SMOOTHING THE PRETEST ESTIMATOR: A MONTE CARLO STUDY FOR HETEROSKEDASTICITY.

by

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Smoothing the Pretest Estimator: A Monte Carlo Study for

Heteroskedasticity

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ABSTRACT

This thesis develops a generalized pretest estimator for heteroskedasticity as a 'smoothed' version of the traditional pretest estimator. Principles from Stein-Rule estimation and Bayesian analysis are used to combine linearly the ordinary least squares estimator(OLS) and the Two-Stage Aitken estimator(2SAE) by developing a weighting system which is a continuous function of the pretest statistic. First, adopting a Bayesian view, the probability that the degree of heteroskedasticity is such that the OLS estimator outperforms the 2SAE is estimated. Second, this probability is used as a weight to combine linearly the OLS estimator and the 2SAE to form the generalized pretest estimator. Several versions of this generalized pretest estimator are developed.

A Monte Carlo study is performed to investigate the mean square error properties of the several 'smoothed' versions of the generalized pretest estimator relative to those of the traditional pretest estimator, the ordinary least squares estimator and the Two-Stage Aitken estimator. The results indicate that the 'smoothed' pretest estimator is an attractive alternative to the traditional pretest estimator used in the context of heteroskedasticity.

DEDICATION

as the tale of time's mind unfolds
we fling our guilt in shame
and swim with burning fury
in the barreness of our expectations
yet our daily fantasies
are too frail to uproot the scales
that shut the door of our perception
we fume and hate in ignorance
and unlike the skillful kingfisher
we dive into arid oceans
that have never nursed a shoal of fishes

and wade back ashore
with a cup of nothingness in our hands

senyo adjibolosoo

TO SABINA

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Finally, to all of you and everyone who contributed in some ways towards the development of my ideas and thoughts, may I say to you that 'The Lord bless you, and keep you: the Lord make His face shine upon you and be gracious unto you: the Lord lift up His countenance upon you and give you peace'.

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CHAPTER I

GENERAL INTRODUCTION

1.1. INTRODUCTION

The purpose of this thesis is to generalize the pretest estimator for heteroskedasticity(by 'smoothing' it) and by means of a Monte Carlo study investigate its properties relative to those of the traditional pretest estimator.
Suppose the presence of heteroskedasticity is suspected and some test, such as the Goldfeld and Quandt F-test, is performed to investigate whether heteroskedasticity exists in the data. If this pretest accepts the null hypothesis of homoskedasticity, the ordinary least squares(OLS) estimator is used for estimation. If, however, the pretest rejects the null hypothesis, the two-stage Aitken estimator (2SAE) is used instead. This methodology defines the traditional pretest estimator for heteroskedasticity.

The traditional pretest estimator is a weighted average of the OLS estimator and the 2SAE where the weighting system is a dichotomous function of the pretest statistic. This thesis develops a 'smoothed' version of this pretest estimator in which the weighting system is a continuous function of the pretest statistic. This is accomplished by borrowing from the Bayesian approach in which the pretest estimator is structured as a weighted average estimator by combining the unrestricted and the restricted least squares estimators using as the weighting system the posterior probability of the null hypothesis. The main problem with this technique is that it requires an informative prior for the case of point null versus composite alternative hypothesis. Without an informative prior, the computation of the posterior odds breaks down making it impossible to calculate the Bayesian pretest estimator.

This thesis circumvents this problem by changing the point null hypothesis into a composite null hypothesis. Although the OLS estimator is dominated by the generalized least squares(GLS) estimator, its estimated version(2SAE) does not outperform the OLS estimator over the whole parameter space. The null hypothesis thus becomes homoskedasticity or heteroskedasticity of sufficiently small degree that the ordinary least squares(OLS) estimator outperforms the two-stage Aitken estimator(2SAE). We develop a means of computing the probability that the degree of heteroskedasticity is such that the ordinary least squares estimator dominates the 2SAE. Assuming that this is the posterior probability associated with the null hypothesis it is used as the weighting system in combining the ordinary least squares estimator and the 2SAE into the generalized(smoothed) pretest estimator as explained earlier. The rest of this chapter discusses in more detail the pretest estimator and its properties and the Stein estimator(a 'smoothed' pretest estimator) that provided the inspiration for the development of the generalized pretest estimator in this thesis.

1.2. THE PRETEST ESTIMATOR

The consequences of incorporating non-sample information into an estimation procedure depend on the quality of the information introduced. Consequently, the researcher may want to test the apriori non-sample information against the data before utilizing it. In this manner his main desire is that his statistical tests may reveal something about the truth and falsity of his apriori information; he either adopts or discards the non-sample information depending upon the results of the statistical test(s) performed. This procedure and rule of estimation is often referred to as the pretest estimator. This pretesting procedure is widely used in a variety of ways in econometrics of which the following are a few examples:

(i). Testing for the presence of heteroskedasticity or serial correlation and selecting either the ordinary least squares estimator or the the generalized least squares estimator based upon the result of the pretest[Greenberg(1980), King and Giles(1984)].

(ii). Including or Excluding a variable or a set of variables into a regression model by performing a preliminary t-test or F-test with the decision to either include or exclude depending upon the outcome of the test[Toro-Vizcarrondo and Wallace(1968, 1969), Wallace(1977)]

(iii). Using the Chow testing procedure to test whether or not a structural change has occurred. This procedure would help him determine whether to pool or not to pool the available data

[Kennedy(1985),pp 87-88].

(iv). Using Almon distributed lags where the polynomial degree is selected on the basis of hypothesis tests[Fomby et al(1984), pp 130].

(v). Checking the compatibility of stochastic sample and prior information before a mixed estimation process is used [Judge and Bock(1978)].

(vi). Using principal components when the number of components chosen to delete is based upon hypothesis testing [Fomby et al(1984),pp 130].

It is common(Judge et al(1985), Chapter 11) to portray the the character of a pretest estimator by means of its risk function(the risk of an estimator is the sum of the mean square errors of its components usually graphed as a function of the extent to which the hypothesis being tested is false). The nature of the risk function depends on a variety of parameters of which the main ones are:

(1). the level of significance($1 - \alpha$) or the pretest critical value,

(2). the regression variance (σ^2),

(3). the number of regressors and the number of restrictions,

(4). the restrictions being tested,

(5). the design matrix, and

(6). the degrees of freedom[Wallace, (1977)].

Risk functions for the restricted, unrestricted, pretest, and Stein estimators are shown in Figure 1.1 for the case of the linear regression model and a set of linear restrictions. On the horizontal axis is the measure of the extent to which the restrictions are false(in this case the non-centrality parameter); the vertical axis measures the risk of these estimators[Kennedy, 1985, pp 161; Wallace, 1977, pp 436].

The mean square error is the sum of the variance of the estimator and the square of the bias. Since unrestricted least squares is always unbiased and has a constant variance regardless of the validity of the restrictions, its risk function is a constant and, therefore, is drawn in the figure as a horizontal line beginning at G. From econometric theory, we know that the restricted least squares estimator has a smaller variance than the unrestricted least squares estimator. If the restrictions are true, it is also unbiased. But if the restrictions become more and more false, it suffers from more and more bias and therefore its risk function is a positively sloped line starting from point J in Figure 1.1.

The pretest estimator has a humped shape as shown in Figure 1.1. This humped shape of the risk function of the pretest estimator is a result of the dichotomous choice between the unrestricted and the restricted least squares estimators after pretesting. If the restrictions imposed are true, in repeated samples the null hypothesis is accepted$(1 - \alpha)\%$ of the time on the basis of the pretests and therefore its risk function is very close to(just above) that of the restricted least squares estimator. At the other extreme, if the restrictions are far

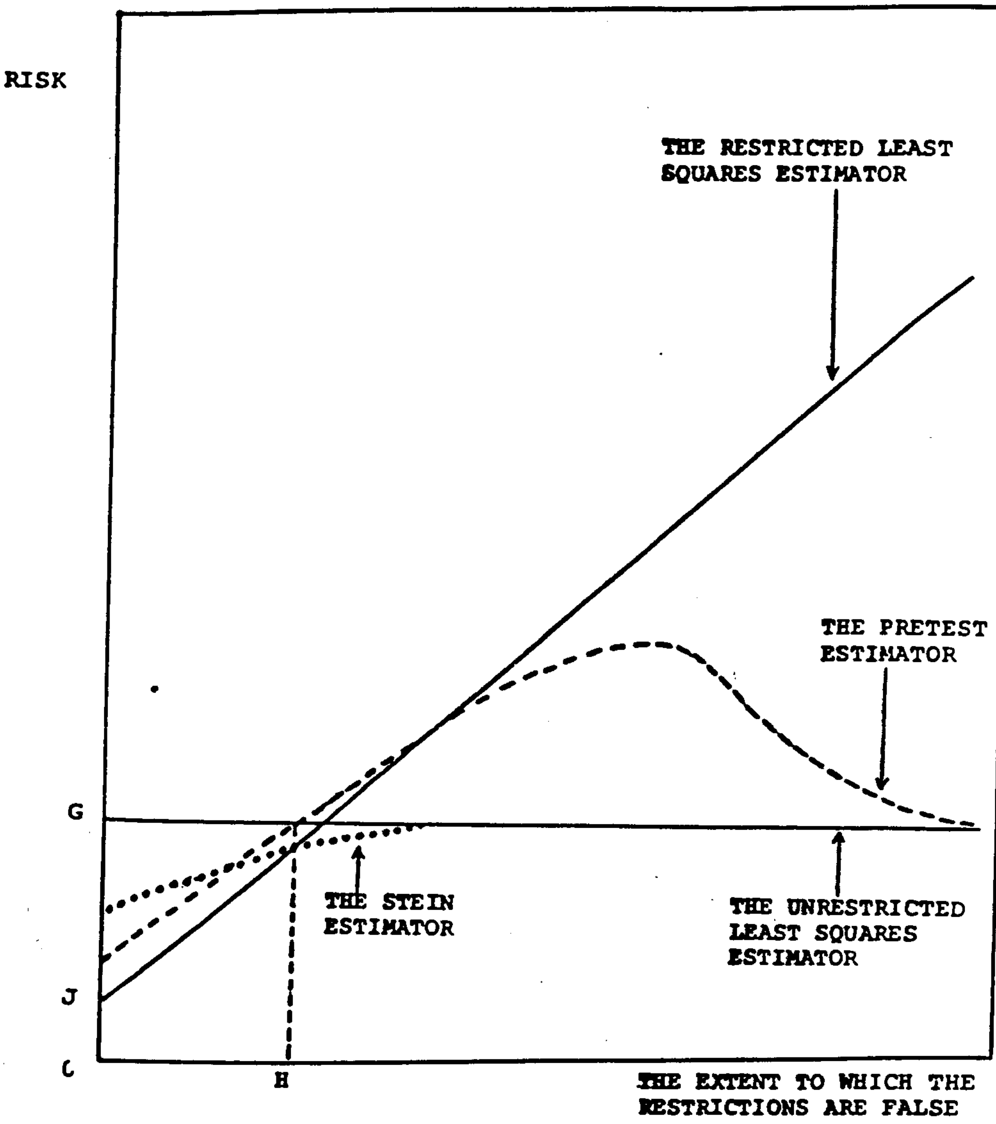

FIGURE 1.1:- THE RISK FUNCTIONS FOR SELECTED ESTIMATORS

from being met(very false), the preliminary tests correctly reject the null hypothesis almost 100% of the time and therefore the risk function of the pretest estimator is very close to(just above) that of the unrestricted least squares estimator. Therefore, the pretest estimator does well when the restrictions are either almost true or very false.

Between these two extremes, the pretest estimator performs very poorly. Suppose, for the purpose of illustration, that the extent to which the restrictions are not met is such that the power of the test is 50%. In this case, the number of times the pretest estimator incorrectly accepts that the restrictions are valid in repeated sampling is equal to the number of times it correctly rejects these restrictions. If it correctly rejects the restrictions imposed, the parameter estimates that result are distributed around the true parameter value; if it incorrectly accepts the restrictions, the estimates it generates are biased. All this is shown in Figure 1.2. In particular, it is seen that the resulting density function for the pretest estimator is such that both its variance and its bias are high, explaining the humped character of the risk function of the pretest estimator[Kennedy, 1985, pp 161 - 162].

For future reference we note that the pretest estimator is often expressed as a weighted average of the restricted and the unrestricted least squares estimators as follows:

$$\beta^{PT} = I_{[0,c)}(U)\beta^{R} + I_{[c,\infty)}(U)\beta^{OLS} \qquad (1.1).$$

where

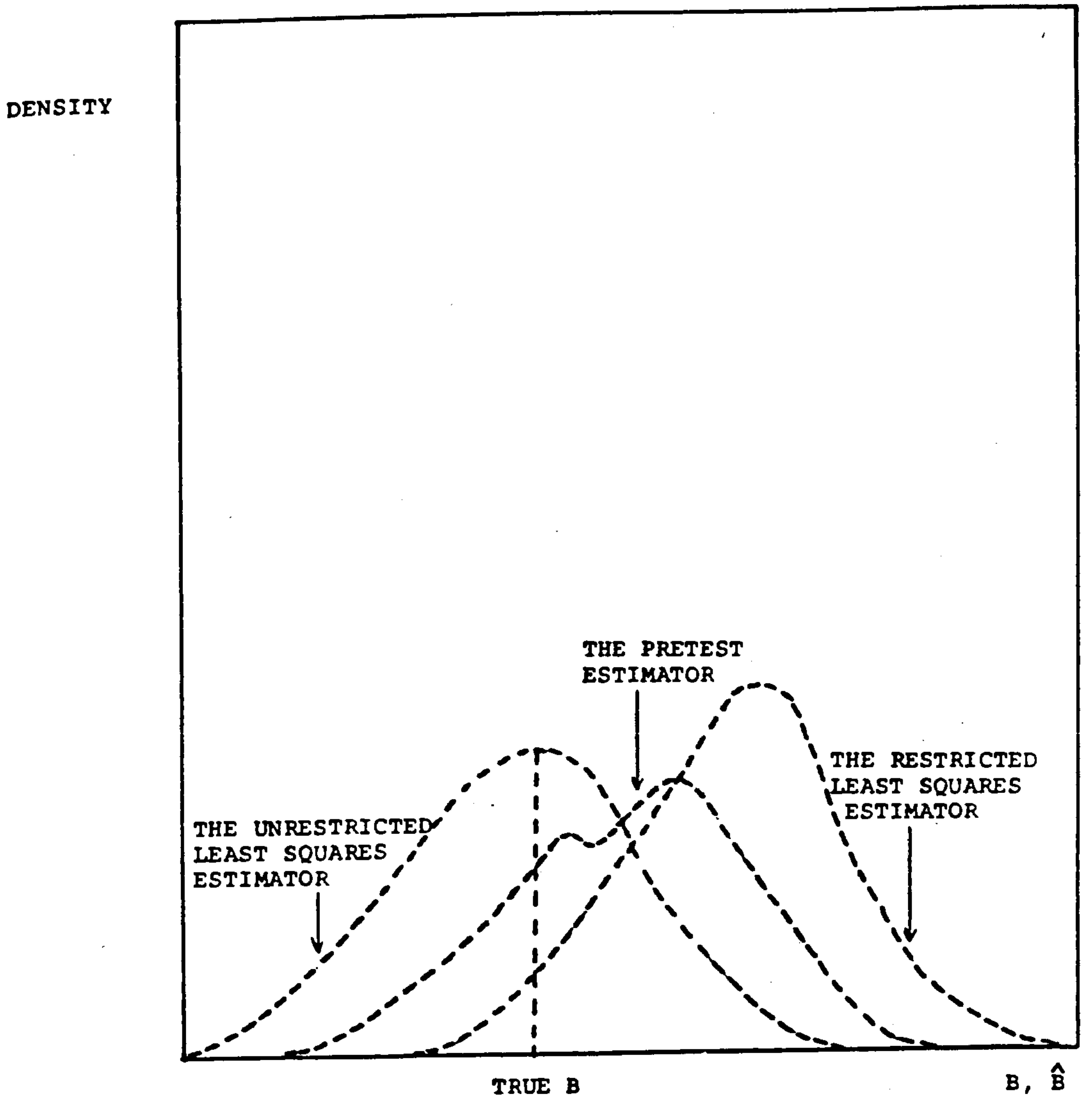

FIGURE 1.2: THE SAMPLING DISTRIBUTIONS OF THE UNRESTRICTED, THE RESTRICTED LEAST SQUARES AND THE PRETEST ESTIMATORS.

β^{PT} = the traditional pretest estimator.

β^{OLS} = the unrestricted least squares estimator

β^{R} = the restricted least squares estimator

$I_{[0,c)}(U)$ and $I_{[c,\infty)}(U)$ are indicator functions taking on the values of one if U, a test statistic, falls in the range subscripted and zero otherwise. Note the dichotomous nature of this weighted average estimator.

Suppose that a researcher, after having formulated his model, tests for the presence of autocorrelation at the 90% level and the null hypothesis is accepted. There is also the possibility that if the test were performed with 80% probability, the null hypothesis could have been rejected. Therefore, according to Zaman(1984, pp 77) "such autocorrelation(or even at lower levels) can cause considerable damage to the OLS results and should, ideally be accounted for". If this is the case, then it follows that most pretest estimators based upon some kind of hypothesis testing are non-optimal. Zaman argues that an answer to this problem is to develop and use shrinkage techniques as evidenced by the James-Stein estimator. With this belief, Zaman(1984), pp 73 has formulated the following heuristic:

"Under as yet unknown but probably quite general regularity conditions, discontinuous functions of the data are inadamissible decision rules".

This heuristic actually rules out a large number of traditional pretest estimators. An ingenious way around this problem is to develop an appropriate shrinkage technique to overcome the discontinuity. This procedure involves the formulation of an estimator that is a convex combination of other selected estimators using a meaningful weighting scheme.

It has been pointed out by many researchers that the pretesting procedure produces an estimator that is inferior to the usual maximum likelihood estimator(MLE) based on the sample information alone over a large portion of the parameter space(as shown in the figure 1.1 above, to the right of point H). Moreover, the pretest estimator possesses an unknown sampling distribution, rendering classical statistical hypotheses testing impossible. The arbitrary selection of the level of significance for the pretesting is also a problem with this estimator. In addition to these negative features of the pretest estimator, Cohen(1965) showed that when the loss function is the squared error loss, the pretest estimator is inadmissible, due to the fact that this estimator is a discontinuous function of the test statistic. Sawa and Horimatsu(1973) made the same observation and comment.

1.3. THE STEIN-ESTIMATORS

From the above discussions, it is clear that the traditional pretest estimator has undesirable risk properties. An alternative to the traditional pretest estimator with better risk properties is a weighted average estimator usually referred to as the James-Stein rule estimator. This estimator utilizes the available prior information to modify the unrestricted least squares estimator in such a way that the resulting estimator dominates the unrestricted least squares over the whole parameter space. This estimator dominates the unrestricted least squares regardless of how correct the prior information is. Note that the weighting system used by the James-Stein rule estimator is a function of the F test statistic(U) utilized to test the set of linear restrictions[Kennedy(1985), pp 161]. The development of these estimator is based on the work of Stein(1965).In this thesis, the Stein-rule estimator is modified to be applicable to the case of heteroskedasticity. The hypothesized set of linear restrictions is replaced by the hypothesis of homoskedasticity implying that the restricted least squares estimator is β^{OLS}, and the unrestricted(i.e., the heteroskedasticity case) estimator is β^{2SAE}. Thus for the pretest estimator in this thesis, the ordinary least squares estimator is employed if the pretest shows that the restriction(homoskedasticity) is true, otherwise, the 2SAE is employed(heteroskedasticity).

Whereas the pretest estimator utilizes the test statistic U to choose either the unrestricted or the restricted least squares estimator, the James-Stein estimator makes use of the test statistic U to combine linearly(in a non-dichotomous fashion) the unrestricted and the restricted least squares estimators into a weighted average estimator. The size of the weights imposed upon each component of this weighted average estimator is a function of the pretesting statistic(U). This weighted average estimator is formally stated as:

$$\theta^S = (\xi/U)\beta^R + (1 - \xi/U)\beta^{OLS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.2).$$

where

θ^S = the Stein estimator,

ξ = a scalar constant which depends on the design matrix and the degrees of freedom.

U = the calculated test statistic,

β^{OLS} = the unrestricted least squares estimator,and

β^R = the restricted least squares estimator.

In the literature the set of linear restrictions most commonly used is a non-stochastic vector for β. But this is not universal; it is not uncommon to shrink the ordinary least squares estimator towards an overall mean rather than a fixed vector, for example. In this thesis, we consider the general case in which the restricted least squares estimator is considered to be stochastic.

If $U = \xi$, the Stein-Rule estimator is identical to the restricted least squares estimator. If the testing statistic(U) is infinite, the Stein-Rule estimator becomes the unrestricted least squares estimator(that is, as the test statistic grows larger in relation to the scalar ξ(i.e., as ξ/U tends towards zero the Stein-Rule estimator gradually approaches the unrestricted least squares estimator). Note that the ratio ξ/U determines the extent to which the unrestricted least squares has to be shrunk towards the restricted least squares estimator

[Judge et al(1980), pp 69 ; Fomby et al(1984), pp 131-134]. Figure 1.3 shows the sampling distribution of the Stein-Rule estimator, illustrating how the Stein estimator dominates the pretest estimator over the whole parameter space.

The Stein-Rule estimator utilizes sample and non-sample information in a superior way than the pretest estimator. Its risk improvement on the maximum likelihood estimator under a variety of loss functions is assured regardless of the correctness of the non-sample information. But unfortunately, these James-Stein Rule estimators have their own problems.

(i). they are highly nonlinear and biased,

(ii). they have unknown small sample distributions,

(iii). they possess covariance matrices depending on unknown population parameters,

(iv). in many cases(e.g., multicollinearity), they improve upon the MLE only if design-related conditions hold [Fomby et al(1984), pp 134],

(v). they depend on the assumption of normally distributed error terms, and

(vi). they are only applicable to the case of a set of linear restrictions and therefore require adaptations for other estimation situations[Efron and Morris(1974)].

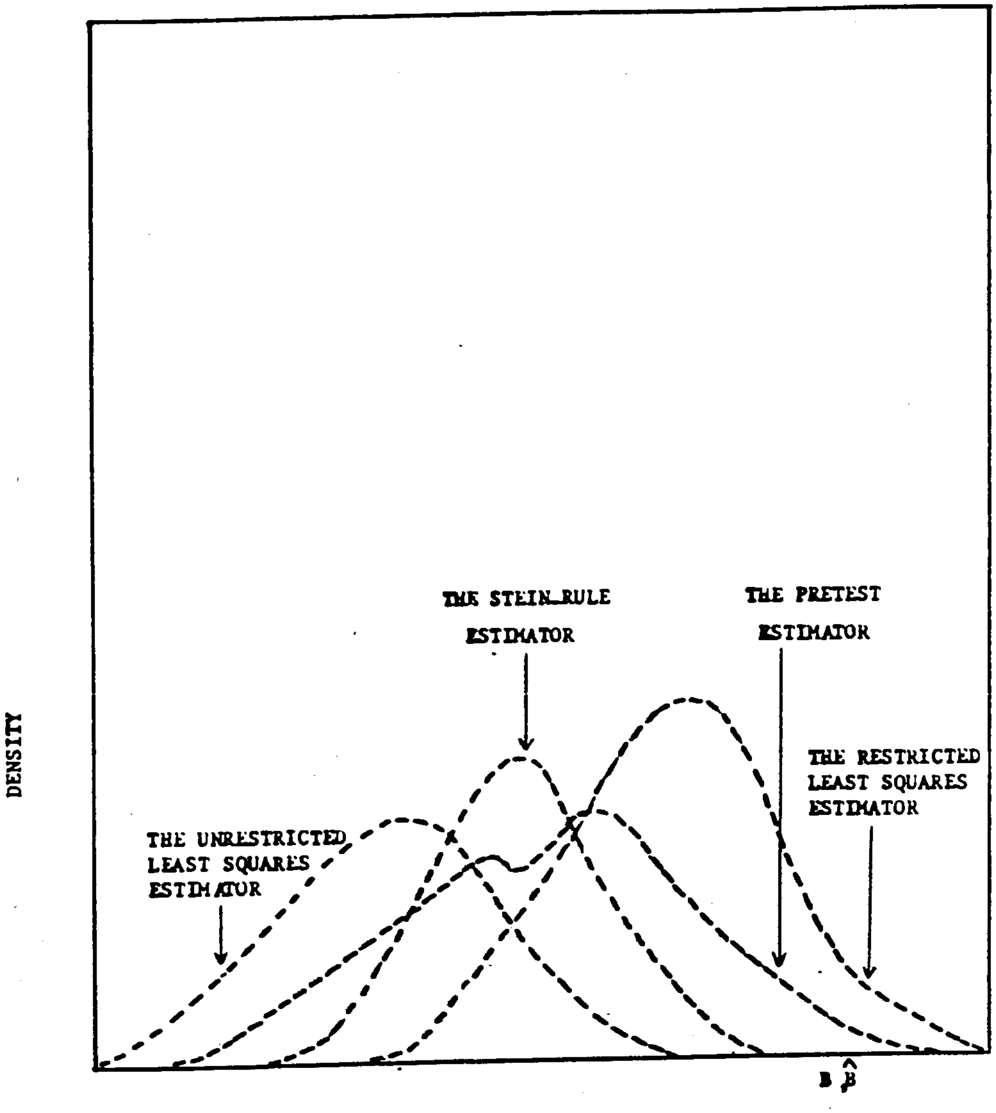

FIGURE 1.3 :- THE SAMPLING DISTRIBUTIONS OF RELATED ESTIMATORS

1.4. GENERALIZING THE PRETEST ESTIMATOR

The purpose of this thesis is to apply the principle of the Stein rule estimator(i.e., using its idea of a continuous weighting system) to the case of heteroskedasticity by a suitable generalization of the pretest estimator. In this way, even though the Stein rule estimator is a continuous function of the data, it is considered to retain the flavour of the pretest estimator. First, a theoretical formulation of the generalized pretest estimator(a weighted average estimator) is undertaken. This estimator is a linear combination of the unrestricted least squares(OLS) and the 2SAE. The Two-Stage Aitken estimator(2SAE) is formulated as follows. First, the ordinary least squares(OLS) regression is run, producing the OLS residuals. Second, these residuals are used to estimate the nature of the heteroskedasticity. Third, the data is transformed to produce an estimating equation for which the ordinary least squares(OLS) estimator is appropriate. This technique is described in more detail later when specific applications are discussed. Unlike the traditional pretest estimator the generalized pretest estimator does not force a dichotomous choice between the estimators. In this respect, it is like the Stein-Rule estimator which develops a weighted average estimation procedure with the weights as a continuous function of the relevant pretest statistic. The aim is to develop a general rule that can improve upon the existing pretest estimators.

Second, Monte Carlo experiments are undertaken to examine the properties of several variants of the generalized pretest estimator, comparing them to those of unrestricted least squares(OLS) and the traditional pretest(BPT) estimators.

The general outline of this thesis is as follows:
Chapter Two provides a review of the existing body of literature on pretesting procedures and the James-Stein rule estimating techniques. It is a general survey of the relevant background material for the thesis. The third chapter contains the theoretical development of the generalized pretest estimator. Chapter Four describes the Monte Carlo experiments to evaluate the performance of the generalized pretest estimator and all the other competing estimators. Chapter Five contains an analysis of the Monte Carlo results. The final chapter contains a summary of the results, suggested topics for future research and a discussion of some limitations of the study.

CHAPTER II

PRETEST ESTIMATORS FOR HETEROSKEDASTICITY

(A REVIEW OF THE LITERATURE)

2.1. PRETEST ESTIMATORS FOR HETEROSKEDASTICITY

The main aim of this chapter is to review the existing relevant literature on pretest estimators in the context of heteroskedasticity. It is well-known that the presence of heteroskedasticity has two undesirable effects on the ordinary least squares(OLS) estimator. First, although the OLS estimator remains unbiased, it is inefficient. Second, its variance-covariance matrix is poorly estimated and so the standard tests of significance have little meaning.

If the true covariance matrix of the error term in a regression model is known to the researcher, he can straightforwardly apply the generalized least squares estimator(GLS) to the data. In this situation, the results are BLUE. Unfortunately the true covariance matrix of the disturbances is rarely known. To circumvent this problem, various techniques have been developed to approximate this covariance matrix. Each such technique results in some variant of the estimating procedure usually referred to as the estimated generalized least squares(EGLS) estimator, sometimes called the Two-Stage Aitken estimator(2SAE). An excellent reference on the numerous versions of this estimating procedure can be found in Chapter 11 of the second edition of Judge et al(1985).

Owing to the fact that uncertainties exist regarding whether or not heteroskedasticity exists in the data, most researchers usually test for its presence. This procedure, as explained in Chapter one is referred to as the pretesting procedure. The resulting estimator is known as the pretest estimator. The general form of the pretest estimator for the case of heteroskedasticity is:

$$\beta^{PT} = I_{[0,c)}(U)\beta^{OLS} + I_{[c,\infty)}(U)\beta^{2SAE} \qquad (2.1).$$

where

β^{PT} = the pretest estimator,

β^{OLS} = the ordinary least squares estimator(OLS), and

β^{2SAE} is the version of the generalized least squares estimator described above. $I_{[0,c)}(U)$ and $I_{[c,\infty)}(U)$ are both indicator functions that assume the value of zero or one. That is, if the value of U, the test statistic lies between zero and C, the indicator function takes the value of one, otherwise zero (i.e., when the value of U lies between C and ∞).

The pretest estimator in equation (2.1) above is a function of many parameters such as β^{2SAE}, β^{OLS}, U, C, etc. Owing to this, its probability density function is a conditional density of β^{OLS}(given that $U \le C$) multiplied by the probability that $U \le C$ added to the conditional density of of β^{2SAE}(given that $U \ge C$) multiplied by the probability that $U \ge C$. In general, the pretest estimator is, therefore, biased [Wallace, 1977].

Most work done on heteroskedasticity pretesting is concerned with cases where the error terms of the first half of the data are assumed to have a constant common variance (σ_1^2) and the error terms of the second half of the data are assumed to have a different constant common variance(σ_2^2). We refer to this case as the 'bivariance' case. The main papers in this area include those of Mandy(1984), Greenberg(1980), Ohtani and Toyoda(1980), Yancey, Judge and Miyazaki(1984), Goldfeld and Quandt(1972), and Sclove, Morris and Radhakrishnan(1972).

Greenberg(1980) formulated the heteroskedasticity pretest estimator as:

$$\beta^{PT}=I_{[0,c_1)}(U)\beta^{2SAE}+I_{[C_1,C_2)}(U)\beta^{OLS}+I_{[C_2,\infty)}(U)\beta^{2SAE} \quad .... (2.2).$$

where

$I_{[a,b]} = 1$ if $U \epsilon (a,b)$

$= 0$ otherwise

That is, $I_{(a, b)}(U)$ is an indicator function that takes on values of one or zero depending upon the outcome of the pretest. β^{OLS} is the ordinary least squares estimator(OLS), and β^{2SAE} is the Two-Stage Aitken estimator.

C_1 and C_2 are the critical values of the two-tailed F-test at some chosen significance level α. This Greenberg heteroskedasticity pretest estimator makes intuitive sense. The underlying principle is described briefly as follows. When heteroskedasticity is suspected, the researcher undertakes a pretest(a two-tailed test) to test whether the errors are homoskedastic, using the Goldfeld and Quandt test statistic U. The hypothesis of equal variances is accepted if $C_1 \leq U \leq C_2$ where

$U = S_1^2/S_2^2$. S_i^2 is the usual estimator of σ_i^2. This test statistic was used by Greenberg to construct the pretest estimator in (2.2). If U assumes values between zero and C_1, the implication is that heteroskedasticity exists in the data and, therefore, the researcher uses the 2SAE . Similarly, if U falls between C_2 and ∞, heteroskedasticity is again implied and the 2SAE is used to estimate the data. However, if U falls between C_1 and C_2, homoskedasticity is implied and the ordinary least squares estimator is chosen for estimation purposes. Greenberg further showed in his analysis that the above heteroskedasticity pretest estimator does not uniformly dominate the unrestricted least squares estimator or the 2SAE estimator over the whole parameter space.

The major observations and conclusions of Greenberg(1980) are summarized in Figure 2.1 below. Greenberg using Monte Carlo results, observed that the ordinary least squares estimator is superior to the other estimators when gamma (i.e., $\gamma = \sigma_1^2/\sigma_2^2$) assumes values that are very close to one. But as soon as gamma takes on larger and larger values the ordinary least squares loses its dominance over the other estimators. The estimated generalized least squares(2SAE) dominates the unrestricted and the Greenberg pretest estimators over a large range of the parameter space. However, the Greenberg pretest estimator based on the Goldfeld and Quandt F-test statistic performs very well when gamma values are very far away from or close to one. For gamma values quite close to one, this pretest estimator

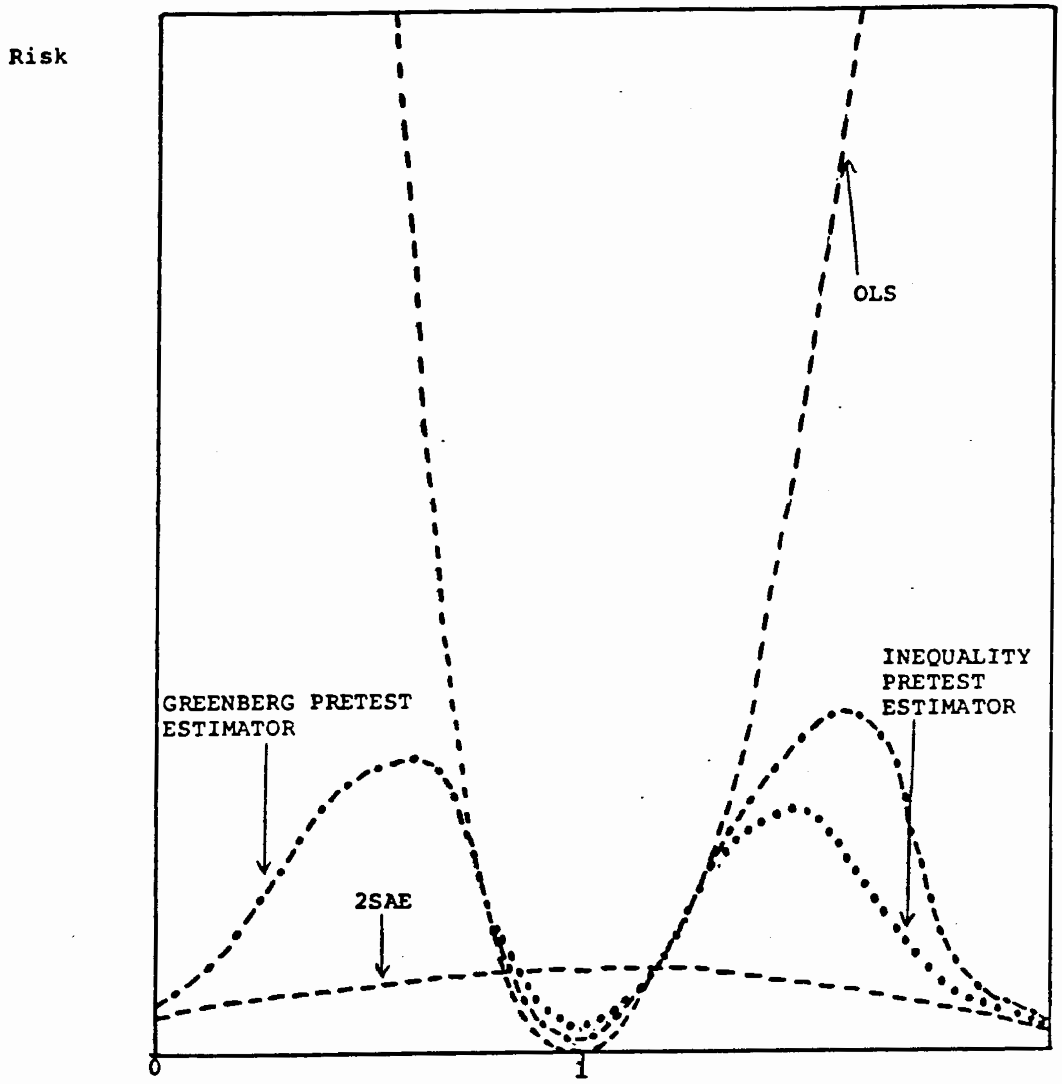

FIGURE 2.1: RISK CHARACTERISTICS OF THE PRETEST AND OTHER CONVENTIONAL ESTIMATORS.

dominates the 2SAE. Since none of these estimators emerges as the 'best' among the rest the choice of estimator depends on a prior information about the variance ratio(γ). If the prior gives a large weight to γ values quite close to 1, which Greenberg(1980), pp 1813 argues will be the case unless the researcher has some specific reason to believe otherwise, both the ordinary least squares and the pretest estimators would be preferred to the 2SAE. A uniform prior on γ, for example, leads to the choice of the 2SAE.

Ohtani and Toyoda(1980) using orthonormal regressors formulated a similar heteroskedasticity pretest estimator. Depending on the outcome of the pretest, either the ordinary least squares estimator or the 2SAE is used in the estimation of the regression coefficients. They derived the mean square error of the pretest estimator. The null hypothesis tested is $\sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $\sigma_1^2 \geq \sigma_2^2$ by their own assumption(i.e., a one-sided test). Using the Goldfeld and Quandt F-test statistic, the heteroskedasticity pretest estimator was developed[see their first equation on pp 153]. Having developed this pretest estimator the authors showed theoretically that the pretest estimator dominates the 2SAE and so that the 2SAE is inadmissible. Their conclusion as summarized on page 155 implies that 'we should not use the 2SAE readily even if we doubt homoskedasticity strongly. It is recommended to test for homoskedasticity prior to estimation of β'.

Mandy(1984) following Greenberg(1980) and Ohtani and Toyoda(1980) examined the inequality pretest estimator for heteroskedasticity without Ohtani and Toyoda's assumption of orthonormal regressors by testing the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 > \sigma_2^2$. The inequality pretest estimator using the pretest statistic selects either the OLS estimator or the 2SAE depending upon whether the test statistic is smaller or larger than the critical value. If the pretest confirms that $\sigma_1^2 > \sigma_2^2$ the OLS estimator is abandoned, otherwise, it is selected for estimating the parameters. He showed that the risk function of the inequality pretest estimator is smaller than that of the Greenberg pretest estimator for all values of gamma greater than one. However, the behaviour of the risk function of the inequality pretest estimator is similar to that of the Greenberg equality pretest estimator. Mandy noted that the reason why the risk function of the inequality pretest estimator is smaller than that of the equality pretest estimator over the parameter space where gamma is greater than one is due to the fact that the inequality pretest estimator possesses a high rejection region "in the upper tail of the distribution and selects the 2SAE more often when gamma is in fact larger than one. As gamma tends towards zero the inequality pretest estimator exhibits the same performance as the OLS estimator and its risk becomes virtually identical to the risk of the OLS estimator... This is also expected since the inequality pretest estimator is constrained to select only the OLS estimator when gamma is less than one".

Mandy's conclusion is that when the non-sample information is correct (i.e., $\sigma_1^2/\sigma_2^2 > 1$, the inequality pretest estimator is superior to the equality pretest estimator. Unfortunately, this is not the case when the non-sample information is not correct.

According to Mandy, "it is important to stress that this inadmissibility of the 2SAE holds only if the researcher is absolutely certain that $\sigma_1^2 > \sigma_2^2$. It is, therefore, unambiguously beneficial to pretest only if one is dealing with a model that rules out the possibility of $\sigma_1^2 < \sigma_2^2$"[Mandy(1984), pp 33]. It should be noted that this quote assumes orthonormality, something Mandy is not clear about. The risk functions of the Greenberg equality pretest and Mandy's inequality pretest estimators are shown with alternative estimators in Figure 2.1 above[Judge et al, (1985), pp 430].

Several Stein rule heteroskedasticity pretest estimators that have been developed are briefly discussed in the following paragraphs. First, note that the James-Stein Positive Rule estimator was developed to solve a sign reversal problem in the use of the Stein-rule estimator. This problem occurs when the test statistic $U \leq \xi$ leading to the shrinkage of the unrestricted least squares beyond the restricted least squares estimator. The occurrence of this usually leads to a problem of sign reversal of the Stein-rule estimator. Kennedy(1985), pp 165-166 notes that 'by truncating this shrinking factor so as to prevent this from happening, an estimator superior to the Stein estimator is created. It is called the Stein positive rule estimator. The name derives from the popular application to zero restrictions: The positive rule estimator prevents the sign of the Stein estimator from differing from that of the unrestricted least squares estimator'. This estimator renders the Stein-rule estimator inadmissible since it has a lower risk. This relationship is shown in Figure 2.2 below. This estimator dominates the maximum likelihood estimator when the number of parameters being estimated is greater than three[Judge et al(1985) pp 82 - 89]. Sclove, Morris and Radhakrishnan(1972) developed a Stein-rule like pretest estimator usually referred to as modified positive-part pretest estimator. In its formulation, Sclove et al, instead of combining the MLE(β^{OLS}) with the restricted least squares estimator to form the pretest estimator, rather combined the James-Stein positve rule estimator and the restricted least squares estimator. This procedure is based upon the fact that since the Stein-rule estimator uniformly dominates the ordinary least squares estimator, its use can lead to the development of a pretest estimator that is superior to the traditional pretest estimator. Applying this information to the context of heteroskedasticity, Sclove et al linearly combined the James-Stein positive rule estimator and the 2SAE into a heteroskedasticity pretest estimator as specified in (2.3).

$$\beta^{PT} = I_{[0,c)}(U)\beta^{+} + I_{[c,\infty)}(U)\beta^{2SAE} \qquad (2.3).$$

where

β^{+} is the Stein-rule positive estimator. This pretest estimator is akin to the Greenberg pretest estimator in (2.2). The only

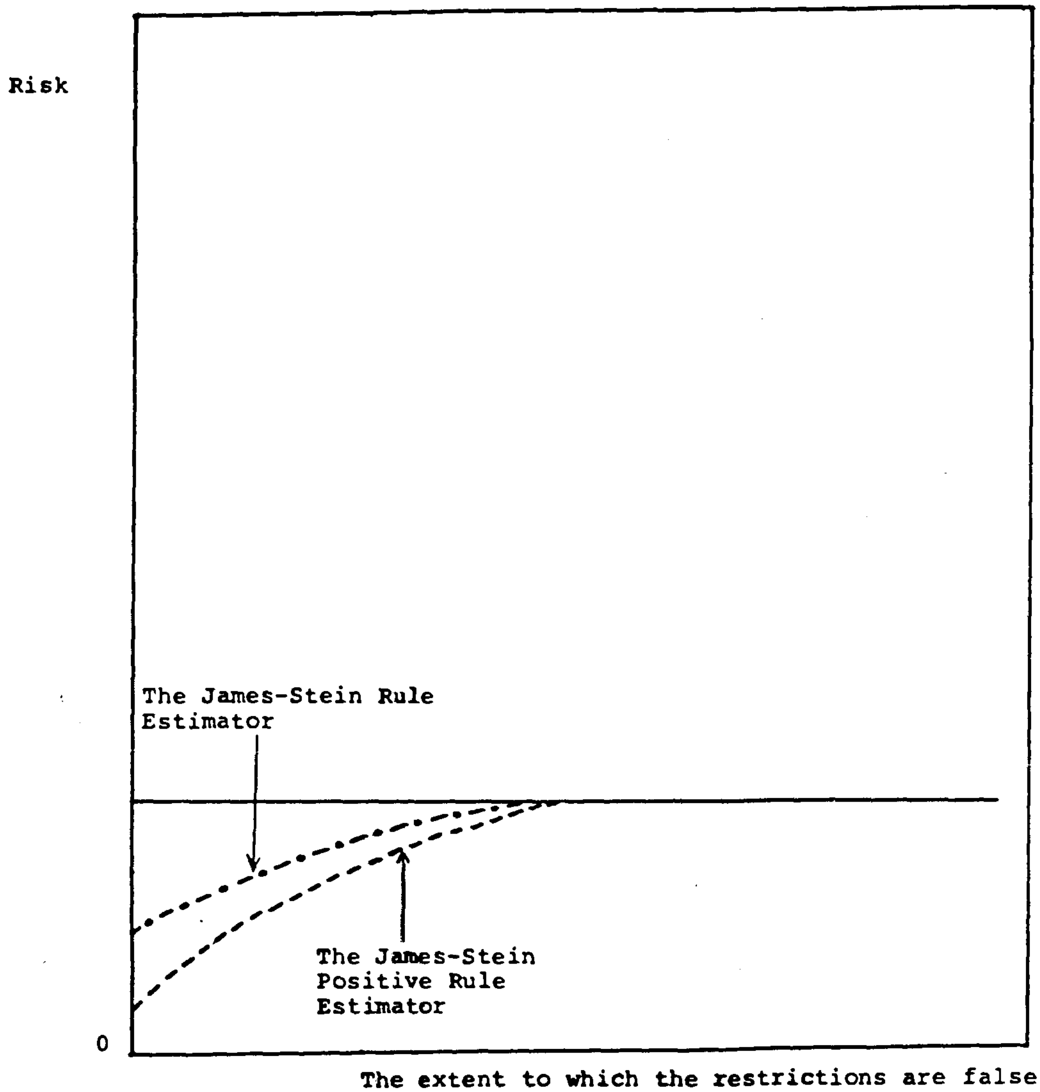

FIGURE 2.2: THE RISK FUNCTIONS OF THE JAMES-STEIN RULE AND THE JAMES-STEIN POSITIVE RULE ESTIMATORS

difference between these two pretest estimators is the replacement of the ordinary least squares estimator in the Greenberg estimator with the James-Stein positive rule estimator. Sclove et al(1972) proved theoretically that the modified positive-part rule estimator dominates the Greenberg pretest estimator under the squared error loss criterion and therefore renders it inadmissible. This pretest estimator has been proved(theoretically) to dominate the traditional pretest estimator(2.7) under the squared error loss criterion rendering it inadmissible. Their risk functions are shown with others in Figure 2.3. Other versions of the heteroskedasticity pretest estimator developed by Yancey et al(1984) are described below.

Yancey et al(1984), following Taylor(1977, 1978), Greenberg(1980), Ohtani and Toyoda(1980), and Mandy(1984), demonstrated that over the parameter space(gamma):

(i). there exists a variety of estimators that uniformly dominate the unrestricted least squares estimator, the Aitken estimator, and the 2SAE(i.e., the James-Stein estimator), and

(ii). that there are alternative types of pretest estimators that possess smaller risks over the whole parameter space than the traditional pretest estimators in the context of heteroskedasticity.

There are two versions of the pretest estimator developed by Yancey et al for the orthonormal case. In the first version of their development, the pretest estimator was derived as a combination of the Stein estimator and the 2SAE(equation 31, pp

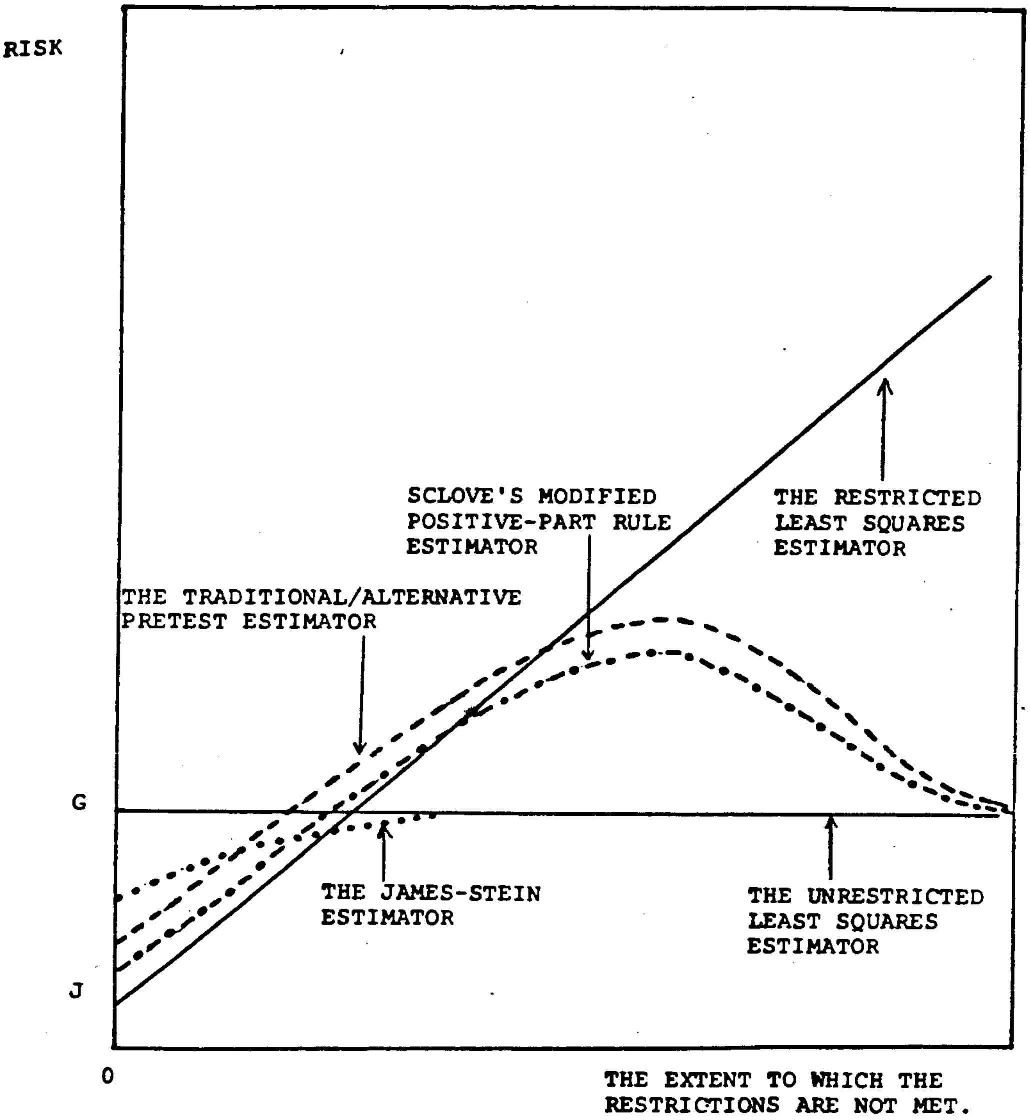

FIGURE 2.3: THE RISK FUNCTIONS OF SOME SELECTED ESTIMATORS.

148 of their paper). In this case, they replaced the unrestricted least squares estimator in the Greenberg pretest estimator with the Stein estimator. This pretest estimator was shown to be superior to the Greenberg equality pretest estimator. The second version of the Yancey et al Stein pretest estimator(equation 33, pp 148 of their paper) combines the Stein estimator and the Stein version of the 2SAE that they developed(S2SAE). This estimator shrinks the 2SAE towards the zero vector instead of the OLS estimator as done by the Stein estimator. In this development, they replaced both the unrestricted and the 2SAE estimators in the Greenberg equality pretest estimator with the Stein estimator and the Stein version of the 2SAE respectively. It has been shown by the authors that the two versions of the Yancey et al Stein pretest estimators dominate the Greenberg equality pretest estimator(hence rendering it inadmissible), and that the second version of the Stein pretest estimator dominates the first version[Yancey et al, pp 149 figure 2].

Even though there are studies that compare the Greenberg equality pretest estimator with either the Sclove et al pretest estimator or the two versions of the Yancey et al Stein versions of the pretest estimators, there have been no attempts to compare the performance of the Sclove et al and Yancey et al heteroskedasticity pretest estimators. In a future study, one may want to compare and contrast the risk functions of these estimators with that of the generalized pretest estimator developed in this thesis.

The best summary on pretest and Stein-rule estimators is found in Judge and Bock(1978). Proofs of the theorems concerning the properties and characteristics of these estimators have been outlined for the general regression and orthonormal models.

2.2. THE BAYESIAN PRETEST ESTIMATOR

Suppose that the researcher is interested in obtaining point estimates of parameters, and is uncertain as to whether the appropriate model is the one with or without the given restrictions. The Bayesian computes the optimal Bayesian point estimate through minimization of the expected loss function which is averaged across both hypotheses. The posterior probabilities of the hypotheses are used as the weighting system. Having computed the posterior probability of the null hypothesis[$P(H_0|y)$] and the posterior probability of the alternative hypothesis[$P(H_1|y)$], the Bayesian obtains the point estimate (β^*) that minimizes the following expected loss function:

$$E[L(\beta,\beta^*)] = P(H_0|y)E[L(\beta,\beta^*)|H_0] + P(H_1|y)E[L(\beta,\beta^*)|H_1] \quad \text{.............(2.4).}$$

According to Judge et al(1985), 'with quadratic loss where the posterior means are optimal, the minimizing value for β^* is a weighted average of the posterior means'. This is computed as:

$$\beta^* = P(H_0|y).E[\beta|H_0] + P(H_1|y).E[\beta|H_1] \quad \text{..................(2.5).}$$

If under the null hypothesis the restrictions are true, then the restricted least squares estimator (β^R) is used. That is, $E[\beta|H_0] = \beta^R$. If, however, the restrictions are false, the unrestricted least squares estimator (β^U) is used and hence $E[\beta|H_1) = \beta^U$(which is referred to earlier as β^{OLS} when an ignorant prior is employed). Assuming that these values are the posterior means, the Bayesian pretest estimator in (2.5) above can be re-written as:

$\beta^* = P(H_0|y)\beta^R + P(H_1|y)\beta^U$............................(2.6).

where $P(H_1|y) = 1 - P(H_0|y)$, and therefore (2.6) becomes

$\beta^* = P(H_0|y)\beta^R + (1 - P(H_0|y)\beta^U$.........................(2.7).

Casting the problem in the non-spherical mold, we treat homoskedasticity as the restriction under consideration. Thus, in this context, the null and the alternative hypotheses refer to homoskedasticity and heteroskedasticity respectively. With this view the generalized pretest estimator can be written(using the principle underlying the Bayesian pretest estimator) as:

$\beta^* = P(H_0|y)\beta^{OLS} + P(H_1|y)\beta^{2SAE}$.........................(2.8).

where $P(H_1|y) = 1 - P(H_0|y)$, and therefore (2.8) becomes

$\beta^* = P(H_0|y)\beta^{OLS} + (1 - P(H_0|y)\beta^{2SAE}$.................(2.9.

Notice that both the Bayesian and the generalized pretest estimators are different from the traditional pretest estimator in that they are continuous functions of the data[Judge et al,(1985) pp 117 - 118].

CHAPTER III

THE THEORETICAL MODEL FORMULATION

3.1. INTRODUCTION

The purpose of this thesis is to develop a weighted average estimator which is comparable to the traditional pretest estimators. This estimator is a linear combination of the OLS and the 2SAE estimators. Unlike the traditional pretest estimators, the weighted average estimator does not lead to a dichotomous choice between the estimators used in the linear convex combination. In this respect, it is like the Stein-Rule estimator which structures a weighted average estimation procedure with the weights as a continuous function of the relevant test statistic.

Suppose that the model under consideration is of the form:

$Y = X\beta + \epsilon$ ....(3.1).

where Y is (N x 1) column vector and so is ϵ(the error term, ϵ, has mean zero and is suspected of being heteroskedastic). X(fixed in repeated samples) is (N x K) and β is (K x 1).

Now, consider the problem of choosing between the OLS and the 2SAE estimators as far as model(3.1) above is concerned. In the econometrics literature, many researchers using regression analysis begin their estimation procedures with some hypothesis testing. If uncertain about the nature of the available data(in this case about the presence or absence of heteroskedasticity), the researcher may test for the presence of heteroskedasticity.

Based upon the outcome of the above pretesting procedure the usual pretest estimator is written as a weighted average of the OLS estimator and the 2SAE. The mathematical formulation of this estimator is specified in equation(1.1) of chapter 1. Depending on the outcome of the pretest the choice made by the pretest estimator between the OLS and the 2SAE estimators is dichotomous.

3.2. THE FIRST FORMULATION

To circumvent the dichotomous choice between these two combined estimators, a generalized pretest estimating technique(GPE) is proposed from which the above mentioned dichotomous pretest procedure could be perceived as a special case. Recall that the Bayesian pretest estimator linearly combines the unrestricted and the restricted least squares estimators by using the posterior odds in favour of the null hypothesis as the weighting system. Casting the problem in the non-spherical error mold and exploiting the Bayesian view and the Stein-Rule estimation principle, the development of the generalized pretest estimator is a two-step procedure. These steps include:

(1). Utilize the sample data to determine(in an objective fashion) the subjective probability(ϕ) that the nature of heteroskedasticity is such that the ordinary least squares estimator dominates(has smaller relative mean square error) the 2SAE.

(2). Use this probability(ϕ) as the weighting scheme to combine the ordinary least squares estimator and the 2SAE into the generalized pretest estimator as:

$$\beta(GPE) = \phi\beta^{OLS} + (1-\phi)\beta^{2SAE} \quad \text{.............................(3.2).}$$

Note that the motivation for doing this comes from the Bayesian view outlined earlier in chapter two.

3.3. THE SECOND FORMULATION

The second development is also a two-step procedure. These steps include:

(1). Based upon a pretesting procedure utilizing the sample data at hand, determine (in an objective fashion) the subjective probability(ϕ) that the nature of heteroskedasticity is such that the ordinary least squares estimator dominates the 2SAE.

(2)Calculate the generalized pretest estimator as:

$$\beta(GPE) = \lambda\beta^{OLS} + (1-\lambda)\beta^{2SAE} \quad \text{............................(3.3).}$$

where λ is chosen to minimize the subjective(based on ϕ from above) expectation of the risk of the 'smoothed' pretest estimator, $\beta(GPE)$.

That is,

$$\text{mintrEV(GPE)} = \phi\text{trV(GPE)}|_{se} + (1 - \phi)\text{trV(GPE)}|_{nse} \quad \text{......(3.4).}$$

where se and nse imply spherical and nonspherical error real worlds respectively.

The value of λ is determined as follows. Suppose that Ω, the variance-covariance matrix of the error vector is known. Then in an OLS world,

$$\beta^{OLS} = (X' X)^{-1} X' Y$$
$$\beta^{GLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \text{ ....(3.5).}$$

$$V(\beta^{OLS}) = \sigma^2 (X' X)^{-1}$$
$$V(\beta^{2SAE}) = \sigma^2 (X' \Omega^{-1} X)^{-1} X' \Omega^{-2} X (X' \Omega^{-1} X)^{-1} \text{ ....(3.6).}$$

$$COV(\beta^{OLS};\beta^{GLS}) = \sigma^2 (X' X)^{-1} \text{ ....(3.7).}$$

Similarly, in a GLS world;

$$\beta^{OLS} = (X' X)^{-1} X' Y$$
$$\beta^{GLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \text{..........................(3.8).}$$

$$V(\beta^{OLS}) = \sigma^2 (X' X)^{-1} X' \Omega X (X' X)^{-1}$$
$$V(\beta^{GLS}) = \sigma^2 (X' \Omega^{-1} X)^{-1} \text{ ....(3.9).}$$

$$COV(\beta^{OLS};\beta^{GLS}) = \sigma^2 (X' \Omega^{-1} X)^{-1} \text{ ....(3.10).}$$

Note that in order to operationalize all these, Ω (which is rarely known) must be estimated as $\hat{\Omega}$ (its unbiased estimator). Using the above definitions, derive the variance of the generalized pretest estimator as:

$$V(GPE) = \lambda^2 V(\beta^{OLS}) + (1-\lambda)^2 V(\beta^{GLS})$$

$$+ 2\lambda(1-\lambda)COV(\beta^{OLS}; \beta^{GLS}) \ldots\ldots\ldots (3.11).$$

Combining (3.3) through (3.7), we obtain,

$$V(GPE)|_{se} = \lambda(2-\lambda)\sigma^2(X'X)^{-1}$$
$$+ (1-\lambda)^2\sigma^2 (X'\Omega^{-1}X)^{-1} X'\Omega^{-2}X(X'\Omega^{-1}X)^{-1} \ldots\ldots\ldots (3.12).$$

Similarly, by combining (3.8) through (3.10), we have,

$$V(GPE)|_{nse} = \lambda^2\sigma^2(X'X)^{-1}X'\Omega X(X'X)^{-1}$$
$$+ (1-\lambda^2)\sigma^2 (X'\Omega^{-1}X)^{-1} \ldots\ldots\ldots (3.13).$$

Substituting(3.12) and (3.13) into equation (3.4), choose λ to minimize the trace of the subjective expectation of the variance of the generalized pretest estimator(GPE). That is ,

$$\min trEV(GPE) = \phi tr[\lambda(2-\lambda)\sigma^2(X'X)^{-1}$$
$$+(1-\lambda)^2\sigma^2 (X'\Omega^{-1}X)^{-1}X'\Omega^{-2}X(X'\Omega^{-1}X)^{-1}]$$
$$+(1-\phi)tr[\lambda^2\sigma^2(X'X)^{-1}X'\Omega X(X'X)^{-1}$$
$$+ (1-\lambda^2)\sigma^2 (X'\Omega^{-1}X)^{-1}] \ldots\ldots\ldots (3.14).$$

That is,

$$\frac{\delta trEV(GPE)}{\delta\lambda} = \phi tr[(2-2\lambda)\sigma^2 (X'X)^{-1} -2(1-\lambda)\sigma^2 (X'\Omega^{-1}X)^{-1}X'$$
$$\Omega^{-2}X(X'\Omega^{-1}X)^{-1}]$$
$$+(1-\phi)tr[2\lambda\sigma^2(X'X)^{-1}X'\Omega X(X'X)^{-1}$$
$$-2\lambda\sigma^2 (X'\Omega^{-1}X)^{-1}]=0 \ldots\ldots\ldots (3.15).$$

expanding and collecting terms together, it turns out that,

$$\lambda tr\{\phi[(X' \Omega^{-1} X)^{-1} X' \Omega^{-2} X (X' \Omega^{-1} X)^{-1} - (X' X)^{-1}]$$
$$+ (1 - \phi)[- (X' \Omega^{-1} X)^{-1} + (X' X)^{-1} X' \Omega(X' X)^{-1} \}$$
$$= tr\{ \phi [(X' \Omega^{-1} X)^{-1} X' \Omega^{-2} X (X' \Omega^{-1} X)^{-1} - (X' X)^{-1}] \}$$

Therefore,

$$\lambda = \{\phi tr[(X' \Omega^{-1} X)^{-1} X' \Omega^{-2} X (X' \Omega^{-1} X)^{-1} -(X' X)^{-1}]\}/$$
$$\{\phi tr[(X' \Omega^{-1} X)^{-1} X' \Omega^{-2} X (X' \Omega^{-1} X)^{-1} -(X' X)^{-1}]$$
$$+(1-\phi)tr[(X' X)^{-1} X' \Omega X(X' X)^{-1} - (X' \Omega^{-1} X)^{-1}]\} \quad \text{..............................(3.16).}$$

and hence dividing both numerator and the denominator by the numerator yields,

$$\lambda = 1 / (1 + \Psi / \chi) \quad \text{.....................................(3.17).}$$

where,

$$\Psi = (1 - \phi)tr[- (X' \Omega^{-1} X)^{-1} + (X' X)^{-1} X' \Omega X(X' X)^{-1}]$$

$$\chi = \phi tr[(X' \Omega^{-1} X)^{-1} X' \Omega^{-2} X (X' \Omega^{-1} X)^{-1} - (X' X)^{-1}]$$

Use this value of λ, evaluated at $\Omega = \hat{\Omega}$, to operationalize the (GPE) as defined in equation 3.3 above.

3.4. OBSERVATION

Note that if the procedure of the traditional pretest estimation is followed, the implication of the above derivations is that:

(a) If $\phi = 1$, it implies that $\lambda = 1$; choose the OLS estimator.

(b) If $\phi = 0$, it implies that $\lambda = 1/\infty \simeq 0$; choose the 2SAE estimator.

However, this dichotomous choice is of little interest; the value of ϕ (which in itself depends on the relevant test statistic) however small it is, will seldom be set equal to zero.

3.5. THE IMPORTANCE AND THE ROLE OF ϕ.

To calculate the pretest estimator the researcher first performs a pretest on the sample information at hand. If the researcher subjectively selects ϕ equal to zero or one on the basis of his pretests, then this implies a corresponding choice of λ of one or zero respectively. Since the critical value of the pretest is chosen subjectively, this selection is subjective. In this sense, the researcher is implicitly choosing the weights as one and zero on the basis of his personal belief (ϕ) as affected by the outcome of the pretest.

Based upon Jeffreys' rule for a theory of inductive inference 'we do accept inductive inference in some sense; we have a certain amount of confidence(ϕ) that it will be right in any particular case, though this confidence does not amount to logical certainty' [Zellner, 1971, pp 8]. Yet, unfortunately, this(certainty) is what most researchers assert when they go through pretesting procedures.

If the researcher is not restricted to a zero-one choice for ϕ, the corresponding choice of λ is no longer dichotomous; it is a continuous function of ϕ (his subjective probability). This way of viewing the pretest procedure suggests a generalization based on using the pretest to produce a value of ϕ which is not restricted to the values of one or zero.

The resulting estimating formula, referred to as the generalized pretest estimator (GPE), is one in which the weights, λ and ϕ are continuous functions of the pretest statistic and this this is in general respect similar to the Stein estimator. This similarity to the Stein estimator raises the hope that its sampling properties may be preferable to those of the traditional pretest estimators. One aim of this thesis is to investigate this by means of a Monte Carlo study.

Note that the introduction of the subjective probability(ϕ) into the analysis is crucial, since this is what circumvents the dichotomy inherent in traditional pretesting procedures. Since ϕ is calculated as an objective function of the pretest statistic, this procedure is consistent with standard practice in classical statistics, in spite of having a Bayesian flavour. The analysis is not really a Bayesian one. All it does is to employ a Bayesian-like justification for structuring the generalized pretest estimator. Therefore, this novel formulation must be viewed as an extension or generalization of the usual pretest methodology.

3.6. COMPUTING PHI FROM THE DENSITY FUNCTION(THE BIVARIANCE CASE)

Since we cannot calculate the generalized pretest estimator without a means of estimating the value of ϕ, the development of a procedure for calculating ϕ is required. The probability(ϕ) that the degree of heteroskedasticity is such that the ordinary least squares estimator outperforms the 2SAE is computed as the area under the posterior distribution of gamma in a specified range of gamma values. Gamma(γ), it will be recalled, is the ratio σ_1^2/σ_2^2 where σ_1^2 and σ_2^2 are the true error variances. The procedure is carefully explained below.

To operationalize the generalized pretest estimator and hence produce a' smoothed' version of the traditional pretest estimator, we must first recall that Taylor(1977, 1978), Greenberg(1980), Ohtani and Toyoda(1980), Mandy(1984) have shown that the ordinary least squares estimator is not only superior to the 2SAE when $\gamma = 1$, but also over some range of gammas quite close to one. In other words, in the neighbourhood of $\gamma_l \leq \gamma \leq \gamma_u$, the ordinary least squares estimator is superior to the 2SAE. Although the generalized least squares estimator dominates the ordinary least squares estimator, its estimated version(2SAE) does not.

Second, this leads us to the notion that the point null hypothesis that $\gamma = 1$ should be respecified as a composite null hypothesis. That is, $H_0 : \gamma_l \leq \gamma \leq \gamma_u$. Thus to estimate ϕ we must devise a technique to calculate the probability (ϕ) that γ lies in the specified range.

In developing these ideas further, we need to have some information about the limiting values of gamma(i.e., γ_l and γ_u). We speculate that perfect knowledge of the critical values of gamma is neither important nor crucial to our development, implying that approximate values for these critical values is sufficient for smoothing the pretest estimator. This speculation is based on the belief that the improved mean square error property of the generalized pretest estimator is due to its smoothing of the traditional pretest estimator; its success stems from the principle of the Stein estimator.

The points of integration, γ_l and γ_u are taken from the table constructed by Taylor(1977, pp 505). This table gives conservative lower and upper gamma values that define the region within which the ordinary least squares estimator outperforms the estimated generalized least squares estimator(2SAE). The table is computed giving the limiting gamma values for the degrees of freedom for cases when $N_1 = N_2$. Taylor(1978, pp 669 - 671) further showed that these values from the table are good approximations to the true values of the lower and upper values of gamma. We, therefore, make use of these values for the Monte Carlo experiments in this thesis.

Zellner(1971, pp 107), using an ignorance prior computed the posterior density function(distribution) of gamma. This posterior distribution of gamma is written as:

$$p(\gamma|y) = C[\gamma^{(N_1-2)/2}|H_1|^{-1/2}]/$$

$$[(Y_1' Y_1+\gamma Y_2' Y_2-H_2' H_1^{-1}H_2)^{(N_1+N_2-K)/2}] \ldots\ldots\ldots\ldots (3.18).$$

where C is the constant of proportionality(or a normalizing constant), N_1 and N_2 are the sizes of the two subsamples, K is the number of independent variables, $H_1 = X_1' X_1+\gamma X_2' X_2$ and $H_2 = X_1' Y_1 + \gamma X_2' Y_2$.

With this relevant information, the probability(ϕ) that the nature of heteroskedasticity is such that the ordinary least squares estimator outperforms the 2SAE is, therefore, computed as the area under the above specified density function between the lower and upper gamma values through univariate numerical integration techniques. Making use of this probability as the weighting system, the two versions of the generalized pretest estimator are calculated as specified in equations (3.2) and (3.3) above. Note that the probability(ϕ) is a continuous function of the available data and hence the generalized pretest estimator is a continuous function of the data. It is hoped that its non-dichotomous nature will make it a superior pretest estimator. Its risk improvement upon the traditional pretest estimator is due to the fact that the weighting system is a continuous rather than a discontinuous function of the sample data.

3.7. COMPUTING PHI FROM THE DENSITY FUNCTION(THE MULTIVARIANCE CASE)

The discussion above refers exclusively to what we have called the bivariance case, in which the variance of the error term takes one of two values. We examine now a variant to which we refer as the multivariance case, in which the variances of all error terms are different.

Suppose that the functional form of heteroskedasticity(for the multivariance case) is known to be:

$$\sigma_t^2 = KX_t^{\delta} \qquad \text{.. ....(3.19).}$$

In this case , H_0: $\delta = 0$(homoskedasticity) and the alternative is H_1: $\delta \neq 0$(heteroskedasticity).

The ordinary least squares estimator is superior to the 2SAE over some range of δ values close to zero. In the region between the lower delta(δ_l) and the upper delta(δ_u) values, the risk of the ordinary least squares estimator is smaller than that of the 2SAE. The above point null hypothesis is replaced by a composite null hypothesis, namely the range of delta values between δ_l and δ_u. In this way, the probability(ϕ) that the null hypothesis is true can be calculated as the area under the posterior density function of delta between δ_l and δ_u.

A common estimating technique used for calculating δ is to regress the logarithm of the squared residuals(using the residuals from the ordinary least squares regression on the original data) on an intercept and the logarithm of the explanatory variable X (i.e., $\ln U_t^2 = \ln K + \delta \ln X_t + V_t$).' A problem with this estimator is that the resulting error term from this regression has non-zero mean and is both heteroskedastic and autocorrelated. However, if the ϵ_t are normally distributed and if $\hat{\epsilon}_t$ converges in distribution to ϵ_t, then, asymptotically, the V_t will be independent with mean and variance given by Harvey(1976)' [Judge et al(1985), pp 440)]. Since the efficiency of the parameter estimates from the model depends on the nature of the estimators of the error variances, a great deal of effort has been put into developing techniques that are used to estimate the heteroskedastic variances in linear models. Among the intensive researches in this area are Nozari(1984), Horn, Horn and Duncan(1975), Chew(1970), Hartley, Rao and Kiefer(1969), Duncan(1966), Mandel(1964), Duncan and Carroll(1962).

In the estimation of the heteroskedastic error model, the selection of the heteroskedastic error structure has been found to be of little importance; the choice of the error structure can be undertaken on the basis of estimation convenience. Surekha and Griffiths(1984) observed that the efficiency of the 2SAE for β rests more on the choice of estimator and sample size than it does on specification of the correct variance structure.

Judge et al(1985, pp 455) concluded that 'in summary, a choice between variance structures such as $\sigma_t^2 = Z_t'\alpha$, $\sigma_t^2 = (Z_t'\alpha)^2$, and $\sigma_t^2 = \exp(Z_t'\alpha)$ is not likely to be very important providing estimators with poor properties are avoided'. Note that $z_t' = [1 \; \ln X_t]$

Taking a Bayesian view, the OLS estimation of δ described above produces a posterior distribution for δ centred at $\hat{\delta}$ and with variance given by $Var(\hat{\delta})$ = the lower right hand element of $4.9348[\Sigma z_t z_t']^{-1}$[Harvey(1976), pp 461 - 466] taking the form of a t-distribution. In our analysis, the probability(ϕ) has been calculated by using Fortran NAG routines to perform the appropriate integral of this density function of $\hat{\delta}$. In performing this integration, the critical values of δ_l and δ_u must be known. For the same reasons as given above for the bivariance case, we speculate that approximate values for δ_l and δ_u will suffice. To obtain these approximate values, a series of mini Monte Carlo studies were performed to compare the risk functions of both the ordinary least squares estimator and the 2SAE. In performing these mini Monte Carlo experiments, different values of δ quite close to zero were used in generating the heteroskedastic data. To this data, we applied both the OLS estimator and the 2SAE and calculated their relative mean square errors. By so doing we obtained approximate values of δ that define the parameter space within which the OLS estimator dominates the 2SAE. Note that in theory this can be done in an actual study so long as the functional form of heteroskedasticity is assumed. Two delta values (δ_l and δ_u) were obtained to represent the boundaries of the region beyond which the estimated generalized least squares estimator(2SAE) begins to outperform(using the MSE criterion) the unrestricted least squares estimator.

3.8. COMPUTING PHI USING THE POSTERIOR ODDS RATIO(THE MULTIVARIANCE CASE)

A second technique suggested for computing ϕ in the multivariance case is a suggestion of Villegas(1986) for calculating the posterior odds ratio in the context of a point null hypothesis versus a composite alternative hypothesis. As noted earlier, for the case of a point null hypothesis versus a composite alternative hypothesis, the Bayesian technique requires an informative prior for the calculation of the posterior odds. This requirement renders the use of the Bayesian approach unpalatable to non-Bayesians; an 'objective' subjective probability for the null hypothesis cannot be calculated.

Villegas(1986) has suggested a way around this dilemma. He proposes truncating the diffuse prior at levels above which and below which everyone can agree that there is zero probability that a parameter would lie. He notes, for example, that the mean height of humans must lie above zero and below that height at which the oxygen content of air can no longer sustain life. This truncation of the diffuse prior allows him to develop the following formula for the posterior odds of a point null hypothesis versus a composite alternative hypothesis in the regression context. Using the results from the OLS regression of the logarithm of the squared residuals on an intercept and the logarithm of the explanatory variables, compute the probability that δ is not equal to zero as:

$$prob(\delta \neq 0) = 1/(1 + e^{(A-B)/2}) \quad \text{.....(3.20).}$$

where

$$A = (mxn)\sigma_2{}^2/\sigma^2 + 2lnmK^2/2\pi + 2ln\pi \quad \text{.....(3.21).}$$

$$B = (mxn)\sigma_1{}^2/\sigma^2 + lnmK^2/2\pi + 2ln2 \quad \text{.....(3.22).}$$

and

m = number of replications,

n = sample size,

K = arbitrary number(set equal to 10), where K/2 is the number of standard deviations from the mean that it is felt reasonable to truancate the diffuse prior,

σ^2 = 4.9348(by Harvey's(1976) calculation),

$\sigma_1{}^2$ = SSE/n from the OLS regression of $\ln\hat{\epsilon}^2$ on a constant and lnX, where $\hat{\epsilon}$ is the residual from the OLS regression of Y on X and a constant, $\sigma_2{}^2 = \Sigma(\hat{\epsilon}_i{}^2 - \hat{\epsilon})^2/n$, from the OLS regression of $\ln\hat{\epsilon}$ on a constant.

CHAPTER IV

STRUCTURING THE MONTE CARLO EXPERIMENTS

4.1. INTRODUCTION

In this chapter, we outline the strategy and structure of the experimental design for the experiments to discover the sampling properties of the generalized pretest estimator(GPE) and other traditional estimators. The design is constructed in such a manner so as to be comparable to previous Monte Carlo studies [Goldfeld and Quandt(1974); Breusch and Pagan(1979); Buse(1984)].

The estimators considered are those discussed at the end of this chapter. The first three estimators(the ordinary least squares estimator, the 2SAE and the traditional pretest estimators) have been included in the Monte Carlo study because they correspond to what researchers most often employ.

4.2. THE MODEL SPECIFICATION FOR HETEROSKEDASTICITY

In keeping with most Monte Carlo Studies examining heteroskedasticity , we assume the data to be generated by a single explanatory variable such that,

$$Y_t = \beta_1 + \beta_2 X_t + U_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1).$$

$$\text{where } U_t \sim N(0, \sigma_t^2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2).$$

With this model specification and the distribution of the error term, two different types of heteroskedasticity structures were considered, mainly the bivariance and the multivariance cases.

In the sampling experiments, with the linear model(4.1) as specified above, we began with a sample size of 20 and then considered the sample size of 40 and 80(by replication). A single explanatory variable was used for simplicity and $\beta_1 = 0.025$ and $\beta_2 = 0.0025$. The choice of the coefficients β_1 and β_2 is irrelevant since the joint distribution of the U_t does not involve them[Breusch and Pagan(1979). Moreover, for the bivariance case the X values for the two halves of the sample are generated so that they are identical. This is in line with Ohtani and Toyoda(1980).

For the bivariance case, the distribution of the error term for each half of the sample is given as,

$U_1 \sim N(0, \sigma_1^2)$...................... ....4.3).

$U_2 \sim N(0, \sigma_2^2)$...................... ....4.4).

For the multivariance case, the distribution of the error term U_t is such that,

$U_t \sim N(0, \sigma_t^2)$.....................................(4.5).

$\sigma_t^2 = \alpha X_t^\delta$..(4.6).

where $\delta = 0$ corresponds to homoskedasticity.

The experiments are repeated for several values of δ between minus two(-2) and two(2).

The log-normal distribution is used to generate the regressor values [Buse,(1984)] . The contention is that 'by and large heteroskedasticity is a cross-section phenomenon and in this context the data are almost invariably skewed. The uniform distribution does not, therefore, seem particularly relevant'[Buse(1984), pp 207-208]. See also Goldfeld and Quandt(1972), and Harvey and Phillips(1974)

The values of the regressor are identical in repeated samples and are drawn from the log-normal distribution with mean 3 and variance 1. Three different sample sizes of 20, 40 and 80 are used and the sample sizes of 40 and 80 are obtained from the sample size 20 by replication. Six hundred different samples for each sample size were generated and used in all the experiments. Lovell(1983) observed that 25 replications gave basically identical results as 50. In view of this, it seems that the number of replications is not the crucial issue in the sampling experiments; the choice of 600 as the number of replications in the current study is arbitrary, but judging by the arbitrary number chosen by others for their Monte Carlo studies, our choice is unexceptionable. All comparisons are carried out using the relative mean square errors of all competing estimators(relative to mean square error of the GLS).

4.3. THE PROCEDURE

In order to operationalize the Monte Carlo experiments described above, Fortran programmes have been written. The subroutines such as AVGE2, VRANC2, TRANSG, CLUHAG, TRANS1, TRANS2 and TRANS3 can easily be derived from their corresponding counterparts and, therefore, have not been included in the appendices. In generating the variables used for all experiments, NAG Fortran Library Routines G05DEF,G05CBF and G05DDF are used. For the calculation of the OLS and the EGLS estimates, other NAG Fortran Library Routines such as F01CKF,F01AAF,and F01CDF are utilized. Other relevant and necessary subroutines have been written to implement the Monte Carlo experiments. The detailed Fortran programmes for the Monte Carlo study in this thesis are provided in appendix A and B.

4.4. AN ARRAY OF COMPETING ESTIMATORS

The general mathematical form of the generalized pretest estimator is given as:

$\beta(GPE) = \zeta\beta^{OLS} + (1 - \zeta)\beta^{2SAE}$ ....(4.7).

where

$\beta(GPE)$ = the generalized(smoothed) pretest estimator.

β^{OLS} = the ordinary least squares estimator.

β^{2SAE} = The Two-Stage Aitken estimator.

For the traditional pretest estimator $\zeta = I_{[0,c)}(U)$, an indicator function taking on the value of one if U, a test statistic, falls in the range subscripted and zero otherwise. In what follows, the various versions of the generalized pretest estimator and all other competing estimators considered here are described. The descriptions are separated into two categories: the bivariance and the multivariance cases.

4.5. THE BIVARIANCE CASE.

The first of these structures we refer to as the bivariance case, in which the data are broken into two sub-sections leading to the specification of the general linear regression model as:

$$Y_1 = \beta_1 + \beta_2 X_1 + U_1 \quad (4.8).$$

$$Y_2 = \beta_1 + \beta_2 X_2 + U_2 \quad (4.9).$$

where the notation is traditional.

$$U_1 \sim N(0, \sigma_1^2) \quad (4.10).$$

$$U_2 \sim N(0, \sigma_2^2) \quad (4.11).$$

with $\sigma_1^2 \neq \sigma_2^2$ and $\sigma_1^2/\sigma_2^2 = \gamma$.

The estimators compared are:

(1). **OLS** :- The ordinary least squares estimator.

(2). **2SAE** :- The OLS estimator is used to compute the variance of the error term for each half of the sample data as S_1^2 and S_2^2. To obtain S_1^2 and S_2^2, two ordinary least squares regressions are run on the two halves of the data separately. From these regressions the residuals from the first and second sub-samples are used to compute S_1^2 and S_2^2 respectively. The first and the second halves of the data are then transformed by dividing both the dependent and the independent variables by the square root of S_1^2 and S_2^2, respectively. Finally, the application of the OLS estimator to this transformed data results in the estimator referred to as the 2SAE.

(3). **BPT** :-This is the pretest estimator(combining the OLS estimator and the 2SAE) that results from using the Goldfeld and Quandt F-test statistic in testing for the presence of heteroskedasticity at the 5% significance level.

(4). **BPTGB** :This estimator is an 'improved' version of BPT, in which the pretest tests not for the existence of heteroskedasticity, but instead for the existence of heteroskedasticity of sufficient magnitude to render the 2SAE superior to the OLS estimator. Taylor(1977, pp 505-6) notes that the statistic $v_1S_1^2/v_2S_2^2\gamma \sim F(v_1, v_2)$ can be used for this purpose, where γ takes on the relevant critical values γ_l and γ_u from the Taylor(1977) table and v_1 and v_2 are the corresponding degrees of freedom. If for γ_u this statistic is less than the critical value obtained from the F-tables, and if for γ_l the inverse of this statistic is greater than the inverse of this critical F-value, ζ takes the value of one and BPTGB is equivalent to the ordinary least squares estimator, otherwise, ζ assumes the value of zero and hence BPTGB is idential to the 2SAE.

(5). **BPTB** :-The probability(ϕ) that the degree of heteroskedasticity is such that the ordinary least squares estimator dominates the 2SAE is computed by integrating Zellner's(1971) posterior density function for γ between the relevant critical γ values taken from Taylor(1977) . Using this probability as ζ, the ordinary least squares estimator and the 2SAE are combined in a non-dichotomous fashion to form this 'smoothed' pretest estimator.

(6). **GPESB** :- In the theoretical model formulation in Chapter three, a weighting system, λ, was developed, calculated as a function of ϕ. Using the value of λ as ζ estimated by using the value of ϕ from (5) above, GPESB combines the ordinary least squares estimator and the 2SAE as another 'smoothed' version of the generalized pretest estimator.

4.6. THE MULTIVARIANCE CASE.

The second heteroskedasticity structure considered in this thesis is the one we have referred to as the multivariance case. For this case the distribution of the error term is such that $U_t \sim (0, \sigma_t^2)$ and $\sigma_t^2 = \alpha X_t^{\delta}$; where $\delta = 0$ corresponds to homoskedasticity. Different values of δ are used for the experiments.

The estimators compared are:

(1). **OLS** :- The ordinary least squares estimator.

(2). **2SAEM** :- In this case, $\hat{\delta}$ is estimated by regressing the logarithm of the squared OLS residual on the logarithm of the explanatory variable. Both the dependent and the independent variables are transformed by dividing through by the square root of $X_t^{\hat{\delta}}$. The 2SAEM is calculated by doing an OLS regression on the transformed variables.

(3). **BPT** :- This is the traditional pretest estimator using the Goldfeld and Quandt F-test statistic, identical to (3) for the bivariance case(i.e., it selects between the OLS estimator and the 2SAE). Note that BPT is included so that we can examine the impact of erroneously assuming that the heteroskedasticity is bivariate rather than multivariate.

(4). **BPT1** :- This is the traditional pretest estimator using the Goldfeld and Quandt F-test statistic, identical to (3) for the bivariance case, however, it selects between the OLS estimator and the 2SAEM.

(5). **BPTT** : This is the traditional pretest estimator using the usual t-test on the estimated value of $\hat{\delta}$.

(6). **BPTGM** :This estimator is an 'improved' version of BPTT, in which the pretest(as for BPTGB) tests for the existence of heteroskedasticity of sufficient magnitude to render the 2SAEM superior to the ordinary least squares estimator. A mini Monte Carlo study is used to estimate the relevant values of $\hat{\delta}$(δ_l and δ_u) and a t-test is employed to test whether $\hat{\delta}$ lies between these values. If this hypothesis is accepted on the basis of the t-test, ζ assumes the value of one and the ordinary least squares estimator is used. However, if the test shows that $\hat{\delta}$

lies outside this region, ζ takes on the value of zero and the 2SAEM is used instead.

(7). **BPTM** :- Assuming that the posterior density of $\hat{\delta}$ is a t-distribution with mean $\hat{\delta}$ and its variance is given by Harvey(1976) as $4.9348(z_t'z_t)^{-1}$, the probability that the degree of heteroskedasticity is such that the ordinary least squares estimator is superior to the 2SAEM is estimated by integrating this t-density function for $\hat{\delta}$ between δ_l and δ_u. Using this probability as ζ, the ordinary least squares estimator and the 2SAEM are linearly combined into the 'smoothed' pretest estimator. It is the multivariance counterpart for BPTB(bivariance).

(8). **GPESM** :This version of the generalized pretest estimator is the multivariance counterpart for GPESB(bivariance). ζ is calculated using the formula for λ given earlier, where ϕ is computed from the integration of the density function for δ as described in (6) above.

(9). **BPE** :- The probability(ϕ) associated with the null hypothesis is calculated via the posterior odds ratio technique developed by Villegas(1986). Setting ζ equal to this value, the ordinary least squares estimator and the 2SAE are combined to form BPE.

(10).**GPEV** :- This is GPESM(as described in 8 above) caculating λ by using the ϕ computed using the posterior odds ratio technique of Villegas.

(11). **BPTBM** :- This is the version of the smoothed pretest estimator that results when the researcher decides to apply the bivariance technique of calculating ϕ to the data suspected of having multiplicative heteroskedasticity. It is accomplished first, by using this technique to calculate the probability(ϕ) that heteroskedasticity is such that the OLS estimator outperforms the 2SAEM. Second, BPTBM linearly combines both the OLS estimator and the 2SAEM by using this value of ϕ.

Note that the difference between GPESM and GPEV is that while GPESM utilizes the ϕ calculated from the integration of the posterior density function for $\hat{\delta}$ in computig the weighting system, λ, GPEV uses the ϕ derived from the method for calculating the posterior odds ratio which was developed by Villegas(1986).

CHAPTER V

COMPARISONS OF RELATIVE MEAN SQUARE ERRORS

5.1. THE BIVARIANCE CASE:-

We begin the analysis of the results with the sample size of 20. In Table 5.1 the values of the relative mean square errors of all competing estimators are presented as a function of the variance ratio $\gamma(\gamma = \sigma_1^2/\sigma_2^2)$. Consider the characteristics of the traditional pretest estimator(BPT). As evident from Table 5.1 and Figure 5.1 respectively, as γ diverges from one the risk function of the traditional pretest estimator(BPT) rises continuously for some time, reaches a maximum and then gradually declines approaching the risk function of the 2SAE in value. The farther γ diverges from one, the hypothesis of equal variances is rejected more often and hence the pretest estimator selects and uses the 2SAE more often than it selects the ordinary least squares estimator. In this way, its risk function continues to decrease in magnitude, approaching the risk function of the 2SAE. As γ tends towards 1(i.e., for γ values between 0.70 and 1.75) the traditional pretest estimator(BPT) has a smaller relative mean square error than the 2SAE. In this region of the parameter space, the relative mean square error of BPT is quite close to that of the ordinary least squares estimator. The explanation of this phenomenon is that in this region(the region between 0.70 and 1.75), the pretest accepts the null hypothesis most often and, therefore, selects the ordinary least

TABLE 5.1

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE BIVARIANCE CASE:- SAMPLE SIZE - 20).

GAMMA	OLS	2SAE	GPESB	BPT	BPTGB	BPTB
0.10	3.0769	1.0604	1.1393	1.2562	1.7569	1.0954
0.15	2.2382	1.0673	1.1608	1.3391	1.7505	1.1021
0.35	1.3147	1.0741	1.1291	1.2100	1.3279	1.0856
0.50	1.1327	1.0751	1.0862	1.1596	1.1432	1.0671
0.80	1.0148	1.0772	1.0328	1.0532	1.0296	1.0482
1.00	1.0000	1.0790	1.0216	1.0388	1.0018	1.0453
1.40	1.0251	1.0827	1.0322	1.0432	1.0232	1.0509
1.50	1.0374	1.0836	1.0388	1.0572	1.0444	1.0535
2.50	1.2143	1.0901	1.1335	1.1604	1.2190	1.0854
6.00	2.0127	1.0926	1.2158	1.3328	1.5749	1.1367
10.0	2.9740	1.0854	1.1813	1.2760	1.8387	1.1458
15.0	4.1895	1.0760	1.1778	1.2257	1.8148	1.1231
20.0	5.4091	1.0682	1.1689	1.1773	1.7562	1.1116

Sample Size = 20
Bivariance Case

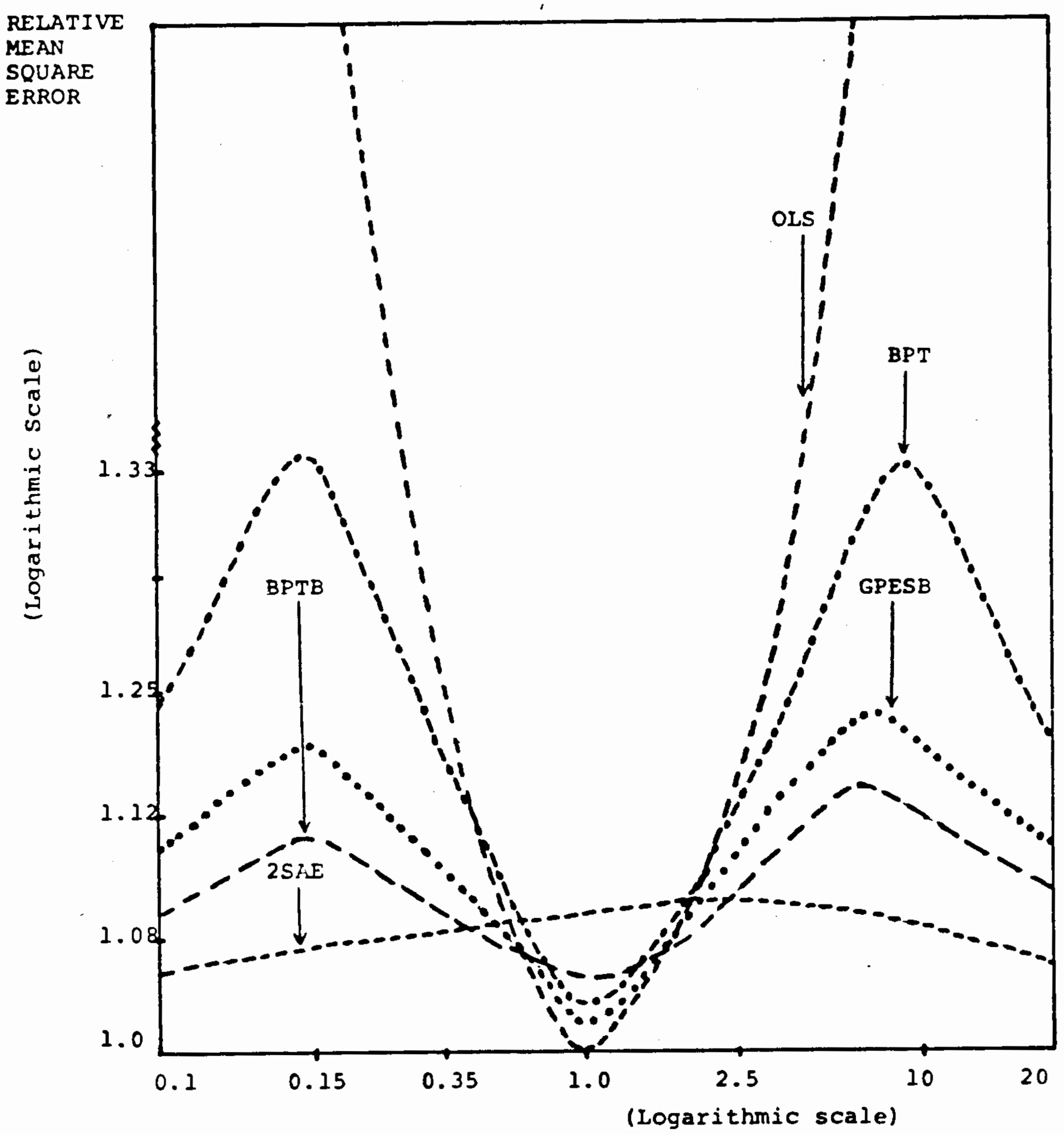

FIGURE 5.1: RELATIVE MSE OF COMPETING ESTIMATORS

squares estimator and uses it to estimate the parameters more often than it uses the 2SAE. In this range, BPT dominates the 2SAE. This finding confirms the conclusions of Taylor(1977, 1978), Greenberg(1980), Toyoda and Ohtani(1980) and Mandy(1984).

The ordinary least squares estimator attains the smallest relative mean square error when γ is 1. In this case, since there is no heteroskedasticity, the ordinary least squares estimator achieves its minimum variance and hence maintains its BLUE properties. Above all, it has a smaller relative mean square error than all the other competing estimators between γ values very close to 0.80 and 1.65. However, the range within which it dominates BPTB is smaller, and still smaller for GPESB. Even though the ordinary least squares estimator possesses this attractive power of retaining its BLUE properties over all the other competing estimators for $\gamma = 1$ its relative mean square error becomes very large either as γ tends towards 0 or ∞. That is, as the null hypothesis becomes more and more false(i.e., the severity of heteroskedasticity increases), even though it maintains its property of unbiasedness, its variance estimator is no longer efficient. Kennedy(1985), pp 96 - 97 notes that 'the higher absolute values of the residuals... indicate a positive relationship between the error variance and the independent variable. With this kind of error pattern, a few additional large positive errors near the right... would tilt the OLS regression line considerably. A few additional large negative errors would tilt it in the opposite direction considerably. In repeated sampling these unusual cases would average out, leaving the OLS estimator unbiased, but the variation of the OLS regression line around its mean will be greater, i.e., the variance of β^{OLS} will be greater'. This explains why the relative mean square error of the ordinary least squares estimator rises continuously as the severity of heteroskedasticity increases. It is, therefore, outperformed by the 2SAE outside the region of the parameter space defined approximately by 0.60 and 1.80.

The mean square error functions of all the pretest estimators(both traditional and smoothed) have the same basic shape for the reasons discussed earlier. The level of significance is 5% as given earlier in the previous Chapter. Note, however, that the smoothed pretest estimators have flatter relative mean square error functions than the traditional pretest estimator. The various versions of the generalized pretest estimator can be viewed as competing alternatives to the traditional pretest estimator because even though they do not outperform the traditional pretest estimator over the whole parameter space, they do outperform it over a large portion of the parameter space. For γ values quite close to one the traditional pretest estimator outperforms all the various versions of the smoothed pretest estimators with exception of GPESB which dominates the traditional pretest estimator over the whole parameter space(more will be said about this below). In view of this, the choice of estimator for estimation purposes depends on one's prior view of the severity of heteroskedasticity. If one's prior is concentrated quite close to one, then the OLS estimator or the traditional pretest estimator would be preferred to the smoothed pretest estimators and the 2SAE.

Greenberg observes that "any a priori information concerning γ possessed by the researcher may be used to determine which estimator has the smallest variance in the range of γ considered to be reasonable. If a prior distribution for γ is available, the choice may be based on Bayes' risk of each estimator" [Greenberg(1980), pp 1811]. This choice of estimator is therefore made easier if the researcher has 'good' prior information about the magnitude of the variance ratio. For example, if the researcher's prior distributuion is concentrated in an area close to $\gamma = 1$, then the best choice is the ordinary least squares estimator. Similarly, if the prior distribution for γ places substantial weight outside the approximate interval $0.50 \leq \gamma \leq 2.0$, then the 2SAE appears to be the best choice.

There exist cases in which the researcher possesses prior knowledge about higher magnitudes of the variance ratio(γ). For example, Taylor(1977) states that it is possible to have large γ values in cross-section data since in this case aggregates of great differences and sizes are under consideration. In these situations, it should be expected that the 2SAE would perform better than the ordinary least squares estimator and should, therefore, be preferred to it[Taylor(1977), pp 504 - 505]. It clearly also should be preferred to the smoothed pretest estimators.

The most interesting case occurs when one's prior belief concerning the degree of heteroskedasticity is completely diffuse, presumably the case of most interest to non-Bayesians. In this case the choice of estimator can be based on various criteria described as follows. First, as noted in Wallace(1976), pp 439, the researcher can consider the overall performance of each estimator by comparing the differences between the relative mean square error functions of all competing estimators integrated over all values of γ. In terms of Figure 5.1 this implies choosing that estimator whose relative mean square error function has the smallest area under it. Using this criterion, Figure 5.1 suggests that the 2SAE is the best choice. Note also that it also suggests that the smoothed pretest estimator is preferred to the traditional pretest estimator. Second, as also noted by Wallace, the comparison of estimators can be undertaken by using the 'minimization of the maximum regret' criterion. In terms of Figure 5.1 this implies choosing the estimator with the smallest maximum height. Once again, Figure 5.1 suggests that the 2SAE is the estimator of choice, and that the smoothed pretest estimator is superior to the traditional pretest estimator.

It seems reasonable, however, that the prior distribution on γ is not diffuse. Wallace, for example, notes that 'such priors may be too conservative in the direction of large θ(which is denoted as γ in this thesis), since the very fact that the investigator is interested in a particular set of restrictions presupposes that he must have a prior belief that θ is small, although this is not always the case'[Wallace T. D.(1977), pp 438]. This would tend to make the choice between the smoothed pretest estimator(BPTB) and the 2SAE less obvious. Clearly the smoothed pretest estimator is an attractive alternative to the 2SAE and seems under reasonable circumstances to be more preferrable to the traditional pretest estimator. Note that θ is the variance ratio and is denoted as γ in this thesis.

It is interesting to note that at the 5% significance level, GPESB dominates the traditional pretest estimator completely over the whole parameter space for the sample size 20. This dominance does not hold for the sample sizes 40 and 80. This suggests that for small sample sizes the traditional pretest estimator should never be employed when significance tests are performed at the 5% level. As the sample size becomes larger, the traditional pretest estimator(BPT) improves because with a larger sample size a 'better' estimate of γ must outweigh the impact of the improvement for GPESB in estimating λ (and γ) as the sample size grows. This is a surprising result.

Table 5.2 and Figure 5.1B show the relative mean square errors and their corresponding graphs for all competing estimators when the F-test was performed using 1% as the significance level. As evident from these results, the relative mean square errors of the traditional pretest estimator(BPT), and its modified version(BPTGB) are larger for high values of γ than they were when the 5% significance level was used, for low values of γ, as illustrated in Figure 5.1B. This result is exactly what would have been expected, following the logic of Toyoda and Wallace(1976). Thus, as the level of significance tends towards zero the risk function of the traditional pretest estimator tends towards that of the restricted least squares estimator. On the other hand, as the chosen level of significance tends towards one, the risk function of this estimator gradually approaches that of the unrestricted(OLS) least squares estimator. It must be stated clearly that at the 1% significance level, GPESB no longer domonates the traditional pretest estimator completely. Note, however, that the level of significance does not affect the performance of any of the other competing estimators, in particular the 'smoothed' pretest estimator. This could be viewed as an advantage of the smoothed pretest estimator relative to the traditional pretest estimator, since a researcher would not have the additional dilemma of arbitrarily choosing an 'optimal' significance level.

TABLE 5.2

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(1% SL).

(THE BIVARIANCE CASE:- SAMPLE SIZE - 20).

GAMMA	OLS	2SAE	GPESB	BPT	BPTGB	BPTB
0.10	3.0769	1.0604	1.1393	1.3041	2.3641	1.0954
0.15	2.2382	1.0673	1.1608	1.5891	2.0957	1.1021
0.35	1.3147	1.0741	1.1291	1.3189	1.3173	1.0856
0.50	1.1327	1.0751	1.0862	1.1442	1.1408	1.0671
0.80	1.0148	1.0772	1.0328	1.0297	1.0160	1.0482
1.00	1.0000	1.0790	1.0216	1.0184	1.0037	1.0453
1.40	1.0251	1.0827	1.0322	1.0382	1.0266	1.0509
1.50	1.0374	1.0836	1.0388	1.0505	1.0389	1.0535
2.50	1.2143	1.0901	1.1335	1.1965	1.2162	1.0854
6.00	2.0127	1.0926	1.2158	1.5144	1.7097	1.1367
10.0	2.9740	1.0854	1.1813	1.6594	2.3715	1.1458
15.0	4.1895	1.0760	1.1778	1.6066	2.4973	1.1231
20.0	5.4091	1.0682	1.1689	1.4488	2.5236	1.1116

Sample Size = 20
Bivariance Case

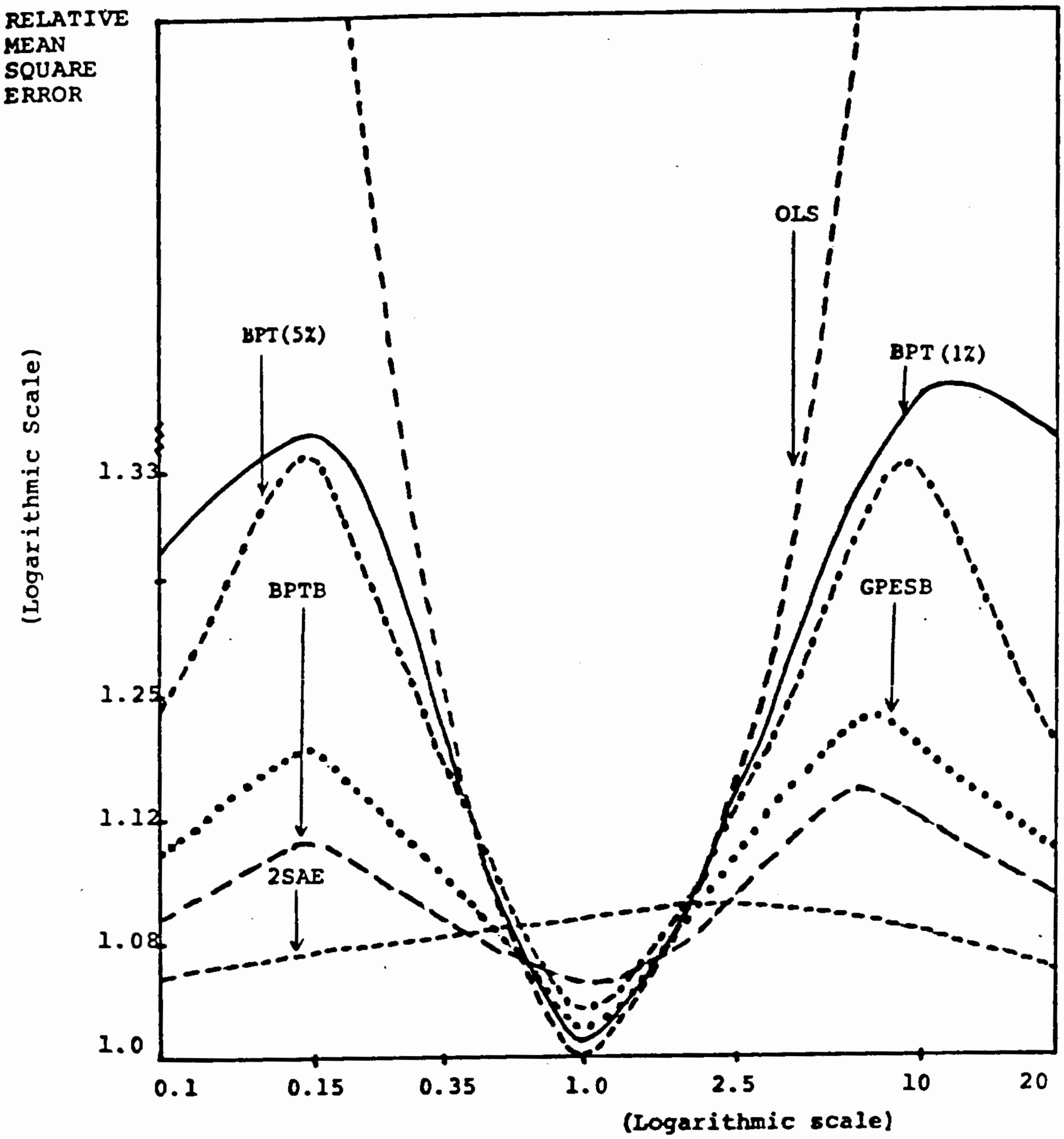

FIGURE 5.1b; RELATIVE MSE OF COMPETING ESTIMATORS

Table 5.3 contains the relative mean square errors of all competing estimators(for the sample size of 40). These are stated as functions of the variance ratio. Figure 5.2 portrays the graphs of the relative mean square errors of these estimators. The shapes and the relationships among these estimators are similar to those discussed above for the sample size of 20 and, therefore, suggest no changes in our conclusions. Note that in general the relative mean square errors of the competing estimators are smaller than those they attained for the sample size of 20. This means that as the sample size increases, the efficiency of each competing estimator improves. This makes sense because as the sample size increases, the variance-covariance estimator of the 2SAE becomes as efficient as that of the GLS estimator.

TABLE 5.3

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE BIVARIANCE CASE:- SAMPLE SIZE - 40).

GAMMA	OLS	2SAE	GPESB	BPT	BPTGB	BPTB
0.10	2.9205	1.0399	1.0520	1.0686	1.1180	1.0396
0.15	2.1353	1.0450	1.0728	1.0802	1.2061	1.0566
0.35	1.2752	1.0707	1.1286	1.1710	1.2458	1.0873
0.50	1.1090	1.0780	1.0966	1.1160	1.1248	1.0830
0.80	1.0077	1.0852	1.0367	1.0511	1.0075	1.0611
1.00	1.0000	1.0874	1.0270	1.0487	1.0000	1.0561
1.40	1.0360	1.0889	1.0477	1.0590	1.0513	1.0645
1.50	1.0506	1.0889	1.0569	1.0687	1.0647	1.0682
2.50	1.2477	1.0851	1.1501	1.1841	1.2279	1.0989
5.00	2.1063	1.0654	1.1258	1.1364	1.1336	1.0864
10.0	3.1342	1.0505	1.0509	1.0505	1.2057	1.0524
15.0	4.4301	1.0393	1.0335	1.0393	1.0427	1.0409
20.0	5.7310	1.0323	1.0338	1.0323	1.0323	1.0329

Sample Size = 40
Bivariance Case

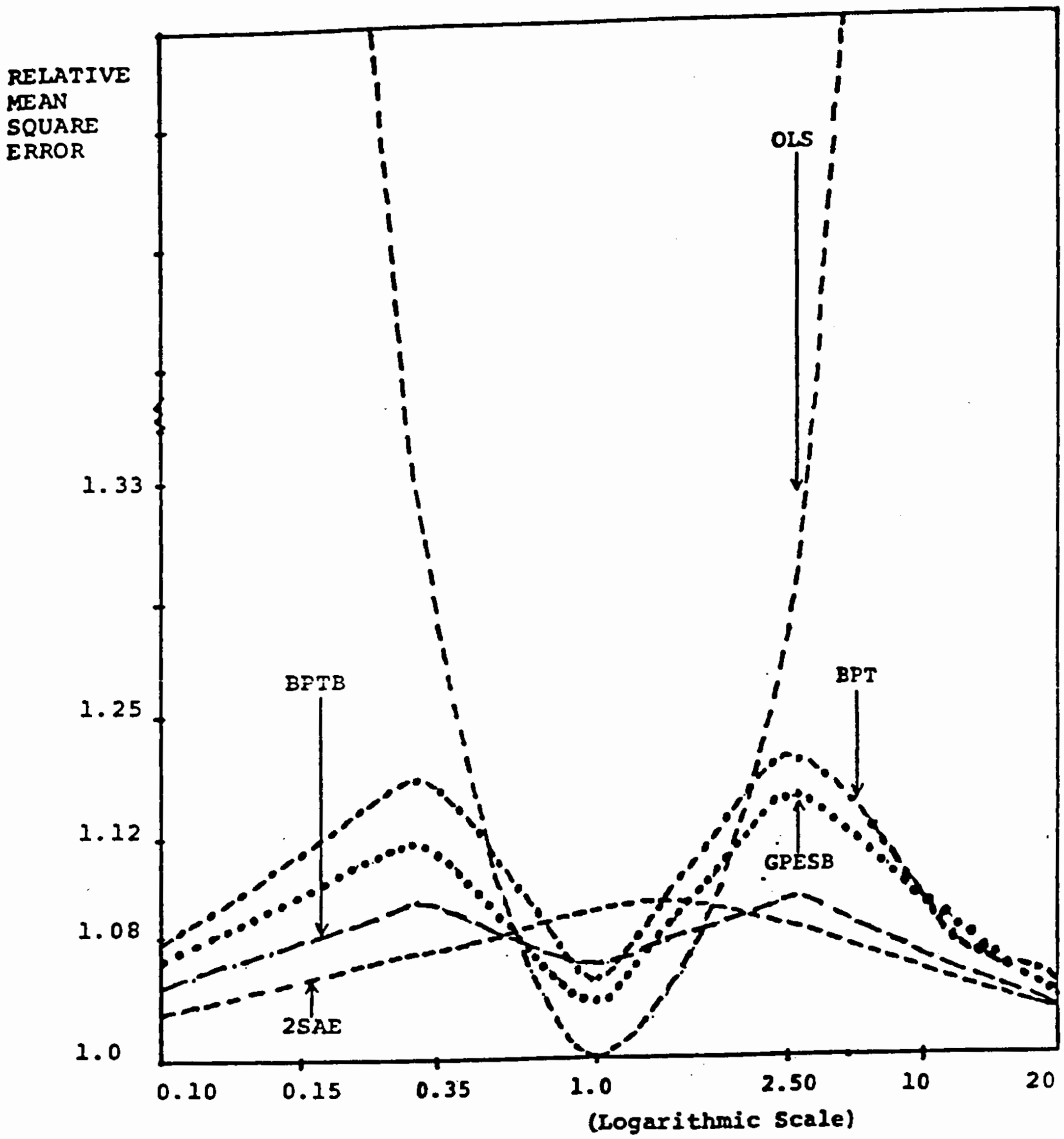

FIGURE 5.2: RELATIVE MEAN SQUARE ERRORS OF COMPETING ESTIMATORS

For the sample size of 40 a corresponding pretest was carried out at 1% significance level. The results are specified in Table 5.4 and the relative mean square error functions are portrayed in Figure 5.2B. Since the conclusions arrived at for the sample size of 20(at the 1% signicance level) do not differ from the results obtained for the sample size of 40, they do not need any further elaboration.

TABLE 5.4

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(1% SL).

(THE BIVARIANCE CASE:- SAMPLE SIZE - 40).

GAMMA	OLS	2SAE	GPESB	BPT	BPTGB	BPTB
0.10	2.9205	1.0399	1.0520	1.0924	1.2890	1.0396
0.15	2.1353	1.0450	1.0728	1.1554	1.5560	1.0566
0.35	1.2752	1.0707	1.1286	1.2261	1.2768	1.0873
0.50	1.1090	1.0780	1.0966	1.1339	1.1230	1.0830
0.80	1.0077	1.0852	1.0367	1.0294	1.0077	1.0611
1.00	1.0000	1.0874	1.0270	1.0135	1.0000	1.0561
1.40	1.0360	1.0889	1.0477	1.0502	1.0354	1.0645
1.50	1.0506	1.0889	1.0569	1.0638	1.0634	1.0682
2.50	1.2477	1.0851	1.1501	1.2091	1.2552	1.0989
5.00	2.1063	1.0654	1.1258	1.2468	1.5489	1.0864
10.0	3.1342	1.0505	1.0509	1.1435	1.4278	1.0524
15.0	4.4301	1.0393	1.0335	1.0391	1.2626	1.0409
20.0	5.7310	1.0323	1.0338	1.0323	1.0490	1.0329

Sample Size = 40
Bivariance Case

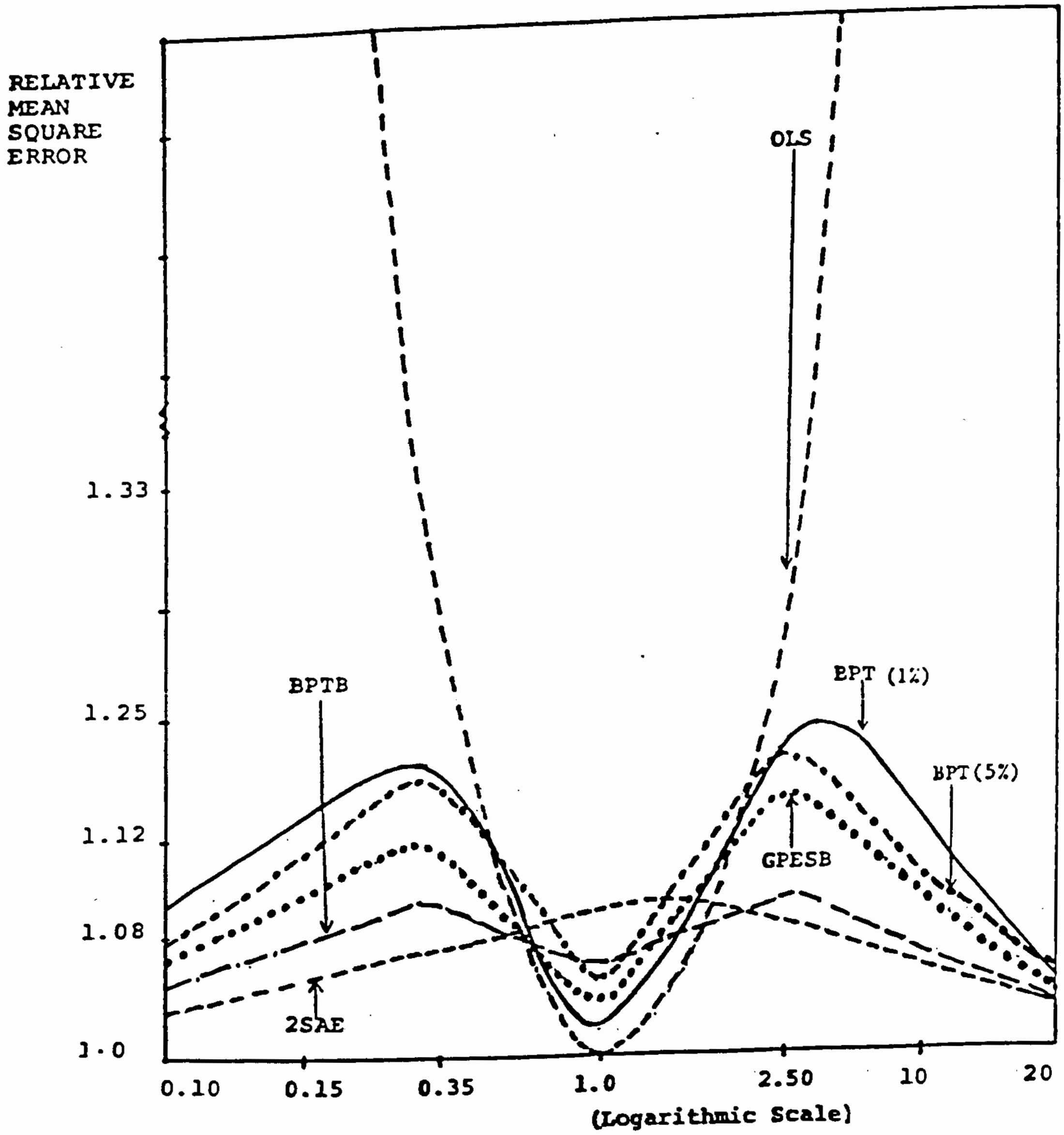

FIGURE 5.2b: RELATIVE MEAN SQUARE ERRORS OF COMPETING ESTIMATORS

Table 5.5 and Figure 5.3 contain the estimates of the relative mean square errors and the graphs of these functions of all competing estimators under consideration for the sample size of 80, respectively. These results confirm the results for the sample sizes of 20 and 40. Though the corresponding graph has been drawn for the sample size of 80, due to the fact that most of these relative MSE estimates for all estimators are very close to each other, the vertical scale was slightly modified. Even though the scale used in drawing the graphs for the sample size of 80 is different from that used for the sample sizes of 20 and 40, the differences in the relative performance of all competing estimators stand out conspicuously.

TABLE 5.5

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE BIVARIANCE CASE:- SAMPLE SIZE - 80).

GAMMA	OLS	2SAE	GPESB	BPT	BPTGB	BPTB
0.10	3.0623	1.0098	1.0244	1.0098	1.0098	1.0069
0.15	2.2290	1.0134	1.0181	1.0134	1.0134	1.0140
0.35	1.3116	1.0211	1.0328	1.0378	1.0211	1.0252
0.50	1.1309	1.0232	1.0230	1.0569	1.0232	1.0216
0.80	1.0143	1.0235	1.0009	1.0193	1.0235	1.0079
1.00	1.0000	1.0226	1.0000	1.0004	1.0226	1.0018
1.40	1.0259	1.0201	1.0061	1.0319	1.0201	1.0036
1.50	1.0384	1.0194	1.0090	1.0447	1.0194	1.0052
2.50	1.2168	1.0136	1.1236	1.0472	1.0136	1.0321
5.00	1.7817	1.0060	0.9382	1.0067	1.0060	1.0000
10.0	2.9881	1.0011	0.9993	1.0011	1.0011	1.0006
15.0	4.2117	1.0010	0.9992	1.0010	1.0010	0.9995
20.0	5.4396	1.0008	1.6701	1.0008	1.0008	1.0076

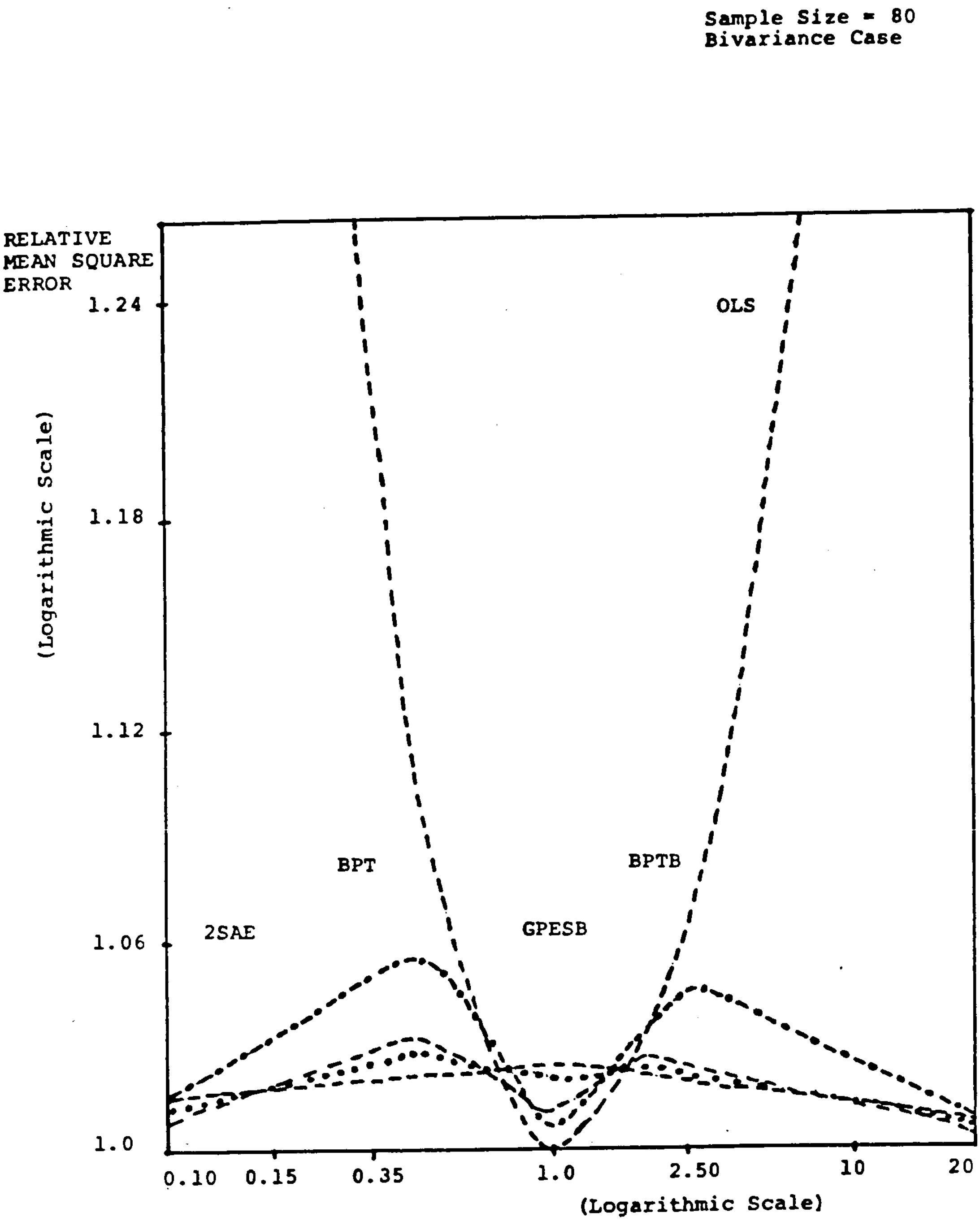

FIGURE 5.3: RELATIVE MSE OF COMPETING ESTIMATORS

The results for the pretests conducted at the 1% significance level for the sample size of 80 are presented in Table 5.6 and Figure 5.3B. Our conclusions are the same as for the sample sizes of 20 and 40.

TABLE 5.6

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(1% SL).

(THE BIVARIANCE CASE:- SAMPLE SIZE - 80).

GAMMA	OLS	2SAE	GPESB	BPT	BPTGB	BPTB
0.10	3.0623	1.0098	1.0244	1.0098	1.0098	1.0069
0.15	2.2290	1.0134	1.0181	1.0134	1.0134	1.0140
0.35	1.3116	1.0211	1.0328	1.0897	1.0211	1.0252
0.50	1.1309	1.0232	1.0230	1.1028	1.0232	1.0216
0.80	1.0143	1.0235	1.0009	1.0065	1.0235	1.0079
1.00	1.0000	1.0226	1.0000	1.0018	1.0226	1.0018
1.40	1.0259	1.0201	1.0061	1.0355	1.0201	1.0036
1.50	1.0384	1.0194	1.0090	1.0456	1.0194	1.0052
2.50	1.2168	1.0136	1.1236	1.0938	1.0163	1.0321
5.00	1.7817	1.0060	0.9382	1.0046	1.0060	1.0000
10.0	2.9881	1.0011	0.9993	1.0011	1.0011	1.0006
15.0	4.2117	1.0010	0.9992	1.0010	1.0010	0.9995
20.0	5.4396	1.0008	1.6701	1.0008	1.0008	1.0076

Sample Size = 80
Bivariance Case

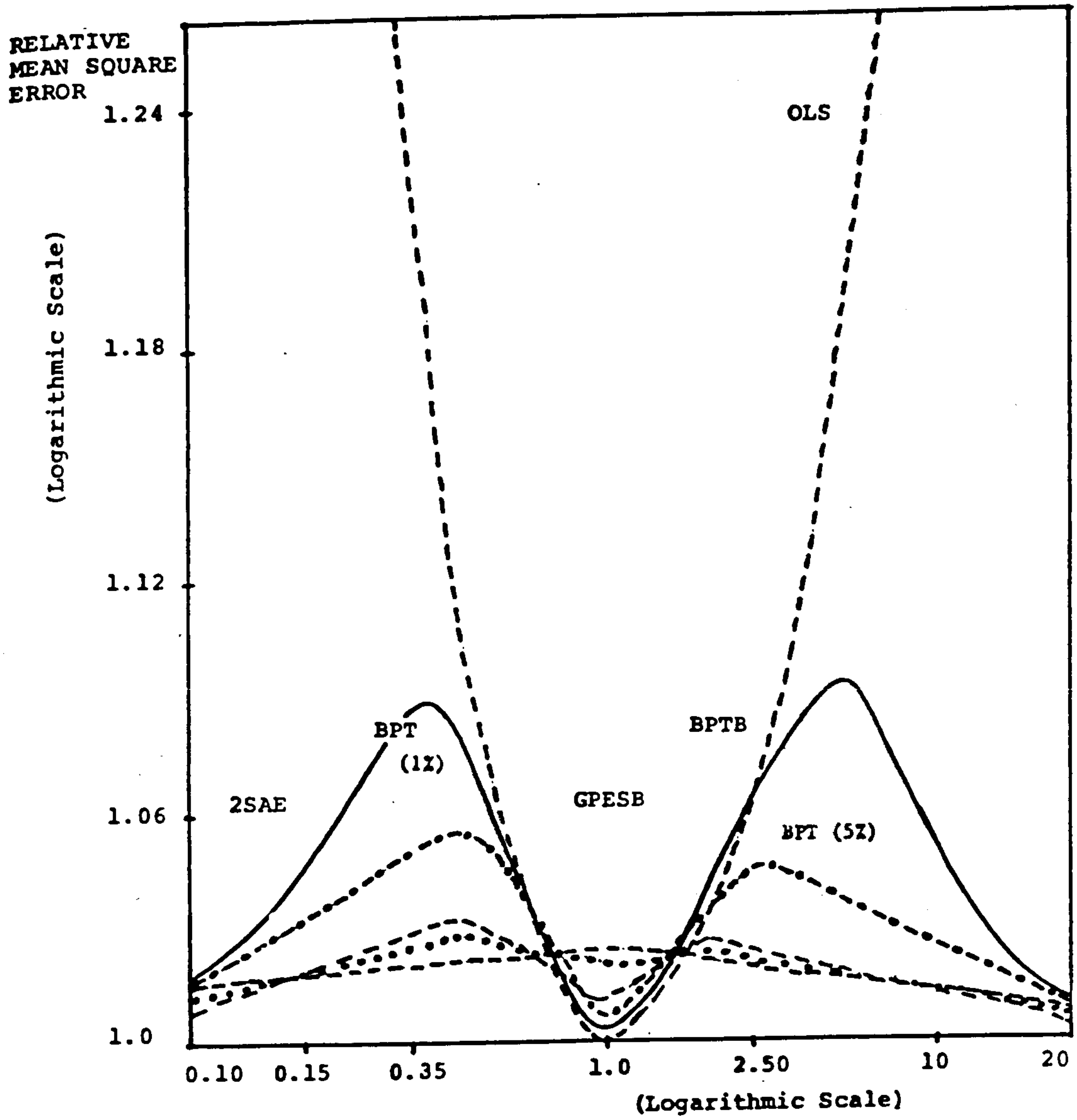

FIGURE 5.38: RELATIVE MSE OF COMPETING ESTIMATORS

In conclusion, Table 5.7 shows the minimum and maximum variance ratios(γ_l and γ_u) that define the boundaries of the parameter space over which the ordinary least squares estimator performs better than the 2SAE as observed from the present Monte Carlo study results. These values are obtained from Tables 5.1, 5.2 and 5.3 respectively.

TABLE 5.7

THE LIMITING VALUES OF THE VARIANCE RATIO.

N	γ_l	γ_u
20	0.50 (0.50)	2.02 (2.09)
40	0.64 (0.61)	1.70 (1.65)
80	0.76 (0.70)	1.40 (1.43)

It is interesting to notice that these values are very close to the values obtained from Taylor(1977), shown in brackets in Table 5.7 above.

Recall that BPTGB was developed by using an F-test suggested by Taylor(1977). This pretest estimator, even though dichotomous, was expected to be an 'improved' version of the traditional pretest estimator. Even though it does not outperform the traditional pretest estimator over the whole parameter space its performance is comparable to that of BPT. Its relative mean square errors have been specified in all the tables above.

The following Table 5.8 contains the mean values and variances of phi with their corresponding gamma values. The characteristics of these ϕ values are what we expect theoretically. That is, at γ values close and equal to 1, we expect larger ϕ values than when γ values are very far away from 1. This is so because at γ value of 1(homoskedasticity), a high weight should be assigned to the null hypothesis. However, as we move away from homoskedasticity to towards higher degrees of heteroskedasticity, the null hypothesis becomes more false and hence the probability(ϕ) associated with it becomes smaller and smaller.

Notice that at γ equal to 1, the ϕ estimator places approximately equal weights on both the null and the alternative hypotheses. This phenomenon is one reason why the risk functions of the various versions of the smoothed pretest estimator are not as close to that of the ordinary least squares estimator as we expected when γ is 1. At the other extreme, the ϕ estimator continuously places small positive weights on the ordinary least squares estimator as evident from Table 5.8. These ϕ values help us give some explanation to the performance and the risk characteristics of the various versions of the smoothed pretest estimator.

TABLE 5.8

THE MEAN AND VARIANCE OF PHI(ϕ).

(FOR ALL SAMPLE SIZES(BIVARIANCE CASE)).

	N = 20		N = 40		N = 80	
GAMMA	MEAN	VAR	MEAN	VAR	MEAN	VAR
0.10	0.0830	0.0076	0.0347	0.0025	0.0046	0.0024
0.15	0.1320	0.0197	0.0276	0.0034	0.0004	0.0025
0.35	0.3273	0.0493	0.2006	0.0383	0.0403	0.0045
0.50	0.4174	0.0462	0.2547	0.0514	0.1777	0.0031
0.80	0.4974	0.0324	0.5131	0.0364	0.4701	0.0257
1.00	0.5106	0.0291	0.5323	0.0297	0.5581	0.0223
1.40	0.4958	0.0322	0.4758	0.0386	0.4974	0.0218
1.50	0.4879	0.0337	0.4538	0.0420	0.4587	0.0166
2.50	0.3895	0.0436	0.2479	0.0466	0.3852	0.0278
5.00	0.2170	0.0310	0.0532	0.0094	0.0135	0.0033
10.00	0.0958	0.0121	0.0049	0.0002	0.0006	0.0195
15.00	0.0571	0.0062	0.0009	0.93E-05	0.0001	0.0129
20.00	0.0400	0.0037	0.0003	0.73E-06	0.0633	0.0036

5.2. THE MULTIVARIANCE CASE:-

Table 5.9 contains the relative mean square errors of all competing estimators for the sample size of 20. As expected (theoretically), the OLS estimator retains its BLUE properties when δ = 0(homoskedasticity). However, as the degree of heteroskedasticity increases, it loses this attractive property and performs very poorly as compared to all the other competing estimators. That is, its relative mean square error increases continuously since it suffers from larger and larger variances. This observation confirms the results of the OLS estimator as described above for the bivariance case. The relative mean square error functions of all competing estimators are shown in Figure 5.4.

Unfortunately, the behaviour of the relative mean square error function of the 2SAEM is not as expected theoretically. We expected that as the degree of heteroskedasticity increases, the relative mean square error of the 2SAEM would take a form similar to the form it takes in the bivariance case. But the Monte Carlo results for the multivariance case do not confirm this observation(Table 5.9 and Figure 5.4). The relative mean square error of the 2SAEM rises continuously as do the relative mean square errors of all the competing estimators. One reason why this occurred may be that the estimator for delta(δ) deteriorates as the degree of heteroskedasticity increases. This high degree of inefficiency in estimating δ also affects the parameter estimates of the 2SAEM. That is, they are poorly estimated. Due to this poor performance of the 2SAEM, all the other competing estimators do not retain the characteristics and the shapes of the relative mean square errors observed for the bivariance case. Like the relative mean square errors of the of the ordinary least squares(OLS) estimator and the 2SAEM the relative mean square errors of all the other competing estimators increase continuously[Table 5.9 and Figure 5.4]. Note that BPE and GPEV perform very poorly. We speculate that the Villegas procedure for estimating the probability(ϕ) that the degree of heteroskedasticity is such that the OLS estimator performs better than the 2SAEM is inefficient. This procedure estimates these probabilities very poorly. These poor estimates of phi and their variances are specified in Table 5.14 below. The information concerning BPTT and BPTG is recorded in Table 5.12.

TABLE 5.9

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- SAMPLE SIZE - 20).

DELTA	OLS	2SAEM	BPE	GPEV	GPESM	BPT	BPTM
-2.00	8.8344	1.3470	8.0945	6.1228	5.7706	1.3554	1.5950
-1.60	4.5903	1.2179	4.3993	3.4827	3.2130	1.3108	1.3712
-1.40	3.3894	1.1883	3.2895	2.7565	2.4804	1.3134	1.2987
-1.20	2.5543	1.1505	2.5140	2.2494	1.9643	1.3086	1.2469
-1.00	1.9751	1.1275	1.9539	1.8410	1.6120	1.3122	1.2014
-0.80	1.5780	1.1347	1.5672	1.5368	1.3841	1.3062	1.1738
-0.60	1.3122	1.1126	1.3069	1.2903	1.2241	1.2514	1.1279
-0.40	1.1427	1.1157	1.1395	1.1318	1.1344	1.1439	1.0959
-0.20	1.0446	1.1059	1.0438	1.0438	1.0776	1.0603	1.0685
0.00	1.0000	1.1067	1.0000	1.0012	1.0653	1.0409	1.0497
0.20	1.0015	1.1207	1.0014	1.0018	1.0841	1.0446	1.0608
0.40	1.0246	1.1478	1.0243	1.0245	1.1266	1.1104	1.0986
0.60	1.0799	1.1296	1.0787	1.0798	1.1230	1.1223	1.1476
0.80	1.1606	1.1392	1.1575	1.1603	1.1419	1.1845	1.1362
1.00	1.2681	1.1289	1.2615	1.2676	1.1410	1.1621	1.1537
1.20	1.4077	1.1727	1.3999	1.4071	1.1896	1.1883	1.2223
1.40	1.5887	1.2034	1.5728	1.5877	1.2212	1.2173	1.2802
1.60	1.8254	1.2427	1.8049	1.8242	1.2605	1.2524	1.3421
2.00	2.5585	1.5183	2.5323	2.5575	1.5384	1.5223	1.6677

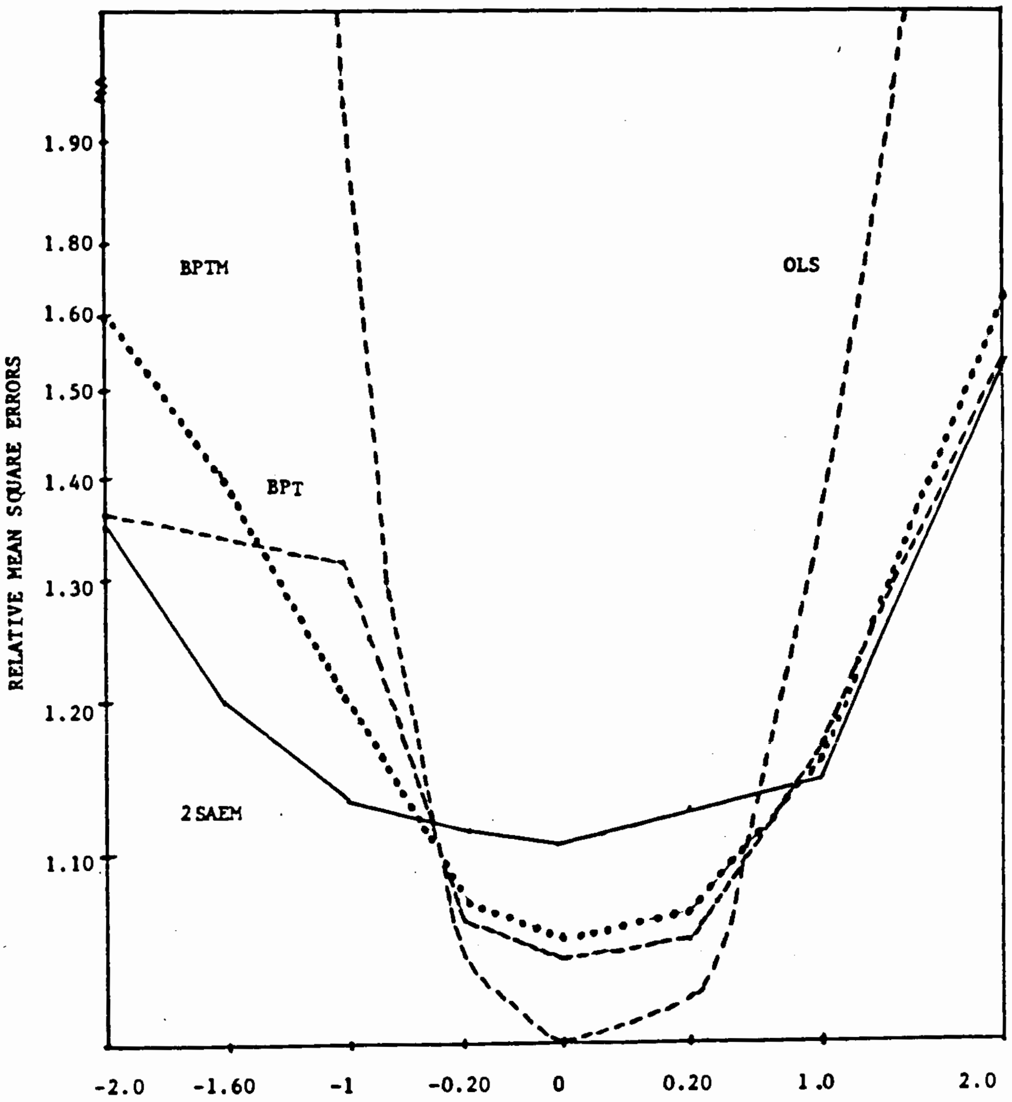

FIGURE 5.4:- RELATIVE MSE OF COMPETING ESTIMATORS

At $\delta = 0$, the traditional pretest estimator(BPT) outperforms the 2SAEM, BPTM and GPESM. Note, however, that since the values of the relative mean square errors of BPE and GPEV are almost always identical to that of the OLS estimator, BPT does not perform better than these versions of the smoothed pretest estimator. As soon as δ departs from zero, the pretest selects the 2SAEM more often than it selects the OLS estimator. By so doing, its relative mean square error gradually approaches that of the 2SAEM. As the severity of heteroskedasticity grows, the relative mean square error of the traditional pretest estimator tends to that of the 2SAEM.

Over a large section of the parameter space the 'smoothed' pretest estimator(BPTM) outperforms the traditional pretest estimator. However, inexplicably, at very high degrees of heteroskedasticity, the traditional pretest estimator begins to attain a smaller relative mean square error than all the versions of the 'smoothed' pretest estimator. This is a matter for further investigation since this observation does not agree with our theoretical expectation. We speculate, however, that this occurs because the smoothed pretest estimators continue to give a small weight(i.e., non-zero) weight to the OLS estimator, whose relative mean square error at that point is extremely large.

A visual inspection of the relative mean square error functions of all estimators in Figure 5.4 reveals that it is very difficult to select a preferred estimator. However, the choice of any of these estimators for the regression analyses can be enhanced if the researcher possesses some apriori knowledge about the degree of heteroskedasticity in the particular data he is dealing with. Judging from our Monte Carlo results, if the degree of heteroskedasticity is very mild(i.e., for δ values quite close to zero), the OLS estimator, BPE, GPEV and the traditional pretest estimators would be favoured over all other competing estimators. On the other hand, for δ values greater or less than zero the 2SAEM, BPTM and GPESM would be preferred to the OLS estimator, BPE, GPEV and the traditional pretest estimator.

The Monte Carlo study results for the sample sizes of 40 and 80 are reported in Tables 5.10 and 5.11. The corresponding graph for the respective relative mean square errors of all competing estimators(for the sample size of 40) are shown in Figure 5.5. All results for the sample sizes of 40 and 80 do not differ qualitatively from those observed and explained for the sample size of 20 above and, therefore, need no further elaboration. Notice, however, that the traditional pretest estimator using the Goldfeld and Quandt F - test(BPT) attains smaller relative mean square error than both BPTM and the pretest estimator using the t - test(BPTT)(Table 5.12) over(almost) the whole parameter space[Harvey(1976), Judge and Bock(1978)]. This suggests that the smoothed pretest estimators suggested in this thesis may not be very useful in the context of this kind of heteroskedasticity. Further, it lends support to the conclusions of Harvey(1976), Judge and Bock(1978) that the Goldfeld and Quandt F - test is superior to the t - test in detecting heteroskedasticity. Note also that as the sample size increases the efficiency of all competing estimators improves. This same conclusion emerged from the discussion of the Monte Carlo study results for the bivariance case above.

TABLE 5.10

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- SAMPLE SIZE - 40).

DELTA	OLS	2SAEM	BPE	GPEV	GPESM	BPT	BPTM
-2.00	8.6286	1.1221	8.5642	8.0204	1.2428	1.1221	1.1305
-1.60	4.4595	1.0759	4.4385	4.3252	1.1939	1.0908	1.0911
-1.40	3.2893	1.0611	3.2854	3.2223	1.1784	1.0713	1.0778
-1.20	2.4793	1.0465	2.4784	2.4614	1.1586	1.0584	1.0673
-1.00	1.9201	1.0493	1.9196	1.9073	1.1450	1.0825	1.0697
-0.80	1.5385	1.0452	1.5382	1.5347	1.1208	1.0914	1.0645
-0.60	1.2850	1.0567	1.2848	1.2834	1.1064	1.1087	1.0721
-0.40	1.1254	1.0484	1.1254	1.1252	1.0619	1.0682	1.0529
-0.20	1.0360	1.0486	1.0361	1.0486	1.0368	1.0149	1.0355
0.00	1.0000	1.0455	1.0000	1.0000	1.0236	1.0000	1.0197
0.20	1.0053	1.0466	1.0052	1.0052	1.0330	1.0263	1.0252
0.40	1.0435	1.0524	1.0434	1.0435	1.0538	1.0630	1.0459
0.60	1.1096	1.0689	1.1095	1.1096	1.0797	1.1191	1.0805
0.80	1.2018	1.0856	1.2016	1.2017	1.0975	1.1085	1.1110
1.00	1.3211	1.1070	1.3209	1.3211	1.1197	1.1165	1.1412
1.20	1.4725	1.1116	1.4723	1.2725	1.1200	1.1161	1.1440
1.40	1.6648	1.1454	1.6645	1.6648	1.1512	1.1493	1.1755
1.60	1.9119	1.1634	1.9112	1.9119	1.1675	1.1678	1.1877
2.00	2.6594	1.2647	2.6586	2.6594	1.2669	1.2647	1.2843

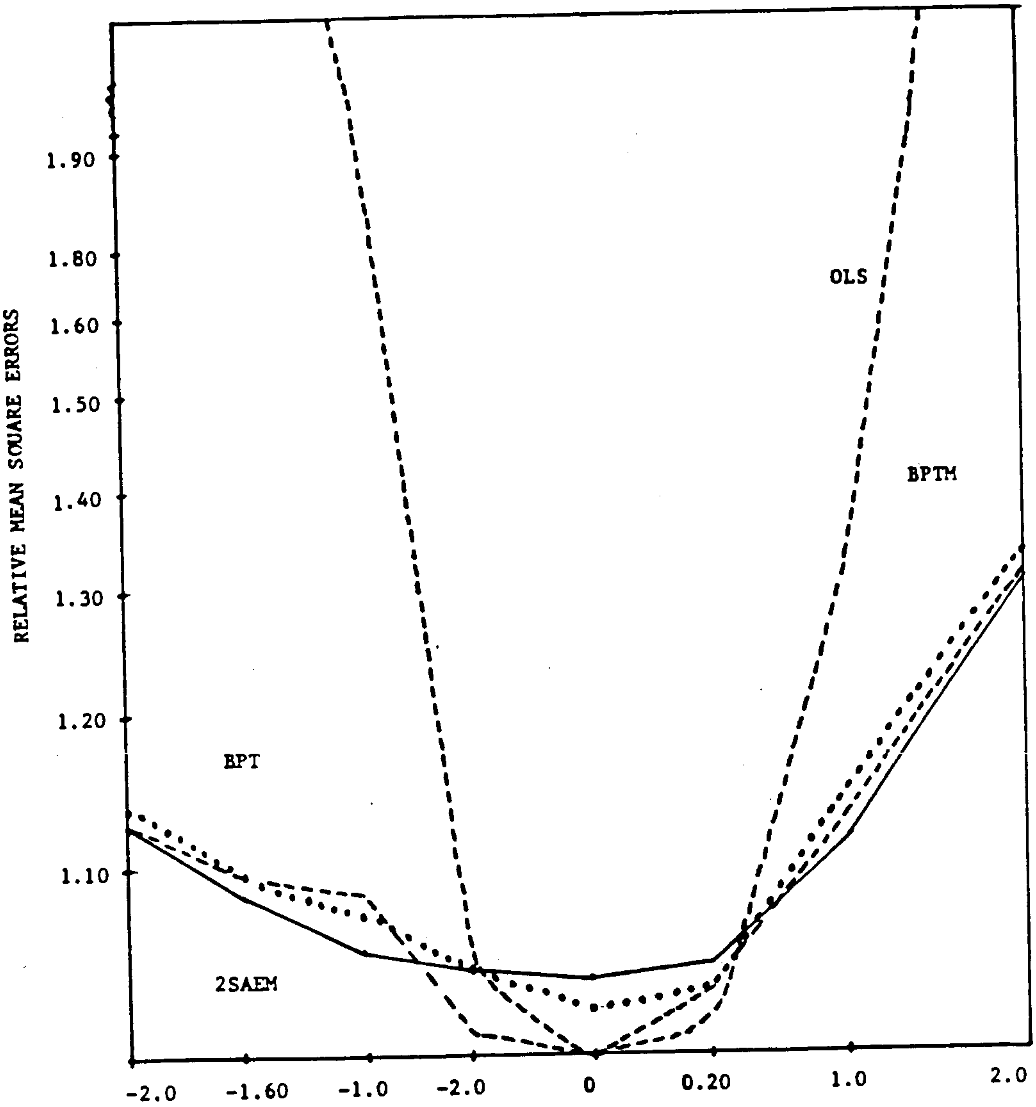

FIGURE 5.5:- RELATIVE MSE OF COMPETING ESTIMATORS

TABLE 5.11

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- SAMPLE SIZE - 80).

DELTA	OLS	2SAEM	BPE	GPEV	GPESM	BPT	BPTM
-2.00	8.0011	1.0295	8.0011	8.0003	1.0299	1.0295	1.0295
-1.60	4.1532	1.0173	4.1532	4.1531	1.0183	1.0173	1.0174
-1.40	3.0696	1.0223	3.0696	3.0695	1.0287	1.0223	1.0230
-1.20	2.3198	1.0259	2.3198	2.3198	1.0416	1.0259	1.0293
-1.00	1.8042	1.0240	1.8042	1.8042	1.0457	1.0260	1.0312
-0.80	1.4555	1.0268	1.4555	1.4555	1.0490	1.0387	1.0364
-0.60	1.2283	1.0241	1.2283	1.2283	1.0423	1.0479	1.0315
-0.40	1.0907	1.0235	1.0907	1.0907	1.0268	1.0297	1.0223
-0.20	1.0202	1.0222	1.0202	1.0202	1.0133	1.0121	1.0139
0.00	1.0000	1.0216	1.0000	1.0000	1.0082	1.0073	1.0090
0.20	1.0181	1.0128	1.0181	1.0181	1.0093	1.0053	1.0091
0.40	1.0657	1.0093	1.0657	1.0657	1.0182	1.0335	1.0228
0.60	1.1377	1.0039	1.1377	1.1377	1.0135	1.0197	1.0243
0.80	1.2322	1.0176	1.2322	1.2322	1.0244	1.0238	1.0382
1.00	1.3507	1.0243	1.3507	1.3507	1.0275	1.0243	1.0379
1.20	1.4984	1.0267	1.4984	1.4984	1.0279	1.0267	1.0336
1.40	1.6844	1.0319	1.6844	1.6844	1.0323	1.0319	1.0353
1.60	1.9231	1.0433	1.9231	1.9231	1.0434	1.0433	1.0451
2.00	2.6495	1.1338	2.6495	2.6495	1.1339	1.1338	1.1348

The rest of the results are stated in Table 5.12 below for BPTT and BPTG for the sample sizes of 20, 40 and 80. A comparison of these results clearly reveals that the traditional pretest estimator performs better than BPTT for the sample sizes 20 and 40. Note that the performance of BPTT improves greatly as the sample size increases. This is evident from Tables 5.11 and 5.12. It is also interesting to note that BPTT performs better than BPE and GPEV over a large region of the parameter space. Similarly, BPTG also performs well compared to the OLS estimator, BPE and GPEV. It must be reiterated again that since none of these estimators exhibits complete dominance over the others, the choice of estimator becomes extremely difficult. However, the existence of a prior information about the severity of heteroskedasticity would be useful to the researcher. This information would aid the researcher to search for the 'best' estimator to use when he suspects that heteroskedasticity exists in the data he is working with.

TABLE 5.12

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- FOR ALL SAMPLES).

	N = 20		N = 40		N = 80	
DELTA	BPTT	BPTG	BPTT	BPTG	BPTT	BPTG
-2.00	2.7095	5.1396	1.1426	1.2157	1.0295	1.0295
-1.60	2.1541	3.2233	1.1452	1.3241	1.0173	1.0173
-1.40	1.9673	2.6842	1.1230	1.3962	1.0223	1.0424
-1.20	1.9027	2.2748	1.1859	1.3984	1.0378	1.0733
-1.00	1.6642	1.8640	1.1866	1.4128	1.0554	1.1015
-0.80	1.4718	1.5586	1.2136	1.3806	1.0636	1.1380
-0.60	1.2790	1.3294	1.2046	1.2416	1.0766	1.1291
-0.40	1.1656	1.1495	1.1212	1.0909	1.0623	1.0842
-0.20	1.0694	1.0439	1.0293	1.0255	1.0156	1.0232
0.00	1.0211	1.0000	1.0140	1.0000	1.0087	1.0000
0.20	1.0461	1.0098	1.0212	1.0247	1.0133	1.0223
0.40	1.0997	1.0673	1.0658	1.0498	1.0486	1.0648
0.60	1.1476	1.0998	1.1218	1.1242	1.0475	1.1039
0.80	1.2173	1.1923	1.1929	1.2117	1.0396	1.1537
1.00	1.2533	1.2975	1.2136	1.3115	1.0293	1.1107
1.20	1.3601	1.4322	1.1958	1.4006	1.0299	1.0690
1.40	1.4282	1.6017	1.2039	1.4312	1.0319	1.0389
1.60	1.4946	1.7818	1.1951	1.4463	1.0433	1.0490
2.00	1.8405	2.4096	1.2856	1.5408	1.1338	1.1338

The following Table 5.13 contains the mean values and the corresponding variances of phi(ϕ) computed by integrating the density function of delta. The values for the three sample sizes are specified in such a way that each delta(δ) for all sample sizes has shown against it its corresponding mean and variance of phi. Note that the major feature of the mean values of phi is their decline as one moves towards higher degrees of heteroskedasticity. That is, for delta values quite close to zero, the probabilities(ϕ) that the degree of heteroskedasticity is such that the OLS estimator performs better than the 2SAEM are larger than at the two extreme ends(i.e., $\delta = -2, 2$) where the phi values decline gradually towards zero. This agrees with our theoretical expectations even though these values are not as large as we thought they should be. Theoretically, one expects that for low degrees of heteroskedasticity, the phi estimator must assign high values to the probability that the null hypothesis is true(i.e., homoskedasticity). Similarly, as the severity of heteroskedasticity increases, the phi estimator must compute phi values that are very small indicating the extent to which the alternative hypothesis is true. Note that very small phi values imply that the alternative hypothesis is more probable than the null hypothesis, hence casting doubts on its validity. Another notable characteristic of the mean phi values is that as the sample size increases, the efficiency of the ϕ estimator improves in that at delta values quite close to zero, the mean phi values are largest for the sample size 80. These values are also larger for the sample size 40 than 20. That is, as the sample size increases, the phi estimator computes larger phi values for the null hypothesis. This observation also agrees with econometric theory.

Finally, it must be noted also that as the sample size increases the phi estimator computes phi values that are smaller as the severity of heteroskedasticity increases. Intuitively, the results specified in Table 5.13 suggest that the efficiency of the smoothed pretest estimator may improve greatly if 'large' sample sizes are used. The corresponding phi values computed by using the posterior odds ratio(Villegas' procedure) are as specified in Table 5.14. Note that these phi values are not encouraging at all. Their values suggest that this technique is an inefficient way of calculating the probability(ϕ). As stated earlier, this poor estimation is probably one of the major reasons that account for the poor performance of both BPE and GPEV. Note that the corresponding probabilities for the combined multivariance/bivariance case are specified in Table 5.20.

TABLE 5.13

THE MEAN AND VARIANCE OF PHI(ϕ).

(FOR ALL SAMPLE SIZES(DENSITY FUNCTION)).

	N = 20		N = 40		N = 80	
DELTA	MEAN	VAR	MEAN	VAR	MEAN	VAR
-2.00	0.0597	0.0054	0.0015	0.0007	0.34E-05	0.12E-08
-1.60	0.0949	0.0103	0.0079	0.0007	0.16E-03	0.21E-05
-1.40	0.1211	0.0128	0.0180	0.0020	0.10E-02	0.42E-04
-1.20	0.1598	0.0162	0.0378	0.0055	0.49E-02	0.38E-03
-1.00	0.2060	0.0197	0.0747	0.0143	0.21E-01	0.36E-02
-0.80	0.2598	0.0222	0.1364	0.0270	0.63E-01	0.14E-01
-0.60	0.3143	0.0226	0.2165	0.0363	0.16E+00	0.35E-01
-0.40	0.3651	0.0206	0.3153	0.0403	0.31E+00	0.58E-01
-0.20	0.4120	0.0174	0.4038	0.0362	0.49E+00	0.57E-01
0.00	0.4408	0.0138	0.4599	0.0275	0.61E+00	0.04E-01
0.20	0.4489	0.0126	0.4670	0.0262	0.62E+00	0.35E-01
0.40	0.4370	0.0143	0.4215	0.0351	0.51E+00	0.60E-01
0.60	0.4079	0.0181	0.3373	0.0436	0.33E+00	0.61E-01
0.80	0.3673	0.0214	0.2404	0.0405	0.17E+00	0.35E-01
1.00	0.3188	0.0236	0.1573	0.0296	0.70E-01	0.12E-01
1.20	0.2723	0.0245	0.0912	0.0158	0.23E-01	0.24E-02
1.40	0.2323	0.0236	0.0488	0.0073	0.63E-02	0.38E-03
1.60	0.1962	0.0017	0.0256	0.0030	0.14E-02	0.34E-04
2.00	0.1435	0.0176	0.0089	0.0008	0.18E-03	0.21E-05

TABLE 5.14

THE MEAN AND VARIANCE OF PHI, (FOR ALL SAMPLE SIZES(POSTERIOR ODDS RATIO)).

	N = 20		N = 40		N = 80	
DELTA	MEAN	VAR	MEAN	VAR	MEAN	VAR
-2.00	0.0227	0.0068	0.21E-02	0.51E-03	0.12E-06	0.17E-11
-1.60	0.0163	0.0025	0.13E-02	0.24E-03	0.96E-07	0.11E-11
-1.40	0.0159	0.0020	0.64E-03	0.41E-04	0.11E-06	0.11E-11
-1.20	0.0135	0.0015	0.34E-03	0.71E-05	0.11E-06	0.19E-11
-1.00	0.0124	0.0012	0.26E-03	0.32E-05	0.17E-06	0.61E-11
-0.80	0.0122	0.0013	0.27E-03	0.35E-05	0.23E-06	0.15E-10
-0.60	0.0118	0.0012	0.29E-03	0.41E-05	0.30E-06	0.28E-10
-0.40	0.0113	0.0091	0.29E-03	0.45E-05	0.37E-06	0.46E-10
-0.20	0.0109	0.0008	0.28E-03	0.45E-05	0.45E-06	0.69E-10
0.00	0.0108	0.0008	0.27E-03	0.45E-05	0.55E-06	0.99E-10
0.20	0.0109	0.0009	0.27E-03	0.49E-05	0.68E-06	0.14E-10
0.40	0.0110	0.0010	0.29E-03	0.61E-05	0.82E-06	0.20E-09
0.60	0.0113	0.0012	0.31E-03	0.83E-05	0.95E-06	0.27E-09
0.80	0.0118	0.0013	0.33E-03	0.11E-05	0.10E-05	0.31E-09
1.00	0.0124	0.0017	0.36E-03	0.13E-04	0.91E-06	0.30E-09
1.20	0.0124	0.0019	0.38E-03	0.17E-04	0.78E-06	0.32E-09
1.40	0.0137	0.0025	0.49E-03	0.21E-04	0.70E-06	0.28E-09
1.60	0.0132	0.0023	0.50E-03	0.25E-04	0.37E-06	0.65E-10
2.00	0.0127	0.0023	0.38E-03	0.11E-04	0.16E-05	0.14E-08

As noted earlier, the relative(relative to GLS) mean square errors of all competing estimators rise continuously. Alarmed by the poor results of the estimators for the cases discussed above, we decided to compute the mean square error of all competing estimators relative to that of the 2SAEM. The results for this experiment for the sample size 20 are shown in Table 5.15 and Figure 5.6. Though the conclusions reached earlier for the sample size 20 do not differ significantly, a few comments are necessary. First, observe from these results that BPTM performs better than the traditional pretest estimator over a relatively large region of the parameter space. However, their relative mean square errors increase continuously. Similarly, for delta values quite close to zero, BPE and GPEV perform better than the traditional pretest estimator. The performance of GPESM is not very encouraging since it outperforms the traditional pretest estimator for only a few values of delta. Second, note also that when the mean square errors are computed relative to that of the 2SAEM, the relative mean square error function of the traditional pretest estimator exhibits the expected characteristics. This occurs because as the degree of severity of heteroskedasticity increases, the pretest selects the 2SAEM more often than it selects the ordinary least squares estimator and hence allowing the relative mean square error to get closer and closer to that of the 2SAEM. For higher degrees of heteroskedasticity, the relative mean square error of the traditional pretest estimator tends gradually towards that of the 2SAEM.

TABLE 5.15

RELATIVE(TO 2SAEM) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- SAMPLE SIZE - 20).

DELTA	OLS	BPE	GPEV	GPESM	BPT	BPTT	BPTM
-2.00	6.5585	6.0092	4.5454	4.2840	1.0062	2.0115	1.1841
-1.60	3.7690	3.6122	2.8596	2.6382	1.0763	1.7687	1.1259
-1.40	2.8523	2.7682	2.3197	2.0874	1.1053	1.6556	1.0929
-1.20	2.2201	2.1851	1.9551	1.7073	1.1376	2.6537	1.0838
-1.00	1.7518	1.7330	1.6328	1.4297	1.1639	1.4761	1.0656
-0.80	1.3907	1.3812	1.3545	1.2199	1.1512	1.2971	1.0345
-0.60	1.1794	1.1746	1.1597	1.1002	1.1247	1.1495	1.0137
-0.40	1.0243	1.0214	1.0144	1.0168	1.0253	1.0448	0.9822
-0.20	0.9446	0.9439	0.9439	0.9744	0.9588	0.9670	0.9663
0.00	0.9036	0.9037	0.9047	0.9626	0.9406	0.9227	0.9485
0.20	0.8889	0.8891	0.8892	0.9674	0.9321	0.9334	0.9465
0.40	0.8926	0.8925	0.8926	0.9816	0.9675	0.9581	0.9572
0.60	0.9560	0.9549	0.9559	0.9942	0.9935	1.0159	0.9744
0.80	1.0188	1.0161	1.0185	1.0024	1.0398	1.0686	0.9975
1.00	1.1233	1.1175	1.1228	1.0107	1.0295	1.1103	1.0220
1.20	1.2004	1.1937	1.1998	1.0146	1.0133	1.1598	1.0423
1.40	1.3201	1.3069	1.3193	1.0148	1.0115	1.1868	1.0638
1.60	1.4689	1.4525	1.4680	1.0144	1.0078	1.2027	1.0800
2.00	1.6851	1.6679	1.6844	1.0132	1.0031	1.2122	1.0984

Table 5.16 and Figure 5.7 contain Monte Carlo results for the sample size 40. Clearly, no estimator dominates the other

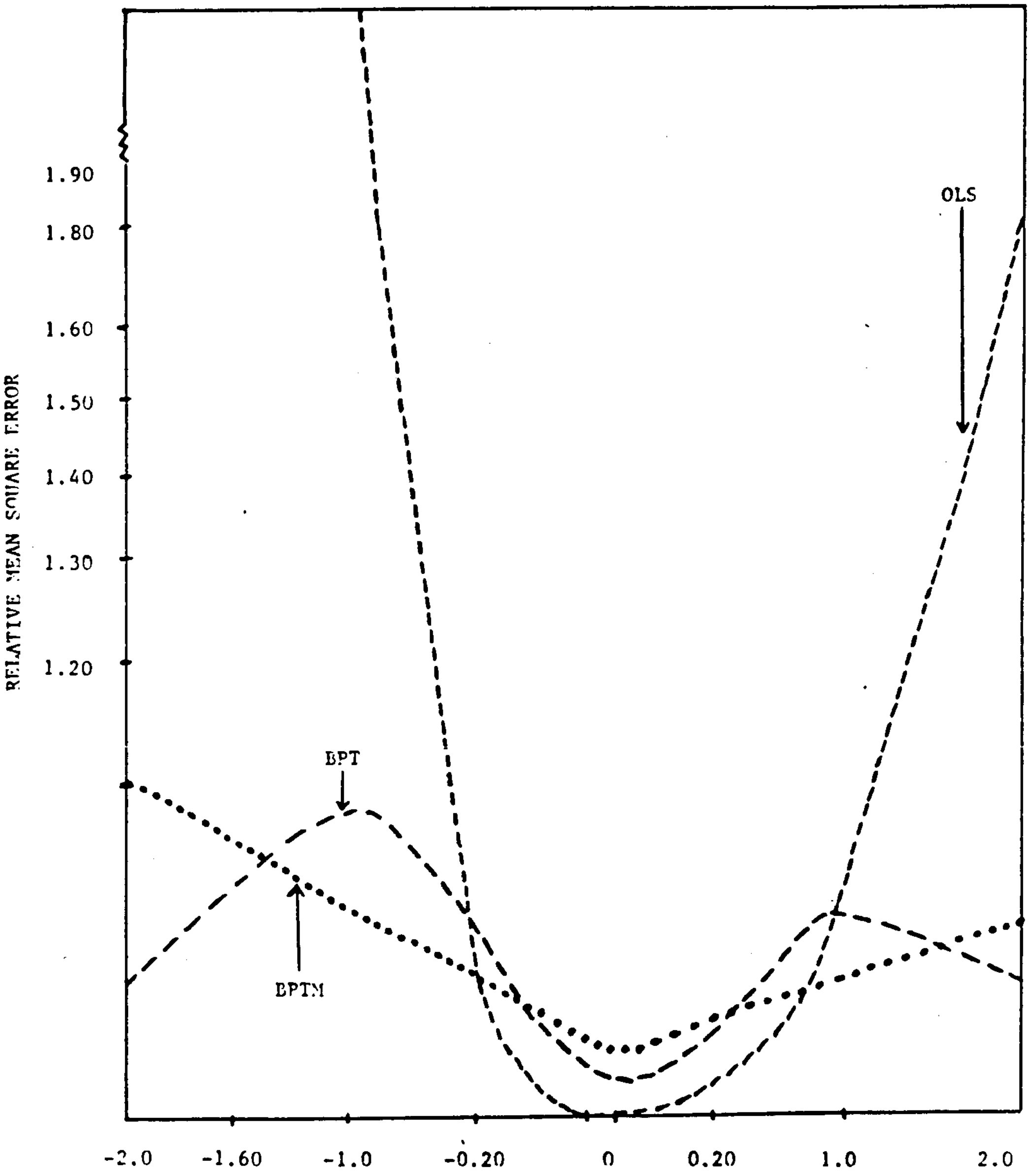

FIGURE 5.6:- RELATIVE MSE OF COMPETING ESTIMATORS

competing estimators. For delta values very close to zero, the OLS estimator, BPE, GPEV and BPT perform better than the other estimators. The performance of BPTM is comparable to that of the traditional pretest estimator in that even though it does not dominate BPT over the whole parameter space, it performs better than it for delta values ranging from -1.20 to -0.40 and from 0.20 to 0.80. As noted earlier for the sample size 20, though the performance of GPESM is not very good it performs better than the OLS estimator, BPE, GPEV and BPTT over a large region of the parameter space.

With the exception of BPE and GPEV, the relative mean square errors of all other competing estimators exhibit the characteristics we expect theoretically. That is, as we move away from very low degree of heteroskedasticity, their relative mean square errors begin to rise continuously up to some point, reach a maximum, and then decline gradually towards one. The relative mean square error of the OLS estimator rises continuously as we move closer to higher degrees of heteroskedasticity. In Figure 5.7, the characteristics of the relative mean square errors of the competing estimators of interest are shown. The results of the sample size 80(as specified in Table 5.17 and Figure 5.7B) do not differ greatly from that discussed above for the sample size 40 and, therefore, do not warrant any further discussions. However, it must be noted that the efficiency of most estimators improved greatly.

TABLE 5.16

RELATIVE(TO 2SAEM) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- SAMPLE SIZE - 40).

DELTA	OLS	BPE	GPEV	GPESM	BPT	BPTT	BPTM
-2.00	7.6896	7.6325	7.1478	1.1076	1.0000	1.0180	1.0075
-1.60	4.1449	4.1253	4.0200	1.1096	1.0138	1.0644	1.0141
-1.40	3.0998	3.0963	3.0369	1.1106	1.0097	1.0583	1.0158
-1.20	2.3691	2.3683	2.3520	1.1071	1.0112	1.1332	1.0198
-1.00	1.8298	1.8294	1.8176	1.0912	1.0316	1.1308	1.0194
-0.80	1.4719	1.4717	1.4684	1.0724	1.0443	1.1611	1.0185
-0.60	1.2160	1.2159	1.2145	1.0470	1.0492	1.1399	1.0145
-0.40	1.0734	1.0735	1.0733	1.0129	1.0189	1.0695	1.0043
-0.20	0.9879	0.9880	0.9880	0.9887	0.9678	0.9816	0.9875
0.00	0.9564	0.9564	0.9564	0.9790	0.9564	0.9699	0.9753
0.20	0.9605	0.9604	0.9604	0.9869	0.9804	0.9756	0.9795
0.40	0.9915	0.9915	0.9915	1.0013	1.0100	1.0127	0.9938
0.60	1.0380	1.0380	1.0381	1.0101	1.0470	1.0495	1.0108
0.80	1.1070	1.1068	1.1070	1.0109	1.0211	1.0988	1.0233
1.00	1.1934	1.1932	1.1934	1.0115	1.0086	1.0962	1.0308
1.20	1.3241	1.3244	1.3246	1.0075	1.0040	1.0757	1.0291
1.40	1.4534	1.4531	1.4535	1.0051	1.0034	1.0511	1.0263
1.60	1.6433	1.6427	1.6433	1.0035	1.0038	1.0272	1.0209
2.00	2.1027	2.1022	2.1028	1.0018	1.0000	1.0166	1.0155

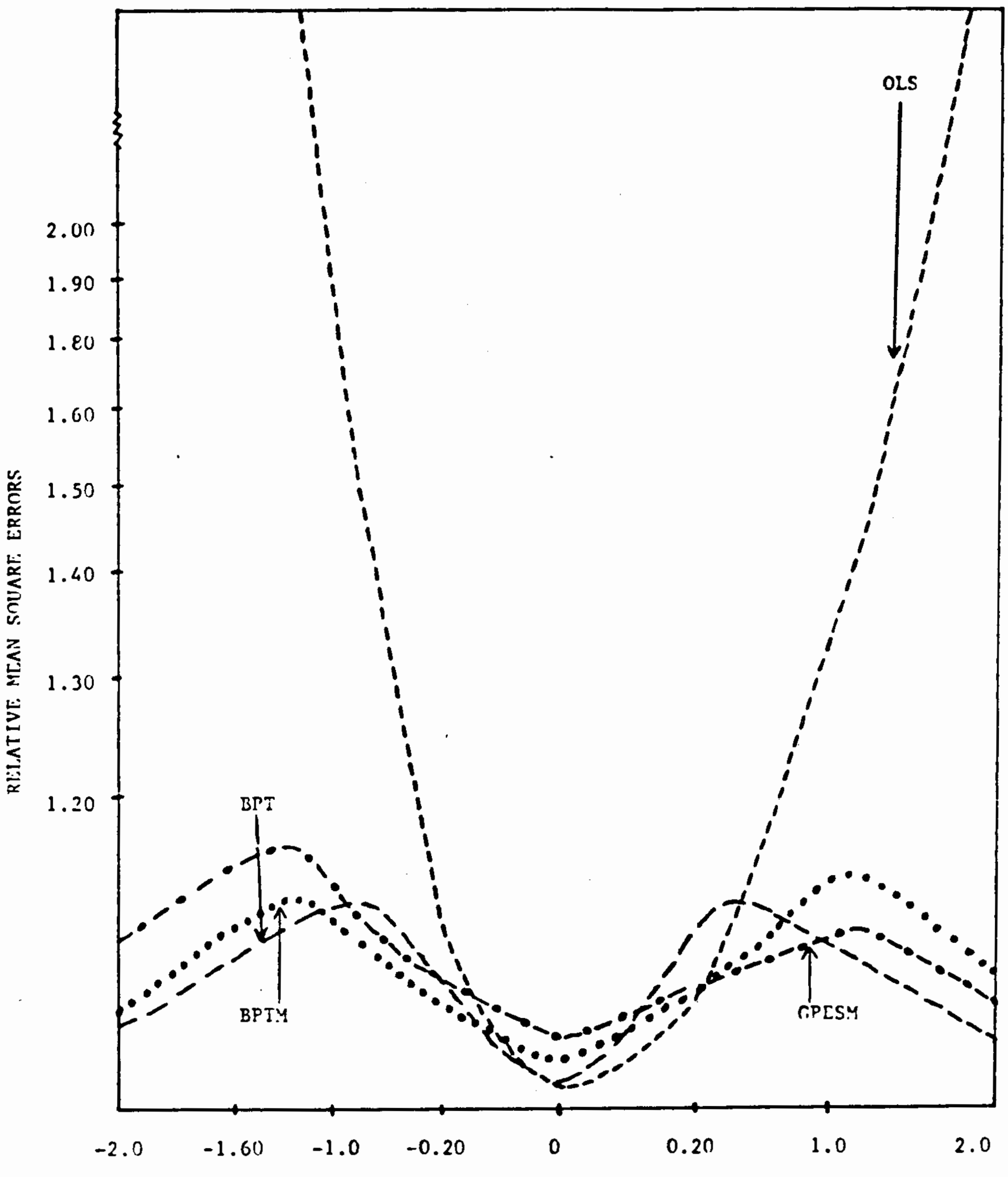

FIGURE 5.7:- RELATIVE MSE OF COMPETING ESTIMATORS

TABLE 5.17

RELATIVE(2SAEM) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE CASE:- SAMPLE SIZE - 80).

DELTA	OLS	BPE	GPEV	GPESM	BPT	BPTT	BPTM
-2.00	7.7718	7.7718	7.7711	1.0004	1.0000	1.0000	1.0000
-1.60	4.0825	4.0825	4.0824	1.0010	1.0000	1.0000	1.0001
-1.40	3.0026	3.0026	3.0025	1.0063	1.0000	1.0000	1.0007
-1.20	2.2612	2.2612	2.2612	1.0153	1.0000	1.0117	1.0033
-1.00	1.7619	1.7619	1.7619	1.0212	1.0020	1.0307	1.0070
-0.80	1.4175	1.4175	1.4175	1.0217	1.0117	1.0358	1.0093
-0.60	1.1993	1.1993	1.1993	1.0178	1.0232	1.0512	1.0071
-0.40	1.0657	1.0657	1.0657	1.0033	1.0060	1.3795	0.9988
-0.20	0.9980	0.9980	0.9980	0.9913	0.9901	0.9935	0.9918
0.00	0.9789	0.9789	0.9789	0.9869	0.9860	0.9874	0.9877
0.20	0.0052	1.0052	1.0052	0.9965	0.9926	1.0005	0.9964
0.40	1.0559	1.0559	1.0559	1.0088	1.0240	1.0390	1.0134
0.60	1.1334	1.1334	1.1334	1.0096	1.0157	1.0435	1.0204
0.80	1.2109	1.2109	1.2109	1.0066	1.0061	1.0216	1.0202
1.00	1.3187	1.3187	1.3187	1.0032	1.0000	1.0049	1.0113
1.20	1.4594	1.4594	1.4594	1.0012	1.0000	1.0031	1.0067
1.40	1.6324	1.6324	1.6324	1.0005	1.0000	1.0000	1.0033
1.60	1.8433	1.8433	1.8433	1.0002	1.0000	1.0000	1.0018
2.00	2.3367	2.3367	2.3367	1.0000	1.0000	1.0000	1.0009

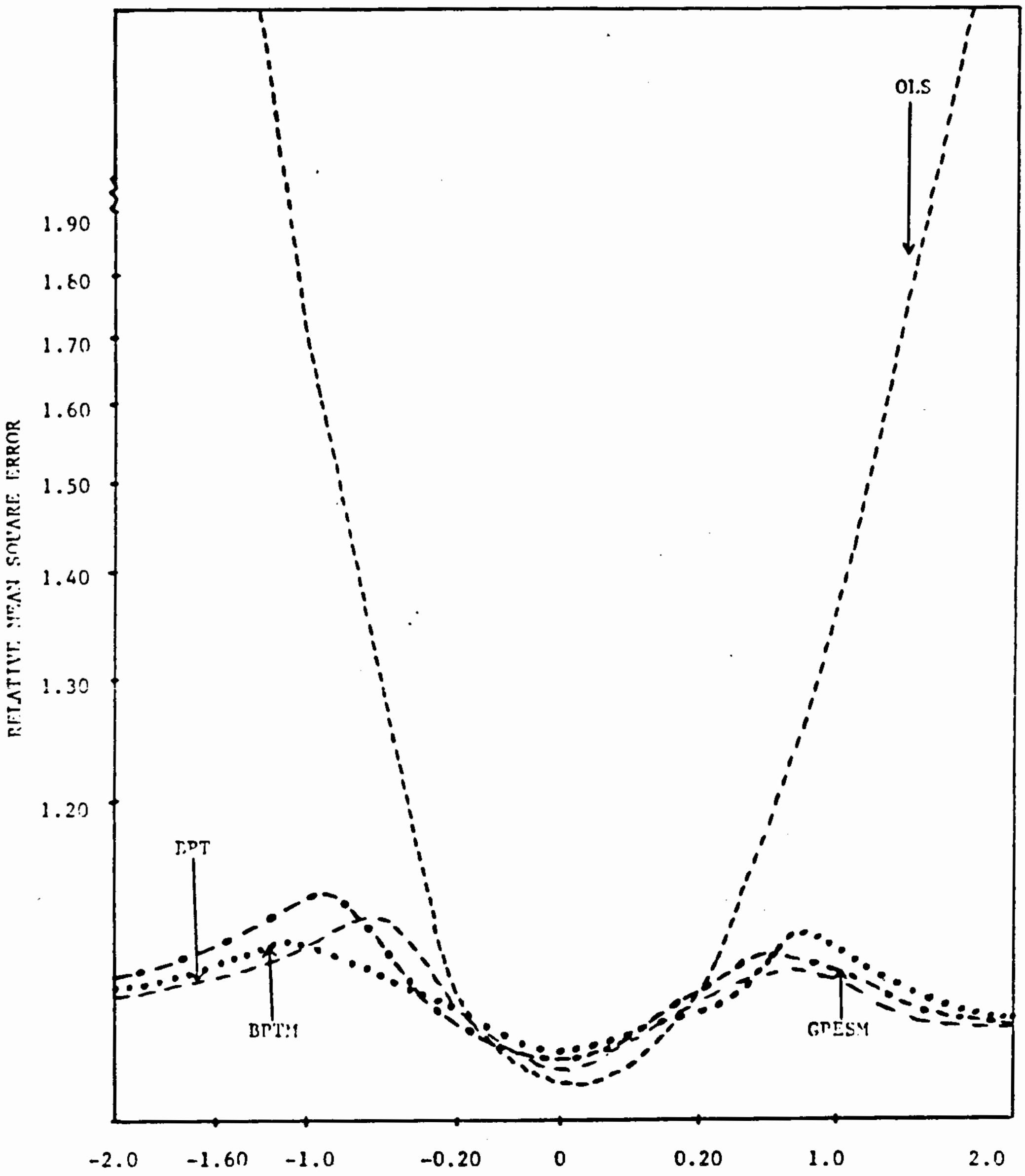

FIGURE 5.7B:- RELATIVE MSE OF COMPETING ESTIMATORS

Finally, owing to the poor results of the estimators for the multivariance case, we decided to see if the successful techniques employed for the bivariance case could be useful in the multivariance context. Thus, we tried using the bivariance technique for estimating ϕ, the probability that the degree of heteroskedasticity is such that the OLS estimator outperforms the 2SAEM, while keeping intact the other dimensions of the estimators for the multivariance case(i.e., ϕ was calculated under the assumption that the heteroskedasticity was of the bivariance type). Judge et al(1985), pp 454 - 455 have shown that in some cases knowledge of the functional form of the heteroskedasticity is not important. Inspired by this result we first tried to employ the ϕ estimation technique of the more successful of the bivariance estimators, namely BPTB, to the data exhibiting multivariance heteroskedasticity. This produced an additional estimator we refer to as BPTBM. We also investigated the use of BPTB itself in this context(i.e., implicitly assuming that the researcher believes heteroskedasticity is of the bivariance rather than the multivariance type). In addition, we evaluated the two traditional pretest estimators, BPT and BPT1(recall that the former uses 2SAE whereas the latter uses the 2SAEM). The results of this experiment are reported in Tables 5.18, 5.19, and Figures 5.8 and 5.9. These results suggest that using the Goldfeld/Quandt F - test helps but using the bivariance transformation hinders. Except for δ values quite close to zero, BPTBM performs better than both traditional pretest estimators, and is comparable to that of the 2SAEM. Note, however, that the relative mean square errors of all competing estimators do not exhibit the expected characteristcs; they do not eventually decline(as expected) as δ grows in size. The results obtained from this experiment suggest that this smoothed pretest estimator(BPTBM) is an appealing alternative to the traditional pretest estimator(BPT1) even if the researcher is not ignorant about the functional form of the existing heterokedasticity. Note also that these results suggest that BPTB should not be preferred to the traditional pretest estimator in any instance since it is completely dominated by the traditional pretest estimator(BPT1). Note also the very poor performance of both the 2SAE and BPT.

TABLE 5.18

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE/BIVARIANCE CASE:- SAMPLE SIZE - 20).

DELTA	OLS	2SAEM	2SAE	BPT1	BPT	BPTBM	BPTB
-2.00	8.8344	1.3470	2.1986	1.3554	2.1956	1.4438	2.0967
-1.60	4.5903	1.2179	2.1261	1.3108	2.2084	1.2991	1.9776
-1.20	2.5543	1.1505	2.1087	1.3088	2.0768	1.2267	1.8479
-1.00	1.9751	1.1275	2.1152	1.3122	1.9455	1.1975	1.7598
-0.80	1.5780	1.1347	2.1295	1.3062	1.8282	1.1854	1.6545
-0.60	1.3122	1.1126	2.1506	1.2514	1.6698	1.1369	1.5414
-0.40	1.1427	1.1157	2.1778	1.1439	1.3929	1.0991	1.4436
-0.20	1.0446	1.1059	2.2114	1.0603	1.1875	1.0759	1.3920
0.00	1.0000	1.1067	2.2529	1.0409	1.0624	1.0594	1.4120
0.20	1.0032	1.1207	2.3050	1.0446	1.1627	1.0728	1.5187
0.40	1.0246	1.1478	2.3722	1.1104	1.3061	1.1139	1.7044
0.60	1.0799	1.1296	2.4608	1.1223	1.6256	1.1175	1.9430
0.80	1.1606	1.1392	2.5794	1.1845	1.8818	1.1373	2.2073
1.00	1.2681	1.1289	2.7392	1.1621	2.3226	1.1279	2.4819
1.20	1.4077	1.1727	2.9556	1.1883	2.7587	1.1708	2.7697
1.40	1.5887	1.2034	3.2491	1.2173	3.1509	1.2059	3.0934
2.00	2.5585	1.5183	4.9326	1.5230	4.9222	1.5601	4.6901

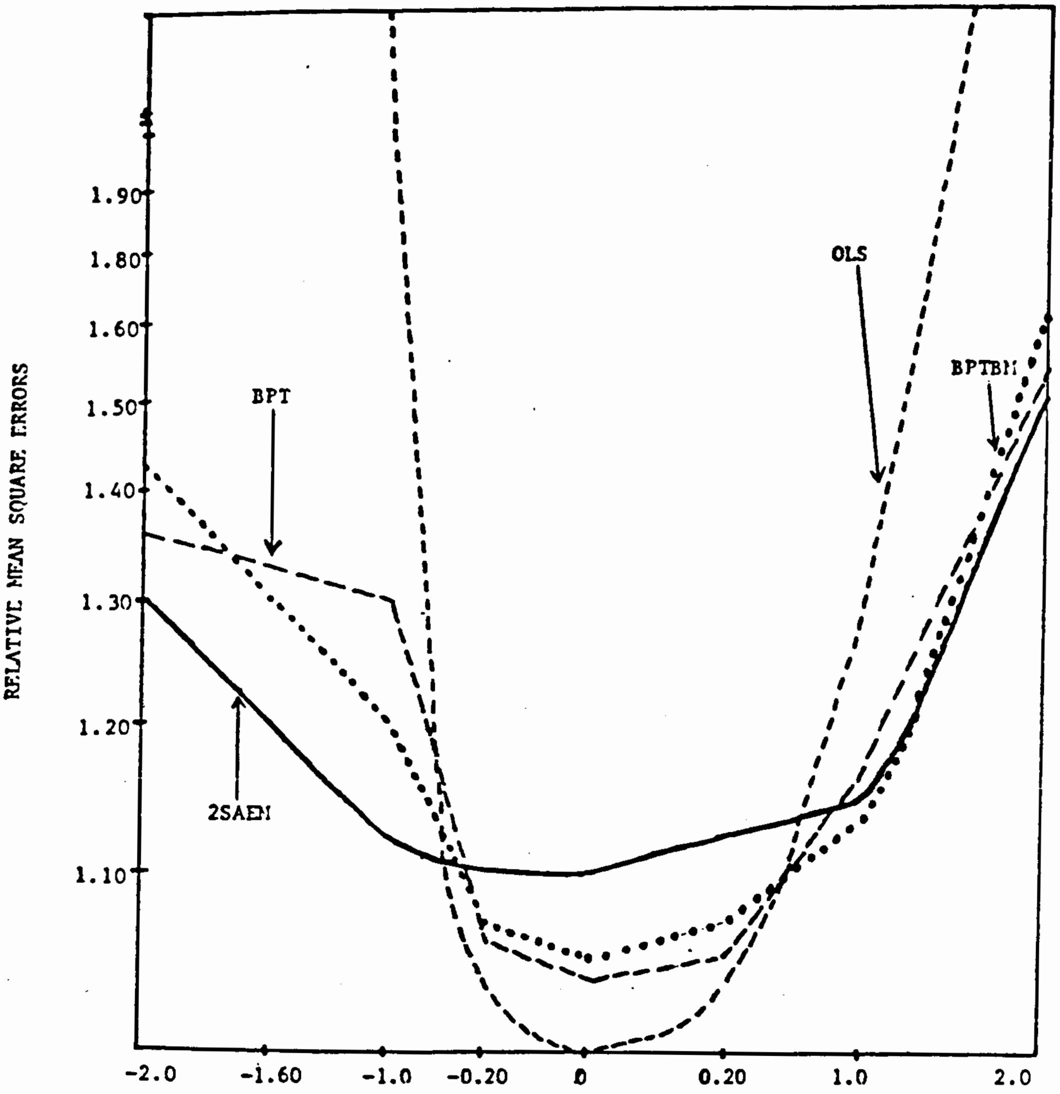

FIGURE 5.8:- RELATIVE MEAN SQUARE ERRORS OF COMPETING ESTIMATORS

TABLE 5.19

RELATIVE(TO GLS) MSE OF COMPETING ESTIMATORS(5% SL).

(THE MULTIVARIANCE/BIVARIANCE CASE:- SAMPLE SIZE - 40).

DELTA	OLS	2SAEM	2SAE	BPT1	BPT	BPTBM	BPTB
-2.00	8.6286	1.1221	2.0137	1.1221	2.0397	1.1226	2.0130
-1.60	4.4595	1.0759	1.9472	1.0908	1.9568	1.0783	1.9424
-1.20	2.4793	1.0465	1.9425	1.0582	1.9929	1.0558	1.9203
-1.00	1.9201	1.0493	1.9597	1.0703	1.9015	1.0615	1.8935
-0.80	1.5385	1.0452	1.9885	1.0909	1.6763	1.0588	1.8131
-0.60	1.2850	1.0567	2.0282	1.0948	1.4625	1.0643	1.6661
-0.40	1.1254	1.0484	2.0783	1.0646	1.1686	1.0480	1.4858
-0.20	1.0360	1.0486	2.1386	1.0143	1.0127	1.0273	1.3398
0.00	1.0000	1.0455	2.2098	1.0000	1.0020	1.0141	1.3216
0.20	1.0005	1.0466	2.2936	1.0234	1.0066	1.0284	1.4890
0.40	1.0435	1.0524	2.3932	1.0633	1.1838	1.0540	1.8057
0.60	1.1096	1.0689	2.5139	1.1111	1.5076	1.0794	2.1800
0.80	1.2018	1.0856	2.6633	1.0973	1.9936	1.0908	2.5182
1.00	1.3211	1.1070	2.8525	1.1148	2.4621	1.1049	2.7830
1.20	1.4725	1.1116	3.0964	1.1149	2.9935	1.1038	3.0121
1.40	1.6648	1.1454	3.4154	1.1498	3.3740	1.1419	3.2569
2.00	2.6594	1.2647	5.1595	1.2647	5.1623	1.4020	4.5592

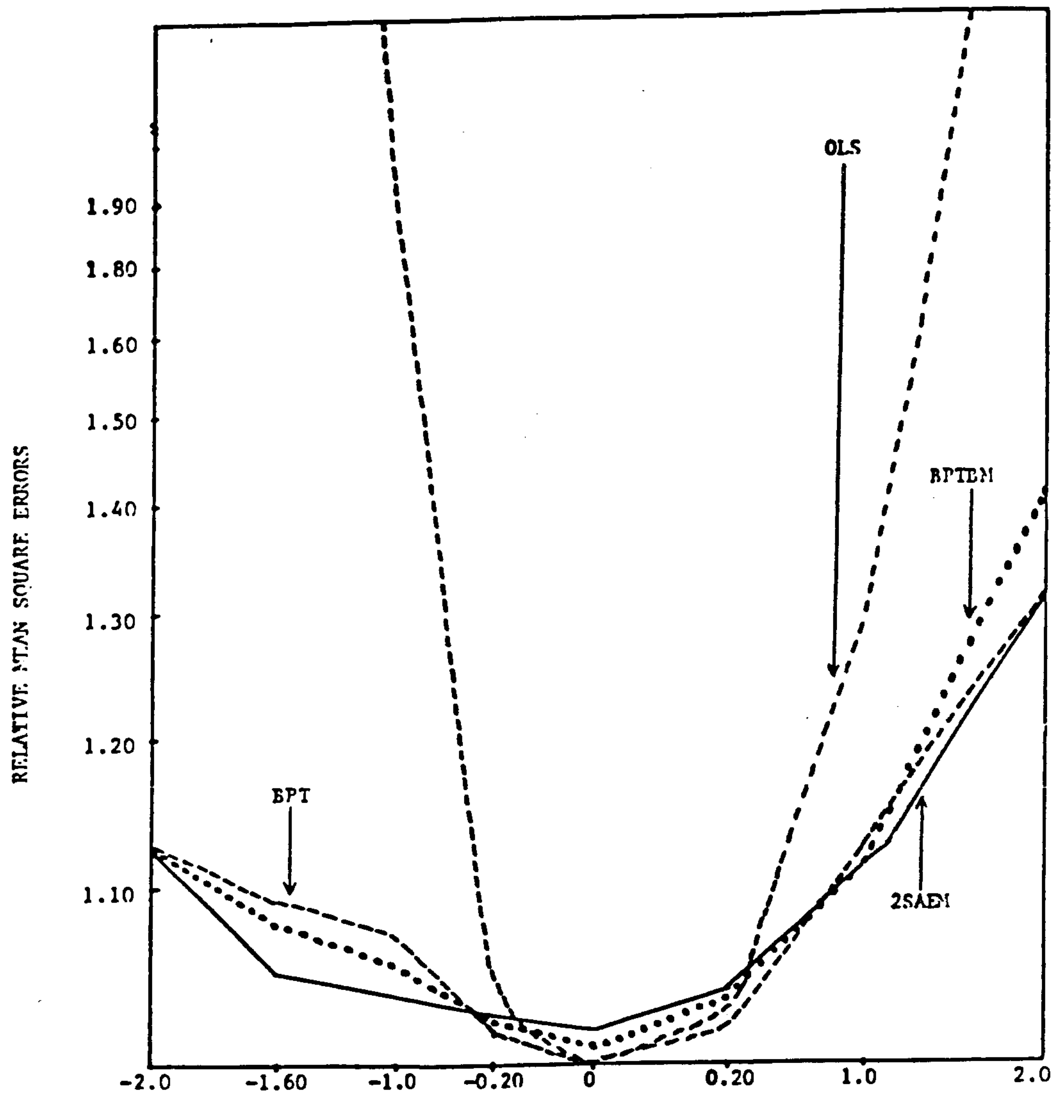

FIGURE 5.9 :- RELATIVE MEAN SQUARE ERRORS OF COMPETING ESTIMATORS

TABLE 5.20

THE MEAN AND VARIANCE OF PHI(ϕ).

(FOR SAMPLE SIZES 20, 40(DENSITY FUNCTION)).

	N = 20		N = 40	
DELTA	MEAN	VAR	MEAN	VAR
-2.00	0.0739	0.0106	0.0006	0.4375E-05
-1.60	0.1047	0.0158	0.0035	0.1202E-03
-1.40	0.1339	0.0212	0.0097	0.7052E-03
-1.20	0.1765	0.0290	0.0253	0.3512E-02
-1.00	0.2341	0.0380	0.0597	0.1275E-01
-0.80	0.3047	0.0456	0.1245	0.3089E-01
-0.60	0.3802	0.0485	0.2288	0.4884E-01
-0.40	0.4457	0.0455	0.3648	0.5185E-01
-0.20	0.4817	0.0405	0.4833	0.4061E-01
0.00	0.4742	0.0390	0.5119	0.3811E-01
0.20	0.4237	0.0442	0.4250	0.4990E-01
0.40	0.3442	0.0506	0.2776	0.5211E-01
0.60	0.2548	0.0485	0.1465	0.3631E-01
0.80	0.1753	0.0370	0.0678	0.1657E-01
1.00	0.1189	0.0227	0.0420	0.6296E-02
1.20	0.0879	0.0112	0.0584	0.5183E-02
1.40	0.0775	0.0049	0.0996	0.6314E-02
1.60	0.0704	0.0025	0.0814	0.6544E-02
2.00	0.0405	0.0013	0.0636	0.3941E-02

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1. SUMMARY

The major conclusions of this study are briefly stated as follows. First, the traditional pretest estimator can be smoothed to attain lower relative mean square errors over certain regions of the parameter space with only a small increase in relative mean square error in a narrorw region of the parameter space close to homoskedasticity. For certain reasonable priors on the degree of heteroskedasticity this makes the smoothed pretest estimator a very attractive alternative to its competitor. Second, the Monte Carlo study experiments in this thesis suggest that even in the presence of prior knowledge about the functional form of the heteroskedasticity in the data, the Goldfeld and Quandt F - test is a superior way of testing for the presence of heteroskedasticity. Finally note, however, that if a prior knowledge about the type(i.e., bivariance or multivariance) of heteroskedasticity exists, regardless of the functional form of heteroskedasticity, using the bivariance technique for calculating ϕ for estimation purposes can be very useful.

6.2. SUGGESTED TOPICS FOR FURTHER STUDY AND RESEARCH

(1). It should be of interest to compare the generalized or 'smoothed' pretest estimator with the Yancey et. al. and Sclove et. al. pretest estimators and to develop a smoothed version of these pretest estimators.

(2). Further investigation of the role of the assumption that the functional form of heteroskedasticity is known seems warranted.

(3). We have not been able to explain why the relative mean square error of the 2SAEM, the traditional pretest estimator and the various versions of the generalized pretest estimator increase continuously for the multivariance case. This, and its implications would be a fruitful area for further research.

(4). The experiment could be extended to examine other forms of heteroskedasticity.

(5). Most tests in this study were conducted at the 1% and 5% significance levels. It would be of interest to investigate the question of the optimal choice of significance level.

(6). In the current study, we assumed that the posterior density function for $\hat{\delta}$ is a t-distribution. This should be investigated. It is quite possible that one of the reasons why the relative mean square errors of all the versions of the smoothed pretest estimator do not decline is that we have made an incorrect assumption about the distribution of $\hat{\delta}$.

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APPENDIX.

APPENDIX A

```
C        HETEROSKEDASTICITY(MULTIVARIANCE)
C                 AVERAGE
         SUBROUTINE AVGE(BHAT,N,AVGEO,AVGE1)
         IMPLICIT REAL*8 (A-H,O-Z)
         REAL*8 BHAT(600,2)
         SUMO = O.DO
         SUM1 = O.DO
         DO 200 I = 1,N
         SUMO = SUMO + BHAT(I,1)
  200    SUM1 = SUM1 + BHAT(I,2)
         AVGEO = SUMO / N
         AVGE1 = SUM1 / N
         RETURN
         END

C                BIASES
         FUNCTION GBIAS(BCAL,BETA)
         IMPLICIT REAL*8 (A-H,O-Z)
         GBIAS = BCAL - BETA
         RETURN
         END

C          VARIANCE OF ANY ARRAY
         FUNCTION SIGMA(ARRAY,M)
         IMPLICIT REAL*8 (A-H,O-Z)
         REAL*8 ARRAY(600)
         CALL AVGE2(ARRAY,M,AVGE)
         SUM = O.DO
         DO 2 I = 1,M
    2    SUM = SUM + (ARRAY(I) - AVGE) * * 2
         SIGMA = SUM / M
         RETURN
         END

C                VARIANCE OF ANY ARRAY
         SUBROUTINE VRANCE(ARRAY,ITIMES,AVGEO,AVGE1,VARO,VAR1)
         IMPLICIT REAL*8 (A-H,O-Z)
         REAL*8 ARRAY(600,2)
         CALL AVGE(ARRAY,ITIMES,AVGEO,AVGE1)
         SUMO = O.DO
         SUM1 = O.DO
         DO 2 I = 1,ITIMES
         SUMO = SUMO + (ARRAY(I,1) - AVGEO) ** 2
    2    SUM1 = SUM1 + (ARRAY(I,2) - AVGE1) ** 2
         VARO = SUMO / (ITIMES - 1.DO)
         VAR1 = SUM1 / (ITIMES - 1.DO)
         RETURN
         END

C                THE OMEGA MATRIX
         SUBROUTINE CALOME(XKEEP,OMEGA,OMEINV,OMEISQ,DELTA,M)
         IMPLICIT REAL*8 (A-H,O-Z)
         REAL*8 XKEEP(20,2),OMEGA(20,20),OMEINV(20,20),OMEISQ(20,20)
         REAL*8 WKSP(20)
         DO 1 I = 1,M
         DO 2 J = 1,M
         OMEGA(I,J) = O.DO
         OMEISQ(I,J) = O.DO
```

```
 2    OMEINV(I,J) = O.DO
      OMEGA(I,I)  = XKEEP(I,2) ** DELTA
 1    OMEINV(I,I) = 1.DO / (DSQRT(XKEEP(I,2)**DELTA))
      DO 4 I=1,M
 4    OMEISQ(I,I)  = OMEINV(I,I) ** 2
      RETURN
      END

C       TRANSPOSE OF X-MATRIX
      SUBROUTINE TRANSP(X,TTX,M,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(20,2),TTX(2,20)
      DO 1 I = 1,M
      DO 1 J = 1,N
 1    TTX(J,I) = X(I,J)
      RETURN
      END

C             OLS RESIDUALS(UHAT)
      SUBROUTINE CLUHAT(X,Y,UHAT,M,BO,B1)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(20),Y(20),UHAT(20)
      DO 1 I = 1,M
      UHAT(I) = Y(I) - BO -B1*X(I)
 1    CONTINUE
      RETURN
      END

C              SORTING THE Xs
      SUBROUTINE SORTX(X,TEMPX,INDICS,M)
      REAL*8 X(20),TEMPX(20),TEMP(20)
      INTEGER INDICS(20),TEMPIN(20)
      DO 1 I = 1,M
 1    TEMPX(I) = X(I)
      IFAIL = O
      CALL MO1AJF(TEMPX,TEMP,INDICS,TEMPIN,M,M,IFAIL)
      RETURN
      END

C       LOG OF THE SQUARES OF UHATS
      SUBROUTINE CUHLOG(UHAT,E,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 UHAT(20),E(20)
      DO 1 I = 1,N
      Q = UHAT(I)**2
 1    E(I) = DLOG(Q)
      RETURN
      END

C         LOG OF THE Xs
      SUBROUTINE CAXLOG(X,W,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 X(20),W(20)
      DO 1 I = 1,N
 1    W(I) = DLOG(X(I))
      RETURN
      END
```

```
C        PROB(PHI) OF NULL HYPOYHESIS
      SUBROUTINE POSTEO(SIGMA1,SIGMA2,NR,K,PHI)
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAM = NR * K ** 2 / 6.2832
      ALPHA = (20.0*NR*SIGMA2)/4.9348 + 2*DLOG(PARAM) + 2.2895
      BETA  = (20.0*NR*SIGMA1)/4.9348 +   DLOG(PARAM) + 1.3863
      DELTA = BETA - ALPHA
      PHI  =  1.DO / (1.DO + DEXP(DELTA))
    6 RETURN
      END

      SUBROUTINE STATIS(ARRAY,ITIMES,SQLBO,SQLB1,BMSEO,BMSE1)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 ARRAY(600,2)
      CALL VRANCE(ARRAY,ITIMES,AVGEO,AVGE1,VARO,VAR1)
       BMSEO = SQLBO / ITIMES
       BMSE1 = SQLB1 / ITIMES
       RETURN
       END

       FUNCTION GQFSTA(UHAT1,UHAT2,M)
       IMPLICIT REAL*8 (A-H,O-Z)
       REAL*8 UHAT1(8),UHAT2(8)

       SUHTQ1 = 0.DO
       SUHTQ2 = 0.DO
       DO 1 I = 1,M
       SUHTQ1 = SUHTQ1 + UHAT1(I) ** 2
    1  SUHTQ2 = SUHTQ2 + UHAT2(I) ** 2
       GQFSTA = SUHTQ2 / SUHTQ1
       RETURN
       END

       SUBROUTINE CALSTA(X,Y,XSTAR,ZET,YSTAR,M,GAMMA)
       IMPLICIT REAL*8 (A-H,O-Z)
       REAL*8 X(20),XSTAR(20),Y(20),YSTAR(20),ZET(20)
       DO 1 I = 1,M
       ZET(I) = 1.DO / DSQRT(X(I) ** GAMMA)
       XSTAR(I) = X(I) * ZET(I)
    1  YSTAR(I) = Y(I) * ZET(I)
       RETURN
       END

       SUBROUTINE RLATIV(AMSEO,AMSE1,BMSEO,BMSE1,RLATO,RLAT1)
       IMPLICIT REAL*8 (A-H,O-Z)
       RLATO = AMSEO / BMSEO
       RLAT1 = AMSE1 / BMSE1
       RETURN
       END

       SUBROUTINE KACONS(X,SIGBAR,GAMMA,ALPHA,M)
       IMPLICIT REAL*8 (A-H,O-Z)
       REAL*8 X(20)
       SUM = 0.DO
       DO 1 I = 1,M
    1  SUM = SUM + X(I) ** GAMMA
       ALPHA = M * SIGBAR / SUM
       RETURN
```

```
      END

C                THE MAIN PROGRAMME

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 TY(12000),Y(20),BETA(2,1),GLS(600,2),G05DDF
      REAL*8 V(20),TV(12000),UHAT(20),EGLS(600,2),BSENYO(600,2)
      REAL*8 TXX(2,2),XINV(2,2),U(20),WKSP(20),G05DEF
      REAL*8 X(20),TTX(2,20),OMEGA(20,20),OLS(600,2),VHAT(20)
      REAL*8 Z(1),XINVTX(2,20),MOVE(11),BHAT(600,2)
      REAL*8 DELTAS(600,2),W(20),E(20),BPT(600,2),GPES(600,2)
      REAL*8 OMEISQ(20,20),GPEV(600,2),OMEINV(20,20)
      REAL*8 TEMPT(2,2),XTEMP(2,2),INVX(2,2),BETAGD(600,2)
      REAL*8 XTEMP3(2,2),XTEMP4(2,2),SABINA(2,2),UHAT1(8)
      REAL*8 XTEMP2(2,2),SENYO(2,2),BPE(600,2),WKSP2(8)
      REAL*8 TWW(2,2),TTW(2,20),WINV(2,2),PART2X(8)
      REAL*8 WINVTW(2,20),HLAMDA(600),PART1X(8),UHAT2(8)
      REAL*8 TEMPX(20),PART1Y(8),PART2Y(8),BETAGU(600,2)
      REAL*8 TTX2(2,8),XINV2(2,2),XIN2TX(8,2),TXX2(2,8)
      REAL*8 YSTAR(20),XSTAR(20),TXO(2,20),TKEEP(2,1)
      REAL*8 RESULT(20),XKEEP(20,2),ZET(20),XSTKIP(20,2)
      REAL*8 YSTKIP(20,1),BPTT(600,2),SLAMDA(600),BPTG(600,2)
      REAL*8 GLAMDA(600),XLOG(20,2),KLAMDA(600),EGLS2(600,2)
      INTEGER INDICS(20)
      REAL*8 G05CAF,DELTA(2),PHI(600),PHIBA(600)

      M = 20
      IM = 20
      N = 2
      READ(5,10) ITIMES
   10 FORMAT(I4)
      READ(5,17) SIGBAR,GAMMA
   17 FORMAT(2F8.4)
      CALL G05CBF(O.DO)
      MHALF = M / 2
      DO 120 I = 1,MHALF

C     Xs FROM LOGNORMAL DISTRIBUTION
      X(I) = G05DEF(3.DO,1.DO)
      XKEEP(I,1) = 1.DO
      XKEEP(I,2) = X(I)
      X(MHALF+I) = X(I)
      XKEEP(MHALF+I,1) = 1.DO
  120 XKEEP(MHALF+I,2) = X(I)
      K = M * ITIMES
      CALL G05CBF(O.DO)
      DO 119 I = 1,K
  119  TV(I) = G05DDF(O.DO,1.DO)
      WRITE(6,9990) ITIMES,SIGBAR,GAMMA
 9990 FORMAT(/' ITIMES =',I4/' SIGBAR =',E16.8/' GAMMA = ',E16.8)
      WRITE(6,35)
      SQOLBO = O.DO
      SQOLB1 = O.DO
      SQEGBO = O.DO
      SQEGB1 = O.DO
      SQEGBC = O.DO
      SQEGBD = O.DO
      SQGLBO = O.DO
```

```
      SQGLB1 = 0.D0
      SQBPB0 = 0.D0
      SQBPB1 = 0.D0
      SQGPB0 = 0.D0
      SQGPB1 = 0.D0
      SQBTB0 = 0.D0
      SQBTB1 = 0.D0
      SQBSB0 = 0.D0
      SQBSB1 = 0.D0
      SQBTT0 = 0.D0
      SQBTT1 = 0.D0
      SQBTG0 = 0.D0
      SQBTG1 = 0.D0
      SQGSB0 = 0.D0
      SQGSB1 = 0.D0
      AVGVAR = 0.D0
      K = 0
      DO 210 L = 1,ITIMES
      IFAIL = 0
      DO 125 II = 1,M
      K = K + 1
125    V(II) = TV(K)
      CALL KACONS(X,SIGBAR,GAMMA,ALPHA,M)
      DO 1510 II = 1,M
1510   U(II) = DSQRT(ALPHA * X(II) ** GAMMA) *  V(II)
      DO 170 II = 1,M
170    Y(II) = 0.025 + 0.0025 * X(II) + U(II)

C  OLS, BPT, BPTT AND BPTG ESTIMATES
      CALL GO2CAF(M,X,Y,RESULT,IFAIL)
301   FORMAT(' OLSs ARE ',2E16.8)
      OLS(L,1) = RESULT(7)
      OLS(L,2) = RESULT(6)
      BPT(L,1) = OLS(L,1)
      BPT(L,2) = OLS(L,2)
      BPTT(L,1) = OLS(L,1)
      BPTT(L,2) = OLS(L,2)
      BPTG(L,1) = OLS(L,1)
      BPTG(L,2) = OLS(L,2)

C             ESTIMATES OF DELTA
      CALL CLUHAT(X,Y,UHAT,M,OLS(L,1),OLS(L,2))
      SIGMA1 = SIGMA(UHAT,M)
      CALL CUHLOG(UHAT,E,M)
      CALL CAXLOG(X,W,M)
      DO 4545 MMM = 1,M
      XLOG(MMM,1) = 1.D0
4545  XLOG(MMM,2) = W(MMM)
      CALL GO2CAF(M,W,E,RESULT,IFAIL)
      DELTAS(L,1) = RESULT(7)
      DELTAS(L,2) = RESULT(6)
      DELTA(1) = DELTAS(L,1)
      DELTA(2) = DELTAS(L,2)
      CALL CLUHAT(W,E,VHAT,M,DELTAS(L,1),DELTAS(L,2))
      SIGMA2 = SIGMA(VHAT,M)
      AVGVAR = AVGVAR + RESULT(8)
      TSTAT = RESULT(10)
      VDLTAO = RESULT(9)
```

```
          VOLTA1 = RESULT(8)

C               THE WEIGHTS(LAMBDA)
          CALL POSTEO(SIGMA1,SIGMA2,1,5,PHI(L))
  205     FORMAT(2X,2E16.8)
          CALL CALOME(XKEEP,OMEGA,OMEINV,OMEISQ,DELTAS(L,2),M)
          CALLTRANSP(XKEEP,TTX,M,N)
          CALL FO1CKF(TXO,TTX,OMEINV,N,M,M,Z,1,1,IFAIL)
          CALL FO1CKF(TXX,TXO,XKEEP,N,N,M,Z,1,1,IFAIL)
          CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
          DO 2000 III = 1,N
          DO 2000 JJJ = 1,N
  2000    TEMPT(III,JJJ) = -XINV(III,JJJ)
          CALL FO1CKF(XTEMP,TTX,XKEEP,N,N,M,Z,1,1,IFAIL)
          CALL FO1AAF(XTEMP,N,N,INVX,N,WKSP,IFAIL)
          CALL FO1CKF(TXO,TTX,OMEGA,N,M,M,Z,1,1,IFAIL)
          CALL FO1CKF(TXX,TXO,XKEEP,N,N,M,Z,1,1,IFAIL)
          CALL FO1CKF(XTEMP2,INVX,TXX,N,N,N,Z,1,1,IFAIL)
          CALL FO1CKF(XTEMP3,XTEMP2,INVX,N,N,N,Z,1,1,IFAIL)
          CALL FO1CDF(XTEMP4,TEMPT,XTEMP3,N,N,IFAIL)
          TRACE1 = 0.DO
          DO 2001 KK = 1,N
  2001    TRACE1 = TRACE1 + XTEMP4(KK,KK)
          TRACEA = (1.DO - PHI(L))*TRACE1
          CALL FO1CKF(XTEMP2,TTX,OMEISQ,N,M,M,Z,1,1,IFAIL)
          CALL FO1CKF(XTEMP3,XTEMP2,XKEEP,N,N,M,Z,1,1,IFAIL)
          CALL FO1CKF(SABINA,XINV,XTEMP3,N,N,N,Z,1,1,IFAIL)
          CALL FO1CKF(XTEMP4,SABINA,XINV,N,N,N,Z,1,1,IFAIL)
          TRACE2 = 0.DO
          DO 2003 III = 1,N
          DO 2203 JJJ = 1,N
  2203    SENYO(III,JJJ) = XTEMP4(III,JJJ) - INVX(III,JJJ)
  2003    CONTINUE
  2013    FORMAT(' XTEMP4',2E16.8)
          DO 2004 III = 1,N
  2004    TRACE2 = TRACE2 + SENYO(III,III)
  2103    FORMAT(' XTEMP4',2E16.8)
          TRACEB = PHI(L)*TRACE2
  7444    FORMAT(' TRACEA AND TRACEB ',2E16.8)
          HLAMDA(L) = 1.DO / (1.DO + TRACEA/TRACEB)

C                   GLS
   35      FORMAT(/)
          CALL CALSTA(X,Y,XSTAR,ZET,YSTAR,M,GAMMA)
           DO 1021 LM=1,M
           XSTKIP(LM,1) = ZET(LM)
           XSTKIP(LM,2) = XSTAR(LM)
  1021     YSTKIP(LM,1) = YSTAR(LM)
           CALL TRANSP(XSTKIP,TTX,M,N)
           CALL FO1CKF(TXX,TTX,XSTKIP,N,N,M,Z,1,1,IFAIL)
           CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
           CALL FO1CKF(XINVTX,XINV,TTX,N,M,N,Z,1,1,IFAIL)
           CALL FO1CKF(BETA,XINVTX,YSTKIP,N,1,M,Z,1,1,IFAIL)
   95      FORMAT(' GLSs ARE ',2E16.8)
          GLS(L,1) = BETA(1,1)
          GLS(L,2) = BETA(2,1)
   30       FORMAT(2X,5F15.8)
```

```
C              EGLS, BPE AND GPEV
         CALL CALSTA(X,Y,XSTAR,ZET,YSTAR,M,DELTAS(L,2))
         DO 1020 LM=1,M
         XSTKIP(LM,1) = ZET(LM)
         XSTKIP(LM,2) = XSTAR(LM)
 1020    YSTKIP(LM,1) = YSTAR(LM)
         CALL TRANSP(XSTKIP,TTX,M,N)
         CALL FO1CKF(TXX,TTX,XSTKIP,N,N,M,Z,1,1,IFAIL)
         CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
         CALL FO1CKF(XINVTX,XINV,TTX,N,M,N,Z,1,1,IFAIL)
         CALL FO1CKF(BETA,XINVTX,YSTKIP,N,1,M,Z,1,1,IFAIL)
         EGLS(L,1) = BETA(1,1)
         EGLS(L,2) = BETA(2,1)
        BPE(L,1) = (PHI(L))*EGLS(L,1) + (1 - PHI(L))*OLS(L,1)
        BPE(L,2) = (PHI(L))*EGLS(L,2) + (1 - PHI(L))*OLS(L,2)
        GPEV(L,1) = (HLAMDA(L))*EGLS(L,1) + (1 - HLAMDA(L))*OLS(L,1)
        GPEV(L,2) = (HLAMDA(L))*EGLS(L,2) + (1 - HLAMDA(L))*OLS(L,2)

C                GPES AND BSENYO
         CALL TRANSP(XLOG,TTX,M,N)
         CALL FO1CKF(TXX,TTX,XLOG,N,N,M,Z,1,1,IFAIL)
         CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
         HARVEY = 4.9348 * XINV(N,N)

         END2  = (0.70 - DELTAS(L,2)) / DSQRT(VDLTA1)
         END1  = (-0.35 - DELTAS(L,2)) / DSQRT(VDLTA1)
         PHIBA(L) = GO1BAF(M-2,END2,IFAIL) - GO1BAF(M-2,END1,IFAIL)
        TRACEA = (1.DO - PHIBA(L)) * TRACE1
        TRACEB = PHIBA(L) * TRACE2
        SLAMDA(L) = 1.DO / (1.DO + TRACEA/TRACEB)
        GPES(L,1) = (SLAMDA(L))*OLS(L,1) + (1 - SLAMDA(L))*EGLS(L,1)
        GPES(L,2) = (SLAMDA(L))*OLS(L,2) + (1 - SLAMDA(L))*EGLS(L,2)
        BSENYO(L,1) = PHIBA(L)*OLS(L,1)+ (1.DO - PHIBA(L))*EGLS(L,1)
        BSENYO(L,2) = PHIBA(L)*DLS(L,2)+ (1.DO - PHIBA(L))*EGLS(L,2)

C             GOLDFELD AND QUANDT F-STATS
        CALL SORTX(X,TEMPX,INDICS,M)
         MM = 12
         DO 5253 KK =1,8
         PART1X(KK) = TEMPX(KK)
         IND        = INDICS(KK)
         PART1Y(KK) = Y(IND)
         PART2X(KK) = TEMPX(MM+KK)
         IND        = INDICS(MM+KK)
 5253    PART2Y(KK) = Y(IND)
        CALL GO2CAF(8,PART1X,PART1Y,RESULT,IFAIL)
        BETAGU(L,1) = RESULT(7)
        BETAGU(L,2) = RESULT(6)
        CALL GO2CAF(8,PART2X,PART2Y,RESULT,IFAIL)
        BETAGD(L,1) = RESULT(7)
        BETAGD(L,2) = RESULT(6)
        CALL CLUHAG(PART1X,PART1Y,UHAT1,8,BETAGU(L,1),BETAGU(L,2))
        CALL CLUHAG(PART2X,PART2Y,UHAT2,8,BETAGD(L,1),BETAGD(L,2))
        FSTATS = GQFSTA(UHAT1,UHAT2,8)

        IF((FSTATS.LT.4.4333).AND.(FSTATS.GT.0.2256)) GOTO 2111
        BPT(L,1) = EGLS(L,1)
        BPT(L,2) = EGLS(L,2)
```

```
 2111  CONTINUE

      TSTAT = DELTAS(L,2) / DSQRT(HARVEY)
       IF((TSTAT.LT.2.101).AND.(TSTAT.GT.-2.101)) GOTO 1100
       BPTT(L,1) = EGLS(L,1)
       BPTT(L,2) = EGLS(L,2)
 1100  CONTINUE

       ALIMIT =-0.35 - 2.101 * DSQRT(HARVEY)
       BLIMIT = 0.70 + 2.101 * DSQRT(HARVEY)
       IF((DEL.GT.ALIMIT).AND.(DEL.LT.BLIMIT)) GOTO 1102
       BPTG(L,1) = EGLS(L,1)
       BPTG(L,2) = EGLS(L,2)
 1102  CONTINUE

C                MSE
       SQOLBO = SQOLBO + GBIAS(OLS(L,1),0.025) ** 2
       SQOLB1 = SQOLB1 + GBIAS(OLS(L,2),0.0025) ** 2

       SQGLBO = SQGLBO + GBIAS(GLS(L,1),0.025) ** 2
       SQGLB1 = SQGLB1 + GBIAS(GLS(L,2),0.0025) ** 2

       SQEGBO = SQEGBO + GBIAS(EGLS(L,1),0.025)**2
       SQEGB1 = SQEGB1 + GBIAS(EGLS(L,2),0.0025)**2

       SQBPBO = SQBPBO + GBIAS(BPE(L,1),0.025) ** 2
       SQBPB1 = SQBPB1 + GBIAS(BPE(L,2),0.0025) ** 2

       SQBTBO = SQBTBO + GBIAS(BPT(L,1),0.025) ** 2
       SQBTB1 = SQBTB1 + GBIAS(BPT(L,2),0.0025) ** 2

       SQBTTO = SQBTTO + GBIAS(BPTT(L,1),0.025) ** 2
       SQBTT1 = SQBTT1 + GBIAS(BPTT(L,2),0.0025) ** 2

       SQBTGO = SQBTGO + GBIAS(BPTG(L,1),0.025) ** 2
       SQBTG1 = SQBTG1 + GBIAS(BPTG(L,2),0.0025) ** 2

       SQGPBO = SQGPBO + GBIAS(GPEV(L,1),0.025) ** 2
       SQGPB1 = SQGPB1 + GBIAS(GPEV(L,2),0.0025) ** 2

       SQGSBO = SQGSBO + GBIAS(GPES(L,1),0.025) ** 2
       SQGSB1 = SQGSB1 + GBIAS(GPES(L,2),0.0025) ** 2

       SQBSBO = SQBSBO + GBIAS(BSENYO(L,1),0.025) ** 2
       SQBSB1 = SQBSB1 + GBIAS(BSENYO(L,2),0.0025) ** 2
  210  CONTINUE
        WRITE(6,2051)
 2051   FORMAT(' FOR GLS '/)
       CALL STATIS(GLS,ITIMES,SQGLBO,SQGLB1,GLMSEO,GLMSE1)

        WRITE(6,35)
        WRITE(6,5556)
 5556   FORMAT(' FOR OLS '/)
       CALL STATIS(OLS,ITIMES,SQOLBO,SQOLB1,OMSEO,OMSE1)
       CALL RLATIV(OMSEO,OMSE1,GLMSEO,GLMSE1,RMSEOO,RMSEO1)
       WRITE(6,205) RMSEOO,RMSEO1

        WRITE(6,35)
```

```
      WRITE(6,2352)
 2352 FORMAT(' FOR EGLS '/)
      CALL STATIS(EGLS,ITIMES,SQEGBO,SQEGB1,EGMSEO,EGMSE1)
      CALL RLATIV(EGMSEO,EGMSE1,GLMSEO,GLMSE1,RMSEGO,RMSEG1)
      WRITE(6,205) RMSEGO,RMSEG1

      WRITE(6,35)
      WRITE(6,2052)
 2052 FORMAT(' FOR BPE '/)
      CALL STATIS(BPE,ITIMES,SQBPBO,SQBPB1,BEMSEO,BEMSE1)
      CALL RLATIV(BEMSEO,BEMSE1,GLMSEO,GLMSE1,RMSBPO,RMSBP1)
      WRITE(6,205) RMSBPO,RMSBP1

      WRITE(6,35)
      WRITE(6,2054)
 2054 FORMAT(' FOR GPEV '/)
      CALL STATIS(GPEV,ITIMES,SQGPBO,SQGPB1,GPMSEO,GPMSE1)
      CALL RLATIV(GPMSEO,GPMSE1,GLMSEO,GLMSE1,RMSGPO,RMSGP1)
      WRITE(6,205) RMSGPO,RMSGP1

      WRITE(6,35)
      WRITE(6,2154)
 2154 FORMAT(' FOR GPES '/)
      CALL STATIS(GPES,ITIMES,SQGSBO,SQGSB1,GSMSEO,GSMSE1)
      CALL RLATIV(GSMSEO,GSMSE1,GLMSEO,GLMSE1,RMSGSO,RMSGS1)
      WRITE(6,205) RMSGSO,RMSGS1

      WRITE(6,35)
      WRITE(6,2053)
 2053 FORMAT(' FOR BPT  '/)
      CALL STATIS(BPT,ITIMES,SQBTBO,SQBTB1,BTMSEO,BTMSE1)
      CALL RLATIV(BTMSEO,BTMSE1,GLMSEO,GLMSE1,RMSBTO,RMSBT1)
      WRITE(6,205) RMSBTO,RMSBT1

      WRITE(6,35)
      WRITE(6,2035)
 2035 FORMAT(' FOR BPTT  '/)
      CALL STATIS(BPTT,ITIMES,SQBTTO,SQBTT1,BTTMSO,BTTMS1)
      CALL RLATIV(BTTMSO,BTTMS1,GLMSEO,GLMSE1,RMBTTO,RMBTT1)
      WRITE(6,205) RMBTTO,RMBTT1

      WRITE(6,35)
      WRITE(6,2045)
 2045 FORMAT(' FOR BPTG  '/)
      CALL STATIS(BPTG,ITIMES,SQBTGO,SQBTG1,BTGMSO,BTGMS1)
      CALL RLATIV(BTGMSO,BTGMS1,GLMSEO,GLMSE1,RMBTGO,RMBTG1)
      WRITE(6,205) RMBTGO,RMBTG1

      WRITE(6,35)
      WRITE(6,2056)
 2056 FORMAT(' FOR BSENYO '/)
      CALL STATIS(BSENYO,ITIMES,SQBSBO,SQBSB1,BSMSEO,BSMSE1)
      CALL RLATIV(BSMSEO,BSMSE1,GLMSEO,GLMSE1,RMSBSO,RMSBS1)
      WRITE(6,205) RMSBSO,RMSBS1

      CALL AVGE2(PHI,ITIMES,AVG)
      CALL VRANC2(PHI,ITIMES,AVG,VAR)
      WRITE(6,35)
```

```
      WRITE(6,9177) AVG,VAR
9177  FORMAT(' THE AVG AND VAR OF PHI ARE ',2E16.8)

      CALL AVGE2(PHIBA,ITIMES,AVG)
      CALL VRANC2(PHIBA,ITIMES,AVG,VAR)
      WRITE(6,35)
      WRITE(6,9177) AVG,VAR
9178  FORMAT(' THE AVG AND VAR OF PHIBA ARE ',2E16.8)
      STOP
      END
```

APPENDIX B

```
C              HETEROSKEDASTICITY(BIVARIANCE)
C     TO AVOID DUPLICATION, OTHER RELEVANT SUBROTINES CALLED IN
C     THIS PROGRAMME ARE SPECIFIED IN APPENDIX-A(REFER TO THEM).

C                   OMEGA MATRIX
      SUBROUTINE CALOME(VARU1,VARU2,OMEGA,OMEINV,OMEISQ,M)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 OMEGA(20,20),OMEINV(20,20),OMEISQ(20,20)
      REAL*8 WKSP(20)
      MHALF = M / 2
      DO 1 I = 1,MHALF
      DO 2 J = 1,M
      OMEGA(I,J) = 0.D0
      OMEISQ(I,J) = 0.D0
    2 OMEINV(I,J) = 0.D0
      OMEGA(I,I)  =  DSQRT(VARU1)
    1 OMEINV(I,I) = 1.D0 / DSQRT(VARU1)
      MHALF = MHALF + 1
      DO 10 I = MHALF,M
      DO 20 J = 1,M
      OMEGA(I,J) = 0.D0
      OMEISQ(I,J) = 0.D0
   20  OMEINV(I,J) = 0.D0
       OMEGA(I,I)  = DSQRT(VARU2)
   10  OMEINV(I,I) = 1.D0 / DSQRT(VARU2)
      DO 4 I=1,M
    4 OMEISQ(I,I)  = OMEINV(I,I) ** 2
      RETURN
      END

C                     PROB(PHI)
      FUNCTION PROGAM(X1,X2,Y1,Y2,N,RAMMA,N1,N2)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X1(10,2),X2(10,2),Y1(10,1),Y2(10,1)
      REAL*8 TTX1(2,10),TTX2(2,10),TTY1(1,10),TTY2(1,10)
      REAL*8 TX1(2,2),TX2(2,2),TY1(1,1),TY2(1,1),Z(1)
      REAL*8 ADDTOP(2,2),X1TY1(2,1),X2TY2(2,1),XYSXY(2,1)
      REAL*8 XYSXYT(1,2),INVTOP(2,2),WSKP(2),MULT(1,2)
      REAL*8 BOTTB(1,1),UHAT1(10),UHAT2(10)

      IFAIL = 0
      CALL TRANS1(X1,TTX1,N1,N)
      CALL TRANS1(X2,TTX2,N2,N)
      CALL TRANS2(Y1,TTY1,N1,1)
      CALL TRANS2(Y2,TTY2,N2,1)
      CALL F01CKF(TX1,TTX1,X1,N,N,N1,Z,1,1,IFAIL)
      CALL F01CKF(TX2,TTX2,X2,N,N,N2,Z,1,1,IFAIL)
      CALL F01CKF(TY1,TTY1,Y1,1,1,N1,Z,1,1,IFAIL)
      CALL F01CKF(TY2,TTY2,Y2,1,1,N2,Z,1,1,IFAIL)

      DO 1 I = 1,N
      DO 1 J = 1,N
      TX2(I,J) = RAMMA * TX2(I,J)
    1 ADDTOP(I,J) = TX1(I,J) + TX2(I,J)

      DET = ADDTOP(1,1)*ADDTOP(2,2) - ADDTOP(1,2)*ADDTOP(2,1)
      DET = 1.D0 / DSQRT(DET)
      TOP = RAMMA ** ((N2-2)/2) * DET
```

```
      BOTTA = TY1(1,1) + RAMMA * TY2(1,1)
      CALL FO1CKF(X1TY1,TTX1,Y1,N,1,N1,Z,1,1,IFAIL)
      CALL FO1CKF(X2TY2,TTX2,Y2,N,1,N2,Z,1,1,IFAIL)

      DO 2 I = 1,N
      DO 2 J = 1,1
    2 XYSXY(I,J) = X1TY1(I,J) + RAMMA * X2TY2(I,J)
      CALL TRANS3(XYSXY,XYSXYT,2,1)
      CALL FO1AAF(ADDTOP,N,N,INVTOP,N,WSKP,IFAIL)
      CALL FO1CKF(MULT,XYSXYT,INVTOP,1,N,N,Z,1,1,IFAIL)
      CALL FO1CKF(BOTTB,MULT,XYSXY,1,1,N,Z,1,1,IFAIL)

      BOTT = (BOTTA - BOTTB(1,1)) ** ((N1+N2-2)/2)
      PROGAM = TOP / BOTT
      RETURN
      END

      FUNCTION PROB(X1,X2,Y1,Y2,N,RAMA1,RAMA2,N1,N2,INTVAL)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X1(10,2),X2(10,2),Y1(10,1),Y2(10,1)
      END1 = RAMA1
      DELT = (RAMA2 - RAMA1) / INTVAL
      SUM = O.DO
      DO 1 I = 1,INTVAL
      END2 = END1 + DELT
      SIDE1 = PROGAM(X1,X2,Y1,Y2,N,END1,N1,N2)
      SIDE2 = PROGAM(X1,X2,Y1,Y2,N,END2,N1,N2)
      SUM = SUM + O.5 * DELT * (SIDE1 + SIDE2)
      END1 = END2
    1 CONTINUE
      PROB = SUM
      RETURN
      END

      FUNCTION PROB2(X1,X2,Y1,Y2,N,RAMA,N1,N2,INTVAL)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X1(10,2),X2(10,2),Y1(10,1),Y2(10,1)
      END1 = O.DO
      INT = RAMA / 8.DO * INTVAL
      DELT = RAMA / INT
      SUM = O.DO
      DO 1 I = 1,INT
      END2 = END1 + DELT
      SIDE1 = PROGAM(X1,X2,Y1,Y2,N,END1,N1,N2)
      SIDE2 = PROGAM(X1,X2,Y1,Y2,N,END2,N1,N2)
      SUM = SUM + O.5 * DELT * (SIDE1 + SIDE2)
      END1 = END2
    1 CONTINUE
      PROB2 = SUM
      RETURN
      END

      FUNCTION CALC(X1,X2,Y1,Y2,N,C,N1,N2,INTVAL)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X1(10,2),X2(10,2),Y1(10,1),Y2(10,1)
      TOP = 8.DO
      VALUE = PROB(X1,X2,Y1,Y2,N,O.DO,TOP,N1,N2,INTVAL)
```

```
      CALC = 1.DO / VALUE
      RETURN
      END

      SUBROUTINE STATIS(ARRAY,ITIMES,SQLBO,SQLB1,BMSEO,BMSE1)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 ARRAY(600,2)
      CALL VRANCE(ARRAY,ITIMES,AVGEO,AVGE1,VARO,VAR1)
       BMSEO = SQLBO / ITIMES
       BMSE1 = SQLB1 / ITIMES
205    FORMAT(2X,2E16.8)
35     FORMAT(//)
       RETURN
       END

       SUBROUTINE CHISQ(UHAB1,UHAB2,M,VARU1,VARU2,CISQD1,CISQD2,F)
       IMPLICIT REAL*8 (A-H,O-Z)
       REAL*8 UHAB1(10),UHAB2(10)
       SUHTQ1 = 0.DO
       SUHTQ2 = 0.DO
       DO 1 I = 1,M
       SUHTQ1 = SUHTQ1 + UHAB1(I) ** 2
1      SUHTQ2 = SUHTQ2 + UHAB2(I) ** 2
       VARU1 = SUHTQ1 / (M - 2)
       VARU2 = SUHTQ2 / (M - 2)
       CISQD1 = (VARU1 / VARU2) * (1.DO / 2.09)
       CISQD2 = (VARU1 / VARU2) * (1.DO / 0.5)
       F = SUHTQ2 / SUHTQ1
       RETURN
       END

C               THE MAIN PROGRAMME

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 TY(12000),Y(20),BETA(2,1),GLS(600,2),G05DDF
      REAL*8 V(20),TV(12000),UHAT(20),EGLS(600,2),G05CAF
      REAL*8 TXX(2,2),XINV(2,2),U(20),WKSP(20),G05DEF
      REAL*8 X(20),TTX(2,20),OMEGA(20,20),OLS(600,2)
      REAL*8 Z(1),XINVTX(2,20),MOVE(11),BHAT(600,2),XVARB1(10,2)
      REAL*8 W(20),E(20),BPT(600,2),GPES(600,2),BSENYO(600,2)
      REAL*8 OMEISQ(20,20),OMEINV(20,20),YVARB1(10,1)
      REAL*8 TEMPT(2,2),XTEMP(2,2),INVX(2,2),BETAGD(600,2)
      REAL*8 XTEMP3(2,2),XTEMP4(2,2),SABINA(2,2),UHAT1(8)
      REAL*8 XTEMP2(2,2),SENYO(2,2),WKSP2(8),YVARB2(10,1)
      REAL*8 TWW(2,2),TTW(2,20),WINV(2,2),PART2X(8),YVARB(20,1)
      REAL*8 WINVTW(2,20),PART1X(8),UHAT2(8),XVARB(20,2)
      REAL*8 TEMPX(20),PART1Y(8),PART2Y(8),BETAGU(600,2)
      REAL*8 TTX2(2,8),XINV2(2,2),XIN2TX(8,2),TXX2(2,8)
      REAL*8 YSTAR(20),XSTAR(20),TXO(2,20),TKEEP(2,1)
      REAL*8 XKEEP(20,2),ZET(20),XSTKIP(20,2),BPTG(600,2)
      REAL*8 YSTKIP(20,1),SLAMDA(600),XVARB2(10,2),PHIBA(600)
      REAL*8 PARB1X(10),PARB2X(10),PARB1Y(10),PARB2Y(10)
      REAL*8 UHAB1(10),UHAB2(10),BEBAVU(600,2),BEBAVD(600,2)
      REAL*8 DELTAS(600,2),RESULT(20),YKEEP(20,1),TEMPU(20)
      INTEGER INDICS(20)

      M = 20
      N = 2
```

```
      READ(5,10) ITIMES
   10 FORMAT(I4)
      READ(5,17)ALPHA,GAMMA,C
   17 FORMAT(3F8.4)
      READ(5,18) GL,GU
   18 FORMAT(2F8.4)
      READ(5,19) FIRST,SECOND
   19 FORMAT(2F10.4)
      CALL GO5CBF(0.DO)
      DO 120 I = 1,10

C       Xs FROM LOGNORMAL DISTRIBUTION
      X(I) = GO5DEF(3.DO,1.DO)
      X(I+10) = X(I)
      XKEEP(I+10,1) = 1.DO
      XKEEP(I+10,2) = X(I+10)
      XKEEP(I,1) = 1.DO
  120 XKEEP(I,2) = X(I)
      WRITE(6,9990) ITIMES,FIRST,SECOND
 9990 FORMAT(/' ITIMES =',I4/' FIRST =',E16.8/' SECOND= ',E16.8)
      K = M * ITIMES
      DO 119 I = 1,K
  119  TV(I) = GO5DDF(0.DO,1.DO)
      SQOLBO = 0.DO
      SQOLB1 = 0.DO
      SQEGBO = 0.DO
      SQEGB1 = 0.DO
      SQGLBO = 0.DO
      SQGLB1 = 0.DO
      SQBTBO = 0.DO
      SQBTB1 = 0.DO
      SQBSBO = 0.DO
      SQBSB1 = 0.DO
      SQBTGO = 0.DO
      SQBTG1 = 0.DO
      SQGSBO = 0.DO
      SQGSB1 = 0.DO
      K = 0
      DO 210 L = 1,ITIMES
      IFAIL = 0
      DO 125 II = 1,M
      K = K + 1
  125  V(II) = TV(K)
       MHALF = M / 2
       SQRTN1 = DSQRT(FIRST)
       SQRTN2 = DSQRT(SECOND)
      DO 1510 II = 1,MHALF
 1510  U(II) = SQRTN1 *  V(II)
       MHALF = MHALF + 1
      DO 1511 II = MHALF,M
 1511  U(II) = SQRTN2 *  V(II)
      DO 170 II = 1,M
       Y(II) = 0.025 + 0.0025 * X(II) + U(II)
  170  YKEEP(II,1) = Y(II)

C              EGLS
       DO 5555 KK =1,10
       PARB1X(KK) = X(KK)
```

```
         PARB1Y(KK) = Y(KK)
         XVARB(KK,2) = X(KK)
         XVARB(KK,1) = 1.DO
         YVARB(KK,1) = Y(KK)
         PARB2X(KK) = X(KK+10)
 5555    PARB2Y(KK) = Y(KK+10)
        CALL GO2CAF(10,PARB1X,PARB1Y,RESULT,IFAIL)
        BEBAVU(L,1) = RESULT(7)
        BEBAVU(L,2) = RESULT(6)
        CALL GO2CAF(10,PARB2X,PARB2Y,RESULT,IFAIL)
        BEBAVD(L,1) = RESULT(7)
        BEBAVD(L,2) = RESULT(6)
        CALL CLUHAG(PARB1X,PARB1Y,UHAT1,10,BEBAVU(L,1),BEBAVU(L,2))
        CALL CLUHAG(PARB2X,PARB2Y,UHAT2,10,BEBAVD(L,1),BEBAVD(L,2))
        CALL CHISQ(UHAT1,UHAT2,10,VARU1,VARU2,CISQD1,CISQD2,F)
        SEVAR1 = DSQRT(VARU1)
        SEVAR2 = DSQRT(VARU2)
        DO 1010 KK = 1,10
        XVARB(KK,2) = X(KK) / SEVAR1
        XVARB(KK,1) = 1.DO / SEVAR1
        YVARB(KK,1) = Y(KK) / SEVAR1
        XVARB(KK+10,1) = 1.DO / SEVAR2
        XVARB(KK+10,2) = X(KK+10) / SEVAR2
 1010   YVARB(KK+10,1) = Y(KK+10) / SEVAR2
         CALL TRANSP(XVARB,TTX,M,N)
         CALL FO1CKF(TXX,TTX,XVARB,N,N,M,Z,1,1,IFAIL)
         CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
         CALL FO1CKF(XINVTX,XINV,TTX,N,M,N,Z,1,1,IFAIL)
         CALL FO1CKF(BETA,XINVTX,YVARB,N,1,M,Z,1,1,IFAIL)
         EGLS(L,1) = BETA(1,1)
         EGLS(L,2) = BETA(2,1)
         SQEGBO = SQEGBO + GBIAS(EGLS(L,1),0.025)**2
         SQEGB1 = SQEGB1 + GBIAS(EGLS(L,2),0.0025)**2

C             OLS, BPT AND BPTG
         CALL TRANSP(XKEEP,TTX,M,N)
         CALL FO1CKF(TXX,TTX,XKEEP,N,N,M,Z,1,1,IFAIL)
         CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
         CALL FO1CKF(XINVTX,XINV,TTX,N,M,N,Z,1,1,IFAIL)
         CALL FO1CKF(BETA,XINVTX,YKEEP,N,1,M,Z,1,1,IFAIL)
        OLS(L,1) = BETA(1,1)
        OLS(L,2) = BETA(2,1)
        BPT(L,1) = OLS(L,1)
        BPT(L,2) = OLS(L,2)
        BPTG(L,1) = OLS(L,1)
        BPTG(L,2) = OLS(L,2)

C                      MSE
         SQOLBO = SQOLBO + (GBIAS(OLS(L,1),0.025) ** 2)
         SQOLB1 = SQOLB1 + (GBIAS(OLS(L,2),0.0025) ** 2)

C              THE WEIGHTS(= LAMBDA)
        CALL CALOME(VARU1,VARU2,OMEGA,OMEINV,OMEISQ,M)
        CALLTRANSP(XKEEP,TTX,M,N)
        CALL FO1CKF(TXO,TTX,OMEINV,N,M,M,Z,1,1,IFAIL)
        CALL FO1CKF(TX,TXO,XKEEP,N,N,M,Z,1,1,IFAIL)
        CALL FO1AAF(TX,N,N,XINV,N,WKSP,IFAIL)
        DO 2000 III = 1,N
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      DO 2000 JJJ = 1,N
2000  TEMPT(III,JJJ) = -XINV(III,JJJ)
      CALL FO1CKF(XTEMP,TTX,XKEEP,N,N,M,Z,1,1,IFAIL)
      CALL FO1AAF(XTEMP,N,N,INVX,N,WKSP,IFAIL)
      CALL FO1CKF(TXO,TTX,OMEGA,N,M,M,Z,1,1,IFAIL)
      CALL FO1CKF(TX,TXO,XKEEP,N,N,M,Z,1,1,IFAIL)
      CALL FO1CKF(XTEMP2,INVX,TX,N,N,N,Z,1,1,IFAIL)
      CALL FO1CKF(XTEMP3,XTEMP2,INVX,N,N,N,Z,1,1,IFAIL)
      CALL FO1CDF(XTEMP4,TEMPT,XTEMP3,N,N,IFAIL)
      TRACE1 = XTEMP4(2,2)
      CALL FO1CKF(XTEMP2,TTX,OMEISQ,N,M,M,Z,1,1,IFAIL)
      CALL FO1CKF(XTEMP3,XTEMP2,XKEEP,N,N,M,Z,1,1,IFAIL)
      CALL FO1CKF(SABINA,XINV,XTEMP3,N,N,N,Z,1,1,IFAIL)
      CALL FO1CKF(XTEMP4,SABINA,XINV,N,N,N,Z,1,1,IFAIL)
      TRACE2 = O.DO
      DO 2003 III = 1,N
      DO 2203 JJJ = 1,N
2203  SENYO(III,JJJ) = XTEMP4(III,JJJ) - INVX(III,JJJ)
2003  CONTINUE
      TRACE2 = SENYO(2,2)

C                   GLS
 35    FORMAT(//)
       DO 1021 LM=1,10
       XVARB(LM,1) = 1.DO / SQRTN1
       XVARB(LM,2) = X(LM) / SQRTN1
       YVARB(LM,1) = Y(LM) / SQRTN1
       XVARB(LM+10,1) = 1.DO / SQRTN2
       XVARB(LM+10,2) = X(LM+10) / SQRTN2
 1021  YVARB(LM+10,1) = Y(LM+10) / SQRTN2
       CALL TRANSP(XVARB,TTX,M,N)
       CALL FO1CKF(TXX,TTX,XVARB,N,N,M,Z,1,1,IFAIL)
       CALL FO1AAF(TXX,N,N,XINV,N,WKSP,IFAIL)
       CALL FO1CKF(XINVTX,XINV,TTX,N,M,N,Z,1,1,IFAIL)
       CALL FO1CKF(BETA,XINVTX,YVARB,N,1,M,Z,1,1,IFAIL)
      GLS(L,1) = BETA(1,1)
      GLS(L,2) = BETA(2,1)
 30      FORMAT(2X,5F15.8)
       SQGLBO = SQGLBO + GBIAS(GLS(L,1),0.025) ** 2
       SQGLB1 = SQGLB1 + GBIAS(GLS(L,2),0.0025) ** 2
       DO 5554 KK =1,10
       XVARB1(KK,2) = X(KK)
       XVARB1(KK,1) = 1.DO
       YVARB1(KK,1) = Y(KK)
       XVARB2(KK,1) = 1.DO
       XVARB2(KK,2) = X(KK+10)
 5554  YVARB2(KK,1) = Y(KK+10)

C                 GPES AND BSENYO
      C = CALC(XVARB1,XVARB2,YVARB1,YVARB2,2,C,10,10,100)
      PHIB1=PROB2(XVARB1,XVARB2,YVARB1,YVARB2,2,GL,10,10,100)
      PHIB2=PROB2(XVARB1,XVARB2,YVARB1,YVARB2,2,GU,10,10,100)
      PHIBA(L) = (PHIB2 - PHIB1) * C
        TRACEA = (1.DO - PHIBA(L)) * TRACE1
        TRACEB = PHIBA(L) * TRACE2
        SLAMDA(L) = 1.DO / (1.DO + TRACEA/TRACEB)
       GPES(L,1) = (SLAMDA(L))*OLS(L,1) + (1 - SLAMDA(L))*EGLS(L,1)
       GPES(L,2) = (SLAMDA(L))*OLS(L,2) + (1 - SLAMDA(L))*EGLS(L,2)
```

```
      BSENYO(L,1) = PHIBA(L)*OLS(L,1) + (1.DO-PHIBA(L))*EGLS(L,1)
      BSENYO(L,2) = PHIBA(L)*OLS(L,2) + (1.DO-PHIBA(L))*EGLS(L,2)

      SQGSBO = SQGSBO + GBIAS(GPES(L,1),0.025) ** 2
      SQGSB1 = SQGSB1 + GBIAS(GPES(L,2),0.0025) ** 2

      SQBSBO = SQBSBO + GBIAS(BSENYO(L,1),0.025) ** 2
      SQBSB1 = SQBSB1 + GBIAS(BSENYO(L,2),0.0025) ** 2

     IF((F.LT.4.4333).AND.(F.GT.0.2256)) GOTO 2111
     BPT(L,1) = EGLS(L,1)
     BPT(L,2) = EGLS(L,2)
2111 CONTINUE

     IF((CISQD1.LT.4.4333).AND.(CISQD2.GT.0.2256)) GOTO 1102
     BPTG(L,1) = EGLS(L,1)
     BPTG(L,2) = EGLS(L,2)
1102 CONTINUE

     SQBTBO = SQBTBO + GBIAS(BPT(L,1),0.025) ** 2
     SQBTB1 = SQBTB1 + GBIAS(BPT(L,2),0.0025) ** 2

     SQBTGO = SQBTGO + GBIAS(BPTG(L,1),0.025) ** 2
     SQBTG1 = SQBTG1 + GBIAS(BPTG(L,2),0.0025) ** 2

210  CONTINUE
205   FORMAT(2X,2E16.8)

      WRITE(6,2051)
2051  FORMAT(' FOR GLS '/)
     CALL STATIS(GLS,ITIMES,SQGLBO,SQGLB1,GLMSEO,GLMSE1)

      WRITE(6,35)
      WRITE(6,5556)
5556  FORMAT(' FOR OLS '/)
     CALL STATIS(OLS,ITIMES,SQOLBO,SQOLB1,OMSEO,OMSE1)
     CALL RLATIV(OMSEO,OMSE1,GLMSEO,GLMSE1,RMSEOO,RMSEO1)

      WRITE(6,35)
      WRITE(6,2352)
2352  FORMAT(' FOR EGLS '/)
      CALL STATIS(EGLS,ITIMES,SQEGBO,SQEGB1,EGMSEO,EGMSE1)
      CALL RLATIV(EGMSEO.EGMSE1,GLMSEO,GLMSE1,RMSEGO,RMSEG1)

      WRITE(6,35)
      WRITE(6,2154)
2154  FORMAT(' FOR GPES '/)
      CALL STATIS(GPES,ITIMES,SQGSBO,SQGSB1,GSMSEO,GSMSE1)
      CALL RLATIV(GSMSEO,GSMSE1,GLMSEO,GLMSE1,RMSGSO,RMSGS1)

      WRITE(6,35)
      WRITE(6,2053)
2053  FORMAT(' FOR BPT  '/)
      CALL STATIS(BPT,ITIMES,SQBTBO,SQBTB1,BTMSEO,BTMSE1)
      CALL RLATIV(BTMSEO,BTMSE1,GLMSEO,GLMSE1,RMSBTO,RMSBT1)

      WRITE(6,35)
      WRITE(6,2045)
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```
 2045 FORMAT(' FOR BPTG  '/)
      CALL STATIS(BPTG,ITIMES,SQBTGO,SQBTG1,BTGMSO,BTGMS1)
      CALL RLATIV(BTGMSO,BTGMS1,GLMSEO,GLMSE1,RMBTGO,RMBTG1)

      WRITE(6,35)
      WRITE(6,2056)
 2056 FORMAT(' FOR BSENYO '/)
 2796 FORMAT(2E16.8)
      CALL STATIS(BSENYO,ITIMES,SQBSBO,SQBSB1,BSMSEO,BSMSE1)
      CALL RLATIV(BSMSEO,BSMSE1,GLMSEO,GLMSE1,RMSBSO,RMSBS1)

      WRITE(6,35)
      CALL VRANC2(PHIBA,ITIMES,AVGE,VAR)
      WRITE(6,2299) AVGE,VAR
 2299 FORMAT(' THE AVERAGE AND VARIANCE OF PHI ARE',2E16.8)
      STOP
      END
```