AN ECONOMIC ANALYSIS OF FULLY FUNDED PENSION PROGRAM

by

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PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

In the
Department
of
Economics

SIMON FRASER UNIVERSITY

Fall 2004

APPROVAL

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ABSTRACT

This paper uses an overlapping generation model to examine how individual choices and welfare are affected with the implementation of a fully funded pension plan. First, I consider the case when the pension is not available and sets this result as a benchmark. I then examine how a uniform pension benefit financed with a flat income tax affects individual behaviour across households that differ in their skill and their preference for leisure. The computational results show that the pension program leads to a distortion in labour supply and generally reduces the level of welfare for all types of individuals.

ACKNOWLEDGEMENTS

First of all, I want to thank Professors David Andolfatto and Professor Steeve Mongrain of the Faculty of Economics for the tremendous support and valuable advice for this paper. I admire them both immensely. I am also very grateful for the help of Professor Ken Kasa and Professor Philip Curry of Faculty of Economics.

Lastly, I want to thank my dearest son Titian Shichao Su, my beloved Husband Xiaoming Su and my parents for all the love and encouragement they have given me; my friends who have always supported and helped me.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

RP	=	retirement pension
L	=	the low skilled people who is the high school dropout
M	=	the median skilled people who is the high school grad
H	=	the high skilled people who is college grad
U	=	utility
SW	=	the indirect utility function
c_1	=	consumption in the first period
l_1	=	leisure in the first period
n_1	=	labour supply in the firs period
c_2	=	consumption in the second period
l_2	=	leisure in the second period
n_2	=	labour supply in the second period
c_3	=	consumption in the third period
l_3	=	leisure in the third period
n_3	=	labour supply in the third period
β	=	discount rate
φ	=	the preference parameter for leisure over consumption
s_1	=	saving in the first period
s_2	=	saving in the second period
τ	=	average value of tax rate
w	=	wage rate
r	=	interest rate

Subscripts

i	=	L, M, and H; each type individual
t	=	1, 2, and 3; three periods

1 INTRODUCTION

This paper uses an overlapping generation model to compute the economic and welfare impact of a tax-financed pension plan in an economy consisting of heterogeneous individuals. When the pension is financed by a flat income tax, it shows this program distorts individual's labour supply decisions and leads to deadweight losses. The higher pension financed by the higher tax rate only brings the lower labour supply and the worse social welfare. Surprisingly, no one benefits from this pension program, even at the lower end of the income distribution. Consequently, this study reveals some doubts concerning this pension programs' efficacy in improving social welfare.

This paper will be structured as follows: section 2 describes the model and provides equations that reflect the equilibrium labour supply and utility. Section 3 discusses the calibration of the parameters within the model to some realistic values based on previous studies. Section 4 provides the results of the model using output from GAUSS. Section 5 concludes the main implication of the results presented in this study.

2 MODEL AND ASSUMPTIONS:

This paper employs a life cycle model over three time periods to estimate the quantitative impact on the individual labour supply and utility different pension program.

The individual maximizes utility over two time-dated goods, consumption and leisure over three time periods. Preferences are represented with the following utility function:

$$U = \ln c_{1i} + \varphi_i \ln l_{1i} + \beta(\ln c_{2i} + \varphi_i \ln l_{2i}) + \beta^2(\ln c_{3i}) \qquad (1)$$

where $i = L,M,H$ denotes household type.

In the first period, "young" from age of 20 to 40, the individual works in full time and supplies elastic labour. In the second period, from 40 to 60, considering the cost, the individual makes a decision regarding working or retiring earlier. In the third period, "old" from 60 to 80, retirement is possible.

Where c_{ti} is the consumption in period t (t=1,2, and 3), l_{ti} is the leisure in period t. This study divides individuals into three types: the low, the intermediate, and the high skilled person, who is the high school dropout, the high school grad and the college grad. They have assorted wage levels consistent with their working abilities. The subscript i =L, M, and H placed on these variables refer to its three possible values: low, medium, and high. Households also have different preferences for consumption and leisure. The parameter φ represents the individual's preference for leisure over consumption. Higher value of φ represents a higher weight being place on leisure relative to consumption.

As is standard, assume that U'>0 and U''<0. Individuals can choose to allocate their fixed amount of time to work or leisure. β is the discount factor. Note that work refers to a participation in the paid labour market and leisure alludes to any other activities, such as homemaker, labour market search, etc. Individuals are endowed with an amount of time equal to 1.

To make the question more focused, this study assumes tax exists both in the first and second periods. As well it introduces the private saving decision. When individuals are in middle age, they have higher wage rate than young age.

The regular retirement pension refers to the old age pension in the third period when workers are assumed to be too old to work.

2.1 Behaviour in the Absence of Pensions

2.1.1 Consumption and leisure in three periods

2.1.1.1 First period:

In the first period, everyone needs to work full time.

Consumption is: $c_{1i} = w_i . n_{1i} - s_1$ (2)

Here, s_1 is the saving in first period and w_i is the wage rate for each type i. n_{1i} is the labour supply in the first period. i = L, M, H.

The time constraint is: $n_{1i} + l_{1i} = 1$ (3)

Equation (5) implies that the individual divides his time into work and leisure in the first period. l_{1i} is the leisure for each type i.

2.1.1.2 Second period:

During the second period, consumption in the second period is:

$$c_{2i} = \lambda w_i n_{2i} + s_1(1+r) - s_2 \qquad (4)$$

Here, r is the real interest rate, and s_2 is savings in the second period. λ equals to 1.5, which implies that the individual in his middle age has a real wage that is 50% higher than when young. The time constraint is:

$$n_{2i} + l_{2i} = 1 \qquad (5)$$

2.1.1.3 Third period:

In the third period, all individuals retire and get the regular old age pension (RP) from the government, and individual consumption equals savings; that is:

$$c_{3i} = s_2(1+r) \qquad (6)$$

Now he retires, therefore: $l_{3i} = 1$ (7)

2.1.1.4 Individual Choice Problem

If the individual continues working, we can combine equations (2),(3),(4), (5), (6), (7), (8) into (1), so that the choice problem is given by:

$$MaxU = \ln(w_i n_{1i} - s_1 + \psi_i \ln(1 - n_{1i}) + \beta(\ln(\lambda w_i n_{2i} + s_1(1+r) - s_2)$$
$$+ \varphi_i \ln(1 - n_{2i})) + \beta\text{^}2(\ln(RP + s_2(1+r))$$

i=L, M, and H. (10)

Then maximize this above equation with respect to n_{1i}, n_{2i} s_{1i}, s_{2i}, to yield the representative agent's labour supply function. The first order condition of this maximization exercise are:

With respect to n_{1i}:

$$w_i /(w_i n_{1i} - s_{1i}) - \varphi_i /(1 - n_{1i}) = 0$$

With respect to n_{2i}:

$$w_i /(\lambda w_i n_{2i} + s_{1i}(1+r) - s_{2i}) - \varphi_i /(1 - n_{2i}) = 0$$

With respect to s_{1i}:

$$\beta(1+r)(w_i n_{1i} - s_{1i}) - (\lambda w_i n_{2i} + s_{1i}(1+r) - s_{2i}) = 0$$

With respect to s_{2i}:

$$\beta(1+r)(\lambda w_i n_{2i} + s_{1i}(1+r) - s_{2i}) - s_{2i}(1+r) = 0$$

Assume that $(1+r)\beta = 1$. This implicitly means, when optimal,

$c_{1i}^* = c_{2i}^* = c_{3i}^* = c_i^*$.

The solutions to this system of equations describe equilibrium labour supply and individual saving behaviour. With the solution obtained, we can construct the indirect utility function:

$$SW = \ln c_{1i}^* + \varphi_i \ln l_{1i}^* + \beta(\ln c_{2i}^* + \varphi_i \ln l_{2i}^*) + \beta\text{^}2(\ln c_{3i}^*)$$

2.2 Government Pension Plan

In this regime, all equations and calculating procedures are similar to the no tax regime except adding the wage tax rate τ and a lump-sum pension payment RP.

2.2.1 Consumption and leisure in three periods:

2.2.1.1 First period:

In the first period, consumption is:

$$c_{1i} = w_i n_{1i}(1-\tau) - s_1 \qquad (11)$$

Where τ is a distortionary wage tax rate.

The time constraint is: $n_{1i} + l_{1i} = 1$ (12)

2.2.1.2 Second period:

During the second period, if the individual continues working, consumption is:

$$c_{2i} = \lambda w_i n_{2i}(1-\tau) + s_1(1+r) - s_2 \qquad (13)$$

The time constraint is: $n_{2i} + l_{2i} = 1$ (14)

2.2.1.3 Third period:

In the third period, all individuals retire and get the regular old age pension (RP) from the government, and the individual consumption equals to the saving plus the RP, that is: $c_{3i} = s_2(1+r) + RP$ (15)

Now he retires in full time, therefore: $l_{3i} = 1$ (16)

2.2.1.4 Government Budget Constraint:

Assume that 25% of the population are type *L* and that 25% are type *H*. The remaining 50% are type *M*. In wage tax regime, the government's budget constraint is:

$$\begin{aligned}&(0.25w_L n_{1L}\tau + 0.5w_M n_{1M}\tau + 0.25w_H n_{1H}\tau)(1+r)^2 \\ &+ (0.25w_L n_{2l} + 0.5w_M * n_{2M} + 0.25w_H n_{2H})\lambda\tau(1+r) \\ &= RP\end{aligned} \qquad (17)$$

2.2.2 Individual Choice Problem

$$\begin{aligned}&MaxU = \ln(w_i n_{1i}(1-\tau) - s_1 + \psi_i \ln(1-n_{1i}) + \beta(\ln(\lambda w_i n_{2i}(1-\tau) + s_1(1+r) - s_2) \\ &+ \varphi_i \ln(1-n_{2i})) + \beta\text{^}2(\ln(RP + s_2(1+r))\end{aligned}$$

i=L,M, and H.

Maximization of this above equation with respect to n_{1i} and n_{2i} s_{1i}, s_{2i} yields the agents' consumption and labour supply functions. The first order conditions of this maximization exercise are:

With respect to n_{1i}:

$$w_i(1-\tau)/(w_i n_{1i}(1-\tau)-s_1)-\varphi_i/(1-n_{1i})=0$$

With respect to n_{2i}:

$$\lambda w_i(1-\tau)/(\lambda w_i n_{2i}(1-t)+s_1(1+r)-s_2)-\varphi_i/(1-n_{2i})=0$$

Where the solutions to this system of equations specify equilibrium labour supply: n_{1i}^* and n_{2i}^*, s_{1i}, s_{2i}.

With respect to s_{1i}:

$$\beta(1+r)(w_i n_{1i}(1-\tau)-s_{1i})-(w_i n_{2i}(1-\tau)+s_{1i}(1+r)-s_{2i})=0$$

With respect to s_{2i}:

$$\beta(1+r)(w_i n_{2i}(1-\tau)+s_{1i}(1+r)-s_{2i})-s_{2i}(1+r)-RP=0$$

When optimal, $c_{1i}^* = c_{2i}^* = c_{3i}^* = c_i^*$

Where the solutions to this system of equations specify equilibrium labour supply: n^*_{1i}, $n^*_{2i}, s^*_{1i}, s^*_{2i}$.

With the equation so indicated, we can construct the indirect utility function,

$$SW = \ln c^*_{1i} + \varphi_i \ln l^*_{1i} + \beta(\ln c^*_{2i} + \varphi_i \ln l^*_{2i}) + \beta\hat{}2(\ln c^*_{3i})$$

3 CALIBRATION:

In this paper, there are seven parameters in the model that need to be calibrated.

The average value of the tax rate, τ, is 0.30, which follows from Andolfatto, Ferrall and Gomme (2000). This is a standard choice for income tax rate of these researches.

The value φ for an average individual is chosen to be 1.4212, which also follows Andolfatto, Ferrall and Gomme (2000). The weight being placed on leisure reflects that in the data, average individuals spend about one third of their discretionary time in the paid labour market and two thirds of their time in leisure activities. This value will be considered to be the median value for this sample parameter. The low value is chosen to be 1.01068 and the high value 1.8249.

w is calibrated to match the earnings distribution over three different education types: high school dropouts, high school graduates and college graduates. Using the data from CANSIM, the college grad earns approximately about 40% higher earnings than high school grad. Thereby, this study sets the wage rate for the college grad at 31 dollars per hour, and for the high school and the drop out at 26 and 17 dollars respectively.

The interest rate r; from CANSIM, it is shown that in the past twenty years, the average interest rate in Canada is 2%, which also consistent with Andolfatto, Ferrall and Gomme (2000). The interest rate for 20 years is $(1+2\%)^{20}-1=0.4859$.

Consequently, the discount factor β in this study is $1/(1+2\%)^{20}=0.6726$.

For working hours, noting that the number of discretionary hours available per year is 16*356=5840, for twenty years, the total hours is 5,840*20=116,800.

With the model so calibrated, one can now examine how the different pension systems will influence the labour supply decision of individuals both with heterogeneous skills and heterogeneous preference for leisure.

Following that, this paper will first study a system without pensions, and set them as a benchmark. Subsequently, it will examine the result of the pension program financed by the flat tax regime.

4 RESULTS

The first case simulated is the economy without tax and without a pension system.

Secondly in flat tax regime, with all other assumptions the same is considered. When set $\tau \neq 0$, it specifies the program that consists of the wage tax and the pension.

The result observed by running the GAUSS program is studied as follows:

4.1 No Pension Plan

4.1.1 Labour supply

Individuals maximize their utility over three periods. Without any distortion, in the first period the labour supply for the college is 0.49. For the high school and the drop out, they are 0.43 and 0.26 respectively. In the second period, the labour supply of the college is 0.54, the high school is 0.48 and the drop out is 0.33. The reason that the individual prefer to work more in the second period is when they are in the middle age, they have higher working skill. Because the wage rate increases, the return for jobs rises also.

4.1.2 Saving

In both first and second periods, all individuals save money to support themselves after retirement in the third period. Because the highest wage rate, the college saves most, in the first period, the saving is 2.28, and in the second period, it is 8.78. The high school is 1.59 and 6.56. The drop out save is just 0.43 and 2.78.

4.1.3 Utility

In the current program neither tax nor pension is attainable. Individuals smooth consumption and leisure over periods using the private saving. For the college grad, the maximum utility he gets is 2.78. The high school is 2.44 and the drop out is 1.25.

All of this result is illustrated in the table 1. This study sets the outcome of no tax as the bench mark.

4.2 Pension Program

4.2.1 Labour supply

When the pension program finances flat tax, the tax people paid in the first period is the pension they attain in the third period. Supposing the consumption and leisure are normal goods at each date. All three types decrease the labour supply and utility compared to no tax regime. First this study sets the tax rate is 0.1. The labour supply of the college falls from 0.494 to 0.492 in the first period, and falls from 0.54 to 0.53 in the second period. The high school decreases from 0.494 to 0.492 in the first period and from 0.488 to 0.485 in the second period. Among the three types, the low skilled decreases his labour supply most. For the college, he drops by 0.37%and 0.31%, he drop out only lowers his labour supply by 1.8% and 1.3%.

4.2.2 Saving

Consistent with individual's labour supply decreasing, all people lower their saving before retirement. Less work causes less wealth they can create. The saving of the college drops by 14% in the first period and 12% in the second period. But for the drop out, he drops by 32% and 18% respectively. He also drops most. For the high school, he changes from 1.59 to 1.34 in the first period, and from 6.56 to 5.67 in the second period.

4.2.3 Utility

Due to the distortion wage tax, it causes deadweight loss on the economy. All three types suffer the utility loss. Set the welfare cost for each type is ε_i. To get this fraction of consumption of each type being willing to pay for having this pension

$$
\begin{aligned}
u(\varepsilon) &= \ln \varepsilon c_{1i}^{0*} + \psi \ln l_{1i}^{0*} + \beta(\ln \varepsilon c_{2i}^{0*} + \psi \ln l_{2i}^{0*}) + \beta^2(\ln \varepsilon c_{3i}^{0*}) \\
&= \ln \varepsilon + \beta \ln \varepsilon + \beta^2 \ln \varepsilon + (\ln c_{1i}^{0*} + \varphi \ln l_{1i}^{0*} + \beta(\ln c_{2i}^{0*} + \psi \ln l_{2i}^{0*}) + \beta^2(\ln c_{3i}^{0*}) \\
&= (1 + \beta + \beta^2)\ln \varepsilon + u_0^* \\
&= u_{0.1}^*
\end{aligned}
$$

program implemented, the calculating procedure is:

First, get the utility of no pension u_0^* and the utility of pension $u_{0.1}^*$.

Therefore the fraction equals to

$$\ln \varepsilon = (u_{0.1}^{*} - u_{0}^{*}) / 1 + \beta + \beta^{2}$$

By running the GAUSS code, the fraction is: for the college, it is 0.92; for the high school, it is 0.91; for the drop out is 0.90.

The surprising thing observed is no type has benefited this program, even the lowest skilled person. The redistributive effects on the economy have not come into view as we supposed to be.

4.2.4 Changing the tax rate

4.2.4.1 Labour supply

Then this study changes the tax rate. One direction is to make it be larger, and another is to vary it to be smaller.

Set the rate is 0.2, 0.3, and 0.4, 0.5 and 0.8. This paper examines when the tax rate increases, the distortion also rises. Changing tax rate from 0 to 0.5, the labour supply of the college turns from 0.49 to 0.47 in the first period; and from 0.54 to 0.52 in the second period. For the high school, he varies from 0.43 to 0.41 in the first period and from 0.48 to 0.46 in the second period; for the drop out also always lowers his labour supply when the tax rate rises, changing from 0.26 to 0.22 in the first period, and from 0.33 to 0.29 in the second period. All individuals have the same trend that is the higher tax rate, the lower the labour supply. When the tax rate is 0.8, for the dropout, he gives up working completely.

If set the rate decreases and change it from 0.1 to 0.05 and to 0.01, the distortion of each type's labour supply becomes less.

When the tax rate is 0.05, for the drop out, his labour supply is 0.2669 in the first period, and 0.3353 in the second period; when tax rate decreases to 0.01, his labour supply increases to 0.2692 and to 0.3357 respectively. When tax rate is 0.05, for the high school, his labour supply is 0.4356 and 0.4869; when at 0.01, the distortion of labour supply shrinks, becoming 0.4366 and 0.4878. It is the same with the college grad.

4.2.4.2 Saving

Consistent with the decreasing of the labour supply, all individual's savings also fall when the tax rate rises. For the college, his saving drops from 0.43 to 0.01, when the tax rate varies from 0 to 0.3 in the first period and from 2.78 to 1.23 in the second period. When the tax rate is 0.4, the drop out even begin to borrow money in his first period. For the high school when the tax rate rises to 0.5, the saving in the first period decreases from 1.59 to 0.36, and lowers from 6.56 to 2.19. When the tax rate is 0.8, the high school will borrow money before his retirement. For the college, when the tax rate rises to 0.5, the saving in the first period drops from 2.2 to 0.7 and from 8.7 to 3.3 in the second period.

4.2.4.3 Utility

Due to distortion of the flat tax, all individual's utility deteriorates. With higher tax rate, the fraction decreases. For the college, when tax rate rises 0.1 to 0.5, the fraction becomes from 0.90 to 0.63; for the high school, it changes from 0.91 to 0.64; for the drop out, it changes from 0.90 to 0.67. Though each type becomes worse off due to the pension when the tax rate rises, the high skilled person is hurt most. Relatively the low skilled person's utility lessens least.

Correspondingly, when the tax rate becomes smaller, the fraction increases. For the college, when tax rate decreases from 0.05 to 0.01, ε increase from 0.95 to 0.99; for the high school, it rises from 0.95 to 0.99; for the drop out, also goes up from 0.96 to 0.99.

4.2.4.4 Consumption:

Due to the flat tax, the difference among three types shrinks. When the tax rate is zero, the college's consumption is as 2.5 times as that of the drop out and is 33% higher than the high school. However when the tax rate is 0.5, the college is as 2.07 times as the drop out and is 32% higher than the high school. The higher tax rate brings the smaller consumption gap.

5 CONCLUSION

By this study, it shows the fully funded pension program financed by the flat tax does not improve social welfare. If it is implemented, both the low skilled and high skilled person is worse off. The only difference is the low skilled person is damaged less than the high skilled by this program. To maximize utility, individuals can use the private saving changes to smooth leisure and consumption over the life cycle. For all individual and the whole economy, no tax combined with no pension program is optimal in this paper. If this pension program is introduced, because the distortion wage tax causes the deadweight loss, it only does harm to all individuals. The results of this study call into question the social desirability of a fully-funded pension plan.

APPENDICES

Appendix 1: Result tables

Table 1: Labour supply when tax rate changes.

	n_{1l}	n_{2l}	n_{1m}	n_{2m}	n_{1h}	n_{2h}
t=0	0.269	0.336	0.437	0.488	0.495	0.495
t=0.1	0.264	0.331	0.434	0.486	0.493	0.539
t=0.2	0.258	0.326	0.431	0.483	0.491	0.537
t=0.3	0.251	0.319	0.428	0.480	0.488	0.534
t=0.4	0.242	0.311	0.423	0.475	0.484	0.531
t=0.5	0.229	0.299	0.417	0.470	0.480	0.527
t=0.8	-0.003	0.088	0.300	0.363	0.392	0.447

Table 2: Saving in the second period:

	s_{1l}	s_{2l}	s_{1m}	s_{2m}	s_{1h}	s_{2h}
t=0	0.437	2.787	1.597	6.569	2.286	8.784
t=0.1	0.294	2.260	1.341	5.674	1.963	7.670
t=0.2	0.153	1.741	1.088	4.785	1.642	6.563
t=0.3	0.017	1.232	0.839	3.906	1.326	5.465
t=0.4	-0.114	0.738	0.596	3.041	1.014	4.382
t=0.5	-0.236	0.264	0.361	2.197	0.711	3.318

Table 3: Utility of three types:

	u_l	u_m	u_h
t=0	1.2535977	2.4452528	2.7846793
t=0.1	1.077237	2.248037	2.5824144
t=0.2	0.88434207	2.030045	2.3583311
t=0.3	0.67127161	1.7861937	2.1069868
t=0.4	0.43289676	1.5091754	1.8205269
t=0.5	0.16154384	1.1878595	1.4869468

Table 4: **Tax rate decreases (t changing from 0.1to 0):**

	t=0.1	t=0.01	t=0.05	t=0
n_{1l}	0.2644	0.2688	0.2670	0.2693
n_{2l}	0.3313	0.3353	0.3336	0.3357
s_{1l}	0.2937	0.4227	0.3651	0.4371
s_{2l}	2.2602	2.7337	2.5226	2.7867
Pension	0.3932	0.0398	0.1979	0.0000
n_{1m}	0.4344	0.4366	0.4357	0.4368
n_{2m}	0.4858	0.4878	0.4870	0.4880
s_{1m}	1.3410	1.5710	1.4686	1.5967
s_{2m}	5.6736	6.4795	6.1207	6.5693
n_{1h}	0.4929	0.4946	0.4939	0.4948
n_{2h}	0.5390	0.5405	0.5399	0.5407
s_{1h}	1.9629	2.2539	2.1243	2.2864
s_{2h}	7.6697	8.6720	8.2259	8.7837
u_l	1.0772	1.2366	1.1673	1.2536
u_m	2.2480	2.4264	2.3490	2.4453
u_h	2.5824	2.7653	2.6860	2.7847

Table 5 when the tax rate increase, the welfare cost of each type:

	t=o	t=0.1	t=0.2	t=0.3	t=0.4	t=0.5
ε_l	1	0.920	0.841	0.760	0.680	0.598
ε_m	1	0.911	0.823	0.733	0.644	0.554
ε_h	1	0.909	0.818	0.727	0.635	0.543

Table 6: **The welfare cost of each type, when the tax rate decreases:**

	t=0.1	t=0.05	t=0.01	t=0
ε_l	0.920	0.960	0.992	1.000
ε_m	0.911	0.956	0.991	1.000
ε_h	0.909	0.955	0.991	1.000

Table 7: **Consumption of each type when the tax rate increases:**

tax rate	t=o	t=0.1	t=0.2	t=0.3	t=0.4	t=0.5
drop out	4.140716	3.752	3.362	2.971	2.578	2.184
high school	9.761311	8.824	7.885	6.944	6.002	5.056
college	13.05165	11.790	10.526	9.261	7.993	6.722

Appendix 2: Result figures

Figure 1: The labour supply in the first period:

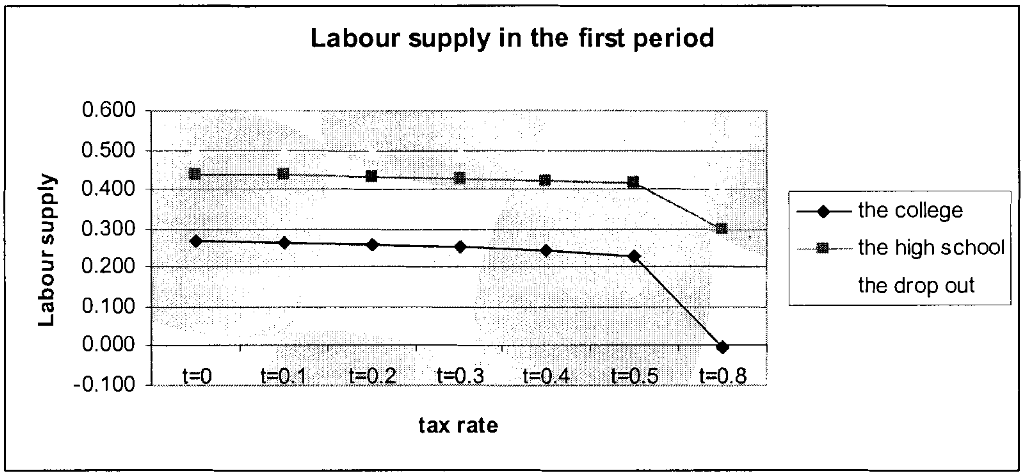


Figure 2: The Labour supply in the second period:

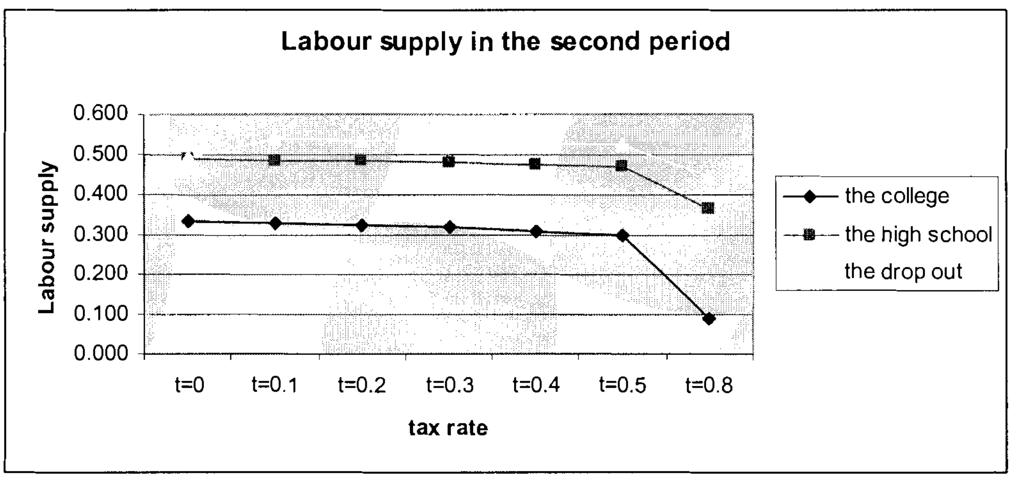


Figure 3: Saving in the first period:

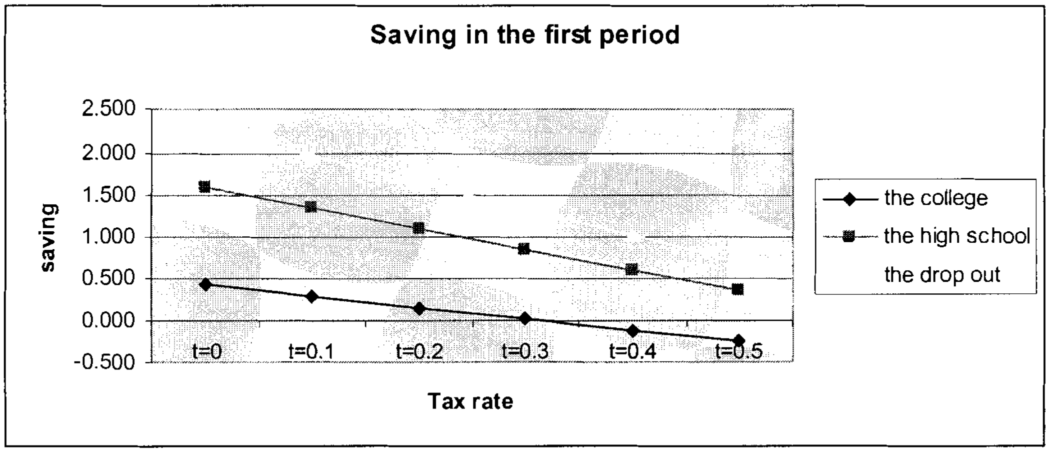


Figure 4: Saving in the second period:

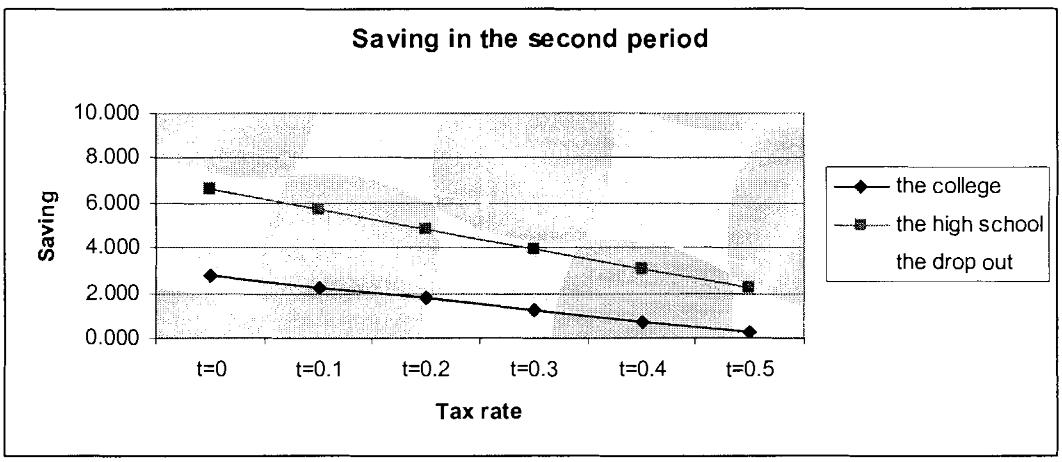


Figure 5: Utility of three types:

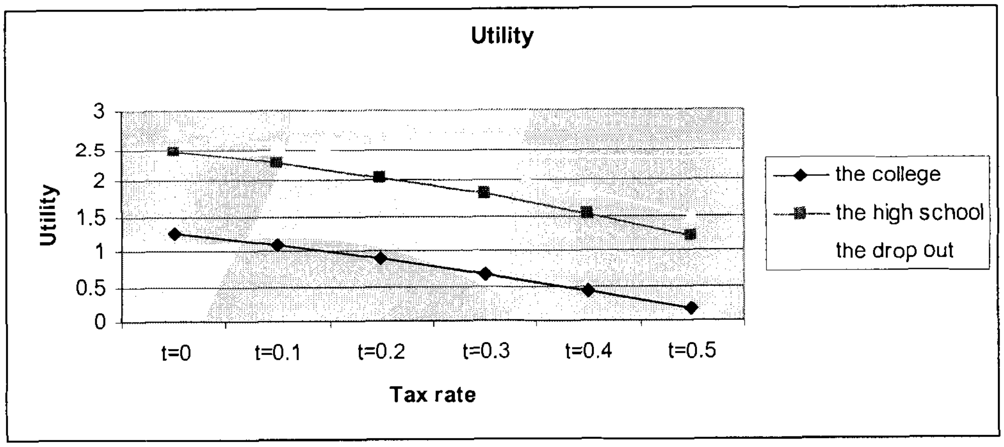


Figure 6: Social cost of each type when the tax rate increases:

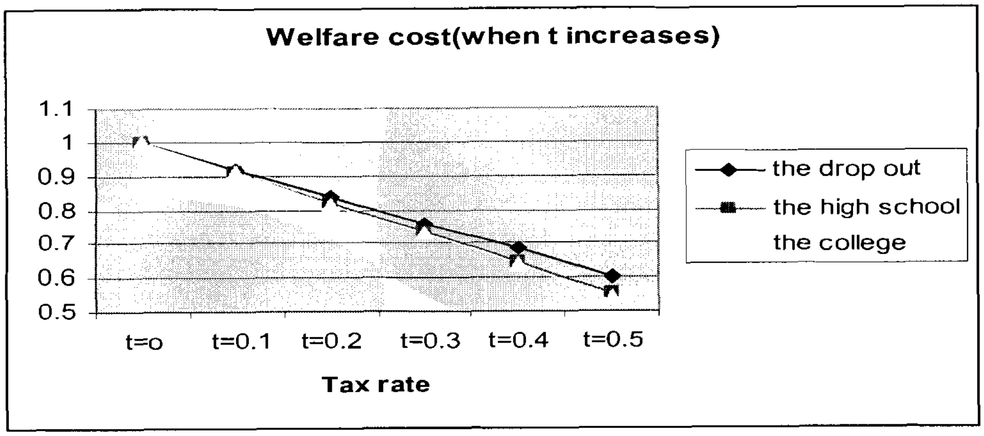


Figure 7: Welfare cost of each type when the tax rate decreases:

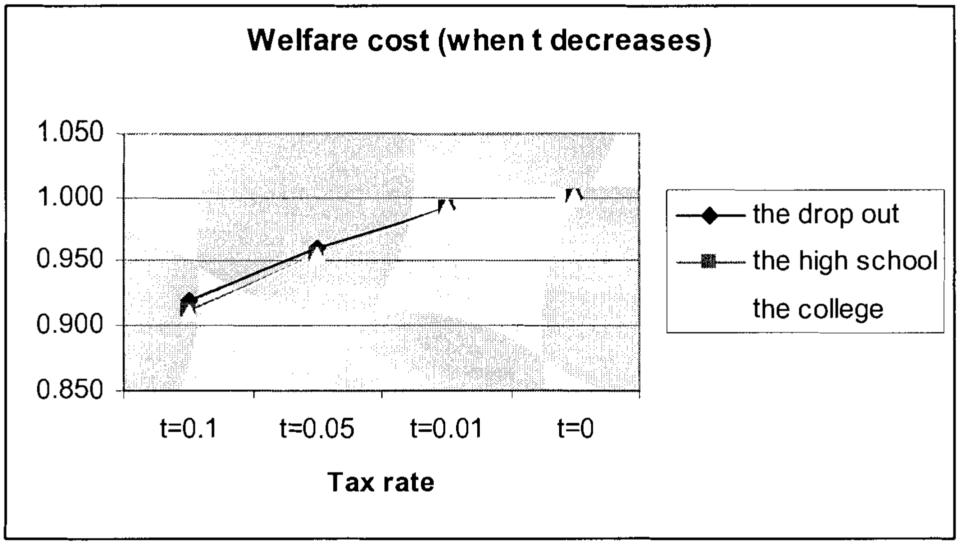
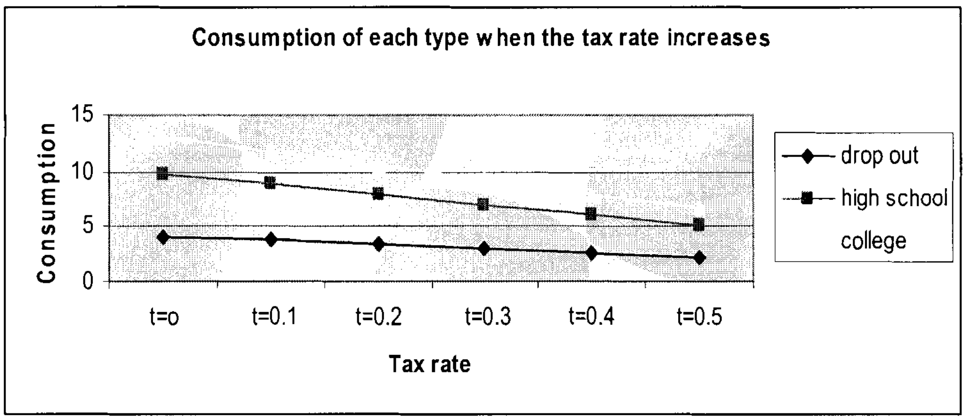


Figure 8: Consumption of each type when the tax rate increases:



Appendix 3: Gauss code

The code for the no pension program in the lump sum tax regime:

```
new;
library nlsys;

nstar = 0.33;
Beta =1/1.4859;
rstar =0.4859;
theta = 1.1;
tstar =0.01;




N = 7;
wage = zeros(3,1);
psi = zeros(3,1);

utilityl = zeros(1,1);
utilitym = zeros(1,1);
utilityh = zeros(1,1);


cpn1l = zeros(1,1);
cpn2l = zeros(1,1);
cpn3l = zeros(1,1);

cpn1m = zeros(1,1);
cpn2m = zeros(1,1);
cpn3m = zeros(1,1);

cpn1h = zeros(1,1);
cpn2h = zeros(1,1);
cpn3h = zeros(1,1);


wage[1] = 17;
wage[2] = 26;
wage[3] = 31;

psi[1] = 3;
psi[2] = 1.5;
psi[3] = 1.2;


      x0 =
nstar|nstar|nstar|nstar|0|nstar|nstar|nstar|nstar|nstar|nstar|nstar|nsta
r;
```

```
      __output = 1;
      { x,f,g,h } = NLSYS(&foc1,x0);

cpn1l = wage[1]*x[1]*(1-tstar) - x[3];
cpn2l = theta*wage[1]*x[2]*(1-tstar) + x[3]*(1+rstar) - x[4];
cpn3l = x[4]*(1+rstar) + x[5];

cpn1m = wage[2]*x[6]*(1-tstar) - x[8];
cpn2m = theta*wage[2]*x[7]*(1-tstar) + x[8]*(1+rstar) - x[9];
cpn3m = x[9]*(1+rstar) + x[5];

cpn1h = wage[3]*x[10] *(1-tstar) - x[12];
cpn2h = theta*wage[3]*x[11]*(1-tstar) + x[12]*(1+rstar) - x[13];
cpn3h = x[13]*(1+rstar) + x[5];

x[5]
=(0.25*wage[1]*x[1]*tstar+0.5*wage[2]*x[6]*tstar+0.25*wage[3]*x[10]*tsta
r)*(1+rstar)^2+theta*(0.25*wage[1]*x[2]*tstar+0.50*wage[2]*x[7]*tstar+0.
25*wage[3]*x[11]*tstar)*(1+rstar);



utilityl =  ln(cpn1l)+psi[1]*ln(1-x[1]) + beta*( ln(cpn2l) +
psi[1]*ln(1-x[2])
) + beta^2*ln(cpn3l);
utilitym =  ln(cpn1m)+psi[1]*ln(1-x[6]) + beta*( ln(cpn2m) +
psi[2]*ln(1-x[7])
) + beta^2*ln(cpn3m);
utilityh =  ln(cpn1h)+psi[1]*ln(1-x[10]) + beta*( ln(cpn2h) +
psi[3]*ln(1-x[11])
) + beta^2*ln(cpn3h);

print;
" Utility of the dropout:  "     utilityl;
" Utility of the high school:  "  utilitym;
" Utility of the college:  "     utilityh;
print;


proc foc1(x);
      local fn1, fn2, fn3, fn4, fn5,fn6,fn7,fn8,fn9,fn10,fn11,fn12,fn13;

      fn1 = (1-tstar)*(wage[1]*((1-x[1])))-(wage[1]*x[1]*(1-tstar)-
x[3])*psi[1];
      fn2 = (1-tstar)*theta*wage[1]*(1-x[2])-
psi[1]*(theta*wage[1]*x[2]*(1-tstar)+x[3]*(1+rstar)-x[4]);
    fn3 = (wage[1]*x[1]*(1-tstar)-x[3])-(theta*wage[1]*x[2]*(1-
tstar)+x[3]*(1+rstar)-x[4]);
    fn4 = theta*wage[1]*x[2]*(1-tstar)+x[3]*(1+rstar)-x[4] -
x[4]*(1+rstar) - x[5];

      fn5 = x[5] -
(1+rstar)^2*tstar*(0.25*wage[1]*x[1]+0.5*wage[2]*x[6]+0.25*wage[3]*x[10]
)+(1+rstar)*tstar*theta*(0.25*wage[1]*x[2]+0.5*wage[2]*x[7]+0.25*wage[3]
*x[11]);
```

```
    fn6 = (1-tstar)*(wage[2]*((1-x[6])))-(wage[2]*x[6]*(1-tstar)-
x[8])*psi[2];
      fn7 = (1-tstar)*theta*wage[2]*((1-x[7]))-
psi[2]*(theta*wage[2]*x[7]*(1-tstar)+x[8]*(1+rstar)-x[9]);
    fn8 = (wage[2]*x[6]*(1-tstar)-x[8])-(theta*wage[2]*x[7]*(1-
tstar)+x[8]*(1+rstar)-x[9]);
    fn9 = theta*wage[2]*x[7]*(1-tstar)+x[8]*(1+rstar)-x[9] -
x[9]*(1+rstar) - x[5];

    fn10 = (1-tstar)*(wage[3]*((1-x[10])))-(wage[3]*x[10]*(1-tstar)-
x[12])*psi[3];
      fn11 = (1-tstar)*theta*wage[3]*((1-x[11]))-
psi[3]*(theta*wage[3]*x[11]*(1-tstar)+x[12]*(1+rstar)-x[13]);
    fn12 = (wage[3]*x[10]*(1-tstar)-x[12])-(theta*wage[3]*x[11]*(1-
tstar)+x[12]*(1+rstar)-x[13]);
    fn13 = theta*wage[3]*x[11]*(1-tstar)+x[12]*(1+rstar)-x[13] -
x[13]*(1+rstar) - x[5];


     retp( fn1|fn2|fn3|fn4|fn5|fn6|fn7|fn8|fn9|fn10|fn11|fn12|fn13);
endp;
```

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