INSTRUCTION OF REPRESENTATION AND SOLUTION IN ALGEBRAIC PROBLEM SOLVING WITH LEARNING DISABLED ADOLESCENTS

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THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in the Faculty

of

Education

SIMON FRASER UNIVERSITY

November 1986

ISBN 0-315-36337-1

APPROVAL

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Degree: Doctor of Philosophy

Title of Thesis: Instruction of Representation and Solution in Algebraic Problem Solving With Learning Disabled Adolescents

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Date approved 21 Nov. 1986

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Title of Thesis/Project/Extended Essay

INSTRUCTION OF REPRESENTATION AND SOLUTION IN ALGEBRAIC PROBLEM SOLVING WITH LEARNING DISABLED ADOLESCENTS

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November 21, 1986

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ABSTRACT

Problem solving in knowledge-rich domains such as algebra is hypothesized to consist of two phases: representation and solution. Experts are better than novices at representation, while there are few differences in solution. Most instruction has focussed on solution; recently there have been calls for instructional research on representation.

Adolescents, and particularly learning disabled adolescents, have difficulty understanding and solving algebra word problems. The purpose of this study was to design, and investigate the effectiveness of, cognitive strategy instruction in representation and solution to improve the algebraic problem solving of learning disabled adolescents.

Two interwoven designs were employed: an experimental-control group design and a single-subject design. Twenty learning disabled adolescents in grades 8 through 10 in two Vancouver area schools were assigned randomly to two groups. On alternate days for three months, the instructed students received individual instruction, to criterion in representation and solution for three problem types. Five surface structures were used with three problem types based on mathematical structure: relational problems, proportion problems, and problems in

two variables and two equations. Instruction for representation and solution in the instructed group was based on cognitive task analysis, and included declarative knowledge, modelling of procedural knowledge by the instructor thinking aloud, guided practice, and independent practice. Students were taught a self-questioning strategy.

Data were derived from problem-solving measures, think-aloud protocols, interviews about metacognition, and classification tasks. These were examined for implications respecting theory in instructional psychology. The data from between-group comparisons demonstrated the superiority of the instructed group on instructed problems, problem solving in general, and in metacognition and schemata for algebraic problem types. Single-subject data showed the instructional students' problem-solving ability increased dramatically over sessions for each problem type. These increases were maintained and transferred to similar problems. Think-aloud protocols showed students applied newly-acquired domain-specific knowledge.

This study affirms the need for instruction in both phases of problem solving, demonstrates that representation can be taught, and that learning disabled adolescents can acquire schemata useful for solving problems. Recommendations were made for future research in instruction leading to the development of expertise in knowledge-rich domains.

ACKNOWLEDGEMENTS

Many individuals have assisted generously as I worked on this dissertation. I wish to express my gratitude to my advisor Dr. Bernice Wong for her knowledgeable guidance and friendship. To Dr. Ron Marx who was always supportive, insightful and constructive, many thanks. Gratitude is extended to Dr. Tom O'Shea who advised me on the mathematical aspects of the study.

The Instructional Psychology Research Group has created an exciting and productive environment. Much of the credit goes to Dr. Phil Winne, one of the most instructive people I know. Thanks also to Dr. Robin Barrow for the education in your outstanding seminars. My fellow graduate students have been caring and challenging. I especially want to thank John Walsh, Joan Cassidy, Dawn Howard, Patrick Keeney, Angela Dereume, and Sharli Orr.

An instructional study requires the cooperation of many people in the field. I acknowledge the assistance of the Coquitlam School Board and two fine special educators, Carlton Olson and Hal Finan. And to twenty enthusiastic adolescents -- I couldn't have done it without you! Thanks!

I want to thank my parents and Deb, Jim, and Sandy for their support and encouragement over the years. Two very special people have helped me in countless ways, have believed in me, and have understood how much this means to me. Pete and Jenny, I can't thank you enough!

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CHAPTER 1

INTRODUCTION

Instructional psychologists study theoretical and practical aspects of instruction and their effects on mental representations, mental processes, and learning outcomes. Instructional psychology has been characterized by a shift toward studying more complex forms of cognitive behavior, the interactionist assumptions that learning occurs as a result of mental constructions of the learner, and a growing interest in the role of knowledge in human behavior (Resnick, 1981). Theory construction in problem solving and investigations of effective instruction for the learning disabled are two areas in which this perspective of instructional psychology is apparent. The purpose of the present investigation is to merge recent work in these two fields to study instruction designed to improve the algebraic problem solving of learning disabled adolescents.

Problem Solving

Problem solving is a widely used but poorly defined term. According to Polya (1981), within mathematical problem solving "to have a problem means: to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim" (p. 117). Usually, a mathematical word problem identifies quantities, describes relationships between them, situates this information in a familiar context, and requires that the student attain a goal.

In the past few years, educators have expressed three related concerns about problem solving. The first is that problem solving is an essential instructional goal in mathematics (The National Council of Teachers of Mathematics, 1980). The second is that students lack expertise in mathematical problem solving (Carpenter, Corbitt, Kepner, Lindquist & Reyes, 1980). The third is an urgent plea that more investigative resources be devoted to cognitive research that will lead to more effective instruction in problem solving (e.g., Davis, 1984; Frederiksen, 1984; Mayer, 1983).

Recent theoretical formulations about the nature of problem solving yield prescriptions that may be instrumental in meeting these concerns. Mayer (1985) reviewed information-processing research on algebra word problems and reasserted there are two phases involved in solving verbal mathematical problems: problem representation and problem solution. Problem representation can be defined as actively transforming an external verbal representation of a problem from words on a page to an internal mental representation consistent with the learner's available knowledge. Problem solution refers to applying the legal operators of mathematics to the internal representation to arrive at an answer that satisfies the goal. Mayer has characterized problem representation as primarily declarative knowledge and problem solution as mainly procedural knowledge. The implications that follow are that to improve performance in the representation phase, there must be instruction in comprehension of difficult linguistic propositions and recognition of problem types or problem schemata. Within the solution phase, there must be instruction in how to use appropriate strategies and efficient algorithms. These implications for instruction remain to be tested empirically.

Reviews of the literature on expert and novice problem solvers show that experts are more adept at problem representation than novices (Glaser, 1984; Hutchinson, 1985) although there are few differences in their problem solutions (Lewis, 1981). However, problem solution is the focus of most instruction (Simon, 1980). It appears that to make problem solving instruction more effective it will be necessary to enable learners to construct the representations essential to good performance (Reif & Heller, 1982).

To observe the development of expertise, it is necessary to instruct learners who are judged to be capable of acquiring the instructed knowledge, but have not yet done so. In the case of algebraic word problems, there are many learning disabled adolescents who fit this description.

Learning Disabilities

Like most research on human learning, the zeitgeist in learning disabilities is cognitive (Reid & Hresko, 1981). Research has shown that the learning disabled are a heterogeneous group, difficult to define, but typically characterized by average intellectual ability and below average achievement in particular curricular areas such as reading and mathematics. They have been described as inactive learners (Torgeson, 1977) and inefficient information processors (Swanson, in press). The definition that has received wide acceptance is the one that appears in the Education for All Handicapped Children Act (U.S. Office of Education, August 23, 1977) and has been adopted by several Canadian provinces. This definition reads:

> "Specific learning disability" means a disorder in one or more of the basic psychological processes involved in understanding or in using language spoken or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell or do mathematical calculations. The term includes such conditions as perceptual handicaps, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.

Although reading is the curricular area which has received the most attention in learning disabilities, researchers and practitioners have found that learning disabled children and adolescents also experience difficulties in mathematics (Cawley, 1985; Fleischner & Garnett, 1983), and that they score lower than normally-achieving peers on measures of formal reasoning and word problem solving (Lee & Hudson, 1981; Skrtic, 1980). While considerable research has been done to improve the effectiveness of reading instruction for the learning disabled, little comparable research has been carried out in mathematics.

A few intervention studies have investigated the effectiveness of particular instructional approaches for mathematics word problem solving. Most of these have provided instruction to learning disabled students in the middle years on one- and two-step problems (e.g., Marzola, 1985; Nuzum, 1983). In a study carried out by Montague (1984), junior high students were taught to solve two-step problems. These problems require that students read and then select two operations from addition, subtraction, multiplication and division to be carried out in the correct order. However, junior secondary mathematics curricula demand that adolescents master abstract algebra problems using symbols to stand for unknowns and generating and solving linear equations. Careful search has yielded no investigations reported in the literature which have carried out systematic instruction on algebraic word problems with learning disabled junior secondary students.

Researchers in learning disabilities have begun to examine the role that metacognition plays in the performance of cognitive tasks (Brown, 1980; Wong, 1985a). According to Flavell (1976), metacognition refers to "one's knowledge concerning one's own cognitive processes and products" (p. 232) and the active monitoring and regulation of these processes. Recently, mathematics educators have asserted that instruction in problem solving would be more effective if it were to incorporate metacognitive elements (Garofalo & Lester, 1985). They argue that such an emphasis would make students more aware of how and when to apply their mathematical knowledge, and more likely to monitor their problem solving to ensure that problem goals were met. The use of self-questioning instructional procedures has been shown to increase comprehension monitoring and prose comprehension (Wong & Jones, 1982; Wong, Wong, Perry, & Sawatsky, in press). Students ask themselves a series of questions that focus their attention on the essential aspects of the task and that help them to be aware of their own lack of understanding when it happens. Incorporating self-questioning into problem-solving instruction in mathematics may also lead to increased metacognition.

The effectiveness of guided instruction in enabling learning disabled adolescents to solve simple word problems has been shown (e.g., Montague, 1984). Guided instruction consists of demonstration, guided practice, independent practice, and feedback followed by assessment. The challenge that remains is to operationalize the recommendations (Mayer, 1985; Reif & Heller, 1982) for teaching both representation and solution for complex, abstract algebraic word problems.

Integrating Problem-Solving Theory and Instructional Research with the Learning Disabled

Algebraic problem-solving abilities are important for high school students to acquire. Unfortunately, they frequently elude the learning disabled. Such abilities are necessary for success in most secondary mathematics and science curricula. As well, post-secondary entrance requirements and program requirements usually include proficiency in mathematics, including word problem solving. Yet, researchers and teachers do not know the role of problem representation, problem solution, or metacognition in developing word problem-solving skills. Nor do they know how effective guided instruction may help adolescents to acquire proficiency with abstract word problems that require representation in unknowns and the use of unknowns to solve equations. Studying the relative contributions of problem representation, problem solution and metacognition to solving word problems successfully should provide educators with information that will enable learning disabled adolescents to become more proficient mathematics problem solvers. It should also help to respond to the recent and widespread concerns that educators have expressed about teaching problem solving generally. Such a study, by extension, will test the validity of hypotheses in instructional psychology about the two phases of problem solving--problem representation and problem solution--and their relevance to classroom instruction.

General Research Questions

This study was designed to answer the following questions:

1. Will the problem-solving ability of learning disabled

adolescents increase over the course of guided instruction in problem representation and problem solution?

2. If present, will these increases be maintained and transferred to tasks with new surface structures and to tasks with new mathematical structure?

3. Will learning disabled students' understanding and metacognitive awareness of word problems increase following guided instruction?

Definitions of Terms Used in this Study

Because many terms associated with problem solving have been defined in various ways, it is necessary to specify the definitions used in this study. The terms which are defined explicitly are: problem solving, problem representation, problem solution, learning disabilities and guided instruction.

Problem solving in this study refers to the entire task of working out algebraic word problems which require the use of symbols to stand for unknowns and the solution of linear equations on a structured worksheet. Problem solving includes problem representation and problem solution.

Problem representation consists of actively transforming an external representation of information into an internal representation. The steps to "get the whole picture" of a verbal problem are to state: (a) the goal of the problem, (b) the information given, (c) the unknowns; and produce: (d) a drawing showing the mathematical relationships, and (e) a symbolic representation (equation). All this is done on a structured worksheet with the students supplying the answers, and must be accurate and thorough to be correct.

Problem solution consists of applying the legal operators of mathematics to the internal representation to arrive at a final answer that meets the goal of the problem. Procedurally, this requires students to state and write in the correct sequence the steps in the manipulations to the equation that are appropriate for each problem. These are (a) clearing brackets, (b) collecting like terms, (c) adding or subtracting whole numbers to isolate the unknown, (d) finding the value of the unknown, (e) meeting the goal of the problem, and (f) checking.

Learning disabled participants in this study are identified by their school district (in accord with British Columbia Ministry of Education guidelines) as having a significant discrepancy between potential, as measured by an individual intelligence test, and achievement, as measured by standardized achievement tests. The relevant area of disability for participation in this study is mathematics.

Guided instruction refers to teaching in small steps by modelling and thinking aloud with student practice after each step, guiding students during initial practice, and providing systematic feedback and corrections. Students are given a high level of successful practice and reach mastery criteria before moving on to new tasks.

Overview

Chapter 2 contains a selective integrative review of problem solving literature from the fields of cognitive psychology, mathematics education, and special education. It concludes with the specific research hypotheses that are tested. A detailed description of the method and procedures constitutes Chapter 3, and the results and discussion appear in Chapters 4 and 5, respectively. The instructional and assessment measures can be found in the Appendices with other information essential for replication.

CHAPTER 2

REVIEW OF LITERATURE

One of the goals of education is to teach problem solving. In mathematics and science, problems are usually well-structured which means they contain all the necessary information and have algorithms which lead to the correct answer (Frederiksen, 1984). However, other areas of the curriculum contain ill-structured problems. In social studies, for example, a problem might ask students how to increase agricultural productivity in the USSR (Voss, Greene, Post, & Penner, 1983). While it is important to educate the next generations so they will have the flexibility to solve ill-structured problems unimagined today (Simon, 1980), there are few guidelines about ways to accomplish this. In fact, despite recent advances in cognitive science, there is little instructional research providing empirical validation of the tasks and procedures used to teach procedures for solving well-structured problems.

The purpose of the first half of this chapter is to review theoretical and empirical literature in cognitive psychology and mathematics education, and on algebraic problem solving with some reference to studies in physics. The second half of this chapter reviews single-subject studies in which learning disabled adolescents are taught to solve simple mathematical word problems. The empirical findings and conceptual aspects of earlier work inform the design of instructional tasks and procedures in this study which investigates teaching well-structured problems in one knowledge domain--algebra--to a sample of students who rarely master such problems--learning disabled adolescents.

Cognitive Theory and Problem Solving

In the past two decades, information-processing theories have replaced earlier analyses of problem solving. Gestalt theorists (Duncker, 1945; Wertheimer, 1959), who focussed on understanding the problem as a whole, were unable to produce verifiable hypotheses. Behaviorist formulations (e.g., Maltzman, 1955), which were concerned with connections between presented problems and overt answers, were not applicable to complex problems. Both internal mental processes and the resulting actions are the foci of information-processing analyses.

Cognitive psychologists build theoretical models of the conceptual knowledge, procedures, and executive processes required for solving particular problems. To test whether they are sufficient explanations, these theories are sometimes implemented as programs run on a computer (e.g., Bobrow, 1968), or operationalized as instruction given to students (e.g., Heller & Reif, 1984). The first computer models like General Problem Solver (Ernst & Newell, 1969) consisted of heuristics or rules-of-thumb which could be applied to many kinds of problems. Gradually these models have come to include declarative and procedural knowledge specific to the problem domain.

Declarative knowledge refers to knowing that, consisting of facts, concepts, and semantic information (Anderson, 1976). Procedural knowledge refers to skills, knowing how (Anderson, 1980). The distinction between the two is sometimes blurred. However, declarative knowledge can usually be expressed verbally, whereas procedural knowledge is frequently difficult to describe verbally and must be acquired by doing. Each type of knowledge is thought to have its own processing characteristics. Declarative knowledge is more flexible in its application and more easily modified, although slow and clumsy to apply. Procedural knowledge can be applied more quickly, but cannot easily be reflected upon or modified, though it can be replaced. (For a detailed discussion of declarative and procedural knowledge and their implications, see Neves and Anderson, 1981.)

Most theories of the architecture of human information processing (e.g., Anderson, 1983) contain several elements including long-term memory (LTM) and working memory or short-term memory (STM). LTM is a repository of nearly permanent knowledge thought to contain both declarative and procedural knowledge. STM contains the information that is being acted on in a given slice of time and maintains an internal representation of what is going on. It has limited capacity (Miller, 1956) which thereby limits the size and complexity of problems with which one can deal.

There are at least two ways to overcome the limited capacity of STM. First, complex knowledge structures can be developed which allow a single modifiable schema to represent a collection of related, organized knowledge. Schemata have been variously defined and discussed in the literature on memory (Bartlett, 1932; Norman & Bobrow 1975; Rumelhart & Ortony, 1977), but certain properties remain constant. For example, a schema represents a prototypical abstraction of a complex concept, derived from many experiences with the complex concept. A schema can guide the organization of incoming information into clusters of knowledge that are instantiations of the schema (Thorndyke & Hayes-Roth, 1979). In addition, a schema can serve as a retrieval mechanism during recall (Mandler & Johnson, 1977). The implication is that an individual with a well-developed schema for a particular type of problem will be able to use that schema with instantiations for particular examples to represent complex cognitive problems in limited STM. Then to be effective, problem-solving instruction should include development of problem schemata.

Second, procedural knowledge, consisting of a series of condition-action statements about how to perform under particular conditions, can be compiled. This means that a series of previously unintegrated productions which follow one another in solving a problem are integrated into a single production that has the effect of the series. Processing is fast and automatic, thereby reducing the load on working memory. Procedural learning occurs only in executing a skill (Anderson, 1983, p. 215). The implication is that an individual who has executed a series of problem-appropriate productions repeatedly will be better at solving that particular type of problem. Hence, to be effective, problem solving instruction should promote development of automatic compiled procedures.

Algebraic Problem Solving: Representation and Solution

An information-processing analysis asks "What cognitive operations and knowledge are necessary to solve this algebraic word problem?" Attempts to answer this question in the past 20 years have produced a model of mathematical and scientific problem solving with two major parts: problem representation, converting a problem from words into an internal representation; and problem solution, applying the legal operators of mathematics to the internal representation to arrive at a final answer (Larkin, McDermott, Simon, & Simon, 1980; Mayer, 1985; Reif & Heller, 1982).

Problem Representation

Problem representation can be broken down into the translation of each proposition in the problem followed by the integration of these propositions into a recognizable whole (Mayer, 1985). This section reviews the research showing the importance of problem representation. It focusses on the role which schemata play in the construction of representations, and raises questions about current conceptions of schemata in the literature on algebraic problem solving.

Evidence to support the sufficiency of translation and integration as a description of representation was sought by Hayes and Simon (1974) using a puzzle called The Tea Ceremony. They designed a computer program that carried out translation, that is, syntactic and semantic analysis of problem text. This was integrated by a second program into a form on which General Problem Solver could carry out problem solution. The authors concluded that the program called UNDERSTAND simulated some of the processes of human understanding for this problem. Because The Tea Ceremony is a knowledge-lean problem, it could not be assumed that these findings applied to problems in a highly-structured knowledge domain like algebra (Glaser, 1984).

Hayes, Waterman and Robinson (1977) hypothesized a third subprocess necessary for describing the way people represent algebraic word problems. Selective attention would occur between translation and integration, enabling a person to make decisions about what is relevant. A computer program ATTEND was compared to human protocols. ATTEND detected problem elements and tagged as relevant the elements referred to most often in the problem.

This model accounted for data when algebra problems were not recognized as familiar. However, when subjects were asked to read a complex problem twice, they changed their judgments of relevance from the first to the second reading. The problem contained information about space, weight, and flight with and against a wind. On first reading, subjects judged nearly every proposition of the problem which contained a number to be relevant. On second reading, they judged as important only those propositions concerned with flying with and against a wind. This finding was not consistent with ATTEND, as there were fewer elements concerned with flying with and against the wind than elements concerned with weight and space. Psychologists began to evoke knowledge structures in the form of schemata to explain how the translated propositions were combined into a coherent representation.

Robinson and Hayes (1978) confirmed that findings about how people worked on the wind problem did not conform to predictions made by ATTEND. They argued that subjects identified it as a 'river current' type of problem for which they had a schema, and then narrowed their attention to a few aspects of the problem specified by the schema. The authors acknowledged, however, that it was impossible to discern whether the subjects actually recognized it as a problem about speed or as a river current problem "<u>since the speed segments are confused with the relevant [current] segments</u>" (Robinson & Hayes, 1978, p. 200, emphasis mine).

The confounding that Robinson and Hayes pointed out has persisted into the 1980's. There are two major systems for classifying algebra word problems: by the form of the underlying equations and by the general form of the story line (Krutetskii, 1976; Mayer, 1981). The form of the underlying equation is based on the mathematical relations present in the problem and refers to the structure of the solution equation. For example, the standard solution for a problem may require the solution of two equations in two unknowns (simultaneous linear equations). The form of the story line refers to contextual or surface details such as mention of river current, money or age. For many problems in the algebra curricula, there is no necessary connection between the problem structure and the surface structure. Age can be associated with simultaneous linear equations or proportion problems.

In some cases, where the equation to solve a problem is derived from a formula such as distance = rate * time, it is difficult to separate the problem structure from the surface structure. However, even in these cases there are several story lines which can be associated with the same problem structure. For example, interest = principal * rate * time and discount = regular price * percent reduction are both rate problems. A person who would not sort standard algebra problems into groups of distance-rate-time, interest, and discount problems but put them into one category of generalized rate problems "would have organized his or her algebra problem solving experience in a very powerful manner" (Silver, 1985, p. 18).

Many of the investigations which provide the strongest arguments for instruction to develop problem schemata confound the mathematical structure and the story line in the same way as Robinson and Hayes (1978). In a series of much-quoted studies, Hinsley, Hayes and Simon (1977) showed that students possessed schemata for familiar types of algebra word problems, and shed some light on the nature of these schemata and their source. They reported that their sample of high school and college students sorted 76 problems from 16 chapters of an algebra text into 16 to 18 highly standardized categories such as 'mixture problems' and 'distance-rate-time problems'. Furthermore, when they were read problems broken into segments, half the subjects categorized the problems after hearing less than one-fifth of the text. For example, after hearing the three words "A river steamer", one subject said, "You are going to compare times upstream and downstream - or, if the time is constant, it will be the distance." In the first two experiments, the role of surface structure and mathematical structure could not be distinguished because they were always combined in the same ways.

In the third experiment protocols were recorded while two graduate students solved nine problems aloud. The standard problem types included 'river current', 'age', and 'work' problems with characteristic matching of cover story to the underlying problem structure. Six problems were nonstandard with a mismatch of cover story and algebraic structure. The standard problems were solved as before. Significantly, the nonstandard problems were solved in a line-by-line fashion, without the use of schemata as described in earlier models (Bobrow, 1968; Hayes & Simon, 1974). Hinsley et al. (1977) suggested that translation-only might be the default option when the problem is not successfully matched to one of the schemata possessed by the subject. The implication is that the subjects' schemata were based on surface features instead of the problem structure. Otherwise, subjects would have been able to overlook the unusual surface features and be guided by the familiar problem structure. The authors concluded that for familiar problems students do have schemata which guide their processing. Given the close match between the 16 chapters of the most widely used high school algebra text (Dolciani, Berman & Wooton, 1973) and the 16 to 18 categories generated by the subjects, it appears that instruction is a source of these story-line based schemata.

Mayer (1981) set out to classify algebra story problems into types and determine the frequency for each problem type. The structure of his classification system is illustrated in Figure 1. Algebra problems are divided into two formats: story problems and non-story problems. Only story problems are considered. These are divided into eight families, one family of non-formula problems and seven families of formula problems. From this point on, Mayer considers primarily the word problems whose mathematical structure and equation are based on a known formula. This makes the system akin to a classification of physics problems.

Problem solving in physics is principle-driven and the subject matter is organized and taught according to principles and the formulae that flow from them (Reif & Heller, 1982). Mathematics, on the other hand is organized according to methods of solution

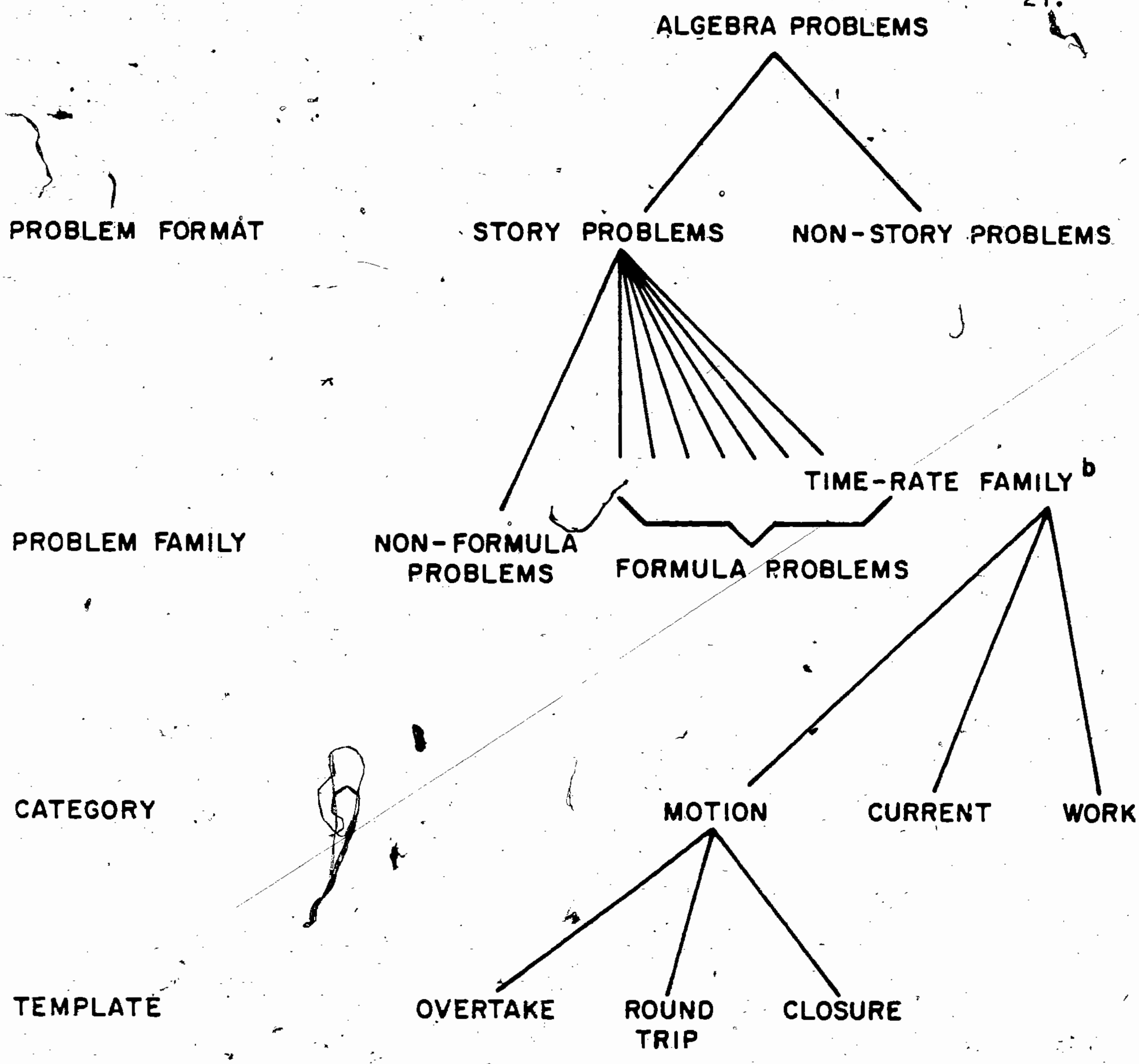


Figure 1

Classification of Algebra Problems Based on Mayer's (1981) System[a]

[a]Figure prepared by author to illustrate Mayer's description of a classification system.

[b]Mayer provides examples for this one of the eight formula-based families of problems.

(Schoenfeld & Herrmann, 1982). An extrapolation of this organization would imply that algebra courses should be organized so that problem solving is taught according to the problem structure instead of the story line. However, after disregarding non-formula problems, Mayer divides each family into categories and each category into templates. A template refers to a specific propositional structure in combination with a particular story line. Mayer (1981) argues that the propositional structure (or mathematical structure) and the surface structure are not mutually exclusive systems, for "in general certain major categories of problems (based on story line) involve characteristic underlying equations" (p. 136). This, as I have argued, is more true for the formula problems which Mayer emphasizes. In the non-formula problems, mathematical structure (such as proportion or linear equations) can be associated with a wide variety of story lines (such as age, money or distance).

Mayer (1981) argues that instruction must be provided at the level of the template because for novices all motion problems may look alike but the solution procedure is different for different templates. He also claims that it will be necessary to teach students to generalize from templates to problem isomorphs in the taxonomy, i.e., problems in which the solution paths map directly on to one another in one-to-one fashion (Simon & Hayes, 1976). However, it might be more effective and parsimonious to teach at the level of isomorphs instead of templates. As Silver (1985) asserts, schemata for isomorphs ought to be a powerful organization of problem knowledge (p. 18). Mayer also recommends investigations to determine how learners make judgments about problem types, and how such schematic knowledge is acquired.

It appears that judgments about problem types are made on the basis of schemata. In an investigation of memory for algebra story problems, Mayer (1982) found that recall was better for schema-relevant than schema-irrelevant information in problems. He also found that memory was better for common problems than for uncommon problems (based on frequency counts in Mayer, 1981). Learners also tended to change uncommon problem types into common problem types.

Mayer also found that the mathematical structure as determined by the propositions in the problem exerted an influence on memory. Specifically, problems that contained relational statements expressing the relationships between variables ("John has $18 more than Mike") were hard to remember. This latter finding substantiates the results of an investigation by Loftus and Suppes (1972) in which they located structural variables that affected difficulty of story problems for sixth graders. The hardest problem in their set was one that contained a relational proposition "Mary is twice as old as Betty was 2 years ago. Mary is 40 years old. How old is Betty?" These findings are bolstered by the outcome of investigations with primary school children (Riley, Greeno, & Heller, 1983) and with college students

(Clement, Lochhead, & Monk, 1981). It appears that the mathematical structure of algebra problems exerts a pervasive influence on students' schemata and problem representation, although texts are organized (Dolciani et al., 1973) and problems frequently categorized by researchers (Hinsley et al., 1977; Mayer, 1981) according to a combination of mathematical structure and surface structure.

As might be expected, research shows that instruction plays a role in schemata formation. In a series of studies employing non-formula problems, Silver (1977, 1981) found a significant correlation between a student's tendency to sort a set of problems by structural similarity and the student's ability to solve those problems. Krutetskii (1976), a Russian psychologist and mathematics educator, has found that highly capable students are able to perceive accurately the mathematical structure of a problem and to generalize rapidly to a class of structurally related problems.

While these studies provide indirect support for the role of instruction, Schoenfeld and Herrmann (1982) explored directly the relationship between problem perception (judgments of similarity) and proficiency. College students' perceptions of structure of mathematical problems were examined before and after a month-long intensive course on mathematical problem solving which emphasized the methods used to solve a variety of problems. These students' perceptions were compared with experts'. Hierarchical clustering analysis of the sorting data showed that before the course students made judgments about problems on the basis of surface structure, words or objects described in the problem. After the course students perceived problem similarity more like the experts, on the basis of the mathematical structure. A control group similar in age and background to the students did not change from categorizing by surface structure to mathematical structure. These data suggest that instruction influences the problem schemata individuals subsequently bring to problem representation. If experts in mathematics sort by problem structure (Schoenfeld & Herrmann, 1982) like experts in physics (Chi, Feltovich, & Glaser, 1981), then maybe algebra problem-solving instruction should include instruction in developing powerful schemata (Silver, 1985) based on the mathematical structure of the problem instead of a combination of mathematical and surface structure (Mayer, 1981). To unravel the effects of mathematical structure and surface structure in the formation of problem schemata, it will be necessary to separate the two and study their influence when they are not confounded.

In summary, it appears that to represent algebra word problems, individuals must translate each proposition and then integrate these into a unified whole which specifies the mathematical relationships among the propositions. Such translation and integration appear to be guided by problem schemata whose major relevant features for this task show the prototypical mathematical features of the problem. Effective instruction in algebra problem solving would be expected to induce individuals to construct relevant schemata based on mathematical structure which would guide problem representation.

Problem Solution

The second major phase of problem solving is problem solution. This refers to applying the legal operators of mathematics to the internal representation to arrive at a final answer. Problem solution is thought to have two major components: strategy application and algorithm automaticity (Mayer, 1985). In generating solutions for algebraic equations, students must be aware of the purpose for which the procedures are used, to find the value of the unknown(s) and then meet the goal(s) of the problem.

Equation solving is a symbol-manipulation task in which the solver must transform a given string of symbols (such as X + X + 3 = 19) into another string of symbols, using certain permitted operations (Lewis, 1981). The goal string must have the unknown for which one is solving on one side by itself, and no occurrence of the unknown on the other side of the equation. To solve an equation, two kinds of knowledge must be used.

First, the solver must know an adequate set of algorithms and operators. If the solver makes an error in adding a quantity to both sides of an equation or in long division, then difficulty will arise. To the extent that these processes can be carried out automatically, without need for direct attention, more space becomes available in working memory for processes that do require attention. Computational algorithms for single- and multi-digit numbers can be done with a pocket calculator if they have not become automatic. However, it is essential for problem solvers to acquire automaticity of the algebraic operators necessary to solve equations for which there is no calculator. The data on algorithm automaticity with calculators (Suydam, 1982; Wheatley, 1980) are analogous to those in reading research where automaticity of word recognition skills has been associated with higher levels of reading comprehension (LaBerge & Samuels, 1974; Perfetti & Hogaboam, 1975).

Second, the solver must know how to choose an appropriate algorithm or operator at a given juncture (Lewis, 1981). The knowledge needed to make this selection can be called strategic knowledge. Problem solution strategies for algebra word problems are rarely taught explicitly to learners. For example, Simon (1980) has observed that students are drilled on how to carry out algebraic operations, such as how to add the same quantity to both sides of an equation, but are rarely taught when to carry out the operations. Algorithms are taught while strategies are overlooked.

Bloom and Broder (1950) found that to enable remedial college students to use appropriate solution strategies on comprehensive examinations, it was necessary to provide models of strategic problem solution. Students who used effective strategies modelled by thinking aloud. The remedial students were asked to list all the differences between a transcript of their own strategy and that of the model. Remedial participants improved their scores and expressed high levels of confidence and optimism about their strategies. These findings suggest that strategic behaviour within problem solution may be amenable to direct instruction when clear demonstration and feedback are incorporated with enough practice.

Problem solution is made up of strategy application and algorithm automaticity. It is hypothesized that efficient instruction in algebra problem solving would entail sufficient modelling, guided practice, and feedback to make the application of procedural knowledge both strategic and automatic.

In summary, information-processing analyses suggest that algebraic problem solving is composed of representation and solution. Most confirmation for these two phases has come from the comparison of computer programs with protocols of competent solvers. The second approach for supporting models of cognition involves inducing hypothesized knowledge structures and cognitive processing in subjects and assessing the sufficiency of the structures and processing. This approach has been used systematically by Reif and his colleagues (Heller & Reif, 1984; Reif & Heller, 1982) in the study of problem solving in physics.

Prescriptions for Problem-Solving Instruction

Effective problem solving in complex domains depends on the content and structure of the knowledge about the particular domain (Glaser, 1984; Hutchinson, 1985). Studies in mathematics (Greeno, 1978), engineering thermodynamics (Bhaskar & Simon, 1977), physics (Chi et al., 1981; Larkin et al., 1980), and social studies (Voss et al., 1983) have shown that the knowledge structures and problem-solving procedures of experts and novices differ in significant ways.

Although studies contrasting experts and novices have yielded insights about problem solving, they are of limited value in designing instruction. The performance of experts is not an optimal example for novices. Rather than emulate the automatic, almost unconscious processing of experts, students must be taught to acquire explicit schemata and use explicit procedures. The task of the instructional designer is to prescribe a theoretical model of good performance by task analysis, rather than describe what experts do (Reif & Heller, 1982). The criterion of validity of such a prescriptive model is that it lead to predictably effective performance when implemented by a human learner, rather than mimicking performance of an expert.

Reif and Heller (1982) demonstrate a prescriptive model. They identify the essential knowledge for good performance in three phases of solving mechanics problems: representation, solution, and assessment of solution. For representation, they lay out the hierarchical, declarative knowledge base for the science of motion (mechanics) which contains individual descriptors, interaction descriptors, interaction laws, and motion principles. Elaborations for each level of the hierarchy are supplied. These are accompanied by procedural knowledge, such as applicability conditions, which enables one to apply the knowledge base. All this knowledge, declarative and procedural, is used for the first task faced by a problem solver, "to redescribe an original problem in a way facilitating the subsequent search for its solution. This can be done by identifying the knowledge relevant to the problem, organizing it effectively, and describing it in convenient symbolic form" (Reif & Heller, 1982, p. 113). The steps they prescribe include generating a basic description containing the specified situation and the goal, and generating a theoretical description including identification of systems, drawing diagrams, and checking that the description is consistent with known motion principles (equations). The construction of a solution for physics problems is carried out by applying procedures such as satisfying constraints, and decomposing the problem. In the simplest case each of these procedures is reduced to the routine application of techniques for solving algebraic equations (Reif & Heller, 1982, p. 117, 119).

Reif and Heller argue that once the task analysis has been carried out, it is essential to show the validity of the model by inducing human subjects to act in accordance with the model and observing whether the resulting performance contains the predicted characteristics and is effective. Then the next step is to test a model of effective problem solving instruction which ensures that students internalize control directions and other knowledge. Heller and Reif (1984) provide empirical validation for their analysis of mechanics problems. They devised a carefully controlled experiment where human subjects were induced to act in accordance with specified alternative models for representing problems. External prompts were available and followed at all times. The resulting performance was observed in detail. The results show that the proposed model is sufficient to generate excellent problem representations, and that these representations result in much better subsequent problem solutions. In contrast to the instructional emphasis on representation, Heller and Reif made no direct effort to induce prescribed problem solutions. The results also show that major components of the model are necessary for good performance. The authors assert there is now a validated basis for teaching problem representation in physics.

The work of Reif and his colleagues has implications for mathematics instruction. Specifically, to design instruction, a task analysis of the declarative and procedural knowledge needed for solving algebra problems is required.

> To teach students how to solve a class of problems, first analyze the knowledge that they need in order to solve that class of problems, and then carry out instruction that will result in their acquisition of the required knowledge. (Greeno, 1980, p. 13)

To make tacit processes explicit, instructors model by thinking aloud (Heller & Hungate, 1985), and to ensure their active participation, students generate those processes themselves and then discuss their performance (Bloom & Broder, 1950). Students need guided practice with the close scrutiny and feedback of a knowledgeable tutor (Heller & Reif, 1984). It makes sense pedagogically to develop qualitative understanding of problems and concepts prior to attempting instruction in specific problem-solving procedures. Lastly, if students are to concentrate on qualitative reasoning, they should be given credit for their representations and solutions, and not just their answers.

Reif's prescriptive analyses provide a plan for bridging the gap between the theory of information processing and the realities of problem-solving instruction. Operationalizing these prescriptions consistent with the nature of algebraic problem solving should provide a sound basis for designing instruction.

It will also be necessary to execute instructional procedures that enable learning disabled students to internalize problem-solving knowledge. The design of such procedures can be informed by both characteristics of the learning disabled and by instructional research that has been effective in teaching problem solving to the learning disabled. These two topics are the subject of the remainder of this chapter.

Problem Solving and Learning Disabilities

Problem solving calls for complex cognitive processing. By definition, learning disabled students have a "disorder in one of the basic psychological processes" (U.S. Office of Education, August 23, 1977). Inefficiencies in reasoning, conceptualizing, evaluating, and memory processes characterize their learning and performance (Reid & Hresko, 1981; Torgeson, 1977). When approaching complex cognitive tasks, the learning disabled tend to lack metacognitive awareness and regulation (Wong, 1985a).

Recently, metacognition has been used to account for the failure of the learning disabled to apply known skills in novel situations (Borkowski & Cavanaugh, 1979; Schneider, 1985). Metacognitive deficiencies may also account for the failure to acquire complex knowledge. Metacognitive comparisons of good and poor readers show that poor readers lack awareness of the features and functions of reading, the ability to monitor comprehension and knowledge, and application of strategies (Canney & Winograd, 1979; Paris & Myers, 1981; Wong & Jones, 1982). The learning disabled have been found deficient in internal storage strategies such as verbal rehearsal (Douglas, 1981) and in focussing on relevant task information (Hallahan & Kneedler, 1979).

Given the nature of problem-solving and the characteristics of the learning disabled, it is not surprising that they experience considerable difficulty in solving mathematics word problems. What is surprising is that the field of special education has not developed a relevant data base nor a substantial theoretical interpretation of mathematical problem solving for instructional purposes regarding learning disabilities (Cawley, 1985).

Investigators at the Learning Disabilities Research Institutes at the University of Kansas and Teachers College Columbia have begun to document the characteristics of learning disabled problem solvers and to develop cognitive interventions. They have used guided instruction procedures and tasks that call for a choice of basic operations and computation. These are the simplest word problems taught to junior and senior high students. Although one must be cautious in generalizing to more complex algebraic problems, there is a knowledge base from which to hypothesize about instructional procedures.

The next two sections contain a review of characteristics of learning disabled adolescent problem solvers followed by a review of cognitive interventions in problem solving for learning disabled adolescents.

Characteristics of Learning Disabled Problem Solvers

Although learning disabilities have tended to be associated with reading problems, investigations and review papers have confirmed that disabilities in mathematics are also widespread (Fleischner & Garnett, 1983; McKinney & Feagans, 1980). In an analysis of the cognitive abilities of grade seven, eight and nine students whose specific learning disabilities pertain to mathematics, Pieper and Deshler (1980) showed that reasoning involving equivalency statements and reintegration was highly correlated with mathematics disabilities.

Comparing learning disabled and non-learning disabled seventh and eighth graders, Skrtic (1980) found a significant difference between the level of formal reasoning attained by the two groups, with the learning disabled lagging behind their non-disabled peers. Additionally, the learning disabled students performed significantly less well on five of seven mathematics subtests of the Woodcock-Johnson Psychoeducational Battery (Woodcock & Johnson, 1977). These studies (Pieper & Deshler, 1980; Skrtic, 1980) confirm the existence of junior high students with learning disabilities in mathematics. Such students perform inadequately on measures of mathematics aptitude and achievement, and are clearly deficient in formal reasoning which plays a major role in problem solving (Revlin & Mayer, 1978).

Adolescents with learning disabilities in mathematics also do poorly on mathematics verbal problems compared to normal peers. Lee and Hudson (1981) showed significant differences in the performance of the two groups on two-step and three-step problems. The learning disabled students had fewer correct answers and lower scores when scoring criteria included choosing the correct operation, making the correct calculation, and answering questions about the information given and the goal of the problem. Analyses of error patterns indicated that learning disabled students made significantly more errors in reasoning and understanding, but there were no differences in computational accuracy if they had chosen the correct operations, in this study by Lee and Hudson (1981). The non learning disabled were better at monitoring their performance. These findings suggest that learning disabled adolescents are poor problem solvers, particularly in regard to problem representation and the metacognitive aspects of solving multi-step word problems.

Instructional Studies with Learning Disabled Adolescents

In a series of interventions, learning disabled adolescents have been taught to solve mathematics word problems. These interventions involved single-subject multiple-baseline designs which facilitate the examination of individual progress. Moreover, the interventions showed a shift from a behavioral perspective (Blankenship & Lovitt, 1976) to a cognitive perspective (cf. Nuzum, 1983), with the problems used becoming more complex, and the age of the students rising from preadolescence to high school. A summary of these studies is presented in Table 1.

In the earliest of these studies, Blankenship and Lovitt (1976) taught seven boys from 9 to 12 years of age one-operation (addition or subtraction) word problems. Because most errors arise in selecting the operation, they tried to reduce "rote computation" habits. Twelve sets of addition and subtraction word problems were gradually increased in difficulty with the introduction of variations in syntax and extraneous information. Three successive techniques--rereading, writing out, and explaining the solution for unsolved problems--enabled each student to reach the criterion (100% in three successive sessions) on all 12 sets. Following mastery, accuracy on a test

Table 1

Overview of Mathematical Problem-Solving Intervention Studies with Learning Disabled Adolescents

Author	Design	Sample	Duration	Strategy	Problems	Outcome	Criteria	Other Features
Blankenship & Lovitt, 1976	single-subject	7 boys 9-12 yr.	80 sessions	1)reread 2)write out 3)explain, for wrong answers	one-step; +,-,both; 12 classes with increasing syntax	all met criteria ave. on posttest 95.6%	for answers, 100% 3 successive sessions	no maintenance no transfer no cognitive model no thinkalouds no calculators
Smith & Alley, 1981	single-subject	3 grade 6	32 sessions	1)read 2)reread 3)decide on operation 4)write 5)compute 6)check 7)answer	one-step; +,-,x,÷ on each worksheet	all learned (no criteria); ave. on posttest 97%	not specified, scored answers only	no maintenance no transfer no cognitive model no thinkalouds no calculators
Nuzum, 1983	single-subject	4 gr. 5 & 6	6 weeks, approx. 24 sessions	1)identify components 2)identify operation 3)identify extran. info. 4)identify two-step 5)check 6)drawing	two-step; +,-,both, problems with extra information	all met process criteria; ave. 82.31% on posttest, 34.75% on pretest	for steps, not answers, 100% 3 consecutive problems	no maintenance no transfer info processing model thinkalouds no calculators

Table 1 (Continued)

Author	Design	Sample	Duration	Strategy	Problems	Outcome	Criteria	Other Features
Marzola, 1985	2-group E and C	60 gr. 5 & 6	6 weeks	E - same as Nuzum (1983) except 5) sort two-step and ext. info.; C - practice only	two-step; +,-, both problems with extra information	all E met criteria; E > C, overall and on all four problem types	for steps, & answers, 100% 3 consecutive problems	no maintenance no transfer info processing model no think-alouds calculators
Montague, 1984	single-subject	6 15-19 yr.	10-19 sessions	1)read 2)paraphrase 3)draw 4)state problem 5)hypothesize 6)estimate 7)calculate 8)self-check	two-step; +,-,x,+ on each worksheet	5 out of 6 reached criterion	for answers, 70% 4 consecutive sessions	maintenance transfer no cognitive model no think-alouds no calculators

containing all problem types averaged 95.6%. The students could discriminate among the classes and produce accurate answers to the addition and subtraction word problems. However, it took 80 instructional sessions to reach these goals. It may have been unnecessary to break the problems into so many classes. The one student for whom baseline data were provided made no errors at all in baseline for 6 of the 12 problem types. The data suggest that students were acquiring a cognitive strategy for solving types of word problems instead of acquiring correct responses.

To teach a cognitive strategy for solving problems, Smith and Alley (1981) applied principles of cognitive behavior modification and self-instruction (Meichenbaum, 1977). Three sixth graders were taught one-step problems involving all four basic operations. Instructional steps drawn from Alley and Deshler's (1979) learning strategies model consisted of making the student aware of current habits, explaining and demonstrating the alternative strategy, the student learning the strategy by verbalizing, the student applying the strategy to controlled materials, and posttesting on classroom materials. The seven steps within the problem-solving strategy based on Kramer's (1970) analysis were: (a) read the problem, (b) reread the problem, identifying what is given and what is asked for, (c) use objects to show the problem and choose the operation, (d) write the numbers to be operated on, (e) complete the computation, (f) check the answer, (g) show the answer. Problems requiring each of the four operations were distributed randomly on each tasksheet. Scores improved from an average of 18% accurate answers on grade three level problems before instruction to an average of 97% accurate answers on grade six level problems following instruction. The number of sessions reported for baseline and instruction was approximately 32, although students reached 100% correct in 1, 3 and 4 instructional sessions, respectively.

Smith and Alley (1981) demonstrated the efficiency and applicability of cognitive guided instructional procedures for problem solving. Students using the strategy produced correct answers and could discriminate among problems requiring each of the four basic operations. The study is limited in that no measures of cognitive processing or knowledge structures were collected, all instruction focussed on procedural knowledge, and there were only three students. However, it served as a model for more process-oriented interventions.

In a study incorporating an information-processing task analysis, Nuzum (1983) taught four learning disabled fifth and sixth graders to solve four types of word problems: addition only, subtraction only, addition or subtraction with extraneous information, and two-step problems. A single-subject design was used including pretreatment baseline, pretreatment interview, intervention, posttreatment baseline and posttreatment interview. Instruction incorporated features of cognitive behavior modification as the teacher provided a verbal explanation, modelled the correct procedure by thinking aloud, and gradually phased out cues. In direct contrast to previous studies, students were required to reach criterion on executing the correct procedure but the answer did not have to be correct. After reaching criterion, the student completed the posttreatment measures followed by instruction in the next phase.

A review of the information processing literature led Nuzum (1983) to divide the knowledge for problem solving into three categories: knowledge of the problem (analysis of component parts and their relationships), task-specific knowledge (identification of problem type), and procedural knowledge. Elements of the three kinds of knowledge for problem solving can be found in the six phases of instruction. In the first phase, the students were taught to identify components and select relevant information for the solution (with addition problems only). Phase two consisted of ascertaining whether the problem was concerned with combining objects (addition) or separating objects (subtraction). Both kinds of problems were included. In the third phase students were instructed to check for extraneous information (which all problems contained). Phase four added the step of identifying two-step problems (when all problems contained two steps). The fifth phase required that the student check for accuracy and the sixth phase required that a diagram be drawn before the computation was done. All students completed all phases showing substantial improvements in problem-solving performance. On a 16-item measure containing the four kinds of problems used in the study, average percentage correct answers improved from 34.8% on pretest to 82.3% on posttest.

Nuzum's (1983) study was clearly an advancement over its predecessors as it employed more complex problems, and attempted to integrate recent theoretical work in information processing. However, it is never specified how the three-component model of knowledge is operationalized in the phases of instruction. There is little specific discussion of how the findings reflect on the merit of the theory.

Most school instruction in problem solving takes place with small groups or entire classrooms. Marzola (1985) adapted Nuzum's (1983) procedures to small group instruction and introduced calculators. A two-group design was used permitting comparisons between experimental and control subjects and causal interpretations of the outcomes. Sixty learning disabled fifth and sixth graders were instructed to criterion in Nuzum's (1983) six phases, except that in phase five students sorted problems with either extraneous information or two steps into two categories. The steps in instruction consisted of verbal explanation, demonstration using the outline card as a guide, the student solving one problem aloud, and the student solving three problems independently. The mastery criterion was changed from Nuzum's in that students were required both to follow the instructions on the card and solve all three problems accurately. The control group received no direct instruction but completed the average number of worksheets completed by the experimental group using calculators.

An analysis of covariance with pretest scores as covariate yielded a significant main effect due to instructional treatment across the four problem types. There was also a significant effect due to problem type, and a significant group by problem type interaction. The experimental group did better than the control group in general and on each of the four problem types. Also, there were different patterns of problem type differences. For example, adjusted posttest performance of the controls was poorest on two-step problems and problems with extraneous information. For the experimental group, adjusted posttest performance was highest on the two-step problems.

Marzola's (1985) investigation is both an advancement and a study with limitations. The consistency between her findings with a two-group design and the findings of the single-subject studies suggest that the outcomes in the quasi-experimental interventions can be attributed to the instruction. Unfortunately, Marzola administered no maintenance or generalization tasks. She did not make a direct connection between the model she adopted and its operationalization in instruction, so no conclusions could be drawn about the relative contribution of the three kinds of knowledge. Although metacognition was not studied directly, it was reported that instructional students verbalized their use of "schemata" during the posttest. A classification task was incorporated as the fifth phase; however, the emphasis throughout Marzola's study was on determining the number and kinds of operations, rather than the mathematical structure of the problems.

Using older students (15 to 19 years) and two-step word problems, Montague (1984) taught six adolescents in a single-subject design. The students had to choose two operations and carry them out accurately in the correct order. An eight-step strategy was used: (a) read the problem aloud, (b) paraphrase the problem, (c) draw a picture, (d) state the problem, (e) hypothesize about the operations, (f) estimate, (g) calculate, (h) self-check. Strategy acquisition was conducted according to the learning strategies model (Deshler, Alley, Warner & Schumaker, 1981). Steps included description of the new strategy, modelling, verbal rehearsal, student practice, and corrective feedback. Five of the six students reached the criterion of seven correct answers on four consecutive ten-item tests. Data were collected on generalization and maintenance tasks. Four students reached the criterion for generalization--five correct answers on a ten-item test of three-step word problems. The same four students maintained criterial performance (seven correct answers on a ten-item test of two-step problems) after an interval of two weeks.

Montague extended previous investigations by instructing older students in two-step problems requiring the four basic operations, and by collecting generalization and maintenance data. The lack of think-aloud measures, metacognitive data, and an integrated theoretical framework prevented Montague from explaining what cognitive acquisitions adolescents made when they improved their problem solving of two-step word problems. The question remains whether these adolescents could then be taught the next type of word problem that appears in the junior and senior secondary curricula--the simplest algebraic word problems requiring the use of unknowns and equations.

Summary

A series of interventions has been conducted with learning disabled adolescents demonstrating the potential of single-subject designs for probing individual progress in problem solving. However, without using think-aloud protocols (Ericsson & Simon, 1984), metacognitive interviews (Meichenbaum, Burland, Gruson, & Cameron, in press), and collecting "cognitive traces" (Winne, 1984) while students work, it is impossible to realize the potential of single subject designs for documenting individual progress and to answer questions about knowledge structures and cognitive processing.

Instructional tasks have been limited to mathematical word problems requiring choice of operation(s) and ensuing calculations. Secondary mathematics curricula demand that adolescents understand and complete more complex algebraic word problems which require the use of unknowns or variables and equations. An extensive theoretical model has been developed of algebraic problem solving. This has been tested by operationalizing it in computer programs which account quite well for protocols of competent solvers. However, the model has not been tested to the limit for algebra problem solving by inducing hypothesized structures and processing in human problem solvers, as

Heller and Reif (1984) did for physics.

The preceding review on interventions shows the applicability and effectiveness of single-subject designs and cognitive behavior modification instructional procedures for teaching learning disabled adolescents to solve simpler mathematics word problems. Components of such effective cognitive instruction include demonstration by thinking aloud, use of self-questioning to acquire strategy, guided practice, independent practice, and frequent feedback and cueing which are gradually phased out. Instruction in both declarative and procedural knowledge should be based on cognitive task analyses of each problem type. To provide stringent tests, maintenance and transfer data are necessary. Each kind of problem learned is valuable in itself only if it is maintained, and is instructionally informative when the limits of near and far transfer have been tested.

<u>A Study That Addresses The Above Research Needs</u>

What is needed is a study instructing learning disabled adolescents to carry out representation and solution of algebraic word problems. Instruction in representation should stress the declarative knowledge essential to schemata for types of word problems classified according to mathematical structure. This is accompanied by the procedural knowledge necessary to use schemata to translate propositions and construct abstract representations. The components of the self-questioning strategy for instructing representation include reading and identifying the goal, knowns, and unknowns; expressing the essential mathematical relationships in a drawing; identifying the type of problem; and generating the equation. After representation has been mastered for a problem type, instruction in solution should concentrate on strategic and algorithmic processing. The components of the self-questioning strategy for instruction in solution include checking that an equation has been written, expanding terms, isolating the unknown, finding the value of the unknown, and ensuring that the answer meets the goal. The final self-question for each of representation and solution instruction directs the student to focus on the mathematical features that characterize the schema for the problem type under instruction. Measures of metacognitive awareness, classification of problems into mathematical types, and think-aloud protocols provide data relevant to questions about cognitive processes and knowledge structures that accompany the development of competence in problem solving. In short, this study attempts to answer pressing instructional questions and reflects on current theoretical formulations of algebraic problem solving.

Research Questions

Research Questions Concerning Instructed Students

1. Will the instructed students' ability to represent word

problems, obtain complete problem solutions, and accurate numerical answers for three problem types increase following the onset of instruction? Will these increases be maintained over time?

2. Will the instructed students show transfer to problem-solving tasks that are similar but distinct from the training tasks in that they contain new surface features (near transfer), in that they contain more complex mathematical structures (far transfer)?

3. Will changes be observed in the students' verbal behavior and understanding while thinking aloud during intervention?

4. Will the intervention differentially affect the students' ability to work out the three types of instructed problems--relational, proportion, and two-variable two-equation problems?

Research Questions On Between-Group Comparisons

5. Will instructed students and non-instructed comparison students show similar outcome scores in algebra problem solving on the instructed problem types, a multiple-choice test and an open-ended problem test?

6. Will instructed students and non-instructed comparison students show similar abilities on qualitative measures

related to algebra problem solving, specifically metacognitive interviews, think-aloud protocols and a problem classification task?

CHAPTER 3

METHOD

This research investigated the effects of instruction in problem representation and problem solution on the mathematical problem solving ability of learning disabled junior high school students. Of interest were students' abilities to represent problems, state or write the problem solution, and obtain the correct numerical answer. Instruction was provided for problems containing three mathematical structures: relational problems, proportion problems, and problems with two variables and two equations. Progression from one problem type to another depended on students' reaching instructional criteria. Five surface structures of age, distance, money, work, and number were combined with each mathematical structure. This systematic variation has made the method complex to describe but provided information about the effects of applying the instructional procedures to more than one type of problem.

Design of the Study

This study employed two interwoven designs. A traditional two-group (instructed vs. non-instructed comparison group) repeated measures design with pretests and posttests was used to investigate the effects of instruction. In order to study the precise changes that took place during the instructional component of the study, a single-subject research design was used. Both were necessary in order to answer the research questions posed in Chapter 2.

Two-Group Design

Students who met the criteria for inclusion in the study were assigned randomly to the instructed or comparison condition (see Figure 2). Analyses of covariance between the posttest scores of the two groups, with pretest scores as covariates, were conducted to compare the outcomes of the intervention with those arising from resource room instruction. Prior to posttesting, members of the comparison group received familiarization with the worksheet used in the instruction-based assessment measures.

Single-Subject Design

A single-subject style of research design was employed to permit close observation of knowledge acquisition and strategy use by individual students throughout the course of intervention. The design adopted features of both an AB design and a multiple-baseline across individuals design (Hersen & Barlow, 1976). These adaptations were necessary to use strategies designed to study behavioral change for the study of knowledge acquisition with complex cognitive tasks. Characteristic of an AB design, baseline data were collected prior to intervention; however, baselines were of the same brief duration of each student due to the obtrusive nature of the baseline task.

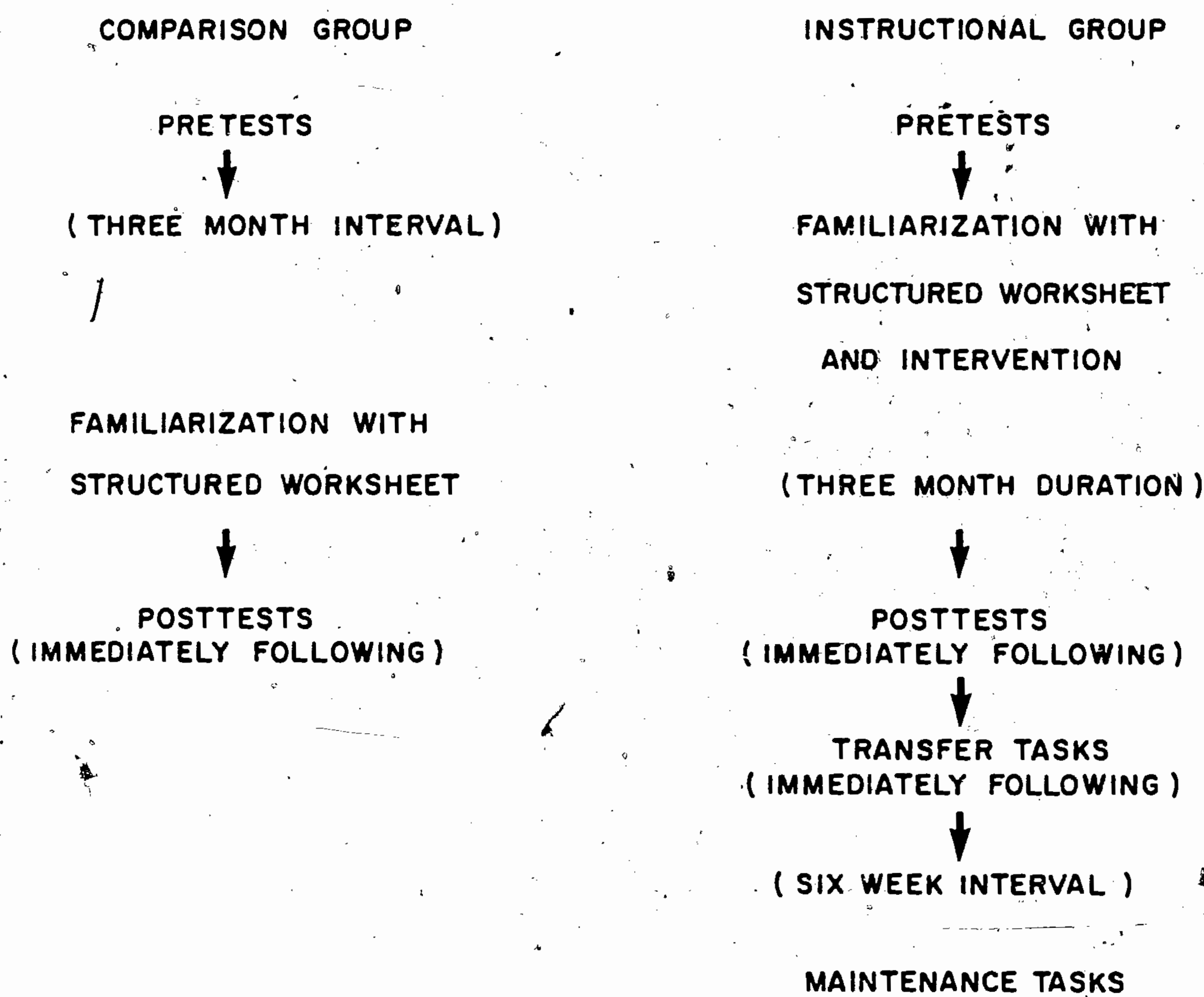


Figure 2

Two-Group Design

Similar to a multiple-baseline design, the intervention was applied across several individuals who exhibited similar target behaviors. Continuous measurement took place during intervention, and baseline and maintenance data were collected.

The students assigned to the instructed condition participated in the intervention individually on alternate days. These sessions were conducted by the investigator, a special educator with seven years' experience, in the learning assistance centres of two junior secondary schools in a suburban school district in the metropolitan Vancouver, British Columbia area. To minimize student frustration, baselines were established in the smallest number of sessions possible. Following establishment of a baseline for a student on a problem type, the instructional treatment was initiated. After the student reached criterion on the representation phase for that problem type, the baseline was established for the solution phase. Once criterion was reached on the solution phase, then these procedures were repeated for the next type, and so on. For each of the three types of word problems, the experimental procedures were ordered as follows: (a) baseline in representation, (b) instruction in representation, (c) baseline in solution, (d) instruction in solution (see Figure 3).

Immediately after the instruction phase had been completed, task-specific measures of near and far transfer were administered. Maintenance measures of representation and solution were collected six weeks later (see Figure 4).

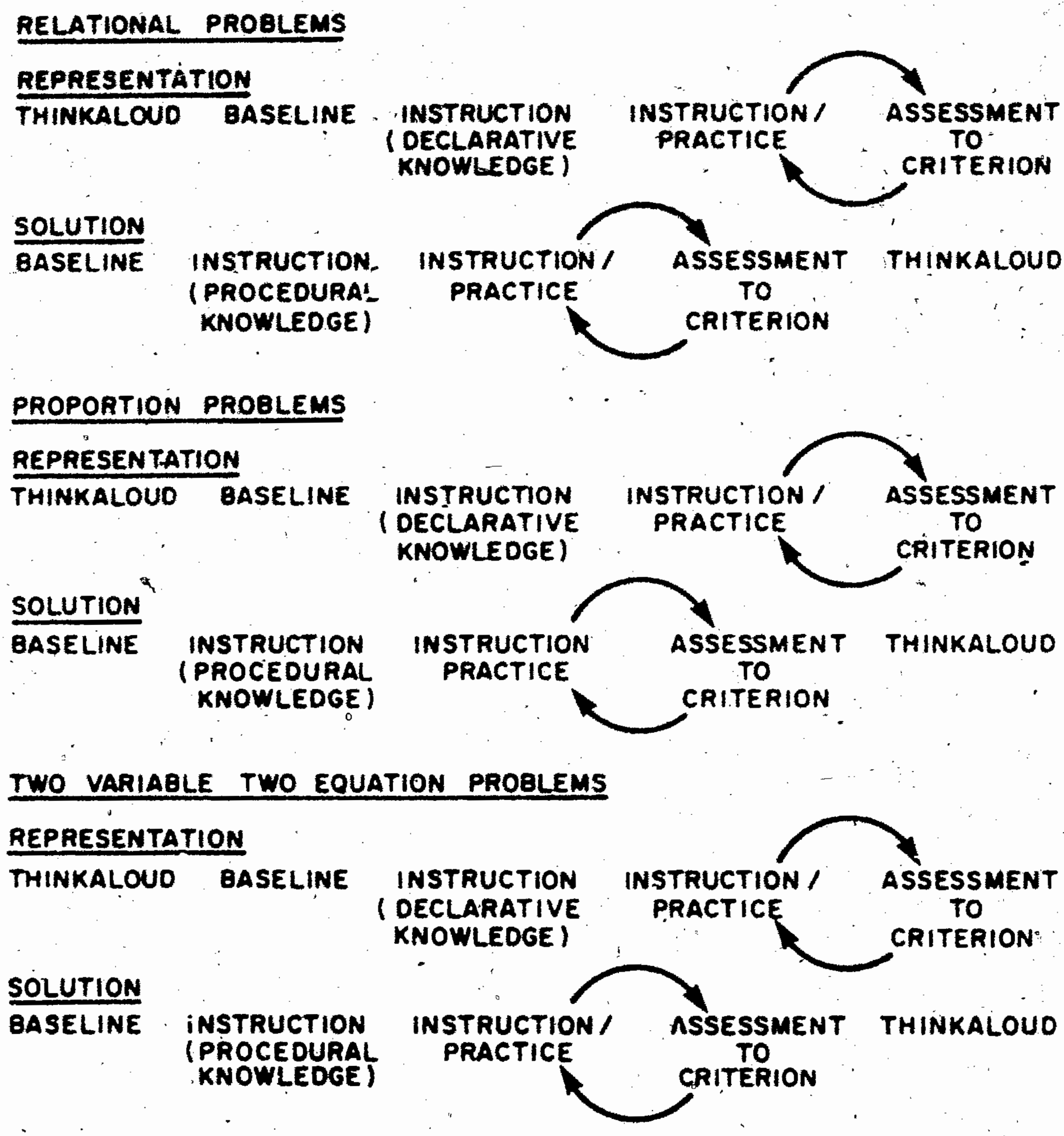


Figure 3

Successive Phases of Single-Subject Design

in Intervention

<u>Pretests</u> (prior to intervention)

- Assessment measure (Criterion-referenced, consisting of instructed problem types)
- Multiple-choice problem test (B.C. Mathematics Achievement Test, Grade 7/8 Applications)
- Open-ended problem test (Q2 B.C. Achievement test)
- Metacognitive interview
- Thinkaloud for a relational problem
- Classification task

<u>Tasksheets</u> (completed during intervention by instructed students)

<u>Posttests</u> (immediately following intervention)

- Assessment measure (Criterion-referenced, consisting of instructed problem types)
- Multiple-choice problem test (B.C. Mathematics Achievement Test, Grade 7/8 Applications)
- Open-ended problem test (Q2 B.C. Achievement test)
- Metacognitive interview
- Thinkaloud for a relational problem
- Classification task

<u>Transfer tasks</u> (immediately following posttests for problem types mastered)

- Near-transfer tasks (criterion-referenced)
  - Relational problems
  - Proportion problems
  - Two-variable two-equation problems
- Far-transfer tasks (criterion-referenced)
  - Relational problems
  - Proportion problems
  - Two-variable two-equation problems

<u>Maintenance</u> (six weeks later, by instructed students)

- Assessment measure (criterion-referenced, consisting of instructed problem types)

Figure 4

Dependent Measures

Students

Selection Criteria

Selection criteria focussed on identifying adolescents studying grade eight mathematics who had a history of learning disabilities, were experiencing particular difficulty in problem solving, but had mastered the basic operations in mathematics.

The first set of criteria required the selection of students identified as learning disabled in accord with school district and provincial guidelines (see Appendix A). These students had been identified as learning disabled during their elementary school years, and all had scored within the normal range on individual intelligence tests, and shown a significant discrepancy between ability and achievement. All had a history of learning disabilities in mathematics on both conceptual and computational tasks. Some had experienced learning difficulties in other areas such as reading and written expression.

Three criteria for prerequisite knowledge were established. These helped to specify the task domain to which criterion-referenced measures of learning outcomes would be applied (Gagne & Beard, 1978). The first criterion was mastery or 80% correct answers on a 10-minute, 35-item test of basic operations with whole numbers (see Appendix B). The second criterion required familiarity or 60% correct answers in a set of five one-step word problems requiring multiplication (see Appendix B). To qualify as experiencing difficulty in solving algebraic word problems, students had to provide the correct numerical answer to no more than 40% of a set of 15 problems designed to require the use of unknowns and equations (see Appendix B). For the second and third criterion measures, calculators and familiarization with their use were provided.

A school district official identified two learning assistance teachers interested in the research. They nominated students who met the first set of criteria. Consent was sought from 25 students and their parents. The students were told what would be expected of them if they participated. A letter was sent to the parents providing information about the study and requesting their signed consent (see Appendix C). Of the 25 families, 21 responded positively. These 21 students completed the criterion tests in the two weeks prior to the start of instruction; all met the second set of criteria. One was later disqualified when identified as an outlier on one of the dependent variables (see Appendix D).

The two junior secondary schools were located in suburban communities near Vancouver. These schools served students from a wide range of socioeconomic and ethnic backgrounds in grades eight, nine and ten. Each school had a learning assistance program in mathematics. In School 1 (see Table 2) the learning assistance students received mathematics instruction in groups of 15. In School 2 the students worked independently on individualized mathematics programs. There were approximately 15 students in the learning centre

Table 2

Characteristics of Students in the Study

Group	S#	Age (Months)	Sex	Grade Placement	School	Score on Basic Operations	Score on 1-Operation Word Problems	Score on Instructional Word Problems
C	1	162	F	8	1	100	80	40
C	2	160	F	8	1	100	100	33
C	3	188	M	10	1	86	80	13
C	4	188	M	10	1	100	60	33
C	5	169	M	9	2	97	100	27
C	6	169	M	9	2	100	100	0
C	7	180	F	8	2	97	60	7
C	8	162	F	9	2	97	80	7
I	1	166	F	8	1	100	100	33
I	2	184	F	10	1	94	80	7
I	3	176	F	9	2	94	80	13
I	4	162	M	9	2	100	80	20
I	5	180	F	10	2	97	60	7
I	6	151	F	8	2	97	80	7
I	7	161	M	8	1	94	80	7
I	8	178	M	8	1	100	80	40
I	9	171	M	9	2	97	100	40
I	10	183	F	10	2	100	60	7
I	11	190	M	9	1	97	80	13
I	12	179	F	9	2	94	60	0

<u>Note</u>: The values for the three scores represent percentages correct.

at one time. Instructional blocks were approximately one hour in both schools. When they were not with the researcher, instructed and comparison students continued with their learning assistance program in mathematics.

Description of Students

The 20 student participants are described in Table 2 according to group, age, sex, school, grade, and performance on criterion measures. The students in each school were assigned randomly to the instructed and comparison conditions to produce groups of 12 and 8 respectively. Randomization was carried out within the restrictions imposed by the students' schedules, as it was necessary that they participate in the intervention during the period they were scheduled to receive learning assistance.

The 12 students assigned to the instructed condition consisted of 7 females and 5 males. Three were registered in grade 10, five in grade 9, and four in grade 8. The results of the criterion tests (see Table 2) indicated that the basic operations scores of the 12 students ranged from 94% to 100% with a mean of 97%. Their scores on accuracy of numerical answers to single-operation word problems ranged from 60% to 100% with a mean of 78.3%. On the criterion measure for word problems requiring variables and equations, their percentage of correct numerical answers ranged from 0 to 40 and their mean was 18%.

The eight students assigned to the control group consisted of 4 females and 4 males. Two were enrolled in grade 10, three in grade 9, and three in grade 8. Results of the criterion tests (see Table 2) indicated that the basic operations scores of the 8 students ranged from 86% to 100%, with a mean of 97%. Their scores on accuracy of numerical answers to one-operation word problems ranged from 60% to 100% with a mean of 82.5%. On the criterion measure for word problems requiring variables and equations, their percentage of correct numerical answers ranged from 0 to 40 with a mean of 20%.

Hotelling's T^2 test was used to compare means of the instructed and comparison groups on the three criterion variables -- basic operations, one-operation word problems, and instructed word problems. (See Table 3 for means and standard deviations.) Results of the Hotelling's T^2 analysis showed no significant differences between the means of the two groups on the criterion measures $F(3,16) = .027$, $p > .05$.

In summary, all participants in the study were junior high students, studying eighth grade mathematics, identified by their schools as requiring learning assistance in mathematics. The students were adequate at basic operations and simple one-step word problems requiring multiplication, but were poor at word problems requiring variables and equations.

Table 3

Summary of Means and Standard Deviations for Instructed and Comparison Groups on Three Pretreatment Criterion Variables

Variable	Comparison Group			Instructed Group		
	Mean	SD	N	Mean	SD	N
Basic Operations	34.00	1.69	8	34.00	0.85	12
1-Operation Word Problems	4.00	0.76	8	3.92	0.67	12
Instructed Word Problems	2.63	2.07	8	2.58	1.93	12

Materials

From the word problems which appear in junior secondary mathematics curricula, three mathematical structures were selected: relational problems, proportion problems, and two-variable two-equation problems. The first two types are typically taught in grade eight and the third type in grade nine. Task analyses of these three problem types appear in Appendix E.

The first type of problem instructed was the relational problem. A representative example is: "Sam has $18 more than Tom. Together they have $82. Find the amount of money each boy has." This type of problem was instructed first because task analysis suggested they are the simplest problems that require representation and the use of a variable for consistent correct solutions. Paige and Simon (1966) found that relational problems required representation of relationships in the problems rather than just translation, that is, step-by-step substitution of algebraic symbols for the English words and phrases. In addition, the equation is typical of those first taught in algebra: "X + (X+18) = $82". Thus the problem solution for relational problems provides a representative introduction to solving equations.

The second type of problem instructed was proportion. An example is: "Brian saved $50 in 18 weeks. How long will it take him to save $350?" These are generally thought to be cognitively more complex as they require that the student set two ratios equal to each other in order to obtain a problem representation with the equation in the form "50/18 = 350/X." This equation involves two equivalent fractions and is not as typical as the equation that characterizes the relational problem.

The third type of problem, requiring representation in two variables and solution of two equations, is sometimes referred to as simultaneous linear equations. An example consists of: "Andrew has 18 coins, some quarters and the rest dimes. The total value of the coins is $3.45. Find the number of each kind of coin." The pair of relevant equations are; "X + Y = 18" and ".25X + .10Y = $3.45". Because of its greater cognitive demands, this type of problem was taught last.

After observation of two expert solvers and theoretical analyses, the task analyses for the three types of instructed word problems evolved, during two one-month pilot studies, each involving two students who met the criteria for inclusion in the study. In each case, I instructed students individually in the general subtasks of word problem representation -- finding the goal, finding what was known, and introducing unknowns. Consistent with Reif and Heller's (1982) work, I found that the students required an integrative framework specific to the particular type of problem. The students produced the diagrammatic representation first, laying out the mathematical relationships inherent in the problem, identifying the features of the problem and/or the problem type, and then moving to the detailed subtasks. Line-by-line translation and detail-by-detail recording became the "default option" when a student couldn't identify the mathematical relationships in the problem as an instance of a particular type of problem. Similar work in the pilot studies led to the decisions to use algorithmic solution procedures for the relational and proportion problems, and systematic trial-and-error for the solution of two-variable two-equation problems. The task analyses for the three types of problems appear in Appendix E.

Training Materials

The study employed training materials consisting of sets of five word problems (see Appendix E). Each set of five problems is referred to as a tasksheet and the form on which the students wrote their answers is called a structured worksheet. Each tasksheet contained one mathematical structure -- relational problems, proportional problems, or problems requiring two equations in two unknowns -- with one problem reflecting each of five contextual details or surface structures of time, money, distance, number, age. The tasksheets were designed to teach students that understanding mathematical structure is necessary for problem representation and problem solution, while surface structure is variable and an unreliable guide.

Problems of each type were drawn from mathematics series used by grade eight mathematics teachers (Ebos, Robinson, & Tuck, 1984; Eicholz, O'Daffer, Brumfiel, Shanks, & Fleenor, 1971; Sobel &

Maletsky, 1974; Stein, 1982). About one-quarter of the problems in the instructional materials were from these sources. The remaining three-quarters were prepared for this study, modelled on those in the texts, but combining each of the five surface structures with each of the three mathematical structures. A total of 200 problems of each mathematical structure were prepared (40 with each surface structure). These were assigned randomly to tasksheets for training materials, baseline tests, and dependent measures for the pretests and posttests. One hundred problems of each mathematical structure were available for use during training having been assigned randomly to 20 tasksheets. The tasksheets were examined by experienced mathematics educators to ensure that they contained no mathematical misconceptions.

The odd-numbered tasksheets were used for training and practice while the even-numbered were used for regular task assessment. All students received the same training materials in the same sequence, but progressed at their own rate. In the early weeks of intervention, instruction took place at one session and assessment at the following session. As students became more proficient, instruction and assessment could usually be completed in one session.

Procedures

To facilitate describing the complex procedures, an overview is given first, followed by detail about each phase of the study. All students received pretests, posttests, and were made familiar with the structured worksheet. For students assigned to the instructed condition, intervention lasted 12 weeks within a span of 15 weeks due to a three week Christmas vacation. A brief review of all previous instruction was conducted in the first post-vacation session. Adjustments in scheduling were necessary when the schools changed from first to second semester, four weeks before the end of instruction. All students received posttest measures at the close of intervention. Transfer tasks were administered to the instructed students following posttesting. Instructed students received maintenance tasks six weeks later.

Procedures applied to all students are described first, followed by procedures unique to instructed students.

Procedures Applied to All Students

Pretesting and Posttesting

Pretesting began immediately following subject selection. Six pretests were administered in six sessions. The assessment measure, consisting of 15 problems, five of each instructed type, had already been administered because it was used as both criterion measure for identifying participants in the study and pretest. The order of the remaining measures was: multiple choice problem test, open-ended problem test, metacognitive interview, thinking aloud for a relational problem, and the classifying task. Although the time taken was recorded, each written test was administered without time limit.

Calculators were provided for the written measures and the thinking aloud. Students were tested individually for the interview and the thinking-aloud protocol, and two at a time for the other measures.

Posttesting was initiated immediately after the last day of instruction. The same schedule and arrangements described for pretesting were used.

Familiarization with Structured Worksheet

All students were introduced to the structured worksheet (see Figure 5) on which the assessment measures for instructed problems were completed in the pretest, posttest, and intervention. The worksheet was divided into two parts, representation and solution, which contained the steps in problem solving derived from cognitive task analysis. Representation of the problem was expressed as "getting the whole picture" on the top half of the worksheet. The lower half of the worksheet was reserved for showing the problem solution. The script for familiarization appears in Appendix F. The pilot studies had shown that the worksheet provided students with effective cues which guided them in completing problems and a language for talking about problems.

The students were provided with explanations for the six components of problem representation that appear on the worksheet: making a drawing or using your own words to show the relationships in the problem; the goal (what you have to find); the unknowns (what you

Problem ________ Date ________ Name ____________________

Exercise ________ Exp. Phase ______ Session ______ Ob. ______

Goal: ____

What I don't know: ______________________________

What I know:

I can write/say this problem in my own words or draw a picture.

Kind of problem: ______________________________

Equation:

____

Solving the equation:

Solution:

[compare to goal]

Check:

Figure 5

Structured Worksheet

don't know in the problem; this is where you introduce a variable); the knowns (the relevant information you know from reading the problem and from your prior knowledge); kind of problem (the class of problem that it fits into or a description of the problem's features; equation (a true statement based on the relationships in the problem and shown in your drawing, including the variable you have introduced).

Problem solution was described as "the steps you carry out on the equation to get to the goal". Brief explanations were provided for the other components of solution; compare your answer to the goal (to insure consistency between the goal and the answer you write in a sentence); checking (your computations and obtained values).

Students in the comparison group were seen individually for three 30-minute sessions. In each session they received the familiarization, were asked to define the terms, and to find the relevant components in particular problems. Then they were given an opportunity to try out the worksheet with those problems. In the first session they used relational problems from the first tasksheet. In the second session they used the first tasksheet for proportion problems, and in the third session, the first tasksheet for two-variable two-equation problems. They did not receive instruction or feedback specific to the problems, only familiarization with the structured worksheet and the meaning of the expressions used on it. These three 30-minute sessions for the comparison group took place in the week prior to posttesting.

Students in the instructional group received familiarization with the meanings of the expressions used on the structured worksheet in the course of the first day of intervention for each type of problem.

The students in the comparison group continued with their learning assistance program in mathematics while the instructed students participated in the intervention. Verbal problem solving was included in the curriculum of each student in the comparison group, and the instructed problem types were part of the problem solving curriculum. In one school, students in learning assistance worked on individualized programs that had been prepared by a special educator. There were about 15 students in the learning assistance centre at a time. In the other school students were instructed in mathematics by a special educator in a group containing about 15 students. Total instructional time was the same for students in the two groups.

Experimental Conditions (Instructional Students)

Baseline

Baseline measures were administered prior to instruction in representation and then again prior to instruction in solution for each type of problem. In all cases students were asked to complete both the representation and solution sections of the structured worksheet. Students were told that the baseline served to determine how much they already knew and as a standard of comparison with their later assessment measures. They were given a set of ten randomly ordered problems of one mathematical structure containing two problems with each surface structure (see Appendix G). The students were provided with calculators, but self-questioning prompts were not made available. The investigator demonstrated the use of the calculator to all students.

In clinical and behavioral interventions, the convention has been to require a minimum of three separate observation points to establish a trend in the baseline data (Hersen & Barlow, 1976, p. 76). However, Kazdin (1982) argues that "requiring the subject to complete a task for assessment purposes may be difficult for an extended baseline." (p. 146), and may jeopardize the intervention. He adds that "the clearest instance of stability would be if behavior never occurs or reflects a complex skill that is not likely to change over time without special training" (Kazdin, 1982, p. 148). Because students in the pilot studies had found the baseline measures disheartening and because the complex knowledge was not likely to change without instruction, only two baseline assessments were administered at the beginning of each phase. To increase reliability, each set contained ten problems rather than five, as in daily assessments. Baseline measures were scored for representation, solution and numerical answer in accord with guidelines presented in the section entitled <u>Scoring Procedures</u>. After finishing a baseline measure, the student was shown his or her obtained scores on a graph. A student who reached the criterion of 80% on baseline in any phase would proceed to the baseline for the next phase.

Think-Aloud Protocols During Intervention

Think-aloud protocols were administered prior to baseline for representation and following attainment of criterion for solution for each type of problem. These were audio recorded on an Aiwa auto-reverse recorder equipped with clip-on microphones. The student was given a problem typed on a card, and asked to think out loud while doing the problem. Consistent with the recommendations of Ericsson and Simon (1984), detailed instructions were employed to ensure that the students understood what was expected of them. At the first administration, students practised thinking aloud while completing a multiplication question (see Appendix H for the instructions). Scoring procedures were developed to rate students' understanding as they thought aloud.

General Procedures Throughout Intervention

Each student met individually with the instructor for one 40-minute session on alternate days. These procedures were followed:

1. The student was reminded of the purpose of the current session, and shown the graph of the results to date.
2. The tasksheet was placed in front of the student with

the prompt card for self-questioning. (The prompt was gradually faded out.)

3. The student read and said from memory the self-questions.
4. The student and the teacher each read the first problem silently.
5. The instructor read aloud and thought aloud for the first word problem, making frequent references to the prompt card for self-questioning. This procedure was repeated for the second problem. (See Appendix I for sample transcript of instruction.)
6. The student read aloud and thought aloud for the third word problem. The instructor prompted, encouraged, and provided feedback specific to the student's verbalizations. Examples are: "Are you certain that you are letting X take the place of the simplest unknown?" "That is correct, it is a relational problem; now, can you tell me what structural features of the problem helped you to identify it?" This procedure was repeated for the fourth problem.
7. The student completed the fifth word problem without interjection from the instructor, aloud in the early stages of the study and then silently. Feedback was provided.

8. The frequent prompts, encouragement and feedback were gradually faded out. (The reader is referred to Appendix I for representative transcripts illustrating the instruction in representation and solution.)
9. After completing a practice tasksheet in this manner, the student completed an assessment tasksheet independently, seeking help only if unable to proceed.
10. The student was shown a graph of progress and informed about the purpose of the next session.

<u>Components of Instruction for Problem Types</u>

A number of scripts were used throughout the study: orientation script, general script about word problems, and specific scripts for representation and solution for each problem type. These can be found in Appendix J.

<u>Instruction in representation for relational problems</u>. On the first day of intervention an orientation script was used to introduce the student to the training experience. The orientation script included a discussion of why we sometimes experience difficulty doing word problems, a description of self-questioning and its relevance to the instruction, and a description of the two phases into which problem solving would be divided: problem representation and problem

solution. The students were informed that these activities would help them to do word problems during intervention and in their mathematics classes. The investigator answered the students' questions and listened to their concerns.

Following the orientation, the general script about word problems (see Appendix J) was used to engage in interactive conversation with the student. The purpose was to assist the student in acquiring general information and vocabulary about word problems and relating these to existing knowledge. The existence of kinds of story problems was related to the existence of kinds of familiar stories (e.g., mysteries and westerns). Declarative information included the importance of the mathematical structure of a word problem (rather than the contextual details) for determining the type of problem and how to proceed. Other topics included the use of a letter to stand for an unknown quantity and the search to find the number so that the letter is no longer necessary.

Then the student was made familiar with the structured worksheet on which the problems were to be completed.

The instructor read through the self-questioning for representation with the student and pointed out the close connection between the self-questions and the top half of the structured worksheet. The student rehearsed until able to say the questions and point to the relevant parts of the worksheet. The self-questions for representation were:

1. Have I read and understood each sentence? Are there any words whose meaning I have to ask?
2. Have I got the whole picture, a <u>representation</u>, for this problem?
3. Have I written down my representation on the worksheet? (goal; unknown(s); known(s); type of problem; equation)
4. What problem features should I focus on in a new problem so I can know whether I can use the representation I have been taught?

A script was followed to present declarative information specific to relational problems (see Appendix J). Students were told to watch for a relational statement, a sentence giving information about one unknown quantity in terms of its relationship to another unknown quantity (e.g., John has $4 more than Sam). They were also informed about guidelines specific to using unknowns to represent quantities in relational problems.

Following this instruction in declarative knowledge, the instructor proceeded as described in the section <u>General Procedures Throughout Intervention</u>. Tasksheets #1 to 20 were used in sequence, odd numbers for practice and even numbers for assessment. Sessions consisting of instruction/practice and assessment (a teach/test cycle) continued until the student reached the criterion of correct representation for 4 out of 5 relational problems on three consecutive assessments. Then baseline was conducted (as has been described) for the solution of relational problems.

Instruction in solution for relational problems. Following baseline for solution, on the first day of instruction in this phase the script to familiarize the student with the worksheet was used (see Appendix F). The instructor read through the self-questioning for solution with the student and pointed out the close connection between the self-questions and the bottom half of the structured worksheet. The student rehearsed until he or she was able to say the questions and point to the relevant parts of the work-sheet. The self-questions for solution were:

1. Have I written an equation?
2. Have I expanded the terms?
3. Have I written out the steps of my solution on the worksheet? (collected like terms; isolated unknown(s); solved for unknown(s); checked my answer with the goal; highlighted my answer)
4. What problem features should I focus on in a new problem so I can know whether I can use the solution I have been taught?

The student was presented with procedural information about the importance of adding/subtracting only terms that are alike, and carrying out addition and subtraction before multiplication and division in the procedure for solving an equation. The need for consistency between representation and solution was emphasized (for script, see Appendix J).

Teaching by thinking aloud proceeded as described in the section <u>General Procedures Throughout Intervention</u>. Students were required to generate both the problem representation and the problem solution for each problem, because the problem solution consisted of solving the equation and answering the goal, which had been generated during the representation. Sessions consisting of instruction/practice and assessment (teach/test cycle) continued until the student reached the criterion for the solution phase of 80% correct representations, solutions, and answers in three consecutive sessions. Tasksheets in the series #1 to 20 were used following on from the last tasksheet employed for assessment in problem representation.

<u>Instruction in representation for proportion problems</u>. Following administration of baseline for proportion problems, on the first day of instruction in this phase, there was a review of the self-questions for representation. The student demonstrated knowledge of the questions and pointed to the relevant parts of the worksheet. Declarative information specific to proportion problems was presented including vocabulary for terms such as ratio, complete ratio, incomplete ratio, equivalent ratios, and "new case". Students were told to watch for a complete ratio and an incomplete ratio that were equivalent. They were reminded to employ a letter to stand for the

unknown term in the "new case" (see script in Appendix J).

Teaching by thinking aloud proceeded as described in the section General Procedures Throughout Intervention. Tasksheets #21 to 40 containing proportion problems were used sequentially. The teach/test cycle continued until the student reached criterion for representation.

Instruction in solution for proportion problems. Following baseline for solution, there was a review of self-questions for solution and the student was required to show that he or she had learned the questions. Procedural information was presented about solving an equation in the special case when each term was in the form of a fraction. Students were shown two procedures: how to use cross-multiplication and how to make one side of the equation equal to 1 by multiplying by the inverse. (Most students chose to cross-multiply, an operation with which many claimed to be familiar.) The need for showing all the steps in one's work was emphasized (see Appendix J).

Following this instruction in procedural knowledge, teaching by thinking aloud proceeded as described in the section General Procedures Throughout Intervention. Tasksheets #21 to 40 continued to be used in sequence with the teach/test cycle continuing to criterion.

Instruction in representation for two-variable two-equation problems. Following administration of baseline for two-variable two-equation problems, self-questions for representation were reviewed as in previous phases. Declarative information specific to these problems was presented including identification of two potential equations. It was pointed out that in each case the more complex equation would be derived from the simpler equation. For example, the value of each kind of coin present could be derived from information about the number of each kind of coin. The need for two variables was discussed. It was emphasized that this type of problem could be identified by the need for two equations and two unknowns (see script in Appendix J).

Teaching by thinking aloud proceeded as described in General Procedures Throughout Intervention. Tasksheets #41 to 60 were used for the teach/test cycle.

Instruction in solution for two-variable two-equation problems. The administration of baseline for solution was followed by a review of self-questions for solution. It was explained that these steps would be necessary for solution of most problems but that for some types of complex problems and for unfamiliar problems another approach would also prove valuable. Procedural knowledge relevant to problem solution by systematic trial-and-error and charting of attempts was presented. The importance of recording all values of variables and outcomes, and of selecting the next values tried in light of previous outcomes was emphasized (see script in Appendix J). Whereas the instruction in solution for the first two problem types taught an algorithm for solving equations, instruction in solution for these problems concentrated on systematic trial-and-error.

Teaching by thinking aloud proceeded as described in General Procedures Throughout Intervention. Tasksheets #41 to 60 continued to be used in sequence.

Transfer

For the instructed students the transfer phase began immediately after posttesting was completed. Students were provided with calculators but no self-questioning prompts. Two measures of transfer were employed: near transfer and far transfer. Each student completed, on structured worksheets, five near-transfer problems and five far-transfer problems of each problem type mastered (relational, proportion, two-variable two-equation).

The near-transfer measures were designed to assess whether students could transfer their learning to problems that were new in surface structure but had the same mathematical structure as the instructed problems. The new surface structures or contextual details consisted of volume, mass, election results, area, and test results. An example of a relational problem with the surface structure volume was: "The volume of the big box was 32 cm^3 more than the volume of the little box. The sum of their volumes was 72 cm^3. Find the volume of each box." (For the problems, see Appendix K.)

The far-transfer measures were designed to assess whether students could transfer their learning to problems that had the same surface structure but new and cognitively more complex mathematical structures which had been derived by cognitive task analysis (for the problems see Appendix K).

Each instructed relational problem contained one of two kinds of relational statements, either: "John had $9 more than Mary", or "John had 3 times as much money as Mary." Each far transfer relational problem contained both kinds of relational statements in combination: "John had $9 more than 3 times as much money as Jane."

Each instructed proportional problem contained two equivalent ratios, one complete and one incomplete: "The ratio of Mary's age to John's is 3:2. If John is 8, how old is Mary?". Each far-transfer proportion problem contained three equivalent ratios, one complete and two incomplete: "The ratio of Jane's age to John's to Frank's is 3:2:1. If John is 18, find Jane's age and Frank's age."

Each instructed two-variable two-equation problem resulted in a pair of equations in two unknowns. Such a problem was "Mike has $1.30 in nickels and quarters. He has 10 coins in all. How many coins of each type does he have?" Each far-transfer problem for two variables and two equations resulted in a pair of equations which appeared to contain three unknowns; however, the third unknown could be obtained by a simple calculation from the information given in the problem. An example of such a problem is: "Sam has $1.55 in nickels, dimes, and quarters. He has 10 coins in all. Nine of the coins are nickels and quarters. How many coins of each kind does he have?" It can be seen that if 9 of the 10 coins are nickels and quarters, then there must be only 1 dime. As with the far-transfer tasks for the other two types of problems, it was reasoned that a student would need to transfer all that was learned in the instructed problems and construct a more complex representation and solution in order to work these transfer problems.

Maintenance

The maintenance data were collected six weeks after the end of instruction by administering the same 15-item pre-post assessment measure (containing five problems of each type of instructional problem) under the same conditions used on the two previous administrations.

Reliability

On a random schedule, audio tapes were made of instruction. These were compared to the scripts to ensure that no important components of instruction were absent or altered in any substantial way.

Dependent Measures

Measures Collected During the Course of Instruction

Problems Completed During Instruction

Assessment measures for the type of instructed problem were collected in successive sessions. These consisted of students' written responses on structured worksheets to the even-numbered tasksheets containing five problems of one mathematical structure. The completed structured worksheets were the principal dependent measures. Detailed criteria used to evaluate the students' worksheets are presented in the section entitled Scoring Procedures.

Evaluation of Understanding at Selected Points Throughout Intervention (Think-Aloud Protocol)

Students were instructed to think out loud, while representing and solving on the structured worksheet, prior to baseline for representation and following attainment of criterion for solution in each problem type. (See Appendix L for the problems which were used.) The purpose was to allow detailed examination of the verbalizations produced by students while doing problem representation and problem solution. These verbalizations were recorded for later scoring in conjunction with the written worksheet and observations made by the investigator. The guidelines used to determine ratings for understanding of problem representation and problem solution are presented in the section entitled Scoring Procedures.

Pre-post Measures

These measures were administered to instructed and comparison students. In all cases students were provided with a calculator and given as much time as they required.

Instructed Problems

This measure was included to assess directly the thoroughness and accuracy with which students completed problems of the same mathematical structures and surface structures as the instructed problems. It consisted of five problems of each type: relational, proportion, and two-variable two-equation. These 15 problems were presented in random order. They had been selected at random with the condition that there be one problem containing each of the 15 combinations of surface structure and mathematical structure (see Appendix B). Each problem was completed on a structured worksheet. Responses were scored according to three sets of criteria described in the section entitled Scoring Procedures.

Multiple Choice Measure (B.C. Mathematics Achievement Tests, Grade 7/8 Applications).

The Ministry of Education in British Columbia has prepared a general mathematics achievement test (Robitaille, Sherrill, Kelleher, Klassen, & O'Shea, 1980) that requires students to apply five skills taught in the mathematics curriculum at grades seven and eight. Three subtests, Analyzing Word Problems, Use of Diagrams, and Solving Problems, were selected for inclusion because they were related to the instruction in this study.

The subtest, Analyzing Word Problems, was selected because its purpose was to determine whether a student was able to analyze a verbal problem and (a) distinguish between what is given and what is found, (b) recognize whether sufficient information is given to solve the problem, and (c) determine what operations are required to solve the problem. It was also intended to assess whether students could translate verbal problems into open sentences. The subtest, Use of Diagrams, was included because it assessed whether students could construct a "model" such as a diagram as an aid to solving a problem, and draw and interpret scale diagrams. The subtest, Solving Problems, was related to the instructed problems in that it assessed whether students could test the appropriateness of an answer to a problem, and solve problems involving percents. See Appendix M for the instructions and problems used in this measure. The three subtests contained 8, 7, and 10 problems respectively. For each problem the students read a brief stem and then marked in the booklet the one of the four options that provided the best answer. Although only multiple choice responses were scored, students were also asked to show their work.

Open-Ended Measure (O2 B.C. Achievement Test)

An assessment test developed by the Learning Assessment Branch (1985) British Columbia Ministry of Education for use in grade 10 was used as a source of relevant word problems. Although many of the problems would be solved better by representing the information in a diagram none was an isomorph of the three instructed types. The students were asked to show their work; however, only the correct numerical answer was scored. This measure of general problem-solving ability included a wide range of problems taught in junior secondary mathematics. Thirteen problems were selected. Two were one-step problems, three were complex variations on relational problems, and four involved measurements of length and area or volume. The remaining four involved adding fractions, permutations, extrapolation from a drawing, and one involved percent and proportion. See Appendix M for the instructions and problems in this measure.

Metacognitive Interview

Metacognition refers to the awareness of knowledge and control and regulation of knowledge (Baker & Brown, 1984). Insufficient self-monitoring by the learning disabled has been alleviated by self-questioning instruction in studies of reading comprehension (Palincsar, 1982; Wong & Jones, 1982). In light of the close relationship between self-questioning and metacognition, a metacognitive interview was used to assess change in metacognition of the instructed group and the comparison group over the period of instruction.

Consistent with Flavell's (1976) recommendations, questions about the students' perceptions of the task of working mathematics word problems, strategies for problem solving, and about themselves as problem solvers were posed in an individual interview (see instrument and flowchart for administration in Appendix M). Each question was posed, and followed by a prompt ("anything else?") to encourage a student to elaborate on a single-item response. If a student failed to respond within ten seconds or demonstrated lack of understanding of the question, a specific probe was used which restated the question in a more concrete and personalized manner.

Classification Task

The ability to recognize problem types and be guided by well developed problem schemata is considered an important indicator of expertise in problem solving (Mayer, 1982; Silver, 1982). A classification task was designed (see Appendix M) consistent with Silver's (1980) recommendation that students be asked to select the most similar problems out of a set. This instrument consisted of 10 sets of three problems. In each set of three problems, two problems had the same underlying mathematical structure (e.g., relational), two problems had the same surface structure (e.g., age). and the third combination bore no planned similarity. In each case, the student chose the two most similar problems and wrote in a few words the reason for the choice. It was possible to discern whether the student could verbalize the similarity between the selected problems.

Scoring Procedures

Some dependent measures were scored simply for correct numerical answer. These measures included the multiple-choice measure (B.C. Mathematics Achievement Tests, Grade 7/8 Applications) and the open-ended measure (Q2 B.C. Achievement Test). The classification task was scored for correct choice of two problems whose similarity was based on mathematical structure. In the following section the criteria and procedures for scoring the complex dependent measures are presented: the instructed problems, the think-aloud protocols, and the metacognitive interview.

Scoring Procedures for Measures Collected During the Course of Instruction

Scoring of Problems Completed During Instruction

Responses to all assessment problems during intervention were scored by the investigator and then verified by a second rater. Disagreements were resolved by referring to established criteria.

For each tasksheet, percentages correct were calculated. When students were required only to represent the problem, only one set of criteria was used -- to decide whether or not the representation for each problem was complete and accurate. When students were required to represent, solve, and give a numerical answer, then three separate sets of criteria were used. These criteria were for the completeness and correctness of the representation, thoroughness and accuracy of the solution, and the accuracy of the numerical answer. In addition to a raw data sheet, a graph displaying daily performance was maintained for each student. Altogether students' completed problems were scored in three areas: representation, solution, and accuracy of numerical answer, yielding three separate scores.

<u>Criteria for representation</u>. In order for a representation to be scored correct, the following criteria had to be met: an accurate and complete statement of the goal of the problem, a succinct statement of what was known after reading and understanding the problem, the introduction of symbols (letters) to represent what was unknown in the problem, and a thorough abstract representation in the form of an equation (which could be accompanied by a drawing) of the mathematical relationships implied in the problem. Students were also required to give a succinct description of the features of the problem or name the type of problem.

<u>Criteria for solution.</u> Problem solution consisted of carrying out algebraic procedures in the correct sequence to obtain the roots

of the equations and meet the goal stated in the problem. The following criteria had to be met for a correct problem solution to a relational or proportion problem: the series of steps known as collecting like terms, isolating the unknown, and solving for the unknown; highlighting the answer that met the stated goal, and evidence of checking the computations and consistency of the answer with what was given in the problem statement. The following criteria had to be met to score a correct problem solution to a two-variable two-equation problem: systematic trial and error with record keeping in a chart for each value tried, the numerical outcome, and a judgment of the resulting error in that outcome. In addition the answer had to be expressed in a brief statement that met the goal, and evidence of checking (could be shown in the chart) was required.

<u>Criterion for answer</u>. The third criterion was that the student have recorded the correct numerical answer.

<u>Scoring Think-Aloud Protocols for Understanding</u>

<u>Rationale.</u> Ericsson and Simon (1984) emphasize that in carrying out protocol analysis consistent with information theory, one is attempting to infer what information was heeded by the subject as input for the observed verbalization. Such protocol analysis is a qualitative process. Yackel and Wheatley (1985) created an instrument to assess the degree of understanding exhibited during problem

solving. However, they were unable to obtain overall quantitative scores because some segments of a subject's protocol were used to generate more than one rating.

The purpose of the present protocol assessment instrument was to obtain overall scores on the degree of understanding exhibited by a student during each of problem representation and solution. The inference of what the student was heeding and understanding was based on written and spoken verbalizations made by the student and an observation checklist (see Appendix N) completed by the investigator.

Consistent with the recommendations made by Ericsson and Simon (1984), the protocol analyses were based on cognitive task analysis of each of the three types of problems. Segments from audiotapes were transcribed and then rated for the degree of understanding shown in problem representation and in problem solution. The six components of representation rated were: understanding of goal; knowledge of necessary explicit relationships; use of an external code; schema for mathematical structure of the problem; abstract coding of implicit relationships; and degree of generalizability. The four components of problem solution rated were: level of abstraction; use of procedural knowledge; computation; and numerical answer. The scoring for each aspect was carried out independently of the others. Each segment of the protocol contributed to only one category. All aspects of the verbalization considered relevant within the task analysis contributed to the score.

Many of the categories yielded by the task analysis are similar to those that Yackel and Wheatley (1985) found to be related to high scores on a test of algebraic problems. The instrument was consistent with the major tenets expressed by Ericsson and Simon (1984) in Protocol Analysis: Verbal Reports as Data but deviates from their recommendations in one major way in that it is used in a quantitative rather than qualitative fashion. Sums were taken across categories and across individuals. This deviation is consistent with the purpose of this study -- to investigate the influence of theory-based instruction in problem representation and problem solution on the degree of understanding exhibited, during think-aloud problem solving by learning disabled adolescents.

Each category was rated 2, 1 or 0, depending on the degree of understanding demonstrated. Two ratings of understanding resulted: one for problem representation, and one for problem solution.

Definitions of Categories of Problem Representation and Their Ratings

Goal. The goal refers to the purpose for which the problem is undertaken. The rating is based on the solver's verbalization of purpose and what is written beside the term "goal" on the worksheet. Scoring:

2 = Accurate goal: The solver shows evidence of having identified the intended goal of the problem before

making the mathematical manipulations which constitute problem solution.

1 = Inaccurate goal: The solver has identified a goal different from the intended goal of the problem.

0 = No goal: The solver has not identified a goal for the problem.

Generalizability. This refers to the extent to which the representation is useful for solving similar problems. The indication used is the extent to which appropriate variables are introduced. These are in the solver's verbalization and written beside the term "what I don't know" on the worksheet.

Scoring:

2 = Integrated: The solver has identified all unknowns required for reaching a generalizable representation and introduced appropriate variables.

1 = Some evidence of integration: Both following conditions must be met: At least one unknown (but not all unknowns) has been identified; at least one appropriate variable (but not all variables) has been introduced.

0 = No evidence of integration: Only one or neither of the following conditions has been met: One unknown (but not all unknowns) has been identified; one appropriate

variable (but not all variables) has been introduced.

Knowledge of needed explicit relationships. This refers to the extent to which the necessary explicit relationships given in the problem information have been stated. The rating is based on the solver's verbalization of what he knows after reading the problem, and what is written beside the term "what I know".

Scoring:

2 = Full Knowledge of Needed Explicit Relationships: The solver has identified all relevant relationships given in the problem.

1 = Partial Knowledge of Needed Explicit Relationships: The solver has identified at least one, but not all, relevant relationships given in the problem.

0 = Lack of Knowledge of Needed Explicit Relationships: The solver has identified none of the relevant relationships given in the problem.

Use of external code. This refers to the extent to which the solver has employed an external code to express the problem in abstract, symbolic notation. The rating is based on the solver's verbalization and written response to "I can write/say this problem in my own words or draw a picture."

Scoring:

2 = <u>High:</u> Drawings or verbalizations showing in symbols the relationships between the elements.

1 = <u>Moderate:</u> Drawings or verbalization showing in symbols the isolated elements without the relationships between them.

0 = <u>None:</u> No drawings or verbalizations (including equations) are presented.

<u>Schema for mathematical structure of problem</u>. This refers to the extent to which the solver identifies (with reasoning) the problem as one with a mathematical structure for which he has a schema. The rating is based on written response to the term "kind of problem" and the unprompted verbalization of the reasoning which led to this identification.

Scoring:

2 = <u>Reasoned use of schema</u>: Name or description of type of problem, based on mathematical structure, is accompanied by reason based on mathematical structure or features of the problem statement.

1 = <u>Use of schema</u>: Name or description of type of problem, based on mathematical structure, is given.

0 = <u>No use of schema:</u> No naming or description of type of problem based on mathematical structure.

Abstract coding of implicit relationships. This refers to the extent to which the solver has employed abstract coding of implicit relationships to form an equation that is accurate and useful for solving similar problems. Verbalizations and what is written beside "equation" on the worksheet will be considered.

Scoring:

2 = *High:* All implicit relationships shown accurately in equations.

1 = *Moderate:* Some, but not all, implicit relationships shown accurately in equations.

0 = *None:* No implicit relationships shown accurately in equations.

Definitions of Categories of Problem Solution and Their Ratings

Level of abstraction in solution. This refers to the format of the first line in which an operation is performed on the equation. The rating is based on the form of the first verbalization and the first line written under "solving the equation".

Scoring:

2 = *High:* Method indicates awareness of class of similar problems by retaining form of equation and including unknowns.

1 = *Moderate:* Some aspects of method could be useful for

solving similar problems. Systematic trial and error with no use of variable will be taken as such an indicator.

0 = <u>Low:</u> Isolated numbers and arithmetic operations, as well as nonsystematic trial and error show lack of abstraction.

<u>Procedural knowledge</u>. This refers to the accuracy with which the algebraic mathematical manipulations or procedural steps are executed in the solution.

Scoring:

2 = <u>High:</u> There are no errors in the order or execution of procedural steps.

1 = <u>Moderate:</u> There is one error in the execution or order of procedural steps.

0 = <u>Low:</u> There are two or more errors in the execution or order of procedural steps.

<u>Computation:</u> This refers to the accuracy with which the computations are carried out and reported in the solution.

Scoring:

2 = <u>High:</u> There are no computational errors in the basic operations of addition, subtraction, multiplication, division.

1 = Moderate: There is one computational error in basic operations.

0 = Low: There are two or more computational errors in basic operations.

Accuracy of answer. This refers to the accuracy of the numerical answer and its consistency with the stated goal in the information provided in the problem.

Scoring:

2 = High: Answer is numerically correct and consistent with the goal.

1 = Moderate: A subgoal has been reached or an answer stated which is not consistent with the goal as stated but is correctly computed given the solver's entries for variables in the problem.

0 = Low: Incorrect numerical answer or no answer.

Scores were found for understanding of problem representation and problem solution. The possible totals were 12 and 8 respectively.

Scoring Pre-Post Measures

Scoring Instructed Problems

This set of 15 problems (five of each of the three types) were scored according to the procedures described in the section entitled Scoring of Problems Completed During Instruction within Scoring Procedures. Three sets of criteria were used for the completeness and correctness of representation, of solution, and the accuracy of the numerical answer.

Scoring Metacognitive Interviews

Rationale. Metacognition refers to the knowledge we have about our cognitive processes and our use of this knowledge to choose ways of dealing with a problem. The students were asked ten structured-interview questions about the task, strategies, and about themselves as problem solvers in accordance with Flavell's (1976) recommendation (see instrument and instructions in Appendix M), and similar to interview questions about comprehension (Myers & Paris, 1978; Paris, 1979). Responses to the interview items were scored on a simple 0-1-2 basis; 0 indicated either no response or no idea about what to do, 1 signaled that some appropriate information was given, and 2 reflected that information was given in a complete and sophisticated manner. Because it is important to be flexible and employ a number of approaches when one encounters difficulty with a word problem, the prompt "anything else" followed single-item responses. Multiple-item responses were scored as more complete and more sophisticated.

Classification system for responses to metacognitive interview about task variables, strategy variables, and person variables.

Question 1 (Task): What are mathematics word problems?

Scoring:

2 = Responses containing reference to the two phases of problem solving, i.e., understanding (representation) and working it out (solution) or reference to problem schemata (types of problems) based on mathematical structure.

1 = Responses containing some appropriate information, such as: problems containing words and numbers, equations put into words.

0 = Responses that indicate no knowledge of abstract dimension such as: don't know, problems containing numbers, have to get the answer.

Question 2 (Person): What makes someone really good at doing word problems:

Scoring:

2 = Responses including two or more of the major steps in representation or solving: read problems carefully, can solve equations, understand the meaning of the whole problem, recognize types of problems.

1 = Responses giving only one of the components in the

previous list.

0 = Responses containing no information or only references to computation: good at them, they ask someone, good at addition, etc., they know what operation to do.

Question 3 (Person): What is the hardest part of doing word problems for you?

Scoring:

2 = Response showing they know that there are different types of problems and some problems requiring representation and solving of equations are more difficult than others.

1 = Some indication that it will be harder for a person to employ unknowns and solve equations than to work simple problems that require only basic operations: solving the equation.

0 = Response containing no information or no awareness of the difficulties posed by the need to use representation: figuring it out, deciding what operation to use, understanding (no specification).

Question 4 (Person): What would help you become a better problem solver?

Scoring:

2 = Response containing specific information that a person could practice or acquire knowledge about the components that posed most difficulty for them: remember the important parts of different kinds of problems, self-test for a specific reason (e.g., to get an equation), other-test for a specific reason; make a representation (whole picture) for problems that person had found hard.

1 = Response containing vague and general information about things that a person could do: practice or study (no explanation), figure out all the words, go over and over problems.

0 = No response or response containing no information: don't know, cheat.

Question 5 (strategy): If you were getting ready to take a word problem test, what would you do that would help you the most to do well on that test?

Scoring:

2 = Responses that indicate two or more productive strategies initiated by the student: ask for help before the test, find out what kind of problems will be on the test, practice the kinds I have learned, self-test, other-test.

1 = Responses that indicate one action initiated by the student from the above list.

0 = No response or no information in response: use a calculator, nothing, try hard (no specifics), skip, ask for help during the test.

Question 6 (task): Are some parts of a word problem, as it is written down, more important than others? How can you tell which parts are the most important?

Scoring:

2 = Complete, sophisticated responses mentioning that one seeks the parts of the word problem that help you to recognize the mathematical structure of the problem, and the type of problem it is.

1 = Responses limited to reading (no specifics), mention of the numbers given in the problem, the words that indicate the operation, the question; or reference to surface structures such as age.

0 = Responses containing no information such as: don't know, yes (no specifics), just guess which parts; or responses holding to the notion that all parts are of equal importance.

Question 7 (Strategy): What do you do if you don't now what a word means in a word problem?

Scoring:

2 = Responses containing a combination of two or more of the following strategies that are used when this problem arises in any reading activity: skip it, recognition, ask, sound it out, guess from context, look it up in a dictionary.

1 = Reference to one strategy listed above.

0 = No response or response containing no information such as: don't know, think, nothing.

Question 8 (Strategy): What do you do if you don't get the "whole picture" or the "whole meaning" of a word problem?

Scoring:

2 = Response that indicates systematic use of prior knowledge about types of problems such as: try to remember a problem that had similar mathematical relationships; represent the equation as a drawing and try to generate the equation from that; try values in the equation as I can express it and keep a record of the results.

1 = Responses that indicate that the student views this misunderstanding as the same as that arising when it is

a word that is not known. Any combination of two strategies like those in the following list; guess, ask, look it up, skip it, reread it trying to understand each word.

0 = Responses containing no information or no response: leave the problem out, guess, anything that would help (no specification).

Question 9 (Strategy): After you have read and understood a word problem, what else must you still do in order to complete the problem successfully?

Scoring:

2 = A complete and sophisticated response including reference to <u>both</u> representation and solution. Reference to representation would include any of the following: finding the goal, figuring out what is known, introducing unknown(s), drawing a picture, representing the problem, writing equation(s). Reference to solution would include any of the following: solving the equation(s), finding the value(s) of the uknown(s), making sure you have reached the stated goal, doing the steps in the right order to solve the equation.

1 = Response which shows some appropriate knowledge of

strategy by referring to <u>one</u> of representation or solution, but not both. See details in paragraph above.

0 = No response or response containing no information that refers to obtaining an equation and solving the equation to obtain a correct answer; check it, carry out the operation (e.g., add).

Question 10 (Task): What is there about a word problem that makes it easy to do?

Scoring:

2 = Complete and sophisticated response that refers to some combination of two or more of the elements in the following list: easy numbers, short problem, clearly stated, no difficult words, familiar type of problem, easy to get the whole picture, easy to solve the equation.

1 = Response that indicates some appropriate knowledge about the task of solving problems, and contains one of the elements listed above.

0 = No response or response that contains no information such as: get it right, know how to do it, know the sign, or a response that refers to the surface structure, such as age.

CHAPTER 4

RESULTS

The first section of this chapter contains the findings for the four research questions about the single-subject data. Graphic and tabular presentations are complemented by summaries of the course of intervention for individual students. The second section presents the results of the analyses of covariance which test the four hypotheses concerning differences between the instructed and comparison groups. Finally, there is a summary of the findings of the study.

Research Questions Relating to Single-Subject Data

> Did the instructed students' ability to represent word problems, obtain complete problem solutions and accurate numerical answers for three problem types increase following the onset of instruction? Were these increases maintained over time?

Twelve instructed students progressed from one problem type to the next after reaching criterion. Because students reached criterion at different rates, there was variation in the number of problem types they learned. However, in every case there was dramatic improvement from baseline to the first assessment after instruction. Every student reached criterion on representation, solution, and answers for each problem type they were taught. Six students (S1, S4, S7, S9, S11, S12) reached criterion for representation, solution, and answers on all three problem types. Four students (S2, S3, S8, S10) reached criterion on two problem types. Two students (S5, S6) reached criterion on only the first problem type. On the maintenance task, five of six students reached criterion on representation, solution, and answers for all three types they had been taught, the other student maintained for two of the three types. Three of the four who learned two types maintained for the two, and the fourth maintained for only one problem type. One of the two students who learned one problem type maintained for that type. The other student did not reach the maintenance criterion for any problems.

The results for individual students on representation, solution, and answers for each problem type are presented in a series of graphs in Figures 6 through 17. Each student received two administrations of baseline (each consisting of 10 questions) prior to instruction in representation and prior to instruction in solution. The numerical answer which is often thought of as the last step in solution was scored separately. A student who reached criterion (80% correct) on both baseline measures for a phase received no instruction in that phase and progressed to the next baseline. There is no instruction phase shown on the graphs (in Figures 6 through 17) when a student reached criterion on baseline and did not receive instruction for that problem type. The number of assessments within instructional phases varied because students continued in a phase until they reached the criterion of 80% correct on three successive assessments. Posttests were administered at the end of three months of instruction. Maintenance was tested six weeks later. The criteria for maintenance and transfer tasks were also set at 80%. This section contains descriptions of the course of intervention for individual students. Each description begins with the instructed student's performance throughout instruction and the problem types mastered (as shown in Figures 6 through 16). If the student modified the instructed strategy by combining or omitting steps, these modifications were noted. Maintenance data and transfer data (as shown in Tables 4, 5, and 6) are discussed along with relevant comments about the student's attitude and attendance.

The descriptions of individual students will be sequenced according to the number of problem types on which the students reached criterion. Descriptions of the six students who reached criterion on three problem types will be presented first, followed by descriptions of students who reached criterion on two problem types, and lastly, descriptions of the two students who reached criterion on only one problem type.

S1 reached criterion on representation, solution and answers for all three types of word problems (see Figure 6). She used trial-and-error haphazardly in the pretest think aloud. On the baseline administrations for representation of relational problems, through trial-and-error she attained 5 out of 10 correct and 7 out of 10 correct numerical answers without representations or solutions.

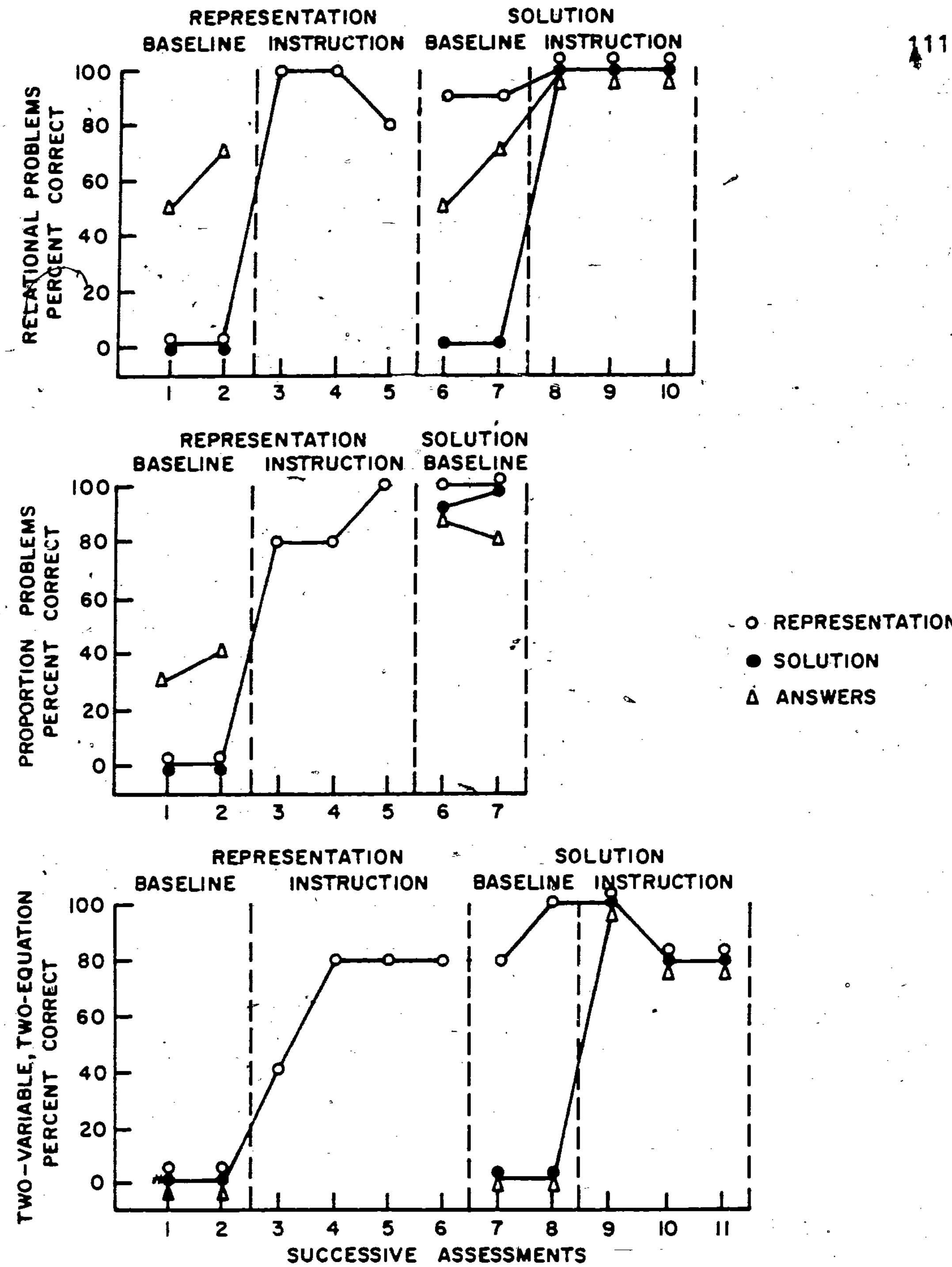


Figure 6

Percent Instructed Problems Correct on

Successive Assessments for Student 1

She commented, "After thinking out loud I have a way that works sometimes, but not all the time." She also used this approach on the baseline measures for representation of proportion problems. On baseline for solution of proportion problems, she reached criterion and reported that she had been taught how to cross-multiply before, but not when until she learned to "get the whole picture". By the end of intervention, she had streamlined the strategy leaving out steps such as the drawing and worked very quickly. She maintained criterial performance on representation, solution, and answers for all problem types (see Table 4). She attained criterion on all measures of near transfer except answers for relational problems where she did not ensure that the answer met the goal. On far transfer she failed to reach criterion on any measures for relational problems but succeeded in reaching criterion for proportion and two-variable two-equation problems.

S2 also reached criterion for all phases of the three problem types (Figure 7). She made no errors in maintenance or near transfer, correctly completing all representations, solutions and answers (Table 4). On far transfer she reached criterion on proportion and two-variable two-equation problems, but not on relational problems. She did not modify the instructed strategy by combining or omitting steps, but used it as it had been taught at all times. In the final meta-cognitive interview, S2 commented on how word problems had "become fun" and how she could not believe she was saying such a thing.

Table 4

Number of Correct Responses for Students 1, 2, 3 and 4 on Pretest, Posttest, Maintenance, and Near-Transfer and Far-Transfer Measures

S#	Problem Type	Pretest	Posttest	Near Transfer	Far Transfer	Maintenance[b]
		R S A[a]	R S A	R S A	R S A	R S A
1	Relational	0 0 1	4 3 4	5 4 3	3 3 3	5 4 4
1	Proportion	0 0 2	5 5 5	5 5 5	5 5 5	5 5 5
1	Two-Variable Two-Equation	0 0 2	4 4 4	5 5 5	5 5 4	5 4 4
2	Relational	0 0 0	5 5 5	5 5 5	2 3 2	5 5 5
2	Proportion	1 1 1	5 5 5	5 5 5	4 4 4	5 5 5
2	Two-Variable Two-Equation	0 0 0	5 5 5	5 5 5	5 5 5	5 5 5
3	Relational	0 0 0	4 5 5	4 4 4	5 5 5	5 5 5
3	Proportion	0 0 2	5 5 5	5 5 5	5 5 5	5 5 5
3	Two-Variable Two-Equation	0 0 0	5 5 5	5 5 5	5 5 5	5 5 5
4	Relational	1 1 1	5 5 5	5 5 5	3 3 3	4 4 4
4	Proportion	0 0 3	5 5 5	5 5 5	0 0 0	5 4 4
4	Two-Variable Two-Equation	0 0 0	5 5 4	4 5 4	0 0 0	3 4 3

Note. All tasks reported on this table have a total possible score of 5

[a]R = Representation, S = Solution, A = Answer.

[b]Maintenance took place 6 weeks later.

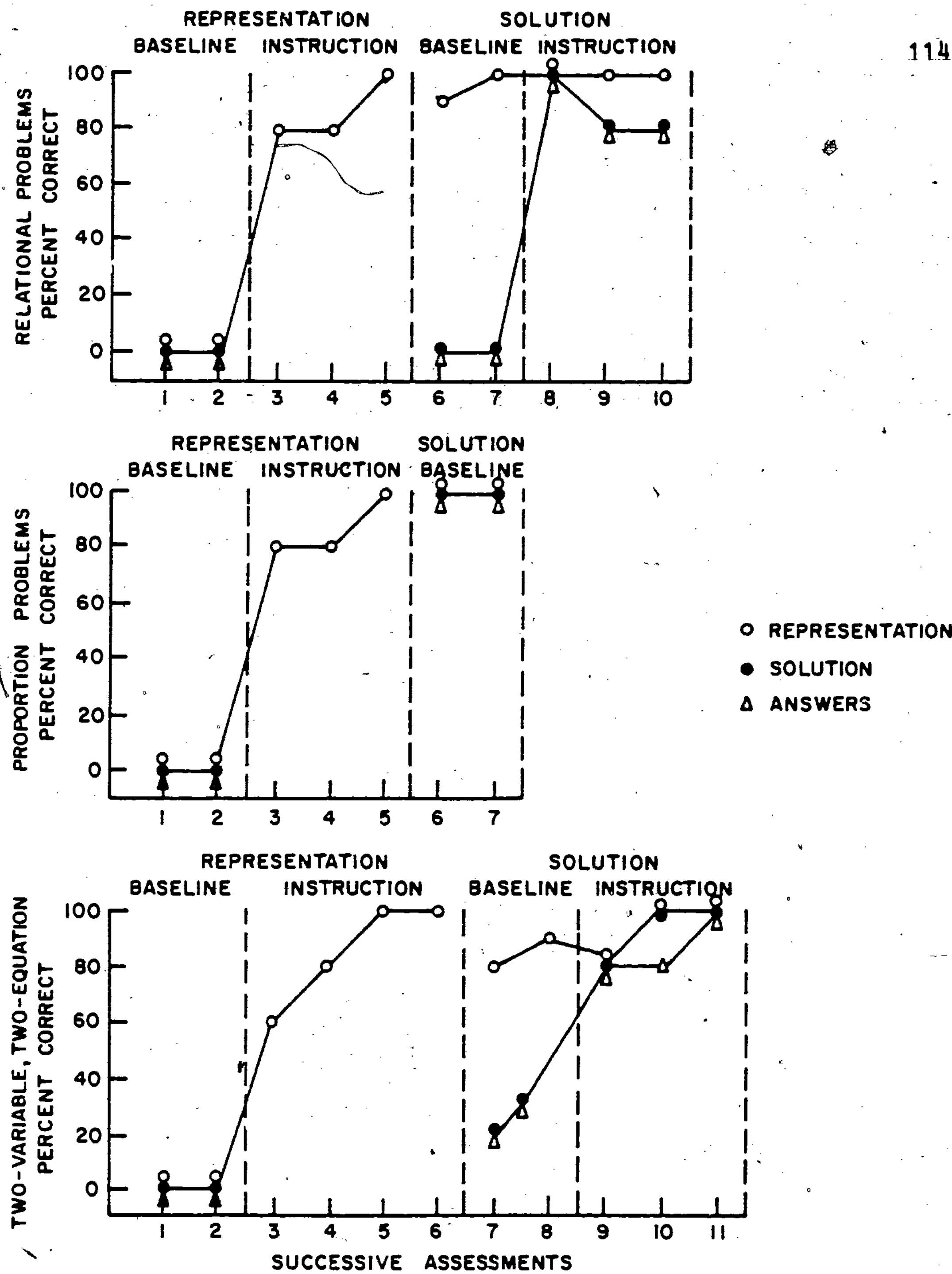


Figure 7

Percent Instructed Problems Correct on

Successive Assessments for Student 2

S3 reached criterion on representation, solution and answers for three problem types, and reached criterion for solution of proportion problems during baseline (Figure 8). She never attained a score below criterion on anything she had been taught, and was proud of this fact. She maintained and transferred all she had learned (Table 4). During maintenance she modified the strategies leaving out steps because she said she was short of time and wanted to finish all the tasks. S3 had a consistently positive attitude toward learning to solve problems, and expressed a desire to be a nurse, "even though I have problems learning in school."

S4 reached criterion on representation, solution, and answers for three problem types (Figure 9). He maintained the three aspects of relational and proportion problems, but only solution for problems in two variables and two equations (Table 4). S4 reached criterion on near transfer for the three types of problems, but did not reach criterion on any far-transfer tasks. This student was so enthusiastic about finally learning to solve word problems that he engaged a tutor and asked the tutor how to do the relational problems from the pretest. As a result he received inflated scores for representation and answers on the baseline for representation in relational problems. At my request he refrained from working on word problems with the tutor during intervention. S4 learned quickly. He was the only student who reached criterion for solution during baseline on relational problems. He was the first to modify the strategies for

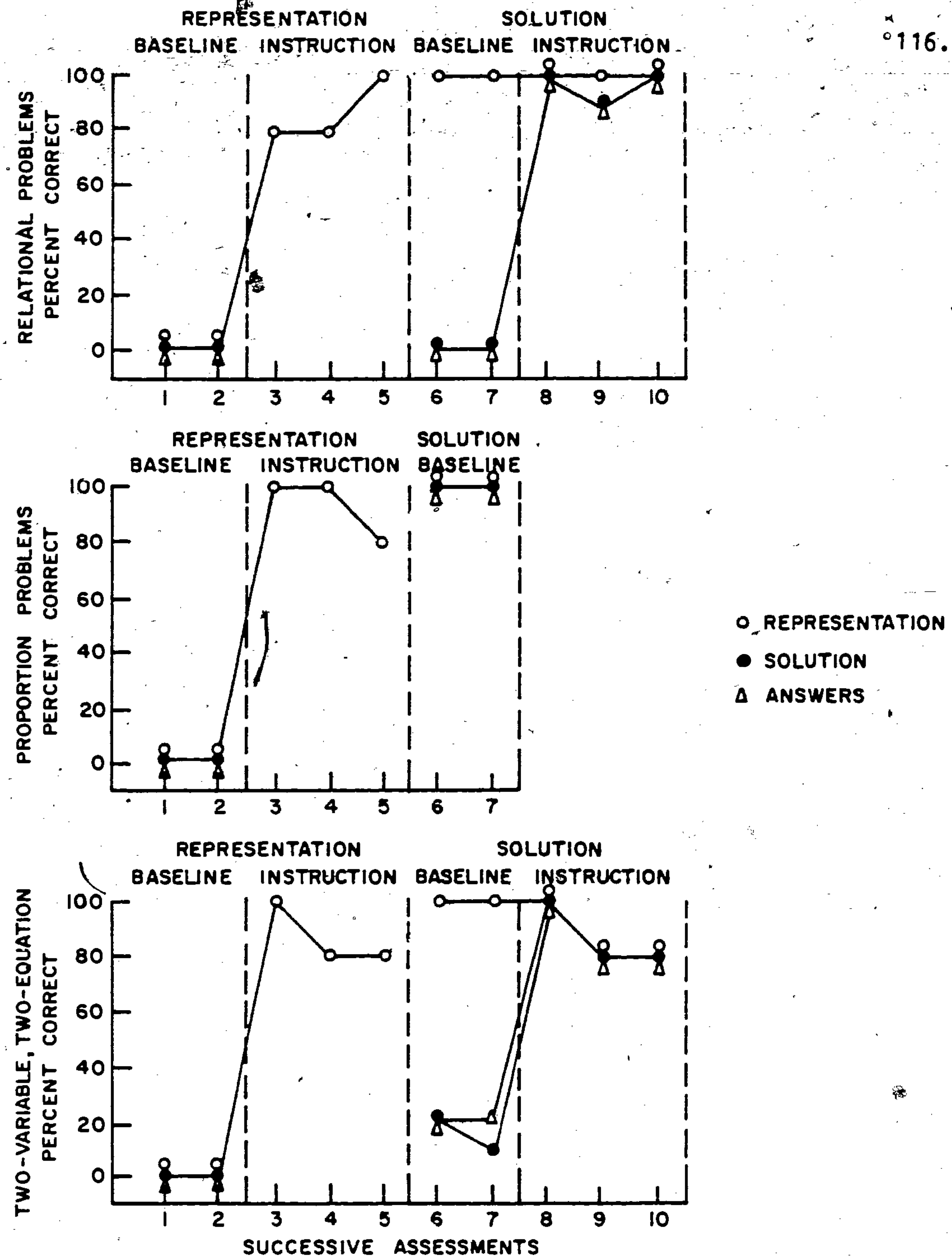


Figure 8

Percent Instructed Problems Correct on

Successive Assessments for Student 3

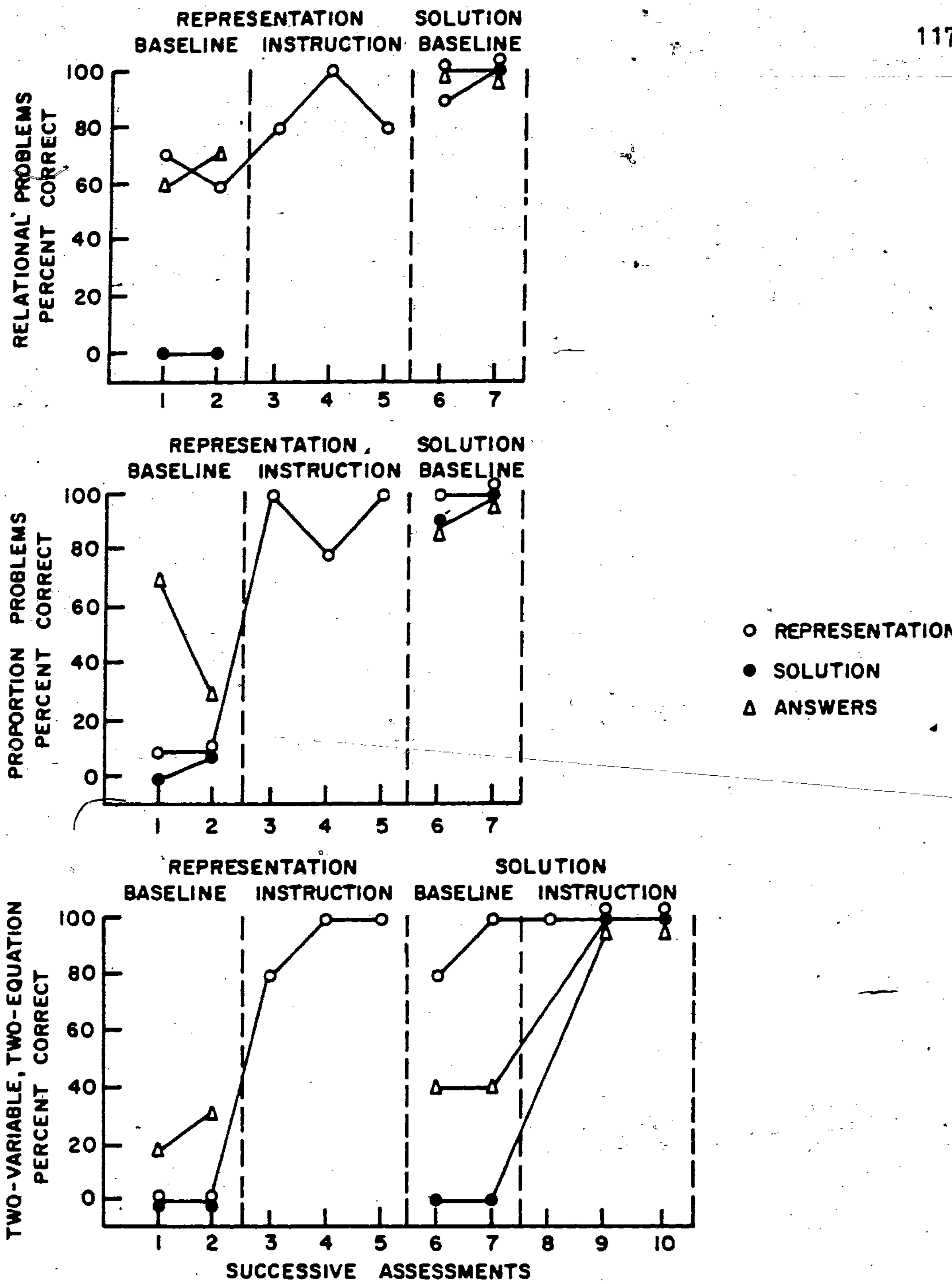


Figure 9

Percent Instructed Problems Correct on

Successive Assessments for Student 4

representation and solution, omitting the drawing, and the statement of givens in representation, and doing up to three steps at once in solution. When semester 2 began, after session #1 for two-variable two-equation problems, S4 was placed in a regular mathematics class. He offered to come to intervention at noon to avoid missing his classes. However, he sometimes confused our two-day schedule. Otherwise, his performance suggests that he would have completed the intervention in less than three months.

S5 was successful with all three problem types (Figure 10). She took great pride in reaching criterion and commented frequently on how much she had learned and how much she enjoyed getting 100% on assessments. The only time she scored less than criterion on instructed tasks, maintenance, or transfer problems was on representation of far-transfer proportion problems (Table 5). Although she developed speed and accuracy, S5 never modified the strategies she had been taught.

S6 worked quickly and reached criterion on all three problem types in spite of a suspension from school in the course of the intervention (Figure 11). She reached criterion during baseline for solution of proportion problems. In far-transfer tasks, she reached criterion for representation, solution, and answers for only proportion problems (Table 5). She worked very quickly with little attention to detail and could not do the far-transfer relational or two-variable two-equation problems. However, she did reach criterion

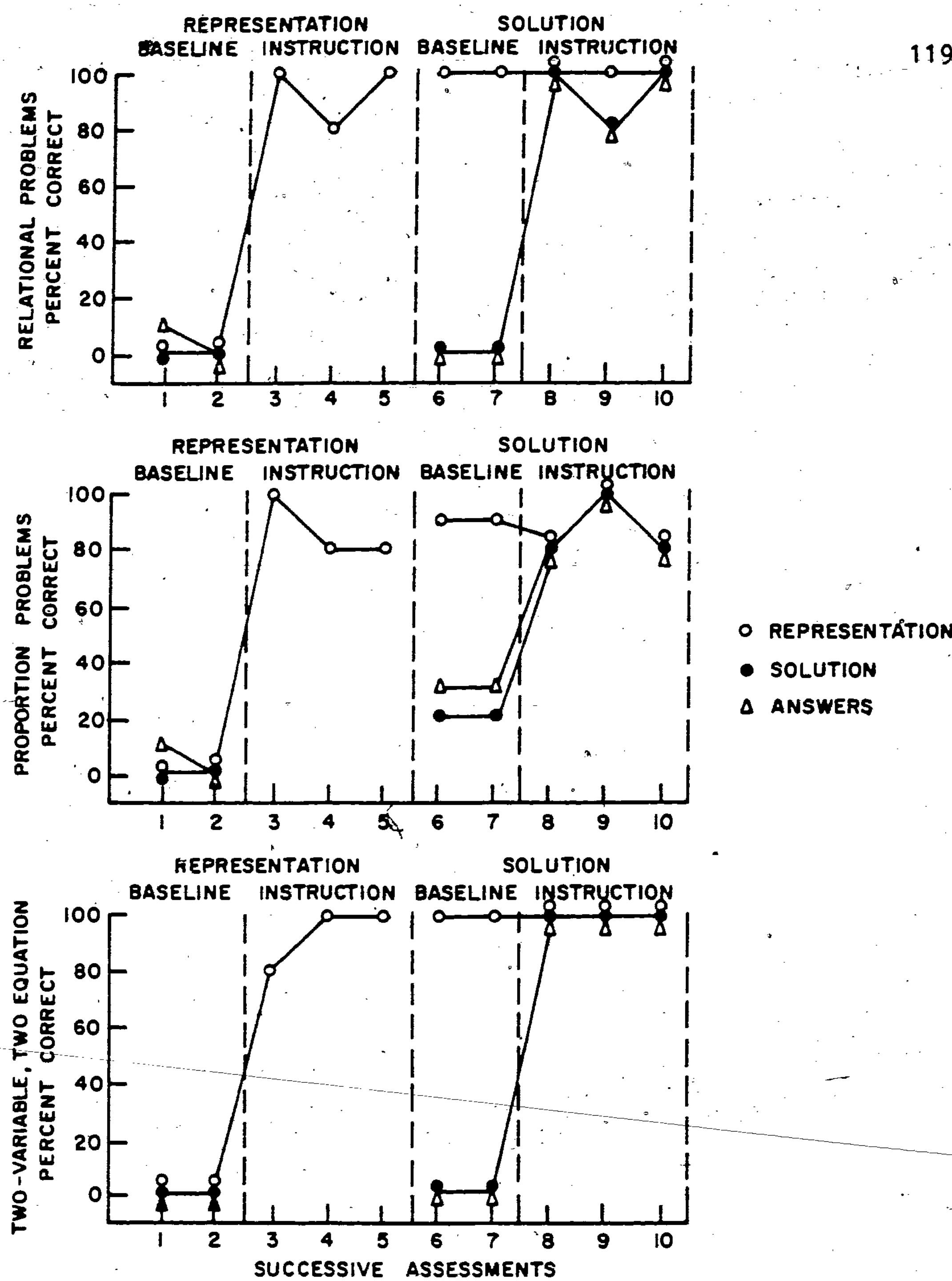


Figure 10

Percent Instructed Problems Correct on

Successive Assessments for Student 5

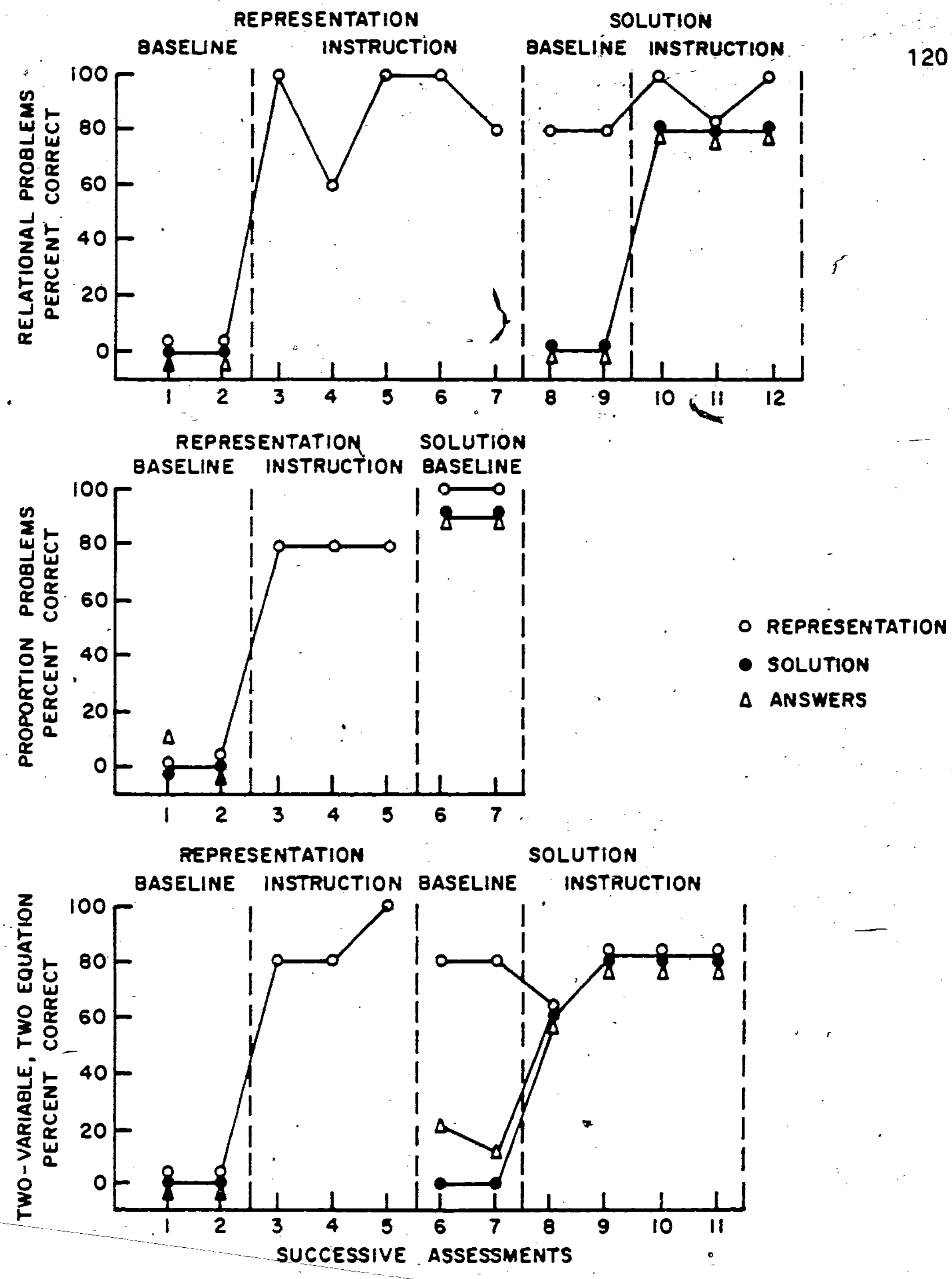


Figure 11

Percent Instructed Problems Correct on

Successive Assessments for Student 6

Table 5

Number of Correct Responses for Students 5, 6, 7 and 8 on Pretest, Posttest, Maintenance, and Near-Transfer and Far-Transfer Measures

S#	Problem Type	Pretest R S A[a]	Posttest R S A	Near Transfer R S A	Far Transfer R S A	Maintenance[b] R S A
5	Relational	0 0 0	5 5 5	5 5 4	5 5 5	5 5 5
5	Proportion	0 0 2	5 4 4	5 5 5	3 4 5	5 4 4
5	Two-Variable Two-Equation	0 0 2	5 5 4	5 5 5	5 5 4	5 5 5
6	Relational	0 0 0	5 5 5	5 5 5	0 0 0	5 4 4
6	Proportion	0 0 1	5 5 5	4 5 4	4 4 4	4 4 4
6	Two-Variable Two-Equation	0 0 0	4 4 4	5 5 4	0 0 0	5 5 4
7	Relational	0 0 0	4 4 4	5 5 3	0 0 0	5 4 4
7	Proportion	0 0 1	5 5 5	5 5 5	5 5 5	5 5 5
7	Two-Variable Two-Equation	0 0 0	0 0 0	- - -[c]	- - -	0 0 1
8	Relational	0 0 1	5 5 5	4 3 3	0 0 0	3 2 3
8	Proportion	0 0 4	5 5 5	5 5 5	5 5 5	5 5 5
8	Two-Variable Two-Equation	0 0 0	0 0 0	- - -	- - -	0 0 0

Note. All tasks reported on this table have a total possible score of 5.
[a]R = Representation, S = Solution, A = Answer.
[b]Maintenance took place 6 weeks later.
[c]Near-transfer and far-transfer tasks were administered only for tasks on which the student received instruction.

on all near-transfer tasks and on all maintenance measures.

S7 reached criterion for representation, solution and answers on two problem types (Figure 12). On the maintenance tasks, he reached criterion on all three aspects for relational and proportion problems (Table 5). For proportion problems, he had 100% for near and far transfer on all of representation, solution, and answers. For relational problems, he reached criterion only on near-transfer representation and solution. He commented while he was attempting the relational far-transfer problems that he was not sure he understood because when he "checked" the problems they did not work. S7 worked very slowly and without confidence in the early sessions, asking for feedback frequently; however, by the end of intervention, he worked quickly and confidently, commenting positively on his own performance.

S8 reached criterion on representation, solution, and answers for two problem types, with criterion for proportion solutions being attained during baseline (Figure 13). He maintained what he had learned for proportion problems but not for relational problems (Table 5). He experienced great difficulty with the transfer tasks. On near transfer he reached criterion on all aspects of proportion but only on representation for relational problems. He obtained zero on far transfer for relational problems and 100% on far transfer for proportion problems. S8 commented frequently during the pretests on his inability to complete word problems. He was easily frustrated and stormed out of the room at one point during the far-transfer task for

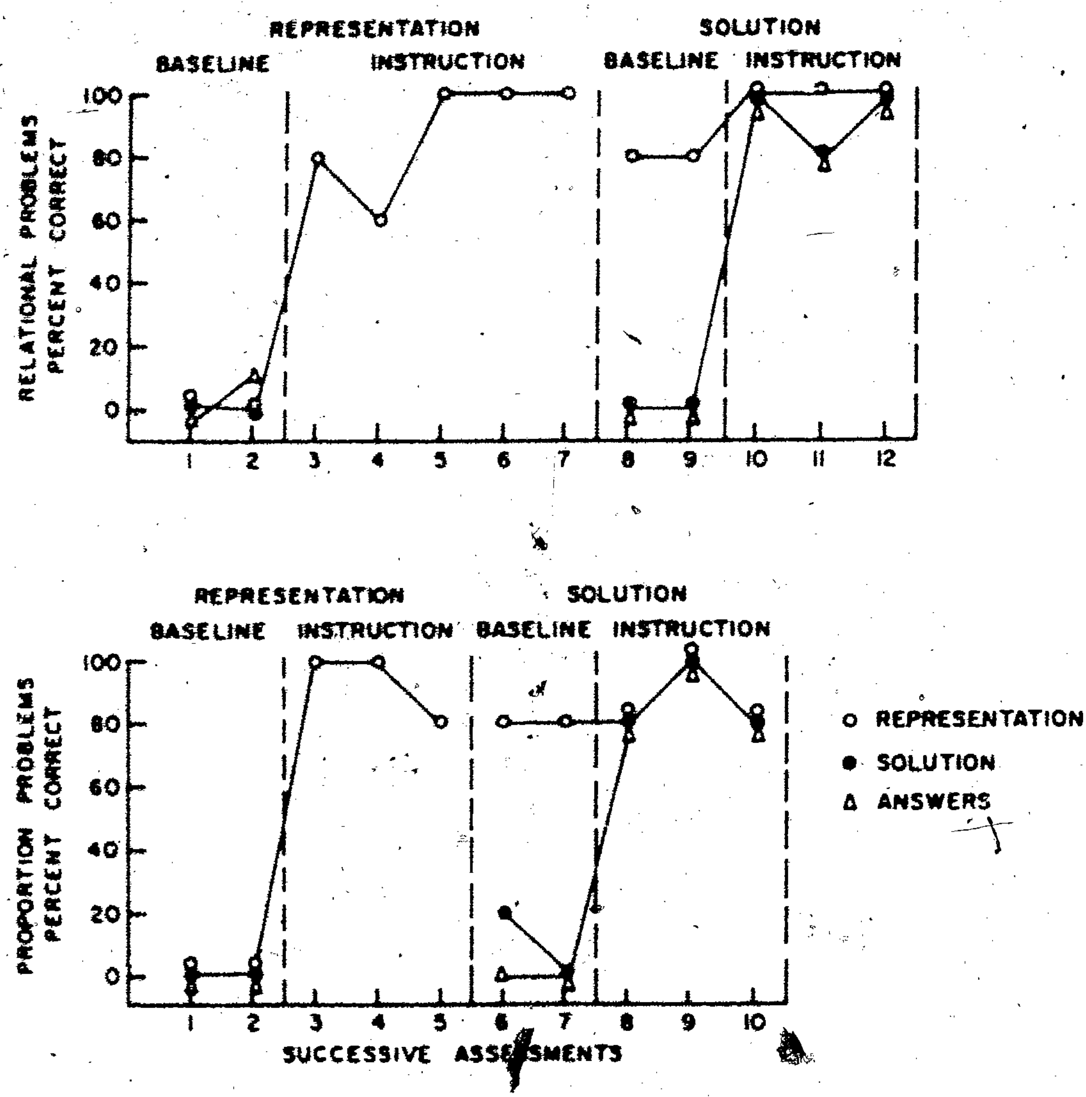


Figure 12

Percent Instructed Problems Correct on

Successive Assessments for Student 7

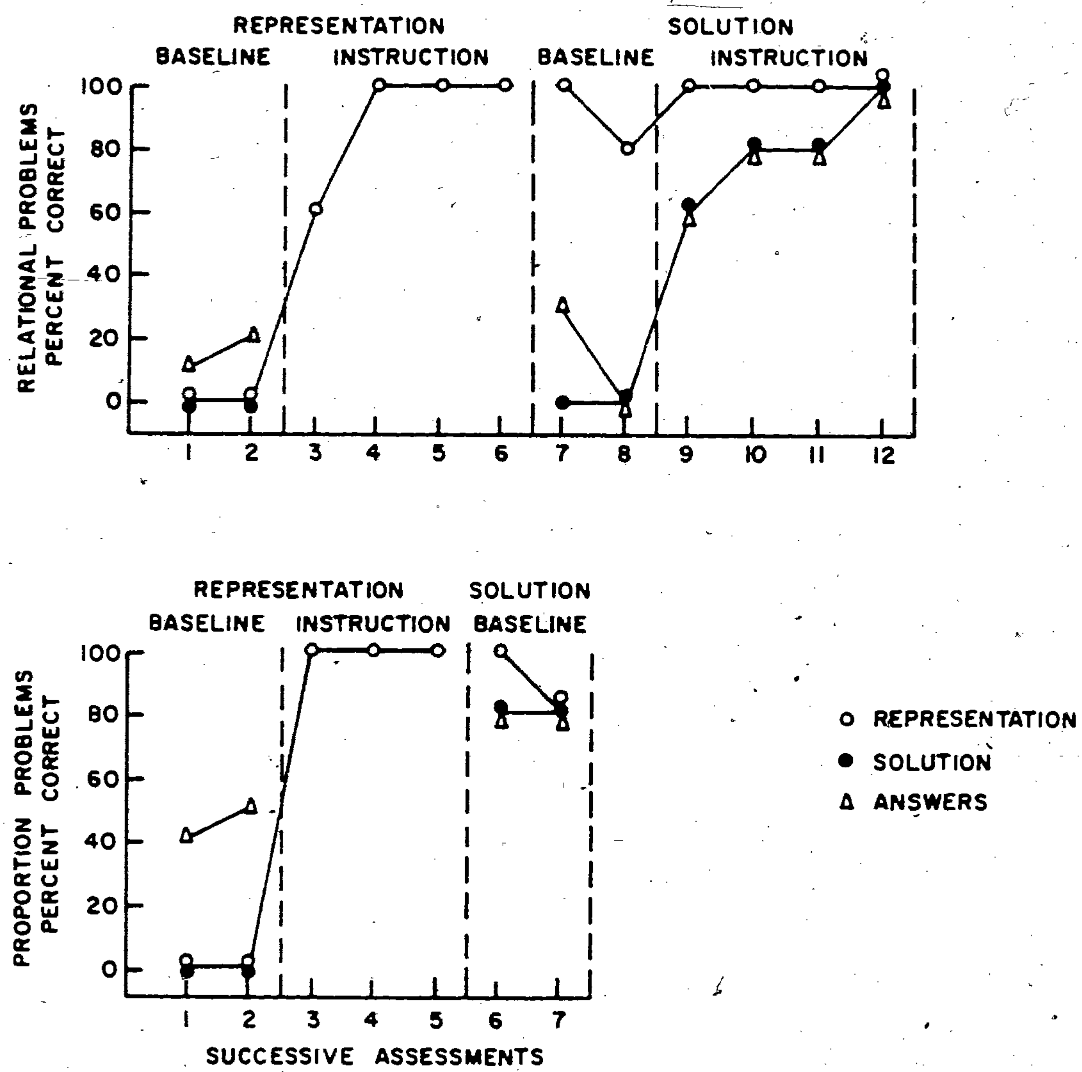


Figure 13

Percent Instructed Problems Correct on

Successive Assessments for Student 8

relational problems. Some days he chose to spend less than 40 minutes in intervention because he did not want to "get behind in [his] other work." This limited the time he spent in intervention but probably minimized his frustration.

S9 reached criterion for representation, solution and answers for two problem types, and attained solution for proportion problems during baseline (Figure 14). He had shown some familiarity with proportion problems during pretesting but said at that time that he had to guess which quantities to put into each ratio. On the maintenance problems he reached criterion for representation, solution, and answers for relational and proportion problems and also scored 60% on all aspects of problems in two-variables and two-equations (Table 6). He scored 100% on all of representation, solution, and answers for relational and proportion problems on near transfer and far transfer. If he had not been ill for three weeks, his prior performance predicts that he would have completed the third problem type. When he returned after his long illness, S9 left out what he considered to be the repetitive parts of representation and reverted to a less detailed listing of steps in solution. Neither of these modifications lessened his understanding or accuracy.

S10 reached criterion on representation, solution and answers for two types of problems, with criterion for solution in proportion reached on the baseline measure (Figure 15). She maintained perfect scores on all aspects of relational and proportion problems (Table 6).

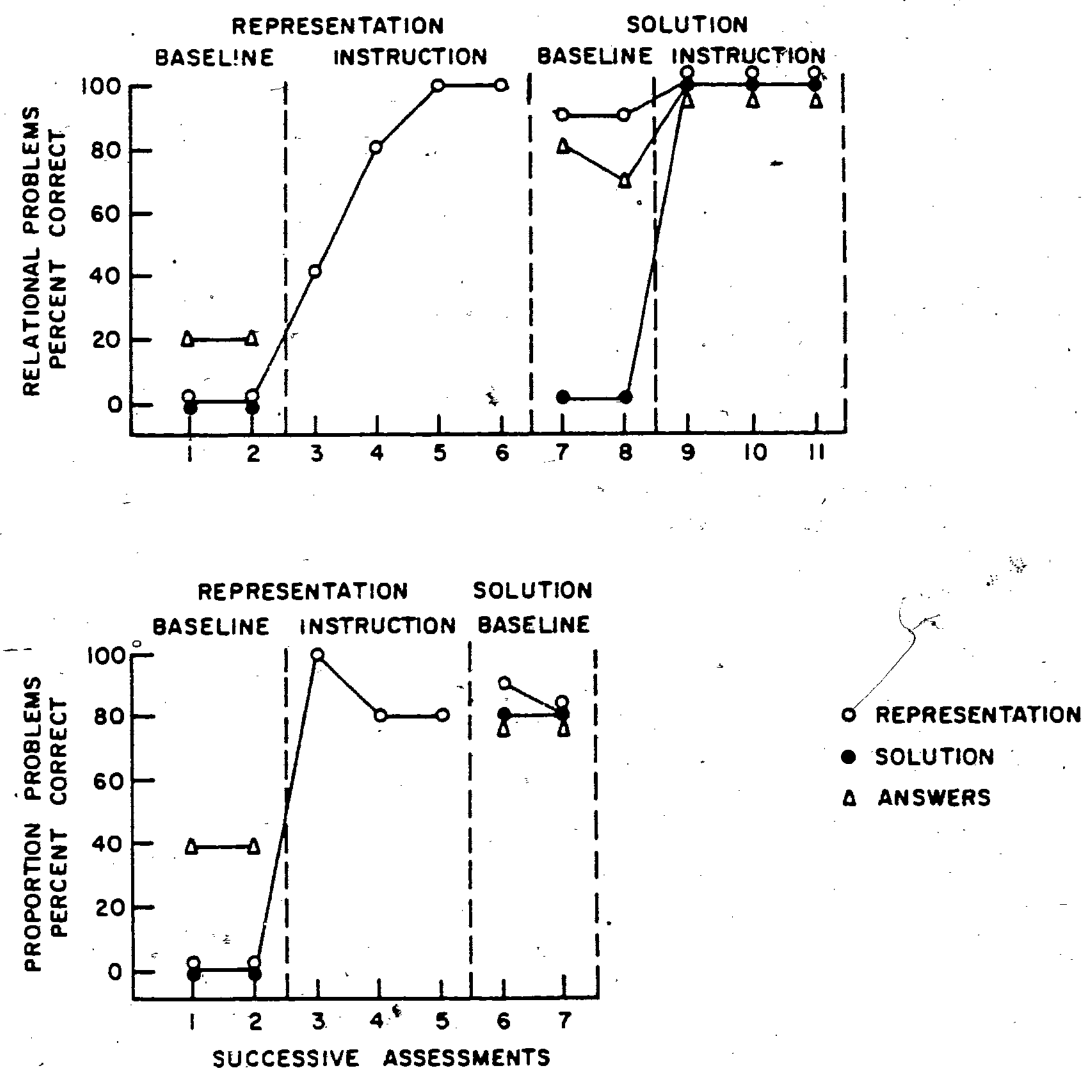


Figure 14

Percent Instructed Problems Correct on

Successive Assessments for Student 9

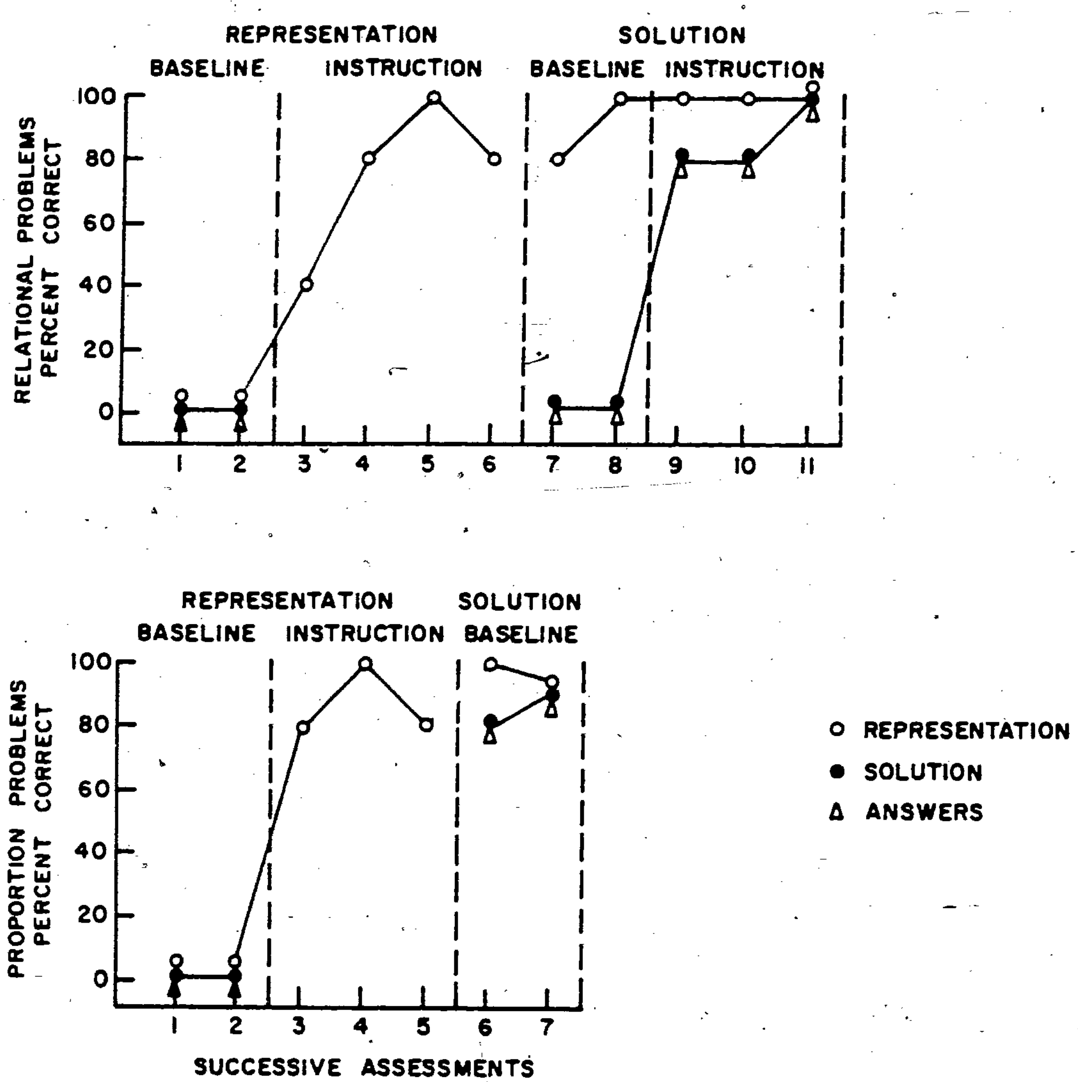


Figure 15

Percent Instructed Problems Correct on

Successive Assessments for Student 10

Table 6

Number of Correct Responses for Students 9, 10, 11 and 12 on Pretest, Posttest, Maintenance, and Near-Transfer and Far-Transfer Measures

S#	Problem Type	Pretest	Posttest	Near Transfer	Far Transfer	Maintenance[b]
		R S A[a]	R S A	R S A	R S A	R S A
9	Relational	0 0 2	5 5 5	5 5 5	5 5 5	4 4 4
9	Proportion	0 0 4	5 5 5	5 5 5	5 5 5	5 5 5
9	Two-Variable Two-Equation	0 0 0	0 0 0	- - -[c]	- - -	3 3 3
10	Relational	0 0 0	5 5 4	5 5 5	0 0 0	5 5 5
10	Proportion	0 0 1	5 5 5	5 5 5	5 5 5	5 5 5
10	Two-Variable Two-Equation	0 0 0	0 0 0	- - -	- - -	3 2 2
11	Relational	0 0 0	5 5 5	5 5 5	5 5 5	5 5 5
11	Proportion	1 1 2	0 4 5	- - -	- - -	0 0 4
11	Two-Variable Two-Equation	0 0 0	0 0 0	- - -	- - -	0 0 0
12	Relational	0 0 0	4 4 4	4 4 4	1 2 2	1 1 1
12	Proportion	0 0 0	0 0 0	- - -	- - -	0 0 0
12	Two-Variable Two-Equation	0 0 0	0 0 0	- - -	- - -	0 0 0

Note. All tasks reported on this table have a total possible score of 5.
[a]R = Representation, S = Solution, A = Answer.
[b]Maintenance took place 6 weeks later.
[c]Near-transfer and far-transfer tasks were administered only for tasks on which the student received instruction.

In addition, she completed some of the problems in two variables and two equations correctly during maintenance, which was the first time she had accomplished this in the entire study. She reached criterion on near-transfer relational and proportion problems and far-transfer proportion problems, but would not try far-transfer relational problems. S10 worked slowly and was prevented from receiving instruction in the third problem type by a suspension from school. She was enthusiastic about the intervention as long as she found the problems easy.

S11 reached criterion on representation, solution, and answers for only the first type of problem--relational (Figure 16). He maintained representation, solution, and answers for relational problems and also obtained 100% on all near-transfer and far-transfer relational problems (Table 6). He worked slowly and hesitantly never completing instruction and assessment in the same session until the last two weeks of intervention. S11 was absent from school for two weeks following the fifth and the ninth assessment sessions. During the second absence he withdrew from school, but agreed to complete the intervention. On the posttest assessment measure he reached criterion for solution and answers on proportion problems which he had not been taught. On the maintenance test he reached criterion for answers on proportion problems. He said that he was trying to apply what he had learned about relational problems.

S12 also reached criterion on representation, solution, and

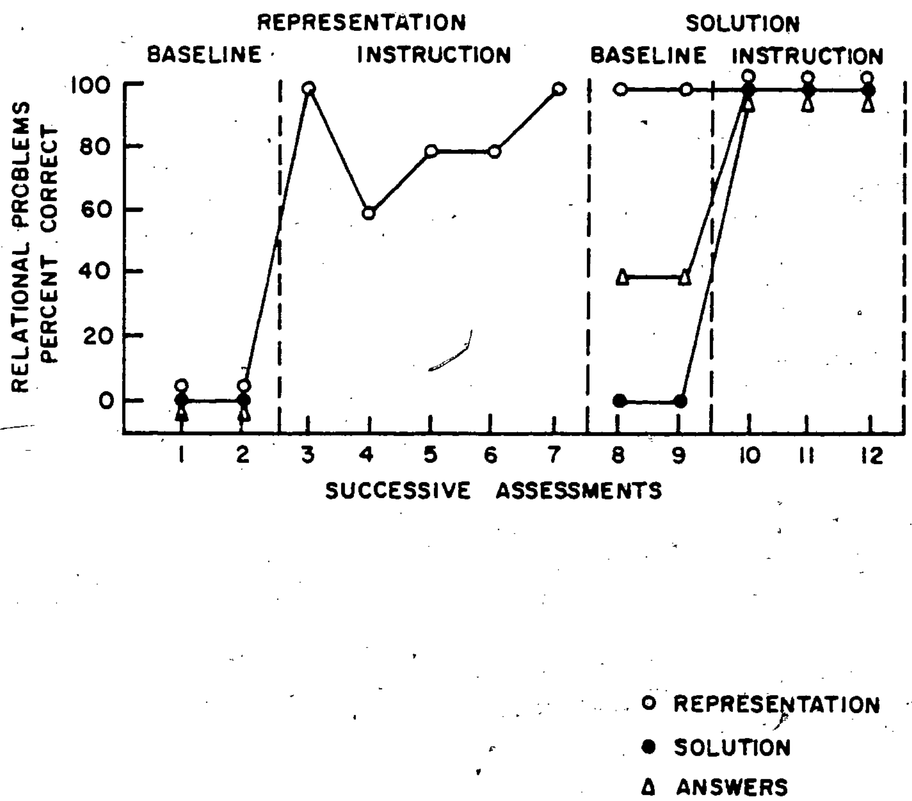


Figure 16

Percent Instructed Problems Correct on

Successive Assessments for Student 11

answers for only one type of problem (Figure 17). She found the representation assessments very difficult and disheartening, and completed eight assessments before reaching criterion on relational problems. Her approach was to try to "remember it all" because it was so difficult to understand. She reached criterion on solution and answers in four assessments. She did not maintain what she had learned; however, she did obtain 80% on all of representation, solution and answers on near-transfer problems completed immediately after posttests (Table 6). S12 did not reach criterion on far transfer. Although she met the criteria for inclusion in the study, the algebraic word problems may have been inappropriate for her. She seemed to lack the necessary prior knowledge, and she was observed to spend little time on-task during intervention and in the learning centre. S12 is the only student who did not reach criterion on any maintenance task.

In summary, the instructed students' ability to represent word problems, obtain problem solutions and correct numerical answers for three problem types increased dramatically following instruction. Two students learned to represent and solve one type of problem, four reached criterion for representation and solution for two types of problem, and six reached criterion on three types of problems. Maintenance data for each problem type are shown in Table 7. Six weeks after the end of instruction, criterial performance was maintained by 10 of the 12 students who had reached criterion on

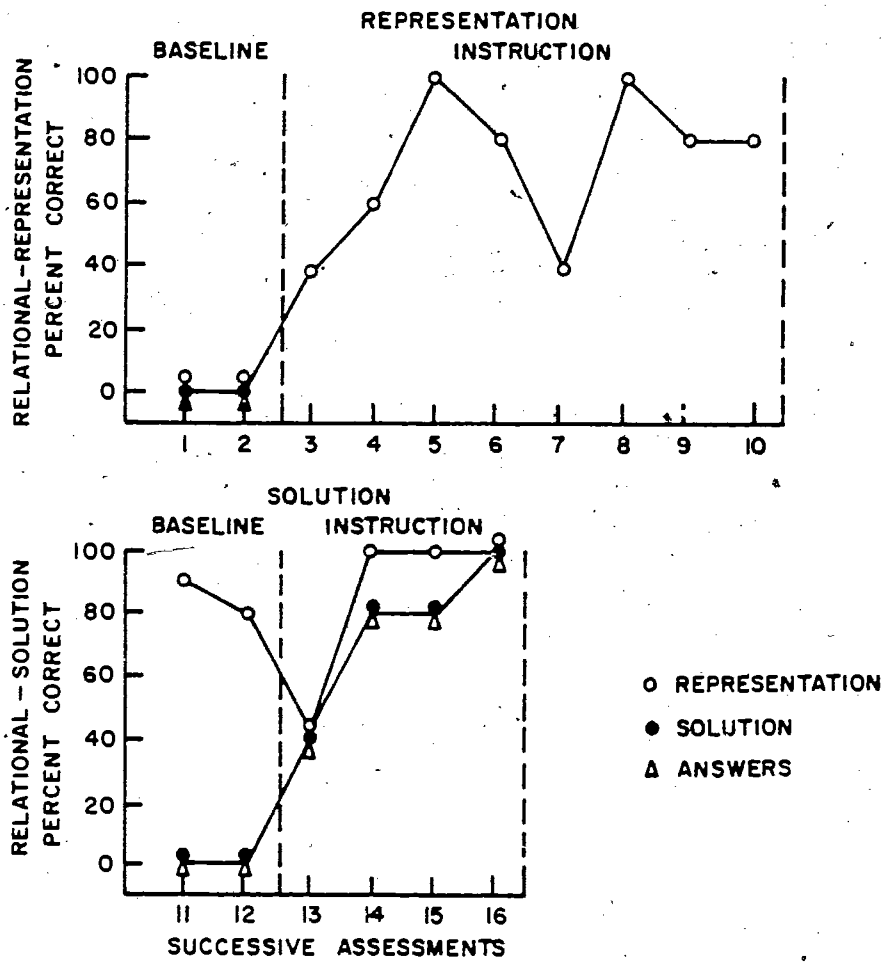


Figure 17

Percent Instructed Problems Correct on

Successive Assessments for Student 12

Table 7

Proportion of Instructed Students Reaching Criterion on Maintenance Problems of Each Type

Type of Problems	Representation	Solution	Answers
Relational	10/12	10/12	10/12
Proportion	10/10	10/10	10/10
Two-Variable Two-Equation	5/6	6/6	5/6

relational problems. For proportion problems, criterion was maintained by all 10 students who had reached criterion during intervention. Five of the six students who had reached criterion for two-variable two-equation problems maintained criterial performance six weeks later. In only three cases out of 28 did a student master a problem type during intervention and not maintain criterial performance on that problem type six weeks later. The students' abilities to do the instructed word problems improved and were maintained over time.

> Did the instructed students show transfer to problem-solving tasks that were similar but distinct from the training tasks? Did they show near transfer, far transfer?

Students received transfer tasks only for the problem types on which they had reached criterion during intervention.

Near Transfer

Near-transfer tasks consisted of problems containing the same mathematical structure as the instructed problems and new surface structures (or contextual details). Table 8 shows the proportion of students reaching criterion on near transfer for representation, solution and answers for each of the instructed problem types. Every student showed near transfer for proportion and two-variable

Table 8

Proportion of Instructed Students Reaching Criterion on Near-Transfer Problems of Each Type

Type of Problems	Representation	Solution	Answers
Relational	12/12	11/12	9/12
Proportion	10/10	10/10	10/10
Two-Variable Two-Equation	6/6	6/6	6/6

two-equation problems. Every student showed near transfer for representation of relational problems. One student did not reach criterion on solution for relational problems and three students did not record the correct numerical answer (consistent with the goal) for relational problems. In summary, the instructed students did show near transfer to problems with the same mathematical structure but different surface structures than the instructed problems.

Far Transfer

Far-transfer tasks consisted of problems containing the same surface structure as the instructed problems and slightly more complex mathematical structure. The relational problems contained two kinds of relational statements rather than one; the proportion statements contained three equivalent ratios, one complete and two incomplete; the two-variable two-equation problems resulted in a pair of equations containing three unknowns, one of which could be obtained by a simple calculation from the information given. Table 9 shows the proportion of students reaching criterion on representation, solution, and answers for each of the instructed problem types. Of the 12 students who reached criterion on representation, solution, and answers for relational problems in instruction, only 4 reached criterion on these aspects in the far-transfer problems. For proportion problems, most students who had reached criterion in instruction reached criterion in far transfer: 8 of 10 on representation, 9 of 10 on solution, and 9

Table 9

Proportion of Instructed Students Reaching Criterion on Far-Transfer Problems of Each Type

Type of Problems	Representation	Solution	Answers
Relational	4/12	4/12	4/12
Proportion	8/10	9/10	9/10
Two-Variable Two-Equation	4/6	4/6	4/6

of 10 on answers. For problems in two variables and two equations, as well, most students who had reached criterion in instruction reached criterion in far transfer: 4 of 6 on representation, 4 of 6 on solution, and 4 of 6 on answers. In summary, the instructed students showed far transfer on proportion and two-variable two-equation problems, but showed little far transfer on relational problems.

> Were changes observed in the students' verbal behavior and understanding while thinking aloud during the course of intervention?

Each student was recorded thinking out loud immediately prior to instruction in representation and immediately following attainment of criterion in solution for each type of problem on which they received instruction. The think-aloud protocols were transcribed and encoded to be rated on ten *a priori* conceptual categories. Six categories contributed to a rating of understanding problem representation. The maximum score for representation was 12: two points for accurate and complete verbalization for each aspect. Four categories contributed to a rating of understanding problem solution, with a maximum score of eight.

Thinking-aloud scores for the verbalizations of individuals are shown in Table 10. For each type of problem, scores for representation and solution increased from before instruction to after instruction. However, the scores on protocols before instruction were similar, for most individuals, across the three types of problems. In

Table 10

Think-Aloud Scores of Individual Instructed Students on Representation and Solution for Three Types of Problems Before and After Instruction

	Relational				Proportion				Two-Variable			
	Pre		Post		Pre		Post		Pre		Post	
S#	R	S[a]	R	S	R	S	R	S	R	S	R	S
1	4	4	11	8	4	4	missing		missing		11	8
2	2	1	10	6	3	0	11	8	------[b]		-----	
3	3	0	9	8	0	4	9	4	------		-----	
4	4	1	11	7	7	4	10	8	7	5	12	8
5	5	0	10	6	----		-----		-----		-----	
6	4	0	12	8	----		-----		-----		-----	
7	4	0	11	8	4	1	1	8	6	4	12	8
8	2	1	11	8	2	4	8	8	-----		-----	
9	10	4	11	7	3	4	9	8	0	5	11	8
10	0	1	10	6	9	0	10	4	-----		-----	
11	0	1	10	8	0	1	9	8	4	0	10	8
12	4	1	11	5	3	0	10	6	4	0	7	8
ps	12	8	12	8	12	8	12	8	12	8	12	8

Note. ps = possible score
[a] R = Representation; S = Solution.
[b] Student did not receive instruction on problem type.

most cases, there was not a gradual improvement on verbalizations before instruction for later problem types. The individual exceptions were S2 and S5.

On representation protocols, there were consistent changes arising following instruction. Before instruction the aspects of representation that were verbalized correctly and completely most frequently were the goal and the knowledge of needed explicit relationships (the information given). Both these aspects required reading and understanding of propositions in the problem statement. There was little evidence of domain-specific problem knowledge. Few students attempted to verbalize even a description of the type of problem, except to identify it by an operation such as "division problem". Before instruction the aspect of solution that was most often verbalized was the choice of an operation and the carrying out of that operation. The equation form was not used and algebraic manipulations were not carried out explicitly. Students lacked this knowledge specific to the domain of algebraic problems. Some used trial-and-error before instruction but none verbalized their reasoning for selecting the next value and none recorded the results of every trial.

The verbalizations immediately following criterion in solution resembled the instructional scripts. Most students were complete and accurate in verbalizing representation except for the frequent omission of the reasoning that supported the choice of problem type.

The next most frequent inadequacy in verbalization of representation was lack of specificity in naming what the unknown would represent. Solutions were verbalized much more fully following instruction with the inclusion of the equation form and explicit steps in solving the equation. All errors in verbalization of solution for relational problems following instruction were in procedure and computation and they consisted either of omissions or errors which the individuals corrected later when the problem resisted solution. For proportion problems, all errors were either a failure to use the equation form or failure to verbalize procedural steps. For two-variable two-equation problems, all students received full scores of eight on their verbalizations of solution.

To summarize, each student demonstrated in the post-instruction verbalizations improvement in understanding of problem representation and problem solution. The major changes in verbalization were the inclusion and utilization of vocabulary and conceptual and procedural knowledge specific to algebraic problem solving. Students spoke of unknowns, variables, and equations. When they gave reasons for their identification of the problems as relational, proportion, two-variable two-equation, the reasons were based on mathematical structure, such as "It has a relational statement, Sam has $18 more than Tom, and I know the total amount they have together, so it must be a relational problem".

Two students (S2 and S5) obtained much higher scores for understanding, before instruction, on later problem types. S2 tried to use all the information she had learned, with relational problems, in her representation and solution of the proportion problem. She also identified it as a proportion problem but could not verbalize a reason. When she obtained an answer of 6300 weeks, she decried it as "too big". Her verbalization before instruction for proportion problems showed much greater understanding than her verbalization before relational problems. On the two-variable two-equation think-aloud protocol before instruction, she again used the steps she had learned, this time to the point of expressing one of the two equations correctly. She used trial-and-error slowly and unsystematically to finally obtain the correct answers.

S5 was also able to represent the proportion problem in her verbalization prior to instruction by reminding herself of each step that had been used for relational problems and altering it to suit the new problem. The other students recognized that the new problem was different when the propositions did not match with those which were familiar. One student said of the proportion problem, "It isn't relational because now you're only doing it with one person and before it was a relationship between two." For most students, this recognition of an unfamiliar mathematical structure derailed attempts to make a drawing and write an equation.

Changes were observed in the students' verbal behavior while thinking aloud during the course of instruction. Most changes were closely related to instruction, reflecting the acquisition of domain-specific algebraic problem-solving language, declarative knowledge, and procedural knowledge. Verbalizations became more abstract, more complete, and more accurate showing increased understanding of representation and solution.

> Did the instruction differentially affect the students' ability to work out relational, proportion, and two-variable two-equation problems?

The study provided data on the question of whether the instructional procedures had the same outcome effects on students' ability to complete three types of algebraic word problems. The answers must be tentative because the outcome effects are confounded by order effects. However, patterns that emerged from the single subject data bear on this question. No student reached criterion on representation for any of the three problems types during baseline assessment. Even after successfully representing previous problem types, students were unable to obtain complete and accurate representations on uninstructed problem types. These data suggest that the representations of all three problem types posed similar difficulties for the students in this study. However, the data on solution suggest that students found the solution for proportion problems to be more straightforward after mastering relational problems and reaching criterion on representation of proportion problems. Four of ten students reached criterion on baseline assessment for solution of proportion problems. On the other hand, no student reached criterion for solution of problems in two variables and two equations on the baseline assessments.

It would appear that instruction in representation specific to the problem type, in combination with the students' prior knowledge of solution (based on instruction in intervention and in their classrooms) had more effect on solution of proportion problems than the other two types. Maintenance data (shown in Table 7) indicate that a slightly higher proportion of students maintained representation and solution for proportion problems following instruction. There was less near transfer and less far transfer for relational problems than for proportion and two-variable two-equation problems (Tables 8 and 9).

All three problem types were learned and maintained due to the intervention, and similarities across problem types have been reported. However, it may be stated tentatively, given the limitations of the study, that there were differential effects, as proportion problems were learned more readily, and relational problems were less likely to be transferred.

Research Questions Relating to Two-Group Data

> Did instructed students and comparison students show similar outcome scores in algebra word-problem solving on the instructed problem types, a multiple-choice problem-solving test, and an open-ended problem-solving test?

Pretests and posttests consisting of the same set of 15 problems (five of each of three problem types) were administered to the comparison and instructed students. Because this was a criterion-referenced measure on which scores were not expected to be normally distributed, non-parametric statistics were employed to answer the questions of whether the two groups had similar outcome scores. Table 11 shows the proportion of students in the comparison and instructed groups who reached criterion on representation, solution, and answers for each type of problem. At pretest no students in either the comparison group or the instructed group reached criterion on any aspect of relational or two-variable two-equation problems. On proportion problems, three comparison students and two instructed students reached criterion for numerical answers. No tests of significance were carried out on the pretest proportions. On the posttest proportions shown in Table 11, Fisher's Exact Test was carried out yielding significance levels. These are shown in Table 12. Differences between instructed and comparison groups were significant on all of representation, solution, and answers for relational, proportion, and two-variable two-equation problems. In every case the instructed group had significantly more students reaching criterion than the comparison group. The similarity of pretest scores and the posttest differences in the number of students reaching criterion permit the conclusion that the two groups did not show similar improvement on the instructed problems. The instructed

Table 11

Proportion of Comparison and Instructed Students Attaining Criterion on Representation, Solution, and Answers on Instructed Problems at Pretest and Posttest

		Pretest			Posttest		
		R	S	A[a]	R	S	A
Relational Problems	Comparison Group	0/8	0/8	0/8	0/8	0/8	0/8
	Instructed Group	0/12	0/12	0/12	12/12	11/12	12/12
Proportion Problems	Comparison Group	0/8	0/8	3/8	0/8	1/8	3/8
	Instructed Group	0/12	0/12	2/12	10/12	11/12	11/12
Two-Variable Two-Equation Problems	Comparison Group	0/8	0/8	0/8	0/8	0/8	0/8
	Instructed Group	0/12	0/12	0/12	6/12	6/12	6/12

[a]R = Representation; S = Solution, A = Answer.

Table 12

Summary of Fisher's Exact Test on Proportion of Instructed and Comparison Students Attaining Criterion at Posttest on Instructed Problems

Type of Problem	Dependent Variable	Significance[a]
Relational	Representation	<.05
Relational	Solution	<.05
Relational	Answer	<.05
Proportion	Representation	<.05
Proportion	Solution	<.05
Proportion	Answer	<.05
Two-Variable Two-Equation	Representation	<.05
Two-Variable Two-Equation	Solution	<.05
Two-Variable Two-Equation	Answer	<.05

[a] *df*=1 in all cases

students showed more improvement than the comparison students.

The three instructed problem types were drawn from the junior secondary curriculum. However, there are many formats on which students are expected to demonstrate their problem-solving abilities. Two assessment measures developed by the British Columbia Ministry of Education represent two of these formats--multiple choice and open ended.

Because this was a pretest-posttest control group design, the data for each of these problem-solving measures were analyzed by means of analyses of covariance (Cook & Campbell, 1979). The covariate employed in each was the pretest score, in one case on the multiple-choice test (B.C. Applications) and in the other case on the open-ended test (Q2). The dependent variable was posttest score, measured on one problem-solving test in each case. Preliminary analyses were carried out in order to ensure that the data met two assumptions required for the analysis of covariance: the assumptions of homogeneity of variance and of parallel regression lines.

Table 13 presents the pretest and posttest means and standard deviations and the adjusted posttest scores for the two groups on B.C. Applications. Pretest scores of the instructed and comparison groups appear fairly similar and quite high. This test was normed on seventh and eighth graders and the placements of the present learning disabled students ranged from grade eight through ten. At the time of posttest, scores among the instructed group had moved closer to the

Table 13

Summary of Means and Standard Deviations for Pretests and Posttests on B.C.A. (Multiple-Choice Problem-Solving Test) and Q2 (Open-Ended Problem-Solving Test)

Group	Time	Test	Mean	Mean (adj)	Standard Deviation
Comparison[a]	Pre	B.C. Applications[b]	14.38		5.04
Comparison	Post	B.C. Applications	15.63	15.64	6.28
Instructed[c]	Pre	B.C. Applications	14.42		4.21
Instructed	Post	B.C. Applications	18.92	18.90	2.64
Comparison	Pre	Q2[d]	3.88		1.96
Comparison	Post	Q2	5.38	5.17	2.50
Instructed	Pre	Q2	3.50		1.45
Instructed	Post	Q2	8.50	8.64	1.45

[a]N=8; [b]possible total=25; [c]N=12; [d]possible total = 13

ceiling of 25. On the other hand, scores of the comparison group had improved only slightly. The adjusted means were 15.64 for the comparison group and 18.90 for the instructed group on the British Columbia Applications posttest.

The analysis of covariance for adjusted posttest scores on the British Columbia Applications test did not meet the required assumptions of homogeneity of variance and of parallel regression lines. A Kruskal-Wallis non-parametric test was applied yielding no significant differences. Inspection of the data suggests that a ceiling effect may have contributed to these findings (Cook and Campbell, 1979).

Table 13 also presents the pretest and posttest means and standard deviations for the Q2 assessment measure and the adjusted posttest means. The pretest scores of the two groups are very similar and quite low as one might expect on a test developed for a tenth grade population. Posttest scores indicate that the instructed group had improved while the comparison group scores were fairly similar to their pretest scores. The adjusted means were 5.17 for the comparison group and 8.64 for the instructed group on the Q2 posttest.

Table 14 is the summary table for the analysis of covariance for adjusted posttest scores on the Q2 assessment measure. This table indicates that the difference noted in the means above was significant, $F(1,17) = 37.79$, $p < .05$. Preliminary analyses indicated that the assumptions of homogeneity of variance and of no

Table 14

Summary of Analysis of Covariance for Posttest Scores on Q2 Open-Ended Problem-Solving Test with Pretest Scores as Covariates

Source of Variation	Sum of Squares	df	MS	F	p
Treatments (adj)	56.90	1	56.90	37.79	<.05
Ss within Treatments (adj)	25.59	17	1.51		

factor-covariate interaction were satisfied. The instructed students had higher posttest scores in algebra word problem solving on the open-ended problem-solving test Q2 than the comparison students.

To answer the research question "Do the two groups have similar outcome scores in problem solving?", the results of the comparisons between the groups on the three relevant measures (instructed problems, multiple-choice problems, and open-ended problems) may be examined using analyses of covariance. In two of the three cases, the tests of instructed problems and Q2, the instructed group had higher outcome scores than the comparison group; however, on the British Columbia Applications multiple-choice test, there was no significant difference between the instructed group and the comparison group.

> Did instructed students and non-instructed comparison students show similar abilities on qualitative measures related to algebra word problem solving, specifically metacognitive interviews, think-aloud protocols, and a problem classification task?

For each of these three measures of thinking related to problem solving, pretests and posttests were administered to the members of the comparison group and the instructed group. Scoring procedures were developed for the metacognitive interviews which yielded interval data with a potential range in scores from 0 to 20. Similarly, scoring procedures were developed to rate the degree of understanding shown in the think-aloud protocols for relational problems.

Understanding of representation was rated out of 12, and understanding of solution was rated out of 8. The findings for the metacognitive interview are presented first, followed by the findings for the think-aloud protocol. Lastly the data for the classification task are presented.

Metacognitive Interviews

The interviews were scored blind by the investigator according to ~~the~~ procedures presented in the section entitled <u>Scoring Procedures</u>. Interrater reliability was established by randomly selecting half of the interviews for each condition (Pretest Comparison, Pretest Instructed, Posttest Comparison, Posttest Instructed) and having these scored blind by an educational psychologist who had been instructed in the scoring procedures. Results of the two scorings were compared and the second scorer was provided with corrective feedback. He then scored the second half of the interviews. Interrater reliability, calculated using Pearson Product Moment correlations on the interview scores, was .95. Analyses were carried out on the scores obtained by the investigator.

Because this was a pretest-posttest control group design, the metacognitive interview data were analyzed by means of an analysis of covariance (Cook & Campbell, 1979). The covariate was the pretest score on the metacognitive interview. The means and standard deviations for the pretest and posttest and the adjusted posttest scores for subjects in each group are presented in Table 15. The scores of the instructed group at pretest, the comparison group at pretest, and the comparison group at posttest were all similar. In contrast the scores of the instructed group at posttest were higher. The adjusted posttest means were 7.67 for the comparison group and 14.39 for the instructed group on the metacognitive interview.

The summary table for analysis of covariance on adjusted posttest scores on the metacognitive interviews is presented in Table 16. The adjusted posttest mean of the instructed group was significantly greater than the adjusted posttest mean of the comparison group, $F(1,17) = 32.38$, $p < .05$. Preliminary analyses indicated that the assumptions of homogeneity of variance and parallel regression lines were met. The instructed students showed more improvement on the metacognitive interview than the comparison students.

Think-Aloud Protocols

The pretest and posttest protocols for a relational problem were scored blind by the investigator. This scoring was done according to the procedures presented in the section entitled Scoring Procedures. As was done for the metacognitive interview, interrater reliability was established by correlating the scores of a second educational psychologist with those obtained by the investigator. Interrater reliability on the scores for understanding representation was .96; interrater reliability on the ratings for understanding solution was

Table 15

<u>Summary of Means, Adjusted Means, and Standard Deviations for Pretest and Posttest Scores on Metacognitive Interviews</u>

	Pretest		Posttest		
Group	Mean[a]	Standard Deviation	Mean	Adj. Mean	Standard Deviation
Comparison (N=8)	6.38	2.13	7.63	7.67	1.69
Instructed (N=12)	6.50	1.57	14.42	14.39	3.18

[a]possible score=20

Table 16

Summary of Analysis of Covariance for Adjusted Posttest Scores on Metacognitive Interview

Source of Variation	Sum of Squares	*df*	MS	F	*p*
Treatments (adj)	216.78	1	216.78	32.38	<.05
Ss within Treatments (adj)	113.80	17	6.69		

.95. The scoring results obtained by the investigator were used in subsequent analyses. A summary of means, adjusted means and standard deviations of these scores is presented in Table 17.

The scores for understanding were analyzed by means of analysis of covariance, as this was a pretest-posttest control group design. Pretest scores served as the covariate and posttest scores as the dependent variable. For understanding representation, the scores of the instructed group at pretest, and the comparison group at pretest, and posttest were similar. The mean of the instructed group at posttest was 10.67 (S.D. = 0.89). out of a possible score of 12. The adjusted posttest means of the comparison and instructed groups were 4.36 and 10.59 for understanding representation. The difference between the adjusted posttest means was significant in an analysis of covariance, $F(1, 17) = 272.66$, $p < .05$ (see Table 18). The requisite assumptions of homogeneity of variance and parallel regression lines were met. The instructed group demonstrated better understanding of representation than the comparison group following instruction.

Findings were similar for the scores on understanding solution. Preliminary analysis showed both assumptions were met. The instructed group at pretest, and the comparison group at pretest and posttest had similar scores. The mean of the instructed group on understanding solution at posttest was 7.50 (SD = 0.67). The possible score was 8. Adjusted means of the instructed and comparison groups were 7.52 and 0.84 at posttest. The analysis of covariance (Table 19) indicated a

Table 17

Summary of Means, Adjusted Means and Standard Deviations for Pretests and Posttests on Ratings of Understanding (Think-Aloud Protocols)

Group	Time	Aspect of Problem Solving	Mean	Mean (adj)	Standard Deviation
Comparison[a]	Pretest	Representation[b]	2.50		1.60
Comparison	Pretest	Solution[c]	1.63		1.51
Comparison	Posttest	Representation	4.25	4.36	0.89
Comparison	Posttest	Solution	0.88	0.84	0.64
Instruction[d]	Pretest	Representation	3.50		2.61
Instruction	Pretest	Solution	1.67		1.40
Instruction	Posttest	Representation	10.67	10.59	0.89
Instruction	Posttest	Solution	7.50	7.52	0.67

[a]N=8; [b]possible total score=12; [c]possible score=8; [d]N=12.

Table 18

Summary of Analysis of Covariance for Adjusted Posttest Scores on Think-Aloud Protocols for Understanding Representation

Source of Variation	Sum of Squares	df	MS	F	p
Treatments (adj)	177.38	1	177.38	272.66	<.05
Ss within Treatments (adj)	11.06	17	0.65		

Table 19

Summary of Analysis of Covariance for Adjusted Posttest Scores on Think-Aloud Protocols for Understanding Solution

Source of Variation	Sum of Squares	*df*	MS	F	*p*
Treatments (adj)	208.68	1	208.68	485.61	<.05
Ss within Treatments (adj)	7.31	17	0.43		

significant difference between the adjusted posttest means, $F(1,17) = 485.61$, $p < .05$. The instructed group was clearly superior following instruction.

Classification Task

Ability to classify problems and identify the basis for the classification is taken as an indication that problem schemata have been acquired. At pretest and posttest comparison and instructional students chose the two problems out of a set of three that they thought were most alike. There were 10 sets of problems. Their score was the number of times that the two problems with the same mathematical structure were selected as most alike. For each group at each testing the mean was expressed as a proportion (see Table 20). The difference was found between each mean and the mean of 0.33 (which could be expected if the students had made their selections randomly). A Z-score was calculated in each case according to the formula:

$$Z = \frac{p-a}{\sqrt{a(1-a)/n}}$$

where p = proportion correct
a = proportion correct if response made randomly
n = number of subjects.

These results appear in Table 20.

For the comparison group at pretest and posttest and for the instructed group at pretest, the mean expressed as a proportion was not significantly different from the mean that would have resulted

Table 20

Summary of Means and Z-Scores for Pretest and Posttest Results on a Problem Classification Task

Group	Time	Mean[a]	Z-Score[b]	Significance
Comparison (N=8)	Pretest	.063	-1.63	NS
Comparison	Posttest	.075	-1.55	NS
Instructed (n=12)	Pretest	.175	-1.16	NS
Instructed	Posttest	.667	+2.46	<.05

[a]Expressed as a proportion of 1; possible score = 10.
[b]Mean that would result from the random selection = 3.3; as a proportion 0.33.

from random selections. On these three occasions, the students were not making their selection of problems that were most alike on a systematic basis. In contrast, the mean for the instructed group on the posttest was significantly different from what would have arisen by chance ($z = 2.46$, $p < .05$).

The reasons the students gave for their selections appear in Table 21. The most frequent justification given by comparison students at pre and posttest and instructed students at pretest referred to the surface structure or contextual details. The most frequent justification given by the instructed students for their posttest choices was the name of the problem type to which they thought the two problems belonged. It is interesting to note that they sometimes mislabelled the problems; knowing the correct name of the problem type was not necessary to selecting by mathematical structure. The students appear to have learned more than verbal labels for types of problems.

In summary, these results indicate that comparison students did not change or improve their classification of algebra word problems, while the instructed students did. The comparison students did not categorize systematically and referred to surface structure at both testings. The instructed students referred to surface structure at the pretest when they did not make their decisions systematically. On the posttest they categorized problems systematically referring to the more sophisticated categorization by mathematical structure.

Table 21

Proportion of Types of Reasons Given for Choices on Classification Task by Instructed and Comparison Students at Pretest and Posttest

	Groups			
	Pretest		Posttest	
Types of Reasons	I	C	I	C
Surface structure: number, money, age, work, distance	.58	.76	.10	.74
Mathematical structure: relationships between elements, name or description of one problem type	.09	.04	.85	.05
Operation: addition, subtraction, division, multiplication	.11	.04	.02	.02
Miscellaneous aspects of problem: e.g., metric, how many, work out the same (no details), etc.	.22	.16	.03	.19

Taking the results across the three qualitative measures related to algebra word problems, it would appear that the comparison and instructed students did not show similar outcomes as a result of their instruction. On the metacognitive interview the instructed students were significantly better in their ability to answer questions about aspects of the task, the strategies and the person (Flavell, 1976) involved in solving word problems. On the think-aloud protocol ratings the instructed students had significantly better understanding of the processes involved in representing and solving word problems. On the classification task the instructed students showed improvement in their ability to categorize problems by using the mathematical structure following instruction. Qualitative changes were also reported in the performance of the instructed students. Following instruction, the instructed students were significantly better than their peers in qualitative aspects of algebra word problem solving.

CHAPTER 5

DISCUSSION

This study merged recent work in cognitive theory in problem solving with intervention research involving the learning disabled. The result was an investigation of the effectiveness of theory-based instruction to improve the algebraic problem solving of learning disabled adolescents.

Instruction was based on cognitive, prescriptive task analysis. Twelve junior high students were taught declarative knowledge and cognitive strategies in two distinct phases (representation and solution) for three types of word problems. None of these three problem types involved the use of a familiar mathematical formula. They were distinguished by mathematical structure unconfounded by cover story (surface structure). This was accomplished by systematically varying the same five cover stories with all three instructed problem types.

A comparison group was familiarized with the structured worksheet used for instruction and assessment. A two-group design was used to compare the improvement of the instructed students with a comparison group on a variety of quantitative and qualitative measures of algebraic problem solving. In addition, a single-subject design was used which facilitated close examination of changes in the instructed students' problem solving during instruction.

Answers to Research Questions

Rather than repeat the answers to the six research questions which were addressed in detail in chapter four, the substance of the answer to each question is briefly restated. These findings are then used to generate implications for research in problem solving, cognitive theory in instruction, and instruction with the learning disabled.

The four research questions addressed by the single-subject design were answered in the affirmative. The instructed students' abilities in representation, solution, and obtaining correct numerical answers increased dramatically following instruction for each problem type. These increases were maintained over time. Moreover, the instructed students showed near transfer to new surface structures for the three problem types. Far transfer occurred less consistently, with relational problems posing the greatest difficulty. The students' think-aloud protocols for representation changed following instruction. There were shifts from reading comprehension of the problems, that is, constructing meaning as in any reading task, to expression and application of knowledge specific to the domain of algebraic problem solving, that is, constructing an algebraically useful representation. For solution, the protocols showed a shift from students' reliance on arithmetic operations and haphazard trial-and-error to their use of algebraic operations and systematic trial-and-error. There was limited evidence that intervention differentially affected the students' mastery of the problem types with relational problems posing the greatest difficulty overall. Because the problem types were always taught in the same order, the difficulty with relational problems could be attributed to their always being taught first.

The two research questions dealing with between-group comparisons demonstrated the superiority of the instructed group. The latter had higher scores on instructed problem types and on an open-ended test than the comparison group, but, because of ceiling effects, not on a multiple-choice test of problem solving. The instructed group also demonstrated better performance and understanding on qualitative measures including metacognitive interviews, think-aloud protocols, and a problem classification task.

Implications for Theory and Research

This study drew on theoretical sources for its framework and design, and its findings reflect on the nature and adequacy of these sources. Hypotheses were drawn from three sources: recent theory about algebraic problem solving and problem isomorphs; the construct of schemata from cognitive instructional psychology; and research on learning disabilities which supplied the constructs of metacognition and guided instruction. The discussion that follows explores the implications of the findings for theory and research in these areas.

Implications for Current Conceptions of Problem Solving

Was There Support for Two Phases of Problem Solving?

The findings of this study provide empirical substantiation for two phases of problem solving: representation and solution. Unlike college-student novices, learning disabled adolescents did not produce adequate solutions to algebraic problems initially (cf. Lewis, 1981). After the learning disabled adolescents had mastered representation of a particular problem type, they were still unable to produce adequate solutions for the same problem type. This was not consistent with Heller and Reif's (1984) finding that it was necessary only to induce excellent representations upon which college physics students would automatically produce similarly high-quality solutions. Such contrasts serve as a reminder that most novice-expert problem-solving studies do not involve true novices, but individuals who have already had years of successful academic experience with complex problem structures and may have automatized many of the algebraic and computational procedures necessary for solution. The learning disabled adolescents in this study had limited and generally unsuccessful experiences with algebraic word problems and the solution of algebraic equations prior to intervention. Hence, the present study provides convincing evidence of the instructability of representation and solution.

It is likely that the learning disabled adolescents in this study had even more to contend with than most other true novices. The extent to which their learning disabilities contributed to their inability to generate solutions following instruction in representation in an empirical question. It could be answered in an investigation which compared normally achieving adolescents to adolescents with learning disabilities in mathematics.

In order to operationalizer Mayer's (1985) recommendations for instruction in problem solving, it was necessary to employ Reif's (Heller & Reif, 1984; Reif & Heller, 1982) work, in teaching representation for physics problems as a model. Mayer's (1985) nonspecific contentions about the necessity of instruction in linguistic propositions and declarative knowledge for representation were supported in a general way. The instruction also confirmed the importance of the acquisition of procedural knowledge which would enable students to act on their knowledge of the problem type and the meaning of expressions such as "consecutive numbers" (Heller & Reif, 1984). Mayer's recommendations for instruction in efficient strategy use and algorithm automaticity received support in the present study. To operationalize instruction in both representation and solution, it was necessary to conduct cognitive task analyses of the knowledge base relevant for each phase, in addition to observing and recording experts thinking aloud. The task analyses yielded the knowledge structures that would be communicated explicitly in the scripts and the procedural knowledge that would be modelled explicitly in the thinking aloud during the intervention. The representations and solutions necessary for competent algebraic solving were induced during practice, providing confirmation of the detailed domain-specific approach taken by Reif and his colleagues. This study went beyond Reif's current published work to demonstrate that functional representations and solutions not only can be induced, but instructed as well. Learning disabled adolescents produced them with only the cues of a structured worksheet, maintained them six weeks later, and showed considerable transfer to new surface structures and some transfer to slightly more complex but related mathematical structures.

In summary, both representation and solution are essential to algebraic problem solving, and both are amenable to intensive instruction that respects the complexity of the domain-specific knowledge and emphasizes the essential aspects of that knowledge.

Does the Concept of Problem Type Need Refinement?

Another current conception of problem solving is that instruction should focus on templates (Mayer, 1981). A template consists of a particular combination of mathematical structure and cover story. For example, according to Mayer, motion problems (within the time-rate family) contain at least three templates: overtake, round trip, and closure. The effectiveness of the present instruction in focussing students' attention on the mathematical structure of the problem suggests that instruction should be be focussed on isomorphs rather than templates. Simon and Hayes (1976) defined isomorphs as problems in which solution paths map directly onto one another in one-to-one fashion. Problem types, as construed in this study, consisted of problems in which mathematical relationships were essentially the same, equations took the same form, and solution paths mapped directly onto one another, while cover stories varied. The notion of isormorph has been refined to reflect the current emphasis on representation. There are more complex variations of each of the instructed problem types, as operationalized in the far-transfer tasks. The students' successes in these tasks with no direct instruction on how to transfer what they had been taught suggests that the variations of surface and mathematical structures may be transferred once the schemata for the mathematical structure has been acquired at the level of isomorph.

Students in this study learned to expect problem types to be distinguishable by mathematical relationships. However, the non-formula problems used in this study are among the simplest algebraic problems in the junior high curriculum, and may not be representative of the formula problems on which Mayer (1981) concentrates. These findings suggest the hypothesis for future research that it would be effective to teach schemata for formula problems that encompass several templates. For example, problems about distance-rate-time, rate of interest, and rate of doing work could be taught as an isomorph for rate in which surface structure varied over a consistent problem structure.

On the basis of data from the present study, naming isomorphs according to their mathematical structure was useful. Some students had difficulty remembering the name "relational" but the "proportion" (sometimes called ratio) and "two- variable two-equation" names were remembered without fail, even at the six-week maintenance test. Students commented that the names helped them to "get the whole picture" because they brought to mind the form the equation would take. One student who had previous negative experiences commented that the problems were easier "now that they are relational" than they had been when they were called age problems. He wondered why they had been "called age problems at all because age doesn't really have anything to do with it." He and several other students continued to have trouble with relational problems about consecutive numbers, confirming Vergnaud's (1982) observation that there are conceptual differences among problems with the same mathematical structure and problem representation. These conceptual differences are probably due to the mathematical meanings of particular domain-specific expressions such as "consecutive numbers". Analyses which would clarify the extent and nature of these conceptual differences should receive more attention in future investigations.

It appears that the isomorph as utilized in this study is an effective and parsimonious vehicle for instruction in algebraic word problems. Naming problems according to their mathematical structure may also be instructive. However, many questions remain for empirical investigation about the feasibility and effectiveness of structuring algebraic curricula in this manner.

What Indications Are There That Students Developed Schemata?

Schemata are hypothetical constructs representing typical abstractions of complex concepts. Information-processing theory posits that instruction which would enable individuals to acquire schemata would facilitate access to knowledge (Thorndyke & Hayes-Roth, 1979) and reduce the load on short term memory (Mandler & Johnson, 1977). What indications are there that students might have developed schemata for the instructed problem types based on mathematical structure instead of either contextual details or a combination of structure and context?

The strongest support for this hypothesis is supplied by data from the classification task. Students were asked to identify which two of three problems were most alike, and the correct answer was the pair with the same mathematical structure. Following instruction, students systematically chose problems with the same mathematical structure, and correctly identified this as the reason for their choices. On the pretest, instructed students and comparison students were unsystematic in their classification of problems and tended to give contextual details as their reason for their choices. Comparison students persisted with these behaviors on the posttest. A trend was observed in the posttest data for instructed students: the two students who learned one problem type averaged 2.5 correct choices, the four students who learned two problem types averaged 6.5 correct choices, and the six students who learned three problem types averaged 8.1 correct choices, out of a possible 10 correct choices. These data suggest that the more problem types students learned, the more they based their classification on problem structure.

Correct classification in the task ranged over all three problem types although only half the instructed students had learned all types. In 85% of the total 120 cases (10 choices made by 12 students), they mentioned the problem structure or type of problem as the reason. However, in one-third of the cases where they referred to structure, they named the problem structure incorrectly. Frequently students who had not been taught two-variable two-equation problems called them relational, and volunteered that they knew they were not relational problems, merely similar. These classification task data enable one to infer that students who received extensive explicit instruction in problem representation and solution appear to have acquired schemata for mathematical problem types based on structure.

Additional support is garnered from the instructed students' high scores on the posttest and the maintenance test where problems of the three types were in random order. Students tended to identify problems of the types which they had learned and solve these immediately, and to identify the unfamiliar which they left to last. They were also able to identify near-transfer and far-transfer problems as altered instances of the instructed problem types. "Like relational only harder" was a frequent description of relational far-transfer problems. Many students did not appear to notice that the cover stories were changed for the near-transfer problems, confirming that their schemata were based on the mathematical structure independent of cover story.

Evidence from this study, which could be interpreted as inconsistent with the view that the students developed schemata based on mathematical structure, came from the post think-aloud protocols. Students frequently did not state spontaneously reasons for their identification of the problem type, although they usually could if asked at the end of the protocol. There appear to be at least two plausible explanations, one methodological and one psychological. The methodological explanation is that there was no cue on the worksheet for giving reasoning about problem type, while the other aspects of representation all were cued, although reasoning had been modelled in the thinking aloud. The psychological explanation arises from the theoretical writings of Ericsson and Simon (1984) who argue that reasons are not heeded or accessible in short term memory and must be reconstructed. It is impossible to determine precisely why there was a lack of explicit verbal reasoning given for classifications of problem type, but it appears that whatever the cause, it is not sufficient to reject the inference that learning disabled students were able to construct schemata for algebraic problem types based on mathematical structure.

To provide a contrast, a composite sketch of students' schemata prior to instruction can be inferred from their answers to pretest problems, think-aloud protocols, classification tasks, and metacognitive interviews. From these data, it can be inferred that they viewed algebraic problems as simple one-operation arithmetic problems made up of "words and numbers", about age, distance, or some other topic. They viewed them as problems requiring the difficult selection of an operation such as addition or multiplication. They appeared to view these problems as a reading comprehension task for which it was essential to construct meaning. They were usually able to write down or say the goal and the given information for the representation and a proposed arithmetic operation for the solution.

On the whole, preinstructional students showed little domain-specific algebraic knowledge and usually named the problem type by operation on the structured worksheet, and by surface structure on the classification task. The comparison students did not appear to construct new schemata based on mathematical structure. On the posttest, they did complete more of the structured worksheet but had not changed from viewing the problems as arithmetic problems to viewing them as algebraic problems requiring the use of unknowns and solution of equations, as the instructed students had.

The findings of this study are consistent with other investigations showing that experts in mathematics (Schoenfeld & Hermann, 1982) and physics (Chi, et al., 1981) may be inferred to have acquired schemata based on underlying problem structure and principles. There have been reports (Krutetskii, 1976; Silver, 1981) that less competent mathematics students classify problems erroneously if they contain "cover stories" that trigger a specific schema. In this study, students with a history of poor performance in mathematics classified items in a sophisticated way after extensive instruction. There is considerable evidence that the learning disabled students in this study formed and used schemata for algebraic problem types based on mathematical structure.

Implications for Metacognitive Research

Did Metacognition Improve as Problem Solving Improved?

A major component of recent interventions that successfully increased the reading comprehension (Wong & Jones, 1982) and problem solving abilities (Montague, 1984) of learning disabled adolescents has been self-questioning. It has been hypothesized that self-questions are effective because they focus attention on the essential aspects of the tasks and strategies, and make it easier for learning disabled adolescents to monitor their own performance (Wong, 1985b). This means that such strategies should lead to increased metacognition: knowledge about and awareness of cognition, and regulation of cognition. Metacognitive knowledge is usually thought to be about the task, strategy, and person (Flavell, 1976). In this study there is direct evidence that students increased their awareness of problem solving as a consequence of intervention. There is also indirect evidence that they increased their regulation and monitoring of problem solving.

The evidence that students increased their awareness and knowledge about problem solving comes from the metacognitive interview questions administered to the instructed and comparison students before and after intervention. The higher scores obtained by the instructed students at posttest are a clear indication. Closer examination of the pretest interview transcripts for both groups shows that the highest scores were obtained on the two questions that were general in nature and required no domain-specific knowledge. These questions were: "What do you do if you don't know what a word means in a word problem?" and "What is there about a word problem that makes it easy to do?" Answers to the question about strategy to use if you don't know what a word means, were scored the same as in metacognitive interviews about reading--two points for any two comprehension strategies from a list of such strategies as: skip it, sound it, use the context, etc. Answers to the question about nature of task that makes word problems easy, were scored the same as one would for any arithmetic word problem--two points for a combination of two items from a list including: easy numbers, short problem, no hard words, etc. The general nature of both questions is apparent. The least complete, accurate, and sophisticated answers on the pretest interview were to the three questions that called for considerable domain-specific knowledge about algebraic word problem solving and its two phases: representation and solution: "What is the hardest part of doing any word problems for you?" (person), "What do you do if you don't get the 'whole picture' or the 'whole meaning' of a word problem?" (strategy) and "What are mathematics word problems?" (task). The domain-specificity of these questions can be contrasted with the general nature of the questions discussed above.

The comparison group's posttest results were the same as their pretests. The instructed students, however, achieved even higher scores for the more general questions about what to do if you do not understand something and what makes a word problem easy. These were still their best two answers. Their most improved answers were to the questions that demanded that the student engage in some task analysis and show knowledge of the relative importance of mathematical structure and of other aspects such as surface structure. These questions were: "What would help you become a better problem solver?" (person), "Are some parts of a word problem as it is written down, more important than others?" (task), and "What do you do if you don't get the 'whole picture' of a word problem?" (strategy). After instruction the learning disabled adolescents showed they were more knowledgeable about person, task and strategy aspects of algebraic word problems.

Metacognitive awareness improved with cognitive problem solving. Before instruction, instructed students and comparison subjects received low scores and showed lack of knowledge and understanding about both metacognitive and cognitive aspects of problem solving. On the posttests the instructed students were significantly better than their peers in the comparison group who received only familiarization with the worksheet. The instructed students were better in metacognitive and cognitive aspects of problem solving following intervention. It is impossible to pinpoint the exact time in the intervention at which this knowledge was acquired; however, it did take place in the course of instruction which emphasized declarative knowledge, think-aloud modelling, and self-questioning. The latter two elements have been recommended for increasing metacognition (Meichenbaum, et al., in press).

There was no direct assessment of metacognitive regulation in this study. Forrest-Pressley and Waller (1984) have identified the dangers of assuming regulation from post-hoc interview data. They found that some students who expressed awareness of metastrategies in reading did not use those strategies to regulate performance. They called these students "mimics", assuming they were saying what they had heard the teacher say without understanding it. There may, however, be a need for direct, explicit instruction (for instance, modelling by thinking aloud) to enable production-deficient students to proceduralize their declarative knowledge about strategies.

Forrest-Pressley and Waller's (1984) investigation was not an instructional study, so they provide no data to answer this question. There are data in the present study to suggest increased metacognitive regulation and monitoring. Students who made errors in the posttest think-aloud protocols frequently did the steps of algebraic manipulation in the wrong order or missed one of the steps. They came back and corrected their error after their numerical answer would not "check". This monitoring, taught in the solution phase and used extensively after instruction, was absent from pretest think-aloud protocols. Similarly, the attention given to writing a sentence containing the final answer and looking back to insure that the answer met the goal of the problem could be taken as an indication of sophisticated metacognitive regulation.

In future studies, it would be informative to assess metacognitive awareness with an interview and corresponding metacognitive regulation with tasks like the hypothetical ones posed in the interview questions. The consistency between awareness and regulation at various points throughout intervention would illuminate the nature of the development of metacognition.

Implications for Intervention Research With the Learning Disabled

How Applicable is Theory Drawn from Instructional Psychology?

One could ask about the generality of the present findings, given the fact that the students were learning disabled. In recent years several writers (e.g. Cawley, 1985; Wong, 1979a, 1979b) have called for the adoption of theory to guide research in learning disabilities. It can be argued that existing theories in established fields generate the framework, methods and hypotheses necessary for such research.

It has not been shown that the learning disabled learn in a way that is fundamentally different from other individuals; rather, learning disabled students appear to be less efficient learners (Reid & Hresko, 1981). Research and instructional approaches have concentrated on executive or metacognitive control (e.g., Wong, et al., in press) since the inefficiencies of the learning disabled are often attributed to production deficiencies, especially in the use of strategies. One critical aspect of a production deficiency is that the person can use the behavior if instructed to do so (Brown & Campione, 1984). Instructional studies have been characterized by direct instruction and considerable practice of efficient cognitive strategies.

Instructional studies have focussed recently on teaching complex knowledge in particular domains such as mathematics problem solving (Montague, 1984) and junior high social studies (Wong, et al., in press) to the learning disabled. Such studies acknowledge the role of prior knowledge and the active construction of knowledge by learners. This research is consistent with current conceptions of cognitive approaches to instructional psychology (e.g., Marx, 1983; Resnick, 1981). Resnick has argued that instructional psychology has been characterized by a shift to the study of complex cognitive tasks, interactionist assumptions emphasizing the mental constructions of the learner, and a growing interest in the role of knowledge in human behavior. This study demonstrates in a convincing manner the effectiveness of enabling students to construct knowledge, in addition to teaching them cognitive strategies, when they are tackling complex cognitive tasks.

This study showed the applicability of existing theory in instructional psychology and current conceptions of problem solving to the learning disabled population. While it is likely that the learning disabled adolescents needed more demonstrations and practice than would have been necessary with normally-achieving adolescents, the cognitive-based instruction was effective. Students not only learned what they were taught, but their general algebraic problem solving ability improved, and they demonstrated maintenance and transfer. As was expected, the concept of metacognition was valuable in understanding the knowledge acquisition that occurred. Also, changes in problem solving could be understood as acquisition and accessing of schemata for various problem types.

Merging hypotheses, methods, and theoretical frameworks from cognitive instructional psychology with guided instruction (which has been shown to be effective with the learning disabled) yielded valuable information about theory and practice. There may be an advantage to using learning disabled individuals in cognitive instructional research. Because they learn like others, only more slowly, it is possible to observe the acquisition of knowledge structures and cognitive processes, as if in slow motion. Whereas, normal learners might show automaticity almost immediately, these students do not.

The benefits of intervention studies with such populations are many--adolescents acquire cognitive competencies which help them reach their academic potential, instructional technology for complex cognitive tasks is enhanced, and researchers acquire a "window" on the development of expertise.

Limitations of the Present Study

This study, like any intervention by one instructor, on particular learning tasks, with a small number of students, has limitations. Therefore, generalizations should be made with caution until replications have been conducted.

All the instruction was done by one experienced special educator, the investigator, who had carried out extensive task analyses on the problems used in the study. The robustness of the instruction will be confirmed when it has been used successfully by other special educators. Three algebra problem types and five cover stories were used. Analyses indicated there were some differences in the effect of the instruction as a function of the problem types. Observation suggested that some cover stories, such as those using the term consecutive numbers, involved more difficult concepts than the others. The instructional procedures may not be equally effective for all algebraic problems. For example, formula problems which tend to confound mathematical structure and cover story may not be as easily taught with instruction that emphasizes schemata based on mathematical structure. In this case, when it is almost impossible to separate the two aspects, students may attribute problem type to the more apparent cover story rather than the complex mathematical structure. Another limiting factor was the small number of subjects and the heterogeneity that exists in any learning disabled sample, despite the criteria used for inclusion. To generalize to learning disabled adolescents generally, it will be necessary to replicate this study with other learning disabled groups.

The implications of this study for normal populations must be drawn tentatively, although it is likely that the instructional procedures would be effective with normally-achieving adolescents. However, explicit and extensive instruction, such as used here, might be unnecessary with normally-achieving adolescents. Replication with normally-achieving adolescents instructed in groups would require some adaptation of the procedure and materials. Such adaptations would include provision for individual practice within the context of a group. For example, students would have to take turns thinking aloud during practice. Modifications of scripts would be required to ask questions of a number of students during the presentation of declarative knowledge in order to hold everyone's attention. If individuals were required to meet mastery criteria, it would be necessary to provide for variable amounts of practice. Marzola (1985) provided a model in her successful adaptation of Nuzum's (1983) instruction for teaching one- and two-step word problems to groups.

In a study with such complex multi-faceted instruction, it is difficult to ascertain which elements were essential to success and which were merely contingent. To answer such questions, it would be necessary to design comparative studies like Heller and Reif's (1984) in which the contribution of a particular element is evaluated by inducing representation with and without the element.

Lastly, no effort was made in the present study to explore motivational aspects of long-term intervention, such as the students' attitude to the instructor, other interpersonal factors, and the role of cognitive motivational constructs such as self-efficacy. Although no data were collected, it was apparent that the students were willing to sustain effort over a long time. Two students who had been suspended from school and one who withdrew from school during the course of the study never lost interest in completing the intervention. Given the current design with its emphasis on cognitive factors, it is impossible to determine the role motivation played in the outcome of the study.

Despite these limitations the long-term intervention was conducted in as rigorous a fashion as possible. The findings are encouraging regarding the possibilities of instruction for the learning disabled and enlightening regarding the application of theories in instructional psychology and problem solving.

Recommendations for Future Research

The improvement in problem solving ability, understanding, metacognition, and the acquisition of schemata for problem types suggest that further research is warranted. The willingess of the teachers and students to cooperate in an intensive five-month venture imply that they viewed the intervention as valuable. It appears that the application of current principles of instructional psychology and conceptions of problem solving is appropriate and fruitful in school settings.

There are two levels at which further research might be done and several important questions to be asked at each level. At the macro level, it is important to demonstrate the adaptation of the material and procedures to small group instruction, and ultimately to whole-class instruction if the findings are to have general relevance for instruction in algebraic problem solving. At the micro level, it is essential to examine the development of individual expertise more closely. The present study tapped the potential of single-subject design, gathering think-aloud protocol data and interview data, to a greater extent than most cognitive interventions. The acquisition of schemata and cognitive processes, however, should be subjected to more fine-grained analysis with more frequent collection of qualitative data, and closer ties among measures of knowledge, metaknowledge, and regulation.

Many questions that remain to be answered have already been alluded to. It appears there are two major types of questions to address. There are questions about variables embedded in the design of this study and questions about links with other theoretical constructs relevant for intervention.

Many features of this study did not vary systematically over the widest possible range. One of these is the domain knowledge. What is the role of representation in areas of mathematical problem solving other than these three types of word problems? Studies on the effect of representation on geometry problem solving (Greeno, 1978) and the solution of simultaneous linear equations using a microcomputer (Lesh, 1985) suggest that the role of representation extends beyond the boundaries of the present study. Extension to both simpler word problem types (such as one-step and two-step problems) and to formula algebraic problems would substantiate the pervasive role of representation in instruction of mathematics.

Studies extending the range of problems and the age of subjects could be carried out at the micro or macro level. Similarly, the generalizability of these findings could be tested by varying the population--is the instruction sufficient for normally-achieving adolescents and low-achievers, and how do gifted mathematical students generate boundaries for identifying the applicability of schemata?

Details of the instruction that present themselves for further investigation include the task analysis. What other operationalizations of instruction in representation and solution would also be sufficient to enable students to generate full and accurate representations and solutions? It has already been suggested that investigations of the scope and sequence of the present steps in each phase be systematically undertaken in the manner recommended by Heller and Reif (1984). In keeping with the character of cognitive instructional psychology, the role of prior knowledge at all steps of acquisition of expertise warrants further investigation. It may be necessary to contrast and refine present methods of assessing knowledge structures, as Leinhardt (in press; Leinhardt & Smith, 1985) has been doing in her studies of students' and teachers' mathematics knowledge structures.

Other lines of research that might help to explain the findings of this study are the roles of self-efficacy and of teachers' knowledge structures. Self-efficacy may have played a mediating role in the students' continuing willingness to participate and the high level of motivation they displayed (Walsh, 1986). Students' comments suggested that the highly structured worksheet and extensive guided practice made them confident that they would learn to solve word problems. The degree of organization and accessibility in a teacher's schemata for problem types may influence the quality of the explanations and feedback provided to students (Herrmann, 1986). Systematic manipulation of one variable at a time could be facilitated by using microcomputers to deliver the guided instruction or tutoring in representation and solution.

To summarize, an intervention that enables students to become better problem solvers raises many questions about why that intervention succeeded, and the extent of the generalizability of the findings.

Summary

Although educators and researchers have expressed concern about the need for instructional research in problem solving, few intensive intervention studies have been conducted to teach algebraic problem solving. The present study merged recent theory in learning disabilities to examine the effectiveness of theory-based instruction. All research questions were substantiated suggesting that the instruction was sufficient to enable learning disabled adolescents to construct adequate representations and solutions for competent algebraic problem solving. Similar improvements were not found in a comparison group who received familiarization with the structured worksheet for problem solving and continued in the learning centre program in mathematics.

Implications of the findings for the adopted theoretical frameworks were discussed. It was inferred that students had acquired well-structured schemata for problem types based on mathematical structure, that such problem isomorphs constituted an appropriate level of instruction, and that metacognition had increased with cognitive acquisitions during the study.

Recommendations were made for further investigations that would shed light on the specific factors most respor ible for the positive outcomes of the instruction, and on the generalization that is warranted.

APPENDIX A

Criteria for Identification as

Learning Disabled

Definition and Criteria for Severe Learning Disabilities Used in School District of Participants

1. Severely learning disabled students (for funding purposes) represent approximately 2% of a "normal" school population.

2. A learning disability is a processing order resulting in a significant discrepancy between estimated learning potential and actual performance. This discrepancy should not be primarily due to other factors such as: sensory impairment, mental handicaps, behavior disorder, environmental or cultural disadvantage, E.S.L.

3. A psychoeducational assessment should identify:

- average or better intellectual ability (within 1 S.D.)
- a specific learning deficit
- a discrepancy of more than 1 S.D. on a standard achievement test (i.e. primary - more than 1 year, Grade 5/6 - more than 2 years, secondary - more than 3 years).

4. Special placements shall be determined by the District Screening Committee, and will require parent approval. Identification for school-based programs should be determined by a school screening committee which includes the area counsellor.

5. An Individualized Education Plan should be developed for each student which includes instructional strategies to address the learning deficit.

6. Service delivery could include placement in a resource room or Skill Development Center, or support service on a regular basis by an itinerant teacher or area counsellor.

Note: These criteria are used in conjunction with the Ministry of Education Special Program Manual, which contains a definition of learning disabilities consistent with that in the Education for All Handicapped Children Act (U.S. Office of Education, August 23, 1977)

APPENDIX B

Criterion Measures

Name: ______________________ Date: ______________________

School: ____________________ Group: ____________________

Time: ____

Instructions: Answer each question. Please show your work.

ADD:

5	9	51	70	698
+8	+7	+69	+83	+759

SUBTRACT:

7	9	21	80	613
-4	-5	- 8	-69	-276

MULTIPLY:

5	8	4	3	6	9
x6	x9	x7	x8	x9	x9

6	7	8	6	5	7
x8	x5	x7	x6	x5	x9

GO TO PAGE 2

PAGE 2

MULTIPLY:

```
  8      7      6      5      3      2
 x8     x7     x7     x4     x7     x9
---    ---    ---    ---    ---    ---
```

```
  4     10
 x8    x 7
---    ---
```

DIVIDE:

$8\overline{)32}$ $9\overline{)72}$ $6\overline{)48}$

$7\overline{)490}$ $5\overline{)325}$

THE END

Name: ______________________ Date: ______________________

School: ______________________ Group: ______________________

Time: ____

Instructions: Answer each word problem. Show your work.

1. The number of gloves in a box is 7. How many gloves will there be in 32 boxes?

2. In one hour a truck driver travels 90 kilometres. How far will he travel in 12 hours, if he continues to drive at the same speed?

3. It costs $23 for one ticket to a WHAM concert. How much will tickets cost for 12 people?

4. A machine packs 47 t-shirts in a minute. Find the number of shirts it will pack in an 8-hour work day?

5. Each boy on our team is 15 years old. There are 8 boys on our team. What is the total age in years of the boys on our team?

ASSESSMENT MEASURE

1. Mary received interest on her money in 1984 and in 1985. In 1985 she received three times as much interest as in 1984. The total of her interest for the two years was $1060. How much interest did she receive in 1985?

2. For every 3 dimes in my savings, there are 5 quarters. If I have 81 dimes, find the number of quarters.

3. The sum of two consecutive odd numbers is 72. Find each number.

4. A woman can build 5 chairs in 4 days. If she works for 72 days, how many chairs can she build?

5. Dan has the job of selling tickets for a school game. The tickets cost 75 cents for each adult and 50 cents for each student. He sold 120 tickets for a total of $82.50. How many tickets of each kind did Dan sell?

6. A picture 6 cm wide and 9 cm high is to be enlarged so that the height will be 18 cm. How wide will it be?

7. Dad and I drove to Calgary. It is 1546 km. He drove 320 km further than I did. How far did I drive?

8. A man invested \$10,000, part at 6% and the rest at 7% per year. The total annual income from these investments is \$680. How much did he invest at each rate?

9. The sum of the ages of Jack, Robin, and Tom is 188 years. Jack is the youngest. Robin is 4 years older than Jack. Tom is 3 years older than Robin. Write an equation stating that the sum of their ages is 188 years. Find the age of each man.

10. The ratio of the number of boys to girls in a class is 5:4. If there are 16 girls, then how many boys must there be?

11. John works 54 hours in two weeks. In the second week he works 3 hours less than he did in the first week. Find the number of hours he worked each week.

12. There were a number of goats and ducks in one pen, 15 in all. There were 44 feet in the pen. How many goats were in the pen?

13. The ratio of Mr. Jones' age to Mrs. Jones' age is 8:7. Mr. Jones is 96 years old. Find Mrs. Jones' age.

14. The sum of a man's age and his son's age is 40. The man has saved $500 a year and his son has saved $50 a year. The total they have saved is $15500. Find the age of the man.

15. I rode a bus and a train for a total of 6 hours. I went at a rate of 50 km/hr on the bus and 60 km/hr on the train. The total distance was 340 km. Find the distance I rode on the bus, and the distance I rode on the train.

APPENDIX C

Letter of Permission

SIMON FRASER UNIVERSITY

FACULTY OF EDUCATION

BURNABY, BRITISH COLUMBIA V5A 1S6
Telephone: (604) 291-3395

22 October 1985

Dear Parent,

I am writing to request your permission to have your son or daughter participate in my project described below.

My project is designed to improve algebra word problem solving in junior high school students. Specifically, I plan to teach students, especially those with mathematics problems, a strategy to use with algebra word problems.

All the information will be analyzed anonymously. The results of the project will help us help students who have difficulty solving word problems. Your son or daughter's participation is completely voluntary. He/she may withdraw from the project at any time during the project. The duration of your son or daughter's participation will depend on his/her individual learning rate. Generally for a high school student to master such a strategy, instruction averages about three months, with three sessions weekly, each session lasting half an hour. Please discuss this matter with your son or daughter as his/her consent to participate is required.

If you have any questions you can call me at 291-3591 during the day (and I will return your call), or at 421-4539 at home in the evening. Thank you for attending to this letter.

Yours sincerely,

Nancy L. Hutchinson, M.A.
Graduate Student
Faculty of Education
Simon Fraser University

Date________________

I, ______________________________, do give permission for my

son/daughter, ______________________________, to participate

in this project.

____
(Signature of parent or guardian)

APPENDIX D

Test for Outliers

Test to Determine Whether Student Was Outlier

<u>Rationale</u>: One student was qualitatively different than all other participants in that he took two or three times as long to complete each criterion measure and each pretest dependent measure. Discussion with his mathematics teacher confirmed that he worked very slowly, demanded perfection of himself, and obtained scores like the other students only when tests were timed. If allowed unlimited time he would score much higher. As an example, on the B.C. Q2 Assessment Test which was open-ended and untimed, he spent three sessions on the 13 problems and obtained a score of 12 out of 15, much higher than the other students.

<u>Procedure</u>: To determine if he was an outlier, his score was removed from the distribution and the standard deviation of the distribution calculated (S.D. = 1.62). His score of 12 was 5.12 standard deviations from the mean of 3.7. This was accepted as evidence that he was an outlier for this distribution. The Z scores of all other students on Q2, calculated according to the same procedures, ranged from -1.57 for the lowest score (1) to 1.31 for the highest score (7).

APPENDIX E

Task Analysis and Tasksheets for the Three Problem Types

E-1. Task Analysis of Relational Problems
E-2. Task Analysis of Proportion Problems
E-3. Task Analysis of Two-Variable Two-Equation Problems
E4. Relational Problems - Exercise 1
E5. Relational Problems - Exercise 2
E6. Proportion Problems - Exercise 21
E7. Proportion Problems - Exercise 22
E8. Two-Variable Two-Equation Problems - Exercise #1
E9. Two-Variable Two-Equation Problems - Exercise #2

Task Analysis of Relational Problems

Problem Representation

1. Read problem aloud.

2. Evoke schema (kind of problem identified by relational statement).

3. Draw and label diagram showing implicit relationships (sum or difference of two unknown but related quantities represented concretely).

4. Write goal.

5. Write unknowns (let X = simplest unknown; elaboration of X = related but more complex unknown).

6. Write knowns (sum or difference, and relational statement).

7. Represent problem in equation (abstract unknowns inserted into equation whose form is isomorphic to diagram).

Problem Solution

8. Check to be certain that an equation has been written.

9. Simplify equation by doing work to remove brackets.

10. Combine like terms (combine unknowns).

11. By applying inverse operation to both sides of equation, remove term added to or subtracted from unknown.

12. By applying inverse operation to both sides of equation, remove term that multiplies or divides the unknown.

13. State equation in terms of X = ...

14. Refer to goal for meaningfulness.

15. Write statement(s) meeting goal(s).

16. Check accuracy of answer(s) by checking consistency with knowns, and substituting into equation.

Notes: The novice may write the goal, unknowns, and knowns prior to evoking schema and making diagram.

The novice will reread the problem frequently, whereas the more experienced solver will carry out quick lookbacks.

Task Analysis of Proportion Problems

Problem Representation

1. Read problem aloud.

2. Evoke schema (kind of problem identified by complete and incomplete ratios comparing two factors).

3. Draw and label diagram showing implicit relationships (given ratio set equal to ratio for new case with labels for all terms, and concrete representation).

4. Write goal.

5. Write unknown (let X = unknown term in incomplete ratio).

6. Write knowns (given ratio, and one term of new case).

7. Represent problem in equation (abstract unknown inserted into equation whose form is isomorphic to diagram).

Problem Solution

8. Check to be certain that an equation has been written.

9. Simplify equation by doing work to remove brackets.

10. Cross-multiply, or apply inverse operation to both sides of equation to reduce one side to 1.

11. By applying inverse operation to both sides of equation, remove term that multiplies or divides the unknown.

12. State equation in terms of X = ...

13. Refer to goal for meaningfulness.

14. Write statement(s) meeting goal(s).

15. Check accuracy of answer(s) by checking consistency with knowns, and substituting into equation.

Notes: The novice may write the goal, unknowns, and knowns prior to evoking schema and making diagram.

The novice will reread the problem frequently, whereas the most experienced solver will carry out quick lookbacks.

Task Analysis of Two-Variable Two-Equation Problems

Problem Representation

1. Read problem aloud.

2. Evoke schema (kind of problem identified by need for two variables and need for two equations).

3. Draw chart showing implicit relationships (sum of two simple unknowns equal to total of simplest factor; sum of two quantities dependent on simple unknowns equal to total of complex factor, with labels for terms).

4. Write goal.

5. Write unknowns. Let X = one unknown, Y = other unknown.

6. Write knowns (the two sums).

7. Represent problem in two equations. First is a statement involving simple unknowns. Second is derived from first equation combined with prior knowledge.

Problem Solution

8. Check to be certain two equations have been written.

9. Set up table with two simple unknowns, their total, two complex terms involving unknowns, and their total across the top, with labels on each line.

10. Fill in the table with terms that sum to the simple total. Complete the table. Check outcome for complex total against that listed at top of column. Describe outcome as low or high or correct.

11. Select new values for unknowns based on whether result was too low or too high. Complete the table and check outcome against that listed at top of column. Describe outcome as low, high or correct.

12. Repeat step 11 until outcome is correct.

13. Refer to goal for meaningfulness.

14. Write statements meeting goals.

15. Check accuracy of answers by checking consistency with knowns and substituting into equation.

Notes: The novice may write the goal, unknowns, and knowns prior to evoking schema and making diagram.

The novice will reread the problem frequently, whereas the most experienced solver will carry out quick lookbacks.

Problem Exercise 1

1. Mike has cycled 13 more km than Sam. Together they have cycled 121 km. Find the number of km Mike has cycled.

2. The sum of two numbers is eight and one of them is two greater than the other. Find the numbers.

3. Jean gave twice as much money to the Red Cross as did Ann. If the sum of their gifts was two dollars and thirteen cents, how much did each give?

4. Ellen is seven years older than her sister, and the sum of their ages is 21 years. How old is Ellen's sister?

5. A grocer packs 70 kilograms of cookies. There are 28 more kilograms of chocolate cookies than peanut butter cookies. Find the number of kilograms of chocolate cookies he packed.

Problem Exercise 2

1. Mr. Smith and Mr. Jones both earned interest on their money. Mr. Smith's interest was twice as much as Mr. Jones'. If the difference between the two men's interest was $510, how much interest did each man receive?

2. Jenny jogged 17 km farther last month than Mark. Their total was 41 km. How far did Jenny jog?

3. Henry is 3 years older than his brother, and the sum of their ages is 25 years. How old is Henry?

4. One of two numbers is six times the other and their sum is one hundred and thirty-three. Find the larger number.

5. Jane and Sam delivered 115 parcels the week before Christmas. Sam delivered 31 fewer than Jane. Find the number of parcels each student delivered.

Problem Exercise 21

1. The ratio of the age of a boy to a girl was 7 : 6. If the age of the girl was 126 months, find the age of the boy.

2. A machine produces 9 items in 5 hours. How many items will it produce in 20 hours?

3. Of every 15 Km I run, I wear weights for 11 Km. If I run 330 km, how many Km will I run wearing weights?

4. Find the regular price of a suit that sold for $41.25 at a 25% reduction.

5. Last baseball season the number of times John went up to bat was 30. In those 30 times at bat, John got 18 hits. What per cent of John's times at bat resulted in hits?

Problem Exercise 22

1. A boy is 2 years old and has received 36 needles. If he continues to need this medication, how old will he be by the time he has received 108 needles?

2. Gwen's car travels 11 kilometres on 2 litres of gas. At this rate, how far can it travel on 46 litres of gas?

3. Mr. Brown picked 6 baskets of apples in 40 minutes. At that rate how many would he pick in 6 hours?

4. Mrs. Wilson pays $144 taxes on a house assessed at $4,800. Using the same tax rate, find the taxes assessed on a house assessed at $5,900.

5. The ratio of the number of objects in set A to the number of objects in set B is 4 to 5. If the objects in set A were grouped by eights instead of by fours, how would you describe this ratio?

Exercise 41

1. I drove my bike and walked for a total of 3 hours. I went at a rate of 30 km/hr on my bike and 5 km/hr walking. The total distance was 65 km. Find how far I walked.

2. John packed 10 crates in all. Each big crate costs $1000. Each small crate costs $600. The total value of the crates he packed was $4,200. How many crates of each size did he pack?

3. The sum of a woman's age and her daughter's age is 35. If the woman has made 3 friends every year and her daughter has made 2 friends every year, find the number of friends the woman has made. In all, the two of them have made 99 friends.

4. A number of cars took the 24 passengers to the picnic. There were some cars that held 6 people and some that held 2 people. If there were 8 cars, how many of them held 6 people? (All the cars were full.)

5. Ann has $75 in $5 bills and $10 bills. She has 12 bills in all. How many $10 bills does she have?

Exercise 42

1. Pete bought pucks and hockey sticks for his team. He bought 15 items in all. The sticks cost $7.00 and the pucks cost $3.00. If he spent $77.00, how many did he buy of each?

2. On Safari John travelled by jeep and on foot. The rate of the jeep was 20 Km/hr. On foot the rate was 4 Km/hr. He travelled 128 Km in 8 hours. How far did he travel in each manner?

3. A grocer mixes tea worth $1.60 per Kg with tea worth $2.20 per Kg, making a blend to sell for $1.80 per Kg. How many Kg of each should he use if he plans to blend 75 Kg?

4. Some children in the class are 7 and some are 8. The difference between the number of 7 year old and the number of 8 year olds is 12. The sum of the ages is 234. Find the number of children of each age. (There are more 7 year olds.)

5. The number of animals in the show ring was 30. Some were cats and some were the owners of the cats. The total number of legs in the ring was 94. How many owners, how many cats were in the show ring?

APPENDIX F

Script for Familiarization with Structured Worksheet

In solving a problem there are two main parts to be done. The first is called representation. This means getting a "whole picture" for the problem. The top half of this worksheet will help you to get a whole picture or an understanding of a problem. It has a place where you can make a drawing showing the relationships among the parts of the problem (point this out*). If you would prefer, you can write the problem in your own words. Most people find it helpful to make a drawing that shows them what is happening in the problem.

After you have read the problem and made your drawing (or written in your own words), then you should write down the "goal". This means what you have to find in the problem. "What I don't know" -- this is where you can introduce an unknown. You could "let X" stand for some number that you don't know.,

"What I know" -- this is your opportunity to write down what information you learned from reading the problem. Do you know the sum of two things? Do you know how much money someone has? Do you know a ratio between two numbers?

"Kind of problem". If you have seen problems like this before, write down the name of this kind of problem. If you have not seen problems like this before, then give this problem-type a name or describe what it is like. "The equation" is the last part of your representation. In an equation, you write down the things that are equal to each other, in this problem. Remember that an equation always contains an equals sign, and is a sentence that is true.

Now, you have written a sentence with an equal sign, telling what things are equal to each other in this problem. This is an equation, a true sentence involving numbers, an unknown, and an equal sign. It is based on the information given in the problem and shown in your drawing.

The bottom half of this worksheet is for the second main part of solving problems -- solution. This means the steps you carry out operating on the equation in order to get to the goal. Show each one of these steps on the worksheet. Remember you want to find the value of the unknown (X, perhaps) that you introduced in the representation. "Compare to the goal" means look back to the goal you have written down for the problem and be sure to write in a sentence the answer(s) that meets goal. "Checking" means that you should check your calculations to be certain there are no errors and check your answer by comparing it to the information you wrote down in the representation. You are making certain that the two are not contradictory, that is, do not disagree with one another. Once you have done all these steps on the bottom half of the worksheet, you have completed the solution.

Representation -- on the top half of the worksheet.

Solution -- on the bottom half of the worksheet.

Problem__________ Date__________ Name__________

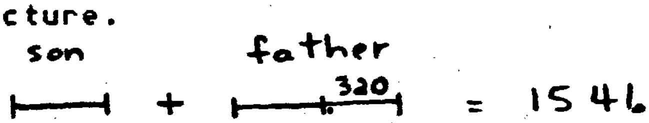
Exercise Relat. Exp.Phase________ Session________ Ob.____

Goal : To find how far the child drove.

What I don't know : Let distance child drove = x km
Distance Dad drove = x + 320 km

What I know: Dad drove 320 km further than son
Total distance driven = 1546 km

I can write/say this problem in my own words or draw a picture.



Kind of problem: Relational

Equation:

$$x + x + 320 = 1546$$

Solving the equation:

$$x + x + 320 = 1546$$
$$2x + 320 = 1546$$
$$2x + 320 - 320 = 1546 - 320$$
$$\frac{2x}{2} = \frac{1226}{2} \qquad x = 613$$

Dad and I drove to Calgary. It is 1546 km. He drove 320 km further than I did. How far did I drive?

Solution:

Child drove 613 km

[compare to goal]

Check:

$$\begin{array}{r} 613 \\ +320 \\ \hline 933 \end{array} \text{ (Dad)} \qquad \begin{array}{r} 933 \\ +613 \\ \hline 1546 \end{array} \checkmark$$

Problem__________ Date__________ Name__________

Exercise Prop. Exp.Phase__________ Session__________ Ob.______

Goal : To find Mrs. Jones' age.

What I don't know : Let Mrs. Jones' age = x

What I know: Ratio of Mr. Jones' to Mrs. Jones' age is 8 : 7.

Mr. Jones is 96 years old.

I can write/say this problem in my own words or draw a picture.

	Ratio	New Case
Mr. Jones	8	96
Mrs. Jones	7	x

Kind of problem: Proportion

Equation:

$$\frac{8}{7} = \frac{96}{x}$$

Solving the equation:

$$\frac{8}{7} = \frac{96}{x}$$

$$\frac{8x}{8} = \frac{96 \cdot 7}{8} \qquad x = 84$$

The ratio of Mr. Jones' age to Mrs. Jones' age is 8 : 7. Mr. Jones is 96 years old. Find Mrs. Jones' age.

Solution: Mrs. Jones is 84 years old.

[compare to goal]

Check: $\frac{8}{7} = \frac{96}{84}$ $8 \times 12 = 96$ ✓ $7 \times 12 = 84$

Problem________ Date________ Name________

Exercise 2-v. 2-e. Exp.Phase________ Session________ Ob.____

Goal : To find the number of goats.

What I don't know : Let x = number of goats
Let y = number of ducks

What I know: Total number of animals = 15.
Total number of feet = 44.
Goats have 4 feet.
Ducks have 2 feet.

I can write/say this problem in my own words or draw a picture.

	goats		ducks		
animals	x	+	y	=	15
feet	4 x	+	2y	=	44

Kind of problem: 2-variable 2-equation

Equation:

$$x + y = 15$$
$$4x + 2y = 44$$

Solving the equation:

x	y	15	4x	2y	44	outcome
8	7	15	32	14	46	too large
7	8	15	28	16	44	✓

Solution: There were 7 goats and 8 ducks.

[compare to goal]

Check: $7 + 8 = 15$ $\quad 7 \times 4 = 28$ $\quad 8 \times 2 = 16$ $\quad 28 + 16 = 44$ ✓

There were a number of goats and ducks in one pen, 15 in all. There were 44 feet in the pen. How many goats were in the pen?

APPENDIX G

RELATIONAL

Baseline Set A

1. Art and Frank worked on Monday. They packed 336 crates in all. Art packed half as many as Frank. How many did Art pack?

2. The sum of two flights made by a pilot is a distance of 260 kilometres. One flight is three times as long as the other. Find the distance of each flight.

3. Find two consecutive integers whose sum is thirty-nine.

4. Find four consecutive integers such that the sum of the second and the fourth is one hundred sixty.

5. Mister C. is 13 years older than Mister B. The total of their ages is 51. Find the age of Mister C.

6. A man is 21 years older than his son. The total of their ages is 101. How old is each?

7. Mary and Tom worked in a grocery store. Together they worked 32 hours last week. Mary worked 4 hours longer than Tom. Find the number of hours Tom worked.

8. Daisy, Lynn and Steve were walking in a marathon to raise money. Altogether they walked 30 kilometres, but Steve walked 3 more than Lynn, and Lynn walked 3 more than Daisy. How many kilometres did each person walk?

9. A coat and hat cost $40.50 and the coat costs sixteen dollars more than the hat. How much does the coat cost?

10. A bottle and a stopper cost one dollar and ten cents, and the bottle costs one dollar more than the stopper. How much did each cost?

RELATIONAL

Baseline Set B

1. Find two consecutive even integers whose sum is fifty-eight.

2. A house in town is worth $1000 more than a house in the country. The value of the two houses is $43,600. Find the value of each house.

3. A woman receives twice as much interest as her husband on her investments. The total interest received by the two of them is $1782. How much interest does the woman receive?

4. Ellen worked 7 days longer at the farm than her sister, and the sum of the days they worked is 21. How many days did each girl work at the farm?

5. The larger of two numbers is four times the smaller. Find the numbers if they add to twenty-five.

6. A man walks three times as far as his son. If the total distance walked by both is 16 kilometres, how far did each walk?

7. Mark has cycled 13 more kilometres than Sam. Together they have cycled 89 km. Find the number of km each boy has cycled.

8. Sandy raked leaves four times as long as Jenny packed them into garbage bags. In total the girls worked 15 hours. How long did each girl spend working?

9. Mrs. Jones is five times as old as her granddaughter. How old is each if the difference between their ages is 36?

10. A boy is seven times as old as his puppy. The sum of their ages is 16. Find the age of each.

PROPORTION

Baseline Set A

1. Joan saved $25 in 9 weeks. At that rate how long will it take her to save $175?

2. A new computer prints 12 words in the time it took the old printer to print 5 words. The old printer printed 795 words in an hour. How many words will new printer print in an hour?

3. On a map, 3/4 cm represents 12 km. What distance is represented by 7/4 cm?

4. Sales tax on a $500 purchase is $20. How much would an item cost if the sales tax was $17?

5. Jim builds models to sell at a craft fair. He builds 3 in 14 hours. How long will it take Jim to build 42 models?

6. On a trip, the ratio of km driven to gallons of gas used is 43 to 2. How many gallons of gas were used in driving 903 km?

7. This old book has been in my family 7 years for every 2 years I have been alive. If I am 36, find the age of the book.

8. The ratio of a man's age to his daughter's age is 10 to 4. The man is 40. How old is his daughter?

9. The number of hits a batter got in 200 times at bat was 55. What percent of his times at bat did he get hits?

10. 96 compared to 180 is the same as 16 compared to what number?

PROPORTION

Baseline Set B

1. What number compared to 56 is the same as 9 compared to 14?

2. Al bought a guitar on sale for $20. He saved 20% of the regular price. What did the guitar cost before the sale?

3. Oranges sell at 5 for 39 cents. How much do 15 oranges cost?

4. At work, John fixes 3 trucks for every 2 cars he fixes. Last week he repaired 12 cars. How many trucks did he repair?

5. 12 compared to 75 is the same as what number compared to 100?

6. A distance of 2 cm on a map represents 120 km. What distance is represented on this map by 5 cm?

7. A motorist travelled 172 km on the Trans Canada Highway in 2 hours. How long will it take him at that rate to travel 301 kilometres?

8. Susie stocked 9 shelves at the grocery store in 50 minutes. At that rate, how long will it take her to stock 81 shelves?

9. In our family, we calculated that my Dad has lived 11 years for every 3 years I have lived. I am 12 years old. How old is my Dad?

10. A man has lived 3.5 years for every 1.5 years his son has lived. If the son is 30, how old is the man?

Type 3

BASELINE A

1. The sum of Steve and Tom's ages is 30. Steve has visited 5 countries every year, and Tom has visited 3 countries every year. The total number of countries they have visited is 106. Find Steve's age and Tom's age.

2. I travelled 760 Km, some at 100 Km/hr and some at 80 Km/hr. The total time it took was 8 hours. Find the distance I travelled at 100 Km/hr.

3. A man can mow a lawn in 24 minutes and trim a shrub in 18 minutes. Yesterday he worked 414 minutes, and completed a total of 19 jobs. How many lawns did he cut?

4. Aunt Alice has saved $50, all in quarters and dimes. She counted her coins and found that she has 380 coins altogether. How many dimes, and quarters does she have?

5. The number of books in a library is 1000. Some were borrowed twice this year. The rest were borrowed 3 times. The total number of loans made was 2700. Find the number of books that were loaned once.

6. I work at a grocery store. I mix cookies worth 96 cents a kg with cookies worth 72 cents a kg to sell at 76 cents a kg. If I require 60 kg of mixed cookies, how many kg of each kind should I have?

7. Robert cycles at 6 km/hr. Frank cycles at 8 km/hr. Robert rides to Frank's house with a message and Frank takes the message the rest of the way. The total distance is 25 km, and the whole operation takes 4 hours. How far did Robert cycle?

8. In a drawer there are 36 stamps. Some are 34 cent stamps, and some are 4 cent stamps. Their total value is $8.64. Find the number of 34 cent stamps.

9. Two men together have a total age of 60 years. Joe has been sick once each year. Frank has been sick twice each year. Altogether they have been sick 85 times. Find the age of each man.

10. A big box contains 8 toys. A small box contains 5 toys. The total number of toys I had was 58. I put them into 7 boxes. Find the number of large boxes and the number of small boxes.

Type 3

BASELINE B

1. Stewart has 25 stamps; some are 15 cent stamps and the rest are 18 cent stamps. The value of all the stamps is $4.05 How many stamps of each kind does he have?

2. Betty sold soft drinks at the school fair. Each big glass cost $1.00 and each small glass cost $0.75. She sold 90 drinks for a total of $71.75. How many soft drinks of each size did Betty sell?

3. Sam and Laura had a total of 19 cavities. Their ages sum to 12. Sam has had 1 cavity each year. Laura has had 2 cavities each year. Find their ages.

4. The number of people at our school fair was 500. Some were children and some were adults. The children paid $0.50 and the adults paid $1.25. Find the number of adults who attended, if we took in $362.50 in admissions.

5. Two boys leave towns 33 km apart. One walks at a rate of 3 km per hour. The other walks at a rate of 6 km per hour. How long did each boy walk, if they met after a total of 8 hours of walking had taken place?

6. Two people together have a total age of 80 years. Nan has lived in 1 house per year. Jen has lived in 2 houses per year. The total number of houses they have lived in is 110. Find the age of Jen.

7. There are more quarters than dimes in a piggy bank. The difference between the number of quarters and dimes is 10. The total value of the coins in the bank is $7.75. Find the number of each coin.

8. There were 6 boxes of candy; a number of them were large and the rest were small. Each large box contained 30 pieces. Each small box contained 20 pieces. Altogether there were 170 pieces of candy. Find the number of small boxes. ?

9. A bird flew with a "wind assist" at 20 km/hr. Then when the wind stopped he continued to fly at 15 km/hr. He travelled 110 km in 6 hours. Find the distance he travelled at each speed.

10. John works 27 hours in total at two jobs. In the first job he makes 7 toys each hour. In the second job he makes 8 toys each hour. Altogether he made 204 toys. How many hours did he work at each job.

APPENDIX H

Think-Aloud Protocols

H-1. Instructions for Think-Aloud Protocol

NAME: ______________________ DATE: ______________________

EXP. PHASE: ______________________ GROUP: ______________________

INTERVIEWER: ______________________ RECORDED: ______________________

PROBLEM: ______________________ TIME: ______________________

INSTRUCTIONS FOR THINK-ALOUD PROTOCOL

I am going to give you a mathematics word problem to read and work on. I will ask you to begin by reading the problem out load. Then I will give you instructions to think it out loud.

Let's try this idea of thinking out loud with a multiplication question. Listen to the instructions. Then you will have an opportunity to ask me any questions you may have.

Here are the instructions: Read the problem out loud. Then try to think out loud. I bet you sometimes do this when you are alone and working on a problem. I am not primarily interested in your answer. I am interested in all you are thinking as you do the question. Don't plan what to say, but let your thoughts be out loud. Do not try to explain anything to me. Pretend no one is here but yourself. Do not tell me about the solution, BUT SOLVE IT.

Here is the multiplication question. Read the question and think out loud while you solve it.

$$\begin{array}{r} 17 \\ \underline{\times 45} \end{array}$$

PROMPTS: "KEEP TALKING" OR "LET ME HEAR YOUR THINKING"

After 15 seconds of silence.

When necessary to encourage students to reveal their thought processes.

ASK for questions and ANSWER any questions the student may have, before repeating instructions.

Here are the instructions: Read the problem out loud. Then try to think out loud. I bet you sometimes do this when you are alone and working on a problem. I am not primarily interested in your answer. I am interested in all you are thinking as you do the problem. Don't plan what to say, but let your thoughts be out loud. Do not try to explain anything to me. Pretend no one is here but yourself. Do not tell me about the solution, BUT SOLVE IT!

APPENDIX I

Representative Transcripts of Instructional Sessions

I-1. Third Session of Instruction in Representation for Proportion Problems with a Student Who Completed Three Problem Types

I-2. First Session of Instruction in Solution for Relational Problems with a Student who Completed Two Problem Types

Excerpt from Third Session of Instruction
for Proportion Problems with a Student
Who Completed Three Problem Types

I: Sandi, I want you to take a look before we start at the questions that you ask yourself for representing the proportion problems?

S: OK. Have I read and understood each sentence? Are there any words whose meaning I have to ask? Have I got the whole picture of representation for this problem? Have I written down my representation on the worksheet? Goals. Unknowns. Knowns. Type of problem and reasons. And the equation. What problem features should I focus on so that I know whether I can use the representations I have been taught?

I: OK. In proportion problems, remember that there will be two ratios that we'll be putting equal to each other. In the problem as you read, you will find one ratio. And then one element out of the second ratio. And the element that you don't know out of the second ratio, remember that you will be putting that equal to X. OK? That will be the unknown. I'll model the first couple problems and then I'll get you to do the next three. And if you'll do them out loud, two out loud, then I can listen to how you're thinking and respond. And you can do the last one silently and I will give you a set of five to do as an assessment. OK? This will be Assessment #3 in representation. So if you get 80% or 100% in the five problems that you'll be doing today, then we'll be going on to solution for proportion. OK. First I have to read and understand each sentence. The earlier steamship run was at the rate of 240 kilometres in 32 hours. The first sentence gives me a ratio between the distance and time, for the earliest steamship run. Then I have some new information, a new case -- how far would I expect the ship to travel in 13 hours? So the distance is going to be what I don't know. And the second ratio. And I know that the time in the new case, or the second ratio, is 13 hours. Then I want to get the whole picture -- a representation, so I put down my ratio and the information for the new case. For the ratio, it will be distance over time. The distance was 240 kilometres and the time was 32 hours. In the new case, I don't know the distance, so it will be X kilometres, but I do know the ship is going to travel for 13 hours. Both in the same units on the top. Both in the same units on the bottom. This is a proportion problem. My goal then is to find the distance the ship would travel in 13 hours. What I don't know. I'll let the distance travelled in 13 hours equal X. And that's the only thing I don't know in this case, isn't it? What I know is that the ratio of distance to time was 240 to 32. Or I could

write it 240 over 32. And I know that time in the new case is 13 hours. My equation will have distance on the top for both ratios and time on the bottom. 240 over 32 = X over 13. Any questions, Sandi? (no questions)
OK. Problem #2. The number of cookies that can be made with the recipe is 2 dozen. This recipe requires 2/3s of a cup of butter. How much butter would be needed to make 72 cookies? I'll have to watch my units because here I have in dozen and here I have in cookies. So I'll change dozen to cookies. So I'm going to have a ratio with number of cookies and cups of butter. What's given is two dozen, which is 24 cookies. And 2/3s of a cup of butter. In the new case the number of cookies is 72 and the amount of butter will be X. We don't know. The goal is to find amount of butter needed for 72 cookies. We'll let X equal the amount of butter needed for 72 cookies. Now, what we know is the ratio of number of cookies to cups of butter is 24 to 2/3s. And you will notice that we have a fraction on the bottom of a fraction. But that's OK. When we're working on solution I'll show you how to work with that. The other information that we have is that you want to make 72 cookies.

S: Could you change the fraction, the 2/3, into decimals points? Is that what I do?

I: Yes. You could do it that way or work with it as a fraction. I'll show you how to do both and then you can decide which way you'd like. The equation, in the equation for a proportion problem, we're setting two ratios equal to each other. So 24 over 2/3s is equal to 72 over X. OK? Any questions. (no questions)

Do you want to do problem 3 thinking out loud Sandi?

S: OK.

I: OK? And if you have any problems you should ask.

S: OK.
(pause)

Wanda picked 15 apples in two minutes. At the rate, how many would she pick in 14 minutes? OK. Ratio is picked apples over minutes. And it would be 15 apples over 2 minutes. In the new case, apples is X, because we don't know how many there are. And minutes is 14. My goal is to find out how many apples Wanda can pick in 14 minutes. What I don't know, we'll let X equal the amount of apples picked in 14 minutes. And what I know, that the ratio of apples to minutes is 15 over 2. In the new case there is 14 minutes. The kind of problem is proportion. And the equation is apples over minutes. That's 15 over 2 is equal to X over 14.

I: Good. Number 4?
(pause)

S: Mrs. Wilson pays $900 taxes on her house assessed at 15,000. Using the same tax rate, find the taxes on a house assessed at 18,400. OK. The ratio is taxes over assessed. So it would be 900 over 15,000. And the new case, we don't know the taxes so they'll be equal X and the assessed value of the house in the new case is $18,400. Find the taxes on assessed house at $18,400. And what I don't know, we'll let X equal tax rate. What I know ...

I: It tells you that, um, using the same rate. So it says assuming the tax rate doesn't change, find the tax on a house assessed at 18,400. So what you've let X be here is not really the tax rate, but the ...

S: Taxes.

I: Taxes. Right.

S: OK. What I know, is the ratio of taxes to assessed is 900 over 15,000. And the new case, the assessed is at 18,400. The kind of problem it is -- proportion.

I: How do you know that it's a proportion problem, Sandi?

S: Because it has ratio and proportion in it.

I: OK. The proportion is when you set two ratios equal to each other, isn't it?

S: Taxes over assessed for the equation is 900 over 15,000 and X over 18,400.

I: And if it's an equation, what would you have to have, Sandi?

S: An equals sign.

I: OK. Very good, Sandi.

Excerpt from Second Session of Instruction in Solution for Relational Problems with a Student Who Completed Two Problem Types

I: Dave, remember we've finished with learning and practicing representation or the first part of problem solving, understanding what's in the problem, and using that information to get a proper equation that expresses the information in the problem. Now we're going to start learning how to solve those equations. When you're solving an equation, you are trying to find the answer or the goals that you wrote down when you understood the problem and did your representation. In order to get that goal, which might be how much money does Sam have, how old is John, or find the cost of the tie. What you have to do is to solve the equation, or find the values of X that satisfy the equation -- find the values of X that make the two sides of the equation equal to each other. OK? So what we'll be doing is working through a series of steps to find what numbers X, or what number in some cases, X actually took the place of. Remember in representation we said that we would let X take the place of something we don't know. Now we're going to figure out what number it is -- that that X actually took the place of. Or in some cases what numbers. In each case when you're solving a problem, the first thing you'll do is the representation. Understand the problem and work through until you have a representation. Because you need the equation in order to do the solving. Then we'll be using a set of self questions that you ask yourself to guide you through the steps in solving an equation....

S: Dad and I drove to Kelowna. It was 1,448 miles. He drove three times as far as I did. How far did I drive?

Find how far the son drove. I guess it's the son isn't it?

Let X equal the son. Let X multiplied by three equal the Dad. I know ... that altogether they drove 1,448 miles. ... And that Dad drove three times as far as the son. ... Equation X plus X multiplied by 3 equals 1,448 kilometres.

I: OK. You have the equation. Do you understand the problem?

S: Yeah.

I: In solving that equation the first step is to ask yourself the question "Have I written the equation?". You know that you have written an equation if what you have written has an equals sign. And values on each side of it. And in each case it does. Sometimes when we're writing an equation, we forget to write an equals sign and what comes behind it. And if you have just written X + X + 3 and stop there, it wouldn't be an equation, would it? What it means to have written an equation is that everything on this side, if you do all of the operations that are indicated here, this will be equal to this side of the equation. OK? If you think of a balance beam. Do you know what a balance beam is like in science? Where the two sides are balanced -- even -- at the same height. They are the same. If you add to one side of the balance beam, it goes down. And if you want to make them even again, you'd have to take some weight off where you add it, or what could you do?

S: Put more on that side.

I: Put more on this side, and it would go down. And what was heavier would come back up. So when you do something to one side of the two arm balance, you have to do it to the other. And that's what happens with an equation. If two things are equal, and then you add more to one side, it won't be equal again until you add the same amount to the other side. You have written an equation, and all the way through when you're solving this, you have to remember the two sides are equal. What I do to the one side -- I must do to the other. What's the second question that you ask yourself, Dave?

S: Have I expanded the terms?

I: Expanding terms usually -- what we mean by that is that you will do the work that is indicated by the brackets. And sometimes you'll find that you have a bracket -- say like this one that has the X in it. Right? And you'll have a number. But in front of the bracket 3 times what's in the bracket. In this case in the example I just wrote, 3 times 3X. When you expand the terms, you'll work that out. So that would be, in this case, 9X. OK? You're familiar with expanding the terms that are inside the bracket? Suppose you had 4 times X plus 2. Do you remember what it would mean to expand the terms? What would we do if we multiplied that out? First we multiply 4 times X and we get 4X, and that we multiply 4 times 2. Right? And all your rules about the signs that you're just working on now in your math class would apply if there are negative signs, and so on. Right? If you had 5 times X - 3. 5 times X would be 5X and 5 times -3 would -15. So expanding your terms is a name for something that you've already learned about. OK? In our case, in the equation we have, inside the bracket we have X times 3. And the simplest way to write that, for solving an equation, would be 3X. So when you expand the term, we would say X + 3X = 1,448. And usually when we're doing the equations, it's important to remember that this is kilometres. But we won't write this in; we don't want to get them mixed in with the other numbers and letters in the equation. X + 3X = 1448. We've expanded the terms as much as we can. Now, what's the next step we have to do? What's the third question to ask?

S: Have I written out my solution on the work sheet?

I: And the first step it asks you?

S: Collect like terms on each side.

I: Do you know what the expression "like terms" means?

S: It means that ~~are~~ alike on each side.

I: Right. So what are the two things are alike on each side of the equation?

S: The left side, there's only the X.

I: There's X and ... the left side of the = sign? Everything to the left of the = sign. You have X and ...

S: Oh, two X's!

I: Yes. X and X
(interrupts)

S: X and X

I: X and 3X

What do we understand there is in front of this X? If it just says X it stands for "1X". Right! So, in combining the like terms, we don't add anything new to the equation. We just do the operations that are indicated in the equation. We do them for the terms that are the same. So, here we have 1X + 3X. We add them together and get 4X. We didn't put anything new in. All we did was combine the same kinds of terms. And it's still equal to 1448. The next thing that we have to do ... what's the next step, Dave?

S: Solve for unknowns?

I: Whoops. We missed one!

S: Isolate the unknowns.

I: OK. Have I isolated the unknowns? What that means is that you want to have the unknown that you're working with, in this case that's X isn't it? On one side of the equation. We don't want a whole bunch of numbers and other things here with it. We want to get it isolated so we can find out what 1X is. Right now we have the X term alone on one side of the equation. Don't we? Because we have 4X equals something.

S: Yeah.

I: So we've pretty well accomplished that. Now, we have to solve for that unknown. We have to find out what X, 1X is. We now know what 4X is. Usually we do this by using the inverse operation of what's here. OK, we have 4 times X. What's the inverse of multiplying -- the opposite?

S: Dividing.

I: Right! Dividing. So what we have to do to find out what 1X is divide this side of the equation by 4. But if we do this for one side of the equation, we have to do it to the other side. So we divide both sides by 4. OK. 4X divided by 4?

S: 1X

I: Will give us 1X. And 1448 divided by 4?

S: 162?

I: 162

S: 362

I: 362. OK. Now we know what 1X is. Now we know what it was that X was standing for up here when we said "Let X equal how far the son drove". As soon as we find the value of X, we have found ..

S: How far the son drove.

I: How far the son drove. Now, what was the goal that we started with?

S: Find far the son drove.

I: Right. Now we've solved for our unknown, we have to check our answer against the goal. Because our goal didn't say to find X, did it? It said to find how far the son drove. So our solution will be that the son drove 362 kilometres. It's often helpful to figure out what the other term is because then you can check to make sure that you're right. Let's find how far Dad drove. What did we say up here what Dad drove, in terms of the unknown?

S: X multiplied by 3.

I: Now you know X. Right? So multiply it by 3, and then you'll know how far Dad drove.

S: Can you make that clear? (can't quite understand)

I: Yes.

S: 1086

I: 1086 kilometres Dad drove. OK. How could you check this, Dave?

S: Add them together.

I: Why don't you do that and we'll see if you're right.

S: On this, or by pen and paper.

I: Whatever you like.

(pause)

And usually what we do where it says check is to write down the two terms -- 362 + 1086 = the total. And put a little check mark to show that you've checked it and you know you're right. There. That's how we solve the equation. Those are steps you do to get the actual answer to, um, the goal. What it was you set out to find. Do you have any questions?

S: No, but I will as soon as I start doing it!

I: I'll do problem #1 -- thinking through out loud. The representation, and then the solution. Stop me if you have any questions.

A shirt and tie cost $12.60. If the shirt was $2.00 more than the tie, find the cost of the tie.

Remember from the representation, my first question I ask myself is "Have I read and understood each sentence?" and the first sentence tells me the total cost of the two items. So I know that, to do my representation, I am adding two things together and getting a total of $12.60. The second sentence tells me that the shirt costs $2.00 more than the tie. That's my relational statement so that I know that the tie is the simplier thing -- the cost of the tie, and the cost of the shirt is the same as the tie and then $2.00 more. OK. Now my representation ... the cost of the tie and the cost of the shirt, which is $2.00 more, together are $12.60. And the last part of the problem tells me that my goal is to find the cost of the tie. What I don't know, I'm going to let X equal the cost of the tie and X +2 will equal the cost of the shirt. I know the total of the two costs is $12.60, and I know that the shirt costs $2.00 more. I know this is a relational problem, because of the relational statement that the shirt costs $2.00 more than the tie. I don't yet know how much each costs. But I know the relationship between them. My equation is going to be the cost of the tie, which is X, plus the shirt, which is X+2. I'll put that in brackets because it all stands for one cost here. And it equals $12.60. OK. In the solution, the first question I ask myself is "Have I written an equation?", and I have. I have an = sign with values on each side. Have I expanded the terms? In this case, it will just be a case of removing the brackets, because there's nothing to multiply the bracket by, X + X + 2 = 1260. The next question I ask myself "Have I written out the steps of my solution on the worksheet?". The first step I'm supposed to write out is to collect like terms on each side. On the left side of the = sign ...

S: 2X

I: Right. I add X + X and I get 2X. I still have 2 there. 2X + 2 = 1260. You notice that each time as you do a step you still write out the full equation, so that you have the whole equation written out with the change it in that you've done. Next, I have to isolate my unknown. That means getting the X term all by itself and preferably, it seems easier, and the custom is to have it on the left side of the equation. Well, that means that I don't want this +2 here, do I? But you remember, that if I do anything to one side of the equation, add anything in or take anything away, I have to take away, or add in, the same amount on the other side of the equation. Usually we remove terms by using the inverse for the operation that's here. I have +2 here, 2 is added on. The inverse of adding is subtracting. So I take away 2 from that side of the equation and 2 from that side. OK? On the left side of the = sign I'll have left the term 2X and on the right side 1260 take away 2. I'll have 1060. 2X is equal to $10.60. What am I trying to find right now? The value of ... 1X. Right? And then I'll use that to find my goal. Now I know what 2 times X is. To find 1X, I'll have to divide both sides of the equation by 2.

Does that make sense to you?

S: Yeah.

I: If you know what 2 times something this, you know that you can find the value of one of them by dividing by 2. So I get X = $5.30. Step 1, we reached the first stage of getting where we want to be. We know the value of the unknown. Now we go back and check our answer with the goal. And the goal was?

S: To find the cost of the tie.

I: And, in fact, we let the cost of the tie equal X. So we've got the answer to that now, don't we? So we write down that the cost of the tie is $5.30. In order to do our checking, it's usually helpful to find the other value, or values involved. So we write down what the cost of the shirt is. The cost of the shirt was X + 2. If X is 530 then we have to add 2 onto that. What must the cost of the shirt be?

S: $7.30.

I: Right. How will we check?

S: Add the $7.30 and the $5.30.

I: And we find we get $12.60. So we know we must be right. OK? If the question was one where it talked about the difference between two things and we set up our representation in our equation with a difference here. Then you'll remember that when you check, you wouldn't add these two things together, but you'd find the difference between them. Right?

S; Uh huh.

I: Because when you're checking, you're really doing, checking what you knew to start with, the total corresponds with what you got after you did your calculations. There. OK. Do you have any questions about that one?

S: No but I probably will when I start doing it!

I: How would you like to try one? OK. Why don't you try this problem? I think you did the representation for it yesterday. #2. The problem about the ages of Mrs. Johnson and her son. If you get stuck, ask, and use the questions to guide you through....

APPENDIX J

Scripts for Instruction

ORIENTATION SCRIPT
(First Day of Intervention)

For the coming weeks we will be working together to improve your ability to do mathematical word problems.

There are a number of reasons why we may have difficulty with word problems. I will tell you about four of these reasons.

1. Sometimes we are so busy thinking about what operation we will have to do (addition, subtraction, multiplication, division) that we fail to read carefully. One thing we will be doing in the coming weeks is reading each sentence carefully to get all the information out of it. This means understanding specific words also. I will suggest you ask me the meaning of words you don't know. And I will help you to write down in a few words what each sentence tells you.

2. Another reason we sometimes have difficulty is that we don't get the whole picture in our minds of what the problem is about. We need to make a representation or picture of the whole thing. This may be in words or pictures. Usually this tells us what kind of problem we are dealing with. You will be practising getting a picture of the whole problem, and categorizing the problem to decide what kind it is.

3. A third cause of difficulty with word problems is not being able to break a problem down into smaller parts. We will be practising working on problems in two stages - Understanding the problem well enough to write an equation (representing) and, later, solving the equation.

4. The last difficulty with word problems is often solving an equation and making certain that your solution allows you to complete your goal for that problem. This is the last thing we will be practising.

In summary, in the coming weeks you will be learning what is called a self-questioning strategy. You will learn to ask yourself a series of questions so that you can solve problems in two phases. First, you will learn to understand or "represent" a type of problem so that you can write an equation. After you have practised and mastered this first phase, then you will concentrate on solving the equation. These two phases will be practised for a number of types of problems.

For each type of problem, I will find out how well you do on a phase. Then, if you need practice, I will demonstrate (show you), you will practice with guidance, and then you will demonstrate (show me) what you have learned.

These activities should help you to be successful with word problems of various types, here and in your classroom. You should use these strategies whenever you are doing word problems.

(In this introduction, I will take time to answer any questions that students may have about my role, their participation, and the project in general.)

General Script about Word Problems

Word problems are sometimes called story problems. Usually there are a number of sentences that tell a story. I'm sure that you have read various kinds of stories -- mysteries, westerns, animal stories. What is your favourite kind of story?

Usually each kind of story has a predictable form. I mean, for example, that once you know a story is a mystery, or a western, you know what form it will take. You may even predict that a number of things will happen. Have you read or seen a mystery? Can you tell me what usually happens at the beginning of a mystery? (Some crime takes place, often a body is found or a murder is committed.) Then what form does the action take in the middle of the story? (You and the detective search for and find clues about the crime or murder.) Can you tell me about the ending or conclusion of a mystery? (The detective figures out who-dunnit and solves the case.) Every story also has contextual details that make it interesting, but do not tell you what kind of story it is. For example, the main character in a mystery can be a man or a woman who is a professional detective, or a young person like you who likes to solve puzzles. Young or old, fat or thin -- this contextual detail won't tell you what type of story it is. The robbery or murder can take place on a train, in an English country garden, or in a school in Vancouver. The story will be a mystery if it has the form of a mystery. Can you think of another contextual detail that won't affect what kind of story it is?

There are a number of types of story problems just as there are types of stories. Each type has its own form and is predictable. You will learn to know what to look for in the form of the story problem so you can decide what type of problem it is. Each story problem also has contextual details. But like the stories, these are not a reliable guide to the type of story problem. A problem could be about money, age or doing work. What else could a story problem be about? (Buying something, distance or travelling somewhere, about test marks, etc.).

Word problems have quantities or amounts in them. These may be made clear to you. Or they may be unknown, and you will have to figure them out. In the simplest story problems, there are three quantities. Two are expressed in numbers -- they are given or known. The third is unknown. You must figure this one out by using what you known. The factors we know are given in numbers in the problem. Or they are things we know from experience. There are 7 days in a week, 100 cm in a metre, and so on.

The quantities we don't know are the ones we are trying to figure out. In order to find the unknowns we begin by letting a letter "stand for" or "take the place of" a number we don't know. If we don't know Sam's age, and this is important to the problem, we say, ... "Let X = Sam's age". The letter X could be any number. We don't know its value yet. By letting X take the place of the unknown, we make it easier to find the quantity we don't know yet. Now we can talk about it and call it by name - like "Mr. X", the stranger whose name we don't known. We figure out the unknown from the things we know in the problem.

We will study three types of word problems, one type at a time. Each type of problem is like a type of story. It has characteristics or features which help you to decide which type of story problem it is. I will explain these "mathematical" features to you so you can recognize them. They are concerned with the mathematical relationships in the problem. For example, a problem may be about the relationship between two quantities. One number is three times as big as another. A boy is 24 years younger than his dad, and so on. Each problem also has contextual details, or a "cover story". It is about age, number, money, distance, or work. But the cover stories will not help you decide what type of problem it is. Each type will have all five cover stories. (Show example of problem sheet with five cover stories.)

Summary:

We will be practising word problems, one type at a time.

It will help you to learn to identify each type of problem by looking at the mathematical relationships among the variables. Cover stories will vary, so they won't help you. Now I will teach you about the worksheet you will be doing your problems on, and the two phases of problem solving -- representation and solution.

Teaching Representation for Relational Problems

The first type of problem we are going to study I have called RELATIONAL. In each of these problems you will find a relational statement. Finding a relational statement will help you to identify a problem as a relational problem. A relational statement is a sentence that tells you about one thing you don't know (or one unknown quantity) in terms of its relationship to something else that you don't know (another unknown quantity). Let me give you an example: a problem may say that I am twice as old as you. Does this problem say how old you are? (student expected to say no) Does it say how old I am? (student expected to say no) But it does tell us that one thing we don't know is twice as much as something we don't know. This is what I mean when I say that one unknown (my age) is expressed in terms of its relationship to another unknown (your age which has not been given either).

Let's consider another example. A relational problem contains the statement that Jane has $4 more than Mary. Do we know how much money Jane has? Do we know how much money Mary has? What do we know? That's right, we know that Jane has $4 more. What do we call this kind of statement? What do we call this kind of problem?

In relational problems we are given some additional information about the quantities. For example, we may be told that the sum of our ages if 45. That means that when we add the two unknown ages together we will get 45. This information will help us to write an equation or a true mathematical sentence about the quantities in the problem. Or we may be told that the difference between two unknowns is 20. In this case we would know that if we write down the larger quantity minus the smaller quantity equals 20, we will get a true mathematical sentence, or equation.

How can we figure out what two things are that we don't know, if we have the relational statement and the total? Remember I told you about using a letter to take the place of something you don't know. In relational problems we usually let the simplest unknown equal X or X or any other letter you like. Let's say that X stands for your age which we don't know. If I am twice as old, I must be 2 times X years old. That is because twice means 2 times and 2 times X is 2X. In relational problems we let the letter stand for the simplest unknown. Then we state the second unknown in terms of its relationship (relational statement) to the first unknown.

Suppose the simplest unknown was X and the other was 30 more. What is 30 more than X? X + 30 is 30 more than X. If a man is 7 years younger than X how old is he? X-7.

The relational statement in a relational problem tells you the relationship between two variables. Sometimes this is a complicated sentence. Try to find the simplest variable and set it equal to X. Then figure out the value of the other variable in relation to X.

Now I will show you how to use the self-questions to represent relational problems on the worksheet.

Teaching Solution for Relational Problems

You have written an equation. You will be working to find the values of the unknowns or letters in this equation. That is your subgoal. Your real goal is to answer the question that you wrote down beside GOAL on the worksheet. You want to find the goal. And write a sentence that answers that question.

The equation that you wrote helps you to do that. But an equation is a special sentence. It says that the left side is equal to the right side. This is like a delicately balanced scale. Think about the balances that you have used in science: (hold out hands to show an equal-arm balance). If you place more weight on one side of a scale, then what must you do to make both sides equal again? Right, add the same amount to the other side. Equations are like this too. Whatever you do to one side you must do to the other side. Because the two sides are equal. If you add anything in to the equation from outside, take anything away, or multiply or divide one side of the equation by a number that you introduce, then you must be certain to do the same thing to both sides of the equation. Remember that the equation is like the equal arm balance.

When you are adding terms that are already in an equation, there are a couple of things you must remember. You can only add terms that are like each other. You have heard the expression that you can't add apples and oranges. Well you can't add X + 2 and get 3. 1X + 2X = 3X and 1 + 2 = 3, but X + 2 is X + 2. That is why we will be trying to isolate the unknowns, that is get all the X's together on one side of the equation. Then we can say 3X = 30, or something like that. After that it is easy to find the value of 1X. Usually we add and subtract before we multiple and divide in solving these equations. You could see in what I was showing you that we need to collect all the X's together in order to find the value of each one.

I want to remind you about operations that are the opposite or obverse of each other. If you add 3 to your money, how can you get back to what you had to start with? Right, you subtract 3. If you multiply what you have by 5, and then want to undo that operation, what must you do? Right, divide by 5. In solving our equations we will frequently be using the opposite operation after we have collected (added, subtracted) like terms.

It will be necessary to check your answer with the goal you stated in the representation, when you have finished. Make a sentence to satisfy that goal. Some people highlight their answer by circling or underlining.

Then you must check your answer against what you wrote down that you knew in the representation. Your answer must be consistent with these findings. Is the relational statement still true when you substitute these values into it? If so you are finished, if not check your solution, if problem not found, check your representation, problem not found, ask for assistance.

Now I will show you how to use the self-questions to find the solution to relational problems on the worksheet.

Teaching Representation for Proportion Problems

The next type of problems we will work on are proportion problems.

They have ratios in them:
You often need to compare one quantity with another. You get 9 questions correct on a recent test. There were 10 questions in all on the test. You compare the number correct to the number of questions 9/10 or 9 : 10.

Both are read as "9 compared to 10" or, for short "9 to 10". 9 and 10 are called the "terms" of the ratio.

Sometimes in a problem we have two ratios which are equivalent ratios. That means that they are equal to each other. Equivalent ratios make up a proportion.

For example:

$$\frac{9}{10} = \frac{18}{20} = \frac{27}{30}$$

To obtain 18/20 from 9/10, you multiple both the numerator (top) and the denominator (bottom) by 2. How do you obtain 27/30 from 9/10?

In proportion problems, you will find one complete ratio such as 4/5, which we will call the given ratio or the ratio. You will also find one incomplete ratio such as something you don't know compared to 25. This will be the new case. Let X take the place of the something you don't known

Let me give you an example: You have obtained 4/5 problems correct. If there had been 25 problems, and you were just as successful, how many problems would you have had correct? 4/5 is equal to X/25. When you have a complete ratio and an incomplete ratio, then you know that it is a proportion problem. 5 is equal to X compared to 25.

Now I will show you how to use the self-questions to represent proportion problems on the worksheets.

Teaching Solution for Proportion Problems

A proportion is a mathematical sentence which states that two ratios are equivalent.

Using the equivalent ratios $\frac{2}{5}$ and $\frac{6}{15}$ we may write the proportion $2/5 = 6/15$. This proportion may also be expressed as $2 : 5 = 6 : 15$. The proportion in both forms is read "2 is to 5 as 6 is to 15".

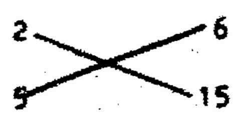
There are four terms in a proportion as shown:

first -> 2 = 6 <- third
second -> 5 = 15 <- fourth

The first term (2) and the fourth term (15) are called the **extremes**. The second term (5) and the third term (6) are called the **means**. Observe that the product of the extremes (2 x 15) is equal to the product of the means (5 x 6).

In the form

$\frac{2}{5} = \frac{6}{15}$ observe that the cross products are equal.



$2 \times 15 = 5 \times 6$

If any three of the four terms of the proportion are known quantities, the fourth may be determined.

Multiply the numerator of the first fraction by the denominator of the second fraction. Multiply the denominator of the first fraction by the numerator of the second fraction. This is called cross-multiplying.

Write one product equal to the other.

Then solve the resulting equation.

You may use another procedure instead of cross multiplying.

If $\frac{4}{5} = \frac{X}{25}$, you may multiply both sides of the equation by the inverse of the complete ratio. In this case, multiple $\frac{4}{5}$ and $\frac{X}{25}$ each by $\frac{5}{4}$. This will give you $1 = \frac{5X}{100}$. This means $5X = 100$

Now you can solve this equation to find the value of X.

Now I will show you how to use the self-questions to find the solution to proportion problems.

Teaching Representation of Two-Variable Two-Equation Problems

These new problems, our third type, are called two-variable, two-equation problems. That is because in order to represent and solve them you will need to use two variables or two unknowns. Let one of the things that you don't know equal X. Let the other equal Y. You will read over the problem and find that there are two relatively simple unknowns, such as the number of dimes and the number of quarters. And you will also find that you know the total of these two unknowns, such as 15 coins in all. That means X + Y = 15. However, there will be more information given in the problem. This additional information will enable you to write a second equation. The unknown quantities in this second equation will be more complex. For example, instead of just having numbers of coins, you may find that you have the total value of the coins. In order to write an equation it will be necessary to use the information given in the problem and information that you already have in your head. You know there are X dimes. How much is each dime worth? Right -- 10 cents. How can you find the value of X dimes? Right -- multiply X by 10 cents. You will get 10X cents. Now how many quarters did we say we had? And how much is each quarter worth? How much are Y quarters worth? You can see that 10X + 25Y cents will equal the total amount of money given in the problem. That will be your second equation.

These new problems will require two variables (X and Y) and two equations -- the simplest equation will involve X and Y. The more complex equation will involve derived values that include X in one case and Y in the other case. There will be two totals given in these problems. And you will have to use two unknowns.

These problems may be about age, work, distance or numbers, as well as money. Suppose you got one book each year and your friend got 2 books each year. We know the total number of books the two of you have and the total of your ages. Then we could use two variables, one for each age. We could write two equations -- one telling a true sentence about the total number of years. The other telling a true sentence about the total number of books. That would be a two-variable two-equation problem.

Let's try some now. Remember to watch for two quantities that will have to be represented by unknowns. Remember to watch for two totals that you can use to write equations.

Teaching Solution for Two Variable Two-Equation Problems

Now, we have two equations, each containing the same unknowns, say X and Y. These are much more complex problems than we have solved before. To do the solution for our first two kinds of problems (relational and proportion) we learned an algorithm that worked for each kind of problem. Sometimes you will come across a complicated problem or a new kind of problem. In these cases you may not know an algorithm or a set of steps for solving the equation. We are going to learn a new procedure that you can always try once you have written the equation(s). This is especially helpful for unfamiliar problems. This procedure is called systematic trial-and-error. You try numbers -- not just any numbers. You choose them for a reason. And you keep a record of the numbers you try, the results you obtain, and the outcome. Did the numbers satisfy the two equations? Was the result too large, or too small?

In order to keep the record it will be necessary to draw a chart.

The chart will have headings for X, Y, and the values in the second equation, along with totals.

We will try this procedure in a two-variable two-equation problem, so you can see how it works.

APPENDIX K

Problems for Transfer Tasks

Transfer Exercise 1

1. Barb's score in spelling was twice as much as her score in math. The total of her scores was 48. Find her score in spelling.

2. The volume of the big box was 32 cm^3 more than the volume of the little box. The sum of their volumes was 72 cm^3. Find the volume of each box.

3. In the class election Chris received 14 votes more than Mark. Together they received 22 votes. How many votes did Mark get?

4. The mass of chemical A was 3 times as much as the mass of chemical B. The total mass was 52 gm. Find the mass of each chemical.

5. The difference between the area of a big room and a small room is 12 m^2. The total of the two areas is 48 m^2. Find the area of the small room.

Transfer Exercise 2

1. The ratio of the mass of the big box to the mass of the small box is 3 : 2. If the mass of the big box is 42 gm, find the mass of the small box.

2. The red container holds 12 m^3 for every 7 m^3 held by the green container. The volume of the green container is 56 m^3. Find the volume of the red container.

3. In the election Jenny got 7 votes for every 6 votes received by Mary. Mary received 18 votes. How many votes did Jenny receive?

4. The ratio of the area of the new rug to the old rug is 5 to 3. The new rug covers 15 m^2. How many m^2 does the new rug cover?

5. Charles got 10 questions right in science for every 3 he got right in math. If he had 18 questions right in math, how many did he have right in science?

Transfer Exercise 3

1. John packed 7 crates in all. Each big crate is 100 m^3. Each small crate is 60 m^3. The total volume of the crates he packed is 540 m^3. How many crates of each size did he pack?

2. In the elections there was a total of 24 votes. Mr. Big Spender spent $5 for each vote he received. Mr. Cheap spent $2 for each vote he received. The total money spent was $81. Find the number of votes received by each man.

3. Farmer Jones has 6 fields in all. Each high field is 60 m^2. Each low field is 50 m^2 The total area is 340 m^2. How many high fields were there?

4. The mass of each box of salt is 30 kg. The mass of each box of sulphur is 20 kg. There are 14 boxes in all, with a total mass of 380 kg. How many boxes of salt were there? How many boxes of sulphur?

5. On test A Sandy received 4 marks on each page, and on test B she received 7 marks on each page. The total number of pages was 15. The total number of marks was 99. Find the number of pages in test B.

Transfer Exercise 4

1. Jane has some money. Sam has $9 more than 3 times as much as Jane. Together they have $41. Find how much money Jane has.

2. A man is 2 years older than twice as old as his son. Find how old the man is if the total of their ages is 38.

3. Jenny jogged 10 km less than 4 times as far as Mark. The total distance they jogged was 40 km. Find the distance each jogged.

4. This week Mary worked 3 hours less than twice as much as she worked last week. Over both weeks she worked 21 hours. How many hours did she work last week?

5. One number is 3 more than 4 times another number. If the sum of the larger number and the smaller number is 33, find the smaller number. Find the larger number.

Transfer Exercise 5

1. The ratio of Mary's age to John's age to Frank's age is 3 :2 : 1. If Mary is 18, find John's age. Then find Frank's age.

2. For every 10 hours Brian works, his father works 25 hours, and his mother works 30 hours. Last month Brian worked 50 hours. Find how long each of his parents worked.

3. The ratio of three numbers is 7 : 5 : 2. The middle number is 35. Find the larger number and the smaller number.

4. Bob has $14 for every $10 Nancy has, and for every $2 Don has. Don has $18. Find how much money Bob has, and how much money Nancy has.

5. A motorist travels 250 km in 5 hours and uses 4 litres of gas. At that rate, how far will she travel in 15 hours, and how many litres of gas will she use?

Transfer Exercise 6

1. Sam has 95 cents in nickels, dimes and quarters. He has 10 coins in all. 9 of these coins are nickels and dimes. How many coins of each kind does he have?

2. There were a number of bicycles, tricycles, and cars going down the street. There were 15 vehicles in all and 45 wheels. 10 of these vehicles were bicycles and tricycles. Find the number of each kind of vehicle.

3. A clothing factory makes blouses, skirts, and dresses. A blouse takes 5 metres of material, a skirt takes 7 metres, and a dress takes 10 metres. The factory uses a total of 66 metres and produces 10 pieces of clothing. 7 of the pieces are blouses and dresses. How many of each kind are produced daily?

4. The sum of the ages of A and B and C is 18. The sum of the ages of A and B is 10. A has read 3 books each year. B has read 2 books each year and C has read 7 books each year. Altogether they have read 80 books. Find the age of A, B and C.

5. A woman traveled by bus, taxi, and small boat for a total cost of $114. The bus cost $1 per km, the taxi cost $3 per km, and the boat cost $5 per km. She travelled a total of 28 km in all. She travelled 8 of those km by bus and taxi. Find the distance she travelled by each vehicle.

APPENDIX L

Problems for Think-Aloud Protocols

Problem for Relational Think-Aloud Protocol

Sam has $15 more than Tom. Together they have $82. Find the amount of money each boy has.

Problem for Proportion Think-Aloud Protocol

Brian saved $50 in 17 weeks. At that rate how long will it take him to save $350?

Problem for Two-Variable Two-Equation Think-Aloud Protocol

Andrew has 18 coins, some quarters and the rest dimes. The total value of the coins if $3.45. Find the number of each kind of coin.

APPENDIX M

Dependent Measures Administered at Pretest and Posttest

M-1. British Columbia Applications

M-2. Q2 (British Columbia Grade 10 Open-Ended Problem-Solving Measure)

M-3. Metacognitive Interview

M-4. Flowchart for Administration of Metacognitive Interview

M-5. Classification Task

M-6. Answer Sheet for Classification Task

BRITISH COLUMBIA

MATHEMATICS ACHIEVEMENT TESTS

GRADE 7/8

APPLICATIONS

REVISED 1980

Name ______________________________

Instructions

1. Do NOT open the test booklet until you are told to do so.
2. Be sure that you have a ruler, a pencil, and an eraser.
3. Do NOT use a compass, a protractor, or a calculator.
4. There are four answer choices for each question. Make a ✓ in the box which corresponds to your answer for each question.
5. Mark only one box per question.
6. If you have no idea of the correct answer, leave the question blank.

David F. Robitaille James M. Sherrill Heather J. Kelleher John Klassen

Province of British Columbia
Ministry of Education
Learning Assessment Branch

PART A — ANALYZING WORD PROBLEMS

(1) You know how much money you had at the start and at the finish of an automobile trip. To find out how much money you spent on the trip, you would:

a. add .... ☐
b. multiply .... ☐
c. divide .... ☐
d. subtract .... ☐

(2) Dorothy earned b cents and spent d cents. How many cents did she have left?

a) bd .... ☐
b) $\frac{b}{d}$ .... ☐
c) b - d .... ☐
d) $\frac{d}{b}$ .... ☐

(3) In 1976, a factory produced 320 radios per day. The factory ran 12 hours a day for 6 days every week. If you were required to find out how many radios were produced in 1 week then which of the following pairs of numbers would you use?

a. 12, 1976 .... ☐
b. 320, 12 .... ☐
c. 12, 6 .... ☐
d. 320, 6 .... ☐

(4) The age, in years, of a tree is represented by T. The age of a smaller tree, 64 years younger, is represented by t. Which of the following shows the difference in their ages?

a. t - T = 64 .... ☐
b. T + t = 64 .... ☐
c. T - t = 64 .... ☐
d. T + 64 = t .... ☐

(5) Read the following problem carefully and then select an open sentence that will help to solve the problem.

91 has two prime factors. If 7 is one, what is the other?

a. 91 x n = 7 .... ☐
b. 91 + 7 = n .... ☐
c. n ÷ 7 = 91 .... ☐
d. 91 ÷ 7 = n .... ☐

(6) Three boys, each with the same amount of money, combined their money and purchased a football for $15.60 and a ball pump for $4.40. They had $40.00 left after their purchases. Which open sentence could be used to determine how much money each boy had to begin with?

a. n ($15.60 + $3.00 + $4.40) = $40.00 .... ☐
b. 3n - ($15.60 + $4.40) = $40.00 .... ☐
c. $40.00 – ($15.60 + $4.40) = n .... ☐
d. n - ($15.60 + $4.40 + $3.00) = $40.00 .... ☐

(7) It is your job to give out programmes to each member of the audience for your class play. The afternoon of the performance you begin with 95 programmes. To find out how many programmes will be left over after the play, you will need to know:

a. The number of people in the cast........................ ☐
b. The number of people in the audience .... ☐
c. The size of the programme .... ☐
d. The name of the play .... ☐

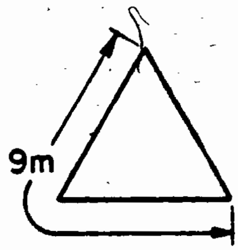
(8) Mr. Roberts travelled a distance of 200 kilometres. His car used 1 litre of gas to go 12 kilometres. Gas costs 22 cents per litre. How much will he spend for gas? Choose the correct number sentence.

a. $\frac{200}{12} \times 0.22 = n$ .... ☐
b. $\frac{200}{0.22} \times 12 = n$ .... ☐
c. $\frac{0.22 \times 12}{200} = n$ .... ☐
d. $\frac{200}{12 \times 0.22} = n$ .... ☐

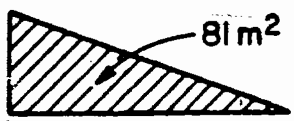
Analyzing Word Problems: ____ out of 8

PART B — USE OF DIAGRAMS

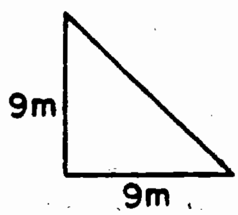
(9) A garden in the shape of a triangle has a perimeter of 24 metres. Each of two sides is 9 metres in length. If we want to calculate the length of the third side, which of the following diagrams would help?

a. 

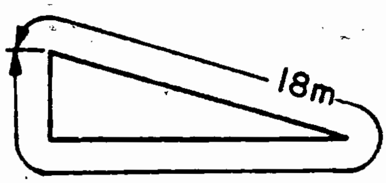
.... ☐

b. 

.... ☐

c. 

.... ☐

d. 

.... ☐

(10) The scale of a map is 1 cm = 80 km. How many centimetres long must a line on the map be to show a distance of 60 km?

a. $\frac{3}{4}$ .... ☐

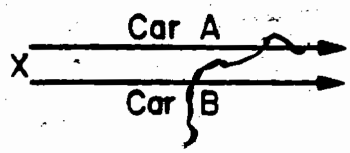
b. $1\frac{1}{3}$ .... ☐

c. 9 .... ☐

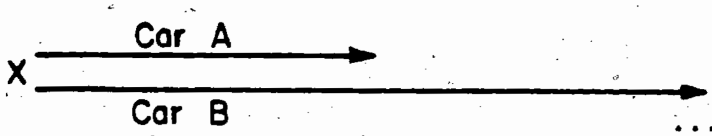
d. 48 .... ☐

(11) Which of the diagrams below best illustrate the problem?

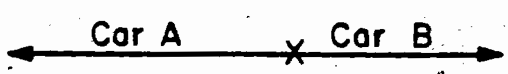
Car A and car B leave Vancouver at the same time and travel along the same road in the same direction. Car A goes twice as fast as Car B. Their position at the end of 2 hours is:

a. 

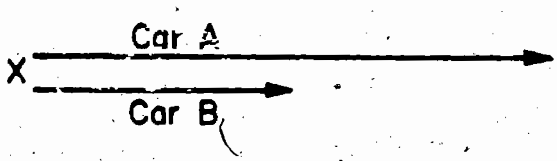
.... ☐

b. 

.... ☐

c. 

.... ☐

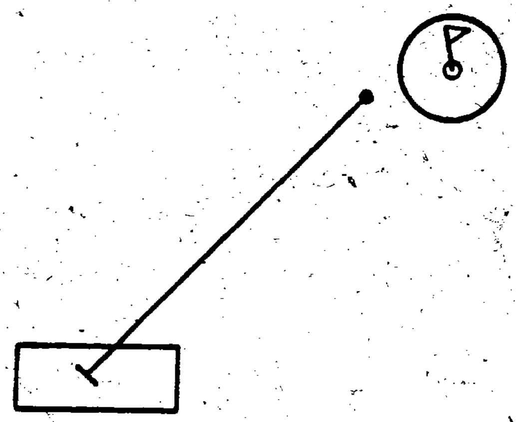
d. 

.... ☐

(12) On a certain map, 3 centimetres represents 60 kilometres. How many kilometres does 6 centimetres represent?

a. 90 .... ☐
b. 100 .... ☐
c. 120 .... ☐
d. 140 .... ☐

(13) Zack Nick Claws, the golfer, hits a golf drive in the World Golf Championship. Using a metric ruler, find out how far he hit the golf ball if the diagram is drawn to a scale of 1 cm = 50 m.

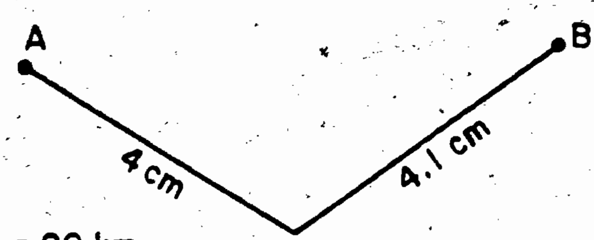


a. 250 m .... ☐
b. 2250 cm .... ☐
c. 45 m .... ☐
d. 450 m .... ☐

(14) The following map is drawn to a scale of 1 cm = 50 km. Using your ruler find the distance from A to B.

a. 4 km .... ☐
b. 200 km .... ☐
c. 400 km .... ☐
d. 50 km .... ☐

(15) The following diagram represents the map of a road joining Town A to Town B. What is the actual road distance from A to B?



SCALE : 1 cm = 20 km

a. 8.1 km .... ☐

b. 81 km .... ☐

c. 162 km .... ☐

d. 16.2 km .... ☐

Use of Diagrams: ____ out of 7

PART E — SOLVING PROBLEMS

(31) If a bank charges $1\frac{1}{2}$ % interest per month, what is the yearly rate of interest?

a. 18% .... ☐
b. 1.5% .... ☐
c. 9% .... ☐
d. 12% .... ☐

(32) Which of the following numbers is about five times as large as 250?

a. 1 000 .... ☐
b. 1 350 .... ☐
c. 255 .... ☐
d. 10 250 .... ☐

(33) How does 105% of a number compare in size with the number?

a. slightly larger .... ☐
b. more than twice .... ☐
c. slightly smaller .... ☐
d. less than half .... ☐

(34) Last baseball season, John went up to bat 30 times. In those 30 times at bat, John got 18 hits. What percent of John's times at bat resulted in hits?

a. 167 .... ☐
b. 60 .... ☐
c. 30 .... ☐
d. 18 .... ☐

(35) A dump truck can hold 18 m^3 of dirt at one time. Approximately how many trips would be required to haul away 3570 m^3 of dirt?

a. 2000 .... ☐
b. 10 .... ☐
c. 100 .... ☐
d. 200 .... ☐

(36) Estimate the most appropriate answer for the following problem.

A department store purchases 298 boxes of baseball gloves. Each box is worth $96.00. If each glove is worth $48.00, about how many baseball gloves were ordered?

a. 500 .... ☐
b. 600 .... ☐
c. 700 .... ☐
d. 750 .... ☐

(37) The total possible score on a test is 50. Jeannie scores 40. Her mark as a percentage is:

a. 80% .... ☐
b. 125% .... ☐
c. 60% .... ☐
d. 90% .... ☐

(38) Bob achieved a mark of 42 out of 50 on his last mathematics test. What percent did he get correct?

a. 42% .... ☐
b. 92% .... ☐
c. 16% .... ☐
d. 84% .... ☐

(39) Ruth budgets her yearly allowance this way: clothes, $80.00; lunches, $50.00; carfare, $20.00; shows, $20.00; miscellaneous, $30.00. What percent of her allowance does she spend for clothes?

a. 25% .... ☐
b. $33\frac{1}{3}$% .... ☐
c. 40% .... ☐
d. 80% .... ☐

Suppose that a rocket ship can travel from the Earth to Mars and back to Earth in 520 days. It takes 45 years and 273 days to reach the planet Pluto. Approximately how many round trips could one rocket ship make between the Earth and Mars while a second ship was traveling to Pluto?

a. 3 .... ☐
b. 30 .... ☐
c. 300 .... ☐
d. 3000 .... ☐

Solving Problems: ____ out of 10

Name: ______________________ Date: ______________________

School: ______________________ Group: ______________________

Time: ____

SHOW ALL OF YOUR WORK FOR EACH OF THE FOLLOWING QUESTIONS IN THIS BOOKLET.

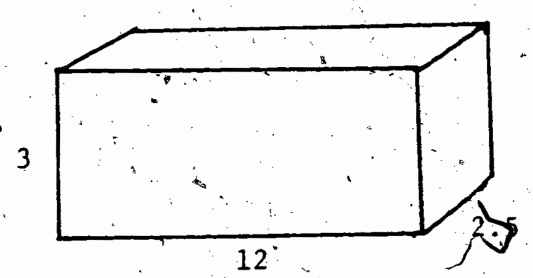
1. A dump truck can hold 15.5 m^3 of gravel at one time. How many trips would be required to haul away 2480 m^3 of gravel?

2. Mr. Jones put a fence around his rectangular garden. The garden is 10 m long and 6 m wide. How many metres of fencing did he use?

3. In a school election with three candidates, Mike received 120 votes, Lawrence received 30 votes, and Lesley received 50 votes. What percent of the total vote did Mike receive?

4. A carpenter cuts a board 10 metres long into lengths of $2\frac{1}{2}$ metres. How many lengths will he get?

5. Find the volume of this box



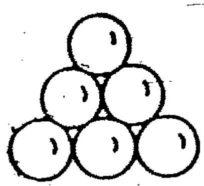
6. Bill's mother bought $2\frac{1}{3}$ dozen chocolate chip cookies, $1\frac{1}{4}$ dozen ginger snaps, and $1\frac{1}{2}$ dozen peanut butter cookies. How many dozen cookies did she buy altogether?

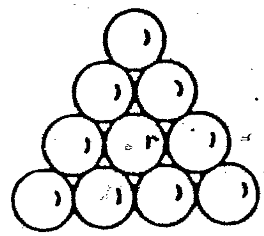
7. Carol owns 3 less than twice as many records as Bob. If she owns 17 records, how many does Bob own?

8. A litre of asphalt paint will cover about 6 m^2 of surface. The paint is sold in cans of 5 litres only. How many cans are needed to paint a driveway 15 m long and 3 m wide?

9. Some candies cost 10 cents each and others cost 25 cents each. Susan bought 3 candies. How many different amounts could Susan have spent?

10. Marbles are arranged in the shape of a triangle on the floor. How many marbles are there in a triangle with 7 marbles in the base?





11. I am a number between 25 and 40. I have a remainder of two when divided by <u>both</u> 6 and 9. Who am I?

12. Susan and Bill went fishing. They both caught two kinds of fish, herring and cod. Bill caught 10 herring. Together they caught 18 herring and 10 cod. Susan caught 15 fish. How many cod did Bill catch?

13. There are 6 fence posts on one side of a garden. The posts are 2 m apart. What is the distance from the first post to the last post?

NAME: ______________________ DATE: ______________________

EXP. PHASE: ______________________ GROUP: ______________________

INTERVIEWER: ______________________ RECORDED: ______________________

INSTRUCTIONS FOR METACOGNITIVE INTERVIEW

I am going to ask you some questions about mathematics word problems. There will be questions about what word problems are, and about people doing word problems. There will also be some questions about what you do when you are working on word problems. I want you to tell me what you actually do, not what you think you are supposed to do, in answering the questions about you.

If you are not certain about the meaning of a question, just let me know. I will say the question again, in a different way, so you are sure about what the question is asking. I will be taking notes while you are talking, and also tape-recording our interview. I am doing these things to help me, so I do not have to remember everything that you say. Do you have any questions?

Let us begin!

Note. See accompanying flowchart for Metacognitive Interview.

METACOGNITIVE INTERVIEW

1. What are mathematics word problems?

PROBE: Please describe them to me, and tell me all you know about them.

(task)

2. What makes someone really good at doing word problems?

PROBE: Think of someone you know who is really good at doing word problems. What is it about them, or what do they do, that makes them so good at it?

(person)

3. What is the hardest part about doing word problems, for you?

PROBE: Think of a time when you found a word problem really hard. What is that made <u>that</u> problem so hard for you, and word problems <u>in general</u>?

(person)

4. What would help you become a better problem solver?

PROBE: Think of that difficult problem again. What would help you to be better at solving it and other hard problems?

(person)

5. If you were getting ready to take a word problem test, what would do do that would help you the most do do well on that test?

PROBE: What kinds of things can help people get ready for problem tests, and what would you do that would help you?

(strategy)

6. Are some parts of a word problem, as it is written down, more important than others? How can you tell which parts are the most important?

PROBE: Think of the parts of a word problem that you have done recently, and ask yourself how you found the important parts in this problem, in other problems.

(task)

7. What do you do if you don't know what a word means in a word problem?

PROBE: Think of a problem that has a tough word in it, and you are not sure of the meaning of that word. What do you do when this happens?

(strategy)

8. What do you do if you don't get the "whole picture" or the "whole meaning" of a word problem?

PROBE: Think of a problem where you can read the words, but you are not sure what they all mean together. What do you do in these situations?

(strategy)

9. After you have read and understood a word problem, what else must you still do in order to complete the problem successfully?

PROBE: What plans do you make, after reading a problem, in order to do it successfully? What does it mean to do a problem successfully?

(strategy)

10. What about a word problem makes it easy to do?

PROBE: Think of an easy word problem, one you can do without any difficulty. Tell me what makes that problem easy to do. Generally, what makes some word problems easy, while others are difficult?

(task)

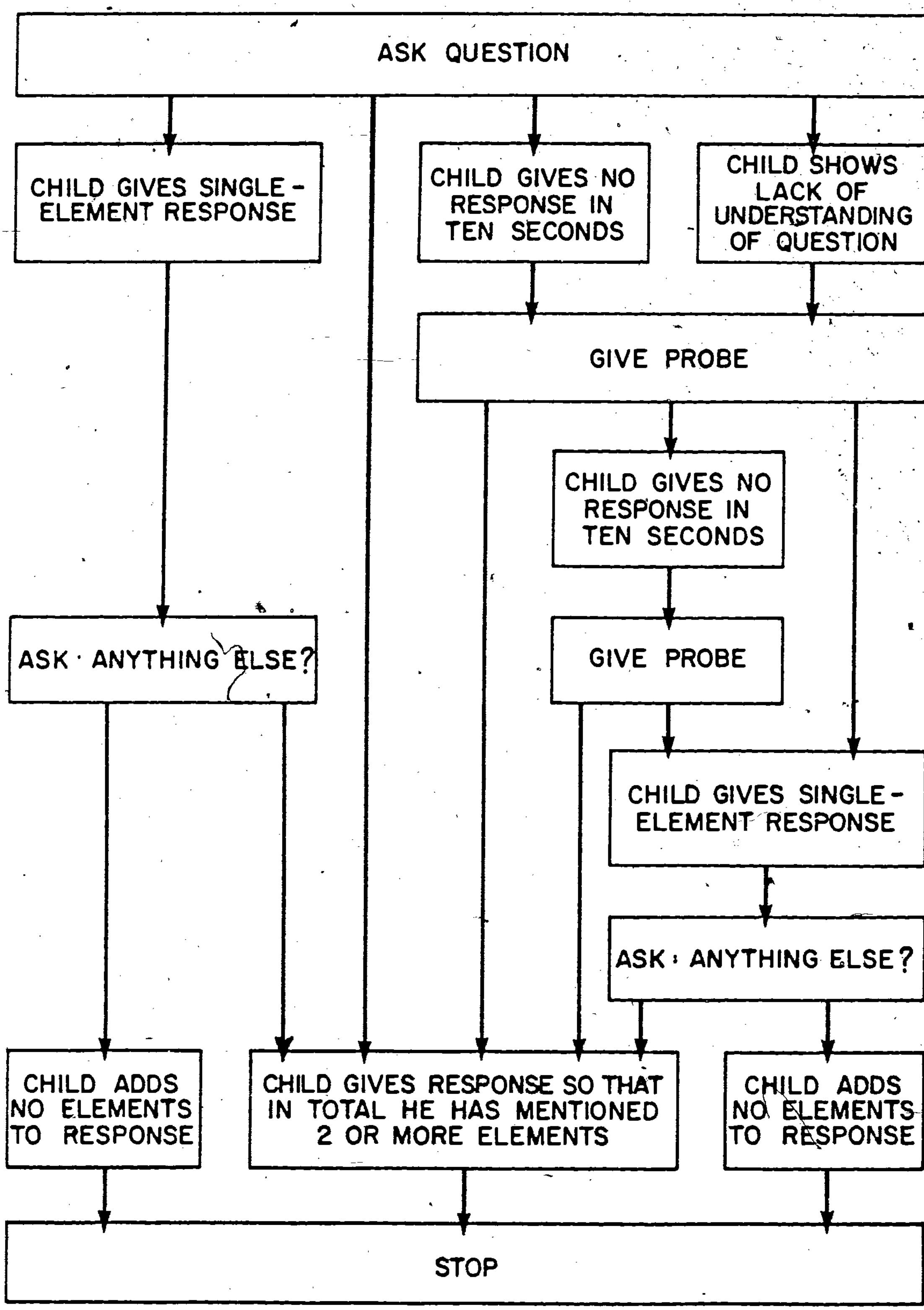
COMMENTS:

Nancy L. Hutchinson (1985)

Adapted from Myers and Paris (1978) and

Paris (1979)

FLOWCHART FOR METACOGNITIVE INTERVIEW



CLASSIFYING PROBLEMS

Choose the <u>two problems</u> in each group that are most alike. In a couple words tell why they are alike. Answer on the accompanying worksheet.

GROUP 1

A. Sharon and John worked a total of 47 hours last week. John worked 5 hours less than Sharon. Find the number of hours each person worked last week.

B. A man can build 7 birdhouses in 5 days. If he works for 45 days, how many birdhouses can he build?

C. The sum of two consecutive odd numbers is 96. Find each number.

ANSWER ON THE WORKSHEET.

Group 2

A. For every 6 dimes in my bank, there are 5 nickels. If I have 78 dimes, find the number of nickels.

B. A picture 14 cm wide and 17 cm high is to be enlarged so that the width will be 70 cm. How high will it be?

C. Frank received his wages for each of the last two weeks. He received twice as much money the first week as the second week. The total amount he recived was $183. How much money did he receive each week?

ANSWER ON THE WORKSHEET.

Group 3

A. The ratio of May's age to Bruce's age is 7:6. May is 56 years old. Find Bruce's age.

B. Jack invested $5000, part at 7% and the rest at 8% per year. The total income from these investments is $360. How much did he invest at each rate?

C. The sum of a man's age and his wife's age is 70. The man has watched 12 movies a year and his wife has watched 15 movies a year. Altogether they have watched 954 movies. Find the man's age.

ANSWER ON THE WORKSHEET.

Group 4

A. I rode a bicycle and a motorcycle for a total of 5 hours. I went at a rate of 8 km per hour on the bicycle and 30 km per hour on the motorcycle. The total distance was 84 km. Find the distance I rode on the motorcycle.

B. Pete and I drove to Kingstown together. It was 752 km. He drove 100 km farther than I did. How far did I drive?

C. The sum of the ages of Tom and Jeff is 99 years. Tom is 7 years older than Jeff. Find the age of each.

ANSWER ON THE WORKSHEET.

Group 5

A. Janice sells two kinds of hats. The wool hats cost $5.00 and the cotton hats cost $7.00. She sold 25 hats for a total of $143.00. How many hats of each kind did Janice sell?

B. The ratio of the number of men to women in a meeting is 7:4. If there are 36 women, then how many men must there be?

C. There were 30 boys and dogs in a park. I counted 84 legs. How many boys were in the park?

ANSWER ON THE WORKSHEET.

Group 6

A. A man flew and drove for a total cost of $500. The flying cost $50 per km and the driving cost $1 per km. The total distance he travelled was 353 km. Find the distance that he flew and the distance that he drove.

B. The sum of a man's age and a woman's age is 56. The man has read 10 books every year and the woman has read 12 books every year. Together they have read a total of 630 books. Find their ages.

C. The sum of Ron and Brian's ages is 81. Ron is 15 years older than Brian. Find their ages.

ANSWER ON THE WORKSHEET.

Group 7

A. At work Fran prunes 7 shrubs in the time Dave prunes 4 shrubs. If Dave prunes 28 shrubs today, how many will Fran prune today?

B. The ratio of a man's age to his son's age is 10:3. If the son is 27, find the man's age.

C. Mr. Smith makes 8 more chairs than Mr. Jones in one week at work. The total made by both is 32 in a week. How many does Mr. Jones make in that week?

ANSWER ON THE WORKSHEET.

Group 8

A. The sum of the distances of two flights made by a pilot is 336 km. One flight is three times as long as the other. Find the distance of each flight.

B. The ratio of two numbers is 7:3. If the smaller number is 57, find the larger number.

C. The sum of two consecutive odd numbers is 72. Find each of the numbers.

ANSWER ON THE WORKSHEET.

Group 9

A. Martha walked 16 km farther than Ted. The total distance they walked was 50 km. How far did Martha walk?

B. On a map 3 cm represents 250 km. What distance is represented by 15 cm?

C. Ms. Lawton saved $35 in 2 weeks. At that rate, how long will it take her to save $700?

ANSWER ON THE WORKSHEET.

Group 10

A. The larger of two numbers is five times the smaller. Find the numbers if their sum is 78.

B. A big box of cereal costs $1.20 and a small box costs $0.80. A man bought 34 boxes for a total cost of $34.00. How many boxes of each size did he buy?

C. A truck costs $2000 more than a car. THe total cost of a truck and a car is $28,500. Find the cost of the truck.

ANSWER ON THE WORKSHEET.

NAME ______________________ DATE ______________________

EXP.PHASE ______________________ GROUP ______________________

SCHOOL ______________________ TIME ______________________

ANSWER SHEET FOR CLASSIFYING PROBLEMS

Choose the <u>two problems</u> in each group that are most alike.

In a couple words tell why they are alike.

<u>Group 1</u>

Problems that are most alike: __________ and __________

Reason: ______________________________

<u>Group 2</u>

Problems that are most alike: __________ and __________

Reason: ______________________________

<u>Group 3</u>

Problems that are most alike: __________ and __________

Reason: ______________________________

<u>Group 4</u>

Problems that are most alike: __________ and __________

Reason: ______________________________

<u>Group 5</u>

Problems that are most alike: __________ and __________

Reason: ______________________________

Group 6

Problems that are most alike: __________ and ______________

Reason: ____

Group 7

Problems that are most alike: __________ and ______________

Reason: ____

Group 8

Problems that are most alike: __________ and ______________

Reason: ____

Group 9

Problems that are most alike: __________ and ______________

Reason: ____

Group 10

Problems that are most alike: __________ and ______________

Reason: ____

APPENDIX N

Observer Checklist

Name of student: ______________________ Problem ________________
Group: ______________ Phase: ______________________________
Session No. ___________ Date: ______________________________

Observer checklist

1. Student seeks help decoding: ____________________

2. Student seeks help with vocabulary: ____________________

3. Student seeks help with a sentence: ____________________

4. Student seeks help with the paragraph: ____________________

5. Help sought in addition to the above: ____________________

6. Student reads aloud: ____________________

7. Student rereads text (specify): ____________________

8. Writes goal: ____________________

9. Lists unknown(s): ____________________

10. Writes all that is known (stated): ____________________

(unstated): ____________________

11. Summarization attempted: ____________________

prompted: ____________________

12. Drawing attempted: ____________________

prompted: ____________________

13. Categorizes problem: ____________________

14. Writes equation: ____________________

15. Expands equation: ____________________

16. Collects like terms: ____________________

17. Isolates variable(s): ____________________

18. Uses inverse operation(s): ____________________

19. Writes value of unknown(s): ____________________

20. Highlights answer: ____________________

21. Compares answer to goal (meaningfulness): ____________________

22. Checks accuracy of calculations: ____________________

APPENDIX O

Raw Data

Raw Data (1 of 2)

```
ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz123456
__________________________________________________________
0121108174114041400501130500000400000000401
0221108160121061400500230700314503301071101
0311110188114040200200160700100200000000301
0411110188118040200200220700022200002020201
0512109169120070500501230901102400000030500
0612109169112020000001090300000100000000101
0722108180107010100101100200000100000000100
0822109162109030100101090300200100000000301
092120816611305122050523094345554441312131055555555333555
10112081611120101001021907444555000090909085555555     000555
11112081781220414004022108555555000101010074333555     000555
122121018411603010010217105555555551515150955555555232444
1311209190114030200202160655504500005091001555         555
1422209179110010000002160844400000000404004444         122
152220917611605020020123104555555551415150844455555555555
16122091711200424006002009555555500010101008555555     555555
171220916211805130040220115555555541515141055555454333000
18222101831100301001001507554555000101009035555555     000555
192221018011405022040219095555445541514130955455555555345
202220815110803010010118085555555444141414035554545540000444
```

Key:

AB = subject number
C = sex; 1 = M; 2 = F
D = school
E = group; 1 = C; 2 = I
FG = grade
HIJ = age (months)
K = card number
LM = pre B.C. Applicat.
NO = pre Q2
P = pre rel. ans.
Q = pre prop. ans.
R = pre two-v. two-e. ans.
ST = pre total ans.
UV = pre classif. task
WX = post B.C. Applic.
YZ = post Q2
a = post rel. rep.
b = post rel. sol.
c = post rel. ans.
d = post prop. rep.
e = post prop. sol.
f = post prop. ans.
g = post two-v. two-e. rep.
h = post two-v. two-e. sol.
i = post two-v. two-e. ans.
jk = post total rep.
lm = post total sol.
no = post total ans.
pq = post classif. task
r = near tran. rel. rep.
s = near tran. rel. sol.
t = near tran. rel. ans.
u = near tran. prop. rep.
v = near tran. prop. sol.
w = near tran. prop. ans.
x = near tran. two-v... rep.
y = near tran. two-v... sol.
z = near tran. two-v... ans.
1 = far tran. rel. rep.
2 = far tran. rel sol.
3 = far tran. rel ans.
4 = far tran. prop. rep.
5 = far tran. prop. sol.
6 = far tran. prop. ans.

NOTE: Subject numbers listed in raw data are translated into student numbers (as found in text) at the end of the raw data.

Raw Data (2 of 2)

```
ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz1234567
____________________________________________________________
012110817420507070000000000007350403040401
022110816020703080001000001083505040406 01
0311110188206080800000000000043004030104 02
041111018820510110000000000004350303010500
051210916920505070000000000001340500010401
061210916920406050000000000001350400010301
072210818020405080000000000004340303010400
082210916220507070000000000004340404000401
09212081662190412000000000000835050404110855454455555441513 13
101120816121707170000000000003330402011007     544555001100910
111120817821906140000000000003350403001108     323555000080708
12212101842200516001100010105330404011208555555555555151515
131120919021608150011000101053404050010 06     555004000050509
142220917922006080000000000004330304001208     111000000010101
152220917621807200000000000004330404001107555555555555151515
161220917121607150000000000003340502010907     444555333121212
171220916221907151100000101143504100411080004445443431 21211
182221018321805150000000000001350300011008     555555322131212
19222101802181016000000000000134030001100855455555445551514 14
20222081512180610000000000000534040401110700054444455414 1312
```

Key:

AB = subject number
C = sex; 1 = M; 2 = F
D = school
E = group; 1 = C 2 = E
FG = grade
HIJ= age (months)
K = card number
LM = post thinkaloud tot.
NO = pre metacog. int.
PQ = post metacog. int.
R = pre rel. rep.
S = pre rel. sol.
T = pre prop. rep.
U = pre prop. sol.
V = pre two-v. two-e. rep.
W = pre two-v. two-e. sol.
XY = pre total rep.
Za = pre total sol.
bc = pre thinkaloud tot.
de = crit. basic op.
fg = crit. one-step prob.
hi = pre thinkaloud rep.
jk = pre thinkaloud sol.
lm = post thinkaloud rep.
no = post thinkaloud sol.
p = far tran. two-v...rep.
q = far tran. two-v...sol.
r = far tran. two-v...ans.
s = mainten. rel. rep.
t = mainten. rel. sol.
u = mainten. rel. ans.
v = mainten. prop. rep.
w = mainten. prop. sol.
x = mainten. prop. ans.
y = mainten. two-v... rep.
z = mainten. two-v... sol.
1 = mainten. two-v... ans.
23 = mainten. total rep.
45 = mainten. total sol.
67 = mainten. total ans.

NOTE: Subject numbers listed in raw data are translated into student numbers (as found in text):

9 in raw data is Student 1 #15 in raw data is Student 3
#10 in raw data is Student 7 #16 in raw data is Student 9
#11 in raw data is Student 8 #17 in raw data is Student 4
#12 in raw data is Student 2 #18 in raw data is Student 10
#13 in raw data is Student 11 #19 in raw data is Student 5
#14 in raw data is Student 12 #20 in raw data is Student 6

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