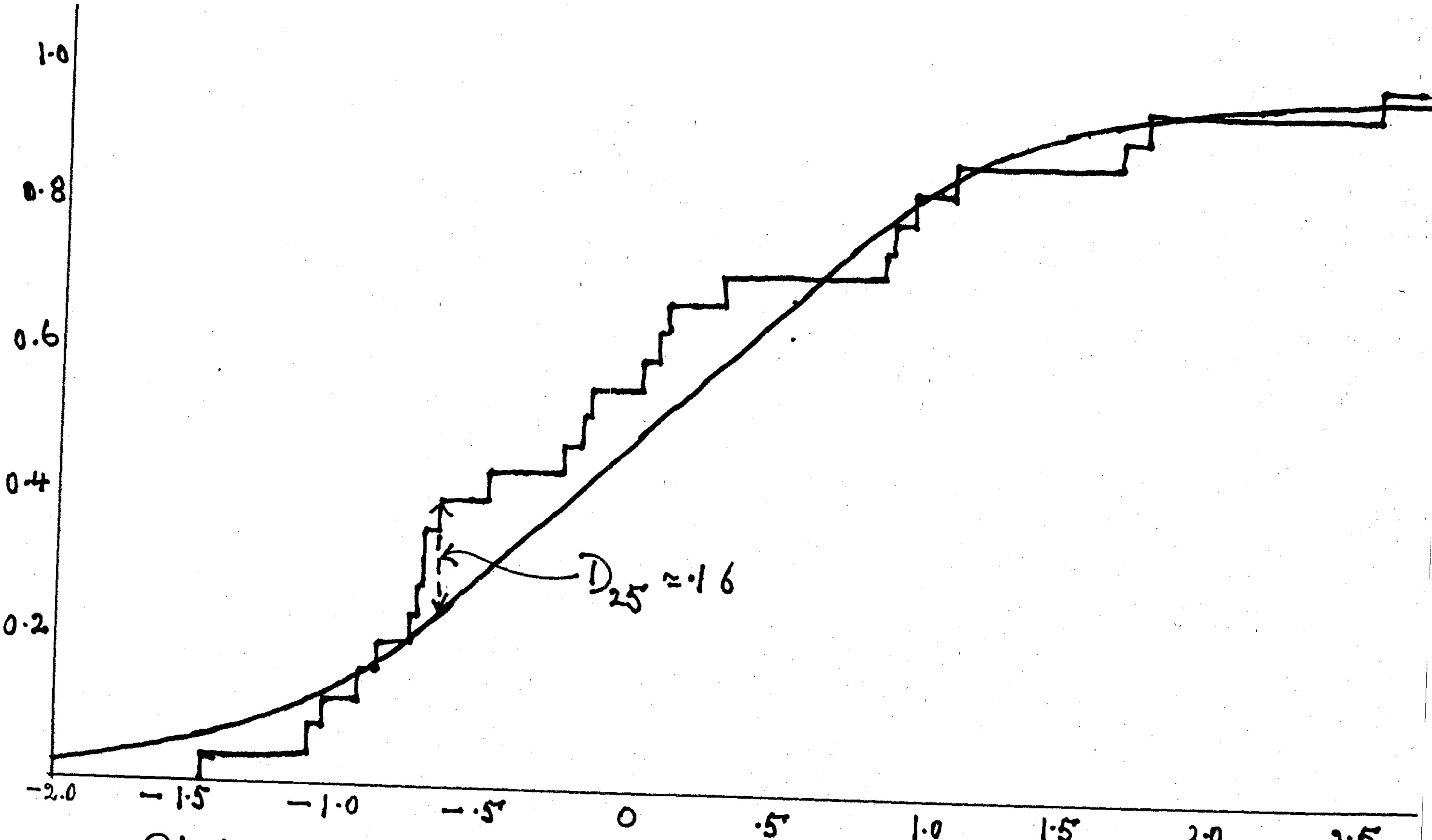
"Carbon aerosols have been identified as a contributing factor in a number of air quality problems. In a chemical analysis of diesel engine exhaust, x = mass(μg/cm²) and y = elemental carbon (μg/cm²) were recorded ("Comparison of Solvent Extraction and Thermal Optical Carbon Analysis Methods: Application to Diesel Vehicle Exhaust Aerosol" *Environ. Sci. Tech.* (1984):231-234). The estimated regression line for this data set is $\hat{y} = 31 + .737x$."[1]

Table 8.1 gives the (x,y) values and the corresponding standardized residuals after fitting a simple linear model to the data set. The EDF and Φ, the standard normal cdf, are shown.

When the Pierce-Kopecky test is applied to the standardized residuals, the values of the EDF statistics are : $\sqrt{n}D_n = 0.783$, $W_n^2 = 0.135$, $U_n^2 = 0.112$, $A_n^2 = 0.778$. Using Table 1.3 from Stephens(1974), $\sqrt{n}D_n$ is not significant at the 10% level; U_n^2 is significant at the 10% level; W_n^2 and A_n^2 are both significant at the 5% level.

---

[1] see Devore and Peck(1986), p488, Problem 11.53



Plot of EDF and Normal CDF for Air Quality DATA.

TABLE 8.2 : Rocket Propellant Data

x_i	y_i	$\hat{y}_i$	ϵ_i	$\hat{\epsilon}_i$
15.50	2158.70	2051.94	106.76	1.11
23.75	1678.15	1745.42	-67.27	-0.70
8.00	2316.00	2330.59	-14.59	-0.15
17.00	2061.30	1996.21	65.09	0.68
5.50	2207.50	2423.48	-215.98	-2.25
19.00	1708.30	1921.90	-213.60	-2.22
24.00	1784.70	1736.14	48.56	0.51
2.50	2575.00	2534.94	40.06	0.42
7.50	2357.90	2349.17	8.73	0.09
11.00	2256.70	2219.13	37.57	0.39
13.00	2165.20	2144.83	20.37	0.21
3.75	2399.55	2488.50	-88.95	-0.93
25.00	1779.80	1698.98	80.82	0.84
9.75	2336.75	2265.58	71.17	0.74
22.00	1765.30	1810.44	-45.14	-0.47
18.00	2053.50	1959.06	94.44	0.98
6.00	2414.40	2404.90	9.50	0.10
12.50	2200.50	2163.40	37.10	0.39
2.00	2654.20	2553.52	100.68	1.05
21.50	1753.70	1829.02	-75.32	-0.78

"A rocket motor is manufactured by bonding an igniter propellant and a sustainer propellant together inside a metal housing. The shear strength of the bond between the two types of propellant is an important quality characteristic. It is suspected that shear strength is related to age in weeks of the batch of sustainer propellant."[2]
Table 8.2 shows 20 observations of shear strength and age of the corresponding batch of propellant. x_i is age in weeks; y_i is shear strength in psi. The fitted regression line is $\hat{y} = 2,627.82 - 37.15x$. The fitted y values, the residuals, and the standardized residuals are also given in Table 8.2.

When the EDF test is done on the standardized residuals the 4 statistics have the values: $\sqrt{n}D_n = 0.864$; $W_n^2 = 0.126$; $U_n^2 = 0.106$; and $A_n^2 = 0.848$.

Using the case 3 Table, $\sqrt{n}D_n$, W_n^2, and U_n^2 are significant at the 10% level; A_n^2 is significant at the 5% level.

----

[2]see Montgomery and Peck(1982), pp11-15, and pp62-65.

EDF for standardized residuals from fitting regression line to the ROCKET PROPELLANT DATA $n = 20$

REGRESSION Equation : $\hat{Y}_i = 2627.82 - 37.15\,X$

$D \simeq 0.188$

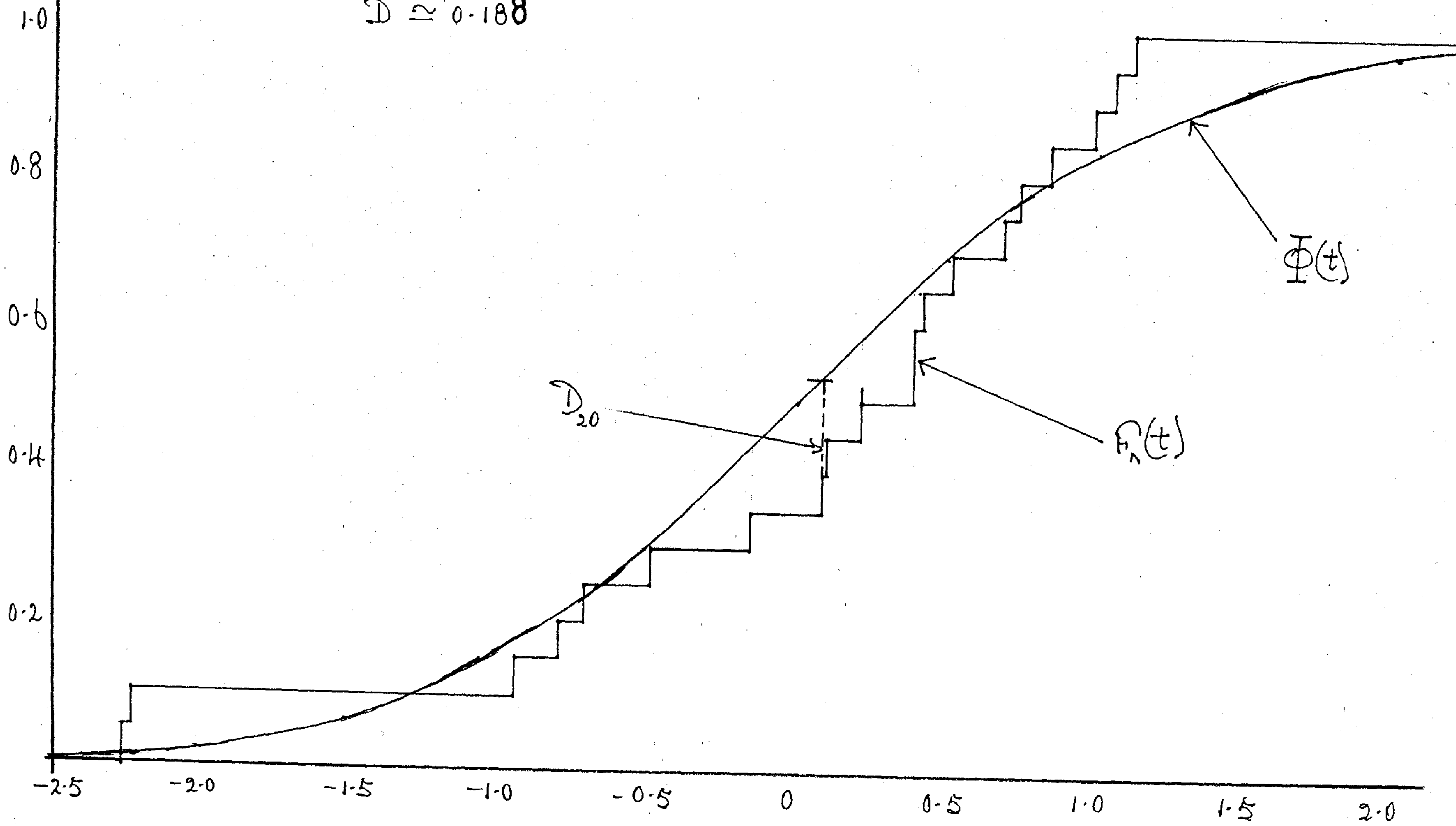


TABLE 8.3 : DNA Sequence Data

L_i	m_i	$\hat{\epsilon}_i$
267	5.9	0.07
234	7.0	-0.57
213	8.1	-0.20
192	9.2	-0.69
184	10.0	0.33
124	16.1	0.98
104	19.2	0.64
89	22.5	1.39
80	24.6	0.69
64	29.5	-0.39
57	32.1	-1.57
51	35.0	-1.66
21	64.0	1.86
18	69.2	-0.95

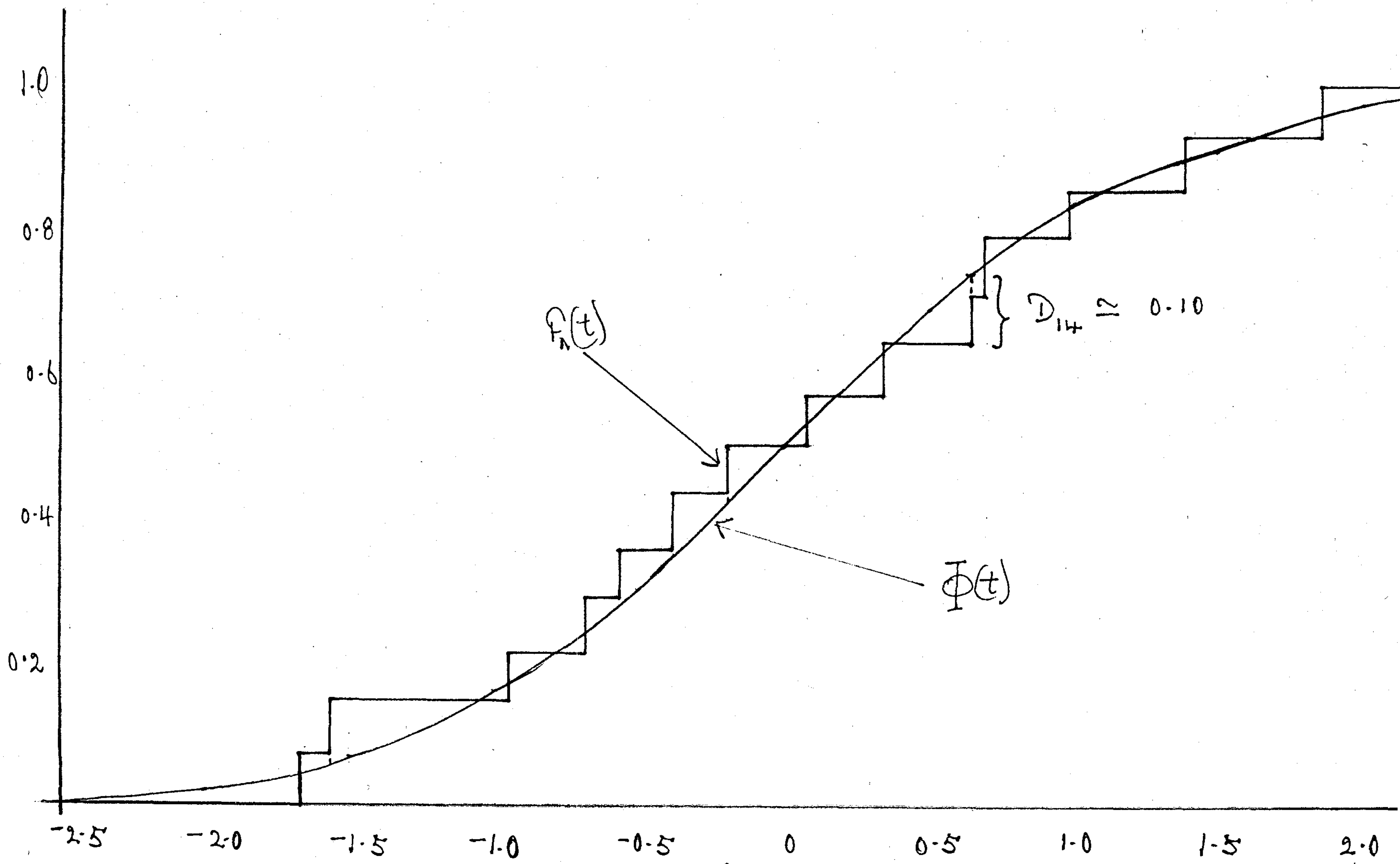
A large DNA molecule is often studied by analysing the fragments generated by several different restriction enzymes. These fragments are then used to construct a restriction map of the whole molecule. Usually the lengths of the fragments are not known very accurately. One technique of estimating the lengths of the fragments is to inject the into an electrophoretic gel and measure their migration distances under a fixed voltage. Duggelby *et al.*(1981) proposed the equation :$m_i = a_0 + a_1 \log L_i + a_2 (\log L_i)^2$ to explain the assumed relationship between migration distance and length of a fragment. Table 8.3 gives the migration distance against known standard lengths of DNA, expressed in base pairs (bp). One bp is approximately 2.7 angstroms depending on the exact base composition. Duggelby used least squares to fit the model to the observations. The fit is very good with R^2= 100%. However, the measurement of the migration distance in the gel is subject to several types of errors.[3] Can the distribution of the total measurement error be considered normal?

The normal probability plot of the standardized residuals seems quite straight. We apply the EDF test to the standardized residuals to test for normality. For the data set in Table 8.3 we get for the values of the 4 EDF statistics: $\sqrt{n}D_n = 0.380$, $W_n^2 = 0.020$, $U_n^2 = 0.019$, and $A_n^2 = 0.162$. From the case 3 Table1.3 of Stephens(1974), none of the 4 statistics is significant, even at the 15% level. In this case, the informal test using the probability plot and the formal test agree in not rejecting normality of the error distribution.

---

[3] see Weir, B.S., ed(1983)

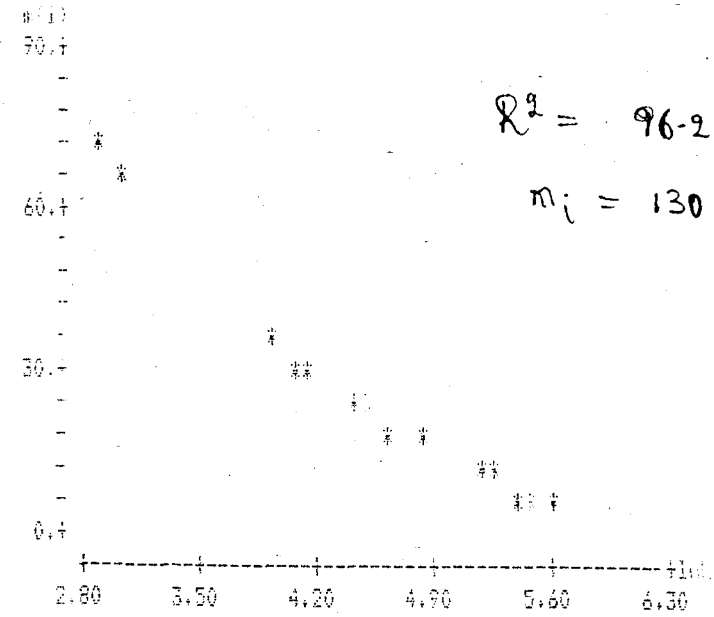
'Fitted regression equation : $\hat{Y}_i = a_0 + a_1 \ln L_i + a_2(\ln L_i)^2$



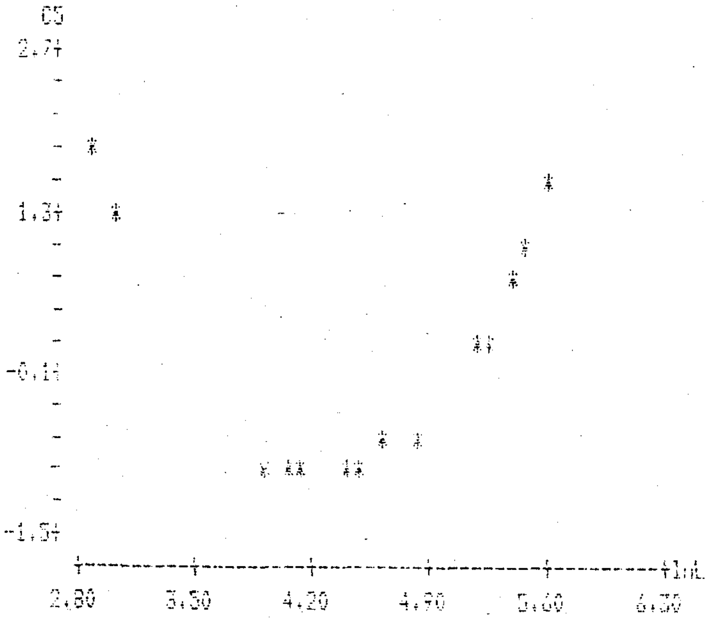
EDF for standardized residuals for the DNA sequence DATA'.

$R^2 = 96.2\ \%$

$m_i = 130 - 23.1 \ln L_i$



Scatter plot of m_i vs $\ln L_i$



Residual plot $\hat{\varepsilon}_i$ vs $\ln L_i$

```
PLOT C5 C12
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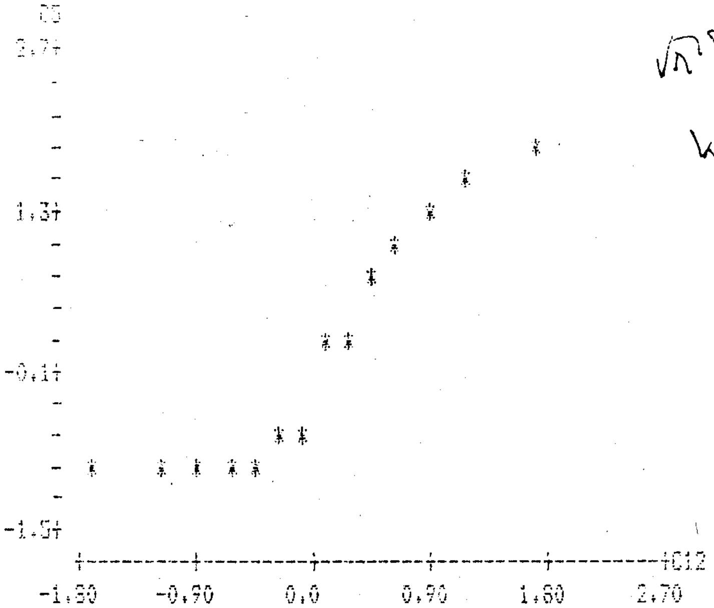

Probability Plot

$\sqrt{n}\,D_n = 0 \cdot 846$ sig at 10%

$W_n^2 = 0 \cdot 132$ sig at 5%

$U_n^2 = 0 \cdot 131$ sig at 5%

$A_n^2 = 0 \cdot 874$ sig at 2·5%

```
MTB >
NOTE:PROBABILITY PLOT OF STD RES WHEN A ST. LINE IN LN L(I)
MTB >
 NOTE: IS FITTED TO M(I) DATA.
MTB >
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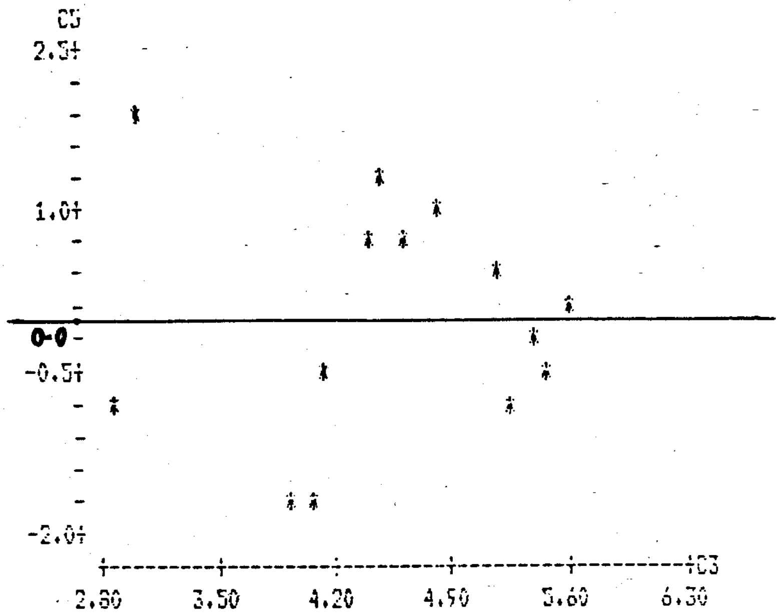
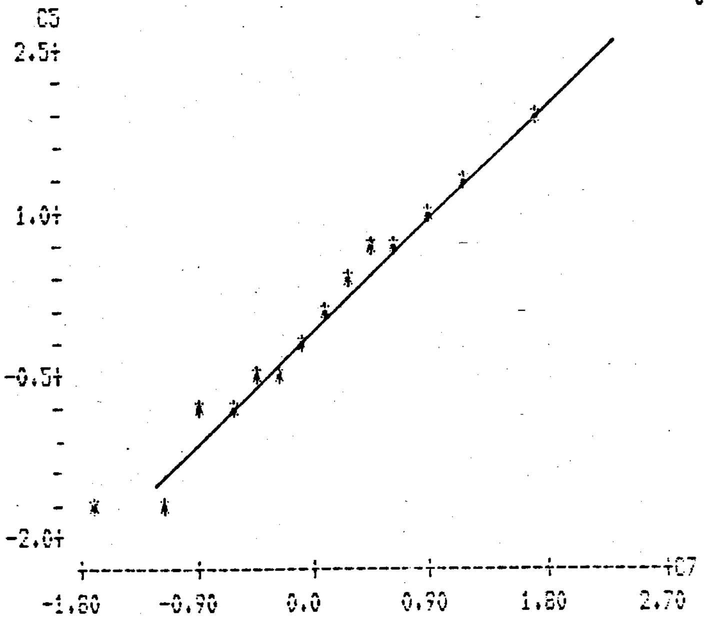


$R^2 = 100\%$

RESIDUAL PLOT when model $\hat{y}_i = a_0 + a_1 \log L_i + a_2 (\log L_i)^2$ is fitted to data

6 >
ot c5 c7



$$\left.\begin{aligned} \sqrt{n}\,D_n &= 0.380 \\ W_n^2 &= 0.020 \\ U_n^2 &= 0.019 \\ A_n^2 &= 0.162 \end{aligned}\right\} \text{not significant}$$

PROBABILITY PLOT for quadratic model

B >

APPENDIX : ROUTINES USED

```
C PROGRAMME TO MONTECARLO THE DBNS OF EDF STATISTICS FOR
C THE PIERCE-KOPECKY RESIDUALS AFTER FITTING A SIMPLE
C LINEAR REGRESSION
C
EXTERNAL REGR
CALL MCFREQ(REGR)
STOP
END
C****************************************** MONTE1
SUBROUTINE REGR(MCLO,DSEED)
DIMENSION S(7),SS(4),Z(100),R(100),E(100),X(100)
REAL Y(100),U(100),V(100)
DOUBLE PRECISION DSEED
COMMON /IPAR/ N
COMMON /RPAR/ C
COMMON /STAT/ S
COMMON /STAT1/ SS
ICASE=3
AL = 2.
C CALL GGNML(DSEED,N,R)
CALL GGUBS(DSEED,N,U)
CALL GGUBS(DSEED,N,V)
DO 11 I= 1,N
R(I) = SIGN(-AL*ALOG(U(I)),V(I)-.5)
C R(I) = U(I)-.5
11 CONTINUE
CALL GENR(N,R,E,X,Y)
DO 40 J = 1,N
CALL MDNOR(E(J),Z(J))
40 CONTINUE
CALL VSRTA(Z,N)
CALL EDF(N,Z,S,ICASE)
SS(1) = S(3)
SS(2) = S(5)
SS(3) = S(6)
SS(4) = S(7)
RETURN
END
C****************************************** GENR
SUBROUTINE GENR(N,R,E,X,Y)
DIMENSION R(1),E(1),X(1),Y(1)
INTEGER N
XSUM = 0.0
YSUM = 0.0
XSQ = 0.0
YSQ = 0.0
PROD = 0.0
C****************************************
C**DATA GENERATED WITH U(-.5,.5) ERRORS**
C****************************************
C**FIT LINEAR MODEL TO QUADRATIC DATA****
```

```
C****************************************
C
RN = N
DO 30 I = 1,N
RI = I
X(I) = SQRT(RI)
Y(I) = X(I)+X(I)*X(I)+R(I)
C
C************************
C*END OF DATA GENERATION*
C************************
C
C********************************
C*FIT MODEL Y(I)=A+B*X(I)+S*E(I)*
C********************************
YSUM = YSUM+Y(I)
XSUM = XSUM+X(I)
YSQ = YSQ+Y(I)*Y(I)
XSQ = XSQ+X(I)*X(I)
PROD = PROD+X(I)*Y(I)
30 CONTINUE
AN = N
BN = 1.0/AN
YMU = YSUM*BN
XMU = XSUM*BN
VY = YSQ-(YMU*YMU)*AN
VX = XSQ-(XMU*XMU)*AN
COV = PROD-(XMU*YMU)*AN
SSQ = (VY-(COV*COV)/VX)*BN
SIGMA = SQRT(SSQ)
S1 = 1.0/SIGMA
BETA = COV/VX
ALPHA = YMU-BETA*XMU
C
C**********************
C*END OF FITTING MODEL*
C**********************
C
C*********************************************
C*COMPUTATION OF THE PIERCE-KOPECKY RESIDUALS*
C*********************************************
C
DO 50 J = 1,N
YFJ = ALPHA+BETA*X(J)
E(J) = (Y(J)-YFJ)*S1
50 CONTINUE
RETURN
END
```

```
C********************************************** QMTCLO
C MAIN PROGRAMME FOR QMTCLO
C PROGRAMME TO MONTE CARLO THE DBNS OF EDF STATISTICS
C FOR THE PIERCE-KOPECKY RESIDUALS AFTER FITTING A
C QUADRATIC MODEL.
C
EXTERNAL REGR
CALL MCFREQ(REGR)
STOP
END
C
C********************************************** REGR
C
SUBROUTINE REGR(MCLO,DSEED)
REAL R(100),X(100),Y(100),YF(100),YR(100),E(100)
REAL Z(100),S(7),SS(4),U(100),V(100)
DOUBLE PRECISION DSEED
COMMON /STAT/ S
COMMON /STAT1/ SS
COMMON /PKRES/ E
COMMON /IPAR/ N
COMMON /IVARS/ J,L,IB,IJOB
COMMON /COEFF/ B(3)
COMMON /IARR/ICHNG(6)
COMMON /RARR/H(6),DET(15)
C
J =3
L =1
IB =3
IJOB =0
ICASE=3
C
C********************************************
C*GENERATION OF DATA WITH U(-.5,.5) ERRORS**
C********************************************
C********************************************
C*GENERATE LINEAR DATA; FIT QUADRATIC MODEL*
C********************************************
C CALL GGNML(DSEED,N,R)
CALL GGUBS(DSEED,N,U)
CALL GGUBS(DSEED,N,V)
RN = N
AL = 2.
DO 42 I = 1,N
RK = I
X(I) = SQRT(RK)
R(I) = -AL*ALOG(U(I))
IF(V(I).LT.0.5)R(I)=-R(I)
C R(I) = U(I)-.5
Y(I) = X(I)+R(I)
42 CONTINUE
C
C*************************
```

```
C*END OF DATA GENERATION*
C************************
C
CALL GPKR(X,R,Y,YF,YR)
DO 11 I = 1,N
CALL MDNOR(E(I),Z(I))
11 CONTINUE
CALL VSRTA(Z,N)
CALL EDF(N,Z,S,ICASE)
SS(1) = S(3)
SS(2) = S(5)
SS(3) = S(6)
SS(4) = S(7)
RETURN
END
C C********************************************GPKR
C*DERIVATION OF THE PIERCE-KOPECKY RESIDUALS*
C*******************************************
C
SUBROUTINE GPKR(X,R,Y,YF,YR)
REAL X(1),R(1),Y(1),YF(1),YR(1),E(1)
COMMON /COEFF/ B(3)
COMMON /PKRES/ E
COMMON /IPAR/ N
COMMON /IVARS/ J,L,IB,IJOB
COMMON /IARR/ ICHNG(6)
COMMON /RARR/ H(6),DET(15)
C
CALL PREG(X,R,Y,IER)
RN = N
SSE = 0.0
DO 13 I = 1,N
YF(I) = B(1)+B(2)*X(I)+B(3)*X(I)*X(I)
YR(I) = Y(I) - YF(I)
SSE = SSE + YR(I)*YR(I)
13 CONTINUE
S0 = SQRT(SSE/RN)
S1 = 1./S0
DO 115 I = 1,N
E(I) = YR(I)*S1
115 CONTINUE
RETURN
END
C
C********************************************
C****END OF DERIVATION OF THE RESIDUALS*******
C********************************************
C
C********************************************
C*FIT MODEL:Y(I) = A+B*X(I)+C*X(I)**2 +S*E(I)*
C********************************************
C******************************************** PREG
C
```

```
SUBROUTINE PREG(X,R,Y,IER)
REAL X(1),R(1),Y(1)
COMMON /IPAR/ N
COMMON /COEFF/ B(3)
COMMON /IVARS/ J,L,IB,IJOB
COMMON /IARR/ ICHNG(6)
COMMON /RARR/ H(6),DET(15)
C
SX1 = 0.0
SX2 = 0.0
SX3 = 0.0
SX4 = 0.0
SY = 0.0
SYX1 = 0.0
SYX2 = 0.0
RN = N
DO 3 I = 1,N
SX1 = SX1+X(I)
SX2 = SX2+X(I)*X(I)
SX3 = SX3+X(I)*X(I)*X(I)
SX4 = SX4+X(I)*X(I)*X(I)*X(I)
SY = SY +Y(I)
SYX1 = SYX1+X(I)*Y(I)
SYX2 = SYX2+X(I)*X(I)*Y(I)
3 CONTINUE
H(1) = RN
H(2) = SX1
H(3) = SX2
H(4) = SX2
H(5) = SX3
H(6) = SX4
B(1) = SY
B(2) = SYX1
B(3) = SYX2
CALL LEQ2S(H,J,B,L,IB,IJOB,ICHNG,DET,IER)
RETURN
END
```

```
C*************************************************** CMTCLO
C MAIN PROGRAMME FOR CMTCLO
C PROGRAMME TO MONTE CARLO THE DBNS OF EDF STATISTICS
C FOR THE PIERCE-KOPECKY RESIDUALS AFTER FITTING A
C CUBIC MODEL.
C
EXTERNAL REGR
CALL MCFREQ(REGR)
STOP
END
C
C************************************************** REGR
C
SUBROUTINE REGR(MCLO,DSEED)
REAL R(100),X(100),Y(100),YF(100),YR(100)
REAL E(100),Z(100),S(16),SS(11)
DOUBLE PRECISION DSEED
COMMON /STAT/ S
COMMON /STAT1/ SS
COMMON /PKRES/ E
COMMON /IPAR/ N
COMMON /IVARS/ J,L,IB,IJOB
COMMON /RARR/ H(10),DET(22)
COMMON /COEFF/B(4)
COMMON /IARR/ ICHNG(8)
C
J =4
L =1
IB =4
IJOB =0
ICASE=3
C
C******************************************
C*********GENERATION OF DATA**************
C******************************************
C*GENERATE QUADRATIC DATA;FIT CUBIC MODEL*
C******************************************
CALL GGNML(DSEED,N,R)
RN = N
DO 42 I = 1,N
RK = I
X(I) = RK/RN
Y(I) = X(I)+X(I)*X(I)+R(I)
42 CONTINUE
C
C*******************************************
C***********END OF DATA GENERATION*********
C*******************************************
C
CALL GPKR(X,R,Y,YF,YR)
DO 11 I = 1,N
CALL MDNOR(E(I),Z(I))
11 CONTINUE
```

```
CALL VSRTA(Z,N)
CALL EDF(N,Z,S,ICASE)
SS(1) = S(3)
SS(2) = S(5)
SS(3) = S(6)
SS(4) = S(7)
RETURN
END
C
C********************************************** GPKR
C*DERIVATION OF THE PIERCE-KOPECKY RESIDUALS**
C**********************************************
C
SUBROUTINE GPKR(X,R,Y,YF,YR)
REAL X(1),R(1),Y(1),YF(1),YR(1),E(1)
COMMON /COEFF/ B(4)
COMMON /PKRES/ E
COMMON /IPAR/ N
COMMON /IVARS/ J,L,IB,IJOB
COMMON /RARR/ H(10),DET(22)
COMMON /IARR/ ICHNG(8)
C
CALL PREG(X,R,Y,IER)
RN = N
SSE = 0.0
DO 13 I = 1,N
YF(I)=B(1)+B(2)*X(I)+B(3)*X(I)*X(I)+B(4)*X(I)*X(I)*X(I)
YR(I) = Y(I) - YF(I)
SSE = SSE + YR(I)*YR(I)
13 CONTINUE
S0 = SQRT(SSE/RN)
S1 = 1./S0
DO 115 I = 1,N
E(I) = YR(I)*S1
115 CONTINUE
RETURN
END
C
C*******************************************
C****END OF DERIVATION OF THE RESIDUALS****
C*******************************************
C
C********************************************************
C*FIT MODEL:Y(I)=A+B*X(I)+C*X(I)**2+D*X(I)**3+S*E(I)****
C********************************************************
C*************************************************** PREG
C
SUBROUTINE PREG(X,R,Y,IER)
REAL X(1),R(1),Y(1)
COMMON /IPAR/ N
COMMON /COEFF/ B(4)
COMMON /IVARS/ J,L,IB,IJOB
COMMON /RARR/ H(10),DET(22)
```

```
COMMON /IARR/ ICHNG(8)
C
SX1 = 0.0
SX2 = 0.0
SX3 = 0.0
SX4 = 0.0
SX5 = 0.0
SX6 = 0.0
SY = 0.0
SYX1 = 0.0
SYX2 = 0.0
SYX3 = 0.0
RN = N
DO 3 I = 1,N
SX1 = SX1+X(I)
SX2 = SX2+X(I)*X(I)
SX3 = SX3+X(I)*X(I)*X(I)
SX4 = SX4+X(I)*X(I)*X(I)*X(I)
SX5 = SX5+X(I)*X(I)*X(I)*X(I)*X(I)
SX6 = SX6+X(I)*X(I)*X(I)*X(I)*X(I)*X(I)
SY = SY +Y(I)
SYX1 = SYX1+X(I)*Y(I)
SYX2 = SYX2+X(I)*X(I)*Y(I)
SYX3 = SYX3+X(I)*X(I)*X(I)*Y(I)
3 CONTINUE
H(1) = RN
H(2) = SX1
H(3) = SX2
H(4) = H(3)
H(5) = SX3
H(6) = SX4
H(7) = H(5)
H(8) = H(6)
H(9) = SX5
H(10) = SX6
B(1) = SY
B(2) = SYX1
B(3) = SYX2
B(4) = SYX3
CALL LEQ2S(H,J,B,L,IB,IJOB,ICHNG,DET,IER)
RETURN
END
```

```
C*******************************************************MONTE3
C PROGRAMME TO MONTECARLO THE DBNS OF EDF STATISTICS
C FOR THE PIERCE-KOPECKY RESIDUALS FROM FITTING A
C FIRST-ORDER AUTOREGRESSIVE PROCESS
EXTERNAL REGR
CALL MCFREQ(REGR)
STOP
END
C*******************************************************
SUBROUTINE REGR(MCLO,DSEED)
DIMENSION S(7),SS(4),Z(120),R(120),E(120),X(120)
REAL Y(120),U(120),V(120)
DOUBLE PRECISION DSEED
COMMON /IPAR/ N
COMMON /RPAR/ C
COMMON /STAT/ S
COMMON /STAT1/ SS
C
ICASE=3
AL =2.
C CALL GGNML(DSEED,N,R)
CALL GGUBS(DSEED,N,U)
CALL GGUBS(DSEED,N,V)
DO 11 I= 1,N
R(I) = -AL*ALOG(U(I))
IF(V(I).LT.0.5)R(I)=-R(I)
C R(I) = U(I)-.5
11 CONTINUE
CALL GENR(N,R,E,X,Y)
M = N-10
DO 40 J = 1,M
CALL MDNOR(E(J),Z(J))
40 CONTINUE
CALL VSRTA(Z,M)
CALL EDF(M,Z,S,ICASE)
SS(1) = S(3)
SS(2) = S(5)
SS(3) = S(6)
SS(4) = S(7)
RETURN
END
C******************************************** GENR
SUBROUTINE GENR(N,R,E,X,Y)
DIMENSION R(1),E(1),X(1),Y(1),W(120)
INTEGER N
C
XSUM = 0.0
YSUM = 0.0
XSQ = 0.0
YSQ = 0.0
PROD = 0.0
C********************************************
C****DATA GENERATED WITH LAPLACE ERRORS*****
```

```
C*******************************************
C
C P = -0.9
C AU = 3.5
C X(1) = 1.5
C AL = AU*(1.-P)
C AS = 2.0
C Y(1) = AL+P*X(1)+AS*R(1)
C DO 30 I = 2,N
C X(I) = Y(I-1)
C Y(I) = AL+P*X(I)+AS*R(I)
C
C *** GENERATION OF AR(2) DATA ***
C
P1 = -0.10D0
P2 = 0.90D0
AU = -2.50D0
AL = AU*(1.-P1-P2)
AS = 1.0D0
X(1) = 1.50D0
W(1) = 0.0D0
Y(1) = AL+P1*X(1)+P2*W(1)+AS*R(1)
X(2) = Y(1)
W(2) = X(1)
Y(2) = AL+P1*X(2)+P2*W(2)+AS*R(2)
DO 30 I = 3,N
X(I) = Y(I-1)
W(I) = X(I-1)
Y(I) = AL+P1*X(I)+P2*W(I)+AS*R(I)
30 CONTINUE
C C*******************************************
C**********END OF DATA GENERATION***********
C*******************************************
C
C*******************************************
C**FIT MODEL Y(I)=AL+P*X(I)+AS*E(I),I=11,N**
C*******************************************
DO 40 I = 11,N
YSUM = YSUM+Y(I)
XSUM = XSUM+X(I)
YSQ = YSQ+Y(I)*Y(I)
XSQ = XSQ+X(I)*X(I)
PROD = PROD+X(I)*Y(I)
40 CONTINUE
AN = N-10
BN = 1.0/AN
CN = 1.0/(AN-2.)
YMU = YSUM*BN
XMU = XSUM*BN
VY = YSQ-(YMU*YMU)*AN
VX = XSQ-(XMU*XMU)*AN
COV = PROD-(XMU*YMU)*AN
SSQ = (VY-(COV*COV)/VX)*CN
```

```
SIGMA = SQRT(SSQ)
S1 = 1.0/SIGMA
BETA = COV/VX
ALPHA = YMU-BETA*XMU
C
C*****************************
C****END OF FITTING MODEL****
C*****************************
C
C********************************************
C*COMPUTATION OF THE PIERCE-KOPECKY RESIDUALS*
C********************************************
C
T = AN*VX
XU = 2.*XMU
DO 50 J = 11,N
YFJ = ALPHA+BETA*X(J)
UJ = XSQ+AN*X(J)*(X(J)-XU)
VJ = SQRT(1.-UJ/T)
E(J-10) = ((Y(J)-YFJ)*S1)/VJ
50 CONTINUE
RETURN
END
```

```
IMPLICIT REAL*8 (A-H,O-Z)
C PROGRAM TO MONTECARLO THE DBNS OF THE EDF STATISTICS
C FROM THE PIERCE-KOPECKY RESIDUALS AFTER FITTING A
C SECOND-ORDER AUTOREGRESSIVE PROCESS :
C y_i = λ+ρ1y_i-1+ρ2y_i-2 +σε_i
EXTERNAL REGR
CALL MCFREQ(REGR)
STOP
END
C********************************************** AR2*
SUBROUTINE REGR(MCLO,DSEED)
DIMENSION S(7),SS(4),Z(120),R(120),E(120),U1(120),V1(120)
REAL*8 X(120),Y(120),W(120),YF(120),YR(120),U(120),V(120)
DOUBLE PRECISION DSEED
COMMON /IPAR/ N
COMMON /STAT/ S
COMMON /STAT1/ SS
C
ICASE=3
AL =2.
C CALL GGNML(DSEED,N,R)
CALL GGUBS(DSEED,N,U1)
CALL GGUBS(DSEED,N,V1)
DO 11 I= 1,N
R(I) = -AL*ALOG(U1(I))
IF(V1(I).LT.0.5)R(I)=-R(I)
C R(I) = U1(I)-.5
11 CONTINUE
CALL GENR(N,R,E,X,Y,W,YF,YR,U,V)
M = N-10
DO 40 J = 1,M
CALL MDNOR(E(J),Z(J))
40 CONTINUE
CALL VSRTA(Z,M)
CALL EDF(M,Z,S,ICASE)
SS(1) = S(3)
SS(2) = S(5)
SS(3) = S(6)
SS(4) = S(7)
RETURN
END
C**********************************************GENR
SUBROUTINE GENR(N,R,E,X,Y,W,YF,YR,U,V)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 X(1),Y(1),W(1),YF(1),YR(1),U(1),V(1)
REAL R(1),E(1)
INTEGER N
C
XSUM = 0.0
WSUM = 0.0
YSUM = 0.0
XSQ = 0.0
WSQ = 0.0
```

```
YSQ = 0.0
PRODXY = 0.0
PRODWY = 0.0
PRODXW = 0.0
C
C**************************************
C**DATA GENERATED WITH LAPLACE ERRORS**
C**************************************
C
C P1 = 0.50D0
C P2 = 0.20D0
C AU = 0.50D0
C AL = AU*(1.-P1-P2)
C AS = 1.0D0
C X(1) = 1.50D0
C W(1) = 0.0D0
C Y(1) = AL+P1*X(1)+P2*W(1)+AS*R(1)
C X(2) = Y(1)
C W(2) = X(1)
C Y(2) = AL+P1*X(2)+P2*W(2)+AS*R(2)
C DO 30 I = 3,N
C X(I) = Y(I-1)
C W(I) = X(I-1)
C Y(I) = AL+P1*X(I)+P2*W(I)+AS*R(I)
C
C*** GENERATION OF AR(1) DATA ***
C
P = -0.90D0
AU = 3.50D0
X(1) = 1.50D0
W(1) = 0.0D0
W(2) = X(1)
AL = AU*(1.-P)
AS = 2.0D0
Y(1) = AL+P*X(1)+AS*R(1)
X(2) = Y(1)
DO 30 I = 3,N
X(I) = Y(I-1)
W(I) = X(I-1)
Y(I) = AL+P*X(I)+AS*R(I)
30 CONTINUE
C
C**************************
C**END OF DATA GENERATION**
C**************************
C
C***************************************************
C*FIT MODEL Y(I)=AL+P1*X(I)+P2*W(I)+AS*E(I),I=11,N**
C***************************************************
DO 40 I = 11,N
YSUM = YSUM+Y(I)
WSUM = WSUM+W(I)
XSUM = XSUM+X(I)
```

```
YSQ = YSQ+Y(I)*Y(I)
WSQ = WSQ+W(I)*W(I)
XSQ = XSQ+X(I)*X(I)
PRODXY = PRODXY+X(I)*Y(I)
PRODWY = PRODWY+W(I)*Y(I)
PRODXW = PRODXW+X(I)*W(I)
40 CONTINUE
AN = DFLOAT(N-10)
BN = 1.0D0/AN
CN = 1.0D0/(AN-3.0D0)
YMU = YSUM*BN
XMU = XSUM*BN
WMU = WSUM*BN
VY = YSQ-(YMU*YMU)*AN
VX = XSQ-(XMU*XMU)*AN
VW = WSQ-(WMU*WMU)*AN
COVXY = PRODXY-(XMU*YMU)*AN
COVXW = PRODXW-(XMU*WMU)*AN
COVWY = PRODWY-(WMU*YMU)*AN
C
C** COMPUTE DENOMINATOR D COMMON TO BETA & GAMMA **
C
D = VX*VW - COVXW*COVXW
C
C** CHECK IF D = 0 *****
C
IF (D)5,3,5
C
C*** IF D = 0 EXIT WITH A MESSAGE
C
3 WRITE(3,4)
4 FORMAT(10X,'THE REGRESSION COEFFICIENTS CANNOT BE FOUND')
C
C*** IF D # 0 COMPUTE THE REGRESSION COEFFICIENTS
C
5 BETA = (COVXY*VW -COVWY*COVXW)/D
GAMMA = (COVWY*VX -COVXY*COVXW)/D
ALPHA = YMU-BETA*XMU-GAMMA*WMU
C
C**************************
C***END OF FITTING MODEL***
C**************************
C
C**********************************************
C**COMPUTATION OF THE PIERCE-KOPECKY RESIDUALS**
C**********************************************
C
A = XSQ*WSQ-PRODXW*PRODXW
B = WMU*COVXW-XMU*VW
C = XMU*COVXW-WMU*VX
T = AN*D
SSE = 0.0D0
TRACE = 0.0D0
```

```
YRT = 0.D0
DO 150 J = 11,N
YF(J) = ALPHA+BETA*X(J)+GAMMA*W(J)
YR(J) = Y(J)-YF(J)
SSE = SSE+YR(J)*YR(J)
UJ = AN*(X(J)*B+W(J)*C-X(J)*W(J)*COVXW)
RJ = AN*(X(J)*X(J)*VW+W(J)*W(J)*VX)
U(J) = (A+2.0D0*UJ+RJ)/T
C
C**********************************************************
C*****UJ = j'th diagonal of X((X'X)inv)X'=j'th lever-****
C*****age. UJ > 0 for X of full rank.TRACE=SUM(UJ)=******
C*****3.0 = # of parameters estimated.UJ measures the****
C*****effect of the j'th predictor variable on the*******
C*****************regressor variable.******************
C**********************************************************
C
TRACE = TRACE+U(J)
YRT = YRT+YR(J)
V(J) = DSQRT(1.0D0-U(J))
150 CONTINUE
RMSE = SSE*CN
S1 = DSQRT(RMSE)
S2 = 1.0D0/S1
DO 155 J = 11,N
E(J-10) = YR(J)*S2/V(J)
155 CONTINUE
RETURN
END
```

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